A LINEAR PROGRAMMING APPROACH

TO

EVALUATING FOREST MANAGEMENT ALTERNATIVES

by

William Edward Kidd, Jr.

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TABLE OF CONTENTS

Tables

Figures

Appendix Tables

Appendix
Table

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INTRODUCTION

Forest managers are faced with the responsibility of making decisions concerning the allocation of the resources at their disposal in such a manner as to maximize specific objectives. These managers, in many instances, are charged with the responsibility of a large capital investment. Fedkiw (1960:2), in emphasizing the capitalistic nature of forestry, gave the example of a 50,000 acre regulated forest enterprise valued at $11,000,000. He assumed twenty full-time employees, making the capital investment per employee approximately $550,000. This compares with capital investments of from $7,000 to $98,000 per employee in other enterprises. For example, the ratio of capital investments per employee is $7,000 for aircraft companies. For telephone systems the ratio is $32,000 and for gas and electric utilities $98,000 (Fedkiw:2).

The successful operation of a forest enterprise is dependent on the use of the available land, labor and capital among alternative investment opportunities in such a manner that each contributes to the objectives of the enterprise. If the stated objective is profit maximization, then each should theoretically contribute to that point at which marginal cost of the resource is equal to marginal revenue received from the use of that resource.

The opportunities presented by forest investment, from regeneration to final harvest, are both numerous and varied. These opportunities range in complexity from rather simple, single treatments to multiple treatments occurring at different times. In addition to the time span of production, the heterogeneous character of the forest greatly compounds the complexity of decisions which the forest manager must make. These opportunities compete for resources not only now but also in the future. Therefore, a method which will allow the examination of a large number of management opportunities in defining an optimum program would be a valuable addition to the forest manager's decision making process. The defined program must be optimum relative to competition for scarce resources both now and in the future as well as fulfill the objectives of management.

This study is concerned with the problems of determining an optimum combination of timber harvest alternatives in such a manner as to maximize objectives, subject to specific constraints. Thus, it is an economic problem and can be examined within the framework of economic theory. It is intended that an optimum plan will be defined that will use available resources in such a manner as to maximize total present worth of a specific forest enterprise.

Objective

The objective of this study is to examine the methodology and the appropriateness of adapting the linear programming model to the scheduling of timber harvest alternatives available on a specific forest enterprise.

Literature Review

Until recently, little attention has been given to the application of linear programming to resource allocation problems of the forest enterprise. Also application, in general, has not been directed to maximizing profit but to maximizing some physical objective.

The literature does not presently contain results of directly comparable studies. Curtis (1962) showed how a linear programming model could be used to maximize wood or dollar yield from a leased property. However, he first determined optimum wood yields relative to compartment scheduling, then simply summed the present dollar value of the acres scheduled.

Coutu and Ellertson (1960) have illustrated a linear programming approach for integrating farm woodlots with the total farm enterprise. The use of linear programming in this manner has been questioned by Stoltenberg and Thomson (1962). Their concern has, in turn, been discussed by Chappelle (1963), Schultz (1963), and Thompson (1963).

Loucks (1964) demonstrated the use of linear programming in the development of an optimal program for sustained yield management. However, he assumed the rationale of volume maximization as the objective. This objective is economically valid only if fixed resources are cost free, or if maximum profit is irrelevant.

Leak (1964) suggested the use of linear programming in the area of forest regulation. He did not, however, demonstrate how the model might be adopted for this application.

The application of linear programming in other types of industry has been more intense than in the forest industry. For example, it has been used to describe the optimum use of resources among the various alternatives available to the farm enterprise (Heady 1963:1). It has also been used in scheduling petroleum refinery operations (Manne 1956), and in determining the optimum combination of fixed machine capacity in other industries.

Characteristics of a Linear Programming Problem

Linear programming has been defined as, "the process of determining a program of activity by finding the optimum solution of a group of restrictive linear equations (Greenwald 1947:3)." Heady (1963:4) stated: "Linear programming generally refers to the computational method used in prescribing production patterns which maximize profits of firms, minimize costs of producing a specific commodity, or related types of aggregative analyses."

According to Heady (1963:2), "A linear programming problem has three quantifiable components: an objective, alternative methods or processes for attaining the objective, and resource restrictions. A problem which has these three components can always be expressed as a linear programming problem." The objective may be one of profit maximization; however, this objective can be modified by certain constraints. Forestry examples of such constraints are maximum allowable cut or minimum area cut (Loucks 1964) and maximum allowable yield (Leak 1964).

Alternative processes can be illustrated by the different opportunities which are present in the management of a forest. For example, a twenty-five year old stand of loblolly pine could be clear-cut and replanted. Another opportunity for the same stand might be to thin now, thin ten years from now, and clear-cut in twenty years. Obviously, other opportunities are present for the same stand.

Resource restrictions refer to the quantity of resources, i.e., land, labor, and capital, available to the management of the enterprise. For example, the resources available on a forest property might be: 20,000 acres of land, 7,000 man hours of labor, 2 log trucks, 1 rubber tired skidder, and a working capital fund of $5,000. In addition to the limitations imposed by resource availability, personal management constraints can restrict the objective of the firm. Examples of personal management constraints are: the maintenance of a share of the market, allowable cut, and a given level of protection from fire, insects, and disease.

APPLICABILITY OF LINEAR PROGRAMMING FOR FOREST RESOURCE ALLOCATION

When considering the application of a recently developed model or method of analysis, the question arises as to whether or not the method is justifiable in view of currently available models. The purpose of this section is to review some of the economic resource allocation models currently available to the forest manager.

One question to be answered in this review is the compatibility of the various models. Another is, of the several possible models which is most applicable to the complex problems presented by the forest firm.

General Review

Pertinent theory is developed, describing models with which the forest manager can evaluate alternatives relative to profit maximization or cost minimization. Marginal analysis, if provided an adequate description of the rate of change of output to input, is capable of describing a situation in which profit will be maximized. Marginal theory is adaptable to the type of continuous production function which is generated by the growth of individual trees, and forest stands. It describes the optimum point of production in terms of maximum net revenue which is commonly referred to in forestry literature as maximum soil expectation value. This type of activity analysis however, assumes that resources will be plentiful and will be available.

Methods of examining the enterprise by ranking the alternatives according to their relative efficiency in the use of capital are available. Fedkiw (1960) presented capital budgeting as a method of allocating capital in the forest enterprise. This is a means of viewing alternatives prior to capital allocation. Management decision determines the actual allocation of capital among the various opportunities.

Budgeting in the forest enterprise has been presented by Barraclough and Gould (1955). Budgeting is process analysis that distributes scarce resources among alternative activities to maximize an objective statement. Budgeting assumes linear relationships and a fixed level of technology.

Linear programming has been used extensively in describing optimum conditions for farming enterprises. It is a method of analysis which allocates scarce resources among alternative processes in order to maximize or minimize some objective statement. Linear programming assumes linear relationships.

Comparison of the Models

It is important to know whether or not we are dealing with models that will provide identical results when applied to a given problem. Thus, the question arises regarding the compatibility of the various models. The answer to the question requires an examination of the relationship of linear programming to the traditional marginal concepts, to capital budgeting, and to budgeting. If linear programming is compatible with each of these other allocation models, then which is the most applicable in a complex situation.

Linear Programming and Marginal Analysis

The relationship between linear programming and marginal analysis has been examined by Dorfman (1951), McConkle and Boles (1954), Smith (1956), Dorfman, et.al. (1958), and Gilson (1960). In each case it has been concluded that linear programming and marginal analysis are compatible in basic theory. According to Dorfman, et.al. (1958:133, 141):

> It would be misleading to contrast the linear programming model with marginal analysis in general. Linear programming is marginal analysis, appropriately tailored to the case of a finite number of activities. 'Traditional' marginal analysis is tailored to the case of a differentiable production function....The point of view, as compared with that of the traditional smooth marginal analysis, involves a shift in the focus of attention. Instead of seeking the optimal combination of inputs and outputs we seek the optimal combination of levels of activities....Putting it roughly, the choice is not among various time rates of input and output, but more directly, among different ways of doing things, each of which implies its own characteristic pattern of input and output rates.

Other authors (McConkle, et.al. 1954; Smith. 1956; Gilson. 1960) follow this same line of reasoning. Thus, it seems that in theory marginal analysis and linear programming are compatible and will, under the same assumptions and constraints, describe the same optimal combination of activities.

It now becomes a question of the applicability of the two models in a complex situation, and perhaps more important, which of the two is most appropriate with a given type of input-output data. Appropriateness in this instance refers to which of the two models is more accessable to the information system, and which is more efficient in the use of time. Of the two techniques, which one can most conveniently handle the personal management constraints imposed within the time allowed for the analysis.

Linear programming is an efficient mathematical model for defining a program which will maximize profit subject to resource restrictions and personal management constraints. In the case of marginal analysis, once personal management constraints are imposed upon the profit maximizing objective, it may become a laborious task to select a combination of activities which will meet the objective. Wu, et.al. (1960:156-157) have examined both techniques of analysis using first a single product firm and then a multiple product firm with multiple constraints.

> In the case of a single-product firm it has been shown that the conventional average variable cost curve and marginal cost curve can be readily constructed from the programming models....When we move to the multiple-product firms with multiple constraints the derivation of the cost curves becomes complicated and cumbersome. It is much easier to construct the supply schedule of the firm by utilizing the concepts of mathematical programming....Our analysis has therefore demonstrated that in all these cases, the familiar concepts of the marginal analysis can be applied to indicate the logical properties of the optimum solution. On the other hand, the traditional tools of marginal analysis may fail to provide a convenient method of arriving at the optimum solution. The availability of techniques for the numerical solution of economic and business problems is indeed the basis of the importance of mathematical programming.

With the linear programming model, the simplex algorithm allows the examination of one activity at a time, determining if this activity will increase profit relative to all constraints imposed. An analysis with this model is limited also in the number of activities and resources or other constraints, if the calculation are performed by hand. However, with the advent of high speed computers, and the programming of the simplex algorithm, this limitation is removed, and the size of the problem is subjected to the ability of the human mind to comprehend the solutions presented.

There remains then the problem of adequate information. Boulding (1960:9) stated:

> Although the basic principal behind linear programming and the marginal analysis is essentially the same, the greater value of the linear technique lies in the fact that the basic limiting inequalities, as well as the utility or maximand function, may be more accessible to the information system.

Finally Dorfman (1951:80) stated:

> In linear programming to a greater extent than in marginal analysis, there is a conscious attempt to state the economic problem in an operationally meaningful way, that is, to work with concepts which correspond to measurable phenomena and to state the problems in terms of the variables with which businessmen and other economic policy-makers actually operate.

Linear Programming and Capital Budgeting

Capital budgeting is that planning method that deals with determining the firm's demand for capital, the firm's supply of capital, and the ultimate allocation of the supply to meet the needs (Dean 1964). It assumes that all other resources will be available in quantities required, capital being the limiting resource. The method requires a determination of the marginal efficiency of capital in use in specific processes, the processes being ranked from the highest to the lowest by rate of return, thus forming a demand schedule for capital.

Linear programming is traditionally a short-run analysis, in which the factors of production are limited. It is activity analysis concerned with fixed levels of technology. Heady (1963:4) stated:

> Linear programming is an efficient way of determining optimum plans only if there are numerous enterprises or processes, and numerous restrictions in attaining a specific objective such as maximizing farm profits, or minimizing production costs. If there are 100 possible enterprises, but only one limiting resource or restriction, the optimum plan will contain only one process."

Thus, capital budgeting with one restricting resource could not be efficiently programmed in the linear programming sense. If one were to program the capital budgeting model, the only activity in the optimum program would be that which provides the greatest return. However, there are some limitations imposed even on the capital budgeting model. The limitations lie in the physical quantity of a particular opportunity and the quantity of capital available. If the restrictions can be defined, then the model is programmable. However, linear programming is not necessary. The activities have already been defined, and the relative productivity of capital determined. It becomes a matter for the decision maker to select those activities of successively lower rates of return at the levels available so that the cumulative cost of those activities selected consumes the available capital supply.

Capital budgeting and linear programming then are not the same analytical technique. Capital budgeting is concerned primarily with the efficient use of one resource, capital. Linear programming, on the other hand, is concerned with the allocation of various scarce resources among alternative processes. It is convenient to consider personal management constraints with linear programming.

Linear Programming and Budgeting

The relationship of linear programming to budgeting has been examined by Heady (1956; 1963) and by Kottke (1961). Kottke (1961) demonstrated that budgeting and linear programming determine the same optimum program. It thus becomes a question of applicability of the two methods. Heady (1963:42-46) in examining the relationship between budgeting and linear programming stated:

> Budgeting and linear programming are not distinctly different procedures, but are the same procedure with differences allowed in (a) the number of opportunities considered, and (b) the calculations involved. Both methods employ the concepts of linearity....where the number of restricting resources and possible enterprises and techniques are large, and particularly if answers must be precise, the efficient procedure is linear programming.

Conclusions

It has been determined that linear programming, marginal analysis, and budgeting are compatible. It has been determined that capital budgeting and linear programming are not the same analytical technique, one dealing with the allocation of capital to ranked alternatives, one dealing with the allocation of several scarce resources among alternative processes. These conclusions were reached by reviewing the models and pointing out the differences between them. Further, it has been concluded that in a more complex situation the linear programming model, programmed for computer solution, is the more applicable method of analysis.

Basic Assumptions of Linear Programming

There are certain assumptions inherent in the use of linear programming that should be examined in the context of the forest enterprise. These assumptions: linearity, additivity, divisibility, and finiteness, are basic to the use of this technique. Heady (1963) examined these assumptions in a general sense and Coutu and Ellertson (1960:6) examined these assumptions relative to the farm-forest enterprise.

Linearity

A linear relationship is assumed between input and output. With a given set of resources, applied within the limits of a given technology, at a specific level, a specific quantity of output will be the result. The linear programming technique further assumes, regardless of the level of production, whether it be 1 or 100 units, that there is a directly proportional relationship between quantity of input and quantity of output. Linearity also assumes constant costs, that is, as the size of the unit treated increases, costs do not decrease as efficiency increases. Heady (1963:5) stated that, "It does not mean that non-linear relationships cannot be analyzed." It simply means that a point of production is selected from a curvilinear set of input-output responses on the same production surface within the economic range of production. Each of the points represents a single production process available to the enterprise, and a number of these processes for the same and different products represent the alternatives available to the enterprise for resource use. Dorfman, et.al. (1958:141) states that "Instead of seeking the optimal combination of inputs and outputs, we seek the optimal combinations of levels of activities."

In the forest enterprise, the concept of a process is a well known principle, although it goes by another term, specifically stand treatment. When the forest manager prescribes treatment for a stand, he is really describing the process of developing a product. For example, he might prescribe the following schedules for a 15 year old stand of loblolly pine:

Stand Treatment Schedule Number 1	Stand Treatment Schedule Number 2
Leave at 15 years	Prune to 16 feet, 120 crop trees at 15 years
Thin to 90 square feet at 25 years	Thin to 100 square feet at 25 years
Thin to 110 square feet at 35 years	Thin to 120 square feet at 35 years
Clearcut and plant at 45 years	Clearcut and plant at 45 years

These are two distinct processes for a given area for a given period of time. Each requires a different quantity of resources and each produces a different return in terms of present worth. These, and all other treatment processes described, form the opportunity schedule to which scarce resources of the forest enterprise can be allocated.

Additivity

Heady (1963:17) stated that, "The activities must be additive in the sense that when two or more are used, their total product must be the sum of their individual products." There can be no interaction between activities, they must be mutually exclusive. Swanson (1958) and Heady (1963) use the example of a crop rotation such as corn-oats-clover, where the fertilizer application for the corn part of the rotation might affect yields of the next crops. Heady (1963:17) stated, "Distinct combinations of grain and forage must be defined as different activities." In the forest enterprise, when describing a process, care must be taken to describe the complete treatment for a given stand in the one process. For example, the process of stand establishment cannot be a relevant process by itself if the area requires some type of site preparation before successful regeneration can be accomplished.

Divisibility

Heady (1963:18) stated:

> It is assumed that factors can be used and commodities can be produced in quantities which are fractional units....This assumption is not a serious limitation, since a program ordinarily can be rounded to include activities produced to the nearest whole unit without causing serious decision-making errors.

Divisibility might be illustrated by a resource of 30 acres of loblolly pine. The optimal program might indicate that only 29.7 acres should be cut. Obviously in installing this activity 30 acres will be treated.

Finiteness

Coutu, et.al. (1960:6) stated: "....the assumption of finiteness means simply that of all possible processes, only a relatively few are considered as alternatives." Heady (1963:18) said:

> It is assumed that there is a limit to the number of alternatives and resource restrictions which need be considered. If the farmer, businessman, or planner had an unlimited number of alternatives, he would never have them programmed, because he could never finish describing additional activities, nor could he ever finish the computational task of determining the most profitable program.

In the forest enterprise, the complexity of the situation is such that one could never describe them all. The best one can do is describe the relevant silviculturally feasible alternatives for the given stands which make up the forest.

In considering the basic assumptions of linear programming, relative to the forest enterprise, there appear to be no serious limitations which would restrict the use of this technique in defining the optimum combination of activities in a given forest enterprise. Thus, in the following sections, the forest enterprise used in this study will be described, and the specific linear programming model will be developed.

DESCRIPTION OF THE FOREST ENTERPRISE

Two approaches may be taken in presenting linear programming as an allocation model applicable to a forest enterprise. The first approach is the application of theory to a real situation with an empirical description; the second is the application of theory to a hypothetical situation. The former approach, with certain necessary adjustments, was selected for this study in the belief that the more realistic approach would be more acceptable to the practicing forester. This section will describe the Seward Forest; the resources available, the product outlet, and the management constraints. These data were developed in an extensive interview with the Director of the Seward Forest, and from the records kept on the operation of the Forest since 1935.

Seward Forest

The Seward Forest is located at Triplett, Virginia, in southeastern Brunswick County in the southside of Virginia. It is on the border dividing the Virginia Piedmont and Coastal Plain. The predominant forest type in the area is loblolly pine (*Pinus taeda* L.).

The Forest was acquired by the University of Virginia through the will of Dr. Walter M. Seward, whose expressed purpose was to leave an "instrumentality for instruction in the science of forestry and in the art of cutting, sawing, manufacturing and marketing the products of the forest." His will further specified that, "a modern and thoroughly equipped sawmill and planing mill, --- be installed....and operated under the direction of the School of Forestry of the University of Virginia --- (Seward 1927).'[1/]

Management Constraints

The following management constraints were determined to exist:

1. The land resource is not available for any use other than wood production.
2. The forest must provide an annual cut of not less than 750,000 board feet of lumbdr per year. No outside purchasing of logs is permitted.
3. All final harvest operations will be contracted.
4. All intermediate harvests will be accomplished by the woods crews employed by the Forest.
5. The objective of the forest manager is to maximize total present worth, within the limits of available fixed resources, subject to the above personal management constraints.

Resources

Land

The Forest consists of approximately 3,500 acres of land, all of which support some type of forest growth. Major timber types present are: pine, pine-hardwood, upland hardwood, and bottomland hardwood. Of the total forested area, detailed management data is available for

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[1/] From the will of Dr. Walter M. Seward 1927.

approximately 1,660 acres, or the part of the Forest consisting of pure pine stands. It is this part of the Forest with which the study is concerned (Table 1).

Site quality data is available which reflects the productive capacity of the land resource.[2/] There are 264 acres of site 70, 1,261 acres of site 80, and 139 acres of site 90 land included on the Forest.

Timber

The timber resource is composed of pure even-aged stands of loblolly pine. The relevant age classes considered in this study ranged from 25 to 75 years (Table 1). The stands are of varying densities, almost all having been thinned at least once, and many, three or four times.

Working Capital

Working capital is that capital available for normal operation of the forest. Such normal operation includes maintenance of operating equipment and roads, planting, and other cultural practices on the forest. It also includes the purchase of additional labor and equipment if needed. There is $33,000 per year available for working capital (Table 2).

