Development and Validation of Reconstruction Algorithms for
3D Tomography Diagnostics

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Abstract

This work reports three reconstruction algorithms developed to address the practical issues encountered in 3D tomography diagnostics, such as the limited view angles available in many practical applications, the large scale and nonlinearity of the problems when they are in 3D, and the measurement uncertainty. These algorithms are: an algebraic reconstruction technique (ART) screening algorithm, a nonlinear iterative reconstruction technique (NIRT), and an iterative reconstruction technique integrating view registration optimization (IRT-VRO) algorithm. The ART screening algorithm was developed to enhance the performance of the traditional ART algorithm to solve linear tomography problems, the NIRT was to solve nonlinear tomography problems, and the IRT-VRO was to address the issue of view registration uncertainty in both linear and nonlinear problems. This dissertation describes the mathematical formulations, and the experimental and numerical validations for these algorithms. It is expected that the results obtained in this dissertation to lay the groundwork for their further development and expanded adaption in the deployment of tomography diagnostics in various practical applications.
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General Audience Abstract

Tomography is a technique to obtain three-dimensional (3D) measurements
noninvasively, and such nonintrusive nature has made it a powerful and indispensable tool
for a wide variety of applications. Regardless of the specific implementation and
application of tomography techniques, they generally involve two steps. In the first step,
2D projections of the target object are captured from different orientations; and in the
second step, the 2D projections obtained in step 1 are fed into a reconstruction algorithm
to obtain the 3D measurements.

This dissertation focuses on the second step, more specifically, the development and
validation of reconstruction algorithms under the context of flow and flame imaging.
Existing reconstruction algorithms encountered various limitations when applied to
turbulent flow and flames due to various factors, such as the limited number of projections
available, scale of the problem, and nonlinear effects. This work reports three
reconstruction algorithms developed to overcome some of these practical issues: an
algebraic reconstruction technique (ART) screening algorithm, a nonlinear iterative
reconstruction technique (NIRT), and an iterative reconstruction technique integrating
view registration optimization (IRT-VRO) algorithm. These new algorithms were
demonstrated to enhance the spatial resolution, computational efficiency, accuracy, and to
address nonlinear effects of tomographic measurements.
This work describes the mathematical formulations, and the experimental and numerical validations of these algorithms. It is expected that the results obtained in this work to lay the groundwork for their further development and expanded adaption in the deployment of tomography diagnostics in various practical applications.
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List of Symbols

\( C \)  
concentration of the target species

\( P \)  
projection

\( PSF \)  
point spread function

\( r \)  
distance of the camera to the target object

\( \theta \)  
azimuth angle of the camera to the target object

\( \phi \)  
elevation angle of the camera to the target object

\( \Gamma \)  
modulation function

\( \alpha \)  
attenuation coefficient

\( l \)  
pathlength of the laser

\( NPSF \)  
nonlinear point spread function

\( e_R \)  
reconstruction error

\( e_{P,VR} \)  
measurement error caused by view registration

\( \Delta \theta \)  
view registration uncertainty
Chapter 1 Introduction

1.1. Overview of 3D tomography diagnostics

Three-dimensional (3D) tomography diagnostics refer to tools and experimental techniques that can image a certain target property in all three spatial directions. In 3D tomography diagnostics, line-of-sight integrated images of the target (named projections) are captured from various orientations. A mathematical inversion is then performed using the captured projections as inputs to obtain a 3D reconstruction of the target property. Different applications involve different target and physical processes, and examples range from the medical imaging of human body using X-ray transmission measurements [1], examination of the internal structure of solids or liquids using electrical capacitance measurements [2], and optical imaging of a chemical species in fluid flows using absorption or emission measurements [3, 4]. Regardless of such wide range of applications and the corresponding physical processes involved, the mathematical background involved remains essentially the same. Hence, the rest of dissertation will be developed under the context of optical imaging of fluid flows, even though the mathematics and algorithm can be extended to other applications straightforwardly.

In the optical imaging of fluid flows, various tomography diagnostic techniques have been successfully demonstrated to measure a range of flow parameters in 2D or 3D. As an incomplete list, these methods include measurements of temperature fields in the exhaust plane of a gas turbine engine using absorption tomography [5-7], measurements of the distribution of chemical species in an internal combustion engine using absorption tomography [8-10], measurements of three-component and 3D velocity fields using
tomographic PIV (particle image velocimetry) [11-15], and measurements of 3D turbulent flame structures using emission tomography [16-20]. Mathematically, all of these tomography problems can be divided into either linear or nonlinear problems. For a linear tomography problem, the projections depend linearly on the property to be reconstructed. In the area of optical imaging of fluid flows, an example of a linear problem involves the imaging of the concentration of a radical (such as CH*) using tomographic chemiluminescence when radiation trapping or self-absorption is negligible [4-8]. Because of the negligible radiation trapping, the projections (i.e., line-of-sight-integrated signals emitted by CH*) are proportional to the sought property (i.e., the concentration of CH*). When radiation trapping is not negligible, the projections will depend nonlinearly on the sought property and the problem becomes nonlinear. Example of other linear problems include tomographic Mie scattering when multiple-scattering is negligible [9, 10] and tomographic particle image velocimetry [11, 12] (which is essentially tomographic Mie scattering with negligible multiple-scattering). For a nonlinear tomography problem, the projections depend nonlinearly on the property to be reconstructed. Besides the example of tomographic chemiluminescence when radiation trapping is not negligible mentioned above, other nonlinear problems include tomographic scattering measurements with appreciably multiple-scattering [15, 16], and tomographic laser induced fluorescence (LIF) measurements with significant absorption [17] or amplified spontaneous emission [18].

1.2. Existing reconstruction algorithms

Various reconstruction algorithms have been developed to solve both linear and nonlinear tomography problems. In general, algorithms proposed in the past could be broadly divided into two categories: analytical and iterative methods.
One of the analytical algorithms, perhaps the best established and most widely used one, is the Filtered Backprojection (FBP) algorithm [21-24]. The FBP algorithm is based on the Fourier slice theorem [25]. This theorem, informally, states that the one-dimensional Fourier transform of a parallel projection is equal to a cross-section of the two-dimensional Fourier transform of the object. Thus using this theorem one can estimate the object by simply performing a two-dimensional inverse Fourier transform. The advantage of the FBP algorithm is that it can be used to solve both linear and nonlinear problems, and consumes relatively modest computational resources. The disadvantage is that the FBP algorithm requires a large number of low noise projections (hundreds and more) for successful reconstructions [26]. However, in many practical problems (such as flow imaging), only a limited number of projections (which also may contain considerable measurement uncertainties) is available, typically ranging from 2 [27] to about 50 [4]. It is practically difficult or infeasible to obtain a large number of projections from a wide angular range due to the transiency of the target, the optical access available, and the cost of equipment.

The iterative algorithms [28-34] have been demonstrated to be more robust and able to generate a high quality reconstruction with limited projections in the presence of noise. Compared to the FBP algorithm, the iterative algorithms require a higher computational cost. The idea of iterative algorithms are based on the conversion of the tomography problem into a minimization problem, for example, to minimize a cost function \( f = |Ax - p| \), where \( x \) represents the discretized sought object, \( p \) the measured projections, \( A \) the weighting matrix, and \( |Ax - p| \) represents the difference between measured and calculated projections. Various techniques are then developed to find a solution to the minimization problem. Here we describe three of them of most relevance to this work: Algebraic
Reconstruction Technique (ART) [17, 35-38], Conjugate Gradient (CG) [28-31] and Simulating Annealing (SA) [6, 39-41].

1.2.1. ART

The ART algorithm was developed by Gordon [42] for solving continuous linear systems. In its basic form, the ART minimize the cost function $f = |Ax - p|$ according to the following equation:

$$x^{k+1} = x^k + a_i \cdot \frac{p_i - \langle a_i, x^k \rangle}{\langle a_i, a_i \rangle}$$  \hspace{1cm} (1.1)

where $x^k$ and $x^{k+1}$ represent the reconstruction of the image vector at the $k$th and $(k + 1)$th iteration; $p_i$ the $i$th element in the projection vector ($p$); $a_i$ the $i$th row of the weighting matrix $A$ (the row corresponding to the $i$th projection); and $\langle , \rangle$ the inner product of two vectors. The iteration starts with an initial guess of $x$, and terminates when a preset criterion is reached. In each iteration, $i$ runs from 1 to $N$ (the length of $p$); therefore consequently, the computational cost of the iteration-based approach will be approximately proportional to the number of view angles.

The following pseudo code summarizes the ART algorithm:

Set the values of all $x$ to 0;

do while (the termination criterion is not satisfied)

Calculate projections according to current image;

Calculate the difference between the calculated and measured projections;

Distribute the difference evenly to the pixels each projection involves;

Based on the difference, modify the values of pixels according to Eq. (1.1);

end do
1.2.2. CG

The CG algorithm [43, 44] is one of the most popular and well known iterative technique for solving the inverse problems. It was originally developed as a direct method but became popular for its properties as an iterative algorithm. The CG algorithm converts the minimization problem of \( f = |Ax - p| \) to an equivalent term \( Q = \frac{1}{2} x^T Ax - x^T p \), where \( Q \) is the new quadratic cost function. The minimizer \( x^* \) of the function \( \phi \) is given as the point where the gradient of the function is equal to zero:

\[
\nabla \phi(x^*) = Ax^* - p = 0 \tag{1.2}
\]

The pseudo code of CG was summarized as following.

Set the values of \( x \) to \( x_0 \);

Compute \( r_0 = Ax_0 - p \), \( b_0 = -r_0 \);

\[\text{do } k = 1, 2, 3, \ldots \text{ until convergence} \]

\[
\alpha_k = \frac{r_k^T r_k}{b_k^T Ab_k}
\]

\[
x_{k+1} = x_k + \alpha_k b_k
\]

\[
r_{k+1} = r_k + \alpha_k Ab_k
\]

\[
\beta_k = \frac{r_{k+1}^T r_{k+1}}{r_k^T r_k}
\]

\[
b_{k+1} = r_{k+1} + \beta_k b_k
\]

end do
1.2.3. SA

The SA algorithm, introduced in 1983 [45], was initially developed for minimizing large scale combinatorial problems. The algorithm was extended to continuous problems shortly afterwards [46]. The algorithm has been extensively demonstrated in various studies as an effective algorithm for large scale and complicated problems, with the global minimum hidden among numerous confusing local minima [6, 39, 40]. The SA algorithm roots from an analogy to the way liquids are annealed, i.e., cooled slowly to arrive at a low energy and crystallize. During the annealing process, the energy of the liquid is lowered gradually such that the system can escape from a local energy minimum due to random thermal fluctuations. On the contrary, if cooled rapidly (i.e., quenched), the liquid is usually forced into a state of local energy minimum. In function minimization, the value of the target function \( f \) is the counterpart of the energy of the liquid under annealing; and a parameter, \( T_{SA} \), is introduced to be the counterpart of the temperature of the liquid. The random thermal fluctuation is implemented by the following Metropolis criterion:

\[
\text{if } \Delta f = f(x_{\text{new}}) - f(x_{\text{old}}) \geq 0, \text{ accept } x_{\text{new}}
\]

\[
\text{else accept } x_{\text{new}} \text{ with probability } P_S = \exp\left(-\frac{\Delta f}{T_{SA}}\right)
\]  

(1.3)

where \( x_{old} \) and \( x_{new} \) represent the variables of \( f \) (either continuous or discrete) in the previous and current iteration. Eq. (1.3) elucidates the essence of the SA algorithm: a new solution \( x_{\text{new}} \) is always accepted if it results in a lower \( f \) (energy); but a new solution (a seeming worse-off solution) is not always rejected if it results in a higher \( f \), and is instead accepted with a certain probability \( P_{SA} \) (random thermal fluctuation). The probability, \( P_{SA} \), decreases with \( T_{SA} \). The SA algorithm converges to the global minimum of \( f \) by “annealing” it, i.e.,
by gradually reducing $T_{SA}$. In contrast, iterative algorithms based on derivative gradient information (i.e., CG) always accept a new solution if it results in a lower $f$, and reject it otherwise. Consequently, these algorithms cannot escape a local minimum once they enter it.

