

BETA BIAS IN LOW-PRICED STOCKS  
DUE TO TRADING PRICE ROUNDING

by

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## Chapter I

### INTRODUCTION

Over the last two decades a large amount of financial research has been devoted to the proposition that the rate of return a security earns is proportional to the rate of return on the market portfolio.<sup>1</sup> Additional research has employed models which explicitly assume such a relationship.<sup>2</sup> Moreover research concerning this proportionality (measured by the security's "beta") has been fruitful enough for practitioners to have adopted the concept. For example, Francis and Archer (1979, p. 67) in their widely adopted textbook state:

In contrast to the traditional practice of pondering over numerous financial ratios for each potential investment, modern risk-return-oriented financial analysts focus their attention on only one statistic for each potential investment -- the beta coefficient. This practice is based on the belief that the beta coefficient subsumes the information contained in all the traditional financial ratios and summarizes it into one single number.

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<sup>1</sup>The precise definition of the market portfolio and the resulting impact on market model parameters is an issue not addressed in this study (cf. Black (1972), Roll (1977), Roll (1978), Ross (1978), Mayers and Rice (1979), and Roll (1979)).

<sup>2</sup>For example, studies which utilize residual analysis such as Kaplan and Roll (1972), Hong, Kaplan and Mandelker (1978) and Keown and Pinkerton (1981).

The assertion that beta reflects the traditional financial ratios is supported by the work of Thompson (1976) in which he, in part, calculated the correlation coefficients between common stock betas and various financial ratios; as well as by the research of others.<sup>3</sup> That two of the major investment advisory services now report a measure of this proportionality\* along with the other information they provide for each security is an indication that practitioners are now using beta coefficients in their analysis of securities. Beta coefficients are not, however, directly observable. They must be estimated, and the substantial use of beta in the investment industry indicates that it is desirable to inquire into biases which may exist and to propose ways to correct them.

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<sup>3</sup>See, for example, Beaver, Kettler and Scholes (1970). This study and others are discussed at length in Lev (1974, pp. 202-210) and more briefly in Thompson (1976).

\*Value Line reports beta coefficients. Moody's, in their Fact Sheets report a measure of performance which is similar to the beta coefficient provided the measure is computed during a period when the market is rising.

### 1.1 BETA ESTIMATION MODELS

The capital asset pricing model (CAPM)<sup>s</sup> derives from capital asset pricing theory and is written as

$$R_{it} = R_{ft} + [R_{mt} - R_{ft}] \beta_i + \epsilon_{it} \quad (1.1)$$

where

$R_{it}$  is the return on security  $i$  in time period  $t$

$R_{ft}$  is the risk free interest rate in time period  $t$  or the return on the minimum variance zero-beta portfolio in time period  $t$

$R_{mt}$  is the market return in time period  $t$

$\beta_i$  is the constant of proportionality (the "beta") for security  $i$

$\epsilon_{it}$  is the stochastic error term for security  $i$  in time period  $t$

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<sup>s</sup>For a complete review of the derivation of the CAPM see Copeland and Weston (1979) or Francis and Archer (1979).

The CAPM assumes that  $\beta_i$  is constant over the time period during which the model is fitted to the data. It requires that what otherwise would be the intercept, be equal to the risk free rate of return or the minimum variance zero-beta portfolio, both of which change over time.

Unlike the capital asset pricing model, the market model is not supported by asset pricing theory; it is a statistical relationship. The market model assumes that the joint distribution of  $R_{it}$  and  $R_{mt}$  is bivariate normal.<sup>6</sup> A result of this assumption is that  $R_{it}$  may be expressed as a linear function of  $R_{mt}$ , i.e., the market model may be written as

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \quad (1.2)$$

where  $\alpha_i$  is the intercept term for security  $i$ .

Another important consequence of the bivariate normal assumption is that the disturbance term,  $\epsilon_{it}$ , is independent of  $R_{mt}$  and therefore  $\text{cov}(\epsilon_{it}, R_{mt}) = 0$ . Further, it can be shown that  $E(\epsilon_{it}) = 0$  and  $\sigma^2(\epsilon_{it})$  is a constant.<sup>7</sup>

<sup>6</sup>See Fama (1976, p. 66)

<sup>7</sup>Fama (1976, pp. 69, 70)

Additional assumptions are that there is no serial correlation in the error terms and that the error terms are not correlated across securities.<sup>8</sup>

The market model also assumes that the slope and intercept parameters are constant over the time period during which the model is fitted to the data. Comparing (1.1) and (1.2) we have for  $\alpha_i$  that

$$\alpha_{it} = R_{ft} - \beta_i R_{ft} = R_{ft}(1 - \beta_i) \quad (1.3)$$

where

$\alpha_{it}$  is the intercept term for security  $i$  in time period  $t$

$R_{ft}$  and  $\beta_i$  are as previously defined

Thus in adopting the market model (1.2) the implication exists that  $R_{ft}$  varies insufficiently to reject  $\alpha_{it}$  from being treated as a constant,  $\alpha_i$ , for each security  $i$ . For this research it will be assumed that the market model is an appropriate model and it will be used exclusively.

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<sup>8</sup>Hawawini (1977, p. 60)

To estimate  $\beta_i$  in the market model ex post prices and dividends are typically used to calculate holding period returns as

$$R_{it} = \frac{P_{it} + D_{it} - P_{i,t-1}}{P_{i,t-1}} \quad (1.4)$$

where

$R_{it}$  is the return on security  $i$  in time period  $t$

$P_{it}$  is the price of security  $i$  at the end of time period  $t$

$D_{it}$  are the dividends received for security  $i$  in time period  $t$

$P_{i,t-1}$  is the price of security  $i$  at the end of the time period just previous to  $t$

or

$$R_{it} = \ln \left( \frac{P_{it} + D_{it}}{P_{i,t-1}} \right) \quad (1.5)$$

where

$\ln$  denotes the natural logarithm

The parameters of the market model are then estimated by ordinary least squares regression (see equation 1.2). Thus  $\hat{\beta}_i$  is the estimated historical (ex post)  $\beta$  for security  $i$ . Of course, estimates of future  $R_{it}$  and  $R_{mt}$  could be used to secure an ex ante estimate of  $\beta_i$ , but these considerations are beyond the scope of this research.

## 1.2 BIASES IN THE ESTIMATED BETA

There are certain criteria which must be met if the ordinary least squares estimates of the beta coefficients are to be unbiased. The model must be appropriate (in particular, it must not be underspecified) and  $E(\epsilon_{it})$  must be zero. These points were addressed in the previous section. A more bothersome point is that the independent variable is usually assumed to be non-stochastic in proofs of unbiasedness. Fama (1976, pp. 85, 86) has asserted that even though  $R_{mt}$  is a random variable, the beta estimate is still unbiased. The purpose here is not to address the ordinary least squares requirements. Rather the purpose of this study examines conditions in the calculation of  $R_{it}$  (see equations (1.4) and (1.5)) which may cause a bias in the estimates of the market model parameters.

### 1.3 DIFFERENCING INTERVAL BIAS

The length of the holding period over which returns are calculated is termed the "differencing" interval. For illustrative purposes, the beta estimate for IBM common stock was calculated for the period 8/26/76 through 8/30/77 for various differencing intervals. The results appear in Table 1.1.

As shown by the table, the level of the estimated beta increases with the length of the differencing interval. This differencing interval effect on the estimation of beta has been examined extensively by, among others, Pogue and Solnik (1974), Levhari and Levy (1977), Hawawini (1977), Smith (1978) and Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980b), and these studies are reviewed in Chapter II.

Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980a) have suggested that the intervaling effect has a more fundamental cause: delayed price adjustments due to friction in the security trading process. One such friction could be less-than-continuous trading. Some widely held stocks are traded almost continuously, but others have infrequent trading. Therefore information that affects the market return today may not affect the price of security until tomorrow or later. A recommended approach for correcting this bias is to include lead and lag terms in the model to estimate

TABLE 1.1

## Estimates of Beta

## IBM Common Stock

Differencing Interval	$\hat{\beta}$
1 Day	.865
3 Days	.930
5 Days	.959
10 Days	.978
15 Days	.992

Period Covered: 8/26/76 - 8/30/77

beta.<sup>9</sup> Another approach is to estimate the amount of the bias from a proxy measure of the expected price-adjustment delay.<sup>10</sup>

#### 1.4 ONE-EIGHTH EFFECT BIAS

Another possible source of bias in the estimates of beta has been termed the "one-eighth effect" (Schwartz and Whitcomb 1977b). Common stocks' prices usually change in multiples of one-eighth of a dollar.<sup>11</sup> A price change of any magnitude will introduce some friction into the trading process and may consequently bias the beta estimates obtained from the rounded ex post prices. While the bias introduced by the rounding is of little consequence for high-priced securities, the bias could be substantial for low-priced securities. It is also possible that the amount of bias introduced in the beta estimates varies with both the differencing interval and the one-eighth effect. The purpose of this dissertation is to determine the degree of bias in the estimates of beta for low-priced securities

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<sup>9</sup>This technique has been proposed by Scholes and Williams (1977), Dimson (1979) and Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980b).

<sup>10</sup>This approach is developed further in Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980a) and is extensively reviewed in Chapter II.

<sup>11</sup>A few securities are traded in integral multiples of one-sixteenth of a dollar.

resulting from the one-eighth effect.

### 1.5 ORGANIZATION OF THE STUDY

In this chapter two beta estimation models have been briefly discussed. Conditions in the data which could cause bias in the estimates of beta coefficients for these models were reviewed with particular emphasis on two conditions in the calculation of  $R_{it}$  from which bias might arise. An examination of one of these, termed the one-eighth effect, was identified as the area of research for this dissertation, and the purpose of the dissertation was explicitly stated.

In Chapter II the relevant literature for this study is reviewed. The first part includes previous studies of the one-eighth effect and the intervaling effect, while the second part of the chapter focuses on the composition and characteristics of common market indexes. The analytical considerations are discussed in Chapter III. The price generating mechanism and the constraint placed upon it by one-eighth price rounding are explicitly stated. Alternative rounding procedures are presented and their implications discussed. In the next section the characteristics of the rounding functions are discussed. Finally, expressions for the amount of bias in beta estimates introduced by the one-

eighth price rounding are derived for both logarithmic returns and holding period (arithmetic) returns.

In Chapter IV the methodology used to secure the results presented in Chapter V is reviewed. The simulation itself is discussed as well as the statistical and ad hoc procedures used to evaluate the results. The results presented in the next chapter also include the results pertinent to two ancilliary issues discussed in Chapter IV, namely, how many replications are needed and how reproducible are the results. Chapter VI summarizes the findings, draws a conclusion, and suggests extensions of the study.

## Chapter II

### LITERATURE REVIEW

Areas of previous research relevant to this study are: (1) the one-eighth effect, (2) the intervaling effect, and (3) the composition and properties of market indices. Each of these will be reviewed in a separate section of this chapter.

#### 2.1 THE ONE-EIGHTH EFFECT

The one-eighth effect has for the most part been ignored in financial research. Schwartz and Whitcomb (1977b) addressed the problem briefly in their investigation of the causes of residual autocorrelation arguing that the one-eighth effect increases the variance of returns. What would be a smooth price series if infinitely small price changes occurred, becomes a lumpy price series as a result of limiting price changes to one-eighth of a dollar. Although a rounding of the price away from the mean is as likely as a rounding of price toward the mean, the squared deviations of price rounding away from the mean will outweigh those toward the mean.<sup>12</sup>

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<sup>12</sup>While Schwartz and Whitcomb did not so state, this variance inflation happens only because they use the logarithms of price relatives as returns. It will not occur if holding period returns are used.

Schwartz and Whitcomb further explained,

Clearly, the lowest price stocks and the least volatile stocks have the greatest 1/8 effect. The lower the price, the greater the return implied by a price change of 1/8; the lower the standard deviation (given a mean of 0), the less likely is a 1/8 return to occur in the absence of rounding; thus for low price, low standard deviation stocks, rounding is more apt to generate abnormally large returns, thereby inflating variance. (1977b, p. 302)

The magnitude of the one-eighth price change effect on variance was investigated by Schwartz and Whitcomb by simulating a series of "true" and "rounded" returns from normal returns distributions with mean zero and selected standard deviations. The "rounded" returns were calculated from the "true" prices which had been rounded to the nearest one-eighth of a dollar. The ratio of the average variance of "rounded" returns to the average variance of returns of the true series of returns was then calculated revealing significant differences between the two variances. Further, the simulation demonstrated that the upward bias in logarithmic return variance introduced by the one-eighth effect diminishes as the holding period is increased.

## 2.2 THE INTERVALING EFFECT

Unlike the one-eighth effect, the intervaling effect has been extensively covered in the literature. For example, Young (1971) and Schwartz and Whitcomb (1977a, b) have

reported that the variance of returns decreases as the length of the differencing interval increases. With respect to the market model, Altman, Jacquillat and Levasseur (1974), Pogue and Solnik (1974), Schwartz and Whitcomb (1977a, b), Levhari and Levi (1977), Hawawini (1977) and Hawawini and Michel (1979) have all shown that the coefficient of determination,  $R^2$ , increases with the length of the differencing interval. That the skewness of return distributions is affected by the differencing interval was noted by Folger and Radcliff (1974) who found returns for the Standard and Poor's composite index between 1948 and 1969 to be positively skewed when measured over yearly intervals but negatively skewed when measured over semi-annual or quarterly intervals.

Francis (1975) also examined the skewness of returns in his empirical investigation of whether or not investors prefer investments which have positively skewed distributions of rates of return. Returns for 113 large mutual funds from January 1960 to December 1968 comprised the sample. Francis (1975, p. 168) commented,

Since annual returns average larger than quarterly returns, the raw third moment of annual returns will be larger than the third moment of quarterly returns. To avoid such scale problems, skewness (that is the third moment normalized by dividing it by the standard deviation cubed) was used to compare monthly, quarterly, and annual returns. However, even in terms of skewness the annual returns of the vast majority of the mutual funds

in the sample were still more positively skewed than their quarterly returns. And, the quarterly returns were likewise more positively skewed than monthly returns. Some funds' skewness statistics even changed from negative monthly values to positive annual values.

Of more direct interest for this study are previous studies which address the effect of the differencing interval on beta. A selected few of these will be reviewed below:

In their empirical study of common stock returns in eight countries, Pogue and Solnik (1974) examined differencing intervals for a day, a week, two weeks, and one month. With one exception, the average beta estimate increased as the differencing interval was lengthened.<sup>13</sup> They attributed this as most likely resulting, "from lags in the adjustment of stock prices to changes in market levels." (Pogue and Solnik 1974, p. 926) The authors further explained that,

Adjustment lags result from the failure of stock prices to fully adjust to market changes during the trading day. Instead the adjustment may be spread over two or more days. This can be caused either by the lack of trading in the stock, or failure to comprehend market conditions when delays in index reporting exist. The result is that the price will "catch up" for previous market activity during later days. ... In an efficient market, where such lags were absent, the expected

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<sup>13</sup>The average beta estimates were the beta estimates averaged across the sample firms in each country. The exception to the direct relationship between the average beta estimate and the length of the differencing interval was Germany where the beta estimate for a one-month differencing interval was less than for a two-week interval.

value of beta should be invariant to the return measurement interval. (1974, pp. 923, 926)

Pogue and Solnik used holding period (arithmetic) returns.

Levhari and Levi (1977) analytically demonstrated that the beta estimate for aggressive securities (true beta greater than one) increases with the differencing interval and decreases for defensive securities (beta less than one). An average risk stock would not change. The authors showed this to be tautological due to the covariances of returns for the n single periods being included in the differencing interval. For empirical verification of their results they selected ten clearly defensive and ten clearly aggressive NYSE stocks and computed each stock's betas for fourteen differencing intervals ranging from one month to thirty months within a sample period of twenty years. The tabulated betas showed weak support for their hypothesis. They generally declined or increased in the predicted manner, but there were numerous exceptions. Levhari and Levi's returns were multiplicative in that returns for differencing intervals of more than a month were calculated from the product of the one-month wealths included in the interval.

Smith (1978) examined the intervaling effect using a sample of monthly returns for two hundred common stocks<sup>14</sup>

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<sup>14</sup>The first two hundred on the CRSP file for which complete data was available.

over a twenty year period (1950-1969). In addition to the full twenty year horizon, subsets of it were also used. They were: (1) three overlapping ten-year horizons, (2) four non-overlapping five-year horizons, and (3) four non-overlapping three-year horizons.

Smith calculated the annualized geometric mean returns as

$$G_i = \left( \prod_i R_{ij} \right)^{12/T} - 1 \quad (2.1)$$

where

$G_i$  is the annualized geometric mean return of security  $i$

$T$  is the number of months per horizon (e.g.,  $T =$  sixty for five years)

$R_{ij}$  is the wealth relative return for security  $i$  in period  $j$  and is given by

$$R_{ij} = \left[ P_{i,mj} + \sum_{t=m(j-1)+1}^{t=mj} D_{it} \right] \div P_{i,m(j-1)+1} \quad (2.2)$$

where

$m$  is the number of months per interval, hence

$T/m$  is the number of intervals

$j$  indexes intervals

$t$  indexes periods of time

$P_{it}$  is the price of security  $i$  at the end of period  
 $t$

$D_{it}$  is the dividend paid during period  $t$  for security  $i$

The market and riskless returns were similarly calculated. Nine differencing intervals ranging from one to twenty months were used.

Smith calculated the beta for each differencing interval for the several horizons (as well as the coefficient of variation for the returns and their relative skewness) and then performed a one-way analysis of variance using the differencing interval as the treatment. Although the hypothesis of equal betas across differencing intervals was

rejected, the observed pattern was not consistent. While the largest average beta was calculated from annual data, the smallest betas were associated with one and twenty month differencing intervals.

Smith, by grouping individual securities into deciles based on beta provided a direct test of the Levhari-Levy proposition that intervaling effects are dependent on the level of systematic risk. He then performed a two-way ANOVA using levels of beta and differencing interval as the two treatments. Smith (1978, p. 321) concluded, "Clearly these findings are supportive of the proposition. Just as important is the single observation that estimates of beta for a given asset ... varied considerably -- as much as fifty percent -- depending on the chosen interval!"

Hawawini (1977 and 1980) has extensively studied the intervaling effect. As to his theoretical work on the effect of beta he concluded,

Systematic risk is invariant to the length  $T$  of the differencing interval over which it is estimated if no correlations are present. In the presence of intertemporal cross-correlations between securities' unit returns and the market portfolio's unit returns, the level of systematic risk may be subject to an intervaling effect whose direction will depend upon the sign and relative magnitude of the intertemporal cross-correlations and market's serial correlations. The level of systematic risk may be either directly or inversely related to  $T$ . (1977, pp. 207, 208)

The above conclusion requires the assumption that returns are additive, that is, returns computed over differencing intervals are equal to the sum of returns over the constituent smaller intervals of the differencing intervals. This is a property of returns computed from the logarithm of price relatives. (Hawawini 1977, p. 204)

Assuming the existence of first order correlations only in the market returns<sup>15</sup> Hawawini (1977, p. 64) derived that

$$\begin{aligned}\Delta\beta_i(T) &> 0 \text{ if } q_{im}^i > 2\rho_m^i \\ &= 0 \text{ if } q_{im}^i = 2\rho_m^i \\ &< 0 \text{ if } q_{im}^i < 2\rho_m^i\end{aligned}\tag{2.3}$$

where

$\Delta\beta_i(T)$  is the incremental value of the slope in the market model for returns measured over differencing intervals of length T for security i

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<sup>15</sup>Hawawini presented empirical evidence to justify this assumption.