2/ Site quality measured by the height of the dominant and co-dominant loblolly pine at 50 years.

Table 1. Land and timber[1/] resource by site quality and age class.

Site	Age	Area	Density	Average DBH	Volume per Acre	
	Years	Acres	Basal Area per Acre	Inches	Std Cords	MBF
	25	144	117	9.0	8.0	5.7
	35	25	108	7.6	18.0	4.8
70	45	78	98	10.7	22.3	9.7
	55	12	100	12.0	0.0	10.0
	75	5	120	10.0	3.0	8.0
	25	30	117	7.5	16.9	4.9
	35	212	113	11.2	3.3	11.2
80	45	474	117	11.0	5.5	11.8
	55	179	105	12.6	3.4	13.7
	65	136	82	13.5	0.8	10.3
	75	230	114	13.9	2.9	19.4
	25	16	145	8.0	30.0	3.0
	35	10	124	11.9	1.5	14.2
90	45	30	125	11.9	3.3	16.9
	55	32	80	12.0	5.0	8.0
	65	36	119	12.2	2.7	15.3
	75	15	116	14.3	0.3	16.6

1/ Even aged stands of loblolly pine.

Table 2. Resources other than land and timber

Resource	Quantity per Year	Unit
Labor	7,673	hours
Working Capital	33,000	dollars
Mill Capacity		
Minimum	750	MBF
Maximum	2,000	MBF
Capital Equipment		
Trucks	2,480	hours
Tractors	3,884	hours
Power saws	4,680	hours
Contract Logging Capital	11,000	dollars

Labor

Labor, in this instance, is the quantity of fixed labor available to the forest manager for woods operations. There is a woods crew on the forest which provides a total of 7,673 hours of labor per year (Table 2).

Capital Equipment

Capital equipment includes the quantity of trucks, tractors, and power saws, expressed in number of hours per year, presently included in the inventory of the Forest (Table 2). There are 2,480 hours of truck time, 3,884 hours of tractor time, and 4,680 hours of power saw time available per year (Table 2).

Contract Logging Capital

Contract logging capital is that capital which, by prior allocation, is assigned to final harvest activities. It is restricted to this use. There is $11,000 per year available to the forest manager for contract logging capital (Table 2).

Sawmill

Included within the resources of the forest enterprise is a small sawmill complete with planing mill. The mill is the conversion and marketing facility for the products of the forest. The market is almost entirely a local one and as previously indicated, amounts to a minimum of 750,000 board feet per year. In order that the local market be maintained, the output of the forest must reach a minimum of 750,000 board feet per year. Maximum allowable output, which is limited by mill size and by working force, is 2,000,000 board feet per year.

PROCEDURES

There are specific procedures to be followed in developing a linear programming model for a forest enterprise. First, the alternatives for resource use must be described. Second in this analysis the response of the stand to treatment must be defined. Third, the technology of the particular enterprise must be determined so that resource requirements necessary to install each of the alternatives can be described. Fourth, the present worth of each alternative must be determined. Finally, a model must be defined which includes the resource and personal management constraints. In this section each of these steps will be considered in describing the linear programming model for the Seward Forest.

Developing Alternatives

When developing timber harvest alternatives for resource use, the forest manager must specify the period of analysis. He must be aware of the limitations imposed by the capacity of the equipment which will perform the calculations. He also must be aware of the time jointness of production which is a characteristic of the development of forest stands.

Period of Analysis

The study examined timber harvest resource use for the next 50 years. Fifty years was chosen simply to insure that each stand would be clear-cut during the analysis. The 50-year period was divided into five 10-year periods.

Computer Limitations

In this study, the number of alternatives examined for resource use was limited by the computer available for the calculations. The machine, an IBM 1620(60K) digital computer, had a total usable capacity of 5600 cells. The program model was limited to a matrix of rows and columns which could not exceed the 5600 cells. This limitation demanded that thoughtful consideration be given to the definition of opportunities in order that only those alternatives would be selected which were most pertinent to the problem.

Time Jointness of Production

Time jointness of production is defined as that time relationship which exists between the application of a resource or group of resources and the resulting output. It is especially important that this relationship be recognized in evaluating alternatives of the forest enterprise, since a continuous input over a long period of time is required to attain a given output. It was recognized that the application of land, labor, and capital now and/or in the future will affect the ultimate yield from a given stand of timber.

The Thinning and Harvesting Alternatives

The alternatives were selected only from those concerned with either thinning or final harvest in the various stands. Other management opportunities are equally applicable and could have been used in combination with the harvest alternatives.

The following method of selecting the pertinent alternatives was used. First, three basic opportunities were recognized for each age class. These three basic alternatives were to leave, thin, or clear. From these were developed succeeding alternatives which were a combination of leaving, thinning, and/or clearing in the various time periods. As a result of this description, alternatives were defined of varying rotation length for stands of each age class (Figure 1). The exception to this method occurred in the case of stands 55 years of age or older. The only alternative described for these older stands was to clear immediately within the initial time period. Seventy-eight alternatives for resource use were developed in this manner (Appendix Table 1).

Response of Forest Stands

Given a specific treatment schedule, the response of a forest stand as the schedule is implemented, must be evaluated. Growth and

Figure 1. Example of defining the alternatives

Present stand age (years)	Basic alternatives(t)	Succeeding alternatives	(t_{10}, t_{20})
		→ CLEAR 55	
	THIN 45	→ THIN 55	→ CLEAR 65
	OR	CLEAR 55	
45			
	LEAVE 45	→ LEAVE 55	→ CLEAR 65
	OR		
	CLEAR 45		

yield data contained in Schumacher and Coile's "Growth and Yield of Natural Stands of the Southern Pines" was used to predict stand development. These data were used because they were considered the best available for predicting both sawtimber and pulpwood volume estimates. Silviculturally feasible stocking levels were specified for stands in which thinning was the treatment scheduled. The stocking levels by site quality are:

Site 70 - - Thin to 80 square feet of basal area per acre.

Site 80 - - Thin to 90 square feet of basal area per acre.

Site 90 - - Thin to 100 square feet of basal area per acre.

The method presented by Schumacher and Coile (1960:18) for predicting stand development, given initial stocking data relative to site, age, number of trees per acre, and basal area per acre, was used.

An assumption, necessary in order to use Schumacher and Coile's method of stand prediction, is that thinnings are distributed proportionately throughout all diameter-height classes. In other words, the average diameter, cubic foot volume per tree, and average board foot-cubic foot ratio remained unchanged after thinning. This assumption is something less than desirable. With present day thinning practices, the several parameters which describe the stand will not remain constant. However, there were no data available by which this change could be predicted. Therefore, this assumption was adopted. In this study, it was further assumed that thinned stands would develop over time as do natural stands of the same density. This assumption was made because the yield tables used do not reflect the growth of thinned stands.

Subsequent Rotations

The linear programming model for the Seward Forest will be required to examine the alternatives valued not only for this rotation but also for subsequent rotations. It was assumed that the initial stocking of subsequent stands would be well stocked (Schumacher and Coile 1960:6). Using the basic alternatives of leave, thin, and clear, the development of subsequent stands was defined (Table 3).[3/]

Resource Requirements

Enumerating resource requirements and product yield in both physical and value terms is required for each alternative. Describing resource use requires that certain production relationships peculiar to the enterprise under analysis be defined. The relationships depend upon the technology adopted by the firm and the efficiency level at which this technology is applied.

[3/] Subsequent rotations were based on maximum soil rent; see page 37.

Table 3. Subsequent rotations by site and by alternative rate

Alternative rate	Site index	Treatment schedule years
	70	Clear at 30 years
3%	80	Clear at 30 years
	90	Thin at 20, Thin at 30, Clear at 40
	70	Clear at 30 years
6%	80	Clear at 30 years
	90	Thin at 20, Clear at 30
	70	
10%	80	Clear at 20 years
	90	

Production Relationships[4/]

Labor requirements per standard cord of pine pulpwood produced from the Seward Forest and delivered to a pulpwood buying yard was 4 man hours. Labor requirements per thousand board feet of sawlogs produced and delivered to the mill location of the Seward Forest was 12 man hours (Table 4).

Truck time per standard cord of wood produced by the woods crew and delivered to a pulpwood buying yard was 1.52 hours. For each thousand board feet of sawlogs, 0.65 hours of truck time was required for delivery to the Seward Forest mill (Table 4).

Tractor time for skidding, per standard cord, was 1.50 hours, and for sawlogs, 1.00 hours per thousand board feet (Table 4).

Power saw time required to fell, limb, and buck a standard cord of pulpwood was 2.60 hours. To fell, limb, and buck a thousand board feet of logs, power saw time required was 1.30 hours (Table 4).

Once the physical production relationships were determined, they were converted to value terms (Table 5).

Product Values

The principal products grown on the Forest are sawtimber and pulpwood stumpage. The gross value of sawtimber produced by either the

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4/ Production relationships were obtained from the Director of the Seward Forest.

Table 4. Production relationships, Seward Forest[1/]

Production factor	Hours required per: Standard cord	MBF
Labor	4.00	12.00
Trucks	1.52	00.67
Tractors	1.50	1.00
Power Saws	2.60	1.30

1/ Obtained from records of the Seward Forest.

Table 5. Fixed and variable costs per unit - Seward Forest[1/]

Production factor	Unit	Unit costs Variable	Fixed	Total
		Dollars	Dollars	Dollars
Labor	Cord	0.00	4.80	4.80
	MBF	0.00	14.80	14.80
Truck	Cord	1.27	0.59	1.86
	MBF	.60	0.23	.83
Tractor	Cord	.83	0.68	1.41
	MBF	.55	0.39	.94
Power saw	Cord	.41	0.75	1.16
	MBF	.38	0.38	.76

1/ See Appendix Table 2 for development.

woods crew or by the contract logger, delivered to the Seward Forest mill, was $52.00 per thousand board feet of logs. The contract logger was paid $20.00 per thousand board feet for all sawlogs delivered to the mill, this being the cost per thousand to the Seward Forest for final harvest. Stumpage value then was $32.00 per thousand board feet.

The contract logger paid the forest enterprise $7.00 stumpage for every standard cord of pulpwood cut from the area upon which he was working under contract. This value became a receipt to the Forest. Pulpwood produced by the woods crew, delivered to a pulpwood buying yard returned a gross value of $15.75 per standard cord to the Forest (Table 6).

Budgeting the Alternatives

After defining the input-output relationships peculiar to the Seward Forest, the next step in developing this study involved a quantitative description of resource requirements and product yields for each of the alternative processes.

All alternatives in the study required an application of resources in the initial time period and a continuing application of resources in each of the four succeeding time periods. It was necessary that the required quantities of land, labor, and capital, including capital equipment, be defined for each of the alternatives included in the study for each of the five time periods. It was also necessary to determine costs and revenues for each alternative in each time period.

Table 6. Prices received for products grown on the Seward Forest, by type of producer

Producer	Price per MBF sawtimber	Price per Standard cord pulpwood
	Dollars	Dollars
Contract logger	52.00[1/]	7.00[3/]
Woods crew	52.00[2/]	15.75[4/]

1/ Delivered to Seward Forest mill, stumpage value = $52.00 - $20.00 logging cost = $32.00 per thousand.

2/ Gross value at the mill of the Seward Forest.

3/ Pulpwood stumpage.

4/ Gross value at the pulpwood buying yard.

Thus, each alternative was examined relative to yield and resources required in the initial and in all subsequent time periods.

For example, the alternative on site 70 of: THIN 35, THIN 45, THIN 55, CLEAR 65, LEAVE 10, has specific product yield and resource requirements for each of the 5 time periods. The alternative required: 33.6 hours of labor, 8.1 hours of truck time, 8.5 hours of tractor time, 14.1 hours of power saw time in the initial time period. Net revenue yield was $124.00 to the fixed resources in the initial time period, the variable costs of the fixed resources being charged against gross revenue.

Resource requirements for each of the 4 succeeding time periods were described in the same manner, with corresponding value yields. However, in the succeeding time periods, neither fixed or variable costs were removed. These costs were charged against revenue in a manner explained in a succeeding section (Appendix Table 3).

Valuation of the Alternatives

Comparative analysis of several opportunities composed of costs and revenues occurring at different times, demands that each opportunity be evaluated on an equal basis with respect to time. Present worth analysis is the generally accepted procedure for comparing alternatives of the type described for the forest enterprise. Present worth analysis requires that the return from an output or series of outputs in future time periods be discounted to a common point by an interest rate, specifically described as an alternative or guiding rate of return. When describing the forest enterprise for analytical purposes, it is necessary to establish the guiding rate which a particular firm employs. For purposes of demonstrating the effect of the alternative rate upon the composition of the optimum program, three such rates were used, 3, 6, and 10 percent. These rates were selected because they are within the range of rates commonly associated with various forest ownership categories U. S. Forest Service (1963:26).

Present Worth Model

The basic formula used in the valuation of the alternatives examined in this study is given as follows:

$$V_0 = \frac{R_1 + Vt_1(1.p)^{r-t_1} \text{ ---- } + Vt_i(1.p)^{r-t_i}}{(1.p)^r} + \frac{\text{Max} \quad \dfrac{R_2 + Vt_1(1.p)^{n-t_1} \text{ ---- } + Vt_i(1.p)^{n-t_i} - C}{(1.p)^n - 1} - C}{(1.p)^r}$$

Where:

R_1 = Value yield of the existing stand when clear cut

R_2 = Stumpage value yield at the end of the next and all succeeding rotations, with logging costs at final harvest charged against revenue

Vt_1 - - Vt_i = Value yield of thinning at time t_1 - t_i

r = Length of time present stand retained

n = Length of the next and succeeding rotations

p = Alternative rate

C = Cost of regeneration

V_0 = Present worth of the alternative

The above model evaluates the present worth of each alternative, given specific treatment regimes for the present stands and subsequent rotations. The costs that are reflected in the V_0 value, are the costs of regeneration at the end of this and all subsequent rotations, the variable cost of any thinning activities in the initial time period, and the costs associated with final harvest in the next and succeeding rotations and the cost of capital.

Developing the Model

This section will describe the linear programming model developed to examine the alternatives defined for the Seward Forest. The programming model was used to describe the best combination of activities relative to: the restrictions imposed by scarce resources, the competition for resources in the initial and four future time periods, and the personal management constraints. The solution of the model provided a program that maximized present worth to the fixed resources of the forest.

Assumptions Incorporated into the Model

The following assumptions were incorporated into the model:

1. For each of the first 10 years, labor, working capital, capital equipment and contract logging capital will be available in the same quantity as in the first year.
2. Working capital and mill requirements in the four succeeding time periods will be available in the same quantity as in the initial time period. All other resources will be purchased in succeeding time periods.

3. Prices and costs will remain stable.

4. The input-output ratios will remain the same. There will be neither an increase nor a decrease in efficiency, technology will remain the same.

5. Instead of the original management constraint of 750,000 MBF per year, the volume output for the basic matrix was set at 1,000 MBF or 10,000 MBF per ten year period to determine if the forest could provide this volume without being forced to do so at a cost.

The Linear Programming Model for the Seward Forest

The linear programming model for the Seward Forest is a system of linear equations which describe the use of specific quantities of resources by the various processes defined for the enterprise. These equations place limits or restrictions upon the quantities of resources which may be used, in aggregate, by the various processes. The complete model with changes which are necessary for evaluation at the different alternative rates is contained in the Appendix (Appendix Table 4).

The representative equation concerned with the land resource is:

$$\underset{\text{Age 25}}{\text{(Land Site 70)}}\ 144 \text{ ac} = \sum_{p=1}^{14} 1x_p + 1x_{109}$$

This equation states that the sum of the acreage (x) used by processes (p) 1 through 14, plus the sum of the acreage remaining unused in process 109, must equal 144 acres. All other activities which did not require this land resource have a process coefficient of zero. Process 109 is the disposal activity which allows the land resource to remain unused.[5/]

The representative equation concerned with the labor resource in the initial time period is:

$$(\text{Labor } t)76{,}730 = \sum_{p=8}^{14} 71.6x_p + \sum_{p=18}^{20} 33.6x_p + \sum_{p=22}^{23} 20.8x_p + \sum_{p=33}^{39} 29.2x_p$$

$$+ \sum_{p=44}^{46} 18.4x_p + \sum_{p=48}^{49} 42.8x_p + \sum_{p=60}^{66} 58.0x_p + \sum_{p=70}^{72} 42.0x_p + \sum_{p=74}^{75} 52.4x_p - 100x_{79} + 1x_{126}$$

This equation states that each acre of processes 8-14 requires 71.6 hours of labor, each acre of processes 18-20 requires 33.6 hours of labor, etc. Processes not in the equation do not require labor. Process 79 is the labor buying activity which provides additional labor in the initial time period. Process 126 is the labor disposal activity. The sum of all the above activities must equal 76,730 hours of labor. The other equations are explained in a similar manner.

The objective statement is:

$$Z \text{ Max} = \sum_{p=1}^{78} Vopx_p + \sum_{p=105}^{108} Vopx_p + \sum_{b=79}^{99} C_b x_b + \sum_{A=100}^{104} C_A x_A + \sum_{d=109}^{156} 0x_d$$

Where Vo is a positive value and all C's are negative values.

The objective equation states that maximum profit is equal to:

1. The sum of the present worth values (Vo) of processes (p) 1 through 78, times the quantity (x) of a particular activity in the program.

5/ See page 45.

2. Plus, the sum of the present worth of the cost of buying activities (b) 79 through 99, times the quantity of the buying activities in the program,

3. Minus the present worth of the costs (C) of the artificial activities (A) 100 through 104, times the quantity of these activities in the program,

4. Plus the sum of the cost of disposing (d) of any of the resources, 109 through 156, times the quantity disposed of.

Profit is maximized subject to the conditions imposed by the complete set of restrictions represented by the linear equalities listed previously.

Equations such as these were described in matrix form for all resources, in all time periods (Appendix Table 4). The management and resource constraints form the rows of the matrix, and the alternative processes form the columns of the matrix. This notion is presented in the context of this problem in the form illustrated in Table 7, which is presented to further clarify the interpretation of the model. The equations describe the restrictions placed upon the model by resources. The columns describe the resource requirements for each process.

The Columns of the Model

Each of the columns describe an activity as a resource user or a resource supplier (Table 7). The coefficients within the column are

Table 7. Example of activities in the basic matrix

Resources Restrictions	Unit	Quantity	1[1]	2[1]	87[2] t	88[2] t10	89[2] t20	90[2] t30	100[3] t	101[3] t10	102[3] t20	103[3] t30	104[3] t40	109[4]	---	156[4]
Land site 70																
Age																
25	Acre	144	1	1												
35	Acre	25														
45	Acre	78												1		
55	Acre	12														
75	Acre	5														
Labor																
t	Hours	76,730														
t10	Hours	0		53.6												
t20	Hours	0		14.4												
t30	Hours	0		13.2												
Capital																
t	Dollars	330,000	20.00		39.00											
t10	Dollars	330,000				94.00										
t20	Dollars	330,000					94.00									
t30	Dollars	330,000	20.00					94.00								
t40	Dollars	330,000		20.00												
Mill Capacity																
t	MBF	10,000	4.2						-1							
t10	MBF	10,000		3.7						-1						
t20	MBF	10,000		1.2							-1					
t30	MBF	10,000	5.3	1.1								-1				
t40	MBF	10,000		11.2									-1			
Trucks																
t	Hours	24,800														
t10	Hours	0		6.0												
t20	Hours	0		.8												
t30	Hours	0		.7												
Tractors																
t	Hours	38,840			-100											
t10	Hours	0		7.2		-100										
t20	Hours	0		1.2			-100									
t30	Hours	0		1.1				-100								
Power Saws																
t	Hours	46,800														
t10	Hours	0		10.8												
t20	Hours	0		1.6												
t30	Hours	0		1.4												
Contract Logging Capital																
t	Dollars	110,000	84.00													
t10	Dollars	110,000														
t20	Dollars	110,000														
t30	Dollars	110,000	106.00													
t40	Dollars	110,000		224.00												

Footnotes

1/ Activity #1 C25, L10, L20, C30, L10
1/ Activity #2 L25, T35, T45, T55, C65
2/ Buying Activities 87-90
3/ Artificial Activities 100-104
4/ Disposal Activities 109-156

a vertical array of the budget developed for the particular process. If the activity uses a resource, the number in the cell is a positive value. If the activity is a resource supplier, then it carries a negative sign. Of those real activities exhibited, other than buying, all of the coefficients are positive, indicating the use of resources by the activities involved. For example, activity number 1, CLEAR 25, LEAVE 10, LEAVE 20, CLEAR 30, requires one acre of site 70 land which supports a 25 year old stand of loblolly pine. In addition, to replant the area, this activity requires at the end of each clear cutting operation, $20.00 worth of working capital. Capital for planting is also needed 30 years from now (t30). The activity uses 4.2 thousand board feet of the available mill capacity in the initial time period, and 5.3 thousand board feet of the available mill capacity 30 years from now, for each acre in the program. It uses contract logging capital at the rate of $84.00 per acre in the initial time period, and $106.00 per acre 30 years from now. The activity has a present worth of $492.97 per acre. All other real activities are interpreted in the same manner (Table 7).

