HEAT TRANSFER FROM A FINNED PIN TO THE AMBIENT AIR

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1. INTRODUCTION

In view of the wide industrial applications of extended surfaces, the study and development of geometrical configurations of extended surfaces were emphasized.

A variety of extended surfaces has been developed to increase the heat transfer rate from a primary surface. Among these are, in general, fins of various shapes and small rods or pins.

Many types of electrical apparatus, heat exchangers, radiators, air conditioners and air-cooled internal combustion engines are equipped with fins to increase the area of heat exchange surface for the dissipation of excess heat. Thus, extensive investigations employing fins have been conducted. In recent years pins have been employed in some cases to replace fins in order to increase the rate of heat transfer. However, no information is available on the heat transfer characteristics of a pin and annular fin combination.

The increased surface area of a finned pin for a given volume of material would promote more efficient heat transfer. Therefore, the objective of this thesis was to carry out an investigation of heat transfer characteristics of a finned pin and to conduct an experimental investigation to verify the theory.

The general procedure followed was:

(1) Derivation of heat flux and temperature distribution equations of a finned pin.

(2) Estimation of optimum dimensions.

(3) Calculation of heat transfer.

(4) Experimental measurements of the temperature distribution and the heat-flow rate of both a finned pin and a plain pin having optimum dimensions.

(5) Comparison of heat transfer characteristics of the finned pin with those of the plain pin.

II. REVIEW OF LITERATURE

The geometrical configurations of extended surfaces play an important role in the basic heat transfer characteristics. To date many investigations on the surface configurations have been conducted. Most of them have been conducted employing various shapes of fins. A few of them have been conducted employing pins. For example, Kays (1), (2), (3), who sponsored a research program for the investigation of compact surfaces, had suggested 94 surface configurations in his papers before 1960.

In the study of heat transfer of extended surfaces many factors that influence heat flow have been explored. This thesis is concerned with the possibility of a new heat exchange surface and only basic heat transfer characteristics will be presented.

Owing to the fact that the configuration of a finned pin is the combination of a pin and annular fins, the basic heat transfer characteristics of a finned pin will be closely related to that of pins and fins.

Therefore, the review of literature will be limited to basic heat transfer characteristics of pins and fins that will be available in the study of a finned pin.

BASIC HEAT TRANSFER EQUATIONS FOR PINS

PINS OF FINITE LENGTH WITH END EFFECT NEGLECTED

The heat transfer equations for a pin of finite length, with heat loss from the free end neglected, or with the free end insulated, were given by Kreith (5) as follows:

Equation of heat flux (heat-flow rate):

$$q_{pin} = \sqrt{phkA}\,(T_0 - T_f)\tanh(mL) = kAm\theta_o \tanh(mL) \qquad (2\text{-}1)$$

Equation of temperature distribution:

$$\frac{T-T_f}{T_o-T_f} = \frac{\theta}{\theta_o} = \frac{\cosh m(L-x)}{\cosh mL} \qquad (2\text{-}2)$$

Where q: Rate of heat flow from the pin to fluid (Btu/hr)

P: Perimeter of the pin (ft)

h: Average convective heat transfer coefficient ($\text{Btu/hr ft}^2\text{F}$)

k: Thermal conductivity of the pin (Btu/hr ft F)

A: Cross-sectional area of the pin (ft^2)

T_o: Root Temperature of the pin (F)

T_f: Temperature of the ambient fluid (F)

m: $m = \sqrt{\frac{hP}{kA}}$

L: Length of the pin (ft)

T: Local temperature (F)

$\theta_o = T_o - T_f$

$\theta = T - T_f$

Pins of Finite Length with End Convection

For the pin of finite length, with heat convection at the free end, the heat transfer equations were given by Kreith (5) as follows:

Equation of heat flux:

$$q = KAM\theta_o \frac{\sinh mL + H \cosh mL}{\cosh mL + H \sinh mL} \qquad (2\text{-}3)$$

Equation of temperature distribution:

$$\frac{\theta}{\theta_o} = \frac{\cosh m(L-x) + H \sinh m(L-x)}{\cosh mL + H \sinh mL} \qquad (2\text{-}4)$$

where $\theta = T - T_f$

$\theta_o = T_o - T_f$

h_e = Heat transfer coefficient at the free end of pin (Btu/hr ft^2F)

$H = h_e/mk$

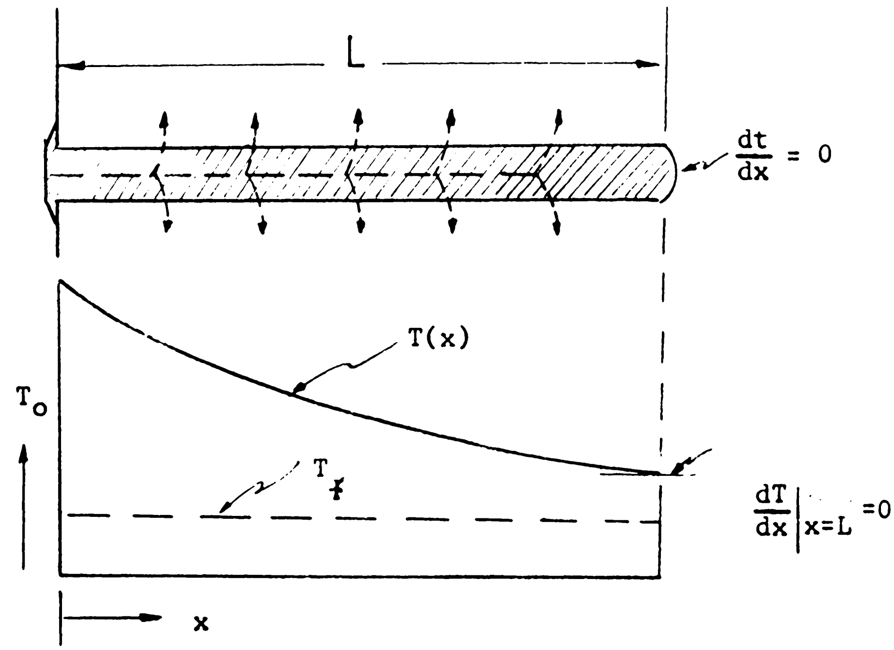
Figure 1 illustrates schematically the temperature distribution for the pins.

Pins of infinite length will not be used in this thesis because they will not be required for finned pins.

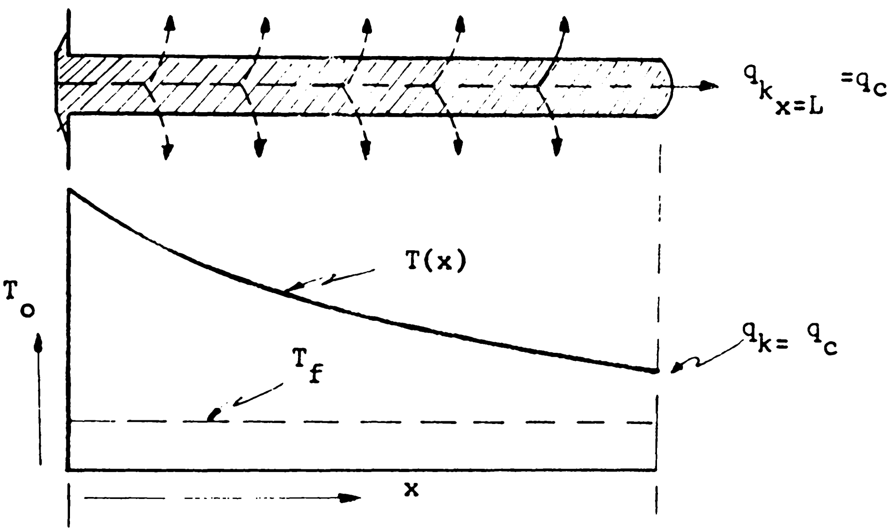
EFFICIENCY AND EFFECTIVENESS

Efficiency of Pins and Straight Rectangular Fins

Schneider (6) defined the fin efficiency as "The ratio of the total heat dissipated by the fin to that which would be dissipated if the entire fin surface were at t_o," where t_o is the root temperature of the fin.



a. The free end of a pin is insulated or heat loss at the free end neglected.



b: The free end of a pin dissipates heat to the ambient fluid.

Figure 1: Schematic sketch of temperature distribution for a pin of finite length. (4) (5)

Schneider also gave the definition in equation form:

$$e = \frac{q}{q_o} = \frac{1}{T_o hS} \int_o^S hT\, dS = \frac{1}{T_o S} \int_o^S T dS, \text{ if } h \text{ is} \qquad (2\text{-}5a)$$

constant

where e = Fin efficiency

$T_o = t_o - t_f = \theta_o$

= Temperature difference if the entire surface were at temperature t_o (F)

S = Surface area (ft^2) = PL

ds = pdx, where p = perimeter

$T = t - t_f = \theta$

= Local temperature difference at the fin surface (F)

The equations for determining the fin efficiency are given in the following forms by integrating Equation 2-5a, changing ds = pdx,

$$T = \frac{T_o \cosh m\,(L-x)}{\cosh mL}$$

For pins and straight rectangular fins with end effects neglected(6)

$$e = \frac{\tanh\ nL}{nL} \qquad (2\text{-}5b)$$

For pins and straight rectangular fins with corrected fin lengths (12)

$$e = \frac{\tanh nL_c}{nL_c} \qquad (2\text{-}6)$$

where e = Fin efficiency

L = Fin length (ft)

L_c = Corrected length of the straight rectangular fin or pin of which the end effect is to be compensated by the corrected length (ft).

$$n = m = \sqrt{\frac{hp}{kA}}$$

Effectiveness of Pins

Effectiveness of pins is defined as the ratio of heat dissipation from a pin to that from the root area without pins, i.e.,

$$E = \frac{q}{q'} = \frac{kAm\theta_o}{Ah\theta_o}\ \frac{\sinh mL + H \cosh mL}{\cosh mL + H \sinh mL}$$

SIMPLIFIED EQUATIONS

Pins with Corrected Length

The simplified heat transfer equations for straight rectangular fins with heat convection at the free end, given by Harper and Brown (12), were:

$$q = kAm\theta_o \tanh mL_c \tag{2-7}$$

$$\frac{\theta}{\theta_o} = \frac{\cosh m\,(L_c - x)}{\cosh mL_c} \qquad L_c = \text{corrected length)} \tag{2-8}$$

These equations are analogous to Equations 2-1 and 2-2.
The heat flux equation for pins was furthermore simplified by introducing the pin efficiency (4)

$$q_{pin} = \theta_o hSe \tag{2-9}$$

where $\theta_o = T_o - T_f$

= Temperature difference across the film at the root of the pin (F)

S = Surface area of the pin in contact with the ambient fluid(ft).

Pins with End Effect Neglected

$$q_{pin} = \sqrt{hpkA}\,(T_o - T_f)\tanh mL \qquad (2\text{-}1)$$

$$= \theta_o \sqrt{hpkA}\, mL \,\frac{\tanh mL}{mL}$$

$$= \theta_o \sqrt{hpkA}\sqrt{\frac{hp}{kA}}\; Le$$

$$= \theta_o\, hpLe = \theta_o hSe$$

where $m = \sqrt{hp/kA}$

$$e = \frac{\tanh mL}{mL}\;; \quad e = \frac{\tanh mL_c}{mL_c}, \text{ for Equation 2-7,} \qquad (2\text{-}5)$$

$S = pL$; $S = pL_c$, for Equation 2-7

From Equation 2-9 it is evident that the heat dissipation from a pin is directly proportional to the temperature difference θ_o , the convective heat transfer coefficient h, the surface area S and the pin efficiency e. Usually θ_o is given for general practice, the terms h, S and e will be the factors that influence heat flow.

Effect of Surface Area on Heat Transfer

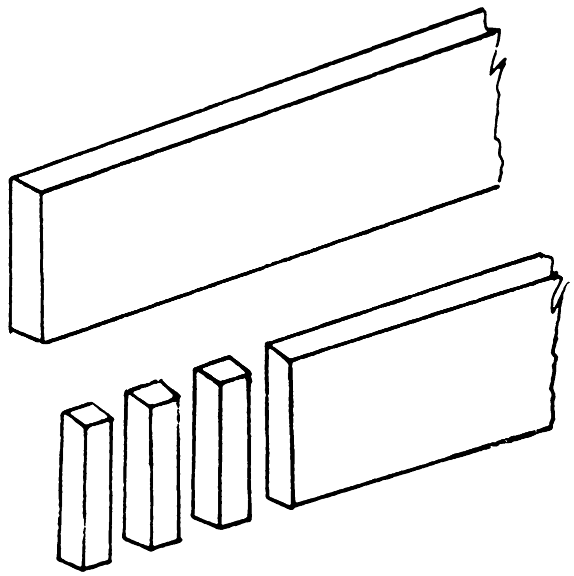
By Equation 2-9, it is seen that a larger surface area will dissipate more heat from a pin to the ambient fluid. However, the surface area is limited by the space available within the heat exchange equipment. Attention has, therefore, been concentrated on the selection of the geometry of surface configurations.

"It is interesting to note that pins will furnish more heat transfer area for equal total cross-section. This is easily visualized by cutting a horizontal rectangular fin into square fins, which doubles the surface area with no increase in cross-section." (Fig. 2-A) (4).

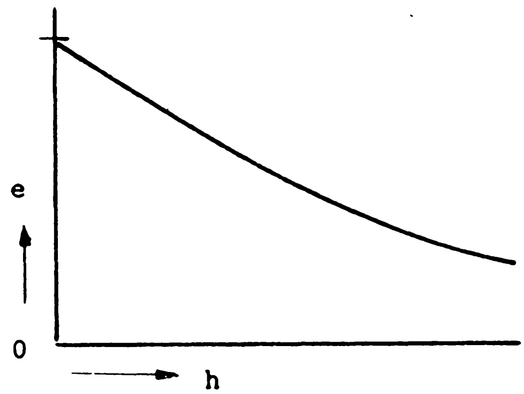
Effect of Pin Efficiency on Heat Transfer:

The pin efficiency must be involved in the simplified heat flux equation because the temperature is not uniform along the entire pin. Heat from the primary surface enters the pin and is dissipated continuously along the length of the pin by transfer to the ambient fluid. The inter-relations between efficiency and heat transfer coefficient are shown on Figure 2-b. It is seen that as the heat transfer coefficient increases the pin efficiency decreases.

"In essence the efficiency factor is a measure of the ability of the pin to transport heat to the dissipating surface." (4)



a. Effective area is doubled by cutting the rectangular fin into pins of square profile (4).



b. Relation between fin efficiency and convective heat transfer coefficient (4).

Fig. 2

<u>Effect of Convective Heat Transfer Coefficient on Heat Transfer</u>

According to the simplified heat flux equation, $q = \Delta T h S e$, it seems that the larger the heat transfer coefficient the greater the heat flux. Actually this is not true because the heat transfer coefficient h is also involved in the efficiency equation.

For a pin,
$$e = \frac{\tanh mL}{mL}$$
$$= \frac{\tanh(\sqrt{ph/kA}\,L)}{\sqrt{ph/kA}\,L}$$
$$= \frac{\tanh(\sqrt{4h/kD}\,L)}{\sqrt{4h/kD}\,L}$$

"The efficiency function is useful for predicting the influence of changes in the various parameters involved, namely, the unit surface conductance h, the conductivity k and the fin dimensions,"(6).

For a pin, a small h and L and a large k and D will increase the pin efficiency, but the usual practice is to use many pins of a small diameter. This will be explained further in the discussion of optimum dimensions.

EMPIRICAL EQUATIONS FOR THE CONVECTIVE HEAT TRANSFER COEFFICIENTS

The convective heat transfer coefficient that will be used in this thesis is for natural convection. For the estimation of the convective heat transfer coefficient of natural convection McAdams(8) recommended the following equations:

For single horizontal cylinder or wire in air at room temperature and atmospheric pressure,

$$Nu = 0.53\ (P_R\ G_R)^{1/4}$$

or,

$$\frac{hD}{k_f} = 0.53\left[\frac{C_p u}{k_f}\ \frac{D^3 \rho^2 \beta g \Delta T}{u^2}\right]^{1/4} \qquad (2\text{-}10)$$

where N_u = Nusselt number

$= hD/k_f$

P_R = Prandlt number

$= c_p\ u/k_f$

G_R = Grashof number

$= D^3 \rho^2 \beta g\ \Delta T/u^2$

D= Diameter of a pin (ft)

u = Viscosity of fluid (lb/hr.ft)

g = Gravitational acceleration (ft/hr^2)

C_p = Specific heat of fluid (Btu/lb_m F)

ρ = Mass density (lb_m/ft^3)

= Coefficient of cubical expansion $= \frac{1}{T_f}$ (1/F)

Simplified forms of Equation 2-10 were given (8) for moderate surface temperatures and air at atmospheric pressure. They are:

$$h = 0.18\ (\Delta T)^{1/3} \qquad (2\text{-}10a)$$

for $10^9 < G_R < 10^{12}$, and

$$h = 0.27\ (\Delta T/D)^{1/4} \qquad (2\text{-}10b)$$

for $10^3 < G_R < 10^9$

where h is in $Btu/hr.ft^2$. F and T is in F.

For vertical plates in air at room temperature and atmospheric pressure, a simplified equation for a plate less than one foot high was also suggested (8). It is

$$h = 0.29\ (\Delta T/L)^{1/4} \qquad (2\text{-}10C)$$

where L is height of a plate.

OPTIMUM CONDITIONS

Optimum Conditions of the Straight Rectangular Fins of Finite Lengths with Heat Convection at the Free End (6)

The heat flux equation

$$q = \sqrt{phkA}(T_o - T_f)\frac{\sinh mL + h \cosh mL}{\cosh mL + H \sinh mL} \qquad (2\text{-}3)$$

can be converted into the following form, by dividing both the numerator and denominator with cosh mL. Thus,

$$q = \sqrt{phkA}(T_o - T_f)\frac{\tanh mL + H}{1 + H \tanh mL} \qquad (2\text{-}11)$$

If the width of the rectangular fin is large compared with the thickness, the perimeter P is approximately equal to 2b, where b denotes the width.

For b = 1 ft, p = 2

$$m = \sqrt{hp/kA} = \sqrt{2h/kt} \qquad (=N \text{ in reference } 6)$$

$$H = h_e/km = \sqrt{ht/2k} \qquad \text{in case } h = h_e$$

Then
$$q = tkm\ \theta_o \frac{\tanh mL + H}{1 + H \tanh mL}$$

or
$$q = 2\delta kN\theta_o \frac{(h/Nk) + \tanh NL}{1 + (h/Nk)\tanh NL} \qquad (\text{Reference } 6)$$

where
$$2\delta = t$$

$$\theta_o = T_o - T_f$$

The above equation suggests that for a fixed δ or t there exists a value of fin length L above which q_{fin} will decrease and below which q_{fin} will increase.

"The limiting condition where it is no longer of advantage to increase the fin length for increasing q_{fin} must, therefore, correspond to the point of vanishing slope, $dq/dL = o$." (6)

Considering t, k, m and θ_o as constant, and differentiating

$$q = tkm\theta_o \frac{\tanh mL + H}{1 + H \tanh mL} \tag{2-13}$$

obtain $\dfrac{dq}{dL} = tkm\theta_o (m \operatorname{sech}^2 m) \dfrac{(1 + H \tanh mL) - (H^2 + H \tanh mL)}{(1 + H \tanh mL)^2}$

Set $\dfrac{dq}{dL} = o$, $\quad H^2 = ht/2k = 1$

which is the limiting condition.

For	$ht/2k < 1$,	$dq/dL =$ positive,	fins have cooling effect on primary surface
	$ht/2k = 1$,	$dq/dL = o$,	no effect on primary surface
	$ht/2k > 1$,	$dq/dL =$ negative,	fins have unfavorable heating effect on primary surface

This analysis suggests that h and t should be small, and k be as large as possible.

