PARAMETER IDENTIFICATION AND CONTROL OF DISTRIBUTED-PARAMETER SYSTEMS

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Dissertation submitted to the Graduate Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY
in
Engineering Mechanics

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August 1981

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ACKNOWLEDGEMENTS

I would like first to thank my major advisor, Professor Leonard Meirovitch for his guidance and support. The learning experience I acquired as a student of Professor Meirovitch has been very valuable.

I would also like to thank my graduate committee members, Professors K. Frair, L. L. Grigsby, L. G. Kraige and M. P. Singh, as well as to former members of the committee, and , for their guidance.

I am very grateful to my dear uncle, , who has been a second father to me throughout my life, for his continuous support and encouragement. Last but not least, I would like to thank my dear parents and , without whose guidance, continuous support, encouragement and love this whole thing would not have been possible.

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INTRODUCTION

1.1 Statement of the Problem

The problem of control of large flexible structures has been receiving a great deal of attention in recent years. Large flexible structures are essentially distributed-parameter systems, their motion being described by coupled sets of partial differential equations. Distributed-parameter systems possess an infinite number of degrees of freedom. Consequently, the solution of the eigenvalue problem associated with distributed systems consists of a denumerably infinite set of eigenvalues and associated eigenfunctions.

In practice, the parameters contained in the equations of motion of distributed-parameter systems are known only approximately. These parameters are functions of the spatial variables. To determine these parameters and the system eigensolution, one must elicit the system response and use this information to deduce the system parameters. This is essentially the problem of parameter identification. Distributed measurements are not within the state of the art. As a result, one is faced with the problem of identifying the parameters that describe a distributed system by using a finite number of discrete measurements.

As the size of the distributed-parameter system increases, so does its flexibility. The result is low frequencies of vibration and high amplitudes. Even though all distributed systems contain a certain amount of damping, and any such damping tends to suppresss the vibratory motion, this damping is relatively slow and it is often necessary

to apply additional forces to drive the amplitudes of vibration to zero. This process is known as controlling the vibratory motion.

The control forces are functions of the displacement and velocity of the distributed system. This is known as feedback control. Ideally, to design the feedback control forces, the response of the actual system must be known at every point of the domain and at every time. This requires distributed measurements. As indicated before, the state of the art does not permit distributed measurements, so that, as in the case of system identification, one must use a limited number of sensors to estimate the state of the distributed-parameter system.

The control forces must be imparted to the distributed system by actuators. To control the distributed system effectively, actuators that operate over the entire distributed domain are required. Such distributed controls are not within the state of the art. Actuators, like sensors, tend to be discrete elements, so that one must use a limited number to implement the control forces.

Because of limitations in the current state of the art, one must use discrete measurements and discrete actuators to identify and control distributed-parameter systems. In addition to the existing problems in identifying and controlling distributed systems, one must answer additional questions, such as the number of sensors and actuators to be used, their location, and how to use these components most efficiently. The subject of this dissertation is the optimal identification and control of distributed systems, with special emphasis on large flexible vibrating structures. The approaches presented in this dissertation

are capable of identifying and controlling the actual distributed system without resorting to discretization.

1.2 Review of Related Work

This section is divided into two parts. In the first part, we present a literature survey for parameter identification methods and in the second part a literature survey for control of distributed-parameter systems. There are very few contributions to the literature on the subject of simultaneous identification and control. They are restricted to simple, low-order systems, and hence they are not pertinent to the distributed systems considered in this dissertation.

There is a large variety of distributed-parameter systems, ranging from chemical processes to high-rise buildings to large flexible spacecraft. It is desirable to construct mathematical models of such systems, so that their behavior can be examined, their response simulated, and any undesirable vibration controlled. When an accurate mathematical model cannot be built, because the system parameters are not well known, one must elicit the system response to extract information about the system. Because of the large variety of dynamical systems, an identification procedure for a certain system may not be applicable to another. As a consequence, the field of parameter identification has become very scattered. As it is stated in Ref. [1], attempts should be made to somehow unify the field of parameter identification. Currently, in the technical community there is no journal that is devoted mainly to the problem of parameter identification.

Surveys of the state of the art in the field of identification of parameters can be found in Refs. [1-12]. These survey papers list references associated with most major identification methods. For vibrating systems, one can divide the identification methods into two categories, namely, time- and frequency-domain. Time-domain identification implies that the output of the dynamical system is fed directly into the identification process. Eigenvalues or other parameters, such as mass, stiffness and damping parameters can be identified. In frequency-domain identification, first the frequency response of the system output is sought. The identification procedure is then based on the frequency response. The general trend in recent years has been towards time-domain analysis, especially for distributed systems and large-scale discrete systems. The main reason for this is the computational difficulties that arise when a frequency domain analysis is conducted for a large-order system. However, the difference between time- and frequency-domain identification is becoming less significant.

The earliest identification schemes for distributed-parameter vibrating systems were based on frequency response methods. The identification procedure was based on discrete models and was implemented by means of near-resonance tests [13]. In near-resonance tests, the distributed system is excited harmonically at various frequencies, and the points at which resonance occurs are identified as corresponding to the system eigenvalues. This kind of analysis has a number of drawbacks. For example, closely-spaced frequencies cannot be isolated. In addition, higher modes are more difficult to excite, so that the identification procedure yields incorrect results for the higher

eigenvalues. Hence, only a very limited number of eigenvalues can be identified.

The next step, after the eigenvalues and eigenvectors have been identified, is to compute the mass, stiffness and damping matrices defining the discrete model [14-19]. References [14-19] attempt to construct a discrete model of a vibrating structure by using a set of measured frequencies and eigenvectors, where the number of identified frequencies is generally lower than the order of the discrete system. It was realized later that to accomplish this task, it is useful to have an initial estimate of the mass, stiffness and damping matrices. The identified eigensolution could then be used to improve the initial estimates of the vibration parameters [20-25].

References [26-32] use time- or frequency-domain identification procedures to determine the vibration parameters of discrete models simulating distributed systems. The identified parameters can be frequencies [26,28,32] or entries of the mass, stiffness and damping matrices [27,29-31]. In Refs. [29-30], the authors assume that the coefficient matrices have tridiagonal form. This assumption is generally used when multistory buildings are analyzed. Reference [31] suggests a method to determine an optimal forcing function for extracting the system parameters efficiently.

The identification of vibration parameters from systems described by nonlinear models is considered in Refs. [33-35]. The identification procedure described in Ref. [35] can also be applied to linear discrete models. Identification of parameters for nonlinear models is generally used for earthquake engineering problems. The identification of

parameters from responses to earthquakes are considered in Refs. [36-38].

As can be expected, work related to the identification of actual distributed systems, by using models consisting of partial differential equations, has appeared less frequently than work related to identification of discrete systems. This is because discrete systems are described by ordinary differential equations, which are easier to handle than partial differential equations. For the identification of the actual distributed system, one must measure the system output at every point along the distributed domain. Such continuous measurements are not within the state of the art. Therefore, the identification procedure must be implemented by using a finite number of sensors. This brings about identifiability and observability problems [39-40]. References [41-44] suggest some guidelines for the placement of the sensors.

References [45-50] consider the identification of the parameters contained in the partial differential equations governing the behavior of distributed systems. They use optimality criteria, frequency response, Green function transformations, Bayesian approach, transient response, and Poisson moment functional expansions, respectively. Reference [49] also examines the problem of optimal location of sensors. The identification schemes described in Refs. [45-50] are described for simple distributed systems, where the number of parameters to be identified is very low. Therefore, the applicability of these methods for the identification of large-order distributed vibrating systems is questionable.

References [51] and [52] attempt to identify the parameters of a distributed system by resorting to spatial discretization and using the

Galerkin and finite element methods, respectively. The identified parameters are the mass, stiffness and damping coefficients. The difference between Refs. [51,52] and previously cited papers using discrete models is that Refs. [51,52] consider the fact that they are dealing with discretized models of continuous sytems, rather than ordinary discrete systems.

An identification procedure developed in Refs. [53-55] identifies the frequencies and mode shapes of discretized models of distributed systems by a time-domain analysis. The method requires a large number of sensors and has been successfully applied to identify complex structures [56,57]. The results of Ref. [57] have been confirmed in Ref. [58], where a time-domain least-squares curve fitting procedure is used.

The identification procedure described in Res. [53-55] is formulated in terms of discretized models. The approach can be extended to distributed systems, as shown in this dissertation.

An important problem in distributed systems is the identification of boundary conditions. Some consideration is given to this problem in Ref. [59], where the boundary conditions of a beam in bending vibration are identified. Another important problem in identification of distributed systems, especially pertinent to spacecraft, is the alteration of the operating environment. If the parameters of a space structure are identified using ground tests and then the spacecraft is launched into orbit, a problem emerges. Because the system parameters are determined in a one-g environment and the spacecraft operates in a zero-g environment, the system eigensolution changes. This problem was investigated in Ref. [60].

Before reviewing the literature related to the control of distributed systems, let us discuss the problem of simultaneous identification and control. As mentioned before, simultaneous identification and control of distributed systems has not received much attention. A limited amount of work has been done for the simultaneous identification and control of discrete systems [61,62]. Adaptive control of distributed systems is considered in Refs. [63,64], but the problems considered in these references are not pertinent to the identification and control problems considered in this disertation.

As indicated before, the equation of motion of a distributed system is in terms of coupled sets of partial differential equations. Control of distributed systems without resorting to discretization, i.e., considering the actual distributed system, has received limited attention [65-69]. References [65-67] do not give an example permitting an assessment as to how their control schemes apply to large order systems. In Refs. [68,69], the authors realize that one must go into the modal configuration space by first converting the partial differential equations into an infinite set of ordinary differential equations. In addition, Refs. [68,69] both note that only a limited number of the system modes can be controlled.

The entire process of controlling a distributed-parameter system consists of three major steps: modeling, control system design and control implementation. In the sequel, we review the literature concerning these three steps. Note that, parameter identification is part of the modeling step.

Almost all of the recent work on the control of distributed systems begins by first discretizing the distributed system in space, and then designing the controls considering a finite-dimensional discretized system. Among common discretization procedures are the Rayleigh-Ritz, Galerkin, lumped-parameter and finite element methods [70-72]. The net effect of discretization is to convert a differential eigenvalue problem into an algebraic one. In addition, partial differential equations are converted into sets of ordinary differential equations.

The solution of the algebraic eigenvalue problem provides the analyst with estimates of the actual eigenvalues and eigenvectors which yield estimates of the eigenfunctions. In addition, one can transform the equations of motion of the discretized system into decoupled form. In the special case in which the differential eigenvalue problem admits a closed-form solution, one can obtain an infinite set of independent ordinary differential equations directly, but such cases are generally of academic interest only.

The eigensolution obtained by solving the algebraic eigenvalue problem is only an estimate of the actual eigensolution. This is because the distributed system is truncated during the discretization process and truncation affects the eigenvalues of the discretized model. Estimates of the higher eigenvalues tend to be less accurate than the estimates of the lower ones. As a result, when one constructs a discretized model of a distributed system, two errors exist: (1) some of the system modes are ignored, and (2) the higher eigenvalues in the discretized model are much less accurate than the actual eigenvalues.

One can minimize the effect of the first of the above problems by increasing the order of the discretized model, such that the truncated modes will have no adverse effect on the system behavior. Care should be taken, however, in increasing the order of the discretized model, because a very high-order model can lead to computational difficulties. The problem of determining the order of discretized models, such that the truncated modes will be insignificant, needs further investigation. So far, most of the work regarding this problem has concentrated on determining which modes should be retained in the discretized model and which of the retained modes should be controlled.

From a structural dynamics point of view, the lower natural frequencies of the uncontrolled system are to be retained in the dynamic model. Other truncation schemes are based on energy considerations [73] and least-volume modes [74]. The approach based on energy considerations in Ref. [73] and the structural dynamicists approach retaining the lower natural frequencies are indeed similar. Reference [75] proposes a truncation scheme based on the premise that the modes to be controlled need not necessarily be the lower ones. The truncation scheme of Ref. [75] is based on the closed-loop poles of the system and although it has some appeal from a controls point of view, it has certain drawbacks from a structural dynamicist's point of view.

The second problem has not been investigated extensively so far. Most control systems consider all the eigenvalues in the discretized model to be exact, which is hardly the case. Some consideration is given to this problem in Ref. [76], where some suggestions are made for

avoiding errors resulting from inaccurate higher modes. Still, more work needs to be done regarding this problem.

As a result of modeling, the infinity of modes that comprise the distributed system can be divided into three categories: modeled controlled, modeled uncontrolled (residual) and unmodeled. The control system is designed based on the modeled modes.

The control system is generally designed in the modal space and in the form of feedback controls. Three feedback control methods have enjoyed popularity in recent years: nonlinear on-off control [77], linear nonoptimal control based on the pole allocation method [78] and linear optimal control [79-80].

Control implementation requires the use of sensors to measure the system output and actuators to impart the control forces on the distributed-parameter system. If continuous sensors and actuators were available, control implementation problems would not exist. Because the state of the art does not permit distributed measurements and controls, one must use discrete sensors and actuators to implement the control forces.

The use of discrete elements for control implementation results in what has been known as "control spillover" and "observation spillover" [81]. It is also shown in Ref. [81] that control spillover alone only degrades the system performance, but a combination of control and observation spillover may lead to instabilities in the closed-loop system, particularly if the sensors and actuators are not collocated.

In most recent papers describing various control techniques, there are no criteria regarding the number and location of sensors and

actuators [82-91]. Reference [92] suggests that the actuators and sensors be collocated, but does not give much detail regarding the locations of these elements.

Some suggestions regarding the problem of actuators locations are made in Refs. [67,93-96], but the applicability of these suggestions to large order vibrating systems is doubtful.

The modal control techniques described in Refs. [81-92] become computationally unattractive as the order of the control system is increased. For example, when linear optimal controls are designed, one must solve a matrix Riccati equation of order twice as large as the number of controlled modes. It is well known that the solution of the Riccati equation exhibits serious computational difficulties for high-order matrices. Nonlinear on-off controls cannot be designed for a large order system, and the pole allocation method can only be implemented when a very small number of modes are controlled and only one actuator is used. Theoretically, one can control an infinite number of modes by a single actuator. In practice, however, this is impossible to implement, because it is impossible to determine gain matrices of infinite dimensions.

A different approach to the control of distributed-parameter systems is known as independent modal-space control [76,77,97-104]. As the name indicates, the various modes are controlled independently, so that one essentially controls a set of independent second-order systems in parallel. As a result, no computational difficulties are encountered and no control spillover into the uncontrolled modeled modes is experienced. Independent modal-space control requires that the number

of actuators be equal to the order of the modeled system. Among advantages of the independent modal-space method is the ease with which the control system is designed, including nonlinear on-off controls [77]. The pole allocation method can also be easily implemented [101]. For optimal control, one must only solve a set of second-order matrix Riccati equations, and for the steady-state case, the solution of the algebraic Riccati equation is available in closed-form [100].

In Refs. [76,77,97-102] the independent modal-space method is described for discretized models of distributed systems. References [103-104] describe the independent modal-space method for the actual distributed system. It is shown in Ref. [104] that the actuators locations no longer represent a serious problem when the independent modal-space-control method is used.

To synthesize the feedback control forces, one must extract the modal quantities from the system output, which can only be measured by a finite number of sensors. This is true for any modal control method, including coupled control. There are two approaches permitting the extraction of modal quantities from the sensors data, namely, observers [105] and modal filters [103]. Let us first consider observers. An observer is a dynamical system, akin to the actual system and one that makes use of the system input and output to estimate the system state. A deterministic observer is known as a Luenberger-type observer [106]. In designing an observer for distributed systems, one only considers the modeled modes. When designing an observer, one must select the number and location of the sensors in advance. In the control systems

described in Refs. [81-92], no guidelines are given as to how many sensors should be used. The use of observers has led to what has come to be known as observation spillover [81]. It is shown in Ref. [99] that if the number of sensors is equal to the order of the discretized model, no observation spillover exists from the modeled modes. Spillover from the unmodeled modes exists, however.

The spillover problems encountered when observers are used can be eliminated if modal filters are used. It is shown in Ref. [103] that, if modal filters are implemented with a sufficient number of sensors, no system discretization is necessary and observation spillover can be eliminated. Moreover, all the modes are accounted for, so that control of the actual distributed system is possible. The number of sensors depends on the mode participation in the overall response and can easily be determined. A comparison of modal filters and observers is given in Ref. [107], where it is concluded that modal filters are superior to observers.

1.3 Outline

The subject of this dissertation is the identification and control of distributed-parameter systems by using discrete sensors and actuators, with special considerations to large flexible structures. The parameter identification and control methods are designed so that the identification and control of the actual distributed system is possible and no truncation or discretization is necessary.

The identification procedure is based on the time-domain approach of Ref. [54], and the control system design is based on the concept of

independent modal-space control described in Refs. [97-104]. These procedures are not restricted to vibrating systems alone. They are applicable to any distributed-parameter system.

In Chapter 2, the equations of motion for distributed-parameter vibrating systems are described. In Chapter 3, the time-domain identification method developed in Ref. [54] is formulated in terms of the actual distributed system. A limited number of eigenvalues and eigenfunctions are identified. Computational aspects of the identification procedure, pertinent to the identification of large flexible structures, are discussed. Chapter 4 develops a scheme where the eigensolution identified by the method of Chapter 3 is used to identify the mass and stiffness distributions. The advantages of determining the mass and stiffness distributions, in addition to the eigensolution, are discussed.

In Chapter 5, first the control system is designed. The design is based on the concept of independent modal-space control. Next, the problem of control implementation is discussed. The control spillover phenomenon is examined qualitatively and quantitatively, in terms of the amount of energy imparted on the distributed system. The concept of modal filters is described, and it is shown that control of the actual distributed system is possible when modal filters are used.

Chapter 6 presents a discussion of the results and makes some recommendations regarding future work in the fields of parameter identification and control of distributed-parameter systems.

CHAPTER 2

EQUATIONS OF MOTION FOR DISTRIBUTED-PARAMETER VIBRATING SYSTEMS

2.1 Introduction

The derivations in this chapter can be found in any standard textbook on vibration analysis. They are included here only for completeness. Throughout this chapter, it is assumed that the system is linear.

2.2 Equations of Motion

The equation of motion for a distributed-parameter system can be written in the form of the partial differential equation [70, Sec. 9-12]

$$Lu(P,t) + \partial[Cu(P,t)]/\partial t + M(P)\partial^2 u(P,t)/\partial t^2 = f(P,t) \qquad (2.1)$$

which must be satisfied at every point P of the domain D, where

$u(P,t)$ = displacement of an arbitrary point P

L = linear differential operator of order 2p, expressing the system stiffness

C = linear differential operator of order 2p, expressing the system damping

$M(P)$ = distributed mass

$f(P,t)$ = distributed force

The displacement $u(P,t)$ is subject to the boundary conditions

$$B_i u(P,t) = 0, \quad i = 1, 2, \ldots, p \qquad (2.2)$$

to be satisfied at every point of the boundary S of the domain D, where B_i $(i = 1, 2, \ldots, p)$ are linear differential operators of order

ranging from zero (in which case the operator is a mere multiplying constant) to 2p - 1.

