NONDIMENSIONAL APPROACH TO THE DESIGN OF OPEN CHANNELS WITH SPATIALLY VARIED FLOW

by

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I. INTRODUCTION

Spatially varied, steady flow is defined by Chow (3) as the flow condition existing in a channel in which water is added or subtracted along the course of flow. This condition is found in roadside gutters, around sewage treatment plants, and drainage and feeding channels in irrigation systems.

This study will be concerned with the case of water being added along the course of flow uniformly. For this case, a large amount of the energy lost during the flow is due to the turbulence created by the inflow impact. This impact loss is variable and has not been accurately defined, therefore, analysis of the problem from the standpoint of the conservation of energy principle is not feasible. The momentum principle, however, does hold true and is the most convenient means of investigating this problem.

Hydraulic structures with spatially varied flow have been designed on the basis of the momentum principle since approximately 1925, when Hinds (6) made what is assumed to be the first correct statement of the fundamental differential equation for spatially varied flow. The basic equation has since been restated with various types of friction terms and different arrangements, but the actual solution has always required an iterative, step-wise computation. This tedious procedure has prompted the use of approximate one-step methods which give an acceptable design, but which lack the high degree of accuracy generally desired in engineering design.

It is the purpose of this study to develop a nondimensional form of the fundamental differential equation which can be solved for a dimensionless profile. From this profile accurate water surface curves may be determined for a wide range of flow conditions.

This study is concerned with one of the most common occurrences of spatially varied flow which is a rectangular channel with a small slope and free overfall at the outlet. Included in the investigation is experimental verification of the nondimensional momentum equation.

II. REVIEW OF LITERATURE

An open channel with uniform inflow throughout its length is difficult to analyze from the standpoint of the principle of conservation of energy. This is due to an unknown energy loss caused by the impact of the inflow on the water already flowing in the channel.

Miller (8) and Stein (9) considered the energy loss due to impact small, relative to the total loss, and developed formulas for the design of washwater troughs for rapid sand filters using the conservation of energy principle. Each formula contained coefficients to account for the impact loss, but each also had serious limitations when considered for general use. Stein's formula did not adequately account for situations where the channel was to be designed with a sloping bottom. While flat channels are advantageous where available head is limited, the wider range of discharge available to sloping channels makes a method that accounts for channel slope more desirable. Miller based his formula on the assumption that the water surface drawdown in the channel was equal to the velocity head at the outlet. This has since been shown (2) to be extremely conservative.

Hinds (6) made laboratory studies as well as field studies on side-channel spillways for large dams. He reached two important conclusions as a result of these studies:

1. Bernoulli's theorem is not conveniently applicable because of the lack of uniformity in the impact loss coefficient.

2. The law of the conservation of linear momentum is directly applicable without an experimental coefficient. This is subject only to a small correction for volume swell due to entrained air and uneven velocity distribution.

Hinds was probably the first to develop an equation based on the momentum principle which accurately describes the water surface profile for spatially varied flow. In the development of this equation he assumed that the increment of inflow per unit length of channel is constant and that the equation was applicable to channels of any shape. He also assumed the inflow to be at right angles with the channel axis and that its momentum could be neglected as could the friction in the channel. By writing an expression for the difference in momentum between two sections of a channel, he developed the following equation.

$$y = \frac{1}{g} \int_0^x \left(V \frac{dV}{dx} + \frac{bV^2}{Q} \right) dx$$

where

y = ordinate to the water surface curve, feet.

g = acceleration of gravity, feet per second per second.

x = length of channel, feet.

dx = distance between the sections being examined, feet.

V = velocity at the upstream section, feet per second.

dV = difference in velocity between the sections being examined, feet per second.

Q = discharge at the upstream section, cubic feet per second.

b = constant inflow per unit length of channel, cubic feet per second per foot.

Hinds intended for this equation to be used in the design of new structures where the relation of Q, V, and x could be set according to the economic factors involved. He developed another form of this equation for use in investigating existing channels where the inflow might not be uniform, namely

$$\Delta y = \frac{Q_1}{g} \frac{(V_1 + V_2)}{(Q_1 + Q_2)} \left(\Delta V + \frac{bV_2 \ \Delta x}{Q_1} \right)$$

in which the subscripts one and two refer to the upstream and downstream sections, respectively, and the delta quantities are finite differences between the sections. This second equation gives the change in surface elevation between the sections under consideration. Solution of this equation and subsequent determination of the water surface curve depend upon the location of a control section where critical values are known. Once the control section has been located the water surface changes may be computed in an iterative fashion, for each succeeding increment of channel length.

Favre and Meyer-Peter (5) rewrote the equation developed by Hinds to include a friction term and a term to account for the increase in velocity due to the component of the inflow in a direction parallel to the channel axis. The basic iterative procedure, however, remained unchanged from that developed by Hinds.

Beij (1) conducted experiments with roof gutters of various cross-sections and developed a series of empirical formulas which were workable as long as the size of the channel under investigation was no larger than a roof gutter (about six inches by six inches). He also developed an equation for spatially varied flow in rectangular channels, which was of the same general form as that developed by Hinds, but he did not get good agreement between his limited experimental data and the solution of his equation.

Camp (2) wrote Hinds' equation to include a friction term using the Darcy-Weisbach friction factor. With this equation he devised a method for computing the water surface profile for channels with sloping as well as vertical side walls. He also collected experimental data which agreed very closely with his computed values.

Li (7) developed a form of Hinds' equation using nondimensional quantities to describe the flow. By using a Froude number to define flow conditions, he developed a method for determining the water surface curve for subcritical and supercritical flow conditions. He also conducted tests with level rectangular channels to demonstrate the validity of the use of the momentum principle, and in sloping rectangular channels to verify his own regime theory. His results compared very favorably with his theoretical computations.

III. THEORETICAL ANALYSIS

As stated previously, the purpose of this investigation is to develop a rapid, yet accurate, method of design for hydraulic structures which must carry spatially varied flow. This will be accomplished in the steps given below:

1. Presentation of the derivation of the fundamental differential equation for spatially varied flow which closely follows that given by Chow (3).

2. Conversion of the equation derived in step one into a nondimensional form which can be solved by the conventional methods developed by Camp (2) and Li (7).

3. Solution of the nondimensional equation for various conditions of surface roughness and slope for rectangular channels with free overfall at the outlet.

4. Presentation of the above solutions in such a manner as to illustrate that for constant conditions of slope and surface roughness, the dimensionless profile is constant over a wide range of discharge.

Fundamental Differential Equation

In the derivation of the momentum equation for spatially varied flow, the following assumptions will be made:

1. The flow is unidirectional.

2. The velocity distribution across the channel section is constant and uniform.

3. The pressure in the flow is hydrostatic.

4. The channel slope is relatively small.

5. The friction loss due to shear along the channel wall may be evaluated by the Manning formula.

6. The effect of air entrainment is neglected.

The symbols used in this derivation are defined below and refer to the channel depicted in Figure 1.

A = the cross-sectional area at section 1, square feet.

dA = change in area between sections 1 and 2, square feet.

D = height of water surface above outfall invert, feet.

F_f = frictional force along the channel, pounds per square foot.

g = acceleration of gravity, feet per second per second.

L = length of channel, feet.

n = Manning's roughness coefficient.

o = refers to outlet condition when used as a subscript.

P = the total pressure on the section under consideration in pounds per square foot.

Q = discharge per unit width of channel at section 1, cubic feet per second.

dQ = discharge increment between sections 1 and 2, cubic feet per second.

R_h = hydraulic radius of the section under consideration, feet.

S_f = friction slope as defined by the Manning formula.

S_o = bottom slope which is equal to sin θ where θ is the angle between the channel bottom and the horizontal.

V = velocity at section 1, feet per second.

dV = change in velocity between sections 1 and 2, feet per second.

w = unit weight of water, pounds per cubic foot.

W = weight of water between sections under consideration, pounds.

x = distance between upstream end and section under consideration, feet.

dx = distance between sections under consideration, feet.

y = depth at section under consideration, feet.

dy = change in depth between sections 1 and 2, feet.

$\bar{z}$ = depth of the centroid of A below the surface of flow, feet.