"It is easily shown that straight rectangular fins are generally advantageous for surface heat-exchange with gases, less effective for forced convection heat-exchange to liquids, and of no advantage for surface condensers and the like. The difference in these three cases show up in the relative magnitude of h. Thus, for a typical 1/8" thick aluminum fin $Nu = 25(0.0625/12)/117 = 0.0011$, which indicates that such a fin should have a pronounced effect, since $Nu << 1$. But for liquids the effect would be much smaller say with $Nu = 0.0011(2500/25) = 0.11$. Finally, for condensing liquids or vapors with h increased ten fold, $Nu = 1.1$, suggests that the primary surface would be more effective without fins at all." (6)

OPTIMUM CONDITIONS OF PINS

The optimum conditions of pins can be determined by the same method as for straight rectangular fins.

OPTIMUM DIMENSIONS OF STRAIGHT RECTANGULAR FINS

Optimum dimensions are the required thickness and length of a rectangular fin for a given amount of material to give the best heat transfer.

For a straight rectangular fin, of which the end effect is to be ignored, and the width (measured along the primary surface) of the fin is given, the profile area is constant for a given amount of material.

$$A_p = tL = \text{profile area} = \text{constant}, \quad L = A_p/t$$

Then

$$q_{fin} = \sqrt{phkA}\,(T_o - T_f)\tanh mL \tag{2-1}$$

$$= \sqrt{2hkt}\,\theta_o \tanh\left(\sqrt{2h/kt}\,L\right) \quad \text{for one foot of width}$$

$$= \sqrt{2hkt}\,\theta_o \tanh\left(A_p\sqrt{2h/kt^3}\right) \tag{2-14}$$

where $m = \sqrt{2h/kt}$, $\theta_o = T_o - T_f$

Differentiating and setting $dq/dt = 0$,

$$0 = dq/dt = \theta_o \sqrt{2hkt}\ \frac{d}{dt}\left(\tanh A_p \sqrt{\frac{2h}{kt^3}}\right) + \left(\tanh A_p \sqrt{\frac{2h}{kt^3}}\right) \frac{d}{dt}\theta_o \sqrt{2hkt}$$

get $\tanh A_p \sqrt{\frac{2h}{kt^3}} = 3A_p \sqrt{\frac{2h}{kt^3}}\ \mathrm{sech}^2\, A_p \sqrt{\frac{2h}{kt^3}}$

Schneider (6) put $A_p \sqrt{\frac{2h}{kt^3}} = \lambda$

$\tanh \lambda = 3\lambda\, \mathrm{sech}^2 \lambda$

Solution by trial and error gives

$$\lambda_{opt} = A_p \sqrt{\frac{2h}{kt^3}} = 1.4192$$

Therefore, $t_{opt} = \left[\frac{2hA_p^2}{(1.4192)^2 k}\right]^{1/3}$ (2-15)

or, $\delta_{opt} = \left[A^2 h/4(1.4192)^2 k\right]^{1/3}$

Where δ = optimum semi-thickness (2-16)

Hence the optimum length becomes

$$L_{opt} = A_p/t_{opt} = A_p/2\delta_{opt} \quad (2\text{-}17)$$

The above two equations were given by Schneider (6).

Usually the designer takes the values of h, k, θ_o and q given, and needs to find the optimum dimensions.

From Equation (2-14), $q_{fin} = \sqrt{2\ hkt}\ \theta_o \ \tanh A_p \sqrt{2h/kt^3}$

Since $\tanh A_p \sqrt{2h/kt^3} = \tanh (1.4192) = 0.889$

$$q_{fin} = 0.889 \sqrt{2hkt}\ \theta_o$$

Optimum thickness (7) , $t_{opt} = \frac{0.6321}{hk} (q/\theta_o)^2$ (2-18)

Equating 2-15 and 2-18

$$\frac{0.6321}{hk}(q/\theta_o)^2 = \left[2hA_p^2/(1.4192)^2 k\right]^{1/3}$$

Optimum profile area (7), $$A_{p_{opt}} = \frac{0.5048}{h^2 k}(q/\theta_o)^3 \qquad (2-19)$$

$A_p = tL$, $$L_{opt} = \frac{0.5048}{h^2 k}(q/\theta_o)^3 \frac{hk}{0.6321}(\theta_o/q)^2$$

Optimum length (7), $$L_{opt} = \frac{0.7979}{h}\left(\frac{q}{\theta_o}\right) \qquad (2-20)$$

From Equation 2-19, " the volume of material is seen to increase as the cube of q . It is therefore necessary to design the fins as small as possible, since to double the heat transfer requires a fin eight times as large instead of two fins of the same size." However there is a limit to spacing the fins closer and closer. "If no interference should be allowed between adjacent boundary layers, the fin spacing distance should be just twice as large as the fully developed boundary-layer thickness"(6)

ANNULAR FINS OF CONSTANT THICKNESS

<u>Basic Equations</u>

The temperature distribution and heat flux equations for an annular fin with heat loss from the outer circumference to be neglected were given by Schneider. (6)

$$\frac{\theta}{\theta_o} = \frac{I_o(Nr)\,K_1(Nr_2) + I_1(Nr_2)K_o(Nr)}{I_o(Nr_1)K_1(Nr_2) + I_1(Nr_2)K_o(Nr_1)} \qquad (2-21)$$

$$q = 2\pi kNtr_1\theta_o \frac{K_1(Nr_1)\,I_1(Nr_2) - K_1(Nr_2)\,I_1(Nr_1)}{K_o(Nr_1)\,I_1(Nr_2) + K_1(Nr_2)\,I_o(Nr_1)} \qquad (2-22)$$

$$= 2\pi kNtr_1\theta_o B$$

Where $N = hp/kA = 2h/kt$

$\theta_o = T_o - T_f$

$$B = \frac{K_1(Nr_1)\,I_1(Nr_2) - K_1(Nr_2)I_1(Nr_1)}{K_o(Nr_1)I_1(Nr_2) + K_1(Nr_2)I_o(Nr_1)}$$

r_1 = Inner radius

r_2 = Outer radius

t = thickness

I_o = Zero-order modified Bessel's function, first kind

K_o = Zero-order modified Bessel's function, second kind

I_1 = First-order modified Bessel's function, first kind

K_1 = First-order modified Bessel's function, second kind

Efficiency

According to definition, the efficiency of an annular fin is:

$$e = q/q_o = \frac{2\pi kNtr_1\theta_o B}{2\pi h(r^2_2 - r^2_1)\theta_o}$$

$$= \frac{2r_1}{N(r^2_2 - r^2_1)}\,\frac{K_1(Nr_1)I_1(Nr_2) - I_1(Nr_1)K_1(Nr_2)}{K_o(Nr_1)I_1(Nr_2) + I_o(Nr_1)K_1(Nr_2)} \qquad (2\text{-}23)$$

Rearrange Equation 2-23. The efficiency of an annular fin is

$$e = \frac{2}{(1 + r_2/r_1)Z}\left[\frac{K_1(R_1Z)I_1(R_2Z) - I_1(R_1Z)\,K_1(R_2Z)}{K_o(R_1Z)I_1(R_2Z) + I_o(R_1Z)\,K_1(R_2Z)}\right] \qquad (2\text{-}24)$$

where A = Design-parameter group

$$= (r_2 - r_1)\sqrt{\frac{2h}{kt}}$$

$$R_1 = \frac{1}{\frac{r_2}{r_1} - 1}$$

$$R_2 = \frac{1}{1- r_1/r_2}$$

$$R_1 Z = \frac{1}{r_2/r_1 - 1}(r_2 - r)\sqrt{2h/kt}$$

$$= r_1\sqrt{2h/kt} = Nr_1$$

$$R_2 Z = \frac{1}{1-r_1/r_2}(r_2-r_1)\sqrt{2h/kt} = Nr_2$$

$$R_2 = \frac{r_2}{r_1} R_1$$

$$\frac{2}{(1+r_2/r_1)Z} = \frac{2r_1}{(r_2+r_1)(r_2-r_1)\sqrt{2h/kt}} = \frac{2r_1}{(r_2^2-r_1^2)\sqrt{2h/kt}}$$

$$= \frac{2r_1}{N(r_2^2-r_1^2)}$$

Equation 2-24 is used for plotting efficiency curves of efficiency versus the design-parameter group, $Z = (r_2-r_1)\sqrt{2h/kt}$.

Efficiency curves of annular fins were plotted in Reference (6), (9) and (13). Gardner (13) and Keller (9) took $(r_2-r_1)\sqrt{2h/kt}$ as a design-parameter group, Schneider (6) took $(r_2-r_1)\sqrt{h/kt}$ as a design parameter group. The curves plotted were for small values of r_2/r_1 for annular fins on tubes having larger diameters. Therefore, efficiency curves will be re-plotted in this thesis for the need of larger r_2/r_1 because of the smaller diameters of pins.

Equations 2-24 is one-dimensional solution for the efficiency of annular fins. Keller and Somers (9) provided a table (table 1) showing the comparison of Gardner's (13) one-dimensional values of efficiency with the two-dimensional values in their paper.

"The one-dimensional solutions previously given for annular fins are accurate for height-to-width ratios of the order of 10 or more." "With height-to-width ratios less than 10 and for annular fins with large curvature, design of fins can be computed with the results presented in this paper." (9)

Effectiveness of an Annular Fin

Murry(17) defined the effectiveness of an annular fin as the ratio of the heat dissipation from the fin to that from the root area without fins, i.e.,

$$E = \frac{q}{q'} = \frac{2\pi kNtr_1\theta_o}{2\pi r_1 th\theta_o} \; \frac{K_1(Nr_1)I_1(Nr_2) - K_1(Nr_2)\,I_1(Nr_1)}{K_o(Nr_1)I_1(Nr_2) + K_1(Nr_2)I_o\,(Nr_1)}$$

$$E = \frac{kN}{h}\left[\frac{K_1(Nr_1)I_1(Nr_2) - K_1(Nr_2)\,I_1(Nr_1)}{K_o(Nr_1)I_1(Nr_2) + K_1(Nr_2)\,I_o(Nr_1)}\right] \qquad (2\text{-}25)$$

Murry (17) suggested that, "For tubes in slowly moving air, investigations carried on in Germany lead one to believe that the addition of the fins has very little influence on the temperature distribution, but that the fin spacing is of great importance. Comparison between theory and experiment shows good agreement until the distance between fins is less than about 1/3 the diameter of the tube. At this point the difference between measured and computed heat transfer appears to be about 12 per cent, with the difference becoming greater as the distance between fins is decreased. As one would expect, the calculated value of the heat transfer is always higher than the measured value. However, with greater distance between the fins, the two values closely approach one another."

$\frac{r_2}{r_1}$	$(r_2-r_1)\left(\frac{2h}{kt}\right)^{1/2}$	Fin Efficiency			
		one dimensional	Two Dimensional		
			$\frac{r_2-r_1}{t}=50$	$\frac{r_2-r_1}{t}=10$	$\frac{r_2-r_1}{t}=\frac{1}{3}$
1.10	0.50	0.920	0.915	0.906	0.685
1.10	2.00	0.470	0.468	0.445	0.150
1.10	5.00	0.195	0.190	0.178	0.038
1.20	0.50	0.920	0.912	0.902	0.665
1.20	2.00	0.465	0.465	0.438	0.140
1.20	5.00	0.190	0.182	0.172	0.035
2.00	0.50	0.900	0.895	0.880	0.600
2.00	2.00	0.390	0.390	0.370	0.126
2.00	5.00	0.150	0.150	0.138	0.032
3.00	0.50	0.875	0.875	0.855	0.560
3.00	2.00	0.340	0.340	0.318	0.110
3.00	5.00	0.120	0.118	0.110	0.027

Table 1. Comparison of one-dimensional and two-dimensional radial cases of annular fins (9)

COMPARISON OF PINS WITH STRAIGHT RECTANGULAR FINS

In addition to the fact that pins furnish more surface area than rectangular fins for a given amount of material, the former are superior in some other respects.

"The heat transfer resulting from air flowing over pins in a transverse manner indicates a continuous function. An average function can only be approximated over a limited Reynold's number range, say 20 to 1, where the function is given by:

$$h = 1.40 \frac{k}{D} (DG/u)^{0.28} (cu/k)^{1/3} \tag{2-26}$$

The heat transfer correlation for staggered pins also indicates no sharp distinction between laminar and turbulent flow. The correlation may be thought of as a gradual transition from near laminar to extreme turbulent flow. The flow across pins creates some degree of turbulence even at very low Reynold's numbers." (Fig. 3a) (4)

For straight fins (4)

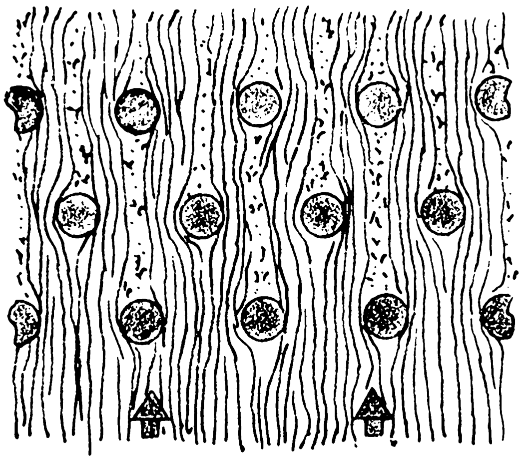
laminar flow (Re=2100) $h = 1.86 \frac{k}{D}\left(\frac{DG}{u}\frac{cu}{k}\frac{D}{L}\right)^{1/3}$ (2-27)

Turbulent flow, $h = 0.027 \frac{k}{D}\left(\frac{DG}{u}\right)^{0.28}\left(\frac{c\,u}{k}\right)^{1/3}$ (2-28)

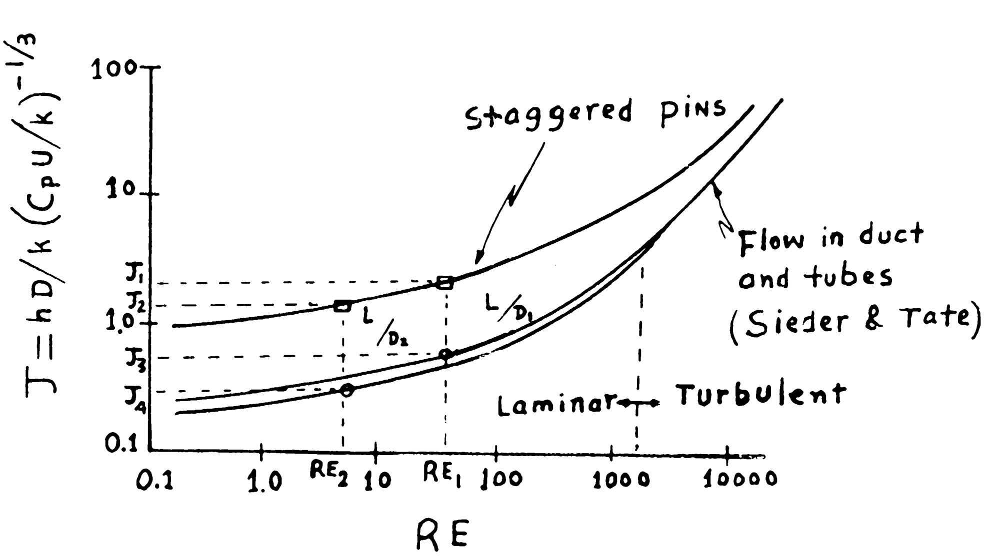
"A heat transfer correlation comparing pins with fins is shown on a logarithmic plot (Fig. 3b)." "The plot indicates that the heat transfer for air flowing over pins(normal to their axes) is higher than that for fin passages at all flow conditions. The difference between the heat transfer capability of pins and fin passages is most striking at very low flow rates and near a Reynold's number of 2000, when the pin can be superior by a factor of 10 to 1."(4)

"The superior heat transfer of pins with respect to fin passages is further improved as the pin diameter becomes smaller. As the pin diameter decreases the heat transfer increases as a result of the factor k/D in Equations 2-26 and 2-27. Countering this heat transfer increase is the factor $\left(\frac{DG}{u}\frac{cu}{k}\frac{D}{L}\right)^{1/3}$ for duct in Equation 2-27, and the factor $\left(\frac{DG}{u}\right)^{0.28}$ for pins in Equation 2-26. It is obvious that a change in D in the factor $\left(\frac{DG}{u}\frac{cu}{k}\frac{D}{L}\right)^{1/3}$ counteracts heat transfer increase more than a similar change of D in the factor $(DG/u)^{0.28}$."

"This phenomenon can be seen pictorially in Fig. 3 where a change from D_1 to D_2 with a corresponding change in Reynold's numbers from Re_1 to Re_2 reflects a counter heat effect from J_1 to J_2 for pins and from J_3 to J_4 for fin passages." (4)



a. Staggered round pin flow pattern



b. Comparison with air flow in tubes

Fig. 3. Air Flow Passing Through Staggered Pins.(4)

III. THEORETICAL INVESTIGATIONS

DERIVATION OF BASIC HEAT TRANSFER EQUATIONS OF A TWO-DISC FINNED PIN

The derivation of mathematical equations will be based on the following assumptions:

1. The temperature over any cross-section of the pin is uniform so that the heat conduction is one dimensional in the solid.
2. The temperature of the ambient fluid is given, constant and uniform.
3. The annular fin is considered very thin. The temperature over any circumferential cross-section is uniform so that the heat conduction is one dimensional. Heat loss from the outer edge to the ambient fluid is neglected.
4. The root temperature of the finned pin is given.
5. The material of the finned pin is isotropic and $k \neq f(t)$ so that the thermal conductivity is constant. Isotropic means k is constant in all directions.
6. The finned pin is of finite length and with heat convection to the ambient fluid over the circumferential surface and at the free end.
7. For simplicity, the heat transfer coefficient over the surface of the finned pin is considered constant, although the coefficient over the pin surface may be different from that over the fin surface.

8. A finned pin may have a number of annular fins on it. In order to facilitate the work of deriving heat transfer equations, a two-disc finned pin was chosen.

