GIRARD DESARGUES,

THE ARCHITECTURAL AND PERSPECTIVE GEOMETRY:

A STUDY IN THE RATIONALIZATION OF FIGURE

by

Mark Schneider

Dissertation submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Environmental Design and Planning

APPROVED:

Dennis J. Kilper, Chairman

Alberto Perez-Gomez

Thomas O. MacAdoo

Gregory K. Hunt

John T. Regan

December, 1983

Blacksburg, Virginia

GIRARD DESARGUES,

THE ARCHITECTURAL AND PERSPECTIVE GEOMETRY:

A STUDY IN THE RATIONALIZATION OF FIGURE

by

Mark Schneider

(ABSTRACT)

Girard Desargues (1951-1662) was a key figure in the transformation of architectural geometry from its ancient and venerated status as transcendental knowledge and supreme reality to a mere technological instrument for the control of building construction practice. As a friend of Rene Descartes and Marin Mersenne, Desargues participated in the development of the mechanistic world-view which accompanied the emergence of experimental science and the renewed interest in mathematics and geometry as axiomatic, deductive systems.

This dissertation examines in detail Desargues' methods of stereotomy (the geometrical basis of architectural stonecutting) and his system of perspective construction without vanishing points beyond the picture-space. Desargues' theorem and other key discoveries for which he is still known in the history of mathematics are discussed as they bear upon his methods of stereotomy and

perspective. Desargues' stereotomy is almost certainly the first attempt at a universal descriptive geometry such as Gaspard Monge finally developed after the French revolution. Desargues' work in this area may thus be seen as a precocious foreshadowing of the engineering geometry in common use today.

The writings of Desargues have been consulted in the original French. Extensive passages are quoted and translated, and a number of illustrations from the original texts are reproduced. Supplementary illustrations are also provided. Appendices list the known architectural works of Desargues, his writings and those of his friend and student Bosse which bear upon the exposition of Desargues' methods.

ACKNOWLEDGMENTS

Authentic, original research in the history and theory of architecture is necessarily an interdisciplinary endeavor which calls for a knowledge of philosophy, world history, literature, the arts, and the requisite foreign languages. I therefore count myself fortunate to have been able to find colleagues and friends, both at VPI and elsewhere, whose erudition in one or more of these several fields exceeds my own. That the College of Architecture and Urban Studies at VPI had the foresight to promote this interdisciplinary study betokens a profound understanding of education.

Foremost among those to whom special thanks are due is Alberto Perez-Gomez who now directs the School of Architecture at Carleton University in Ottawa. Without his great understanding of the history of geometry in architecture and unfailing support, this dissertation could not have been brought to its present form. Special thanks are also due to , professor emeritus of mathematics at New York University for his critical reading of the text and helpful suggestions. I would also like to thank for helpful discussions in the early stages of research.

Among the faculty in residence at VPI, sincerest thanks must be extended to Dennis Kilper who served as committee chairman, but above all, for his foresight and able assistance in providing the context for a truly interdisciplinary study. Thanks are also due to Tom MacAdoo who not only taught me Latin but also brought his broad erudition to bear upon a number of difficult issues. Sincere thanks are also extended to of the Foreign Language department who helped with a number of difficult passages in Desargues' "abominable", 17th century French prose style.

Hardest of all to give adequate recognition are the many fine architects at VPI who know how to let learning happen. Tom Regan, Gregory Hunt, and deserve special mention in this regard.

Thanks must also be extended to the students in my seminars at VPI and the University of Houston with whom I have had many helpful talks about the contents of this dissertation and related matters.

CONTENTS

INTRODUCTION

The time for a book-length study of the French geometer, architect and engineer Girard Desargues (1591-1662) in English is long overdue. To date there has been no extended critical study of his contributions to architectural and perspective geometry in any language, and his writings remain almost completely untranslated. But why study Desargues? Ridiculed by his critics as a dilettante bungler in geometry, attacked as a propounder of stone cutting methods too chimerical and extravagant to be used by masons, eclipsed in his own time by Descartes and Fermat, all but forgotten in the history of science and mathematics, is he not one of those shadowy, second-rate figures better forgotten than exhumed for an academic autopsy? Not at all. Desargues was a geometer of the first rank, and his contributions to the theory of projective geometry are enough to ensure him a place in the history of mathematics. But history, and above all that history which seeks to understand the fundamental intentionality of Western science, is ill served when it inquires only into the lives and works of the leading lights who shaped the apparently unqualified success-story which modern science and mathematics have long been considered to represent. As a friend of Mersenne and Descartes, Desargues was present at the birth of mechanism--the world picture upon which experimental science was founded. Desargues was a mechanist at a time when there was no better way to make enemies. The life and work of Desargues can help us understand the birth of mechanism.

Two of the things we least understand about ourselves today are what we have become in becoming the scientists we are, and what we were like before we took the mechanistic turn. Our own age is characterized by a profound and systematic distrust of experience. To find out what and how things really are, we turn to science. And yet, somewhat paradoxically, science has at the same time become increasingly pragmatic and operational. Some philosophers have even gone so far as to propose a "pragmatic theory of truth": we say a certain view is true when, by means of it, we are able to obtain useful results. As Edmund Husserl has eloquently shown in The Crisis In European Sciences, the original commitment of science to explaining to man the totality of what it has been categorically abrogated. And yet, where, if not in mechanism, is the modern scientist to seek the grounding for his work? Leibniz, the great scientist-philosopher in an age of scientist-philosophers, could say: "When I seek for the ultimate reasons of mechanism and the laws of motion I am surprised to discover that they are not to be found in mathematics and that we must turn to metaphysics." But today, metaphysics is a completely bankrupt and discredited adventure; indeed, even Heidegger himself has said that metaphysics is dead.

That the death of metaphysics should coincide with the "death of God", with the dissolution of the symbolic understanding of the world order, with the development of pragmatic and instrumentalist theories of truth, with the loss of certainty in mathematics (most paradigmatically represented by Gödel's proof that all consistent formal systems are necessarily incomplete), and with the development of Non-Euclidian geometries is not a matter of chance. And yet, as long as the historian continues to seek the explanation for these events only in the local causes of surrounding events, or in the chronology of external events over time, the real processes at work will remain elusive. History in the fullest sense is *historia*--an *inquiry*. But the one-sided emphasis commonly placed today upon the diachronic study of facts: what happened when, decapitates history. History in the fullest sense always simultaneously involves an inquiry back into the self. And 'self' does not here mean just what is unique or peculiar to the historian as "individual". Rather, what is called for is an inquiry into the nature and grounding of that common sensibility which the historian, as a person who has a world, must presume to share with those about whom he writes, however different particular theoretical interpretations of human nature may have been at different times.

Considered diachronically, the history of Western philosophy seems to be a succession of different philosophies--as many in fact as there have been philosophers. What indeed could be more disparate than the Platonic theory of Ideas and the logical positivist program for the elimination of metaphysics? When the synchronic dimension is added however, it begins to be evident, as Husserl has pointed out, that there has been only one philosophy which consists of the permutations in Western man's attempt to understand his own nature as a being to whom the world is always already given in idealized form. Fundamental to our nature as persons is not just that we perceive, but that we perceive birds, mountains, summer heat and winter wind. In human perception, the <u>what</u>--the essence and idea--is always and inseparably present. And yet, what it seems we have never quite clearly understood is how science represents the extension of this pre-reflective idealization upon which our experience depends into a systematic and <u>reflective</u> method for understanding and controlling the world by means of reason. Thus the ontological grounding of science in human experience and its dependence upon that grounding has remained ambiguous to the present day. On the one hand it has seemed that experience gives the mere appearance of things, and that the discovery of the truth and the reality behind the appearance requires a scientific investigation. On the other hand, at least after Hume's skepticism and Kant's attempt to salvage what might be saved for the province of pure reason, reality as noumenon (permutation of the Platonic Idea, meaning a thing conceived by reason), was considered in principle unknowable, except for the certainty of its bare existence. Today, science continues to fail to find its rightful place in a true cosmological synthesis. Thus Hesserl has said that the crisis of science is its loss of meaning for the world as lived.

Among the first of the sciences to emerge in Western thought was geometry; not the deductive geometry of Euclid, but geometry as the idealization of significant figure. For a long time, geometry was the paradigm of knowledge as idealization, and this is how it was understood by Plato. But in Greek times, geometry was conceptually a part of mathematics, and mathematics was not just calculation, or reckoning with numbers, but ta *mathēmatica*--what could be learned. And, if we remember that, for Plato, all knowledge was a recollection of Forms as essences already imprinted upon the mind, we begin to understand how his doctrine of epistemic seeing as a recognition (re-cognition) seizes upon and deals in a very profound way with the fact that all seeing involves the simultaneous and inseparable forthcoming of an essence--of a 'what-it-is', even in those cases where we later find that we were mistaken in our initial perception. Thus geometry was a primordial symbol of knowing as a measuring and delimiting--of finding the bounds--an act symbolically represented by drawing a figure, which Heidegger has called the making of a 'Riss-gestalt'--the 'cutting out' of the figure in its 'what' and 'how' from the field of the page by means of a line.

A full understanding of the significance of the work of Desargues requires his accomplishments to be situated in the progressive unfolding of the idea of geometry as a form and symbol of knowledge and reality which began in Greek times and continued until the symbolic crisis of the 19th century. What makes such a study imperative in this case is that, as an architect-geometer, Desargues was among the first, if not the very first, to adopt the mechanistic world view and thus to reject the symbolic understanding of architectural geometry which had been universal up to his time and which in fact continued to dominate the field for a long time after him. Thus the present essay has two objectives. On the one hand it aims to present a technical exposition of Desargues' work on stone cutting and perspective in the light of the accomplishments in the theory of projective geometry. The second objective is explicitly theoretical--to offer an interpretation of the significance of Desargues' work in the development of Western thought, while at the same time, making explicit the philosophical basis of that interpretation. Only by combining these two objectives, it seems to me, can one write what is truly history.

CHAPTER 1

GIRARD DESARGUES: THE GEOMETER, THE ARCHITECT, THE BROUILLEUR

THE EARLY YEARS

Desargues was born at Lyons, being there baptized in the parish church of Sainte-Croix March 2, 1591[1] The view held by Poudra and others that he was born in 1593 stems from an error in Baillet's biography of Descartes where it is wrongly stated that Desargues was three years older than Descartes who was born in 1596.[2] The father of the geometer (also Girard) married Jeanne Croppet in Condrieu, thereafter moving to Lyons, where he was employed as an investigator to the bailiff (1574), tax collector (1583-91) and royal notary (1585-91). In Lyons, he produced a large family (four daughters: Marie, Clemence, Francoise and Catherine) and (four sons: Fleury, Philippe, Antoine and Girard). It is probable that Girard was the youngest of the family.

Nothing is known of Desargues' childhood or education beyond the few remarks scattered through his publications. He always stated that his geometrical ideas were of his own invention, and that he had acquired the basics of geometry from reading Euclid and Apollonius. After the baptismal record, the next positive indication we have of his activities is Bosse's mention that in February 1630 Desargues received a privilege for the publication of several writings.[3] Baillet is of the opinion that by this time Desargues held a place of esteem in the entourage of Cardinal Richelieu who, Baillet says, employed him in an engineering capacity at the siege of La Rochelle in 1628.[4] Baillet also indicates that it was during the preparations for the siege that Desargues first met Descartes who had come out to view this vast defensive project designed for and successful in preventing an English invasion. A popular history gives the following description of the fortifications:

> On the land side they had constructed a line of redoubts and batteries connected by trenches, which extended over a distance of three leagues, and on the side towards the sea they built, after several ineffectual attempts, the celebrated dike, measuring 747 fathoms, bristling with cannon, pierced in the centre to give passage to the tides, defended from every attack by a floating barricade of beams and ships all bound together, and guarded by 25 large vessels, 12 galleys, and 45 boats and barges: it took Metezeau, the architect, and Thiriot, the mason, six months to carry out the huge undertaking, in the teeth of the enemy's opposition.[5]

Adam and Tannery, editors of the complete works of Descartes, maintain, however, that it was not until circa 1637 that Desargues met Descartes through Marin Mersenne (1588-1648)[6] That Desargues, at least by 1638, held a position of some influence with Richelieu is suggested by Poudra on the grounds that Desargues was successful in obtaining a pension for Descrates (who refused it) in order to draw him back from Holland where he had retired to a life of of quiet and contemplation.[7] It is also suggested by the fact that in his will of 1656, Desargues granted to his brother Antoine a life-time annuity of 1,200 livre per year.[8] If Desargues was the youngest son of such a large family, it is unlikely that his inheritance could have accounting for such resources. Most probably, therefore, he had been employed by Richelieu in some capacity and perhaps augmented his income by investments, a skill for which his family background would have equipped him, and a form of income which is suggested by the capacity to grant a life-time annuity.[9]

We know that sometime before 1636, Desargues entered the cadre of savants with whom Marin Mersenne surrounded himself, for in 1635-6 Mersenne published his monumental <u>Le Harmonie Universelle</u> (The Universal Harmony) which contains a brief paper by Desargues on: <u>Un methode aisee pour apprendre et enseigner a lire et escrire la musique</u> (An Easy Method for Learning and Teaching the Reading and Writing of Music).[10] Mersenne, who has been aptly described as adopting for himself the role of secretary to the savants of Europe,[11] played a key role in promoting Mechanism as a world view. In constant communication with most of the leading minds of his time, with Galileo, Torricelli, the Huygens, Cavendish, Gassendi, Descartes, Fermat, the Pascals and Thomas Hobbes, to cite only a few of the most illustrious, he set himself the task of promoting experimental method as the <u>only</u> true key to what could be known of the physical world, rejecting metaphysical speculation, and thus, to some extent, even the work of Descartes.[12] We also know that Desargues had made the acquaintance of Gassendi before the latter left Paris in 1632, and it is quite possible that he made this contact through Mersenne who mentions the meeting in a letter.[13] Thus, by entering the Mersenne circle, Desargues had ready access to the leading ideas of his time.

Desargues' <u>Easy Method</u> is his only known writing which does not deal with geometry and its application. Here perhaps we have an indiction that Desargues had been under the influence of Mersenne during the period in which his ideas on geometry were taking their definitive form. Later we shall examine the relations between Mersenne, Descrates and Desargues in greater detail in order both to fix Desargues' philosophical position more exactly and to form a better idea of why his geometrical theories developed as they did.

THE YEARS OF GEOMETRICAL PRODUCTION

At approximately this time (1631-6), Desargues began to tutor the young Blaise Pascal (1623-1662) in geometry. Again, a precise date cannot be established, but we do know that the elder Pascal (Etienne, 1588-1651) had moved his family to Paris from Clermont in the Auvergne in 1631.[14] That Desargues was now actively engaged in geometry is indicated by his 1636 publication of an Example de l'une des manieres universelles du S.G.D.L. touchant la pratique de la perspective sans emploier aucun tiers point, de distance ny d'autre nature, qui soit hors du champ de l'ouvrage (Example of one of the Universal Methods of (Desargues) Touching upon the Practice of Perspective without the use of a Vanishing Point, or Point of other Nature, Which is Outside the Picture Space). The title of this work already gives a clear indication of Desargues' interest in the universal. Later it will be shown that Desargues quest for universality is associated with his concept of geometrical truth, according to which only what is universal may be considered true or a proper subject of inquiry for a scholar.

The influence of Desargues' thinking on the young Pascal is directly acknowledged in the latter's 1640 publication (at age 16) of an Essay pour les Coniques (Essay on Conics). In this work which gives what has since become known as Pascal's Theorem, or the "theorem of the mystic hexagram", as Pascal sometimes called it, we find the following appreciation of Desargues:

> Nous demontrerons aussi cette propriete, dont le premier inventeur est Mr. Desargues Lyonnois, un des grands esprits de ce temps, & des plus verses aux Mathematiques, & entr'autres aux Coniques, dont les escripts sur cette matiere, quoy qu'en petit nombre, en ont donne un ample tesmoignage a ceux qui en auront voulu recevoir l'inteligence: & veux bien advouer que je doibs le peu que j'ay trouve sur cette matiere a ses escrits, & que j'ay tasche d'imiter autant qu'il m'a este possible sa methode sur ce subjet,...[15]

> We shall likewise demonstrate this property of which the original inventor is Mr. Desargues of Lyons who is one of the great minds of this time and versed as well in mathematics, among others that of conics, whose writings on this matter, though small in number, have given ample testimony of his ability to those who had desired to become aware of it, and will not object to allowing, as I believe, the little which I have found on this matter in his writings and which I have attempted to imitate as far as it is possible for me of his method on this subject,...[16]

Thus Pascal deftly asserts both the originality of his own idea and the debt he owes to Desargues. Interestingly, Descartes at first refused to believe that this paper was by Pascal, insisting that it must be by Desargues himself.[17] The "little bit" which Desargues had by this time published on conics was a very great deal, for in 1639 he brought out his largest known work, the Brouillon project d'une atteinte aux evenemens des rencontres du Cone avec un Plan (Rough draft of an attempt at the result of the intersection between a cone and a plane). In this work, Desargues proves, among other things, his celebrated involution theorem (for which see Chapter III) which, in modern form, states that any transversal meets a conic section and the opposite sides of a quadrilateral inscribed in it in four pairs of points which are four pairs in involution. This work caused something of a furor in the Mersenne circle and in fact marks the beginning of a critical attitude toward the work of Desargues which was to trouble him for the rest of his life. One criticism was that Desargues refused to use the traditional language of geometry. For example, in describing the relations between the various points on a line he makes use of an extended plant metaphor. Thus, for example, a line cut by other lines is called a trunk (tronc), a point of intersection between lines is called a knot (noeud), and a line segment passing through a knot is called a branch (rameau). Another criticism was that Desargues did not seem to be able to form a clear idea of the kind of reader he was writing for. These problems were pointed out to Desargues in a letter which Descartes wrote to him after reading his work on conics:

> Vous pouves avoir deux desseins, qui sont fort bons et fort luables, mais qui ne requierent pas tous deux mesme facon de proceder. L'un est d'escrire pour les doctes, et de leur enseigner quelques nouvelles proprietz de ces sections qui ne leur soient pas encore connues; et l'autre est d'escrire pour cette matiere qui n'a pu jusqui'ici estre entendue

que de fort peu de personnes, et qui est neanmoins fort utile pour la Perspective, l'Architecture, &c., devinne vulgaire et facile a tous deux qui la voudrount estudier dans votre livre. Si vous avez le premier, il ne me semble pas qu'il soit necessaire d'y emploier aucun nouveau terme: car les doctes, estant deja accoutumes a ceux d'Apollonius, ne les changeront pas aisement pour d'autres, quoique meilleurs, et ainsi les votres ne serviroient qu'a leur rendre vos demostrations plus difficiles et a les detourner de les lire. Si vois avez le second, il est certain que vous termes qui sont francois et dans l'invention desquels on remarque de l'esprit et de la grace, seront bien mieux recus, par des personnes non preoccupees que ceux des Anciens; et mesme, ills pourront servir d'attrait a plusiers pour leur faire lire vos Escrits, ainsi qu'ils lisent ceux qui traitent des armoiries, de la chasse, de l'Architecture, &c. sans vouloir estre ni chasseurs, ni architects, seulement pour en savoir parler en mots propres. Mais si vous avec cette intention, il faut vous resondre a composer un gros livre, et a y expliquer tout si amplement, si clairement et si distinctement que ces Messiuers, qui n'etudient qu'en baillant et qui ne peuvent se peiner l'imagination pour entendre une proposition de Geometrie, ni tourner les feuillets pour regarder les lettres d'une figure, ne trouvent rein en vostre discours, qui leur semble plus malaise a comprendre qu'est la description d'un palais enchante dans un roman, Et a cet effect il me semble que, pour rendre vos demonstrations plus triviales, il ne seroit pas hors de propos d'user des terms et du calcul de l'Arithmetique, ainsi que j'ai fait en ma Geometrie; car il y a bien plus de gens qui scavent ce qu'est multiplication, qu'il n'y en a qui scavent ce que c'est que composition de raisons, etc.[18]

You may have two objectives which are very good and very laudable, but which do not both require the same manner of proceeding. The one is to write for the educated, and to show them some new properties of these (conic) sections which are as yet unknown to them. The other is to write for the curious who are not educated and to show them that this matter which, up to the present time, has been understood by only a very few persons, but is nevertheless very useful in perspective, architecture, etc., should become popular and easy for all those who would

> study it in your book.
>
> If you have the first (as your objective), it does not seem to me that it should be necessary to employ any new terms, because the educated, being already accustomed to those of Apollonius, will not easily exchange them for others, even if they are better, and thus your terms only render your demonstrations more difficult for them and distract them. If you have the second (as your objective), it is certain that your terms which are French, and in the invention of which one notes both spirit and charm, will be much better received than those of the ancients by persons not (already) prejudiced (as are the educated). And for the same reason, your terminology could serve to attract many to read your writings in the same way that they read those who treat of arms, the hunt, architecture, etc.; (that is), without desiring to be either hunters or architects, but only to know how to speak of these things in the right terms. But if this is your intention, it will be necessary for you to resolve to compose a large book and therein to explain all at ample length, with great clarity and distinctness, so that these gentlemen who never study without yawning and who have never strained their imaginations to understand a geometrical proposition, nor bothered to turn pages in order to study the letters accompanying a figure, will find nothing in your discourse which appears to them to be harder to understand than the description of an enchanted castle in a romance. And to this end it seems to me that in order to render your demonstrations more commonly intelligible it would not be inappropraite to use the terms and calculations of arithmetic as I have done in my Geometry, for there are a great many more people who understand multiplication than who understand a system of ratios, etc.

What Desargues thought of this appraisal is perhaps indicated by the fact that he never adopted Descartes' suggestions. In none of his words does he resort to any form of calculation to explain his demonstrations. In every case, he continues to make use of the most elaborate ratio equations. Likewise, he continues to invent names rather than use the terms of the classical geometers, and he certainly never attempts to write for the merely "curious" with whom Descartes somewhat ironically suggests his terms might find their happiest reception.

Today we are perhaps too ready to favor Desargues' intransigence in these matters, regarding it as a mere foible of genius; however, it seems to me that the situation is in fact more complex than such interpretations will permit one to recognize. In fact, when Desargues' inability or unwillingness to identify the nature of his audience is properly understood, we have an important clue to his philosophical alignment with the new mechanistic world view which Mersenne was by this time promoting with every resource available to him. This however is a matter on which discussion must be deferred until a more complete picture of Desargues' theoretical activities has been developed.

In 1638, Mersenne asked Desargues to referee a dispute between Descartes and Pierre de Fermat (1601-1665) on the proper treatment of tangents to curves. Fermat had read Descartes' Dioptrics and had taken rather vigorous exception to certain parts. Since Desargues was well regarded by both men, he was an obvious choice for the role of mediator--a role which Mersenne frequently had to find someone to fill as he attempted to maximize the interchange of ideas between so many brilliant and often difficult personalities. Indeed we are fortunate that in this case Mersenne chose Desargues, for, in discharging his responsibility, Desargues produced the only letter of his to have survived to the present day. This letter is of great value in helping to establish Desargues' philosophical position, for in it he expresses his favorable view of Descartes' Meditations on First Philosophy:

> Touchant les autres objections de Mr de Fermat contre Mr des Cartes vous scavez que je vous dy au commencement sur le peu que j'en veis entre vos mains que ne ne trouvoy pas que Mr de Fermat entreprit cette objection de bonne sorte, a mon sentiment qui s'accommode mieux au Meditations de Mr des Cartes que d'aucun autre, veu mesmes la conformite que je trouve de plusieurs observations que j'ay faictes avec ce qu'il escrit & dont j'etens ce me semble a peu prez tout ce que j'ay veu de luy hors sa Geometrie, et j'en suis jusques icy passablement satisfaict, et surtout de sa facon de conduire ses raisonnements.[19]

> Touching upon the other objections of Mr. de Fermat contra Mr. des Cartes, you know what I told you at the beginning (of this letter) about the little (difference) that I see between your sides, nor that it will not be found that Mr. de Fermat undertook this objection in good faith according to my feeling which agrees better with the Meditations of Mr. des Cartes than with any other, seeing likewise the conformity with what he wrote that I find in several observations which I have made, and that I understand, it seems to me a little better, that

> of his which I saw outside of his Geometry, and I am up to now passably satisfied with it, and, above all, by his manner of conducting his reasonings.

This passage contains at least three important clues to the philosophicl position of Desargues. First is the indication that besides Descartes' Geometry, Desargues knew the Meditations and perhaps some other works by Descartes. Second, Desargues states that he is "passably satisfied" with the results of Descartes' philosophical investigations. Third, even if not completely satisfied by the results, Desargues is in full agreement with Descartes' method of philosophizing. For his part, Descartes held an equally favorable view of Desaruges' judgement in philosophical matters. In a letter to Mersenne in which he announces that he will soon send his Synopsis ou Abrege des six Meditations (Synopsis or Abridgment of the Six Meditations), he states that:

> Je serai bien aise que Monsieur Desargues soit aussi un de mes Juges, s'il lui plait d'en prendre la paine, et je me fie plus a lui qu'en trois theologiens.[20]

> I would be very glad if Mr. Desargues would be one of my critics if it would please him to take the trouble for it, and I have more faith in him than in three theologians.

In 1640, Desargues published two works, one dealing with stone cutting in architecture and the other with the calibration of sundials. The first of these, the

Brouillon project d'example d'une maniere universelle du S.G.D.L. touchant la pratique du trait a preuves pour la coupe des pierres en l'Architecture . . . (Rough draft of an exmaple of a universal method by (Desargues) touching upon the practice of a previous work concerning stone-cutting in architecture), is of great importance in the history of descriptive geometry and is given a full analysis in Chapter II of the present work. This work of Desargues constitutes the first known attempt at a universal system for finding the true shape of any oblique projection of a solid onto a plane surface. As such, it represents a key step in the development of a complete method of descriptive geometry which was finally accomplished by Gaspard Monge just after the French Revolution. Desargues' other work of 1640, the Maniere universelle de poser le style aux rayons du soleil en quel conque endroit possible, avec la regle, le compas, l'esquerre et le plomb (Universal method for situating the gnomon to the rays of the sun in any possible locality, using the rule, the compass, the square and the plumbline), apparently has not survived. Its content was probably similar to Bosse's 1643 explanation of this method.[21]

The method for stone-cutting shows many of the same idiosyncrasies which Descartes had criticized in the work on conics. Again, Desargues introduces many new terms, eg: "axle," "cross-axle," "counter-axle," "plane perpendicular to the face and level," etc. In this case however there is a greater justification for novel language, since, on the one hand, terms known to practicing masons (who, then, as now, were not particularly consistent in such matters) would have been unfamiliar to scholars, and, on the other, because there simply are no terms in classical geometry for the parts of arches or vaults, or for the geometrical constructions required in determining their shapes. It is in fact only much later when descriptive geometry finally becomes a system completely independent of the kind of object described, that its terms achieve a clarity and simplicity comparable to those of pure geometry.

Desargues' method of stone-cutting works and is indeed a brilliant invention, but, at the same time, it must be noted that without the author's personal tutoring no mason of the time would have been likely to understand it. It was in fact only with Bosse's 1643 publication of a Pratique de trait a preuves de Mr. Desargues Lyonnois pour la coupe des pierres en l'Architecture that Desargues' method received the kind of explanation a common worker might understand. What Bosse supplies is an extended, step-by-step demonstration of the procedures, and in this way he fills the prescription Descartes had given for writing this sort of work.

THE YEARS OF PUBLIC CRITICISM AND SCANDAL

We turn now to the attacks which Desargues brought upon himself as a result of publishing his views on a method of perspective which seemed to him to both plagarize and misrepresent his own ideas. In examining these developments, two points should be kept in view. First, unlike Mersenne, who knew how to be politic in matters of published criticism (he was, for example, able to obtain an impartial and ultimately advantageous evaluation from Gassendi in his dispute with Robert Fludd)[22], Desargues is grossly impolitic. Desargues attacks the person and grants his enemy no chance of suing for peace. Thus he receives from his opponent only what his harsh terms will allow for: a personal and vindictive counterattack. Second, ours is perhaps not a time in which it is particularly easy to appreciate either the love of scandal or the cut-throat tactics which often prevailed in the popular press of 17th century Paris.

What happened was this: In 1642, the house of Melchior Tavernier and Francois l'Anglois brought out an anonymous publication on perspective called La Perspective pratique necessaire a tous paintres, sculpteurs, graveurs, architects, orphevres, bordeurs, et autres se servans du dessine.[23] (The practical perspective necessary to all painters, sculptors, engravers, architects, goldsmiths, embroiderers, and other designers). In this work, which was actually by Jean Dubreuil (1602-1670), Desargues thought his ideas had been both plagiarized and corrupted. In actual fact, Dubreuil did credit Desargues in the preface, but what seems to have particularly irritated Desargues was that in addition to violating his rights to publication, the book credited him with a perspective method which was full of errors. Taton suggests that the motive for this publication may have been rivalry between the house of Tavernier and Bosse/Desargues' publisher Des-Hayes who was about to release Bosse's explanation of Desargues' perspective method.[24] In any event, Desargues' response was to print two broadsides in placard form and post them in the streets of Paris. one of these he called Erreur incroyable ... (Incredible error...), and the other Faute et faussetes enormes ... (Enormous faults and duplicities...) While no copies of these placards are known, Poudra was able to identify a few quotations from them in other publications relating to the dispute.[25] Of these, one will serve to illustrate what the titles already suggest of Desargues' tone and frame of mind in in this matter. It also suggests that the real basis for the intensity of Desargues' reaction may have been a certain duplicity on Dubreuil's part. Having at some previous time negatively criticized the method, Dubreuil nonetheless used it to his own advantage in this book. Desargues' references in the passage to "five years" agrees roughly with the 1636 publication of his perspective method:

> Qu'il a cinq annees que l'enuie pu avec sa langue persuader que cette maniere universelle que'il s'attribue ne valoit rien, elle a tant fait qu'on a fourre dans ce livre de la perspective pratique une figure de l'example qu'il dit sien, alteree et falsifiee par les griffes mesquines de l'envie.[26]

> Even though for five years he (Dubreuil) has had the desire to be able to convince by his way of talking that this universal method which he attributes to himself is good for nothing, it is nevertheless the case that in this book on perspective practice there appears a figure (illustrating Desargues' method) which he calls his own, falsified though it is by the shabby claws of envy.

That Desargues might suppose he could post remarks like these in the streets of Paris without receiving a counterattack in kind is quite amazing. First, Dubreuil himself replied in a pamphlet also published by Tavernier and l'Anglois.[27] In this, Debreuil stated that he had been wrong to claim that the perspective method was Desargues' invention, because it had already appeared in a work by Vauzelard and in another by Aleaume written in 1628. To add further fuel to fire, he declared that all of Desargues' works were, for all practical purposes, completely without value. One can easily imagine Desargues' state of mind upon seeing this in print. Now he was being accused of plagiarism, with a dose of incompetence added for good measure. Desargues responded in April, 1642 with a 15 page pamphlet giving detailed explanations of the errors in Dubreuil's text.[28] Continuing to fan the flames from his side of the fire, Desargues referred to Dubreuil as a "maladroit copyist" in his pamphlet.

From this point, the publishers Tavernier and l'Anglois took a more open and active part in the dispute. They began to collect all of the negative criticism they could find on the work of Desargues and proceeded to publish it in a series of broadsides bearing the title: Advis charitables sur les diverses oeuvres et feuilles volantes du Sieur Girard Desargues, publiees sous les titres ...[29] (Charitable advice regarding the diverse works and loose sheets of Mr. Girard Desargues of Lyons, published under the titles...). These criticisms apparently went through several "editions" because the content differs in extant copies, suggesting that, as the publishers accumulated more material, they expanded their format.

None of the criticisms in the Charitable Advice bring to light theoretical defects in Desargues' methods, and it would be pointless to dwell on them at any length, were it not for the fact that they do show, if only indirectly, how the public might react to a person who at one and the same time appeared to deny the value of a scholastic education and also proposed to tell artists and craftsmen how to solve their geometrical problems without being himself skilled in any of the arts and crafts. In examining these criticisms, one begins to see how Desargues, in following Mersennes' rejection of metaphysical explanation in favor of experiment and practical techniques, was in fact placing himself in an extremely vulnerable position with respect to his critics. In fact, it would seem that Desargues himself had, at best, a very unclear idea about how to fill this role of self-tutored savant and practical craftsman's assistant which he had identified for himself, at least partly as a result of his relations with Mersenne and Descartes.

In general, the criticisms of the Charitible Advice are of following kinds:

1) Desargues had no skill in any of the arts in which he pretends to give instruction.

2) Desargues writes in an obscure and convoluted style which cannot be understood by the common workers he proposes to instruct.

3) Desargues makes pretentious and unsupported claims for the universality of his methods.

4) Desargues has frequently taken his methods from other sources which he does not credit.

5) Desargues vainly wishes to be counted among the learned but has no educational credentials which would entitle him to such respect.

6) The arts and sciences are no easy things to master; therefore, Desargues could not have so quickly devised universal methods for practices in which he has no skill.

7) Desargues will not be guided either by nature or by the methods of exposition and analysis perfected by the great scholars of earlier times.

8) Though he pretends to give methods for craftsmen, he insists that they must be judged, not by the craftsmen themselves, but by scholars and geometers who have no practical experience in such matters.

9) Desargues' methods are only ideas and, as such, are not adapted to the exigencies of practice.

Desargues' response to the Charitable Advice appeared in two forms. The first was a placard published

December 16, 1642 with the title: Reponses a causes & moyens d'opposition (Responses to the motives and methods of opposition). Except for a brief passage quoted in a later criticism, this work has been lost.[30] At this time, Bosse was also preparing his explanations of Desargues' stereotomy and sundial method for publication in 1643. Into these the two authors now inserted several items by way of response to the criticism. The most interesting and informative of these are Desargues' Reconnaissances in which he acknowledges that he is the author of the idea which Bosse presents. Here Desargues' tone is by turns sarcastic and witty. He shows quite clearly that, from a theoretical standpoint, his methods are impeccable. At the same time, he has collected signed documents from the masonic officials which make it possible for him to accuse Tavernier of libel. Each of the two Reconnaissances begins with a similar statement:

> Ie soub-signe confesse avoir veu ce que M. Bosse a mis dans ce volume-cy, de la pratique de trait pour la coupe des pierres en l'Architecture, reconnois que tout y est conforme a ce qu'il a volu prendre la patience d'en onyr et conceuoir de mes pensees,...[31]

> I the undersigned (Desargues) acknowledge having seen what M. Bosse has put into this volume on the practice of stone-cutting in architecture, recognizing that all here conforms to that which he has desired to have the patience to hear and understand of my thought,...

In the Reconnaissance to the work on stone-cutting, Desargues then passes directly to an attack upon the criticisms contained in the various documents already cited. His critics had called him a "nouveau Maistre et nouveau Doctor" (new master and upstart doctor) referring to his lack of a scholastic education, and so Desargues replies in kind:

> Et ce vieux docteur, apres s'estre longument escrime contra son ombre, au suject de madite maniere...[32]
>
> And this senile doctor (Tavernier), after having so long fenced with his shadow on the subject of my method...

Desargues then takes up the issue of libel. In the Charitable Advice, Tavernier had atempted to show that the masonic guild had conclusively rejected Desargues' method of stone-cutting. According to Tavernier's story, Charles Bressi, one of Desargues' students, had asked the masonic officials to be allowed to execute his master's piece (a work designed to show that a mason has enough skill to warrant the title of Mastermason) according to the procedures of Desargues' method. Tavernier claimed that the masons had rejected Bressi's request on the grounds that Desargues' method"ne pouuoit legitiment tenir rang parmy les receuables dans la pratique, pour estre par trop chimerique et extrauagant,..."[33] (could not rightfully hold a place among those acceptable in the practice because it is too chimerical and extravagant). Since the story was false, Desargues obtained sworn testimony to this effect, thus hoping to conclusively discredit Tavernier.[34] But it remained for Bosse and Desargues to argue the positive values of the method, since the masons themselves were willing or able to testify only that Bressi had made no such request and that they had not, therefore, condemned Desargues' method. Bosse's text, which is in fact only the first of two projected volumes in which the full scope of Desargues' method was to have been explained, shows evidence of the pains taken to avoid further criticism (See <u>Chapter II</u> below). In this respect it must be judged a success, though it does not contain any discussion of the geometrical theory which Desargues considered the true basis of his work. In fact, what Bosse produced, shows, on more than one occasion, that traditional techniques of stereotomy may be combined with or even substituted for those of Desargues.

In acknowledging the treatise on sundials, Desargues explains two matters which are of considerable importance to an understanding of his works. The first of these concerns what he intended by the expression "brouillon project" which so frequently appears in his titles. In French, the word "brouillon", as applied to a written work is somewhat ambiguous. It can mean either a "rough draft," or a "blundering"; that is, a crude piece of writing.[35] By extension, it can also mean "mischief making." Now, as we have seen, all of these definitions are in some degree applicable to the writings of Desargues, and it is possible that, at least before he began to be so harshly criticized, he intended all of these significations. Obviously however, the term is not a happy choice for a work which might receive negative criticism, since it can be so easily turned against the author. What then was Desargues' intention? In the passage I quote, Desargues is on the defensive; he says:

> ...ie lui ay dit qu'un simple brouillard et encore seulement d'un projet, qu en une autre matiere on nommeroit un esquis ou esbauche, n'est un ouurage a examiner en detail, comme alors qu'il paraistra pour acheue; que les scauants n'en considereront que les fonds de la pensee;...[36]

> ...I had said it was a simple collection of loose papers (brouillard), and again, a mere schematism, which, in other circumstances, one would call a sketch or rough draft, which is not a work to be examined in detail as though it were already finished; for the savants will only consider the basis of the idea...

Thus we see that Desargues thought of his papers as sketches giving only the kernel of an idea and in such a way that a scholar might understand it. For Desargues, then, a "brouillon" is a synopsis or abstract which on many points merely provides an outline of what would have to be said by way of explanation in the more extended work for which it provides the sketch. In the last analysis, it would appear that Desargues was writing for a person very like himself--a self-tutored savant with a somewhat utopian conception of the relation between theory and practice. It is probable that Desargues intended his own papers to circulate only among those few friends and scholars who were interested in his ideas. He printed his papers only in small quantities and this, in part, accounts for the virtual disappearance of original copies. There is no indication that Desargues himself ever intended to write the books which would give the polished and detailed accounts of his ideas alluded to in the quoted passage, and, were it not for Bosse's efforts, we would today know even less of his ideas.

The second and more important point which Desargues makes in his acknowledgement concerns the place of universality in geometrical inquiry. Along with "brouillon project", the expression "maniere universelle" is the one which occurs most frequently in his titles. It has already been suggested that the universal is associated with the idea of truth in the work of Desargues. Indeed, what is universally true is what is non-contingently true. Here we learn that, in Desargues' view, the universal is the only proper mode of inquiry for the scholar:

> Ie veux dire qu'au lieu d'examiner et explique mon projet, suiuant ma pensee et facon de conceuoir ces matieres dans l'vniuersel; comme c'est l'vnique facon legitime de faire des sçauants, et la raison de la vouloir; il l'a examinee selon so facon de conceuoir dans le particulier . . . (et) il combat seulement son explication et non pas ma pensee, a laquelle il n'est point encore arriue.[37]

> I want to say that instead of examining and explicating my sketch, following my thought and way of conceiving these matters in the universal, as is the only legitimate way of proceeding for savants, and the reason to desire it, he has examined it according to his way of conceiving in the particular . . .(thus) he combats only his explication, and not my thought at which he has not yet arrived.

If, at this point, Desargues and Bosse thought they were gaining the upper hand in the dispute, their optimism was to be short-lived. On December 29, 1644, the house of Tavernier and l'Angloise brought out a new critique of Desargues. This, by Jacques Curabelle was, called Examen des oeuvres du Sieur Desargues[38] (Examination of the works of Mr. Desargues). Today, nothing is known of Curabelle beyond his involvement in the ongoing attacks on Desargues which emanated from this publisher. Curabelle had received a privilege to publish a comprehensive work on architecture dealing with, among other things, sundials stereotomy and perspective. Throughout his critique of Desargues, Curabelle takes the position of defending traditional methods. He argues that Desargues' "errors" are to be attributed to a lack of practical experience:

> Si le dit sieur eust cognu et pratique la chose dont il a voulu parler, il ne fust sans doute tombe dans de telles erreurs, la pratique estant necessarie a ayder et fortifier nos sens; confirmer ou infirmer ce que la speculation de nostre esprit auroit produit . . . [39]

> If the said sir had known and practiced the matter of which he has wanted to speak, he no doubt would not have fallen into such errors, practice being necessary to aid and fortify our sensibility; to confirm or invalidate that which the speculation of reason would produce . . .

Curabelle's publication was also announced via placards posted in the streets of Paris. For his part, Desargues threatened to bring suit against Curabelle to force retraction, and various documents pertaining to this matter were exchanged, though, at this point, the exact sequence of events becomes difficult to follow. On April 2nd, Desargues distributed a placard called Le Honte de Sieur Curabelle[40] (The disgrace of Mr. Curabelle), and Curabelle responded on April 8th with Calomnieuses faussetez contenues dans un affice du sieur G. Desargues[41] . . . (Calumnious falsehoods contained in a placard of Mr. G. Desargues . . .) On April 18th, Desargues responded with a pamphlet: Sommation faite au sieur Curabelle[42] (Demand made to Mr. Curabelle) and Curabelle reparteed with Foiblesse pitoyable du Sr. G. Desargues employee contre l'Examen fait de ses oeuvres[43] (Contemptible foibles of Mr. G. Desargues employed against the examination made of his works).

According to the story as Curabelle gives it in the last-mentioned item, Desargues offered to bet 100,000 livre that Curabelle could not show his methods defective. Did Desargues actually wager such a large sum? It has not been possible to consult two rare documents which might shed light on this point.[44] Curabelle claimed to have accepted the bet for 10 pistoles, and this is the only sum mentioned by Desargues in his later writings. In an extended innuendo about the relation between cowardice and the size of one's "army", Curabelle implies that Desargues wanted to call geometers from Spain and Holland to judge the formal debate which was to decide the winner of the bet. This raises interesting questions about what contacts Desargues might have had in these countries. Descartes was now in Holland, and this almost certainly accounts for that reference. We know of no contacts in Spain, though Pierre de Fermat was in Toulouse, and thus within 50 miles of the Spanish border.

What transpired from this point cannot be accurately determined from Curabelle's obviously slanted story. As he would have it, Desargues finally appeared in a state of great agitation to sign the documents formalizing the terms of the debate. At this time, Desargues stipulated that he would have no further communication with Curabelle except by signed letter. Each party to the dispute was to supply two judges, one a mason, and the other a geometer. As a part of the contest, sample stones were to be cut according to each method. The text does not make clear whether the contestants themselves were to do the cutting. In the event of a tie decision, a fifth judge was to be selected, either by lot, or by the other judges. According to Curabelle, from this point on, Desargues employed every imaginable device to gain concessions favorable to his position. None of the documents we have indicate that the debate ever took place, and there does not seem to be any indication of how the matter was eventually resolved.

In 1646, the provosts of Lyons asked Desargues to make proposals for a municipal hotel in his native city. In their first letter, after briefly explaining their project, the provosts point out that, in as much as this is to be a public work, it is appropriate for them to consult these knowledgeable in architecture and that they consider Desargues capable in this area.[45] Other documents indicate that a scheme for this project had already been drawn up to the town surveyor, Simon Maupin and revised by the architect Jacques Lemercier. This, for one reason or another, did not prove to be satisfactory.

Records indicate that at least 400 livre were paid to Desargues and Lemercier for their work on this project, and it is possible that the two men in fact worked in collaboration.[46] Since Lemercier had been one of Richelieu's chief architects (he designed both a large chateau and the town of Indre-et-Loire for the cardinal) it may be that Desargues First Met him through Richelieu.[47]

In 1647, Bosse published his explanation of Desargues' perspective method. In his Reconnaisance to this work, Desargues gives the most definitive statement of his views on the relation between geometrical theory and practical work:

> ...et auoue franchement que ie n'eus iamais de goust a l'estude on recherche, n'y de la physique, n'y de la geometrie, sinon en tant qu'elles peuuent serùir a l'esprit, d'vn moyen d'arriuer a quelque sorte de connaissance de causes prochaines des effets de choses qui se puissent reduire en acte effectif, au bien et commodite de la vie qui soit en vsage pour l'entretien et conseruation de la sante; soit en leur application pour la pratique de quelque art, et m'estant aperceu qu'vne bonne partie d'entre les pratiques des arts, est fondee en la geometrie ainsi qu'en une baze assuree; entre autre celle de la coupe des pierres en l'architecture,...[48]

> ... and (I) avow frankly that I have never had the taste for study or research, either in physics or in geometry, except in so far as it might serve the intellect as a means of arriving at some sort of knowledge of the proximate effects of things which prove capable of reduction to practical instruments for the goodness and commodiousness of life. which will reside in their use for the conservation of well-being and in their application to the practice

> of some art. And it is apparent to me that a good part of the practical methods in the arts are founded on geometry, and thus upon a solid basis; among others, that of stone-cutting in architecture, . . .

Here Desargues aligns himself directly with Descartes who expresses himself in such strikingly similar terms that one would assume Desargues had read Descartes' Discourse on Method in which the following statement occurs:

> But as soon as I had acquired some general notions concerning Physics, . . . I believed that I could not keep them concealed without greatly sinning against the law which obliges us to procure, as much as in us lies, the general good of all mankind. For they caused me to see that it is possible to attain knowledge which is very useful to life, and that, instead of that speculative philosophy which is taught in the schools, we may find a practical philosophy by means of which, knowing the force and action of fire, water, air, the stars, heavens and all other bodies that environ us, as distinctly as we know the different crafts of our artists, we can in the same way employ them in all those uses to which they are adapted, and thus render ourselves the masters and possessors of nature. This is not merely to be desired with a view to the invention of an infinity of arts and crafts which enable us to enjoy without any trouble the fruits of the earth and all the good things which are to be found there, but also principally because it brings about the preservation of health, which is without doubt the chief blessing and the foundation of all other blessings in this life.[49]

Thus, both men share the view that all inquiry is to be motivated by and directed toward an instrumental objective which is to develop tools and techniques that will improve the lot of mankind.

Desargues concludes his acknowledgement with a statement about the kind of person who may properly judge of his work. Because this passage is of such very great importance in understanding how practice begins to be thought of as controlled by a technology which is developed and dominated by the savant, it is here quoted at length. It is, says Desargues:

> . . . gens d'authorite, non suspects, et bien entendus en la geometrie, qui seuls peuuent estre juges capables de ces choses, et non pas les massons, comme il voudroit faire acroire, en quoy son humeur peruerse ne veut pas seulement affronter le public; mais aussi contredire la verite mesme, en ce qu'elle a prononce que la disciple n'est point dessus son maistre. Car non plus que les medecins, pour se rendre scauants en leur profession, ne vont n'y a l'ecole n'y a la lecon des apoticaries qui effectuent leurs ordonnances; mais au contraire les apoticaires pour se rendre capables de leur profession, vont a l'ecole et a la lecon des medecins, en quoy les medicins sont maistres, et les apoticaires disciples; aussi les geometres, pour s'auancer en cette science, ne vont n'y a l'ecole, n'y a la lecon des massons, mais au contraire, les massons pour se rendre habiles aux traits geometriques necessaires a la pratique de leur art, et deuenir plus capables de faire chef'd'oeuure pour leur maistrise, vont a l'ecole et a la lecon des geometres, en quoy de mesme, les geometres sont maistres, et les massons disciples, et estant question de juger, si une ordonnonance de medecine est bien conceue dans les lois de cette science, il ne serait pas plus ridicule de proposeret soustenir qu'il faut des apoticaires et non des medecins, pour en juger, sous prétexte que ce sont les apoticaires qui préparent les drogues, et mettent les ordonnances des medecins à exécution; qu'il est extrauagent de dire et soustenir qu'il faut des massons et non des géometres pour iuger de la précision et briefueté demonstratiue d'un trait géometric, pour l'apareil de la coupe des pierres en l'architecutre, sous prétexte que ce sont les massons qui manuellement

tracent, posent et massonnent lesdites pierres, ou qui aprennent de memoire et effectuent les regles de la pratique du trait, que les geometres ont inuentees a cet effect.[50]

. . . men of unimpeachable authority and well learned in geometry who alone can be capable judges of these matters, and not the masons, as he (Curabelle) would have it, and in the proposing of which, his perverse humor not only insults the public but also contradicts the truth itself, for the disciple is never above the master. For it is not the doctors who, in order to become expert in their profession, attend either the schools or the lessons of the apothecaries who carry out their orders. On the contrary, it is the apothecaries who, in order to render themselves skilled in their profession, attend the schools and lessons of the doctors, where the doctors are the masters and the apothecaries the disciples. Likewise, the geometers, in order to advance themselves in that science, attend not the schools and lessons of the masons. On the contrary, the masons, in order to render themselves skilled in geometrical matters necessary to the practice of their art, and in order to become more capable of cutting the master's piece for their masterships, attend the schools and lessons of the geometers, in which they are the masters and the masons the disciples. And when it is a question of judging whether a prescription of medicine is well conceived according to the laws of this science, it would be ridiculous to propose that it is the apothecaries and not the doctors who should be the judges on the pretext that it is apothecaries who prepare the drugs and carry out the prescriptions of the doctors. In the same way, it is extravagant to say that it is the masons and not the geometers who should judge the precision and conciseness of a geometric treatise on the cutting and dressing of stones in architecture on the pretext that it is the masons who manually trace, cut and place said stones, or who memorize and implement the rules of the method which the geometers have invented for this purpose.

From the modern point of view about the relation between theory and practice in architecture, Desargues' position will certainly seem to be correct and just, for we have become thoroughly habituated to this way of viewing the matter. However the opposition which Desargues received from Curabelle and others indicates that in his own time, the domination of practice by the geometer-architect was not widely accepted. Desargues' efforts to instrumentalize geometry represent the beginnings of a fundamental change in the attitude toward science in Western culture. In the concluding chapter of the present work, this change, which gave us the technology we have today, will be the subject of an extended study.

THE ARCHITECTURAL ACTIVITIES OF DESARGUES

The year 1648 was something of a turning point in the life of Desargues. It was at this time that he returned to his natal city (Lyons) where he appears to have made his residence during most of his remaining years. From this time on, there is also a marked decrease in publication. At the same time, he seems to have increased his design and building activities, though, unfortunately, it is not always possible to assign a date to these. To what extent these changes may be attributed to his struggles in Paris cannot be determined on the basis of existing information.

The architectural activities of Desargues fall into two classes. The first consists of the buildings or parts of buildings which he himself designed; the second, of his contributions to method and graphic representation. There are a number of references in old sources to architectural works which were built according to Desargues' designs (see Appendix II below). These structures however all appear to have been either destroyed or altered beyond recognition, though site-visits, which could not be undertaken for this writing, might lead to some unexpected discoveries. The main sources of information about Desargues' architectural activities are the various works of Bosse on architecture and perspective. In some cases, Bosse specifically credits Desargues as the source of an idea, design, or technique. In other cases, it is possible to be reasonably certain of Desargues' authorship.

To begin with the executed works, we may cite the house for Monsieur Roland (Fig. 1-1) in the rue de Clery in Paris.[51] The plan shows the ground floor and a large, interior courtyard which can be reached from the street through a passage below the first floor. The main entrance to the house is located in a corner of the courtyard, where a flight of steps leads into an oval vestibule. At the back of the vestibule, a central stair rises to a landing at which the stair divides into two flights which carry up to the second level. The number of steps suggests a height of about 16 feet for the lower level.

The scale on the drawing (in toise)[52] indicates that the courtyard is approximately 42 feet square. The square containing the stair is about 27 feet on a side. Bosse also illustrates the stair of this house in his <u>Traite des Maineres de dessiner les Ordres</u> (Treatise on the methods of designing the architectural orders) (see Fig. 1-2). Bosse points out that the stair was not actually built as he shows it:

> Le plan d'Escalier cy dessoubs fig'4 est de la pencee de feu Mons' Desaruges, lequel a este construit dans un Bastime nouf, an quartier Mont-marthe rue de Clery, non tout a fait semblable, car pour faire la rampe du millieu plus large e on a rendu celle des Costez plus esbroites, Et les Paliers Ireguliers, ainsi que celuy fig'f; et les marches de la rampe du millieu courbes comme fig'd, Et en fin, d'autres particularitez de telle nature, que ledit Desargues desaprouuoit, lesquelles ny seroient pas, sans quelque mestinteligence, qui fit que le bourgeois en consina la conduite a des personnes qui n'entendoiènt a fonds cette matiere.[53]

> The plan of the stair below in fig. 4 (see my Fig. 1-2) is the idea of the late Mr. Desargues which was constructed in a new building in the Mont-marthe quarter, rue de Clery. It was not constructed exactly as shown because, in order to make the flight in the center larger, it was necessary to render those at the sides more compressed and the landings irregular as shown in fig. f; and the steps of the middle flight were curved as shown in fig. d. And finally (it was also changed) in regard to other particulars of a like nature of which the said Desargueswould have disapproved, and which would not have occured were it not for the misunderstanding which resulted when the owner entrusted the construction to persons who did not understand the fundamental principles of the matter.

What emerges from Bosse's text begins to be an indication of the kinds of concerns Desargues had in architectural design. From this and other remarks, it is possible to reconstruct something of Deargues' conception of architecture. A basic concern which Bosse's criticism here and elsewhere implies is that architecture must have a clear goemetry which is revealed without impairment in the built form. Any compromise which sacrifices the clarity of the goemetric expression is to be rejected. Here again we also have a clear statement of the way in which the architect's geometry is to dominate practice. It is not permissible for the workers to alter dimensions in order to faiclitate construction as they did in this case because Desargues was not able to supervise the work. These concerns are also evident in Bosse's discussion of the proper treatment of balustrades (Fig. 1-3). Bosse shows three staircases in his plate. Of these, the one on the right (Bosse's Fig. 1) shows all of the "errors" he is concerned to correct. The two on the left (Bosse's Figs. 2 and 3) show the balustrade correctly handled. The caption and text to Bosse's Fig. 1 read:

> Le manuuais Effet des ressaultz ou ruptures, manque dentendre ce que deuant.
> Plusieurs ont voulu depuis quelques années coriger ces resaults duBas et Cime dajpuy a et b, mais itz ne l'ont pu faire, qu'en rendant les pilastres ic et

ba jnegaux en hauteur, ou bien l'ajpuy et les balustres, et des jregularitéz aux paliers, ou des marches jnegales en hauteur et largeur fig. 1.[54]

The unpleasant effect of breaks or ruptures, defective in conception, here as before (in the text).

For several years, some have desired to correct these breaks in the base and rail at (a) and (b), but they have not been able to do it except by making the pilasters (ic) and (ba) unequal in height or else by making the bases or the balusters unequal, or by transferring the the irregularities to the landings, or by making the steps unequal in height and width as in Fig. 1.

There then follows a passage in which it is explained that a lack of space is no justification for bad design or corrupt geometry:

Ceux qui disent que cette pratique estoit connüe de tout temps, et que ce quelle ne sest pas pratiquée'c vient du manque de place, seront s'il leur plaist cette reflexon, qu'encore quil n'en manquast, ny a Luxembourg, ny au Palais Cardinal, ny en une infinite d'autres splandides Bastiments, on n'a laisse dy faire ces fautes, et au contraire feu Mr. Desargues les a fait cuiter par sa conduitte a l'hostel de l'hopital, et en des lieux assez reserréz, et de plus aussi, Mr. Rougetz professeurs en l'Art de Bastir, ausquels jávois expliqué cette facille pratique.[55]

Those who say that this practice was characteristic of all times and that they continue to use it when there is a lack of space could make, if it pleases them, this reflection: neither at Luxenbourg nor at the Palais Cardinal, nor in an infinity of other splended buildings were these faults permitted. On the contrary, the late Mr. Desargues eliminated them through his supervision of hall of the Hospital and in places equally restricted, as likewise did Mr. Rougetz, professor of the art of building, to whom I have explained this fine practice.