<u>The buying activities</u>. The buying activities were included in order that required resources could be furnished in all time periods. All buying activities have both positive and negative coefficients which indicate that these activities use available resources to supply additional resources of another kind to the program. For example, buying tractor time requires $39.00 worth of capital in the initial

time period to supply the program with 100 additional hours of tractor time. The cost of this activity to the program is \$39.00 per 100 hours of tractor time purchased. Ten years from now, \$94.00 worth of capital is required to furnish 100 additional hours of tractor time to the program in the second time period. This purchase will cost the program \$73.00 per 100 hours of tractor time. The reason for this difference can be found in the objective statement - maximize returns to the fixed resources. The V_0 value of each activity in the basic matrix reflects the removal of variable costs in the initial time period. In order that this cost would not be included twice, only the fixed costs of the resource bought in the initial time period were entered for the buying activities. In succeeding time periods, neither the variable nor the fixed costs of resources have been charged to the V_0 value. Therefore, the \$94.00 per 100 hours is the sum of the fixed and variable costs for buying additional tractor time in the second and subsequent periods.

The costs of \$39.00 and \$73.00 are deducted from present worth as the resources are purchased. The explanation of the difference which exists is based in part on the explanation of the difference charged to working capital. However, in addition, to keep the analysis in the context of present worth evaluation, all costs are the present worth of future expenditures, discounted, at the appropriate alternative rate.

The artificial activities. The artificial activities are included to make the program more flexible. In this instance, it allows the addition of mill capacity at zero cost to the program. An alternative to build additional mill capacity was not included. However, additional output from the forest was considered possible. Therefore, instead of including a stumpage selling activity, mill capacity could be increased to allow for increased output from the forest, because the artificials provide this opportunity. The additional production from the forest was sold to the mill.

These artificial activities were included in the original matrix at a high negative value (cost). This was done so that once the basic plan had been determined, changes could be made. The changes to be made would be in mill capacity at different time periods to discover if there was another combination of activities which would provide a greater total present worth and greater output from the forest.

The disposal activities. Disposal activities were included so that resources could remain idle. Non-use of all or part of a resource supply is just as valid an alternative as is the use of that resource in a production process. These activities can also be used to force activities into the optimum program by assigning high negative costs to them.

Heady (1963:57) stated that, "we do not wish to force a program which just exactly uses all resources. Such a plan may not be mathematically or physically possible. Even if it is mathematically possible, it need not be the most profitable one." Thus, by including disposal activities as part of the alternatives for resource use, the model becomes more versatile and more valid in an economic sense.

Changes in the Basic Matrix

Once the solution of the basic matrix was obtained and the resources which were restricting the program were identified, changes were made in the basic matrix so that additional solutions could be made to determine if adjustments were possible that would increase present worth.

How the Changes Were Made

The linear programming model for the Seward Forest included several activities which were not used in the basic solution. These activities were prohibited from entering the initial program by assigning high costs to them. The activities treated in this manner were all buying activities in the initial time period and artificial activities in all time periods. Both groups of activities allowed an increase in the quantity of resources available to the enterprise. The buying activities allowed setting the fixed resources in the initial time period at a zero quantity level so that the forest manager could examine the needs of the forest enterprise prior to the allocation of resources.

Mill capacity, which was originally set at a zero cost level, could be changed so that the alternative of non-use of mill capacity could be eliminated from the program by assigning a high cost to disposal. All resources could be treated in this manner. Artificial activities which were originally set at a high negative cost could be set at a zero cost level so that mill capacity could be increased over the entering 10,000 MBF per time period level.

Relevance of these Changes to the Forest Manager

Any changes in constraints means a change in the situation that is being analyzed, because either more or less of the "production resources" are assumed to be available. Further, the personal management constraints are either relaxed or intensified. To the forest manager who is evaluating opportunities before resources are committed, this is an invaluable aid to decision making. There will be a new program defined which is optimum for the constraints imposed upon the enterprise and in each instance he can determine the cost of the constraints.

These changes can further serve the forest manager in decision making, if for instance, he would like to determine forest treatment, thinning and harvesting schedules in a regulatory sense, when an increase or decrease in allowable cut is the alternative being examined. He can program these changes and then observe changes in stand treatment and in growing stock levels before resources are committed, and before any treatment has been initiated. The changes would further serve in evaluating the opportunities with respect to a minimum or maximum annual flow of products from the forest.

The Linear Programming Model and the Forest Manager

What will the practicing forester see in the linear programming model developed for the Seward Forest? A model may be thought of as a representation or an orderly description of something. In this case the model is an orderly description of the timber harvest alternatives available on a specific forest property.

The timber on the specific forest property is loblolly pine ranging in age from 25 to 75 years old. The timber occurs on land of three different site qualities. The model also includes the personal constraints on timber harvest imposed by the forest manager as well as constraints imposed by limited resources.

The solution of the linear programming model will provide the schedule of timber harvest activities which will maximize the present worth of the forest property subject to the personal and resource constraints imposed. Information of this type should be very useful to the practicing forester.

RESULTS AND DISCUSSION

The basic matrix, described by the system of equations in the preceding section, was solved. The solution described the optimum program of timber harvest and provided information about: resources which were not used, the activities not included in the optimum program, the resources that were exhausted in the optimum program, and the value of acquiring additional resources.

Specific information which the forest manager needs in the operation of the forest enterprise is available. The program is a listing of activities, each of which describes the best treatment schedule for present and future forest stands for five 10-year periods. Resource requirements are defined for the installation of the treatments scheduled for each of the time periods. Age class distribution can be determined for any time within the 50 year period. Stand treatment, i.e., clear, thin, or leave, for each age class in each period can be determined in terms of a given site and in terms of specific stands. This information, obtained from the solution of the basic matrix, will be discussed in detail.

Additional solutions were made to determine if adjustments could be made to the basic matrix which would define a program in which total present worth would be greater than that resulting from the solution of the basic matrix. Further, the additional solutions which were made involved changing the constraints imposed upon the original matrix. These changes will be described and the derived optimal programs will be discussed both individually and comparatively.

Finally, additional solutions were made which involved two changes in the alternative rate, one at 6 percent, and one at 10 percent. The identical constraints imposed upon each of the solutions involving the 3 percent rate were imposed for each of the four solutions of the 6 and 10 percent rates. The results of all solutions at the 3, 6, and 10 percent rates will be discussed comparatively.

The Basic Optimum Program

The solution of the basic matrix defined an optimum program in which the best combination of activities, subject to the constraints imposed upon the model were enumerated (Table 8).

Constraints Imposed upon the Basic Matrix

This solution allowed the use of fixed resources in the initial time period. Buying of additional resources was prohibited in the initial time period. The program was allowed to purchase resource requirements for use in future time periods. Mill capacity was set at an upper limit of 10,000 MBF per ten year period. That is, the cut in any of the five 10-year periods could be less than or equal to, but not greater than 10,000 MBF. The program was allowed to waste mill capacity and other resources if non-use was profitable.

Table 8. Basic optimum program, 3% alternative rate, solution one
Total Present Worth
$1,402,341.30

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t_{10}	t_{20}	t_{30}	t_{40}			
			Site 70					
13	T25	T35	L45	C55	L10	Acre	144	593.00
20	T35	T45	T55	C65	L10	"	25	497.70
23	T45	T55	C65	L10	L20	"	78	512.20
24	C55	L10	L20	C30	L10	"	112	718.00
25	C75	L10	L20	C30	L10	"	5	932.00
			Site 80					
38	T25	T35	L45	C55	L10	"	11	623.10
39	T25	T35	C45	L10	L20	"	19	651.80
45	T35	T45	C55	L10	L20	"	212	718.40
48	T45	C55	L10	L20	C30	"	143	903.50
49	T45	T55	C65	L10	L20	"	331	832.00
50	C55	L10	L20	C30	L10	"	179	1018.00
51	C65	L10	L20	C30	L10	"	136	879.00
105	L75	C85	L10	T20	C30	"	230	1202.60
			Site 90					
64	T25	T35	L45	C55	L10	"	16	1085.70
70	L35	C45	L10	T20	T30	"	10	1162.10
74	T45	C55	L10	T20	T30	"	30	1315.10
76	C55	L10	T20	T30	C40	"	32	1074.00
77	C65	L10	T20	T30	C40	"	13	1708.00
80	Buy Labor				t10	Hours	14,660	-0.93
81	"	"			t20	"	980	-0.67
82	"	"			t30	"	1,650	-0.49
84	Buy Trucks				t10	"	1,290	-0.95
85	"	"			t20	"	336	-0.68
86	"	"			t30	"	290	-0.51
88	Buy Tractor				t10	"	1,650	-0.73
89	"	"			t20	"	210	-0.52
90	"	"			t30	"	334	-0.39
92	Buy Saw				t10	"	2,170	-0.38
93	"	"			t20	"	340	-0.27
94	"	"			t30	"	520	-0.20
96	Buy Logging Cap.				t10	$	177,800	-0.0223
97	"	"	"		t20	"	199,000	-0.0166
98	"	"	"		t30	"	108,000	-0.0124
99	"	"	"		t40	"	83,500	-0.0092

Value of an Additional Unit of Resource

Mill Capacity	t_{10}	$12.85/MBF
	t_{20}	$ 4.80/MBF
Contract Logging Cap.	t	$ 3.17/$

The Program

The program describes a specific plan of action which the forest manager should follow for each of five 10-year periods. Each activity included within the optimum program is a definitive silvicultural prescription for a specific area of land for the next 50 years.

In the basic solution, the plan of action dictates that in the first ten year period, 1965-1974, the forest manager must arrange to thin 278 acres of loblolly pine; clear and plant 337 acres of loblolly pine; and leave 278 acres of loblolly pine for additional volume growth (Fig. 2 A).

At the beginning of this 10 year period, the loblolly pine stands will be distributed over the total land resource by age class in the following manner: 25 year old stand, 190 acres; 35 year old stands, 247 acres; 45 year old stands, 582 acres; 55 year old stands, 223 acres; 65 year old stands, 172 acres; and 75 year old stands, 250 acres. Within the initial 10 year period, the forest manager must arrange to: thin 190 acres of the 25 year old stands; thin 237 acres of the 35 year old stands, leaving 10 acres for additional volume growth; thin 582 acres of the 45 year old stands; clear and plant 223 acres of the 55 year old stands; clear and plant 149 acres of the 65 year old stands, leaving 23 acres for additional volume growth; clear and plant 5 acres of the 75 year old stands, leaving 245 acres for additional volume growth (Figure 2 B).

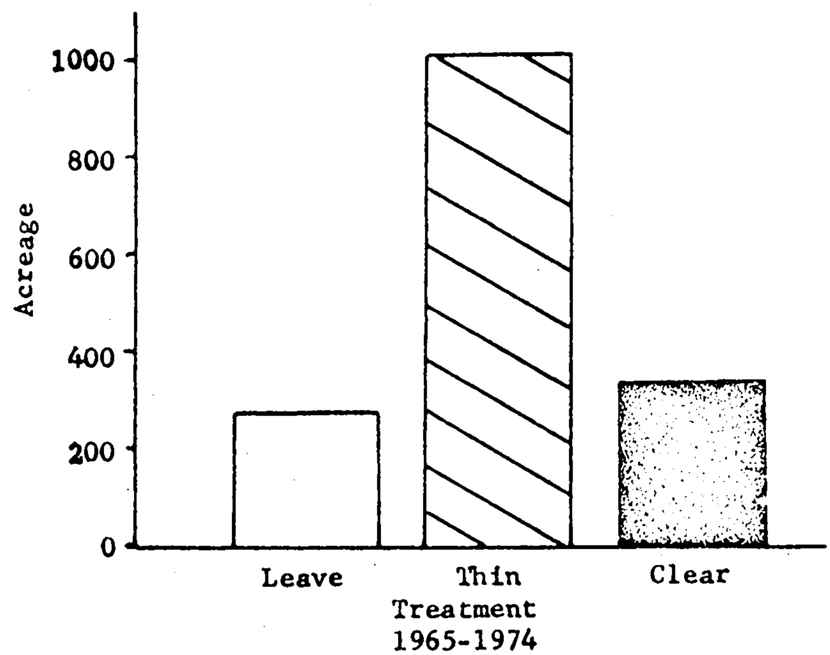

Figure 2 A. Schedule of treatments for the first time period - basic solution

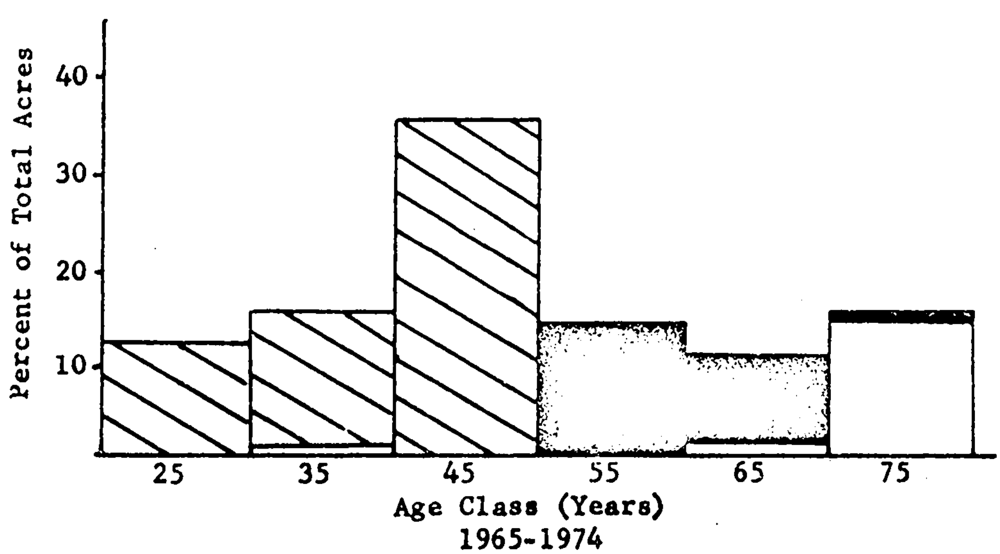

Figure 2 B. Age class distribution and schedule of treatment for first time period - basic solution

Within the next ten years, the forest manager must plan to thin all of the 144 acres of 25 year old loblolly pine, all of the 25 acres of 35 year old loblolly pine, and all of the 78 acres of 45 year old loblolly pine on the site 70 land resource. He must also clear and plant, all of the 12 acres of 55 year old loblolly pine and the 5 acres of 75 year old loblolly pine which stands on the site 70 land resource (Table 9).

On land of site quality 80, he must plan to thin all of the 30 acres of 25 year old loblolly pine, all of the 212 acres of 35 year loblolly pine, and all of the 474 acres of 45 year old loblolly pine. He must also plan to clear and plant, all of the 315 acres of 55 and 65 year old loblolly pine (Table 9).

On site 90, he must plan to thin 16 acres of 25 year old loblolly pine, all of the 30 acres of 45 year old loblolly pine, and clear and plant, all of the 32 acres of 55 and 13 acres of the 65 year old loblolly pine (Table 9).

During the first 10 year period, activities in the optimum program will require: 40,757 hours of labor, $18,573 of operating capital, 6,004 hours of truck time, 7,170 hours of tractor time, 10,648 hours of power saw time, and $110,000 of contract logging capital. The forest will provide 7,935 MBF of sawtimber to the mill (Table 9).

In all succeeding time periods, a specific plan of action such as that described above is outlined for the forest manager if he wishes to maximize present worth within the constraints imposed. This plan

Table 9. Cutting schedule, 1965-1974, by site and age class and resource requirements, 3 percent altern tive rate, solution one

Site index	Age years	Operation: Thin acres	Operation: Clear acres	Operation: Leave acres
	25	144	0	0
	35	25	0	0
70	45	78	0	0
	55	0	12	0
	75	0	5	0
	Subtotal	247	17	0
	25	30	0	0
	35	212	0	0
	45	474	0	0
80	55	0	179	0
	65	0	136	0
	75	0	0	230
	Subtotal	716	315	230
	25	16	0	0
	35	0	0	10
	45	30	0	0
90	55	0	32	0
	65	0	13	23
	75	0	0	15
	Subtotal	46	45	48
	Total	1009	377	278

Resource Requirements: Resource	Unit	Quantity
Labor	Hours	40,757
Operating Capital	Dollars	18,513
Trucks	Hours	6,004
Tractors	Hours	7,170
Power Saws	Hours	10,648
Contract Logging Capital	Dollars	110,000

of action can be developed by examining the treatment schedules for each time period (Table 8). Resource requirements for each time period can also be developed in a similar manner (Table 8 and Appendix Tables 17, 18, 19 and 20).

Resources not exhausted. The quantity of labor, operating capital, and capital equipment which was available in the first time period but which remained unused, indicated that these resources were not restricting the program. They were available in more than adequate quantities (Appendix Table 17). This would suggest that the forest manager make some adjustments, i.e., examine other opportunities of using these resources.

Resources exhausted - value of additional quantities. On the other hand, mill capacity and contract logging capital were exhausted, mill capacity 10 and 20 years from now contract logging capital in the initial time period (Table 8). The solution provides information on the relative profitability of the purchasing of additional quantities of these resources. For example, the value per thousand board feet of additional mill capacity 10 years from now is $12.85 and 20 years from now, $4.80 (Table 8). For each additional unit of mill capacity provided in the second and third time periods, there will be an addition to total present worth of the amounts given.

In the initial time period the program was not allowed to purchase additional contract logging capital. The value of each additional dollar of contract logging capital to the program was $3.17 (Table 8). This indicates a great need for this resource since its original cost was $0.0300 per dollar (Table 8). Logging capital can provide additional volume to the mill. It is this fund and no other that provides for the cost of final harvest.

Additional Solutions

Once the resources which were restricting the program in the original solution were identified, it became obvious that it might be worthwhile to examine some specific changes in the basic matrix. These changes involved constraints placed upon the optimum program. Additional solutions were made to determine if adjustments could be made to the basic matrix which would increase total present worth.

Solution Two

To observe the effect on present worth of the land and mill capacity restrictions, the basic model was changed with the quantity of labor, trucks, tractors, power saws, and contract logging capital was set at zero. Land, operating capital, and mill capacity remained at the level specified in the basic model. The buying activities provided labor, trucks, tractors, power saws, and contract logging capital in all five time periods, using operating capital as the source of funds. The change removed all limitations imposed by the "operating resources", i.e., those resources, other than operating capital, which were required to provide volume to the mill. Mill capacity remained with an upper limit of 10,000 MBF.