Consider pin 7 in Figure 4. The heat flux equation is given by Equation 2-3.

$$q_{7-o} = kAm\theta_{7-o} \frac{\sinh mc + H_1 \cosh mc}{\cosh mc + H_1 \sinh mc}$$

$$q_{7-o} = kAm\theta_{7-o} B_1 \qquad (3-1)$$

Where q_{7-o} = Heat flux from pin 7

C = Length of pin 7

$\theta_{7-o} = T_{7-o} - T_f$

T_{7-o} = Root temperature of pin 7

$m = \sqrt{ph/kA} = \sqrt{4h/kD_1}$

$H_1 = h/mk$ (assume h = he)

$$B_1 = \frac{\sinh mC + H_1 \cosh mc}{\cosh mC + H_1 \sinh mc}$$

By Equation 2-22, the heat flux from fin 6 is

$$q_{6-o} = 2\pi kNtr_1\theta_{6-o} \frac{K_1(Nr_1)I_1(Nr_2) - I_1(Nr_1)K_1(Nr_2)}{K_o(Nr_1)\,I_1(Nr_2) + I_o(Nr_1)K_1(Nr_2)}$$

$$= B_2\theta_{6-o} \qquad (3-2)$$

Where q_{6-o} = heat flux from fin 6

$\theta_{6-o} = T_{6-o} - T_f$

T_{6-o} = Average root temperature of fin 6

$N = \sqrt{Ph/kA} = \sqrt{2h/kt}$

$$B_2 = 2\pi kNtr_1 \frac{K_1(Nr_1)I_1(Nr_2) - I_1(Nr_1)K_1(Nr_2)}{K_o(Nr_1)I_1(Nr_2) + I_o(Nr_1)\,K_1(Nr_2)}$$

where $r_1 = D/2$

$r_2 = OD/2$

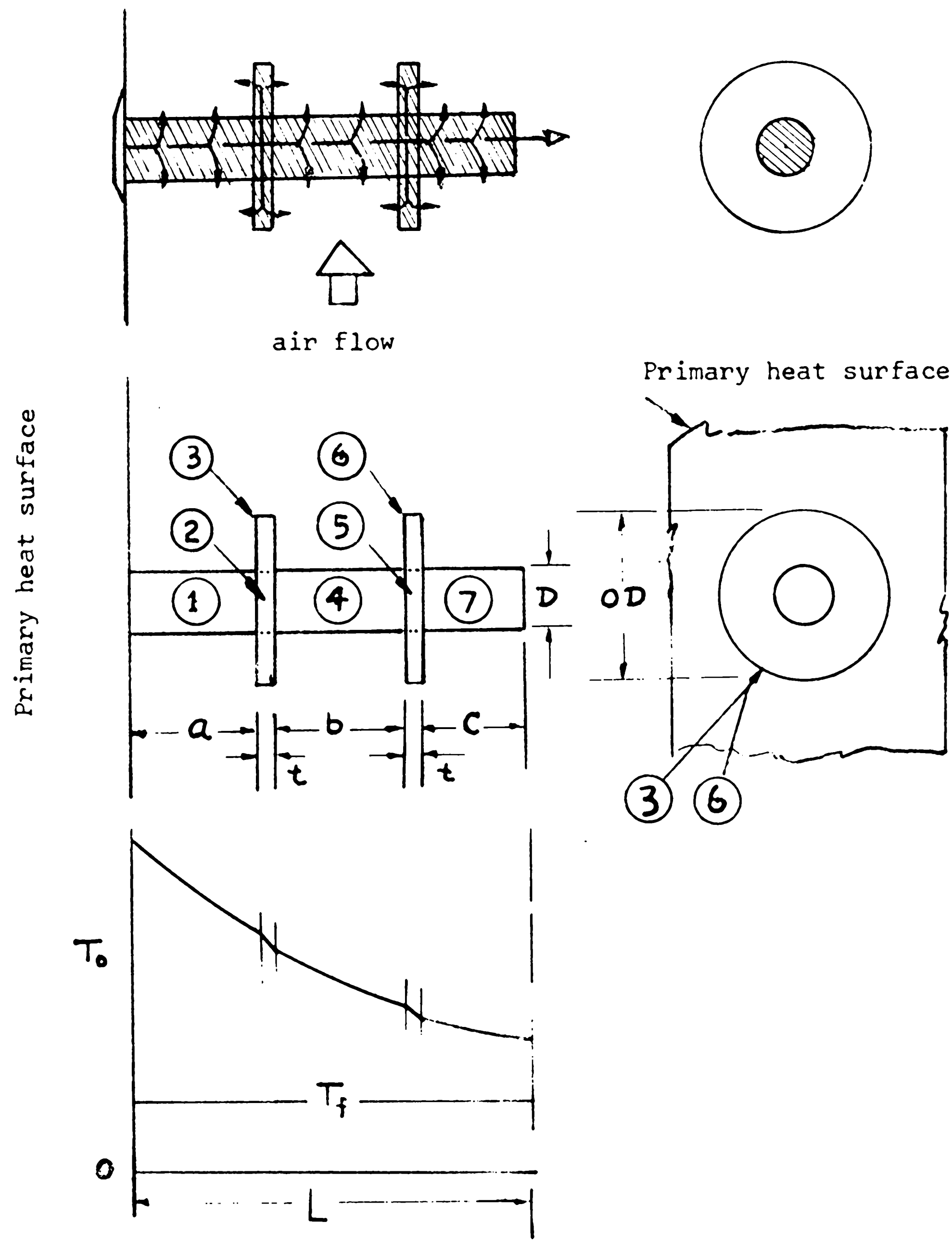


Fig. 4. The Two-disc Finned Pin

For determining the heat flux in pin 5, imagine that if pin 5 were surrounded by a fictitious fluid at the same temperature T_f, and this fictitious fluid would absorb heat equal to the amount q_{6-o} from pin 5, then

$$q_{6-o} = h_i S\, \theta_{6-o}$$

$$= h_i(\pi D_1 t)\theta_{6-o}$$

where h_i = Fictitious heat transfer coefficient

$\theta_{6-o} = T_{6-o} - T_f$

= Average temperature difference at the root of fin 6

t = Length of Pin 5 = thickness of Fin 6.

By Equation 3-2,

$$q_{6-o} = B_2\theta_{6-o}$$

Therefore $B_2\theta_{6-o} = h_i\ (\pi D_1 t)\theta_{6-o}$

$$h_i = B_2/\pi D_1 t$$

Now, pin 5 can be dealt with as if it were an individual pin with the following boundary conditions:

$q_{5-t} = q_{7-o}$, at $x = t$ first boundary condition

$\theta = \theta_{5-o}$, at $x = 0$ second boundary condition

where q_{5-t} is heat flux from the end of pin 5

From Equation 3-1, and the first boundary condition,

$$q_{5-t} = -kA\frac{dT}{dx}\bigg|_{x=t} = kAm\, B_1\theta_{7-o}$$

At $x = t$, $T = T_{5-t}$, $\theta = \theta_{5-t} = \theta_{7-o}$

$$\frac{dT}{dx}\bigg|_{x=t} = \frac{d\theta}{dx}\bigg|_{x=t} = -\frac{kAmB_1\theta_{7-o}}{kA}$$

That is $\frac{d\theta_{5-t}}{dx} = -mB_1\theta_{5-t}$ (3-3)

Since the heat balance for a differential element of pin 5 is

$$-kA\frac{dT}{dx}\Big|_x = -kA\frac{dT}{dx}\Big|_{x+dx} + h_i\ pdx\ (T - T_f)$$

$$-kA\frac{dT}{dx} = -kA\left[\frac{dT}{dx} + \frac{d^2T}{dx^2}dx\right] + h_i\ pdx\ (T-T_f)$$

$$\frac{d^2T}{dx^2} = \frac{h_iP}{kA}(T-T_f)$$

or $$\frac{d^2\theta}{dx^2} - m_i^2\theta = 0, \quad \left(m_i = \sqrt{\frac{h_iP}{kA}} = \sqrt{\frac{4h_i}{kD_1}}\right)$$

The solution of the above differential equation is

$$\theta = c_1 e^{m_ix} + C_2 e^{-m_ix}, \quad \frac{d\theta}{dx} = m_ic_1e^{m_ix} - m_ic_2e^{-m_ix}$$

For Pin 5, at $x = t$, $\theta = \theta_{5-t}$

$$\theta_{5-t} = c_1 e^{m_it} + c_2e^{-m_it}$$

$$\frac{d\theta_{5-t}}{dx} = m_ic_1e^{m_it} - m_1c_2e^{-m_it}$$

Substitute $\theta_{5-t} = c_1e^{m_it} + c_2e^{m_it}$ in Equation 3-3.

$$\frac{d\theta_{5-t}}{dx} = -mB_1(c_1 e^{m_it} + c_2e^{-m_it})$$

Then $m_ic_1e^{-m_it} - m_ic_2e^{-m_it} = -mB_1(c_1e^{m_it} + c_2e^{-m_it})$

$$c_1e^{m_it} - c_2e^{-m_it} = -\frac{mB_1}{m_i}(C_1e^{m_it} + c_2e^{-m_it})$$

$$c_2e^{-m_it} - c_1e^{m_it} = \frac{mB_1}{m_i}(c_1e^{m_it} + c_2e^{-m_it}) \quad (3\text{-}4)$$

The second boundary condition is

at $x = 0$, $\theta = \theta_{5-o}$ for Pin 5

From $\theta = c_1 e^{m_i x} + c_2 e^{-m_i x}$

$$\theta_{5-o} = c_1 + c_2 \text{ , } c_1 = \theta_{5-o} - c_2 \qquad (3\text{-}5)$$

Solving Equation 3-4 and 3-5 simultaneously

$$c_2 e^{-m_i t} - (\theta_{5-o} - c_2) e^{m_i t} = \frac{mB_1}{m_i}\left[(\theta_{5-o} - c_2) e^{m_i t} + C_2 e^{-m_i t} \right]$$

$$c_2 \left[e^{-m_i t} + e^{m_i t} + \frac{mB_1}{m_i} (e^{m_i t} - e^{-m_i t}) \right] = \theta_{5-o} (e^{m_i t} + \frac{mB_1}{m_i} e^{m_i t})$$

$$c_2 = \frac{\theta_{5-0} (e^{m_i t} + \frac{mB_1}{mi} e^{m_i t})}{e^{m_i t} + e^{-m_i t} + \frac{mB_1}{m_i} (e^{m_i t} - e^{-m_i t})}$$

$$c_1 = \theta_{5-o} - C_2$$

$$= \theta_{5-o} \left[1 - \frac{e^{m_i t} + \frac{mB_1}{m_i} e^{m_i t}}{e^{m_i t} + e^{-m_i t} + \frac{mB_1}{m_i} (e^{m_i t} - e^{-m_i t})} \right]$$

$$= \theta_{5-o} \left[\frac{e^{-m_i t} - \frac{mB_1}{m_i} e^{-m_i t}}{e^{m_i t} + e^{-m_i t} + \frac{mB_1}{m_i} (e^{m_i t} - e^{-m_i t})} \right]$$

Substitute C_1 and C_2 in $\theta = c_1 e^{m_i x} + c_2 e^{-m_i x}$

$$\theta = \theta_{5-o} \left[\frac{(e^{-m_i t} - \frac{mB_1}{m_i} e^{-m_i t}) e^{m_i x} + (e^{m_i t} + \frac{mB_1}{m_i} e^{m_i t}) e^{m_i x}}{e^{m_i t} + e^{-m_i t} + \frac{mB_1}{m_i} (e^{m_i t} - e^{-m_i t})} \right]$$

Therefore, the temperature distribution of Pin 5 is

$$\frac{\theta}{\theta_{5-o}} = \frac{\cosh m_i (t-x) + H_2 \sinh m_i (t-x)}{\cosh m_i t + H_2 \sinh m_i t} \qquad (3\text{-}6)$$

Where $H_2 = \frac{m}{m_i} B_1$

For Pin 5 at $x = t$, $\theta = \theta_{5-t} = \theta_{7-o}$

By Equation 3-6

$$\frac{\theta_{5-t}}{\theta_{5-o}} = \frac{\theta_{7-o}}{\theta_{5-o}} = \frac{\cosh m_i(t-t) + H_2 \sinh m_i(t-t)}{\cosh m_i t + H_2 \sinh m_i t}$$

$$= \frac{1}{\cosh m_i t + H_2 \sinh m_i t} \qquad (3\text{-}7)$$

$$\theta_{7-o} = \frac{\theta_{5-o}}{\cosh m_i t + H_2 \sinh m_i t}$$

$$= B_3 \theta_{5-o} \qquad (3\text{-}8)$$

Where $B_3 = \dfrac{1}{\cosh m_i t + H_2 \sinh m_i t}$

By differentiating Equation 3-6

$$\frac{d\theta}{dx} = - m_i \theta_{5-o} \frac{\sinh m_i(t-x) + H_2 \cosh m_i (t-x)}{\cosh m_i t + H_2 \sinh m_i t}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = -m_i \theta_{5-o} \frac{\sinh m_i t + H_2 \cosh m_i t}{\cosh m_i t + H_2 \sinh m_i t}$$

$$q_{5-o} = -kA \left.\frac{d\theta}{dx}\right|_{x=0}$$

$$= kAm_i \theta_{5-o} \frac{\sinh m_i t + H_2 \cosh m_i t}{\cosh m_i t + H_2 \sinh m_i t}$$

$$q_{5-0} = kAm_i \theta_{5-0} B_4$$

Where $B_4 = \dfrac{\sinh m_i t + H_2 \cosh m_i t}{\cosh m_i t + H_2 \sinh m_i t}$ (3-9)

For Pin 4

Let q_{4-b} = heat flux from the end of Pin 4 at x = b

θ_{4-b} = Temperature difference at the end of Pin 4 at x = b

Then $q_{4-b} = q_{5-0}$

$\theta_{4b} = \theta_{5-0}$

By equation 3-9, the heat balance at x = b is

$$-kA \left.\frac{dT}{dx}\right|_{x=b} = kAm_i\, B_4\, \theta_{5-0}$$

At x = b,

$$T = T_{4-b},\ \theta = \theta_{4-b}$$

$$\left.\frac{dT}{dx}\right|_{x=b} = \left.\frac{d\theta}{dx}\right|_{x=b} = \frac{d\theta_{4-b}}{dx}$$

Hence $- kA \dfrac{d\theta_{4-b}}{dx} = kAm_i\, B_4\theta_{5-0}$

$$\frac{d\theta_{4-b}}{dx} = -m_i B_4\theta_{5-0} \tag{3-10}$$

Recall that the heat balance equation for a differential element of a pin is

$$\frac{d^2\theta}{dx^2} - m^2\,\theta = 0$$

and the solution is

$$\theta = c_1\, e^{mx} + c_2\, e^{-mx}$$

At $x = b$

$\theta = \theta_{4-b}$

Then
$$\theta_{4-b} = c_1 e^{mb} + c_2 e^{-mb}$$

$$\frac{d\theta}{dx} = mc_1 e^{mx} - mc_2 e^{-mx}$$

$$\frac{d\theta_{4-b}}{dx} = mc_1 e^{mb} - mc_2 e^{-mb}$$

Substitute $\theta_{5-0} = \theta_{4-b} = c_1 e^{mb} + c_2 e^{-mb}$ in Equation 3-10

$$\frac{d\theta_{4-b}}{dx} = -m_i B_4 (c_1 e^{mb} + c_2 e^{-mb})$$

Hence,
$$mc_1 e^{mb} - mc_2 e^{-mb} = -m_i B_4 (c_1 e^{mb} + c_2 e^{-mb})$$

$$c_2 e^{-mb} - c_1 e^{mb} = \frac{m_i}{m} B_4 (c_1 e^{mb} + c_2 e^{-mb}) \tag{3-11}$$

At $x = 0$, $\theta = \theta_{4-o}$ for Pin 4, Second boundary condition

From $\theta = c_1 e^{mx} + c_2 e^{-mx}$

At
$$x = 0, \ \theta_{4-o} = c_1 + c_2 \tag{3-12}$$

$$c_1 = \theta_{4-o} - c_2$$

Solve Equation 3-11 and 3-12 simultaneously

$$c_2 e^{-mb} - (\theta_{4-o} - c_2) e^{mb} = \frac{m_i}{m} B_4 \left[(\theta_{4-o} - c_2) e^{mb} + c_2 e^{-mb} \right]$$

$$c_2 \left[e^{-mb} + e^{mb} + \frac{m_i}{m} B_4 (e^{mb} - e^{-mb}) \right] = \theta_{4-o} (e^{mb} + \frac{m_i}{m} B_4 e^{mb})$$

$$c_2 = \theta_{4-0}\ \left(e^{mb} + \frac{m_i}{m} B_4 e^{mb}\right) \Big/ \left[e^{mb} + e^{-mb} + \frac{m_i}{m} B_4\ (e^{mb} - e^{-mb})\right]$$

$$c_1 = \theta_{4-0} - c_2$$

$$= \theta_{4-0}\left[1 - \frac{e^{mb} + \frac{m_i}{m} B_4 e^{mb}}{e^{mb} + e^{-mb} + \frac{m_i}{m} B_4\ (e^{mb} - e^{-mb})}\right]$$

$$= \theta_{4-0}\left[\frac{e^{mb} - \frac{m_i}{m} B_4\ e^{mb}}{e^{mb} + e^{-mb} + \frac{m_i}{m} B_4 (e^{mb} - e^{-mb})}\right]$$

So, $\theta = c_1\ e^{mx} + c_2 e^{-mx}$

$$= \theta_{4-0}\left[\frac{\left(e^{-mb} - \frac{m_i}{m} B_4 e^{-mb}\right) e^{mx} + \left(e^{mb} + \frac{m_i}{m} B_4 e^{mb}\right) e^{-mx}}{e^{mb} + e^{-mb} + \frac{m_i}{m} B_4\ (e^{mb} - e^{-mb})}\right]$$

$$\theta = \theta_{4-0}\ \frac{\left[e^{m(b-x)} + e^{-m(b-x)}\right] + H_3\left[e^{m(b-x)} - e^{-m(b-x)}\right]}{e^{mb} + e^{-mb} + H_3\ (e^{mb} - e^{-mb})} \qquad (3\text{-}13)$$

Where $H_3 = \frac{m_i}{m} B_4$

Thus, the temperature distribution of Pin 4 is

$$\frac{\theta}{\theta_{4-o}} = \frac{\cosh m\ (b-x) + H_3 \sinh m(b-x)}{\cosh mb + H_3 \sinh mb} \qquad (3\text{-}14)$$

at $x = b$, $$\frac{\theta_{4-b}}{\theta_{4-o}} = \frac{\theta_{5-o}}{\theta_{4-o}} = \frac{1}{\cosh mb + H_3 \sinh mb} = B_5 \qquad (3\text{-}15)$$

Since $\left.\frac{dT}{dx}\right|_{x=0} = \left.\frac{d\theta}{dx}\right|_{x=0}$ at $x = 0$

By differentiating Equation 3-14,

$$\frac{dt}{dx} = \frac{d\theta}{dx} = \theta_{4-o}\left[\frac{-me^{m(b-x)} + me^{-m(b-x)} + H_3(-me^{m(b-x)} - me^{-m(b-x)})}{e^{mb} + e^{-mb} + H_3\ (e^{mb} - e^{-mb})}\right]$$

$$\left.\frac{dT}{dx}\right|_{x=0} = -m\theta_{4-o}\left[\frac{e^{mb} - e^{-mb} + H_3(e^{mb} + e^{-mb})}{e^{mb} + e^{-mb} + H_3\,(e^{mb} - e^{-mb})}\right]$$

Therefore, the heat flux is

$$q_4 = -kA\left.\frac{dT}{dx}\right|_{x=0}$$

$$= kAm\theta_{4-o}\,\frac{\sinh mb + H_3\cosh mb}{\cosh mb + H_3\sinh mb}$$

$$q_{4-o} = kAm\theta_{4-o}\,B_6 \qquad (3\text{-}16)$$

Where $B_6 = \dfrac{\sinh mb + H_3\cosh mb}{\cosh mb + H_3\sinh mb}$

Next, Pin 2 comes into consideration. Because the conditions of Pin 2 are similar to that of Pin 5, Pin 2 can, therefore, be dealt with in the same pattern as heat transfer equations of Pin 5.

The heat dissipation from Pin 3 is obviously

$$q_{3-0} = B_2\,\theta_{3-o}$$

Since the term B_2 does not change by making Fin 3 with exactly the same dimensions as Fin 6 for the same heat transfer coefficient h.

Put q_{2-t} = Heat flux from the end of Pin 2

Then $q_{2-t} = q_{4-0}$

By equation 3-16,

$$q_{2-t} = -kA\left.\frac{dT}{dx}\right|_{x=t} = kAmB_6\theta_{4-o} \qquad (3\text{-}17)$$

At $x = t$, $T = T_{2-t}$, $\theta = \theta_{2-t} = \theta_{4-0}$

$$\left.\frac{dT}{dx}\right|_{x=t} = \left.\frac{d\theta}{dx}\right|_{x = t}$$

Hence, $$-kA \left.\frac{d\theta}{dx}\right|_{x=t} = kAm\ B_6\ \theta_{4-o}$$

$$\theta_{2-t} = \theta_{4-0};\quad \frac{d\theta_{2-t}}{dx} = -mB_b\ \theta_{2-t}$$

Following the same pattern used in deriving Equations 3-3 to 3-9, an equation analogous to Equation 3-6 is of the following form:

$$\frac{\theta}{\theta_{2-o}} = \frac{\cosh m_i\ (t-x) + H_4 \sinh\ (t-x)}{\cosh m_i t + H_4 \sin\ m_i t} \qquad (3\text{-}18)$$

Where $H_4 = \frac{m}{m_i} B_6$

By Equation 3-18, at $x = t$

$$\frac{\theta_{2-t}}{\theta_{2-0}} = \frac{\theta_{4-0}}{\theta_{2-o}} = \frac{1}{\cosh m_i t + H_4 \sinh m_i t} \qquad (3\text{-}19)$$

$$\theta_{4-o} = \frac{\theta_{2-o}}{\cosh m_i t + H_4 \sinh m_i t}$$

$$\theta_{4-o} = B_7\ \theta_{2-o} \qquad (3\text{-}20)$$

Where, $B_7 = \dfrac{1}{\cosh m_i t + H_4 \sinh m_i t}$

The heat flux of Pin 2 is analogous to Equation 3-9.

$$q_{2-o} = kAm_i\ B_8\ \theta_{\ 2-o} \qquad (3\text{-}21)$$

$$\text{Where } B_8 = \frac{\sinh m_i t + H_4 \cosh m_i t}{\cosh m_i t + H_4 \sinh m_i t}$$

At last, refer to Pin 1.