Taken as instruction for architects, this discussion would certainly appear to be an instance of Bosse's tendency to excessive didacticism. At this time, certainly, any architect of significance could lay out a stair in a way that would avoid unequal steps and the grosser irregularities which Bosse censors. At the same time, however, we must remember that Bosse's text is directed primarily towards the practical workers rather than the architect. His aim is to insure that the geometry of the architect's plan is carried out in practice by workers who might be willing to sacrifice geometry to the exigencies of construction. Here we might also consider that one of the motivations behind Desargues' seemingly altruistic concern to aid the common worker was, quite possibly, the fear that, unless he did so the precise geometrical relations he wanted would fail to appear in the constructed work and that, as a result, what was actually built would be taken as further evidence that his geometrical theory was faulty.

Bosse's criticisms also imply a preference for the plastically continuous forms which were beginning to be popular in his time, but which were by no means as universal in France as in Italy. Thus the 1642 stair by Mansart in the Chateau of Maisons (Fig. 1-4), for example, shows just the change in height at each turn which

Desargues and Bosse object to.[56] To justify his preference for continuous balustrades, Bosse argues that the French, unlike the ancient Greeks, are obliged to attend to such matters because the French must build in more compact spaces:

> On ne voit point que les Grecs ayent fait de si beaux Degrez que ceux que nous faisons et pouuons faire a present, la cause estant comme je croy, que leur usage et commodité de Bastir ne les y obligeoit, car le plus beau et majeshieux de leurs Bastiments, estoit in l'Exterieur et en leurs bas departments, lieux que nous nommons Portiques, Courts, perrons, vestibules, Salons, Sales, &c; Mais comme neus faisons d'ordinaire de trés Riches departments en nez premiers Estages, cela nous oblige d'en faire qui y coniuennent, Estánt sans contredit l'on des considerables membras ou partie des Bastiments, qui ne fait pas que l'on ny en fasse d'autres petits, pour seruir de decharges ou de degagements.[57]

> One does not find that the Greeks have made such fine stairs as those which we make and are able to make at present, the reason being, as I believe, that their custom and the capaciousness with which they built did not oblige them to it, for the finest and most majestic (aspects) of their buildings consisted of the exteriors and of what we call the proticos, courts, entry stairs, vestibules, salons and great halls of their lower floors. But, as we customarily make very rich quarters in our first floors, this obliges them to make what is brought together there in such a way that there is no contradiction between the considerable number of building parts.

The argument here is interesting. Bosse obviously shares with his contemporaries the view that the art of Greece represents an ideal to be emulated. At the same time, he knows that the Greeks never treated balustrades as he proposes to treat them. No matter; the Greeks would have done the same thing, if it had been their custom to build in confined spaces as the French do.

Many of the staircase designs which Desargues undertook were small-scale projects which lent themselves too and in fact demanded treatment as unified wholes rather than as articulated complexes of connected parts. We are indeed fortunate to have Bosse's engraving of just such a project (Fig. 1-5).[58] The entry stair to the Chateau de Vizille is a geometrical tour de force which is scarcely comprehensible in Bosse's engraving. Desargues' design surely approaches the lower scale limit of what is possible in a masonry balustrade. The text reads:

> Apres que feu Monsieur Desargues eut donne l'Order de ce Perron, estant a Paris, (..) ayant a l'Hostel de Thurenne de cette Ville, un Escalier a abattre contraint par la place et Estage, il le fit construire de la mesme sorte, ne le pouuant mieux ce qui est neammoins tres estime, des entendus en ces sortes d'ouurages. A coste Fig'A est en perspective un coin de ce Perron eu l'on voit que l'arrestede lajpuy (edfg) rampe parallelem a celle des Marches, et tous les membres d'entre elle, son juperfecon n'est que en ces Saults de (de), (rf) sus l'apuy.[59]
>
> After the late Mr. Desargues had designed this stair, being at Paris, he there had a stair brought down at the Hostel de Thurenne. Constrained by the place and the height, he even so had it constructed in the same way, not being able to improve upon it. Nevertheless, it is much esteemed by those who are knowledgeable in this kind of work.
> At the side in Fig. A is a perspective drawing of a corner of this stair. One sees that the railing (cdfg) slopes parallel to the steps and that among all of the parts, the irregularity consists only of the small jumps between (de) and (rf).

What Desargues has done by means of a clever beveling of the handrail is to eliminate the abrupt change in level which would otherwise occur between the two sections of balustrade. The exact nature of this joint can only be guessed at from Bosse's illustration, so I have made a more readable reconstruction of it in (Fig. 1-5-a). As a result of Desargues' design, the outer edge of the handrail forms a continuous line around the turning of the stair. This small detail is in fact a symbol or icon of the principle of oblique projection which is fundamental to descriptive geometry. As we shall see in the following chapters, this oblique, triangular stone might well serve as a summary statement of Desargues' discoveries in the area of projective geometry.

Bosse's engraving shows that he had a great deal of trouble with the perspective representation of Desargues' corner. It is important to realize that here we have an example of the gap which still existed in his time between drawing and geometry. Desargues' system of stereotomy made it possible to construct solids which could not yet be accurately represented in perspective except by drawing from the already-constructed object. It only became possible for theory to dominate practice when the stage had been reached at which the architect could describe (draw) and thus prescribe in drawings any buildable form.

For this, the complete system of Mongian descriptive geometry was necessary. Prior to it, as Bosse disdainfully suggests, what could be built was in part often determined by the bricolage of the builders at the site. This situation is is no way comparable to those modern cases in which, due either to some oversight in the plans, or to a failure to undestand the working drawings, builders will sometimes improvise a solution to a problem. Desargues, in his time, was very much closer than we are to that period in which a degree of improvisation was the norm, and was so at least partly because it was not always possible to draw what was desired. Today we can perhaps scarcely comprehend the risks taken in the cause of architecture when building was the primary means of finding out what could be built.

One of the most remarkable of Desargues' contributions to graphic technique in art and architecture is his system of perspective. This is given a full analysis in Chapter III of the present work. Bosse himself was well-versed in perspective and in fact taught a course on the subject at the Academie Royale. His tenure there, however, was terminated in 1661 following a dispute over the merits of Desargues' method which he had been teaching in his classes. Again the issue, or at least one of them, seems to have been the relation between theory and practice. Bosse took the position that Desargues' method was both theoretically sound and a sufficient guide for practice.[60] His opponents maintained that no theory could, by itself, be a complete and satisfactory guide.[61] All of this begins to sound very much like Desargues' dispute with Curabelle (cf. p. 22). Here again we encounter what might be characterized as a conflict between Cartesian rationalism and the more popular practicalism of the time. The suppressed premise, if one may call it that, in the arguments of Bosse's opponents is that man is not, by nature, perfect, and so no purely geometrical theory can give an adequate account of his true nature. Applied to the specific case of perspective drawing, this would mean that visual experience, as represented in a picture, has no exact mathematics or geometry. Thus, all theories must be qualified by practice, through which the exceptions to the rules are discovered. Desargues, for his part, was so convinced of the correctness and excellence of his method that he offered a prize of 1,000 francs to anyone in the Academie who could devise a method better than his own.[62]

In addition to the perspective method itself, Desargues developed several techniques for checking and implementing architectural designs. One of these is shown in (Fig. 1-6). This plate which is also from

Bosse's Traite des Manieres de dessiner les Ordres[63] shows (at the top) a church entry designed by Desargues which is said to employ the Corinthian order. This plan, and the drawing below it, are used to illustrate a method for checking the visual effect of a facade. It is suggested that the architect should view a large drawing or model of his proposed facade from the vantage points an observer would occupy when looking at the real building. The objective is to have a means of making adjustments in the proportions according to what will actually be seen. This, of course, touches upon the same issue already raised in the Renaissance by Alberti and Palladio, concerning whether the dimensions established by proportional systems should be adjusted to compensate for the angle of view.

LESSONS FROM SHADOWS

The writings of Desargues' contemporaries contain a number of references to an earlier work of his which has since been lost. This was said to have been called Lecon de Tenebres[64] (Lessons from Shadows) because, in it, Desargues apparently made extensive use of the geometry of shadows to illustrate the principles of projection. It is likely that Bosse used some of these ideas in his later works. As an example of the probable content of this lost work, we cite Bosse's discussion of methods for projecting paintings and drawings onto curved surfaces such as cylindrical and spherical vaults. This is essentially a species of anamorphic perspective projection. At this time, anamorphic perspective was beginning to become a popular form of "perspective magic" about which more will be said in the concluding chapter of this work. In architecture, the need for anamorphic perspective developed with the desire to have a means of accurately projecting scenes conceived in the flat onto the curved ceilings of large rooms. A plate from Bosse's _Moyen universel de pratiquer la perspective_ (Universal perspective practice)(1653) shows one method of projecting a perspective grid onto a cylindrical vault (Fig. 1-7). As if to illustrate a "lesson from shadows," the candle is used as a point source of light which projects the shadows of the strings (fm) and (fn), etc. onto the surface of the vault.[65] While the details of this method are too complex to discuss here, it will be seen that the strings themselves are meant to represent a (partial) grid of squares hung vertically, and thus seen in perspective by the viewer looking up from point (o). When this system is used to project a painting conceived in the flat, the observer at (o) will experience the illusion that the scene rises vertically from the impost line of the vault (gc), even though the scene itself is actually painted on the curved surface of the vault. The illusion will only be complete for an observer standing at (o). An example of this delightful deception is shown in (Fig. 1-8).[66] The reader is left to determine where the real architecture ends and the painting begins.

Though Desargues himself never discusses anamorphic art in those works of his which we have, there are a number of rather curious circumstances which connect him with this form. The first of these consists of the attribution to him of what appears to have been an anamorphic projection very much like the example in (Fig. 1-8). According to Piganoil, this was in the Old Church of The Carmelites in Paris, which had been designed by Philippe de Champaigne. Piganoil tells us that:

> Les curieux et les connaisseurs, regardent avec une attention particulière un morceau de Perspective dont Desargues, habile mathématicien, avait donné le trait à Champaigne; c'est un crucifix entouré de la sainte Vierge et saint Jean. Ce groupe paroît être sur un plan vertical quoiqu'il soit sur un plan horizontal.[67]

> The curious and the connoisseurs noted with particular attention a bit of perspective which Desargues, able mathematician, has given to the work of Champaigne. It is a crucifix framed by Saint Vierne and Saint Jean. The group appears to be in a vertical plane, although it is (actually) in a horizontal plane.

The church was completed in 1628, which could make this Desargues' first involvement in an architectural project. The date also coincides approximately with the time during which he appears to have been working on the Lessons from Shadows. Taken together, these events suggest that Desargues' remarkable discoveries in projective geometry may have had their origin in experiments with anamorphic perspective.

A second curiosity is that of the two French writers who, at this time published extended works on anamorphic perspective, Jean Francois Niceron (1613-1646), in 1638 and 1646, and Jean Dubreuil (1602-1670), in 1646, both mention the work of Desargues in their publications.[68] Further, both men were, like Desargues, living in Paris, and Niceron was, like Mersenne, a member of the order of Minimes and was, in fact, one of Mersenne's students.[69] Desargues' quarrel with Dubreuil has already been discussed. What adds yet another curiosity to the list is the fact that both began with favorable veiws of Desargues, and both reversed them in the wake of the disputes about Desargues' methods. Niceron's position on Desargues was again reversed when Mersenne posthumously edited his work for republication.[70] While all of this is evidently circumstantial, it does suggest an important if unclarified connection between Mersenne, Desargues, Niceron and the development of anamorphic perspective in France.

The discussion of Desargues' architectural activities raises once again the question of his primary line of endeavor and source of income. If we have anything approaching a complete record of his work in architecture, he could not possibly have derived a sustaining income from his work in this area. Likewise, it appears that his tutoring activities were hardly extensive enough to be a major source of revenue. A few documents pertaining to his later years suggest, not as some writers have proposed, that he was a practicing engineer, but that he was a kind of savant-inventor of ingenious mechanical devices. Further, it would seem that if Desargues did indeed have significant practical experience in civil engineering, he almost certainly would have reported this to his own advantage in the course of defending himself against his critics. Most probably, therefore, his main income was either from investments or from some sort of pension through Richelieu for services rendered during or after the siege of La Rochelle, as already suggested.

Foremost among the indications of Desargues' ability to invent machines is the design attributed to him of a pumping mechanism for the water fountains at the Chateau de Beaulieu. This is said to have employed an epicycloidal wheel, according to the descriptions given by Christian Huygens (1629-1695) who saw it in operation and by P. de la Hire who repaired it. In a letter to his father, Huygens says:

> Pour des fontaines il n'y en a point, que par les moyens de pompes, qui vont par une belle machine de fabrique de Monsieur des Argues. Un mulet y fait tourner une grande qui par le bas, est taillée ondes qui en passant sur un rouleau le font baisser et hausser, et en mesme temps le bras auquel est attaché le piston de la pompe. De sorte que n'y ayant aucune roue à dents, cela fait que l'entretien de la machine couste très peu.[71]

> For the fountains they can get nothing except with pumps which operate by means of a clever machine designed by Mr. des Argues. A mule turns a large wheel that is notched with teeth on the inside and which passes over a roller gear, causing the latter to move up and down, at the same time moving an arm which is attached to the piston of the pump. It is of the sort which has no paddle-wheel, and this explains why it is so inexpensive to maintain.

This attribution would make Desargues the inventor of the epicycloidal gearing system rather than Romer for whom Leibniz claimed it. Desargues is also said to have had a demonstration showing the impossibility of perpetual motion.

Near the end of his life, Desargues was again in Paris, though we do not know whether he was once more making it his residence. In 1658 he made a will there, though another had been drawn up at Lyons in 1957. In 1660, Constantin Huygens made note of a gathering in

Paris at which Desargues made a vigorous defense of his thesis that geometrical points have real existence:

> Le Frère mande de Paris qu'il s'est trouvé dans un assemblée chez Monsieur de Montmor ou il y avoit plus de 30 beux esprits ensemble, dans laquelle ne fut traité autre chose si non an punctum geometricum sit ens revera existens, ce que Monsieur des Argues que connoissez ayant soustenu par un long discours, il se suscita un adversaire qui se mit à luy contredire avec un furie si grande qu'à touts coups, il sembloit se mettre en posture de luy sauter au cou, et autre chose ne se traita pour lors.[72]

> Brother writes from Paris that he happened to be in a gathering at the house of Mr. de Montmore where there were more than thirty fine minds gathered in which no other subject was treated than whether an punctum geometricum sit ens revera existens (whether the geometric point is a really existing thing), which Mr. des Argues, whom you know, had sustained by a long discourse. At length, he aroused an adversary who started to contradict him with such great fury that all at once it seemed as if he were positioning himself to spring at this throat, and no other matters were discussed at that time.

One of Desargues' wills was opened October 8, 1661 at Lyons, but it is not known whether he died there or in Paris. In addition to the life annuity of 1,200 livre (yearly) for his brother Antoine, Desargues left 2,000 livre to his life-long friend and assistant A. Bosse.

CHAPTER II

DESARGUES' METHOD OF STEREOTOMY

A detailed exposition of Desargues' method of stereotomy is no simple matter to write or for a reader to follow. What makes such an exposition a matter of more than mere esoteric interest however is that it reveals the cause of the difficulties which Desargues' contemporaries had in understanding his procedures. Desargues' method is based upon the use of an oblique coordinate system in three dimensions, rather than upon the modern rectangular coordinate system. As such, his method depends upon a system of variable relations which are extremely hard to visualize or even comprehend. A detailed exposition will also make it possible to understand more clearly the transitional state of geometry in Desargues' time. The conceptual framework of geometry, as Desargues inherited it, still owed a great deal to Greek and Medieval cosmology. There was still a strong tendency to think of geometry in symbolic rather than analytical terms. While Desargues showed no desire to deal with the ancient tradition of geometry as a cosmological symbol, he did find himself obliged to struggle against the practical limitations which "the state of the art" imposed upon the development of a universal system of steretomy.

The NATURE OF STEREOTOMY

Today, stereotomy is a branch of descriptive geometry which may be defined as the practical science of constructing geometrical projections of all kinds. The basic aim of modern descriptive geometry is to enable a designer to draw the true shape of any surface of an object which stands in an oblique relation to both plan and elevation. By its very nature, descriptive geometry is thus predictive and prescriptive. It is a method for determining the precise shape of every surface of an object <u>before making it</u>. Thus descriptive geometry is also the antithesis of the mediaeval masonic tradition[1] so frequently criticized by Desargues and later writers on the art of stone cutting. Stereotomy, as Desargues conceived of it, would leave no shape to chance or to discovery by the hand in the act of making. For Desargues, a universal stereotomy was the key to the rationalization and prescriptive determination of architectural geometry. In his work, we find the door first opened to the modern view that <u>doing</u>, the making of the building, is not a justifiable form of thinking in architecture. Now it is to be the savant, as geometer-architect, who does the thinking--on paper. The workers are merely to carry out the savant's thoughts with absolute and unfailing precision just as the apothecary does when he fills the doctor's prescription.

STONE CUTTING PROBLEMS IN RENAISSANCE AND BAROQUE ARCHITECTURE

It will be helpful to precede the study of Desargues' method of stereotomy with a brief examination of the kinds of stone cutting problems which occurred in Renaissance and Baroque architecture. As we have said, the basic task of stereotomy is to find the true shapes of oblique surfaces. It is customary for the masons to cut a building stone from a rough block by using templates, as shown in (Fig. 2-1). The techniques of stereotomy are used to determine the precise shapes of these templates before cutting begins, so that, when the stones are assembled in a vault, arch, dome, or some other form of construction, they will fit perfectly without further cutting. Even when vaults were merely roughed out in brick and then plastered in order to arrive at the finished surface, the brickwork itself often required a wooden centering to support the construction. In such cases, the geometrical intersection problems occurred in the support structure rather than in the masonry itself, but they still had to be resolved with a certain degree of accuracy. This is one reason why all of the treatises on stereotomy mention the fact that the method can be applied to carpentry as well as masonry. It was, of course, Desargues' personal ambition to "err" on the side of completeness in order to better demonstrate the universality of his method, and, in that way, to establish for it the status of absolute and unqualified truth.

In (Fig. 2-2) a range of basic intersection problems given by Desargues' pupil Abraham Bosse are shown.[2] Among these, the spiral vault on an inclined spiral base plane is noteworthy (bottom right). This is also a case where wall stones might be laid on a slope rather than horizontal. The "trompe" shown in (Fig. 2-2, bottom left) and the two shown at the left in (Fig. 2-3) are basically alcoves cantilevered over an interior corner formed by the intersection of two walls.[3] This type of construction is said to have been a specialty of Desargues and would indeed have provided the occasion for a tour de force in stereotomy. The example on the extreme left in (Fig. 2-3) is in fact a conical vault which slopes downward in the direction of the corner. The middle figure in the same plate shows a variation in which the trompe has a projecting corner and a pointed quasi-Gothic vault. In some cases, the trompe was made even more elaborate by building it over a projecting, curved stairway. In such cases, the steps themselves formed the vault supporting the trompe. De L'Orme devotes considerable space in his chapters on stereotomy to just such a case, and, judging from the drawing (see Fig. 2-4), it would appear that he drove his engraver completely to distraction with it--a fact which once again points up the gap between theory and practice which existed in his time.[4] The reverse of the trompe is the coin or "quoin" vault--an arch in a corner where two walls intersect. An example of the quoin is shown on the extreme right in (Fig. 2-3). In cases like these, the axis of the vault usually bisected the angle between the two walls.

Intersections between domes and vaults of various kinds provided some of the most complex problems in stereotomy. Domes might be spherical or else ellipsoidal as shown in (Fig. 2-5).[5] When such domes were constructed of stone, it was necessary to determine the precise shape of each voussoir. In the case of the spherical vault, this is not too difficult since all of the stones in a single course can be cut from the same templates. In the case of the prolate ellipsoid (figure generated by rotating an ellipse about its long axis), however, the stones within each course are continually changing shape around the curve. Such domes require a much greater number of templates for their construction, and there is a much greater chance of error.

The torus or "doughnut" has also been used as an architectural form. A most remarkable example of this form is the incomplete torus vault of the roof of S.S. Giovanni e Remigio in Curignano (1755-1763) designed by Benedetto Alfieri (1699-?) which terminates at each end in a spherical dome (Fig. 2-6).[6] The illustration shows the swirling spectacle seen when looking back toward the main entrance from the altar. The small windows up in the vault are oval in oval dormers. Here the intersection between the dormer and vault presents the problem of determining the shape of the intersection between a torus and an oval vault.

As a last example, we might consider a somewhat fanciful study for a dome which appears in De l'Orme's _De l'Architecture_ (Fig. 2-7).[7] Here a spherical dome is combined with a spiral. The drawing suggests that the intention is to support the dome entirely by means of the ribs and the spiral, leaving the spaces in the skeleton open for glazing.

A paradox which becomes evident is that Gothic vaulting systems are often more complex than those of the Renaissance and Baroque, yet attempts at systematic methods of stereotomy do not appear until the Baroque period. Here indeed is a reflection which should give pause to anyone inclined to seek simple cause-effect explanations for historical developments. Clearly, Desargues did not develop his system of stereotomy because he felt called upon to do so by new and pressing problems in masonry construction. Rather, it would be more accurate to say that, because he felt a new and pressing need to <u>prescribe</u> precise shapes in masonry, he tried to develop a universal system of stereotomy. The difference here is one of <u>intention</u>. Gothic masons did not understand architecture as a process of building according to a set of "working drawings" in which every detail was precisely determined in advance of construction. Thus they felt no need for a universal system of stereotomy and it would be naive to explain the lack of such systems in the Gothic as being due to an ignorance of the geometrical principles required. True, they did not know the requisite principles, but that is not why they failed to develop a universal system of stereotomy. The full force behind any such development must be looked for in sources which run deeper. In the last chapter, the work of Desargues will be situated in the broader course of that destining which has transformed architectural geometry from a cosmological symbol into a technological tool.

DESARGUES' METHOD OF STEREOTOMY:

With this much established concerning the nature of stereotomy and the kinds of problems it was used to solve, it is possible to turn to a detailed study of the method proposed by Desargues himself. Since all of Desargues' terms are explained in the body of this text, I have not provided a glossary for his words, however, the reader who is unfamiliar with the basic terms for the parts of a vault may wish to consult Appendix III where they are illustrated. What Desargues was aiming for was a method which would be universal, which is to say, one which would make it possible to solve all problems in stereotomy by means of a single set of operations. There is, of course, a price to be paid for the universality of a method, and we have already seen how Desargues was criticized for proposing a method "too chimerical and extravagant to hold a rightful place among those used by practicing masons." As will be shown, what, in a sense, justifies this criticism is that Desargues' method is vastly counterintuitive. His method is, to be sure, deductively sound and is in fact a master-piece of geometrical reasoning. But it is the very complexity of the reasoning involved which makes it less than ideal as a practical tool for the mason at the building site. One might say that it is a geometer's method rather than a mason's, and here too the conflict between theory and practice is evident. In most instances, it would probably have been easier for a mason to memorize the several case-by-case methods needed to do the same job. The older methods were certainly easier to visualize without the aid of complex drawings. Indeed, in order to explain Desargues' method for the reader of the present study, it has been necessary to draw seven fairly complex perspective illustrations as a supplement to the drawings given by Desargues and Bosse. This is not unusual in a theoretical text on descriptive geometry, but it is unusual for a practical method seriously intended for use by workers at the building site. As late as 1930 practical handbooks on stone cutting for masons continued to be published, and they were indeed <u>practical</u> handbooks. For example, the sixth edition of <u>Practical Masonry: A Guide to the Art of Stone Cutting</u> by William Purchase still treated the subject without any use of descriptive geometry at all!

In 1643 Abraham Bosse, engraver and pupil of Desargues, published an explanation of his teacher's method of stereotomy which was quite evidently an attempt to counteract the criticism to which Desargues had been subjected, partly as a result of his own brief 1640 publication of the method.[8] Because of its compactness and because Desargues did not explain the geometrical principles upon which his method depended, his own exposition is hard to follow, even for someone with a background in descriptive geometry and a favorable disposition toward the author. Bosse, on the other hand, is clearly trying to write for the practical mason whose main concern is with the practical problems of stone cutting rather than the theory of descriptive geometry. Thus Bosse proceeds a step at a time through the method, giving a separate illustration for each operation and using special symbols to indicate what information is carried from one plate to the next. So great is his concern to remove all possible grounds for criticism that at one point he even cautions the reader that, inasmuch as the pages of the text have been printed on both sides under different conditions of temperature and humidity, slight variations in line length from plate to plate may be found.[9]

Because of the systematic nature and clarity of Bosse's exposition, it is followed here in explaining the mechanics of Desargues' method. My procedure has been to use the illustrations from Bosse's text wherever possible and to supply brief explanations in the language of modern geometry to go with them. These explanations are in no sense translations or paraphrases of Bosse's text, though they do cover the essence of his directions for each procedure. At some places in the sequence, I have omitted steps where it has appeared that Bosse is moving more slowly than necessary, or where he digresses to cover alternative procedures which are not essential to an understanding of the basic method. For the sake of compactness and continuity, as many steps as possible have been grouped together on a single page. Where the operations become more involved, It has been necessary to increase the size of the format to accommodate them.

After the method has been explained, it is next compared to modern descriptive geometry in order to determine to what extent the procedures developed by Desargues are the same as those of the generalized system of descriptive geometry which was finally codified by Gaspard Monge some 150 years later and which has remained substantially unchanged to the present day.[10] This represents only the first stage of the investigation of the geometrical principles behind Desargues' method. In the following chapter the study of these underlying principles is completed by means of an examination of the relations between Desargues' methods of stereotomy and perspective construction, and his study of conic sections. The present chapter concludes with an examination of Desargues' own exposition of his method of stereotomy. The discovery, in 1943, of a complete printed copy of the method of stereotomy now makes it possible to correct certain errors made by Poudra in his 1854 reconstruction of the method from a handwritten copy without plates which had been made by La Hire.

Bosse developed his exposition in close association with Desargues, and a comparison of their writings on stereotomy indicates that Bosse is giving an accurate account of Desargues' ideas. In some places Bosse provides alternative procedures which are closer to older methods of stereotomy such as that of De L'orme, but in most cases he indicates that Desargues approves of the alternatives.[11] It is not of course always possible to tell whether any idea originated with Desargues or whether it was invented by Bosse and approved by Desargues. In some cases, Bosse's exposition covers details which Desargues does not discuss anywhere in his own writings, and thus we cannot determine for certain whether their treatment was, in every instance, the invention of Desargues himself. This however is a point which need not detain us, since it is clear that the basic principles of the method are due to Desargues, being ultimately based upon that same remarkable study of conic sections which gave us two of the most fundamental theorems of projective geometry--the theorem of involution and the theorem of perspective triangles, about which we will say more in the following chapter.[12]

THE DESCRIPTIVE SYSTEM OF DESARGUES' METHOD

First it is necessary to become familiar with the coordinate system Desargues uses to specify the key relationships of any arch or vault for which the shapes of the templates for the voussoirs are to be found. This requires the introduction of a few terms which have not yet been discussed. In (Fig. 2-8), which reproduces one of Bosse's introductory plates, three different kinds of vaults are shown.[13] Throughout this plate and the following, corresponding points are assigned the same letter or number. The top figure illustrates a horizontal vault; the middle a vertical "vault" (ie: a tower), and the bottom a vault which slopes from one end to the other. The oddity of considering a tower as a kind of vault stems from the fact that for the purpose of stereotomy, towers and vaults may be considered identical. In the case of the top figure, both face planes (FPOVQL and fpovql) are skewed. The face plane FPOVQL is also sloped. In the middle figure, both face planes are horizontal. In each figure, there is a line EAa running the length of the vault. Desargues calles this the axle of the vault. Just as the axle of a wheel is always parallel to the rim surface, so too, the axle of a vault is always parallel to the intrados and extrados surfaces. In Desargues' notation system, the axle is always assigned the ellipse as an identifying symbol which marks each end.

The numbers 23456 in the top and bottom drawings of (Fig. 2-8) identify a plane which Desargues calls the plane of the right arch. This plane is always perpendicular to the axle of the vault and represents with one might call a "true" cross-section of the vault. In the case of vaults which slope from one end to the other, the plane of the right-arch will not always be vertical because it is perpendicular to the axle which follows the slope of the vault. The bottom figure in Bosse's plate is somewhat misleading because it does not look as if the plane of the right-arch is perpendiular to the axle.

The axle of the vault also lies in an imaginary vertical plane called the route-plane, so-named because it represents the route one follows in going through the vault. In (Fig. 2-8), line AO represents the intersection between the route-plane and the face-plane of the vault. Lines FPQL and fpql are called the level in the face. As the name indicates, these lines are level across the face of a vault and define a horizontal plane which Desargues calls the level-plane. The level lines also represent the intersection between the face-plane and the level-plane. The level-plane must not, however, be imagined as extending between the two level-lines at opposite ends of a vault. This is because, in the case of a sloping vault, as shown at the bottom in (Fig.2-8), each level line will occur at a different elevation; thus a plane connecting them would not be level. The plane which follows the slope of the vault (DCcd), Desargues calls the path-plane. In (Fig. 2-9), all of these reference planes are shown. If the vault floor has no slope, the level-plane and the path-plane will occupy the same position.

Desargues' method requires one further plane which is also shown in (Fig. 2-9). This called the plane perpendicular to the face and level planes. As the name implies, this plane stands perpendicular to both the face-plane and to the level-plane. In the case of a vault in which the face-plane is not skewed, the plane perpendicular to the face and level planes will occupy the same position as the route-plane.

It is necessary to note one further feature of the system of reference planes which Desargues points out in his writing. All of these planes may be imagined as moved to any place in the work, as long as they remain parallel in the new positions to the positions they held formerly.[14] In discussing the definition of the level plane by means of the level line in the face, we saw one instance of this phenomenon. Since, in the case of a vault which slopes from front to back, the level line in the face will occur at a different height within each face, in effect, two different level-planes have been defined. Since, in Desargues' method, such duplications of planes are unnecessary, he chooses to think rather of a single plane which has been moved to a different position in the work--in this case to a different elevation.

We turn next to Desargues' system of angle measurement, by means of which the disposition of a particular vault is specified in preparation for doing the stereotomy. The angles to be measured occur between the various planes which have been defined above. In (Fig. 2-10), which reproduces another of Bosse's plates, we see three possible relations between the path-plane, the face-plane and the slope of the vault itself relative to level.[15] In the illustration at the top, the path-plane is level and the vault has no slope. The points HAH define the face-plane, and points BAN define the level-plane. Line BAB is the level line in the face. In the two illustrations at the bottom, the path-plane and the vault both slope. This is the kind of vault one might find over a stairway, and for this reason Bosse has shown the outline of steps on the profile of the path-plane. Angle HBD is the angle between the face-plane and the path-plane, as measured in the plane perpendicular to the face and level planes (rather than in the route-plane, as might be supposed). Angle HBG is the angle between the face-plane and the level plane, again measured in the plane perpendicular to the face and level planes. Angle PAN is the angle between the level line in the face BAB and the route-plane AN, as measured in the level-plane. Angle PAC is the angle between the level line in the face BAB and the route-plane, but measured this time in the path-plane instead of in the level-plane.

It is necessary to point out that in (Fig. 2-10) the slope of the vault is incorrectly shown. In this figure, the slope is accounted for by the slope of the top joint surface of the first arch stone above the impost. This would create inordinate and pointless difficulties in finding the template shapes for the voussoirs of the arch.[16] In the method which follows, as well as in

Desargues' text, the slope occurs on the impost surface itself.

In (Fig. 2-11), another of Bosse's plates is given[17]. The top two illustrations show three archways through a wall as seen in plan view, with the section occuring at the level of the impost. In each case, the angle PAN is again the angle between the level line in the face and the route-plane, as measured in the level-plane. In Figures 2 and 3 of this plate, the passage through the wall is oblique to the faces, and this results in an archway with skewed face planes. The bottom three figures merely show three possible conditions of slope or batter in an exterior wall and the measurement of it by means of the angle HBG.

THE SAMPLE PROBLEM

Before introducing the procedures for doing stereotomy following Desargues' method, it will be helpful to have a clear statement of the problem which will be used to illustrate the method. Bosse does not give a preliminary description of the problem, feeling, apparently, that the problem will be easier to understand if it is explained a step at a time along with the method. In (Fig. 2-12), I have illustrated the problem with a perspective drawing. We are dealing with an oblique passageway through a wall of which the face toward the viewer is battered. In addition, the path-plane and the vault over the path both slope at the same angle, with the higher elevation being on the side of the wall toward the viewer. In the illustration, the wall through which the vault passes is shown as a transparent volume so that the position of the archway within it may be seen. Perspective grids have been drawn on some of the planes to make the relationships easier to see.

One point which it is particularly important to note about the sample problem is that the shape of the arch on the face-plane of the wall is an arc of a circle. This means that a section through the arch in the plane of the right-arch (ie: perpendicular to the axle) will yield an arc of an ellipse. This follows from the fact that the archway is part of a cylinder which is elliptical in cross section, and of which the face-plane represents that oblique section of the elliptical cylinder which produces a circle. (For any elliptical cylinder, there are two angles of section which produce a circle.)

Three important angles of the method are shown on the level plane in front of the vault. Angle PAN is the angle between the route-plane AN, and the face-plane PA, as measured in the level-plane. Angle HBG is the angle between the face-plane HB and the level-plane BG. The

Greek cross (✠) is Bosse's symbol for the right-angle. Here it represents the fixed right-angle between the face-plane and the plane perpendicular to the face and level planes. A fourth angle of importance, which could not be shown clearly in the perspective, is that between the face-plane and the path-plane, as measured in the plane perpendicular to the face and level. This can easily be imagined by looking at the perspective drawing. In the method which follows, it will be labeled HBD.[18]

THE PROCEDURES OF THE METHOD

Normally, one would begin with the templates for the montants, but since this is one of the more difficult parts of the method, we begin instead by finding the templates for the vault itself.

The drawings necessary to follow steps one through eleven are shown in (Fig. 2-13). This page can be folded out along side the book, as an aid to following the text.

STEP 1: The angles at the top of the drawing have already been explained. They represent the given information. Draw a line such as PB to represent the level-line in the face of the vault as seen in plan view. Draw a second line AN to some convenient point in PB to represent the route at the level. The angle PAN is then the horizontal angle between the face-plane PB and the route-plane AN as represented on the level-plane.

STEP 2: Draw a line NB perpendicular to the level line in the face AB so that it intersects the route at the level AN at some convenient point N. The line NB represents the plane perpendicular to the face and level planes as it appears on the level-plane, as seen in plan view.

STEP 3: Take the angle HBG, which is the vertical angle between the level-plane and the face-plane as seen in side elevation, and place it so that the line HB of the face-plane coincides with the line NH representing the route-plane, and so that the apex of the angles lies at point B. Now you have a composite drawing which is part plan view and part side elevation.

STEP 4: Take the angle HBD, which is the vertical angle between the face-plane HB and the path-plane BD, as seen in side elevation, and place it so that its apex coincides with point B, and so that the leg HB representing the face-plane coincides with the line HN. Note that the side-elevation of the path-plane BD will appear above the side elevation of the level-plane BG because we see both only as extended beyond the front face of the archway HB.

STEP 5: Take the length BN with the dividers, and turn the arc NG about the point B so as to transfer the length BN in the plane perpendicular to the face and level planes to the route-plane BG.

STEP 6: Draw a line GD perpendicular to the level-plane BG so that it intersects the path-plane BD as at point D.

STEP 7: From the point D on the path-plane, construct line DH perpendicular to the line of the face-plane NBH. All of the work done so far has been for the purpose of finding this point H.

STEP 8: Through the points A and H draw a straight line AH. This line will represent the sub-axle in its proper position, as projected onto the face-plane of the vault. From this it follows that our drawing is now a composite of three different views: plan, side elevation, and face-plane elevation. In the face-plane elevation, point A is the place where the axle pierces this plane, PAB is the level line in the face, and AH the sub-axle. The sub-axle is always marked with its end symbol and the letter "S".

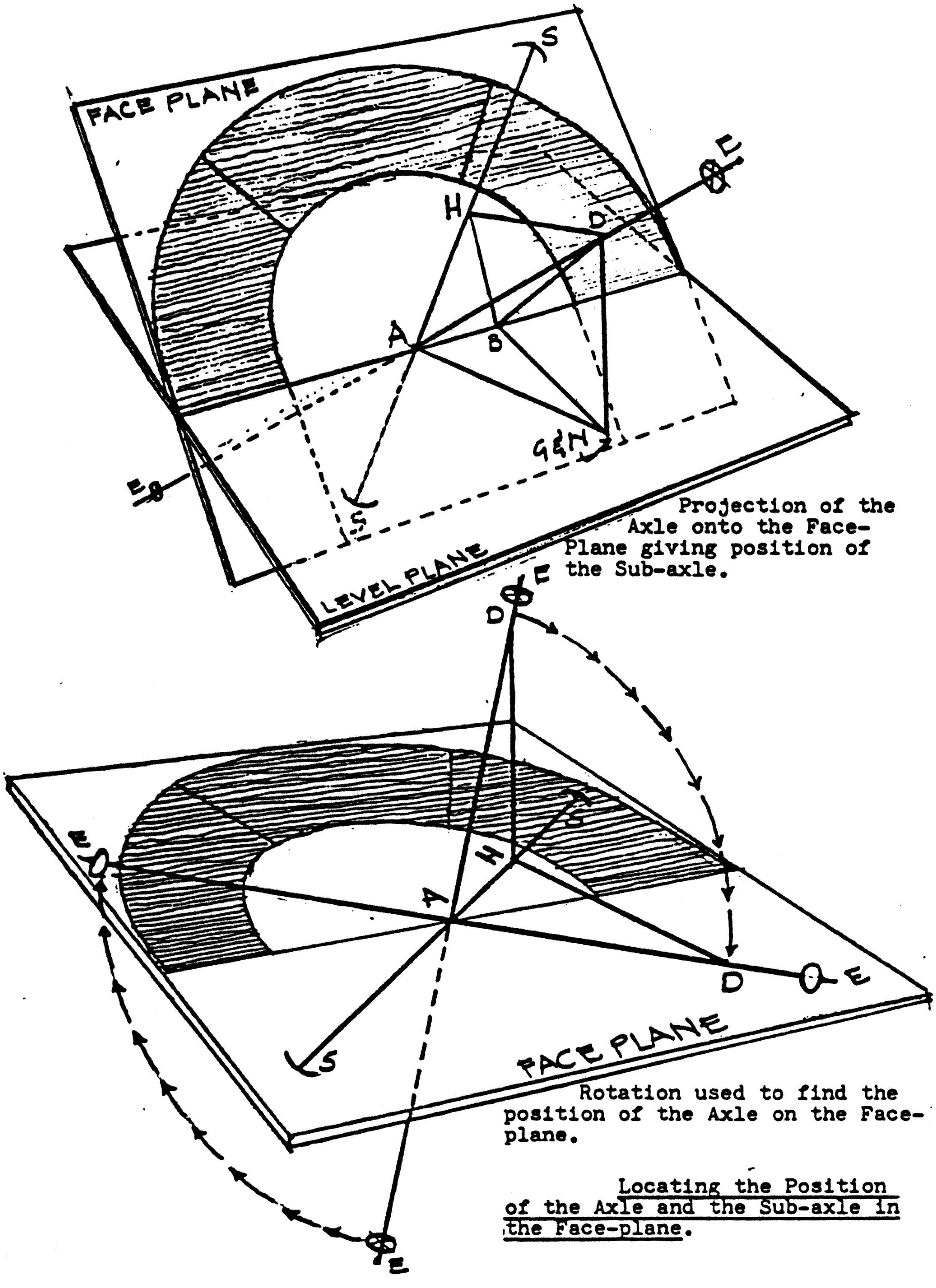
STEP 9: Draw a line HK perpendicular to the sub-axle HA at point H.

STEP 10: Open the dividers to length HD and transfer it to line HK by turning the arc DK about the point H.

STEP 11: Draw a straight line AK through the points A and K. The line AK represents the axle of the vault rotated about point A so that lies in the face-plane.

COMMENTARY ON STEPS 1 THROUGH 11

Having completed the first stage of the method, we will now examine the principles involved in determining the position of the axle and sub-axle on the face-plane of the arch. By definition, the sub-axle SS is the projection of the axle EE onto the face-plane. In the perspective drawing at the top of (Fig. 2-14), I have shown the lines involved in the projection of the axle onto the face-plane. The axle EE comes through the face-plane at point A. Since, by definition, the sub-axle SS also passes through the axle at point A, A will also be one of the points in the sub-axle. To establish the position of the axle it is then necessary to establish the location of one other point in it. To do this, we pick some convenient point in the axle such as D and find its projection (point H) onto the face-plane. The various constructions of the method have enabled us to do this by giving us (1) line length AB which represents the horizontal distance in the face-plane which the axle covers in going from point A to point D (in three space)



Projection of the Axle onto the Face-Plane giving position of the Sub-axle.

Rotation used to find the position of the Axle on the Face-plane.

<u>Locating the Position of the Axle and the Sub-axle in the Face-plane.</u>

Figure 2-14 Locating the Position of the Axle and Sub-axle. (SAMPLE: All figures are bound at rear of text.)

and (2), the line length BH which represents the vertical distance in the face-plane covered by the axle between points A and D. Thus, in the face-plane, length AB gives the horizontal or "X" coordinate of point H, and length BH gives the vertical or "Y' coordiante of point H. Having found point H in the face-plane, the sub-axle can then be drawn by means of a straight line passing through points A and H.

In the drawing at the bottom of (Fig. 2-14), I have shown (again in perspective) the effect of the operations for finding the position of the axle EE in the face-plane. For any given arch, the axle EE, and the sub-axle SS stand in a fixed angular relation to each other. This angle is HAD in both drawings. Now the objective of the construction pertaining to the axle is to preserve this angular relation between the axle and the sub-axle when the axle is rotated about point A so that it lies in the face-plane together with the sub-axle. This is accomplished by using a construction procedure which has the effect of rotating triangle HAD about the sub-axle AH as a hinge so that point D is brought out of three-space into the face-plane without changing its angular or distance relations to the sub-axle AH. In the drawing, this rotation is indicated by arrows pointing in the direction of the rotation. Why it is necessary to establish the positions of the axle and the sub-axle in the face-plane cannot be explained until the part of the method which applies to finding the shapes of the templates for the voussoirs has been covered. We turn to this next.

STEPS 12 TO 17: FINDING THE SHAPES OF THE VOISSOIR TEMPLATES:

For this work, the necessary illustrations are again given on fold-out pages (see Figs. 2-15, 2-16, 2-17). Because of the complexity of the drawings, only two illustrations could be included on a page. They are reproductions from an original copy of Bosse's text.

STEP 12:

All of the work done so far has been for the purpose of finding the positions of the axle and sub-axle on the face-plane. This being done, the construction lines used may be removed. Two additional lines are drawn, as shown in the bottom figure. They are the cross-axle TT which is perpendicular to the sub-axle SS, and the counter-axle CC which is perpendicular to the axle EE.

STEP 13:

Draw lines such as F2 and P3 from the ends of all of the voussoir joint lines so that they are perpendicular to the sub-axle at points as 2 and 3, etc. Next (bottom figure), from the points thus-located in the sub-axle, draw perpendiculars to the axle, such as 2k and 3h. The points thus-located will be used to determine the shape of the right-arch, as well as the shapes of the joint and intrados templates.

STEP 14:

Now, to find the head-angles for the templates of the joints such as VR, take the length of this line in the face-place with the dividers. Trace the projection of points V and R to the axle where they appear as z and b. Place a point of the dividers on z and turn an arc of radius VR to intersect line 9b in r. Through the points thus located, draw line zr. The angle Azr is then the head-angle of the joint template for line VR of the face-plane.

STEP 15:

To find the head-angles for the intrados templates as OV in the face-plane, exactly the same procedure is followed as for the joints, except that one takes the length OV and turns it off between the lines 4d and 7z which represent the projection of the points O and V onto the axle.

STEP 16:

The previous operation was for finding the shape of the intrados templates. If, instead, you want to find the shape of the right-arch (the shape of the arch in a section perpendicular to the axle), then, instead of drawing lines from the sub-axle perpendicular to the axle, draw them perpendicular to the counteraxle CC. For example, having drawn F2 and P3 perpendicular to the sub-axle SS, draw lines 21 and 3n perpendicular to the counter-axle CC at points 1 and n.

STEP 17:

Next, take the distances such as F2 and P3 and transfer them to the corresponding perpendiculars to the counter-axle, where they will appear as 1f, np, etc. When all of the points such as f and p have thus been located, it is only necessary to connect them as shown in the bottom figure to get the shape of the right-arch. For the purpose of this method, it is not necessary to find the curves of the intrados and extrados in the right-arch.

THE SYSTEM OF PROJECTION IN DES ARGUES' METHOD

Having seen how the shapes of the voussoir templates are found according to the method, we are now in a position to understand the system of projection which is fundamental to Desargues' method. In (Fig. 2-18), I have shown the four key lines of the method--the axle EE, the sub-axle SS, the counter-axle CC and the cross-axle TT together with the invariant, 90 degree angles which exist between them. The counter-axle CC is always perpendicular to the axle EE, and it always lies in the plane of the right-arch which is perpendicular to the axle. The cross-axle TT is the line of intersection between the face-plane and the plane of the right-arch. The cross-axle is also perpendicular to the axle. The sub-axle SS lies in the face plane and is perpendicular to the cross-axle TT.

Now the principle which is fundamental to the method is this: If the angular relations which exist between the axle, sub-axle, cross-axle and counter-axle _in three space_ are preserved when the lines are rotated into the _two space_ of a single plane represented by a drawing on a sheet of paper, then one can work with the lines _on paper just as if they existed in three-space_. How this works will be easier to see if we go through a single projection operation as represented in the three-space of the perspective drawing. In the perspective drawing of (Fig. 2-18), take, for example, the joint line OM. This line lies in the face-plane as does the sub-axle CC. If, on the face-plane, we draw perpendiculars 04 and M5 to the sub-axle from the ends of joint line OM, we will clearly have the projection 4-5 of the joint line OM onto the sub-axle. Note that this was also the first projection operation given in step 13 of the method, except that in the method we were, of course, working in the two-space of the sheet of paper. Next, suppose we take the length 4-5 on the sub-axle (again looking at the situation in the three-space of Fig. 2-18) and project it from the subaxle onto the axle by means of the pendiculars to the axle, 4d and 5h. Now we have a second projection of the joint line. This, it will be noted, was also the second operation in step 13 of the method.

The key to the method now depends upon understanding the relations between these two projections. If we were working directly in three-space, as, for example, with a model, we could obtain the length h-d directly by projecting from M to h and from 0 to d at right-angles to the axle. But, the very nature of our problem is such that we cannot do this because we have no way of establishing the angular relation between the joint line MO and the axle. In fact, these two lines do not even need to lie in the same plane. The position of any joint line in the face-plane may be thought of as consisting of two length components which we will call <u>rise</u> and <u>run</u>.[19] Rise is the difference in height between the end points of a joint line as measured in the face-plane above the position of the cross-axle which is taken as a line with zero rise. The projection of the joint-line onto the sub-axle gives the rise of the joint-line, if any. Since the cross-axle is perpendicular to the sub-axle, it has zero rise, and its projection onto the sub-axle will thus be a point. The same will obviously be true for any joint line in the face plane which is parallel to the cross-axle.

Run, on the other hand, is the longitudinal component of the position of a joint line, as measured along the length of the axle. Thus, the projection from the sub-axle to the axle gives the corresponding run on the axle for a joint-line whose rise has been established by projection onto the sub-axle. Since the cross-axle is also perpendicular to the axle, it will have zero run as well as zero rise, and its projection onto the axle will also be a point. All joint-lines parallel to the cross-axle will also have zero run on the axle.

Having seen how the projection of the joint-line onto the axle gives the run component of its position, it remains to be explained how this length can be used to determine the head-angle of a joint template. In the small perspective drawing at the bottom of (Fig. 2-18), hd is the run length on the axle which has been determined by the two projection operations already discussed. Lines dn and hM have been constructed perpendicular to the axle at points h and d, as described in the step 13 of the method. Length OM is the true length of the joint line as measured in the face-plane. Arc M has been turned about point d and intersects line hM in M. This operation has the effect of establishing both the true width hM of the joint templates and the true head-angle hdM. By isolating the run component of the position of the joint line, we have, in effect, brought the axle and the joint line into a common plane which enables us to draw true lengths and angles.

It may be helpful to point out that Desargues' system of projection may also be thought of as a proportioning system which makes it possible to find unknown quantities (lengths and angles) as proportions of known quantities.[20] This can be shown by means by an equation which gives the angle between any joint line and the axle of the vault as a function of:

1. The fixed angle between the axle and the sub-axle.

2. The variable angle between the cross-axle and the joint line in the face-plane.

The equation is:

$$\text{Runangle}° = \frac{\text{Jointangle}°}{90°} \times (A° + 90°),$$

where the variable joint angle is between 0 degrees and 90 degrees above the cross-axle, and A is the angle less than or equal to 90° between the axle and the sub-axle. The ratio between the joint angle and the vertical limit of 90° above the cross-axle (which is the position of the sub-axle) establishes the proportional part of the angle between the axle and the sub-axle which a given joint line makes with the axle.

The principle behind the method of finding the head angles and true widths of the intrados templates (step 15) is exactly the same as that for the joint templates, so it requires no further discussion. The same principle is also involved in determining the true shape of the right-arch (steps 16 and 17), except that this operation requires projection of the end-points of the joints such as O and M, first onto the sub-axle, and then onto the <u>counter-axle</u>. The reason for the projection onto the counter-axle can be seen in (Fig. 2-19). The sub-axle SS lies in the plane of the face-plane, and the counter-axle lies in the plane of the right-arch which is always perpendicular to the axle. The sub-axle SS, the counter-axle CC and the axle all lie together in a single longitudinal plane running from the front to the back of the arch. The counter-axle may therefore be thought of as the projection of the sub-axle from the face-plane onto the plane of the right-arch. Again it will be helpful to note that if we were working directly in three-space, we could project the vault directly from the face-plane onto the plane of the right-arch in a single step. In (Fig. 2-19), this direct projection is shown for the head of a single voussoir. In the figure, RVLQ is the head of the voussoir template in the face-plane, and rvlq is its direct projection onto the plane of the right-arch by means of lines Rr, Vv, Ll, and Qq. The arrows on these lines indicate the direction of the projection. However, since we were working in the two-space of the paper (when we use the procedure of Deargues' method), we must once again go through the step of an intermediate projection onto the sub-axle SS. This sequence of projection operations is shown in (Fig. 2-19) for a single joint in the face-plane. The end points R and V of the joint line are first projected onto the sub-axle SS, there giving the points 9 and 7 respectively. Points 9 and 7 are then projected onto the counter-axle, there giving points b and z.

The rest of the procedure (Step 17 of the method) may be imagined in the following way, again referring to (Fig. 2-19). In the face-plane, construct lines br and zv perpendicular to the counter-axle at b and z. Transfer line length 9R in face-plane to br in the plane of the right-arch. Do the same with 7V, carrying it to line zv in the plane of the right-arch. The points r and v then locate the projection of joint line RV from the face-plane to the plane of the right-arch. It will be seen that, in the perspective drawing of (Fig. 2-19), the projection of line RV by way of the sub-axle and counter-axle, following the rules of Desargues' method, brings line RV to exactly the same place (rv) in the plane of the right-arch as the direct projection in three-space did. This is what would be expected to happen if the projections are correclty conceived. When the projection process has been completed for the end-points of all the joint lines, the shape of the right-arch may be constructed by connecting the set of projected points. Desargues' method does not require the intrados and extrados curves to be drawn for the right-arch, though this could be done by taking more points in the curves on the face-plane and projecting them into the plane of the right-arch along with the end-points of the joint lines.[21]

THE PRINCIPLE OF ROTATION

— The method of projection which Desargues uses depends upon the concept of the rotation of planes about hinge lines so as to bring several planes and the lines which they contain into a single plane which corresponds to the sheet of paper upon which the drawing is being made. A similar principle, usually called rebattement in French texts is also fundamental to modern descriptive geometry. While the historical origin of this principle is difficult to establish, it is quite certain that Desargues was the first to exploit its full potential for solving problems in stereotomy.

The modern principle of rotation or unfolding of planes has already been described at the beginning of this chapter, but at that point we were not yet in a position to compare it with Desargues' method. In order to make this comparison, it is first necessary to be clear about two points which are fundamental to the rotation of planes in modern descriptive geometry:

1. The projection of any point from one plane to another is always by means of a straight line which is perpendicular to the joint line along which the planes have been unfolded so as to make them both lie in the plane of the drawing.

2. An auxiliary oblique projection plane into which a projection is made must be perpendicular to some other plane of known position out of which the projection is made. At the top left of (Fig. 2-20), a point P is seen projected orthogonally onto two planes (1-1 and 2-2) which stand at right angles to each other and intersect at a hinge line.[22] At the top right of (Fig. 2-20), the two planes have been rotated or unfolded so that they both lie flat in the plane of the page. It will be seen that the traces A and B of the two projection lines on these two planes form a straight line P_1-P_2 perpendicular to the hinge line when the two planes have been unfolded. If the location of the projection of point P onto plane 1-1 is known, one coordinate of its projection onto plane 2-2 is found by drawing a line from P_1 perpendicular to the hinge line and into plane 2-2. The other coordinate is found by making the distance from the hinge line to point P_2 in plane 2-2 equal to the distance from the point P to its projection P_1 onto plane 1-1. This illustrates principle (1) above. This same principle is also illustrated at the bottom of (Fig. 2-20) which shows the projection of point P onto three auxiliary, oblique planes 3-3, 4-4, and 5-5. The unfolded drawing shows that, in every case,

the projection line is perpendicular to the hinge line which it crosses.

The drawings at the bottom of (Fig. 2-20) also illustrate the second principle of projection listed above. Any plane into which a projection is made must stand at right-angles to the plane from which the projection is made. Thus plane 2-2 is perpendicular to plane 1-1, and plane 5-5 is perpendicular to plane 2-2. Note however that plane 5-5 is oblique to plane 1-1. Thus, one cannot project from plane 1-1 into plane 5-5, but one can project from 1-1 to 2-2, and, from the point P_2 thus established in plane 2-2, one can project to plane 5-5 and thus locate P_5. There is in fact no limit to the number of successive oblique projections one may make, so long as these two principles are observed. And, the power of the method lies in the fact that one can go on making oblique projections long after exceeding one's capacity to visualize the complete set of projection planes as they stand in relation to each other. The system of projection is in this way analogous to the rules of inference in logic which permit one to obtain valid conclusions so remote from the premises of the argument that they could not be arrived at directly by inspecting the premises themselves.[23]

Desargues' method employs these same two principles of projection, though he does not explain this to the reader. Thus, we are right to conclude that, in this fundamental respect, his method is consistent with the generalized method of descriptive geometry which Monge developed some 150 years after him. Desargues' use of the first of the two principles is demonstrated by his requirement that all projections, as for, for example, of joint lines onto the sub-axle, counter-axle and axle (steps 13-17 of the method) are always to be made at right-angles to the line onto which they are projected. Each one of these lines is, in fact, a hinge line representing the intersection of two planes standing at right-angles to each other, though this fact is, for reasons to be discussed below, somewhat obscured by other features of his method.[24] In (Fig. 2-21), the key, right-angle relations of Desargues' method are shown in perspective. Triangle BAC is a part of a plane containing the axle EE, the counter-axle CA (only half of which is shown) and the sub-axle SS. Plane BAC is perpendicular to the face-plane, which it intersects along the sub-axle SS. Points B, A, E and D define the plane of the right-arch which is perpendicular to triangle BAC along line BA. Thus line CA meets the 90 degree angle requirement for a hinge line between triangle BAC and the plane of the right-arch.

