TESTING FOR NEUTRALITY AND RATIONALITY WITH AN OPEN-ECONOMY MODEL: THE CASE OF CANADA

by

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ABSTRACT

CHAPTER I

INTRODUCTION

A. Policy Activists and Non-Activists

In his presidential address to the American Economic Association in 1976, Franco Modigliani stated that "There are in reality no serious analytical disagreements between leading monetarists and leading nonmonetarists."[1] Few economists today would seriously doubt the accuracy of Modigliani's statement. Yet, economists today are far from reaching a concensus as to how stabilization policies should be conducted. According to Modigliani, there are two schools of thought on the proper role of stabilization policies, which he termed 'monetarist' and 'nonmonetarist.' In Modigliani's words,

> Nonmonetarists accept what I regard to be the fundamental practical message of *The General Theory*: that a private enterprise economy using an intangible money *needs* to be stabilized, *can* be stabilized, and therefore *should* be stabilized by appropriate monetary and fiscal policies. Monetarists by contrast take the view that there is no serious need to stabilize the economy; that even if there were a need, it could not be done, for stabilization policies would be more likely to increase than to decrease instability....[2]

It is clear from the above passage that the current controversy between the two schools of thought is centered on the desirability and the effectiveness of stabilization policies. On the one hand, the nonmonetarists favor an activist policy to stabilize the economy. On the other hand, the monetarists believe that active stabilization policies would do more harm than good to the economy and favor instead a passive (non-activist) policy. The purpose of this chapter is to trace the history of the current debate and to consider both sides of the debate briefly.

B. The Phillips Curve and Policy Activism

The current debate on policy activism can be traced back to the seminal paper by A. W. Phillips in 1958.[3] In his paper, Phillips investigated the relationship between money wage inflation and excess demand in the labor market by using United Kingdom data from 1861 to 1957. Phillips posited a relationship between money wage inflation and excess demand in the labor market:

$$\dot{W} = a\left(\frac{d-s}{s}\right), \qquad \text{(I.1)}$$

where $\dot{W}$ is the rate of change of the money wage, d and s are the demand and supply for labor respectively, and $a>0$ is an adjustment parameter. Since there was no good measure of excess demand in the labor market, Phillips used the unemployment rate as a proxy. What Phillips found was a negative nonlinear relationship between money wage inflation and unemployment. This negative relationship between money wage inflation and unemployment is, of course, the now famous Phillips curve.

In a subsequent extension of Phillips' work, Richard Lipsey[4] attempted to provide some theoretical underpinnings for the Phillips curve and also formalized some of Phillips' verbal discussion into empirical relationships. Lipsey modified Phillips' basic relationship between wage inflation and unemployment by adding the percentage rate of change of the unemployment rate $\dot{U}_t$ and the percentage rate of change of the price level $\dot{P}_t$ as explanatory variables. The reason for including the percentage rate of change of the unemployment rate was to take into consideration the observed time series relationship between money wage inflation and the unemployment rate, which takes the form of short-run "loops" around the estimated long-run equation. However, the reason for the inclusion of the percentage rate of change of the price level as an additional explanatory variable was less clear. Lipsey suggested that attempts will be made by workers or unions to push up money wages if the change in the price level threatened to lower real wages. Using standard regression techniques, Lipsey obtained empirical results which confirmed Phillips' basic findings of a nonlinear inverse relationship between money wage inflation and unemployment for the United Kingdom.[5]

The Phillips curve was introduced to the United States by Samuelson and Solow.[6] They assumed that the price level was a mark-up over the wage rate by a fairly constant proportion, and hence, they obtained a negative relationship between price inflation and unemployment. Furthermore, Samuelson and Solow put the discussion of the Phillips curve into a policy context, suggesting that if full employment is incompatible with price stability, the policy makers are faced with a "menu" of choices along the Phillips curve where a lower unemployment rate can only be obtained at the expense of a higher inflation rate. Hence, provided the Phillips curve relationship is stable and a social welfare function could be chosen, then policy variables can be manipulated deliberately to attain an optimal point on the Phillips curve. This optimal point on the Phillips curve then represents the lowest attainable disutility on the social indifference contour.

Considerable other efforts were devoted to theoretical and empirical investigations of the Phillips curve.[7] For example, some investigators incorporated variables such as corporate profit rates, union membership, the lagged rate of change of costs of living, vacancy rates and a host of other variables into the original two-variable Phillips curve. While the inclusion of additional explanatory variables helped to obtain better statistical fits in some cases, the inclusion of such explanatory variables was often not grounded in firm theoretical analysis. Furthermore, the inclusion of variables such as profit rates, the rate of change of prices, and union membership introduced simultaneous equation bias into a single equation estimate.[8] Nevertheless, the empirical evidence obtained by the inclusion of additional explanatory variables was mixed, and no firm conclusions could be reached.

Furthermore, much of the empirical work on the Phillips curve has two common problems. The first is serial correlation, and the second is the temporal stability of the estimated relationship. Unfortunately, these two problems were often ignored.

Rowley and Wilton[9] have shown that researchers working with quarterly models but specifying their dependent and explanatory variables as four-quarter moving averages will have an error term that has the same moving average process as the dependent and explanatory variables. In these cases, first order serial correction will be of little help. They reestimated three empirical studies of the Phillips curve with GLS and compared the results with Ordinary Least Squares estimates. They found that in a large number of cases, those coefficients that were significant with ordinary least squares estimate were no longer significant with Generalized Least Squares. Their conclusion is worth quoting here:[10]

> Only the profit variable retains significance in even half of the GLS equations. Unemployment and price increases, perhaps the two most common explanatory variables in all wage research, are significant in only two of eight individual cases. One can only speculate whether the various authors would have advanced the Phillips curve model had they been faced with GLS estimates rather than the OLS estimates.

With respect to the second problem, most authors implicitly assumed that the structure of their Phillips curve was a stable relationship over time. With a few exceptions such as Oi,[11] Lucas and Rapping[12] for the United States, and Riddell[13] for Canada, this assumption has never been subjected to statistical test. The overwhelming evidence, either by casual observations, or by formal statistical tests, was that the Phillips curve relationship was temporally unstable. The terms of the tradeoff between inflation and unemployment had progressively worsened over time. As a result, the usefulness of the Phillips curve as a guide to policy actions is diminished; for it cannot be used as a reliable basis for predictions since it is not clear which Phillips curve, if any, is faced by the policy makers.

From 1967 to 1969, the inflation rate accelerated, but unemployment rate remained largely unchanged--contrary to the prediction of a trade-off between unemployment and inflation as suggested by the Phillips curve. However, this was viewed as a minor setback until a more sophisticated Phillips curve could be found. Empirical research with the Phillips curve in the late 1960s and early 1970s has been towards more and more sophisticated formulation of the Phillips curve. Such research has consisted mainly of refitting the wage equation by adding new variables to the basic wage-unemployment equation, generating new data for empirical research, or experimenting with different lag structures.

The most notable example is a series of papers in 1971 and 1972 by Robert Gordon for the Brookings Institute.[14] The novel features of Gordon's wage equation were the inclusion of a distributed lag of the gap between the change in product and consumer prices, and the replacement of an aggregate unemployment rate by three partial labor market variables: (i) unemployment dispersion, (ii) disguised unemployment rate, (iii) unemployment of hours. Based on his results, Gordon concluded that the wage inflation of the late 1960's could be satisfactorily explained by his modified Phillips curves. In a later paper in 1977,[15] Gordon reestimated the same wage equation, and despite apparent substantial differences in the two sets of coefficients, Gordon did not investigate the stability of his equation between the two periods. When Gordon replaced his three unemployment variables with a variable measuring output gap and another variable measuring the rate of change of the output gap, he obtained slightly better results. On the basis of these results, Gordon concluded that demand for labor is the main determinant of money wages and that actions by workers have little impact.

However, Gordon did not provide any formal theoretical reasoning for the link between excess commodity demand and money wage changes. Furthermore, the relative importance of excess aggregate demand variables versus labor market variables in determining money wage changes cannot be assessed from Gordon's regression results. A proper test should include both sets of variables in a common regression equation and testing to determine whether the coefficients on the labor market variables are simultaneously zero. Unfortunately, Gordon did not carry out such a test.

It is clear that Gordon's wage equation fitted the historical data well, but what is not so clear is whether his equation will continue to perform well in the future. The conclusion reached by Oi after critically reviewing several models of wage and price determination, including Gordon's, may well summarize the feelings of a number of economists in the profession. In Oi's words,[16]

> The correspondence between the underlying theory of a wage or price equation and the empirical variables appearing in the econometric models appears to be quite weak. Goodness of fit and plausible t-values based on shaky time series data appear to be more important than adherence to an accepted theory of wage and price dynamics in determining just which variables should, or should not, be included in the final wage and price equations. An example of the flexible character of the theory is provided by the corporate profits variable. The profits variable was popular in the wage equation prior to 1968, but when

profits failed to track the wage explosion of 1967-70, it was quickly discarded, and the recent wage equations do not even mention "ability to pay" and "liquidity" as determinants of short run wage changes.

C. Natural Rate of Unemployment and the Vertical Phillips Curve

The discussion in the last section has shown that the empirical foundation of the Phillips curve appears to be rather weak. However, policy discussions in the 1960s were largely based on a stable Phillips curve tradeoff. A few economists, notably Milton Friedman, did question the validity of a stable Phillips Curve. However, his skepticism stemmed from theoretical rather than empirical grounds. Friedman noted that if both workers and employers were free of money illusion, it implied that the Phillips curve must be vertical. However, this theoretical vertical Phillips curve must be reconciled with the observed inverse time series relationship between inflation and unemployment. Phelps[17] and Friedman[18] were apparently successful in providing such a theoretical framework.

Friedman started by distinguishing between anticipated and unanticipated inflation, and by noting that the supply and demand for labor depend on the real wage rather than the nominal wage. From an equilibrium position where inflation is fully and accurately anticipated and thus fully incorporated into all wage contracts, suppose nominal aggregate demand starts growing due to an increase in money supply for example. This increase in aggregate demand in turn produces a proportionate rise in prices and wages. Workers will initially interpret this as a rise in the real wage since they have come to expect constant prices, and consequently they will increase their labor supply. The employers, however, cannot initially distinguish the price rise to be an increase in general price level or a relative price rise for their product, but will, in general, interpret it to be an increase in the relative price for their product. To the employers, this is a fall in the real wage they must pay in terms of their product, and they are willing to hire more labor. In Friedman's words, "...the simultaneous fall ex post in real wages to the employers and rise ex ante in real wages to employees is what enabled employment to increase."[19]

The initial impact of an unanticipated increase in nominal aggregate demand is a decrease in unemployment, and this produces the observed trade-off of the Phillips curve. However, when employers and workers come to realize that prices in general are rising, and in reality, real wages have not changed, this will lead to an upward revision of anticipated inflation rate and a decrease in employment. The upward revision of the anticipated inflation rate shifts the Phillips curve upward until the anticipated inflation rate equals the now higher actual inflation rate. Hence, there is not a stable Phillips curve but a host of them, each corresponding to a given level of inflationary expectations. In equilibrium, anticipated inflation equals actual inflation, and this is consistent with only one level of unemployment which Friedman called the "natural rate of unemployment." The natural rate of unemployment is consistent with any level of price inflation, positive, negative, or zero. Furthermore, the natural rate of unemployment is determined by real phenomena such as market frictions, real income, tax rates, etc. When these real phenomena change, the natural rate is likely to change also. Thus, the natural rate should not be regarded as fixed or constant. In Friedman's words,

> The "natural rate of unemployment," in another word, is the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actural structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the costs of mobility, and so on.[20]

Friedman's natural rate of unemployment has far reaching implications. First, it implies that any active policies to reduce the unemployment rate below the natural rate can have only a temporary effect on the unemployment rate before anticipated inflation catches up with the actual inflation rate. Second, it implies that any policy attempts to reduce unemployment permanently below its natural rate without affecting the real variables that determine the natural rate can only lead to ever accelerating inflation. Hence, given flexible prices and wages, expansionist policy is both unnecessary and harmful since it can only increase inflation without permanently lowering the unemployment rate.

In view of the importance of Friedman's natural rate hypothesis for economic policy, it was subjected to empirical tests in order to verify its validity. Most early tests of the natural rate hypothesis consisted of estimating a Phillips curve relationship of the following form:

$$\dot{W}_t = c + f(U_t,\ H_t) + c_1\dot{P}^e_t, \tag{I.2}$$

where $\dot{P}^e_t$ is the inflation expected to hold at time t, and H_t is a vector of other variables that may be of interest. If the natural rate hypothesis is correct, the coefficient on the inflationary expectations variable should be unity. However, the price expectations variable is not directly observable, it must be related to some other observable variables, or researchers must resort to using survey data on price expectations in the empirical work. Since Friedman did not specify an expectations generating mechanism, it is up to each individual researcher to specify his own expectations generating mechanism. The result is that any hypothesis testing includes a joint test of the underlying model and the expectations generating mechanism. Two such expectations generating mechanisms that are generally used in most empirical studies are the adaptive expectations and a distributed lag of past inflation rates. The adaptive expectations hypothesis states that expectations are revised on the basis of the differene between the current rate of inflation and the expected rate. It is generally written in the following form:

$$\dot{P}^e_t = \lambda\dot{P}_t + (1-\lambda)\dot{P}^e_{t-1},\ 0 \leq \lambda \leq 1. \tag{I.3}$$

This expectations generating mechanism can be combined with the expectatins-augmented Phillips curve (1.2) to form a Phillips curve in terms of only observable variables.

The other mechanism is to posit the expected inflation rate as a distributed lag of past inflation rates,

$$\dot{P}_t^e = \sum_{i=1}^{n} v_i \dot{P}_{t-i}, \quad v_i \geq 0, \tag{I.4}$$

and with a side constraint that $\sum_{i=1}^{n} v_i = 1$. We will defer a detailed discussion of this class of test for the natural rate hypothesis until chapter II. Here, we will report some of the results obtained from estimating a variant of equation (I.2). Gordon,[21] Solow,[22] and Turnovsky and Wachter,[23] found the coefficient to be less than unity. Turnovsky,[24] Vanderkamp,[25] and later Gordon[26] found a unit coefficient, while Lucas and Rapping,[27] and Riddell[28] obtained mixed results. Although the empirical results were mixed, most economists believe that the long-run Phillips curve is steeper than the short-run Phillips curve, but there is no concensus as to whether it is vertical or not. Policy activists still maintain that the long-run Phillips curve is not vertical and thus can be exploited by the policy makers.

D. Rational Expectations and the Phillips Curve Analysis

The expectations generating mechanisms inroduced in the last section are often criticized on the ground that they are ad hoc and may bear no relation to the true underlying process generating inflation. Thus, they may lead to systematic errors in expectations. This in turn implies that a policy of ever accelerating the inflation rate can lead to a persistent decrease in the level of unemployment. This is clearly inconsistent with Friedman's verbal discussion of the natural rate hypothesis. Indeed, Lucas[29] argued that the hypotheses of adaptive expectations and the natural rate of unemployment are mutually contradictory.

A number of economists, in particular, Robert Lucas, Thomas Sargent, and Robert Barro believe that systematic errors in expectations are inconsistent with the rationality of economic agents. Instead, they believe that economic agents, when forming expectations of future variables, will make the most efficient use of all available and relevant information. This has come to be known as "rational expectations" in the literature and its origin is attributed to Muth.[30] According to Muth,

> ...expectations, since they are informed predictations of fuure events, are essentially the same as the predictations of the relevant theory....The hypothesis can be rephrased a little more precisely as follows: the expectations of firms (or,more generally, the subjective probability distributions of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the 'objective' probability distributions of outcomes).

It is important to note that rational expectations hypothesis does not assert that agents' expectations are always correct, but that they need only to be correct on the average, thus implying all expectational errors are random with zero mean. Also, it does not assert that agents have all the information, only that they form optimal predictions based on the information available to them.

The idea that economic agents make optimal predictions based on the information available to them is uncontroversial. However, rational expectations should not be confused with rational expectations models which combine rational expectations with a macroeconomic model which embodies the natural rate hypothesis. As Lucas pointed out, the assumption of rational expectations does lead to a consistent formulation of the natural rate hypothesis. However, the implications of rational expectations for the Phillips curve tradeoff are rather drastic and very controversial.[31] We will briefly consider two of these implications in this chapter. First, it implies that systematic countercyclical policies will be totally ineffective since rational economic agents would have already incorporated into their expectations the policy rule followed by the authority. In other words, it denies that the unemployment rate can be persistently depressed below its natural rate by ever accelerating the rate of inflation. To make this point clearer, let us consider a simple concrete example.

Consider the following simple aggregate model which is similar to Fischer's model:[32]

$$y_t = a_1(P_t - E_{t-1}P_t) + u_{1t}, \tag{I.5}$$

$$y_t = M_t - P_t - u_{2t}, \tag{I.6}$$

$$M_t = g_o + g_1 M_{t-1} + u_{3t}, \tag{I.7}$$

$$E_{t-1}P_t = E(P_t | I_{t-1}), \tag{I.8}$$

where y_t is the real output measured as deviation from some potential level, P_t is the price level, M_t is the nominal money supply, E is the expectations operator, u_{1t}, u_{2t}, and u_{3t} are white noise stochastic processes assumed to be distributed independently of each other.[33]

Equation (I.5) is a simple aggregate supply curve incorporating the natural rate hypothesis. Equation (I.6) is a velocity equation intended to account for the aggregate demand side of the model. Equation (I.7) is the money supply equation where the policy parameters g_o and g_1 are assumed to be under the control of the monetary authority. Finally, equation (I.8) is a formal statement of rational expectations. It says that rational expectations are mathematical expectations of P_t given the information available at time (t-1), denoted by I_{t-1}.

Equating (I.5) and I.6) and solving for P_t, we obtain

$$P_t = \left(\frac{a_1}{1+a_1}\right)E_{t-1}P_t + \left(\frac{1}{1+a_1}\right)M_t - \left(\frac{1}{1+a_1}\right)(u_{1t} + u_{2t}). \qquad \text{(I.9)}$$

Take expectations on both sides of equation (I.9) conditional on the information available at period (t-1), subtract from equation (I.9) and substitute into equation (I.5), the reduced form solution for real output is

$$y_t = \left(\frac{a_1}{1+a_1}\right)(M_t - E_{t-1}M_t) - \left(\frac{a_1}{1+a_1}\right)u_{2t} + \left(\frac{1}{1+a_1}\right)u_{1t}. \qquad \text{(I.10)}$$

Since public's expectations are rational, $(M_t - E_{t-1}M_t) = u_{3t}$, hence, equation (I.10) can be rewritten as

$$y_t = \left(\frac{a_1}{1+a_1}\right)(u_{3t} - u_{2t}) + \left(\frac{1}{1+a_1}\right)u_{1t}. \qquad \text{(I.11)}$$

According to equation (I.11), systematic policy cannot affect y_t since the policy parameters g_o and g_1 do not appear in the reduced form equation for y_t. What this implies is that monetary policies, to the extent that they can be anticipated, cannot have an effect on real output. Hence, a deliberate policy to accelerate the rate of inflation cannot increase the level of output <u>even in the short-run</u> if expectations are rational.

The second implication of the rational expectations models was well articulated by Lucas.[34] Lucas questioned the relevance of quantitative policy evaluation from econometric models estimated from past data, since the parameters from such models are not likely to be invariant under arbitrary, i.e. non-random, policy changes. To continue with our example, first note that

$$E_{t-1}P_t = E_{t-1}M_t, \tag{I.12}$$

and

$$E_{t-1}M_t = g_o + g_1M_{t-1}. \tag{I.13}$$

Substituting equation (I.13) into (I.12) and (I.12) into (I.9), the reduced form solution for the price level is

$$P_t = b_o + b_1M_t + b_2M_{t-1} - (\frac{1}{1+a_1})(u_{1t} + u_{2t}), \tag{I.14}$$

where

$$b_o = \frac{a_1g_o}{1+a_1}, \tag{I.15}$$

$$b_1 = \frac{1}{1+a_1}, \qquad \text{(I.16)}$$

$$b_2 = \frac{a_1 g_1}{1+a_1}. \qquad \text{(I.17)}$$

Suppose, the monetary authority has been following the monetary rule according to equation (I.7) for an extended period of time, and that estimates for b_o, b_1, and b_2 are also obtained. Now, if the monetary authority wishes to affect P_t by alternate choices of the policy parameters g_o and g_1, it is clear from (I.15) and (I.17) that b_o and b_2 will not remain invariant with changes in g_o and g_1. This consideration led Lucas to conclude that the outcome of policy simulation based on the assumption that the parameters b_o and b_2 in equation (I.14) are fixed will be incorrect and misleading.

E. Criticisms of Rational Expectations and Rational Expectations Models

Reactions to rational expectations and models which embody the rational expectations and natural rate hypothesis are mixed. Critics of rational expectations have correctly pointed out that rational expectations models do not provide a theory about the learning process of economic agents. They merely assert that economic agents would eventually learn of the policy rule followed by the authority and thus converge to the rational expectations solutions. Rational expectations models are equally silent on how agents learn with costly information. Maximizing behavior on the part of economic agents will not necessarily lead to rational expectations solutions since it depends on the costs and benefits from accurate predictions.[35] On the other hand, Lucas[36] pointed out that the important point is not whether the postulate of rational expectations is descriptively true; rather, the postulate of rational expectations is based on the belief that whatever policies are pursued, economic agents will not leave large, obvious economic profit unclaimed.

Another line of argument is that rational expectations models typically assume flexible prices and wages in the sense that prices and wages adjust each period to clear all markets. Both Modigliani[37] and Gordon[38] have argued that the experience in the 1970s has refuted the notion that the economy is essentially self-correcting. Furthermore, Fischer,[39] Phelps and Taylor[40] have shown that the existence of long-term contracts without wage indexation can destroy the conclusions of rational expectations models, thus leaving a role for stabilization policies. Barro[41] countered Fischer's argument by pointing out that Fischer-type contracts are Pareto suboptimal. McCallum[42] also considered several form of "sticky" prices, and in each case, the conclusions from the rational expectations models again emerged.

The above considerations demonstrate that the question of policy effectiveness is not likely to be resolved by purely theoretical arguments, instead, formal econometric evidence would have to be called upon to shed more light on the debate.

At the present time, there is a growing empirical literature on the rational expectations-natural rate hypothesis. The empirical results amassed so far are encouraging to the rational expectations-natural rate hypothesis, but by no means conclusive. Among the more well known empirical studies which support the conclusions of the rational expectations-natural rate hypothesis are Barro,[43] McCallum,[44] Lucas,[45] Sargent,[46] Neftci and Sargent,[47] Neftci,[48] and Leiderman.[49] Other empirical studies by Small,[50] Cuddington,[51] Froyen and Waud,[52] found only weak evidence if any, to support the rational expectations-natural rate hypothesis.

By far the vast majority of the empirical research dealing with rational expectations-natural rate hypothesis has been based upon closed economy macroeconomic models. Empirical tests in the context of an open economy macromodel have not yet received a great deal of attention. In particular, the behavior of price level and real output under different monetary and exchange rate regimes has not yet been examined. Furthermore, the effects of foreign monetary policies on the behavior of domestic price level and real output have only been recently investigated by Leiderman.[53] Further empirical study in this area can therefore serve a useful purpose. The purpose of this dissertation is to fill this gap in the empirical literature.

F. Structure of the Dissertation

This dissertation consists of six chapters. Chapter two presents a review of the empirical methodologies that have been used in the literature to test the implications of the rational expectations-natural rate hypothesis. In chapter three, a simple prototype model of a small open economy and the rest of the world is developed. The model of the small economy is then solved to obtain the reduced form solutions for real output, the exchange rate, and the price level under flexible exchange rates, and for real output, the domestic money supply, and the domestic price level under fixed exchange rates. Similarly, the rest of the world model is also solved to obtain reduced form solutions for real output, the price level, and the interest rate. Chapter four discusses the testable implications of the model developed in chapter three and presents the form of the model to be used as the basis of the empirical investigation. Chapter five carries out the empirical tests of the implications derived in chapter four using Canada as the small open economy and the United States as the rest of the world. Finally, chapter six concludes the dissertation with a summary of the empirical results from chapter five and with suggested avenues for future research.

Footnotes - Chapter I

1. See Modigliani (1977).

2. See Modigliani (1977), p. 1, italics in the original.

3. See A. W. Phillips (1958).

4. See Richard Lipsey (1960).

5. See Lipsey (1960), equations 10 and 13.

6. See Samuelson and Solow (1960).

7. The huge volume of work devoted to the Phillips curve is ably reviewed by Morris Goldstein, and by Santomero and Seater. The conclusions reached by these review authors suggest that at least for the United Kingdom, United States, and Canada, an inverse relationship does exist between price inflation or wage inflation and unemployment. But, the empirical Phillips countries are also found to be temporally unstable. See Goldstein (1972), and Santomero and Seater (1978).

8. Single equation estimation that includes the level of unionization as an explanatory variable introduces possible simultaneous equation bias. As Schmidt and Strauss has shown, causality not only runs from unionization to higher wages, but also from higher wages to unionization. For example, if unions are successful in gaining higher wages for their members, more workers are induced to join unions. See Schmidt and Strauss (1976).

9. For a partial listing of the empirical works using a four quarter moving average, see Rowley and Wilton (1973).

10. See Rowley and Wilton (1973), p. 385.

11. Walter Oi tested Gordon's 1972 wage equation for stability of the coefficients and found that they were stable. See Oi (1976).

12. Lucas and Rapping (1969) divided their sample period into 1903-29, 1930-45, and 1946-65, and using a chi-square test, they decisively rejected the hypothesis that the coefficients were equal in all sub-periods.

13. Working with Canadian data, Riddell (1979) found his Phillips curves to be unstable.

14. See Gordon (1971, 1972, and 1977). Gordon's empirical studies are not subjected to the criticisms of Rowley and Wilton since he used quarterly rate of change in his empirical work.

15. See Gordon (1977). Compare equations 3 and 5 with 4 and 6 in table 3.

16. Walter Oi (1976), p. 45.

17. See Phelps (1967).

18. See Friedman (1968, 1975, 1977).

19. Friedman (1968), p. 10. Italics in the original.

20. Friedman (1968), p. 8.

21. See Gordon (1970).

22. See Solow (1968).

23. See Turnovsky and Wachter (1972).

24. See Turnovsky (1972).

25. See Vanderkamp (1966, and 1972).

26. Gordon reestimated his 1971 wage equation, with revised official data and found a unit coefficient on his price expectations variables. See Gordon (1977), table 3, equation 2.

27. See Lucas and Rapping (1969).

28. Riddell used several different specifications for his Phillips curve and found a unit coefficient on the price expectations variable for two of his specifications when they were estimated separately for two sub-periods, 1953-65, and 1965-1973. See Riddell (1979), table VIII, equations 25, 26, 27, and 28. For the entire sample period 1953-1973, the coefficient is greater than unity in all cases.

29. See Lucas (1972).

30. See John Muth (1961).

31. The proponents of the rational expectations view are not necessarily the same people as the monetarists as defined by Modigliani. However, they also favor a passive (non-activist) policy.

32. See Fischer (1977).

33. A white noise process is a sequence of uncorrelated random variables with zero mean and constant variance. Let u_t be a white noise process, then $Eu_t = 0$, $Eu_t u_{t-s} = 0$ for $s \neq 0$, and σ_u^2 for $s = 0$.

34. See Lucas (1976).

35. See the conclusion by DeCanio (1979).

36. See Lucas (1975).

37. See Modigliani (1977).

38. See Gordon (1976).

39. See Fischer (1977).

40. See Phelps and Taylor (1977).

41. See Barro (1977b).

42. See McCallum (1977).

43. See Barro (1977, 1978).

44. See McCallum (1976).

45. See Lucas (1973).

46. See Sargent (1976).

47. See Neftci and Sargent (1978).

48. See Neftci (unpublished).

49. See Leiderman (1979, 1980a, and 1980b).

50. See Small (1979).

51. See Cuddington (1980).

52. See Froyen and Waud (1980).

53. See Leiderman (1980b).

CHAPTER II

REVIEW OF THE LITERATURE

The purpose of chapter I was to trace the development of the debate on policy activism and to point out the areas of disagreements where further research is warranted. The purpose of this chapter is to review critically the existing literature on testing rational expectations models. Some of the empirical literature was briefly mentioned in chapter I, but will be examined in greater detail in the present chapter. Since the focus of this dissertation is on the effectiveness of policy activism in an open-economy macroeconomic setting with rational expectations, other empirical studies which incorporate rational expectations but are not directly concerned with the question of the effectiveness of policy activism will not be discussed.

Before presenting our review, it is instructive to first review the implications of the rational expectations-natural rate hypothesis for empirical test. As noted in Chapter I, the assumption of rational expectations does lead to a consistent formulation of the natural rate hypothesis. The resulting model implies that the time path of real output or unemployment is independent of systematic monetary or fiscal actions. This property of the rational expectations model is known as the neutrality hypothesis. In other words, the neutrality of real output or unemployment to systematic monetary or fiscal policies is equivalent to the existence of a natural rate of output or unemployment. It follows that empirical tests of the natural rate hypothesis must consist of (1) testing of the rationality assumption, and (2) testing of the neutrality hypothesis. Empirical tests of these two hypotheses are commonly referred to as tests of the rational expectations-natural rate hypothesis.

However, it should also be noted that the finding of neutrality of real output or unemployment with respect to monetary policies implies that expectations are formed rationality. For, unless expectations are rational, it is possible for systematic monetary policies to have a real effect on real output or unemployment. On the other hand, the finding of rationality alone does not guarantee the neutrality of real output or unemployment with respect to monetary policies. The natural rate hypothesis has also to be tested for explicitly.

In the following sections, we examine the different methodologies that have been used in the literature to test the rational expectations-natural rate hypothesis.

A. Early Tests[1]

Early attempts to empirically verify the natural rate hypothesis simply estimated a Phillips curve type relationship with price expectations included as an extra explanatory variable. The general relationship estimated was[2]

$$\dot{W}_t = c_o + f(U_t, H_t) + c_1\dot{P}^e_t, \qquad \text{(II.1)}$$

where all the variables are as defined in chapter I. If the natural rate hypothesis is correct, c_1 should be unity.

The maintained hypothesis that c_1 be unity if there is no long run trade-off between wage inflation and unemployment comes from the following argument.[3] Assume that price inflation equation is a "mark-up" of wage inflation according to

$$\dot{P}_t = \dot{W}_t + h(G_t),$$

where G_t is a vector of variables that may potentially contribute to determining the inflation rate.[4] In the long-run, any sustained inflation will be fully anticipated so that $\dot{P}^e = \dot{P}$, and unemployment will be at its natural rate level. Substituting the above into equation (II.1) yields

$$\dot{W} = c_o + f(U, H) + c_1\dot{W} + c_1 h(G), \qquad \text{(II.1a)}$$

where the variables without their time subscripts denote their steady state values. Upon rearranging terms, we obtain

$$\dot{W} = \left(\frac{c_o}{1-c_1}\right) + \left(\frac{1}{1-c_1}\right)f(U,H) + \left(\frac{c_1}{1-c_1}\right)h(G).$$

Since the slope of the Phillips curve, $\left(\frac{1}{1-c_1}\right)\left(\frac{\partial f}{\partial U}\right)$, approaches negative infinity as c_1 approaches unity, the long-run Phillips curve is vertical at the natural rate.

The difficulty in estimating equation (II.1) is that $\dot{P}^e_t$ is not directly observable, and empirical researchers have attempted to relate it to some observable variables.[5] The usual approach is to hypothesize that $\dot{P}_t^e$ is a distributed lag of past inflation rates:

$$\dot{P}_t^e = \sum_{i=1}^{n} v_i \dot{P}_{t-i}, \quad v_i \geqslant 0. \tag{II.2}$$

Substituting (II.2) into (II.1), the estimated form is then

$$\dot{W}_t = c_o + f(U_t, H_t) + c_1 \sum_{i=1}^{n} v_i \dot{P}_{t-i}. \tag{II.3}$$

However, the crucial parameter c_1 in equation (II.3) cannot be directly estimated since there are n + 1 parameters (c_1 and v_i, i = 1, 2, 3,...,n) but only n terms in equation (II.2); therefore, only n separate regression coefficients can be estimated.

To overcome this problem, a side constraint of

$$\sum_{i=1}^{n} v_i = 1 \tag{II.4}$$

is usually added. To justify this side constraint, researchers[6] have appealed to rational expectations, and the following argument is usually given. Suppose that the inflation rate has been a constant k percent, $\dot{P}_{t-i} = k$, for i = 1, 2,...,n. If expectations are formed rationally, then $\dot{P}_t^e$ should be equal to k, and this is possible only if $\sum_{i=1}^{n} v_i = 1$.

Sargent[7] has shown, however, that this side constraint implies that the inflation rate displays strong serial correlation, a fact that is not borne out by an examination of the time series of inflation for the United States. More importantly, Sargent has demonstrated that as long as the inflation rate can be approximated by a covariance stationary stochastic process and the v_i's in equation (II.2) are nonnegative, then, v_i's must sum to less than unity. Thus, Sargent concluded that imposing equation (II.4) as a side constraint would lead to serious underestimation of c_1 in equation (II.1).[8]

According to Sargent,[9] the restriction that should be placed on the v_i's in equation (II.2) is not that they should sum to unity, rather, in order to form a rational forecast of the inflation rate, the v_i's in equation (II.2) should correspond to the actual inflation process. In particular, if the actual inflation rates evolve according to the nth order autoregressive process,

$$\dot{P}_t = \sum_{i=1}^{n} q_i \dot{P}_{t-i} + e_t, \qquad \text{(II.5)}$$

where e_t is a white noise stochastic process, then a rational forecaster would use the above autoregressive scheme to forecast the inflation rate and would accordingly forecast by

$$\dot{P}_t^e = \sum_{i=1}^{n} q_i \dot{P}_{t-i}. \qquad \text{(II.6)}$$

The one period ahead forecast obtained would have the property of minimum mean-squared-error. Thus, according to Sargent, the valid constraint to impose is that $v_i = q_i$, $\overline{i} = 1, 2, 3, \ldots, n$.

