A MODEL FOR THE INVESTIGATION OF COST VARIANCES: THE FUZZY SET THEORY APPROACH

by

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DEDICATION

To my wife, , to my daughter, , and to my father, , and mother, , for their encouragement and support.

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CHAPTER ONE

Introduction

The subject of the dissertation is the cost variance investigation problem. This introductory chapter has three objectives. First, it provides a general description of the problem. Second, it explains--in general terms--the methodology to be used. Finally, an outline of the remaining chapters of the dissertation is provided.

1.1 Nature of the Problem

In a standard costing system, once the deviations of the actual performance from the standard performance are isolated, they are subject to investigation. The investigation is necessary because management, in order to take corrective actions, needs to know not only the size of the variances, but also " . . . where the variances are originated, who is responsible for them, and what caused them to arise."[1]

On the other hand, in real life situations, the actual performance is rarely (if ever) equal to the standard performance. That is, a variance is likely to result for every cost item. Consequently, in practice, so many variances occur that it would be impractical to investigate all of them. The investigation process may also be extremely costly and hence, for small variances, unprofitable for the business enterprise.

As a result of these two conflicting factors--the necessity of investigating cost variances and the impracticality of investigating all

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[1]National Association of Accountants, Report No. 22: The Analysis of Manufacturing Cost Variances. (New York: NAA, 1952, p.2)

the variances--management is confronted with the question of which variances to investigate.[2]

In making the decision of which variances to investigate, management has employed two criteria, namely, the absolute size of the variance and the relative size of the variance to the standard cost. For example, rules such as "investigate all variances exceeding $1,000, or 20% of the standard cost, whichever is lower," are widely used by managers. Such rules are based upon the intuition of the manager.

Recently, several quantitative models for solving the cost variance investigation problem have been proposed by researchers. These models are divided into two groups. The first group includes models that do not take into consideration the costs and benefits of the investigation. This group of models is exemplified by the Zannetos, Jeurs, Luh, Koehler, and Probst models.[3] The second group of models includes models that consider both the costs and benefits of the investigation. The

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[2]As noted by Horngren the question of which variances are worthy of investigation is the ". . . most troublesome aspect of feedback," C. T. Horngren, Introduction to Management Accounting, 5th ed. (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1981), p. 186.

[3]Z. A. Zannetos, "Standard Cost as a First Step to Probabilistic Control: A Theoretical Justification, an Extension and Implications", The Accounting Review (April 1964), pp. 296-304. D. A. Jeurs, "Statistical Significance of Accounting Variances", Management Accounting (October 1967), pp. 20-25. F. Luh, "Controlled Costs: An Operational Concept and Statistical Approach to Standard Costing", The Accounting Review (January 1968), pp. 123-132. R. W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control", Management Accounting (October 1968), pp. 35-41. F. R. Probst, "Probabilistic Cost Controls: A Behavioral Dimension", The Accounting Review (January 1971), pp. 113-118.

models in this group include the models suggested by Bierman, Fouraker, Jaedicke, Kaplan, Dyckman, and Duvall.[4]

These models are reviewed and evaluated in Chapter Three. In evaluating the models, three criteria are used, namely, answering the cost-benefit question, capturing the essence of the real problem, and the feasibility of the model.

As will be shown in Chapter Three, the first group of models suffers basically from ignoring the costs and benefits of the investigation. The models in the second group fail to handle the problem of imprecision that surrounds the cost variance investigation decision. Moreover, these latter models suffer from the lack of applicability; they call for precise measures that cannot be obtained. Furthermore, the models are based on the "law of the excluded middle"; i.e. the models do not allow for the transition between what is in-control and what is out-of-control. Stated differently, the models assume that the performance can only be either in-control or out-of-control. This assumption is an oversimplification of the real problem. Finally, because of the demand for high levels of precision in measuring the variables and in describing how they are related, the models lose part

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[4] H. Bierman, Jr., L. E. Fouraker, and R. K. Jaedicke, "The Use of Probability and Statistics in Performance Evaluation", The Accounting Review (July 1961), pp. 409-417. R. S. Kaplan, "Optimal Investigation Strategies with Imperfect Information", Journal of Accounting Research (Spring 1969), pp. 32-43. T. R. Dyckman, "The Investigation of Cost Variances", Journal of Accounting Research (Autumn 1969), pp. 215-244. R. M. Duvall, "Rules for Investigating Cost Variances", Management Science, Vol. 13 (1967), pp. B-631 - B-641.

of their relevance to the real world through ignoring some relevant items just because these items are incapable of precise measurement.

Taking these deficiencies into consideration, the objective of this dissertation is the development of a new cost variance investigation model; a model that takes into account the costs and benefits of the investigation; the imprecision surrounding the investigation decision; and the fact that there are intermediate states between the in-control and out-of-control states. Moreover, the model is intended to reduce the need for precise measures that are difficult, if not impossible, to obtain.

1.2 Methodology

In developing the new model, the calculus of fuzzy set theory will be utilized. Fuzzy set theory was introduced by Zadeh[5] in 1965 to provide a mathematical framework wherein imprecise phenomena in decision making can be dealt with in a precise and rigorous manner. Moreover, the theory represents a vehicle for reducing the need for precise measurements. Finally, fuzzy set theory, through its basic concept of membership function, relaxes the "law of the excluded middle" assumed by the traditional quantitative models. Consequently, the theory appears to be well suited for handing the cost variance investigation problem.

The model to be developed will be applied, through a pilot study, to an actual cost variance investigation problem encountered by a manufacturing firm. Such an application may provide an insight into the

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[5] Lotfi A. Zadeh, "Fuzzy Sets", Information and Control, Vol. 8 (1965), pp. 338-353.

problems that may arise when the model is applied in real world situations. This is important to the cost variance investigation and the fuzzy set theory because (1) as indicated by Koehler, Anthony, and Magee available cost variance investigation models have not been used in practice and (2) as noted by Flonder and Kickert the literature on fuzzy set theory lacks practical applications.[6]

1.3 Outline of the Study

The dissertation is comprised of seven chapters. In the next chapter, Chapter Two, an introduction to the basic elements of fuzzy set theory is provided. The chapter also discusses some of the theory's applications in the decision making area. Chapter Three includes a review of the available cost-variance investigation models. Also, in Chapter Three, some of the problems encountered by these models are identified and discussed. The new model for the cost-variance investigation decision is presented in Chapter Four. The model is extended in Chapter Five. Chapter Six is devoted to discussing the actual application of the proposed model. Finally, in Chapter Seven, summary and implications for future research are presented.

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[6]See R. W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control", p. 35; R. N. Anthony, "Some Fruitful Directions for Research in Management Accounting", in N. Dopuch and L. Revsine (eds.), Accounting Research 1960-1970: A Critical Evaluation (Center for International Education and Research in Accounting: University of Illinois, 1973), p. 52; R. P. Magee, "A Simulation Analysis of Alternative Cost Variance Investigation Models", The Accounting Review (July 1976), p. 529; W. Kickert, Fuzzy Theories on Decision-Making (Netherland: Martinus Nijhoff Social Sciences Division, 1978), p. 158; and P. Flonder, "An Example of A Fuzzy System", Kybernetes, Vol. 6 (1977), p. 229.

CHAPTER TWO

Fuzzy Set Theory

As explained earlier, the objective of the dissertation is the development of a new model for the cost-variance investigation decision. The model to be developed is based on fuzzy set theory introduced by Zadeh in 1965. This chapter provides a discussion of the basic elements of the theory. Also included is a discussion of some of the theory's applications in the decision-making area.

2.1 Elements of Fuzzy Set Theory

A close look at human-centered systems (systems with human interaction) reveals that fuzziness is a major source of inexactness in many humanistic systems.[1] For example, statements and terms such as : "we expect our sales to be much higher than $10,000," "President Carter won the Wisconsin primary with a comfortable margin," "if the inflation rate is very high, the government spending should be substantially reduced," "young man," "experienced manager," "good-looking girl," and "high profit," are common in humanistic systems.[2]

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[1]See, for example, L. A. Zadeh, "Fuzzy Sets", Information and Control, Vol. 8 (1965), pp. 338-353, and L. A. Zadeh, "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes", IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-3 (1973), pp. 28-44. Political systems, economic systems, and educational systems are examples of humanistic systems.

[2]Fuzziness in those statements and terms lies with the imprecise meaning of the underlined words.

Since 1965 Zadeh has argued that there is a need for the differentiation between fuzziness and randomness.[3] According to him, fuzziness has to do with classes with no sharp transitions between membership and nonmembership.[4] Randomness, however, deals with the uncertainty (in terms of probability) concerning the occurrance or nonoccurrance of an event.

An example may clarify the difference between uncertainty and fuzziness. The term "cool weather", for example, involves fuzziness; there is no sharp transition between what is cool and what is not. However, the question about the probability of having rain tommorrow involves uncertainty. The event is well described, either it rains or it does not; the uncertainty lies with the occurrance or nonoccurrance of the event.[5]

Moreover, Zadeh has argued that applications of the conventional quantitative techniques to the study of humanistic systems are often unsuccessful because: (1) they are not equipped to handle fuzziness which is a major source of inexactness in many humanistic systems, and

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[3]For references, see footnote number 1.

[4]L. A. Zadeh, "Fuzzy Sets", p. 339.

[5]Note that the uncertainty, as measured by probability, can be related to fuzzy events. For example, one may ask about the probability of having cool weather tomorrow.

(2) they call for high levels of precision that cannot be obtained.[6]

In addition to not being obtainable, the high levels of precision required by the conventional quantitative methods are--according to Zadeh--unnecessary for an effective study of the systems.[7] This is mainly because people--unlike computers--can understand and respond to imprecise instructions. For example (from Yager and Basson) people can respond to the fuzzy command: "tall people in the back and short people in the front"; however, computers as well as the conventional quantitative techniques are not capable of handling such an instruction.[8]

To reflect the distinction between randomness and fuzziness and to lessen the requirement of precise numerical inputs to decision analysis, Zadeh introduced the fuzzy set theory. The underlying philosophy of the theory is to provide a mathematical framework, wherein imprecise phenomena in humanistic systems can be dealt with in a precise and systematic manner.

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[6] See, for example, L. A. Zadeh, "The Linguistic Approach and Its Application to Decision Analysis", in Y. C. Ho and S. K. Mitter (eds.) Directions in Large-Scale Systems (N.Y.: Plenum Press, 1976), p. 340. A. Kaplan also considers the demand for precise measures to be one of the major problems encountered by models. See A. Kaplan, The Conduct of Inquiry (California: Chandler Publishing Company, 1964), p. 283.

[7] L. A. Zadeh, "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes", p. 29.

[8] R. R. Yager and D. Basson, "Decision Making with Fuzzy Sets", Decision Sciences, Vol. 6 (1975), p. 590.

According to Zadeh, a fuzzy set is a class of objects with no sharp boundaries to separate those objects belonging to the class from those not belonging.[9] For example, the set "old women" and the set "tall men" are fuzzy sets. Similarly, classes described by adjectives such as large, small, substantial, significant, important, etc., are fuzzy sets.

Mathematically, a fuzzy set $\underset{\sim}{A}$ of the universe E is defined by:

$$\underset{\sim}{A} = \int [u_{\underset{\sim}{A}}(x)/x], \qquad (1)$$

where $u_{\underset{\sim}{A}}(x)$, called the membership or compatibility function, represents numerically the degree to which an object, x, belongs to the set $\underset{\sim}{A}$.[10]

The membership function is similar to the characteristic function for a set in the ordinary set theory. However, instead of treating the presence or absence in a binary manner, as in the ordinary set theory, the membership function in the fuzzy set theory takes its values in the closed interval from zero to one.

That is, in the ordinary set theory the characteristic function for a set A [denoted by $u_A(x)$] can take on only the value 1 or 0. An element $x_i \varepsilon E$ can be either a member of A [$u_A(x_i) = 1$] or a nonmember of A [$u_A(x_i) = 0$]. However, in the fuzzy set theory, the compatibility function for a fuzzy set $\underset{\sim}{A}$ [denoted by $u_{\underset{\sim}{A}}(x)$] takes any value in the closed interval from zero to 1. The closer x_i is to satisfying the requirements of the set $\underset{\sim}{A}$ the nearer $u_{\underset{\sim}{A}}(x)$ is to 1, and vice versa.

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[9] L. A. Zadeh, "Fuzzy Sets", p. 339.

[10] The intergration sign in the context of fuzzy set theory indicates the union. That is, $\underset{\sim}{A}$ is the union of its constituent singletons $[u_{\underset{\sim}{A}}(x)/x]$.

Thus, an element $x_i \varepsilon E$ may be a nonmember of $\underset{\sim}{A}$ $[u_{\underset{\sim}{A}}(x_i) = 0]$, could be slightly a member of $\underset{\sim}{A}$ $[u_{\underset{\sim}{A}}(x_i)$ near zero], may more or less be a member of $\underset{\sim}{A}$ $[u_{\underset{\sim}{A}}(x_i)$ close to .5], could be strongly a member of $\underset{\sim}{A}$ $[u_{\underset{\sim}{A}}(x_i)$ near 1], or finally, could be a member of $\underset{\sim}{A}$ $[u_{\underset{\sim}{A}}(x_i) = 1]$.[11]

An example of a fuzzy set may clarify the concept of membership function. Let

$$E = \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110\}$$

be the set of possible weather temperatures. Then the fuzzy set $\underset{\sim}{A}$ of E representing "moderate temperatures" can be subjectively defined by:

$$\underset{\sim}{A} = \{(.5/50), (.8/60), (1.0/70), (.8/80)\}$$

As another example, consider the fuzzy set $\underset{\sim}{A}$ = {young person} whose membership function is defined by

$$u_{\underset{\sim}{A}}(x) = [1 + (.04x)^2]^{-1}.$$

Thus, the compatibility of a person who is 25 years old with the set "young person" is 0.5 while the compatibility of a person who is 50 years old with the same set is 0.2. The fuzzy set representation of the above membership function is provided in Figure 1.

A fuzzy set, $\underset{\sim}{A}$, is classified as normal if its height is equal to 1, while it is classified as subnormal if its height is less than one. The points in the universe, E, at which $u_{\underset{\sim}{A}}(x) > 0$ constitute the support

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[11] Stated differently, in fuzzy set theory "... the concepts of yes/no, true/false, black/white, and good/bad are replaced by concepts where partial truth and partial falsity reside and where transitions between what 'is' and what 'is not' are admitted." J. P. Gigch and L. L. Pipino, "From Absolute to Probable and Fuzzy in Decision Making", Kybernetes, Vol. 9 (1980), p. 51.

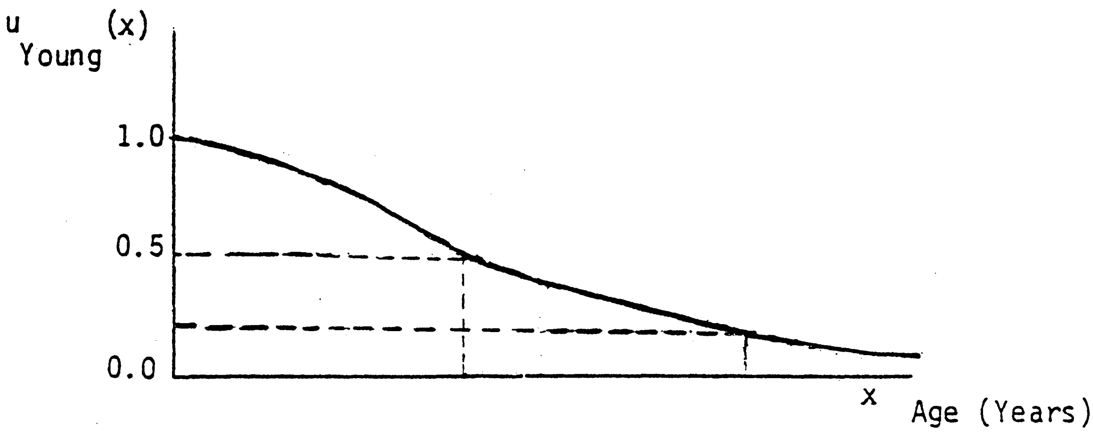

Figure 1. Compatibility Function for "Young Person".

of the fuzzy set $\underset{\sim}{A}$, while the points at which $u_{\underset{\sim}{A}}(x) = 0.5$ are the crossover points of $\underset{\sim}{A}$.

Reflecting the distinction between the membership function and the characteristic function, the mathematical techniques of the fuzzy set theory are different from those of the ordinary set theory. This will be shown below.

The calculus of the fuzzy set theory is based on several operations, namely, the intersection, union, and complementation. Let E be a universal set, let $\underset{\sim}{A} \subseteq E$ with $u_{\underset{\sim}{A}}(x) \varepsilon M = [0, 1]$, and let $\underset{\sim}{B} \subseteq E$ with $u_{\underset{\sim}{B}}(x) \varepsilon M = [0, 1]$, then the intersection ($\underset{\sim}{A} \cap \underset{\sim}{B}$), union ($\underset{\sim}{A} \cup \underset{\sim}{B}$) and complementation ($\overline{\underset{\sim}{A}}$) are given, respectively, by:

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \int [u_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x)/x], \tag{2}$$

$$\underset{\sim}{A} \cup \underset{\sim}{B} = \int [u_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x)/x], \tag{3}$$

and

$$\overline{\underset{\sim}{A}} = \int [u_{\overline{\underset{\sim}{A}}}(x)/x], \tag{4}$$

where the membership functions are defined by:

$$u_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = u_{\underset{\sim}{A}}(x) \wedge u_{\underset{\sim}{B}}(x), \tag{5}$$

$$u_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = u_{\underset{\sim}{A}}(x) \vee u_{\underset{\sim}{B}}(x), \tag{6}$$

and

$$u_{\overline{\underset{\sim}{A}}}(x) = 1 - u_{\underset{\sim}{A}}(x).^{12} \tag{7}$$

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[12] The symbols "$\wedge$" and "$\vee$" denote the operations of taking the minimin and the maximum, respectively.

As an example, let $E = \{x_1, x_2, x_3, x_4\}$. Moreover, let $\underset{\sim}{A}$ and $\underset{\sim}{B}$ be two fuzzy subsets of E defined by

$$\underset{\sim}{A} = \{(.2/x_1), (0/x_2), (.8/x_3), (1/x_4)\},$$

$$\underset{\sim}{B} = \{(.8/x_1), (.5/x_2), (.1/x_3), (0/x_4)\}.$$

Then, using equations (2)-(7) the intersection $(\underset{\sim}{A} \cap \underset{\sim}{B})$, union $(\underset{\sim}{A} \cup \underset{\sim}{B})$, and complementation $(\overline{\underset{\sim}{A}})$ are given by:

$$(\underset{\sim}{A} \cap \underset{\sim}{B}) = \{.2/x_1), (0/x_2), (.1/x_3), (0/x_4)\},$$

$$(\underset{\sim}{A} \cup \underset{\sim}{B}) = \{(.8/x_1), (.5/x_2), (.8/x_3), (1/x_4)\},$$

and

$$\overline{\underset{\sim}{A}} = \{(.8/x_1), (1/x_2), (.2/x_3), (0/x_4)\}.$$

It should be noted that, in the case of ordinary sets--where $M = \{0, 1\}$--the minimum and maximum operators are equivalent to the boolean product (.) and the boolean sum $(\dot{+})$ operators, respectively. However, as can be verified by the above example, these equivalence relations do not hold in the case of fuzzy sets where $M = [0, 1]$. Moreover, in the case of fuzzy sets, unlike in ordinary set theory, the negation is not complementary. That is, as can be verified by the above example,

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq E$$

$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \emptyset.$$ [13]

[13] The other properties of an ordinary set hold in the case of fuzzy sets. That is, fuzzy sets have the following properties: commutativity, associativity, idempotence, distributivity, and involution. DeMorgan's Theorem also holds under the fuzzy set theory.

As a result the algebra is no longer in the sense of ordinary set theory; the structure is that of a vector lattice.

In addition to the intersection, union, and complementation operations, other fuzzy set operations are defined in the Appendix to this chapter.

2.2 Applications of Fuzzy Set Theory in the Decision Making Area[14]

Bellman and Zadeh[15] were the first to provide a methodology for decision making in a fuzzy environment. According to them, if $\underset{\sim}{G}$ is a fuzzy goal with a membership function $u_{\underset{\sim}{G}}(x)$ and if $\underset{\sim}{C}$ is a fuzzy constraint with a membership function $u_{\underset{\sim}{C}}(x)$, then $\underset{\sim}{D}$ is a fuzzy decision which results from the intersection of $\underset{\sim}{G}$ and $\underset{\sim}{C}$ with a membership

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[14] Since its introduction, fuzzy set theory has attracted many researchers. As a result, the literature on the theory has grown rapidly. For example, in 1966 only two papers on fuzzy set theory were published. In 1977, Gains and Kohout published a bibliography on fuzzy set thoery and related topics containing 1164 references of which 763 were classified as fuzzy. See B. R. Gains and L. J. Kohout, "The Fuzzy Decade: A Bibliography of Fuzzy Systems and Closely Related Topics", International Journal of Man-Machine Studies, Vol. 9 (1977), pp. 1-68. A more recent bibliography has been published by Kandel and Yager. It contains 1799 papers on fuzzy set theory and its applications. See A. Kandel and R. Yager, "A 1979 Bibliography on Fuzzy Sets, Their Applications, and Related Topics", in M. M. Gupta, R. K. Ragade, and R. R. Yager (eds.), Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 621-744. The applications of the theory cover many fields such as system theory, automata theory, decision theory, switching theory, pattern recognition, and medicine. Since this dissertation is intended to apply fuzzy set theory to a decision problem, namely, the cost variance investigation decision, some of the theory's applications in the decision making area are reviewed in this section.

[15] R. E. Bellman and L. A. Zadeh, "Decision-Making in a Fuzzy Environment", Management Science, Vol. 17 (1970), pp. B-141 - B-164.

function $u_{\tilde{D}}(x) = u_{\tilde{G}}(x) \wedge u_{\tilde{C}}(x)$. The optimal decision is any alternative in X which maximizes $u_{\tilde{D}}(x)$, where X is the set of the available alternatives. Using the above concepts of fuzzy goal, fuzzy constraint, and fuzzy decision Bellman and Zadeh suggest a method to solve the problem of multistage decision making for finite state fuzzy systems.

Jain[16] presents a decision model for selecting the optimal alternative under three situations, namely, (1) the knowledge about the system is fuzzy with non-fuzzy utilities; (2) the utilities associated with the alternatives are fuzzy with non-fuzzy system; and (3) fuzzy knowledge about the system with fuzzy utilities. In these three situations, the utilities associated with each alternative constitute a fuzzy set. To obtain the optimal alternative Jain uses the concept of maximizing set which is introduced by Zadeh.[17]

Watson, Weiss, and Donnell[18] present a method that takes into consideration the imprecision that surrounds the probabilities and utilities used in a decision tree. Since the probabilities and utilities are assumed to be fuzzy, the resulting expected utilities for the available actions are also fuzzy. The problem then is to determine which alternative is preferable. That is, to what extent do the

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[16] R. Jain, "Decisionmaking in the Presence of Fuzzy Variables", IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-6 (1976), pp. 698-703.

[17] L. A. Zadeh, "On Fuzzy Algorithm", Memo No. ERL-M325, Electronics Research Laboratory, University of California, 1972.

[18] S. R. Watson, J. J. Weiss, and M. L. Donnell, "Fuzzy Decision Analysis", IEEE Transactions on Systems, Man and Cybernetics, Vol. SMC-9 (1979), pp. 1-9.

membership functions for the alternatives imply that one alternative is better than the others. This problem is known in fuzzy set theory as the problem of deriving the "degree of truth" of an implication. The implication problem is discussed by Zadeh.[19]

Baas and Kwakernaak[20] propose a method to deal with multiple-aspect decision making in the presence of fuzziness. Their method is based on straightforward rating and ranking method, where the weights and ratings are represented by fuzzy variables. With fuzzy ratings and fuzzy weights the final ratings of the alternatives become fuzzy. To obtain a single optimal alternative, the concept of fuzzy sets induced by mapping is used by Baas and Kwakernaak. The concept of fuzzy sets induced by mapping is discussed by Zadeh.[21] Fuzzy multiple-aspect decision making is also considered by Yager, Jain, and Van Velthoven.[22]

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[19] L. A. Zadeh, "A Theory of Approximate Reasoning", Memo UCB/ERL M77/58, University of California, Berkeley, 1977, p. 27.

[20] S. M. Baas, and H. Kwakernaak, "Rating and Ranking of Multiple-Aspect Alternatives Using Fuzzy Sets", Automatica, Vol. 13 (1977), pp. 47-58.

[21] L. A. Zadeh, "Fuzzy Sets", pp. 346.

[22] R. R. Yager, "Multiple Objective Decision-Making Using Fuzzy Sets", International Journal of Man-Machine Studies, Vol. 9 (1977), pp. 375-382. R. Jain, "A Procedure for Multi-Aspect Decision Making Using Fuzzy Sets", International Journal of Systems Science, Vol. 8 (1977), pp. 1-7. G. Van Velthoven, "Fuzzy Models in Personnel Management", Proceedings of International Congress of Cybernetics and Systems, Bucharest (1975).

Fuzzy group decision making is introduced by Blin, and Blin and Whinston.[23] The preference of a group (A) is defined as a fuzzy relation with a membership function ($u_{\underset{\sim}{R}}$) that associates with each pair (a_i, a_j) in A x A its compatibility with the group preference. Moreover, a method is suggested to obtain a final group decision from the fuzzy group preference. The proposed method is based on the concept of an α-level set introduced by Zadeh.[24]

In the mathematical programming area (decision making under constraints), Zimmermann[25] extends the conventional linear programming problem into fuzzy linear programming through the fuzzification of the objectives and constraints. Then, he applies his method of fuzzy linear programming to a decision on the size and structure of a truck fleet. Zimmermann[26] also extends the linear programming with several objective functions into a fuzzy environment.

A more general method for decision making (i.e., the linguistic approach) is suggested by Zadeh.[27] The linguistic approach to decision

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[23] J. M. Blin, "Fuzzy Relations in Group Decision Theory", Journal of Cybernetics, Vol. 4 (1974), pp. 17-22. J. M. Blin and A. B. Whinston, "Fuzzy Sets and Social Choice", Journal of Cybernetics, Vol. 3 (1974), pp. 28-36.

[24] L. A. Zadeh, "Fuzzy Sets", p. 342.

[25] H. J. Zimmermann, "Description and Optimization of Fuzzy Systems", International Journal of General Systems, Vol. 2 (1976), pp. 209-215.

[26] H. J. Zimmermann, "Fuzzy Programming and Linear Programming with Several Objective Functions", Fuzzy Sets and Systems, Vol. 1 (1978), pp. 45-56.

[27] L. A. Zadeh, "The Linguistic Approach and Its Application to Decision Analysis", pp. 339-369.

making uses words, in place of numbers, to describe the values of variables, the relations between variables, as well as the values of probabilities.[28] According to Zadeh, "The rationale for ... employing words in place of numbers is that verbal characterizations are intrinsically approximate in nature and hence are better suited for the description of systems and processes which are as complex and as ill-defined as those which relate to human judgment and decision making."[29]

The central concept in the linguistic approach is the linguistic variable; a variable whose values are words or sentences instead of numbers.[30] Formally, a linguistic variable is defined by a quintuple $[X, T(X), U, G, M(x)]$, where

X = name of the variable,

$T(X)$ = collection of the variable linguistic values,

U = universe of discourse,

G = semantic rule which associates with each $x \varepsilon T(X)$ its meaning $M(x)$,

and

$M(x)$ = meaning of the linguistic value x.

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[28]The linguistic approach to decision making is not a qualitative method of making decisions. Instead, it is a combination of the quantitative and qualitative methods. Words are used when numbers are not adequate. Then, numbers are used to make the words more precise. Ibid., pp. 340-341.

[29]L. A. Zadeh, "The Linguistic Approach and Its Application to Decision Analysis", p. 340.

[30]Ibid.

For example, the variable "temperature", whose universe of discourse may be defined by the temperatures between zero and 110 degrees, is called a linguistic variable if it takes on values such as cool, moderate, hot, ..., etc. In this case, and T(X) are given by:

$U = \{0, \ldots, 110\}$,

and

$T(X) = \{\text{cool, moderate, hot, } \ldots\}$.

The linguistic values in T(X) are defined by fuzzy sets of the universe U. That is, each value in T(X) is described by a compatibility function which associates with every $u \in U$ its compatibility with the linguistic value. Figure 2 is the fuzzy set theory representation of the linguistic variable "temperature". The figure shows the interrelationships between the concepts of linguistic variable, linguistic values, and the meanings of the linguistic values.

Two other important concepts in the linguistic approach are the relational assignment equation and the fuzzy restriction concepts. Propositions related to a linguistic variable, such as "X is x," where x is the linguistic value of the linguistic variable X, can be translated into relational assignment equations by $R_X(u) = x$.[31] In this case, x

[31]Relational assignment equations can also be obtained for propositions involving two or more interrelated linguistic variables, such as, conjunctive, disjunctive, and conditional propositions. Moreover, propositions involving a linguistic truth value, such as true, very true, more or less true, etc., can also be translated into relational assignment equations through the use of the rules of truth-functional modifications. Ibid., pp. 346-350.

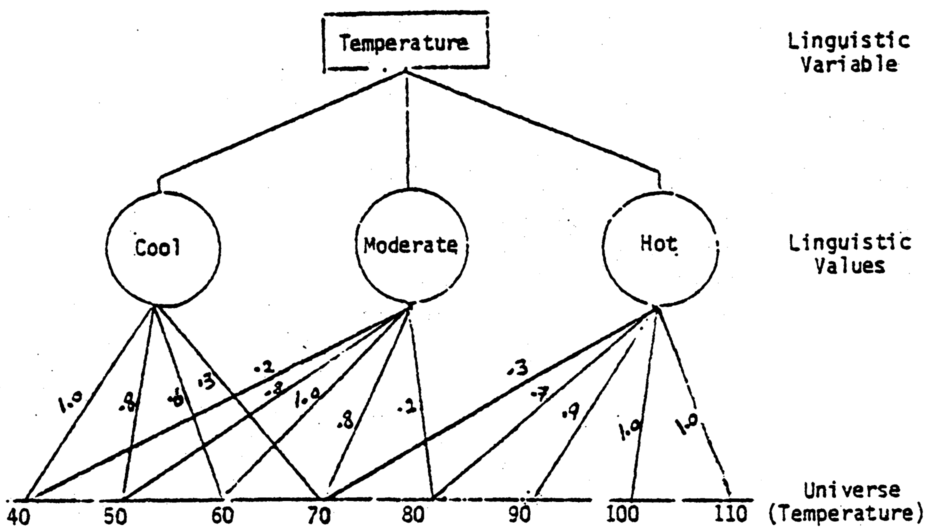

Figure 2. The Linguistic Variable "Temperature".

can be considered as a fuzzy restriction on the values that may be assigned to the base variable u.

The above concepts are used by Zadeh to solve the problem of optimization under multiple criteria. The linguistic approach is well suited for solving the multicriteria problem because

> ... when more than one criterion of performance is involved, the trade-offs between the criteria are usually poorly defined. In such cases ... linguistic characterizations of trade-offs or preference relations provide a more realistic conceptual framework for decision analysis than the conventional methods employing binary-valued preference relations.[32]

2.3 Summary

As indicated in Chapter One, the objective of the dissertation is to develop a new model for the cost-variance investigation problem. Since the calculus of fuzzy set theory is to be utilized in developing the new model, this chapter is devoted to the discussion of the basic elements of fuzzy set theory.

The central concept in the fuzzy set theory is the membership function which associates with each element in the universe its compatibility with the fuzzy set. Unlike the characteristic function for an ordinary set, the membership function takes its values in the closed interval from zero to one; the closer the element is to satisfying the requirements of the set, the closer its grade of membership is to one, and vice versa.

Because it allows for varying degrees of membership--through the concept of compatibility function--fuzzy set theory provides a means

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[32] Ibid., p. 354.

for handling fuzziness which is a major source of inexactness in many humanistic systems.

Since it was introduced, fuzzy set theory has been applied to many fields such as decision theory, automata theory, pattern recognition, and system theory. Because this dissertation is intended to apply fuzzy set theory to a decision problem, namely, the cost-variance investigation decision, some of the theory's applications in the decision making area are also reviewed in the chapter.

CHAPTER THREE

Cost Variance Investigation Models

Several quantitative models for solving the cost-variance investigation problem have been suggested by researchers. These models are reviewed and evaluated in this chapter. Since any evaluation should be based on certain standards, the first section of the chapter is devoted to discussing some of the criteria to be used in evaluating the existing cost-variance investigation models.

3.1 Criteria for Evaluating Cost-Variance Investigation Models

In evaluating the existing cost-variance investigation models, three criteria are used. First, a model should take into consideration the costs and benefits of the investigation. This criterion is initiated by the cost-benefit philosophy of management accounting which states that actions should be taken only if their potential benefits exceed their incremental costs.[1] The cost-variance investigation action is no exception; it should be subject to the cost-benefit test. Consequently, any cost variance investigation model should consider both the costs and benefits of the investigation.[2]

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[1] As noted by Horngren "The cost-benefit theme provides innate appeal to both the hard-headed manager and the theoretician. Managers have been employing the cost-benefit test for years even though they may not have expressed it as such. Instead, they may have referred to the theme as 'having to be practical' despite what theory may say." C. T. Horngren, Introduction to Management Accounting, 5th ed. (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1981), p. 16.

[2] Horngren also indicates that any model for the cost-variance investigation decision must "... answer the cost-benefit question." Ibid., p. 187.

Second, a cost-variance investigation model--like any other model--should capture the essence of the real problem.[3] If a reasonable degree of isomorphism (similarity) between the model and the problem is not preserved then "The Mathematical solution of [the] problem formulated in terms of the model may not be even an approximate to the corresponding problem in the real case."[4]

It should be noted that realism in the model is related directly to the realism of the assumptions introduced in the model. Over-simplified assumptions--or as indicated by A. Kaplan "... having too much of a good thing,"[5]--may lead to contrived isomorphism between the model and the problem. Consequently, the results obtained from the model are more likely to be false.

It is inappropriate to construe the above statement as urging that models should be based on no assumptions; assumptions are necessary to make the model solvable. Instead, what is needed are models "... whose assumptions can be met without distorting the conditions existing in the problem."[6]

The third criterion for evaluating cost-variance investigation models is the feasibility of the model. Or, stated differently, a cost-variance investigation model should be implementable.

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[3]This does not mean that the model should be a reproduction of reality.

[4]A. Kaplan, The Conduct of Inquiry (California: Chandler Publishing Company, 1964), p. 281.

[5]Ibid.

[6]J. P. Gigch and L. L. Pipino, "From Absolute to Probable and Fuzzy in Decision Making", Kybernetes, Vol. 9 (1980), p. 51.