That Desargues also observes the second principle of projection is evident from his rule of drawing lines perpendicular to the sub-axle and counter-axle. There is, however, one additional matter which needs to be explained in this connection. If a point to be projected into some other plane lies <u>on</u> a hinge line, then it may be regarded as lying in <u>either</u> of the two planes to which the hinge line is common, and in such cases, the rule of drawing prependicular to the hinge line does not apply to a hinge line <u>in which a point lies</u>. It is this principle which Desargues observes in projecting points from the sub-axle in the face-plane to the counter-axle in the plane of the right-arch. In other words, referring again to (Fig. 2-21), Desargues makes no projections <u>onto</u> the plane of triangle BAC, and, since all points to be projected from the sub-axle SS to the plane of the right-arch lie <u>in</u> the line of the sub-axle, they are not drawn perpendicular to the sub-axle in plane BAC, but are rather regarded as lying in that plane at the start. Thus we see that in regard to the two fundamental principles of projection, Desargues is entirely consistent with modern methods and remarkably in advance of his time.

THE COORDINATE REFERENCE SYSTEM

It is in the area of the coordinate reference system by means of which the relations between projection planes are established and controlled that Desargues' method differs significantly from that of modern descriptive geometry. Modern descriptive geometry is based, as we have already noted, upon a three-dimensional rectangular coordiante system. With a rectangular coordinate system, the location of any point in space is established by giving its X, Y and Z coordinates, and this is also essentially what happens when a point is projected onto the two basic planes of descriptive geometry--the horizontal projection plane and the vertical projection plane. This is sometimes mistakenly called a three-dimensional, Cartesian coordinate system.[25] But, a Cartesian coordinate system may be either rectangular or oblique, and it is the older, oblique system which Descartes himself used, and which we also find in Desargues' method of stereotomy. In an oblique coordinate system (Fig. 2-22), any point J on a curve such a J-J'-J" is established relative to some other reference point D by specifying the distances A and E, and the angle DZJ which is fixed only for a partciular description of a particular curve. In Desargues' system of stereotomy, the only fixed reference plane is the level-plane. The positions of all of his other lines and planes change according to the particular characteristics of the vault being described. Thus, for example, the path-plane may slope, in which case it will not coincide with the level-plane. Likewise, the face-plane may be sloped and skewed, so it too has no fixed relation to the level plane. Even the plane of the right-arch is variable in position relative to the level-plane, because the method covers cases in which a vault slopes from the front to the back of the arch-way.

What is of greatest interest about the system of oblique coordinates, as compared to the rectangular coordinates of modern geometry, is the relative dependence, in the case of oblique coordinates, of the descriptive system upon the thing being described. By the time Monge gave the world a universal method of descriptive geometry, the descriptive reference system had been completely separated from the unique characteristics of the object being described.[26] In the Mongean system, the horizontal, frontal and side projection planes, each standing at right-angles to the other two, are conceived of as an invariant reference system with regard to which the obliquity of all other lines and planes is determined. In this way, the Mongean system establishes a three-dimensional grid or matrix, extended indefinitely in space, in which any object to be described can be situated and in reference to which its shape can be determined. This difference in the degree of independence of the descriptive system is also a remarkable index of the separation of geometry from reality and of space from place. As Ernest Cassirer points out in The Philosophy of Symbolic Forms, in mythical thought, each number and each geometric figure has its own unique essence. This can be seen in the myth of the creation of the world, as presented in Plato's Timaeus, where each of the four primordial elements, earth, air, fire, and water, is associated with a particular geometric figure. Earth, for example, is symbolized by the square and the cube. Oblique coordinate systems are a vestige of this way of understanding geometry. That Desargues was willing to debate about whether geometrical points really existed, and that he was worried about the problem of imagining infinity, are factors which indicate the transitional nature of his geometrical thought.

DESARGUES' OWN EXPLANATION OF HIS METHOD OF STEREOTOMY:

Having to this point examined Desargues' method of stereotomy and compared it with certain aspects of modern descriptive geometry, we will conclude the present chapter by looking at certain differences between Desargues' own account of his method and those given by Bosse and Poudra. Aside from its compactness, the primary difference in Desargues' exposition is that he chooses an even more complex case than Bosse to illustrate it. In 1943, William Ivins, then director of the Print Room at the Metropolitan Museum of Art published the original plates which Desargues' had used to illustrate his method.[27] Up to that time, it had been thought that all of the original printed copies had been lost. It is indeed remarkable that after all these years, a perfect copy should turn up, not in France, but in the United Sttes. Ivins reported that he made the discovery quite by accident, while looking through the books which had been willed to the Museum by the late W.G. Beatty. It was found bound at the end of a 1547 French translation of Vitruvius.

Since the hand-written La Hire copy from which Poudra worked in compiling his 1864 edition of the works of Desargues contained no illustrations, Poudra was forced to reconstruct them on the basis of the text and the method, as explained by Bosse. In so-doing, he overlooked or ignored some rather important points. In (Fig. 2-23), I have reproduced Desargues' original Plate I side-by-side with Poudra's reconstruction of it.[28] Both plates show a vault in perspective as seen from the front. In Desargues' original plate, however, it will be seen that line QAP, which connects the top-front inner corners of the imposts, slopes upward from left to right, whereas in Poudra's reconstruction, this line is shown as level. Thus, in Desargues' Plate I, the imposts are each at a different level, giving the case in stereotomy which is called an "arc rampant" (rampant or sloping arch), as distinguished from the "arc in plain cintre" (level arch) which Poudra shows in his plate. Poudra's arch is a simple circular or Roman arch, whereas Desargues is actually illustrating his method with a rampant arch which is elliptical both in the section represented by the face-plane and in the section taken in the plane of the right-arch. It would appear that Poudra chose to ignore this feature of Desargues' sample problem because it is clearly indicated in the text of the La hire manuscript as he reproduces it:

> . . . dont la figure d'vene porte, en la face plate dUn mur a talus, pour un descente biaise, ayant l'arc rampant, ou tous les joints sont en ligne droicte et ou ceux de face, front, ou teste ne tendent pas tous ensemble a un mesme poinct ou but.[29]

> . . . of which the figure (is that of) a portal in the face plane of a wall with a sloped and a skewed descent, having a rampant arch in which all the joints lie in a straight line (ie: are parallel to the axle), and in which (the joint lines) in the face-plane do not all come together in a single point (on the axle).

As it turns out, Poudra's oversight, whether intentional or inadvertent, does not seriously damage his account of Desargues' method, because the method, being a "universal" method in the best 17th century sense of the word, is equally well suited to handling the case Poudra attributes to Desargues, as it is to solving the problem Desargues is actually considering. One would like to think that Poudra realized this, but we shall never know for sure. Other, relatively minor confusions result from this difference in examples, but it is of much greater importance to turn now to the geometrical foundations of Desargues' method. In this we go beyond descriptive geometry to the very foundations of projective geometry and the theory of perspective.

CHAPTER III

THE GEOMETRICAL FOUNDATIONS OF DESARGUES' "UNIVERSAL METHODS"

The aim of the present chapter is to bring together the two main strands of Desargues' work in applied geometry--the practical methods for stone-cutting, and perspective construction--and to examine them in the light of his work in the theory of projective geometry which served as their basis.

Desargues does not tell us as much as we should like to know about the connections between his discoveries in projective geometry and the various practical methods which are based upon them. In fact, with the exception of two brief discussions of the theoretical basis of his perspective method, he gives us little more than hints on this point. His hinting is both persistent and consistent, and it deserves to be taken seriously. At the beginning of his paper on stereotomy he says, rather cryptically, that:

> . . . pour donner a entendre cette maniere de trait aux contemplatifs, il n'y auroit qu'une seule proposition de trois lignes qu'ils ont desia veüe en un brouillon de coupe de cone . . .[1]

> . . . to explain this method of treatment to intellectuals, it would only require a single proposition of three lines which they have already seen in a rough draft on conic sections . . . .

The work to which Desargues is referring can only be his 1639 Brouillon project d'une atteinte aux evenemens des rencontres du Cone avec un Plan: (Rough draft of a discovery with regard to the consequences of the intersections of a cone with a plane). It is in this paper that his famous theorem of involution is given. Likewise, at the very end of this paper, referring back to the one on stone-cutting he says:

> Touchant la coupe des Pierres de taille:
> En une mesme face de mur les arestes des pierres de taille sont communement de un mesme ordonnance entre elles & l'essieu de l'ordonnance de'entre les planes des joincts qui passent à ces arestes.[2]

> Touching upon the cutting of stones:
> In a given wall face, the edges of the cut stones share among themselves a single ordonnance which is also the axle of the ordonnance of the joint planes which pass through these edges.

In the specialized language which Desargues devised for his paper on conic sections, an "ordonnance" is defined as follows:

> Pour donner à entendre de plusieurs lignes droictes, qu'elles sont toutes entre elles ou bien paralleles, ou bien inclinées à mesme poinct, il est icy dit que toutes ces droicts sont d'une mesme ordonnance entre elles, par où l'on concevra de ces plusieurs droictes qu'en l'une aussi bien qu'en l'autre de ces deux especes de position, elles tendent comme toutes à un mesme endroict.[3]

> To indicate with regard to several straight lines, either that they are all parallel to each other, or that they are all directed to a single point, it is here said that all of these lines are of a single ordonnance, by which one will imagine that in the one as well as the other of these two kinds of positions, the several lines stretch to a single point.

Thus, what Desargues is directing the reader's attention to in the brief comment on joint lines in vaults, is that the joint lines in the face plane of a vault have a point on the axle of the vault as a common intersection, and that the joint lines which run through the vault parallel to the axle meet the axle at infinity. Later, this important clue will be discussed in detail.

Desargues' most explicit statements about the connections between his study of conic sections and his practical methods are found in two brief papers on perspective. For this reason, we will begin with a study of the perspective method. Desargues' perspective method is also important because, taken together with his method of sundial calibration and his method of stonecutting, it shows how far he got in the development of a complete system of descriptive geometry--a program which is also hinted at by the titles which all contain the expression "manieres universal" (universal methods).

THE PERSPECTIVE METHOD

Of the several presentations given by Desargues of his perspective method, the clearest is the 1636 <u>Example de l'une des manieres universelles du S.G.D.L. touchant la pratique de la perspective sans emploier aucun tiers point, de distance ny d'autre nature, qui soit hours du champ de l'ouvrage</u>[4] (Example of one of the universal methods of S.G.D.L. touching upon the practice of perspective without employing a vanishing point or a point of other nature which will be outside the field of the work space). The exposition which follows is therefore based primarily upon this latter work. Some of Desargues' later writings on the subject are more helpful in explaining the relations between the perspective method and projective geometry.[5] They are also more detailed, showing how, for example, to find the true size of an angle seen in perspective. The relevant parts of these other essays will be discussed later in this chapter. In the presentation of the method which follows, no attempt has been made to give a translation or paraphrase of Desargues' text. Where possible, terminology has been modernized to conform with contemporary usage. In a few cases, it has been necessary to invent terms for which there are no equivalents in modern perspective practice. Where these are not self-explanatory, a parenthetical note follows the term. A copy of Desargues' original plate is used for the illustration.[6] (See Fig. 3-1 which folds out as an aid to following the text.)

THE ILLUSTRATION

The plate contains three drawings. At the top, right is a plan view of the perspective situation. In the center is a perspective drawing together with the construction lines of the method. At the top left is a similar set of construction lines shown removed from the picture-space of the perspective. The letters in all three figures coincide. It is assumed that the subject, which is a kind of pavilion with a sunken floor, is seen by a single eye which remains stationary through observations of different parts of the subject to be drawn.

THE PLAN VIEW

The square likm is the plan of the pavilion. Line x gives the height of the corners of this pavilion (17 feet above grade and one foot below grade). The scale is three toise long (a toise = approx. 6 feet). one toise of the scale has been divided into feet. line ts gives the height of the eye above the ground-plane of the subject (here 4-1/2 feet). The ground-plane is assumed level. Line ab is the edge of the picture-space, as seen in plan. (The picture-space occurs in the picture-plane, but, unlike the picture-plane, it has specified boundaries)

OPERATIONS ON THE PLAN

Draw line ag perpendicular to the picture-plane ab. From the corners of the pavilion, draw lines mr, lh, kn, el5 and ig perpendicular to ag and thus parallel to the picture-plane ab. At point b in the picture-plane, draw bq parallel to ag. Scale all of the lines drawn to ag using the plan scale and record this data. Thus, line kn, for example, is marked as having a length of 7-1/2 feet. Also, scale the width of the picture-space (12 feet). The distance of 17 feet between points r and a on line ag indicates that the picture-plane stands 17 feet in front of point r. Point t is the station-point. It is 24 feet in front of the picture-plane.

OPERATIONS ON THE PERSPECTIVE

Draw base line AB as long as desired. This line represents the intersection between the ground-plane and the picture-plane. At the ends of AB, draw lines AF and BE perpendicular to AB. These will then be the limits of the picture-space within the picture-plane. Next, divide line AB into as many parts as are contained in its representation (ab) in the plan. In this case ab contains 12 parts. Further subdivide one of the feet which have been marked on the base-line of the perspective into inches. (Desargues subdivides the 8th foot.) Using the scale which has now been constructed on the picture-plane, mark the height of the eye (4-1/2 feet) on lines AF and BE and draw the line FE across the picture-space. This line FE will be the horizon line. On line FG, mark the point of the eye G at seven feet to the right of F, just as it is shown in plan. Point G will be the vanishing point of a one-point perspective toward which all lines perpendicular to the picture plane and parallel to the ground-plane converge. Lastly, draw line GC perpendicular to AB.

DEVELOPING THE SCALE OF DISTANCE

A scale of distances is a scale which makes it possible to determine the location in a perspective drawing of any line on the ground-plane which is parallel to the picture-plane and at a specified distance from it. Any such line will lie between the base-line AB and the horizon line FE.

To construct the scale of distances, proceed as follows: On the rectangle ACGF in the picture-plane, draw the diagonals AG and CF. Through the point at which AG and CF intersect, draw line HD parallel to line AB. Line HD represents a line in the ground-plane which is as far behind the picture-plane as the station-point is in front of it. Next, draw the diagonal HG in the rectangle TGFH. Where diagonal HG intersects diagonal FC of the larger rectangle ACGF, draw line QN parallel to AB. Line QN then reprsents a line in the ground-plane twice as far behind the picture-plane as the station-point f is in front of it. Next, draw diagonal QG, and, where QG intersects FC, draw VS parallel to AB. Line VS then represents a line on the ground-plane three times as far behind the picture-plane as the station-point is in front of it. In the figure at the top-left of the plate, these same operations have been carried out in reverse (left to right).

Lastly, divide the segment AC of the ground-line AB into parts equal in number to the distance of the station-point from the picture-plane (here 24 feet), and if necessary, further subdivide one of these units into inches. This completes the scale of distances.

THE SCALE OF MEASURES

The complement to the scale of distances is the scale of measures which makes it possible to determine the length of any line parallel to the picture-plane and at a specified distance from it. To construct the scale of measures, it is only necessary to draw lines from the vanishing-point G to the one-foot increments along the base-line AB. For greater accruacy one of these units is divided into inches. By using the scale of measures together with the scale of distances, the length of any line parallel to the picture-plane can be determined as in the following demonstration.

DEMONSTRATION

The principle by which the perspective method operates is this: Any desired point in a two-point perspective can be located in a corresponding one-point perspective which is drawn to the same scale from the same station-point. Thus, the two-point perspective of the pavilion will be constructed by locating its key points in a corresponding one-point perspective and then connecting the points thus-located with lines which would ordinarily run to vanishing points beyond the picture-space.

To locate, for example, plan point m in the perspective (where it appears as point M), do as follows: First, note that line ag in plan corresponds to line AG in the perspective. On the perspective, draw a line from F to the 17 foot mark on the line AC. (The line thus-drawn is shown with dots in the figure.) This line (F-17) intersects the line AG at point R. Point R in the perspective thus corresponds to point r in the plan. At R in AG, draw a line parallel to AB and long enough so that it runs through the scale of measures. From the plan it has been determined that point m lies 1-1/2 feet to the right of point r. Therefore, in the scale of measures on the perspective, measure 1-1/2 feet in the line drawn from point R and lay this off to the right of point R in the same line. This gives the location of point M in the perspective. The same procedure is followed for all points which lie from 0 to 24 feet beyond the picture-plane. For a point which lies beyond 24 feet and thus lies off the scale of distances, one of two procedures may be followed: (1) at the start of work, a scale of distances long enough to cover all cases can be constructed. (2) alternatively, one can make the following modification in the procedure already given. Suppose the location of plan point k in the perspective is desired. From the plan it is determined that k is located 29 feet beyond the picture-plane, thus exceeding the scale of distances by 5 feet. To locate k in the perspective, draw a line from point G to the 5 foot mark on AC (24 + 5 = 29). Now, find the point where the line G-5 intersects line HD (rather than AG as before). From the point where G-5 intersects HD, draw a line to point F on FG. Where this line to F intersects

AG, draw a line parallel to AB. This parallel to AB will now correpond to line kn in plan. As before, extend it into the scale of measures to find the length, in perspective, which corresponds to 7-1/2 feet--the indicated length in plan. The length is laid off, this time to the left of line AG, thus giving the location of point K in the perspective.

One last example of the use of the scale of distances will help to clarify the technique for points still further from the picture-plane. Suppose a point in line AG 53 feet behind the picture-plane is desired. From the introductory discussion, we know that line HD is 24 feet behind the picture-plane, and that QN is 48 feet behind the picture-plane. The point desired is 48 + 5 feet behind the picture-plane. Draw a line from G to the 5 foot mark on the line AC. Then find the intersection of G-5 with QN which is the 48 foot distance line. From the point of intersection between G-5 and QN, draw a line to point F, and where the line to F intersects AG, we have the point in line AG which is 53 feet behind the picture-plane.

SUMMARY

The rules for this part of the operation are as follows:

1) Determine the distance from the picture-plane to the desired point and mark the same distance in line ag.

If the point does not lie on ag, also determine its distance to ag in a line parallel to the picture-plane.

2) If the distance to the desired point exceeds the value available on AC, determine the nearest lower value for which there is a horizontal line (such as HD, QN, etc.) in the distance-scale and subtract this value from the distance to the desired point.

3) Draw a line from G to the length on AC obtained as remainder in step (2).

4) From the point where the line to the remainder distance on AC intersects the line of the next lower distance in the scale (HD, QN, etc.), draw a line to point F.

5) The desired distance in line AG is then the point where the line to F intersects AG. If the desired point does not lie in AG, use the scale of measures to locate its lateral displacement.

VERTICAL LENGTHS

The distance and measure scales are also used to determine the lengths of vertical lines in the perspective. Exactly the same procedure is used to locate the base point of a vertical line on the ground plane as was used to find other points in the ground plane. When the position of the base point has been established, a horizontal line is drawn from it into the scale of measures to determine the desired length. This length is then laid off on a vertical line extending from the base point. If the line extends below the grade of the ground-plane, the length is laid off below the base point.

This completes the method.

ANALYSIS OF THE PERSPECTIVE METHOD

Having examined the procedures of Desargues' perspective method, we turn next to an analysis of the principles by which it operates. It has already been pointed out that the method is based on the fact that for any two-point perspective, there is a corresponding one-point perspective of the same scale and station-point in which coordiantes of all points in the two-point perspective can be established. Today, it is customary to make a much sharper division between one and two-point perspective techniques than it was in Desargues' and earlier times, and, for this reason, people are sometimes surprised to learn about the relationship upon which Desargues' method depends. Yet the drawings in many old treatises show that an understanding of this key relation developed quite early. It is clearly indicated for example in Plate 36 of Jacques Androuet du Cerceau's

1576 Lecons de perspective positive (Fig. 3-2).[7] It would in fact seem that the realization came about quite naturally as a result of drawing the diagonals of a grid of squares in a one-point perspective. It would thus appear that, in regard to this point, Desargues merely availed himself of a relationship which was already common knowledge in his time.

Just how Desargues' scale of distances works will be easier to understand if we look at a perspective situation from the side, as shown in (Fig. 3-3). Line wC is the ground plane, and FA is the picture-plane. Point G is the location of the observer's eye, and thus point C is the station-point. Distance AC is the horizontal distance from the observer's station-point to the picture-plane. This distance may be assigned a value of 24 feet as Desargues does in his sample problem, but it can be any desired distance. If we take a corresponding point V in the ground-plane 24 feet on the other side of the picture-plane, we see that a line drawn from the observer's eye at C to this point V has the effect of dividing the vertical distance AF on the picture-plane exactly in half, giving a part to part ratio of 1 to 1 and a part to whole ratio of 1 to 2. This is why Desargues is able to establish the position of his first distance line HT simply by drawing the diagonals of the rectangle

ACGF and then drawing a horizontal line through their point of intersection. The diagonals automatically give a point in perspective which is as far behind the picture-plane as the station-point is in front of it, provided, of course, that the ground plane is level.

Suppose now that Desargues' construction procedure is carried several steps further. If a line is drawn to the observer's eye at C from point W, which is twice as far (48 feet) from the picture-plane as the observer's eye, this line will pass through the picture-plane at point Q and it will divide the top half HF in the part to whole ratio of 1 to 3. If the operation is repeated for a point 72 feet removed from the picture-plane, the remaining segment QF of the picture-plane is divided in the part to whole ratio of 1 to 4, and so on. Thus, there turns out to be an invariant relationship between the station-point distance AC and the segments of the picture-plane cut off by lines drawn to multiples of the station-point distance on the ground plane. One way of expressing this relationship is to say that the remainder of the picture-plane between a previous division line (if any) and the horizon line FG is divided in a part to whole ratio which corresponds to the reciprocal of the number by which the station-point distance AC is multiplied in order to obtain the distance on the ground-plane from the station-point C to the point observed. Thus we may write:

$$\text{Remainder Ratio} = \frac{1}{n},$$

but it would be more convenient to think of this ratio in terms of the fraction of the whole distance AF from the base line AC to the horizon line FG. A function for this ratio can be obtained by multiplying the dominator of the remainder ratio by the denominator of the next previous ratio:

$$R = \frac{1}{(n-1)n},$$

where R is the part-to-whole ratio between the segment of the picture-plane cut off by a given line and the entire length AF, and n is the number by which the distance AC is multiplied to give the distance from the station-point C to the point observed. Thus, for example, in the case of a point W which is 48 feet beyond the picture plane, the total distance from the station-point is 3 x 24 = 72 feet, in which case, n = 3 and R, the ratio of QH to FA, is $\frac{1}{(3-1)3}$ or $\frac{1}{6}$. Taking the distance AC from the picture-plane to the station-point at a unit value of 1, we can extrapolate the ratios for the integer multiples of the unit value as follows:

n=AC=1	1x1	2x1	3x1	4x1	5x1	6x1	7x1	8x1	
R= $\frac{1}{(n-1)n}$	$\frac{1}{0}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{20}$	$\frac{1}{30}$	$\frac{1}{42}$	$\frac{1}{56}$	etc.

Fractional distances will follow the same rule. It will be noted that these ratios apply to any perspective situation with a vertical picture-plane and a horizontal ground-plane. Given these conditions, the only variables are the distance from the station-point to the picture-plane and the height of the observer's eye above the picture-plane.

Another interesting property of the series of integer multiples of the picture-plane to station-point distance AC is that any neighboring pair of numbers gives the ratio into which the line to the higher divides the picture-plane. This is shown for one case by the fractions in the right margin of (Fig. 3-3). Consider for example the line from the eye point G to the 7th multiple of AC on the ground-plane. The part-to-whole ratio into which this line divides the picture-plane is 6 to 7.

From this analysis, it will be evident that Desargues has devised a simple geometric method of finding the ratios we have just expressed with numbers. We turn now to a study of the basis of his method in his study of conic sections.

PERSPECTIVE AND CONIC SECTIONS

In any perspective situation, the several lines drawn from the observer's eye to the various points in the subject may be thought of as forming a cone which is sometimes called the cone of vision, though it should not be confused with the visual field as discussed in modern perceptual psychology. In (Fig. 3-4), the basic perspective situation is shown, using Desargues' pavilion as the subject. The apex of the cone of vision S is at the observer's eye. The picture-plane is one section through the cone of vision, and the various planes of the subject may also be thought of as sections of this cone. For the purpose of understanding the basic relationship, it does not matter whether the image formed on the picture-plane is shaped like a conic section. What is important is that each plane of the subject, such as K,M,st,fr has its corresponding projection on the picture-plane, and that these projections obey the laws of conic sections. This relationship is shown for a single plane of the subject K,M,st,fr in the drawing at the bottom left of (Fig. 3-4). The laws of perspective projection are thus the same as those governing the relations between conic sections, and any properties which remain invariant through the change of one conic section into another will also remain invariant through the perspective projection of plane figures. How Desargues proved this relationship will be discussed in detail after his perspective theorm has been explained.

DESARGUES' PERSPECTIVE THEOREM

The drawing in (Fig. 3-5) duplicates the one Desargues uses to illustrate his 1648 statement of his perspective theorem.[8] In it, we are looking at the perspective situation from the side. Since the parts are named on the drawing they will not be defined here. However several points of clarification are in order which go beyond anything which Desargues explicitly states.[9] First, it will be noted that the picture-plane cg is inclined rather than perpendicular to the subject-plane pf. There are two reasons for this condition. First, Desargues indicates later in his text that this drawing is to be thought of as covering a number of different cases, and we indeed would expect that a universal method will be able to handle a sloped picture-plane. This condition is not as unusual as it might at first appear to be because, in terms of the geometry involved, it is really the same as a case in which the ground plane slopes--a condition frequently met with in perspective drawing. The second reason is that Desargues wants to include a number of different eye-points such as k and k', from which the visual rays are to pass through identical points q,i,e, in the picture-plane gc. If the picture-plane were vertical, point k would have to lie at a great distance form points a and g--a graphical inconvenience which the universality of his method allows him to avoid. Also, for reasons of graphic convenience, lines k'g and cf' slope in order to make it possible to include in one drawing the case in which the angle between the picture-plane and the subject-plane is different, yet the visual rays from the eye-point k' still pass through the same points q,i,e on the picture-plane. Thus the drawing actually includes three different cases, and the objective is to show how the different segments of the subject planes cf and cf' are related to one another when their projections onto the picture-plane are held constant through changes in the slope of the picture-plane and the place of the observer's eye.

Desargues' own statement of his perspective theorem is quite compact. It is paraphrased below in the language of modern geometry.

DESARGUES' "PERSPECTIVE PROPOSITION"

IF: (1) Two lines gak and gk' intersecting at a point g are respectively parallel to two other lines cf and cf' intersecting in a point c, and if (2) a straight line cg connects points c and g, and if (3) two lines aed and aqb, coming from the same point a on ga, intersect line cf in points d and b, and line cg in points q and e respectively, and if (4) two other lines kef and kqt, coming from one and the same point K on line gk, also intersect line gc in points e and q, lines aed and aqb in points e and q, and line cf in f and t respectively, THEN: (1) the segments cd, cb, db produced in the line cf are respectively proportional to the segments cf, ct, ft, which are also in line cf, and: (2) the ratio between the corresponding members of the two sets of segments (cd, cb, db) and (cf, ct, ft) will be as ga is to gk on line gak, and: (3) the same will be true for corresponding segments produced in the line cf' by lines k'ef' and k'qt' from point k' on the line gk'; that is, the ratio between corresponding members of the two sets of segments (cf', ct', ft') and (cd, cb, db) will be as gk' is to ga, and: (4) $\frac{cd}{cb} = \frac{cf}{ct} = \frac{ec}{eg} \times \frac{gq}{qc}$ and: (5) $\frac{db}{eq} = \frac{dc}{ec} \times \frac{ab}{aq}$ and: (6) if from points a and k, respectively, two lines ahi and ksi are drawn intersecting at point i on cg, then: $\frac{iq}{ie} = \frac{aq}{ab} \times \frac{hb}{hd} \times \frac{ad}{ae}$ and: (7) conversely, if two pairs of intersecting parallels such as (ga, cf), and (gk', cf') contain proportional line segments so that, for example, cd:cf=ct':cf', then lines such as ad and k'f' will intersect in common points such as e in cg.

ANALYSIS OF THE PERSPECTIVE THEOREM

Desargues first presents his theorem without any reference to perspective. We are thus given a pure statement of geometric relations, and it is only in the section following the one just given that he explains how it applies to perspective. In the following analysis, the numbers at the start of each paragraph refer to the conclusion numbers in the paraphrase.

(1,2,3) In Desargues' illustration, the segments cd, cb, db of the eye-point a are respectively twice as long as the segments cf, ct, ft of the projection from eye-point k. It also turns out that eye-point k is twice as far from point g on the picture-plane cg as eye-point a is. Thus:

$$\frac{ga}{gk} = \frac{cd}{cf} = \frac{cb}{ct} = \frac{bd}{ft} \qquad \text{and, in this case: } \frac{ga}{gk} = \frac{1}{2}.$$

Also, taking the third eye-point k', we have:

$$\frac{gk'}{ga} = \frac{cf'}{cd} = \frac{ct'}{cb} = \frac{f't'}{bd}$$

These equations give us a fundamental relationship which has already been pointed out in the discussion of the scale of distances in Desargues' perspective method (cf: p. 16). The length of a segment such as tf which is cut off on the subject-plane cf by two lines such as kt and kf is a function of the distance of the eye-point from the picture-plane.

(4) The reasoning of this conclusion is based on relations between pairs of similar triangles. Consider first the slightly simplified equation:

$$\frac{cd}{cb} = \frac{ec}{eg} \times \frac{gq}{qc}$$

In (Fig. 3-5) it will be seen that cb and cd are sides of two pairs of similar triangles, aeg and ced being one pair, and aqg and cqb being the other pair. By definition, corresponding sides of similar triangles are proportional, and there is but one ratio for all three sides. Thus, the ratio between the hypotenuses of a pair of similar triangles is the same as the ratio between corresponding legs. In the equation above, ec:eg is the ratio between corresponding sides of similar triangles aeg and ced, and gq:qc is the ratio between corresponding sides of similar triangles aqg and cqb. If the above equation is now rewritten in an equivalent form:

$$\frac{cd}{cb} = \frac{ec}{eg} \div \frac{qc}{gc}$$

it is evident that conclusion (4) depends upon the fact that the ratio between two sides cd and cb, each taken from one of the pairs of similar triangles, is equal to the result obtained by dividing the ratio of the pair from which cd is taken by the ratio of the pair from which cb is taken.

The same reasoning is applied to cf and ct which belong respectively to two pairs of similar triangles keg,cef and kqg,cqt. The pair keg,cef to which cf belongs has a side in common with the pair aeg,ced, and the pair kqg,cqt to which ct belongs has a side in common with the pair aqg,cqb. Thus the ratio within each pair of triangles to which cf and ct belong respectively is the same as the ratio within each pair to which cd and cb respectively belong. Hence (4):

$$\frac{cd}{cb} = \frac{cf}{ct} = \frac{ec}{eg} \times \frac{gq}{qc}$$

which amounts to a statement that proportional relations on the subject-plane are preserved when only the eye-point to picture-plane distance is changed in a perspective situation.

(5) The task is to show why:

$$\frac{db}{eq} = \frac{dc}{ec} \times \frac{ab}{aq}$$

The lines pertaining to this equation are shown in (Fig. 3-6a).

Line rb has been drawn parallel to line ad, in consequence of which triangles qrb and qae are similar. Since they are similar, their corresponding sides will all have the same ratio, thus er:eq = ab:aq. As a result of drawing rb parallel to ad, triangles crb and ced are also similar and thus, db:er = dc:ec. Given these two equations, we may write:

$$\frac{db}{er} \times \frac{er}{eq} = \frac{dc}{ec} \times \frac{ab}{aq},$$

which may be simplified to:

$$\frac{db}{eq} = \frac{dc}{ec} \times \frac{ab}{aq},$$

which is conclusion (5). The purpose of this conclusion is to show that the proportional relation between a subject length db and its projection eq on the picture-plane cg is a function of the ratio between the eye-point to subject distance ab and the eye-point to picture-plane distance aq. Desargues is here using a theorem of Menelaus which he elsewhere attributes to Ptolemy.[10] Since the theorem is also used in (6), it will be discussed below.

(6) The task is to show why it true that:

$$\frac{iq}{ie} = \frac{aq}{ab} \times \frac{hb}{hd} \times \frac{ad}{ae}$$

The lines pertaining to this equation are given in (Fig. 3-6b). The first step given by Desargues in his demonstration is to establish that:

$$\frac{iq}{ie} = \frac{iq}{ic} \times \frac{ic}{ie},$$

which will be obvious upon inspection. The second step consists of showing that:

$$\frac{iq}{ic} = \frac{aq}{ab} \times \frac{hb}{hc} \text{ and that: } \frac{ic}{ie} = \frac{hc}{hd} \times \frac{ad}{ae}$$

both of which follow from the theorem of Menelaus already mentioned. To understand this theorem, consider first a special case (Fig. 3-6c) in which two straight lines ah and ce are drawn

to two other straight lines da and dc so that da and dc are bisected at points e and h respectively. This results in two pairs of congruent triangles: eia,hic, and edc, adh. From the numerical values assigned to the figure, it will be evident that:

$$\frac{ic}{ie} = \frac{hc}{hd} \times \frac{ad}{ae} \quad \text{or:} \quad \frac{ic}{ie} = \frac{1}{1} \times \frac{2}{1} = \frac{2}{1}\ .$$

It turns out that this same equation also holds true for cases in which lines ce and ah do not bisect the lines to which they are drawn, and that it holds true even for cases in which they divide the lines to which they are drawn in different ratios. This generalized form of the relationship is called the theorem of Menelaus and is the one which Ptolemy used in his Almagest where Desargues appears to have learned of it.[11] It is this theorem which Desargues bases his two equations upon, as will be evident by comparing (Figs. 3-6b and 3-6c). And, having already established that:

$$\frac{iq}{ie} = \frac{iq}{ic} \times \frac{ic}{ie}\ ,$$

it is possible to obtain by substitution:

$$\frac{iq}{ie} = \frac{aq}{ab} \times \frac{hb}{hc} \times \frac{hc}{hd} \times \frac{ad}{ae}\ ,$$

which simplifies to:

$$\frac{iq}{ie} = \frac{aq}{ab} \times \frac{hb}{hd} \times \frac{ad}{ae}\ ;$$

the equation desired. This last equation explains a projection qie on the picture-plane cg as a function of the subject bhd and the proportional distances aq:ab and ad:ae between the eye-point a, the picture-plane cg and the plane of the subject cf. Thus the five basic conclusions of the theorem consider different sets of proportional relationships which hold true in any perspective situation. Conclusion (7) states the converse of the proposition.

DESARGUES' THEOREM

Desargues' perspective proposition, as given above is a specialized application of certain relationships which also appear in a more general theorem which is usually called Desargues' Theorem in projective geometry.[12] This theorem and his theorem of involution form the conceptual basis from which all of his practical methods are derived. In modern form, Desargues' Theorem may be expressed as follows: Two triangles are in central perspective if the lines joining corresponding vertices of the triangles are at the center of vision, and if two triangles are in central perspective, then their corresponding sides (extended if necessary) intersect in three collinear points. Further, if the two triangles lie in different planes, the three collinear points define the line of intersection between the two planes.[13] In (Fig. 3-7a), Desargues' theorem is illustrated for two dimensions; that is, triangles ABC and A'B'C' are considered as being in the plane of the page. It will be seen that the corresponding sides intersect in the three points N,L,M. The line defined by these three points is often called a Desargues' line and will be referred to by this term in the remainder of the present work. Point V is the point of central projection. (It may be considered as coresponding to the eye-point in a perspective situation.) The lines CC'V, AA'V, and BB'V connect corresponding vertices of the two triangles with point V and may be considered as representing the visual rays of the perspective situation.

In (Fig. 3-7b), Desargues' theorem is illustrated for three dimensions. This drawing may be regarded as a perspective representation in which (Fig. 3-7a) has been folded along the Desargues line NLM so that the triangle ABC lies in a plane which is oblique to the plane of the page. As a result of the folding operation, point V will also necessarily move out of the plane of the page. It will also be clear that, if (Fig. 3-7b) is taken as representing a normal perspective sitaution, then V will be the eye-point, the plane of triangle A'B'C' will be the picture-plane, the plane of triangle ABC will be the subject or ground-plane, and the Desargues line NLM will be the line of intersection between the picture-plane and the subject-plane. Alternatively, the plane of triangle ABC may be taken as a vertical plane of the subject, such as the wall of a building, but this merely amounts to a change in the orientation of the viewer located at V and has no effect upon the geometry of the projection relations. By the same line of reasoning, it will be seen that in fact the plane of triangle ABC could be the plane of a subject in any orientation whatsoever relative to the viewer's vertical and horizontal reference. Thus, the two planes are capable of representing the perspective projection of any plane of any subject, regardless of position. For, the plane of the triangle ABC may be rotated through 360 degrees about the Desargues line MLN, and both planes together, in whatever position relative to each other, may be rotated through 360 degrees about the visual ray VW (or about any visual ray).

In the presentation of his theorem, Desargues proposes a method of projection by which triangles that are in perspective in three dimensions may be brought into a single plane (that of the paper) for comparison. At the very end of the single-page proof of the theorem which bears his name, he remarks that the method he has just given is such that:

> . . . par ce moyen se passer de celle du relief en luy substituant celle d'une seul plan.
>
> . . . by this means, one passes from a figure in relief, substituting for it a figure in a single plane.[14]

To accomplish this, Desargues resorts to certain conclusions which can be drawn from the assumption that parallel lines meet at infinity. If parallel lines meet at infinity, a cone and a cylinder are, as he puts it, two species of the same genus, and thus any cylinder of finite length may be regarded as a segment of a cone with its apex at infinity. Thus parallel and central projection are also related to each other as two species of the same genus.[15]

Desargues' own proof of this theorem is very compact, but, unlike his perspective proposition, it does not lend itself to translation because many of the key relationships are not clearly expressed.[16] The drawing which accompanies the proof is also difficult to read. In (Fig. 3-8), his original drawing is shown at the top. The three main triangles upon which the proof depends are indicated with heavy outlines. In the course of the proof, parts of this drawing must be visualized in both two and three dimensions. As an aid to visualizing the planes which are to be imagined in three dimensions, a small perspective rendering is given at the bottom center. This is supplemented by line drawings of the transversals of Menelaus configuration for each plane. The equation for each Menelaus configuration used in the proof is shown below the figure to which it corresponds. Desargues observes the rule that points considered as lying the plane of the page are identified by lower-case letters, and those to be considered as lying in a different plane are assigned capital letters.

Triangles abl and DEK are in perspective from point H. Their corresponding sides intersect in three points c, f, and g on line cg. Desargues here uses a variation of the perspective triangle relationship in which two of the corresponding sides intersect each other (at points f and g). Thus only the remaining pair of sides ab and DE need to be extended to establish the Desargues line cfg. In the course of the proof, triangles abl and DEK are first considered as lying in one plane and then, for the proof in three dimensions, in two planes. For the proof in two dimensions, Desargues makes extensive use of the transversals of Menelaus. For this proof he uses the converse of Menelaus theorem[17] to show that because:

$$\frac{gD}{gK} \text{ X } \frac{fK}{fE} = \frac{cD}{cE} \text{ ,}$$

points c, f, and g must lie in the same straight line which is the Desargues line for triangles abl and DEK. The reasoning is as follows: By hypothesis:

$$(1)\quad \frac{gD}{gK} = \frac{lH}{lK} \times \frac{aD}{aH} \quad \text{and,}$$

$$(2)\quad \frac{fK}{fE} = \frac{lK}{lH} \times \frac{bH}{bE} \ .$$

These two equations may be multiplied together to obtain:

$$(3)\quad \frac{gD}{gK} \times \frac{fK}{fE} = \frac{aD}{aH} \times \frac{bH}{bE} \ .$$

Also by hypothesis:

$$(4)\quad \frac{aD}{aH} \times \frac{bH}{bE} = \frac{cD}{cE} \ .$$

Then, by substitution in (3), we obtain:

$$(5)\quad \frac{gD}{gK} \times \frac{fK}{fE} = \frac{cD}{cE}$$

and thus the points c, f, and g must lie in a single straight line. In other words, given three Menelaus configurations related as shown in (Fig. 3-8), it is possible to deduce that there is a fourth composed of corresponding diagonals of the other three and such that it contains the three points c,f,g in one of <u>its</u> diagonals.

Thusfar it has been assumed that triangles abl and DEK lie in the same plane. If we assume that they lie in two planes, as shown in the perspective at the bottom of (Fig. 3-8), then point H, from which the two triangles are in perspective will lie in yet a third plane HEbl. The proof in three dimensions consists of showing that in this case too, the corresponding sides of triangles abl and DEK will intersect in the Desargues line cfg. This proof will not be discussed here. Having given it, Desargues points out that by means of parallel projection, it is possible to create a figure in two dimensions which, as he puts it, corresponds point to point, line to line, and proof to proof to a figure in three dimensions, so that the properties of the figures can be reasoned about from a drawing in a single plane. In other words, if triangle abl is imagined as lying in the plane of the paper, it is possible by means of this projection method, to create a third triangle which lies in the plane of the page with abl and yet at the same time retains all of the perspective relations which exist between triangles abl and DEK which are being imagined in two different planes. This parallel projection operation must not be confused with a rotation of triangle DEK about line cfg as a hinge or axle in order to bring it into the plane of the page. Rather, we are dealing with an orthogonal projection in which line Hh, Dd, Ee and Kk are parallel to each other and at the same time perpendicular to the plane of the page onto which the projection of triangle DEK is being made in order to produce triangle dek. It must also be realized that the distance from point h to point H is necessarily arbitrary because the true lengths of lines such such as aH in three dimensions are unknown. What makes it possible for Desargues to project as he does is that his method preserves the proportional relations which are essential to the perspective situation; for example: aD:aH = ad:ah. Thus a drawing is created in the plane of the page which preserves all of the perspective and hence proportional relations which a model in three dimensions would have.[18]

INVOLUTION

It is in Desargues' theorem of involution that we find his most explicit statement of the relations which remain constant through central projection. An involution is a relation which may occur in a set of elements which are conjugate to each other in pairs. In (Fig. 3-9a), there are two conjugate pairs of points AB and A'B'.[19] The necessary and sufficient condition for their being in involution from point O is that: OA x OB = OA' x OB'. Any number of points on a line may be in involution as long as the condition that all products of conjugate pairs be equal is met.

In modern form, Desargues' involution theorem may be stated as follows: <u>Any transversal meets a conic and the opposite sides of a (complete) quadrilateral inscribed in the conic in four pairs of points which are four pairs in involution.</u>[20] In (Fig. 3-9b), the theorem is illustrated for the circle, but it applies to all conic sections. Points DCEB define the quarilateral in the plane of the conic section represented by the circle. One pair of opposite sides EB and DC intersect at F. A second pair, ED and BC, intersect at A. Since the theorem is based on a <u>complete</u> quadrilateral, there is a third pair of sides, EC and BD, which intersect at point O. Now suppose that line PM is the transversal which meets all of the sides of the quadrilateral: EB at P, DC at Q, BC at I, BD at G, EC at H, and ED at K. This transversal also intersects the circle at L and M. The transversal will then contain four pairs of points in involution: PQ, IK, GH, and LM.

Again using the transversals of Menelaus, Desargues also proves (Fig. 3-9c) that if four points AB and A'B' are two pairs in involution, and they are projected from point P onto points A_1B_1, $A_1'B_1'$ of another line, this second set of four points will also be in involution. If now it is imagined that (Fig. 3-9c) reprsents the side view of a cone having its apex at P, it will be seen that the involution of points on a line is a property which persists through projective transformation. And, since central projection is also the fundamental principle of perspective, it will likewise be seen that involution is an invariant property through the projection of a plane of the subject onto the picture-plane in a perspective situation.

The connection between Desargues' involution theorem and his theorem about perspective triangles will become visually evident if Fig. 3-9b is mapped onto a simplified version of his drawing for the perspective triangles theorem. This is shown in (Fig. 3-9d). The circle with its inscribed quadrilateral BCDE may be thought of as lying in the plane of the page. It thus forms the base of a cone which has its apex at H. The ellipse represents a section of the same cone in a plane oblique to the page and intersecting the plane of the circle at points C and D. The transversal and diagonals of the quadrilateral in the plane of the ellipse have been omitted to avoid cluttering the drawing.[21]

CONICS, PERSPECTIVE AND STEREOTOMY

Vaults may be of two basic kinds--cylindrical and conical.[22] A cylindrical vault may be regarded as a segment of a cone with its apex at an infinite distance from the vault. A conical vault may be regarded as a segment of a cone with its apex at a finite distance from the vault. In either case, the face-plane and the plane of the right-arch may be regarded as two sections of the cone or cylinder. When the face-plane is not parallel to the plane of the right-arch, one or the other of these two planes may be thought of as corresponding to the picture-plane of a perspective situation, in which case the other plane section of the vault will correspond to the plane of the subject in the perspective situation. Further, these two planes, whether taken as the face-plane and the plane of the right arch in stereotomy, or as the picture-plane and a plane of the subject in a perspective situation, will always intersect in a Desargues line. These statements represent the basic connections between Desargues' method of stereotomy, his method of perspective, and his study of conic sections. In what follows, they will be explored in detail.

Today, we ordinarily tend to think of orthogonal projection and perspective projection as two different things. Desargues did not. Instead, he reasoned that orthogonal (or parallel)[23] projection is merely a case of perspective (or central) projection in which the point of projection lies at an infinite distance from the planes in which the projection occurs. As shown in Fig. 3-10), Desargues' theorem applies to parallel projection as well as to central or perspective projection. In the dawing at the top of the figure, we see four triangles in perspective projection from point V. In perspective projection, there is one case (represented by triangles BDA and B'D'A') in which two corresponding sides BD and B'D' do not appear to intersect the Desargues line NLM. This is the case where BD and B'D' are parallel to the Desargues line. This, however, will not be a real exception if one holds the view that parallel lines intersect in a common point at infinity. This was Desargues' view.[24]

The drawing at the bottom of (Fig. 3-10) shows four triangles in parallel projection. In this case, the point of projection V will lie at an infinite distance. However, in this case too, it will be seen that the sides of the two triangles ABC and A'B'C' define a Desargues line NLM just as in the case of central projection. In parallel projection, there is likewise the one apparently exceptional case (triangles BDA and B'D'A) which have a side parallel to the Desargues line, but here again, these sides may be taken as intersecting the Desargues line in a point at infinity.

While the Desargues line is defined by the intersection of corresponding sides of triangles, it is also the line of intersection between corresponding sides of any figure in projection and is so regardless of whether the projection is parallel or central. This relationship between triangles and other figures in projection can be shown (Fig. 3-11) by the projection of two hexagons together with their diagonals. In the figure, we see two hexagons in perspective projection from V. The corresponding diagonals and sides intersect the Desargues line NM in points such as N, L, and K. It will also be noted that if the parallel sides of the hexagon in what appears to be a ground plane are extended beyond the Desargues line, they intersect in points such as VL and VC which can be regarded as defining the horizon line of a perspective. For reasons of convenience, the smaller hexagon was drawn in true shape, but it could equally well be regarded as a perspective of an irregular hexagon seen in the one perspective view which makes it look regular.[25] If it is imagined that the figure represents a perspective situation in which the observer's eye is at V, then the larger hexagon is in the subject-plane, the smaller is in the picture-plane, and the Desargues line NM represents the intersection between the two planes.

As an alternative reading, consider what cases in stereotomy (Fig. 3-11) might represent. One possibility is that the two hexagons and the lines joining their vertices represent a conical vault (carried around through

360 degrees), with the larger hexagon lying in a skewed face-plane. (This is easier to see if the page is turned so that the Desargues line TT is vertical.) Regarded in this way, point V lies at a finite distance, and the line EE joining the centers of the hexagons becomes the axle of the vault. If the smaller of the two hexagons is taken as lying in the plane of the right-arch, then the Desargues line will be the cross-axle (defined previously as the line of intersection between the face-plane and the plane of the right-arch; (cf. Chapt. II). Further, the diagonals of the hexagons will represent the voissoir joint lines.

The projected hexagons should make it easier to recognize the relationship between perspective, stereotomy and conic sections in Desargues' thinking, for it will be evident that the two hexagons may also be regarded as two sections of a cone with its apex at V. This relationship is what Desargues is hinting at toward the end of his 1639 publication on conics when he says that: "In a given wall-face, the arrises of the cut stones share among themselves a single ordonnance which is also the axle of the ordonnance of the joint planes which pass through these arrises."[26] The diagonals of the hexagons correspond to the arrises or joint lines in the face-plane and the plane of the right-arch. The center of each hexagon is the ordonnance of the set of joint-lines. And, the axle, which connects the centers of the two hexagons, has on it point V which is the ordonnance of the lines of the joint planes which stretch toward it. The positions of the sub-axle SS and the counter-axle CC are also represented in (Fig. 3-11). The sub-axle is defined as a line in the face-plane perpendicular to the cross-axle TT. The counter-axle is defined as a line in the plane of the right-arch perpendicular to the cross-axle TT and to the axle EE.

UNIVERSALITY:

The several methods of Desargues are "universal" on at least two levels. At one level, each method is designed to handle all cases within its domain by the application of a single set of rules. In the case of the perspective method, this means that all one and two-point perspectives are constructed according to the same method. As we saw, one of the key steps in doing this was to locate the points for a two-point perspective in a corresponding one-point perspective. In the case of stereotomy, there is one method for all kinds of vaults. Likewise, there is one system for all sundial calibrations. At a second level, each practical method is based upon the application of the properties of conic sections to the descriptive problems within the domain of each method. Thus the relations between conic sections become the basis for solving problems in several fields between which the major differences are in subject matter rather than in underlying geometry. It is to Desargues that we must credit the first recognition of the geometrical relations between these apparently diverse fields.

DESARGUES LINES AND MODERN DESCRIPTIVE GEOMETRY.

Now that the concept of the Desargues line has been introduced, it is necessary to draw attention to one feature of Desargues' descriptive system which, perhaps more than any other, sets it apart from Mongean descriptive geometry. If a distinction is not made between the Desargues line and the hinge line between projective planes in descriptive geometry, confusion is sure to be the result. Both lines are indeed lines of intersection between planes, but this is the extent of the relation. Any attempt to apply the rules for operating with one to the use of the other will produce erroneous results. In modern descriptive geometry, an object which is given an orthogonal projection onto a plane normally does not lie in a plane which has a hinge-line in common with the plane receiving the projection, and the projection lines

<u>must</u> be perpendicular to the plane receiving the projection. In the case of the Desargues line, on the other hand, the figure projected <u>always</u> lies in a plane which has the Desargues line in common with the plane receiving the projection and the projection lines <u>need not</u> be perpendicular to <u>either</u> plane. The differing results produced by each system are shown in (Fig. 3-12). In the top drawing, triangle ABC is shown projected onto two typical descriptive planes 1-1 and 2-2. The line HH is the hinge line between the two planes. The line DD is the Desargues line for the intersection between plane 1-1 and the plane of triangle ABC. In the bottom figure, the two descriptive planes have been unfolded along the hinge line HH. It will be seen that the Desargues line for these two triangles does not coincide with the hinge line HH.

The difference between the Desargues line and the hinge line of descriptive geometry is fundamental to the difference between the two conceptual systems involved. In Mongean descriptive geometry, the reference system is, as we have remarked elsewhere, completely separated from the object being described. Thus, in the Mongean system, the object described is thought of a something surrounded by and independent of the descriptive planes, whereas in Desargues' system, the planes coincide with the planes of the object itself. For this reason, Desargues' system of stereotomy is most successful in dealing with objects which can be defined completely by a projection of a two-dimensional figure from one plane into another. Obviously, vaults, sundials and perspectives are well suited to this treatment. With the beginning of the Industrial Revolution however, the number the problems which could be given this sort of description began to be outweighed by others which required a different approach. Examples would be cast machine and building parts. One might say that Desargues' stereotomy is the descriptive geometry of masonry construction, whereas the descriptive geometry of Monge is for the Industrial Age. The far-reaching implications of the differences between the systems will be considered in the following chapter.

CHAPTER IV

FROM SYMBOL TO INSTRUMENT OF TECHNOLOGY: THE ROLE OF DESARGUES IN THE INSTRUMENTALIZATION OF ARCHITECTURAL GEOMETRY

To understand the full significance of the work of Desargues as a precocious step toward the instrumentalization of geometry, it is necessary to situate his accomplishments in the process through which geometry was transformed from a cosmological symbol into a tool for the control of practice. This means that it is necessary to survey, if only in brief, the changes in the attitude toward architecture and geometry from the earliest written documents up to the time of Desargues and then again from his time to the present. While developments in the attitude toward geometry in architecture will be emphasized, what is said is not and cannot be limited to that field. Here under discussion is the gradual transition from a time in which geometry was considered the most real and essential reality of all things to a time when it came to have no reality at all, the latter condition being most decisively realized with the discovery of Non-Euclidian geometries. As we saw in

Chapter I, in 1660, it was still possible for Desargues to get into a heated debate "an punctum geometricum sit ens revera existens" (whether the geometric point is a really existing thing). Properly understood, that amounted to a debate about whether geometry is reality, for such disputes fall ultimately within the shadow of the reinterpretation of Plato and Aristotle along Christian lines by the scholastic philosophers, beginning in Mediaeval times. Plato had raised the question about whether the Form, Idea, or essence (eidos) is a reality which exists independently of the objects in the physical world which exemplify it. Throughout the Dialogues arguments are given for such a view, and it was an idea which the Christian philosophers could be sympathetic to for reasons which will be discussed below. Well into Renaissance times, scholastics continued to debate what came to be called the realist/nominalist controversy--whether class names denote essences which have an existence which is independent of the physical objects which make up the class. The view that geometry is the essential reality of things derives from the Platonic doctrine of ideal essences, but how it does so is a matter which requires a detailed exposition.

THE MEDIEVAL AND GOTHIC UNDERSTANDING OF ARCHITECTURAL GEOMETRY:

When, circa 1250, the master mason Villard de Honnecourt so engagingly addressed those who might look into his Sketch Book by saying:

> Wilars de Honecort v(os) salue (et) si proie a tos ceus qui de ces engiens ouverront, c'on trovera en cest livre q(u)' il proient por s'arme (et) qui'il lor soviengne de lui. Car en cest livre puet o(n) trover grant consel de le grant force de maconerie (et) des engiens de carpenterie, (et) si troveres le force de le portraiture, les trais, ensi come li ars de iometrie le (com)ma(n)d(e)(et) ensaigne,[1]

> Villard de Honnecourt greets you and bids all those who work with the devices found in this book to pray for his soul and to remember him. For in this book one will find strong advice on the great power of masonry and the devices of carpentry. You will also find the power of portrayng (with) drawings--in the way that the art of geometry commends and teaches,

stone-cutting and building in general were very much processes of discovery by doing. Such rules and procedures as then existed were memorized and passed on by word of mouth from master to apprentice.