The following pseudo code summarizes the SA algorithm:

Initialize parameters;

**do while** ($T_{SA} > 10^{-6}$)

**do** $i = 1$ to the number of variables

Change the value of the $i$th variable;

Recalculate the projections passing through the pixel that corresponding to the $i$th variable;

Recalculate $f$;

Accept or reject the change according to the Metropolis criterion;

**end do**

Reduce the temperature $T_{SA}$;

**end do**

To summarize, existing reconstruction algorithms can be broadly divided into analytical and iterative methods. FBP represents a classic example of the analytical methods. The analytical algorithms enjoy the advantage of computational efficiency, but are limited to the problems in which a large number of (i.e., hundreds and more) projections with low noise is available. This latter requirement restricts their application in the 3D tomography diagnostics of flow and combustion measurements, in which typically only a limited number of projections (i.e., 5 to 50) are available. Therefore, in flow and
combustion diagnostics, iterative techniques become the method of choice because of their ability to provide acceptable reconstruction with fewer and noisy projections. For iterative algorithms, ART, CG and SA are three representative techniques of most relevance to this work. The iterative techniques are more computationally costly, but they can solve tomographic problems with fewer projections. For example, the ART algorithm has been demonstrated to be able to yield a high fidelity of reconstruction with only 5 to 10 views [42]. The disadvantage of ART is that it is limited only to solve the linear equation systems, thus cannot be applied in the nonlinear tomography problems. CG and SA enjoy the flexibility of being applicable to both linear and nonlinear problems. However, in practice, the problem may involve a large number of variables (e.g., on the order of millions for the problems examined in this work) and the projections are measured with certain uncertainty. As a result, it is not always feasible to apply the existing CG and SA algorithm to practical flow and combustion diagnostics due to efficiency, robustness, and accuracy, either for linear or nonlinear problems [39]. For example, practical problems often feature many confusion local minima and gradient-based techniques (i.e., CG) can only converge to one of these many local minima within the measurement uncertainty [34]. The SA algorithm is developed to overcome this limitation to find the global minimum or a solution sufficiently close to it [45]. However, the computational cost of those techniques is dramatically higher than those based on gradients and can become prohibitive for problems with practical scale.

1.3. Contributions and organization of the dissertation

Based on the above understanding of existing techniques, this work was aimed to address some of the issues encountered in practical tomography problems. The rest of this dissertation is organized as the following, together with the main contributions:
Chapter 2 introduces a modified ART algorithm, named ART screening algorithm, for solving linear tomography problems. The main contribution of this new algorithm is that it extends the capability of traditional ART by performing a pre-screening of the computational domain, leading to enhanced spatial resolution and accuracy. This technique so far is applicable to only linear problems (but there is no fundamental limitation for it to be applied in nonlinear problems). As an example, the ART screening algorithm was applied and demonstrated in the 3D measurements of CH radicals of flames based on volumetric laser-induced fluorescence (VLIF). This problem is a linear tomography problem because 1) CH radicals only exist in a thin layer near the flame front at low concentration, as a result, the attenuation of the excitation pulse due to absorption was negligible; 2) the LIF signal of CH is easily saturated [47], thus the sensitivity of the measurement to the spatial variation of the excitation laser pulses was minimized. This chapter will describe the mathematical formulation of the ART screening algorithm, and then demonstrates its use in the CH VLIF problem to demonstrate its capabilities and usefulness.

Chapter 3 reports the development and validation of a reconstruction algorithm for nonlinear tomography problems, motivated by a range of practical applications involving absorption, scattering, or radiation trapping. The major contribution is the development of the nonlinear iterative reconstruction technique (NIRT) algorithm, a nonlinear iterative algorithms that can be applied to solve a range of 3D tomography problem where nonlinear effects are non-negligible. Algorithms developed in the past are not readily extendable to such nonlinear problems due to several challenges, such as computation cost, limited view angles available, and the measurement uncertainty. To address these challenges, this
Chapter therefore developed the NIRT algorithm. The NIRT algorithm was validated both experimentally and numerically on a controlled cell, and was then demonstrated on the turbulent flows in this chapter.

Chapter 4 describes an algorithm developed for addressing the uncertainty of view registration. View registration is a key step in tomography diagnostics, both linear and nonlinear problems. View registration determines the orientations and location of the optical sensors relative to the target object, which are key inputs to the subsequent tomographic reconstruction. Uncertainty in view registration propagates through the reconstruction process and eventually impact the accuracy and resolution of the 3D measurements. The main contribution here is the development of an iterative reconstruction algorithms integrating view registration optimization (code named IRT-VRO) that can reduce the impact of view registration uncertainty on the accuracy of the tomographic reconstruction. Compare to the established practice that performs view registration and tomographic reconstruction as two separate processes, the IRT-VRO method integrates them and optimize them holistically. This chapter describes the development of the algorithm, followed by the experimental and numerical validations of the IRT-VRO algorithm using both controlled cell experiments and flow measurements.

Finally, Chapter 5 summarizes this dissertation and outlines possible future research directions.

To summarize, the major contributions and their logical relations of this work are:

1) The development of the ART screening algorithm to extend the capability of established ART algorithm. More specifically, the ART screening algorithm can reduce the computational cost, enhance the spatial resolution, or improve
the reconstruction accuracy compared to the established ART. The ART screening algorithm is only applicable to linear problems at the stage of this work.

2) The development of a nonlinear reconstruction algorithm, NIRT, to solve large scale nonlinear tomography problems in 3D.

3) The development of an IRT-VRO algorithm that optimizes the view registration process and tomography reconstruction holistically. The IRT-VRO algorithm is applicable to both linear and nonlinear tomography problems, and can significantly reduce the impact of view registration uncertainty on reconstruction accuracy.
Chapter 2 Algorithms for linear tomography diagnostics

2.1. Background

This chapter describes the development of the ART screening algorithm to enhance the computational efficiency, spatial resolution, or fidelity of solving linear tomographic problems. The algorithm will be described and is experimentally validated under the context of volumetric laser-induced fluorescence (LIF) measurements. Therefore, this chapter first provides the necessary background in LIF measurements.

Among the numerous flow and flame diagnostic techniques, LIF (laser-induced fluorescence) represents a well-established and indispensable tool [48-50]. LIF can enable measurements of properties critical to flow diagnostics, such as the flow topography [51, 52], reaction zone characteristics [53-55], mixture fraction [54, 56, 57], and temperature [58]. However, LIF-based diagnostics have been predominately restricted to measurements at a point, along a line, or across a two-dimensional (2D) region [59-61], while turbulent flows are inherently three-dimensional (3D). Therefore, it has been long desired to extend the capability of LIF-based diagnostics to provide volumetric information.

One possible strategy to enable volumetric measurements involves a scanning planar LIF (PLIF) method [62-70] wherein the probe laser sheet is rapidly scanned across multiple spatial locations and then PLIF images are recorded sequentially at these locations, which are eventually compiled to form a 3D measurement. Even though the strategy is conceptually straightforward and has been demonstrated as early as the 1980s [62, 67, 68], obtaining 3D measurements with sufficient temporal and spatial resolution remains challenging even with recent advancement in high speed lasers (and the corresponding
camera and optomechanical technologies). For example, recent measurements have reported temporal resolution on the order of 1 kHz, limited by the speed of the laser and scanner [63-66]. The spatial resolution, in principle, can be comparable to that of the PLIF in all three directions. However, in practice, the spatial resolution has been limited to be on the order of 1 mm in the direction of the scan due to experimental issues (e.g., the accuracy of the scanning mechanism and the tradeoff between the spatial resolution and the size of the measurement volume) [63-66].

Another approach to obtaining instantaneous 3D LIF measurements without the need of scanning (and this approach is termed VLIF, volumetric LIF) involves the combination of tomography with LIF [71-74]. Here, one excites the target species volumetrically, and the LIF signals are emitted volumetrically. The volumetric fluorescence is then captured simultaneously by multiple cameras arranged at different orientations to provide the inputs for subsequent 3D tomography reconstruction. The VLIF approach has been demonstrated to enable 3D flame measurements with higher temporal resolution [73] and spatial resolution on the order of 0.4 mm in all three directions [74].

The VLIF problem could be either the linear or nonlinear tomography problem, based on if the excitation laser was significantly absorbed by the target flow field or not. This chapter shows a linear VLIF diagnostic, applying to the CH-based combustion measurement. CH radicals are the product of combustion and exist in a thin layer near the flame front at low concentration, as a result, the attenuation of the excitation pulse due to absorption can be negligible. The ART screening algorithm was developed to improve the performance of solving linear tomography problems in this work, though there is no fundamental limitation to its application in nonlinear tomography.
2.2. Mathematical formulation and algorithm description

This section introduces the mathematical formulation of the linear tomography problem and the description of the algorithm using the volumetric signal of CH radical as an example. The formulation and the algorithm can be extended to other linear tomography with little or no modification. The goal in this formulation was to measure the instantaneous 3D distribution of CH concentration at a given time \( t \), denoted as \( C(x, y, z, t) \). After discretizing \( C \) in the computational domain into voxels, the projection \( P \) measured on a given camera is then related to \( C \) in the following way:

\[
P(r, \theta, \phi; t) = \sum_{i_x, i_y, i_z} \text{PSF}(x_i, y_i, z_i; r, \theta, \phi) \cdot C(x_i, y_i, z_i, t) \cdot I(x_i, y_i, z_i, C)
\]

(2.1)

where \( r, \theta, \phi \) are parameters that specify the location and orientation of the camera, \( \text{PSF} \) the point spread function for a voxel located at \( (x_i, y_i, z_i) \) on the camera, and \( I \) the intensity of the excitation laser pulse in the voxel. In general, \( I \) depends on both on the spatial location due to the spatial non-uniformity of pulses generated by the laser and also on the sought \( C \) itself due to absorption by the target LIF species.

The use of the CH radical and the choice of the particular transition offered a critical advantage in this work to avoid such complications. For the particular CH transition used in this work, the Einstein B coefficient is large, and it thus is easily saturated [47]. Furthermore, CH radicals only exist in a thin layer near the flame front at low concentration (in contrast to OH radicals); as a result, the attenuation of the excitation pulse due to absorption was negligible (and likewise, fluorescence trapping was minimal), and the sensitivity of the measurement to the spatial variation of the excitation laser pulses was minimized. With this understanding, Eq. (2.1) can be simplified to:
\[ P(r, \theta, \phi; t) = \sum_{i, j, k} PSF(x_i, y_j, z_k; r, \theta, \phi) \cdot F(x_i, y_j, z_k, t) \]  \hspace{1cm} (2.2)

where \( F \) is product of \( C \) and \( I \), which is proportional to \( C \). Solving for \( F \) from Eq. (2.2) is now equivalent to linear tomographic problems studied in the past such as those based on absorption spectroscopy [5, 75] or chemiluminescence [17, 19, 35], and solution techniques developed for those problems can be applied. After solving Eq. (2.2), \( C \) can be decoupled from \( F \), if needed, with a measurement of the laser intensity profile (to yield the spatial distribution of the concentration). But in our case (and many other cases using CH-based LIF), knowing \( F \) is sufficient and knowing \( C \) is of less importance because either \( F \) or \( C \) will be binarized in further data processing to obtain flame front properties such as location, surface area, curvature, etc. With the above understanding, we solved for \( F \) from Eq. (2.2) using a variation of the ART (algebraic reconstruction technique) algorithm that was previously developed for tomographic chemiluminescence [18, 76].

With the above understanding of the tomography problem, the ART screening algorithm, shown in the flow chart of Figure 2-1, was developed. The major variation from the standard ART algorithm [77] was to add an additional screening step to exploit flow and flame features typically encountered in practice, i.e., that the target species (e.g., the CH radical in this case) usually only occupies a fraction of the entire measurement volume. The screening algorithm therefore first finds the regions where target species actually is present, and then only performs the tomographic reconstruction over those regions (instead of over the entire volume as in traditional ART). The screening step was shown in the orange box in Figure 2-1, the rest of the flow chart corresponds to the steps in the traditional ART algorithm. As to be shown later, exploiting this feature enables the reduction of
computational cost, enhancement of the spatial resolution of the reconstruction with the given cameras, or improvement of the reconstruction accuracy.

Figure 2-1 Flow chart of the ART screening algorithm

To further explain the benefits of this variation, we first need to examine the scale of Eq. (2.2) when written in matrix form. The projections were organized into a vector (i.e., $P$ in Eq. (2.2)) having a length on the order of $7 \times 10^5$ (a total of five cameras with $\sim 200 \times 700$ effective pixels per camera). The sought distribution of CH (i.e., $F$ in Eq. (2.2)) was similarly organized into a vector voxel by voxel, and its length was on the order of $9 \times 10^5$
with the 64×64×224 discretization. Therefore, first, the dimension of the PSF was on the order of $7 \times 10^5 \times 9 \times 10^5$ elements. Storage and process of a problem on this scale was both memory and computationally expensive. Second, the length of $F$ (i.e., the number of unknowns) was larger than that of $P$ (i.e., the number of equations), seemingly resulting in an underdetermined system. However, due to the aforementioned fact that the flame usually only occupies a small fraction of the entire measurement volume, many of the elements in $F$ must be zero. Therefore, the screening algorithm can quickly exclude those zero elements in $F$ before applying the ART algorithm to solve Eq. (2.2) with fewer unknowns. The screening algorithm was based on the idea that if the projection is zero on a pixel, then the concentration of the target species on all voxels that could contribute to this pixel (which were found using a ray tracing program as detailed in [78]) must be zero. As an example, this screening was able to determine that $\sim 8 \times 10^5$ of the elements in $F$ were zero, thusly reducing the number of unknowns to be solved by ART from $\sim 9 \times 10^5$ to $\sim 1 \times 10^5$. As a result, this screening step significantly reduced computational cost and enhanced the spatial resolution (by enabling us to solve for $F$ on a finer discretization than we could otherwise apply). The spatial resolution could be further increased by the use of additional cameras and the increase in resolution would be inversely proportional to the number of cameras according to the Fourier Slice Theorem. After the screening process, the ART algorithm solves for $F$ iteratively according to the following scheme:

$$D^i(k) = P(k) - P^i(k)$$

(2.3)

$$F(l)^{i+1} = F(l)^i + \frac{D(k)^i}{\|PSF\|} PSF(k,l)$$

(2.4)
where \( I \) represents the index of the iteration, \( k \) the index of the pixel, and \( \| PSF \| \) the norm of \( PSF \). The first step of the iteration, as shown in Eq. (2.3), calculates the difference \( (D) \) between the measured projection at the \( k \)th pixel \( (P(k)) \) and the calculated projection based on the \( F \) obtained in the \( I \)th iteration \( (P^I(k)) \). The second step then obtains an updated \( F \) by correcting the \( F \) obtained in the \( I \)th iteration according to Eq. (2.4). The correction is performed voxel by voxel.