Sargent's suggestion is merely an extension of the early tests of the natural rate hypothesis by providing a more plausible way to model expectations. However, it is an important extension since, unlike the early tests, it requires that the way expectations are assumed to be formed be compatible with the actual inflation process during the sample period. Thus, equation (II.6) embodies the notion that economic agents are aware of the actual inflationary process during the sample period.

Even with this important extension, this type of test represents a joint test of the natural rate hypothesis and a hypothesis on the way expectations are formed. The danger here is, of course, that the data may incorrectly reject the natural rate hypothesis because of an incorrect specification of the process generating expectations. furthermore, the estimate for the crucial parameter c_1 depends on the variables that are included in the function $f(U_t, H_t)$. As pointed out in chapter I, there is no concensus as to what variables should or should not be included. Thus, a misspecification of the function $f(U_t, H_t)$ can lead to a rejection of the natural rate hypothesis even though the true value of c_1 is unity.

Lastly, using equation (II.6) to generate expectations of inflation is not strictly rational in Muth's sense. Strictly speaking, rational expectations in the sense of Muth is defined as $E_{t-1}x_t = E(x_t|\Phi_{t-1})$, where x is the variable to be forecasted and Φ_{t-1} denotes all the relevant information available through time t-1. The information set Φ_{t-1} will typically include the past history of the variable being forecasted and also past history of other endogeneous and exogeneous variables in the structural model. Since past history of the variable being forecasted is a proper subset of the entire information set, the expectations of a variable conditional only on its past history has become known as "partly," "partial," or "weak" rational expectations. Charles Nelson[10] has shown that if the information set contains other relevant information besides the past history of the variable being forecasted, then "partial," "partly," or "weak" expectations is a misspecification of the process of the expectations formation and least squares estimates will be inconsistent. However, equation (II.6) can be modified easily to include past values of other variables. This is essentially the approach taken by McCallum in his paper which we will examine next.

B. McCallum's Instrumental Variable Method

McCallum's model[11] consists of three equations, a Phillips curve, an aggregate demand curve and a price adjustment equation. All of his equations are specified in logarithmic form, with $\dot{x}$ denoting the first difference of the logarithm of the variable x. McCallum's Phillips curve is specified as

$$\dot{W}_t = a_1 ED_{t-1} + a_2 \dot{P}^e_{t+1}, \qquad \text{(II.7)}$$

where $\dot{W}_t$ is the proportionate change of money wages, ED_{t-1} is the relative level of excess demand in the relevant labor market in period t - 1 and $\dot{P}^e_{t+1}$ is the expected price level change for period t + 1, with expectations formed in period t. Labor market excess demand is assumed to be a linear function of a productivity measure and the real wage. Specifically,

$$ED_t = b_o + b_1(y_t - n_t) + b_2 w_t \quad (II.8)$$

where y_t, n_t, and w_t are real output, the population, and the real wage respectively. Eliminating ED_{t-1} in equation (II.7) by using equation (II.8) and adding an error term gives McCallum's wage equation as

$$\dot{W}_t = d_o + d_1(y_{t-1} - n_{t-1}) + d_2 w_{t-1} + a_2 \dot{P}^e_{t+1} + e_{1t}, \quad (II.9)$$

where $d_o = a_1 b_0$,

$d_1 = a_1 b_1 > 0,$

$d_2 = a_1 b_2 < 0,$

and e_{1t} is an error term. To complete the model, McCallum specified an aggregate demand function and a price adjustment equation.[12] The complete model is simultaneous but recursive, with equation (II.9) forming the lowest block. Thus, assuming the errors across the equations are independent, equation (II.9) can be estimated in isolation from the rest of the equations in the model.[13] However, equation (II.9) contains an unobservable variable $\dot{P}^e_{t+1}$. Hence, in order to estimate equation (II.9), the expectation formation process must be specified. McCallum assumed that expectations are formed rationally in Muth's sense, i.e.,

$$\dot{P}^e_{t+1} = E(\dot{P}_{t+1} | \Phi_t), \quad (II.10)$$

where Φ_t is the information set used to forecast the inflation rate. By the assumption of rational expectations, Φ_t includes all the information on the past history of the variables in the model:

$$\Phi_t = (y_{t-1} - n_{t-1}, \ldots, P_{t-1}, P_{t-2}, \ldots, \dot{P}_{t-1}, \dot{P}_{t-2}, \ldots,$$

$$\dot{G}_{t-1}, \dot{G}_{t-2}, \ldots, W_{t-1}, W_{t-2}, \ldots, M_{t-1}, M_{t-2}, \ldots).$$

The variables $\dot{P}_t$, $\dot{G}_t$, and $\dot{M}_t$ are the inflation rate, the rate of change of nominal government spending, and the rate of change of the nominal money stock respectively. These variables are from the aggregate demand equation and the price adjustment equation.

McCallum further assumed that the inflation rate for period (t+1) can be decomposed into an anticipated and an unanticipated component:

$$\dot{P}_{t+1} = \dot{P}^e_{t+1} + u_t, \qquad \text{(II.11)}$$

where u_t represents expectations error and is assumed to be a white noise stochastic process uncorrelated with Φ_t.

Substituting equation (II.11) for $\dot{P}^e_{t+1}$ in equation (II.9), we obtain the wage inflation equation in terms of only observable variables:

$$\dot{W}_t = d_o + d_1(y_{t-1} - n_{t-1}) + d_2 w_{t-1} + a_2 \dot{P}_{t+1} + e_{2t}, \qquad \text{(II.9a)}$$

where

$$e_{2t} = e_{1t} - a_2 u_t.$$

The variables $\dot{P}_{t+1}$ and e_{2t} are now correlated and least squares estimates will be inconsistent.[14] However, equation (II.9a) can be consistently estimated by the instrumental variable technique if we find an instrumental variable for $\dot{P}_{t+1}$ which is uncorrelated with e_{2t}.

To obtain an instrumental variable for $\dot{P}_{t+1}$, McCallum assumed that expectations in (II.10) are log linear in the variables in Φ_t, so that we can write

$$E(\dot{P}_{t+1}|\Phi_t) = \sum_{i=1}^{n} c_i X_i, \qquad \text{(II.12)}$$

where X_i, i = 1, 2,...,n are variables selected from Φ_t. Substituting (II.12) into (II.11), we obtain

$$\dot{P}_{t+1} = \sum_{i=1}^{n} c_i X_i + u_t. \qquad \text{(II.13)}$$

The instrumental variable for $\dot{P}_{t+1}$ can now be obtained as the least squares predicted value of $\dot{P}_{t+1}$:

$$\hat{\dot{P}}_{t+1} = \sum_{i=1}^{n} \hat{c}_i X_i. \qquad \text{(II.13a)}$$

When $\dot{P}_{t+1}$ in equation (II.9a) is replaced with its predicted value $\hat{\dot{P}}_{t+1}$, equation (II.9a) can be consistently estimated with ordinary least squares since, by construction, e_{2t} is uncorrelated with $\hat{\dot{P}}_{t+1}$.

To test the natural rate hypothesis, the two-step regression method as described in the last paragraph is utilized and if the hypothesis that $a_2 = 1$ cannot be rejected in equation (II.9a), then the natural rate hypothesis is also not rejected. McCallum used quarterly data from the United States for his empirical work and used several different sets of variables for his first stage regression, thus obtaining several different instrumental variables for $\dot{P}_{t+1}$. His purpose was to determine whether the estimates of equation (II.9a) are sensitive to the addition of first stage regressor variables. Although the estimates of equation (II.9a) do not seem to be very sensitive to the addition of more first stage regressor variables,[15] McCallum's empirical results are nevertheless mixed. In only two out of seven cases does he accept the hypothesis that $a_2 = 1$.

McCallum also considered the possibility that expectations may only be "partly" rational in the sense that the information used by economic agents is limited only to past inflation rates, i.e. the information set is $\phi_t = (\dot{P}_{t-1}, \dot{P}_{t-2}, \ldots)$, where ϕ_t is a proper subset of Φ_t. Again using the two-step regression method, McCallum found that a_2 in equation (II.9a) is not statistically different from unity for the four cases that he considered.

In general, McCallum's results are encouraging to the natural rate hypothesis, but by no means overwhelming. However, since his procedure is model-specific, the reservations about this type of test mentioned at the end of section A must be kept in mind when interpreting his results. What is more disturbing is that McCallum did not provide an explanation nor try to reconcile his two sets of results obtained from the two assumptions about the ways expectations are formed. The two different sets of results demonstrate vividly the difficulties with testing the natural rate hypothesis with a Phillips curve type of relationship. A joint test of the natural rate hypothesis and the way expectations are assumed to be formed cannot settle the question of the value of the critical parameter a_2, since the value of a_2 is sensitive to the ways expectations are modeled. McCallum's results favored the "partly" rational expectations over the rational expectations in Muth's sense. This result may be defended on theoretical ground that information gathering is not costless, and economic agents may find it advantageous to base their forecasts on only a portion of the relevant information. However, this assertion is itself an empirical question. If expectations are assumed to be only partly rational, but economic agents actually used a larger set of information, then the estimates for equation (II.9a) obtained from the assumption that expectations are only partly rational will be inefficient,[16] and McCallum's results may be invalidated on econometric grounds. One way to empirically test whether other relevant information are used for forming expectations is to regress the residuals obtained from the regression used to obtain predicted values for the partly rational case on a set of variables that may be important in forming expectations and testing to see if any of these variables are statistically significant. If any one of these variables is found to be statistically significant, partly rational expectations can be rejected in favor of rational expectations in

Muth's sense. McCallum did not carry out this type of investigation, and his results can only be viewed as inconclusive.

C. Lucas's Econometric Testing of the Natural Rate Hypothesis

The preceding two sections have demonstrated some of the difficulties with empirical research on the natural rate hypothesis. The heart of the difficulty is in modeling the process of forming price expectations. Any test of the natural rate hypothesis using a Phillips curve type relationship represents a joint test of the natural rate hypothesis and the process of price expectations formation. Some economists have tried to circumvent this joint test by using survey data on price expectations. However, their efforts were hindered by both the scarcity and the unknown quality of the survey data.[17] Furthermore, the empirical tests described in the last two sections were not in the same spirit as Friedman's verbal discussion of the natural rate hypothesis. According to Friedman, it is not unemployment that is causing inflation, rather, it is unexpected movements in inflation which are causing movements in the observed unemployment rate. Hence, a proper empirical model should have the unemployment rate as the dependent variable.

In view of these difficulties, several economists have explored alternative methods to test the rational expectations-natural rate hypothesis. In particular, Lucas[18] has advanced a model where the aggregate supply curve explicitly embodies the natural rate assumption and economic agents are assumed to form their expectations rationally in Muth's sense. In this way, testable implications are derived from the reduced form of the model which differ from those discussed in the previous two sections.

Lucas's model was in log-linear form, and his aggregate supply function which incorporated explicitly the natural rate hypothesis is given as

$$y_t = a(P_t - P_t^e), \qquad 0 < a < 1, \tag{II.14a}$$

where P_t^e is an index of expected future prices, and y_t is the log of real output measured as deviation from its natural rate. The aggregate demand curve is

$$y_t + P_t = x_t, \tag{II.14b}$$

where x_t is the logarithm of nominal GNP, and it is viewed as a shift parameter accounting for variations in demand. The policy rule[19] for nominal GNP specified by Lucas takes the following form

$$x_t = r_1 x_{t-1} + r_2 x_{t-2} + e_t, \tag{II.14c}$$

where e_t is a white noise stochastic process. To close the system, Lucas defined the expectation of the future price level to be the rational expectations, i.e.,

$$P_t^e = E_t(P_{t+1} | \Phi_t) + u_t, \tag{II.14d}$$

where E_t is the mathematical expectation of P_{t+1} conditional on all the information available up to the end of time t, and u_t is another white noise stochastic process assumed to be distributed independently from

e_t. Given equations (II.14a) to (II.14d), we can solve the system of equations to obtain the reduced form solution to price and real output as

$$P_t = ((1+a)/D)x_t + (r_2 a/D)x_{t-1} + (a/(1+a))u_t, \qquad \text{(II.15)}$$

and

$$y_t = [(a(1+a)(1-r_1) - a^2 r_2)/D]x_t - (ar_2/D)x_{t-1} - (a/(1+a))u_t \quad \text{(II.16)}$$

respectively, where

$$D = (1+a)[1+a(1-r_1)]-a^2 r_2.$$

Note that the policy parameters (r_1, r_2) appear in the reduced form equation for real output. However, equation (II.16) does imply a natural rate since the <u>unconditional</u> mean of y_t is zero.[20]

The important part of Lucas's equation (II.16) is in its implications for testing the natural rate hypothesis. The type of tests discussed in sections A and B would amount to a linear restriction that the coefficients on x_t and x_{t-1} in equation (II.16) summed to zero. This is because, under the natural rate hypothesis, a once and for all demand shift has no effect on real output in the long run. However, from equation (II.16), it is clear that, under the rational expectations version of the natural rate hypothesis, no such restriction is implied. The reason is that a demand shift implies a change in the policy parameters r_1 and r_2. Since these parameters are embedded in the coefficients of the real output equation, the time series property of the real output must be recomputed, and therefore this policy shift cannot be evaluated by simply summing parameters implied by some previous policy.

To conduct the proper test, Lucas proposed the following steps. First find a period in which policy has been stable in the sense that it can be described in a simple stable stochastic structure and obtain least squares estimates $\hat{r}_1$, and $\hat{r}_2$ for r_1, and r_2 in equation (II.14c). Next, estimate equation (II.16) with ordinary least squares and obtain estimates for the coefficients x_t and x_{t-1}, call them $\hat{b}_1$ and $\hat{b}_2$ respectively. Reestimate equation (II.16) with the restrictions that $r_1 = \hat{r}_1$ and $r_2 = \hat{r}_2$ and obtain new estimates for the coefficients in equation (II.16). Under the assumption of rational expectations, a stable demand policy followed by the authorities would be anticipated by the economic agents and thus be fully reflected in the coefficients b_1 and b_2. Therefore, we would not expect a significant loss of explanatory power due to the restrictions. If there is a significant loss of explanatory power due to the restrictions, we would reject the natural rate hypothesis. An alternative method is to estimate equations (II.14c) and (II.16) simultaneously as a system of two equations with the restrictions above imposed across the two equations. The same test for loss of explanatory power would apply in this case.

Lucas made it quite clear that the proper test for the rational expectations–natural rate hypothesis restricts the relationship of the policy parameters to the reduced form parameters of the real output equation. It cannot be tested on a behavioral relationship alone.

Hence, empirical tests on a behavioral relationship alone are not valid tests of the rational expectations-natural rate hypothesis. The proper test must take into consideration the dependence of the reduced form parameters of the real output equation to the policy parameters. It involves first estimating the policy rule and the reduced form equation for real output without any restrictions. The reduced form equation for real output is estimated a second time with the parameters being restricted by the policy parameters.[21]

The difficulty with Lucas's testing procedure is that the cross-equation restrictions implied by rational expectations are rather complicated. Hence, except for relatively simple models like the one described above, this approach is not very practical. Furthermore, the testing procedure proposed by Lucas also represents a joint test of the rational expectations assumption and the neutrality hypothesis.[22] In the event that the joint hypothesis is rejected, it is desirable to test the rational expectations hypothesis separately from the natural rate hypothesis because the rejection of one hypothesis does not imply the rejection of the other.

Recently, Leiderman[23] tested a fixed exchange rate open economy model with the methodology similar to that of Lucas's. However, the rational expectations hypothesis was tested separately from that of the neutrality hypothesis. Leiderman's reduced form equations are

$$P_t = b_o + \sum_{i=1}^{4} b_{1i}P_{t-i} + \sum_{j=1}^{4} b_{2j}P^*_{t-j} + u_{1t}, \qquad \text{(II.17a)}$$

$$P_t^* = c_0 + \sum_{i=1}^{4} c_{1i}P_{t-i} + \sum_{j=1}^{4} c_{2j}P_{t-j}^* + u_{2t}, \qquad \text{(II.17b)}$$

$$y_t = a_0 + a_1(P_t - {}_{t-1}P_t^e) + a_2(P_t - P_t^*)$$

$$+ a_3(P_t^* - {}_{t-1}P_t^{*e}) + \sum_{i=1}^{4} a_{4i}y_{t-i} + u_{3t}, \qquad \text{(II.17c)}$$

where P_t^*, P_t, and y_t are the log of the foreign price level, the domestic price level, and domestic real output respectively, and ${}_{t-1}X_t^e$, $(X_t = P_t, P_t^*)$ is the rationally formed expectations conditional on the information at $(t - 1)$.

The interesting feature of Leiderman's real output equation is that when purchasing power parity is not a necessary condition in the model, as assumed by Leiderman, the relative price of domestic good to foreign good may affect domestic real output. Hence, to the extent that domestic monetary authority can affect the price of domestic good with monetary policies and therefore, the relative price of domestic good to foreign good, domestic real output is not independent of monetary policies. Such a version of the natural rate hypothesis is termed the "weak" open economy natural rate hypothesis by Leiderman. However, when $a_2 = 0$, Leiderman's domestic real output equation will exhibit the strict neutrality of closed economy model.

Now combine the unanticipated domestic and foreign price terms with the error term in equation (II.17c), taking the first differences of equations (II.17a) to (II.17c) and substituting the resulting expressions of foreign and domestic price inflation into the real output equation. The system of equations to be estimated becomes[24]

$$\dot{P}_t = b'_o + \sum_{i=1}^{4} b_{1i}\dot{P}_{t-1} + \sum_{j=1}^{4} b_{2j}\dot{P}^*_{t-j} + e_{1t}, \quad \text{(II.18a)}$$

$$\dot{P}^*_t = c'_o + \sum_{i=1}^{4} c_{1i}\dot{P}_{t-1} + \sum_{j=1}^{4} c_{2j}\dot{P}^*_{t-j} + e_{2t}, \quad \text{(II.18b)}$$

$$\dot{y}_t = f_o + \sum_{i=1}^{5} f_{1i}\dot{P}_{t-1} + \sum_{i=1}^{5} f_{2i}\dot{P}_{t-i} + \sum_{i=1}^{4} a_{4i}\dot{y}_{t-i} + e_{3t}, \quad \text{(II.18c)}$$

where

$$\begin{aligned}
f_{11} &= a_2(1 + b_{11} - c_{11}),\\
f_{1i} &= a_2(b_{1i} - b_{1i-1} - c_{1i} + c_{1i-1}),\ i = 2, 3, 4,\\
f_{15} &= a_2(c_{14} - b_{14}), \qquad \text{(II.19)}\\
f_{21} &= a_2(1 + b_{21} - c_{21}),\\
f_{2j} &= a_2(b_{2j} - b_{2j-1} - c_{2j} + c_{2j-1}),\ j = 2, 3, 4,\\
f_{25} &= a_2(c_{24} - b_{24}).
\end{aligned}$$

Using quarterly data from Italy, Leiderman estimated his system of equations three times with the Full Information Maximum Likelihood estimation technique. The first time, the system of equations was estimated without any restrictions. When the system of equations was estimated the second time, the rational expectations hypothesis was tested alone by imposing the cross-equation restrictions implied by

(II.19). The results indicate that the restrictions cannot be rejected at the 10% and 5% level of significance. When the system of equations was estimated the third time, the joint hypothesis of rational expectations and the strict natural rate hypothesis were tested by imposing the rational expectations restrictions (II.19) and the neutrality restriction ($a_2 = 0$). Again, the results indicate that these restrictions cannot be rejected. Hence, Italian real output is found to be strictly neutral with respect to the domestic and foreign price ratio.

Leiderman's results appear to be consistent with the rational expectations-natural rate hypothesis. It should be emphasized that this conclusion was arrived at by testing each of the hypotheses of rational expectations and neutrality separately and jointly. As such, this represents a very important contribution to the empirical literature on the rational expectations-natural rate hypothesis. However, Leiderman's empirical results are only valid if the vector autoregressive process describing the domestic and foreign inflation rate equations are statistically stable during the sample period. This was not explicitly tested for by Leiderman. Furthermore, when $a_2 = 0$, f_{1i}, f_{2j}, i, j = 1, 2, 3, 4, 5 in (II.19) are zero regardless of whether the restrictions implied by (II.19) are valid or not. A more appropriate testing procedure would appear to involve testing for the significance of a_2 before testing for the rational expectations restrictions implied by (II.19).

D. Barro's Unanticipated Money Growth Model

Lucas's paper discussed in the last section pointed out several important considerations in conducting the proper test for the rational expectations-natural rate hypothesis. In particular, Lucas pointed out that the proper test involves specifying a real output equation and a policy equation and then testing for the cross equation restrictions implied by the rational expectations-natural rate hypothesis.

In a series of papers, Barro[25] attempted to empirically test the theoretical implications of the rational expectations-natural rate hypothesis by examining the effects of anticipated and unanticipated money growth rates on the level of output, unemployment, and pirces. Barro specified his empirical model with Lucas's criteria by including an unemployment equation and an explicit policy rule. Barro specified his unemployment equation as

$$UN_t = b_o + \sum_{i=0}^{2} b_{i+1}(DM_{t-i} - E_{t-i}DM_{t-i}) + b_4 MIL_t + b_5 MINW_t + e_t, \qquad \text{(II.20)}$$

where e_t is an error term, and UN_t is the unemployment rate defined by Barro to be the natural logarithm of $\left(\frac{U_t}{1-U_t}\right)$, where U_t is the annual average unemployment rate in the total labor force which includes military personnel. The money supply variable DM_t is the annual growth rate of the nominal money supply and is defined as $DM_t = \log(M_t) - \log(M_{t-1})$. The two other explanatory variables are MIL_t, which is a measure of military conscription, and $MINW_t$, which is intended to measure the impact of the minimum wage on the unemployment rate.[26] By writing the unanticipated change of money growth as $(DM_{t-i} - E_{t-i}DM_{t-i})$, i = 0, 1, 2, Barro was assuming that expectations for time (t - i) are based on information available up to time (t-i-1) and also partial information in time (t - i).

The policy rule adopted by Barro was

$$DM_t = a_o + \sum_{i=1}^{2} a_i DM_{t-i} + a_3 FEDV_t + a_4 UN_{t-1} + u_t, \qquad (II.21)$$

where u_t is an error term. The inclusion of Un_{t-1} is intended to account for any countercyclical behavior on the part of the monetary authority. The variable $FEDV_t$ is a measure of actual real government expenditures, FED_t, relative to the normal level of real government expenditures, i.e. $FEDV_t = \log FED_t - \log\overline{FED}_t$, where $\log\overline{FED}_t$ is the normal level of real government expenditures.[27]

The difficulty in estimating equation (II.20) is in quantifying the notions of anticipated and unanticipated money growth. Barro viewed the anticipated money growth as "the prediction that could have been obtained by exploiting the systematic relationship between money and this set of independent variables,"[28] where the set of independent variables are two lagged values of money growth, $FEDV_t$ and UN_t, as given in equation (II.21). In other words, given the monetary authority's reaction function (II.21), rational economic agents would forecast future money growth with the systematic part of equation (II.21).

The unanticipated money growth is then taken to be that part of money growth that cannot be predicted from the systematic part of the money growth equation, i.e., $DM_t - E_tDM_t = u_t$. Anticipated money growth is obtained empirically as the best linear prediction once equation (II.21) is estimated by ordinary least squares. Unanticipated money growth, denoted by Barro as DMR_t, is the residuals from this regression.

Using DMR as explanatory variables, Barro tested the implication from the rational expectations-natural rate hypothesis that only unanticipated money growth affects unemployment or real output. Barro found that for annual United States data, the unanticipated money growth variables have very significant explanatory power in his unemployment and real output regressions. To test that only unanticipated money growth affects the real variables, the DMR_t variables in both the unemployment and real output equations were replaced by the actual money growth rates DM_t's. In both instances, the fit of the regression deteriorated considerably. When both DMR_t's and DM_t's were included in the same regression, an F-test on the coefficients of the DM_t's that they were all zero cannot be rejected. On the basis of these results, Barro's tests appear to support the rational expectations-natural rate hypothesis favorably.

Barro's empirical tests were extended by Leiderman[29] to include a separate test for the neutrality hypothesis and the rational expectations assumption instead of Barro's joint test of the rational expectations-neutrality hypothesis.[30] Leiderman retained Barro's money growth equation (II.21), but modified the unemployment equation as

$$UN_t = b_o + b_{11}DM_t - b_{12}\hat{DM}_t + b_{21}DM_{t-1} - b_{22}\hat{DM}_{t-1}$$

$$+ b_{31}DM_{t-2} - b_{32}\hat{DM}_{t-2} + b_4MIL_t + b_5MINW_t + e_t. \qquad (II.22)$$

The hypothesis of neutrality imposes the following restrictions on the parameters of equation (II.22):

$$b_{11} = b_{12},$$
$$b_{21} = b_{22}, \qquad (II.23)$$
$$b_{31} = b_{32}.$$

To derive the implications for rational expectations, $\hat{DM}_t$, $\hat{DM}_{t-1}$, and $\hat{DM}_{t-2}$, are eliminated from equation (II.22) by the repeated substitutions from the deterministic part of equation (II.21), and we obtain

$$UN_t = f'_o + f'_1DM_{t-1} + f'_2DM_{t-2} + f'_3DM_{t-3} + f'_4DM_{t-4}$$

$$+ f'_5FEDV_t + f'_6FEDV_{t-1} + f'_7FEDV_{t-2} + f'_8UN_{t-1}$$

$$+ f'_9UN_{t-2} + f'_{10}UN_{t-3} + b_4MIL_t + b_5MINW_t + e_{1t} \qquad (II.22a)$$

where the following cross-equation restrictions between equations (II.21) and (II.22a) are implied by rational expectations:

$$f'_o = b_o + (b_{11} - b_{12} - b_{22} - b_{32})a_o,$$

$$f'_1 = (b_{11} - b_{12})a_1 + b_{21},$$

$$f'_2 = (b_{11} - b_{12})a_2 - b_{22}a_1 + b_{31},$$

$$f'_3 = -(a_2b_{22} + b_{32}a_1),$$

$$f'_4 = -b_{32}a_2,$$

$$f'_5 = (b_{11} - b_{12})a_3, \qquad \text{(II.24)}$$

$$f'_6 = -b_{22}a_3,$$

$$f'_7 = -a_{32}a_3,$$

$$f'_8 = (b_{11} - b_{12})a_4,$$

$$f'_9 = -b_{22}a_4,$$

$$f'_{10} = -b_{32}a_4.$$

To test the various hypotheses, the testing procedure similar to that discussed in section C was used. The system of equations (II.21) and (II.22a) was first estimated without any restrictions. The assumption of rational expectations was tested by estimating the system of equations a second time with the restrictions (II.24) imposed. Finally, the joint hypothesis of rational expectations and neutrality was tested by estimating the system of equations the third time with both the rational expectations restrictions (II.24) and the neutrality restrictions (II.23) imposed. In each instance, Leiderman found that the restrictions were not rejected by the sample data. Thus, Leiderman's results with the different testing procedures provided support to Barro's original conclusion that only unanticipated money growth affects the unemployment rate or real output.

Although Barro's results appear to be favorable to the rational expectations-natural rate hypothesis, his empirical results have also received the greatest amount of criticism. The criticism may be broadly divided into two groups: those that are aimed at Barro's specification of the money growth equation, and those that are aimed at Barro's empirical methodology in general.

First, on Barro's specification of the money growth equation, McCallum[31] noted that economists do not have a lot of experience in analyzing the behavior of the policy rule. Hence, any attempt to model the authorities' policy rule must necessarily involve estimating an ad hoc policy equation where the validity of the included variables is determined by tests of statistical significance or some other measure of goodness of fit. The results from estimating an ad hoc policy rule are most likely to be sensitive to the specification of the policy rule. Kantor[32] further noted that with Barro's procedure, "one is in effect being asked to accept a joint hypothesis. The appropriateness of the measure of anticipated money growth is judged by its ability to help predict unemployment."[33]

Furthermore, Blinder[34] and Weintraub[35] contended that given the Federal Reserve's propensity to stabilize the interest rate in the post World War II period, a short run interest rate should be included as an explanatory variable in the money growth equation. Weintraub went on to argue that the inclusion of UN_{t-1} in the money growth equation cannot be defended on logical ground. The reason is that if the monetary authority understands that only unanticipated money growth affects unemployment or real output, and if the policy is to control unemployment or real output, then the monetary authority would not follow a money growth rule which changes in response to unemployment in a predictable way.[36] Hence, any observed relationship between money growth and UN_{t-1} must be regarded as accidental. Weintraub further argued that if the monetary authority does change the money rule in a predictable way in response to UN_{t-1}, then it implies irrational behavior on the part of the monetary authority. However, a money rule such as Barro's may not necessarily imply that the monetary authority is behaving irrationally. It is possible that the monetary authority understands that any money rule that is being followed will come to be expected by the general public and will have no effect on unemployment or real output. Therefore, a money rule such as Barro's may reflect the monetary authority's attempts to stabilize some other macroeconomic goals other than the unemployment rate or real output. The upshot of the whole argument here is that without an explicit objective function of the monetary authority, the question of the appropriate monetary rule cannot be settled.

On Barro's empirical methodology in general, his results do not appear to be robust with respect to different specifications of the money growth equation. As an example, consider Small's[37] reestimations of Barro's money growth and unemployment equations. Small included a new variable, $DFEDV_t$, in his money growth equation and obtained a new series for DMR. This new variable is Barro's $FEDV_t$ variable multiplied by a zero-one dummy variable intended to take account of the temporary increase in federal government spending during the war years.[38] When this new DMR series was used in the unemployment equation, the r^2 fell from .78 to .53, and only the DMR_{t-1} variable retained statistical significance. When the money growth equation was expanded to include a measure of unanticipated increases in federal government expenditures and a measure of the cost of the war per year, the unemployment equation estimated with the residuals from this expanded money growth equation was statistically insignificant.

This last piece of empirical evidence from Small appears to be damaging to the rational expectations-natural rate hypothesis. Since the theory predicts that only unanticipated money growth affects unemployment or real output, while anticipated money growth does not, one money growth rule is as good as any other when economic agents form their expectations rationally. However, it is important to note that an incorrect empirical specification of the monetary rule may invalidate the empirical result. Blinder suggested that Barro should further check the robustness of his results with different specifications of the money rule and, therefore, different DMR_t series.

However, Blinder's suggestion seems to present another problem. With several statistically plausible money growth equations, it is not clear which one properly represents the "true" money growth equation which the economic agents are assumed to know and use to form their expectations.

Another difficulty with Barro's empirical methodology is that his method of obtaining the unanticipated component of the money growth rates implies that economic agents are forming their expectations for time t not only with all the relevant information available up to t-1 period, but also information in t, t+1, t+2,... periods. This is because the regression coefficients in equation (II.21) were obtained by using the sample observations for the whole period. When the coefficients were used to obtain the predicted value for time t, information that was not available at time t would be used to obtain the predicted value for time t since the coefficients "contain" later information.[39] A procedure that avoids the use of later information to generate earlier predictions is to obtain $\hat{DM}_t$ from a regression that contains observations only up to t-1 time period. In this procedure, there are as many regressions as there are predicted values, with each regression containing one additional observation. Barro reported that there were only minor differences in his results but did not report his results. A recent paper by Grossman[40] also used this procedure to generate the anticipated and unanticipated components of his policy rule. Grossman also found no essential difference in the results that he obtained.

Grossman's paper is of interest for another reason. Grossman modeled his policy rule as an autoregressive time series. Since this form of the policy rule can be interpreted as a very general reduced form equation, Grossman did not have to model explicitly the behavior of the monetary authority. This is an improvement over Barro's explicit money growth equation. However, a more general class of models can be used to model the policy rule with the use of the general Autoregressive Integrated Moving Average (ARIMA) models. Unfortunately, with the use of an autoregressive policy rule, Grossman's reduced form equation for unemployment is indistinguishable from one which embodies Keynesian policy implications. We will return to this "observational equivalence" problem again in the next section.

Finally, both McCallum[41] and Blinder[42] have questioned the validity of MIL_t as an included variable in the unemployment equation. Barro himself conceded that the influence of MIL_t on unemployment is not well established.[43]

The criticisms of Barro's procedure we dwell on in this section are not intended to diminish his original and insightful contribution to testing the natural rate hypothesis. Rather, they serve only to indicate the difficulties involved in such a test and point out the direction that future empirical research should take. In the next section, we will examine Sargent and Neftci's empirical procedure for testing the rational expectation—natural rate hypothesis. We will also indicate the ways that the criticisms mentioned in this section could be overcome.

E. Neftci and Sargent's Breakpoint Test

Neftci and Sargent's[44] model for their test of rationality and neutrality differs from the models we have examined so far in one important aspect; Neftci and Sargent's model is based entirely on the statistical properties of economic time series. The important advantage in using the time series approach to modeling is that the resulting models are compatible with any structural economic model one could imagine, therefore, many of the criticisms in the last section do not apply to this class of models.

A very general time series model for real output which embodies the rational expectations-natural rate hypothesis may be written as

$$y_t = a(L)y_{t-1} + b(L)(M_t - {}_{t-1}M_t^e) + e_t, \qquad \text{(II.25)}$$

and the policy rule may be written as

$$M_t = c(L)y_t + d(L)M_{t-1} + u_t, \qquad \text{(II.26)}$$

where y_t is real output, M_t is the money supply, a(L), b(L), c(L), and d(L) are polynominals in the lag operator L; e.g., $a(L) = \sum_{i=0}^{\infty} a_i L^i$, $L^i X = X_{t-i}$. The two error terms e_t and u_t are assumed to be independent, zero mean white noise processes.