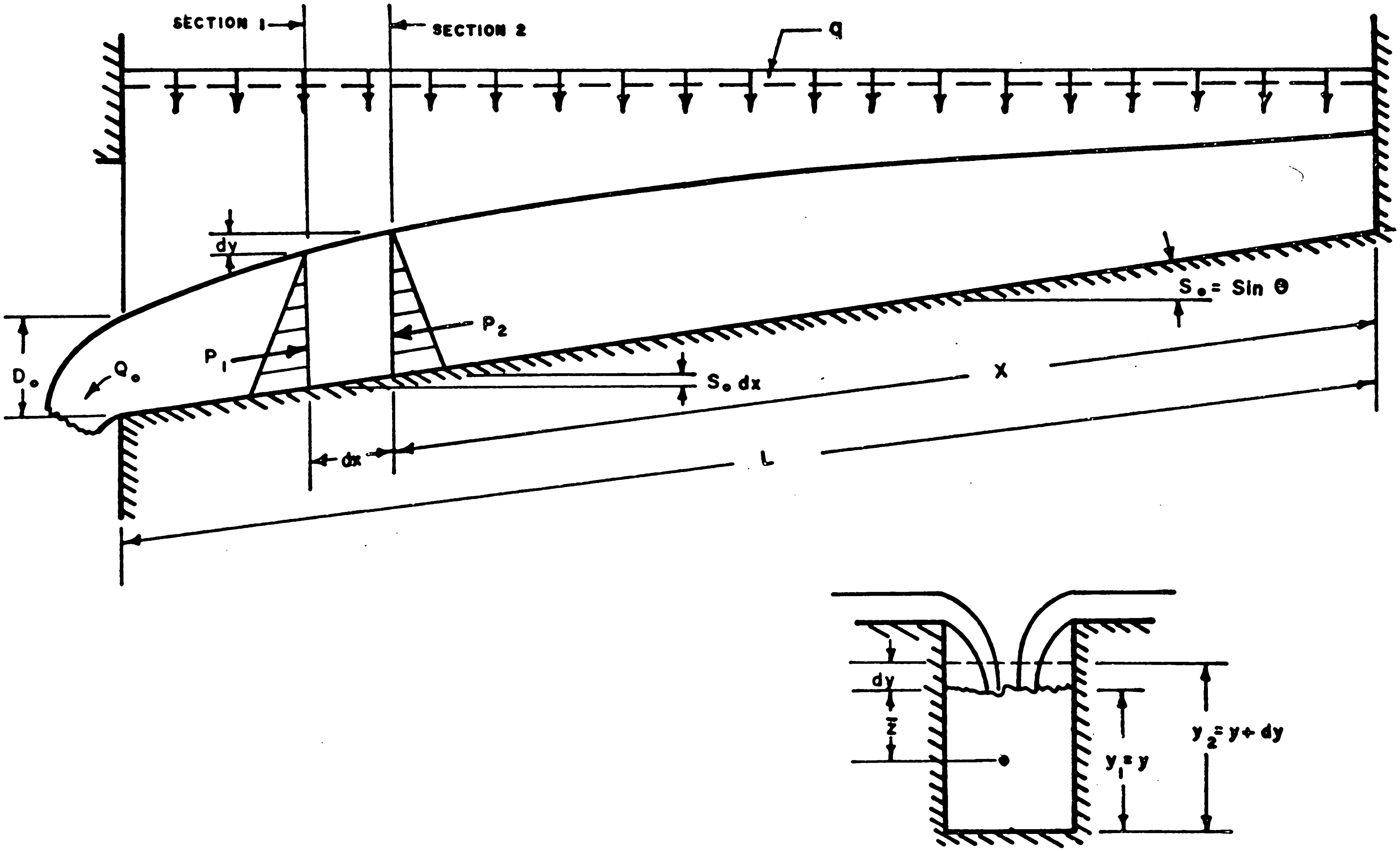

FIGURE 1. DIAGRAM USED IN THE DERIVATION OF THE EQUATION FOR SPATIALLY VARIED FLOW

The momentum past section 1 per unit time is

$$\frac{w}{g} QV$$

The momentum past section 2 is

$$\frac{w}{g} (Q - dQ)(V - dV)$$

The difference in momentum between the sections is

$$\frac{w}{g} QV - \frac{w}{g} \; (QV - QdV - VdQ) \; = \; \frac{w}{g} \; (QdV + VdQ)$$

where the product of the differentials has been dropped.

The component of the weight of water, between the sections, in the direction of flow, is

$$W \sin \theta \; = \; wS_o (A + \frac{dA}{2}) \, dx \; = \; wS_oAdx$$

where the product of differentials has been dropped.

The frictional force along the channel wall is

$$F_f \; = \; w (A + \frac{dA}{2}) \; S_f \, dx \; = \; wAS_f dx$$

where the product of differentials has been dropped and the friction slope, S_f, is given by the Manning formula as

$$S_f \; = \; \frac{V^2 \, n^2}{2.22 \, R_h^{4/3}}$$

The hydrostatic pressure between the sections is

$$P = P_1 - P_2 = w (\bar{z} + dy) A - w \bar{z} A = w A \, dy$$

The change in momentum between the sections is equal to the sum of the external forces acting on the body of water between the sections, or

$$\frac{w}{g}\,(QdV + VdQ) = wAdy + wAS_o\,dx - w\,AS_f dx$$

Dividing both sides of the equation by wA and solving for dy gives

$$dy = \frac{1}{g}\left(VdV + \frac{V}{A}\,dQ\right) + (S_f - S_o)\,dx \qquad (\;)$$

Nondimensional Equation

Equation (1) was nondimensionalized after the substitutions given below were made.

An average area is used because of the variations of discharge with length and is defined as

$$(Q_1 + Q_2) \; / \; (V_1 + V_2)$$

Also, V was taken as V_2, Q was taken as Q_1 and the differentials were considered to be finite increments. Remembering that $Q = VA$ and factoring Q_1/A from the momentum term gives equation (1) as

$$\Delta y = \frac{Q_1 (V_1 + V_2)}{g \; (Q_1 + Q_2)} \left(\Delta V + \frac{V_2 \Delta Q}{Q_1} \right) + (S_f - S_o) \, \Delta x \qquad (2)$$

For the purpose of nondimensionalizing equation (2) the following dimensionless variables are defined:

$$\bar{D} = D/D_o \qquad \bar{V} = V/V_o$$

$$\bar{x} = x/L \qquad \bar{Q} = Q/Q_o = Q/V_oD_o$$

The above expressions may be rewritten as

$$D = \bar{D}\, D_o \qquad V = \bar{V}\, V_o$$

$$X = \bar{X}\, L \qquad Q = \bar{Q}\, V_oD_o$$

and substituted into equation (2) with y being replaced by D.

$$\Delta \bar{D}\, D_o = \frac{\bar{Q}\, V_oD_o \; (\bar{V}_1 V_o + \bar{V}_2 V_o)}{g\, (\bar{Q}_1 V_oD_o + \bar{Q}_2 V_oD_o)} \left(\Delta \bar{V}\, V_o + \frac{\bar{V}_2 V_o \, \Delta \bar{Q} V_oD_o}{\bar{Q}_1 \, V_oD_o} \right)$$

$$+ \; (S_f - S_o) \, \Delta \bar{x}\, L$$

Simplifying and solving for $\Delta \bar{D}$ gives

$$\Delta \bar{D} = \frac{V_o^2}{gD_o} \frac{(\bar{V}_1 + \bar{V}_2)}{(\bar{Q}_1 + \bar{Q}_2)} \left(\Delta \bar{V} + \frac{\bar{V}_2 \Delta \bar{Q}}{\bar{Q}_1} \right) + (S_f - S_o) \Delta \bar{x} \frac{L}{D_o} \qquad (3)$$

where S_f is defined by the Manning formula as

$$S_f = \frac{\bar{V}_2^2 V_o^2 n^2}{2.22 \; K^{4/3} \bar{D}_2^{4/3} D_o^{4/3}}$$

In the expression for S_f, K was defined as the hydraulic radius of the critical section divided by D_o.

In equation (3), $\bar{Q}$, $\bar{V}$ and $\bar{x}$ vary from an initial value of unity, at the outlet, to zero at the upstream end. Knowing these initial conditions the equation may be solved for $\Delta \bar{D}$ by trial, beginning at the outlet and proceeding upstream in equal increments of length.

In the manner just described, equation (3) was solved on the IBM 7040 computer (Appendix C) for various values of discharge, channel slope and roughness. Those computations yielded the curves plotted in Figures 2, 3, and 4.

Figure 2 shows a dimensionless flow profile for each of four Manning roughness coefficients in a channel with zero slope. Figures 3 and 4 also have dimensionless profiles for four different Manning numbers, but the slopes have been changed to 0.01 and 0.02 respectively.

