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Hypothesis Testing Procedures for Non-Nested Regression Models

by

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(ABSTRACT)

Theory often indicates that a given response variable should be a function of certain explanatory variables yet fails to provide meaningful information as to the specific form of this function. To test the validity of a given functional form with sensitivity toward the feasible alternatives, a procedure is needed for comparing non-nested families of hypotheses. Two hypothesized models are said to be non-nested when one model is neither a restricted case nor a limiting approximation of the other. These non-nested hypotheses cannot be tested using conventional likelihood ratio procedures. In recent years, however, several new approaches have been developed for testing non-nested regression models.

A comprehensive review of the procedures for the case of two linear regression models was presented. Comparisons between these procedures were made on the basis of asymptotic distributional properties, simulated finite sample performance and computational ease. A modification to the Fisher and McAleer JA-test was proposed and its properties investigated. As a compromise between the JA-test and the Orthodox F-test, it was shown to have an exact non-null distribution. Its properties, both analytically and empirically derived, exhibited the practical worth of such an adjustment.

A Monte Carlo study of the testing procedures involving non-nested linear regression models in small sample situations ($n \leq 40$) provided information necessary for the formulation of practical guidelines. It was evident that the modified Cox procedure, $\tilde{N}$, was most powerful for providing correct inferences. In addition, there was strong evidence to support the use of the adjusted J-test

(AJ) (Davidson and MacKinnon's test with small-sample modifications due to Godfrey and Pesaran), the modified JA-test (NJ) and the Orthodox F-test for supplemental information. Under nonnormal disturbances, similar results were yielded.

An empirical study of spending patterns for household food consumption provided a practical application of the non-nested procedures in a large sample setting. The study provided not only an example of non-nested testing situations but also the opportunity to draw sound inferences from the test results.

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I. Introduction

In recent years, the study of non-nested hypothesis testing has received a great deal of attention in the statistical and econometric literature. In general, non-nested hypotheses arise when the researcher wants to test a given null hypothesis against an alternative hypothesis which belongs to a separate parametric family. A common problem of this nature is determining whether a given set of data was sampled from one distributional family or from another. For instance, a researcher may be interested in seeing whether a sample of data follows a log-normal distribution or an exponential distribution. This example however does not represent the only type of application of non-nested hypotheses.

In regression studies, a similar situation occurs when the researcher wants to test the validity of one model against a specified alternative model. Specifically, the researcher may be interested in testing whether the given model is appropriate in terms of its functional specification: the form of f in the following model,

$$\underline{y} = f(\underline{\beta}; \underline{x}_1, \underline{x}_2, \ldots, \underline{x}_k) + \underline{\varepsilon}. \tag{1.1}$$

Models involving different functional forms provide one type of non-nested situation. Another way in which non-nested models arise is when the models have the same functional specification, such as a linear form, but contain different regressor variables. However, not all cases of testing model validity, even of these types, come under the heading of non-nested hypotheses.

If one of the models represents a restricted case (or subset) of the alternative model, then the test of this null model versus the specified alternative can be accomplished using the classical likelihood ratio (LR) approach. The following pair of linear regression models represents this "nested" case

$$H_0\colon \underline{y} = X_1\underline{\beta}_1 + \underline{\varepsilon}_1 , \qquad\qquad H_0\colon \underline{\beta}_2 = \underline{0}$$

$$H_1: \underline{y} = X_1\underline{\beta}_1 + X_2\underline{\beta}_2 + \underline{\varepsilon}_2 \,, \qquad H_1: \underline{\beta}_2 \neq \underline{0} \tag{1.2}$$

and can be tested using the following F-test:

$$F_{01} = \frac{(SSE_{(H_0)} - SSE_{(H_1)}) \,/\, rank(X_2)}{MSE_{(H_1)}} \tag{1.3}$$

In general, such nested cases are indicative of the variable selection process in which a test of hypothesis is performed to see if an additional explanatory variable, or set of variables, should be included in the model. Even in the most general variable selection situation, the tests performed on these models are designed to address the validity or correctness of the given model specification. The purpose is to detect misspecifications in the given model in the form of biases resulting from the exclusion of important explanatory variables or the use of an incorrect functional form. A particular case is when the alternative model is some higher order equation in the explanatory variables. For instance, the researcher wants to determine whether a linear form or a quadratic form is the more appropriate functional specification for a particular response model; i.e. (for $t = 1, 2, \ldots, n$),

$$H_L: y_t = \alpha_0 + \beta_0 x_t + \varepsilon_{0t} \tag{1.4}$$

$$H_Q: y_t = \alpha_1 + \beta_1 x_t + \gamma_1 x_t^2 + \varepsilon_{1t} \tag{1.5}$$

This testing of model specification can be handled using classical techniques since the models under test are of the nested form.

Suppose, however, that the alternative model of interest to this researcher is a semi-log model in lieu of a quadratic model; i.e.,

$$H_{SL}: y_t = \alpha_2 + \beta_2 \ln x_t + \varepsilon_{2t} \tag{1.6}$$

In this case, the null model (H_L: linear form) is not a restricted case of the alternative model (H_{SL}: semi-log form), or vice versa. In other words, these two models are not nested, or "non-nested." When one model is neither a restricted case nor a limiting approximation of the other, and vice-versa, then the two models are said to be non-nested. Consider the following hypothesized models:

$$H_1: \underset{\sim}{y} = X_1\underset{\sim}{\beta}_1 + \underset{\sim}{\varepsilon}_1, \qquad (\underset{\sim}{\beta}_1, \sigma_1^2) \in \Omega_1 \tag{1.7}$$

$$H_2: \underset{\sim}{y} = X_2\underset{\sim}{\beta}_2 + \underset{\sim}{\varepsilon}_2, \qquad (\underset{\sim}{\beta}_2, \sigma_2^2) \in \Omega_2 \tag{1.8}$$

If $\Omega_1 \cap \Omega_2 \neq \Omega_1$ and $\Omega_1 \cap \Omega_2 \neq \Omega_2$, then these two models are non-nested, or "separate." This definition allows for overlap of some of the regressor variables in the models under consideration as long as the space spanned by the columns of X_1 is not a subset of the space spanned by the columns of X_2, and vice-versa.

Hypotheses involving non-nested regression models cannot be tested using the classical Neyman-Pearson likelihood ratio approach. Not only are the testing procedures no longer appropriate, but the interpretation of such tests must also be modified. In the classical hypothesis testing situation, when there is sufficient evidence from the data to "reject H_0" the alternative is then accepted as "correct." This conclusion may be invalid in the non-nested case. Here, the alternative model represents the "direction" in which high sensitivity is desired in the test. In other words, the alternative is specified as representative of the type of model (if not a specific model) against which high power of the test is required. The specified alternative is usually another candidate model. However, since the rejection of the maintained hypothesis is not indicative of the alternative's validity, this is not a necessary condition.

Consequently, in order to also test the alternative model as being valid, the hypotheses must be reversed and the test repeated with the alternative as the maintained hypothesis. Therefore, four possible outcomes can be obtained from the pair of tests for a given pair of models:

(1) Accept first model, reject second model;

(2) Reject first model, accept second model;

(3) Reject both models;

or (4) Accept both models.

In the case of (1) or (2), the test has yielded a decisive inference as to which model specification is "correct." However, outcomes (3) and (4) imply inconclusive results. Outcome (3) indicates that neither model is adequate, while outcome (4) may be interpreted as the case in which the data do not provide enough information to distinguish between the two models. In other words, outcome (4) implies that both models perform equally well in terms of their ability to explain the behavior of the response variable. In either of these cases, however, further investigation of alternative models is warranted. Notice, that in order to test k hypothesized models, k(k-1) pairwise tests need to be performed.

From the emphasis given to the possible outcomes for a given pair of tests, it is clear that the purpose of these procedures is to evaluate the validity, or "truth," of the models considered, not just in choosing which better fits the data. If model discrimination was the only concern there would be no need for such hypothesis tests. The necessary decision could be made on the basis of comparisons of R^2, adjusted R^2, MSE, C_p, and other measures of fit. The PRESS statistic would be a useful tool in model selection, particularly if prediction capability is important.

Since these tests are designed for evaluating the validity of the functional form for a given response model in the presence of a specified alternative, it is no surprise that many relevant applications of non-nested model testing deal with economic modeling. Model specification concerns are very important to the economist since theory often indicates that a particular response variable should be a function of certain explanatory variables yet fails to provide meaningful information as to the specific form of this function. In demand analyses involving the general Engel curve, theory designates a small set of functional forms which are all feasible, and for a particular data set for a given response, one of those forms should be close enough to the true underlying relationship to be considered valid. Consequently, without a priori information, a method is needed to judge among the various functional forms.

Applications also exist in other fields. In early work on non-nested hypothesis testing (i.e., Cox, 1962), tests of competing quantal response models were used as examples. Therefore, the usefulness of non-nested hypothesis testing can extend to any field in which competing models arise in the modeling of the behavior of a response variable.

By their very nature, non-nested hypotheses presented a need for new testing procedures. The first work in this area was that of Cox (1960, 1962) who proposed an asymptotic test which was a modification to the classical likelihood ratio test. His test statistic considered the difference between the value of the log-likelihood ratio for the two hypotheses under test and an estimate of the expected value of this log-likelihood ratio under the assumption that the maintained hypothesis, H_0, was true. Pesaran (1974) and Pesaran and Deaton (1978), respectively, formulated the Cox test for testing linear and nonlinear regression models. Since then, other tests have been proposed. It turns out that these tests are either linearized or slightly modified versions of the Cox test. (See Fisher, 1983 and MacKinnon, 1983 for examples of such discussions.) For several of the resulting tests which are asymptotic in nature, small sample corrections have been suggested (Godfrey and Pesaran, 1982, 1983). Although the approaches taken to handle the problem differ, they all rely heavily on the asymptotic properties of maximum likelihood estimation and are thus quite similar, often equivalent at least in large sample. Recently, the use of a parametric and a non-parametric bootstrap approach and an empirical moment generating function approach were applied to the general concept of non-nested hypothesis testing (Aguirre-Torres and Gallant, 1983; Epps et al, 1982; Loh, 1985).

Most of the procedures for testing non-nested hypotheses, non-nested regression models in particular, are asymptotic in nature. Specifically, only two of the more commonly used tests, the Fisher and McAleer JA-test and the Atkinson (NA) test, have exact null distributions. Consequently, a void is created about the usefulness of these tests in practice. In particular, since the sample sizes in many applications are relatively small ($20 \leq n \leq 50$), large sample approximations become questionable. Some Monte Carlo experiments and real-data examples have been performed by many of the originators of these tests, and a summary of these results will be presented in Chapter II. These studies have by no means been comprehensive, and thus there is still much to be learned about the appropriateness of the tests in small samples.

In Chapter II, a formal discussion of the more commonly employed non-nested procedures is presented and includes an examination of the asymptotic distributions of the test statistics, asymptotic power comparisons under local alternatives and equivalencies among the various tests and their underlying approaches. Because of the asymptotic nature of most of these tests, this discussion would not be complete without investigation into the analytic and simulated power comparisons of the tests in the context of linear regression models with nontransformed dependent variables. These past studies bring to light some of the serious flaws in the various procedures. Based on examination of the the apparent advantages as well as flaws of the procedures, a modified version of the Fisher and McAleer JA-test is proposed and its properties examined in Chapter III.

There is still much to be learned about the relative performance of the testing procedures in small sample situations under varying conditions. Therefore, some of the cases warranting investigation are addressed in a Monte Carlo study. In particular, this study examines hypothesis testing situations involving models which are linear in functional form and non-nested only in the choice of regressor variables. Not only is this study concerned with violations in the classical assumption of normal disturbance terms but also with uncovering the usefulness of the test procedures in cases where both models under test are incorrectly specified. Both the layout and results of these experiments are presented in Chapter IV. Comparisons are made on the basis of average estimated power and type I error probabilities as well as a measure of concordance among the tests in replications. From these, some guidelines and warnings for use in practical applications of testing non-nested regression models are formulated. Such rules can be used to guide the researcher to correctly interpret test results under cases involving varying numbers of regressor variables and degrees of collinearity both within and between the models under consideration.

In Chapter V, an empirical study for modelling weekly household food expenditures provides a real data setting involving a large sample of cross-sectional data as well as non-nested functional forms. An examination of several widely endorsed functional specifications of the general Engel curve is made for the purpose of selecting the most appropriate model in analyzing food expenditure patterns. In many demand analyses, choosing the most appropriate specification of the Engel curve as well as evaluating the validity of the model are main concerns of the econometrician. Therefore, this study should provide a good practical example. Other useful aspects of the study are empirical comparisons between the non-nested procedures and the Box-Cox formulation (Box and Cox, 1964), where applicable.

Consequently, there are several phases of this study which have different immediate objectives; the main interest, however, is in pooling together the wealth of information regarding tests of non-nested regression models in such a manner that it is useful to the applied researcher. In particular, discussion of both the analytic and simulated power comparisons of the tests leads to practical warnings about the interpretations of the test results in applications. Since the choice of functional form is important to the economist in particular, the study of both small sample time series data and large sample cross-sectional data provides helpful results. The recommendations for use of the tests as well as topics warranting further study are summarized in Chapter VI. By utilizing this information, the researcher can gain greater confidence in the results obtained from testing hypotheses involving non-nested regression models.

II. Tests of Non-Nested Regression Models

2.1 Approaches to the Non-Nested Problem

Although the classical testing procedures cannot be used to evaluate the "truth" of the maintained model in the non-nested case, the new procedures proposed for testing model specification in terms of functional form align themselves closely with the classical theory. In all cases they represent either modified versions of traditional nested tests or else asymptotically valid applications of these tests under an induced nesting scheme. These methods encompass the two main approaches to testing non-nested models which Fisher (1983) formally termed as the centered log-likelihood ratio (CLR) criterion (also referred to as the modified log-likelihood ratio criterion-MLR) and the artificial nesting (AN) criterion. Strong similarities and, in some cases, equivalencies exist among the tests resulting from these two approaches.

The CLR criterion is credited to Cox (1960,1962); his work marks the origination of non-nested testing procedures. His work addressed the general problem of testing between separate distributional families (i.e., Y has probability density function (pdf) $f_0(y; \underline{\alpha}_0)$ versus Y has pdf $f_a(y; \underline{\alpha}_a)$), of which testing between two non-nested regression models is just one specific form. Cox's approach was to develop a test based on the log-likelihood ratio (llr) between the two hypotheses. However, unlike the usual nested situations, the standard Likelihood Ratio (LR) test is not valid here. LR testing procedures require a distributional assumption to formulate the appropriate likelihood, but in the general case of non-nested hypotheses, it is this distributional assumption that is being tested. Thus Cox derived the asymptotic null distribution of the log-likelihood ratio using the asymptotic properties of Maximum Likelihood estimators (MLE's). The result is a test statistic which is asymptotically distributed as standard normal, based on the comparison of the value of the log-likelihood ratio and its expected value under the maintained hypothesis. Consequently, procedures based on the CLR criterion represent modifications of the classical LR tests, thus also the name modified log-likelihood ratio (MLR).

The alternative non-nested testing approach is rooted in the concept of "artificially" nesting the two models or their likelihoods in some fashion so that nested procedures can then be applied validly (usually, asymptotic validity only). This nesting is accomplished through the use of a mixing parameter, λ, and is commonly formulated as either an exponential combination of the likelihoods or a linear combination of the models (i.e., the pdf's) themselves. If the artificial model is a linear combination of the pdf's, it would be of this form:

$$\underline{y} = (1 - \lambda)f_0(\underline{\alpha}_0) + \lambda f_a(\underline{\alpha}_a). \tag{2.1}$$

However, the resulting artificial model is plagued by an identification problem. In general, λ is not identifiable (unless the values of $\underline{\alpha}_0$ and $\underline{\alpha}_a$ are known a priori) and thus there are too many parameters to be estimated. In other words, the above equation could be estimated although it would not be clear how to "separate" the estimated products of parameters of the form $\lambda\alpha_{ij}$ into the appropriate pieces without having a priori information. Therefore, the parameters are not identifiable. Consequently, the term "artificial" nesting is derived from the replacement of the parameters from the alternative model by estimated values. This replacement circumvents the problem in that λ, although still not identifiable, can be estimated and its value tested. Therefore, the tests derived under the AN approach are asymptotically valid applications of the usual t-test (or LR test) on the value $\lambda = 0$ or 1, which implies the truth of H_0 or H_a, respectively. Different choices of nesting formulation and parameter estimators result in a variety of tests constructed in this manner.

Interestingly enough, Cox (1960) also suggested a simplified approach aimed at handling non-nested regression models. This approach advocates the simple difference between two variance estimates (from the two models) as the basis of the test and therefore can be regarded as a linearization of the Cox CLR criterion. Fisher (1983) demonstrated that the procedures derived under the AN approach are direct realizations of this simplified approach. Consequently, the resulting tests derived under both approaches are closely related. Then, why might the AN procedures be employed in place of the Cox test? Generally, these procedures have test statistics whose values can be read directly from the output of any conventional regression package and under some circumstances yield exact null distributions.

Furthermore, there is another tie which binds the two approaches together philosophically. The concept of nesting the alternative models can also be traced back to the derivations of the test procedures under the Cox approach, as demonstrated by Atkinson (1971). However, a clear distinction exists between the two approaches. Under the CLR criterion, there is no estimation of the mixing parameter as in the AN approach. The value of λ under the maintained hypothesis is assumed ($\lambda = 0$ or 1) and then the centered log-likelihood ratio (cllr) is employed to see if the data provide sufficient evidence to disprove the validity of the maintained model. This procedure is indeed different from the AN approach which estimates the value of λ and then tests to see if the parameter value is either 0 or 1, as indicated by the maintained hypothesis. Therefore, due to the similarity of these approaches, it is understandable that the tests resulting from the two approaches are also quite similar.

Some general comments regarding these two approaches can be made. In general, the AN procedures are advantageous in that they tend to be easier to compute. However, with the software packages available now, this advantage is not particularly salient. Also, the AN approach can lead to the development of exact tests, particularly in the case of two linear regression models. These tests are exact in the sense that their null distribution is known. As will be shown later, however, the performance of these exact tests in terms of power is not necessarily better than that of their asymptotic counterparts based on the CLR approach. Thus, there do not appear to be any concrete reasons at the onset to prefer procedures based on one approach over the other. Therefore, procedures developed under both approaches will be examined in depth.

2.2 An Overview of the Tests for Non-Nested Linear Regression Models

Based on the two approaches a number of tests for non-nested linear regression models have been developed. The Cox test along with its modified versions were derived on the basis of the CLR criterion. On the other hand, the J-test due to Davidson and MacKinnon (1981) and the JA-test due to Fisher and McAleer (1981) are the result of the AN approach, with nesting applied linearly in the models. These tests as well as any small-sample adjustments for testing the following models are presented in Table II.1:

$$H_1:\ \underline{y} = X_1\underline{\beta}_1 + \underline{\varepsilon}_1, \quad \underline{\varepsilon}_1 \sim N(\underline{0}, \sigma_1^2) \tag{2.2}$$

$$H_2:\ \underline{y} = X_2\underline{\beta}_2 + \underline{\varepsilon}_2, \quad \underline{\varepsilon}_2 \sim N(\underline{0}, \sigma_2^2). \tag{2.3}$$

Allowing for the overlap and/or exact collinearities between various regressor variables in the two models, these models can be reexpressed as:

$$H_1:\ \underline{y} = X_1\,\underline{\beta}_1 + \underline{\varepsilon}_1 = X\underline{\beta} + Z_1\,\underline{\gamma}_1 + \underline{\varepsilon}_1$$

$$H_2:\ \underline{y} = X_2\,\underline{\beta}_2 + \underline{\varepsilon}_2 = X\underline{\beta} + Z_2\,\underline{\gamma}_2 + \underline{\varepsilon}_2 \tag{2.4}$$

where X is $n \times k_0$, $\underline{\beta}$ is $k_0 \times 1$, Z_1 is $n \times k_1$, $\underline{\gamma}_1$ is $k_1 \times 1$, Z_2 is $n \times k_1$ and $\underline{\gamma}_2$ is $k_2 \times 1$. In this case, no columns of Z_1 can be obtained as a linear combination of columns of Z_2, and vice-versa. It is necessary to make the distinction between the overlapping portion of the models ($X\underline{\beta}$) and those "separate" pieces ($Z_1\,\underline{\gamma}_1$ and $Z_2\,\underline{\gamma}_2$, respectively) in determining the appropriate form of the testing procedures. Specifically, the overlapping portion is always included as part of the maintained model, with the alternative model being treated as only including the non-nested set of independent or explanatory variables. This approach is a viable means of dealing with the situation, since the results from the testing will then be more conservative in terms of rejecting the null model which is maintained as being correct in the development of the testing procedures. In addition, it is also reasonable from the standpoint that the test of interest is to see which functional form, in the non-nested portions specifically, is best able to explain the response variable's behavior.

For the purposes of testing the model in H_1 maintained against the model in H_2 as in (2.4), the two models actually employed in the procedures are

$$H_1:\ \underset{\sim}{y} = X_1 \underset{\sim}{\beta}_1 + \underset{\sim}{\varepsilon}_1 = X\underset{\sim}{\beta} + Z_1 \underset{\sim}{\gamma}_1 + \underset{\sim}{\varepsilon}_1$$

$$H_2:\ \underset{\sim}{y} = Z_2 \underset{\sim}{\gamma}_2 + \underset{\sim}{\varepsilon}_{Z2}\ , \tag{2.5}$$

where $\underset{\sim}{\varepsilon}_{Z2} = X\underset{\sim}{\beta} + \underset{\sim}{\varepsilon}_2$. The procedures given in Table II.1 are the embodiment of the non-nested approaches for these hypothesized models. However, there is an alternative to using non-nested testing procedures. Prior to Cox's work and its actual application to regression models in the 1970's, a method, the Orthodox F-test, was used which employed the straightforward combination of the hypothesized models and then applied likelihood ratio theory for testing. This Orthodox F-test is first given consideration.

2.2.1 Orthodox F-test

The Orthodox F-test is simply a classical F-test used to judge whether or not a particular subset of the regressor variables in a comprehensive model has coefficients significantly different from zero. The corresponding comprehensive model for the hypothesized models given in (2.4) would be of the form

$$\underset{\sim}{y} = X\underset{\sim}{\beta} + Z_1\underset{\sim}{\gamma}_1 + Z_2\underset{\sim}{\gamma}_2 + \underset{\sim}{\varepsilon}, \tag{2.6}$$

or in compact form,

$$\underset{\sim}{y} = X^*\underset{\sim}{\beta}^* + \underset{\sim}{\varepsilon}.$$

Table II.1: Non-Nested Testing Procedures for Linear Regression Models

Test	Criterion	Test Statistic
Cox (N) test	$T_{12} = \frac{n}{2}\log\left[\frac{\hat{\sigma}^2_{Z2}}{\hat{\sigma}^2_{21}}\right]$	$N_{12} = \dfrac{\frac{n}{2}\log\left[\frac{\hat{\sigma}^2_{Z2}}{\hat{\sigma}^2_{21}}\right]}{\left[\frac{\hat{\sigma}^2_1}{\hat{\sigma}^4_{21}}\, \underline{y}'P_1M_{Z2}M_1M_{Z2}P_1\underline{y}\right]^{1/2}}$
$\bar{N}$-test		$\bar{N}_{12} = \dfrac{1/2\,(n-k_2)\,\log\left[\frac{\bar{\sigma}^2_{Z2}}{\bar{\sigma}^2_{21}}\right]}{\left[\frac{\bar{\sigma}^2_1}{\bar{\sigma}^4_{21}}\,\underline{e}_{211}'\underline{e}_{211} + 1/2\,\bar{\sigma}^2_1\,tr(B^2)\right]^{1/2}}$ where $B = M_{Z2} - P_1M_{Z2}P_1 - \left[\frac{tr(M_1M_{Z2})}{n-k_0+k_1}\right]M_1$ and $\underline{e}_{211} = M_1M_{Z2}P_1\underline{y}$
W-test		$W_{12} = \dfrac{(n-k_2)\left[\bar{\sigma}^2_{Z2} - \bar{\sigma}^2_{21}\right]}{\left[4\,\bar{\sigma}^2_1\underline{e}_{211}'\underline{e}_{211} + 2\,\bar{\sigma}^4_1\,tr(B^2)\right]^{1/2}}$

Table II.1: Non-Nested Testing Procedures for Linear Regression Models (cont'd)

Test	Criterion	Test Statistic
Atkinson's (NA) test	$TA_{12} = \dfrac{-\,y'M_1P_{Z2}P_1y}{\hat{\sigma}^2_{21}}$	$NA_{12} = \dfrac{-\,y'M_1P_{Z2}P_1y}{\left[\hat{\sigma}^2_1\; y'P_1P_{Z2}M_1P_{Z2}P_1y\right]^{1/2}}$
Linearized Cox (NL) test	$TL_{12} = \dfrac{n}{2}\left[\dfrac{\hat{\sigma}^2_{Z2} - \hat{\sigma}^2_{21}}{\hat{\sigma}^2_{21}}\right]$	$NL_{12} = \dfrac{1/2\; y'[P_{Z2} - P_1P_{Z2}P_1]y}{\left[\hat{\sigma}^2_1\; y'P_1P_{Z2}M_1P_{Z2}P_1y\right]^{1/2}}$
J-test	$TJ_{12} = \dfrac{y'M_1P_{Z2}y}{y'P_{Z2}M_1P_{Z2}y}$	$J_{12} = \dfrac{y'M_1P_{Z2}y}{\left[\hat{\sigma}^2_J\; y'P_{Z2}M_1P_{Z2}y\right]^{1/2}}$
AJ-test		$AJ_{12} = \dfrac{y'M_1[P_{Z2}y - p_1M_1y]}{\left[\hat{\sigma}^2_{AJ}[P_{Z2}y - p_1M_1y]'[P_{Z2}y - p_1M_1y]\right]^{1/2}}$
JA-test	$TJA_{12} = \dfrac{y'M_1P_{Z2}P_1y}{y'P_1P_{Z2}M_1P_{Z2}P_1y}$	$JA_{12} = \dfrac{y'M_1P_{Z2}P_1y}{\left[\hat{\sigma}^2_{JA}\; y'P_1P_{Z2}M_1P_{Z2}P_1y\right]^{1/2}}$

Then, in order to test H_1 as the maintained hypothesis against H_2, the corresponding test in terms of the comprehensive model would be H_1: $\underline{\gamma}_2 = \underline{0}$ versus H_2: $\underline{\gamma}_2 \neq \underline{0}$:

$$F_{12} = \frac{(\hat{\underline{\beta}}'^{*} X'^{*} \underline{y} - \hat{\underline{\beta}}'_1 X'_1 \underline{y}) \,/\, k_2}{\underline{y}'(I_n - X^{*}(X'^{*} X^{*})^{-1} X'^{*})\underline{y} \,/\, (n - k_0 - k_1 - k_2)} \overset{H_1}{\sim} F_{(k_2,\, n - k_0 - k_1 - k_2)} \qquad (2.7)$$

where the $\hat{\underline{\beta}}$ are the corresponding MLE's of the $\underline{\beta}$ for the indicated models. If H_1 were rejected, it would imply that at least one regressor variable exclusive to Z_2 was useful in modelling the response y, beyond the modelling capability already provided by those variables in X_1 .

The interpretation of this comprehensive model testing approach is similar to that of the original non-nested hypotheses in that two tests must be performed so that each model is given the role of the maintained hypothesis (i.e., must also test H_0: $\underline{\gamma}_1 = \underline{0}$ versus H_a: $\underline{\gamma}_1 \neq \underline{0}$). However, in using a comprehensive model, the temptation exists of using models which include some variables from each hypothesized model. It would appear that if a mixture of the hypothesized models provided a theoretically feasible solution to the modelling problem at hand, then the intent is truly not testing model specification. Consequently, variable selection criterion would provide more pertinent results.

If, however, theoretical considerations specify the alternatives as separate, then the advantages of using the orthodox F-test are that it is an exact test and it can be easily implemented by researchers in other fields. Its distribution under H_2 as well as under H_1 is exact. One possible drawback is the estimation of what may become a large comprehensive model thwarted by multicollinearity. This problem could have a major impact on the test results manifested as deficiencies in terms of power when compared to its non-nested counterparts. Therefore, it is easy to understand the desire to develop better procedures for testing hypotheses concerning non-nested regression models. The following non-nested tests have more intuitive appeal for testing correct functional form.

2.2.2 The Cox Test and its Small Sample Modifications (N,W,$\tilde{N}$)

The first non-nested procedure was developed by Cox (1960,1962). His approach (CLR) to testing hypotheses involving separate distributional families of hypotheses was based on the centered log-likelihood ratio (cllr): the difference between the maximized value of the log-likelihood and its expected value under the null hypothesis. So, in order to test the following pair of hypotheses:

$$H_0\text{: } Y \text{ has pdf } f_0(y, \underline{\alpha}_0)$$

$$H_a\text{: } Y \text{ has pdf } f_a(y, \underline{\alpha}_a) \tag{2.8}$$

the corresponding cllr is given by:

$$\begin{aligned} T_0 &= [L_0(\hat{\underline{\alpha}}_0) - L_a(\hat{\underline{\alpha}}_a)] - E_0\,[L_0(\hat{\underline{\alpha}}_0) - L_a(\hat{\underline{\alpha}}_a)\,]|_{\underline{\alpha}_0 = \hat{\underline{\alpha}}_0} \\ &= \hat{L}_{a0} - n \underset{n\to\infty}{plim_0}[\frac{\hat{L}_{a0}}{n}]\,|_{\underline{\alpha}_0 = \hat{\underline{\alpha}}_0} \\ &= \hat{L}_{a0} - E_0(\hat{L}_{a0})|_{\underline{\alpha}_0 = \hat{\underline{\alpha}}_0} \end{aligned} \tag{2.9}$$

where $\hat{L}_{a0} = [L_0(\hat{\underline{\alpha}}_0) - L_a(\hat{\underline{\alpha}}_a)]$, $L_j(\hat{\underline{\alpha}}_j)$ denotes the maximized log-likelihood corresponding to H_j, evaluated at the MLE of $\underline{\alpha}_j$, $\hat{\underline{\alpha}}_j$, and $plim_j$ denotes the probability limit under H_j ($j = 0, a$). In the expression for T_0 , MLE's are used in place of unknown parameter values. Then by using the asymptotic properties associated with MLE's, Cox derived the asymptotic null distribution of this statistic. Under the necessary regularity conditions, he showed that

$$T_0 \underset{asympt.}{\overset{H_0}{\sim}} N(0, V_0(T_0)) \tag{2.10}$$

$$where\ \ V_0(T_0) = V_0(L_{a0}) - (1/n)\underline{\eta}\, Q^{-1}\underline{\eta} \tag{2.11}$$

where Q is the asymptotic information matrix associated with $\underline{\alpha}_0$ given by

$$Q = -\underset{n\to\infty}{plim_0}\frac{1}{n}\ \frac{\partial^2(L_0(\underline{\alpha}_0))}{\partial\underline{\alpha}_0\ \partial\underline{\alpha}'_0} \tag{2.12}$$

and

$$\underline{\eta} = n\frac{\partial[\ \underset{n\to\infty}{plim_0}(\hat{L}_{a0}/n)\ ]}{\partial\underline{\alpha}_0} \tag{2.13}$$

Based on this result, a test can be formulated which is an asymptotically valid standard normal test.

It is from this general work that Pesaran (1974) derived the explicit formulation of the test statistic for the case of two linear regression models. In terms of the models given in (2.4), the regularity conditions on the Cox test are (White, 1982):

(a) X_1 and X_2 are non-nested and non-stochastic;

(b) $\lim\limits_{n\to\infty}\frac{1}{n}X'_1X_1 = \Sigma_{11}$, $\lim\limits_{n\to\infty}\frac{1}{n}X'_2X_2 = \Sigma_{22}$,
$\lim\limits_{n\to\infty}\frac{1}{n}Z'_1Z_1 = \Sigma_{Z11}$ and $\lim\limits_{n\to\infty}\frac{1}{n}Z'_2Z_2 = \Sigma_{Z22}$ exist and are non-singular;

(c) $\lim\limits_{n\to\infty}\frac{1}{n}X'_1Z_2 = \Sigma_{1Z2}$ and $\lim\limits_{n\to\infty}\frac{1}{n}Z'_1X_2 = \Sigma_{Z12}$ exist and are non-null.

Based on assumption (c), the Cox test cannot handle the situation where X_1 and X_2 are orthogonal to one another. In other words, if the two models are "completely" separate, this test falls apart.

For the models in (2.4) under the assumption of normality on the disturbance terms, the log-likelihoods are readily obtained

$$L_0(\underline{\alpha}_0) = -\frac{n}{2}\log(2\pi\sigma_1^2) - \frac{1}{2\sigma_1^2}(\underline{y} - X_1\underline{\beta}_1)'(\underline{y} - X_1\underline{\beta}_1) \tag{2.14}$$

where $\underline{\alpha}'_0 = (\underline{\beta}'_1, \sigma_1^2)$ and

$$L_a(\underline{\alpha}_a) = -\frac{n}{2}\log(2\pi\sigma^2_{Z2}) - \frac{1}{2\sigma^2_{Z2}}(\underline{y} - Z_2\underline{\gamma}_2)'(\underline{y} - Z_2\underline{\gamma}_2) \tag{2.15}$$

where $\underline{\alpha}'_a = (\underline{\gamma}'_2, \sigma^2_{Z2})$. Then for these models of interest, the specific expression for the maximized log-likelihood ratio is given by

$$\hat{L}_{a0} = \frac{n}{2}\log[\frac{\hat{\sigma}^2_{Z2}}{\hat{\sigma}^2_1}] \tag{2.16}$$

where $\hat{\sigma}^2_i = \frac{\underline{e}'_i\underline{e}_i}{n}$, the MLE of σ^2_i, and $\underline{e}_i =$ the ML residual vector under H_i using projection matrix $P_i = X_i(X'_iX_i)^{-1}X'_i$ for the full models, and similarly $\hat{\sigma}^2_{Zi} = \frac{\underline{e}'_{Zi}\underline{e}_{Zi}}{n}$, the MLE of σ^2_{Zi}, and $\underline{e}_{Zi} =$ the ML residual vector under H_{Zi} using projection matrix $P_{Zi} = Z_i(Z'_iZ_i)^{-1}Z'_i$ for the partial alternative models (for $i = 1, 2$).

Next, in order to obtain the expression for the centered log-likelihood ratio, the expected value of this ratio under H_1 must be estimated. A necessary piece of the expression is the asymptotic expection of the variance estimate for the alternative (partial) model under the assumption that H_1 is true. Since, $AsyE_1(\hat{\sigma}^2_{Z2}) = \sigma^2_{21} = \sigma^2_1 + \underline{\beta}'_1(\Sigma_{11} - \Sigma_{1Z2}\Sigma^{-1}_{Z22}\Sigma_{Z21})\underline{\beta}_1$, the following expression is derived for the expected value of the llr is obtained

$$n\,\underset{n\to\infty}{plim_0}[\frac{\hat{L}_{a0}}{n}] = \frac{n}{2}\log[\frac{\sigma^2_{21}}{\sigma^2_1}] \tag{2.17}$$

Consequently, by replacing the unknown quantities in (2.17) with their MLE's and combining (2.16) and (2.17), the estimated centered log-likelihood ratio (cllr), or the numerator of the Cox test, can be written

$$T_0 = T_{12} = \frac{n}{2}\log[\frac{\hat{\sigma}^2_{Z2}}{\hat{\sigma}^2_{21}}] \tag{2.18}$$

This cllr measures the difference between the estimated error variance in the alternative model (H_2) and the estimated expected value of that same estimated error variance given that the maintained model (H_1) was indeed true. In other words, the numerator of the Cox test, T_0, measures the validity of the maintained model against the specified alternative on the basis of how well the null model can predict the performance of the alternative, in terms of estimated error variances. In order to evaluate the numerator, the necessary MLE's are given as (for $i = 1, 2$)

$$\hat{\sigma}^2_{21} = \hat{\sigma}^2_1 + (1/n)\hat{\beta}'_1 X'_1 M_{Z2} X_1 \hat{\beta}_1 \ , \tag{2.19}$$

$$\hat{\beta}_i = (X'_i X_i)^{-1} X'_i \underline{y} \ , \tag{2.20}$$

$$M_i = I_n - X_i (X'_i X_i)^{-1} X'_i = I_n - P_i \tag{2.21}$$

and

$$M_{Zi} = I_n - Z_i (Z'_i Z_i)^{-1} Z'_i = I_n - P_{Zi}. \tag{2.22}$$

Similarly, the formulation for $V_1(T_{12})$ was derived. (See Pesaran (1974) for a detailed development.) With unknown quantities replaced with consistent estimates, the estimated variance of T_{12} is given by:

$$\hat{V}_1(T_{12}) = \frac{\hat{\sigma}^2_1}{\hat{\sigma}^4_{21}} \hat{\beta}'_1 X'_1 M_{Z2} M_1 M_{Z2} X_1 \hat{\beta}_1. \tag{2.23}$$

Then, the resulting test for these two linear regression models is given by:

$$\begin{aligned} N_{12} &= \frac{T_{12}}{[\hat{V}_1(T_{12})]^{1/2}} \\ &= \frac{n/2 \log[\hat{\sigma}^2_{Z2}/\hat{\sigma}^2_{21}]}{[\frac{\hat{\sigma}^2_1}{\hat{\sigma}^4_{21}} \hat{\beta}'_1 X'_1 M_{Z2} M_1 M_{Z2} X_1 \hat{\beta}_1]^{1/2}} \end{aligned} \tag{2.24}$$

and under H_1 is asymptotically N(0,1). This test is defined as long as $\lim_{n \to \infty} (X'_1 M_{Z2} M_1 M_{Z2} X_1) \neq 0$. This condition implies that the test is not appropriate for cases where the models are nested or where the two sets of explanatory variables are orthogonal to one another. However, by its nature, the Orthodox F-test is applicable and is exact in both of these cases.