If, today, the role of geometry in architecture has been reduced to mere techniques for the instrumentalization of technology and proportional systems are devalued to the status of "games" or condemned as mere "number mysticism," it is all the more important for us to realize that these were not always the prevailing attitudes. In Villard's time, architecture was masonry and masonry was geometry, as the following passage from the Constitutions of Masonry (circa 1400) clearly states:

> . . . and they (the Egyptians) took their sons to Euclid to govern them at his own will and he taught them the craft of masonry and gave it the name of Geometry because of the parting of the ground that he taught to the people in the time of the making of the walls and ditches aforesaid to close out the water (of the flooding Nile).[2]

The Cooke MS also contains an older version of the story from The Old Book of Charges to masons:

> Good men, for this cause and this manner masonry took first beginning. It befell sometime that great lords had not so great possessions that they might not advance their free begotten children for they had so many. Therefore they took counsel how they might their children advance and ordain them honestly to live; and sent after wise masters of the worthy science of Geometry (that) they through their wisdom should ordain them some honest living.
>
> Then one of them that had the name which was called Euclid, that was most subtle and wise founder, ordained an art and called it masonry.[3]

Clearly, we are here confronted with a concept of geometry which differs in important ways from the modern understanding of the subject. And yet the "myth" of the Old Book of Charges, if transparently a fiction in terms of historical fact, is even so a true expression of the relations between geometry, masonry and architecture as conceived at the time Charges were given. According to the story, certain great lords in ancient times were blessed with a capacity to begat children at a rate which exceeded their ability to support them. Confronted with this difficulty, they sought the advice of wise men; men distinguished above all by their knowledge of geometry. If we consider Plato's discussions of the import of geometry in the *Timaeus*, the *Republic* and at other places, it is not at all hard to see why geometrical knowledge should have been the first mark of wisdom. Consider, for example, the following passage from Book VII of the *Republic* (527); Socrates is talking with Glaucon:

> The knowledge at which geometry aims is knowledge of the eternal, and not of aught perishing and transient.
>
> That, he replied, may be readily allowed, and is true.
>
> Then, my noble friend, geometry will draw the soul towards truth, and create the spirit of philosophy, and raise up that which is now unhappily allowed to fall down.
>
> Nothing will be more likely to have such an effect.
>
> Then nothing should be more sternly laid down than that the inhabitants of your fair city should by all means learn geometry.[4]

For both Plato and for the Medieval mason, geometry was not just that narrowly circumscribed branch of deductive reasoning about figure exemplified by Euclid's *Elements*. As Heidegger has clearly shown, for the Greeks, and particularly for Plato, geometry was above all "ta mathēmata"--all things that can be *learned*.[5]

The Old Book of Charges explains that among the wisest of the wise was Euclid, and here it is crucial to note the sequence of events: Euclid, already master of geometry (knowledge or wisdom in the deepest Platonic/ scholastic sense), ordained an art and called it masonry. Euclid ordained (set into order) the art of masonry; he did not just say "Do it!", he told how to do it. Our task must now be to try to understand something of the "how" of this ordination in terms of which the Medieval mason conceived his calling as a master-builder in the image of geometry as knowledge of the eternal, the One and the imperishable, which is to say, of absolute reality. Among the clues which can help here is the interpretation of the word "geometry" given in the Cook MS itself. Geometry, it is said, is "measure of the earth," and that would seem to be obvious enough since that is what the word literally means. If however we resist the temptation to gloss over these words, they can help us to understand the Medieval view about the relation between geometry and masonry. Foremost among our questions should certainly be one which addresses the meaning of "earth" in this definition. Shall we understand "earth" as mere generalized substance, as though it were a certain, measurable amount of "mud" between here and there? Certainly it was sometimes desirable to know the amount of earth to be excavated, but that, just as certainly, is not what was intended by the author of our definition. In the Cooke MS we read:

> Marvel ye not that I said that all sciences live only by the science of Geometry. For there is not artificial nor handicraft that is wrought by man's hand but it is wrought by geometry; and a notable cause. For if a man work with his hands he worketh with some manner of tool and there is no instrument of material things in this world but it come from the kind of earth, and to earth it will return again. And there is no instrument, that is to say, a tool to work with, but it hath some proportion more or less. And proportion is measure and the tool or instrument is earth. And geometry is the measure of earth wherefore I say that men live all by Geometry. For all men here in this world live by the labor of their hands.[6] (Italics mine)

The modern understanding will be that "earth" has no specific intention--that it stands for something like Cartesian space, and yet there is no reason to believe that the author of the text had any such intention. Here, "earth' stands for that which is in-formed (given a form) when form is given to matter. In Platonic/ scholastic terms, one might say that this "earth" is not measured mud-substance at all, but "Earth" as a member of the four primary elements--earth, air, fire, and water as they are defined in Platonic cosmology. This is why the writer of the text tells us that there is no instrument of material things in this world which does not come from Earth or which, upon destruction, will not return to Earth.

But there is more. The second key word in this pasage is "measure". Proportion, we are told, is measure and the tool is earth. Thus we are being told that a thing comes into being through geometry and is therefore proportioned earth. The reality of the thing lies in the order with which the marker has invested the chaotic, un-formed primary Earth by the application of geometry as the "just measure" or proportion. In Platonic cosmology, proportion as just measure is everywhere manifest in the harmony of the universe and in the human body and the well-ordered soul:

> And (musical) harmony, whose motions are akin to the revolutions of the soul within us has been given by the Muses to him whose commerce with them is guided by intelligence not for the sake of irrational (un-ratioed) pleasure, but as an ally against the inward discord that has come into the revolution of the soul . . .[7]

Despite the references to Euclid in the texts we have been considering, it is evident that the Medieval mason did not think of geometry as a deductive system of axioms, postulates and proofs. As Lon Shelby has pointed out, the Medieval masons' education was such that in general he would have lacked the language skill in Latin and Greek to read the as yet unvernacularized texts on mathematics and geometry.[8] More importantly however, even if he could read them, he would have had no reason to do so, because the idea of geometry as a deductive system was completely foreign to his needs and to the way he understood his work. For the medieval mason, masonry was geometry precisely because geometry was what one did in building. To engage in masonry was fundamentally to give form to matter, where form (the just measure) was considered the idea and essence of the thing made--its truth and its reality.

Most discussions of early masonic lodge books have tended to emphasize only the purely practical significance of the texts as handbooks for obtaining results. As Shelby has pointed out, they did indeed have a practical and prescriptive rather than a theoretical orientation which is evident in the basic mode of exposition which they employ: "If you want to solve _that_ problem, do it _this_ way."[9] But it is necesary to consider these handbooks, not in isolation, but in the context of the other masonic writings with which they are contemporary. To fail to do so is like attempting to understand the thinking of the best modern architects just on the basis of the technical handbooks which are commonly found in architectural offices. When however the early mason's handbooks are considered in the light of the ideas about geometry expressed in the _Cooke MS_ and other such writings, then even the practical intentions of the handbooks themselves appear in a different light, for it becomes evident that for the medieval mason, practice as an art of making has transcendental implications quite different from the modern idea of 'technology'. Because the Nile flooded, Euclid taught those aspects of the highest order of knowledge (episteme, in the Platonic sense) that had a direct bearing upon the need to live which the circumstances of the flood (perhaps here conceived with reference to the archetypal flood of the Bible) brought to light. Indeed, following the Platonic metaphor of birth as a descent of the soul into a condition of chaos and forgetfulness, the 'flood' of the Nile might itself be understood as a submergence of the soul in a chaotic medium which is inimical to its survival. What Euclid taught was measure, understood not just as the precise specification of spatial extension, but as the very archetype of all human intellection. As Gadamer puts it at the end of his profound study of Plato's dialectic: "It appears that it is <u>the</u> human task to constantly be limiting the measureless with measure."[10] At the highest level, masonry was wisdom--symbol of the primordial nature of human understanding as the giving of form to chaotic matter--as the measuring of the as yet boundless.

The intentions of the early masonic texts can only be grasped through an understanding of the world-view in which they were grounded. Foremost among the documents of that view are of course the Bible and the vast scholastic literature which attempts to reconcile the teachings of the Bible with the views of Plato, Aristotle and the Neo-Platonists. A more direct and specific influence however may be the Ten Books on Architecture by Vitruvius. Writing circa 50 B.C. with the expressed aim of bringing together in one volume the ideas on architecture scattered through a number of earlier sources, Vitruvius was able to provide medieval architects with a direct link to Pre-Christian thought. The oldest extant manuscript of the Ten Books dates form about 900 A.D., and, in all, some 55 hand-written copies have come down to us. Throughout Roman and medieval times, there are references to Vitruvius' text and there were copies in the libraries of a number of monasteries.[11]

A number of Vitruvius' Platonically tinged ideas seem to find an echo in the early masonic manuscripts, though it is impossible, and in any event, not ultimately important to determine whether this is due to direct tranmission or to the general take-over of Platonic and Neo-Platonic thought into the medieval Christian world-view. Foremost among the Vitruvian concepts which may be cited in this regard is his treatment of drawings as ideas. As Vitruvius develops it, the distinction is reminiscent of the one Plato makes between Matter and

Form (hylē and eidos). This becomes explicit in the discussion of dispositio where Vitruvius tells us that:

> Dispositio autem est rerum apta conlocatio elegansque e compositionibus effectus operis cum qualitate.
>
> Species dispositionis, quae graece dicitur ideai, sunt hae, ichnographica, orthographia, scaenographia . . .[12]
>
> Dispositio however is the fitting arrangement of things and the elegent effect of the work resulting from putting them together with qualitas.
>
> The kinds of dispositio which are called in Greek ideai (ideas) are ground plan, elevation and scaenographia (drawings for stage sets).

Thus a drawing is an idea, form or archetype of the arrangement of a building, and the building itself is then, in a sense, formed matter. Further, in so far as the drawing or idea may precede the coming to be of the building and/or survive its destruction, drawings, like ideas, would appear to have at least a quasi-eternal status analogous to that of the Platonic Forms. But the Platonic treatment of the Idea as the highest level of reality and as a thing apart from its physical manifestation forces us to consider what Vitruvius really means when he says that plan and elevation are ideas. Since Vitruvius was not a philosopher, it would be gratuitous to suppose that his statement represents a carefully developed philosophical position. On the other hand, given his frequent references to Plato, it is improbable that he had not at least heard something of the Platonic and Neo-Platonic views on the subject. While this is a difficult question which cannot be fully resolved within the scope of the present study, it is so important for the development of Western thought about architecture that it must be given some consideration. We might begin by considering what Plato would have said about the relation between architectural drawings and ideas. For Plato, all physical objects are but imperfect copies of ideas or essences. Thus a drawing is also, in a sense, a copy--perhaps the first copy--of the essence. Indeed, the ambiguity of the relation between idea, notation system and reality has persisted in Western thought to the present day. Architects, for example, still debate about whether a thing (idea or drawing) is architecture if it is not built. And, in his Sketch of a New Aesthetic of Music[13] (1910) Busoni proposed that any act of notating music is ipso facto a case of transcription.[14] In other words, even the first writing out or composing of a musical work is, in his view, the transcription of a musical idea or essence which, by its very nature, transcends the scope of the notation system. Those who have tried to learn a complex and unknown musical work from the printed page alone might agree with this.

Vitruvius offers no explicit basis for distinguishing between prior intentions and the built result, and this is at least in part because in his time it was still normal for architects to make even major changes at the building site as both problems and possibilities were revealed in the course of construction. Thus the intention was always something which was embodied in the building itself rather than in the drawings made before construction was started. In a sense, architects always worked with a full-sized model which was the building itself. And yet architects then as now must have had the experience of looking at a completed work only to discover things which might have been better done in another way. Thus it was always possible that a new idea might better represent the essence of the building than the building itself. In this way, architecture was, and, under the best of circumstances, still is a dialectical process.

On the basis of the foregoing considerations, it will be evident that there exist three basic possibilities for the interpretation of Vitruvius' treatment of plans as "ideas". 1) Vitruvius understands "ideas" in a distinctly Platonic sense in which they are the forms and essence which the built reality but imperfectly exemplifies. 2) In Vitruvius, the "idea" is opposed to reality, which only the physical building can present to experience. 3) Vitruvius' understanding of the ontological priority of the "idea" as truth and essence is clouded by the same ambiguity which has affected the whole of Western thought about ideas and reality since the time of Plato.

In the present and concluding chapter, there will be occasion to further examine all three of these interpretations. I will present evidence which suggests that (3) above is the most accurate view.

ATTITUDES TOWARDS ARCHITECTURAL GEOMETRY IN THE RENAISSANCE:

The view that a drawing is not the reality of the thing drawn appears in Renaissance architectural theory. During this period, the development of perspective drawing added a new dimension to questions about the relations between idea, drawing and architectural reality. In his 1465 Treatise Filarete makes this point, but with the somewhat curious qualification that a perspective drawing shows what is true in drawing:

> You can say that it (a perspective drawing) is false for it shows you a thing that is not. This is true; nevertheless, it is true in drawing, for drawing itself is not true but a demonstration of the thing you are drawing or what you wish to show. Therefore it (perspective) is true and perfect for this, and without it, the art of painting or sculpture cannot be done well.[15]

Filarete's position will be less than transparent to the modern reader. A perspective drawing, according to his point of view, shows or represents depth, but does not have it--the drawing is flat like the sheet of paper on which it is made. Thus the drawing shows depth where in fact there is none--i.e. in the picture-space on the sheet of paper. Thus he says that the drawing shows you a thing that is not. Further, the drawing is a "demonstration" of the thing you wish to show--it is not the thing itself, but a demonstration of it. And yet the perspective may be true in the drawing, which means that it may be correctly or incorrectly constructed according to the laws of perspective representation.

Filarete was much closer than we are to the discovery of perspective drawing. Indeed, he offers the opinion that "Pippo di Ser Burnellesco" must have discovered perspective by looking into a mirror.[16] A mirror also shows you a thing that is not in so far as it shows depth where there is none--you may be able to walk through the looking glass, but you cannot step into it! To recapture Filarete's wonder at perspective drawing, it is necessary to overcome our own indifference to it. In Filarete's time, the possibility of perspective drawing was so novel and unexpected, so contrary to the factual flatness of the drawing, that it was regarded both as an illusion and as a representation of reality. And yet for Filarete, the perspective, like the mirror, has its own truth, a view which can perhaps only be proposed when truth is taken as "correctness" rather than as the disclosedness of being. This is a point we shall return to in due course.

Alberti, who wrote his Ten Books On Architecture shortly before Filarete composed his Treatise, offers a similar view on perspective:

> Between the design of the painter and that of the architect, there is this difference, that the painter by the exactness of his shades, lines and angles, endeavours to make the parts seem to rise from the canvass, whereas the architect, without any regard to the shades, makes his relieves (reliefs) from the design of his platform (plan), as one that would have his work valued, not by the apparent perspective.[17]

In other words, the architect designs in the three dimensions, not by means of perspective drawing, but by means of the plan. In the plan, no attempt is made to recreate the visual experience which the finished building will create. The attitude that perspective representation was the creation of an illusion and therefore, except in free-hand sketching, unfit to hold a rightful place among the kinds of drawings used in architectural design persisted well into the Baroque period and even later.

The symbolic value attached to geometry in medieval times was carried over into the Renaissance with little change even though the architect was becoming more of a designer and thus less of a builder and maker. Filarete is quite explicit about the symbolic import of geometry:

> As everyone knows, man was created by God; the body, the soul, the intellect, the mind, and everything was produced in perfection by him. The body was organized and measured and all its parts proportioned according to their qualities and measure . . .
>
> Since man is made with the measure stated above, he decided to take the measures, members, proportions, and qualities from himself and to adapt them to this method of building . . . When a man is well formed and every member is in harmony with every other, then we say that he is well-proportioned. . . . The square and every other measure is derived from man.[18]

Filarete goes on to explain how even the standard of measure in Italy during his time, the braccio (1.91 feet), is derived from the length of a man's arm. He then turns to an explanation of how man, by building in his own image, emulates God:

> Now you will understand how the form of the building is derived from the form and measure of man. . . . The edifice too have all other members in harmony with the "face" (as man does). . . . God wishes that man, just as he was made in His image, should make something similar to himself. In this way, man participates in God by making something in his image through the use of his God-given intellect.[19]

Filarete was, of course, not the only Renaissance architect to attach transcendental symbolic value to

Geometry. Similar remarks can be found in the writings of Alberti, Philibert De l'Orme and many others. All took similar ideas in Vitruvius as their starting point and elaborated upon them in some degree with the aid of Platonic and Neo-Platonic cosmology. One of the most moving passages on the symbolic significance of geometry in architecture and life is found in De L'Orme's Architecture where he explains how the architect, in starting to draw a building, literally makes the sign of the Cross with the first two lines he places upon his sheet. Here the idea that the creation of architecture is a fundamentally scared act which by its very nature calls forth the gesture of consecration is explicitly stated. This passage is such a remarkable compendium of a Renaissance architect's understanding of the relation between geometry, architecture and the world that I quote and translate it here in nearly complete form:

> Nous disons donc que les Architects & maistres Macons ne scauroient bien commencer vn oeuure, soit pour faire vn plan ainsi qui'ils le desirent, ou pour faire modelles ou pour commencer a trasser & marquer les fondements, que premier ils ne tierent sur un ligne droicte, vne autre perpendicularie, ou traict de-equierre (comme l'appellent les ouuriers) soit simplement, ou dedans la circonference d'vn cercle. . . . il faut tousiours commencer par vne ligne perpendiculariement tiree sure vne droicte: laquelle reppresente & figure vn charactere de Croix, qui est si admirable, que ie ne puis passer outre sans escrire ce que i'en ay appris de Marcile Ficin, & autres escellens Philosophes, qui disent que la figure de deux lignes droictes, qui s'entrecouppent

par le milieu a angles droicts, & representent le characterе de la croix, a tant este honnoree & estimee des Anciens (voire long-temps auparauant l'aduenement de Iesus-Christ) que les Egyptiens, comme chose tres-saincte, tres-sacree & miraculeuse, l'auoient engrauee sur la poitrine de l'idole Serapis, laquelle ils adoroient pour leur Dieu. Il se trouue d'auantage que les Arabes tres-scauants en la cognoissance d'Astrologie & tout Philosophie, faisoient plus de cas de ce signe de la croix que de tous autres, & l'auoient en si grand estime & reuerence, qu'ils luy attribuoient plus de force, vertu & heur, qu'a toutes autres figures & characters, voire iusqua le tenir auec trest-grand honeur & sainctete en leurs maisons, & lieux sacrez. Mais laissons a part l'honneur & reuerence que nous deuons tous auoir en general a ceste Croix, pour la satisfaction qui a este faicte pour nous en icelle par la mort de Iesus-Christ nostre seul iustificateur, & la prenons & considerons comme vne des primieres & parfaictes figures de Geometrie. Nous la trouuerons en egales longueurs & angles bien droicts, ansi que Dieu autheur de toutes choses, l'a faicte & ordonnee premierement, en creant le ciel & la terre, & la mettant au milieu de la circonference de ses oeuures. Car apres auoir cree de sa seule parole, toute la machine de l"Vniuers, sous vne forme ronde & spherique, il diuisa la circonference d'icelle en quatre parties egales, moyennant deux lignes droictes qui s'entrecouppent au centre & millieu, ou si vous voulez, au poinct de la diuision, qui est la terre. lesdictes parties sont figurees par une croix, & diuisent tout l'vniuers par leurs extremitez, en quatre parties, appelex Orient, occident, Midy, & Septentrion. . . . Quand les estoiles sont venues aux exremitez de la figure ainsi croisee, ou, si vous voulez, de la croix du monde, par le mouuement vniuersel du ciel, elles ont trop plus grande force & vertu qu'ailleurs, comme nous le voyons journellement aduenir: de sorte que s'll se trouue vne Eclipse de Soleil ou de Lune, ou bien quelque grand conjunction des Planetes, qui nous promettent fertilite guerre, mortalite, cherte de viures, ou bien changement de Monarchie, ou Religion, comme nous la voyons a present: si telle constellations se trouuent aux extremitez du signe de la Croix, ou si vous voulez, aux angles du ciel & monde (ainsi appelez d'acuns) elles ont effect merueilleux & incroyable: voire beaucoup plus que si elles se faisoient ou rencontroient aux lieux mottoyans & qui sont entre lesdicts angles. Autant en peut-on-dire des estoilles fixes, quand elles setrouuent iustement leuer, coucher, ou tenir le milieu du ciel auecques les deux luminaires ou Planettes, au temps des susdictes Esclypses & conionctions. Qui n'est autre chose qu'ester droictement sur le poinct d'Orient, Occident, Midy, & Septentroin; ou bien en la premiere, septiesme, dixiesme, ou quatriesme maison du ciel, ainsi que parlent les Mathematicies. . . . Parquoy il n'est de merueilles si lesdicts Egyptiens colloquoient ledit charactere de la Croix au lieu le plus eminent & singulier de tout le corps de leur dieu Serapis, qui est la poictrine, au milieu de laquelle reside le coeur, fource & fountaine de la vie. Parauanture pour figurer que la vie & le salut deuoit auenir aux hommes, par la mort d'vn seul mediateur Iesus-Christ, qui seroit attache au bois, portant figure de Croix, qui est la primere que Dieu son pere a figure au monde.[20]

Thus we say that architects and master masons will find no better way to begin a work, whether they are drawing a plan of what is desired, making models, or drawing and making foundations, than if at the start they first draw a horizontal line and then a second line perpendicular to it--the draught of the square as the workers call it--either by itself or within the circumference of a circle. . . . It is always necessary to begin with one line drawn perpendicular to another that is horizontal, which drawing then represents and gives figure to the Cross-- a circumstance so miraculous that I cannot pass further without writing what I have learned of it from Marsilio Ficino and other excellent philosophers who say that the figure consisting of two straight lines which intersect each other in the middle at right angles was honored by the ancient Egyptians a long time indeed before the coming of Jesus Christ. They considered it as a thing most sacred and miraculous. It was carved on the chest of the idol Serapis whom they worshipped as their God. One finds further that the Arabs who were most knowledgeable in astronomy and all of philosophy fashioned more examples of this sign of the Cross than of any other and expressed greater esteem for it than for all other symbols because they attributed greater power, vertu and fortune to it than to all other sacred figures in their houses and holy places. But let us put aside the respect and reverence which we all generally have toward the Cross because of the Peace which has been rendered us through the death of Jesus Chrrist our sole redeemer and pass now to considering it as one of the first and most perfect figures of geometry. We shall discover that its equal lengths and right angles are the same as those which God, creator of all things primordially made and ordained in creating heaven and earth and which he placed in the very center of his creation. For, after having created by his Word alone the whole framework of the universe within a round and spherical form, he divided its circumference into four equal parts by means of two straight lines which intersected at the center, or, if you will, at the point of division, which is the place of the earth. These parts are configured by a cross and divide the whole universe into four parts by means of their extremities. These parts are called East (Orient), West (Occident), South (Midy) and North (Septentrion). When the stars have arrived at the points of the cross, or, if you will, at the points of the Cross of the world by means of the universal movement of the heavens, they have very much greater force and vertu than at other positions such as we daily see them in, so that when there is an eclipse of the sun or moon, or else some grand conjunction of planets, these promise us fertility, strife, death, famine, or change of rule or religion such as we see at present. If such constellations are found at the extremities of the sign of the cross, or, if you will, at the angles of heaven and earth, as some say, they have a much more marvelous and incredible effect than if they occur in positions which are between said angles. One can say the same about the fixed stars when they are found lying upon or holding a place in the sky with the two luminous bodies (sun and moon) or with planets at times of said eclipses or conjunctions, which is no different from being directly on the points of East, West, South, or North; in other words, in the first, seventh, tenth or fourth house. The casters of nativities speak in the same way. . . . For which reason it is not surprising that the Egyptians placed the figure of the cross in the most eminent and singular place on the whole body of their God

> Seraphis, which is the breast wherein resides the heart, force and fountain of life. In former times it symbolized that life and health would come to man through the death of a single mediator Jesus Christ who would be attached to wood bearing the figure of the cross which is the first figure that God his father drew upon the world.

De l'Orme's cosmological synthesis is remarkable for its capacity to draw together various strands of doctrine and technique from architecture, geometry, astrology, Christianity and Platonic cosmology, and to present them in a unified world view. The tone of the text is eloquent and even poetic. There is no feeling of contrivance about it. In the Renaissance, it was still possible to live the universal harmony as the unquestionable, given order of things, and this passage shows how profoundly that self-evidence might be experienced even by a man who was not a philosopher, but who, as architect, consulted the philosophers.

Concerning the symbolism of the square, a further point is worthy of mention. It does not seem to be generally recognized that the traditional diagram of the houses used in astrology (Fig. 4-1a) corresponds with the geometrical figure used by Gothic masons to generate the proportions of the cathedrals (Fig. 4-1b). This figure also corresponds to the method for doubling the area of a square given by Plato in the <u>Meno</u> (81 & ff.) where Socrates attempts to show by questioning a slave boy that all knowledge is recollection.[21] In Platonic cosmology, the square and the cube are symbols for Earth, one of the four primary elements.[22]

De l'Orme's cosmology is also noteworthy as an example of history as "historia." When the old word "historia" is translated as "story," the fact that an historia was kind of inquiry and exposition is lost sight of. An historia was above all a collocation and idealization of a diversity with the aim of discovering its underlying unity and essence. This is also evident in the Renaissance concept of the historia as a kind of painting. Alberti explains that in composing an historia, the painter selects the best features of man, city, and world and assembles them harmoniously in a single picture-space, even though such perfections might never be found together in nature:

> The early painter Demetrius failed to obtain the highest praise because he was more devoted to presenting the likeness of things than to beauty. Therefore, excellent parts should all be selected from the beautiful bodies. . . . Although this is the most difficult of all, because the merits of beauty are not all to be found in one place, but are dispersed here and there in many. . . .Zeuxis, the most eminent, learned and skillful painter of all, when about to paint a pannel to be publicly dedicated in the temple of Lucina at Croton did not set about his work trusting rashly in his own talent like all painters do now; but, because he believed that all the things he desired (in order) to achieve beauty not only could not be found by his intuition,

> but were not to be discovered even in nature in one body alone, he chose from all the youth of the city five outstandingly beautiful girls, so that he might represent in his painting whatever feature of feminine beauty was most praiseworthy in each of them.[23]

The Renaissance idea of art as mimesis (from Aristotle) does not imply the mere, quasi-photographic copying of the nature in situ. Rather, guided by his capacity to recognize the ideal perfection or essence of things, the artist selects only the best examples. It is by this process of selecting the ideal that an historia is created. That the artist is, nevertheless, to look to nature rather than to invent his forms indicates that nature is thought to function as a vehicle of recognition and recollection of the ideal forms of beauty.

Alberti's Platonic treatment of ideas is also evident in his attitude toward the relation between design and the building as a material object made from the design:

> It is the property and business of the design to appoint to the edifice and all its parts their proper places, determinate number, just proportion and beautiful order, so that the whole form of the structure be proportionable. <u>Nor has this design anything that makes its nature inseparable from matter</u>; for we see that the same design is in a multitude of buildings which have all the same <u>form</u> and are exactly alike as to the situation of their parts and the disposition of their lines and angles, and we can in our thought and imagination contrive perfect <u>forms</u> of buildings entirely separate from <u>matter</u> by settling and regulating in a certain order the disposition and conjunction of lines and angles. Which being granted, we shall call the design a firm

> and graceful preordering of the lines and angles conceived in the mind.[24] (Italics mine)

Here we have Alberti's interpretation of the Vitruvian doctrine that drawings are ideas. If Vitruvius is less than explicit about the Platonic overtones of his view, Alberti is not. For Alberti, architecture is informed matter. The design gives the form and the essence; it exists independently of the matter because it can be used for more than one building and because designs can be invented without building at all.

THE MANNERIST PERIOD: 1500-1600

It is in the Mannerist period that a significant change in the attitude toward geometry in architecture begins to be clear. In Italy, Mannerism coincided with a time of economic decline and foreign invasions. The idea that a return to Greek and Roman values would bring about a definitive rebirth of high culture and extricate man from the Dark Ages begins to be questionable in the light of these events.

At the beginning of the 15th century, there begins to be an interset in the creation of perspective illusions in architecture. At about the same time, there is also a growing interest in anamorphic perspective as a form of entertainment. Techniques of practical geometry were regarded as secrets--not so much in the sense of masonic secrets, but in the sense of the secret order of nature; its hidden meaning and structure. Thus Sebastian Serlio, for example, begins his book on geometry with the following admonishment to the reader:

> How needful and necessary the most secret art of geometry is for every artificer and workman, as those that for a long time have studied and wrought without the same can sufficiently witnesse, who since that time have attained unto any knowledge of the said arte, do not only laugh and smile at their former simplicities, but in truth may very well acknowledge that all whatsoever had been formerly done by them, was not worth the looking on.[25] (Italics mine)

One of the defining characteristics of Mannerism is undoubtedly the opening of a gap between what a thing is and what it appears to be.[26] At this time perspective effects began to be used within the building itself to create a variety of visual illusions. As early as circa 1490, Fullio Lombardo used life-sized perspective scenes on the exterior walls of the Scuola di S. Marco in Venice to create the appearance fo space where there was none.[27] At S. Satiro in Milan, Bramante used the same technique in a much more dramatic way to create the illusion of a choir behind the alter.[28] In this case, the illusory perspective depth of the false choir corresponds exactly to the perceived depth of the two real transepts. To these might be added the many examples of forced perspective in which the bounding surfaces of a space are made to converge even though they look parallel. This results in a space which (depending upon the position of the viewer) looks either longer or shorter than it actually is. Michelangelo's treatment of the piazza at the Capitol in Rome is a well-known example of this effect. Another is the somewhat later (1663) Scala Regia by Bernini at the Vatican.

The perspective illusions devised in the Mannerist period represent the first definitive break with the idea that geometry is the underlying truth and reality of all things. While such developments may appear rather suddenly, they cannot be satisfactorily explained by looking for causes in the proximate social and cultural environment. What gets overlooked in such treatments is that man comes to any local situation already formed--above all by the langauge he speaks, with its attendant set of philosophical problems. While the languages of the peoples we have been studying: Greek, Latin, Italian, French and English are in certain respects very different, they all belong to the Indo-European group, and three of them (Latin, French, and Italian) are much more closely related. The most serious studies of the problem of the relation between idea and reality have shown that the developments we have been considering must be understood as the realization of a tendency which already existed in Greek thought at the time of Plato. This matter will now be examined in detail.

Originally, the Greek word which Plato used for the Form or idea (eidos) meant that which is seen--the form, shape or figure.[29] The "eidos" was what was given in experience--and particularly in visual experience. There was no connotation of deception or disssembling. And yet, very early in Greek experience, certainly prior to Plato's time, the form as essence began to be distinguished from "mere" appearance, so that what a thing truly _was_ no longer coincided with what _appeared_ to be. Thus "eidos" and "idea" came to mean the look of a thing as opposed to its reality or essential nature.[30] This understanding of "eidos" and "idea" is, of course directly contrary to Plato. Plato himself had attached to ideas the status of Form and essence, or such at least is the position he repeatedly cross-examines in the _Dialogues_.[31] To understand how this tranformation in the treatment of appearance came about and to see how Mannerist perspective experiments derive from it as a reactivation of the inquiry and questioning which the ambiguity of the relation between idea and reality provoked, it is first necessary to understand how ideas came to be taken for essences. As Gadamer has pointed out, fundamental to

Greek experience at this stage, and, indeed, to the whole of Western thought from its inception in Greek philosophy is the recognition of a primordial discrepancy between essence and phenomenon.[32] Things are not always what they appear to be. Arising equiprimordially from this very same ground--from the possibility of a discrepancy between truth and appearance--is the fundamental human propensity to limit the measureless with measure, where "to measure" means not just to find out how big something is, but to "take its measure"; that is, to find out *what* one is *really* dealing with.[33] Today this old meaning of "measure" can perhaps still be heard in the expression "to size up", as when we say of a person that he is "sizing up the situation". To size up a situation is to compare one's capacities with the demands which circumstances may place upon one. We extend the same metaphor when we go on to say of a person that he was not "equal to the task before him."

In his *Introduction to Metaphysics*, Heidegger gives the following account of the transformation of "eidos" from appearance to essence:

> In the end, the word *idea*, *eidon*, "idea", came to the fore as the decisive and predominant name for being (*physis*). Since then the interpretation of being as idea has dominated all Western thinking throughout the history of its transformations down

> to the present day. . . . The word _idea_ means that which is seen in the visible aspect (a thing) offers. What is offered is the appearance _eidos_ of what confronts us . . . . In the appearance, the present, the essent, presents its what and how...Thus _ousia_ (substance, essence, reality) can signify both: the presence of something present _and_ this present thing in the what of its appearance.
>
> Herein is concealed the source of the subsequent distinction between _existentia_ and _essentia_. . . . In the appearance there lies first the standing-out-of concealment, the simple _estin_. In the appearance is disclosed second, that which appears, that _which_ stands, the _ti estin_. Thus the _idea_ and _eidos_ are used in an extended sense, not only for that which is visible to the physical eye but for everything that can be perceived.[34]

What Heidegger here points to is that the possibility of taking the _idea_ as the _essence_ stems from a fundamental ambiguity in the way things disclose themselves to us in experience. This same ambuguity is found in our own verb "to see." When a person says "I see a bird", he is using "see" in a primarily visual sense, though he is also reporting the essence of what he is seeing--namely that is a bird. On the other hand, we sometimes say "I see what you mean". Here, "see" does not refer to a sensory accomplishment but to an understanding of an intention or idea. This same ambiguity runs through a whole range of related English words. It is also found in the case of "to perceive" for example. We normally suppose that perception is primarily an affair of the senses, and yet we can also say "I began to _perceive_ that his _intention_ was to defraud the government." And we can also say in perfectly idomatic English "I began to realize (to make real) that his intention was to defraud the government." The systematic ambiguity becomes even more interesting when we compare English with other languages, but the point is by now obvious. Such seeing and insight depends in part upon what is seen in the visual sense, and yet at the same time it necessarily goes beyond the visual to something not visible--to the essence and idea. Indeed one of the things we least understand about our every-day "seeing" is what part depends upon "idea" and understanding, and what part depends upon pure visual perception, if, indeed, there is any such thing. Perhaps the first mistake is to look for parts!

Thus Heidegger tells us that Plato was able to elevate ideas to the status of essences because of the inherent ambiguity in the nature of perception which always involves both appearance and the simultaneous recognition of the "what-it-is"--the essence. Here it might be added that this is presumably a problem only for those beings who speak a language. While it is certain, for example, that cats see birds, it is just as certain that they cannot see that something is a bird. There is more truth than fiction in the view that language is a sixth sense.

Heidegger goes on to explain that:

> . . . . as soon as the essence of being resides in the whatness (idea), whatness as the being of the essent becomes that which is most beingful in the essent. It becomes the actual essent, *ontōs on*. Being as *idea* is exalted, it becomes true being, while being itself, previously dominant, is degraded to what Plato calls *mē on*, what really should not be and really *is* not, because in the realization it always deforms the idea, the pure appearance, by incorporating it in matter. The *idea* now becomes a *paradeigma*, a model. . . . the cleft has opened between the idea as what really is, the prototype and archetype, and what actually is not, the copy or image . . . appearing is now the emergence of the copy. Since the copy never equals its prototype, what appears is mere appearance, actually an illusion, a deficiency.[35]

Now it was precisely this world view which treats the idea as essence and perfection over and against which the thing itself stands as an imperfect copy which was taken over into the medieval Christian understanding and later carried forward into the Renaissance. Except that, under the Christian reinterpretation, the idea and the essent as form and archetype is held to have been created by God and to exist, above all, in the mind of God. Thus the *Idea* becomes something which mere mortals can never hope to fully understand.[36]

Mannerism accepts the gap thus opened between appearance and reality, and, in a sense, even begins to wallow in it. Above all, the Mannerist perspective illusions in architecture play upon the gap between appearance and reality which was opened in Platonic cosmology. At this point, man the maker begins to emulate his creator in a new way. If the ideas and essences of things reside in the mind of God, and the mind of God is essentially inscrutable, then the world which he created contains but also conceals the ideas upon which it was formed, and truth and appearance are forever after irreconcilable. This world view is duplicated and lived by the architect creating perspective illusions. Geometry as truth no longer coincides with the geometry of appearance.

And yet at this piont the break with geometry as transcendental symbol is still not complete. The most serious Mannerist and Baroque perspective illusions are still created with the aim of bringing about experiences which have transcendental implications. The tapered stair tunnel of the Scala Regia was intended to make the Pope appear larger than life when he passed at the small end. At the same time, looking at his "flock" from his vantage point, the Pope would see very small people. Thus, while an illusion is created, it preserves symbolic order, and, in the case of the Scala, a convention from pre-perspectivist painting is reactivated in which the importance of a person is indicated by his size.

THE BAROQUE PERIOD: 1600-1700

We have now virtually arrived at the time when the work of Desargues begins to appear. And yet, as has already been explained (cf. Chapter I), his views were not typical of the prevailing attitude toward the symbolic value of geometry in architecture. While the Baroque period is characterized by an intense and often acrimonious conflict between symbolic and mechanistic world views, architects, almost without exception, retained the older symbolic view of Geometry. In architecture and masonry, geometry continued to be considered a secret, not a secret in the professional sense of a trade secret, but in the cosmological sense of hidden truth about the order and form of the universe. When Mathurin Jousse published his text on stereotomy in 1643, it bore the title "Secret d'Architecture de'covverant Fidelement les Traits geometriques, couppes, et Derobemens necessaries dans les bastiments"[37] (The Secret of Architecture, Faithfully Revealing the Geometrical Methods and Hidden Factors (derobemens) Necessary in Building).

The period from 1600 to 1700 in Europe is in fact marked by a struggle for ascendancy between three competing world-views: naturalism, mechanism and scholasticism, and it was out of the struggle between these that modern experimental science eventually emerged as the preferred mode of inquiry in all matters. At the beginning of the Baroque period, the differences between these world views and the superiority of the scientific were not as self-evident as we take them to be today. To begin to understand the Baroque experience, it is necessary to identify the salient features of each of these three world views.

Naturalism may be characterized as the doctrine that nature--the macrocosm--has a soul, the nature and structure of which is duplicated in man--the microcosm. This point of view is very explicit in the writings of the English Hermeticist Robert Fludd (1574-1637) who attributes the idea to Martianus Capella:

> Martianus has a threefold interpretation of the word 'world'. (1) As an archetype whose substance is incorporeal, invisible, intellectual and sempiternal; after whose model and divine image the beauty and form of the real world was constructed (as Boethius says): and this world remains permanently in the divine mind. (2) As noncelestial body, i.e. the greater world that is bounded and contained by the concavity of the Primum Mobile: and this world is, by the will of God, eternal. (3) As man, who is called the lesser world, and is said to be perpetual in form but corruptible in body. We differ little from Martianus' opinion in treating the world as duplex: the Macrocosm and the Microcosm. The first is to be distinguished from man, the Microcosm, in that it designates the entire space of prime matter as a world, a Cosmos, or a Macrocosm: for the spiritual light, or the spirit of God, encircles both waters in its embrace. This portion of the abyss is composed of a circle of manifold lights and darknesses, divided into three regions according to their degrees of purity and impurity. The highest is

> the region of the world where the igneous spirit is prepared, and the primary substance of light contained, extending inwards from the sphere of the Trinity as far as the sphere of the stars. . . . These (regions) the ancient Philosophers call the regions of the Macrocosm, and the Scriptures call (them) the Heavens. But they call the highest one the Empyrean or fiery heaven, for it is filled with spiritual fire or the substance of light. And it was into that third heaven, which he called Paradise, that Saint Paul was taken in the spirit. The scriptures generally call the middle region the ethereal or 'lucid' heaven, or simply 'heaven.'[38]

Most architects of the Renaissance and Baroque periods were naturalists in some degree, though the extent to which they accepted the full range of occult doctrines and practices which tended to accompany this view, from magic and alchemy to astrology and geomancy is not always clear from their writings. We have already seen something of De l'Orme's commitment to astrology. Alberti, on the other hand, had certain doubts about it:

> I must confess, though I have no such faith in the professors of this science (astrology), and (in) the observers of times and seasons, as to believe their art can influence the fortune of anything, yet I think they are not to be despised when they argue for the happiness or adversity of such stated times as these from the disposition of the heavens.[39]

And yet what Alberti appears to doubt is not the truth of astrological predictions per se, but man's capacity to take advantage of them by modifying his actions according to what the stars portend. Alberti also frequently indicates his acceptance of the macrocosm/microcosm analogy:

> We consider that an edifice is a kind of body, consisting, like all other bodies, of design and matter; the first is produced by thought, the other by Nature; so that one is produced by the application & contrivance of the mind, and the other by due preparation and choice.[40] (Italics mine)

Filarete also accorded a high place to astrology, as did his patron Francesco Sforza who, for example, employed three astrologers to determine the most favorable moment for taking possession of the Castello at Porta Giovia.[41]

If naturalism was one point of view carried over from Renaissance into Baroque times, scholasticism was another. Scholastic philosophy and theology had been associated with the church since medieval times, and remained so. While today the scholastics are commonly taken less than seriously because of their often elaborate deductive arguments and proofs of God's existence, one of their primary concerns was to establish the ideal relation between reason and faith. Scholastic treatments of this issue were both subtle and varied--so much so in fact, that only a few of the main lines of thought can be presented here. Parallel to the problem of the relation between reason and faith or revelation was the question of the relation between theology and philosophy. From Roman times there had been a tendency to use reason and argument for persuasive purposes and to defend Christianity against its critics. There was also a tendency to try to understand intellectually what was asserted dogmatically as revelation in the Bible and other sources. Thus, from its very start, Christian theology entered into a sometimes ill-defined alliance with philosophy. The situation was further complicated by the desire to take over as much as possible of Plato and Aristotle into the Christian world view. Thus, for the medieval philosopher and theologian alike, there was a whole range of master problems which called for the use of reason, and it is also on this account that it is correct to characterize scholastic philosophy as anti-operational, where we mean by this a world view in which the most important truths are sought in the area of ideas and revelation rather than in the experimental probing of the physical world.

From what has already been said of the great influence of the Platonic theory of ideas, it will be evident that the anti-operationalism of scholastic philosophy stems in part at least from that source. This is very evident in the synthesis of reason and relevation proposed by St. Thomas Aquinas (1224-1274):

> Furthermore, that which is introduced into the soul of the student by the teacher is contained in the knowledge of the teacher--unless his teaching is fictitious, which it is improper to say of God. Now the knowledge of the principles that are known to us naturally has been implanted in us by God; for God is the author of our nature. These principles,

> therefore, are also contained by the divine Wisdom. Hence, whatever is opposed to them is opposed to the divine Wisdom, and, therefore, cannot come from God. That which we hold by faith as divinely revealed, therefore, cannot be contrary to our natural knowledge.[42]

Thus, according to the scholastic view point, the important things for man to know are either implanted in his mind by God, in which case he will discover them by the natural light of reason, or else, in the case of truths about God which exceed the capacity of human reason, they are given by divine revelation which proposes them for belief.[43] Sensible things:

> . . . from which the human reason takes the origin of its knowledge retain within themselves some sort of trace of a likeness to God. This is so imperfect however that it is absolutely inadequate to manifest the substance of God. For effects bear within themselves, in their own way, the likeness of their causes, since an agent produces its like; *yet an effect does not always reach to the full likeness of its cause.*[44] (Italics mine.)

Here St. Thomas indicates quite clearly why, from the scholastic point of view, inquiry into sensible things is really pointless. In that way man will never come to know the highest truth which rests with God. Further, since the highest truth of the divine mind is largely incomprehensible to mortals, it must remain a matter for revelation.

One of the few scholastic philosophers to rebel against the predominant emphasis placed on reason by most of his colleagues was Roger Bacon (c.1212-c.1292), one of the early champions of experiment:

> Reasoning draws a conclusion and makes us grant the conclusion, but does not make the conclusion certain, nor does it remove doubt so that the mind may rest on the intuition of truth, unless the mind discovers it by the path of experience.[45]

Bacon was imprisoned for teaching such "novelties" as these, and it is evident from the tone of his text and the treatment he received that his contemporaries were largely unprepared to accept such views.

The scholastic point of view was carried over into the 17th century, especially in France, where it formed the basis of the education provided in the schools of the church. Both Mersenne and Descartes received this form of indoctrination at La Fleche. We have already seen how Descartes rebelled against scholastic speculative metaphysics (See <u>Chapter I</u>), preferring instead to involve himself in experimental studies which would produce results of practical value to man. Mersenne's rejection of both naturalism and scholasticism was even more intense. As Francis Bacon (1561-1626) had already put it:

> I have made the whole of science my domain and I would purge it of two species of brigands: the one (the scholastics) corrupts it with their frivolous disputes, their refutations, and their verbosity; the other (the naturalists) with their deluded experiments, their hearsay and their frauds.[46]

As the term suggests, 'mechancism' is a world view which directly opposes naturalism, or vitalism, or animism, as the naturalist view is sometimes called. Mechanism flatly denies that the world has a soul or that it is a living being. Mersenne argues against naturalism in the following way:

> Ie desirerois fort qu'ils m'explicassent comment il est possible qu'il y ait un ame universelle, et par consequent que mon ame soit la mesme que la vostre, et que celle d'un beuf, d'un moucheron, d'une rose, d'une pierre, et neantmoins que ie ressente aucun mouvement de tous ceux que mon ame exerce en tous ces corps . . . Vostre ame seroit la mienne, elle auroit donc une pareille puissance sur moy que sur vous: et si vous blessois, ou si ie yous tuois, ie n'en serois tout soul coupable, vostre ame en seroit aussi la cause, puis qu'elle seroit aussie bien le principe, et la cause de mes actions, comme des vostres.[47]

> I strongly desire that they (the naturalists) will explain to me how it is possible that there is a universal soul, in consequence of which my soul is the same as yours, an that of an ox, a gnat, a rose, or of a stone, and that nevertheless, I do not experience any of the movement which my soul incites in all of these bodies. . . . Your soul will be mine. It will thus have equal force in you and me, and if I injure you or kill you, I alone will not be responsible, since your soul will also be the cause, for the rason that it will be as much the cause and principle of my actions as of yours.

Such arguments, of course, would have carried little weight with the naturalists who could have replied that the organization of the world soul is beyond the scope of rational comprehension.

In attempting to understand how the mechanistic world view was able to gain the foothold by which it became the foundation of modern empirical science, it is easy to over-estimate the role of reasoned arguments such as the one given by Mersenne. Above all, it was the capacity of the new, mechanistic view to produce a certain kind of physical result through experiment and operation that decided matters. Today, it is almost impossible to understand the confusion and ambiguity which existed at that time. Today we commonly invoke experimental science as the standard against which naturalistic thought is judged to be "mere superstition", and yet, at the beginning of the 17th century, there was no such clear-cut standard to appeal to. Johannes Kepler (1571-1630) is an outstanding example of how this ambiguity was sometimes experienced and lived. While Kepler, the scientist, did indeed make observations and calculations, he also held a thoroughly Neo-Platonic conception of knowledge and used it in developing his account of planetary motion. Kepler's view of knowledge is indeed very much like that of St. Thomas, in that he too believes that ideas are pre-existent in the mind of God and implanted in the human soul at birth. In later life, the soul may recollect these ideas which it perceives by instinct.[48] It is this theory of knowledge which Kepler uses on more than one occasion to explain his own scientific acocmplishments:

> To find suitable relationships among the sense data (of observation) is to find such relations between them as can be brought into correspondence with the real and true harmonies of which an archetype is to be found in the soul.[49]

Or again, explaining his discoveries in terms of both his theory of knowledge <u>and</u> his theory of astrological influences:

> The astrologers will look in vain to find in my horoscope the causes of the fact that in 1596 I discovered the relationships between the planetary orbits, in 1604 the laws of optics, and in 1618, the reasons why the eccentricities of each planet are just <u>so</u> large, and not larger nor smaller. All this has not come into me with the constellations of the sky, but was imprinted on me, on this small lit-up flame of me--when I entered independent life. Indeed the only effect of my birth constellation was the fact that this little flame and my inborn predisposition has ceaselessly been driving my mind to untiring work, and to increasing my lust for knowledge. In short, the constellations have not created my mind with all its spiritual powers--they have only awakened it![50]

Kepler's theory of universal order is clearly derived from Pythagoras. The hypothesis which he set out to prove was that the ratios between the planetary orbits were the same as those of the intervals of the untempered, diatonic, musical scale. This was not an hypothesis founded upon observation but upon the long-established doctrine of the universal harmony and upon the sense of universal order which Kepler felt within himself. In

Kepler's view, it was only by means of a consummate sensitivity both to the order within himself and to the order without, that man could attain to the highest level of understanding:

> It is the nature of man as such that lends to the planetary radiations their effect on itself (on the nature of man) just a the sense of hearing, endowed with the faculty of discerning chords, lends to music such power that it incites him who hears it to dance.[51]

Thus, while the great diapasons of the planetary harmonies fell below the threshold of human hearing, they might yet be perceived by those who knew how to listen to living. This was a point Pythagoras himself had made.

When Kepler began his study of planetary motion, he was at first alarmed to discover that the orbits were elliptical rather than circular. This meant that the planets did not follow the course of the most perfect of Platonic forms--the circle. To resolve this problem, Kepler had recourse to geometry and the theory of proportions. From his study of geometry, he found that the circle and ellipse are related as conic sections. From this it followed that the elliptical orbits of the planets were not different <u>in kind</u> from the circle. Thus, as a symbol, the ellipse could hold a rightful place with the most perfect of figures. Through a careful study of the frequency ratios of the diatonic scale generated by a Pythagorean monochord, Kepler was at last able to establish that at aphelion (orbit point most distant from the sun) and at pherihelion (orbit point nearest the sun), the ratios of the planetary orbits did indeed correspond to the musical scale. That for Kepler this discovery seemed to vindicate a whole world view is perhaps best expressed in the Harmonia Mundi where he explains that:

> . . . since God had established nothing without geometrical beauty which was not bound by some other prior law of necessity, we may easily infer that the periodic times have got their due lengths and thereby the mobile bodies have got their bulks from something which is prior in the archetype . . . (but) they seem disproportionate.
>
> But if you compare the extreme intervals of the planets with one another, some harmonic light begins to shine. For the extreme diverging intervals of Saturn and Jupiter make slightly more than an octave (1:2) and the converging, a mean between the major and minor sixths (3:5 and 5:8) . . .[52] (Italics mine.)

One of the leading problems in the philosophy of science today is still the question of the origin and foundation of scientific hypotheses. A second major problem is that of understanding the criteria by which one hypothesis is discarded in favor of another. That they are simply accepted or rejected on the basis of a "critical" experiment is not a view which agrees with what actually happens. When Kepler discovered that the planetary orbits were elliptical rather than circular, he did not discard either his harmonic theory or the idea that the orbits in some way partook of the most perfect of figures. Instead he modified his auxiliary hypothesis about perfect figures so that the ellipse could be included, and he did so without making an ad hoc adjustment in it. He did not simply say that the ellipse is "really" the most perfect of figures; that would have been ad hoc. Instead, he sought in the geometry of conic sections a relation between the ellipse and the circle which would provide a rationale for admitting the ellipse without violating the basic tenets of his cosmology. Both Thomas Kuhns and Imry Lakatos have shown that such procedures are common in the development of scientific theories, though we are far from fully clear about the "rules" for making adjustments which are not ad hoc.[53] Thus, it is interesting to see that, for Kepler, what had to be done was obvious. For Kepler, what could rightfully be counted as an hypothesis for observation invariably took its grounding in a cosmology and world view which were beyond question. That science has largely relinquished the aim of explanation in favor of correct results (operationalism) makes it all the more imperative that we try to understand the original thinking behind the mechanistic world view which set in motion the developments leading to our own positivistic and operational reduction of the scientific enterprise.

The nature of our inquiry has made it necessary to look at some key developments in the field of science. Now our findings must once again be brought to bear more directly upon the question of changes in the symbolic role of geometry in architecture during the 17th century. Thus we return to Mersenne, who, in addition to being a major figure in the development of mechanism, had some interesting things to say about how the mechanistic world view was to carry over into architecture. What is most striking about Mersenne is that he seems to have understood exactly what the long-term effect of mechanism would be on the field of architecture. To get at Mersenne's views on this matter, we might imagine ourselves posing the following question: "Sir, you have explained that experiment is necessary if we are to understand the order of the world, for, as you say, God neither reveals nor conceals the true nature of things--all is there for man to understand if he will but reasonably inquire." "Where then do you place the greater emphasis--in thought or in experiment?" Only if we accept the 17th century propensity to regard "thinking through" as a bonafide form of "experiment" can we fully understand Mersenne's answer:

> La perfection de l'entendement ne consiste pas dans la Pratique, mais dans la contemplation; et . . . ce que tombe dans la Pratique est beaucoup moins excellent, que ce qui n'y peut tomber, car encore que Dieu soit admirable dans la creation des estres corporels, et des intellectuels, il est neantmoins plus admirable infiniment dans la contemplation de soy-mesme . . . , ce pourquoi la theorie surpasse autant la pratique, que le Ciel la terre.[54]

> The perfection of the understanding consists not in practice but in comtemplation, and . . . that which falls within the realm of practice is very much less excellent than that which does not fall there, for even though God is admirable in the creation of both corporeal and intellectual beings, he is nevertheless infinitely more admirable in the contemplation of Himself. This is why theory as much surpasses practice as Heaven surpasses earth.

And so it develops that even the arch-mechanist of all time has more than tinge of Platonism in his world view! God contemplating himself beholds the divine Idea of the universe at a higher and more perfect level than when he looks into the universe itself. In emulating God's own order, man, the scientist, is constrained to place theory above practice.

When Desargues entered the Mersenne circle, he also entered the ferment of thought our question to Mersenne has just revealed. In his writings, Desargues never makes an attempt to develop the transcendental implications of geometry. He will only say that theory must rule over practice. The source of this view in Mersenne's mechanistic philosophy is by now evident, but it becomes even more so when we consider what Mersenne had to say about architecture:

> S'il n'y avoit point en de Theorie, il n'y auroit en nulle pratique. Et bien que l'edifice soit plus utile pour loger quand il est fait, et que la figure face plus d'effet quant elle est descrite, que quand les bastiment est seulement dans l'imagination, et dans l'idee de l'Architecte, ou que la figure, qui demeure dans l'esprit du Geometre, neantmoins l'edifice, et la figure n'ont pas moins d'excellence dans l'idee des maistres, que quand on le expose aux yeux.[55]

> If there had been no theory, there would no practice. And even though the building is more useful for lodging when it is built and the flat figure more effective when drawn, than is the case when the building is only in the imagination and idea of the architect and the figure remains in the geometer's imagination; nevertheless, the building and the figure are no less excellent as things in the minds of their masters than when exposed to the eyes.

Thus, for Mersenne, what makes a real building better than one which is merely imagined by the architect is reduced to purely pragmatic considerations: The real building is <u>more</u> (note!) useful than the imaginary one because the real building helps to keep the rain out. In this way, the mechanistic world view opens the door to the domination of architectural practice by theory. That this attitude is brought about by a transformation of the Platonic theory of Ideas can now perhaps be seen more clearly. It is likewise no mere coincidence that when the 17th century begins to exalt the <u>Idea</u> as a thing with inherently greater value than practice, we also see a rebirth of pure geometry and mathematics as theoretical deductive disciplines.

The interaction between scholasticism, mechanism and naturalism produced an extremely diversified and complex pattern of attitudes toward geometry in the 17th century. There was an increase in the number of practical handbooks on perspective and stonecutting, and a number of these omitted all reference to the symbolic and philosophical import of geometry. The mechanistic world view now made it possible to free both geometry and mathematics from their time-honored role as symbolic mediators between macrocosm and microcosm. Thus it is interesting but hardly surprising that Descartes' algebraic treatment of geometry, from which all discussion of symbolic import is omitted, should have developed within the shadow and spirit of the Mersenne circle. Considered in the light of the predominant role Descartes' mathematical approach to the subject was to play in the future development of geometry, Desargues' life-long refusal to apply mathematics to geometry represents a curious anachronism, especially given his involvement with both Mersenne and Descartes.