### 2.3. Validation of the algorithm

After introducing the ART screening algorithm, this section describes its experimental validations. Figure 2-2 schematically illustrates the experimental arrangement used for the demonstrating and validation tests from the top view. The setup involved four major components: 1) a burner that generated the target flame; 2) a laser system that generated ns pulses to excite the CH radicals in the flame; 3) focusing and expansion optics that shaped the excitation beam into the desired dimension to illuminate the desired measurement region volumetrically; and 4) five intensified cameras that collected the VLIF projections from different angular positions.
The burner consisted of a primary tube (5 mm ID), from which issued chemically pure, premixed CH\textsubscript{4} and air and a secondary tube (18 mm ID, concentric with the inner tube) for stabilizing a low-speed, lean CH\textsubscript{4}-air pilot flame. The central tubes were surrounded by a 150 mm ID air coflow with a flow rate 250 SLPM (standard liters per minute). To generate the stable laminar flames, the flow rates of the fuel and air were set to be 0.352 and 2.41 SLPM, respectively, producing a flame with equivalence ratio of $\phi = 1.4$. To generate the turbulent flames, CH\textsubscript{4} issued from the central tube at 1.11 SLPM (and air flow rate was adjusted correspondingly so that $\phi = 1.05$). As shown in Figure 2-2, the laser system consisted of a 10-Hz pump laser (Spectra-Physics GCR-170 Nd:YAG laser) and a dye laser (Lumonics HD300 dye laser) followed by an Inrad Autotracker III for frequency doubling the dye laser beam. The pulse energy generated by the laser system was $\sim$10mJ/pulse at the target excitation wavelength of 314.415 nm, and the pulse duration was $\sim$10 ns. This wavelength was chosen following the work described in [23], to excite the overlapped Q\textsubscript{2}(6) and Q\textsubscript{2}(2) transitions from the CH C\textsuperscript{2}$\Sigma^+$--X\textsuperscript{2}$\Pi(v'=0,v''=0)$ band. The excitation slab
was formed by a simple telescope, consisting of a -0.1-m focal length lens to expand the frequency-doubled dye beam followed a 1-m focal length lens to collimate the beam. Two vertical knife edges were then used to clip the beam and control the thickness of the illumination slab.

Based on the configuration of the burner and optical path, a Cartesian coordinate system was established such that the origin was defined at the center of the burner, the X axis was defined along the propagation direction of the laser, the Y axis was defined to be perpendicular to the propagation direction of the laser, and the Z axis was defined to be along the direction of the flow. The VLIF signals were captured by five intensified cameras, as shown in Figure 2-2. Cameras 1, 3, and 5 were Photron SA-Zs (CMOS), and 4 was a Photron SA-5 (CMOS), all equipped with the same intensifier (LaVision highspeed IRO). Camera 2 was a PI-Max intensified (CCD) camera (Princeton Instruments). All cameras were aligned in the X-Y plane (i.e., in a coplanar fashion), and therefore their orientations were completely specified by \( \theta \), defined as the angle formed between the optical axis of each camera and the positive X axis in the anti-clockwise direction. All five cameras were equipped with the same UV lens (Cerco with f=100 mm and f/2.8). A close up lens (UV coated) was applied in front of the UV lens on each camera (with focal length = 200 mm, 300 mm, 200 mm, 250 mm, and 250 mm for cameras 1 through 5, respectively) to optimize the field of view and signal level. This setup resulted in a depth-of-field of ~5mm, sufficiently large to cover the field of interest (although the reconstruction algorithm used here did not require sharply focused images). According to our tests, the use of close up lens resulted in a ~2.5\( \times \) signal boost compared to the use of extension rings (while maintaining the same field of view and spatial resolution). All the cameras and
intensifiers were synchronized with the laser at a frame rate of 10 Hz with an intensifier gate time of 100 ns. The models of cameras, lenses, and corresponding parameters were taken into consideration in the reconstruction. Prior to any measurement, a calibration target was placed on the burner to determine the orientation and location of the cameras with a view registration program [79, 80]. The angular orientations of cameras 1 through 5 were found to be $\theta = 236^\circ, 270^\circ, 307^\circ, 52^\circ, \text{and } 114^\circ$, respectively. The error of the view registration was within 0.6° (and a 0.5° error caused ~8% of average reconstruction error according to our numerical simulations). This work did not rely on the so-called parallel-beam imaging assumption; therefore all cameras provided useful information to the reconstruction even though some of them are aligned in almost opposite positions.

Figure 2-3a shows a set of sample VLIF projections to better illustrate the nature of the experiments. Figure 2-3a shows the single-shot VLIF projections captured simultaneously on a steady laminar cone flame by cameras 1-5. For comparison, Figure 2-3b shows a single-shot PLIF measurement along the central plane of the same flame.
VLIF and PLIF images were captured at different times (due to our lack of equipment needed to perform these measurements simultaneously). However, as to be detailed later in Section 4, these measurements are still comparable and valuable for validation purposes, owing to the stability and repeatability of the laminar flame generated by this burner. Figure 2-3a and b clearly contrast the volumetric and planar nature of VLIF and PLIF. The PLIF signal was only generated from regions where the 3D flame interacts with the probe laser sheet, i.e., the edge of the cone as shown in Figure 2-3b. In contrast, the VLIF signal was generated and integrated volumetrically from the entire target volume, as shown in Figure 2-3a.

Figure 2-4 Comparison of signal intensity between VLIF and PLIF on the cone flame.

Figure 2-4 further illustrates such contrast by comparing the VLIF and PLIF signals (both captured by camera 2) at two flame heights of $Z = 5$ and 19 mm. The PLIF signal shows two distinct peaks corresponding to the left and right edges of the cone flame and an almost zero signal elsewhere. The VLIF signal shows two peaks (again corresponding
to the left and right edges of the cone flame) and also a non-zero signal in between due to its volumetric and integrating nature.

Figure 2-5 shows a demonstration and validation of the ART screening algorithm by directly comparing the 3D measurements obtained from the ART screening algorithm to a 2D PLIF measurement. Figure 2-5a shows the 3D VLIF reconstruction of the cone flame using those projections measured in Figure 2-3a. Figure 2-5b shows the reconstruction at $Y = 0$ mm (i.e., the central slice of the cone flame), and Figure 2-5c shows the comparison against the PLIF measurement. Only half of the cone flame is shown in Figure 2-5c due to symmetry (though note that the reconstruction algorithm did not make any assumption of symmetry). To compare the VLIF and PLIF results quantitatively, the images shown in Figure 2-5c were binarized based on the background noise level of the raw projections to extract the flame front cone radius $r$ (i.e., the distance from the Z axis to the location of the flame front) at different heights, as shown in Figure 2-6a. Note that all our work was performed without any assumption of symmetry, i.e., even the cone flame case here was processed without assuming any a priori information of its symmetry.
Figure 2-5 (a) 3D VLIF measurement. (b) The central slice of the VLIF measurement at Y=0. (c) Comparison of VLIF and PLIF.

Figure 2-6 Comparison of flame front location extracted from VLIF and PLIF.

Figure 2-6b then shows the differences in $r$ determined from the PLIF and VLIF measurements. Here, we focus on two observations drawn from these data. First, the data in Figure 2-6 show good agreement between the PLIF and VLIF measurements, providing
an experimental validation of the VLIF measurements. We extracted \( r \) from 200 frames of the flame images, taken at different times, and the variation of \( r \) was within 0.02 mm, significantly smaller than the difference seen in Figure 2-6b. Second, the comparison between PLIF and VLIF also illustrate the differences in their spatial resolution. The pixel size of the PLIF image as shown in Figure 2-5c was about 0.05 mm/pixel, significantly finer than the voxel size of the VLIF (defined as the dimension of the measurement volume divided by the discretization) of 0.15 mm/voxel. Comparison with PLIF measurements suggested that the spatial resolution of the CH-based VLIF technique for flame surface characterization was about 0.4 mm (estimated as the voxel size of 0.15 mm plus the range of difference seen in Figure 2-6b). Therefore, with the current status of the VLIF technique demonstrated in this work, it still cannot compete with PLIF in terms of resolution in a plane, though the advantages of VLIF are also clear: it offers more imaging elements than PLIF (a total of \(~9\times10^5\) VLIF voxels in Figure 2-5a versus \(~1.4\times10^5\) PLIF pixels in Figure 2-3b) and these images are offered in 3D. Projections from more views or improved algorithms are needed to improve the spatial resolution of the VLIF technique.

After the above fundamental study, we describe two more experiments performed on stable laminar flames to further illustrate characteristics of the VLIF measurements and also the application of the ART screening algorithm. The first experiment involved VLIF on half of the cone flame, as shown in Figure 2-7 and Figure 2-8, to illustrate capability of VLIF to target an arbitrary measurement region. In this experiment we adjusted location of one of the knife edges, so that only half of the cone flame was illuminated. Figure 2-7 shows a set of VLIF projections; the difference is apparent when compared to those shown in Figure 2-3a, where the entire flame was illuminated. Figure 2-8a shows the 3D
reconstruction, Figure 2-8b shows the central slice at $Y = 0$ mm of the 3D reconstruction, and Figure 2-8c shows a direct comparison of the VLIF reconstruction against a PLIF measurement (again, only of half of the PLIF image is shown). This simple demonstration illustrates the ability of VLIF to target an arbitrary region in the flame, an important advantage compared to other volumetric techniques such as tomographic chemiluminescence (TC) [81]. The TC technique utilizes nascent flame emissions that are generated throughout the entire flame, and therefore it is practically difficult to selectively target a specific portion of the flame. VLIF utilizes LIF photons that are generated by the probe laser slab, and therefore it is straightforward to target a selected region in the flame by adjusting the slab location and dimensions.

![VLIF projection measurements of half of the cone flame.](image)

Figure 2-7 VLIF projection measurements of half of the cone flame.
Figure 2-8 (a) 3D VLIF measurements on half of the cone flame. (b) The central slice of VLIF measurement at \( Y=0 \). (c) Comparison of VLIF and PLIF measurements.

The second experiment involved VLIF on a more complicated V-shaped laminar flame, as shown in Figure 2-9, created by placing a tungsten rod at the exit of the \( \text{CH}_4 \)-air jet. Figure 2-9a-d shows the projections, and Figure 2-9e shows the 3D reconstruction. As seen from Figure 2-9e, the 3D reconstruction correctly captured the overall V-flame shape, and the fact that one branch was taller than the other. Figure 2-10a shows a 2D slice of the VLIF reconstruction at \( Y = 1 \) mm, to be compared with the projection measured along the \( Y \) axis (i.e., measured by camera 2), as shown in Figure 2-10b. To facilitate the comparison, the contour of Figure 2-10a was extracted via binarization and overlaid on the projection shown in Figure 2-10b. The idea behind this comparison was that even though the projection was line-of-sight averaged, some 3D information may still be inferred from salient features of the flame, and such information then can be used to validate the VLIF measurement. In the example shown in Figure 2-10a and b, the plane at \( Y = 1 \) mm was chosen because the left branch of the flame (i.e., the shorter branch) was at its largest extent on this plane, and this largest extent should be captured by the projection because of its
line-of-sight nature. This idea was confirmed by the close overlap seen in Figure 2-10b between the contour and projection on the left branch. Note that the contour and projection did not overlap on the right branch, because at $Y = 1$ mm, the right branch was not at its largest extent. The largest extent of the right branch occurred on the plane at $Y = -1.5$ mm, as shown in Figure 2-10c. And as expected, the contour extracted at $Y = -1.5$ mm overlapped closely with the projection on the right branch, as shown in Figure 2-10d.

Figure 2-9 (a)-(e) VLIF projection measurements on a V flame. (f) 3D VLIF reconstruction.
Figure 2-10 (a) Planar slice extracted the VLIF reconstruction at Y=1 mm. (b) The contours of (a) overlaid with the projections. (c) Planar slice extracted from the VLIF reconstruction at Y=-1.5 mm. (d) The contours of (c) overlaid with the projections.