The program. An optimum program was defined (Table 10). As a direct result of releasing resource restrictions, total present worth was increased by $41,580.00 over the basic optimum program. The reason for this increase lies in the amount of the present stand cut in the early time periods. An additional 14 percent of the area of the present stand was clear-cut in the initial time period over solution one, and by the second time period, an additional 14 percent was cut over solution one. Sixty-two percent of the present stand had been cleared by the second time period in solution two. None of the original stand remained after the third time period (Table 11). There was an increase in the initial time period of 215 acres clear-cut over the first solution in the same time period (Table 12). This is the results of freeing the restriction on contract logging capital. There was an increase in total volume cut of approximately 2.3 million board feet, the majority of it occurring in the initial time period (Table 12). The increase in area clear-cut and in volume is the result of removing the restriction on contract logging capital. A greater percentage of the value of the present stand harvested in the initial time period means that a lower percentage of the dollar value of the present stand is being discounted, thus the reason for the $41,580 increase in total present worth.

Table 10. Optimum program, 3 percent alternative rate, solution two
Total Present Worth
$1,443,921.60

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t_{10}	t_{20}	t_{30}	t_{40}			
			Site 70					
9	T25	T35	C45	L10	L20	Acre	144	627.50
15	C35	L10	L20	C30	L10	"	25	511.40
21	C45	L10	L20	C30	L10	"	78	606.00
24	C55	L10	L20	C30	L10	"	12	718.00
25	C75	L10	L20	C30	L10	"	5	932.00
			Site 80					
39	T25	L35	C45	L10	L20	"	30	651.80
41	L35	C45	L10	L20	C30	"	212	786.00
47	C45	L10	L20	C30	L10	"	74	965.00
49	T45	T55	C65	L10	L20	"	400	832.00
50	C55	L10	L20	C30	L10	"	179	1018.00
51	C65	L10	L20	C30	L10	"	136	879.00
105	L75	C85	L10	L20	C30	"	188	1202.60
106	L75	L85	C95	L10	L20	"	42	1014.50
			Site 90					
54	L25	C35	L10	T20	T30	"	16	1083.00
68	L35	T45	C55	L10	T20	"	10	1117.50
74	T45	C55	L10	T20	T30	"	30	1315.00
76	C55	L10	T20	T30	C40	"	32	1074.00
77	C65	L10	T20	T30	C40	"	36	1708.00
78	C75	L10	T20	T30	C40	"	15	1786.00
79	Buy Labor				t	Hours	29,883	-1.20
80	" "				t10	"	9,680	-0.93
81	" "				t20	"	1,160	-0.67
82	" "				t30	"	2,740	-0.49
83	Buy Trucks				t	"	4,880	-0.39
84	" "				t10	"	850	-0.95
85	" "				t20	"	273	-0.68
86	" "				t30	"	442	-0.51
87	Buy Tractors				t	"	5,380	-0.39
88	" "				t10	"	1,100	-0.73
89	" "				t20	"	290	-0.52
90	" "				t30	"	517	-0.39
91	Buy Saw				t	"	8,620	-0.29
92	" "				t10	"	1,580	-0.38
93	" "				t20	"	480	-0.27
94	" "				t30	"	794	-0.20
95	Buy Logging Cap.				t	$	166,700	-0.0300
96	" " "				t10	"	185,500	-0.0223
97	" " "				t20	"	197,600	-0.0166
98	" " "				t30	"	81,900	-0.0124
99	" " "				t40	"	102,800	-0.0092

Value of Additional Resources

Mill Capacity t $19.79/MBF
t_{10} $ 7.12/MBF

Table 11. Cumulative percentage of present stands clear-cut by time period and solution number - 3 percent alternative rate

Solution[1/] Number	Time Period 1	2	3	4	5	Total Present Worth	Volume Cut MBF
1	22	48	86	98	98	$1,402,341.30	37,762
2	36	62	100	100	100	$1,443,921.60	39,496
3	42	77	100	100	100	$1,511,719.70	46,289
4	76	98	100	100	100	$1,576,176.50	58,258

1/ 1. Basic solution
2. Removed restrictions on contract logging capital
3. Minimum cut, 750 MBF per year - no upper limit
4. Removed all restrictions except land

Table 12. Sawtimber volume harvested and present worth of enterprise by alternative rate and solution

Alternative rate	Solution no.	Sawtimber volume cut MBF t	t10	t20	t30	t40	Total	Present worth
		MBF	MBF	MBF	MBF	MBF	MBF	Dollars
3%	1	7,935	10,000	10,000	5,543	4,284	37,762	1,402,341.30
	2	10,000	10,000	9,933	4,336	5,227	39,496	1,443,921.60
	3	12,283	8,836	7,500	10,170	7,500	46,289	1,511,719.70
	4	19,713	4,566	482	28,465	5,041	58,258	1,576,176.50
6%	1	7,935	10,000	10,000	5,215	5,176	38,326	951,996.55
	2	10,000	10,000	9,633	1,204	4,853	39,973	990,836.00
	3	11,833	7,500	7,500	7,500	7,500	41,833	1,023,389.70
	4	21,855	1,398	62	13,249	1,245	37,809	1,286,443.90
10%	1	7,935	10,000	10,000	2,358	6,275	36,568	725,552.24
	2	10,000	10,000	10,000	1,204	5,452	36,656	772,448.84
	3	9,073	7,500	7,500	7,500	7,500	39,073	695,463.55
	4	22,044	908	2,843	87	2,843	28,725	1,196,382.70

Throughout the remainder of this paper, the discussion will to a large extent be centered upon the rate of clearing of the present stands. More rapid removal of the present stand results in value yields of these stands being discounted for a lesser period of time. This in turn results in the value yield of the next and succeeding rotations being discounted for a lesser period of time. In other words, as the discount period decreases value yield increases. This in effect provides a measure of how quickly the new stands will be generated relative to effect on income of the discounting of future rotations.

<u>Resources exhausted - value of additional quantities</u>. The value of additional resources changed from the value indicated in the basic solution. Since this program was allowed to purchase all of the contract logging capital required, within the limitations of the operating capital available, the value of additional contract logging capital is zero for this solution. Mill capacity was exhausted in the first and second time periods. The value of additional mill capacity in the initial time period is $19.79 per MBF, and for the second time period, $7.12 per MBF (Table 10). Since other resources were not restricting the program, this implies that there is a definite need for increasing mill capacity in the initial and in the succeeding time period.

Solution Three

As indicated, the basic solution did not contain the managerial constraint of cutting at least 750,000 board feet per year. This constraint is contained in solution three. In addition, solution three allowed the buying of additional resources in all time periods. The model was adjusted in this solution and did not allow the wasting of mill capacity. This set of constraints had the effect of imposing one limitation upon the program other than that of limited land. This limitation was the requirement placed upon the forest to produce a minimum of 7,500 MBF per ten year period, however more than 7,500 MBF could be produced.

The Program

The significance of this solution and the resulting program lies in the requirement that the forest produce a given flow of sawtimber for the next 50 years. This requirement places the solution in the context of a simplified regulation model, compatible with the objective of profit maximization with a constraint of a minimum sustained annual yield.

Upon solution, an optimum program was defined that fulfilled the objective statement (Table 13). The 7,500 MBF minimum was exceeded in the initial time period and in the second and fourth time periods (Table 16). This solution resulted in a greater total present worth than that in either the basic solution or that in solution two. The

Table 13. Optimum program, 3 percent alternative rate, solution three
Total Present Worth
$1,511,719.70

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t10	t20	t30	t40			
			Site 70					
9	T25	T35	C45	L10	L20	Acre	144	627.50
19	T35	T45	C55	L10	L20	"	25	513.90
21	C45	L10	L20	C30	L10	"	78	606.00
24	C55	L10	L20	C30	L10	"	12	718.00
25	C75	L10	L20	C30	L10	"	5	932.00
			Site 80					
39	T25	L35	C45	L10	L20	"	30	651.80
41	L35	C45	L10	L20	C30	"	212	786.00
47	C45	L10	L20	C30	L10	"	108	965.00
48	T45	C55	L10	L20	C30	"	366	903.50
50	C55	L10	L20	C30	L10	"	179	1018.00
51	C65	L10	L20	C30	L10	"	136	879.00
52	C75	L10	L20	C30	L10	"	63	1270.00
106	L75	L85	C95	L10	L20	"	167	1014.50
			Site 90					
65	T25	L35	C45	L10	T20	"	16	1066.30
67	C35	L10	T20	T30	C40	"	10	1244.00
73	C45	L10	T20	T30	C40	"	30	1440.00
76	C55	L10	T20	T30	C40	"	32	1074.00
77	C65	L10	T20	T30	C40	"	36	1708.00
78	C75	L10	T20	T30	C40	"	15	1786.00
80	Buy Labor				t10	Hours	2,270	-0.93
81	" "				t20	"	1,720	-0.67
82	" "				t30	"	4,000	-0.49
84	Buy Trucks				t10	"	400	-0.95
85	" "				t20	"	405	-0.68
86	" "				t30	"	430	-0.51
88	Buy Tractors				t10	"	430	-0.73
89	" "				t20	"	420	-0.52
90	" "				t30	"	528	-0.39
92	Buy Saw				t10	"	700	-0.38
93	" "				t20	"	720	-0.27
94	" "				t30	"	774	-0.20
96	Buy Logging Cap.				t10	$	174,300	-0.0223
97	" " "				t20	"	148,700	-0.0166
98	" " "				t30	"	96,500	-0.0124
99	" " "				t40	"	149,800	-0.0092
100	Mill Capacity				t	MBF	4,783	0
101	" "				t10	"	1,336	0
103	" "				t30	"	2,670	0

increase over solution two was $67,798.10, and over the basic solution an increase of $109,378.00. This increase resulted from removing the upper limit constraint on mill capacity.

This increase can be explained since the present stand was removed faster than in either of the two preceeding solutions. An additional 6 percent was cleared in the initial time period and an additional 15 percent in the second time period as compared with solution two (Table 11). This solution generated a 6.8 million board foot increase over solution two, a third of which occurred in the initial time period (Table 12). It also generated an 8.5 million board foot increase over solution one, one-half of which occurred in the first time period. There was an increase in the initial time period of 112 acres and in the second time period of 132 acres of clear cutting of the present stand, generating a greater percentage of the value of the present stand which is not discounted. Seventy-seven percent of the present stand had been cleared by the second time period. This means that the greatest discount period possible for 77 percent of the present stand is ten years.

Solution Four

In solutions one and two, the value of additional mill capacity indicated the need to increase volume output of this facility - indirectly indicating the need to increase the cut in the early time periods. All constraints, except land, were removed. Mill capacity was allowed to increase by setting the artificial activities which provided additional capacity at a zero cost. This is comparable to selling the wood on the open market. Provisions were not made to build additional manufacturing facilities.

<u>The Program</u>

There was a drastic change in the composition of the optimum program (Table 14). There was an increase in the volume cut and to a lesser degree an increase in total present worth (Tables 11 and 12).

Total present worth was maximum for all solutions, the most obvious reason being that the greatest volume, both in a physical sense and in terms of present dollar value was generated in the initial time period (Table 12). The increase in total volume cut over the preceding solution was approximately 12,000 MBF, and over all solutions prior to that approximately 20,000 MBF. The increase in total present worth over the basic solution was $173,835.20. The percentage of the present stand cleared in the initial time period increased by 54 percent, and by the second time period, 98 percent of the original stand had been cleared (Tables 11 and 12).

<u>Comparison of the Four Solutions</u>

A comparison of the four solutions developed above shows that as the constraints upon the program were relaxed, first allowing the purchase of required operating resources, and then increasing the

Table 14. Optimum program, 3 percent alternative rate, solution four
Total Present Worth $1,576,176.50

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t10	t20	t30	t40			
	Site 70							
8	T25	C35	L10	L20	C30	Acre	144	611.20
15	C35	L10	L20	C30	L10	"	25	511.40
21	C45	L10	L20	C30	L10	"	78	606.00
24	C55	L10	L20	C30	L10	"	12	718.00
25	C75	L10	L20	C30	L10	"	5	932.00
	Site 80							
39	T25	L35	C45	L10	L20	"	30	651.80
41	L35	C45	L10	L20	C30	"	212	786.00
47	C45	L10	L20	C30	L10	"	474	965.00
50	C55	L10	L20	C30	L10	"	179	1018.00
51	C65	L10	L20	C30	L10	"	136	879.00
52	C75	L10	L20	C30	L10	"	230	1270.00
	Site 90							
60	T25	C35	L10	T20	T30	"	16	1094.80
67	C35	L10	T20	T30	C40	"	10	1244.00
73	C45	L10	T20	T30	C40	"	30	1440.00
76	C55	L10	T20	T30	C40	"	32	1074.00
77	C65	L10	T20	T30	C40	"	36	1708.00
78	C75	L10	T20	T30	C40	"	15	1786.00
81	Buy Labor				t20	Hours	1,720	-0.67
82	" "				t30	"	4,230	-0.49
85	Buy Trucks				t20	"	405	-0.68
86	" "				t30	"	483	-0.51
89	Buy Tractors				t20	"	430	-0.52
90	" "				t30	"	584	-0.39
93	Buy Saw				t20	"	720	-0.27
94	" "				t30	"	784	-0.20
95	Buy Logging Cap.				t	$	279,000	-0.0300
96	" " "				t10	"	91,300	-0.0223
97	" " "				t20	"	8,400	-0.0166
98	" " "				t30	"	194,100	-0.0124
99	" " "				t40	"	99,900	-0.0092
100	Mill Capacity				t	MBF	9,713	0
103	" "				t30	"	18,456	0

allowable cut, total cut and total present worth increased. The reason for this increase can be determined. As the constraints were lifted, a greater and greater percentage of the present stand was clear cut in the first 10 years. This percentage increased from 22 percent in solution one to 76 percent in solution four. In the second ten year period of solution four, 98 percent of the original stand had been cleared and the new rotation initiated as compared to 48 percent for solution one (Tables 11 and 15). As the constraints were relaxed, total volume cut increased. It increased in the first time period by a significant amount (Table 12).

This comparison provides some insight into the cost of the constraints imposed. For instance, total present worth of a program in which contract logging capital was unlimited was $41,580 over one with limited contract logging capital and limited mill capacity (solution two over solution one). When the mill capacity limitation was removed, total present worth increased by $132,255 (solution four over solution two). This represents the cost of maintaining a limit on output from this forest enterprise. In fact, this indicates the cost incurred in solution two by not allowing the old stands to be cut.

Table 15, which is a comparison of stand treatment in the aggregate, by solutions and by time periods within a solution, contains data which shows a greater percentage of the stands being thinned in the early time periods than in the latter time periods. A high

Table 15. Acreage by type of treatment, time period, and solution - 3 percent alternative rate

Time Period	Treatment	Solution No. 1	2	3	4
	Thin	1009	604	581	190
1	Clear	377	592	704	1262
	Leave	278	468	379	212
	Thin	836	554	169	0
2	Clear	413	446	578	372
	Leave	414	664	917	1292
	Thin	70	83	123	123
3	Clear	640	626	382	30
	Leave	916	955	1159	1551
	Thin	315	129	123	139
4	Clear	528	509	581	1139
	Leave	821	1026	960	386
	Thin	40	56	16	16
5	Clear	418	483	701	479
	Leave	1168	1125	947	1169

percentage of thinning occurs as long as the solutions are limited by volume output and resource availability. This occurred because as contract logging capital is exhausted additional stands cannot be harvested. But as these limitations are removed, the area thinned decreases, volume being provided by harvesting the present stands. In the latter time periods, and solutions, the area thinned becomes less and less. This occurs because in the next and succeeding rotations, only a small percentage of the land area is defined as site quality 90, and this is the only site upon which the stands are thinned in the next and succeeding rotations.

Marginal Value Product of the Land Resource

The marginal value product of the land resource, expressed as a percentage of the entering price of the activity which uses that resource represents the relative need of each optimum program for additional land of the same site, with stands of the same age, other things being equal.

As the restrictions on mill capacity and on contract logging capital were relaxed, there was a general increase in this ratio for each land classification (Table 16). In solution three the forest was forced within its capabilities to provide an annual flow of 750 MBF or greater. For this solution, the ratio of MVP to original activity price was greater than one. This indicates a great need for additional land resource of the same quality with the same age stand.

Table 16. Marginal value product, of timber harvest activities, expressed as percent of entering price, by site, age class and solution - 3 percent alternative rate

Site	Age class	Solution 1	2	3	4
	Years	Percent	Percent	Percent	Percent
	25	95	74	108	100
	35	89	82	109	81
70	45	83	71	99	99
	55	5	69	99	99
	75	3	69	99	99
	25	93	89	118	100
	35	82	84	106	99
	45	85	81	106	99
80	55	4	69	99	99
	65	8	70	99	99
	75	71	99	124	99
	25	93	85	117	99
	35	77	88	105	97
	45	76	76	105	97
90	55	13	69	108	96
	65	0	67	105	97
	75	0	66	104	97

In solution number four, the optimum program was composed of those activities which provided maximum present worth. In this solution the ratio was nearly 1, indicating nearly optimum conditions, that is, that point where marginal cost equals marginal revenue (Table 16).

Generally, these ratios indicate a greater need for the younger age class stands, the ratio decreasing in almost every instance from the land resource supporting younger age to older age classes (Table 16). The one exception is site 80, age 75, in solutions 1, 2, and 3. The ratio in these instances increased.

One possible explanation for this is the number of alternatives defined for each age class. The number of alternatives for each age class declines from a high of 14 for the 25 year old stands to a low of 1 for the 55, 65, and 75 year stands, except for 5 alternatives for 230 acres of 75 year old trees on site 80. There is an indication then that more alternatives for these older age stands should have been included in the program, especially in the basic solution in which the MVP of the stands is either zero as in the case of site 90, or less than 14 percent of the entering price of the activity.

Changes in the Alternative Rate

A change in the alternative rate means a change in one of the basic assumptions of the firm and presents a new situation for analysis. To the forest enterprise which uses maximum soil expectation value as a guide to establishing the rotation, changing the alternative rate has the effect of changing the length of the rotation. However, changes in rotation in this study were not considered until after the present stands had been removed and the area replanted. The rotation for the present stands was set by competitive action - that is - the alternatives were defined for each age class so that there was an array of rotation length for each age class (Appendix Table 1). The program was allowed to select the rotation over which the present stands could profitably be carried relative to the fixed quantity of resources and relative to output requirements in the various time periods. Thus, as a result of the change in the alternative rate, resource requirements for that part of each alternative which was concerned with the present stands were not changed. As the alternative rate was changed, the rotations dictated by each alternative rate required resources in different proportions.

Solutions Involving the 6 Percent Alternative Rate

The basic matrix was adjusted to accommodate the changes dictated by the 6 percent rate (Appendix Table 5). Optimum programs were defined for each of four solutions alternative resource situations as in the case of the 3 percent alternative rate (Appendix Tables 6, 7, 8, and 9).

Comparison of the Four Solutions at the 6 Percent Alternative Rate

Total present worth again increased as the constraints upon the program were removed. The increase from solution one to solution two was $38,840.33 because the contract logging capital restriction was removed. The increase from solution one to solution four was $334,447.35. This resulted from removing the restrictions on volume output, and the restrictions placed upon the operating resources.

At each succeeding solution total volume cut increased with the exception of solution four. This decrease in volume was a direct result of cutting the majority of the present stand, 89 percent, in the initial time period. The character of the growing stock changed and merchantable growth per acre decreased because a large percentage of the area was occupied by young stands.

The percentage of the present stand which was cut in the early - first two - time periods, greatly increased from solution one through four (Table 17). This increase though small in the first three solutions was rather large in the fourth solution, with an additional 45 percent of the present stand cut (solution four over solution three). This in part explains the large increase in present worth from solution one to solution four. The removal of a great part of the present stand in the initial time period generated a large portion of the capital value of the standing timber as value which was not discounted, also increasing the present value of the succeeding rotations because the discount period is less.