Let us consider the associated eigenvalue problem consisting of the differential equation [70, Sec. 5-4]

$$L\phi_r = \Omega_r M\phi_r, \quad r = 1, 2, \ldots \tag{2.3}$$

and the boundary conditions

$$B_i\phi_r = 0, \quad i = 1, 2, \ldots, p; \quad r = 1, 2, \ldots \tag{2.4}$$

The solution of Eqs. (2.3-2.4) consists of a denumerably infinite set of eigenvalues Ω_r ($r = 1, 2, \ldots$) and associated eigenfunctions ϕ_r. For most distributed-parameter vibrating systems the operator L is self-adjoint and positive definite. The implication is that all the eigenvalues are positive. We shall order them so that $\Omega_1 \leq \Omega_2 \leq \ldots$. The eigenvalues are related to the natural frequencies ω_r of the undamped oscillation by $\Omega_r = \omega_r^2$ ($r = 1, 2, \ldots$). In addition, eigenfunctions possess the orthogonality property and they can be normalized so as to satisfy

$$\int_D M\phi_r\phi_s dD = \delta_{rs}, \quad r,s = 1, 2, \ldots \tag{2.5}$$

where δ_{rs} is the Kronecker delta. It follows from Eqs. (2.3) and (2.5) that

$$\int_D \phi_s L\phi_r dD = \Omega_r\delta_{rs}, \quad r,s = 1, 2, \ldots \tag{2.6}$$

The eigenfunctions ϕ_r constitute a complete set. Hence, any function u(P) in the Hilbert space K_B^{2p} consisting of all the functions that are 2p times differentiable and satisfy the boundary conditions

(2.4) has the unique representation

$$u(P) = \sum_{r=1}^{\infty} u_r \phi_r(P) \tag{2.7}$$

where the coefficients u_r are given by

$$u_r = \int_D M\phi_r u dD \tag{2.8}$$

Equations (2.7) and (2.8) represent the so-called expansion theorem [70, Sec. 5-4].

Using the expansion theorem, the solution of Eqs. (2.1)-(2.2) can be represented by an infinite series of space-dependent eigenfunctions, $\phi_r(P)$ multiplied by time-dependent generalized coordinates $u_r(t)$ of the form

$$u(P,t) = \sum_{r=1}^{\infty} \phi_r(P) u_r(t) \tag{2.9}$$

Introducing Eq. (2.9) into Eq. (2.1), multiplying both sides of the result by ϕ_s and integrating over D, we obtain

$$\ddot{u}_r(t) + \sum_{s=1}^{\infty} c_{rs}\dot{u}_s(t) + \omega_r^2 u_r(t) = f_r(t), \quad r = 1, 2, \ldots \tag{2.10}$$

where

$$c_{rs} = \int_D \phi_r C \phi_s dD, \quad r,s = 1, 2, \ldots \tag{2.11}$$

are damping coefficients and

$$f_r(t) = \int_D \phi_r f(P,t) dD, \quad r = 1, 2, \ldots \tag{2.12}$$

are modal control forces.

Equation (2.10) represents an infinite set of simultaneous ordinary differential equations. Indeed, as seen from Eq. (2.10), damping produces coupling of the generalized coordinates $u_1(t)$, $u_2(t)$, ... This coupling must be regarded as internal to the system. The internal coupling disappears when damping is of the proportional type, i.e., when the operator C is a linear combination of the operator L and the mass function M [70, p. 431]. Hence, when

$$C = g_1 L + g_2 M \tag{2.13}$$

where g_1 and g_2 are arbitrary constants, the damping coefficients reduce to

$$c_{rs} = c_r \delta_{rs} = 2\zeta_r \omega_r \delta_{rs}, \quad r,s = 1, 2, \ldots \tag{2.14}$$

where ζ_r ($r = 1, 2, \ldots$) are modal damping factors, so that from the infinite series on the left-hand side of Eq. (2.10) only one term survives, namely, the term for which $s = r$. Equation (2.10) then becomes

$$\ddot{u}_r(t) + c_r \dot{u}_r(t) + \omega_r^2 u_r(t) = f_r(t), \quad r = 1, 2, \ldots \tag{2.15}$$

The above derivation is based on the assumptions that the closed-form eigensolution of the differential eigenvalue problem is available and that damping is of the proportional type. These assumptions generally do not apply to many real-life systems. For example, there still is a considerable amount of controversy regarding the nature of damping in distributed-parameter systems. Various models for damping exist [108], and new models are being introduced as more light is shed on the subject [109-110]. All types of damping tend to suppress the vibratory motion [111], but the exact nature of damping for a certain distributed

system is yet unknown. It is generally believed that damping in a structure is caused by more than one factor.

When describing the identification procedure, we assume that damping is of the proportional type. The reasons for this assumption is explained in the next chapter. For the control system design, damping will be ignored, so that the design will be based on undamped systems. It should be noted that elimination of damping from the equations of motion when designing a control system represents a conservative approach.

Distributed-parameter systems possessing closed-form eigensolutions are very rare and generally of academic interest only. To obtain the eigensolution for systems not admitting closed-form solutions, one must discretize the system in space. This can be done by expanding the distributed dependent variable into a finite series of eigenfunctions [70-72].

An approximate solution of the eigenvalue problem is likely to contain accurate information about the lower modes only. As described in Section 1.2, estimates of the higher eigenvalues tend to be less accurate than estimates of the lower ones. The use of inaccurate information to control a certain mode may result in sensitivity problems. Such problems have yet to be investigated.

Throughout this dissertation, only the lower modes will be retained in the mathematical model. Consequently, only the lower modes will be identified and controlled. Higher modes require more energy to excite, have lower amplitudes and damp out faster than lower modes, so that the

contribution of higher modes to the overall response tends to be unimportant.

The numerical examples in Chapters 4 and 5, illustrating the identification procedure and control implementation, are described for systems admitting closed-form eigensolutions. The reasons why such systems are chosen are that, for the identification procedure, the availability of a closed-form solution enables us to compare the identified system with the actual one and, for the control system design the problem of determining whether the estimate of an eigenvalue is accurate enough is eliminated.

CHAPTER 3

IDENTIFICATION OF THE EIGENSOLUTION

3.1 Introduction

To model a distributed system, the parameters contained in the equations of motion must be known accurately. In many practical cases, this information is not available, so that one must find ways of obtaining it. A frequently used approach is to elicit the system response and use this information to deduce the system parameters. This is the essence of parameter identification.

The parameter identification methods pertinent to the vibration of distributed systems can be divided into two approaches. The first is to consider a discretized or discrete model, and identify the parameters contained in the discrete model [13-35,51-58]. The second approach is to identify the eigensolution and/or parameters contained in the original equations of motion, thus identifying the actual distributed system [45-50]. In this dissertation, we adopt the second approach. The identification procedure will be carried out in two steps. The first step of the identification process is the identification of the eigensolution, and is the subject of this chapter. The identification of the mass and stiffness distributions will be described in Chapter 4.

The identification method used here is an extension of the time-domain method described in Ref. [54].

3.2 The ITD Method

The ITD (Ibrahim Time Domain) method is a time-domain identification method for identifying the parameters of a vibrating system [53-55]. The method has been developed for discrete systems and lumped-parameter models of distributed systems. It makes use of the free response to identify a limited number of eigenvalues and associated eigenvectors. Following is a brief description of the method.

The equations of motion for a discrete nongyroscopic system, in the absence of external disturbances, can be written in the matrix form

$$M\ddot{\underset{\sim}{x}}(t) + C\dot{\underset{\sim}{x}}(t) + K\underset{\sim}{x}(t) = \underset{\sim}{0} \tag{3.1}$$

where M, C and K are the m × m mass, damping, and stiffness matrices, respectively, and $\underset{\sim}{x}(t)$ is an m-dimensional configuration vector. For discrete vibrating systems $\underset{\sim}{x}(t)$ describes the displacements of the masses, and in the case of systems discretized by the lumped-parameter method, $\underset{\sim}{x}(t)$ describes the displacements of the lumped masses. The eigenvalue problem associated with Eq. (3.1) is given by

$$(\lambda_r^2 M + \lambda_r C + K)\underset{\sim}{p}_r = \underset{\sim}{0}, \quad r = 1, 2, \ldots, 2m \tag{3.2}$$

where λ_r and $\underset{\sim}{p}_r$ ($r = 1, 2, \ldots, 2m$) are the eigenvalues and associated eigenvectors. They can be real or complex, where in the latter case they occur in pairs of complex conjugates. For the special case when damping is of the proportional type, i.e., when

$$C = g_1 K + g_2 M \tag{3.3}$$

where g_1 and g_2 are constants, the eigenvectors $\underset{\sim}{p}_r$ ($r = 1, 2, \ldots, 2m$) are real. In this case, the eigenvectors can be normalized so as to

satisfy

$$\begin{aligned} \underset{\sim}{p}_r^T \underset{\sim}{M} \underset{\sim}{p}_s &= \delta_{rs} \\ \underset{\sim}{p}_r^T \underset{\sim}{C} \underset{\sim}{p}_s &= 2\xi_r \omega_r \delta_{rs}, \quad r,s = 1, 2, \ldots, 2m \\ \underset{\sim}{p}_r^T \underset{\sim}{K} \underset{\sim}{p}_s &= \omega_r^2 \delta_{rs} \end{aligned} \tag{3.4}$$

where ξ_r and ω_r are the damping ratio and the frequency of undamped oscillation associated with the r'th mode, respectively. The eigenvalues λ_r are related to ξ_r and ω_r by the relation

$$\lambda_r = -\xi_r \omega_r \pm i\omega_r \sqrt{1 - \xi_r^2}, \quad r = 1, 2, \ldots, m \tag{3.5}$$

where $i^2 = -1$.

The time-domain identification consists of determining the eigenvalues and eigenvectors of the system described by Eq. (3.1) from the free response of the system. For systems vibrating freely, the response at any time t can be expressed as

$$\underset{\sim}{x}(t) = \sum_{j=1}^{2m} \underset{\sim}{p}_j e^{\lambda_j t} \tag{3.6}$$

If the response of the system is measured at 2m different instances, the following matrix relationship can be written

$$[\underset{\sim}{x}(t_1)\ \underset{\sim}{x}(t_2)\ \cdots\ \underset{\sim}{x}(t_{2m})] = [\underset{\sim}{p}_1\ \underset{\sim}{p}_2\ \cdots\ \underset{\sim}{p}_{2m}]\begin{bmatrix} e^{\lambda_1 t_1} & e^{\lambda_1 t_2} & \cdots & e^{\lambda_1 t_{2m}} \\ e^{\lambda_2 t_1} & \cdots & \cdots & e^{\lambda_2 t_{2m}} \\ \vdots & & & \\ e^{\lambda_{2m} t_1} & \cdots & \cdots & e^{\lambda_{2m} t_{2m}} \end{bmatrix}$$

or (3.7)

$$X = P\Lambda \tag{3.8}$$

where X and P are $m \times 2m$ matrices and Λ is a $2m \times 2m$ matrix.

The response of the system, measured a time increment Δt later, can be expressed as

$$[\underset{\sim}{x}(t_1 + \Delta t)\ \underset{\sim}{x}(t_2 + \Delta t)\ \cdots\ \underset{\sim}{x}(t_{2m} + \Delta t] = [\underset{\sim}{p}_1\ \underset{\sim}{p}_2\ \cdots\ \underset{\sim}{p}_{2m}]$$

$$\times \begin{bmatrix} e^{\lambda_1(t_1+\Delta t)} & e^{\lambda_1(t_2+\Delta t)} & \cdots & e^{\lambda_1(t_{2m}+\Delta t)} \\ e^{\lambda_2(t_1+\Delta t)} & e^{\lambda_2(t_2+\Delta t)} & \cdots & e^{\lambda_2(t_{2m}+\Delta t)} \\ \vdots & & & \vdots \\ e^{\lambda_{2m}(t_1+\Delta t)} & \cdots & \cdots & e^{\lambda_{2m}(t_{2m}+\Delta t)} \end{bmatrix} \tag{3.9}$$

or

$$Y = P\Lambda_1 \tag{3.10}$$

where the notation is obvious. Comparing Eqs. (3.7)-(3.10), one can

verify that

$$\Lambda_1 = D\Lambda \tag{3.11}$$

where D is a $2m \times 2m$ diagonal matrix of the form

$$D = \text{diag}\left[e^{\lambda_1 \Delta t} \quad e^{\lambda_2 \Delta t} \quad \ldots \quad e^{\lambda_{2m} \Delta t}\right] \tag{3.12}$$

Using the above analysis, the response of the system measured yet another time increment Δt later can be expressed as

$$Z = PD^2\Lambda \tag{3.13}$$

where

$$Z = [\underset{\sim}{x}(t_1 + 2\Delta t) \quad \underset{\sim}{x}(t_2 + 2\Delta t) \quad \ldots \quad \underset{\sim}{x}(t_{2m} + 2\Delta t)] \tag{3.14}$$

Equations (3.8), (3.10), (3.11) and (3.13) can be combined to give

$$\begin{bmatrix} X \\ \hline Y \end{bmatrix} = \begin{bmatrix} P \\ \hline PD \end{bmatrix}\Lambda$$

$$\begin{bmatrix} Y \\ \hline Z \end{bmatrix} = \begin{bmatrix} P \\ \hline PD \end{bmatrix}D\Lambda \tag{3.15}$$

Introducing the notation

$$A = \begin{bmatrix} X \\ \hline Y \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} Y \\ \hline Z \end{bmatrix}, \quad E = \begin{bmatrix} P \\ \hline PD \end{bmatrix} \tag{3.16a,b,c}$$

one arrives at the relation [53]

$$\hat{A}A^{-1}E = ED \tag{3.17}$$

Because D is a diagonal matrix, Eq. (3.17) can be described as 2m relations for each column of the matrix E. Letting

$$E = [\underset{\sim}{E}_1 \quad \underset{\sim}{E}_2 \quad \cdots \quad \underset{\sim}{E}_{2m}] \tag{3.18}$$

we can write

$$\hat{AA}^{-1} \underset{\sim}{E}_j = e^{\lambda_j \Delta t} \underset{\sim}{E}_j, \quad j = 1, 2, \ldots, 2m \tag{3.19}$$

Equation (3.19) represents an eigenvalue problem for the matrix $\hat{AA}^{-1}$. Considering Eqs. (3.9), (3.10) and (3.12), we obtain

$$\underset{\sim}{E}_r = \begin{bmatrix} \underset{\sim}{p}_r \\ \hline e^{\lambda_r \Delta t} \underset{\sim}{p}_r \end{bmatrix}, \quad r = 1, 2, \ldots, 2m \tag{3.20}$$

so that the eigenvalues of the matrix $\hat{AA}^{-1}$ are $e^{\lambda_r \Delta t}$ $(r = 1, 2, \ldots, 2m)$, and the upper halves of the eigenvectors $\underset{\sim}{E}_r$ of $\hat{AA}^{-1}$ are the eigenvectors $\underset{\sim}{p}_r$ $(r = 1, 2, \ldots, 2m)$ belonging to the original vibrating system described by Eq. (3.2).

It is shown in Ref. [54] that by introducing the notation

$$e^{\lambda \Delta t} = \beta \pm i\gamma = e^{(-\xi\omega \pm i\omega\sqrt{1-\xi^2})\Delta t} \tag{3.21}$$

where we note that the subscripts have been dropped, and considering Eqs. (3.5), we can write

$$\begin{aligned} -\xi\omega &= \frac{1}{2\Delta t} \ln(\gamma^2 + \beta^2) \\ \omega\sqrt{1 - \xi^2} &= \frac{1}{\Delta t} [\tan^{-1}(\gamma/\beta) + k\pi], \quad k = 0, 1, 2, \ldots \end{aligned} \tag{3.22}$$

from which we obtain

$$\xi = \left[\frac{\ell n(\gamma^2 + \beta^2)^2}{4(\tan^{-1}(\gamma/\beta) + k\pi)^2} + 1 \right]^{-1/2}$$

$$\omega = \frac{1}{\Delta t}\left[\frac{[\ell n(\gamma^2 + \beta^2)]^2}{4} + [\tan^{-1}(\gamma/\beta) + k\pi]^2 \right]^{1/2} \tag{3.23}$$

The choice of the parameter k places constraints on the choice of the time increment Δt [54].

3.3 Extension of the ITD Method to Distributed Systems

In the preceding section, the ITD method was described for discrete systems. In this section, we extend the method to distributed systems. Distributed systems contain an infinite number of modes. However, it is possible to identify only a limited number of the eigenvalues and associated eigenfunctions. On physical grounds, it seems reasonable to identify the lower modes.

In the absence of controls or other external forces,

$$f_r(t) = 0 \quad r = 1, 2, \ldots \tag{3.24}$$

So that Eq. (2.12) reduces to that of a free vibration problem.

The response of a certain mode can be expressed in real quantities only, but for reasons that will become evident shortly we represent the time-dependent part of the displacement (for underdamped modes) as

$$u_r(t) = e^{\lambda_r t} + e^{\bar{\lambda}_r t}, \quad r = 1, 2, \ldots \tag{3.25}$$

so that considering Eq. (2.9), the displacement of a certain point P_i along the distributed domain can be expressed as

$$u(P_i,t) = \sum_{r=1}^{\infty} \phi_r(P_i)\left(e^{\lambda_r t} + e^{\bar{\lambda}_r t}\right) \quad i = 1, 2, \ldots \tag{3.26}$$

where we note that implicit in Eq. (3.26) is the assumption of proportional damping.

Assuming that we wish to identify the first m modes, the contribution of the higher modes (m + 1, m + 2, ...) should be eliminated from the response data, or should be ignored. Two approaches are proposed:

(a) The use of a low-pass filter. When a low-pass digital filter is used, frequency components higher than the cutoff frequency can easily be filtered out, <u>especially for systems in free vibration</u> [112]. However, because of the aliasing problem associated with digital filters, some of the higher modes coinciding with the periodic bandwidth of the filter will not be filtered. If m is chosen sufficiently large, the modes which cannot be filtered because of aliasing will have very low amplitudes, and their contribution to the overall motion can be ignored.

(b) Choose m large enough so that, even without using a digital filter, the higher modes not included in the identified model will have little influence on the overall response. Again, in this case, the contamination by the higher modes is either ignored or treated as measurement noise.