2.4. Demonstration on turbulent flows

After the validations of the ART screening algorithm on the laminar flame measurement, this section describes its demonstration on the turbulent flow measurements. Figure 2-11 and Figure 2-12 show a set of sample 3D VLIF measurements from a turbulent jet flame. The Reynolds number for the jet was 3355 (based on jet diameter and jet-exit properties). Figure 2-11a shows a set of projection measurements captured simultaneously on all five cameras during a single shot. Figure 2-11b shows the 3D reconstruction using the measured projections as inputs. As can be seen, the reconstruction successfully resolved the turbulent wrinkles and flame detachment as one can expect from observation of the measured projections. Figure 2-12a shows the 2D slice at Y = 0 mm (i.e., the central slice), taken out of the 3D VLIF reconstruction (which also elucidate the turbulent wrinkles and
flame detachment more clearly). The contour of this slice was then extracted and overlapped on the projection measured along the Y axis (i.e., by camera 2), as shown in Figure 2-12b. As can be seen, the contour and the projection only overlap on an overall level. The reason is that because of the 3D nature of the turbulent flame, the projection in Figure 2-12b was the result of many superimposed features across different Y planes along the line-of-sight. As a result, no 2D slice at any particular plane can overlap entirely with the projection, as confirmed by the results shown in Figure 2-12c, where the projection was compared to the contour extracted from another 2D slice taken out of the VLIF measurement at Y = 1.05 mm.

Figure 2-11 (a) VLIF projections from a turbulent flame. (b) 3D VLIF reconstruction.
Figure 2-12(a) Planar slice of the VLIF reconstruction at $Y=0$ mm. (b) The contour of (a) overlaid with the projection. (c) The contour of a planar slice at $Y=1.05$ mm overlapped with the projection.

Lastly, the comparisons shown in Figure 2-12 do not provide a sufficient condition to prove the correctness of the reconstruction but only a necessary condition. Further analysis and experiments are being planned to provide further validations, e.g., the comparison of a statistical quantity obtained via PLIF and VLIF, and/or the direct comparison of PLIF and VLIF measurements obtained simultaneously. It should be noted too that although this work focused on the application of the VLIF technique to obtain 3D flame fronts and not 3D concentration fields as such (due to the nature of the distribution of CH radicals in combustion flows), the tomographic VLIF technique is applicable to the instantaneous measurements of chemical species, as demonstrated in other recent efforts [71-73].

2.5. Summary

This chapter reports the development of the ART screening algorithm and its validation and demonstration using volumetric laser-induced fluorescence (VLIF) based
on CH radical, a key flame marker used in combustion diagnostics. The results showed that based on the ART screening algorithm, single-shot VLIF measurements with sufficient accuracy can be obtained in a sizable volume (a volume of 9.3×9.3×32.7 mm$^3$ was demonstrated in this work) with an excitation pulse energy of ~10 mJ. Validation and demonstration measurements were performed on multiple laminar and turbulent flames. With a total of five intensified cameras, 3D measurements were demonstrated over a total of ~10$^6$ voxels with a voxel size of 0.15 mm. Comparison with PLIF measurements suggested that the spatial resolution of the ART screening algorithm based VLIF technique to characterize the flame surface was about 0.4 mm. Though such spatial resolution cannot compete with that of PLIF (on the order of 0.05 mm/pixel in this work), the advantage of VLIF is clear: it offers more imaging elements than PLIF and these images are offered in 3D. The ART screening algorithms was directly validated by a comparison of the 3D measurements obtained from the ART screening algorithm against PLIF measurements. The results demonstrate that ART algorithm enabled improved computational efficiency, spatial resolution, and reconstruction accuracy when compared to traditional ART algorithm.
Chapter 3 Algorithms for nonlinear tomography diagnostics

3.1. Background

Many optical tomography problems encountered in practice are nonlinear, for example, due to significant absorption, multiple-scattering, or radiation trapping. Nonlinear problems pose several unique challenges, and reconstruction algorithms developed for linear problems are not easily extendable to solve nonlinear problems. As mentioned in Chapter 1, the analytical algorithms (i.e., FBP) and several iterative algorithms (i.e., CG and SA) can be used to solve the nonlinear tomography problems. However, these existing algorithms confront several challenges in practical 3D flow diagnostics when nonlinear effects are not negligible. These challenges include the computational cost caused by the nonlinearity (which was compounded by the large scale of the problems when they are 3D), the limited view angles available in many practical applications, and the measurement uncertainty.

More specifically, for the FBP algorithm, it requires a large number of projections from a wide range of perspectives (hundreds and more). However, in many practical problems (such as flow imaging), it is practically difficult or infeasible to obtain a large number of projections from a wide angular range due to the transiency of the target, the optical access available, and the cost of equipment. For the iterative CG algorithm, it cannot always converge to the correct solution given the many local confusing minima when the problem involves a large number of variables (e.g., on the order of millions for the problems examined in this work) within the measurement uncertainty [34]. To overcome this limitation, global iterative techniques such as SA [6, 39, 40] and genetic algorithm
(GA) [82] have been adopted to solve the tomography problems. These algorithms, at least in principle, can find the global minimum or a solution sufficiently close to it [45]. However, the computational cost of those techniques is dramatically higher than those based on gradients and can become prohibitive for problems with practical scale.

Based on the above understanding of past work, this chapter describes a new approach, termed NIRT (Nonlinear Iterative Reconstruction Technique), to solve nonlinear tomography problems. The NIRT approach solves the nonlinear problem iteratively in a manner analogous to the solution of linear tomography problem using ART. The NIRT algorithm is demonstrated to solve nonlinear tomography problems while retaining the advantages of ART for linear problems. For example, the NIRT algorithm was able to solve nonlinear problem with limited number of projections, with robustness in the presence of uncertain projections, and with good computational efficiency. In the rest of this chapter, we first detail the mathematical formulation of the NIRT algorithm, and then report its numerical and experimental validation.
3.2. Mathematical formulation and algorithm description

This section introduces the mathematical formulation of nonlinear tomography and describes the NIRT algorithm. Figure 3-1 schematically illustrates the mathematical formulation. As can be seen, the 3D distribution of the sought property is denoted as $C$ (e.g., the concentration distribution of a chemical species or particulates) and discretized into voxels under a Cartesian coordinate system. An imaging system collects the signal emitted by the target property to form an image on the camera chip (defined as a projection and denoted as $P$). As an example, the dashed red and brown lines illustrate that the signals emitted by two different voxels arrive on two sets of pixels on the camera chip and that these two sets of pixels can overlap. The projection $P$ depends on two factors: the parameters of the imaging system and the modulation (attenuation or magnification) of the signal by the target property itself. The parameters of the imaging system include the distance and orientation of the imaging system, specified by $r$ (distance), $\theta$ (azimuth angle), and $\phi$ (inclination angle). The effects of all these parameters are reflected in the point

![Figure 3-1 Mathematical formulation of a nonlinear tomography problem.](image-url)
spread function (PSF) of the imaging system [76, 83]. The modulation of the target signal could be caused by a variety of physical processes depending on the specific signal generation and propagation mechanisms. As a simple example shown in Figure 3-1, when the signal generation involves a laser (e.g., in LIF or Mie scattering measurements, respectively), the signal could be attenuated by the absorption or scattering of the illumination laser or magnified by amplified spontaneous emission. When the signal generation does not involve a laser (e.g., in chemiluminescence measurements), the signal could be attenuated by radiation-trapping.

Regardless of the specific signal generation and modulation mechanism, the relationship between the measured projection $P$ and the sought property can be mathematically capsulated into the following equation:

$$P = PSF \cdot (\Gamma \circ C)$$

(3.1)

where $P$ presents the projection vector formed by organizing the measured projections pixel by pixel, $PSF$ the point spread function in matrix form, $C$ the vector formed by organizing the discretized sought property voxel by voxel, and lastly $\Gamma$ a modulation function representing the signal modulation mechanism. In Eq. 3.1, the operator ‘$\circ$’ represents the Hadamard product and ‘$\cdot$’ matrix and vector product. Note that the Hadamard product is associative but the matrix product is not. This work uses bold letters to symbolize vectors or matrices formed by discretizing their corresponding continuous functions. The tomographic problem is to solve for $C$ with $P$ measured at different locations and orientations. Based on Eq. 3.1, it can be seen that when $\Gamma$ does not depend on $C$ or depends linearly on $C$, the tomographic problem is a linear problem. This work focuses on nonlinear problems where $\Gamma$ depends nonlinearly on $C$. 
To better elucidate the problem and describe the NIRT algorithm, Eq. 3.1 is expanded into its element form as shown in Eq. 3.2:

\[
\begin{bmatrix}
    P_1 \\
    P_2 \\
    \vdots \\
    P_m \\
    \vdots \\
    P_M
\end{bmatrix} =
\begin{bmatrix}
    PSF_{1,1} & PSF_{1,2} & \cdots & PSF_{1,n} & \cdots & PSF_{1,N} \\
    PSF_{2,1} & PSF_{2,2} & \cdots & PSF_{2,n} & \cdots & PSF_{2,N} \\
    \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
    PSF_{m,1} & PSF_{m,2} & \cdots & PSF_{m,n} & \cdots & PSF_{m,N} \\
    \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\
    PSF_{M,1} & PSF_{M,2} & \cdots & PSF_{M,n} & \cdots & PSF_{M,N}
\end{bmatrix}
\begin{bmatrix}
    \Gamma_1C_1 \\
    \Gamma_2C_2 \\
    \vdots \\
    \Gamma_nC_n \\
    \vdots \\
    \Gamma_NC_N
\end{bmatrix}
\]

(3.2)

where \(m\) and \(M\) are the index and total number of pixels on the projection, respectively; \(P_m\) is measured projection on the \(m\)th pixel; \(n\) and \(N\) are the index and total number of voxels in the measurement domain, respectively; \(PSF_{m,n}\) is the value of PSF matrix for the \(n\)th voxel on the \(m\)th pixel, and \(C_n\) and \(\Gamma_n\) are the values of the sought property and the nonlinear modulation function for the \(n\)th voxel, respectively. Since the PSF matrix does not depend on \(C\), established ART algorithm [84] can be applied to solve for \(\Gamma \circ C\) voxel by voxel. However, in general, \(\Gamma \circ C\) cannot be decoupled by ART to obtain \(C\) when \(\Gamma\) depends on \(C\) nonlinearly. As an example, \(\Gamma\) assumes the following exponential form according to the Beer-Lambert relationship for a variety of signal generation processes (such as absorption, scattering, or radiation trapping):

\[
\begin{bmatrix}
    \Gamma_1 \\
    \Gamma_2 \\
    \vdots \\
    \Gamma_n \\
    \vdots \\
    \Gamma_N
\end{bmatrix} =
\begin{bmatrix}
    I_{0,1} \cdot e^{-\alpha C_{d1}} \\
    I_{0,2} \cdot e^{-\alpha C_{d1}} \\
    \vdots \\
    I_{0,n} \cdot e^{-\alpha C_{d1}} \\
    \vdots \\
    I_{0,N} \cdot e^{-\alpha C_{d1}}
\end{bmatrix}
\]

(3.3)
where $\alpha$ represents the attenuation coefficient (either absorption or scattering coefficient), $l$ the pathlength propagated by the illumination laser and/or the signal photons before reaching the imaging system, $C_l$ the value of $C$ along $l$, and $I_{0,n}$ the incident laser intensity on the $n$th voxel. Here, $\Gamma$ depends not only on $C$ nonlinearly but also on $l$, both may vary from voxel to voxel. The established ART algorithm is unable to address such dependence in general.

The NIRT algorithm was developed to address these challenges and solve the nonlinear tomography problem described above. The first step of the development was to cast Eq. 3.1 into a different form as shown below:

$$P = \text{PSF} \cdot \text{diag}(\Gamma) \cdot C$$

(3.4)

by converting $\Gamma$ from a vector to a diagonal matrix (i.e., diag($\Gamma$)) with the elements of vector $\Gamma$ on the main diagonal. The second step was to define a new point spread function, $NPSF$, by combining the first two terms of right hand in Eq. 3.4. The specific element form of $NPSF$ is shown below:

$$NPSF = \begin{bmatrix}
PSF_{1,1} \Gamma_1 & PSF_{1,2} \Gamma_2 & \cdots & PSF_{1,n} \Gamma_n & \cdots & PSF_{1,N} \Gamma_N \\
PSF_{2,1} \Gamma_1 & PSF_{2,2} \Gamma_2 & \cdots & PSF_{2,n} \Gamma_n & \cdots & PSF_{2,N} \Gamma_N \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
PSF_{m,1} \Gamma_1 & PSF_{m,2} \Gamma_2 & \cdots & PSF_{m,n} \Gamma_n & \cdots & PSF_{m,N} \Gamma_N \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
PSF_{M,1} \Gamma_1 & PSF_{M,2} \Gamma_2 & \cdots & PSF_{M,n} \Gamma_n & \cdots & PSF_{M,N} \Gamma_N
\end{bmatrix}$$

(3.5)

Conceptually, $NPSF$ represents the nonlinear point spread function that takes both the geometric PSF and the nonlinear modulation $\Gamma$ into account. Now Eq. 3.1 can be rewritten into:

$$P = NPSF \cdot C$$

(3.6)