$q_{im}^1$  is the security's first order q-ratio for unit returns of security  $i$  and the market unit returns and is given by

$$q_{im}^1 = \frac{\rho_{im}^{+1} + \rho_{im}^{-1}}{\rho_{im}} \quad (2.4)$$

where  $\rho_{im}^{+1}$  and  $\rho_{im}^{-1}$  are the first order intertemporal cross-correlations coefficient between security  $i$ 's unit returns and the market unit returns when security  $i$ 's unit returns respectively lead and lag the market's unit returns

$\rho_{im}$  is the contemporaneous cross-correlation coefficient between security  $i$ 's unit returns and the market's unit returns

$\rho_m^1$  is the first order serial correlation coefficient for the market returns (1977, p. 204)

Hawawini's (1977 and 1980) empirical work supported his theoretical derivations. Using a random sample of fifty NYSE securities over the period January 1, 1970 to December 3, 1973 and three market indexes, he found all but one of

the securities to have positive intertemporal cross-correlations of more than twice the value of the market first order serial correlation. Further, the beta of each of these securities was directly related to the length of its differencing interval. The single security with positive cross-correlations less than twice the value of the market first order serial correlations had a calculated beta inversely related to the length of the differencing interval.

Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980a) have suggested three approaches to reducing or eliminating the differencing interval bias in beta. The first, also proposed by Scholes and Williams (1977) and Dimson (1979), is to account for delayed stock price adjustments by adding leading and lagged terms to the estimating equation.<sup>16</sup> The second approach proposed by Cohen et al estimates the asymptotic value of beta as the differencing interval increases without bound. This is accomplished by varying the length of the differencing interval. Their third approach is to estimate the difference between the observed beta for any differencing interval and the asymptotic beta as a function of the expected delayed price adjustment;<sup>17</sup> Cohen et al

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<sup>16</sup>Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980b) gives the development of their first approach.

<sup>17</sup>The basis for this third approach is given by Propositions

proposed the inverse of the value of shares outstanding as a proxy.

Because the estimation of the asymptotic value of beta (Cohen, Hawawini, Maier, Schwartz and Whitcomb's second approach) is particularly relevant to the research proposed here it will now be reviewed in detail.

The authors' sample was fifty NYSE listed stocks randomly selected from a stratified sample. The population of common stocks which remained listed on the NYSE from January 1, 1970 to December 31, 1973 was divided into deciles of market value of shares outstanding as of the last trading day of 1971 and five stocks were randomly drawn from each decile. Their returns were the log of price relatives adjusted for cash and stock dividends.

To estimate the intervaling effect on each security's beta coefficient, a two-pass regression analysis was employed. First the market model was used to estimate the beta coefficient for each of the fifty sample securities for

fourteen differencing interval lengths as

$$r_{iLT} = \alpha_{iL}^{(1)} + \beta_{iL}^{(1)} r_{MLT} + \epsilon_{iLT}^{(1)} \quad (2.5)$$

where

$r_{iLT}$  is the return for security  $i$  in period  $T$  and for  
a differencing interval  $L$

$\alpha_{iL}^{(1)}$  is the first-pass intercept term for security  $i$   
and differencing interval  $L$

$\beta_{iL}^{(1)}$  is as  $\alpha_{iL}^{(1)}$  above except is the slope coefficient

$r_{MLT}$  is the market return in period  $T$  for differenc-  
ing interval  $L$

$\epsilon_{iLT}^{(1)}$  is the first-pass error term for security  $i$  in  
period  $T$  for differencing interval  $L$

$i = 1, \dots, 50$  and is the index for the security

$L = 1, \dots, 6, 8, 10, 12, 14, 15, 16, 18, 20$  and is the differencing interval expressed in days

$T = 1, \dots, 960/L$  and is the time period as measured in lengths of  $L$ , e.g.,  $T=2$  for  $L=2$  encompasses  $T=3$  and  $T=4$  for  $L=1$

Next the intervaling effect was estimated by regressing the estimated beta coefficients for each security obtained from the first pass against the inverse of the length of the corresponding differencing intervals raised to the positive power  $n$  as

$$\hat{\beta}_{iL}^{(1)} = \alpha_i^{(2)} + \beta_i^{(2)} \left( \frac{1}{L^n} \right) + \epsilon_{iL}^{(2)} \quad (2.6)$$

where

$\hat{\beta}_{iL}^{(1)}$  is the slope coefficient estimated in the first-pass regression above for security  $i$  and differencing interval  $L$

$\alpha_i^{(2)}$  is the second-pass intercept term for security  $i$

$\beta_i^{(2)}$  is the second-pass slope coefficient for security i

$\epsilon_{iL}^{(2)}$  is the second-pass error term for security i and differencing interval L

i, L are as in the first-pass regression

n is a positive number

If it is true that the strength of the delayed price adjustment is reduced as the length of the differencing interval increases, and hence in the limit as the length of the differencing interval increases without bound, the OLS estimation of beta will approach an asymptote that is a consistent estimation of true beta, then equation (2.6) is appropriate. That is, the OLS estimation of beta, the dependent variable in (2.6), for a differencing interval of L approaches an asymptote, the intercept in (2.6), as the independent variable, L, increases without bound.

If there is no intervaling effect, then  $\hat{\beta}_i^{(2)}$  should not be significantly different from zero. If there is an intervaling effect, securities whose betas increase with increasing differencing intervals will have negative slopes,  $\hat{\beta}_i^{(2)}$ , so that the right hand side of equation (2.6) will

increase with increasing L and approach the intercept term for large L. Those securities whose betas decrease with increasing differencing interals will have positive slopes so that as L increases and the second right hand side term of (2.6) becomes smaller, the entire right hand side will also become smaller. The absolute value of  $\hat{\beta}_j^2$  will be larger the stronger the intervaling effect.

From the results of their empirical work, Cohen et al noted that using the second pass regression to estimate the asymptotic value for beta results in substantial adjustments to betas estimated from short period (e.g., daily) data. Additionally, a cross-sectional regression showed that the proportional difference between unadjusted one-day beta estimates and estimated asymptotic betas is significantly related to the logarithm of value of shares outstanding. They observed a close association between the inferred asymptotic betas obtained using the intervaling effect -- thinness<sup>18</sup> relationship to correct a stock's short period beta<sup>19</sup> and the estimated asymptotic betas obtained from the second pass regressions.

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<sup>18</sup>As derived from the proxy variable, "market value of shares outstanding."

<sup>19</sup>Their "third" approach, noted earlier.

### 2.3 MARKET INDEXES

It is an obvious characteristic of studies utilizing the market model or (similar models) that they employ a market index series. There are a number of these readily available, and each has its peculiar characteristics and composition. Since this study uses a market index, it is appropriate to review the different indexes in common use and the issues relevant to them.

As with studies of the differencing interval effect, there are numerous articles on market indexes in the literature. Since the choice of market indexes to be used in this study is not crucial to the study and is fairly limited by practical considerations,<sup>20</sup> only a few selected articles will be reviewed here.

Lorie and Hamilton (1971) have reviewed three important issues arising in the construction of indexes:

1. Selecting stocks for inclusion (the sample).
  2. Determining the relative importance or weight of each included stock.
- 

<sup>20</sup>The indexes to be used should be reasonably available as daily quotes over the same extended period. They should also be representative of the market as a whole. The latter rules out such indexes as the Dow-Jones Utility Index or using the returns of a specialized mutual fund as an index.

3. Combining or averaging included stocks (primarily an issue of geometric mean versus arithmetic averages).

They also described the major indexes in current use.<sup>21</sup>

On the first issue Lorie and Hamilton (1971, pp. 79, 81) explained that the sample size of modern, popular indexes ranges from the thirty stocks comprising the Dow-Jones Industrials to all those listed on an exchange, as in the New York Stock Exchange and American Stock Exchange indexes.<sup>22</sup> Stocks of relatively few companies constitute a large proportion of the value of the stocks of all companies. Consequently if large companies are considered more important than small, as is true when one is interested in changes in the market value of all stocks, this concentration of value contributes to the power of small samples. Moreover, there is a tendency of all stocks to move together. Hence when the purpose of the index is to

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<sup>21</sup>The American Stock Exchange Index has changed since the article was written. Their new index is a value-weighted index and is more fully described in American Stock Exchange, Inc. (1973). The new index was begun in September 1973 and was calculated back to 1969 (Coleman 1980). The new index, like the old, is calculated from all securities listed on the American Stock Exchange, and does not include dividends.

<sup>22</sup>Additionally, the Center for Research on Security Prices (CRSP) now computes daily and monthly indexes which are based on all stocks in both the New York and American Stock Exchanges.

represent changes in the value of all stocks, small samples can be used with very great confidence. For example, the Dow-Jones Composite Average, based on only sixty-five stocks, included stocks having a value equal to 27.5 percent of all stocks listed on the NYSE in December 1970. The stocks included in the Standard and Poor's 500 Composite Index constituted about 80 percent of the value of all NYSE listed stocks in 1970.

On the second issue, Lorie and Hamilton noted,

The reason for weighting is to insure that the index reflects the relative importance of each stock in a way suited to the index. The most common ways of weighting stocks are in accordance with market value or by assigning equal weights to relative price changes.<sup>23</sup> The former method is appropriate for indicating changes in the aggregate market value of stocks represented by the index while the latter is more appropriate for indicating movements in the prices of typical or average stocks. Changes in general market value are more important for studies of relationships between stock prices and other things in the national economy. Value-weighted indexes also have the desirable property ... (that) it is possible for all investors to hold portfolios in which the individual stocks have a relative importance equal to the relative magnitude of the

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<sup>23</sup>By equal weighting, Lorie and Hamilton meant an index based on the assumption that equal dollar amounts are invested in each stock. They did not mean the process used in constructing the Dow-Jones Averages by which prices of included stocks are added up and divided by the number of stocks (adjusted for stock splits). (p. 81, fn. 1)

One of the CRSP indexes is an equally weighted index. There are no popular, readily-available equally-weighted indexes since the demise of the first ASE Index.

values of all outstanding shares.

On the other hand, indexes based upon equal weighting are better indications of the expected change in prices of stocks selected at random. For some purposes, such an index is a more appropriate benchmark than a value-weighted index.<sup>24</sup> (1971, pp. 81-82)

All of the most widely used indexes are based on arithmetic means except the Value Line Index. Lorie and Hamilton explained that if there is any variation through time in the prices making up the index, there will be a difference between the value of an arithmetic mean of the prices and the geometric mean. The index based on the geometric mean will increase more slowly and decrease more rapidly than an index based on the arithmetic mean. The degree of divergence will be greater the greater the variability in the constituent prices.

The authors addressed the claim that the arithmetic mean is biased upward and the geometric mean has a downward bias. They illustrated the former point by supposing a \$10

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<sup>24</sup>Cootner (1966, pp. 99-100) cautioned that they still are not appropriate for a portfolio chosen at random. He restricts their usefulness to, "an investment advisor with a relatively small clientele who intends to select ten individual stocks to 'outperform the market' without giving any portfolio advice. In this case he is trying to demonstrate his ability to choose the stocks which will perform best and his customers can, and would be expected to, invest equally in all. In such a case the selective ability of the advisor is measured by a single unweighted arithmetic index. Any portfolio decisions are made by the customer and are a different matter from the choice of stocks."

stock goes to \$20 and a \$20 stock goes to \$10. The arithmetic average of the relative changes is +25 percent. If they then return to their original prices, the arithmetic average of the relative changes is still 25 percent even though the total value of the stocks is unchanged. A geometric mean adjusts for this. They asserted that a property of arithmetic averages, "is that over long periods of time, an arithmetic index will outperform most of the components. This is due in part, however, to the economic characteristics of stock prices. Since there is a lower, but no upper, limit to price changes, their distributions will not be symmetric." (1971, p. 85)

Lorie and Hamilton gave a brief description of each popular index, and candidate indexes to be used in this study will be reviewed at the end of this chapter.

Cootner (1966) has rebutted three popularly held notions about stock market indexes (which he termed "fallacies"). He asserted that the downward bias in a geometric mean is a tautological result. As long as there is any variability among the constituents of the index, a geometric index will grow more slowly or decline more swiftly than the corresponding identically constituted arithmetic index. This will always be true. It does not depend upon any particular performance of the stock market. As previously men-

tioned, the only popular geometric index is the Value Line Index. About this index Cootner (1966, p. 95) remarked, "Indeed for almost all relevant uses, the Value Line Index is a very poor choice."

An opposing notion, mentioned by Lorie and Hamilton (1971) and illustrated above, is that the arithmetic mean is upward biased. Cootner noted that to a close approximation, the change in the logarithm of price is symmetrically distributed.<sup>25</sup> Therefore, stocks are equally as likely to rise to N times their initial value as to fall to 1/N of that initial value.

The result is that the distribution is asymmetrical; it will have a longer "tail" of large price increases than of large price decreases. Now this asymmetry means... , that the average of these price changes will not coincide with the most frequent outcome. ...this means that a much larger number of stocks will rise by less than the average for the market than will rise by more than the average. ...The point is that this is not bizarre behavior -- it is to be expected and is no indictment of the Dow or any other arithmetic index. (1966, pp. 98-99)

Cootner further cautioned that a geometric index is unsuitable as a performance index for portfolios. Since two or more stocks chosen as random will of necessity tend to behave like their average (the more stocks, the more like the average of all stocks), substantially more than half of

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<sup>25</sup>This is just a more precise statement of the Lorie and Hamilton (1971) explanation above.

all random portfolios will outperform a geometric index.

Finally, Cootner addressed the argument than an unweighted index is preferable since individual portfolios are not weighted. Though doubting the latter point, he argued that all outstanding shares must be held by someone. If it is assumed that all portfolios hold either equal dollar amounts or an equal number of shares of every stock in the portfolio, then, "there must be many more portfolios with AT&T than with Lukens Steel." (1966, p. 99) Therefore the expected outcome for a portfolio chosen at random is as a weighted index.

Latane et al (1971) have examined the question of portfolio allocation and reallocation implicit in the methodology used in each index. They sub-divided the popular indexes into those that are comparable to a buy-and-hold strategy, a periodic reallocation and a continuous allocation<sup>26</sup> and exercised each type through hypothetical situations.

For their empirical work, they constructed seven monthly indexes for a thirty-five year period and 233 stocks that were continuously listed on the NYSE from January 1926

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<sup>26</sup>Of the generally available indexes, only one uses a periodic allocation (the United Press International Market Indicator, which is reallocated at the end of each trading day) and one corresponds to continuous reallocation (the Value Line average).

through December 1960. The seven indexes included no reallocation (buy-and-hold), periodic reallocations of twelve months, six months, three months, two months, and one month, and a continuous reallocation index. (The first six used equally weighted arithmetic means, and the last was an equally weighted geometric mean.)

The index value at the end of the thirty-five year period was largest for the one month reallocation and declined as the reallocation period was lengthened to about one-third of that value for a buy-and-hold strategy. However, the continuous reallocation index was only about one-seventh of the one month reallocation index value. Hence their work gives strong support to Cootner's assertion that the geometric mean index will always be lower than a comparable index based on the arithmetic mean. Latane et al concluded that their results, ..."clearly indicate that the reallocation procedure employed by particular stock market indexes can make a substantial difference in the level and movements of these indices. The difference depends on whether individual stock prices tend to follow trends or move in a random fashion." (1971, p. 83)

Fisher (1966) introduced at least two concepts not widely discussed. The first is that of an investment performance index, an index that includes cash dividends (as well as capital changes and changes in price quotations).

Fisher argued that indexes of price alone may give a fairly accurate picture of investment performance in the short run (because in the short run dividends are only a small part of investment performance), but are not very useful in the long run.

His second new concept was that of an index using both an arithmetic and a geometric mean. Fisher tried an arithmetic average with the index based on link (wealth) relatives.<sup>27</sup> He found the rate of return indicated by this index for the period January 1926 through December 1960 was considerably higher<sup>28</sup> than that he had previously gotten using other methodology.<sup>29</sup> He then tried an approximation of a geometric index and found that rates of return computed from it showed downward bias that was somewhat greater in magnitude than the upward bias found in the arithmetic index. As

<sup>27</sup>Fisher's procedure was to compute an average rate of return for a single month by dividing the price of each stock (adjusted for dividends, splits, etc., that have taken place during the month) by the price of the stock at the end of the previous month, sum these results, divide by the number of stocks and subtract one. The link (wealth) relative is one plus the return. To find the rate of return for more than one month, he multiplied successive link relatives together and then took the appropriate root.

<sup>28</sup>It is interesting that part of the bias he thought existed in his arithmetic mean index, he supposed might be from non-synchronous closing prices of the constituent stocks. This effect was previously discussed.

<sup>29</sup>As reported in Fisher and Lorie (1964)

a result, he built an index from monthly returns that was a weighted average of arithmetic mean returns and geometric mean returns.<sup>30</sup>

#### 2.4 DESCRIPTION OF SELECTED INDEXES

##### 2.4.1 The Dow-Jones Industrial Average and The Dow-Jones Composite Average

The Dow-Jones Industrial Average is probably the most familiar of stock price measures. It is an unweighted average<sup>31</sup> of the price of thirty industrial stocks. These thirty stocks are large, well established companies. In 1970 they comprised 23.8 percent of the market value of all common stocks on the New York Stock Exchange. Therefore it may be argued that these "blue chip" stocks are not representative of an average portfolio and as a result are poor measures of market performance. However, for some purposes, as when one is interested in changes in the market value of all stocks (as opposed to considering each company to be equally important), this concentration of value is very helpful. Although the stocks in this average are not value-weighted, the selection process produces about the same

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<sup>30</sup>The weights were .56 for the arithmetic average and .44 for the geometric average.

<sup>31</sup>However, as explained later, it takes on some of the characteristics of both a value-weighted and a price-weighted index.

results.