One aspect of implementation is the obtainability of the data required to operate the model. If the data are not obtainable, then, as indicated by Kaczka, the model builder "... may find he has given birth to a model which is incapable of reasonably answering the questions it will be asked."[7]

The obtainability of the data--and consequently the feasibility of the model--is affected by the level of precision required by the model. As noted by A. Kaplan "Models are often improperly exact: they call for measures that we cannot in fact obtain."[8]

Note that undue emphasis on exactness may affect adversely the relevance of the model to the real world. Models are likely to ignore some relevant items just because these items are incapable of precise measurement or the inclusion of them may complicate the model. This can be explained by the Principle of Incompatability which is stated by Zadeh as follows:

> . . . as the complexity of a system increases our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost exclusive characteristics.[9]

Again, it is not appropriate to interpret the above statement as urging that models should be imprecise. Instead, what is meant by the

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[7] E. E. Kaczka, "Computer Simulation", Decision Sciences, Vol. 1 (1971), p. 177.

[8] A. Kaplan, The Conduct of Inquiry, p. 283.

[9] L. A. Zadeh, "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes", IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-3 (1973), p. 28.

statement is that if a choice between precision and relevance is to be made, ". . . there is in fact no choice but to go for the [relevance]."[10]

3.2 Review of Cost-Variance Investigation Models

The available cost variance investigation models can be divided into two groups. The first group includes models that do not take into consideration the costs and benefits of the investigation action, while the second includes models that encompass both the costs and the benefits of the investigation decision.

The first group of models is exemplified by the Zannetos, Jeurs, Luh, Koehler, and Probst models.[11] This group of models is based on the classical statistical theory. According to these models, the expected value and standard deviation of the cost distribution are assumed to be known when the process is in control. Then, the probability that any given cost observation could have come from the assumed distribution is calculated. The investigation is undertaken if the calculated probability falls below a given level.

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[10] A. Kaplan, The Conduct of Inquiry, p. 284.

[11] Z. A. Zannetos, "Standard Cost as a First Step to Probabilistic Control: A Theoretical Justification, an Extension and Implications", The Accounting Review (April 1964), pp. 296-304. D. A. Jeurs, "Statistical Significance of Accounting Variances", Management Accounting (October 1967), pp. 20-25. F. Luh, "Controlled Costs: An Operational Concept and Statistical Approach to Standard Costing", The Accounting Review (January 1968), pp. 123-132. R. W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control", Management Accounting (October 1968), pp. 35-41. F. R. Probst, "Probabilistic Cost Controls: A Behavioral Dimension", The Accounting Review (January 1971), pp. 113-118.

This process is facilitated through the use of control charts such as the simple $\overline{X}$-Chart suggested by Shewhart.[12] Essentially the $\overline{X}$-Chart consists of control limits placed on either side of the standard. Variations falling outside the control limits are attributed to controllable factors, however, variations inside the limits are considered the result of noncontrollable factors. Consequently, an investigation is warranted only when the cost observation falls outside a control limit.

The second group of models extends the probability analysis to include the costs and benefits of the investigation action. In the accounting literature three models can be identified, namely, the Bierman, Fouraker, and Jaedicke (BFJ) model; the Kaplan model; and the Dyckman model.[13]

In the BFJ model, the cost distribution is assumed to be known when the process is in control. Moreover, the model assumes the costs (C) and the benefits (L) of the investigation action to be known. Probability analysis is then used to compute the probability (P) that any given cost observation could have come from the known in-control distribution. The decision rule is to investigate the variance if the cost of investigation is less than (1 - P)L.

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[12] W. A. Shewhart, The Economic Control of the Quality of Manufactured Profit (New York: MacMillan, 1931).

[13] H. Bierman, Jr., L. E. Fouraker, and R. K. Jaedicke, "The Use of Probability and Statistics in Performance Evaluation", The Accounting Review (July 1961), pp. 409-417. R. S. Kaplan, "Optimal Investigation Strategies with Imperfect Information", Journal of Accounting Research (Spring 1969), pp. 32-43. T. R. Dyckman, "The Investigation of Cost Variances", Journal of Accounting Research (Autumn 1969), pp. 215-244.

According to the BFJ model, the expected benefit is defined as the ". . . present value of the costs that will be incurred in the future if an investigation is not made now."[14] This method of estimating the expected benefits is criticized by Kaplan for ignoring the fact that future actions may be taken to correct an out-of-control situation.[15] The BFJ model is also criticized by Dyckman for not considering prior information.[16]

The second model in this group is the multiperiod Bayesian model developed by Kaplan. The model assumes a two-state (in-control, out-of-control), two-action (investigate, do not investigate) system. The system is described by a Markovian chain. Bayes' theorem is then employed to calculate the posterior state probabilities. Moreover, dynamic programming is used to calculate the break-even probability (q_i^*). When the number of periods increases, the break-even probabilities converge to a limiting value, q^*, which is the steady state value. This q^* is then used to determine whether a cost variance is worthy of investigation. The decision is made to investigate the variance if the posterior probability for the period is less than or equal to q^*.

It should be noted that Kaplan's model does not include a consideration of the implementation and information costs. Moreover, the distributions of the in-control and out-of-control costs must be

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[14] H. Bierman, Jr., L. E. Fouraker, and R. K. Jaedicke, "The Use of Probability and Statistics in Performance Evaluation", pp. 414-415.

[15] R. S. Kaplan, "The Significance and Investigation of Cost Variances: Survey and Extensions", Journal of Accounting Research (Autumn 1975), p. 319.

[16] T. R. Dyckman, "The Investigation of Cost Variances", p. 221.

known. However, Kaplan's model does consider the implications of future decisions.

The third model in this group is the single-period Bayesian model developed by Dyckman. In his paper "The Investigation of Cost Variances",[17] Dyckman discusses three situations: (a) A process once corrected remains in the in-control state; otherwise changes between states are not permitted, (b) A process is in the state of control initially but may move to the out-of-control state with probability P at the start of any period. However, the process cannot shift from the out-of-control state to the in-control state, and (c) Changes in both directions are possible with the process being in-control initially.

For situation (a), Dyckman presents a revision of the BFJ formulation. According to Dyckman, the investigation decision is based on the priors, the observed cost level, and the conditional probability distribution rather than being based on only the most recent cost observation.

For situations (b) and (c), Dyckman provides a model similar to Kaplan's. However, instead of using dynamic programming to obtain the break-even probability, Dyckman uses the equation $f^{*}(\theta_1) = \frac{L - C}{L}$, where C and L represent the costs and benefits of the investigation, respectively.

Because it is similar to Kaplan's model, Dyckman's model is restricted by the same limitations. Dyckman's model also is faced

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[17] Ibid., pp. 215-244.

with a new difficulty, namely, ignoring the effects of future decisions.[18]

In addition to the three models which appeared in the accounting literature and are discussed above, the second group of models includes the Duvall model which appeared in the management science literature.[19]

In his model, Duvall allows the in-control costs to be described by a continuous probability distribution. Moreover, the model assumes that any deviation from the standard can be divided into two parts, namely, deviations caused by non-controllable factors and deviations caused by controllable factors. The two types of deviations are assumed to be normally distributed and statistically independent. Duvall suggests a method to obtain the parameters of the two distributions. In addition, Duvall assumes the savings from the investigation process to be directly related to the size of the controllable variance. After obtaining a cost variance, the conditional probability of the controllable deviations is determined. Then, the expected net savings are calculated. The decision is to investigate the variance if the expected net savings are greater than zero.

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[18] Y. Li, "A Note on the Investigation of Cost Variances", Journal of Accounting Research (Autumn 1970), p. 282.

[19] R. M. Duvall, "Rules for Investigating Cost Variances", Management Science, Vol. 13 (June 1967), pp. B-631 - B-641. A similar model (but not directly related to the cost-variance investigation problem) appeared in the statistical literature. See, G. A. Bather, "Control Charts and the Minimization of Costs", Journal of the Royal Statistical Society, Vol. 25 (1963), pp. 49-70.

Duvall's model is criticized by Kaplan for not taking into consideration the fact that actions may be taken in the future.[20] Kaplan also questions the validity of Duvall's procedure for using only 25 observations to calculate the parameters of the controllable cost distribution.[21] The model is also criticized by Kaplan for not being consistent; the model assumes stationarity, however, it ignores previous information.[22] Finally, Dyckman criticizes the model for assuming that the total deviation is equal to the sum of the controllable and noncontrollable deviations.[23]

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[20] R. S. Kaplan, "The Significance and Investigation of Cost Variances: Survey and Extensions", p. 328.

[21] Kaplan states, "Just how we know that 25 is the right number of observations to be taken for estimating sample means and standard deviations is never discussed. Nor are we told what to do if some of the early observations indicate an out-of-control situation. Must we still wait until we get the full 25 observations before taking action?" Ibid., p. 329. Duvall's method of estimating the mean and standard deviation of the controllable cost distribution is also criticized by Dyckman who states that the method ". . . does not appear to be operationally helpful, and it delays decisions over substantial time periods." T. R. Dyckman, "The Investigation of Cost Variances", pp. 240-241.

[22] According to Kaplan, if stationarity is assumed, then the decision of investigation should be based on all the observations. R. S. Kaplan, "The Significance and Investigation of Cost Variances: Survey and Extension", p. 328. Dyckman also criticizes Duvall's model for basing the investigation decision on only the most recent observation. See T. R. Dyckman, "The Investigation of Cost Variances", p. 241.

[23] Dyckman states: "The . . . model is unsatisfactory in that it permits negative costs if a long enough period is assumed. A model of the [multiplicative] form is perhaps more intuitively appealing." T. R. Dyckman, "The Investigation of Cost Variances", p. 236.

3.3 Deficiencies of the Models

Although it may be hard to believe, several authors have noted that managers do not seem to be using the investigation models which have appeared in the academic literature. For example, Koehler reported that ". . . in some general inquiry from some prominent corporations, I was unable to find a single use of statistical procedures for variance control."[24]

Anthony also reported that ". . . few if any managers believe that statistical techniques . . . are worth the effort to calculate them."[25] Finally, Magee said: "While researchers have suggested various methods of making this decision . . . it seems that these methods are seldom used, at least formally, in practice."[25]

Different reasons for the lack of application of these models have been suggested by several authors. According to Koehler, the models are rarely used in practice because of the lack of statistical knowledge among accountants.[27] On the other hand, Anthony attributes this lack of application to the inappropriateness of the models; the

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[24] R. W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control", p. 35.

[25] R. N. Anthony, "Some Fruitful Directions for Research in Management Accounting", in N. Dopuch and L. Revsine (eds.), Accounting Research 1960-1970: A Critical Evaluation (Center for International Education and Research in Accounting: University of Illinois, 1973), p. 52.

[26] R. P. Magee, "A Simulation Analysis of Alternative Cost Variance Investigation Models", The Accounting Review (July 1976), p. 529.

[27] R. W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control", p. 35.

stochastic processes assumed by these models are not similar to the actual processes.[28]

Magee states:

> There are a number of possible explanations for managers not choosing to use the existing 'sophisticated' cost investigation models, even if the stochastic cost process assumed by the models is a reasonable approximation of reality.[29]

According to him, the models are not used in practice because they do not capture the real world problem. For example, the models ignore important factors such as: (1) the cost of operating a more complex model, (2) the manner in which the decision maker's performance is evaluated, and (3) the shape of the decision maker's utility function.

Besides the deficiencies already noted and based on the criteria discussed in the first section of this chapter, other deficiencies in the models can be identified.

The first group of models does not meet the first criterion for cost-variance investigation models; the models in this group fail to consider the costs and benefits of the investigation.

In addition to this major problem, the first group of models suffers from being based purely on objective evidence; ". . . objective evidence may be lacking, too expensive, or a new process may be involved, etc."[30] Second, the models ignore the information from

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[28] R. N. Anthony, "Some Fruitful Directions for Research in Management Accounting", p. 52.

[29] R. P. Magee, "A Simulation Analysis of Alternative Cost Variance Investigation Models", p. 532.

[30] M. Onsi, "Quantitative Models for Accounting Control", The Accounting Review (April 1967), p. 324.

previous observations of the process. Third, there is no consideration of the possible implications of future actions. Fourth, the models do not consider the implementation costs and information requirements.

The second group of models meets the test of addressing the cost-benefit question. However, the models in this group suffer from not capturing the real world problem (over-simplifying assumptions) and from the lack of applicability. In support of these contentions, the following observations regarding the models are offered.

First, the models assume that the inexact knowledge about the state of the system may be attributed to uncertainty (i.e., randomness). For example, Kaplan reports: "The situation is complicated because the states must be inferred from observation of the outputs of the system, in the form of operating reports. But outputs of the system do not bear a unique relationship to the states. Thus, merely observing the outputs does not give complete state information so that there is some uncertainty about the optimal action."[31]

However, as Kaplan's statement may imply, the sort of inexactness that comes into play here is better represented by the notion of fuzziness (or imprecision) than that of randomness (or uncertainty); the knowledge about the system is not complete because there is no precise relationship between the outputs and the states of the system. In support of this claim, consider what is reported by Kaplan and by Duvall:

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[31]R. S. Kaplan, "Optimal Investigation Strategies with Imperfect Information", pp. 33-34.

> Presumably $f_1(x)$ is such that most of the probability is concentrated about low costs and $f_2(x)$ has most of its mass at a higher cost. However, in the general formulation of the problem, there is the possibility of obtaining high costs when in control as well as low costs when out of control.[32] [Emphasis added].

> For small values of x, $\lambda(x)$ should be near zero indicating the greater likelihood that the observation was obtained while the system was in control. Conversely, for large values of x (i.e., costs), $\lambda(x)$ should be large indicating that large costs are more likely to arise from an out of control situation.[33] [Emphasis added].

> A deviation which occurs when the activity is being properly performed will be referred to as noncontrollable deviation. Similarly, a deviation which occurs due to nonstandard performance will be referred to as a controllable deviation. It may be that the nonstandard performance was due to causes which were, in fact, noncontrollable, and in these cases the terminology is inappropriate.[34] [Emphasis added].

> . . . Table 1 gives [the] cost deviations for twelve days when it has been determined that close to standard conditions prevailed. If we average these twelve deviations, we get $w^* = \ldots = \$.0724$ which is close to zero, our assumed value for u_w, and indicates that the standards are probably about right.[35] [Emphasis added].

> If [the] probability is small it is assumed that nonstandard procedures made the actual deviation very large and an investigation is undertaken.[36] [Emphasis added].

These statements include imprecise or ill-defined terms such as large, most, small, high, low, more likely, probably, close, near, properly performed, etc. It is believed that these terms, and not the randomness

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[32] Ibid., p. 34.

[33] Ibid.

[34] R. M. Duvall, "Rules for Investigating Cost Variances", p. B-631.

[35] Ibid., p. B-632.

[36] Ibid., p. B-633.

or uncertainty, are the reasons for the inexact knowledge about the state of the system. Stated differently, the inexact knowledge about the state of the system is attributed to the fact that we are dealing with fuzzy classes--classes that do not have crisp boundaries that separate those objects belonging to the class and those not belonging.

Second, the models are based on the "law of the excluded middle"; they do not allow for the transition between what is "in" what is "out". In other words, the models assume that the performance can only be either in-control or out-of-control. This is an oversimplification of the real problem. Kaplan, for example, reports:

> There are at least two other assumptions that may limit the application of this approach. One is the heroic simplification of the process to a two-state system, in-control and out-of-control, with sudden transitions between the states. While we have seen that this model is frequently used in the quality control literature, the process by which a controllable cost process suddenly moves from in control to out of control is rather difficult to articulate. Intuitively it is more appealing to consider a process that gradually drifts away from standards through an evolutionary process of neglect and lack of proper supervision. In such a case, the forced dichotomy between in control and out of control may be an unrealistic aggregation of reality. One solution is to expand the number of states to allow for <u>varying degrees of out-of-controlness</u>. For example, we might allow <u>S states (S = 5 or 10, say)</u> with state 1 representing <u>perfectly</u> in contol, state 2 representing <u>slight</u>

deterioration, and state 5 being well out of control.[37] [Emphasis added].

Third, the above models do not allow the decision maker to express his perceived imprecision of his estimates. Stated differently, the above models assume that the accuracy and precision of the information required for the analysis is fixed. This can be exemplified by the following statement: ". . . certain variables which are a part of the control system will be treated as parameters, i.e., their levels will be assumed fixed (although perhaps unknown). These parameters [include] the accuracy and precision of the information."[38]

However, in real life situations, the accuracy and precision of the information are not always fixed. That is, in real world, some human judgments are "harder" than others. This fact is implied by Dyckman's statement: ". . . the estimation of the parameters p and g in this case is more difficult than estimating the parameter p in the transition

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[37] R. S. Kaplan, "The Significance and Investigation of Cost Variances: Survey and Extensions", pp. 322-323. This quote also supports the fact that we are dealing with fuzziness rather than randomness. The terms perfectly in-control, slight deterioration, and well out-of-control represent fuzzy classes. In the Duvall model, a different dichotomy is used, namely, the dichotomization between controllable and noncontrollable deviations. This dichotomy may also not capture the real world problem. For example, Duvall reports: "It may be that the nonstandard performance was due to causes which were, in fact, noncontrollable, and in these cases the terminology is inappropriate." R. M. Duvall, "Rules for Investigating Cost Variances", p. B-631. Kaplan also considers the dichotomy between controllable and noncontrollable deviations to be one of the major problems encountered by the Duvall model because one may not be able ". . . to separate out the effects of normal fluctuations (noise) from changes in the level of the process". R. S. Kaplan, "The Significance and Investigation of Cost Variances: Survey and Extensions", p. 330.

[38] T. R. Dyckman, "The Investigation of Cost Variances", p. 217.

matrix P."[39] The same fact is also recognized by Bierman and Dyckman, who write "Reliable estimates of the investigation and correction costs . . . should be easy to obtain. On the other hand, the benefits from an investigation are more difficult to determine with precision."[40]

Fourth, the models are based on the implied assumption that each of the judgments called for in the analysis can be represented accurately by a single precise number. Stated differently, the models demand high levels of precision in measuring the variables and in describing how they are related. However, in most cases these levels of precision are not attainable.

For example, the models require exact values of the benefits obtained from the investigation. However, as indicated by Duvall, the precise determination of the benefits resulting from the investigation is usually difficult to attain.[41]

Moreover, the determination of the precise shapes of cost distributions is necessary for the analysis. This can be exemplified by the following statement: "In order to use Bayes' theorem, the conditional probability of the observed cost under each state must be known."[42] However, in real life situations, the shapes of the distributions are known only imprecisely. This fact is recognized by Dyckman who writes: "Suppose . . . that the cost distributions in question are

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[39] Ibid., p. 228.

[40] H. Bierman, Jr., and T. Dyckman, Management Cost Accounting (New York: The MacMillan Company, 1971), p. 504.

[41] R. M. Duvall, "Rules for Investigating Cost Variances", p. 638.

[42] T. R. Dyckman, "The Investigation of Cost Variances", p. 222.

normal or <u>approximately</u> normal."[43] [Emphasis added]. The same fact is also implied by Kaplan who states: "One alternative might be to compute optimal policies under a number of different cost distributions . . . to determine the sensitivity of the optimal policy to the precise form of these cost distributions."[44]

Fifth, as a result of being based on precise measures, the models are likely to lose much relevance to the real-world problem. In other words, the models ignore some relevant items just because these items are incapable of precise measurement and/or the inclusion of them may lead to the complexity of the models.

To support the above claim, consider, for example, the following statements:

> Finally, an investigation, once initiated, may fail to disclose a situation requiring adjustment; or alternatively, adjustment, once undertaken, may fail to restore the desired state. This last embellishment will be ignored . . .[45]

will

> An additional assumption . . . is that a full investigation will always reveal the cause of an out-of-control situation, which can then be immediately corrected.[46]

> The probability that the process will be discovered to be out of control in period n is given by the expression:

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[43] Ibid., p. 222.

[44] R. S. Kaplan, "Optimal Investigation Strategies with Imperfect Information", pp. 42-43.

[45] T. R. Dyckman, "The Investigation of Cost Variances", p. 217.

[46] Ibid., p. 218.

$$\sum_{k=1}^{n} \left\{(1 - P)^{k-1} \; p[(1 - N_X)(x_{c,n}; E(X/\theta_2), \sigma(X/\theta_2)] \right.$$

$$\left[\prod_{k}^{n-k} N_X[x_{c,k}; E(X/\theta_2), \sigma(X/\theta_2)]\right.$$

$$\left.\left.\prod_{1}^{k-1} \Big((1 - N_X)(x_{c,n}; E(X/\theta_1)[\sigma(X/\theta_1)])\Big)\right]\right\}.$$

> . . . Due to the complexity of this expression, the estimate of p discussed earlier may be satisfactory.[47]

From these statements it is clear that some relevant items are ignored because they are incapable of precise measurement and/or their inclusion may increase the complexity of the model.

Sixth, the demand of precise measurements may well explain the uneasiness of the potential users of the models. This uneasiness concerns the decision maker's ability to provide precise measurements necessary for the models to give reliable answers.

3.4 Summary

Two groups of cost-variance investigation models have appeared in the accounting literature. In this chapter, these two groups of models are reviewed and evaluated.

The evaluation of the models is based on three criteria, namely,

(1) answering the cost-benefit question;

(2) capturing the real-world problem, i.e., reality of assumptions; and

(3) feasibility of the model, i.e., the obtainability of the data required to operate the model.

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[47] Ibid., p. 225.

As shown in the chapter, the first group of models does not answer the cost-benefit question. The second group of models, although meeting the first criterion, fails to capture the real-world problem (over-simplified assumptions). For example, the models assume that inexactness--whatever its nature--can be equated with randomness. However, the sort of inexactness involved in the investigation decision is better represented by fuzziness (or imprecision). Second, the models are based on the assumption of a two-state system, in-control and out-of-control. This assumption is an oversimplification of the process. Third, the models assume a fixed level of accuracy and precision. However, some human judgments are "harder" than others.

The models in the second group also suffer from the lack of applicability. They require precise numerical inputs to the analysis; however, in real-life situations, the required level of precision is not obtainable.

As a result of the insistence on precise measurements the models are likely to lose part of their relevance to the real-world problem through ignoring some relevant items. Furthermore, the demand for precise numerical values may be the reason for the uneasiness of the potential users of these models.

CHAPTER FOUR

Application of Fuzzy Set Theory to the Cost Variance Investigation Problem

In this chapter a new model for the cost variance investigation decision that may overcome some of the problems encountered by the available cost variance investigation models is developed. A brief discussion of the cost variance investigation process is provided first.

4.1 Cost Variance Investigation Process

In a standard costing system, performance reports are prepared regularly to report on performance during different periods. For any period, the performance report indicates the standard performance, actual performance, and the variance. The variance specifies the conditions of the performance which are known as the states of the performance. The set of the performance states will be represented by:

$$S_t = \left\{ s_1, \ldots, s_i, \ldots, s_n \right\} . \tag{1}$$

Faced with a performance report, the decision maker should make an investigation decision. The set of permissible decisions is represented by:

$$D_t = \left\{ d_1, \ldots, d_j, \ldots, d_m \right\} . \tag{2}$$

Associated with each $d_j \varepsilon D_t$ and $s_i \varepsilon S_t$, is an output state, $s_k \varepsilon S_{t+1}$. The set of the output states, denoted by:

$$S_{t+1} = \left\{ s_1, \ldots., s_k, \ldots., s_n \right\}, \tag{3}$$

is related to S_t through a transformation function which is represented by:

$$S_{t+1} = f(S_t, D_t). \tag{4}$$

Moreover, with each $s_i \varepsilon S_t$, $d_j \varepsilon D_t$, and $s_k \varepsilon S_{t+1}$ there is a return or objective function that measures the effectiveness of the decision being made and the output arising from that decision. The return function will be represented by:

$$B = b(S_t, D_t, S_{t+1}). \tag{5}$$

For any $s_i \varepsilon S_t$, the net benefits associated with the different combinations of the decisions and the output states can be represented in matrix form as illustrated in Figure 3. In the figure, B_{ijk} represents the net benefit that may be obtained if the new state is s_k given that the current state is s_i and decision d_j is made.

4.2 Fuzzy Set Theory Approach to the Cost Variance Investigation

As was seen in Chapter Three, the available cost variance investigation models fail to handle the imprecision surrounding the cost-variance investigation problem. Stated differently, according to the available models, the states of the performance are defined by characteristic functions that take only the values of zero or 1. Moreover, most of the available models dichotomize between completely

D_t	S_{t+1}				
	s_1	......	s_k	......	s_n
d_1	B_{i11}	....	B_{i1k}	....	B_{i1n}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
d_j	B_{ij1}	....	B_{ijk}	....	B_{ijn}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
d_m	B_{im1}	....	B_{imk}	....	B_{imn}

Figure 3. The net benefits matrix for a single input state.

out-of-control and completely in-control states.[1] Since such a cichotomization does not capture the real world problem, the need for allowing for varying degrees of out-of-controllness was recognized by R. Kaplan.

Allowing for different degrees of out-of-controllness as well as for fuzziness that surrounds the investigation decision can be achieved by using the calculus of fuzzy set theory which will be utilized in developing the new model.

In the new model, in contrast to what was assumed by the available models, the set of the performance states, S_t, may include not only the two states, in-control and out-of-control, but also intermediate states. For example, S_t may include, in addition to the extreme states, the more or less out-of-control state.

Moreover, in the new model, every state $s_i \varepsilon S_t$ will be described by a fuzzy set represented by:

$$s_i = \left\{ u_{s_i}(v_t)/v_t \right\} . \tag{6}$$

The membership function $u_{s_i} : u \rightarrow [0,1]$ associates with every variance in the universe $V = \{0,1,2,...\}$ its compatibility with the state s_i. For example, the in-control state may be described by:

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[1]R. Kaplan, "The Significance and Investigation of Cost Variances: Survey and Extensions", Journal of Accounting Research (Autumn 1975), pp. 322-323. In the Duvall model, a different dichotomy is used, namely, the dichotomization between controllable and noncontrollable deviations. However, as was shown in Chapter Three, footnote No. 37, such a dichotomy may not capture the real world problem. Also, the Duvall model, like the other models, is subject to the limitation of not handling fuzziness that surrounds the investigation decision.

$$u_{in}(v) = \begin{cases} 1, & \text{if } 0 \leq v \leq 100 \\ \dfrac{1}{1 + \left(\dfrac{v - 100}{100}\right)^2} & \text{if } v > 100 \end{cases} \tag{7}$$

Thus, the compatibility of a variance of \$400 with the in-control state is approximately 0.1, while the compatibility of a variance of \$800 with the same state is 0.02. Figure 4 is an example of three compatibility functions for the three states in-control, more or less out of control, and out-of-control.

As was also mentioned in Chapter Three, the net benefits from the investigation are difficult to determine with precision. Consequently, it is more appropriate to assume that the net benefits can only be determined fuzzily (imprecisely).

In the new model the estimates of the net benefits, B_{ijk}, will be allowed to take the form: approximately \$800, near \$1,000, etc. The estimates may also take the form of linguistic values such as high, very high, medium, low, and very low net benefits, where these values are fuzzy sets represented by:

$$B_{ijk} = \left\{ u_{B_{ijk}} (b_L)/b_L \right\} . \tag{8}$$

The compatibility function $u_{B_{ijk}} : u \rightarrow [0,1]$ associates with every net benefit b_L in the universe $B = \{0,1,2,\ldots\}$ its compatibility with the fuzzy set B_{ijk}. For instance, low (L), medium (M), and high (H) net

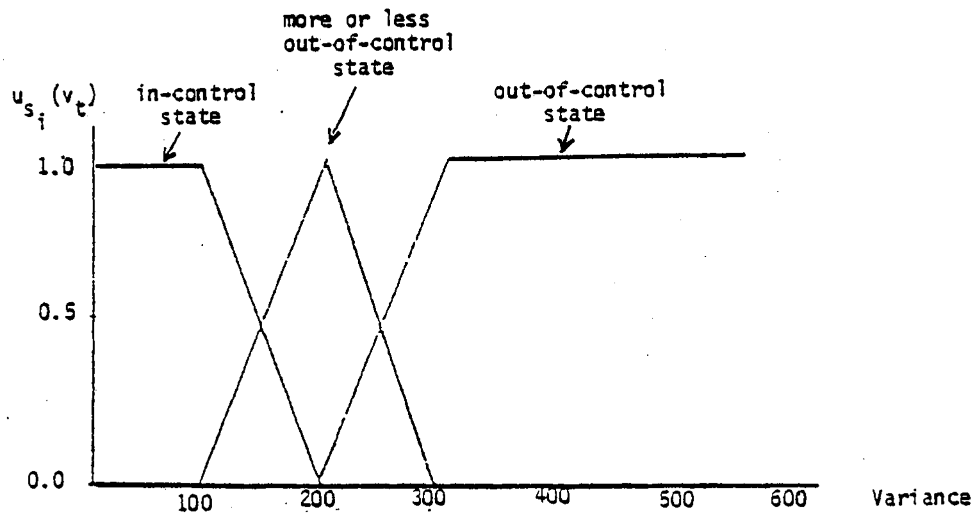

Figure 4. An example of compatibility functions for three performance states

benefits may be subjectively defined by:

$$L = \{(.4/100), (1.0/200), (.5/300)\} ,$$

$$M = \{(.2/200), (.4/300), (.7/400), (1.0/500), (.7/600), (.4/700), (.2/800)\},$$

$$H = \{(.5/700), (1.0/800), (.5/900), (.2/1000)\} .$$

These fuzzy sets can be diagrammed as in Figure 5.

As indicated earlier, the set S_{t+1} is related to the set S_t through a transformation function. This function can be deterministic, stochastic, or fuzzy. At this level of the discussion, the transformation function is assumed to be deterministic. (This assumption will be relaxed in Chapter Five.)

Under the above assumption, the net benefits associated with the different combinations of the input states and the decisions can be represented in matrix form as in Figure 6. In the figure, B_{ij} represents the net benefits that may result when the state of performance is s_i and decision d_j is made.

Since the states of the performance and the net benefits associated with the different combinations of the states and decisions are assumed to be known fuzzily, one cannot use the statistical decision theory to obtain the optimal decision. This is because the statistical decision theory is not equipped to handle fuzziness or imprecision. Taking this into consideration, a method based on fuzzy set theory will be suggested to obtain the optimal alternative.

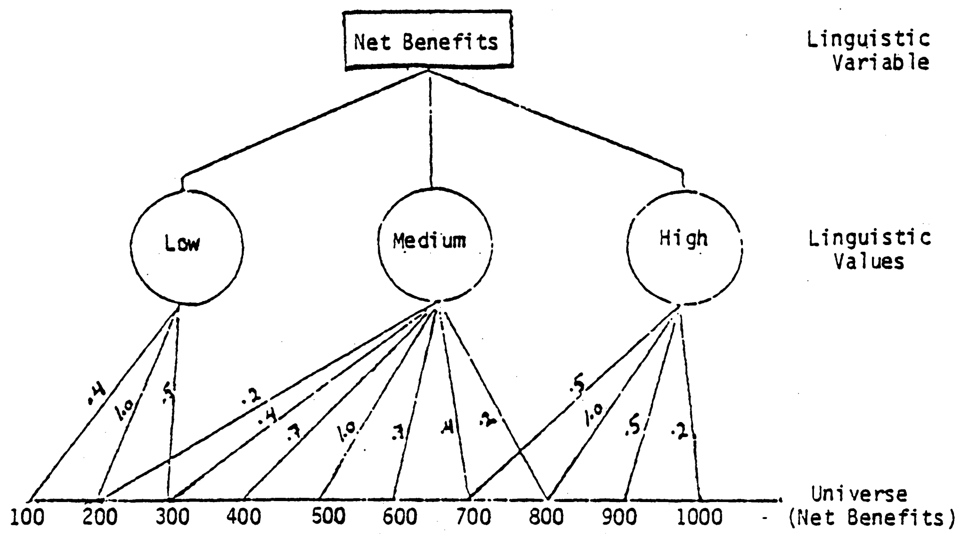

Figure 5. The Linguistic Variable "Net Benefits".

D_t	S_t				
	s_1	......	s_i	......	s_n
d_1	B_{11}	.....	B_{i1}	.....	B_{n1}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
d_j	B_{1j}	.....	B_{ij}	.....	B_{nj}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
d_m	B_{1m}	.....	B_{im}	.....	B_{nm}

Figure 6. Net benefits associated with the different combinations of input states and decisions.

The method to be presented is based on formulating a fuzzy set

$$D_o = \left\{ u_{D_o} (d_j)/d_j \right\} \tag{9}$$

representing a fuzzy optimal decision set. The grade of membership in this set, $u_{D_o}(d_j)$, reflects the relative merit of the decision d_j. The optimal alternative is, then, the decision having the highest grade of membership in the set D_o.

To obtain D_o, one first obtains the net benefits associated with every alternative. The extension principle[2] can be used to induce for each decision, d_j, a fuzzy net benefit which reflects the fuzzy knowledge about the state of the performance. The net benefits associated with decision $d_j \varepsilon D_t$ is given by:

$$B_j = \left\{ u_{B_j} (B_{ij})/B_{ij} \right\} , \tag{10}$$

where

$$u_{B_j} (B_{ij}) = \max_{B_{ij} \varepsilon f^{-1}(B_j)} \quad u_{s_i}(v_t). \tag{11}$$

It should be noted that the set B_j is a fuzzy set of fuzzy sets. This set should be reduced to a fuzzy set of nonfuzzy net benefits. This can be accomplished through the use of the operation of fuzzification.

As defined by Zadeh, the operation of fuzzification has the effect of ". . . transforming a fuzzy (or nonfuzzy) set A into an

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[2]The extension principle is mainly a principle that allows the domain of a mapping to be extended from points in the universe into fuzzy sets of the universe. A more detailed discussion of the extension principle is provided in the next chapter.

approximating set $\underset{\sim}{A}$ which is more fuzzy than A."[3] Two types of fuzzification are available, namely, the S-fuzzification and the G-fuzzification. Only the S-fuzzification, in which every point in the support of the nonfuzzy (or fuzzy) set is fuzzified, will be discussed.

The basis for S-fuzzification is the point fuzzification which maps points in X into fuzzy sets K(x). The fuzzy sets K(x) are called the kernel of the fuzzification.