However, apart from equations (II.25) and (II.26), another relationship can be derived for y_t and M_t. To see this, first note that

$$M_t - {}_{t-1}M_t^e = -c(L)y_{t-1} - d(L)M_t = u_t, \; d_o = -1. \qquad \text{(II.27)}$$

Substituting (II.27) into (II.25), we obtain

$$y_t = i(L)y_{t-1} + j(L)M_t + e_t, \qquad \text{(II.28)}$$

where

$$\begin{aligned} i(L) &= -a(L)b(L)c(L), \\ j(L) &= -b(L)d(L). \end{aligned} \qquad \text{(II.29)}$$

It is clear that equation (II.28) is an alternative, but observationally equivalent version of equation (II.25). For a given policy regime, both equations (II.25) and (II.28) will fit the data equally well since they have an identical error term. Yet, equation (II.28) suggests that policy can be manipulated to minimize the deviation of real output from some desired level, while equation (II.25) presents no such scope for monetary policy.

The above analysis suggests that for a given policy regime, equations (II.25) and (II.28) are indistinguishable empirically unless some restrictions on i(L) or j(L) can be identified. McCallum[45] and Weber[46] have considered the case where $b(L) = b_o$. In this case, testable restrictions can be identified for i(L) and j(L). However, when $b(L) \neq b_o$, no identifying restrictions for i(L) and j(L) are possible.

To get around this problem when b(L) is restricted to $b(L) = b_o$, Neftci and Sargent proposed identifying distinct periods in which there occurs a change in policy regime and then testing whether equation (II.25) or equation (II.28) is invariant across such change in policy regime. If equation (II.28) is invariant across changes in policy regimes, then Keynesian policy implications are obtained. If, on the other hand, equation (II.25) is invariant across changes in policy regimes (as it should be since equation (II.25) depends on money innovations), then we obtain the strong neutrality result that one deterministic feedback rule for policy is as good as any other.

Thus, the first thing to do is to identify a change in policy. One way in which this may be done is by estimating equation (II.26) over several subperiods of the sample and using a Chow-test to determine if the coefficients obtained from different subperiods regressions are equal. When a break is determined,[47] the whole sample is divided into two subsamples, the first sample containing observations up to the break point and the second sample contains observations from after the break point. Equations (II.25) and (II.28) are then estimated with ordinary least squares first with observations from the first sample only and then again with only the observations from the second sample. Using the Chow test, the equality of the two sets of coefficients estimated from the two subperiods for each equation is then determined. The unanticipated component of the policy rule which is used in estimating equation (II.25) is obtained from the residuals of estimating equation (II.26). However, it should be noted that it is incorrect to obtain the residuals from a regression of equation (II.26) estimated from the entire sample since we have determined that there is a change in the policy regime. The residuals to be used as the unanticipated component of the policy rule must be obtained from two regressions, one containing only the observations from the first sample, the other containing observations from the second sample. This reflects the idea that economic agents will take into consideration the change in policy regime when forming their expectations.[48]

Neftci and Sargent found a break in the policy rule around 1964.I, and of the two equations, equation (II.25) was found to be the stable one across this break in the sample.

The use of the Chow-test to determine the stability of equations (II.25), (II.26) and (II.28) is not entirely satisfactory since an arbitrary break point must be determined before the Chow-test could be utilized. In a paper by Neftci,[49] he used the more satisfactory Quandt procedure to search for the break point. Assuming there is a break in the sample, the Quandt procedure essentially attempts to identify the "most likely" break point by estimating the log likelihood function from each successive regression. In other words, a regression is estimated successively with sample size t, t+1, t+2,..., etc., and the log likelihood function is calculated with each successive estimation. The "most likely" candidate for the sample break point is found when the log likelihood function is maximized.[50] Using the Quandt procedure, Neftci estimated equations (II.25), (II.26), and (II.28) over the interval from 1956.IV to 1967.I. The "most likely" break point for equation (II.26) was determined to be 1962.IV,[51] and of the two equations (II.25) and (II.28), the one that was invariant to policy changes should not have a maximum value for its log likelihood function around that neighborhood. Neftci's empirical results indicated that equation (II.25) was more stable than equation (II.28), thus confirming the earlier results of Neftci and Sargent.

The works of Neftci and Sargent and of Neftci alone represent a major improvement in the empirical efforts to test the rational expectations-natural rate hypothesis since they do not depend on ad hoc behavioral assumptions underlying a policy rule or an unemployment equations. However, their results depend on an accurate specification of the lag structure. The Quandt procedure is also more satisfactory than the Chow-test since it does not require an arbitrarily chosen breakpoint. However, the one disadvantage of using the Quandt or Chow-test is that it cannot detect multiple breakpoints in the sample, and therefore the empirical results are preconditioned on the assumption that there is only one break point in the sample period.

F. Lucas's International Output-Inflation Model

The important point about Sargent and Neftci, and Neftci's articles is that in the absence of any prior information on the lag structure of a general time series reduced form model, the natural rate hypothesis can be tested only by examining the behavior of the economy under different aggregate demand policy. The alternative is to examine a cross-section of countries where aggregate demand policies show great variations across countries. This is the rationale underlying Lucas's empirical test.[52]

Lucas postulated that suppliers are located in a large number of scattered, competitive markets. Demand for goods in each period is distributed unevenly over individual markets, leading to relative as well as general price movements. Letting Z index the markets, y_{nt} represent the log of the normal output which is common to all markets, and $y_{ct}(Z)$ represent the log of the cyclical output which varies from market to market, supply from market Z is

$$y_t(Z) = y_{nt} + y_{ct}(Z). \tag{II.30}$$

Normal output is assumed to follow a trend line

$$y_{nt} = a + bt, \tag{II.31}$$

and cyclical output is assumed to vary with perceived relative prices and its own lagged value:

$$y_{ct}(Z) = r[P_t(Z) - E(P_t|\Phi_t(Z))] + \lambda y_{c,t-1}(Z) \tag{II.32}$$

where $P_t(Z)$ is the actual price in Z at time t and $E(P_t|\Phi_t(Z))$ is the expected current general price level conditional on the information available in market Z at time t. The actual current general price level P_t is unknown but is assumed to be normally distributed with mean $\overline{P}_t$ and variance σ^2. The observed price in market Z is assumed to be the sum (in logs) of

$$P_t(Z) = P_t + Z, \tag{II.33}$$

where Z is the percentage deviation of the price in Z from the average price P_t and assumed to be distributed with mean zero and variance σ_z^2.

Since supply decisions depend on the perceived relative price in market Z, suppliers must forcast P_t given all the available information in market Z. By using linear projection theory,[53] it can be shown that

$$E(P_t|\Phi_t(Z)) = (1 - \theta)P_t(Z) + \theta\overline{P}_t, \qquad (II.34)$$

where $\theta = \sigma_z^2/(\sigma^2 + \sigma_z^2)$.

Substituting equations (II.31) and (II.32) into (II.30), we obtain

$$y_t(Z) = y_{nt} + \theta r(P_t(Z) - \overline{P}_t) + \lambda y_{c,t-1}(Z). \qquad (II.35)$$

By averaging over the markets, the aggregate supply curve is

$$y_t = y_{nt} + \theta r(P_t - \overline{P}_t) + \lambda(y_{t-1} - y_{n,t-1}). \qquad (II.36)$$

The slope of the aggregate supply curve depends on θ which is a ratio of the variance σ^2 and σ_z^2. The larger is σ_z^2 relative to σ^2, the larger is the response of aggregate output to unexpected aggregate price changes, for the rational suppliers are likely to interpret the unexpected price change to be a change in relative prices. On the other hand, if σ^2 is large relative to σ_z^2, the slope of the aggregate supply curve is nearly vertical since in this case the individual price changes are certain to reflect general price changes.

To complete the model, the aggregate demand curve is given as

$$y_t + P_t = x_t, \qquad (II.37)$$

where x_t is the log of the nominal GNP and is assumed to be a shift parameter. The policy rule is

$$x_t = x_{t-1} + \delta + \varepsilon_t, \qquad \text{(II.38)}$$

where ε_t is normally distributed with mean zero and variance σ_x^2, and δ is the average rate of demand expansion.

Using the method of undetermined coefficients, the solution for y_{ct} is (we are ignoring the solution for P_t)

$$y_{ct} = - \pi\delta + \pi\Delta x_t + \lambda y_{c,t-1}, \qquad \text{(II.39)}$$

where

$$\pi = \theta r/(1 + \theta r),$$

or

$$\pi = \sigma_z^2 r/[(1 - \pi)^2\sigma_x^2 + \sigma_z^2(1 + r)]$$

upon further manipulation.

To empirically test his model, Lucas assumed σ_z^2 and r were relatively stable across countries, and thus concentrated on the relationship between π and σ_x^2. Using annual time series data from 1953-1963 for 18 countries, Lucas found that countries with large variance in nominal aggregate demand have a small slope coefficient of the aggregate supply curve, thus confirming the prediction from his theory.

Although Lucas's results do not reject the rational expectations-natural rate hypothesis as he formulated it, Vining and Elwertowski[54] provided empirical evidence that there is a positive correlation between σ_z^2 and σ^2. This casts doubts on Lucas's assumption of the stability of σ_z^2 and the interpretations of his results altogether. However, Cukierman and Wachtel[55] were able to reconcile the findings of Vining and Elwertowski by modifying Lucas's original model with the inclusion of a market specific demand function. This modification introduces individual market equilibrium condition for each of the markets. As a result, both σ_z^2 and σ^2 are positive functions of σ_x^2. This explains the positive correlation between σ_z^2 and σ^2 observed by Vining and Elwertowski.[56].

Froyen and Waud[57] incorporated the modification suggested by Cukierman and Wachtel in a reexamination of Lucas's empirical evidence. With this modification, the testable form of Lucas's equation becomes

$$y_{ct} = \alpha + \pi' \Delta x_t + \lambda y_{c,t-1}, \qquad \text{(II.40)}$$

where

$$\pi' = \frac{r}{(\sigma_x^2/\sigma_w^2) + (1 + r)},$$

where σ_w^2 is the variance of the market specific demand shock. Assuming that σ_w^2 and r are relatively stable across countries, and denoting $\sigma_{\dot{p}}^2$ as the variance of the inflation rate, Froyen and Waud have shown the following three testable implications:[58] (1) π' and σ_x^2 should be negatively correlated; (2) σ_x^2 and $\sigma_{\dot{p}}^2$ should be positively correlated; (3) π' and $\sigma_{\dot{p}}^2$ should be negatively correlated.

Froyen and Waud tested the above three hypotheses using annual time series data from 1956-1976 for ten of the same countries examined by Lucas, and quarterly data for four of the countries. Two types of tests were conducted, cross-country comparisons and intracountry comparisons overtime, where cross-country comparisons were also done for various subperiods. Their results may be summarized as follows. First, the proposition that π' and σ_x^2 should be negatively correlated did not receive general support from either the cross-country tests or intracountry tests overtime. Second, the proposition that σ_x^2 and $\sigma_{\dot{p}}^2$ should be positively correlated did not receive support from intracountry tests over time, but did receive some supports from cross-country tests when a later subperiod was examined. Finally, the proposition that π' and $\sigma_{\dot{p}}^2$ should be negatively correlated was supported only by cross-country tests, and the results from intracountry tests overtime being inconclusive.

The results from cross-country comparisons are rather disappointing to the natural rate hypothesis. Further evidence was also provided by Taylor[59] who examined ten of the same countries as Lucas for the period 1956-1976 and found an inverse relationship between the variance of real output and the variance of the price level. This is consistent with the prediction from a long term wage contract model where policy makers can trade off higher variance of the price level for lower variance of the real output. However, the tradeoffs found by Taylor appeared to be unstable.

The evidence from cross-country comparisons must be interpreted with care. First, since many of the results are obtained from estimating a specific model, the results may be conditional on the particular formulation of the natural rate hypothesis. Second, all the countries examined are assumed to have the same structure but different aggregate demand policies. In particular, the models have abstracted from explicit open-economy considerations. This is clearly inappropriate since many of the countries considered are small open economies. Thus, given the degree of openness of a country and the exchange rate regime, the output response for a given aggregate demand policy may very well be very different across countries because of differences in the structure of the economies. This last point is worth emphasizing in light of Leiderman[60] and Turnovsky's[61] theoretical findings. The theoretical models of Leiderman and Turnovsky indicate that for a small open economy, real output may not be neutral with respect to domestic or foreign monetary policies if these policies were successful in affecting the relative price of domestic to foreign goods. Hence, although international comparisons may provide many useful information, future research in this area should explicitly consider the open-economy aspect of many of the countries and the possible differences in the structure of the economies.

G. Sargent's Application of Granger Causality and Sims's Exogeneity Tests to the Natural Rate Hypothesis

The idea of using time series models to test the natural rate hypothesis was introduced in the next to last section. The advantage in using time series models is that the rational expectations-natural rate hypothesis can be formulated in such a way that it is essentially independent of any particular macroeconomic structural model, being compatible with a variety of structures one could imagine. Yet, economically meaningful hypotheses can be tested. In this section, we will examine in more detail the time series approach to testing the natural rate hypothesis.

A particularly useful concept in testing the natural rate hypothesis with time series model is that of causality. According to Granger,[62] a variable Y_t is said to cause another variable X_t if we are better able to predict X_t with past information on X_t and Y_t than with past information on X_t alone.

There are two methods to make the above definition operational. The first method is based on a test of goodness of fit, and the following two equations are typically estimated:

$$X_t = \sum_{i=1}^{n_1} a_i X_{t-i} + \sum_{j=1}^{n_2} b_j Y_{t-j} + u_{1t}, \qquad \text{(II.41)}$$

$$X_t = \sum_{i=1}^{n_1} c_i X_{t-i} + u_{2t}. \qquad \text{(II.42)}$$

The test that Y_t does not cause X_t is a test that $b_j = 0$, $j = 1,2,\ldots,n_2$. Similarly, reverse causality can be tested by interchanging X_t and Y_t in equations (II.41) and (II.42).

The second method for testing causality was proposed by Ashley, Granger, and Schmalensee.[63] This method involves comparing the post sample forecasting performance of the univariate and bivariate models of interest. To be more specific, let $MSE(X_t)$ be the mean-square-error of one-step-ahead post sample forecasts of the univariate model for X_t, and $MSE(X_t, Y_t)$ be the mean-square-error of one-step-ahead post sample forecasts of the bivariate model for X_t. The Y_t is said to cause X_t if $MSE(X_t, Y_t) < MSE(X_t)$. The method of Ashley, Granger, and Schmalensee is more satisfactory than the goodness of fit test since it is closer in spirit to Granger's definition of causality. However, most of the empirical work employing the Granger causality test has used the goodness of fit test rather than the test based on comparing post sample forecasting performance.

Sims elaborated on Granger's paper and demonstrated the relationship between the concept of strict econometric exogeneity and Granger's concept of causality. According to Sims,

> Y_t can be expressed as a distributed lag function of current and past X_t with a residual which is not correlated with any values of X_t, past or future, if, and only if, Y_t does not cause X_t in Granger's sense.[64]

The proof of Sims's theorem is given both in Sargent[65] and Sims,[66] and, therefore, will not be repeated here. However, what Sims has shown was that X_t will be econometrically exogeneous with respect to Y_t if Y_t does not cause X_t in Granger's sense. Sims's test for exogeneity of X_t with respect to Y_t is to regress Y_t on past, present, and future X_t and then test the null hypothesis that the coefficients on future X_t's are zero. For example,

$$Y_t = \sum_{i=-n}^{m} g_i X_{t-i} + e_t,$$

if Y_t does not cause X_t in Granger's sense, or, if X_t is econometrically exogeneous with respect to Y_t, then $g_i = 0$ for $i < 0$.

Sargent's[67] empirical test for the natural rate hypothesis was based on the observation that if the rational expectations-natural rate hypothesis is true, then variables measuring government policies cannot systematically affect the unemployment rate. In other words, variables measuring government policies do not Granger cause unemployment and, therefore, will not have predictive power in a reduced form equation for unemployment or real output. This hypothesis can be tested directly using either Granger's causality test or Sims's exogeneity test.

Sargent's statistical formulation of the unemployment rate takes the form of a univariate Wold representation:

$$UN_t = \sum_{i=0}^{\infty} a_i e_{t-i}, \quad \sum_{i=0}^{\infty} a_i^2 < \infty, \qquad \text{(II.43)}$$

where UN_t is the unemployment rate and e_t is a white noise stochastic process. Assuming invertibility, the unemployment rate equation can be written in its autoregressive form:

$$UN_t = \sum_{i=1}^{\infty} g_i UN_{t-i} + e_t.$$

Let Φ_t be the set of observations on all variables that may potentially contribute to predicting unemployment. Φ_t may include endogeneous variables or exogeneous variables in a structural macroeconomic model, and may also include unemployment itself. Sargent then advanced the following formulation of the natural rate hypothesis:

$$E(e_t | \Phi_{t-1}) = 0. \qquad (II.44)$$

Equation (II.44) says that the innovation in the unemployment rate is statistically independent of each component of Φ_{t-1}, and so cannot be predicted on the basis of the information in Φ_{t-1}. This implies that the forecast of UN_t based on lagged values of UN_t alone cannot be improved upon by taking into account variables in Φ_{t-1} other than lagged values of UN_t. In other words, variables in Φ_{t-1} do not Granger cause UN_t. This version of the natural rate hypothesis implies that the government policies cannot tradeoff a lower unemployment rate for a higher expected inflation rate even in the short run.

To see this, suppose money supply is determined by the following feedback rule

$$M_t = f(\Phi_{t-1}), \qquad (II.45)$$

where f can be a very complicated function determining M_t. Equation (II.44) implies

$$E(UN_t | M_t, UN_{t-1}, UN_{t-2}, \ldots) = E(UN_t | UN_{t-1}, UN_{t-2}, \ldots).$$

So, taking account of M_t will not improve the forecast of UN_t once lagged UN_t's are taken into account. This is equivalent to UN_t not being Granger caused by money supply or any other policy variables.

To empirically test Sargent's version of the natural rate hypothesis, UN_t is regressed on its past values and other variables selected from Φ_{t-1} to represent policy variables. So letting X_t be any "real" variable and Y_t be any causal variable, Sargent estimated the two equations (II.41) and (II.42), and tested whether the lagged values of the causal variable are significant.

Alternatively, using Sims's test, a variable representing policy measure is regressed on past, present, and future values of a chosen "real" variable. The test for economic exogeneity of the "real" variable is a test that the coefficients of the future "real" variables are zero.

Sargent used quarterly United States data for his estimation, and used four lagged values of the dependent variable and six lagged causal variables in his regressions. Sargent's measures of real variables were the unemployment rate (UN_t), real GNP (RGNP), and the labor force participation rate (N_f). His measures of policy and nominal variables were the nominal wage in manufacturing (W), the GNP deflator (P), the money supply measured as M1 (M), the federal, state, and local government surplus (SUR), SUR in constant 1958 dollars (RSURP), federal, state, and local government purchases (G), and G in constant 1958 dollars (RG).

Of the causal candidates that Sargent considered, he found that nominal wage (W) caused the unemployment rate according to both the Granger and Sims's tests, and money supply (M) caused the unemployment rate according to Granger's test.

Sargent's results indicated that Granger's causality tests seem to reject the natural rate hypothesis more readily than Sims's exogeneity tests. Part of the reason may be that the results of causality tests using either Granger or Sims's procedure are sensitive to the number of lags included. In addition, Feige and Pearce[68] have also found that the results with Sims's procedure are sensitive to the number of future variables included and the type of filter used to remove serial correlation from the data. Nevertheless, Sargent's results were mixed but cannot be judged to be overwhelmingly against the natural rate hypothesis.

The bivariate Granger causality test for the natural rate hypothesis was extended by Leiderman[69] to include multivariate causal models. The advantage in using multivariate causal models is that it reduces the possibility of spurious causality due to omitted variables. The test is implemented by including several different measures of fiscal and monetary policies in the reduced form equation for real output or unemployment:

$$X_t = \sum_{i=1}^{k_o} a_{oi} X_{t-i} + \sum_{i=1}^{k_1} a_{1i} Y_{1t-i} + \dots + \sum_{i=1}^{k_n} a_{ni} Y_{nt-1} + e_t, \quad \text{(II.46)}$$

where X_t is any "real" variable, and Y_{jt}, $j = 1, 2, \dots, n$ are causal candidates measuring the fiscal and monetary variables. In this case, output exogeneity requires that all the causal variables have zero coefficients.

The data employed by Leiderman was Italian quarterly data corresponding to a fixed exchange rate period. His variables measuring fiscal and monetary policies included both foreign (United States) and domestic (Italy) variables. Using four lagged values of the dependent variable, and alternatively four and six lagged values for his causal variables, Leiderman found that only six lagged values of the Italian domestic credit component of the money supply Granger caused Italian real output. Hence, Leiderman's results were very supportive of the natural rate hypothesis.

Cuddington[70] further extended Sargent's Granger causality test of the natural rate hypothesis by examining a multivariate, simultaneous - equation version of the Granger causality test. This version of the Granger causality test treats all the "real" variables in the model as a block, and the test for the natural rate hypothesis is a test that the "real" variables are block exogeneous with respect to the policy variables. As noted by Cuddington, Sargent's use of single equation estimation method assumes a recursive model structure. However, with the multivariate, simultaneous equations version of the Granger causality test, the assumption of model recursivity is not necessary. Furthermore, it also reduces the possibility of spurious causality due to omitted variables.

Cuddington specified his model to be estimated as

$$\begin{bmatrix} UN_t \\ R_t \\ RGNP_t \\ N_{ft} \end{bmatrix} = F(L) \begin{bmatrix} UN_{t-1} \\ R_{t-1} \\ RGNP_{t-1} \\ N_{ft-1} \end{bmatrix} + G(L)Y_{t-1} + n_t, \qquad (II.47)$$

where R_t is a measure of long term interest rate which Cuddington included in his block of "real" variables, Y_{t-1} is lagged causal variables measuring fiscal and monetary actions, and F(L) and G(L) are polynominals in the lag operator defined as

$$F(L) = \begin{bmatrix} f_{11}(L) & f_{12}(L) & f_{13}(L) & f_{14}(L) \\ f_{21}(L) & \cdots & \cdots & f_{24}(L) \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ f_{41}(L) & \cdots & \cdots & f_{44}(L) \end{bmatrix} ,$$

$$G(L) = \begin{bmatrix} g_1(L) \\ g_2(L) \\ g_3(L) \\ g_4(L) \end{bmatrix} .$$

Cuddington first reestimated Sargent's single equation tests but included eight lags each of all the "real" variables in the equation. Using alternatively M, RSUR, G, and RG as his causal variables, Cuddington found that UN_t was not exogeneous with respect to M, RG, and G, and $RGNP_t$ was not exogeneous with respect to G. When equation (II.47) was estimated jointly as a system of simultaneous equations, the real variables were not block exogeneous to all the causal candidates considered.

Similar results were obtained by Sims[71] when he considered a six variables vector autoregressive model for the United States and West

Germany. Sims's model was more general than Cuddington's model and has the following form:

$$\begin{bmatrix} M_t \\ RGNP_t \\ UN_t \\ W_t \\ P_t \\ PM_t \end{bmatrix} = H(L) \begin{bmatrix} M_{t-1} \\ RGNP_{t-1} \\ UN_{t-1} \\ W_{t-1} \\ P_{t-1} \\ PM_{t-1} \end{bmatrix} + \underline{e_t},$$

$$H(L) = \begin{bmatrix} h_{11}(L) & h_{12}(L) & \cdots\cdots & h_{16}(L) \\ h_{21}(L) & h_{22}(L) & \cdots\cdots & h_{26}(L) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ h_{61}(L) & h_{62}(L) & \cdots\cdots & h_{66}(L) \end{bmatrix},$$

PM_t is the import price index, and each variable on the right hand side has lag length of four. Sims treated all the variables in his model as endogeneous variables, and it is a more general model than Cuddington's model since the nominal variables are also determined in the system of equations. Sims tested for block exogeneity of the real variables, this amounts to a test of the restrictions that $h_{21}(L) = h_{31}(L) = h_{24}(L) = h_{25}(L) = h_{26}(L) = h_{34}(L) = h_{35}(L) = h_{36}(L) = 0$. In neither West Germany nor the United States was the test passed.

The results of testing for the natural rate hypothesis with time series models and Granger causality tests are rather disappointing. In the single equation tests, we have several cases of non-neutrality of the real variables with respect to the nominal or policy variables. The evidence from simultaneous equations tests is even more discouraging. In all cases, the hypothesis of block exogeneity of the real variables is uniformly rejected.

Although the results from time series models are rather unfavorable to the natural rate hypothesis, they are by no means definitive. Further empirical research with time series models are needed. We will indicate the possible areas for future research by way of summarizing the advantages and disadvantages of using time series models. In general, the chief advantage of using time series models to test the natural rate hypothesis is that the resulting model can be treated as a very general reduced form model. Hence, the empirical results are not dependent on the particular structure of the model or on the particular formulation of the natural rate hypothesis. Furthermore, a multivariate simultaneous equations time series model such as Sims's does not require a priori restrictions (except for the lag length) to achieve identification as in a simultaneous structural model. Yet, economically meaningful hypotheses can be formulated and tested with such a time series model.

However, there are several drawbacks with time series model tests of the natural rate hypothesis. First, the model formulated and tested by Sargent was a joint test of the rational expectations-natural rate hypothesis. Unlike Barro's or Leiderman's models, a separate test for the rational expectations hypothesis and the natural rate hypothesis is not possible with such a time series model. Thus, when the results reject real output or unemployment exogeneity, we are forced to reject the joint hypothesis without any information as to which of the constituent parts of the joint hypothesis may be valid.

Second, hypothesis testing with time series models is sensitive to the number of lags included in the model.[72] Furthermore, since economic theory does not provide any guide as to what variables should or should not be included in a time series model, hypothesis testing is also sensitive to omitted relevant variables in the system. Since this last point is of some importance, we will elaborate it here. The multiequations, simultaneous time-series models approach, such as Cuddington's or Sims's, reduces the possibility of spurious causality due to omitted variables, but does not eliminate such a possibility. This is because when deciding the variables to be included in the system of equations, the researchers have chosen those variables which they considered to be the most important for the problem at hand. However, this is but a small subset of all the potentially important variables, and in practice, it is impossible to include all the variables. Hence, the danger of omitting relevant variables also remains.

Finally, to estimate a model such as Sims's vector autoregressive model requires a large data sample. Unless the data set is large, the degrees of freedom can be exhausted rather rapidly in hypothesis testing. Given the caveats about time series model tests of the natural rate hypothesis, future empirical research may take the form of estimating time series models with different lag structures, with the lag structure perhaps be determined by the method proposed by Hsiao.[73] The robustness of the results should also be checked with several different sets of variables. Future empirical tests should also employ the causality test proposed by Ashley, Granger, and Schmalensee, since their method is closer to the spirit of Granger's definition of causality.

H. Concluding Remarks

In this chapter, we have given an overview of the different methodologies that have been used to test the rational expectations-natural rate hypothesis. Basically, we can divide the tests into two approaches. The first approach consists of testing a specific model, such as the approach taken by McCallum, Lucas, Barro, Grossman, and Leiderman. The second approach consists of testing a statistical definition of the natural rate hypothesis, it is not dependent on any particular structural macroeconomic model, but is compatible with a number of structural models. Such is the approach taken by Neftci and Sargent, Sargent, Cuddington, Sims, and Leiderman. Judging from the empirical literature reviewed in this chapter, the second approach provides several advantages over the first approach. For example, with a specific model, the natural rate hypothesis may not be rejected by the data. However, the observations collected may be consistent with other models, including models that are not classical in nature.

Furthermore, the approach taken by Barro and Grossman depends critically on the accurate decomposition of the policy rule into its anticipated and unanticipated components. As noted by McCallum, the behavior of the policy rule is something economists do not have a lot of experience with. So, instead of a joint test of the natural rate hypothesis and the way in which expectations are assumed to be formed, we have a joint test of the natural rate hypothesis, the rational expectations hypothesis, and the validity of the hypothesized policy rule.

Given the above considerations, the empirical tests in this dissertation will take the second approach in testing the rational expectations-natural rate hypothesis via a statistical formulation of the theory which is independent of any particular structural macroeconomic model. Although a structural model is not important for the empirical tests of the dissertation, a simple model will, however, suggest clearly the types of neutrality propositions in an open economy model. For example, the effects of domestic and foreign monetary policies on unemployment or output will be examined under a fixed exchange rate regime and a flexible exchange rate regime. In this way, we can ascertain what policies, if any, are likely to affect the domestic unemployment rate or real output under a fixed exchange rate regime and a flexible exchange rate regime. Clearly, this is a less ad hoc and a more satisfactory way of providing the rationale for the empirical tests. The task of chapter III is the construction of a simple open economy macroeconomic model.

Footnotes - Chapter II

1. This corresponds to Sargent's layer 1 test in Sargent (1976c).

2. This is the same equation as (I.2) in chapter I transposed here for convenience of exposition.

3. The exposition here follows that of Sargent (1971), footnote 3, p. 721.

4. Included in G_t may be such variables as unit labor cost, gross national product gap. See for example Cukierman (1974).

5. See for example Solow (1974), Gordon (1970, 1971).

6. See for example Solow (1974).

7. See Sargent (1971, 1976c).

8. Solow (1974) obtained results where c_1 were consistently below unity.

9. This corresponds to Sargent (1976c) layer 2 test.

10. See Nelson (1975).

11. See McCallum (1976).

12. McCallum's aggregate demand and price adjustment equations were

$$\dot{y}_t = g_o + g_1\dot{M}_t + g_2\dot{G}_t - (g_1 + g_2)\dot{P}_t + g_3\dot{C}_{t-1} + e_{2t},$$

$$\dot{P}_t = h_o + h_1(y_t - q_t) + h_2W_t + h_3t + h_4P_{t-1} + e_{3t},$$

where q_t is a measure of potential output, and all other variables are as defined in the text. See McCallum (1976).

13. For this reason, the aggregate demand and the price adjustment equations will not be discussed.

14. Consistency is a large sample property. Briefly, it can be described as follows. Let $\hat{b}$ be the estimator of b. The estimator of $\hat{b}$ is said to be a consistent estimator of b if $\underset{n\to\infty}{\text{Plim}}\ \hat{b} = b$.

15. The exception is when P_{t+1} was used as its own instrument and equation (II.9a) was estimated with ordinary least squares. In this case, the estimate for a_2 fell to about half the value of its other estimates. See McCallum (1976), option V, p. 49.

16. This is similar to the problem with errors-in-variables in econometrics.

17. The most popular survey data on price expectations are the Livingston survey data. Brown and Shlomo (1981) found that the Livingston survey were not rational in the sense that the respondents to the survey underutilized some relevant information.

18. See Lucas (1972).

19. Lucas defined a policy as a "a (possibly randomized) rule giving the current value of x_t as a function of the state of the system." See Lucas (1972), p. 55.

20. The unconditional mean of y_t is the mean of x_t and u_t. The mean of u_t is zero. To see that the mean of x_t is also zero, note that from equation (II.14c) we can rewrite the policy as

$$(1-r_1L-r_2L^2)x_t = e_t,$$

where L is the lag operator i.e. $L^i x_t = x_{t-i}$. Rearranging terms, we obtain

$$x_t = \left(\frac{1}{1-r_1L-r_2L^2}\right)e_t,$$

the mean of which is zero.

21. Lucas was assuming that the error terms were independent across equations. If this was not the case, the policy rule and the reduced form real output equation must be estimated as a system of two equations.

22. To be fair to Lucas, his particular formulation of the natural rate hypothesis does not lend itself to separate tests of the rational expectations and the neutrality hypothesis. This is because according to his formulation, the assumption of rational expectations implies the existence of a natural rate. Hence, Lucas's test for the natural rate hypothesis reduces to a test of the rational expectations assumption.

23. See Leiderman (1979).

24. We have simplified Leiderman's system of equations by excluding his seasonal dummies.

25. See Barro (1977, 1978, 1981), see also Barro and Rush (1980).

26. Both MIL_t and $MINW_t$ are real variables expected to affect the real output and unemployment. Barro provided two justifications for the inclusion of the MIL_t variable besides the direct effect on employment. A selective draft provides incentives for eligible civilians to seek low draft probability such as remaining in school rather than entering the labor force. Second, if draft probability is highest for unemployed persons, this would reduce the unemployment rate, for labor force participants would likely choose to be employed rather than unemployed.

27. The inclusion of $FEDV_t$ in the money growth equation is intended to capture the revenue motive of the government in money creation. Barro justified the inclusion of this variable by arguing that short-run deviation of government expenditures from its normal level can be financed in a less costly way by creating money rather than expanding the "tax-raising industry." The normal level of government expenditures was generated by an adaptive scheme,

$$\log\overline{FED}_t = d\log FED_t + (1-d)\log\overline{FED}_{t-1},$$

with the adaptive coefficient d = .2. Barro reported that there were little sensitivity in the unemployment equation for d between the interval from .15 to .3. See Barro (1977), p. 104, ftn. 5.

28. See Barro (1977), p. 101.

29. See Leiderman (1980a).

30. Barro also carried out a crude test of the rational expectations restrictions. Barro compared the implied coefficients obtained by substituting the estimated coefficients from equation (II.21) into equation (II.20) with those that would have been obtained by estimating them directly. See Barro (1977), table 3, p. 110.

31. See McCallum (1979b).

32. See Kantor (1979).

33. See Kantor (1979), p. 1432.

34. See Blinder's comments to the paper by Varro and Rush in Fischer (1980a).

35. See Weintraub's comments to the paper by Barro and rush in Fischer (1980a).

36. See Weintraub's comments to Barro and Rush's paper in Fischer (1980a). However, it should be noted that this does not imply that the monetary rule should be completely unsystematic.

37. See Small (1979).

38. Small (1979) only treated 1950-52, 1965-70 as war years. See Barro's reply to Small in Barro (1979).

39. Barro maintained that his procedure was correct "If individuals have information about the money growth structure beyond that conveyed in prior observations (for example, from the experiences of other countries or on theoretical grounds), ..." See Barro (1977), p. 106.