In view of the above, one can replace the upper limit on the summation in Eq. (3.26) by m, so that we have the relation

$$u(P_i,t) = \sum_{r=1}^{m} \phi_r(P_i)\left(e^{\lambda_r t} + e^{\bar{\lambda}_r t}\right), \quad i = 1, 2, \ldots, m \tag{3.27}$$

where we note that m sensors are used. Introducing the notation

$$\underset{\sim}{u}_I(t) = [u(P_1,t) \quad u(P_2,t) \quad \ldots \quad u(P_m,t)]^T \tag{3.28a}$$

$$\underset{\sim}{b}_r = [\phi_r(P_1) \quad \phi_r(P_2) \quad \ldots \quad \phi_r(P_m)]^T \tag{3.28b}$$

$$\underset{\sim}{b}_{r+m} = \underset{\sim}{b}_r, \quad r = 1, 2, \ldots, m \tag{3.28c}$$

one can express the displacement of the distributed system, measured at the sensors locations, as

$$\underset{\sim}{u}_I(t) = \sum_{r=1}^{2m} \underset{\sim}{b}_r e^{\lambda_r t} \tag{3.29}$$

where we note that we have made use of the relations

$$\underset{\sim}{b}_{r+m} = \underset{\sim}{b}_r$$

$$r = 1, 2, \ldots, m \tag{3.30a,b}$$

$$\lambda_{r+m} = \bar{\lambda}_r$$

Equation (3.29) corresponds to Eq. (3.6). Indeed, the vectors $\underset{\sim}{x}(t)$ in Eq. (3.6) and $\underset{\sim}{u}_I(t)$ in Eq. (3.29) represent displacements for the discrete and distributed systems, respectively. The vector $\underset{\sim}{p}_r$ in Eq. (3.6) is the rth eigenvector of the discrete system and $\underset{\sim}{b}_r$ in Eq. (3.29) is a vector with components equal to the values of the rth eigenfunction at the sensors locations. Even though Eqs. (3.6) and (3.29) represent different models, they are mathematically equivalent. Therefore, the time domain identification scheme described in Section 3.2 can be applied to identify the lowest m modes of a distributed system. The

identification procedure need not be repeated here. Indeed, the identified eigenvalues $e^{\lambda_r \Delta t}$ $(r = 1, 2, \ldots, 2m)$, will be the eigenvalues of the matrix $\hat{A}A^{-1}$, where

$$A = \begin{bmatrix} u(P_1,t_1) & u(P_1,t_2) & \cdots & u(P_1,t_{2m}) \\ u(P_2,t_1) & u(P_2,t_2) & \cdots & u(P_2,t_{2m}) \\ \vdots & & & \vdots \\ u(P_m,t_1) & u(P_m,t_2) & \cdots & u(P_m,t_{2m}) \\ u(P_1,t_1+\Delta t) & u(P_1,t_2+\Delta t) & \cdots & u(P_1,t_{2m}+\Delta t) \\ u(P_2,t_1+\Delta t) & u(P_2,t_2+\Delta t) & \cdots & u(P_2,t_{2m}+\Delta t) \\ \vdots & & & \vdots \\ u(P_m,t_1+\Delta t) & \cdots & \cdots & u(P_m,t_{2m}+\Delta t) \end{bmatrix} \qquad (3.31a)$$

$$\hat{A} = \begin{bmatrix} u(P_1,t_1+\Delta t) & u(P_1,t_2+\Delta t) & \cdots & u(P_1,t_{2m}+\Delta t) \\ u(P_2,t_1+\Delta t) & u(P_2,t_2+\Delta t) & \cdots & u(P_2,t_{2m}+\Delta t) \\ \vdots & & & \vdots \\ u(P_m,t_1+\Delta t) & u(P_m,t_2+\Delta t) & \cdots & u(P_m,t_{2m}+\Delta t) \\ u(P_1,t_1+2\Delta t) & u(P_1,t_2+2\Delta t) & \cdots & u(P_1,t_{2m}+2\Delta t) \\ u(P_2,t_1+2\Delta t) & u(P_2,t_2+2\Delta t) & \cdots & u(P_2,t_{2m}+2\Delta t) \\ \vdots & & & \vdots \\ u(P_m,t_1+2\Delta t) & u(P_m,t_2+2\Delta t) & \cdots & u(P_m,t_{2m}+2\Delta t) \end{bmatrix} \qquad (3.31b)$$

The natural frequencies ω_r, and damping ratios ξ_r $(r = 1, 2, \ldots, m)$

can be computed from Eqs. (3.23). The upper halves of the eigenvectors of $\hat{A}A^{-1}$ are the vectors $\underset{\sim}{b}_r$ and the lower halves are $\underset{\sim}{b}_r e^{\lambda_r \Delta t}$ (r = 1, 2, ..., m).

3.4 Identification of the Eigenfunctions

Generally, the concern has been mainly with identifying the eigenvalues of a dynamical system. However, a complete identification of a distributed system requires knowledge of its eigenfunctions, in addition to that of its eigenvalues.

As stated earlier, eigenfunctions are continuous functions of the spatial variables, so that distributed measurements are required to identify them. As a result of the identification procedure described in this chapter, the only information available about a certain eigenfunction is its magnitude at the measurement points. Our task is to identify the eigenfunctions from a limited number of measurements.

One method of estimating the eigenfunctions from the vectors $\underset{\sim}{b}_r$ (r = 1, 2, ..., m) is interpolation. Interpolation is the approximation of the shape of a function in a given interval using values on the boundaries of the interval. Among common interpolation methods are finite element techniques [71], splines [113], Lagrange and Chebychev polynomials [114] and Gregory-Newton formulas [114]. The choice of the interpolation method affects the quality of the estimation.

It is intuitively clear that as the number of measurements increases the quality and accuracy of the estimation improves. Increasing the number of measurements also increases the order of the eigenvalue problem, however.

Eigenfunctions belonging to the lower modes are smoother than eigenfunctions belonging to the higher modes. Because it is easier to estimate a smooth curve than a wrinkled one using a limited number of measurements, eigenfunctions belonging to the lower modes will be identified with higher accuracy than eigenfunctions belonging to the higher modes. This is not an important drawback, because we are interested in identifying the eigenfunctions belonging to the lower modes in the first place.

It was stated earlier that damping was assumed to be of the linear, proportional type. As a result of this assumption, in the identification problem for distributed systems, the vectors $\underset{\sim}{b}_r$ ($r = 1, 2, \ldots, 2m$) corresponding to the upper halves of $\underset{\sim}{E}_r$, i.e., the eigenvectors of $\hat{A}A^{-1}$, are considered to be real quantities.

If damping is not of the proportional type, then the vectors $\underset{\sim}{b}_r$ are not necessarily real quantities. This implies complex eigenfunctions. The problem of having complex eigenfunctions has not been investigated yet. In addition, the linearity assumption for damping is not fully justified. Some people consider the proportional damping assumption as a secondary error, because they consider the assumption of linear damping as a primary error. In any case, more work must be done to shed some light on the nature of damping for distributed systems. In view of the above, we shall ignore the imaginary parts of $\underset{\sim}{b}_r$ ($r = 1, 2, \ldots, 2m$) and identify the eigenfunctions by using the real parts of $\underset{\sim}{b}_r$. Actually, by examining the complex part of $\underset{\sim}{b}_r$ one can determine how good the proportional damping assumption is. A similar argument can be made

about the identified eigenvalues. Indeed, it can be shown that for proportional damping

$$\xi_r = \frac{1}{2}(g_1\omega_r + g_2/\omega_r), \quad r = 1, 2, \ldots \tag{3.32}$$

After ξ_r and ω_r are determined by using Eqs. (3.23), one can then use a few of the identified eigenvalues (at least two), to determine g_1 and g_2 and examine the validity of the proportional damping assumtpion.

It should also be noted that the eigenfunctions identified in this chapter are orthogonal, but are not in normalized form. To convert these eigenfunctions into normalized form, one must know the mass and stiffness distributions. These distributions will be identified by a method presented in the next chapter.

3.5 Computational Aspects

One problem that surfaces during identification of distributed systems is the presence of measurement noise. Whereas in discrete systems noise occurs in the form of measurement error only, in the case of distributed systems there also exists the problem of contamination of the output by the higher modes not included in the model. It was suggested earlier that low-pass filters be used to eliminate this spillover. Even then some problems exist. When low-pass filters are used and one goes from Eq. (3.26) to Eq. (3.27), an assumption is made that

$$\begin{aligned} \omega_m &\leq \omega_c \\ \omega_{m+1} &> \omega_c \end{aligned} \tag{3.33}$$

where ω_c is the cutoff frequency of the digital filter. Because ω_m is

not known in advance, it is very likely that Eqs. (3.33) will not be satisfied. For the case in which $\omega_m > \omega_c$, there are less modes to identify than the number of measurement points. This results in the appearance of noise modes [54]. As indicated in Ref. [54], these noise modes can be detected and separated from the system modes through a series of identification tests. For the case when $\omega_{m+1} < \omega_c$, the previously described problem of contamination of the output by ω_{m+1}, and any other frequencies higher than ω_m, but lower than ω_c, emerges. Reference [54] suggests a determinant check to determine whether Eqs. (3.33) are satisfied. Nevertheless, when digital filters are used, all the frequency components above the cutoff frequency cannot be eliminated, due to the aliasing problem. Some of the higher modes will not be filtered. But because these higher modes are likely to have very low amplitudes and tend to damp out quickly, their contamination can be regarded as measurement error.

To eliminate the noise problem and improve the accuracy of the identification procedure, Ref. [54] proposes some modifications. The first suggestion is to measure the system output at more than 2m instances and use a least squares minimization procedure to identify the eigensolution. Because the output is measured at more than 2m instances, A and $\hat{A}$ [Eqs. (3.31)] become rectangular, so that the eigensolution of the matrix

$$G = \hat{A}A^T(AA^T)^{-1} \tag{3.34}$$

is required. This is a method in which the eigenvalue problem remains of order 2m, so that the computational burden is not increased. In

addition, errors due to the contamination of the response by the higher modes and by measurement noise are minimized. The second suggestion is to use an overspecified model, with more sensors than the number of modes to be identified. Reference [54] indicates that this approach results in cancellation of some of the effects of measurement noise. Because the number of measurement points is increased, the order of the matrix $\hat{A}A^{-1}$ becomes higher than 2m, so that a larger order eigenvalue problem must be solved. One advantage of this approach is that there will be more points known along the domain of the distributed system, so that the eigenfunctions can be interpolated with higher accuracy. The disadvantage of the approach is that, as the order of the eigenvalue problem increases, computational difficulties arise. It should be noted that the identification scheme described here requires the eigensolution of a real general matrix, so that, an eigensolution may not be possible for large-order systems (> 120). Another problem is that the eigenvalues of the matrix $\hat{A}A^{-1}$ tend to be very closely spaced. This problem may cause additional computational difficulties and may result in inaccurate eigenvalues.

Yet another problem that arises during identification is the optimal location of sensors [41-44]. For the identification problem considered here, it is recommended that the sensors be located more closely at places where the mass or stiffness distribution changes rapidly. An optimal sensor location scheme is really not necessary.

CHAPTER 4

IDENTIFICATION OF THE MASS AND STIFFNESS DISTRIBUTIONS

4.1 Introduction

In this chapter, a method is presented for the identification of the parameters entering into the equations of motion of distributed systems. These parameters are the mass, stiffness and damping distributions, which are in general continuous functions of the spatial variables.

For discrete systems, the idea of identifying the mass and stiffness parameters by using the eigensolution is considered in Refs. [15-25]. The general concepts in this chapter are similar to those in Refs. [15-25], but the approach is quite different.

Identification of the actual distributed system is considered in Refs. [45-50]. In Ref. [47], identification of parameters is carried out by expanding these parameters in terms of finite series. The identification is implemented by Green's function transformation. In this chapter, the mass, stiffness and damping distributions are expanded in a finite series of known functions of the spatial variables, multiplied by undetermined coefficients. Then, use is made of the least-squares method, in conjunction with the orthogonality of the eigenfunctions, to identify the undetermined coefficients. The eigenvalues and eigenfunctions are identified by the method described in Chapter 3.

4.2 The Identification Procedure

It is assumed here that the nature of the equations of motion is known, in the sense that the form of the mass, stiffness and damping operators [see Eq. (2.1)] is known. However, explicit expressions for the mass, stiffness and damping distributions contained in these operators are not known. Generally, the mass operator is merely a function representing the mass distribution itself. The stiffness operator is a differential operator of order 2p, and contains the stiffness at any point. For example, for the bending vibration of a beam the stiffness operator is

$$L = \frac{d^2}{dx^2}\left[EI(x)\frac{d^2}{dx^2}\right] \tag{4.1}$$

and the stiffness at a point is

$$K(x) = EI(x) \tag{4.2}$$

where E is the modulus of elasticity and I is the mass moment of inertia, in which x is the spatial variable. It was assumed earlier that damping is of the linear and proportional type. This implies that the damping operator is a linear combination of the mass and stiffness operators. As a result, one must only identify the mass and stiffness distributions. The constants g_1 and g_2 that appear in Eqs. (2.13) and (3.3) can easily be determined from the identified eigenvalues.

Because the mass and stiffness distributions are continuous functions of the spatial variables, one can expand them in terms of a series of known functions of the spatial variables multiplied by undetermined coefficients. This idea is very similar to that behind the expansion theorem [72], but the expansion functions here need not be

orthogonal. For an exact representation of the mass and stiffness distributions, an infinite number of terms are required in the expansions. However, one must truncate the series for computational purposes. We can expand the mass and stiffness distributions as

$$M(P) = \sum_{r=1}^{\ell_1} d_r M_r(P)$$

$$K(P) = \sum_{r=1}^{\ell_2} e_r \theta_r(P) \qquad (4.3a,b)$$

where $M_r(P)$ and $\theta_r(P)$ are functions from a complete set, d_r and e_r are undetermined coefficients and ℓ_1 and ℓ_2 are the number of terms in the mass and stiffness distributions, respectively. The functions $M_r(P)$ and $\theta_r(P)$ can be global or local functions.

The stiffness operator can also be expressed in terms of a finite series as follows:

$$L = \sum_{r=1}^{\ell_2} e_r L[\theta_r(P),P] = \sum_{r=1}^{\ell_2} e_r L_r \qquad (4.4)$$

where we note that the stiffness operator is expressed in terms of expansion functions $\theta_r(P)$. As mentioned before, the mass operator is the mass distribution itself.

We assume that q eigenvalues and eigenfunctions are known. The orthogonality relations, Eqs. (2.5) and (2.6) indicate that the eigenfunctions can be normalized with respect to the mass and stiffness distributions. Because the mass and stiffness distributions are not known, the amplitudes of the identified eigenfunctions are not unique. They can only be identified to within a multiplicative constant.

Hence, we write

$$\phi_r' = a_r \phi_r, \quad r = 1, 2, \ldots, q \tag{4.5}$$

where ϕ_r' and ϕ_r ($r = 1, 2, \ldots, q$) are the identified and actual eigenfunctions and a_r are the multiplicative constants. Introducing Eqs. (4.5) into the orthogonality conditions, Eqs. (2.5) and (2.6), we obtain

$$\begin{aligned} \int_D M\phi_r'\phi_s' dD &= a_r a_s \int_D M\phi_r\phi_s dD = a_r^2 \delta_{rs} \\ \int_D \phi_r' L \phi_s' dD &= a_r a_s \int_D \phi_r L \phi_s dD = a_r^2 \omega_r^2 \delta_{rs} \end{aligned} \qquad r,s = 1, 2, \ldots, q \tag{4.6}$$

Considering Eqs. (4.3a), (4.4) and the orthogonality relations, we can write

$$\begin{aligned} \int_D M\phi_r'\phi_s' dD &= \sum_{j=1}^{\ell_1} d_j \int_D M_j \phi_r'\phi_s' dD \\ \int_D \phi_r' L \phi_s' dD &= \sum_{j=1}^{\ell_2} e_j \int_D \phi_r' L_j \phi_s' dD \end{aligned} \qquad r,s = 1, 2, \ldots, q \tag{4.7}$$

where we note that the integrands on the right sides of Eqs. (4.7) contain known or identified functions. Denoting these integrands by

$$U_{rsj} = \int_D \phi_r' M_j \phi_s' dD, \quad r,s = 1, 2, \ldots, q; \; j = 1, 2, \ldots, \ell_1 \tag{4.8a}$$

$$V_{rsj} = \int_D \phi_r' L_j \phi_s' dD, \quad r,s = 1, 2, \ldots, q; \; j = 1, 2, \ldots, \ell_2 \tag{4.8b}$$

and considering Eqs. (4.7) and (4.8), we obtain

$$\sum_{j=1}^{\ell_1} d_j U_{rsj} = a_r^2 \delta_{rs}$$

$$\sum_{j=1}^{\ell_2} e_j V_{rsj} = a_r^2 \omega_r^2 \delta_{rs} \qquad r,s = 1, 2, \ldots, q \qquad (4.9a,b)$$

Equations (4.9) represent q^2 relations for each of the three sets of the unknown coefficients d_j ($j = 1, 2, \ldots, \ell_1$), e_j ($j = 1, 2, \ldots, \ell_2$) and a_r ($r = 1, 2, \ldots, q$). Because the distributed system is self-adjoint (see Section 2.2), not all of the q^2 relations are independent. This is so because

$$U_{rsj} = U_{srj}, \quad V_{rsj} = V_{srj} \qquad (4.10)$$

In view of Eqs. (4.10), the number of independent relations for the unknown coefficients is reduced to $q(q + 1)/2$, so that one is faced with the problem of solving two sets of $q(q + 1)/2$ equations for two sets of $q + \ell_1$ and $q + \ell_2$ unknowns. Because q of the unknowns a_r ($r = 1, 2, \ldots, q$) are the same in both sets of undetermined coefficients, one can solve for the first set of $q + \ell_1$ ($q + \ell_2$) unknowns by using one of Eqs. (4.9), and solve for the remaining ℓ_2 (ℓ_1) coefficients of the second set, by using the remaining relation in Eqs. (4.9). We assume that Eqs. (4.9a), which contain the coefficients associated with the mass distribution, will be solved first. Note that, the choice as to which set of coefficients are to be solved for first is arbitrary.

Defining the matrices U and T of orders $q(q + 1)/2 \times \ell_1$ and $q(q + 1)/2 \times q$ as

$$U = \begin{bmatrix} U_{111} & U_{112} & \cdots & U_{11\ell_1} \\ U_{121} & U_{122} & \cdots & U_{12\ell_1} \\ \vdots & & & \vdots \\ U_{qq1} & U_{qq2} & \cdots & U_{qq\ell_1} \end{bmatrix} \tag{4.11a}$$

$$T = \begin{bmatrix} -1 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & 0 & \cdots & -1 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & -1 & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & -1 \end{bmatrix} \tag{4.11b}$$

and introducing the vectors

$$\underset{\sim}{d} = \begin{bmatrix} d_1 & d_2 & \cdots & d_{\ell_1} \end{bmatrix}^T, \quad \underset{\sim}{a} = [a_1^2 \quad a_2^2 \quad \cdots \quad a_q^2]^T \tag{4.12}$$

we can write Eqs. (4.8a) as

$$[T \,\vdots\, U] \begin{bmatrix} \underset{\sim}{a} \\ \cdots \\ \underset{\sim}{d} \end{bmatrix} = \underset{\sim}{0} \tag{4.13}$$

Note that, the right side of Eq. (4.13) is zero. Because of this, one cannot obtain a unique solution of Eq. (4.13). To solve Eq. (4.13)

uniquely, we must have some additional information about the distributed system. One such piece of additional information can be knowledge about the mass distribution at a certain point P_0 along the domain of the distributed-parameter system. If the value of the mass distribution is known at point P_0, by considering Eq. (4.3a), we can write

$$\sum_{j=1}^{\ell_1} d_j M_j(P_0) = M(P_0) \tag{4.14}$$

The availability of Eq. (4.14) enables us to add one row to Eq. (4.12), so that, Eq. (4.12) becomes

$$\left[\begin{array}{c|c} \underset{\sim}{T} & \underset{\sim}{U} \\ \hline \underset{\sim}{0}^T & \underset{\sim}{M}^T \end{array}\right] \left[\begin{array}{c} \underset{\sim}{a} \\ \hline \underset{\sim}{d} \end{array}\right] = \left[\begin{array}{c} \underset{\sim}{0} \\ \hline M(P_0) \end{array}\right] \tag{4.15}$$

where

$$\underset{\sim}{M} = \left[M_1(P_0) \quad M_2(P_0) \quad \cdots \quad M_{\ell_1}(P_0)\right]^T \tag{4.16}$$

The matrix on the left side of Eq. (4.15) is of order $[q(q+1)/2 + 1] \times (q + \ell_1)$. We shall denote it by U*. For most cases it is not square, and it generally defines an overspecified model. We can then use a least-squares or pseudo-inverse type approach to determine $\underset{\sim}{a}$ and $\underset{\sim}{d}$. For cases when an overspecified model is encountered, one can convert U* into a square matrix by ignoring some of the orthogonality relations. It turns out that such an approach is not necessary. Indeed, because the othogonality conditions are consistent, having more rows than columns does not imply that the mass distribution is identified as a least-squares fit. The actual mass distribution satisfies all

of the orghogonality relations, so that having more rows than columns in the matrix U* merely implies redundancy. Redundancy occurs if the ℓ_1 terms in the expansion of the mass distribution can describe it exactly. For cases when the mass distribution cannot be represented exactly by a series of ℓ_1 terms, we need as many orthogonality relations as possible to obtain an accurate description of the mass distribution. Note that, we cannot determine the number of terms necessary to describe the mass distribution in advance. In view of the above, it is advisable to retain all of the available orthogonality relations in Eq. (4.13). Using more equations than unknowns is equivalent to identifying a larger number of coefficients in the expansion of the mass distribution than ℓ_1, in which the surplus coefficients are identified as zero. If U* is changed into a square matrix, the terms in the expansion of the mass distribution higher than ℓ_1 are totally ignored.

