

AN EXPERIMENT IN PROGRAMMED BUSINESS MATHEMATICS
AT CHURCHLAND HIGH SCHOOL
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by

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CHAPTER I

THE PROBLEM

Background for the Study

Today, more than ever before, schools are overcrowded. This overcrowding further complicates the problem of handling individual differences in the classroom. Educators are vainly seeking ways to cope with these individual differences.

For centuries, the common method of teaching has been that of lecturing to the student and assigning him outside reading. The student did not usually know whether he understood the material he was studying until the next day, sometimes even days and weeks later when he was tested. It is possible that the student's lack of knowledge of his progress caused anxiety and depression until he took a test. Continued frustration was likely to ensue until his test was finally returned with the grade marked on it.

In recent years, men like Skinner and other educational psychologists have been studying new applications of old theories and laws of learning. Making new applications of the law of "effect"¹ and the reinforcement theory,² authors are now writing texts which are

¹Charles E. Skinner, Elementary Educational Psychology, (New York: Prentice-Hall, Inc., 1950), p. 227.

²J. M. Stephens, Educational Psychology, (New York: Henry Holt and Company, Inc., 1956), p. 279.

entirely new in the mechanics of presentation. The basis of these mechanics is called programmed instruction.

The entire text in a programmed-instruction type of presentation is written in frames. Each frame contains three parts. It (a) presents one minute bit of instruction, (b) asks a question based upon the understanding of the instruction, and (c) provides a reinforcement (the answer). After the learner finds himself to be right in his response, he proceeds to the next frame where he is given more information which builds upon the information presented in the previous frame.

Statement of the Problem

Huffman has recently completed a business mathematics text in programmed form.¹ His text was developed for post-high school students and college freshmen. The problem for this study is to adjust "Part I" of Dr. Huffman's programmed mathematics text to a level of comprehension suitable for high school students of low ability. This adjustment would enable a high school student of low ability to work at his own rate of speed. Programmed instruction would also enable the fast learner to work ahead of the class and cover more material than he would ordinarily if he were forced to

¹Harry Huffman, College Business Mathematics, Part I, "Rebuilding the Fundamentals," (New York: The Center for Programmed Instruction, 1961)

progress with students of average ability. The adjusted text would provide the teacher with a tool for taking care of individual differences in the classroom.

Three Needs for the Study

The Need to Provide for Individual Differences.—Bernard¹

indicates that in a classroom, regardless of the grade level, the teacher will be faced by perhaps thirty or more students; and each student will have many individual difficulties in learning. The teacher's words of advice and instruction will be quickly grasped by some. Some will not even know what he is speaking about. Some of the students will quickly and eagerly perform the assigned tasks with precision and confidence in their results. Others will struggle along, very unsure, frustrated, and dejected at the thought of making a low mark on their work. Still others will not even attempt to do the work because they know they can never attain the goal set up for them by the teacher.

Of course, in many instances, teachers do everything possible to handle these individual differences. Smith² said that there is

¹Harold W. Bernard, Psychology of Learning and Teaching, (New York: McGraw-Hill Book Company, Inc., 1954), p. 222.

²Henry P. Smith, Psychology in Teaching, (New York: Prentice-Hall, Inc., 1954), p. 292.

probably no school in the country that fails to make "some" provision for individual differences. During the past two decades, the trend has been toward making more and more provisions in the classroom for these individual differences. With increasing emphasis on improved professional training of our teachers and with continual improvement of instructional materials, we can expect this trend to persist.

Increased Computations Necessary in Business.—Today, there is a growing demand for statistical records in business and industry, which is partly caused by local and Federal legislation concerning tax records, social security, and other regulations. To do these computations, there is an ever increasing influx of young people into our modern business offices who are often ill-prepared to meet the demands of their employers. Business needs more young people who have developed computational skills.

The Need for Business Mathematics in Vocational Training.—With the emphasis on mass education today, some students are being "pushed" through our high schools with insufficient knowledge of basic mathematics. The teacher of business education feels obligated to help students in his program supplement their knowledge of business mathematics. Upon graduation from high school, students going out to earn a living in the clerical field need a good business mathematics background. Employers in our communities expect business education

¹Ibid.

students whom they hire to have a practical working knowledge of business mathematics. An important characteristic of programmed instruction is that each student works at his own rate of speed. Therefore, the business mathematics student, using programmed instruction would not hold back the class if he is a slow learner. If he is a fast learner, he will not waste his time waiting for the class to catch up with him.

Objectives of the Study

As a basis for the study, two major objectives were established as follows:

1. The first objective of the experiment was to adjust Huffman's programmed business mathematics text to a level of comprehension of high school students of below average ability so that their completed work, without a teacher's help, would be approximately 95 per cent accurate.
2. The second objective of the study was to determine whether students could retain the material covered in the programmed text. This objective was achieved through intermediate tests given to each student-editor throughout the unit.

Delimitations of the Study

The study was delimited as follows:

1. The student-editors who participated in the study were restricted to seniors attending Churchland High School.
2. Each student-editor was a business education major.
3. A panel of only three student-editors worked on the programmed unit at any one time.
4. A total of fifteen student-editors (five panels) participated in the study.
5. The programmed unit in business mathematics was a review of the fundamentals of addition, subtraction, multiplication, division, fractions, and decimals.
6. Students with intelligence quotients ranging from 79 to 100 participated in the study.

Definition of Terms

Cue (or Prompt): Cues are devices used within a frame to give hints as to what response is needed to successfully complete the question or problem in the frame.

Frame: Each frame is divided into three parts: (a) a step of instruction, (b) a step to determine understanding of the instruction, and (c) a reinforcement (provision for immediate answer to the question).

Original Program: A series of 432 frames on the fundamentals of business mathematics, developed by Huffman for use by post-high school students and college freshmen.

Programmed Instruction: This is a method of instruction in which a body of knowledge is broken down into small steps, each containing a minute fact or bit of instruction. Each step builds on the knowledge of the previous step. A student should be able to handle a programmed text with no assistance.

Panel: A panel is composed of three student-editors.

Revised Program: The program adjusted by the writer from Huffman's original program.

Student-Editor: The student-editor was a senior attending Churchland High School, enrolled as a business major, and was below average ability (Intelligence quotients ranged from 79 to 100).

Section: The programmed unit used in this study was broken down into six sections: addition, subtraction, multiplication, division, fractions, and decimals.

CHAPTER II

REVIEW OF LITERATURE

An Introduction to the Review

In this chapter the writer will summarize the early developments of Pressey and Peterson in programmed learning. Recent developments by B. F. Skinner will be discussed and a comparison of Skinner's and Pressey's methods will be made. Finally, the writer will give a brief analysis of the advantages and disadvantages of programmed learning as discussed by various educators.

Early Developments in Programmed Learning

Professor John Blyth of Hamilton College points out that programmed instruction combines: (a) the Socratic method--teaching by asking questions and (b) the Cartesian method--analyzing a problem into its smallest parts and proceeding from the simple to the complex.¹

Contributions of Sidney L. Pressey

Sidney L. Pressey seems to have invented the first teaching machine in 1924. His machine used a multiple-choice keyboard. The machine was about the size of an ordinary portable typewriter, but

¹Jay W. Stein, "Machine Teachers for Your School?" The Rotarian, No. 3 (Evingston, Illinois: September, 1961), p. 37.

much simpler.¹ It had a small window through which the student could read a partial statement. Next to the statement would be found four possible word choices, one of which would correctly complete the partial statement. The student pushed a key corresponding to what he thought was the right answer. If he was right, the machine automatically turned up to the next question. If he pushed the wrong key, the machine did not advance to the next question. When he finally pushed the right key, the machine then advanced to the next question.

Even though the chief purpose of Pressey's first machine was to test, he maintained that it could also be used as a teaching device.²

Contributions of J. C. Peterson

One of S. L. Pressey's former students and his brother, J. C. Peterson, contributed a simplified variant to Pressey's multiple-choice response apparatus. They used a self-scoring "chemo card."

¹Sidney L. Pressey, "A Simple Apparatus Which Gives Tests and Scores--and Teaches," School and Society, XXIII (March 20, 1926), as reprinted in Teaching Machines and Programmed Learning, (eds.) A. A. Lumsdaine and Robert Glaser (Washington: National Education Association, 1960), p. 35.

²B. F. Skinner, "Teaching Machines," Science, CXXVIII (October, 1958), as reprinted in Teaching Machines and Programmed Learning, (eds.) A. A. Lumsdaine and Robert Glaser (Washington: National Education Association, 1960), p. 138.

"This was used in an early experimental comparison to assess the value of immediate knowledge to test results which it provided."¹ Peterson called his device the "Self-Instructor and Tester."² In one of his articles, Dr. Peterson stated:

In its simplest form this device consists of one or more sheets of multiple-choice questions whose correct and incorrect answers are treated with two matched moisture-sensitive inks which turn to contrasting colors when moistened. In using this device, the learner reads each question, chooses from its alternative answers the one he deems correct, and touches it with a strip of moistened felt. If he chooses the correct answer, it changes immediately to a predetermined color, say blue; if incorrect, it changes to a different predetermined color, say red. This change of color simultaneously records his response and shows the learner whether his choice of answers was correct or incorrect.³

Peterson's contention was that his self-instructor and tester could "be used both as a means of guidance in learning and as a means of improving speed and accuracy in scoring tests of what has already been learned."⁴

¹A. A. Lumsdaine and Robert Glasser, (eds.), "Pressey's Self-Instructional Test-Scoring Devices," Teaching Machines and Programmed Learning, (Washington: National Education Association, 1960), p. 33.

²J. C. Peterson, "The Value of Guidance in Reading Information," Transactions of the Kansas Academy of Science, XXXIV, 1931, as reprinted in Teaching Machines and Programmed Learning, (eds.) A. A. Lumsdaine and Robert Glaser (Washington: National Education Association, 1960), p. 52.

³Ibid.

⁴Ibid.

B. F. Skinner's Recent Developments
in Programmed Learning

B. F. Skinner, an educational psychologist at Harvard, developed a programmed teaching machine in which the answer is constructed by the learner. Under a glass window, on Skinner's machine, the learner reads an item to which he must make a response. Next to the glass window is an open window where the learner can write his response. Then he pulls a lever which reveals the correct answer, and at the same time pushes his answer under a sheet of glass so that he cannot erase.

Skinner's "response-construction" type of programmed learning has been adapted to programmed texts. In the text form of program, instructional matter is broken down into very small steps called frames. In each frame is presented a bit of information; a question is asked pertaining to the information; and a space is provided for the learner's written response. The answer to the question appears under the frame. The learner keeps the answer covered with a mask until he has written his response to the question. He then slides the mask down and checks his response with the correct answer. Then the learner proceeds to the next frame and repeats the process.

A Comparison of Skinner's Method with Pressey's
Method of Programmed Learning

Skinner's method¹ features "short-answer" questions requiring a constructed (usually written or drawn) response. He uses programmed material structured in small, well-ordered steps made sufficiently easy so as to assure the maximum of correct responses from the student. Skinner's method serves chiefly as a learning instrument.

Pressey's principles,² on the other hand, present multiple-choice questions which evoke "recognition" type responses. Pressey used programmed materials with varied sequences which permitted some errors in responses. This method was chiefly used as a testing device. It could be used, however, to teach. In this case it was used primarily to supplement classroom instruction in standard textbooks.³

Some Advantages and Disadvantages
of Programmed Learning

One of the most bitter arguments against programmed learning was summed up by Kvaraceus who said,

Unfortunately few personal, social, economic or
political problems that are met in everyday living tend

¹Elaine Exton, "Teaching Machines, Fad or Here to Stay?"
American School Board Journal, CLI (September, 1960), p. 19.

²Ibid.

³Ibid.

to be clear, simple or categorical. . . . These levels are still outside the reach of most self-learning devices and thereby place a low ceiling on what is to be mastered."¹

In contrast to what Kvaraceus may think, many people feel that the opportunity to deal with these personal, social and economic problems may be even greater when using programmed instruction. Huffman said,

Because students are required to be active . . . the teacher thus has a more creative roll. . . . The teacher typically will conduct discussions from time to time and provide reviews with students. When discussions are conducted, all students will be confident because they will have a common background from which to draw in order to participate in discussions.²

It is during these discussion periods that personal and social problems are brought out. Therefore, students should be able to grow more from a personal and social standpoint because they will have a basis from which to contribute to class discussion.

A second big fear caused by programmed instruction is that the machines may completely take the place of teachers in the classroom. People who follow this line of thought state their argument somewhat as follows:

We should be concerned with the threat of emotional desolation that may be artificially

¹Wm. C. Kvaraceus, "Future Classroom--An Educational Automat?" Educational Leadership, XVIII, No. 5 (February, 1961), p. 289

²Harry Huffman, "Seventeen Q's and A's About Programmed Instruction," Business Teacher, XXXIV, No. 4, (February, 1962), p. 11.

produced by crowding the teacher out of the classroom and learning laboratory, thereby converting immature human beings into mere intellectual machines.¹

Many educators seem to follow Exton's line of thought:

Teaching machines will not replace the teacher. . . . Many of the more routine aspects of coaching the student and testing his ability can be turned over to teaching machines . . . this will free the teacher to give more of her time to the individual students, including the gifted and retarded, and to projects and activities utilizing more fully the abilities of a skilled instructor.²

Another argument against programmed learning was that students with superior ability may be held back with small steps used in programmed units. One article in Know-How stated:

Writing responses is more generally useful to pupils of lower academic ability than their more able classmates. Contradictory though it may seem, students who are superior in verbal skill and better able to handle abstractions can actually be hindered by having to write responses to each frame in a text. Why? Because each frame advances knowledge only fractionally, but rapid learning students often like to move ahead in large steps.³

Of course the answer to this problem would be to use a branching program. In this type of program, when a student gets a right answer to a problem, he is told to skip several frames. If on the other hand, another student was to miss the same problem, he would

¹Kvaraceus, Op. Cit., p. 292.

²Exton, Op. Cit., p. 19.

³"Programmed Learning—What Teachers are Saying About It and Doing about It," Know-How, (New York: Associated Public School Systems, May-June, 1962), p. 5.

have to continue in the same sequence which gives more drill on the type of problem missed. Wittig explains his success with the branching type program in an experiment at Sligo Junior High School, Montgomery County, Maryland.

I have been using a branch-type program. . . . Once I began . . . the difference in the abilities of students became quickly apparent. Several pupils sped rapidly ahead, while their classmates lagged behind at various stages.¹

There were many advantages of the program suggested for the learner. One argument for the slow learner was stated by a social psychologist:

Perhaps the most important aspect of the teaching devices mentioned above is that they offer students alienated from the educational world an educational experience devoid of public shame: If the student makes a mistake, he knows it and the teacher knows it, but other students will not. Thus, the opportunities to "like learning" are enormously increased.²

Sierles, a teacher of economics at Samuel J. Telden High School, Brooklyn, New York, gives a digest of problems and solutions of the slow learner as follows:

- (a) The slow learner understands more than he is able to express orally or in writing.

Solution: Problems and questions can be presented in the form of multiple-choice and other objective-type questions which he is able to grasp.

¹Lester Wittig, "Unscramble Ability Levels," Know-How, (New York: Associated Public School Systems, May-June, 1962), p. 7.

²Herbert E. Krugman, "Education and the New Learning Devices," N. E. A. Journal, LXI, No. 4, (April, 1962), p. 56.

- (b) His motivation for learning is limited as compared with that of other pupils.

Solution: To stimulate interest and attention, a short-answer test is given to the class at the end of the lesson. (Tests are an effective stimulus learning for all pupils, but the immediacy of testing is essential for slow students.)

- (c) This type of pupil is success starved.

Solution: The learning problems and tests are so framed as to guarantee frequent success.

- (d) He is a poor reader and finds learning from textbooks too demanding for him.

Solution: He is given a printed course of study in which the units of work do not require prolonged concentration and discussion.

- (e) Slow learners find it difficult to follow an extended classroom discussion.

Solution: The course of study presents the subject in the way a tutor would teach an individual pupil.

- (f) The slow learner is easily bored and prone to daydreaming.

Solution: The modified program keeps him busy throughout the lesson. Interest is stimulated because of the anticipated test at the end of the lesson.¹

From the literature available to this writer, it appears that the advantages of programmed instruction, if used properly, far outweigh the disadvantages.

¹Samuel Sierles, "The Slow Learner Can Learn!" The Clearing House, XXXVI, No. 6 (February, 1962), pp. 361-2.

CHAPTER III

THE METHOD OF PROCEDURE

Introduction to the Procedure

College Business Mathematics, Part I is a programmed unit in "Rebuilding the Fundamentals" of business mathematics. This textbook was developed by Dr. Harry Huffman, Head, Business Education Department, Virginia Polytechnic Institute, Blacksburg, Virginia. One of the purposes for Huffman's programmed text was to provide post-high school and college students with a tool for remedial work in the fundamentals of business mathematics. The purpose of the writer in the study was to adjust the program to a level of comprehension in order for high school students of low ability to do remedial work in business mathematics.

In order to familiarize himself with Huffman's program, the writer worked through the original unit. The unit was comprised of 432 frames broked down into six sections. The various sections are outlined below.

Speed and Accuracy in Addition (58 frames).

Speed and Accuracy in Subtraction (23 frames).

Speed and Accuracy in Multiplication (100 frames).

Speed and Accuracy in Division (59 frames).

Speed and Accuracy in Using Common Fractions (87 frames).

Speed and Accuracy in Using Decimals (105 frames).

Construction of Tests for the Programmed Unit

After carefully working through the programmed unit, the writer constructed six tests to measure retention of concepts presented in the six sections of the programmed unit. The tests were sent to Dr. Huffman, author of the original program, for criticism. Huffman's suggestions were incorporated into the tests. (The six tests as reconstructed by Huffman are included in Appendix C.)

Preparations for the Experiment

Preparation of the Programmed Unit.--In order to facilitate revisions, additions, and deletions of the original frames as the writer deemed necessary, each frame in the programmed unit was pasted to the bottom of a 5" x 8" card. The answer to the question or problem to be solved was pasted to the top of the succeeding card. The cards were placed face down so that the student could not see the answer until he had listed his response on the answer sheet.

Selection of Student-Editors.--The fifteen student-editors participating in the experiment were selected from three classes of the writer's office training program. Only those students were picked whose intelligence quotient was 100 or below. Permission was obtained from the school principal to include personal data of student-editors in the study. (See Appendix B)

Composition and Scheduling of the Panels.--The fifteen students participating in the experiment were divided into five panels; three students to a panel. One student was selected from each of three office training classes to make up one panel.

The writer estimated that from 15 to 20 hours would be needed for a student to complete the program and the six tests required after each section of the programmed unit.

The office training class was set up on a five-hour rotational basis. Four rotation periods were set aside for those students participating in the study, a maximum of 20 hours. If a student finished before the four rotation periods were up, he was assigned to an optional work station, therefore, wasting no time at the end of the programmed unit.

Preparation of the Answer Sheet.--The original program provided space for a response after each question on a frame. Because the experiment was set up on cards to facilitate revisions and adjustments, all students had to use the same set of cards. They were not permitted to write responses on the cards. Therefore, an answer sheet was provided for each student. (See Illustration on the following page.)

The answer sheet called for information in addition to the response. At the top of the answer sheet, spaces were provided for the student's name, the date, and the number of the first and last frames completed during the period.

The body of the answer sheet was divided into five columns. The columns included a space for frame numbers, responses, work space for problems, and a space for comments on each frame.

Procedure for Collecting the Data

Procedures for the Student-Editor

Editing the Programmed Unit.—The student-editor studied the information contained on each frame. He read the question pertaining to the frame and recorded his response on the answer sheet. In the column provided, the student also recorded the frame number. If it was necessary for the student to make calculations, a space was so provided on the answer sheet. All calculations had to be next to the appropriate response. The student was not allowed to do any calculations on scrap paper. After recording the answer, the student dropped the card, face down, in a tray supplied for that purpose. He then picked up the next card which was also face down. At the top of this card the correct response was supplied to the previous frame. If the response was wrong, a red cross mark was made next to the response on the answer sheet. After checking his response, the student was encouraged to make comments, applicable to the frame just completed, in a space provided on the answer sheet. After making his comment, the student went to the next frame.

Taking Intermediate Tests to Measure Retention.—At the end of each section of the programmed unit (addition, subtraction, etc.), the student was administered a test to measure retention of concepts covered in that section. The results of the tests were used to aid the writer in revising the program.

Procedure of the Researcher

Motivating Student-Editors.—In order to get the student to do the best job possible on the program, the researcher explained to the student that he would receive remedial value from participating in the study. The student was told that he was participating in an "important" experiment which would have lasting value to him and other students. Honesty was stressed. The student-editor was informed that any wrong response on the answer sheet would not count against him; that in fact, it would be of much value in the experiment. Copying of responses without reading the information in the frame would tend to lessen his understanding of the concepts in which case he might be unable to make a high score on the intermediate tests.

Orienting the Student as to Procedure.—The researcher made it a point to go over the first few frames with the student in order to make sure that he fully understood the correct procedure.

Cataloging Incorrect Responses.—Each day the answer sheets of student-editors were collected by the researcher. The answer sheet was checked for errors. A card catalog was set up to keep a count of errors made in each frame.

When the researcher discovered an incorrect response, he checked the catalog to see if the answer had been missed by a previous student. If so, he put the editor's assigned number by the response missed. If an incorrect response had not been recorded on that frame, the researcher made out a card for the frame indicating the error and the number of the student making the incorrect response. This card was inserted into the catalog of incorrect responses.

A record was also kept on a "student comment sheet" of all comments made by an individual student.

After all errors had been recorded, the answer sheets and comments were stored in individual folders for each student.

Analyzing Incorrect Responses.—As a panel completed a particular section of the programmed unit, incorrect responses were compared and analyzed to determine the cause of the errors. Comments by students were also considered in the analysis.

Writing New Frames.—Data gathered from the analysis of incorrect responses were used as a basis for making changes, additions, or deletions of frames in the original program.

As new frames were written, they were typed on a master set for duplication. A copy was forwarded to Dr. Huffman, author of the original program, for criticism.

Another copy was cut into strips and pasted to colored cards. Colored cards were used to furnish the researcher with a means of immediate identification of changes or additions to the original program.

Number System for New Frames.—The original frames in the programmed unit were numbered from 1 through 432. A method was devised to insert new frames without changing the original frame number. The writer also included in the numbering system lettered symbols to indicate whether the new frame preceded or followed a particular original frame and when the change was made. Illustration 2, on the following page, interprets the symbols used in the revised program.

ILLUSTRATION 2

Symbols Utilized on Revised Frames

Symbols	Interpretation of Symbols
A, B, C, or D	A capital letter immediately following the frame number indicates which panel of students prompted the change in the original frame or the addition of the new frame.
c	A "c" (for cue) preceding the frame number, indicates that a word in the original frame has been underscored, put in quotes, or typed in uppercase letters to act as a prompt.
f	An "f" (for following) preceding the original frame number indicates that this frame has been added to follow the original frame in order to strengthen concepts in the preceding frame.
Hyphen and a number (-1, -2, -3, -4, etc.)	A "hyphen and number" following the original frame number and panel symbol, indicates how many and the sequence of frames that have been added to the original frame. (This symbol is used only when more than one frame is needed in addition to the original or revised frame.)
p	A "p" (for precedes) preceding the original frame number indicates that additional frames have been added to precede the original frame in order to introduce the original or strengthen the concept of the original frame.
x	An "x" preceding the frame number indicates that the original frame has been removed and that this revised frame has been substituted.

CHAPTER IV

THE ANALYSIS

Introduction to the Analysis

The analysis of this experiment consists of five parts corresponding to the five successive panels of student-editors participating in the study. Each part includes three tables. The purpose of the first table is to give a breakdown of the errors made by each student in the various sections of the programmed unit and to reflect the percentage of errors made by the individual student on the entire program. In addition, the table reveals the total number of errors for each section made by that particular panel, the total responses possible, and the per cent of errors for each section by the panel. Beginning with Panel Two, this table also reflects the decrease or increase in percentage of errors made.

The second table in each part of the analysis shows the results of the intermediate tests. This table reflects each panel member's individual grade and the average grade of the panel. Beginning with Panel Two this table also reflects the decrease or increase in scores of the intermediate tests.

The third table found in each part of the analysis, with the exception of Panel Five, gives a breakdown, by section, of the number of changes made in the original frames of the program. This table also

reveals the total number of frames and responses added to the program as a result of an analysis of each successive panel.

The programmed unit underwent four revisions. After each panel completed the unit, with exception to Panel Five, the writer analyzed the errors of each student and made such revisions in the program as he determined necessary to decrease the error rate of succeeding panels.

Examples of significant changes made in the frames of the original program and those frames which were added to the program are shown in illustrations throughout the analysis.

Analysis of the Work of Panel One

Table I reflects the per cent of errors for each section of the Programmed Unit in Business Mathematics to be above the maximum five per cent error rate of responses set up for this experiment. Only one student-editor, Rosie, came within the established error rate.

As shown in Table II, the average grade of the panel on the six intermediate tests was 78.2. Carolyn had the lowest average with 65.0 and Rosie was highest with 81.8. Considering 70 as a passing grade, Carolyn failed three sections of the intermediate tests. Her lowest score was 28 on the decimals section. Rosie failed only one section, making 69 on the fractions section. The lowest section average was in decimals with 61.3. This section reflected an error rate of 7.84 per cent for Panel I. The next lowest average grade was

TABLE I

Number and Per Cent of Errors
Made by Panel One

Student	Addition (110)*	Sub- traction (31)*	Multipli- cation (197)*	Division (101)*	Fractions (154)*	Decimals (187)*	Total Errors (780)*	% Errors in 708 Possible Responses by Student
Carolyn	7	5	35	21	18	29	115	14.74
Cheryl	8	0	9	6	10	10	43	5.51
Rosie	14	1	8	2	0	5	30	3.85
Total Errors	29	6	52	29	28	44	188	
Total Possible Responses for Three Students	330	93	591	303	462	561	2340	
% Errors in Total Possible Responses by Section	8.79	6.45	8.80	9.57	6.06	7.84	8.03	

*Number of actual responses required of each student.

TABLE II

Test Scores for Members of Panel One

<u>Name</u>	<u>Addition</u>	<u>Subtraction</u>	<u>Multiplication</u>	<u>Division</u>	<u>Fractions</u>	<u>Decimals</u>	<u>Average</u>
Carolyn	79	70	64	64	85	28	65.0
Cheryl	86	100	73	89	92	86	87.6
Rosie	79	100	73	100	69	70	81.8
Average	81.3	90.0	70.0	84.3	82.0	61.3	78.2

in the multiplication section which reflected next to the highest per cent of errors.

Table III shows that 18 frames were added in this revision. However, nine of these added frames were consecutive. They were in the sequence which taught short cuts in multiplying fractions. (See Illustration 19.)

The majority of changes were made by rewording the original frames. Other frames were revised by giving cues or prompts to important words and terms. Still other changes were made in the structure of the frames in order to separate statements of fact from questions and problems within the frames. Table III shows that 36 frames were revised in these ways.

The average intelligence quotient of student-editors on Panel One was 94.5. The average English grade for the first three years in high school was "D." The average mathematics and algebra grades were "D minus." These individual scores and those on "Sequential Tests of Educational Progress," and others can be found in Appendix B.

The First Revision

Building Speed and Accuracy in Addition

When to Round to Tens, Hundreds, or Thousands.--Throughout the sequence of frames which dealt with rounding numbers in order to

TABLE III

Number of Changes and Additions
Made in the First Revision

<u>Section</u>	<u>Number Changes Made in Original Frames</u>	<u>Total Number Frames Added</u>	<u>Total Number of Responses Added</u>
Addition	8	2	4
Subtraction	3	3	4
Multiplication	13	0	2
Division	4	0	0
Fractions	5	10	17
Decimals	3	3	5
Totals	36	18	32

estimate totals, there appeared eleven frames with problems of the following nature:

$$\begin{array}{r} 10,295 \\ 4,625 \\ 5,132 \\ 3,276 \\ 87 \\ \hline 267 \end{array}$$

The problem in this type of frame was to have students round, "uniformly," to either tens, hundreds, or thousands and add the rounded numbers in order to obtain an estimated total.

Of eleven frames in this type of problem, the first panel of three students missed a total of seven problems. Each student on the panel missed at least one problem in this sequence.

After analyzing errors of Panel One, it was determined that the reason for missing these problems was lack of "uniformity" in rounding to tens, hundreds, or thousands. Therefore, the writer added two frames, placing them in what he considered strategic sequence. He also revised one frame which presented sufficient information, but lacked in clarity.

Illustration 3 shows the added Frame f9A. This frame was added to emphasize to the student "when" to round a group of figures to tens.

Illustration 4 shows Frame pl9A. This frame was added to emphasize to the student "when" to round a group of figures to hundreds.

ILLUSTRATION 3

When to Round to Tens

ADDED FRAME

94	Most of the numbers in the problem to the	90	f9A
16	left are in tens. Therefore, all of the	20	
74	numbers in the problem should be rounded	70	
123	to <u>tens</u> as shown at right.	120	
<u>2,416</u>		<u>2,420</u>	
	(estimated answer)	2,720	

All the numbers in the following problem are rounded to _____? (tens, hundreds, thousands)

824	The <u>estimated</u> answer is _____.
62	
73	
<u>65</u>	

tens 1020

ILLUSTRATION 4

When to Round to Hundreds

ADDED FRAME

When estimating a group of numbers and most of the numbers fall between 100 and 999, all of the numbers should be rounded to hundreds. p19A

78	In the problem to the left, all the numbers should be rounded to _____? (tens, hundreds, or thousands)
196	
721	
1,436	
<u>121</u>	The ESTIMATED total is _____.

hundreds 2500

The original Frame 24 sufficiently explained the concept of "when" to round figures to thousands. However, in Problem One, Cheryl indicated that she did not fully understand what was being presented. She stated that, "I did not understand what 'inclusive' means here."

Therefore, the writer attempted to clarify the frame in three ways. He began by breaking the first relatively long sentence into two short sentences. Since each sentence stated a fact, they were set apart by a space between them in order to reduce confusion.

Secondly, in revising the sentences, the writer excluded the word "inclusive" which gave Cheryl some trouble.

Third, the question in the frame was separated from the statement of fact preceding it.

Illustration 5 shows Frame 24 as it originally appeared and how it appeared after it was revised.

Angular Addition to Increase Accuracy.—Students in Panel One made a high percentage of errors in angular addition. However, the cause of the error was not through lack of understanding the concept of angular addition. Incorrect responses resulted from errors in addition.

