

MINIMIZATION OF BLENDING LOSSES  
TO DETERMINE OPTIMAL STANDARD ENRICHMENTS  
OF NUCLEAR FUEL,

by

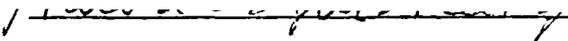
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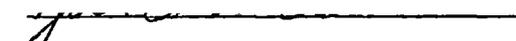
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## CHAPTER I

### INTRODUCTION

Within the scenarios for future expansion of the present United States enrichment capacity, alternative enrichment technologies leading to a limited number of enrichment values are proposed. The gas centrifuge is the most prominent example. Cost-effective gas centrifuge facility design implies the production of a limited number of enrichment values. The early employment of these facilities leads to a need to blend these enrichment values to meet existing demands for a wide range of different enriched fuels.

The requirement is to produce a wide range of products from a limited number of ingredients. The optimal design of such a blending system includes consideration of both the mechanical blending processes and the economic penalties resulting from the nonlinearities associated with enrichment costs. The design parameters of the mechanical processes for blending ingredient enrichments to produce fuels for existing reactors are not defined and are assumed here to be independent of both ingredient and product enrichments. The economic penalties of blending ingredient enrichments are analyzed using a linear program with the objective of determining the identities of the ingredients that minimize these penalties as a function of the number of ingredients produced.

#### A. Background

Presently there are 60 commercial light water nuclear reactors in

operation; each of which is to some extent custom designed and built. While based on a fundamental design, the reactors have design and construction modifications to satisfy continually changing regulations. The nuclear power industry may be unique in having evolved entirely under federal regulations.<sup>(1)</sup> As a result, a dynamic and highly nonstationary regulatory environment leads to substantial modification in the implementation of the fundamental reactor design. The simultaneous evolution of regulation and design is generally viewed as industry growth and as therefore acceptable. Nevertheless, the diversity in implemented reactor design format carries associated customization costs.

Considerable interest has recently been generated in the concept of standardization of nuclear reactor systems. This interest is in part motivated by a desire to contain rising energy costs, but is to a greater extent a reflection of prevailing views of the appropriate pattern of further nuclear energy industry growth.<sup>(1)</sup>

The concept of standardization can be applied to the entire reactor installation or to specific common components of nuclear power plants. When considering component standardization, attention can be restricted to new installations or can include both equipment for new plants and replacement items for existing reactors. The impact that standardization will have upon existing reactor installations, however implemented, must be investigated.

Through standardization of nuclear power plants, a utility applicant can reference and thereby gain advanced approval of all equipment, structures, and interfaces necessary for plant construction.<sup>(1,2)</sup> An-

other advantage to using a standardized system is an estimated shortening of plant regulatory reviews by as much as 6 to 7 months and a savings of between 40 and 50 man-years of utility and construction work during PSAR (Preliminary Safety Analysis Report) preparation.<sup>(3)</sup>

The reactor fuel is an attractive component for the implementation of standardization. In particular, standardized fuel enrichments have the advantage of being very cost effective and new enrichment technologies have the appeal of potentially lower capital costs relative to those of the presently used gaseous diffusion process. In the case of new reactors, standardized fuel enrichments can have numerous advantages, including manufacturing cost, licensing, startup testing, interchangeability in the event of fuel assembly damage, and uniformity of shielding requirements. The use of standardized fuel for existing reactors may not be practical. However, the concept of using standardized fuel enrichments and blending them to produce the required enrichments for existing reactors would assure continued service to existing reactors while permitting standard fuel for new reactors. For example, if 2 percent and 4 percent by weight of uranium-235 are standardized enrichments and if uranium at an enrichment of 3 percent is needed, the blending of one kilogram of 2 percent enriched material with one kilogram of 4 percent enriched material will yield two kilograms of 3 percent enriched fuel. Thus fuel requirements for existing reactors need not necessarily block the use of standardized fuel enrichments.

The existing nuclear industry is not going to be immediately standardized. Meanwhile, needed and beneficial technology should be aggressively pursued, and interim procedures for fuel supply must be

considered. The use of a fixed set of selected enrichment values is viewed as a possible first step in fuel standardization and, in turn, in reactor design standardization. An analysis of blending selected enrichments is a necessary input to the evaluation of standardization of enrichments.

#### B. Problem Focus

It should be noted, that the requirement to blend standard enrichments can produce technological and quality assurance problems. The standard enrichments must be homogeneously blended with a high degree of assurance, precision, and repeatability in either the gaseous, liquid, or solid state. The physical process and related costs are not available in existing literature, which suggests that these parameters be eliminated from any present-day optimization analysis of enrichment blending. Otherwise the uncertainty reflected in these parameters can render results and trends of such analysis indistinguishable.

Due to the nonlinearity in the cost of separative work required in producing enriched products, the optimal choice of a standard set of enrichments and their associated quantities necessary to meet existing commercial light water reactor demands are not readily clear. It is this determination toward which this affect is directed. The necessary requirement is that the actual cost of physically blending the standard enrichments, and assuring the proper homogeneous blend, must not affect the optimization procedure. If these operational blending costs are not a function of the values of the ingredient being blended, then the optimization achieved ignoring these oper-

ational costs will be identical to that achieved when considering operational costs. For example, if the operational cost of blending A percent enriched material with B percent enriched material is the same as that of blending C percent enriched material with D percent enriched material, where A, B, C, and/or D are not necessarily equal, then the operational blending costs are not a function of the values of the blending ingredients. The operational costs can then be ignored.

Without actual knowledge of the blending process and/or procedure, it is reasonable to assume that the physical blending and quality assurance costs are constant for blending different valued ingredients. It may be questionable, however, whether the cost associated with the effect to obtain confidence in the variance from the desired blended product for blended ingredients of narrowly differing enrichments (such as 0.1 percent difference) is the same as to obtain confidence in the variance for blending ingredients of widely differing enrichments (up to 3 percent difference or so). It will be assumed that the cost to obtain this confidence is constant and it is upon this assumption that the resulting optimization analysis rests. At this time, it is reasonable to state that not only the relationship between this confidence cost and the difference in blending ingredient values are unknown, but also the blending process and other associated costs are unknown. The analysis is hence performed under the assumption stated above and the trends of the analysis considered appropriate until more detailed blending information is available.

### C. Approach

A mixed integer linear programming model is developed which will determine the identities of the enrichment values, the quantity of each enrichment value required to optimally blend the fuel for existing reactors and the blending penalty associated with using the selected enrichments, given the following data:

- 1) N, the number of ingredient enrichments to be blended;
- 2) a variety of ingredient enrichment levels, the amount of separative work needed to produce one unit of the associated level, and their associated production costs;
- 3) the number, enrichment levels, and associated amounts of the enrichments demanded by the nuclear industry in a given time frame; and
- 4) the maximum amount of separative work available in that time frame.

The model is applied to a reference case corresponding to the fuel requirements for the expected reactor population (800 MWe and greater) in 1980 and is analyzed for several values of N, the number of selected enrichments.

### D. Results

From the analysis of the results obtained for various values of N, it is found that the cost penalty associated with blending is small, even for a small number of selected enrichments. In fact, as few as 4 enrichment values can be used with a cost penalty of only 0.18 percent. Another result is that the costs of blending decrease rapidly as the

number of selected enrichments increases. It is also found that the amount of separative work is the major contributor to the total enrichment cost. Finally, it is found that certain enrichment values occur in the optimal set of selected enrichments for many values of  $N$ . Implications include the economic attractiveness of enrichments from new technologies, the extension possibilities to mixed oxide fuels, and the optimal use of residual material during maintenance.

## Chapter II

### ENRICHMENT AND BLENDING LOSSES

There are several isotopes of uranium found in nature. The two principal ones are uranium-235 and uranium-238. Because of the favorable ratio of fission to absorption probabilities of uranium-235 with thermal neutrons, it is necessary to enhance the concentration of uranium-235 from its natural 0.711 weight percent to the 2 to 4 percent desired for fuel in light water reactors. The gaseous diffusion process is presently used to accomplish this enrichment. However, there is a definite potential for use of the laser and/or gas centrifuge enrichment processes.

Uranium enrichment is defined as the process of separating a quantity of feed uranium into an enriched product component containing a higher concentration of uranium-235 and a depleted tails component containing a lower concentration of uranium-235. The total quantities of uranium in feed, product, and waste streams are symbolically referred to as "F", "P", and "W", respectively. Similarly, the weight fraction of uranium-235 in each stream will be symbolized by "Z" with the appropriate subscript (i.e.  $Z_F$ ,  $Z_P$ , or  $Z_W$ ). The overall separation operation is defined by the following mathematical relationships:

- a) total uranium material flow balance,

$$F = P + W;$$

- b) uranium-235 material flow balance,

$$F \cdot Z_F = (P \cdot Z_P) + (W \cdot Z_W); \text{ and}$$

c) "separative work balance",

$$SW = P \cdot V(Z_P) + W \cdot V(Z_W) - F \cdot V(Z_F)$$

which can be written

$$SWU = V(Z_P) - V(Z_W) - F/P \cdot (V(Z_F) - V(Z_W))$$

where  $SWU = SW/P$  and  $V(Z)$  is a defined "value function"

given by  $V(Z) = (2Z - 1) \cdot \ln(Z/(1 - Z))$ .<sup>(4,7)</sup>

As can be seen from the equation, the separative work balance possesses the same units as the quantities of total uranium.

An important concept in uranium enrichment is separative work.

Separative work is a measure of the quantity of physical work required to accomplish a given enrichment. The effect of a unit of separative work may be understood by noting that it takes approximately 1.78 units of separative work applied to 3.99 kilograms of natural uranium feed to produce one kilogram of uranium product containing 2.00 percent by weight uranium-235 while stripping the uranium tails to 0.28 percent uranium-235, using the gaseous diffusion process. It should be noted, however, that in deriving the separative work unit for the gaseous diffusion process the stage enrichment factor, which is equal to the separation factor minus one, is assumed to be a very small positive number, but in the laser and centrifuge processes that assumption is not valid. Consequently, for laser and centrifuge enrichment the equation for separative work does not really measure the work "done" by these systems. It can still be used to describe the capacity of laser or centrifuge enrichment plants and to compare their capacities to the projected needs.

The waste or depleted stream from the enrichment plant is commonly

called "tails" and the uranium-235 content of the tails is referred to as the "tails assay". In specifying the quantity of separative work necessary to accomplish the production of a specific quantity of enriched material, a variety of feed streams and tails enrichments are available. Alternatively, for a fixed waste stream, a variety of alternative combinations of feed streams and separative work will yield the required product. Thus, there is a trade off between the use of feed and the use of separative work in the production of the enriched material. This trade off is subject to economic optimization.

For example, Figure 1 shows the effects of changing the tails assay on the amount of separative work required to produce 3.0 percent by weight enriched uranium using natural uranium feed and on the associated amount of feed material required. Figure 1 uses as a base point the projected 1980 tails assay of 0.28 percent<sup>(5)</sup>, and expresses the effects of different tails assays on separative work and feed in percent units. The effect of separative work and feed on the enriched material can be seen to be that increasing tails assays increases the requirement for feed material and decreases the requirement for separative work. These relationships are independent of separative work costs and feed costs. Consider the total cost relationship for the enrichment trade-off shown in Figure 1 including both separative work and feed cost in the production of 3.00 percent by weight enriched uranium, using natural uranium feed costing \$31.64 per kilogram and \$76.00 per unit charge of separative work. It can be seen that the total cost is relatively insensitive to the tails assay over a considerable range. To both the uranium enrichers and uranium producers, the tails assay adopted is of greater economic

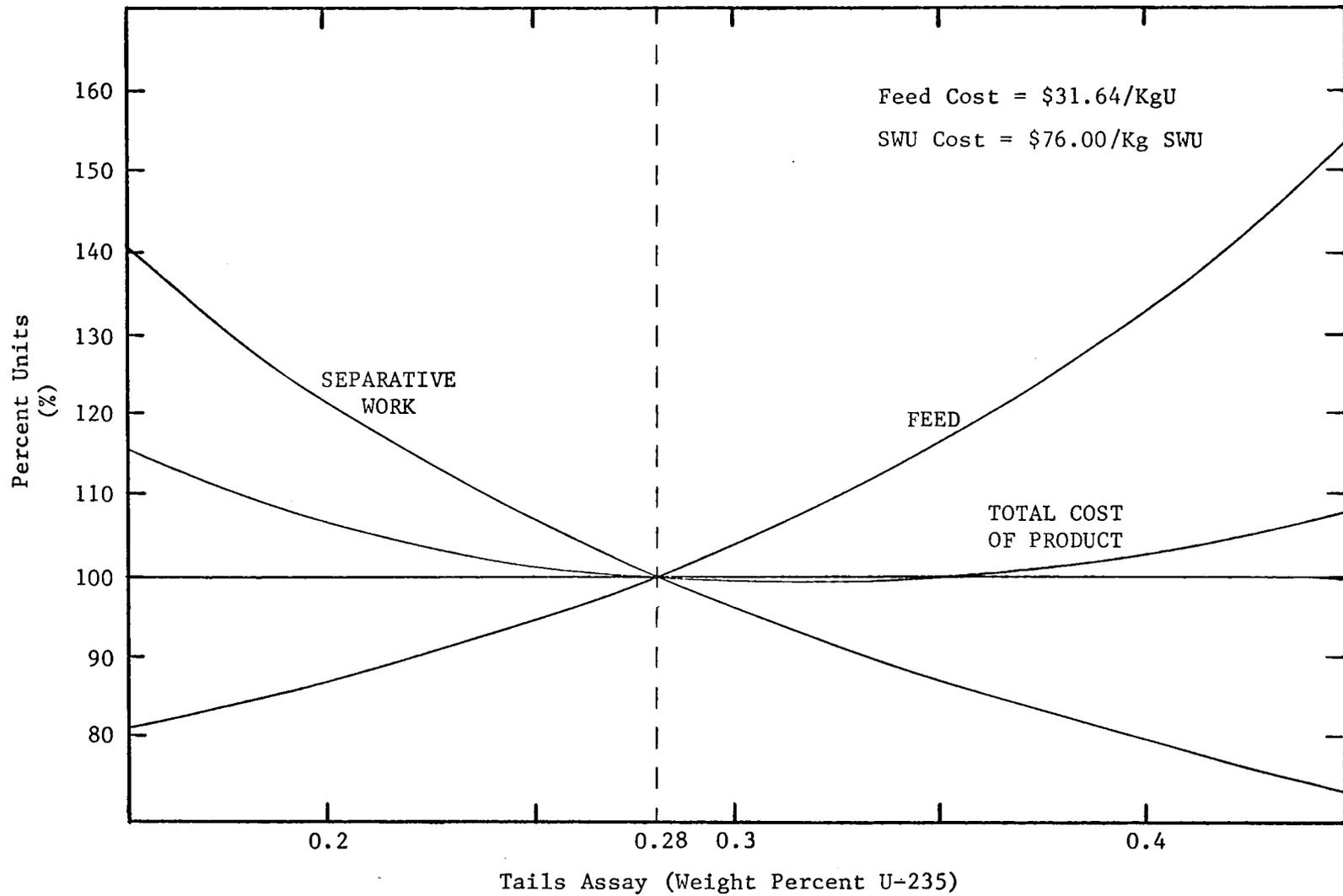


Figure 1. The Effects Various Tails Assays Have on the Amount of Feed and Separative Work Required to Produce Three Percent Uranium Product. (5,6)

significance than it is to the enriched uranium customers.<sup>(6)</sup>

On inspection of the total cost curve in Figure 1, it may be noted that the minimum cost point occurs at a tails assay greater than 0.28 percent uranium-235. This observation leads to the conclusion that the tails assay used is slightly suboptimal for the user of enriched uranium. This would be the case if the \$76.00 charge for separative work was, in fact, independent of the standard tails assay. However, this is not the case. The level of plant operation affects the cost of separative work which in turn is affected by the magnitude of the demand for separative work; and the demand is affected by the tails assay selected.<sup>(6)</sup>