Upon obtaining the solution for the mass distribution and the normalization factors for the eigenfunctions, we can obtain the solution for the stiffness distribution by using Eq. (4.9b), where we are faced with only ℓ_2 unknowns, namely, the coefficients of the terms in the expansion of the stiffness distribution. We can represent Eq. (4.9b) in matrix form as

$$\underset{\sim}{V}\underset{\sim}{e} = \underset{\sim}{z} \tag{4.17}$$

where

$$V = \begin{bmatrix} V_{111} & V_{112} & \cdots & V_{11\ell_2} \\ V_{121} & V_{122} & \cdots & V_{12\ell_2} \\ \vdots & & & \vdots \\ V_{qq1} & V_{qq2} & \cdots & V_{qq\ell_2} \end{bmatrix} \tag{4.18}$$

is a $[q(q + 1)/2] \times \ell_2$ matrix, and

$$\underset{\sim}{e} = \begin{bmatrix} e_1 & e_2 & \cdots & e_{\ell_2} \end{bmatrix}^T \tag{4.19}$$

$$\underset{\sim}{z} = \begin{bmatrix} a_1^2\omega_1^2 & 0 & 0 & \cdots & a_{q-2}^2\omega_{q-2}^2 & 0 & 0 & a_{q-1}^2\omega_{q-1}^2 & 0 & a_q^2\omega_q^2 \end{bmatrix}^T$$

are vectors of dimensions ℓ_2 and $q(q + 1)/2$. By inverting Eq. (4.17), we can determine the coefficients of the functions in the expansion of the stiffness distribution, thus identifying the stiffness distribution. The discussion presented earlier in connection with rectangular matrices is equally valid for the case of identification of the stiffness distribution.

4.3 General Considerations

The identification method described in this chapter requires the inversion of two matrices. These matrices are in general rectangular and correspond to overspecified models. Both the pseudo-inverse and least-squares approaches require the inversion of a symmetric positive definite matrix. With the exception of very ill-conditioned matrices, the inversion of a positive definite symmetric matrix is a very stable

operation, so that dealing with rectangular matrices does not represent a computational problem.

It was assumed that the value of the mass distribution was known at least at one point along the domain of the distributed-parameter system. The idea behind this assumption is that an average value of the mass distribution is available. For cases when the mass distribution is known at more than one point, one can include this information in the problem formulation as well. In fact, it is recommended that one include as much information as possible into the least-squares identification scheme.

In the above analysis, no prior knowledge of the mass and stiffness distributions was assumed. In most cases, however, the analyst has a reasonably good initial estimate of the mass and stiffness distributions. Inclusion of the initial estimate in the identification procedure is likely to increase the accuracy of the identification. Equations (4.3) can be augmented by the addition of the initial estimates as

$$M(P) = M_0(P) + \sum_{r=1}^{\ell_1} d_r M_r(P)$$

$$K(P) = K_0(P) + \sum_{r=1}^{\ell_2} e_r \theta_r(P) \qquad (4.20a,b)$$

where $M_0(P)$ and $K_0(P)$ represent the initial estimates of the mass and stiffness distributions. In view of the above, Eq. (4.4) becomes

$$L = L_0 + \sum_{r=1}^{\ell_2} e_r L_r \qquad (4.21)$$

where $L_0 = L_0[K_0(P),P]$ represents the stiffness operator that acts on the initial estimate of the stiffness distribution. Substituting Eqs. (4.20a) and (4.21) into the orthogonality relations, Eqs. (2.5) and (2.6), and considering Eqs. (4.6)-(4.10), we obtain

$$\sum_{j=1}^{\ell_1} d_j U_{rsj} = a_r^2 \delta_{rs} - U_{rs0}$$

$$r = 1, 2, \ldots, q, \; s = r, r+1, \ldots, q \qquad (4.22)$$

$$\sum_{j=1}^{\ell_2} e_j V_{rsj} = a_r^2 \omega_r^2 \delta_{rs} - V_{rs0}$$

where

$$U_{rs0} = \int_D M_0 \phi_r' \phi_s' dD$$

$$V_{rs0} = \int_D \phi_r' L_0 \phi_s' dD \qquad (4.23)$$

In addition, the information about the mass distribution at point P_0 can be expressed as

$$M(P_0) = M_0(P_0) + \sum_{j=1}^{\ell_1} d_j M_j(P_0) \qquad (4.24)$$

We note from the above that in the presence of initial conditions the left sides of Eqs. (4.15) and (4.17) remain unchanged. Only the vectors on the right sides of Eqs. (4.15) and (4.17) change, which implies that the identification procedure can be carried out for different initial estimates without added computational effort.

One may question the desire to identify the actual distributed system. The justification for this approach is that one obtains a complete representation by identifying the actual system. Another

justification for identifying the mass distribution will become evident in Chapter 5, during control implementation. In Chapter 5, modal filters are developed to extract modal displacements and velocities from the system output. To implement the modal filters, one must know the mass distribution, in addition to the eigensolution associated with the controlled modes.

One way of checking the accuracy of the identification procedure is to solve the eigenvalue problem associated with the identified mass and stiffness distributions, and compare the computed eigensolution with the eigensolution identified earlier. The identification is not complete if the two eigensolutions do not match. In such cases, one or more of the following changes can be implemented:

(a) Increase or decrease the number of terms in the expansions of the mass and stiffness distributions.

(b) Change the functions used to expand these distributions.

(c) Use the identified mass and stiffness distributions as the new initial estimates.

(d) Conduct another time-domain analysis to identify the eigenfunctions.

(e) Change the number of identified eigenfunctions used for the orthogonality conditions.

As can be seen from the above, a great deal of variables can be changed during the identification process. One is well advised to perform a series of sensitivity analyses to determine whether the identification results are accurate.

4.4 Illustrative Example

As an illustration of the identification methods described in Chapters 3 and 4, let us consider the problem of identification of the parameters of a tapered beam in axial vibration. The mass and stiffness distributions are given by

$$M(x) = 2(1 - x)$$
$$0 < x < 1 \tag{4.25}$$
$$EA(x) = 2(1 - x)$$

where we note that the beam length is chosen as 1. The stiffness operator is

$$L = -\frac{d}{dx}\left[EA(x)\frac{d}{dx}\right] \tag{4.26}$$

The boundary conditions are

$$B_1(0) = 1$$
$$\tag{4.27}$$
$$B_2(1) = EA\frac{d}{dx}$$

This particular problem is chosen as a numerical example because its closed-form eigensolution is available, so that the identified eigensolution can be compared with the actual one. In addition, the eigenvalues are closely spaced, and the mass and stiffness distributions vanish at one end, which make the identification of this beam difficult.

It can be shown that the associated eigenvalue problem lends itself to a closed-form solution. The transcendental equation takes the form

$$J_0(\omega_r) = 0, \quad r = 1, 2, \ldots \tag{4.28}$$

where J_0 is the zeroth order Bessel function of the first kind and ω_r is the frequency of undamped oscillation. The solution of Eq. (4.28) can be found in any textbook or mathematical tables (see, for example Ref. [115], p. 469, table 9.5). It can also be shown that the normalized eigenfunctions have the form

$$\phi_r(x) = \frac{J_0[\omega_r(1 - x)]}{J_1(\omega_r)}, \quad r = 1, 2, \ldots \tag{4.29}$$

where J_1 is the first order Bessel function of the first kind.

Damping was assumed to be of proportional type and the proportionality constants g_1 and g_2, Eq. (2.6) were chosen as

$$g_1 = 0.002, \quad g_2 = 0.1 \tag{4.30}$$

Let us first identify the eigensolution associated with the beam by using the ITD method. It was assumed that only the first 20 modes contribute to the overall motion. The identification procedure was carried out by using 12, 15 and 18 sensors. The response was obtained for a unit impulse, applied at $x = 0.63$. It can be shown that the unit impulse has the effect of an initial modal velocity on each mode. The time increment was chosen as $\Delta t = 0.01$ s, and the identification procedure was implemented by using 6m time points so that the matrices A and $\hat{A}$ are of order $2m \times 6m$ ($m = 12, 15, 18$). The sensors locations were taken as

$$x_i^s = \frac{i}{m}, \quad i = 1, 2, \ldots, m; \quad m = 12, 15, 18 \tag{4.31}$$

Table 4.1 shows the identified modes for the case when 12, 15 and 18

sensors are used and it compares them with the actual eigenvalues. From Table 1, we conclude that even for the case in which 12 modes are identified and 20 modes contribute to the system response, the lower modes are identified with sufficient accuracy. The accuracy of the estimation improves as the number of sensors is increased.

Let us now compare the accuracy of the identified eigenfunctions. Figure 4.1 compares the identified eigenfunction belonging to the eighth mode, $\phi_8(x)$, with the actual one. It is seen that there almost is no difference between the identified and actual eigenfunctions. A similar result was obtained for the eigenfunctions belonging to the lower modes.

For the identification of the eigenfunctions, the sensors data was interpolated by using cubic splines [113]. The reason why cubic splines are used will become evident during the second part of the identification process.

Next, let us identify the mass and stiffness distributions. We assume that they can be expanded in the form

$$M(x) = \sum_{r=1}^{\ell_1} d_r x^{r-1}$$

$$K(x) = \sum_{r=1}^{\ell_2} e_r x^{r-1} \qquad (4.32)$$

We also assume that the value of the mass distribution is known at the point $x = 0.5$. Then using Eqs. (4.7), (4.13) and (4.18) the coefficients d_r and e_r were determined. Tables 4.2 and 4.3 compare the identified coefficients d_r and e_r for different numbers of sensors and

different numbers of terms ℓ_1 and ℓ_2. Clearly, as the number of sensors is increased, the identified mass and stiffness distributions approach the actual ones. It is also observed that by using a lower number of terms in the expansions the coefficients d_r and e_r are identified with greater accuracy. Note that only the first four identified eigenfunctions are used to identify the mass and stiffness distributions, so that $q = 4$.

Figures 4.2 and 4.3 compare the exact and identified mass and stiffness distributions for different numbers of sensors and for 4 terms in the expansion. We note that even when the coefficients of d_r and e_r do not agree with their actual values exactly the identified mass and stiffness distributions are very close to the actual ones.

It should be noted that the V_{rsj} terms defined by Eq. (4.8b) require integrations involving the eigenfunctions and the stiffness operator. Because the stiffness operator is a differential operator, the identified eigenfunctions must have continuous derivatives through order $2p$ ($p = 1$ for our case). This is one reason why cubic splines are used as interpolation functions. Indeed, cubic splines interpolate the data points such that the estimate of the displacement of the beam will have continuous first and second derivatives. Because one can design $2p + 1$ ($p = 1, 2, \ldots$) order splines and the design can be in one or two dimensions [113], the use of splines as interpolation functions is fully justified.

Table 4.1

Identified Eigenvalues for Different Numbers of Sensors

r	m = 12		m = 15		m = 18		Exact	
	ξ_r	ω_r	ξ_r	ω_r	ξ_r	ω_r	ξ_r	ω_r
1	0.0232	2.4048	0.0231	2.4048	0.0232	2.4048	0.0232	2.4048
2	0.0146	5.5201	0.0146	5.5201	0.0146	5.5201	0.0146	5.5201
3	0.0145	8.6538	0.0144	8.6537	0.0144	8.6537	0.0144	8.6537
4	0.0159	11.791	0.0160	11.792	0.0160	11.792	0.0160	11.792
5	0.0250	14.963	0.0182	14.931	0.0183	14.931	0.0183	14.931
6	0.0212	18.077	0.0208	18.071	0.0208	18.071	0.0208	18.071
7	0.0238	21.180	0.0235	21.211	0.0235	21.212	0.0236	21.212
8	0.0264	24.375	0.0264	24.352	0.0264	24.353	0.0264	24.352
9	0.0284	27.506	0.0293	27.495	0.0293	27.493	0.0293	27.493
10	0.0664	33.436	0.0353	30.548	0.0322	30.636	0.0322	30.635
11	0.0670	36.870	0.03606	33.965	0.0353	33.776	0.0353	33.776
12	0.0065	38.921	0.0341	37.157	0.0382	36.916	0.0382	36.917

Table 4.1 (Continued)

Identified Eigenvalues for Different Numbers of Sensors

r	m = 12		m = 15		m = 18		Exact	
	ξ_r	ω_r	ξ_r	ω_r	ξ_r	ω_r	ξ_r	ω_r
13			0.0362	42.342	0.0409	40.048	0.0413	40.058
14			0.0879	46.275	0.0447	43.203	0.0444	43.200
15			0.0242	49.589	0.0448	46.384	0.0474	46.341
16					0.0417	50.094	0.0505	49.483
17					0.0893	54.163	0.0536	52.624
18					0.0370	58.991	0.0567	55.766
19							0.0596	58.907
20							0.0629	62.048

Table 4.2

Identified Mass and Stiffness Coefficients*

Mass Coefficients

m	12	15	18	Exact
d_1	2.0000	2.0007	2.0002	2.0000
d_2	-1.9997	-2.0022	-2.0005	-2.0000
d_3	-0.0004	0.0017	0.0004	0.0000

Stiffness Coefficients

m	12	15	18	Exact
e_1	2.0033	1.9990	1.9997	2.0000
e_2	-2.0104	-1.9965	-1.9989	-2.0000
e_3	0.0028	-0.0025	-0.0008	0.0000

*3 terms in the expansion ($\ell_1 = \ell_2 = 3$)

4 eigenfunctions are used ($q = 4$)

Table 4.3

Identified Mass and Stiffness Coefficients*

Mass Coefficients

m	12	15	18	Exact
d_1	2.0062	1.9986	1.9996	2.0000
d_2	-2.0337	-1.9908	-1.9974	-2.0000
d_3	0.0587	-0.0184	-0.0050	0.0000
d_4	-0.0320	0.0109	0.0029	0.0000

Stiffness Coefficients

m	12	15	18	Exact
e_1	1.9964	1.9997	1.9997	2.0000
e_2	-1.9490	-2.0042	-2.0001	-2.0000
e_3	-0.1115	0.0127	0.0018	0.0000
e_4	0.0596	-0.0083	-0.0015	0.0000

*4 terms in the expansion ($\ell_1 = \ell_2 = 4$)

4 eigenfunctions are used ($q = 4$)

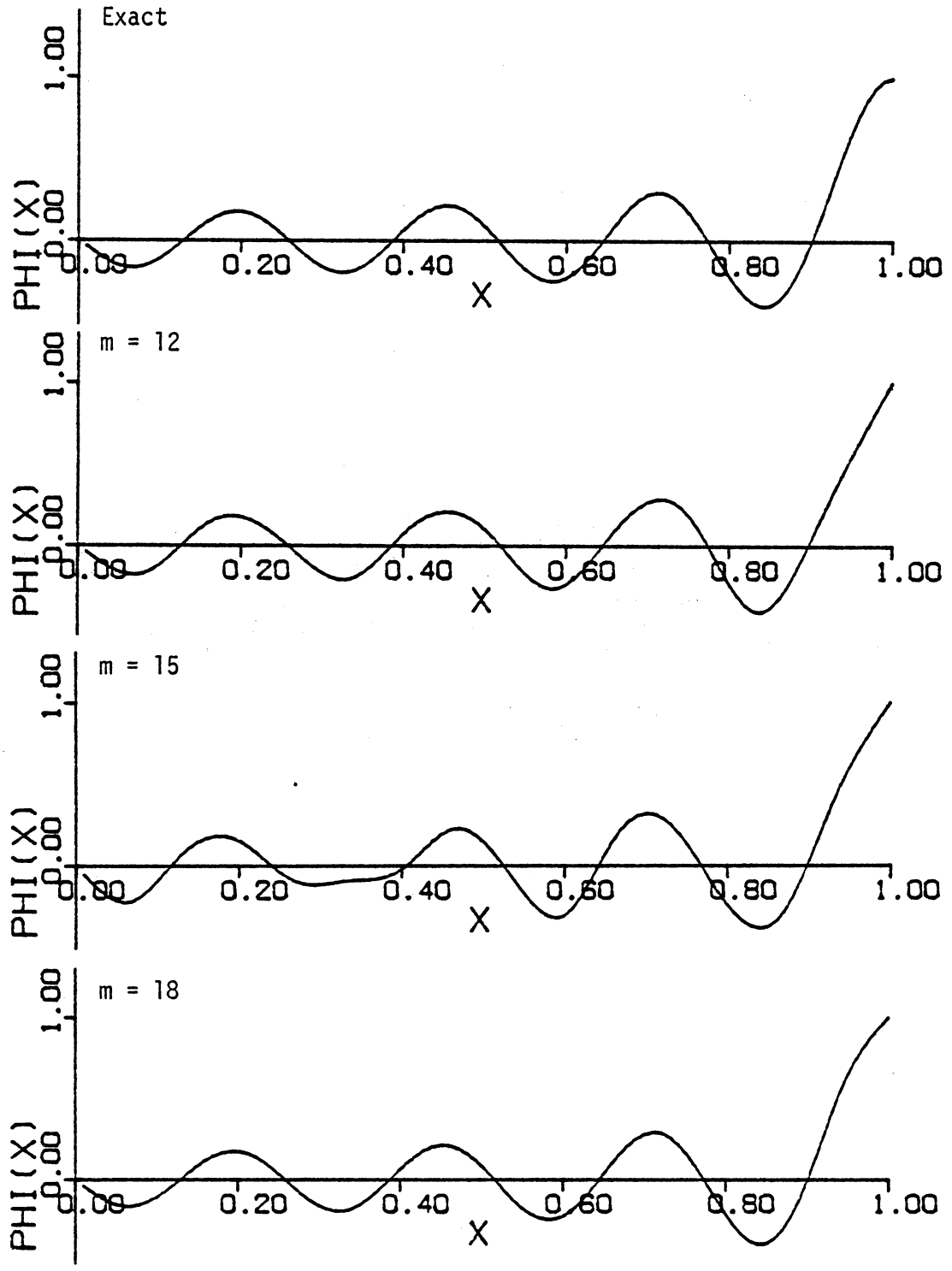

Figure 4.1 Exact and identified eighth eigenfunction, $\phi_3(x)$.