Table 17. Cumulative percentage of the present stand cut by time period and solution number - 6 percent alternative rate

Solution Number	Time Period 1	2	3	4	5	Total Present Worth	Volume Cut MBF
1	22	62	95	98	98	$ 951,996.55	38,326
2	36	67	99	100	100	$ 990,836.88	39,973
3	44	73	86	88	100	$1,023,389.70	41,833
4	89	100	100	100	100	$1,286,443.90	37,809

All solutions except number three had either an upper limit on volume output, or as in solution four, were unlimited in volume output. Solution three, however, was required to produce a minimum of 7,500 MBF in each 10 year period. This resulted in a greater percentage of the present stand being retained for a longer period of time than in any of the other solutions (Table 17). This retention of the present stand was necessary since its removal would result in less than the required volume being present in several of the 5 time periods with which this analysis is concerned (Table 12).

Generally, as restrictions were removed (i.e., those on contract logging capital in the initial time period, and finally those on both contract logging capital and on mill capacity in solution four) the removal of the present stand and generation of the new stands progressed at a rather rapid rate (Tables 17 and 18). This resulted in a greater total present worth being described for the optimum program defined.

Solutions Involving the 10 Percent Alternative Rate

Again the basic matrix was adjusted to accommodate the changes which were dictated by the 10 percent alternative rate (Appendix Tables 10, 11, and 12). Optimum programs were defined for each of the four solutions, each of which had the constraints described for the 3 and 6 percent rates (Appendix Tables 13, 14, 15, and 16).

Table 18. Acreage treated in each time period by type of treatments and solution number - 6 percent alternative rate

Time Period	Treatment	Solution No. 1	2	3	4
1	Thin	1009	571	355	190
	Clear	372	595	736	1474
	Leave	283	498	573	0
2	Thin	317	396	92	0
	Clear	702	522	485	190
	Leave	645	746	1087	1474
3	Thin	45	103	328	139
	Clear	462	537	1211	0
	Leave	1157	1024	1125	1525
4	Thin	10	46	133	16
	Clear	515	605	761	1474
	Leave	1139	1013	770	174
5	Thin	46	0	0	0
	Clear	649	522	692	190
	Leave	969	1142	972	1474

Comparison of the Four Solutions at the 10 Percent Alternative Rate

The four solutions are presented comparatively (Tables 19 and 20). The most noticeable change at the 10 percent rate is the drastic reduction in present worth for the third solution. As the restrictions were relaxed, total present worth increased with the exception of solution number three. As the restrictions were relaxed, the percentage of the present stand removed in the early time periods increased with the exception of solution three. This delayed removal of the present stand will help to explain the reduction in present worth. The treatment of the present stand which is dictated by the program defined as a result of solution three requires that a greater percentage of it be retained over the second, third, and fourth time periods, than any other solution (Table 20). This occurs so that the required minimum output of 7,500 MBF per ten year period will be available. The next rotation dictates that the stands be held for only 20 years, then clear-cut. An examination of the volumes provided in the unrestricted fourth solution at the 10 percent rate in Table 12, show that by quickly moving into the required next and succeeding rotations, the attainment of the required volume is not possible. This is because the greatest percentage of the area is occupied by young growth. Again the character of the growing stock has been drastically changed with an accompanying reduction in merchantable volume growth. Thus, implementing solution three would

Table 19. Cumulative percentage of the present stand cut by time period and solution number - 10 percent alternative rate

Solution Number	Time Period 1	2	3	4	5	Total Present Worth	Volume Cut MBF
1	21	63	86	88	96	$ 725,552.24	36,568
2	39	77	100	100	100	$ 772,448.84	36,656
3	27	54	68	86	100	$ 695,463.55	39,073
4	91	100	100	100	100	$1,196,382.70	28,725

Table 20. Acreage treated in each time period, type of treatment and by solution number - 10 percent alternative rate

Time Period	Treatment	Solution No. 1	2	3	4
	Thin	1009	557	716	144
1	Clear	353	867	445	1520
	Leave	302	240	503	0
	Thin	168	145	395	0
2	Clear	691	638	450	144
	Leave	805	881	819	1520
	Thin	144	0	421	0
3	Clear	739	1026	675	1520
	Leave	781	638	568	144
	Thin	144	0	144	0
4	Clear	719	638	758	144
	Leave	801	1026	762	1520
	Thin	0	0	0	0
5	Clear	883	1026	906	1520
	Leave	781	638	758	144

cost the firm $30,088.69 over solution one, and $500,919.15 over solution four to maintain an annual cut of 750 MBF or more.

Comparison of all Solutions at 3, 6, and 10 Percent

It is interesting to compare the action taken in the forest enterprise of firms or individuals with different alternative rates with the same resource base and identical personal management constraints. This section will develop a comparison of this action relative to the length of time the present stand is retained in each of the alternative rates, the scheduled treatment of the stands in the aggregate and the volume removed in each case. A comparison of present worth would not be very meaningful among alternative rates. The expectation is that as the discount rate increases the present worth of a future sum would decrease. This is the pattern followed by increasing the alternative rate in this study.

Solution 1

Solution one provides an insight into what happened in the first time period when the upper limit on mill capacity was 10,000 MBF and the limit on contract logging capital was $110,000.00. In each instance the same quantity of the present stand was thinned and for all practical purposes approximately the same area was cleared. In each instance the same volume was provided to the mill. All contract logging capital available was used in the initial time period.

Generally speaking, in these programs which were the most limited of all programs, there was very little difference in the rate of clearing the present stands. The greatest percentage of the present stand was held for the longest period of time and thus the slower rate of clearing occurred in the 10 percent solution (Table 12).

In total more thinning occurred in the 3 percent solution than in either the 6 or 10 percent solutions (Table 22). In the latter time periods, especially after the new stand has been generated, there is very little thinning. The explanation for this action lies in the small amount of acreage on the forest which was included in the site 90 classification of the land resource. This was the only site for which the next and succeeding rotations dictated a thinning regime. Furthermore, this thinning occurred in only the 3 and 6 percent rates. The 10 percent rate dictated clearing at age 20 on this site.

Approximately the same volume was harvested from forests treated under the various alternative rates in this solution (Table 12). However, the differences when they occurred, did so essentially in the fourth and fifth time periods, after the land resource had been regenerated to a new stand (Table 22).

Solution 2

This solution provides an insight into how quickly the old stand will be removed and the new stand regenerated when some of the restrictions are removed. In this case, contract logging capital was

Table 21. Cumulative percentage of present stands cut by solution, alternative rate, and time period; total present worth and total volume harvested by solution and time period

Solution no.	Alternative rate	Time period 1	2	3	4	5	Total volume harvested	Total present worth
	Percent	Percent	Percent	Percent	Percent	Percent	MBF	Dollars
	3	22	48	86	98	98	37,762	1,402,341.30
1	6	22	62	95	98	98	38,326	951,996.55
	10	21	63	86	88	96	36,568	725,552.24
	3	36	62	100	100	100	39,496	1,443,921.60
2	6	36	67	99	100	100	39,973	990,836.00
	10	52	77	100	100	100	36,656	772,448.84
	3	42	77	100	100	100	46,289	1,511,719.70
3	6	44	73	86	88	100	41,833	1,023,389.70
	10	27	54	68	86	100	39,073	695,463.55
	3	76	98	100	100	100	58,258	1,576,176.50
4	6	89	100	100	100	100	37,809	1,286,443.90
	10	91	100	100	100	100	28,725	1,196,382.70

Table 22. Acreage treated by time period, treatment and alternative rate, solution one

Time Period	Treatment	3%	6%	10%
	Thin	1009	1009	1009
1	Clear	377	372	353
	Leave	278	283	302
	Thin	836	317	168
2	Clear	413	702	691
	Leave	414	645	805
	Thin	70	45	144
3	Clear	640	462	739
	Leave	916	1157	781
	Thin	315	10	144
4	Clear	528	515	719
	Leave	821	1139	801
	Thin	40	46	0
5	Clear	418	649	883
	Leave	1168	969	781

not restrictive in the initial time period for all alternative rates. Consequently, a greater proportion of the sites occupied by the old stand were cleared in the early time periods, most all of the present stands having been cleared by the third time period (Table 21).

Total volume cut, increased slightly from the 3 to the 6 percent rate, but decreased by approximately 3 million board feet for the 10 percent rate (Table 12). This can in part be explained by the fact that an additional 16 percent of the area of the present stands was clear cut in the initial time period (initial time period at 10 percent over the same period at 3 and 6 percent). By the second time period, 77 percent had been cleared. In comparison, 62 and 67 percent was cleared in the 3 and 6 percent solutions (Table 21). This would mean that in the latter time periods, there would be less volume available from that forest with an alternative rate of 10 percent. Greater percentages of new stands would be generated earlier and these new stands were not allowed to develop over as long a period of time as were those at the 3 and 6 percent rates (Table 21).

In this solution, nearly 10,000 MBF was cut in each of the first three time periods for each alternative rate. The manner in which the stands were treated to provide this volume is not at all consistant in each period among alternative rates (Table 23). Stand treatment schedules are dictated by the restrictions imposed upon the model and by the rotation in the next and succeeding time periods. These rotations are dictated by the alternative rate. The schedule

Table 23. Acreage treated by time period, treatment and alternative rate, solution two

Time Period	Treatment	3%	6%	10%
	Thin	604	571	557
1	Clear	592	595	867
	Leave	468	498	240
	Thin	554	396	145
2	Clear	446	522	638
	Leave	664	746	881
	Thin	83	103	0
3	Clear	626	537	1026
	Leave	955	1024	638
	Thin	129	46	0
4	Clear	509	605	638
	Leave	1026	1013	1026
	Thin	56	0	0
5	Clear	483	522	1026
	Leave	1125	1142	638

in the latter time periods depends upon the treatment of the old stands in the early time periods.

Solution 3

This solution is indicative of the volumes available in the next and succeeding rotations resulting from rotations dictated by increasing the discount rate and setting the rotations at maximum soil rent. This solution required a minimum output of 7,500 MBF per ten year period. It is noted that the rate of cutting of the present stand decreases as the alternative rate increases, and that total volume cut decreases (Table 21). The explanation for this can be developed by examining the rotations which follow the removal of the present stand. This solution required a minimum output of 7,500 MBF per ten year period. The 3 percent rate allows the new stands to develop for 30 years on sites 70 and 80, and, with two thinnings to 40 years on site 90. The 6 percent rate allows the new stands to develop for 30 years on sites 70 and 80, and with one thinning to 30 years on site 90. The 10 percent rate allows the new stands to develop for only 20 years on all sites with no intermediate thinnings providing volume. Consequently, large differences in volume available at harvest in succeeding rotations was accrued among the three alternative rates. For example, the difference in volume between the 3 percent and 10 percent rate from site 70 was 4.7 MBF per acre; on site 80, 7.1 MBF per acre, and on site 90, 15.5 MBF per acre. Thus, the 3 percent alternative rate allows the cutting of present stands in the early time periods because in the next and succeeding rotations, the stands could and were allowed to develop the required volume. Because of the greater volume it is possible at 6 percent to cut a greater percentage of the present stand than at 10 percent. The differences in volume produced at 6 percent is greater than that at 10 percent. Since the 10 percent rate dictates that clear cutting of the succeeding stands must be accomplished 20 years after establishment, considerably less volume would be available in the latter time periods of this analysis. Thus a greater percentage of the present stand is carried for a longer period of time at 10 percent than in either of the preceeding rates in order that the required volume can be produced (Table 21).

In general, treatment of the stands required that, in the 10 percent alternative rate, greater acreages be thinned early, and in general, less acreage left for additional volume growth (Table 24).

Total volume cut decreased from the 3 percent rate to the 10 percent rate by some 7,216 MBF, 4.6 MBF of which can be attributed to differences in treatment in the first two time periods which in turn can be attributed to the alternative rate (Tables 12 and 24).

Solution 4

This solution for all alternative rates illustrates the effect upon the derived programs of changing the alternative rate when the

Table 24. Acreage treated by time period, treatment and alternative rate, solution three

Time Period	Treatment	3%	6%	10%
	Thin	581	355	716
1	Clear	704	736	445
	Leave	379	573	503
	Thin	169	92	395
2	Clear	578	485	450
	Leave	917	1087	819
	Thin	123	328	421
3	Clear	382	211	675
	Leave	1159	1125	568
	Thin	123	133	144
4	Clear	581	761	758
	Leave	960	770	762
	Thin	16	0	0
5	Clear	701	692	906
	Leave	947	972	758

model is not restricted by a given quantity of "operating resources", or by personal management constraints. The most noticeable change is in the treatment of the present stands (Table 21). With the exception of the insignificant 2 percent of the old stand remaining in the second time period at 3 percent, all of the present stand was cut by the second time period, moving directly into the next rotation (Table 21). The significance of this is that given land as fixed, and all necessary resources and no personal management constraints, the most profitable situation is to generate the new stands as quickly as possible. The rate of clearing of the present stands explains the great difference in total volume cut as the alternative rate increases. As in prior explanations, there was more volume available in the succeeding time periods as a direct result of carrying the new stands longer at the lower alternative rate (Table 12).

General Comparative Discussion

As the study proceeded from solution one through four, restrictions were removed from the model to observe the effect of these restrictions upon the derived optimum program. As the study proceeded from solution one through solution four, increasingly greater percentages of the original stand were removed in the early time periods, allowing the new stands to be generated. With the exception of solution three, which required a minimum quantity of volume output, there was a general trend to clear more of the original stand earlier

Table 25. Acreage treated by time period, treatment and alternative rate, solution four

Time Period	Treatment	3%	6%	10%
	Thin	190	190	144
1	Clear	1262	1474	1520
	Leave	212	0	0
	Thin	0	0	0
2	Clear	372	190	144
	Leave	1292	1474	1520
	Thin	123	139	0
3	Clear	30	0	1520
	Leave	1551	1525	144
	Thin	139	16	0
4	Clear	1139	1474	144
	Leave	386	174	1520
	Thin	16	0	0
5	Clear	479	190	1520
	Leave	1169	1474	144

as the constraints were relaxed. There was also a tendency to either increase volume output or for volume output to remain about the same in any given solution except for the wide range of differences observed in solution four. As long as the programs were committed to a specific set of constraints, volume did not vary greatly among alternative rates. However, as the constraints were relaxed, volume output tended to increase but finally decrease at the 10 percent rate. This is the compound effect of constraints and alternative rate. As long as resources were restricting, output was limited. The present stands were retained for a longer period of time with more volume being available in the latter time periods for all alternative rates. When the alternative rate increased, and as restrictions were relaxed, the present stands were cleared at a more rapid rate. There was a continuing decrease in volume available in latter time periods at these higher alternative rates.

Solution four exhibited the full effect of changing the alternative rate, without restricting resources or volume. These wide differences of volume production can be explained by noting the required duration of the succeeding rotation which is dictated by the alternative rate. The succeeding stands were allowed to develop for a longer period of time at the 3 percent rate, to a lesser extent at the 6 percent rate, and to a much lesser extent at the 10 percent rate. The differences in the volume developed in these succeeding rotations was rather large. For example, the difference between the

3 percent and 10 percent rate on site 70 was 4.7 MBF per acre, on site 80, 7. 1 MBF, and on site 90, 15.5 MBF, a rather large difference in each case.

There are at least two other points that can be developed, both of which resulted as residual information from the several different solutions of the model. First, there is evidence which suggests that the rotation age for a given site, age, and initial stocking level cannot be predetermined if there are several alternatives competing for limited resources and if there are volume constraints involved. This evidence suggests that several different rotations can be optimal for a given site occupied by a specific age stand. Note the solution for the basic matrix in which the present stand, age 25 and 45 on site 80 was assigned more than one alternative for each of the age class. Eleven acres of the 25 year old stand were clear-cut at 55 years, while nineteen acres were cut at 45 years. One hundred, forty-three acres of the 45 year old stand were clear-cut at 45 years. This is the result of examining the total management of the forest, not just the management of stands, with the purpose of maximizing some objective statement for the forest. The parts must contribute to the whole. What may be optimal management for a single stand, may not be optimal when viewed in terms of the total enterprise. Dual treatment of present stands occurred in every solution except the one in which resources and volume output were not restricting in any time period. The optimum program defined in this instance was the same as it would have been if the activities of maximum present worth had been selected from those representing each age class on each site. Thus, when resources or volume are not restricting, the treatment that is optimum for a given stand when viewed by itself, can also be considered optimum in the total enterprise sense.

CONCLUSIONS

Applicability of the Model

It is concluded that linear programming theory is applicable to evaluating the timber harvesting alternatives presented by the forest enterprise to determine an optimum program which will maximize total present worth subject to a given set of constraints. In this study, a specific forest enterprise was described. The objectives of management were defined. The linear programming model was developed which provided for the particular constraints of the forest manager and which provided for time jointness in the use of resources and the output of products. The resulting model was solved and a feasible optimum program was defined. Changes were made in the basic model. The solutions provided additional feasible optimum programs.

The compatibility of the optimum program with established concepts of forest management is significant. Regulation is one of the aspects of the management of forests considered by foresters to be of prime importance. However, many foresters are oriented to volume rather than value. Thus, a prime objective of many foresters is to maximize wood yields, fully expecting this objective to maximize income. This rationale is valid only if fixed resources are free or if profit maximization is irrelevant. The optimum program described in this study may be considered a regulatory plan to maximize present worth. If it should happen to correspond to a regulatory plan aimed at some physical objective, it would be purely coincidental.

Contribution of the Study to Forest Management

The prime contribution of this study is that it serves as a basis for a regulatory model based upon the economic principle of maximizing profit subject to the specific constraints of the resource owner. With modifications to the mill capacity category, and with additional machine capacity, specific products, such as veneer logs, stave bolts, pulpwood, poles, and piling, and just wood can be incorporated into the model.

Decision making is a difficult process for the forest manager who is able to comprehend to a degree, the complex situation in which the modern forest firm finds itself. This study provides the manager with a means of describing a real situation and arriving at conclusions which can be described numerically, based upon those economic principles which deal with the allocation of scarce resources among alternative uses. It provides the forest manager with a decision making tool that insures an optimum program in an economic sense, given a set of constraints and an objective.

The study provides the forest manager with a means of evaluating the cost of choosing one alternative course of action over another, or in another sense of evaluating the cost of imposing specific constraints upon the profit maximizing objective.

Critical Review

The application of the theory of linear programming, once understood, is a straight forward, step by step, procedure. With certain adjustments, concerned with describing and valuing the processes which are characteristic of forest management alternatives, its applicability to allocation problems of the forest enterprise can be of significant value to the forest manager. This is especially true in the instance of the very complex situation.

It is not the application, in a mechanical sense, that is so difficult, but the availability of the required necessary data and management's evaluation of demand and of technology in distant time periods. These are perhaps the most important and thus can be the weakest parts of the analysis. Linear programming is simply an allocation model which, when accurately defined, will provide an optimum solution.

This review then really divides naturally into two parts, (1) a review of the basic information and of the assumptions used to describe the processes, and (2) a review of the model regarding the adequacy of the alternatives describing the forest enterprise. Necessarily the review of the basic information will be further separated into a discussion of (1) the assumptions incorporated relative to stand development, and (2) management's evaluation of future demand and of future technology, which must be especially critical and sensitive in an enterprise where yield in any time period is jointly dependent upon continuous inputs over a long period of time.

The review of the model described for the Seward Forest will be generally concerned with the simplification of the model and with the need for viewing additional opportunities for stands occupying the sites in succeeding periods.

Basic Information and Assumptions

In this study, the basic information was assumed to be perfect in that management had the perception of a prophet in the evaluation of future conditions. Therefore, more emphasis was placed upon examining the methodology of adopting the general linear programming model to evaluating forest management alternatives than was placed upon the precision of management's evaluation of the future. Assumptions included within the study will be critically reviewed.