The DJI is computed by adding the prices of the component stocks and dividing by the number of stocks adjusted for stock splits and for stock dividends greater than ten percent. The method of adjustment is rather pragmatic -- a new divisor is calculated such that the average is unaffected on the transition date. Hence, each new stock split or dividend reduces the divisor. One result is that there is no equivalence between points in the average and dollars and cents. Another result is that the change in the divisor reduces the importance of the split stock relative to that of other stocks. This gives it somewhat of the characteristic of a price-weighted index. "The major criticisms of the methodology are this somewhat whimsical system of implicit price weights, the possibility of bias resulting from the adjustments, and the failure to adjust for small stock dividends." (Lorie and Hamilton 1971, p. 86) The Dow-Jones Composite Average is very similar to the DJI except that its sample includes, in addition to the thirty stocks in the DJI, twenty transportation industry common stocks and fifteen utilities. Both technically are averages rather than indexes since they are not indexed to a particular value at a given point in time. (Lorie and Hamilton 1971)

#### 2.4.2 The Standard and Poor's 500 Composite Index

This index of 500 stocks includes 425 industrials, twenty railroads, and fifty-five utilities. Hence its coverage is much broader than Dow-Jones averages. The relative importance of the prices (the weighting) is determined by the number of shares outstanding.<sup>32</sup>

The aggregate market value of the stocks in the index is expressed as a percentage of the average market value in 1941-43. This percentage is divided by ten, which was selected as the value of the index in the base period. This was done in order to make the index in line with the actual average of stock prices. (Lorie and Hamilton 1971, p. 87)

The S&P 500 was first published in 1957. Relying on a previous index,<sup>33</sup> it has been extended back to 1928 on a daily basis.

(This) composite index has several advantages. The coverage is broad, and the weighting is explicit. Moreover, no adjustments for splits is necessary. Critics have argued that the index is dominated by large companies, and that value weights can create an upward bias. These criticisms are much less universal than those aimed at the Dow-Jones Industrials. (Lorie and Hamilton 1971, p. 87)

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<sup>32</sup>Such a weighting is often termed a value-weighting, even though the weight itself is the number of shares outstanding. The total contribution to the index, the number of shares outstanding times the market price, is the value outstanding of the stock. An opposing idea is equal weighting. See fn. 23 for an explanation of equal weighting. (Lorie and Hamilton 1971)

<sup>33</sup>The Standard and Poor's 425 Index.

#### 2.4.3 The New York Stock Exchange Composite Index

The New York Stock Exchange Composite Index is a value-weighted index comprised of all the common stocks listed on the New York exchange. The index was set to a level of fifty on December 31, 1965 to make it comparable to the actual stock price average on that date of \$53.33. It is available on a daily basis beginning May 28, 1964. (Lorie and Hamilton 1971)

#### 2.4.4 The American Stock Exchange Market Value Index

This index is almost identical to the New York Stock Exchange Index except that the component assets are all the common stocks, American Depository Receipts (ADR's) and warrants listed on the American Stock Exchange. It is a value-weighted index and its level was set at \$100.00 at the close of August 31, 1973. It is available on a daily basis from January 2, 1969. Prior to the establishment of this market value index, the American Stock Exchange published a price level index (an unweighted index). The price level index was first begun April 29, 1966 and was calculated back to October 1962. (American Stock Exchange, Inc. 1973; Coleman 1980)

## 2.5 SUMMARY

The literature relevant to this study has been reviewed in this chapter. First the Schwartz and Whitcomb article on serial correlations in market model residuals, was reviewed. Next reviewed were several studies of the intervaling effect. There was general agreement in the studies that there was an intervaling effect, but lack of agreement as to its magnitude and direction under varying circumstances.

The remaining sections of the chapter were devoted to market indexes. The literature reviewed first covered issues arising in the construction of these indexes, i.e., what stocks to include in the sample, how to weight them, and what kind of average to use on them. Ancilliary issues were also discussed. These included such considerations as how to treat stock dividends or stocks that have split. Chapter II concluded with a description of five of the most commonly used indexes.

## Chapter III

### ANALYTICAL CONSIDERATIONS

In the first section of this chapter the price generating process for this study is developed from the market model and the one-eighth price rounding is explicitly stated. Three alternative rounding procedures are developed in the next section, and the implications of each are discussed. The characteristics of the rounding functions are discussed in the following section. Finally, the amount of bias introduced by one-eighth price rounding when both (arithmetic) holding period returns and logarithmic returns are used, is derived.

#### 3.1 THE PRICE CONSTRAINT

Assuming the market model (equation 1.2)

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \quad (3.1)$$

where

$R_{it}$  is the return on security  $i$  in time period  $t$

$R_{mt}$  is the market return in time period  $t$

$\beta_i$  is the constant of proportionality (the "beta")  
for security i

$\alpha_i$  is the intercept term for security i

$\epsilon_{it}$  is the stochastic error term for security i in  
time period t

and given a return-price relationship such as

$$P_{it} = P_{i,t-1} (1+R_{it}) \quad (3.2)$$

where

$P_{it}$  is the price of security i at the end of period  
t

$P_{i,t-1}$  is the price of security i at the end of the  
period previous to t

$R_{it}$  is the holding period return on security i in  
time period t

and then substituting (3.1) in (3.2), we have as a "true"

price generating process

$$P_{it} = P_{i,t-1} (1 + \alpha_i + \beta_i R_{mt} + \varepsilon_{it}) \quad (3.3)$$

It is the true price that would prevail if the true  $\alpha_i$ ,  $\beta_i$  and  $\varepsilon_{it}$  were known and price change is not constrained.

From equation (3.3) it is evident that the observed  $P_{it}$  will not change from the observed  $P_{i,t-1}$  unless one or more of  $P_{i,t-1}$ ,  $\alpha_i$ ,  $\beta_i$ ,  $R_{mt}$  or  $\varepsilon_{it}$  are sufficiently large to change  $P_i$  by at least one-eighth of a dollar,<sup>34</sup> or to change  $P_i$  by an amount such that investors perceive the change to be at least one-eighth of a dollar. Furthermore, if  $P_{it}$  as given by (3.3) has any value different from a multiple of one-eighth of a dollar, the observed prices will be different from the "true" prices indicated by (3.3). As indicated in Chapter I, if the observed betas are calculated from an ex post series of stock prices, a bias may be introduced in the estimated betas, a result of the one-eighth rounding constraint on price movements.

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<sup>34</sup>Or one-sixteenth of a dollar for those securities traded in one-sixteenth dollars.

In the presence of the one-eighth constraint on price movements, the observed price will be a function of (1) the market-generated price (equation 3.3) and (2) a rounding adjustment of such magnitude to bring the price to a multiple of one-eighth of a dollar, specifically

$$P'_{it} = P_{it} + \Delta_{it} \quad (3.4)$$

where

$P'_{it}$  is the observed price of security  $i$  at the end of period  $t$

$P_{it}$  is the market-generated price of security  $i$  at the end of period  $t$  in the absence of a price change constraint i.e., as given by (3.3)

$\Delta_{it}$  is a price adjustment for security  $i$  at the end of time period  $t$  such that  $P'_{it}$  will be a multiple of one-eighth of a dollar

### 3.2 THE ROUNDING PROCEDURES

It will be assumed that the rounding rule for rounding price to a multiple of one-eighth of a dollar is an unbiased rule. That is, the mean of the rounding function is zero. If the rounding rule were biased, then over a period of time price would gradually move either up or down as a result of the bias regardless of other price movement. It would be necessary for the market to occasionally "correct" the price to eliminate the accumulated rounding error. Such a correction has not been observed in reported financial research. It is more logical to assume that any correction occurs synchronously with trades, thus resulting in an unbiased rounding rule.

Excluding biased rounding rules precludes investors' perceiving a price increase differently (with respect to rounding) from a price decrease of equal magnitude. For example, investors might perceive a price increase to be to the next multiple of one-eighth of a dollar before "true" price gets one-sixteenth shy of that level, but perceive a "true" price decrease of more than one-sixteenth of a dollar to still be the original price. The common rounding rule of rounding to the nearest integral multiple of the minimum unit of trading is an example of an unbiased rounding rule.<sup>35</sup> The simplest rounding procedure that can be con-

structed using this rule is to assume that investors ignore any previous rounding adjustments to price. With this assumption, (3.2) becomes, letting the superscript (1) indicate values under this first rounding procedure

$$P_{it}^{(1)} = P_{i,t-1}^{(1)'} (1+R_{it}) \quad (3.5)$$

Substituting (3.5) into (3.4), the observed price is

$$P_{it}^{(1)'} = P_{i,t-1}^{(1)'} + (P_{i,t-1}^{(1)'}) (R_{it}) + \Delta_{it}^{(1)} \quad (3.6)$$

Equation (3.6) specifies that the return given by the market model (3.1) is to be earned on the actual invested amount at the beginning of period t (the observed price,  $P_{i,t-1}^{(1)'}$ ) and that the amount of this return is to be added to the actual invested amount, the resultant sum of which is then to be rounded to the nearest integral multiple of one-eighth of a dollar to determine the observed price for period t,  $P_{it}^{(1)'}$ .

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<sup>35</sup>There must also be a suitable rule for ties such as rounding up if the preceding digit is even and rounding down if it is odd.

The alternative rounding procedures to be reviewed below differ in these particular aspects.

There is the possibility that the rounding adjustments to price reflect past over-adjustments. For example, consider an initial price of one dollar and "true" upward price movements in the next two periods of slightly more than one-sixteenth of a dollar each. If the above rounding procedure is followed, the closing price at the end of the second day will be \$1-1/4, whereas investors may perceive the price to be only \$1-1/8.

There are at least two ways of implementing a rounding adjustment which reflects past rounding adjustments. One way is to readjust each period the observed price in period  $t-1$  in the amount by which it was originally adjusted. It then follows that, as in the previous rounding procedure,  $\Delta_{it}$  rounds price to the nearest integral multiple of one-eighth of a dollar. Letting the superscript '(2)' notation stand for this particular rounding procedure, (3.4) becomes

$$p_{it}^{(2)} = p_{it}^{(2)} + \Delta_{it}^{(2)} \quad (3.7)$$

and (3.2) adapted to this procedure becomes

$$P_{it}^{(2)} = (P_{i,t-1}^{(2)'} - \Delta_{i,t-1}^{(2)}) (1+R_{it}) \quad (3.8)$$

Setting  $t = t-1$  in (3.7) and then substituting (3.7) in (3.8)

$$P_{it}^{(2)} = P_{i,t-1}^{(2)} (1+R_{it}) \quad (3.9)$$

which can be recognized as identical to (3.2), a result which is expected when one considers that this procedure converts the last period's adjusted price to a "true" price. Finally, substituting (3.9) in (3.7) obtains

$$P_{it}^{(2)'} + P_{i,t-1}^{(2)} + (P_{i,t-1}^{(2)}) (R_{it}) + \Delta_{it}^{(2)} \quad (3.10)$$

Equation (3.10) specifies that the return given by the market model (3.1) is to be earned on the calculated "true"

invested amount at the beginning of period  $t$  (not the actual invested amount) and that the amount of this return is to be added to the calculated "true" invested amount, the resultant sum of which is then to be rounded to the nearest integral multiple of one-eighth of a dollar to determine the observed price for period  $t$ ,  $P_{it}^{(2)}$ . A comparison with (3.6) shows that calculated ("true") previous prices have replaced actual invested amounts.

A different rounding procedure which utilizes past rounding adjustments to compute the present rounding adjustment is obtained by adjusting the current price rather than the previous price for past over or under adjustments. For each price rounding adjustment to reflect all past rounding adjustments, it is necessary only that each adjustment reflect the previous adjustment (since the previous adjustment reflects all rounding adjustments prior to it, etc.). Therefore (3.4) becomes for this procedure

$$P_{it}^{(3)}' = P_{it}^{(3)} + \Delta_{it}^{(3)} - \Delta_{i,t-1}^{(3)} \quad (3.11)$$

where

$P_{it}^{(3)}$ ' is the observed price of security  $i$  at the end of period  $t$  and is an integral multiple of one-eighth of a dollar

$P_{it}^{(3)}$  is the market-generated price of security  $i$  at the end of period  $t$  as given by (3.3)

$\Delta_{it}^{(3)}$  is a price adjustment for security  $i$  at the end of period  $t$  of such size that  $P_{it}^{(3)}$ ' will be an integral multiple of one-eighth of a dollar, i.e.,  $\Delta_{it}^{(3)}$  price rounds  $(P_{it}^{(3)} - \Delta_{i,t-1}^{(3)})$  to the nearest one-eighth of a dollar

$\Delta_{i,t-1}^{(3)}$  is the price adjustment for security  $i$  made in the period just prior to period  $t$  and is of such size that  $(P_{i,t-1}^{(3)} - \Delta_{i,t-2}^{(3)})$  was rounded to the nearest integral multiple of one-eighth of a dollar.  $\Delta_{i,t-1}^{(3)}$  is undefined at  $t=0$

Thus,  $P_{it}^{(3)}$ ' has the same role as  $P_{it}^{(2)}$ ', but for any given observation of price may have a different value. Procedurally,  $P_{it}^{(3)}$  is first decreased (increased) by any upward (downward) adjustment made to  $P_{i,t-1}^{(3)} - \Delta_{i,t-2}^{(3)}$  in the preceding period to accomplish the one-eighth rounding. Then the resultant sum is adjusted by  $\Delta_{it}^{(3)}$  to the nearest integral multiple of one-eighth of a dollar.

In the presence of the one-eighth constraint on prices, equation (3.2) expressed in the notation for this procedure becomes

$$P_{it}^{(3)} = P_{i,t-1}^{(3)'} (1+R_{it}) \quad (3.12)$$

Equation (3.12) specifies that the end-of-period "true" (not rounded) price is calculated from the return given by the market model (equation 3.1) applied to the actual investment at the beginning of period  $t$ ; that is, the observed price at the end of the previous period. Substituting (3.12) into (3.11) gives

$$\begin{aligned} P_{it}^{(3)'} &= P_{i,t-1}^{(3)'} - \Delta_{i,t-1}^{(3)} \\ &\quad + (P_{i,t-1}^{(3)'}) (R_{it}) + \Delta_{it}^{(3)} \end{aligned} \quad (3.13)$$

But from (3.11), replacing  $t$  by  $t-1$  and  $t-1$  by  $t-2$

$$P_{i,t-1}^{(3)'} - \Delta_{i,t-1}^{(3)} = P_{i,t-1}^{(3)} - \Delta_{i,t-2}^{(3)} \quad (3.14)$$

Substituting (3.14) into (3.13)

$$\begin{aligned} P_{it}^{(3)'} &= P_{i,t-1}^{(3)} - \Delta_{i,t-2}^{(3)} \\ &+ (P_{i,t-1}^{(3)'}) (R_{it}) + \Delta_{it}^{(3)} \end{aligned} \quad (3.15)$$

Equation (3.15) specifies that the return given by the market model (3.1) is to be applied to the actual investment at the beginning of the period (the observed price at the end of the preceding period). But the dollar amount to which the dollar return is to be added, to obtain the current "true" price, is the "true" price at the end of the previous period readjusted by the rounding to price for the period previous to that. The resultant sum is then adjusted by  $\Delta_{it}^{(3)}$  to the nearest integral multiple of one-eighth of a dollar. Equation (3.15) then describes a second possible rounding procedure which utilizes information about past rounding adjustments.

In summary, the three rounding procedures reviewed above are

$$(1) \quad P_{it}^{(1)\prime} = P_{i,t-1}^{(1)\prime} + (P_{i,t-1}^{(1)\prime}) (R_{it}) + \Delta_{it}^{(1)} \quad (3.6)$$

$$(2) \quad P_{it}^{(2)\prime} = P_{i,t-1}^{(2)\prime} + (P_{i,t-1}^{(2)\prime}) (R_{it}) + \Delta_{it}^{(2)} \quad (3.10)$$

$$(3) \quad P_{it}^{(3)\prime} = P_{i,t-1}^{(3)} - \Delta_{i,t-2}^{(3)} \\ + (P_{i,t-1}^{(3)\prime}) (R_{it}) + \Delta_{it}^{(3)} \quad (3.15)$$

They all derive from the market model and a premise that observed price is equal to a "true" price (calculated from the market model) plus a rounding adjustment. They differ in the amount of investment to which the return is applied and in the beginning investment to which the amount of return is added to get a true ending price which is then rounded to the nearest one-eighth. The first and third procedures apply the rate of return to the actual amount of investment at the beginning of the period (the observed price). The second method applies it to the "true" price. In the first method the amount of return for the period is added to the observed beginning price. In the second procedure, it is added to the "true" beginning price; and in the

third, it is added to the "true" beginning price adjusted by the previous period's rounding adjustment. In all three procedures the role of the rounding function is the same. It rounds the sum of the other terms on the right hand side to the nearest one-eighth of a dollar giving as a result the observed price at the end of period  $t$ .

### 3.3 CHARACTERISTICS OF THE ROUNDING FUNCTION

It will be assumed that the  $P_{it}$  of equation (3.4) or the sums which replace it in (3.6), (3.10) or (3.15) are continuous and that ties (as to which integral multiple of one-eighth of a dollar is nearer) are extremely unlikely. With the above assumptions,  $\Delta_{it}$  has a uniform continuous distribution between minus one-sixteenth and plus one-sixteenth of a dollar and is zero everywhere else. Its first moment is

$$\mu_{1i} = c \int_{-1/16}^{1/16} x dx = \frac{c}{2} \left[ x^2 \right]_{-1/16}^{1/16} = 0 \quad (3.16)$$

where

$\mu_{1i}$  is the first moment of the frequency distribution of  $\Delta_{it}$

c is a constant and is the frequency distribution of  $\Delta_{it}$ , i.e.,  $f(x)$

x is the variable representing the axis along which  $\Delta_{it}$  is distributed

The second moment of  $\Delta_{it}$  is

$$\begin{aligned} \mu_{2i} &= c \int_{-1/16}^{1/16} x^2 dx = \frac{c}{3} x^3 \Big|_{-1/16}^{1/16} \\ &= c \left[ \frac{(1/16)^3 - (-1/16)^3}{3} \right] = \frac{c}{3} \left( \frac{1}{8} \right)^3 \end{aligned} \quad (3.17)$$

It can be further shown that all odd moments of  $\Delta_{it}$  will be zero and all even moments will be non-zero, a property that follows naturally from the symmetry of  $\Delta_{it}$ .