The point fuzzification can be extended from mapping elements in X into mapping fuzzy subsets of X. Let X be a set with each $x \varepsilon X$ being mapped into a fuzzy set K(x). If A is a fuzzy subset of X, then A can be mapped into a fuzzy set F(A;K) which is given by:

$$F(A;K) = \int_X u_A(x) \cdot K(x), \qquad (12)$$

where $u_A(x)$ is the membership function of A; K(x) is the kernel of fuzzification; and $u_A(x) \cdot K(x)$ is the product of $u_A(x)$ and K(x).[4]

It should be noted that the operation of fuzzification provides a means for handling situations in which the elements in the initial set X are fuzzy sets of another set. For example, if A is a fuzzy set of X and each $x \varepsilon X$ is a fuzzy set of another set, say Y, then A can be expressed as a fuzzy set of Y as follows:

$$A = \left\{ u_A(y)/y \right\}, \qquad (13)$$

where

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[3] L. A. Zadeh, "A Fuzzy Set Theoretic Interpretation of Linguistic Hedges", Journal of Cybernetics, Vol. 2 (1972), p. 17.

[4] Ibid., p. 18. The integration sign indicates the union.

$$u_A(y) = \max_{y \varepsilon f^{-1}(A)} [u_A(x) \cdot u_x(y)], \tag{14}$$

$u_A(x)$ is the membership function of A,

and

$u_x(y)$ is the membership function of x.

As an example, let X, Y be two sets defined by

$X = \{x_1, x_2\}$,

and

$Y = \{y_1, y_2, y_3\}$.

Moreover, let K be a mapping from X to fuzzy sets of Y such that

$K(x_1) = \{(.3/y_1), (.6/y_2), (.5/y_3)\}$,

and

$K(x_2) = \{(.6/y_1), (.5/y_2), (.8/y_3)\}$.

Finally, let A be a fuzzy set of X defined by

$\underset{\sim}{A} = \{(.5/x_1), (.3/x_2)\}$.

Then, by applying equations (13) and (14), A can be expressed by:

$$A = \{.5[(.3/y_1), (.6/y_2), (.5/y_3)]\}, \{.3[(.6/y_1), (.5/y_2), (.8/y_3)]\}$$

$$= \{(.18/y_1), (.3/y_2), (.25/y_3)\}.$$

As was mentioned earlier, set B_j is a fuzzy set of fuzzy sets. The operation of fuzzification can be used to reduce this set B_j into a fuzzy set of nonfuzzy net benefits, B_{jL}, which is given by:

$$B_{jL} = \left\{u_{B_{jL}}(b_L)/b_L\right\}, \tag{15}$$

where

$$b_L \varepsilon B_{ij},$$

and

$$u_{B_{jL}}(b_L) = \max_{b_L \varepsilon f^{-1}(B_{jL})} [u_{B_j}(B_{ij}) \cdot u_{B_{ij}}(b_L)] \; . \qquad (16)$$

The optimal decision may now be selected either on the basis of the maximum net benefit, b_L, or on the basis of the highest grade of membership, $u_{B_{jL}}(b_L)$, in the sets B_{jL}'s. These two methods of selecting the optimal decision may lead to an improper decision. If the choice is based on the maximum net benefits associated with the decisions, there may be only a small possibility of receiving this net benefit. On the other hand, if the choice is based on the highest grade of membership, it may happen that the net benefit associated with the highest grade of membership is very small.

As an illustration, consider two decisions 1 and 2 whose B_{1L} and B_{2L} are given by:

$$B_{1L} = \{(.2/1000), (.6/3000), (.1/10000)\},$$

$$B_{2L} = \{(.8/1000), (.9/3000), (.5/5000)\}.$$

If the decision is based upon only the maximum net benefits, then the decision maker would choose decision 1 to obtain the \$10,000 net benefit. However, the \$10,000 has a compatibility of only .1 with B_{1L}. On the other hand, if the decision is based on the highest possibility of obtaining any net benefit, then the decision maker would choose decision 2 as the optimal decision since it has the greatest possibility of obtaining a net benefit. However, the net benefit associated with the highest $u_{B_{2L}}$ is only $3,000.

If, however, the decision maker is attempting to maximize the net benefits to be derived in the long run, he would consider both the value of the net benefit and the possibility of obtaining the net benefit. That is, the selection of the optimal decision should be based on both the maximum b_L's associated with the decisions and the grades of membership associated with the net benefits.

This can be accomplished by using the concept of a maximizing set for a function f introduced by Zadeh[4] and extended by Jain[5] to the case where the function f is replaced by a set Y. According to Jain, ". . . the maximizing set M(Y) of a set Y [is] a fuzzy set such that the grade of membership of a point $y \varepsilon Y$ in M(Y) represents the degree to which y approximates to sup Y in some specified sense."[6]

To use the concept of a maximizing set, one needs to form the set Y which consists of all possible net benefit values. Mathematically, the set Y is given by:

$$Y = \bigcup_{j=1}^{m} b_L \varepsilon B_{jL}. \qquad (17)$$

Then, the maximizing set for decision $d_j \varepsilon D_t$ (denoted by B_{jm}) is given by:

[5] L. A. Zadeh, "On Fuzzy Algorithm", Memo No. ERL-M325, Electronic Research Laboratory, University of California (1972), p. 10.

[6] R. Jain, "Decisionmaking in the Presence of Fuzzy Variables", IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-6 (1976), pp. 698-703.

[7] Ibid., p. 700.

$$B_{jm} = \left\{ u_{B_{jm}}(b_L)/b_L \right\} , \tag{18}$$

where

$$u_{B_{jm}}(b_L) = (b_L/b_{max})^n , \tag{19}$$

$$b_{max} = \sup Y, \tag{20}$$

and n is an integer chosen depending on the application.

After obtaining the maximizing set B_{jm}, a fuzzy set B_{jo} is formed by the intersection of the sets B_{jm} and B_{jL}. The membership function of the fuzzy set

$$B_{jo} = \left\{ u_{B_{jo}} (b_L)/b_L \right\} \tag{21}$$

is given by:

$$u_{B_{jo}} (b_L) = u_{B_{jL}} (b_L) \wedge u_{B_{jm}} (b_L) \quad . \tag{22}$$

Now, the fuzzy set $D_o = \{u_{D_o}(d_j)/d_j\}$ representing the fuzzy optimal decision space can be formulated. The grade of membership, $u_{D_o}(d_j)$, of each decision $d_j \varepsilon D_t$ is given by:

$$u_{D_o}(d_j) = \bigvee_L u_{B_{jo}}(b_L). \tag{23}$$

The optimal decision, d_o, is then the one having the highest grade of membership in the set D_o. That is, the optimal alternative is the one which has

$$u_{D_o}(d_o) = \bigvee_j u_{D_o}(d_j) \quad . \tag{24}$$

4.3 Numerical Example

Let $S_t = \{s_1, s_2, s_3\}$, where s_1, s_2, and s_3 represent the in-control, more or less out of control, and out-of-control states, respectively. Moreover, let $D_t = \{d_1, d_2, d_3\}$, where d_1, d_2, and d_3 represent the investigation, exploratory investigation, and no-investigation actions, respectively. Let the net benefits matrix be given by:[8]

Decision	State		
	s_1	s_2	s_3
d_1	VL	M	VH
d_2	L	M	H
d_3	L	L	VL

where L, VL, M, H, and VH are defined as follows:

VL = {(1.0/100), (.4/200)} ,

L = {(.4/100), (1.0/200), (.5/300)} ,

M = {(.4/300), (.7/400), (1.0/500), (.7/600), (.4/700)} ,

H = {(.5/700), (1.0/800), (.5/900)} , and

VH = {(.5/900), (1.0/1000)} .

Moreover, let a performance report indicate a variance of \$500. Assume, also, that the indicated variance has compatibilities of .4, .5, and .6

[8] VL, L, M, H, and VH denote very low, low, medium, high, and very high net benefits, respectively.

with the in-control, more or less out of control, and out-of-control states. That is, $u_{s_1}(500) = .4$, $u_{s_2}(500) = .5$, and $u_{s_3}(500) = .6$.

Using equations (10) and (11), one can obtain the sets B_1, B_2, and B_3 as follows:

$$B_1 = \{(.4/VL), (.5/M), (.6/VH)\} ,$$

$$B_2 = \{(.4/L), (.5/M), (.6/H)\} ,$$

$$B_3 = \{(.4/L), (.5/L), (.6/VL) = (.5/L), (.6/VL)\}.$$

Using the definitions of VL, L, M, H, and VH, the sets B_1, B_2, and B_3 can be rewritten as follows:

$$\begin{aligned} B_1 = \ & \{(.4/[(1.0/100), (.4/200)]), \\ & (.5/[(.4/300), (.7/400), (1.0/500), (.7/600), (.4/700)]), \\ & (.6/[(.5/900), (1.0/1000)])\} , \end{aligned}$$

$$\begin{aligned} B_2 = \ & \{(.4/[(.4/100), (1.0/200), (.5/300)]), \\ & (.5/[(.4/300), (.7/400), (1.0/500), (.7/600), (.4/700)]), \\ & (.6/[(.5/700), (1.0/800), (.5/900)])\} , \end{aligned}$$

$$\begin{aligned} B_3 = \ & \{(.5/[(.4/100), (1.0/200), (.5/300)]), \\ & (.6/[(1.0/100), (.4/200)])\} . \end{aligned}$$

The sets B_1, B_2, and B_3 which are fuzzy sets of fuzzy subsets can be reduced to the sets B_{1L}, B_{2L}, and B_{3L}, respectively, by using Eqs. (15) and (16) as follows:

$$\begin{aligned} B_{1L} = \ & \{(.4/100), (.16/200), (.2/300), (.35/400), (.5/500), \\ & (.35/600), (.2/700), (.3/900), (.6/1000)\} , \end{aligned}$$

$$B_{2L} = \{(.16/100), (.4/200), (.2/300), (.2/300), (.35/400), (.5/500), (.35/600), (.2/700), (.3/700), (.6/800), (.3/900)\}$$

$$= \{(.16/100), (.4/200), (.2/300), (.35/400), (.5/500), (.35/600), (.3/700), (.6/800), (.3/900)\},$$

$$B_{3L} = \{(.2/100), (.5/200), (.25/300), (.6/100), (.24/200)$$
$$= (.6/100), (.5/200), (.25/300)\} .$$

Using Eq. (15), the set Y can be obtained as follows:

$$Y = \{100, 200, 300, 400, 500, 600, 700, 900, 1000\} \cup \{100, 200, 300, 400, 500, 600, 700, 800, 900\} \cup \{100, 200, 300\}$$
$$= \{100, 200, 300, 400, 500, 600, 700, 800, 900, 1000\} .$$

It should be noted that b_{max} = Sup Y = 1000. Then using n = 2 and Eqs. (18) and (19), the maximizing set for decision d_1 can be obtained as follows:

$$B_{1m} = \left\{\left((\tfrac{100}{1000})^2/100\right), \left((\tfrac{200}{1000})^2/200\right), \left((\tfrac{300}{1000})^2/300\right), \left((\tfrac{400}{1000})^2/400\right), \left((\tfrac{500}{1000})^2/500\right), \left((\tfrac{600}{1000})^2/600\right), \left((\tfrac{700}{1000})^2/700\right), \left((\tfrac{900}{1000})^2/900\right), \left((\tfrac{1000}{1000})^2/1000\right)\right\}$$

$$= \{(.01/100), (.04/200), (.09/300), (.16/400), (.25/500), (.36/600), (.49/700), (.81/900), (1.0/1000)\} .$$

Similarly, the sets B_{2m} and B_{3m} are obtained as follows:

$$B_{2m} = \{(.01/100), (.04/200), (.09/300), (.16/400), (.25/500), (.36/600), (.49/700), (.64/800), (.81/900)\} ,$$

$$B_{3m} = \{(.01/100), (.04/200), (.09/300)\} .$$

Now, Eqs. (21) and (22) can be used to obtain the set B_{1o} as follows:

$$\begin{aligned} B_{1o} = & \{(.4\wedge.01/100), (.16\wedge.04/200), (.2\wedge.09/300), \\ & (.35\wedge.16/400), (.5\wedge.25/500), (.35\wedge.36/600), \\ & (.2\wedge.49/700), (.3\wedge.81/900), (.6\wedge1.0/1000)\} \\ = & \{(.01/100), (.04/200), (.09/300), (.16/400), (.25/500), \\ & (.35/600), (.2/700), (.3/900), (.6/1000)\} . \end{aligned}$$

Similarly, the sets B_{2o} and B_{3o} are given by:

$$B_{2o} = \{(.01/100), (.04/200), (.09/300), (.16/400), (.25/500), (.35/600), (.3/700), (.6/800), (.3/100)\},$$

$$B_{3o} = \{(.01/100), (.04/200), (.09/300)\} .$$

To obtain the grade of membership of d_1 in the set D_o, Eq. (23) may be used as follows:

$$u_{D_o}(d_1) = (.01 \vee .04 \vee .09 \vee .16 \vee .25 \vee .35 \vee .2 \vee .3 \vee .6) = .6.$$

Similarly, $u_{D_o}(d_2) = .6$ and $u_{D_o}(d_3) = .09$. That is, $D_o = \{(.6/d_1), (.6/d_2), (.09/d_3)\}$. The optimal decision is then either d_1 or d_2.

4.4 Summary

In this chapter a new model for the cost-variance investigation decision is proposed. The model is intended to overcome the problems encountered by the existing cost-variance investigation models.

The proposed model allows for varying degrees of out-of-controllness. That is, the model is not subject to the limiting assumption of a two-state system. Moreover, the proposed model allows for the imprecision surrounding the states and the net benefits through the use of the compatibility function concept.

Furthermore, the model allows the decision maker to express his perceived imprecision about his judgments. Stated differently, the model is not subject to the limiting assumption of constant level of precision.

Finally, the model does not require a high level of precision in measuring the variables. That is, the model reduces the need for precise measures that are difficult to obtain in real life situations.

The proposed model is based on the assumption that the performance is described by a deterministic function. This assumption will be relaxed in the next chapter.

CHAPTER FIVE

Extension of the Model

As mentioned earlier, the transformation function, $S_{t+1} = f(S_t, D_t)$, can be deterministic, stochastic, or fuzzy. In the previous chapter, the deterministic transformation function was assumed. In this chapter, the model is extended to cover the stochastic and fuzzy transformation function cases.

5.1 Stochastic Transformation Function

In this section, the performance is assumed to be stochastic whose state at time t+1 is a probability distribution over S_{t+1} which is conditioned on S_t and D_t. That is, for each input state $s_i \varepsilon S_t$, the probabilities that the performance will be in state $s_k \varepsilon S_{t+1}$ when decision $d_j \varepsilon D_t$ is taken are represented in matrix form as in Figure 7. In the figure, Q_{ijk} represents the probability that the new state will be s_k given that the current state is s_i and decision d_j is made.

As mentioned in Chapter Three, the precise values of the probabilities (i.e. the precise shapes of the probability distributions) are not obtainable. Moreover, the fact that some probability judgments are "harder" than others should be captured within the decision analysis. Consequently, it is more appropriate to assume that the probability values can only be determined imprecisely.

In the new model, the decision maker is allowed to express his judgments about the probabilities by using statements like "about 30%", "approximately 40%", and "roughly 20%". The judgments may also take the form of linguistic values such as: "quite likely", "almost

D_t	S_{t+1}				
	s_1	......	s_k	......	s_n
d_1	Q_{i11}	....	Q_{i1k}	....	Q_{i1n}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
d_j	Q_{ij1}	....	Q_{ijk}	....	Q_{ijn}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
d_m	Q_{im1}	....	Q_{imk}	....	Q_{imn}

Figure 7. Probabilities matrix for a single input state.

certainly", "better than even", "pretty likely", etc. Each fuzzy probability can be defined by a fuzzy set represented by:

$$Q_{ijk} = \left\{ u_{Q_{ijk}} (q_k)/q_k \right\}, \tag{1}$$

where the compatibility function $u_{Q_{ijk}}(q_k)$ associates with each $q_k \varepsilon \{0,.1,.2,...,1\}$ its compatibility with the fuzzy set Q_{ijk}. For example, the probability judgment of "about 30%", can be subjectively defined by:

$$Q = \{(.1/.20),\ (.6/.25),\ (1.0/.30),\ (.6/.35),\ (.1/.40)\}.$$

For each change in the performance (i.e., for every combination of $d_j \varepsilon D_t$, $s_i \varepsilon S_t$, and $s_k \varepsilon S_{t+1}$) there is a corresponding net benefit. Given $s_i \varepsilon S_t$ to be the input state, the net benefits resulting from the different combinations of $d_j \varepsilon D_t$ and $s_k \varepsilon S_{t+1}$ can be represented in matrix form as shown in Figure 3, Chapter 4. The net benefit, B_{ijk}, will be assumed to be defined fuzzily as in the deterministic case. That is,

$$B_{ijk} = \left\{ u_{B_{ijk}} (b_k)/b_k \right\}, \tag{2}$$

where the compatibility function, $u_{B_{ijk}}(b_k)$, associates with each $b_k \varepsilon B = \{0,1,...\}$ its compatibility with the fuzzy set B_{ijk}.

The above situation (fuzzy probabilities, fuzzy net benefits, and fuzzy states) can be dealt with by first obtaining the expected net benefit (denoted by EB_{ij}) associated with every $d_j \varepsilon D_t$ given s_i to be the input state.

In the probability theory, the expected value of an action (denoted, for example, by EB_{ij}) is given by:

$$EB_{ij} = \sum_{k=1}^{n} B_{ijk} \cdot Q_{ijk} \text{ ,} \tag{3}$$

where the B_{ijk}'s are real numbers and Q_{ijk}'s are points in the interval [0,1].

The basic question is how to calculate the expected net benefit, EB_{ij}, for each $d_j \varepsilon D_t$ given s_i to be the input state when the net benefits and the probabilities are assumed to be fuzzy. Before suggesting a method to calculate the expected net benefits, a brief discussion of the extension principle is provided.

As defined by Zadeh, the extension principle is a principle that ". . . allows the domain of the definition of a mapping or a relation to be extended from points in U to fuzzy subsets of U."[1] Let f be a mapping from X to W s.t. f(x) = w, where $x \varepsilon X$ and $w \varepsilon W$. Moreover, let A be a fuzzy set of X given by:

$$A = \{u_A(x)/x\}.$$

Then, by the extension principle, the image of A under f is a fuzzy set of W defined by:

$$f(A) = \left\{ u_{f(A)}(w)/w \right\}, \tag{4}$$

where

$$w = f(x), \tag{5}$$

and

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[1]L. A. Zadeh, "The Concept of a Linguistic Variable and Its Application to Approximate Reasoning", Part I, Information Sciences, Vol. 8 (1975), p. 236.

$$u_{f(A)}(w) = \max_{x \varepsilon f^{-1}(w)} u_A(x) \quad . \tag{6}$$

The extension principle can be applied to cases where the function f is an n-ary function. For example, let f be a mapping from the Cartesian product X x Y to a space W s.t. $w = f(x,y)$, where $x \varepsilon X$, $y \varepsilon Y$ and $w \varepsilon W$. Moreover, let A and B be fuzzy sets of X and Y, respectively, defined by:

$$A = \{u_A(x)/x\} \ ,$$

and

$$B = \{u_B(y)/y\} \ .$$

Then, the image of (A,B) under f is a fuzzy set of W defined as:

$$f(A,B) = \{u_{f(A,B)}(w)/w\} \ , \tag{7}$$

where

$$w = f(x,y), \tag{8}$$

and

$$u_{f(A,B)}(w) = \max_{(x,y) \varepsilon f^{-1}(w)} \min[u_A(x), \ u_B(y)]. \tag{9}$$

As an example, let X = Y = R and let A and B be fuzzy sets of X and Y, respectively, defined by:

$$A = \underset{\sim}{2} = \text{approximately } 2 = \{(.6/1), \ (1/2), \ (.3/3)\},$$

and

$$B = \underset{\sim}{6} = \text{approximately } 6 = \{(.8/5), \ (1/6), \ (.7/7)\} \ .$$

Moreover, let f : X x Y $\rightarrow$ W s.t. $w = x + y$, where x, y, and w denote the generic points in X, Y, and W, respectively. Then, using Equations (7)-(9), one can obtain $\underset{\sim}{2} + \underset{\sim}{6}$ as follows.

$$\underset{\sim}{2} + \underset{\sim}{6} = \{(.6/6), (.8/7), (1/8), (.7/9), (.3/10)\} .$$

The extension principle can be used to extend the definition of the expected value given in Equation (3) from points in B and Q into fuzzy sets of B and Q. Let Q_{ijk}'s be fuzzy probabilities described by compatibility functions $u_{Q_{ijk}}(q_k)$'s, where q_k is the generic point associated with Q_{ijk}. Moreover, let $B_{ijk}(b_k)$'s be fuzzy net benefits described by compatibility functions $u_{B_{ijk}}(b_k)$'s, where b_k is the generic point associated with B_{ijk}. As a direct application of the extension principle, the expected net benefit, EB_{ij}, is given by:

$$EB_{ij} = \left\{ u_{EB_{ij}}(b_L)/b_L \right\} , \tag{10}$$

where

$$b_L = \sum_{k=1}^{n} q_k \cdot b_k, \tag{11}$$

and

$$u_{EB_{ij}}(b_L) = \max_{(q_k, b_k)\varepsilon f^{-1}(EB_{ij})} \min_k \left\{ u_{Q_{ijk}}(q_k);\ u_{B_{ijk}}(b_k) \right\} . \tag{12}$$

The above expression for EB_{ij} which is obtained by a direct application of the extension principle, does not take into account the fact that the Q_{ijk}'s are related to each other through the condition $\sum_{k=1}^{n} q_k = 1$. To account for such a condition, the expression for EB_{ij} given above needs to be changed to the following:

$$EB_{ij} = \left\{ u_{EB_{ij}}(EB_{ij})/b_L \right\}, \tag{13}$$

where

$$b_L = \begin{cases} \sum_{k=1}^{n} q_k \cdot b_k, & \text{if } \sum_{k=1}^{n} q_k = 1 \\ 0, & \text{elsewhere} \end{cases} \tag{14}$$

and

$$u_{EB_{ij}}(EB_{ij}) = \begin{cases} \max\limits_{(q_k,b_k)\varepsilon f^{-1}(EB_{ij})} \min\limits_{k} \left\{ u_{Q_{ijk}}(q_k); u_{B_{ijk}}(b_k) \right\}, \\ \text{if } \sum_{k=1}^{n} q_k = 1 \\ 0, \quad \text{elsewhere} \end{cases} \tag{15}$$

Once the expected net benefits for the decisions are obtained, one can construct a table as the one in Figure 8. In the figure, EB_{ij} represents the expected net benefit that may result when the state of the performance is s_i and decision d_j is made.

After obtaining the expected net benefits, it becomes possible to reduce the solution of the problem under consideration to that of the deterministic case; the expected net benefits can be treated as the net benefits were treated in the deterministic problem. That is, to obtain the optimal alternative the following steps are to be taken:

(1) Obtain the expected net benefits associated with each decision d_j. Using the fuzzy knowledge about the state of the performance, the expected net benefits associated with decision d_j (denoted EB_j) is

D_t	S_t				
	s_1	......	s_i	......	s_n
d_1	EB_{11}	....	EB_{i1}	....	EB_{n1}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
d_j	EB_{1j}	....	EB_{ij}	....	EB_{nj}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
d_m	EB_{1m}	....	EB_{im}	....	EB_{nm}

Figure 8. Expected net benefits associated with the different combinations of input states and decisions.

obtained by:

$$EB_j = \left\{ u_{EB_j}(EB_{ij})/EB_{ij} \right\} , \tag{16}$$

where

$$u_{EB_j}(EB_{ij}) = \max_{EB_{ij} \varepsilon f^{-1}(EB_j)} u_{s_i}(v_t) \quad . \tag{17}$$

(2) Reduce the set EB_j, which is a fuzzy set of fuzzy sets, to a fuzzy set of nonfuzzy benefits. This can be accomplished by obtaining the fuzzy set EB_{jL} which is given by:

$$EB_{jL} = \left\{ u_{EB_{jL}}(b_L)/b_L \right\} , \tag{18}$$

where

$$b_L \varepsilon EB_{ij} \quad ,$$

and

$$u_{EB_{jL}}(b_L) = \max_{b_L \varepsilon f^{-1}(EB_{jL})} [u_{EB_j}(EB_{ij}) \cdot u_{EB_{ij}}(b_L)] \; . \tag{19}$$

(3) Obtain the maximizing set. To do this one first obtains the set Y which is given by:

$$Y = \bigcup_{j=1}^{m} b_L \varepsilon EB_{jL} \quad . \tag{20}$$

Then the maximizing set for decision $d_j \varepsilon D_j$ (denoted by EB_{jm}) is given by:

$$EB_{jm} = \left\{ u_{EB_{jm}}(b_L)/b_L \right\} , \tag{21}$$

where

$$u_{EB_{jm}}(b_L) = (b_L/b_{max})^n , \tag{22}$$

$$b_{max} = \sup Y , \tag{23}$$

and n is an integer.

(4) Obtain the intersection of the sets EB_{jL} and EB_{jm}. This can be done by obtaining the set EB_{jo} which is given by:

$$EB_{jo} = \left\{ u_{EB_{jo}}(b_L)/b_L \right\} , \tag{24}$$

where

$$u_{EB_{jo}} = u_{EB_{jL}}(b_L) \wedge u_{EB_{jm}}(b_L). \tag{25}$$

(5) Formulate the fuzzy optimal decision set. This can be accomplished by obtaining the set D_o which is given by:

$$D_o = \left\{ u_{D_o}(d_j)/d_j \right\}, \tag{26}$$

where

$$u_{D_o}(d_j) = \bigvee_L u_{EB_{jo}}(b_L) \quad . \tag{27}$$

The optimal decision is d_j which has the highest grade of membership in the set D_o.

5.2 Numerical Example I

Let $S_t = \{s_1, s_2\}$, where s_1 and s_2 denote the in-control and out-of-control states, respectively. Let $D_t = \{d_1, d_2\}$, where d_1 and d_2 represent the investigate and do-not-investigate actions, respectively.

Let the Q_{ijk} and B_{ijk} matrices for s_1 and s_2 be given by:

S_t	D_t	Q_{ijk} s_1	Q_{ijk} s_2	B_{ijk} s_1	B_{ijk} s_2
s_1	d_1	Q_1	Q_2	B_1	B_2
	d_2	Q_3	Q_4	B_3	B_4
s_2	d_1	Q_5	Q_6	B_5	B_6
	d_2	Q_7	Q_8	B_7	B_8

where

Q_1 = ".3" = {(.8/.2), (1/.3), (.6/.4)},

Q_2 = ".7" = {(.8/.6), (1/.7), (.6/.8)},

$Q_3 = Q_5$ = likely = {(.5/.8), (.8/.9), (1/1)},

$Q_4 = Q_6$ = unlikely = {(1/0), (.8/.1), (.5/.2)},

Q_7 = ".6" = {(.6/.4), (1.0/.6), (.6/.8)},

Q_8 = ".4" = {(.6/.2), (1.0/.4), (.6/.6)},

B_1 = low = {(.4/100), (1.0/200), (.5/300)},

$B_2 = B_5 = B_8$ = very high = {(.5/900), (1.0/1000)},

B_3 = high = {(.5/700), (1.0/800), (.5/900)},

$B_4 = B_7$ = very low = {(1.0/100), (.4/200)},

B_6 = medium = {(.7/400), (1/500), (.4/600)}.

Finally, assume that a performance report indicates a variance of \$600 whose compatibilities with s_1 and s_2 are .8 and .4, respectively.

Using Equations (13)-(15), one can obtain the sets EB_{11}, EB_{21}, EB_{12}, and EB_{22} as follows:

$$EB_{11} = \{(.4/580), (.5/620), (.4/640), (.5/660), (.6/680), (.5/690), (.5/720), (.4/730), (.4/740), (1/760), (.5/780), (.5/790), (.4/820), (.6/840), (.5/860)\},$$

$$EB_{21} = \{(.5/800), (.5/820), (.4/840), (.5/850), (.5/860), (.4/870), (.5/880), (.5/900), (.4/920), (.7/940), (.8/950), (.4/960), (1/1000)\},$$

$$EB_{12} = \{(.5/580), (.4/600), (.5/640), (.4/650), (.5/660), (.4/680), (.5/700), (.8/730), (.5/740), (.4/760), (1/800), (.5/820), (.4/830), (.5/900)\},$$

$$EB_{22} = \{(.5/260), (.6/280), (.4/340), (.4/360), (.5/420), (1/460), (.4/480), (.4/520), (.5/580), (.4/620), (.6/640), (.4/680)\}.$$

Using Equations (16) and (17) and the fact that $u_{s_1}(600) = .8$ and $u_{s_2}(600) = .4$, one can obtain the sets EB_1 and EB_2 as follows:

$$EB_1 = \{(.8/EB_{11}), (.4/EB_{21})\},$$

and

$$EB_2 = \{(.8/EB_{12}), (.4/EB_{22})\}.$$

Using the definitions of EB_{11}, EB_{21}, EB_{12}, and EB_{22}, and Equations (18) and (19), one can obtain the sets EB_{1L} and EB_{2L} as follows:

$$EB_{1L} = \{(.32/580), (.4/620), (.32/640), (.4/660), (.48/680), (.4/690), (.4/720), (.32/730), (.32/740), (.8/760), (.4/780), (.4/790), (.32/820), (.48/840), (.4/860), (.2/800), (.2/820), (.16/840), (.2/850), (.2/860), (.16/870), (.2/880), (.2/900), (.16/920), (.28/940), (.32/950), (.16/960), (.4/1000)\}$$

$$= \{(.32/580), (.4/620), (.32/640), (.4/660), (.48/680), (.4/690), (.4/720), (.32/730), (.32/740), (.8/760), (.4/780), (.4/790), (.32/820), (.48/840), (.4/860), (.2/800), (.2/850), (.16/870), (.2/880), (.2/900), (.16/920), (.28/940), (.32/950), (.16/960), (.4/1000)\}$$

$$EB_{2L} = \{(.4/580), (.32/600), (.4/640), (.32/650), (.4/660), (.32/680), (.4/700), (.64/730), (.4/740), (.32/760), (.8/800), (.4/820), (.32/830), (.4/900), (.2/260), (.24/280), (.16/340), (.16/360), (.2/420), (.4/460), (.16/480), (.16/520), (.2/580), (.16/620), (.24/640), (.16/680)\}$$

$$= \{(.4/580), (.32/600), (.4/640), (.32/650), (.4/660), (.32/680), (.4/700), (.64/730), (.4/740), (.32/760), (.8/800), (.4/820), (.32/830), (.4/900), (.2/260), (.24/280), (.16/340), (.16/360), (.2/420), (.4/460), (.16/480), (.16/520), (.16/620)\}.$$

From the definitions of EB_{1L} and EB_{2L}, one can easily verify that sup Y = 1000. Then, using b_{max} = sup Y = 1000, n = 2, and Equations (21) and (22), one can obtain the maximizing sets EB_{1m} and EB_{2m} as follows:

EB_{1m} = {(.34/580), (.38/620), (.41/640), (.43/660), (.46/680), (.47/690), (.52/720), (.53/730), (.55/740), (.57/760), (.61/780), (.62/790), (.67/820), (.70/840), (.74/860), (.64/800), (.72/850), (.76/870), (.77/880), (.81/900), (.85/920), (.88/940), (.90/950), (.92/960), (1/1000)}

EB_{2m} = {(.34/580), (.36/600), (.41/640), (.42/650), (.43/660), (.46/680), (.49/700), (.53/730), (.55/740), (.58/760), (.64/800), (.67/820), (.69/830), (.81/900), (.07/260), (.08/280), (.12/340), (.13/360), (.18/420), (.21/460), (.23/480), (.27/520), (.38/620)} .

Now, Equations (24) and (25) can be used to obtain the sets EB_{1o} and EB_{2o} as follows:

EB_{1c} = {(.32/580), (.38/620), (.32/640), (.4/660), (.46/680), (.4/690), (.4/720), (.32/730), (.32/740), (.57/760), (.4/780), (.4/790), (.32/820), (.48/840), (.4/860), (.2/800), (.2/850), (.2/870), (.2/880), (.2/900), (.16/920), (.28/940), (.32/950), (.16/960), (.4/1000)},

EB_{2o} = {(.33/580), (.32/600), (.4/640), (.32/650), (.4/660), (.32/680), (.4/700), (.53/730), (.4/740), (.32/760), (.64/800), (.4/820), (.32/830), (.4/900), (.07/260), (.08/280), (.12/340), (.13/360), (.18/420), (.21/460), (.16/480), (.16/520), (.16/620)}.

Using Equations (26) and (27) and the above definitions of EB_{1c} and EB_{2o}, one can obtain the set D_o as follows:

$$D_0 = \left\{(.57/d_1),\ (.64/d_2)\right\}.$$

The optimal decision is then d_2.

5.3 Fuzzy Transformation Function

In this section the fuzzy transformation function case is discussed. More specifically, in what follows, the performance will be described by a fuzzy relation[2] R: $S_t \times D_t \sim S_{t+1}$, where S_t, D_t, and S_{t+1} are the set of input states, the set of available decisions, and the set of output states, respectively.

For any given decision $d_j \varepsilon D_t$, the relation R is described by the fuzzy set

$$R(S_t,\ S_{t+1}\,||\,d_j) = \{u_R(s_i,\ s_k\,||\,d_j)/(s_i,\ s_k\,||\,d_j)\}, \tag{28}$$

where $s_i \ \varepsilon\ S_t$, $s_k \ \varepsilon\ S_{t+1}$, $d_j \ \varepsilon\ D_t$, and $u_R(s_i,\ s_k\,||\,d_j)$ is the membership function which associates with every pair $(s_i,\ s_k)\ \varepsilon\ S_t \times S_{t+1}$ its compatibility with the fuzzy set R. In matrix form, this relation can be represented as in Figure 9. In the figure, u_{ijk} stands for $u_R(s_i,\ s_k\,||\,d_j)$.

[2]A fuzzy relation $\underset{\sim}{R}$ between two sets E_1 and E_2 is a fuzzy subset of their Cartesian product $E_1 \times E_2$. Mathematically $\underset{\sim}{R}$ is defined by:

$$\underset{\sim}{R} = \int_{(x,y)\ \varepsilon\ E_1 \times E_2} (u_{\underset{\sim}{R}}(x,y)/(x,y)),$$

where $x \varepsilon E_1$, $y \varepsilon E_2$, and $u_{\underset{\sim}{R}}(x,y)$ is the membership function of the fuzzy relation $\underset{\sim}{R}$. L. A. Zadeh, "Fuzzy Sets", Information and Control, Vol. 8 (1965), p. 345.

d_j \ S_{t+1} / S_t	s_1	……	s_k	……	s_n
s_1	u_{ij1}	....	u_{ijk}	....	u_{ijn}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
s_i	u_{ij1}	....	u_{ijk}	....	u_{ijn}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
s_n	u_{nj1}	....	u_{njk}	....	u_{njn}

Figure 9. Fuzzy relation $R(S_t, S_{t+1} || d_j)$.