First, there is the assumption that stand parameters will not change after thinning. The assumed principal result of thinning is not so much the maintenance of vigor, or of up grading quality, but the immediate yield resulting from the inputs of labor and capital. Specific information needs to be developed for each operation so that the change in these parameters can be evaluated. Once this change can be described, it will be reflected in both present dollar value, and in incremental increases.

The assumption that the thinned stand will react as do natural stands of the same density is not acceptable in a silvicultural sense. This would mean that the parameters of the thinned natural stand with a higher initial density would be the same as the unthinned natural stand of lower initial density. This is probably not so.

The assumption that prices will remain constant would not be acceptable in a realistic analysis. Based upon historical data and upon a recent analysis of future demand for wood, one observes that the price of stumpage relative to the price of all competing commodities (index years 1957-59) from 1920 to 1963, has experienced a relatively fast rate of increase (U.S.F.S. 1965). This in the face of a total annual lumber consumption rate which has remained relatively steady for the past 30 years. Future outlook is that consumption of wood will increase, based on population increase. However, in this study, sawtimber and pulpwood stumpage was considered to be the product mix. The national outlook for lumber consumption by the year 2000 is a 43 percent increase over the 1962 level of consumption. Demand for pulpwood, plywood and veneer is predicted to increase by 2.7 times over the 1962 consumption level. Assuming static prices may very well be too conservative in view of the preceding discussion, and in view of the following information. The U. S. F. S. "Timber Trends in the U. S." assumes that gross national product will increase 3.5 times, per capita income will nearly double, and total use of raw materials will roughly double. It was assumed that prices of timber products in relation to competing materials would remain at about the levels of the 1950's which was the high point for stumpage prices U.S.F.S. (1965).

Static technology was assumed. A given efficiency level within a given state of technology was assumed unchanging. In a competitive economy, new and better ways of doing things can be considered one of the focal points of the vigor of the economy. Price may determine resource allocation, but innovation which leads to a general increase in efficiency in an industry and its adoption determines success or failure of individual firms within an industry.

The Linear Programming Model for the Seward Forest

The model developed for the Seward Forest is at best a simplified description of the alternatives available for resource use. The simplification, however, was the direct result of limited machine capacity. For instance, the only rotations viewed for the next and succeeding occupancy of the sites were those, selected from several arrays of alternatives, which provided maximum present worth in each array. In the instance where volume output and where resources were not restricting, these rotations are valid. However, in the case of output constraints and resource restrictions, the rotation described by the alternative of maximum present worth may not be optimum. Too, several stands of older age classes were assigned only one treatment schedule. In an effort to economize on machine capacity, some of the defined treatments for the stands were omitted as being probably not relevant.

The omission of alternatives, which are pertinent in an output sense, provides fewer choices for available output in critical periods of volume need. This may result in a lower present worth being defined than would have been if the alternatives were included. Thus it would appear that it would be very desirable to view additional alternatives. Specifically several alternatives for each stand, both for the existing stands, and for succeeding stands which occupy the sites.

In viewing this model and its application several refinements which might be incorporated become apparent.

1. The model should describe the complete array of alternatives available to the forest enterprise, i.e., regeneration by the different methods, cultural practices which reflect intermediate treatments, and harvesting activities. The product mix should be described completely, sawtimber, pulpwood, veneer, logs, poles, piling, etc., with transfer activities included to store production between time periods.
2. The model should be described in a manner so that there will be more flexibility in land treatment in the succeeding rotations, i.e., more than one rotation for a given site.
3. The alternatives should be described in terms of management's evaluation of the future, i.e., the price of an alternative should reflect the imposition of a probability statement relative to management's evaluation of the future. To make this evaluation of the effect of the future more meaningful, perhaps it should be a series of long-run views with short-run

adjustments. For example, a long-run view of management's evaluation of the future with an analysis of how this evaluation might affect growing stock levels in future time periods.

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Appendix Table 1. Method of defining the alternatives

Present stand age (years)	Basic alternative (t)	Succeeding alternatives t_{10}	t_{20}	t_{30}	t_{40}
	CLEAR 25				
		CLEAR 35	CLEAR 45	CLEAR 55	
	THIN 25	THIN 35	THIN 45	THIN 55 →	CLEAR 65
25		LEAVE 35 — CLEAR 45	LEAVE 45 — CLEAR 55	LEAVE 55 →	CLEAR 65
		CLEAR 35	CLEAR 45	CLEAR 55	
	LEAVE 25	THIN 35	THIN 45	THIN 55 →	CLEAR 65
		LEAVE 35	CLEAR 45	CLEAR 55	
			LEAVE 45	LEAVE 55 →	CLEAR 65
		THIN 45	CLEAR 55		
			THIN 55 → CLEAR 65		
		CLEAR 45	CLEAR 55		
	LEAVE 35	THIN 45	THIN 55 → CLEAR 65		
		LEAVE 45	CLEAR 55		
			LEAVE 55 → CLEAR 65		
35		CLEAR 45			
	THIN 35	THIN 45	CLEAR 55		
			THIN 55 → CLEAR 65		
	CLEAR 35				
	CLEAR 45				
		CLEAR 55			
45	THIN 45	THIN 55 → CLEAR 65			
	LEAVE 45	CLEAR 55			
		LEAVE 55 → CLEAR 65			
	CLEAR 55				
55					
	LEAVE 55	CLEAR 65			
65	CLEAR 65				
75	CLEAR 75				

Appendix Table 2. Determination of costs per hour for equipment

Tractors, Farmall 300		
Assume:		
Delivery price	$3000.00	
Depreciation period	5 years	
Depreciation hours	10,000 hours	
Average investment	$1800.00	
Fixed costs per hour:		
Interest and insurance @ 5% of average investment	.09	
Depreciation	.30	
Total fixed costs per hour	$0.39	
Operating costs per hour:		
Gas, oil, etc.	.25	
Repairs	.35	
Total var. costs per hour	$0.55	
Total fixed and variable cost per hour		$0.94
Trucks, International 2½ ton		
Assume:		
Delivery price	$3000.00	
Depreciation period	5 years	
Depreciation hours	10,000 hours	
Average investment	$1800.00	
Fixed costs per hour:		
Interest and insurance @ 5% of average investment	.09	
Depreciation	.30	
Total fixed costs per hour	$0.39	
Operating costs per hour:		
Gas, oil, etc.	.55	
Repairs	.30	
Total var. costs per hour	$0.85	
Total fixed and variable cost per hour		$1.24
Power saw		
Assume:		
Delivery price	$ 350.00	
Depreciation period	1 year	
Depreciation hours	1,000 hours	
Average investment	$ 350.00	
Fixed costs per hour:		
Interest @ 3% of average investment	.0066	
Depreciation	.22	
Maintenance	.0625	
Total fixed costs per hour	$0.2891	or $0.29
Operating costs per hour:		
Gas, oil, etc.	.20	
Total var. costs per hour	$0.20	
Total fixed and variable cost per hour		$0.49

Appendix Table 3. Method of budgeting the alternatives

Site 70 Stand Age 35

Activity:		THIN 35	THIN 45	THIN 55	CLEAR 65	LEAVE 10
Yields	Cords:	4.8	1.6	0.8	2.4	0
	MBF:	1.2	0.7	0.9	10.6	0

Revenue @ 35:

4.8 cords @ $15.75		$ 75.60
1.2 MBF @ $52.00		62.40
	Total gross revenue	$138.00

Resource Requirements and Costs @ 35:

		Hrs/ Unit	No. Units	Total hrs/ unit	Fixed costs unit	Vari. costs unit	Total vari. costs	Total costs f & v
Labor	cords	4.00	4.8	19.2	1.20			23.04
	MBF	12.00	1.2	14.4	1.20			17.28
Truck	cords	1.52	4.8	7.3	1.86	1.27	6.10	8.93
	MBF	0.67	1.2	0.8	0.83	0.60	0.72	1.00
Tract.	cords	1.50	4.8	7.3	1.41	0.83	3.98	6.77
	MBF	1.00	1.2	1.2	0.94	0.55	0.66	1.13
Power saws	cords	2.60	4.8	12.5	1.16	0.41	1.97	5.57
	MBF	1.30	1.2	1.6	0.76	0.38	0.46	0.91
							$13.89	$64.63

Revenue @ 35 less variable costs in this time period = $124.11

This procedure was duplicated for each time period, i.e., THIN 45, THIN 55, with gross revenues at those periods of $61.60 and $59.40 respectively. Upon reaching CLEAR 65, the following calculations completed the budget development for this alternative.

Revenue @ 65

2.4 cords @ $ 7.00		$ 16.80
10.6 MBF @ $52.00		551.20
	Total gross revenue	$568.00

Resource Requirements and Costs @ 65:

Contract logging capital ($20.00/MBF)(10.6MBF) = $212.00

		Resources & Restrictions										Site 70	Age 25				
		Disposal	Age	Ba/ac	Time	Unit	Quantity	1	2	3	4	5	6	7	8	9	10
Land and timber	Site 70	109	25	117		Acres	144	1	1	1	1	1	1	1	1	1	1
		110	35	108		Acres	25										
		111	45	98		Acres	78										
		112	55	100		Acres	12										
		113	75	120		Acres	5										
	Site 80	114	25	117		Acres	30										
		115	35	113		Acres	212										
		116	45	117		Acres	474										
		117	55	105		Acres	179										
		118	65	82		Acres	136										
		119	75	114		Acres	230										
	Site 90	120	25	145		Acres	16										
		121	35	124		Acres	10										
		122	45	125		Acres	30										
		123	55	80		Acres	32										
		124	65	119		Acres	36										
		125	75	116		Acres	15										
Labor		126			t	Hours	76,730								71.6	71.6	71.6
		127			t10	Hours	0			53.6	53.6	53.6				13.2	13.2
		128			t20	Hours	0				14.4	14.4	71.2	71.2			15.6
		129			t30	Hours	0					13.2		54.0			
Capital		130			t	Dollars	330,000	20.00							37.13	37.13	37.13
		131			t10	Dollars	330,000		20.00						20.00		
		132			t20	Dollars	330,000			20.00						20.00	
		133			t30	Dollars	330,000	20.00			20.00		20.00				20.00
		134			t40	Dollars	330,000		20.00			20.00		20.00	20.00		
Mill cap.		135			t	MBF	10,000	4.2							1.3	1.3	1.3
		136			t10	MBF	10,000		9.6	3.7	3.7	3.7			6.3	.7	.7
		137			t20	MBF	10,000			8.9	1.2	1.2	5.5	5.5		8.8	1.3
		138			t30	MBF	10,000	5.3			9.8	1.1	9.9	4.5			10.0
		139			t40	MBF	10,000		5.3			11.2		11.0	5.3		
Trucks		140			t	Hours	24,800								22.2	22.2	22.2
		141			t10	Hours	0			6.0	6.0	6.0				2.3	2.3
		142			t20	Hours	0				0.8	0.8	5.7	5.7			0.9
		143			t30	Hours	0					0.7		3.0			
Tractors		144			t	Hours	38,840								22.3	22.3	22.3
		145			t10	Hours	0			7.2	7.2	7.2				2.5	2.5
		146			t20	Hours	0				1.2	1.2	7.5	7.5			1.3
		147			t30	Hours	0					1.1		4.5			
Power saws		148			t	Hours	46,800								38.1	38.1	38.1
		149			t10	Hours	0			10.8	10.8	10.8				4.0	4.0
		150			t20	Hours	0				1.6	1.6	10.6	10.6			1.7
		151			t30	Hours	0					1.4		5.9			
Cont. log. cap		152			t	Dollars	110,000	84.00									
		153			t10	Dollars	0		192.00						126.00		
		154			t20	Dollars	0			178.00						176.00	
		155			t30	Dollars	0	106.00			196.00		198.00				200.00
		156			t40	Dollars	0		106.00			224.00		220.00	106.00		
					Cj	Dollars	3%	492.97	493.98	508.64	475.84	465.65	443.00	501.50	611.23	627.46	604.68
							6%	375.12	304.89	281.54	241.18	216.64	190.53	193.86	455.44	433.80	398.49
							10%	342.05	197.74	156.54	124.48	124.84	74.92	78.69	379.27	339.57	311.67

		Site 70	Age 25				Site 70	Age 35				Site 70 Age 45		70 Age 55
	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Disposal														
109	1	1	1	1										
110					1	1	1	1	1	1				
111											1	1	1	
112														1
113														
114														
115														
116														
117														
118														
119														
120														
121														
122														
123														
124														
125														
126	71.6	71.6	71.6	71.6				33.60	33.60	33.6		20.8	20.8	
127	13.2	13.2	13.2			46.8	46.8		14.80	14.8			14.0	
128	15.6	15.6					13.2			14.0				
129	13.2													
130	37.13	37.13	37.13	37.13	20			13.89	13.89	13.89	20.00	3.45	3.45	20.00
131								20.00				20.00		
132				20.00		20.00			20.00				20.00	
133			20.00		20.00		20.00			20.00		20.00		20.00
134	20.00	20.00						20.00			20.00			
135	1.3	1.3	1.3	1.3	4.5			1.2	1.2	1.2	8.5	1.6	1.6	10.8
136	0.7	0.7	0.7			2.5	2.5	6.2	0.7	0.7		9.4	1.1	
137	1.3	1.3		8.6		8.3	0.9		8.2	0.9			11.0	
138	1.1		11.2		5.3		9.5			10.6	5.3			5.3
139	10.9	12.2						5.3				5.3		
140	22.2	22.2	22.2	22.2				8.1	8.1	8.1		1.7	1.7	
141	2.3	2.3	2.3			8.1	8.1		2.9	2.9			1.0	
142	0.9	0.9					1.5			7.5				
143	0.7													
144	22.3	22.3	22.3	22.3				8.5	8.5	8.5		2.2	2.2	
145	2.5	2.5	2.5			8.8	8.8		3.1	3.1			1.4	
146	1.3	1.3					1.8			2.1				
147	1.1													
148	38.1	38.1	38.1	38.1				14.1	14.1	14.1		3.1	3.1	
149	2.5	2.5	2.5			14.2	14.2		5.1	5.1			1.9	
150	1.3	1.3					2.8			3.3				
151	1.4													
152					90.00						170.00			216.00
153								124.00				188.00		
154				172.00		166.00			164.00				220.00	
155			224.00		106.00		190.00			212.00	106.00			106.00
156	218.00	244.00						106.00				106.00		
3%	572.18	569.35	592.99	583.66	511.37	491.28	454.53	480.44	513.89	497.66	605.97	547.85	512.23	717.07
Cj 6%	370.60	367.21	388.28	401.43	393.52	267.24	223.78	325.71	315.44	281.26	488.12	361.57	288.62	600.02
10%	327.51	325.55	305.20	317.54	360.45	145.85	113.92	250.52	216.21	188.42	455.10	257.09	173.98	566.95

	70 Age 75						Site 80	Age 25						
	25	26	27	28	29	30	31	32	33	34	35	36	37	38
Disposal														
109														
110														
111														
112														
113	1													
114		1	1	1	1	1	1	1	1	1	1	1	1	1
115														
116														
117														
118														
119														
120														
121														
122														
123														
124														
125														
126									29.2	29.2	29.2	29.2	29.2	29.2
127				63.6	63.6	63.6				13.6	13.6	13.6	13.6	13.6
128					22.0	22.0	78.0	78.0			22.4	22.4	22.4	
129						18.8		26.0				18.4		
130	20.00	20.00							12.32	12.32	12.32	12.32	12.32	12.32
131			20.00						20.00					
132				20.00	20.00					20.00				
133	20.00	20.00					20.00				20.00			20.00
134			20.00			20.00		20.00	20.00			20.00	20.00	
135	14.2	4.3							1.0	1.0	1.0	1.0	1.0	1.0
136			10.1	3.9	3.9	3.9			8.2	1.4	1.4	1.4	1.4	1.4
137				12.7	1.7	1.7	5.7	5.7		11.7	1.6	1.6	1.6	
138	5.3	8.9			14.5	1.5	14.5	2.0			13.8	1.4		16.0
139			8.9			16.6		15.1	8.9			16.3	18.0	
140									7.2	7.2	7.2	7.2	7.2	7.2
141				9.0	9.0	9.0				3.5	3.5	3.5	3.5	3.5
142					1.7	1.7	7.4	7.4			2.3	2.3	2.3	
143						1.3		2.1				1.5		
144									7.5	7.5	7.5	7.5	7.5	7.5
145				10.2	10.2	10.2				4.0	4.0	4.0	4.0	4.0
146					2.3	2.3	9.3	9.3			2.8	2.8	2.8	
147						1.8		2.8				2.0		
148									12.5	12.5	12.5	12.5	12.5	12.5
149				16.0	16.0	16.0				6.2	6.2	6.2	6.2	6.2
150					3.2	3.2	13.6	13.6			4.2	4.2	4.2	
151						2.5		3.9				2.7		
152	284.00	86.00												
153			202.00						164.00					
154				254.00						234.00				
155	106.00	178.00			290.00		290.00				276.00			320.00
156			178.00			332.00		302.00	178.00			326.00	360.00	
3%	932.47	563.46	603.05	694.30	655.94	589.92	588.70	555.46	633.30	657.74	627.26	596.48	591.83	623.08
Cj 6%	814.62	402.91	362.91	378.46	322.21	249.93	246.88	205.64	412.41	381.14	330.88	295.41	290.50	321.65
10%	781.55	360.16	234.06	206.12	162.20	130.56	94.16	81.14	301.48	243.36	202.98	202.16	199.39	195.54

Appendix Table 4. Basic Matrix (Continued) Page Number 111

	80 Age 25			Site 80 Age 35					Site 80 Age 45			80 Age 55	80 Age 65	80 Age 75
	39	40	41	42	43	44	45	46	47	48	49	50	51	52
Disposal														
109														
110														
111														
112														
113														
114	1													
115		1	1	1	1	1	1	1						
116									1	1	1			
117												1		
118													1	
119														1
120														
121														
122														
123														
124														
125														
126	29.2					18.4	18.4	18.4		42.8	42.8			
127				54.0	54.0		22.4	22.4			17.6			
128					18.0			26.8						
129														
130	12.32	20.00				3.14	3.14	3.14	20.00	7.06	7.06	20.00	20.00	20.00
131			20.00			20.00				20.00				
132	20.00			20.00			20.00				20.00			
133		20.00			20.00			20.00	20.00			20.00	20.00	20.00
134			20.00			20.00				20.00				
135	1.0	10.1				1.4	1.4	1.4	14.1	3.3	3.3	15.4	12.8	20.3
136			16.1	4.4	4.4	13.4	1.8	1.8		14.5	1.4			
137	14.0			15.1	1.5		15.7	2.2			16.6			
138		8.9			16.6			16.6	8.9			8.9	8.9	8.9
139			8.9			3.9				8.9				
140	7.2					1.5	1.5	1.5		3.4	3.4			
141				3.4	3.4		1.5	1.5			1.2			
142					1.0			1.7						
143														
144	7.5					2.0	2.0	2.0		4.5	4.5			
145				4.9	4.9		2.0	2.1			1.7			
146					1.5			2.4						
147														
148	12.5					2.7	2.7	2.7		6.4	6.4			
149				6.5	6.5		2.8	2.8			2.3			
150					2.0			3.2						
151														
152		202.00							282.00			308.00	256.00	256.00
153			322.00			268.00				290.00				
154	280.00			302.00			314.00	332.00			332.00			
155		178.00			332.00				178.00			178.00	178.00	178.00
156			178.00			178.00				178.00				
Cj 3%	651.80	750.26	785.73	726.96	660.13	756.70	718.42	655.35	965.26	903.49	831.98	1018.16	879.46	1270.16
6%	332.71	589.71	449.99	391.91	314.05	497.16	401.18	325.48	804.71	632.63	506.43	857.61	718.91	1109.61
10%	224.91	546.96	328.70	208.27	151.60	348.26	236.12	180.47	761.96	475.26	336.73	814.86	676.16	1066.96