### 3.4 BETA BIAS IN HOLDING PERIOD RETURNS

For holding period returns (vis-a-vis logarithmic returns, to be discussed in the next section), rearranging (3.2) gives

$$R_{it} = \frac{P_{it} - P_{i,t-1}}{P_{i,t-1}} \quad (3.18)$$

and therefore for the observed returns,  $R'_{it}$

$$R'_{it} = \frac{P'_{it} - P'_{i,t-1}}{P'_{i,t-1}} \quad (3.19)$$

Assuming that the data has been standardized to a mean of zero, the observed beta,  $b'_i$ , will be given by

$$b'_i = \frac{\sum_t R_{mt} R'_{it}}{\sum_t R_{mt}^2} = \frac{\sum_t R_{mt}}{\sum_t R_{mt}^2} \left( \frac{P'_{it} - P'_{i,t-1}}{P'_{i,t-1}} \right) \quad (3.20)$$

substituting (3.4) into (3.20)

$$b'_i = \frac{\sum_t R_{mt} \left( \frac{p_{it} + \Delta_{it} - p_{i,t-1} - \Delta_{i,t-1}}{p_{i,t-1} + \Delta_{i,t-1}} \right)}{\sum_t R_{mt}^2} \quad (3.21)$$

But in the absence of the one-eighth price constraint, the beta that would be observed,  $b_i$ , would be

$$b_i = \frac{\sum_t R_{mt} \left( \frac{p_{it} - p_{i,t-1}}{p_{i,t-1}} \right)}{\sum_t R_{mt}^2} \quad (3.22)$$

From (3.21) and (3.23), the change in the observed beta due

to the one-eighth price constraint is

$$b'_i - b_i = \frac{\sum_{mt}^R \left[ \frac{(P_{i,t-1})(\Delta_{it}) - (P_{it})(\Delta_{i,t-1})}{(P_{i,t-1})(P_{i,t-1} + \Delta_{i,t-1})} \right]}{\sum_{mt}^R} \quad (3.23)$$

But from (3.16)

$$E(\Delta_{it}) = E(\Delta_{i,t-1}) = 0 \quad (3.24)$$

Therefore, if  $\Delta_{it}$  is statistically independent of  $P_{i,t-1}$ ,  $R_{mt}$ , and  $\Delta_{i,t-1}$  (and similarly,  $\Delta_{i,t-1}$  independent of  $P_{it}$ ,  $R_{mt}$ , and  $P_{i,t-1}$ ) then, using (3.24) in (3.23), gives

$$E(b'_i - b_i) = 0 \quad (3.25)$$

If  $\Delta_{it}$  is not statistically independent of  $P_{i,t-1}$ , then the bias introduced to beta by the one-eighth rounding of price

(and using holding period returns) is that given by equation (3.23).

### 3.5 BETA BIAS IN LOGARITHMIC RETURNS

For logarithmic returns,

$$R_{it} = \ln \left( \frac{P_{it}}{P_{i,t-1}} \right) \quad (3.26)$$

where  $\ln$  indicates the natural logarithm. Solving (3.26) for  $P_{it}$ , (3.2) becomes

$$P_{it} = e^{R_{it}} \cdot P_{i,t-1} \quad (3.27)$$

and the three rounding procedures become

$$(1) \quad P_{it}^{(1)'} = e^{R_{it}} \cdot P_{i,t-1}^{(1)'} + \Delta_{it}^{(1)} \quad (3.28)$$

$$(2) \quad P_{it}^{(2)'} = e^{R_{it}} \cdot P_{i,t-1}^{(2)'} + \Delta_{it}^{(2)} \quad (3.29)$$

$$(3) \quad P_{it}^{(3)'} = e^{R_{it}} \left[ P_{i,t-1}^{(3)} - \frac{\Delta_{i,t-1}^{(3)}}{e^{R_{it}}} \right] + \Delta_{it}^{(3)} \quad (3.30)$$

The holding period return expressions for the three rounding procedures (equations 3.6, 3.10, 3.15) provided for two separate uses of price: (1) a price to be multiplied by the rate of return to get the dollar amount of return for the period, and (2) an invested quantity to which the dollar return for the period was added. The price was not necessarily the same for these two uses. No such separation exists in (3.28 - 3.30). However it could be obtained by

using the series expansion for  $e^{R_{it}}$ , namely

$$e^{R_{it}} = 1 + R_{it} + \frac{R_{it}^2}{2!} + \frac{R_{it}^3}{3!} + \dots \infty \quad (3.31)$$

Moreover, it can readily be seen that using the first order approximation to (3.31) in (3.28 - 3.30) produces identically the holding period expressions for rounding rule procedures (3.6, 3.10, 3.15).

The lack of such a separation while complicating the interpretation of (3.30), does not appear to alter the role of  $\Delta_{it}$  in any way. Therefore it is assumed that  $\Delta_{it}$  retains the properties previously discussed, namely that it has, (1) a uniform continuous distribution between minus one-sixteenth and plus one-sixteenth of a dollar and is zero everywhere else, (2) all odd moments equal to zero, and (3) non-zero even moments.

Using (3.26) the observed returns,  $R'_{it}$  are

$$R'_{it} = \ln \left( \frac{P'_{it}}{P'_{i,t-1}} \right) \quad (3.32)$$

and again assuming that the data has been standardized to a mean of zero the bias equation (3.23), for logarithmic returns becomes

$$b'_i - b_i = \frac{\sum_{mt}^t \left[ \ln \left( \frac{p_{it} + \Delta_{it}}{p_{i,t-1} + \Delta_{i,t-1}} \right) \right] - \sum_{mt}^t \left[ \ln \left( \frac{p_{it}}{p_{i,t-1}} \right) \right]}{\sum_{mt}^t} \quad (3.33)$$

But

$$\ln \left( \frac{p_{it} + \Delta_{it}}{p_{i,t-1} + \Delta_{i,t-1}} \right) - \ln \left( \frac{p_{it}}{p_{i,t-1}} \right) = \ln \left[ \frac{1 + \frac{\Delta_{it}}{p_{it}}}{1 + \frac{\Delta_{i,t-1}}{p_{i,t-1}}} \right] \quad (3.34)$$

Substituting (3.34) into (3.33)

$$b'_i - b_i = \frac{\sum_{mt} R_{mt} \cdot \ln \left( \frac{1 + \frac{\Delta_{it}}{P_{it}}}{1 + \frac{\Delta_{i,t-1}}{P_{i,t-1}}} \right)}{\sum_{mt} R_{mt}^2} \quad (3.35)$$

For small x

$$\ln(1 + x) \cong x \quad (3.36)$$

Both  $\frac{\Delta_{it}}{P_{it}}$  and  $\frac{\Delta_{i,t-1}}{P_{i,t-1}}$  are small for high-prices stocks.  
Therefore, using (3.36) in (3.35), for a first order approximation

$$b'_i - b_i = \frac{\sum_{mt} R_{mt} \left( \frac{\Delta_{it}}{P_{it}} - \frac{\Delta_{i,t-1}}{P_{i,t-1}} \right)}{\sum_{mt} R_{mt}^2} \quad (3.37)$$

If  $\Delta_{it}$  is statistically independent of  $R_{mt}$  and  $P_{it}$ , and

recalling that

$$E(\Delta_{it}) = E(\Delta_{i,t-1}) = 0 \quad (3.24)$$

we have

$$E(b'_i - b_i) = 0 \quad (3.38)$$

to a first order approximation. If  $\Delta_{it}$  is not statistically independent, then (3.37) gives, to a first order approximation, the bias introduces by the one-eighth price rounding.

For low-priced stocks, price is small compared to  $\Delta_{it}$  and as a result  $\frac{\Delta_{it}}{P_{it}}$  is not small, and the first order approximation,  $\ln(1+x) = x$ , is not appropriate. The series expansion of  $\ln(1+x)$  is, for  $-1 < x \leq 1$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots \infty \quad (3.39)$$

Applying (3.39) to (3.35) results in the following terms in the numerator of (3.35)

$$\sum_t R_{mt} \left\{ \left[ \frac{\Delta_{it}}{P_{it}} - \frac{\Delta_{it}^2}{P_{it}^2} + \frac{\Delta_{it}^3}{P_{it}^3} - \dots \right] - \left[ \frac{\Delta_{i,t-1}}{P_{i,t-1}} - \frac{\Delta_{i,t-1}^2}{P_{i,t-1}^2} + \frac{\Delta_{i,t-1}^3}{P_{i,t-1}^3} - \dots \right] \right\} \quad (3.40)$$

From equations (3.16) and (3.17) and the discussion following them, the odd moments of  $\Delta_{it}$  are zero but the even moments are not. Therefore even if  $\Delta_{it}$  is statistically independent of  $R_{mt}$  and  $P_{it}$  the numerator of (3.35) does not vanish when expectations are taken and we have for low-priced stocks that

$$E(b'_i - b_i) \neq 0 \quad (3.41)$$

It may be concluded that for low-priced stocks the one-eighth price rounding does introduce a bias in the observed beta coefficients. Its direction is not clear. But that it

exists (and always exists for low-priced stocks) gives a firm foundation to this study. Its magnitude may be small -- perhaps too small to be material -- but that is a result to be determined in this study.

### 3.6 SUMMARY

The rounding procedures must be unbiased. There are at least three plausible unbiased rounding procedures which can be developed, and that has been done above. The forms applicable to holding period (arithmetic) returns provide for the separation of price into two distinct uses: (1) the investment to which the rate of return is to be applied to compute the amount of return, and (2) the amount of investment to which the result of (1) is to be added; and the price need not be the same for each purpose. The forms for logarithmic returns do not provide for this separation in the use of price.

The rounding function itself has a uniform continuous distribution between plus and minus one-sixteenth of a dollar, and has zero odd moments and non-zero even moments. Because of these moment characteristics, when returns are calculated from the usual holding period expression (3.18), the amount of bias introduced into estimates of beta by the one-eighth rounding is zero if the rounding variable is sta-

tistically independent of the other random variables; otherwise it is non-zero. However, if the logarithmic expression for returns is used (3.26), then the beta bias is always non-zero. If the rounding variable is statistically independent of the other random variables, then the bias enters only from the even moments of the rounding function and is a second and higher (even) order effect. Otherwise the bias is a first order effect.

## Chapter IV

### THE RESEARCH METHODOLOGY

In this chapter the design of the simulation program, the techniques, and the statistical procedures used to produce the results described in Chapter V are reviewed. The procedures followed in the simulation program are discussed in some detail, as are the statistical procedures which were used to aid in the general analysis of results from the simulation. The techniques used to address the question of how many replications were needed are described next. A tangential but important issue is what disturbance term to use when generating the "true" price series from the market model. The technique used to determine this is described in the next section. The last section of the chapter describes tests used to determine how well results can be reproduced; that is, if the simulation program is presented with different random number streams, will the beta estimates be materially different?

#### 4.1 OVERVIEW OF THE SIMULATION RUN

For each simulation<sup>36</sup> a starting point in the market index price level series was randomly chosen. Using this price and subsequent prices selected from the index series for a specified differencing interval, market returns were calculated to the end of the predetermined time period.<sup>37</sup> Using the specified "true" beta and "true" alpha for this particular simulation, the market returns described above, and an initial market price for the simulated security, a price series for the security was calculated from the price generating process described in the previous chapter for

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<sup>36</sup>The terminology used here is that a simulation includes the exercise of all steps associated with one pass through the market price level time series. One or more simulations constitute a simulation run. Simulations in which the controlling parameters differ only with the starting point of the market index price level series serve as replications for the statistical analysis.

<sup>37</sup>For simplicity a simulation is described in this overview as if it were completed in its entirety before another simulation was begun. In actuality each simulation was completed only to the point of determining the required observations from the market index series and then subsequent calculations were performed for all simulations in the run. This procedure is more fully explained in the next section.

arithmetic returns, .

$$P_{it} = P_{i,t-1} (1 + \alpha_i + \beta_i R_{mt} + \epsilon_{it}) \quad (3.3)$$

or the equivalent of (3.3) for logarithmic returns,

$$P_{it} = P_{i,t-1} \cdot e^{\alpha_i + \beta_i R_{mt} + \epsilon_{it}} \quad (4.1)$$

The "true" price series for the hypothetical security was then rounded to the nearest one-eighth of a dollar using the second rounding procedure developed in Chapter III, namely, for arithmetic returns

$$P^{(2)}_t = P^{(2)}_{i,t-1} + (P^{(2)}_{i,t-1})(R_{it}) + \Delta^{(2)}_{it} \quad (3.10)$$

or the equivalent of (3.10) for logarithmic returns

$$p_{it}^{(z)} = e^{R_{it}} \cdot p_{i,t-1}^{(z)} + \Delta_{it}^{(z)} \quad (3.29)$$

Returns for the security, either arithmetic or logarithmic, were then calculated from the rounded price series and an ordinary least squares regression (OLS) was performed on the series of returns using the corresponding series of market returns and the market model, equation (1.2). Hence the observed alpha and beta were obtained for the simulation. To provide the desired number of replications the simulation was repeated using a different randomly chosen starting point in the index series each time while holding fixed the other specified parameters, namely, differencing interval, initial price, "true" alpha and "true" beta. These parameters were also varied and the desired number of replications was provided for each combination of the specified parameters by using a randomly chosen starting point in the market index price level series for each replication.

#### 4.2 THE SIMULATION PROGRAM

The simulation program produced a file containing the results of each simulation including: the observed alpha and beta, the replication number of the observation, the specified differencing interval, "true" alpha, "true" beta, initial price, type of return used, and the market index used. To produce this file, a series of constants which governed the operation of the simulation program was specified at the beginning of the program as follows:

The random starting point in the market index price level series was governed by a specified seed for the uniform random deviate generator<sup>38</sup> and a specified "range" constant which multiplied the generator output (in the interval 0-1). An offset was also provided so that the range would not begin prior to the specified number of observations of the offset constant. (However, the offset feature was not used in the results reported here.) The length of the period, measured from the random starting point, for which observations were to be selected from the market series at the specified differencing interval, was also specified. For example, a specified offset of 500, range of 1,000, length of 3,000 and differencing interval of two, produced

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<sup>38</sup>The random deviate generator is described in Helwig and Council (1979, p. 443) and Lewis, Goodman and Miller (1969).

for each replication a series of 1,500 index prices randomly beginning somewhere between the 501st through the 1,501st observation of the actual market series.

The additional parameters specified for a run were the market index to be used, the type of return(s) (arithmetic or logarithmic), initial price(s), "true" alpha(s), "true" beta(s), differencing interval(s), and number of replications. Additionally, an option was included which allowed a "true" alpha to be estimated from an assumed annual risk-free interest rate and the "true" beta and differencing interval in use at that point. The calculation used was that given by equation (1.3) assuming that  $\alpha_i$  is constant over t for each combination of "true" beta and differencing interval. More precisely, the expression for the specified true alpha was,

$$\alpha_i = (R_f / 365) (INT) (1 - \beta_i) \quad (4.2)$$

where

$R_f$  is the assumed annual risk-free interest rate

INT is the differencing interval

$\beta_i$  is the assumed "true" beta

All but the first of these additional parameter lists were executed in hierachial fashion in the order listed; that is, the specified number of replications was performed for each of the specified differencing intervals, all of which in turn were done for each of the specified betas, all of which in turn were done for each of the specified alphas, and so on. The results, therefore, for all desired combinations of the parameters for a particular market index would be obtained in a single run. In practice, however, this would have made the run entirely too large.

The first part of the simulation program, using the parameters described above, computed the observations from the market index series which were required to perform all the simulations specified by the parameters. (There typically would be, of course, many duplications.) This file was then sorted by observation number and match-merged<sup>39</sup> with a copy of the market index file from which all holidays had been dropped.<sup>40</sup> The resulting file was then sorted by index, type of return, initial price, alpha, beta, interval, replication number, and observation number. Thus it was the

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<sup>39</sup>In match-merging, observations in two or more files that have the same value of a common matching variable, in this case the observation number, are joined to produce a new observation. Thus each entry in the file of required observations additionally has the market index price level for that observation after this step has been performed.

<sup>40</sup>In this study only trading days were recognized.

same as the original required observations file except that the corresponding market index price levels had been added.

Next, for each series, the market return for each period was calculated similarly to either (1.4) or to (1.5), depending upon whether the returns were to be arithmetic or logarithmic respectively. An appropriate standard error of regression was computed to be used as the standard deviation of the random disturbance term (this computation is discussed in section 4.5 of this chapter), and an ending price for the period was calculated from (3.3) or (4.1) using the disturbance term, the market return for the period, the specified "true" alpha and beta, and the "true" (not rounded) price for the previous period. The ending price was rounded to the nearest one-eighth of a dollar and the security's return for the period was computed using this rounded price and the rounded price for the previous period in (1.4) or (1.5). These steps were repeated for every holding period in each series (replication).

After all returns for every series were computed, various descriptive statistics (mean, standard deviation, maximum, minimum and range) for price, rounded price, rounded return and market return using the replications within each combination of parameters were calculated. The statistics for price were particularly useful in determining if the

price series for a particular combination of parameters was well-behaved. Finally, the series of security returns for each replication was regressed on the corresponding market returns series (recall that at this point the corresponding market returns are part of the same file) using the market model (1.2). The resulting observed alphas and betas along with the specified parameters which produced them and the replication number were added to the output file of the simulation run.

#### 4.3 THE STATISTICAL ANALYSIS

The output files from the simulation runs or combinations of these files produced various descriptive statistics and were used for several statistical tests. Two such tests, (1) determining the appropriate number of replications to be used, and (2) examining the repeatability of the experiment, will be separately described in following sections of this chapter. The most pertinent of various other procedures used will be described in this section.

For each replication, the bias in the observed beta (i.e., "true" beta minus observed beta) was computed. For each unique combination of specified index, type of return, initial price, "true" alpha and beta, and differencing interval; the mean, standard deviation, minimum, and maximum

of the beta bias, observed beta, and observed alpha were computed. A one-sample Student's t test<sup>\*1</sup> was performed to test the null hypothesis of zero beta bias. The probability of  $|T| > |t|$  was computed and listings were obtained and sorted by: (1) the mean of the beta bias, (2) the probability  $|T| > |t|$  for the beta bias, and (3) the standard deviation of the beta estimates for each combination of the specified parameters. These listings were then examined to see which combinations were causing extreme results. An n-way analysis of variance,<sup>\*2</sup> with interactions, was also performed using as factors the specified parameters which changed during the simulation run and the amount of bias in the beta estimates as the response variable.

#### 4.4 THE DETERMINATION OF THE APPROPRIATE NUMBER OF REPLICATIONS

Following simulation runs using 150 replications, the replications for each combination of specified parameters were divided into groups of the first 10 replications, the next 20, the next 30, 40, and 50. Then for each combination of specified parameters, a one-way analysis of variance<sup>\*3</sup>

\*1This test is discussed in a variety of standard statistical tests, for example, Ott (1977, p. 103).

\*2For a discussion of this procedure, see any standard statistical text such as Ott (1977, p. 430).

\*3See, for example, Ott (1977, p. 354).

was run using the number of replications as the factor and amount of bias in the beta estimates as the response. A Duncan's multiple range test,<sup>\*\*</sup> as extended to unequal cell sizes by Kramer (1956), was then performed to see if there were significant differences in the means for the differing number of replications. For some combinations of parameters this procedure was repeated for groups of replications of sizes 5, 10, 15 and 20, and for sizes of 50 and 100.

An additional procedure used to check the stabilization of the beta estimates was to compute a cumulative average for each replication of each combination of specified parameters. The average amount of bias in the beta estimates versus replication number was then plotted for each combination and visually inspected to identify the point at which the beta estimates became relatively stable.