40. Grossman also used what he called the sequential method with unequal weights. Under this scheme, the sample size that is used to estimate the policy rule is kept constant. To be more specific, the regression for the policy rule uses a sample size with observations from 1 to t-1 to obtain the predicted value for period t. Next, the first observation is dropped from the sample but the observation in time t is added in the regression to obtain the predicted value for period t+1. This procedure is repeated for t+2, t+3,..., periods and so on, with every regression, the current observation is added but dropping the earliest observation. This scheme implies that current information is more valuable than more distant past information in forming expectations. Grossman reported that his results using this scheme is not significantly different from his other results. See Grossman (1979).

41. See McCallum (1979b).

42. See Blinder's comments to Barro and Rush's paper in Fischer (1980a).

43. Small (1979) also shown that the variable MIL_t was responsible for overestimating the natural rate of unemployment for the United States after 1970.

44. See Neftci and Sargent (1978).

45. See McCallum (1979a).

46. See Weber (1981).

47. Note that the Chow test cannot determine the exact breakpoint. It can only determine the equality of the coefficients once a breakpoint is arbitrarily chosen.

48. Neftci and Sargent did not report how they obtained their residuals for their regressions.

49. See Neftci (unpublished).

50. For a discussion of the Quandt procedure, see Neftci (unpublished).

51. Neftci reported that the break was at 1962.II. However, from his table 1, the maximum value for the log likelihood function for his money supply equation was at 1962.IV.

52. See Lucas (1972).

53. For a discussion of the linear projection theory, see Sargent (1979), chapters 10 and 13.

54. See Vining and Elwertowski (1976).

55. See Cukierman and Wachtel (1979).

56. To be more specific, let σ_w^2 be the variance of the individual market demand shock, Cukierman and Wachtel (1979) shown that

$$\sigma_z^2 = \frac{\sigma_w^2}{(1+\theta r)^2}, \quad \sigma^2 = \frac{\sigma_x^2}{(1+\theta r)^2}, \quad \theta = \frac{\sigma_w^2}{(\sigma_x^2+\sigma_w^2)}.$$

Cukierman (1979) shown that

$$\frac{\partial\sigma^2}{\partial\sigma_x^2} = \frac{1+r\theta+2r\theta(1-\theta)}{(1+r\theta)^3} > 0,$$

and

$$\partial\sigma_z^2/\partial\sigma_x^2 = \frac{2r\theta^2}{(1+r\theta)^3} > 0.$$

57. See Froyen and Waud (1980).

58. The second testable implication was shown by Cukierman and Wachtel (1979) as

$$\partial \sigma_p^{\bullet 2} / \partial \sigma_x^2 = \frac{2}{(1+\theta r)^3} (1+\theta r+2\theta r(1-\theta)) > 0.$$

59. See Taylor (1980).

60. See Leiderman (1979).

61. See Turnovsky (1981).

62. See Granger (1969).

63. See Ashley, Granger, and Schmalensee (1980).

64. This is Sims's theorem 2 in Sim (1972).

65. See Sargent (1979), pp. 286-87.

66. See Sims (1972), pp. 550-51.

67. See Sargent (1976a).

68. See Feige and Pearce (1979).

69. See Leiderman (1980c).

70. See Cuddington (1980).

71. See Sims (1980).

72. See the results obtained by Leiderman (1980c) and Hsiao (1981).

73. See Hsiao (1981).

CHAPTER III

A PROTOTYPE MODEL

A. Introduction

This chapter develops a simple prototype model of a small open economy and the rest of the world where both models embody the rational expectations-natural rate hypothesis. The purpose of the models is mainly illustrative and is intended to determine the types of testable restrictions rational expectations-natural rate hypothesis will impose on the reduced form of this type of model.

The plan of this chapter is as follows. Section B presents several preliminary assumptions needed to construct and simplify the exposition of the models. Section C presents the supply side of the small economy model, and section D presents the demand side of the small open economy. Section E presents the aggregate model of the small open economy and the rest of the world, followed by the solutions for the reduced form of the models.

B. The General Framework of the Prototype Model

In this section, we will present some preliminary assumptions needed to construct the model. The supply and the demand side of the model will be discussed in more detail in the next two sections. The model will first be specified in level (unlog) form, but will be transformed into log-linear form for the purpose of obtaining the reduced form solutions.

The modeling strategy in this chapter follows that of Flood[1] where a small open economy and the rest of the world are explicitly modeled. In this way, the small economy assumption is retained, but the structure of the rest of the world is assumed to be included in the information set of the domestic economic agents. All current information is available to economic agents, and all expectations about future variables are formed rationally in Muth's sense. Since the structure of the rest of the world is included in the information set of the economic agents of the small country, the same expectations for the foreign variables are held by the domestic economic agents and the economic agents of the rest of the world.

Furthermore, because the rest of the world is explicitly modeled, the rest of the world variables are exogeneous to the small open economy by the small country assumption, but they are not assumed to be fixed. Instead, they will vary according to the stochastic structure of the economy of the rest of the world. This allows us to examine the effects of both the domestic and foreign monetary policies on the behavior of the real output and the price level of the small open economy under different monetary and exchange rate regimes.

Domestic economic agents are assumed to produce only domestic goods, but their utility depends on both the domestic and the foreign goods, which are assumed to be imperfect substitutes in consumption. As a result, international trade exists to satisfy the demand of domestic economic agents, but purchasing power parity is not a necessary condition in the model. Furthermore, part of the domestically produced goods are consumed domestically and part are exported. The price of the domestic goods is endogeneously determined by domestic demand and supply conditions and by exogeneous foreign aggregate demand policies. On the other hand, the domestic economy is sufficiently competitive in the market for its imports such that the price of its imports is exogeneously determined by the rest of the world.

To simplify exposition, the following is a list of all the variables to be used for the remainder of this chapter:

P_t = price of domestic goods in domestic currency units;

P_t^* = price of foreign goods in foreign currency units;

S_t = spot exchange rate, defined as the domestic price of one unit of foreign currency;

B_t = nominal quantity of one period domestic bonds in domestic currency unit;

B_t^* = nominal quantity of one period foreign bonds in foreign currency units;

i_t = domestic nominal interest rate;

i_t^* = foreign nominal interest rate;

W_t = nominal wage;

N_t^s = quantity of labor supply;

N_t^d = quantity of labor demand;

y_t = real domestic goods;

y_t^* = real foreign goods;

M_t = nominal domestic money balances;

M_t^* = nominal foreign money balances;

Z_t = the relative price of domestic goods to foreign goods in domestic currency units, $(\frac{P_t}{S_t P_t^*})$;

Q_t = domestic price level, defined as $P_t^{\theta}(S_t P_t^*)^{1-\theta}$;

Y_t = nominal labor income;

K_t = capital stock;

t = time subscript

${}_{t-i}X^e_{t+j} = E(X_{t+j} | I_{t-i})$, rationally formed expectation of the variable X for period (t+j) conditional on the information available at period (t - i).

C. The Supply Side of the Small Economy Model

It is assumed that the domestic economic agents are all identical with identical preferences. For convenience, we will refer to a "representative" domestic economic agent throughout this chapter.

The development of the supply side of the model follows closely that of Fisher[2] and Gray.[3] As part of our maintained hypothesis, we assume that the labor market is characterized by wage contracts.[4] All labor contracts have the duration of one period and wage contracts for any period (t) are negotiated at the end of period (t-1). It is also assumed that the nominal wage is set with the goal of equating expected supply of labor to the expected demand. Specifically, given the domestic economic agent's expectations about the price level in period (t), his expected labor supply is described as

$$ {}_{t-1}N_t^{s^e} = \left(\frac{W_t}{{}_{t-1}Q_t^e}\right)^{\gamma_1}, \qquad \text{(III.1a)} $$

where the domestic price level is defined as

$$ Q_t = P_t^{\theta}(S_t P_t^*)^{1-\theta}, \quad 0 \leq \theta \leq 1, \qquad \text{(III.2)} $$

which is a geometrically weighted average of the price of domestic goods and the domestic price of foreign goods. The weights (θ) and ($1-\theta$) are fixed and are intended to reflect the relative expenditures share of the domestic and foreign goods in the domestic price index. The use of this price index to deflate the nominal wage reflects our assumption that our domestic economic agents consumes both domestic and foreign goods. Hence, his real wages must be able to command real purchasing power over both types of goods.

Firms' expected demand for labor is given by

$$ {}_{t-1}N_t^{d^e} = \left(\frac{W_t}{{}_{t-1}P_t^e}\right)^{-\gamma_2}, \qquad \text{(III.1b)} $$

where to the employers, the relevant real wage is that wage rate evaluated by the product price at which they sell their products.

In order to ensure that the labor market is expected to clear, W_t must be set to equate expected labor demand and expected labor supply. Thus, equating equations (III.1a) and (III.1b), we obtain

$$W_t = ({}_{t-1}Q^e_t)^{\frac{\gamma_1}{\gamma_1+\gamma_2}} ({}_{t-1}P^e_t)^{\frac{\gamma_2}{\gamma_1+\gamma_2}} . \qquad \text{(III.3)}$$

There are two important implications arising from our assumption of wage contracts. First, once wage contracts have been negotiated, the supply of labor becomes perfectly elastic and the level of actual employment for each period is completely demand determined by the employers at the negotiated wage rate. The workers, therefore, supply the amount of labor demanded by the employers and equation (III.1a) is dropped from the model.

Second, the domestic economic agent's consumption plans for period (t) are contingent upon whether he is able to sell the labor services he had originally planned. However, since labor contracts are negotiated at the end of (t - 1) for period (t), the domestic economic agent must therefore make his labor supply transactions before his consumption transactions. This is in the spirit of Clower's[5] dual decision hypothesis, and it is a consequence of our assumption of labor contract.

Once labor contracts have been negotiated, information for period (t) becomes available and production decisions are made. For convenience, firms' production function is assumed to take the form of a Cobb-Douglas function with fixed capital and variable labor as its two inputs:[6]

$$y_t = \overline{K}^{\alpha} N_t^{\beta}, \quad 0 < \alpha, \beta < 1. \qquad \text{(III.4)}$$

Furthermore, the actual employment is determined by the realized real wage in period (t), i.e.,

$$N_t = \left(\frac{W_t}{P_t}\right)^{-\gamma_2} . \qquad \text{(III.5)}$$

Combining equations (III.2), (III.3), (III.4), (III.5) and taking logs on both sides, we obtain the firms' aggregate supply function as

$$\ln y_t = c_o + c_1(\ln P_t - \ln_{t-1}P_t^e) + c_2(1-\theta)_{t-1}Z_t^e, \qquad \text{(III.6)}$$

where

$$c_o = \alpha \ln \overline{K},$$

$$c_1 = a_2\beta,$$

$$c_2 = \frac{c_1\gamma_2}{\gamma_1+\gamma_2} .$$

Equation (III.6) states that domestic real output responds positively to unanticipated changes in the price of the domestic goods and to anticipated changes in the relative price of domestic goods to foreign goods. The former is a feature of closed economy models embodying the natural rate hypothesis. The second term is a consequence of workers and employers being concerned with different prices in determining real wages. However, given our assumptions, this is entirely consistent with economic agents' optimizing behavior in an open economy.

D. The Demand Side of the Small Economy Model

On the demand side, the domestic economic agent is assumed to consume both domestic and foreign goods, but holds only domestic and foreign bonds, and domestic nominal money balances in his portfolio. In other words, we are assuming away currency substitution in the model. The foreign and domestic bond markets are assumed to be perfectly integrated, and the two types of bonds are assumed to be perfect substitutes. Hence, interest rate parity must always obtain, i.e.,

$$(1 + i_t) = (1 + i_t^*) \frac{{}_tS^e_{t+1}}{S_t} . \qquad \text{(III.6)}$$

The domestic economic agent is assumed to maximize a utility function of the following form:

$$U = U(y_t, y_t^*, y_{t+1}, y_{t+1}^*, M_t, P_t, S_tP_t^*; N_t, N_{t+1}^e). \qquad \text{(III.7)}$$

Note that nominal domestic money balances are included in the utility function to reflect that nominal money balances yield positive utility by the various functions that they perform.[7] The services performed by nominal domestic money balances depend on the price of the domestic goods and the domestic price of the foreign goods. Since these are the prices which involve transaction with nominal domestic money balances. Hence, these two prices are also included in the utility function.

We will restrict the admissible form of the above utility function by making additional assumptions. We will assume that the utility function is strictly convex and differentiable. The utility function is also assumed to be weakly separable in its labor supply arguments. In this case, the marginal rate of substitution between consumption and leisure will not be affected by changes in the labor supply. Thus, as noted by Mackay and Weber,[8] the commodity demand functions can be written as functions of prices and realized labor income rather than prices and constrained labor supply.

Period (t)'s budget constraint is given as

$$Y_t = P_t y_t + S_t P_t^* y_t^* + M_t + B_t + S_t B_t^*, \qquad \text{(III.8)}$$

which states that the sum of the expenditures on consumption, the holding of domestic nominal money balances, and the holding of domestic and foreign bonds, must be equal to the realized income for that period. The holding of domestic and foreign bonds may be either positive or negative depending on whether the economic agent is a lender or borrower.

Period (t+1)'s budget constraint is given as

$$_tY_{t+1}^{e+} + (1+i_t)B_t + (1+i_t^*)(_tS_{t+1}^{e}B_t^*) + M_t =$$

$$(_tP_{t+1}^{e})y_{t+1} + {}_t(S_{t+1}^{e})(_tP_{t+1}^{*e})y_{t+1}^*, \qquad \text{(III.9)}$$

which states that the sum of the expected income from sale of labor services in (t+1),[9] the expected return (or repayment) from holding foreign bonds, the return (or repayment) from holding domestic bonds, and the nominal domestic money balances carried over from period (t) must be equal to the expected expenditures on period (t+1)'s consumption bundle. Period (t+1)'s budget constraint implies that the economic agent does not intend to leave a bequest nor to be in debt at the end of period (t+1). This ensures that the economic agent will not be able to borrow an unlimited amount and, hence, demand an unlimited amount of goods each period.

Equations (III.8) and (III.9) can be combined to form a wealth constraint by substituting for $(1+i_t^*)_t S_{t+1}^e$ in equation (III.9) with equation (III.6) and combining the resulting expression with equation (III.8). Using the current domestic goods as the numeraire good, we divide through by P_t to obtain

$$\frac{{}_tY_{t+1}^{e+}}{(1+i_t)P_t} + \frac{Y_t}{P_t} - \frac{i_t}{(1+i_t)}\left(\frac{M_t}{P_t}\right) = \frac{1}{(1+i_t)P_t}\left[({}_tP_{t+1}^e)y_{t+1} + ({}_tS_{t+1}^e)({}_tP_{t+1}^{*e})y_{t+1}^*\right] + y_t + \left(\frac{S_tP_t^*}{P_t}\right)y_t^*. \qquad \text{(III.10)}$$

Maximizing the utility function (III.7) subject to the wealth constraint, the following implicit demand functions are obtained:

$$D_j = D\left(\frac{{}_tY_{t+1}^{e+}}{(1+i_t)P_t}, \frac{Y_t}{P_t}, \frac{i_t}{(1+i_t)}, \frac{S_tP_t^*}{P_t}, \frac{{}_tP_{t+1}^e}{P_t(1+i_t)}, \frac{({}_tS_{t+1}^e)_tP_{t+1}^{*e}}{(1+i_t)P_t}\right), \qquad \text{(III.11)}$$

where $D_j = y_t, y_t^*, y_{t+1}, y_{t+1}^*, \left(\frac{M_t}{P_t}\right), B_t, B_t^*$.

We will assume that the utility function is almost homogeneous of degree k in M_t, P_t, and $S_t P_t^*$.[10] As demonstrated in Dusansky and Kalman,[11] this implies that the demand for commodities will be homogeneous of degree zero in prices and labor income while the demand for domestic nominal money balances will be homogeneous of degree one in prices and labor income.

With a view of specifying the (I-S) and the (L-M) functions for the aggregate model, we will restrict our discussion for the remainder of this section to a discussion of the demand for domestic money function and the demand for current domestic output. Anticipating the development in the next section, we will assume that the demand functions are log-linear in the variables except for the variable $(\frac{i_t}{1+i_t})$. The variable $(\frac{i_t}{1+i_t})$ reflects the optimal marginal rate of substitution between current domestic goods (y_t) and holding of domestic real money balances $(\frac{M_t}{P_t})$.[12] For our purposes we will approximate $(\frac{i_t}{1+i_t})$ with i_t since it can be well approximated for small i_t.[13]

Based on the result of equation (III.11), we postulate the following two demand equations:

$$\ln y_t = a_1' \ln\left(\frac{S_t P_t^*}{P_t}\right) + a_2' \ln\left(\frac{{}_tP_{t+1}^e}{P_t(1+i_t)}\right) + a_3'\left(\frac{({}_tS_{t+1}^e)\,{}_tP_{t+1}^{*e}}{(1+i_t)P_t}\right)$$

$$+ a_4' \ln\left(\frac{Y_t}{P_t}\right) + a_5' i + a_6' \ln\left(\frac{{}_tY_{t+1}^{e+}}{P_t(1+i_t)}\right), \qquad \text{(III.12)}$$

$$\ln M_t = (1-b_1)\ln P_t + b_1 \ln(S_t P_t^*) - b_2 \ln\left(\frac{{}_tP_{t+1}^e}{P_t(1+i_t)}\right)$$

$$- b_3 \ln\left(\frac{({}_tS_{t+1}^e)\,{}_tP_{t+1}^{*e}}{(1+i_t)P_t}\right) + b_4 \ln\left(\frac{Y_t}{P_t}\right) - b_5 i_t$$

$$+ b_6 \ln\left(\frac{{}_tY_{t+1}^{e+}}{P_t(1+i_t)}\right). \qquad \text{(III.13)}$$

The demand for current domestic goods, equation (III.13), is a function of the three "relative" prices, real current labor income, expected future labor income, and the nominal domestic interest rate. The three "relative" prices are the current relative price of foreign to domestic goods, $\left(\frac{S_t P_t^*}{P}\right)$; the expected relative price of domestic goods to current domestic goods, $\left(\frac{{}_tP_{t+1}^e}{P_t(1+i_t)}\right)$; and the expected relative price of foreign goods to current domestic goods, $\left(\frac{({}_tS_{t+1}^e)\,{}_tP_{t+1}^{*e}}{P_t(1+i_t)}\right)$. Assuming that the current foreign goods and the future consumption of foreign and domestic goods are substitutes for the current consumption of domestic goods, we obtain the positive effects postulated for these variables.

The current real labor income is postulated to have its conventional positive effect on the demand for current domestic goods. However, we will assume that the expected future labor income to have a negligible effect on the demand for current domestic goods,[14] i.e., $a_6' \approx 0$. Finally, we will also assume that the current nominal domestic interest rate has a negligible effect on the demand for current domestic goods,[15] $a_5' \approx 0$.

The demand for nominal domestic money balances, equation (III.13), treats the nominal money balances as one form of asset in which wealth may be held.[16] Since the demand for money is a demand for real money balances, both the price of the domestic goods and the domestic price of the foreign goods are included. The coefficients of these two price variables sum to one, implying that the demand for nominal money balances is homogeneous of degree one in prices. However, economic literature on the demand for money in the open economy suggests that the more appropriate deflator for the nominal domestic money balances is the domestic price index (Q_t).[17] Thus, assuming $b_1 \approx (1-\theta)$, we can substitute Q_t for the two price variables in equation (III.13).

With respect to the rest of the variables in equation (III.13), expected real income for period (t+1) is assumed to have a negligible effect as before. The nominal domestic interest rate is postulated to have a negative effect since it represents the opportunity cost of holding nominal domestic money balances relative to holding foreign and domestic bonds. The current real income is postulated to have its conventional positive effect.

The expected relative price of domestic goods to current domestic goods, $\left(\frac{{}_tP^e_{t+1}}{P_t(1+i_t)}\right)$, and the expected relative price of foreign goods to current domestic goods, $\left(\frac{({}_tS^e_{t+1})\,{}_tP^{*e}_{t+1}}{P_t(1+i_t)}\right)$, are unconventional in the demand for money function. Assuming current domestic nominal money balances are substitutes for current consumption of domestic goods and future consumption of foreign and domestic goods, we postulate negative effects for these variables on the demand for nominal domestic money balances.

This concludes the discussion of the supply and demand sides of the small open economy. In the next section, the aggregate models of the small open economy and the rest of the world will be presented.

E. The Aggregate Model

The discussion in sections C and D leads us to specify the following five equations for the aggregate model for the small open economy. For notational ease, we have written the model in log-linear form and have assumed that X = lnX, and $\ln(1+i_t) \approx i_t$. Appropriate stochastic error terms have also been added.

(I-S)

$$y_t = a_1(S_t+P_t^*-P_t) + a_2({}_tP_{t+1}^e-P_t-i_t) + a_3({}_tS_{t+1}^e+{}_tP_{t+1}^{*e}-P_t-i_t) + u_{1t}, \tag{III.14}$$

(L-M)

$$M_t = Q_t - b_2({}_tP_{t+1}^e-P_t-i_t) - b_3({}_tS_{t+1}^e+{}_tP_{t+1}^{*e}-P_t-i_t) + b_4y_t - b_5i_t + u_{2t}, \tag{III.15}$$

(Domestic Real Output)

$$y_t = c_1(P_t - {}_{t-1}P_t^e) + c_2(1-\theta)({}_{t-1}Z_t^e) + u_{3t}, \tag{III.16}$$

(Interest Rate Parity)

$$i_t = i_t^* + ({}_tS_{t+1}^e - S_t), \tag{III.17}$$

(Domestic Price Index)

$$Q_t = \theta P_t + (1-\theta)(S_t + P_t^*), \tag{III.18}$$

where u_{1t}, u_{2t}, and u_{3t} are assumed to be white noise processes.

The (I-S) and the (L-M) curves are obtained by aggregating equations (III.12) and (III.13) over the economic agents, and the domestic real output equation is the same as equation (III.6). Real current income in the (I-S) curve has been combined with the real output, and the constant term in the domestic real output equation has been omitted for convenience.[18] Several points about the real output equation deserved to be emphasized. In particular, the neutrality property of the domestic real output equation which forms the focal point of our empirical study will be discussed more fully here.

Equation (III.16) indicates that in addition to unanticipated movements in the price of domestic goods, domestic real output is also affected by the anticipated relative price of domestic to foreign goods. This is at variance with the strict natural rate hypothesis advanced in the literature for the closed economy, but it is entirely consistent with the open economy version of the natural rate hypothesis advanced by Leiderman[19] and Turnovsky.[20] We will adopt the terminology used by Leiderman and call equation (III.16) the weak open economy version of the natural rate hypothesis. According to the weak open economy version of the natural rate hypothesis, perfectly anticipated domestic and foreign monetary policies may potentially affect domestic real output to the extent that they affect the relative price of domestic to foreign goods. However, there are two cases where anticipated changes in the domestic and foreign prices will have no effect on domestic real output. The first case is when $\theta = 1$. This corresponds to the situation where the domestic workers, when formulating their labor supply decisions, completely ignore the effect of foreign price changes on the purchasing power of their wage incomes. However, this is unlikely for it implies irrational behavior on the part of the domestic economic agents. The second case is when both the domestic and foreign goods are perfect substitutes in consumption. In this case, the law of one price will hold, and hence the second term in equation (III.16) will vanish.

To complete the aggregate model, the rest of the world also must be modeled. We will assume that the rest of the world is so large in relation to the domestic economy that we may treat the rest of the world as a closed economy.[21] Accordingly, the following three equations describe the rest of the world:

(I-S)

$$y_t^* = a_1^*({}_tP_{t+1}^{*e} - P_t^* - i_t^*) + u_{1t}^*, \qquad \text{(III.19)}$$

(L-M)

$$M_t^* = P_t^* - b_1^* i_t^* + b_2^* y_t^* - b_3^*({}_tP_{t+1}^{*e} - P_t^* - i_t^*) + u_{2t}^*, \qquad \text{(III.20)}$$

(Real Output)

$$y_t^* = c_1^*({}_tP_t^* - {}_{t-1}P_t^{*e}) + u_{3t}^*, \qquad \text{(III.21)}$$

where u_{1t}^*, u_{2t}^*, and u_{3t}^* are white noise stochastic processes. Equations (III.19) to (III.21) are derived analogously to equations (III.14) to (III.18).

The solution method to obtain the reduced form equations is given in appendix A; in the text we will limit our attention to only the reduced form equations. The reduced form solutions to the rest of the world model are

$$P_t^* = J_1^* M_t^* + \sum_{i=1}^{\infty} R_i^* [J_2^* ({}_tM_{t+i}^{*e}) + J_3^* ({}_{t-1}M_{t+i-1}^{*e})] + e_{1t}^{*'}, \qquad \text{(III.22)}$$

$$i_t^* = K_1^* M_t^* + \sum_{i=1}^{\infty} R_i^* [K_2^* ({}_tM_{t+i}^{*e}) + K_3^* ({}_{t-1}M_{t+i-1}^{*e})] + e_{1t}^{*'}, \qquad \text{(III.23)}$$

$$y_t^* = c_1^* [J_1^* (M_t^* - {}_{t-1}M_t^{*e}) + J_2^* \sum_{i=1}^{\infty} R_i^* ({}_tM_{t+1}^{*e} - {}_{t-1}M_{t+1}^{*e}) + e_{1t}^{*'}] + u_{2t}^{*'},$$

(III.24)

where

$$R_i^* = \left(\frac{J_1^*}{J_2^*}\right)\left(\frac{J_2^*}{(1-J_3^*}\right)^i,$$

and the J^*'s and the K^*'s are functions of the structural parameters of the rest of the world.

Given the solutions for the rest of the world variables, the reduced form equations for the domestic economy can be obtained. Unfortunately, these reduced form equations are rather cumbersome. Consequently, in the text we present the solutions in general form and leave the exact form of these solutions to appendix B. For the flexible exchange rate period, the endogeneous variables are y_t, P_t, and S_t; the exogeneous variables are M_t, M_t^*, i_t^*, P_t^* and their expected values. To maintain a fixed exchange rate, we assume that the domestic monetary will buy or sell sufficient foreign and domestic reserves to keep the exchange rate fixed. However, these actions will affect the domestic monetary base. Hence, the fixed exchange rate period, the endogeneous variables are y_t, P_t, and M_t; the exogeneous variables are $\overset{*}{M}_t$, $\overset{*}{i}_t$, $\overset{*}{P}_t$, and their expected values.

However, since the structure of the rest of the world is assumed to be included in the information set of the domestic economic agents, they are aware of the processes which determine the foreign price level and interest rate. These processes will be taken into consideration when forming expectations of the foreign price level and the foreign interest rate. This implies that we can substitute equations (III.22) and (III.23) for $\overset{*}{P}_t$ and $\overset{*}{i}_t$ and their expectations into the reduced form equations of the endogeneous variables of the domestic economy. Therefore, for the flexible exchange rate period, the reduced form equations of the domestic economy are functions solely of the current foreign and domestic money supply and the expected values of these variables. For the fixed exchange rate period, the reduced form equations of the domestic economy are functions solely of current and expected foreign money supply. In particular, the reduced form solutions for the domestic economy under flexible exchange rate are

$$\begin{aligned} P_{t,flex} = P(&M_t, \overset{*}{M}_t, {}_tM^e_{t+1}, {}_tM^e_{t+2}, \cdots, {}_{t-1}M^e_t, \\ &{}_{t-1}M^e_{t+1}, {}_{t-1}M^e_{t+2}, \cdots, {}_tM^{*e}_{t+1}, {}_tM^{*e}_{t+2}, \cdots, \\ &{}_{t-1}M^{*e}_t, {}_{t-1}M^{*e}_{t+1}, {}_{t-1}M^{*e}_{t+2}, \cdots) + U_{1t}, \end{aligned} \qquad \text{(III.25)}$$

where

$$U_{1t} = J_5 \overset{*'}{e}_{1t} + J_6 \overset{*'}{e}_{2t} + e'_{1t};$$

$$S_t = S(M_t, \overset{*}{M}_t, {}_tM^e_{t+1}, {}_tM^e_{t+2}, \ldots , {}_{t-1}M^e_t,$$
$${}_{t-1}M^e_{t+1}, {}_{t-1}M^e_{t+2}, \ldots , {}_t\overset{*}{M}{}^e_{t+1}, {}_t\overset{*}{M}{}^e_{t+2}, \ldots,$$
$${}_{t-1}\overset{*}{M}{}^e_t, {}_{t-1}\overset{*}{M}{}^e_{t+1}, {}_{t-1}\overset{*}{M}{}^e_{t+2}, \ldots) + U_{2t}, \qquad \text{(III.26)}$$

where

$$U_{2t} = E_5 \overset{*'}{e}_{1t} + E_6 \overset{*'}{e}_{2t} + e'_{2t};$$

and

$$y_{t,flex} = Y(M_t - {}_{t-1}M^e_t, \overset{*}{M}_t - {}_{t-1}\overset{*}{M}{}^e_t, {}_tM^e_{t+1} - {}_{t-1}M^e_{t+1},$$
$${}_tM^e_{t+2} - {}_{t-1}M^e_{t+2}, \ldots , {}_t\overset{*}{M}{}^e_{t+1} - {}_{t-1}\overset{*}{M}{}^e_{t+1},$$
$${}_t\overset{*}{M}{}^e_{t+2} - {}_{t-1}\overset{*}{M}{}^e_{t+2}, \ldots) + c_2(1-\theta)_{t-1}Z^e_t$$
$$+ c_1U_{1t} + u_{3t}, \qquad \text{(III.27)}$$

where

$${}_{t-1}Z^e_t = Z({}_{t-1}\overset{*}{M}{}^e_t, {}_{t-1}\overset{*}{M}{}^e_{t+1}, {}_{t-1}\overset{*}{M}{}^e_{t+2}, \ldots), \qquad \text{(III.28)}$$

and J's and E's are functions of the structural parameters of the small domestic economy.

Similarly, for the fixed exchange rate period, the reduced form solutions are

$$P_{t,fixed} = P'(M_t^*,\ {}_{t-1}M_t^{*e},\ {}_{t-1}M_{t+1}^{*e},\ {}_{t-1}M_{t+2}^{*e},\ \ldots,$$

$$ {}_tM_{t+1}^{*e},\ {}_tM_{t+2}^{*e},\ \ldots,\ \overline{S}) + U_{3t}, \qquad \text{(III.29)}$$

where $\overline{S}$ denotes the fixed exchange rate, and

$$U_{3t} = K_4 e_{1t}^{*'} + K_5 e_{2t}^{*'} + e_{3t}^{'},$$

$$M_t = M(M_t^*,\ {}_{t-1}M_t^{*e},\ {}_{t-1}M_{t+1}^{*e},\ {}_{t-1}M_{t+2}^{*e},\ \ldots,$$

$$ {}_tM_{t+1}^{*e},\ {}_tM_{t+2}^{*e},\ \ldots,\ \overline{S}) + U_{4t}, \qquad \text{(III.30)}$$

where

$$U_{4t} = F_4 e_{1t}^{*'} + F_5 e_{2t}^{*'} + e_{3t}^{'},$$

and

$$y_{t,fixed} = Y'(M_t^* - {}_{t-1}M_t^{*e},\ {}_tM_{t+1}^{*e} - {}_{t-1}M_{t+1}^{*e},\ {}_tM_{t+2}^{*e} - {}_{t-1}M_{t+2}^{*e},$$

$$\ldots,\ \overline{S}) + c_2(1-\theta)\,{}_{t-1}Z_t^e + c_1 U_{3t} + u_{3t}, \qquad \text{(III.31)}$$

where ${}_{t-1}Z_t^e$ is given by equation (III.28),[22] and the K's and F's are also functions of the structural parameters of the domestic economy.

We will end our discussion of the aggregate models for the domestic economy and the rest of the world at this point. The discussion of the reduced form solutions obtained here and their implications for empirical tests are left to the next chapter since they logically belong together. In chapter IV, the empirical form of the models will also be presented.

Footnotes - Chapter III

1. See Flood (1979).

2. See Fischer (1977).

3. See Gray (1976).

4. The existence of labor contracts has been justified by Azariadis (1975), and Baily (1977).

5. See Clower (1969).

6. The assumption of a Cobb-Douglas production function is strictly for convenience since it is linear in logs.

7. The inclusion of real money balances in the utility function follows that of Patinkin (1965). See also Mackay and Weber (1977) for a discussion of the utility function where nominal money balances and the prices are entered as separate arguments.

8. See Mackay and Weber (1977).

9. We have denoted the expected income for period (t+1) as ${}_tY_{t+1}^{e+}$ to distinguish it from expectations of other variables. The expectations of variables such as the exchange rate, the domestic and foreign prices are formed rationally, conditional on the information available at the time expectations are formed. This includes the stochastic structure of the domestic and foreign economies and the policy rules of the domestic and foreign monetary authorities. However, expectations for income in period (t+1) are formed on a different information set since they are conditional on the quantity of employment the economic agent is able to obtain in period (t+1) after negotiating a labor contract at the end of period (t).

10. See Mackay and Weber (1977) for a discussion of a utility function that is almost homogeneous of degree k.

11. See Dusansky and Kalman (1974).

12. Hirshleifer (1970), p. 145, defines this variable as the equivalent y_t sacrifice per unit of real liquidity (M_t/P_t).

13. The alternative is to approximate $\ln(\frac{i_t}{1+i_t})$ with $\ln(i_t)$. However, since we will also be approximating $\ln(1+i_t)$ with i_t, the above approximation of $\ln(\frac{i_t}{1+i_t})$ will result in having both $\ln(i_t)$ and i_t in the model and this will complicate the solution method. Furthermore, since we will not be solving for i_t directly, rather, we will eliminate i_t with the assumption of interest rate parity, we decided to approximate $\ln(\frac{i_t}{1+i_t})$ with i_t.