For any enrichment level required, the relation between feed, product, and waste streams, the enrichment of these streams, and the separative work required to process them is specifically defined and can be applied to any level of product enrichment. The costs of using the enrichment process are the costs of providing the feed material and the costs of the separative work. Thus, for any enrichment,  $z$ , the total unit cost,  $TC_z$ , is equal to the unit cost of feed,  $CF$ , times the amount of feed material used,  $F_z$ , plus the unit cost of separative work,  $CSW$ , times the quantity of separative work utilized,  $SWU_z$ . Because the relation between product enrichment level and separative work is essentially exponential, cost per unit of product is an exponential function of product enrichment for a fixed feed quantity and fixed tails assay. Consequently, the cost of producing two specific enrichment levels and blending them to produce a third enrichment level is greater than the cost of producing the third level directly. The difference between these two costs plus the costs associated with a particular fuel blending

operation is called the "blending loss". Consider the following illustration using \$76.00 per kilogram separative work unit, \$31.64 per kilogram of natural uranium feed, and a 0.28 percent tails assay. It costs \$261.46 to produce one kilogram of 2.00 percent enriched fuel and \$689.88 to produce one kilogram of 4.00 percent enriched fuel. Blending these two yields a cost of \$951.34 per two kilograms of the 3.00 percent enriched fuel plus the cost associated with the blending operation. The difference of \$4.74 plus the blending operation cost per kilogram is the "blending loss". If a set of selected enrichments are produced, the elements of this set should be chosen in order to minimize the total blending losses.

## CHAPTER III

### MODEL FORMULATION

#### Mathematical Model

Existing commercial light water reactors use fuel at a variety of enrichment values. The objective of the study is to develop a methodology that identifies a set of selected enrichments from which the needed quantities of any number of fuel enrichments can be blended at minimum cost. If the elements of the set of selected enrichments are defined to be ingredients and the required enrichments are labelled products, the problem of identifying the elements of the set of selected enrichments can be formulated as an optimization problem. In order to construct a model of this problem, the following variables are defined:

$X_{ij}$  = the quantity of ingredient  $i$  used to blend product  $j$ , ( $X_{ij} \geq 0$ ),

$C_i$  = the total unit cost of producing fuel material at the  $i^{\text{th}}$  enrichment level directly (the total unit cost includes both unit feed costs and unit separative work costs),

$U_i$  = the amount of separative work required to produce one unit of material of the  $i^{\text{th}}$  enrichment,

$S_i$  = weight percent of uranium-235 in the  $i^{\text{th}}$  ingredient enrichment,

$P_j$  = weight percent of uranium-235 in the  $j^{\text{th}}$  product enrichment,

$D_j$  = the demand for product material of the  $j^{\text{th}}$  enrichment,

SC = total separative capacity available,

$M$  = the number of product enrichments demanded ( $M \geq N$ ),

$I_i$  = use coefficient; equal to zero if ingredient  $i$  is not used at all and equal to one if it is used to blend product material,

$N$  = the number of ingredient enrichments used to blend the product material,

$F_i$  = amount of feed required to produce one unit of material of the  $i^{\text{th}}$  enrichment,

TF = total amount of feed material available, and

EC = the total enrichment cost.

The objective is to minimize the total enrichment cost subject to the constraints imposed by the product-fuel demand, availability of feed, availability of separative work, and blending. The total enrichment cost is

$$EC = \sum_j^M \sum_i^N C_i X_{ij} \quad (3.1)$$

Five basic sets of constraints must be satisfied. The first set assures that the blending of enrichments yields the correct enrichment levels. Thus, the enrichment of the  $j^{\text{th}}$  product,  $P_j$ , is equal to the summation over all ingredients of the products of the quantities blended and their enrichment levels, divided by the summation of the quantities of the  $i^{\text{th}}$  ingredient enrichment used. This can be expressed as

$$\frac{\sum_i^N S_i X_{ij}}{\sum_i^N X_{ij}} = P_j \quad (3.2)$$

or

$$\sum_i^N (P_j - S_i) X_{ij} = 0. \quad (3.3)$$

The next set of constraints requires that the quantities of the desired products satisfy the demand for the given year:

$$\sum_i^N X_{ij} \geq D_j. \quad (3.4)$$

The third type of constraint restricts the total amount of separative work to be within production limits (i.e., enrichment plant capacity is not exceeded):

$$\sum_j^M \sum_i^N X_{ij} U_i \leq SC. \quad (3.5)$$

The fourth type of constraint limits the total amount of feed material to within mining and milling output capacity and enrichment plant input capacity for a given year:

$$\sum_i^N F_i \leq TF. \quad (3.6)$$

The final set of constraints restricts the number of ingredients that can be utilized to produce the desired products. Thus the summation of the use coefficients,  $I_i$ , must be equal to the total number of ingredient enrichments allowed in the blending process:

$$\sum_i^N I_i = N. \quad (3.7)$$

The summation of the amounts of the  $i^{\text{th}}$  ingredient utilized in the production of the  $M$  products is equal to the total amount of the  $i^{\text{th}}$  ingredient consumed by the enriching process. This can be expressed as

$$\sum_j^M X_{ij} = A_i \quad (3.8)$$

where

$A_i$  = the total amount of the  $i^{\text{th}}$  ingredient utilized in the enriching process.

The amount of the  $i^{\text{th}}$  ingredient consumed in the enriching process must be less than or equal to the availability of the  $i^{\text{th}}$  ingredient,  $AV_i$ , times its use coefficient,  $I_i$ ;

$$A_i \leq AV_i \cdot I_i \quad (3.9)$$

where

$I_i = 0$  when ingredient not used and

$I_i = 1$  when ingredient used in the blending process.

Substituting Eq. (3.9) into Eq. (3.8) yields the following equation:

$$\sum_j^M X_{ij} \leq AV_i \cdot I_i \quad (3.10)$$

The availability of an  $i^{\text{th}}$  ingredient,  $AV_i$ , is limited by the enrichment plant capacity. In this study, it should be noted that the value of  $AV_i$  is assigned a constant value,  $AV$ , such that  $AV$  is equal to a value greater than the total demand for any  $i^{\text{th}}$  ingredient used in the blending process. This simplification of  $AV_i$  to  $AV$  is an attempt to reduce the complexity of the model and help in the determination of the optimal enrichment sets when ingredient availability is unlimited.

The formulation of this mixed integer problem is identical to the formulation of a linear programming problem except for the zero/one integer variables of the use coefficient. MISTIC (Mixed Integer Search Technique Internally Controlled),<sup>(25)</sup> a MPS III proprietary procedure for solving optimization problems having a mixture of integer variables

(restricted to values of zero and one) and continuous variables, is employed to solve the mixed integer problem. See Appendix III for additional information about MISTIC.

#### B. Reference Case

Sample calculations to demonstrate application of the model are performed using the following reference case. The reference case selected consists of the quantities and enrichment levels of reactor fuels required for planned existing commercial light water reactor refueling schedules and for new commercial light water reactor installation schedules in the United States in 1980.<sup>(8,9)</sup> It is assumed that all the reactors on line in 1980 are rated at an electric capacity of 800 MW or greater and that recycled uranium or plutonium is not available. It is further assumed that the gaseous diffusion process will still be the main process of uranium enrichment (the centrifuge and laser enriching processes are still a few years from commercial feasibility). Finally, the blending process is assumed to occur using the enriched uranium hexafluoride gas components at the enrichment plant and the costs associated with this particular blending technique are assumed to be negligible.

The enrichment values for each reactor type are chosen according to the readily-available literature.<sup>(8,9)</sup> Refueling quantities of fuel are estimated at one-third core. It is conservatively estimated that two full core quantities of fuel [one initial startup core and replacement fuel (refuel enrichments) equalling one additional core] are required at the time of a new reactor fueling. The estimated distribution of required assays are shown in Table 1. The identified 39 different fuel

Table 1.

Enrichment Levels and Their Demands for the Year 1980. (8,9)

Enrichment (%)	Demand (Metric Tons)	Enrichment (%)	Demand (Metric Tons)
1.69	48.77	2.93	31.03
1.80	145.20	2.94	39.00
1.90	257.67	2.96	172.62
2.10	569.64	2.99	55.33
2.15	40.33	3.00	62.79
2.17	35.30	3.03	18.07
2.19	136.67	3.05	31.03
2.20	169.00	3.10	522.49
2.25	33.67	3.17	16.33
2.34	88.23	3.20	94.06
2.40	5.33	3.22	27.40
2.45	70.74	3.23	93.43
2.50	19.20	3.30	264.34
2.54	31.33	3.35	46.67
2.60	397.00	3.40	31.87
2.62	416.43	4.00	47.83
2.63	292.77	4.08	8.63
2.66	226.07	4.94	6.90
2.70	29.00		
2.82	61.30		
2.90	79.95		

enrichments are a conservative simplification of the over 50 separate fuel enrichments utilized in commercial light water reactors. <sup>(9)</sup>

For each of the model parameters, representative values are obtained. They are as follows:

- 1) The projected unit price for separative work, CSW, is \$76.00 per kilogram separative work unit. <sup>(14)</sup>
- 2) The projected unit price for natural uranium feed material, CF, is \$31.64 per kilogram (0.711 percent by weight uranium-235). <sup>(19)</sup>
- 3) The tails assay,  $Z_W$ , is 0.28 percent. <sup>(5)</sup> This proposed increase of the tails assay by NRC from the present 0.20 to 0.28 percent by 1980 results in a 14 percent increase in the amount of feed required and a 16 percent decrease in the amount of separative work needed. The overall cost would be "only slightly affected". <sup>(15,16)</sup>
- 4) The available amount of separative work, SC, projected for the year 1980 is between 13.8 and 14.5 million separative work units. <sup>(5,15)</sup>

The corresponding values for the parameters  $C_i$ ,  $U_i$ , and  $F_i$ , the unit cost of production, the separative work, and the feed material required to produce the  $i^{\text{th}}$  enrichment fuel material, are calculated using FM-3 <sup>(11)</sup> and illustrated in Figures 2, 3, and 4. Using these parameter values, the model is analyzed to determine the optimal enrichment set for a specified N, which will satisfy the nuclear fuel demands for the year 1980 while minimizing blending losses. By varying the values of N, optimal sets of enrichment levels of varying sizes can be

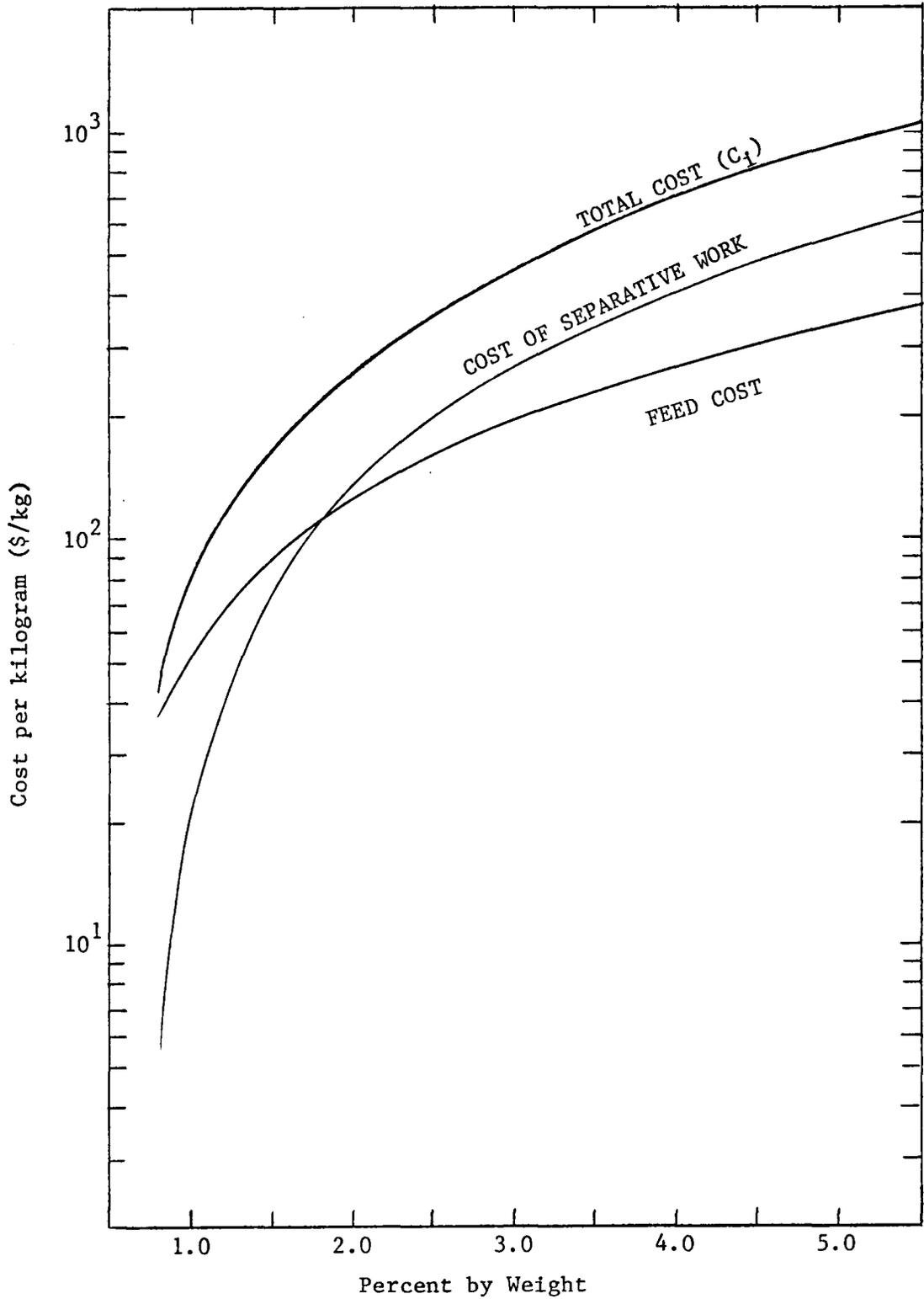


Figure 2. SWU Cost, Feed Cost, and Total Cost for Producing One Kilogram of a Desired Enrichment. (11)