#### 4.5 THE DETERMINATION OF AN APPROPRIATE STANDARD ERROR

In the generation of a price series using (3.3) or (4.1) the disturbance term was generated using a normal random deviate generator (Helwig and Council 1979, p. 444). The mean was set at zero, but the appropriate standard deviation to use was open to question. If the value chosen was too small, then there was little value in using a disturb-

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<sup>\*\*</sup>See, for example, Ott (1977, p. 392).

ance term. If too large, the price series became ill-behaved.

Using daily price data from another study,<sup>45</sup> average standard errors of regression of the market model were computed for 92 over-the-counter stocks, 58 New York Stock Exchange stocks, and 43 American Stock Exchange stocks. The average price of each stock for the first 100 days was also computed. The results are shown Table 4.1 below. From Table 4.1 it can be seen that the average standard error of regression varies considerably. The range of average prices indicates that the stocks traded on the New York Stock Exchange are higher priced than are the others. Higher priced stocks would have their standard errors inflated less by one-eighth price rounding, so the average of the New York Stock Exchange group was chosen as the appropriate standard error for one day differencing intervals. Since the standard error varies with the differencing interval<sup>46</sup> it was necessary to devise an appropriate adjustment to the one day differencing interval standard error for longer differencing intervals.

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<sup>45</sup>Keown and Pinkerton (1981).

<sup>46</sup>See Schwartz and Whitcomb (1977b) and Smith (1978).

Table 4.1

## Average Standard Errors And Prices

Number of Stocks	Exchange	Average Std. Err.	Average Minimum	Price Maximum
92	OTC	.0268	1.24	34.67
58	NYS	.0242	4.48	58.31
43	ASE	.0309	2.22	43.90
193		.0269		

It can be shown that where the number of observations,  $n$ , is sufficiently large compared to the number of parameters to be estimated in a regression  $k$ , so that  $n-k \approx n-1$  then

$$\frac{SE_n}{SE_1} \cong \sqrt{\frac{1-R^2_n}{1-R^2_1}} \quad (4.3)$$

where

$SE_n$  is the standard error of regression at a differencing interval of length  $n$

$SE_1$  is the standard error of regression at a differencing interval of length 1

$R^2_n$  is the coefficient of determination at a differencing interval of length  $n$

$R^2_1$  is the coefficient of determination at a differencing interval of length 1

In their Table I, Cohen, Hawawini, Maier, Schwartz and Whitcomb (1980a) reported  $R^2$  for differencing intervals of 1, 2, 3, 4, 5, 10, 15, and 20 days. Additionally Smith (1978) reported  $R^2$  for differencing intervals of two months (about

42 trading days), three months (about 63 trading days), 12 months (about 243 trading days) and others. Using the Cohen et al data extended by the Smith data in equation (4.3) provides the adjustments shown in Table 4.2. Using an equation of the form

$$y = a \log x + b \quad (4.4)$$

and adjusting slightly so that  $SE_n/SE_1 = 1$  at  $n = 1$ , and using the previously discussed standard error at  $n = 1$ , we have

$$SE_n = .0242 [1 - (.05644) (\ln(INT))] \quad (4.5)$$

where

$SE_n$  is the standard error of regression for a differencing interval of length  $n$

$\ln(INT)$  is the natural logarithm of the differencing interval

The result of (4.5) then becomes the appropriate standard deviation to use with the normal random deviate generator to produce the disturbance term for the price series generation.

#### 4.6 EXAMINATION OF THE REPRODUCIBILITY OF THE EXPERIMENT

For a few selected combinations of the specified parameters, simulation runs were made using the following variations:

1. the "control" run
2. same as (1) except using a different seed for the random number generator which picks the starting point in the market index series
3. same as (1) except using a different seed for the random number generator which creates the disturbance term.

Paired difference t tests<sup>\*7</sup> were performed on each pair of the above.<sup>\*8</sup> The paired differences were also tested for

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\*7For a discussion of this test, see Ott (1977), p. 244).

\*8Multiple pair-wise tests suffer a degradation in their ability to avoid type I errors overall. However for three pairs the degradation is not large, an  $\alpha$  of about .035 being required for each individual test in order to give an overall error rate of .10.

Table 4.2

## Standard Error Adjustments

Differencing Interval	Ratio SE /SE
1	1.00000
2	.96868
3	.96217
4	.94646
5	.92619
10	.88356
15	.86850
20	.81776
42	.77848
63	.75834
243	.68994

normality using the Kolmogorov-Smirnov D-statistic<sup>49</sup> and the differences were further tested using the non-parametric tests, Wilcoxon's signed-rank test and Fisher's sign test.<sup>50</sup>

It should be noted that the question of reproducibility is closely allied with the question of how many replications are required for the beta estimates to stabilize, which was discussed in Section 4.4. If the estimates do not stabilize sufficiently in the number of replications which were made, then a different random number stream will likely produce different beta estimates.

#### 4.7 SUMMARY

In Section 4.1 the simulation program has been conceptually discussed. Briefly, the simulation procedure was to determine which observations in the market index price level series were required to perform the specified simulation run, then get them, return each series to its original order, perform the required regressions and place each simulation's alpha and beta estimates in a file along with other information describing the simulation. Section 4.2 described how this was done. Section 4.3 reviewed the

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<sup>49</sup>See Stephens (1974).

<sup>50</sup>These two tests are discussed in Ott (1977, pp. 307, 313) and in Hollander and Wolfe (1973, pp. 27, 39) as well as in various other statistical texts.

descriptive statistics produced from the simulation runs and the various orders into which they were sorted to facilitate analysis. Statistical inference about the population mean of the amount of bias in the beta estimates was also introduced.

The use of Duncan's multiple range test and also plots of the cumulative average for the replications to address the question of how many replications were needed was presented in Section 4.4. In Section 4.5 the determination of the appropriate standard deviation of the price-generating model's random disturbance term was discussed. Data from another study was used to estimate an appropriate standard error of regression for a one-day differencing interval.  $R^2$ 's reported in two other studies were used to estimate the relationship between the standard error for a one-day differencing interval and the standard errors for larger intervals, and an empirical equation was derived to implement this estimation. Lastly, the statistical procedures used to test the robustness of the simulation runs to changes in the random number streams were described in Section 4.6.

## Chapter V

### THE RESULTS

The results of two procedures intended to resolve the questions of how many replications to use, are presented in the first section of this chapter. The next section addresses the very important question of how reproducible are the results. If the random number streams are different, will the results be materially different? The results of a number of runs and procedures are reviewed in order to answer this question. The third section of this chapter contains the principal results of this study, namely the amount of bias in the observed betas for different combinations of the specified parameters. Results are presented for a broad set of specified parameters (for logarithmic returns) and then contrasted with results for a set which is similar but uses different choices for the assumed "true" alpha. The results for logarithmic returns are also contrasted with the results for holding period (arithmetic) returns.

#### 5.1 THE DETERMINATION OF THE APPROPRIATE NUMBER OF REPLICATIONS

The results from Duncan's multiple range tests in general were of little help in establishing the appropriate

number of replications to be used. Eighteen runs were produced from the possible combinations of the following specified parameters:

- Initial Prices: .5, 1, 1.5
- True Betas : .2, 2
- Intervals : 10, 20, 60

True alpha's were calculated from (4.2) at  $R_f = .06$  annually. For logarithmic returns, with the S&P 500 as the market index, no starting point offset, starting point range of 1,000 trading days and a series length of 3,450 trading days only four of the combinations produced any significant differences in the means of the observed betas for replications of 50, 40, 30, 20 and 10 in number. These four are shown in Table 5.1 below. The vertical bars span treatment classes<sup>51</sup> which are not significantly different from each other at an alpha level of .05.

The results in Table 5.1 suggest, although not strongly,<sup>52</sup> that of the number of replications compared at this point either 40 or 50 replications is preferred. Since these are at the upper end of the alternatives, an

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<sup>51</sup>In this particular analysis the treatments are the different number of replications used each time.

<sup>52</sup>Recall that 14 of the 18 combinations indicate that there is no statistically significant difference (at an alpha level of .05) in the means of observed betas for numbers of replications of 10, 20, 30, 40 or 50.

TABLE 5.1

## Significant Differences in Observed Beta

<u>Values of Parameters</u>	<u>Number of Replications</u>	<u>Mean Observed Beta</u>
Initial Price: 1	50	.214
True Beta : .2	40	.210
Interval : 10	20	.208
	10	.175
	30	.167
Initial Price: 1	40	.215
True Beta : .2	20	.211
Interval : 20	50	.191
	30	.176
	10	.156
Initial Price: 1	10	.263
True Beta : .2	50	.197
Interval : 60	40	.188
	30	.178
	20	.165
Initial Price: 1.5	50	.209
True Beta : .2	30	.206
Interval : 20	40	.204
	20	.184
	10	.159

additional run was made using replications of 50 and 100. None of the means of the observed betas for the combinations of specified parameters were significantly different for the additional replications.

Lack of a statistical difference in the means of observed betas for the different numbers of replications can be explained by one of two possible reasons: (1) the between treatments sum of squares is small, hence the sample means of the observed betas are close together for the differing numbers of replications, or (2) the within treatments sum of squares is large, hence the difference in sample means of the observed betas for differing numbers of replications may be sizeable. The first illustration in Table 5.2 shows the range of mean observed betas having the largest variation for the different numbers of replications for a specified combination of parameters, the last illustration shows the smallest. The smallest variation is within acceptable limits; but the largest is larger than desirable, albeit perhaps not prohibitively so.

Since the use of Duncan's multiple range test did not give a clear indication of the appropriate number of replications to use, the cumulative average procedure described

TABLE 5.2

## Range of Means of Observed Betas

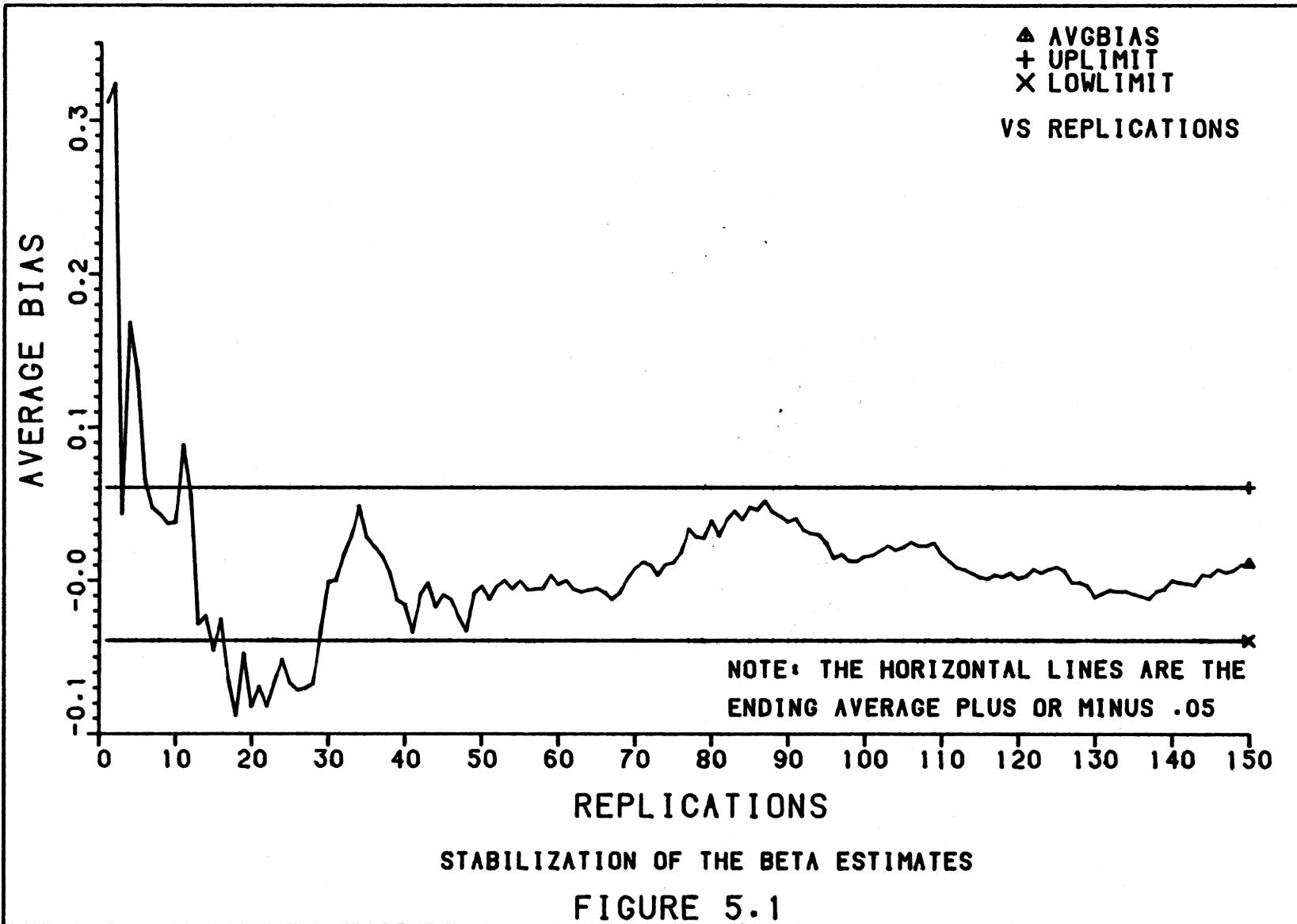
Values of Parameters	Number of Replications	Mean Observed Beta
Initial Price: 0.5 True Beta : 2.0 Interval : 60	30	2.23
	50	2.18
	100*	2.17
	40	2.14
	50*	2.11
	20	2.09
	10	1.94
Initial Price: 1.0 True Beta : 2.0 Interval : 60	10	2.22
	30	2.17
	50*	2.16
	20	2.13
	100*	2.12
	40	2.12
	50	2.11
Initial Price: 1.5 True Beta : 2.0 Interval : 10	50*	2.04
	10	2.03
	20	2.02
	50	2.02
	40	2.02
	30	2.01
	100*	2.01

\*From a run using replications of 50 and 100.  
 Remainder are from a run of replications of  
 10, 20, 30, 40 and 50. Therefore, each of the  
 50 replications use different random number  
 streams for the starting point.

in Section 4.4 of Chapter IV was used for all of the combinations of specified parameters mentioned earlier in this section, as well as various others. The number of replications was 150. Figure 5.1 is an illustration of these plots and is for an initial price of \$0.50, a true beta of 2.0, and an interval of 20 trading days. From visual inspection of these plots, it was concluded that the number of replications required for the bias estimate to stabilize was quite variable, but for all combinations of parameters it was reasonably well accomplished<sup>53</sup> within the 150 replications. For a few combinations of specified parameters, 255 replications were tried, and 500 was tried for one combination. No new information was obtained from these extended runs. Therefore it was assumed that 150 replications was an appropriate number to use for subsequent simulation runs.

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<sup>53</sup>A reasonable stabilization was assumed to be when the mean of the bias in beta came within  $\pm .05$  of its value at 150 replications and stayed within that range for the remainder of the replications.



## 5.2 EXAMINATION OF THE REPRODUCIBILITY OF THE EXPERIMENT

Table 5.3 displays the results of the three simulation runs mentioned in Section 4.6 of Chapter IV.<sup>54</sup> It can be seen that for many of the parameter combinations the results either do not differ greatly between runs or the amount of bias is so small that it is not material that the results differ.<sup>55</sup> However, the variation in results for several of the combinations is disturbingly large as revealed in the box in Table 5.3. Other large variations are indicated by an asterisk.

The entry with the largest variation in bias in Table 5.3 was studied further. Runs were made at 255 replications instead of 150. Also a differencing interval of two trading days was used in addition to the ten trading day interval which was previously examined. The results are shown in Table 5.4. Runs 4, 5 and 6 correspond to Runs 1, 2 and 3 of Table 5.3 in their use of random number generator seeds. Run 7 used only 150 replications, but employed different random number streams than Runs 1-3. Runs 4-7 show a disturbing amount of variation in the mean amount of bias

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<sup>54</sup>Run 1 is the control run. Run 2 has a different seed for the random number generator which picks the starting point in the market index series. Run 3 is the same as Run 1 except that the seed for the random number generator for the disturbance term is different.

<sup>55</sup>For an example of the latter, see the first entry in Table 5.3.

TABLE 5.3

Mean Amount of Bias in Beta  
(150 Replications)

Initial Price	True Beta	Interval	Mean Amount of Bias		
			Run 1	Run 2	Run 3
0.5	.2	10	.007	-.009	-.008
0.5	.2	20	.000	-.002	.004
*0.5	.2	60	-.026	.008	-.007
*0.5	2	10	.056	.018	.107
0.5	2	20	-.039	.028	-.028
0.5	2	60	-.153	-.162	-.154
1.0	.2	10	.000	-.005	-.007
1.0	.2	20	.005	-.005	.005
1.0	.2	60	.009	.010	.010
1.0	2	10	-.024	-.009	-.030
*1.0	2	20	-.039	-.043	-.076
1.0	2	60	-.134	-.144	-.129
1.5	.2	10	.000	.000	-.012
1.5	.2	20	.000	.001	-.002
1.5	.2	60	.002	.009	-.002
1.5	2	10	-.021	-.043	-.038
*1.5	2	20	-.050	-.060	-.031
1.5	2	60	-.141	-.146	-.139

\*Displays large between-run variation in mean amount of bias

obtained. However, at an interval of ten trading days, the results displayed in Table 5.4 are all within the range established by the results in Table 5.3, and the variation is somewhat less. At an interval of two trading days the variation is substantially smaller, being about 11%.

To further examine the question of reproducibility, paired difference tests were employed on each pair of Runs 1, 2 and 3, and the results of these tests as well as the results of tests to see if the paired differences are normally distributed, are presented Table 5.5. The t tests indicate insufficient evidence in the data to reject the null hypothesis,  $H_0$ , of zero differences at any reasonable significance level. However the Kolmogorov-Smirnov tests indicate that the paired differences of Runs 2 and 3 clearly should not be assumed normal, and the differences of Runs 1 and 3 could only marginally be considered as normal. Therefore the non-parametric Wilcoxon signed-rank test and Fisher's sign test was run on the paired differences. The results of both of these tests indicated that there was insufficient evidence in the data to reject  $H_0$  at any reasonable level of significance.<sup>56</sup>

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<sup>56</sup>Recall, however, that the paired difference tests are tests for a location difference in the two sets of readings. Differences between the readings for each pair can be material, but if one is not somewhat systematically higher or lower than the other, the paired difference test will indicate a failure to reject  $H_0$ : the differences are

TABLE 5.4

Mean Amount of Bias in Beta  
(255 Replications)

Initial Price: 0.5

True Beta : 2.0

Mean Amount of Bias

<u>Interval</u>	<u>Run 4</u>	<u>Run 5</u>	<u>Run 6</u>	<u>Run 7*</u>
10	.068	.023	.060	.098
2	.405	.429	.431	.453

\*150 Replications

TABLE 5.5  
Paired Difference and Normal Tests

	<u>Runs 1-2</u>	<u>Runs 1-3</u>	<u>Runs 2-3</u>
<u>T tests</u>			
t statistic <sup>1</sup>	.97	-.14	-.70
p-value	.34	.87	.50
<u>Kolmogorov-Smirnov Tests</u>			
D statistic <sup>2</sup>	.95	.92	.79
p-value	.49	.14	<.01
<u>Wilcoxon Signed Rank Test</u>			
T* statistic <sup>1</sup>	-1.20	-.20	.37
p-value	.23	.84	.71
<u>Fisher's Sign Test</u>			
B* statistic <sup>1</sup>	-.94	-.47	.94
p-value	.48	.81	.48

Note: The p-values are for two-sided tests.