The Cox test is constructed so that a two-sided N(0,1) test can be used to verify the appropriateness of the maintained model. The sign of the test statistic is also informative. Under H_1, $E(T_{12}) < 0$. Therefore, a significant negative N_{12} statistic implies that the null model is not the "truth" and that the true model is in the direction, in some sense, of the specified alternative. In other words, the alternative model performed better (i.e. explained more variation in the dependent variable) than the maintained model projected it would. If the statistic is positively significant, this can be interpreted as the null model once again being incorrect, but that the true model is in a direction opposite to the specified alternative. Similarly, the maintained model's projection of how well the alternative model would perform was not met. In either case, however, it is clear that a significantly nonzero test statistic indicates that the maintained model could not explain that which the alternative could and therefore is not valid.

Cox developed the test such that the alternative model is representative of the type of model against which high power is desired. This alternative model can be thought of as the type of functional form misspecification that the researcher wants to be able to detect with high sensitivity. Once again, it is useful to reiterate that the non-nested procedures are designed to test for correct model specification not to choose the "best" model from a set of candidate models. As Pesaran and Deaton (1978, p. 692) expressed it, "The N (Cox) test is not a measure of the relative fit; it is a measure of whether a given hypothesis can or cannot explain the performance of an alternative hypothesis against the evidence."

It should also be remembered that this test result is asymptotic in nature, so in general its finite sample null distribution is not known. However, Pesaran's (1974) limited simulation work showed that the normal approximation was good even in samples as small as n = 20. Also, its power appears good relative to the F-test, particularly in cases where the correlation between the explanatory variables in the alternative models is strong (i.e., the canonical correlations between X_1 and Z_2) and even in small samples (n = 20,40). However, the estimated probability of making a type I error, or the estimated size of the test, appears to be understatement of the nominal level from the standard normal distribution. This result is due to finite sample bias in the numerator of the test statistic. It was this apparent problem with the Cox test that led Godfrey and Pesaran (1983) to the small sample modified versions of this test: the W and $\tilde{N}$ tests.

Examination of the numerator, T_{12}, of the Cox test sheds much light on the conditions which create a bias large enough to force the test to over-reject a true null hypothesis. Another way to look at the form of T_{12} (2.18) is as follows:

$$T_{12} = (n/2)\log\left[\frac{\underline{e}'_{Z2}\,\underline{e}_{Z2}}{\underline{e}'_1\,\underline{e}_1 + \underline{e}'_{21}\,\underline{e}_{21}}\right] \tag{2.25}$$

where $\underline{e}_{21} = M_{Z2}P_1\underline{y}$, which is the residual vector from the regression of $\hat{\underline{y}}_1$ on Z_2 . $\underline{e}_{21}$ corresponds to the residual vector from the regression of the estimated "true" y's, given H_1 as true, on the non-nested portion of the H_2 model.

Under H_1, it is desired that N_{12} have expectation of zero. In order for this to be the case, $E_1\,[\hat{\sigma}^2_{Z2} / \hat{\sigma}^2_{21}]$ should equal one. Equivalently, $E_1(z_1)$ should be zero where

$$z_1 = \underline{e}'_{Z2}\underline{e}_{Z2} - (\underline{e}'_1\underline{e}_1 + \underline{e}'_{21}\underline{e}_{21}). \tag{2.26}$$

Under H_1, Godfrey and Pesaran (1982) show that:

$$\begin{aligned} E_1(z_1) &= \sigma_1^2[tr(P_{Z2}P_1) - k_2] \\ &= -\sigma_1^2[\sum_{i=1}^{s}(1-\rho_i^2) + \max(k_2 - (k_0 + k_1), 0)] \end{aligned} \tag{2.27}$$

where the ρ_i^2 are the squared canonical correlations given by the $s = \min(k_0 + k_1, k_2)$ non-zero solutions to:

$$|X'_1 Z_2 (Z'_2 Z_2)^{-1} Z'_2 X_1 \ - \ \rho_i^2 X'_1 X_1| = 0. \tag{2.28}$$

From this result, it is clear that the Cox test will tend to over-reject under any of the following situations:

(i) the correlation between the two sets of regressor variables, X_1 and Z_2, is low or moderate (i.e., the ρ_i^2 are small),

(ii) the true model does not fit the data well (i.e., σ_1^2 is large),

(iii) the number of regressor variables in the true model is smaller than the number of regressor variables in the false alternative model.

Based on these, Godfrey and Pesaran derived a modified Cox test based on the unbiased criterion

$$\begin{aligned} \tilde{z}_1 &= z_1 - \tilde{\sigma}_1^2[tr(P_{Z2}P_1) - k_2] \\ &= (n - k_2)(\tilde{\sigma}_2^2 - \tilde{\sigma}_{21}^2) \end{aligned} \tag{2.29}$$

The $\sim$'s indicate unbiased estimates instead of the MLE's. The unbiased estimates referred to here coincide the Ordinary Least Squares (OLS) estimates in the case of linear regression models:

$$\tilde{\sigma}_i^2 = \frac{\underline{y}' M_i \underline{y}}{(n - k_0 - k_i)}$$

$$\tilde{\sigma}_{Zi}^2 = \frac{\underline{y}' M_{Zi} \underline{y}}{(n - k_i)}$$

$$and \ \tilde{\sigma}_{21}^2 = \frac{\underline{e}'_{21}\underline{e}_{21} + \tilde{\sigma}_1^2 \, tr(M_1 M_{Z2})}{(n - k_2)}$$

Based on $\tilde{z}_1$, a test was derived in a fashion similar to the Cox test. Not only was the bias in the numerator eliminated, but also the test was adjusted for a variance estimate of the cllr that was consistent but tended to underestimate the true variance. The test, denoted $\tilde{N}_{12}$, is also an asymptotically standard normal test:

$$\tilde{N}_{12} = \frac{\tilde{T}_{12}}{(\tilde{V}_1(\tilde{T}_{12}))^{1/2}} = \frac{(1/2)(n - k_2)\log(\tilde{\sigma}^2_{Z2}/\tilde{\sigma}^2_{21})}{[(\tilde{\sigma}^2_1/\tilde{\sigma}^4_{21})(\underline{e}'_{211}\,\underline{e}_{211} + (1/2)\tilde{\sigma}^2_1 tr(B^2_{12}))]^{1/2}} \tag{2.30}$$

where $\underline{e}_{211} = M_1 M_{Z2} P_1 \underline{y}$, the residual vector from the regression of $\underline{e}_{21}$ on X_1, and

$$B_{12} = M_{Z2} - P_1 M_{Z2} P_1 - \frac{tr(M_1 M_{Z2})}{(n - k_0 - k_1)} M_1. \tag{2.31}$$

Asymptotically, $\tilde{N}_{12}$ has a N(0,1) distribution under H_1.

The modifications made to the Cox (N_{12}) test statistics involved the use of:

(i) unbiased estimators of σ^2_{Z2} and σ^2_{21} instead of MLE's;

(ii) adjustment of the variance (shifted upward by the magnitude of $(1/2)(\tilde{\sigma}^4_1/\tilde{\sigma}^4_{21})tr(B^2_{12})$);

(iii) $(n - k_2)/2$ in place of $(n/2)$.

Consequently, the asymptotic null distribution of this modified version of the Cox test remains N(0,1). In addition, simulation studies performed by Godfrey and Pesaran show improvements in terms of its observed size (more in line with the nominal level) without any significant reduction in power. At the same time, Godfrey and Pesaran presented a Wald-type test (W-test) that was essentially an adjusted Cox test:

$$W_{12} = \frac{(n - k_2)(\tilde{\sigma}^2_2 - \tilde{\sigma}^2_{21})}{[2\tilde{\sigma}^4_1 tr(B^2_{12}) + 4\tilde{\sigma}^2_1 \underline{e}'_{211}\underline{e}_{211}]^{1/2}} \tag{2.32}$$

This test is based directly on $\tilde{z}_1$ and is similar to a Wald test if we consider $g(\underline{v}) = \sigma_2^2 - \sigma_{21}^2$ and are testing H_1: $g(\underline{v}) = 0$. It too is an asymptotic standard normal test due to the Lindberg-Feller Central Limit Theorem. Also, as in the case of the $\tilde{N}$-test, Godfrey and Pesaran's simulation work shows much promise in the small sample performance of the W-test in terms of power and size for a variety of cases involving unequal numbers of regressors, varying degrees of collinearity between the models, and two skewed non-normal distributions on the disturbance terms.

Therefore, it appears that the CLR criterion provides the basis for some well-behaved non-nested testing procedures, especially when adjustments for finite sample size are used. However, there are two other procedures which are rooted heavily in the original Cox approach. These procedures as well as others which are offshoots of the CLR based tests are the next topic for review.

2.2.3 Atkinson's CLR Test (NA) and the Linearized Cox Test (NL)

Atkinson's (1970) work to develop an alternative procedure to discriminate between two separate families of hypotheses (i.e., pdf's or models) brought into focus a parallel interpretation of the Cox test statistic. His approach is rooted in Cox's suggestion to work from an exponentially combined form of the likelihoods or pdf's. In order to test the pair of hypotheses given in (2.8), or as in Atkinson's intention to discriminate (choose) between the two, the combined pdf would be of the form

$$f_\lambda(y) = \frac{\{f_0(y, \underline{\alpha}_0)\}^{1-\lambda} \; \{f_a(y, \underline{\alpha}_a)\}^{\lambda}}{\int \{f_0(y, \underline{\alpha}_0)\}^{(1-\lambda)} \; \{f_a(y, \underline{\alpha}_a)\}^{(\lambda)} \; \partial\lambda} \tag{2.33}$$

Consequently, the hypothesized model in (2.4) can be reexpressed as testing H_0: $\lambda = 0$ versus H_1: $\lambda = 1$ in the context of the combined pdf. Atkinson's method was to construct an asymptotically normal test statistic formulated as the LR test on λ while the $\underline{\alpha}_0$ and $\underline{\alpha}_a$ were treated as nuisance parameters.

The resulting test statistic can be interpreted as the "derivative of the log-likelihood with respect to the parameter of interest adjusted for regression on the partial derivative with respect to the nuisance parameter, divided by the appropriate standard error" (Atkinson, 1970, p. 332). The adjustment to the derivative is made to guarantee the asymptotic unbiasedness of this statistic no matter the estimate of the nuisance parameters. The work of Bartlett and Neyman led Atkinson to the decision of using the MLE's of the nuisance parameters under the null hypothesis. This application leads to the general expression of TA_0 in a form that is quite similar to the expression for T_0, specifically for the hypotheses given in (2.8):

$$TA_0 = L_0(\hat{\underline{\alpha}}_0) - L_a(\hat{\underline{\alpha}}_{a0}) - E_0[L_0(\hat{\underline{\alpha}}_0) - L_a(\hat{\underline{\alpha}}_{a0})]|_{\underline{\alpha}_0 = \hat{\underline{\alpha}}_0} \tag{2.34}$$

where $\underline{\alpha}_{a0} = \underset{n \to \infty}{plim_0}\, \underline{\alpha}_a$, and correspondingly, $\hat{\underline{\alpha}}_{a0} = \underset{n \to \infty}{plim_0}\, \hat{\underline{\alpha}}_a|_{\underline{\alpha}_0 = \hat{\underline{\alpha}}_0}$. Clearly, the only difference between Cox's CLR and Atkinson's CLR criterion is the evaluation of the entire statistic under the null hypothesis in the case of Atkinson's work. These two statistics are asymptotically equivalent under the null hypothesis and result in asymptotically equivalent tests for the case of linear regression models. However, their finite sample behavior would be expected to be somewhat different. To illustrate, consider the form of this asymptotically normal test statistic for the case of two linear regression models as given in (2.4):

$$NA_{12} = \frac{-\underline{y}'M_1P_{Z2}P_1\underline{y}}{[\hat{\sigma}_2^2\,\underline{y}'P_1P_{Z2}M_1P_{Z2}P_1\underline{y}]^{1/2}} \overset{H_1}{\underset{asympt.}{\sim}} N(0,1). \tag{2.35}$$

Notice the difference between this test statistic and N_{12} as given in (2.24). The numerators are not only different, but also the NA-test uses $\hat{\sigma}^2_{Z2}$ as its error variance estimate while the Cox test employs $\hat{\sigma}_{12}/\hat{\sigma}^4_{21}$ in the estimate of the asymptotic variance of the cllr.

As indicated here, this test, as derived by Atkinson, is asymptotically valid only. However, Fisher (1983) goes on to show that for the case of testing two linear non-nested regression models, the test statistic is distributed as a beta variate with 1/2 and $(n - k_1 - 1)/2$ degrees of freedom. This proof was accomplished by employing the work of Graybill and Milliken (1969, 1970) to express the NA_{12} test statistic as the ratio of a chi-square to the sum of that chi-square and an independent chi-square with 1 and $(n - k_1 - 1)$, respectively.

In order to compare their performance in finite samples, Atkinson performed a series of Monte Carlo studies for the case of testing the exponential distribution against the log normal for sample sizes of n = 20, 50, 100, 150, and 250. From the examination of the moments of the empirical distributions of the statistics, he found that in both cases the approach to the asymptotic normal distribution was rather slow. Generally, on the basis of the first two moments, NA_{12} was preferable; however, large values of the third and fourth moments offset this advantage. It should again be noted that this comparison was for just one case, and that there is still much to be learned about its small sample performance in the case of two non-nested linear regression models.

Along this line, Pereira (1977b) asserted that Atkinson's CLR criterion yielded inconsistent tests in some instances. However, Fisher and McAleer (1981) provided a proof of consistency in the cases of both linear and nonlinear regression models. Therefore, this test is one to be considered, particularly in light of its being less biased in small samples than the unadjusted Cox test with its apparent size larger than the nominal level.

The one remaining testing procedure which is an offspring of the Cox criterion is the linearized Cox test. This statistic, presented by Fisher and McALeer (1981), is simply based on an approximation of T_{12} using a linearized estimate, and is given by

$$TL_{12} = \frac{n}{2}\{\frac{\hat{\sigma}^2_{Z2}}{\hat{\sigma}^2_{21}} - 1\} \tag{2.36}$$

The linearization of the Cox criterion and the numerator of the Wald-type test, W, are indeed similar. Based on this similarity and on the use of unbiased estimates in the W-test, it is reasonable to suspect that the linearized Cox test's performance relative to the W-test would be similar to that of the unadjusted Cox test (N) and the $\tilde{N}$-test. Based on TL_{12}, a consistent, asymptotically normal test statistic is obtained for the linear models given in (2.4) and is given by

$$NL_{12} = \frac{1/2\underset{\sim}{y}'\{P_{Z2} - P_1 P_{Z2} P_1\}\underset{\sim}{y}}{\{\hat{\sigma}_1^2\, \underset{\sim}{y}' P_1 P_{Z2} M_1 P_{Z2} P_1 \underset{\sim}{y}\}^{1/2}} \tag{2.37}$$

Although the behavior of this statistic previously has not been studied in the case of finite samples, several relationships can be formed regarding these three versions of the Cox CLR test. Fisher (1983) noted that the numerators of the unadjusted tests were related in the following manner:

$$TA_{12} \geq TL_{12} \geq T_{12} \tag{2.38}$$

This relationship provides some insight as to the linearized test being more conservative than the other tests under certain conditions. If the alternative model fits much better than it should, the unadjusted Cox test is more likely to reject the null hypothesis than the NL test. On the other hand, if the alternative model is fitting much worse than it should, Atkinson's version is more likely to reject the null hypothesis. Consequently, under these conditions, NL may accept the null hypothesis, while the others may reject. Therefore, it may be reasonable in practical applications to compute and compare the results of all three test statistics.

These tests are asymptotically equivalent under the null hypothesis. However, under the alternative hypothesis, the relationship among them is unknown. A hint of their relative performance can be seen in this expression which follows from (2.38):

$$\underset{n\to\infty}{plim_2} \frac{TA_{12}}{n} \geq \underset{n\to\infty}{plim_2} \frac{TL_{12}}{n} \geq \underset{n\to\infty}{plim_2} \frac{T_{12}}{n} \tag{2.39}$$

Without information on the behavior of the variance estimates in the denominators of the test statistics under the alternative hypothesis, no concrete information regarding their relative power is obtained, even asymptotically. Consequently, in order to judge these three versions of the Cox test as to their practical usefulness, Monte Carlo studies for finite samples need to be engaged. However, it is expected that the Atkinson version will have a more reasonable size than the unadjusted

Cox test. Also, the relative ease in computing the linearized Cox test, once the necessary regressions have been performed, make it worth further investigation.

In conclusion, the Cox CLR-based tests are rooted in the classical likelihood ratio theory and provide a feasible approach to testing non-nested hypotheses. All the tests are essentially testing to see if $\hat{\sigma}^2_{22}/\hat{\sigma}^2_{21} = 1$. The bottom line, in other words, is the investigation of the ability of the maintained model to predict the fitting ability of the alternative model through examining the ratio of two error variance estimates. However, under the Cox CLR approach, this comparison is accomplished through testing whether or not the residuals from the regression of X_1 on $\underset{\sim}{y}$ are asymptotically uncorrelated with the difference between the fitted values from the two alternative models (MacKinnon, 1983). Although indeed a reasonable approach, it would appear simpler to test the relationship directly. According to MacKinnon, this approach is indeed the intention of the testing procedures derived under the Artificial Nesting approach.

2.2.4 Tests Derived Under the AN Approach (J,AJ,JA)

In general, the AN approach involves the formulation of a nested model from the two or more individual models under investigation. If the artificially nested model is properly constructed, then the resulting test is simply a traditional test of hypothesis (classical LR test) on the appropriate parameter, or set of parameters. However, the artificial nesting procedure often leads to the situation where not all of the parameters to be estimated are identifiable. Thus the various AN tests were developed through the application of various means of circumventing the identification problem. Two such tests are the J- and JA-tests which are asymptotically equivalent.

To see how these tests are constructed for the case of two non-nested linear equations, consider the exponentially weighted combination of the models in (2.4) using ξ as a mixing parameter

$$y = (1 - \xi)\frac{\sigma^2}{\sigma_1^2}X_1\beta_1 + \xi\frac{\sigma^2}{\sigma_{Z2}^2}Z_2\gamma_2 + \varepsilon \tag{2.40}$$

where $Var(\varepsilon) = \sigma^2 I_n$ for $\sigma^{-2} = (1 - \xi)\sigma_1^{-2} + \xi\sigma_{Z2}^{-2}$. Then, in the univariate case, this can be transformed into what appears to be a straightforward linear combinatiion of the deterministic portions of the models, by substituting λ for $\xi\sigma^2/\sigma_{Z2}^2$:

$$y = (1 - \lambda)X_1\beta_1 + \lambda Z_2\gamma_2 + \varepsilon \tag{2.41}$$

Then corresponding to the testing of H_1 as the maintained hypothesis against H_2 in (2.4) is the equivalent testing of H_1: $\lambda = 0$ against H_2: $\lambda \neq 0$ in the above artificially formulated model. It is evident that this model cannot be estimated directly, since λ is not identifiable as long as the values of the parameter vectors β_1 and γ_2 are unknown. Consequently, if the test to be performed is on the value of λ from the combined model, a way to force λ to be estimable is necessary. The J and JA tests accomplish this through the replacement of $Z_2\gamma_2$ (from the alternative hypothesized model) with a consistent estimate. Since different consistent estimators are used in these two tests, they may be asymptotically equivalent although not equivalent in small samples. Importantly, the use of any consistent estimator for $Z_2\gamma_2$ would yield an alternative asymptotically valid test. However, the two tests discussed here are two of the more reasonable members of a much larger class of AN testing procedures.

Davidson and MacKinnon (1981) proposed the use of $\hat{y}_2 = P_{Z2}y$, the predicted value of $Z_2\gamma_2 = E_2(y)$ under the alternative hypothesis, an OLS/ML (consistent) estimator. Correspondingly, to test H_1 against H_2 in (2.4), the following model is estimated and a test on the significance of λ is used to make the inference regarding the maintained model:

$$y = (1 - \lambda)X_1\beta_1 + \lambda\hat{y}_2 + \varepsilon \tag{2.42}$$

From this estimated model, the t-test on the significance of the value of λ is of the form:

$$J_{12} = \frac{\hat{\underline{\gamma}}'_2 Z'_2 M_1 \underline{y}}{(\hat{\sigma}_J^2 \hat{\underline{\gamma}}'_2 Z'_2 M_1 Z_2 \hat{\underline{\gamma}}_2)^{1/2}} \underset{asympt.}{\overset{H_1}{\sim}} N(0,1), \tag{2.43}$$

where $\hat{\sigma}_J^2$ is the variance estimate based on the SSE from the regression in (2.42) and has $(n - k_0 - k_1 - 1)$ df.

This test is very easily computed using a regression procedure such as PROC REG in SAS. Since it only requires the estimation of four regressions for testing any given pair of hypothesized models, it can be implemented readily. (See section 2.4.) However, the J-test, similar to the unadjusted Cox test, is biased in the numerator under the maintained hypothesis. Consequently, it too has the tendency to over reject a true null model, particularly when the number of regressors in a false alternative is larger than that in the true null.

Once again, such a problem was investigated by Godfrey and Pesaran (1982) in order to derive a similar adjustment for small sample bias in the numerator as employed in the case of the Cox test. The adjusted J-test, or AJ-test, maintained its nature of being a t-test on the value of λ with the adjustment applied to the estimator of the predicted value from the alternative model. Unfortunately, the Monte-Carlo work of Godrey and Pesaran showed only minimal improvement over the J-test and, in some cases, it performed with less satisfaction in terms of power. This result was the case even though its observed size was brought closer to the nominal level.

The J_{12} test statistic was derived asymptotically through the limiting distribution of $n^{-1/2}\underline{e}'_1\hat{\underline{y}}_2$ (Godfrey and Pesaran, 1982). The bias in the numerator of the test statistic comes from its expectation under H_1 which is of the form:

$$E_1(n^{-1/2}\underline{e}'_1\hat{\underline{y}}_2) = n^{-1/2}\sigma_1^2[\sum_{i=1}^{s}(1-\rho_i^2) + \max(k_2 - (k_0 + k_1), 0)] \tag{2.44}$$

The bias is similar to that in the cllr of the Cox test (2.27). Godfrey and Pesaran asserted that given the use of the appropriate adjustment in the model to be estimated, namely the use of $\hat{\underline{y}}_2 - p_{12}\underline{e}_1$ in place of $\hat{y}_2$ in the combined model (2.41), the resulting test would have expectation zero under the maintained hypothesis, H_1. Specifically, the AJ-test is based on the estimation of the following model with the resulting t-statistic being an asymptotically valid test on the significance of λ:

$$\underline{y} = (1 - \lambda)X_1\underline{\beta}_1 + \lambda(\hat{\underline{y}}_2 - p_{12}\underline{e}_1) + \underline{\varepsilon} \tag{2.45}$$

$$where \quad p_{12} = \frac{k_2 - tr(P_1 P_{Z2})}{n - k_0 - k_1} \tag{2.46}$$

An equally popular AN testing procedure is the JA-test proposed by Fisher and McAleer (1981). This procedure is identical to the unadjusted J-test except for the consistent estimator used for $Z_2\underline{\gamma}_2$. Their approach reflects Atkinson's since the estimator used is one that estimates $E_1(\hat{\underline{y}}_2)$ (which is essentially, for the case at hand, $\underset{n\to\infty}{plim_1}[\hat{\underline{y}}_{Z2}]$) and is given by $\hat{\underline{y}}_{21} = P_{Z2}P_1\underline{y}$. The corresponding t-test on λ for the estimation of (2.41) using this new consistent estimator is the JA-test, which is appealing in that its null distribution is an exact t-distribution; i.e.,

$$JA_{12} = \frac{\hat{\underline{y}}'_{21}M_1\underline{y}}{(\hat{\sigma}^2_{JA}\ \hat{\underline{y}}'_{21}M_1\hat{\underline{y}}_{21})^{1/2}} \overset{H_1}{\sim} t_{(n - k_0 - k_1 - 1)} \tag{2.47}$$

However, the JA-test is not exact under the alternative hypothesis. Even though the nominal size of the test should be maintained, no information regarding its relative power under the alternative is gained analytically. From the studies of Godfrey and Pesaran (1983), it is evident that the nominal size of the JA-test appears to actually be, in general, an over-statement of the "true" probability of making a type I error. In addition, the test lacks power to make the correct inference in the case where the number of regressors in the true null hypothesized model is larger than that in the false alternative model. This last result seems counter-intuitive since it would be expected that given two models which fit the given data set relatively well the procedures would tend to "lean" toward the model with the larger number of parameters. However, this emphasizes the difference between the J-test and the JA-test (and in a similar fashion, the N-test and the NA-test) in terms of the degree of conservativeness in the projected variance estimate from the alternative model.

Like the J-test, the JA-test has inherent problems when the number of regressors in the competing models are not equal. One definitive advantage to using these AN tests is their ready extension to hypothesized models involving different transformations on the dependent variable. Such procedures will be discussed within the context of the empirical study in Chapter V. Although these procedures are generally simpler to compute, they are not necessarily better. Therefore, further investigation of these tests derived under the AN approach as well as those derived under the CLR approach is warranted.

For completeness, the general form and properties of the class of AN procedures should be discussed. Pesaran (1982b) investigated this class whose member tests are based on the substitution of any linear $\tilde{\underline{\gamma}}_2$ into (2.42) which are of the form $R\underline{y}$ where R is a $k_2 \times n$ matrix satisfying the following conditions:

R.1 $\underset{n\to\infty}{plim_1}(R\underline{\varepsilon}_1) = \underset{n\to\infty}{plim_2}(R\underline{\varepsilon}_{Z2}) = \underline{0}$;

R.2 $\lim_{n\to\infty}(RX_1) = D_1$, where D_1 is a finite, non-zero matrix;

R.3 $\lim_{n\to\infty}(RZ_2) = D_{Z2}$, where D_{Z2} is a $k_2 \times k_2$ positive semi-definite matrix and $D_{Z2}\underline{\gamma}_2 \neq \underline{0}$.

If conditions R.1 and R.2 are met by a given R, then the resulting t-test on λ from the combined model in (2.42) is an asymptotically valid standard normal test under the maintained hypothesis and is of the form:

$$t_\lambda(R) = \frac{\underline{y}'R'Z'_2M_1\underline{y}}{[\hat{\sigma}^2_R(\underline{y}'R'Z'_2M_1Z_2R\underline{y})]^{1/2}} \tag{2.48}$$

where

$$\hat{\sigma}^2_R = \frac{1}{n - k_0 - k_1 - 1}[\underline{y}'M_1\underline{y} - \frac{(\underline{y}'M_1Z_2R\underline{y})^2}{\underline{y}'R'Z'_2M_1Z_2R\underline{y}}] \tag{2.49}$$

If in addition to conditions R.1 and R.2, condition R.3 is met by a given $\tilde{\gamma}_2$, then the resulting test of H_1 maintained against H_2 is consistent. From this general framework, it should be clear that for the J-test, $R = (Z'_2Z_2)^{-1}Z'_2$ and for the JA-test, $R = (Z'_2Z_2)^{-1}Z'_2P_1$. Although there are a variety of tests which could be formed in the manner discussed here, the J- and JA-tests are both intuitively appealing and more powerful (as will be shown in section 2.3.1) in the presence of certain alternative models.

With the groundwork for the various non-nested testing procedures now laid, it is important to see how they actually perform relative to one another. Therefore, comparisons among these testing procedures, both asymptotically and in finite samples, will be made.

2.3 Comparisons: Analytic and Simulated

The fundamental aspects of the developments of the various non-nested testing procedures for the case of two linear regression models have been presented in the previous section along with some of their properties. The purpose of this section is to provide comparable information about the various tests in terms of their distributional properties and actual performance. First, consideration is given to asymptotic properties and other analytic comparisons which can be made. Then statements regarding the relative performance of the tests as witnessed in past simulation studies will be noted.

2.3.1 Analytic Comparisons

Since most of the testing procedures are only asymptotically valid, analytic comparisons are primarily limited to asymptotic properties. To start, all the test statistics derived under the two non-nested approaches are asymptotically distributed standard normal under the maintained hypothesis (excluding the Orthodox F-test). All these tests are consistent, with correct size asymptotically. As noted, the JA- and F-tests are the only two test statistics possessing exact finite-sample null distributions. In addition, through investigation of regularity conditions (White, 1983), it is clear that the tests remain valid asymptotically in the presence of non-normal disturbances.

The Cox procedures (N, NA and NL) are asymptotically equivalent under the maintained hypothesis, although the only information regarding their behavior under the alternative was provided by Fisher (1983) and given in (2.39). But as previously indicated, the estimated variances are not the same for these three testing procedures and therefore no concrete information is gained regarding their relative asymptotic power. In terms of finite samples, Pereira (1977b) showed that the Atkinson procedure was in general less biased in the first and second moments of the test statistic than the Cox procedure under the maintained hypothesis. However, the Cox procedure demonstrated closer agreement with the limiting distribution in the higher moments measuring skewness and kurtosis. (It is interesting to note at this point that Pereira stated that the Cox test was still practical since corrections could more readily be made to correct for the bias in the first and second moments, which is precisely what Godfrey and Pesaran did in the development of the W- and $\tilde{N}$-tests.)

Also, by construction, any AN testing procedure, specifically the J- and JA-tests, are asymptotically equivalent under the null hypothesis. This equivalency relies solely on the consistency of the estimate of $Z_2 \underline{\gamma}_2$ employed. It appears that the respective tests are essentially equivalent on the basis of limiting behavior under the maintained hypothesis. However, since the power of the tests when the alternative is true is important to the researcher, it would be useful to compare the power under the limiting distributions in some manner.

Since the limiting distributions of the various test statistics are not available under the alternative hypothesis, except for the case of the Orthodox F-test, power comparisons are not directly obtainable. Consequently, Pesaran (1983) investigated the Cox, Orthodox F- and J-tests on the basis of power under local alternatives. Such power is a means of comparing the asymptotic efficiency of the procedures. Given the null model H_1 as in (2.4), local alternatives are defined as a sequence of alternatives H_{2n} which approach H_1 as n approaches infinity. Specifically, Pesaran defined them by

$$H_{2n}: \underline{y} = X_1 B \underline{\gamma}_2 + n^{-1/2} \Delta \underline{\gamma}_2 + o(n^{-1/2}) \underline{1} + \underline{\varepsilon}_{Z2} \qquad (2.50)$$

where $o(.)$ denotes the small order relation, $\underline{1}$ is a $n \times 1$ vector of 1's and B and Δ are $(k_0 + k_1) \times k_2$ and $n \times k_2$ nonzero matrices of constants, with the restriction on Δ such that

$$\lim_{n \to \infty} \frac{\Delta' M_1 \Delta}{n} = W_2 \qquad (2.51)$$

exists and is nonzero. These limiting requirements on Δ guarantee that as $n \to \infty$, $H_{2n} \to H_1$ at a rate so that the asymptotic power will be strictly larger than the probability of a type I error while it is bounded away from one.

Under this structure of local alternatives, Pesaran derived the following results regarding the asymptotic power of the testing procedures. Specifically, for the Orthodox F-test, its asymptotic power under local alternatives, denoted by P_F, is given by

$$\begin{aligned} P_F &= \lim_{n \to \infty} Pr[F_{12n} \geq \chi^2_{(k_2),1-\alpha} | H_{2n}] \\ &= Pr[\chi^{2\prime}_{(k_2),\eta_F} \geq \chi^2_{(k_2),1-\alpha}] \end{aligned} \qquad (2.52)$$

where $\eta_F = \underline{\gamma}'_2 W_2 \underline{\gamma}_2 / \sigma^2_2$. For the square of the unadjusted Cox test, N^2_{12n} its limiting power under local alternatives is given by

$$\begin{aligned} P_N &= \lim_{n \to \infty} Pr[N_{12n2} \geq \chi^2_{(1),1-\alpha} | H_{2n}] \\ &= Pr[\chi^{2\prime}_{(1),\eta_N} \geq \chi^2_{(1),1-\alpha}] \end{aligned} \qquad (2.53)$$

where $\sigma^2_2 \eta_N = \underline{\gamma}'_2 W_2 \underline{\gamma}_2$. Similarly, the limiting power of the square of the J-test, J^2_{12n}, under local alternatives is the same as that for N^2_{12n}. Consequently, the Cox N-test and the J-test are asymptotically equivalent under both the maintained hypothesis as well as under local alternatives.

Therefore, $P_N = P_J$ can be compared to P_F since both are based on the noncentral χ^2-distributions with the same non-centrality parameters ($\eta_F = \eta_N = \eta_J$) but different degrees of freedom (df). On the basis of work by Das Gupta and Perlman(1974), Pesaran stated that the power function of such noncentral χ^2 tests is strictly decreasing in df. Therefore, it is concluded that

$$P_N \;=\; P_J \;\geq\; P_F \tag{2.54}$$

with equality holding only when the number of non-overlapping variables between the models, k_2 is one. Correspondingly, as the number of non-overlapping variables increases, the more powerful the non-nested procedures become compared to the Orthodox F-test, at least in large samples.

Now consideration is given to the family of AN procedures addressed by Pesaran (1982b) based on the class of consistent linear estimators of the form $\tilde{\underline{\gamma}}_2 = R\underline{y}$ meeting the conditions specified by R.1-R.3 discussed in Section 2.2.4. Under local alternatives as defined in (2.47), this family of t-tests has asymptotic distributions given by

$$t_{\lambda}^2(R) \;\underset{asympt.}{\overset{H_{2n}}{\sim}}\; \chi^{2\prime}_{(1),\,\eta^2(R)} \tag{2.55}$$

where

$$\eta^2(R) \;=\; \frac{[\underline{\gamma}'_2 D_{Z2}' W_2 \underline{\gamma}_2]^2}{\sigma_2^2\, \underline{\gamma}_2' D_{Z2}' W_2 D_{Z2} \underline{\gamma}_2} \tag{2.56}$$

From this general form of the asymptotic power function of this family of AN procedures under local alternatives, the power can be maximized in terms of maximizing the value of the non-centrality parameter based on values of R. Pesaran showed that maximum asymptotic local power is achieved for this class of tests when $\eta^2(R) = \underline{\gamma}_2' W_2 \underline{\gamma}_2 / \sigma_2^2$. In addition, Pesaran (1982b) proved that both the J- and JA-tests meet this requirement for achieving maximum asymptotic power under local alternatives, and therefore the procedures are not only asymptotically equivalent under the null hypothesis but also under local alternatives.

From these results, it would seem reasonable that the non-nested procedures would be expected to have higher power for making the correct inference on the basis of a pair of competing models than the Orthodox F-test, at least asymptotically. This result will not necessarily hold though, depending on the form of the alternative model since the comparisons made here deal only with local alternatives. To evaluate the finite sample behavior of these testing procedures, Monte Carlo studies must be considered.

2.3.2 Simulated Comparisons

There have been several Monte Carlo studies conducted to evaluate the relative performance of the non-nested testing procedure in finite samples. The most comprehensive of these studies was that performed by Godfrey and Pesaran (1983) in the context of examining the behavior of the small-sample modifications to the Cox test in the case of two linear regression models. They controlled for characteristics of the competing models such as the number of regressor variables in the competing models, the amount of collinearity between the models, the quality of fit of the true model, presence of a lagged dependent variable and non-normal disturbances (namely, χ^2 and log-normal). From these, useful information regarding the testing procedures is obtained. More limited studies were conducted by Atkinson (1971), Pesaran (1974, 1982a), Davidson and MacKinnon (1983) and Sawyer (1983).

The two main criteria for evaluating the performance of these tests are the estimated type I error probability (size of the test) and estimated power. Here the power of the test is really related to the pair of tests on each pair of competing models and is the probability of rejecting the false null when it is maintained and accepting the true null when it is maintained. In other words, it measures the ability of the test to lead the researcher to the correct inference regarding a pair of models.

Since the F-test and the JA-test are both exact under the null, it is expected that the estimated type I error probability ($\hat{\alpha}$) will align itself closely with the nominal level. When the true model is maintained as the null hypothesis, $\hat{\alpha}$ is the proportion of times the true null is incorrectly rejected. For the F-test, the nominal and observed levels appear to be in agreement. However, in the case of the JA-test, the observed size tends to be somewhat less than the designated significance level, but not extremely so. This result may be indicative of how the conservative estimate of the error variance for the alternative model actually works in practice. However, it is important to remember that all the results discussed here are restricted to a limited number of model conditions. Specifically, it is when the maintained model has a smaller number of regressors than the alternative that the $\hat{\alpha}$ tends to be smaller than the nominal level in the case of the JA-test.

The bias in the N- and J-tests (in regards to expectation of the test statistic under the maintained hypothesis) manifests itself in observed type I error probabilities which exceed the nominal level. Based on large sample approximations, a test on the true finite sample size of these tests will often reject the hypothesis that the type I error probability equals the nominal level. In terms of the other two CLR tests, the NA- and NL-tests, not much is known based on simulation. However, Atkinson (1971) showed through simulated testing between the log-normal and exponential distributional assumptions that the NA procedure was indeed less biased in terms of the first and second moments than the unadjusted Cox procedure.

When small-sample bias adjustments as suggested by Godfrey and Pesaran (1983) are applied to the Cox (N) and J-tests, the observed type I error probability is more in line with the nominal level of the test based on the asymptotic distribution. Particularly, $\hat{\alpha}$ is better behaved in the case of the W- and $\tilde{N}$-tests. Taking into account the range of the observed significance levels of the tests under the maintained hypothesis, the ability of the testing procedures to lead to a correct inference regarding a given pair of models is examined.

In general, (based primarily on the results of Godfrey and Pesaran, 1982 and 1983), the power of the unadjusted Cox test as well as the W- and $\tilde{N}$-tests are fairly large. For most cases, the power of the Orthodox F-test, which was adversely affected by increased amounts of collinearity between the models, tended to be less than that of any of the three Cox procedures. Unfortunately, no information about the NA- and NL-tests was compiled in the case of two linear regression models. Supplementary to this study is information regarding the reinforcement of the trends in terms of asymptotic power under local alternatives between the N- and F-tests in finite samples (Pesaran, 1983). Also, in this study, it becomes evident that the size and power of the Cox N-test, although far off in samples of size 20, rapidly approach their asymptotic levels.

Turning to the AN procedures, the J- and AJ-test tended to have reasonably large power. One difficulty with the J-test as indicated from this study is its tendency to over-reject the true null when the number of regressors in the false alternative is larger than in the null. Consequently, guarded use of the J-test is advisable in practice, even though its power is fairly good. On the other hand, its adjusted version, AJ, has a more reasonable $\hat{\alpha}$ but at the expense of reduced power in some cases (Godfrey and Pesaran, 1982).