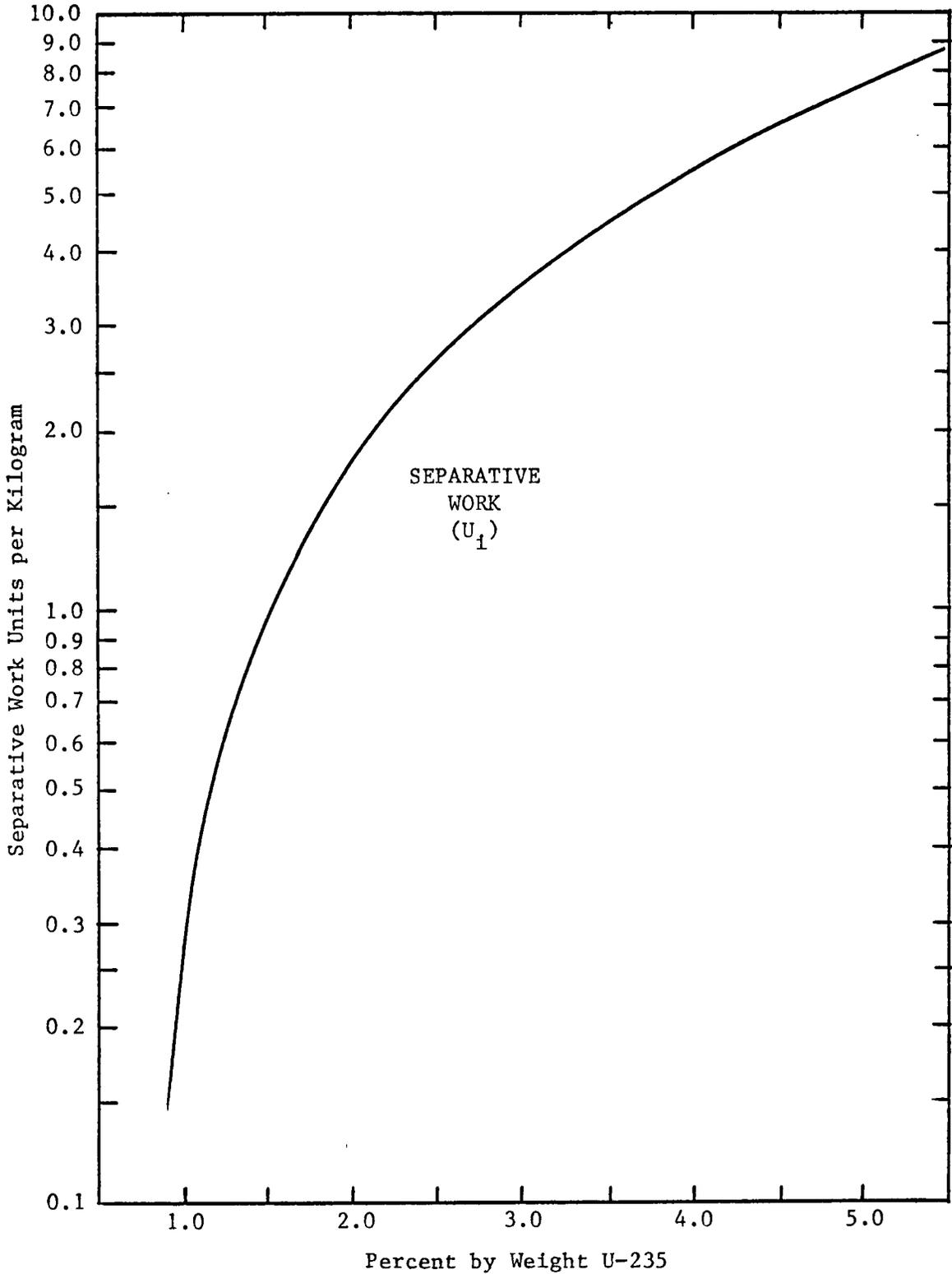


Figure 3. The Amount of Separative Work Required to Produce One Kilogram of a Desired Enrichment. (11)

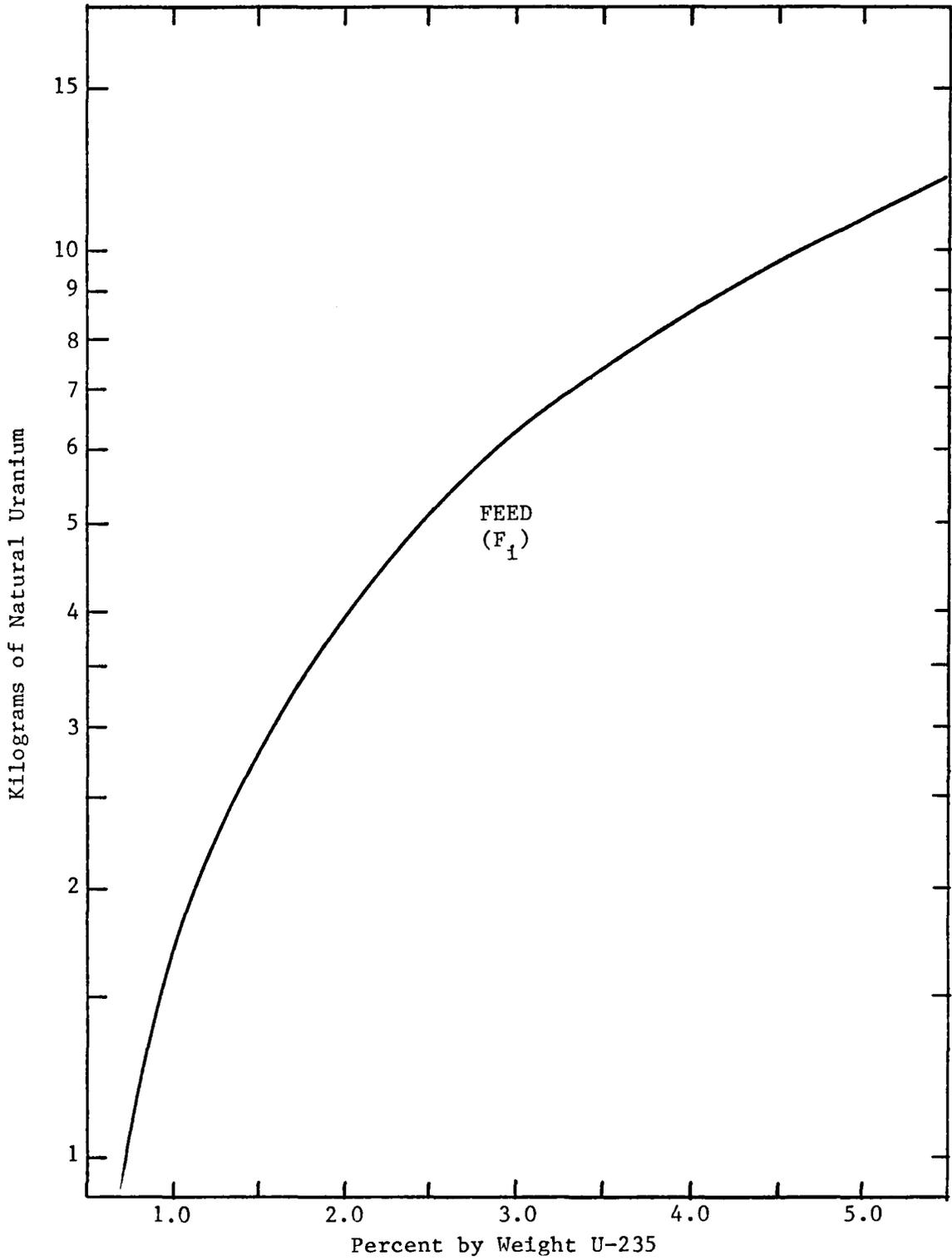


Figure 4. The Amount of Natural Uranium Feed Material Required to Produce One Kilogram of a Desired Enrichment. (11)

identified and compared.

## CHAPTER IV

### RESULTS AND CONCLUSIONS

In evaluating the results of the model, it is important to define the number of significant figures that are to be responsibly carried in order to properly display the trends of the optimization analysis. The development of the optimization model for blending selected enrichments is much like the definition, design, construction, and testing of experimental apparatus. With high precision digital computers, the error associated with operation of the model can be rendered negligible compared to typical experimental error. Also, input data for a computer model can be actual or hypothetical. For the problem studied here, input parameters have values, such as cost, which are rapidly changing with time. To evaluate trends, it is important to use input values that are in the correct range. Any input value for the model can be considered as precise as desired by considering the value as a snapshot precisely correct at some point in time.

For the case where the trend rapidly approaches an asymptote over a defined range, it is desirable to maintain a sufficient number of significant figures to observe the functional form and the relationship to the asymptote. For that reason, the number of significant figures carried will be that number that demonstrates the effects under observation. Justification for the number of significant figures will relate to the concept of the validity of the input to any degree of precision at some instant in time. Since the exact instant in time is not known, the precision of the result of the model is only valid for that unknown point.

Thus, the significant figures considered for the numerical result must be many fewer than the significant figures used to establish the trend. However, the appropriate view of both trend and result are important.

It should be remembered that the major objectives of the effect are to establish a model and to check its validity through application to a reference case. The resulting model can then be used to obtain results to any degree of accuracy, dependent upon the ability of the user to obtain precise time-related input. The second objective is to observe trends for the reference case application, realizing that the trends observed are valid but the absolute value of the results are strongly related to the accuracy of the input. It is not claimed here that the input is precise due to the inability to obtain exact and timely cost data.

Sample calculations to demonstrate the application of the model are performed to investigate the previously defined reference case. The results obtained in determining various optimal sets of N blended enrichments are summarized in Tables 2 and 3.

Examination of the model solutions displayed in Tables 2 and 3 reveals several significant results. Although available as ingredients, enrichments less than the lowest product enrichment demanded are not utilized in the blending procedure. It appears that this is because the cost of the blending complement for an enrichment less than the lowest demanded product enrichment must have a correspondingly high enrichment value. Such a blend is not cost effective under exponential enrichment costs.

An anticipated result is that in each case the highest and lowest enrichment demanded is always the upper and lower bound of each of the

Table 2.

Cost, Feed, and Separative Work for Optimal Blending Plans.

Number of Ingredients	Total Feed (MTU)	Separative Work Units (x10 <sup>7</sup> )	Total Enrichment Cost (\$ x10 <sup>9</sup> )	Total Marginal Cost* (x10 <sup>7</sup> )
2	25742.4	1.42883	1.90040	4.37260
3	25742.2	1.38956	1.86447	0.77935
4	25742.2	1.37573	1.86004	0.33683
5	25742.0	1.37287	1.85786	0.11844
6	25742.0	1.37222	1.85736	0.06900
7	25742.2	1.37204	1.85723	0.05601
8	25742.0	1.37181	1.85705	0.03779
9	25742.0	1.37164	1.85692	0.02512
10	25742.0	1.37158	1.85688	0.02039
11	25742.0	1.37148	1.85680	0.01258
12	25742.0	1.37145	1.85678	0.01049
13	25742.1	1.37140	1.85675	0.00730
14	25742.0	1.37138	1.85673	0.00535
15	25742.2	1.37136	1.8567]	0.00403
16	25742.2	1.37135	1.85671	0.00358
17	25742.2	1.37134	1.85670	0.00295
18	25742.2	1.37133	1.85669	0.00204
19	25742.2	1.37132	1.85669	0.00128
21	25742.2	1.37132	1.85668	0.00114
22	25742.1	1.37132	1.85668	0.00094
23	25742.1	1.37131	1.85668	0.00071
24	25742.1	1.37131	1.85668	0.00058
25	25742.1	1.37131	1.85668	0.00049
30	25742.1	1.37131	1.85668	0.00026
35	25742.1	1.37131	1.85667	0.00005
39	25742.1	1.37130	1.85667	0.00000

\* Marginal costs are the difference between the blending plan cost and the cost of producing the 39 products directly.

Table 3a.

Ingredients and Quantities Blended for Various Numbers of Selected Enrichments  
(2-10 Enrichments)

Ingredients	Number of Ingredients									
	Quantity (tonnes)	2	3	4	5	6	7	8	9	10
1.69	3358.87	1011.57	927.58	280.71	280.71	280.71	117.91	117.91	117.91	
1.90							538.41	333.73	333.73	
2.10				1178.46	1178.46	1154.90		823.20	614.06	
2.20							896.52		403.70	
2.34								342.31		
2.45									206.15	
2.60							491.17			
2.61			2207.39							
2.62				1698.45	1698.45		1425.98	1361.57	303.27	
2.66		3345.94								
2.90							491.14	429.09	429.09	429.09
3.10			1499.17	1476.52	1168.85	918.54	918.54	918.54	918.54	
3.40					367.61	367.61	367.61	367.61	367.61	
4.94	1364.55	365.92	89.28	89.28	29.35	29.35	29.35	29.35	29.35	

Table 3b.