The third step was to solve Eq. 3.6 iteratively according to the following scheme:
\[ D_m^q = P_m - P_m^q \]  
\[ C_n^{q+1} = C_n^q + \beta \frac{D_m^q \cdot NPSF_{m,n}^q}{\| NPSF_q \|^2} \]  

where \( q \) represents the index of the iteration step, \( D \) the difference between the measured \( (P_m) \) and calculated projection \( (P_m^q) \) at the \( m \)th pixel in the \( q \)th iteration, \( \beta \) a relaxation factor set to be 0.01 in this work, and \( \| NPSF_q \| \) the L2 norm of \( NPSF \) in the \( q \)th iteration. As can be seen from the Eqs. 3.7 and 3.8, the NIRT algorithm accounts for the modulation of the signal in each step of the iteration to address the nonlinearity (at the cost of increased computational cost compared to the linear ART algorithm, since the \( NPSF \) matrix needs to be updated in every iteration). To address the signal modulation’s dependence on \( l \) (i.e., the propagation pathlength), a ray-tracing module based on the Monte Carlo method [78] was employed to track the signal modulation on a voxel by voxel bases. This ray-tracing module is an important component of the NIRT algorithm, and it enables the NIRT algorithm to be applicable to a wider range of tomographic problems compared to past algorithms. For instance, past algorithms typically were developed under the so-called parallel beam or fan beam assumption. These assumptions have limited range of validity in practice, primarily in two aspects. First, these assumptions become invalid in practice when the signals emitted from different voxels overlap (as illustrated in Figure 3-1) due to focusing and zooming. Second, algorithms based on these assumptions are typically limited to two-dimensional (2D) problems or 3D problems that can be decouple into a series of 2D problems, while many practical problems are truly 3D due to signal overlapping and pathlength dependent signal modulation, and cannot be decoupled into a series of 2D problems.
Lastly, the iteration terminates when the change in $C$ during two consecutive iterations is less than a preset value. More specifically, the following termination criterion as described in [85] was employed in this work:

$$\left| \sum_{n=1}^{N} C_n^q - \sum_{n=1}^{N} C_{n-1}^q \right| \leq \Delta \beta \sum_{n=1}^{N} C_n^q$$

where $\Delta$ is a small number that controls the speed of convergence. The value of $\Delta$ was empirically suggested to be in a range of 0.0001% to 0.1% from previous work [85], and was preset to be 0.1% in this work. The results from this work showed that the reconstruction accuracy was insensitive to the choice of $\Delta$ (the reconstruction accuracy changes within 0.5% when $\Delta$ varied between 0.001% to 0.1%). Note that there is no extra constraint placed on $C$ by the algorithm besides that it should be positive. The above development of the NIRT algorithm was summarized in the flow chart of Figure 3-2.
In summary, the development of the NIRT algorithm relied on two key enabling techniques to overcome the challenges of the nonlinear tomography problems. The first enabling technique involved the derivation and definition of NPSF, the nonlinear point spread function, which allowed the development of the iterative scheme to decouple the nonlinear problem. The second enabling technique involved the application of a ray-tracing module to account for dependence of signal modulation on pathlength. The rest of this
paper will describe the experimental and numerical validation of the algorithm, and then
the demonstration of the algorithm in an application involving volumetric LIF measurement in turbulent flows.

3.3. Experimental and numerical validations

The NIRT algorithm has been first extensively validated numerically using a range of phantoms. Here, we will focus on the description of the numerical validation on a phantom for which an accompanying experimental validation was performed, and only briefly discuss the numerical validation on other phantoms. This validation involved a phantom with a uniform distribution of the target species, which can be experimentally realized with excellent control and accuracy as shown in Figure 3-3 (a). Figure 3-3 (a) shows the setup of the validation experiment from the top view. The setup used a cubical cell, with a dimension of $50 \times 50 \times 50 \text{ mm}^3$, filled with a fluorescence dye solution (a well stirred mixture of ethanol and Rhodamine 6G) to experimentally create a uniform phantom.

![Figure 3-3 Panel (a): experimental setup from the top view. Panel (b): the spatial profile of illumination laser intensity at X=25 mm.](image)

When illuminated volumetrically by a laser at a wavelength of 532 nm (generated by a pulsed Nd:YAG and expanded by a series of lenses as shown), the dye solution absorbs
the illumination laser as it propagates through the solution and emits LIF photons, resulting in a nonlinear problem as described above. The dye solution emits LIF signals volumetrically, which were captured by a total of 7 cameras (6 Photron SA4s and 1 SA6) from different orientations. The goal of this validation was to compare the reconstructed distribution of the dye concentration to the uniform distribution known a priori.

To facilitate the following discussion, a right-handed Cartesian coordinate system was defined as shown in the figure: the origin was defined as the center point of the cubical cell, the X axis was defined along the propagation direction of the illumination laser (which propagated perpendicularly into the dye cell), and the Z axis was defined to be out of the paper as shown in Figure 3-3 (a). In the experiment, all 7 cameras were aligned in the X-Y plane and therefore their orientations were completely specified by $\theta$, defined as the angle formed by the optical axis of a given camera relative to the positive X direction as shown. The focal length and f-number of the lenses used on all cameras were 105 mm and 2.8, respectively. Each lens was equipped with a 532 ± 10 nm OD4 notch filter to block any interference due to the scattering or reflection of the illumination laser. Prior to any measurement, a calibration target was used in view registration program to determine the orientations of cameras [79]. The orientations of camera 1 through 7 were determined to be $\theta = 90.0^\circ$, 127.5$^\circ$, 213.8$^\circ$, 240.2$^\circ$, 272.2$^\circ$, 316.9$^\circ$ and 48.9$^\circ$, respectively, with an accuracy estimated to be within 0.6$^\circ$.

The profile of the illumination laser pulses (i.e., $I_0$ in Eq. 3.3) was needed to perform the tomographic reconstruction. To obtain $I_0$, another dye cell, with a length and height both of 50 mm and a thickness of 0.5 mm, was fabricated. This cell was used to hold a thin layer of the same dye solution, and was then placed at X=25 mm, i.e., where the laser
entered the target measurement volume. The thin cell was illuminated by the laser, and an image of the LIF signal emitted was captured as shown in Figure 3-3(b). Due to the thinness of the dye solution in the cell, the integration effects in the dye were neglected and the image shown in Figure 3-3(b) was used to represent the intensity profile of the illumination laser (i.e., $I_0$).

Figure 3-4 Experimental results obtained from the controlled dye cell. Panel (a): projection of the volumetric LIF signal from the top view. Panel (b): intensity of signal along the laser propagation direction at three different $Y$ locations ($Y = -5, 0$ and $5$ mm). Panel (c): the reconstructed concentration distribution at three difference planes ($Z = 10, 20$ and $30$ mm). Panel (d): reconstructed dye concentration along three lines.

To confirm the absorption, a camera was placed to capture the projection of the LIF signal emitted by the dye cell from the top view, as shown in Figure 3-4(a). If the absorption was negligible, the LIF signal captured from this view should be constant along the $X$ direction (i.e., the propagation direction of the laser) due to the uniformity of the dye concentration. Figure 3-4(b) shows the intensity of the signal along the $X$ direction at three
Y locations (Y = -5, 0 and 5 mm, illustrated by the dashed lines in panel a). As seen in Figure 3-4 (b), signal decreased appreciably as the laser propagated into the dye solution, confirming that the absorption was not negligible. A curve fitting on the data shown in Figure 3-4 (b) also confirmed that the signal decreased exponentially according to Beer-Lambert relationship as shown in Eq. 3.3. The absorption coefficient, $\alpha$, was estimated to be around 0.006 mm$^{-1}$ for this sample.

Based on the projections obtained on all 7 cameras (each with 800 × 800 pixels, resulting $M \approx 4.5 \times 10^7$ as shown in Eq. 3.2), a tomography reconstruction was performed using the NIRT algorithm to obtain the 3D distribution of dye concentration (which should be ideally a uniform distribution because the dye solution was well mixed). In this volumetric LIF problem, the two-level LIF model [86] was used and the fluorescence yield was set as a constant value for all particles in the concentration field (i.e., the goal is essentially to measure a relative concentration field, which is what many LIF measurements are aimed at). The computational domain was taken to be a volume of 40 × 40 × 40 mm$^3$ center around the origin, smaller than the volume of the dye cell to exclude non-ideal effect around the edges and corners of the cell. The computational domain was then discretized into 120 × 120 × 120 voxels (i.e., $N \approx 1.8 \times 10^6$ as shown in Eq. 3.2), resulting in a nominal resolution of 0.33 mm in all three directions. The nominal resolution was calculated by dividing the dimension of the measurement volume with the discretization (i.e., 40 mm/120 = 0.33 mm). This nominal resolution represents the highest possible resolution that can be obtained. The actual resolution will be worse than the nominal resolution due to the factors such as measurement and reconstruction uncertainty. Our previous work [71] have quantified the spatial resolution by comparing the volumetric
LIF with a planar LIF (PLIF) measurement on a turbulent flow. Based on the comparison, the actual spatial resolution was estimated to be 0.71 mm. Also note that $M$ is larger than $N$ in this work, resulting in an over-determined system (i.e., more equations than unknowns). In practice, we always prefer to design the experiments to make the problem over-determined. The number of equations is controlled by the number cameras and their field of view, and the number of unknowns by the discretization scheme.

Figure 3.5 Numerical validation using a uniform phantom. Panel (a): reconstructed distribution on the central plane (i.e., $Y=0$) using simulated projections with 4% artificial noises added. Panel (b): reconstructed distribution along 3 lines.

Figure 3.4(c) shows the result of the reconstruction on three planes (corresponding to $Z = 10, 20$ and 30 mm). As seen, the reconstruction shows the expected uniform distribution. To examine the reconstruction closely and quantitatively, Figure 3.4(d) shows the reconstructed concentration along three lines shown by the dashed line in Figure 3.4(c). As seen from Figure 3.4(d), the reconstructed concentration agreed well with the expected uniform distribution (shown as dotted lines). To quantify the reconstruction accuracy, an average reconstruction error ($e_R$) was defined according to Eq. 3.10:
\[
    e_R = \frac{\sum_{n=1}^{N} |C_n^{\text{rec}} - C_n^{\text{true}}|}{\sum_{n=1}^{N} C_n^{\text{true}}}
\]

where \(C_n^{\text{rec}}\) and \(C_n^{\text{true}}\) represent the reconstructed and true concentration distribution on the \(n\)th voxel, respectively. For results shown in Figure 3-4 (d), \(e_R\) was on the order of 4%.

Numerical validation was performed parallel with the above experiments, and such numerical validation also provided further insights to the \(e_R\) calculated above. For the numerical validation, a uniform phantom was created to simulate the distribution of the dye in the cell. Projections from the same seven orientations as used in the experiments were calculated using the laser profile shown in Figure 3-3(b). To simulate the experimental uncertainty, a total of 4% of noise was artificially added to the projections. This noise level was chosen based on an analysis of the experiments. Three primary sources of uncertainty were identified in the experiments: the uncertainty with the camera (such as background noises, camera nonlinearity and/or nonuniformity et al., estimated to be about 1%), the shot-to-shot variation of the laser system (estimated to be about 1.1%), and the accuracy of our view registration process. As aforementioned, the accuracy of the view registration was estimated to be within 0.6°, and such an uncertainty translated into about 1.6% uncertainty in the projection. The overall uncertainty from all three sources was therefore estimated to be around 4%. Gaussian noise was added to the numerical projection data to simulate the background noise and the shot-to-shot variation. As for the view registration noise, the noisy projections were simulated using the angle sets with measurement uncertainty. Based on the simulated projections, reconstructions were performed and the results summarized in Figure 3-5. Figure 3-5 (a) shows the reconstructed
concentration at a slice corresponding to $Y = 0$ mm using the simulated projections. Figure 3-5 (b) shows the reconstructions along three lines in comparison to the phantom (shown in black dash lines), with $e_R$ calculated to be 4.01%, 4.03% and 4.53%, respectively. These results were in good agreement with those obtained experimentally, supporting the accuracy of the algorithm and the analysis of the experimental uncertainty.

Lastly, to further elucidate the difficulties associated with nonlinear tomography problems, numerical studies were performed to compare the accuracy and computational cost of three reconstruction techniques: NIRT, ART, and a minimization technique. When the ART algorithm was used, it was used to solve the problem as if it were a linear problem (i.e., the absorption is neglected). In the minimization technique, the tomography problem was converted into a problem to find a distribution that can reproduce the projections with minimum difference. Due to the complexity of this minimization process, a stochastic minimization algorithm based on simulated annealing (SA) [40] was used. Figure 3-6
compares the $e_R$ and computing time when these techniques were applied on the same uniform phantom. All simulations were performed with 4% artificial noise added to the projections, and all computations were performed on a same computational workstation with 16-Core Intel Xeon 2.6GHz CPU (only 1 core is used in all calculations) and 512GB memory. As can be seen, both NIRT and SA achieved the same level of accuracy, indicating that their ability to converge to the globally optimal solution for complicated tomographic problems is comparable. Both NIRT and the minimization technique SA yielded good reconstruction accuracy with $e_R$ below 5%, illustrating the effectiveness of NIRT method and also the flexibility of the minimization technique and the prowess of SA to minimize large scale and complicated functions. In contrast, ART failed to solve the problem correctly and resulted in an $e_R \approx 60\%$ due to its inability to address the nonlinearity in the problem.

In terms of the computational cost, the computing time of NIRT (~7,200 seconds) was on the same order of magnitude as that of ART (~5,760 second) because the NPSF needed to be updated in each iteration in NIRT and required extra computation time. The SA algorithm, in contrast, cost approximately 30× more computing time than NIRT (~219,600 seconds). In conclusion, the NIRT algorithm was capable of performing 3D nonlinear tomography with reconstruction fidelity as methods based on minimization techniques, while with computational cost on the same order as ART.