The JA-test, however, with its correct size also had problems in terms of power when there was an unequal number of regressors in the competing models. Overall, the power of the JA-test tended to be lower than the power of the other procedures. This result was emphasized in the cases where the false null model with $k_0 + k_1$ paramters was maintained against the true alternative model with $k_2 > k_0 + k_1$ parameters. Consequently, the J-test and the JA-test have problems when the number of regressors are unequal in the models, although their difficulties pull in opposite directions.

Therefore, it would appear on the basis of the information presented here that the adjusted Cox tests (W, $\tilde{N}$) and the F-test tend to be more reasonable for use in practice based on the designated evaluation criteria. Again, it should be reemphasized that the statements presented here indicate observed trends based on results compiled under a limited set of model conditions. Also, not all the procedures were used in the studies, so the information is not complete. Finally, in all the cases studied, one of the two competing models was the correct model, although this may not be the case in practice. Even though these summaries are informative, they do not provide the researcher with the full picture. There is still much to be learned about the practical use of non-nested hypothesis testing procedures for linear regression models.

2.4 Computational Information

Since most of the testing procedures for non-nested regression models are asymptotic in nature and their performance characteristics can be quite variable depending on the condition of the models under test, some consideration should be given to the ease, or lack thereof, in which they can be computed. By their nature, the AN test statistics can be calculated quite readily, without any additional computations, within the framework of regression packages such as PROC REG in SAS. Other test statistics, particularly those employing modifications for finite sample biases, need some additional work to obtain the necessary formulation. In this section, the basic steps for computing each of the test statistics as well as associated observed significance levels (p-values) will be outlined.

Table II.4 contains a brief list of steps for testing the pair of models in (2.4); i.e., to test H_1 maintained against H_2 as well as H_2 maintained against H_1 in order to test model validity. Some gain in terms of reduction in the number of regressions can be achieved when there is no overlapping portion between the models. Generally, if X were null in (2.4), then only one regression would be required in place of every pair of "full" and "partial" (non-nested piece only) regressions indicated.

By examining the information given in Table II.4, several points become obvious. First, small sample adjustments to the Cox and J-tests require substantially more effort to obtain the value of the test statistic. In cases where the data set is not particularly small ($30 \leq n \leq 40$), the additional matrix manipulations may not be very manageable in terms of computing time and storage allocation. However, this condition does not pose a problem for genuinely small data sets since a package with matrix operations could handle the computations. Particularly, PROC MATRIX (soon to be replaced with PROC IML) can perform the additional calculations (e.g., B and $tr[B^2]$) without any difficulty. Also, FORTRAN matrix operators could be programmed to do the necessary operations. Consequently, it may require some added effort, but the obstacles encountered in the computation of the finite sample adjusted test statistics are not insurmountable.

Second, the Cox-derived (asymptotic) procedures and the Orthodox F-test require a fair number of regressions as well as some further manipulation. These are by no means as complicated as the matrix manipulations discussed above. In fact, they involve scalar calculations which could even be done by hand. The scalar components of the test statistics can be stored from output of the formal regression packages and then a short series of statements programmed to put them together properly. Once the test statistics themselves have been computed for any of the procedures addressed previously, associated p-values are easy to obtain using the probability functions in SAS.

Finally, the J- and JA-tests can be computed through the performance of four and six regressions, respectively, within the framework of the regression packages. Since the tests are really t-tests regarding the significance of one of the regressor coefficients, standard output in regression estimation packages; even the p-values associated with the hypothesis tests can be read directly from the PROC REG output. Therefore, from the layman's perspective, these tests are very appealing given their relatively easy usage.

A macro in PROC MATRIX (SAS) to compute this series of test statistics for a pair of linear regression models with non-transformed dependent variables is provided in Appendix D. For alternative programming of these testing procedures for use in larger samples, refer to the section in Chapter V wherein the calculation of the non-nested hypothesis tests is discussed.

Table II.4 Computational Outline

Test	Number of Regressions	Additional Calculations?		Basic Steps in Computation
		Scalar	Matrix	
(1) Cox Test (N) (2.24)	8	Yes	No	1. Regress X_1 and X_2 on $\underline{y}$ and retain SSE_1, SSE_2, $\hat{\underline{y}}_1$ and $\hat{\underline{y}}_2$; 2. Regress Z_1 and Z_2 on $\underline{y}$ and retain SSE_{Z1} and SSE_{Z2}; 3. Regress Z_1 on $\hat{\underline{y}}_2$ and Z_2 on $\hat{\underline{y}}_1$ and retain SEE_{12}, SEE_{21} and residual vectors $\underline{e}_{12}$ and $\underline{e}_{21}$, respectively; 4. Regress X_1 on $\underline{e}_{21}$ and X_2 on $\underline{e}_{12}$ and retain SSE_{211} and SSE_{112}; 5. Compute: $N_{ij} = \dfrac{(n/2)\ \log\ [SSE_{zj}/SSE_{ji}]}{[n\,SSE_i(SSE_{jii}/SSE_{ji}^2)]^{1/2}}.$
(2) $\bar{N}$ -Test (2.30)	8	Yes	Yes	1-4. as in Cox (N) test; 5. Compute $tr\,(B_{12}^2)\ tr\,(B_{21}^2)$, (see(2.31)), $tr\,(M_1M_{z2})$ and $tr\,(M_2M_{zi})$; 6. Compute $\bar{\sigma}_{21}^2$ and $\bar{\sigma}_{12}^2$ using $\hat{\sigma}_{ji}^2 = [SSE_{ji} + MSE_i\ tr\,(M_jM_{zi})]/(n - k_j)$; 7. Compute: $\bar{N}_{ij} = \dfrac{[(n - k_j)/2]\ log\ [MSE_{zj}/\bar{\sigma}_{ji}^2]}{\left[\frac{MSE_i}{\bar{\sigma}_{ji}^4}\left(SSE_{jii} + \frac{MSE_i}{2}\ tr\,(B_{ij}^2)\right)\right]^{1/2}}.$

Table II.4 Computational Outline

Test	Number of Regressions	Additional Calculations? Scalar	Matrix	Basic Steps in Computation
(3) W-Test (2.32)	8	Yes	Yes	1-6. in $\hat{N} - test$; 7. Compute: $W_{ij} = \dfrac{(n - k_j)(MSE_j - \hat{\sigma}^2_{ji})}{[2(MSE_i^2)\, tr\,(B_{ij}^2) + 4(MSE_i)SSE_{jii}]^{1/2}}$
(4) Atkinson's (NA) Test (2.35)	6	Yes	Yes	1. Regress X_1 and X_2 on $\underset{\sim}{y}$ and retain SSE_1, SSE_2, $\hat{\underset{\sim}{y}}_1$, $\hat{\underset{\sim}{y}}_2$, $\underset{\sim}{e}_1$ and $\underset{\sim}{e}_2$; 2. Regress Z_1 on $\hat{\underset{\sim}{y}}_2$ and Z_2 on $\hat{\underset{\sim}{y}}_1$ and retain $\hat{\underset{\sim}{y}}_{12}$ and $\hat{\underset{\sim}{y}}_{21}$, respectively; 3. Regress X_1 on $\hat{\underset{\sim}{y}}_{21}$ and X_2 on $\hat{\underset{\sim}{y}}_{12}$ and retain SSE^R_{211} and SSE^R_{IDD} ; 4. Compute $num_{12} = \underset{\sim}{e}'_1\hat{\underset{\sim}{y}}_{21}$ and $num_{21} = \underset{\sim}{e}'_2\hat{\underset{\sim}{y}}_{12}$; 5. Compute: $NA_{ij} = \dfrac{-num_{ij}}{\left[\left(\frac{SSE_i}{n}\right)SSE^R_{jii}\right]^{1/2}}$.

Table II.4 Computational Outline

Test	Number of Regressions	Additional Calculations? Scalar	Matrix	Basic Steps in Computation
(5) Linearized Cox (NL) Test (2.37)	8	Yes	No	1. Regress X_1 and X_2 on y and retain SSE_1, SSE_2, $\hat{y}_1$ and $\hat{y}_2$; 2. Regress Z_1 and Z_2 on y and retain SSE_{Z1}, SSE_{Z2}; $SSReg_{Z1}$ and $SSReg_{Z2}$; 3. Regress Z_1 on $\hat{y}_2$ and Z_2 on $\hat{y}_1$ and retain $\hat{y}_{12}$, $\hat{y}_{21}$, $SSReg_{12}$ and $SSReg_{21}$; 4. Regress X_1 on $\hat{y}_{21}$ and X_2 on $\hat{y}_{12}$ and retain SSE^R_{211} and SSE^R_{122}; 5. Compute: $NL_{ij} = \dfrac{(1/2)[SSReg_{zj} - SSReg_{ji}]}{\left[\left(\frac{SSE_i}{n}\right)SSE^R_{jii}\right]^{1/2}}$

Table II.4 Computational Outline

Test	Number of Regressions	Additional Calculations? Scalar	Matrix	Basic Steps in Computation
(6) J-Test (2.43)	4	No	No	1. Regress Z_1 and Z_2 on y and retain $\hat{y}_{Z1}$ and $\hat{y}_{Z2}$; 2. Regress $[X_1 \mid \hat{y}_{Z2}]$ and $[X_2 \mid \hat{y}_{Z1}]$ on y and J_{ij} = t-statistic on the coefficient of $\hat{y}_{zj}$.
(7) AJ-Test (2.45-2.46)	6	Yes	Yes	1. Regress Z_1 and Z_2 on y and retain $\hat{y}_{z1}$ and $\hat{y}_{z2}$; 2. Regress X_1 and X_2 on y and retain e_1 and e_2; 3. Compute $tr(P_1 P_{z2})$ and $tr(P_2 P_{z1})$; 4. Construct vectors $\hat{y}_{2p1}$ and $\hat{y}_{1p2}$ of the form: $\hat{y}_{jpi} = \hat{y}^2_{zj} - \dfrac{k_j - tr(P_i P_{zj})}{n - k_0 - k_i} e_i$ 5. Regress $[X_1 \mid \hat{y}_{2p1}]$ and $[X_2 \mid \hat{y}_{1p2}]$ on y and AJ_{ij} = t-statistic on the coefficient of $\hat{y}_{jpi}$.

Table II.4 Computational Outline

Test	Number of Regressions	Additional Calculations? Scalar	Matrix	Basic Steps in Computation
(8) JA-Test (2.47)	6	No	No	1. Regress X_1 and X_2 on y and retain $\hat{y}_1$ and $\hat{y}_2$; 2. Regress Z_1 on $\hat{y}_2$ and Z_2 on $\hat{y}_1$ and retain $\hat{y}_{12}$ and $\hat{y}_{21}$, respectively; 3. Regress $[X_1 \| \hat{y}_{21}]$ and $[X_2 \| \hat{y}_{12}]$ on y and JA_{ij} = t-statistic on the coefficient of $\hat{y}_{ji}$.
(9) Orthodox-Test (2.7)	3	Yes	No	1. Regress $X^* = [X \| \tilde{Z}_1 \| Z_2]$ on y and retain SSE; 2. Repress X_1 and X_2 on y and retain SSE_1 and SSE_2; 3. Compute: $F_{ij} = \dfrac{(SSE_i - SSE)/k_j}{SSE/(n - k_0 - k_i - k_j)}$.

III. The NJ-Test: A Modified JA-Test

3.1 Introduction and Motivation

From the dicsussion of the various testing procedures derived under the two non-nested approaches (CLR and AN), it is clear that one of the two tests having a known, finite-sample null distribution - namely Fisher and McAleer's JA-test - has some appealing characteristics. Having an exact null distribution is in itself an advantage of this test over the others since it insures the correct size of the test (i.e., the nominal significance level of the test is maintained). Also, the test statistic is unbiased under the null hypothesis. By contrast, the simulated results, both in past studies and that in Chapter IV, showed that maintaining the nominal size was difficult for the asymptotic tests, except where tedious small-sample modifications were employed. In addition, the computation process for obtaining the JA-test involves only six regressions and can be read directly from the computer output of any standard regression package. Correct size and ease of computation are both sound reasons for promoting the use of the JA-test in practice.

However, there is another side to the story. As discussed briefly by Godfrey and Pesaran (1983), both of the AN procedures have problems in terms of power when the number of regressor variables in the competing models is unequal. The power referred to here is the ability of making the correct inference from the pair of tests performed on a given pair of competing models. In particular, the JA-test with its "conservative" estimate of the fit from the alternative model tends to favor the null too much when it has fewer parameters. In other words, when a false null model with k_1 ($k_0 = 0$) parameters is maintained against the true model with $k_2 > k_1$ parameters, it seems to be fairly difficult to reject using the JA-test relative to the other tests. Although this conclusion is based on observed behavior under a limited set of model conditions, the pattern appeared to be consistent. Consequently, Godfrey and Pesaran dropped the AN procedures from further Monte-Carlo investigation.

The concern with the JA-test then is its reduced power stemming from the derivation of the entire test statistic, even the estimates from the alternative model, under the maintained hypothesis. Similar to Atkinson's test in this approach, the question becomes, is this "conservative edge" really a drawback, or can it be used to the researcher's advantage. Too much power of the test in rejecting the null hypothesis (i.e., each individual test of hypothesis, not each pair) may be dangerous when there are multiple hypothesized models to be considered. In such a case, the researcher would be lucky if at least one of the models was close enough to the true underlying relationship to be considered valid. Therefore, the issue becomes "How much power is good?" when considering the possibility of both models under test being false. This issue is addressed in depth in the Monte-Carlo study discussed in Chapter IV. But for immediate purposes, it raises some doubt in the use of this power argument as a means of dismissing the practical use of a test such as the JA-test. If there were a means to modify the JA-test to improve the power in these circumstances while retaining its exact null distributional properties, a useful testing procedure may possibly result.

On a parallel note, the properties of the Orthodox F-test are also worthy of attention. As it is argued by some, the Orthodox F-test is really not a non-nested hypothesis testing procedure since its formulation of a comprehensive model is not based on the nesting schemes suggested by Cox. However, it can be used to provide meaningful information regarding the appropriateness of a given model formulation. Specifically, it has an exact known distribution under both the maintained and alternative hypothesis. Because of this property, power in terms of the ability of the test to reject the false null model given that the alternative is true (i.e., one half of the inference process) can be investigated analytically. This F-test is indeed the only test which provides such information. However, possible problems with the power of the test caused by strong collinearity between the non-nested sets of regressors cannot be ignored.

Both of these testing procedures, the JA-test and the Orthodox F-test, have their strengths and weaknesses. Therefore, the proposed modification to the JA-test involves the incorporation of the more appealing aspects of the F-test into the JA-test formulation. Thus the resulting test statistic represents a compromise between the two testing procedures.

3.2 Proposed Modification and its Impact on the Test

The proposed modification to the JA-test is the substitution of the estimated error variance from the artificially nested model,

$$\underline{y} = X_1 \underline{\beta}_1^* + \lambda \hat{\underline{y}}_{21} + \underline{\varepsilon}^*, \tag{3.1}$$

with that from the comprehensive model approach,

$$\underline{y} = X_1 \underline{\beta}_1 + Z_2 \underline{\gamma}_2 + \underline{\varepsilon}, \tag{3.2}$$

in the JA-test statistic. This new JA-test, named NJ, for testing H_1 against H_2 (2.4) is of the form:

$$\begin{aligned} NJ_{12} &= JA_{12} \left[\frac{\hat{\sigma}_{JA}}{\hat{\sigma}_F}\right] \\ &= \frac{\underline{y}'M_1P_{Z2}P_1\underline{y}}{\{(\underline{y}'P_1P_{Z2}M_1P_{Z2}P_1\underline{y})[\underline{y}'M\underline{y}/(n - k_0 - k_1 - k_2)]\}^{1/2}} \end{aligned} \tag{3.3}$$

In this expression $\underline{y}'M\underline{y}$ denotes the unrestricted error sum of squares (SSE) from the regression of the set of regressors $[X|Z_1|Z_2]$ on $\underline{y}$.

What is the motivation for making such an adjustment, and what has been gained, if anything, in terms of actual test performance? To see the possible advantages, examination of the properties of such a test is warranted. In particular, of utmost importance is the finite sample distributional properties of the test statistic under both the maintained and alternative hypotheses. It will be helpful to also examine properties on the basis of asymptotic theory in order to make analytic comparisons between the NJ-test and the remaining non-nested testing procedures which lack known finite sample behavior. However, before the actual distributional nature of the NJ-test is addressed, some background on the distributional properties of the JA-test will be presented.

3.2.1 The Distribution of the JA-test Under the Maintained Hypothesis

The JA-test was derived by Fisher and McAleer (1981) as an asymptotically valid (standard normal) test for hypotheses involving two non-nested regression models. Pesaran (1982b) was the first to realize that the resulting test statistic followed an exact t-distribution under the maintained hypothesis when it was a linear regression model. This result is based on the application of Graybill and Milliken's work (1969,1970) regarding the distribution of quadratic forms involving idempotent matrices which contain random elements. The results used in the proof of the exact distribution of the JA test statistic under H_1 are based on Theorems 3.1 and 3.2 in Graybill and Milliken (1969). These are given in Appendix A.1.

Because this theory will be used in the development of the exact distribution of the NJ test statistic, it is worthwhile to take an in-depth look at its application to the JA-test. The following development was presented by Fisher (1983) to show that the square of the JA test statistic follows a central F-distribution with $(1, n - k_0 - k_1' - 1)$ degrees of freedom under the maintained hypothesis H_1 . This result will also bring to light one motivating factor for making the proposed modification. Consider the form of JA_{12}^2:

$$JA_{12}^2 = \frac{[\underset{\sim}{y}' M_1 P_{Z2} P_1 \underset{\sim}{y}]^2}{\hat{\sigma}_{JA}^2 \{\underset{\sim}{y}' P_1 P_{Z2} M_1 P_{Z2} P_1 \underset{\sim}{y}\}}, \tag{3.4}$$

in which

$$\hat{\sigma}^2{}_{JA} = \frac{\underset{\sim}{y}' M_1^* \underset{\sim}{y}}{n - k_0 - k_1 - 1}. \tag{3.5}$$

For computing the SSE for $\hat{\sigma}_{JA}^2$, the matrix of regressors $X_1^* = [X | Z_1 | P_{Z2} P_1 \underset{\sim}{y}]$ is regressed against the vector of dependent variable observations $\underset{\sim}{y}$. Since the set of regressor variables contains the consistent estimator of $Z_2 \underset{\sim}{\gamma}_2$ under expectation from H_1, namely, $P_{Z2} P_1 \underset{\sim}{y}$, it is clear that the quadratic forms for SSE and regression sums of squares (SSR) under the JA artificial model involve matrices containing random elements. All testing procedures constructed under the general AN framework have the same difficulty which stems from the use of consistent estimators of the form $R\underset{\sim}{y}$. (For additional remarks concerning the general AN family, see Section (3.4).) Consequently, it is necessary to apply the theory of Graybill and Milliken (1969) to obtain the distributional properties of the resulting test statistic. The JA_{12}^2 test statistic can be reexpressed in the following form:

$$JA_{12}^2 = \frac{\underset{\sim}{y}'[P_0 - P_1]\underset{\sim}{y}}{\{\underset{\sim}{y}'[I_n - P_0]\underset{\sim}{y}/(n - k_0 - k_1 - 1)\}} \tag{3.6}$$

where P_0 is the orthogonal projection onto a subspace, ω_0 , (of the space spanned by $X = [X|Z_1|Z_2] = [X_1|Z_2]$), which is defined to be the direct sum of ω_1 (span of X_1) and the span of $P_{Z2}P_1\underset{\sim}{y}$. Fisher represents this subspace as

$$\omega_0 = \omega_1 \oplus S[P_{Z2}P_1\underset{\sim}{y}] = \omega_1 \oplus S[M_1P_{Z2}P_1\underset{\sim}{y}], \tag{3.7}$$

since M_1 is orthogonal to the projection matrix, P_1 , onto ω_1.

From this, an additional subspace, ω_3, is considered which is defined to contain all scalar multiples of $M_1P_{Z2}P_1\underset{\sim}{y}$. Then, since ω_1 and ω_3 are othogonal to one another, the projection matrix, P_3 onto ω_3, is equivalent to $[P_0 - P_1]$. In particular, P_3 representing the orthogonal projection onto ω_3 can be expressed as :

$$\begin{aligned} P_3 = P_0 - P_1 &= M_1P_{Z2}P_1\underset{\sim}{y}[\underset{\sim}{y}'P_1P_{Z2}M_1P_{Z2}P_1\underset{\sim}{y}]^{-1}\underset{\sim}{y}'P_1P_{Z2}M_1 \\ &= \underset{\sim}{q}[\underset{\sim}{q}'\underset{\sim}{q}]^{-1}\underset{\sim}{q}' \end{aligned} \tag{3.8}$$

where $\underset{\sim}{q} = M_1P_{Z2}P_1\underset{\sim}{y}$.

Consequently, both the numerator and denominator of the JA-test are quadratic forms containing random elements in their designated projection matrices. Therefore, the distributions of both of the quadratic forms can be shown to be central χ^2 variates under H_1 through the application of Graybill and Milliken's Theorem 3.1 (hereafter referred to as Theorem A.3.1). The theorem is relevant since the elements of both $P_3 = P_0 - P_1$ and $I_n - P_0$ are Borel functions of the random vector $P_1\underset{\sim}{y} = K\underset{\sim}{y}$. Also there exists a constant matrix $L = M_1 = I_n - P_1$ whose rows are contained in the orthogonal complement of the row space of K (i.e., $LK = 0$) such that all the necessary conditions are met by both quadratic forms, i.e.,

$$(1)\ [P_0 - P_1] = M_1[P_0 - P_1]M_1 \qquad [I_n - P_0] = M_1[I_n - P_0]M_1;$$

$$(2)\ [P_0 - P_1] = [P_0 - P_1]^2 \qquad [I_n - P_0] = [I_n - P_0]^2; \tag{3.9}$$

$$(3)\ tr[P_0 - P_1] = 1 \qquad tr[I_n - P_0] = n - k_0 - k_1 - 1;$$

and

$$(4)\ \underset{\sim}{\beta}'_1X'_1[P_0 - P_1]X_1\underset{\sim}{\beta}_1 = 0 \qquad \underset{\sim}{\beta}'_1X'_1[I_n - P_0]X_1\underset{\sim}{\beta}_1 = 0.$$

Then, invoking Theorem A.3.1 from Graybill and Milliken, the following hold:

$$\frac{\underset{\sim}{y}'[P_0 - P_1]\underset{\sim}{y}}{\sigma_1^2} \overset{H_1}{\sim} \chi^2_{(1)} \tag{3.10}$$

$$\frac{\underset{\sim}{y}'[I_n - P_0]\underset{\sim}{y}}{\sigma_1^2} \overset{H_1}{\sim} \chi^2_{(n-k_0-k_1-1)}. \tag{3.11}$$

In addition, since $[P_0 - P_1][I_n - P_0] = 0$, the numerator and denominator quadratic forms of the JA-test are independent by Theorem 3.2 of Graybill and Milliken (hereafter referred to as Theorem A.3.2). Therefore,

$$JA_{12}^2 \overset{H_1}{\sim} F_{(1,n-k_0-k_1-1)}, \tag{3.12}$$

or equivalently,

$$JA_{12} \overset{H_1}{\sim} t_{(n-k_0-k_1-1)}.$$

However, the exact nature of the distribution of the JA test statistic does not hold under the alternative hypothesis, H_2 . In particular, $\mu'_2[I_n - P_0]\mu_2 \neq 0$, or any other constant under H_2. Therefore, even if an exact distribution holds under H_2 for the numerator and the denominator quadratic forms, the denominator would be noncentral in nature. At a minimum, an alternative estimator of the error variance is needed if any exact distributional properties are to be obtained under H_2 using this approach. The use of the comprehensive model approach may be advantageous here. Therefore, a closer look at these alternative variance estimators would be beneficial.

3.2.2 The Error Variance Estimators

In the general hypothesis testing framework, when a hypothesis H_1 is maintained against hypothesis H_2, the test statistic is (constructed using information) based on the assumption that the null hypothesis H_1 is true. In the situation at hand, the choice of the error variance estimator must necessarily give consideration to the maintained hypothesis. An estimator which is unbiased for σ_1^2 when expectation is taken under H_1 would be ideal from this standpoint. For both of the estimators, $\hat{\sigma}_{JA}^2$ and $\hat{\sigma}_F^2$, this property holds. For the case of the JA-test, this result follows directly from (3.11). This result is readily observable for the Orthodox F-test, where the error variance estimator is based on the unrestricted SSE from the comprehensive model in (3.2). In fact,

$$\hat{\sigma}_F^2 \overset{H_1}{\sim} \frac{\sigma_1^2 \chi^2_{(n - k_0 - k_1 - k_2)}}{(n - k_0 - k_1 - k_2)}. \tag{3.13}$$

Given this information, either estimator is reasonable for use in the testing procedure. However, it is interesting to compare the variance associated with the two estimators as a basis to compare their null behavior. Since both are unbiased, the variance is equivalent to the Mean Squared Error (MSE) of the estimator. For the JA derived estimator,

$$MSE_1[\hat{\sigma}_{JA}^2] = Var_1[\hat{\sigma}_{JA}^2] = \frac{2\sigma_1^4}{(n - k_0 - k_1 - 1)}, \tag{3.14}$$

whereas for the comprehensive approach,

$$MSE_1[\hat{\sigma}_F^2] = Var_1[\hat{\sigma}_F^2] = \frac{2\sigma_1^4}{(n - k_0 - k_1 - k_2)}. \tag{3.15}$$

This result implies that under H_1, $MSE_1[\hat{\sigma}_F^2] \geq MSE_1[\hat{\sigma}_{JA}^2]$, and that as the number of non-overlapping regressor variables in the alternative hypothesized model increases, the disparity between them becomes larger. Therefore, under H_1, both estimators provide unbiased estimates of the error variance with the JA derived estimator having smaller variance in general.

Given these comparisons under H_1, attention now centers on the behavior of these variance estimators under the alternative hypothesis, H_2. As indicated previously, Graybill and Milliken's work will not be applicable in an unconditional sense since $\underline{\mu}'_2[I_n - P_0]\underline{\mu}_2 \neq \lambda$, a constant. In particular,

$$\begin{aligned} \underline{\mu}'_2[I_n - P_0]\underline{\mu}_2 &= \underline{\beta}'_2 X'_2[I_n - P_0]X_2\underline{\beta}_2 \\ &= \underline{\beta}'_2 X'_2\{M_1 - \frac{M_1 P_{Z2} P_1 \underline{y}\, \underline{y}' P_1 P_{Z2} M_1}{\underline{y}' P_1 P_{Z2} M_1 P_{Z2} P_1 \underline{y}}\}X_2\underline{\beta}_2 \end{aligned} \tag{3.16}$$

The second form of the SSE is a direct application of Pesaran's derivation for the general family of AN procedures given in (2.49). This estimator does not follow a χ^2 distribution, central or otherwise, under the alternative hypothesis.

On the other hand, the comprehensive model approach yields an estimator for the true error variance under both the maintained and alternative hypotheses, and more specifically,

$$\hat{\sigma}_F^2 \overset{H_2}{\sim} \frac{\sigma_2^2 \chi^2_{(n - k_0 - k_1 - k_2)}}{(n - k_0 - k_1 - k_2)}. \tag{3.17}$$

Then two arguments for making the proposed modification have been presented:

(1) $\hat{\sigma}^2_F$ is unbiased for the true error variance under both H_1 and H_2;

(2) $\hat{\sigma}^2_F$ has an exact central χ^2 distribution under both H_1 and H_2.

These arguments also provide information about the behavior of the denominator quadratic form of the NJ-test but not about the distributional properties of the test statistic itself.

3.2.3 The Distribution of the NJ-test

Recall that the JA-test followed an exact distribution under H_1 and that

$$NJ^2_{12} = JA^2_{12}\left[\frac{\hat{\sigma}^2_{JA}}{\hat{\sigma}^2_F}\right]$$

Therefore, since $\hat{\sigma}^2_F$ follows a central χ^2 distribution under both H_1 and H_2, all that is necessary to formulate the exact distribution of the NJ^2_{12} test statistic is to show that $\underline{y}'[P_0 - P_1]\underline{y}$ and $\underline{y}'M\underline{y}$ are independent. By Graybill and Milliken's Theorem A.3.2, this is easily shown since the matrix $M = I_n - X^*(X^{*\prime}X^*)^{-1}X^{*\prime}$ can be considered a constant Borel function of $P_1\underline{y}$. Then since $L'ML = M'_1MM_1 = M$, $M = M^2$, $tr(M) = n - k_0 - k_1 - k_2$ and $\mu'_2M\mu_2 = 0$, the only condition left to be shown is that $[P_0 - P_1]M = 0$. This is indeed the case since by (3.8) the product can be examined in the following manner:

$$\begin{aligned}
[P_0 - P_1]M &= \underline{q}[\underline{q}'\underline{q}]^{-1}\underline{y}'P_1P_{Z2}M_1M \\
&= \underline{q}[\underline{q}'\underline{q}]^{-1}\underline{y}'P_1[P_{Z2} - P_{Z2}P_1][I_n - P] \\
&= \underline{q}[\underline{q}'\underline{q}]^{-1}\underline{y}'P_1[P_{Z2} - P_{Z2}P_1 - P_{Z2}P + P_{Z2}P_1P] \\
&= \underline{q}[\underline{q}'\underline{q}]^{-1}\underline{y}'P_1[P_{Z2} - P_{Z2}P_1 - P_{Z2} + P_{Z2}P_1]
\end{aligned}$$

since $P = X^*[X^{*\prime}X^*]^{-1}X^{*\prime}$ and $X^* = [X_1 | Z_2]$. Then,

$$[P_0 - P_1]M = \underline{q}[\underline{q}'\underline{q}]^{-1}\underline{y}'P_1 0 = 0.$$

Therefore, since $\underset{\sim}{y}'[P_0 - P_1]\underset{\sim}{y}$ and $\underset{\sim}{y}'M\underset{\sim}{y}$ are independent, central χ^2 random variables, then

$$NJ_{12}^2 \overset{H_1}{\sim} F_{(1,\, n-k_0-k_1-k_2)}, \tag{3.18}$$

or equivalently,

$$NJ_{12} \overset{H_1}{\sim} t_{(n-k_0-k_1-k_2)}. \tag{3.19}$$

As indicated previously, the exactness of the distribution of the test statistic under H_1 is valuable since it maintains the nominal size of the test. However, in order to evaluate the power, in some sense, of this non-nested testing procedure, the distribution of the NJ_{12}^2 test statistic under the alternative hypothesis is needed. Under H_2, $\underset{\sim}{y}'[P_0 - P_1]\underset{\sim}{y}$ and $\underset{\sim}{y}'M\underset{\sim}{y}$ remain independent of one another and $\hat{\sigma}_F^2$ still follows a central χ^2 distribution. However, the issue of the distribution of the numerator quadratic form under H_2 is still unresolved. To examine its distribution, it is first necessary to make an amendment to the Graybill and Milliken Theorem 3.1 and the proof thereof.

As evidenced by its application to the JA-test, Theorem A.3.1 provides a means to obtain an unconditional χ^2 distribution for quadratic forms based on matrices with random elements. However, this result is rooted in the fact that the conditional distribution of a quadratic form $\underset{\sim}{y}'A\underset{\sim}{y} \mid K\underset{\sim}{y} = (K\underset{\sim}{y})^*$ is the same non-central chi-square for all possible values of $K\underset{\sim}{y} = (K\underset{\sim}{y})^*$. (See Appendix A.1). The necessary condition for this unconditional distribution is that $\underset{\sim}{\mu}'A\underset{\sim}{\mu} = \lambda$, where λ is a constant (and the resulting noncentrality parameter is $\lambda/2$).

If condition (4) does not hold with probability one, but the remaining conditions of Theorem A.3.1 do, then the distribution of the quadratic form $\underset{\sim}{y}'A\underset{\sim}{y}$ would still be non-central chi-square but one which is conditional on $K\underset{\sim}{y}$. In addition, the value of the noncentrality parameter is a random variable conditional on $K\underset{\sim}{y}$. Consequently, a somewhat weaker distributional result is obtained.

By applying this result to the distributional development of $\underset{\sim}{y}'[P_0 - P_1]\underset{\sim}{y}$ under H_2, the following theorem can be stated for the statistic of the NJ-test:

Theorem 1. For testing H_1 maintained against H_2 as defined in (2.4),

$$H_1:\ \underset{\sim}{y} = X_1 \underset{\sim}{\beta}_1 + \underset{\sim}{\varepsilon}_1 = X\underset{\sim}{\beta} + Z_1 \underset{\sim}{\gamma}_1 + \underset{\sim}{\varepsilon}_1$$

$$H_2:\ \underset{\sim}{y} = X_2 \underset{\sim}{\beta}_2 + \underset{\sim}{\varepsilon}_2 = X\underset{\sim}{\beta} + Z_2 \underset{\sim}{\gamma}_2 + \underset{\sim}{\varepsilon}_2$$

the distribution of the NJ test statistic under the alternative hypothesis, H_2, conditional on $P_1\underset{\sim}{y} = \hat{\underset{\sim}{y}}_1$ is

$$NJ_{12}^2 \mid_{P_1\underset{\sim}{y} = \zeta} \overset{H_2}{\sim} F'_{(1,\, n - k_0 - k_1 - k_2),\, \lambda_{NJ}} \tag{3.20}$$

where

$$\lambda_{NJ} = \frac{1}{2\sigma_2^2} \frac{\{\underset{\sim}{\beta}'_2 X_2 M_1 P_{Z2} P_1 \underset{\sim}{y}\}^2}{[\underset{\sim}{y}' P_1 P_{Z2} M_1 P_{Z2} P_1 \underset{\sim}{y}]}. \tag{3.21}$$

Proof: Under H_2, $\underset{\sim}{y}'[P_0 - P_1]\underset{\sim}{y}$ are independent with $\underset{\sim}{y}'M\underset{\sim}{y} \overset{H_2}{\sim} \sigma_2^2 \chi^2_{(n - k_0 - k_1 - k_2)}$. The amendment to Theorem A.3.1 of Graybill and Milliken (1969) can be applied to $\underset{\sim}{y}'[P_0 - P_1]\underset{\sim}{y}$ under the assumption that H_2: $\underset{\sim}{y} = X_2\underset{\sim}{\beta}_2 + \underset{\sim}{\varepsilon}_2$ is the true model and since conditions (1)-(3) hold with probability one. This allows for the distribution of $\underset{\sim}{y}'[P_0 - P_1]\underset{\sim}{y}$ conditional on $P_1\underset{\sim}{y} = \zeta$ to be $\chi^{2}_{(1)}{}'$ with the noncentrality parameter conditioned on $P_1\underset{\sim}{y}$ given to be:

$$\begin{aligned}\lambda_{NJ} &= \frac{1}{\sigma_2^2} \underset{\sim}{\mu}'_2 [P_0 - P_1] \underset{\sim}{\mu}_2 \\ &= \frac{1}{\sigma_2^2} \frac{\{\underset{\sim}{\beta}'_2 X'_2 M_1 P_{Z2} P_1 \underset{\sim}{y}\}\, \{\underset{\sim}{y}' P_1 P_{Z2} M_1 X_2 \underset{\sim}{\beta}_2\}}{\{\underset{\sim}{y}' P_1 P_{Z2} M_1 P_{Z2} P_1 \underset{\sim}{y}\}}\end{aligned}$$

Consequently, the random variable NJ_{12}^2 conditional on $P_1\underset{\sim}{y} = \zeta$ follows a non-central F distribution with $(1, n - k_0 - k_1 - k_2)$ degrees of freedom and noncentrality parameter, λ_{NJ}, given in (3.21).

Through Theorem 1 and earlier discussion, distributional information about the NJ test statistic has been obtained. At first glance, the distribution of NJ_{12}^2 under H_2 being exact only when conditional on $P_1\underset{\sim}{y} = \zeta$ does not appear to be a very meaningful result. However, from a practical point of view, the motivation for examining the non-null behavior of the test statistic is to gain information about the power of the testing procedure. In this sense, it is not particularly detrimental to this power issue to examine properties conditional on $\hat{\underset{\sim}{y}}_1$, the fitted values of $\underset{\sim}{y}$ under H_1, the maintained hypothesis. In any real applications of this procedure for non-nested linear regression models, the fitted values are readily available, and can be considered "fixed" for a given set of data. Therefore, it is not unreasonable to examine the power based on these observed values. An analogy can be made to the case where σ^2 is a nuisance parameter in the test on the population mean μ of a normal distribution, although an unconditional result was obtainable in this particular situation when s^2 was used to estimate the unknown population variance.

Although the noncentrality parameter of the numerator $\chi^{2\prime}$ is a random variable itself, general comments concerning the non-null behavior of the test statistic can be made by investigating the distribution based on the noncentrality parameter evaluated at $\underset{\sim}{y} = E_2(\underset{\sim}{y}) = X_2\underset{\sim}{\beta}_2$. This substitution by no means yields the expectation of the noncentrality parameter under H_2, but it can provide a basis from which to gain insight about the power of the testing procedure. Utilizing this substitution,

$$\begin{aligned}\bar{\lambda}_{NJ} &= \lambda_{NJ}\big|_{\underset{\sim}{y} = E_2(\underset{\sim}{y})} \\ &= \frac{\{\underset{\sim}{\beta}'_2 X'_2 M_1 P_{Z2} P_1 X_2 \underset{\sim}{\beta}_2\}^2}{\underset{\sim}{\beta}'_2 X'_2 P_1 P_{Z2} M_1 P_{Z2} P_1 X_2 \underset{\sim}{\beta}_2}\ .\end{aligned} \tag{3.22}$$

By making use of the conditional distribution under H_2, without the above simplification, the power of the NJ-test can be discussed in the following manner:

$$\begin{aligned} Power_{NJ} &= P[\text{ Reject false } H_1 \text{ maintained against true } H_2] \\ &= P[NJ_{12}^2 > F_{(1,n-k_0-k_1-k_2),1-\alpha}] \qquad (3.23) \\ &= P[F'_{(1,n-k_0-k_1-k_2),\lambda_{NJ}} > F_{(1,n-k_0-k_1-k_2),1-\alpha}]. \end{aligned}$$

This expression is not the power associated with making the correct inference regarding a pair of tests on a given pair of hypothesized models, H_1 and H_2. This definition of power is concerned only with the probability that the non-nested testing procedure leads to a correct rejection of a false null model maintained against the true model. Therefore, it relates to only one half of the actual inferential process for a given pair of hypothesized models.