DEVELOPMENTS IN THE EIGHTEENTH CENTURY:

The mechanistic view of things made it both possible and necessary to explore geometry and mathematics as deductive systems. New experiments in physics called for more advanced methods of calculation. Toward the end of the 17th century, motion studies had helped to spur the development of the calculus which continued to be refined throughout the 18th century. Thus it was that mathematics and geometry began to enter into a new form of mediation between man and world--symbolic order was being replaced by the calculated, deductive order of experimental science. Liebniz (1646-1716) had even dreamed of a "universal characteristic" which would allow all matters amenable to deductive reason to be resolved by calculation:

> Whence it is manifest that if we could find characters or signs appropriate for expressing all our thoughts as definitely and exactly as arithmetic expresses numbers or geometric analysis expresses lines, we could in all subjects in so far as they are amenable to reasoning accomplish what is done in arithmetic and geometry. . . . moreover we should be able to convince the world of what we should have found or concluded, since it would be easy to verify the calculation either by doing it over or by trying tests similar to that of casting out nines in arithmetic. And if someone would doubt my results, I should say to him: "Let us calculate sir", and thus by taking pen and ink, we should soon settle the question.[56]

Here Leibniz appears to anticipate a very high-powered system of symbolic logic--one perhaps even stronger than the combination of the quantifier calculus and modal logic can provide us with today. And yet it is important to note that for all his zeal to calculate, Leibniz adds the caveat: "in so far as they are amenable to reason." Leibniz appears to have understood that there were certain important things in the affairs of life which could not be decided by calculation. Like Newton, he was a devoutly religious person, and, while he subscribed to the mechanistic view of nature, he did so with the kinds of reservations which would have been unacceptable to a man like Mersenne:

> The Cartesians rightly felt that all particular phenomena of bodies are produced mechanically, but they failed to see that the sources of mechanism in turn arise in some other cause.[57]

Therefore:

> When I seek for the ultimate reasons of mechanism and the laws of motion I am 'surprised' to discover that they are not to be found in mathematics and that *we must turn to metaphysics*.[58] (Italics Mine)

Thus, while mechanism became the fundamental assumption of experimental science, few were yet willing to accord it the status of an ultimate account of the universe. As Leibniz clearly states, mechanism itself needs to be explained, and this can only be accomplished by appealing to a transcendent order which gives direction and purpose (telos) to mechanism itself. So it is that while, in one sense, mechanism rejected the burden of symbolism inherent in Platonic and Neo-Platonic cosmology, and scientists now founded their experimental activities on a mechanist view of nature's "surface structure," they continued to develop their world picture within a framework conditioned by Greek philosophy. For this reason, the symbolic value of geometry was not at once discarded, but was instead adapted and transformed. Indeed, as late as Kant's Critique of Pure Reason (1781) we have a definite assertion that space, as defined by Euclidian geometry, is an a priori given in human nature, such that no experience in possible without it. For Kant, to be a person is necessarily to have the world which Euclidian geometry describes.[59] Kant, of course, defined the scope of his own investigations in terms of both the work of Descartes and Hume. On the one hand, he wanted to find the limits of pure reason, for which, following Descartes, so many unsupportable accomplishments had been claimed. On the other, he wanted to show that skepticism, especially as exemplified by Hume, sells man short in terms of what reason may accomplish. Thus, for example, in the Refutation of Idealism he undertakes to remodel Descartes' proof of the existence of the world external to the self in such a way that it will be immune to the doubt to the skeptic.

And yet it was in Kant's own time that the a priori nature and necessity of Euclidian geometry began to be called into question. A series of developments led rather swiftly to the discovery that Non-Euclidian geometries could be constructed and that these geometries were both logically self-consistent and sometimes as accurately descriptive of the space of experience as Euclidian goemetry. Once this was recognized and accepted, Euclidian and indeed all geometry lost its last vestige of transcendental symbolic value. If man could invent geometries at will which were consistent with space as experienced, then Euclidian geometry had to be regarded as merely one of many such possible inventions. Far from being a precondition of all experience, and, at least in that sense, transcendental, Euclidian geometry now came to be seen as a merely useful construction of man, having no more rightful claim to necessity than the various money systems used throughout the world as a basis of exchange.

The locus of the problem which led to the discovery of Non-Euclidian geometries was Euclid's fifth postulate, the parallel axiom, which states that:

> If a straight line falling on two (other) straight lines makes the interior angles on the same side less than two right angles, then the two straight lines if extended will meet on the side of the straight line on which the angles are less than two right angles.[61]

From Greek times, this axiom had seemed less self-evident than the others; indeed Euclid himself appears to have been less than satisfied with it because he introduces it only after he has proved as many theorems as he can without it. Two strategies had been used to give the axiom a better basis. One was to attempt to deduce it from the other axioms, and this proved to be impossible, as one would expect in the case of a true axiom. Indeed, if it were deducible from the others, it would not be an axiom but a conclusion. The other strategy was to attempt to replace the parallel axiom with a more self-evident one, and this too proved to be impossible in as much as the substitute axioms always made some equally obscure assumption.

Euclid had cast his axiom in a form which avoided any assertion about infinity (he does not say, for example, that parallel lines will never meet, even if extended to infinity). The Greek world view opposed the idea of an infinitely extended universe (Aristotle, for example, offers five arguments against it). Indeed, even though we speak of "Euclidian space", it would appear that the Greeks themselves never completely separated the idea of space from that of place. There is no word for "space" in classical Greek. The Greek geometers also held the view that in order to be considered real, a geometrical figure had to be constructible. Thus a parallel axiom stated in terms of infinity would have been considered unacceptable on two counts; first, because it implied the extension of lines beyond the finite world, and, second, because it referred to a line which in principle could not be constructed.

The development of perspective drawing raised the puzzle about infinity in another way: what is the ontological status of the vanishing point to which all parallel lines appear to converge? Indeed it was Desargues' own work on perspective which led him to postulate that parallel lines intersect at infinity, though he always maintained that infinity was hard to imagine.[62] By the middle of the 18th century however, the concept of the 'limit' had been rigorized and accepted through the work of men such as d'Alembert and Cauchy, so that infinity in geometry no longer posed an unprecedented idea.[63] It could now be argued that the idea of infinity was an idealization of the same generic order as the geometric point, line, and plane, since these too were ideals which things of the physical world could approach but never perfectly exemplify.

And yet the parallel postulate continued to seem less than self-evident to the most careful and rigorous geometers. By 1817, Carl Friedrich Gauss (1777-1855) was ready to sum up the situation in this way:

> I am becoming more and more convinced that the (physical) necessity of our (Euclidian) geometry cannot be proved, at least not by human reason nor for human reason. Perhaps in another life we will be able to obtain insight into the nature of space, which is now unattainable. Until then, we must place geometry not in the same class with arithmetic, which is purely a priori, but with mechanics.[64]

Here Gauss reveals a characteristically romantic nostalgia and pessimism which was to pervade the literature in all fields as the symbolic crisis of the 19th century gathered force. One is reminded of the paintings of Caspar David Friedrich--Götterdämmerung had arrived. Shortly after Gauss' statement, Nicolai Ivanovich Lobatchevsky (1793-1856) and Janos Bolyai (1802-1860) published Non-Euclidian geometries.

Turning to practical geometry and the geometry of architecture in the 18th century and after, we find developments in the attitude toward symbolism which parallel the developments in pure geometry. The industrial revolution spurred the desire to develop technologies able to guide and control practice. It was during this period that structural engineering as the mathematical science of statics was developed There was likewise a concerted effort to develop academies for instruction in a whole range of practical sciences. In France, one of the most notable of these was the Ecole Polytechnique where Gaspard Monge developed and later taught the first complete system of descriptive geometry. This was just after the French revolution. Monge's work was in many respects a synthesis of techniques created by his predecessors, but at the same time it provided the first truly universal method. In this sense, it may be considered the definitive realization of the project Desargues had attempted 160 years earlier. It also resembles the work of Desargues in another respect: all mention of the symoblic aspect of geometry is eliminated. Thus, in a sense, the work of Monge also completes the spiritual mission which Desargues had taken up under the influence of Mersenne and Descartes.

Monge was at times very explicit in his condemnation of all attempts to "contaminate" geometry with metaphysical considerations of any kind. A stenographic record was kept of some of his lectures at the Ecole and in one of these we find the following remark:

> Au reste les geometres connaissent parfaitement la nature des raisonnements qu'ils emploient: ils savent pour chacun d'eux jusqu'a quel point ils

> peuvent y avoir confiance. La severite exageree que des metaphysiciens qui n'etaient pas geometres ont a plusieurs reprises essaye d'introduire dans la geometrie et dans l'analyse n'a jamais fait faire un pas a la science, et elle a quelque fois retarde ses progres in occupant les geometres a des disputes frivoles et en les forcant d'epuiser leurs forces contre fantomes.[66]
>
> Moreover, the geometers know perfectly well the nature of the reasonings they employ; they know how far each (argument) can be trusted. The exaggerated rigor which metaphysicians who are not geometers have on a number of occasions attempted to introduce into geometry and analysis have never made a contribution to the science, and they have sometimes retarded its progress by occupying the geometers in frivolous disputes and by forcing them to exhaust their forces against phantoms.

Monge's position is entirely instrumentalist and pragmatic in outlook--what matters above all is to obtain useful, practical results. All that can reasonably be required of the practical geometer is that he know when the method he uses will fail to yield the correct result. This attitude of course continues to the present day and in fact extends beyond descriptive geometry to the whole field of practical sciences which we call "technology". The least asked questions remain those which point toward a metaphysical inquiry which seeks to find the most profound basis upon which we might appraise our own technological intentions. One of the aims of the present study is to make a modest contribution toward the reactivation of that inquiry.

Architects writing about the symbolic aspects of geometry in the 18th and 19th centuries were not as quick as the engineers and practical geometers to abandon the transcendental implications of their subject.

THE PRESENT SITUATION:

If Le Corbusier may be counted among the last of architects to attach an explicit symbolic significance to geometry, it must also be admitted that he does so with a certain tendency to suppress overt references to this aspect of his subject. In the two volumes of the Modulor, considerable space is given to showing the practical value of a measure system based upon human proportions and dimensions. Corbusier knew who the opposition was, and he always pretended to accept the practical viewpoint of his time, whether arguing for the Modulor, or any of his other proposals for the revision of architecture. Thus he concludes his Introduction to the Modulor by emphasizing the practical usefulness of his system and by alluding somewhat enigmatically to the visual harmony which might result from its employment:

> In the construction of objects of domestic, industrial, or commercial use, such as are manufactured, transported, and bought in all parts of the world, modern society lacks a common measure capable of ordering the dimensions of that which contains and that which is contained: capable in other words, of offering a solid pledge of satisfaction to supply and demand. To offer such a measure is the purpose of our enterprise. That is its raison d'etre: to bring order.
>
> And if, over and above that, our efforts were to be crowned with (visual) harmony?
>
> . . . who knows . . .[67]

That Corbusier personally attached transcendent value to proportion in architecture cannot be denied. In actual fact, his own real views seem to have been nearly the reverse of those quoted above: The raison d'etre for the Modulor is to create visual harmony and to relate the proportions of artifacts to the proportions of the human body; if, over and above that, the Modulor might provide a universal measuring system . . . who knows . . . Obviously, Corbusier did not feel at all comfortable about presenting himself to the world in a way that revealed his own true priorities; to do so would be to risk criticism as a "number mystic". Indeed, this is precisely the charge which Norberg-Schulz leveled against him in his Intentions in Architecture:

> In letting all the measures of the building correspond to values from the (modulor) scales, Le Corbusier believes to attain the desired order, at the same time as the dimensions remain 'human'. The latter idea is new, in so far as previously theories of proportion only tried to manifest the human order, without using the real size of man as a basic measure. Not only because of the varying size of human beings is the idea hardly convincing. That measures derived from the human body are considered more pleasing than other *is a typical case of 'number-mysticism.'*[68] (Italics mine)

Denying as it does all symbolic value to proportional systems,this passage succinctly summarizes the prevailing modern attitude toward the subject, even among architects. In fairness however it should be pointed out that

Norberg-Schulz no longer appears to hold this view. It might indeed be argued that the quoted passage represents a misunderstanding of Corbusier, because the latter did not really maintain that it is human proportions per se which are pleasing. Rather, what is said to be pleasing is the visual agreement of proportionally related parts in an artifact, or, one might add, in man himself. It might also be pointed out that, as has already been shown, the Renaissance architect did indeed have the idea of proportions derived from the human body. If however, one fails to grasp the full import of the Renaissance and Baroque idea of the macrocosm/microcosm relation, one will also fail to see the fundamental idea that God's economy insures that the same ratios will appear in man and world. The relation of architectural proportions to the human body was also in some degree assured by the use of measuring systems based upon the human body--a point noticed by Corbursier. Filarete's unit of measure, for example, is the braccio (1.91 ft.)--an arm's length which is also three times the height of a man's head, and so on.[69]

The root of many of these confusions lies in our own superficial understanding of symbolism. Experimental psychology has tended to reduce symbols to mere signs--something which stands for something else according to a convention or code. Even after the work of Levi-Strauss, myth remains a form of life which is considered for the most part to fall outside the province of rigorous scientific study. The same confusion also appears in another form. From time to time there have been attempts to prove that the geometry of art and life are identical. A recent study, for example, has shown that vibrating strings and bodies such as planets in gravitational fields obey similar laws.[70] Thus Kepler's search for the universal harmony is presumed vindicated by modern physics. Other studies have attempted to jusify the old number cosmologies with evidence that nature does indeed "prefer" certain geometric shapes and proportions.[71] What lends to all such post-facto "justifications" a certain superficiality is not that they err in matters of scientific fact but that they attempt to justify man's symbolic understanding of the universe by means of <u>causal</u> explanations. What was most moving to historic man was the mystery of an unexplained <u>synchronicity</u> in all things. But, when the magician shows how the trick is done, the magic is lost. It is precisely this wonder which the post-facto vindications of pre-causal world views lose sight of and fail to account for.

At this hour it remains an open question whether man will reach the most profound understanding of himself by means of his present mode of scientific inquiry. Here it would be well to remember two points made by Leibniz, who, perhaps more than any other philosopher, had the gift for asking ultimate questions. We have already seen that while Leibniz insisted upon mechanistic explanation in the scientific study of all phenomena, he also saw a higher level of inquiry which questioned the very foundations of the causal and mechanistic world view: "When I seek for the ultimate reasons of mechanism I must turn to metaphysics." In other words, it is necessary to ask for the foundations of science itself. For Lebiniz however, this question was only a corollary to a yet more fundamental question: "Why is there _something_ rather than _nothing_?" In trying to answer this question, we are still learning to be Leibniz's contemporaries. Therein we may also regain our wonder.

NOTES TO CHAPTER I

In view of the great length of many 17th century French titles, all tites are here signaled by quotation marks rather thanan underscore.
For complete descriptions of the works of Desargues and Bosse mentioned in this chapter but not cited in the notes, see Appendix II.

1. This and the following information about the Desargues family was collected by René Teton, for which see René Taton, "L'Oeuvre mathématique de G. Desargues" (Paris, Presses Universitairies de France, 1951), pp. 11-12.

2. A. Baillet, "La Vie de Monsieur des Cartes" (Paris, 1691) Vol. 2, p. 131.

3. A. Bosse, "La pratique du trait à preuves de Mr Desargues Lyonnoise pour la coupe des pierres en l'Architecutre (Paris, Des-Hayes, 1643). See the unnumbered page immediately preceding the "Avant-Propos." The date given is February 1630.

4. Baillet, op. cit., Vol I, p. 157.

5. Fr. Funck-Brentano, ed., "The National History of France" (New York, AMS Press, 1967) Vol 4, pp. 52-53.

6. Taton, op cit., pp. 12-13.

7. M. Poudra, "Oeuvres de Desargues" (Paris, Leiber, 1864) Vol. I, p. 41.

8. Taton, op. cit., p. 60.

9. According to the information Taton was able to obtain in the records at Lyons, the Desargues family was always associated with the legal and mercantile classes. Taton, op. cit., p. 12.

10. Marin Mersenne, "Harmonie vniverselle" (Paris, S. Cramoisy, 1636-7) Vol II, pp. 440-442. This work contains a number of errors in pagination.

11. Robert Lenoble, "Mersenne ou la Naissance du Méchanisme" (Paris, Librarie Philosophique J. Vrin, 1943) p. 35.

12. Ibid., pp. 50-51. For a list of Mersenne's contacts see also Cornelis de Waard and Rene Pintard, ed., "Correspondance du P. Marin Mersenne, Religieux Minime" (Paris, Presses Universitaires, 1945). The biography in Vol. I, pp. XIX-LV lists and identifies a great number of these.

13. Mersenne to Peiresc, Sept. 1635, in Tamizey de Larrouque, "Les Correspondants de Peiresc" (Paris, 1891), Vol. XIX, p. 138.

14. Morris Kline, "Mathematical Thought from Ancient to Modern Times" (New York, Oxford U. Press, 1972) p. 295.

15. Taton, op cit., pp. 34-35.

16. All translations preceded by the French originals are mine. In those few cases where an established English translation is used, only that is given, along with a citation in the notes.

17. Taton, op. cit., p. 34.

18. Ibid., pp. 185-186. Letter of Descartes to Desargues, June 19, 1639.

19. Desargues to Mersenne, April 4, 1638. The complete letter is reproduced in Taton, op. cit., pp. 80-87. I quote lines 180-190.

20. Descartes to Mersenne, December 24, 1640. Quoted in Taton, op. cit., p. 41.

21. A. Bosse, "La Maniere universelle de Mr. Desargues, Lyonnois, pour poser l'essieu & placer les heures et autres choses aux cardans au Soleil" (Paris, 1643).

22. Lenoble, op. cit., p. 28.

23. Jean Dubreuil, "La perspective pratique nécessarie à tous peintres, graveurs, sculpteurs, architectes, orfevres, brodeurs, tapissiers et autres se servant du dessein, par un Parisien religieux de la Companie de Jésus" (Paris, 1642-9) Three vol.

24. Taton, op. cit., p. 51.

25. Poudra, "Oeuvres" I, pp. 497-498.

26. Ibid., p. 497.

27. No author, "Diverses methodes universelles et nouvelles en tout ou en partie pour faire des perspectives . . . le tout avec un tres grande justesse, promptitude et facilite. Tirées pour la plupart du conteun du livre de la perspective pratique. Ce que servir de plus de response aux deux affiches du sieur Desargues contra ladite perspective pratique" (Paris, Tavernier et l'Anglois, 1642) 14 pages.

28. Girard Desargues, "Six erreurs des pages 87. 118. 124. 128. 132. & 134. du livre intitulé 'la Perspective pratique necessarie a tous peintres . . .'" (Paris, 1642) 11 pages, three plates. Though lost by Poudra ("Oeuvres," I, p. 498). Taton has located two copies in the Bibliothèque National, on which see Taton, op. cit., p. 51, note 1.

29. No author, "Advis charitables sur les diverses oeuvres et feuilles volantes du Sieur Girard Desargues, Lyonnois, publiées sous les titres: . . . , Mis au jour: pour satisfaire au desir qu'il en a témoigné publiquement . . . , (Paris, Tavernier et l'Anglois, 1642) 17 pages. The entire text is reproduced in Poudra, "Oeuvres", II, pp. 251-381.

30. The excerpt is given in Poudra, "Oeuvres", I, p. 498.

31. The "Reconnaisances" will, of course, be found in the original works by Bosse for which they were written. They are also given in Poudra, "Oeuvres", I, pp. 467-494. This passage appears on p. 469.

32. Poudra, "Oeuvres", I, p. 473.

33. Ibid., II, p. 304.

34. Ibid., I, p. 476.

35. See Randle Cotgrave, "A Dictionarie of the French and English Tongves" (London, 1611).

36. Poudra, "Oeuvres," I, p. 481.

37. Ibid., I, pp. 483-484.

38. Jacques Curabelle, "Examen des oeuvres du Sieur Desargues, Lyonnois" (Paris, l'Anglois, 1644) 81 pages. Excerpts are reproduced in Poudra, "Oeuvres," II, pp. 381-389.

39. Poudra, "Oeuvres", II, p. 385.

40. Girard Desargues, "La Honte du S. J. Curabelle, que refuse de maintenir à peine d'une somme & dire de Juges, Geometres & Jurez Massons de Paris si besoin est" (Paris, 1644) Placard. Lost. According to Taton, op. cit., p. 69, note 7, its content is reproduced in Desargues "Recit au vray . . ." (Paris, 1648), pp. 28-29.

41. Jacques Curabelle, "Calomnieuses faussetez contenues dans un affiche du sieur G. Desargues, Lyonnois, in--titulee 'La Honte de S. Curabelle . . . '" (Paris, 1644) placard. Lost. Content reproduced in Desargues "Recit au vray", pp. 29-30.

42. Girard Desargues, "Sommation fait au Sieur Curabelle, au sujet de ses affiches calomnieuses" (Paris, 1644). Listed in the catalogue of the Bibliothèque Mazarine, but not in file. Poudra reconstructed two passages; see "Oeuvres", I, p. 499.

43. Jacques Curabelle, "Foiblesse pitoyable du Sr. G. Desargues employée contra l'Examen fuit de ses oeuvres" (Paris, 1644) 9 pages. Reproduced in Poudra, "Oeuvres", II, pp. 388-426.

44. The "Recit au vray" and the "Sommation . . ."

45. Poudra reproduces the letters and other information in "Oeuvres", I, pp. 46-53.

46. As a standard against which to judge the values of the various sums of money mentioned in connection with Desargues, it is helpful to learn that S. Maupin's yearly wage as town surveyor was apparently 300 livre. See Poudra, "Oeuvres", I, p. 47, item 1.

47. For illustrations of the two projects for Richelieu and a brief discussion of Lemercier's architecture, see Anthony Blunt, "Art and Architecture in France, 1500 to 1700" (Baltimore, Md., Penguin Books, 1957) plate 92 and pp. 114-119.

48. Poudra, "Oeuvres," I, p. 487.

49. E. S. Haldane and G.R.T. Ross, trans., "The Philosophical works of Descartes" (New York, Cambridge U., 1967). Vol. I, pp. 119-120.

50. Poudra, "Oeuvres" I, pp. 491-492.

51. The illustration is from Blunt, op. cit., p. 141.

52. One toise equals six feet.

53. A. Bosse, "Traité des maneres de dessiner les ordres de l'Architecture Antique en tovtes levrs parties" (Paris, by the author, 1664) plate XXXIX.

54. Ibid.

55. Ibid.

56. Blunt, op. cit., plate 100.

57. Bosse, "Traité . . .", plate XXXIV, last paragraph.

58. Ibid., plate XL.

59. Ibid.

60. Various documents indicate Bosse's position in the dispute. Bosse discusses his differences with the Academie in "Le Peintre converty aux precises et universelles Regels de son art" (Paris, 1667). Passages form this are quoted in Poudra, "Oeuvres", II, pp. 67-95. See also the minutes of the "Academie Royale" for the dates: 2/23/1657, 7/3/1660, 7/31/1660, 8/5/60 and 5/7/1661 in the Collection Bibliothèque de l'Ecole des Beaux Arts, Paris. The issue of plagiarism seems again to have been involved. In this case, J. Le Bicheur was accused by Bosse of both using and corrupting Desargues' perspective method. Bosse also relates some of the particulars in an open letter to Desargues (July, 1655) which is reproduced in Poudra, "Oeuvres", II, pp. 49-66.

61. See A. Bosse, "Traité . . .", in particular the section titled "Avis donné par A. Bosse a ceux qui pretendent coriger les regles de perspective par des licences et des Regles de bien-sceance visionnarie" (no page or plate no.) In the plate just before this, Le Bicheur's errors are identified.

62. Extract from a letter of Desargues to Bosse (July 25, 1657), quoted from F. P. Charles Burgoing and reproduced in Poudra, "Oeuvres", I, pp. 503-506.

63. Bosse, "Traité", plate XLIII.

64. References to the lost "Leçons de Ténèbres" are discussed at length in Taton, op. cit., pp. 44-49. The most important of these is in Desargues' own "Six erreurs des pages . . .", where he refers to a second work on conic sections. (This would be in addition to the 1639 "Brouillion project d'une atteinte au evenemens des recontres du Cone avec un Plan.) A brief critical discussion by Gregoire Huret in "Optique de portraiture et de peinture, en deux parties . . . " (Paris, 1670, pp. 157-158 and reproduced in Poudra, "Oeuvres", II, pp. 210-217, describes the "Leçons" as giving a method showing that all conic sections are perspective projections of a circle. This is supported by a letter from Oldenburg to Leibniz (April 16, 1673), for which see C. I. Gerhardt, "Der Breifwechsel von G.W. Leibniz mit Mathematikern" (Berlin, 1899) pp. 40-41, 65 and 78.

65. A. Bosse, "Moyen vniversel de pratiquer la perspective svr les tablevx, ou surfaces irregulieres . . ." (Paris, by the author, 1653) plate 15.

66. The fresco is in S. Ignazio at Rome.

67. Piganoil de la Force, "Description de Paris, de Versailles . . . nouv. ed." (Paris, 1742) Vol 5, p. 346.

68. Jean-Francois Niceron, "La perspective curiuse ou Magic artificiele des effets marveilleux. La catoptrique, par la reflexion des miroirs plats, cylindriques & coniques. La dioptrique par la refraction des crystaux . . ." (Paris, Pierre Billaine, 1638). Later editions in 1646, 1652 and 1663. Niceron also painted a large anamorphic

perspective in the monastary of the Minimes at the Place Royale in Paris where Desargues certainly must have seen it. Descartes is also known to have been interested in Niceron's work. Only the example at Rome still exists.

Jean Dubreuil, "La perspective pratique nécessarie à tous peintres, graveurs, sculpteurs, arhcitectes, orfevres, brodeurs, tapissier et autres se servant du dessein par un Parisien religieux de la Companie de Jésus" (Paris, 1642-49 3 Vol. Second edition, 1651. Dubreuil seems to have had a number of things in common with Desargues. Most notable is the remark in his preface that: "However excellent a painter may be, he must follow these (perspective) rules or end up appealing only to the ignorant."

69. Lenoble, op. cit., p. 20.

70. Taton, op. cit., p. 17.

71. Christian Huygens to Lodewijk Huygens, October 29, 1671, in Oeuvres de Christian Huygens . . . " (La Haye, 1888-1950), Vol. 7, p. 112.

72. Ibid., Vol. 3, p. 182.

NOTES TO CHAPTER II

1. Paul Frankl, The Gothic: Literary Sources and Interpretations Through Eight Centuries (Princeton, Princeton U. Pres,s 1960) p. 132.

2. A. Bosse, La Pratique du Trait a prevves de Mr. Desargues Lyonnois Pour la Coupe des Pierres en l'Architecture (Pierre Des-Hayes, Paris, 1643), Plate 3.

3. Ibid, plates 78, 79, and 67.

4. Philibert De l'Orme, De l'Architecture (Ferrand, Rouen, 1648), p. 89.

5. This illustration is from: William R. Purchase, Practical Masonry, A guide to the Art of Stone Cutting (Gautheir-Villars et Cie, Paris, 1917), p. 83.

6. The illustration is from: R. Pommer, Eighteenth Century Architecture in the Piedmont (N.Y. Univ. Press, N.Y., 1967), plate 148.

7. De l'Orme, op Cit., p. 108.

8. Bosse's work is: La Pratique du Trait a Preuves de Mr. Desargues Lyonnis, Pour la Coupe des Pierres en l'Architecture (Paris, Pierre Des Hayes, 1643). Desargues' own pamphlet on stereotomy is: Brouillon project d'example d'une maniere universelle du S.G.D.L. touchant la pratique du trait a preuves pour la coupe des pierres en l'Architecture; et de l'esclaircissement d'une maniere de reduire au petit pied en Perspective comme en Géométral, et de tracer tous Qudrans plats d'heures egales au Soleil (Paris, 1640). The expression "brouillon project" ("rough draft") was frequently used by Desargues and appears to indicate that he intended these papers as preludes to more complete and polished expositions in book form. These, of couse, were never written. Bosse indicates that his own work is the first volume of a projected, two volume series on stereotomy (cf: p. 50), but the second volume does not appear to have been either written or published.

9. Bosse, op. cit., commentary to Plate 31.

10. Gaspard Monge, Lecons de Géométrie Descriptive (Paris, 1795).

11. Philibert De l'Orme, De l'Architecture (Rouen, Ferrand, 1648), Books III and IV.

12. I am referring to Desargues' Brouillon project d'une atteinte aux evenemens desrencôntres du Cone avec un Plan (Paris, 1639).

13. Bose, op. cit., Plate 2.

14. Unless otherwise indicated, the page references given to Desargues' writings are to the collection by M. Poudra: Oeuvres de Desargues (Paris, Leiber, 1864) 2 Vol. Hereafter, the following citation practice will be observed for this work: Poudra, Oeuvres (abbreviated title of the work by Desargues being referred to) Vol. #, page. Thus, the citation for this reference is: Poudra, Oeuvres (Coupe des Pierres) Vol. I, p. 323.

15. Bosse, op cit., Plate 8.

16. I have not been able to determine whether the method of sloping the vault, as Bosse illustrates it here, was ever used in practice. Much of the stone work in buildings of this period was rendered, and it is thus often only during renovation that the true configuration of the structural masonry can be examined.

17. Bosse, op. cit., Plate 9.

18. Bosse's care in explaining the early steps of the method is evident in plates 12 through 16. Each of the given angles is shown by means of an adjustable square at the top of the plate. After the angle has been used in the method, the square which records it is shown with dotted rather than solid lines.

19. The terms "rise" and "run" are most frequently used in architecture to describe the slope of stairs and roofs. I have used them here because they seem better suited to describing slopes which are not directly related to a true vertical or horizontal line than any others in common use.

20. Desargues never discusses the application of mathematics to his method of stereotomy. The equation is offered here because, for modern readers, it may help to clarify the relations involved in this aspect of his method.

21. At least in the stereotomy of Desargues' time, it seems that the curves of the vault in the plane of the right-arch were generally not drawn because the shape of the right-arch was used only to establish the angles between an intradoes template for a voussoir and the adjoining joint template. This meant that the curve of the intrados and extrados surfaces of a voussoir had to be determined from the shape of the head templates alone--a practice which was not always followed in later times.

22. These illustrations are based on: Steve M. Slaby, Fundamentals of Three-Dimensional Descriptive Geometry, 2nd. Ed. (New York, John Wiley & Sons, 1976) pp 19-20.

23. And, of course, one can go on making projections or deductions until one has completely lost one's way and hasn't the faintest idea about where one is or why one should be there in the first place. This too is a "beauty" of the method.

24. Primarily by his use of an oblique coordinate system.

25. A good discussion of oblique coordinates and their origin in the geometry of Apollonius is found in: Michael Sean Mahoney, The Mathematical Career of Pierre De Fermat (1605-1665) (Princeton N.J., Princeton U. Press, 1973), pp. 80-83 and 113-117. Fermat was a contemporary of Desargues and Descartes. His work was known to Desargues through Father Merin Mersenne who made great efforts to bring the leading minds of 17th century Europe together in a concerted effort to apply experimental and deductive methods to the burning questions of the time in science and mathematics. A good introduction to the concept of oblique coordinates is also found in: Morris Kline, Mathematical Thought from Ancient to Modern Times (New York, Oxford U. Press, 1972) Pp. 302-324. This volume also contains material on Desargues and Descartes, and on the application of mathematics to geometry in the 17th century.

26. Two discussions of Monge's role in the development of descriptive geometry by Rene Taton are worthy of examination. His brief L'Historie de la Géométrie Descriptive (Universitie de Paris, 1954) is a transcript of a lecture tracing the development of the principles of descriptive geometry from The Sketchbook of Villard de Honnecourt through Durer and Desargues, to the work of Monge. Taton's book: L'Oeuvre Scientifique de Monge (Paris, French U. Press, 1951) is probably the most extended account of Monge's contribution to mathematics and science to be published in modern times. On Desargues, Taton's L'Oeuvre Mathematique de G. Desargues (University de Paris, 1951) is indispensable, containing as it does, the fruits of Taton's extensive efforts to locate new material on Desargues in France. it also contains a corrected version of Desargues' work on conic sections.

27. William Ivins presented his discoveries in two articles: (1) "Two First Editions of Desargues", Bulletin of the Metropolitan Museum of Art, 2nd Series, Vol. 1, 1942, pp. 33-45 and (2) "A note on Girard Desargues", Scripta Mathematica, Vol. 9, 1943, Pp 33-48. Only the first of these two articles reproduces all of the plates for Desargues' method of stereotomy. Only sample pages of the text are reproduced. In addition to these articles, Ivins has published two other works which contain information on Desargues: (1) Art and Geometry, A Study in Space Intuitions (New York, Dover, 1946), and (2) "A Note on Desargues' Theorem", Scripta Mathematica.

28. Poudra, Oeuvres. The plate appears in the Appendix at the end. Desargues' original Plate I is reproduced in both of the Ivins' articles mentioned above in Note 27: "Two first . . ." and "A Note on Girard Desargues."

29. Poudra, Oeuvres (Coupe des Pierres), Vol. 1, p. 316.

NOTES TO CHAPTER III

1. Poudra, *Oeuvres*, I, pp. 319-20.

2. René Taton, *L'Oeuvre Mathematique de G. Desargues* (Paris, Presses Universitaries de France, 1951), p. 180. Taton reproduces an extensively corrected and footnoted version of Desargues' 1639 *Brouillon* on conic sections. The corrections are based on a copy of the original discovered in the Bibliotheque Nationale by Pier Moisy.

3. Ibid., p. 100.

4. Poudra, *Oeuvres*, pp. 55-84.

5. The other two works on perspective are: *Proposition fondamentale de la pratique de la perspective* (Poudra, *Oeuvres*, I, pp. 402-9) and *Perspective adressée aux theoriciens* (Poudra, *Oeuvres*, I, pp. 439-462). Both works were published by Abraham Bosse in: *Maniere universelle de Mr. Desargues pour pratiquer la perspective par petit pied, comme le geometral* (Paris, P. Des-Hayes, 1648), *Aux theoriciens* gives a refined version of the method first published in 1636 in the *Example de l'une des manieres . . .* (Poudra, *Oeuvres*, I, pp. 56-84), but contains nothing on the geometrical principles involved. There is also some general discussion of perspective at the beginning of the 1640 *Brouillon* on stereotomy (Poudra, Oeuvres, I, pp. 305-58), but this contains nothing except the hint cited in (1) concerning the geometrical principles. The 1636 *Example* contains a few general statements at the end concerning the basis of the method, but these are covered more precisely in the *Proposition fondamentale*.

6. The plate was reproduced by William Ivins in two articles: "A Note on Girard Desargues," *Scripta Mathematica*, Vol. 9, 1943, p. 45 and "Two First Editions of Desargues," *Bulletin of the Metropolitan Museum of Art*, 2nd Series, Vol. 1, 1942, p. 38.

7. Pierre Descargues, Perspective (New York, Abrams Inc., 1977), Plate 48.

8. Poudra, Oeuvres, I, pp. 402-9.

9. Desargues never mentions that three different cases are being shown, yet there appears to be no other way to understand the drawing.

10. In the 1639 Brouillon on conics in Taton, Op. cit., p. 126.

11. A translation of Ptolemy's Almagest is found in: Great Books of the Western World, ed. R. M. Hutchins (Chicago, Wm. Benton, 1952), Vol. 16, pp. 1-478. The theorem is stated and proved on p. 26.

12. The theorem is the first of three appended to the Proposition fondamentale de la perspective (Poudra, Oeuvres, I, pp. 413-15.) It is also found in Taton, op. cit., pp. 206-7.

13. See for example A.F. Horadam. A Guide to Undergraduate Projective Geometry (Sydney Aus., Pergamon Press, 1970), p. 73.

14. Poudra, Oeuvres, I, p. 415.

15. Desargues makes this point in the 1639 Brouillon on conics, for which see Taton, Op. cit., p. 134.

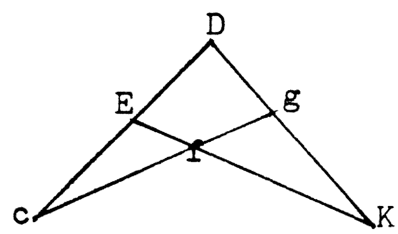
16. For example, Desargues gives the entire proof without mentioning triangles at all.

17. Menelaus' theorem and its converse can be given in various ways, depending upon which segments are compared. Desargues' conclusion is:

$$\frac{gD}{gK} \times \frac{fK}{fE} = \frac{cD}{cE}$$

which is equivalent to:

$$\frac{gD}{gK} \times \frac{fK}{fE} \times \frac{cE}{cD} = 1$$



For Desargues' proof the theorem may be taken as stating that if a transversal cfg cuts the sides of a triangle KDE in c, f, g respectively, then it determines upon them segments which are such that:

$$\frac{gD}{gK} \times \frac{fK}{fE} \times \frac{cE}{cD} = 1$$

The converse of the theorem then states that if on the sides ED, Ek, Dk respectively of the triangle KDE, points c,f,g are taken so that the above relation holds, then c, f, and g will be collinear, which is what Desargues is seeking to prove. See Luigi Cremona, Elements of Projective Geometry (Oxford, Oxford U. Press, 1913), pp. 107-9 for a discussion of Menelaus' theorem and its converse.

18. A good discussion of Desargues' theorem is found in: William Ivins, "A Note on Desargues' Theorem," Scripta Mathematica.

19. They are said to be conjugate through point O.

20. The phrasing follows Cremona, Op. cit., p. 144. However Cremona only mentions three pairs in involution. Figs. 12 and 13 are based on Morris Kline, Mathematical Thought from Ancient to Modern Times (New York, Oxford U. Press, 1972), pp. 292-3. Prof. Kline mentions the fourth pair of points. Fig. 12 is a somewhat modified version of Desargues' own drawing from the 1639 Brouillon on conics which appears in Taton, Op. cit. p. 142.

21. On account of its complexity, Desargues' proof of the involution theorem is not discussed here. The combination of the two drawings is meant to suggest how the theorem of Menelaus would be used in this proof.

22. Conical vaults were sometimes used to create forced-perspective effects in passageways.

23. Strictly speaking, orthogonal and parallel projection are not identical. In orthogonal projection, but not in parallel projection, the projection lines must be perpendicular to the plane receiving the projection. Thus, in a sense, orthogonal projection is a special kind of parallel projection.

24. Of course the view did not originate with Desargues.

25. This point shows how much the reading of a drawing depends upon the percipient's ability to understand the delineator's intention. For someone not familiar with Western conventions, a perspective representation of a house for example might appear to be an elevation or orthogonal projection of a building which was actually built in perspective distortion. Likewise, any orthogonal projection might be seen as a perspective representation from an angle at which the shape of the object represented exactly nullifies the perspective convergence. This indeed is why people in certain primitive cultures find it impossible to undertand photographs upon first seeing them. Likewise with depth-clues such as shading; if shading is mistaken for a darkening of the object in certain areas, it cannot function as an indication of relief. "Perspective play" on reversals and distortions of this sort was popular in the 17th century, on which see: Fred Leeman Hidden Images (New York, Abrams Inc., 1976). At a deeper level they are associated with plays such as the one by Bernini in which the plot is based on creating the illusion that the audience is watching a play being played to another audience.

26. Taton, Op. cit. p. 180.

NOTES TO CHAPTER IV

1. Villard de Honnecourt, "The Sketchbook of Villard de Honnecourt", ed. Theodore Bowie (New York, Geo. Wittenborn, 1959) Plate 2. See also Lon R. Shelby, 'The Geometrical Knowledge of Mediaeval Master Masons', Speculum, 47 (1972) pp. 395-421. This passage is translated on p. 395. While I follow Shelby and Bowie in parts, I have opted for a more literal reading to avoid "modernizing" Villard's idea. This practice is also followed by Paul Frankl, "The Gothic: Literary Sources and Interpretations Through Eight Centuries" (Princeton, J.N., Princeton U. Press, 1960) p.41.

2. The passage is quoted in John Harvey, "The Mediaeval Architect" (London, Wayland, 1972) p. 197.

3. Ibid., p. 199.

4. Republic VII, 526: B. Jowett, trans. "The Dialogues of Plato" (New York, Random House, 1920) 2 Volumes. See Vol I, p. 786.

5. Martin Heidegger, "What is a Thing", trans. W.B. Barton Jr. and Vera Deutsch (Chicago, Henry Regnery, 1967) pp. 69-76.

6. Harvey, Op. Cit., p. 193.

7. Timaeus, 47D.: Francis M. Cornford, Trans. "Plato's Cosmology: The 'Timaeus' of Plato" (New York, Bobbs-Merrill, 1937), p. 158.

8. Shelby, Op. cit.

9. Ibid., p. 415.

10. Hans-Georg Gadamer, 'Plato's Unwritten Dialectic' in: "Eight Hermeneutical Studies on Plato", Trans. P.C. Smith (New Haven, Yale U. Press, 1980), p. 155.

11. Frankl, Op. Cit., pp. 88-89.

12. Ibid., pp. 92-93.

13. Ferruccio Busoni, 'Sketch of a New aesthetic of Music' in: "Three Classics in the Aesthetic of Music" (No ed.) (New York, Dover, n.d.) pp. 75-102.

14. Ibid., pp. 84-85.

15. Filaete (Antonio Averlino), "Treatise on Architecture", Trans. John R. Spencer (New Haven, Yale U. Press, 1965) 2 Vol. See Vol I, p. 305.

16. Ibid.

17. Leone Battista Alberti, "Ten Books on Architecture", Trans. James Leoni (London, Alec Tiranti, 1955) p. 22.

18. Filarete, Op. Cit., pp. 6-8.

19. Ibid., p. 11.

20. Philibert De l'Orme, "De l'Architecture" (Rouen, Ferrand, 1648) pp. 31-33.

21. Examples of the use of this figure in astrology can be found in many sources. For a French example, see: Marin Mersenne, "Les Prelvdes de l'Harmonie vniuerselle" (Paris, Henry Gusnon, 1634) pp 2-20. Here Mersenne casts a horoscope of the "perfect musician." For discussion of the figure in a masonic context, see Shelby, Op. Cit. In the Meno (81 & ff.) Plato makes Socrates prove that all knowledge is recollection by eliciting from a slave boy the understanding that this figure gives a method for doubling the area of a square. The point is illustrated with a diagram in Jowett's translation, Op. Cit., Vol. 1, p. 362.

22. Timaeus, 53C-55C & ff. Cornford, Op. Cit., pp. 211-224.

23. Leon Battista Alberti, "On Painting and On Sculpture: The Latin Texts of 'De Pictura' and 'De Statua'", Trans. Cecil Grayson (London, Phaidon, 1972), p. 99.

24. Alberti, "Ten Books", pp. 1-2.

25. Sebastiano Serlio, "The Five Books of Architecture" (New York, Dover, 1982) 1st Book, 1st page.

26. As noted, for example, by Colin Rowe in 'Mannerism and Modern Architecture', in his: "The Mathematics of the Ideal Villa and Other Essays" (Cambridge, Mass. MIT Press, 1982) p. 34.

27. Illustrated in: Fred Leman, "Hidden Images" (New York, Abrams, 1976) Plates 30 & 31.

28. Ibid., Plates 26 & 27.

29. See, for example: Liddell and Scott, "An Intermediate Greek-English Lexicon" (New York, Harper & Bros., 1897) p. 226.

30. Ibid., p. 375.

31. It is commonly said that while Plato separated matter and form, Aristotle did not. After Gadamer's penetrating study of the function of dialectic in Greek philosophy from an hermeneutical point of view, this oversimplification must be discounted. See Gadamer, Op. Cit., 'Idea and Reality in Plato's Timaeus' (pp. 156-193) and 'Amicus Plato Magic Amica Veritas' (pp. 194-218).

32. Gadamer, Op. Cit., p. 206.

33. Ibid., p. 155.

34. Martin Heidegger, "Introduction to Metaphysics", Trans. Ralph Manheim (New Haven, Yale U. Press, 1959), pp. 180-181.

35. Ibid., p. 184.

36. Ibid., p. 193.

37. Mathurin Jousse, "Secret d'Architecture de'couverant Fidelement les Traits geometriques, couppes et derobemens dans les bastiments" (La Fleche, George Griveav, 1642).

38. Robert Fludd, "History of the Macrocosm". The passage is translated in: Joscelyn Godwin, "Robert Fludd: Hermetic Philosopher and Surveyor of Two Worlds" (Boulder Col. Shambhala, 1979) pp. 14-15.

39. Alberti, "Ten Books", p. 35.

40. Ibid., 'Preface', last page.

41. See Spencer's footnote #14 in: Filarete, Op. Cit., p. 22.

42. St. Thomas Aquinas "Summa Contra Gentiles", 'Book I: God', in Herman Shapiro, ed., "Medieval Philosophy" Selected Readings from Augustine to Buridan" (New York, Random House, 1964) pp. 346-347. This is from 'Chapter 6: That To Give Assent to the Truths of Faith is no Foolishness Even Though They are Above Reason.'

43. Ibid., p. 339.

44. Ibid., p. 348.

45. Ibid., p. 305.

46. Robert Lenoble, "Mersenne ou le Naissance du Mechanisme" (Paris, Librarie Philosophique J. Vrin, 1943) p.11.

47. Ibid., p. 156.

48. See the article by Arthur Beer: 'Kepler's Astrology and Mysticism' in : A. Beer & P. Beer, ed. "Kepler: Four Hundred Years" (New York, Pergamon, 1975) p. 408.

49. J. Kepler "Weltharmonik," p. 206, quoted in: Rudolf Hasse, 'Kepler's Harmonies, between Pansophia and Mathesis Universalis', in: Beer & Beer, Op Cit., p. 527.

50. J. Kepler, "Harmonice", quoted in: Arthur Beer," Kepler's Astrology and Mysticism," in Beer & Beer, Op. Cit., p. 424.

51. Ibid., p. 412.

52. J. Kepler, "The Harmonies of the World", trans. Charles G. Wallis, in "Great Books of the Western World (Chicago, 1952) Vol. 16, p. 1030.

53. See: Thomas Kuhn, "The Structure of Scientific Revolutions" (Chicago, U. Chicago Press, 1962) and: Imre Lakatos & Alen Musgrave, ed. "Criticism and the Growth of Knowledge" (Cambridge, Eng., Cambridge U. Press, 1970). See esp. Lakatos' article: 'Falsification and the Methodology of Scientific Reserach Programs', pp. 91-197.

54. Marin Mersenne, "Les Preludes de l'Harmonie vniverselle", p. 171. Quoted in Lenoble, Op. Cit., p. 362.

55. Mersenne, "Questions harmoniques," pp. 235-236. See Lenoble, Op. Cit. p. 362, Note 3.

56. Gottfried Wilhelm von Leibniz, 'Preface' to: "General Science," quoted in "Leibniz: Selections", ed. Philip Wiener (New York, Scribners, 1951) p. 15.

57. Leibniz: Letter to Schulemburg (Dutens, Vol. III, p. 332). See: Leibniz, "Basic Writings" Trans. George R. Montgomery (LaSalle, Ill., Open Court, 1962) P. XI, Note (*).

58. Leibniz: Letter to Remond de Montmort (Erdman, Opera Philosophica, p. 702). See "Basic Writings," Ibid., P. XI, (Note *).

59. Immanuel Kant, "Critique of Pure Reason", Trans. Norman Kemp Smith (New York, St. Martins, 1965), p. 71 & ff.

60. Ibid., pp. 244-252.

61. Morris Kline, "Mathematics: The Loss of Certainty" (New York, Oxford U. Press, 1980) p. 78.

62. Girard Desargues, "Brovillon Proiect d'vne Atteinte avx evenemens desrencontres du Cone avec un plan" (1639). For the reproduction of the original text, see: René Taton, "L'Oeuvre mathematique de G. Desargues" (Paris, Presses Universitaires de France, 1951) p. 99.

63. Kline, Op. Cit. p. 175.

64. Morris Kline, "Mathematical Thought from Ancient to Modern Times" (New York, Oxford U. Press, 1972) p. 872. The letter is from Gauss to Bolyai, Dec. 17, 1799.

65. Kline, Ibid., pp. 73-74.

66. René Taton, "L'Oeuvre Scientifique de Monge" (Paris, Presses Universitaires de France, 1951) p. 92.

67. Le Corbusier, "The Modulor" Trans. Peter de Francia & Anna Bostock (Cambridge Mass., MIT Press, 1954) pp. 20-21.

68. Christian Norberg-Schulz "Intentions in Architecture" (Cambridge, Mass., MIT Press, 1965) p. 93.

69. Filarete, Op. Cit. pp. 9-10.

70. D. G. King-Hele, 'From Kepler's Heavenly Harmony to Modern Earthly harmonics", in Beer & Beer, Op. Cit., pp. 497-517.

71. See, for example, Matila Ghyka, "The Geometry of Art and Life" (New York, Dover, 1977).

BIBLIOGRAPHY OF LITERATURE CITED

Note: For a complete listing of the works of Desargues and of those works by Bosse dealing with Desargues' methods, see Appendix IV.

Alberti, L. B., "On Painting and On Sculpture", Trans. C. Grayson, New York, 1972.

__________, "Ten Books on Architecture", Trans. J. Leoni, London, 1955.

Aquinas, St. Thomas, "Summa Contra Gentiles", Medieval Philosophy, ed. H. Shapiro, New York, 1964, pp. 331-367.

Baillet, A., "La Vie de Monsieur des Cartes", 2 vols. Paris, 1691.

Beer, A., "Kepler's Astrology and Mysticism", Kepler: Four Hundred Years, ed. A. Beer and P. Beer, New York, 1975, pp. 399-426.

Bosse, A., "La Maniere universelle de Mr. Desargues, Lyonnois, pour poser l'essieu . . .", Paris, 1643.

_____, "Moyen vniversel de pratiquer la perspective svr les tablevx, ou surfaces irregulieres . . .", Paris, 1653.

_____, "La pratique de trait a preuves de Mr. Desargues Lyonnois pour la coupe des pierres . . .", Paris, 1643.

_____, "Traite des manieres de dessiner les ordres de l'Architecture . . .", Paris, 1664.

Blunt, A. "Art and Architecture in France, 1500 to 1700", Baltimore, 1957.

Busoni, F., "Sketch of a New Aesthetic of Music", Three Classics in the Aesthetics of Music, no. ed., New York, no date.

Cassirer, E. The Philosophy of Symbolic Forms, 3 vol., New Haven, 1955.

Le Corbusier, "The Modulor", 2 vols., Cambridge, Mass., 1954.

Cornford, F. M., "Plato's Cosmology: The Timaeus of Plato With a Running Commentary", New York, no date.

Cotgrave, R., "A Dictionairie of the French and English Tongves", London, 1611.

Cremona, L., "Elements of Projective Geometry", Oxford, 1913

Descargues, P., "Perspective", Trans. I. M. Paris, New York, 1977.

Descartes, R., "The Philosophical works of Descartes" Trans. E.S. Haldane and G.R.T. Ross, 2 vols., New York, 1967.

"Diverse methodes universelles et nouvelles en tout ou en partie pour faire des perspectives . . ." (no author), Paris, 1642.

Dubreuil, J., "La perspective pratique necessarie a tous peintres . . .", 3 vols., Paris, 1642-49.

Filarete (Antonio Averlino), "Treatise on Architecture", trans. J.R. Spencer, 2 vols, New Haven, 1969.

Frankl, P. "The Gothic: Literary Sources and Interpretations Through Eight Centuries", Princeton, 1960.

Gadamer, H. G., "Amicus Plato Magis Amica Veritas", <u>Dialogue and Dialectic</u>, New Haven, 1980, pp. 194-218.

________. "Idea and Reality in Plato's Timaeus", <u>Dialogue and Dialectic</u>, New Haven, 1980, pp 156-193.

Ghyka, M., "The Geometry of Art and Life", New York, 1977.

Godwin, J., "Robert Fludd: Hermetic Philosopher and Surveyer of Two Worlds", Boulder, 1979.

Harvey, J., "The Mediaeval Architect", London, 1972.

Hasse, J., "Kepler's Harmonies, Between Pansophia and Mathesis Universalis", Kepler: Four Hundred Years, ed. A. Beer and P. Beer, New York, 1975, pp. 519-533.

Heidegger, M., "Introduction to Metaphysics", New Haven, 1959.

_________ "What is a Thing?", Chicago, 1967.

Honnecourt, V. de, "The Sketchbooks of Villard de Honnecourt", ed T. Bowie, New York, 1959.

Horadam, A. E., "A Guide to Undergraduate Projective Geoemtry", Sydney, 1970.

Husserl, E., "The Crisis of European Sciences and Transcendental Phenomenology", Evanston, 1970.

Husserl, E., L'Origine de la Geometrie", Paris, 1974.

Ivins, W., "A Note on Desargues 'Theorem'", Scripta Mathematica, vol. 13, 1947, pp. 202-210.

_________, "A Note on Girard Desargues", Scripta Mathematica, vol. 9, 1943, pp. 33-48.

_________, "Art and Geometry, a Study in Space Intuitions", New York, 1946.

_________, "Two First Editions of Desargues", Bulletin of the Metroplitan Museum of Art, 2nd. series, Vol. 1, 1942, pp. 33-45.

Jousse, M., "Secret d'Architecture de' couverant Fidelement . . .", La Fleche, 1642.

Kant, I., "Critique of Pure Reason", Trans. N. K. Smith, New York, 1965.

Kepler, J., "The Harmonies of the World", Great Books of the Western World, vol. 16, Chicago, 1952.

King-Hale, D.G., "From Kepler's Heavenly Harmony to Modern Earthly Harmonics", Kepler: Four Hundred Years, ed A. Beer and P. Beer, New York, 1975, pp. 497-517.

Kline, M., "Mathematics: The Loss of Certainty", New York, 1980.

__________, "Mathematical Thought from Ancient to Modern Times", New York, 1972.

Kuhn, T., "The Structure of Scientific Revolutions", Chicago, 1962.

Lakatos, I., "Falsification and the Methodology of Scientific Rsearch Programs", Criticism and the Growth of Knowledge, ed. I. Lakatos and A. Musgrave, Cambridge, Eng. 1970, pp. 91-197.

Leeman, F. "Hidden Images", New York, 1976.

Leibniz, G. W. "Basic Writings", Trans. G. R. Montgomery, LaSalle, 1962.

__________, "Preface to General Science", Leibniz: Selections, ed. P. Wiener, New York, 1951, pp. 12-17.

Lenoble, R., "Mersenne ou le Naissance du Mechanisme", Paris, 1948.

Liddell, H. G. and Scott, "An Intermediate Greek-English Lexicon", New York, 1897.

L'Orme, P. de, "De l'Architecture", Rouen, 1648.

Mahoney, M. S., "The Mathematical Career of Pierre de Fermat, (1601-1665)", Princeton, 1973.

Mersenne, M. "Correspondance", ed. C. deWaard and R. Pintard, 7 vols., Paris, 1945.

__________, "Harmonie vniverselle", 2 vol., Paris, 1636-7.

__________, "Les Prelvdes de l'Harmonie vuniuerselle", Paris, 1634.

Monge, G. "Lecons de Geometrie Descriptive", Paris, 1795.

"The National History of France", ed. Fr. Funck-Brentano, 7 vols., New York, 1957.

Niceron, J. F., "La perspective curiuse ou Magic artificiele . . .", Paris, 1638.

Norberg, Schulz, C., "Intentions in Architecture", Cambridge Mass., 1965.

"Oeuvres de Christian Huygens", La Haye, 1888-1950.

Peiresc, "Les Correspondantes de Peiresc", ed. T. de Larrouque, Paris, 1891.

Plato, "Meno", The Dialogues of Plato, trans. B. Jowett, New York, 1937, Vol. I, pp. 349-383.

Plato, "The Republic", The dialogues of Plato, trans. B. Jowett, New York, 1937, Vol. I, pp. 591-852.

Pommer, R., "Eighteenth Century Architecture in the Piedmont", New York, 1967.

Poudra, N. G., "Oeuvres de Desargues", 2 vols, Paris, 1864.

Ptolemy, "Almagest", Great Books of the Western World, vol. 16, Chicago, 1952, pp. 1-478.

Purchase, W. R., "Practical Masonry, A Guide to the Art of Stone Cutting", Paris, 1917.

Rowe, C., "Mannerism and Modern Architecture", The Mathematics of the Ideal Villa and Other Essays, Cambridge, Mass., 1982, pp. 29-59.

Serlio, S., "Five Books on Architecture", New York, 1982.

Shelby, L. R., "The 'Secret' of the Medieval Masons", On Pre-Modern Technology and Science: A Volume of Studies in Honor of Lynn White Jr., ed. B. S. Hall and D. C. West, Malibu Cal., 1976, pp. 201-219.

________. "Medieval Mason's Templates", Journal of the Society for Architectural Historians, 30, 1971, pp. 140-54.

________. "The Geometrical Knowledge of Mediaeval Master Masons", Speculum, 47, 1972, pp. 395-421.

________. "The Education of Mediaeval Master Masons", Mediaeval Studes, 32, 1976, pp. 1-26.

Slaby, S. M., "Fundamentals of Three-Dimensional Descriptive Geometry", New York, 1976.

Taton, R. "L'Historie de la Geometrie Descriptive", Paris, 1954.