For every change in the performance (i.e., for every combination of $d_j \varepsilon D_t$, $s_i \varepsilon S_t$, and $s_k \varepsilon S_{t+1}$) there is a corresponding net benefit. Given d_j, the net benefits resulting from the different combinations of $s_i \varepsilon S_t$ and $s_k \varepsilon S_{t+1}$ can be represented in matrix form as shown in Figure 10. The net benefit, B_{ijk}, is assumed to be defined fuzzily as in the deterministic and stochastic cases. That is, B_{ijk} is given by:

$$B_{ijk} = \{u_{B_{ijk}}(b_L)/b_L\}, \tag{29}$$

where the compatibility function, $u_{B_{ijk}}(b_L)$, associates with each $b_L \varepsilon B = \{0, 1, \ldots\}$ its compatibility with the fuzzy set B_{ijk}.

This situation, fuzzy transformation function and fuzzy net benefits, can be dealt with by first obtaining the fuzzy net benefits (denoted B_j) associated with each decision. Before suggesting a method to obtain B_j, a brief discussion of the compositional rule of inference is provided.

As defined by Zadeh, the compositional rule of reference is an extension of the process of inferring the value of y given y = f(x) and x = a.[3] More specifically, let

$X = \{x_1, \ldots, x_n\}$,

$Y = \{y_1, \ldots, y_m\}$,

R = fuzzy relation from X to Y whose compatibility function is $u_R(x, y)$,

and

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[3]L. A. Zadeh, "The Concept of a Linguistic Variable and Its Application to Approximate Reasoning", Part III, Information Sciences, Vol. 9 (1975), p. 57.

d_j S_{t+1} / S_t	s_1 ...... s_k ...... s_n
s_1	B_{ij1} .... B_{ijk} .... B_{ijn}
$\vdots$	$\vdots$ $\vdots$ $\vdots$
s_i	B_{ij1} .... B_{ijk} .... B_{ijn}
$\vdots$	$\vdots$ $\vdots$ $\vdots$
s_n	B_{nj1} .... B_{njk} .... B_{njn}

Figure 10. Net Benefits associated with the different combinations of $s_i \varepsilon S_t$ and $s_k \varepsilon S_{t+1}$ for a given d_j.

A = fuzzy set of X whose compatibility function is $u_A(x)$.

Then, by the compositional rule of inference, one can infer the fuzzy set B (fuzzy set of Y) from R and A. The compatibility function for the set B is given by:

$$u_B(y) = \max_{x \in X}[u_A(x) \wedge u_R(x, y)]. \tag{30}$$

As an example, let X and Y be defined by:

$X = \{x_1, x_2, x_3\}$,

and

$Y = \{y_1, y_2, y_3\}$.

Moreover, let A be a fuzzy set of X defined as:

$\underset{\sim}{A} = \{(.6/x_1), (.9/x_2), (.6/x_3)\}$.

Finally, let R be a fuzzy relation from X to Y described by:

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} .6 & .9 & .3 \\ .5 & .8 & .6 \\ .4 & .8 & .9 \end{bmatrix} \end{array}$$

Then, by the compositional rule of inference, the fuzzy set B (a fuzzy set of Y) is given by:

$B = \{(.6/y_1), (.8/y_2), (.6/y_3)\}$.

The compositional rule of inference can be used to obtain, for any decision $d_j \in D_t$, the set S_{t+1} given S_t. By applying Equation (30), the set S_{t+1} is given by:

$$S_{t+1} = \{u_{S_{t+1}}(s_k || s_i, d_j)/s_k\}, \tag{31}$$

where

$$u_{S_{t+1}}(s_k || s_i, d_j) = \max_{s_i}[u_{S_t}(s_i) \wedge u_R(s_i, s_k || d_j)], \tag{32}$$

$$u_{S_t}(s_i) = u_{s_i}(v_t), \tag{33}$$

and $u_R(s_i, s_k || d_j)$ is defined as above.

Once S_{t+1} is obtained, one can determine the fuzzy net benefits associated with each decision, namely B_j. By the extension principle, B_j is given by:

$$B_j = \{u_{B_j}(B_{ijk})/B_{ijk}\}, \tag{34}$$

where

$$u_{B_j}(B_{ijk}) = \max_{B_{ijk} \varepsilon f^{-1}(B_j)} u_{S_{t+1}}(s_k || s_i, d_j). \tag{35}$$

It should be noted that the set B_j is a fuzzy set of fuzzy net benefits. To reduce B_j into a fuzzy set of nonfuzzy net benefits, the operation of fuzzification can be used. By the operation of fuzzification, the reduced set, denoted B_{jL}, is given by:

$$B_{jL} = \{u_{B_{jL}}(b_L)/b_L\}, \tag{36}$$

where

$$b_L \varepsilon B_{ijk},$$

and

$$u_{B_{jL}}(b_L) = \max_{b_L \varepsilon f^{-1}(B_{jL})} [u_{B_j}(B_{ijk}) \wedge u_{B_{ijk}}(b_L)]. \tag{37}$$

Now, the maximizing set, B_{jm}, can be obtained. First, one obtains the set Y which is given by:

$$Y = \bigcup_{j=1}^{m} b_L \varepsilon B_{jL}. \tag{38}$$

Then, the maximizing set is given by:

$$B_{jm} = \{u_{B_{jm}}(b_L)/b_L\}, \tag{39}$$

where

$$u_{B_{jm}}(b_L) = (b_L/b_{max})^n, \tag{40}$$

$$b_{max} = \sup Y, \tag{41}$$

and n is an integer.

The intersection of the sets B_{jL} and B_{jm} (denoted B_{jo}) is given by:

$$B_{jo} = \{u_{B_{jo}}(b_L)/b_L\}, \tag{42}$$

where

$$u_{B_{jo}}(b_L) = u_{B_{jL}}(b_L) \wedge u_{B_{jm}}(b_L). \tag{43}$$

The fuzzy optimal decision set can be formulated by obtaining the set

$$D_o = \{u_{D_o}(d_j)/d_j\}, \tag{44}$$

where

$$u_{D_o}(d_j) = \bigvee_{L} u_{B_{jo}}(b_L). \tag{45}$$

finally, the optimal decision is obtained. The optimal decision is d_j which has the highest grade of membership in the set D_o.

5.4 Numerical Example II

Let $S_t = S_{t+1} = \{s_1, s_2\}$ where s_1 and s_2 represent the in-control and out-of-control states, respectively. Moreover, let $D_t = \{d_1, d_2\}$, where d_1 and d_2 represent, respectively, the investigation and no-investigation actions. Let the transformation functions and the net benefits matrices be given by:

d_1		S_{t+1}: s_1	s_2
S_t	s_1	1.	.4
	s_2	.6	.5

d_1		S_{t+1}: s_1	s_2
S_t	s_1	L	VL
	s_2	VH	VL

d_2		S_{t+1}: s_1	s_2
S_t	s_1	.7	.6
	s_2	.4	.9

d_2		S_{t+1}: s_1	s_2
S_t	s_1	H	VL
	s_2	H	L

where VL, L, H, VH are defined as:[4]

VL = {(1/100), (.4/200)},

L = {(.4/100), (1.0/200), (.5/300)},

H = {(.5/700), (1.0/800), (.5/900)},

and

VH = {(.5/900), (1.0/1000)}.

---

[4] VL, L, H, and VH indicate Very Low, Low, High, and Very High net benefits, respectively.

Finally, let a performance report indicate a variance of \$1000 whose compatibilities with the input states are .5 and .8, respectively. That is, $u_{s_1}(1000) = 0.5$ and $u_{s_2}(1000) = 0.8$.

By Equations (31)-(33) the set S_{t+1} for d_1 and d_2 are obtained as follows:

$$u_{S_{t+1}}(s_k \| s_i, d_1) = \begin{bmatrix} .5 & .8 \end{bmatrix} \begin{bmatrix} 1 & .4 \\ .6 & .5 \end{bmatrix} = \begin{bmatrix} .6 & .5 \end{bmatrix}$$

$$u_{S_{t+1}}(s_k \| s_i, d_2) = \begin{bmatrix} .5 & .8 \end{bmatrix} \begin{bmatrix} .7 & .6 \\ .4 & .9 \end{bmatrix} = \begin{bmatrix} .5 & .8 \end{bmatrix}$$

Then, by Equations (34) and (35) the fuzzy net benefits associated with the different decisions are obtained as follows:

$$B_1 = \{(.6/L), (.6/VH), (.5/VL), (.5/VL)\}$$
$$= \{(.6/L), (.6/VH), (.5/VL)\},$$
$$B_2 = \{(.5/H), (.5/H), (.8/VL), (.8/L)\}$$
$$= \{(.5/H), (.8/VL), (.8/L)\}.$$

Using the definitions of VL, L, H, and VH the sets B_1 and B_2 can be rewritten as:

$$B_1 = \{.6[(.4/100), (1.0/200), (.5/300)],$$
$$.6[(.5/900), (1.0/1000)],$$
$$.5[(1/100), (.4/200)]\},$$
$$B_2 = \{.5[(.5/700), (1.0/800), (.5/900)],$$
$$.8[(1.0/100), (.4/200)],$$
$$.8[(.4/100), (1.0/200), (.5/300)]\}.$$

It should be noted that B_1 and B_2 can be reduced into fuzzy sets of nonfuzzy net benefits as follows:

$$B_{1L} = \{(.24/100), (.6/200), (.3/300),$$
$$(.3/900), (.6/1000),$$
$$(.5/100), (.2/200)\}$$
$$= \{(.5/100), (.6/200), (.3/300),$$
$$(.3/900), (.6/1000)\}$$
$$B_{2L} = \{(.25/700), (.5/800), (.25/900),$$
$$(.8/100), (.32/200),$$
$$(.32/100), (.8/200), (.4/300)\}$$
$$= \{(.25/700), (.5/800), (.25/900),$$
$$(.8/100), (.8/200), (.4/300)\}.$$

By the inspection of B_{1L} and B_{2L}, one can see that B_{max} = \$1000. Then, using n = 2 and Equations (39) and (40), the maximizing sets are obtained by:

$$B_{1m} = \{(.01/100), (.04/200), (.09/300),$$
$$(.81/900), (1/1000)\}$$
$$B_{2m} = \{(.49/700), (.64/800), (.81/900),$$
$$(.01/100), (.04/200), (.09/300)\}.$$

Now, the sets B_{1O} and B_{2O} can be obtained. By Equations (42) and (43), B_{1O} and B_{2O} are given by:

$$B_{1O} = \{(.01/100), (.04/200), (.09/300), (.3/900), (.6/1000)\},$$
$$B_{2O} = \{(.25/700), (.5/800), (.25/900), (.01/100),$$
$$(.04/200), (.09/300)\}.$$

The fuzzy optimal decision set is given by $D_0 = \{(.6/d_1), (.5/d_2)\}$ and the optimal decision is d_1.

5.5 Summary

In this chapter, the new cost variance investigation model presented in the previous chapter is extended to cover the cases in which the performance is described by: (1) stochastic transformation function, and (2) fuzzy transformation function. Numerical examples are also provided.

CHAPTER SIX

Pilot Study

In this chapter the proposed cost variance investigation model is applied--through a pilot study--to an actual cost variance investigation problem encountered by a manufacturing firm. The study consists of two parts, namely, the computerization and the application of the model. These two parts are discussed in the chapter. The objectives of the study are presented first.

6.1 The Objectives of the Pilot Study

The major objective of the study is to demonstrate the feasibility (i.e., applicability) of the model as well as to provide insight into the problems that may arise when the model is applied to real world situations. This is important to cost variance investigation and fuzzy set theory because (1) as indicated by Koehler, Anthony, and Magee available cost variance investigation models have not been used in practice, and (2) as noted by Flonder and Kickert the literature on fuzzy set theory lacks practical applications.[1]

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[1] See R. W. Koehler, "The Relevance of Probability Statistics to Accounting Variance Control", Management Accounting (October 1968), p. 35; R. N. Anthony, "Some Fruitful Directions for Research in Management Accounting", in N. Dopuch and L. Revsine (eds.) Accounting Research 1960-1970: A Critical Evaluation (Center for International Education and Research in Accounting: University of Illinois, 1973), p. 59; R. P. Magee, "A Simulation Analysis of Alternative Cost Variance Investigation Models", The Accounting Review (July 1976), p. 529; W. Kickert, Fuzzy Theories on Decision-Making (Netherland: Martinus Nijhoff Social Sciences Division, 1978), p. 158; and P. Flonder, "An Example of A Fuzzy System", Kybernetes, Vol. 6 (1977), p. 229.

In the application, the model will be used to obtain optimal investigation decisions for some cost variances that have occurred in the manufacturing company. These decisions will be compared to the actions suggested by the decision maker. The objective of such comparison is to examine whether there are systematic differences between the two sets of actions.[2] This may provide an input for further research.

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[2]Note that the comparison is not intended to see whether the model results in the same actions made by the decision maker. In fact, as in the case of other decision models, the conflict between the two sets of actions is possible and even expected. Horngren, for example, states, "A fascinating aspect of [using decision models] is the possible conflict between what the decision model might indicate as optimal action and what the manager might perceive as optimal." C. T. Horngren, Cost Accounting--A Managerial Emphasis, 5th edition (N. J.: Prentice-Hall, Inc., 1982), p. 761. The reasons for the conflict are many. First, the decision maker may not act rationally all the time. G. M. Becker and C. G. McClintock, "Value: Behavioral Decision Theory", Annual Review of Psychology, Vol. 18 (1967), pp. 240-241. Second, the decision maker may have other information which may affect his decision. D. J. White, "The Nature of Decision Theory", in D. J. White and K. C. Bowen (eds.), The Role and Effectiveness of Theories of Decision in Practice (London: Hodder and Stoughton, 1975), p. 11. In fact, as argued by Demski, "... the optimal decision determined by means of the [formal] model is one--but not necessarily the only--determinant of the decision maker's ultimate choice." J. S. Demski, Information Analysis (California: Addison-Wesley Publishing Company, Inc., 1972), p. 27. Third, individual decision makers may act in a way to optimize their measured performance rather than acting in the best interest of their organizations. C. T. Horngren, Cost Accounting--A Managerial Emphasis, p. 761. Examples of situations in which individual decision makers made decisions that were not in the best interest of their organizations have been reported in the accounting literature. See, for example, Y. Ijiri, R. K. Jaedicke, and K. E. Knight, "The Effects of Accounting Alternatives on Management Decisions", in R. K. Jaedicke, Y. Ijiri, and O. Nielsen (eds.), Research in Accounting Measurement (Chicago: AAA, 1966), pp. 186-199. Fourth, the conflict may occur because of problems in the experimental settings. For example, White indicates: "If a theory is not in conflict with present observational knowledge, it does not mean it is true. Neither does the fact that it conflicts with observational knowledge mean that it is false, since errors will be made in recording observations or the results might be distorted because the operations involved in verification actually change the conditions we may think are fixed." D. J.

6.2 Computerization of the Model

Three computer programs were written: Fuzzy-9, Fuzzy-10, and Fuzzy-11. These three programs correspond to the three cases discussed in Chapters Four and Five, namely, the deterministic, stochastic, and fuzzy transformation function cases. The listings of these programs are presented in Appendix I to this chapter.

The programs were validated through the use of the data appearing in the numerical illustrations discussed in Chapters Four and Five. The

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White, Decision Theory (Chicago: Aldine Publishing Company, 1966), p. 120. One major problem with studies that use businessmen as subjects (such as the current study) is the absence of a system of rewards and punishments. As a result of such absence, the subject may not act in an experimental setting as he acts in real situations. "The subject had no reason to perform well, except for the intrinsic interest of the task and his pride in his performance. While both of these may be very strong motivating forces to some, they are difficult for the experimenter to measure." J. G. Birnberg and R. Nath, "Laboratory Experimentation in Accounting Research", The Accounting Review (Jan. 1968), p. 45. Fifth, the conflict may occur because of the inability of human decision maker to correctly process complex data necessary for optimal decisions and their use of simplified heuristics that may result in biases. "To reduce cognitive strain, they resort to simplified decision strategies, many of which lead them to ignore or misuse relevant information." P. Slovic and S. Lichtenstein, "Comparison of Bayesian and Regression Approaches to the Study of Information Processing in Judgment", Organizational Behavior and Human Performance (November 1971), p. 724. Finally, one may argue that the human decision makers are difficult to predict and that they display inconsistency in their actions. Bowen, for example, indicates: "There is ... something paradoxical about the human decision maker. Whatever his stated framework for choice, he still retains the freedom to make decisions which are not consistent with that framework for choice, he still retains the freedom to make decisions which are not consistent with that framework. Again, he can order preferences differently on two occasions when faced with what is apparently the same situation and the same data." K. C. Bowen, "Models and Decision Process", in D. J. White and K. C. Bowen (eds.), The Role and Effectiveness of Theories of Decision in Practice, p. 394.

computer programs produced results identical to the results obtained by the manual solution of the illustrations.[3] Consequently, it was concluded that the computer programs were correct.

An input-output flow diagram required for the computerized model is presented in Figure 11. As shown in the diagram, the inputs to the model include:

(1) The amount of the cost variance;

(2) Number of performance states with each state being described by a compatibility function;

(3) Number of available decisions;

(4) Type and shape of the transformation function; and

(5) Estimates of the net benefits with each net benefit being described by a compatibility function.

The outputs of the computer model include:

(1) A summary of the input data; and

(2) An optimal decision.[4]

The first input to the model, the amount of the cost variance, can be obtained from the performance reports that are prepared for a certain level of management. The performance reports usually include, in

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[3]The outcome obtained from each computer program is presented at the end of the corresponding program in Appendix I to this chapter. In validating the programs, the intermediate results obtained by the programs were also compared to those obtained by the manual solution of the illustrations. The programs produced results identical to those obtained by the manual solution.

[4]As defined in Chapters Four and Five, the optimal decision is the decision $d_j \varepsilon D_t$ which has the highest compatibility $u_{D_o}(d_j)$ with the fuzzy optimal decision set D_o.

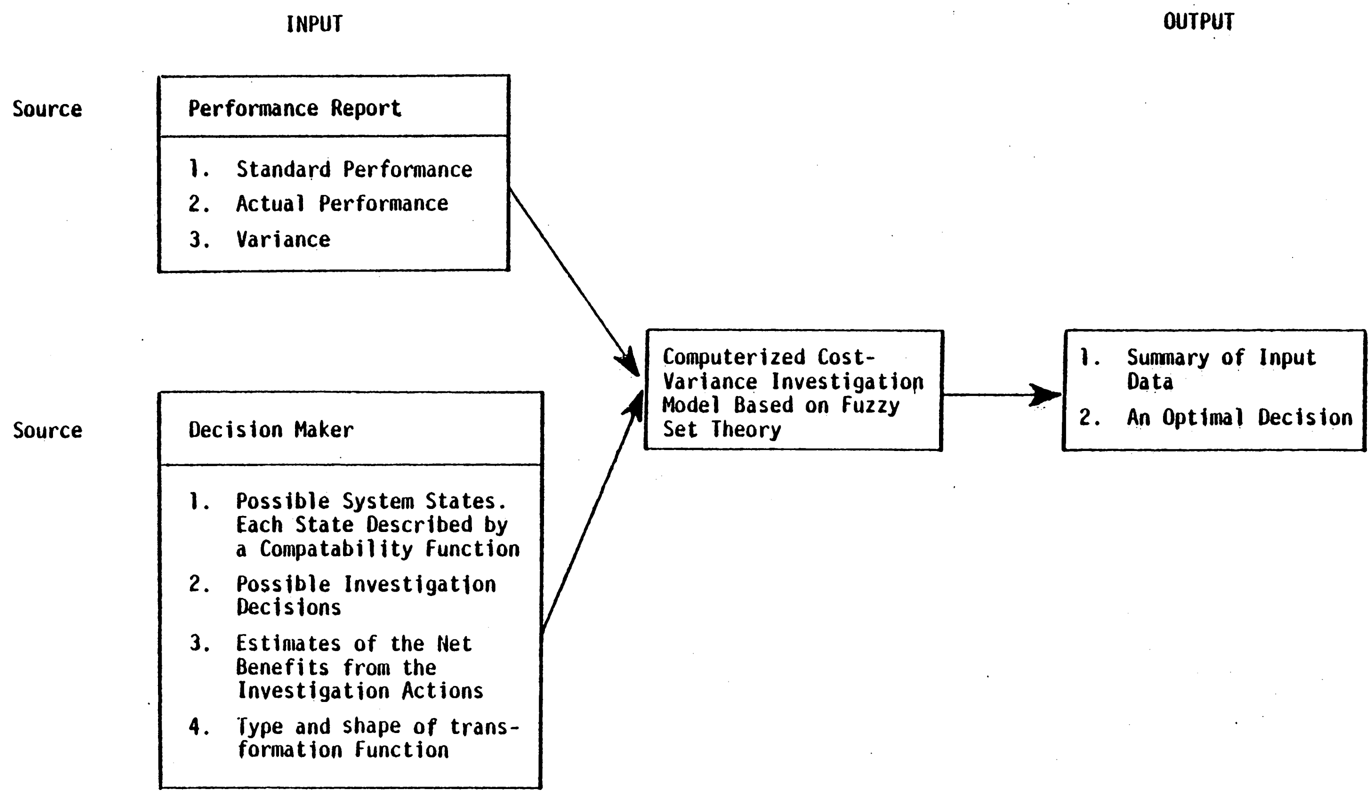

Figure 11. Input-Output Flow Diagram for the Computerized Cost-Variance Investigation Model.

addition to the standard cost and actual cost, the amount of the variance.

The remaining inputs to the model are provided by the decision maker. That is, the decision maker determines the number of states, the number of available decisions, the type and shape of the transformation function, and the estimates of the net benefits. Moreover, the decision maker provides compatibility functions to describe the states, the estimates of the net benefits, and the transformation function.

As indicated by Watson, Weiss, and Donnell[5], there are two modes in which the decision maker can provide the required membership functions. In the first mode, the decision maker provides the membership functions at the time of the decision. In the second mode, membership functions are stored in the computer and, at the time of the decision, the decision maker selects the appropriate membership function. Regardless of the mode being used, the membership functions are determined subjectively; the closer an element is to satisfying the requirements of a set, the closer its grade of membership to one, and vice versa.

Some authors, such as Watson, Weiss, and Donnell[6], and Watanabe[7], have attacked the subjectivity involved in determining the membership functions and called for empirical derivation of the membership functions, i.e., more precise membership functions. However, one must

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[5]S. R. Watson, J. J. Weiss, and M. L. Donnell, "Fuzzy Decision Analysis", IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-9 (1979), p. 6.

[6]Ibid., p. 6.

[7]S. Watanabe, "A Generalized Fuzzy-Set Theory", IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-8 (1978), pp. 756-760.

realize that subjectivity is the essence of fuzziness, and a methodology to handle fuzzy problems is needed. Moreover, other decision making techniques (such as Bayesian Theory and Utility Theory) also involve subjectivity. Finally, as pointed out by Zadeh, during the 1977 IEEE Conference on Decision and Control, "... it is not in keeping with the spirit of fuzzy set approach to be too concerned about the precision of [the grades of membership]".[8]

Several researchers, in reaction to the above criticism, have suggested methods for deriving empirical membership functions.[9] Most of these methods are variations of the traditional method for measuring the strength of belief (i.e., asking the subject to indicate on a scale based on semantic differential how strongly he agrees or disagrees with a given statement).

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[8]This was reported in S. R. Watson, J. J. Weiss, and M. L. Donnell, "Fuzzy Decision Analysis", pp. 2-3.

[9]See, for example, T. Saaty, "Measuring the Fuzziness of Sets", Journal of Cybernetics, Vol. 4 (1974), pp. 53-61; M. Kocken and A. Badre, "On the Precision of Adjectives Which Denote Fuzzy Sets", Journal of Cybernetics, Vol. 4 (1974), pp. 49-59; H. M. Hersh and A. Caramazza, "A Fuzzy Set Approach to Modifiers and Vagueness in Natural Language", Journal of Experimental Physhology: General, Vol. 105 (1976), pp. 254-276; H. M. Hersh, A. Caramazza, and H. H. Brownell, "Effects of Context on Fuzzy Membership Functions", in M. Gupta, R. Ragade, and R. Yager (eds.), Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 389-408; H. H. Brownell and A. Caramazza, "Categorizing with Overlapping Categories", Memory and Cognition, Vol. 6 (1978), pp. 481-490; P. J. MacVicar-Whelan, "Fuzzy Sets, The Concept of Height, and the Hedge Very", IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-8 (1978), pp. 507-511; and M. Nowakowska, "Fuzzy Concepts: Their Structure and Problems of Measurement", in M. Gupta, R. Ragade, and R. Yager (eds.), Advances in Fuzzy Set Theory and Applications, pp. 361-387.

To summarize, there are two approaches for obtaining the required membership functions; the subjective approach and the empirical approach. The position adopted in this study follows the position of Zadeh, who indicates that: ". . . the grades of membership are subjective, in the sense that their specification is a matter of definition rather than objective experimentation or analysis."[10]

6.3 Actual Application

As mentioned earlier, the second phase in the pilot study is to apply the model to an actual cost variance investigation problem faced by a manufacturing firm. A brief description of the company is provided first. Then the procedures employed in collecting the data used in the application are presented. The results of the application are discussed in Section 6.4.

The Company

The firm used in this study is the Blacksburg, Virginia branch of a manufacturing company. The branch manufactures heavy equipment and its operations encompass six production departments (bonding, auto lines, press, finish, flange, and plating departments) and two service departments (inspection and service packing departments).

The branch has been using a standard costing system for about four years. Initially, the branch used standards developed by the corporate office. Because these standards were very high for branch operations,

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[10] L. A. Zadeh, "A Fuzzy Set Theoretic Interpretation of Linguistic Hedges", Journal of Cybernetics, Vol 2 (1972), p. 5. In Appendix II to Chapter Six, the application is replicated by using empirical membership functions.

the engineering department in the branch began developing new standards for the branch. The process of replacing the corporate office standards by special standards is continuing. That is, the standards used by the branch at the time of this application were mixed standards; some of the standards were corporate standards and some were set by the branch. The departments that used corporate standards were the auto lines, press, and finish departments.

The data related to the actual costs are accumulated by the accounting department, which also prepares weekly and monthly performance reports that include the standard cost, actual cost, and the variance for material, labor, and overhead costs used by the several departments. The reports are presented to the heads of the cost centers (production and service departments), the controller, and the plant manager. The company does not use any of the available cost-variance investigation models, but rather it bases its investigation decisions on the managers' intuition and judgment.

In addition to the standard costing system, the branch employs a flexible budgeting system. That is, after cost variances are calculated, they are divided into two components: variances caused by the deviation from planned production and variances caused by the deviation from the standard costs for the actual production. The second kind of variances is divided into efficiency and price (or rate) variance.

Collecting the Data

The performance reports used in this study are those presented to the controller of the branch. They include data related to the costs of the different resources (such as material, labor, and overhead) used within the different production and service departments.

Forty-eight variances relating to the total direct labor cost in the eight departments were obtained. These variances covered a period of six months, January 1980 through June 1980. A summary of these variances is presented in Figure 12.

The remaining data required for the model were obtained through several meetings with the controller.[11] In these meetings I explained the model to the controller, queried him about it, and provided him with choices related to the required data. The controller provided answers and made choices.

After explaining the meaning of the input states and output states to the controller and after telling him that he could use more than the two extreme states, in-control and out-of-control, he elected to use three states, in-control, more or less out of control, and out-of-control. The controller was also told that the decision space could include actions such as exploratory investigation in addition to the two decisions of investigate and do-not investigate. He elected to use the two-decision space.

The controller was also informed about the three types of transformation functions--deterministic, stochastic, and fuzzy. He was told that he could use any of these functions. The controller elected to use the fuzzy transformation function. That is, he viewed the

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[11]The researcher met with the controller eight times. Each meeting lasted for about 90 minutes. The meetings were conducted over a period of two months, mid June to mid August 1980.

January 1980

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 9044	$ 7057	$ 1987 U
Auto Lines**	31764	22301	9463 U
Press**	16295	10686	5609 U
Finish**	26778	19941	6837 U
Flange	23342	18330	4962 U
Plating	28837	36701	7862 F
On-Line Pack	26888	23007	3881 U
Serv Pack	8303	4131	4172 U

February 1980

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 7680	$ 7419	$ 261 U
Auto Lines**	29596	21890	7706 U
Press**	18196	14171	4025 U
Finish**	24229	17212	7017 U
Flange	22353	16949	5404 U
Plating	25023	29046	4023 F
On-Line Pack	23532	18808	4724 U
Serv Pack	10051	4022	6029 U

*U and F indicate unfavorable variances, respectively.

**Corporate standards were used in these departments.

Figure 12. Summary of the Variances.

March 1980

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 7329	$ 6918	$ 411 U
Auto Lines**	23457	17300	6157 U
Press**	19726	14184	5542 U
Finish**	26115	21110	5005 U
Flange	20179	14656	5523 U
Plating	25246	29515	4269 F
On-Line Pack	26985	21656	5329 U
Serv Pack	6223	2747	-3476 U

April 1980

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 6765	$ 4198	$ 2567 U
Auto Lines**	24019	15908	8111 U
Press**	23268	14254	9014 U
Finish**	29375	19219	10156 U
Flange	17245	10589	6656 U
Plating	23737	22899	838 U
On-Line Pack	27943	18678	9245 U
Serv Pack	12196	3743	8453 U

*U and F indicate unfavorable and favorable variances, respectively.

**Corporate standards were used in these departments.

Figure 12 (Con't). Summary of the Variances.

May 1980

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 5803	$ 4180	$ 1623 U
Auto Lines**	15635	10816	4817 U
Press**	19081	11545	7536 U
Finish**	26973	18173	8800 U
Flange	11406	7368	4038 U
Plating	16680	19705	3225 F
On-Line Pack	21277	15124	6153 U
Serv Pack	7729	2812	4917 U

June 1980

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 4850	$ 3973	$ 877 U
Auto Lines**	21258	15799	5459 U
Press**	15017	9691	5326 U
Finish**	24027	16596	7431 U
Flange	14196	8911	5285 U
Plating	18580	20292	1712 F
Inspection	20317	13999	6318 U
Serv Pack	8132	3065	5067 U

*U and F indicate unfavorable and favorable variances, respectively.

**Corporate standards were used in these departments.

Figure 12 (Con't). Summary of the Variances.

relationship between the input states and the output states to be a fuzzy relationship.

The fuzzy transformation function, as provided by the controller, is presented in Figure 13. This function was obtained by asking the controller to associate with every combination of input state, $s_i \in S_t$, output state, $s_k \in S_{t+1}$, and decision, $d_j \in D_t$, a number between zero and one representing the strength of the relationship between s_i, s_k, and d_j. (Note that there were 18 combinations--3 input states X 2 decisions X 3 output states.) The stronger the relationship, the closer the number is to one, and vice versa.[12] For example, as can be seen from Figure 13, the controller assigned 0.9 for the combination of

current state (s_i) = in-control (s_1),

action (d_j) = investigate (d_1),

and

output state (s_k) = in-control (s_1),

while he assigned 0.1 for the combination of

current state (s_i) = in-control (s_1),

action (d_j) = investigate (d_1),

and

output state (s_k) = out-of-control (s_3).

The controller viewed the relationship between the elements in the first

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[12]In the language of possibility theory, this number can be interpreted as the possibility that the output state will be s_k given that the current state is s_i and decision d_j is made. The greater the possibility, the closer the number is to one, and vice versa. See L. A. Zadeh, "Fuzzy Sets as a Basis for a Theory of Possibility", <u>Fuzzy Sets and Systems</u>, Vol. 1 (1978), pp. 3-28.

Input State, $s_i \in S_t$

$s_i = s_1$ = in-control

Decision**	Output State, $s_k \in S^*_{t+1}$		
$d_j \in D_t$	s_1	s_2	s_3
d_1	.9	.4	.1
d_2	.8	.4	.3

$s_i = s_2$ = more or less out-of-control

Decision**	Output State, $s_k \in S^*_{t+1}$		
$d_j \in D_t$	s_1	s_2	s_3
d_1	.8	.5	.2
d_2	.4	.8	.7

$s_i = s_3$ = out-of-control

Decision**	Output State, $s_k \in S^*_{t+1}$		
$d_j \in D_t$	s_1	s_2	s_3
d_1	.7	.4	.3
d_2	.1	.6	.9

*s_1, s_2, and s_3 indicate in-control, more of less out-of-control, and out-of-control states, respectively.

**d_1 and d_2 indicate investigate and do-not investigate actions, respectively.

Figure 13. Fuzzy Relation $S_{t+1} = f(s_t, D_t)$.

the first combination to be strong, while he considered the relationship between the elements in the second combination to be weak.

With respect to the expected net benefits, the controller was told that his estimates of the net benefits resulting from the different combinations of decisions and states could take the form of "about $5000," or the form of linguistic values such as "very large". He chose the linguistic value approach because, he felt, it would be easier to use.

The estimated net benefits--in linguistic form--for the different combinations of decisions, input states, and output states are presented in Figure 14.[13] The letters S, VS, VVS, M, L, VL, and VVL stand for small, very small, very very small, medium, large, very large, and very very large, respectively. These values were defined, by the controller, as follows:

VVS = {(1/1000), (.5/2000)},

VS = {(.7/1000), (1/2000), (.7/3000)},

S = {(.6/2000), (.9/3000), (1/4000), (.6/5000)},

M = {(.2/3000), (.5/4000), (.7/5000), (1/6000), (.7/7000), (.5/8000), (.2/9000)},

L = {(.6/6000), (.8/7000), (1/8000), (.8/9000), (.6/10000)},

VL = {(.6/8000), (.8/9000), (1/10000), (.8/11000), (.6/12000)},

VVL = {(.4/9000), (.6/10000), (.8/11000), (1/12000)}.

The above definitions of the linguistic net benefits were derived as follows. First, a set of possible net benefit values (B) was

[13]The estimates of the net benefits were provided by the controller based on his experience as well as his expectations; the branch did not keep any records related to the costs and benefits that might result from the investigation action.

Input State, $s_i \varepsilon S_t$

$s_i = s_1$ = in-control

Decision** $d_j \varepsilon D_t$	Output State, $s_k \varepsilon S^*_{t+1}$		
	s_1	s_2	s_3
d_1	S	VS	VVS
d_2	M	S	VS

$s_i = s_2$ = more or less out-of-control

Decision** $d_j \varepsilon D_t$	Output State, $s_k \varepsilon S^*_{t+1}$		
	s_1	s_2	s_3
d_1	L	S	VS
d_2	L	S	S

$s_i = s_3$ = out-of-control

Decision** $d_j \varepsilon D_t$	Output State, $s_k \varepsilon S^*_{t+1}$		
	s_1	s_2	s_3
d_1	VL	S	VS
d_2	VVL	M	VS

*s_1, s_2, and s_3 indicate in-control, more or less out-of-control, and out-of-control states, respectively.