It was observed that the three frames used for practice in angular addition were relatively long problems for the caliber of students who participated in this experiment. For example, in Frame 47, the student had to add eight numbers with five digits in each

ILLUSTRATION 5

When to Round to Thousands

ORIGINAL FRAME

When rounding to thousands, round numbers less than 500 24
to 0 thousands and round numbers from 500 to 1,499 inclusive
to 1 thousand. Similarly round numbers between 1,500 and
2,499 inclusive to _____(no.) thousands.

2

REVISED FRAME

When rounding to thousands, round numbers from 1 through x24A
499 down to zero thousands.

When rounding to thousands, round numbers from 500 through
1,499 to 1 thousand.

Similarly, round numbers from 1,500 through 2,499 to
_____ thousands.

2

figure. In Frame 48, there were nine numbers with seven digits in each figure. Frame 70 used eight numbers with six digits each.

In these three frames of angular addition, there were a total of six wrong responses by Panel One. Each student made no less than one incorrect response. Therefore, the problems in angular addition were shortened to increase the chances of correct responses. This would have the effect of creating positive motivation. An example of these revisions is shown in Illustration 6.

Building Speed and Accuracy in Subtraction

Approximating Answers in Subtraction.—The purpose of Frame 62 was to elicit a recall of the term "rounding." In Panel One, Carolyn commented that she "...just couldn't think of what the word was." The writer checked and discovered that the term had not been used since Frame 24. Also, due to week-ends and a couple of absences, a period of eight days had elapsed since she had used the word in the programmed unit. Therefore, a prompt was added to Frame 62 as shown in Illustration 7.

Rounding the Smaller Number First.—The purpose of Frames 62 through 75 was to present the concept of rounding the subtrahend before rounding the minuend and to round them to the same extent. The student was required to explain the procedure by using such terms and phrases as: "smaller number, larger number, same extent, and to the same extent."

ILLUSTRATION 6

Shortening Length of Problem
in Angular Addition

ORIGINAL FRAME

Add the problem at the right by angular addition;	35467	47
verify the correctness of each column by adding	46843	
in the reverse direction.	56789	
	46246	
	78965	
	54175	
	68987	
	<u>67624</u>	

<u>455,096</u>	<u>455,096</u>
----------------	----------------

REVISED FRAME

Add the problem at the right by angular	3546	x47A
addition; verify the correctness of each	4684	
column by adding in the reverse direction.	5678	
	4624	
Show the total of each column of figures in	<u>7896</u>	
<u>angular</u> fashion on the work space of your		
answer sheet.		

<u>26,428</u>	<u>26,428</u>
---------------	---------------

ILLUSTRATION 7

Using Introductory Sentence as Prompt

ORIGINAL FRAME

Estimates of differences in subtraction are made by _____ numbers as you did in addition. Since 62
only two numbers are involved, care must be taken in
choosing how to round the _____ numbers.
(no.)

rounding two

REVISED FRAME

You remember that in estimating addition problems the _____ numbers were rounded. x62A

Likewise, estimates of differences in subtraction are made
by _____ numbers as you did in addition.

Since only two numbers are involved in subtraction, care must
be taken in choosing how to round the _____ numbers.
(no.)

rounding two

In Frame 63, the student was asked to show how to round the larger number to the same extent as the smaller number in the problem given. In Frame 68 the student was asked to show the rounding of both minuend and subtrahend and to estimate the difference.

Because this sequence was broken by a review frame in angular addition and the student was allowed to practice the complete concept only one time before his train of thought was broken, the writer believed these to be the reasons for Carolyn's wrong response in Frames 73 and 74.

Therefore, another practice problem was added before breaking the thought pattern in this concept. The change is shown in illustration 8.

In addition to the errors in Frames 73 and 74, as a result of failing to round properly, problems missed on the intermediate test on subtraction further bore out the writer's conviction that the student should be provided additional prompts before allowing him to estimate. Therefore, Illustration 9 shows two frames which were added to precede frame 71.

The first frame makes use of prompts to stress further the concept in rounding smaller and larger numbers in subtraction. In the second frame, the writer used the answer as a prompt to show exactly how the subtrahend and minuend were rounded to get the correct estimate. Completion of these frames should enable the student to work the remaining problems in this series without error.

ILLUSTRATION 8

Practice in Rounding in Subtraction

ADDED FRAME

Show the rounded numbers
of the problem at the right.

Problem

Rounded To

f69A

Show estimated answer.

548,752
- 8,236

Estimated Answer: ----->

549,000
- 8,000
541,000

ILLUSTRATION 9

Adding Frames which Utilize
Prompts to Stress Concept

ADDED FRAMES

When estimating the difference between \$1,626.25 and \$895.60, we first round the SMALLER figure to _____ hundred. p71A-1

Therefore, the LARGER figure must also be rounded to hundreds and would be read as _____ hundred.

9 16

In the problem at the right, show how each figure would be rounded to find the estimated difference.	<u>Problem</u> \$1,626.25 <u>- 895.60</u>	<u>Round To</u> _____	p71A-2
--	---	--------------------------	--------

Estimated answer: ----->

\$1,600.00 <u>- 900.00</u> \$ 700.00
--

Building Speed and Accuracy
in Multiplication

Adding a Prompt to Elicit a Term.--It was the intention of Frame 87, as shown in Illustration 10, to elicit the word "annexed." In Panel One, Carolyn responded with the word "added." The wording of the frame was not changed. However, the writer inserted a cue by means of an underscore in the frame to prompt the student on the term "annexed." There were no further wrong answers for this frame by any of the students on succeeding panels.

Substituting a Statement for an Implication.--In her comments in Panel One, Carolyn had this to say about the original Frame 92, as it is shown in Illustration 11. "The question doesn't ask what the incorrect partial product should be."

As the question was stated in the original frame, it "implied" that the student should put down the correct partial product. However, the student did not comprehend the implication.

Frame 92 was revised, as shown in Illustration 11, to state more clearly exactly what response is desired. There were no further questions or comments as to the intent of the question in this frame.

Defining Terminology.--Three responses were asked for in the original Frame 93, as shown in Illustration 12. Student-editors in Panel One made four incorrect responses on this frame.

The revision was made on the basis of the comment made by Cheryl in Panel One. She commented, "the word 'audit' has not been used before. I am not sure what it means."

ILLUSTRATION 10

The Addition of a Prompt to Elicit a Term

ORIGINAL FRAME

$$10 \times 89 = 890$$

87

In mathematics when 0 is placed to the right of a number, it is said to be annexed. Thus, one zero _____ to 42 results in 420.

annexed

REVISED FRAME

$$x0 \times 89 = 890$$

c87A

In mathematics when 0 is placed to the right of a number, it is said to be annexed.

Thus, one zero _____ to 42 results in 420.

annexed

ILLUSTRATION 11

Substituting a Statement for
an Implication for Clarity

ORIGINAL FRAME

Which partial product, if any is in error? _____ 92

$$\begin{array}{r} 6,742 \\ \times 78 \\ \hline 53936 \\ 45194 \\ \hline 505,876 \end{array}$$

Obtain the correct answer: _____

45,194 should be 47,194 525,876 is the correct answer.

REVISED FRAME

Is one of the partial products below in error? _____ (yes/no) x92A

$$\begin{array}{r} 6,742 \\ \times 78 \\ \hline 53\ 936 \\ 451\ 94 \\ \hline 505,876 \end{array}$$

The partial product _____ should read _____

The CORRECT answer to the problem is _____

**

Yes

45,194 should read 47,194525,876 (is the correct answer)

Since understanding of terminology is a secondary purpose in this unit, the frame was revised to define the term "audit" as it appears in Illustration 12.

Revision of Preceding Frame to Elicit Correct Response in Next Frame.—In the original Frame 97, the question was asked, "Are \$ signes omitted?" Carolyn made this comment: "The question asked, 'Are dollar signs omitted?' It didn't ask if they were supposed to be, or if they were omitted in this problem. It wasn't clearly understood."

Cheryl also gave the wrong response to the question in this frame. In order to make the student more secure in his response, the researcher inserted additional information in the preceding frame which would explain why dollar signs were not used on invoices. The reason for using the original Frame 96, as shown in Illustration 13, was that it presented a simulated invoice. After this revision, no error was made on Frame 97 by members of succeeding panels.

Determining When to Round to Tens, Hundreds, or Thousands.—Frames 102 through 134 were used to develop the concept of estimating the product to a multiplication problem. Panel I made 15 errors on 28 frames requiring the rounding of factors.

Upon analyzing the work of the panel, it was concluded that the errors made were due in part to the fact that the student did not understand when to round to tens, hundreds, or thousands. (The reader will remember that this problem was encountered in addition.)

ILLUSTRATION 12

The Revision of a Frame to Define a Term

ORIGINAL FRAME

Check each partial product below and correct errors, if any. 93

$$\begin{array}{r} \$947.62 \\ \times 827 \\ \hline 663334 \\ 189424 \\ \hline 758196 \\ \hline 783,771.74 \end{array}$$

The audited answer is _____

189,424 should be 189,524 758,196 should be 758,096
Answer: \$783,681.74

REVISED FRAME

Check each partial product below and correct errors, if any. x93A

$$\begin{array}{r} \$947.62 \\ \times 827 \\ \hline 6,633.34 \\ 18\ 942\ 4 \\ \hline 758\ 196 \\ \hline 783,771.74 \end{array}$$

AUDIT means to check and verify
or correct.

The audited answer is _____

189,424 should read 189,524
758,196 should read 758,096
AUDITED answer is \$783,681.74

ILLUSTRATION 13

The Revision of a Preceding Frame to Elicit
the Correct Response in the Next Frame

ORIGINAL FRAME

Audit this invoice:

96

Sold to:			
<u>San Pedro College</u>		September 6, 19__	
<u>Los Angeles, California</u>			
BORDON FURNITURE COMPANY			
6	Double pedestal desks	127.50	765.00
6	Swivel Chairs	75.40	<u>452.40</u>
			1,217.40

No Errors

REVISED FRAME

Audit this invoice:

x96A

Sold To:			
<u>San Pedro College</u>		September 6, 1961	
<u>Los Angeles, California</u>			
BORDON FURNITURE COMPANY			
6	Double pedestal desks	127.50	765.00
6	Swivel Chairs	75.40	<u>452.40</u>
			1,217.40

NOTE: To SAVE TIME, dollar signs should be omitted from an invoice.

No errors

The writer added information to three frames located in strategic positions throughout this series to develop an understanding of this concept.

The original Frame 106, as shown in Illustration 14, was chosen to develop the concept of "when" to round to hundreds because it was the first frame in which the student was elicited to round to hundreds. This practice was followed throughout the revision of these frames. Revised Frame 106 appears in Illustration 14 and shows the explanation of rounding to hundreds.

The concept of when to round to tens was developed in Frame 111. The revision appears in Illustration 15.

The concept of rounding to thousands was explained in Frame 132. The revised form of this frame is shown in Illustration 16.

Determining When a Product is Reasonable.—In the original Frame 129, the student was required to determine if a product was reasonable. There was no explanation of what constituted a reasonable product prior to this frame. Carolyn missed the question.

Cheryl and Rosie missed a similar question on the multiplication test. Therefore, Frame 129 was revised, as shown in Illustration 17, to explain to the student to what extent an answer may be considered reasonable.

Using a Prompt to Elicit the Correct Response.—The purpose of the original Frame 162 was to motivate the student to figure out the

ILLUSTRATION 14

Determining When to Round to Hundreds

ORIGINAL FRAME

The other technique for estimating the product is based on 106
 rounding each factor both up and down. 687 lies between
 600 and 700. 426 lies between _____ and _____.

400 500

REVISED FRAME

The other technique for estimating the product is based on x106A
 rounding each factor both up and down. If the number to be
 rounded is from 100 through 999, round up or down to even hundreds.

687 lies between 600 and 700

426 lies between _____ and _____.

400 500

ILLUSTRATION 15

Determining When to Round to Tens

ORIGINAL FRAME

Obenchain has weekly take-home pay of \$93.78. What would be the lower estimate of his total take-home pay for 42 weeks? 111

\$3,600

REVISED FRAME

If a factor is less than 100, that factor is rounded up and down to even tens. x111A

Campbell has weekly take-home pay of 93.78. What would be the lower estimate of his total take-home pay for 42 weeks? _____

\$3,600

ILLUSTRATION 16

Determining When to Round to Thousands

ORIGINAL FRAME

For the problem $1,247 \times 87 =$, the lower estimate is _____ 132
 and the upper estimate is _____. The exact product is _____.
 (Hint: Round 1,247 to thousands and 87 to tens.)

80,000 180,000 108,489

REVISED FRAME

If a factor to be rounded is in the thousands it should $\times 132A$
 be rounded to even thousands.

In the problem $2,375 \times 65$: 2,375 is rounded to 2,000 for
 the lower estimate and 65 is rounded to 60 for the lower estimate.

For the problem $1,247 \times 87 =$, the lower estimate is _____.

The upper estimate is _____.

The exact product is _____.

80,000 180,000 108,489

ILLUSTRATION 17

Determining When a Product is Reasonable

ORIGINAL FRAME

Computation on an invoice: 129

2,869 A-1516 tubes 4.86 15,943.34

By estimation, determine whether the extension is (reasonable/
unreasonable). _____

unreasonable

REVISED FRAME

If the product of two factors falls between the upper x129A
and lower estimates, it is considered to be reasonable.

Computation on an invoice:

2,869 A-1516 tubes 4.86 15,943.34

Estimate the extension. Is the extension reasonable or
unreasonable? _____

unreasonable

procedure for a shortcut of multiplying by 998. Cheryl was not able to determine the correct procedure. After discovering that she had made the wrong response, she asked the writer to explain the procedure to her. After the explanation the student made the following comment: "The 'B' part needs to be explained. It does not say that 1,341 should be multiplied by 2 then subtracted from 1,000 times itself."

Cheryl and Rosie also missed a similar problem on the test regarding the 998 shortcut rule. Therefore, the frame was revised, as shown in Illustration 18, to give an additional prompt as to the procedure to use for this shortcut method.

Building Speed and Accuracy with Common Fractions

Developing a Shortcut Method to Find the Product of Fractions.

--In Frame 303 the student was to make an estimation in which the procedure involved multiplying $\frac{1}{4} \times 24$. On this frame and succeeding frames of this kind, the writer, in studying the answer sheets of Panel One, observed that the students, in arriving at the answer, were making various kinds of unnecessary cancellations in their computation. This procedure took an undue amount of time with the result that Cheryl got confused to the point of giving an incorrect response.

Speed was one of the primary concerns of the programmed unit. Therefore, an effort was made to show the student a much simpler and

ILLUSTRATION 18

Using a Prompt to Elicit the Correct Response

ORIGINAL FRAME

Using the shortcut for multiplying by 999 and a similar one 162
for 998, do these problems:

$$\begin{array}{l} \text{(a)} \quad 999 \times 1,245 = \underline{\hspace{2cm}} \\ \text{(b)} \quad 998 \times 1,341 = \underline{\hspace{2cm}} \end{array}$$

1,243,755 1,338,318

REVISED FRAME

Using the shortcut for multiplying by 999 and a similar one x162A
for 998 do the problems below.

(Hint: The shortcut rule for 998 is similar to the rule for 98.)

$$\begin{array}{l} \text{(a)} \quad 999 \times 1,245 = \underline{\hspace{2cm}} \\ \text{(b)} \quad 998 \times 1,341 = \underline{\hspace{2cm}} \end{array}$$

1,243,755 1,338,318

much faster method of performing the particular calculation in their heads. The process used to present this particular method, as shown in Illustration 19, was somewhat long. However, the success of succeeding students with this procedure caused the writer to hesitate in attempting to shorten the explanation any further. A total of nine frames were used to develop the procedure and these added frames were situated to precede Frame 303.

In the process of developing new frames to precede Frame 303, the researcher took the opportunity to review the student on terminology of fractions.

Defining the Term "Inverted."--Before completing Frame 313, Cheryl wanted to be sure that she knew what the term "inverted" meant. Therefore, she asked the instructor to define it for her. This frame was not missed by any student on Panel One. However, the writer decided, in anticipation of other students not being sure of the term, to give a definition which would suit the purpose of this frame. The revised frame showing the definition of "invert" is shown in Illustration 20.

Decimal Fractions

The Placement of Decimals in Converted Fractions.--The intent of Frames 352 and 353 was to demonstrate where the decimal falls when converting a common fraction such as $\frac{1}{15}$ or $\frac{3}{32}$ to a 3-place decimal. In this type of problem, one or more zeros may precede the first

ILLUSTRATION 19*

Developing a Shortcut Method to
Find the Product of Fractions

ORIGINAL FRAME

303

When estimating the product of fraction problems like $\frac{1}{3} \times 17\frac{1}{2}$,
think $\frac{1}{3} \times 18 = 6$ for the estimated product.

$$\frac{1}{4} \times 24\frac{1}{3} = \underline{\hspace{2cm}} \text{ (estimated product).}$$

6

FRAMES ADDED TO PRECEDE THE ORIGINAL FRAME SHOWN ABOVE

p303A-1

When multiplying a proper fraction like $\frac{1}{3} \times 15$, we need only
to divide 15 by 3, the denominator of the proper fraction.

EXAMPLE: $\frac{1}{3} \times \frac{15}{1} = 5$

It would be easier and much faster to say $15 \div 3 = 5$; or $3 \overline{)15}^5$.

Find the answer to this problem by the fast method.

$$\frac{1}{4} \times 12 = \underline{\hspace{2cm}}. \text{ (Show your work on the answer sheet.)}$$

$$4 \overline{)12}^3$$

The numerator of the fraction $\frac{1}{3}$ is .

p303A-2

1

*This illustration is continued on the next page.

(ILLUSTRATION 19, Continued)*

When multiplying a number, such as 15, by a fraction with a numerator of 1, such as $\frac{1}{3}$, we simply divide 15 by the _____ of the fraction. p303A-3

denominator ("3" will do.)

If we wish to multiply a number by a proper fraction whose numerator is 1, we simply _____ the number by the _____ of the fraction. p303A-4

divide denominator

The quick way to multiply $\frac{1}{4} \times 12$ is to divide _____ by _____. p303A-5
(No.) (No.)

12 by 4

Work the following problem the quick way and show your work. p303A-6

$$\frac{1}{6} \times 18 = \underline{\hspace{2cm}}$$

3

Work the following multiplication problems the quick way in your head. p303A-7

$$\frac{1}{2} \times 14; \quad \frac{1}{3} \times 9; \quad \frac{1}{8} \times 24; \quad \frac{1}{9} \times 45.$$

7 3 3 5

*This illustration is continued on the next page.

(ILLUSTRATION 19, Continued)

When estimating products of mixed numbers, such as $17\frac{1}{2}$, p303A-8
round the mixed number to the nearest whole number.

$17\frac{1}{2}$ is rounded to _____.

18

When estimating, if the fractional part of the mixed p303A-9
number is $\frac{1}{2}$ or more, it is rounded up. If it is less
than $\frac{1}{2}$ it is rounded down.

$17\frac{1}{3}$ is rounded to _____.

$17\frac{5}{8}$ is rounded to _____.

$17\frac{1}{2}$ is rounded to _____.

17 18 18

REVISED FRAME

When estimating the product of the fraction problem x303A

$\frac{1}{3} \times 17\frac{1}{2}$ we round $17\frac{1}{2}$ to 18 and multiply by $\frac{1}{3}$.

We think $\frac{1}{3} \times 18 = 6$ for the estimated product. Use the quick way
of multiplying after rounding the mixed number and then estimate.

$\frac{1}{4} \times 24\frac{1}{3} = \underline{\hspace{2cm}}$. (estimated product.)

6

ILLUSTRATION 20

Revising a Frame to Define the Term Inverted

ORIGINAL FRAME

When $\frac{1}{3}$ is inverted it becomes $\frac{3}{1}$. 313

When $\frac{3}{8}$ is changed to $\frac{8}{3}$, we say that the $\frac{3}{8}$ has been _____.

inverted

REVISED FRAME

Invert means to turn upside down when dividing fractions. x313A

When $\frac{1}{3}$ is inverted it becomes $\frac{3}{1}$.

When $\frac{3}{8}$ is changed to $\frac{8}{3}$, we say that the $\frac{3}{8}$ has been _____.

inverted

significant digit in the decimal fraction. Frame 352 simply indicated that the decimal would be placed to the left of the first significant digit.

All three students in Panel One missed at least one of the frames. Work on the student's answer sheets indicated that they did not comprehend the actual process by which the placement of the decimal was determined.

Therefore, Frame 352 was revised to illustrate more fully the actual process that may be taken to determine the position of the decimal.

As shown in Illustration 21, the revised frame presented the explanation in three steps, aided by visual illustrations in each step. After this revision, these two frames were not missed by anyone on succeeding panels.

Alignment of Decimals.—The purpose of Frames 373 and 374 was to determine if the student understood the method of lining up decimals when the minuend and subtrahend has unlike number of digits to the right of the decimal in a subtraction problem. Neither frame indicated how the procedure should take place.

Carolyn missed four of six possible answers in these two frames. Upon analyzing the answer sheet of this student, evidence pointed to the fact that the student lined up all subtraction problems on the left digit. Therefore, two frames were added to

ILLUSTRATION 21

The Placement of Decimals
in Converted Fractions

ORIGINAL FRAME

Convert $\frac{1}{15}$ to a 3-place decimal. $15 \overline{) 1000}^{66\frac{2}{3}}$ 352

Because 3 decimal places are required, the decimal for $66\frac{2}{3}$ will be placed _____ place(s) to the left of $66\frac{2}{3}$ as follows: $.066\frac{2}{3}$, which may be rounded to _____.

one .067

REVISED FRAME

To convert $\frac{1}{15}$ to a 3-place decimal we think: $1 \div 15$. x352A

The steps are: (1) $15 \overline{) 1}$ Locate the decimal and add three
(2) $15 \overline{) 1.000}$ 0's to carry three places.
 $.066\frac{2}{3}$ Which may be rounded to _____.
(3) $15 \overline{) 1.000}$

.067

precede Frame 373, as shown in Illustration 22. The purpose of the added frames was to explain how decimals should be aligned before subtracting.

Placement of the Decimal in Multiplication Problems.---Frames 377, 378, and 379 developed the concept of marking off decimal places in multiplication problems when both factors included decimal fractions.

The purpose of Frame 380 was to test the concept learned in the three previous frames. Frame 380, as it appears in its original form in Illustration 23, gave two factors "without" decimals and also gave the product. Next the frame gave the same two factors "with" decimals and asked the student to supply the correct product. Actually, all the student had to do was copy the product of the first problem and count off five decimal places.

Carolyn did not make this connection. Instead, she went to the trouble of multiplying the factors and finally made an error in the computation. This made the answer wrong regardless of the fact that she located the decimal in the correct position.

Since the purpose of this frame was not to test the computational ability of the student, but rather to test the concept of counting off decimal places, Frame 380 was revised, as shown in Illustration 23, to include a prompt to the effect of cuing the student that all that is necessary is to count off the correct number of decimal places in the first product which was given in this same frame.

ILLUSTRATION 22

The Alignment of Decimals before Subtracting

ADDED FRAMES

Study the following subtraction problem in decimals: p373A-1

$$825.6 - 614.35$$

In which number do we have the most digits to the right of the decimal?

614.35

When putting the decimals under one another in the p373A-2
 problem $825.6 - 614.35$, a zero must be added to
 the top figure as shown below:

$$\begin{array}{r} 825.60 \\ -614.35 \\ \hline \end{array}$$

The answer to this problem is _____.

211.25

ILLUSTRATION 23

Placement of the Decimal in
Multiplication Problems

ORIGINAL FRAME

If the product of 15 and 135 is 2025, then the product of 1.5 and 1.35 is _____ 380

2.025

REVISED FRAME

If the product of 15 and 135 is 2025, then the product of 1.5 and 1.35 is _____ x380A

(HINT: the answer is the same except for counting off decimal places.)

2.025

Analysis of the Work of Panel Two

Table VI reveals that changes were made in 12 frames on the basis of errors made by Panel Two. Eleven frames were supplemented in this revision resulting in a total of 26 additional responses.

No changes or additions were made in the section on subtraction. However, the reader will observe in Table IV that the highest per cent of errors for Panel Two were made in this section of the programmed unit. He will also see that there was a 4.03 per cent increase in errors of the subtraction section over Panel One. The statistics as they appear here may be somewhat misleading to the reader. There are two causes for the apparent misrepresentation.

One cause of the relatively high percentage of errors here is that Kathy made seven incorrect responses. However, two of these were made in Frame 70 which was a review of angular addition and should not be considered as errors in subtraction.

The second reason for the disproportionate figure was that there were three responses in Frame f69A-1. This problem was one in which the student was required to give the rounded forms of the subtrahend and the minuend and then give the correct remainder for the rounded figure. When either the subtrahend or minuend was rounded incorrectly, the third response resulted in a cumulative error.

Taking these two factors into consideration, the "true" per cent of errors in this section would be approximately 6.66 per cent

TABLE IV

Number and Per Cent of Errors
Made by Panel Two

Student	Addition (114)*	Sub- traction (55)*	Multipli- cation (199)*	Division (101)*	Fractions (171)*	Decimals (192)*	Total Errors (312)*	% Errors in 812 Possible Responses by Student
Alice	2	0	3	5	1	9	18	2.22
Kathy	2	7	9	7	9	12	46	5.67
Kay	14	4	25	9	13	18	81	9.98
Total Errors	18	11	35	19	23	39	145	
Total Possible Responses for Three Students	342	105	597	303	513	576	2436	
% Errors in Total Possible Responses by Section	5.26	10.48	5.86	6.27	4.48	6.77	5.95	
% Increase or Decrease	- 3.53	+ 4.03	- 2.94	- 3.30	- 1.58	- 1.07	- 2.08	

*Number of actual responses required of each student.

TABLE V

Test Scores for Members of Panel Two

<u>Name</u>	<u>Addition</u>	<u>Subtraction</u>	<u>Multiplication</u>	<u>Division</u>	<u>Fractions</u>	<u>Decimals</u>	<u>Average</u>
Alice	86	70	82	89	69	70	77.6
Kathy	86	90	82	89	85	70	83.6
Kay	92	100	82	78	92	77	86.8
Average	88.0	86.7	82.0	92.7	82.0	72.3	84.0
Increase or Decrease	+6.7	-3.3	+12.0	+8.4	0.0	+11.0	+5.8

TABLE VI

Number of Changes and Additions
Made in the Second Revision

<u>Section</u>	<u>Number Changes Made In Original Frames</u>	<u>Total Number Frames Added</u>	<u>Total Number of Responses Added</u>
Addition	0	5	15
Subtraction	0	0	0
Multiplication	4	2	3
Division	0	4	8
Fractions	4	0	0
Decimals	4	0	0
Totals	12	11	26

of the total responses. This true figure was more in line with the average percentage (5.95%) of the total errors of Panel Two.

Results of the intermediate subtraction tests for Panel Two indicated correct responses by all members of the panel on this specific type of problem. The writer's interpretation here was that students understood the concept of rounding subtraction at the end of that section. Therefore, no changes were made in the section on subtraction during this revision.

Table IV reflects an error rate of 5.86 per cent in the multiplication section. The writer attempted to improve the presentation of concepts in this section to increase understanding. Some of the significant changes in this section are recorded in the analysis of frames of Panel Two.

Attention may also focus on the percentage of errors made by Panel Two in the section of decimal fractions. These errors were relatively scattered and the researcher could not seem to "put his finger" on any one factor contributing to the cause. Since, in the first revision an attempt was made to cut down on errors by several changes in frame wording and developing additional frames in some areas, the writer decided to settle for inserting cues such as "underlining" key words in this section. These changes were of such an insignificant nature that no space will be devoted to showing them here.

The division section indicates that Panel Two had an average error rate of 6.27 per cent. This high rate was due, in part, to confusion on terminology. Some of the significant revisions to correct this situation are discussed in the analysis of frames in this section.

The addition section for Panel Two exceeded the maximum error rate set up for this experiment by only .26 per cent. The weakness in student responses seemed to center around incorrect rounding procedures when estimating. The significant revisions made to strengthen this weakness are shown in the analysis of frames.

Table V indicates that Panel Two had an average increase over Panel One of 5.8 points in correct problems on the intermediate tests. There was an increase in scores for all sections except for subtraction.

Alice was the only student to fail a test with a 69 in fractions. The average score for the panel was 84.0. Multiplication had the highest average increase in grade, however, this section netted a decrease of only 2.94 per cent in error rate.

The average intelligence quotient of Panel Two was 86.3 as compared with 94.3 of Panel One. This was the lowest average intelligence quotient of any panel participating in the study.

The average error rate for Panel two was 5.95 per cent, a decrease of 2.08 per cent as compared with Panel One. Two students, Kathy and Kay exceeded the maximum error rate.

The Second RevisionBuilding Speed and Accuracy
in Addition

Rounding Four, Five, and Six-digit Figures to Thousands.--In the first revision, the writer tried to develop the concept of when to round to tens, hundreds, or thousands. Although the students seemed to be catching on to the concept, they still had trouble when they were given a problem which included a combination of thousands, ten-thousands, or hundred-thousands to round and then estimate the total.

There were six frames of this type which presented different combinations of thousands. Of the six students on Panels One and Two, who had worked these frames, three of the students missed at least one frame. Kay missed five responses. There were a total of nine incorrect responses in this series. Cheryl, Alice, and Kathy gave all correct responses in this sequence.

Of those students who responded incorrectly in the sequence of frames which stressed rounding numbers to thousands, the tendency was to round all numbers to the extreme left digit. Therefore, the writer developed three additional frames as shown in Illustration 24. These frames were inserted in the programmed unit to follow the original Frame 7.

The purpose in developing Frame f7B-1 was to determine if the students knew "when" to round to thousands.

ILLUSTRATION 24

Rounding Four, Five, and Six-Digit
Figures to Thousands

ADDED FRAMES

65 6,792 12,376 <u>422</u>	If you were asked to estimate the total of the figures at the left, would you round to tens, hundreds, or thousands? _____	f7B-1
-------------------------------------	--	-------

Thousands

Sue's boss asked her to estimate the following sales to the nearest thousand dollars: f7B-2

\$23,015.23	\$42,611.32	\$61,456.
-------------	-------------	-----------

Sue rounded the figures to the nearest thousand dollars like this:

\$23 thousand	A. Is she correct? _____
43 thousand	B. What is the estimated total? _____
<u>61</u> thousand	

A. yes
 B. \$127,000

Round the following figures to thousands: f7B-3

1,892 is rounded to	_____
12,136 is rounded to	_____
3,462 is rounded to	_____
108,526 is rounded to	_____

<u>2,000</u>	<u>12,000</u>	<u>3,000</u>	<u>109,000</u>
--------------	---------------	--------------	----------------

Frame f7B-2 was developed to "show" students that when rounding numbers with five digits to "thousands" the number is not rounded to the extreme left, but to the fourth digit of that particular number.