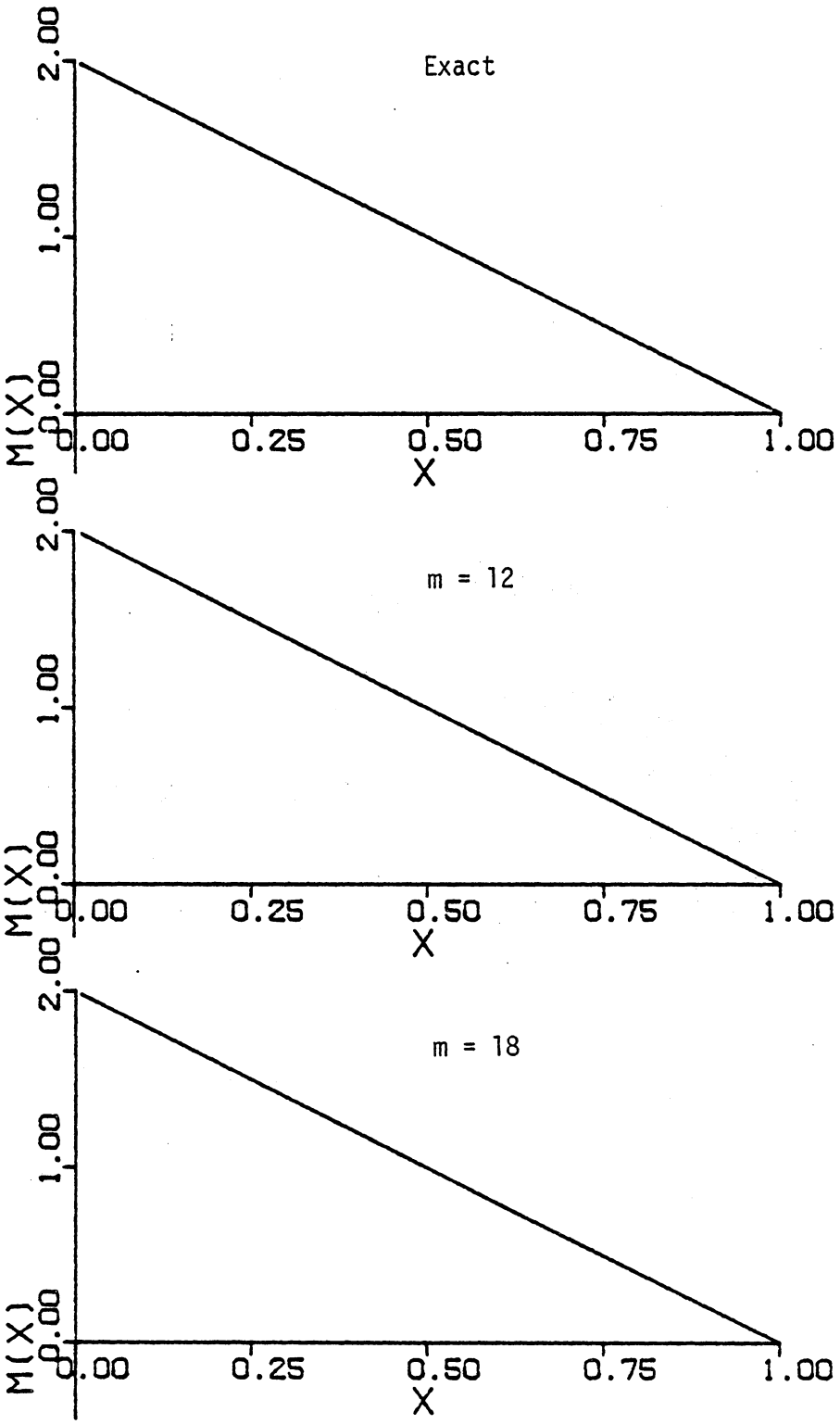

Figure 4.2 Exact and identified mass distribution.

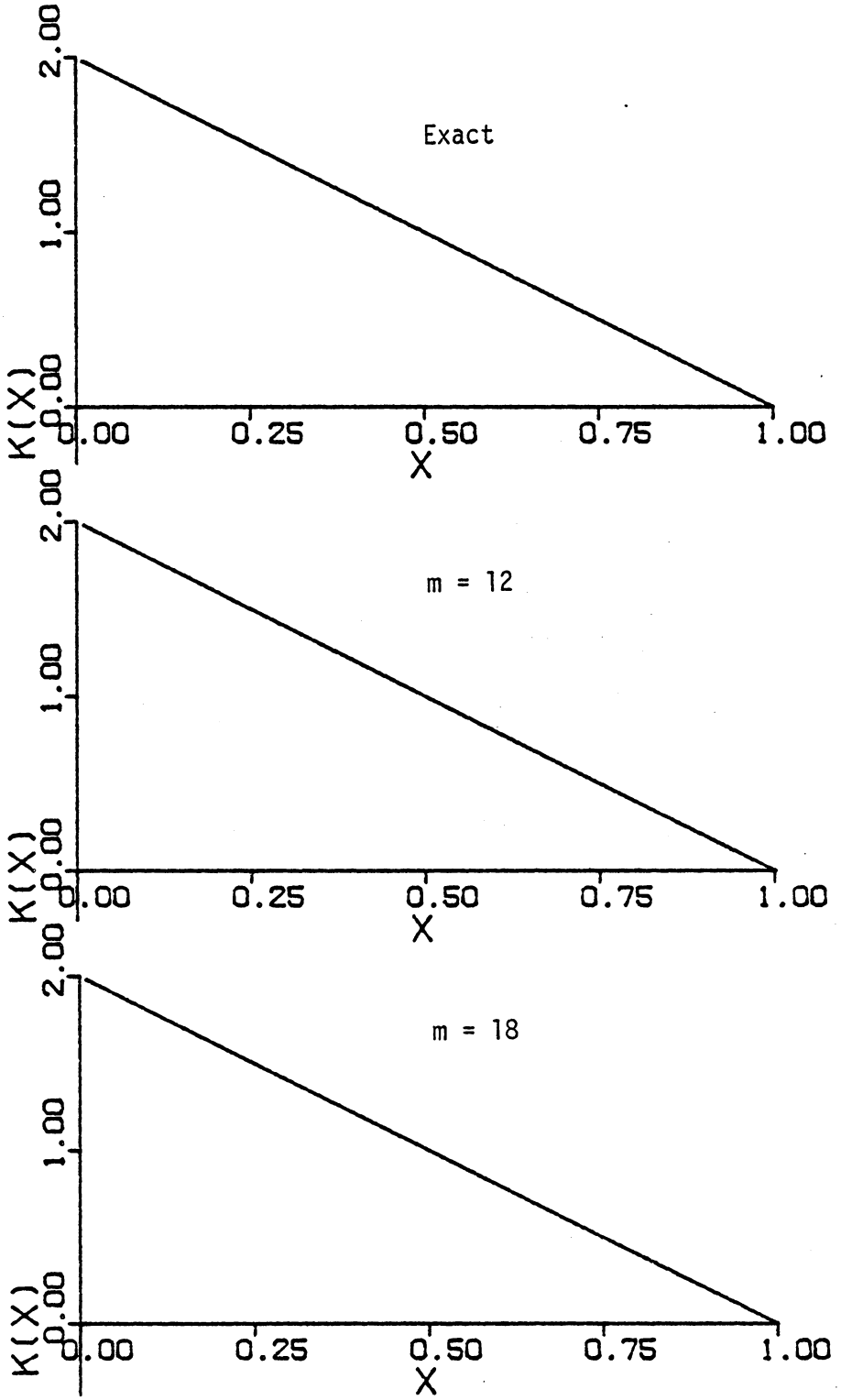

Figure 4.3 Exact and identified stiffness distribution.

CHAPTER 5

CONTROL DESIGN AND IMPLEMENTATION

5.1 Introduction

Modeling is the first step in a control process. It is assumed here that an accurate model of the distributed system is available. We also assume that a large number of eigenvalues and eigenfunctions are known with sufficient accuracy. The second and third steps of a control process consist of control system design and control implementation.

The control system design proposed here is based on the concept of independent modal-space control. This approach is adopted because independent modal-space control has distinct advantages over other control methods both from design and computational viewpoints [116]. In designing the control system, the damping properties of the distributed system are ignored. Note that elimination of damping during control system design is a conservative approach, because damping tends to suppress the vibratory motion.

Control implementation consists of two parts. The first is to extract the displacements and velocities of the controlled modes, and the second is to compute the actual control forces to act on the distributed system. Clearly, control implementation depends on the control system design.

To control a distributed-parameter system, one should have distributed measurements and should apply distributed forces. Because distributed sensors and actuators are not commercially available, one must implement the control forces by means of discrete sensors and

actuators. The use of discrete sensors and actuators results in what has come to be known as observation and control spillover [81].

The purpose of this chapter is to use discrete sensors and actuators in such a way as to minimize the spillover effects. To this end, a new mechanism for extracting modal coordinates from the sensors output, referred to as modal filters, is introduced. It is shown here that by using modal filters one can eliminate observation spillover, thus being able to control the actual distributed system. The control spillover effect is investigated qualitatively and quantitatively, and a method of minimizing this spillover by properly selecting the actuator locations is described.

5.2 Independent Control in the Modal Space

In the absence of damping, the modal state equations, Eqs. (2.15) become

$$\ddot{u}_r(t) + \omega_r^2 u_r(t) = f_r(t), \quad r = 1, 2, \ldots \tag{5.1}$$

Equations (5.1) have the appearance of an infinite set of independent second-order differential equations and in the absence of feedback control forces they indeed are. We refer to such decoupling as internal. If feedback control forces are present, however, and the modal feedback forces $f_r(t)$ depend on all the modal coordinates

$$f_r(t) = f_r(u_1, \dot{u}_1, u_2, \dot{u}_2, \ldots), \quad r = 1, 2, \ldots \tag{5.2}$$

then Eqs. (5.1) are coupled through the feedback controls. Hence, in this general case Eqs. (5.1) are internally decoupled but externally coupled, so that the equations are not independent. In the special case

in which $f_r(t)$ depends on u_r and $\dot{u}_r$ alone

$$f_r(t) = f_r(u_r, \dot{u}_r), \quad r = 1, 2, \ldots \tag{5.3}$$

Eqs. (5.1) become both internally and externally decoupled. Equations (5.3) imply that the modal control force f_r is designed independently of any coordinates other than u_r and $\dot{u}_r$. This is the essence of the independent modal-space control method. Independent modal-space control permits both linear and nonlinear control.

The design of the feedback control forces represents no computational difficulties whatsoever when the independent modal-space control method is used [73,77,97-104]. For example, in the case of the optimal control problem, a closed-form solution exists for the steady-state control gains [100]. Moreover, nonlinear on-off controls can be designed with relative ease [77] and the pole allocation method can be implemented readily. A comparison of independent modal-space control and coupled controls is given in Ref. [116], where it is shown that independent modal-space control is clearly superior to other control methods from design and computational viewpoints. Further justifications for the use of the independent modal-space control method will be presented later in this chapter.

In deciding which modes to control, physical considerations described in Sections 1.2 and 2.2 dictate that the lower modes be controlled. We propose to control the lowest n modes. As a result, we distinguish between two sets of modes, controlled and uncontrolled (residual). It should be noted that if a certain higher mode is excited

by external distrurbances, it too can be included in the set of controlled modes.

5.3 Distributed Sensors and Actuators

The independent modal-space control method, as well as any other feedback control method, requires the modal displacements $u_r(t)$ and modal velocities $\dot{u}_r(t)$ ($r = 1, 2, \ldots$). From the second part of the expansion theorem, Eqs. (2.8), we can write

$$u_r(t) = \int_D M(P)\phi_r(P)u(P,t)dD$$

$$\dot{u}_r(t) = \int_D M(P)\phi_r(P)\dot{u}(P,t)dD \qquad , \quad r = 1, 2, \ldots \tag{5.4}$$

Equations (5.4) can be regarded as modal filters [103]. They permit the extraction of $u_r(t)$ and $\dot{u}_r(t)$ from the measurements of displacement $u(P,t)$ and velocity $\dot{u}(P,t)$ at every point P of the domain D and at all times t. Having the generalized modal displacement $u_r(t)$ and velocity $\dot{u}_r(t)$, one can generate the generalized modal controls, thus being able to regulate every single mode.

Because the lowest n modes are controlled, one only needs to extract the amplitudes of the first n modal coordinates from the displacement and velocity profiles. The implication is that knowledge of only the first n eigenfunctions is required. Because the lower eigenfunctions of distributed systems are known more accurately than the higher eigenfunctions, modal filters can be implemented with relatively good accuracy.

Only n modes are controlled, which implies that there exists an infinity of uncontrolled modes. The question arises whether these residual modes are truly unmodeled. If the eigensolution associated with every mode is known, by using modal filters one can extract the amplitudes for each mode. Controlling n modes implies that only the n lowest modal displacements and velocitities need to be extracted from measurements by means of modal filters, and the higher modes play no particular role. The implication of the above is that all the system modes can be regarded as being modeled. This means that, by letting $n \to \infty$, control of the actual distributed-parameter system is possible when modal filters are used. In addition, observation spillover problems encountered when observers are used do not exist when modal filters are implemented.

The generalized modal controls $f_r(t)$ are only abstract forces and not actual control forces. The actual control force applied on the system is the distributed force $f(P,t)$, as indicated by the right side of Eq. (2.1). Hence, the question of extracting the generalized modal controls $f_r(t)$ from the actual distributed control $f(P,t)$ remains. It can be verified that the actual distributed control can be synthesized from the generalized modal controls $f_r(t)$ by writing

$$f(P,t) = \sum_{r=1}^{\infty} M(P)\phi_r(P)f_r(t) \tag{5.5}$$

Indeed, multiplication of both sides of Eq. (5.5) by $\phi_s(P)$, integration over the domain D and use of the orthonormality relations (2.5) yield Eqs. (2.12). Because only the first n modes are controlled, we select

the modal forces for the uncontrolled modes as

$$f_r(t) = 0 \quad r = n + 1, n + 2, \ldots \tag{5.6}$$

The first n modal control forces are determined by the techniques described by Refs. [76-77,97-104]. As a result no control spillover into the uncontrolled modes occurs.

It is clear from the above that if displacement and velocity profiles are available at all times, and the control forces can be applied at every point of the distributed domain, all the modes of a distributed system can be controlled. Our task, however, is to implement the modal filters and the control forces by means of discrete sensors and actuators, as distributed measurements and controls are not within the state of the art.

5.4 Spatial Interpolation of Discrete Measurements

Let us assume that there are k sensors capable of measuring displacements, velocities, slopes of displacements and slopes of velocities at the discrete points $P = P_j$ $(j = 1, 2, \ldots, k)$. Then, the question can be posed simply as to how large should the number of measurements be for an accurate estimate of $u(P,t)$ and $\dot{u}(P,t)$. This is not a new question and has been explored repeatedly, perhaps most recently in connection with the finite element method [71]. Indeed, the domain D can be divided into a given number of "finite elements" and the displacement can be measured at the nodal points. Then, using various interpolation functions, the entire displacement pattern $u(P,t)$ and velocity pattern $\dot{u}(P,t)$ can be estimated with sufficiently good accuracy. As far as the question of the number of measurements is concerned, this

is the same question as how many nodes must be taken in a finite element approximation of the system. Of course, the answer depends on the desired accuracy [71]. In our case, the accuracy should be such as to permit identification of the contribution of the first n modal displacements $u_r(t)$ ($r = 1, 2, \ldots, n$) to the overall motion $u(P,t)$. It should be intuitively clear that the number of measurements to achieve the desired accuracy depends on the participation of the various modes in the overall motion, and this number is problem dependent. The specific numerical example presented later illustrates how the accuracy of the displacement estimation is influenced by the number of measurements used. The implication is that for a given set of modes a certain number of sensors can estimate the modal coordinates and velocities for the controlled modes accurately, and moreover it is possible to determine that number with relative ease.

The modal filters estimate the modal coordinates from the outputs of a limited number of sensors. The displacements and velocities for the controlled modes can be estimated accurately from these limited measurements. Therefore, it can be said that having the right number of sensors is equivalent to having the output of the distributed system at every point of the domain. It follows that all the statements made earlier about modal filters hold when the right number of sensors is used. Therefore, control of the actual distributed system is possible and there is no model truncation. In addition, the problem of observation spillover, which arises when a limited number of sensors is used, is virtually eliminated. This is because modal filters process the

sensors data, so that discrete measurements are interpolated so as to yield equivalent distributed measurements.

Let us assume that the domain of the distributed system has been divided into K elements. Denoting the displacement in the ith element by $u_i(P,t)$ ($i = 1, 2, \ldots, K$), the modal filter equations, Eqs. (5.4), become

$$u_r(t) = \sum_{i=1}^{K} \int_{D_i} M(P)\phi_r(P)u_i(P,t)dD$$

$$\dot{u}_r(t) = \sum_{i=1}^{K} \int_{D_i} M(P)\phi_r(P)\dot{u}_i(P,t)dD \qquad , \quad r = 1, 2, \ldots, n \qquad (5.7)$$

where D_i ($i = 1, 2, \ldots, K$) is the domain of extension of the ith element. If the interpolation functions are chosen following the same guidelines as in the finite element method, one can approximate the displacement in the ith element by

$$\hat{u}_i(P,t) = \underset{\sim}{L}^T \underset{\sim}{y}_i(t) \qquad (5.8)$$

where $\hat{u}_i$ is the estimate of $u(P,t)$ in the ith interval, $\underset{\sim}{L}$ is a vector of interpolation functions from the finite element method and $\underset{\sim}{y}_i(t)$ is a vector containing the measured quantities such as displacements and slopes at the boundaries of the ith interval. For example, for one-dimensional systems, one can choose the components of $\underset{\sim}{L}$ as Hermite cubics [72].

Introducing Eq. (5.8) into Eqs. (5.7), we obtain

$$\hat{u}_r(t) = \sum_{i=1}^{K} \int_{D_i} M(P)\phi_r(P)\underset{\sim}{L}^T \underset{\sim}{y}_i(t)dD$$

$$\hat{\dot{u}}_r(t) = \sum_{i=1}^{K} \int_{D_i} M(P)\phi_r(P)\underset{\sim}{L}^T \dot{\underset{\sim}{y}}_i(t)dD \qquad r = 1, 2, \ldots, n \qquad (5.9)$$

where $\hat{u}_r$ and $\hat{\dot{u}}_r$ are the estimated modal quantities. Note that in Eqs. (5.9) none of the terms depends on time except $\underset{\sim}{y}_i(t)$ and $\dot{\underset{\sim}{y}}_i(t)$, so that the integrations can be carried out as off-line computations. Introducing the notation

$$\underset{\sim}{I}_{ir} = \int_{D_i} M(P)\phi_r(P)\underset{\sim}{L}dD, \quad i = 1, 2, \ldots, K; \; r = 1, 2, \ldots, n \qquad (5.10)$$

where the integrals can be computed before the control process begins, the estimated modal coordinates become

$$\hat{u}_r(t) = \sum_{i=1}^{K} \underset{\sim}{I}_{ir}^T \underset{\sim}{y}_i(t)$$

$$\hat{\dot{u}}_r(t) = \sum_{i=1}^{K} \underset{\sim}{I}_{ir}^T \dot{\underset{\sim}{y}}_i(t) \qquad r = 1, 2, \ldots, n \qquad (5.11)$$

The computation of $\underset{\sim}{I}_{ir}$ can be facilitated by performing the integrations in terms of local coordinates associated with each interval. Because on-line integrations are eliminated, modal filters can easily be implemented.