14. The effect of expected real income on the demand for current domestic goods and domestic nominal money balances cannot be ascertained in the context of our model. As an example, assume that the economic agent expects with certainty that he will be able to negotiate a new labor contract at the end of period (t) and that he will be employed in period (t+1). This will most likely have a positive income effect on current consumption. On the other hand, if the domestic economic agent is faced with great uncertainty as to whether he will be able to negotiate a new labor contract for period (t+1), he may reduce the current consumption of domestic and foreign goods. Since the theory of consumer demand under this type of uncertainty is not well developed, we will assume that the effect of expected real labor income on the demand for current domestic goods and the demand for nominal domestic money balances to be negligible.

15. Recall that i_t is the approximation for $(\frac{i_t}{1+i_t})$, which represents the optimal marginal rate of substitution between y_t and (M_t/P_t). This can be interpreted as the implicit real future consumption foregone by holding real money balances rather than interest bearing bonds. A change in the nominal interest rate will affect the choice between holding nominal money balances and holding bonds. But, the effect of a change of nominal interest rate on the choice between consumption of current domestic goods and the holding real money balances is unclear. Empirical, Lovell (1975) p. 125, reported that much of the empirical evidence suggests that the effect of the interest rate on the demand for current consumption is negligible. However, Weber, (1970) found a positive effect of the interest rate on consumption. Finally, note that a change in the domestic nominal interest rate does have a negative effect on the demand for domestic goods. But, the effect is indirect through changes in the expected relative price of future to current domestic goods, and the expected relative price of future foreign goods to current domestic goods.

16. See Friedman (1956).

17. See for example Turnovsky (1977, 1981).

18. Real output should therefore be interpreted as deviation from some "normal" level.

19. See Leiderman (1979).

20. See Turnovsky (1981).

21. Since the rest of the world is modeled as a closed economy, the only policy that the monetary authority of the rest of the world can pursue is monetary policy of setting the money supply.

22. It should be noted carefully that we are saying that the reduced form solutions to ${}_{t-1}Z_t^e$ in <u>general</u> form are the same for both the fixed and the flexible exchange rate regimes. However, the exact solutions for ${}_{t-1}Z_t^e$ are different under the fixed and the flexible exchange rate regimes. For purpose of exposition, we will treat the reduced form solutions in <u>general</u> form as if they were the same.

CHAPTER IV

THE EMPIRICAL MODELS

A. Introduction

In the previous chapter, we derived the reduced form solutions for all the endogeneous variables under both flexible and fixed exchange rate regimes. The purpose of this chapter is to develop an empirical model to be used as the basis for empirical testing. This chapter consists of four sections. In section B, we summarize the results in chapter III and discuss in very general terms the methods of empirically testing the model for different monetary and exchange rate regimes. Although we review all of the results we have obtained in chapter III, our discussion concentrates on the testable implications of the reduced form equations for both the domestic real output and the domestic price level under fixed and flexible exchange rate regimes. In section C, we develop the explicit empirical model and examine the testable implications of the assumptions of rational expectations and the natural rate hypothesis. In this section, we also consider ways in which these implications may be tested. This chapter concludes with some general remarks in section D.

B. Summary and Discussion of the Reduced Form Equations

We begin by summarizing all the reduced form equations obtained in chapter III. For the rest of the world, the reduced form equations for the foreign price level, the foreign interest rate, and the foreign real output are

$$P_t^* = J_1^* M_t^* + \sum_{i=1}^{\infty} R_i^* [J_2^* ({}_tM_{t+i}^{*e}) + J_3^* ({}_{t-1}M_{t+i-1}^{*e})] + e_{1t}^{*'}, \qquad \text{(IV.1)}$$

$$i_t^* = K_1^* M_t^* + \sum_{i=1}^{\infty} R_i^* [K_2^* ({}_tM_{t+i}^{*e}) + K_3^* ({}_{t-1}M_{t+i-1}^{*e})] + e_{2t}^{*'}, \qquad \text{(IV.2)}$$

$$y_t^* = c_1^* [J_1^* (M_t^* - {}_{t-1}M_t^{*e}) + J_2^* \sum_{i=1}^{\infty} R_i^* ({}_tM_{t+i}^{*e} - {}_{t-1}M_{t+i}^{*e})$$

$$+ e_{1t}^{*'}] + u_{3t}^*, \qquad \text{(IV.3)}$$

where

$$R_i^* = \left(\frac{J_1^*}{J_2^*}\right)\left(\frac{J_2^*}{(1-J_3^*)}\right)^i.$$

The J^*'s and K^*'s are functions of the structural parameters of the rest of the world (their definitions are given in appendix A and c_1^* is the same structural parameter as in equation (III.21).

Equations (IV.1) to (IV.3) exhibit two important properties of rational expectations models. First, the reduced form solutions to the nominal variables P_t^* and i_t^* are functions of current and expected rest of the world money supply, with expectations formed in periods (t) and (t+1). Hence, both the foreign price level and the foreign interest rate will depend on the money rule chosen by the rest of the world.[1]

Second, the reduced form equation for the foreign real output is a function of only unanticipated foreign money supply shocks and the term $({}_tM^{*e}_{t+i} - {}_{t-1}M^{*e}_{t+i})$, which is the revision of expectations concerning the foreign money supply for period (t+i) between periods (t) and (t-1) given the new information obtained during period (t). The reduced form equation for the foreign real output can be shown to have the familiar neutrality result of closed economy models which embody the rational expectations-natural rate hypothesis, and this can be shown as follows. Let I^*_{t-1} be the information set which economic agents used for forecasting M^*_t. In particular, let I^*_{t-1} be

$$I^*_{t-1} = (\underline{G}^*_{t-1}, \underline{G}^*_{t-2}, \ldots , \underline{G}^*_{t-n}), \qquad \text{(IV.4)}$$

and

$$I^*_t = (\underline{G}^*_t, I^*_{t-1}), \qquad \text{(IV.5)}$$

where $\underline{G}^*_{t-1} = (g^*_{1t-i}, g^*_{2t-i}, \ldots , g^*_{kt-i})$, $i = 0, 1, \ldots n$, is a vector of variables at time (t-i) that are useful in forecasting M^*_t. Then

$${}_tM^{*e}_{t+i} = E(M^*_{t+i} | I^*_t) = E(M^*_{t+i} | \underline{G}^*_t, I^*_{t-1}), \qquad \text{(IV.6)}$$

and

$$ {}_{t-1}M^{*e}_{t+i} = E(M^{*}_{t+i} \mid I^{*}_{t-1}), \tag{IV.7} $$

where E is the expectation operator.

Applying recursive projection theory, we can rewrite (IV.6) as

$$ {}_{t}M^{*e}_{t+i} = E(M^{*}_{t+i} \mid I^{*}_{t-1}) + E[M^{*}_{t+i} - E(^{*}_{t+i} \mid I^{*}_{t-1}) \mid \underline{G}^{*}_{t} - E(\underline{G}^{*}_{t} \mid I^{*}_{t-1})], \tag{IV.6a} $$

or

$$ {}_{t}M^{*e}_{t+i} - {}_{t-1}M^{*e}_{t+i} = E[M^{*}_{t+i} - E(M^{*}_{t+i} \mid I^{*}_{t-1}) \mid \underline{G}^{*}_{t} - E(\underline{G}^{*}_{t} \mid I^{*}_{t-1})]. \tag{IV.6b} $$

The right hand side of equation (IV.6b) can further be rewritten as

$$ E[M^{*}_{t+i} - E(M^{*}_{t+i} \mid I^{*}_{t-1}) \mid \underline{G}^{*}_{t} - E(\underline{G}^{*}_{t} \mid I^{*}_{t-1})] = [\underline{G}^{*}_{t} - E(\underline{G}^{*}_{t} \mid I^{*}_{t-1})]\underline{D^{*T}} \tag{IV.8} $$

using linear projection theory. The column vector $\underline{D^{*T}} = (d^{*}_{1}, d^{*}_{2}, \ldots, d^{*}_{k})^{T}$ can be interpreted as regression coefficients of the regression of $[M^{*}_{t+i} - E(M^{*}_{t+i} \mid I^{*}_{t-1})]$ on $[\underline{G}^{*}_{t} - E(\underline{G}^{*}_{t} \mid I^{*}_{t-1})]$.[2]

Combining equations (IV.6d) and (IV.8), we obtain

$$ {}_{t}M^{*e}_{t+i} - {}_{t-1}M^{*e}_{t+i} = [\underline{G}^{*}_{t} - E(\underline{G}^{*}_{t} \mid I^{*}_{t-1})]\underline{D^{*T}}, \tag{IV.9} $$

which states that the forecasts for M^* are revised between periods (t) and (t-1) according to the unanticipated new information obtained when $\underline{G}_t^*$ is observed. Projecting both sides of equation (IV.9) on I_{t-1}^*, we have

$$E[({}_tM_{t+i}^{*e} - {}_{t-1}M_{t+i}^{*e})|I_{t-1}^*] = E([\underline{G}_t^* - E(\underline{G}_t^*|I_{t-1}^*)]\underline{D^{*T}}|I_{t-1}^*)$$
$$= 0, \qquad \text{(IV.10)}$$

since each term on the right hand side of equation (IV.10) is zero by the orthogonality condition. Thus, we find that foreign real output is a function solely of random shocks.[3] This is, of course, the familiar conclusion of models which embody the rational expectations-natural rate hypothesis.

For the domestic economy, the endogeneous variables under flexible exchange rates are y_t, P_t, and S_t; the exogeneous variables are M_t, M_t^*, P_t^*, i_t^*, and their expected values. For the fixed exchange rate period, the endogeneous variables are y_t, P_t, and M_t; the exogeneous variables are M_t^*, P_t^*, i_t^*, $\overline{S}$, and their expected values. The reduced form equations for these endogeneous variables are obtained by substituting equations (IV.1) and (IV.2) for P_t^* and i_t^*. This reflects that the structure of the rest of the world is included in the information set of the domestic economic agents, and the stochastic processes that determine P_t^* and i_t^* will be taken into consideration when forming expectations of these variables. In particular, the reduced form equation for domestic real output under flexible exchange rate is (in general form)

$$y_{t,flex} = Y(M_t - {}_{t-1}M_t^e,\ M_t^* - {}_{t-1}M_t^{*e},\ {}_tM_{t+1}^e - {}_{t-1}M_{t+1}^e,$$
$${}_tM_{t+2}^e - {}_{t-1}M_{t+2}^e,\ \ldots,\ {}_tM_{t+1}^{*e} - {}_{t-1}M_{t+1}^{*e},$$
$${}_tM_{t+2}^{*e} - {}_{t-1}M_{t+2}^{*e},\ \ldots) + c_2(1-\theta)\,{}_{t-1}Z_t^e$$
$$+ c_1U_{1t} + u_{3t}, \qquad \text{(IV.11)}$$

where

$${}_{t-1}Z_t^e = Z({}_{t-1}M_t^{*e},\ {}_{t-1}M_{t+1}^{*e},\ {}_{t-1}M_{t+2}^{*e},\ \ldots), \qquad \text{(IV.12)}$$

$$U_{1t} = J_5e_{1t}^{*'} + J_6e_{2t}^{*'} + e_{1t}^{'}, \qquad \text{(IV.13)}$$

J_5 and J_6 are defined in appendix B and θ is a fixed weight reflecting the relative expenditures share of foreign and domestic goods in the domestic price index.

For the fixed exchange rates, domestic real output is

$$y_{t,fixed} = Y'(M_t^* - {}_{t-1}M_t^{*e},\ {}_tM_{t+1}^{*e} - {}_{t-1}M_{t+1}^{*e},\ {}_tM_{t+2}^{*e} - {}_{t-1}M_{t+2}^{*e},$$
$$\ldots) + c_2(1-\theta)\,{}_{t-1}Z_t^e + c_1U_{3t} + u_{3t}, \qquad \text{(IV.14)}$$

where

$$U_{3t} = K_4e_{1t}^{*'} + K_5e_{2t}^{*'} + e_{3t}^{'}, \qquad \text{(IV.15)}$$

K_4 and K_5 are defined in appendix B. The form of the domestic real output equation is not completely general, however; the present form will facilitate exposition later.

The general reduced form solution for the expected relative price of domestic to foreign goods, ${}_{t-1}Z_t^e$, is the same for both the flexible and the fixed exchange rate regimes and is therefore given by equation (IV.12). Note that the general reduced form solution for ${}_{t-1}Z_t^e$ is a function solely of expected foreign money supply. This implies that domestic monetary policies will be totally ineffective in affecting the domestic real output for both of the exchange rate regimes. On the other hand, foreign monetary policies, to the extent that they can influence the relative price of domestic to foreign goods, can have a real effect on the domestic real output for both the flexible and the fixed exchange rate regimes.

Under flexible exchange rates, domestic real output is a function of both foreign and domestic money supply shocks, revisions of expectations for both future foreign and domestic money supply, and expected relative price of domestic to foreign goods. For the fixed exchange rates case, domestic real output is a function of only foreign money supply shock, revisions of expectations of future foreign money supply, and the expected relative price of domestic to foreign goods. Domestic money supply does not appear as an exogeneous variable because it is assumed to be endogeneous under the fixed exchange rate period. Since we have shown that revisions of expectations are on the basis of expectational errors,[4] we can combine them with the error term and rewrite domestic real output as

$$y_{t,flex} = c_2(1-\theta)\,{}_{t-1}Z_t^e + \varepsilon_t, \qquad \text{(IV.16)}$$

and

$$y_{t,fixed} = c_2(1-\theta)_{t-1}Z^e_t + \varepsilon'_t. \qquad \text{(IV.17)}$$

Note that the mean values of domestic real output are the same under both exchange rate regimes. However, the variances may be different due to the difference in the error terms since now the error term in equation (IV.16) contains unanticipated domestic money shocks while the error in equation (IV.17) does not. This suggests that the proper empirical tests should be conducted under separate exchange rate regimes, unless the potential heteroscedasticity is taken into account.

The presence of the expected relative price of domestic to foreign goods implies that the foreign monetary authority may affect domestic real output by affecting the relative price of domestic to foreign goods. This non-neutrality is a result of open economy models where purchasing power parity is not assumed.[5] Furthermore, this is consistent with our model where workers form their expectations with the domestic price index while the relevant price for the producers is the price of the domestic goods that they sell. This suggests that our empirical tests need to distinguish between testing for neutrality and testing for rationality of expectations. Furthermore, the rejection of neutrality of domestic real output with respect to different monetary rules does not imply the rejection of rationality of expectations since it is possible for the anticipated relative price of domestic to foreign goods to affect domestic real output. On the other hand, rationality of expectations does not necessarily lead to neutrality of the domestic real output with respect to domestic or foreign monetary expansions.

The reduced form equations for the price of domestic goods and the exchange rate under flexible exchange rates are

$$P_{t,flex} = P(M_t, \overset{*}{M}_t, {}_tM^e_{t+1}, {}_tM^e_{t+2}, \cdots , {}_{t-1}M^e_t, {}_{t-1}M^e_{t+1}, \cdots , {}_tM^{*e}_{t+1}, {}_tM^{*e}_{t+2}, \cdots , {}_{t-1}M^{*e}_t , {}_{t-1}M^{*e}_{t+1}, \cdots) + U_{1t}, \quad \text{(IV.18)}$$

$$S_t = S(M_t, \overset{*}{M}_t, {}_tM^e_{t+1}, {}_tM^e_{t+2}, \cdots , {}_{t-1}M^e_t, {}_{t-1}M^e_{t+1}, \cdots , {}_tM^{*e}_{t+1}, {}_tM^{*e}_{t+2}, \cdots , {}_{t-1}M^{*e}_t , {}_{t-1}M^{*e}_{t+1}, \cdots) + U_{2t}, \quad \text{(IV.19)}$$

where

$$U_{2t} = E_5\overset{*'}{e}_{1t} + E_6\overset{*'}{e}_{2t} + e'_{2t}, \quad \text{(IV.20)}$$

E_5 and E_6 are defined in appendix B, and U_{1t} is given by equation (IV.13).

The reduced form equations for P_t and M_t under fixed exchange rates are

$$P_{t,fixed} = P'(\overset{*}{M}_t, {}_{t-1}M^{*e}_t , {}_{t-1}M^{*e}_{t+1}, \cdots , {}_tM^{*e}_{t+1}, {}_tM^{*e}_{t+2}, \cdots ; \overline{S}) + U_{3t}, \quad \text{(IV.21)}$$

$$M_t = M(\overset{*}{M}_t, {}_{t-1}\overset{*e}{M}_t, {}_{t-1}\overset{*e}{M}_{t+1}, \ldots, {}_t\overset{*e}{M}_{t+1}, {}_t\overset{*e}{M}_{t+2},$$

$$\ldots ; \overline{S}) + U_{4t}, \qquad \text{(IV.22)}$$

where

$$U_{4t} = F_4\overset{*'}{e}_{1t} + F_5\overset{*'}{e}_{2t} + \overset{'}{e}_{4t}, \qquad \text{(IV.23)}$$

F_4 and F_5 are defined in appendix B, U_{3t} is given by equation (III.15), and $\overline{S}$ denotes the fixed exchange rate. However, for the purpose of the empirical work, the domestic price level is more useful than the price of domestic goods since no reliable data exist for the price of domestic goods. Substituting equations (IV.1), (IV.18), and (IV.19) into equation (III.18) in chapter III, we obtain the reduced form equation to the domestic price level under flexible exchange rate as

$$Q_{t,flex} = Q(M_t, \overset{*}{M}_t, {}_t\overset{e}{M}_{t+1}, {}_t\overset{e}{M}_{t+2}, \ldots, {}_{t-1}\overset{e}{M}_t,$$

$${}_{t-1}\overset{e}{M}_{t+1}, \ldots, {}_t\overset{*e}{M}_{t+1}, {}_t\overset{*e}{M}_{t+2}, \ldots,$$

$${}_{t-1}\overset{*e}{M}_t, {}_{t-1}\overset{*e}{M}_{t+1}, \ldots; \theta) + V_{1t}, \qquad \text{(IV.24)}$$

where

$$V_{1t} = U_{1t} + U_{2t} + \overset{*'}{e}_{1t}. \qquad \text{(IV.25)}$$

Similarly, by substituting equations (IV.1) and (IV.21) into equation (III.20), we obtain

$$Q_{t,fixed} = Q'(M^*_t, {}_tM^{*e}_{t+1}, {}_tM^{*e}_{t+2}, \ldots , {}_{t-1}M^{*e}_t ,$$

$$ {}_{t-1}M^{*e}_{t+1}, \ldots ; \overline{S}, \theta) + V_{2t}, \qquad \text{(IV.26)}$$

where

$$V_{2t} = U_{3t} + e^{*'}_{1t}. \qquad \text{(IV.27)}$$

The reduced form equations to the nominal variables again exhibit the property of models which embody the rational expectations hypothesis since the time paths of these variables depend on the money rule chosen. Since our empirical tests will be conducted with the domestic price level equations, our discussion here will concentrate on them.

Under flexible exchange rates, the domestic price level is a function of current and expected foreign and domestic money supply, while under the fixed exchange rates it is a function of current and expected foreign money supply only since domestic money supply is assumed to be endogeneous under the fixed exchange rate regime.

Note that the error terms are again different for the two exchange rate regimes.[6] One reason for the difference in the error terms is simply that the stochastic process determining the price of domestic goods is different for the two exchange rates regimes. This suggests once again that the empirical tests should be conducted separately for the two exchange rate regimes unless the potential heteroscedasticity is once again taken into account. In the next section, we will develop an explicit empirical model and describe the tests for neutrality and rationality.

C. Implications for Empirical Testing of the Model

As indicated in the last section, the proper empirical tests must be carried out separately under the two exchange rate regimes unless the potential heteroscedasticity is taken into account. We will first consider the testable implications from the reduced form equations for domestic real output. Since the reduced form equations for domestic real output are the same for both exchange rate regimes except for the error terms, our discussion in this section will treat them as one if they were the same.

To develop an empirical counterpart to the domestic real output equation, there are several methodological issues that must be resolved. First, our development of the domestic real output equation in chapter III implied that actual output adjusted to its equilibrium value instantaneously. In actuality, however, domestic real output may adjust to its equilibrium value only gradually because of adjustment costs. Findings by Barro,[7] Grossman,[8] Leiderman,[9] Lucas,[10] Sargent,[11] and Neftci and Sargent[12] provide empirical support for this lagged adjustment in real output. Hence, in order to avoid misspecifying the empirical model and biasing the empirical results, we include lagged domestic real output terms, yielding

$$y_t = f_{o\,t-1}Z_t^e + \sum_{i=1}^{q} f_i y_{t-i} + \varepsilon_t, \tag{IV.28}$$

where $f_o = c_2(1 - \theta)$.

Second, expected relative price is unobservable, but its solution is given by equation (IV.12), which expresses ${}_{t-1}Z_t^e$ as a function of expected future foreign money supply conditional on the information available at period (t - 1). Since expected future foreign money supply is also unobservable, equation (IV.28) cannot be estimated unless we can relate expected foreign money supply to observable variables. One way that this may be done is to assume that foreign money supply is set according to some feedback rule of the form

$$\overset{*}{M}_t = \overset{*}{M}(\overset{*}{\phi}_{t-1}) + \overset{*}{v}_t, \tag{IV.29}$$

where $\overset{*}{\phi}_{t-1}$ represents all relevant lagged exogeneous variables used in the foreign money supply rule. Rational economic agents would then forecast future foreign money supply with equation (IV.29). By Wold's[13] chain rule of forecasting, such forecasts would be unbiased and forecasts of all future values can be expressed as a function of distributed lag of the same variables contained in $\overset{*}{\phi}_{t-1}$. We will restrict the variables in $\overset{*}{\phi}_{t-1}$ to contain only lagged exogeneous variables. The inclusion of lagged endogeneous variables would make the forecasting problem very difficult since we would then have to forecast simultaneously for the current and future values of the included endogeneous variables.

The final step in making equation (IV.28) empirically operational involves substituting the solution to forecasting future money supply into equation (IV.12) and then substituting the resulting expression into equation (IV.28,[14] yielding

$$y_t = f_o F^*(\overset{*}{\phi}_{t-1}) + \sum_{i=1}^{q} f_i y_{t-i} + \varepsilon_t. \qquad \text{(IV.28a)}$$

Since the variables in $\overset{*}{\phi}_{t-1}$ can be determined empirically from equation (IV.29), we can now conduct tests for neutrality and rationality. We will first discuss the test for neutrality.

There are several methods that have been used in the literature to test for neutrality. Barro's[15] method involves regressing real output on money innovations and other real variables. We have reviewed the problems with measuring money innovations in chapter two, and since Saidi and Barro,[16] and Wogin[17] have conducted tests along this line for Canada, we will adopt a different methodology in testing for neutrality.

Neutrality in the context of our model corresponds to requiring that $f_o F^*(\overset{*}{\phi}_{t-1}) = 0$. This is equivalent to requiring that domestic real output be econometrically exogeneous with respect to the variables in $\overset{*}{\phi}_{t-1}$. In other words, neutrality will hold if none of the variables in $\overset{*}{\phi}_{t-1}$ "cause" domestic real output in Granger's sense. In addition, we can impose an even stronger set of neutrality conditions by requiring that domestic real output be econometrically exogeneous with respect to all the nominal variables in the model. As noted by

Sargent,[18] if domestic real output is econometrically exogeneous with respect to all the nominal variables in the model, this also implies that expectations are formed rationally. This is true for the closed economy models as well as for our model, for unless expectations are formed rationally, systematic monetary policies can affect real output. However, for our model, rejection of neutrality does not allow us to reject rationality. This is because the expected relative price of domestic to foreign goods may affect domestic real output. Consequently, even if we reject neutrality we must test for rationality of expectations by other means.

To test for neutrality, we will consider not only bivariate causal models, but also multivariate causal models in which several of the causal candidates are included in the regression equation. This reduces the possibility of spurious causality due to omitted variable. Furthermore, two methods of conducting the Granger causality tests will be employed. We will first perform the direct Granger causality tests by estimating the following two equations with ordinary least squares:

$$y_t = \sum_{i=1}^{n_1} f_{1i} y_{t-i} + \sum_{i=1}^{n_2} f_{2i} G_{t-i} + e_t, \tag{IV.29b}$$

$$y_t = \sum_{i=1}^{n_1} g_i \, y_{t-i} + e_t', \tag{IV.29c}$$

where G_t may be any causal or group of causal candidates. Hence, if $f_{2i} = 0$ for $i = 1, 2, 3, \ldots n_2$, then G_t does not cause y_t in Granger's sense.

The second method of analyzing causality was proposed by Ashley, Granger, and Schmalensee.[19] Their method of analyzing causality involves comparing the forecasting performance of equations (IV.29b) and (IV.29c). To be more specific, let MSE_1 be the mean-square-error of one-step-ahead postsample forecasts of the univariate domestic real output equation, and MSE_2 be the mean-square-error of one-step-ahead forecasts of the model when a vector of causal variables is included. The causal variable G_t is said to cause y_t if $MSE_2 < MSE_1$.

To assess whether MSE_2 is significantly smaller than MSE_1 statistically, Ashley, Granger, and Schmalensee proposed the following indirect test. Let s_1^2 and $\bar{e}_1$ be the sample variance and the sample mean respectively of the postsample forecast errors of the univariate domestic real output equation. Similarly, let S_2^2 and $\bar{e}_2$ be the sample variance and the sample mean respectively of the postsample forecast errors from an equation where causal variables are included. Since

$$MSE_1 = s_1^2 + \bar{e}_1,$$

$$MSE_2 = s_2^2 + \bar{e}_2,$$

therefore,

$$MSE_1 - MSE_2 = s_1^2 - s_2^2 + e_1^2 - e_2^2.$$

It is clear that $MSE_2 < MSE_1$ if one of the following four conditions is satisfied: (i) $s_2^2 < s_1^2$ and $\bar{e}_2 < \bar{e}_1$, (ii) $s_2^2 < s_1^2$ and $\bar{e}_2 = \bar{e}_1$, (iii) $s_2^2 = s_1^2$ and $\bar{e}_2 < \bar{e}_1$, (iv) $\bar{e}_1 - \bar{e}_2 < s_1^2 - s_2^2$.

Now consider the following regression equation,

$$\text{Diff}_t = a + b(\text{NSUM}_t) + \text{error}_t, \qquad \text{(IV.30)}$$

where

$$\text{Diff}_t = e_{1t} - e_{2t},$$

$$\text{NSUM}_t = (e_{1t} + e_{2t}) - (\bar{e}_1 + \bar{e}_2),$$

and where e_{1t} is the postsample forecast errors from the univariate domestic real output equation and e_{2t} is the postsample forecast errors from the equation which includes causal variables. After some algebraic manipulations, the least squares estimates are

$$\hat{b} = \frac{s_1^2 - s_2^2}{(1/n) \sum_{t=1}^{n} (\text{NSUM}_t)^2},$$

and

$$\hat{a} = \bar{e}_1 - \bar{e}_2,$$

where n is the number of observations, $\hat{a}$ and $\hat{b}$ are orthogonal by construction.

Since the denominator of $\hat{b}$ is a positive constant, $\hat{b}$ has the same sign as $(s_1^2 - s_2^2)$. The estimate $\hat{a}$ is the difference of the two mean errors, and $\hat{a} \gtreqless 0$ as $\bar{e}_1 \gtreqless \bar{e}_2$. When both $\bar{e}_1$ and $\bar{e}_2$ are positive, $\hat{a} \gtreqless 0$ also implies $\bar{e}_1^2 \gtreqless \bar{e}_2^2$. Thus, in this case $\text{MSE}_2 < \text{MSE}_1$

significantly when $\hat{a}$ and $\hat{b}$ are both significantly positive, and this can be tested with a F-test. Therefore, the hypothesis to be tested is H_o: $\hat{a} \leq 0$, and $\hat{b} \leq 0$, against the alternate hypothesis that $\hat{a} > 0$ or $\hat{b} > 0$.

Consider next the case when both $\bar{e}_1$ and $\bar{e}_2$ are negative. In this case, $a \gtreqless 0$ implies $\bar{e}_1^2 \lesseqgtr \bar{e}_2^2$. Hence $MSE_2 < MSE_1$ significantly requires that $\hat{a}$ is significantly negative and $\hat{b}$ is significantly positive. In this case the null hypothesis is H_o: $\hat{a} \geq 0$ and $\hat{b} \leq 0$, against the alternative hypothesis that $\hat{a} < 0$ or $\hat{b} > 0$, where again this can be tested with a F-test.

When $\bar{e}_t$ and $\bar{e}_t$ are of the opposite sign, we modified equation (IV.30) to

$$SUM_t = a' + b'(NDIFF_t) + error'_t, \qquad \text{(IV.30a)}$$

where

$$SUM_t = e_{1t} + e_{2t},$$
$$NDIFF_t = (e_{1t}-e_{2t}) - (\bar{e}_1-\bar{e}_2).$$

It is easily verified that

$$\hat{b}' = \frac{s_1^2 - s_2^2}{(1/n)\sum_{t=1}^{n}(NDIFF_t)^2},$$

and

$\hat{a}' = \bar{e}_1 + \bar{e}_2$.

The least squares estimators $\hat{a}'$ and $\hat{b}'$ are again orthogonal. The denominator of $\hat{b}'$ is a positive constant and $\hat{b}'$ has the same sign as $(s_1^2 - s_2^2)$. When $\bar{e}_1 > 0$ and $\bar{e}_2 < 0$, $\hat{a}' = 0$ when $\bar{e}_1 = \bar{e}_2$, and $\hat{a}' \gtrless 0$ as $\bar{e}_1 \gtrless -\bar{e}_2$. Hence, $\hat{a}' \stackrel{>}{\underset{<}{=}} 0$ also implies $\bar{e}_1^2 \stackrel{>}{\underset{<}{=}} \bar{e}_2^2$. Thus, in this case $MSE_2 < MSE_1$ significantly when $\hat{a}'$ and $\hat{b}'$ are both significantly positive. The hypothesis to be tested in this case is the same as when both $\bar{e}_1$ and $\bar{e}_2$ are positive. Therefore, the null hypothesis is H_o: $\hat{a}' \leq 0$ and $\hat{b}' \leq 0$, against the alternative hypothesis that $\hat{a}' > 0$ or $\hat{b}' > 0$.

When $\bar{e}_1 < 0$ and $\bar{e}_2 > 0$, $\hat{a}' = 0$ when $\bar{e}_1 = \bar{e}_2$, and $\hat{a}' \gtrless 0$ as $-\bar{e}_1 \lessgtr \bar{e}_2$. Hence, $\hat{a}' \stackrel{>}{\underset{<}{=}} 0$ also implies $\bar{e}_1^2 \stackrel{<}{\underset{>}{=}} \bar{e}_2^2$. Since this case is equivalent to the case when both $\bar{e}_1^2$ and $\bar{e}_2^2$ are negative, $MSE_2 < MSE_1$ significantly requires that $\hat{a}'$ is significantly negative and $\hat{b}'$ is significantly positive. Therefore, the null hypothesis is H_o: $\hat{a}' \geq 0$ and $\hat{b}' \leq 0$ against the alternate hypothesis that $\hat{a}' < 0$ or $\hat{b}' > 0$.

We now turn to the discussion of testing for rationality with the domestic real output equation. Rationality implies that domestic economic agents are aware of the stochastic processes generating foreign and domestic money supply and forecast the future values of foreign and domestic money supply with these processes. Hence, a change in the stochastic process generating the foreign money supply would produce a change in the coefficients in the function $F^*(\phi^*_{t-1})$ in equation (IV.28a) when expectations are formed rationally, but the same coefficients would remain invariant if expectations are not formed rationally. What we have established is that the assumption of rational expectations places testable restrictions on the coefficients of the domestic real output equation when there is a change in the foreign money supply rule. One way to conduct the test is to empirically determine a breakpoint in the foreign money rule within a given exchange rate regime, and then test for the equality of the coefficients of the domestic real output equation before and after this break in the foreign money rule. This is in the spirit of Neftci and Sargent,[20] and Neftci's[21] tests. Alternatively, if we can find a period where the foreign money rule can be described by a stable stochastic process, Lucas,[22] McCallum,[23] Weber,[24] Hansen and Sargent[25] have shown that rational expectations implies cross-equation restrictions between the foreign money rule and domestic real output equation. Unfortunately, the type of test proposed by Hansen and Sargent is rather complicated. Given the limited scope of this dissertation, we will follow the simpler methodology used by Neftci and Sargent in conducting our test for rationality.

To conduct the break point test, we first estimate an empirical counterpart to equation (IV.29) within a given exchange rate regime and use Quandt's[26] procedure to determine the most likely breakpoint. After the most likely break point has been determined, we can then use Chow's[27] test to determine the equality of the coefficients in $F^*(\phi^*_{t-1})$ of equation (IV.28a) before and after this breakpoint.

With respect to testing for rationality with the domestic price equations, Neftci and Sargent's breakpoint test can again be applied. For fixed exchange rates, domestic price level is a function of current and expected foreign money supply with expectations formed in both periods (t) and (t-1). Substituting the solution to forecasting foreign money supply and equation (IV.29) into equation (IV.26), the reduced form price level equation becomes

$$Q_{t,fixed} = g'(\phi^*_{t-1}) + V_{2t}. \qquad \text{(IV.31)}$$

Rationality in this instance requires that a shift in the foreign money rule produces changes in the coefficients of equation (IV.31). The Quandt procedure can again be used to determine the most likely break point in the foreign money rule, and the Chow test can be used to determine the equality of the coefficients of equation (IV.31) before and after the break point.

For the flexible exchange rates case, the reduced form equation for domestic price level is a function of the current foreign and domestic money supply, and expected foreign and domestic money supply, with expectations formed in periods (t) and (t-1). Here, we need to specify the stochastic process for domestic money supply. Assume that the domestic money supply under flexible exchange rates is set according to

$$M_t = m(\phi_{t-1}) + v_t, \qquad (IV.32)$$

where ϕ_{t-1} contains all the relevant lagged exogeneous variables useful in forecasting the domestic money supply. Domestic economic agents would then forecast future domestic money supply with equation (IV.32), and the chain rule of forecasting again enables us to express expected domestic money supply as a function of distributed lag of the same variables as in ϕ_{t-1}.