The purpose of developing Frame f7B-3 was to give students repetitious practice in rounding 4, 5, and 6-digit numbers to thousands.

Practice in Rounding Up or Down.---It was observed that Rosie and Kay would round aimlessly either up or down. Frames 10, 11, and 12 developed the concept of when to round up or down. However, the student was required to use the "round up or down" technique in estimating the total of succeeding frames. He was not able to diagnose the cause of the wrong response. He could not comprehend that his answer was wrong simply because various numbers were rounded either up or down incorrectly.

Therefore, the writer decided that the student should have more repetition in rounding individual numbers up or down without having to estimate their total. To this end, Frames f12B-1 and f12B-2 were written. These new frames were inserted into the programmed unit to follow the original Frame 12. This revision is shown in Illustration 25.

ILLUSTRATION 25

Practice in Rounding Up or Down

ADDED FRAMES

Show how the following figures would be rounded up or
down for estimating:

f12B-1

426 is rounded to _____
 475 is rounded to _____
 550 is rounded to _____
 514 is rounded to _____

400 500 600 500

Round the following figures as you would for
estimating in addition:

f12B-2

342 is rounded to _____
 605 is rounded to _____
 799 is rounded to _____
 850 is rounded to _____
 990 is rounded to _____

300 600 800 900 1000

Building Speed and Accuracy
in Multiplication

Defining the Term "Extension."—In her comments in Panel One, Rosie stated: "I didn't know what extension was." Since most students who worked on this unit had previously taken General Business and Bookkeeping, the writer decided at that time to disregard this statement. However, after studying the answer sheets of Panel Two, the writer found a comment made by Kathy which was similar to Rosie's. She commented: "Don't know what extending means!" At this point the writer decided that it may be expedient to define the term "extension."

Frame 98 was revised as shown in Illustration 26. Here the writer defined the term extension. Until this time column headings had not been shown on simulated invoices in the unit. For this reason, a visual illustration was given to this effect on the revised Frame x98B.

Rewording a Frame for Clarity.—Kathy made the following statement concerning Frame 107: "At first did not understand what 'itself' was; not very clear."

Therefore, Frame 107 was revised, as it appears in Illustration 27, in order to clarify the statement in this frame.

The Concept of a Reasonable Estimate.—The reader will remember that Frame 129 was rewritten, as shown in Illustration 17, in the first revision of this unit for the purpose of informing the student what constituted a reasonable estimate.

ILLUSTRATION 26

Defining the Term Extension

ORIGINAL FRAME

Computation on an invoice.

98

5 adding machines 462.00 _____

Put in extension here _____↑

The product of 5 and \$462 is extended to the (right/left). _____

1.580 yes

REVISED FRAME

On an invoice, EXTENSIONS are made by multiplying the cost of each item by the number of items purchased.

x98B

Computation on an invoice:

Quantity	Item	Price Per Item	Amount
5	Adding Machines	462.00	

(extension here)

Write the correct extension for the above invoice on the answer sheet.

The product of 5 and \$462 is extended to the _____ (right/left?) .

2.310 right

ILLUSTRATION 27

Rewording a Frame for Clarity

ORIGINAL FRAME

687 X 426 = (Do not solve) 107
 The factor 687, in the problem above, itself lies between _____ and _____. Likewise 426 lies between _____ and _____.

600 700 400 500

REVISED FRAME

687 X 426 = (Do not solve) x107B
 In the problem above, the factor 687 lies between _____ and _____.
 Likewise 426 lies between _____ and _____.

600 700 400 500

Even after the revision, Alice and Kay, in Panel Two, were unable to determine if an estimate was reasonable on the test. Alice also gave a wrong response to the revised frame.

This concept was explained only one time in the first revision. There was no effort made to determine if the student actually understood the definition of a "reasonable estimate." Therefore, two frames were devised for the purpose of increasing an understanding of this concept as required in Frame x129A. These frames, as they appear in Illustration 28, were added to the unit to precede Frame x129A.

Building Speed and Accuracy in Division

Confusion Caused by Unfamiliar Terms.—In the section on division, sub-section "b" and "c" dealt with "rounding numbers to measure accuracy in division" and "handling the trial divisor in long division," respectively. In developing these concepts, in Frames 205 through 239, it was necessary to understand thoroughly the terms "divisor" and "dividend."

In the original unit, three frames (195, 196, and 215) were utilized to familiarize students with "dividend" and "divisor." All students, in Panels One and Two, gave correct responses to these frames. Therefore, the writer assumed that the terms were understood. However, several students remarked orally that the terms confused them. The writer was not overly concerned about their remarks at

ILLUSTRATION 28

The Concept of a Reasonable Estimate

ADDED FRAMES

If the product of two factors falls between the upper and lower estimates, it is considered REASONABLE. p129B-1

A product of two factors is considered reasonable when it falls _____ the upper and lower estimates.

between

When a product of two factors falls between the _____ and _____ estimates it is considered reasonable. p129B-2

upper lower

this time because they responded correctly to those frames eliciting responses which referred to "divisor" and "dividend."

When studying the comments of Panel Two, the researcher became interested in two remarks jotted down by Kathy. In Frame 209 she stated, "I get all mixed up when they say divisor and dividend." Her next remark, in Frame 215, was "Those crazy words again!" Those "crazy words" to which she made reference were divisor and dividend. At this time, the experimenter gave more thought to the matter of "divisor" and "dividend." After analyzing the frames in sub-sections "b" and "c" in the section on division, the following facts were discovered:

- (1) In these sections, 15 frames used either "divisor" or "dividend" as the key word in eliciting the correct response in a problem frame.
- (2) Of the 15 frames, there were "six" in which 12 incorrect responses were made.

Upon the basis of the above analysis, four frames were written to provide thorough familiarization of the key terms in this section. These frames, f195B-1 through 195B-4, as they were inserted in the program, are shown in Illustration 29. They follow the original Frame 195 because at this point the original program attempted to clarify the terms.

ILLUSTRATION 29

Confusion Caused by Unfamiliar Terms

ADDED FRAMES

In the Problem: $25 \overline{) 150}$ f195B-1

Do not round:

(a) _____ is the divisor. (b) _____ is the dividend.

(a) 25 (b) 150

In the problem: $789 \div 36 =$ f195B-2

Do not round:

(a) _____ is the divisor. (b) _____ is the dividend.

(a) 36 (b) 789

In the problem: $9726 \overline{) 478768}$ f195B-3

(a) 478768 is the _____ (divisor or dividend?)

(b) 9726 is the _____ (divisor or dividend?)

(a) dividend (b) divisor

In the problem: $478928 \div 382426 =$ f195B-4

(a) 478928 is the _____ (divisor or dividend?)

(b) 382426 is the _____ (divisor or dividend?)

(a) dividend (b) divisor

Analysis of the Work of Panel Three

Panel Three reflected an average error rate of 5.01 per cent as revealed in Table VII, only .01 per cent above the maximum rate set for this experiment. All individual sections except subtraction and multiplication fell within the maximum error limit. There was a decrease of errors in every section, the greatest occurring in subtraction which dropped 3.81 per cent from that of Panel Two.

Substantial revisions were made in the multiplication section. Several of these changes and additions may be seen in the analysis of frames. Errors were still being made in the section on subtraction, the same sequence of problems giving trouble here as in Panels One and Two. Two additional frames were written to follow the original frame 64 to increase understanding in rounding the minuend to the same extent as the subtrahend when estimating in subtraction. These frames were not shown in the analysis, however, because of their similarity to a like change shown in the first revision. (See Illustration 8.)

As shown in Table VIII, Panel Three had an average decrease of 1.7 in grades on the intermediate tests. A probable reason for this decrease is that at this point a new set of tests were substituted which the writer believed to be superior in measurement of student retention than were those in the original set. These tests were a revision of the writer's original tests made by Huffman, author of the programmed unit.

TABLE VII

Number and Per Cent of Errors
Made in Panel Three

Student	Addition (129)*	Sub- traction (35)*	Multipli- cation (202)*	Division (109)*	Fractions (171)*	Decimals (192)*	Total Errors (838)*	% Errors in 838 Possible Responses by Student
Bev	0	1	12	6	8	3	30	3.58
Pat	9	6	21	5	12	16	69	8.28
Peggy	5	0	10	4	2	6	27	3.22
Total Errors	14	7	43	15	22	25	126	
Total Possible Responses for Three Students	387	105	606	327	513	576	2514	
% Errors in Total Possible Responses by Section	3.62	6.67	7.10	4.59	4.29	4.34	5.01	
% Increase or Decrease	- 1.64	- 3.81	+ 1.24	- 1.68	- .19	- 2.43	- .94	

*Number of actual responses required of each student.

TABLE VIII

Test Scores for Members of Panel Three

<u>Name</u>	<u>Addition</u>	<u>Subtraction</u>	<u>Multiplication</u>	<u>Division</u>	<u>Fractions</u>	<u>Decimals</u>	<u>Average</u>
Bev	100	100	73	95	88	90	91.0
Pat	76	75	75	80	88	60	75.7
Peggy	79	90	70	85	88	85	82.8
Average	85.0	83.3	72.7	86.7	88.0	78.3	82.3
Increase or Decrease	-3.0	-34.4	-9.3	-6.0	+6.0	+6.0	-1.7

TABLE IX

Number of Changes and Additions
Made in the Third Revision

<u>Section</u>	<u>Number Changes Made in Original Frames</u>	<u>Total Number Frames Added</u>	<u>Total Number of Responses Added</u>
Addition	3	25	60
Subtraction	0	3	4
Multiplication	2	18	30
Division	6	10	23
Fractions	3	12	23
Decimals	2	0	0
Totals	16	68	140

Pat was the only student in Panel Three whose error rate was above the limit of five per cent set for this study. This can be accounted for, in part, due to her relatively low ability. Pat, with an intelligence quotient of 79, was probably the less able of the fifteen student-editors participating in this study. She was very unsure of herself. She was afraid to advance from frame to frame without constantly seeking help from the writer. As reflected in Table IX, 16 of the original frames were revised. Also, 68 frames were added to the program which resulted in 140 more responses.

The Third Revision

Building Speed and Accuracy in Addition

Determining the Position of a Digit.—In conversing with student-editors in Panel Three concerning the programmed unit, the writer became aware that they were not fluent with such terms as unit's position, ten's position, hundred's position, and others.

When trying to explain rounding to tens, hundreds, or thousands, it was necessary to indicate positions by saying "so many digits to the left, or so many digits to the right."

Since reference to positions of digits were used throughout the programmed unit, three frames were developed to introduce the significant positions in a number. They were placed in the unit to follow the original Frame 5. As shown in Illustration 30, the

ILLUSTRATION 30

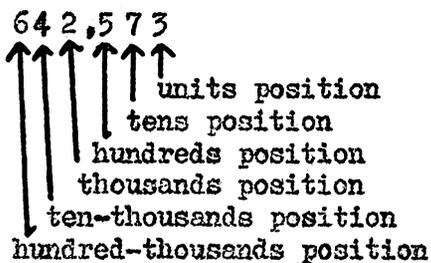
Determining the Position of a Digit

ADDED FRAMES

In order to ESTIMATE accurately, we must be able to round numbers accurately. f5C-1

In order to ROUND accurately, we must be able to tell whether a number is rounded to tens, hundreds, thousands, etc.

Study the number below:



In the number at the left:

7 is in the _____ position
5 is in the _____ position
2 is in the _____ position
4 is in the _____ position

tens hundreds thousands ten-thousands

926,485 In the number at the left: f5C-2

6 is in the _____ position
8 is in the _____ position
4 is in the _____ position

Thousands tens hundreds

843,562 In the number at the left: f5C-3

_____ is in the hundreds position
_____ is in the ten-thousands position
_____ is in the thousands position
_____ is in the tens position

5 4 3 6

first frame, f5C-1, gives a visual presentation to positions of various digits. Frame f5C-2 elicits the student to indicate the "position" of certain digits in a given number. Frames f5C-3 asks the student to identify a digit in a certain position.

Rounding to Zero-Thousands.--Bev was the "fault finder" and "conscientious objector" in Panel Three. Although she did not always go to the inconvenience of writing her comments out, she would let it be known to anyone within hearing range when she came across something in the unit which did not meet with her approval. In the majority of cases, her remarks were unfounded, as the writer would soon point out. They were made simply to draw attention to the fact that she was doing something "different." However, upon one occasion she made a "just" complaint. Her comment was to the effect that the unit told about "stuff" that was not needed.

The information to which Bev referred was one sentence in Frame x24A dealing with rounding to "zero"-thousands. Feeling sure that she was wrong, the writer attempted to show Bev the practice material for rounding to zero-thousands. However, as she pointed out, there was no practice given in this theory throughout the entire section.

After he was convinced that there was no provisions for rounding to zero-thousands, the writer revised Frame x24A as shown in

ILLUSTRATION 31*

Rounding to Zero Thousands

ORIGINAL REVISED FRAME

When rounding to thousands, round numbers from 1 through 499 down to zero thousands. x24A

When rounding to thousands, round numbers from 500 through 1,499 to 1 thousand.

Similarly, round numbers from 1,500 through 2,499 to _____ thousands.

2

REVISED FRAME

When rounding to thousands, round numbers from 1 through 499 down to ZERO thousands. Study the problem below: x24AC

1,450	Which number in the problem to the left
2,652	would be rounded down to ZERO thousands? _____
326	
4,630	
<u>764</u>	
10,000 (estimate)	

326

*This illustration is continued on the next page.

(ILLUSTRATION 31, Continued)*

ADDED FRAMES

52,826		f24C-1
7,123		
201	Study the problem at the left. Show on	
75,326	the answer sheet the figures which would	
302	be rounded to ZERO thousands.	
962		
<u>1,026</u>		
137,000		

201 302

When rounding to thousands, round numbers from 500 through 1,499 to 1 thousand. f24C-2

Therefore, we would round numbers from 1,500 through 2,499 to _____ thousand.

2 thousand

When rounding to THOUSANDS: f24C-3

- 4,600 would round to 5,000
- 5,200 would round to 5,000
- 8,900 would round to _____
- 3,100 would round to _____
- 6,500 would round to _____

9,000 3,000 7,000

*This illustration is continued on the next page.

(ILLUSTRATION 31, Continued)

ADDED FRAMES (CONTINUED)

When rounding to THOUSANDS:

f24C-4

611 rounds to 1,000	}	Likewise	{	833 rounds to _____
346 rounds to 0				421 rounds to _____
512 rounds to 1,000				37 rounds to _____

1,000 0 0

Round the following figures to thousands:

- | | | |
|-----------|------------|------------|
| (a) 1,200 | (f) 5,321 | (k) 22,422 |
| (b) 600 | (g) 725 | (l) 1,000 |
| (c) 3,811 | (h) 11,450 | (m) 25 |
| (d) 465 | (i) 8,511 | (n) 833 |
| (e) 500 | (j) 15,112 | (o) 311 |

- | | | |
|-----------|------------|------------|
| (a) 1,000 | (f) 5,000 | (k) 22,000 |
| (b) 1,000 | (g) 1,000 | (l) 1,000 |
| (c) 4,000 | (h) 11,000 | (m) 0 |
| (d) 0 | (i) 9,000 | (n) 1,000 |
| (e) 1,000 | (j) 15,000 | (o) 0 |

Illustration 31. The total revision was effected through six frames.

The information in Frame x24A was broken down into two frames (x24AC and f24C-2). The theory dealing with rounding to zero-thousands was stated in Frame f24C-2. Frames f24C-1 and f24C-4 stressed drill in rounding to zero-thousands. Frame f24C-3 stressed drill in rounding to "significant" thousands. Frame f24C-5 combines the two theories.

Techniques in Building Speed and Accuracy.—About the first of February, Dr. Huffman, author of the original programmed unit, sent to the researcher a set of six intermediate tests which he prepared for this unit. Until this time, the writer had been using tests which he had prepared in testing students in Panels One and Two.

Several of the questions on Dr. Huffman's tests required the student to list various techniques to increase speed in addition, and those which would increase accuracy.

No specific statements had been made in the original unit to explain this to the student. After several students took Dr. Huffman's tests, the writer realized that the caliber student with which he was working, in most cases, did not have the ability to rationalize to this extent. It would be necessary to "explain" to these students why they were learning the various concepts and techniques in this unit. Although the writer had explained to the student that the programmed unit would help to increase their speed and accuracy in the areas covered, in an attempt to motivate them, they were not capable of distinguishing between these concepts.

Building speed and accuracy in the mathematical processes were two of the primary objectives of the unit. If the student knows "why" he is doing a particular activity, he might be motivated to do a better job. Therefore, a total of 17 frames were programmed to explain to the student why he was participating in this experiment (to develop speed and accuracy in the mathematical processes).

Four of the frames programmed to explain to the student techniques which increase "accuracy" in addition appear in Illustration 32. Four frames were also developed to teach techniques to obtain "speed" in addition. These are shown in Illustration 33.

Building Speed and Accuracy in Multiplication

A Shortcut Method for Multiplying by 998.—The reader may recall that in the first revision, Frame 162 was revised. This was because Cheryl made the wrong response to the frame and then on the intermediate test she and Rosie missed a problem which tested the knowledge of that frame, the shortcut for multiplying by 998. As shown in Illustration 18, the frame was revised at that time to give only a hint as to the procedure involved.

After the revision was made, Kay, in Panel Two, made an incorrect response to the revised frame. All three students in this panel (Alice, Kathy, and Kay), missed problem on the intermediate test. Although no student in Panel Three made an incorrect

ILLUSTRATION 32

Techniques to Increase Accuracy in Addition

ADDED FRAMES

One way we have learned to increase accuracy in addition is to estimate. f35C-1

Accuracy is also increased when we use REVERSE ADDITION.

When we use reverse addition we increase _____.

accuracy

Accuracy is increased by either estimating or reverse addition. f35C-2

reverse

Why do we use estimating and reverse addition? _____ f35C-3

To increase accuracy

Accuracy may be increased by either _____ or _____ addition. f35C-4

estimating
reverse



ILLUSTRATION 33*

Techniques for Increasing Speed in Addition

ADDED FRAMES

By using combinations of 10, we can increase SPEED in addition. Speed in addition may be obtained by of 10. f38C-1

combinations

6)	The problem at left is an example of	f38C-2
4)	adding by _____	
7)		
3)	Adding by combinations of 10 helps to	
8)	increase _____ in addition.	
2)		
<u>30</u>		

combinations of 10 speed

Adding by _____ of 10 helps increase speed. Another method we learned previously to help increase speed is _____ addition. f38C-3

combinations horizontal

*This illustration is continued on the next page.

(ILLUSTRATION 33, Continued)

ADDED FRAMES CONTINUED

$$\begin{array}{r}
 (a) \quad 5) \\
 \quad 5) \\
 \quad 7) \\
 \quad 3) \\
 \quad 2) \\
 \quad 8) \\
 \hline
 \quad 30
 \end{array}$$

$$(b) \quad 10 + 7 + 8 + 9 + 1 = 35$$

f38C-4

Problem "a" at left is an example of
adding by _____ of _____.

Problem "b" above is an example of
_____ addition.

Both of these methods shown here help
increase _____ in addition

combinations of 10
horizontal
speed _____

response to Frame x162A in the revised programmed unit, Bev and Peggy, of that panel, missed the problem on the test.

To summarize, after the revision was made, only one incorrect response was made in the revised unit, however, five students missed a similar problem on the test.

Therefore, the writer revised Frame x162A to show more explicitly the procedure involved in the shortcut method for multiplying by 998. The original frame included drill for the 999 shortcut method, plus eliciting the procedure for the 998 shortcut method.

The researcher split the frame in order that the drill for the 999 method could be put into separate frames to follow Frame 161 which developed the procedure for that method.

Frame x162AC was utilized to demonstrate the shortcut method for 998. Additional drill was provided for this technique in Frame f162C-1 as shown in Illustration 34.

Building Speed and Accuracy in Division

Showing the Easy Way to Estimate.--The purpose of Frame 204 was to make students realize the most efficient way to round when estimating in division. This frame received more written comments from student-editors than any one frame previous to the time of this revision. However, only two students out of the nine in Panels One through Three made incorrect responses to the frame. Probably the

ILLUSTRATION 34*

A Shortcut Method for Multiplying by 998

REVISED ORIGINAL FRAME

Using the shortcut for multiplying by 999 and a similar x162A
one for 998 do the problems below.

(HINT: The shortcut rule for 998 is similar to the rule for 98.)

(a) $999 \times 1,245 = \underline{\hspace{2cm}}$

(b) $998 \times 1,341 = \underline{\hspace{2cm}}$

1,243,755 1,338,318

REVISED FRAME

To multiply 998 x 231: x162AC

(1) Annex 3 zeroes to 231 like this: \longrightarrow 231,000

(2) Multiply 231 x 2 and subtract results like this: $\underline{\hspace{1cm}} - 462$

The answer is: \longrightarrow

Using the same shortcut shown above, multiply 998 x 321 =

Show your work on the answer sheet.

320,358

*This illustration is continued on the next page.

(ILLUSTRATION 34, Continued)

 ADDED FRAME

Using the shortcut method for 993, multiply the following and show your work.

f162C-1

$$998 \times 652 = \underline{\hspace{2cm}}$$

$$998 \times 411 = \underline{\hspace{2cm}}$$

$$998 \times 625 = \underline{\hspace{2cm}}$$

650,696

410,178

621,754

reason most students made correct responses may be summed up in Pat's comment; "Had to guess, but I got it right." There were three other comments as follows.

Carolyn, who responded incorrectly came up with her "usual" comment on such occasions; "The question was not clear."

Kathy, who also had an incorrect response, commented: "It was asking too many questions here. I can't keep it straight."

Kay commented: "Not very clear."

The frame was not revised previously because only two students had responded incorrectly. The frequency and context of comments on the frame was the basis used for the revision. The frame was revised to include a visual aid to help clarify the questions. The revision of Frame 204 appears in Illustration 35.

Steps in Estimating Division.--The purpose of Frames 204 through 214 was to develop the techniques of rounding numbers in division in order to make logical estimates. Frames 205, 206, and 209 developed the procedure for estimating. Frames 207 through 212 gave drill in this procedure. In the six drill frames, there were a total of 12 incorrect responses from the nine students who had previously edited this unit. Three of the incorrect responses were made by students in Panel Three.

There were three comments by two student-editors pertaining to errors made in calculations. On Frame 209, Bev's comment was,

ILLUSTRATION 35

Showing the Easy Way to Estimate

ORIGINAL FRAME

Which problem can be done most easily in the head? 204

- (a) Two digits followed by 0's \div 1 digit followed by 0's.
 (b) 3 or more digits followed by zeros \div 2 digits followed by 0's.
 (c) 1 digit with 0's \div 2 digits with 0's.

(answer) _____

(a) _____

REVISED FRAME

Which problem can be done most easily in the head? x204C

- (a) $600 \overline{) 2,400}$ Two digits followed by 0's,
divided by one digit followed by 0's.
 (b) $620 \overline{) 2,490}$ Three or more digits followed by 0's,
divided by two digits followed by 0's.
 (c) $620 \overline{) 2,000}$ One digit followed by 0's,
divided by two digits followed by 0's.

(a) _____

"Forgot to strike out zeros." Pat made comments on two frames. On Frame 208 she remarked, "I took off one zero to many." And on Frame 211 she said, "I was just dividing straight out."

In analyzing errors of other students, the writer found that comments made by Bev and Pat could very well have been made for wrong responses of their co-editors in similar frames.

Toward the end of the section on division, two frames furnished review for rounding and estimating quotients. Of the nine students who had completed this unit, there were five incorrect responses to the two review frames.

After some consideration, the writer made a decision to use visual aids in programming the procedure for rounding and estimating quotients.

There are three major parts in the estimation procedure:

- (A) Rounding the Divisor and Dividend
- (B) Striking Out Zeros to Make Division Easier
- (C) Dividing

In the original unit, one frame, 205, was utilized to "show" how the figures were rounded. No practice was given in rounding before striking out zeros was introduced in the next frame.

Therefore, five frames were added to follow the original Frame 205, as shown in Illustration 36. The purpose of these frames were to make sure that the student fully understood the two steps in

ILLUSTRATION 36*

Rounding to Make Estimating Easy

ADDED FRAMES

Study the division problem below to see how it has been rounded for estimating. f205C-1

$$7,681 \overline{) 475,263} \text{ is rounded to: } 8,000 \overline{) 480,000}$$

- (a) The divisor has been rounded to ONE digit followed by 0's.
- (b) The dividend has been rounded to TWO digits followed by 0's.

To estimate in division round the divisor to _____ digit followed by zeros and the dividend to _____ digits followed by zeros.

$$\underline{1 \quad 2}$$

When we want to estimate in division, we round the divisor to _____ digits followed by zeros and round the dividend to _____ digits followed by zeros. f205C-2

$$\underline{1 \quad 2}$$

To estimate in division, the problem $781 \overline{) 46,513}$ would be rounded like this: $\longrightarrow 800 \overline{) 47,000}$ f205C-3

In the same way, the problem $672 \overline{) 18,416}$ would be rounded like this: $\longrightarrow \overline{\hspace{2cm}}$ (Show how problem would be rounded)

$$\underline{700 \overline{) 18,000}}$$

*This illustration is continued on the next page.

(ILLUSTRATION 36, Continued)

ADDED FRAMES CONTINUED

Show how the problems below would be rounded to estimate in division.

f205C-4

415 $\overline{) 3,672}$ is rounded to: $\underline{\hspace{2cm}}$

58 $\overline{) 46,203}$ is rounded to: $\underline{\hspace{2cm}}$

62 $\overline{) 187,301}$ is rounded to: $\underline{\hspace{2cm}}$

400 $\overline{) 3,700}$ 60 $\overline{) 46,000}$ 60 $\overline{) 190,000}$

Show how the problems below would be rounded to estimate in division.

f205C-5

81 $\overline{) 235}$ $\underline{\hspace{2cm}}$

193 $\overline{) 47,845}$ $\underline{\hspace{2cm}}$

44 $\overline{) 8,342}$ $\underline{\hspace{2cm}}$

80 $\overline{) 240}$ 200 $\overline{) 48,000}$ 40 $\overline{) 8,300}$

rounding before he proceeded to strike out zeros. The steps in this part of the procedure in estimating are:

- (1) Round the divisor to "one" digit followed by 0's.
- (2) Round the dividend to "two" digits followed by 0's.

The original Frame 206 tells the procedure for striking out zeros:

- (1) Strike out all zeros in the divisor.
- (2) Strike out the same amount of zeros in the dividend.

Two frames, f206C-1 and f206C-2, were added to give drill on these two steps. The frames are shown in Illustration 37. The third procedure, dividing the rounded figures, was practiced in the original Frames 207 and 208.

Original Frame 209 was revised to bring all of the steps in the technique together. This was to make it possible for the student to "see" the complete series of five steps in uninterrupted sequence and without having to guess. (See Illustration 38.) Practice was provided in that frame while looking at the steps involved.

Frames f209C-1, 2, and 3 were used to give additional drill in the foregoing concepts. These frames appear in Illustration 39.

Building Speed and Accuracy with Common Fractions

The purpose of Frames 241 through 258 was to familiarize the student with the definition of terms in common fractions. When the

ILLUSTRATION 37

Striking-out Zeros to
Make Estimating Easy

ADDED FRAMES

Further steps to make estimating still more simple is to f206C-1
STRIKE OUT all of the 0's in the divisor and then strike
out the SAME AMOUNT of 0's in the dividend.

Study the combined steps for estimating $246 \overline{) 18,362}$ below:

(a) Round divisor and dividend to: $200 \overline{) 18,000}$

(b) Strike out 0's and divide: $2\cancel{00} \overline{) 18,0\cancel{00}} \begin{matrix} 90 \\ \hline \end{matrix}$ (est. answer)

Show how to estimate: $382 \overline{) 24,123}$

$4\cancel{00} \overline{) 24,0\cancel{00}} \begin{matrix} 60 \\ \hline \end{matrix}$

$52 \overline{) 14,625}$ is estimated like this: $5\cancel{0} \overline{) 15,0\cancel{00}}$ f206C-2

(a) $43 \overline{) 20,262}$ is estimated like this: $\overline{\hspace{2cm}}$

(b) $78 \overline{) 2,836}$ is estimated like this: $\overline{\hspace{2cm}}$

(a) $4\cancel{0} \overline{) 20,0\cancel{00}} \begin{matrix} 500 \\ \hline \end{matrix}$

(b) $8\cancel{0} \overline{) 2,80\cancel{0}}$

ILLUSTRATION 38

Combined Steps of Estimating

ORIGINAL FRAME

The steps for estimating answers in division are: 209

- (1) round divisor to one digit followed by 0's.
- (2) round dividend to two digits (or less) followed by 0's.
- (3) obtain answer mentally.

Problem

$$321 \overline{) 64,256} \quad \longrightarrow \quad \overline{\hspace{2cm}}$$

Estimate answer! ↑

$$\underline{300 \overline{) 64,000} \quad \text{Answer} = 210 \text{ or } 213}$$

REVISED FRAME

Below we have a breakdown of the steps for estimating x209C
in division:

- (1) Round divisor to one digit followed by 0's
- (2) Round dividend to two digits followed by 0's.
- (3) Strike out all 0's in the divisor.
- (4) Strike out the same amount of 0's in the dividend.
- (5) Obtain estimated answer mentally.

Problem: $321 \overline{) 64,265}$ is rounded to: $\overline{\hspace{2cm}}$ (estimated ans.)

$$\underline{300 \overline{) 64,000} \quad \text{213 or 213.3}}$$

ILLUSTRATION 39

Additional Drill for Estimating

ADDED FRAMES

When estimating in division: f209C-1

- (1) Round _____ to ONE digit followed by 0's.
- (2) Round _____ to TWO digits followed by 0's.

- (1) divisor (2) dividend

To estimate in division after rounding we: f209C-2

- (1) Strike out _____ 0's in the divisor.
- (2) Strike out the _____ of 0's in the dividend.
- (3) Obtain the estimated answer mentally.

- (1) all (2) same amount

The combined steps in estimating division are: f209C-3

- (1) Round the divisor to _____ digit followed by 0's.
- (2) Round the _____ to two digits followed by 0's.
- (3) Strike out all 0's in the _____.
- (4) Strike out0's in the dividend.
- (5) obtain the estimated answer _____.

one
 dividend
 divisor
 the same amount of
 mentally

terms "numerator" and "denominator," were first introduced in Frames 243 and 244, all students, except Carolyn and Cheryl in Panel One, responded correctly. Cheryl missed Frame 245 and Carolyn made an incorrect response in 246 and 247. Carolyn and Kathy responded incorrectly to Frame 248. Four students, at least one from each panel, made incorrect responses to Frame 249. In other words, there were six incorrect responses in the two frames 248 and 249.