These profiles generated on the computer varied slightly for different values of discharge. For a channel 100 feet long and three feet wide,

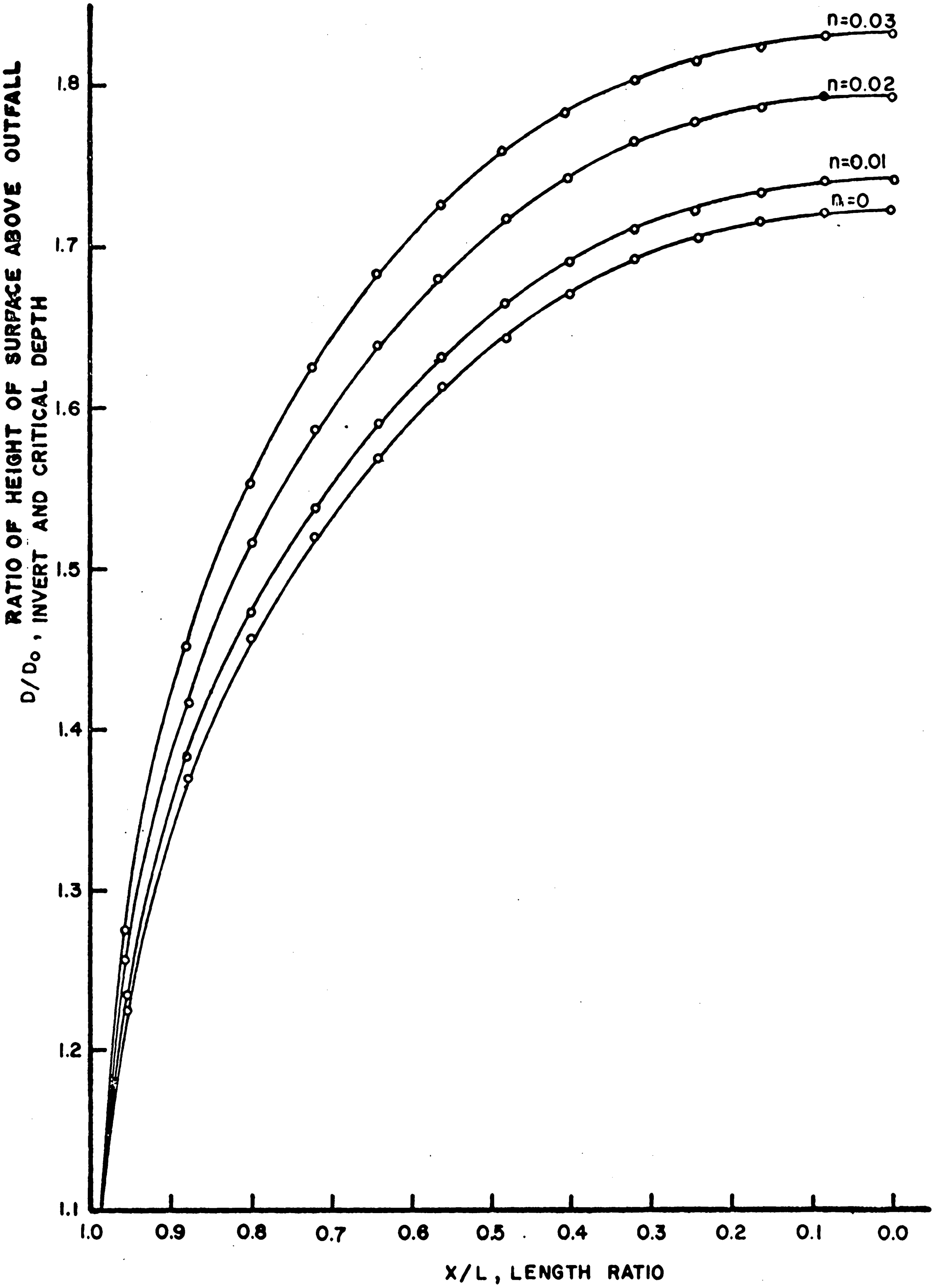

FIGURE 2. DIMENSIONLESS FLOW PROFILES FOR RECTANGULAR CHANNELS WITH A BOTTOM SLOPE OF ZERO

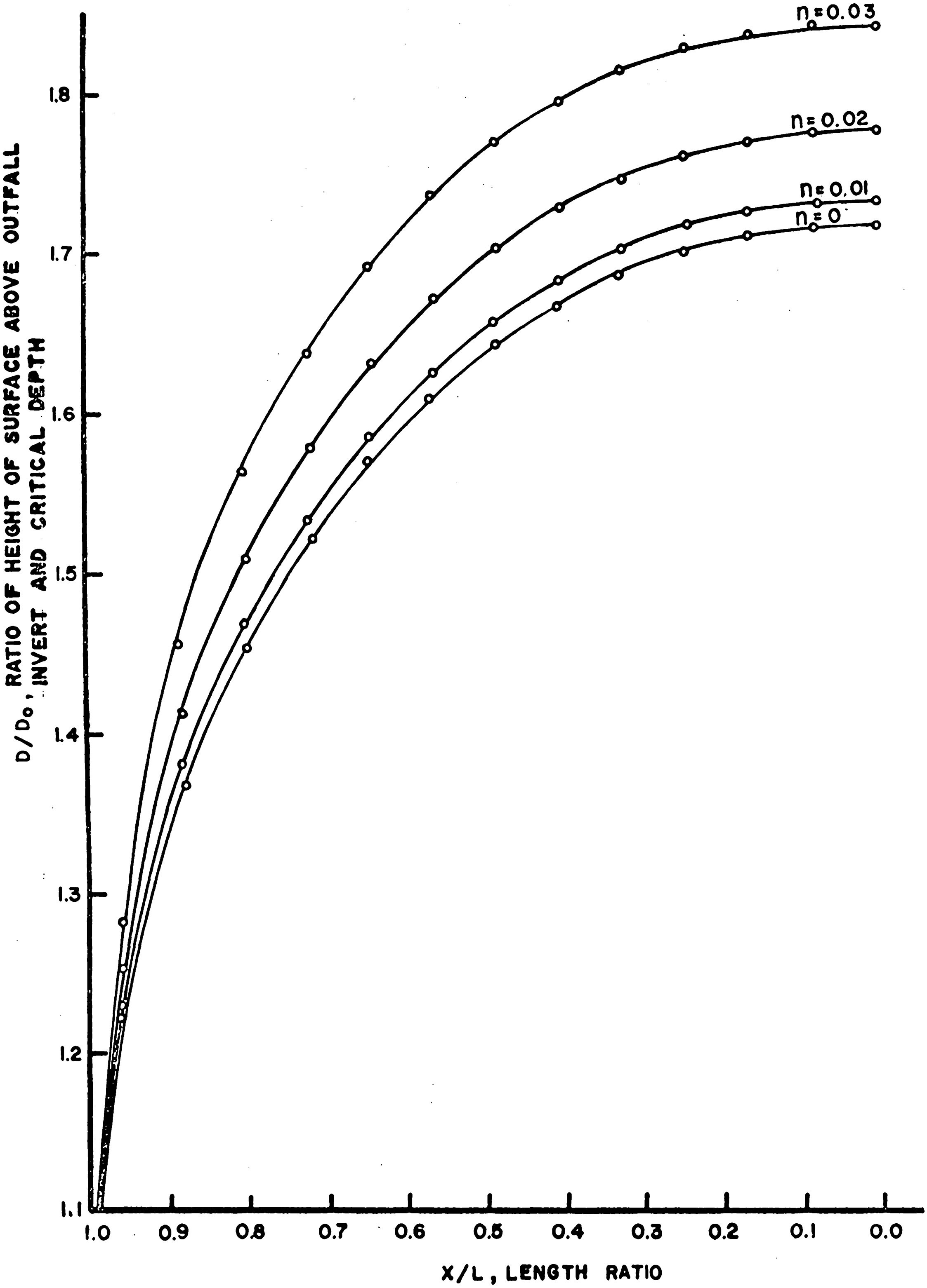

FIGURE 3. DIMENSIONLESS FLOW PROFILES FOR RECTANGULAR CHANNELS WITH A BOTTOM SLOPE OF 1.0 %

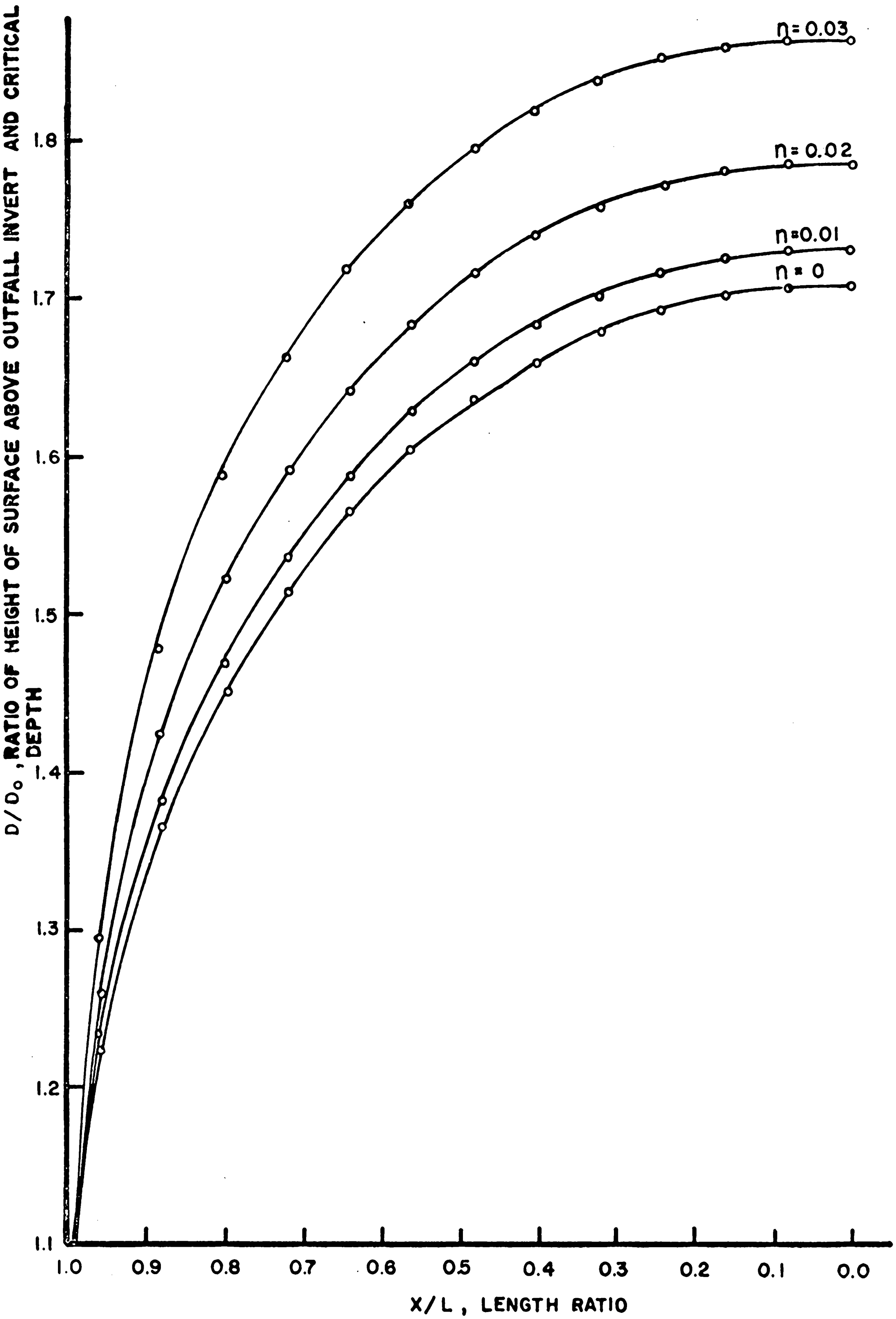

FIGURE 4. DIMENSIONLESS FLOW PROFILES FOR RECTANGULAR CHANNELS WITH A BOTTOM SLOPE OF 2.0%

the upstream depth ratios decreased as the total discharge was increased from 20 to 50 cubic feet per second. The amount of the reduction ranged between two and five percent depending on the slope and Manning number used. This difference is probably due to the evaluation of the friction slope by the Manning formula in which the hydraulic radius is approximated by $K\bar{D}$ where K is the hydraulic radius of the critical section divided by the critical depth. As the discharge increases, the critical depth also increases, thereby making the value of K smaller, which in turn increases the friction loss computed by the Manning formula. This could be remedied by developing a variable hydraulic radius, but not without altering the equation in such a way as to make the initial conditions unknown.

An example problem demonstrating the use of the curves is shown in Appendix A.

IV. EXPERIMENTAL INVESTIGATION

Description of Apparatus

In order to demonstrate the validity of the water surface profiles computed from the dimensionless curves, an open channel with spatially varied flow was set up in the laboratory (Figure 5). The channel was constructed with varnished plywood with an assumed Manning coefficient of 0.01 (4), a length of ten feet, a