Substituting the solutions to forecasting foreign and domestic money supply for expected foreign and domestic money supply, and equation (IV.29) and (IV.32) for M_t and $\overset{*}{M}_t$ respectively into equation (IV.24), we obtain

$$Q_{t,flex} = g(\phi_{t-1}, \overset{*}{\phi}_{t-1}) + V_{1t}. \qquad (IV.33)$$

Rationality in this case implies that when there is a change in the foreign money rule, but no change in the domestic money supply rule, the coefficients of the variables of $\overset{*}{\phi}_{t-1}$ should be unequal before and after this change in the foreign monetary regime, while the coefficients of ϕ_{t-1} should be invariant. On the other hand, when there is a change in the domestic money supply rule but no change in the foreign money supply rule, the coefficients of ϕ_{t-1} should be unequal while the coefficients of $\overset{*}{\phi}_{t-1}$ should be invariant before and after this change in the domestic monetary regime.

To be more specific, the empirical test involves the following: First, we estimate empirical counterparts to equations (IV.29) and (IV.32) to determine the most likely break point for each of the money rules using the Quandt's procedure. Second, we choose a sample period where only one money rule has undergone a shift. For simplicity, suppose it is the domestic money supply rule that has undergone a shift in this sample period. We then estimate equation (IV.33) and use the Chow test to determine the equality of the coefficients of ϕ_{t-1} before and after this shift in the domestic money supply rule. Finally, equation (IV.33) is estimated a second time to determine the equality of the coefficients of $\overset{*}{\phi}_{t-1}$ before and after this shift in the domestic money supply rule. We can accept the hypothesis of rational expectations only if the coefficients of ϕ_{t-1} are unequal while the coefficients of $\overset{*}{\phi}_{t-1}$ are equal before and after this change in the domestic money supply rule.

In describing the above test for rationality, we have implicitly assumed that the variables in ϕ_{t-1} and $\overset{*}{\phi}_{t-1}$ are mutually exclusive. However, it is possible that ϕ_{t-1} contains some of the variables as in $\overset{*}{\phi}_{t-1}$. In particular, suppose $\overset{*}{\phi}_{t-1}$ contains only lagged $\overset{*}{M}_t$, while ϕ_{t-1} contains both lagged M_t and $\overset{*}{M}_t$. In this case, $Q_{t,flex}$ is a function of lagged M_t and $\overset{*}{M}_t$. When there is a shift in the domestic money supply rule, but no change in the foreign money supply rule, <u>all</u> the parameters in $Q_{t,flex}$ should be unequal before and after this change in the domestic money supply rule, yet this is consistent with rational expectations. On the other hand, when there is a change in the foreign money supply rule, it is also possible that the parameters associated with lagged M_t^* in the domestic money supply rule also change but there is no change in the parameters associated with M_t. In this case, we would expect the parameters in $Q_{t,flex}$ associated with lagged M_t^* to be unequal before and after this change in the foreign money supply rule, but those parameters associated with lagged M_t in the domestic price equation to be invariant.

Our test for rationality depends very crucially on our ability to determine a breakpoint for the foreign or the domestic money rule. If no breakpoint can be determined, we need another testing procedure for rationality. To illustrate the alternative testing procedure, consider the domestic price level equation for the flexible exchange rate period. This equation should contain the same explanatory variables as the domestic and the foreign money supply equations. Hence, we can test for rationality by testing whether the same variables in the domestic money supply equation and the foreign money supply equation are also included in the domestic price level equation for the flexible exchange rate period. Similarly, the domestic price level equation for the fixed exchange rate period should contain only the same explanatory variables as the foreign money supply equation. In particular, domestic money supply variables should not be included in this equation, and this is a testable implication. Note that the test is to determine whether any relevant variables that are useful in determining the foreign and domestic money rules are omitted from the domestic price level equations. It is possible that the domestic price level is determined by other additional variables that are not useful in determining the foreign and domestic money supply. In this case, it should be clear that this will not invalidate our tests for rationality.

D. Concluding Remarks

In this chapter, we developed an empirical counterpart to the theoretical model developed in chapter three. In the next chapter, we will conduct the empirical tests described in this chapter using Canada as the small domestic economy and United States as the rest of the world.

The choice of United States as the rest of the world is governed by the availability of data since it is not possible to obtain monthly seasonally unadjusted data for world money supply, interst rates, and price levels, although seasonally adjusted yearly data are available for these variables from the International Financial Statistics. One may also argue that since United States is the single most important trading partner of Canada, United States monetary policy will probably have a very important impact on the economic activities of Canada. Furthermore, since domestic economic agents probably will not have detailed information about world money supply, interest rates, and price level, it is reasonable to believe that they will use United States money supply, interest rates and price level as proxies for the same world variables. And, if the movements of United States money supply, interest rates, and price level are positively correlated with the movements of world money supply, interest rates, and price level, we may view using monthly United States observation of these variables as "instruments" for the unobservable monthly world observation in our econometric work.

Finally, since the data are obtained from two different official government sources, there are several problems associated with them. In particular, the procedure for deseasonalizing the data may both be inadequate and different for the two official government sources. Hence, before conducting the empirical tests, the next chapter will begin with a discussion of the data problems and ways in which these problems may be overcome.

Footnotes - Chapter IV

1. For a discussion of the reduced form equations similar to that obtained in this chapter, see Sargent and Wallace (1973).

2. See Sargent (1979), pp. 206-8.

3. It should be noted that it is the summation of revisions of expectations for periods $i = 1, 2, 3, \ldots, \infty$ that appears in the reduced form foreign real output equation. However, for any arbitrary period (j), the revision in expectations is a function of random shocks, hence, the sum of the revisions of expectations for periods $i = 1, 2, 3, \ldots, \infty$ is the sum of random shocks.

4. Of course in this case, the information set of the economic agents would include the structure of the rest of the world.

5. Leiderman (1979) arrived at similar result with a different framework.

6. Whether this difference in the error terms can be attributed to our assumption of rational expectations or not cannot be determined, unless we postulate another expectations formation scheme and compare the property of the error terms obtained with rational expectations assumption. It is clear that if these error terms have the same property with alternate assumption of expectations formation scheme, then we cannot attribute the difference in error terms to the rational expectations assumption. We will not pursue our inquiry along this line in this dissertation.

7. See Barro (1978, 1979), Barro and Rush (1980).

8. See Grossman (1979).

9. See Leiderman (1980b).

10. See Lucas (1973).

11. See Sargent (1976a).

12. See Neftci and Sargent (1978).

13. See Wold (1973).

14. This implies that economic agents forecast the relative price of domestic to foreign goods by forecasting future foreign money supply.

Note that even though we have expectations forming in periods (t) and (t-1), equations (IV.24) is still valid. An example would perhaps make this clear. Let the reduced form solution to y_t be given as

$$y_t = a_1 \sum_{i=1}^{\infty} {}_t x_{t+i}^{e} + a_2 \sum_{i=0}^{\infty} {}_{t-1}x_{t+i}^{e} + u_{1t} \tag{A1}$$

where y_t is any arbitrary endogeneous variable and x_t is any arbitrary exogeneous variable. Further, assume x_t evolves according to the following first order autoregressive process,

$$x_t = dx_{t-1} + u_{2t}, \tag{A2}$$

then,

$${}_t x_{t+j}^{e} = \sum_{i=1}^{j} d^{1+i} x_{t-1}, \text{ and,} \tag{A3}$$

$${}_{t-1}x_{t+j} = \sum_{i=0}^{j} d^{1+i} x_{t-1}. \tag{A4}$$

Substituting (A3) and (A4) into (A1), we have

$$y_t = \sum_{j=0}^{\infty} v_j x_{t-1}, \text{ where}$$

$$v_0 = a_2 d,$$

$$v_j = (a_1 + a_2)d^{1+j}, \text{ for } j = 1, 2, 3, \ldots, \infty.$$

Hence, equation (al) can be expressed as a general function of x_{t-1}.

15. See For example Barro (1978).

16. See Saidi and Barro (unpublished).

17. Wogin (1980).

18. See Sargent (1976a).

19. See Ashley, Granger, and Schmalensee (1980).

20. See Neftci and Sargent (1978).

21. See Neftci (unpublished).

22. See Lucas (1973).

23. See McCallum (1979a).

24. See Weber (1981).

25. See Hansen and Sargent (1980).

26. See Quandt (1958).

27. See Chow (1960).

CHAPTER V

EMPIRICAL RESULTS

A. Introduction

This chapter presents the empirical results of the testing of the hypotheses developed in chapter IV and is divided into five sections. Section B gives a general discussion of the data used and the problems associated with the data. Section C presents the empirical results of testing for neutrality and rationality with the domestic real output equation under both the flexible and the fixed exchange rate periods. Section D presents the empirical results of testing for rationality with the domestic price level equation, and section E concludes the chapter with a summary of the empirical results.

B. The Data

The data used in the empirical work are monthly data from Canada and United States. Since the data are obtained from different sources, possible bias and error may be introduced into the regression results because of different seasonal adjustment procedure or incomplete seasonal adjustment.[1] To guard against such possible biases, it is desirable to obtain seasonally unadjusted time series and remove the seasonality from these time series with a procedure common to all the series. Therefore, we obtained the following seasonally unadjusted time series: Un_t, Un^*_t, Q_t, Q^*_t, M_t, M^*_t, i_t, i^*_t, and S_t. A description of these series and their sources are contained in Appendix C. The new variables introduced here are Un_t, and Un_t^*, which are the total civilian unemployment rates for Canada and United States respectively. These two variables will be used as our "real" variables since we are unable to obtain seasonally unadjusted time series for real GNP or the index of industrial production to use as the measure of real output.

To adjust for seasonality in our time series, we follow the procedure suggested by Sims[2] and used by Sargent.[3] We first take the Fourier transform of each series and examine the periodograms. Of the nine series, Un_t, Un_t^*, M_t, and M_t^*, exhibit pronounced seasonal peaks around the seasonal frequency and its harmonics. No seasonal pattern for the remaining five series was discernable. To obtain the seasonally adjusted series, the Fourier coefficients of each series are set to zero in a band of length $(\pi/54)$ at the seasonal frequency and its harmonics, and the inverse transform is taken.[4] This procedure completely removes the power at the seasonal frequency and its harmonics. The seasonally adjusted time series are then used for our estimation in the next section.

C. Empirical Results with the Unemployment Equation

In this section, we present the empirical results of testing for neutrality with the unemployment equation using the Granger causality methodology discussed in chapter IV.[5] We first present the direct Granger causality test for whether the various causal candidates are significant determinants of the Canadian unemployment rate. We next present the Granger causality test proposed by Ashley, Granger, and Schmalensee[6] by comparing the forecasting performance of the univariate Canadian unemployment rate equation with the performance of the equations where various causal candidates are included. Our sample contains observations for 1953-1979 and is divided into three subsamples: 1953-1962, 1963-1970, and 1971-1979. The first and third subsamples correspond to the flexible exchange rate periods while the second subsample corresponds to the fixed exchange rate period. For each exchange rate period, the sample observations are further divided into insample and postsample observations. The insample observations are used for estimating and conducting the direct Granger causality tests while the postsample observations are used for forecasting. We feel that about 60 observations are needed to obtain adequate estimates of the equations, hence 30 observations from the first flexible exchange rate period 1953-1962, 20 observations from the fixed exchange rate period 1963-1970, and 25 observations from the second flexible exchange rate period are left for forecasting purpose.

To conduct the direct Granger test, for each exchange rate period, we estimate the following two equations for each causal candidate or group of causal candidates:

$$Un_t = \sum_{i=1}^{n_1} f_{1i} Un_{t-i} + \sum_{j=1}^{n_2} f_{2j} G_{t-j} + e_t, \qquad (V.1)$$

and

$$Un_t = \sum_{i=1}^{n_1} f_{3i} Un_{t-i} + e_t', \qquad (V.2)$$

where G_t may be any causal candidate or group of causal candidates. The direct Granger test that G_t does not cause Un_t is a test that $f_{2j} = 0$ for $j = 1, 2, 3, \ldots, n_2$. Our test for neutrality, or equivalently, the natural rate hypothesis is a test that the causal candidates do not Granger cause Un_t.

For each exchange rate period, we first obtain the "best" univariate Canadian unemployment equation. This is done by choosing the equation with the smallest standard error of regression, and the residuals from that regression are then examined to determine if they are free of serial correlation. The test for serial correlation used in the text is the "Q-statistic" proposed by Box and Pierce,[7] and it is calculated as

$$Q = n \sum_{i=1}^{m} r_i^2, \qquad \text{(V.3)}$$

where n is the number of observations and the r_i, $i = 1, 2, \ldots, m$, are the residual autocorrelations. This statistic is distributed as chi-squared with m−q degrees of freedom, where q is the number of parameters estimated and m = 50 for all our calculations.

Using this technique, the best univariate Canadian unemployment equations are 12 lags for the period 1953–1962, 8 lags for the period 1963–1970, and 2 lags for the period 1971–1979.[8] For the bivariate regressions, we use six lags each of the causal variables. For the multivariate regressions, we include alternatively six lags of (i) the foreign and domestic money supply, (ii) the foreign and domestic price level, (iii) the foreign and domestic interest rate, (iv) three lags each of all the domestic variables and the exchange rate, (v) three lags each of all the foreign variables and the exchange rate, and (vi) two lags each of all the variables, foreign and domestic. The number of lagged causal variables is decreased as more causal candidates are included because in several cases we could not have enough degrees of freedom to conduct the direct Granger tests otherwise.

All regressions are estimated by ordinary least squares, and all variables, except the foreign and domestic interest rates, are in logs. Since the direct Granger test is a time series test which requires that the time series be stationary, we also include a constant and a linear time trend with each regression. Only summary statistics are presented in the text since the estimates of the regressions are of no great interest to us.

Table 1 presents the summary statistics for the univariate Canadian unemployment rate equation for each of the exchange rate periods. The values of the Q-statistics indicate that the residuals from each regression do not depart significantly from white noise. Tables 2, 3, and 4 present the summary statistics of the bivariate and multivariate regression results together with the results of the direct Granger tests for each of the exchange rate period. The results in tables 2, 3, and 4 clearly suggest that for all exchange rate periods, Canadian unemployment rate is econometrically exogeneous with respect to all the causal variables considered. In particular, none of the causal candidates, either singly or as a group, contribute

Table 1

Summary Statistics for Univariate Canadian Unemployment Rate Equations

Period	Equation	SSE[a]	SER[b]	$\bar{R}^2$	Q-Statistic[c]	F-Statistic[d]
1953-1962	Un_t = f(12 lagged Un_t)	.159	.055	.961	22.3 (36, 51.0)	125.
1963-1970	Un_t = f(8 lagged Un_t)	.206	.062	.844	18.2 (40, 55.8)	39.9
1971-1979	Un_t = f(2 lagged Un_t)	.083	.036	.946	27.1 (46, 62.8)	384.

[a]Sum of squared errors.

[b]Standard error of regression.

[c]Box-Pierce statistic for serial correlation of the residuals. The first number in the parenthesis is the degrees of freedom, the second number is the critical Chi-squared value at .05 level of significance.

[d]F-Statistic that the coefficients of the independent variables are not significant.

Table 2

Summary Statistics For Bivariate and Multivariate Models, 1953-1962

Regression of $Un_t = \sum_{i=1}^{12} f_{1i} Un_{t-i} + \sum_{j=1}^{n_2} f_{2j} G_{t-j} + f_{1o} + f_{2o}(t) + e_t$

Causal Variable (G)	SSE[a]	SER[b]	Q-Statistic[c]	$\bar{R}^2$	F-Statistic[d]
(2.A) 6 lags of Un_t^*	.145	.056	23.9	.960	.191
(2.B) 6 lags of Q_t	.141	.056	21.1	.961	.248
(2.C) 6 lags of Q_t^*	.149	.057	19.2	.959	.137
(2.D) 6 lags of M_t	.142	.056	25.1	.961	.237
(2.E) 6 lags of M_t^*	.121	.051	30.2	.967	.632
(2.F) 6 lags of i_t	.145	.056	29.5	.960	.193
(2.G) 6 lags of i_t^*	.136	.054	22.0	.963	.340
(2.H) 6 lags of S_t	.130	.053	25.3	.964	.447
(2.I) 6 lags each of Q_t, and Q_t^*	.129	.057	21.1	.959	.115
(2.J) 6 lags each of M_t, and M_t^*	.105	.051	32.4	.967	.259
(2.K) 6 lags each of i_t, and i_t^*	.124	.056	21.2	.961	.140

[a]Sum of squared errors.

[b]Standard error of regression.

[c]Box-Pierce statistic for serial correlation of the residuals.

[d]F-statistic that the coefficients of all the causal variables are insignificant.

Table 2 (continued)

Causal Variable (Y)	SSE[a]	SER[b]	Q-Statistic[c]	$\bar{R}^2$	F-Statistic[d]
(2.L) 3 lags each of Q_t, M_t, i_t, S_t	.132	.057	25.9	.958	.104
(2.M) 3 lags each of Q^*_t, M^*_t, i^*_t, Un^*_t, S_t	.087	.049	24.8	.970	.168
(2.N) 2 lags each of all variables	.079	.047	25.6	.972	.127

[a]Sum of squared errors.

[b]Standard error of regression.

[c]Box-Pierce statistic for serial correlation of the residuals.

[d]F-statistic that the coefficients of all the causal variables are insignificant.

Table 3

Summary Statistics For Bivariate and Multivariate Models, 1963-1970

Regression of $Un_t = \sum_{i=1}^{8} f_{1i} Un_{t-i} + \sum_{j=1}^{n_2} f_{2j} G_{t-j} + f_{1o} + f_{2o}(t) + e_t$

Causal Variable (G)	SSE[a]	SER[b]	Q-Statistic[c]	$\bar{R}^2$	F-Statistic[d]
(3.A) 6 lags of Un^*_t	.206	.062	17.7	.848	.359
(3.B) 6 lags of Q_t	.187	.062	19.7	.841	.243
(3.C) 6 lags of Q^*_t	.193	.063	16.2	.836	.158
(3.D) 6 lags of M_t	.195	.064	20.5	.834	.137
(3.E) 6 lags of M^*_t	.186	.062	20.5	.842	.260
(3.F) 6 lags of i_t	.195	.064	18.8	.834	.136
(3.G) 6 lags of i^*_t	.193	.064	16.0	.835	.153
(3.H) 6 lags of S_t	.190	.063	15.4	.838	.198
(3.I) 6 lags each of Q_t, and Q^*_t	.169	.063	18.0	.836	.146
(3.J) 6 lags each of M_t, and M^*_t	.168	.063	18.0	.836	.152
(3.K) 6 lags each of i_t, and i^*_t	.182	.066	15.2	.824	.091

[a]Sum of squared errors.

[b]Standard error of regression.

[c]Box-Pierce statistic for serial correlation of the residuals.

[d]F-statistic that the coefficients of all the causal variables are insignificant.

Table 3 (continued)

Causal Variable (Y)	SSE[a]	SER[b]	Q-Statistic[c]	$\bar{R}^2$	F-Statistic[d]
(3.L) 3 lags each of Q_t, M_t, i_t, S_t	.183	.066	15.2	.824	.087
(3.M) 3 lags each of Q^*_t, M^*_t, i^*_t, Un^*_t, S_t	.165	.065	18.6	.828	.083
(3.N) 2 lags each of all variables	.156	.064	21.6	.833	.081

[a]Sum of squared errors.

[b]Standard error of regression.

[c]Box-Pierce statistic for serial correlation of the residuals.

[d]F-statistic that the coefficients of all the causal variables are insignificant.

Table 4

Summary Statistics For Bivariate and Multivariate Models, 1971-1979

$$\text{Regression of } Un_t = \sum_{i=1}^{2} f_{1i} Un_{t-i} + \sum_{j=1}^{n_2} f_{2j} G_{t-j} + f_{1o} + f_{2o}(t) + e_t$$

Causal Variable (G)	SSE[a]	SER[b]	Q-Statistic[c]	$\bar{R}^2$	F-Statistic[d]
(4.A) 6 lags of Un_t^*	.075	.036	28.1	.946	.409
(4.B) 6 lags of Q_t	.076	.036	31.9	.946	.395
(4.C) 6 lags of Q_t^*	.073	.036	34.5	.947	.523
(4.D) 6 lags of M_t	.069	.035	27.0	.950	.785
(4.E) 6 lags of M_t^*	.063	.033	29.6	.955	1.26
(4.F) 6 lags of i_t	.064	.034	29.7	.954	1.15
(4.G) 6 lags of i_t^*	.063	.033	26.1	.954	1.21
(4.H) 6 lags of S_t	.078	.037	28.1	.944	.287
(4.I) 4 lags each of[e] Q_t, and Q_t^*	.074	.037	42.2	.945	.332
(4.J) 6 lags each of M_t, and M_t^*	.518	.031	25.2	.958	.865
(4.K) 6 lags each of i_t, and i_t^*	.051	.032	25.5	.959	.884

[a]Sum of squared errors.

[b]Standard error of regression.

[c]Box-Pierce statistic for serial correlation of the residuals.

[d]F-statistic that the coefficients of all the causal variables are insignificant.

[e]4 lags each of Q_t, and Q_t^* are used here since the Q-statistic indicates the presence of serial correlation with 6 lags of Q_t, and Q_t^*

Table 4 (continued)

Causal Variable (Y)	SSE[a]	SER[b]	Q-Statistic[c]	$\bar{R}^2$	F-Statistic[d]
(4.L) 3 lags each of Q_t, M_t, i_t, S_t	.057	.034	27.8	.954	.646
(4.M) 3 lags each of Q^*_t, M^*_t, i^*_t, Un^*_t, S_t	.054	.034	37.9	.954	.515
(4.N) 2 lags each of all variables	.044	.031	27.7	.962	.733

[a]Sum of squared errors.

[b]Standard error of regression.

[c]Box-Pierce statistic for serial correlation of the residuals.

[d]F-statistic that the coefficients of all the causal variables are insignificant.

significantly to explaining variations in Canadian unemployment rate once lagged Canadian unemployment rates are included.

Tables 5, 6, and 7 report summary statistics on testing for causality with the Ashley, Granger and Schmalensee methodology. Estimates for $\hat{a}$ and $\hat{b}$ are only included in those cases where the MSEs of the bivariate and multivariate regressions are smaller than the univariate Canadian rate equation. Correction for first order serial correlation has been applied where the Durbin-Watson statistic indicates serial correlation.

Table 5 pertains to the first flexible exchange rate period (1953-1962). From the table, it is clear that with the exception of equations (5.A), (5.C), the MSEs of postsample one-step-ahead forecasts of the bivariate and multivariate models are larger than the MSE of the univariate Canadian unemployment rate equation. Hence, they cannot be judged to be improvements over the univariate Canadian unemployment rate model. For equation (5.A) in table 5, there is a 3% reduction in MSE when six lagged foreign unemployment rates are included. The F-statistic to test the null hypothesis of no causality (H_0: $\hat{a} \geq 0$ and $\hat{b} \leq 0$) against the alternative hypothesis of no causality is 1.83, which is not significant at the .05 level of significance.[9] Thus, we cannot reject the null hypothesis of no causality.

For equation (5.C) in table 5, there is a 34% reduction in MSE by the inclusion of six lagged values of Q_t^*. The estimate $\hat{a}$ is significant but of the wrong sign (in the sense that it does not satisfy H_0) and $\hat{b}$ is not significant but is of the right sign.[10] When

Table 5**

Summary Statistics for Postsample Forecasting, 1953-1962

Dependent Variable: Un_t

	Independent Variables	MSE[a]	$\bar{e}$[b]	$s^2(e)$[c]	$\hat{a}$	$\hat{b}$	rho	$\bar{R}^2$	D-W
	12 lagged Un_t	.293	-.030	.209					
(5.A)	12 lagged Un_t 6 lagged Un^*_t	.285	-.027	.219	-.002 (1.24)	.030 (1.40)	-.559 (3.63)[d]	.033	2.05
(5.B)	12 lagged Un_t 6 lagged Q_t	.334	.040	.181					
(5.C)	12 lagged Un_t 6 lagged Q^*_t	.193	-.006	.196	-.026 (11.8)[d]	-.0007 (.079)	.654 (4.65)[d]	.000	2.13
(5.D)	12 lagged Un_t 6 lagged M_t	.352	-.338	.246					
(5.E)	12 lagged Un_t 6 lagged M^*_t	.311	-.026	.254					
(5.F)	12 lagged Un_t 6 lagged i_t	.305	.037	.191					
(5.G)	12 lagged Un_t 6 lagged i^*_t	.416	.040	.263					
(5.H)	12 lagged Un_t 6 lagged S_t	1.57	.093	.727					
(5.I)	12 lagged Un_t 6 lagged Q_t 6 lagged Q^*_t	.426	.051	.171					
(5.J)	12 lagged Un_t 6 lagged M_t 6 lagged M^*_t	.313	-.026	.256					

Table 5 (continued)

	Independent Variables	MSE[a]	$\bar{e}$[b]	$s^2(e)$[c]
(5.K)	12 lagged Un_t 6 lagged i_t 6 lagged i^*_t	.578	.052	.322
(5.L)	12 lagged Un_t 3 lags each of Q_t, M_t, i_t, S_t	.969	.079	.353
(5.M)	12 lagged Un_t 3 lags each of Q^*_t, M^*_t, i^*_t Un^*_t, S_t	11.7	.317	1.73
(5.N)	12 lagged Un_t 2 lags each of all variables	9.74	.284	1.74

**The absolute value of the t-statistics are in parentheses

[a]Mean-squared-error X 100.

[b]The mean of one-step-ahead forecast error.

[c]The sample variance of one-step-ahead error X 100.

[d]Significant at the .05 level.

the regression is reestimated with $\hat{a}$ set to zero, the F-statistic to test the null hypothesis of no causality (H_0: $\hat{a} = 0$ and $\hat{b} \leq 0$) against the alternative hypothesis of causality is 240.8, which is significant at the .05 level. Hence, the null hypothesis of no causality can be rejected.

These results indicate that the foreign price level Granger causes the Canadian unemployment rate and that the reduction in MSE comes entirely from $\bar{e}_2^2 < \bar{e}_1^2$. It is difficult to reconcile the result of equation (5.C) in table 5 with that of equation (2.C) in table 2. One may argue that the F-value for the insample Granger causality test is biased toward zero by the inclusion of too many insignificant variables. On the other hand, comparing postsample forecasting errors may very well be a more powerful test for detecting causal relationship between variables.

Although the result of equation (5.C) indicates that the foreign price level Granger causes the Canadian unemployment rate, it is consistent with "weak" neutrality as formulated in our model in chapter IV. According to the weak version of the natural rate hypothesis, it is possible for the foreign price level to have a systematic effect on the domestic real output to the extent that these changes in the foreign price level alter the relative price of domestic to foreign goods.

Table 6 presents the results for the fixed exchange rate period, 1963-1970. The MSEs of equations (6.C) and (6.E) are both 7% smaller

Table 6**

Summary Statistics for Postsample Forecasting, 1963-1970

Dependent Variable: Un_t

	Independent Variables	MSE[a]	$\bar{e}$[b]	$s^2(e)$[c]	$\hat{a}$	$\hat{b}$	rho	$\bar{R}^2$	D-W
	8 lagged Un_t	.208	.020	.177					
(6.A)	8 lagged Un_t 6 lagged Un^*_t	.301	-.027	.242					
(6.B)	8 lagged Un_t 6 lagged Q_t	.211	-.017	.192					
(6.C)	8 lagged Un_t 6 lagged Q^*_t	.193	-.025	.138	-.005 (.278)	.660 (.923)		.000	2.07
(6.D)	8 lagged Un_t 6 lagged M_t	.375	.036	.259					
(6.E)	8 lagged Un_t 6 lagged M^*_t	.194	.006	.200	.014 (3.08)[d]	-.031 (.584)		.000	1.82
(6.F)	8 lagged Un_t 6 lagged i_t	.309	.024	.267					
(6.G)	8 lagged Un_t 6 lagged i^*_t	.246	.027	.184					
(6.H)	8 lagged Un_t 6 lagged S_t	1.34	.070	.885					
(6.I)	8 lagged Un_t 6 lagged Q_t 6 lagged Q^*_t	2.01	.110	.844					
(6.J)	8 lagged Un_t 6 lagged M_t 6 lagged M^*_t	.475	.029	.414					

Table 6 (continued)

	Independent Variables	MSE[a]	$\bar{e}$[b]	$s^2(e)$[c]
(6.K)	8 lagged Un_t 6 lagged i_t 6 lagged i^*_t	.386	.037	.261
(6.L)	8 lagged Un_t 3 lags each of Q_t, M_t, i_t, S_t	1.20	.031	1.16
(6.M)	8 lagged Un_t 3 lags each of Q^*_t, M^*_t, i^*_t Un^*_t, S_t	2.21	-.120	.922
(6.N)	8 lagged Un_t 2 lags each of all variables	10.6	-.242	5.00

**The absolute value of the t-statistics are in parentheses

[a]Mean-squared-error X 100.

[b]The mean of one-step-ahead forecast error.

[c]The sample variance of one-step-ahead error X 100.

[d]Significant at the .05 level.

than the MSE of the univariate Canadian unemployment rate equation while the rest have larger MSEs.

For equation (6.C) in table 6, both the estimates $\hat{a}'$ and $\hat{b}'$ are not significant. However, $\hat{b}'$ is of the wrong sign (again in the sense that it does not satisfy H_0). When equation (6.C) is reestimated with $\hat{b}'$ set to zero, the F-statistic to test the null hypothesis of no causality (H_0: $\hat{a}' \leq 0$ and $\hat{b}' = 0$) against the alternative hypothesis of causality is .426, which is not significant at the .05 level. Therefore, we cannot reject the null hypothesis of no causality.

For equation (6.E) in table 6, the estimate $\hat{a}$ is significant but of the wrong sign (since $\hat{a}$ does not satisfy H_0). The F-statistic to test the null hypothesis of no causality (H_0: $\hat{a} = 0$ and $\hat{b} \leq 0$) against the alternative hypothesis of causality is 4.74, which is significant at the .05 level. It appears that we can reject the null hypothesis of no causality in favor of causality from the foreign money supply to the Canadian unemployment rate. However, this result is at odds with that obtained from equation (6.C). Both equations (6.C) and (6.E) indicated a 7% reduction in MSE, yet there is no evidence of causality with equation (6.C). This anomaly may be explained by observing that equation (6.E) has a larger variance of forecast errors than the univariate Canadian unemployment rate equation, but its mean forecast error is smaller. Hence, the indicated reduction in MSE from equation (6.E) resulted entirely from $\bar{e}_2^2 < \bar{e}_1^2$. However, the evidence of causality from the difference in the mean errors alone is not entirely convincing, since a small difference in the mean errors may result in a large difference in the squared mean errors. We would also want to have a smaller variance of forecast errors as additional evidence of causal relationship. In other words, a smaller MSE is a necessary but not sufficient condition for causality. A necessary condition would require that both the squared mean error and the variance of forecast errors be smaller than the univariate model. Given these considerations, together with the result of equation (6.C) in table 6 and the result from the goodness of fit test (equation (3.E) in table 3), we conclude that equation (6.E) is not a significant improvement over the univariate Canadian unemployment rate model. Accordingly, we retain the null hypothesis of no causality.

Table 7 pertains to the second flexible exchange rate period 1971-1979. When compared to the results of the direct Granger causality tests in table 4 for the same period, three exceptions may be noted. First, equation (7.A) in table 7 indicates that there is a reduction of 47% in MSE when six lagged Un_t^* are included. The F-statistic to test the null hypothesis of no causality (H_0: $\hat{a} \geq 0$ and $\hat{b} \leq 0$) against the alternative hypothesis of causality is 2.88 which is significant at the .05 level. Therefore, the null hypothesis is rejected. This suggests that the foreign unemployment rate Granger causes the Canadian unemployment rate. However, it does not destroy our neutrality proposition since by neutrality, we mean that the real variable be exogeneous with respect to the nominal variables.

Equation (7.G) in table 7 indicates a 16% in MSE when six lagged values of i_t^* are included. Similarly, equation (7.L) in table 7

Table 7**

Summary Statistics for Postsample Forecasting, 1971-1979

Dependent Variable: Un_t

	Independent Variables	MSE^a	$\bar{e}^b$	$s^2(e)^c$	$\hat{a}$	$\hat{b}$	rho	$\bar{R}^2$	D-W
	2 lagged Un_t	.156	-.024	.102					
(7.A)	2 lagged Un_t 6 lagged Un^*_t	.083	-.00005	.086	-.025 (29.5)[d]	.035 (2.05)[d]	-.606 (3.81)[d]	.122	2.30
(7.B)	2 lagged Un_t 6 lagged Q_t	.187	-.019	.157					
(7.C)	2 lagged Un_t 6 lagged Q^*_t	.173	-.012	.164					
(7.D)	2 lagged Un_t 6 lagged M_t	.384	-.050	.139					
(7.E)	2 lagged Un_t 6 lagged M^*_t	.335	.037	.204					
(7.F)	2 lagged Un_t 6 lagged i_t	.170	.003	.177					
(7.G)	2 lagged Un_t 6 lagged i^*_t	.131	.022	.085	-.002 (.163)	.678 (.883)		.000	1.63
(7.H)	2 lagged Un_t 6 lagged S_t	.212	-.028	.141					
(7.I)	2 lagged Un_t 6 lagged Q_t 6 lagged Q^*_t	.383	-.019	.359					
(7.J)	2 lagged Un_t 6 lagged M_t 6 lagged M^*_t	.305	.008	.311					

Table 7 (continued)

	Independent Variables	MSE[a]	$\bar{e}^{b}$	$s^2(e)^c$	$\hat{a}$	$\hat{b}$	rho	$\bar{R}^2$	D-W
(7.K)	2 lagged Un_t 6 lagged i_t 6 lagged i^*_t	.309	.037	.181					
(7.L)	2 lagged Un_t 3 lags each of Q_t, M_t, i_t, S_t	.135	.023	.085	-.001 (.092)	.193 (.523)		.000	1.98
(7.M)	2 lagged Un_t 3 lags each of Q^*_t, M^*_t, i^*_t Un^*_t, S_t	.202	.011	.198					
(7.N)	2 lagged Un_t 2 lags each of all variables	1.27	.092	.443					

**The absolute value of the t-statistics are in parentheses

[a]Mean-squared-error X 100.