The students seemed to be confused on two points:

- (1) Which part of the fraction (numerator or denominator) represented the whole.
- (2) The relationship of two fractions as to their comparative size.

Use of Pie-Chart in Defining Numerator and Denominator.---The writer decided that this confusion may best be decreased by visual aids. Frame 243 introduced the terms of numerator and denominator and their respective relationship with the parts and the whole of a fraction. Therefore, three frames were added, as shown in Illustration 40, utilizing pie-charts as a visual aid in explaining the terms.

Use of Bar-Graphs in Comparing Size of Fractions.---The original Frame 248 introduced the relationship of two fractions as to their comparative size. Two frames were added to give visual illustration to this concept in the form of bar-graphs. These frames, f248C-1 and f248C-2, appear in Illustration 41.

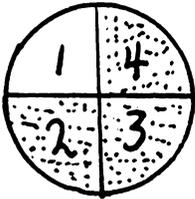
ILLUSTRATION 40*

Use of Pie-Chart in Defining
Numerator and Denominator

ADDED FRAMES

The circle below is divided into 4 parts.

f243C-1

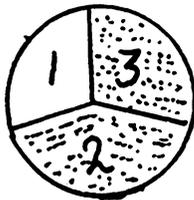


There are _____ parts in the whole circle.

One part of the circle is white, therefore $\frac{1}{4}$
of the circle is white.

Likewise, 3 parts of the circle are shaded. Therefore,
_____ of the circle is shaded.

$$\frac{4}{4}$$



f243C-2

$\frac{1}{3}$ ← Numerator: Represents a part taken
from the circle.
 $\frac{1}{3}$ ← Denominator: Represents the total parts
in the circle.

What part of the fraction above represents the total number of
parts in the circle? (numerator or denominator?) _____

The white part of the circle above is represented by the
_____ (numerator or denominator?)

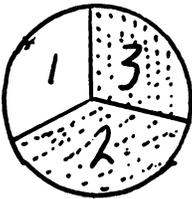
denominator numerator

*This illustration is continued on the next page.

(ILLUSTRATION 40, Continued)

 ADDED FRAMES CONTINUED

f2430-3



The fraction representing the white part of the circle is $\frac{1}{3}$.

The fraction representing the shaded parts of the circle is _____.

$\frac{2}{3}$

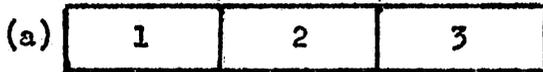
ILLUSTRATION 41

Use of Bar-Graphs in Comparing
Size of Fractions

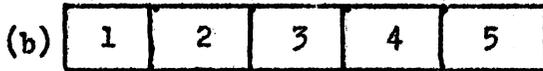
ADDED FRAMES

f248C-1

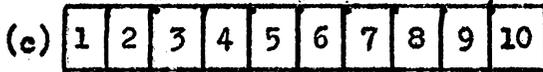
Below a, b, and c, represent 3 gold bars, all the same size.



Here $\frac{1}{3}$ represents one part.



Here ____ represents one part.

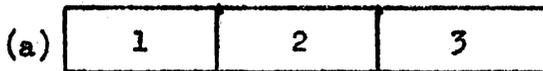


Here ____ represents one part.

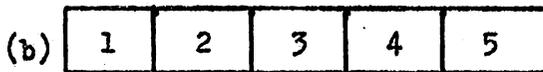
If a greedy person could have one part of anyone of the gold bars, from which bar would he take a part? (a,b, or c?) _____

$\frac{1}{5}$ $\frac{1}{10}$ (a)

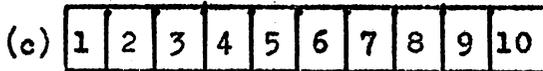
f248C-2



Bar "a" is divided into larger parts than bar "b"



Therefore $\frac{1}{3}$ is larger than $\frac{1}{5}$.



Is $\frac{1}{5}$ larger or smaller than $\frac{1}{10}$?

(answer) _____

larger

Analysis of the Work of Panel Four

As revealed in Table X, Panel Four was the first group of student-editors whose average error rate came within the limits set up for this study with a 4.26 per cent average. However, the errors of two students were just above the maximum.

The only section of the unit in which the errors of the panel exceeded the error limit was decimals with a panel average of 5.90 per cent. This average was an increase of 1.56 per cent over the preceding panel.

Table XI reflects an average increase of 1.5 correct problems in the intermediate tests for Panel Four. In four sections, test scores increased over the previous panel. Test scores in the addition and decimal sections dropped slightly. All student-editors in Panel Four passed the entire sequence of tests with a range from 70 to 95.

As illustrated in Table XII, seven new frames were added to the program in decimals and fractions. Five of these frames were added in the decimal section.

The average intelligence quotient for Panel Four was 90.7 as compared with 89.3 for Panel Three. Verna's and Martha's intelligence quotient was 85 and 87 respectively, and Cindy's was 100, the highest of any student participating in the experiment.

TABLE X

Number and Per Cent of Errors
Made by Panel Four

Student	Addition (189)*	Sub- traction (39)*	Multipli- cation (232)*	Division (132)*	Fractions (194)*	Decimals (192)*	Total Errors (978)*	% Errors in 978 Possible Responses by Student
Cindy	2	1	4	2	2	0	11	1.13
Verna	3	1	15	9	19	16	63	6.50
Martha	14	2	11	2	4	18	51	5.26
Total Errors	19	4	30	13	25	34	125	
Total Possible Responses for Three Students	567	117	696	396	582	576	2934	
% Errors in Total Possible Responses by Section	3.35	3.42	4.31	3.28	4.30	5.90	4.26	
% Increase or Decrease	- .27	- 3.25	- 2.79	- 1.31	+ .01	+ 1.56	- .75	

*Number of actual responses required of each student.

TABLE XI

Test Scores for Members of Panel Four

<u>Name</u>	<u>Addition</u>	<u>Subtraction</u>	<u>Multiplication</u>	<u>Division</u>	<u>Fractions</u>	<u>Decimals</u>	<u>Average</u>
Cindy	86	90	80	95	94	85	83.3
Verna	86	80	80	85	88	70	81.5
Martha	76	85	70	95	88	75	81.5
Average	82.7	85.0	76.6	91.7	90.0	76.7	83.8
Increase or Decrease	-2.3	+1.7	+3.9	+5.0	+2.0	-1.6	+1.5

TABLE XII

Number of Changes and Additions
Made in the Fourth Revision

<u>Section</u>	<u>Number Changes Made in Original Frames</u>	<u>Total Number Frames Added</u>	<u>Total Number of Responses Added</u>
Addition	0	0	0
Subtraction	0	0	0
Multiplication	0	0	0
Division	1	0	0
Fractions	1	2	5
Decimals	6	5	5
Totals	8	7	10

The Fourth RevisionBuilding Speed and Accuracy
with Common Fractions

Estimating for Accuracy.—The purpose of Frames 318 through 324 was to teach estimation of quotients in common fractions. There were six frames in this series on which the first four panels made a total of 16 incorrect responses. An analysis of the answer sheets revealed that students were not rounding fractions to whole numbers properly before estimating. Usually, the error resulted from one of two causes.

- (1) The student would always round "any" fraction up, regardless of its relationship to one-half.
- (2) The student would always drop "any" fraction regardless of its relationship to one-half.

The original Frame 318 indicated by visual illustration that fractions would be rounded either up or down. The frame did not "specifically" state, however, that the number would be rounded either up or down depending on the size of the fraction.

Frame 318 was revised to state that mixed numbers would be rounded to whole numbers as shown in Illustration 42.

Also, two frames were added to follow the revised frame. The additions may be seen in Illustration 43. Frame f318D-1 was designed to give drill in the procedure. The purpose of Frame f318D-2 was to

ILLUSTRATION 42

The Importance of Rounding

ORIGINAL FRAME

The estimation of answers to the division of fractions is just as important as estimating products to multiplication problems. 318

The problem $15\frac{1}{3} \div 4\frac{2}{3} =$ rounded becomes $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}}.$

The estimated answer is $\underline{\hspace{1cm}}.$

$$\underline{15 \div 5 = 3}$$

REVISED FRAME

The estimation of answers to the division of fractions is just as important as estimating products to multiplication problems. x318D

In division, just as in multiplication, we must round mixed numbers to whole numbers in order to estimate quickly.

To estimate in division, we must round mixed numbers to numbers.

whole

ILLUSTRATION 43

When to Round Up or Down

ADDED FRAMES

When the fractional part of a number is LESS than $\frac{1}{2}$ f318D-1
 the fraction is DROPPED and the number stays the same.

EXAMPLE: $5\frac{1}{8}$ is rounded to 5.

When the fractional part of a number is $\frac{1}{2}$ or OVER
 add 1 to the whole number.

EXAMPLE: $5\frac{7}{8}$ is rounded to 6.

Round the following fractions: $4\frac{1}{2}$, $3\frac{1}{3}$, $6\frac{3}{4}$

Answers: _____

5 3 7

The problem $19\frac{3}{4} \div 5\frac{1}{3}$ would be rounded to $20 \div 5$. f318D-2

The estimated answer is 4.

The problem $15\frac{1}{3} \div 4\frac{2}{3}$ would be rounded to _____ \div _____

The estimated answer is _____.

15 \div 5 = 3

make sure that the student actually wrote out the rounded numbers before estimating. Throughout the remainder of the series, he was not required to do this.

Decimal Fractions

Converting Mixed Decimals to Common Fractions.—The purpose of Frames 363 through 371 was to teach converting mixed decimals to common fractions. Seven of 12 students in Panels One through Four gave correct responses to all four frames. However, the remaining five students made a total of 14 errors.

There were only two comments on this series of frames. Verna, in Panel Four, commented about Frame 369; "I don't know how to do it without looking." Pat, in Panel Three, said, "I got mixed up."

The only example given of the procedure was in Frame 368. This was a rather involved process considering the ability of most of the student-editors. The difficulty was due partially to the crowded state of the original frame. From oral remarks of the students, some of the difficulty seemed to lie in the fact that they could not determine how to arrive at the following:

$$\frac{43\frac{3}{4} \times 4}{100 \times 4} = \frac{175}{400}$$

The writer revised Frame 368 by showing how the "4" in the multiplicand of the numerator cancelled out with the "4" in the

multiplier of the numerator. This revision appears in Illustration 44. In addition to the revision, three frames were developed to follow Frame 368. The purpose of these frames was to show a part of a problem done and have the student complete the process of converting and give the final common fraction. In each of the succeeding frames, the student was required to work a larger portion of the problem. The added frames follow Frame 368 as they appear in Illustration 45.

Analysis of the Work of Panel Five

Student-editors in Panel Five completed the unit after the final revision with an average error rate of 2.83 per cent out of a possible 988 responses. Table XIII shows that there was a total of 84 errors made, an average of 28 for each student in Panel Five. All students scored within the maximum error limit set for the study. Marie had the highest rate of errors, 4.35 per cent.

Errors in all sections of the unit were within the error limit. The section on subtraction had the highest rate of errors with 4.27 per cent. The next highest was decimals with 3.38 per cent of errors. Fractions had the lowest error rate with 1.84 per cent of incorrect responses in that section. The per cent of decrease in errors for Panel Five on the final revision of the programmed unit as compared with the previous panel was 2.83 per cent.

ILLUSTRATION 44

Converting Mixed Decimals to Fractions

ORIGINAL FRAME

$.43\frac{3}{4}$ is converted to a common fraction as follows: 368
 $.43\frac{3}{4} = \frac{43\frac{3}{4}}{100} = \frac{43\frac{3}{4} \times 4}{100 \times 4} = \frac{175}{400}$
 $\frac{175}{400}$ is then reduced; $\frac{175}{400} = \frac{7}{16}$
 What common fraction is $.56\frac{1}{4}$ _____.

$\frac{9}{16}$

REVISED FRAME

$.43\frac{3}{4}$ is converted to a common fraction like this: x368D

$$.43\frac{3}{4} = \frac{43\frac{3}{4}}{100} = \frac{\frac{175}{4} \times \frac{1}{4}}{100 \times 4} = \frac{175}{400} = \frac{7}{16}$$

The common fraction for $.56\frac{1}{4}$ is _____. (show all work)

$\frac{9}{16}$

ILLUSTRATION 45

Drill in Converting Decimals to Fractions

ADDED FRAMES

Finish converting $.11\frac{4}{6}$ into a common fraction. (Show work) f368D-1

$$.11\frac{4}{6} = \frac{11\frac{4}{6}}{100} = \frac{\frac{70}{6} \times 6}{100 \times 6} = \underline{\hspace{2cm}}$$

$$\frac{7}{60}$$

Finish Converting $.3\frac{1}{3}$ into a common fraction. f368D-2

$$.3\frac{1}{3} = \frac{3\frac{1}{3}}{10} = \frac{\frac{10}{3}}{10} = \underline{\hspace{2cm}} \quad (\text{show all work})$$

$$\frac{1}{3}$$

Finish converting $.12\frac{6}{7}$ into a common fraction. f368C-3

$$.12\frac{6}{7} = \frac{12\frac{6}{7}}{100} = \underline{\hspace{2cm}} \quad (\text{show all work})$$

$$\frac{9}{70}$$

Table XIV reflects that Panel Five had an average increase of 3.5 correct answers over Panel Four. Fractions was the only section which showed an average decrease in test score. The average increase in scores from Panel One through Panel Five on the intermediate tests was 9.1. The only section which reflected a decrease from Panel One was subtraction. Panel One had the highest average in subtraction than any other panel. The highest average grade for Panel Five was in division with 93.3. This was the highest average grade of any panel on any section of the tests.

Students on Panel Five had the second highest average intelligence quotient (93.7) of the preceding panels. Panel One had the highest average with 94.3. (See Appendix B.) However, the error rate for Panel One was 8.03 per cent as compared with only 2.83 per cent for Panel Five.

The intelligence quotients of Gwin and Carol were 94 and 88 respectively. Marie had the highest intelligence quotient in Panel Five with 99, however, she made the highest percentage of errors, 4.35 per cent, as compared with Gwin, 2.53 per cent and Carol, 1.62 per cent. All of Marie's mental ability tests and subject matter grades compared favorably with those of Gwin and Carol. The probable reason for Marie's relatively high error rate may be explained in the fact that she was absent due to illness two times, a week each time, during the period in which she participated in the study.

TABLE XIII

Number and Per Cent of Errors
Made by Panel Five

Student	Addition (189)*	Sub- traction (39)*	Multipli- cation (232)*	Division (132)*	Fractions (199)*	Decimals (197)*	Total Errors (988)*	% Errors in 988 Possible Responses by Student
Gwin	5	1	3	5	3	8	25	2.53
Carol	1	3	7	2	0	3	16	1.62
Marie	10	1	10	5	8	9	43	4.35
Total Errors	16	5	20	12	11	20	84	
Total Possible Responses for Three Students	567	117	696	396	597	591	2964	
% Errors in Total Possible Responses by Section	2.82	4.27	2.87	3.03	1.84	3.38	2.83	
% Increase or Decrease	- .53	+ .85	- 1.44	- .25	- 2.46	- 2.52	- 1.43	

*Number of actual responses required of each student.

TABLE XIV

Test Scores for Members of Panel Five

<u>Name</u>	<u>Addition</u>	<u>Subtraction</u>	<u>Multiplication</u>	<u>Division</u>	<u>Fractions</u>	<u>Decimals</u>	<u>Average</u>
Gwin	100	90	70	95	94	80	88.2
Carol	72	80	90	90	81	80	82.2
Marie	92	90	95	95	94	95	93.5
Average	88.0	86.7	83.0	93.3	89.7	83.0	87.3
Increase or Decrease	+5.3	+1.7	+6.4	+1.6	- .3	+6.3	+3.5

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

This research involved a programmed text of 432 frames, entitled Fundamentals of College Business Mathematics, written by Dr. Harry Huffman. The text was published in 1961 by the Center for Programmed Instruction, New York, New York, for experimental tryout with both post-high school students and college freshmen.

As a basis for this study, two major objectives were established. The first objective for the experiment was to adjust Huffman's programmed business mathematics text to a level of comprehension of high school students of below average ability, so that their completed work, without a teacher's help, would be approximately 95 per cent accurate.

The second objective of the study was to determine whether students could retain the material covered in the programmed text. This objective was achieved by means of intermediate tests given to each student throughout the unit. The case study approach was utilized in attaining the two objectives of this experiment.

Four revisions of Huffman's original programmed text were necessary to accomplish the objectives of the study. These revisions took place between September 15, 1961 and May 4, 1962.

Five panels of student-editors, each panel comprised of three students of below average ability, participated in the research.

The Results of Panel One on the Original Program.---Panel One completed the original program before any revision took place. A count of errors made by members of Panel One revealed that an average of 8.03 per cent of errors were made on 780 possible responses in the original program. The average test score on six intermediate tests for Panel One was 78.2 out of a possible 100 points.

The errors made on the programmed unit, intermediate tests, and comments made by student-editors in Panel One were used as a basis to make the first revision. Revisions to the original programmed unit were in the following forms: (a) Additional frames were added to strengthen understanding of concepts; (b) frames were split to reduce density of content to be learned in individual frames; (c) individual frames were revised for clarification; and (d) prompts were added to insure correct responses.

As a result of the first revision, 18 additional frames were added which included a total of 32 responses. A total of 36 minor changes were made in the original frames.

Results of Panel Two on the First Revision.---As a result of the first revision, Panel Two worked through the program with an average error rate of 5.95 per cent of a possible 812 responses. The average score for the panel on six intermediate tests was 84.

The results of the work of Panel Two provided the basis for the second revision. The wording of 12 original frames was slightly

TABLE XV

Summary of Changes and Additions Made
in Each Section of the Program

(Original Program of 432 Frames)

<u>Section</u>	<u>Number Changes Made in Original Frames</u>	<u>Total Number Frames Added</u>	<u>Total Number of Responses Added</u>
Addition	11	32	79
Subtraction	3	6	8
Multiplication	19	20	35
Division	11	14	31
Fractions	13	24	45
Decimals	15	8	10
Totals	72	104	208

TABLE XVI

A Summary of Additions of Frames and Number of Responses Required in Successive Revisions of the Programmed Unit

<u>Revision</u>	<u>Frames</u>	<u>Responses</u>
First Revision	20	32
Second Revision	11	26
Third Revision	66	140
Fourth Revision	7	10
Total Added	104	208
Total Frames in Original Program	432	780
Total Frames and Responses in the Revised Program	536	988

altered or prompts added. Eleven new frames were added which included 26 responses.

Results of Panel Three on the Second Revision.---Panel Three worked through the program after the second revision was made. The panel had an average error rate of 5.01 per cent on 838 responses. Intermediate test results revealed an average score of 82.3. These data were the basis for a third revision. Sixteen changes were made in the original frames and 68 new frames were added with a total of 140 additional responses.

Results of Panel Four on the Third Revision.---After the third revision, Panel Four worked through the programmed unit making 4.26 per cent errors on their 978 responses. The average score on intermediate tests was 83.8.

Since Panel Four came within the error limit of 5 per cent set up for this study, only minor revisions were made on two of the weakest sections of the unit, fractions and decimals. Eight of the original frames underwent minor word changes and addition of prompts. Seven new frames were written containing a total of 10 additional responses.

Results of Panel Five on the Fourth Revision.---Panel Five read the programmed unit after it had been revised four times. The panel had an error rate of 2.83 per cent on a possible 988 responses. The average score of the panel on six intermediate tests was 87.3.

Results of the Total Experiment.--There were a total of 104 new frames added to the original program of 432 frames. This represented a 24.1 per cent increase of frames in the total program. The new frames added to the program required an additional 208 responses.

There was a total decrease in errors from Panel One to Panel Five of 5.20 per cent. The average score on the six intermediate tests increased 9.1 points from the beginning to the end of the experiment.

Conclusions

Upon the basis of the analysis of data in this experiment, the following conclusions were drawn.

1. The error rate of successive panels dropped from 8.03 to 2.83 per cent. This represents a total decrease of 5.20 per cent from the beginning to the end of the experiment. Therefore, it can be concluded that senior students, in high school, of below average ability can complete the adjusted programmed unit with an average error rate of less than 5 per cent.

2. Test score average for the panels rose from 78.2 to 87.3, an increase of 9.1 points from the beginning to the end of the experiment. Therefore, it can be concluded that students of below average ability can successfully perform on the retention

tests used to measure the result of the programmed unit on fundamentals of business mathematics.

Recommendations

As a result of observations made during the experiment, the following recommendations are made:

1. Due to the fact that there is a vast difference in the amount of drill necessary for various individuals to grasp ideas and concepts, it is recommended that some thought be given to a branching program for those individuals of lower ability. This would enable the fast learners to advance at a more rapid pace without being required to do unnecessary drill.

2. Since some errors were made by students in the final panel to work through the program, it is recommended that the program be adjusted further to decrease these errors.

3. That this adjusted program be tested with a larger body of students to determine where further adjustments may be necessary.

4. That business educators be made aware of the value of programmed business mathematics as a tool to help take care of individual differences in the business curriculum.

5. That, due to the shortage of programmed material in the field, business teachers should be encouraged to attend work shops in programming subject matter in an effort to discover those teachers with an ability to program instructional material.

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APPENDICES

Appendix A

**Student Comments on Fundamentals of
of Business Mathematics**

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel One)

FrameStudent's Comments

ADDITION

- 4 I missed because I didn't look at the problem carefully before beginning to add.
- 9 I missed because I didn't think about question or study it carefully.

SUBTRACTION

- 62 I just couldn't think what the word was.
- 63 I didn't understand question. It wasn't clear.
- 70 I don't understand angular addition. It isn't explained clearly enough.

MULTIPLICATION

- 87 The problem isn't written clearly enough to understand which answer is wanted.
- 90 The question was written so it could be understood which answer was wanted; the figure or what the figure was called.
- 92 The answer card tells what the partial product should be, but the question doesn't ask for what the incorrect partial product should be.
96. It didn't explain what audit was. But I knew anyway.
- 97 The question asked "Are dollar signs omitted?" It didn't ask if they were suppose to be or if they were omitted in this problem. It wasn't clearly understood.
- 100 There were only two sections to audit. But three answers were given.

's Comments on Fundamentals of Mathematics, (Continued)

MULTIPLICATION (Continued)

- 114 Didn't clearly understand question.
143 Typeographical error.
145 Doesn't give the answer for the total amount of pay.
151 Don't understand the shortcut to multiplying by 9.

DIVISION

- 182 I didn't give the answer to all three problems.
190 The question was not clear.
204 The question was not clear.

FRACTIONS

- 246 The question wasn't made clear.

DECIMALS

- 404 I didn't quite understand the question.
407 The question was not made clear enough.
409 It didn't say which side of the 1 to put the zeros.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel One)

FrameStudent's Comments

ADDITION

- 14 I don't understand the second part of the question. What does sum mean here?
- 21 It is hard to compare the figures with each other.
- 24 I did not understand what inclusive means here.
- 46 Was just a mistake in adding.

MULTIPLICATION

- 93 The word "audit" has not been used before. I am not sure what it means.
- 153 I thought it meant to give the other also.
- 162 The "B" part needs to be explained. It does not say that 1,341 should be multiplied by 2 then subtracted from 1,000 times itself.

DIVISION

- 185 I thought it meant to show the solution for both A and B.
- 198 The problem is worked out wrong, it should be 31.

FRACTIONS

- 249 I thought you wanted to know which fraction was divided into the most parts.
- 313 I did not know just what inverted means.

DECIMALS

- 349 I thought in part "B" where the line is over 1,000, it meant for me to supply the answer.
- 353 It should state that the answer be rounded.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel One)

<u>Frame</u>	<u>Student's Comments</u>
ADDITION	
16	If you round these amounts out, you get more than 30 dollars. In the preceding problems, I was taught to round off.
17	I don't understand the problem.
25	I didn't read the problem carefully enough.
47	My mistake was in my addition. I didn't understand when it said add in the reverse direction. It should have said use reverse addition.
MULTIPLICATION	
96A	Figures on invoice should be headed.
98	I didn't know what extension was.
99	Why doesn't this frame come before 98?
111	The wording of the problem was not clear enough. I believe that it should be omitted because you have the same question in Frame 115.
142	The problem didn't state to move the decimal over. I didn't know to do so until I got it wrong.
DECIMALS	
353	The problem didn't say to round off the answer.
359	The problem didn't state whether to round off or not to. In some problems the answer rounds off, in some they don't.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS**(Panel Two)****Frame****Student's Comments****ADDITION**

53

Carelessness.

57

Carelessness.**DECIMALS**

400

Could use more practice in estimating.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Two)

FrameStudent's Comments

ADDITION

- 54 Had to copy! Instead of doing it in my head.
- 55 Had to copy problem. Directions state not to.
- 56 Had to copy.
- 57 Had to copy problem. Does not explain horizontal addition.

SUBTRACTION

- 61 I thought that I was suppose to take first number on right and round off each and every number all the way to the left. Therefore, I had a large number of 6,000 instead of 5,000.
- 69A-1 I rounded off wrong in larger number. Now I understand.

MULTIPLICATION

- 98 Don't know what "extending" means.
- 107 At first did not understand what "itself" was; not very clear.
- 108 Rounded to closest 100. Did not know about "up" and "down."
- 154 Did not have short-cut for "14."

DIVISION

- 204 It is asking too many questions here. I can't keep it straight.
- 209 I get all mixed up when they say divisor and dividend.
- 215 Those crazy words again.

's Comments on Fundamentals of Mathematics, (Continued)

FRACTIONS

x305A Did not understand A and B parts.

322 Do not understand what I did wrong.

DECIMALS

329 Misunderstood. Took it for granted that decimal fractions had no denominator anyhow.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Two)

FrameStudent's Comments

ADDITION

- 2 Change additions to addition problems.
- 11 Make one sentence instead of two.
- 13 Second part not clear.
- 16 Express more clearly around the area of barely sufficient; it is questionable.
- 19 Explain better than more than one sentence, one that isn't so long because you lose your train of thought.
- x24A Change structure as (less than 499 to zero thousands.)
Make it clearer in some way that you want to round the two numbers separately in the second part. I thought it was meant together.
- 28 Rephrase. You lose your train of thought.
- 45 Confusing.
- 49 Confusing. You don't know exactly what the question wants.

SUBTRACTION

- 62 The last part of the question is not too clear.

MULTIPLICATION

- 115 Too confusing. You don't know what the question wants.
- 145 Haven't caught on to the annexing of zeros in 50 and 500 yet.
- 171 The problem isn't stressed clearly.

's Comments on Fundamentals of Mathematics, (Continued)

DIVISION

- 190 Confusing question.
- 204 Not very clear
- 216 Make the question clearer. (Does it want both or just one?)

FRACTIONS

- 341 Explain more fully. I don't understand exactly this process.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Three)

FrameStudent's Comments

ADDITION

24 See researcher's comment at bottom of page.

SUBTRACTION

x70A Carelessness.

MULTIPLICATION

116 Not thinking.

131 Didn't carry right.

x132A Carelessness.

157 Carelessness.

DIVISION

187 Divided by 3 instead of 2.

209 Forgot to strike out zeros.

227 I was trying to go too fast.

RESEARCHER'S COMMENT

24 Bev indicated orally that Frame 24 told about rounding to zero-thousand but that she was never required to do so. It was her opinion that the frame should not be cluttered with "stuff" as she put it, that was not needed. After checking the material, the writer found that she was somewhat justified in her criticism.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Three)

<u>Frame</u>	<u>Student's Comments</u>
48	This gets me confused.
55	This makes it hard to add the right numbers.
SUBTRACTION	
70	This is the same stuff that confused me before. (Instructor's note. It came hard to her in the addition section.)
MULTIPLICATION	
x111A	I forgot to round the bottom number.
x132A	Multiplied wrong at first.
146	I didn't know.
154	I divided wrong, but I know how to do it.
204	I had to guess but I got it right.
DIVISION	
208	I took off one zero too many.
211	I was just dividing straight out.
226	I was not looking at the cents.
FRACTIONS	
249	Is the answer right?
287	I just didn't divide right.
302	I was not thinking about $2/3$ being as much as a half.
319	I know how but I didn't do it the right way.

's Comments on Fundamentals of Mathematics, (Continued)

DECIMALS

- 339 I put too many zeros but I got it wrong anyway.
- 341 Went over too far.
- 369 I got mixed up.
- 416 Divided wrong.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Three)

FrameStudent's Comments

ADDITION

15 Carelessness.

x24A Should be clarified a little more.

29 Carelessness.

52 Carelessness.

SUBTRACTION

60 Needs clarifying.

x62A Clarify last part.

MULTIPLICATION

94 Carelessness.

x105B Carelessness.

117 Wasn't too clear.

146 Clarify.

154 Carelessness.

164 Carelessness.

166 Needs to be clarified.

171 Mistake on card. Sum should be 1,796,895, not 1,795,895.

's Comments on Fundamentals of Mathematics, (Continued)

DIVISION

196 Clarify.

226 Please check answer on card. I think it's \$400 off.

FRACTIONS

No comments.

DECIMALS

389 Please check answer.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Four)

<u>Frame</u>	<u>Student's Comments</u>
x14A	Here I made the mistake of not rounding off \$6,643.72 to \$7,000 and 843.79 to 1,000.
29	Careless.
x48A	The explanation on this problem was not clear. When I first read it I had to read it over 3 or 4 times before I understood it.
SUBTRACTION	
x70A	Carelessness.
MULTIPLICATION	
x96A	Carelessness.
x105B	Do not understand.
x111A	Not clear.
x129A	Do not understand.
DIVISION	
209	Rounded off wrong.
210	Rounded wrong.
FRACTIONS	
260	Carelessness.
279	Carelessness.
302	Carelessness.
DECIMALS	No comments.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Four)

FrameStudent's Comments

ADDITION

6 Carelessness on my part.

x24A Took a while to understand what you wanted

27 Carelessness.

SUBTRACTION No comments.

MULTIPLICATION

84 Not reading carefully. Thought it wanted the answer. Going too fast.

90 Thought it wanted the total again.

p92C Thought it wanted correct answer.

x92A I thought this problem was much clearer than other.

108 I have always rounded to nearest hundred. Wasn't used to what the word down meant there.

x111A I thought it meant weekly.