Since the Orthodox F-test based on the comprehensive model approach is the only other testing procedure which provides an exact non-null distribution of its corresponding test statistic (under H_2), it is the only non-nested test to which analytic power comparisons can be made from the NJ-test on a finite sample basis. The only realm on which to make comparisons between the performance of the NJ-test and the remaining asymptotically valid tests is in the context of large samples. Therefore, power comparisons must be made on the basis of asymptotic power under local alternatives.

3.2.4 Asymptotic Power Under Local Alternatives for the NJ-test

As presented in section (2.3), Pesaran (1983) defined a sequence of so called local alternatives H_{2n}, given in (2.50), which approach the maintained model H_1, given in (2.4), as n approaches infinity; i.e.,

$$H_{2n}: \underline{y} = X_1 B \underline{\gamma}_2 + n^{-1/2}\Delta\underline{\gamma}_2 + o(n^{-1/2})\underline{1} + \underline{\varepsilon}_{Z2} \qquad (2.50)$$

where $o(.)$ denotes the small order relation, $\underline{1}$ is a $n \times 1$ vector of 1's and B and Δ are $(k_0 + k_1) \times k_2$ and $n \times k_2$ nonzero matrices of constant, with the restriction on Δ such that

$$\lim_{n \to \infty} \frac{\Delta' M_1 \Delta}{n} = W_2 \tag{2.51}$$

exists and is nonzero. On the basis of local alternatives constructed in this manner, the asymptotic power of the NJ^2-test can be formulated.

<u>Theorem 2.</u> Under local alternatives as defined in (2.50) with condition (2.51) met, the asymptotic power of the NJ^2 -test, P_{NJ} , is given by:

$$\begin{aligned} P_{NJ} &= \lim_{n \to \infty} P[NJ^2_{12n} \geq \chi^2_{(1),1-\alpha} \mid H_{2n}] \\ &= P[\chi^{2\prime}_{(1),\eta_{NJ2}} \geq \chi^2_{(1),1-\alpha}] \end{aligned} \tag{3.24}$$

$$\text{where } \eta_{NJ2} = \frac{\underline{\gamma}'_2 W_2 \underline{\gamma}_2}{\sigma^2_2}.$$

<u>Proof:</u> For any pair of hypothesized models H_1 and H_{2i}, whether H_{2i} is a local alternative of H_1 or not, $\hat{\sigma}^2_{F,2i}$ follows a central χ^2 distribution. Therefore when the comprehensive model formulation is employed for a sequence of local alternatives to H_1 as in (2.50), this result is still valid. Then, for the local alternatives H_{2n} ,

$$\frac{\hat{\sigma}^2_{F,2n}}{\sigma^2_{2n}} = \frac{\underline{y}' M_{2n} \underline{y} / (n - k_0 - k_1 - k_2)}{\sigma^2_2} \overset{H_{2n}}{\sim} \chi^2_{(n - k_0 - k_1 - k_2)}.$$

Therefore, $\operatorname{plim}_{H_{2n}, n \to \infty} (\hat{\sigma}^2_{F,2n} / \sigma^2_{2n}) = 1$. Consequently, the asymptotic power of the NJ^2 -test under local alternatives is based on the numerator χ^2 distribution and is given by:

$$P_{NJ} = \lim_{n \to \infty} P[\underline{y}'[P_0 - P_1]\underline{y} \geq \chi^2_{(1),1-\alpha} | H_{2n}]$$
$$= P[\chi^{2'}_{(1),\eta_{NJ}2} \geq \chi^2_{(1),1-\alpha}]$$

where $\eta_{NJ}2 = \eta_{JA}2 = \dfrac{\underline{\gamma}'_2 W_2 \underline{\gamma}_2}{\sigma_2^2}$,
as shown by Pesaran (1982b, 1983).

As expected, $P_{NJ} = P_{JA}$ since the tests are asymptotically equivalent under the assumption of local alternatives. In other words, the asymptotic power of the NJ- and JA-tests are equivalent under local alternatives. Therefore, the comparisons under local alternatives made between the JA-test and the others are equivalent to such comparisons made between the NJ-test and others. At this point, both the finite sample and asymptotic properties of the modified JA-test have been derived. So, it is now important to make use of this information about the behavior of the NJ-test to judge whether ot not it performs well relative to the other non-nested testing procedures. Detailed results for addressing this issue can be resolved in a practical way in the Monte Carlo study of Chapter IV. Therefore, the comparisons to be presented in this next section are based on theoretical considerations.

3.3 Comparisons With Other Testing Procedures

3.3.1 Estimated Variances Under H_1 for the JA- and NJ-tests

The numerator of the JA and NJ test statistics are identical. Therefore, any differences in their inferential ability will stem from the denominators: the estimated error variance. Under the alternative hypothesis, the JA derived error variance estimator does not follow an exact finite sample distribution and is clearly not unbiased. However, using the amended version of Theorem A.3.1 (Graybill and Milliken, 1969), it can be seen that

$$\hat{\sigma}^2_{JA} \mid_{P_1\underline{y} = \zeta} \overset{H_2}{\sim} \chi^{2\prime}_{(n - k_0 - k_1 - 1), \lambda_{JA}} \tag{3.25}$$

where

$$\lambda_{JA} \mid_{P_1\underline{y} = \hat{\underline{y}}_1^{\bullet}} = \frac{1}{2\sigma_2^2} \underline{\beta}'_2 X_2 \{M_1 - M_1 P_{Z2}\zeta[\zeta' P_{Z2} M_1 P_{Z2}\zeta]^{-1}\zeta' P_{Z2} M_1\} X_2 \underline{\beta}_2. \tag{3.26}$$

Consequently, even on a conditional basis, the JA-test will not be exact under the alternative since its denominator is a non-central, as well as conditional, χ^2. Therefore, the only other way to compare the NJ-test to its parent test is on the basis of their error variance estimates under H_1. As was indicated in Section 3.2.2, under H_1 the two variance estimates are unbiased, with that estimator from the JA-test having smaller variance in general. However, for a given sample of data, the two estimators will not yield the same estimates. Therefore, it is worthwhile to compare the estimates in terms of their magnitudes under certain model conditions. In particular, each of the corresponding SSE's can be written in a more telling form:

$$\hat{\sigma}^2_{JA} = \underline{y}'\{M_1 - M_1 P_{Z2} P_1 \underline{y}[\underline{y}' P_1 P_{Z2} M_1 P_{Z2} P_1 \underline{y}]^{-1} \underline{y}' P_1 P_{Z2} M_1\}\underline{y}/(n - k_0 - k_1 - 1), \tag{3.27}$$

and in a similar fashion for the F-test while employing more of the results about partitioned matrices of the form $X = [X_1 | Z_2]$, the following expresssion for the F-test's estimated variance is obtained:

$$\hat{\sigma}^2_F = \underline{y}'\{M_1 - M_1 Z_2 (Z'_2 M_1 Z_2)^{-1} Z'_2 M_1\}\underline{y}/(n - k_0 - k_1 - k_2) \tag{3.28}$$

It is clear that the SSE under either formulation of the models will be a reduction from the simple H_1 SSE. In addition, the SSE from the F-test will be smaller in general. But to what degree depends on the number of non-overlapping variables, as well the relative fit and other characteristics common to both models. More importantly is the fact that the different degrees of freedom prevent the making of any clear cut comparisons of the two variance estimators.

3.3.2 Analytic Power Comparisons Between the NJ- and F-tests

Since the Orthodox F- and NJ-tests are the only two which have exact finite sample distributions under the alternative hypothesis, H_2 , they are the only two for which power can be discussed in a finite sample setting. Because both tests have the same central χ^2 denominator, namely the SSE from the comprehensive model, the real comparisons of power must be based on the numerator χ^2 's: particularly their degrees of freedom and noncentrality parameters.

Notice that the Orthodox F-test has the following distribution under H_2 :

$$F_{12} = \frac{[SSE_1 - SSE]/k_2}{SSE/(n - k_0 - k_1 - k_2)} \overset{H_2}{\sim} F'_{(k_2, n - k_0 - k_1 - k_2), \lambda_F}$$

$$\begin{aligned} \text{where } \lambda_F &= \frac{1}{2\sigma_2^2} \mu'_2[M_1 - M]\mu_2 \\ &= \frac{1}{2\sigma_2^2} \beta'_2 X'_2 M_1 X_2 \beta_2 \, . \end{aligned} \tag{3.30}$$

Therefore, if the power of the comprehensive F-test is defined in the same manner as the power of the NJ-test, the following expression for power is formulated:

$$\begin{aligned} Power_F &= P[\text{ Reject false } H_1 \text{ maintained against true } H_2] \\ &= P[F_{12} > F_{(k_2, n - k_0 - k_1 - k_2), 1-\alpha}] \\ &= P[F'_{(k_2, n - k_0 - k_1 - k_2), \lambda_F} > F_{(k_2, n - k_0 - k_1 - k_2), 1-\alpha}]. \end{aligned} \tag{3.31}$$

In order to examine the power of these two tests it is important to realize that the characteristics of the models play a great role in determining the magnitude of the noncentrality parameter as well as the number of degrees of freedom. Such characteristics include the number of regressors (overlapping and not) in the alternative models, the amount of collinearity between the competing sets of regressor variables and the quality of fit on the true model itself. Consequently, it is not possible to draw any absolute conclusions about one test having greater power than the other uniformly. In addition to the varying model characteristics influencing power, it is important to keep in mind that the noncentrality parameter of the NJ^2-test is really a random variable, which complicates even more the comparability of the power of the two tests. In other words, even though the above model characteristics may be known, the actual value of the noncentrality parameter for the NJ-test, and thus power, varies with the actual observed $\underline{y}$ (i.e., $\underline{\varepsilon}$).

In the case of the NJ-test, then, the substitution of $E_2(\underline{y})$ for $\underline{y}$ in the conditional portions of the noncentrality parameter will be used to simplify the examination of the behavior of the noncentrality parameters, and thus power, in general. At this point, however, there is no evidence about how misleading such results may be in regard to the true conditional value of the noncentrality parameter. In order to evaluate the magnitude of such discrepancies, the observed differences between the resulting power under the two approaches are computed in the Monte Carlo study. In the limited context of the simulation design presented in Chapter IV, some empirical evidence was obtained through the calculation of the power of the NJ-test in the detection of a false null model maintained against the true alternative using both the observed $\underline{y}$ (in $P_1\underline{y}$) as well as the expectation of $\underline{y}$ under H_2 (i.e., $X_2\underline{\beta}_2$). Then as a measure of the similarity of these two methods, the squared deviation between the two calculations of power was computed for each repetition of the process and an average squared deviation between the power and "expected" power was computed. Although these results are not presented until Chapter IV, they do indicate that the discrepancy between the "expected" and the true power was minimal. Therefore, since this substitution could provide helpful comparison information, the current discussion of power centers on the "expected" power using $\bar{\lambda}_{NJ}$ in the case of the NJ testing procedure.

As far as the power comparisons are concerned, it is clear that not only are the noncentrality parameters unequal in general, but also the numerator degrees of freedom from the two tests can be quite different depending on the number of non-overlapping regressors in the (true) alternative model. Specifically, as k_2 becomes larger, the difference in the numerator degrees of freedom $(k_2 - 1)$ will also increase. What influence does this have on the overall power of the tests. To see its effect, the results of Das Gupta and Perlman (1974) should be utilized.

Their work showed that the power function of a test based on a non-central χ^2, with a fixed value of the noncentrality parameter, is strictly decreasing in degrees of freedom. Therefore, in the case where $\lambda_{NJ} = \lambda_F$, this result implies that the power of the NJ-test would become increasingly larger than that of the F-test as the number of non-overlapping variables in the alternative model, k_2, increases above 1. However, based on the differences in the noncentrality parameters, it is unknown how likely this is to be the case. It would be useful, then, to examine the model conditions which lead to such results as well as those which lead to the contrary.

It is therefore necessary to evaluate and compare power under given cases of the model characteristics. Since there are an infinite number of possible model conditions to consider, a limited number of cases will be examined by imposing certain constraints on the forms of the models as well as the controlling parameters indicated above. Clearly, the numbers of regressors, k_0, k_1, and k_2, the true error variance σ_2^2 and the collinearity between the models (as controlled through the squared canonical correlation between the two sets of regressors) are characteristics of the model which greatly influence the power and thus should be controlled when examining power. In addition, for this investigation of power, the design structure for the small sample Monte Carlo study (Chapter IV) will be invoked on the models as a means of obtaining a reasonable number of cases which are representative of the more general setting. The exact structure of the true and false competing model generation is discussed in Section 4.2.3. However, the implications of the imposed structure which concern this examination of power are that the regressors within each model are independent of one another, the amount of collinearity between the competing sets of regressor variables is controlled through the squared canonical correlation ρ^2, the fit of the true model is controlled through its R^2 and that there are no overlapping variables in the two competing models (i.e., $k_0 = 0$). The formulation of the variance-covariance structure between the sets of regressor variables for this class of models is given in Appendix B.1.

For the purpose of investigating power, it is necessary to look at the noncentrality parameters from the two tests under this model framework. The derivations of the noncentrality parameters for both the NJ- and F-tests are given in Appendix B.2, and the results are as follows:

$$\lambda_{NJ} = \frac{1}{2\sigma_2^2} \sum_{j=1}^{s} \beta_{2j}^2 \tag{3.32}$$

where $s = \min(k_1, k_2)$, and

$$\lambda_F = \frac{1}{2\sigma_2^2} \sum_{j=1}^{k_2} \beta_{2j}^2 \quad . \tag{3.33}$$

Specifically, without a finite sample of regressor variables specified, these results on the form of the noncentrality paramet are derived by substituting the variance-covariance matrices in for the X'_iX_j matrices. Such a substitution, although necessary for obtaining some basis for comparison, will pull the power down quite a bit from what it actually would be. Notice the similarity between the two noncentrality parameters under the constrained class of competing models. One disconcerting aspect, however, is that past empirical evidence has indicated that the amount of collinearity between the competing models has a strong influence on the resulting power. This apparent influence gets completely "washed out" in this design for the models. Clearly, this result implies that the use of this design on the competing models leaves some unanswered questions with respect to this feature's effect on power. On the other hand, the value of the noncentrality parameter and the corresponding power can be viewed as being averaged over all levels of ρ^2, the strength of the collinearity between the competing models.

Also, it is interesting to note that the only difference between these two noncentrality parameters is whether or not all of the β_{2j}'s are included in the summation or only those which correspond to regressor variables which were non-independent of regressors in the true model.

Although this may seem puzzling at first, it is reasonable when the nature of the two testing procedures in terms of numerator χ^2's is considered.

First consider the form of the quadratic form in the numerator of the NJ-test, which can be expressed as

$$\underset{\sim}{y}'\underset{\sim}{q}[\underset{\sim}{q}'\underset{\sim}{q}]^{-1}\underset{\sim}{q}'\underset{\sim}{y}\ , \qquad \underset{\sim}{q} = M_1 P_{Z2} P_1 \underset{\sim}{y}$$

where $Z_2 = X_2$ since $k_0 = 0$ in the design class under consideration. Any pieces of information in the H_2 model which are independent of the regressors in the H_1 model (i.e., orthogonal regressors) are eliminated from increasing the quadratic form's sum of squares. This condition is a direct result of the NJ (and JA) approach in that it forces the alternative model to explain the behavior of the estimated expected value of the dependent vector $\underset{\sim}{y}$ under the assumption that the maintained model is indeed valid. Therefore, in the resulting regression, the fitted values $\hat{\underset{\sim}{y}}_{21} = P_1 P_2 \underset{\sim}{y}$ (since $P_2 = P_{Z2}$ in this setting) depend only on the information coming from those regressors in the alternative model which are non-orthogonal to the regressors in H_1. Consequently, even though the resulting quadratic form is based on the projection of $\underset{\sim}{y}$ onto $\underset{\sim}{q}$, $\underset{\sim}{q}$ also will exclude the explanatory information of those orthogonal regressors through the above original projection of X_2 onto $\hat{\underset{\sim}{y}}_1 = P_1 \underset{\sim}{y}$.

On the other hand, the F-test is based on the projection of $\underset{\sim}{y}$ onto the full comprehensive model (regressors $[X_1 | X_2]$). Therefore, the explanatory information from all the regressor variables in X_2, which is in addition to that from those regressors in X_1, is included in that quadratic form in the numerator. Consequently, the difference in the formulation of the tests manifests itself in this apparent difference in the noncentrality parameters. However, having such orthogonal regressors between models is highly unlikely in practical situations, and thereby indicates another limitation in this particular formulation of the models.

Consequently, the information gained concerning the relative power of these two testing procedures is by no means comprehensive, but it does provide some general trends regarding the influence of various model characteristics on the power. Based on this formulation, power curves have been constructed for various combinations of the number of regressors in the two models, R^2 for the true model and ρ^2 . However, in order to actually compute the power based on the "F-test" construction, it is necessary to specify the true values of the β_{2j}'s as well as the sample size n. For this situation (as in the simulation design), all the β_{2j}'s have been set to 1 and two sample sizes, n = 20 and 40, were used. Several of these power curves are given in Figures 3.1-3.6.

Based on the examination of the plots, it is evident that the NJ-test tends to be more powerful than the F-test when the number of non-overlapping variables in the true models is less than or equal to the that in the false alternative. However, the conservative approach used in the JA formulation as it applies to the numerator quadratic form of the NJ-test makes itself evident in terms of the F-test having greater power than the NJ-test when the true alternative model has more regressors than the false null model. Although this reduction in power is not great for most cases and the overall power of the NJ-test tends to be at least comparable with that of the F-test in a larger more general class of conditions on the models, it still warrants close consideration when it comes to practical use of the procedure.

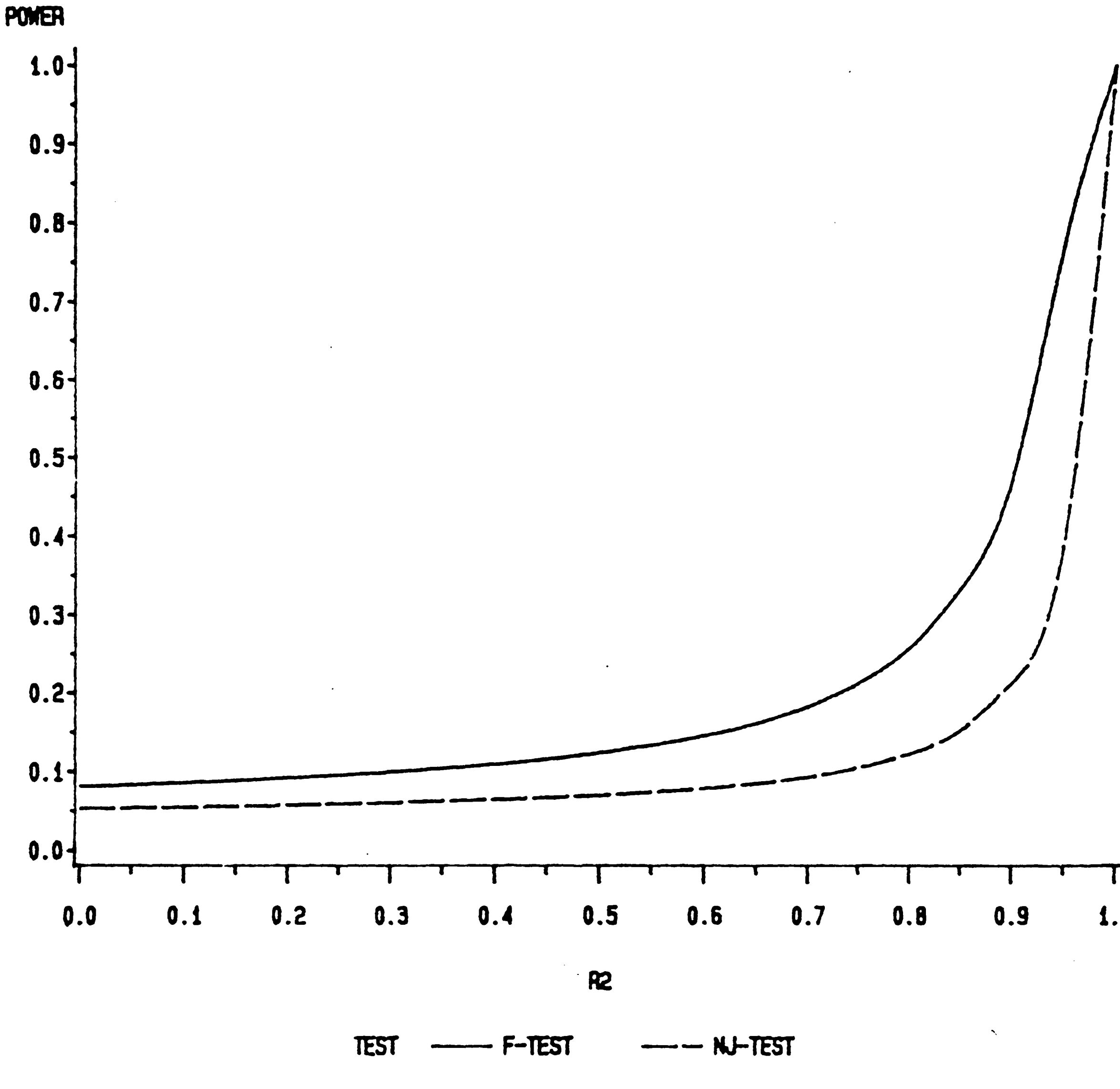

FIGURE 3.1 POWER CURVES FOR THE NJ AND F TESTS
POWER VS R2
N = 20 K = (2, 6)

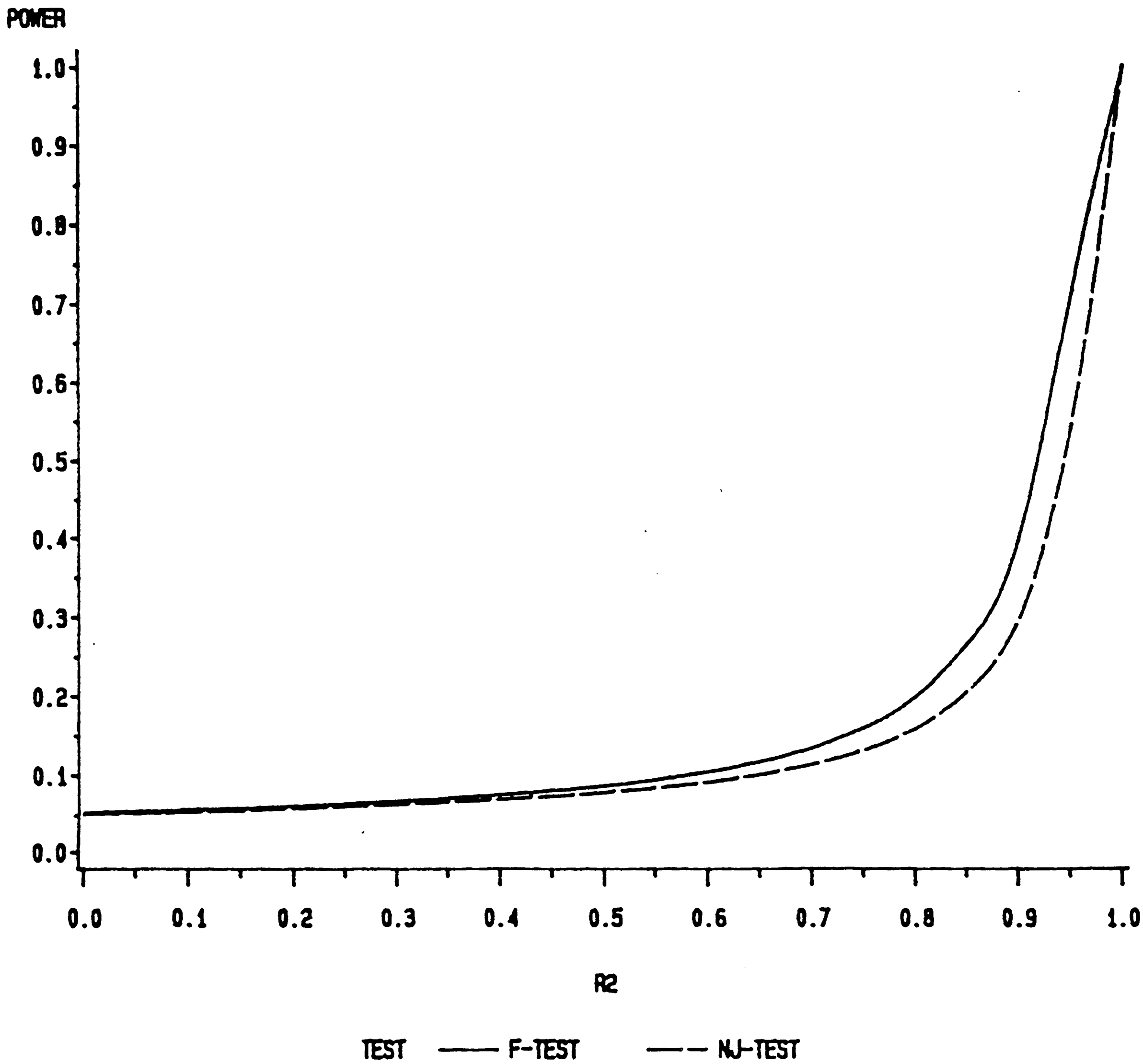

FIGURE 3.2 POWER CURVES FOR THE NJ AND F TESTS
POWER VS R2
N = 20 K = (2, 4)

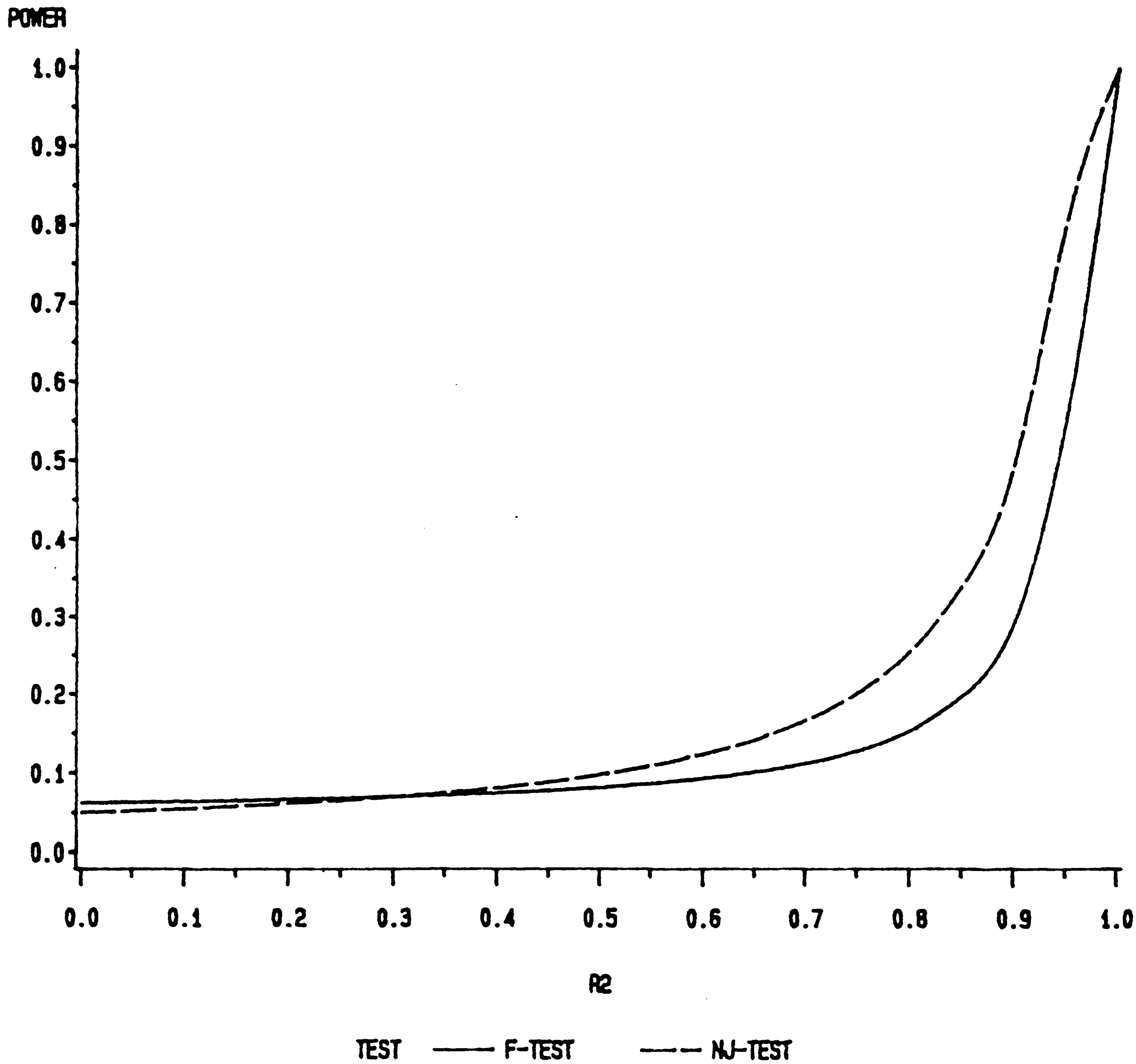

FIGURE 3.3 POWER CURVES FOR THE NJ AND F TESTS
POWER VS R2
N = 20 K = (4, 4)

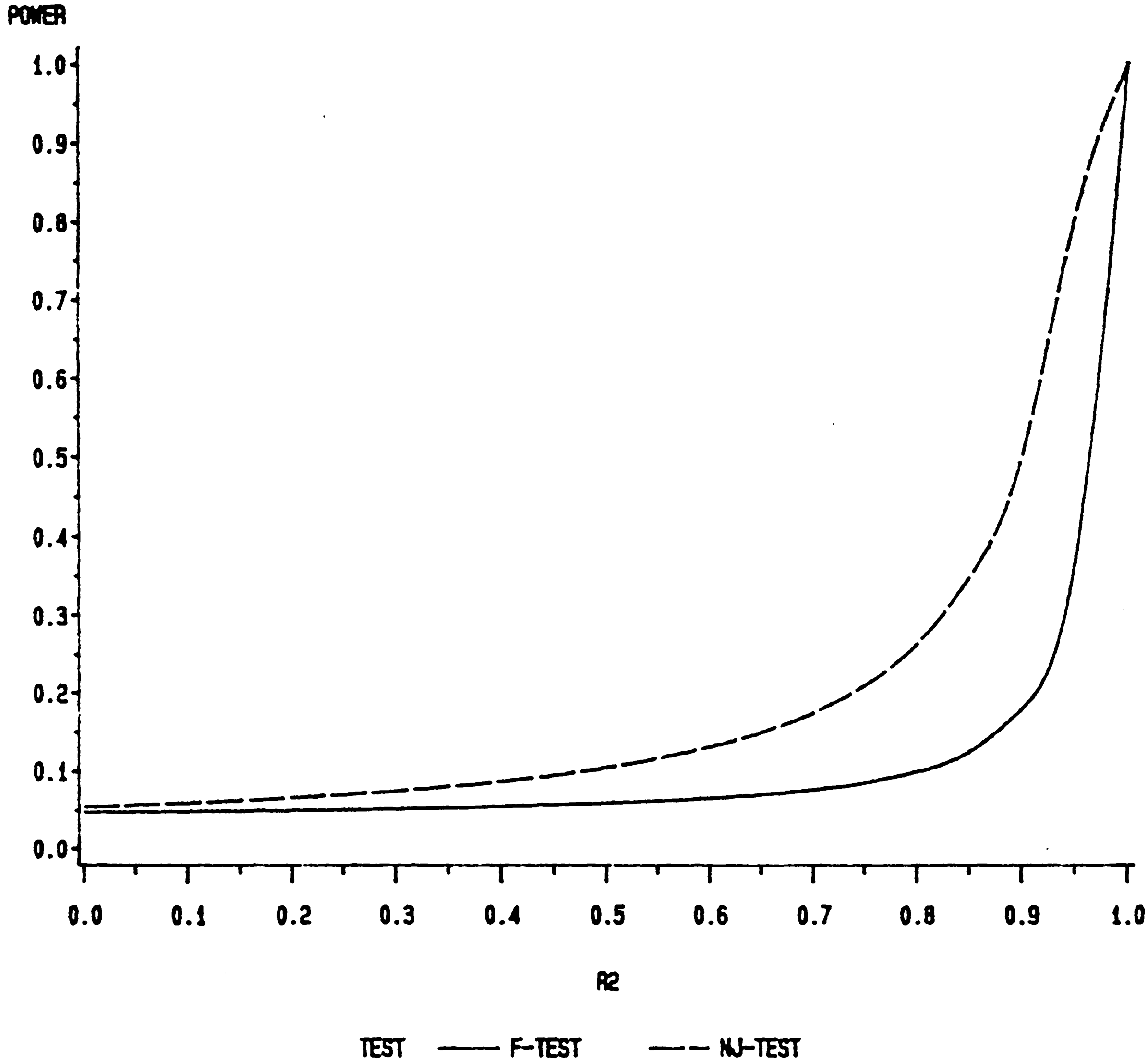

FIGURE 3.4 POWER CURVES FOR THE NJ AND F TESTS
POWER VS R2
N = 20 K = (6, 4)

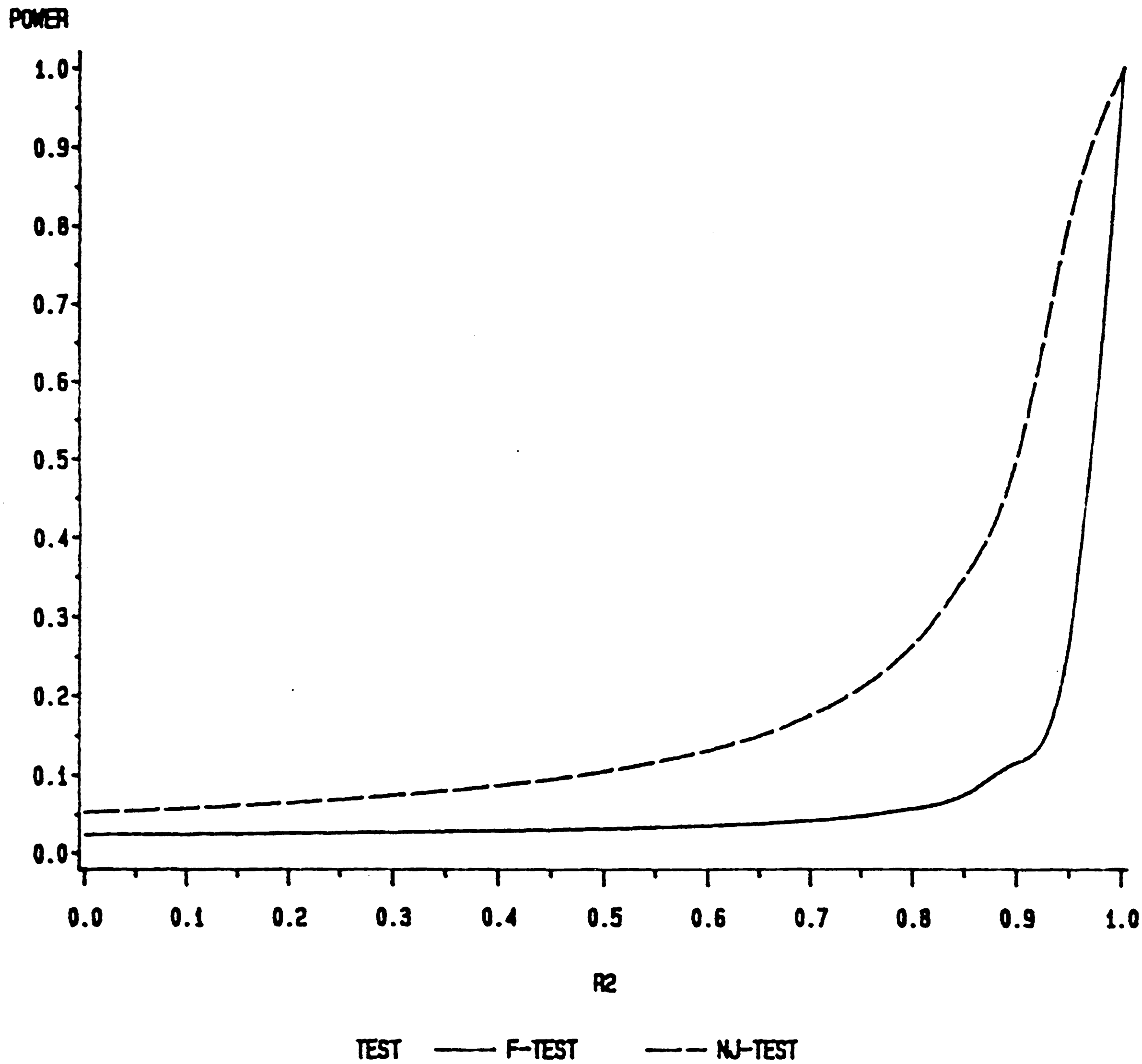

FIGURE 3.5 POWER CURVES FOR THE NJ AND F TESTS
POWER VS R2
N = 20 K = (6, 2)

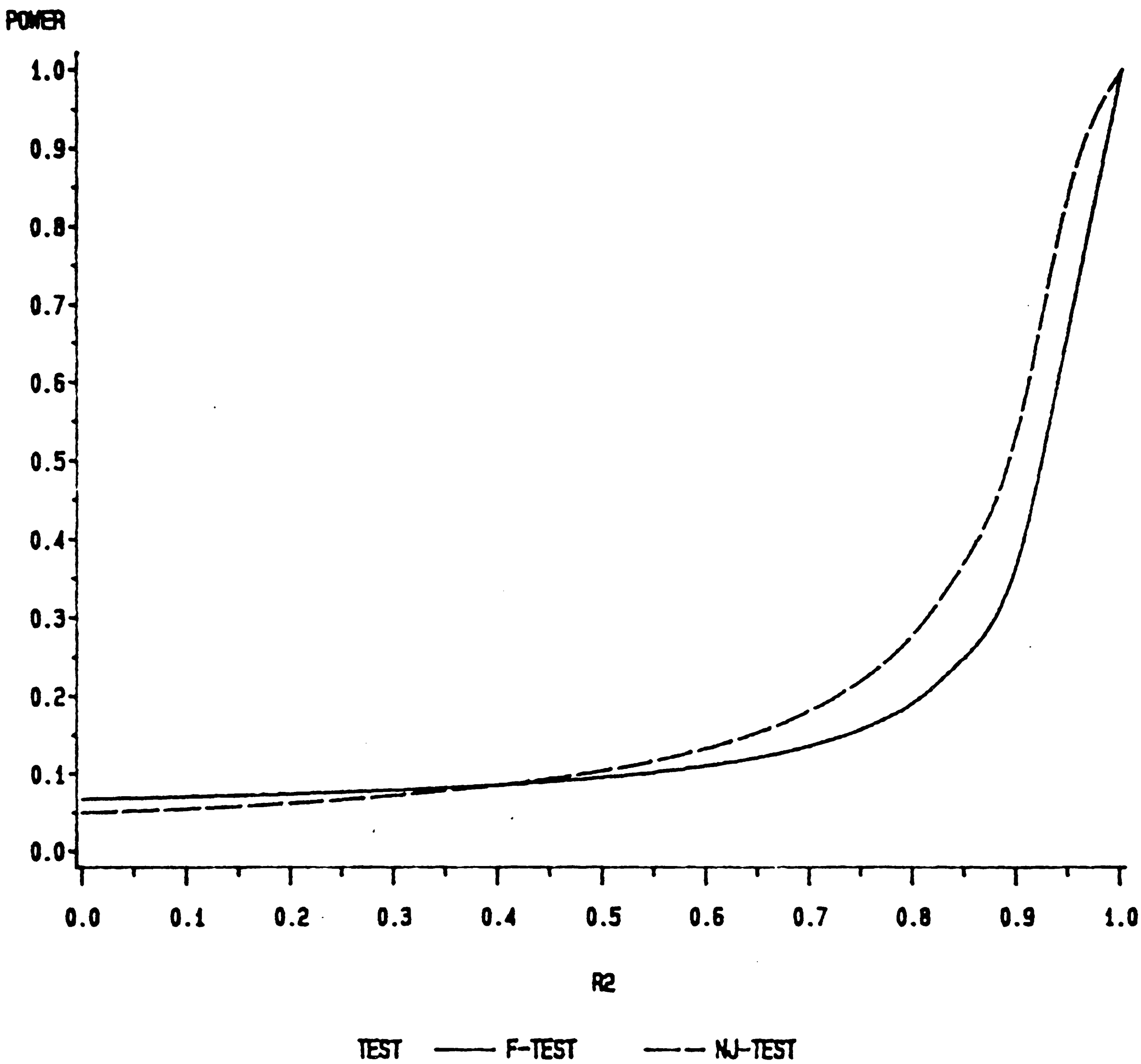

FIGURE 3.6 POWER CURVES FOR THE NJ AND F TESTS
POWER VS R2
N = 40 K = (4, 4)

Once again, it is important to point out that these comparisons are based on a rather limited set of cases on a somewhat restrictive set of models. Therefore, the above results can be considered indicators of the general trends in the behavior of the power under varied model conditions for both the NJ- and F-tests. Clearly, there is still much to be learned about the relative power of these tests. In particular, it is the power of making a correct inference based on a pair of tests on a given pair of competing models, and not that of rejecting the false model only, which is truly of interest. The Monte Carlo study discussed in Chapter IV provides a supplement to and a reinforcement of the trends in power observed here. 3.3.3 Asymptotic Comparisons

Since all of the non-nested testing procedures for the case of two linear regression models are only asymptotic in nature, with the exception of the NJ- and F-tests, it is necessary to make analytic comparisons between the various procedures through the use of asymptotics. This situation induces the return to the concept of local alternatives and the asymptotic power of the various procedures under them. As it was indicated in section 3.2.4, the asymptotic power of the NJ-test was equivalent to that of the JA-test. From this, some comparisons between the NJ-test and the others in this asymptotic setting are apparent (refer to Section 2.3). First,

$$P_{NJ} = P_{JA} = P_J = P_N \tag{3.34}$$

under all cases of local alternatives. In addition, since its asymptotic power under local alternatives is equivalent to that of the J- and JA- tests, the NJ-test like the other two achieves maximum local power for Pesaran's general class of AN testing procedures. (Even though the NJ-test is a modified version of one of the general family of AN procedures.) Then, in regards to the Orthodox F-test, it is evident (from Pesaran, 1982b,1983) that

$$P_{NJ} \geq P_F \tag{3.35}$$

with the equality holding when the number of additional regressors in the local alternative sequence k_{2n} is only one and strict inequality holding when $k_{2n} > 1$. Clearly, in terms of its asymptotic power, the NJ-test has lost nothing relative to the JA-test and in some cases has gained power over the Orthodox F-test. Based on these results and the finite sample comparisons given in section 3.3.2, it appears that the NJ-test as a compromise between the JA- and F-tests may have some real advantages.