5.5 Discrete Actuators and Spillover into the Uncontrolled Modes

Let us consider the problem of controlling the lowest n modes by means of discrete actuators. The actuator forces can be treated as

distributed by writing

$$f(P,t) = \sum_{j=1}^{n} F_j(t)\delta(P - P_j) \tag{5.12}$$

where $\delta(P - P_j)$ is a spatial Dirac delta function applied at $P = P_j$. Note that we considered n forces $F_j(t)$ (j = 1, 2, ..., n) in recognition of the fact that the modal controls must be independent. Indeed, introducing Eqs. (5.12) into Eqs. (2.12), we obtain

$$f_r(t) = \sum_{j=1}^{n} \int_D \phi_r(P)F_j(t)\delta(P - P_j)dD = \sum_{j=1}^{n} \phi_r(P_j)F_j(t)$$

$$r = 1, 2, \ldots, n \tag{5.13}$$

Equations (5.13) have a unique solution provided the matrix

$$B = \begin{bmatrix} \phi_1(P_1) & \phi_1(P_2) & \cdots & \phi_1(P_n) \\ \phi_2(P_1) & \phi_2(P_2) & \cdots & \phi_2(P_n) \\ & & & \\ \cdots & \cdots & \cdots & \cdots \\ \phi_n(P_1) & \phi_n(P_2) & \cdots & \phi_n(P_n) \end{bmatrix} \tag{5.14}$$

in nonsingular. Introducing the n-vectors

$$f(t) = [f_1(t) \quad f_2(t) \quad \cdots \quad f_n(t)]^T$$

$$F(t) = [F_1(t) \quad F_2(t) \quad \cdots \quad F_n(t)]^T \tag{5.15a,b}$$

where the first is the generalized control vector and the second the actual control vector, we conclude that the actual controls can be synthesized from the generalized controls by writing

$$\underset{\sim}{F}(t) = B^{-1} \underset{\sim}{f}(t) \tag{5.16}$$

It is clear from Eqs. (5.14)-(5.16) that the independent modal-space control method requires the same number of actuators as controlled modes.

One drawback of having discrete actuators is that they cause control spillover. Indeed, if we introduce Eq. (5.12) into Eq. (2.12) we obtain

$$f_r(t) = \int_D \phi_r(P) \sum_{j=1}^{n} F_j(t)\delta(P - P_j)dD$$

$$= \sum_{j=1}^{n} \phi_r(P_j)F_j(t), \quad r = 1, 2, \ldots \tag{5.17}$$

Of course, the first n of Eqs. (5.17) are identical to Eqs. (5.13). On the other hand, because $f_r(t) \neq 0$ ($r = n + 1, n + 2, \ldots$), control spillover into the uncontrolled modes does occur.

As a result of control spillover, part of the energy imparted to the distributed-parameter system is pumped into the uncontrolled modes, which tends to degrade the control system performance [104].

First, we wish to explore the control spillover effect in a qualitative manner. Actuators designed to control lower modes tend to act in a random fashion on the more "wrinkled" higher modes and cancel each other's effect on these higher modes. Hence, whereas control spillover into the uncontrolled modes does exist, the importance may not be great. There are other reasons arguing against undue importance of the control spillover. As mentioned in Chapter 2, higher modes have higher frequencies of oscillations, so that for a given amount of energy imparted to a system, higher-frequency modes tend to have lower

amplitudes, i.e., they are more difficult to excite. It should be further remembered that any physical system has a certain amount of damping, and any such damping tends to have a larger effect on higher modes. Hence, even before the controlled modes are completely regulated, the residual modes are likely to be insignificant.

Next, let us explore the control spillover effect in a quantitative manner. The work done on the distributed-parameter system during the control process can be given as

$$W = \int_D \int_{u_0}^{u_f} f(P,t)du(P,t)dD \tag{5.18}$$

where u_0 and u_f are the initial and final displacements, respectively. Substitution of Eqs. (5.12) and (2.9) into Eq. (5.18) yields

$$W = \int_D \sum_{r=1}^{\infty} \int_{u_{r0}}^{u_{rf}} \sum_{j=1}^{n} F_j(t)\delta(P - P_j)\, \phi_r(P)du_r(t)dD$$

$$= \sum_{r=1}^{\infty} \int_{u_{r0}}^{u_{rf}} \sum_{j=1}^{n} F_j(t)\, \phi_r(P_j)du_r(t) \tag{5.19}$$

where u_{r0} and u_{rf} are the initial and final displacements of the $\underline{r}$th mode. Let us introduce the vectors corresponding to the controlled and residual modes

$$\underset{\sim}{u}_C = [u_1 \quad u_2 \quad \cdots \quad u_n]^T,$$

$$\underset{\sim}{u}_R = [u_{n+1} \quad u_{n+2} \quad \cdots]^T \tag{5.20a,b}$$

as well as the $\infty \times n$ matrix B_R corresponding to the residual modes

$$B_R = \begin{bmatrix} \phi_{n+1}(P_1) & \phi_{n+1}(P_2) & \cdots & \phi_{n+1}(P_n) \\ \phi_{n+2}(P_1) & \phi_{n+2}(P_2) & \cdots & \phi_{n+2}(P_n) \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \tag{5.21}$$

Considering Eqs. (5.14), (5.15), (5.20) and (5.21), Eq. (5.19) can be expressed in the matrix form

$$W = \int_{\underset{\sim}{u}_0}^{\underset{\sim}{u}_f} \underset{\sim}{F}^T \left[B^T \vdots B_R^T \right] \begin{bmatrix} d\underset{\sim}{u}_C \\ \hline d\underset{\sim}{u}_R \end{bmatrix} \tag{5.22}$$

The energy imparted to the distributed-parameter system can be divided into two parts

$$W = W_C + W_R \tag{5.23}$$

where W_C and W_R represent the energy going into the controlled and residual modes, respectively. Clearly, W_R is the spillover energy. From Eq. (5.22) it can be concluded that

$$W_C = \int_{\underset{\sim}{u}_{CO}}^{\underset{\sim}{u}_{Cf}} \underset{\sim}{F}^T B^T d\underset{\sim}{u}_C, \quad W_R = \int_{\underset{\sim}{u}_{RO}}^{\underset{\sim}{u}_{Rf}} \underset{\sim}{F}^T B_R^T d\underset{\sim}{u}_R \tag{5.24a,b}$$

where $\underset{\sim}{u}_{CO}$, $\underset{\sim}{u}_{Cf}$, $\underset{\sim}{u}_{RO}$ and $\underset{\sim}{u}_{Rf}$ are the initial and final displacements of the controlled and residual modes.

5.6 Design of Actuators Locations

In this section, we consider the effect of actuators locations on the amount of work done. Our goal is to select actuators locations such

that the work done to control the distributed parameter system is minimized.

The relationship between the actual control forces $\underset{\sim}{F}(t)$ and the generalized control forces $\underset{\sim}{f}(t)$ is given by Eq. (5.16), or

$$\underset{\sim}{F}(t) = B^{-1}\ \underset{\sim}{f}(t) \tag{5.25}$$

Introducing Eq. (5.25) into Eq. (5.24a) we obtain

$$W_C = \int_{\underset{\sim}{u}_{C0}}^{\underset{\sim}{u}_{Cf}} \underset{\sim}{f}(t)^T B^{-T} B^T d\underset{\sim}{u}_C(t) = \int_{\underset{\sim}{u}_{C0}}^{\underset{\sim}{u}_{Cf}} \underset{\sim}{\hat{f}}^T(t) d\underset{\sim}{u}_C(t) \tag{5.26}$$

Recalling the definitions of $\underset{\sim}{f}(t)$ and $\underset{\sim}{u}_C(t)$, Eqs. (5.15a) and (5.20a), we can write

$$W_C = \sum_{r=1}^{n} \int_{u_{r0}}^{u_{rf}} f_r(t) du_r(t) \tag{5.27}$$

Equation (5.27) has significant implications. It indicates that the work done to control the controlled modes does not depend on the actuator locations. Of course, this statement holds true if the matrix B is nonsingular, which is the case because no two actuators are located on the same point. The fact that the actuators locations are immaterial is yet another advantage of the independent-modal space control method as opposed to other control methods.

Because W_C does not depend on the actuator locations, we have the freedom of selecting the actuators locations such that the amount of energy going into the uncontrolled modes is minimized. To this end, let us examine qualitatively the energy that goes into the uncontrolled modes. The object in controlling a distributed-parameter system is to suppress the vibratory motion. This requires the control of a certain

number of modes. As mentioned earlier, generally the lower modes are controlled and the higher modes are left uncontrolled. Furthermore, it is known that higher modes possess high frequencies so that they require more energy to excite. Therefore, if a sufficient number of modes is controlled, the energy that is pumped into the residual modes loses significance and the design of actuator locations no longer represents a serious problem. It should also be noted that most actual systems have a certain amount of internal damping, which tends to drive the higher modes to equilibrium faster than the lower modes.

Next, let us investigate quantitatively the energy that goes into the residual modes. Introducing Eq. (5.25) into Eq. (5.24b) and differentiating, we obtain

$$W_R = \int_0^{t_f} \underset{\sim}{f}(t)^T B^{-T} B_R^T \dot{\underset{\sim}{u}}_R(t) dt \tag{5.28}$$

where t_f is the final time. We wish to express Eq. (5.28) in terms of $\underset{\sim}{f}(t)$. Introducing the infinite-dimensional residual force vector

$$\underset{\sim}{f}_R = [f_{n+1} \quad f_{n+2} \quad \ldots]^T \tag{5.29}$$

and the infinite-dimensional matrix

$$\Lambda_R = \text{diag}[\omega_{n+1}^2 \quad \omega_{n+2}^2 \quad \ldots] \tag{5.30}$$

and considering Eqs. (5.1), (5.15)-(5.17) and (5.20)-(5.21) we conclude that

$$\ddot{\underset{\sim}{u}}_R(t) + \Lambda_R \underset{\sim}{u}_R(t) = \underset{\sim}{f}_R(t) = B_R B^{-1} \underset{\sim}{f}(t) \tag{5.31}$$

The response problem associated with Eq. (5.31) can be shown to be

$$\underset{\sim}{u}_R(t) = \int_0^t \text{diag}\left[\frac{\sin \omega_r(t - \tau)}{\omega_r}\right] B_R B^{-1} \underset{\sim}{f}(\tau)d\tau,$$

$$r = n + 1, n + 2, \ldots \tag{5.32}$$

where diag $[\sin \omega_r(t - \tau)/\omega_r]$ is an infinite-dimensional diagonal matrix. The time derivative of $\underset{\sim}{u}_R(t)$ is

$$\frac{d\underset{\sim}{u}_R(t)}{dt} = \int_0^t \text{diag}[\cos \omega_r(t - \tau)] B_R B^{-1} \underset{\sim}{f}(\tau)d\tau \tag{5.33}$$

where diag$[\cos \omega_r(t - \tau)]$ is an infinite-dimensional diagonal matrix. The substitution of Eqs. (5.33) into Eq. (5.28) yields

$$W_R = \int_0^{t_f} \underset{\sim}{f}(t)^T B^{-T} B_R^T \left\{ \int_0^t \text{diag}[\cos \omega_r(t - \tau)] B_R B^{-1} \underset{\sim}{f}(\tau)d\tau \right\} dt \tag{5.34}$$

Equation (5.34) describes the energy pumped into the residual modes in terms of the modal control forces, where we note that the modal control forces are functions of the controlled modes.

Equation (5.34) is very cumbersome and it involves the infinite-dimensional B_R matrix. To gain some insight into the problem of actuator distribution, let us first consider the special case in which only one mode is controlled. Only one actuator is needed so that, $\underset{\sim}{f}(t)$ and B become scalars and B_R becomes an infinite-dimensional vector. Introduction of the values

$$B = \phi_1(P_1) = b_1$$

$$B_R = [\phi_2(P_1) \quad \phi_3(P_1) \quad \ldots]^T = [b_2 \quad b_3 \quad \ldots]^T \tag{5.35}$$

$$\underset{\sim}{f} = f$$

into Eq. (5.34) yields

$$W_R = \int_0^{t_f} f(t) \frac{1}{b_1} B_R^T \left\{ \int_0^t \text{diag}[\cos \omega_r(t - \tau)] B_R \frac{1}{b_1} f(\tau) d\tau \right\} dt \tag{5.36}$$

Because B_R is a vector, we conclude that

$$B_R^T \text{diag}[\cos \omega_r(t - \tau) B_R = \sum_{r=2}^{\infty} b_r^2 \cos \omega_r(t - \tau) \tag{5.37}$$

is a scalar, so that introducing Eq. (5.37) into Eq. (5.36) we obtain

$$W_R = \sum_{r=2}^{\infty} W_r \tag{5.38}$$

where

$$W_r = \frac{b_r^2}{b_1^2} \int_0^{t_f} f(t) \left\{ \int_0^t \cos\left[\omega_r(t - \tau) \right] f(\tau) d\tau \right\} dt \quad r = 2, 3, \ldots \tag{5.39}$$

in which W_r ($r = 2, 3, \ldots$) can be considered as the energy that goes into each residual mode. We therefore have an infinite sum to be minimized with only one parameter that can be varied, namely, the location of the single actuator.

All the W_r's ($r = 2, 3, \ldots$) cannot be minimized. One way of minimizing a specific W_r is to place the actuator at one of the nodes of the rth eigenfunction

$$b_r = \phi_r(P) = 0 \tag{5.40}$$

As a result $W_r = 0$, so that no energy goes into the rth mode. To determine which W_r to be minimized, energy considerations discussed earlier suggest that W_2 should be set equal to zero, because the second mode is the easiest one to excite among the residual modes.

The above development can be easily extended to the more general case where more than one mode is controlled. Indeed, assuming that there are n controlled modes, and introducing the vectors [see Eq. (3.28b)]

$$\underset{\sim}{b}_r = [\phi_r(P_1) \quad \phi_r(P_2) \quad \ldots \quad \phi_r(P_n)]^T, \quad r = n + 1, n + 2, \ldots \tag{5.41}$$

it can be shown that the energy pumped into the uncontrolled modes takes the form

$$W_R = \sum_{r=n+1}^{\infty} W_r \tag{5.42}$$

where

$$W_r = \int_0^{t_f} \underset{\sim}{f}^T(t) P_r \left\{ \int_0^t \cos \omega_r(t - \tau) \underset{\sim}{f}(\tau) d\tau \right\} dt$$

$$r = n + 1, n + 2, \ldots \tag{5.43}$$

and

$$P_r = B^{-T} \underset{\sim}{b}_r \underset{\sim}{b}_r^T B^{-1}, \quad r = n + 1, n + 2, \ldots \tag{5.44}$$

Equation (5.43) enables us to monitor the amount of energy that goes into each residual mode.

We are again faced with the problem of minimizing an infinite sum with only n parameters that can be varied. It seems reasonable to minimize W_{n+1}, i.e., the energy that goes into the first residual mode. This does not necessarily guarantee that W_R is minimized. One way of minimizing W_{n+1} is to set

$$\underset{\sim}{b}_{n+1} = \underset{\sim}{0} \tag{5.45}$$

so that W_{n+1} becomes identically zero. Equation (5.45) is satisfied if

and only if the actuators are placed on the nodes of the (n + 1)-st eigenfunction. Note that the (n + 1)-st eigenfunction has n nodes.

The whole control process can be summarized as follows: the first n modes are controlled by the independent modal-space control method, the (n + 1)-st mode vibrates freely with no external disturbances, and the remaining uncontrolled modes vibrate, while the n actuators excite them in a random fashion. It was stated earlier that Eq. (5.43) enables the analyst to assess the amount of energy that goes into each residual mode. The amount of energy that goes into each controlled mode is given by Eq. (5.27), so that one has the capability of monitoring the amount of energy that is pumped into each and every mode of the distributed-parameter system. This enables one to make a rational decision as to the number of modes in need of active control.

5.7 Computational Considerations

The control implementation techniques described in this chapter are extremely easy to apply. Modal filters can be implemented readily and since the independent modal-space control method is used, imparting the control forces to the distributed system does not represent a serious problem.

An advantage of modal filters is that they can be implemented for any kind of control method including nonlinear controls and coupled controls. Another advantage is that when modal filters are used, the system output is multiplied by the eigenfunctions and integrated over the distributed domain. But integration is a smoothing operation, so that any error associated with the discretization of the spatial domain

and errors arising from the use of discrete sensors tend to cancel each other out during the integration. This surely adds to the accuracy in estimating modal coordinates. In addition, observation spillover, a possible significant problem when observers are used can be eliminated when a sufficient number of sensors are used [107]. Moreover, when modal filters are used the selection of the sensors locations no longer represents a problem.

The actual control forces are obtained from the modal control forces by Eq. (5.16). Equation (5.16) requires the inverse of the matrix B, Eq. (5.14), which is of order n. Many stable algorithms exist for the computation of a matrix inverse [72], so that this operation does not represent a significant problem. Note that B is guaranteed to be nonsingular because no two actuators are located at the same point.

The actuators locations do not represent a serious problem since the energy required to control the controlled modes is independent of the actuators locations. This is another advantage of independent modal-space control over other control methods.

5.8 Illustrative Example

As an illustration of the control design and implementation techniques described in this chapter, let us consider the independent modal-space control of the bending vibration of a uniform beam hinged at both ends. Choosing for convenience unit bending stiffness, and mass per unit length and a beam length of 10, the stiffness and mass operators of Eq. (2.1) and the boundary operators of Eq. (2.2) become

$$L = \frac{d^4}{dx^4}, \quad M = 1$$

$$B_1(0) = B_1(10) = 1, \quad B_2(0) = -B_2(10) = \frac{d^2}{dx^2} \tag{5.46}$$

The eigenvalue problem (2.3)-(2.4) admits a closed-form solution consisting of the eigenvalues and eigenfunctions

$$\lambda_r = \omega_r^2 = \left(\frac{r\pi}{10}\right)^4, \quad \phi_r(x) = \frac{1}{\sqrt{5}} \sin \frac{r\pi x}{10}, \quad r = 1, 2, \ldots \tag{5.47}$$

Modal Filter Design

Introducing Eqs. (5.47) into the modal filter equations, Eqs. (5.4), we obtain

$$u_r(t) = \frac{1}{\sqrt{5}} \int_0^{10} u(x,t) \sin \frac{r\pi x}{10}\, dx, \quad r = 1, 2, \ldots$$

$$\dot{u}_r(t) = \frac{1}{\sqrt{5}} \int_0^{10} \dot{u}(x,t) \sin \frac{r\pi x}{10}\, dx, \quad r = 1, 2, \ldots \tag{5.48}$$

Our task is to approximate $u(x,t)$ and $\dot{u}(x,t)$ along the full length of the beam by means of measurements from a finite number of sensors. We place k sensors at equal intervals along the beam, so that we have k - 1 subdomains. Equal intervals are chosen here only for convenience. In fact, for complicated geometries it is advisable not to choose equal intervals.

The sensors measure the displacements (and velocities) and the slopes of the displacements (and velocities) continuously in time. At any time t_0, measurements of the displacement and slope at both ends of a given subdomain are used to approximate $u(x,t_0)$ and $\dot{u}(x,t_0)$ in the subdomain in question. To this end, it is advisable to use Hermite

cubics as interpolation functions [72]. As soon as the displacement and velocity patterns are estimated, one can use the modal filters, Eqs. (5.48) to determine the modal displacements $u_r(t_0)$ and modal velocities $\dot{u}_r(t_0)$ ($r = 1, 2, \ldots$).