**d_1 and d_2 indicate investigate and do-not investigate actions, respectively.

Figure 14. Estimated Net Benefits Associated with the Different Combinations of Input States, Decisions, and Output States.

obtained. This was done by determining the smallest and largest possible net benefits. The controller elected to use \$1,000 and \$12,000 as the lowest and highest possible net benefits, respectively.[14] Then, by using an increment of \$1,000, the set of possible net benefits was determined as B = {\$1000, \$2000, ..., \$11000, \$12000}. (An increment of less than \$1,000--such as \$500--can be used.)

Second, the controller was asked to provide a compatibility function for each linguistic value. More specifically, he was asked to associate with each possible net benefit a number between zero and one representing the compatibility of the net benefit value with a certain linguistic value. As an example, the controller was asked to assign a number from zero to 1 for each net benefit in B = {1000, 2000, ..., 12000} representing its compatibility with very very small net benefits. He assigned 1 for \$1000, .5 for \$2000, and zero for every value greater than \$2000. Thus, the controller provided the following compatibility function for very very small net benefits:

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[14]In determining the lowest and highest possible net benefits, the controller was asked to consider the smallest and the largest variances over the period. The largest variance was \$10,156 and the smallest variance was \$216. Then, he was asked to adjust these values to the cost of investigation and additional benefits from the investigation--such as the effect of the investigation on the employees' behavior--to get the lowest and highest possible net benefits. The adjustments were based on the experience of the controller as well as his expectation; no attempt was made to determine the exact costs and benefits. This method may not provide precise measures of the expected net benefits. However, one must remember that "The difficulty of measuring likely benefits may be the major barrier to the implementation of [the existing cost variance investigation] models." [Emphases added]. C. T. Horngren, Cost Accounting: A Managerial Emphasis, p. 876.

$$u_{VVS} = \begin{cases} 1.0, & \text{if} \quad \text{net benefit} = 1000 \\ .5, & \text{if} \quad \text{net benefit} = 2000 \\ 0.0, & \text{if} \quad \text{net benefit} > 2000 \end{cases}$$

Stated differently, he defined very very small net benefits as follows:

VVS = {(1.0/1000), (.5/2000), (0/3000), (0/4000), (0/5000), (0/6000), (0/7000), (0/8000), (0/9000), (0/10000), (0/11000), (0/12000)}.

This definition of VVS can be shortened as follows:

VVS = {1/1000), (.5/2000)}.

The second step was repeated with different linguistic values of net benefits to obtain the compatibility functions for the remaining linguistic net benefits.

Regarding the compatibility functions for the states, the controller was told that he had two options for providing the compatibilities of the variances with the different states. The first option was to derive (either subjectively or empirically) membership functions for the states. Then, variances could be plugged into the formula to obtain their compatibilities with the states.[15] (The problem with this approach is the need to revise the derived functions when there is a shift in either the standards or the variances.) The second option was to provide directly--at the time of the decision--the compatibilities of the variances with the different states.

The controller selected the second option (i.e., he preferred to provide directly the compatibility of each variance with the different

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[15] As an example, let the in-control and out-of-control states be described by the following compatibility functions:

$$u_{in}(v) = \begin{cases} 1, & \text{if} \quad 0 \leq v \leq 100 \\ \dfrac{1}{1 + (\frac{v - 100}{100})^2}, & \text{if} \quad v > 100 \end{cases}$$

and

$$u_{out}(v) = \begin{cases} \dfrac{1}{1 + (\frac{v - 400}{400})^2}, & \text{if} \quad 200 \leq v \leq 400 \\ 1, & \text{if} \quad v > 400. \end{cases}$$

Then, the compatibilities of a variance of \$200 with the two states are $u_{in}(200) = .50$ and $u_{out}(200) = .80$, while the compatibilities of a variance of \$300 with the two states are $u_{in}(300) = .20$ and $u_{out}(300) = .94$.

states).[16] The compatibilities obtained from the controller for the 48 variances used in this study, with the different input states, are presented in Figure 15, column 3.

The above data were used with the computer program (Fuzzy-11) to obtain the optimal decision for every variance of the 48 variances used in the study. (The cost of running the program was only $1.04.) The results are presented in Figure 15. The figure also includes the decisions suggested by the controller for the 48 variances.

6.4 Discussion of the Results

By examining Figure 15, one can see that there were 12 differences (related to variances 4, 5, 6, 12, 15, 18, 19, 25, 27, 28, 35, and 43) between the actions indicated by the model and those suggested by the controller. (For reasons to explain these difference, the reader is referred to footnote No. 2, this chapter.)

In order to provide insight into these 12 differences and to achieve the second objective of the application--examining whether there are any systematic differences between the actions indicated by the

[16]The controller felt that by doing so he would have more flexibility to reflect his state of mind at the time of the decision. Consider, as an example, the second variance in Figure 15. The amount of the variance ($9,463) is considered to be large when compared to other variances. In spite of its size, this variance was classified as in control by the controller. (The variance was assigned compatibility of .9 with the in-control state and .2 with the out-of-control state.) When asked about this, the controller responded that the standard was difficult to meet because it was set by the corporate office. (The variance was expected to be large because the standard was very high.) As another example, consider variance 28. Although this variance was the largest variance ($10,156), the controller did not classify it as out-of-control because the standard was high; the variance was assigned a compatibility of 0.7 with the in-control state and a compatibility of only 0.5 with the out-of-control state.

Var. No.	Var. Amt.[1]	Compatability with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	A or C[3]
1	1987 U	.8	.6	.5	D	D	A
2	9463 U	.9	.4	.2	D	D	A
3	5609 U	.7	.7	.8	D	D	A
4	6837 U	.7	.7	.8	I	D	C
5	4962 U	.6	.8	.8	D	I	C
6	7862 F	.7	.7	.7	I	D	C
7	3881 U	.8	.7	.6	D	D	A
8	4172 U	.5	.8	.9	I	I	A
9	261 U	.9	.3	.2	D	D	A
10	7706 U	.9	.3	.2	D	D	A
11	4025 U	.7	.7	.7	D	D	A
12	7017 U	.7	.7	.6	I	D	C
13	5404 U	.6	.7	.8	I	I	A
14	4023 F	.8	.6	.7	D	D	A
15	4724 U	.6	.7	.7	D	I	C
16	6029 U	.4	.8	.9	I	I	A
17	411 U	.9	.4	.3	D	D	A
18	6157 U	.8	.5	.4	I	D	C
19	5542 U	.7	.6	.6	I	D	C
20	5005 U	.7	.6	.6	D	D	A
21	5523 U	.6	.7	.7	I	I	A
22	4269 F	.8	.7	.7	D	D	A
23	5329 U	.7	.8	.8	D	D	A
24	3476 U	.6	.8	.8	I	I	A

[1] U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2] I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3] A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 15. Decisions for the 48 Variances.

Var. No.	Var.[1] Amt.	Compatability with States			Decisions[2]		
		In-Control	More or less out of control	Out of Control	Suggested by Controller	Indicated by Model	A or C[3]
25	2567 U	.7	.5	.4	I	D	C
26	8111 U	.8	.4	.3	D	D	A
27	9014 U	.7	.5	.5	I	D	C
28	10156 U	.7	.5	.5	I	D	C
29	6656 U	.6	.5	.7	I	I	A
30	838 U	.7	.5	.7	D	D	A
31	9245 U	.6	.7	.7	I	I	A
32	8453 U	.5	.8	.9	I	I	A
33	1623 U	.8	.4	.4	D	D	A
34	4819 U	.9	.5	.3	D	D	A
35	7536 U	.7	.6	.5	I	D	C
36	8800 U	.7	.6	.5	D	D	A
37	4032 U	.6	.7	.7	I	I	A
38	3225 F	.7	.7	.7	D	D	A
39	6153 U	.8	.8	.8	D	D	A
40	4917 U	.5	.8	.9	I	I	A
41	877 U	.9	.4	.4	D	D	A
42	5459 U	.9	.5	.3	D	D	A
43	5326 U	.7	.6	.5	I	D	C
44	7431 U	.7	.6	.5	D	D	A
45	5285 U	.6	.7	.7	I	I	A
46	1712 F	.7	.8	.7	D	D	A
47	6318 U	.6	.8	.8	I	I	A
48	5067 U	.5	.9	.9	I	I	A
						Total	36 12

[1]U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 15 (Con't). Decisions for the 48 Variances.

model and those suggested by the decision maker--the variances were classified in three different ways. First, the variances were divided into extreme and non-extreme variances. The extreme variances included 12 variances: 2, 8, 9, 10, 16, 17, 32, 34, 40, 41, 42, and 48.[17] The remaining 36 variances were classifed as non-extreme variances. The results for the extreme and non-extreme variances are reproduced in Figures 16 and 17, respectively.

By examining Figure 16, one can see that actions indicated by the model and actions suggested by the controller were the same for the extreme cases. However, as shown in Figure 17, there were 12 differences between the actions indicated by the model and those suggested by the controller for the non-extreme cases.

Second, the variances were divided into favorable and unfavorable variances. Five variances (6, 14, 22, 38, 46) were classifed as favorable. The remaining 43 variances were unfavorable. Figures 18 and 19

[17] A variance is considered to be an extreme case if it has a compatibility of .9 (or more) with either the in-control or out-of-control state. Also, to be classified as an extreme case, the variance should have a large difference (of at least .4) between its compatibility with the in-control and out-of-control states. For example, variance number 2 is considered to be an extreme case because it has a compatibility of .9 with the in-control state, while its compatibility with the out-of-control state is only .2. Similarly, variance number 8 is classified as an extreme case because it has compatibilities of .5 and .9 with the in-control and out-of-control states, respectively. Note that the amount of the variance, the standard, as well as the attainability of the standard should be reflected in the compatibilities of the variance with the different states. The attainability of the standard may well explain why some large variances (such as variance 2) are classified as in control. The attainability of the standard also explains why variance 28 ($10,156) was not classified as an extreme out-of-control. The standard was difficult to attain; consequently, the variance was expected to be large.

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	A or C[3]
2	9463 U	.9	.4	.2	D	D	A
8	4174 U	.5	.8	.9	I	I	A
9	261 U	.9	.3	.2	D	D	A
10	7706 U	.9	.3	.2	D	D	A
16	6029 U	.4	.8	.9	I	I	A
17	411 U	.9	.4	.3	D	D	A
32	8453 U	.5	.8	.9	I	I	A
34	4819 U	.9	.5	.3	D	D	A
40	4917 U	.5	.8	.9	I	I	A
41	877 U	.9	.4	.4	D	D	A
42	5459 U	.9	.5	.3	D	D	A
48	5067 U	.5	.9	.9	I	I	A

Total 12 0

[1] U indicates 'unfavorable' variance.

[2] I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3] A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 16. Results for Extreme Variances

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	A or C[3]
1	1987 U	.8	.6	.5	D	D	A
3	5609 U	.7	.7	.8	D	D	A
4	6837 U	.7	.7	.8	I	D	C
5	4962 U	.6	.8	.8	D	I	C
6	7862 F	.7	.7	.7	I	D	C
7	3881 U	.8	.7	.6	D	D	A
11	4025 U	.7	.7	.7	D	D	A
12	7017 U	.7	.7	.6	I	D	C
13	5404 U	.6	.7	.8	I	I	A
14	4023 F	.8	.6	.7	D	D	A
15	4724 U	.6	.7	.7	D	I	C
18	6157 U	.8	.5	.4	I	D	C
19	5542 U	.7	.6	.6	I	D	C
20	5005 U	.7	.7	.6	D	D	A
21	5523 U	.6	.7	.7	I	I	A
22	4269 F	.8	.7	.7	D	D	A
23	5329 U	.7	.8	.8	D	D	A
24	3476 U	.6	.8	.8	I	I	A
25	2567 U	.7	.5	.4	I	D	C
26	8111 U	.8	.4	.3	D	D	A
27	9014 U	.7	.5	.5	I	D	C
28	10156 U	.7	.5	.5	I	D	C
29	6656 U	.6	.5	.7	I	I	A
30	838 U	.7	.5	.7	I	I	A
31	9245 U	.6	.7	.7	I	I	A
33	1623 U	.8	.4	.4	D	D	A
35	7536 U	.7	.6	.5	I	D	C
36	8800 U	.7	.6	.5	D	D	A

[1]U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 17. Results for Non-Extreme Variances.

Var. No.	Var. Amt.[1]	Compatibility with States			Decisions[2]		
		In-Control	More or less out of control	Out of Control	Suggested by Controller	Indicated by Model	A or C[3]
37	4032 U	.6	.7	.7	I	I	A
38	3225 F	.7	.7	.7	D	D	A
39	6153 U	.8	.8	.8	D	D	A
43	5326 U	.7	.6	.5	I	D	C
44	7431 U	.7	.6	.5	D	D	A
45	5285 U	.6	.7	.7	I	I	A
46	1712 F	.7	.8	.7	D	D	A
47	6318 U	.6	.8	.8	I	I	A
						Total	24 12

[1]U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 17 (Con't). Results for Non-Extreme Variances.

summarize the results for the favorable and unfavorable variances, respectively.

As can be seen from Figure 18, for the favorable variances (except for variance 6), the actions indicated by the model were the same decisions suggested by the controller; do-not investigate. In the case of variance 6, the controller elected the investigation action. For the unfavorable variances, as shown in Figure 19, there were 11 differences between what the model indicated as optimal action and what the controller perceived as optimal action.

Third, the variances were divided into variances that occurred in departments using corporate standards and variances that occurred in departments using branch standards. The first group occurred in the auto lines, press, and finish departments. Consequently, this group of variances included 18 variances (2, 3, 4, 10, 11, 12, 18, 19, 20, 26, 27, 28, 34, 35, 36, 42, 43, and 44). The remaining 30 variances occurred in departments using branch standards. The results for the two types of variances are presented in Figures 20 and 21, respectively.

As can be seen from Figure 20, there were 8 differences related to variances (4, 12, 18, 19, 27, 28, 35, 43) occurring in departments that used standards set by the corporate office. For the variances from branch standards, as shown in Figure 21, there were only 4 differences between the actions indicated by the model and those suggested by the controller.

To summarize, there were 12 differences between what the model indicated as optimal actions and what the decision maker perceived as optimal actions. None of these differences were related to extreme

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	A or C[3]	
6	7862	.7	.7	.7	I	D		C
14	4023	.8	.6	.7	D	D	A	
22	4269	.8	.7	.7	D	D	A	
38	3225	.7	.7	.7	D	D	A	
46	1712	.7	.8	.7	D	D	A	
						Total	4	1

[1]All variances are 'favorable'.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 18. Results for Favorable Variances.

Var. No.	Var. Amt.[1]	Compatibility with States			Decisions[2]		
		In-Control	More or less out of control	Out of Control	Suggested by Controller	Indicated by Model	A or C[3]
1	1987	.8	.6	.5	D	D	A
2	9463	.9	.4	.2	D	D	A
3	5609	.7	.7	.8	D	D	A
4	6837	.7	.7	.8	I	D	C
5	4962	.6	.8	.8	D	I	C
7	3881	.8	.7	.6	D	D	A
8	4172	.5	.8	.9	I	I	A
9	261	.9	.3	.2	D	D	A
10	7706	.9	.3	.2	D	D	A
11	4025	.7	.7	.7	D	D	A
12	7017	.7	.7	.6	I	D	C
13	5404	.6	.7	.8	I	I	A
15	4724	.6	.7	.7	D	I	C
16	6029	.4	.8	.9	I	I	A
17	411	.9	.4	.3	D	D	A
18	6157	.8	.5	.4	I	D	C
19	5542	.7	.6	.6	I	D	C
20	5005	.7	.6	.6	D	D	A
21	5523	.6	.7	.7	I	I	A
23	5329	.7	.8	.8	D	D	A
24	3476	.6	.8	.8	I	I	A
25	2567	.7	.5	.4	I	D	C
26	8111	.8	.4	.3	D	D	A
27	9014	.7	.5	.5	I	D	C
28	10156	.7	.5	.5	I	D	C
29	6656	.6	.5	.7	I	I	A
30	838	.7	.5	.7	D	D	A
31	9245	.6	.7	.7	I	I	A

[1] All variances are 'unfavorable'.

[2] I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3] A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 19. Results for Unfavorable Variances.

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	A or C[3]	
32	8453	.5	.8	.9	I	I	A	
33	1623	.8	.4	.4	D	D	A	
34	4819	.9	.5	.3	D	D	A	
35	7536	.7	.6	.5	I	D		C
36	8800	.7	.6	.5	D	D	A	
37	4032	.6	.7	.7	I	I	A	
39	6153	.8	.8	.8	D	D	A	
40	4917	.5	.8	.9	I	I	A	
41	877	.9	.4	.4	D	D	A	
42	5459	.9	.5	.3	D	D	A	
43	5326	.7	.6	.5	I	D		C
44	7431	.7	.6	.5	D	D	A	
45	5285	.6	.7	.7	I	I	A	
47	6318	.6	.8	.8	I	I	A	
48	5067	.5	.9	.9	I	I	A	
						Total	32	11

[1]All variances are 'unfavorable'.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respecitvely.

[3]A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 19 (Con't). Results for Unfavorable Variances.

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	A or C[3]	
2	9463 U	.9	.4	.2	D	D	A	
3	5609 U	.7	.7	.8	D	D	A	
4	6837 U	.7	.7	.8	I	D		C
10	7706 U	.9	.3	.2	D	D	A	
11	4025 U	.7	.7	.8	D	D	A	
12	7017 U	.7	.7	.6	I	D		C
18	6157 U	.8	.5	.4	I	D		C
19	5542 U	.7	.6	.6	I	D		C
20	5005 U	.7	.6	.6	D	D	A	
26	8111 U	.8	.4	.3	D	D	A	
27	9014 U	.7	.5	.5	I	D		C
28	10156 U	.7	.5	.5	I	D		C
34	4819 U	.9	.5	.3	D	D	A	
35	7536 U	.7	.6	.5	I	D		C
36	8800 U	.7	.6	.5	D	D	A	
42	5459 U	.9	.5	.3	D	D	A	
43	5326 U	.7	.6	.5	I	D		C
44	7431 U	.7	.6	.5	D	D	A	
						Total	10	8

[1]U indicates 'unfavorable' variance.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 20. Results for Variances Related to Corporate Standards.

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	A or C[3]
1	1987 U	.8	.6	.5	D	D	A
5	4962 U	.6	.8	.8	D	I	C
6	7862 F	.7	.7	.7	I	D	C
7	3881 U	.8	.7	.6	D	D	A
8	4172 U	.5	.8	.9	I	I	A
9	261 U	.9	.3	.2	D	D	A
13	5404 U	.6	.7	.8	I	I	A
14	4023 F	.8	.6	.7	D	D	A
15	4727 U	.6	.7	.7	D	I	C
16	6029 U	.4	.8	.9	I	I	A
17	411 U	.9	.4	.3	D	D	A
21	5523 U	.6	.7	.7	I	I	A
22	4269 F	.8	.7	.7	D	D	A
23	5329 U	.7	.8	.8	D	D	A
24	3474 U	.6	.8	.8	I	I	A
25	2567 U	.7	.5	.4	I	D	C
29	6656 U	.6	.5	.7	I	I	A
30	838 U	.7	.5	.7	D	D	A
31	9245 U	.6	.7	.7	I	I	A
32	8453 U	.5	.8	.9	I	I	A
33	1623 U	.8	.4	.4	D	D	A
37	4032 U	.6	.7	.7	I	I	A
38	3225 F	.7	.7	.7	D	D	A
39	6153 U	.8	.8	.8	D	D	A
40	4917 U	.5	.8	.9	I	I	A

[1] U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2] I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3] A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 21. Results for Variances Related to Branch Standards.

Var. No.	Var. Amt.[1]	Compatibility with States			Decisions[2]		
		In-Control	More or less out of control	Out of Control	Suggested by Controller	Indicated by Model	A or C[3]
41	877 U	.9	.4	.4	D	D	A
45	5285 U	.6	.7	.7	I	I	A
46	1712 F	.7	.8	.7	D	D	A
47	6318 U	.6	.8	.8	I	I	A
48	5067 U	.5	.9	.9	I	I	A
						Total	26 4

[1]U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 21 (Con't). Results for Variances Related to Branch Standards.

variances. One of these differences was related to a favorable variance, and 11 were related to unfavorable variances. Eight of these 11 differences were related to corporate standards, and three were related to variances occuring in departments using branch standards.

Based on these results, one may conclude that special care should be exercised when the model is applied to situations where the standards are set by corporate office. It could be that such care was not exercised by the controller. That is, because of being unrewarded for his performance during the study, the controller might not spend enough time and/or efforts to provide accurate estimates of the inputs (such as the compatibilities of the variances with the different performance states) required for the model. There was also the learning factor; it was the first time for the controller to be exposed to the idea of fuzzy sets and the concept of membership function. This learning factor may be demonstrated by examining Figure 22 which gives the results classified by months. As one can see from the figure, there were three differences between actions indicated by the model and actions suggested by the controller in the first month, January. (These differences relate to variances 4, 5, and 6.) However, there was only one difference in each of the last two months, May and June. (They are related to variances 35 and 43.)

Taking these results into consideration, the application was replicated, eighteen months later, by using a new set of variances

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	A or C[3]	
January:								
1	1987 U	.8	.6	.5	D	D	A	
2	9463 U	.9	.4	.2	D	D	A	
3	5609 U	.7	.7	.8	D	D	A	
4	6837 U	.7	.7	.8	I	D		C
5	4962 U	.6	.8	.8	D	I		C
6	7862 F	.7	.7	.7	I	D		C
7	3881 U	.8	.7	.6	D	D	A	
8	4172 U	.5	.8	.9	I	I	A	
						Total	5	3
February:								
9	261 U	.9	.3	.2	D	D	A	
10	7706 U	.9	.3	.2	D	D	A	
11	4025 U	.7	.7	.7	D	D	A	
12	7017 U	.7	.7	.6	I	D		C
13	5404 U	.6	.7	.8	I	I	A	
14	4023 F	.8	.6	.7	D	D	A	
15	4724 U	.6	.7	.7	D	I		C
16	6029 U	.4	.8	.9	I	I	A	
						Total	6	2

[1]U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 22. Results for the 48 Variances Classified by Months.

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	A or C[3]	
March:								
17	411 U	.9	.4	.3	D	D	A	
18	6157 U	.8	.5	.4	I	D		C
19	5542 U	.7	.6	.6	I	D		C
20	5005 U	.7	.6	.6	D	D	A	
21	5523 U	.6	.7	.7	I	I	A	
22	4269 F	.8	.7	.7	D	D	A	
23	5329 U	.7	.8	.8	D	D	A	
24	3476 U	.6	.8	.8	I	I	A	
						Total	6	2
April:								
25	2567 U	.7	.5	.4	I	D		C
26	8111 U	.8	.4	.3	D	D	A	
27	9014 U	.7	.5	.5	I	D		C
28	10156 U	.7	.5	.5	I	D		C
29	6656 U	.6	.5	.7	I	I	A	
30	838 U	.7	.5	.7	D	D	A	
31	9245 U	.6	.7	.7	I	I	A	
32	8453 U	.5	.8	.9	I	I	A	
						Total	5	3

[1]U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 22 (Con't). Results for the 48 Variances Classified by Months.

Var. No.	Var. Amt.[1]	Compatibility with States			Decisions[2]			
		In-Control	More or less out of control	Out of Control	Suggested by Controller	Indicated by Model	A or C[3]	
May:								
33	1623 U	.8	.4	.4	D	D	A	
34	4819 U	.9	.5	.3	D	D	A	
35	7536 U	.7	.6	.5	I	D		C
36	8800 U	.7	.6	.5	D	D	A	
37	4032 U	.6	.7	.7	I	I	A	
38	3225 F	.7	.7	.7	D	D	A	
39	6153 U	.8	.8	.8	D	D	A	
40	4917 U	.5	.8	.9	I	I	A	
						Total	7	1
June:								
41	877 U	.9	.4	.4	D	D	A	
42	5459 U	.9	.5	.3	D	D	A	
43	5326 U	.7	.6	.5	I	D		C
44	7431 U	.7	.6	.5	D	D	A	
45	5285 U	.6	.7	.7	I	I	A	
46	1712 F	.7	.8	.7	D	D	A	
47	6318 U	.6	.8	.8	I	I	A	
48	5067 U	.5	.9	.9	I	I	A	
						Total	7	1

[1] U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2] I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3] A and C indicate that the decisions are 'in agreement' and 'in conflict', respectively.

Figure 22 (Con't). Results for the 48 Variances Classified by Months.

covering a period of six months (July 1981 - December 1981).[18] A summary of these variances is provided in Figure 23. The compatibilities of the variances with the different performance states as well as the decisions suggested by the controller are presented in Figure 24, columns 3 and 4, respectively.

The variances and their compatibilities with the different states were then used with the data presented earlier (the fuzzy transformation function and the estimated net benefits for the different combinations of states and decisions) and the computer program to obtain optimal investigation decisions for the variances. The results are provided in Figure 24, column 5.

By examining Figure 24, one can see that there are no differences between the decisions indicated by the model and those suggested by the controller. This may suggest that the differences between the two sets of actions when one kind of standard is being used will be less than the number of differences that may occur when two kinds of standards are being used by the company. Further research, however, is needed before generalizing this result.

The first objective of the application--as mentioned earlier--is to demonstrate the applicability of the model and to provide insight into the problems that may arise when the model is applied in actual

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[18] As indicated earlier, the process of replacing corporate standards by branch standards was continuing at the time of the application. As a result, during the new period the branch used more branch standards and less corporate standards than it used during the period covered in the application. Moreover, during the replication the controller was asked to exercise more attention in providing the required data.

July 1981

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 9020	$ 8852	$ 168 U
Auto Lines	28655	24827	3828 U
Press	17419	13201	4218 U
Finish	38200	28014	10186 U
Flange	26829	17796	9033 U
Plating	29685	26963	2722 U
On-line Pack	34993	28321	6672 U
Serv Pack	6343	3263	3080 U

August 1981

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 9394	$ 9851	$ 457 F
Auto Lines	37552	29539	8013 U
Press	16934	10829	6105 U
Finish	38103	28483	9620 U
Flange	26451	17054	9397 U
Plating	35329	29020	6309 U
On-Line Pack	41374	34988	6386 U
Serv Pack	5832	2817	3015 U

*U and F indicate unfavorable and favorable variances, respectively.

Figure 23. Summary of a New Set of Variances.

September 1981

Department	Actual Cost	Standard Cost	Variance*
Bonding	$10099	$10855	$ 765 F
Auto Lines	37420	27393	10027 U
Press	17848	12010	5838 U
Finish	40332	30409	9923 U
Flange	32380	22625	9755 U
Plating	34911	32340	2571 U
On-Line Pack	39940	43597	3657 F
Serv Pack	7302	3968	3334 U

October 1981

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 9564	$11477	$ 1913 F
Auto Lines	26927	22388	4539 U
Press	13426	10072	3354 U
Finish	34666	26222	8444 U
Flange	27709	18480	9229 U
Plating	26520	25350	1170 U
On-Line Pack	27601	30072	2471 F
Serv Pack	3456	2229	1227 U

*U and F indicate unfavorable and favorable variances, respectively.

Figure 23. Summary of a New Set of Variances.

November 1981

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 7782	$ 8757	$ 975 F
Auto Lines	24292	20981	3311 U
Press	11617	8350	3267 U
Finish	30819	22175	8644 U
Flange	25971	16327	9644 U
Plating	22668	22485	183 U
On-Line Pack	30838	29821	1017 U
Serv Pack	6703	4560	2143 U

December 1981

Department	Actual Cost	Standard Cost	Variance*
Bonding	$ 7172	$ 8456	$ 1284 F
Auto Lines	22294	19865	2429 U
Press	8691	6659	2032 U
Finish	22370	15808	6562 U
Flange	26758	17360	9398 U
Plating	20690	21731	1041 F
On-Line Pack	25960	26497	537 F
Serv Pack	6180	4365	1815 U

*U and F indicate unfavorable and favorable variances, respectively.

Figure 23. Summary of a New Set of Variances.

Var. No.	Var. Amt.[1]	Compatibility with States			Decisions[2]	
		In-Control	More or less out of control	Out of Control	Suggested by Controller	Indicated by Model
July:						
1	168 U	1.0	.1	.1	D	D
2	3828 U	.9	.3	.2	D	D
3	4218 U	.5	.6	.6	I	I
4	10186 U	.5	.6	.8	I	I
5	9033 U	.6	.7	.8	I	I
6	2722 U	.8	.3	.2	D	D
7	6672 U	.6	.7	.8	I	I
8	3080 U	.4	.6	.8	I	I
August:						
9	457 F	1.0	.1	.0	D	D
10	8013 U	.5	.5	.6	I	I
11	6105 U	.4	.6	.8	I	I
12	9620 U	.5	.6	.7	I	I
13	9397 U	.5	.6	.8	I	I
14	6309 U	.4	.5	.7	I	I
15	6386 U	.5	.6	.7	I	I
16	3015 U	.3	.7	.9	I	I
September:						
17	756 F	1.0	.0	.0	D	D
18	10027 U	.4	.5	.6	I	I
19	5838 U	.5	.6	.7	I	I
20	9923 U	.5	.6	.7	I	I
21	9755 U	.5	.7	.8	I	I
22	2571 U	.9	.8	.2	D	D
23	3657 F	1.0	.0	.0	D	D
24	3334 U	.4	.6	.8	I	I

[1] U and F indicate 'unfavorable' and 'favorable variances, respectively.

[2] I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

Figure 24. Results for the New Set of Variances.

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model
October:						
24	1913 F	1.0	.0	.0	D	D
26	4539 U	.8	.6	.3	D	D
27	3354 U	.7	.6	.5	D	D
28	8444 U	.5	.6	.7	I	I
29	9229 U	.4	.6	.8	I	I
30	1170 U	.9	.7	.1	D	D
31	2471 F	1.0	.0	.0	D	D
32	1227 U	.5	.6	.7	I	I
November:						
33	975 F	1.0	.0	.0	D	D
34	3311 U	.8	.7	.2	D	D
35	3267 U	.7	.6	.5	D	D
36	8644 U	.5	.6	.7	I	I
37	9644 U	.5	.6	.7	I	I
38	183 U	.9	.1	.0	D	D
39	1017 U	.9	.1	.1	D	D
40	2143 U	.5	.6	.7	I	I
December:						
41	1284 F	1.0	.0	.0	D	D
42	2429 U	.8	.6	.2	D	D
43	2032 U	.6	.5	.4	D	D
44	6562 U	.5	.6	.7	I	I
45	9398 U	.5	.6	.7	I	I
46	1041 F	1.0	.0	.0	D	D
47	537 F	1.0	.0	.0	D	D
48	1815 U	.7	.5	.4	D	D

[1] U and F indicate 'unfavorable' and 'favorable variances, respectively.

[2] I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

Figure 24 (Con't). Results for the New Set of Variances.

situations. As the application may indicate, the proposed model can be applied in real world situations. However, several problems may be encountered when such an application is attempted.

First, the model allows for (1) different transformation functions (deterministic, stochastic, and fuzzy); (2) many investigation actions; and (3) many performance states. As a result of such flexibility, a choice must be made with respect to the number of actions available, number of performance states, and type of transformation function.

In the application two actions (investigate and do-not investigate), three states (in-control, more or less out of control, and out-of-control), and the fuzzy transformation function were used. Some users, however, may prefer to add a third action (such as exploratory investigation) to the basic two actions--investigate and do-not investigate. Others may elect to use more than three performance states. Finally, the deterministic or stochastic transformation function can be used in place of the fuzzy transformation function. The choice is highly affected by the users preferences.

Second, the compatibilities of the variances with the different states are needed. There are two ways in which these compatibilities can be obtained. One way is to derive membership functions for the states. Then variances can be plugged into the derived functions to obtain their compatibilities with the states. The problem with this approach is the need to revise the derived functions when there are major shifts in either the standards or variances.

The second way is to allow the decision maker to provide directly--at the time of the decision--the compatibilities of the variances with the different states. This method may provide the decision maker with more flexibility to reflect his/her state of mind at the time of the decision. However, if this approach is to be used, care should be exercised especially when standards are set by the corporate office (i.e., when mixed standards are used by the company).

Third, there is the problem of estimating the net benefits for the different combinations of decisions, input states, and output states. These net benefits are difficult to determine with precision. However, one must remember that the new model--as the rest of fuzzy decision models--is mainly to reduce the need for precise measures that are difficult, if not impossible, to obtain.

In the application, single estimates of the net benefits were obtained from the controller. These estimates were in linguistic form. It is possible to use the form of "approximately $500" to describe the net benefits. Moreover, the net benefits can be broken into their components of different costs and benefits. The components are then combined--by using the extension principle discussed in chapters four and five--to obtain the estimates of the net benefits.

Finally, there is the problem of using empirical versus subjective membership functions. The position adopted in the application is that of Zadeh who considers the derivation of compatibility functions to be a matter of definition rather than empirical experimentation.

To examine the effect of using the empirical approach rather than the subjective approach, the application was replicated in Appendix II to Chapter Six by using empirical membership functions. The Hersh and Carmazza method for deriving empirical membership functions was used to derive membership functions for (1) the expected net benefits, and (2) the fuzzy transformation function.[19] The derived membership functions were then used with the remaining data and the computer program to obtain the optimal decisions for the 48 variances used in the study.

As shown in the appendix, the empirical membership functions were quite different from the subjective ones. However, only four decisions (out of the 48 decisions) were reversed when the empirical membership functions were used in place of the subjective membership functions. This may indicate that the model is not very sensitive to the shape of the membership functions. However, additional research is needed before generalizing this finding. (See the appendix for a full description of the replication of the study.)

6.5 Summary

In this chapter, the proposed cost variance investigation model is applied to an actual cost variance investigation problem encountered by a manufacturing firm. The main objective of the application is to demonstrate the applicability of the model and to provide insight into the problems that may arise when the model is applied in actual situations. A second objective is to compare the decisions indicated by the model to those suggested by the decision maker for some cost variances to determine if there are any systematic differences between the two sets of actions.

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[19] H. M. Hersh and A. Caramazza, "A Fuzzy Set Approach to Modifiers and Vagueness in Natural Language", pp. 254-276. See, also, H. M. Hersh, A. Caramazza, and H. H. Brownell, "Effects of Context on Fuzzy Membership Functions", pp. 389-408.

As indicated in the chapter, the major problem that may arise when the model is applied to real situations is the problem of using subjective versus empirical membership functions. Subjective membership functions were used in the application. In Appendix II to this chapter, the study is replicated by using empirical membership functions. As shown in the appendix, the empirical membership functions differ from the subjective ones. However, only four decisions (out of the 48 decisions) are reversed when empirical functions are used in place of the subjective ones. This may indicate that the model is not sensitive to the shapes of the membership functions; however, further research is needed before generalizing this result.