However, there is still much to be investigated which can only be grasped through the use of simulation studies. Of particular interest is the comparison between the observed performance of the JA- and the NJ-tests. In other words, empirical evidence must be compiled in order to judge whether or not the proposed modification accomplished what it was intended to do in terms of increased power in cases where the JA-test tended to be conservative. Since the remaining comparisons can only be made on the basis of empirical evidence, the Monte Carlo study and the results thereof is the next topic presented.

IV. A Monte Carlo Study of Finite Sample Performance

4.1 Introduction and Objectives

The purpose of this Monte Carlo study is to produce a practical comparison of the non-nested testing procedures in the case of linear regression models. Such a comparison is needed in order to formulate guidelines which will enable the researcher to employ those non-nested testing procedures which will yield the most reliable results. However, as was indicated in the review of the testing procedures in Section 2.3, the inferential ability of each test is a function of the characteristics of the models under test. Therefore, under various conditions regarding the formulation of the competing models, comparisons will be made on the basis of observed power, observed type I error probabilities, and rankings based on p-values among the testing procedures. The basic underlying design for this study is the simulation work embarked upon by Godfrey and Pesaran (1982, 1983). In the context of this basic design, the current study will emcompass a larger number of testing procedures as well as the effects of other model features on the performance of the tests.

There are two main objectives in addition to an investigation into the influence of basic model characteristics. One is the performance of the test procedures under the violation of the normality assumption on the distribution of the disturbance terms. The performance will be evaluated under both skewed and symmetric distributions for competing models constructed with varying numbers of regressor variables, quality of fit on the true model and degrees of collinearity between the competing sets of regressor variables. Through these results, evidence about the robustness of the tests to violations of the normality assumption on the disturbances will be compiled.

Similarly, the worth of these tests lies in the ability to produce a correct inference regarding a pair of models, and not just in its ability to discriminate between the two models where one is indeed correct. Therefore, it is a parallel objective to evaluate how adequately the testing procedures can correctly indicate the presence of two false models. This issue is of particular importance in practical situations, such as the demand analysis in Chapter V, where the researcher has more than two models to investigate.

4.2 Design Structure

4.2.1 Control Parameters

The structure of this study relies heavily on the work of Godfrey and Pesaran (1982,1983) which was mainly concerned with the performance of the Cox test, its small-sample adjusted versions and the Orthodox F-test in the case of two competing linear regression models. Also, in their 1982 study, the main goal was to evaluate the improvement accomplished by the small-sample adjustment to the J-test (AJ-test). In the more comprehensive study, some limited comments were made regarding the performance of the J- and JA-tests in the preliminary stages of their work. However, due to their apparent deficiencies in cases having unequal numbers of regressor variables in the competing models they were quickly dismissed from further investigation. With respect to the model parameters to be controlled within the study and the methodology by which the competing models are constructed, the Godfrey and Pesaran study provides a sound foundation on which to build.

In their study, two non-normal distributions for the disturbance terms were considered: the chi-square with 2 degrees of freedom and the log-normal based on the transformation of a normal with mean zero. Both distributions are skewed and were adjusted so that they would have mean zero and a variance which yielded the appropriate R^2 for the true model. Therefore, this study will employ these two skewed distributions as well as two symmetric distributions: the truncated normal and the Student-t with 3 degrees of freedom. Since one has a shorter and the other a heavier tail than the corresponding normal, some additional information regarding the robustness of the various non-nested testing procedures to the normality assumption can be obtained.

Since the ability of the testing procedures to make a correct inference when both models under test are incorrect is to be examined, the generation of a second false model will be added to the study. Therefore, information regarding this aspect of performance capability will be gained by evaluating the proportion of times the procedures correctly indicate that both models are invalid.

In addition, this study will employ all of the procedures discussed in section 2.2 as well as the modified JA-test. Therefore, the Atkinson (NA) and Linearized Cox (NL) tests will be given the same consideration as the other test procedures examined in the previous Godfrey and Pesaran studies. Of particular interest is the performance of the modified JA-test (NJ) in small samples for making inferences based on a pair of tests compared to the F-test. From Chapter III, it is known analytically that the power of the NJ-test for rejecting a false null model maintained against the true model will exceed that of the F-test under particular model conditions; however, greater power in the resulting inference is not guaranteed since the previous power measure does not take into account the probability of a type I error for the reversed test. In addition, it will be useful to compare the NJ-test's power with that of the JA-test in order to determine the conditions under which the NJ-test will be an improvement over the unmodified test.

By use of the Godfrey and Pesaran approach, the control parameters for the simulation study as well as the associated levels of interest for each are given as:

sample size:	n = 20, 40
fit of the true model:	R^2 = 0.75, 0.90
collinearity between true model and its alternatives as measured by the squared canonical correlations between the two sets of regressor variables:	ρ^2 = 0.25, 0.50, 0.75, 0.90
number of regressor variables: (where $k_0 = 0$)	(k_1, k_2, k_3) = (2,4,6), (4,2,6), (6,2,4), (4,4,4)
distribution on the disturbance terms:	$N(0, \sigma_\varepsilon^2)$, $truncN(0, \sigma_\varepsilon^2)$ (10%), $t_{(3)}$, $\chi^2_{(2)}$, ln = exp(N(0,1)).

Each individual experiment consists of generating the true model and two false alternatives under the constraint of the control parameter values and performing all ten test procedures on the three pairwise combinations of the models, 1000 times each for samples of size 20 and 500 times each for samples of size 40. It is obvious that a complete investigation of all combinations of the control levels would involve 5 sets (one for each distributional assumption) of a $2^2 \times 4^2$ factorial design (i.e., 320 experiments). Employing this full design would yield an unmanageable as well as expensive Monte Carlo study. Consequently, in order to make this study more reasonable, fractional factorial designs will be used within the context of each of the distributional studies. The fractions are selected such that all "main" effects as well as some of the pairwise interactions between the control parameters are estimable. This setup provides the means to evaluate the relative importance of the various model characteristics, as controlled through the above parameters, on the performance of any of the ten given testing procedures.

The fractional designs for each of the distributional cases are selected on a somewhat sequential basis. Primarily, the classical case (normal case) will yield a standard by which to measure the robustness of the tests. Therefore, in the normal case, a one-half fraction was employed as well as some additional runs in which the sample size was held at 20 and the number of regressor variables held at 4 in all three models. For the other distributional cases, some direct comparisons were made to the normal case through experimental runs with the same control parameter settings. However, a one-fourth fraction of a $2^2 \times 4^2$ factorial design for which the sample size, n, was held at 20 represents the formally specified design in each case. The actual set of experimental runs for these cases will be presented in section 4.3.2.

Regarding the issue of the number of replications in each sample size, the main consideration in using only 500 replications for samples of size 40 was expense. However, in terms of the stability of the results as well as the estimated standard errors on all observed performance criteria, there was not a significant loss by reducing the number of replications for samples of size 40 from 1000 to 500. As an example of this, Appendix C contains a comparable pair of runs based on samples of size 40 using both 500 and 1000 replications.

4.2.2 Comparison Criteria

The purpose of this study is to obtain practical guidelines (for the use in actual applications) regarding the credibility of the results from the ten tests so that the most meaningful results will be obtained. Therefore, the feasibility of this study must be judged through the criteria to be used for making comparisons among the testing procedures. Of primary concern is the power of the tests in finite samples, i.e., the ability of the test to indicate the correct inference for the given pair of models under test. In addition, the size of the tests is important, particularly as it influences power. Consequently, for the case of testing the true model with one of the false alternative models, the following comparison criteria will be employed:

1. Power (and its standard error): To be calculated as the proportion of replications within each experiment for which a correct decision regarding the pair of models under test is made. (i.e., reject the false and accept the true)

2. Type I error probability (and its standard error): To be calculated as the proportion of replications within each experiment for which the true model is incorrectly rejected when it was maintained against a false model.

3. Kendall's coefficient of concordance (Kendall, 1939): To be calculated for each pair of models tested within each experiment from assigned rankings on the p-values associated with the rejection of the false model when it is maintained against the true model. To see the practical side of using this measure, the individual replications can be viewed as judges who rank the ten testing procedures on the basis of how likely they are to detect the presence of a misspecified maintained model (the p-values). To be used as a measure of agreement among individual replications, this statistic is aimed at evaluating the stability of the relative performance of the testing procedures.

To take into account and to assess a penalty for type I errors, the p-value is set to one whenever the corresponding test rejects the true model when it is the maintained hypothesis.

4. Average rankings of the tests: To be calculated as the p-value rankings within each replication as computed for Kendall's concordance coefficient. To serve as relative "power" rankings among the testing procedures.

5. Analytically computed power (and its standard error): To be analytically calculated - for the NJ- and F-tests only - as the power of rejecting a false model when it is maintained against the true model within each replication, and averaged over all replications. To compare the two procedures, the proportion of replications for which the computed power of the F-test exceeded that of the NJ-test is recorded.

For the case involving two false models under test, the proportion of replications for which each of the four possible inferential outcomes from a pair of tests regarding two specified models is computed. In particular, comparisons can be made on the basis of how often the testing procedures yield the correct inference, indicating the need for further investigation in the search for the correct model. In addition, warnings can be drawn for practical use concerning which test procedures tend to "lean" toward a false model which possesses certain model characteristics, such as larger R^2 or larger number of regressor variables.

Clearly, valid analyses can be made using the comparison-oriented data as indicated above under the designated sets of experimental runs for each distributional case. From these analyses, meaningful comparisons can be drawn. Therefore, this Monte Carlo study, as this layout indicates, should go a long way toward providing the information necessary to formulate practical guidelines. Only the mechanics behind the model building procedure given the various control parameters remain to be presented.

4.2.3 Model Generation

As indicated by the control parameters for the simulation study, the format for the generation of the models under test is that of Godfrey and Pesaran (1983). Within each replication of each experimental run, one "true" model and two false linear regression models are generated. In particular, for a given set of parameters $\{n, R^2, \rho^2, (k_1, k_2, k_3)\}$ the true model, H_1, is of the following form:

$$H_1: \quad y_t \; = \; \sum_{i=1}^{k_1} x_{1it} \; + \; \varepsilon_t, \quad for\ t = 1, 2, ..., n \tag{4.1}$$

where $\varepsilon_t \sim \ iid\,(0, \sigma_\varepsilon^2)$ and $x_{1it} \sim iidN(0,1) \ \forall \ i,t$.

The variance of the disturbance terms, ε_t, is generated so that the coefficient of determination, R^2, is maintained at the specified level through the following expression:

$$\sigma_\varepsilon^2 \; = \; \frac{k_1(1 - R^2)}{R^2} \tag{4.2}$$

Accordingly, the distribution used to generate the disturbance terms depends on the assumption made. For the normal case, $\varepsilon_t \sim iid\,N(0, \sigma_\varepsilon^2)$. Similarly, for the truncated normal based on the p.d.f. of a $N(0, \sigma_\varepsilon^2)$ with tails cut-off at $\pm \ 1.6449\sigma_\varepsilon$, the appropriate variance is maintained by using this formulation:

$$\varepsilon_t \; = \; \frac{\sigma_\varepsilon}{[0.6230336]^{1/2}} u_t, \quad where\ u_t \sim \ TruncN(0,1), \tag{4.3}$$

since $Var(u_t) \; = \; 1 - 2\{\frac{1.6449}{0.90(2\pi)^{1/2}} \exp[-\frac{1}{2}(1.6449)^2]\} \; = \; 0.6230336$. Once again, the transformation of the Student-t (with 3 df) variates into a mean 0, variance σ_ε^2 disturbances only require a straightforward scaling of the data; i.e.,

$$\varepsilon_t = \frac{\sigma_\varepsilon}{[3]^{1/2}} u_t, \quad where\ u_t \sim iid\, t_{(3)} \tag{4.4}$$

In the case of the skewed distributions, transformations must be made not only to achieve the appropriate variance but also a mean of zero. As indicated by Godfrey and Pesaran, the necessary transformations for the chi-square and log-normal deviates are as follows:

For the log-normal case,

$$\varepsilon_t = \exp\{\gamma_0 u_t\} - \gamma_1 \tag{4.5}$$

where $u_t \sim iid\, N(0,1)$; $\gamma_0^2 = \log\{1/2 + 1/2\,(1 + 4\sigma_\varepsilon^2)^{1/2}\}$; $\gamma_1 = \exp\{\gamma_0^2/2\}$;

and for the chi-square distributional assumption,

$$\varepsilon_t = \frac{1}{2}\sigma_\varepsilon\,\{u_t - 2\} \tag{4.6}$$

where $u_t \sim iid\ \chi^2_{(2)}$.

Then for the two false models, H_2 and H_3, the regressor variables are generated in order to have the squared canonical correlation between themselves and those in the true model be the specified value of ρ^2. Through this parameter value, the strength of the collinearity between the models is regulated. Correspondingly, the regressor variables in $H_j,(j = 2,3)$ are generated as follows:

$$x_{jit} = \begin{cases} \dfrac{\rho}{(1-\rho^2)^{1/2}} x_{1it} + v_{jit}, & for\ i = 1, 2, \ldots, \min(k_1, k_j) \\ v_{jit}, & for\ i = k_1 + 1,\ k_1 + 2, \ldots,\ k_j;\quad if\ k_j > k_1 \end{cases} \tag{4.7}$$

where $v_{jit} \sim iid\, N(0,1)$. Consequently, under this construction process, the models generated will reflect the model characteristics dictated by the control parameter values.

The simulation program involves the random deviate generation through the use of IMSL subroutines in FORTRAN and the remaining model construction, test procedure calculations and comparison criterion calculations are performed within the framework of PROC MATRIX in SAS. A copy of the programs for the normal and nonnormal cases are contained in Appendix D. Since the layout of the study has been presented in detail, the results of the study are next examined and appropriate comparisons drawn.

4.3 Results and Practical Conclusions

Within this section of results, the normal disturbance case is given much consideration. Following that discussion and its implications regarding the usefulness of the various tests in the case of two invalid models, the remaining four distributional studies will be presented. Parallels to the behavior under the normal case will be made in regard to the influence of the other control factors. Then the presence of non-normality in the disturbances will be used to make statements evaluating the apparent robustness of some of the test procedures. Once all the results have been highlighted, practical conclusions will be made regarding these ten procedures in terms of their relative inferential ability in the case of non-nested linear regression models.

4.3.1 The Normal Disturbance Case

For the case of $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, a total of 50 experimental runs were made: a 32-run one-half fraction of the $2^2 \times 4^2$ factorial design in the control parameters, 10 additional runs to complete the $n=20$, $k_t = 4$ analysis over all levels of $R^2 \times \rho^2$, as well as several other runs. With this design, all of the 2-way interactions between the control parameters as factors were estimable. An analysis of variance could be performed on the effects of the control parameters on the power, size and rankings of the various test procedures. This information will provide a good basis for making comparisons.

Table IV.2.2 contains the results from all 50 experimental runs, with a listing of the control parameter levels for all experiments given in Table IV.2.1. (All tables for Chapter IV are located following the text.) Based on the results given in Table IV.2.2, the general behavior of each of the testing procedures is discussed individually. These highlights on each testing procedure are designed to point out those aspects of the test's performance which vary with given model characteristics. Following these individual comments, the tests are examined in terms of their relative performance with one another in order to judge which tests yield the most reliable inferences. The concept of the test's "performance" is used instead of power since the asymptotics of most of the testing procedures does not guarantee the maintainence of a nominal size for the tests in general.

As it will become clear through the discussions which follow, having different numbers of regressor variables in the competing models plays an important part in the determining a test's performance. In fact, almost all of the testing procedures show either a direct or inverse relationship between the signed difference in the number of (non-overlapping) regressors in the true and false models and the observed power of the test. In many practical applications involving the testing of non-nested regression models, the number of regressors in the competing models will be equivalent with the form or transformation of the regressor variables being the primary concern of the researcher. Consequently, the cases where the number of (non-overlapping) regressors in the competing models are the same will be given particular attention. (It is important to note that this equal number of regressors does indeed represent an equal number of non-overlapping regressor variables since one can compare models with overlapping pieces on the basis of $\underset{\sim}{y} - \hat{\underset{\sim}{y}}_0$, the dependent variable observations after first extracting the overlapping influence of those k_0 regressors, in place of the original observations, $\underset{\sim}{y}$.)

Kendall's coefficient of concordance is computed in order to obtain a measure of how much agreement there is among the testing procedures relative to one another across all the replications in a given experiment. The power and type I error probabilities obtained from each experiment will be used in the formulation of guidelines for real applications. Therefore, the guidelines which are set forth will only be as reliable as the empirical results on which they are founded. In addition to measuring the stability of the experiments themselves, some insight into the concept of power is gained. The rankings used to compute the concordance coefficient are based on the p-values associated with the detection of the false null. Consequently, since the rejection of a false model when it is maintained against the the true model is one of the necessary steps in making the correct inference concerning the pair of models, such p-values can provide a general idea of the actual power of the procedure. This result is not the actual power, so it seems that a more meaningful piece of information is how the procedures compare relative to one another. The average, taken over all the replications in a given experimental run, of the p-value rankings for a given procedure can be thought of as a relative, as well as a scaled, measure of power. Based on the observed values of Kendall's concordance coefficient and its significance over the various experimental settings, the results from the various runs indicate consistency in the relative performance of the testing procedures.

Also, it should be noted that there an instances throughout the discussion of the simulation results in which statistical analyses, such as ANOVA's, repeated measures designs and even paired t-tests, are employed. However, due to the nature of the simulation study, each experimental really yields two observations for cases in which the true model is tested with one or the other false alternative. As a result, the basic assumptions governing most of the procedures listed above are violated (i.e., there are dependencies among some of the observations). These analyses, in turn, are to serve only as a means to discuss the results in a more formal setting. Consequently, information coming from those procedures must not be taken as absolute, but rather as an indication of trends. Clearly, if the results are taken at their true worth, they can provide additional insight or stronger evidence to support the apparent behavior trends (as extended to real data applications) which are observable in the results.

4.3.1.1 Performance of the Tests on an Individual Basis

As indicated, consideration is given to the testing procedures on their merits individually. Specifically, the strong trends in their behavior as affected by changes in model characteristics are highlighted. An indication as to their overall ability to detect the presence of two incorrectly specified models is also given.

<u>The Cox (N) test:</u> As previous evidence has shown, its power in terms of small p-values for the rejection of the false null when it is maintained against the true alternative is large, even when the p-values are penalized for rejecting the true null (i.e., the p-values associated with the rejection of the false null model is set to one when a type I error has occurred on the reversed test. This result is evidenced by small average p-value rankings throughout the experimental runs. In terms of the power of making the correct inference, it is not so promising due to the large observed probabilities of making a type I error. For samples of size 20 the Cox test falsely rejected the true maintained model an average of 13.2% of the time instead of the nominal 5%, with that being reduced to an average of 8.4% with samples of size 40. As the evidence, both theoretical and empirical, of Godfrey and Pesaran (1983) showed, this was as expected due to biases in the test statistic under the maintained hypothesis.

In addition, when attention is given to cases involving two incorrectly specified models, the unmodified Cox test indicates that further investigation is warranted on average more than 50% of the time. In some instances, it did even better than that due to its bias toward rejecting the null model, even when it was correct, so that much more when the model is false. When it was testing between false models which both fitted poorly and had relatively few regressor variables, it tended to be much less effective at detection. Even with the positive aspects of the test's performance taken into account, the small sample modifications are indeed necessary in order to give credibility to the practical use of the Cox approach.

The W- and $\bar{N}$- tests Given the past simulation work of Godfrey and Pesaran, it is of no surprise that these small sample adjusted versions of the Cox test retain the attributes of high power while bringing the observed significance levels much closer to the nominal levels. When comparing the two procedures, which are very closely related, it appears that in terms of the p-value measure of power for rejecting the false null model, the $\bar{N}$- test fares better in terms of its average ranking being much smaller than that of the W-test, and in fact is often very close to that of the unadjusted Cox test. This seems to be related to the fact that the W-test, as a Wald-type test, is more conservative in terms of rejecting the maintained hypothesis. Evidence for this conjecture is also obtained through the comparison of the observed significance levels of the two procedures. The average size of the $\bar{N}$-test was 0.0493, whereas the average observed size of the W-test was 0.0398, about a 1% difference overall.

Once again, if consideration is given to the cases involving incorrectly specified models, both procedures are able to indicate the need for further investigation over 50% of the time. In this situation, the W-test tends to do slightly better than the $\bar{N}$, with both doing a much better job than the unadjusted Cox-test in the cases of relatively poor fit on those incorrect models. For the most part, the W-test relies mainly on neither model being able to reject the other, more so than the $\bar{N}$-test, as a detection mechanism for questionable models. Clearly, the small- sample adjustments to the Cox test are a great improvement over the original procedure on all points of performance.

Atkinson's (NA) test: This is the first test to be addressed which has not received much attention in terms of empirical power studies. It becomes quite clear that the test suffers from poor power and relatively large observed significance levels, which is surprising when the conservative nature of this test in its "bias" toward the null model is considered. However, in cases where the true model has fewer regressor variables than the competing alternative, it has the ability to detect such a case, much more so than some of its competitors. Corresponding to its conservative nature, the rankings on the p-values for rejecting the false null model are clearly quite poor.

In the situation involving two false models, the Atkinson test indicates that both models are questionable over 50% of the time, on average, which is due mainly to the case in which neither model is able to "reject" the other. In the cases where the false models involve (2,4) and (2,6) regressor variables, the Atkinson test detects questionable models (i.e., either both rejected or both accepted) an average of 62.3% of the time (which is quite good given the tendency of most procedures to choose the model with more regressors), with the test rejecting both models only 3.9% of the times.

Linearized Cox (NL) test: This test seems to be plagued by large observed significance levels ranging from a minimum of 0.05 to 0.173, in a similar manner as its predecessor. As in the case of the unadjusted Cox test, its ranking in terms of the p-values with which it rejects the false null are fairly small (generally between 2.5 and 4.5 on the 10 scale ranking). In addition, this test indeed suffers from reduced power in cases where the true and alternative models have an equal number of regressor variables. Each of the testing procedures tends to show an interaction effect between R^2 and ρ^2 in the equal k case, but it is much more pronounced in the results of the NL-test.

If its behavior under the situation of two incorrect models is addressed, the NL-test indicates the presence of questionable models more than half of the time, in general. Even though all of the testing procedures tend to exhibit a reduced ability in detection as the collinearity between the models increases, the NL-test is affected to a stronger degree. Overall, this testing procedure, although appealing with its ease of calculation, is too volatile to be of practical use.

The J-test: As Godfrey and Pesaran indicated on the basis of their empirical work, the J-test exhibits a tendancy toward overrejecting the true null model in the presence of an alternative which has more regressors. This result is demonstrated clearly through the results of this study. In particular, consider the contrast between observed power of 0.957 ($\hat{\alpha}$ = 0.043) in Experiment 2 and of 0.970 ($\hat{\alpha}$ = 0.030) in Experiment 22 when the true model had 6 regressor variables and the false model had only 2 and the cases where the true model, having only 2 regressors was maintained against an alternative model of 6: i.e., observed power of 0.678 ($\hat{\alpha}$ = 0.239) in Experiment 5 and that of 0.744 ($\hat{\alpha}$ = 0.190) in Experiment 23. These indeed bear out the inherent problems with the J-test when the models contain different numbers of parameters. How it performs under the "equal k" case will be addressed in the next section.

However, if its performance in the case of two false models under test is examined, the J-test once again is often misled because of different numbers of parameters in the models. It has a tendency to reject the model with fewer regressor variables in favor of another false model which contains a larger number of regressors. In addition, as the collinearity between the models increases, its ability to detect the two models as being false diminishes as with the other procedures. Overall, it tends to detect the presence of the incorrect models just about 50% of the time, which is again not a favorable result in terms of promoting the use of the procedure in practical applications. This result then encourages the investigation into the advantages, if there are any substantial ones, of incorporating Godfrey and Pesaran's adjustment to the test.

The AJ-test: As the work of Godfrey and Pesaran (1982) showed, the observed significance levels of the test in practice are brought down in magnitude dramatically by using their adjustment to the J-test. In fact, they have been brought down well below the nominal level of 0.05 to an overall average for the experimental runs of 0.024. In terms of observed power, the AJ-test tends to do quite well under most situations. However, its power can become quite small in cases where the R^2 is low and the collinearity between models is increased. Particularly, in situations where the true model also has fewer regressors (i.e., in addition to R^2 low and ρ^2 high), the reduction in power is intensified. Interestingly enough, this observation did not coincide with an increase in the observed significance level as was the case with the unadjusted J-test.

Turning to the situation in which both models are incorrectly specified, the AJ-test did a much better job of detection than the unadjusted test when the fit on the competing models was relatively poor. Based on cases of $R^2 = 0.50$ and 0.70 for the equal k case, the AJ-test detected questionable models an average of 71.6% of the time, whereas the J-test detected them only an average of 45.3% of the time. This improved ability in detection is rooted in the large proportion of times neither model was rejected, which is a direct result of the adjustment designed to decrease the bias in the J test statistic under the maintained hypothesis. When this information is considered altogether, the AJ-test is a vast improvement over the J-test and is fairly well behaved with the exception of cases where the fit was relatively poor and the collinearity between competing models large.

The JA-test As far as the procedures derived under the AN approach are concerned, the JA-test stands in striking contrast to the J-test. Both test procedures are plagued with inherent difficulties when the competing models involve different numbers of regressors due to their construction. However, the similarity stops there. In terms of relative behavior, the J-test favors the alternative to the same degree as the JA-test favors the maintained model. This result is quite obvious by the very small observed significance levels in the case of the JA-test. In general, its $\hat{\alpha}$ averaged 0.025, well below the nominal level of the test, and it reached a maximum of 0.04 over all 50 experimental runs. Since the JA-test results in observed significance levels being so far below the nominal level it is not taking full advantage of its stated probability for making type I errors. In other words, the rejection region for the test could be made larger and still have the nominal size maintained. Therefore, this feature is manifested in terms of the JA-test's power being generally less than that of the other tests under the majority of model conditions investigated. In addition, its conservative approach yields p-value (associated with the rejection of the false null) rankings being on average 7.5 on the 1-10 scale, which is relatively poor in comparison to its competitors.

It is interesting to see how the two testing procedures from the AN approach can yield such different inferences. The JA-test, as is evidenced here and in the work of Godfrey and Pesaran, has the tendency to exhibit high power when the true model has fewer parameters than a false alternative, and very low power when the situation is reversed. (This result is the opposite of the J-test.) Particularly, its power is greatly affected when the difference $|k_1 - k_2|$ grows in magnitude. By examining the JA-test's performance on those experimental runs examined under the J-test, the following is observed: When the two variable model was true and tested against an alternative with 6 variables, the observed power was 0.852 ($\hat{\alpha}$ = 0.026) for Experiment 23 and 0.787 ($\hat{\alpha}$ = 0.024) for

Experiment 5; whereas for cases involving a true 6 regressor model versus a 2 variable model, the observed powers were 0.304 ($\hat{\alpha}$ = 0.022) for Experiment 2 and 0.322 ($\hat{\alpha}$ = 0.026) for Experiment 22. Overall, the JA-test's power is not as large as the majority of its competitor's, and is highly unpredictable when there are different numbers of regressors in the competing models.

Under the situation where both models are incorrectly specified, the JA-test is able to detect questionable models about 63.8% of the time overall. In particular, when the relative fits are poor on the models under consideration, the JA-test does a better job of detection. As in the case of Atkinson's test, the detection of questionable models relies mainly on the non-rejection of both models when maintained against one another. Although the test fares well in terms of the detection of false models, it still is deficient in terms of power. Therefore, it will be interesting to see how much improvement is accomplished by the modified JA-test, NJ.

The NJ-test: This testing procedure represents the combination of the JA-test and the Orthodox F-test. Consequently, its performance should tend to lie "in between," in some sense, the performance of the JA- and F-tests on which it is based. Of particular interest is to see whether the observed significance levels for this modified JA-test are more closely aligned with the nominal level of the test. If this has been obtained, then the power of the test should be increased above that of the JA-test, if not in general, at least for some of the cases in which the JA-test was lacking. Clearly, the observed significance levels from these experimental runs lie close to the nominal 0.05 level, with an average over all the experimenatal runs being 0.054. In addition, its power is larger than that of the JA-test in general. Specifically, in the cases where the number of regressors in the true model is larger than that in the alternative, the NJ-test yields substantial increases in power for most situations. When the number of regressors are the same, the two tests tend to perform equally well in terms of power. On the other hand, a slight price is paid as far as power is concerned in the cases where the true model has fewer regressors than its alternative by using the NJ-test in place of the JA-test. However, the magnitude of the power improvement for other cases more than compensates for this slight reduction in power. When consideration is given to the p-value rankings, the NJ-test comes out only slightly better than the unmodified test, with an average ranking of about

7. Therefore, it is clear that the modification is slight enough not to change the basic nature of the testing procedure, but significant enough to improve its power where needed most.

Considering the situaiton in which both models under test are misspecified, the NJ-test detects the problem an average of 61% of the time. In this aspect, the NJ-test's detection ability is quite similar to that of the unmodified JA-test, although it has a tendency to be slightly less helpful. Once again the conservative approach in the design of the JA-test carries over to the NJ-test in that the detection of questionable models relies primarily on the case where neither model is able to reject the other. On the basis of these empirical results, it is clear that the NJ-test is slightly less sensitive to the unequal regressor case. However, in terms of overall power, it is not the most powerful of the procedures investigated here. Since it does have an exact non-null distribution, it is worth consideration. Also, the modification to the NJ-test involved the use of the error variance estimator from the comprehensive model approach, which brings up the issue of how well this modified JA-test performs relative to the Orthodox F-test.

Orthodox F-test: As the only procedure which is truly not rooted in the Cox non-nested approaches, the Orthodox F-test warrants investigation into its performance and comparisons made with the other procedures on a relative basis. First, since this test has an exact null distribution, as the JA- and NJ-tests do, its observed significance levels over the range of experiments should remain close to the nominal 0.05 level. The overall average size of the F-test in the study was 0.0494, which is indeed in line with the theory. Therefore, it is important to see how well the test performs in terms of its power to make the correct inference. Overall, the power of the F-test tends to remain close to that of the NJ-test, with the exception of those cases in which it is not adversely affected by the false alternative model under test having fewer regressor variables than the true model. In such instances, the F-test's power remains relatively large. On the other hand, the F-test's power trails that of the NJ-test when the quality of fit of the true model is low. Another interesting fact about the F-test is that its power is larger than that of the NJ-test when the collinearity between the competing models is very small. This result should be of no surprise based on the difference in the "nesting" methods used to derive the two testing procedures. (See Section 3.3.2 for further discussion on this point.) Further attention will be given later to the performance of the F-test relative to that of the NJ-test. However, on its own merit, the F-test is farirly powerful, but is often outdone by its non-nested competitors. In terms of its p-value rankings, it has an average ranking of about 7.4. It does slightly better in terms of this p-value ranking as well as the power to make the correct inference when the sample size is increased to 40.

However, another issue is how this nesting approach fares in terms of detecting the presence of two misspecified models. By the nature of the comprehensive model formulation, a more accurate ability for detecting invalid models would be expected in general. In the experimental runs discussed in this study based on samples of size 20, the F-test detected the inappropriate models 70.7% of the time on average, which was larger than that for any other test. In paricular, the F-test was able to detect the presence of the false models a larger proportion of the time (0.853, 0.789 on average) when the quality of fit was relatively poor (in particular, the cases where the true model had $R^2 = 0.50, 0.70$). Also, it does not seem to fall into the trap of uniform distribution into the four inferential categories when the number of regressors in the competing false models were equal: on average, it detected the misspecified models 72.7% of the time. By weighing the evidence regarding this "non non-nested" procedure, it can be quite useful in practical applications, particularly if it is used in conjunction with some of the non-nested procedures.

Now that all the procedures have been discussed briefly in an overall context and their strong/weak points highlighted, some useful statements can be made that go a long way toward setting forth practical guidelines. However, investigation into several of the specifics of the model characteristics and their effects on the resulting inferences is warranted before such recommendations can be made.

Based on the information gained from the overview of the various testing procedures as far as observed power, observed significance levels and the p-value rankings on the rejection of the false model maintained against the true model, several tests have been deemed inappropriate for the situations involving such small samples. In particular, the unadjusted Cox (N) test, the Linearized Cox (NL) test and possibly the J-test can all be ruled out of the lineup of practical testing procedures for the small sample cases on the basis of their large observed significance levels. In addition, Atkinson's (NA) test is also one that can be eliminated from further investigation on the basis of its overall low power.

Therefore, the remaining tests, as well as the J-test, will be examined under the situation involving equal numbers of (non-overlapping) regressor variables in the competing models, which will be referred to as the "equal k" case.

4.3.1.2 Equal k Case

Some motivation was presented in the earlier part of this section for investigating the case of equal numbers of regressors. For many practical applications, as in the empirical demand study in Chapter V, it is a functional form issue and not one of variable selection which brings about non-nested regression models. Since it is a situation which arises often in the empirical applications of these procedures and the primary objective of this study is to formulate guidelines for such situations, the "equal k" case deserves special attention.