158 I was right the first time.

DIVISION

222 There was to large.

FRACTIONS

306 I thought you wanted the answer.

DECIMALS

369 Don't know how to do it without looking.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Four)

Student's Comments

This student could not be encouraged to give comments.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS**(Panel Five)**

<u>Frame</u>	<u>Student's Comments</u>
ADDITION	
f35C-3	Carelessness.
f39C-3	Didn't know it.
SUBTRACTION	
68	Didn't understand.
MULTIPLICATION	
	No comment.
f205C-1	Did not understand it.
FRACTIONS	
	No comments.
DECIMALS	
	No comments.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Five)

FrameStudent's Comments

ADDITION

7	Didn't understand that it was multiple choice.
13	Couldn't understand first part.
17	I thought it was "at least." It was close to it,
x29C	Didn't estimate the total sum.
x41A	Carelessness.
44	Don't understand it.

SUBTRACTION

64	Don't understand.
f64C-1	Didn't know.
76	Didn't know for sure if you were asking for smaller or larger.

MULTIPLICATION

x92A	Not typed out right. The six in the answer is in the wrong place.
103	Didn't understand.
108	Carelessness.
115	Don't understand this stuff.
116	Don't understand.
f146C	Carelessness.
f143C-2	Carelessness.

's Comments on Fundamentals of Mathematics, (Continued)

DIVISION

233 Didn't do it right

FRACTIONS

258 Carelessness.

264 My answer is almost the same thing.

f285C-2 Didn't understand.

DECIMALS

369 Couldn't work it.

STUDENT COMMENTS ON FUNDAMENTALS OF MATHEMATICS

(Panel Five)

<u>Frame</u>	<u>Student's Comments</u>
ADDITION	
f24C-1	Carelessness.
x27C	Carelessness.
31	This problem was rather hard to understand.
39	Didn't know was supposed to add problem.
42	Carelessness.
f45C-2	Didn't know.
56	Carelessness.
57	Carelessness.
SUBTRACTION	
72	Carelessness.
MULTIPLICATION	
c87A	Didn't understand.
p92C	Carelessness.
x95B	Carelessness.
100	Carelessness.
103	Didn't understand.
109	Carelessness.
x111A	Didn't round 42.
166	Didn't understand question at first.

's Comments on Fundamentals of Mathematics (Continued)

DIVISION

- 202 Carelessness.
212 Didn't understand.
223 Didn't understand.

FRACTIONS

- 257 Carelessness.
278 Carelessness.
311 Didn't realize it was division.

DECIMALS

- 345 Carelessness.
401 Didn't know how to work.

Appendix B

Personal Data of Student-Editors

Student _____ (Panel One) _____

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programmed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	D
English II	Sophomore	D
English III	Junior	D
General Mathematics	Freshman	D

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programmed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	106
		Non Language Factor	90
		TOTAL I. Q. SCORE	98
1959 Sophomore	STEP ¹	Mathematics	16-49
		Reading	43-63
	SCAT ²	Verbal	31-54
		Quantitative	36-63
1960 Junior	STEP	Mathematics	38-58
		Reading	58-74
	SCAT	Verbal	43-63
		Quantitative	30-56

¹Sequential Tests of Educational Progress

²School and College Ability Tests

PERSONAL DATA

Student _____ (Panel One)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programmed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	D
English II	Sophomore	D
English III	Junior	D
Algebra I	Freshman	E - B (Repeat)
Algebra II	Junior	E

Mental aptitude tests and scores in those areas related to Mathematics and those areas that may indicate an aptitude for programmed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	109
		Non-Language Factor	64
		TOTAL I. Q. SCORE	86
1959 Sophomore	STEP ¹	Mathematics	16-49
		Reading	54-71
	SCAT ²	Verbal	42-62
		Quantitative	22-24
1960 Junior	STEP	Mathematics	33-66
		Reading	49-67
	SCAT	Verbal	31-55
		Quantitative	20-41

¹Sequential Tests of Educational Progress

²School and College Ability Tests

PERSONAL DATA

Student _____ (Panel One)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programmed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	B
English II	Sophomore	C
English III	Junior	C
General Mathematics	Freshman	C-
Algebra I	Sophomore	D-

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programmed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	101
		Non Language Factor	96
		TOTAL I. Q. SCORE	99
1959 Sophomore	STEP ¹	Mathematics	9-39
		Reading	59-75
	SCAT ²	Verbal	42-62
		Quantitative	74-90
1960 Junior	STEP	Mathematics	6-33
		Reading	44-62
	SCAT	Verbal	25-50
		Quantitative	81-94

¹Sequential Tests of Educational Progress

²School and College Ability Tests

PERSONAL DATA

Student _____ (Panel Two)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	C
English II	Freshman	C-
English III	Junior	B-
Algebra I	Freshman	D+

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	93
		Non Language Factor	79
		TOTAL I. Q. SCORE	86
1959 Sophomore	STEP ¹	Mathematics	- - -
		Reading	71-84
	SCAT ²	Verbal	57-75
		Quantitative	25-49
1960 Junior	STEP	Mathematics	6-33
		Reading	71-92
	SCAT	Verbal	58-77
		Quantitative	17-37

¹ Sequential Tests of Educational Progress

² School and College Ability Tests

PERSONAL DATA

Student _____ (Panel Two)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programmed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	D
English II	Freshman	D
English III	Junior	E (Repeated C)
Algebra I	Freshman	E (Repeated D)

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programmed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	54
		Non Language Factor	79
		TOTAL I. Q. SCORE	62
1959 Sophomore	STEP ¹	Mathematics	33-56
		Reading	58-74
	SCAT ²	Verbal	31-55
		Quantitative	41-68
1960 Junior	STEP	Mathematics	58-81
		Reading	49-67
	SCAT	Verbal	58-77
		Quantitative	41-68

¹Sequential Tests of Educational Progress²School and College Ability Tests

PERSONAL DATA

Student (Panel Two)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programed learning,

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	C
English II	Freshman	C-
English III	Junior	B-
General Mathematics	Freshman	C
Algebra I	Sophomore	D-

Mental aptitude tests and scores in these areas relating to mathematics and those areas that may indicate an aptitude for programed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	102
		Non Language Factor	87
		TOTAL I. Q. SCORE	93
1959 Sophomore	STEP ¹	Mathematics	- - -
		Reading	43-63
	SCAT ²	Verbal	31-54
		Quantitative	32-58
1960 Junior	STEP	Mathematics	9-34
		Reading	54-71
	SCAT	Verbal	23-45
		Quantitative	30-56

¹Sequential Tests of Educational Progress²School and College Ability Tests

PERSONAL DATA

Student (Panel Three)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	D-
English II	Freshman	D
English III	Junior	D
Algebra I	Freshman	D-

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	95
		Non Language Factor	85
		TOTAL I. Q. SCORE	90
1959 Sophomore	STEP ¹	Mathematics	11-43
		Reading	29-46
	SCAT ²	Verbal	1-05
		Quantitative	22-44
1960 Junior	STEP	Mathematics	38-58
		Reading	28-44
	SCAT	Verbal	6-21
		Quantitative	10-27

¹ Sequential Tests of Educational Progress

² School and College Ability Tests

PERSONAL DATA

Student (Panel Three)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	E (Repeated D)
English II	Freshman	D
English III	Junior	D+
General Mathematics	Freshman	D

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	80
		Non Language Factor	78
		TOTAL I. Q. SCORE	79
1959 Sophomore	STEP ¹	Mathematics	9-39
		Reading	7-18
	SCAT ²	Verbal	10-21
		Quantitative	29-54
1960 Junior	STEP	Mathematics	4-29
		Reading	5-14
	SCAT	Verbal	3-10
		Quantitative	20-41

¹Sequential Tests of Educational Progress

²School and College Ability Tests

PERSONAL DATA

Student _____ (Panel Three)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	B
English II	Sophomore	C+
English III	Junior	A-
Algebra I	Freshman	A-
Plane Geometry	Sophomore	B

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	107
		Non Language Factor	91
		TOTAL I. Q. SCORE	99
1959 Sophomore	STEP ¹	Mathematics	55-76
		Reading	87-94
	SCAT ²	Verbal	84-93
		Quantitative	74-90
1960 Junior	STEP	Mathematics	68-89
		Reading	90-99
	SCAT	Verbal	90-97
		Quantitative	66-86

¹Sequential Tests of Educational Progress

²School and College Ability Tests

PERSONAL DATA

Student (Panel Four)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	B
English II	Freshman	C
English III	Junior	C-
Algebra I	Freshman	C
Algebra II	Junior	C
Plane Geometry	Sophomore	C

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	107
		Non Language Factor	88
		TOTAL I. Q. SCORE	100
1959 Sophomore	STEP ¹	Mathematics	49-72
		Reading	39-59
	SCAT ²	Verbal	31-54
		Quantitative	74-90
1960 Junior	STEP	Mathematics	38-58
		Reading	36-54
	SCAT	Verbal	25-50
		Quantitative	41-68

¹Sequential Tests of Educational Progress

²School and College Ability Tests

PERSONAL DATA

Student _____ (Panel Four) _____

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programmed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	D
English II	Sophomore	D
English III	Junior	D
General Mathematics	Freshman	C+

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programmed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	93
		Non-Language Factor	76
		TOTAL I. Q. SCORE	85
1959 Sophomore	STEP ¹	Mathematics	61-80
		Reading	67-81
	SCAT ²	Verbal	38-57
		Quantitative	32-58
1960 Junior	STEP	Mathematics	50-68
		Reading	74-94
	SCAT	Verbal	34-61
		Quantitative	41-68

¹Sequential Tests of Educational Progress

²School and College Ability Tests

PERSONAL DATA

Student _____ (Panel Four)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programmed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	C
English II	Sophomore	D
English III	Junior	C+
General Mathematics	Freshman	B+

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programmed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	91
		Non-Language Factor	82
		TOTAL I. Q. SCORE	87
1959 Sophomore	STEP ¹	Mathematics	24-61
		Reading	4-14
	SCAT ²	Verbal	13-31
		Quantitative	32-58
1960 JUNIOR	STEP	Mathematics	9-34
		Reading	8-20
	SCAT	Verbal	2-8
		Quantitative	30-56

¹Sequential Tests of Educational Progress

²School and College Ability Tests

PERSONAL DATA

Student _____ (Panel Five)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	B
English II	Freshman	C
English III	Junior	B
Algebra I	Freshman	B-

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	115
		Non Language Factor	82
		TOTAL I. Q. SCORE	99
1959 Sophomore	STEP ¹	Mathematics	49-72
		Reading	80-92
	SCAT ²	Verbal	87-94
		Quantitative	56-77
1960 Junior	STEP	Mathematics	62-86
		Reading	90-99
	SCAT	Verbal	88-96
		Quantitative	37-64

¹ Sequential Tests of Educational Progress

² School and College Ability Tests

PERSONAL DATA

Student _____ (Panel Five)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programmed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	C-
English II	Sophomore	D-
English III	Junior	D
General Mathematics	Freshman	D

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programmed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	91
		Non-Language Factor	97
		TOTAL I. Q. SCORE	94
1959 Sophomore	STEP ¹	Mathematics	-----
		Reading	29-46
	SCAT ²	Verbal	-----
		Quantitative	-----
1960 Junior	STEP	Mathematics	4-29
		Reading	16-31
	SCAT	Verbal	7-24
		Quantitative	54-75

¹Sequential Tests of Educational Progress

²School and College Ability Tests

PERSONAL DATA

Student (Panel Five)

Subjects and grades related to mathematics or those subjects that may indicate an aptitude for programmed learning.

SUBJECT	CLASS	TERMINAL GRADE
English I	Freshman	B
English II	Freshman	B
English III	Junior	C
Mathematics (Business)	Sophomore	D
Algebra I	Freshman	E
Geometry	Sophomore	E

Mental aptitude tests and scores in those areas relating to mathematics and those areas that may indicate an aptitude for programmed learning.

YEAR	TYPE OF TEST	AREA	SCORE OR RANGE
1958 Freshman	I. Q.	Language Factor	92
		Non Language Factor	34
		TOTAL I. Q. SCORE	88
1959 Sophomore	STEP ¹	Mathematics	12-44
		Reading	35-54
	SCAT ²	Verbal	46-67
		Quantitative	15-36
1960 Junior	STEP	Mathematics	04-29
		Reading	40-58
	SCAT	Verbal	43-63
		Quantitative	30-56

¹Sequential Tests of Educational Progress²School and College Ability Tests

Appendix C

**Intermediate Tests for Fundamentals
of Business Mathematics**

BUSINESS MATHEMATICS

Scoring Procedure: Each problem is counted as 4 points.

List two techniques to increase speed in addition:

1. _____
2. _____

List three techniques to increase accuracy in addition:

3. _____
4. _____
5. _____

By means of rounding, estimate which of the following problems are obviously added incorrectly. Put an X in the parentheses before each obviously incorrect problem.

$$\begin{array}{r}
 6. \\
 () \ \$19,463 \\
 \quad \underline{27,954}
 \end{array}$$

$$\begin{array}{r}
 7. \\
 \ \$38,421 \\
 \quad \underline{31,872}
 \end{array}$$

$$\begin{array}{r}
 8. \\
 \ \$91,428 \\
 \quad \underline{4,639}
 \end{array}$$

$$\begin{array}{r}
 9. \\
 () \ \$6.75 \\
 \quad 8.98 \\
 \quad 6.24 \\
 \quad 10.15 \\
 \quad 2.79 \\
 \quad \underline{34.91}
 \end{array}$$

$$\begin{array}{r}
 10. \\
 \ \$198.60 \\
 \quad 214.77 \\
 \quad 391.50 \\
 \quad 98.75 \\
 \quad 1.47 \\
 \quad \underline{805.09}
 \end{array}$$

$$\begin{array}{r}
 11. \\
 \ \$5,126.40 \\
 \quad 426.30 \\
 \quad 8,097.30 \\
 \quad 988.40 \\
 \quad \underline{9,143.42} \\
 \quad 21,781.82
 \end{array}$$

Go back over problems 6 - 11 which do not have an X indicating they are incorrect and verify the correctness of the remaining problems. If the problem is correct, place a ✓ in the parentheses. If an answer is incorrect, draw a line through it and write the correct answer below it, but do not place any mark in the parentheses.

12. Without recopying, find the total of this sales record:

Sales Record									
1.60	1.25	1.98	2.17	5.45	9.50	6.20	1.95	10.40	_____

A report for the Ace Grocery Store appears below, which has not been added. Examine the report and without recopying, find:

13. Estimated total for February 1 - 7 \$ _____
 14. Estimated total for February 8 - 14 \$ _____
 15. Estimated total for February 15 - 21 \$ _____
 16. Estimated total for February 22 - 28 \$ _____
 17. Estimated total sales of canned goods for February \$ _____
 18. Estimated total sales of meat for February \$ _____
 19. Estimated total sales of produce for February \$ _____
 20-24. Obtain the exact totals for each week and the total for February.

ACE GROCERY STORE
Sales for February

<u>February 1 - 7</u>	
Canned Goods	\$456.89
Meat	789.01
Produce	567.10
TOTAL FIRST WEEK:	_____ (20.)
<u>February 8 - 14</u>	
Canned Goods	548.13
Meat	729.70
Produce	722.17
TOTAL SECOND WEEK:	_____ (21.)
<u>February 15 - 21</u>	
Canned Goods	600.10
Meat	625.00
Produce	674.90
TOTAL THIRD WEEK:	_____ (22.)
<u>February 22 - 28</u>	
Canned Goods	680.25
Meat	950.50
Produce	731.25
TOTAL FOURTH WEEK:	_____ (23.)
TOTAL SALES FOR FEBRUARY:	_____ (24.)

25. Add by angular addition:

\$31,376
13,687
44,246
68,926
3,197
84,770
23,092
12,903

BUSINESS MATHEMATICS

Scoring Procedure: Each problem is counted as 5 points.

1. Describe two steps in verifying a subtraction problem:

(1) _____

(2) _____

2. How much is the larger number rounded, when the difference to a subtraction problem is estimated? _____

By means of rounding, estimate which of the following problems are obviously SUBTRACTED incorrectly. Place an X in the parentheses before each obviously incorrect problem.

3.
 \$9,826.75

$$\begin{array}{r} 847.15 \\ \underline{} \\ \$9,979.60 \end{array}$$

4.
 \$87,826.25

$$\begin{array}{r} 925.37 \\ \underline{} \\ \$86,900.88 \end{array}$$

5.
 \$526.47

$$\begin{array}{r} 19.58 \\ \underline{} \\ \$406.89 \end{array}$$

6.
 \$61,431.42

$$\begin{array}{r} 29,862.41 \\ \underline{} \\ \$31,529.01 \end{array}$$

7.
 \$125,650.75

$$\begin{array}{r} 1,967.43 \\ \underline{} \\ \$122,683.32 \end{array}$$

8.
 \$156.43

$$\begin{array}{r} 98.76 \\ \underline{} \\ \$ 57.67 \end{array}$$

Go back over problems 3 - 8 that do not have an X indicating they are incorrect and verify the correctness of these remaining answers. Place a ✓ in the parentheses of those that are correct. For any remaining incorrect problem, draw a line through the answer and write the correct answer below; do not place an X in the parentheses, however.

9. The Overstreet Corporation has deposits and withdrawals as follows:

January 1	Deposit	\$16,724.56
January 15	Withdrawal	8,500.00
February 6	Deposit	22,438.46

Estimate the bank balance: \$ _____

Below are a list of deposits and a list of withdrawals:

Deposits: \$21,425.40, \$9,986.50, \$1,046.40

Withdrawals: \$8,000, \$15,500, \$6,000

10. What are the estimated total deposits? _____
11. What are the estimated total withdrawals? _____
12. What is the estimated difference between the deposits and withdrawals? _____
- 13-14. What is the estimated total in hundred thousands for the problem below? \$ _____ (13.)

\$324,815
 651,892
 598,432
 215,785
 245,008
 749,581
 120,038
986,161

14. Now add the same problem by angular addition. (Show your work.)

15. Add the problem below without recopying:

$$\$145.62 + 69.80 + 62.75 = \$ \underline{\hspace{2cm}}$$

Without recopying, complete these subtractions:

16. $\$72.45 - 14.30 =$ Est. Answer \$ _____ Exact Answer \$ _____
17. $\$207.80 - 145.53 =$ Est. Answer \$ _____ Exact Answer \$ _____
18. $\$16,450 - 986.00 =$ Est. Answer \$ _____ Exact Answer \$ _____
19. $\$125,450 - 17,850 =$ Est. Answer \$ _____ Exact Answer \$ _____
20. $\$256,148.67 - 49,987.78 =$ Exact Answer \$ _____

BUSINESS MATHEMATICS

Scoring Procedure: Each problem is counted as 5 points.

Determine the lower and upper estimates to the following problem:

$$792 \times 47$$

1. The lower estimate is _____.
2. The upper estimate is _____.

Below are some extensions taken from an invoice of a wholesale grocery company. By estimation, determine which extension is obviously wrong.

	Quantity	Item	Unit Price	Total	Estimates	
					Lower	Upper
3.	71 cases	beans	2.64 each	\$198.00	_____	_____
4.	18 cases	peaches	8.40 each	151.20	_____	_____
5.	12 cases	tomatoes	2.88 each	134.56	_____	_____

The obviously incorrect extension is:

\$ _____

Extend this invoice and find the total:

Sold to:		January 9, 196-	
J. B. Jones			
107 Sky View Drive			
THE OFFICE SUPPLY CO.			
6.	130 Reams	Duplicating paper	1.21 each _____
7.	12 Quires	Four-Hole Stencils	2.14 each _____
8.	TOTAL		_____

9. Campbell pays \$67.50 a month in rent. He estimates that he will pay \$500 in rent during the next 12 months. Determine whether Campbell's estimate is (reasonable/unreasonable) _____.

Figure the following extensions taken from the invoice of a local hardware store. (Hint: Use short-cut methods for multiplying.)

	<u>QUANTITY</u>	<u>ITEM NO.</u>	<u>UNIT PRICE</u>	<u>TOTAL</u>
11.	250	A-01	\$ 77.00	_____
12.	17	#875	50.00	_____
13.	99	P-120	76.00	_____
14.	87	L-122	998.00	_____

15-16. Solve the problem below and verify your answer by reverse multiplication. Show all work.

$$74 \times 53$$

17. Show by division the verification of the following multiplication problem:

$$384 \times 26 = 9,984$$

18. (a) $398 \times 100 =$

(b) $1,000 \times 1,428 =$

19. (a) $419 \times 200 =$

(b) $672 \times 60 =$

20. (a) $235 \times 76 =$

BUSINESS MATHEMATICS

Scoring Procedures: Each problem is counted as 5 points.

1. How are the divisor and dividend rounded to perform an estimation to a division problem? _____

2. How do you verify the answer to a division problem? _____

By means of rounding estimate which of the problems below are obviously incorrect. Place an X in the parentheses before the problem if it is obviously incorrect.

3. $\frac{22}{98)2,156}$
 ()

4. $\frac{53}{115)60,095}$
 ()

5. $\frac{1,086}{81)87,966}$
 ()

6. $\frac{79}{478)37,762}$
 ()

7. $\frac{883}{1,359)112,797}$
 ()

8. $\frac{364}{1,182)4,302,484}$
 ()

Go back over problems 3 - 8 that do not have an X indicating they are incorrect and verify the correctness of these remaining answers. Place a \checkmark in the parentheses of those that are correct. For any remaining incorrect problem, draw a line through the answer and write the correct answer below; do not place any mark in the parentheses, however.

9-10. Show how the zeros are cancelled in these problems and determine the answers:

9. $400)800,000$

10. $18,000)36,000,000$

11. The Deep Creek Power Plant burns 5,000 tons of coal per day. The plant has 25,000 tons of coal on hand. How many days will the coal on hand last? _____ (No. of days)

12. Mr. Moore owns 1,332 acres of land. Each year he pays \$102 in taxes on this land. Estimate the amount of tax Mr. Moore pays per acre. \$ _____

Test 4, page 2

13. At Churchland High School \$1,933.50 was collected for accident insurance. Each student paid \$1.50 for the insurance. Exactly how many students bought insurance at Churchland High School?
 (No.)

14.-15. Find the estimated and the exact answer.

14. Estimated

)

15. Actual

 489)15,648

16. $67,000 \div 1,000 =$

17. $42,800 \div 10 =$

18. $8,600 \div 200 =$

19. $125,000 \div 25,000 =$

20. $6,000,000 \div 30 =$

BUSINESS MATHEMATICS

Scoring Procedure: Each problem is counted 6½ points.

Reduce to lowest terms

1. $\frac{25}{100} =$

2. $\frac{375}{1,000} =$

3. $\frac{16}{64} =$

4. When Beck retired from farming, he divided his land among his three sons. Bill received $19\frac{1}{3}$ acres, Jim received $14\frac{1}{2}$ acres, and John received $10\frac{7}{8}$ acres.

How many acres of land did Mr. Beck own before retiring? _____

5. If a grocery store had $19\frac{5}{8}$ pounds of cheese at the beginning of the day and $5\frac{11}{12}$ pounds at the end, how much was sold? _____

6. Multiply: (Show all work.)

$$\frac{27}{32} \times \frac{8}{9} =$$

7. Multiply: (Show all work.)

$$5\frac{5}{6} \times 4\frac{2}{7} =$$

8. $12 \frac{2}{3} \times 4 \frac{1}{4} = \underline{\hspace{2cm}}$

9. $\frac{1}{5} \times 50 \frac{1}{8} = \underline{\hspace{2cm}}$

10. $7 \frac{2}{3} \times 2 \frac{1}{2} = \underline{\hspace{2cm}}$

11. If Smith drove $140 \frac{1}{2}$ miles on $7 \frac{7}{8}$ gallons of gasoline, estimate the number of miles per gallon he obtained. $\underline{\hspace{2cm}}$

12. If Wilson wished to divide $12 \frac{1}{2}$ lbs. of tomatoes into 3 equal packages, how much would each package contain? $\underline{\hspace{2cm}}$

13. Add:

$$\begin{array}{r} 17 \frac{1}{2} \\ 15 \frac{3}{4} \\ 14 \frac{7}{8} \\ \hline \end{array}$$

14. Subtract:

$$\begin{array}{r} 104 \frac{1}{8} \\ 75 \frac{1}{4} \\ \hline \end{array}$$

15. Multiply:

$$7 \frac{1}{2} \times \frac{1}{5} =$$

Divide:

$$14 \frac{1}{2} \div 1 \frac{1}{2} =$$

BUSINESS MATHEMATICS

Scoring Procedure: Allow 5 points for each problem.

Change these fractions to decimals:

1. $\frac{27}{100} =$ 2. $\frac{41}{1,000} =$ 3. $\frac{3}{20} =$

Convert these fractions to 3-place decimals:

4. $\frac{1}{8} =$ 5. $\frac{1}{7} =$ 6. $\frac{5}{6} =$

Write the common fraction equivalent and reduce:

7. $.8 =$ 8. $.75 =$ 9. $.375 =$

10. On June 15 at the end of the day, a tally sheet of trucks hauling gravel, showed for truck A, 45.6 tons; B, 147.123; C, 98.45 tons; and D, 291 tons. The total is _____.
11. If Swanson bought a plot of land 50.02 ft. by 75.1 ft., what is the total sq. ft. of land he owns? _____
12. Subtract 16.5 from 75.14.
13. The cost of operating a papermaking machine for 17.4 hours was \$1,393.39. Find the cost per hour to the nearest cent.
\$ _____

If 47.55 yards of brand A-210 material are needed to drape windows, and the material is priced at \$6.35, answer the questions below:

14. What is the lower estimate? _____
15. What is the upper estimate? _____
16. What is the exact cost of material? _____

Test 6, page 2

17. Estimate the product of $7,596.825 \times .0078 =$ _____.
18. If the product of 1,589 and 642 = 1,020,138, then the product of 1.589 and 64.2 = _____.
19. If the answer to $986,004 \div 1,347 = 732$, then the answer to $986,004 \div 13.47 =$ _____.
20. Estimate the answer:

$$125.6 \overline{)54.636}$$

Appendix D

**The Original Programmed Unit in Fundamentals
of Business Mathematics, Part I**

BUILDING SPEED AND ACCURACY IN ADDITION

(a) Approximating Answers to Addition

(a)	(b)	1
17,368	23,140	
<u>18,229</u>	<u>41,600</u>	
63,597	64,740	

Which one of these is added incorrectly? _____

(a) _____

(a)	(b)	2
23,774	75,710	
41,659	<u>8,304</u>	
<u>716</u>	84,014	
94,049		

Which one of these additions is incorrect? _____

(a) _____

(a)	(b)	(c)	3
7,770	6,070	5,770	
1,700	2,700	5,700	
<u>4,070</u>	<u>4,770</u>	<u>4,070</u>	
13,540	13,540	13,540	

Which one of these sums is incorrect? _____

What should the answer be? _____

(c) 15,540

(a)	(b)	4
\$52,750	\$82,749	
<u>30,120</u>	<u>14,625</u>	
\$82,870	\$87,374	

Which addition is obviously incorrect? _____

To discover the error, was it necessary to add all the digits in the two numbers? (yes/no)

(b) no

Without adding, estimate the approximate sum of these numbers: 5
\$375.49, \$815.75, \$1,276.43. Estimated sum is: \$_____.

Any answer between 2300 and 2500 is acceptable

(b) Rounding Numbers to Test Accuracy

A business person or one good in arithmetic estimates the sum of \$375.40 + \$815.75 + \$1,276.43 by rounding them to 4 hundred + 8 hundred + 13 hundred, which totals approximately _____ hundred. 6

25

Numbers are usually rounded to the nearest 10's 100's, and 1000's depending on the purpose. If estimates are made mentally, rounding is kept very simple. Mentally \$5,143.85 and \$7,243.50 can be most easily rounded to (tens, hundreds, or thousands) _____. 7

Thousands

Executives often ask their accountants for an estimate to the nearest thousand dollars of the sales for the week. What estimate would the accountant give for these sales: \$17,043.75, \$3,105.80, \$10,256.40? Estimate _____ 8

\$30,000

Mentally \$17.45, \$32.49, and \$67.53 can be most easily rounded to (tens, hundreds, or thousands) _____. 9

Tens

When rounding numbers, care must be taken to round up or round down properly. In hundreds, \$536.42 would be rounded down to 5 hundred dollars, because \$536.42 is nearer \$500 than \$600. \$420.84 rounds down to _____. 10

\$400

In hundreds, \$487.42 is rounded up to \$500 for estimation procedures. \$187.79 is rounded up to _____ for estimation procedures. 11

\$200

350 is exactly half way between 3 and 4 hundred; and generally business people round it up to 4 hundred for estimation. 150 is rounded up to for estimation. 12

200

For estimation in addition, examine these numbers: \$375.49, \$815.75, and \$1,276.73. How many of the amounts are in hundreds of dollars? 13

Because _____ (No.) of three amounts are in hundreds, it is desirable to round them all to _____.

2 2 hundreds

For estimation in addition, \$6,643.72, \$1,104.15, and \$843.79 can be rounded to 7 thousand dollars, 1 thousand dollars, and 1 thousand dollars because _____ (No.) of the three amounts are _____.

The estimated sum is _____.

2 thousands 9 thousand

Estimate the sum to the nearest thousand dollars: \$8,746.53, \$5,074.21, \$11,921.43, and \$2,146.40. 15

\$28,000

Here is a list of amounts you expect to pay: \$7.93, \$1.95, \$6.15, \$10.89, and \$3.19. Which of the amounts of money below would be barely sufficient to make the payments? 16

- (a) four ten-dollar bills
- (b) six five-dollar bills
- (c) twenty one-dollar bills

(b)

Stone owns goods worth \$2,357.62, \$12,675.59, \$5,176.40, and \$1,920.50. Is he worth at least \$23,000? (yes/no) 17

No

Russell has \$9,175.50 in one bank, \$2,876.00 in a second, and \$5,016.75 in a third. Could he make an \$18,000 cash payment on a new business? (yes/no) 18

No

When a group of numbers is being rounded to hundreds, numbers less than 50 are rounded to 0 hundreds and numbers between 50 and 149 are rounded to 1 hundred. Of these numbers, 149, 100, and 47, the number _____ will be rounded to 0 hundreds. 19

47

Numbers between 50 and 149 are rounded to 1 hundred when rounding numbers to hundreds. Select the number from this list that would not be rounded to 1 hundred: 75, 130, 42, and 103. _____ 20

42

If \$205.00, \$462.00, and \$23.50 are each rounded to hundreds of dollars, \$_____ would be rounded to 0 hundred dollars. 21

\$23.50

When estimating the sum of \$205.00, \$462.40, and \$23.50, each number would be rounded to the nearest _____. The estimated sum is _____ 22

Hundred \$700

What is the estimated sum of \$267.35, \$410.50, \$835.62, \$10.75, \$506.70, and \$17.45? _____ 23

\$2,000

What is the estimated sum in hundreds?