Put q_{1-a} = Heat flux of Pin 1
at $x = a$

Then $q_{1-a} = q_{2-o}$

By Equation 3-21

$$- kA \left.\frac{dT}{dx}\right|_{x=a} = kAm_i B_8 \theta_{2-o}$$

At $x = a$

$$\theta_{2-o} = \theta_{1-a}$$

$$\left.\frac{dT}{dx}\right|_{x = a} = \left.\frac{d\theta}{dx}\right|_{x = a}$$

$$\left.\frac{d\theta}{dx}\right|_{x=a} = -m_i B_8 \theta_{2-o} \qquad (3\text{-}22)$$

Following the same procedure used in deriving Equations 3-10 to 3-16,

The temperature distribution equation of Pin 1 analogous to Equation 3-13 to 3-15 is

$$\theta = \theta_{1-o} \frac{(e^{-ma} - \frac{m_i}{m} B_8 e^{-ma}) e^{mx} + (e^{ma} + \frac{m_i}{m} B_8 e^{ma}) e^{-mx}}{e^{ma} + e^{-ma} + \frac{m_i}{m} B_8 (e^{ma} - e^{-ma})}$$

$$\theta = \theta_{1-o} \frac{e^{m(a-x)} + e^{-m(a-x)} + H_5 \left[e^{m(a-x)} - e^{-m(a-x)}\right]}{e^{ma} + e^{-ma} + H_5 (e^{ma} - e^{-ma})} \qquad (3\text{-}23)$$

or, $$\frac{\theta}{\theta_{1-o}} = \frac{\cosh m(a-x) + H_5 \sinh m\,(a-x)}{\cosh ma + H_5 \sinh ma} \qquad (3\text{-}24)$$

$$\frac{\theta_{1-a}}{\theta_{1-o}} = \frac{\theta_{2-o}}{\theta_{1-o}} = \frac{1}{\cosh ma + H_5 \sinh ma} = B_9 \qquad (3\text{-}25)$$

Where $H_5 = \frac{m_i}{m} B_8$

Since the heat flux from the entire finned pin is equal to q_{1-o}

Hence $q_{\text{finned pin}} = q_{1-o} = -kA \left.\frac{dT}{dx}\right|_{x=o}$

$$q_{\text{finned pin}} = -kA \left.\frac{d\theta}{dx}\right|_{x=o}$$

Differentiating Equation 3-23

$$\frac{d\theta}{dx} = \theta_{1-o}\left[\frac{-me^{m(a-x)} + me^{-m(a-x)} + H_5\,(-me^{m(a-x)} - me^{-m(a-x)})}{e^{ma} + e^{-ma} + H_5(e^{ma} - e^{-ma})}\right]$$

At x = 0,

$$\left.\frac{d\theta}{dx}\right|_{x=o} = -m\theta_{1-o}\left[\frac{e^{ma} - e^{-ma} + H_5\,(e^{ma} + e^{-ma})}{e^{ma} + e^{-ma} + H_5(e^{ma} - e^{-ma})}\right]$$

$$= -m\theta_{1-o}\,\frac{\sinh ma + H_5 \cosh ma}{\cosh ma + H_5 \sinh ma}$$

Therefore

$$q_{\text{Finned pin}} = -\,kA \left.\frac{d\theta}{dx}\right|_{x=0}$$

$$= kAm\theta_{1-o}\,\frac{\sinh ma + H_5 \cosh ma}{\cosh ma + H_5 \sinh ma} \qquad (3\text{-}26)$$

Where $H_5 = \frac{m_i}{m} B_8$

Calculation operators ------------------- (3-26a)

$$B_9 = \frac{1}{\cosh ma + H_5 \sinh ma} = \frac{\theta_{2-o}}{\theta_{1-o}}$$

$$B_8 = \frac{\sinh m_i t + H_4 \cosh m_i t}{\cosh m_i t + H_4 \sinh m_i t}$$

$$B_7 = \frac{\theta_{4-o}}{\theta_{2-o}} = \frac{1}{\cosh m_i t + H_4 \sinh m_i t}$$

$$B_6 = \frac{\sinh mb + H_3 \cosh mb}{\cosh mb + H_3 \sinh mb}$$

$$B_5 = \frac{1}{\cosh mb + H_3 \sinh mb} = \frac{\theta_{5-o}}{\theta_{4-o}}$$

$$B_4 = \frac{\sinh m_i t + H_2 \cosh m_i t}{\cosh m_i t + H_2 \sinh m_i t}$$

$$B_3 = \frac{1}{\cosh m_i t + H_2 \sinh m_i t} = \frac{\theta_{7-o}}{\theta_{5-o}}$$

$$B_2 = 2\pi k N t r_1 \frac{K_1(Nr_1)\, I_1(Nr_2) - I_1(Nr_1) K_1(Nr_2)}{K_o(Nr_1) I_1(Nr_2) + I_o(Nr_1) K_1(Nr_2)} = q_6/\theta_{6-o}$$

$$B_1 = \frac{\sinh mc + H_1 \cosh mc}{\cosh mc + H_1 \sinh mc}$$

$$H_5 = \frac{m_i}{m} B_8$$

$$H_4 = \frac{m}{m_i} B_6$$

$$H_3 = \frac{m_i}{m} B_4$$

$H_2 = \frac{m}{m_i} B_1$

$H_1 = h/mk$

$h_i = B_{2/\pi Dt}$

$N = \sqrt{2h/kt}$ for the annular fins

$m = \sqrt{4h/kD}$

$m_i = \sqrt{4h_i/_{kD}}$

$r_2 = OD/2$

$r_1 = D/2$

I_o = Zero order modified Bessel Function, 1st kind

K_o = Zero order Modified Bessel Function, 2nd kind

I_1 = First order Modified Bessel Function, 1st kind

K_1 = First Order Modified Bessel Function, 2nd kind

The basic heat transfer equations for a finned pin with more than two annular fins on it can be derived by the same method.

OPTIMUM DIMENSIONS

Pins

For a straight rectangular fin with heat convection at the free end, a simplified heat transfer equation, as first used by Harper and Brown (12), is

$$q = kAm\theta_c \tanh mL_c \qquad (2\text{-}7)$$

The above equation was obtained by letting

$$hA(T_L - T_f) = h(2\Delta L)(T_L - T_f)$$

where, $2\Delta L = \Delta s$, for one foot of fin width with $t \ll w$, and $L_c = L + \Delta L$ = corrected length.

Equation 2-7 will be applicable to pins by putting

$$h\left(\frac{\pi D^2}{4}\right)(T_L - T_f) = h\pi D(\Delta L)(T_L - T_f) \text{ or } \Delta L = D/4 \qquad (3\text{-}27)$$

Theoretically, the temperature over ΔL is not uniform so that both the equation $hA(T_L - T_f) = h(2\Delta L)(T_L - T_f)$

and $h\left(\frac{\pi D^2}{4}\right)(T_L - T_f) = h\pi D(\Delta L)(T_L - T_f)$

are only approximately correct and result in some errors in some cases.

However, Equation 2-7 can be applied to pins without error by imagining that the fictitious part of a pin, i.e., ΔL, would be another small pin having end effect neglected and having $(T_L - T_f)$ as its root temperature. Thus, by Equation 2-1 the heat balance at the free end of the actual pin will be such that

$$hA\theta_L = kAm\theta_L \tanh m(\Delta L)$$

$$\Delta L = \frac{1}{m} \tanh^{-1}\left(\frac{h}{km}\right) \qquad (3\text{-}27a)$$

Where $m = \sqrt{4h/kD}$

θ_L = Temperature difference at the free end of the actual pin (F).

With $L_c = L + \Delta L$, where ΔL is from Equation 3-27. Equation 2-7 will be as accurate as Equation 2-3 and has been checked by numerical computations.

For a given volume of material of a pin, $\frac{\pi}{4}D^2L$ is constant, i.e., D^2L = constant.

Now, for a pin using corrected length $D^2L_c = c$, $L_c = \frac{c}{D^2}$.

By Equation 2-7

$$q = kAm\theta_o \tanh mL_c$$

$$= \theta_o \frac{\pi}{2} \sqrt{hkD^3} \tanh\left(\sqrt{\frac{4h}{kD}}\ \frac{c}{D^2}\right)$$

$$q = \frac{\theta_o \pi}{2} \sqrt{hkD^3} \tanh c \sqrt{\frac{4h}{kD^5}} \qquad (3\text{-}28)$$

$$\frac{dq}{dD} = \frac{\theta_o \pi}{2}\left[\sqrt{hkD^3}\ \frac{d}{dD} \tanh c\sqrt{\frac{4h}{kD^5}} + \tanh c\sqrt{\frac{4h}{kD^5}}\ \frac{d}{dD}\sqrt{hkD^3}\right]$$

put $\frac{dq}{dD} = 0$

$$0 = hkD^3\left[\frac{c}{2}\left(\frac{4h}{kD^5}\right)^{-1/2} \frac{4h}{k}\left(\frac{-5}{D^6}\right) \operatorname{sech}^2 c\sqrt{\frac{4h}{kD^5}}\right] + \left[\frac{1}{2}(hkD^3)^{-1/2}(hk)(3D^2) \tanh c\sqrt{\frac{4h}{kD^5}}\right]$$

Multiply both sides by $\sqrt{hkD^3}$

$$0 = -\frac{5}{2}(hkD^3)\ c\left(\frac{4h}{kD^6}\right)\left(\frac{4h}{kD^5}\right)^{-1/2} \operatorname{sech}^2 c\sqrt{\frac{4h}{kD^5}} + \frac{3}{2}(hkD^2)\tanh c\sqrt{\frac{4h}{kD^5}}$$

$$0 = -\frac{5}{2}\left(\frac{4h}{kD^5}\right) c\left(\frac{4h}{kD^5}\right)^{-1/2} \operatorname{sech}^2 c\sqrt{\frac{4h}{kD^5}} + \frac{3}{2}\tanh c\sqrt{\frac{4h}{kD^5}}$$

$$\tanh c\sqrt{\frac{4h}{kD^5}} = \frac{5}{3}\ c\sqrt{\frac{4h}{kD^5}} \operatorname{sech}^2 c\sqrt{\frac{4h}{kD^5}}$$

By trial and error or by computer

$$c\sqrt{\frac{4h}{kD^5}} = 0.9193$$

$$\frac{4h}{kD^5} = \left(\frac{0.9193}{c}\right)^2$$

Hence, the optimum diameter for maximum heat transfer is

$$D_{opt} = \left[\frac{4hC^2}{(0.9193)^2 k}\right]^{1/5} \qquad \text{(ft.)} \qquad (3\text{-}29)$$

$$L_{c_{opt}} = \frac{c}{D^2_{opt}} \qquad \text{(ft.)} \qquad (3\text{-}30)$$

For a pin with end effect neglected or insulated

$$L_{opt} = \frac{c}{D^2_{opt}} \qquad \text{(ft.)} \qquad (3\text{-}31)$$

For a pin with heat convection at the free end

$$L_{opt} = L_{c_{opt}} - \Delta L_{opt} \qquad (3\text{-}32)$$

Where, $L_{c_{opt}}$ is calculated from Equation 3-30.

ΔL_{opt} is calculated from Equation 3-27 or 3-27a with

$$m = \sqrt{\frac{4h}{kD_{opt}}} \quad \text{and } \tanh^{-1}\left(\frac{h}{km}\right) \text{ in radians.}$$

Usually q_{pin}, θ_o, h and k are given to a designer, and it is necessary to find the optimum dimensions of pins in terms of q_{Pin}, θ_o, h and k.

Since $\tanh c\sqrt{\frac{4h}{kD^5}} = \tanh 0.9193 = 0.7256$

by Equation 3-28

$$q = \frac{\pi\theta_o}{2}\sqrt{hKD^3}\tanh C\sqrt{\frac{4h}{KD^5}}$$

$$= \frac{0.7256}{2}\theta_o\pi\sqrt{hkD^3}$$

$$hkD^3 = \left(\frac{2q}{0.7256\theta_o\pi}\right)^2$$

$$D = \left(\frac{2}{0.7256}\right)^{2/3}\left(\frac{1}{hk}\right)^{1/3}\left(\frac{q}{\theta_o}\right)^{2/3}$$

$$D_{opt} = 0.9165\left(\frac{1}{hk}\right)^{1/3}\left(\frac{q}{\theta_o}\right)^{2/3} \qquad (3\text{-}33)$$

Equate Equations 3-29 and 3-33

$$\left[\frac{4hc^2}{(0.9193)^2k}\right]^{1/5} = 0.9165\left(\frac{1}{hk}\right)^{1/3}\left(\frac{q}{\theta_o}\right)^{2/3}$$

$$\frac{4\ hc^2}{(0.9193)^2k} = (0.9165)^5\left(\frac{1}{hk}\right)^{5/3}\left(\frac{q}{\theta_o}\right)^{10/3}$$

$$c^2 = \frac{(0.9193)^2(0.9165)^5}{4}\frac{1}{h^{8/3}\ k^{2/3}}\left(\frac{q}{\theta_o}\right)^{10/3}$$

$$c = \frac{(0.9193)(0.9165)^{5/2}}{2h^{4/3}\ k^{1/3}}\left(\frac{q}{\theta_o}\right)^{5/3} \qquad (3\text{-}34)$$

By Equation 3-31, 3-33 and 3-34

$$L_{c_{opt}} = \frac{c}{D^2_{opt}}$$

$$= \frac{(0.9193)(0.9165)^{5/2}}{2h^{4/3}\,k^{1/3}}\left(\frac{q}{\theta_o}\right)^{5/3}\frac{(hk)^{2/3}}{(0.9165)^2}\left(\frac{\theta_o}{q}\right)^{4/3}$$

Therefore, $$L_{c_{opt}} = 0.4213\left(\frac{k}{h^2}\right)^{1/3}\left(\frac{q}{\theta_o}\right)^{1/3} \qquad (3\text{-}35)$$

$$L_{opt} = L_{c_{opt}} - \Delta L_{opt} \qquad (3\text{-}36)$$

Where $L_{c_{opt}}$ is from Equation 3-35

ΔL_{opt} is from Equation 3-27 or 3-27a

with $m = \sqrt{\frac{4h}{kD_{opt}}}$, $\tanh^{-1}\left(\frac{h}{km}\right)$ in radians.

Annular Fins

By definition of fin efficiency, $e = q/q_o$

where q_o = Heat flux from a fin if the entire fin were at temperature T_o

$$= 2\pi(r_2^2 - r_1^2)h\theta_o$$

$$q = q_o e\,, \quad \frac{q}{\theta_o} = 2\pi h(r_2^2 - r_1^2)\,e \qquad (3\text{-}37)$$

In the above equation, usually the terms q, θ_o, h and r_1 are given, and the terms $(r_2^2 - r_1^2)$ and e will determine the optimum dimensions of annular fins, i.e., the values of r_2/r_1 and t that make $(r_2^2 - r^2)$ e maximum for a given **volume** of material will be the optimum dimensions. Therefore, efficiency curves of annular fins are basic data for designers.

By Equation 2-24

$$e = \frac{2}{\left(1 + {}^{r_2}/_{r_1}\right) Z} \quad \frac{K_1(R_1Z)\, I_1(R_2Z) - I_1(R_1Z)K_1(R_2Z)}{K_o(R_1Z)\, I_1(R_2Z) + I_o(R_1Z)K_1(R_2Z)}$$

curves are plotted by putting r_2/r_1 = 2 to 7 (any adequate number can be chosen if necessary) with the design-parameter group, $z = (r_2 - r_1)\sqrt{2h/kt}$, as abscissa and efficiency e as ordinate. (Fig. 5).

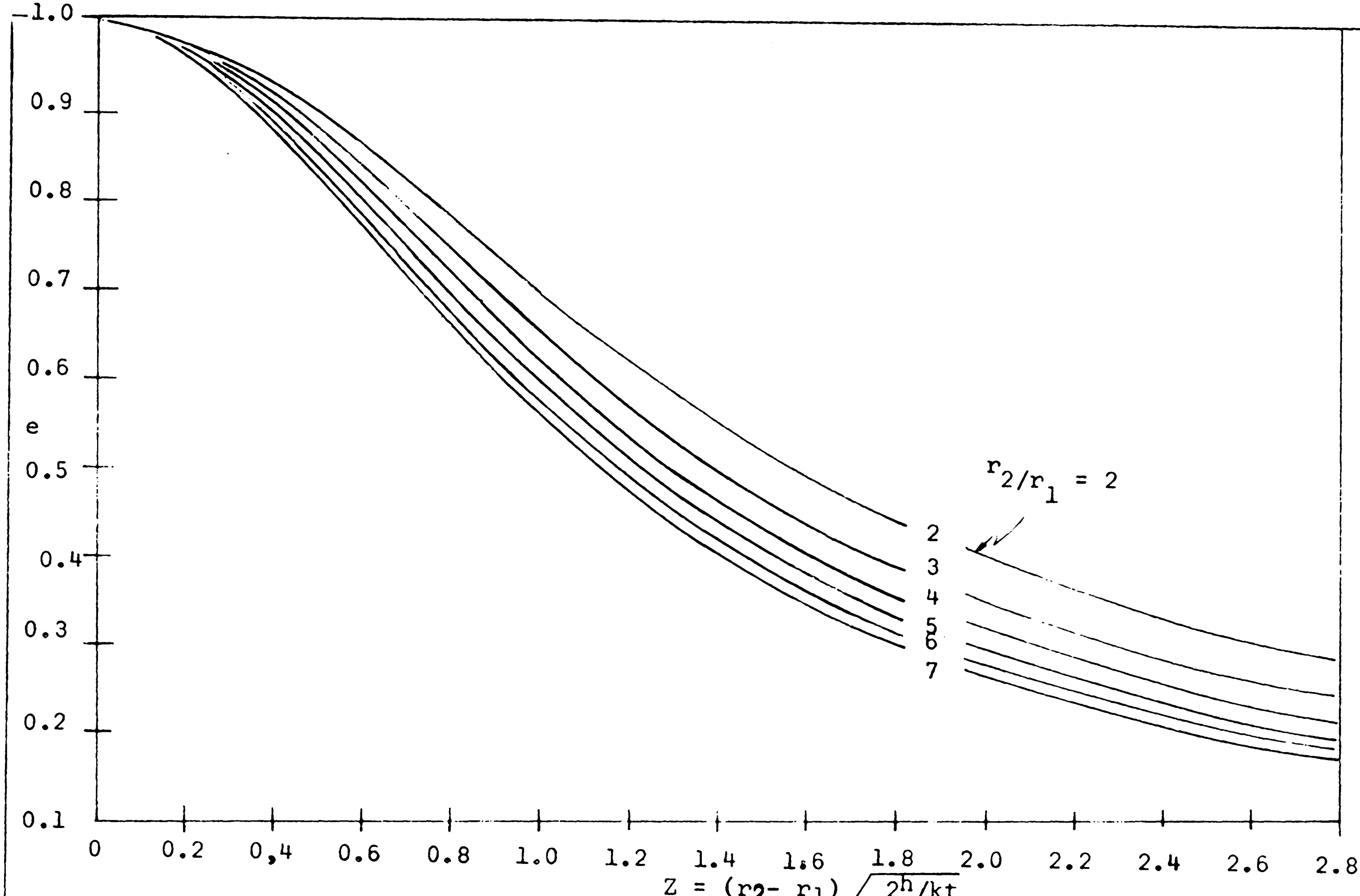


Fig. 5 Efficiency curves of annular fins of constant thickness (See calculated data, Table 7

By Equation 3-37 and with the aid of Figure **5**, optimum dimensions of an annular fin can be estimated for two cases that follow:

(1) For given values of q, θ_o and h
Assign values of r_2/r_1, calculate corresponding values of e by Equation 3-37, check the corresponding values of Z from efficiency curves and then corresponding values of t are determined by $Z = (r_2 - r_1)\sqrt{2h/kt}$. The value of t and the value of r_2/r_1 that result in least volume of material will be the optimum dimensions, (r_1 being given).