________, "L'Oeuvre mathematique de G. Desargues", Paris, 1951.

________, "L'Oeuvre scientifique de Monge", Paris, 1951.

APPENDIX I: THE WRITINGS OF DESARGUES AND BOSSE

The standard reference for the writings of Desargues remains Poudra's "Oeuvres de Desargues", and a number of libraries in the United States have circulating copies of this. Poudra's expositions and commentaries however do not for the most part make a significant contribution to the understanding of Desargues' methods. The expositions are dated and may cause the reader almost as much difficulty as the original texts. Even so, as William Ivins has pointed out, we owe a very great debt to Poudra for collecting every scrap of information he could find concerning Desargues, including some items which are no longer easily available to the scholar.

René Taton's "L'Oeuvre mathematique de G. Desargues" (1951) is a helpful work, containing as it does the fruits of his search for new material on Desargues in France. An updated biography is also included. Like Poudra, however, Taton presumes to share with Desargues a common intentionality. For this reason, his work never makes a serious question of Desargues' role in the transformation of geometry from a cosmological symbol to a technological instrument. Taton, for the most part, leaves the mathematical texts to speak for themselves, so that the reader who wants, for example, to understand Desargues' very difficult "Rough Draft Upon the Consequences of the Intersection Between a Cone and a Plane", will find here little more to guide him than a more authentic version of the text than Poudra was able to provide from La Hire's copy.

Below are listed the known writings of Desargues and those by Bosse which bear upon the exposition of Desargues' methods. This listing follows substantially the one given by Taton in "L'Oeuvre." To date, no major work of exposition or evaluation has been published on Desargues in any language. This bibliography may be considered to contain most of the primary source material which the interested student can expect to locate, short of extended searches in the libraries and private collections of France. Because of its thoroughness, Taton's research would suggest that, for the most part, further new documents are likely to come to light only through future corrections of cataloging errors in existing collections. This has already happened in several cases, due to the fact that Desargues' pamphlets were bound with other works and thus "lost".

<u>1. Works Published During Desargues' Lifetime:</u>

<u>1636</u>

1. "Une methode aisee pour apprendre et enseigner a lire

et escrire la musique" (in Mersenne: Harmonie universelle . . . , vol. I, Paris, 1636, in-fol., Book VI, Prop. I, pp. 332-342.) Some copies of this work have a different pagination.

2. "Exemple de l'une des manieres universelles du S.G.D.L. touchant la pratique de la perspective sans emploier aucun tiers point, de distance ny d'autre nature, qui soit hors du champ de l'ouvrage," Paris, May 1636, with privilege, 12 pages, 32 x 22 cm., 1 plate, printed double. Poudra published this (Oeuvres, I, pp. 55-84), but based his version on Bosse: "Maniere universelle de M. Desargues pour pratiquer la perspecitve . . .", Paris, 1648, pp. 321-334; pl. 150. Ivins came upon a copy of this by accident as curator of the Print Room at the Metropolitan Museum for which see "Two First Editions of Desargues", Bulletin of the Metropolitan Museum of Art, 2nd. series, vol. 1, 1942, pp. 33-45. Ivins reproduces two sample pages of the text and the two plates which appear in that copy. Taton is of the opinion that the second plate is actually the fifth plate to Desargues' 1640 study of perspective for which see below.

1639

3. "Brouillon project d'une atteinte aux evenemens des rencontres du Cone avec un Plan, Paris, 1639, 30 p., 19 X 26 cm., x pl.?. Poudra bases his text on a hand-written

copy made by La Hire, for which see "Oeuvres", vol. I, pp. 103-241. Taton reproduces an extensively corrected version based on a copy of the original discovered in the Bibliotheque Nationale by Pierre Moisy, for which see: Taton, "L'Oeuvre mathematique de G. Desragues", Paris, 1951, pp. 87-200. There is also a German translation: Max Zacharias; "Erster Entwurf eines Versuchs Über die Ergebnisse des Zusammentreffens eines Kegels mit einer Ebene", Leipzig, 1922.

3. bis. "Atteinte aux Evenements des contrarietez d'entre les actions des puissances ou forces, Paris, 1639, 2 pages and two plates, 19 X 26 cm., x pl.?.

3. bis. "Advertissement", Paris, 1639, 4 p., 19 X 26 cm. This contains errata relative to the two preceding texts. No example is known at the present time, but Poudra gives it, cf., "Oeuvres", as does Zacharias (see above).

1640

4. "Brouillon project d'exemple d'une maniere universelle du S.G.D.L. touchant la pratique du trait a preuves pour la coupe des pierres en l'Architecture: Et de l'esclaircissement d'une maniere de reduire au petit pied in Perspective comme en Geometral, & de tracer tous Quadrans plats d'heures egales au Soleil," Paris, August, 1640, 4 p., 32 X 22 Cm., 5 pl. Text reproduced in Poudra,

"Oeuvres", I, pp. 305-358. Poudra reconstructs the illustrations on the basis of the text with some errors. Ivins discovered a copy containing the original illustrations. These are reproduced along with two sample pages of the text in: "Two First Editions of Desargues", Bulletin of the Metropolitan Museum of Art, 2nd series, vol. 1, 1942, pp. 33-45.

5. "Lecons de tenebres", Paris, 1640?, in-fol.? This work is lost, but several of Desagues' contemporaries refer to it. The title, "Lessons from Shadows", appears to refer to Desargues' use of shadow projection to study the properties of conic sections.

6. "Maniere universelle de poser le style aux rayons du soleil en quelconque endroit possible,avec la regle, le compas, l'esquerre et le plomb", Paris, end of 1640?

1642

7. "Erreur incroyable . . .", Paris, January, 1642, placard. This is lost, but Poudra published several passages from it which had been reproduced in a pamphlet against Desargues: "Diverses Methodes universelles et nouvelles . . .", Paris, 1642 ("Oeuvres, I, pp. 497-498.)

8. "Faute et faussetes enormes . . .", same date?, placard. Lost. Passages reproduced: 'Oeuvres", I, pp. 497-498.

9. "Six erreurs des pages 87. 118. 124. 128. 132. & 134 du Livre intitule la Perspective Practique . . .", Paris, April, 1642, in -4°, 14 p., 3 pl. This work does not appear to have been known to Poudra. Two copies exist in the Bibl. Nat.: Vp 2.879 and Vz 1.135.

10. "Reponse d causes et moyens d'opposition . . .", Paris, Dec. 16, 1642, placard. This is lost, but Poudra was able to reconstruct a passage from it, for which see "Oeuvres," I, p. 498.

1643

11. "Reconnaissance de Monsieur Desargues", Paris, July 20, 1643, in A. Bosse: "La Pratique de trait a preuves de Mr. Desargues Lyonnois, pour la Coupe des Pierres en l'Architecture", Paris, 1643, in-8°, pp. 51-55. Reproduced in Poudra, "Oeuvres", I, pp. 469-478.

12. "Reconnaissance de Monsieur Desargues", Paris, Sept. 1643, in A Bosse: "La Maniere universelle de Mr. Desargues, Lyonnois, pour poser l'essieu et placer les heures et autres choses aux cardans au Soleil", Paris, 1643, in-8°, pp. 25-28. Reproduced in Poudra: "Oeuvres", I, pp. 479-485.

13. "Livret de perspective adresse aux theoriciens", Paris, 1643? in 8°? Lost. Poudra reproduces some passages: "Oeuvres", I, pp. 439-462. Reproduced in A. Bosse: "Maniere universelle de Mr. Desargues pour pratiquer la

perspective", Paris, 1648, starting at page 313-pagination defective.

1644

14. "La Honte du S. J. Curabelle, qui refuse de maintenir a peine d'une somme & au dire des Juges, Geometres, & Jurez Massons deParis si besoin est", Paris, April 2, 1644, placard. Lost, but reproduced in a later writing by Desargues: "Recit au vray. . . .", Paris, 1644, pp. 28-29.

15. "Sommation faite au Sieur Curabelle, au sujet de ses affiches calomnieuses", Paris, April 18, 1644, in-4°? Listed in the catalog of the Bibl. Mazarine, but apparently lost. Poudra reconstructs two passages: "Oeuvres", I, p. 499.

16. "Recit au vray de ce qui a este la cause de faire cet escrit", May-June? 1644, in-8°, 38 pages. Not known to Poudra; two copies exist: Bibl. Nat.: Res V. 9033 and Bibl. Mazarine: n° 56.595, 8e piece.

1647

17 and 18. "Reconnaisance de Monsieur Desargues, Paris, Oct. 1, 1647 may be taken together with various theorems on perspective included in A. Bosse: "Maniere universelle de Mr. Desargues pour pratiquer la Perspective par petit-pied comme le Geometral. Ensemble les places . . .", Paris, 1648, in 8°, pp. 13-16 and 335-343. These various

passages have been reproduced in Poudra, "Oeuvres", I, pp. 401-422. The "reconnaissance" for the work on perspective appears in Poudra, "Oeuvres," I, pp. 486-493.

1657

19. "Lettre de Mr. Desargues a Monsieur Bosse", Paris, July 25, 1657, in-8°, 3 p. Only one passage of this letter was known to Poudra; cf. "Oeuvres", I, pp. 504-505.

Those who have access to European libraries and wish further details on their holdings of works by Desargues should consult Taton's"L'Oeuvre Mathematique de Desargues".

2. Works by Bosse Inspired by Desargues:

1. "La Pratique du trait a preuves de Mr. Desargues, Lyonnois, pour la coupe des pierres en l'Architecture", Paris, 1643. There is also a German translation: "Kunstrichtige und probmassige Zeichnung zum Steinhauen in der Baukunst", Nuremberg, 1699. This went through several editions, with different titles.

2. "La Maniere universelle de Mr. Desargues, Lyonnois, pour poser l'essieu & placer les heures et autres choses aux cardans au Soleil", Paris, 1643. This is apparently Bosse's development of (6) above. There is also an English translation by D. King: "Universal way of

dyaling: or . . . directions for placing the axeltree and marking the hours in Sundyals . . ., London, 1659.

3. "Maniere universelle de Mr. Desargues pour pratiquer la perspective par petit-pied, comme le Geometral. Ensemble les places et proportions des fortes et foibles touches, teintes ou Couleurs", Paris, 1648. A second part is published under the title: "Moyen universel de pratiquer la perspective sur les tableaux ou surfaces irregulieres. Ensembles quelques particularitez concernant cet art, & celuy de la Gravure en Taille-douce", Paris, 1653. There are Dutch translations of both of these works, dating from shortly after the death of Descartes (1660) and Desargues (1661). This would suggest the possibility of some connection between Descartes' friends in the north and Desargues.

Letters:

Aside from the published letter to Bosse (cf., 19 above), the only known letter of Desargues is to Mersenne, April 4, 1638. This is reprinted in Taton, "L'Oeuvre Mathematique de G. Desargues", Paris, 1951, pp. 80-86. A letter from the same year to Desargues", Paris, 1951, pp. 80-86. A letter from the same year to Descartes has been lost.

APPENDIX II: THE ARCHITECTURAL WORKS OF DESARGUES

The following is a list of the architectural works in which Desargues is known or thought to have had a hand. The degree of his involvement in some projects cannot be determined, and he may have been involved in others of which there is no record. Most of the works have been either destroyed or altered beyond recognition, but it is possible that a search through Paris and Lyon might turn up something unexpected. The list is based on Taton, Poudra, and Bosse.

PARIS:

1. A stair in the Hotel de l'Hospital.
2. A Stair at the Hotel Turenne on the Rue Neuve-Saint-Louis. According to Piganoil de la Force, this could "no longer be admired" because it had been included in the convent of the Daughters of Saint-Sacrement: "Description de Paris, de Versailles . . .", new edition, Paris, 1742, vol. 4, p. 262.
3. Stairs in Maison of M. Vedeau de Grammont.
4. Stairs in the Hotel Rolin in the rue Clery.

5. House with a monumental staircase in the rue de Bernardins. See: Bauchal: "Dictionnaire des architects francais," Paris, 1887, pp. 177-178.

6. Perspective designs (anamorphic perspectives) for the vaults of the old Church of the Carmelites designed circa 1628 by Philippe de Champaigne.

7. A grand staircase in the second courtyard of the Palais-Royal. Destroyed. See Sauval: "Histoire et recherches des antiquites de la ville de Paris . . .", vol. II, Paris, 1724, p. 160.

8. Staircase and possibly the entire house of Monsieur Roland in the rue Clery. (See Fig. 1-1 of this work).

LYON:

1. An hotel de ville, apparently started by Simon Maupin, but revised in some degree by Desargues and Jacques Le Mercier (circa 1646).

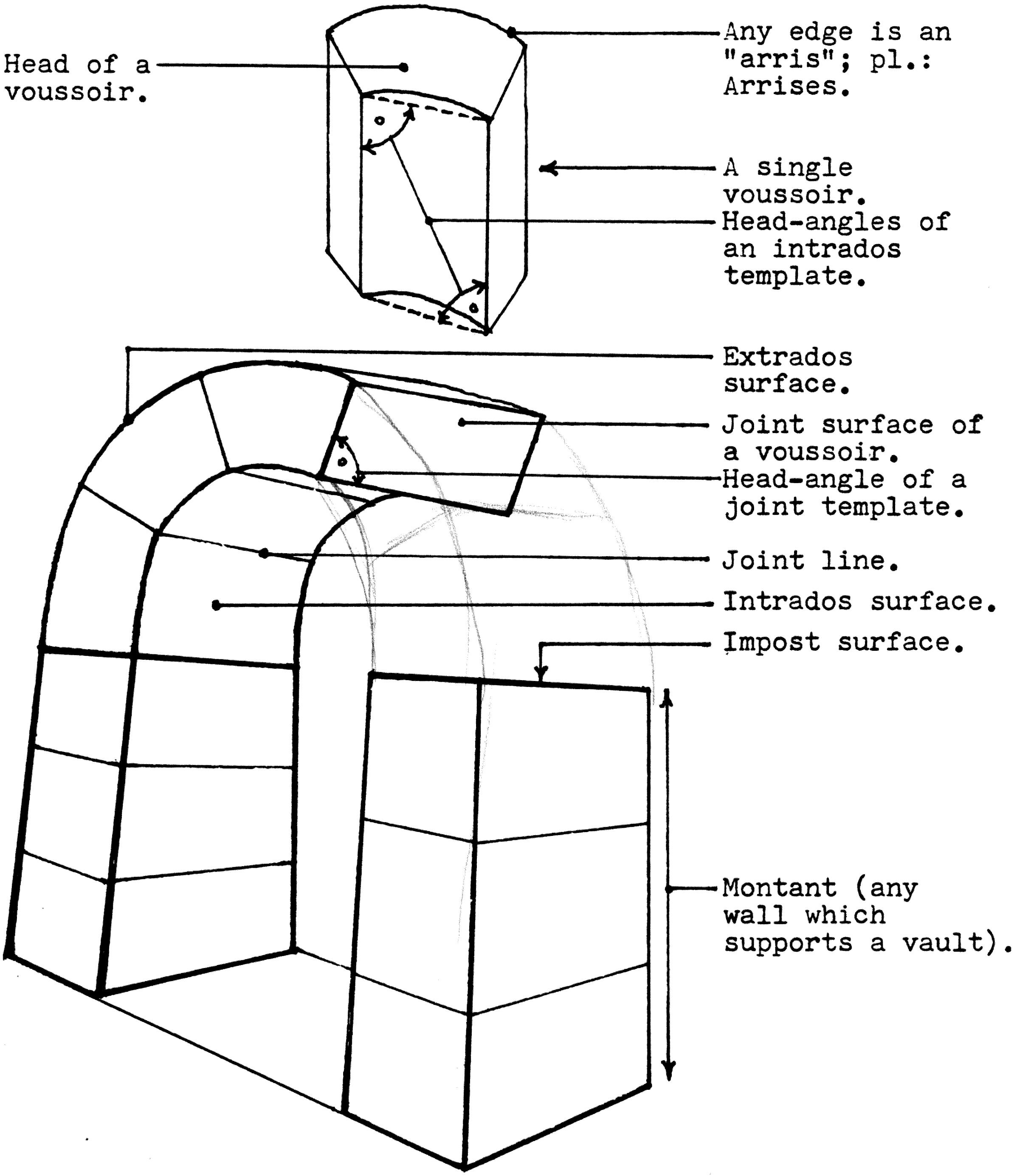
2. The Hotel de l'Europe (1651).

3. A trompe to support a house in a corner at the entry to the Pont de Pierre over the river Saone on the Quay Villeroy. This is highly praised in the "Dictionnaire universel et francois . . ." (Dictionnaire de Trevous), new ed., vol. 8, Paris, 1771, p. 213. This bridge appears to have been destroyed in the 19th century.

4. Steps in the court yard across from the Chateau de Vizille (1653). (See Fig. 1-5 of the present study.)

5. Collaboration with the sculptor of Lyon, Jacques Mimerel on an epitaph for Grenoble (1654).

6. Plan of a church entrance with the Corinthian order. (See Fig. 1-6).



APPENDIX III: The Terms for the Parts of a Vault, as used in This Essay.

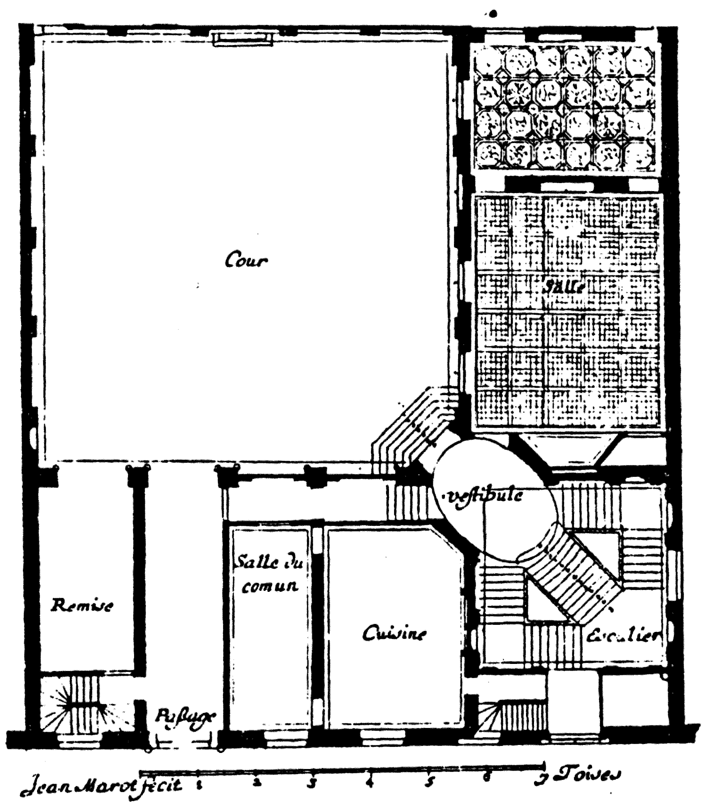


Figure 1-1 House of Monsieur Roland

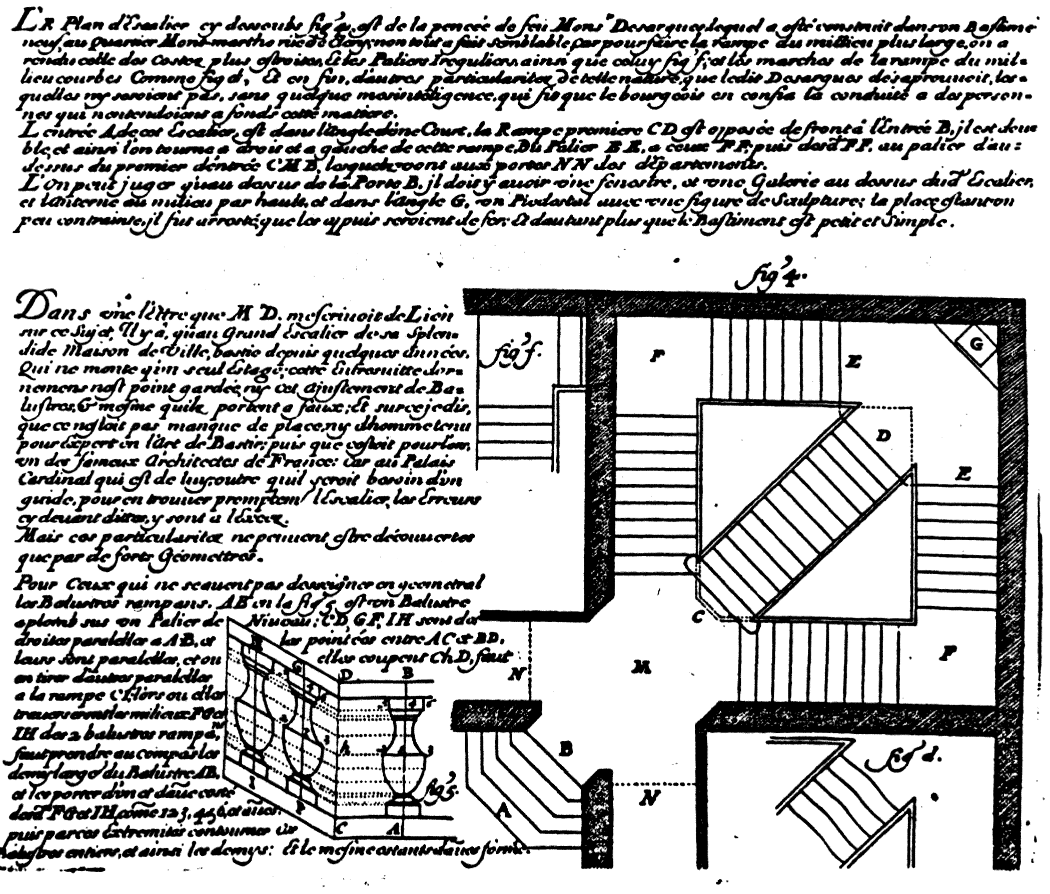


Figure 1-2 Bosse's Engraving of Desargues' Design for the Stair in the House of Monsieur Roland

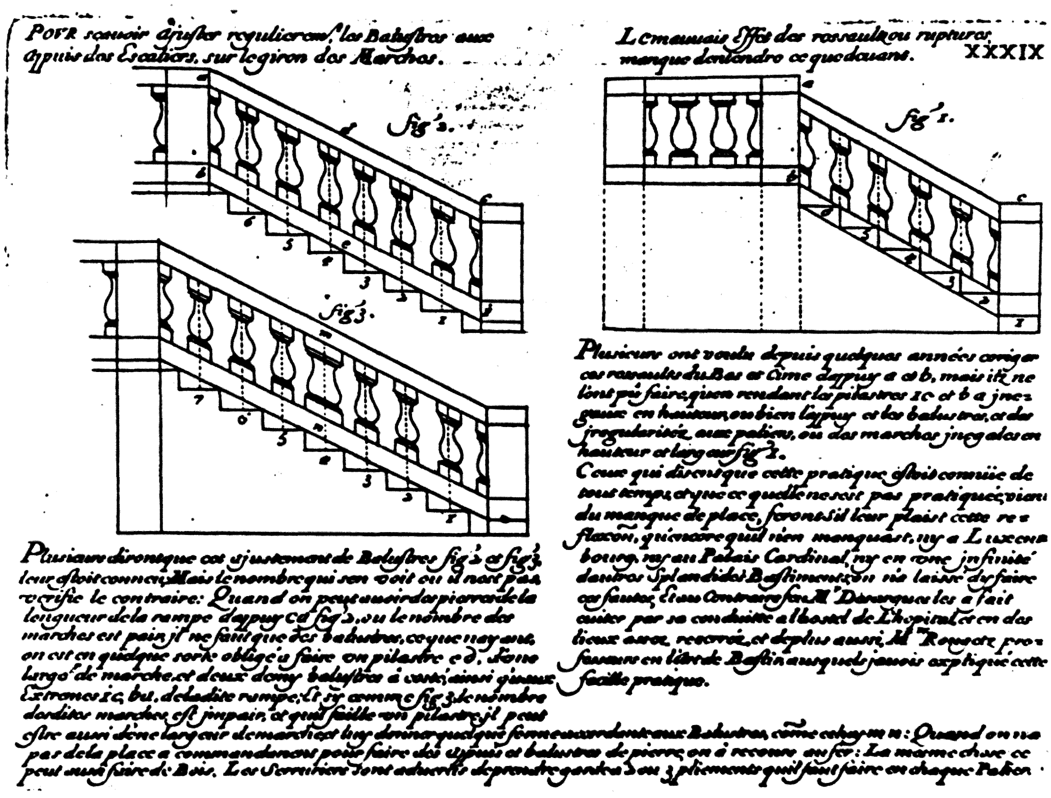


Figure 1-3 Bosse's Illustrations of Improper (Fig. 1) and Proper (Fig. 2 & 3) Treatment of Staircases.

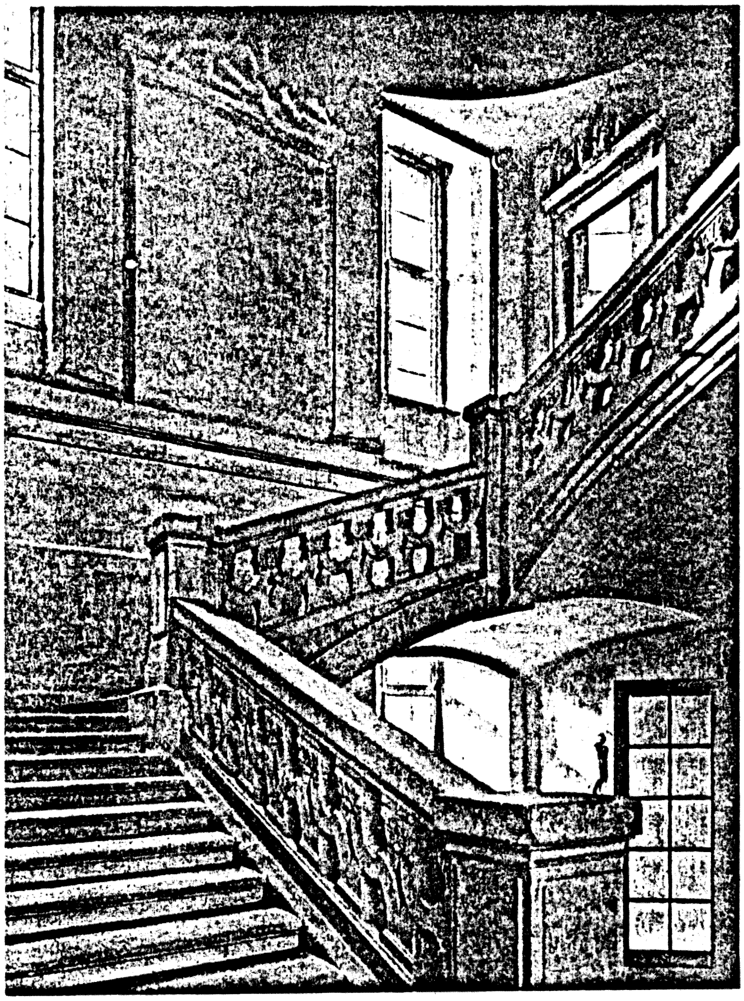


Figure 1-4 Staircase in the Chateau of Maisons, Francois Mansart, 1642-6

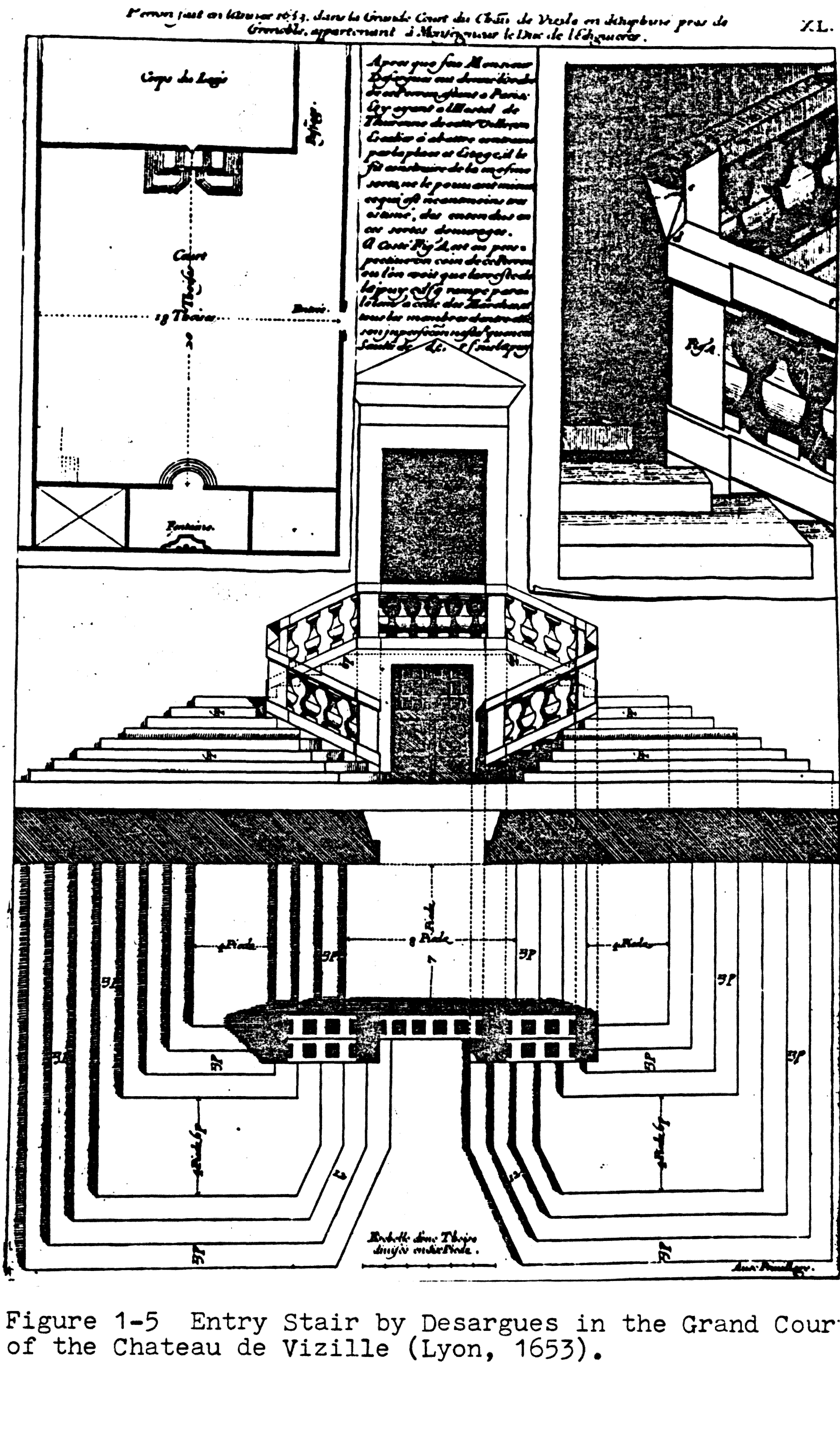


Figure 1-5 Entry Stair by Desargues in the Grand Court of the Chateau de Vizille (Lyon, 1653).

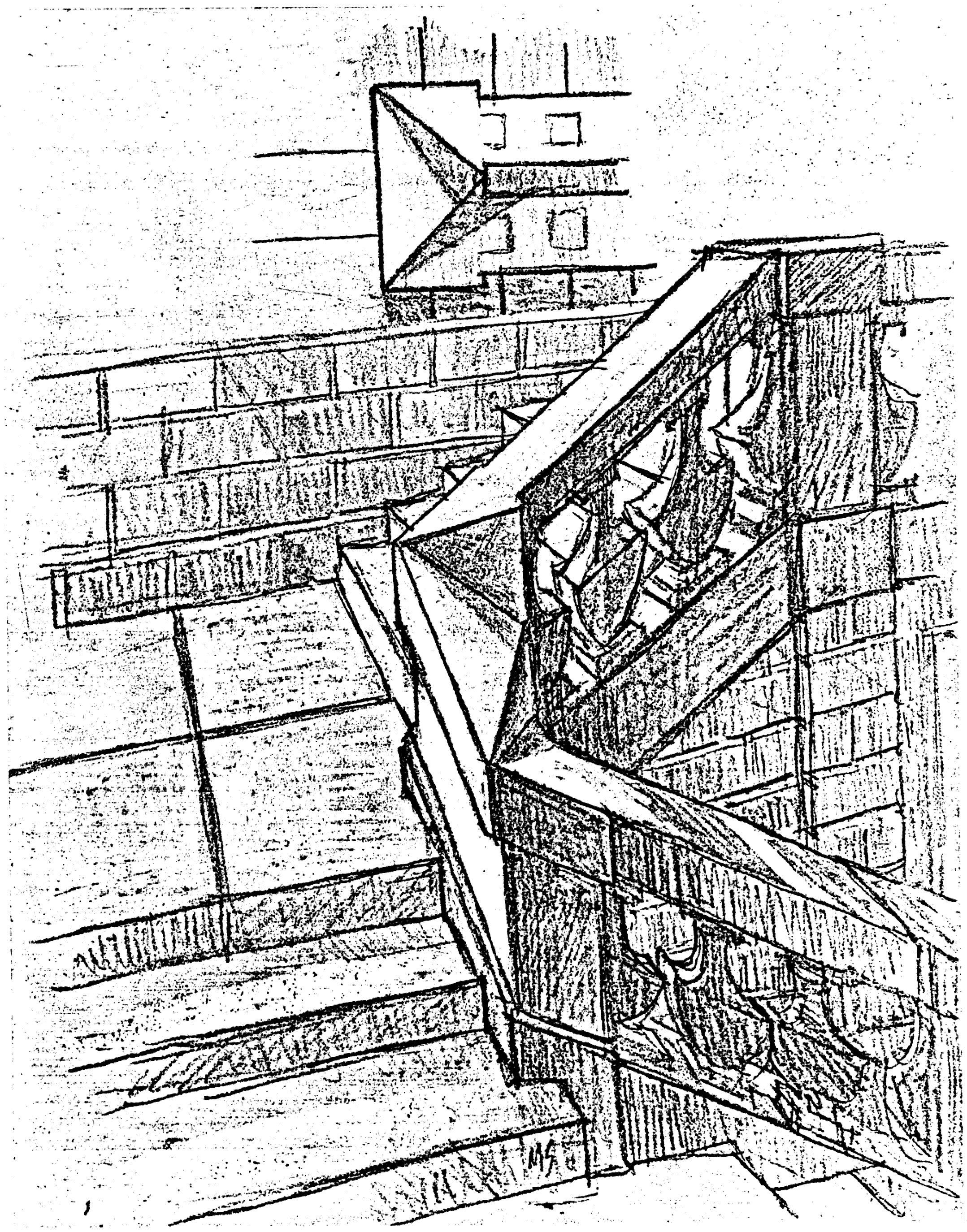


Figure 1-5a Reconstruction of Desargues' Method of Joining the Balusters at the Turning of a Stair so that the Line of the Balustrade is Continuous.

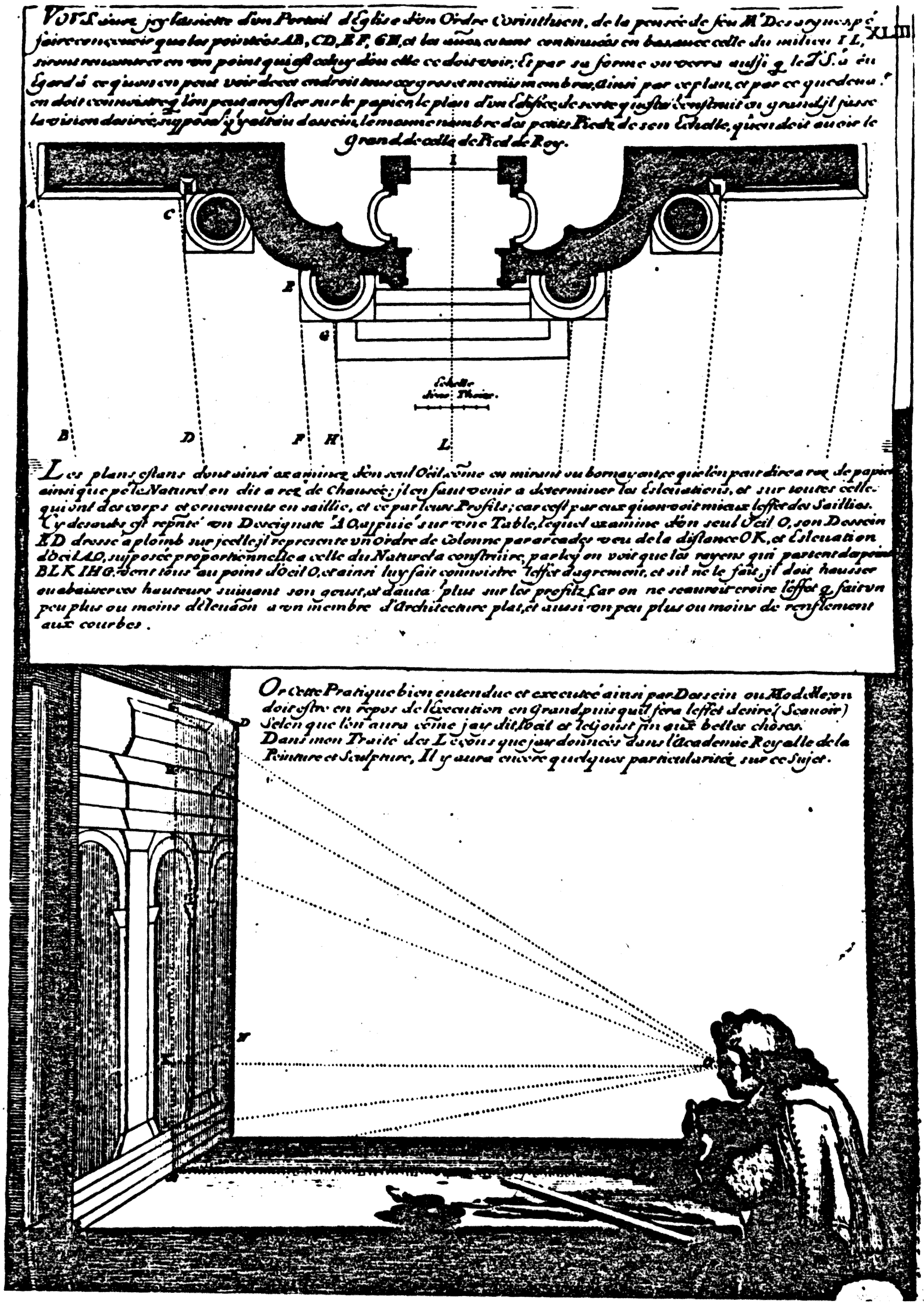


Figure 1-6 Plan of a Church Entry by Desargues (top). Method of Making Optical Corrections in Elevations of Buildings (bottom).

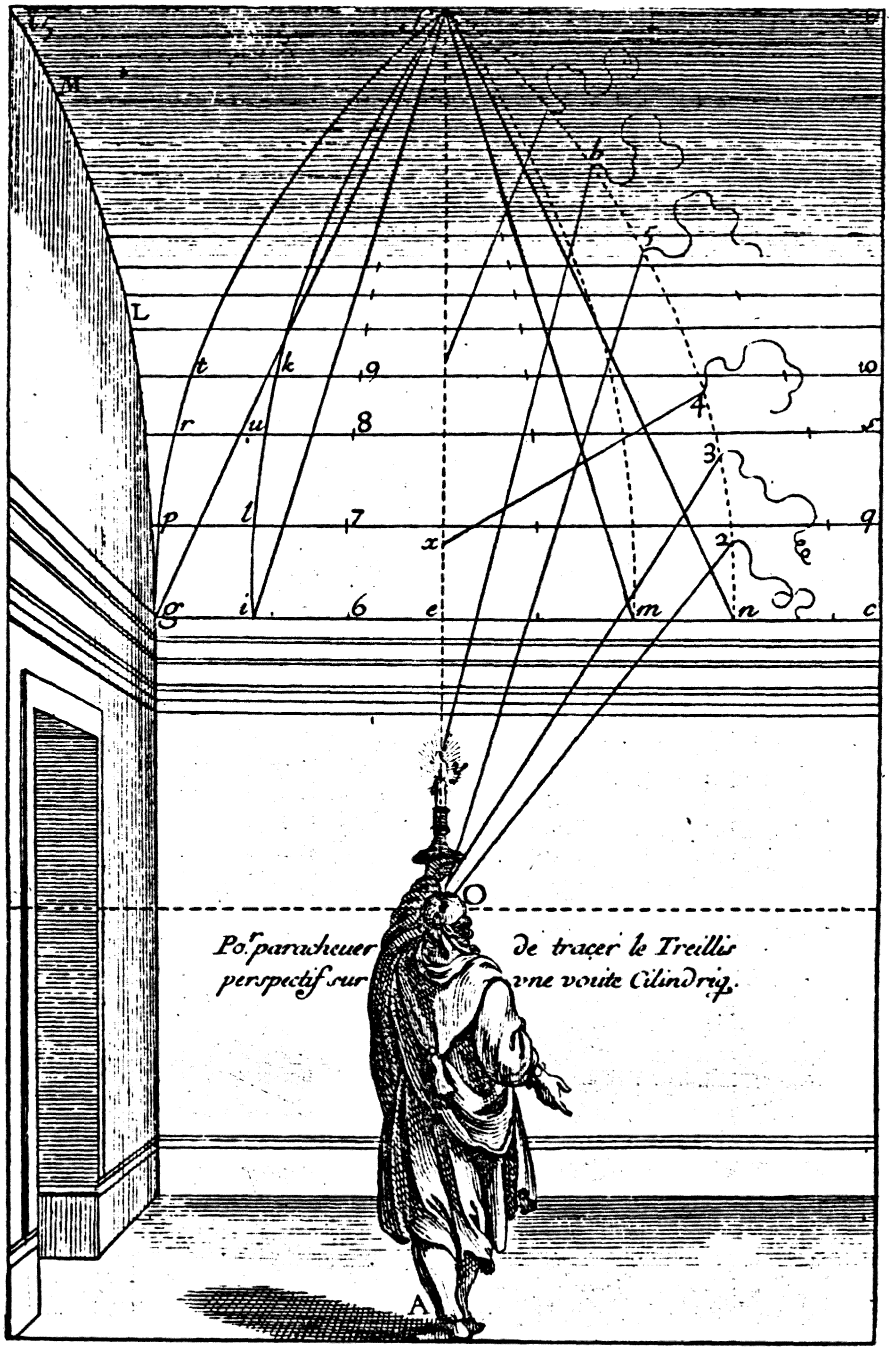


Figure 1-7 Bosse's Illustration of a Method for Projecting a scene Conceived in the Flat onto a Cylindrical Vault so that it will look Vertical to an Observer at Point O.

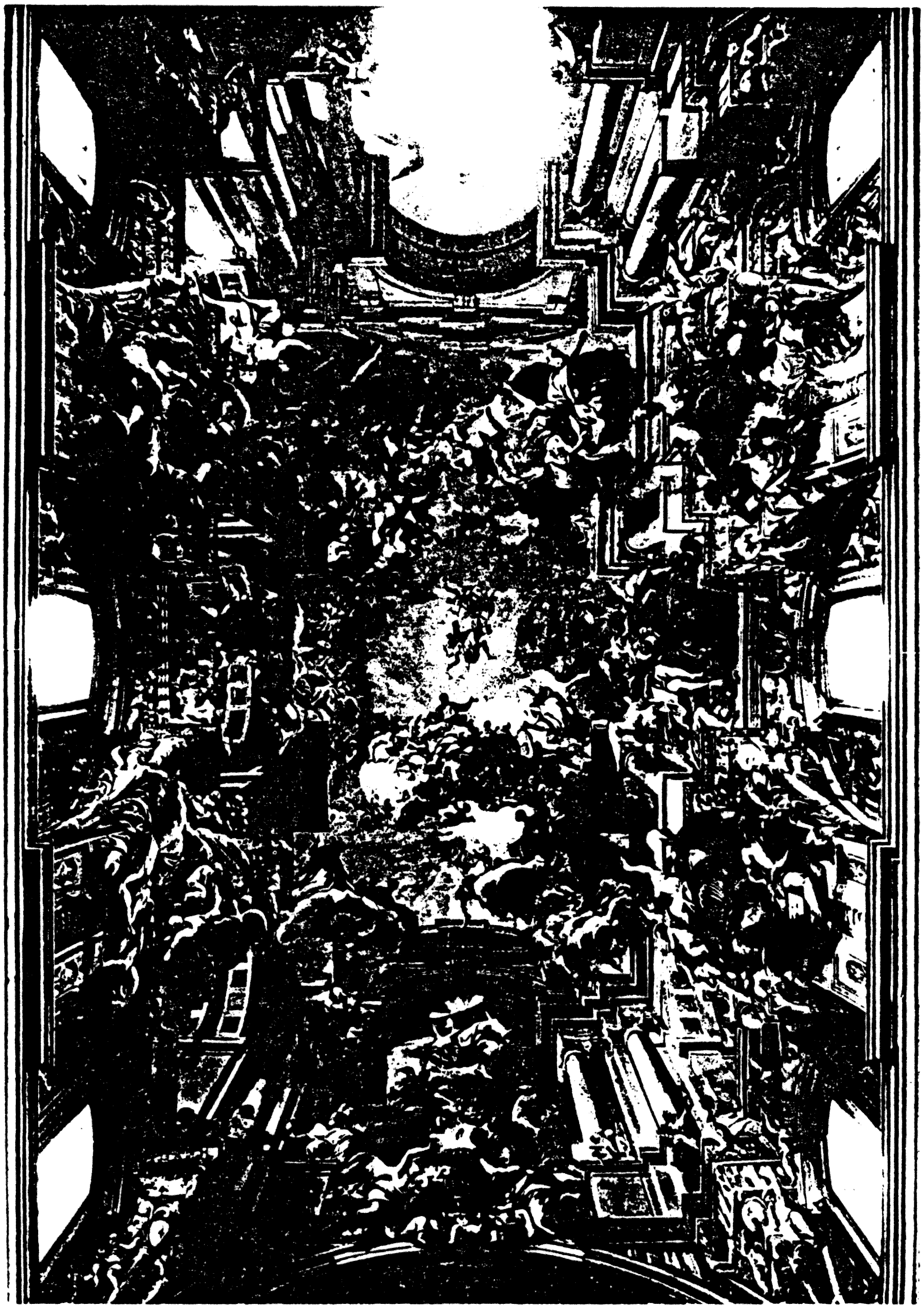


Figure 1-8 Anamorphic projection of a scene onto a cylindrical vault. Andrea Pozzo: "Entrance of St. Ignatius into Paradise," 1694.

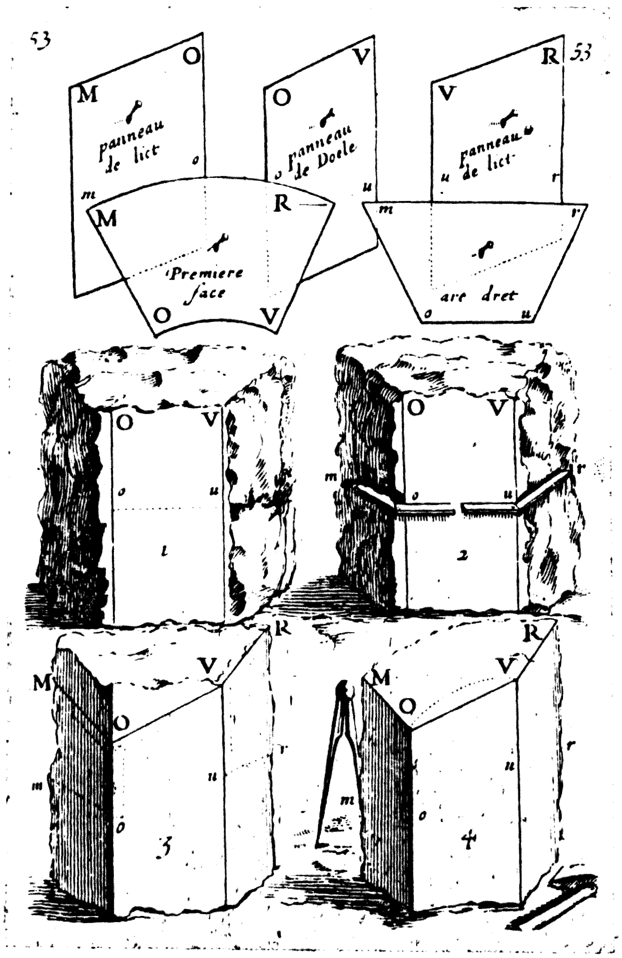


Figure 2-1 Bosse's Illustration of the use of Templates in Stone-Cutting (1643).

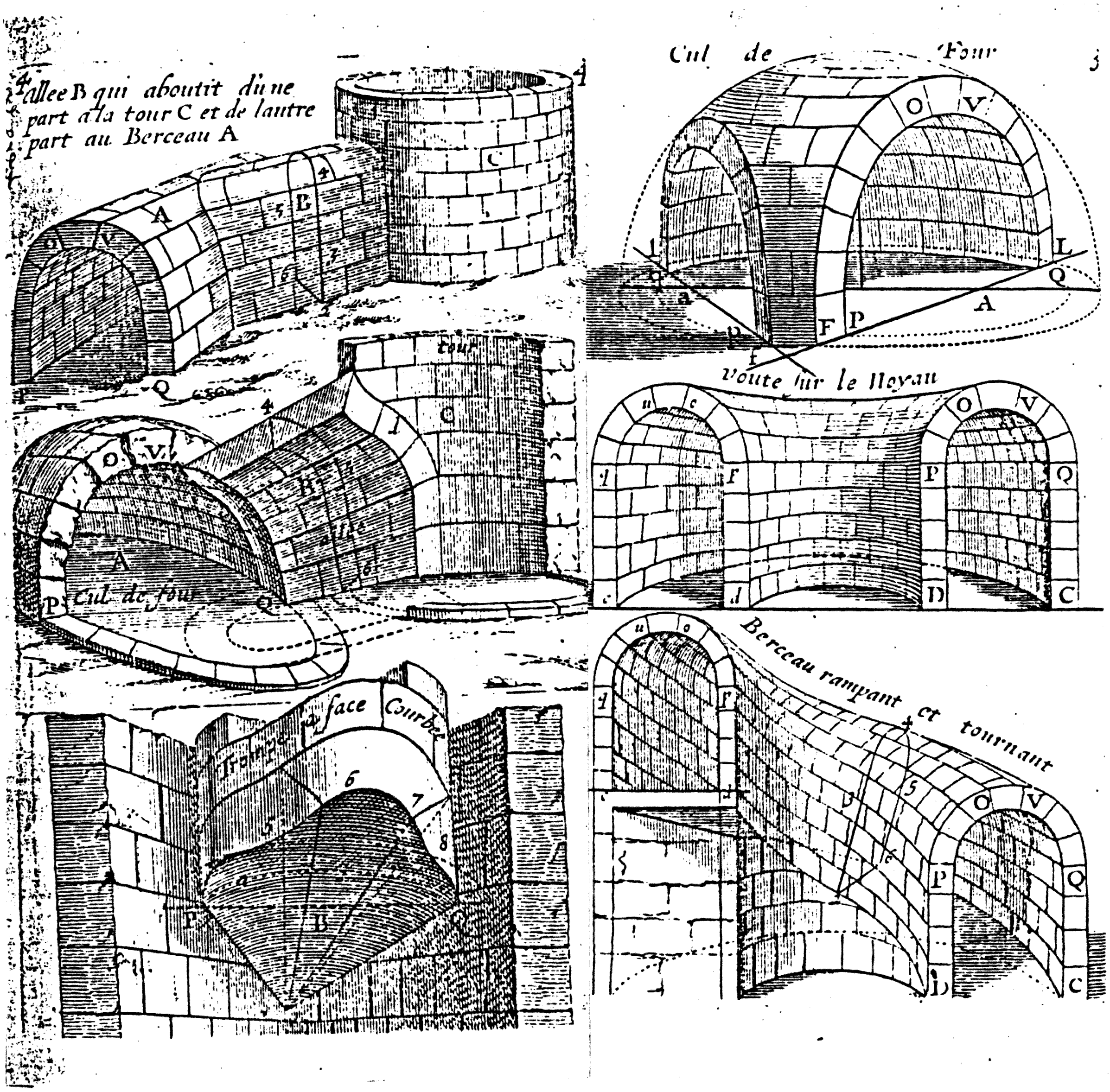


Figure 2-2 Intersection Problems in Stereotomy, as Given by Bosse.

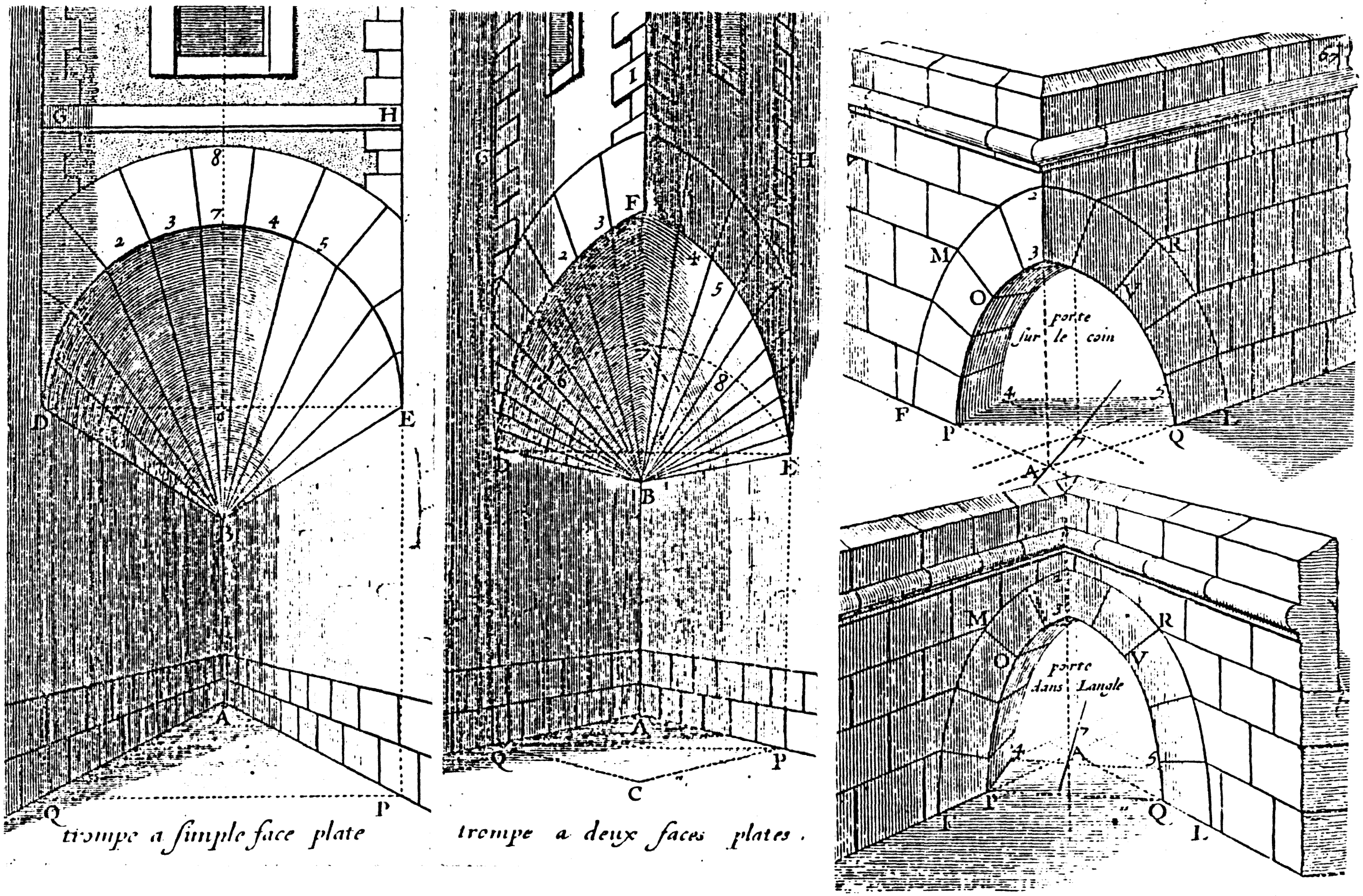


Figure 2-3 Trompes and Quoins by Bosse.