Ingredients and Quantities Blended for Various Numbers of Selected Enrichments

(11-19 Enrichments)

Ingredients	Quantity (tonnes)	Number of Ingredients								
		11	12	13	14	15	16	17	18	19
1.69	117.91	117.91	117.91	48.77	48.77	48.77	48.77	48.77	48.77	48.77
1.80				145.20	145.20	145.20	145.20	145.20	145.20	145.20
1.90	333.73	333.73	333.73	257.67	257.67	257.67	257.67	257.67	257.67	257.67
2.10	614.06	614.06	614.06	614.06	614.06	614.06	614.06	614.06	614.06	614.06
2.20	403.70	403.70	403.70	403.70	388.60	388.60	358.52	358.52	358.52	358.52
2.34							100.26	102.68	102.68	
2.40					147.56	147.56	77.38			
2.45	206.15	206.15	206.15	206.15				98.98	98.98	
2.60					585.03	585.03	585.03	561.01	561.01	
2.62	1303.27	1265.35	1265.35	1265.35						
2.63					779.08	570.39	570.39	570.39	570.39	570.39
2.66						270.67	270.67	247.82	247.82	
2.82		179.24	179.24	179.24	154.16			68.55	68.55	
2.90	429.09					154.17	154.17	108.47	108.47	

Table 3b. (continued)

Ingredients and quantities Blended for Various Numbers of Selected Enrichments  
(11-19 Enrichments)

Ingredients	Quantity (tonnes)	Number of Ingredients								
		11	12	13	14	15	16	17	18	19
2.96			384.56	384.56	384.56	384.56	322.58	322.58	322.58	322.58
3.10	918.54	821.77	682.57	682.57	682.57	682.57	682.57	682.57	682.57	586.17
3.20										192.81
3.30			417.59	417.59	417.59	417.59	417.59	417.59	417.59	321.18
3.40	333.60	333.60	55.21	55.21	55.21	55.21	55.21	55.21	55.21	55.21
4.00	55.73	55.73	55.73	55.73	55.73	55.73	55.73	55.73	55.73	55.73
4.94	7.63	7.63	7.63	7.63	7.63	7.63	7.63	7.63	7.63	7.63

Table 3c.

Ingredients and Quantities Blended for Various Numbers of Selected Enrichments  
(21-25, 30, 35, 39 Enrichments)

Ingredients	Number of Ingredients								
	Quantity (tonnes)	21	22	23	24	25	30	35	39
1.69	48.77	48.77	48.77	48.77	48.77	48.77	48.77	48.77	48.77
1.80	145.20	145.20	145.20	145.20	145.20	145.20	145.20	145.20	145.20
1.90	257.67	257.67	257.67	257.67	257.67	257.67	257.67	257.67	257.67
2.10	614.06	595.41	595.41	595.41	595.41	595.41	569.64	569.64	569.64
2.15							57.98	40.33	40.33
2.17								35.30	35.30
2.19		186.53	186.53	186.53	186.53	186.53	154.32	136.67	136.67
2.20	358.52	190.65	190.65	190.65	190.65	190.65	190.65	169.00	169.00
2.25								33.67	33.67
2.34	102.68	102.68	102.68	102.68	102.68	102.68	102.68	90.65	88.23
2.40									5.33
2.45	73.65	82.18	82.18	82.18	82.18	82.18	82.18	82.18	70.74
2.50	38.00								19.20
2.54		42.00	42.00	42.00	42.00	42.00	42.00	42.00	31.33

Table 3c. (continued)

Ingredients and Quantities Blended for Various Numbers of Selected Enrichments  
(21-25, 30, 35, 39 Enrichments)

Ingredients	Number of Ingredients								
	Quantity (tonnes)	21	22	23	24	25	30	35	39
2.60	548.32	397.00	397.00	397.00	397.00	397.00	397.00	397.00	397.00
2.62		416.43	416.43	416.43	416.43	416.43	416.43	416.43	416.43
2.63	699.57	292.77	292.77	292.77	292.77	292.77	292.77	292.77	292.77
2.66		247.82	247.82	247.82	247.82	247.82	247.82	247.82	226.07
2.70	125.89							29.00	29.00
2.82	61.30	68.55	68.55	68.55	68.55	68.55	68.55	61.30	61.30
2.90	199.65	108.47	108.47	108.47	108.47	108.47	87.71	79.95	79.95
2.93								31.03	31.03
2.94							62.27	39.00	39.00
2.96		322.58	227.97	227.97	227.97	227.97	172.62	172.62	172.62
2.99							55.33	55.33	55.33
3.00	269.23		132.45	132.45	132.45	132.45	70.02	70.02	62.79
3.03									18.07
3.05							41.87	41.87	31.03

Table 3c. (continued)

Ingredients and Quantities Blended for Various Numbers of Selected Enrichments  
(21-25, 30, 35, 39 Enrichments)

Ingredients	Quantity (tonnes)	Number of Ingredients						
		21	22	23	24	25	30	35
3.10	548.33	586.17	548.33	548.33	548.33	527.39	527.39	522.49
3.17								16.33
3.20	192.81	192.81	192.81	192.81	114.62	114.62	105.49	94.06
3.22							27.40	27.40
3.23					111.70	111.70	93.43	93.43
3.30	321.18	321.18	321.18	321.18	287.68	264.34	264.34	264.34
3.35						46.67	46.67	46.67
3.40	55.21	55.21	55.21	55.21	55.21	31.87	31.87	31.87
4.00	47.83	55.73	55.73	47.83	47.83	47.83	47.83	47.83
4.08	8.63			8.63	8.63	8.63	8.63	8.63
4.94	6.90	6.90	6.90	6.90	6.90	6.90	6.90	6.90

optimal enrichment sets. This simple result, however, leads to further observations about the solutions. Note the change in the model solution generated by permitting a single additional ingredient enrichment ( $N+1$ ). The model responds to this relaxation in the constraint upon the number of ingredients in either of two ways.

One response is exemplified by increasing the number of ingredients from  $N = 13$  to  $N+1 = 14$ . The selection of ingredients for the optimal  $N+1$  set remains the same as that to the optimal  $N$  set except for a choice to avoid blending two ingredient enrichments to obtain a third. Instead, the third enrichment is produced directly, thereby reducing the production of the two original ingredients. In the case of 14 ingredients, 1.80 percent enriched fuel is produced directly causing a reduction in the production of 1.69 percent and 1.90 percent enriched fuels from 117.913 and 333.727 metric tons, utilized in the 13 ingredients case, to 48.770 and 257.670 metric tons, respectively. The model makes this adjustment because it offers greater marginal savings in total production costs than does any other potential alteration to the  $N = 13$  solution.

The other response is exemplified by increasing the number of ingredients from  $N = 11$  to  $N+1 = 12$ . For this change, the set of ingredients selected for  $N+1$  remains unchanged from that of  $N$ , above and below a range of enrichments; however, within the range the choice of ingredients as well as their quantities are adjusted. In going from 11 to 12 ingredients, the 1303.274 metric tons of 2.62 percent, 429.091 metric tons of 2.90 percent, and 918,544 metric tons of 3.10 percent enrichments are replaced by 1265.345 metric tons of 2.62 percent, 179.238 metric tons of 2.82 percent, 384.556 metric tons of 2.96 percent and 821.771 metric

tons of 3.10 percent enrichments. The model makes this adjustment because it yields a greater marginal savings in the total production cost than does any other potential alteration of the N ingredient solution.

Thus, relaxing the constraint upon the number of ingredient enrichments causes the model to search for the adjustment of the still feasible but now suboptimal solution that will yield the greatest marginal reduction in the total cost of the products. This search locates one of two types of adjustments both of which eliminate some blending while leaving the majority of the solution intact. As N is increased from 2 to 39, the adjustments occur within the highest and lowest enrichment values first and then among the intermediate values. This reflects the fact that the exponential separative work costs yield greater marginal cost reduction when requirements for higher enrichments are reduced and that the use of either a high or low enrichment value as an ingredient requires the production of a higher enrichment valued ingredient.

Another result is that certain of the potential ingredients occurred in most or all of the optimal solutions. In particular, aside from 1.69 percent and 4.94 percent, each of the enrichment values of 1.90, 2.10, 3.10, and 3.40 percent occurred in most of the solutions. This phenomenon is attributable to two factors. Primary among these is the set of quantities of the product enrichments required. The distribution of necessary product quantities acts to stabilize the intermediate enrichment values in the solutions and to weigh blending costs. The second

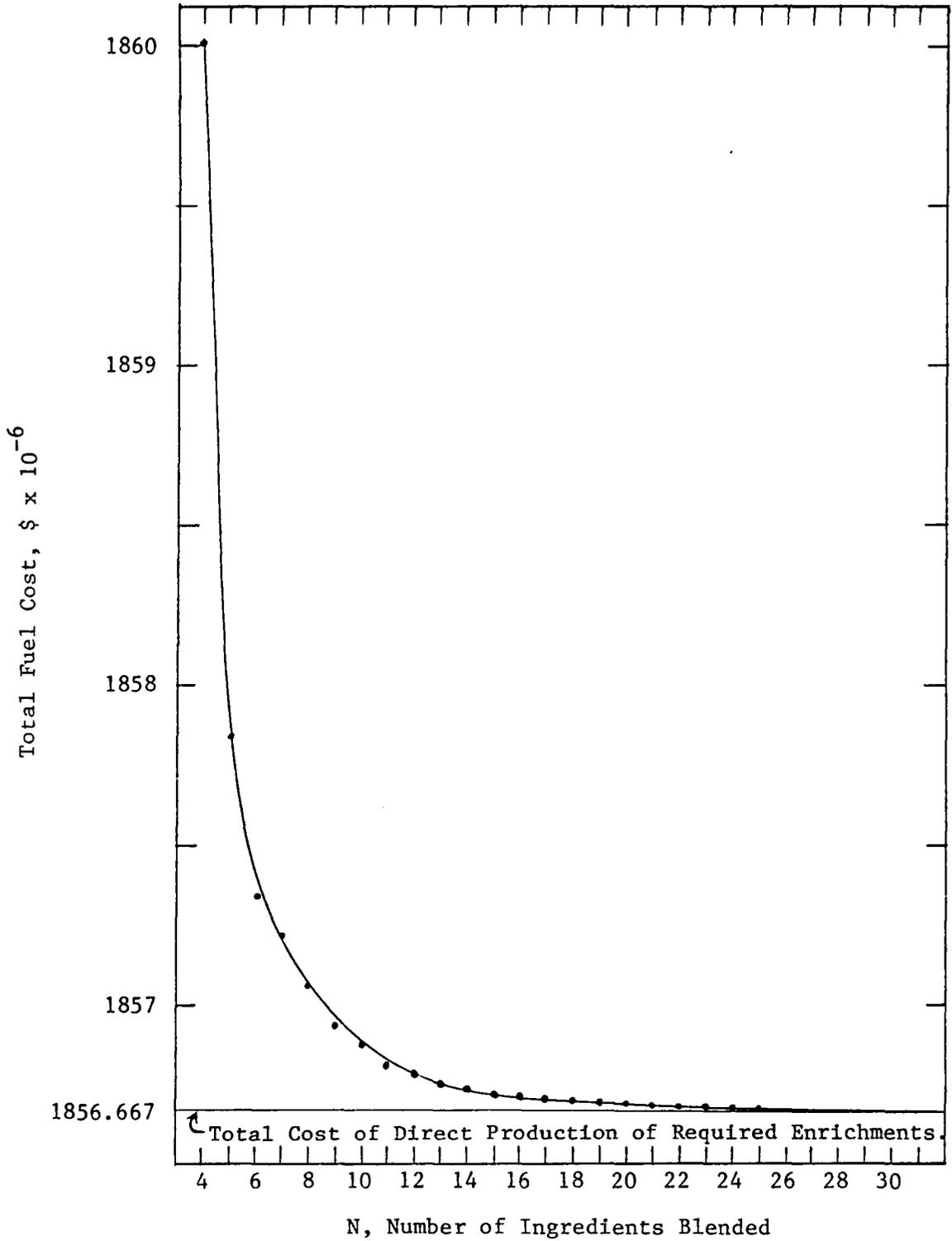


Figure 5. Total Enrichment Costs for the Optimal N Enrichment Sets.

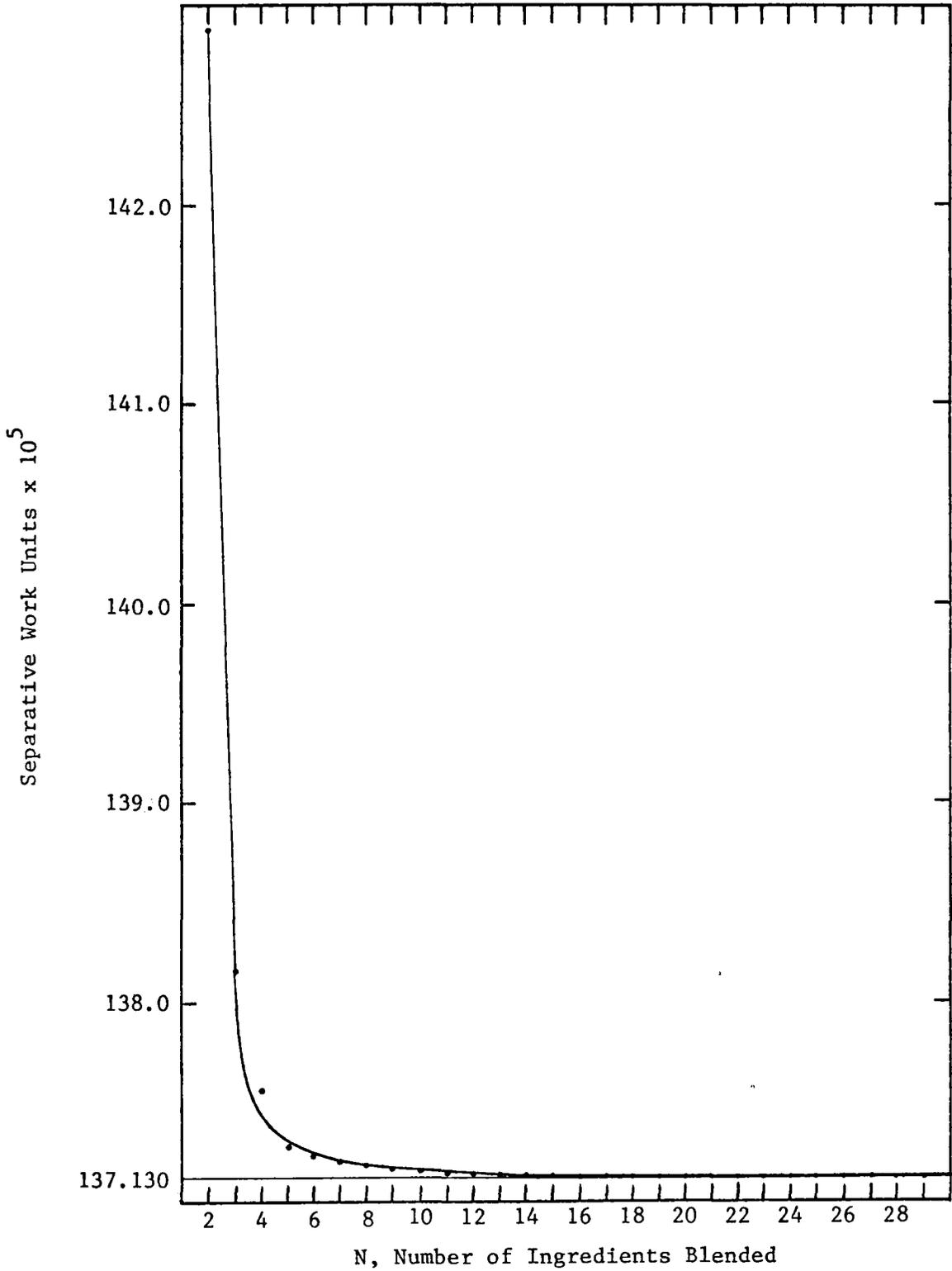


Figure 6. Total Amount of Separative Work for the Optimal N Enrichment Set.