(2). For given values of h and a given volume of material with r_1 given, choose the value of t according to commercial gauge of sheet metal. The values of r_2/r_1 and t, that result in maximum heat transfer, will be optimum dimensions for a given volume of material.

The optimum conditions of an annular fin are somewhat like that of a straight rectangular fin. Equation 3-37 can be put into a simplified form,

q/θ_o = hS e, which is the simplified form for a pin or a straight rectangular fin with end effect neglected. For a given value of h, the surface $S = 2\pi(r^2 - r_1^2)$ and efficiency e are factors that influence heat flow.

For given values of r_2/r_1, if t is doubled, the volume of material will be doubled without increasing the surface (except the slight increase in outer-edge area), and the design parameter group Z decreases ($1/\sqrt{2}$ times its original value), resulting in a slight increase in efficiency, as seen from Figure 5. It is meant that the heat transfer increase slightly with the volume doubled. But with two thinner fins of thickness t, The surface is approximately two times that of one fin with thickness 2t. Thinner fins are, therefore, preferable. Moreover, for a given value of t, the larger value of r_2/r_1 of an annular fin can be replaced by a smaller value with increase of heat transfer by making two fins out of one for the same volume of material. Since both the thickness t and the ratio r_2/r_1 are limited either because of the spacing of finned pins within heat exchangers or because of difficulties in manufacture of very thin sheet metal, more fins with small thickness and reasonable values of r_2/r_1 will be better than fewer fins of large size for the same volume of material.

<u>Finned Pins</u>

The optimum dimensions for extended surfaces of a given volume of material are the dimensions that produce maximum heat transfer. For given values of h, k and ΔT the heat transfer of a finned pin is a function of pin diameter and length, fin thickness and outside diameter, number and spacing of fins, i.e., $q=f(D,L,t,OD,N,S)$. The optimum dimensions of a finned pin are very complicated. The purpose of this thesis is to compare a plain pin having optimum dimensions with a finned pin which was the same pin with annular fins added. For the given finned pin with two fins of the same size and equally spaced on the pin having optimum dimensions, the optimum dimensions are approximately those of the constituting parts, i.e., the pin and annular fins.

EFFICIENCY AND EFFECTIVENESS OF THE

TWO-DISC FINNED PIN

By Equation 3-26

$$q_{\text{finned pin}} = kAm\theta_{1-o}\left(\frac{\sinh ma + H_5 \cosh ma}{\cosh ma + H_5 \sinh ma}\right)$$

Let q_o = heat dissipation from the two disc finned pin if the entire finned pin were at θ_{1-o}

$$q_o = \left[\pi D_1(L-2t)+2\,\frac{\pi}{4}\left(D_2^2 - D_1^2\right)+ A\right]h\,\theta_{1-o}$$

Let q' = heat dissipation from the root area if the two-disc finned pin is removed.

$$q' = Ah\theta_{1-o}$$

(Actually θ_{1-o} will rise if the two-disc finned pin is removed. θ_{1-o} is used for an approximation) (17)

Therefore, the efficiency of the two-disc finned pin is

$$e = \frac{q}{q_o} = \frac{kAm\theta_{1-o}\left[\dfrac{\sinh ma + H_5 \cosh ma}{\cosh ma + H_5 \sinh ma}\right]}{\left[\pi D_1(L-2t) + \frac{\pi}{2}(D_2^2 - D_1^2) + A\right]h\theta_{1-o}} \qquad (3\text{-}38)$$

$$\text{or, } e = \frac{KAm\left[\dfrac{\sinh ma + H_5 \cosh ma}{\cosh ma + H_5 \sinh ma}\right]}{\pi\left[D_1(L-2t) + 1/2(D_2^2 - D_1^2) + A\right]h}$$

The effectiveness of the two-disc finned pin is

$$E = \frac{q}{q'} = \frac{kAm\theta_{1-o}\left[\dfrac{\sinh ma + H_5 \cosh ma}{\cosh ma+H_5 \sinh ma}\right]}{Ah\theta_{1-o}} \qquad (3\text{-}39)$$

$$\text{or, } E = \frac{km}{h}\left[\frac{\sinh ma + H_5 \cosh ma}{\cosh ma + H_5 \sinh ma}\right]$$

SAMPLE CALCULATION OF OPTIMUM DIMENSIONS

In the estimation of optimum dimensions, the thermal conductivity k and heat transfer coefficient h must be known. However, the temperature distribution and h of the finned pin are both unknown. The calculation of optimum dimensions is, therefore, based on the heat transfer coefficient of the pin, which constitutes the major part of the finned pin.

Given data for the two-disc finned pin:

Material: Carbon steel, SAE 1020

Diameter of the pin: 3/8 in. (1/32 ft.)

Thickness of annular fins: 0.015 in.

k = 0.124 cal/sec. cm.°c, at room temperature (P.55, Metals Handbook, ASM)

$$= \frac{0.124(0.003968)}{\frac{1}{3600}\,\frac{1}{30.48}\,1.8} = 30 \text{ BTU/hr.ft. F}$$

$h_c = 0.27\ 100(32)/1^{\ 1/4} = 2.03$ Btu/hr.ft.^{2}F., (Equation $h = 0.27(\Delta T/D)^{1/4}$)

where 100°F = average temperature difference (assumed)

$h_r = 0.171e\left[\left(\frac{640}{100}\right)^4 - \left(\frac{540}{100}\right)^4\right] / 640-540$, which can be neglected for low temperature range and polished surface

If average T_f = 80°F for ambient free air

Average pin surface temperature T = 180°F., $h = h_c + h_r = 2.03$

(1) Optimum Dimensions of the Pin

By Equation 3-33

$$D_{opt} = 0.9165\left(\frac{1}{hk}\right)^{1/3}\left(\frac{q}{\theta_o}\right)^{2/3}$$

$$1/32 = 0.9165\left[\frac{1}{2.03(30)}\right]^{1/3}\left(\frac{q}{\theta_o}\right)^{2/3}$$

$$\frac{q}{\theta_o} = \left[\frac{1}{32(0.9165)}\right]^{3/2}\left[2.03(30)\right]^{1/2}$$

$$= 0.006295\ (7.8038)$$

$$= 0.04913$$

By Equation 3-35

$$L_{c_{opt}} = 0.4213\left(\frac{k}{h^2}\right)^{1/3}\left(\frac{q}{\theta_o}\right)^{1/3}$$

$$= 0.4213\left(\frac{30}{2.03^2}\right)^{1/3} (0.04913)^{1/3}$$

$$= 0.4213\ (1.9381)\ (0.3663)$$

$$= 0.2991 \text{ ft.}$$

By Equation 3-27

$$\Delta L = D/4 = \frac{1}{32(4)} = 0.0078 \text{ ft.}$$

or by Equation 3-27a

$$\Delta L = \frac{1}{m} \tan^{-1}\left(\frac{h}{km}\right) = \frac{1}{2.943} \tan^{-1} \frac{2.03}{30(2.943)}$$

$$\Delta L = 0.0078 \text{ ft.}$$

By Equation 3-36

$$L_{opt} = L_{c\ opt} - \Delta L = 0.2991 - 0.0078 = 0.2913$$

Use L_{opt} = 3 1/2", i.e., 0.2917 ft.

(2) Optimum Dimensions of the Annular Fins

As observed from Fig. 5, the rate of change of e with respect to the design-parameter group $Z = (r_2 - r_1)\sqrt{2h/kt}$ is higher between the points e = 0.6 to e = 0.8. For a given value of t, the ratio r_2/r_1 that makes e around 0.7 may be tried first, because the change of e is greater in this part with same change of r_2/r_1, i.e., the change of q is greater with same change of volume.

For h = 2.03 BTU/hr. ft.^{2}F

k = 30 BTU/hr. ft. F

$$Z = (r_2/r_1 - 1)r_1 \sqrt{\frac{2(2.03)12}{30t}} = r_1(r_2/r_1 - 1)\ (1.275)\ \sqrt{1/t}$$

Where r_1 is in ft.

t is in inches.

Given $r_1 = 1/64$ ft.

$t = 0.015$ in.

Assume $r_2/r_1 = 6$, trying to obtain a value of e around 0.7

Then $Z = 5(1/64)\ (1.275)\ \sqrt{1/0.015} = 0.814$

From the curve,

$e = 0.66$

Thus the given volume of each fin

$$V = \pi(r_2^2 - r_1^2)t = \pi\left[\left(\frac{r_2}{r_1}\right)^2 - 1\right] r_1^{\ 2}\ t$$
$$= 3.14(35)(1/64)^2\ (0.015/12)$$
$$= 0.0000336\ \text{ft.}^3 = 3.36 \times 10^{-5}\ \text{ft.}^3$$

For this given volume $V = \pi(35)r_1^2\ (0.015/12)\ \text{ft}^3$

If $r_2/\ r_1 = 5$, $\pi(24)r_1^2 t = \pi(35)r_1^2\ (0.015/12)$

$$t = \frac{35}{24}(0.015/12)\ \text{ft.}$$
$$= 0.0219\ ''$$

If $r_2/r_1 = 7$

Similarly $$t = \frac{35}{43}(0.015/12)\ \text{ft.}$$
$$= 0.0109\ ''$$

By Equation 3-37

$$q/\theta_o = \pi(r_2^2 - r_2^2)he$$

$$= 2\pi\left[\left(\frac{r_2}{r_1}\right)^2 - 1\right] r_1^2 \ he$$

$$= 2\pi\left[\left(\frac{r_2}{r_1}\right)^2 - 1\right]\left(\frac{1}{64}\right)^2 (2.03)\ e$$

The value of q/θ_o are tabulated for different r_2/r_1 and t of the same volume

r_2/r_1	t (in)	$z=(r_2 r_1)\frac{1}{64}(1.275)\sqrt{\frac{1}{t}}$	e	$\frac{q}{\theta_0} = 2\pi\left[(r_2/r_1)^2-1\right](\frac{1}{64})^2(2.03)e$
5	0.0219	0.538	0.837	0.063
6	0.015	0.814	0.660	0.702 max
7	0.0109	1.145	0.482	0.072

The result shows that t = 0.015"

$$r_2 = 6r_1 = 6(3/16) = 1.125"$$

are the optimum dimensions of each annular fin, because the heat transfer is approximately maximum at these dimensions. Since one dimensional solution for annular fins are accurate for height-to-width ratios more than 10 (9) the above calculation is accurate with the ratio

$$\frac{r_2 - r_1}{t} = \frac{1.125 - 3/16}{0.015} = 62.5 > 10$$

(3) Optimum dimensions of the finned pin constructed by the same pin and two fins of same size equally spaced on the pin:

pin diameter = 3/8" = 1/32'

pin length = 3.5" = 0.2917'

fin thickness = 0.015" = 0.015/12'

O.D. = 2.25" = 6/32' for annular fins

Volume of the finned pin

$$= 0.000223 + 2(.0000336)$$

$$= 0.000257 \text{ ft.}^3 = 2.57 \times 10^{-4}$$

SAMPLE CALCULATION OF HEAT TRANSFER

Given data:

$h = 2.03$ Btu/hr.ft.^{2}F

$k = 30$ Btu/hr.ft. F

$D = 3/8" = 1/32'$

$L = 3.5" = 0.2917'$, $L_c = 0.2991'$

$A = \frac{\pi}{4}(1/32)^2 = 0.000767$ ft^2

$m = 4h/kD = 2.943$

$kA = 30(0.000767) = 0.023$

$kAm = 0.023(2.943) = 0.0677$

$H = h/km = 2.03/30(2.943) = 0.023$

$t = 0.015"$

$OD = 2.25" = 6/32'$

$T_f = 80°F$

$T_m = 180°F$

$T_m - T_f = 100°F = \theta_m$

$q/\theta_o = 0.04913$ for the plain pin having optimum dimensions

$q/\theta_o = 0.072$ for each annular fin having optimum dimensions

$a = b = c = 0.0964'$ (a, b, and c can be unequal)

where $a + b + c + 2t = 0.2917'$

(1) Heat-Flow Rate from the Plain Pin to Free Air

By the temperature distribution curve (Fig. 1), the mean temperature difference θ_m is equal to $\frac{1}{L}\int_o^L \theta dx$.

By Equation 2-8

$$\frac{\theta}{\theta_o} = \frac{\cosh m(L_c - x)}{\cosh mL_c}, \quad \text{by substituting} \quad \theta = \theta_o \frac{\cosh m(L_c-x)}{\cosh mL_c}$$

in $\theta_m = \frac{1}{L_c}\int_o^{L_c} \theta dx$,

$$\theta_m = \frac{\theta_o}{L_c}\int_o^{L_c} \frac{\cosh m\,(L_c-x)}{\cosh\; mL_c}\, dx$$

$$= \frac{\theta_o}{L_c}\int_o^{L} \frac{e^{m(L_c-x)} + e^{-m\,(L_c-x)}}{e^{mL_c} + e^{-mL_c}}\, dx$$

$$= \frac{\theta_o}{mL_c}\left[\frac{-e^{m(L_c-x)} + e^{-m(L_c-x)}}{e^{mL_c} + e^{-nL_c}}\right]_0^{L_c}$$

$$= \frac{\theta_o}{mL_c}\left[\frac{e^{mL_c} - e^{-mL_c}}{e^{mL_c} + e^{-mL_c}}\right]$$

So, $\theta_m = \left(\frac{\theta_o}{mL_c}\right)\tanh mL_c$

or, $\frac{\theta_m}{\theta_o} = \frac{\tanh mL_c}{mL_c}$

which is the same to the equation of pin efficiency.

By substituting known data to the above equation

$$\frac{100}{\theta_o} = \frac{\tanh 2.943(0.2991)}{2.943(0.2991)}$$

$$\theta_o = \frac{100(0.88)}{0.7064} = 124.6^\circ F.$$

Therefore $T_o = 124.6 + 80 = 204.6^\circ$ F.

By $q/\theta_o = 0.04913$

$q = 0.04913\ (124.6) = 6.1216$ BTU/hr.

Checking by $q = kAm\theta_o \dfrac{\sinh mL + H \cosh mL}{\cosh mL + H \sinh mL}$

$$= 0.0677(124.6)\ \frac{\sinh 2.943(0.2917)+0.023 \cosh 2.943(0.2917)}{\cosh 2.943(0.2917)+ 0.023 \sinh 2.943(0.2917)}$$

$$= 0.0677(124.6)\ \frac{\sinh 0.86 + 0.023 \cosh 0.86}{\cosh 0.86 + 0.023 \sinh 0.86}$$

$$= 0.0677\ (124.6)\ \frac{0.97 + 0.023(1.3932)}{1.3932+ 0.023\ (0.97)}$$

$$= 0.0677(124.6)\ \left(\frac{1.022}{1.4155}\right)$$

$$= 6.12 \text{ BTU/hr.}$$

(2) Heat-Flow Rate from the two-disc finned pin to ambient free air.

$$ma = mb = mc = 2.943(0.0964) = 0.284$$

$$H_1 = h/mk = 2.03/2.943(30) = 0.023$$

$$mL = 2.943(0.2917) = 0.86$$

By Equation 3-26a

$$\text{Operator } B_1 = \frac{\sinh mc + H_1 \cosh mc}{\cosh mc + H_1 \sinh mc}$$

$$= \frac{\sinh 0.284 + 0.023 \cosh 0.284}{\cosh 0.284 + 0.023 \sinh 0.284}$$

$$= \frac{0.2878 + 0.023(1.0406)}{1.0406 + 0.023(0.2878)}$$

$$= \frac{0.3117}{1.0473} = 0.298$$

$$\theta_{7-c}/\theta_{7-o} = \frac{1}{1.0473}$$

$$B_2 = q_6/\theta_{6-o} = 0.072 \text{ (See calculation for annular fin)}$$

$$h_i = B_2/\pi Dt = \frac{0.072}{3.14(1/32)\ (0.015/12)}$$

$$= 587$$

$$m_i = \sqrt{\frac{4h_i}{kD}} = \sqrt{\frac{4(587)}{30(1/32)}}$$

$$= 50$$

$$m/m_i = 2.943/50 = 0.0589$$

$$m_i/m = 50/2.943$$

$$H_2 = \frac{m}{m_i} B_1 = 0.0589\ (0.298) = 0.0176$$

$$B_3 = \theta_{7-o}/\theta_{5-o} = \frac{1}{\cosh m_i t + H_2 \sinh m_i t}$$

$$= \frac{1}{\cosh 50\left(\frac{0.015}{12}\right) + 0.0176 \sinh 50\left(\frac{0.015}{12}\right)}$$

$$= \frac{1}{\cosh 0.0625 + 0.0176 \sinh 0.0625}$$

$$= \frac{1}{1.0020 + 0.0176\ (0.0625)}$$

$$= \frac{1}{1.0031}$$

$$B_4 = \frac{\sinh m_i t + H_2 \cosh m_i t}{\cosh m_i t + H_2 \sinh m_i t}$$

$$= \frac{\sinh 0.0625 + 0.0176 \cosh 0.0625}{\cosh 0.0625 + 0.0176 \sinh 0.0625}$$

$$= \frac{0.0625 + 0.0175\ (1.0020)}{1.0020 + 0.0175\ (0.0625)}$$

$$= \frac{0.0801}{1.0031}$$

$$= 0.0798$$

$$H_3 = \frac{m_i}{m} B_4 = \frac{50}{2.943}\ (0.0798)$$

$$= 1.356$$

$$B_6 = \frac{\sinh mb + H_3 \cosh mb}{\cosh mb + H_3 \sinh mb}$$

$$= \frac{0.2878 + 1.356(1.0406)}{1.0406 + 1.356\ (0.2878)}$$

$$= \frac{1.6994}{1.4309} = 1.188$$

$$H_4 = \frac{m}{m_i} B_6 = 0.0589\ (1.188)$$

$$= 0.07$$

$$B_5 = \frac{\theta_{5-o}}{\theta_{4-o}} = \frac{1}{\cosh mb + H_3\ mb}$$

$$= \frac{1}{1.4309}$$

$$B_8 = \frac{\sinh m_i t + H_4 \cosh m_i t}{\cosh m_i t + H_4 \sinh m_i t}$$

$$= \frac{0.0625 + 0.07\ (1.0020)}{1.0020 + 0.07(0.0625)}$$

$$= \frac{0.1325}{1.0064}$$

$$= 0.1318$$

$$B_7 = \frac{\theta_{4-o}}{\theta_{2-o}} = \frac{1}{\cosh m_i t + H_4 \sinh m_i t} = \frac{1}{1.0064}$$

$$H_5 = \frac{m_i}{m} B_8 = \frac{50}{2.943}(0.1318) = 2.235$$

$$B_9 = \frac{\theta_{2-o}}{\theta_{1-0}} = \frac{1}{\cosh ma + H_5 \sinh ma}$$

$$= \frac{1}{\cosh 0.284 + 2.235 \sinh 0.284}$$

$$= \frac{1}{1.0406 + 2.235(0.2878)}$$

$$= \frac{1}{1.6838}$$

By Equation 3-26

$$q_{\text{finned-pin}} = kAm\theta_{1-o} \times \frac{\sinh ma + H_5 \cosh ma}{\cosh ma + H_5 \sinh ma}$$

$$= 0.0677\ \theta_{1-o} \frac{\sinh 0.284 + 2.235 \cosh 0.284}{\cosh 0.284 + 2.235 \sinh 0.284}$$

$$= 0.0677\ \theta_{1-o} \frac{0.2878 + 2.235\ (1.0406)}{1.0406 + 2.235\ (0.2878)}$$

$$= 0.0677\ \theta_{1-o} \frac{2.6135}{1.6838}$$

$$= 0.0677\ (1.5522)\ \theta_{1-o}$$

$$= 0.1051\ \theta_{1-o}$$

If the root temperature θ_{1-o} is equal to the root temperature of the plain pin, i.e., 124.6°F

$$q_{finned\ pin} = 0.1051(124.6) = 13.0955\ \text{Btu/hr.}$$

SAMPLE CALCULATION OF TEMPERATURE DISTRIBUTION

Temperature calculation is shown in Table 2 and Table 3.