Regarding the second objective and as shown in the chapter, there are 12 differences between the decisions indicated by the model and those suggested by the controller. Most of these differences are related to variances that occurred in departments using corporate standards.

The application was replicated by using a new set of variances covering a period of six months (July 1981 - December 1981). During this period the branch used less corporate standards than it used during the application. Moreover, during the replication the controller was asked to exercise more care in providing the required data. As shown in the chapter there were no differences between the decisions indicated by the model and those suggested by the controller for the new set of variances. This may suggest that the differences between the two sets of actions when one kind of standards being used will be less than the number of differences when the two kinds of standards are being used. However, further research is needed before generalizing this result.

CHAPTER SEVEN

Summary and Implications for Further Research

7.1 Summary

The problem of when to investigate cost variances has received much attention in the accounting literature. The discussion of the problem in terms of management's judgment and intuition has been rejected and several quantitative models for solving the problem have been suggested by researchers. These models are reviewed and evaluated in Chapter Three of this dissertation.

The models are divided into two groups. The first group includes models that do not consider the costs and benefits of the investigation action, and the second group are models that take into consideration both the costs and the benefits of the investigation decision. The first group of models is exemplified by the Zannetos (1964), Jeurs (1967), Luh (1968), Koehler (1968), and Probst (1971) models. The second group includes the Bierman, Fouraker, and Jaedicke (BFJ) model (1961), the Kaplan model (1969), the Dyckman model (1969), and the Duvall model (1967).

In evaluating the available cost variance investigation models, three criteria are used. First, the model should consider the costs and benefits of the investigation action. Second, the model should capture the essence of the real world problem (i.e., reality of the assumptions). Third, the model should be feasible (i.e., obtainability of the data required to operate the model).

As shown in Chapter Three, the first group of models suffers basically from ignoring the costs and benefits of the investigation action. Moreover, the models in this group encounter additional problems. First, they are based on purely objective evidence which may be unrealistic. Second, they ignore the information from previous observations of the process. Third, there is no consideration of the possible implications of future actions. Finally, the models do not consider the implementation costs and information requirements.

The models in the second group consider the costs and benefits of the investigation decision. However, they fail to capture the essence of the real world problem. For example, the models assume that inexactness, whatever its nature, can be equated with randomness; however, this is an erroneous assumption. Second, the models are based on the assumption of a two-state system. This assumption is an unrealistic aggregation of reality. Third, the models assume constant level of accuracy and precision. However, some judgments are harder than others.

In addition to not capturing the essence of the real world problem, the models in the second group suffer from the lack of applicability (i.e., feasibility). As shown in Chapter Three, the models require precise numerical inputs to the analysis; however, in practice the required level of precision is difficult, if not impossible, to attain.

As a result of demanding precise measurements, the models lose part of their relevance to the actual problem through ignoring some relevant items just because these items cannot be precisely measured. Moreover, the insistence on demanding precise numerical values may be the reason for the uneasiness of the potential users of these models.

Taking this into consideration, the objective of this dissertation is the development of a new cost variance investigation model that may overcome some of the above problems. The model utilizes the calculus of fuzzy set theory which was introduced by Zadeh in 1965. A brief discussion of the basic elements of fuzzy set thoery is provided in Chapter Two. The chapter also includes a discussion of some of the theory's applications in the decision making area.

As defined by Zadeh, fuzzy sets are classes of objects with no sharp transition between membership and nonmembership. For example, the classes of "experienced managers", "young men", "corporations with high profits", "large variances", and "good-looking girls" are fuzzy sets.

The basic concept in fuzzy set theory is the membership (compatibility) function for a fuzzy set which associates with every element in the universe its compatibility with the fuzzy set. Unlike the characteristic function for an ordinary set, the membership function takes its values in the closed interval from zero to one. That is, instead of treating the presence or absence in a binary manner, the fuzzy set theory allows for gradual membership.

Reflecting the distinction between the membership function and the characteristic function, the calculus of fuzzy set theory is different from the calculus of the ordinary set theory. Moreover, because it allows for varying degrees of membership, through the concept of compatibility function, the calculus of fuzzy set theory is well suited for dealing with fuzziness which is a major source of inexactness in many humanistic systems.

The new cost-variance investigation model is presented in Chapter Four. First, the problem of cost-variance investigation is formulated as follows:

$$\begin{tikzcd} & D_t \arrow[d] & \\ S_t \arrow[r] & \boxed{S_{t+1} = f(S_t, D_t)} \arrow[r] \arrow[d] & S_{t+1} \\ & B = b(S_t, D_t, S_{t+1}) & \end{tikzcd}$$

where

S_t = set of input states = $\{s_1, \ldots, s_i, \ldots, s_n\}$,

D_t = set of available decisions = $\{d_1, \ldots, d_j, \ldots, d_m\}$,

S_{t+1} = set of output states = $\{s_1, \ldots, s_k, \ldots, s_n\}$,

and

B = set of possible net benefits resulting from the different combinations of $s_i \in S_t$, $s_k \in S_{t+1}$, and $d_j \in D_t$.

The set of output states is related to the set of input states through a transformation function $S_{t+1} = f(S_t, D_t)$. This transformation function can be deterministic, probabilistic, or fuzzy function. (The deterministic case is discussed in Chapter Four while the remaining two cases are discussed in Chapter Five.)

Methods are suggested to obtain the optimal decision for the three cases of transformation functions. These methods are based on formulating a fuzzy optimal decision set

$$D_o = \{u_{D_o}(d_j)|d_j\},$$

where the membership function $u_{D_o}(d_j)$ represents the compatibility of each decision with the optimal decision set D_o (i.e., it reflects the relative merit of the decision). It is worth noting that the knowledge about the state of the performance, the possible net benefits, and the possibility of obtaining the net benefits are reflected in the optimal decision set. The optimal decision is the decision having the highest compatibility with the fuzzy optimal decision set.

Numerical examples are provided in Chapters Four and Five to illustrate how one can formulate the set D_o and obtain the optimal decision for the three types of transformation functions.

In addition to allowing for different transformation functions, the proposed model allows for varying degrees of out-of-controllness. That is, in contrast to what was assumed by the available cost variance investigation models, the new model allows the set of performance states, S_t, to include not only the two states, in-control and out-of-control, but also intermediate states. For example, S_t may include, in addition to the extreme states, the more or less out-of-control state.

Moreover, in the proposed model, each state $s_i \in S_t$ is described by a fuzzy set whose membership function associates with every variance in the universe $V = \{0, 1, 2, \ldots\}$ its compatibility with the state s_i. That is, the model allows the states to be defined imprecisely.

The model also provides for the imprecision surrounding the net benefits from the investigation action. As an example, the estimates of the net benefits are allowed to take the form: approximately \$800, near \$1,000, etc. The estimates may also take the form of linguistic values such as high, very high, medium, low, and very low. Any of these values is described by a fuzzy set, B_j, whose compatibility function $u_{B_j}(b_k)$ associates with every net benefit $b_k \varepsilon B = \{0, 1, ...\}$ its compatibility with the fuzzy set B_j.

Finally, the proposed model allows the decision maker to express his judgment about probabilities by using statements like "approximately 30%", "about 30%", or "roughly 30%". The judgments may also take the form of linguistic values such as: "quite likely", "almost certainly", "better than even", "pretty likely", etc. Any of these values is defined by a fuzzy set, P_k, whose compatibility function $u_{P_k}(p)$ associates with every $p \varepsilon \{0, .1, .2, ..., 1\}$ its compatibility with the fuzzy set P_k.

As indicated earlier, available cost variance investigation models have not been used in practice. Similarly, the literature on fuzzy set theory lacks practical applications. Taking this into consideration, the proposed model was applied--through a pilot study--to an actual cost variance investigation problem encountered by a manufacturing company. First, the model was computerized. Three computer programs were written. These programs correspond to the three cases of transformation functions. (Copies of the programs appear in Appendix I to Chapter Six.) The programs were validated by using data from the numerical illustrations discussed in Chapters Four and Five. The results obtained by the programs were as those obtained by the manual solution of the illustrations.

Second, the new model was applied to an actual cost-variance investigation problem encountered by a branch (located in Blacksburg, Virginia) of a manufacturing company. Variances related to direct labor cost over a period of six months were obtained. The controller of the branch elected to use the fuzzy transformation function, three performance states (in-control, more-or-less-out-of-control, and out-of-control), and two decisions (investigate and do-not-investigate). The controller provided the estimates of the net benefits--in linguistic form--for the different combinations of decisions and performance states. He also provided compatibility functions for the linguistic net benefits. Finally, the controller provided the transformation function and the compatibilities of the variances with the different performance states. The data were then used with the computer program to obtain the optimal investigation decision for every variance of the 48 variances used in the study.

One problem that may be encountered in applying the model to real situations is the problem of deriving the required compatibility functions. As indicated in Chapter Six, there are two approaches to deriving compatibility functions, namely, the subjective and empirical approaches. The position adopted in the application is that of Zadeh who considers the derivation of compatibility functions to be a matter of definition rather than empirical experimentation.

To examine the effect of using the empirical approach rather than the subjective approach, the application was replicated in Appendix II to Chapter Six by using empirical membership functions. The Hersh and Caramazza method for deriving empirical membership functions was used to derive membership functions for (1) the expected net benefits, and (2) the fuzzy transformation function. The derived membership functions were then used with the remaining data and the computer program to obtain the optimal decisions for the 48 variances used in the study.

As shown in the appendix, the empirical membership functions were quite different from the subjective membership functions. However, only 4 decisions (out of the 48 decisions) were reversed when the empirical membership functions were used in place of the subjective membership functions. This may indicate that the model is not very sensitive to the shape of the membership functions. However, additional research is needed before generalizing this finding.

7.2 Implications for Future Research

Fuzzy set theory was introduced by Zadeh as a means to provide a mathematical framework wherein imprecise phenomena in decision making can be dealt with in a precise and rigorous manner. The theory also reduces the need for precise measurements that are difficult, if not impossible, to obtain in real world situations.

Since its introduction, fuzzy set thoery has been applied to many fields such as system theory, decision theory, automata theory, medicine, and pattern recognition. In this dissertation the theory is applied to an accounting problem, namely, the cost variance investigation problem. This lays the groundwork for further research to investigate how fuzzy set theory can be used in solving other accounting problems.

Available cost variance investigation models have not been used in practice. Similarly, the literature on fuzzy set theory lacks practical applications. In the dissertation the proposed cost variance investigation model is applied to an actual cost variance investigation problem. Further research involving actual applications of cost variance investigation models and fuzzy decision theories is needed.

In the application part of the study, the sensitivity of the model to the shape of membership functions was examined. Examining the sensitivity of the model to the other variables (such as the number of states, number of decisions, and the compatibilities of the variances with the different performance states) represents another area for future research. (A simulation study may be used in such research.)

Finally, extending the model to cover the cost variance investigation decision over many periods is another avenue for future research. Such an extension may be achieved by employing both the fuzzy set theory and dynamic programming.

Bibliography

Anthony, R. N. "Some Fruitful Directions for Research in Management Accounting," in N. Dopuch and L. Revsine, eds., Accounting Research 1960-1970: A Critical Evaluation (Center for International Education and Research in Accounting: University of Illinois, 1973), pp. 37-68.

Asai, K., H. Tanaka, and T. Okuda, "On Discrimination of Fuzzy States in Probability Space," Kybernetes, Vol. 6 (1977), pp. 185-192.

Aubin, J. P., "Fuzzy Core and Equilibria of Games Defined in a Strategic Form," in Y. C. Ho and S. K. Mitter, eds., Directions in Large-Scale Systems (New York: Plenum Press, 1976), pp. 371-388.

Bandyopadhyay, R., "Use of Taxonomies of Information and Decision--A Case Study," in D. J. White and K. C. Bowen, eds., The Role and Effectiveness of Theories of Decision in Practice (London: Hodder and Stoughton, 1975), pp. 207-224.

Baas, S. M. and H. Kwarkernaak, "Rating and Ranking of Multiple-Aspect Alternatives Using Fuzzy Sets," Automatica, Vol. 13 (1977), pp. 47-58.

Bather, G. A., "Control Charts and the Minimization of Costs," Journal of the Royal Statistical Society, Vol. 25 (1963), pp. 49-70.

Becker, G. M. and C. G. McClintock, "Value: Behavioral Decision Theory," Annual Review of Psychology, Vol. 18 (1967), pp. 239-286.

Bellman, R. E. and L. A. Zadeh, "Decision-Making in a Fuzzy Environment," Management Science, Vol. 17 (1970), pp. B-141 - B-164.

Bezdek, J. C., B. Spillman, and R. Spillman, "A Fuzzy Relation Space for Group Decision Theory," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 255-268.

Bierman, H., Jr., and T. Dyckman, Management Cost Accounting (New York: The MacMillan Company, 1971).

Bierman, H., Jr., L. E. Fouraker, and R. K. Jaedicke, "The Use of Probability and Statistics in Performance Evaluation," The Accounting Review (July 1961), pp. 409-417.

Birnberg, J. G. and R. Nath, "Laboratory Experimentation in Accounting Research," The Accounting Review (January 1968), pp. 38-45.

Blin, J. M., "Fuzzy Relations in Group Decision Theory," Journal of Cybernetics, Vol. 4 (1974), pp. 17-22.

Blin, J. M. and A. B. Whinston, "Fuzzy Sets and Social Choice," Journal of Cybernetics, Vol. 3 (1974), pp. 28-36.

Braae, M. and D. A. Rutherford," Fuzzy Relations in a Control Setting," Kybernetes, Vol. 7 (1978), pp. 185-188.

Bowen, K. C., "Models and Decision Processes," in D. J. White and K. C. Bowen, eds., The Role and Effectiveness of Theories of Decision in Practice (London: Hodder and Stoughton, 1975), pp. 392-401.

Brown, J. G., "A Note on Fuzzy Sets," Information and Control, Vol. 18 (1971), pp. 32-39.

Brownell, H. H. and A. Caramazza, "Categorizing with Overlapping Categories," Memory and Cognition, Vol. 6 (1978), pp. 481-490.

Butnariu, D., "Fuzzy Games: A Description of the Concept," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 181-192.

Chang, R. L. P. and R. Pavlidis, "Fuzzy Decision Tree Algorithms," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-7 (1977), pp. 28-35.

Chang, S. S. L., "Fuzzy Dynamic Programming and the Decision Making Process," Proceedings 3rd Princeton Conf. on Information Sciences and Systems (1969), pp. 200-203.

Chang, S. S. L., "Application of Fuzzy Set Theory to Economics," Kybernetes, Vol. 6 (1977), pp. 203-207.

Chang, S. S. L. and L. A. Zadeh, "On Fuzzy Mapping and Control," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-2 (1972), pp. 30-35.

Chesley, G. R., "Elicitation of Subjective Probabilities: A Review," The Accounting Review (April 1970), pp. 235-245.

Cooley, J. W. and J. O. Hicks (Jr.), "A Linguistic Approach to the Evaluation of Internal Control Systems," Working Paper, Virginia Polytechnic Institute and State University (July 1979).

Davis, D. R., L. Duckstein, and C. C. Kisiel, "Model Choice and Evaluation from the Decision Viewpoint," in D. J. White and K. C. Bowen, eds., The Role and Effectiveness of Theories of Decision in Practice (London: Hodder and Stoughton, 1975), pp. 341-351.

De Luca, A. and S. Termini, "A Definition of a Nonprobabilistic Entropy in the Setting of Fuzzy Sets Theory," Information and Control, Vol. 20 (1972), pp. 301-312.

De Luca, A. and S. Termini, "On the Convergence of Entropy Measures of a Fuzzy Set," Kybernetes, Vol. 6 (1977), pp. 219-227.

Demski, J. S., Information Analysis (California: Addison-Wesley Publishing Company, Inc., 1972).

Demski, J. S. and G. A. Feltham, Cost Determination: A Conceptual Approach (Iowa: The Iowa State University Press, 1976).

Dimitrov, V. D., "Social Choice and Self Organization under Fuzzy Environment," Kybernetes, Vol. 6 (1977), pp. 153-156.

Dubois, D. and H. Prade, "Comment on 'Tolerance Analysis using Fuzzy Sets' and a Procedure for Multiple Aspect Decision Making," International Journal of Systems Science, Vol. 9 (1978), pp. 357-360.

Dubois, D. and H. Prade, "Operations on Fuzzy Numbers," International Journal of Systems Science, Vol. 9 (1978), pp. 613-626.

Dubois, D. and H. Prade, Fuzzy Sets and Systems: Theory and Applications (New York: Academic Press, 1979).

Duvall, R. M., "Rules for Investigating Cost Variances," Management Science, Vol. 13 (1967), pp. B-631 - B-641.

Dyckman, T. R., "The Investigation of Cost Variances," Journal of Accounting Research (Autumn 1969), pp. 215-244.

Elmaghraby, S. E., "The Role of Modeling in IE Design," Journal of Industrial Engineering, Vol. 19 (1968), pp. 292-305.

Flachs, J. and M. A. Pollatschek, "Further Results on Fuzzy-Mathematical Programming," Information and Control, Vol. 38 (1978), pp. 241-257.

Flonder, P., "An Example of A Fuzzy System," Kybernetes, Vol. 6 (1977), p. 229.

Franksen, O. I., "On Fuzzy Sets, Subjective Measurements and Utility," International Journal of Man-Machine Studies, Vol. 11 (1979), pp. 521-545.

Gaines, B. R., "Foundations of Fuzzy Reasoning," in M. M. Gupta, G. N. Saridis and B. R. Gaines, eds., Fuzzy Automata and Decision Processes (New York: Elsevier North-Holland, Inc., 1977), pp. 19-75.

Gaines, B. R. and L. J. Kohout, "The Fuzzy Decade: A Bibliography of Fuzzy Systems and Closely Related Topics," International Journal Man-Machine Studies, Vol. 9 (1977), pp. 1-68.

Gigch, J. P. and L. L. Pipino, "From Absolute to Probable and Fuzzy in Decision Making," Kybernetes, Vol. 9 (1980), pp. 47-55.

Gitman, I., "Organization of Data; A Model and Computational Algorithm that Uses the Notion of Fuzzy Sets," Ph.D. Dissertation, McGill University, Montreal, Canada, 1970.

Good, I. J., "Amount of Deciding and Decisionary Effort," Information and Control, Vol. 4 (1961), pp. 271-281.

Gottinger, Hans-Werner, "Toward a Fuzzy Reasoning in the Behavioral Science," Cybernetica, Vol. 26 (1978), pp. 113-135.

Gupta, M. M., R. K. Ragade and R. R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979).

Hagg, C., "Possibility and Cost in Decision Analysis," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 81-86.

Hamacher, H., H. Leberling and H. J. Zimmermann, "Sensitivity Analysis in Fuzzy Linear Programming," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 269-281.

Henerson, M. E., L. L. Morris, and C. T. Fitz-Gibbon, How to Measure Attitude (California: Sage Publications, Inc., 1978).

Hersh, H. M. and A. Caramazza, "A Fuzzy Set Approach to Modifiers and Vagueness in Natural Language," Journal of Experimental Psychology: General, Vol. 105 (1976), pp. 254-276.

Hersh, H. M., A. Caramazza and H. H. Brownell, "Effects of Context on Fuzzy Membership Functions," in M. Gupta, R. Ragade and R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 389-408.

Hersh, H. M., "Fuzzy Reasoning: The Integration of Vague Information," Doctoral Dissertation, The Johns Hopkins University, 1976.

Hisdal, E., "Conditional Possibilities: Independence and Noninteraction," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 283-297.

Hisdal, E., "Possibilistically Dependent Variables and a General Theory of Fuzzy Sets," in M. Gupta, R. Ragade and R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 215-234.

Horngren, C. T., Cost Accounting: A Managerial Emphasis, 5th ed. (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1982).

Horngren, C. T., Introduction to Management Accounting, 5th ed. (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1981).

Ijiri, Y., R. K. Jaedicke, and K. E. Knight, "The Effects of Accounting Alternatives on Management Decisions," in R. K. Jaedicke, Y. Ijiri, and O. Nielsen, eds., Research in Accounting Measurement (Chicago: American Accounting Association, 1966), pp. 186-199.

Jain, R., "Tolerance Analysis using Fuzzy Sets," International Journal of Systems Science, Vol. 7 (1976), pp. 1393-1401.

Jain, R., "Decisionmaking in the Presence of Fuzzy Variables," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-6 (1976), pp. 698-703.

Jain, R. "A Procedure for Multiple-Aspect Decision Making Using Fuzzy Sets," International Journal of Systems Science, Vol. 8 (1977), pp. 1-7.

Jeurs, D. A., "Statistical Significance of Accounting Variances," Management Accounting (October 1967), pp. 20-25.

Kacprzyk, J., "A Branch and Bound Algorithm for the Multistage Control of a Nonfuzzy System in a Fuzzy Environment," Control and Cybernetics, Vol. 7 (1978), pp.

Kacprzyk, J., "Decision-Making in a Fuzzy Environment with Fuzzy Termination Time," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 169-179.

Kacprzyk, J., "A Branch and Bound Algorithm for the Multi-Stage Control of a Fuzzy System in a Fuzzy Environment," Kybernetes, Vol. 8 (1979), pp. 139-147.

Kaczka, E. E., "Computer Simulation," Decision Sciences, Vol. 1 (1971), pp. 174-192

Kandel, A., "On the Minimization of Incompletely Specified Fuzzy Functions," Information and Control, Vol. 26 (1974), pp. 141-153.

Kandel, A., "On Fuzzy Statistics," in M. Gupta, R. Ragade and R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 181-199.

Kandel, A. and R. Yager, "A 1979 Bibliography on Fuzzy Sets, Their Applications, and Related Topics," in M. Gupta, R. Ragade, and R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 621-744.

Kaplan, A., The Conduct of Inquiry (California: Chandler Publishing Company, 1964).

Kaplan, R. S., "Optimal Investigation Strategies with Imperfect Information", Journal of Accounting Research (Spring 1969), pp. 32-43.

Kaplan, R. S., "The Significance and Investigation of Cost Variances: Survey and Extensions," Journal of Accounting Research (Autumn 1975), pp. 311-337.

Kaufman, A., Introduction to the Theory of Fuzzy Subsets (New York: Academic Press, 1975).

Kickert, W., Fuzzy Theories on Decision-Making (Netherland: Martinus Nijhoff Social Sciences Division, 1978).

Kling, R., "Fuzzy Planner: Reasoning with Inexact Concepts in a Procedural Problem-Solving Language," Journal of Cybernetics, Vol. 3 (1974), pp. 1-16.

Kochen, M. and A. Badre, "On the Precision of Adjectives which Denote Fuzzy Sets," Journal of Cybernetics, Vol. 4 (1974), pp. 49-59.

Koehler, R. W., "The Relevance of Proabaility Statistics to Accounting Variance Control," Management Accounting (October 1968), pp. 35-41.

Kwakernaak, H., "An Algorithm for Rating Multiple-Aspect Alternatives Using Fuzzy Sets," Automatica, Vol. 5 (1979), pp. 615-616.

Lee, E. T. and L. A. Zadeh, "Note on Fuzzy Languages," Information Sciences, Vol. 1 (1969), pp. 421-434.

Li, Y., "A Note on the Investigation of Cost Variances," Journal of Accounting Research (Autumn 1970), pp. 282-283.

Loo, S. G., "Measures of Fuzziness," Cybernetica, Vol. 20 (1977), pp. 201-210.

Loo, S. G., "Fuzzy Relations in the Social and Behavioral Sciences," Journal of Cybernetics, Vol. 8 (1978), pp. 1-16.

Luh, F., "Controlled Costs: An Operational Concept and Statistical Approach to Standard Costing," The Accounting Review (January 1968), pp. 123-132.

Macvicar-Whelan, P. J., "Fuzzy Sets, The Concept of Height, and the Hedge Very," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-8 (1978), pp. 507-511.

Magee, R. P., "A Simulation Analysis of Alternative Cost Variance Investigation Models," The Accounting Review (July 1976), pp. 529-544.

McCloskey, M. E. and S. Glucksburg, "Natural Categories: Well Defined or Fuzzy Sets?," Memory and Cognition, Vol. 6 (1978), pp. 462-472.

Mizumoto, M. and K. Tanaka, "Some Properties of Fuzzy Sets of Type 2," Information and Control, Vol. 31 (1976), pp. 312-340.

Mizumoto, M. and K. Tanaka, "Some Properties of Fuzzy Numbers," in M. Gupta, R. Ragade and R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 153-164.

Nahmias, S., "Fuzzy Variables," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 97-110.

Nahmias, S., "Fuzzy Variables in a Random Environment," in M. Gupta, R. Ragade and R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 165-180.

Naplatanoff, N. and D. Lakov, "Decision Making in Vague Conditions," Kybernetes, Vol. 6 (1977), pp. 91-93.

National Association of Accountants, Report No. 22: The Analysis of Manufacturing Cost Variances (New York: NAA, 1952).

Negoita, D. V. and D. A. Ralescu, "Representation Theorems for Fuzzy Concepts," Kybernetes, Vol. 4 (1975), pp. 169-174.

Negoita, C. V. and A. C. Stefanescu, "On the State Equation of Fuzzy Systems," Kybernetes, Vol. 4 (1975), pp. 213-214.

Negoita, C. V. and D. A. Ralescu, "On Fuzzy Optimization," Kybernetes, Vol. 6 (1977), pp. 193-195.

Nguyen, H. T., "On Conditional Possibility Distributions," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 299-309.

Nguyen, H. T., "Toward a Calculus of the Mathematical Notion of Possibility," in M. Gupta, R. Ragade and R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 235-246.

Nowakowska, M., "New Ideas in Decision Theory," International Journal of Man-Machine Studies, Vol. 11 (1979), pp. 213-234.

Nowakowska, M., "Fuzzy Concepts: Their Structure and Problems of Measurement," in M. Gupta, R. Ragade and R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 361-387.

Okuda, T., H. Tanaka and K. Asai, "A Formulation of Fuzzy Decision Problems with Fuzzy Information Using Probability Measures of Fuzzy Events," Information and Control, Vol. 38 (1978), pp. 135-147.

Onsi, M. "Quantitative Models for Accounting Control," The Accounting Review (April 1967), pp. 321-330.

Orlovsky, S. A., "On Programming with Fuzzy Constraint Sets," Kybernetes, Vol. 6 (1977), pp. 197-201.

Orlovsky, S. A., "Decision Making with Fuzzy Preference Relation," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 155-167.

Pollatschek, M. A., "Hierarchical Systems and Fuzzy-Set Theory," Kybernetes, Vol. 6 (1977), pp. 147-151.

Probst, F. R., "Probabilistic Cost Controls: A Behavioral Dimension," The Accounting Review (January 1971), pp. 113-118.

Ragade, R. K., "Fuzzy Sets in Communication Systems and in Consensus Formation Systems," Journal of Cybernetics, Vol. 6 (1976), pp. 21-38.

Ragade, R. K., "Fuzzy Interpretive Structural Modeling," Journal of Cybernetics, Vol. 6 (1976) pp. 189-211.

Ragade, R. K., "Fuzzy Games in the Analysis of Options," Journal of Cybernetics, Vol. 6 (1976), pp. 213-221.

Saaty, T., "Measuring the Fuzziness of Sets," Journal of Cybernetics, Vol. 4 (1974), pp. 53-61.

Saaty, T., "Exploring the Interface between Hierarchies,, Multiple Objectives, and Fuzzy Sets," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 57-68.

Santos, E. S., "Fuzzy and Probabilistic Programs," Information Sciences, Vol. 10 (1976), pp. 331-345.

Shewhart, W. A., The Economic Control of the Quality of Manufactured Profit (New York: MacMillan, 1931).

Slovic, P. and S. Lichtenstein, "Comparison of Bayesian and Regression Approaches to the Study of Information Processing in Judgment," Organizational Behavior and Human Performance (November 1971), pp. 649-744.

Spillman, B., J. Bezdek and R. Spillman, "Coalition Analysis with Fuzzy Sets," Kybernetes, Vol. 8 (1979), pp. 203-211.

Stallings, W., "Fuzzy Set Theory Versus Bayesian Statistics," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-7 (1977), pp. 216-219.

Tanaka, H., T. Okuda and K. Asai, "On Fuzzy-Mathematical Programming," Journal of Cybernetics, Vol. 3 (1974), pp. 37-46.

Tanaka, H., T. Okuda and K. Asai, "Fuzzy Information and Decision in Statistical Model," in M. Gupta, R. Ragade and R. Yager, eds., Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 303-320.

Van-Velthoven, G., "Fuzzy Models in Personnel Management," Proceedings of International Congress of Cybernetics and Systems, Bucharest (1975).

Watanabe, S., "A Generalized Fuzzy-Set Theory," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-8 (1978), pp. 756-760.

Watson, S. R., J. J. Weiss and J. L. Donnell, "Fuzzy Decision Analysis," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-9 (1979), pp. 1-9.

Wenstop, F., "Application of Linguistic Variables in the Analysis of Organizations," Doctoral Dissertation, University of California, Berkeley, 1975.

Wenstop, F., "Fuzzy Set Simulation Models in a Systems Dynamics Perspective," Kybernetes, Vol. 6 (1976), pp. 209-218.

Wenstop, F., "Deductive Verbal Models of Organizations," International Journal of Man-Machine Studies, Vol. 8 (1976), pp. 293-311.

White, D. J., Decision Theory (Chicago: Aldine Publishing Company, 1969).

White, D. J., "The Nature of Decision Theory," in D. J. White and K. C. Bowen, eds., The Role and Effectiveness of Theories of Decision in Practice (London: Hodder and Stoughton, 1975), pp. 3-18.

Woodhead, R. G., "On the Theory of Fuzzy Sets to Resolve Ill-Structured Marine Decision Problems," Dept. of Naval Architecture, University of Newcastle (1972).

Yager, R. R. "Multiple Objective Decision-Making Using Fuzzy Sets," International Journal of Man-Machine Studies, Vol. 9 (1977), pp. 375-382.

Yager, R. R., "Fuzzy Decision Making Including Unequal Objectives," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 87-95.

Yager, R. R., "Fuzzy subsets of Type II in Decisions," Journal of Cybernetics, Vol. 10 (1980), pp. 137-159.

Yager, R. R., "Fuzzy Sets, Probabilities, and Decision," Journal of Cybernetics, Vol. 10 (1980), pp. 1-18.

Yager, R. R. and D. Basson, "Decision Making with Fuzzy Sets," Decision Sciences, Vol. 6 (1975), pp. 590-600.

Zadeh, L. A. "Fuzzy Sets," Information and Control, Vol. 8 (1965), pp. 338-353.

Zadeh, L. A., "Probability Measures of Fuzzy Events," Journal Math. Anal. Appl., Vol. 23 (1968), pp. 421-427.

Zadeh, L. A., "Quantitative Fuzzy Semantics," Information Sciences, Vol. 3 (1971), pp. 159-176.

Zadeh, L. A., "A Fuzzy Set Theoretic Interpretation of Linguistic Hedges," Journal of Cybernetics, Vol. 2 (1972), pp. 4-34.

Zadeh, L. A., "On Fuzzy Algorithm," Memo No. ERL-M325, Electronics Research Laboratory, University of California (1972).

Zadeh, L. A., "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes," IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-3 (1973), pp. 28-44.

Zadeh, L. A., "The Concept of a Linguistic Variable and Its Application to Approximate Reasoning," Part I, Information Sciences, Vol. 8 (1975), pp. 199-249.

Zadeh, L. A., "The Concept of a Linguistic Variable and Its Application to Approximate Reasoning," Part II, Information Sciences, Vol. 8 (1975), pp. 301-357.

Zadeh, L. A., "The Concept of a Linguistic Variable and Its Application to Approximate Reasoning," Part III, Information Sciences, Vol. 9 (1975), pp. 43-80.

Zadeh, L. A., "Calculus of Fuzzy Restrictions," in L. A. Zadeh, K. S. Fu, K. Tanaka and M. Shimura, eds., Fuzzy Sets and Their Applications to Cognitive and Decision Processes (New York: Academic Press, 1975), pp. 1-38.

Zadeh, L. A., "The Linguistic Approach and Its Application to Decision Analysis," in Y. C. Ho and S. K. Mitter, eds., Directions in Large-Scale Systems (New York: Plenum Press, 1976), pp. 339-370.

Zadeh, L. A., "A Theory of Approximate Reasoning," Memo UCB/ERL-M77/58, University of California, Berkeley (1977).

Zadeh, L. A. "Fuzzy Sets as a Basis for a Theory of Possibility," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 3-28.

Zadeh, L. A., "PRUF--a Meaning Representation Language for Natural Languages," International Journal of Man-Machine Studies, Vol. 10 (1978), pp. 395-460.

Zadeh, L. A., K. Fu, K. Tanaka and M. Shimura, eds., Fuzzy Sets and Their Applications to Cognitive and Decision Processes (New York: Academic Press, 1975).

Zannetos, Z. A., "Standard Cost as a First Step to Probabilistic Control: A Theoretical Justification, an Extension and Implications," The Accounting Review (April 1964), pp. 296-304.

Zimmermann, H. J., "Description and Optimization of Fuzzy Systems," International Journal of General Systems, Vol. 2 (1976), pp. 209-215.

Zimmermann, H. J., "Fuzzy Programming and Linear Programming with Several Objective Functions," Fuzzy Sets and Systems, Vol. 1 (1978), pp. 45-56.

APPENDICES

Appendix to Chapter Two

Fuzzy Set Operations and Fuzzy Relations

In addition to intersection, union, and complementation operations, other fuzzy set operations can be defined:[1]

Algebraic Product: The algebraic product of $\underset{\sim}{A}$ and $\underset{\sim}{B}$ (denoted by $\underset{\sim}{A}\cdot\underset{\sim}{B}$) is defined by:

$$\underset{\sim}{A}\cdot\underset{\sim}{B} = \int_{x\varepsilon E} (u_{\underset{\sim}{A}\cdot\underset{\sim}{B}}(x)/x) \ ,$$

where

$$u_{\underset{\sim}{A}\cdot\underset{\sim}{B}}(x) = u_{\underset{\sim}{A}}(x) \cdot u_{\underset{\sim}{B}}(x) \quad .$$

Algebraic Sum: The algebraic sum of two fuzzy sets (denoted by $\underset{\sim}{A} \hat{+} \underset{\sim}{B}$) is defined by:

$$\underset{\sim}{A} \hat{+} \underset{\sim}{B} = \int_{x\varepsilon E} (u_{\underset{\sim}{A}\hat{+}\underset{\sim}{B}}(x)/x) \ ,$$

where

$$u_{\underset{\sim}{A}\hat{+}\underset{\sim}{B}}(x) = u_{\underset{\sim}{A}}(x) + u_{\underset{\sim}{B}}(x) - u_{\underset{\sim}{A}}(x) \cdot u_{\underset{\sim}{B}}(x) \quad .$$

It should be noted that if M = {0,1}, that is, if we are in the case of ordinary sets, then:

$$\underset{\sim}{A} \cap \underset{\sim}{B} = \underset{\sim}{A}\cdot\underset{\sim}{B}$$

$$A \cup B = A \hat{+} B$$

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[1]The definitions in this appendix are mainly from L. A. Zadeh, "Fuzzy Sets", Information and Control, Vol. 8 (1965), pp. 338-353.