[b]The mean of one-step-ahead forecast error.

[c]The sample variance of one-step-ahead error X 100.

[d]Significant at the .05 level.

indicates a 13% reduction in MSE when three lagged values each of the domestic variables and the exchange rate are included. The F-statistic to test the null hypothesis of no causality (H_0: $\hat{a}' \geq 0$ and $\hat{b}' \leq 0$) against the alternative hypothesis of causality is .403 for equation (7.G) and .141 for equation (7.L) respectively. Both the F-statistics are not significant at the .05 level, therefore, we cannot reject the null hypothesis of no causality for both these two cases.

Our results with comparing postsample forecasting performance indicate that for the first flexible exchange rate period, we can reject the strict version of the natural rate hypothesis in favor of the weak open economy version of the natural rate hypothesis. This is because the foreign price level appears to Granger cause the Canadian unemployment rate. For the fixed exchange rate period, the results are consistent with those of the goodness of fit tests. We have no evidence of causality from any of the variables considered to the Canadian unemployment rate. For the second flexible exchange rate period, the foreign unemployment rate appears to Granger cause the Canadian unemployment rate. This is not inconsistent with either the strict or the weak version of the natural rate hypothesis since the natural rate hypothesis does not preclude real variables from affecting the Canadian unemployment rate. Since the only nominal variable that appears to Granger cause the Canadian unemployment rate is the foreign price level in the first flexible exchange rate period, this can hardly be considered as overwhelming evidence against the strict version of the natural rate hypothesis.

D. Test of Rational Expectations with the Domestic Price Equation

The results in the last section indicate that the Canadian unemployment rate is not exogeneous with respect to the foreign price level. The next task is to test for rationality of expectations. To test for rationality with the procedure described in chapter IV requires that in the equation

$$Un_t = \sum_{i=1}^{n_1} f_{1i} Un_{t-i} + \sum_{j=1}^{n_2} f_{2j} P^*_{t-j} + e_t$$

$f_{2j} \neq 0$. Unfortunately, the results from the direct Granger test indicate that this was not the case. Hence, there is no meaningful way to test for rationality with the domestic unemployment rate equation. Instead, we will attempt to test for rationality of expectations with the domestic price level equation.

The analysis in chapter IV indicates that for the flexible exchange rate period, the reduced form equation for the domestic price level is a function of the same variables as the foreign and domestic money rules. For the fixed exchange rate period, the reduced form equation for the domestic price level is a function only of the same variables as the foreign money rule. Hence, rationality of expectations implies that for the flexible exchange rate period, when there is a change in the process determining the foreign and domestic money rules, the process determining the domestic price level should also change. Similarly, for the fixed exchange rate period, when there is a change in the process determining the foreign money rule, then the domestic price level equation should also change.

In the event that we are unable to detect any change in the foreign or domestic money rules, we will adopt the alternate strategy of testing for the rationality of expectations by examining whether the correct variables are included in the domestic price level equation as in the foreign and/or domestic money rules. In particular, we will examine whether the domestic price level equation for the fixed exchange rate period is a function of the same variables as the foreign money rule, while the domestic price level equation for the flexible exchange rate period is a function of the same variables as the foreign and domestic money rules. The first task therefore is to determine the foreign and domestic money rules for the two exchange rate periods.

The strategy we follow in determining the foreign and domestic money rules is similar to that of last section. We first choose the best univariate model for the foreign and domestic money rules for the two exchange rate periods based on the standard error of regression and white noise residuals by estimating the following equations:

$$M_t^* = d_o + \sum_{i=1}^{k_1} d_i M_{t-i}^* + v_t^*, \qquad \text{(V.10)}$$

and

$$M_t = e_o + \sum_{j=1}^{k_2} e_j M_{t-j} + v_t, \qquad \text{(V.11)}$$

where we have also included a linear time trend with each regression. Table 8 presents the summary statistics for estimating the foreign and domestic money rules over the entire subsample period for the two exchange rate regimes.

Table 8

Summary Statistics For Estimating Foreign and Domestic Money Rules

Period	Equation	SSE[a]	SER[b]	Q-Statistic[c]	$\bar{R}^2$	F-Statistic[d]
1953-1962	$M^*_t = f(14 \text{ lagged } M^*_t)$	.001	.004	28.3	.991	712.
1953-1962	$M_t = f(16 \text{ lagged } M_t)$	.011	.012	39.9	.977	241.
1963-1970	$M_t = f(8 \text{ lagged } M_t)$	.001	.004	28.8	.998	6058.
1971-1979	$M^*_t = f(13 \text{ lagged } M^*_t)$	.002	.006	22.4	.998	3370.
1971-1979	$M_t = f(9 \text{ lagged } M_t)$	.012	.012	22.9	.997	2704.

[a]Sum of squared errors.

[b]Standard error of regression.

[c]Box-Pierce statistic for serial correlation.

[d]F-statistic that the coefficients of all the independent variables are not significant.

We next add several lags of other variables that may be important in determining the money rules to the best univariate foreign and domestic money rule. For the foreign money rule for both the exchange rate periods, we tried several lags of foreign unemployment rate, and for the domestic money rule, we tried several lags of domestic unemployment rate and several lags of foreign money. The reason for including the unemployment rate is to attempt to account for any countercyclical behavior on the part of the foreign and domestic monetary authorities. The inclusion of lagged foreign money in the domestic money rule may be justified on the ground that when pruchasing power parity is not a necessary condition in the model, changes in the foreign money supply can be expected to affect the domestic economy through their impact on the relative price of domestic to foreign goods. Hence, the domestic monetary authority may be expected to offset any undesirable foreign influence with monetary policy. Table 9 presents summary statistics of testing whether other variables are important determinants of the foreign and domestic money rules other than their own lagged values. The bivariate models presented in table 9 are chosen because they have the smallest standard error of regression.

The results in table 9 indicate that no other variables are significant determinants of the foreign and domestic money rules once their own lagged values are included. We, therefore, take the specifications in table 8 as being adequate representations of the foreign and domestic money rules for the two exchange rate periods.

Table 9

F-Tests That No Other Variables Are Significant Determinants of Foreign and Domestic Money Rules

				Stability Tests	
Period	Equation	Null Hypothesis	F-Statistic	Chow	Farley-Hinich
1953-1962	M^*_t = f(14 lagged M^*_t, 9 lagged Un^*_t)	9 lagged Un^*_t = 0	1.03		
	M_t = f(16 lagged M_t, 9 lagged Un_t)	9 lagged Un_t = 0	1.01		
	M_t = f(16 lagged M_t, 16 lagged M^*_t)	16 lagged M^*_t = 0	1.34		
	M^*_t = f(14 lagged M^*_t)			.309	.368
	M_t = f(16 lagged M_t)			.409	.563
1963-1970	M^*_t = f(8 lagged M^*_t 10 lagged Un^*_t)	10 lagged Un^*_t = 0	1.03		
	M^*_t = f(8 lagged M^*_t)			.817	.563
1971-1979	M^*_t = f(13 lagged M^*_t 9 lagged Un^*_t)	9 lagged Un^*_t = 0	.448		
	M_t = f(9 lagged M_t, 12 lagged Un_t)	12 lagged Un_t = 0	.771		
	M_t = f(9 lagged M_t, 13 lagged M^*_t)	13 lagged M^*_t = 0	.475		
	M^*_t = f(13 lagged M^*_t)			.365	.767
	M_t = f(9 lagged M_t)			.590	.492

Table 9 also reports the statistics pertaining to the structural stability of the foreign and domestic money rules for each of the exchange period. Since we cannot be sure that there is a shift in the domestic or foreign money rule during the sample period, we cannot use the Quandt's procedure to find the most likely breakpoint. Instead, we apply a very general test proposed by Farley and Hinich[11] to determine if there is a discrete change in the slope coefficients of the foreign and domestic money rules during each of the sample period. Furthermore, for each of the sample period, we have also divided the sample observations into two equal halves and use the Chow test to determine the equality of the coefficients between the two halves. From the results, we are unable to reject the null hypothesis that the slope coefficients of each of the money rule are equal for each of the sample period.[12]

Since we are unable to detect a breakpoint in any of the money rule, the break point test for rationality of expectations as described in chapter IV is precluded. However, we will adopt an alternate strategy in testing for rationality of expectations by examining whether the relevant variables are included in determining the domestic price level equations.

Since the foreign and domestic money rules are given by equations (V.10) and (V.11), the reduced form equation for the domestic price level equation under the flexible exchange rate period can be written as

$$Q_{t,flex} = g_{3o} + \sum_{i=1}^{q_1} g_{3i} M_{t-i} + \sum_{j=1}^{q_2} g_{4j} M^{*}_{t-j} + \sum_{k=1}^{6} g_{5k} Un_{t-k} + V_{1t}, \tag{V.12}$$

where $q_1 = 16$, $q_2 = 14$ for the first flexible exchange rate period, and $q_1 = 9$, $q_2 = 13$ for the second flexible exchange rate period. Similarly, the domestic price level equation for the fixed exchange rate period is

$$Q_{t,fixed} = g'_{6o} + \sum_{i=1}^{8} g'_{6i} M^{*}_{t-i} + \sum_{j=1}^{6} g'_{7j} Un_{t-j} + V_{2t}. \tag{V.13}$$

Note that by substituting equations (V.10) and (V.11), we have eliminated M^{*}_t and M_t from equation (V.12) and M^{*}_t from equation (V.13). Therefore, the test for the inclusion of the relevant variables is formally equivalent to the direct Granger causality test. We have also included six lagged values of the Canadian unemployment rate in each of the domestic price level equations. This is because these variables appear as exogeneous variables in the reduced form equation for the Canadian unemployment rate model and therefore should also appear as exogeneous variables in the reduced form equations for the domestic price level.

Since preliminary results indicate the presence of serial correlation, we examined the pattern of residual correlations and found that a simple first order autoregressive filter was sufficient to remove the serial correlation. We, therefore, estimated a first order autoregressive filter for each of the sample periods, and transformed all the variables in the same sample period with the estimated filter for that period. The results of estimating equations (V.12) and (V.13) and testing for various restrictions are presented in table 10.

The results in table 10 are rather disappointing, none of the restrictions that we have tested can be rejected at any reasonable level of significance. Furthermore, when we included six lagged values of the domestic money supply in the domestic price level equation for the fixed exchange rate, the result indicates that they are not significant. However, in view of all the results in table 10, this cannot be interpreted as evidence that domestic economic agents are rational.[13] One possible explanation for these results is that all the explanatory variables are highly correlated, and hence their individual effects cannot be distinguished. In view of the results, no firm conclusion could be reached from our tests for rationality of expectations with the domestic price level equations.

E. Conclusion

This chapter presents the empirical results of testing for neutrality with the domestic unemployment rate equation and rationality with the domestic price level equation. To test for neutrality, we employed the direct Granger test and the Granger test proposed by Ashley, Granger, and Schmalensee. To analyze rationality, we have to abandon the break point test since we are unable to detect any break point in the foreign and domestic money rules. We tested instead for inclusion or exclusion of relevant variables in the domestic price level equation. The following is a summary of our results.

Table 10

Empirical Results with the Domestic Price Level Equations

Period		F-Statistic[a]	lagged $M^*_t = 0$	lagged $M_t = 0$	lagged $Un_t = 0$
1953-1962	$Q_t = f(14 \text{ lagged } M^*_t, 16 \text{ lagged } M_t, 6 \text{ lagged } Un_t)$	2.07	.503	.298	.212
1963-1970	$Q_t = f(8 \text{ lagged } M^*_t, 6 \text{ lagged } Un_t)$	54.8	.725	.553	.111
1971-1979	$Q_t = f(13 \text{ lagged } M^*_t, 9 \text{ lagged } M_t, 6 \text{ lagged } Un_t)$	39.6	.197	.213	.215

[a]F-statistic that all the coefficients of the independent variables of the domestic price level equations are not significant.

The filters used to transform the variables are:

1953-1962: (1-.952L)
1963-1970: (1-.805L)
1971-1979: (1-.897L)

(1) The direct Granger causality tests indicate that Canadian unemployment rate is exogeneous with respect to all the causal candidates for all the exchange rate periods. However, the results are not consistent with the postsample Granger tests.

(2) For the period 1953-1962, we find that Canadian unemployment rate is not exogeneous with respect to the foreign price level. However, this result is not inconsistent with weak neutrality since foreign price changes may affect the relative price of foreign to domestic good and hence, Canadian unemployment even though this movement in the relative price may be fully anticipated.

(3) For the fixed exchange rate period 1963-1970, the postsample Granger tests are consistent with the direct Granger tests. We find no evidence that the Canadian unemployment rate is Granger caused by any of the variables considered.

(4) For the period 1971-1979, the Canadian unemployment rate is Granger caused by the foreign unemployment rate. However, it does no damage to our neutrality proposition since we are primarily concerned with the exogeneity of Canadian unemployment rate with respect to the nominal variables in the model. We also have no strong evidence that the foreign interest rate and the domestic variables as a group Granger cause the Canadian unemployment rate.

(5) The results with testing for rationality with the domestic price level equations are inconclusive for two reasons. First, we are unable to find a breakpoint in any of the money rules considered, hence we cannot carry out the breakpoint test. Second, because of the high

degree of correlation among variables, we are unable to determine the best domestic price equation for each exchange rate period. Hence, our test for alternate specifications of the domestic price level equation is also inconclusive.

Footnotes - Chapter V

1. For a discussion of the problems associated with incomplete seasonal adjustment, see Sims (1974).

2. Sims (1974).

3. Sargent (1976a).

4. The frequency bands as fraction of cycle that are zeroed out are: .0802, .0833, .0864; .1636, .1667, .1698; .2469, .2500, .2531; .3302, .3333, .3364; .4136, .4167, .4198; .4969, .5000. The degrees of freedom are adjusted according to the method suggested by Sims. See Sims (1974).

5. Since our domestic real output equation does not involve lagged foreign or domestic money innovations, it is therefore not subject to the criticism that it is observational equivalent to a real output equation which has Keynesian policy implications. For a discussion of the observational equivalence problem and ways to distinguish a classical real output equation from a Keynesian real output equation, see McCallum (1979) and Weber (1981).

6. See Ashley, Granger, and Schmalensee (1980).

7. The Box-Pierce "Q-Statistic" is a test for overall whiteness of the residuals. It is not intended to detect specific departure from whiteness of each residual autocorrelation.

8. Our procedure for choosing the best models will not necessarily lead to the same structural form for the same variable for the different exchange rate periods. For the purpose of testing for neutrality with the Granger methodology, the stochastic structure is not of great concern to us since we are approximating whatever the stochastic process with an autoregressive process. What is of concern is whether nominal variables Granger cause the real variable regardless of the structure the real variable happens to take.

9. As noted by Ashley, Granger, and Schmalensee, this F-test is in essence four tailed and it does not take into account the signs of the estimated coefficients. Ashley, Granger, and Schmalensee suggested performing a F-test but report a significance level equal to half that obtained from the table. For our purpose and throughout this chapter we will report a 5% level of significance.

10. Note that since $s_1^2 > s_2^2$, the estimate $\hat{b}$ should be positive. However, it is negative possibly due to the serial correlation correction.

11. The test proposed by Farley and Hinich involves modeling the coefficient in a linear model as a deterministic function of a constant and a linear time trend,

$$X_t = a_o + b_t Y_t + e_t,$$

where $b_t = b_o + ct$. Substituting into the above equation, we obtain

$$X_t = a_o + b_o Y_t + cY_t(t) + e_t$$

The test for a shifting slope coefficient is a test that $c \neq 0$. This method can be extended in a straight forward way to testing slope coefficients of a multiple regression including the constant. See Farley and Hinich (1970).

12. Our results need to be interpreted with care. As noted by Farley, Hinich, and McGuire (1975), shifts in the parameters are difficult to detect, unless the sample is large or the shift is great. Otherwise, the tests for a shift in the slope coefficients are not very powerful.

13. We have also estimated the domestic price level equations with six lagged values of the domestic price level included. When the six lagged values of the domestic price level are included, the F-statistics that all the variables except the lagged values of the domestic price level are insignificant are $F = .391$ for the first flexible exchange rate period; $F = .390$ for the fixed exchange rate period; and $F = .215$ for the second flexible exchange rate period. These results again reject our test for rationality of expectations. One possible explanation is that the contemporary value of domestic and foreign money supply are important determinants of the domestic price level. Hence, when lagged values of the domestic price level are included, the effect of lagged domestic and foreign money variables are fully captured in the lagged domestic price variables, and therefore, the inclusion of lagged values of domestic and foreign money variables do not contribute significantly to explaining the domestic price level once the lagged values of the domestic price level are included. Feige and Pearce (1976), Brillembourg and Khan (1979) also obtained similar results for the United States.

CHAPTER VI

CONCLUSION

The vast majority of the empirical tests of the rational expectations - natural rate hypothesis have been conducted in the context of a closed economy model. This dissertation extended the empirical test to a small open economy model under both the fixed and the flexible exchange rate regimes. For the two exchange rate regimes, we investigated the effects of domestic and foreign monetary policies on the behavior of the domestic real output and the domestic price level. The empirical methodology we employed was the Granger causality test. However, in addition to the conventional goodness of fit test, we also employed the Granger causality test based on a comparison of the forecasting performance of the univariate model against bivariate and multivariate models. This chapter gives a summary of the empirical results and concludes the dissertation with suggestions for future research.

The most important result that emerged from the theoretical model was that for a small open economy, anticipated changes in the relative price of domestic to foreign goods may have a systematic effect on the domestic real output if purchasing power parity does not hold. This result was obtained for both the flexible and the fixed exchange rate periods. Following Leiderman, we called this the weak open economy version of the natural rate hypothesis. However, the reduced form solution for the relative price of domestic to foreign goods is in terms of only the foreign money supply. Hence, the time path of the domestic real output is independent of domestic monetary policies, but systematic foreign monetary policies may have a real effect on the domestic real output by influencing the relative price of domestic to foreign goods.

Using monthly data from Canada and the United States to represent the domestic small open economy and the rest of the world respectively, the above two implications from the theoretical model were tested with bivariate and multivariate causal models employing the two methods of Granger causality test. The empirical results with the Canadian unemployment rate instead of the real output are as follows.

Based on the goodness of fit criterion, we found that for all the exchange rate periods, the domestic unemployment rate was neutral with respect to all the causal candidates considered. However, these results were not collaborated by the results based on a comparison of the forecasting performance of the univariate model against the bivariate and multivariate models. Based on the latter criterion, we found that for the first flexible exchange rate period, the Canadian unemployment rate was Granger caused by the foreign price level. However, this result is not inconsistent with the weak open economy version of the natural rate hypothesis. For the fixed exchange rate period, the two methods of Granger causality test yield the same result. We found no evidence that the Canadian unemployment rate was Granger caused by any of the causal candidates considered. For the second flexible exchange rate period, the domestic unemployment rate was Granger caused by the foreign unemployment rate. This result did no damage to the natural rate hypothesis, however, since the natural rate hypothesis does not preclude real variables from having a systematic effect on the domestic unemployment rate.

Our overall results were consistent with our hypotheses. As expected, we found no evidence that the domestic unemployment rate was Granger caused by any of the domestic variables. Furthermore, the non-neutrality of the domestic unemployment rate with respect to the foreign price level is not inconsistent with our theory. The non-neutrality of the domestic unemployment rate with respect to the foreign unemployment rate was unexpected, but this is not inconsistent with the natural rate hypothesis.

Our tests for rationality of expectations with the domestic price level equations were rather disappointing. We had to abandon the breakpoint test since we were unable to detect any breakpoint for the foreign or domestic money supply rules for all the exchange rate periods. Furthermore, for each exchange rate period, we were unable to determine whether the relevant variables were included in the domestic price level equations as implied by the rational expectations assumption.

Our empirical results indicated that the Canadian unemployment rate was affected by systematic foreign monetary policies while domestic monetary policies were ineffective. Our empirical results also suggested that as far as the domestic unemployment rate was concerned, the Canadian economy was more insulated from systematic foreign monetary policies and changes in the foreign unemployment rate with a fixed exchange rate regime rather than with a flexible exchange rate regime. Unfortunately, our analysis did not provide a convenient framework for policy discussion since we have not explicitly accounted for the effects of unanticipated domestic and foreign price changes or unanticipated domestic and foreign monetary expansions on the domestic unemployment rate. Hence, the question of the optimal exchange rate regime or the proper role for domestic monetary policies cannot be addressed unless we have a richer framework for analysis.[1]

Future empirical research in testing for neutrality in an open economy may take the form of replicating the results of this dissertation with the experience of other small open economies. The empirical model may also be estimated as a vector autoregressive model to serve as an additional check on the robustness of the results obtained from single equation estimation.

With respect to testing for rationality of expectations with the domestic price level equation, it is apparent that more work needs to be done on the specification of the domestic price level equation. The method of Hansen and Sargent[2] may also prove to be a promising avenue for empirical research on the rationality of expectations.

Footnotes - Chapter VI

1. Cox (1980) and Weber (1980) have considered the question of the optimal exchange rate regime and the optimal domestic monetary policies when the domestic real output is subjected to only unanticipated shocks.

2. See Hansen and Sargent (1980).

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Appendix A

This appendix provides detailed solution method to obtain the reduced form solutions to P_t^*, i_t^*, and y_t^*. The complete model for the rest of the world is given by the following three equations:

(I-S)

$$y_t^* = a_1^*({}_tP_{t+1}^{*e} - P_t^* - i_t^*) + u_{1t}^*, \tag{A.1}$$

(L-M)

$$M_t^* = P_t^* - b_1^* i_t^* + b_2^* y_t^* - b_3^*({}_tP_{t+1}^{*e} - P_t^* - i_t^*) + u_{2t}^*, \tag{A.2}$$

(Real Output)

$$y_t^* = c_1^*(P_t^* - {}_{t-1}P_t^{*e}) + u_{3t}^*. \tag{A.3}$$

To solve for the reduced form solutions for P_t^*, i_t^*, and y_t^*, it is convenient to rewrite the system of equations in matrix form,

$$\begin{bmatrix} 1 & a_1^* & a_1^* \\ b_2^* & -(b_1^* - b_3^*) & 1 + b_3^* \\ 1 & 0 & -c_1^* \end{bmatrix} \begin{bmatrix} y_t^* \\ i_t^* \\ P_t^* \end{bmatrix} = \begin{bmatrix} a_1^* {}_tP_{t+1}^{*e} + u_{1t}^* \\ M_t^* + b_3^* {}_tP_{t+1}^{*e} + u_{2t}^* \\ -c_1^* {}_{t-1}P_t^{*e} + u_{3t}^* \end{bmatrix} .$$

Solving for P_t^*, we obtain

$$P_t^* = [a_1^* M_t^* + (a_1^* b_3^* + (b_1^* - b_3^*) a_1^*)_t P_{t+1}^{*e} + (a_1^* b_2^* c_1^* + (b_1^* - b_3^*) c_1^*)_{t-1} P_t^{*e}$$

$$+ e_{1t}^*] / |D^*|,$$

where

$$|D^*| = a_1^*(1 + b_2^* c_1^* + b_1^*) + c_1^*(b_1^* - b_3^*),$$

$$e_{1t}^* = a_1^* u_{2t}^* - (b_1^* - b_3^*) u_{1t}^* - (b_1^* - b_3^* + a_1^* b_2^*) u_{3t}^*.$$

Rewriting P_t^* in simpler form, we have

$$P_t^* = J_1^* M_t^* + J_2^*({}_t P_{t+1}^{*e}) + J_3^*({}_{t-1} P_t^{*e}) + e_{1t}^{*\prime} \qquad \text{(A.4)}$$

where

$$J_1^* = \frac{a_1^*}{|D^*|},$$

$$J_2^* = \frac{(a_1^* b_3^* + (b_1^* - b_3^*) a_1^*)}{|D^*|},$$

$$e_{1t}^{*\prime} = \frac{e_{1t}^*}{|D^*|}.$$

To solve for ${}_{t-1}P_t^{*e}$ in equation (A.4), we take expectation on both sides of (A.4) conditional on the information available at period (t-1). Upon rearranging terms, we obtain

$$ {}_{t-1}P_t^{*e} = \left(\frac{J_1^*}{1-J_3^*}\right) {}_{t-1}M_t^{*e} + \left(\frac{J_2^*}{1-J_3^*}\right) {}_{t-1}P_{t+1}^{*e}, \tag{A.5} $$

where we have made use of the condition that $E_{t-1}({}_tP_{t+1}^{*e}) = {}_{t-1}P_{t+1}^{*e}$. Increment equation (A.5) by j = 1, 2, 3, ... ∞ and substitute repeatedly into equation (A.5), the solution to ${}_{t-1}P_t^{*e}$ is

$$ {}_{t-1}P_t^{*e} = \left(\frac{J_1^*}{1-J_3^*}\right) \sum_{i=1}^{\infty} \left(\frac{J_2^*}{1-J_3^*}\right)^{i-1} ({}_{t-1}M_{t+i-1}^{*e}), \tag{A.6} $$

since

$$ \lim_{j\to\infty} \left(\frac{J_2^*}{1-J_3^*}\right)^j {}_{t-1}P_{t+j}^{*e} \to 0, $$

$$ \text{for } \left(\frac{J_2^*}{1-J_3^*}\right) = \frac{b_1^*}{1+b_1^*}, \text{ and } 0 < \frac{b_1^*}{1+b_1^*} < 1. $$

To solve for ${}_tP_{t+1}^{*e}$, first increment equation (A.4) by 1 and take expectation conditional on the information at period (t). Upon rearranging terms, we obtain

$$ {}_tP_{t+1}^{*e} = \left(\frac{J_1^*}{1-J_3^*}\right) {}_tM_{t+1}^{*e} + \left(\frac{J_2^*}{1-J_3^*}\right) {}_tP_{t+2}^{*e}. \tag{A.7} $$

Increment equation (A.7) by j = 2, 3, ... , ∞, and substitute repeatedly into equation (A.7), the solution to ${}_tP^{*e}_{t+1}$ is

$$ {}_tP^{*e}_{t+1} = \left(\frac{J_1^*}{1-J_3^*}\right) \sum_{i=1}^{\infty} \left(\frac{J_2^*}{1-J_3^*}\right)^{i-1} ({}_tM^{*e}_{t+i}), \tag{A.8} $$

where again

$$ \lim_{j\to\infty} \left(\frac{J_2^*}{1-J_3^*}\right)^{j-1} ({}_tP^{*e}_{t+j}) \to 0. $$

Substituting equations (A.6) and (A.8) into (A.4), the reduced form solution for P_t^* is

$$ P_t^* = J_1^* M_t^* + \sum_{i=1}^{\infty} R_i^* [J_2^*({}_tM^{*e}_{t+i}) + J_3^*({}_{t-1}M^{*e}_{t+i-1})] + e^{*'}_{1t}, \tag{A.9} $$

where

$$ R_i^* = \left(\frac{J_1^*}{J_2^*}\right)\left(\frac{J_2^*}{1-J_3^*}\right)^i. \tag{A.10} $$

Similarly, solving equations (A.1) to (A.3) simultaneously for i_t^*, we obtain

$$ i_t^* = K_1^* M_t^* + K_2^*({}_tP^{*e}_{t+1}) + K_3^*({}_{t-1}P^{*e}_t) + e^{*'}_{2t}, \tag{A.11} $$

where

$$K_1^* = \frac{-(c_1^*+a_1^*)}{|D^*|},$$

$$K_2^* = \frac{a_1^* c_1^* b_2^* + a_1^* - c_1^* b_3^*}{|D^*|},$$

$$K_3^* = \frac{c_1^* + b_3^* c_1^* - a_1^* b_2^* c_1^*}{|D^*|},$$

$$e_{2t}^{*\prime} = \frac{(c_1^* b_2^* + 1 + b_3^*)u_{1t}^* - (c_1^* + a_1^*)u_{2t}^* + (a_1^* b_2^* - 1 - b_3^*)u_{3t}^*}{|D^*|}.$$

Substituting equations (A.6) and (A.8) into (A.11), the reduced form solution for i_t^* is

$$i_t^* = K_1^* M_t^* + \sum_{i=1}^{\infty} R_i^* [K_2^* ({}_tM_{t+i}^{*e}) + K_3^* ({}_{t-1}M_{t+i-1}^{*e})] + e_{2t}^{*\prime}. \qquad (A.12)$$

Finally, subtract equation (A.6) from equation (A.9), and substitute the resulting expression into equation (A.3), the reduced form solution for y_t^* is

$$y_t^* = c_1^* [J_1^* (M_t^* - {}_{t-1}M_t^{*e}) + J_2^* \sum_{i=1}^{\infty} R_i^* ({}_tM_{t+1}^{*e} - {}_{t-1}M_{t+1}^{*e}) + e_{1t}^{*\prime}] + u_{2t}^*.$$

Appendix B

This appendix provides detailed solution method to the reduced form model of the small economy. The model for the small economy is given by the following five equations:

(I-S)

$$y_t = a_1(S_t+P^*_t-P_t) + a_2({}_tP^e_{t+1}-P_t-i_t) + a_3({}_tS^e_{t+1}+{}_tP^{*e}_{t+1}-P_t-i_t)+u_{1t}, \quad (B.1)$$

(L-M)

$$M_t = Q_t - b_2({}_tP^e_{t+1}-P_t-i_t) - b_3({}_tS^e_{t+1}+{}_tP^{*e}_{t+1}-P_t-i_t) + b_4y_t - b_5i_t + u_{2t}, \quad (B.2)$$

(Domestic Real Output)

$$y_t = c_1(P_t-{}_{t-1}P^e_t) + c_2(1-\theta)({}_{t-1}Z^e_t) + u_{3t}, \quad (B.3)$$

(Interest Rate Parity)

$$i_t = i^*_t + ({}_tS^e_{t+1}-S_t), \quad (B.4)$$

(Domestic Price Index)

$$Q_t = \theta P_t + (1-\theta)(S_t+P^*_t), \quad (B.5)$$

where u_{1t}, u_{2t}, and u_{3t} are assumed to be white noise processes.