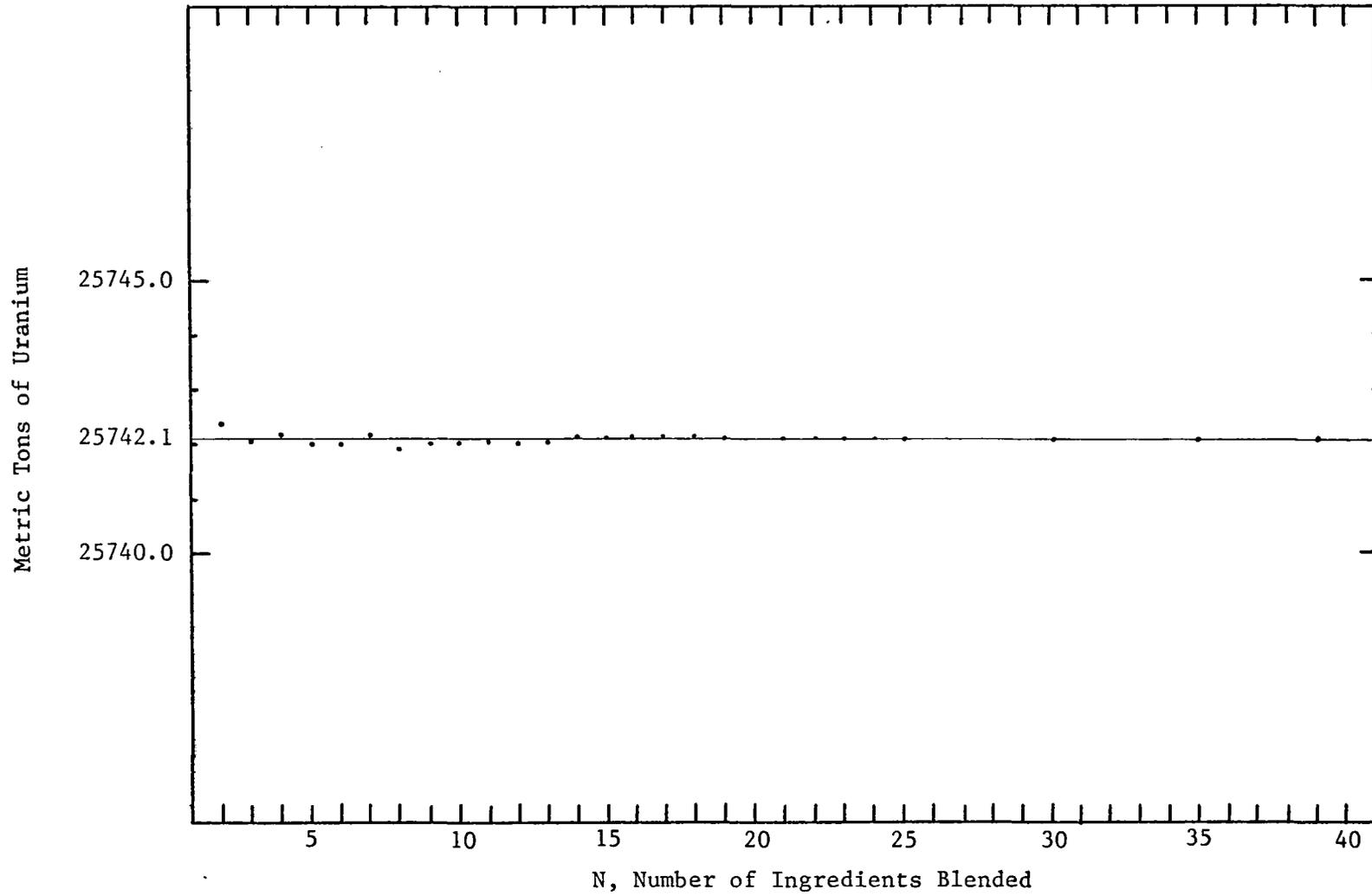


Figure 7. Feed Material Necessary to Produce Optimal N Enrichment Sets.

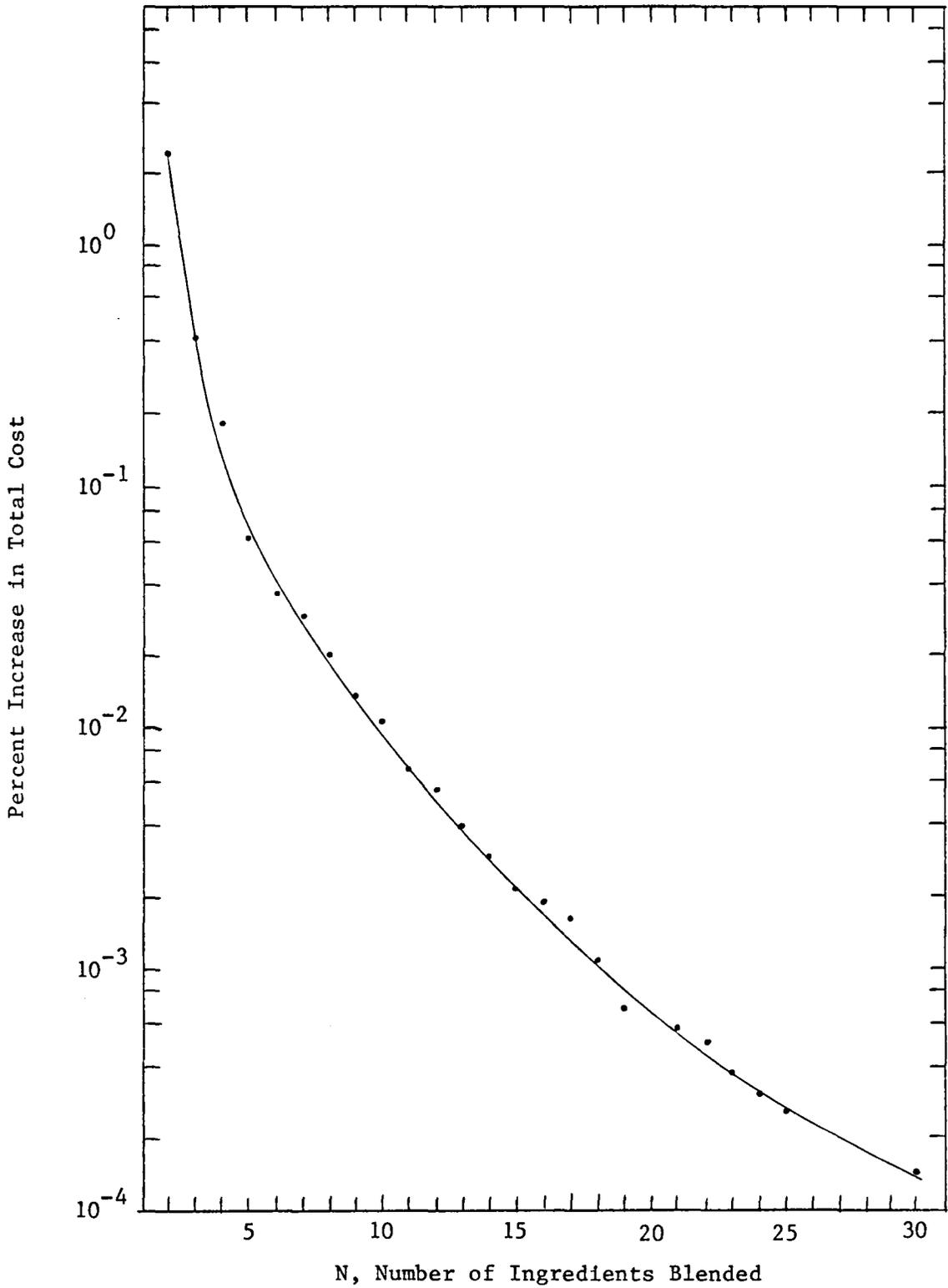


Figure 8. Percent Increase in Total Enrichment Costs for the Optimal N Enrichment Sets.

reason for the consistent choice of certain ingredients is the need to use enrichment values lower and higher than the bounding values for blending. That is, the lowest and highest values are not cost effective blending ingredients because of their distance from most of the product enrichment values. The gaps between 1.69 percent and 2.10 percent and between 3.40 percent and 4.94 percent make 1.69 percent and 4.94 percent marginally unattractive but necessary blending ingredients. The use of 1.90, 2.10, 3.10, and 3.40 percent enriched ingredients is therefore a cost effective alternative to the use of the extreme enrichment values as ingredients.

The amount of natural uranium feed required to satisfy the demands for the enrichment procedure for any  $N$ , the amount of optimal selected enrichments, is presented in Figure 7 and Table 2. As can be seen, the demand for natural uranium feed is relatively constant over the whole range of  $N$ . This constancy means that environmentally and economically there would be no significant increase in the demands for exploration, mining, milling or conversion of the uranium ore, due to the blending procedure. This constant demand for natural uranium feed reflects a constant feed cost and a greater importance of SWU costs with regard to the total enrichment cost of each of the optimal enrichment sets.

Figure 6 displays the amount of separative work necessary to satisfy the enrichment demands for each of the optimal enrichment sets. Note that the demand for separative work decreases sharply with the initial increase in  $N$  and approaches asymptotically the 39 product solution. This reflects a decrease in dependence of intermediate product enrichments on the higher and lower ingredient enrichments. Note also that

SWU requirements level off at  $N = 18$ . This indicates that for  $N$  greater than or equal to 18, the demands for separative work are approximately equal to the separative work demanded when producing the product enrichments directly.

The total enrichment cost for each of the optimal enrichment sets is presented in Figure 5 and Table 2. Note the rapidity with which the total enrichment costs decrease as the value of  $N$  increases. This rapid decline in the total enrichment cost is due to the rapid decay of blending losses as  $N$  increases, see Figure 8. These costs reach zero when the number of ingredients equals the number of products, e.g., when the products are produced directly and no blending is used. For as few as four ingredients, the marginal blending cost is only 0.18 percent of the total direct production cost. Thus, the increased fuel cost under blending of four ingredients exceeds the direct production cost by less than two tenths of one percent. In terms of dollars, this is over 3 million. Nevertheless, it is a small fraction of the total fuel costs. The margin is less than \$21,000 for 21 ingredients. Thus, the extent to which blending costs are incurred can be controlled and balanced against the benefits of using selected enrichments.

Shadow prices,<sup>(17,18)</sup> an integral part of the linear programming technique, are evaluated. From this evaluation it is evident that the trends resulting from the evaluation of the arbitrary reference case input data are not significantly sensitive to the number of product enrichments, the magnitude of the blending costs, and the relative quantities of demanded material. The conclusions associated with the results obtained here can be considered valid for a range of cases

similar to the reference case. The trends are not affected by a change in demand as long as the new demand is reflected in the evaluation of all values of N. Variations in SWU costs affect the magnitude of the marginal costs but do not affect the reported trends.

The results indicate that blending is a reasonable approach for providing fuel for existing commercial light water reactors while pursuing reactor component standardization. In fact, the marginal blending losses (or blending costs) may be offset by the economic benefits of using standardized fuel in new reactors or by the advantages of reduced capital requirements for alternate enrichment technologies. The results also demonstrate that the model is an effective tool for analyzing fuel blending problems. The model can therefore be used to investigate blending when standardized reactor design fixes the set of fuel enrichment levels or to evaluate strategies for blending mixed oxide fuels under spent fuel reprocessing.

## CHAPTER V

### RECOMMENDATION FOR FUTURE WORK

The model provides substantial insight into the consequences of using blending as a means towards achieving standardization of nuclear fuels in existing reactors. However, there are several areas of investigation that should be explored.

One area for investigation deals with the determination of an optimal blending method and the cost associated with it. At present, the model does not examine the physical mechanisms by which the enrichment ingredients are blended; nor does it adjust the total enrichment cost to incorporate the associated cost of using a particular blending method or facility.

As previously discussed, there are  $M$  different fuel enrichments utilized in existing commercial light water reactors. The exact number of these  $M$  fuel enrichments and their amounts depends on the reference material used. A more up-to-date selection of enrichment levels and the associated amounts of these enrichments demanded by commercial light water reactors should be incorporated into the model.

In this study,  $AV_i$ , the availability of the  $i^{\text{th}}$  ingredient enrichment level, is equal to a constant value,  $AV$ , greater than the largest demand for any  $i^{\text{th}}$  ingredient. Thus  $AV$  has no influence on the selection of ingredient enrichment levels in the optimal set of blended ingredients. By determining the availability of any or all  $i^{\text{th}}$  ingredient enrichment levels,  $AV_i$ , and incorporating them into the model,

the constraint on the availability of ingredients can be explored and studied.

The effects uranium recycle will have on the enrichment process and the selection of blending ingredient enrichment levels should be investigated. Recycled uranium has an assay of approximately 0.80 percent (except with extended burnup) and natural uranium 0.711 percent. Utilizing the limited amount of recycled uranium as a feed input into the enrichment process, the total enrichment cost associated with a given product or products will be less than if the same product was produced using natural uranium as a feed input. Exploration into possible modes of utilizing the recycled uranium in the enrichment process will give rise to possible cost advantages that should be determined.

At present, there is research being done dealing with the production and utilization of mixed oxide fuels in an attempt to use plutonium reserves in light-water moderated reactors. Mixed oxide fuels contain a large fraction of uranium oxide and a smaller fraction of plutonium oxide (up to 3.0 percent of the mixed oxide). Mixed oxide fuel performs as well as uranium fuel with some minor core physics changes. The present model can be modified to handle the blending operation of the mixed oxide fuels, selection of optimal blending ratios, the selection of possible standardized enrichment sets, and plutonium recycle.