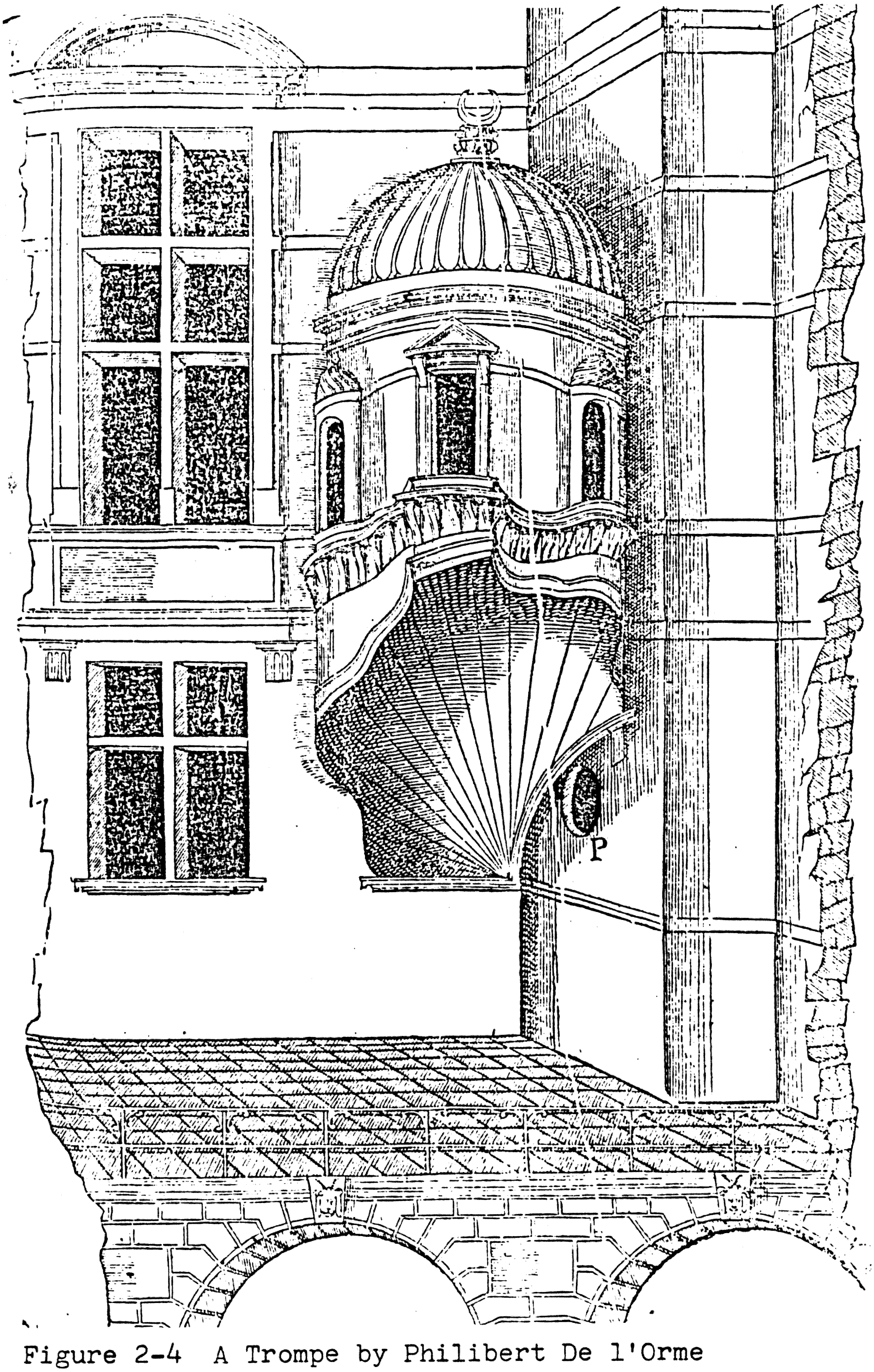


Figure 2-4 A Trompe by Philibert De l'Orme

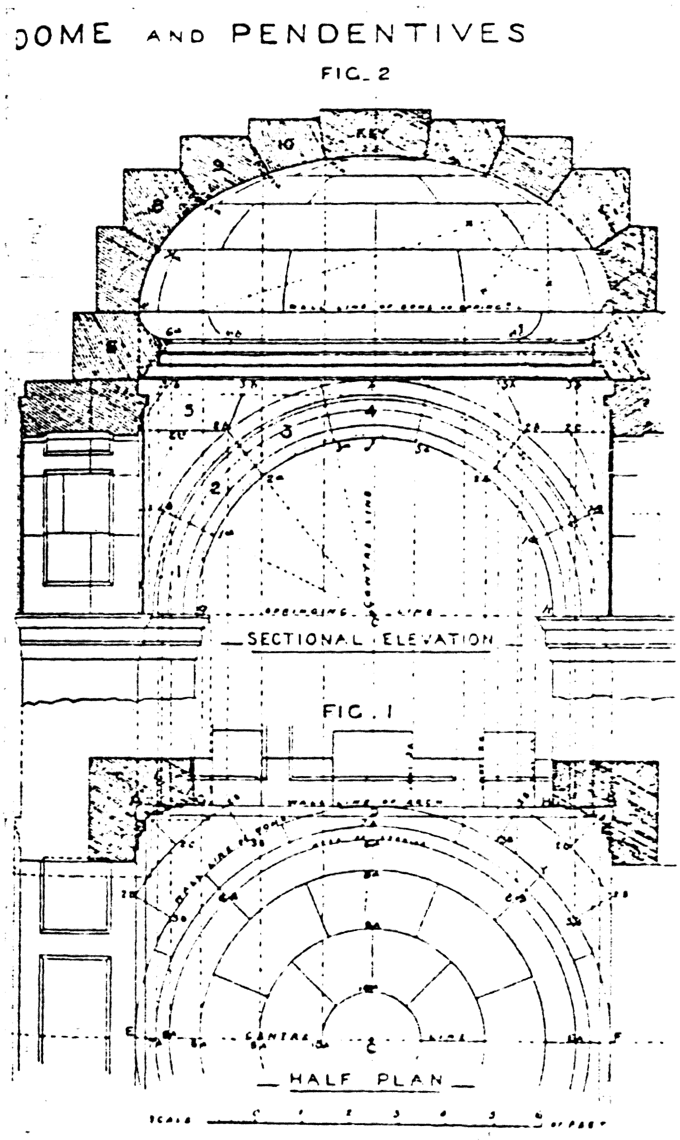


Figure 2-5 An Ellipsoidal Dome by William R. Purchase (Circa 1890).

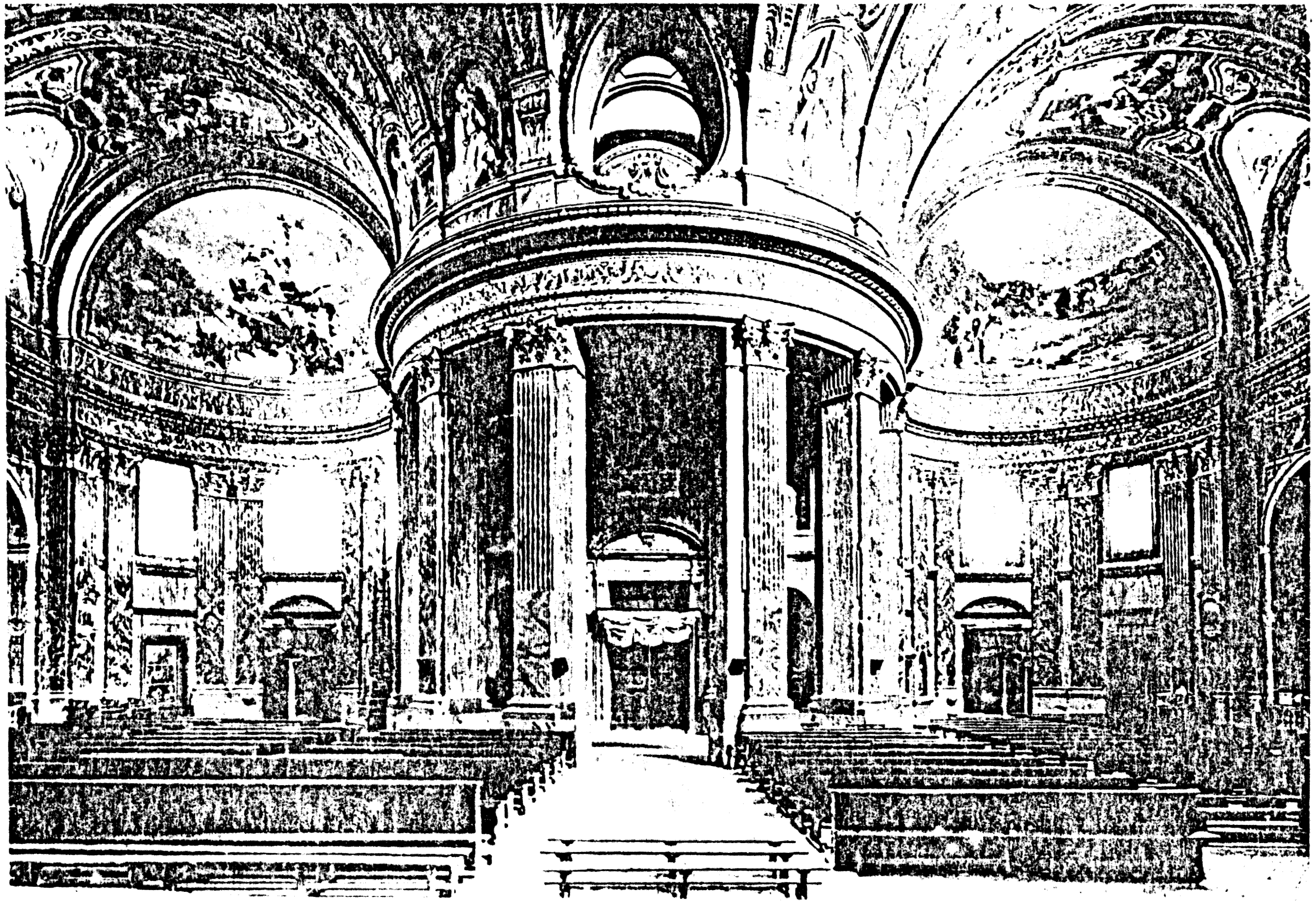


Figure 2-6 SS. Giovanni e Remigio in Carignano, Benedetto Alfieri.

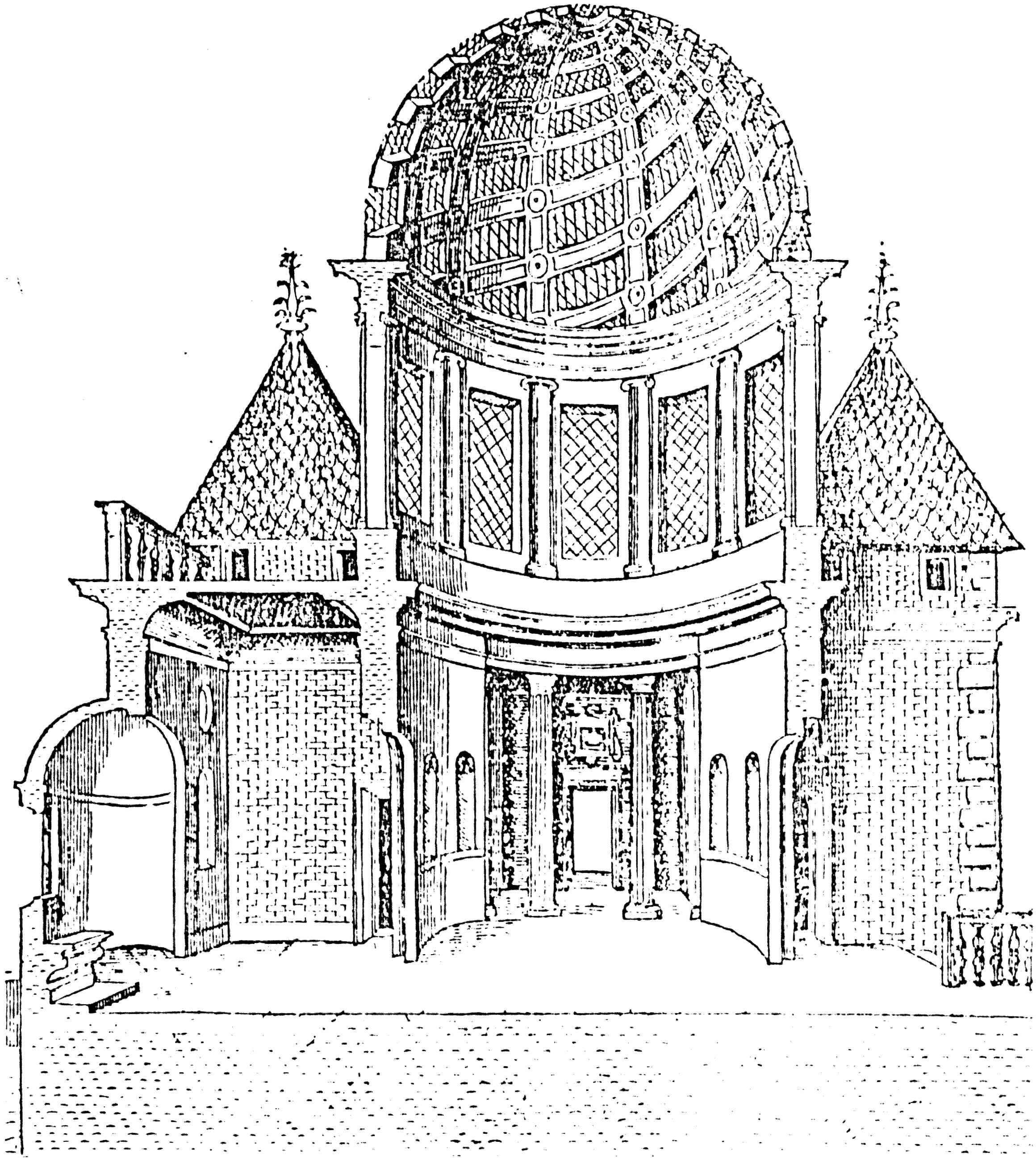


Figure 2-7 Philibert De l'Orme: Spherical dome with Spiral

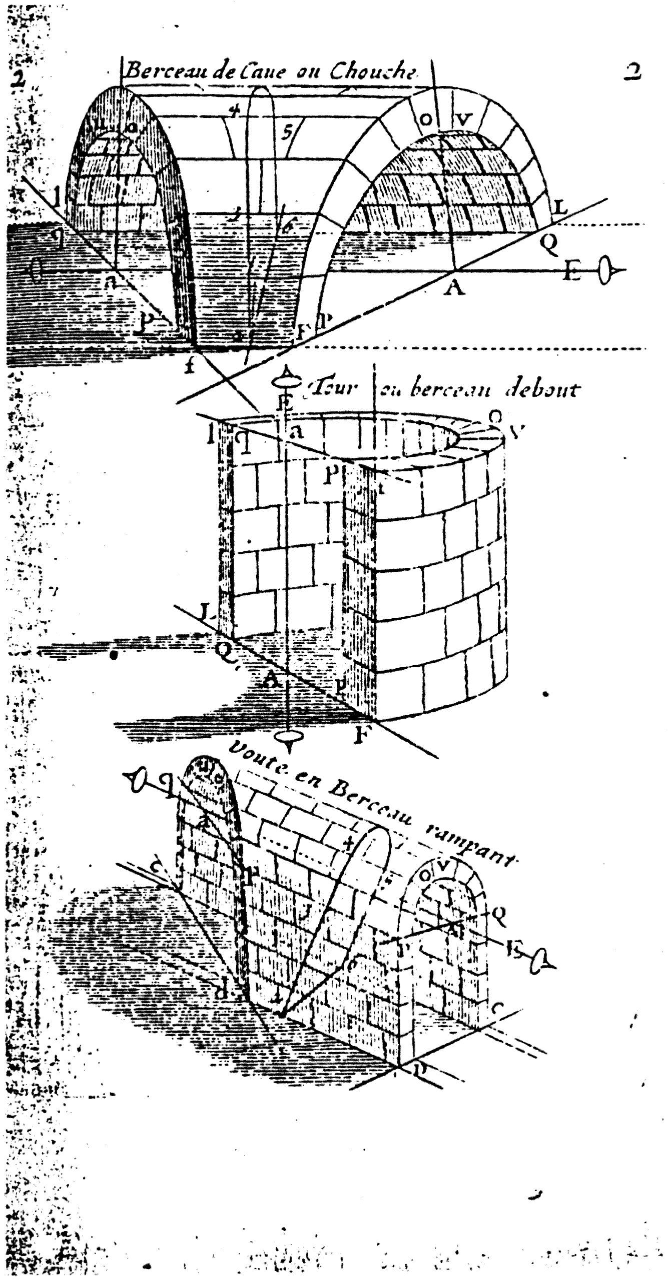


Figure 2-8 The Descriptive System of Desargues' Method.

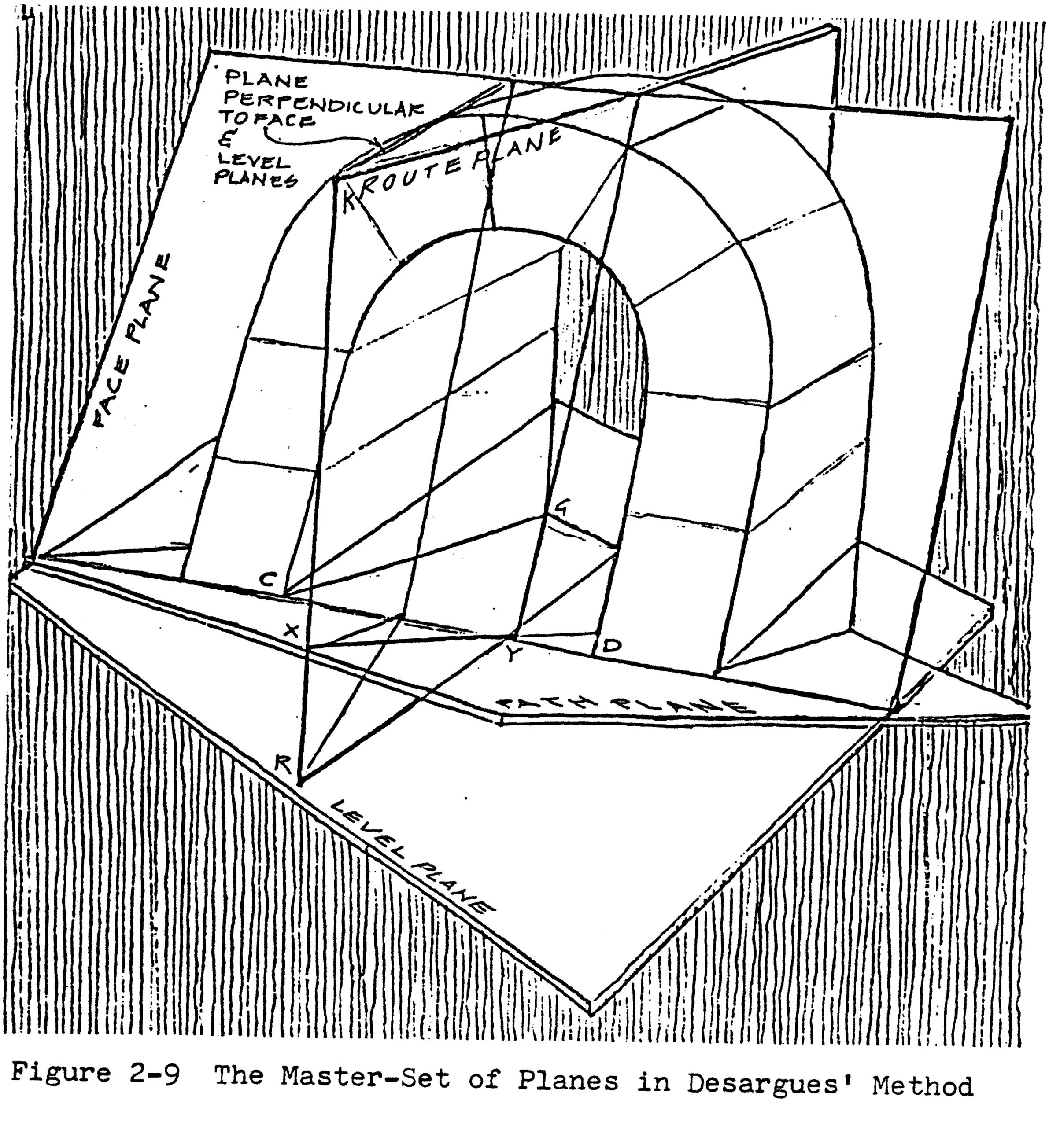


Figure 2-9 The Master-Set of Planes in Desargues' Method

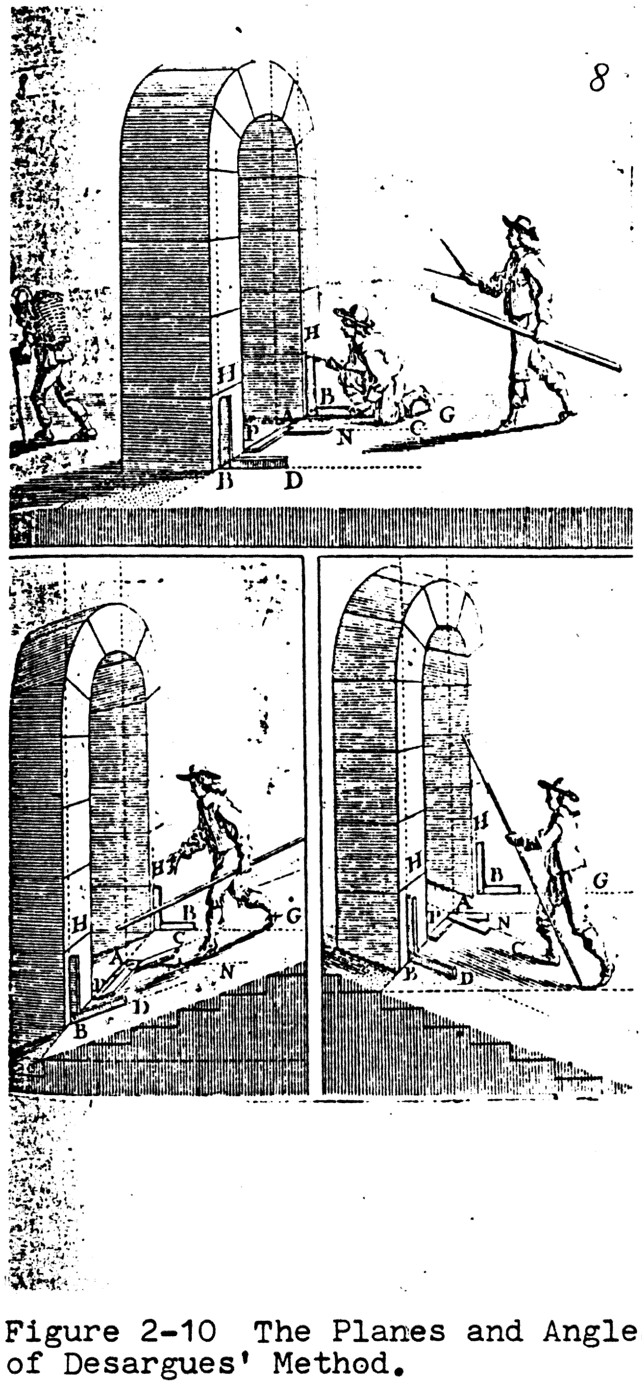


Figure 2-10 The Planes and Angles of Desargues' Method.

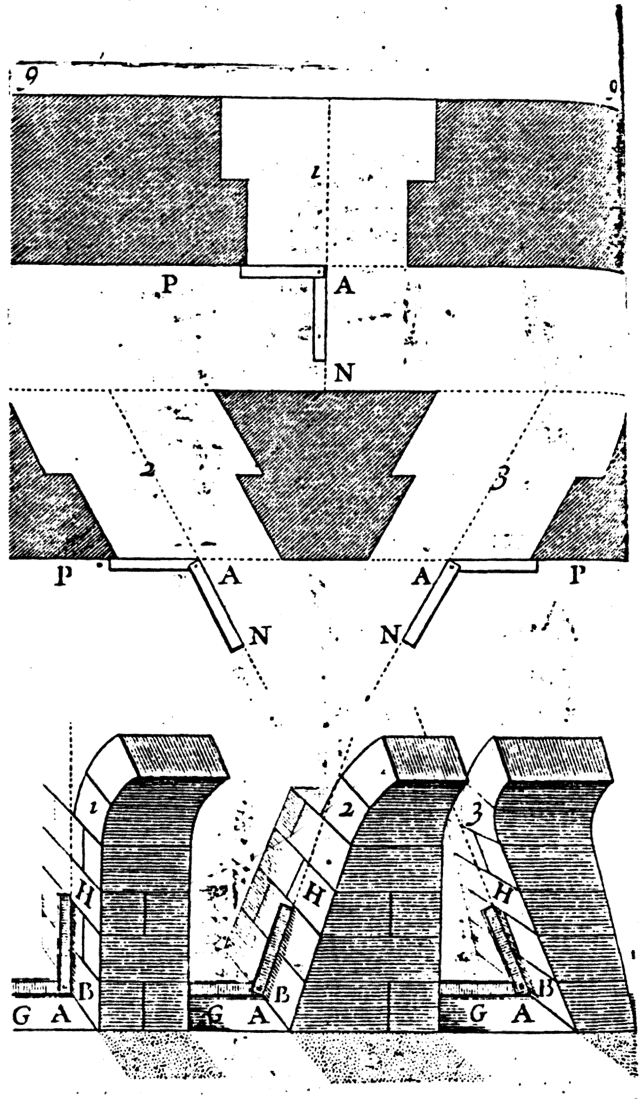


Figure 2-11 The System of Angles in Various Kinds of Vaults.

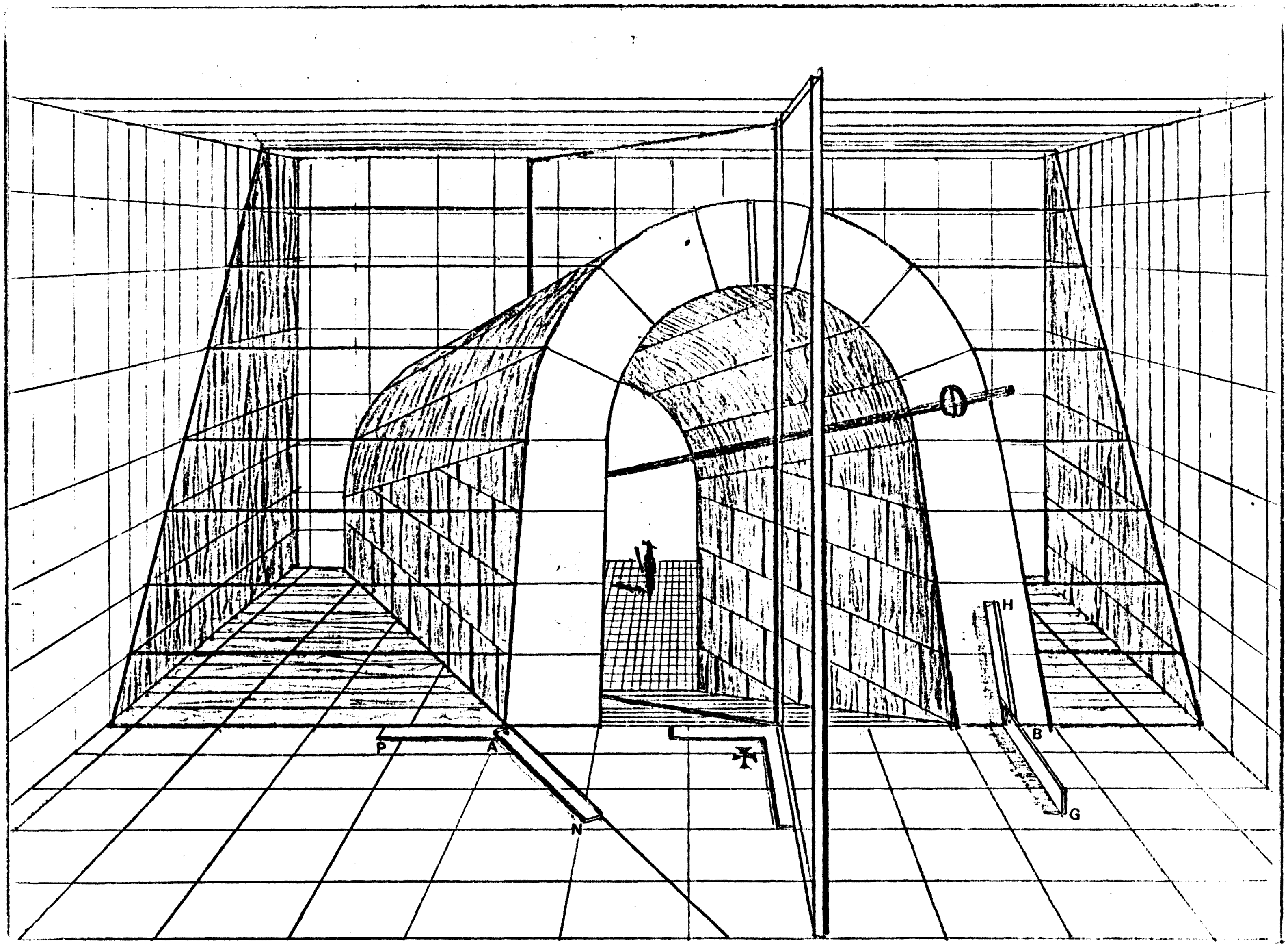
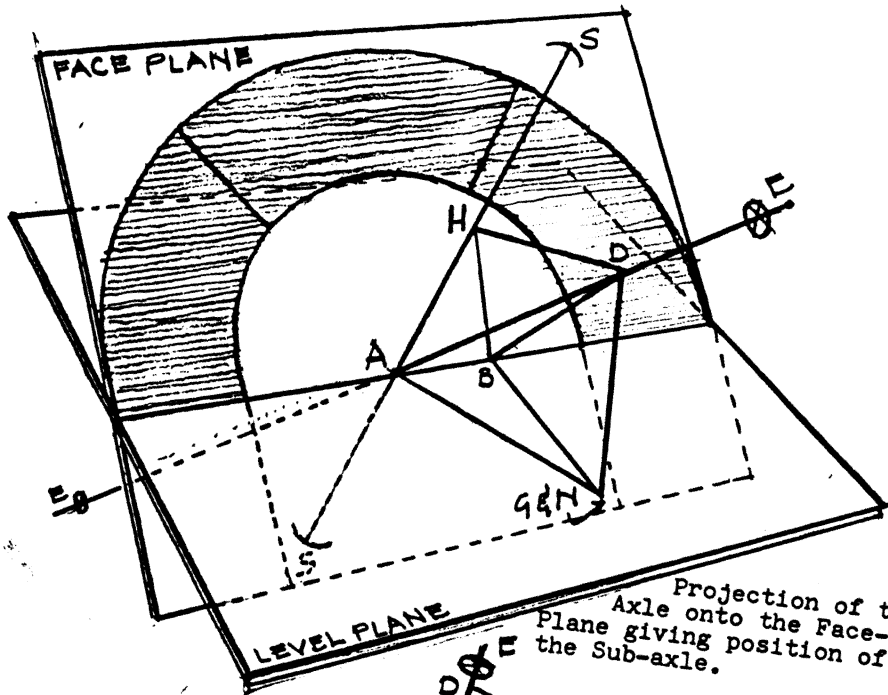
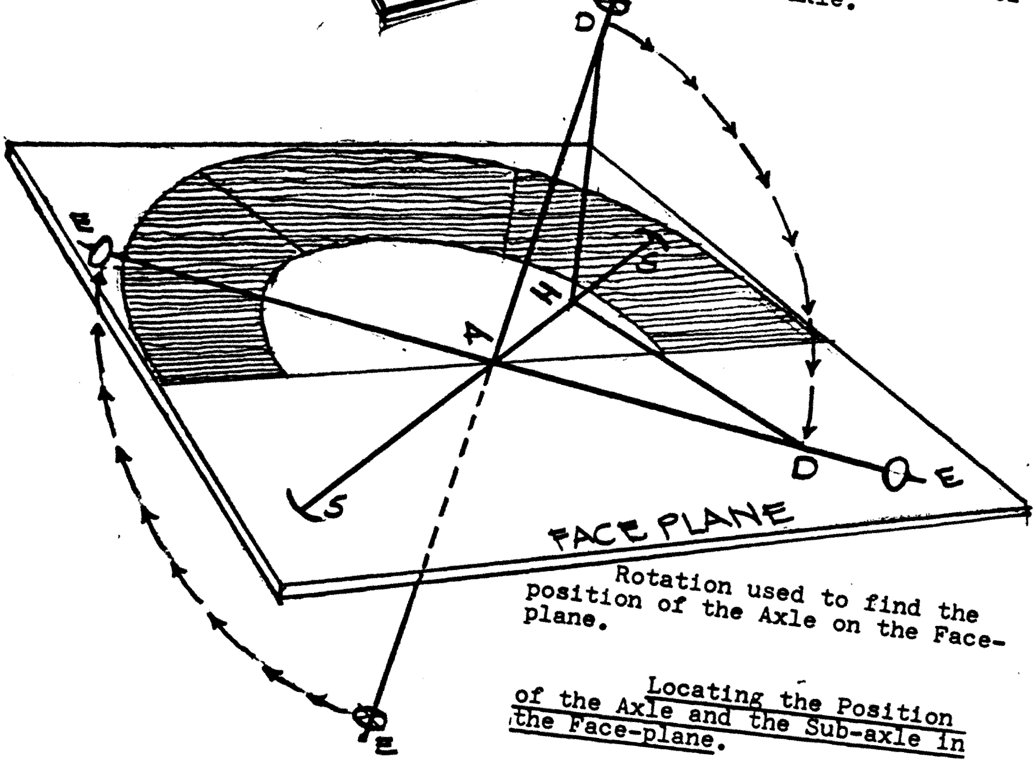


Figure 2-12 The Problem Used by Bosse to Explain Desargues' Method.

Figure 2-13 The Given Angles, and the Constructions for Steps 1-11 of Desargues' Method (foldout)



Projection of the Axle onto the Face-Plane giving position of the Sub-axle.



Rotation used to find the position of the Axle on the Face-plane.

Locating the Position of the Axle and the Sub-axle in the Face-plane.

Figure 2-14 Locating the Position of the Axle and Sub-axle.

12 13

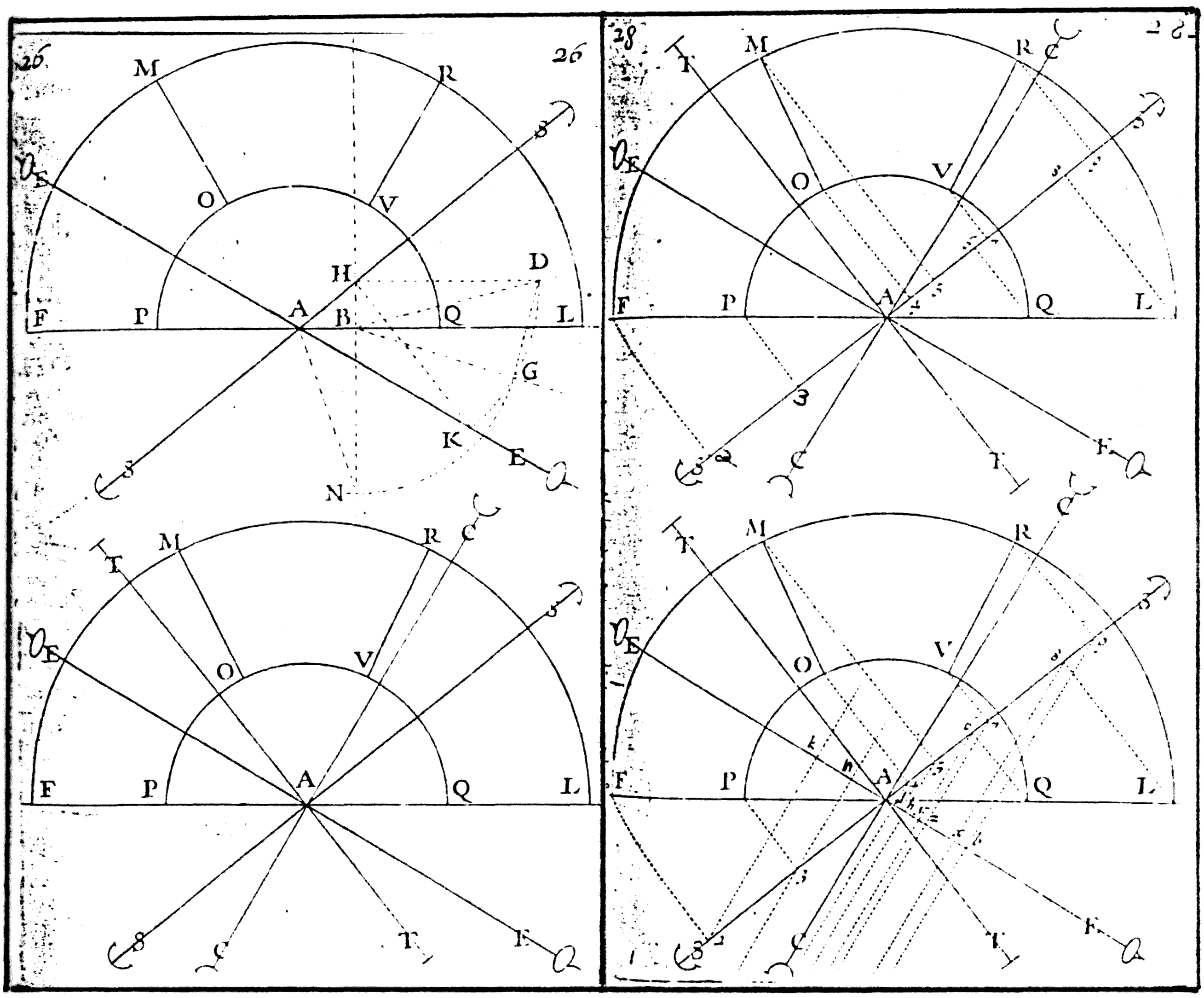


Figure 2-15 Steps 12 and 13 of Desargues' Method.

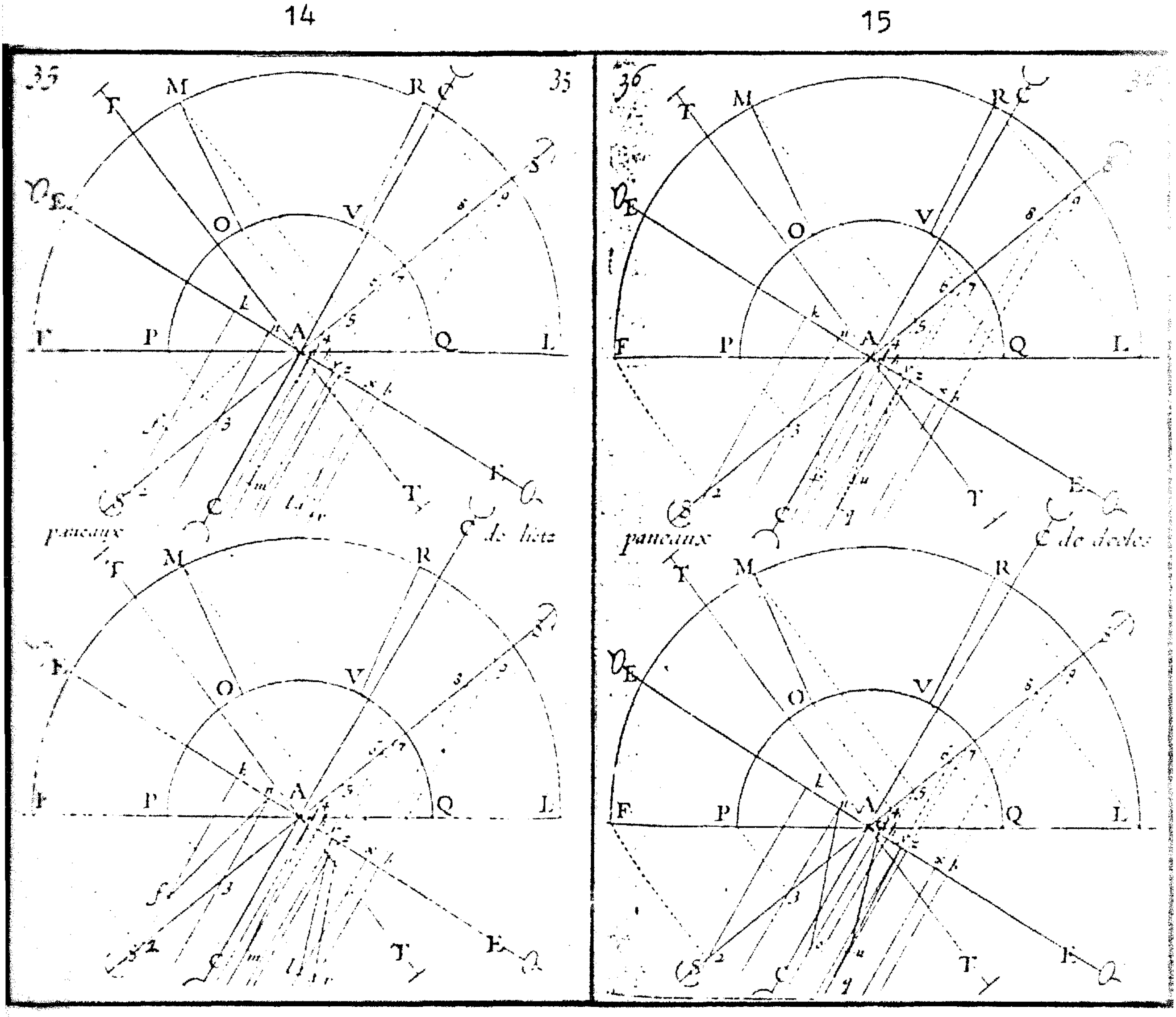


Figure 2-16 Steps 14 and 15 of Desargues' Method

16 17

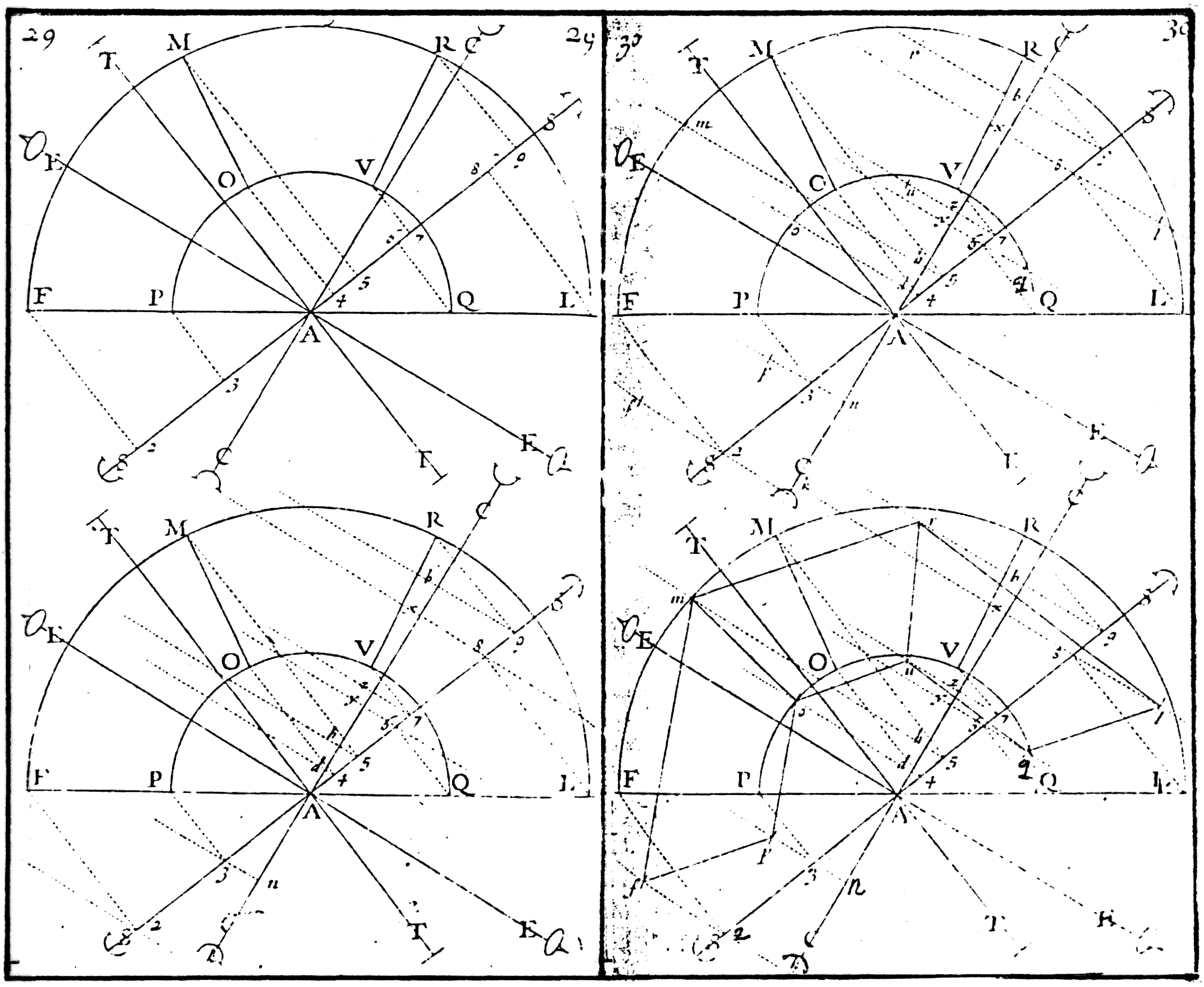


Figure 2-17 Steps 16 and 17 of Desargues' Method

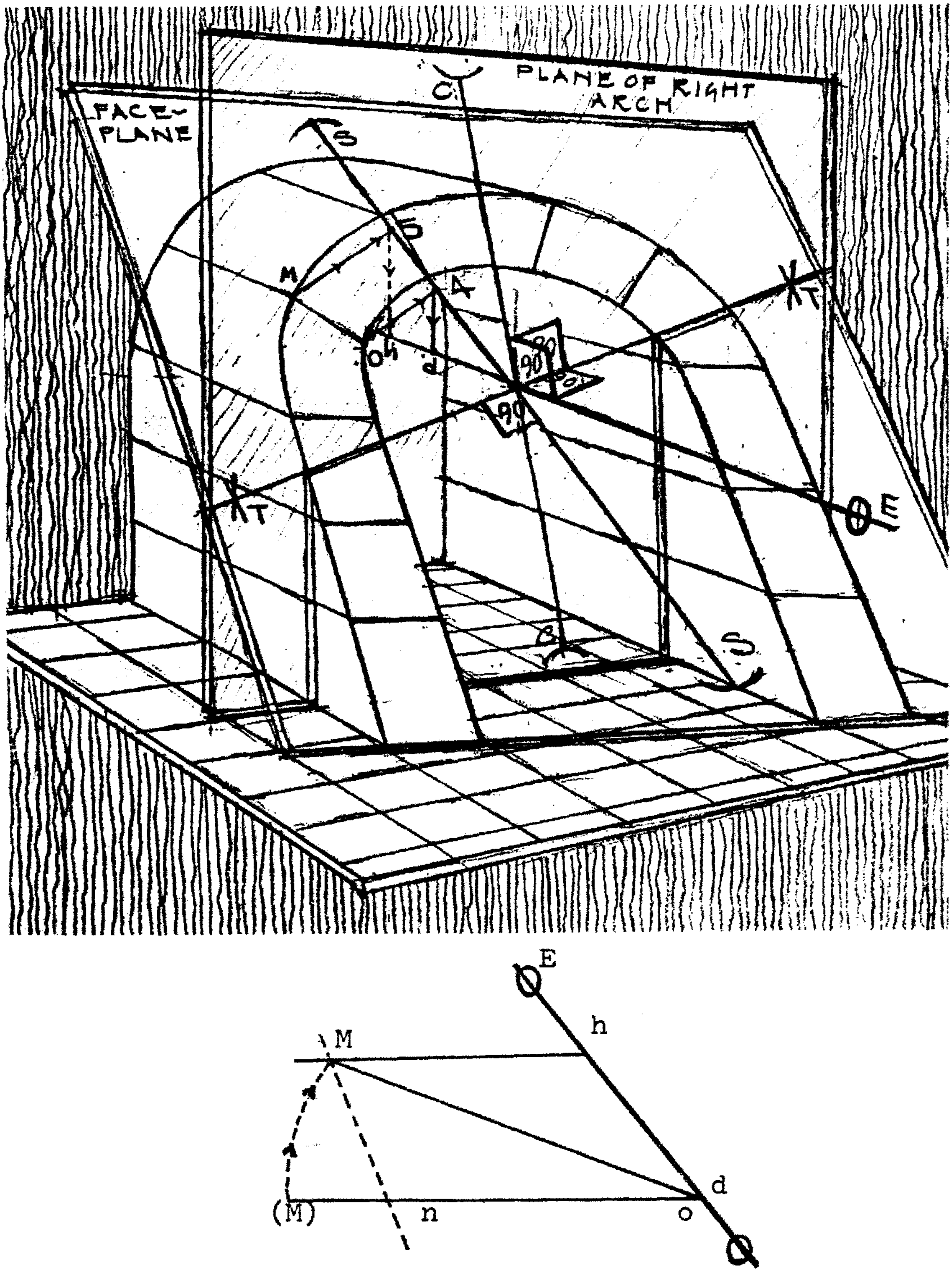


Figure 2-18 The Projection of a Joint Line onto the Axle and Subaxle as seen in Perspective

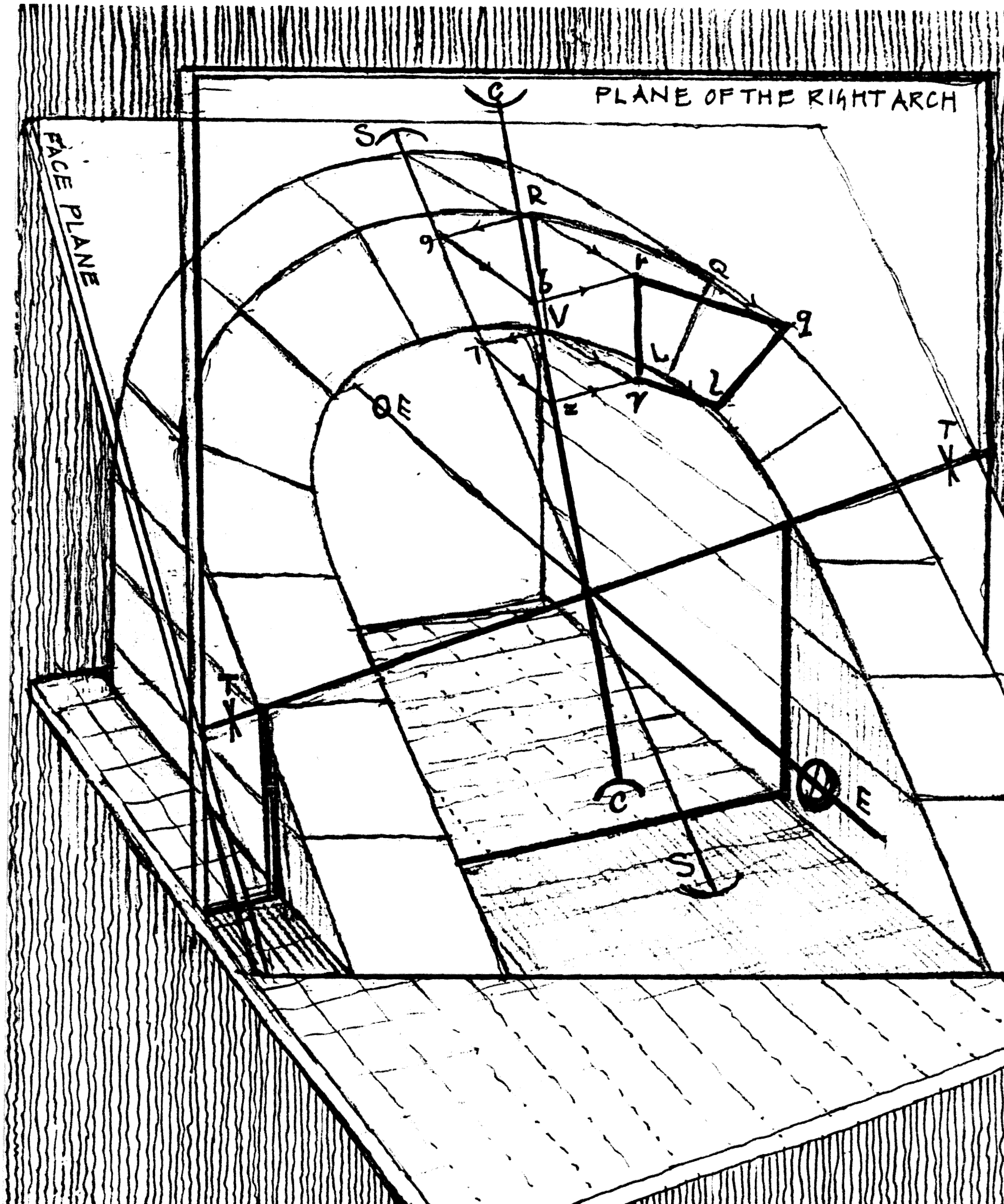


Figure 2-19 The Projection of Voussoir Heads Into the Plane of the Right-Arch

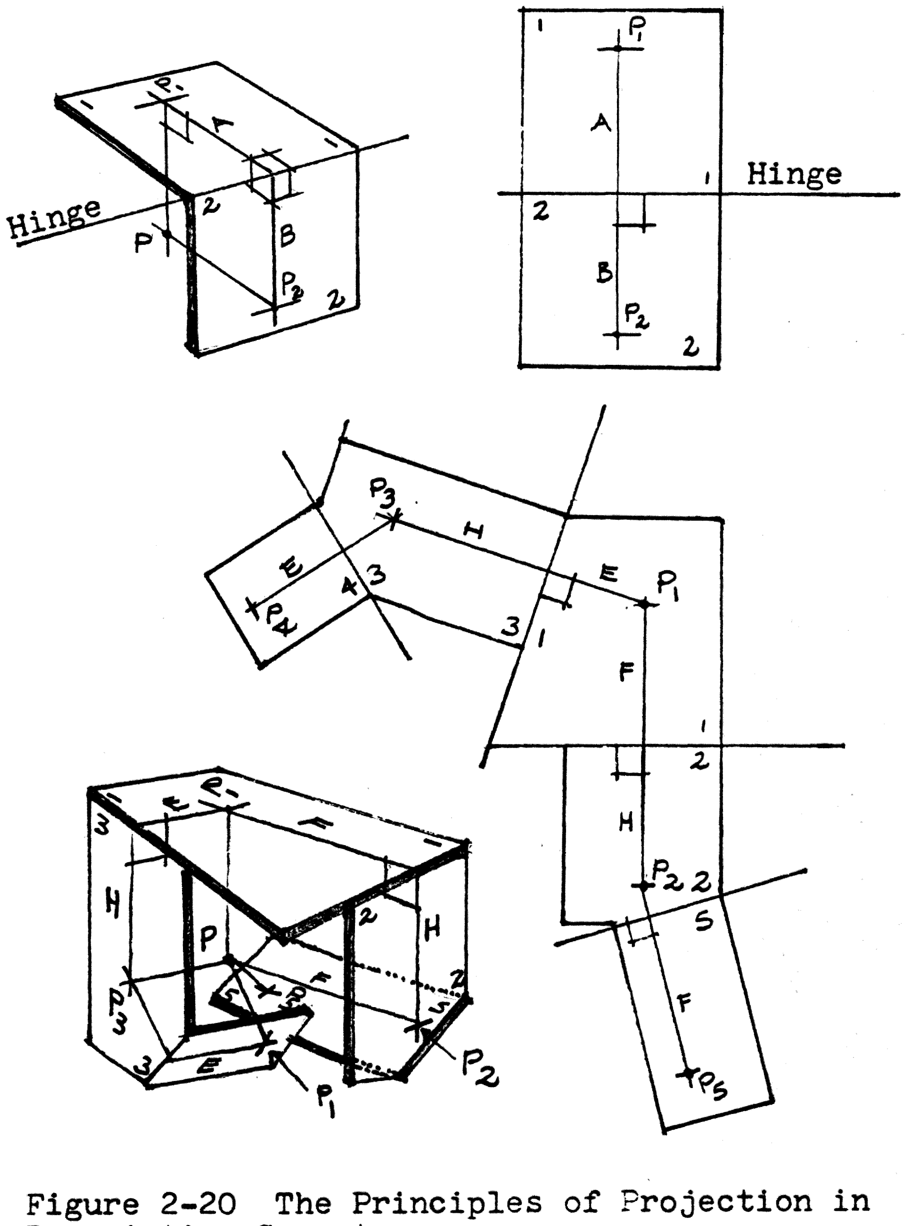


Figure 2-20 The Principles of Projection in Descriptive Geometry

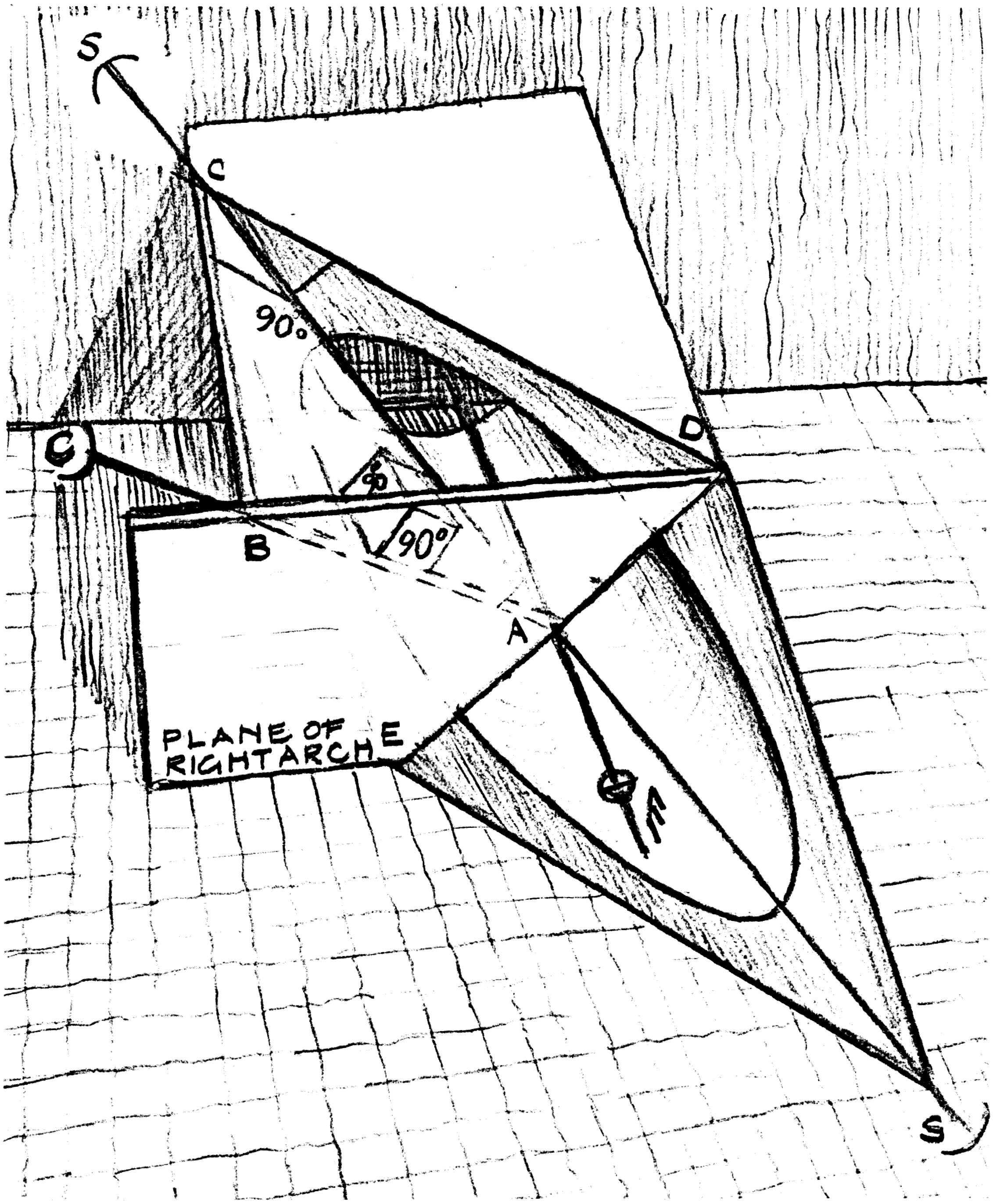


Figure 2-21 The Angular Relations Between the Projection-Planes in Desargues' Method of Stereotomy.