width of six inches and a bottom slope that could be varied from zero to two percent. Water was added to the channel over two level side weirs from two side troughs approximately the same size as the channel itself. These side troughs were fed at the upstream end by a head box with a total volume of approximately 30 cubic feet. In an attempt to cut down the velocity added to the flow by the impact of the inflow, aluminum vanes (Figure 6) were placed on the weirs, perpendicular to the channel axis, at two-inch intervals.

The depth of flow was measured by means of manometer tubes (Figure 7) placed at one-foot intervals along the bottom of the channel 1.5 inches from the side in an attempt to minimize the effect of the inflow impact on the depth readings. Previous measurements taken with a manometer placed in the center of the channel were consistently higher than the profile measured in the flume with a hand rule.

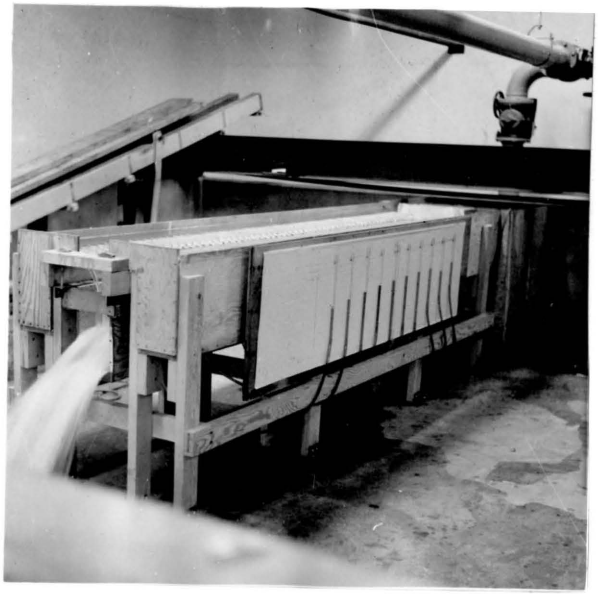

Figure 5. Spatially Varied Flow Apparatus

Figure 6. Aluminum Vanes Used to Minimize the Velocity Added to the Flow by the Side Troughs

Measurement of Discharge

The total discharge was measured by weighing the volume of water passing through the channel in one minute. This weight was checked each time the profile was taken and all total discharge figures are the average of at least ten measurements. The uniformity of the inflow was checked at one foot intervals along each weir by recording the time required to fill a can (Figure 8) held under the weir and then recording its weight to determine the volume of water collected. Figure 9 shows a comparison of theoretical values of uniform inflow and the values observed during the experimentation. Data from these flow measurements are given in Table 1, of Appendix B.