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## APPENDIX I

### NUCLEAR FUEL CYCLE

The transformation of uranium from uranium bearing ore into reactor fuel and its return back to the earth in the form of solid waste is shown in Figure 9. As indicated in the figure, the nuclear fuel cycle begins with the exploration and mining of the uranium bearing ore. The low grade ore is then mechanically and chemically processed to obtain approximately 85 percent pure uranium oxide,  $U_3O_8$  ("yellow cake"). It takes up to 500 kilograms of low grade ore to yield one kilogram of "yellow cake". The "yellow cake" is further purified and chemically converted into gaseous uranium hexafluoride,  $UF_6$ . The uranium hexafluoride is processed in the enrichment plant where it is mechanically enriched in the lighter uranium-235 isotope. In the fuel fabrication plant the uranium hexafluoride is chemically converted into uranium dioxide,  $UO_2$  and formed into pellets. The uranium pellets are loaded into zircalloy or stainless steel tubes which are assembled into matrices known as fuel assemblies. From the fabrication plant, the fuel assemblies are shipped to nuclear fuel consumers (reactors). At the nuclear power reactor, the fuel assemblies are located into the reactor core where the uranium-235 is fissioned and the heat energy produced is transformed, through steam generation and turbines, to electrical energy. Since only a fraction of the uranium-235 is fissioned or transmuted, the spent fuel contains residual uranium-235, radioactive fission products, and plutonium. After sufficient cooling, the spent fuel is shipped to a reprocessing plant where the fuel is separated into plutonium and uranium for recycle and

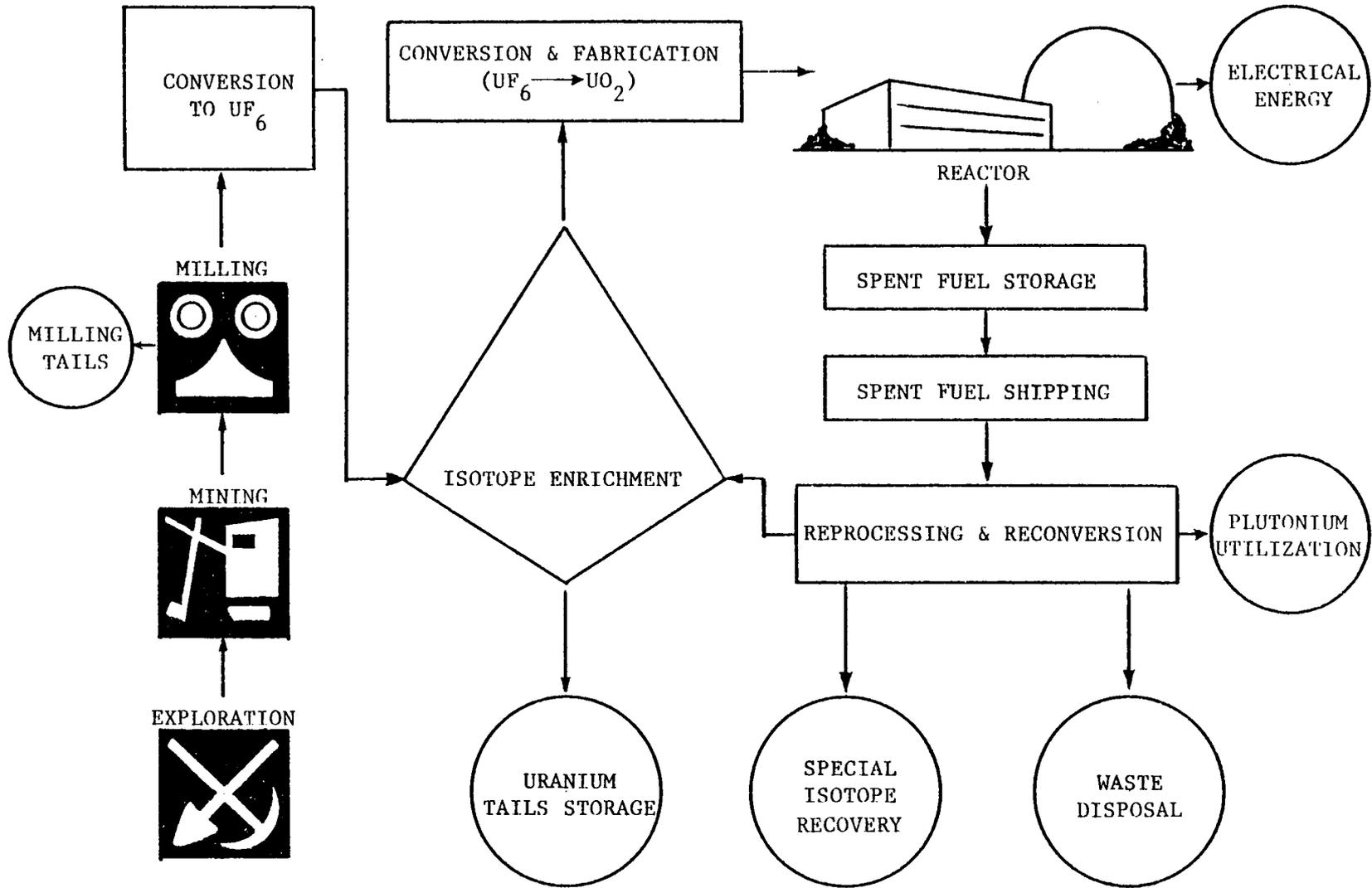


Figure 9. Nuclear Fuel Cycle.

solid wastes which are disposed of in a manner which precludes release to the environment.

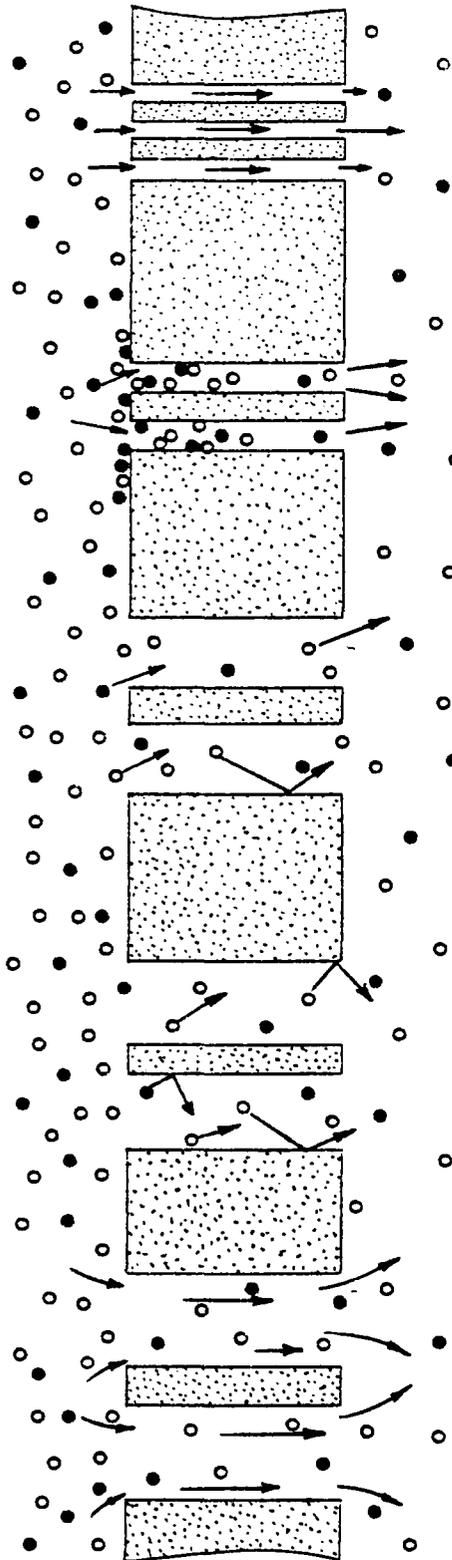
## APPENDIX II

### ENRICHMENT TECHNOLOGIES

Isotopes of an element are identical chemically; therefore, separation and concentration of a particular isotope must be done by physical means, utilizing slight differences in the atoms or molecules in which they are contained. Any physical phenomenon in which a difference in mass, light absorption, or velocity affects the distribution of the molecules or atoms can potentially be employed to separate isotopes. Further discussion of separation techniques in the enrichment of natural uranium into a larger fraction of uranium-235 is presented below.

#### Gaseous Diffusion

The gaseous diffusion separation process is based upon the small differences between the average molecular velocities of the gas molecules containing uranium-235 and uranium-238. Due to this difference, the molecules containing the lighter uranium-235 strike the porous membrane walls of the containment vessel more frequently than the heavier molecules of uranium-238. Because of the frequency of collisions of the lighter uranium-235 compounds, on the porous membrane (or "barrier"), there is a higher rate of diffusion of the uranium-235 compound across the "barrier" (see Figure 10). The result is a slightly enriched diffused stream of the lighter uranium-235 compound compared with the undiffused stream. Graham in 1846 first discovered this separation technique which was later explained theoretically by Maxwell's kinetic theory of gases. (6)



#### CAPILLARY FLOW

In very fine pores, the gas condenses and flows as a liquid to evaporate at the low pressure face. Nonseparative.

#### SURFACE FLOW

Molecules adsorb at the high pressure face, migrate as a surface film, and desorb at the low pressure face. This mode of flow can occur in all sizes of pores, but it is especially serious in small pores which have a high ratio of surface/volume. Partially nonseparative.

#### PURE KNUDSEN FLOW

Molecules move completely independently through the pores, colliding only with the pore walls. Separative.

#### MIXED KNUDSEN FLOW

Some collisions occur between molecules within the pores. The effect tends to make both types of molecules move through the pore at the same rate and reduces the separation.

#### VISCOUS FLOW

Viscous flow occurs in relatively large pores and leaks. Collision between molecules within the pores is very frequent and the molecules move as a group rather than independently. Nonseparative.

Figure 10. Modes of Molecular Flow Through Capillary Media.

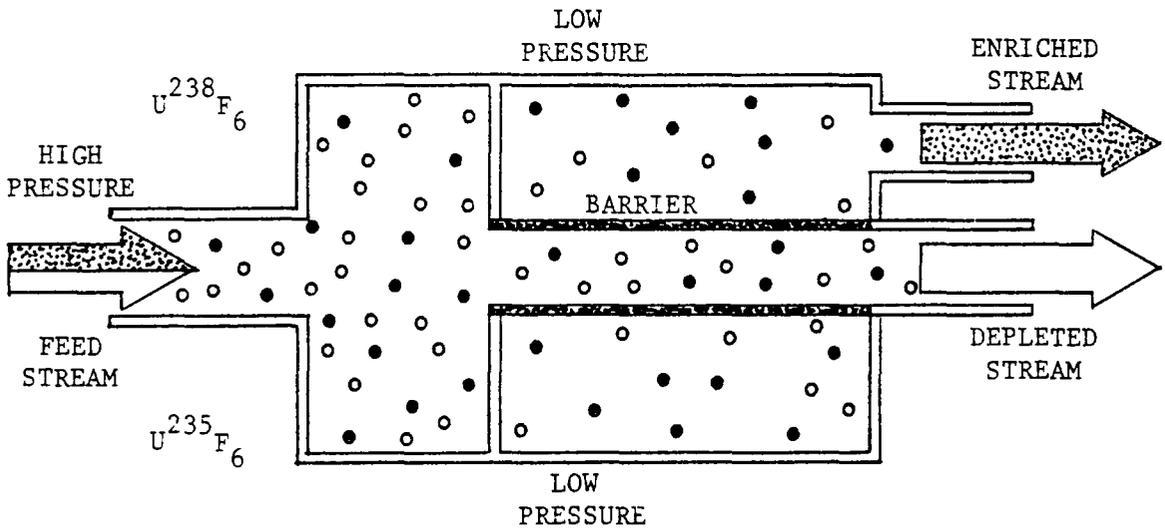
Uranium hexafluoride ( $UF_6$ ) is the only uranium compound which is gaseous at convenient temperatures and pressures. Because fluorine has only one naturally occurring isotope, separation of the  $UF_6$  is due only to the mass difference between uranium-235 and uranium-238. Utilizing this gas, the maximum theoretical separation factor obtainable by gaseous diffusion is 1.00429. With the theoretical separation factor so close to unity, the enrichment gain achieved per single stage is very small. This necessitates the utilization of a cascade consisting of a number of stages to obtain a significant degree of enrichment (Figure 11). By utilizing larger equipment to permit a very large throughput of uranium hexafluoride, the amount of separative work performed per stage can be quite large.

A term frequently mentioned in discussing isotope separation techniques is separative work. Separative work, as applied to the separation of uranium isotopes, is a measure of the work required to separate a given uranium-235 concentration in a feed stream into a product stream which is more enriched, and a waste stream depleted in uranium-235. Separative work can thus be used as a "unit" of enriching services performed by isotope separation facilities. Separative work has the units of mass and is usually expressed in kilogram units.