x	Corresponding position at finned pin (approximate)	$\theta=\theta_o \dfrac{\cosh m(L-x)+H \sinh m(L-x)}{\cosh mL + H \sinh mL}$	θ
0	0	given	124.6
$\frac{1}{6}L$	mid-point of a	$124.6 \dfrac{\cosh 0.717+0.023 \sinh 0.717}{\cosh 0.86+0.023 \sinh 0.86}$	113.0
$\frac{2}{6}L$	end-point of a	$124.6 \dfrac{\cosh 0.575+0.023 \sinh 0.575}{\cosh 0.86 + 0.023 \sinh 0.86}$	104.0
$\frac{3}{6}L$	mid-point of b	$124.6 \dfrac{\cosh 0.43 + 0.023 \sinh 0.43}{\cosh 0.86 + 0.023 \sinh 0.86}$	97.0
$\frac{4}{6}L$	end-point of b	$124.6 \dfrac{\cosh 0.285 + 0.023 \sinh 0.86}{\cosh 0.86 + 0.023 \sinh 0.86}$	91.5
$\frac{5}{6}L$	mid-point of c	$124.6 \dfrac{\cosh 0.142+0.023 \sinh 0.142}{\cosh 0.86 + 0.023 \sinh 0.86}$	89.0
L	end	$124.6 \dfrac{1}{\cosh 0.86 + 0.023 \sinh 0.86}$	88.0

Table 2. Calculated Temperature Distribution of the Plain Pin

Position	Notation Previously Used	Equation	θ
root	θ_{1-o}	Given	124.6
mid-point of a		$\theta_{1-o}\dfrac{\cosh m(a-x)+H_5 \sinh m(a-x)}{\cosh ma + H_5 \sinh ma}$	98.3
End-point of a	$\theta_{1-t}, \theta_{2-o}$	$\theta_{1-o}\dfrac{1}{\cosh ma + H_5 \sinh ma}$	74.0
root of b	$\theta_{2-t}, \theta_{4-0}$	$\theta_{2-o}\dfrac{1}{\cosh m_i t + H_4 \sinh m_i t}$	73.5
mid-point of b		$\theta_{4-o}\dfrac{\cosh m(b-x)+H_3 \sinh m(b-x)}{\cosh mb+ H_3 \sinh mb}$	61.8
end-point of b	$\theta_{4-t}, \theta_{5-o}$	$\theta_{4-o}\dfrac{1}{\cosh mb + H_3 \sinh mb}$	51.4
root of c	$\theta_{5-t}, \theta_{7-o}$	$\theta_{5-o}\dfrac{1}{\cosh m_i t + H_2 \sinh m_i t}$	51.2
mid-point of c		$\theta_{7-o}\dfrac{\cosh m(c-x)+H_1 \sinh m(c-x)}{\cosh mc + H_1 \sinh mc}$	49.6
end	θ_L	$\theta_{7-o}\dfrac{1}{\cosh mc + H_1 \sinh mc}$	48.9

Table 3. Calculated Temperature Distribution of the Two – Disc Finned Pin

IV. EXPERIMENTAL INVESTIGATION

OBJECTIVE OF INVESTIGATION

It has been the intention of this investigation to verify the basic heat transfer equations derived in this thesis by comparing experimental and theoretical results. The ultimate objective is to determine the heat transfer characteristics of finned pins.

EXPERIMENTAL PROCEDURE

Set-up of Heat Supply Circuit

Since the ammeter and voltmeter were placed between the wattmeter and the auto-transformer, (wiring system shown Fig. 7) there was no effect on wattmeter reading. The potential circuit of the wattmeter was connected directly across the heater. By disconnecting the load , i.e., by turning off switch S the potential and current circuits of the wattmeter continued to carry a current and, therefore, the wattmeter itself consumed electric energy. The net power taken by the load was the reading when switch S was on, minus the reading when switch S was off.

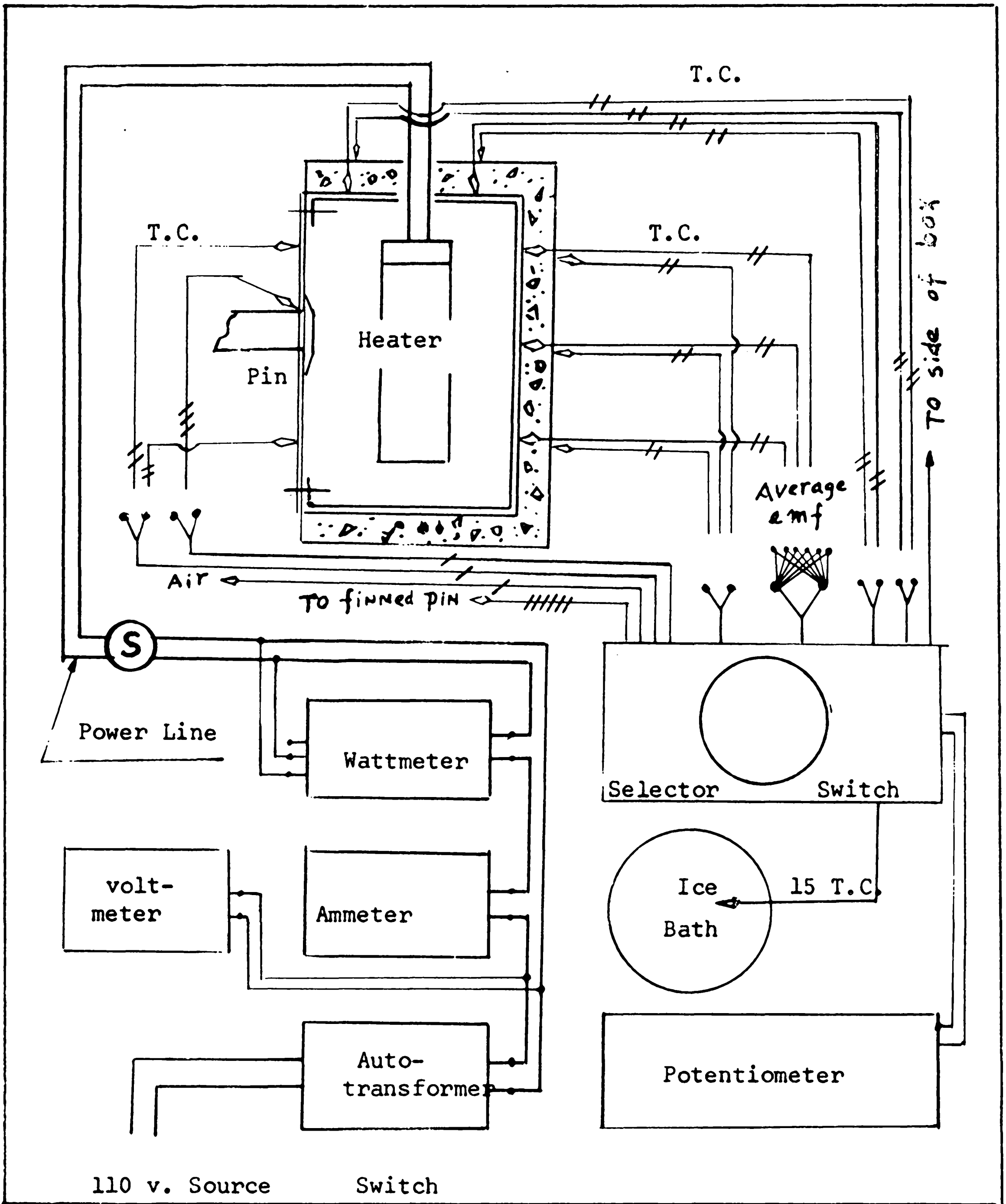


Fig. 6 Schematic Sketch of Measuring Apparatus

Construction of Heat Box, Pins and Finned Pins

Heat Box - Three front plates were cut to size and holes were drilled in the plates and box so that each plate could be screwed to the box front, (Fig. 7). Insulation around the box (except front plate) was accomplished by putting plaster of asbestos in the space between the heat box and a larger wooden box.

Pins - Three pin holes were drilled in one front plate. The pins were slipped in and fastened on the back of plate by silver soldering.

Finned Pins - Starting with one annular fin slipped on the pin, the end of the pin was heated by gas flame and soft solder was applied around the circumferential contact line. The pins had to be fastened to another front plate by silver soldering before the fastening of annular fins, otherwise, soft solder would be melted because of the higher temperature required for silver solder.

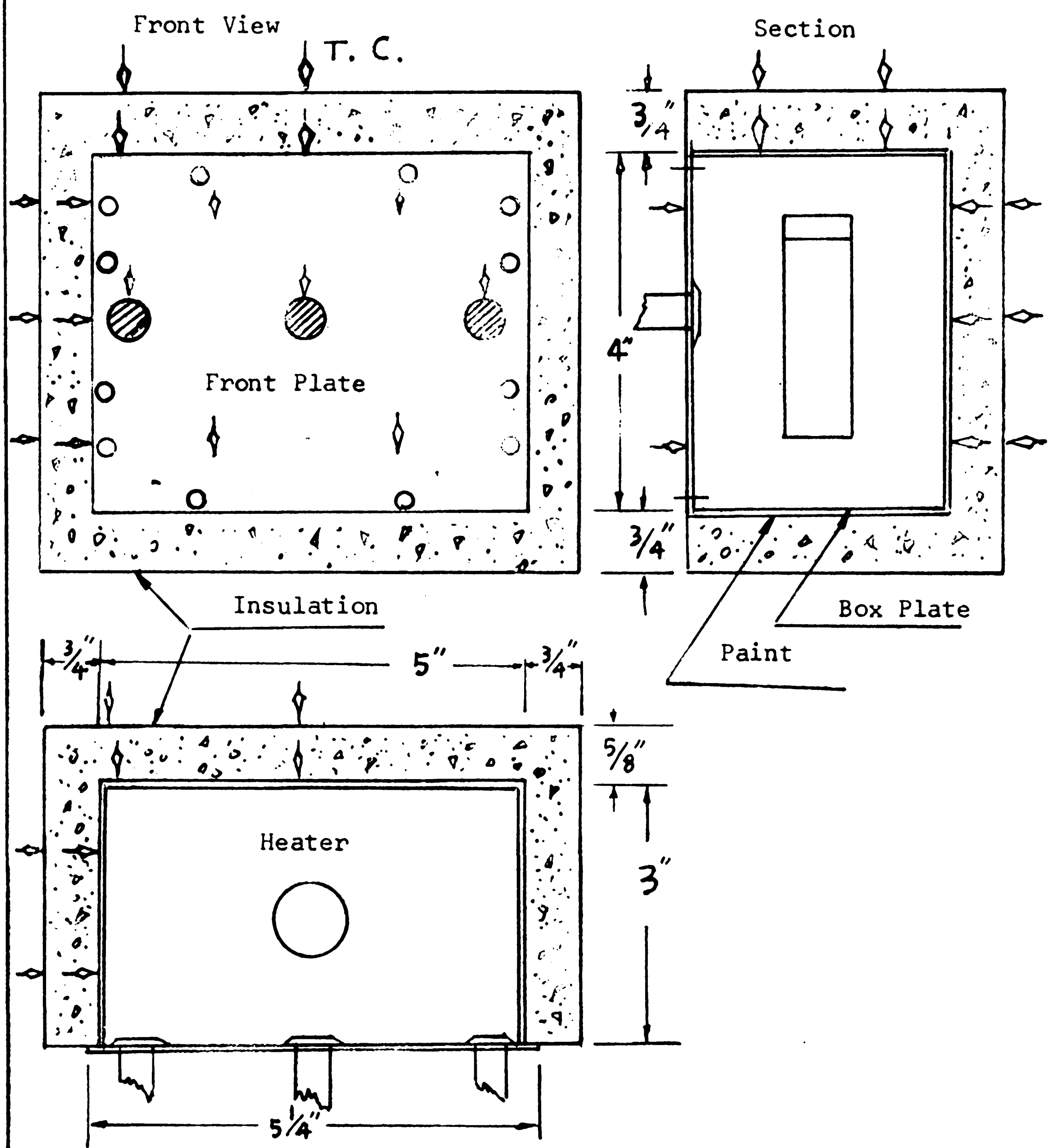


Fig. 7 Sketch of Heat-Box

Set-up of Measuring Circuit

(1) Construction and Calibration of Thermocouples:

Thermocouples were made of copper-constantan with junctions formed with soft solder which can be worked to temperatures up to 300° F.

The thermocouples were calibrated at the steam point only and it was found that the thermocouple emf at the steam point was consistent with the standard emf-temperature table. Because the temperatures to be measured were 32° F to 280° F. and because the desired measurement was temperature difference rather than absolute temperature, detailed calibration was not necessary.

(2) Thermocouple Circuit:

Each thermocouple circuit was completed by inserting a selector switch and a potentiometer on the copper wire between the hot and cold junctions. Parallel multi-circuits were used where an average temperature was required (See Fig. 8).

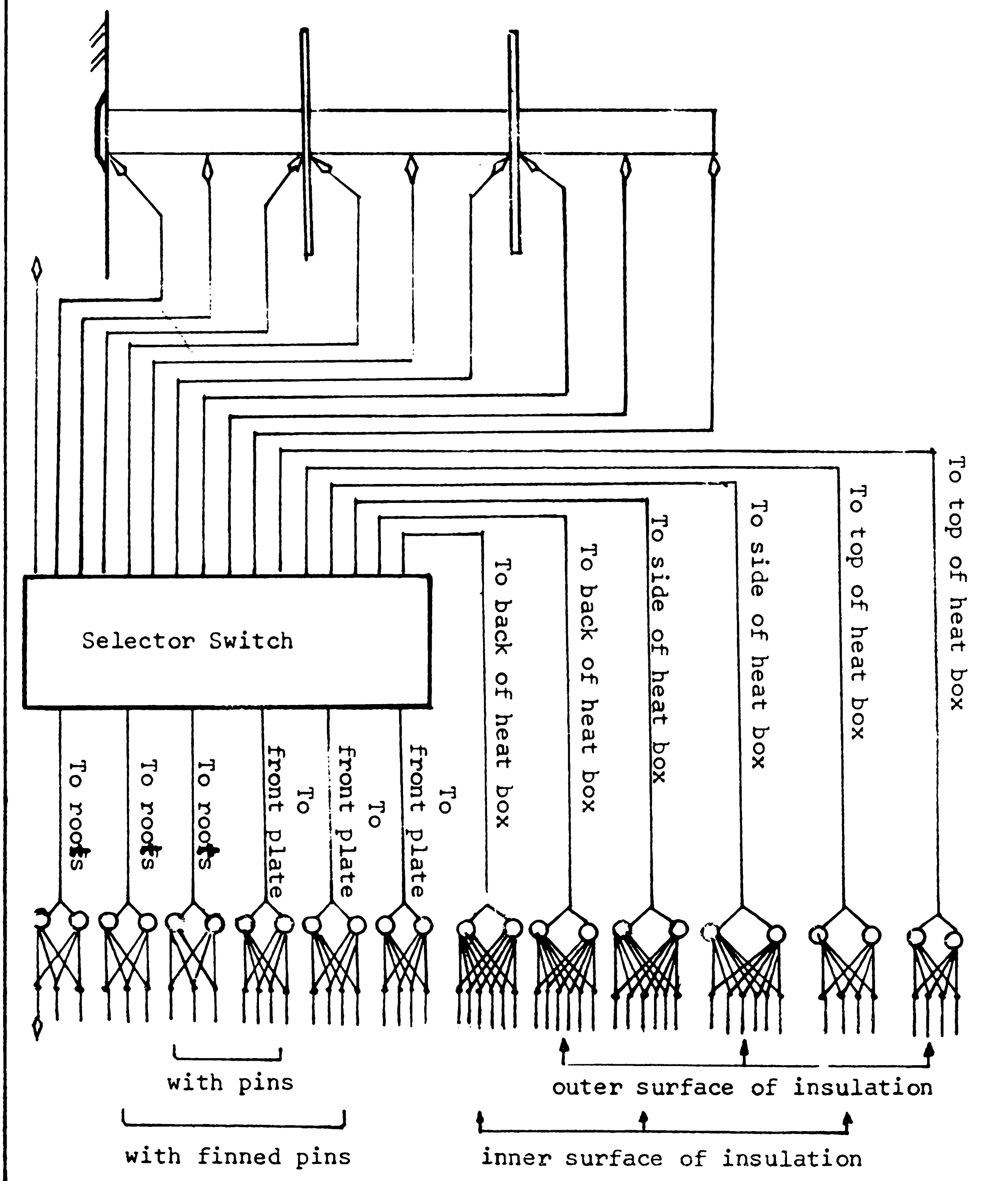


Fig. 8 Measuring Thermocouples (Hot Junctions only)

Attachment of the Thermocouples

The thermocouple hot junctions were attached to metallic surfaces by soldering, but for the inner and outer surfaces of the insulation, the hot junctions were put in contact with the surfaces (next to the inner insulation surface , the ready-made box had a layer of existing paint; thermocouples could not be soldered.) (See Fig. 7).

Experimental Operation

(1) By putting the front plate on the heat box and switching on the power supply circuit, the box was heated up gradually until the three thermocouples with their hot junctions at the roots of the pins had a reading of average emf corresponding to the given root temperature T_o. T_o was determined by $\theta_o + T_f$, where T_f was measured by the thermocouple with the hot junction in ambient air and θ_o was a predetermined constant temperature difference.

Several hours were required before the root temperature reached steady state. When T_o reached the predetermined value and did not change with time, the heat box and plain pins were then at static conditions. The power supply and temperature measurements from each thermocouple circuit were made.

(2) The front plate with plain pins was then replaced by the front plate with no pins. Following the same method as described in (1), the temperature rise of the box because of the removal

of pins was measured with heat supply unchanged.

(3) The front plate with three finned pins was then substituted for the plain plate. The power supply and temperatures were measured in the same way as described in (1).

(4) The front plate with the finned pins was then replaced by the plain plate again. The heat box was heated with the same heat supply as for the finned pins. The temperature rise of the box, because of the removal of the finned pins, was measured, following the same procedure.

(5) For measuring the temperature along the plain pin or the finned pin, single-circuit thermocouples were employed. Measurements of temperature were made when the root temperature of one pin or one finned pin was maintained at predetermined value at static condition. If the heat supply by keeping one pin at θ_o was slightly different from the case by keeping three pins at average θ_o, it was immaterial. The determination of the temperature distribution was dependent on θ_o, h and k.

List of Material

The following material was used in this experiment:

Thermocouple Wire.

Matched copper-constantan wires, obtained from the Mechanical Engineering Department, Virginia Polytechnic Institute were used for constructing thermocouples.

Steel Bar

SAE 1020, square bars, obtained from and cut into round rods by the machine shop, Mechanical Engineering Department, Virginia Polytechnic Institute were used as pins.

Steel Plate.

SAE 1020, 0.015 inch thickness, obtained from and cut into discs by the machine shop, Mechanical Engineering Department, Virginia Polytechnic Institute, was used as annular fins.

Steel Plate

About 1/16" thickness, obtained from the Mechanical Engineering Department, V.P.I. was used as front plate of the heat box. The front plate was used as the primary surface.