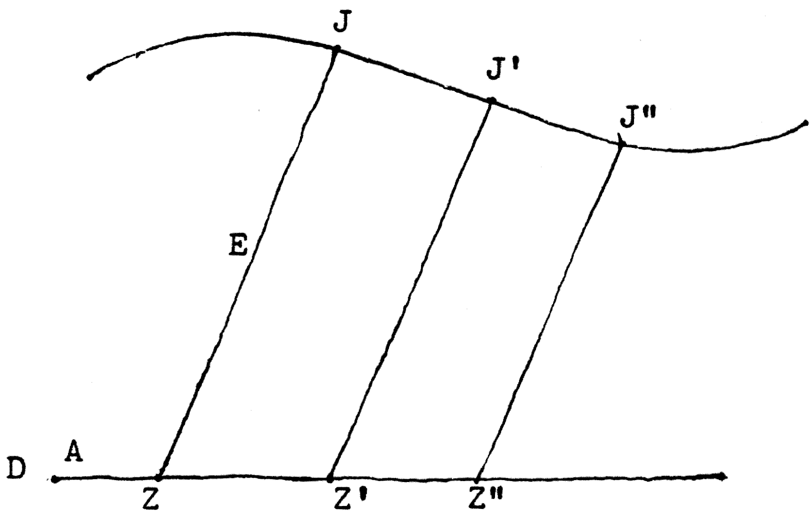
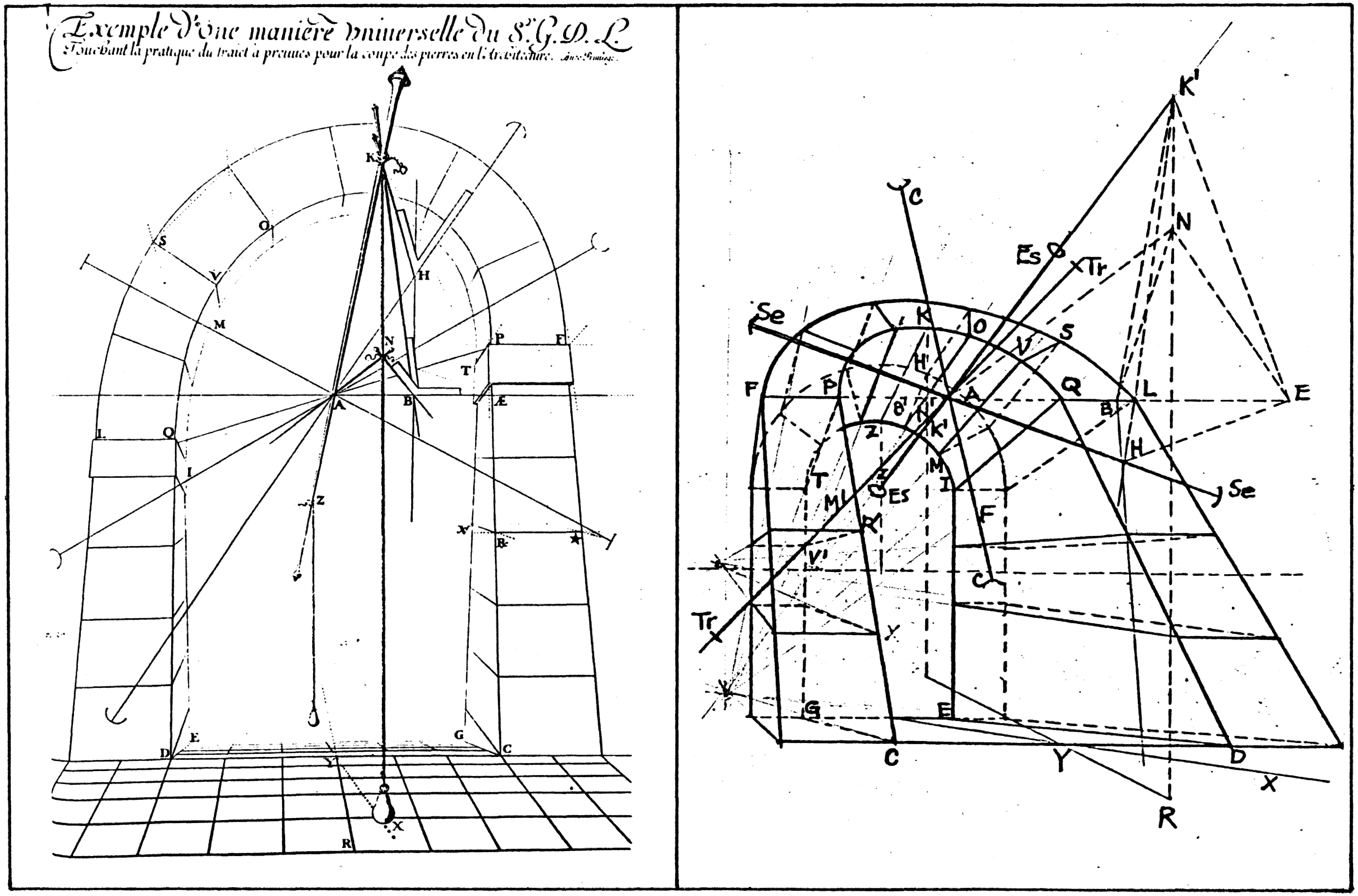
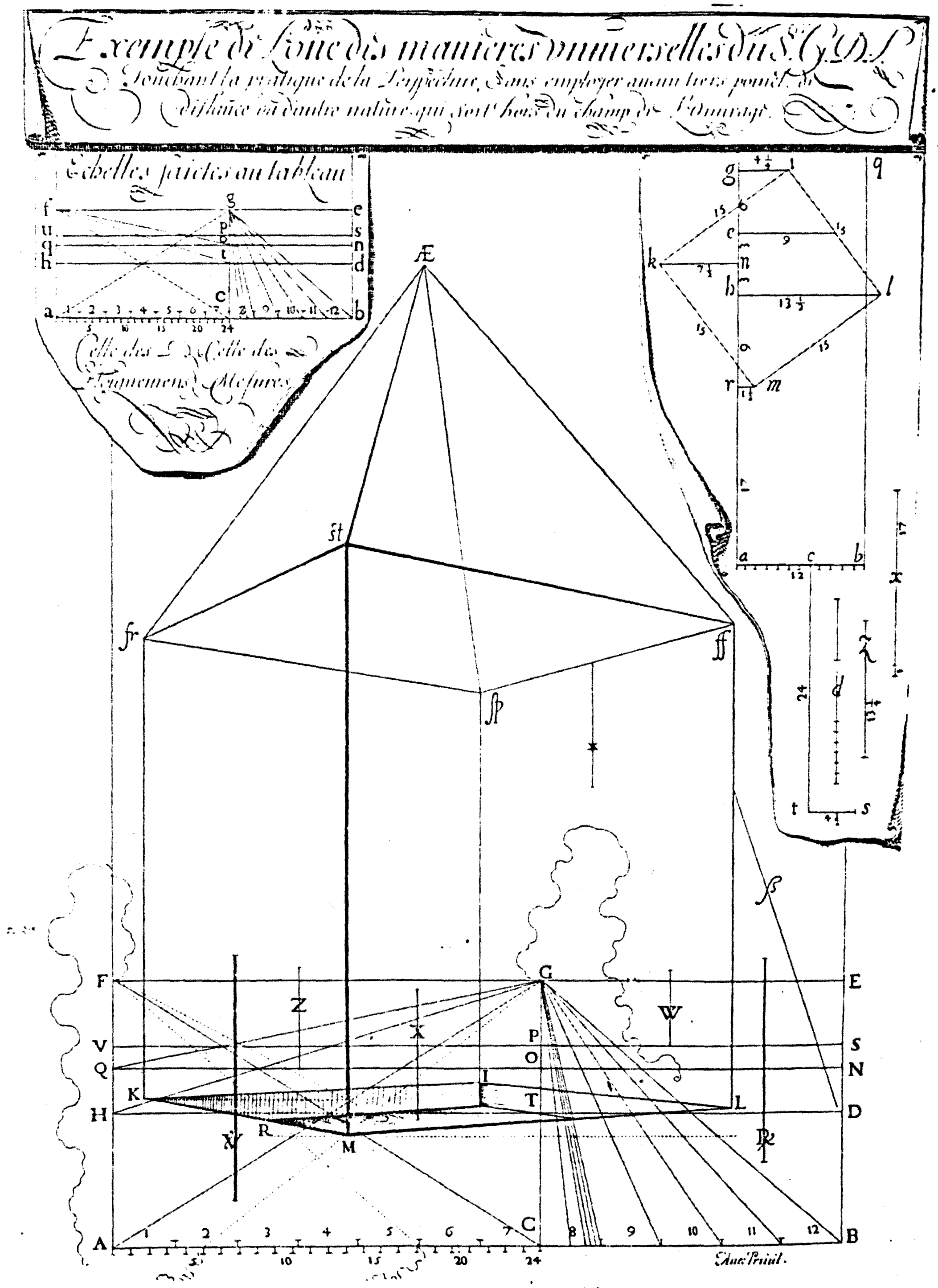


Figure 2-22 A System of Oblique Coordinates.



Desargues' Original First Plate. Poudra's Reconstruction From the Text.
Figure 2-23



EXEMPLE DE L'UNE DES MANIERES. PLATE I

Figure 3-1 Desargues' Illustration for his Perspective Theorem

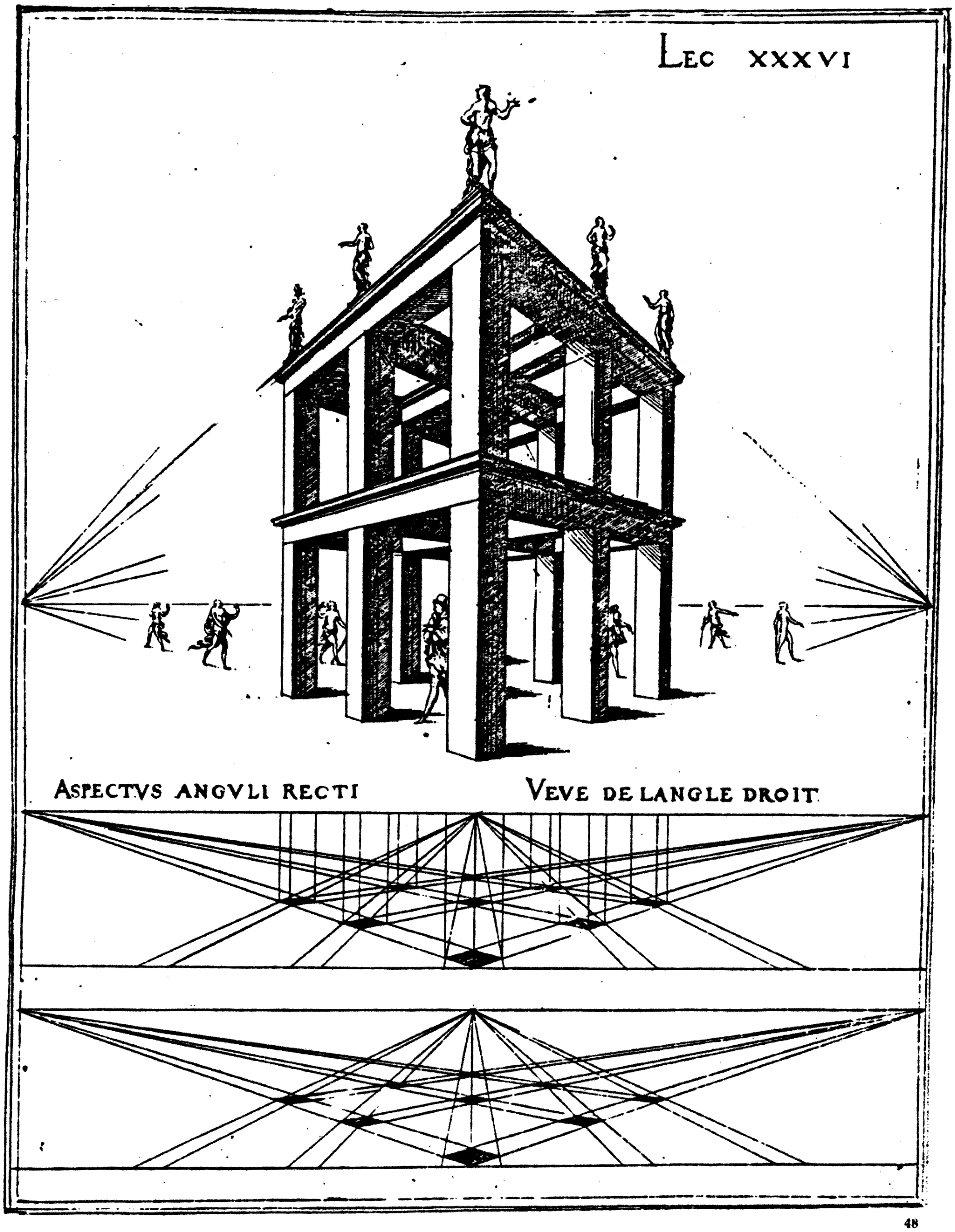
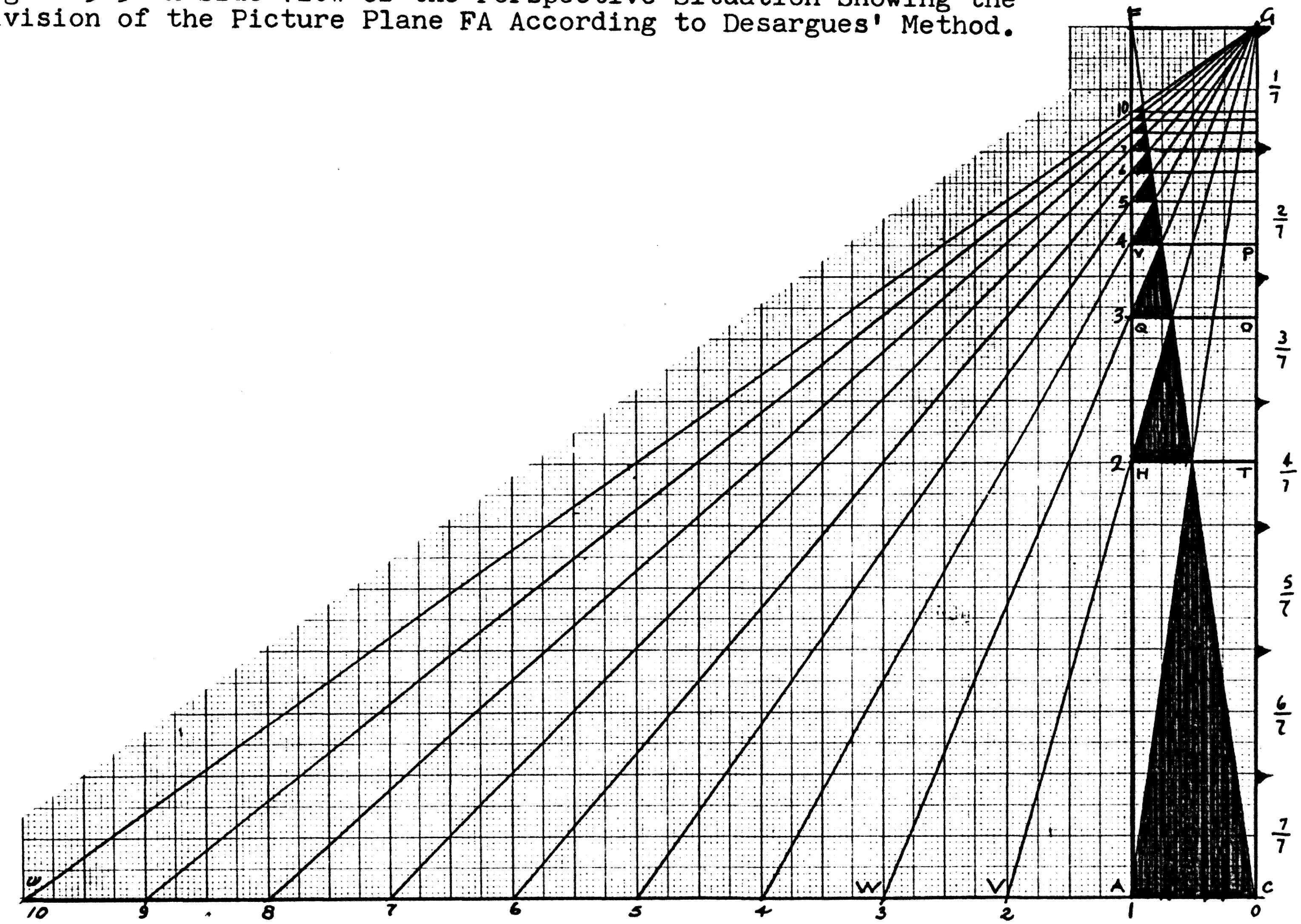


Figure 3-2 Relatedness of one and two-point perspective as illustrated in Androuet du Cerceau's 1576 Lecons de perspective positive.

Figure 3-3 A Side View of the Perspective Situation Showing the Division of the Picture Plane FA According to Desargues' Method.



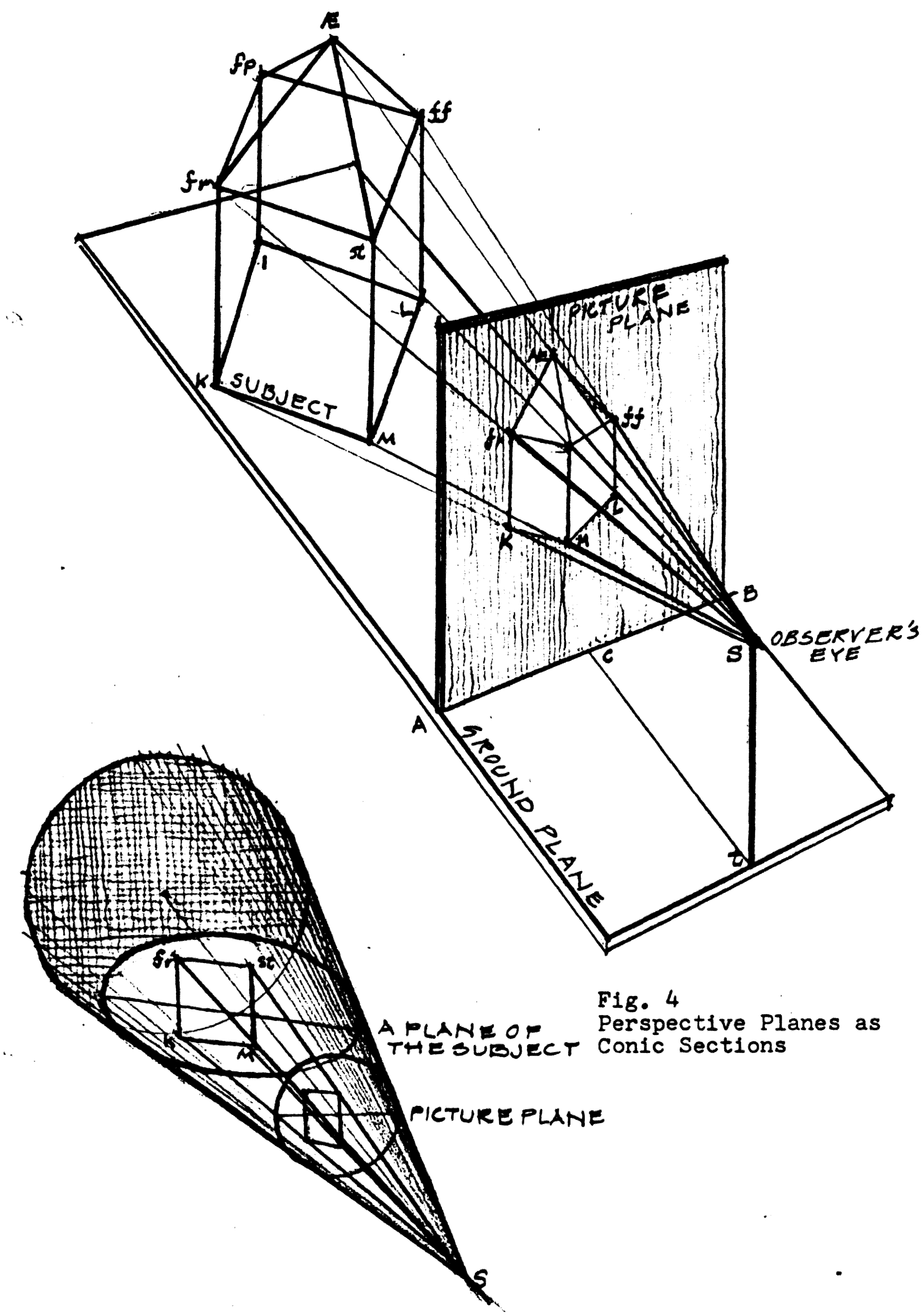


Fig. 4
Perspective Planes as Conic Sections

Figure 3-4 Perspective planes as Conic Sections

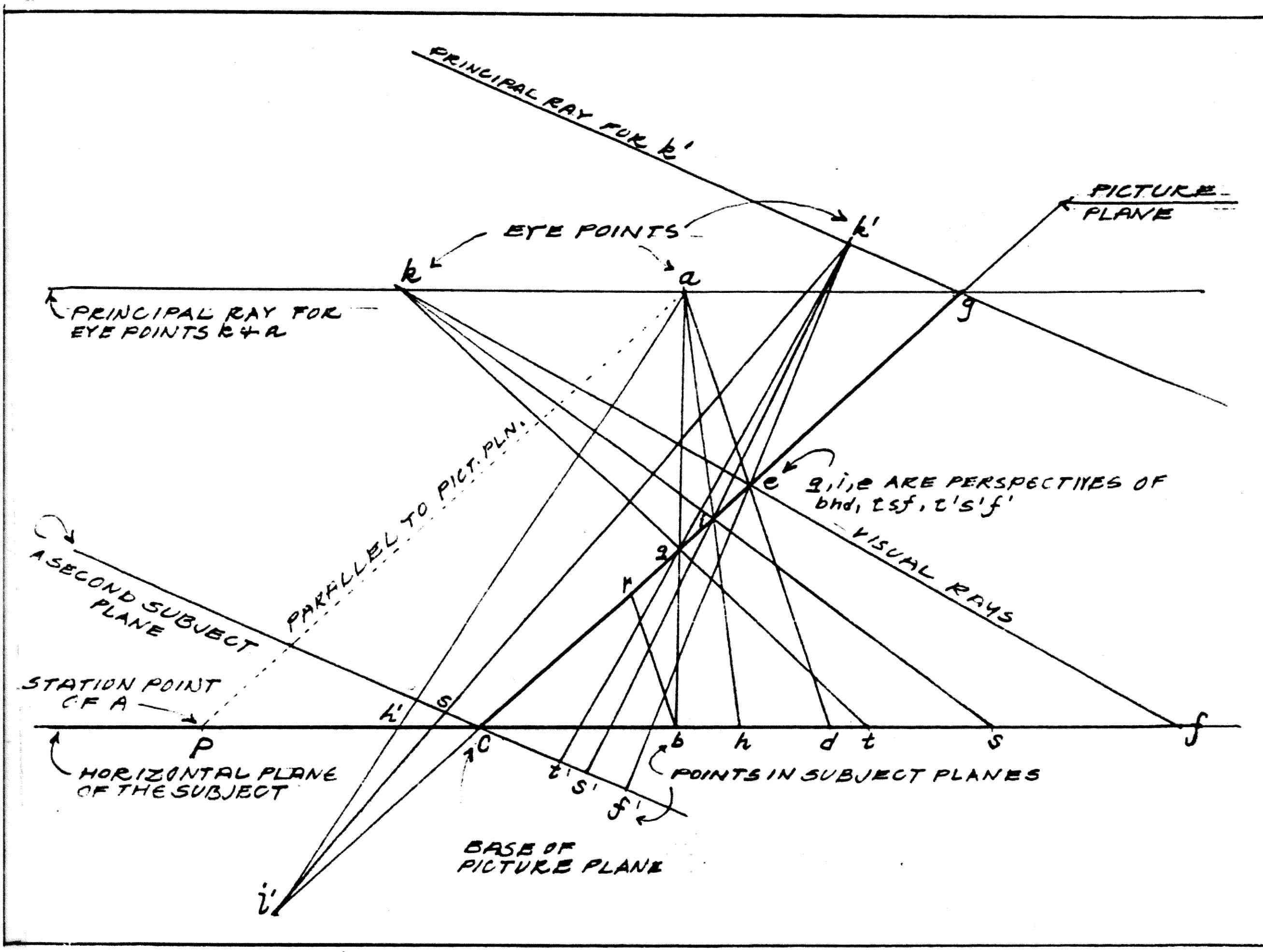


Figure 3-5 Desargues' Perspective Theorem

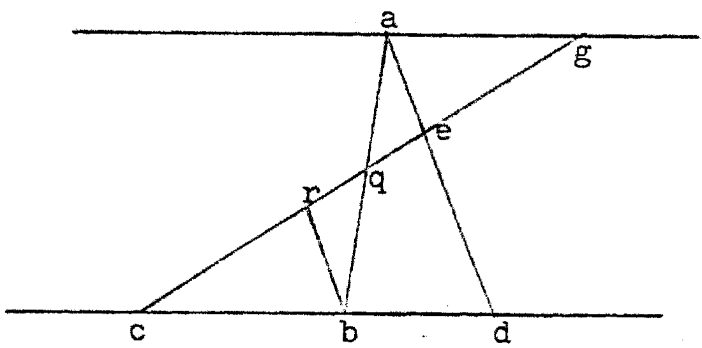


Figure 3-6a

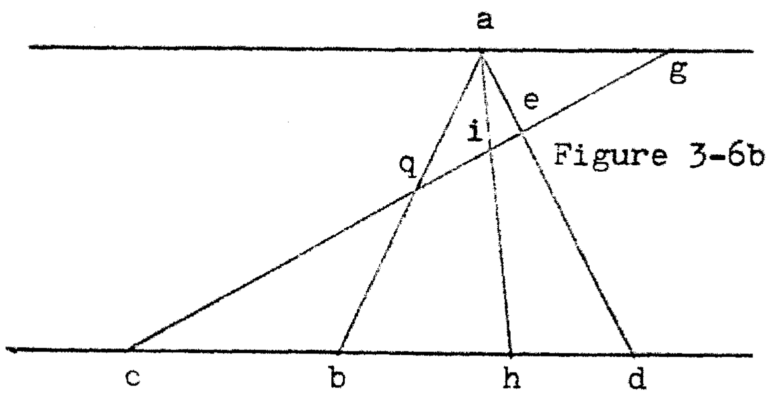


Figure 3-6b

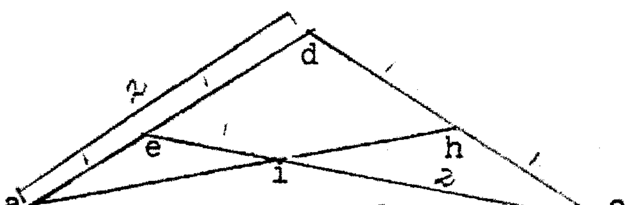


Figure 3-6c

Figure 3-6 Analysis of Desargues' Perspective Theorem

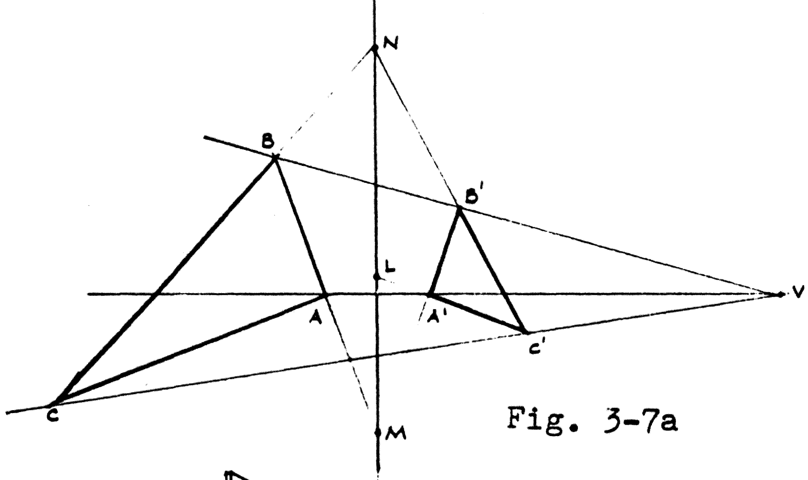


Fig. 3-7a

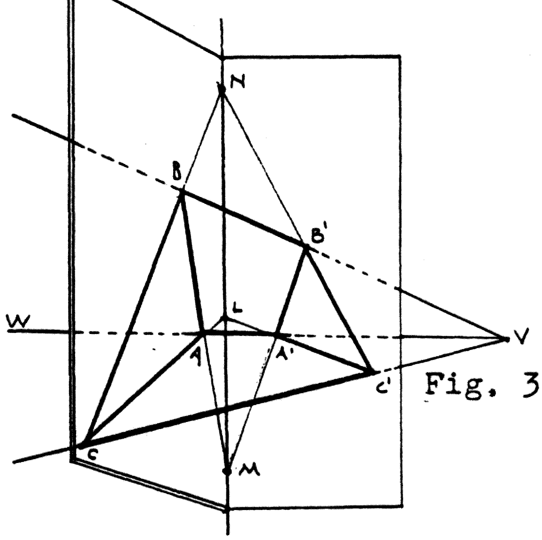


Fig. 3-7b

Figure 3-7 Desargues' Theorem in Two and Three Dimensions

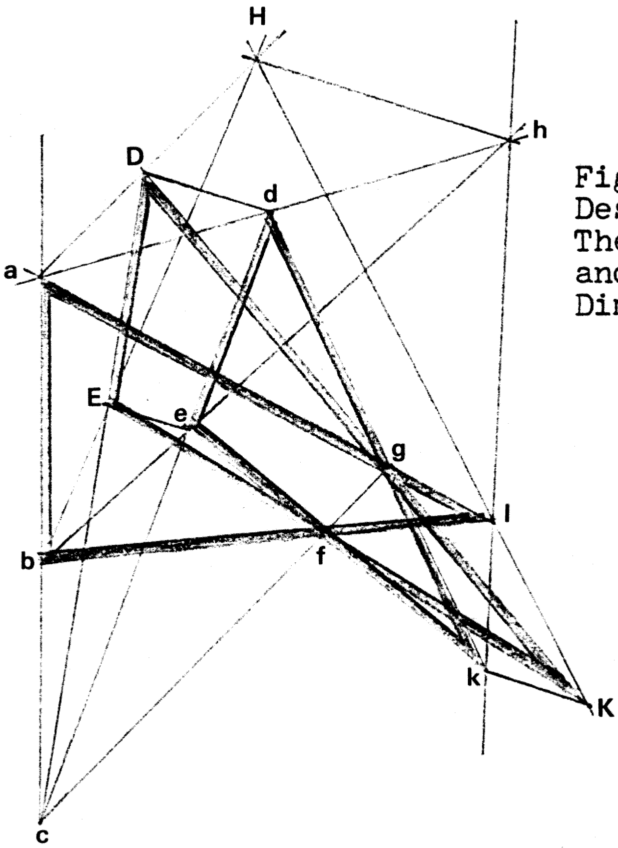
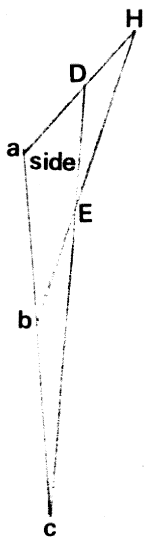
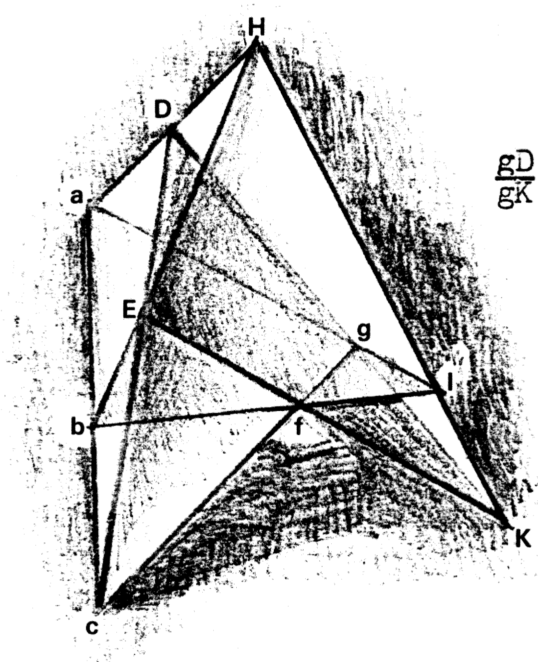
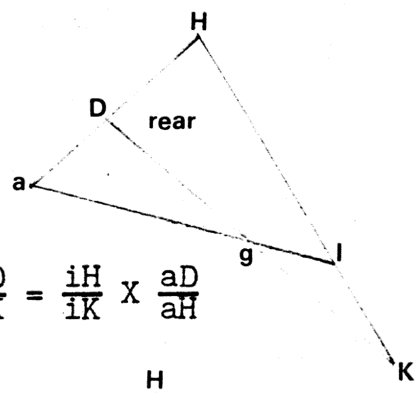


Figure 3-8
Desargues'
Theorem in Two
and Three
Dimensions

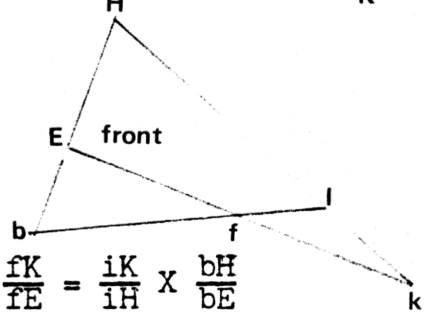


$$\frac{cD}{cE} = \frac{aD}{aH} \times \frac{bH}{bE}$$





$$\frac{gD}{gK} = \frac{iH}{iK} \times \frac{aD}{aH}$$



$$\frac{fK}{fE} = \frac{iK}{iH} \times \frac{bH}{bE}$$

Figure 3-9a

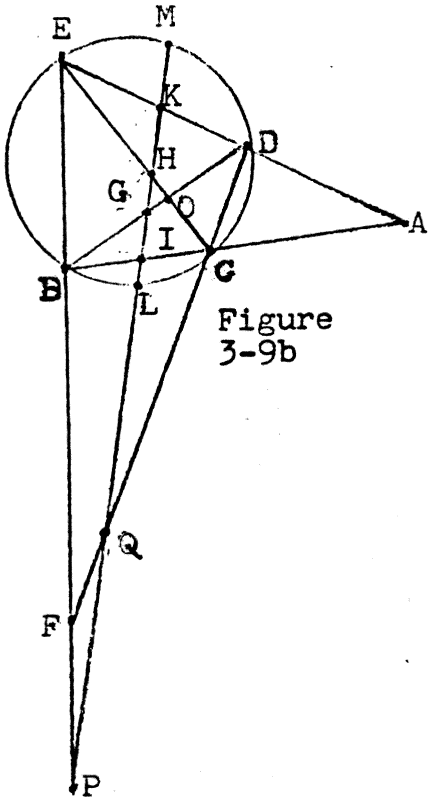


Figure 3-9b

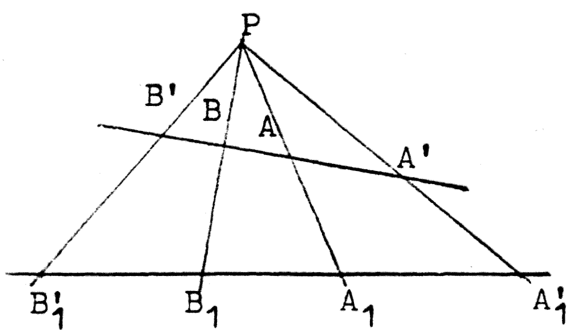


Figure 3-9c

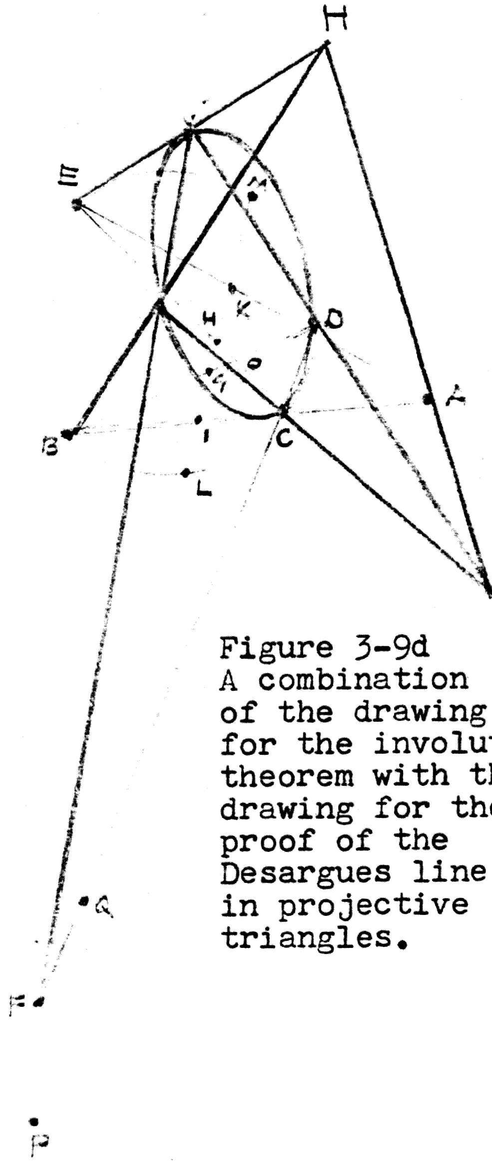


Figure 3-9d
A combination of the drawing for the involution theorem with the drawing for the proof of the Desargues line in projective triangles.

Figure 3-9 Desargues' Theorem of Involution

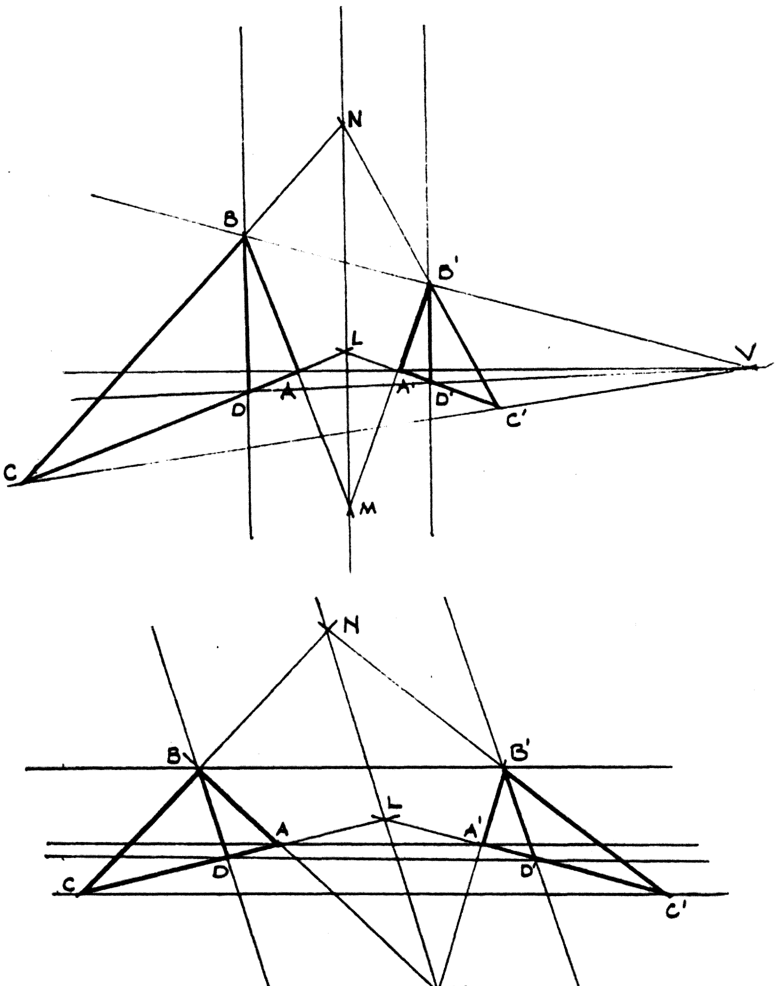


Figure 3-10 Parallel and Perspective Projection

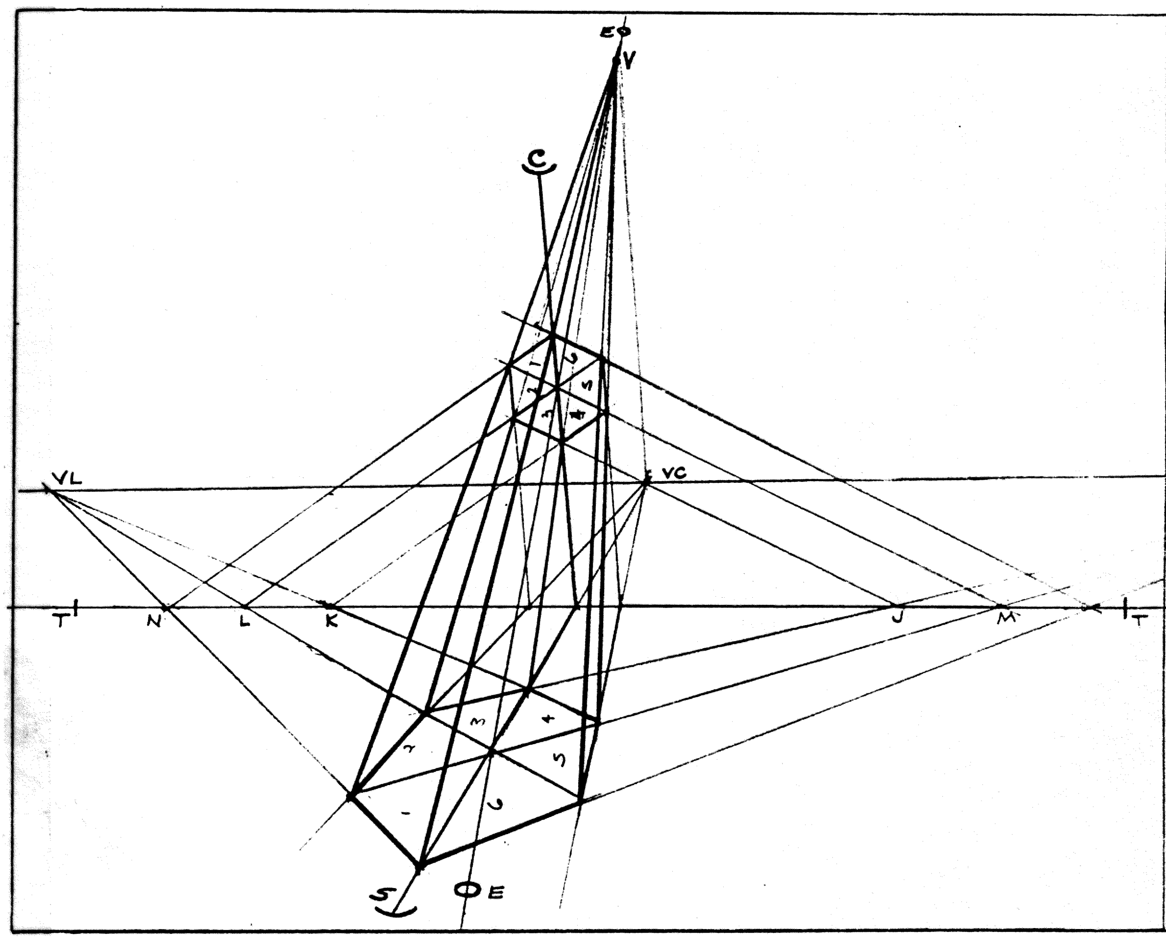


Figure 3-11 The Desargues Line in Perspective and Stereotomy

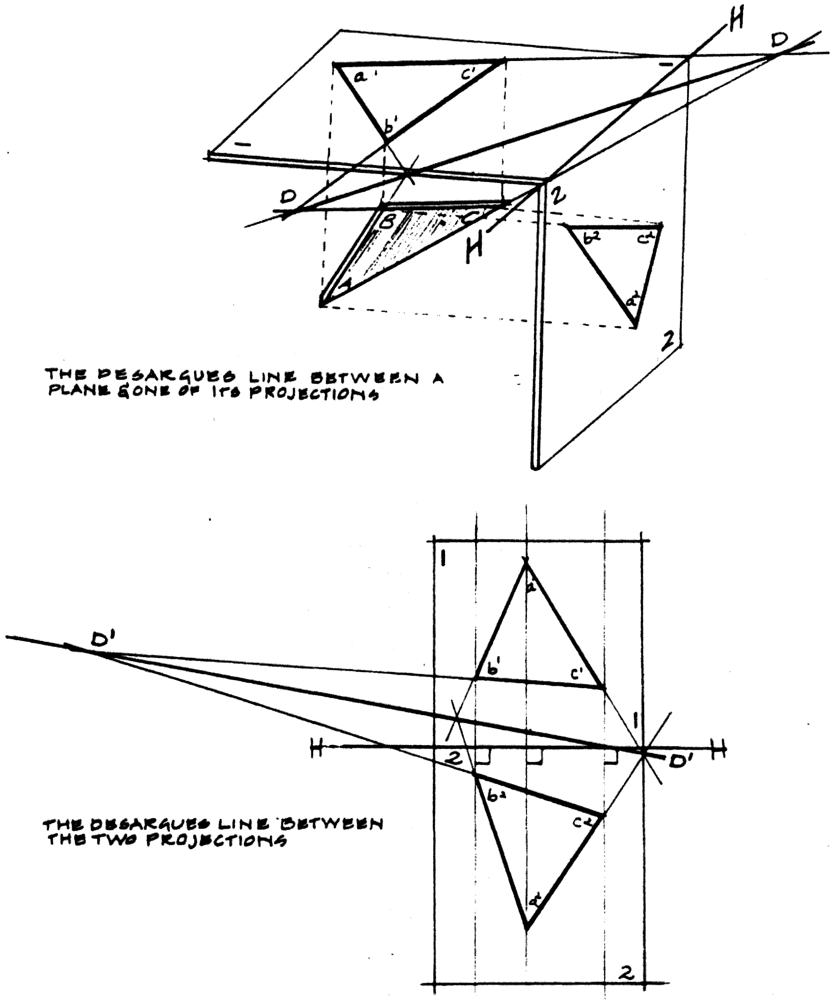


Figure 3-12 The Incompatibility of Desargues Lines with the Hinge Lines of Descriptive Geometry.

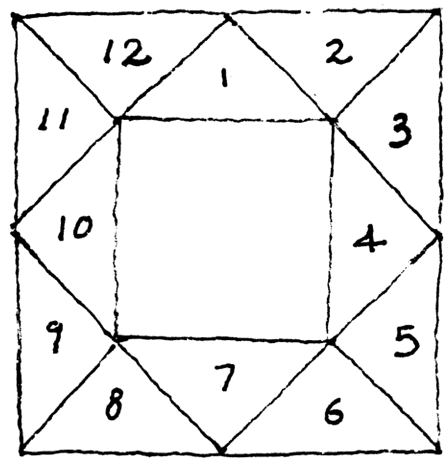


Fig. 1-a: Diagram for the casting of horoscopes. Numbers indicate houses.

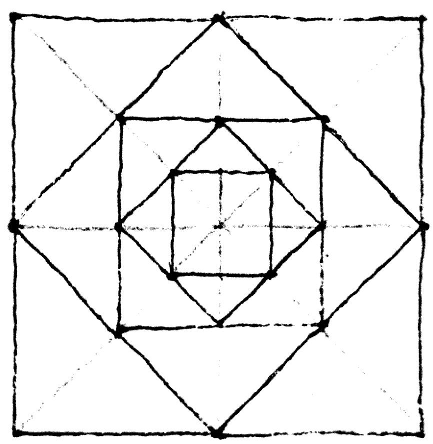


Fig. 1-b: The mason's 'secret': geometric basis of medieval & Gothic architectural proportions as given by Villard, Roriczer, et al.

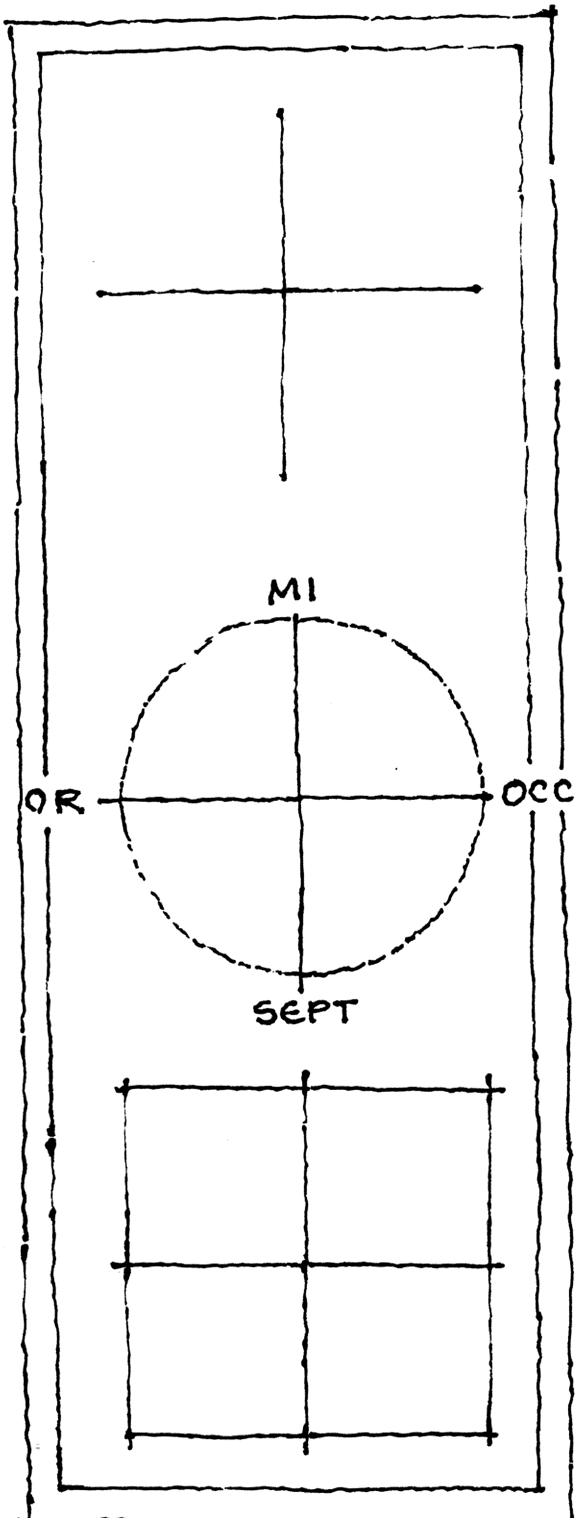


Fig. 1-c: De l'Orme's illustration of the "draught of the square".

Figure 4-1 Cosmological Symbols in Renaissance Geometry

The three page vita has been removed from the scanned document. Page 1 of 3

The three page vita has been removed from the scanned document. Page 2 of 3

The three page vita has been removed from the scanned document. Page 3 of 3