#### Gaseous Centrifuge

Today, the gas centrifuge process is one of the most attractive alternatives to gaseous diffusion for providing new separation capacity. The appeal of the gas centrifuge process is based upon the single stage comparison of the ideal separation factor for uranium-235 and uranium-

## Gaseous Diffusion Stage.



## Stage Arrangement.

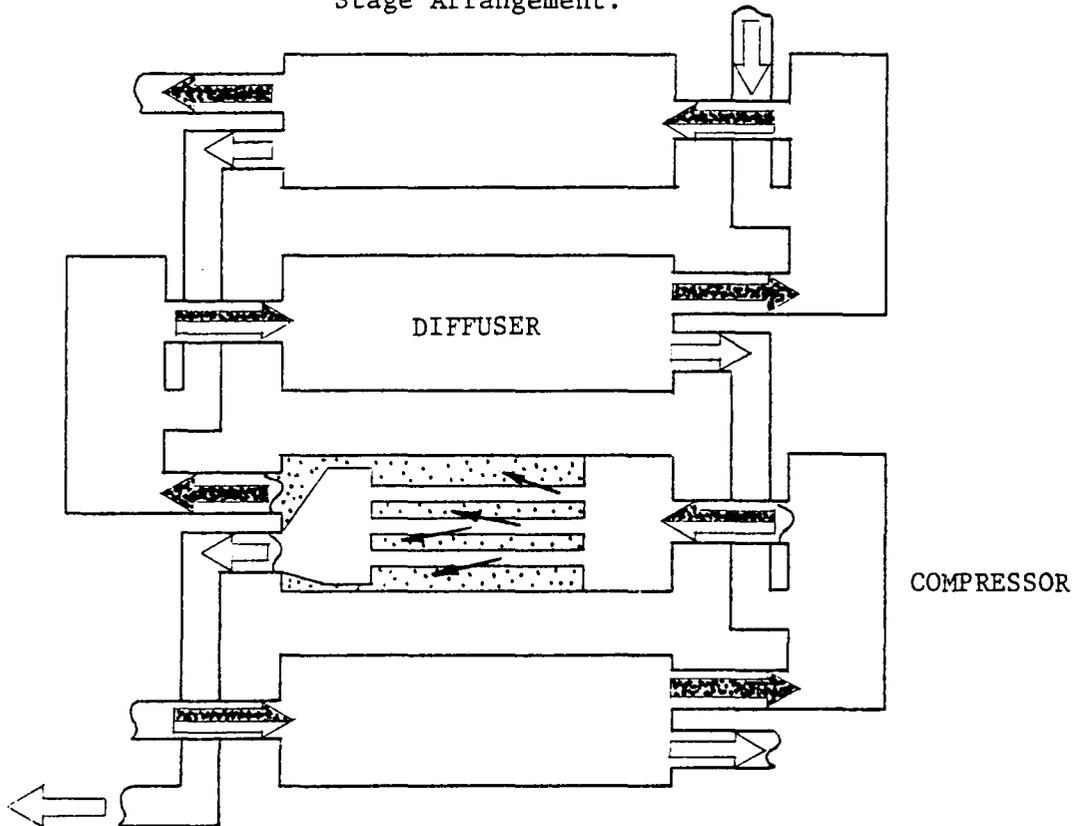


Figure 11. Gaseous Diffusion Stage and Stage Arrangement. (4)

238. In the gaseous diffusion case, the ideal separation factor is 1.0043 compared to 1.055 for a centrifuge having a peripheral speed of 300 meters per second. (10)

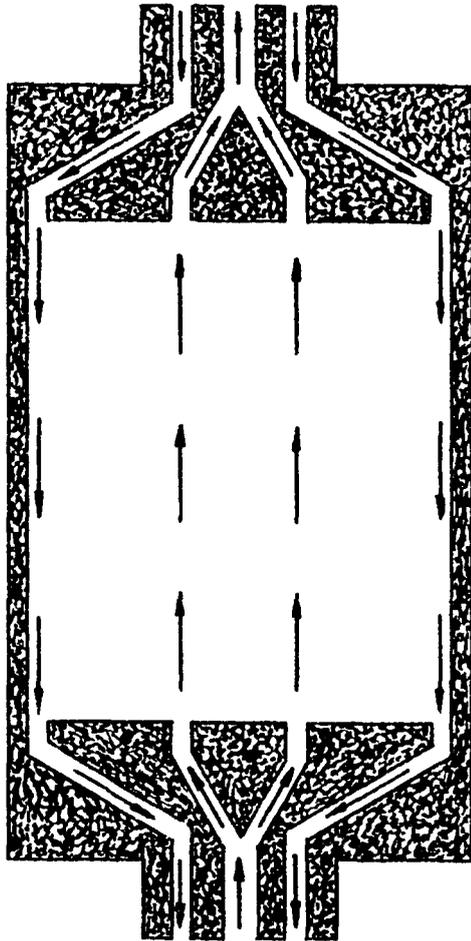
The countercurrent centrifuge method suggested by H. C. Urey and the theory principally developed by K. Cohen is of the most interest. (12) This method employs an axial convective circulation (axial counter-current flow) of uranium hexafluoride gas in a hollow vertical cylinder rotating about its axis at a high angular velocity. Due to the axial rotation, the lighter isotope becomes enriched in the vicinity of the wall, forming an axial concentration gradient.

Countercurrent centrifuges are of two basic types; individual streamed and internal recirculation (see Figure 12). The advantages of the countercurrent centrifuge are:

- 1) Both isotope fractions can be extracted from the periphery without difficulty, due to the high pressure in this area; (10)
- 2) A high separation factor can be obtained from each single unit, thus reducing the amount of gas to be transported between units; (10) and
- 3) The reduction in the number of stages to achieve a given enrichment. (7,10)

In theory, the gas centrifuge process is thermodynamically reversible, making it possible to operate a centrifuge plant with far less power than required for a diffusion cascade. Another advantage is that the equivalent capacities can be constructed at lower capital costs with comparable unit costs, as shown in Table 4. The gase centrifuge process,

Individual Streamed



Internal Recirculation

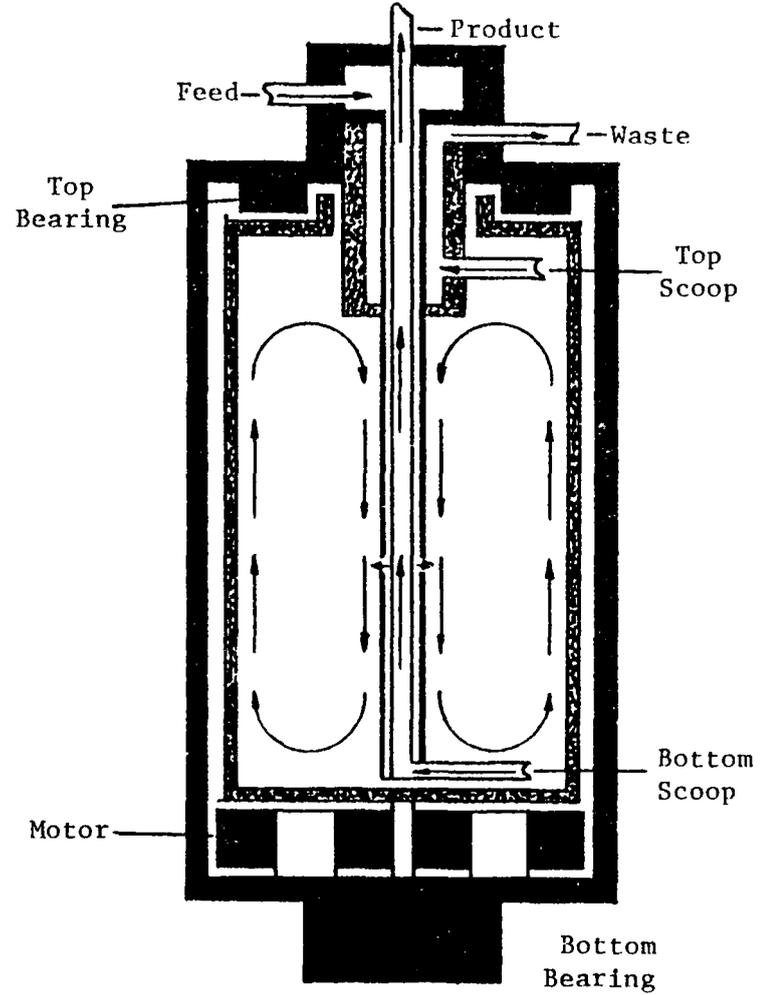


Figure 12. General Types of Countercurrent Centrifuge. (10)

Table 4.  
 Economic Comparison Between  
 Gaseous Diffusion and Gaseous Centrifuge. (13)

	Gas Centrifuge	Gaseous Diffusion
Plant Size ( $\times 10^3$ SWU yr.)	3,100	9,000
Capital Cost ( $\$10^9$ )*	.13*	3.16
Specific Investment Cost ( $\$/\text{SWU yr.}$ )	365	351
Unit Cost ( $\$/\text{SWU}$ )		
Financing	43	35
Operating	27	44
	70	79

\* Including working capital and interest during construction (calculated in 1974 dollars).

however, loses its economic appeal as the number of different enrichment levels it can produce increases.

Comparing a 3.1-million-SWU/yr. gas centrifuge plant with a 9-million-SWU/yr. gaseous diffusion plant the smaller gas centrifuge plant can produce enrichments at a cost of \$70 per SWU compared to \$79 per SWU for the larger diffusion plant (both calculated in 1974 dollars). Furthermore, it is predicted that even a 1-million-SWU/yr. gas centrifuge plant would produce enrichments at only \$78 per SWU, still a dollar less than the product of the larger gaseous diffusion plant.<sup>(13)</sup>

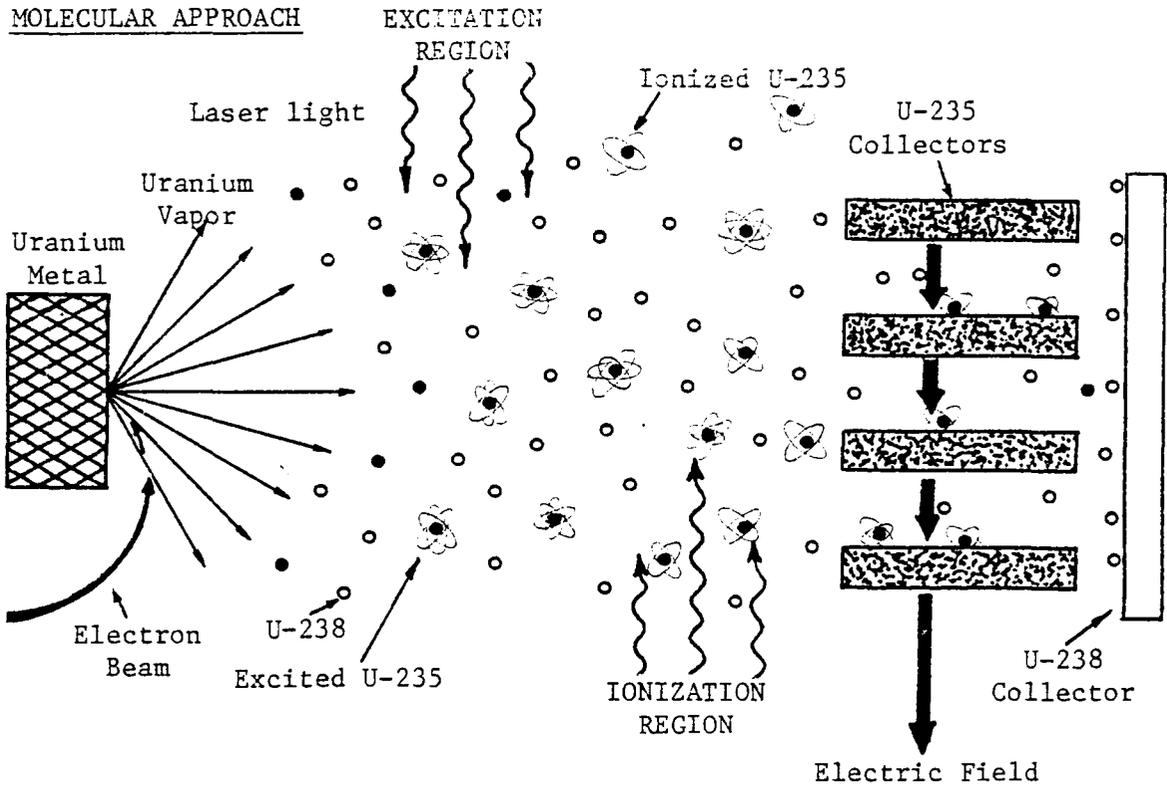
Besides the economic advantages of the gas centrifuge process, low capital requirements and small size, the centrifuge plant uses only one tenth of the electricity consumed by a gaseous diffusion plant.<sup>(13)</sup> Thus the centrifuge process has a lower environmental impact than the gaseous diffusion process.

#### Laser Enrichment

Laser enrichment has the advantage of nearly complete separation of uranium-235 and uranium-238 in a single stage. Currently laser enrichment research is following two modes of development. One deals with utilizing atomic uranium (metallic uranium) as feed, the other molecular uranium (uranium hexafluoride). The basic ideas for both modes of development are very much the same; but the engineering problems they present are very different.

In the atomic approach to laser enrichment ( see Figure 13) uranium metal is vaporized at a very high temperature (on the order of 2300<sup>o</sup> C). The uranium vapor containing atoms of uranium-235 and

MOLECULAR APPROACH



ATOMIC APPROACH

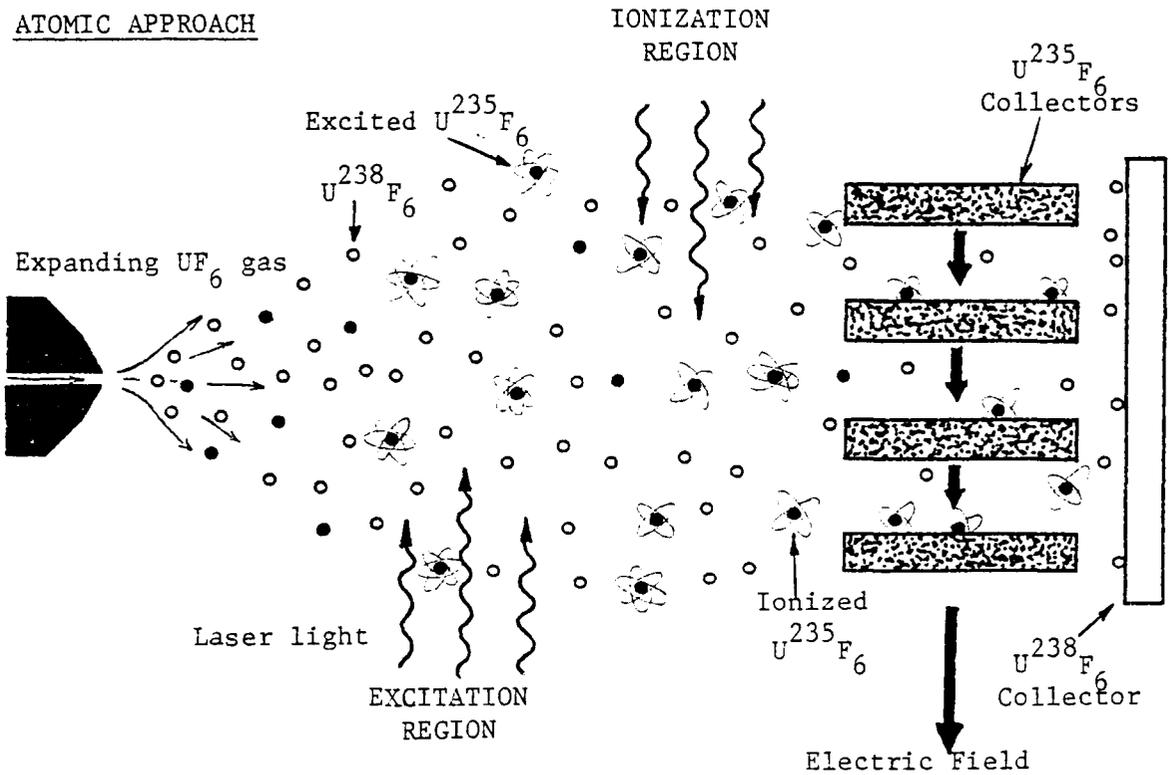


Figure 13. Atomic and Molecular Approach to Laser Enrichment.

uranium-238 is then exposed to the light from a tunable dye laser of just the right wave length to excite the atoms of uranium-235 but not those of uranium-238. The vapor is then illuminated by the light from a second tunable dye laser which ionizes the excited uranium-235 but not the unexcited uranium-238. An electric field is then applied to sweep the charged uranium-235 ions onto collecting plates.