T\* and B\* are the large sample approximations for the T and B statistic.

<sup>1</sup>H<sub>0</sub>: The difference is 0.

<sup>2</sup>H<sub>0</sub>: The distribution is normal.

The paired difference tests were repeated for another set of specified parameters, namely specified "true" betas of 0.5, 1, 2 and 3 and differencing intervals of 20, 40, 60 and 80 trading days. Only an initial price of 0.5 was used, other parameters remaining unchanged. These combinations generally tended to generate larger amounts of bias than previous combinations. The bias means are given in Table 5.6. As in Table 5.3, the entry having the largest between-run variation is within the box and others with large variations are denoted with an asterisk. The results of the paired difference tests and the tests for a normal distribution are given in Table 5.7. The t tests indicate that there is insufficient evidence in the data to reject  $H_0$ : the differences are zero, at any reasonable significance level. The normality tests indicate that the differences between Runs 8 and 9 are only marginally normally distributed, but the results of the two non-parametric paired difference tests agree with the t test in failing to reject  $H_0$  at any reasonable significance level.

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zero.

TABLE 5.6

Mean Amount of Bias in Beta  
(150 Replications)

<u>"True"</u> Beta	<u>Interval</u>	<u>Mean Amount of Bias</u>		
		<u>Run 8</u>	<u>Run 9</u>	<u>Run 10</u>
0.5	20	.003	.007	0
0.5	40	-.009	.005	.002
0.5	60	-.008	.001	-.016
0.5	80	.010	.006	-.031
1.0	20	.001	-.008	-.013
1.0	40	-.021	-.009	-.018
1.0	60	-.008	-.007	-.016
1.0	80	-.012	-.005	-.005
*2.0	20	-.047	-.086	-.114
*2.0	40	-.191	-.179	-.244
*2.0	60	-.168	-.202	-.156
2.0	80	-.164	-.134	-.165
3.0	20	1.165	1.132	1.170
3.0	40	1.139	1.147	1.166
3.0	60	1.046	1.002	1.012
*3.0	80	1.032	1.062	1.099

All initial prices are 0.5 dollars.

\*Displays large between-run variation in mean amount of bias.

TABLE 5.7  
Paired Difference and Normal Tests

	<u>Runs 8-9</u>	<u>Runs 8-10</u>	<u>Runs 9-10</u>
<u>T tests</u>			
t statistic <sup>1</sup>	.36	.74	.52
p-value	.72	.47	.61
<u>Kolmogorov-Smirnov Tests</u>			
D statistic <sup>2</sup>	.89	.95	.97
p-value	.07	.50	.75
<u>Wilcoxon Signed Rank Tests</u>			
T* statistic <sup>1</sup>	.10	-.67	-.57
p-value	.91	.50	.57
<u>Fisher's Sign Tests</u>			
B* statistic <sup>1</sup>	10	9	11
p-value	.45	.80	.21

Note: The p-values are for two-sided tests.

T\* is the large sample approximation  
for the T statistic

<sup>1</sup>H<sub>0</sub>: The difference is 0

<sup>2</sup>H<sub>0</sub>: The distribution is normal.

### 5.3 THE AMOUNT OF BIAS IN THE OBSERVED BETAS

#### 5.3.1 Results For Logarithmic Returns

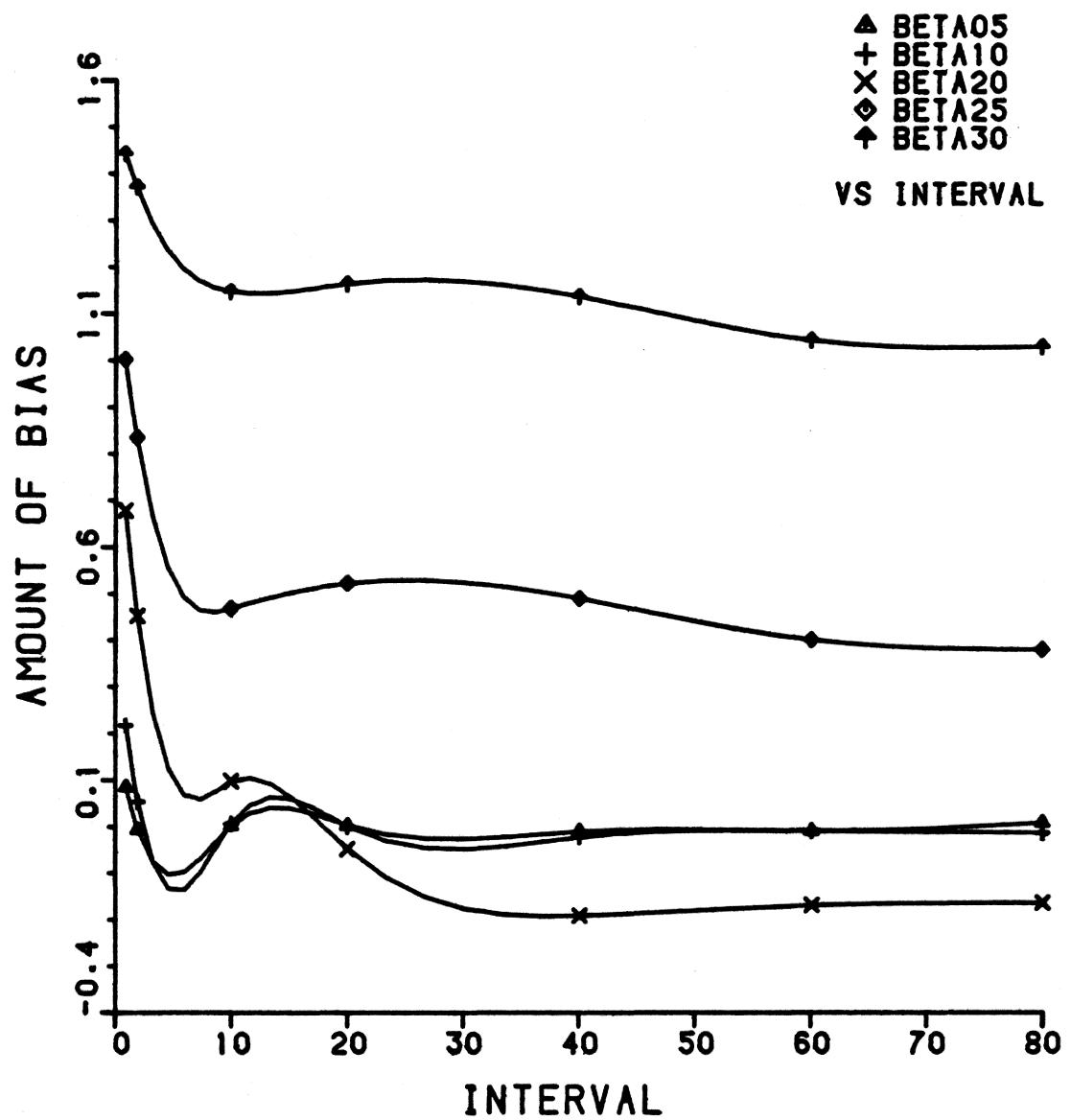
Figure 5.2 shows the amount of bias<sup>57</sup> existing in calculated beta coefficients at several differencing intervals for various levels of "true" beta. The differencing intervals were 1, 2, 10, 20, 40, 60, and 80 trading days. The levels of "true" beta used were 0.5, 1.0, 2.0, 2.5, and 3.0. The market index used was the S&P 500, the returns were logarithmic, the initial price was 0.5 dollars, and the alpha values used were calculated from (4.2) using an annual interest rate of 6%.

It can be seen from Figure 5.2, that the bias at "true" betas of 0.5 and 1.0 is almost zero except at a differencing interval of one where it is slightly positive. For an assumed "true" beta of 2.0 the bias is decidedly positive at small differencing intervals and declines for increasing differencing intervals, becoming zero between ten and twenty days and is negative thereafter. Biases for "true" betas of 2.5 and 3.0 are decidedly positive over the range of differencing intervals examined.

Figure 5.3 displays the same data as Figure 5.2. The amount of bias, however, is plotted versus the "true" beta (at different levels of differencing interval) instead of

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<sup>57</sup>The amount of bias is the "true" beta minus the observed beta from the simulation run.



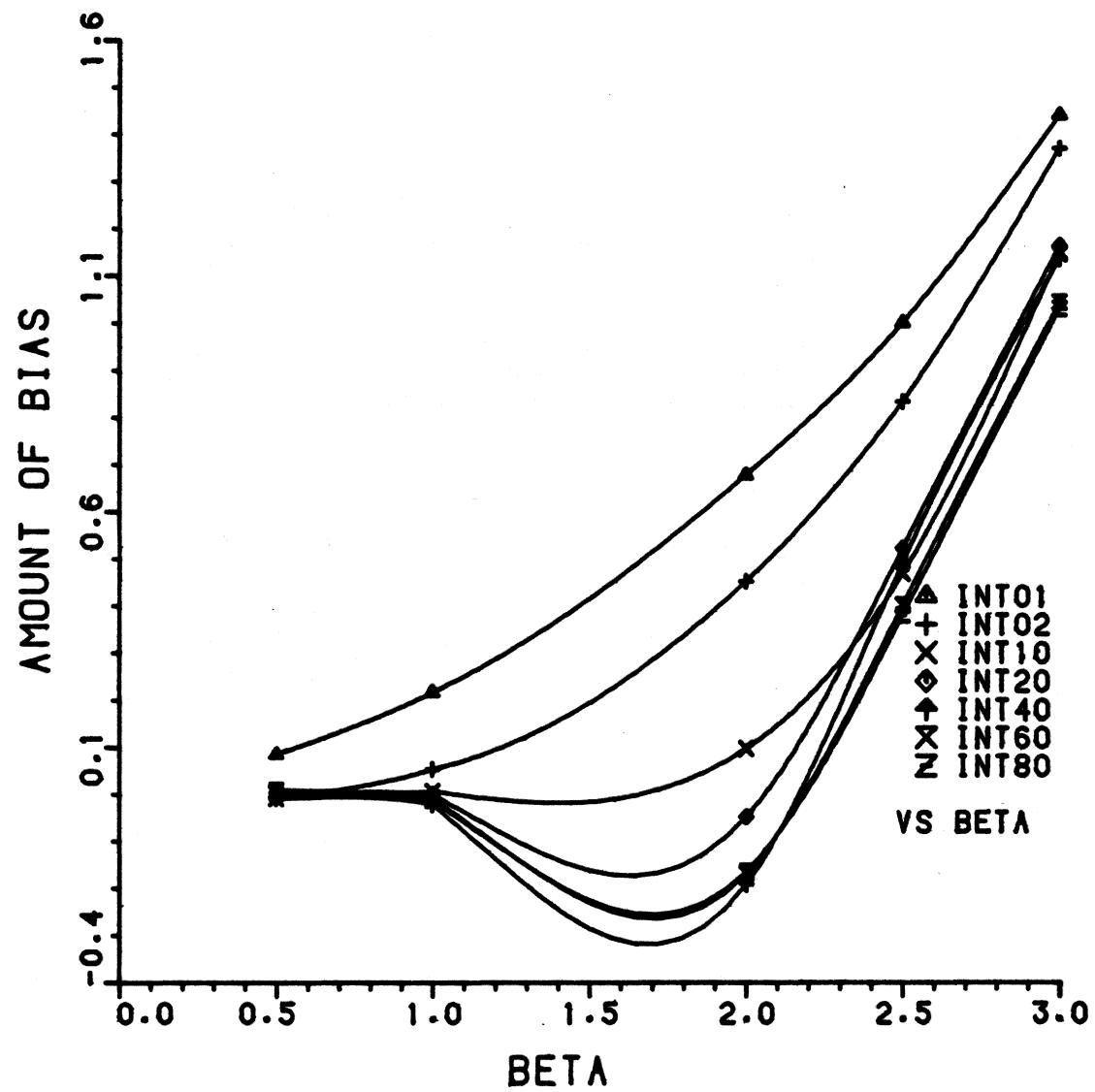
THE AMOUNT OF BETA BIAS  
AT VARIOUS LEVELS OF BETA  
FOR DIFFERENT INTERVALS

FIGURE 5.2

versus differencing interval (at different levels of beta). From Figure 5.3 it can be seen that the amount of bias is substantially greater at the higher betas and at the very short differencing intervals.

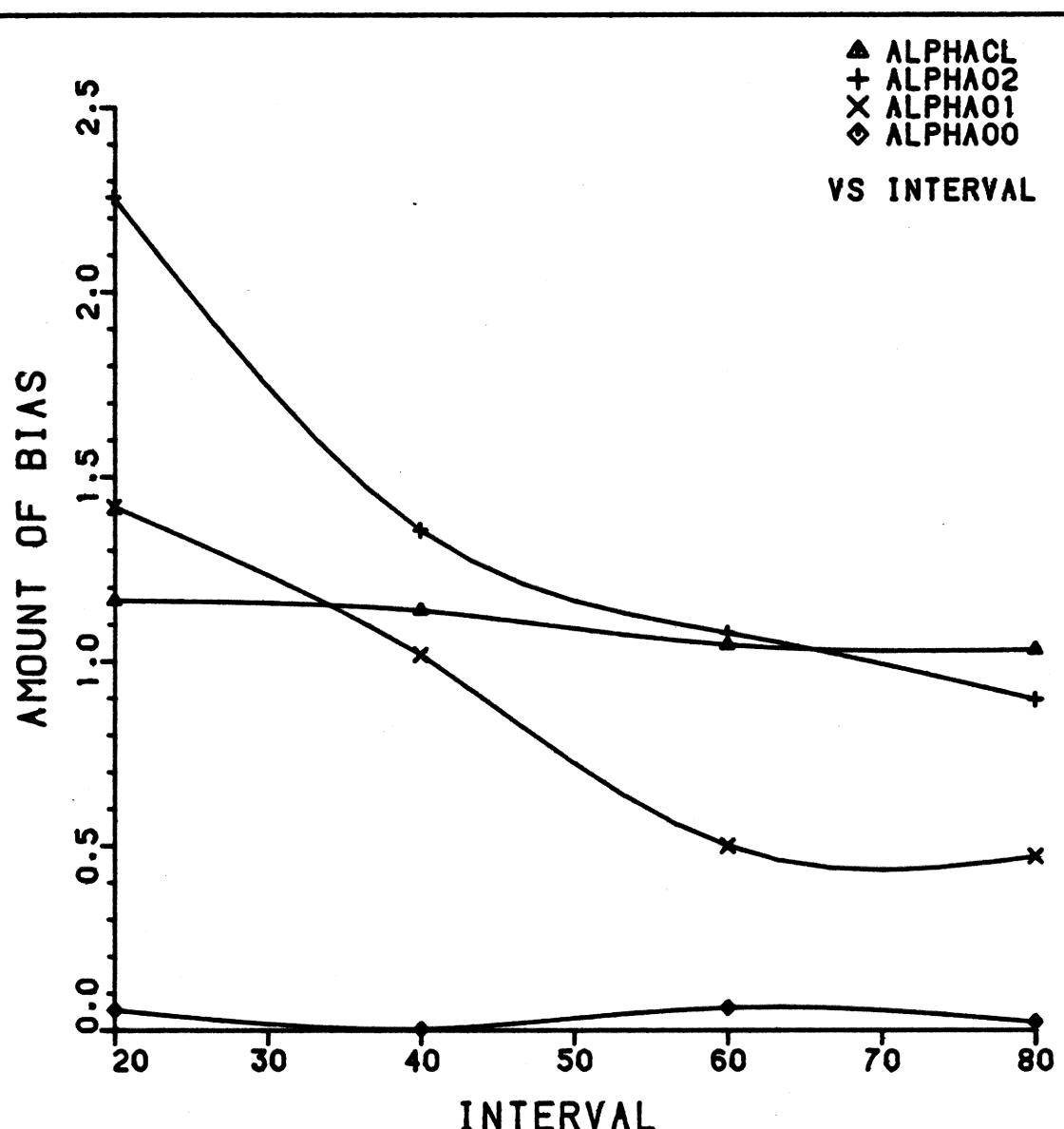
The amounts of bias shown in Figures 5.2 and 5.3 as described above could be due to factors other than the 1/8th price rounding. One such possibility, that it is due to the choice of the "true" alpha, was explored more fully and the results are shown in Figures 5.4 and 5.5.

Figure 5.4 is similar to Figure 5.2 except that the curves are only at a "true" beta of 3.0 and only for differencing intervals of 20 to 80. Additionally, curves for "true" alphas of 0, -0.1, and -0.02 have been added to the curve for a calculated value of alpha which was included in Figure 5.2. This figure shows that the amount of bias does vary considerably with the specified value of alpha, being near zero for an alpha of zero and about 2.25 for an alpha of -0.02 and differencing interval of 20 trading days. The data for Figure 5.5 was similar to that for Figure 5.4 except that the initial price specified was one dollar and the choices of alpha did not include zero. The results are similar to Figure 5.4 except that the biases are shifted toward lower values.