30

\$673.47
 924.50
 99.40
 102.50
 17.42
624.60

Estimated sum _____

\$2,400

Shriver Road Construction Company has these debts. Estimate the total in thousands: 31

Building	\$125,000
Land	17,500
Machinery	1,200
Trucks	<u>8,900</u>

Estimated sum _____

\$153,000

Each of the six divisions of Western Empire Electric Company has stock-piles of coal. Estimate the total in thousands of tons. 32

# 1	10,600 tons	# 4	51,200 tons
# 2	19,800 tons	# 5	8,900 tons
# 3	31,200 tons	# 6	4,100 tons

Estimate _____

126,000

The process of checking the sum by adding a column of figures or a row of figures in reverse direction is known as verification by _____ 37
The addition of a column of figures from the top down and from the bottom up is also known as verification by _____.

Reverse addition reverse addition

(d) Using Combinations of 10 to Provide Speed

Do these addition problems:

38

$$\begin{array}{r} 5 \\ \underline{5} \end{array} \quad \begin{array}{r} 7 \\ \underline{3} \end{array} \quad \begin{array}{r} 8 \\ \underline{2} \end{array} \quad \begin{array}{r} 9 \\ \underline{1} \end{array} \quad \begin{array}{r} 4 \\ \underline{6} \end{array}$$

Because all the sums are _____, the pairs of numbers are examples of combinations of 10. Is $5 + 8$ a combination of 10? _____

$$\underline{10 \quad 10 \quad 10 \quad 10 \quad 10 \quad \text{No}}$$

Add by combinations of 10.

39

$$\begin{array}{r} 25 \\ \underline{55} \end{array} \quad \begin{array}{r} 27 \\ \underline{53} \end{array} \quad \begin{array}{r} 26 \\ \underline{54} \end{array} \quad \begin{array}{r} 51 \\ \underline{29} \end{array} \\ \begin{array}{r} 55 \\ \underline{55} \end{array} \quad \begin{array}{r} 87 \\ \underline{23} \end{array} \quad \begin{array}{r} 15 \\ \underline{95} \end{array} \quad \begin{array}{r} 78 \\ \underline{32} \end{array} \\ \begin{array}{r} 55 \\ \underline{95} \end{array} \quad \begin{array}{r} 78 \\ \underline{32} \end{array} \quad \begin{array}{r} 22 \\ \underline{88} \end{array} \quad \begin{array}{r} 47 \\ \underline{43} \end{array}$$

These problems are easy to add because there are many pairs of numbers that form _____.

$$\underline{340 \quad 300 \quad 300 \quad 280 \quad \text{Combinations of 10}}$$

Observing pairs of numbers that form 10's increases accuracy in adding. Add these by watching for combinations of 10.

40

$$\begin{array}{r} 85 \\ \underline{53} \end{array} \quad \begin{array}{r} 56 \\ \underline{85} \end{array} \quad \begin{array}{r} 57 \\ \underline{56} \end{array} \quad \begin{array}{r} 59 \\ \underline{58} \end{array} \quad \begin{array}{r} 83 \\ \underline{19} \end{array} \\ \begin{array}{r} 55 \\ \underline{55} \end{array} \quad \begin{array}{r} 54 \\ \underline{54} \end{array} \quad \begin{array}{r} 84 \\ \underline{33} \end{array} \quad \begin{array}{r} 81 \\ \underline{82} \end{array} \quad \begin{array}{r} 67 \\ \underline{41} \end{array}$$

$$\underline{290 \quad 270 \quad 230 \quad 280 \quad 210}$$

Add these rows of numbers by watching for combinations of 10. Verify by reverse addition.

41

$$\underline{\quad\quad} 7 + 3 + 6 + 5 + 4 + 1 + 8 + 7 + 2 + 3 = \underline{\quad\quad\quad}$$

$$\underline{\quad\quad} 6 + 5 + 5 + 9 + 6 + 1 + 4 + 7 + 6 + 3 = \underline{\quad\quad\quad}$$

$$\underline{\quad\quad} 9 + 9 + 1 + 8 + 2 + 6 + 5 + 4 + 5 + 9 = \underline{\quad\quad\quad}$$

46 46 52 52 58 58

(e) Angular Addition to Increase Accuracy

(A)		(B)			
98765	9	8	7	6	5
96865	9	6	8	6	5
76854	7	6	8	5	4
99876	9	9	8	7	6
86974	8	6	9	7	4
97774	9	7	7	7	4
67989	6	7	9	8	9
<u>87987</u>	<u>8</u>	<u>7</u>	<u>9</u>	<u>8</u>	<u>7</u>

Which seems more difficult, the addition of A or the addition of the separate problems in B? 42

Many people are more accurate on B than A, because B appears like five easy problems, whereas A seems like a difficult problem. Which one involves the "carrying" in addition? _____

A A

The process of adding a row of figures from left to right and from right to left to check the sum is known as verification by _____ 43

Reverse addition

98,765	In the addition problem to the left, the sum of the first	44
96,865	column is 44. The sum of the second column is _____.	
76,854		
99,876		
86,974		
97,774		
67,989		
<u>87,987</u>		
44		

54

- (a) Estimate the sum in ten thousands.
- (b) Obtain the exact answer by the angular method.

48

\$62,432.63
 24,371.97
 15,600.84
 59,124.85
 75,576.48
 64,383.66
 52,948.90
 79,002.01
18,239.71

\$450,000 \$451,681.05

When preparing a temporary financial report, an accountant frequently obtains sub-totals. Therefore the name applied to this process would be _____ addition.

49

Sub-total

(f) Accuracy By Means of Sub-total Addition

At the right is shown an example of sub-total addition. Note that 222 is the sum of 65, _____, and _____. 265 is the sum of 96, _____, and _____.

65	
73	
84	
	222
96	
78	
91	
	265
74	
68	
56	
87	
	285
	772

50

73 84 78 91

The type of addition shown at the right is frequently employed by office workers to obtain quarterly sub-totals. The yearly total is obtained by adding the quarterly s_____s.

65	
73	
84	
	222
96	
78	
91	
	265
74	
68	
56	
	198
87	
59	
72	
	218
	903

51

Sub-totals

Jan 75
 Feb 63
 Mar 84 _____ 1st Quarter
 Apr 72
 May 65
 Jun 72 _____ 2nd Quarter
 Jul 63
 Aug 74
 Sep 42 _____ 3rd Quarter
 Oct 56
 Nov 89
 Dec 71 _____ 4th Quarter
 _____ Year

Find the total of the twelve numbers at the left by sub-total addition. 52

Check each sub-total by reverse addition.

(Even though the problem is short, to obtain practice, use sub-total addition).

222 209 179 216 826

Jan 65
 Feb 28
 Mar 73 _____ 1st Quarter
 Apr 29
 May 68
 Jun 45 _____ 2nd Quarter
 Jul 47
 Aug 78
 Sep 53 _____ 3rd Quarter
 Oct 46
 Nov 59
 Dec 16 _____ 4th Quarter
 _____ Year

Find the total of the twelve numbers at the left by sub-total addition: Check the sub-totals by reverse addition. 53

166 142 178 121 607

(g) Horizontal Addition to Increase Speed

Amounts horizontally arranged require skill in horizontal addition. 54
 Without recopying add the ten amounts below:

.47	.23	.50	.65	.75	.96	.84	.23	.47	.86	<u>Total</u>
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	--------------

This method is known as _____ addition.

\$5.96 horizontal

The special skill in horizontal addition involves care in avoiding 55
 confusion in the units', tens', hundreds', and other places. Without
 recopying add these two numbers:

$$75,475 + 86,498 =$$

161,973

Without recopying add these two numbers horizontally. 56

$$\$147,650.90 + \$237,486.45 = \$ \underline{\hspace{2cm}}$$

Verify by adding again horizontally.

$$\text{TOTAL} = \$ \underline{\hspace{2cm}}$$

\$385,137.35 \$385,137.35

Horizontally add the row of amounts below. Avoid confusing the units', 57
 tens', and other positions.

1.95	2.95	6.95	4.50	3.95	7.95	6.95	1.50	1.95	3.95	4.75	6.50	<u>Total</u>
------	------	------	------	------	------	------	------	------	------	------	------	--------------

Verify by reverse addition. Total = _____.

The procedure for adding across a row of numbers is known as _____
 addition.

\$53.85 \$53.85 horizontal

When you arrive at this point, ask your teacher for directions for completing Panel 1. In the absence of specific directions, complete Panel 1 following the instructions on it carefully. The letter in the upper right of Panel 1 is _____.

58

A

BUILDING SPEED AND ACCURACY IN SUBTRACTION

(a) Approximating Answers in Subtraction

Suppose \$67.87 was given as the answer to the difference between \$97.43 and \$19.56. Without subtracting estimate whether this difference is correct: (correct/incorrect) _____ 59

Incorrect

A quick estimate of the difference between \$97.43 and \$19.56 is obtained by thinking \$100 minus \$20 is \$80. An exact answer, represented as \$67.87, must be (correct/incorrect). _____ 60

Incorrect

Burman had \$5,245.67 in his checking account. After writing a check for \$1,995.39, he had approximately: 61

- (a) \$2,000
- (b) \$3,000
- (c) \$4,000

(b)

Estimates of differences in subtraction are made by _____ numbers as you did in addition. Since only two numbers are involved, care must be taken in choosing how to round the _____ numbers. 62

Rounding two

(b) Rounding Numbers to Measure Accuracy in Subtraction

After rounding the smaller number, then round the larger to the same extent. If the smaller is in hundreds, then round the larger to hundreds. 63

<u>Problem</u>	<u>Rounded</u>	
1,427	?	In the problem to the left, the rounded larger number is _____.
<u>256</u>	<u>300</u>	

1400

When estimating the difference in a subtraction problem, first round the smaller number. 64

<u>1,427</u>	
<u>256</u>	In the problem to the left, the rounded smaller number is _____.

300

<u>Problem</u>	<u>Rounded</u>		
1,427	1,400		65
<u>256</u>	<u>300</u>		

For the subtraction problem above, the rounded answer is _____.

1,100

The reason for rounding the larger number to the same extent as the smaller is illustrated below: 66

<u>Problem</u>		<u>Rounded to the same extent</u>		<u>Not rounded to the same extent</u>
1,427	Each	<u>1,400</u>	Rounded to thousands	<u>1,000</u>
<u>256</u>	rounded to	<u>300</u>	Rounded to hundreds	<u>300</u>
1,171	hundreds	<u>1,100</u>		<u>700</u>

The closer estimate to the actual answer is the one (rounded to the same extent/not to the same extent.) _____.

Rounded to the same extent

When estimating answers in subtraction, first round the _____ 67
 and then round the _____ to the same extent.

Smaller number larger number

Complete the estimate of the difference Problem Rounded 68
 for the problem at the right. The number 3,264.45
 first rounded is the _____ . - 576.38 _____

Estimated
 answer

Smaller number 600 3,300 2,700

The larger number in a subtraction estimation should be rounded to the _____ 69
 _____ as the smaller.

Same extent

Add these problems by angular addition. Check each by reverse addition. 70

- | | |
|----------------|----------------|
| (A) | (B) |
| 427,436 | 424,815 |
| 709,425 | 860,109 |
| 918,920 | 573,386 |
| 611,593 | 215,895 |
| 813,739 | 236,046 |
| 745,206 | 729,374 |
| 586,140 | 207,438 |
| <u>527,083</u> | <u>965,161</u> |

5,339,542 4,212,224

The estimated difference between \$1,526.95 and \$685.43 is which of the answers below: 71

- (A) 800
- (B) 900
- (C) 1,000

A

What is the estimated sum: 175 feet, 560 feet, 204 feet, 168 feet, 240 feet? 72

1,400 feet

Which of the answers below represents the estimate of the difference of \$6,179.43 and \$268.40? 73

- (A) 5,000
- (B) 5,800
- (C) 5,900

C

Which of the answers below represents the estimate of the difference of these two amounts: \$6,010.50 and \$125.43? 74

- (A) 5,000
- (B) 5,900
- (C) 6,000

B

Barnes made two bank deposits of \$6,728.95 and \$4,376.40 and a withdrawal of \$2,980.50. In thousands of dollars, estimate the balance of his account. 75

\$8,000

(c) Verification of Subtraction

Complete this subtraction: 215 76

$$\begin{array}{r} 215 \\ -164 \\ \hline \end{array}$$

 Difference

If you add the difference and the smaller number you should obtain 215, which is the _____.

51 larger number

The first step in verification of subtraction is to add the difference and the _____ 77

Smaller number

The second step in verification of subtraction is to compare the result of adding the difference and the smaller number with the _____ 78

Larger number

An illustration of verification of subtraction is shown below: 79

<u>Problem</u>	<u>Thought process</u>		
4,291	4,291		
<u>1,118</u>	<u>1,118</u>	(1) Add	(2) Compare
3,073	3,073		
	4,291		

The thought process includes two steps: (1) Add _____; (2) Compare the result with the _____.

The difference and the smaller number larger number

Complete the verification of the difference to this subtraction problem. 80
Write the proof number below the dotted line to show the thought process.

$$\begin{array}{r}
 6,524 \\
 \underline{1,437} \\
 5,087 \\
 \dots\dots
 \end{array}$$

6,524

When you arrive at this point, ask your teacher for directions for 81
completing Panel 2. In the absence of specific directions, complete
Panel 2 following the instructions on it carefully. The letter in
the upper right of Panel 2 is _____.

B

BUILDING SPEED AND ACCURACY IN MULTIPLICATION

(a) Techniques of Multiplication

The terms frequently used to describe a multiplication problem are shown to the right.

Factor	6	82
Factor	<u>X 9</u>	
Product	54	

The numbers to be multiplied are called _____.
The answer is known as the _____.

factors product

In multiplication the numbers to be multiplied are commonly called _____ and the answer is called the _____.

factors product

Ten 89's is often written 10 X 89. Examine (a), (b), and (c) below: 84

(a)	(b)	(c)
Ten 47's	10 X 47	$\begin{array}{r} 47 \\ \times 10 \\ \hline \end{array}$

All yield (the same/different) answers. _____

the same

If ten 1's is 10 and ten 2's is 20, then ten 89's = _____ 85
and ten 137's = _____.

890 1,370

If ten 89's is 890, then 10 X 89 = _____.

890

10 X 89 = 890

87

In mathematics when 0 is placed to the right of a number, it is said to be annexed. This, one zero _____ to 42 results in 420.

annexed

100 X 73 = 7,300

88

73 is changed to 7,300 by _____ ing / two zeros to 73.

annexing

The rule for multiplying a whole number by ten is to annex _____ (No.) zero(s). A similar rule for multiplying by 100 is to annex _____ (No.) zero(s); for multiplying by 1,000 is to annex _____ (No.) zero(s). 89

one zero two zeros three zeros

A review of the special techniques of multiplication is illustrated below: 90

429	←	factor	
X 658	←	factor	
3432	←		<u>partial products</u>
2145	←		
2574	←		
<u>282,282</u>	←	product	

3,432, 2,145, and 2,574 are _____.

partial products

In the problem at the right, eight 429's results in the partial product _____. Which partial product contains an error? _____. The correct partial product is _____.

429
X 658
3432
2245
<u>2574</u>
<u>282,282</u>

91

3432 2,245 should be 2,145

Which partial product, if any, is in error? _____

92

$$\begin{array}{r} 6,742 \\ \times 78 \\ \hline 53936 \\ 45194 \\ \hline 505,876 \end{array}$$

Obtain the correct answer: _____

45,194 should be 47,194 525,876 is the correct answer.

Check each partial product below and correct errors, if any.

93

$$\begin{array}{r} \$947.62 \\ \times 827 \\ \hline 663334 \\ 189424 \\ 758196 \\ \hline 783,771.74 \end{array}$$

The audited answer is _____.

189,424 should be 189,524 758,196 should be 758,096 Answer: \$783,681.74

Audit these products:

94

$$\begin{array}{l} \$59.72 \times 28 = \$1,672.16 \\ \$77.85 \times 79 = \$6,152.15 \end{array}$$

\$6,152.15 should be \$6,150.15

Audit these products. Place a check mark beside each correct answer. Show corrected answers below for incorrect items:

95

- | | | |
|------------------|----------------------|---------------------|
| (a) 17 X 5 = 85 | (e) 125 X 15 = 1,125 | (i) 98 X 12 = 976 |
| (b) 92 X 7 = 644 | (f) 135 X 8 = 1,180 | (j) 59 X 19 = 1,121 |
| (c) 87 X 4 = 348 | (g) 250 X 13 = 3,250 | (k) 50 X 80 = 400 |
| (d) 75 X 4 = 300 | (h) 175 X 12 = 2,100 | (l) 33 X 55 = 1,815 |

Correct answers:

All are correct except (e) 1,875, (f) 1,080, (i) 1,176, and (k) 4,000.

(b) Auditing and Extending Invoices

Audit this invoice:

96

Sold to: San Pedro College
Los Angeles, California
September 6, 19__

BORDON FURNITURE COMPANY

6	Double pedestal desks	127.50	765.00
6	Swivel chairs	75.40	452.40
			<u>1,217.40</u>

No errors

Computation on an invoice.

97

4 Electric typewriters 395.00 _____

Put in the total for
the four typewriters

Are the \$ signs omitted? (yes/no) _____

1,580 yes

Computation on an invoice.

98

5 Adding machines 462.00 _____

Put in extension here

The product of 5 and \$462 is extended to the (right/ left). _____

2,310 right

The process of calculating the amount of each item on an invoice is known as extending the invoice. Underscore the extension below. 99

Invoice

25	boxes	Letterheads	5.10	127.50
----	-------	-------------	------	--------

127.50	

Extend this invoice. Find the total. 100

Sold to	<u>Oct. 24, 19--</u>
Belden Bros.	
Portland, Oregon	
ALLSTATE PRINTERS	
65 boxes	#10 envelopes 6.20
140 reams	Engraved letterheads 8.75

403.00	1,225	1,628	

Briefly glancing at this invoice computation, an accountant would be disturbed. 101

Computation on an invoice.

672	#4,238 beakers	1.50	100.80
-----	----------------	------	--------

What do you think is the reason? _____

Incorrect extension or incorrect decimal place.

687 X 426 = (Do not solve)

107

The factor 687, in the problem above, itself lies between _____ and _____. Likewise 426 lies between _____ and _____.

600 700 400 500

In the problem 687 X 426 = the factor 687 rounded down to the nearest hundred is _____ and the factor 426 rounded down to the nearest hundred is _____.

600 400

The lower estimate of the multiplication problem: 687 X 426 = is found by multiplying the factors rounded down to the nearest hundreds. The _____ estimate would be 600 X 400, which equals _____.

lower 240,000

The lowest estimate of the multiplication problem: 687 X 426 = is found by rounding both _____ down and multiplying. The lower estimate is _____.

factors 240,000

Obenchain has weekly take-home pay of \$93.78. What would be the lower estimate of his total take-home pay for 42 weeks? _____

\$3,600

In the multiplication problem 687 X 426 = the first factor 687 rounded up to the nearest hundred is _____ and the factor 426 rounded up to the nearest hundred is _____.

700 500

The upper estimate of the multiplication problem $687 \times 426 =$ 113
is found by multiplying the factors rounded up to the nearest hundreds.
The _____ estimate would be 700×500 , which equals _____.

upper 350,000

The upper estimate of the multiplication problem $687 \times 426 =$ is 114
found by rounding up the _____ and multiplying. The upper estimate
is _____.

factors 350,000

Obenchain's weekly take-home pay = \$93.78 115
Lower estimate for 42 weeks = $\$90 \times 40 = \$3,600$
Upper estimate for 42 weeks = = _____
In 42 weeks, Obenchain will earn between \$ _____ and \$ _____.

\$5,000 \$3,600 and \$5,000

What would be the range of earnings for Osgood during 33 weeks, 116
if his weekly take-home pay is \$86.49.
Between \$ _____ and \$ _____.

\$2,400 and \$3,600

When determining the lower estimate of the product of 572 and 117
78, round down 572 to 500 and round down 78 to 70. 572 is rounded
down to the nearest _____ and 78 is rounded down to the
nearest _____.

hundreds tens

If the factors in 572×78 are rounded down to 500 and 70, then 118
the lower estimate of the product is _____.

35,000

When deriving the upper estimate of the product of 572 and 78, round up 572 to 600 and round up 78 to 80. 572 is rounded up to the nearest _____, and 78 is rounded up to the nearest _____.

119

hundred ten

If the factors in 572 X 78 are rounded up to 600 X 80, then the upper estimate of the product is _____.

120

48,000

Examine the thought process for estimating the product of 572 X 78.

121

572	is between	500	and	600
<u>X 78</u>	is between	<u>70</u>	and	<u>80</u>
The product is between		35,000	and	48,000

The lower limit of the estimated product of 572 X 78 is _____, and the upper limit is _____.

35,000 48,000

Show the thought process for obtaining the lower estimate for this problem:

122

429	is between	_____	and	500
<u>X 59</u>	is between	_____	and	<u>60</u>
				30,000

The product is between _____ and 30,000.
The lower estimate is _____.

400 50 20,000 20,000

Show thought process for obtaining the upper estimate for the same problem.

123

429	is between	400	and	_____
<u>X 59</u>	is between	<u>50</u>	and	_____
		<u>20,000</u>		

The product is between 20,000 and _____.
The upper estimate is _____.

<u>500</u>	<u>60</u>	<u>30,000</u>	<u>30,000</u>
------------	-----------	---------------	---------------

Show the thought process for obtaining the lower estimate for this problem:

124

341	is between	_____	and	400
<u>X 29</u>	is between	_____	and	<u>30</u>

The product is between _____ and 12,000.
The lower estimate is _____.

<u>300</u>	<u>20</u>	<u>6,000</u>	<u>6,000</u>
------------	-----------	--------------	--------------

Show the thought process for obtaining the upper estimate for this problem:

125

341	is between	300	and	_____
<u>X 29</u>	is between	<u>20</u>	and	_____

The product is between 6,000 and _____.
The upper estimate is _____.

<u>400</u>	<u>30</u>	<u>12,000</u>	<u>12,000</u>
------------	-----------	---------------	---------------

Show the complete thought process for obtaining the upper and lower estimates for this problem:

126

341	is between	_____	and	_____
<u>X 29</u>	is between	_____	and	_____

The product is between _____ and _____.

<u>300</u>	<u>400</u>
<u>20</u>	<u>30</u>
<u>6,000</u>	<u>12,000</u>

Once you understand the procedure for obtaining the lower and upper estimates of the product of two factors, you should obtain them mentally. For the problem $341 \times 29 =$, the lower estimate is _____ and the upper estimate is _____.

6,000 12,000

For the problem $341 \times 29 =$, the lower estimate is _____ and the upper estimate is _____. The exact product is _____.

6,000 12,000 9,889

Computation on an invoice: 129

2,869 A-1516 tubes 4.86 15,943.34

By estimation, determine whether the extension is (reasonable/unreasonable).

unreasonable

For the problem $639 \times 85 =$, the lower estimate is _____ and the upper estimate is _____, the exact product is _____. Does the exact product fall between the estimates? (yes/no).

48,000 63,000 54,315 yes

For the problem $757 \times 849 =$, the lower estimate is _____ and the upper estimate is _____. The exact product is _____. Does the exact product fall between the estimates? (yes/no).

560,000 720,000 642,693 yes

For the problem $1,247 \times 87 =$, the lower estimate is _____, and the upper estimate is _____. The exact product is _____. (Hint: Round 1,247 to thousands and 87 to tens.)

80,000 180,000 108,489

	<u>Length</u>		<u>Width</u>	<u>Area (Sq. ft.)</u>	
(a)	97'	X	62'	6,014	
(b)	282'	X	146'	41,172	
(c)	329'	X	189'	82,181	
(d)	868'	X	719'	624,092	133

Which of these areas is obviously incorrect? _____

(c) should be 62,181

Estimating the product of two factors provides an indication of the size of the exact answer. This skill will be useful later in _____ the product of decimal factors. 134

estimating

(d) Multiplication Shortcuts

We will now examine a few shortcuts that are commonly used in arithmetic calculations. The first shortcuts are based on the multiplication of 10, 100, and 1,000. When multiplying by factors 10, 100, 1,000 and so on, you simply annex as many _____ as are in the factor. 135

zeros

Because 5 is $\frac{1}{2}$ of 10, a convenient way to multiply by 5 is to annex one zero to the other factor, and divide by 2. 136

Problem	Thought process
5 X 2685	$\begin{array}{r} 13,425 \\ 2) 26,850 \end{array}$

To multiply by 5, annex one _____ and divide by _____.

zero two

A shortcut to multiplying by 5 is to _____ one zero and divide by _____. 137

annex two

What is the shortcut to multiply by 5? 138

Annex one zero to the other factor and divide by 2.

Use the shortcut for multiplying by 5 for these problems: 139

- (a) 5 X 7,284
- (b) 5 X 6,292
- (c) 5 X 143,764

36,420 31,460 718,820

Using the shortcut for multiplying by 5, do these problems mentally: 140

- (a) $5 \times 9,768$
- (b) 5×379
- (c) $5 \times 143,767$

48,840 1,895 713,835

To save time, dollar signs (are/are not) included on invoice calculations. 141

are not

Computations on an invoice. 142

5	#2687 desks	239.40	_____
5	#4000-5 chairs	79.40	_____
5	A-CD cabinets	148.90	_____

Complete these extensions

1,197 397 744.50

The same shortcut can be used when multiplying by 50, 500, and so on. Because 50 is $\frac{1}{2}$ of one hundred, you annex 2 zeros and divide by 2; because 500 is $\frac{1}{2}$ of _____, you annex _____ zeros and divide by _____.

1000 3 2

Using the shortcuts for multiplying by 50 and 500, do these problems mentally: 144

- (a) $50 \times 8,397$
- (b) $500 \times 4,673$

419,850 2,336,500

<u>Hours</u>		<u>Rate of Pay</u>	<u>Gross Pay</u>	145
50	X	3.70	_____	
50	X	1.75	_____	
50	X	2.10	_____	
50	X	1.90	_____	

Compute pay

185.00 87.50 105.00 95.00

Because 25 is $\frac{1}{4}$ of 100, a shortcut to multiplying by 25 is to annex _____ zeros and divide by _____.

two four

Using the shortcut for multiplying by 25, do these problems: 147

- 25 X 363 =
- 25 X 369 =
- 25 X 370 =

9,200 9,225 9,250

Because 250 is $\frac{1}{4}$ of 1,000, you can use a shortcut to multiplying by 250 by annexing _____ (No.) zeros and dividing by _____.

250 X 763 =

three four 190,750

Cost Sheet for Printing Shop

149

<u>Hours</u>		<u>Cost of Machine Per Hour</u>	<u>Total</u>
25	X	\$ 62.	_____
25	X	87.	_____
250	X	39.	_____
250	X	134.50	_____

Complete calculations

1,550 2,175 9,750 33,625

What are the shortcut rules for multiplying by 25 and 250? _____ 150

For 25, annex 2 zeros and divide by 4; for 250, annex 3 zeros and divide by 4.

Calculating machines themselves employ shortcuts to multiplying by 9, 99, 999, and so on. The shortcut for multiplying by 9 is to subtract the other factor from 10 times itself. 151

Example: $9 \times 153 = 1530 - 153 =$ _____

1,377

The shortcut to multiplying by 9 is to _____ the other factor 152
from _____ (No.) times itself.

subtract 10

Using the shortcut for multiplying by 9, do these problems. The first 153
one is partially completed.

$9 \times 237 = 2370 - 237 =$ _____
 $9 \times 1,763 =$ _____ $-$ _____ $=$ _____
 $9 \times 9,876 =$ _____ $-$ _____ $=$ _____

2,133 15,867 88,884

Trucking Department Calculation

154

<u>Load</u> <u>(Tons)</u>		<u>Distance</u> <u>(Miles)</u>	<u>Ton-Miles</u>
9	X	187	_____
25	X	363	_____
14	X	250	_____
9	X	327	_____

(Use shortcut procedures)

1,683 21,575 3,500 7,443

The shortcut for multiplying by 99, similar to the one for 9, is to subtract the other factor from 100 times itself. 155

Example: $99 \times 873 = 87,300 - 873 = \underline{\hspace{2cm}}$

86,427

Describe an easy way to multiply by 99. _____ 156

Subtract the other factor from 100 times itself.

99 X 147 = _____
 99 X 625 = _____
 99 X 1,437 = _____

14,553 61,875 142,263

The shortcut for multiplying by 98 is similar to that for multiplying by 99: 158

Subtract twice the other factor from _____ (No.) times itself.

Example: $98 \times 324 = 32,400 - 648 = \underline{\hspace{2cm}}$

100 31,752

$$98 \times 124 = \underline{\hspace{2cm}}$$

$$98 \times 5,107 = \underline{\hspace{2cm}}$$

159

$$\underline{12,152} \qquad \underline{500,486}$$

Using the shortcuts, do these problems:

160

$$99 \times 836 = \underline{\hspace{2cm}}$$

$$98 \times 2,043 = \underline{\hspace{2cm}}$$

$$\underline{82,764} \qquad \underline{200,214}$$

The shortcut for multiplying of 999, similar to those for 9 and 99, is to subtract the other factor from (No.) times itself.

161

$$\underline{1,000}$$

Using the shortcut for multiplying by 999 and a similar one for 998, do these problems:

162

$$(a) 999 \times 1,245 = \underline{\hspace{2cm}}$$

$$(b) 998 \times 1,341 = \underline{\hspace{2cm}}$$

$$\underline{1,243,755} \qquad \underline{1,338,318}$$

Weight Memorandum Report

163

<u>Quantity</u>	<u>Item</u>	<u>Unit Weight</u>	<u>Total Weight</u>
98 crates	oranges	146 lbs.	<u> </u>
999 boxes	apples	68 lbs.	<u> </u>
25 bags	onions	54 lbs.	<u> </u>

$$\underline{14,308} \qquad \underline{67,932} \qquad \underline{1,350}$$

Invoice--Accounting Copy

164

<u>Quantity</u>	<u>Item</u>	<u>Price</u>	<u>Total</u>
426 ea	A-01	\$ 99	_____
178 ea	#675	250	_____
998 ea	PP-1	65	_____
25 ea	675A	87	_____

42,174 44,500 64,870 2,175

(e) Verification of Multiplication

We have previously studied estimation of the product in multiplication. 165
 As in addition and subtraction, another technique for increasing
 accuracy is that of _____.

Verification

As you will remember, many individual steps are involved in most 166
 multiplication problems. So, you will expect the verification procedure
 also to have _____ steps.