We will first solve the model assuming flexible exchange rates. To do so, it is convenient to substitute equation (B.4) into equations (B.1) and (B.2), substitute equation (B.5) into (B.2), and rewrite the system of equations in matrix form,

$$\begin{bmatrix} 1 & -a' & a \\ b_4 & b_1' & b_2' \\ 1 & 0 & -c_1 \end{bmatrix} \begin{bmatrix} y_t \\ S_t \\ P_t \end{bmatrix}$$

$$= \begin{bmatrix} a_1 P_t^* + a_2({}_tP_{t+1}^e) + a_3({}_tP_{t+1}^{*e}) - a_2({}_tS_{t+1}^e) - (a_2+a_3)i_t^* + u_{1t} \\ (1-\theta)P_t^* + b_2({}_tP_{t+1}^e) - (b_2-b_5)\,{}_tS_{t+1}^e + b_3({}_tP_{t+1}^{*e}) \\ \quad - (b_2+b_3-b_5)i_t^* + M_t + u_{2t} \\ (c_2(1-\theta)-c_1)\,{}_{t-1}Z_t^e - c_1({}_{t-1}S_t^e + {}_{t-1}P_t^{*e}) + u_{3t} \end{bmatrix}$$

Solving for P_t, we obtain

$$P_t = J_1 M_t + J_2({}_tP_{t+1}^e) + J_3({}_tP_{t+1}^{*e}) + J_4({}_tS_{t+1}^e) + J_5 P_t^*$$

$$+ J_6 i_t^* + J_7({}_{t-1}S_t^e + {}_{t-1}P_t^{*e}) + J_8({}_{t-1}Z_t^e) + e_{1t}', \qquad \text{(B.6)}$$

where

$$J_1 = \frac{-a'}{|D|},$$

$$J_2 = \frac{-(a' b_3 + b_1' a_2)}{|D|},$$

$$J_3 = \frac{-(a' b_3 + b_1' a_3)}{|D|},$$

$$J_4 = \frac{a'(b_2 - b_5) + b_1' a_2}{|D|},$$

$$J_5 = \frac{(a' b_1 - b_1' a_1)}{|D|},$$

$$J_6 = \frac{a'(b_2 + b_3 - b_5) + b_1'(a_2 + a_3)}{|D|},$$

$$J_7 = \frac{-c_1(b_1' + a' b_4)}{|D|},$$

$$J_8 = \frac{(c_2(1-\theta) - c_1)(b_1' + a' b_4)}{|D|}$$

$$a' = (a_1 + a_2 + a_3),$$

$$b_1' = (1 - \theta - b_2 - b_3 + b_5),$$

$$b_2' = (\theta + b_2 + b_3),$$

$$e'_{1t} = \frac{-(a'u_{2t}+b'_1u_{1t}-(b'_1+a'b_4)u_{3t})}{|D|},$$

$$|D| = -(b'_2a'+a'b'_1) - c_1(b'_1+a'b_4).$$

Next, we need to obtain solutions for ${}_tP^e_{t+1}$, ${}_tS^e_{t+1}$, ${}_{t-1}P^e_t$, ${}_{t-1}S^e_t$, and ${}_{t-1}Z^e_t$. First, substitute equation (B.4) into equation (B.1), then add and subtract $a_2({}_tP^{*e}_{t+1})$, $a_2P^*_t$, and $a_3P^*_t$ from (B.1), we obtain,

$$y_t = -a'Z_t + a_2({}_tZ^e_{t+1}) - (a_2+a_3)(i^*_t-{}_tP^{*e}_{t+1}-P^*_t) + u_{1t}. \qquad (B.7)$$

Update equations (B.3) and (B.7) by $j \geq 1$, and take expectations conditional on information available at period (t), we have

$${}_ty^e_{t+j} = -a'({}_tZ^e_{t+j}) + a_2({}_tZ^e_{t+j+1}) - (a_2+a_3)({}_ti^{*e}_{t+j}-{}_tP^{*e}_{t+j+1}+{}_tP^{*e}_{t+1}), \qquad (B.8)$$

$${}_ty^e_{t+j} = c_2(1-\theta){}_tZ^e_{t+j}. \qquad (B.9)$$

Equating equations (B.8) and (B.9) and rearranging terms, we obtain

$${}_tZ^e_{t+j} = \left(\frac{a_2}{A}\right){}_tZ^e_{t+j+1} - \frac{a_2+a_3}{A}\left({}_ti^{*e}_{t+j}-{}_tP^{*e}_{t+j+1}+{}_tP^{*e}_{t+j}\right), \qquad (B.10)$$

where

$$A = a' + c_2(1-\theta).$$

The foreward solution for ${}_tZ^e_{t+j}$ is

$$ {}_tZ^e_{t+j} = -\left(\frac{a_2+a_3}{A}\right) \sum_{k=0}^{\infty} \left(\frac{a_2}{A}\right)^k ({}_ti^{*e}_{t+j+k} - {}_tP^{*e}_{t+j+k+1} + {}_tP^{*e}_{t+j+k}), \qquad (B.11) $$

after imposing the terminal condition that

$$ \lim_{n\to\infty} \left(\frac{a_2}{A}\right)^n ({}_tZ^e_{t+j+n}) \to 0,\ n \geq 1. $$

Next, substitute (B.3), (B.4), and (B.5) into (B.2) and update (B.2) by j and take expectations on both sides conditional on the information at period (t), we obtain

$$ \begin{aligned} {}_tM^e_{t+j} = {}_tP^e_{t+j} - (1-\theta)\,{}_tZ^e_{t+j} &- b_2({}_tP^e_{t+j+1} - {}_tP^e_{t+j} - {}_ti^{*e}_{t+j} \\ &- {}_tS^e_{t+j+1} + {}_tS^e_{t+j}) - b_3({}_tS^e_{t+j+1} + {}_tP^{*e}_{t+j+1} - {}_tP^e_{t+j} \\ &- {}_ti^{*e}_{t+j} - {}_tS^e_{t+j+1} + {}_tS^e_{t+j}) + b_4c_2(1-\theta)\,{}_tZ^e_{g+j} \\ &- b_5({}_ti^{*e}_{t+j} + {}_tS^e_{t+j+1} - {}_tS^e_{t+j}). \end{aligned} \qquad (B.12) $$

Add and subtract $b_2({}_tP^{*e}_{t+j+1})$, $b_2({}_tP^{*e}_{t+j})$, and $b_3({}_tP^{*e}_{t+j})$, and rearranging terms, (B.12) can be written as

$$ \begin{aligned} {}_tS^e_{t+j} = B_1({}_tM^e_{t+j}) &+ B_2({}_tZ^e_{t+j}) + B_3({}_tZ^e_{t+j+1}) + B_4({}_tP^{*e}_{t+j+1}) \\ &+ B_5({}_tP^{*e}_{t+j}) + B_6({}_ti^{*e}_{t+j}) + B_7({}_tS^e_{t+j+1}), \end{aligned} \qquad (B.13) $$

where

$$B_1 = \frac{1}{1+b_5} ,$$

$$B_2 = \frac{-(\theta+b_2+b_3+b_4c_2(1-\theta))}{1+b_5} ,$$

$$B_3 = \frac{b_2}{1+b_5} ,$$

$$B_4 = \frac{b_2+b_3}{1+b_5} ,$$

$$B_5 = -\frac{1+b_2+b_3}{1+b_5} ,$$

$$B_6 = -\frac{b_2+b_3-b_5}{1+b_5} ,$$

$$B_7 = \frac{b_5}{1+b_5} .$$

Substituting for ${}_tZ^e_{t+j}$ and ${}_tZ^e_{t+j+1}$ with equation (B.11), the solution for ${}_tS^e_{t+j}$ is

$$
{}_tS^e_{t+j} = B_1 \sum_{L=0}^{\infty} (B_7)^L {}_tM^e_{t+j+L} + \Big(\frac{B_2(a_2+a_3)}{A} \sum_{k=0}^{\infty} \Big(\frac{a_2}{A}\Big)^k
$$

$$
- \frac{B(a_2+a_3)}{a_2} \sum_{k=1}^{\infty} \Big(\frac{a_2}{A}\Big)^k\Big) \sum_{L=0}^{\infty} (B_7)^L \big({}_ti^{*e}_{t+j+k+L}
$$

$$
- {}_tP^{*e}_{t+j+k+L+1} + {}_tP^{*e}_{t+j+k+L}\big) + B_4 \sum_{L=0}^{\infty} (B_7)^L {}_tP^{*e}_{t+j+L+1}
$$

$$
- B_5 \sum_{L=0}^{\infty} (B_7)^L {}_tP^{*e}_{t+j+L} - B_6 \sum_{L=0}^{\infty} (B_7)^L {}_ti^{*e}_{t+j+L}, \qquad (B.14)
$$

where

$$
\lim_{n\to\infty} (B_7)^n {}_tS^e_{t+j+n} = \lim_{n\to\infty} \Big(\frac{b_5}{1+b_5}\Big)^n {}_tS^e_{t+j+n} \to 0, \; n \geq 1.
$$

To obtain the reduced form solution for ${}_tP^e_{t+j}$, note that

$$
{}_tP^e_{t+j} = {}_tZ^e_{t+j} + {}_tS^e_{t+j} + {}_tP^{*e}_{t+j}. \qquad (B.15)
$$

Therefore, the reduced form solution for ${}_tP^e_{t+j}$ is obtained by substituting (B.11) and (B.14) into (B.15).

Now, the reduced form solution for P_t is obtained by substituting the solutions for ${}_tP^e_{t+j}$, ${}_tS^e_{t+j}$, ${}_tZ^e_{t+j}$, equations (A.9), (A.11), and the expected values for equations (A.9), (A.11), and the expected values for equations (A.9), (A.11) into equation (B.6) and noting that

${}_{t-1}({}_{t}X^{e}_{t+j})^{e} = {}_{t-1}X^{e}_{t+j}$ for any variable X. The exact solution for P_t is

$$
\begin{aligned}
P_t = {} & H_1 M_t + H_2 M^{*}_t + H_3(\Sigma)\,{}_{t}M^{e}_{t+j} + H_4(\Sigma)\,{}_{t-1}M^{e}_{t+j-1} + H_5(\Sigma)\,{}_{t}M^{*e}_{t+j} \\
& + H_6(\Sigma)\,{}_{t-1}M^{*e}_{t+j-1} + H_7(\Sigma)\,{}_{t}M^{*e}_{t+j+k} + H_8(\Sigma)\,{}_{t}M^{*e}_{t+j+k-1} \\
& + H_9(\Sigma)\,{}_{t-1}M^{*e}_{t+j+k-1} + H_{10}(\Sigma)\,{}_{t-1}M^{*e}_{t+j+k-2} + H_{11}(\Sigma)[h_1({}_{t}M^{*e}_{t+j+k+L}) \\
& + h_2({}_{t}M^{Ie}_{t+j+k+L-1}) + h_3({}_{t}M^{*e}_{t+j+k+1-2}) + h_4({}_{t-1}M^{*e}_{t+j+k+L-1}) \\
& + h_5({}_{t-1}M^{*e}_{t+j+k+L-2}) + h_6({}_{t-1}M^{*e}_{t+j+k+L-3})] + U_{1t}, \qquad \text{(B.16)}
\end{aligned}
$$

where

$$H_1 = J_1,$$

$$H_2 = (J_5 J^{*}_1 + J_6 K^{*}_1),$$

$$H_3(\Sigma) = (J_2 + J_4)\left(\frac{B_1}{B_7}\right) \sum_{j=1}^{\infty} B_7^{j},$$

$$H_4(\Sigma) = \left(\frac{J_7 B_1}{B_7}\right) \sum_{j=1}^{\infty} B_7^{j},$$

$$H_5(\Sigma) = (J_3+J_2 + J_2^*J_5 + J_6K_2^*)\left(\frac{J_1^*}{J_2^*}\right)\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j,$$

$$-\left(\frac{J_2K_1^*(a_2+a_3)}{a_2}\right)\sum_{j=1}^{\infty}\left(\frac{a_2}{A}\right)^j - \left(\frac{(J_2+J_4)K_1^*B_6}{B_7}\right)\sum_{j=1}^{\infty}B_7^j,$$

$$H_6(\Sigma) = (J_5J_3^*+J_6K_3^*+J_7)\left(\frac{J_1^*}{J_2^*}\right)\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j - \left(\frac{J_7K_1^*B_6}{B_7}\right)\sum_{j=1}^{\infty}B_7^j$$

$$+\left(\frac{J_8K_1^*(a_2+a_3)}{a_2}\right)\sum_{j=1}^{\infty}\left(\frac{a_2}{A}\right)^j,$$

$$H_7(\Sigma) = \frac{J_2+J_4}{B_7}\left(\frac{B_3K_1^*(a_2+a_3)}{a_2}\sum_{j=1}^{\infty}\left(\frac{a_2}{A}\right)^j + \frac{(B_4-K_2^*B_6)J_1^*}{J_2^*}\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j\right)\sum_{k=1}^{\infty}B_7^k$$

$$-\frac{J_2(K_2^*-1)(a_2+a_3)J_1^*}{a_2J_2^*}\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j\sum_{k=1}^{\infty}\left(\frac{a_2}{A}\right)^k,$$

$$H_8(\Sigma) = \frac{J_2+J_4}{B_7}\left(\frac{B_2K_1^*(a_2+a_3)}{a_2}\sum_{j=1}^{\infty}\left(\frac{a_2}{A}\right)^j - \frac{(K_3^*B_6+B_5)J_1^*}{J_2^*}\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j\right)\sum_{k=1}^{\infty}B_7^k$$

$$-\frac{J_2(K_3^*+1)(a_2+a_3)J_1^*}{a_2J_2^*}\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j\sum_{k=1}^{\infty}\left(\frac{a_2}{A}\right)^k,$$

$$H_9(\Sigma) = \frac{J_7}{B_7}\left(\frac{B_3 K_1^*(a_2+a_3)}{a_2}\sum_{j=1}^{\infty}\left(\frac{a_2}{A}\right)^j + \frac{(B_4-K_2^*B_6)J_1^*}{J_2^*}\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j\right)\sum_{k=1}^{\infty}B_7^k$$

$$- \frac{J_8(K_2^*-1)(a_2+a_3)J_1^*}{a_2}\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j\sum_{k=1}^{\infty}\left(\frac{a_2}{A}\right),$$

$$H_{10}(\Sigma) = \frac{J_7}{B_7}\left(\frac{B_2 K_1^*(a_2+a_3)}{a_2}\sum_{j=1}^{\infty}\left(\frac{a_2}{A}\right)^j - \frac{(K_3^*B_6+B_5)J_1^*}{J_2^*}\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j\right)\sum_{k=1}^{\infty}B_7^k$$

$$- \frac{J_8(K_3^*+1)(a_2+a_3)J_1^*}{a_2J_2^*}\sum_{j=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^j\sum_{k=1}^{\infty}\left(\frac{a_2}{A}\right),$$

$$H_{11}(\Sigma) = \frac{(a_2+a_3)J_1^*}{a_2B_7J_2^*}\sum_{j=1}^{\infty}\left(\frac{a_2}{A}\right)^j\sum_{k=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^k\sum_{L=1}^{\infty}B_7^L,$$

$h_1 = -(J_2+J_4)B_3(K_2^*-1),$

$h_2 = B_2(K_2^*-1)-B_3(1+K_3^*),$

$h_3 = B_2(1+K_3^*),$

$h_4 = -J_7B_3(K_2^*-1),$

$h_5 = B_2(K_2^*-1)-B_3(1+K_3^*),$

$$h_6 = B_2(1+K_3^*),$$

$$U_{15} = J_5 e_{1t}^{*'} + J_6 e_{2t}^{*'} + e_{1t}^{'}.$$

From equation (B.16), we see that P_t is a general function of M_t, M_t^*, and their expected values with expectations formed in periods (t) and (t-1). To obtain the reduced form solution for y_t, take expectations on both sides of (B.16) conditional on the information available at period (t-1) and subtract from (B.16), we obtained

$$y_t = c_1[H_1(M_t - {}_{t-1}M_t^e) + H_2(M_t^* - {}_{t-1}M_t^{*e}) + H_3(\Sigma)({}_tM_{t+j}^e - {}_{t-1}M_{t+j}^e)$$

$$+ H_5(\Sigma)({}_tM_{t+j}^{*e} - {}_{t-1}M_{t+j}^{*e}) + H_7(\Sigma)({}_tM_{t+j+k}^{*e} - {}_{t-1}M_{t+j+k}^{*e})$$

$$+ H_8(\Sigma)({}_tM_{t+j+k-1}^{*e} - {}_{t-1}M_{t+j+k-1}^{*e})$$

$$+ H_{11}(\Sigma)(h_1({}_tM_{t+j+k+L}^{*e} - {}_{t-1}M_{t+j+k+L}^{*e})$$

$$+ h_2({}_tM_{t+j+k+L-1}^{*e} - {}_{t-1}M_{t+j+k+L-1}^{*e})$$

$$+ h_3({}_tM_{t+j+k+L-2}^{*e} - {}_{t-1}M_{t+j+k+L-2}^{*e}))]$$

$$+ c_2(1-\theta)_{t-1}Z_t^e + c_1U_{1t} + u_{3t}, \qquad \text{(B.17)}$$

where, for the flexible exchange rate, the solution for ${}_{t-1}Z_t^e$ is

$$ {}_{t-1}Z_t^e = A_1(\Sigma)_{t-1}M_{t+k}^{*e} + A_2(\Sigma)_{t-1}M_{t+k+i}^{*e}, \tag{B.18} $$

$$ A_1(\Sigma) = -\left(\frac{(a_2+a_3)K_1^*}{A}\right)\sum_{k=0}^{\infty}\left(\frac{a_2}{A}\right)^k, $$

$$ A_2(\Sigma) = -\left(\frac{a_2+a_3}{A}\right)\sum_{k=0}^{\infty}\left(\frac{a_2}{A}\right)^k\left[\left(\frac{K_2^*J_1^*}{1-J_3^*} - \frac{J_1^*}{J_2^*}\right)\sum_{i=1}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^i + \left(\frac{J_1^*(K_3^*-1)}{1-J_3^*}\right)\sum_{i=0}^{\infty}\left(\frac{J_2^*}{1-J_3^*}\right)^i\right]. $$

To obtain the reduced form solution for S_t, we first solve (B.1) to (B.5) simultaneously to obtain

$$ S_t = E_1M_t + E_2({}_tP_{t+1}^e) + E_3({}_tP_{t+1}^{*e}) + E_4({}_tS_{t+1}^e) + E_5P_t^* $$

$$ + E_6i_6^* + E_7({}_{t-1}S_t^e + {}_{t-1}P_t^{*e}) + E_8({}_{t-1}Z_t^e) + e_{2t}', \tag{B.19} $$

where

$$ E_1 = \frac{-(c_1+a')}{|D|}, $$

$$ E_2 = \frac{(c_1b_4+b_2')a_2 - (c_1+a')b_2}{|D|}, $$

$$ E_3 = \frac{(c_1b_4+b_2')a_3 - (c_1+a')b_3}{|D|}, $$

$$E_4 = \frac{(c_1+a')(b_2-b_5) - a_2(c_1b_4+b_2')}{|D|} ,$$

$$E_5 = \frac{(c_1b_4+b_2')a_1 - (c_1+a')b_1}{|D|} ,$$

$$E_6 = \frac{(c_1+a')(b_2+b_3-b_5) - (a_2+a_3)(c_1b_4+b_2')}{|D|} ,$$

$$E_7 = \frac{(b_2'-a'b_4)}{|D|} ,$$

$$E_8 = \frac{(b_2'-a'b_4)(c_2(1-\theta)-c_1)}{|D|} ,$$

$$e_{2t}' = \frac{(c_1b_4+b_2')u_{1t} - (c_1+a')u_{2t} - (b_2'-a'b_4)u_{3t}}{|D|} .$$

Substituting the solutions for ${}_tP^e_{t+j}$, ${}_tS^e_{t+j}$, ${}_tZ^e_{t+j}$, (A.9), (A.11), and the expected values for (A.9), (A.11) into (B.19), we obtain the reduced form solution for S_t as

$$S_t = G_1M_t + G_2M^*_t + G_3(\Sigma)_tM^e_{t+j} + G_4(\Sigma)_{t-1}M^e_{t+j-1} + G_5(\Sigma)_tM^{*e}_{t+j}$$

$$+ G_6(\Sigma)_{t-1}M^{*e}_{t+j-1} + G_7(\Sigma)_tM^{*e}_{t+j+k} + G_8(\Sigma)_tM^{*e}_{t+j+k-1}$$

$$+ G_9(\Sigma)_{t-1}M^{*e}_{t+j+k-1} + G_{10}(\Sigma)_{t-1}M^{*e}_{t+j+k-2} + G_{11}(\Sigma)[g_1({}_tM^{*e}_{t+j+k+L})$$

$$+ g_2({}_tM^{*e}_{t+j+k+L-1}) + g_3({}_tM^{*e}_{t+j+k+L-2}) + g_4({}_{t-1}M^{*e}_{t+j+k+L-2})$$

$$+ g_5({}_{t-1}M^{*e}_{t+j+k+L-2}) + g_6({}_{t-1}M^{*e}_{t+j+k+L-3})] + U_{2t}. \qquad \text{(B.20)}$$

where $U_{25} = E_5 e^{*'}_{1t} + E_6 e^{*'}_{2t} + e'_{2t}$.

The G_1, G_2, $G_i(\Sigma)$, $i = 3, 4, \ldots, 11$, and g_j, $j = 1, 2, \ldots, 6$ have exactly the same form as H_1, H_2, $H_i(\Sigma)$, $i = 3, 4, \ldots, 11$, and h_j, $j = 1, 2, \ldots, 6$ except with E_k's replacing J_k's, $k = 1, 2, \ldots,$ 8.

For the fixed exchange rate period, the endogeneous variables are y_t, P_t, M_t, the exogeneous variables are P^*_t, i^*_t, S_t, and their expected values. Rewriting equations (B.1) to (B.5) in matrix form for the fixed exchange rate period, we have

$$\begin{bmatrix} 1 & 0 & a' \\ b_4 & -1 & b'_2 \\ 1 & 0 & -c_1 \end{bmatrix} \begin{bmatrix} y_t \\ M_t \\ P_t \end{bmatrix}$$

$$= \begin{bmatrix} a'S_t + a_2P^*_t + a_3({}_tP^{*e}_{t+1}) - a_2({}_tS^e_{t+1}) - (a_2+a_3)i^*_t + u_{1t} \\ b'_1St + b_2({}_tP^e_{t+1}) - (b_2-b_5)_tS^e_{t+1} + b_3({}_t(P^{*e}_{t+1}) - (b_2+b_3-b_5)i^*_t+u_{2t} \\ (c_2(1-\theta)-c_1)_{t-1}P^e_t - c_2(1-\theta)({}_{t-1}S^e_t+{}_{t-1}P^{*e}_t) + u_{3t} \end{bmatrix}$$

Solving for P_t, we obtain,

$$P_t = K_1({}_tP^e_{t+1}) + K_2({}_tP^{*e}_{t+1}) + K_4P^*_t + K_5i^*_t + K_6({}_{t-1}P^e_t) + K_7({}_{t-1}P^{*e}_t)$$

$$+ K\overline{S} + e'_{3t}, \qquad \text{(B.21)}$$

where

$$K_1 = \frac{a_2}{|C|},$$

$$K_2 = \frac{a_3}{|C|},$$

$$K_3 = \frac{-a_2}{|C|},$$

$$K_4 = \frac{a_1}{|C|},$$

$$K_5 = \frac{-(a_2+a_3)}{|C|},$$

$$K_6 = \frac{-(c_2(1-\theta)-c_1)}{|C|},$$

$$K_7 = \frac{c_2(1-\theta)}{|C|},$$

$$K_8 = \frac{a'}{|C|},$$

$K = (K_3+K_7+K_8)$,

${}_{t-i}S^e_{t+j} = \overline{S}$, for any i and j by the assumption of a fixed exchange rate,

$$e'_{3t} = \frac{u_{1t}-u_{3t}}{|C|},$$

and a', b'_2 are as defined before, and

$|C| = c_1 + a'$.

The solution for ${}_{t-1}P^e_t$ is

$${}_{t-1}P^e_t = \left(\frac{K_2}{1-K_6}\right)\sum_{j=0}^{\infty}\left(\frac{K_1}{1-K_6}\right)^j {}_{t-1}P^{*e}_{t+j+1} + \left(\frac{K_4+K_7}{1-K_6}\right)\sum_{j=0}^{\infty}\left(\frac{K_1}{1-K_6}\right)^i {}_{t-1}P^{*e}_{t+j}$$

$$+ \left(\frac{K_5}{1-K_6}\right)\sum_{j=0}^{\infty}\left(\frac{K_1}{1-K_6}\right)^j {}_{t-1}i^{*e}_{t+j} + \left(\frac{K}{1-K_6}\right)\sum_{j=0}^{\infty}\left(\frac{K_1}{1-K_6}\right)^j\overline{S}, \quad (B.22)$$

after assuming that

$$\lim_{n\to\infty}\left(\frac{K_1}{1-K_6}\right)^n {}_{t-1}P^e_{t+n}\to\infty,\ n \geq 1.$$

The solution for ${}_tP^e_{t+1}$ is

$${}_tP^e_{t+1} = \left(\frac{K_2}{1-K_6}\right)\sum_{j=0}^{\infty}\left(\frac{K_1}{1-K_6}\right)^j {}_tP^{*e}_{t+j+2} + \left(\frac{K_4+K_7}{1-K_6}\right)\sum_{j=0}^{\infty}\left(\frac{K_1}{1-K_6}\right)^j {}_tP^{*e}_{t+j+1}$$

$$+ \left(\frac{K_5}{1-K_6}\right)\sum_{j=0}^{\infty}\left(\frac{K_1}{1-K_6}\right)^j {}_ti^{*e}_{t+j+1} + \left(\frac{K}{1-K_6}\right)\sum_{j=0}^{\infty}\left(\frac{K_1}{1-K_6}\right)^j\overline{S}, \quad (B.23)$$

after assuming that

$$\lim_{n\to\infty} \left(\frac{K_1}{1-K_6}\right)^{n-1} {}_tP^e_{t+n} \to \infty, \quad n \geq 2.$$

Substituting equations (B.22), (B.23), (A.9), (A.11), and the expected values for 9A.9), (A.11) into equation (B.21), we obtain the reduced form solution for P_t under the fixed exchange rate period as

$$P_t = R_1 M^*_t + R_2(\Sigma)_t M^{*e}_{t+i} + R_3(\Sigma)_{t-1} M^{*e}_{t+i-1} + R_4(\Sigma)_t M^{*e}_{t+i+j}$$

$$+ R_5(\Sigma)_{t-1} M^{*e}_{t+i+j} + R_6(\Sigma)\overline{S} + U_{3t}, \qquad \text{(B.23)}$$

where

$$R_1 = (K_5 K^*_1 + K_4 J^*_1),$$

$$R_2(\Sigma) = \left(\frac{K_2 J^*_1}{J^*_2} + K_4 J^*_1 + K_5 K^*_2 J^*_1\right) \sum_{i=1}^{\infty} \left(\frac{J^*_2}{1-J^*_3}\right)^i + (K_5 K^*_1) \sum_{i=1}^{\infty} \left(\frac{K_1}{1-K_6}\right)^j,$$

$$R_3(\Sigma) = (K_4 J^*_3 + K_5 K^*_3 + K_7)\left(\frac{J^*_1}{J^*_2}\right) \sum_{i=1}^{\infty} \left(\frac{J^*_2}{1-J^*_3}\right)^i + \left(\frac{K_6 K_5 K^*_1}{K_1}\right) \sum_{i=1}^{\infty} \left(\frac{K_1}{1-K_6}\right)^i,$$

$$R_4(\Sigma) = \left(\frac{K_1 J^*_1}{(1-K_6) J^*_2}\right) \sum_{j=0}^{\infty} \left(\frac{K_1}{1-K_6}\right)^j \Big[(K_4 + K_7 + K^*_3 K_5) \sum_{i=1}^{\infty} \left(\frac{J^*_2}{1-J^*_3}\right)^i$$

$$+ (K_2 + K_5 K^*_2) \sum_{i=2}^{\infty} \left(\frac{J^*_2}{1-J^*_3}\right)^{i-1}\Big],$$

$$R_5(\Sigma) = (\frac{K_6}{1-K_6}) \sum_{j=0}^{\infty} (\frac{K_1}{1-K_6})^j [(\frac{J_1^*}{J_2^*})(K_2K_5^*+K_2) \sum_{i=1}^{\infty} (\frac{J_2^*}{1-J_3^*})^i$$

$$+ (\frac{J_1^*}{1-J_3^*})(K_5K_3^*+K_4+K_7) \sum_{i=0}^{\infty} (\frac{J_2^*}{1-J_3^*})^i] ,$$

$$R_6(\Sigma) = (K_1+K_6)(\frac{K}{1-K_6}) \sum_{j=0}^{\infty} (\frac{K_1}{1-K_6})^i,$$

$$U_{3t} = K_4 e_{1t}^{*'} + K_5 e_{2t}^{*'} + e_{3t}^{'}.$$

To obtain the reduced form solution for y_t for the fixed exchange rate period, take expectations on both sides of (B.23) conditional on the information available at period (t-1) and subtract from (B.23). After substituting the resulting expression into (B.3), we obtain

$$y_t = c_1[R_1(M_t^* - {}_{t-1}M_t^{*e}) + R_2(\Sigma)({}_tM_{t+i}^{*e} - {}_{t-1}M_{t+i}^{*e})$$

$$+ R_4(\Sigma)({}_tM_{t+i+j}^{*e} \; {}_{t-1}M_{t+i+j}^{*e})] + c_1U_{3t} + c_2(1-\theta)_{t-1}Z_t^e + u_{3t}, \quad \text{(B.24)}$$

where for the fixed exchange rate, the solution for ${}_{t-1}Z_t^e$ is

$$ {}_{t-1}Z_t^e = Z_1(\Sigma)_{t-1}M_{t+i}^{*e} + Z_2(\Sigma)_{t-1}M_{t+i+j}^{*e} + Z_3(\Sigma)\overline{S}, \quad \text{(B.25)}$$

where

$$Z_1(\Sigma) = \left(\frac{K_1^* K_5}{1-K_6}\right) \sum_{i=0}^{\infty} \left(\frac{K_1}{1-K_6}\right)^i - \left(\frac{J_1^*}{1-J_3^*}\right) \sum_{i=0}^{\infty} \left(\frac{J_2^*}{1-J_3^*}\right)^i,$$

$$Z_2(\Sigma) = \left(\frac{J_1^*}{1-J_3^*}\right)\left(\frac{1}{1-K_6}\right) \sum_{j=0}^{\infty} \left(\frac{K_1}{1-K_6}\right)^j \left[\left(\frac{K_2(1-J_3^*)}{J_2^*} + K_5 K_2^*\right) \sum_{i=1}^{\infty} \left(\frac{J_2^*}{1-J_3^*}\right)^i\right.$$

$$\left. + (K_4 + K_7 + K_3^* K_5) \sum_{i=0}^{\infty} \left(\frac{J_2^*}{1-J_3^*}\right)^i\right],$$

$$Z_3(\Sigma) = \left(\frac{K}{1-K_6}\right) \sum_{j=0}^{\infty} \left(\frac{K_1}{1-K_6}\right)^j.$$

Note that the reduced form solution for ${}_{t-1}Z_t^e$ for the fixed exchange rate period is a general function of expected future foreign money supply, which is the same general form as the reduced form solution for ${}_{t-1}Z_t^e$ for the flexible exchange rate period.

Solving (B.1) to (B.5) simultaneously for M_t, we obtain

$$M_t = F_1({}_tP_{t+1}^e) + F_2({}_tP_{t+1}^{*e}) + F_4P_t^* + F_5i_t^* + F_6({}_{t-1}P_t^e)$$

$$+ F_7({}_{t-1}P_t^{*e}) + F'\overline{S} + e_{4t}', \qquad \text{(B.26)}$$

$$F_1 = \frac{-b_2(c_1+a')}{|C|},$$

$$F_2 = \frac{a_3(b_4c_1+b_2') - b_3(c_1+a')}{|C|},$$

$$F_3 = \frac{-a_2(b_4c_1+b_2') + (c_1+a')(b_2-b_5)}{|C|},$$

$$F_4 = \frac{a_2(b_4c_1+b_2')}{|C|},$$

$$F_5 = \frac{-(a_2+a_3)(b_4c_1+b_2') + (c_1+a')(b_2+b_3-b_5)}{|C|},$$

$$F_6 = \frac{-(b_2'-a'b_4)(c_2(1-\theta)-c_1)}{|C|},$$

$$F_7 = \frac{(b_2'-a'b_4)(c_2(1-\theta))}{|C|},$$

$$F_8 = \frac{-a'(b_4c_1+b_2')}{|C|},$$

$$F' = (F_3+F_7+F_8),$$

$$U_{4t} = \frac{(c_1b_4+b_2')u_{1t} - (c_1+a')u_{2t} - (b_2'-a'b_4)u_{3t}}{|C|},$$

$|C|$, a', b_1', b_2' are as defined before.

Substituting equations (B.22), (B.23), (A.9), (A.11), and the expected values for (A.9), (A.11) into equation (B.26), we obtain the reduced form solution for M_t under the fixed exchange rate as

$$M_t = T_1 M_t^{*} + T_2(\Sigma)_t M_{t+i}^{*e} + T_3(\Sigma)_{t-1} M_{t+i-1}^{*e} + T_4(\Sigma)_t M_{t+i+j}^{*e}$$

$$T_5(\Sigma)_{t-1} M_{t+i+j}^{*e} + T_6(\Sigma)\overline{S} + U_{4t}.$$

where $U_{4t} = F_4 e_{1t}^{*'} + F_5 e_{2t}^{*'} + e_{4t}^{'}$.

The T_1, $T_i(\Sigma)$, i = 2, 3, ..., 6 have exactly the same form as R_1, $R_j(\Sigma)$, j = 2, 3, ... 6 except with F_k's replacing K_k's, k = 1, 2, ..., 8.

Appendix C

The data series are obtained from the following sources:

<u>Domestic Price and Foreign price</u>

a. Canadian Index of Consumer Prices (1967=100)
Monthly, 1950-1979, seasonally unadjusted.
Source: Business Condition Digest.

b. United States Index of Consumer Prices (1967=100)
Monthly, 1950-1979, seasonally adjusted.
Source: Business Condition Digest.

<u>Domestic and Foreign Unemployment Rates</u>

a. Canadian Total Unemployment Rate
Monthly, 1950-1979, seasonally unadjusted.
Source: Bank of Canada Data Bank.

b. United States Total Unemployment Rate
Monthly, 1950-1979, seasonally unadjusted.
Source: Business Condition Digest.

<u>Domestic and Foreign Money Supply</u>

a. Canadian M1 (currency outside banks and Chartered Banks deposits)
Monthly, 1953-1979, seasonally unadjusted.
Source: Bank of Canada Bank.

b. United States M1 (currency outside banks and demand deposits)
Monthly, 1950-1979, seasonally unadjusted.
Source: Federal Reserve Bulletin.

<u>Domestic and Foreign Interest Rates</u>

a. Canadian Government 3-Month Bill Rate
Monthly, 1950-1979.
Source: CANSIM Data Bank.

b. United States Government 3-Month Treasury Bill Rate
Monthly, 1950-1979.
Source: Business Condition Digest.

<u>Exchange Rate</u>

a. Spot Exchange Rate, C$/US$1
Monthly average of daily closing rate, 1950-1979.
Source: Bank of Canada Data Bank.

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TESTING FOR NEUTRALITY AND RATIONALITY WITH AN OPEN-ECONOMY MODEL: THE CASE OF CANADA

by

Francis W. Ahking

(ABSTRACT)

A small open economy model which incorporated the rational expectations - natural rate hypothesis was constructed. The resulting model indicated that to the extent that foreign monetary policies may affect the relative price of domestic to foreign goods, the domestic unemployment rate was not neutral with respect to foreign monetary policies.

Using Canada as the small open economy and the United States as the rest of the world, the weak open economy version of the natural rate hypothesis was empirically tested for both the flexible and the fixed exchange rate periods. The empirical methodology employed was the Granger causality test. The results based on the goodness of fit test indicated that for all the exchange rate regimes, the Canadian unemployment rate was not Granger caused by any causal variables considered. However, the results based on a comparison of the postsample forecasting performance were different. For the first flexible exchange rate period, 1953-1962, we found that the Canadian unemployment rate was Granger caused by the foreign price level. However, this result was consistent with the weak open economy version

of the natural rate hypothesis. We also found that the foreign unemployment rate Granger caused the Canadian unemployment rate in the second flexible rate period, 1970-1979. But, this did not damage the natural rate hypothesis since the natural rate hypothesis does not preclude real variables from having a systematic effect on the Canadian unemployment rate.

The tests for rationality with the Canadian price level equations for both the flexible and the fixed exchange rate periods were inconclusive. First, we were not able to detect a breakpoint in the foreign and domestic money rules. The alternate tests of rationality which examined whether the relevant variables were included in the Canadian price level equations were also inconclusive because of a high degree of multicollinearity between the variables.