List of Apparatus

The following apparatus was used in this investigation:

Wattmeter

Weston type, rated current, 1 amp. voltage, low range 0-74, high range 75-150, power range 0-150 watts. Total resistance 5764 and 11528 ohm, manufactured by Daystrom, Inc., Weston Instrument Division, Newark, New Jersey. Used for measuring heat supply.

Ammeter

Range 0-3 amp., Weston type, manufactured by Daystrom, Inc. used in conjunction with wattmeter for checking current.

Voltmeter

Used in conjunction with wattmeter for checking voltage.

Auto transformer

Type W5MT, manufactured by General Radio Company, Cambridge, Mass. Input voltage 115 v. output voltage 0-135 v. Used for regulating electric power to heater.

Heater

Cartridge type, 135 watt, 115 volt, used as the heat generating source.

Selector Switch

Manufactured by Minneapolis-Honeywell Regulator Company, Philadelphia, Pa. Used to connect desired thermocouple circuit.

Potentiometer

Number 8662, Portable precision type, manufactured by Leeds and Northrup Company, Philadelphia, Pa., used for measuring emf between the hot junction and cold junction of a thermocouple.

Vacuum Bottle

Obtained from the Mechanical Engineering Department, V.P.I., used as ice bottle for the reference junctions of thermocouple circuits.

Heat Box

A ready-made painted steel box of 3" x 5" x 4" size, obtained from the Mechanical Engineering Department, V. P.I., used as primary heat surface for the pins or finned pins.

DATA AND RESULTS

Notation :

Q = Heat supplied by the heater, watts

θ_o = Average root temperature difference of the plain pins, °F

θ_p = Average temperature difference between front plate and air, °F

θ_b = Average temperature difference between inner and outer surfaces of insulation on the back of box, °F

θ_s = Average temperature difference between inner and outer surfaces of insulation on the sides of box, °F

θ_{Tb} = Average temperature difference between the inner and outer surfaces of insulation on the top and bottom of box, °F

T_f = Average temperature of the ambient air, °F

q_{p1} = Heat dissipation from the front plate without pins or finned pins, BTU/hr

q_{p2} = Heat dissipation from the front plate with pins or finned pins, BTU/hr

q_{Tb1}, q_{s1}, q_{b1} : Heat loss through insulation on top and bottom, sides and back of the heat box without pins or finned pins, BTU/hr

q_{Tb2}, q_{s2}, q_{b2} : Heat loss through insulation on top and bottom, sides and back of the heat box with pins or finned pins, BTU/hr

a. Measurement of temperature of the heat box with three plain pins

$\theta_o = 124.6°F.$

Q (w)	T_f (°F)	θ_o (°F)	θ_p (°F)	θ_{Tb} (°F)	θ_s (°F)	θ_b (°F)
36	82	206.6-82	210.8-82	210-109	191-89	197-97
36	85	209.6-85	214.3-85	210-109	193-90	198-98
36	85	209.6-85	214-85	210-110	194-91	197.5-99
36	81	205.6-81	210-81	209-107	191-88	196.5-97.5
36	76	200.6-76	205.5-76	205-104	188-86	193-95.5
36	84	208.6-84	212-84	210-110	192-89	197-99
Average		124.6	129	101	102.7	98.6

b. Measurement of temperatures of the heat box without pins

Q = 36 watts

Q (w)	T_f (°F)	θ_o (°F)	θ_p (°F)	θ_{Tb} (°F)	θ_s (°F)	θ_b (°F)
36	83	247-83	247-83	232-115	212-99	216-108
36	79	242-79	242-79	229-112	211-96	213-106
36	81	244-81	244-81	230-113	211-98	214-107
36	84	248-84	248-84	232-114	213-101	217-109
36	84	248-84	248-84	231-113	214-102	217-109
36	78	241-78	241-78	229-110	210-96	212-105
Average		163.5	163.5	117.7	113	107.7

c. Measurement of energy consumed within the wattmeter by turning off switch S(disconnect the load): Energy loss within wattmeter = 0.6 watts

Table 4. Measured Data of Heat Flow Rate from the Plain Pins to Still Air at Room Temperature

(a) Measurement of temperatures of the heat box with three finned pins

$\theta_o = 124.6^{o}F.$

Q (w)	T_f (°F)	θ_o (°F)	θ_p (°F)	θ_{Tb} (°F)	θ_s (°F)	θ_b (°F)
46.5	82	206.6-82	224.3-82	230-115	212-95	217-105
46.5	83	207.6-83	225-83	231-116	214-97	218-105
46.5	83	207.6-83	224.5-83	231-116	214-96.5	218-105
46.5	83	207.6-83	225-83	232-116	214-97	219-106
46.5	75	199.6-75	217-75	227-112	207-90	210-97
46.5	75.5	200.1-75.5	217.4-75.5	227-113	207.5-91	211.5-98
Average		124.6	142	115	117	113

(b) Measurement of temperatures of the heat box without finned pins

Q = 46.5 watts

Q (w)	T_f (°F)	θ_o (°F)	θ_p (°F)	θ_{Tb} (°F)	θ_s (°F)	θ_b (°F)
46.5	72	277-72	276-72	261-115	241-95	240-99
46.5	72	277-72	276-72	261-115	241-96	240-99
46.5	75	280-75	279-75	263-116	244-97	243.5-102.7
46.5	75	280-75	279-75	263-116	244-98	243.5-103
46.5	79	284-79	283-79	266-119	247-98	246-107
46.5	78.5	284-79	283-79	266-120	247-99	246-108
Average		205	204	146.5	146.8	140

(c) Measurement of Energy Consumed within the wattmeter by turning off switch S

Energy loss within wattmeter = 0.9 watt

Table 5. Measured Data of Heat Flow Rate from the Finned Pins to Still Air at Room Temperature

(a) For the plain pin

T_f	θ_0	θ_1	θ_2	θ_3	θ_4	θ_5	θ_L
83	124.6	103.0	90.0	80.5	73.5	71.0	69.0
83	124.6	103.5	91.0	81.0	74.0	70.5	68.0
83	124.6	103.5	92.0	81.0	75.0	71.0	69.0
82	124.6	102.0	91.0	80.5	74.0	71.0	70.5
82	124.6	103.0	90.5	81.5	74.0	71.0	69.0
82	124.6	103.0	91.0	81.0	74.0	72.0	69.0
Average		103	91	81	74	71	69

(b) For the finned pin

T_f	θ_o	θ_1	θ_2	θ_3	θ_3'	θ_4	θ_5	θ_5'	θ_L
81°F	124.6	98.0	81.5	80.0	64.0	56.0	56.0	51.5	49.0
81	124.6	98.0	81.5	80.0	63.5	56.0	55.0	50.0	49.0
81	124.6	98.0	81.0	80.0	63.0	56.0	55.0	51.0	48.0
81	124.6	97.5	81.0	80.0	63.0	56.0	55.0	51.0	49.0
81	124.6	98.0	81.0	80.0	62.0	56.0	55.0	51.5	49.0
81	124.6	97.5	81.0	80.0	63.0	57.0	55.0	51.5	49.5
Average		98	81	80	63	56	55	51	49

Table 6. Measured Data of Temperature Distribution

Calculation of Heat Transfer

(1) Heat dissipation from the plain pins to air

Given data:

Area of front plate, $A_p = \frac{4(5.25)}{144} = 0.146 \text{ ft}^2$

For top and bottom, $\frac{A}{L} = 2 \frac{3(5)}{144} \frac{4(12)}{3} = 3.33$

For sides, $\frac{A}{L} = 2 \frac{3(4)}{144} \frac{4(12)}{3} = 2.67$

For the back, $\frac{A}{L} = \frac{4(5)}{144} \frac{8(12)}{5} = 2.66$

From Equation 2-10C

For the heat-box without pins, convective heat transfer coefficient between front plate and air

$$h_{p1} = 0.29 \left(\frac{\Delta T}{L}\right)^{1/4}$$

$$= 0.29\left(\frac{163.3}{4/12}\right)^{1/4} = 1.362$$

For the heat-box with pins, convective heat transfer coefficient between front plate and air

$$h_{p2} = 0.29 \left(\frac{129}{4/12}\right)^{1/4} = 1.287$$

Heat loss through insulation of the box without pins

$q_{Tb1} = 3.33\ (117.7)k = 392k$

$q_{S1} = 2.67\ (113)k = 302k$

$q_{b1} = 2.66(107.7)k = 286k$

Heat loss through insulation = (392 + 302 + 286)k = 980k Btu/hr.

Heat loss from front plate, $q_{P1} = A_p h_{p1} \theta_{P1} = 0.146(1.362)(163.5) = 32.5$ Btu/hr.

Net heat supply = heat loss from front plate + heat loss through insulation in static condition

Therefore, (36 - 0.6)3.412 = 32.5 + 980k (1WaH = 3.412 Btu/hr)

$$k \simeq \frac{(36 - 0.6)\ 3.412 - 32.5}{980} = 0.09$$

Heat loss through insulation of the box with pins

$q_{Tb2} = 3.33\ (101)k = 3.33(101)0.09 = 30.27$ Btu/hr

$Q_{S2} = 2.67\ (102.3)\ 0.09 = 24.58$ Btu/hr.

$q_{b2} = 2.66\ (98.6)\ 0.09 = 23.6$ Btu/hr.

$q_{P2} = A_p h_{p2} \theta_{P2} = 0.146(1.287)(129) = 24.24$ Btu/hr.

Heat dissipation from the plain pins to air

= net heat supply

- heat loss through insulation

- heat loss from front plate

Thus, heat dissipation from the three plain pins to air

$$\begin{aligned} q &= (36-0.6)3.412 - (30.27+24.58+23.6) - 24.4 \\ &= 120.78-120.69 \\ &= 18.09 \text{ Btu/hr.} \end{aligned}$$

(2) Heat dissipation from the Finned Pin to Air

For the box without finned pins,

$$h_{P1} = 0.29\left(\frac{204(12)}{4}\right)^{1/4} = 1.44$$

$$q_{p1} = 0.146(1.44)(204)$$

$$= 42.9 \text{ Btu/hr.}$$

$$q_{Tb1} = 3.33(147)k = 489k$$

$$q_{S1} = 2.67\ (146)k = 390k$$

$$q_{b1} = 2.66\ (139.2)k = 370k$$

$$(46.5 - 0.9)3.412 = 42.9+(489+390+370)k$$

$$k = \frac{112.69}{1249} = 0.09 \text{ (approximate)}$$

For the box with finned pins

$$h_{P2} = 0.29\left(\frac{142(12)}{4}\right)^{1/4} = 1.317$$

$$q_{P2} = 0.146\ (1.317)\ 142 = 27.3 \text{ Btu/hr.}$$

$$q_{Tb2} = 3.33\ (115)\ 0.09 = 34.47 \text{ Btu/hr.}$$

$$q_{s2} = 2.67\ (117)0.09 = 28.12 \text{ Btu/hr.}$$

$$q_{b2} = 2.66(113)0.09 = 27.05 \text{ Btu/hr.}$$

Heat dissipation from the finned pins to air

= net heat supply - heat loss through insulation

- heat loss from front plate

$$q = (46.5 - 0.9)\ 3.412 - (34.47+28.12+27.05) - 27.3$$

$$= 155.59 - 116.94$$

$$= 38.65 \text{ Btu/hr.}$$

V. DISCUSSION

EXPERIMENTAL ACCURACY

The data obtained from the power supply apparatus employed in this investigation were reproducible, and the temperature measuring instrument as well as the copper-constantan thermocouples were accurate for the range of temperature measured.

The heat box, having the heater at its center, created slightly higher temperature at the root position where the central pin was to be attached. Steel plates were placed inside the box to make the root temperature uniform. This was done for both cases, (front plate without pins, with pins or with finned pins). The temperature at the root of the central pin still was consistently high. The average root temperature was measured by a multi-thermocouple circuit for measuring average emf. Since the root temperatures differed from each other slightly (1 to 5°F), measurement of average emf was considered acceptable.

As to the attachment of thermocouples, the tighter or looser contact with the outer surface of insulation made the measured emf waver slightly. This error was expected to be reduced by the average of repeated measurements. Thermocouple attachments elsewhere were stable.

Measurements were carried out at a room temperature around 80°F. In case of room temperature change during measurement, the pre-determined root temperature had to be adjusted to kepp θ_o unchanged.

Heat dissipation from pins or finned pins is dependent upon h, k and θ_o. For a given θ_o, although slight changes of room temperature result in changes in the pin temperature, the changes in h and k are so small that they can be neglected.

The average temperature measurement of insulation was an approximation. It was insufficient to measure the true average temperature, but since the desired measurement was temperature difference, there was no appreciable effect on accuracy. Same errors caused by an insufficient number or improper position of thermocouples in hot and cold temperature measurements had little effect on temperature difference.

COMPARISON OF THE TWO-DISC FINNED PIN WITH THE PLAIN FIN

Temperature Distribution

The temperature distribution curves plotted from calculated and measured data are shown in Figure 9. Curve 1, the calculated temperature distribution, is higher than Curve 2, the measured temperature distribution. The measured temperatures are considered comparatively accurate because all temperatures were measured on the pin itself. Since the temperature distribution is dependent only upon h and k, and since the value of k is accurate, it is concluded that a small h made the calculated θ greater than measured θ. The value of h was calculated by neglecting h_r, heat transfer coefficient by radiation, which might be noticeable because of the larger emmissivity of the pin surface which was not highly polished. A smaller average h results in less heat dissipation along the pin, and therefore, the rate of temperature drop with respect to distance along the pin was smaller than the actual pin, i.e., the calculated curve is higher than the measured curve.

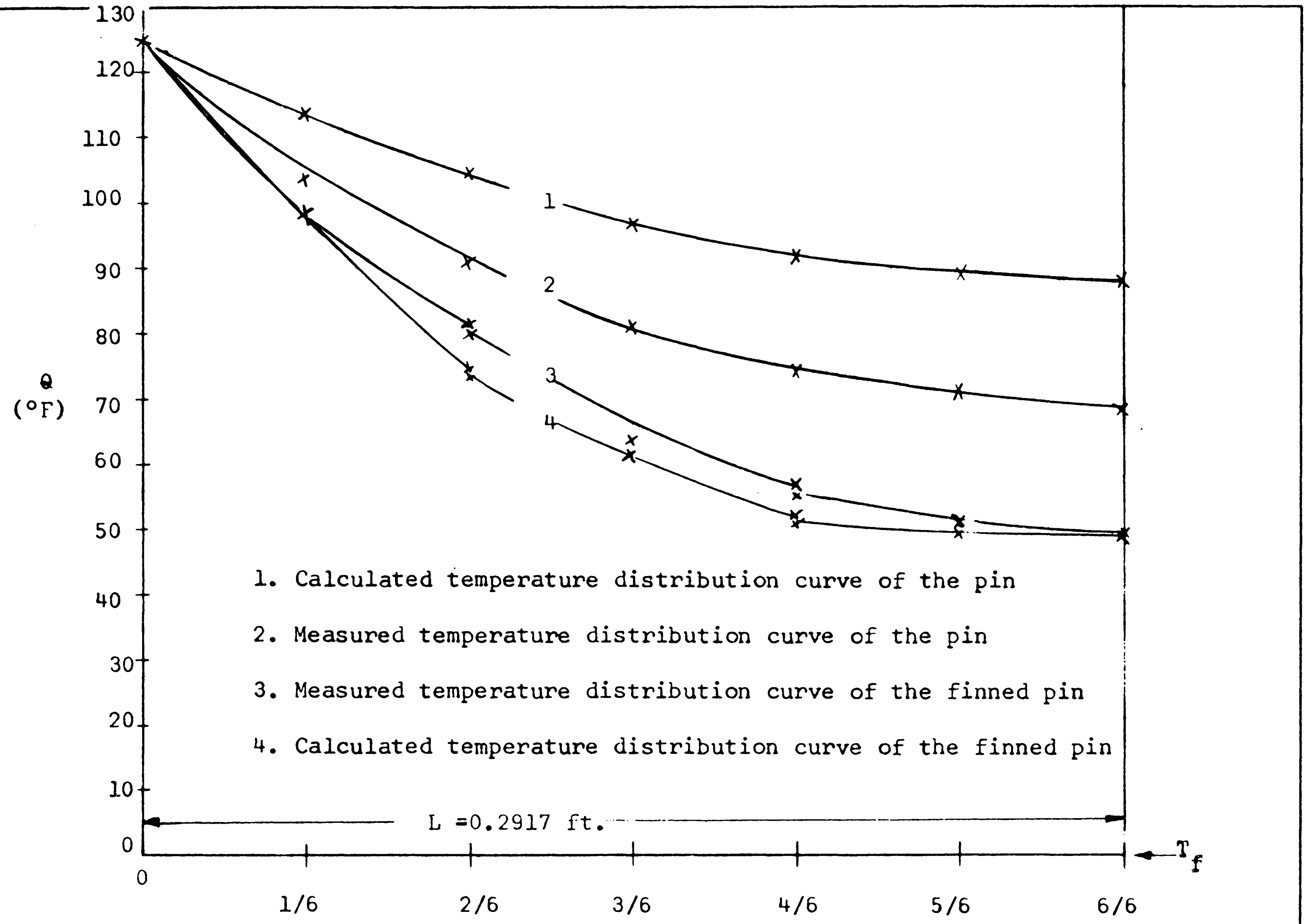


Figure 9. Calculated and Measured Temperature Distribution Curves

Curve 4, the calculated temperature distribution of the finned pin is slightly different from Curve 3, the measured temperature distribution. This indicates that the average value of h used in the calculations was slightly higher than the actual h. Even if the average h used in calculations were identical to the actual h, the calculated curve would approach, but not exactly coincide, with the measured curve for the calculations were usually made by using an average h, while the local true h varied appreciably for the finned pin which reduced the temperature much more than the plain pin.

For a given root temperature difference θ_o, a smaller average h results in smaller calculated heat dissipation. This was why the measured heat dissipation was so close to the calculated value in this investigation. A slightly higher calculated dissipation would make the measured heat dissipation relatively lower, which might be more reasonable because of the additional resistance to heat transfer of soldered contact between fins and pins, and between the pins and front plate.

Heat-Flow Rate

For the plain pin,

Calculated heat-flow rate	6.12 Btu/hr per pin
Measured heat-flow rate	18.09 Btu/hr per three pins
Per cent error	$\frac{3(6.12)-18.09}{3(6.12)} = 1.47\%$

For the finned pins,

Calculated heat-flow rate 13,0955 Btu/hr per finned pin

Measured heat-flow rate 38.65 Btu/hr per three finned pins

percent error $\frac{3(13.0955)-38.65}{3(13.0955)} = 1.62\%$

Measured heat-flow rate from the plain pin per cubic ft. volume=18.09/0.000223(3)=$\frac{81000}{3}$ Btu/hr.

Measured heat-flow rate from the finned pin per cubic ft. volume=38.65/0.000257(3)=$\frac{150000}{3}$ Btu/hr.

Per cent increase in heat-flow rate from the finned pin with same volume of plain pin, $\frac{150000-81000}{81000} = 85\%$

<u>Effectiveness</u> (using measured data)

For the front plate without pins, $h_p = 1.362$ Btu/hr ft^{2}F

For the front plate without pins, $\theta_p = 163.5$ F

For the front plate without finned pins $h_p = 1.44$ Btu/hr.ft^{2}F

For the front plate without finned pins, $\theta_p = 205$ F

Heat dissipation from the root area without plain pins

$$q' = Ah_p\theta_p = 0.000767\ (1.362)(163.3) = 0.172 \text{ Btu/hr}$$

Heat dissipation from the root area without finned pins,

$$q' = Ah_p\theta_p = 0.000767\ (1.44)\ (205) = 0.228 \text{ Btu/hr}$$

Effectiveness of the plain pins $= \frac{18.09}{3(0.172)} = 35$

Effectiveness of the finned pins $= \dfrac{38.65}{3(0.228)} = 56.5$

Per cent increase in effectiveness of the two-disc finned pins $= \dfrac{56.5 - 35}{35} = 61\%$

Limiting Condition

Suppose the volume of a finned pin is equal to m+1 pins, the increase of surface of the finned pin ceases when the surface of the finned pin is equal to the surface of the pins.