3.4. Demonstration on turbulent flows

After the above experimental and numerical validations, this section reports an application of the NIRT algorithm on the 3D measurement of iodine ($I_2$) vapor using a technique we codenamed VLIF (volumetric LIF). The experimental setup was similar to
that shown in Figure 3-3a, with two major differences. The first major difference was that the controlled cell was replaced by a turbulent nitrogen ($N_2$) jet flow seeded with $I_2$ vapor. The exit diameter of the jet was 6.35 mm. The $I_2$ vapor was introduced into the flow by heating solid iodine crystals in a water bath to a temperature of 200 °F. The mole concentration of the $I_2$ vapor in the target flow was estimated to be 4%. A rod (with a diameter of 3.18 mm) was placed at the exit of the jet, to increase the complexity of the flow structures (and also to create an easily recognizable pattern of the flow to facilitate the discussion of the results). The second difference was that the 10 Hz Nd:YAG laser was replaced by a 5 kHz pulsed laser at a wavelength of 527 nm (Photonics Industries DM20–527). The pulse duration of the 5 kHz laser was ~300 ns and the pulse energy was 24 mJ. Similar to the dye cell measurement, the laser pulses generated were expanded to illuminate the flow volumetrically, exciting the seeded $I_2$ molecules to generate LIF photons volumetrically.

Figure 3-7 A set of example projections on the turbulent flow

Base on this new setup, Figure 3-7 shows a set of example projections of the target flow captured by camera 1 through 7, and also an example 2D laser profile measured using
the thin dye cell mentioned before. All images shown here were single-shot measurements. The projections displayed a turbulent jet flow with an overall V shape generated by the rod placed at the exit of the jet. All projections had a resolution of 800 × 800 pixels, and each pixel corresponded to a physical dimension of 0.05 × 0.05 mm. The overall size jet flow was about 40 mm in each direction. The absorption of the illumination laser by the seeded iodine was estimated to be ~5% as it propagated through the measurement volume, not as large as the absorption observed in the dye cells but large enough to preclude the use of linear algorithms such as ART.

Based on the measured projections, 3D distribution of the I₂ vapor concentration in the jet flow was reconstructed using NIRT. These reconstructions were performed in a computational domain of 40×40×40 mm³ encompassing the entire target flow. The computational domain was discretized into 120×120×120 voxels (~1.8×10⁶), resulting in a nominal spatial resolution of 0.33 mm. Figure 3-8 shows an example reconstruction obtained using the projections shown in Figure 3-7. Figure 3-8 (a) shows a 3D rendering of the relative I₂ concentration, illustrating that the reconstruction captured the overall V-shape of the flow. To further examine the reconstruction, Figure 3-8 (b) shows three planar slices of the reconstruction at three different Y locations (Y = -5, 0 and 5 mm). As seen from Figure 3-8 (b), the 3D reconstruction also captured the flow feature at a more detailed level. For example, the slice at Y = 0 corresponded to the central plane of the jet flow, and therefore the overall I₂ concentration on this slice should be higher than the other two slices due to less entrainment, as clearly captured by the 3D reconstruction shown in Figure 3-8 (b).
Figure 3-8 Application of NIRT on 3D measurement of I$_2$ concentration in turbulent flows using VLIF. Panel (a): 3D rendering of the reconstruction. Panel (b): three planar slices of the 3D reconstruction. Panel (c): a planar slice taken out of 3D reconstruction at $Y = 5$ mm. Panel (d): projection measured by camera 5.

To further elucidate the usefulness of the 3D reconstruction, Figure 3-8 (c) and (d) show a comparison between the 3D reconstructed I$_2$ distribution against a measured projection. Figure 3-8 (c) shows a planar slice of the 3D reconstruction at $Y = 5$ mm, and Figure 3-8 (d) shows the projection measured by camera 5 (which was placed in the same view orientation as Figure 3-8 (c)). The images shown in Figure 3-8 (c) and (d) differ apparently, due to the fact that the result shown in Figure 3-8 (c) was spatially resolved in 3D while that in Figure 3-8 (d) was line-of-sight-integrated. Such difference was further displayed by comparing the contours of the flow. The white line shown in Figure 3-8 (c) shows the contour of the flow extracted from the 3D reconstruction, and this contour was overlaid on the line-of-sight-integrated measurements shown in Figure 3-8 (d).
Similar to the validation studies described in Section 3.3, parallel numerical simulations were performed for these measurements conducted in turbulent flows to provide further insights. In these simulations, a phantom distribution resembling that target turbulent flow was created. Based on the phantom, projections at the same orientations used in the experiments were computed with artificial noises added to simulate measurement uncertainty. Then these simulated projections were used in the NIRT algorithm. As aforementioned, the noises considered included the uncertainty with camera (estimated to be about 1%), the shot-to-shot variation of the laser system (estimated to be about 1.1%), and the accuracy of our view registration process. Figure 3-9 examines the last noise source more closely by showing the $e_R$ obtained under various level of uncertainty caused by view registration error. As seen, at our current level of view registration accuracy of $\Delta \theta = 0.6^\circ$, the error in the projection ($e_{P, VR}$) caused by $\Delta \theta$ was about 2.7% (which was larger than that for the simpler uniform distribution examined in Section 3.3). In general, our results suggest that the impact of $\Delta \theta$ on $e_{P, VR}$ increases as the complexity of the distribution increases, motivating the improvement of the view registration process as to be further elaborated at the end of this section. With an $e_{P, VR}$ of 2.7% and a combined uncertainty of 2.1% due to the background noise registered on the camera and the shot-to-shot variation, a total of 4.8% artificial noise was added to the computed projections to simulate the experiments. The $e_R$ obtained was ~8% as shown in Figure 3-9, significantly larger than that obtained for the simpler uniform phantom discussed in Section 3.3 due to the increased complexity in the target distribution. Before further discussion on such reconstruction accuracy and on possible ways to improve it, we
wanted to first point out that a reconstruction with an $e_R$ of 8% is actually quite accurate already and resolve most of the detailed features as illustrated in Figure 3-10.

Figure 3-9 Reconstruction error using simulated projections based on turbulent phantoms.

Figure 3-10 shows a more detailed comparison between the turbulent phantom and the reconstruction using simulated projections (with 4.8% artificial noise added) across several planes and along several lines. In each panel, the first column shows phantom distribution of I$_2$ vapor (i.e., the true answer) across a plane, the second column shows the I$_2$ distribution reconstructed by NIRT across the same plane, and the third column compares the phantom and the reconstruction along three lines (the dashed line). These three lines were picked so that the $e_R$ along the first line (8.9%) was about the same as the overall 3D $e_R$ of 8% (as shown in Figure 3-10a), the $e_R$ along the second line (5.6%) was significantly smaller than the overall 3D $e_R$, and the $e_R$ along the third line (10.8%) was significantly larger than the overall 3D $e_R$. As seen, at a reconstruction accuracy within 8%, the reconstruction was able to capture the overall features of the flow and also most of the detailed features.
Lastly, we conclude this discussion with a discussion of possible approaches that can further improve the reconstruction accuracy of nonlinear tomography problems. As mentioned above, the reconstruction algorithm need to be able to address at least three sources of noises encountered in practice: uncertainty with the camera, the laser system, and the view registration process. The first two sources need to be addressed primarily by experimental equipment and are not discussed further here. The last source can be addressed both by experimental approaches (e.g., design of a new view registration procedure to reduce $\Delta \theta$) and also by the reconstruction algorithm. One possible approach involves modifying the NIRT algorithm to include the orientations of the cameras (i.e., $\theta$’s) as variables too, so that the view registration errors are accounted for during the reconstruction process. Our preliminary study shows that this approach along could reduce $\Delta \theta$ from $= 0.6^\circ$ to an equivalent of $0.3^\circ$, leading to an improved $e_R$ of ~6.6% from ~8% as shown in Figure 3-9.
3.5. Summary

In summary, this chapter reports the development and validation of a reconstruction algorithm for nonlinear tomography problems, motivated by a range of practical applications involving absorption, scattering, or radiation trapping. Algorithms developed in the past are not readily extendable to such nonlinear problems due to several challenges, such as computation cost, limited view angles available, and the measurement uncertainty. To address these challenges, this work therefore developed a nonlinear iterative reconstruction technique (NIRT). The NIRT algorithm was demonstrated and validated using both experimental tests and numerical simulations. Results show that the NIRT...
algorithm is able to solve large scale nonlinear tomography problems in 3D effectively with high fidelity.
Chapter 4 Algorithms integrating view registration and tomographic reconstruction

4.1. Background

From the discussion of tomography problems in chapter 2 and 3, it is apparent that solving tomography problems (either linear or nonlinear) consists of two steps. In the first step, a calibration measurement is needed to determine the locations and orientations of cameras relative to the target object, and this calibration step is usually referred to as view registration (VR). In the second step, projections of the target captured from the locations and orientations determined in step 1 are fed into a reconstruction algorithm to reconstruct the 3D distribution of the target object.

As a result, the overall performance of tomography measurements depends on two issues: 1) the accuracy of the VR methods [87, 88], and 2) the performance of the reconstruction algorithms to process projection data contaminated with noise (both due to VR error and other measurement errors). Past efforts have been addressing these two issues separately. Efforts to address the first issue aimed to improve VR methods and reduce the uncertainties in the determination of the camera location and orientation. Existing VR methods typically involve the use a calibration target (either a cylinder or a plate) with known patterns, and the analysis of the distortion and magnification of the images of these patterns to determine the location and orientation of cameras. For example, with the use of a cylindrical calibration target, the calibration target has known angular scale affixed around the circumference [17, 35, 81]. This cylindrical calibration target was placed on the center of the measurement object. The view angles can be determined from analyzing the
magnification and distortion of the angular scales seen on the images captured by the cameras. With the use of a plate calibration target, the calibration plate has known dimensions and patterns (i.e., squares with known size) [71, 73, 79, 89]. When images of this calibration plate are captured by cameras from different views, the pattern shows different degree of distortion and magnification on each camera, and these information can be analyzed by a view registration program [90, 91] to determine the orientation and location of each camera [92, 93].

However, regardless of the choice of calibration target or VR program, the camera location and orientation can only be determined with a finite accuracy due to practical factors such as limited depth-of-field (DOV) of camera lenses and other measurement error [6, 94]. For example, the uncertainties of VR method using the plate calibration target was reported to be 0.6° based on recent work [79, 94]. The location and orientation of the cameras is one of the two inputs needed by the tomography algorithm (with the other being the projections), and the error from the VR process therefore will propagate in the tomographic process and impact the accuracy of the final 3D reconstruction.

Past work has addressed the issue of VR uncertainty separately from the tomography reconstruction. For example, research efforts have been invested to research new VR procedure and VR algorithms to reduce uncertainty. Meanwhile, research efforts have also been invested to research new reconstruction algorithms that are more resistant to VR error and noisy projection data, including the design of convergence condition [17, 76, 85, 95] and the use of regularization [96-98].

This work therefore investigated a new approach by addressing both issues together, i.e., integrating the VR and the tomographic reconstruction holistically. More specifically,
this work developed an improved iterative reconstruction algorithm to include the orientations of the cameras as variables too (together with the projections themselves), so that they can be adjusted and optimized during the reconstruction process. This algorithm is therefore code named IRT-VRO, iterative reconstruction technique integrating view registration optimization. In contrast, in past efforts, the orientations of the cameras were treated as parameters (i.e., preset as constants in the tomographic algorithms) as described in chapter 2 and 3.

The rest of this chapter is organized as follows: section 4.2 describes the mathematical formulation and introduces the IRT-VRO algorithm. Section 4.3 reports the experimental and numerical validations of IRT-VRO algorithm on a set of experiments using a controlled cell. Section 4.4 reports the experimental and numerical validations of IRT-VRO algorithm on turbulent flows. Finally, Section 4.5 summarizes this chapter.