In particular, the experimental runs to be considered in this section encompass competing models based on 4 regressor variables each and for samples of data of 20 observations. Then, within this context, overall performance of the tests can be compared for the equal k case. In addition, it provides a feasible setting for investigating the influence of R^2 for the true model and ρ^2, the collinearity between competing models, on the inferential ability of the various procedures. Therefore, the tests will be examined on the basis of observed power and significance levels for models under the conditions: R^2 = 0.50, 0.70, 0.75, 0.90 and ρ^2 = 0.25, 0.50, 0.75, 0.90 (which correspond to Experiments 4, 6, 10, 16, 33-40 and 47-50).

Using the results from these experimental runs, the overall stability of the procedures should first be addressed. In terms of observed significance levels over all the combinations of R^2 and ρ^2, the size of all the testing procedures under examination (i.e., excluding the N-, NA- and NL-tests), definitely exhibit a stability which was not always present in situations involving models with unequal numbers of regressors. In particular, the J-test has an observed significance level which can be much larger than the nominal level under the unequal k cases. In the equal k case, however, the significance levels observed when the J-test was used still tend to exceed the nominal level when the fit of the true model is relatively poor or when the collinearity between models is very low, but not to the same degree. What is seen, then, is that even under the equal k case, the J-test, although much better behaved, is still overly influenced by the bias in the numerator of the test statistic under H_1 and thus yields an observed size of the test larger than the nominal level much too frequently.

For the remaining tests, which are either exact under the null or representative of small sample modifications to the Cox and J-test, the levels are well within reasonable bounds of the nominal size. In particular, when the observed significance levels from the various tests are considered as dependent variables in a two-way ANOVA with R^2 and ρ^2 as treatments, neither the interaction effect, $R^2 \times \rho^2$, nor either main effect (R^2 or ρ^2) are significant for any of these remaining procedures with the exception of the $\bar{N}$ test. (See Appendix E.1.) For this adjusted Cox test, the significance level appears to be influenced by R^2, with the borderline significance of the other two effects. If Duncan's multiple range test is used to test for significant differences in the mean observed significance levels over R^2 and ρ^2 , not all the means are determined to be equal. On the other hand, it appears that the mean observed levels are centered within a reasonable proximity of the nominal 0.05 level, even though they fluctuate slightly as the levels of R^2 and ρ^2 vary. Since any differences among the average observed type I error probabilities over levels of R^2 represent a reduction on the observed size from the nominal level in general, such variations resulting from changes in R^2 and ρ^2 do not provide evidence against the stability of the $\bar{N}$-test's behavior.

Once again, it should be pointed out that the $\tilde{N}$-, NJ- and F-tests all tend to maintain the size close to the nominal level. With the W-, AJ- and JA- tests, however, the specified level is an overstatement of the true type I error probabilities (as estimated by the Monte Carlo study).

Since all of these 6 testing procedures are acceptable from the perspective of significance levels, the power properties of these procedures should be examined in general. A good starting point is to use the same two-way ANOVA approach that was employed in the analysis of the observed significance levels. The analysis determined that all of the effects, all of the effects, interaction and main, were significant regarding the influence of R^2 and ρ^2 on the power of these tests. (See Appendix E.2.) Therefore, two very meaningful pieces of information can be used to determine which procedures work best under various model conditions. This can be quite helpful for making practical guidelines since the fit on the competing models as well as the degree of the collinearity between the competing sets of regressors can be computed from the researcher's data. The computed R^2's and ρ^2's can be used as "ball-park" estimates of the model characteristics surrounding the correctly specified model, whether or not it is one of the models under investigation. This information can then be used in conjuction with the trends observed here to provide the researcher with the best procedure(s) to be used in his particular application.

Considering the power of the tests across these 16 combinations of $R^2 \times \rho^2$ under the equal k case with samples of size 20, statements regarding the relative power of the procedures can be made. It is clear that the $\tilde{N}$ test exhibits that largest power for all levels of ρ^2 when the true model has a coefficient of determination of 0.50 or 0.70. Once the R^2 on the true model reaches 0.75, any marked differences in power among the various procedures have been greatly diminished.

For the lower three levels of collinearity between the models, given $R^2 = 0.75$, the AJ-test demonstrated slightly larger power than the $\tilde{N}$-test, with the AJ-test's power tapering off a bit quicker than that of $\tilde{N}$ as ρ^2 was increased. Then once the true model reached an $R^2 = 0.90$, the $\tilde{N}$- and AJ-tests both generally exhibit larger power over all the collinearity levels. These two tests are quite comparable in terms of power for this case in that no meaningful advantage is gained by either test.

This result does not imply that the other testing procedures are not useful since they do not achieve the largest observed power. In particular, the NJ- and JA-tests perform well when the R^2 is at least 0.75 and the collinearity between the competing models' regressors is at least as large as $\rho^2 = 0.50$. With consideration to the W-test, although the $\tilde{N}$- test will beat the W-test in terms of power in most instances, the W-test has large power when the fit of the true model is quite good and the collinearity between the models is low.

The collinearity between the models seems to have a positive impact on the power of the test given that the fit on the true model is fairly good up to a point. (See displays of means in Table IV.3.1.) However, when the canonical correlation between the two sets of regressor variables was 0.90, the power of all the testing procedures experience a drop. It is the AN and Orthodox F procedures which are most adversely affected by this increase in collinearity, particularly for R^2 of 0.75. The two adjusted Cox procedures also reflect this relationship, although the $\tilde{N}$-test seems to be the most resistent to it. Therefore, it appears that under this design, the small sample adjustments to the asymptotically valid procedures seem to come out front-runners in the power race.

Next, the issue of detecting two misspecified models is addressed since it is to play a part in determining the practical worth of the testing procedures. The equal k case, by the construction of the models under this Monte Carlo design, should provide some interesting results in this situation. Since the two false models generated will have the same number of regressors with the same collinearity structure with respect to the same true model, it would be of no surprise to find that the procedures uniformly (or randomly) distributed their outcomes over the individual replications across the four possible inferences. In such a case, it would be expected that each of the four possible inferences would occur approximately 25% of the time. Consequently, in this case, the procedures can be judged as to whether or not they really can distinguish between the models on the basis of information other than the observed R^2. To evaluate this, a chi-square test of independence for the two by two contingency tables created by each procedure for all 16 experimental runs will provide useful information toward this end. Here the two-way table was constructed based on proportion of times each test did and did not reject for testing H_2 versus H_3 and H_3 versus H_2.

The F-test, as well as the NJ- and JA-tests, tend to do a much better job at indicating the presence of the false models. Also, on the basis of the chi-square tests of independence between the outcomes on each of the tests, H_1 versus H_2 and H_2 versus H_1, in each case, it is clear that these "AN" procedures (used loosely for the F-test) rely on information other than their relative fit to determine whether the sample evidence is in favor of an alternative. The F-test rejected the null hypothesis of independence in all but one of the 16 experiments, namely Experiment 40, with $R^2 = 0.90$ and $\rho^2 = 0.75$. The JA- and NJ-tests also did quite well with the JA-test rejecting the hypothesis of independence in 14 of the 16 runs (exceptions were Experiments 37, 47, each with $\rho^2 = 0.25$) and the NJ-test rejecting in 13 of the 16 runs (exceptions were Experiments 36, 37, 47). The other testing procedures did not do quite so well: W-test rejected in 11 of 16 runs (exceptions: 10, 34, 37, 39, 47); $\bar{N}$ rejected in 10 of 16 runs (exceptions: 4, 10, 33, 34, 37, 47); and the AJ-test rejected a disappointing 9 of 16 (exceptions: 4, 10, 34, 37, 39, 47, 48).

On the other hand, since the two models are so close in terms of model characteristics, then the situation in which neither model provides sufficient evidence to reject the other may not be very informative at all. It really is indicative of a lack of distinguishability between the models. But if this is what is being indicated by the outcome, isn't it really an indication that further investigation as to the correct model specification is warranted. What the empirical results indicate is that for situations involving equal numbers of regressors, the more conservative approach may be more appropriate, at least from the perspective of detecting false models.

If the information regarding the case of true versus false as well as that of false versus false is pooled together, some interesting ideas about practical use of the tests has been gathered. First, the $\bar{N}$ - and AJ-tests indeed yield larger observed power in general. Therefore, if it is known a priori that one of the two models under consideration should be close enough to being the true model that it can be considered a correct specification, then these tests are sure to lead to the correct inference with a high degree of certainty.

On the other hand, if there are multiple models to be considered, the procedures which tend to be slightly more conservative, the NJ- and F-tests (JA-, too) may also be worth consideration. In other words, several procedures may be applied to the data and then information about their "sensitivity" toward rejecting the null used to formulate the most reasonable inference. More will be said in regards to a practical set of guidelines in section 4.3. (As a sidenote: the procedures above being conservative implies that the procedure requires stronger evidence in order to reject the null in the presence of the specified alternative.)

However, before the equal k case is put aside, it is important to keep in mind its usefulness due to the frequency with which it will be encountered in practice. Based on the above results, the six procedures discussed in depth - $\bar{N}$, W, AJ, JA, NJ and F - all have potential for practical use, as long as their limitations are kept in perspective.

4.3.1.3 Analytic Power Comparisons for the NJ- and F-tests

At this point in the discussion of the Monte Carlo results, many comments regarding the behavior of the NJ- and F-tests, in their own right and relative to one another, have been made. The main purpose of this section is to examine how the analytic power for rejecting the false model in the presence of the true model relates to the observed power of making the correct two-test inference for the same pair of models. Also, in the case of the NJ-test, empirical evidence is compiled in order to see whether the use of $\bar{\lambda}_{NJ} = \lambda_{NJ}$ evaluated at $E_2(\underline{y})$ in the analytic power computation yields results which are close enough to the actual power based on the observed value of λ_{NJ} to make it a reasonable tool for making comparisons.

The latter issue is the first one to be discussed. Overall, the average squared deviation between the true power and the "expected" power (i.e., based on $\bar{\lambda}_{NJ}$) is about 0.0055, or in terms of a standardized deviation, about 0.07, or 7%. The average squared deviation between the two measures is decreasing in the sample size, n, as well as in R^2 and ρ^2, on average. In other words, the closer the estimated $\hat{\underline{y}}_{21}$ under H_2 is to the true $\underline{y}$, the smaller the discrepancy between the two measures of power. The $\bar{\lambda}_{NJ}$ yields a larger power than the true λ_{NJ}, due to the presence of the disturbances (i.e., variability around the true mean of $\underline{y}$). Therefore, for making general comparisons as was done in Section 3.3.2, the $\bar{\lambda}_{NJ}$ provides basic information about the behavior of the power curve for the NJ-test, although it tends to overstate the true power conditional on $P_1\underline{y} = \zeta$; nevertheless this overstating is only slight in most cases.

However, when the analytic power for the NJ-test is compared to that of the F-test, it is done using power computations with the true noncentrality parameter, not its "expected" value. One built-in comparison tool is the actual proportion of replications for a given experiment for which the analytic power of the F-test exceeded that of the NJ-test. Another way to compare the analytic power of these procedures more formally is through a repeated measures design on these analytic powers from the experimental runs, with between-subject (or crossed) effects being n, ρ^2, R^2 and $k_{12} = (k_1, k_2)$, including all two-way interactions. This analysis yielded significance on all between-subject effects, except for the two-way interactions involving k_{12}. (See Table IV.3.2.) Therefore, if the observed analytic powers are averaged over the various effects, the powers are fairly close overall, with the dramatic exception being based on the different numbers of regressor variables in the competing models. As the power curves in Figures 3.1-3.6 showed, the power of the NJ-test is larger, in general, when the number of variables in the true model is less than or equal to that in the false alternative. The F-test gives larger power when the reverse is the case. This result seems to parallel the trends in the observed power for making the correct two-test inference.

Investigating the interaction effect between R^2 and ρ^2 , averaged over the other model characteristics, points out several trends. When the R^2 for the true model is relatively poor (0.50, 0.70), the analytic power for both tests decreases as ρ^2 increases, but the NJ-test holds a slight edge over its competitor. (Recall that these are equal k cases, so tests perform as expected based on the observed inferential power.) Once the fit for the true model has reached $R^2 = 0.75$, the F-test has much higher power, on average, than the NJ-test when the squared canonical correlation between the sets of regressor variables for the true and false models is only 0.25 (i.e., 0.854 compared to 0.795). But as ρ^2 increases, both procedures show substantial reductions in power, although the F-test trails off more quickly. In particular, both tests experienced sharp reductions in power when ρ^2 reached 0.90. This result implies that the average analytic power of the tests become quite similar for large ρ^2 given $R^2 = 0.75$. Then for the case in which the true model fit is quite good, $R^2 = 0.90$, there is no real difference in terms of power.

Referring back to the power curves of Chapter III, it was clear that n, $(k_1 - k_2)$ and the R^2 had a significant influence, in a statistical sense, on the analytic power for the true model. However, since the noncentrality parameters derived in Chapter III were computed using variance-covariance matrices in place of X'_iX_j, the influence of ρ^2 on power was not visible. (Refer back to Appendix B.2.) In practice, though, the degree of collinearity between the models under test is an important factor in determining the true analytic power of both the NJ- and F- tests. By examining the form of the noncentrality parameters, it is clear that the observed matrices X'_iX_j, $i \neq j$ for a given sample play an important role in the power of the testing procedure, and the structure of these matrices depend heavily on the magnitude of ρ^2. As the Monte Carlo results indicate, the analytic power for testing a false alternative against the true model reflects the influence of all the control parameter values, including ρ^2.

As the results indicated, the behavior of the analytic power for one-half of the testing process parallels the behavior of the empirical power for making the correct inference on a given pair of models. Specifically, if the Pearson product-moment correlations between these two power measures are computed over all experimental runs, the following are obtained:

$\hat{\rho}(\hat{P}_{NJ}, power_{NJ}) = 0.94602$ (0.96237 for equal k cases only)

$\hat{\rho}(\hat{P}_F, power_F) = 0.94008$ (0.95377 for equal k cases only)

However, this does not consider the equality of, or the magnitude of the differences in, the actual computed powers. In checking for equality, an interesting observation can be made. For the NJ-test, the actual observed empirical power for making the correct inference is almost always greater than the corresponding analytically computed power. The only exceptions coincide with cases in which the analytic power exceeded 0.95. Then the observed power of making the correct inference was brought back down to approximately 0.95, which reflects the significance level on the reversed test. For the F-test, similar results were observed. In some cases, the increase from the indicated analytic power for rejecting the false model in favor of the true to the observed probability of making a correct inference for the pair of models was quite dramatic. Consequently, this analytic power can be considered a lower bound for the empirical power of making the correct inference for a given pair of models unless this power exceeds $(1 - \alpha)$, which serves as an upper bound for the power.

Further investigation can be made into how these tests perform, but much useful information has already been obtained. In particular, the motivation for making comparisons between the analytic power for detecting the true alternative and that of the inference itself is to gain insight into the latter through something that is computable, or at least estimable. If the value of the non-centrality parameter corresponding to each half of the testing procedure can be estimated from the data, these analytically computed powers for the rejection of a false maintained model in favor of the true alternative can be used to see which of these two procedures may be more useful in a particular application. Quite simply, the added information can only help in drawing the correct conclusion based on the non-nested testing results.

4.3.1.4 Some Additional Comments

There are many ways to analyze the results from this Monte Carlo study which are still untapped. However for the purposes of this study, sufficient information has been drawn from the experimental outcomes to formulate some practical guidelines. A parallel purpose for this study is to evaluate the robustness of these testing procedures to the violation of the normality assumption on the disturbance term. The results from that portion of this study will be presented and discussed in the next section. After they have also been reviewed, the implications of the study regarding practical use of the procedures can be drawn and incorporated with the information from cases involving the classical assumptions.

Before the normal case is considered closed, it is worthwhile to consider the asymptotic properties of some of the procedures. In many situations, samples of size 30 are considered large enough to use limiting approximations. As all statisticians are aware, the quality of the approximation is often dependent on other factors. As the discussion based on the normal disturbance case experiments revealed, a sample of size 40 in the case of non-nested linear regession models is generally not large enough to have the asymptotically valid procedures behave well. In this case, the behavior of the test is measured in terms of its agreement with its asymptotic distribution. Support fo this contention can be gleaned from the Cox (N) test and Linearized Cox (NL) test in particular, and the J-test to some extent. With the increase from a sample size of 20 up to 40, many improvements resulted. However, a warning about assuming that the asymptotic results hold approximately even in samples of size 40 is necessary. (For evidence regarding the behavior of the Cox and J-tests in samples of size 60, see Godfrey and Pesaran, 1982, 1983).

4.3.2 The Non-Normal Disturbance Case

The purpose of this portion of the Monte Carlo study is to evaluate how robust these non-nested testing procedures are to a violation of the normality assumption on the disturbance terms. White (1982), in his discussion of the regularity conditions for the Cox (N) test, addressed the asymptotic validity of the Cox procedure under the case of non-normal disturbances. Since all the procedures discussed thus far are essentially asymptotically equivalent, this asymptotic robustness should hold for the other procedures. However, the main concern here is whether or not this asymptotic property will hold in finite samples as small as 20.

In the work of Godfrey and Pesaran (1983), they investigated the case of skewed non-normal distributions on the disturbances. Once again it is necessary to point out that their work dealt primarily with the W, $\bar{N}$ and F test procedures. Therefore, in the current study, an extension of their work regarding robustness is presented here. This extended study will incorporate all ten of the procedures discussed thus far and an additional pair of non-normal distributions on the disturbance terms, both of which symmetric: the Student t and a truncated normal (10% in tails). (The model generation for the non-normal cases is described in section 4.2.3.)

Since the goal of this part of the study is to evaluate the robustness, and not the relative inferential ability, of the tests under this particular violation of the classical assumptions, only a limited number of situations were examined. Consequently, the simulation was designed to handle all four non-normal distributions simultaneously as well as a normal case. This approach is employed so that the normal and non-normal models will be comparably based on the same sets of generated regressor variables. (The program for this portion is contained in Appendix D.)

Eight experimental runs based on 500 replications each were used since they formed a one-half fraction of a $2^2 \times 4$ factorial design with factors: R^2 at levels 0.75, 0.90; ρ^2 at levels 0.25, 0.50, 0.75, 0.90; and (k_1, k_2, k_3) at levels (4,2,6) and (4,4,4). The sample size has held at 20 for all of the experiments which are given by the following sets of model conditions of the form $\{R^2, \rho^2,(k_1, k_2, k_3)\}$:

NN1: 0.75, 0.25, (4,2,6)	NN5: 0.75, 0.50, (4,4,4)
NN2: 0.90, 0.25, (4,4,4)	NN6: 0.75, 0.75, (4,2,6)
NN3: 0.90, 0.50, (4,2,6)	NN7: 0.75, 0.90, (4,2,6)
NN4: 0.90, 0.75, (4,4,4)	NN8: 0.90, 0.90, (4,4,4)

The results for these experiments are presented in Table IV.3.3 in a test-by-test manner since the robustness of each procedure is determined independently of the other procedures and not on any relative basis. A glance across the different testing procedures and the various non-normal distributions for the experiments outlined above reveals no severe drops in power under the non-normal disturbance models nor increases in observed significance levels. Slight variations are present as would be expected, particularly in the case of either a skewed or heavy-tailed distribution. In order to determine where any significant variation which would question the robustness of a given procedure was coming from, a comparison criterion is needed.

One way to analyze the results in terms of detecting significant differences between the non-normal cases and the normal is through the use of contrasts within the repeated measures framework for analysis of variance. Specifically, the different error distributions would represent levels of the within model effect, and the usual model conditions of R^2, ρ^2, and their interaction would represent the between model effects. Under all ten of the testing procedures, there was no significant effect which corresponded to the different error distributions, when the dependent measure was power or significance level.

Therefore, in order to see if the robustness of the various procedures to the normality assumption is in actuality dependent on the type of non-normal error distribution invoked, the four non-normal distributions for the error terms must be compared to the normal case one at a time. To accomplish this, a series of paired t-tests was performed on the basis of the power of the tests. (See Table IV.3.4.) Once again, there were not many significant differences in terms of power, except in a few instances. In particular, the heavy-tailed Student t distribution yielded power which was significantly different from that of the normal case for several of the procedures. The "AN" procedures: J, AJ, JA, NJ and F, were the ones most affected in terms of power reduction.

Similarly, when the case of two misspecified models is addressed, some of the procedures yielded significant changes in the proportion of times "both models" and "neither model" was rejected. These occurred under the t and chi-square distributional assumptions generally. Overall, it safe to say that although not completely resistent to the presence of non-normal error terms, these testing procedures remained quite robust given the small sample size of 20. However, the "AN" procedures (including the F-test) were less robust to the presence of the heavy-tailed symmetric distribution and the chi-square distribution than were the Cox based approaches. In addition, the asymptotic theory indicates asymptotic validity of all the procedures, so things can only improve with increased sample sizes. In particular, although it too experienced slight changes in power and significance level, the $\bar{N}$-test seems to be the most robust, at least for the distributions investigated here.

4.3.3 Comparisons and Summary Information

When both the normal and non-normal cases are considered simultaneously, the recommendations made previously regarding the six procedures which performed quite adequately in small samples- $\tilde{N}$, W, AJ, JA, NJ and F -have still been upheld. However, it would seem that those tests which were conservative in their approach are the ones which show the largest discrepancies between the normal and non-normal disturbance cases; i.e., the JA, NJ and F testing procedures. This result is still not enough reason to rule out the possible use of these procedures as a means of providing supporting information into the inference making process. The practical guidelines which will now be discussed indicate when the added information from the conservative tests may be desired.

4.4 Practical Guidelines and Warnings

When deciding which of the non-nested testing procedures to use, the researcher should keep a few facts in mind. First, even when the sample is of size 40, those procedures which are only asymptotically valid will not necessarily be close enough to the nominal size to be worth practical consideration. Therefore, the unadjusted Cox (N) test and the Linearized Cox (NL) test are not suitable for small sample situations. Although the Atkinson (NA) test does have an exact finite sample null distribution, it suffers in general from poor power as well as relatively large observed significance levels. This test, too, is not suitable for samples of size 20 and 40, at least by the evidence this study provided. There is sufficient evidence in terms of large significance levels for also eliminating the J-test, in its unmodified form, from the pool of applicable testing procedures. Therefore, warnings concerning the realization of asymptotic behavior in the finite sample setting have led to a sizable reduction in the number of procedures which could be used with some confidence in small samples.

As a result, there are essentially two types of procedures to consider in small sample situations ($20 \leq n \leq 50$). The first being the small sample adjusted versions of the Cox and J-tests. The empirical evidence from this study showed that the $\bar{N}$-test possesses the best power properties overall and its observed significance level is well in line with the nominal level. The other tests in this category are the Wald-type adjusted version of the Cox test, W, and the AJ-test, the modified version of Davidson and MacKinnon's J-test. The W-test enjoys fairly large power in general and is noted for a reasonably large detection rate regarding the situation involving two misspecified models. On a complementary note regarding the Adjusted J-test (AJ), its observed significance level is often smaller than the nominal size and yet it exhibits large power when the fit of the true model is indeed quite good and the number of regressors equal across models. Clearly all three of these procedures have their advantages for use in practice.

The second set represents the "AN" procedures which posess exact null distributions: JA, NJ and F. Recall that the JA- and F-tests both detected situations involving two misspecified models a large percent of the time, overall. In addition, the NJ- and F-tests have exact distributional properties under the alternative which allows lower bounds on the probability of the test yielding a correct inference to be estimated and thus added performance information obtained. The NJ-test tended to exhibit larger power than the F-test under the equal k case. All of these represent sound reasons to consider using the procedures in practical situations. However, a warning should be stated regarding the low power of the JA-test when the true model has more regressor variables than its competition and small observed significance levels which are consistently below the nominal level.

Interestingly enough, these two groups represent differing opinions regarding how much evidence is needed by an alternative model to warrant the rejection of the maintained model, thus deeming it misspecified. The latter group of tests are clearly of a conservative nature when it comes to rejecting the null hypothesis. This contention is supported by the JA- and NJ-tests which will not easily reject a model with only two variables when it is tested against a correct model specification with many more variables. This is not to say that small sample modified procedures require little in order to reject the null - only that they are tempered versions of tests which were biased away from the null model. This difference can be a useful piece of information for making practical guidelines.

In order to make any worthwhile guidelines to be used in practice, two separate cases of models under test must be addressed: equal and unequal numbers of variables across models.

Case 1. Guidelines for the case of equal numbers of regressors across models under test: For this case, the $\bar{N}$-test leads to the largest power in general, although the AJ-test is just as appealing in terms of power when the models have relatively large coefficients of determination (i.e., $R^2 \geq 0.75$). As a supplement, the F- or NJ-test should be used also to cover the possibility that both models being tested are misspecified. (This result would most likely be the case if these two procedures were able to reject neither of the hypothesized models.)

Case 2. Guidelines for the case of unequal numbers of regressors in the competing models (i.e., $k_1 \neq k_2$): In this case, the importance of handling the situation differently from the equal k case is directly related to the magnitude of $|k_1 - k_2|$. If the numbers of non-overlapping variables in the two models are very different, then the following procedure will be essential if the probability of making the correct inference is to be "maximized."

Again, the $\bar{N}$ procedure will provide a good starting point due to its good power properties, although it might not do too well at detecting the case of both models being invalid. Two interesting scenerios can result on the basis of the $\bar{N}$ -test's results:

Scenario 1: Suppose you have two models under test having regressor variables such that $k_1 < k_2$ and the initial test result from the $\bar{N}$-test indicates that the model with k_2 variables is valid. The question arises as to whether or not this is just a result of increased R^2 on the model due to more regressors. Therefore, in order to gain some support for this inference, a more stringent test should be applied to see if the resulting inferences agree. In this scenario, a more stringent test would be either the NJ- or JA-test (although the JA-test may be unreasonable based on its power in such cases). Clearly, the NJ-test would be a test whose reduced power in cases involving the true model having the larger number of regressors would require "more evidence" in some sense in order the reject the model with fewer parameters.

If the NJ- and $\bar{N}$-tests yield the same inference, namely that the k_2 model is indeed valid, extra support for the original inference was obtained. On the other hand if they disagree, further investigation is warranted using other non-nested procedures, such as the F and W-tests, as well as other criteria.

Scenario 2: Suppose you have the same two models with $k_1 < k_2$, but this time the initial inference resulting from the $\bar{N}$-test is that the model with k_1 regressors is the valid model. In this case additional support of the $\bar{N}$'s inference would come from a "less stringent" procedure such as the AJ- or J-test (once again, as in the case of JA and NJ, the unmodified test's size is generally much larger than the nominal level making it almost certain that it would reject the "smaller" model). As the Monte Carlo study evidenced, the AJ-test, although less biased than the unmodified J-test, has fairly large power for rejecting the smaller model in favor of a larger one.

If the AJ-test yields the same inference as the $\bar{N}$-test, then even more evidence has been compiled to conclude that the k_1 model was indeed valid. If, however, the two procedures yield different results, as in the case involving the NJ-test, further investigation is warranted, not only by using other non-nested procedures such as the F-test, but through the comparison of other model criteria.

Based on the empirical evidence from this Monte Carlo study, these guidelines should help the researcher use the non-nested testing procedures with greater confidence. Clearly, if the results from the study extend to the more general arena, which they no doubt will, then the use of these simple recommendations in small sample settings should improve the probability of making the correct inference regarding the validity of the models under test. Therefore, from a practical perspective, the Monte Carlo study has accomplished what it set out to do.

Table IV.2.1: Experimental Runs for the Normal Deviate Case

Run	n	R^2	ρ^2	(k_1, k_2, k_3)
1	20	0.75	0.25	(4,2,6)
2	20	0.75	0.25	(6,2,4)
3	20	0.75	0.50	(2,4,6)
4	20	0.75	0.50	(4,4,4)
5	20	0.75	0.75	(2,4,6)
6	20	0.75	0.75	(4,4,4)
7	20	0.75	0.90	(4,2,6)
8	20	0.75	0.90	(6,2,4)
9	20	0.90	0.25	(2,4,6)
10	20	0.90	0.25	(4,4,4)
11	20	0.90	0.50	(4,2,6)
12	20	0.90	0.50	(6,2,4)
13	20	0.90	0.75	(4,2,6)
14	20	0.90	0.75	(6,2,4)
15	20	0.90	0.90	(2,4,6)
16	20	0.90	0.90	(4,4,4)
17	40	0.75	0.25	(2,4,6)
18	40	0.75	0.25	(4,4,4)
19	40	0.75	0.50	(4,2,6)
20	40	0.75	0.50	(6,2,4)
21	40	0.75	0.75	(4,2,6)
22	40	0.75	0.75	(6,2,4)
23	40	0.75	0.90	(2,4,6)
24	40	0.75	0.90	(4,4,4)
25	40	0.90	0.25	(4,2,6)
26	40	0.90	0.25	(6,2,4)
27	40	0.90	0.50	(2,4,6)
28	40	0.90	0.50	(4,4,4)
29	40	0.90	0.75	(2,4,6)
30	40	0.90	0.75	(4,4,4)
31	40	0.90	0.90	(4,2,6)
32	40	0.90	0.90	(6,2,4)

Table IV.2.1: (continued)

Run	n	R^2	ρ^2	(k_1, k_2, k_3)
33	20	0.50	0.25	(4,4,4)
34	20	0.50	0.50	(4,4,4)
35	20	0.50	0.75	(4,4,4)
36	20	0.50	0.90	(4,4,4)
37	20	0.75	0.25	(4,4,4)
38	20	0.75	0.90	(4,4,4)
39	20	0.90	0.50	(4,4,4)
40	20	0.90	0.75	(4,4,4)
41	20	0.75	0.50	(4,2,6)
42	20	0.90	0.25	(4,2,6)
43	20	0.90	0.90	(6,2,4)
44	40	0.90	0.50	(2,4,6)
45	40	0.90	0.50	(6,2,4)
46	40	0.90	0.75	(4,2,6)
47	20	0.70	0.25	(4,4,4)
48	20	0.70	0.50	(4,4,4)
49	20	0.70	0.75	(4,4,4)
50	20	0.70	0.90	(4,4,4)

Table IV.2.2 Results of Normal Deviate Experiments

Experiment: 01

Parameters: n = 20 $R^2 = 0.75$ $\rho^2 = 0.25$ $(k_1, k_2, k_3) = (4,2,6)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.881682 (0.01131)	0.674905 (0.027126)
$Power_{NJ}$	0.605316 (0.08198)	0.776474 (0.030767)
$Power_F > Power_{NJ}$	0.8060	0.1690
$Power_{\overline{NJ}}$	0.648731 (0.07082)	0.785694 (0.040897)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.013740	0.017374

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.896 (.00966)	0.104 (.00966)	2.1925	0.789 (.01291)	0.211 (.01291)	2.7255
W	0.956 (.00649)	0.033 (.00565)	5.7910	0.932 (.00796)	0.032 (.00557)	6.9975
$\bar{N}$	0.957 (.00642)	0.038 (.00605)	2.7010	0.936 (.00774)	0.049 (.00683)	2.7010
NA	0.520 (.01581)	0.071 (.00813)	8.8670	0.842 (.01154)	0.078 (.00848)	7.0775
NL	0.907 (.00919)	0.093 (.00919)	2.5530	0.829 (.01191)	0.171 (.01191)	3.2990
J	0.962 (.00605)	0.037 (.00597)	3.9310	0.827 (.01197)	0.171 (.01191)	4.6270
AJ	0.973 (.00513)	0.015 (.00385)	5.2915	0.947 (.00709)	0.029 (.00531)	5.2495
JA	0.515 (.01581)	0.022 (.00464)	9.1045	0.823 (.01208)	0.028 (.00522)	7.2700
NJ	0.744 (.01381)	0.042 (.00635)	7.1750	0.886 (.01006)	0.054 (.00715)	6.5560
F	0.950 (.00690)	0.035 (.00581)	7.4085	0.851 (.01127)	0.064 (.00774)	8.4970
Kendall's W	0.769899 (.000)			0.506009 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 01 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.436	0.010	0.483	0.071
W	0.019	0.805	0.095	0.081
$\bar{N}$	0.031	0.680	0.180	0.109
NA	0.012	0.759	0.083	0.146
NL	0.414	0.013	0.490	0.083
J	0.180	0.113	0.661	0.046
AJ	0.020	0.771	0.161	0.048
JA	0.002	0.853	0.073	0.072
NJ	0.012	0.820	0.087	0.081
F	0.033	0.780	0.119	0.068

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 02

Parameters: $n = 20$ $R^2 = 0.75$ $\rho^2 = 0.25$ $(k_1, k_2, k_3) = (6,2,4)$ $m = 1000$

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.797130 (0.02046)	0.673709 (0.025820)
$Power_{NJ}$	0.445039 (0.07837)	0.609523 (0.065880)
$Power_F > Power_{NJ}$	0.8610	0.5220
$Power_{\overline{NJ}}$	0.479578 (0.07297)	0.647423 (0.072540)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.007555	0.026509

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.855 (.01114)	0.145 (.01114)	2.4650	0.853 (.01120)	0.147 (.01120)	2.3215
W	0.937 (.00769)	0.029 (.00531)	6.1775	0.909 (.00910)	0.025 (.00494)	6.7570
$\bar{N}$	0.951 (.00683)	0.034 (.00573)	3.0215	0.938 (.00763)	0.037 (.00597)	3.0895
NA	0.307 (.01459)	0.099 (.00945)	8.6095	0.571 (.01566)	0.095 (.00928)	7.9875
NL	0.860 (.01098)	0.140 (.01098)	2.6855	0.867 (.01074)	0.133 (.01074)	2.7985
J	0.957 (.00642)	0.043 (.00642)	3.7360	0.925 (.00833)	0.075 (.00833)	3.9015
AJ	0.963 (.00597)	0.016 (.00397)	4.8270	0.947 (.00709)	0.020 (.00443)	5.0360
JA	0.304 (.01455)	0.022 (.00464)	9.2315	0.547 (.01575)	0.025 (.00494)	8.6565
NJ	0.571 (.01566)	0.050 (.00690)	7.0920	0.782 (.01306)	0.046 (.00663)	6.5430
F	0.918 (.00868)	0.047 (.00670)	7.1675	0.848 (.01136)	0.044 (.00649)	7.9090
Kendall's W	0.711844 (.000)			0.644823 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 02 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.350	0.019	0.463	0.168
W	0.011	0.800	0.105	0.084
$\bar{N}$	0.020	0.723	0.153	0.104
NA	0.016	0.796	0.081	0.107
NL	0.319	0.025	0.474	0.182
J	0.108	0.290	0.496	0.106
AJ	0.007	0.808	0.126	0.059
JA	0.004	0.866	0.063	0.067
NJ	0.014	0.835	0.077	0.074
F	0.019	0.799	0.110	0.072

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 03

Parameters: n = 20 R^2 = 0.75 $\rho^2 = 0.50$ $(k_1, k_2, k_3) = (2,4,6)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.756183 (0.02153)	0.690914 (0.024772)
$Power_{NJ}$	0.825962 (0.01733)	0.779198 (0.020923)
$Power_F > Power_{NJ}$	0.0600	0.0400
$Power_{\overline{NJ}}$	0.837298 (0.01534)	0.765011 (0.034349)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.002032	0.015466

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.845 (.01145)	0.154 (.01142)	2.2565	0.791 (.01286)	0.209 (.01286)	2.6860
W	0.936 (.00774)	0.041 (.00627)	6.5165	0.923 (.00843)	0.038 (.00605)	7.3520
$\tilde{N}$	0.924 (.00838)	0.066 (.00786)	2.3470	0.937 (.00769)	0.045 (.00656)	2.2840
NA	0.917 (.00873)	0.056 (.00727)	8.2205	0.910 (.00905)	0.067 (.00791)	6.8150
NL	0.889 (.00994)	0.107 (.00978)	3.4035	0.859 (.01101)	0.140 (.01098)	3.5160
J	0.883 (.01017)	0.106 (.00974)	4.9325	0.800 (.01266)	0.187 (.01234)	5.2610
AJ	0.934 (.00786)	0.034 (.00573)	7.3495	0.931 (.00802)	0.022 (.00464)	6.4005
JA	0.933 (.00791)	0.025 (.00494)	6.2795	0.936 (.00774)	0.017 (.00409)	5.8930
NJ	0.922 (.00848)	0.052 (.00702)	7.5325	0.909 (.00910)	0.048 (.00676)	7.6960
F	0.884 (.01013)	0.059 (.00745)	6.1775	0.882 (.01021)	0.043 (.00642)	7.0965
Kendall's W		0.541307 (.000)			0.464313 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 03 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.636	0.002	0.209	0.153
W	0.052	0.590	0.138	0.220
$\bar{N}$	0.113	0.388	0.213	0.286
NA	0.065	0.422	0.199	0.314
NL	0.561	0.004	0.252	0.183
J	0.444	0.040	0.342	0.174
AJ	0.066	0.542	0.181	0.211
JA	0.024	0.558	0.176	0.242
NJ	0.033	0.536	0.182	0.249
F	0.044	0.721	0.092	0.143

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 04

Parameters: n = 20 $R^2 = 0.75$ $\rho^2 = 0.50$ $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.596498 (0.02493)	0.600657 (0.022390)
$Power_{NJ}$	0.748413 (0.02947)	0.755419 (0.026989)
$Power_F > Power_{NJ}$	0.1000	0.0930
$Power_{\overline{NJ}}$	0.789605 (0.02000)	0.771000 (0.035493)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.009073	0.015690

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.869 (.01067)	0.131 (.01067)	2.0470	0.850 (.01130)	0.149 (.01127)	2.2015
W	0.900 (.00949)	0.041 (.00627)	7.0455	0.928 (.00818)	0.036 (.00589)	7.1035
$\bar{N}$	0.919 (.00863)	0.058 (.00740)	2.6620	0.930 (.00807)	0.054 (.00715)	2.5680
NA	0.838 (.01166)	0.088 (.00896)	7.6535	0.862 (.01091)	0.078 (.00848)	7.6245
NL	0.901 (.00945)	0.098 (.00941)	3.0945	0.898 (.00958)	0.101 (.00953)	3.1095
J	0.930 (.00807)	0.059 (.00745)	4.0440	0.935 (.00780)	0.063 (.00769)	4.0820
AJ	0.925 (.00833)	0.024 (.00484)	5.9700	0.935 (.00780)	0.027 (.00513)	5.8965
JA	0.862 (.01091)	0.023 (.00474)	7.2585	0.871 (.01061)	0.028 (.00522)	7.2990
NJ	0.875 (.01046)	0.057 (.00734)	6.7015	0.901 (.00945)	0.047 (.00670)	6.5950
F	0.822 (.01210)	0.046 (.00663)	8.5370	0.827 (.01197)	0.041 (.00627)	8.5205
Kendall's W	0.619672 (.000)			0.613459 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 04 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.563	0.008	0.215	0.214
W	0.114	0.488	0.193	0.205
$\bar{N}$	0.178	0.334	0.250	0.238
NA	0.088	0.417	0.237	0.258
NL	0.466	0.023	0.255	0.256
J	0.328	0.096	0.296	0.280
AJ	0.099	0.502	0.199	0.200
JA	0.048	0.540	0.193	0.219
NJ	0.066	0.503	0.202	0.229
F	0.090	0.613	0.140	0.157