Then, $$\frac{\text{surface of the finned pin}}{\text{surface of the m+1 pins}} = \frac{s_1 + ns_2}{s_1 + ms_1}$$

Where n = number of annular fins

s_2 = surface of each annular fin with outer edge area neglected

s_1 = surface of each pin with end area neglected

Since volume of annular fins = volume of m pins

$$n\frac{\pi}{4}(D_2^{\,2} - D_1^{\,2})\,t = m\frac{\pi}{4}D_1^{\,2}L, \quad D_2^{\,2} - D_1^{\,2} = \frac{mD_1^{\,2}L}{nt}$$

$$\frac{nS_2}{mS_1} = \frac{(2n)\frac{\pi}{4}(D_2^{\,2} - D_1^{\,2})}{m\pi D_1 L} = \frac{\frac{n\pi}{2}\left(\frac{mD_1^{\,2}L}{nt}\right)}{m\pi D_1 L} = \frac{D_1}{2t}$$

When $\dfrac{nS_2}{mS_1} = \dfrac{D_1}{2t} = 1$, the surface of the finned pin is equal to the surface of m+1 pins having the same volume. Since the heat dissipation from a finned pin depends mainly on increased surface, $\dfrac{D}{2t} = 1$ is approximately the limiting condition that a finned pin has no more increase in heat dissipation. The value of $\dfrac{D}{2t}$ should be as large as possible in determining the inter-relation between the dimensions of the pins and annular fins.

SUMMARY

It has been shown that the experimental accuracy was within acceptable limit. Even with a higher h used for calculation for the given root temperature the experimental results were acceptable. For example, from the measured temperatures the value of h was approximately 2.66 for the plain pin (corresponding to an emissivity of 0.45) by a trial-and-error calculation. With this value of h, the calculated heat-flow rate equals 6.5 Btu/hr., and the experimental accuracy equals $\frac{3(6.5) - 18.09}{3(6.5)} = 7.2\%$, which is acceptable.

The basic heat transfer equations derived in this thesis have been proven to be logical and the pattern of process for deriving these mathematical equations can be used by others with confidence, and can be applied to any finned pin regardless of the number and spacing of annular fins, natural or forced convection.

VI. CONCLUSIONS

The object of this investigation was accomplished since: (1) the heat exchange capability and effectiveness of a finned pin were found to be much better than those of a plain pin, and (2) the basic heat transfer equations derived for a finned pin were verified by the experiment.

The increase in heat transfer by employing a finned pin depends on the dimensions, spacing and number of annular fins. In general, the thickness of annular fins should be much smaller compared to diameter of the pin. If a finned pin were constructed of a pin and a bank of annular fins, the increase in heat transfer and pin effectiveness could be much higher.

It is expected that the finned pins can be applied to practical use. No doubt where pins can be used, finned pins can be employed to improve heat transfer capability. It is also possible for the designer to use finned pins as substitute for ordinary fins of various type with or without re-designing the primary heat surface.

VII. RECOMMENDATIONS

It is recommended for the coming investigators of finned pins that:

1. A cylindrical heat-box be used because
   (1) The heat loss throug insulation can be accurately measured because no corner effect exists and (2) the temperature will be uniform circumferentially on the cylindrical surface, with a heater at center.
2. A finned pin be constructed of a pin and a bank of annular fins.
3. Perform experiments on a group of staggered finned pins.
4. Forced convection be employed.
5. Average heat transfer coefficient of annular fins be pre-determined experimentally, if possible.
6. Mathematical solution of optimum dimensions for finned pins be derived, if possible.
7. Experiment be made in an air conditioned room.

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Professor R. S. Lecky, Mathematics Department

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$r_2/r_1 = 2,\quad r_1 = 1,\quad r_2 = 2R_1 = 2,\quad \frac{2}{(1+r_2/r_1)Z} = \frac{2}{3Z}$

Z	R_1Z	R_2Z	$K_1(R_1Z)$	$I_1(R_2Z)$	$I_1(R_1Z)$	$K_1(R_2Z)$	$K_0(R_2Z)$	$I_0(R_1Z)$	B	$\frac{2}{3Z}$	e
0	0	0		0	0			1.000			1.000
0.2	0.2	0.4	4.7760	0.2040	0.1005	2.1844	1.7527	1.010	0.293	2/0.6	0.977
0.4	0.4	0.8	2.1844	0.4329	0.2040	0.8618	1.1145	1.0404	0.558	2/1.2	0.930
0.6	0.6	1.2	1.3028	0.7147	0.3137	0.4346	0.7775	1.0920	0.773	2/1.8	0.860
0.8	0.8	1.6	0.8618	1.0848	0.4329	0.2406	0.5653	1.1665	0.930	2/2.4	0.776
1.0	1.0	2.0	0.6019	1.5906	0.5652	0.1399	0.4210	1.2661	1.035	2/3.0	0.691
1.2	1.2	2.4	0.4346	2.2981	0.7147	0.0837	0.3185	1.3937	1.107	2/3.6	0.615
1.4	1.4	2.8	0.3208	3.3011	0.8861	0.0511	0.2437	1.5534	1.150	2/4.2	0.548
1.6	1.6	3.2	0.2406	4.7343	1.0848	0.0316	0.1880	1.7500	1.165	2/4.8	0.485
1.8	1.8	3.6	0.1826	6.7927	1.3172	0.0198	0.1459	1.9896	1.177	2/5.4	0.436
2.0	2.0	4.0	0.1399	9.7595	1.5906	0.0125	0.1139	2.2796	1.180	2/6.0	0.394
2.2	2.2	4.4	0.1079	14.0462	1.9141	0.00792	0.0893	2.6291	1.178	2/6.6	0.357
2.4	2.4	4.8	0.0837	20.2528	2.2981	0.00506	0.0702	3.0493	1.172	2/7.2	0.326
2.6	2.6	5.2	0.0653	29.2543	2.7554	0.00324	0.0554	3.5533	1.163	2/7.8	0.298
2.8	2.8	5.6	0.0511	42.3283	3.3011	0.00208	0.0438	4.1573	1.156	2/8.4	0.275

Table 7. Calculated data for plotting efficiency curves of annular fins of constant thickness

$\frac{r_2}{r_1} = 3$ $\quad R_1 = \frac{1}{2}$ $\quad R_2 = 3R_1 = \frac{3}{2}$, $\quad \frac{2}{(1 + r_2/r_1)Z} = \frac{1}{2Z}$

Z	R_1Z	R_2Z	$K_1(R_1Z)$	$I_1(R_2Z)$	$I_1(R_1Z)$	$K_1(R_2Z)$	$K_0(R_1Z)$	$I_0(R_1Z)$	B	$\frac{1}{2Z}$	e
0.0	0.00	0.00	∞	0	0	∞	∞	1.00		∞	1.000
0.2	0.10	0.30	9.8538	0.1517	0.0500	3.0560	2.427	1.0025	0.3910	1/0.4	0.977
0.4	0.20	0.60	4.7760	0.3137	0.1005	1.3028	1.7527	1.0100	0.7329	1/0.8	0.916
0.6	0.30	0.90	3.0560	0.4971	0.1517	0.7165	1.3725	1.0226	0.9967	1/1.2	0.831
0.8	0.40	1.20	2.1844	0.7147	0.2040	0.4346	1.1145	1.0404	1.1792	1/1.6	0.737
1.0	0.50	1.50	1.6564	0.9817	0.2579	0.2774	0.9244	1.0635	1.2928	1/2.0	0.646
1.2	0.60	1.80	1.3028	1.3172	0.3137	0.1826	0.7775	1.0920	1.3557	1/2.4	0.565
1.4	0.70	2.10	1.0503	1.7455	0.3719	0.1227	0.6605	1.1263	1.3846	1/2.8	0.495
1.6	0.80	2.40	0.8618	2.2981	0.4329	0.0837	0.5653	1.1665	1.3921	1/3.2	0.435
1.8	0.90	2.70	0.7165	3.0161	0.4971	0.0577	0.4867	1.2130	1.3872	1/3.6	0.385
2.0	1.00	3.00	0.6019	3.9534	0.5652	0.0402	0.4210	1.2661	1.3741	1/4.0	0.344
2.2	1.10	3.30	0.5098	5.1810	0.6375	0.0281	0.3656	1.3262	1.3582	1/4.4	0.309
2.4	1.20	3.60	0.4346	6.7927	0.7147	0.0198	0.3185	1.3937	1.3409	1/4.8	0.279
2.6	1.30	3.90	0.3725	8.9128	0.7973	0.0140	0.2782	1.4693	1.3235	1/5.2	0.255
2.8	1.40	4.20	0.3208	11.7056	0.8861	0.0099	0.2437	1.5534	1.3062	1/5.6	0.233

$\frac{r_2}{r_1} = 4$, $R_1 = \frac{1}{3}$, $R_2 = 4R = \frac{4}{3}$, $\frac{Z}{\frac{(1+r_2)Z}{n}} = \frac{2}{5Z}$

Z	R_1Z	R_2Z	$K_1(R_1Z)$	$I_1(R_2Z)$	$I_1(R_1Z)$	$K_1(R_2Z)$	$K_0(R_1Z)$	$I_0(R_1Z)$	B	$\frac{2}{5Z}$	e
0.18	0.06	0.24	16.5637	0.1209	0.0300	3.9191	2.9329	1.0009	0.4407	2/0.90	0.979
0.36	0.12	0.48	8.1688	0.2470	0.0601	1.7447	2.2479	1.0036	0.8294	2/1.80	0.922
0.54	0.18	0.72	5.3447	0.3838	0.0904	1.0083	1.8537	1.0081	1.1343	2/2.70	0.840
0.72	0.24	0.96	3.9191	0.5375	0.1209	0.6447	1.5798	1.0145	1.3496	2/3.60	0.750
0.90	0.30	1.20	3.0560	0.7147	0.1517	0.4346	1.3725	1.0226	1.4861	2/4.50	0.660
1.08	0.36	1.44	2.4760	0.9235	0.1829	0.3034	1.2075	1.0327	1.5620	2/5.40	0.579
1.26	0.42	1.68	2.0590	1.1733	0.2147	0.2156	1.0721	1.0446	1.5977	2/6.30	0.507
1.44	0.48	1.92	1.7447	1.4758	0.2470	0.1557	0.9584	1.0584	1.6059	2/7.20	0.446
1.62	0.54	2.16	1.4994	1.8449	0.2800	0.1138	0.8614	1.0742	1.5977	2/8.10	0.394
1.80	0.60	2.40	1.3028	2.2981	0.3137	0.0837	0.7775	1.0920	1.5801	2/9.00	0.351
1.98	0.66	2.64	1.1420	2.8569	0.3483	0.0623	0.7043	1.1119	1.5570	2/9.90	0.315
2.16	0.72	2.88	1.0083	3.5480	0.3838	0.0465	0.6399	1.1339	1.5323	2/1.08	0.284
2.34	0.78	3.12	0.8955	4.4049	0.4204	0.0348	0.5829	1.1580	1.5070	2/1.17	0.258
2.52	0.84	3.36	0.7993	5.4691	0.4581	0.0262	0.5321	1.1843	1.4823	2/1.26	0.235
2.70	0.90	3.60	0.7165	6.7927	0.4971	0.0198	0.4867	1.2130	1.4586	2/1.35	0.216

$$\frac{r_2}{r_1} = 5, \quad R_1 = \frac{1}{4} = 0.25 \quad R_2 = 5R_1 = 1.25, \quad \frac{2}{(1 + r_2/r_1)Z} = \frac{1}{3Z}$$

Z	R_1Z	R_2Z	$K_1(R_1Z)$	$I_1(R_2Z)$	$I_1(R_1Z)$	$K_1(R_2Z)$	$K_0(R_1Z)$	$I_0(R_2Z)$	B	$\frac{1}{3Z}$	e
0.2	0.05	0.25	19.9097	0.1260	0.0250	3.7470	3.1142	1.0006	0.5831	1/0.6	0.972
0.4	0.10	0.25	9.8538	0.2579	0.0501	1.6564	2.4271	1.0025	1.0752	1/1.2	0.896
0.6	0.15	0.75	6.4775	0.4020	0.0752	0.9496	2.0300	1.0056	1.4291	1/1.8	0.794
0.8	0.20	1.00	4.7760	0.5652	0.1005	0.6019	1.7527	1.0100	1.6509	1/2.4	0.688
1.0	0.25	1.25	3.7470	0.7553	0.1260	0.4035	1.5415	1.0157	1.7656	1/3.0	0.588
1.2	0.30	1.50	3.0560	0.9817	0.1517	0.2774	1.3725	1.0226	1.8135	1/3.6	0.504
1.4	0.35	1.75	2.5591	1.2555	0.1777	0.1960	1.2327	1.0309	1.8163	1/4.2	0.432
1.6	0.40	2.00	2.1844	1.5906	0.2040	0.1399	1.1145	1.0404	1.7950	1/4.8	0.374
1.8	0.45	2.25	1.8915	2.0040	0.2307	0.1010	1.0129	1.0513	1.7636	1/5.4	0.327
2.0	0.50	2.50	1.6564	2.5167	0.2579	0.0739	0.9244	1.0635	1.7254	1/6.0	0.288
2.2	0.55	2.75	1.4637	3.1554	0.2855	0.0544	0.8466	1.0771	1.6867	1/6.6	0.256
2.4	0.60	3.00	1.3028	3.9534	0.3137	0.0402	0.7775	1.0920	1.6480	1/7.2	0.229
2.6	0.65	3.25	1.1668	4.9525	0.3425	0.0299	0.7159	1.1084	1.6119	1/7.8	0.207
2.8	0.70	3.50	1.0503	6.2058	0.3719	0.0222	0.6605	1.1263	1.5740	1/8.4	0.187

$r_2/r_1 = 6$, $R_1 = 1/5 = 0.2$, $R_2 = 6R_1 = 1.2$, $\dfrac{2}{(1+r_2/r_1)Z} = \dfrac{2}{7Z}$

Z	R_1Z	R_2Z	$K_1(R_1Z)$	$I_1(R_2Z)$	$I_1(R_1Z)$	$K_1(R_2Z)$	$K_0(R_1Z)$	$I_0(R_1Z)$	B	$\frac{2}{7Z}$	e
0.1	0.02	0.12	49.9547	0.0601	0.0100	8.1688	4.0285	1.0001	0.3472	2/0.7	0.992
0.3	0.06	0.36	16.5637	0.1829	0.0300	2.4760	2.9329	1.0009	0.9803	2/2.1	0.934
0.5	0.10	0.60	9.8538	0.3137	0.0501	1.3028	2.4271	1.0025	1.4635	2/3.5	0.836
0.7	0.14	0.84	6.9615	0.4581	0.0702	0.7993	2.0972	1.0049	1.7762	2/4.9	0.725
0.9	0.18	1.08	5.3447	0.6227	0.0904	0.5266	1.8537	1.0081	1.9467	2/6.3	0.618
1.1	0.22	1.32	4.3092	0.8146	0.1107	0.3602	1.6620	1.0121	2.0194	2/7.7	0.525
1.3	0.26	1.56	3.5880	1.0426	0.1311	0.2553	1.5048	1.0170	2.0275	2/9.1	0.446
1.5	0.30	1.80	3.0560	1.3172	0.1517	0.1826	1.3725	1.0226	2.0043	2/10.5	0.382
1.7	0.34	2.04	2.6470	1.6510	0.1725	0.1310	1.2587	1.0291	1.9647	2/11.9	0.330
1.9	0.38	2.28	2.3227	2.0598	0.1935	0.0976	1.1596	1.0364	1.9140	2/13.3	0.288
2.1	0.42	2.52	2.0590	2.5628	0.2147	0.0722	1.0721	1.0446	1.8637	2/14.7	0.254
2.3	0.46	2.76	1.8405	3.1840	0.2361	0.0537	0.9943	1.0536	1.8146	2/16.1	0.225
2.5	0.50	3.00	1.6564	3.9534	0.2579	0.0402	0.9244	1.0635	1.7683	2/17.5	0.202
2.7	0.54	3.24	1.4994	4.9081	0.2800	0.0302	0.8614	1.0742	1.7254	2/18.9	0.185

$r_2/r_1=7$, $R_1 = 1/6$, $R_2= 7R_1 = 7/6$, $\frac{2}{(1+r_2/r_1)Z} = \frac{1}{4Z}$

Z	R_1Z	R_2Z	$K_1(R_1Z)$	$I_1(R_2Z)$	$I_1(R_1Z)$	$K_1(R_2Z)$	$K_0(R_1Z)$	$I_0(R_1Z)$	B	$\frac{1}{4Z}$	e
0.18	0.03	0.21	33.2715	0.1056	0.0150	4.5317	3.6235	1.0002	0.7010	1/0.72	0.974
0.36	0.06	0.42	16.5637	0.2147	0.0300	2.0590	2.9329	1.0009	1.2988	1/1.44	0.902
0.54	0.09	0.63	10.9749	0.3309	0.0450	1.2186	2.5310	1.0020	1.7376	1/2.16	0.804
0.72	0.12	0.84	8.1688	0.4581	0.0601	0.7993	2.2479	1.10036	2.0164	1/2.88	0.700
0.90	0.15	1.05	6.4775	0.6008	0.0752	0.5558	2.0300	1.0056	2.1647	1/3.60	0.601
1.08	0.18	1.26	5.3447	0.7636	0.0904	0.3973	1.8537	1.0081	2.2276	1/4.32	0.516
1.26	0.21	1.47	4.5317	0.9522	0.1056	0.2905	1.7062	1.0111	2.2334	1/5.04	0.443
1.44	0.24	1.68	3.9191	1.1733	0.1209	0.2156	1.5798	1.0145	2.2063	1/5.76	0.383
1.62	0.27	1.89	3.4405	1.4346	0.1362	0.1620	1.4697	1.0183	2.1866	1/6.48	0.337
1.80	0.30	2.10	3.0560	1.7455	0.1517	0.1227	1.3725	1.0226	2.1079	1/7.20	0.293
1.98	0.33	2.31	2.7402	2.1171	0.1673	0.0939	1.2857	1.0274	2.0847	1/7.92	0.263
2.16	0.36	2.52	2.4760	2.5628	0.1829	0.0722	1.2075	1.0327	1.9975	1/8.64	0.231
2.34	0.39	2.73	2.2518	3.0989	0.1987	0.0557	1.1367	1.0384	1.9459	1/9.36	0.207
2.52	0.42	2.94	2.0590	3.7453	0.2147	0.0433	1.0721	1.0446	1.8969	1/10.18	0.186
2.70	0.45	3.15	1.8915	4.5256	0.2307	0.0336	1.0129	1.0513	1.8514	1/10.80	0.171

ABSTRACT

Extensive investigations employing fins as extended surfaces have been conducted, but relatively little experimental work has been conducted with pins, no information is available on the heat transfer characteristics of a pin and annular fin combination, i.e., a finned pin.

The increased surface area of a finned pin would promote more heat transfer. In this thesis a theoretical investigation of the basic heat transfer characteristics of a finned pin, and an experimental investigation to verify the theoretical result were conducted.

1. Theoretical investigation consists of:

   (1) Optimum dimensions
   (2) Sample calculation of optimum dimensions
   (3) Derivation of heat transfer equations
   (4) Sample calculations of heat flow-rate and temperature distribution

2. Experimental investigation consists of:

   (1) Set-up of experimental equipment
   (2) Measurements of heat flow-rate and temperature distribution
   (3) Comparison of theoretical results with measured results

3. Conclusions: The conclusions were based on the comparison of the two-disc finned pin with the plain pin.

   (1) For the two-disc finned pin employed in this thesis:

Increase in heat-flow-rate:	85%
Increase of effectiveness:	61%

   (2) In general, the increase in heat flow-rate depends on the material, dimensions, temperature difference and spacing of annular fins.