Many individual

By reversing the factors in the multiplication problem $78 \times 29 =$, you 167
 can verify the product.

(1)	(2)
Problem	Reversed factors
78	29
<u>X29</u>	<u>X78</u>

The factors in (1) are _____ in (2).

Reversed

Verification of the product of two factors can be made by _____ 168
 the factors and multiplying.

Reversing

Show the verification of the multiplication problem below by reversing 169
 the factors.

<u>Problem</u>	<u>Verification</u>
65	
<u>X 78</u>	
520	
455	
<u>5,070</u>	

$78 \times 65 = 5,070$

Show verification of this product:

170

Verification

$$\begin{array}{r}
 1,789 \\
 \times 187 \\
 \hline
 334,543
 \end{array}$$

$$\begin{array}{r}
 187 \\
 \times 1,789 \\
 \hline
 1683 \\
 1496 \\
 1309 \\
 187 \\
 \hline
 334,543
 \end{array}$$

Show the verification of 4,923 X 365 = 1,795,895. Show reverification if necessary. 171

Verification

Reverification

$$\begin{array}{r}
 4923 \\
 \times 365 \\
 \hline
 24615 \\
 29538 \\
 14769 \\
 \hline
 1,796,895
 \end{array}$$

$$\begin{array}{r}
 365 \\
 \times 4923 \\
 \hline
 1095 \\
 730 \\
 3285 \\
 1460 \\
 \hline
 1,796,895
 \end{array}$$

Miller and Boyd's Department Store owns these assets: building \$275,000, store equipment \$185,000, merchandise for sale \$95,000, and truck \$3,000. Estimate the total in thousands. 172

558 thousand

Ross opened a bank account with a deposit of \$717.42. After he had written two checks for \$98.50 and \$121.00, he calculated his bank balance and obtained \$397.92. Does it appear that his calculations are correct? (yes/no) 173

No (The estimated bank balance is about \$500.)

Sales Invoice

174

187 PKGS	A-1 BISCUITS	.15	28.05
243 CASES	TOASTIES	2.56	622.08
49 BOXES	EVAP MILK	8.60	421.40
87 CASES	CATSUP	7.60	666.20

Place check marks by correct extensions.
Correct errors in extensions which are wrong.

661.20

Estimate the difference in ten thousands:

175

\$786,543.57 - \$28,186.27

(Choose)

- (a) \$700,000
- (b) \$759,000
- (c) \$760,000

(c)

In the amounts below, a plus sign (+) indicated a deposit, and a minus sign (-) designates a withdrawal. Estimate in thousands the balance.

176

\$6,126.72 +
 1,929.40 +
 3,126.87 -

Estimated balance \$ _____

\$5,000

Machine Utilization Report

177

<u>Machine No.</u>	<u>Items Per Minute</u>		<u>Number of Minutes</u>	<u>Item-Minutes</u>
16	16	X	500	8,000
29	49	X	1,427	66,923
47	30	X	75	6,000
48	89	X	120	10,680

Verify extensions and correct errors.

Machine No. 29 should be 69,923

Another method of verification is to divide the product by one of the factors. The result should be the other factor. 178

Illustration

Verification

179 X 224 = 40,096

$$\begin{array}{r}
 224 \\
 179 \overline{)40096} \\
 \underline{358} \\
 429 \\
 \underline{358} \\
 716 \\
 \underline{716} \\
 0
 \end{array}$$

If 40,096 is divided by 224, the result should be (No.).

179

Show by division the verification of the following multiplication: 179
 1,789 X 657 = 1,175,373.

Verification

$$\begin{array}{r}
 657 \\
 1789 \overline{)1175373} \\
 \underline{10734} \\
 10197 \\
 \underline{8945} \\
 12523 \\
 \underline{12523} \\
 0
 \end{array}$$

Show by division the verification of the following multiplication:
 1,246 X 695 = 865,970

180

Verification

$$\begin{array}{r}
 \underline{695} \\
 1246 \overline{)865970} \\
 \underline{7476} \\
 11837 \\
 \underline{11214} \\
 \underline{6230} \\
 \underline{6230}
 \end{array}$$

When you arrive at this point, ask your teacher for directions for completing Panel 3. In the absence of specific directions, complete Panel 3 following the instructions on it carefully. The letter in the upper right of Panel 3 is _____.

181

C

BUILDING SPEED AND ACCURACY IN DIVISION

(a) Division Techniques

To find the number of 10's in 100, we think $10 \overline{)100}$. Examine (a), (b), and (c) below: 182

(a)	(b)	(c)
$10 \overline{)100}$	$100 \div 10 =$	$\frac{100}{10}$

The answer to each problem is _____.

10

Examine these problems: (a) $100 \div 10 = 10$ tens $\div 1$ ten = 10 183
 (b) $2,350 \div 50 = 235$ tens $\div 5$ tens = 47

The rule for dividing a number ending in zeros by a divisor ending in zero is to cross off the ending zero in each case and _____.

Divide

Examine these problems: 184
 (a) $2,300 \div 100 = 23$ hundreds $\div 1$ hundred = 23
 (b) $6,600 \div 300 = 66$ hundreds $\div 3$ hundreds = 22

The rule for dividing a number ending in two zeros by a divisor ending in two zeros is to cross off the _____ (No.) ending zeros in each case and _____.

Two divide

<u>Problem</u>	<u>Solution</u>
(a) $2,300 \overline{)460,000}$	$2,300 \overline{)460,000}$
(b) $3,400 \overline{)68,000}$	

Show solution

$\frac{20}{3400 \overline{)68,000}}$

Show how the zeros are removed in these problems and determine the answers: 186

(a) $200 \overline{)400,000}$

(b) $9,000 \overline{)18,000,000}$

(a) $\begin{array}{r} 2000 \\ 200 \overline{)400,000} \end{array}$ (b) $\begin{array}{r} 2,000 \\ 9,000 \overline{)18,000,000} \end{array}$

$42,000 \div 200 = 187$

$158,000 \div 2,000 =$

$6,118,000 \div 300 =$

$\begin{array}{r} 210 \\ 79 \overline{)20,370} \end{array}$

Assume that you were to determine the number of lengths of rope 81 feet long on a reel of 2,592 feet of rope. An extremely elementary way to determine the number would be to cut lengths of 81 feet from the reel and to _____ them. 188

Count

Cutting and counting to determine the number of 81 foot lengths in 2,592 feet of rope is a simple explanation for division. Division is actually measuring the number of 81 foot pieces there are in 2,592 feet. Finding how many 14 pound bags can be gotten from 692 pounds of apples is actually a _____ process. 189

Measuring or division

When you determine the number of equal parts that can be obtained from something, you are using the _____ idea of division. 190

Measuring

How many 81's are there in:

191

- (a) 81 _____
- (b) 162 _____
- (c) 243 _____

- (d) 810 _____
- (e) 1,620 _____
- (f) 2,430 _____

1 2 3 10 20 30

Assuming that you don't want to go to the trouble of cutting and counting 81-foot lengths, you can say:

192

- (a) one 81-foot piece is 81 feet
- (b) two 81-foot pieces is 162 feet
- (c) three 81-foot pieces is 243 feet
- (d) thirty 81-foot pieces is 2430 feet

The combination of (b) and (d) equals 2,592 feet, so we can say there are _____ pieces of 81-foot rope in 2,592 feet.

32

Using the measuring idea of division, determine the number of 63's in 1,323.

193

- One 63 is 63.
- Two 63's is 126.
- Twenty 63's in 1,260.

Are there at least twenty 63's in 1,323? (yes/no) _____, twenty-one 63's? (yes/no) _____, twenty-two 63's? (yes/no) _____.

Yes _____ yes _____ no _____

A rough measure of the number of 81's in 2,592 is to say how many 80's there are in 2,600. There are roughly (3/30/300). _____

194

30

Now that we recognize division as a measuring device, let's examine the process of division of whole numbers. In the problem $2,592 \div 81$, the divisor is _____ and the dividend is _____.

81 2,592

In the problem $2,592 \div 81$, the divisor is _____. Because numbers composed of two or more digits are difficult to use as a divisor, it is customary to use a trial divisor of one digit. When a divisor is composed of two or more digits, a/an _____ is used to obtain the answer.

81 trial divisor

A few steps in the problem at the right are completed. The 8 in 81 is used as a trial divisor. There are 3 eights in 25, so 3 is the first digit in the _____.

3
81)2,592
 243

197

Answer

The division problem at the right is completed. The 2 in the quotient is obtained by dividing 16 by _____. The 8 is known as the _____.

32
81)2592
 243
 162
 162

198

8 trial divisor

51)3825 =

199

75

\$5,000 ÷ 25 = ? (Don't divide!)

200

Would you estimate the answer as:

- (a) \$20
- (b) \$200
- (c) \$2,000

(b) \$200

\$68,000 ÷ 200 = ? (Don't divide!)

201

Is the answer:

- (a) \$34
- (b) \$340
- (c) \$3,400

(b) \$340

\$750,000 ÷ 5,000 = ? (Don't divide!)

202

Estimate the answer: _____

\$150

(a)

(b)

203

3)63

31)651

Which problem above can be done easily in the mind? _____

(a)

(b)

71)22,081

70)21,000

Which problem above can be done most quickly? _____

(a)

(b)

(b) Rounding Numbers to Measure Accuracy in Division

Which problem can be done most easily in the head? 204

- (a) two digits followed by 0's + 1 digit followed by 0's
- (b) 3 or more digits followed by zeros + 2 digits followed by 0's
- (c) 1 digit with 0's + 2 digits with 0's

(a) _____

Examine the problem and its rounded form below: 205

<u>Problem</u>	<u>Rounded</u>
$78 \overline{)4936}$	$80 \overline{)4900}$

The divisor 78 has been rounded to _____, which is one digit followed by a zero; and the dividend 4,936 has been rounded to _____, which is two digits followed by zeros.

$80 \overline{)4,900}$

$80 \overline{)4,900}$ is the rounded form of $78 \overline{)4,936}$. 206

$80 \overline{)4,900}$ can be thought of as $80 \overline{)4900}$, which is roughly 61. Thus, we can say an estimate of the quotient of $78 \overline{)4,926}$ is _____.

61

<u>Problem</u>	<u>Rounded for Estimation</u>
$37 \overline{)17,438}$	$40 \overline{)17,000}$

What is the estimated answer? _____

400 or 425

Problem

Rounded for Estimation

208

137)437,678 _____

100)440,000

Estimate the answer! _____

4,400

The steps for estimating answers in division are:

209

- (1) Round divisor to one digit followed by 0's
- (2) Round dividend to two digits (or less) followed by 0's
- (3) Obtain answer mentally

Problem

Rounded Form

321)64,256 _____

) _____

Estimate answer! _____

300)64,000 Answer = 2100 or 2133

Estimate the answer:

210

795)315,789 _____

) _____

Rounded form

800)320,000 Answer = 400

Estimate the answers to these problems:

211

- (a) 643,752 ÷ 431 =
- (b) 1,179 ÷ 38 =

1,600 300

1,674 ÷ 71 = 23 and remainder 41.

212

What is the estimated answer? _____)

24
70)1,700

Cost Department

213

<u>Pieces Work Completed</u>		<u>Number of Employees</u>	<u>Average No. of Pieces</u>
4,756	÷	41	116
13,309	÷	59	221
7,917	÷	87	121

Which average is obviously incorrect? _____

121 should be 90

Personnel Department

214

<u>Departmental Absences</u>		<u>No. of Employees</u>	<u>Average</u>
A 1,512		378	4
B 4,505		905	6
C 273		39	7

Which average is most likely to be incorrect according to estimation? _____

B

The verification of the answer 23 R 41 for $71 \overline{)1,674}$ is shown below: 215

(a)
$$\begin{array}{r} 71 \text{ divisor} \\ 23 \text{ answer} \\ \hline 213 \\ 142 \\ \hline 1,633 \end{array}$$

(b)
$$\begin{array}{r} 1,633 \\ 41 \text{ remainder} \\ \hline 1,674 \end{array}$$

The steps of verification are (a) multiply the divisor by the _____;
 (b) add the _____ to the product of the divisor and the answer.
 The sum should equal the _____.

Answer remainder dividend

Find the answer and remainder:

Estimate answer:

216

$$71 \overline{)1,674}$$

$$\overline{\hspace{2cm}}$$

23 41 remainder 24

Study the attempt to complete $74\overline{)2,176}$ below.

221

(a) $74\overline{)2176}$ ³

(b) $74\overline{)2176}$ ²⁹
148
696
666
30 remainder

Because 222 is too large in (a), we must abandon Step (a) and begin Step (b) for which we obtained 29 as an answer and 30 as a _____.

Remainder

$74\overline{)2176}$ ²⁹ and R 30 is in (reasonable agreement/complete disagreement) 222
with the estimated quotient of 31.

Reasonable agreement

The rounding of the divisor is helpful in providing us with a 223
_____.

Trial divisor

The rounding of both the divisor and dividend helps us to estimate 224
the _____.

Answer

\$1.95	\$.95	\$10.25	225
3.95	7.09	.40	
1.10	1.45	6.20	

Determine the estimated sum in dollars. \$ _____

\$32

\$125.42 +	\$300.00 +	226
79.60 +	50.00 +	
100.00 -	100.00 -	

Determine the estimated balance of this account. \$ _____
 Calculate the exact balance. \$ _____

\$355 \$355.02

The estimated answer for 63)3717 is _____ and the actual answer is _____ 227

61 59

The estimated answer for 91)2370 is _____ and the actual answer is _____ 228

27 26 and 4 remainder

The estimated answer for 421)87672 is _____ and the actual answer is _____ 229

220 208 remainder

Cost Accounting Computation 230

\$28,435.69 X 379 = 10,777,126.51

Determine whether the product given lies between the upper and lower estimates.

Lower estimate _____ Upper estimate _____

\$6,000,000 \$12,000,000

The estimated answer for $48 \overline{)1655}$ is _____.

231

$$\begin{array}{r}
 34 \\
 48 \overline{)1655} \\
 \underline{144} \\
 215 \\
 \underline{192} \\
 23 \text{ remainder}
 \end{array}$$

This problem involves (whole numbers only/decimals and whole numbers).

34 whole numbers only

In the problem $48 \overline{)1655}$, we use 5 as the trial divisor because 48 rounds to 50. Divisors of 45, 46, 47, and 49 all would be rounded to _____, which results in _____ being the _____ divisor for all of them.

50 5 trial

The estimated answer for $37 \overline{)1542}$ is _____. The trial divisor for $37 \overline{)1542}$ is _____.

233

37 4

Study the attempt to complete $37 \overline{)1542}$.

234

$$\begin{array}{r}
 3 \\
 37 \overline{)1542} \\
 \underline{111} \\
 43
 \end{array}$$

The 3 in the answer is too small because 43 is larger than _____.

37

Study the attempt to complete $\overline{37)1542}$ below:

235

$$(a) \begin{array}{r} \underline{3} \\ 37)1542 \\ \underline{111} \\ 43 \end{array}$$

$$(b) \begin{array}{r} 4 \\ \underline{4} \\ 37)1542 \\ \underline{148} \end{array}$$

Because the answer was too small in (a), we must abandon Step (a) and begin Step (b).

Complete Step (b). Answer _____ Remainder _____

$$\underline{41 \qquad 25 \text{ remainder}}$$

$$\begin{array}{r} 41 \text{ actual answer} \\ 37)1542 \\ \underline{148} \\ 62 \\ \underline{37} \\ 25 \text{ remainder} \end{array}$$

$$\begin{array}{r} 37 \text{ estimated answer} \\ 37)1542 \end{array} \qquad 236$$

Does the actual answer appear to be reasonable? (yes/no) _____

Yes

The estimated answer to $\overline{77)3716}$ is _____. The actual answer is _____ 237

$$\underline{46 \qquad 48 \ 20 \text{ remainder}}$$

The estimated answer to $\overline{663)31161}$ is _____. The actual answer is _____ 238

$$\underline{44 \qquad 47}$$

During 25 working days, 56 employees produced 1,728,400 lbs. of corrugated paper. 239

(a) Determine the number of man-days. (Hint: multiply the no. of days by the no. of employees.) _____

(b) Determine the number of lbs. per man-day. _____

1400 1,234 800 remainder

When you arrive at this point, ask your teacher for directions for completing Panel 4. In the absence of specific directions, follow the instructions on it carefully. The letter in the upper right of Panel 4 is _____.

D

BUILDING SPEED AND ACCURACY WITH COMMON FRACTIONS

(a) Meaning and Terminology of Fractions

Examine these statements:

241

(a) Two out of three voted "yes."

(b) $\frac{2}{3}$'s voted "yes."

(c) Of the 60 present, 40 voted "yes."

(d) Of the 60 present, 10 voted "yes."

Statement _____ does not fit the pattern of the other three.

(d) _____

Many ideas in business are converted to fractions, such as $\frac{2}{3}$'s for 2 out of 3. The idea of 14 out of 16 could be converted to the fraction

242

_____.

$\frac{7}{8}$ ths _____

$\frac{7}{8}$ _____ numerator
_____ denominator

243

Of 8 parts, how many parts are represented by the numerator? _____

7

$\frac{2}{3}$ _____ 244

Does the numerator designate how many parts in all? (yes/no) _____
Which represents the number of parts taken from the whole?
(numerator/denominator) _____

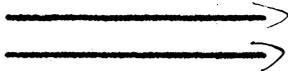
No numerator

Fraction

Name of Part

245

$\frac{9}{10}$



Numerator

denominator

Which of these parts of a dollar is the smallest: \$ $\frac{1}{8}$ $\frac{1}{3}$ $\frac{1}{5}$? 251

\$ $\frac{1}{8}$

The more pieces a dollar is divided into the (smaller/larger) the value of the piece. 252

Smaller

$\frac{1}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$

253

The denominator in each case is _____. The size of the denominator in $\frac{1}{8}$ is (the same as/different from) that in $\frac{3}{8}$. _____

8 the same as

$\frac{1}{8} + \frac{5}{8} + \frac{7}{8} + \frac{3}{8} + \frac{3}{8} =$ _____. Their sum is obtained by placing the sum of the numerators over 8, which would be $\frac{19}{8}$. Because $\frac{19}{8}$ represents 2 wholes and 3 eights remaining, we write $2\frac{3}{8}$ for the sum. If we had gotten $\frac{21}{8}$, we would have changed it to _____.

$2\frac{3}{8}$ $2\frac{5}{8}$

$\frac{19}{8}$ is called an improper fraction because the numerator is larger than the denominator, $\frac{25}{4}$ would be known as a/an _____ fraction. 255

Improper

If $\frac{19}{8}$ is called an improper fraction because the numerator is larger than the _____, then $\frac{3}{8}$ could be called a _____ fraction because the denominator is larger than the _____.

Denominator proper numerator

$\frac{19}{20}$ is called a/an _____ fraction because the numerator is _____ than the _____.

Proper smaller denominator

Because in $\frac{3}{4}$ the denominator is larger than the _____, it is called a _____ fraction. Because in $\frac{175}{9}$ the numerator is _____ than the denominator, it is known as a/an _____ fraction.

Numerator proper larger improper

(b) Addition and Subtraction of Fractions

Because $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$, then we must consider all eights to be 259

(the same/different) size. _____

Because $\frac{1}{3} + \frac{1}{5} + \frac{1}{3}$ does not equal a certain number of thirds or fifths,

then it must be that thirds and fifths are of (the same/different) size(s).

Same different

Because the _____ of fractions indicate the size of the parts to be 260
added or subtracted, it is necessary that the fractions have the same

_____.

Denominator denominator

$\frac{1}{3}$ and $\frac{1}{5}$ can be added or subtracted only if they are converted to like 261

fractions, which means fractions with the same (denominators/numerators).

_____. Fractions with the same denominators are known as
_____ fractions.

Denominators like

Because $\frac{2}{35}$ and $\frac{17}{35}$ have the _____ denominators, they are called 262

_____ fractions.

Same like

Because $\frac{4}{15}$ and $\frac{7}{15}$ have the _____ denominators, they are 263

known as _____ fractions.

Likewise, because $\frac{5}{16}$ and $\frac{3}{8}$ have different denominators, they are

designated as _____ fractions.

Same like unlike

Because $\frac{17}{15}$ and $\frac{7}{10}$ have _____ denominators, they are called _____ fractions. 264

Different unlike

The fractions $\frac{1}{3}$ and $\frac{1}{5}$ are converted to like fractions by thinking of _____ 265
a number into which both 3 and 5 will divide evenly. What is the smallest number into which 3 and 5 will divide evenly? _____

15

What is the smallest number into which these will divide evenly? 266
(a) 3, 4, 6, 12 _____
(b) 2, 4, 8, 2 _____

(a) 12 (b) 8

What is the smallest number into which these will divide evenly? 267
(a) 2, 3, 4, 8 _____
(b) 9, 3, 15, 5 _____

(a) 24 (b) 45

When adding or subtracting the fractions $\frac{1}{3}$ and $\frac{1}{5}$, 15 is chosen 268
as the least common denominator because it is the smallest number into which both 3 and 5 will divide.

For the fractions $\frac{1}{3}$ and $\frac{1}{5}$, 15 is the _____
denominator.

Least common

The least common denominator for adding the fractions $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{1}{3}$ is 12. The least common denominator for the fractions $\frac{7}{8}$, $\frac{3}{4}$, and $\frac{1}{3}$ is _____.

12

$\frac{1}{3} = \frac{?}{15}$ The missing numerator is determined by thinking of what you can multiply both the numerator and denominator by to obtain 15ths. It is _____ because $15 \div 3 =$ _____.

5 5

$\frac{1}{3} = \frac{5}{15}$ because $\frac{1 \times 5}{3 \times 5} =$ _____ 271

$\frac{5}{15}$

$\frac{1}{5} = \frac{\quad}{15}$ Fill in numerator 272

3

The procedure for adding $\frac{1}{3}$ and $\frac{1}{5}$ is shown below:

273

$$\begin{aligned} \frac{1}{3} &= \frac{5}{15} \\ \frac{1}{5} &= \frac{3}{15} \\ &= \frac{8}{15} \end{aligned}$$

Show how to add $\frac{1}{4}$ and $\frac{1}{3}$.

$$\begin{aligned} \frac{1}{4} &= \frac{3}{12} \\ \frac{1}{3} &= \frac{4}{12} \\ &= \frac{7}{12} \end{aligned}$$

Because $\frac{1}{2}$ and $\frac{4}{8}$ of a dollar represent the same amount of money, they are called equivalent fractions. Because $\frac{1}{3}$ and $\frac{5}{15}$ represent the same part of a whole, they are called _____ fractions.

274

Equivalent

$\frac{3}{7}$ and $\frac{6}{14}$ are _____ fractions.

275

Equivalent

If the answer to a problem involving fractions is $\frac{29}{58}$, we always find an equivalent fraction with the smallest denominator possible. For example:

276

$$\frac{\frac{29}{58}}{2} = \frac{1}{2}$$

$\frac{29}{58}$ is equivalent to/different from $\frac{1}{2}$. _____

Equivalent to

When an answer such as $\frac{29}{58}$ is changed to $\frac{1}{2}$, we say that we are 277
reducing $\frac{29}{58}$ to its lowest terms. $\frac{13}{39} = \underline{\hspace{2cm}}$. When we write
 the answer, we have ed to its lowest terms.

$\frac{1}{3}$ reduced

Observe one procedure for reducing $\frac{125}{1000}$ to its lowest terms: 278

$$\begin{array}{r} 1 \\ 5 \\ 25 \\ \hline 125 \\ 1000 \\ 200 \\ 40 \\ 8 \end{array} = \frac{1}{8}$$

In each step both the numerator and denominator were both divided by ____.

5

Reduce these fractions to their lowest terms. 279

$\frac{375}{1000}$ _____ $\frac{5625}{10000}$ _____

$\frac{3}{8}$ $\frac{9}{16}$

A mixed number is the combination of a whole number and a fraction. 280
 Which one of these is a mixed number?

- (a) $\frac{7}{8}$ (b) $\frac{17}{2}$ (c) $5\frac{2}{3}$ _____

Which is an improper fraction? _____

$5\frac{2}{3}$ $\frac{17}{2}$

Because $6\frac{7}{8}$ is composed of a whole number and a fraction, it is known as a/an _____.

Mixed number

The least common denominator for the fractions at the right is $7\frac{1}{2}$ 282

The sum of the three mixed numbers is _____.

$$\begin{array}{r} 4\frac{2}{3} \\ 10\frac{5}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 24 \\ 22\frac{19}{24} \\ \hline \end{array}$$

$$\begin{array}{r} 86\frac{7}{8} \\ - 25\frac{3}{4} \\ \hline \end{array}$$

What is the common denominator? _____

What is the difference? _____

283

$$\begin{array}{r} 8 \\ 61\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 31\frac{3}{8} = 31\frac{3}{8} \\ - 14\frac{3}{4} = 14\frac{6}{8} \\ \hline \end{array}$$

The subtraction problem is partially completed. Before the problem can be completed 1 must be borrowed from 31. There are _____ (No.) eights in 1.

284

$$\begin{array}{r} 8 \\ \hline \end{array}$$

The subtraction problem at the right is partially complete. $\frac{3}{8}$ was changed to

$\frac{11}{8}$ because the borrowed 1 and $\frac{3}{8}$ together make $\frac{8}{8} + \frac{3}{8} =$ _____.

The answer is _____.

$$\begin{array}{r} 31\frac{3}{8} = 31\frac{3}{8} = 30\frac{11}{8} \\ - 14\frac{3}{4} = 14\frac{6}{8} = 14\frac{6}{8} \\ \hline \end{array}$$

285

$$\begin{array}{r} \frac{11}{8} \\ 16\frac{5}{8} \\ \hline \end{array}$$

The common denominator for the subtraction problem at the right is _____.

$$42 \frac{5}{8}$$

286

The difference between the two _____ numbers is _____.

$$\begin{array}{r} 42 \frac{5}{8} \\ - 17 \frac{11}{12} \\ \hline \end{array}$$

$$\frac{24 \text{ mixed } 24 \frac{17}{24}}{\underline{\hspace{10em}}}$$

A brief review for adding mixed numbers is shown at the right.

$$75 \frac{1}{3} = \frac{4}{12}$$

287

The equivalent fraction in 12ths for $\frac{3}{4}$ is _____.

$$16 \frac{1}{2} = \frac{6}{12}$$

The sum of the mixed numbers is _____.

$$24 \frac{5}{6} = \frac{10}{12}$$

$$28 \frac{3}{4} = \frac{\hspace{1em}}{12}$$

$$\frac{\frac{9}{12} \qquad 145 \frac{5}{12}}{\underline{\hspace{10em}}}$$

The common denominator for the fractions at the right is _____.

$$42 \frac{1}{2}$$

288

The sum of the mixed numbers is _____.

$$65 \frac{1}{3}$$

$$12 \frac{1}{4}$$

$$5 \frac{5}{6}$$

$$\frac{24 \frac{1}{3}}{\underline{\hspace{1em}}}$$

$$\frac{12 \qquad 150 \frac{1}{4}}{\underline{\hspace{10em}}}$$

(c) Multiplication of Fractions

A common business problem might be to determine one half of three quarters of a ton, which would be written as $\frac{1}{2}$ of $\frac{3}{4}$. How would

289

one-third of one-half gross be written? _____

$\frac{1}{3}$ of $\frac{1}{2}$

(a) $\frac{1}{2}$ of $\frac{1}{2}$ is computed as $\frac{1}{2} \times \frac{1}{2}$. $\frac{1}{2} \times \frac{1}{2} =$ _____ 290

(b) $\frac{2}{3}$ of $\frac{3}{4}$ would be computed as _____. $\frac{2}{3} \times \frac{3}{4} =$ _____

$\frac{1}{4}$ $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

$1 \times 1 =$ _____ 291
 $\frac{7}{8} \times \frac{4}{21} = \frac{1}{6}$
 $2 \times 3 =$ _____

Compute this problem also by cancellation.

$\frac{27}{32} \times \frac{8}{9} =$

$\frac{3}{4}$

Attempt to multiply these mixed numbers without changing them to improper fractions. $6 \frac{1}{4} \times 2 \frac{2}{5} = ?$ 292

Is this easy to do? _____

No

Multiplying directly two mixed numbers such as $6\frac{1}{4}$ and $2\frac{2}{5}$ is too complicated so we change them to improper fractions. In an improper fraction the _____ is larger than the _____.

Numerator denominator

$6\frac{1}{4}$ is converted to an improper fraction by thinking 6 is $\frac{24}{4}$; 294
 $\frac{24}{4} + \frac{1}{4} = \frac{25}{4}$

Convert $2\frac{2}{5}$ to an improper fraction: _____

$2 = \frac{10}{5}$; $\frac{10}{5} + \frac{2}{5} = \frac{12}{5}$

When the mixed numbers in the problem $6\frac{1}{4} \times 2\frac{2}{5} =$ are converted to 295

improper fractions, we can cancel and multiply as shown:

$$6\frac{1}{4} \times 2\frac{2}{5} = \frac{24}{4} \frac{5}{1} \times \frac{12}{5} \frac{3}{1} = \frac{15}{1} = 15$$

Complete:

$$7\frac{1}{2} \times 3\frac{1}{5} =$$

24

(a) $3\frac{3}{4} \times 2\frac{2}{5} =$ 296

(b) $5\frac{5}{6} \times 4\frac{2}{7} =$

(a) 9 (b) 25

Richards, an executive, estimated the answer to $79 \frac{3}{4}$ ft. X $3 \frac{1}{5}$ ft. to 297

be about 240 sq. ft.

Do you think he would also have to calculate the exact sq. ft.? _____

"yes" is a good answer

As in all arithmetic, it is possible to estimate fairly accurately 298
the product of the multiplication of fractions. Fractions are rounded
to the nearest whole numbers. For the problem $7 \frac{1}{5}$ X $6 \frac{2}{3}$, the mixed

numbers are rounded to 7 X $7 = 49$. The actual answer is _____,
which is only _____ (No.) away from the estimate,

48 1

The mixed numbers in $8 \frac{1}{3}$ X $4 \frac{1}{5}$ may be rounded to 8 X 4 , which equals 299

_____.
The actual product is _____, which is only _____ (No.) from the estimate.

32 35 3

$2 \frac{1}{8}$ is rounded down to 2. 300

$2 \frac{7}{8}$ is rounded up to 3.

What is $6 \frac{1}{5}$ rounded to? _____

What is $6 \frac{11}{12}$ rounded to? _____

6 7

Round these: 301

(a) $14 \frac{2}{3}$ _____

(b) $17 \frac{1}{4}$ _____

(c) $12 \frac{1}{2}$ _____

(d) $58 \frac{11}{15}$ _____

15 17 13 59

First estimate the product to these problems; then compute the exact product.