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 05

Parameters: n = 20 R^2 = 0.75 ρ^2 = 0.75 (k_1, k_2, k_3) = (2,4,6) m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.475189 (0.01790)	0.419554 (0.016794)
$Power_{NJ}$	0.586030 (0.02073)	0.531912 (0.021291)
$Power_F > Power_{NJ}$	0.0110	0.0140
$Power_{\overline{NJ}}$	0.596471 (0.01988)	0.538639 (0.032168)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000689	0.012708

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.816 (.01226)	0.166 (.01177)	2.2535	0.731 (.01403)	0.263 (.01393)	3.0430
W	0.794 (.01280)	0.044 (.00649)	7.7410	0.771 (.01329)	0.034 (.00573)	7.8115
$\bar{N}$	0.851 (.01127)	0.070 (.00807)	2.5385	0.833 (.01180)	0.069 (.00802)	2.5905
NA	0.828 (.01194)	0.068 (.00796)	6.3075	0.842 (.01154)	0.057 (.00734)	4.9480
NL	0.827 (.01197)	0.123 (.01039)	3.5935	0.771 (.01329)	0.199 (.01263)	3.8660
J	0.788 (.01293)	0.115 (.01009)	5.0680	0.678 (.01478)	0.239 (.01349)	5.6940
AJ	0.795 (.01277)	0.028 (.00522)	7.1890	0.755 (.01361)	0.039 (.00613)	6.6250
JA	0.807 (.01249)	0.033 (.00565)	5.9870	0.787 (.01295)	0.024 (.00484)	5.6700
NJ	0.793 (.01282)	0.050 (.00690)	7.3390	0.762 (.01347)	0.048 (.00676)	7.3270
F	0.699 (.01451)	0.042 (.00635)	6.9985	0.649 (.01510)	0.053 (.00709)	7.4250
Kendall's W	0.478547 (.000)			0.409325 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 05 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.554	0.009	0.258	0.179
W	0.049	0.576	0.160	0.215
$\bar{N}$	0.136	0.336	0.233	0.295
NA	0.080	0.348	0.227	0.345
NL	0.446	0.018	0.307	0.229
J	0.379	0.057	0.364	0.200
AJ	0.076	0.521	0.195	0.208
JA	0.038	0.519	0.203	0.240
NJ	0.042	0.495	0.216	0.247
F	0.050	0.726	0.095	0.129

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 06

Parameters: n = 20 $R^2 = 0.75$ $\rho^2 = 0.75$ $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.330075 (0.01146)	0.327821 (0.010821)
$Power_{NJ}$	0.546323 (0.02245)	0.542773 (0.021765)
$Power_F > Power_{NJ}$	0.0060	0.0090
$Power_{\overline{NJ}}$	0.578697 (0.01979)	0.575413 (0.035441)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.003173	0.015173

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.853 (.01120)	0.122 (.01035)	1.9185	0.868 (.01071)	0.118 (.01021)	1.9185
W	0.747 (.01375)	0.039 (.00613)	7.6670	0.769 (.01333)	0.035 (.00581)	7.7835
$\bar{N}$	0.827 (.01197)	0.066 (.00786)	2.9060	0.855 (.01114)	0.050 (.00690)	2.8190
NA	0.773 (.01325)	0.080 (.00858)	6.1360	0.782 (.01306)	0.079 (.00853)	6.1400
NL	0.846 (.01142)	0.101 (.00953)	3.2060	0.866 (.01078)	0.087 (.00892)	3.1365
J	0.845 (.01145)	0.054 (.00715)	4.1670	0.875 (.01046)	0.040 (.00620)	4.0560
AJ	0.760 (.01351)	0.028 (.00522)	6.6830	0.799 (.01268)	0.016 (.00397)	6.6530
JA	0.767 (.01337)	0.027 (.00513)	6.5070	0.775 (.01321)	0.025 (.00494)	6.6255
NJ	0.748 (.01374)	0.055 (.00721)	6.8385	0.764 (.01343)	0.050 (.00690)	6.8675
F	0.535 (.01578)	0.053 (.00709)	8.9850	0.557 (.01572)	0.049 (.00683)	9.0005
Kendall's W	0.606749 (.000)			0.637769 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 06 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.429	0.026	0.255	0.290
W	0.088	0.361	0.271	0.280
$\bar{N}$	0.189	0.203	0.281	0.327
NA	0.111	0.277	0.294	0.318
NL	0.309	0.068	0.286	0.337
J	0.193	0.147	0.306	0.354
AJ	0.071	0.406	0.254	0.269
JA	0.073	0.383	0.269	0.275
NJ	0.073	0.362	0.276	0.289
F	0.064	0.614	0.161	0.161

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 07

Parameters: n = 20 R^2 = 0.75 $\rho^2 = 0.90$ $(k_1, k_2, k_3) = (4,2,6)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.715245 (0.02222)	0.129463 (0.001113)
$Power_{NJ}$	0.180046 (0.00688)	0.242237 (0.005917)
$Power_F > Power_{NJ}$	0.9970	0.0020
$Power_{\overline{NJ}}$	0.185135 (0.00682)	0.259158 (0.011075)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000101	0.004570

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.903 (.00936)	0.095 (.00928)	1.8970	0.670 (.01488)	0.208 (.01284)	2.6570
W	0.909 (.00910)	0.046 (.00663)	5.0470	0.393 (.01545)	0.051 (.00696)	7.7250
$\bar{N}$	0.928 (.00818)	0.049 (.00683)	2.9050	0.551 (.01574)	0.049 (.00683)	3.8280
NA	0.062 (.00763)	0.091 (.00910)	8.7335	0.528 (.01579)	0.091 (.00910)	4.1285
NL	0.913 (.00892)	0.085 (.00882)	2.4825	0.613 (.01541)	0.167 (.01180)	3.3745
J	0.960 (.00620)	0.038 (.00605)	4.1260	0.508 (.01582)	0.132 (.01071)	4.9810
AJ	0.931 (.00802)	0.031 (.00548)	5.6165	0.446 (.01573)	0.031 (.00548)	6.7845
JA	0.076 (.00838)	0.034 (.00573)	9.4915	0.454 (.01575)	0.019 (.00432)	6.1765
NJ	0.260 (.01388)	0.069 (.00802)	7.7695	0.424 (.01564)	0.062 (.00763)	6.9730
F	0.881 (.01024)	0.058 (.00740)	6.9505	0.201 (.01268)	0.069 (.00802)	8.3720
Kendall's W		0.820392 (.000)			0.455658 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 07 (continued)

Testing H_2 vs H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.295	0.001	0.696	0.008
W	0.057	0.262	0.603	0.078
$\bar{N}$	0.108	0.138	0.693	0.061
NA	0.011	0.732	0.045	0.212
NL	0.251	0.002	0.734	0.013
J	0.152	0.006	0.838	0.004
AJ	0.097	0.165	0.708	0.030
JA	0.001	0.822	0.045	0.132
NJ	0.011	0.719	0.136	0.134
F	0.102	0.277	0.611	0.010

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 08

Parameters: n = 20 R^2 = 0.75 $\rho^2 = 0.90$ $(k_1, k_2, k_3) = (6,2,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.690759 (0.02327)	0.370026 (0.015298)
$Power_{NJ}$	0.130732 (0.00433)	0.189887 (0.006159)
$Power_F > Power_{NJ}$	1.0000	0.9210
$Power_{\overline{NJ}}$	0.134844 (0.00435)	0.210482 (0.010257)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000060	0.003706

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.887 (.01002)	0.113 (.01002)	2.1865	0.890 (.00990)	0.101 (.00953)	1.8350
W	0.919 (.00863)	0.030 (.00540)	5.8720	0.792 (.01284)	0.022 (.00464)	5.9255
$\bar{N}$	0.942 (.00740)	0.031 (.00548)	3.0495	0.874 (.01050)	0.027 (.00513)	3.2795
NA	0.022 (.00464)	0.114 (.01006)	8.8065	0.141 (.01101)	0.087 (.00892)	8.3000
NL	0.886 (.01006)	0.114 (.01006)	2.4305	0.887 (.01002)	0.098 (.00941)	2.6575
J	0.967 (.00565)	0.033 (.00565)	3.7225	0.955 (.00656)	0.024 (.00484)	3.6035
AJ	0.940 (.00751)	0.023 (.00474)	4.8370	0.859 (.01101)	0.018 (.00421)	5.1450
JA	0.030 (.00540)	0.030 (.00540)	9.5265	0.130 (.01064)	0.019 (.00432)	9.2815
NJ	0.211 (.01291)	0.047 (.00670)	7.6920	0.296 (.01444)	0.046 (.00663)	7.5865
F	0.882 (.01021)	0.048 (.00676)	6.8910	0.594 (.01554)	0.041 (.00627)	7.3860
Kendall's W		0.808056 (.000)			0.763464 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 08 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.144	0.043	0.747	0.066
W	0.021	0.491	0.447	0.041
$\dot{N}$	0.036	0.388	0.520	0.056
NA	0.010	0.836	0.054	0.100
NL	0.124	0.070	0.743	0.063
J	0.072	0.153	0.748	0.027
AJ	0.028	0.458	0.483	0.031
JA	0.000	0.879	0.049	0.072
NJ	0.008	0.826	0.093	0.073
F	0.053	0.533	0.398	0.016

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 09

Parameters: n = 20 R^2 = 0.90 $\rho^2 = 0.25$ $(k_1, k_2, k_3) = (2,4,6)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.997474 (0.00010)	0.992037 (0.000849)
$Power_{NJ}$	0.993060 (0.00320)	0.995256 (0.000422)
$Power_F > Power_{NJ}$	0.2360	0.1410
$Power_{\overline{NJ}}$	0.997972 (7.9E-0)	0.992267 (0.001131)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.003115	0.000776

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.896 (.00966)	0.104 (.00966)	2.3355	0.877 (.01039)	0.123 (.01039)	2.3575
W	0.954 (.00663)	0.046 (.00663)	6.6485	0.953 (.00670)	0.047 (.00670)	7.8110
$\bar{N}$	0.950 (.00690)	0.050 (.00690)	1.9565	0.942 (.00740)	0.058 (.00740)	1.9655
NA	0.921 (.00853)	0.067 (.00791)	9.5450	0.931 (.00802)	0.066 (.00786)	9.2730
NL	0.919 (.00863)	0.081 (.00863)	3.2590	0.900 (.00949)	0.100 (.00949)	3.6615
J	0.927 (.00823)	0.073 (.00823)	4.4840	0.889 (.00994)	0.111 (.00994)	4.4430
AJ	0.976 (.00484)	0.024 (.00484)	5.9985	0.973 (.00513)	0.027 (.00513)	5.1775
JA	0.955 (.00656)	0.027 (.00513)	7.0650	0.966 (.00573)	0.028 (.00522)	6.2835
NJ	0.935 (.00780)	0.060 (.00751)	7.6910	0.947 (.00709)	0.053 (.00709)	7.5640
F	0.937 (.00769)	0.063 (.00769)	6.0355	0.949 (.00696)	0.051 (.00696)	6.4635
Kendall's W		0.684640 (.000)			0.666159 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 09 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.563	0.005	0.263	0.169
W	0.022	0.746	0.106	0.126
$\bar{N}$	0.061	0.577	0.178	0.184
NA	0.037	0.633	0.136	0.194
NL	0.499	0.009	0.294	0.198
J	0.402	0.067	0.367	0.164
AJ	0.041	0.657	0.164	0.138
JA	0.012	0.774	0.094	0.120
NJ	0.023	0.730	0.118	0.129
F	0.055	0.757	0.090	0.098

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 10

Parameters: n = 20 R^2 = 0.90 $\rho^2 = 0.25$ $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.989644 (0.00069)	0.988397 (0.000868)
$Power_{NJ}$	0.977132 (0.00680)	0.979313 (0.005716)
$Power_F > Power_{NJ}$	0.3180	0.2920
$Power_{\overline{NJ}}$	0.984878 (0.00290)	0.978227 (0.005781)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.002679	0.001542

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.898 (.00958)	0.102 (.00958)	2.2580	0.895 (.00970)	0.105 (.00970)	2.2715
W	0.963 (.00597)	0.037 (.00597)	7.5810	0.957 (.00642)	0.043 (.00642)	7.5420
$\bar{N}$	0.959 (.00627)	0.041 (.00627)	1.9720	0.952 (.00676)	0.048 (.00676)	1.9970
NA	0.875 (.01046)	0.080 (.00858)	9.2445	0.860 (.01098)	0.099 (.00945)	9.2820
NL	0.910 (.00905)	0.090 (.00905)	3.2035	0.899 (.00953)	0.101 (.00953)	3.2615
J	0.951 (.00683)	0.049 (.00683)	3.9995	0.943 (.00734)	0.057 (.00734)	4.0430
AJ	0.982 (.00421)	0.018 (.00421)	4.9530	0.985 (.00385)	0.015 (.00385)	4.8860
JA	0.910 (.00905)	0.023 (.00474)	8.1290	0.910 (.00905)	0.025 (.00494)	8.0740
NJ	0.947 (.00709)	0.045 (.00656)	6.4745	0.945 (.00721)	0.052 (.00702)	6.3865
F	0.958 (.00635)	0.042 (.00635)	7.1990	0.941 (.00745)	0.058 (.00740)	7.2565

	H_1 vs H_2	H_1 vs H_3
Kendall's W	0.751564 (.000)	0.748962 (.000)

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 10 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.588	0.001	0.204	0.207
W	0.039	0.647	0.153	0.161
$\bar{N}$	0.082	0.510	0.194	0.214
NA	0.036	0.620	0.171	0.173
NL	0.536	0.005	0.226	0.233
J	0.388	0.093	0.264	0.255
AJ	0.050	0.623	0.162	0.165
JA	0.011	0.729	0.126	0.134
NJ	0.021	0.672	0.149	0.158
F	0.067	0.682	0.131	0.120

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 11

Parameters: $n = 20$ $R^2 = 0.90$ $\rho^2 = 0.50$ $(k_1, k_2, k_3) = (4,2,6)$ $m = 1000$

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.995799 (0.00021)	0.913317 (0.009873)
$Power_{NJ}$	0.865508 (0.04485)	0.976900 (0.002384)
$Power_F > Power_{NJ}$	0.8880	0.0310
$Power_{\overline{NJ}}$	0.873191 (0.04058)	0.973435 (0.003514)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.001252	0.001182

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.933 (.00791)	0.067 (.00791)	2.0770	0.868 (.01071)	0.132 (.01071)	2.1695
W	0.968 (.00557)	0.032 (.00557)	6.5785	0.951 (.00683)	0.045 (.00656)	8.2435
$\bar{N}$	0.966 (.00573)	0.034 (.00573)	1.9480	0.944 (.00727)	0.056 (.00727)	2.0485
NA	0.582 (.01561)	0.069 (.00802)	9.1195	0.910 (.00905)	0.085 (.00882)	7.9270
NL	0.935 (.00780)	0.065 (.00780)	2.9505	0.891 (.00986)	0.109 (.00986)	3.8470
J	0.976 (.00484)	0.024 (.00484)	3.9070	0.920 (.00858)	0.080 (.00858)	4.0360
AJ	0.982 (.00421)	0.018 (.00421)	4.9460	0.969 (.00548)	0.029 (.00531)	5.0255
JA	0.593 (.01554)	0.020 (.00443)	9.3855	0.960 (.00620)	0.029 (.00531)	6.3480
NJ	0.907 (.00919)	0.039 (.00613)	7.1935	0.935 (.00780)	0.061 (.00757)	6.5205
F	0.966 (.00573)	0.034 (.00573)	6.9075	0.937 (.00769)	0.051 (.00696)	8.8345
Kendall's W		0.852636 (.000)			0.691372 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 11 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.536	0.003	0.421	0.040
W	0.087	0.449	0.322	0.142
$\dot{N}$	0.155	0.280	0.423	0.142
NA	0.029	0.550	0.149	0.272
NL	0.493	0.006	0.450	0.051
J	0.284	0.021	0.670	0.025
AJ	0.085	0.382	0.453	0.080
JA	0.013	0.666	0.145	0.176
NJ	0.035	0.587	0.215	0.163
F	0.097	0.487	0.343	0.073

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 12

Parameters: n = 20 R^2 = 0.90 $\rho^2 = 0.50$ $(k_1, k_2, k_3) = (6,2,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.991713 (0.00056)	0.951975 (0.004612)
$Power_{NJ}$	0.706068 (0.07825)	0.898331 (0.024377)
$Power_F > Power_{NJ}$	0.9390	0.5440
$Power_{\overline{NJ}}$	0.718396 (0.07333)	0.897148 (0.027646)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.001335	0.005359

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.898 (.00958)	0.102 (.00958)	2.3170	0.872 (.01057)	0.128 (.01057)	2.2475
W	0.956 (.00649)	0.044 (.00649)	6.7320	0.954 (.00663)	0.044 (.00649)	7.0850
$\bar{N}$	0.957 (.00642)	0.043 (.00642)	2.2355	0.948 (.00702)	0.051 (.00696)	2.1710
NA	0.285 (.01428)	0.098 (.00941)	8.8475	0.704 (.01444)	0.100 (.00949)	8.6155
NL	0.899 (.00953)	0.101 (.00953)	2.8000	0.887 (.01002)	0.113 (.01002)	3.3410
J	0.959 (.00627)	0.041 (.00627)	3.9555	0.956 (.00649)	0.044 (.00649)	3.8870
AJ	0.973 (.00513)	0.027 (.00513)	4.8570	0.976 (.00484)	0.023 (.00474)	4.8150
JA	0.289 (.01434)	0.031 (.00548)	9.4810	0.711 (.01434)	0.022 (.00464)	8.9270
NJ	0.788 (.01293)	0.054 (.00715)	7.2405	0.911 (.00901)	0.054 (.00715)	6.3915
F	0.956 (.00649)	0.044 (.00649)	6.5525	0.951 (.00683)	0.047 (.00670)	7.5195
Kendall's W	0.799244 (.000)			0.737635 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 12 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.367	0.028	0.499	0.106
W	0.036	0.592	0.276	0.096
$\bar{N}$	0.065	0.472	0.343	0.120
NA	0.025	0.690	0.154	0.131
NL	0.323	0.032	0.528	0.117
J	0.147	0.158	0.623	0.072
AJ	0.030	0.600	0.305	0.065
JA	0.007	0.762	0.141	0.090
NJ	0.024	0.719	0.172	0.085
F	0.057	0.626	0.262	0.055

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 13

Parameters: n = 20 $R^2 = 0.90$ $\rho^2 = 0.75$ $(k_1, k_2, k_3) = (4,2,6)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.989979 (0.00061)	0.670006 (0.028178)
$Power_{NJ}$	0.724474 (0.04677)	0.883949 (0.012939)
$Power_F > Power_{NJ}$	0.9840	0.0000
$Power_{\overline{NJ}}$	0.731980 (0.04473)	0.883458 (0.015005)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000480	0.002920

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.916 (.00878)	0.084 (.00878)	2.0690	0.872 (.01057)	0.128 (.01057)	2.0505
W	0.957 (.00642)	0.043 (.00642)	6.2230	0.949 (.00696)	0.037 (.00597)	8.5175
$\tilde{N}$	0.959 (.00627)	0.041 (.00627)	1.9455	0.950 (.00690)	0.048 (.00676)	2.1380
NA	0.361 (.01520)	0.074 (.00828)	8.8940	0.917 (.00873)	0.079 (.00853)	6.8660
NL	0.914 (.00887)	0.086 (.00887)	3.2120	0.915 (.00882)	0.085 (.00882)	3.9305
J	0.970 (.00540)	0.030 (.00540)	3.8900	0.933 (.00791)	0.066 (.00786)	4.1145
AJ	0.978 (.00464)	0.022 (.00464)	4.9180	0.969 (.00548)	0.019 (.00432)	5.4830
JA	0.342 (.01501)	0.028 (.00522)	9.5375	0.967 (.00565)	0.021 (.00454)	5.5465
NJ.	0.862 (.01091)	0.050 (.00690)	7.5540	0.938 (.00763)	0.048 (.00676)	6.9955
F	0.951 (.00683)	0.049 (.00683)	6.7750	0.878 (.01035)	0.038 (.00605)	9.3580
Kendall's W		0.840798 (.000)			0.706294 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 13 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.520	0.000	0.472	0.008
W	0.180	0.143	0.570	0.107
$\bar{N}$	0.260	0.053	0.605	0.082
NA	0.026	0.514	0.121	0.339
NL	0.472	0.001	0.514	0.013
J	0.322	0.002	0.672	0.004
AJ	0.202	0.076	0.678	0.044
JA	0.019	0.629	0.107	0.245
NJ	0.050	0.484	0.243	0.223
F	0.202	0.166	0.600	0.032

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 14

Parameters: n = 20 R^2 = 0.90 $\rho^2 = 0.75$ $(k_1, k_2, k_3) = (6,2,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.984420 (0.00142)	0.886878 (0.012347)
$Power_{NJ}$	0.528865 (0.06637)	0.754695 (0.038861)
$Power_F > Power_{NJ}$	0.9920	0.7380
$Power_{\overline{NJ}}$	0.537079 (0.06497)	0.759852 (0.041602)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000373	0.005985

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.898 (.00958)	0.102 (.00958)	2.2855	0.879 (.01032)	0.121 (.01032)	2.0505
W	0.959 (.00627)	0.040 (.00620)	6.3765	0.957 (.00642)	0.042 (.00635)	6.4645
$\bar{N}$	0.958 (.00635)	0.042 (.00635)	2.2650	0.948 (.00702)	0.051 (.00696)	2.1875
NA	0.131 (.01067)	0.086 (.00887)	8.8195	0.548 (.01575)	0.114 (.01006)	8.6030
NL	0.905 (.00928)	0.095 (.00928)	2.7670	0.882 (.01021)	0.118 (.01021)	3.4605
J	0.967 (.00565)	0.033 (.00565)	3.9090	0.964 (.00589)	0.036 (.00589)	3.8180
AJ	0.973 (.00513)	0.027 (.00513)	4.8570	0.972 (.00522)	0.027 (.00513)	4.8345
JA	0.117 (.01017)	0.031 (.00548)	9.5975	0.512 (.01581)	0.026 (.00503)	9.2020
NJ	0.713 (.01431)	0.047 (.00670)	7.5600	0.871 (.01061)	0.065 (.00780)	6.8445
F	0.952 (.00676)	0.048 (.00676)	6.5815	0.920 (.00858)	0.068 (.00796)	7.5350
Kendall's W	0.820769 (.000)			0.765695 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 14 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.283	0.021	0.619	0.077
W	0.037	0.463	0.406	0.094
$\bar{N}$	0.071	0.328	0.502	0.099
NA	0.008	0.717	0.107	0.168
NL	0.230	0.038	0.643	0.089
J	0.134	0.102	0.720	0.044
AJ	0.040	0.415	0.486	0.059
JA	0.005	0.784	0.095	0.116
NJ	0.012	0.726	0.149	0.113
F	0.062	0.495	0.401	0.042

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 15

Parameters: n = 20 R^2 = 0.90 $\rho^2 = 0.90$ $(k_1, k_2, k_3) = (2,4,6)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.547739 (0.02095)	0.473753 (0.022584)
$Power_{NJ}$	0.669368 (0.02060)	0.597687 (0.025185)
$Power_F > Power_{NJ}$	0.0000	0.0000
$Power_{\overline{NJ}}$	0.672553 (0.02043)	0.597505 (0.029251)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000040	0.004799

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.866 (.01078)	0.127 (.01053)	1.9450	0.767 (.01337)	0.223 (.01317)	2.7230
W	0.864 (.01085)	0.038 (.00605)	9.0585	0.828 (.01194)	0.043 (.00642)	8.7005
$\bar{N}$	0.912 (.00896)	0.057 (.00734)	2.2680	0.879 (.01032)	0.066 (.00786)	2.3010
NA	0.882 (.01021)	0.076 (.00838)	6.4615	0.885 (.01009)	0.057 (.00734)	5.2105
NL	0.885 (.01009)	0.086 (.00887)	4.6380	0.833 (.01180)	0.143 (.01108)	4.2285
J	0.877 (.01039)	0.076 (.00838)	4.7795	0.741 (.01386)	0.202 (.01270)	5.4830
AJ	0.904 (.00932)	0.020 (.00443)	6.3300	0.835 (.01174)	0.041 (.00627)	6.1590
JA	0.905 (.00928)	0.029 (.00531)	5.3715	0.857 (.01108)	0.029 (.00531)	5.2495
NJ	0.869 (.01067)	0.059 (.00745)	7.3225	0.832 (.01183)	0.044 (.00649)	7.4180
F	0.792 (.01284)	0.048 (.00676)	6.8215	0.710 (.01436)	0.058 (.00740)	7.5270
Kendall's W	0.550230 (.000)			0.494089 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 15 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.589	0.004	0.243	0.164
W	0.088	0.444	0.218	0.250
$\dot{N}$	0.229	0.182	0.298	0.291
NA	0.151	0.173	0.306	0.370
NL	0.442	0.016	0.317	0.225
J	0.377	0.044	0.386	0.193
AJ	0.125	0.374	0.256	0.245
JA	0.075	0.328	0.295	0.302
NJ	0.096	0.339	0.283	0.282
F	0.076	0.663	0.112	0.149

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 16

Parameters: n = 20 R^2 = 0.90 ρ^2 = 0.90 (k_1, k_2, k_3) = (4,4,4) m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.388175 (0.01614)	0.386459 (0.015112)
$Power_{NJ}$	0.650878 (0.02277)	0.650652 (0.021003)
$Power_F > Power_{NJ}$	0.0000	0.0000
$Power_{\overline{NJ}}$	0.660467 (0.02174)	0.656654 (0.024736)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000264	0.005406

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.880 (.01028)	0.107 (.00978)	1.7970	0.894 (.00974)	0.100 (.00949)	1.7300
W	0.847 (.01139)	0.043 (.00642)	8.4565	0.875 (.01046)	0.040 (.00620)	8.4530
$\bar{N}$	0.894 (.00974)	0.067 (.00791)	2.4135	0.912 (.00896)	0.063 (.00769)	2.3615
NA	0.862 (.01091)	0.078 (.00848)	6.0505	0.873 (.01053)	0.083 (.00873)	6.1100
NL	0.875 (.01046)	0.084 (.00878)	3.9195	0.885 (.01009)	0.085 (.00882)	3.9795
J	0.897 (.00962)	0.032 (.00557)	4.2315	0.911 (.00901)	0.027 (.00513)	4.2020
AJ	0.873 (.01053)	0.027 (.00513)	6.2585	0.895 (.00970)	0.023 (.00474)	6.2350
JA	0.879 (.01032)	0.029 (.00531)	5.8215	0.901 (.00945)	0.021 (.00454)	5.7370
NJ	0.843 (.01151)	0.047 (.00670)	6.7940	0.863 (.01088)	0.053 (.00709)	6.8480
F	0.613 (.01541)	0.054 (.00715)	9.2735	0.644 (.01515)	0.050 (.00690)	9.3440
Kendall's W	0.663162 (.000)			0.690333 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 16 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.489	0.012	0.241	0.258
W	0.159	0.226	0.296	0.319
$\bar{N}$	0.339	0.068	0.292	0.301
NA	0.238	0.119	0.319	0.324
NL	0.328	0.055	0.299	0.318
J	0.211	0.140	0.319	0.330
AJ	0.140	0.230	0.306	0.324
JA	0.152	0.206	0.313	0.329
NJ	0.143	0.217	0.313	0.327
F	0.083	0.554	0.183	0.180

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 17

Parameters: n=40 R^2 = 0.75 ρ^2=0.25 (k_1, k_2, k_3)=(2,4,6) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.998682 (0.00001)	0.997956 (2.6E-05)
$Power_{NJ}$	0.994975 (0.00142)	0.997484 (0.000178)
$Power_F > Power_{NJ}$	0.2420	0.2160
$Power_{\overline{NJ}}$	0.999085 (0.00001)	0.993012 (0.001272)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.001403	0.000901

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.902 (.01331)	0.098 (.01331)	2.7890	0.872 (.01496)	0.128 (.01496)	2.8440
W	0.952 (.00957)	0.048 (.00957)	3.1870	0.950 (.00976)	0.050 (.00976)	3.7470
$\bar{N}$	0.954 (.00938)	0.046 (.00938)	2.3820	0.948 (.00994)	0.052 (.00994)	2.2640
NA	0.928 (.01157)	0.068 (.01127)	9.6380	0.942 (.01046)	0.058 (.01046)	9.4880
NL	0.922 (.01200)	0.078 (.01200)	2.8950	0.892 (.01389)	0.108 (.01389)	3.0060
J	0.926 (.01172)	0.074 (.01172)	6.1050	0.864 (.01535)	0.136 (.01535)	6.3280
AJ	0.978 (.00657)	0.022 (.00657)	7.4360	0.972 (.00739)	0.028 (.00739)	7.1640
JA	0.966 (.00811)	0.026 (.00712)	7.8050	0.974 (.00712)	0.026 (.00712)	7.4700
NJ	0.936 (.01096)	0.064 (.01096)	7.8590	0.952 (.00957)	0.048 (.00957)	7.9470
F	0.950 (.00976)	0.050 (.00976)	4.9040	0.954 (.00938)	0.046 (.00938)	4.7420
Kendall's W	0.779068 (.000)			0.718034 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 17 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.814	0.000	0.120	0.066
W	0.168	0.350	0.200	0.282
$\bar{N}$	0.200	0.304	0.208	0.288
NA	0.082	0.444	0.192	0.282
NL	0.794	0.000	0.140	0.066
J	0.700	0.006	0.206	0.088
AJ	0.150	0.432	0.200	0.218
JA	0.060	0.492	0.186	0.262
NJ	0.078	0.454	0.200	0.268
F	0.120	0.492	0.164	0.224

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 18

Parameters: n=40 R^2 = 0.75 ρ^2=0.25 (k_1, k_2, k_3)=(4,4,4) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.995387 (0.00011)	0.995709 (0.000103)
$Power_{NJ}$	0.983879 (0.00626)	0.984008 (0.007167)
$Power_F > Power_{NJ}$	0.3340	0.3580
$Power_{\overline{NJ}}$	0.990715 (0.00320)	0.980143 (0.008384)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.002093	0.001814

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.916 (.01242)	0.084 (.01242)	2.4770	0.902 (.01331)	0.098 (.01331)	2.5340
W	0.962 (.00856)	0.038 (.00856)	3.9430	0.958 (.00898)	0.042 (.00898)	4.1340
$\bar{N}$	0.952 (.00957)	0.048 (.00957)	2.2400	0.952 (.00957)	0.048 (.00957)	2.1750
NA	0.934 (.01111)	0.056 (.01029)	9.5750	0.926 (.01172)	0.060 (.01063)	9.5710
NL	0.920 (.01214)	0.080 (.01214)	2.7190	0.916 (.01242)	0.084 (.01242)	2.7000
J	0.940 (.01063)	0.060 (.01063)	4.8970	0.942 (.01046)	0.058 (.01046)	4.8590
AJ	0.980 (.00627)	0.020 (.00627)	6.0220	0.974 (.00712)	0.026 (.00712)	6.0720
JA	0.972 (.00739)	0.018 (.00595)	8.1780	0.954 (.00938)	0.032 (.00788)	8.1660
NJ	0.946 (.01012)	0.046 (.00938)	7.1150	0.946 (.01012)	0.050 (.00976)	7.0670
F	0.938 (.01080)	0.062 (.01080)	7.8340	0.962 (.00856)	0.038 (.00856)	7.7220
Kendall's W		0.787540 (.000)			0.775867 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 18 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.804	0.000	0.090	0.106
W	0.190	0.226	0.308	0.276
$\bar{N}$	0.224	0.188	0.322	0.266
NA	0.114	0.390	0.274	0.222
NL	0.776	0.000	0.100	0.124
J	0.652	0.020	0.164	0.164
AJ	0.142	0.336	0.270	0.252
JA	0.080	0.454	0.266	0.200
NJ	0.086	0.396	0.284	0.234
F	0.150	0.370	0.244	0.236

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 19

Parameters: n = 40 $R^2 = 0.75$ $\rho^2 = 0.50$ $(k_1, k_2, k_3) = (4,2,6)$ m = 500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.997324 (4.7E-0)	0.958182 (0.002168)
$Power_{NJ}$	0.861180 (0.02974)	0.987752 (0.000360)
$Power_F > Power_{NJ}$	0.9760	0.0420
$Power_{\overline{NJ}}$	0.873556 (0.02570)	0.982356 (0.001271)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.001510	0.000800

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.916 (.01242)	0.084 (.01242)	2.4350	0.886 (.01423)	0.114 (.01423)	2.1910
W	0.946 (.01012)	0.054 (.01012)	3.9530	0.960 (.00877)	0.040 (.00877)	5.7330
$\bar{N}$	0.942 (.01046)	0.058 (.01046)	2.2860	0.954 (.00938)	0.046 (.00938)	1.9420
NA	0.806 (.01770)	0.060 (.01063)	9.3320	0.944 (.01029)	0.056 (.01029)	8.6550
NL	0.924 (.01186)	0.076 (.01186)	2.6540	0.924 (.01186)	0.076 (.01186)	3.1910
J	0.958 (.00898)	0.042 (.00898)	4.7120	0.926 (.01172)	0.074 (.01172)	4.4660
AJ	0.970 (.00764)	0.030 (.00764)	5.7900	0.976 (.00685)	0.024 (.00685)	6.0160
JA	0.822 (.01712)	0.034 (.00811)	9.0710	0.974 (.00712)	0.026 (.00712)	6.6560
NJ	0.914 (.01255)	0.050 (.00976)	7.5570	0.954 (.00938)	0.046 (.00938)	6.7300
F	0.950 (.00976)	0.050 (.00976)	7.2100	0.956 (.00918)	0.044 (.00918)	9.4200
Kendall's W		0.825987 (.000)			0.740819 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 19 (continued)

Testing H_2 vs H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.656	0.000	0.336	0.008
W	0.310	0.076	0.494	0.120
$\bar{N}$	0.358	0.064	0.480	0.098
NA	0.078	0.356	0.262	0.304
NL	0.618	0.000	0.366	0.016
J	0.470	0.002	0.520	0.008
AJ	0.258	0.090	0.570	0.082
JA	0.062	0.412	0.250	0.276
NJ	0.092	0.332	0.322	0.254
F	0.262	0.112	0.544	0.082

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 20

Parameters: n = 40 R^2 = 0.75 $\rho^2 = 0.50$ $(k_1, k_2, k_3) = (6,2,4)$ m = 500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.995881 (8.9E-0)	0.979521 (0.000886)
$Power_{NJ}$	0.687027 (0.06448)	0.918192 (0.013768)
$Power_F > Power_{NJ}$	0.9840	0.7040
$Power_{\overline{NJ}}$	0.709447 (0.05733)	0.920258 (0.017433)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.003608	0.005280

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.896 (.01367)	0.104 (.01367)	2.5930	0.908 (.01294)	0.092 (.01294)	2.2320
W	0.954 (.00938)	0.046 (.00938)	4.6240	0.964 (.00834)	0.036 (.00834)	5.4890
$\bar{N}$	0.946 (.01012)	0.054 (.01012)	2.2960	0.960 (.00877)	0.040 (.00877)	2.1510
NA	0.532 (.02234)	0.060 (.01063)	8.9680	0.894 (.01378)	0.068 (.01127)	8.9770
NL	0.908 (.01294)	0.092 (.01294)	2.5850	0.926 (.01172)	0.074 (.01172)	2.7450
J	0.950 (.00976)	0.050 (.00976)	4.2920	0.952 (.00957)	0.048 (.00957)	4.1250
AJ	0.976 (.00685)	0.024 (.00685)	5.6020	0.972 (.00739)	0.028 (.00739)	5.3640
JA	0.528 (.02235)	0.034 (.00811)	9.3780	0.916 (.01242)	0.024 (.00685)	8.6750
NJ	0.788 (.01830)	0.044 (.00918)	7.5590	0.944 (.01029)	0.046 (.00938)	6.9500
F	0.960 (.00877)	0.040 (.00877)	7.1030	0.956 (.00918)	0.044 (.00918)	8.2920
Kendall's W	0.799241 (.000)			0.801133 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 20 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.512	0.002	0.438	0.048
W	0.144	0.236	0.468	0.152
$\bar{N}$	0.186	0.192	0.476	0.146
NA	0.040	0.514	0.236	0.210
NL	0.480	0.002	0.462	0.056
J	0.322	0.036	0.592	0.050
AJ	0.104	0.318	0.498	0.080
JA	0.028	0.554	0.234	0.184
NJ	0.040	0.498	0.274	0.188
F	0.142	0.326	0.444	0.088

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 21

Parameters: n=40 R^2 = 0.75 ρ^2=0.75 (k_1, k_2, k_3)=(4,2,6) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.990784 (0.00025)	0.733452 (0.012201)
$Power_{NJ}$	0.676123 (0.03344)	0.897427 (0.004722)
$Power_F > Power_{NJ}$	1.0000	0.0000
$Power_{\overline{NJ}}$	0.685934 (0.03216)	0.899872 (0.006846)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000479	0.003636