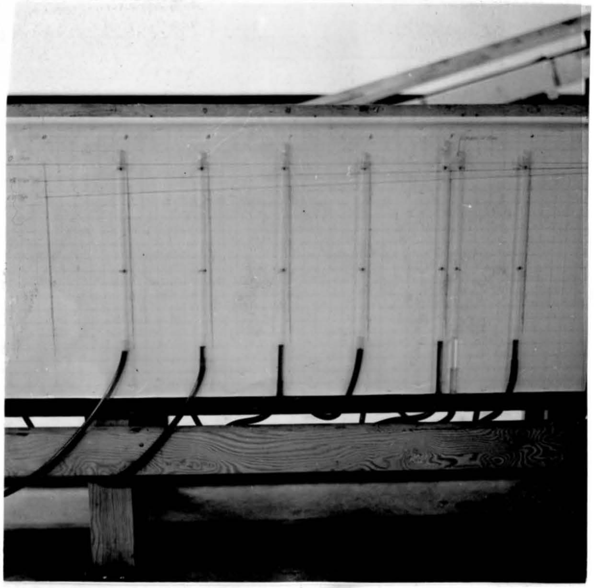

Figure 7. Manometer Tubes Used to Measure the Depth of Flow in the Flume

Figure 8. Measuring Can Used to Check the Inflow Along the Weirs

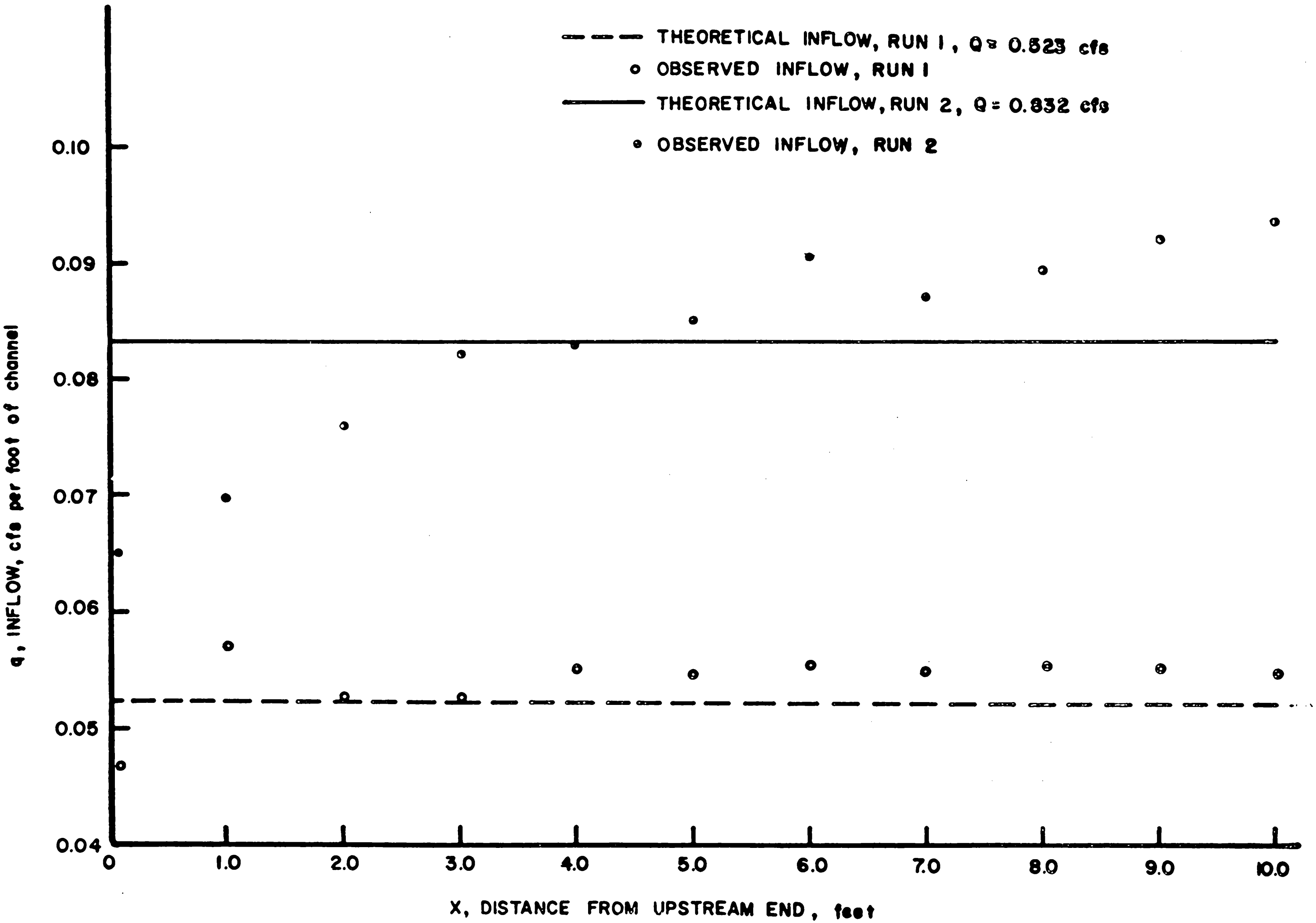

FIGURE 9. COMPARISON OF THEORETICAL AND OBSERVED VALUES OF INFLOW

V. RESULTS

The water surface profile was measured for two rates of flow at slopes of zero, one and two percent. These profiles are shown in Figures 10, 11, and 12. Also shown in these figures are the corresponding theoretical profiles computed from the dimensionless curves of Figures 2, 3, and 4. Each of the observed values plotted represents an average of the ten measurements taken at each discharge. Data collected during these measurements are shown in Table 2, of Appendix B.