Denoting the estimations of $u(x,t_0)$ and $\dot{u}(x,t_0)$ in the $\underline{i}$th subdomain by $\hat{u}_i(x,t_0)$ and $\hat{\dot{u}}_i(x,t_0)$, we can write the estimated modal displacements and velocities in the form

$$\hat{u}_r(t_0) = \frac{1}{\sqrt{5}} \sum_{i=1}^{k-1} \int_{\frac{10(i-1)}{k-1}}^{\frac{10i}{k-1}} \hat{u}_i(x,t_0) \sin \frac{r\pi x}{10}\, dx, \quad r = 1, 2, \ldots$$

$$\hat{\dot{u}}_r(t_0) = \frac{1}{\sqrt{5}} \sum_{i=1}^{k-1} \int_{\frac{10(i-1)}{k-1}}^{\frac{10i}{k-1}} \hat{\dot{u}}_i(x,t_0) \sin \frac{r\pi x}{10}\, dx, \quad r = 1, 2, \ldots \tag{5.49}$$

Considering Eqs. (5.8)-(5.10) we conclude that

$$\underset{\sim}{I}_{ir} = \frac{10}{k-1} \int_0^1 \sin\left[\frac{r\pi i}{k-1} - \frac{r\pi\xi}{k-1}\right] \underset{\sim}{L}\, d\xi, \quad i = 1, 2, \ldots, k-1;$$

$$r = 1, 2, \ldots \tag{5.50}$$

where

$$\xi = i - \frac{x}{h} \tag{5.51}$$

is the local coordinate associated with the $\underline{i}$th interval and h is the length of each interval. The Hermite cubics have the form [72]

$$L_1 = 3\xi^2 - 2\xi^3, \quad L_2 = \xi^2 - \xi^3$$

$$L_3 = 1 - 3\xi^2 - 2\xi^3, \quad L_4 = -\xi + 2\xi^2 - \xi^3 \tag{5.52}$$

so that

$$\underset{\sim}{L} = [L_1 \quad L_2 \quad L_3 \quad L_4]^T \tag{5.53}$$

The accuracy of the estimated modal displacements improves as the number of sensors is increased. This is true because higher eigenfunctions are more "wrinkled", so that more sensors are better able to sense their contributions.

The modal filters can be simulated on a digital computer as follows: arbitrary displacement and velocity patterns F(x) and G(x) are fed into the computer, and exact modal displacements and velocities are computed from Eqs. (5.48). Next, the discrete (in space) values $F(x_i)$ and $dF(x_i)/dx$ (i = 1, ..., k) are taken as sensors outputs. Then, the $\underset{\sim}{I}_{ir}$ vectors are computed from Eqs. (5.50), and the modal displacements $\hat{u}_r(t_0)$ and the modal velocities $\dot{\hat{u}}_r(t_0)$ are determined by means of Eqs. (5.11). Table 5.1 compares the exact modal displacements u_r and the estimated ones for different numbers of sensors. The estimates are of the function

$$F(x) = \frac{1}{\sqrt{5}} \sum_{i=1}^{15} \frac{1}{(2i)^2} \sin \frac{i\pi x}{10} \tag{5.54}$$

and we note that F(x) satisfies all the boundary conditions. As Table 5.1 clearly shows, the accuracy of the estimation improves as the number of sensors is increased.

Control System Design

Let us consider the optimal control of the beam. We confine ourselves to the steady-state case of the Riccati equation for the modal control gains. The performance index associated with the rth mode is taken as

$$J_r = \int_0^{t_f} [\dot{u}_r^2(t) + \omega_r^2 u_r^2(t) + R_{\eta r}^* f_r^2(t)]dt, \quad r = 1, 2, \ldots, n \tag{5.55}$$

where $R_{\eta r}^*$ is a gain parameter. Then, the performance index for all the controlled modes can be expressed as

$$J = \sum_{r=1}^{n} J_r \tag{5.56}$$

where J can be minimized by minimizing each and every J_r independently. As a result, one must solve n sets of 2 × 2 Riccati equations, whose closed-form solution is readily available [103].

Four modes are controlled and the optimal gain parameter $R_{\eta r}^*$ is taken as $R_{\eta r}^* = 20$ (r = 1, 2, 3, 4). The initial excitation is chosen to be a unit impulse applied at an arbitrary point x_0, with the initial displacement and velocity taken as zero. The unit impulse excitation can be expressed as

$$f_e(x,t) = F_0 \delta(x - x_0)\delta(t) \tag{5.57}$$

It can easily be verified that the unit impulse has the effect of an initial modal velocity

$$\dot{u}_r(0) = F_0 \phi_r(x_0), \quad r = 1, 2, \ldots \tag{5.58}$$

on every mode.

Control Implementation

Four actuators and sensors at nine locations measuring displacements, slopes, velocities and angular velocities were used for the control implementation. It was observed that by using nine sensors,

the modal filters estimated $\hat{u}_r(t)$ and $\hat{\dot{u}}_r(t)$ ($r = 1, 2, 3, 4$) with sufficient accuracy. The sensors locations were taken as

$$x_j^s = \frac{5(j - 1)}{4}, \quad j = 1, 2, \ldots, 9 \tag{5.59}$$

The actuator locations were varied so as to permit a comparison of responses for different actuators locations. The point at which the unit impulse was applied was chosen as $x_0 = 6.3$.

Using the above initial conditions and control gain parameters the response of the beam was simulated as follows: the equations of motion for the controlled modes can be shown to be

$$\ddot{\underset{\sim}{u}}_C(t) + \Lambda_C \underset{\sim}{u}_C(t) = \underset{\sim}{f}(t) \tag{5.60}$$

where Λ_C is a diagonal matrix containing the first n eigenvalues. The equation describing the motion of the residual modes is given in Eq. (5.31) and involves infinite-dimensional vectors and matrices. They have to be truncated for computational purposes. Five residual modes were retained, so that Eq. (5.30) becomes

$$\ddot{\overline{\underset{\sim}{u}}}_R(t) + \overline{\Lambda}_R \overline{\underset{\sim}{u}}_R(t) = \overline{B}_R B^{-1} \underset{\sim}{f}(t) \tag{5.61}$$

where the overbars indicate that the vectors and matrices are finite dimensional. Equations (5.60) and (5.61) can be combined to give

$$\begin{bmatrix} \ddot{\underset{\sim}{u}}_C(t) \\ \ddot{\overline{\underset{\sim}{u}}}_R(t) \end{bmatrix} + \begin{bmatrix} \Lambda_C & 0 \\ 0 & \overline{\Lambda}_R \end{bmatrix} \begin{bmatrix} \underset{\sim}{u}_C(t) \\ \overline{\underset{\sim}{u}}_R(t) \end{bmatrix} = \begin{bmatrix} 1 \\ \overline{B}_R B^{-1} \end{bmatrix} \underset{\sim}{f}(t) \tag{5.62}$$

At a certain time t_0, the displacements and velocities at the sensors locations were calculated by using

$$u(x_j^s,t_0) = \sum_{r=1}^{9} \phi_r(x_j)u_r(t_0)$$

$$\frac{du}{dx}(x_j^s,t_0) = \sum_{r=1}^{9} \frac{d\phi_r(x_j)}{dx} u_r(t_0)$$

$$j = 1, \ldots, m \qquad (5.63)$$

$$\dot{u}(x_j^s,t_0) = \sum_{r=1}^{9} \phi_r(x_j)\dot{u}_r(t_0)$$

$$\frac{d\dot{u}}{dx}(x_j^s,t_0) = \sum_{r=1}^{9} \frac{d\phi_r(x_j)}{dx} \dot{u}_r(t_0)$$

The quantities obtained from Eqs. (5.63) were then fed into the modal filters, Eqs. (5.48), and the estimated modal coordinates $\hat{u}_r(t_0)$ and $\hat{\dot{u}}_r(t_0)$ ($r = 1, 2, 3, 4$) were computed. The generalized control forces $f_r(t)$ were determined to be [103]

$$f_r(t_0) = \omega_r\left(\omega_r - \sqrt{\omega_r^2 + R_{\eta r}^{*-1}}\right)\hat{u}_r(t_0)$$

$$- \left[2\omega_r\left(-\omega_r + \sqrt{\omega_r^2 + R_{\eta r}^{*-1}}\right) + R_{\eta r}^{*-1}\right]^{1/2} \hat{\dot{u}}_r(t_0) \quad r = 1, 2, 3, 4 \qquad (5.64)$$

After the generalized control forces were determined, Eqs. (5.62) were integrated using a transition matrix approach to obtain the values of the modal coordinates for the next time step.

It was observed that when nine sensors were used, the response of the modal coordinates obtained by using modal filters were identical to the response obtained by directly integrating Eqs. (5.62), without using modal filters. This is a clear indication that one can select the

number of sensors so that observation spillover can be eliminated, when one uses modal filters.

Figures 5.1a,b,c-5.6a,b,c show the displacement of the beam at various times for the actuator locations

$$2.0 \quad 4.0 \quad 6.0 \quad 8.0 \tag{5.65a}$$

$$1.5 \quad 3.5 \quad 6.5 \quad 8.5 \tag{5.65b}$$

$$1.3 \quad 2.6 \quad 3.9 \quad 5.2 \tag{5.65c}$$

and at times t = 10, 20, 30, 40, 50 and 60 sec. Note that the actuators locations given by Eq. (5.65a) are the nodes of the fifth eigenfunction

$$\phi_5(x) = \frac{1}{\sqrt{5}} \sin \frac{\pi x}{2} \tag{5.66}$$

so that they correspond to the scheme proposed earlier. It is clear that the displacement of the beam decays with time, which justifies the earlier statement concerning the magnitude of control spillover into residual modes. Note that the system parameters were chosen so that the first natural frequency of the beam is very low. Large structures are characterized by low natural frequencies. Because of this, during the first part of the control process the sign of the displacement of the beam remains the same as the initial displacement. As time unfolds, the displacement in certain segments of the beam changes sign. In addition, there is little difference in the response of the beam for different actuator locations. It is also observed that the amplitudes in Figs. 5.1a-5.6a are lower than those in Figs. 5.1b-5.6b and 5.1c-5.6c. These observations provide some justification for the earlier statements regarding the importance of actuators locations, and for the proposed

scheme for the selection of these locations, at least for the example considered.

Note that the response in Figs. 5.1-5.6 was obtained ignoring the internal damping present in the distributed-parameter system. The inclusion of a minimal amount of damping into the equations of motion should minimize the amplitude differences in Figs. 5.1-5.6.

Table 5.1

Estimated Modal Displacements

u_r for F(x), Eq. (5.54)

k \ r	1	2	3	4	5
Exact	0.25000	0.06250	0.02778	0.01563	0.01000
2	0.40970	0.09852	0.01517	0.01232	0.00328
3	0.26718	0.08492	0.04269	0.01742	0.00639
4	0.25336	0.06762	0.03452	0.01881	0.01141
5	0.25007	0.06334	0.02903	0.01689	0.01093
6	0.25026	0.06288	0.02815	0.01586	0.01081
7	0.25015	0.05274	0.02805	0.01422	0.00865
8	0.25010	0.06134	0.02667	0.01456	0.00898
9	0.24906	0.06160	0.02691	0.01479	0.00920
10	0.25000	0.06248	0.02709	0.01496	0.00935
11	0.25000	0.06248	0.02775	0.01557	0.00947

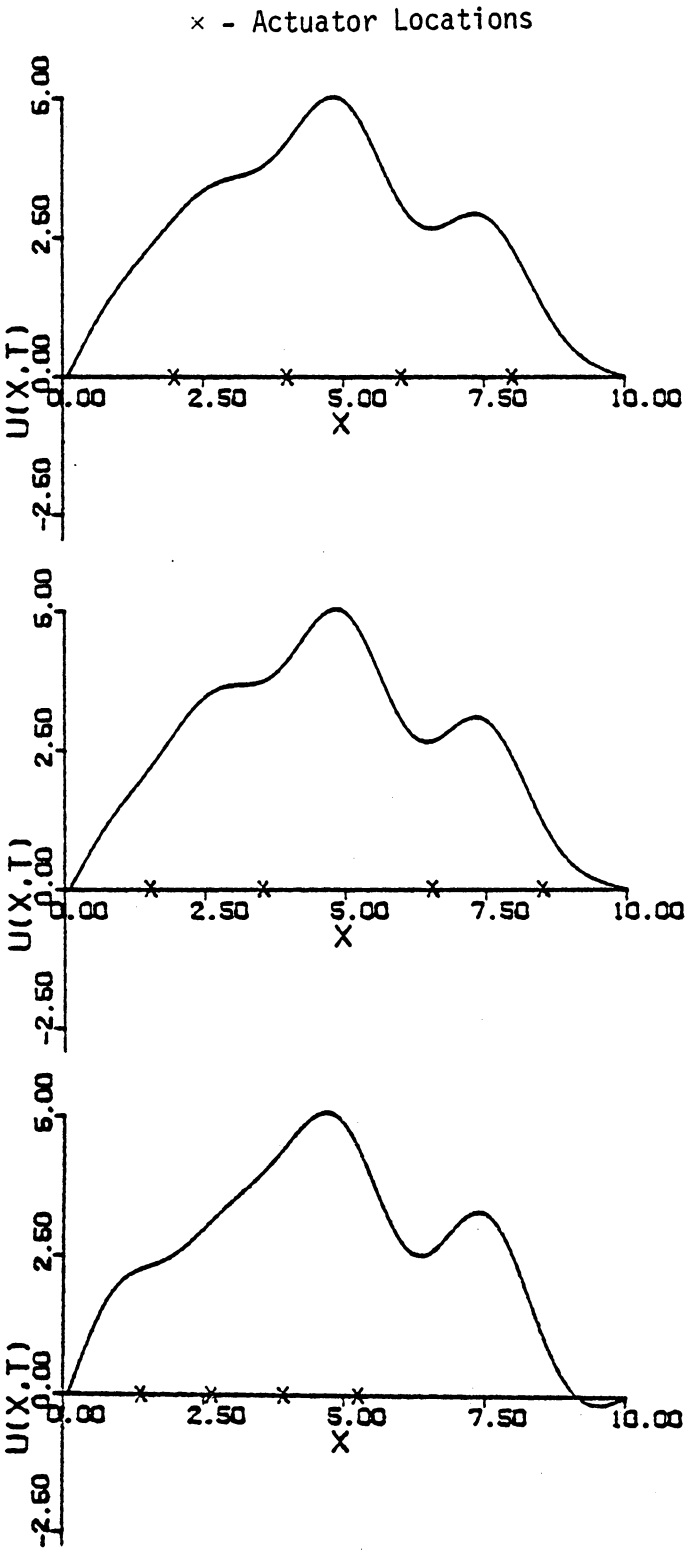

Figure 5.1a-c Response of beam at t = 10 sec.

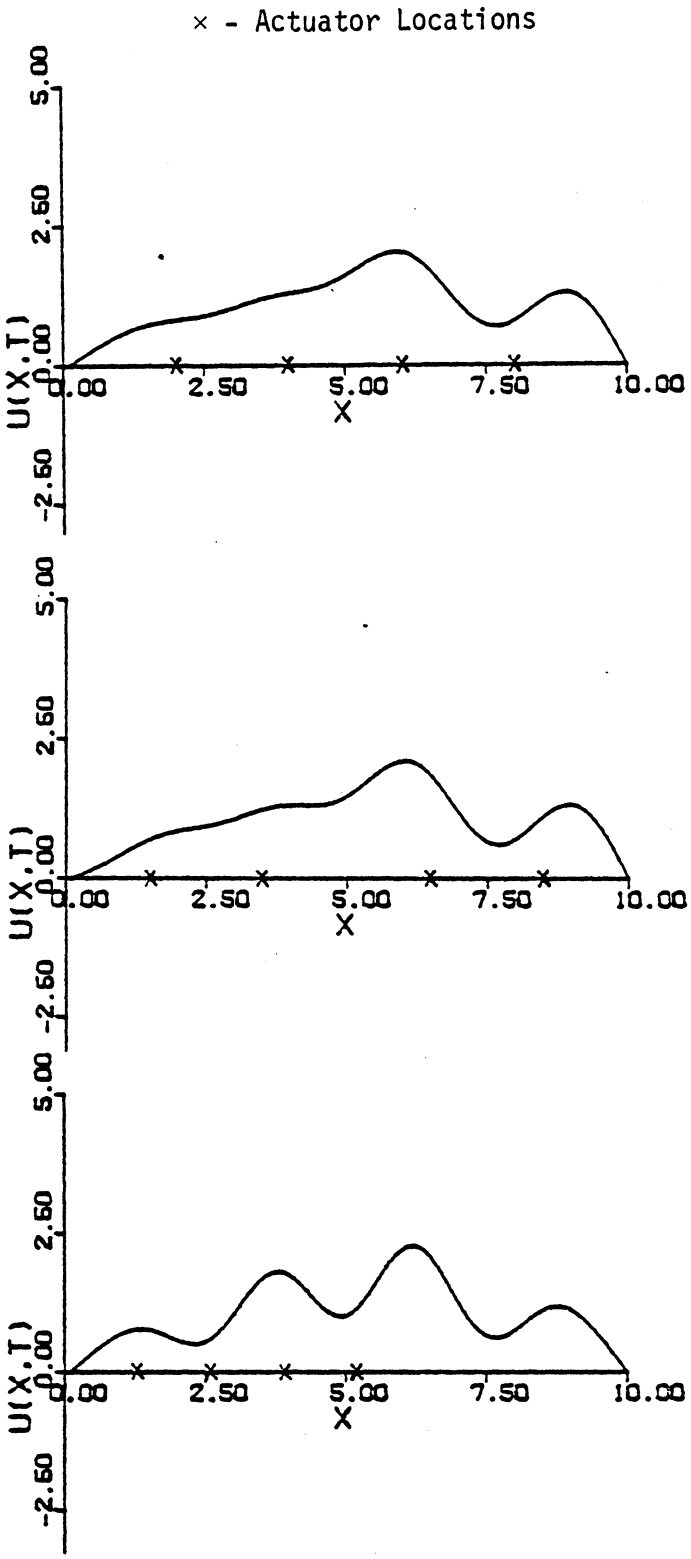

Figure 5.2a-c Response of beam at t = 20 sec.