However, this is not true for $M = [0,1]$, except in a very few trivial cases. It should be noted that $\cup$ is not distributive with respect to $(.)$ and $(\hat{+})$ and likewise $\cap$. However, the operation $(.)$ is distributive with respect to $\cup$ and $\cap$. The operation $(\hat{+})$ is also distributive with respect to $\cup$ and $\cap$.

Disjunctive Sum: The disjunctive sum of two fuzzy subsets (denoted $\underset{\sim}{A} \oplus \underset{\sim}{B}$) is defined by: $\underset{\sim}{A} \oplus \underset{\sim}{B} = (\underset{\sim}{A} \cap \underset{\sim}{B}) \cup (\underset{\sim}{\bar{A}} \cap \underset{\sim}{B})$. This operation corresponds to "fuzzy disjunctive or".

Difference: The difference between two fuzzy subsets (denoted $\underset{\sim}{A} - \underset{\sim}{B}$ is defined by $\underset{\sim}{A} - \underset{\sim}{B} = \underset{\sim}{A} \cap \underset{\sim}{\bar{B}}$.

Equality: The two fuzzy subsets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are equal (denoted $\underset{\sim}{A} = \underset{\sim}{B}$) if and only if $u_{\underset{\sim}{A}}(x) = u_{\underset{\sim}{B}}(x)$, $\forall\ x \in E$.

If there is at least one $x \in E$ such that $u_{\underset{\sim}{A}}(x) \neq u_{\underset{\sim}{B}}(x)$ then $A \neq B$.

Inclusion: A fuzzy subset $\underset{\sim}{A}$ is included in the fuzzy subset $\underset{\sim}{B}$ (denoted $\underset{\sim}{A} \subset \underset{\sim}{B}$) if $u_{\underset{\sim}{A}}(x) \leq u_{\underset{\sim}{B}}(x)$, $\forall\ x \in E$.

The set $\underset{\sim}{A}$ is strictly included in $\underset{\sim}{B}$ (denoted as $\underset{\sim}{A} \subset\subset \underset{\sim}{B}$) if $u_{\underset{\sim}{A}}(x) \leq u_{\underset{\sim}{B}}(x)$, $\forall\ x \in E$, with at least one $u_{\underset{\sim}{A}}(x)$ and $u_{\underset{\sim}{B}}(x)$ between which there exists a strict relation.

Convex Combination: The convex combination of two fuzzy subsets $\underset{\sim}{A}$ and $\underset{\sim}{B}$ is given by:

$$\alpha \underset{\sim}{A} + (1 - \alpha)\, \underset{\sim}{B} = \int_{x \in E} (\alpha u_{\underset{\sim}{A}}(x) + (1 - \alpha) u_{\underset{\sim}{B}}(x)/x), \quad 0 \leq \alpha \leq 1.$$

Concentration: The concentration of $\underset{\sim}{A}$ is given by:

$$\mathrm{CON}\ (\underset{\sim}{A}) = \int_{x \in E} [u^2_{\underset{\sim}{A}}(x)/x]$$

Dilation: The dilation of $\underset{\sim}{A}$ is given by:

$$DIL(\underset{\sim}{A}) = \int_{x \varepsilon E} (u_{\underset{\sim}{A}}^{.5}(x)/x)$$

Contrast Intensification: The contrast intensification of $\underset{\sim}{A}$ is given by:

$$INT(\underset{\sim}{A}) = \begin{cases} CON(\underset{\sim}{A}) & , x \text{ such that } u_{\underset{\sim}{A}}(x) < 0.5 \\ DIL(\underset{\sim}{A}) & , x \text{ such that } u_{\underset{\sim}{A}}(x) \geq 0.5 \end{cases}$$

This has the property of increasing membership if greater than 0.5 and decreasing membership if less than 0.5.

Blurring: The blurring of a fuzzy subset $\underset{\sim}{A}$ is given by:

$$BLR(\underset{\sim}{A}) = \begin{cases} DIL(\underset{\sim}{A}) & , x \text{ such that } u_{\underset{\sim}{A}}(x) < 0.5 \\ CON(\underset{\sim}{A}) & , x \text{ such that } u_{\underset{\sim}{A}}(x) \geq 0.5 \end{cases}$$

This has the property of decreasing membership if greater than 0.5 and increasing membership if less than 0.5. The level of 0.5 is chosen arbitrarily.

Fuzzy Relations: Let x be an element of set E_1 and y be an element of set E_2. Then, the set of ordered pairs (x,y) defined their Cartesian product set $E_1 X E_2$. A fuzzy relation $\underset{\sim}{R}$ between E_1 and E_2 is a fuzzy subset of their Cartesian product $E_1 x E_2$ Mathematically, a fuzzy relation is defined by:

$$\underset{\sim}{R} = \int_{(x,y) \varepsilon E_1 x E_2} (u_{\underset{\sim}{R}}(x,y)/(x,y))$$

where

$u_{\underset{\sim}{R}}(x,y)$ is the membership function of the relation $\underset{\sim}{R}$.

<u>Composition of two Fuzzy Relations</u>: Let $x \varepsilon E_1$, $y \varepsilon E_2$, $z \varepsilon E_3$ and let $\underset{\sim}{R}_1$ and $\underset{\sim}{R}_2$ be two fuzzy relations on $E_1 x E_2$ and $E_2 x E_3$, respectively. Then, the max-min composition of $\underset{\sim}{R}_1$ and $\underset{\sim}{R}_2$ (denoted as $\underset{\sim}{R}_1 \circ \underset{\sim}{R}_2$) is a fuzzy relation on $E_1 x E_3$. The max-min composition is given by:

$$\underset{\sim}{R}_1 \circ \underset{\sim}{R}_2 = \int_{(x,z) \varepsilon E_1 x E_3} (u_{\underset{\sim}{R}_1 \circ \underset{\sim}{R}_2}(x,z)/(x,z)),$$

where

$$u_{\underset{\sim}{R}_1 \circ \underset{\sim}{R}_2}(x,z) = \bigvee_{y \varepsilon E_2} (u_{\underset{\sim}{R}_1}(x,y) \bigwedge u_{\underset{\sim}{R}_2}(y,z))$$

The max-product composition is given by:

$$\underset{\sim}{R}_1 \cdot \underset{\sim}{R}_2 = \int_{(x,z) \varepsilon E_1 x E_2} (u_{\underset{\sim}{R}_1 \cdot \underset{\sim}{R}_2}(x,z)/(x,z))$$

where

$$u_{\underset{\sim}{R}_1 \cdot \underset{\sim}{R}_2}(x,z) = \bigvee_{y \varepsilon E_2} (u_{\underset{\sim}{R}_1}(x,y) \cdot u_{\underset{\sim}{R}_2}(y,z))$$

A relation $\underset{\sim}{R}$ is said to be:

Reflexive if $u_{\underset{\sim}{R}}(x,x) = 1$, $\forall x \varepsilon E X E$;

Antireflexive if $u_{\underset{\sim}{R}}(x,x) = 0$, $\forall x \varepsilon E X E$;

Symmetric if $u_{\underset{\sim}{R}}(x,y) = u_{\underset{\sim}{R}}(y,x)$, $\forall (x,y) \varepsilon E X E$;

Antisymmetric if $u_{\underset{\sim}{R}}(x,y) \neq u_{\underset{\sim}{R}}(y,x)$, $\forall (x,y) \varepsilon E X E$;

Max-Min transitivity if $u_{\underset{\sim}{R}}(x,z) \geq \max_{y \varepsilon E} \min (u_{\underset{\sim}{R}_1}(x,y), u_{\underset{\sim}{R}_2}(y,z))$, $\forall x,y,z, \varepsilon E$;

Min-Max transitivity if $u_{\underset{\sim}{R}}(x,z) \geq \min_{y \varepsilon E} \max (u_{\underset{\sim}{R}_1}(x,y), u_{\underset{\sim}{R}_2}(y,z))$, $\forall x,y,z, \varepsilon E$.

A fuzzy relation is said to be a pre-order if it is reflexive and has max-min transitivity. It is a similarity relation if it is reflexive, symmetric and has a max-min transitivity. It is a resemblance relation if it is reflexive and symmetric.

Appendix I to Chapter Six

Computer Programs

```
PROGRAM NAME:        FUZZY-9
PROGRAMMER:          AWNI M. ZEBDA

DECTIONARY OF VARIABLES:
------------------------

VB(I,J,L)       =    MATRIX TO STORE THE VALUE OF THE BENEFITS
CB(I,J,L)       =    MATRIX TO STORE THE COMPATABILITY OF THE
                     BENEFITS
VBD(J,I,L)      =    MATRIX TO STORE THE BENEFITS ASSOCIATED WITH
                     THE DECISIONS
CBD(J,I,L)      =    MATRIX TO STORE THE COMPATABILITY OF THE
                     BENIFITS ASSOCIATED WITH THE DECISIONS
VARC(I)         =    VECTOR TO STORE THE COMPATABILITY OF THE
                     VARIANCE WITH THE INPUT STATES
CBDMAX(J,I,L)=       MATRIX TO STORE THE COMPATATABILITY OF THE
                     BENEFITS IN THE MAXIMIZING SETS ASSOCIATED
                     WITH THE DECISIONS
CBDD(J,I,L)     =    MATRIX TO STORE THE RESULT OF THE INTERSECTION
                     OF CBD AND CBDMAX
CBMAXJ(J)       =    VECTOR TO STORE THE COMPATABILITY OF THE
                     DECISIONS WITH THE OPTIMAL DECISION SET
I               =    AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT
                     TO REPRESENT A PARTICULAR INPUT STATE
J               =    AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT
                     TO REPRESENT A PARTICULAR DECISION
L               =    AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT
                     TO REPRESENT A PARTICULAR ELEMENT IN A FUZZY SET
------------------------------------------------------------------------
```

```
                        C
                        C      PROGRAM LISTING:
                        C      ----------------
                        C
                        C      DIMENSION:
                        C      ----------
ISN 0002                       DIMENSION VB(3,3,5),CB(3,3,5),VBD(3,3,5),CBD(3,3,5)
ISN 0003                       DIMENSION VARC(3),CBDMAX(3,3,5),CBDD(3,3,5),CBMAXJ(3)
                        C
                        C      READ IN DATA:
                        C      -------------
                        C
ISN 0004                       READ(5,10)NSTAT,NDEC,NELEM
                        C      WRITE(6,1000)NSTAT,NDEC,NELEM
                        C1000  FORMAT(5X,'NSTAT=',I3,5X,'NDEC=',I3,5X,'NELEM=',I3)
ISN 0005                   10  FORMAT(3I2)
ISN 0006                       READ(5,20) (VARC(I),I=1,NSTAT)
                        C      WRITE(6,1001) (VARC(I),I=1,NSTAT)
                        C1001  FORMAT(////1H ,'VARC',//,3F6.3)
ISN 0007                   20  FORMAT(3F3.1)
ISN 0008                       DO 30 J=1,NDEC
ISN 0009                       DO 40 I=1,NSTAT
ISN 0010                       READ(5,50)((VB(J,I,L),L=1,NELEM),(CB(J,I,L),L=1,NELEM))
                        C      WRITE(6,1002)J,I,(VB(J,I,L),L=1,NELEM)
                        C      WRITE(6,1002)J,I,(CB(J,I,L),L=1,NELEM)
ISN 0011                   40  CONTINUE
ISN 0012                   30  CONTINUE
                        C1002  FORMAT(////,4X,'I=',I3,2X,'J=',I3,5F6.2)
ISN 0013                   50  FORMAT(5F6.2,5F3.1)
                        C
                        C      OBTAINING BENEFITS ASSOCIATED WITH DECISIONS:
                        C      ---------------------------------------------
                        C
ISN 0014                       DO 60 J=1,NDEC
ISN 0015                       DO 70 I=1,NSTAT
ISN 0016                       DO 80 L=1,NELEM
```

```
ISN 0017          VBD(J,I,L)=VB(J,I,L)
ISN 0018          CBD(J,I,L)=VARC(I)*CB(J,I,L)
ISN 0019       80 CONTINUE
ISN 0020       70 CONTINUE
ISN 0021       60 CONTINUE
ISN 0022          KKK=1
ISN 0023          KK=1
ISN 0024          K=1
ISN 0025       90 TEMP=VBD(KKK,KK,K)
ISN 0026          DO 100 I=1,NSTAT
ISN 0027          DO 110 L=1,NELEM
ISN 0028          IF(VBD(KKK,I,L).NE.TEMP) GO TO 120
ISN 0030          CBD(KKK,I,L)=AMAX1(CBD(KKK,I,L),CBD(KKK,I,K))
ISN 0031          CBD(KKK,I,K)=CBD(KKK,I,L)
ISN 0032      120 CONTINUE
ISN 0033      110 CONTINUE
ISN 0034      100 CONTINUE
ISN 0035          K=K+1
ISN 0036          IF(K.LE.NELEM) GO TO 90
ISN 0038          KK=KK+1
ISN 0039          IF(KK.LE.NSTAT) GO TO 90
ISN 0041          KKK=KKK+1
ISN 0042          IF(KKK.LE.NDEC) GO TO 90
C
C                 OBTAINING THE MAXIMUM BENEFIT , (B MAX):
C                 ------------------------------------------
C
ISN 0044          BMAX=0.0
ISN 0045          DO 130 J=1,NDEC
ISN 0046          DO 140 I=1,NSTAT
ISN 0047          DO 150 L=1,NELEM
ISN 0048          IF(VBD(J,I,L).GT.BMAX) BMAX=VBD(J,I,L)
ISN 0050      150 CONTINUE
ISN 0051      140 CONTINUE
ISN 0052      130 CONTINUE
C
C                 OBTAINING THE MAXIMIZING SET,B(JM), AND THE SET B(JD):
C                 --------------------------------------------------------
C
```

```
      DO 160 J=1,NDEC
      DO 170 I=1,NSTAT
      DO 180 L=1,NELEM
      CBDMAX(J,I,L)=(VBD(J,I,L)/BMAX)**2
      CBDO(J,I,L)=AMIN1(CBDMAX(J,I,L),CBD(J,I,L))
  180 CONTINUE
  170 CONTINUE
  160 CONTINUE
C
C     OBTAINING THE OPTIMAL DECISION SET AND THE OPTIMAL DECISION:
C     ------------------------------------------------------------
C
      HCOMP=0.0
      WRITE(6,220)
      DO 190 J=1,NDEC
      CBMAXJ(J)=0.0
      DO 200 I=1,NSTAT
      DO 210 L=1,NELEM
      IF(CBDO(J,I,L).GT.CBMAXJ(J)) CBMAXJ(J)=CBDO(J,I,L)
  210 CONTINUE
  200 CONTINUE
      IF(CBMAXJ(J).GT.HCOMP) JJ=J
      IF(CBMAXJ(J).GT.HCOMP) HCOMP=CBMAXJ(J)
      WRITE(6,221) J,CBMAXJ(J)
  190 CONTINUE
      WRITE(6,230) JJ
  220 FORMAT(////,10X,'OPTIMAL DECISION SET ',//,9X,'------------------
     *---',//,13X,'DEC.',5X,'COMP.',//)
  221 FORMAT(12X,I3,4X,F6.3)
  230 FORMAT(////,10X,'OPTIMAL DECISION',3X,'--->',I2)
      STOP
      END
```

OPTIMAL DECISION SET

DEC.	COMP.
1	0.600
2	0.600
3	0.090

OPTIMAL DECISION ---> 1 OR 2

PROGRAM NAME: FUZZY-10
PROGRAMMER: AWNI M. ZEBDA

DECTIONARY OF VARIABLES:

```
VB(J,I,K,L)          = MATRIX TO STORE THE VALUE OF THE BENEFITS
CB(J,I,K,L)          = MATRIX TO STORE THE COMPATABILITY OF THE
                       BENEFITS
VQ(J,I,K,L)          = MATRIX TO STORE THE VALUE OF THE
                       PROBABILITIES
CQ(J,I,K,L)          = MATRIX TO STORE THE COMPATABILITY OF THE
                       PROBABILITIES
VBQ(J,I,K,L,L)       = MATRIX TO STORE THE RESULT OF MULTIPLYING
                       VB BY VQ
CBQ(J,I,K,L,L)       = MATRIX TO STORE THE COMPATABILITY OF THE
                       RESULT OF MULTIPLYING VB BY VQ
VEB(J,I,K,L**K)      = MATRIX TO STORE THE EXPECTED BENEFITS
CEB(J,I,K,L**K)      = MATRIX TO STORE THE COMPATABILITY OF THE
                       EXPECTED BENEFITS
VEBD(J,I,K,L**K)     = MATRIX TO STORE THE EXPECTED BENEFITS
                       ASSOCIATED WITH THE DECISIONS
CEBD(J,I,K,L**K)     = MATRIX TO STORE THE COMPATABILITY OF THE
                       EXPECTED BENEFITS ASSOCIATED WITH THE
                       DECISIONS
CEBDMX(J,I,K,L**K)   = MATRIX TO STORE THE COMPATABILITY OF THE
                       EXPECTED BENEFITS IN THE MAXIMIZING SETS
                       ASSOCIATED WITH THE DECISIONS
CEBDO(J,I,K,L**K)    = MATRIX TO STORE THE RESULT OF THE INTERSECTION
                       OF CEBD AND CEBDMX
CEBMXJ(J)            = VECTOR TO STORE THE COMPATABILITY OF THE
                       DECISIONS WITH THE OPTIMAL DECISION SET
VARC(I)              = VECTOR TO STORE THE COMPATABILITY OF THE
                       VARIANCE WITH THE INPUT STATES
I                    = AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT
                       TO REPRESENT A PARTICULAR INPUT STATE
J                    = AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT
                       TO REPRESENT A PARTICULAR DECISION
K                    = AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT
                       TO REPRESENT A PARTICULAR OUTPUT STATE
L                    = AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT
                       TO REPRESENT A PARTICULAR ELEMENT IN A FUZZY
                       SET
```

```
                C
                C       PROGRAM LISTING:
                C       ----------------
                C
                C       DIMENSION:
                C       ----------
                C
ISN 0002                DIMENSION VB(3,3,3,5),CB(3,3,3,5),VQ(3,3,3,5),CQ(3,3,3,5)
ISN 0003                DIMENSION VBQ(3,3,3,5,5),CBQ(3,3,3,5,5),VEB(3,3,5,125)
ISN 0004                DIMENSION VEBD(3,3,5,125),CEBD(3,3,5,125),CEB(3,3,5,125)
ISN 0005                DIMENSION CEBDMX(3,3,5,125),CEBDO(3,3,5,125)
ISN 0006                DIMENSION VARC(3),CEBMXJ(3),KD(3)
ISN 0007                DATA KD/3*0/
                C
                C       READ IN DATA:
                C       -------------
                C
ISN 0008                READ(5,10)NSTAT,NDEC,NOSTAT,NELEM
ISN 0009             10 FORMAT(4I2)
ISN 0010                READ(5,20) (VARC(I),I=1,NSTAT)
ISN 0011             20 FORMAT(2F3.1)
ISN 0012                DO 30 J=1,NDEC
ISN 0013                DO 35 I=1,NSTAT
ISN 0014                DO 40 K=1,NOSTAT
ISN 0015                READ(5,45)((VB(J,I,K,L),L=1,NELEM),(CB(J,I,K,L),L=1,NELEM))
                C       WRITE(6,1002)J,I,K,(VB(J,I,K,L),L=1,NELEM)
                C       WRITE(6,1002)J,I,K,(CB(J,I,K,L),L=1,NELEM)
ISN 0016             40 CONTINUE
ISN 0017             35 CONTINUE
ISN 0018             30 CONTINUE
                C1002 FORMAT(////,4X,'I=',I3,2X,'J=',I3,3X,'K=',I3,5F8.2)
ISN 0019             45 FORMAT(5F8.2,5F3.1)
ISN 0020                DO 50 J=1,NDEC
ISN 0021                DO 55 I=1,NSTAT
ISN 0022                DO 60 K=1,NOSTAT
ISN 0023                READ(5,65)((VQ(J,I,K,L),L=1,NELEM),(CQ(J,I,K,L),L=1,NELEM))
                C       WRITE(6,1002)J,I,K,(VQ(J,I,K,L),L=1,NELEM)
                C       WRITE(6,1002)J,I,K,(CQ(J,I,K,L),L=1,NELEM)
ISN 0024             60 CONTINUE
ISN 0025             55 CONTINUE
ISN 0026             50 CONTINUE
ISN 0027             65 FORMAT(5F8.2,5F3.1)
```

```
                     C
                     C       MULTIPLYING THE BENEFITS BY THE PROBABILITIES:
                     C       ------------------------------------------------
                     C
ISN 0028                     DO 70 J=1,NDEC
ISN 0029                     M=1
ISN 0030                     DO 80 I=1,NSTAT
ISN 0031                     M=1
ISN 0032                     DO 90 K=1,NOSTAT
ISN 0033                     M=1
ISN 0034                 100 CONTINUE
ISN 0035                     DO 110 L=1,NELEM
ISN 0036                     VBQ(J,I,K,M,L)=VQ(J,I,K,M)*VB(J,I,K,L)
ISN 0037                     CBQ(J,I,K,M,L)=AMIN1(CQ(J,I,K,M),CB(J,I,K,L))
                     C       WRITE(6,998) J,I,K,M,L,VBQ(J,I,K,M,L)
                     C   998 FORMAT(/,10X,'VBQ ...',/,3X,'J=',I3,2X,'I=',I3,2X,'K=',I3,2X,
                     C      * 'M=',I3,2X,'L=',I3,2X,5F8.2)
                     C   998 FORMAT(5X,'VEB ...',3X,'J=',I3,2X,'I=',I3,2X,'N4=',
                     C      *I3,2X,'L=',I3,2X,2F8.2)
ISN 0038                 110 CONTINUE
ISN 0039                     M=M+1
ISN 0040                     IF(M.LE.NELEM) GO TO 100
ISN 0042                  90 CONTINUE
ISN 0043                  80 CONTINUE
ISN 0044                  70 CONTINUE
                     C
                     C       OBTAINING EXPECTED BENEFITS ASSOCIATED WITH DECISIONS:
                     C       -------------------------------------------------------
                     C
ISN 0045                     DO 120 J=1,NDEC
ISN 0046                     K=1
ISN 0047                     DO 120 I=1,NSTAT
ISN 0048                     K=1
ISN 0049                     DO 120 N4=1,NELEM
ISN 0050                     K=1
ISN 0051                     DO 120 N3=1,NELEM
ISN 0052                     DO 120 N2=1,NELEM
ISN 0053                     DO 120 N1=1,NELEM
```

```
ISN 0054          M=1
ISN 0055          MM=2
ISN 0056          MMM=3
ISN 0057          A1=VBQ(J,I,M,N4,N3)
ISN 0058          A2=VBQ(J,I,MM,N4,N2)
ISN 0059          A3=VBQ(J,I,MMM,N4,N1)
ISN 0060          VEB(J,I,N4,K)=A1+A2+A3
ISN 0061          GO TO (2,3),NOSTAT
ISN 0062        3 CONTINUE
ISN 0063          TEMP1=AMIN1(CBQ(J,I,M,N4,N3),CBQ(J,I,MM,N4,N2))
ISN 0064          CEB(J,I,N4,K)=AMIN1(CBQ(J,I,MMM,N4,N1),TEMP1)
ISN 0065        2 CEB(J,I,N4,K)=TEMP1
          C       WRITE(6,998) J,I,N4,K,VEB(J,I,N4,K),CEB(J,I,N4,K)
ISN 0066          K=K+1
ISN 0067      120 CONTINUE
ISN 0068          L3=NELEM**3
ISN 0069          DO 180 J=1,NDEC
ISN 0070          DO 180 I=1,NSTAT
ISN 0071          DO 180 N4=1,NELEM
ISN 0072          DO 180 L=1,L3
ISN 0073          VEBD(J,I,N4,L)=VEB(J,I,N4,L)
ISN 0074          CEBD(J,I,N4,L)=VARC(I)*CEB(J,I,N4,L)
          C       WRITE(6,1003)J,I,N4,L,VEBD(J,I,N4,L),CEBD(J,I,N4,L)
ISN 0075      180 CONTINUE
          C1003   FORMAT(5X,'I=',I3,2X,'J=',I3,3X,'N4=',I3,3X,'L=',I3,3X,
          C      *2F6.2)
ISN 0076          M1=1
ISN 0077          M2=1
ISN 0078          M3=1
ISN 0079          M4=1
ISN 0080      220 TEMP=VEBD(M4,M3,M2,M1)
ISN 0081          DO 230 I=1,NSTAT
ISN 0082          DO 230 N4=1,NELEM
ISN 0083          DO 230 L=1,L3
ISN 0084          IF(VEBD(M4,I,N4,L).NE.TEMP) GO TO 260
ISN 0086          CEBD(M4,I,N4,L)=AMAX1(CEBD(M4,I,N4,L),CEBD(M4,1,N4,M1))
ISN 0087          CEBD(M4,M3,M2,M1)=CEBD(M4,I,N4,L)
ISN 0088      260 CONTINUE
ISN 0089      230 CONTINUE
```

```
                 C
                 C       OBTAINING THE MAXIMUM BENEFIT (B MAX):
                 C       ----------------------------------------
                 C
ISN 0090                 M1=M1+1
ISN 0091                 IF(M1.LE.L3) GO TO 220
ISN 0093                 M2=M2+1
ISN 0094                 IF(M2.LE.NELEM) GO TO 220
ISN 0096                 M3=M3+1
ISN 0097                 IF(M3.LE.NSTAT) GO TO 220
ISN 0099                 M4=M4+1
ISN 0100                 IF(M4.LE.NDEC) GO TO 220
ISN 0102                 BMAX=0.0
ISN 0103                 DO 270 J=1,NDEC
ISN 0104                 DO 270 I=1,NSTAT
ISN 0105                 DO 270 N4=1,NELEM
ISN 0106                 DO 270 L=1,L3
ISN 0107                 IF(VEBD(J,I,N4,L).GT.BMAX) BMAX=VEBD(J,I,N4,L)
ISN 0109             270 CONTINUE
ISN 0110                 WRITE(6,999)BMAX
ISN 0111             999 FORMAT(//,10X,'  BMAX=',F8.3)
                 C
                 C       OBTAINING THE MAXIMIZING SET, EB(JM), AND THE SET EB(JO):
                 C       ------------------------------------------------------------
                 C
ISN 0112                 DO 310 J=1,NDEC
ISN 0113                 DO 310 I=1,NSTAT
ISN 0114                 DO 310 N4=1,NELEM
ISN 0115                 DO 310 L=1,L3
ISN 0116                 CEBDMX(J,I,N4,L)=(VEBD(J,I,N4,L)/BMAX)**2
ISN 0117                 CEBDO(J,I,N4,L)=AMIN1(CEBDMX(J,I,N4,L),CEBD(J,I,N4,L))
ISN 0118             310 CONTINUE
                 C
                 C       OBTAINING THE OPTIMAL DECISION SET AND THE OPTIMAL DECISION:
                 C       ---------------------------------------------------------------
                 C
ISN 0119                 HCOMP=0.0
ISN 0120                 WRITE(6,400)
ISN 0121                 DO 350 J=1,NDEC
```

```
ISN 0122          CEBMXJ(J)=0.0
ISN 0123          DO 360 I=1,NSTAT
ISN 0124          DO 360 N4=1,NELEM
ISN 0125          DO 360 L=1,L3
ISN 0126          BZ=CEBDO(J,I,N4,L)
ISN 0127          IF(CEBDO(J,I,N4,L).GT.CEBMXJ(J)) CEBMXJ(J)=BZ
ISN 0129      360 CONTINUE
ISN 0130          KD(J)=J
ISN 0131          IF(CEBMXJ(J).GT.HCOMP) JJ=J
ISN 0133          IF(CEBMXJ(J).GT.HCOMP) HCOMP=CEBMXJ(J)
ISN 0135          WRITE(6,390) J,CEBMXJ(J)
ISN 0136          IF(KD(J).NE.JJ) KK=KD(J)
ISN 0138      350 CONTINUE
ISN 0139          GO TO (1),JJ
             C    IF(JJ.EQ.2) GO TO 7
ISN 0140        1 WRITE(6,420)   JJ
             C    7 WRITE(6,420)   JJ
             C    WRITE(6,421)   JJ,KK
ISN 0141      400 FORMAT(//,10X,'OPTIMAL DECISION SET ',//,9X,'--------------------
                 *--',//,13X,'DEC.',5X,'COMP.',//)
ISN 0142      390 FORMAT(12X,I3,6X,F6.3)
             C 421 FORMAT(//,10X,'OPTIMAL DECISION',3X,'--->',2X,I2,2X,'OR',2X,I2)
ISN 0143      420 FORMAT(//,10X,'THE OPTIMAL DECISION IS',3X,'--->',5X,'   (   ',I2,'
                 *   )')
ISN 0144          STOP
ISN 0145          END
```

OPTIMAL DECISION SET

DEC.	COMP.
1	0.578
2	0.640

THE OPTIMAL DECISION IS ---> (2)

PROGRAM NAME: FUZZY-11
PROGRAMMER: AWNI M. ZEBDA

DECTIONARY OF VARIABLES:

VB(J,I,K,L)	=	MATRIX TO STORE THE VALUE OF THE BENEFITS
CB(J,I,K,L)	=	MATRIX TO STORE THE COMPATABILITY OF THE BENEFITS
RC(J,I,K)	=	MATRIX TO STORE THE FUZZY TRANSFORMATION FUNCTION
VARC(I)	=	VECTOR TO STORE THE COMPATABILITY OF THE VARIANCE WITH THE INPUT STATES
VBD(J,I,K,L)	=	MATRIX TO STORE THE NET BENEFITS ASSOCIATED WITH THE DECISIONS
CBD(J,I,K,L)	=	MATRIX TO STORE THE COMPATABILITY OF THE NET BENEFITS ASSOCIATED WITH THE DECISIONS
CBDMAX(J,I,K,L)	=	MATRIX TO STORE THE COMPATABILITY OF THE BENEFITS IN THE MAXIMIZING SETS ASSOCIATED WITH THE DECISIONS
CBDO(J,I,K,L)	=	MATRIX TO STORE THE INTERSECTION OF CBD AND CBDMAX
CBDMXJ(J)	=	VECTOR TO STORE THE COMPATABILITY OF THE DECISIONS WITH THE OPTIMAL DECISION SET
TEMP(J,I,K)	=	MATRIX TO STORE THE RESULT OF FINDING THE MINIMUM OF RC AND VARC
ROSDC(J,K)	=	MATRIX TO STORE THE FUZZY TRANSFORMATION FUNCTION GIVEN VARC
I	=	AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT TO REPRESENT A PARTICULAR INPUT STATE
J	=	AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT TO REPRESENT A PARTICULAR DECISION
K	=	AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT TO REPRESENT A PARTICULAR OUTPUT STATE
L	=	AN INDEX USED AS A COUNTER, AND AS A SUBSCRIPT TO REPRESENT A PARTICULAR ELEMENT IN A FUZZY SET

```
              C      DIMENSION:
              C      ---------
ISN 0002             DIMENSION VB(3,3,3,5),CB(3,3,3,5),VBD(3,3,3,5),CBD(3,3,3,5)
ISN 0003             DIMENSION VARC(3),CBDMAX(3,3,3,5),CBDO(3,3,3,5),CBMAXJ(3)
ISN 0004             DIMENSION RC(3,3,3),ROSDC(3,3),TEMP(3,3,3)
              C
              C      READ IN DATA:
              C      -------------
              C
ISN 0005             READ(5,10)NSTAT,NDEC,NOSTAT,NELEM
              C      WRITE(6,1000)NSTAT,NDEC,NOSTAT,NELEM
              C1000  FORMAT(5X,'NSTAT=',I3,5X,'NDEC=',I3,5X,'NOSTAT=',I3,5X,'NELEM=',I3
ISN 0006          10 FORMAT(4I2)
ISN 0007             READ(5,20) (VARC(I),I=1,NSTAT)
              C      WRITE(6,1001) (VARC(I),I=1,NSTAT)
              C1001  FORMAT(////1H ,'VARC',//,3F6.3)
ISN 0008          20 FORMAT(3F3.1)
              C
ISN 0009             DO 30 J=1,NDEC
ISN 0010             DO 40 I=1,NSTAT
ISN 0011             DO 45 K=1,NOSTAT
ISN 0012             READ(5,50)((VB(J,I,K,L),L=1,NELEM),(CB(J,I,K,L),L=1,NELEM))
              C      WRITE(6,1002)J,I,K,(VB(J,I,K,L),L=1,NELEM)
              C      WRITE(6,1002)J,I,K,(CB(J,I,K,L),L=1,NELEM)
ISN 0013          45 CONTINUE
ISN 0014          40 CONTINUE
ISN 0015          30 CONTINUE
              C1002  FORMAT(////,4X,'I=',I3,2X,'J=',I3,2X,'K=',I3,2X,5F8.2)
ISN 0016          50 FORMAT(5F8.2,5F3.1)
ISN 0017             DO 51 J=1,NDEC
ISN 0018             DO 52 I=1,NSTAT
ISN 0019             READ(5,53)(RC(J,I,K),K=1,NOSTAT)
              C      WRITE(6,1001)J,I,(RC(J,I,K),K=1,NOSTAT)
ISN 0020          52 CONTINUE
ISN 0021          51 CONTINUE
ISN 0022          53 FORMAT(3F4.2)
              C1001  FORMAT(////,4X,'J=',I3,2X,'I=',I3,4X,3F8.2)
```

```
                  C
                  C      OBTAINING BENEFITS ASSOCIATED WITH DECISIONS:
                  C      ----------------------------------------------
                  C
ISN 0023                 DO 54  J=1,NDEC
ISN 0024                 DO 55  K=1,NOSTAT
ISN 0025                 DO 56  I=1,NSTAT
ISN 0026                 TEMP(J,I,K)=AMIN1(VARC(I),RC(J,I,K))
ISN 0027                 WRITE(6,1003)J,I,K,TEMP(J,I,K)
ISN 0028              56 CONTINUE
ISN 0029              55 CONTINUE
ISN 0030              54 CONTINUE
ISN 0031            1003 FORMAT(////,4X,'J=',I3,2X,'I=',I3,2X,'K=',I3,4X,F8.2)
ISN 0032                 DO 57  J=1,NDEC
ISN 0033                 DO 58  K=1,NOSTAT
ISN 0034                 RODC1=0.0
ISN 0035                 DO 59  I=1,NSTAT
ISN 0036                 IF(TEMP(J,I,K).GT.RODC1) RODC1=TEMP(J,I,K)
ISN 0038              59 CONTINUE
ISN 0039                 ROSDC(J,K)=RODC1
ISN 0040                 WRITE(6,1004)J,K,ROSDC(J,K)
ISN 0041              58 CONTINUE
ISN 0042              57 CONTINUE
ISN 0043            1004 FORMAT(////,4X,'J=',I3,2X,'K=',I3,4X,F8.2)
ISN 0044                 DO 60  J=1,NDEC
ISN 0045                 DO 70  I=1,NSTAT
ISN 0046                 DO 75  K=1,NOSTAT
ISN 0047                 DO 80  L=1,NELEM
ISN 0048                 VBD(J,I,K,L)=VB(J,I,K,L)
ISN 0049                 CBD(J,I,K,L)=ROSDC(J,K)*CB(J,I,K,L)
                  C      WRITE(6,1002)J,I,VBD(J,I,L)
                  C      WRITE(6,1002)J,I,CBD(J,I,L)
ISN 0050              80 CONTINUE
ISN 0051              75 CONTINUE
ISN 0052              70 CONTINUE
ISN 0053              60 CONTINUE
ISN 0054                 M1=1
ISN 0055                 M2=1
ISN 0056                 M3=1
ISN 0057                 M4=1
```

```
ISN 0058        90 TEMP1=VBD(M4,M3,M2,M1)
ISN 0059           DO 100 I=1,NSTAT
ISN 0060           DO 105 K=1,NOSTAT
ISN 0061           DO 110 L=1,NELEM
ISN 0062           IF(VBD(M4,I,K,L).NE.TEMP1) GO TO 120
ISN 0064           CBD(M4,I,K,L)=AMAX1(CBD(M4,I,K,L),CBD(M4,I,K,M1))
ISN 0065           CBD(M4,M3,M2,M1)=CBD(M4,I,K,L)
ISN 0066       120 CONTINUE
ISN 0067       110 CONTINUE
ISN 0068       105 CONTINUE
ISN 0069       100 CONTINUE
ISN 0070           M1=M1+1
ISN 0071           IF(M1.LE.NELEM) GO TO 90
ISN 0073           M2=M2+1
ISN 0074           IF(M2.LE.NOSTAT) GO TO 90
ISN 0076           M3=M3+1
ISN 0077           IF(M3.LE.NSTAT) GO TO 90
ISN 0079           M4=M4+1
ISN 0080           IF(M4.LE.NDEC) GO TO 90
              C
              C    OBTAINING THE MAXIMUM BENEFIT , (B MAX):
              C    ------------------------------------------
              C
ISN 0082           BMAX=0.0
ISN 0083           DO 130 J=1,NDEC
ISN 0084           DO 140 I=1,NSTAT
ISN 0085           DO 145 K=1,NOSTAT
ISN 0086           DO 150 L=1,NELEM
ISN 0087           IF(VBD(J,I,K,L).GT.BMAX) BMAX=VBD(J,I,K,L)
ISN 0089       150 CONTINUE
ISN 0090       145 CONTINUE
ISN 0091       140 CONTINUE
ISN 0092       130 CONTINUE
              C
```

```
                  C     OBTAINING THE MAXIMIZING SET,B(JM), AND THE SET B(JO):
                  C     -------------------------------------------------------
                  C
ISN 0093                DO 160 J=1,NDEC
ISN 0094                DO 170 I=1,NSTAT
ISN 0095                DO 175 K=1,NOSTAT
ISN 0096                DO 180 L=1,NELEM
ISN 0097                CBDMAX(J,I,K,L)=(VBD(J,I,K,L)/BMAX)**2
ISN 0098                CBDO(J,I,K,L)=AMIN1(CBDMAX(J,I,K,L),CBD(J,I,K,L))
ISN 0099            180 CONTINUE
ISN 0100            175 CONTINUE
ISN 0101            170 CONTINUE
ISN 0102            160 CONTINUE
                  C
                  C     OBTAINING THE OPTIMAL DECISION SET AND THE OPTIMAL DECISION:
                  C     ------------------------------------------------------------
                  C
ISN 0103                HCOMP=0.0
ISN 0104                WRITE(6,220)
ISN 0105                DO 190 J=1,NDEC
ISN 0106                CBMAXJ(J)=0.0
ISN 0107                DO 200 I=1,NSTAT
ISN 0108                DO 205 K=1,NOSTAT
ISN 0109                DO 210 L=1,NELEM
ISN 0110                IF(CBDO(J,I,K,L).GT.CBMAXJ(J)) CBMAXJ(J)=CBDO(J,I,K,L)
ISN 0112            210 CONTINUE
ISN 0113            205 CONTINUE
ISN 0114            200 CONTINUE
ISN 0115                IF(CBMAXJ(J).GT.HCOMP) JJ=J
ISN 0117                IF(CBMAXJ(J).GT.HCOMP) HCOMP=CBMAXJ(J)
ISN 0119                WRITE(6,221) J,CBMAXJ(J)
ISN 0120            190 CONTINUE
ISN 0121                WRITE(6,230) JJ
ISN 0122            220 FORMAT(////,10X,'OPTIMAL DECISION SET ',//,9X,'-------------------
                       *---',//,13X,'DEC.',5X,'COMP.',//)
ISN 0123            221 FORMAT(12X,I3,4X,F6.3)
ISN 0124            230 FORMAT(////,10X,'OPTIMAL DECISION',3X,'--->',I2)
ISN 0125                STOP
ISN 0126                END
```

```
OPTIMAL DECISION SET
--------------------
    DEC.      COMP.

     1       0.600
     2       0.500


OPTIMAL DECISION    ---> 1
```

APPENDIX II TO CHAPTER SIX

The Empirical Approach to Membership Functions

As indicated in Chapter Six, membership functions can be obtained either subjectively or empirically. The subjective approach was used in the application. In this appendix, empirical membership functions are derived for (1) the linguistic net benefits, and (2) the fuzzy transformation function. The derived functions are then used with the remaining data (presented in Chapter Six) to obtain the optimal investigation decisions for the 48 variances used in the application. The results are presented at the end of this appendix.

Empirical Membership Functions for the Net Benefits

The estimated net benefits--in linguistic form--for the different combinations of decisions, input states, and output states are presented in Figure 13, Chapter Six. The figure includes small (S), very small (VS), very very small (VVS), medium (M), large (L), very large (VL), and very very large (VVL) net benefits values. The empirical membership functions for these net benefits are presented in Figure 25. The figure also includes the corresponding subjective functions which were presented in Chapter Six.

The empirical functions for the linguistic net benefits were derived as follows. First, a set of possible net benefits values (B) was obtained. This was done as in Chapter Six where B was determined as:

$$B = \{1000, 2000, \ldots, 11000, 12000\}.$$

Empirical:

VVS = {(1/1000), (.9/2000), (.75/3000), (.5/4000)},

VS = {(.9/1000), (1/2000), (.85/3000), (.8/4000), (.45/5000)},

S = {(.9/1000), (1/2000), (1/3000), (.85/4000), (.75/5000), (.15/6000)},

M = {(.7/3000), (.75/4000), (.9/5000), (1/6000), (.9/7000), (.9/8000), (.8/9000)},

L = {(.3/6000), (.4/7000), (.8/8000), (.9/9000), (1/10000), (.95/11000), (.9/12000)},

VL = {(.75/7000), (.8/8000), (.9/9000), (.9/10000), (1/11000), (.95/12000)},

VVL = {(.1/7000), (.7/8000), (.8/9000), (.85/10000), (.9/11000), (1/12000)}.

Subjective:

VVS = {(1/1000), (.5/2000)}

VS = {(.7/1000), (1/2000), (.7/3000)}

S = {(.6/2000), (.9/3000), (1/4000), (.6/5000)}

M = {(.2/3000), (.5/4000), (.7/5000), (1/6000), (.7/7000), (.5/8000), (.2/9000)}

L = {(.6/6000), (.8/7000), (1/8000), (.8/9000), (.6/10000)}

VL = {(.6/8000), (.8/9000), (1/10000), (.8/11000), (.6/12000)}

VVL = {(.4/9000), (.6/10000), (.8/11000), (1/12000)}.

Figure 25. Membership Functions for Expected Net Benefits: Empirical vs. Subjective.

Second, compatibility functions for the linguistic net benefits were obtained.[1] This was achieved as follows. The controller was presented a sequence of cards (stimuli). On each card a certain linguistic value (such as "large net benefit") paired with a certain net benefit value (such as $3000) was recorded. Then, the controller was asked to answer the question of whether he thought that the linguistic value described the net benefit value. (He was instructed to use 'yes' if his answer was positive and 'no' if his answer was negative.) The controller was also asked to give a confidence rating for his answer; he was asked to put a mark on a scale from 1 to 5 with 5 indicating complete confidence and 1 indicating a pure guess.

Each linguistic value was paired twice with every net benefit value. That is, two cards including the same pair of linguistic net benefit and a certain net benefit value were presented to the controller. Consequently, the number of cards was 168 (12 possible net benefits × 7 possible linguistic net benefits × 2 cards for every combination of possible net benefits and possible linguistic value). The cards were presented in random.

---

[1] The method used in deriving the compatibility functions for the linguistic net benefits is based on the method suggested by Hersh and Caramazza for deriving empirical membership functions. See H. M. Hersh and A. Caramazza, "A Fuzzy Set Theory Approach to Modifiers and Vagueness in Natural Language", Journal of Experimental Psychology: General, Vol. 105 (1976), pp. 254-276. The method was also used in H. M. Hersh, A Caramazza, and H. H. Brownell, "Effects of Context on Fuzzy Membership Functions", in M. Gupta, R. Ragade, and R. Yager (eds.), Advances in Fuzzy Set Theory and Applications (Amsterdam: North-Holland Publishing Company, 1979), pp. 389-408.

For every card, the compatibility of the net benefit value with the linguistic net benefit was calculated by suing the following formula:

$$\text{Compatibility} = .5 + d(r/10), \qquad (1)$$

where

$d = 1$ (if answer was 'yes'),

$d = -1$ (if answer was 'no'),

and

r = confidence ratio $(1 \leq r \leq 5)$.[2]

For example, if the answer was 'yes' and the confidence rating was 5 (complete confidence), then the grade of membership is 1.0. On the other hand, if the answer was 'no' and r was 5, then the grade of membership is zero.

As indicated above, two cards containing the same pair of linguistic net benefit and net benefit value were presented to the controller. For each card, a grade of membership was calculated by Equation (1). The average of the two values (one for each card) was obtained. This average was used to represent the compatibility of the

---

[2] Hersh and Caramazza say, "Although there is no clear theoretical justification for this mapping, the resulting scale seemed to reflect the important conceptual ideas implicit in fuzzy set theory. The extremes of definite membership and nonmembership correspond to judgments that were rated highly confident. Likewise, judgments rated as purely guessing corresponded to the crossover points in a fuzzy set." H. M. Hersh and A. Caramazza, "A Fuzzy Set Approach to Modifiers and Vagueness in Natural Language", pp. 267-268. Hersh and Caramazza go on to say, "Inherent in this transformation is the assumption that equal confidence intervals reflect equal differential grades of membership. [The result of the study] tends to support this assumption," p. 268.

net benefit value (appearing on the two cards) with the accompanying linguistic net benefit. Figure 26 contains a summary of the controller's responses and the calculated compatibilities for the cards.

Empirical Membership Function for the Transformation Function

The empirical fuzzy transformation function is provided in Figure 27. (The figure also includes the corresponding subjective function.) The empirical function was derived as follows. The controller was presented a sequence of cards. Every card contained a combination of $s_i \in S_t$, $d_j \in D_t$, and $s_k \in S_{t+1}$. (For example, one card contained in-control as the input state, investigate as the decision, and out-of-control as the output state.) The controller was asked to respond with 'yes' or 'no' whether he thought that the output state would be s_k given that the current state was s_i and decision d_j was made. He was also asked to give a confidence rating for his response by putting a mark on a scale from one to five with the five indicating a complete confidence and the one representing a pure guess.

There was 18 combinations of input states, decisions, and output states (3 input states × 2 decisions × 3 output states). Every combination was presented twice to the controller. That is, the number of cards used was 36 cards. The cards were presented in random. A summary of the different combinations as well as the responses of the controller is provided in Figure 28.

For every card (i.e., for every combination) a grade of membership was calculated by using Equation (1). (The calculated grades of membership appear in the last solumn of Figure 28.) Moreover, for every two

Linguistic Net Benefit	Net Benefit Value	Answer	Confidence Rating	Membership Grade
VVS	$ 1000	yes	5	1.0
VVS	1000	yes	5	1.0
VVS	2000	yes	4	0.9
VVS	2000	yes	4	0.9
VVS	3000	yes	3	0.8
VVS	3000	yes	2	0.7
VVS	4000	yes	2	0.7
VVS	4000	no	2	0.3
VVS	5000	no	5	0.0
VVS	5000	no	5	0.0
VVS	6000	no	5	0.0
VVS	6000	no	5	0.0
VVS	7000	no	5	0.0
VVS	7000	no	5	0.0
VVS	8000	no	5	0.0
VVS	8000	no	5	0.0
VVS	9000	no	5	0.0
VVS	9000	no	5	0.0
VVS	10000	no	5	0.0
VVS	10000	no	5	0.0
VVS	11000	no	5	0.0
VVS	11000	no	5	0.0
VVS	12000	no	5	0.0
VVS	12000	no	5	0.0
VS	1000	yes	4	0.9
VS	1000	yes	4	0.9
VS	2000	yes	5	1.0
VS	2000	yes	5	1.0
VS	3000	yes	4	0.9
VS	3000	yes	3	0.8
VS	4000	yes	3	0.8
VS	4000	yes	3	0.8
VS	5000	yes	2	0.7
VS	5000	no	3	0.2
VS	6000	no	5	0.0
VS	6000	no	5	0.0
VS	7000	no	5	0.0
VS	7000	no	5	0.0

Figure 26. Summary of Data Used in Calculating Empirical Membership for the Linguistic Net Benefits.

Linguistic Net Benefit	Net Benefit Value	Answer	Confidence Rating	Membership Grade
VS	$ 8000	no	5	0.0
VS	8000	no	5	0.0
VS	9000	no	5	0.0
VS	9000	no	5	0.0
VS	10000	no	5	0.0
VS	10000	no	5	0.0
VS	11000	no	5	0.0
VS	11000	no	5	0.0
VS	12000	no	5	0.0
VS	12000	no	5	0.0
S	1000	yes	4	0.9
S	1000	yes	4	0.9
S	2000	yes	5	1.0
S	2000	yes	5	1.0
S	3000	yes	5	1.0
S	3000	yes	5	1.0
S	4000	yes	4	0.9
S	4000	yes	3	0.8
S	5000	yes	2	0.7
S	5000	yes	3	0.8
S	6000	no	3	0.2
S	6000	no	4	0.1
S	7000	no	5	0.0
S	7000	no	5	0.0
S	8000	no	5	0.0
S	8000	no	5	0.0
S	9000	no	5	0.0
S	9000	no	5	0.0
S	10000	no	5	0.0
S	10000	no	5	0.0
S	11000	no	5	0.0
S	11000	no	5	0.0
S	12000	no	5	0.0
S	12000	no	5	0.0
M	1000	no	5	0.0
M	1000	no	5	0.0
M	2000	no	5	0.0
M	2000	no	5	0.0

Figure 26 (Con't). Summary of Data Used in Calculating Empirical Membership for the Linguistic Net Benefits.

Linguistic Net Benefit	Net Benefit Value	Answer	Confidence Rating	Membership Grade
M	$ 3000	yes	2	0.7
M	3000	yes	2	0.7
M	4000	yes	3	0.8
M	4000	yes	2	0.7
M	5000	yes	4	0.9
M	5000	yes	4	0.9
M	6000	yes	5	1.0
M	6000	yes	5	1.0
M	7000	yes	4	0.9
M	7000	yes	4	0.9
M	8000	yes	4	0.9
M	8000	yes	4	0.9
M	9000	yes	3	0.8
M	9000	yes	3	0.8
M	10000	no	5	0.0
M	10000	no	5	0.0
M	11000	no	5	0.0
M	11000	no	5	0.0
M	12000	no	5	0.0
M	12000	no	5	0.0
L	1000	no	5	0.0
L	1000	no	5	0.0
L	2000	no	5	0.0
L	2000	no	5	0.0
L	3000	no	5	0.0
L	3000	no	5	0.0
L	4000	no	5	0.0
L	4000	no	5	0.0
L	5000	no	5	0.0
L	5000	no	5	0.0
L	6000	no	5	0.0
L	6000	yes	1	0.6
L	7000	no	5	0.0
L	7000	yes	3	0.8
L	8000	yes	3	0.8
L	8000	yes	3	0.8
L	9000	yes	4	0.9
L	9000	yes	4	0.9

Figure 26 (Con't). Summary of Data Used in Calculating Empirical Membership for the Linguistic Net Benefits.

Linguistic Net Benefit	Net Benefit Value	Answer	Confidence Rating	Membership Grade
L	$10000	yes	5	1.0
L	10000	yes	5	1.0
L	11000	yes	5	1.0
L	11000	yes	4	0.9
L	12000	yes	4	0.9
L	12000	yes	4	0.9
VL	1000	no	5	0.0
VL	1000	no	5	0.0
VL	2000	no	5	0.0
VL	2000	no	5	0.0
VL	3000	no	5	0.0
VL	3000	no	5	0.0
VL	4000	no	5	0.0
VL	4000	no	5	0.0
VL	5000	no	5	0.0
VL	5000	no	5	0.0
VL	6000	no	5	0.0
VL	6000	no	5	0.0
VL	7000	yes	2	0.7
VL	7000	yes	3	0.8
VL	8000	yes	3	0.8
VL	8000	yes	3	0.8
VL	9000	yes	4	0.9
VL	9000	yes	4	0.9
VL	10000	yes	4	0.9
VL	10000	yes	4	0.9
VL	11000	yes	5	1.0
VL	11000	yes	5	1.0
VL	12000	yes	4	0.9
VL	12000	yes	5	1.0
VVL	1000	no	5	0.0
VVL	1000	no	5	0.0
VVL	2000	no	5	0.0
VVL	2000	no	5	0.0
VVL	3000	no	5	0.0
VVL	3000	no	5	0.0
VVL	4000	no	5	0.0
VVL	4000	no	5	0.0

Figure 26 (Con't). Summary of Data Used in Calculating Empirical Membership for the Linguistic Net Benefits.

Linguistic Net Benefit	Net Benefit Value	Answer	Confidence Rating	Membership Grade
VVL	$ 5000	no	5	0.0
VVL	5000	no	5	0.0
VVL	6000	no	5	0.0
VVL	6000	no	5	0.0
VVL	7000	no	5	0.0
VVL	7000	no	3	0.2
VVL	8000	yes	2	0.7
VVL	8000	yes	2	0.7
VVL	9000	yes	3	0.8
VVL	9000	yes	3	0.8
VVL	10000	yes	4	0.9
VVL	10000	yes	3	0.8
VVL	11000	yes	4	0.9
VVL	11000	yes	4	0.9
VVL	12000	yes	5	1.0
VVL	12000	yes	5	1.0

Figure 26 (Con't). Summary of Data Used in Calculating Empirical Membership for the Linguistic Net Benefits.

Input State, $s_i \in S_t$ $\quad$ $s_i = s_1$ = in-control

Decision** $d_j \in D_t$	Output State, $s_k \in S^*_{t+1}$		
	s_1	s_2	s_3
d_1	1.0	0.1	0.0
d_2	0.9	0.1	0.1

Decision** $d_j \in D_t$	Output State, $s_k \in S^*_{t+1}$		
	s_1	s_2	s_3
d_1	.9	.4	.1
d_2	.8	.4	.3

Input State, $s_i \in S_t$ $\quad$ $s_i = s_2$ = more or less out-of-control

Decision** $d_j \in D_t$	Output State, $s_k \in S^*_{t+1}$		
	s_1	s_2	s_3
d_1	0.9	0.1	0.15
d_2	0.25	1.0	0.7

Decision** $d_j \in D_t$	Output State, $s_k \in S^*_{t+1}$		
	s_1	s_2	s_3
d_1	.8	.5	.2
d_2	.4	.8	.7

Input State, $s_i \in S_t$ $\quad$ $s_i = s_3$ = out-of-control

Decision** $d_j \in D_t$	Output State, $s_k \in S^*_{t+1}$		
	s_1	s_2	s_3
d_1	0.9	0.9	0.3
d_2	0.1	0.45	0.95

Decision** $d_j \in D_t$	Output State, $s_k \in S^*_{t+1}$		
	s_1	s_2	s_3
d_1	.7	.4	.3
d_2	.1	.6	.9

Empirical $\qquad\qquad$ Subjective

---

* s_1, s_2, and s_3 indicate in-control, more or less out-of-control, and out-of-control states, respectively.

** d_1 and d_2 indicate investigate and do-not investigate actions, respectively.

Figure 27. Fuzzy Transformation Function: Empirical vs. Subjective.

Current State[1]	Decision[2]	Output State[1]	Answer	Confidence Rating	Grade of Membership
IC	I	IC	yes	5	1.0
IC	I	IC	yes	5	1.0
IC	I	MOLOC	no	4	0.1
IC	I	MOLOC	no	4	0.1
IC	I	OC	no	5	0.0
IC	I	OC	no	5	0.0
IC	D	IC	yes	4	0.9
IC	D	IC	yes	4	0.9
IC	D	MOLOC	no	4	0.1
IC	D	MOLOC	no	4	0.1
IC	D	OC	no	4	0.1
IC	D	OC	no	4	0.1
MOLOC	I	IC	yes	4	0.9
MOLOC	I	IC	yes	4	0.9
MOLOC	I	MOLOC	no	4	0.1
MOLOC	I	MOLOC	no	4	0.1
MOLOC	I	OC	no	3	0.2
MOLOC	I	OC	no	4	0.1
MOLOC	D	IC	no	3	0.2
MOLOC	D	IC	no	2	0.3
MOLOC	D	MOLOC	yes	5	1.0
MOLOC	D	MOLOC	yes	5	1.0
MOLOC	D	OC	yes	2	0.7
MOLOC	D	OC	yes	2	0.7
OC	I	IC	yes	4	0.9
OC	I	IC	yes	4	0.9
OC	I	MOLOC	yes	4	0.9
OC	I	MOLOC	yes	4	0.9
OC	I	OC	no	2	0.3
OC	I	OC	no	2	0.3

[1] IC, MOLOC, and OC stand for in-control, more or less out-of-control, and out-of-control states, respectively.

[2] I and D stand for the investigate and do-not investigate actions, respectively.

[3] Calculated by using Equation (1).

Figure 28. Summary of Data Used in Calculating Empirical Membership Function for the Fuzzy Transformation Function.

Current State[1]	Decision[2]	Output State[1]	Answer	Confidence Rating	Grade of Membership
OC	D	IC	no	4	0.1
OC	D	IC	no	4	0.1
OC	D	MOLOC	no	4	0.1
OC	D	MOLOC	yes	3	0.8
OC	D	OC	yes	5	1.0
OC	D	OC	yes	4	0.9

[1] IC, MOLOC, and OC stand for in-control, more or less out-of-control, and out-of-control states, respectively.

[2] I and D stand for the investigate and do-not investigate actions, respectively.

[3] Calculated by using Equation (1).

Figure 28 (Con't). Summary of Data Used in Calculating Empirical Membership Function for the Fuzzy Transformation Function.

cards containing the same combination a grade of membership was calculated by finding the average of the two values calculated for the two cards. The average represented the compatibility of the combination in the transformation function appeared in Figure 27.

The empirical membership functions for the linguistic net benefits and transformation function were used with the data presented in Chapter Six (i.e., the variances, compatibilities of the variances with the different states, and expected net benefits for the different combinations of input states, decisions, and output states) and the computer program (Fuzzy 11) to obtain the optimal investigation decision for every variance of the 48 variances used in the study. The results are presented in Figure 29.

To provide insight into the sensitivity of the model to the shapes of the membership functions, the optimal decisions indicated by the model for the 48 variances (when the subjective approach for deriving compatibility functions is used) are also presented in Figure 29.

When the empirical membership functions for the linguistic net benefits and transformation function are compared to the subjective ones appeared in Chapter Six and reproduced in Figures 25 and 27, one can see that the two types of membership functions are quite different. (See Figures 25 and 27.) However, as can be seen from Figure 29, only four decisions (out of the 48 decisions) are reversed when empirical functions are used in place of the subjective functions. This may indicate that the proposed model is not very sensitive to the shapes of the membership functions. Further research, however, is needed before generalizing this finding.

Var. No.	Var. Amt.[1]	Compatibility with States			Decisions[2]		
		In-Control	More or less out of control	Out of Control	Empirical Approach	Subjective Approach	NR or R[3]
1	1987 U	.8	.6	.5	I or D	D	NR
2	9463 U	.9	.4	.2	D	D	NR
3	5609 U	.7	.7	.8	I	D	R
4	6837 U	.7	.7	.8	I	D	R
5	4962 U	.6	.8	.8	I	I	NR
6	7862 F	.7	.7	.7	I or D	D	NR
7	3881 U	.8	.7	.6	I or D	D	NR
8	4172 U	.5	.8	.9	I	I	NR
9	261 U	.9	.3	.2	D	D	NR
10	7706 U	.9	.3	.2	D	D	NR
11	4025 U	.7	.7	.7	I or D	D	NR
12	7017 U	.7	.7	.6	I or D	D	NR
13	5404 U	.6	.7	.8	I	I	NR
14	4023 F	.8	.6	.7	I or D	D	NR
15	4724 U	.6	.7	.7	I	I	NR
16	6029 U	.4	.8	.9	I	I	NR
17	411 U	.9	.4	.3	D	D	NR
18	6157 U	.8	.5	.4	I or D	D	NR
19	5542 U	.7	.6	.6	I or D	D	NR
20	5005 U	.7	.6	.6	I or D	D	NR
21	5523 U	.6	.7	.7	I	I	NR
22	4269 F	.8	.7	.7	I or D	D	NR
23	5329 U	.7	.8	.8	I	D	R
24	3476 U	.6	.8	.8	I	I	NR

[1] U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2] I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3] R and NR indicate that the decision was 'reversed' and 'not reversed', respectively.

Figure 29. Decisions for the 48 Variances: Empirical Approach vs. Subjective Approach.

Var. No.	Var. Amt.[1]	Compatibility with States: In-Control	Compatibility with States: More or less out of control	Compatibility with States: Out of Control	Decisions[2]: Empirical Approach	Decisions[2]: Subjective Approach	Decisions[2]: NR or R[3]
25	2567 U	.7	.5	.4	I or D	D	NR
26	8111 U	.8	.4	.3	I or D	D	NR
27	9014 U	.7	.5	.5	I or D	D	NR
28	10156 U	.7	.5	.5	I or D	D	NR
29	6656 U	.6	.5	.7	I	I	NR
30	838 U	.7	.5	.7	I or D	D	NR
31	9245 U	.6	.7	.7	I	I	NR
32	8453 U	.5	.8	.9	I	I	NR
33	1623 U	.8	.4	.4	I or D	D	NR
34	4819 U	.9	.5	.3	D	D	NR
35	7536 U	.7	.6	.5	I or D	D	NR
36	8800 U	.7	.6	.5	I or D	D	NR
37	4032 U	.6	.7	.7	I	I	NR
38	3225 F	.7	.7	.7	I or D	D	NR
39	6153 U	.8	.8	.8	I or D	D	NR
40	4917 U	.5	.8	.9	I	I	NR
41	877 U	.9	.4	.4	D	D	NR
42	5459 U	.9	.5	.3	D	D	NR
43	5326 U	.7	.6	.5	I or D	D	NR
44	7431 U	.7	.6	.5	I or D	D	NR
45	5285 U	.6	.7	.7	I	I	NR
46	1712 F	.7	.8	.7	I	D	R
47	6318 U	.6	.8	.8	I	I	NR
48	5067 U	.5	.9	.9	I	I	NR
						Total	44 NR, 4 R

[1]U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]R and NR indicate that the decision was 'reversed' and 'not reversed', respectively.

Figure 29 (Con't). Decisions for the 48 Variances: Empirical Approach vs. Subjective Approach.

In order to examine if there are differences between the decisions suggested by the controller and those indicated by the model when empirical membership functions were used, the two sets of decisions are compared in Figure 30. As can be seen from the figure, there were only 5 cases in which the two sets of actions were in conflict. (None of these cases was related to either extreme or favorable variances. One of these cases was related to a variance from a standard set by corporate office--variance number 3. The remaining four cases were related to variances from branch standards.)

This may suggest that using the empirical approach for deriving membership functions may reduce the conflict between actions indicated by the model and actions made by the decision maker. (Note that there were 12 differences when the subjective approach was used.) However, further research is needed before generalizing this result.

Var. No.	Var. Amt.	Compatability with States: In-Control	More or less out of control	Out of Control	Decisions[2]: Suggested by Controller	Indicated by Model	C or NC[3]
1	1987 U	.8	.6	.5	D	I or D	NC
2	9463 U	.9	.4	.2	D	D	NC
3	5609 U	.7	.7	.8	D	I	C
4	6837 U	.7	.7	.8	I	I	NC
5	4962 U	.6	.8	.8	D	I	C
6	7862 F	.7	.7	.7	I	I or D	NC
7	3881 U	.8	.7	.6	D	I or D	NC
8	4172 U	.5	.8	.9	I	I	NC
9	261 U	.9	.3	.2	D	D	NC
10	7706 U	.9	.3	.2	D	D	NC
11	4025 U	.7	.7	.7	D	I or D	NC
12	7017 U	.7	.7	.6	I	I or D	NC
13	5404 U	.6	.7	.8	I	I	NC
14	4023 F	.8	.6	.7	D	I or D	NC
15	4724 U	.6	.7	.7	D	I	C
16	6029 U	.4	.8	.9	I	I	NC
17	411 U	.9	.4	.3	D	D	NC
18	6157 U	.8	.5	.4	I	I or D	NC
19	5542 U	.7	.6	.6	I	I or D	NC
20	5005 U	.7	.6	.6	D	I or D	NC
21	5523 U	.6	.7	.7	I	I	NC
22	4269 F	.8	.7	.7	D	I or D	NC
23	5329 U	.7	.8	.8	D	I	C
24	3476 U	.6	.8	.8	I	I	NC

[1]U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]C and NC indicate that the decisions are 'in conflict' and 'not in conflict', respectively.

Figure 30. Decisions Suggested by Controller vs. Decisions Indicated by Model When Empirical Membership Functions Were Used.

Var. No.	Var. Amt.	Compatability with States			Decisions[2]		
		In-Control	More or less out of control	Out of Control	Suggested by Controller	Indicated by Model	C or NC[3]
25	2567 U	.7	.5	.4	I	I or D	NC
26	8111 U	.8	.4	.3	D	I or D	NC
27	9014 U	.7	.5	.5	I	I or D	NC
28	10156 U	.7	.5	.5	I	I or D	NC
29	6656 U	.6	.5	.7	I	I	NC
30	838 U	.7	.5	.7	D	I or D	NC
31	9245 U	.6	.7	.7	I	I	NC
32	8453 U	.5	.8	.9	I	I	NC
33	1623 U	.8	.4	.4	D	I or D	NC
34	4819 U	.9	.5	.3	D	D	NC
35	7536 U	.7	.6	.5	I	I or D	NC
36	8800 U	.7	.6	.5	D	I or D	NC
37	4032 U	.6	.7	.7	I	I	NC
38	3225 F	.7	.7	.7	D	I or D	NC
39	6153 U	.8	.8	.8	D	I or D	NC
40	4917 U	.5	.8	.9	I	I	NC
41	877 U	.9	.4	.4	D	D	NC
42	5459 U	.9	.5	.3	D	D	NC
43	5326 U	.7	.6	.5	I	I or D	NC
44	7431 U	.7	.6	.5	D	I or D	NC
45	5285 U	.6	.7	.7	I	I	NC
46	1712 F	.7	.8	.7	D	I	C
47	6318 U	.6	.8	.8	I	I	NC
48	5067 U	.5	.9	.9	I	I	NC
						Total	5 / 43

[1]U and F indicate 'unfavorable' and 'favorable' variances, respectively.

[2]I and D indicate 'investigate' and 'do-not investigate' actions, respectively.

[3]C and NC indicate that the decisions are 'in conflict' and 'not in conflict', respectively.

Figure 30 (Con't). Decisions Suggested by Controller vs. Decisions Indicated by Model Whem Empirical Membership Functions Were Used.

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A MODEL FOR THE INVESTIGATION OF COST VARIANCES:

THE FUZZY SET THEORY APPROACH

by

Awni M. Zebda

(ABSTRACT)

Available cost-variance investigation models are reviewed and evaluated in Chapter Three of this study. As shown in the chapter, some models suffer from ignoring the costs and benefits of the investigation. Other models, although meeting the cost-benefit test, fail to capture the essence of the real-world problem. For example, they fail to handle the imprecision (fuzziness) surrounding the investigation decision. They are also based on the unrealistic assumptions of (1) a two-state system, and (2) constant level of accuracy and precision. In addition, the models suffer from the lack of applicability. They require precise numerical inputs to the analysis that are difficult, if not impossible, to attain.

This dissertation provides a new cost-variance investigation model that may overcome some of these problems. The new model utilizes the calculus of fuzzy set theory which was introduced by Zadeh in 1965 as a means for dealing with fuzziness. The theory is also intended to reduce the need for precise measures that are difficult to obtain. Consequently, the theory seems to be well suited for handling the investigation problem. (Chapter Two provides a summary of the theory and its applications in the decision making area.)

The new model is presented in Chapter Four and extended in Chapter Five. The performance is assumed to be described by a transformation function, $S_{t+1} = f(S_t, D_t)$, where S_t, D_t, and S_{t+1} represent the sets of the input states, available decisions, and output states, respectively. The transformation function can be deterministic, stochastic, or fuzzy.

Methods are suggested to obtain the optimal decision for the three cases of transformation functions. These methods are based on formulating a fuzzy optimal decision set

$$D_o = \{u_{D_o}(d_j)\ d_j\},$$

where $u_{D_o}(d_j)$ represents the compatibility (i.e., relative merit) of decision d_j with the optimal decision set. The optimal decision is the decision having the highest compatibility with the fuzzy optimal decision set.

In addition to allowing for different transformation functions, the new model allows for varying degrees of out-of-controllness. The model also provides for the fuzziness (imprecision) surrounding (1) the states of performance, (2) the net benefits from the investigation, and (3) the probabilities. This is done by employing the basic concept in fuzzy set theory, namely, the membership function concept.

The new model was examined (in Chapter Six) for feasibility. First, the model was computerized. Then, it was applied to an actual investigation problem encountered by a manufacturing company. As the application may indicate, the new model can be applied to real-world situations.