							Site 90	Age 25							
		53	54	55	56	57	58	59	60	61	62	63	64	65	66
	Disposal														
	109														
	110														
	111														
	112														
	113														
	114														
	115														
	116														
	117														
	118														
	119														
	120	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	121														
	122														
	123														
	124														
	125														
	126								58.0	58.0	58.0	58.0	58.0	58.0	58.0
	127			108.8	108.8	108.8				35.6	35.6	35.6	35.6		35.6
	128	14.0			37.2	37.2	142.0	142.0			26.0	26.0			26.0
	129	32.6	14.0			22.00		24.8	14.0			24.8			
	130	20.00							20.80	20.80	20.80	20.80	20.80	20.80	20.80
	131		20.00						20.00						
	132			20.00	20 00					20.00				20.00	
	133						20.00				20.00		20.00		
	134	20.00				20.00		20.00				20.00			20.00
	135	8.3							2.6	2.6	2.6	2.6	2.6	2.6	2.6
	136		20.4	7.8	7.8	7.8			15.3	2.5	2.5	2.5	2.5		2.5
	137	0.5		19.0	2.9	2.9	11.0	11.0		18.6	2.0	2.0		22.5	2.0
	138	2.6	0.5		20.0	1.0	23.3	2.0	0.5		23.3	2.0	28.6		
	139	19.1	2.6	0.5		24.2		23.8	2.6	0.5		23.8		0.5	25.7
	140								11.9	11.9	11.9	11.9	11.9	11.9	11.9
	141			11.0	11.0	11.0				3.9	3.9	3.9	3.9		3.9
	142	3.3			2.8	2.8	11.2	11.2			2.1	2.1			2.1
	143	3.5	3.3			1.4		1.6	3.3			1.6			
	144								12.7	12.7	12.7	12.7	12.7	12.7	12.7
	145			13.6	13.6	13.6				4.6	4.6	4.6	4.6		4.6
	146	3.5			3.8	3.8	14.8	14.8			2.8	2.8			2.8
	147	4.3	3.5			2.0		2.3	3.5			2.3			
	148								20.8	20.8	20.8	20.8	20.8	20.8	20.8
	149			25.8	25.8	25.8				6.9	6.9	6.9	6.9		6.9
	150	5.9			5.4	5.4	20.8	20.8			3.9	3.9			3.9
	151	6.3	5.9			2.6		7.8	5.9			7.8			
	152	166.00													
	153		408.00						306.00						
	154			156.00						372.00				460.00	
	155				416.00		466.00				466.00		572.00		
	156	382.00				484.00		476.00				476.00			514.00
	3%	909.79	1082.66	1088.02	1019.18	961.72	979.90	865.62	1094.83	1065.07	1034.57	922.32	1085.72	1066.26	908.07
Cj	6%	667.74	677.31	602.08	515.06	457.31	419.51	338.81	741.33	641.61	567.94	487.86	580.72	629.46	478.75
	10%	600.46	441.65	332.48	266.68	260.58	162.34	189.46	553.93	428.98	366.36	361.29	365.46	410.03	357.79

			Site 90	Age 35			Site 90 Age 45			90 Age 55	90 Age 65	90 Age 75		
		67	68	69	70	71	72	73	74	75	76	77	78	79	80
	Disposal														
	109														
	110														
	111														
	112														
	113														
	114														
	115														
	116														
	117														
	118														
	119														
	120														
	121	1	1	1	1	1	1								
	122							1	1	1					
	123										1				
	124											1			
	125												1		
	126				42.0	42.0	42.0		52.4	52.4				-100	
	127		74.4	74.4		24.8	24.8			24.4					-100
	128	14.0		58.0			24.0	14.0			14.0	14.0	14.0		
	129	32.6			14.0			32.6	14.0		32.6	32.6	32.6		
	130	20.00			5.98	5.98	5.98	20.00	7.68	7.68	20.00	20.00	20.00	120.00	
	131				20.00				20.00						120.00
	132		20.00			20.00				20.00					
	133			20.00			20.00								
	134	20.00						20.00			20.00	20.00	20.00		
	135	17.4			3.4	3.4	3.4	21.1	4.2	4.2	14.2	26.3	28.0		
	136		6.1	6.1	19.1	2.1	2.1		21.9	2.0					
	137	0.5	24.2	4.8		21.6	2.0	0.5		23.2	0.5	0.5	0.5		
	138	2.6		23.0	0.5		23.3	2.6	0.5		2.6	2.6	2.6		
	139	19.1	0.5		2.6	0.5		19.1	2.6	0.5	19.1	19.1	19.1		
	140				2.8	2.8	2.8		3.6	3.6					
	141		4.6	4.6		1.6	1.6			1.5					
	142	3.3		3.4			1.3	3.3			3.3	3.3	3.3		
	143	3.5			3.3			3.5	3.3		3.5	3.5	3.5		
	144				3.9	3.9	3.9		5.0	5.0					
	145		6.6	6.6		2.3	2.3			2.0					
	146	3.5		5.0			2.0	3.5			3.5	3.5	3.5		
	147	4.3			3.5			4.3	3.5		4.3	4.3	4.3		
	148				5.2	5.2	5.2		6.8	6.8					
	149		8.7	8.7		7.8	7.8			2.9					
	150	5.9		5.1			2.6	5.9			5.9	5.9	5.9		
	151	6.3			5.9			6.3	5.9		6.3	6.3	6.3		
	152	348.00						422.00			284.00	526.00	560.00		
	153				382.00				438.00						
	154		484.00			432.00				464.00					
	155			460.00			466.00								
	156	382.00						382.00			382.00	382.00	382.00		
	3%	1244.39	1117.54	1006.10	1162.10	1062.58	951.08	1439.59	1315.02	1148.92	1074.49	1707.89	1786.49	-120.00	- 93.00
Cj	6%	1002.34	599.26	625.97	780.74	615.10	496.38	1197.54	906.24	681.71	832.44	1465.84	1544.44	-120.00	- 67.00
	10%	935.06	314.02	230.97	567.41	388.29	304.54	1130.26	667.36	441.85	765.16	1395.56	1477.16	-120.00	- 46.00

Appendix Table 4. Basic Matrix (Continued) Page Number 114

	Buy Labor			Buy Truck				Buy Tractor				Buy Saws		
	81	82	83	84	85	86	87	88	89	90	91	92	93	94
Disposal														
109														
110														
111														
112														
113														
114														
115														
116														
117														
118														
119														
120														
121														
122														
123														
124														
125														
126														
127														
128	-100													
129		-100												
130			39.00				39.00				29.00			
131				123.00				94.00				49.00		
132	120.00				123.00				94.00				49.00	
133		120.00				123.00				94.00				49.00
134														
135														
136														
137														
138														
139														
140			-100											
141				-100										
142					-100									
143						-100								
144							-100							
145								-100						
146									-100					
147										-100				
148											-100			
149												-100		
150													-100	
151														-100
152														
153														
154														
155														
156														
Cj 3%	- 67.00	- 49.00	- 39.00	- 95.00	- 68.00	- 51.00	- 39.00	- 73.00	- 52.00	- 39.00	- 29.00	- 38.00	- 27.00	- 20.00
Cj 6%	- 37.00	- 21.00	- 39.00	- 69.00	- 38.00	- 21.00	- 39.00	- 52.00	- 29.00	- 16.00	- 29.00	- 27.00	- 15.00	- 9.00
Cj 10%	- 18.00	- 7.00	- 39.00	- 47.00	- 18.00	- 7.00	- 39.00	- 36.00	- 14.00	- 5.00	- 29.00	- 19.00	- 7.00	- 3.00

Appendix Table 4. Basic Matrix (Continued) Page Number 115

		Buy Logging Capital					Artificial Activities					Site 80 75 Yr. Lob.		
	95	96	97	98	99	100	101	102	103	104	105	106	107	108
Disposal														
109														
110														
111														
112														
113														
114														
115														
116														
117														
118														
119											1	1	1	1
120														
121														
122														
123														
124														
125														
126														
127														
128														
129														
130	3.00													
131		3.00									20.00			
132			3.00									20.00		
133				3.00									20.00	
134					3.00						20.00			20.00
135						-1								
136							-1				26.0			
137								-1				31.0		
138									-1				35.0	
139										-1	8.9			35.0
140														
141														
142														
143														
144														
145														
146														
147														
148														
149														
150														
151														
152	-100.00													
153		-100.00									520.00			
154			-100.00									620.00		
155				-100.00									700.00	
156					-100.00						178.00			700.00
3%	- 3.00	- 2.23	- 1.66	- 1.24	- 0.92	- 99.9	- 99.9	- 99.9	- 99.9	- 99.9	1202.58	1014.47	836.20	622.29
Cj 6%	- 6.00	- 3.35	- 1.87	- 1.04	- 0.58	- 99.9	- 99.9	- 99.9	- 99.9	- 99.9	782.38	517.93	325.41	181.76
10%	- 10.00	- 3.86	- 1.49	- 0.57	- 0.22	- 99.9	- 99.9	- 99.9	- 99.9	- 99.9	523.65	240.49	104.63	40.37

Appendix Table 4. Basic Matrix (Continued) Activity Identification

Site 90 Age 45 (Cont'd)

74 T45 C55 L10 T20 T30

75 T45 T55 C65 L10 T20

Site 90 Age 55

76 C55 L10 T20 T30 C40

Site 90 Age 65

77 C65 L10 T20 T30 C40

Site 90 Age 75

78 C75 L10 T20 T30 C40

Buying Activities

79 Buy labor now t

80 Buy labor t10

81 Buy labor t20

82 Buy labor t30

83 Buy truck t

84 Buy truck t10

85 Buy truck t20

86 Buy truck t30

87 Buy tractor now t

88 Buy tractor t10

89 Buy tractor t20

90 Buy tractor t30

91 Buy saw not t

92 Buy saw t10

93 Buy saw t20

94 Buy saw t30

95 Buy Cont. logging Capital t

96 Buy Cont. logging Ca Capital t10

97 Buy Cont. logging Capital t20

98 Buy Cont. logging Capital t30

99 Buy Cont. logging Capital t40

100 Artificial t

101 Artificial t10

102 Artificial t20

103 Artificial t30

104 Artificial t40

Site 80 Age 75

105 L75 C85 L10 L20 C30

106 L75 L85 C95 L10 L20

107 L75 L85 L95 C105 L10

108 L75 L85 L95 L105 C115

Appendix Table 4. Basic Matrix (Continued) Activity Identification

Site 80 Age 25 (Cont'd)

39 T25 L35 C45 L10 L20

Site 80 Age 35

40 C35 L10 L20 C30 L10

41 L35 C45 L10 L20 C30

42 L35 T45 C55 L10 L20

43 L35 T45 T55 C65 L10

44 T35 C45 L10 L20 C30

45 T35 T45 C55 L10 L20

46 T35 T45 T55 C65 L20

Site 80 Age 45

47 C45 L10 L20 C30 L10

48 T45 C55 L10 L20 C30

49 T45 T55 C65 L10 L20

Site 80 Age 55

50 C55 L10 L20 C30 L10

Site 80 Age 65

51 C65 L10 L20 C30 L10

Site 80 Age 75

52 C75 L10 L20 C30 L10

Site 90 Age 25

53 C25 L10 T20 T30 C40

54 L25 C35 L10 T20 T30

55 L25 T35 C45 L10 T20

56 L25 T35 T45 C55 L10

57 L25 T35 T45 T55 C65

58 L25 L35 T45 C55 L10

59 L25 L35 T45 T55 C65

60 T25 C35 L10 T20 T30

61 T25 T35 C45 L10 T20

62 T25 T35 T45 C55 L10

63 T25 T35 T45 T55 C65

64 T25 T35 L45 C55 L10

65 T25 L35 C45 L10 T20

66 T25 T35 T45 L55 C65

Site 90 Age 35

67 C35 L10 T20 T30 C40

68 L35 T45 C55 L10 T20

69 L35 T45 T55 C65 L10

70 L35 C45 L10 T20 T30

71 T35 T45 C55 L10 T20

72 T35 T45 T55 C65 L10

Site 90 Age 45

73 C45 L10 T20 T30 C40

Appendix Table 4. Basic Matrix (Continued) Activity Identification

Site 70 Age 25

1 C25 L10 L20 C30 L10

2 L25 C35 L10 L20 C30

3 L25 T35 C45 L10 L20

4 L25 T35 T45 C55 L10

5 L25 T35 T45 T55 C65

6 L25 L35 T45 C55 L10

7 L25 L35 T45 T55 C65

8 T25 C35 L10 L20 C30

9 T25 T35 C45 L10 L20

10 T25 T35 T45 C55 L10

11 T25 T35 T45 T55 C65

12 T25 T35 T45 L55 C65

13 T25 T35 L45 C55 L10

14 T25 L35 C45 L10 L20

Site 70 Age 35

15 C35 L10 L20 C30 L10

16 L35 T45 C55 L10 L20

17 L35 T45 T55 C65 L10

18 T35 C45 L10 L20 C30

19 T35 T45 C55 L10 L20

20 T35 T45 T55 C65 L10

Site 70 Age 45

21 C45 L10 L20 C30 L10

22 T45 C55 L10 L20 C30

23 T45 T55 C65 L10 L20

Site 70 Age 55

24 C55 L10 L20 C30 L10

Site 70 Age 75

25 C75 L10 L20 C30 L10

Site 80 Age 25

26 C25 L10 L20 C30 L10

27 L25 C35 L10 L20 C30

28 L25 T35 C45 L10 L20

29 L25 T35 T45 C55 L10

30 L25 L35 T45 C55 L10

31 L25 L35 T45 C55 L10

32 L25 L35 T45 T55 C65

33 T25 C35 L10 L20 C30

34 T25 T35 C45 L10 L20

35 T25 T35 T45 C55 L10

36 T25 T35 T45 T55 C65

37 T25 T35 T45 L55 C65

38 T25 T35 L45 C55 L10

INSTRUCTIONS FOR CHANGING BASIC MATRIX

Appendix tables 5, 10, 11, and 12 contain data for the changes which must be made in the basic matrix in order that an analysis can be made at 6 and 10 percent. When making the changes, certain co-efficients must be removed from and other coefficients must be inserted into the basic matrix. The changes are made in the following manner:

1. Remove all coefficients not directly concerned with the first rotation.
2. Insert the coefficients indicated in the pertinent table.
3. Select the proper Cj value from the bottom of the basic matrix.

Appendix Table 5. Insertions in basic matrix necessary for solutions at 6 percent alternative rate - site 90 - next rotation
LEAVE 10 - THIN 20 - CLEAR 30

Resource and time period	Quantity	Unit
Activity numbers 53, 67, 73, 76, 77, and 78		
Labor t10	14.00	hours
Capital t30	20.00	dollars
Mill t20	3.60	MBF
t30	13.40	MBF
Trucks t20	3.30	hours
Tractors t20	3.50	hours
Power saws t20	5.90	hours
Contract logging capital t30	268.00	dollars
Activity numbers 54, 60, 70, and 74		
Labor t30	14.00	hours
Capital t40	20.00	dollars
Mill t30	3.60	MBF
t40	13.40	MBF
Trucks t30	3.30	hours
Tractors t30	3.50	hours
Power saws t30	5.90	hours
Contract logging capital t40	268.00	dollars
Activity numbers 55, 61, 65, 68, 71, and 75		
Mill t40	3.60	MBF
Capital buying activities 95, 96, 97, 98, and 99		
Working capital t	6.00	dollars
t10	6.00	dollars
t20	6.00	dollars
t30	6.00	dollars
t40	6.00	dollars

Appendix Table 6. Basic optimum program, 6 percent alternative rate, solution one

Total Present Worth $951,996.55

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t10	t20	t30	t40			
			Site 70					
9	T25	T35	C45	L10	L20	Acre	36	433.80
13	T25	T35	L45	C55	L10	"	108	388.30
19	T35	T45	C55	L10	L20	"	25	315.40
22	T45	C55	L10	L20	C30	"	78	361.60
24	C55	L10	L20	C30	L10	"	12	600.00
25	C75	L10	L20	C30	L10	"	5	815.00
			Site 80					
38	T25	T35	L45	C55	L10	"	30	321.70
44	T35	C45	L10	L20	C30	"	87	497.20
45	T35	T45	C55	L10	L20	"	125	401.20
48	T45	C55	L10	L20	C30	"	474	632.60
50	C55	L10	L20	C30	L10	"	179	858.00
51	C65	L10	L20	C30	L10	"	136	719.00
106	L75	L85	C95	L10	L20	"	230	517.90
			Site 90					
65	T25	L35	C45	L10	T20	"	16	628.50
70	L35	C45	L10	T20	C30	"	10	780.70
74	T45	T55	C65	L10	T20	"	30	906.20
76	C55	L10	T20	C30	L10	"	32	832.00
77	C65	L10	T20	C30	L10	"	113	1466.00
80	Buy Labor				t10	Hours	5,480	-0.67
81	" "				t20	"	630	-0.37
82	" "				t30	"	180	-0.21
84	Buy Trucks				t10	"	690	-0.69
85	" "				t20	"	149	-0.38
86	" "				t30	"	132	-0.21
88	Buy Tractors				t10	"	800	-0.52
89	" "				t20	"	150	-0.29
90	" "				t30	"	140	-0.16
92	Buy Saw				t10	"	1,070	-0.27
93	" "				t20	"	260	-0.15
94	" "				t30	"	235	-0.09
96	Buy Logging Cap.				t10	$	192,200	-0.0335
97	" " "				t20	"	199,500	-0.0187
98	" " "				t30	"	103,800	-0.0104
99	" " "				t40	"	118,700	-0.0058

Appendix Table 7. Optimum program, 6 percent alternative rate, solution two
Total Present Worth $990,836.88

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t10	t20	t30	t40			
			Site 70					
9	T25	T35	C45	L10	L20	Acre	144	433.80
15	C35	L10	L20	C30	L10	"	25	393.50
21	C45	L10	L20	C30	L10	"	78	488.10
24	C55	L10	L20	C30	L10	"	12	600.00
25	C75	L10	L20	C30	L10	"	5	815.00
			Site 80					
28	L25	T35	C45	L10	L20	"	30	378.50
42	L35	T45	C55	L10	L20	"	212	391.90
47	C45	L10	L20	C30	L10	"	77	805.00
48	T45	C55	L10	L20	C30	"	397	632.60
50	C55	L10	L20	C30	L10	"	179	858.00
51	C65	L10	L20	C30	L10	"	136	719.00
105	L75	C85	L10	L20	C30	"	79	782.40
106	L75	L85	C95	L10	L20	"	151	517.90
			Site 90					
54	L25	C35	L10	T20	C30	"	16	677.00
69	L35	T45	T55	C65	L10	"	10	626.00
74	T45	C55	L10	T20	C30	"	30	906.00
76	C55	L10	T20	C30	L10	"	32	832.00
77	C65	L10	T20	C30	L10	"	36	1466.00
78	C75	L10	T20	C30	L10	"	15	1544.00
79	Buy Labor				t	Hours	28,888	-1.20
80	" "				t10	"	16,000	-0.67
81	" "				t20	"	1,740	-0.37
82	" "				t30	"	260	-0.21
83	Buy Trucks				t	"	4,650	-0.39
84	" "				t10	"	1,360	-0.69
85	" "				t20	"	307	-0.38
86	" "				t30	"	151	-0.21
87	Buy Tractors				t	"	5,140	-0.39
88	" "				t10	"	1,770	-0.52
89	" "				t20	"	340	-0.29
90	" "				t30	"	160	-0.16
91	Buy Saw				t	"	8,230	-0.29
92	" "				t10	"	2,520	-0.27
93	" "				t20	"	540	-0.15
94	" "				t30	"	271	-0.09
95	Buy Logging Cap.				t	$	167,500	-0.0600
96	" " "				t10	"	175,700	-0.0335
97	" " "				t20	"	190,800	-0.0187
98	" " "				t30	"	109,200	-0.0104
99	" " "				t40	"	97,000	-0.0058