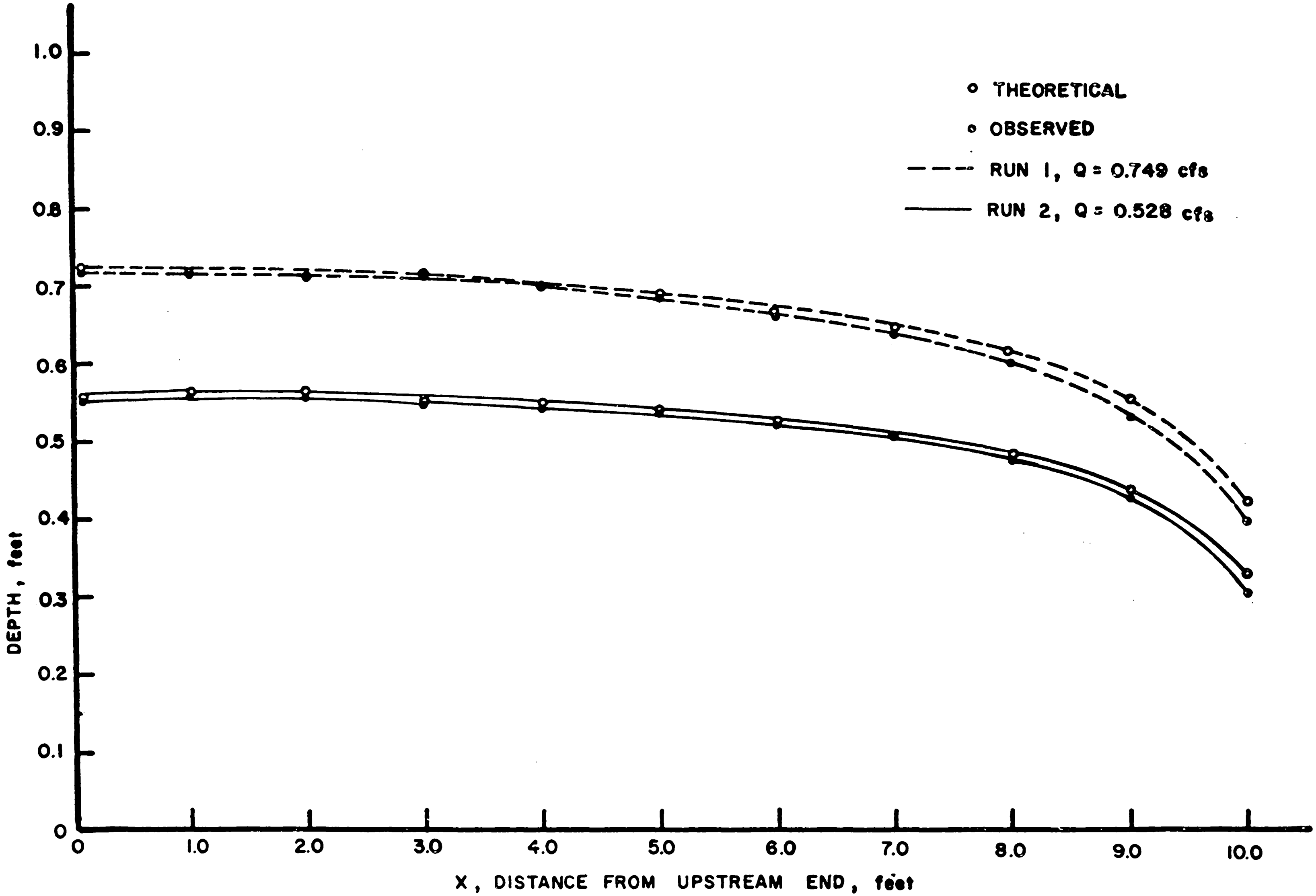

FIGURE 10. DEPTH OF FLOW IN A LEVEL RECTANGULAR CHANNEL

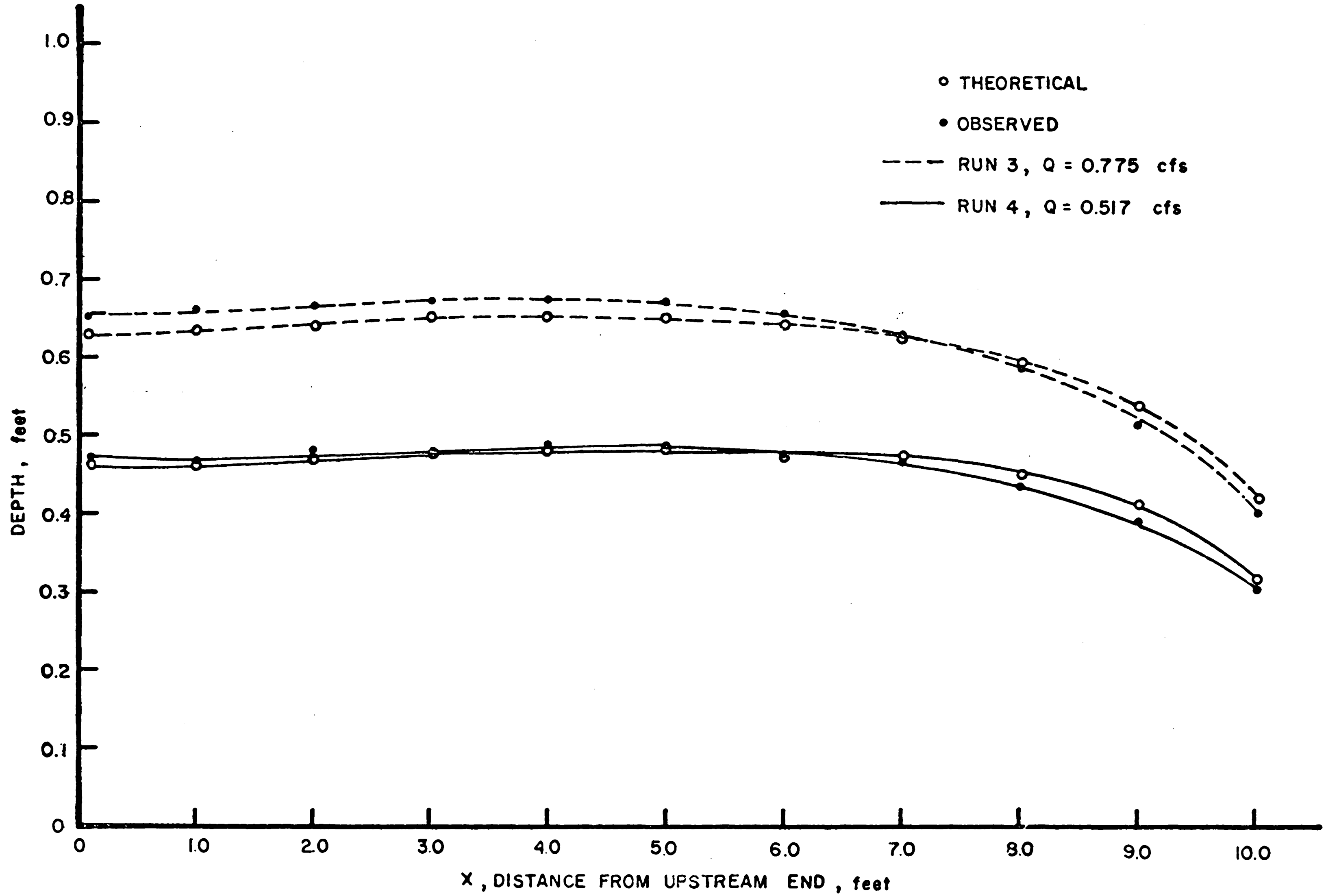

FIGURE 11. DEPTH OF FLOW IN A RECTANGULAR CHANNEL WITH A BOTTOM SLOPE OF 1.0 %

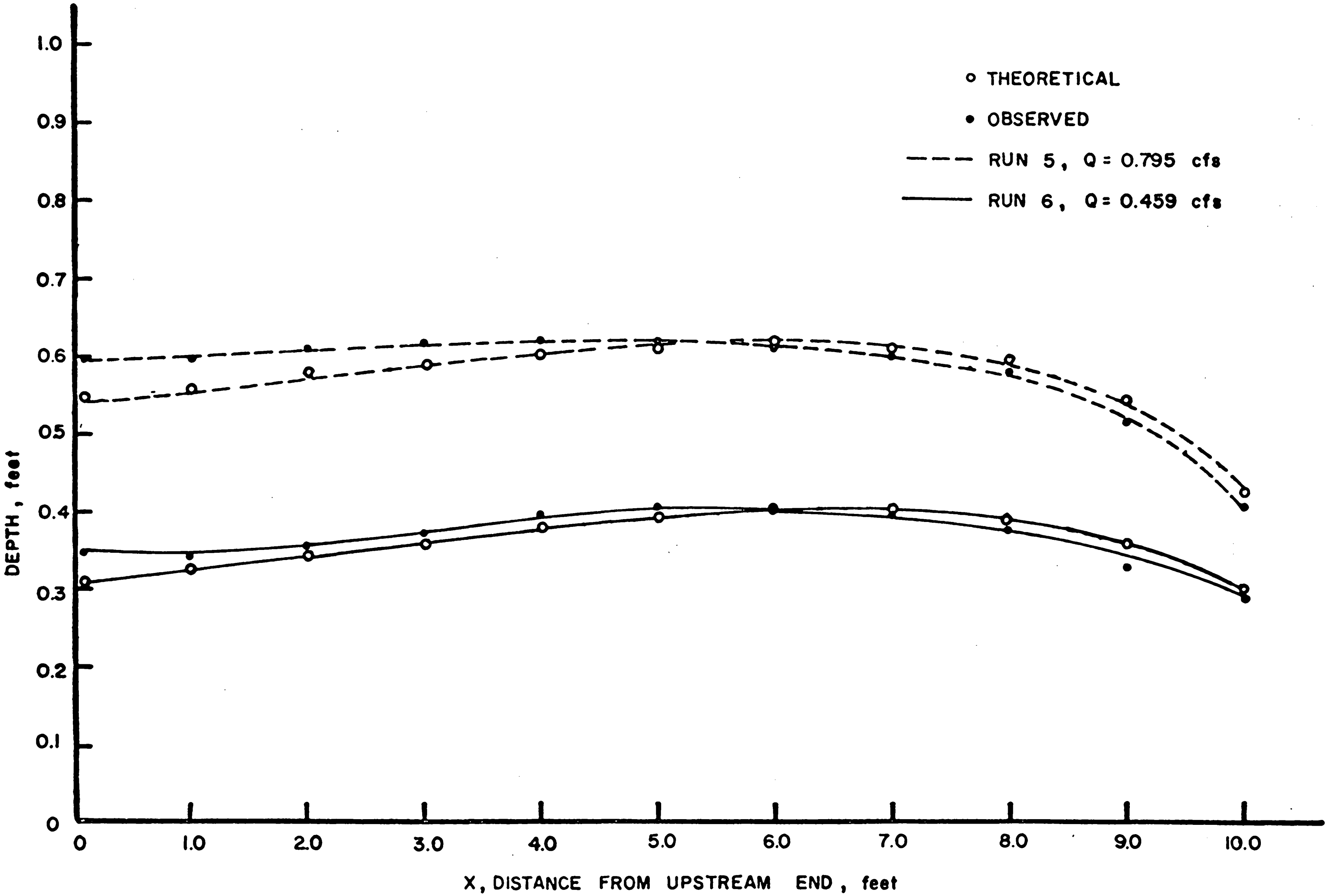

FIGURE 12. DEPTH OF FLOW IN A RECTANGULAR CHANNEL WITH A BOTTOM SLOPE OF 2.0 %

VI. DISCUSSION

The comparisons made in Figures 10, 11, and 12 show the observed depths to be less than the theoretical depths in every case. This difference is negligible for the case of zero slope. For the cases of one and two percent slopes, however, the difference is not negligible, and is probably due to the following factors:

1. The comparison of observed and theoretical inflow, in Figure 9, shows the observed inflow to be considerably less than the theoretical uniform inflow. This deficiency alone could cause the low profile obtained at the higher slopes. The good results obtained at zero slope, however, indicate that there are other sources of error.

2. In the theoretical analysis, the inflow was assumed to be unidirectional and at right angles to the channel centerline. In cases of an "infinite" reservoir or where the contributing areas are very much larger than the channel itself, this assumption is a reasonable and valid one. However, in the case of the experimental apparatus, the limited areas of the side troughs probably cause the velocity component of the inflow in the downstream direction to be appreciably high. This velocity component could conceivably have a "helping" effect on the flow in the channel, which outweighs the energy loss due to impact.