The molecular approach utilizes much the same ideas as the atomic approach except that molecules of uranium hexafluoride, instead of uranium metal, are exposed to laser light. The absorption of the laser light by the molecule of uranium hexafluoride increases its internal energy. The molecular and atomic approach differ in that, typically, internal energies of molecular vibration are approximately 100 times smaller than the internal energies of atoms. Thus infrared (very low energy "photons") lasers are used to excite molecules, whereas visible or ultraviolet lasers are used for atomic excitation.

The uranium hexafluoride gas is mixed with either helium or nitrogen gas and forced at supersonic speeds (Mach 3 to Mach 5) through a nozzle. Upon exit from the nozzle the gas mixture supersonically expands decreasing the gas temperature. Under proper supersonic expansion conditions, the molecular vibration of the uranium hexafluoride decreases while retaining the physical properties of independent molecules containing uranium-235 will be excited. Further illumination of the excited molecules by a second laser will cause ionization of those molecules. The ionized molecules are then swept by an electric field onto collecting plates. (26)

Becker Trennduse or "Nozzle Process

The Becker process utilizes the pressure gradient developed in a curved, expanding, supersonic jet to achieve separation of a gas mixture of uranium hexafluoride and hydrogen carrier gas. A schematic of the separation nozzle stage is shown in Figure 14. The gas mixture is forced at high velocity between a pair of paring blads, and a semi-circular groove having a radius of approximately one-tenth millimeter (0.039 inch). Through a combination of pressure and centrifuge effects, the heavier isotope of uranium hexafluoride is concentrated in the vicinity of the groove's wall. The expanding jet of gas is divided into two fractions by the second blade, one enriched in the lighter uranium hexafluoride component and the other enriched in the heavier uranium hexafluoride component. Connecting large numbers of separation nozzle stages together to form a cascade is necessary to obtain desired separation.

The Becker process is a high-pressure process. The machined tolerance of the groove and blade must be one micron (39 millionths of an inch) in order to function correctly.

The major disadvantage of the Becker process is the relatively high power requirements due to the large volume of light gas that must be recompressed between stages. This is due to the fact that the efficiency of the Becker process per stage varies with the percentage of uranium hexafluoride, the greater the degree of enrichment but the less material that can be enriched. (7,23)

Fenn Shock Process

In this aerodynamic process (Figure 14), a high powered blower and nozzle direct a supersonic stream of gas containing uranium hexafluoride against hollow metal probes. The shock wave formed ahead of the probe acts as a separation zone. The heavier isotope concentrates in the stagnant gas behind the shock wave and is collected through the hollow probes. A number of probes may be placed in the gas stream from a single nozzle to increase the degree of separation per stage. The fact that this process concentrates the heavier isotope of uranium hexafluoride instead of the lighter isotope creates complications; removal of a large amount of uranium-238 from the feed gas will only cause a small increase in the percentage of uranium-235 in the remaining gas. The economic feasibility of the Fenn shock process is reduced due to this significant characteristic of enriching the uranium hexafluoride in the heavier isotope of uranium. (23)

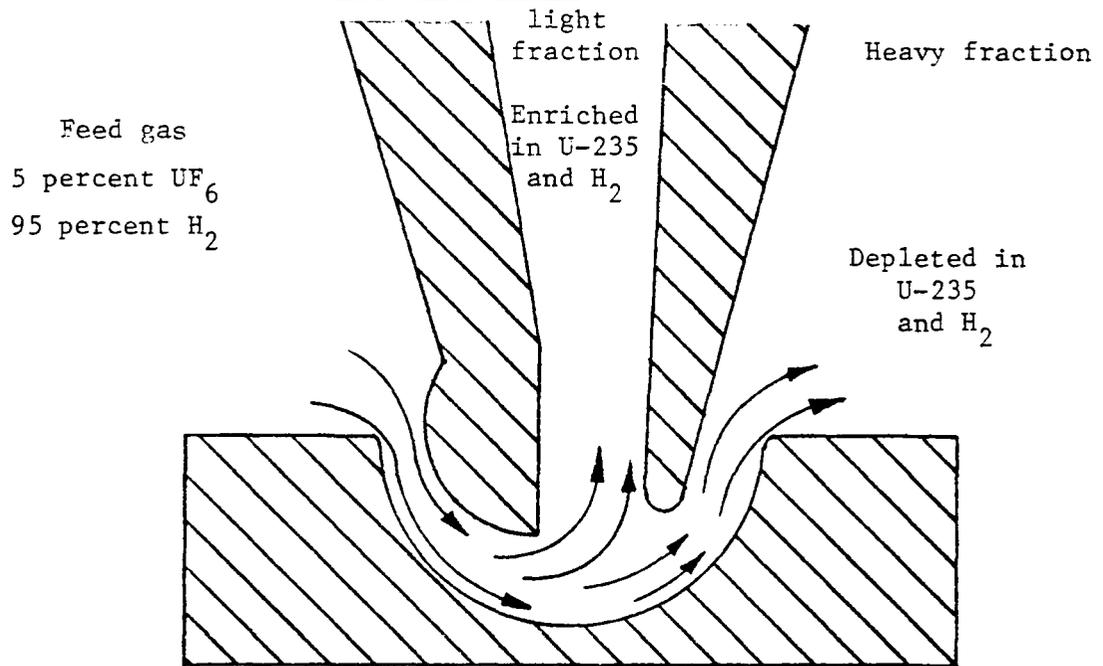
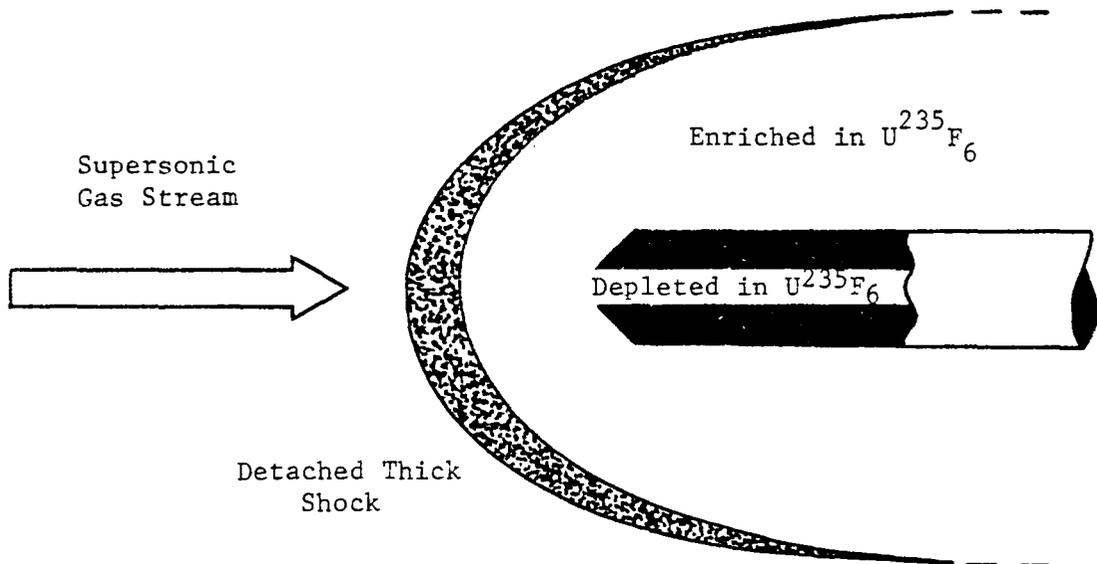
BECKER TRENDUSE OR "NOZZLE" PROCESSFENN SHOCK PROCESS

Figure 14. Becker Trenduse and Fenn Shock Process.

## APPENDIX III

### MIXED INTEGER PROCEDURE

#### MISTIC<sup>(25)</sup>

MISTIC, Mixed Integer Search Technique Internally Controlled, is an MPS III proprietary procedure for solving optimization problems having a mixture of continuous and integer variables. The integer variables are restricted to values of zero and one. The basic method used is the branch-and-bound algorithm.<sup>(23)</sup>

Formulation of the mixed integer problem is identical to a linear programming problem except that a unique identifying character must be contained in the zero/one integer variable names. The integer variables must also be defined in the bounds section of the input data as having an upperbound of 1.0. Continuous variables may be bounded according to the normal MPS conventions.

MISTIC incorporates a branch-and-bound enumeration procedure for finding an optimal solution from a finite number of feasible solutions. The basic idea of the branch-and-bound technique is the following. Assume that the value of the objective function for the best feasible solution identified thus far, sometimes called the continuous optimal solution, is available; and that the objective function is to be minimized. Once the continuous problem is solved, a branch-and-bound algorithm is employed to generate partial solutions (nodes) of the mixed integer problem. The nodes are partitioned into several subsets, and, for each subset, a lower bound is obtained for the value of the

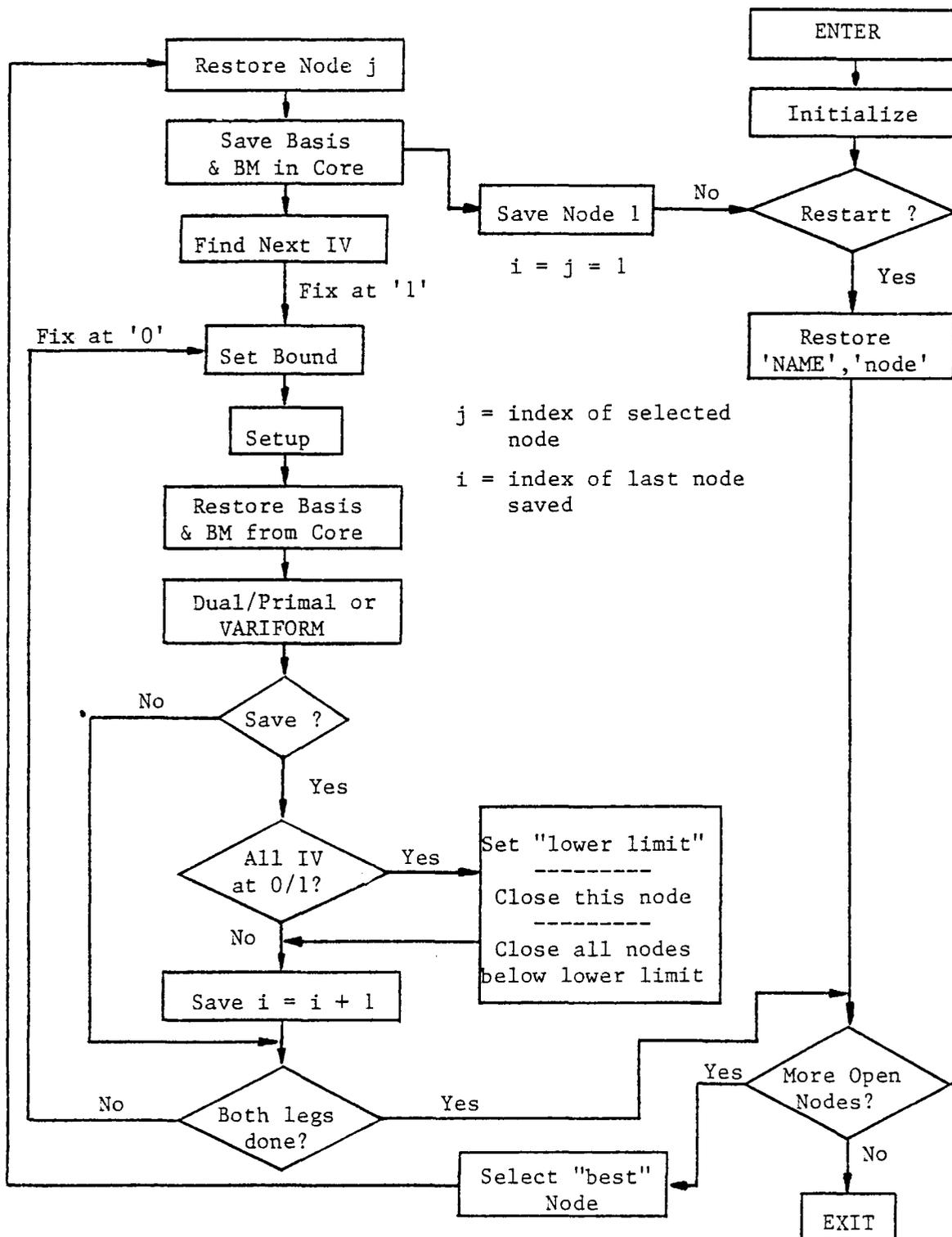


Figure 15. MISTIC Loop.

objective function of the nodes within that subset. To eliminate a large number of possible nodes from consideration, those subsets whose lower bounds exceed the current upper bound on the objective function are excluded. The remaining subsets are then partitioned further into several subsets. In turn, these lower bounds are obtained and used to exclude some of these subsets from consideration. From all of the remaining subsets, another one is selected for further partitioning, etc. This process is repeated until the reduced solution space is exhausted or a feasible solution is found for which the corresponding value of the objective function is no greater than the lower bound for any subset. (17,24,25) A flow chart of the MISTIC loop is shown in Figure 15.

The following control program demonstrates the use of MISTIC.

```

PROGRAM
INITIALZ
MOVE(XDATA,'ENRICH')
MOVE(XPBNAME,'BLEND')
CONVERT('SUMMARY')
SETUP('MIN','BOUND','MIX')
MOVE(XOBJ,'MINZ','AV...I')
MOVE(XRHS,'LIMIT')
VARIFORM
XPARAMAX = 0.
MISTIC
MISTIC('SOLUTION',XR1C2)
EXIT
PEND

```

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MINIMIZATION OF BLENDING LOSSES TO DETERMINE OPTIMAL STANDARD  
ENRICHMENTS OF NUCLEAR FUEL

by

John Scott Lorber Junior

(ABSTRACT)

Identities, quantities, and costs associated with producing a set of selected enrichments and blending them to provide fuel for existing reactors are investigated using an optimization model constructed with appropriate constraints. Selected enrichments are required for either nuclear reactor fuel standardization or potential uranium enrichment alternatives such as the gas centrifuge. Using a mixed-integer linear program, the model minimizes present worth costs for a 39-product-enrichment reference case. For four ingredients, the marginal blending cost is only 0.18 percent of the total direct production cost. Natural uranium is not an optimal blending ingredient. Optimal values reappear in most sets of ingredient enrichments.