Appendix Table 8. Optimum program, 6 percent alternative rate, solution three

Total Present Worth $1,023,389.70

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t10	t20	t30	t40			
			Site 70					
7	L25	L35	T45	T55	C65	Acre	107	193.90
12	T25	T35	T45	L55	C65	"	37	367.20
20	T35	T45	T55	C65	L10	"	25	281.30
21	C45	L10	L20	C30	L10	"	78	488.10
24	C55	L10	L20	C30	L10	"	12	600.00
25	C75	L10	L20	C30	L10	"	5	815.00
			Site 80					
37	T25	T35	T45	L55	C65	"	30	290.00
41	L35	C45	L10	L20	C30	"	212	450.00
47	C45	L10	L20	C30	L10	"	211	805.00
48	T45	C55	L10	L20	C30	"	263	632.60
50	C55	L10	L20	C30	L10	"	179	858.00
51	C65	L10	L20	C30	L10	"	136	719.00
52	C75	L10	L20	C30	L10	"	2	1110.00
106	L75	L85	C95	L10	L20	"	211	517.90
108	L75	L85	L95	L105	C115	"	17	181.80
			Site 90					
59	L25	L35	T45	T55	C65	"	16	339.00
70	L35	C45	L10	T20	C30	"	10	780.70
73	C45	L10	T20	C30	L10	"	30	1198.00
76	C55	L10	T20	C30	L10	"	32	832.00
77	C65	L10	T20	C30	L10	"	36	1466.00
78	C75	L10	T20	C30	L10	"	15	1544.00
80	Buy Labor				t10	Hours	1,260	-0.67
81	" "				t20	"	13,090	-0.37
82	" "				t30	"	6,340	-0.21
84	Buy Trucks				t10	"	260	-0.69
85	" "				t20	"	1,454	-0.38
86	" "				t30	"	381	-0.21
88	Buy Tractors				t10	"	280	-0.52
89	" "				t20	"	1,620	-0.29
90	" "				t30	"	555	-0.16
92	Buy Saw				t10	"	400	-0.27
93	" "				t20	"	2,390	-0.15
94	" "				t30	"	817	-0.09
95	Buy Logging Cap.				t	$	106,400	-0.0600
96	" " "				t10	"	148,300	-0.0335
97	" " "				t20	"	131,100	-0.0187
98	" " "				t30	"	139,500	-0.0104
99	" " "				t40	"	150,000	-0.0058

Appendix Table 9. Optimum program, 6 percent alternative rate, solution four

Total Present Worth $1,286,443.90

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t10	t20	t30	t40			
			Site 70					
8	T25	C35	L10	L20	C30	Acre	144	455.40
15	C35	L10	L20	C30	L10	"	25	393.50
21	C45	L10	L20	C30	L10	"	78	488.10
24	C55	L10	L20	C30	L10	"	12	600.00
25	C75	L10	L20	C30	L10	"	5	815.00
			Site 80					
33	T25	C35	L10	L20	C30	"	30	412.40
40	C35	L10	L20	C30	L10	"	212	589.70
47	C45	L10	L20	C30	L10	"	474	805.00
50	C55	L10	L20	C30	L10	"	179	858.00
51	C65	L10	L20	C30	L10	"	136	719.00
52	C75	L10	L20	C30	L10	"	230	1110.00
			Site 90					
60	T25	C35	L10	T20	C30	"	16	741.30
67	C35	L10	T20	C30	L10	"	10	1002.00
73	C45	L10	T20	C30	L10	"	30	1198.00
76	C55	L10	T20	C30	L10	"	32	832.00
77	C65	L10	T20	C30	L10	"	36	1466.00
78	C75	L10	T20	C30	L10	"	15	1544.00
81	Buy	Labor			t20	Hours	1,720	-0.37
82	"	"			t30	"	550	-0.21
85	Buy	Trucks			t20	"	405	-0.38
86	"	"			t30	"	52	-0.21
89	Buy	Tractors			t20	"	430	-0.29
90	"	"			t30	"	99	-0.16
93	Buy	Saw			t20	"	720	-0.15
94	"	"			t30	"	72	-0.09
95	Buy	Logging	Cap.		t	$	321,900	-0.0600
96	"	"	"		t10	"	27,900	-0.0335
98	"	"	"		t30	"	264,800	-0.0104
99	"	"	"		t40	"	24,800	-0.0058

Appendix Table 10. Insertions in basic matrix necessary for solutions at 10 percent alternative rate - site 70 - next rotation
CLEAR 20

Resource and time period	Quantity	Unit
Activity numbers 1, 15, 21, 24, and 25		
Capital t20	20.00	dollars
t40	20.00	dollars
Mill t20	0.60	MBF
t40	0.60	MBF
Contract logging capital t20	12.00	dollars
t40	12.00	dollars
Activity numbers 2, 8, 18, and 22		
Capital t30	20.00	dollars
Mill t30	0.60	MBF
Contract logging capital t30	12.00	dollars
Activity numbers 3, 9, 14, 16, 19, and 23		
Capital t40	20.00	dollars
Mill t40	0.60	MBF
Contract logging capital t40	12.00	dollars

Appendix Table 11. Insertions in basic matrix necessary for solutions at 10 percent alternative rate - site 80 - next rotation
CLEAR 20

Resource and time period	Quantity	Unit
Activity numbers 26, 40, 47, 50, 51, and 52		
Capital t20	20.00	dollars
t40	20.00	dollars
Mill t20	1.80	MBF
t40	1.80	MBF
Contract logging capital t20	36.00	dollars
t40	36.00	dollars
Activity numbers 27, 33, 41, 44, and 48		
Capital t30	20.00	dollars
Mill t30	1.80	MBF
Contract logging capital t30	36.00	dollars
Activity numbers 28, 34, 39, 42, 45, and 49		
Capital t40	20.00	dollars
Mill t40	1.80	MBF
Contract logging capital t40	36.00	dollars

Appendix Table 12. Insertions in basic matrix necessary for solutions at 10 percent alternative rate - site 90 - next rotation
CLEAR 20

Resource and time period	Quantity	Unit
Activity numbers 53, 67, 73, 76, 77, and 78		
Capital t20	20.00	dollars
t40	20.00	dollars
Mill t20	3.60	MBF
t40	3.60	MBF
Contract logging capital t20	72.00	dollars
t40	72.00	dollars
Activity numbers 54, 60, 70, 74		
Capital t30	20.00	dollars
Mill t30	3.60	MBF
Contract logging capital	72.00	dollars
Activity numbers 55, 61, 65, 68, 71, and 75		
Capital t40	20.00	dollars
Mill t40	3.60	MBF
Contract logging capital t40	72.00	dollars
Capital buying activities 95, 96, 97, 98, and 99		
Working capital t	10.00	dollars
t10	10.00	dollars
t20	10.00	dollars
t30	10.00	dollars
t40	10.00	dollars

Appendix Table 13. Basic optimum program, 10 percent alternative rate, solution one

Total Present Worth $725,552.24

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t10	t20	t30	t40			
			Site 70					
11	T25	T35	T45	T55	C65	Acre	144	327.50
19	T35	T45	C55	L10	C20	"	25	216.20
22	T45	C55	L10	C20	L10	"	78	257.10
24	C55	L10	C20	L10	C20	"	12	567.00
25	C75	L10	C20	L10	C20	"	5	782.00
			Site 80					
33	T25	C35	L10	C20	L10	"	30	301.50
44	T35	C45	L10	C20	L10	"	69	348.30
45	T35	T45	C55	L10	C20	"	143	236.10
48	T45	C55	L10	C20	L10	"	474	475.30
50	C55	L10	C20	L10	C20	"	179	815.00
51	C65	L10	C20	L10	C20	"	89	676.00
106	L75	L85	C95	L10	C20	"	202	240.50
107	L75	L85	L95	C105	L10	"	28	104.60
			Site 90					
65	T25	L35	C45	L10	C20	"	16	410.00
70	L35	C45	L10	C20	L10	"	10	567.40
74	T45	C55	L10	C20	L10	"	30	667.40
76	C55	L10	C20	L10	C20	"	32	765.00
77	C65	L10	C20	L10	C20	"	36	1396.00
80	Buy Labor				t10	Hours	5,470	-0.46
81	"	"			t20	"	2,240	-0.18
82	"	"			t30	"	1,900	-0.07
84	Buy Trucks				t10	"	610	-0.47
85	"	"			t20	"	129	-0.18
86	"	"			t30	"	100	-0.07
88	Buy Tractors				t10	"	720	-0.36
89	"	"			t20	"	180	-0.14
90	"	"			t30	"	158	-0.05
92	Buy Saw				t10	"	880	-0.19
93	"	"			t20	"	180	-0.07
94	"	"			t30	"	201	-0.03
96	Buy Logging Cap.				t10	$	192,400	-0.0386
97	"	"	"		t20	"	196,200	-0.0149
98	"	"	"		t30	"	43,900	-0.0057
99	"	"	"		t40	"	60,000	-0.0022

Appendix Table 14. Optimum program, 10 percent alternative rate, solution two

Total Present Worth $772,448.84

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t10	t20	t30	t40			
			Site 70					
1	C25	L10	C20	L10	C20	Acre	47	342.10
9	T25	T35	C45	L10	C20	"	97	339.60
15	C35	L10	C20	L10	C20	"	25	360.50
21	C45	L10	C20	L10	C20	"	78	455.10
24	C55	L10	C20	L10	C20	"	12	567.00
25	C75	L10	C20	L10	C20	"	5	782.00
			Site 80					
26	C25	L10	C20	L10	C20	"	30	360.20
41	L35	C45	L10	C20	L10	"	212	329.00
47	C45	L10	C20	L10	C20	"	40	762.00
48	T45	C55	L10	C20	L10	"	396	475.30
49	T45	T55	C65	L10	C20	"	38	336.70
50	C55	L10	C20	L10	C20	"	179	815.00
51	C65	L10	C20	L10	C20	"	136	676.00
106	L75	L85	C95	L10	C20	"	230	240.50
			Site 90					
53	C25	L10	C20	L10	C20	"	16	600.50
68	L35	T45	C55	L10	C20	"	10	314.00
74	T45	C55	L10	C20	L10	"	30	667.40
76	C55	L10	C20	L10	C20	"	32	765.00
77	C65	L10	C20	L10	C20	"	36	1396.00
78	C75	L10	C20	L10	C20	"	15	1477.00
79	Buy Labor				t	Hours	27,073	-1.20
80	"	"			t10	"	2,680	-0.46
83	Buy Trucks				t	"	3,720	-0.39
84	"	"			t10	"	310	-0.47
87	Buy Tractors				t	"	4,250	-0.39
88	"	"			t10	"	370	-0.36
91	Buy Saw				t	"	6,660	-0.29
92	"	"			t10	"	560	-0.19
95	Buy Logging Cap.				t	$	166,300	-0.1000
96	"	"	"		t10	"	196,300	-0.0386
97	"	"	"		t20	"	200,000	-0.0149
98	"	"	"		t30	"	24,000	-0.0057
99	"	"	"		t40	"	34,500	-0.0022

Appendix Table 15. Optimum program, 10 percent alternative rate, solution three

Total Present Worth $695,463.55

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t10	t20	t30	t40			
			Site 70					
7	L25	L35	T45	T55	C65	Acre	144	787.00
20	T35	T45	T55	C65	L10	"	25	188.40
23	T45	T55	C65	L10	C20	"	78	174.00
24	C55	L10	C20	L10	C20	"	12	567.00
25	C75	L10	C20	L10	C20	"	5	782.00
			Site 80					
37	T25	T35	T45	L55	C65	"	30	199.40
43	L35	T45	T55	C65	L10	"	119	151.60
46	T35	T45	T55	C65	L10	"	93	180.50
48	T45	C55	L10	C20	L10	"	450	475.30
49	T45	T55	C65	L10	C20	"	24	336.70
50	C55	L10	C20	L10	C20	"	179	815.00
51	C65	L10	C20	L10	C20	"	136	676.00
106	L75	L85	C95	L10	C20	"	128	240.50
107	L75	L85	L95	C105	L10	"	45	104.60
108	L75	L85	L95	L105	C115	"	57	40.40
			Site 90					
64	T25	T35	L45	C55	L10	"	16	365.50
69	L35	T45	T55	C65	L10	"	10	231.00
73	C45	L10	C20	L10	C20	"	30	113.00
76	C55	L10	C20	L10	C20	"	32	765.00
77	C65	L10	C20	L10	C20	"	36	1396.00
78	C75	L10	C20	L10	C20	"	15	1477.00
80	Buy	Labor			t10	Hours	12,100	-0.46
81	"	"			t20	"	16,480	-0.18
82	"	"			t30	"	7,770	-0.07
84	Buy	Trucks			t10	"	930	-0.47
85	"	"			t20	"	1,388	-0.18
86	"	"			t30	"	432	-0.07
88	Buy	Tractors			t10	"	1,260	-0.36
89	"	"			t20	"	1,660	-0.14
90	"	"			t30	"	647	-0.05
92	Buy	Saw			t10	"	1,740	-0.19
93	"	"			t20	"	2,320	-0.07
94	"	"			t30	"	849	-0.03
95	Buy	Logging	Cap.		t	$	33,000	-0.1000
96	"	"	"		t10	"	130,500	-0.0386
97	"	"	"		t20	"	124,100	-0.0149
98	"	"	"		t30	"	137,000	-0.0057
99	"	"	"		t40	"	108,400	-0.0022
100	Art.				t	MBF	15,730	0

Appendix Table 16. Optimum program, 10 percent alternative rate, solution four

Total Present Worth $1,196,382.70

Act. no.	Activity description					Unit	Activity level	Price per unit
	t	t_{10}	t_{20}	t_{30}	t_{40}			
			Site 70					
8	T25	C35	L10	C20	L10	Acre	144	379.30
15	C35	L10	C20	L10	C20	"	25	360.50
21	C45	L10	C20	L10	C20	"	78	455.10
24	C55	L10	C20	L10	C20	"	12	567.00
25	C75	L10	C20	L10	C20	"	5	782.00
			Site 80					
26	C25	L10	C20	L10	C20	"	30	360.20
40	C35	L10	C20	L10	C20	"	212	547.00
47	C45	L10	C20	L10	C20	"	474	762.00
50	C55	L10	C20	L10	C20	"	179	815.00
51	C65	L10	C20	L10	C20	"	136	676.00
52	C75	L10	C20	L10	C20	"	230	1067.00
			Site 90					
53	C25	L10	C20	L10	C20	"	16	600.50
67	C35	L10	C20	L10	C20	"	10	935.00
73	C45	L10	C20	L10	C20	"	30	1130.00
76	C55	L10	C20	L10	C20	"	32	765.00
77	C65	L10	C20	L10	C20	"	36	1396.00
78	C75	L10	C20	L10	C20	"	15	1477.00
92	Buy Saw				t10	Hours	10	-0.19
95	Buy Logging Cap.				t	$	327,100	-0.1000
96	"	"	"		t10	"	18,100	-0.0386
97	"	"	"		t20	"	56,800	-0.0149
98	"	"	"		t30	"	1,700	-0.0057
99	"	"	"		t40	"	56,800	-0.0022
100	Art.				t	MBF	12,044	0

Appendix Table 17. Resources available, used, remaining - basic optimum program - solution one, 3 percent alternative rate

Resource		Unit	Amount Available	Amount Used	Amount Remaining
Site 90	Age 65	Acre	36	13	23
Site 90	Age 75	Acre	15	0	15
Labor	t	Hours	76,730	40,757	35,973
Capital	t	Dollars	330,000	18,513	311,487
Capital	t10	Dollars	330,000	35,398	294,602
Capital	t20	Dollars	330,000	20,745	309,254
Capital	t30	Dollars	330,000	16,717	313,283
Capital	t40	Dollars	330,000	10,863	319,137
Mill Capacity	t	MBF	10,000	7,935	2,065
Mill Capacity	t30	MBF	10,000	5,543	4,457
Mill Capacity	t40	MBF	10,000	4,284	5,716
Trucks	t	Hours	24,800	6,004	18,796
Tractors	t	Hours	38,840	7,170	31,670
Saws	t	Hours	46,800	10,648	36,152
Contract Logging Capital	t	Dollars	110,000	110,000	0

Appendix Table 18. Resources available, used, remaining - solution two, 3 percent alternative rate

Resource		Unit	Amount Available	Amount Used	Amount Remaining
Capital	t	Dollars	330,000	67,978	262,022
Capital	t10	Dollars	330,000	28,977	301,023
Capital	t20	Dollars	330,000	20,701	309,299
Capital	t30	Dollars	330,000	17,354	312,646
Capital	t40	Dollars	330,000	12,741	317,259
Mill Capacity	t	MBF	10,000	10,000	0
Mill Capacity	t10	MBF	10,000	10,000	0
Mill Capacity	t20	MBF	10,000	9,923	77
Mill Capacity	t30	MBF	10,000	4,336	5,664
Mill Capacity	t40	MBF	10,000	5,227	4,773

Appendix Table 19. Resources available, used, remaining - solution three, 3 percent alternative rate

Resource		Unit	Amount Available	Amount Used	Amount Remaining
Labor	t	Hours	76,730	28,612	48,118
Capital	t	Dollars	330,000	26,289	303,711
Capital	t10	Dollars	330,000	20,765	309,235
Capital	t20	Dollars	330,000	15,495	314,565
Capital	t30	Dollars	330,000	20,733	309,267
Capital	t40	Dollars	330,000	18,512	311,488
Trucks	t	Hours	24,800	5,550	19,250
Tractors	t	Hours	38,840	5,899	32,941
Saws	t	Hours	46,800	8,889	37,911
Contract Logging Capital	t	Dollars	110,000	110,000	0

Appendix Table 20. Resources available, used, remaining - solution four, 3 percent alternative rate

Resource		Unit	Amount Available	Amount Used	Amount Remaining
Labor	t	Hours	76,730	12,115	64,615
Capital	t	Dollars	330,000	39,662	290,338
Capital	t10	Dollars	330,000	10,180	319,820
Capital	t20	Dollars	330,000	4,178	325,822
Capital	t30	Dollars	330,000	35,213	294,787
Capital	t40	Dollars	330,000	12,580	317,420
Mill Capacity	t10	MBF	Unlimited	4,566	0
Mill Capacity	t20	MBF	50,000	482	49,518
Mill Capacity	t40	MBF	Unlimited	5,041	0
Trucks	t	Hours	24,800	3,575	21,225
Tractors	t	Hours	38,840	4,040	34,800
Saws	t	Hours	46,800	6,195	40,605
Contract Logging Capital	t	Dollars	110,000	110,000	0

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ABSTRACT

of

A LINEAR PROGRAMMING APPROACH

TO

EVALUATING FOREST MANAGEMENT ALTERNATIVES

by

William Edward Kidd, Jr.

Thesis submitted to the Graduate Faculty of the

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in

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Major

FOREST ECONOMICS

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ABSTRACT

The methodology and the appropriateness of adapting the linear programming model to the evaluation of timber harvest alternatives of a specific forest enterprise was examined. The use of linear programming to describe a program in which profit is maximum rather than one of several other economic allocation models was justified. The basic model, using 3 percent as the alternative rate, described the alternative thinning and harvesting opportunities for the Seward Forest at Triplett, Virginia. The optimum program had to satisfy the restrictions imposed by scarce resources and by personal management constraints. The solution of the model described a course of action for the forest manager for the next 50 years. The initiation of the optimum plan would result in maximizing total present worth to the fixed resources of the Forest. Changes were made in the constraints on the model to demonstrate their effect upon the combination of activities which comprise the optimum program and the effect of these constraints on present worth. Additional solutions at 6 percent and 10 percent alternative rates were made to demonstrate the change which occurs in the activities that describe the optimum program at successively higher alternative rates.