302

(a) $14 \frac{2}{3} \times 2 \frac{1}{4} =$

(b) $5 \frac{2}{3} \times 1 \frac{1}{2} =$

(c) $12 \frac{2}{3} \times 3 \frac{3}{5} =$

Estimate: 30	12	52
Actual: 33	$8 \frac{1}{2}$	$45 \frac{3}{5}$
<hr style="border: 0.5px solid black;"/>		

When estimating the product of fraction problems like $\frac{1}{3} \times 17 \frac{1}{2}$, think $\frac{1}{3} \times 18 = 6$ for the estimated product.

303

$\frac{1}{4} \times 24 \frac{1}{3} =$ _____ (estimated product).

6

Estimate the products of these:

304

(a) $\frac{1}{5} \times 74 \frac{1}{2} =$

(b) $81 \frac{3}{5} \times \frac{1}{3} =$

(c) $65 \frac{1}{5} \times \frac{1}{8} =$

15 27 8

For the problem $\frac{2}{5} \times 29 \frac{3}{4}$, estimate the answer by thinking:

305

(a) $\frac{2}{5}$ is less than $\frac{1}{2}$

(b) $\frac{1}{2} \times 30 = 15$

(c) The estimated product is less than _____.

15

Estimate the product for this problem: $\frac{3}{8} \times 241 \frac{1}{2} =$ 306
The estimated product is less than .

121

Estimate the products: 307

(a) $\frac{3}{10} \times 89 \frac{1}{2} =$

(b) $\frac{5}{12} \times 195 \frac{1}{8} =$

Each product is less than: 45 98

For the problem $\frac{7}{8} \times 864 \frac{3}{4}$, estimate the answer by thinking: 308

(a) $\frac{7}{8}$ is a little less than 1

(b) $1 \times 865 = 865$

(c) The estimated product is less than .

865

Estimate the product: $\frac{5}{6} \times 125 \frac{3}{8} =$ 309

Less than 125

Estimate the products: 310

(a) $\frac{2}{3} \times 39 \frac{7}{8} =$

(b) $\frac{11}{12} \times 4,876 \frac{1}{2} =$

Less than 40 Less than 4,877

(d) Division of Fractions

Examine these two problems:

311

(a) $5 \div \frac{3}{1} =$

(b) $5 \times \frac{1}{3} =$

The answer for (a) is _____ and for (b) is _____.
They have (the same/different) answers.

$1 \frac{2}{3}$ $1 \frac{2}{3}$ same

Examine these two problems:

312

(a) 5 divided into thirds

(b) $5 \times \frac{3}{1} =$

or
 $5 \div \frac{1}{3} =$

The answer for (a) is _____ and for (b) is _____.
They have the same/different answers.

15 15 same

When $\frac{1}{3}$ is inverted it becomes $\frac{3}{1}$.

313

When $\frac{3}{8}$ is changed to $\frac{8}{3}$, we say that the $\frac{3}{8}$ has been _____.

Inverted

Because $5 \div \frac{1}{3} =$ and $5 \times \frac{3}{1} =$ have the same answers, we can state

314

a rule for division of fractions:
Invert the divisor and multiply.

Complete this division: $16 \div \frac{2}{3} = 16 \times \frac{3}{2} =$ _____

To divide fractions, invert the (divisor/dividend) and multiply. _____ 315

Divisor

When division problems contain mixed numbers, first change the mixed numbers to improper fractions. Then _____ the divisor and multiply. 316

Invert

In the problem $12 \frac{1}{2} \div 3 \frac{3}{4} =$, it is necessary to change the mixed numbers to _____; then invert the _____ and _____.

Complete the division: $12 \frac{1}{2} \div 3 \frac{3}{4} =$

$12 \frac{1}{2} \div 3 \frac{3}{4} =$

Improper fractions divisor multiply $3 \frac{1}{3}$

The estimation of answers to the division of fractions is just as important as estimating products to multiplication problems. The problem $15 \frac{1}{3} \div 4 \frac{2}{3} =$ rounded becomes _____ \div _____. The estimated answer is _____.

$15 \div 5 = 3$

The estimated answer for $15 \frac{1}{3} \div 4 \frac{2}{3} =$ is _____. The actual answer is _____.

$3 \frac{2}{7}$

The estimated answer for $52 \frac{1}{2} \div 7 \frac{1}{4} =$ is figured: $53 \div 7 = 8$ approx. 320
The actual answer is _____.

$7 \frac{7}{29}$

Find the estimated and actual answers:

321

(a) $17 \frac{1}{2} \div 8 \frac{1}{6} =$ Est. _____ Act. _____

(b) $127 \frac{3}{4} \div 19 \frac{7}{8} =$ Est. _____ Act. _____

$$\begin{array}{r} 2 \quad 2 \frac{1}{7} \quad 6 \quad 6 \frac{68}{159} \\ \hline \end{array}$$

Estimate and complete these divisions:

322

(a) $4 \frac{1}{2} \div 3 \frac{3}{4} =$ Est. _____ Act. _____

(b) $2 \frac{1}{2} \div 3 \frac{3}{4} =$ Est. _____ Act. _____

$$\begin{array}{r} 1 \quad 1 \frac{1}{5} \quad \frac{3}{4} \quad \frac{2}{3} \\ \hline \end{array}$$

When estimating the answer of $7 \frac{1}{4} \div \frac{1}{8}$, think:

323

(a) Round $7 \frac{1}{4}$ to 7

(b) $7 \div \frac{1}{8} = 7 \times 8 = 56$

The estimated answer for $12 \frac{1}{15} \div \frac{1}{3} =$ _____.

36

Estimate the answers for these problems:

324

(a) $89 \frac{2}{3} \div \frac{1}{2} =$

(b) $4 \frac{3}{4} \div \frac{1}{6} =$

180 30

When estimating the answer of $173 \frac{1}{2} + \frac{7}{8}$, think:

325

(a) $\frac{7}{8}$ is approximately 1

(b) $174 + 1 = 174$

The estimated answer of $26 \frac{1}{3} + \frac{5}{6} =$ _____

26 approximately

Estimate the answers for these problems:

326

(a) $12 \frac{1}{5} + \frac{11}{12} =$ approx. _____

(b) $36 \frac{5}{8} + \frac{7}{9} =$ approx. _____

12 37

When you arrive at this point, ask your teacher for directions for completing Panel 5.

327

In the absence of specific directions, complete Panel 5 following the instructions on it carefully. The letter in the upper right of Panel 5 is _____.

F

DECIMAL FRACTIONS

(a) The Meaning of Decimals

The calculating machines of modern business are not equipped to handle 328
 the fractions that we have been just studying, known as common fractions.
 They can, however, compute with decimal fractions.
 Modern calculating machines can easily compute with _____ fractions.

Decimal

The denominators of decimal fractions are not written, but understood. 329
 For example:

	<u>Common Fraction</u>	=	<u>Decimal Fraction</u>
(a)	$\frac{5}{10}$	=	.5
(b)	$\frac{23}{100}$	=	.23
(c)	$\frac{6}{100}$	=	.06

The denominators of _____ fractions are understood.

Decimal

The common fraction $\frac{5}{10}$ and the decimal fraction .5 are both read 330
 5 tenths.

Observe: $\frac{5}{10}$ = .5

1 zero

1 place to the
 right of the point

The tenths in decimals are indicated by _____ (No.) place(s) to the right
 of the point.

1

The common fraction $\frac{23}{100}$ and the decimal fraction .23 are both read 23 hundredths. 331

Observe: $\frac{23}{100} = .23$

2 zeros

2 places to the right of the point

The hundredths in decimals are indicated by (No.) place(s) to the right of the point.

$\frac{2}{100}$

In $\frac{5}{10} = .5$, there are as many zeros in the denominator of the common fraction 332

fraction $\frac{5}{10}$ as there are places to the right of the decimal in .5.

There are (No.) zeros in the denominator of $\frac{23}{100}$, and there (No.) places to the right of the decimal in .23.

$\frac{2}{100}$

For every zero in the denominator of the common fraction $\frac{248}{1000}$, there 333

should be (the same/a different) number of places to the right of the decimal in the equivalent decimal fraction.

$\frac{248}{1000} =$ _____ (decimal fraction)

Same $\frac{248}{1000} = .248$

Change these common fractions to decimal fractions: 334

$\frac{125}{1000} =$

$\frac{75}{100} =$

$\frac{1,897}{10,000} =$

$\frac{125}{1000} = .125$ $\frac{75}{100} = .75$ $\frac{1,897}{10,000} = .1897$

The equivalent decimal fraction for $\frac{3}{100}$ is .03. 335

The equivalent decimal fraction for $\frac{6}{100}$ is ____.

.06

When a whole number precedes a decimal, the point is read as "and." 336
Therefore 2.4 is read as 2 _____ 4 tenths.

And

.37 is read 37 _____ . 337
.165 is read 165 _____ .
5.027 is read 5 _____ 27 _____ .

Hundredths thousandths and thousandths

The way in which $\frac{3}{100}$ is interpreted indicates how it can be changed to 338
a decimal.

$\frac{3}{100}$ is read 3 _____ and is written _____ as a decimal.

Hundredths .03

$\frac{41}{10,000}$ is read 41 ten _____, which means as a decimal it has 339

(No.) places to the right of the decimal. As a decimal

$\frac{41}{10,000}$ is written _____.

Thousandths 4 .0041

(b) Converting Common Fractions to Decimals

Convert these common fractions to decimals:

340

- (a) $\frac{578}{1000} =$
- (b) $\frac{35}{1000} =$
- (c) $\frac{8}{1000} =$

.578 .035 .008

$\frac{35}{10}$ is equivalent to 3.5

341

$\frac{483}{100} =$ _____ . (Decimal)

4.83

Common Fraction

Decimal Fraction

342

- (a) $\frac{3}{10,000} =$
- (b) $\frac{35}{10} =$
- (c) $\frac{387}{100} =$

.0003 3.5 3.87

To convert $\frac{1}{8}$ to a decimal you must find a common fraction whose denomi-

343

nator is 10, 100, or 1000. Fractions whose denominators are 10, 100, or 1000 can be immediately written as _____ fractions.

Decimal

To convert $\frac{1}{8}$ to a decimal fraction. Think $\frac{1}{8}$ is how many thousandths. 344

Follow these two steps:

(a)
$$\begin{array}{r} 125 \\ 8 \overline{)1,000} \end{array}$$

(b) $\frac{1}{8} = \frac{125}{1000} = \underline{\hspace{2cm}}$ (Decimal)

$$\begin{array}{r} .125 \\ \hline \end{array}$$

Convert $\frac{1}{16}$ to a decimal. Follow these two steps: 345

(a)
$$16 \overline{)100000}$$
 (Use as many zeros as necessary to divide evenly)

(b) $\frac{1}{16} = \frac{\hspace{1cm}}{10000} = \underline{\hspace{2cm}}$ (Decimal)

$$\begin{array}{r} 625 \\ \hline .0625 \end{array}$$

To convert $\frac{1}{6}$ to a common fraction, think $\frac{1}{6} = \frac{?}{1,000}$ 346

$$6 \overline{)1000} \begin{array}{l} 166 \\ 2 \\ 3 \end{array}$$

Additional zeros in the denominator will yield repeating 6's. So, if we want 3 decimal places, we stop with:

$\frac{1}{6} = \frac{166}{1000} \frac{2}{3} = \underline{\hspace{2cm}}$ (Decimal)

$$\begin{array}{r} .166 \frac{2}{3} \\ \hline \end{array}$$

Convert $\frac{1}{9}$ to a 3-place decimal: 347

(a) $\frac{1}{9} = \frac{\hspace{1cm}}{1,000}$

(b)
$$9 \overline{)1000}$$

(c) $\frac{1}{9} = \frac{\hspace{1cm}}{1000} = \underline{\hspace{2cm}}$ (Decimal)

$$\begin{array}{r} 111 \frac{1}{9} \\ \hline \end{array} \quad \frac{111}{1000} \frac{1}{9} = \underline{\hspace{2cm}} \frac{1}{9}$$

If $\frac{1}{9} = .111 \frac{1}{9}$ as a decimal, then $\frac{2}{9}$ will be twice as large.

348

$\frac{2}{9} = \underline{\hspace{2cm}}$ (Decimal)

$.222 \frac{2}{9}$

A short way to change $\frac{2}{9}$ to a decimal is to divide 9 into 2 with as many zeros as we want decimal places after it.

(a) $9 \overline{)2,000}$

(b) $\frac{2}{9} = \frac{\hspace{1cm}}{1000} = \underline{\hspace{1cm}}$ (Decimal.)

$222 \frac{2}{9}$ $.222 \frac{2}{9}$

The rule for converting any common fraction into a decimal is to divide the denominator into the numerator with as many zeros as places wanted in the decimal.

350

To convert $\frac{2}{3}$ to a 3-place decimal you would divide 2,000 by .

3

To convert $\frac{2}{3}$ to a 3-place decimal you complete:

351

$3 \overline{)2.000} = .666 \frac{2}{3}$

A decimal is placed so that there are 3 decimal places as shown here:

$.666 \frac{2}{3}$, which may be rounded to .

$.667$

Convert $\frac{1}{15}$ to a 3-place decimal.

352

$$\begin{array}{r} 66 \frac{2}{3} \\ 15 \overline{)1000} \end{array}$$

Because 3 decimal places are required, the decimal for $66 \frac{2}{3}$ will be placed (No.) place(s) to the left of $66 \frac{2}{3}$ as follows:

$.066 \frac{2}{3}$, which may be rounded to _____.

One .067

$\frac{3}{32}$ converted to a 3-place decimal is _____.

353

.094

As a common fraction, .7 is written _____; and as a common fraction .70 is written _____.

354

$$\frac{7}{10} \qquad \frac{70}{100}$$

Observe that $\frac{70}{100} = \frac{7}{10}$

355

Therefore, we can say that .70 is (equal to/different from) .7 _____.

Equal to

$\frac{2}{25}$ converted to a 3-place decimal is .080, which is often immediately

356

written as .08.

Is .08 (equal to/less than) .080? _____.

Equal to

$\frac{3}{16}$ converted to a 3-place decimal is _____.

357

.187 $\frac{1}{2}$ or .188

Convert these common fractions to 3-place decimals:

358

(a) $\frac{1}{8} =$

(c) $\frac{5}{8} =$

(b) $\frac{3}{8} =$

(d) $\frac{7}{8} =$

.125 .375 .625 .875

Convert these common fractions to 3-place decimals:

359

(a) $\frac{1}{7} =$

(b) $\frac{3}{7} =$

(c) $\frac{126}{500} =$

.142 $\frac{6}{7}$.429 .252

$\frac{2}{350}$ converted to a 3-place decimal is _____.

360

.005 $\frac{5}{7}$ or .006

(c) Converting Decimals to Common Fractions

Decimal fractions are converted to common fractions by writing them as common fractions and reducing the result. 361

$$.5 = \frac{5}{10} = \underline{\hspace{2cm}} \text{ (Reduced fraction)}$$

$$\frac{1}{2}$$

The common fraction equivalent for .125 is $\frac{125}{1000} = \underline{\hspace{2cm}}$. (Reduced) 362

$$\frac{1}{8}$$

The common fraction equivalent for .179 is $\underline{\hspace{2cm}}$. 363

$$\frac{179}{1000}$$

Determine the common fraction equivalents for: 364

- | | |
|----------|-----------|
| (a) .2 = | (c) .25 = |
| (b) .8 = | (d) .75 = |

$$\frac{1}{5} \quad \frac{4}{5} \quad \frac{1}{4} \quad \frac{3}{4}$$

T O N S
(Decimals)

T O N S
(Common Fractions)

365

.875

.500

.125

.625

.375

$\frac{7}{8}$ $\frac{1}{2}$ $\frac{1}{8}$ $\frac{5}{8}$ $\frac{3}{8}$

We already know that $\frac{3}{4}$ is equivalent to $\frac{9}{12}$, because $\frac{9}{12}$ reduces to $\frac{3}{4}$. 366

We can also change $\frac{3}{4}$ back to $\frac{9}{12}$ as follows:

$$\frac{3 \times 3}{4 \times 3} =$$

$\frac{9}{12}$

Multiplying the numerator and denominator of a common fraction by the same number produces a/an _____ fraction. 367

Equivalent

.43 $\frac{3}{4}$ is converted to a common fraction as follows: 368

$$.43 \frac{3}{4} = \frac{43}{100} \frac{3}{4} = \frac{43}{100} \frac{3 \times 4}{4 \times 4} = \frac{175}{400}$$

$\frac{175}{400}$ is then reduced; $\frac{175}{400} = \frac{7}{16}$

What common fraction is $.56 \frac{1}{4}$? _____.

$\frac{9}{16}$

(d) Adding, Subtracting, and Multiplying Decimals

When adding and subtracting decimals, care should be taken to put the decimals one under another. Copy and add: 372

1.5 + 17.25 + 16 + 18.75 + 6.25 + 1.7 = _____

(Note that 16 has an understood decimal, 16.)

61.45

Subtract the following: 373

(a) 12.5 - 6.75 = _____

(b) 42 - 17.5 = _____

(c) 28.12 - 5.4 = _____

5.75 24.5 22.72

Subtract the following: 374

(a) 1,672.05 - 829.5 = _____

(b) 1000 - 128.47 = _____

(c) 621.55 - 149 = _____

842.55 871.53 472.55

(a)
 $\frac{1}{2} \times \frac{1}{2}$

(b)
 $.5 \times .5$

375

Should the answer be the same? (yes/no) _____

Yes

(a) $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(b) $.5 \times .5 = .25$

376

$\frac{1}{4}$ is (equivalent to/different from) .25. _____

When multiplying a number with 1 decimal by another number with 1 decimal, you count off _____ (No.) decimal places in the answer.

Equivalent to 2

$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$

$.1 \times .1 = .01$

377

$\frac{1}{100}$ is (equivalent to/different from) .01. _____

When multiplying a number with 1 decimal by another number with 1 decimal, you point off _____ (No.) decimal places in the answer.

Equivalent to 2

(a) $\frac{1}{100} \times \frac{1}{10} = \frac{1}{1000}$

(b) $.01 \times .1 = .001$

378

$\frac{1}{1000}$ is (equivalent to/different from) .001. _____

When multiplying a number with 2 decimals by another number with 1 decimal, you point off _____ (No.) decimal places in the answer.

Equivalent to 3

The general rule for pointing off the decimal place in the product of two decimals is to place a decimal in the product by counting from right to left as many places as are in the two factors. 379

When multiplying a number with 3 decimals by another with 2 decimals, you point off _____ (No.) decimals in the product.

5

If the product of 15 and 135 is 2025, then the product of 1.5 and 1.35 is 380 _____.

2.025

If the product of 1,007 and 121 is 121847, then the product of 1.007 and 1.21 is _____ 381

1.21847

Cost Department Computation 382

7.037 tons of sulphite pulp at \$122.50 per ton is equal to \$ _____
(Actual answer)

\$862.03

(e) Estimating the Product of Decimals

The estimate of the product of decimals is particularly helpful in checking the decimal point. Look at the problem: 383

10.261 X 1,036 = ?

Because 10.261 may be rounded to 10 and 1.036 may be rounded to 1, then the estimated answer is _____.

10

When the factors are not close to 10 and 1 as in the problem, 10.261 X 1,036 =, you may use the same procedure you would use for the multiplication of whole numbers. When estimating the answer for the multiplication of whole numbers, you obtained _____ and _____ estimates. 384

Low high

Study this estimation procedure: 385

25.83 is between 20 and 30
X 6.37 is between 6 and 7

164.5371 is between _____ and _____

The low estimate for 25.83 X 6.37 is _____.
The high estimate is _____.

120 and 210 120 210

Suppose the product 16.45371 was obtained for the problem, 25.83 X 6.37. We know that the answer is (correct/incorrect) _____ 386

The answer should lie between _____ and _____.

Incorrect 120 210

Suppose the product 1,645.371 was obtained for the problem, $25.83 \times 6.37 =$. This answer is (reasonable/unreasonable). 387

It should lie between and .

Unreasonable 120 210

Another procedure for estimating the answer to the problem 25.83×6.37 388 is to round to the nearest whole numbers and then multiply.

$26 \times 6 = 156$

Which of the following is closest to 150?

- (a) 1,645.371
- (b) 16.45371
- (c) 164.5371

(c) 164.5371

Using the procedure for rounding to the nearest whole numbers, obtain 389 an estimate to the product $81.04 \times 3.7 =$. The estimate is . The exact product is .

324 299.848

Production Department Computation 390

Formula A 1.075 tons of B-196 at \$269.50 a ton.

Formula B 11.386 tons of B-203 at \$20.23 a ton.

Estimate which formula is the cheaper?

Estimate the dollars of savings?

B is cheaper by about \$50

Using the procedure for rounding to the nearest whole numbers, obtain the estimate for $62.5 \times 7.09 =$. The estimate is _____. The exact product is _____.

441 443.125

To estimate the product of decimals less than 1, round to the nearest tenth, hundredth, thousandth, and so on.

.21, .213, .2134, and .21346 all should be rounded to the nearest tenth, which would be _____.

.2

Determine the value of these decimals to the nearest hundredth.

.05 .048 .0483 .04838

If all are rounded to the nearest hundredth, they would be _____.

.05

An engineer made these three measurements:

.0004523" .00045" .0005037"

If they are each rounded to the nearest ten thousandth, they would all round to _____.

.0005"

(a) (b) (c) (d)
.05 .0036 .035 .00036

The decimal that rounds to .004 is _____.

.0036

(a) .06389 (b) .0701 (c) .00638 (d) .05499 396

The decimal that rounds to .06 is _____.

(a) .06389

The estimate for 626.743 X .0048 is found as follows: 397

626.743 rounds to 627
X .0048 rounds to .005
3.135 estimated product

According to the estimate, in the exact product (with the point omitted) 30131664, where should the point be placed. _____

3.0131664

The estimate of 1,500.21 X .00014 is found as follows: 398

1500.21 rounds to 1500
X .00014 rounds to .0001
.1500 estimated product

According to the estimate, in the exact product (with the point omitted) 2100294, where should the point be placed? _____

.2100294

72.374 rounded is 72
X .0093 rounded is .01 399

The estimated answer is _____.

.72

1,199.735 rounds to 1200
X .0008736 rounds to X.001

400

An estimate of the answer is _____.

1.2

\$15,301.50 rounds to _____
X .0075 rounds to _____

401

The estimated answer is _____.

\$15,000 .01 \$150

Estimating the product of two decimals is particularly helpful in
verifying the position of the _____.

402

Decimal point

(f) Dividing Decimals

If a dollar is divided into parts, each equal to $\frac{1}{100}$ th of a dollar, there would be 100 No. equal parts. 403

If five dollars is divided into parts each equal to $\frac{1}{20}$ th of a dollar, there would be 100 No. parts.

100 100

The rule for dividing by a common fraction is to invert the divisor and multiply. 404

Invert multiply

- (A) $\$1 \div \frac{1}{100} =$ 405
 (b) $\$5 \div \frac{1}{20} =$

$\$1 \times \frac{100}{1} = 100$ $\$5 \times \frac{20}{1} = 100$

Is $1 \div \frac{1}{100}$ equivalent to $1 \div .01$? 406

Is $5 \div \frac{1}{20}$ equivalent to $5 \div .05$?

Yes yes

If $1 \div \frac{1}{100} =$, then $1 \div .01 =$. 407

If $5 \div \frac{1}{20} =$, then $5 \div .05 =$.

100 100 100 100

$$\frac{1}{10} \div \frac{1}{1000} = 100$$

$$.1 \div .001 = 100$$

408

These problems have answers (different from/equivalent to) one another. _____

Equivalent to

In the problem $.1 \div .001 = 100$, if two zeros were annexed to the dividend $.1$, it would become _____ which would be read as 100 _____.

.100 thousandths

Is $.100 \div .001$ (equivalent to/different from) $.1 \div .001$? _____ 410

Equivalent to

$.100 \div .001$ may be read: 100 thousandths divided by 1 _____, which equals _____. We can conclude there must be at least as many decimal places in the dividend as there are in the _____.

Thousandth 100 divisor

In order to complete the division of decimals, there must be at least as many decimals in the _____ as there are in the _____.

Dividend divisor

$2.5 \overline{)60}$ 413

The divisor has _____ No. decimal place(s).

The dividend has _____ No. decimal place(s).

Therefore _____ No. decimal place(s) must be annexed to the divisor.

1 0 1

Problem $2.5 \overline{)60}$

Solution
$$\begin{array}{r} 24. \\ 2.5 \overline{)60.0} \\ \underline{50} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

414

In the solution, a zero is annexed to 60 so that there will be at least as many decimal places in the _____ as in the _____. Then the division is completed as if the problem contained whole _____.

Dividend divisor numbers

When there are as many decimal places in the dividend as in the divisor, 415 you can complete a division problem as if it contained only _____.

Whole numbers

Complete this problem by annexing 1 zero and dividing: $.15 \overline{)22.5}$ 416

$$\begin{array}{r} 150. \\ .15 \overline{)22.50} \\ \hline \end{array}$$

Complete this problem: $2.5 \overline{)67.5}$ (Show the determination of the decimal point.) 417

$$\begin{array}{r} 27. \\ 2.5 \overline{)67.5} \\ \hline \end{array}$$

Complete these problems: $3.5 \overline{)805}$ $.35 \overline{)80.5}$ $.35 \overline{)8.05}$ 418

$$\begin{array}{r} 230. \\ 3.5 \overline{)805.0} \\ \underline{70} \\ 105 \\ \underline{105} \\ 00 \\ \underline{00} \\ 00 \end{array}$$

$$\begin{array}{r} 230. \\ .35 \overline{)80.50} \\ \hline \end{array}$$

$$\begin{array}{r} 23. \\ .35 \overline{)8.05} \\ \hline \end{array}$$

Examine this problem: $3.5 \overline{)8.05}$

419

When there are more decimal places in the dividend than divisor, locate the decimal in the answer as follows:

$$3.5 \overline{)8.05}$$

The answer is _____.

2.3

Complete these problems: $27 \overline{)1.431}$ $2.7 \overline{).1431}$ $27 \overline{)14.31}$ 420

$\begin{array}{r} .53 \\ 2.7 \overline{)1.431} \\ \underline{1\ 35} \\ 81 \\ \underline{81} \end{array}$	$\begin{array}{r} .053 \\ 2.7 \overline{).1431} \end{array}$	$\begin{array}{r} .53 \\ 27 \overline{)14.31} \end{array}$
--	--	--

(g) Estimating Answers to the Division of Decimals

When dividing by decimals greater than 1, use the same procedure as you did with _____ numbers to determine an estimated answer. 421

Whole _____

37)156 rounds to _____ 422

The estimated answer is _____. The same procedure is followed for the problem:

26.4)175.32 rounds to 30)180

The estimated answer is _____.

40)160 4

19.95)102.47 rounds to _____ 423

20)100 5

Obtain the estimate and the actual answer for 50.24 ÷ 31.4 = 424

Estimate _____ Actual answer _____

1 1.6

Cost Department Computation 425

Cost \$822.68 + Hours 13.1 Cost Per Hour \$ _____

Actual Cost

Estimated answer is \$ _____.

62.8 80

$.176\overline{)16.5}$ rounds to $.2\overline{)17}$

426

The estimated answer is _____.

85

$.00987\overline{)267.437}$ rounds to $.01\overline{)267}$

427

The estimated answer is _____.

26,700

Study this problem: \longrightarrow $.176\overline{)16.59}$

428

Obtain the correct answer to the nearest 2 decimal places:

Here is the solution: \longrightarrow

$$\begin{array}{r}
 94.261 \\
 .176\overline{)16.590\ 000} \\
 \underline{15\ 84} \\
 750 \\
 \underline{704} \\
 46\ 0 \\
 \underline{35\ 2} \\
 10\ 80 \\
 \underline{10\ 56} \\
 240 \\
 \underline{176}
 \end{array}$$

The answer rounded to two decimals is _____.

94.26

In order to obtain a correct answer to 2 decimal places, it may be necessary to carry the division out to 3 places to determine whether the third place is No., or greater, which would increase the second-place digit by No.

429

5 1

Estimate the answer: $1.74 \overline{)26.437}$

430

Obtain the actual answer correct to two decimal places. Answer _____

13 15.19

Estimate the answer. _____

$627.4 \overline{)24.634}$

431

Obtain the actual answer correct to two decimal places. _____

.04 .04

When you arrive at this point, ask your teacher for directions for completing Panel 6. In the absence of specific directions, complete Panel 6 following the instructions on it carefully. The letter in the upper right of Panel 6 is _____.

432

F

ABSTRACT

An experiment to adjust a programmed college business mathematics text to the level of comprehension suitable for high school students of low ability was prompted by the writer's awareness of the need for a similar text for high school business majors. The writer was especially concerned with those students who lack ability to attend college, but who go into business offices seeking employment--positions which invariably require a knowledge of the fundamentals of business mathematics.

The case study approach was used in conducting the experiment. The research involved a programmed text of 432 frames, entitled Fundamentals of College Business Mathematics, by Dr. Harry Huffman.

Two major objectives were established as a basis for this study. The first objective was to adjust Huffman's programmed business mathematics text to a level of comprehension of students of below average ability, so that their completed work, without a teacher's help, would be approximately 95 per cent accurate.

The second objective of the study was to determine whether students could retain the material covered in the programmed unit. This objective was achieved by means of intermediate tests given to each student throughout the unit.

Four revisions of Huffman's original programmed text were necessary to accomplish the objectives of the study. Five panels of student-editors, each panel comprised of three students of below average

ability, participated in the research. Revisions to the original programmed text were made on the basis of incorrect responses by individual students working through the unit. Written and oral comments by these students were also taken into consideration in making revisions.

Analysis of the data revealed the following results of the total experiment. A total of 104 new frames were added to the original program of 432 frames. This represented a 24.1 per cent increase of frames in the total unit which required an additional 208 responses.

There was a total decrease in errors from Panel One to Panel Five of 5.20 per cent. The average score on the six intermediate tests increased 9.1 points from the beginning to the end of the experiment.

Results of the experiment lead the writer to conclude that:

1. The error rate of successive panels dropped from 8.03 to 2.83 per cent. This represents a total decrease of 5.20 per cent from the beginning to the end of the experiment. Therefore, it was concluded that senior students, in high school, of below average ability can complete the adjusted programmed unit with an average error rate of less than 5 per cent.

2. Test score average for the panels rose from 78.2 to 87.3, an increase of 9.1 points, from the beginning to the end of the experiment. Therefore, it was concluded that students of below average ability can successfully perform on the retention tests used to measure the result of the programmed unit on fundamentals of business mathematics.