3. The friction has been evaluated in the Manning Formula which was assumed to be applicable to spatially varied flow situations. This assumption may not be valid because of the turbulence due to the impact of the inflow.

VII. SUMMARY AND CONCLUSIONS

A differential equation based on the principle of conservation of momentum was developed which describes the water surface profile in a rectangular channel with spatially varied flow. This equation was altered so as to describe the surface profile in terms of dimensionless ratios. The nondimensional equation was solved on the IBM 7040 computer for various conditions of slope, surface roughness and quantity of flow. When slope and roughness were held constant, the solutions were the same for all values of the flow.

A rectangular, wooden channel with spatially varied flow was constructed in an attempt to verify the results of the nondimensional equation solutions. The channel was ten feet long, six inches wide and had a variable bottom slope. The water surface profile was recorded for two rates of flow at slopes of zero, one and two percent.

From the theoretical and experimental investigations conducted during this study, the following conclusions can be drawn:

1. For a given condition of slope and surface roughness, the dimensionless flow profile for spatially varied flow in a rectangular channel varies only slightly over a wide range of discharge.

2. The nondimensional flow profile offers a rapid, safe design for hydraulic structures involving open channels subject to conditions of spatially varied flow.

3. Frictional forces in spatially varied flow are distinct and significant according to the theoretical analysis and experiments conducted using a slope of zero. The theoretical analysis concerning slopes greater than zero also showed distinct and significant frictional forces. The experiments conducted for these slopes varied, however, over such a broad range as to be inconclusive with regard to frictional forces.

VIII. SUGGESTIONS FOR FURTHER STUDY

It is suggested that further experimentation be performed utilizing slopes greater than zero and cross-sectional shapes other than rectangular.

It is strongly urged that data be taken from existing structures and that profile and flow measuring devices be incorporated in the design of new structures expected to have spatially varied flow.

Work should also be done with the purpose of developing an easy method for locating the critical section in cases of spatially varied flow in channels with steep slopes and submerged outlets.

A study should also be made on the evaluation of frictional forces in channels where spatially varied flow occurs. A possible solution might be found in the determination of a set of computed Manning numbers which can be applied to situations where turbulence occurs in a smooth channel with subcritical flow.

IX. ACKNOWLEDGEMENTS

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XII. APPENDIX A: EXAMPLE PROBLEM

Plot the flow profile and determine the channel depth required for a wash water trough that must carry 100 cubic feet per second. The channel is to be concrete with a Manning roughness coefficient of 0.015. The width will be 3.0 feet and the length is to be 80.0 feet. The bottom will be horizontal. Include a six inch freeboard in the required depth.

Solution:

$$\text{Critical Depth} = \sqrt[3]{q^2 / g} = \sqrt[3]{(100 / 3)^2 / 32.2}$$

$$= 3.25 \text{ feet}$$

x (distance from up-stream end, feet)	x/L	D/D_o (from Fig. 2)	D (feet)
0	0.000	1.770	5.76
10	0.125	1.762	5.73
20	0.250	1.750	5.69
30	0.375	1.725	5.62
40	0.500	1.683	5.48
50	0.625	1.624	5.28
60	0.750	1.540	5.01
70	0.875	1.422	4.63
80	1.000	1.000	3.25

Required Depth = 5.76 + 0.50 = 6.26 feet

For channels with sloping inverts it must be remembered that D is the elevation of the water surface above the outfall invert. When the actual depth at any point is desired, the product of the bottom slope and the distance from the outfall to that point must be subtracted from the calculated D.

XIII. APPENDIX B: DATA

TABLE 1. DATA FOR INFLOW UNIFORMITY CHECK

Measuring Container Width = 0.538 feet   Discharge = Weight / (Time (w) 0.538)

Run No. 1. Total Discharge = 0.523 cfs

Weir No. 1	X = 0	X = 1	X = 2	X = 3	X = 4	X = 5	X = 6	X = 7	X = 8	X = 9	X = 10
Weight, lbs.	8.4	8.0	7.3	8.4	7.9	7.5	8.2	8.1	7.9	7.9	8.1
Time, sec.	10.6	8.8	8.1	9.3	8.5	8.3	8.5	8.6	8.3	8.5	8.8
Discharge, cfs/ft.	0.024	0.027	0.027	0.027	0.028	0.027	0.029	0.028	0.029	0.028	0.027
Weir No. 2											
Weight, lbs.	8.2	9.5	8.2	8.1	7.8	8.1	8.2	8.3	8.0	7.7	7.8
Time, sec.	10.8	9.5	9.5	9.4	8.4	8.7	9.0	9.2	8.6	8.2	8.5
Discharge, cfs/ft.	0.023	0.030	0.026	0.026	0.028	0.028	0.027	0.027	0.028	0.028	0.027
Total, Run No. 1.	0.047	0.057	0.053	0.053	0.056	0.055	0.056	0.055	0.057	0.056	0.054

Run No. 2. Total Discharge = 0.832 cfs.

Weir No. 1	X = 0	X = 1	X = 2	X = 3	X = 4	X = 5	X = 6	X = 7	X = 8	X = 9	X = 10
Weight, lbs.	8.5	8.5	8.1	8.5	8.9	8.5	8.3	8.6	8.5	8.5	8.7
Time, sec.	7.4	7.0	5.9	5.8	6.0	5.5	5.2	5.7	5.4	5.2	5.0
Discharge, cfs/ft.	0.034	0.036	0.041	0.044	0.044	0.046	0.048	0.045	0.047	0.049	0.052
Weir No. 2											
Weight, lbs.	8.2	7.9	8.3	7.8	8.1	8.7	8.2	8.8	8.3	8.6	8.2
Time, sec.	7.8	7.0	7.0	6.0	6.2	6.6	5.7	6.2	5.8	5.8	5.8
Discharge, cfs/ft.	0.031	0.034	0.035	0.039	0.039	0.039	0.043	0.042	0.043	0.044	0.042
Total, Run No. 2.	0.075	0.070	0.076	0.083	0.083	0.085	0.091	0.087	0.090	0.093	0.094

TABLE 2. FLOW PROFILES FOR A RECTANGULAR CHANNEL WITH SPATIALLY VARIED FLOW

Run No. 1 Total Discharge = 0.528 cfs Slope = 0.0 Depth in hundredths of a foot.

Reading No.	X = 0	X = 1	X = 2	X = 3	X = 4	X = 5	X = 6	X = 7	X = 8	X = 9	X = 10
1	55	56	55	54	53	53	51	51	47	43	30
2	55	55	55	55	54	53	51	50	46	43	30
3	55	55	55	55	53	53	51	49	46	43	30
4	55	57	56	55	54	53	51	50	47	43	30
5	55	56	56	55	54	53	51	50	47	43	30
6	55	56	55	55	54	53	51	50	47	45	30
7	55	55	56	55	55	53	51	51	47	43	30
8	55	56	55	55	55	53	52	50	47	42	30
9	55	55	55	54	53	53	51	50	46	43	30
10	55	55	55	55	55	53	51	49	47	43	30
Averages	55.0	55.6	55.3	54.8	53.9	53.0	51.1	50.0	46.7	42.9	30.0

Run No. 2. Total Discharge = 0.749 cfs Slope = 0.0

Reading No.	X = 0	X = 1	X = 2	X = 3	X = 4	X = 5	X = 6	X = 7	X = 8	X = 9	X = 10
1	71	71	71	70	69	68	66	65	59	52	39
2	71	71	71	70	70	69	66	65	60	53	39
3	71	71	71	71	70	68	65	64	60	53	39
4	71	71	71	71	70	68	66	64	60	53	39
5	71	71	71	71	70	68	65	64	60	53	39
6	71	71	71	71	69	68	66	65	60	53	39
7	71	71	71	71	70	69	66	65	60	53	39
8	70	71	71	70	70	68	65	64	60	53	39
9	71	71	72	71	70	69	66	64	60	53	39
10	71	71	71	71	69	68	66	65	60	53	39
Averages	70.9	71.0	71.1	70.7	69.7	68.7	65.7	64.6	59.9	52.9	39.0

Run No. 3. Total Discharge = 0.517 cfs Slope = 1.0

Reading No.	X = 0	X = 1	X = 2	X = 3	X = 4	X = 5	X = 6	X = 7	X = 8	X = 9	X = 10
1	46	48	48	48	49	49	48	46	43	39	30
2	47	47	48	48	49	49	47	46	44	39	30
3	48	48	48	49	49	49	48	46	44	39	30
4	47	48	48	48	49	49	48	46	44	38	30
5	48	47	50	49	49	49	48	46	44	39	30
6	47	47	49	48	49	49	47	46	44	38	30
7	48	47	48	48	49	49	48	46	43	39	30
8	48	47	49	49	49	49	47	47	44	39	30
9	46	49	48	49	49	49	48	47	44	39	30
10	48	47	49	49	49	49	48	47	44	39	30
Averages	47.3	47.2	48.5	48.5	49.0	49.0	47.7	46.3	43.8	39.2	30.0