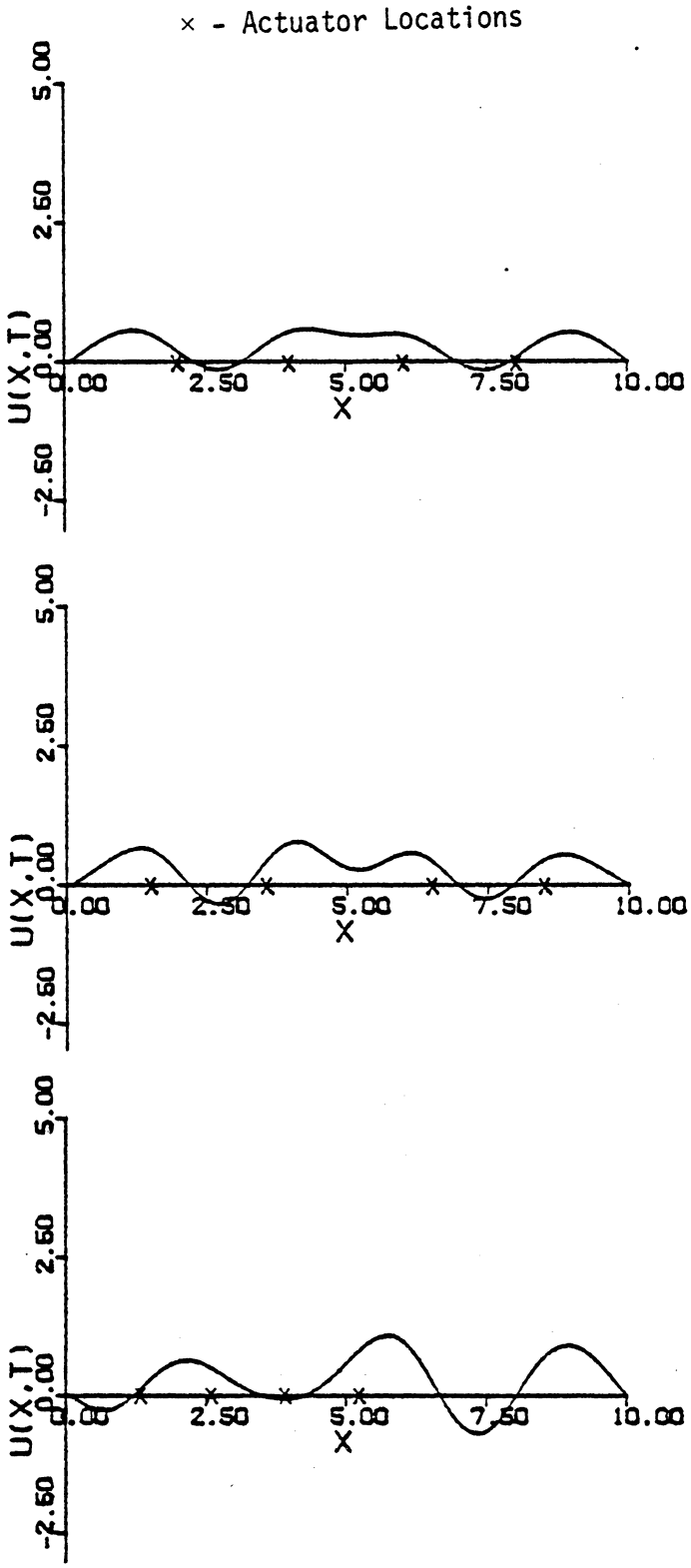

Figure 5.3a-c Response of beam at t = 30 sec.

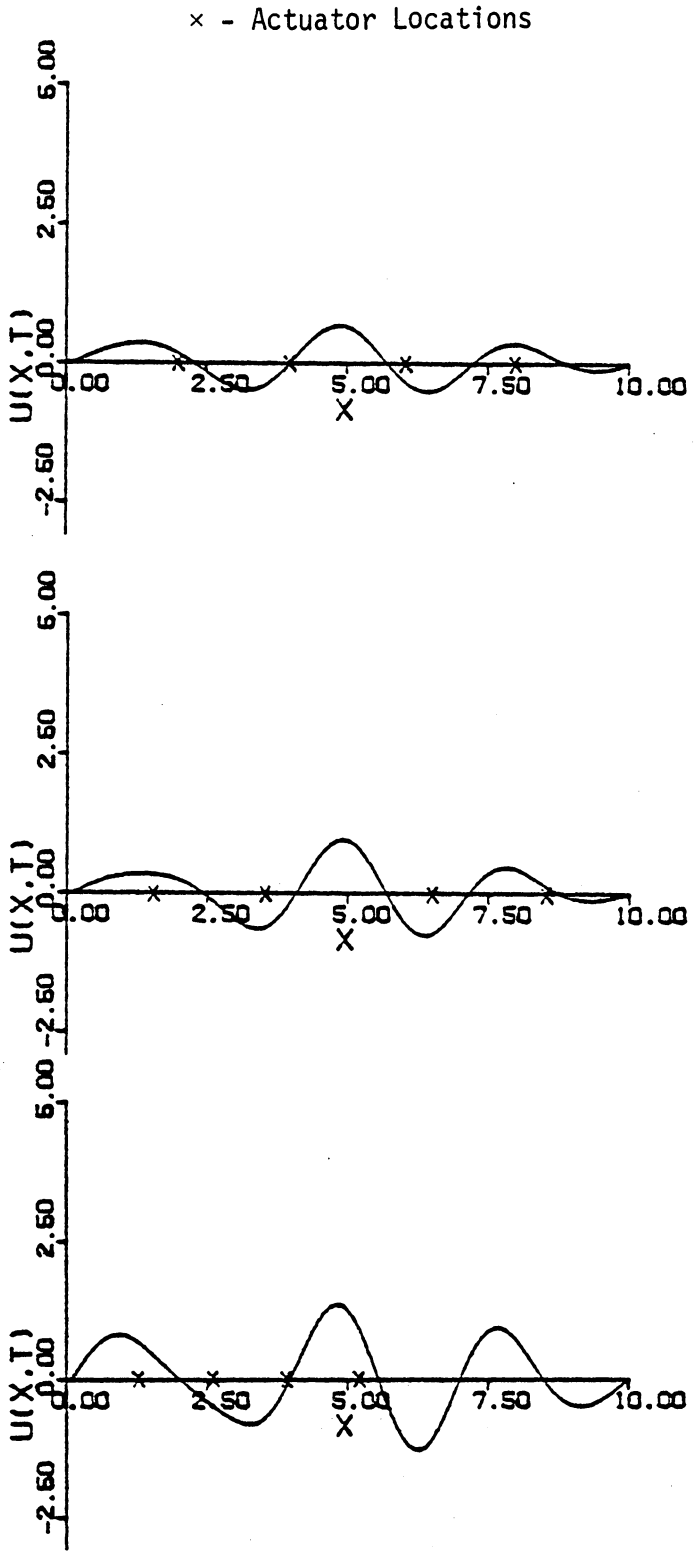

Figure 5.4a-c Response of beam at t = 40 sec.

× - Actuator Locations

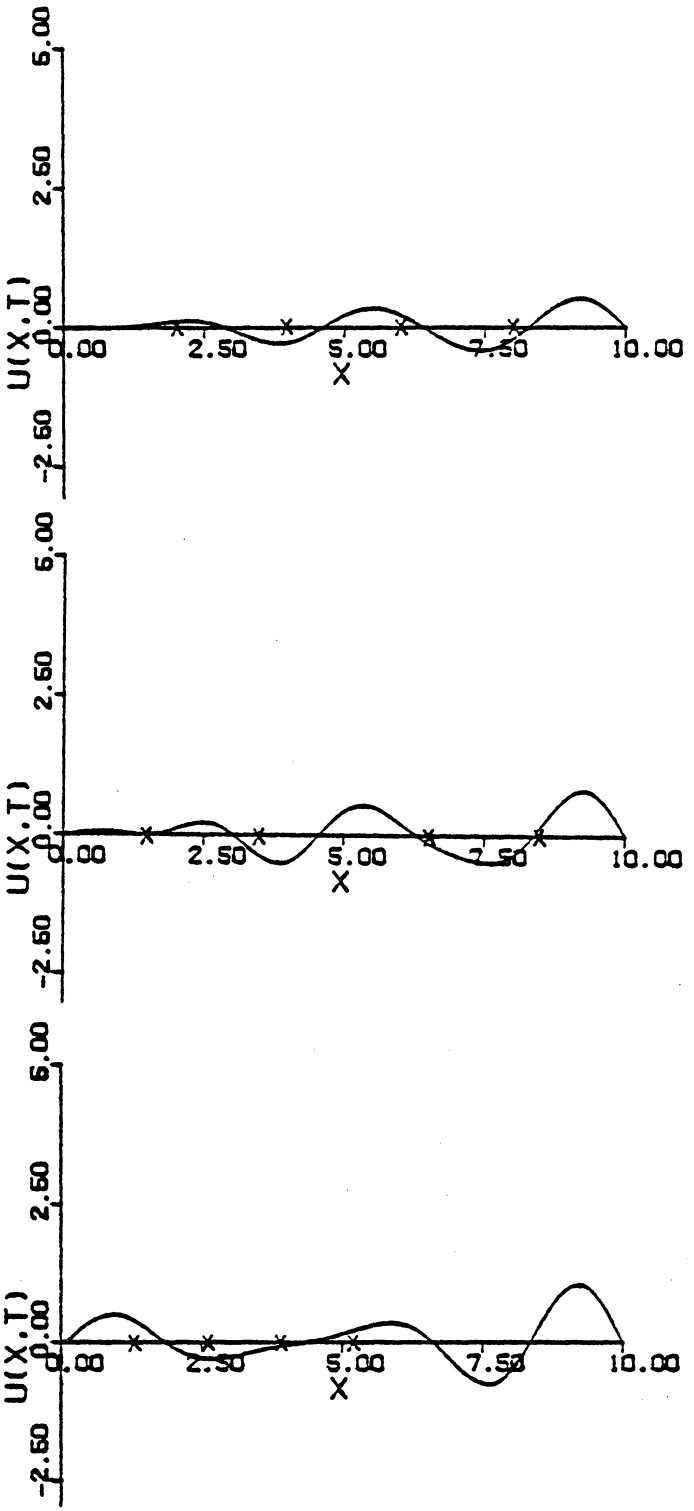

Figure 5.5a-c Response of beam at t = 50 sec.

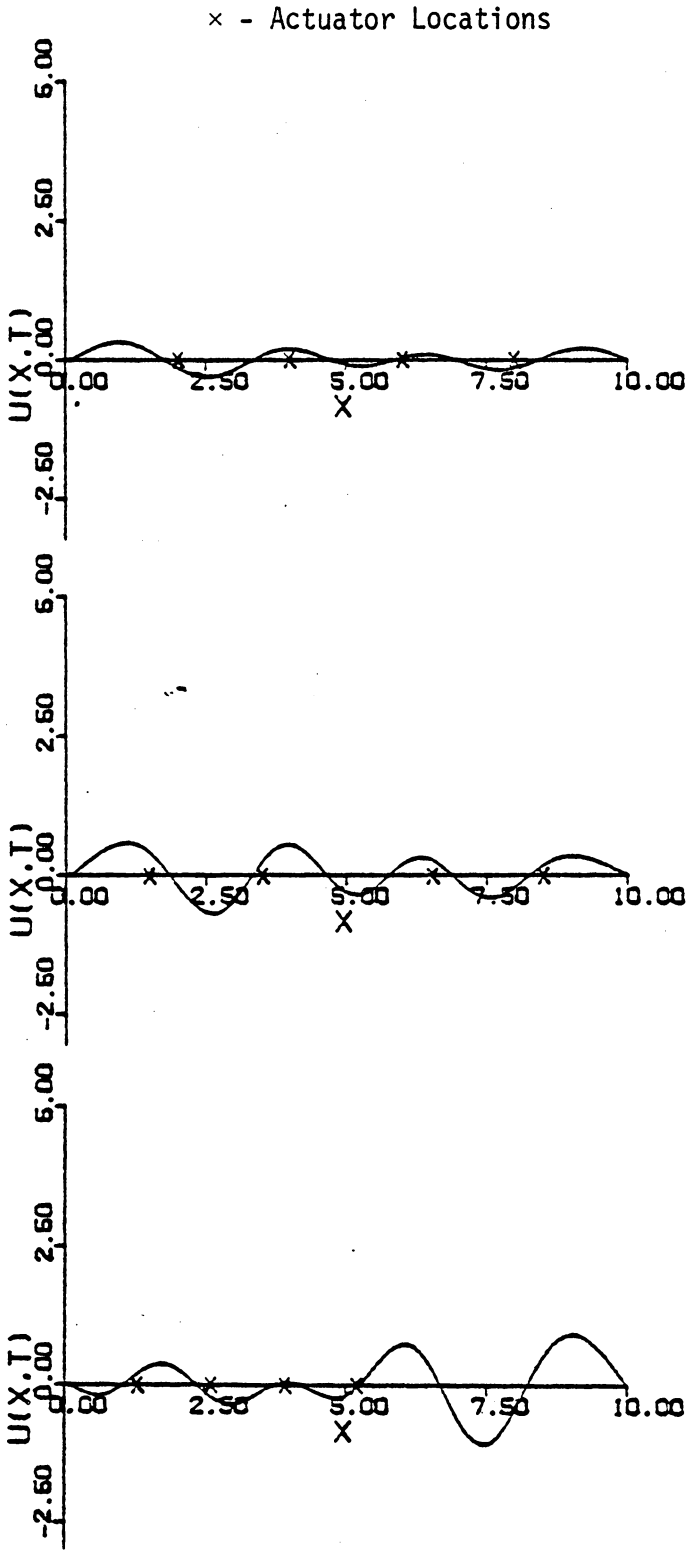

Figure 5.6a-c Response of beam at t = 60 sec.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

In this dissertation, we have presented two methods, one for the identification and one for the control of distributed-parameter systems. The two methods are independent of each other and are implemented using discrete measurements and discrete controls.

The main idea behind the approaches proposed in this dissertation is to identify and control the actual distributed system, without resorting to discretization or truncation. Because the motion of distributed-parameter systems is governed by partial differential equations, distributed-parameter systems essentially possess an infinite number of degrees of freedom. A truncated model cannot account for the entire infinity of the modes, especially when a limited number of measurements is used. According to the methods described in this dissertation the output of the sensors is interpolated to obtain displacement and velocity profiles. The implication is that, for a given number of sensors, one can assume that distributed measurements are available. This number can be determined with relative ease. As a result, one can identify and control the actual distributed system.

Let us now consider the results of the identification and control methods independently and make some suggestions regarding future work. As can be seen from the example presented in Chapter 4, the identification method works very well, provided a sufficient number of sensors is available. The first part of the identification method can be plagued by computational difficulties for large order systems. To identify the

eigenvalues and the values of the eigenfunctions at the sensors locations, the eigensolution of a real general matrix is required. For large order matrices (> 120), this is not an attractive task from a computational point of view. The procedure can be improved by developing an identification method that extracts the eigenvalues and eigenfunctions without the burden of computing the eigensolution of a large-order real general matrix. In addition, an identification method that works in the presence of external disturbances is highly desired.

The second part of the identification method does not exhibit any computational difficulties and is easy to apply. Identification, in general, is a synthesizing operation. We know the input and the output at given points, and we must infer the system parameters from these. How this information is used depends on the analyst. In addition to making the necessary assumptions, one must make sure that all the available information is included in the identification process.

The field of parameter identification has become very broad and scattered. Because different analysts are interested in different aspects of a certain problem, and there exists infinitely many dynamical systems, a very large number of identification algorithms have been proposed. As stated in Ref. [1], some attempts should be made to unify this field.

The control implementation scheme described in this dissertation is extremely easy to apply and exhibits virtually no computational difficulties. Modal filters, used to extract the modal quantities from sensors data, are very easy to design and implement. They can eliminate observation spillover when a sufficient number of sensors is used.

Futhermore, modal filters can be implemented with any kind of control method and through their use control of the actual distributed system is possible.

The consequences of control implementation using discrete actuators are discussed qualitatively and quantitatively, and some guidelines are suggested for the placement of actuators. Expressions are developed for the energy imparted to the distributed-parameter system. It is shown that, as a result of using discrete actuators, control spillover occurs, so that part of the energy imparted to the distributed system is pumped into the uncontrolled modes. As can be seen from the theoretical development and numerical example, the placements of actuators does not represent a serious problem when the independent modal space control method is used. This is because the work done to control the controlled modes is independent of the actuators locations, and if a sufficient number of modes is controlled, energy pumped into the uncontrolled modes loses its significance.

A few questions arise during the control system design. The design process assumes that the eigensolution associated with the controlled modes is known accurately. Of course, when the eigensolution is obtained by a discretization method, not all the eigenvalues are known, and of the known ones some are inaccurate. Computed lower modes tend to be more accurate than computed higher modes. A measure to determine if the eigensolution associated with a certain mode is estimated or identified accurately is highly desirable. It is recommended here that more work be done on methods enabling one to determine whether the

identification or computation of a certain eigenvalue and eigenfunction is sufficiently accurate.

Another problem that arises in designing controls is the lack of a criterion to determine the number of modes in need of control. It is suggested here that a criterion be based on the amount of energy that goes into each mode, but more work must be done in this area.

The identification and control methods described in this dissertation are independent of each other. A more sophisticated method for the simultaneous identification and control of distribured-parameter systems is highly desirable. Simultaneous identification and control has received some consideration in recent years [61-62], but these investigations are not pertinent to distributed systems.

One problem in need of further investigation is damping. It is known that damping suppresses the vibratory motion, but a better understanding of damping and its effect on the system modes should aid in determining the number of modes in need of control.

It is clear from the developments and numerical examples in this dissertation that, to identify and control the distributed system effectively, a large number of sensors and actuators is required. This is a very logical conclusion. More sensors produce more information about the system behavior, so that the interpolation process yields more accurate results. A similar statement can be made about the number of actuators. More actuators approximate the distributed forces better, so that less energy gets wasted by being pumped into the uncontrolled modes. It should be noted that, if distributed sensors and actuators were

available, control implementation would not represent a problem. With advances in technology, distributed sensors are likely to be available in the not too distant future. The problem of distributed actuators is more complicated, but in some problems electroplate devices can provide the answer.

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PARAMETER IDENTIFICATION AND CONTROL OF DISTRIBUTED-PARAMETER SYSTEMS

by

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(ABSTRACT)

Two methods, one for the identification and one for the control implementation of distributed-parameter systems are presented. The methods are designed to identify and control the actual distributed system, without resorting to discretization. They are implemented using discrete sensors and actuators.

The identification process is carried out in two steps. First, the eigensolution of the distributed system is identified. The lowest frequencies and associated eigenfunctions are identified using an extension of a time-domain approach developed for discrete systems. The extension to distributed systems is carried out in this dissertation. To this end, the sensors output is interpolated to identify the eigenfunctions. Next, the parameters contained in the equations of motion are identified. The motion of distributed-parameter systems is described in terms of partial differential equations, so that these parameters are in general continuous functions of the spatial variables. For vibrating systems, these parameters ordinarily represent the mass, stiffness and damping distributions. These distributions are expanded in terms of finite series of known functions of the spatial variables multiplied by undetermined coefficients. Then, using the identified eigensolution and assuming that the general nature of the equation of motion is known, use

is made of the least squares method, in conjunction with the eigenfunctions orthogonality to compute the undetermined coefficients, thus identifying the actual distributed system.

The control system design is based on the concept of independent modal-space control. Implementation of the independent modal-space control method requires that the number of actuators be equal to the number of controlled modes. Because the actuators are discrete elements, control spillover into the uncontrolled modes is experienced. The effect of control spillover is to pump part of the energy imparted to the distributed system into the uncontrolled modes. It is shown that when the independent modal-space control method is used, the energy required to control the controlled modes does not depend on the actuators locations, so that the placement of the acturators does not represent a serious problem, as it can for coupled controls.

A new concept in extracting modal coordinates from the system output, namely modal filters, is introduced. Modal filters extract the modal quantities from the sensors data by interpolating the output of the sensors to obtain continuous displacement patterns and by performing certain weighted integrations over the distributed domain. If the interpolation functions are chosen following the same guidelines as in the finite element method, the integrations can be carried out as off-line computations, which facilitates the control implementation. It is shown that when modal filters are used, control of the actual distributed system is possible and no spatial discretization is necessary. In addition, observation spillover, a possible significant problem when observers are used, is eliminated.

Two numerical examples are presented to illustrate the identification and control methods. The methods described in this dissertation are in terms of vibrating systems, with special emphasis on large flexible structures. However, these methods are applicable to any distributed-parameter system.