4.2. Mathematical formulation and algorithm description

This section introduces the mathematical formulation and describes the IRT-VRO algorithm. As mentioned in previous chapters, the tomographic problem is to solve for $C$ with $P$ measured at different locations and orientations in the following equations:

$$ P = \text{PSF} \cdot C \quad (\text{Linear}) $$

$$ P = \text{PSF} \cdot (\Gamma \circ C) \quad (\text{Nonlinear}) $$

In Eq. 4.1 and 4.2, the $\text{PSF}$ matrix depends on the geometry parameters, but not $C$, and the $\text{PSF}$ matrix is pre-calculated once $\theta$ and $\phi$ are determined in a view registration process. In our iterative reconstruction algorithms described in chapter 2 and 3, the $\text{PSF}$ is set as a constant in the iteration loop once it was calculated. And the sought distribution $C$ is
iteratively corrected until the difference between the calculated and measured projections is minimized. The difference is denoted as $D$, and defined in the following equation:

$$D = [P^c(\theta, \phi) - P^m]^2$$ (4.3)

where $P^c$ represents the calculated projection and $P^m$ the measured projection. Thus, the entire reconstruction process using the existing algorithms will contain the uncertainty of the VR, because the correctness (or the accuracy) of the PSF matrix depends on the orientations of the cameras. As an example, our simulations [99] have shown that a 0.6° uncertainty in the VR process can lead to an ~8% of reconstruction error. Therefore, new methods are motivated to reduce the impact of VR error on the reconstruction quality.
The IRT-VRO algorithm was developed to address this research need by integrating the VR process with the tomographic reconstruction. The IRT-VRO algorithm is based on the IRT reconstruction algorithms and Simulating Annealing (SA) technique, and is summarized in Figure 4-1. First, the reconstruction was performed by an IRT algorithm (either the ART or the NIRT algorithm as described in previous chapters for linear and nonlinear problems, respectively) with an initial guess of the angles ($\theta$, $\phi$) of the cameras.
The results obtained from the VR process were used as the initial guess in this work. Based on this initial guess, a reconstruction was performed and $D$ corresponding to this reconstruction was recorded. Second, a new set of angles $(\theta', \phi')$ were generated by applying a random offset to the current angle set, so that the new angle values were uniformly distributed in intervals centered around the current angle values. Third, the new angles $(\theta', \phi')$ were used in the IRT algorithm to perform the reconstruction again and a new $D'$ was recorded. Fourth, the new angles $(\theta', \phi')$ were either accepted or rejected as a better estimation than the initial guess by comparing $D$ and $D'$ according to the Metropolis criterion in the SA algorithm. Steps 2 to 4 were repeated until a termination criterion was achieved.

As seen from the above description, the essence of the IRT-VRO algorithm is to consider both the orientations of the cameras and the projections as variables in tomographic reconstruction. In contrast, the orientations of the cameras were preset parameters in past efforts. Therefore, as a result, the IRT-VRO was able to find a globally optimal solution to the problem, whereas the past methods can only find a locally optimal solution (the optimal solution under the preset orientations). More specifically, the IRT-VRO algorithm seeks the globally optimal solution by iteratively evaluating the output $D$ of the IRT algorithms at different estimated sets of angles. The IRT-VRO algorithm accepts or rejects a candidate angle set according to the Metropolis criterion as described in the SA technique. By repeating this process, the IRT-VRO algorithm can find both the best angle set and the reconstruction to minimize $D$ globally.

The above advantage enabled by the IRT-VRO algorithm comes at the cost of increase computational cost. The major reason for the increase in computational cost is that in the
IRT-VRO algorithm, a new PSF matrix needs to be computed in every iteration of the algorithm because the PSF matrix depends on the camera orientations and the camera orientations were adjusted in every iteration. In contrast, in past algorithms, the PSF matrix only needs to be computed once. As discussed in earlier chapters, the computation of the PSF matrix is computationally expensive because of its large dimension of the 3D problems of interest to flow and combustion diagnostics. Therefore, it is an algorithm worth consideration when the improvement of reconstruction accuracy merits the increased computational cost.

4.3. Experimental and numerical validations

![Experimental setup and sample measurement projection](image)

Figure 4-2 Panel (a): Experimental setup. Panel (b): a sample measurement projection captured by camera 5.

The IRT-VRO algorithm was validated both experimentally and numerically in this work. This section describes the experimental and numerical validation on a series of experiments using a controlled cubical cell. Figure 4-2a shows the experimental setup from the top view. The cubical cell was the same one as used in Chapter 3, with a dimension of
50 × 50 × 50 mm³, filled with a fluorescence dye solution (a well stirred mixture of ethanol and Rhodamine 6G) to experimentally create a uniform distribution. When illuminated volumetrically by a laser at a wavelength of 532 nm (generated by a pulsed Nd: YAG and expanded by a series of lenses as shown), the dye solution absorbs the illumination laser as it propagates through the solution and emits LIF photons, which were captured by a total of 5 cameras (4 Photron SA4s and 1 SA6) from different orientations. To facilitate the following discussion, a right-handed Cartesian coordinate system was defined as shown in the figure: the origin was defined as the center point of the cubical cell, the X axis was defined along the propagation direction of the illumination laser (which propagated perpendicularly into the dye cell), and the Z axis was defined to be out of the paper as shown in Figure 4-2a.

In the experiment, all 5 cameras were aligned in the X-Y plane and therefore their orientations were completely specified by θ, defined as the angle formed by the optical axis of a given camera relative to the positive X direction as shown. The focal length and f-number of the lenses used on all cameras were 105 mm and 2.8, respectively. Each lens was equipped with a 532 ± 10 nm OD4 notch filter to block any interference due to the scattering or reflection of the illumination laser. Prior to any measurement, a view registration program using a calibration target was used to determine the orientation of each camera [79, 100]. The orientations of camera 1 through 5 were determined to be θ = 270.0°, 311.8°, 341.9°, 73.7° and 111.1°, respectively, with an accuracy estimated to be within 0.6°.

From the results shown in Chapter 3, the tomography problem involved in the above experimental setup is a nonlinear problem due to the non-negligible absorption of the
illumination laser by the dye solution. The goal in these experiments was to reconstruct the dye concentration distribution from its LIF signal projections from different orientations. Figure 4-2b shows an example projection of the illuminated cell captured by camera 5. The LIF projection seen in Figure 4-2b is not uniform because of the non-uniform spatial distribution and the absorption of the illuminating laser. Such non-uniformity was characterized beforehand in the experiments as described in chapter 3, and accounted for in the tomography algorithm.

Based on the measured projections, both the NIRT algorithm and IRT-VRO algorithms were applied to solve the problem and obtain the concentration distribution of the dye in the solution (again, which is known to be uniform). Thus, the goal of this validation experiment was to quantitatively compare the reconstruction quality (i.e., defined as the difference between the reconstructed distribution of the dye concentration to the uniform distribution known \textit{a priori}) between a past algorithm and the new IRT-VRO algorithm.

![Figure 4-3 Panels (a): reconstruction obtained by NIRT. Panel (b): reconstruction obtained by IRT-VRO.](image-url)
Figure 4-3a shows the reconstruction results by using the NIRT algorithm.

Figure 4-3a shows the concentration distribution at three Y locations (Y = -10 mm, Y = 0 mm and Y = 10 mm) at the same Z plane (Z = 20 mm). As seen, the NIRT algorithm successfully reconstructed the overall uniform concentration distribution. To quantify the reconstruction accuracy, an average reconstruction error \( e_R \) (defined as the overall difference between the reconstructed distribution and the correct answer as in previous chapters) was calculated to be 3.99 %, 4.2 % and 4.06 %, respectively along the three locations shown here. Two major factors causing such reconstruction error included the reconstruction algorithm and measurement uncertainties. First, according to our numerical simulations, the reconstruction algorithm was able to solving the problem within 0.1% accuracy given noise-free projections. Second, measurement uncertainties primarily
included the uncertainty with the camera (such as background noises, camera nonlinearity and/or nonuniformity et al.), the shot-to-shot variation of the laser system, and the accuracy of the view registration process. Among these measurement uncertainties, the camera uncertainty was estimated to be about 1%, and the shot-to-shot variation of the laser system was estimated to be about 1.1%. These uncertainties cannot be reduced without investment in better equipment. The uncertainty introduced by the view registration process, on the other hand, can be reduced by the new IRT-VRO algorithm.

Therefore, the IRT-VRO algorithm was applied and demonstrated to reduce the impact from the VR uncertainty. The same projections were used in the IRT-VRO algorithm to reconstruct the dye concentration distribution.

Figure 4-3b shows the reconstruction results obtained by the IRT-VRO algorithm using the same projections as used in the NIRT algorithm. To be directly comparable with the results of NIRT, Figure 4-3b also shows the results at the same Y, Z locations (i.e., Y = -10 mm, Y = 0 mm and Y = 10 mm; Z = 20 mm). The reconstruction errors at these three locations were
2.87%, 3.14% and 2.97%, respectively (compared to 3.99 %, 4.2 % and 4.06 % for the results obtained from the NIRT algorithm). Therefore, the IRT-VRO algorithm was demonstrated to outperform the NIRT algorithm with a ~30% improvement of the reconstruction accuracy. However, as mentioned earlier, such improved accuracy was obtained at the cost of additional computational cost. And the computation time required by the IRT-VRO method here was about 50x more than that required by the NIRT algorithm.

The IRT-VRO algorithm also determined the optimized angle set $\theta_{\text{opt}}$ was found to be: 271.13°, 311.60°, 341.12°, 73.81° and 111.72°, respectively. The changes between the optimized angle set $\theta_{\text{opt}}$ with the initial angle set $\theta$ from the view registration were: 1.13°, -0.20°, -0.78°, 0.11° and 0.62°, respectively, illustrating the ability of the new algorithm to optimize camera orientations as an integral part of the tomographic reconstruction.

![Figure 4-4](image)

Figure 4-4 Panels (a): reconstruction obtained by NIRT. Panel (b): reconstruction obtained by IRT-VRO.

Numerical validation was also performed in parallel with the above experiments, and such numerical validation provided further insights to the improvement in reconstruction accuracy (as reflected in the $e_R$ calculated above). For the numerical validation, a uniform phantom was created to simulate the distribution of the dye in the cell. Projections from the same 5 orientations as used in the experiments were calculated. To simulate the experimental uncertainties, a total of 4% of noise was artificially added to the projections. More specifically, the uncertainties with the camera was estimated to be about 1%, the shot-to-shot variation of the laser system was estimated to be about 1.1%, and the accuracy of the view registration was estimated to be within 0.6°, and such an uncertainties translated
into about 1.6% uncertainties in the projection. The overall uncertainty from all three sources was therefore estimated to be around 4%. Gaussian noise was added to the numerical projection data to simulate the background noise and the shot-to-shot variation. As for the view registration noise, the noisy projections were simulated using the angle sets with measurement uncertainties (which was set to be 0.6°). Based on the simulated projections, reconstructions were performed by both the NIRT and IRT-VRO algorithms and the results summarized in

**Figure 4-4.**

Figure 4-4a shows the reconstructed concentration obtained by NIRT along three locations in comparison to the phantom, with $e_R$ calculated to be 4.01%, 4.03% and 4.53%, respectively.
Figure 4-4b shows the results obtained by IRT-VRO along the same three locations, with $e_R$ calculated to be 3.54%, 2.5% and 3.59%, respectively. These results were in good agreement with those obtained experimentally, supporting both the superiority of the IRT-VRO algorithm and the analysis of the experimental uncertainty.

Figure 4-5 Panel (a): comparison of $\Delta \theta$ before and after applying the IRT-VRO algorithm. Panel (b): comparison of the convergence curve of NIRT and IRT-VRO.
Figure 4-5 shows further comparison between the NIRT and IRT-VRO algorithms to elucidate the advantage of the IRT-VRO algorithm in terms of the reconstruction accuracy. First, Figure 4-5a shows the comparison between the initial angle uncertainties ($\Delta \theta$) fed into the NIRT and the IRT-VRO algorithm. In Figure 4-5a, the black line represents the initial $\Delta \theta$, which equals either 0.6° or -0.6°, corresponding to the uncertainty in the VR process. The red line represents the $\Delta \theta$ corresponding to the camera orientations optimized by the IRT-VRO algorithm. As seen, the angle uncertainty $\Delta \theta$ was significantly reduced by the IRT-VRO algorithm, from ±0.6° to 0.37°, 0°, 0.06°, -0.61° and 0°, for
cameras 1 to 5, respectively.

Figure 4-5b further compares the convergence curves of NIRT (black-solid) and IRT-VRO (red-dash). As seen, $e_R$ does not decrease monotonically in the NIRT algorithm due to the existence of the noise and was shown to be 4.2% at the end of the 50th iteration. In contrast, $e_R$ decreases monotonically in the IRT-VRO algorithm and converges to 2.61% at the 39th iteration. The black-dashed line shown in Figure 4-5b represents the $e_R$ calculated without considering the view registration noise while other uncertainties sources remained unchanged, and therefore the result of the black-dashed line represents the theoretically best value of $e_R$ that could be achieved with a perfect VR process. As seen, even though the IRT-VRO algorithm did not converge to the best optimization, it reached a level closer to the black-dashed line than NIRT, resulting in a 37.8% improvement of the reconstruction accuracy. The comparison of the
convergence curves further illustrates the superiority of IRT-VRO in processing 3D reconstruction with measurement uncertainties in the VR process.

Figure 4-6 Panel (a): the effect of $\Delta\theta$ on the measurement error. Panel (b): reconstruction error obtained by NIRT and IRT-VRO under various levels of $e_{VR}$.

After comparing the NIRT and IRT-VRO algorithms with the fixed view registration uncertainty (i.e., $\Delta\theta=\pm0.6^\circ$),
Figure 4-6 further compares the NIRT and IRT-VRO algorithms under various levels of view registration uncertainties.

Figure 4-6a first quantifies the effect of the view registration uncertainties on the error of projection measurement based on the cubical cell phantom used in the experiments. The measurement error caused in the view registration procedure, denoted as $e_{VR}$, was defined by:

$$e_{VR} = \frac{\sum |P - P^N|}{\sum P}$$

where $P$ and $P^N$ represent the measured projections without and with view registration noise, respectively. As seen in
Figure 4-6a, $e_{VR}$ increases significantly with the increase of $\Delta \theta$ and equals to 2% when $\Delta \theta$ reaches 1° on the cubical cell phantom. But note that the relationship between $\Delta \theta$ and $e_{VR}$ is expected to be different for phantoms with different structures. In general, the more symmetric the phantom structure is, the less effect of $\Delta \theta$ on $e_{VR}$ will be. As a simple example, for a phantom with perfectly symmetric cone shape, $\Delta \theta$ does not cause any measurement error on projections. Therefore, the impact of VR error is expected to be more significant on practical turbulent flow patterns than on a simple uniform or symmetric distribution, and the enhancement of reconstruction quality brought about by the IRT-VRO is also expected to be more dramatic.