Run No. 4. Total Discharge = 0.775 cfs Slope = 1.0

Reading No.	X = 0	X = 1	X = 2	X = 3	X = 4	X = 5	X = 6	X = 7	X = 8	X = 9	X = 10
1	65	67	67	68	68	67	66	63	59	52	40
2	65	66	66	67	67	67	66	63	59	52	40
3	65	66	67	67	68	68	66	63	59	52	40
4	65	66	66	66	67	67	66	62	59	52	40
5	65	65	66	67	68	69	66	63	59	52	40
6	65	66	66	67	67	68	65	63	59	52	40
7	65	66	66	66	67	67	66	63	59	53	40
8	65	66	67	66	66	66	65	62	59	52	40
9	65	65	66	66	67	66	65	62	59	52	40
10	65	66	66	66	67	67	65	62	59	52	40
Averages	65.0	65.9	66.3	66.6	67.2	67.2	65.6	62.6	59.0	52.0	40.0

Run No. 5. Total Discharge = 0.459 cfs Slope = 2.0

Reading No.	X = 0	X = 1	X = 2	X = 3	X = 4	X = 5	X = 6	X = 7	X = 8	X = 9	X = 10
1	34	33	35	36	39	40	40	39	37	33	28
2	34	33	36	36	39	40	40	39	37	34	28
3	34	34	36	36	39	40	39	39	37	33	28
4	34	34	36	37	39	40	39	39	37	33	28
5	35	34	35	37	39	40	39	39	37	34	28
6	34	34	35	36	39	40	39	39	37	33	28
7	34	34	36	37	39	40	39	39	37	33	28
8	34	34	36	37	40	41	39	39	38	34	28
9	34	34	36	36	39	40	40	39	37	34	28
10	34	34	35	37	40	40	40	39	37	34	28
Averages	34.1	33.8	35.6	36.5	39.2	40.1	39.4	39.0	37.1	33.5	28.0

Run No. 6. Total Discharge = 0.795 cfs Slope = 2.0

Reading No.	X = 0	X = 1	X = 2	X = 3	X = 4	X = 5	X = 6	X = 7	X = 8	X = 9	X = 10
1	59	60	60	61	62	62	61	59	57	50	40
2	60	60	61	61	62	62	61	59	57	51	40
3	59	59	61	62	62	62	61	59	57	50	40
4	59	60	61	62	62	63	62	59	57	51	40
5	59	59	61	62	62	62	61	59	57	51	40
6	59	60	61	62	62	63	62	59	57	51	40
7	60	60	61	62	63	62	62	60	57	51	40
8	59	60	61	62	62	62	62	60	57	51	40
9	59	60	60	61	62	63	61	60	57	51	40
10	59	60	61	61	62	62	61	59	57	51	40
Averages	59.2	59.8	60.8	61.6	62.1	62.3	61.4	59.3	57.0	50.8	40.0

XIV. APPENDIX C:

PROGRAM FOR IBM 7040 ELECTRONIC COMPUTER

```
$JOB    7        826140203 HUBBARD, L. D.
$IBJOB
$IBFTC D1
    1 READ 5,DIS,WIDTH,BSLOPE,QT,R,GD
C       DIS = TOTAL LENGTH OF THE CHANNEL
C       WIDTH = CHANNEL WIDTH
C       BSLOPE = SLOPE OF THE CHANNEL BOTTOM
C       QT = UNIFORM INFLOW PER FOOT OF CHANNEL
C       R = MANNING ROUGHNESS COEFFICIENT
C       GD = NUMBER OF CHANNEL LENGTH INCREMENTS TAKEN
    5 FORMAT(1X,6F9.4)
      DIMENSION DV(30),D(30),DDELT(30),CHECK(30),DQ(3
C       DV = VELOCITY AT POINT OF CONSIDERATION DIVIDED
C       VELOCITY
C       D = DEPTH AT POINT OF CONSIDERATION DIVIDED BY
C       DDELT = CHANGE IN DEPTH BETWEEN SECTIONS UNDER
C       DIVIDED BY THE CRITICAL DEPTH
C       CHECK = DDELT, USED IN TRIAL SOLUTION FOR DDELT
C       DQ = CHANGE IN FLOW BETWEEN SECTIONS UNDER CONS
C       BY THE FLOW AT THE CRITICAL SECTION
C       DEPTH = CRITICAL DEPTH
C       AREA = AREA OF CRITICAL SECTION
C       RAD = HYDRAULIC RADIUS OF CRITICAL SECTION DIVI
C       DEPTH
C       Q = DISCHARGE AT THE CRITICAL SECTION
```

```
C       V = CRITICAL VELOCITY
C       C2, C3, C5, =CONSTANT TERMS CALCULATED TO SHOI
        DEPTH=(((QT*DIS)/WIDTH)**2/32.2)**(0.33333333
        AREA=DEPTH*WIDTH
        RAD=AREA/(DEPTH*(2.*DEPTH+WIDTH))
        Q=QT*DIS
        V=Q/AREA
        C2=(R*R*V*V)/(2.22*DEPTH**1.333)
        C3=DIS/DEPTH
        C5=(BSLOPE/QD)*C3
        I=1
        DV(I)=1.0
        D(I)=1.0
        DDELT(I)=0.0
        DQ(I)=1.0
        K=QD
        DO20I=1,K
        J=I+1
        DDELT(J)=0.0
C       AFTER THE INITIAL CONDITIONS HAVE BEEN DEFINEC
C       ITERATIVE PROCEDURE TAKES PLACE.
C       A VAL1E OF DDELT IS ASSUMED AND THE DIMENSIONL
C       PUTED. THE VALUE OF CHECK IS THEN COMPUTED ANC
C       COMPARED. WHEN THE DIFFERENCE BETWEEN THEM IS
C       TOLERANCE, THAT VALUE IS PRINTED AND THE PROCE
C       THE NEXT LENGTH INCREMENT.
```

```
12 DDELT(J)=DDELT(J)+0.C01
   D(J)=D(I)+DDELT(J)
   DQ(J)=DQ(I)-(1./QD)
   DV(J)=DQ(J)/D(J)
   CHECK(J)=(DQ(I)*(DV(I)+DV(J))/(DQ(I)+DQ(J)))*((
  1/DQ(I))*(DQ(I)-DQ(J)))+C2*DV(J)**2/(D(J)**(1.33
  2-C5
   A=CHECK(J)-DDELT(J)
   IF(A-0.001)20,20,12
20 CONTINUE
   PRINT 25,DIS,WIDTH,SSLOPE,QT,R,QD
25 FORMAT(1H1,21X,86HDIMENSIONLESS WATER SURFACE P
  1Y VARIED FLOW IN A RECTANGULAR CHANNEL///33X,18
  2,F10.4, 6H  FEET/33X,17H CHANNEL WIDTH = ,F10.4
  3CHANNEL SLOPE = ,F10.4/33X,18H UNIFORM INFLOW =
  4C FEET PER SECOND PER FOOT OF CHANNEL/33X,33H M
  5EFFICIENT = ,F10.4/33X,36H THE CHANNEL LENGTH I
  6.4,2X,8HSECTIONS////20X,  7H DDELT ,5X, 7H DEPT
  75X,14HCRITICAL DEPTH,5X,17HCRITICAL VELOCITY/)
   I=1
   PRINT30,D(I),DV(I),DEPTH,V
30 FORMAT(30X,F10.4,5X,F10.4,5X,F10.4,5X,F10.4)
   DO40I=1,K
   J=I+1
   PRINT 35,DDELT(J),D(J),DV(J)
```

```
   35 FORMAT(15X,F10.4,5X,F10.4,5X,F10.4)
   40 CONTINUE
      GO TO 1
   80 STOP
      END
$ENTRY D1
$IBSYS
```

Abstract

A dimensionless equation is developed which describes the flow profile in rectangular channels with spatially varied flow. This equation is solved for various slopes and rates of discharge.

The results show that when the slope and roughness are constant the dimensionless profiles are also constant over a very wide range of discharge. Once the dimensionless profile is established the water surface curve may be rapidly and accurately determined.

Tests were conducted in the laboratory which reasonably verified the validity of the dimensionless profiles.