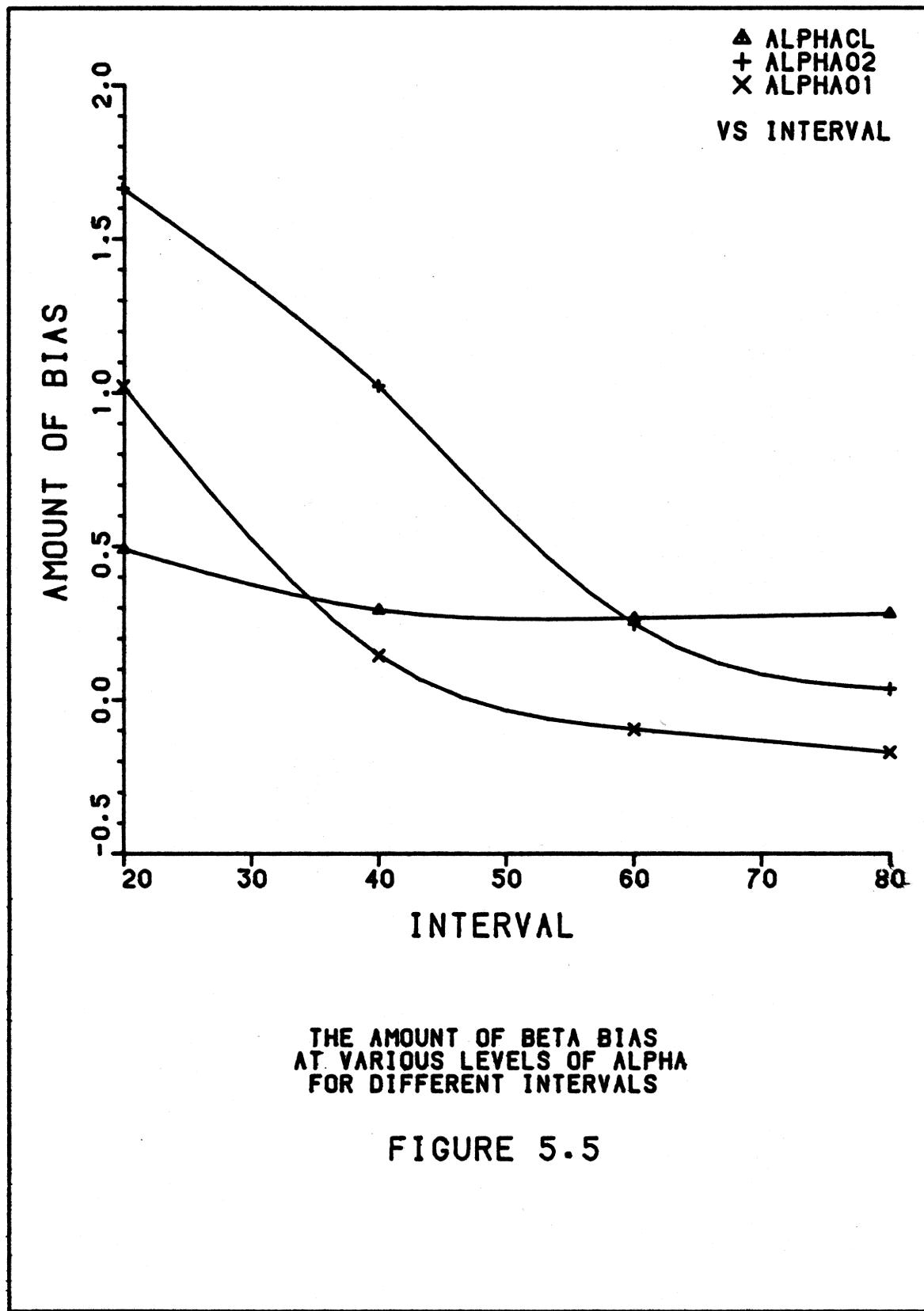
THE AMOUNT OF BETA BIAS AT  
VARIOUS LEVELS OF INTERVAL  
FOR DIFFERENT BETAS

FIGURE 5.3



THE AMOUNT OF BETA BIAS  
AT VARIOUS LEVELS OF ALPHA  
FOR DIFFERENT INTERVALS

FIGURE 5.4



The results from each of the replications which form the means plotted in Figure 5.4 and 5.5 vary considerably. Therefore, there is a question of the significance of the differences in their means which are illustrated in Figures 5.4 and 5.5. Although they visually appear to be different, they may not be statistically different. For the data producing the results in Figures 5.4, an analysis of variance was performed using the replications as observations, amount of bias as the response, and the four choices of alpha as the four treatment levels. The analysis was first performed using the four levels of interval as the second factor in a two-way analysis of variance. Since the interaction term (cross effect) was found to be highly significant (p-value of .0001), the analysis was repeated as a one-way ANOVA<sup>58</sup> at each of the four levels of interval. In each analysis the null hypothesis of no difference in the bias means for each choice of alpha, was rejected at the .0001 significance level. Additionally, a Duncan's multiple range test for each ANOVA showed all means to be significantly different from each other at the .05 significance level except for the two means at a differencing interval of 60 trading days with calculated and -0.02 choices for alpha. (The latter can be observed in Figure 5.4 as the two most closely spaced

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<sup>58</sup>Analysis of variance.

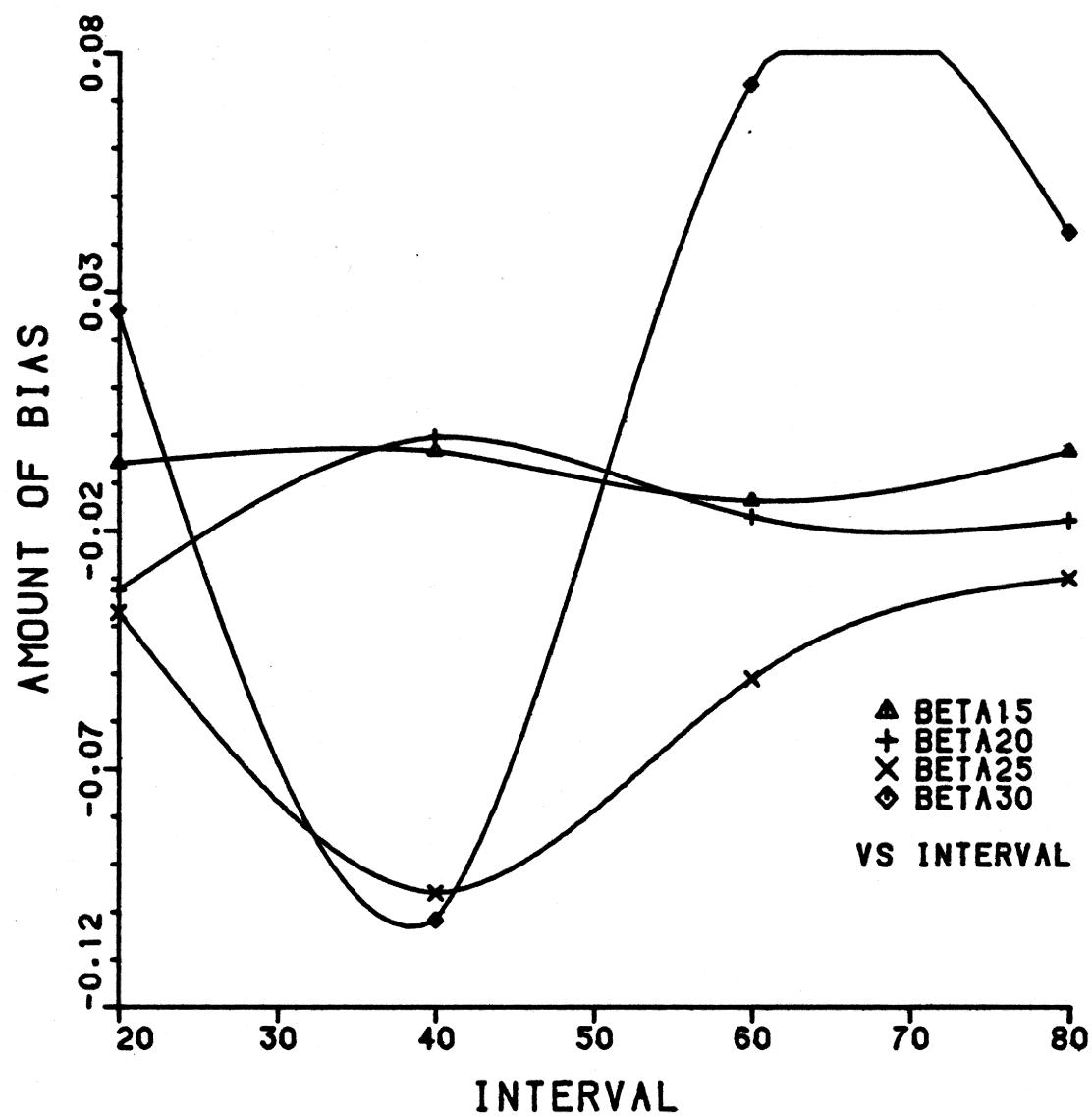
responses.)

### 5.3.2 Results For Holding Period (Arithmetic) Returns

Figures 5.6 and 5.7 display the results for holding period returns. Figure 5.6 is a plot of the amount of bias versus interval at different levels of beta and roughly corresponds to Figure 5.2 for logarithmic returns. However, the initial price was one dollar instead of .5 and curves for differencing intervals less than 20 are not shown. "True" alphas were calculated from (4.2) using an annual risk-free rate of .06, and the index used was the S&P 500.

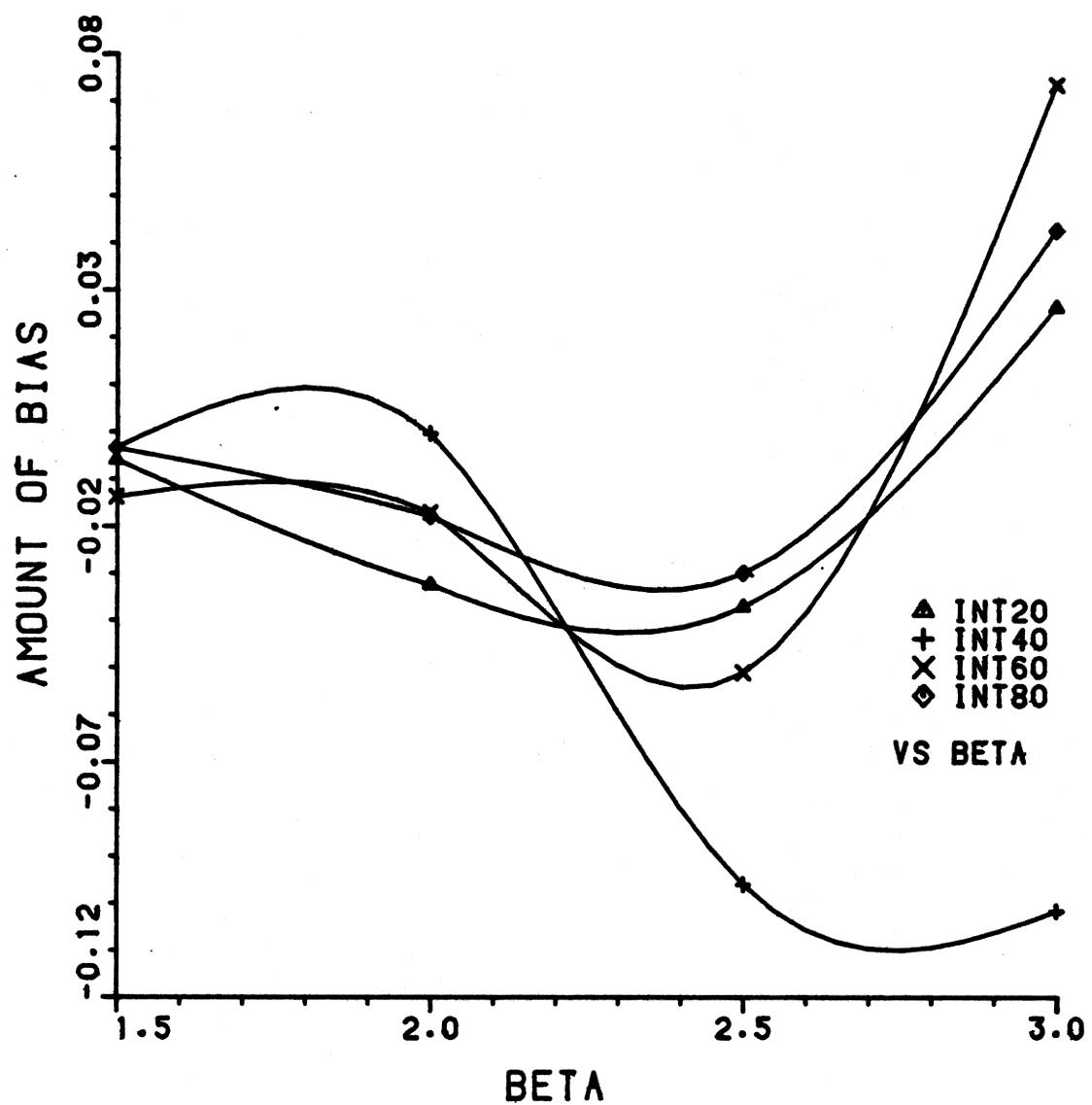
Figure 5.6 is directly comparable to Figure 5.8, which is for logarithmic returns. In Comparing Figure 5.6 and 5.8, it can be seen that the amount of bias for holding period returns is considerably less than for logarithmic returns and is considerably more erratic, thus raising the question of the bias pictured in Figure 5.6 being almost a spurious result.

Figure 5.7 displays the same data as Figure 5.6. However, the amount of bias has been plotted versus "true" beta at different levels of the differencing interval. The almost negligible amount of bias for most combinations of beta and interval is again evident, as is the erratic behavior at some combinations. Figure 5.7 may be compared to



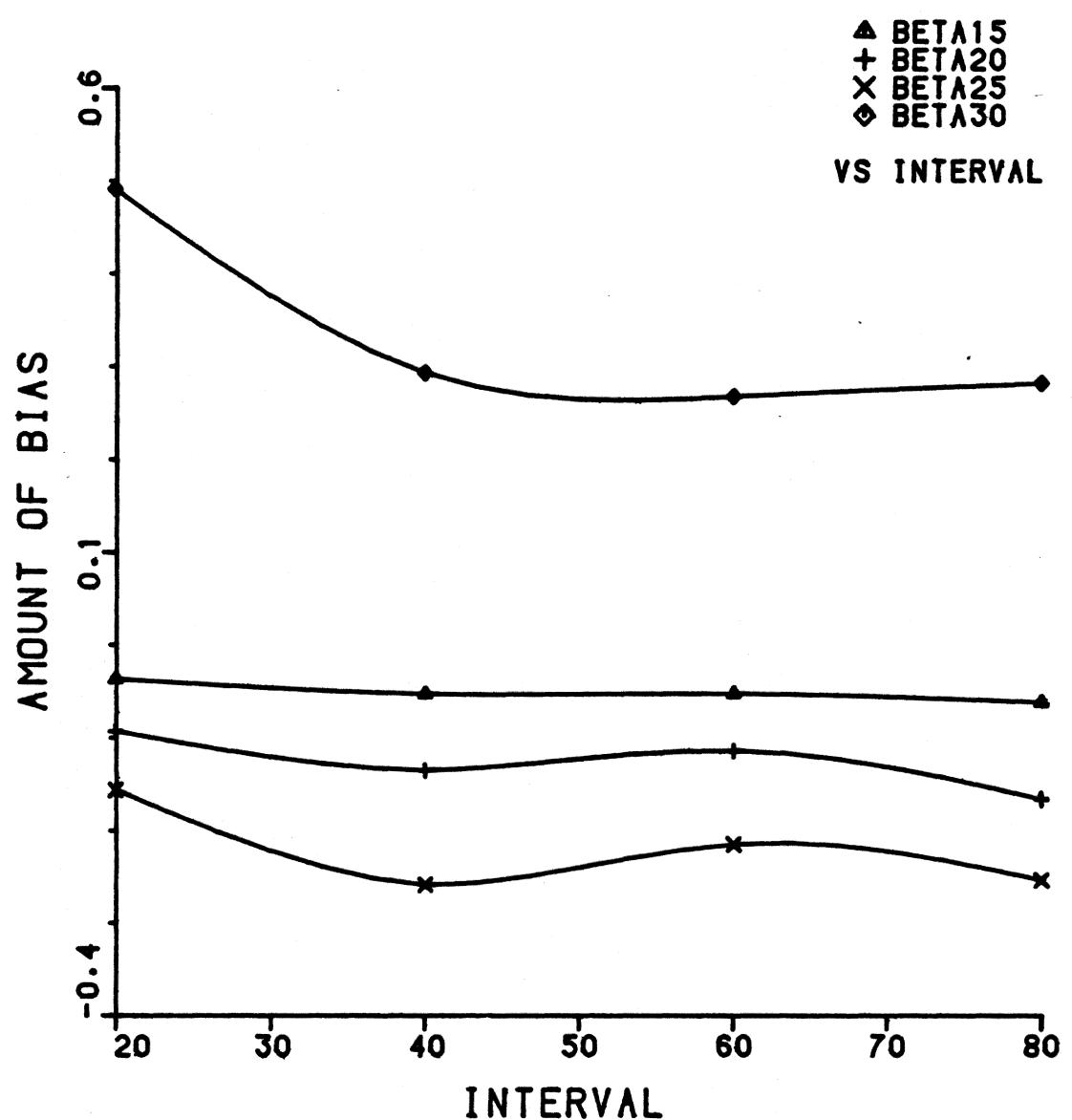
THE AMOUNT OF BETA BIAS  
AT VARIOUS LEVELS OF BETA  
FOR DIFFERENT INTERVALS

FIGURE 5.6



THE AMOUNT OF BETA BIAS AT  
VARIOUS LEVELS OF INTERVAL  
FOR DIFFERENT BETAS

FIGURE 5.7



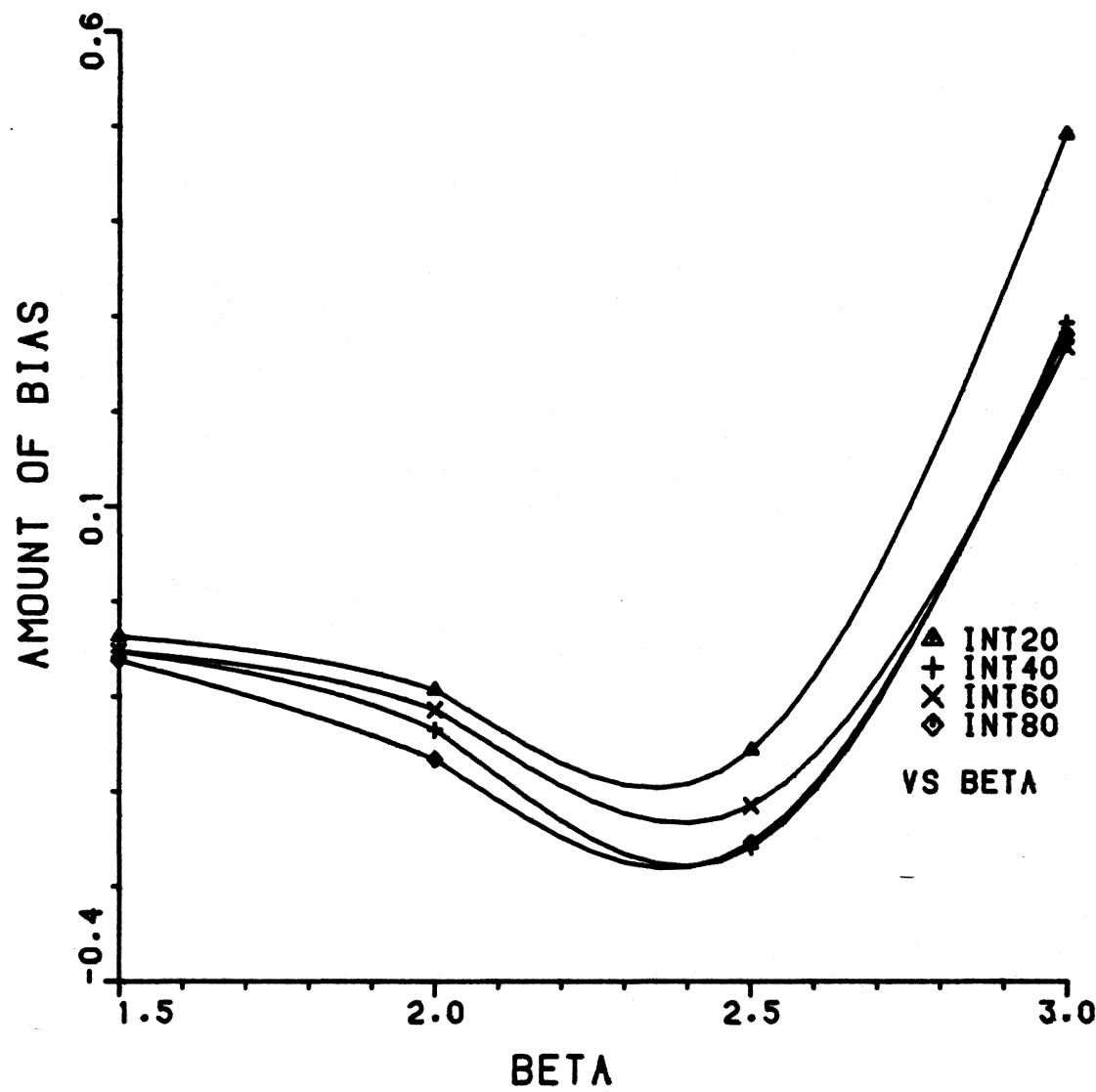
THE AMOUNT OF BETA BIAS  
AT VARIOUS LEVELS OF BETA  
FOR DIFFERENT INTERVALS

FIGURE 5.8

Figure 5.9, which is for the same combinations of parameters for logarithmic returns.