Figure 4-6b shows the $e_R$ obtained under various levels of $e_{VR}$. These results suggest several key observations. First, $e_R$ increases significantly as $e_{VR}$ increases both with the NIRT and IRT-VRO results, illustrating the effects of the VR error on the reconstruction accuracy and that neither method was able to completely remove such effects. Second, the IRT-VRO algorithm outperforms NIRT under each level of $e_{VR}$ and the advantage of IRT-VRO becomes more obvious when $e_{VR}$ becomes larger, showing the ability and superiority of the IRT-VRO algorithm to address VR uncertainties.
After the above experimental and numerical validations on the controlled cubical cell, the IRT-VRO algorithm was also validated on turbulent flows. The experimental setup was shown in Figure 4-7, with two major differences compared to Figure 4-2a. The first major difference was that the controlled cell was replaced by a turbulent nitrogen (N\textsubscript{2}) jet flow seeded with I\textsubscript{2} vapor as shown. The exit diameter of the jet was 6.35 mm. The I\textsubscript{2} vapor was introduced into the flow by heating solid iodine crystals in a water bath to a temperature of 200 °F. The mole concentration of the I\textsubscript{2} vapor in the target flow was estimated to be 4%. A rod (with a diameter of 3.18 mm) was placed at the exit of the jet, to increase the complexity of the flow structures (and also to create an easily recognizable pattern of the flow to facilitate the discussion of the results). The second difference was that a second laser (label as the PLIF laser) was used to generate a simultaneous PLIF measurement for the validation purpose. More specifically, the PLIF measurements provided a 2D image at a certain cross section of the 3D flow, which can be compared against the 3D reconstruction.
at the same cross section. The PLIF laser (Photonics Industries DM20 - 527 DH) generated laser pulses at a wavelength of 527 nm, with a duration of 440 ns, a pulse energy of 12 mJ, and a repetition rate of 5 kHz. The PLIF laser pulses were focused into sheets with a thickness less than 0.8 mm. The PLIF laser was aligned perpendicular to the optical axis of camera 1. To best utilize the cameras available to this project, the VLIF laser, PLIF laser, and camera 1 were configured in such a way that camera 1 captured the VLIF and PLIF signal sequentially in two consecutive frames, and the temporal separation between these two frames was 0.2 ms. As a result, even though both the VLIF and PLIF techniques were able to generate single-shot measurements and therefore can be applied to highly turbulent flows, experiments performed in this work were performed under moderate turbulence levels (with a Reynolds number of 2,000 defined based on the jet exit diameter). Such that the flow can be approximately considered frozen during the 0.2 ms separation to make the PLIF and VLIF measurements comparable.

Figure 4-8 Panel (a): a set of example VLIF projections captured by camera 1 through 5 and the corresponding PLIF measurement captured by camera 1. All projections display

Figure 4-8a shows a set of example VLIF projections captured by camera 1 through 5 and the corresponding PLIF measurement captured by camera 1. All projections display
an incompletely developed jet flow with two branches generated by the rod placed at the exit of the jet. As mentioned earlier, the VLIF projections were measured 0.2 ms after the PLIF image, and this delay was negligible for this flow (with a Reynolds number of 2,000 defined based on the jet exit diameter) so that the PLIF and VLIF measurements were considered to be simultaneous. All projections had a resolution of 600 × 600 pixels, and each pixel corresponded to a physical dimension of 0.06 × 0.06 mm. Both the PLIF image and VLIF projections were single-shot measurements. Note that the PLIF image in Figure 4-8a appeared clearer and sharper than the VLIF projection because the VLIF projections were line-of-sight integrated.

The VLIF projections shown in Figure 4-8a were used to reconstruct the 3D distribution of the I\(_2\) concentration by using NIRT and IRT-VRO algorithms, and the results were compared to the PLIF measurement. Figure 4-8b shows a 3D rendering of the relative I\(_2\) concentration obtained by the IRT-VRO algorithm, illustrating that the reconstruction captured the overall V-shape of the flow. Figure 4-9 shows the comparisons between the results of NIRT and IRT-VRO with PLIF. Figure 4-9a and b show the central slice (the same location as the PLIF measurement) of the reconstructions by NIRT and IRT-VRO, respectively. As seen, both algorithms reconstruct the flow structure in agreement with PLIF. Figure 4-9c and d quantify the comparisons of NIRT and IRT-VRO with PLIF at three Z locations (Z = 10 mm, 15 mm and 18 mm). It can be seen that \(e_R\) was shown to be 11.79%, 12.87% and 11.90% at three locations for NIRT while it was 8.71%, 8.40% and 5.65% for IRT-VRO, suggesting that the reconstruction error \(e_R\) was larger in the turbulent flow measurement than that in the uniform cell measurement and IRT-VRO outperforms NIRT on the turbulent flow case by improving a 41% of \(e_R\).
Figure 4.9 Panel (a): reconstruction of I2 flow by NIRT. Panel (b) reconstruction of I2 flow by IRT-VRO. Panel (c): quantitatively comparison of VLIF by NIRT with PLIF at three different Z locations (Z = 18, 15 and 10 mm). Panel (d) quantitatively comparison of VLIF by IRT-VRO with PLIF at three different Z locations (Z = 18, 15 and 10 mm).

Similar to the analysis in the cubical cell,
Figure 4-10 studies the performance of the IRT-VRO under various view registration conditions. First,

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Numerical results from NIRT and IRT-VRO under different view registration uncertainties.}
\end{figure}

Figure 4-10a shows the effect of the view registration uncertainty ($\Delta\theta$) on $e_{VR}$, in which a nearly quadratic relationship was observed. By comparing the results with those obtained in cubical cell case (the hollow points), $e_{VR}$ caused by the same $\Delta\theta$ is larger on the turbulent case than the cubical cell case, suggesting that the impact of $\Delta\theta$ on $e_{VR}$ increases as the complexity of the distribution increases.

Figure 4-10b shows the $e_R$ obtained under various $e_{VR}$. Similar conclusions can be drawn that IRT-VRO outperforms NIRT under each level of view registration uncertainty, and the advantage of IRT-VRO becomes more obvious when $e_{VR}$ becomes larger. For example, the IRT-VRO improved $\sim 50\%$ of the reconstruction accuracy (by decreasing $e_R$ from 12\% to 6.2\%) when $\Delta\theta = \pm 1^\circ$; but only improved $\sim 29\%$ of the reconstruction accuracy (by decreasing $e_R$ from 5.9\% to 4.2\%) when $\Delta\theta = \pm 0.3^\circ$. 

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4.4. Summary

In summary, this chapter reports the development and validation of an improved reconstruction algorithm that considers the view registration optimization and tomographic reconstruction holistically. The method can be applied to both linear and nonlinear tomography problems, by combining to either a linear iterative algorithm or the NIRT algorithm. Existing reconstruction algorithms considered the view registration and tomographic reconstruction separately, and therefore can only reach a locally optimized solution to the problem. The IRT-VRO algorithm was developed to consider them holistically and thereby improve the reconstruction accuracy. The algorithm was validated both numerically and experimentally using both controlled cell measurements and turbulent flows, respectively. The results shown that the new algorithm was able to correct the view registration error and significantly enhance the reconstruction accuracy.
Chapter 5  Conclusions and future work

This chapter summarizes the principal findings and conclusions derived from this dissertation, and provides some recommendations for future work.

5.1 Conclusions

In summary, this dissertation introduced three reconstruction algorithms for 3D tomography diagnostics. These algorithms were developed to address technical issues encountered in practical tomography problems, including computation cost, limited view angles available, nonlinearity, and the measurement uncertainty.

More specifically, chapter 2 focused on the development of linear reconstruction algorithm. A new algorithm, the ART screening, is developed, demonstrated, and validated under the context of 3D measurement of CH radicals using volumetric laser-induced fluorescence (VLIF). The ART algorithm exploits the fact that the target species is not present everywhere in the measurement domain. Therefore, a screening can be performed to first quickly identify the regions where the target species is present, and only solve the tomography problem in these regions, thus resulting in improved computational efficiency, accuracy, or spatial resolution. The algorithm was validated both numerically and then experimentally on laminar flames, and then finally demonstrated on turbulent flames. The results show that the ART screening algorithm enabled single-shot VLIF measurements with sufficient accuracy in a sizable volume (a volume of 9.3×9.3×32.7 mm³ was demonstrated in this work). With a total of five intensified cameras, 3D measurements were demonstrated over a total of ~10⁶ voxels with a voxel size of 0.15 mm. Comparison with PLIF measurements suggested that the spatial resolution of the ART screening algorithm
based VLIF technique to characterize the flame surface was about 0.4 mm. Though such spatial resolution cannot compete with that of PLIF (on the order of 0.05 mm/pixel in this work), the advantage of VLIF is clear: it offers more imaging elements than PLIF and these images are offered in 3D and it offers more than image elements than established ART algorithm.

Chapter 3 reported the development and validation of a new reconstruction algorithm (NITR) for nonlinear tomography problems. The NIRT algorithm solves the nonlinear problem iteratively in a manner analogous to the solution of linear tomography problem using ART. The NIRT algorithm is demonstrated to solve nonlinear tomography problems while retaining the advantages of ART for linear problems. For example, the NIRT algorithm was able to solve nonlinear problem with limited number of projections, with robustness in the presence of uncertain projections, and with good computational efficiency. The chapter detailed the mathematical formulation of the NIRT algorithm, and reported its numerical and experimental validation in both controlled cell measurements and flow measurements.

Chapter 4 developed an improved iterative reconstruction algorithm aiming to reduce the impact of view registration uncertainty on the reconstruction fidelity (this algorithm is code named IRT-VRO, iterative reconstruction technique integrating view registration optimization). This algorithm integrates the view angles with measurement uncertainty as variables instead of presetting them as constants in the tomography algorithm, so that the view registration error can be corrected and optimized during the reconstruction iteration. The IRT-VRO algorithm was validated numerically and experimentally using both controlled cell measurements and in flow measurements. The results showed that the IRT-
VRO algorithm outperformed existing algorithms in each tested case, and that the advantage of the IRT-VRO became more significant when the view registration uncertainty became larger or when the target flow became more complicated.

After the above numerical and experimental validation, we expected to see expanded applications of these algorithms in practical flow and combustion diagnostics.

5.2 Future work

Future work is suggested in two major aspects: 1) evaluate the capabilities and limitations of those algorithms on the diagnostics of highly turbulent flows; 2) further improve the algorithms to address other challenges in practical measurements, such as the issue of finite focal-depth typically encountered in volumetric measurements.

First, in this work, validation and evaluation of the reconstruction algorithms have been studied primarily through the use of controlled cells, laminar flows, or moderately turbulent flows. Given the potential impact of 3D measurements in the study of turbulent flows and flames, it is desired to experimentally and directly validate the reconstruction algorithms and quantify its accuracy in practical situations with high turbulence. One of our ongoing work involves such evaluation of the ART screening algorithm on the 3D measurement of highly turbulent flames. The experimental setup and the preliminary tests were shown in Figure 5-1 and Figure 5-2. The goal of the experiment is to direct compare the 2D and 3D LIF applied to highly turbulent flames to illustrate the capabilities and limitations of VLIF obtained by ART screening algorithm. To accomplish these goals, planar LIF (PLIF) and VLIF measurements were simultaneously performed on turbulent flames based on the CH radical. As seen from Figure 5-1, the PLIF measurements imaged a planar cross-section of the target flames across a 2D field-of-view (FOV) of 50×50 mm.
The VLIF measurements imaged the same region in the target flame with a 3D FOV of 50×50×5 mm, with 5 mm being the thickness of the measurement volume. The VLIF signals generated in this volume were captured by five intensified cameras from different perspectives, based on which the ART screening algorithm was performed to obtain the 3D reconstruction of the CH radical (as a marker of the flame front). The PLIF measurements were then compared to a cross-section of the VLIF measurement to demonstrate the feasibility and accuracy of the reconstruction in highly turbulent flames. Figure 5-2 shows a set of sample VLIF and PLIF projections of the highly turbulent flames. Figure 5-2a-e shows the measured VLIF projections by cameras 1-5 and Figure 5-2f shows the simultaneous PLIF measurements by camera 6. The 3D reconstruction and further data analysis are ongoing.

Figure 5-1 Schematic of the experimental setup
Second, the issue of finite focal-depth is a common problem encountered in most of tomography measurements [101, 102]. Theoretically, the camera can only focus on a single plane at certain distance, and any object away from this plane would be blurred. This issue would significantly affect the measurement quality and thus the reconstruction accuracy. Therefore, new reconstruction algorithms are suggested to take the issue of finite focal-depth into account and thus to de-blur the out of focus images.
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