Only five of the sixteen combinations of specified parameters illustrated in Figures 5.6 and 5.7 have a mean amount of bias which is statistically different from zero at the .05 level (as indicated by a one-sample t test). Seven of the 16 are significant at the .10 level. This contrasts markedly with the data for logarithmic returns displayed in Figures 5.8 and 5.9 where the mean bias for all combinations of specified parameters is statistically different from zero at the .0005 level and 15 of the 16 combinations have p-values of .0001 or less.

To further test the significance of the mean amount of bias for the holding period returns, a two-way ANOVA was performed using the amount of bias for each of the replications as the response and the "true" betas and differencing intervals as the two factors. The levels of the factors were those of the data displayed in Figures 5.6 and 5.7, that is, assumed "true" betas of 1.5, 2, 2.5 and 3, and differencing intervals of 20, 40, 60 and 80 trading days. The interaction term was significant (p-value of .0004). Therefore the analysis was unfolded and a one-way ANOVA performed at each level of differencing interval. The results are given in Table 5.8 and may be conveniently reviewed in con-



THE AMOUNT OF BETA BIAS AT  
VARIOUS LEVELS OF INTERVAL  
FOR DIFFERENT BETAS

FIGURE 5.9

junction with Figure 5.6. At a differencing interval of 20, none of the means for different betas are significantly different from each other. At an interval of 40, the bias means for "true" betas of 1.5 and 2 are significantly different from those of 2.5 and 3. At 60, 3 is significantly different from the rest; and at 80 trading days, 3, 1.5 and 2 are not significantly different from each other, as are not 1.5, 2 and 2.5.

#### 5.4 SUMMARY OF RESULTS

The first section of this chapter discussed the results obtained from two different procedures which addressed the problem of how many replications, each with a different randomly chosen starting point in the series of market index levels, are needed for each combination of specified parameters in the simulation runs. The use of Duncan's multiple range test to examine the bias means for groups containing differing numbers of replications (but which were otherwise identical) did not clearly answer the question. Therefore, plots of the cumulative average bias for each of the replications from the first through the 150th<sup>59</sup> were visually examined for a number of combinations

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<sup>59</sup>In a few cases, the 255th; and in one case, the 500th replication.

TABLE 5.8

## Holding Period Returns

## One-Way ANOVA Results

<u>Differencing Interval</u>	<u>P-Value of Overall F Ratio</u>	<u>Beta Groupings<sup>1</sup></u>
20	.15	<u>3, 1.5, 2, 2.5</u>
40	.0002	<u>2, 1.5, 2.5, 3</u>
60	.0008	<u>3, 1.5, 2, 2.5</u>
80	.12	<u>3, 1.5, 2, 2.5</u>

<sup>1</sup>From Duncan's multiple range test at the alpha = .05 level. The horizontal bar encompasses groups whose means are not significantly different from each other.

of specified parameters. A satisfactory number of replications was found to be 150.

Next the reproducibility of the experiment was examined. The results of different runs using different random number seeds for the random number generators controlling the starting point in the series of market index price levels and the random disturbance term in the price generating function were compared by visual inspection and statistically. Statistically the differences were found to be not significant. Visually the results indicated that the reproducibility of the means of the amount of bias for each group of 150 replications is about  $\pm 0.1$  or better.

The last section of this chapter reviewed the principal findings of the study, namely, the amount of bias in the observed betas. Bias was observed to be extremely large for some combinations of specified parameters, particularly for large betas (2.5 and 3.0) with logarithmic returns and calculated "true" alphas. However, it was also pointed out that the amount of bias could be almost eliminated by using a different choice for the specified "true" alpha. An additional result was that for arithmetic holding period returns the amount of bias was almost negligible for a set of combinations of specified parameters which had produced some sizeable biases for logarithmic returns.

## Chapter VI

### CONCLUSION

In this chapter a brief review of the study is presented, followed by a review of results obtained and the problems encountered which affect these results. Finally a few extensions of the study will be presented.

#### 6.1 OVERVIEW OF THE STUDY

The restriction of the trading price of a security to multiples of one-eighth (or one-sixteenth) of a dollar creates friction in the trading process of stocks. Another source of friction, examined by other researchers, was found to produce systematic bias in the observed betas of securities. Moreover the one-eighth effect itself has been examined briefly by Schwartz and Whitcomb using a simulation technique and was found to increase the variance of returns.

In the present study a simulation procedure was developed to examine the effect on observed betas calculated from price series constrained by one-eighth price rounding. In the simulation a series of unconstrained prices was constructed using the market model, an assumed "true" beta, an assumed "true" alpha, a series of market prices, a generated random disturbance term, and an assumed initial price. The

unconstrained prices were then rounded according to an assumed rounding rule<sup>60</sup> and an observed beta was calculated for the rounded series using the market model and ordinary least squares regression. The observed beta and assumed "true" beta were then compared to obtain the amount of bias introduced by the price rounding. The simulation was then repeated for additional replications for the same specified parameters (but differing in their random starting points in the market price series) and for different specified parameters. Various descriptive statistics for the observed bias were computed and tests undertaken to evaluate the bias.

## 6.2 REVIEW OF THE RESULTS

The magnitude of the random disturbance term used in the price generating function was found to be critical to the results. If too small, the result was as though no random disturbance term was used and the resulting biases were different from the biases when an appropriate disturbance term was used. If too large, the effect was disrupting to the generated price series in the sense that the series very

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<sup>60</sup>The rounding rule applied was the second procedure developed in Chapter III. This procedure adjusts the rounding each period by the amount of rounding in the previous period (and hence the rounding for all previous periods). It was previously shown that this is equivalent to using the unrounded price for the previous period in the price generating function.

quickly went to near zero or grew to very large prices. An appropriate magnitude<sup>61</sup> to use was determined from actual prices for a large number of firms as described in Chapter IV.

The appropriate number of replications to use for each combination of specified parameters was examined. Multiple comparisons did not provide a clear answer within a range of replications considered viable for the experiment. Therefore a plot of the cumulative simple average of the bias after each succeeding replication was visually reviewed and was found to become reasonably stable within 150 replications.

The amount of bias in the beta estimates indicated by the simulation procedure was found to be quite large for some betas of 2.5 or larger, differencing intervals of one or two days, logarithmic returns, and values of alpha calculated from a correspondence with CAPM. However, it was found that the bias could be changed substantially by the choice of the assumed "true" alpha. This was an unfortunate result since nothing in the study pointed to any particular value of alpha as being a proper choice. It may be concluded that this approach to determining the amount of bias

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<sup>61</sup>More precisely the appropriate standard deviation for the normal distribution generated by the random deviate generator. The mean of the distribution was zero.

in beta estimates from logarithmic returns gives an estimate for the bias which is a function of the choice of alpha, a somewhat arbitrary choice. The corresponding amount of bias for holding period returns was found to be considerably smaller than for logarithmic returns and for the majority of combinations of specified parameters examined, was not significantly different from zero.

### 6.3 EXTENSIONS OF THE STUDY

Several extensions of this study come to mind. First, considerably more investigation could be undertaken to determine if the little or no bias introduced by one-eighth price rounding holds true across a wide range of specified parameters for holding period returns. It would be useful if consistent non-zero results for bias were found in the combinations of market model parameters.

The CAPM could be used as the return generating model instead of the market model. Since the CAPM does not have an intercept, the problem of an appropriate choice of alpha would thus be avoided. However, results using CAPM might be of very limited usefulness. The use of the market model is widespread, and the CAPM has other problems associated with it.

A more direct approach to the problem is to perform additional analytical work to determine why the choice of alpha is so critical to the simulation and what an appropriate choice should be in order to eliminate bias from this source. A somewhat allied approach to the problem would be to construct artificial market indexes with carefully controlled (and simple) characteristics. The bias responses to choices of alpha for indexes with these known characteristics could conceivably indicate the nature of the problem and the solution for it.

## LITERATURE CITED

Altman, Edward I.; Jacquillat, Bertrand; and Levasseur, Michel. 1974. "Comparative Analysis of Risk Measures: France and the United States." Journal of Finance 29 (December): 1495-1511.

American Stock Exchange, Inc. 1973. "Amex Introduces New Market Value Index System." American Investor (September).

Beaver, William; Kettler, Paul; and Scholes, Myron. 1970. "The Association Between Market Determined and Accounting Determined Risk Measures." The Accounting Review 45 (October): 654-682.

Black, F. 1972. "Capital Market Equilibrium With Restricted Borrowing." Journal of Business 45 (July): 444-454.

Cohen, K. J.; Hawawini, G. A.; Maier, S. F.; Schwartz, R.A.; and Whitcomb, D. K. 1980a. "Estimating and Adjusting For The Intervaling Effect Bias In Beta." Working Paper.

\_\_\_\_\_. 1980b. "Friction In The Trading Process And The Estimation Of Systematic Risk." Working Paper as reported in CHMSW ibid.

Coleman, Janice. Business letter dated November 7, 1980.

Cootner, Paul H. 1966. "Stock Market Indexes: Fallacies and Illusions." Commercial and Financial Chronicle, (September); as reprinted in Modern Developments In Investment Management 2nd ed. James Lorie and Richard Brealy, eds, Hinsdale, Illinois: Dryden Press (1978): 94-100.

Copeland, Thomas E, and Weston, J. Fred. 1979. Financial Theory and Corporate Policy. Reading, Mass: Addison-Wesley.

Dimson, Elroy. 1979. "Risk Measurement When Shares Are Subject To Infrequent Trading." Journal of Financial Economics 7 (June) : 197-226.

Fama, Eugene F. 1976. Foundations of Finance. New York: Basic Books.

Fisher, Lawrence. 1966. "Some New Stock-Market Indexes." Journal of Business 34 (January): 191-218.

Fisher, Lawrence, and Lorie, James H. 1964. "Rates of Return on Investments in Common Stocks." Journal of Business 37 (January): 1-21.

Fogler, H. R., and Radcliff, R. C. 1974. "A Note On Measurement Of Skewness." Journal of Financial and Quantitative Analysis (June): 485-489.

Francis, J. C. 1975. "Skewness And Investors' Decisions." Journal of Financial and Quantitative Analysis 10 (March): 163-172.

Francis, Jack Clark, and Archer, Stephen H. 1979. Portfolio Analysis. 2nd ed. Englewood Cliffs: Prentice-Hall.

Hawawini, Gabriel A. 1977. "On The Time-Behavior Of Financial Parameters: An Investigation Of The Intervaling Effect." (Ph.D. dissertation, New York University).

\_\_\_\_\_. 1980. "Intertemporal Cross-Dependence In Securities Daily Returns And The Short-Run Intervaling Effect On Systematic Risk." Journal of Financial and Quantitative Analysis 15 (March): 139-149.

Hawawini, Gabriel A., and Michel, P. 1979. "An Assessment of Risk in Thinner Markets: The Belgian Case." Journal of Economics and Business 31 (Spring/Summer): 196-201.

Helwig, Jane T., and Council, Kathryn A., eds. 1979. SAS Users Guide. 1979 ed. Cary, North Carolina: SAS Institute, Inc.

Hollander, Myles, and Wolfe, Douglas A. 1973. Nonparametric Statistical Methods. New York: John Wiley & Sons, Inc.

Hong, H.; Kaplan, R. S.; and Mandelker, G. 1978. "Pooling vs. Purchase: The Effect of Accounting for Mergers on Stock Prices." The Accounting Review 53 (January): 31-47.

Kaplan, R. S., and Roll, R. 1972. "Investor Evaluation of Accounting Information: Some Empirical Evidence." Journal of Business 45 (April): 225-257.

- Keown, Arthur J., and Pinkerton, John M. 1981. "Merger Announcements and Insider Trading Activity: An Empirical Investigation." Journal of Finance (forthcoming).
- Kramer, Clyde Young. 1956. "Extensions of Multiple Range Tests to Group Means With Unequal Numbers of Replications." Biometrics 12 (September): 307-310.
- Latane, Henry A.; Tuttle, Donald L.; and Young, William E. 1971. "Market Indexes and Their Implications For Portfolio Management." Financial Analysts Journal 27 (September-October): 75-85.
- Lev, Baruch. 1974. Financial Statement Analysis. Englewood Cliffs: Prentice-Hall.
- Levhari, David, and Levy, Haim. 1977. "The Capital Asset Pricing Model and The Investment Horizon." The Review of Economics and Statistics 59 (February): 92-104.
- Lewis P. A. W.; Goodman A. S.; and Miller, J. M. 1969. "A Pseudo-Random Number Generator for the System 360." IBM Systems Journal 8 (no. 2): 136-146.
- Lorie, James H., and Hamilton, Mary T. 1971. "Stock Market Indexes." in an unreferenced article as reprinted in Modern Developments In Investment Management 2nd ed. James Lorie and Richard Brealey, eds. Hinsdale, Illinois: Dryden Press (1978): 78-93.
- Mayers, D., and Rice, E. 1979. "Measuring Portfolio Performance and the Empirical Content of Asset Pricing Models." The Journal of Financial Economics 7 (March): 191-235.
- Ott, Lyman. 1977. An Introduction to Statistical Methods and Data Analysis. North Scituate, Mass.: Duxbury Press.
- Pogue, Gerald A., and Solnik, Bruno H. 1974. "The Market Model Applied To European Common Stocks: Some Empirical Results." Journal of Financial and Quantitative Analysis 9 (December): 917-944.
- Roll, R. 1977. "A Critique of the Asset Pricing Theory's Tests; Part I: On Past and Potential Testability of the Theory." Journal of Financial Economics 4 (March): 129-176.

- \_\_\_\_\_. 1978. "Ambiguity When Performance Is Measured By The Securities Market Line." Journal of Finance 33 (September): 1051-1069.
- \_\_\_\_\_. 1979. "A Reply to Mayers and Rice." Journal of Financial Economics 7 (March): 241-254.
- Ross, Stephen A. 1978. "The Current Status of the Capital Asset Pricing Model (CAPM)." Journal of Finance 33 (June): 885-901.
- Scholes, Myron, and Williams, Joseph. 1977. "Estimating Betas From Nonsynchronous Data." Journal of Financial Economics 5: 309-327.
- Schwartz, Robert A., and Whitcomb, David K. 1977a. "The Time-Variance Relationship: Evidence of Autocorrelation of Common-Stock Returns." Journal of Finance 32 (March): 41-55.
- \_\_\_\_\_. 1977b. "Evidence On The Presence and Causes of Serial Correlation In Market Model Residuals." Journal of Financial and Quantitative Analysis 12 (June): 291-313.
- Smith, Keith V. 1978. "The Effect of Intervaling On Estimating Parameters Of The Capital Asset Pricing Model." Journal of Financial and Quantitative Analysis 13 (November): 313-332.
- Stephens, M. A. 1974. "EDF Statistics for Goodness of Fit and Some Comparisons." Journal of the American Statistical Association 69 (September): 730-737.
- Thompson, Donald J., II. 1976. "Sources of Systematic Risk in Common Stocks." Journal of Business 49 (April): 173-188.
- Young, W. E. 1971. "Random Walk of Stock Prices: A Test of the Time-Variance Function." Econometrica 39 (September): 797-812.

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BETA BIAS IN LOW-PRICED STOCKS  
DUE TO TRADING PRICE ROUNDING

by

Walter Lewis Young, Jr.

(ABSTRACT)

Stocks and similar securities are normally traded in prices which are integral multiples of one-eighth of a dollar (a few are traded in one-sixteenths of a dollar). This price constraint may introduce a bias in the estimates of beta for low-priced securities, and the purpose of this dissertation is to examine the bias introduced from this source.

The research methodology briefly consists of constructing a price series for a hypothetical stock by computing "true" prices from an assumed "true" beta and alpha, the series of returns generated from a market index, and a random disturbance term. The constructed price series is rounded to the nearest one-eighth of a dollar and an "observed" beta for this rounded price series is calculated.

The "observed" beta is compared to the "true" beta to observe the degree of bias. Replications are made which differ in their randomly chosen starting point in the market index series; and the experiment is repeated for various "true" betas and alphas within the range of interest, for different intervals between price observations, and for different initial prices.

Chapter I provides an introduction to the study. In Chapter II the relevant literature for this study is reviewed. The first part includes previous studies of the one-eighth effect and the intervalling effect, while the second part of the chapter focuses on the composition and characteristics of common market indexes. The analytical considerations are discussed in Chapter III. The price generating mechanism and the constraint placed upon it by one-eighth price rounding are explicitly stated. Alternative rounding procedures are presented and their implications discussed. In the next section the characteristics of the rounding functions are discussed. Finally, expressions for the amount of bias in beta estimates introduced by the one-eighth price rounding are derived for both logarithmic returns and holding period (arithmetic) returns.

In Chapter IV the methodology used to secure the results presented in Chapter V is reviewed. The simulation itself is discussed as well as the statistical and ad hoc

procedures used to evaluate the results. The results presented in the next chapter also include the results pertinent to two ancillary issues discussed in Chapter IV, namely, how many replications are needed and how reproducible are the results. Chapter VI summarizes the findings, draws a conclusion, and suggests extensions of the study.