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.950 (.00976)	0.050 (.00976)	2.0110	0.888 (.01412)	0.112 (.01412)	1.8680
W	0.966 (.00811)	0.034 (.00811)	4.3310	0.956 (.00918)	0.036 (.00834)	7.2150
$\bar{N}$	0.966 (.00811)	0.034 (.00811)	2.0990	0.956 (.00918)	0.042 (.00898)	2.1410
NA	0.578 (.02211)	0.052 (.00994)	9.1400	0.944 (.01029)	0.050 (.00976)	7.2920
NL	0.952 (.00957)	0.048 (.00957)	2.7080	0.916 (.01242)	0.084 (.01242)	3.4170
J	0.980 (.00627)	0.020 (.00627)	4.5420	0.912 (.01268)	0.088 (.01268)	4.4450
AJ	0.984 (.00562)	0.016 (.00562)	5.7780	0.966 (.00811)	0.028 (.00739)	6.5000
JA	0.590 (.02202)	0.016 (.00562)	9.4660	0.974 (.00712)	0.018 (.00595)	5.9270
NJ	0.852 (.01590)	0.044 (.00918)	7.8290	0.948 (.00994)	0.044 (.00918)	6.6900
F	0.954 (.00938)	0.046 (.00938)	7.0960	0.906 (.01306)	0.054 (.01012)	9.5050
Kendall's W	0.889995 (.000)			0.694716 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 21 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.606	0.000	0.386	0.008
W	0.420	0.012	0.532	0.036
$\bar{N}$	0.480	0.008	0.494	0.018
NA	0.062	0.288	0.236	0.414
NL	0.576	0.000	0.414	0.010
J	0.476	0.000	0.520	0.004
AJ	0.344	0.006	0.632	0.018
JA	0.046	0.354	0.236	0.364
NJ	0.102	0.246	0.334	0.318
F	0.282	0.016	0.684	0.018

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 22

Parameters: n=40 R^2 = 0.75 ρ^2=0.75 (k_1, k_2, k_3)=(6,2,4) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.992241 (0.00045)	0.942457 (0.003346)
$Power_{NJ}$	0.508021 (0.04144)	0.761019 (0.021437)
$Power_F > Power_{NJ}$	1.0000	0.9440
$Power_{\overline{NJ}}$	0.522467 (0.03934)	0.758911 (0.029792)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000837	0.010501

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.922 (.01200)	0.078 (.01200)	2.3570	0.924 (.01186)	0.076 (.01186)	1.8470
W	0.960 (.00877)	0.040 (.00877)	4.6900	0.948 (.00994)	0.052 (.00994)	5.5710
$\bar{N}$	0.964 (.00834)	0.036 (.00834)	2.2600	0.950 (.00976)	0.050 (.00976)	2.2620
NA	0.320 (.02088)	0.068 (.01127)	8.8940	0.772 (.01878)	0.088 (.01268)	8.8380
NL	0.918 (.01228)	0.082 (.01228)	2.5210	0.922 (.01200)	0.078 (.01200)	2.9580
J	0.970 (.00764)	0.030 (.00764)	4.2470	0.966 (.00811)	0.034 (.00811)	4.0220
AJ	0.974 (.00712)	0.026 (.00712)	5.6590	0.974 (.00712)	0.026 (.00712)	5.2760
JA	0.322 (.02092)	0.026 (.00712)	9.6320	0.800 (.01791)	0.024 (.00685)	9.1690
NJ	0.694 (.02063)	0.050 (.00976)	7.8050	0.898 (.01355)	0.062 (.01080)	7.2890
F	0.964 (.00834)	0.036 (.00834)	6.9350	0.948 (.00994)	0.048 (.00957)	7.7680
Kendall's W	0.849362 (.000)			0.825855 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 22 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.342	0.004	0.600	0.054
W	0.110	0.186	0.606	0.098
$\dot{N}$	0.146	0.132	0.614	0.108
NA	0.012	0.636	0.150	0.202
NL	0.314	0.010	0.622	0.054
J	0.222	0.028	0.718	0.032
AJ	0.104	0.206	0.646	0.044
JA	0.010	0.674	0.138	0.178
NJ	0.016	0.608	0.204	0.172
F	0.116	0.256	0.606	0.022

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 23

Parameters: n=40 R^2 = 0.75 ρ^2=0.90 (k_1, k_2, k_3)=(2,4,6) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.491368 (0.00835)	0.458676 (0.008765)
$Power_{NJ}$	0.604140 (0.00887)	0.569309 (0.009978)
$Power_F > Power_{NJ}$	0.0000	0.0000
$Power_{\overline{NJ}}$	0.608579 (0.00872)	0.570021 (0.014575)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000065	0.005945

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.860 (.01553)	0.114 (.01423)	1.9180	0.792 (.01817)	0.184 (.01735)	2.4160
W	0.832 (.01674)	0.042 (.00898)	8.0680	0.832 (.01674)	0.042 (.00898)	8.2670
$\bar{N}$	0.866 (.01525)	0.054 (.01012)	2.3790	0.874 (.01486)	0.056 (.01029)	2.3690
NA	0.834 (.01666)	0.044 (.00918)	6.5510	0.842 (.01633)	0.062 (.01080)	5.3920
NL	0.838 (.01649)	0.086 (.01255)	4.0030	0.822 (.01712)	0.132 (.01515)	3.9350
J	0.826 (.01697)	0.080 (.01214)	5.4960	0.744 (.01954)	0.190 (.01756)	6.1250
AJ	0.832 (.01674)	0.028 (.00739)	7.8850	0.838 (.01649)	0.036 (.00834)	7.6320
JA	0.852 (.01590)	0.012 (.00487)	6.4490	0.852 (.01590)	0.026 (.00712)	6.2850
NJ	0.822 (.01712)	0.040 (.00877)	7.3650	0.826 (.01697)	0.052 (.00994)	7.5910
F	0.744 (.01954)	0.046 (.00938)	4.8860	0.732 (.01983)	0.040 (.00877)	4.9880
Kendall's W	0.551513 (.000)			0.511522 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 23 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.564	0.002	0.244	0.190
W	0.110	0.236	0.308	0.346
$\bar{N}$	0.208	0.138	0.302	0.352
NA	0.138	0.146	0.324	0.392
NL	0.434	0.004	0.308	0.254
J	0.428	0.014	0.336	0.222
AJ	0.110	0.280	0.288	0.322
JA	0.100	0.182	0.336	0.382
NJ	0.108	0.192	0.330	0.370
F	0.056	0.508	0.204	0.232

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 24

Parameters: n=40 R^2 = 0.75 ρ^2=0.90 (k_1, k_2, k_3)=(4,4,4) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.363204 (0.00770)	0.368023 (0.006869)
$Power_{NJ}$	0.585926 (0.01146)	0.593233 (0.010717)
$Power_F > Power_{NJ}$	0.0000	0.0000
$Power_{\overline{NJ}}$	0.598820 (0.01097)	0.604089 (0.017261)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000332	0.008016

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.908 (.01294)	0.062 (.01080)	1.4510	0.890 (.01401)	0.074 (.01172)	1.4450
W	0.840 (.01641)	0.040 (.00877)	7.8280	0.828 (.01689)	0.062 (.01080)	8.0850
$\bar{N}$	0.896 (.01367)	0.040 (.00877)	2.3940	0.876 (.01475)	0.062 (.01080)	2.4390
NA	0.838 (.01649)	0.064 (.01096)	6.1120	0.832 (.01674)	0.076 (.01186)	6.2210
NL	0.878 (.01465)	0.060 (.01063)	3.2730	0.858 (.01563)	0.076 (.01186)	3.3400
J	0.890 (.01401)	0.024 (.00685)	4.2470	0.868 (.01515)	0.042 (.00898)	4.2360
AJ	0.864 (.01535)	0.018 (.00595)	7.0780	0.850 (.01598)	0.036 (.00834)	6.9960
JA	0.852 (.01590)	0.020 (.00627)	6.3470	0.848 (.01607)	0.040 (.00877)	6.2830
NJ	0.834 (.01666)	0.046 (.00938)	6.9090	0.828 (.01689)	0.066 (.01111)	6.7590
F	0.616 (.02177)	0.038 (.00856)	9.3610	0.644 (.02143)	0.064 (.01096)	9.1960
Kendall's W		0.739765 (.000)			0.728012 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 24 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.398	0.012	0.284	0.306
W	0.160	0.100	0.362	0.378
$\bar{N}$	0.260	0.042	0.332	0.366
NA	0.198	0.080	0.344	0.378
NL	0.260	0.034	0.336	0.370
J	0.212	0.052	0.354	0.382
AJ	0.128	0.122	0.362	0.388
JA	0.150	0.116	0.356	0.378
NJ	0.148	0.110	0.364	0.378
F	0.078	0.456	0.226	0.240

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 25

Parameters: n=40 R^2 = 0.90 ρ^2=0.25 (k_1, k_2, k_3)=(4,2,6) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	1.000000 (0.00000)	0.999975 (0.000000)
$Power_{NJ}$	0.974396 (0.01250)	0.999922 (1.1E-06)
$Power_F > Power_{NJ}$	0.9000	0.2860
$Power_{\overline{NJ}}$	0.974932 (0.01196)	0.999922 (0.000001)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000536	0.000000

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.944 (.01029)	0.056 (.01029)	3.4540	0.918 (.01228)	0.082 (.01228)	3.1130
W	0.964 (.00834)	0.036 (.00834)	3.4770	0.966 (.00811)	0.034 (.00811)	5.4730
$\bar{N}$	0.964 (.00834)	0.036 (.00834)	3.3080	0.970 (.00764)	0.030 (.00764)	2.7070
NA	0.856 (.01572)	0.052 (.00994)	9.5820	0.946 (.01012)	0.054 (.01012)	9.6890
NL	0.950 (.00976)	0.050 (.00976)	3.4090	0.934 (.01111)	0.066 (.01111)	3.1580
J	0.968 (.00788)	0.032 (.00788)	3.7250	0.948 (.00994)	0.052 (.00994)	3.8170
AJ	0.982 (.00595)	0.018 (.00595)	4.4810	0.986 (.00526)	0.014 (.00526)	4.6490
JA	0.872 (.01496)	0.020 (.00627)	8.9270	0.980 (.00627)	0.020 (.00627)	7.7520
NJ	0.950 (.00976)	0.044 (.00918)	7.2570	0.960 (.00877)	0.040 (.00877)	6.4220
F	0.936 (.01096)	0.064 (.01096)	7.3800	0.970 (.00764)	0.030 (.00764)	8.2200
Kendall's W	0.718071 (.000)			0.678645 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 25 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.772	0.000	0.218	0.010
W	0.202	0.292	0.320	0.186
$\bar{N}$	0.240	0.252	0.338	0.170
NA	0.076	0.486	0.164	0.274
NL	0.744	0.000	0.244	0.012
J	0.470	0.014	0.502	0.014
AJ	0.156	0.338	0.388	0.118
JA	0.048	0.550	0.168	0.234
NJ	0.070	0.494	0.208	0.228
F	0.170	0.346	0.326	0.158

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 26

Parameters: n=40 R^2 = 0.90 ρ^2=0.25 (k_1, k_2, k_3)=(6,2,4) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	1.000000 (0.00000)	1.000000 (0.000000)
$Power_{NJ}$	0.936777 (0.02883)	0.991811 (0.003094)
$Power_F > Power_{NJ}$	0.9680	0.7840
$Power_{\overline{NJ}}$	0.950333 (0.01829)	0.989443 (0.005109)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.004118	0.001678

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.952 (.00957)	0.048 (.00957)	3.1600	0.912 (.01268)	0.088 (.01268)	3.0760
W	0.976 (.00685)	0.024 (.00685)	4.3970	0.956 (.00918)	0.044 (.00918)	5.6820
$\bar{N}$	0.972 (.00739)	0.028 (.00739)	3.0080	0.960 (.00877)	0.040 (.00877)	2.7050
NA	0.742 (.01959)	0.048 (.00957)	9.4030	0.906 (.01306)	0.066 (.01111)	9.5220
NL	0.956 (.00918)	0.044 (.00918)	3.1390	0.926 (.01172)	0.074 (.01172)	2.9770
J	0.978 (.00657)	0.022 (.00657)	3.3730	0.958 (.00898)	0.042 (.00898)	3.3770
AJ	0.990 (.00445)	0.010 (.00445)	4.4770	0.978 (.00657)	0.022 (.00657)	4.4240
JA	0.748 (.01944)	0.012 (.00487)	9.2380	0.940 (.01063)	0.024 (.00685)	8.7280
NJ	0.948 (.00994)	0.028 (.00739)	7.4100	0.948 (.00994)	0.052 (.00994)	6.7590
F	0.954 (.00938)	0.046 (.00938)	7.3950	0.944 (.01029)	0.056 (.01029)	7.7500
Kendall's W		0.764144 (.000)			0.746831 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 26 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.608	0.000	0.316	0.076
W	0.104	0.438	0.312	0.146
$\bar{N}$	0.120	0.410	0.322	0.148
NA	0.058	0.612	0.160	0.170
NL	0.596	0.000	0.326	0.078
J	0.298	0.096	0.540	0.066
AJ	0.074	0.542	0.302	0.082
JA	0.038	0.672	0.152	0.138
NJ	0.058	0.616	0.178	0.148
F	0.088	0.504	0.294	0.114

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 27

Parameters: n=40 R^2 = 0.90 ρ^2=0.50 (k_1, k_2, k_3)=(2,4,6) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.999998 (0.00000)	0.999997 (0.000000)
$Power_{NJ}$	1.000000 (0.00000)	0.999999 (0.000000)
$Power_F > Power_{NJ}$	0.0540	0.0360
$Power_{\overline{NJ}}$	1.000000 (0.00000)	0.999999 (0.000000)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000000	0.000000

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.918 (.01228)	0.082 (.01228)	2.7450	0.920 (.01214)	0.080 (.01214)	2.5060
W	0.948 (.00994)	0.052 (.00994)	7.1450	0.944 (.01029)	0.056 (.01029)	7.8150
$\bar{N}$	0.932 (.01127)	0.068 (.01127)	2.6230	0.942 (.01046)	0.058 (.01046)	2.3090
NA	0.940 (.01063)	0.060 (.01063)	9.7280	0.940 (.01063)	0.060 (.01063)	9.6870
NL	0.926 (.01172)	0.074 (.01172)	4.7950	0.928 (.01157)	0.072 (.01157)	4.9280
J	0.954 (.00938)	0.046 (.00938)	4.8370	0.942 (.01046)	0.058 (.01046)	4.7800
AJ	0.978 (.00657)	0.022 (.00657)	6.0160	0.974 (.00712)	0.026 (.00712)	5.8080
JA	0.978 (.00657)	0.022 (.00657)	6.3730	0.974 (.00712)	0.026 (.00712)	6.2310
NJ	0.946 (.01012)	0.054 (.01012)	7.2840	0.944 (.01029)	0.056 (.01029)	7.4740
F	0.958 (.00898)	0.042 (.00898)	3.4540	0.968 (.00788)	0.032 (.00788)	3.4620
Kendall's W	0.586124 (.000)			0.656564 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 27 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.954	0.000	0.020	0.026
W	0.708	0.002	0.122	0.168
$\bar{N}$	0.804	0.000	0.080	0.116
NA	0.626	0.020	0.118	0.236
NL	0.940	0.000	0.024	0.036
J	0.928	0.000	0.040	0.032
AJ	0.636	0.024	0.148	0.192
JA	0.554	0.036	0.148	0.262
NJ	0.602	0.024	0.144	0.230
F	0.484	0.070	0.204	0.242

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 28

Parameters: n=40 R^2 = 0.90 ρ^2=0.50 (k_1, k_2, k_3)=(4,4,4) m=500

	H_1 *vs* H_2	H_1 *vs* H_3
$Power_F$	0.999980 (0.00000)	0.999981 (0.000000)
$Power_{NJ}$	0.999995 (0.00000)	0.999993 (0.000000)
$Power_F > Power_{NJ}$	0.0560	0.0540
$Power_{\overline{NJ}}$	0.999998 (0.00000)	0.999996 (0.000000)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000000	0.000000

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.942 (.01046)	0.058 (.01046)	2.1950	0.944 (.01029)	0.056 (.01029)	2.1960
W	0.960 (.00877)	0.040 (.00877)	7.1010	0.958 (.00898)	0.042 (.00898)	7.2460
$\bar{N}$	0.964 (.00834)	0.036 (.00834)	2.0020	0.960 (.00877)	0.040 (.00877)	2.0630
NA	0.938 (.01080)	0.062 (.01080)	9.8070	0.946 (.01012)	0.054 (.01012)	9.7760
NL	0.948 (.00994)	0.052 (.00994)	3.7770	0.950 (.00976)	0.050 (.00976)	3.6990
J	0.972 (.00739)	0.028 (.00739)	3.7320	0.972 (.00739)	0.028 (.00739)	3.7190
AJ	0.984 (.00562)	0.016 (.00562)	5.0140	0.972 (.00739)	0.028 (.00739)	5.1050
JA	0.976 (.00685)	0.024 (.00685)	7.1710	0.972 (.00739)	0.028 (.00739)	7.1120
NJ	0.950 (.00976)	0.050 (.00976)	6.3030	0.956 (.00918)	0.044 (.00918)	6.2120
F	0.948 (.00994)	0.052 (.00994)	7.8980	0.946 (.01012)	0.054 (.01012)	7.8720
Kendall's W		0.755970 (.000)			0.751467 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 28 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.954	0.000	0.020	0.026
W	0.816	0.010	0.082	0.092
$\bar{N}$	0.850	0.010	0.068	0.072
NA	0.718	0.020	0.138	0.124
NL	0.942	0.000	0.030	0.028
J	0.912	0.000	0.040	0.048
AJ	0.742	0.018	0.110	0.130
JA	0.674	0.026	0.146	0.154
NJ	0.710	0.018	0.132	0.140
F	0.578	0.038	0.192	0.192

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 29

Parameters: n=40 R^2 = 0.90 ρ^2=0.75 (k_1, k_2, k_3)=(2,4,6) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.998322 (3.2E-0)	0.998437 (2.1E-05)
$Power_{NJ}$	0.999451 (5.6E-0)	0.999513 (3.3E-06)
$Power_F > Power_{NJ}$	0.0000	0.0000
$Power_{\overline{NJ}}$	0.999469 (5.2E-0)	0.999374 (5.0E-06)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000000	0.000001

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.916 (.01242)	0.084 (.01242)	2.0390	0.922 (.01200)	0.078 (.01200)	1.9780
W	0.948 (.00994)	0.052 (.00994)	8.5960	0.950 (.00976)	0.050 (.00976)	8.9370
$\bar{N}$	0.940 (.01063)	0.060 (.01063)	1.9130	0.962 (.00856)	0.038 (.00856)	1.7350
NA	0.942 (.01046)	0.058 (.01046)	9.7160	0.952 (.00957)	0.048 (.00957)	9.4830
NL	0.936 (.01096)	0.064 (.01096)	7.1330	0.930 (.01142)	0.070 (.01142)	6.9290
J	0.960 (.00877)	0.040 (.00877)	4.4470	0.936 (.01096)	0.064 (.01096)	4.6130
AJ	0.978 (.00657)	0.022 (.00657)	5.8080	0.980 (.00627)	0.020 (.00627)	5.8090
JA	0.980 (.00627)	0.020 (.00627)	5.3130	0.980 (.00627)	0.020 (.00627)	5.2170
NJ	0.948 (.00994)	0.052 (.00994)	6.8850	0.954 (.00938)	0.046 (.00938)	7.0970
F	0.956 (.00918)	0.044 (.00918)	3.1500	0.954 (.00938)	0.046 (.00938)	3.2020
Kendall's W		0.801661 (.000)			0.820608 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 29 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.960	0.000	0.016	0.024
W	0.846	0.000	0.056	0.098
$\bar{N}$	0.908	0.000	0.036	0.056
NA	0.852	0.000	0.050	0.098
NL	0.946	0.000	0.022	0.032
J	0.934	0.000	0.034	0.032
AJ	0.816	0.004	0.066	0.114
JA	0.808	0.000	0.062	0.130
NJ	0.828	0.000	0.060	0.112
F	0.550	0.052	0.140	0.258

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 30

Parameters: n=40 R^2 = 0.90 ρ^2=0.75 (k_1, k_2, k_3)=(4,4,4) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.996093 (1.4E-0)	0.995317 (9.1E-05)
$Power_{NJ}$	0.999615 (4.6E-0)	0.999596 (1.8E-06)
$Power_F > Power_{NJ}$	0.0000	0.0000
$Power_{\overline{NJ}}$	0.999663 (3.7E-0)	0.999394 (6.7E-06)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000000	0.000003

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.924 (.01186)	0.076 (.01186)	1.9390	0.928 (.01157)	0.072 (.01157)	1.9240
W	0.946 (.01012)	0.054 (.01012)	8.1320	0.968 (.00788)	0.032 (.00788)	8.3940
$\bar{N}$	0.940 (.01063)	0.060 (.01063)	1.8570	0.944 (.01029)	0.056 (.01029)	1.8850
NA	0.936 (.01096)	0.064 (.01096)	9.5870	0.958 (.00898)	0.042 (.00898)	9.5550
NL	0.934 (.01111)	0.066 (.01111)	5.8730	0.958 (.00898)	0.042 (.00898)	5.7290
J	0.958 (.00898)	0.042 (.00898)	3.4370	0.966 (.00811)	0.034 (.00811)	3.4050
AJ	0.964 (.00834)	0.036 (.00834)	4.9770	0.976 (.00685)	0.024 (.00685)	4.9710
JA	0.962 (.00856)	0.038 (.00856)	5.1770	0.974 (.00712)	0.026 (.00712)	5.1850
NJ	0.942 (.01046)	0.058 (.01046)	5.8870	0.970 (.00764)	0.030 (.00764)	5.7120
F	0.948 (.00994)	0.052 (.00994)	8.1340	0.956 (.00918)	0.044 (.00918)	8.2400
Kendall's W		0.776631 (.000)			0.795968 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 30 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.970	0.000	0.018	0.012
W	0.894	0.000	0.064	0.042
$\bar{N}$	0.928	0.000	0.040	0.032
NA	0.896	0.000	0.064	0.040
NL	0.940	0.000	0.036	0.024
J	0.918	0.000	0.046	0.036
AJ	0.864	0.000	0.084	0.052
JA	0.874	0.002	0.076	0.048
NJ	0.878	0.000	0.072	0.050
F	0.694	0.010	0.180	0.116

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 31

Parameters: n=40 R^2 = 0.90 ρ^2=0.90 (k_1, k_2, k_3)=(4,2,6) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.999997 (0.00000)	0.814289 (0.010823)
$Power_{NJ}$	0.755406 (0.02357)	0.946053 (0.002474)
$Power_F > Power_{NJ}$	1.0000	0.0000
$Power_{\overline{NJ}}$	0.758268 (0.02302)	0.945292 (0.002867)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000040	0.000585

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.926 (.01172)	0.074 (.01172)	2.8670	0.890 (.01401)	0.110 (.01401)	1.8270
W	0.946 (.01012)	0.054 (.01012)	3.5890	0.954 (.00938)	0.046 (.00938)	8.5760
$\bar{N}$	0.944 (.01029)	0.056 (.01029)	2.7330	0.934 (.01111)	0.066 (.01111)	2.2490
NA	0.284 (.02019)	0.070 (.01142)	8.9150	0.936 (.01096)	0.064 (.01096)	7.3100
NL	0.926 (.01172)	0.074 (.01172)	2.9270	0.914 (.01255)	0.086 (.01255)	4.9870
J	0.974 (.00712)	0.026 (.00712)	4.2650	0.928 (.01157)	0.072 (.01157)	3.9300
AJ	0.980 (.00627)	0.020 (.00627)	5.3090	0.972 (.00739)	0.028 (.00739)	5.4040
JA	0.286 (.02023)	0.024 (.00685)	9.6370	0.976 (.00685)	0.024 (.00685)	4.9890
NJ	0.874 (.01486)	0.052 (.00994)	7.8810	0.946 (.01012)	0.052 (.00994)	6.2130
F	0.962 (.00856)	0.038 (.00856)	6.8770	0.904 (.01319)	0.060 (.01063)	9.5150
Kendall's W		0.792332 (.000)			0.715773 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 31 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.628	0.000	0.372	0.000
W	0.488	0.000	0.512	0.000
$\bar{N}$	0.538	0.000	0.462	0.000
NA	0.016	0.374	0.086	0.524
NL	0.600	0.000	0.400	0.000
J	0.488	0.000	0.512	0.000
AJ	0.456	0.000	0.544	0.000
JA	0.010	0.456	0.082	0.452
NJ	0.136	0.148	0.382	0.334
F	0.402	0.000	0.598	0.000

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 32

Parameters: n = 40 R^2 = 0.90 $\rho^2 = 0.90$ $(k_1, k_2, k_3) = (6,2,4)$ m = 500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.999999 (1.9E-1)	0.999639 (1.3E-06)
$Power_{NJ}$	0.574959 (0.04268)	0.838531 (0.013327)
$Power_F > Power_{NJ}$	1.0000	1.0000
$Power_{\overline{NJ}}$	0.579020 (0.04214)	0.837247 (0.015404)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000072	0.002710

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.920 (.01214)	0.080 (.01214)	3.0120	0.928 (.01157)	0.072 (.01157)	2.2500
W	0.942 (.01046)	0.058 (.01046)	3.9750	0.948 (.00994)	0.052 (.00994)	5.3980
$\bar{N}$	0.944 (.01029)	0.056 (.01029)	2.8320	0.946 (.01012)	0.054 (.01012)	2.1120
NA	0.056 (.01029)	0.080 (.01214)	8.8480	0.640 (.02149)	0.076 (.01186)	8.8740
NL	0.918 (.01228)	0.082 (.01228)	3.0350	0.928 (.01157)	0.072 (.01157)	2.8320
J	0.970 (.00764)	0.030 (.00764)	3.8080	0.970 (.00764)	0.030 (.00764)	4.0440
AJ	0.972 (.00739)	0.028 (.00739)	5.0480	0.978 (.00657)	0.022 (.00657)	5.1800
JA	0.060 (.01063)	0.030 (.00764)	9.6340	0.630 (.02161)	0.026 (.00712)	9.5690
NJ	0.746 (.01949)	0.060 (.01063)	7.8710	0.930 (.01142)	0.058 (.01046)	7.7440
F	0.938 (.01080)	0.062 (.01080)	6.9370	0.950 (.00976)	0.050 (.00976)	6.9970
Kendall's W		0.768393 (.000)			0.839603 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 32 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.300	0.000	0.674	0.026
W	0.162	0.024	0.766	0.048
$\bar{N}$	0.188	0.010	0.756	0.046
NA	0.004	0.700	0.078	0.218
NL	0.262	0.004	0.706	0.028
J	0.212	0.004	0.772	0.012
AJ	0.152	0.026	0.794	0.028
JA	0.000	0.724	0.074	0.202
NJ	0.012	0.620	0.170	0.198
F	0.144	0.042	0.804	0.010

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 33

Parameters: n = 20 $R^2 = 0.50$ $\rho^2 = 0.25$ $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.326868 (0.01060)	0.329654 (0.011297)
$Power_{NJ}$	0.358386 (0.02963)	0.353577 (0.028283)
$Power_F > Power_{NJ}$	0.3720	0.3870
$Power_{\overline{NJ}}$	0.447937 (0.02239)	0.449312 (0.061484)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.021163	0.041080

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.790 (.01289)	0.204 (.01275)	2.6570	0.778 (.01315)	0.212 (.01293)	2.7420
W	0.633 (.01525)	0.026 (.00503)	6.6645	0.623 (.01533)	0.026 (.00503)	6.6345
$\bar{N}$	0.725 (.01413)	0.039 (.00613)	3.9370	0.703 (.01446)	0.049 (.00683)	4.0720
NA	0.532 (.01579)	0.071 (.00813)	6.6540	0.502 (.01582)	0.072 (.00818)	6.7425
NL	0.821 (.01213)	0.173 (.01197)	3.1070	0.810 (.01241)	0.171 (.01191)	3.0780
J	0.833 (.01180)	0.121 (.01032)	3.9420	0.811 (.01239)	0.131 (.01067)	3.9585
AJ	0.661 (.01498)	0.028 (.00522)	6.1475	0.664 (.01494)	0.025 (.00494)	6.0220
JA	0.494 (.01582)	0.022 (.00464)	7.6215	0.448 (.01573)	0.018 (.00421)	7.6640
NJ	0.551 (.01574)	0.046 (.00663)	6.6495	0.521 (.01581)	0.038 (.00605)	6.6435
F	0.535 (.01578)	0.057 (.00734)	7.6325	0.546 (.01575)	0.051 (.00696)	7.4430

Kendall's W	0.417972 (.000)	0.403787 (.000)

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 33 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.422	0.013	0.272	0.293
W	0.016	0.827	0.085	0.072
$\bar{N}$	0.022	0.740	0.123	0.115
NA	0.023	0.770	0.105	0.102
NL	0.375	0.023	0.289	0.313
J	0.218	0.229	0.278	0.275
AJ	0.022	0.801	0.102	0.075
JA	0.001	0.851	0.080	0.068
NJ	0.015	0.824	0.086	0.075
F	0.034	0.807	0.087	0.072

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 34

Parameters: n=20 R^2 = 0.50 ρ^2=0.50 (k_1, k_2, k_3)=(4,4,4) m=1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.228990 (0.00512)	0.232136 (0.005518)
$Power_{NJ}$	0.318421 (0.01738)	0.326490 (0.017962)
$Power_F > Power_{NJ}$	0.1790	0.1530
$Power_{\overline{NJ}}$	0.388506 (0.01279)	0.412722 (0.044707)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.012447	0.029475

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.799 (.01268)	0.167 (.01180)	2.3385	0.794 (.01280)	0.173 (.01197)	2.3640
W	0.495 (.01582)	0.031 (.00548)	7.2440	0.510 (.01582)	0.034 (.00573)	7.1420
$\bar{N}$	0.628 (.01529)	0.048 (.00676)	4.3650	0.628 (.01529)	0.048 (.00676)	4.3135
NA	0.541 (.01577)	0.075 (.00833)	5.8460	0.541 (.01577)	0.089 (.00901)	5.9375
NL	0.804 (.01256)	0.130 (.01064)	2.9165	0.792 (.01284)	0.141 (.01101)	2.9755
J	0.789 (.01291)	0.091 (.00910)	3.7945	0.759 (.01353)	0.084 (.00878)	3.7655
AJ	0.529 (.01579)	0.025 (.00494)	6.8775	0.546 (.01575)	0.018 (.00421)	6.6895
JA	0.496 (.01582)	0.027 (.00513)	6.9645	0.491 (.01582)	0.030 (.00540)	6.9955
NJ	0.505 (.01582)	0.048 (.00676)	6.6005	0.506 (.01582)	0.054 (.00715)	6.6885
F	0.379 (.01535)	0.042 (.00635)	8.0655	0.378 (.01534)	0.046 (.00663)	8.1285
Kendall's W	0.460645 (.000)			0.461304 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 34 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.373	0.035	0.289	0.303
W	0.026	0.701	0.131	0.142
$\bar{N}$	0.048	0.566	0.185	0.201
NA	0.021	0.602	0.172	0.205
NL	0.300	0.053	0.316	0.331
J	0.192	0.228	0.273	0.307
AJ	0.027	0.711	0.130	0.132
JA	0.006	0.718	0.125	0.151
NJ	0.015	0.698	0.136	0.151
F	0.030	0.766	0.099	0.105

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 35

Parameters: n = 20 R^2 = 0.50 ρ^2 = 0.75 (k_1, k_2, k_3) = (4,4,4) m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.132940 (0.00109)	0.132278 (0.001043)
$Power_{NJ}$	0.210386 (0.00558)	0.209081 (0.005608)
$Power_F > Power_{NJ}$	0.0630	0.0770
$Power_{\overline{NJ}}$	0.250259 (0.00489)	0.260972 (0.018586)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.003420	0.012573

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.651 (.01508)	0.147 (.01120)	2.2955	0.645 (.01514)	0.134 (.01078)	2.2195
W	0.276 (.01414)	0.025 (.00494)	7.5960	0.295 (.01443)	0.024 (.00484)	7.7395
$\bar{N}$	0.404 (.01552)	0.054 (.00715)	5.1430	0.414 (.01558)	0.043 (.00642)	5.0905
NA	0.374 (.01531)	0.072 (.00818)	5.1300	0.383 (.01538)	0.076 (.00838)	5.0750
NL	0.600 (.01550)	0.116 (.01013)	3.0275	0.595 (.01553)	0.104 (.00966)	3.0075
J	0.503 (.01582)	0.066 (.00786)	3.6750	0.499 (.01582)	0.051 (.00696)	3.6215
AJ	0.324 (.01481)	0.020 (.00443)	7.0665	0.336 (.01494)	0.021 (.00454)	7.1930
JA	0.326 (.01483)	0.028 (.00522)	6.6665	0.340 (.01499)	0.019 (.00432)	6.5845
NJ	0.307 (.01459)	0.046 (.00663)	6.6420	0.319 (.01475)	0.054 (.00715)	6.6360
F	0.185 (.01229)	0.042 (.00635)	7.7695	0.164 (.01171)	0.049 (.00683)	7.8330
Kendall's W	0.445028 (.000)			0.475183 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 35 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.161	0.125	0.359	0.355
W	0.008	0.737	0.121	0.134
$\bar{N}$	0.033	0.588	0.186	0.193
NA	0.016	0.604	0.179	0.201
NL	0.099	0.216	0.346	0.339
J	0.048	0.379	0.283	0.290
AJ	0.006	0.728	0.134	0.132
JA	0.007	0.715	0.132	0.146
NJ	0.007	0.707	0.143	0.143
F	0.029	0.827	0.074	0.070

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 36

Parameters: n = 20 $R^2 = 0.50$ $\rho^2 = 0.90$ $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.080146 (0.00012)	0.079501 (0.000127)
$Power_{NJ}$	0.116817 (0.00088)	0.114934 (0.000904)
$Power_F > Power_{NJ}$	0.0370	0.0390
$Power_{\overline{NJ}}$	0.131279 (0.00082)	0.135519 (0.003819)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000422	0.002668

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.382 (.01537)	0.107 (.00978)	2.3470	0.361 (.01520)	0.112 (.00998)	2.4865
W	0.113 (.01002)	0.022 (.00464)	7.9160	0.111 (.00994)	0.026 (.00503)	7.8000
$\bar{N}$	0.194 (.01251)	0.034 (.00573)	5.9025	0.194 (.01251)	0.041 (.00627)	5.9390
NA	0.189 (.01239)	0.070 (.00807)	4.4595	0.193 (.01249)	0.077 (.00843)	4.4725
NL	0.312 (.01466)	0.089 (.00901)	3.1720	0.281 (.01422)	0.095 (.00928)	3.2645
J	0.239 (.01349)	0.042 (.00635)	4.0340	0.230 (.01331)	0.043 (.00642)	4.0515
AJ	0.152 (.01136)	0.022 (.00464)	7.2030	0.159 (.01157)	0.024 (.00484)	7.0725
JA	0.161 (.01163)	0.023 (.00474)	6.3130	0.170 (.01188)	0.026 (.00503)	6.2860
NJ	0.147 (.01120)	0.043 (.00642)	6.4890	0.145 (.01114)	0.052 (.00702)	6.5375
F	0.069 (.00802)	0.041 (.00627)	7.1720	0.068 (.00796)	0.050 (.00690)	7.0900
Kendall's W		0.411922 (.000)			0.387528 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 36 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.010	0.375	0.312	0.303
W	0.000	0.833	0.081	0.086
$\bar{N}$	0.003	0.685	0.159	0.153
NA	0.014	0.698	0.148	0.140
NL	0.006	0.493	0.252	0.249
J	0.001	0.640	0.180	0.179
AJ	0.000	0.807	0.096	0.097
JA	0.000	0.786	0.110	0.104
NJ	0.010	0.788	0.102	0.100
F	0.030	0.889	0.037	0.044

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 37

Parameters: n = 20 R^2 = 0.75 $\rho^2 = 0.25$ $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.760629 (0.02171)	0.761759 (0.020599)
$Power_{NJ}$	0.780885 (0.04121)	0.768994 (0.046905)
$Power_F > Power_{NJ}$	0.3020	0.3310
$Power_{\overline{NJ}}$	0.833585 (0.02383)	0.780747 (0.054765)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.015298	0.020123

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.850 (.01130)	0.149 (.01127)	2.2765	0.859 (.01101)	0.141 (.01101)	2.2375
W	0.938 (.00763)	0.040 (.00620)	6.6425	0.943 (.00734)	0.038 (.00605)	6.6515
$\bar{N}$	0.932 (.00796)	0.059 (.00745)	2.7120	0.935 (.00780)	0.054 (.00715)	2.7240
NA	0.802 (.01261)	0.082 (.00868)	8.2190	0.779 (.01313)	0.077 (.00843)	8.3045
NL	0.872 (.01057)	0.127 (.01053)	3.0310	0.892 (.00982)	0.107 (.00978)	2.8640
J	0.900 (.00949)	0.098 (.00941)	4.2400	0.912 (.00896)	0.086 (.00887)	4.1780
AJ	0.951 (.00683)	0.032 (.00557)	5.5175	0.956 (.00649)	0.029 (.00531)	5.5065
JA	0.815 (.01229)	0.031 (.00548)	7.9095	0.790 (.01289)	0.027 (.00513)	8.0180
NJ	0.874 (.01050)	0.058 (.00740)	6.5810	0.863 (.01088)	0.052 (.00702)	6.6585
F	0.893 (.00978)	0.060 (.00751)	7.8875	0.902 (.00941)	0.056 (.00727)	7.8575
Kendall's W	0.600893 (.000)			0.631397 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 37 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.530	0.007	0.214	0.249
W	0.031	0.706	0.128	0.135
$\bar{N}$	0.065	0.597	0.166	0.172
NA	0.034	0.658	0.155	0.153
NL	0.470	0.011	0.242	0.277
J	0.322	0.139	0.260	0.279
AJ	0.034	0.693	0.124	0.149
JA	0.012	0.750	0.121	0.117
NJ	0.024	0.705	0.140	0.131
F	0.051	0.763	0.084	0.102

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 38

Parameters: n = 20 R^2 = 0.75 ρ^2 = 0.90 $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.150102 (0.00174)	0.151878 (0.001817)
$Power_{NJ}$	0.276436 (0.00749)	0.281069 (0.007898)
$Power_F > Power_{NJ}$	0.0020	0.0000
$Power_{\overline{NJ}}$	0.293253 (0.00735)	0.298256 (0.014088)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000692	0.005794

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.704 (.01444)	0.097 (.00936)	1.7920	0.721 (.01419)	0.088 (.00896)	1.7295
W	0.417 (.01560)	0.041 (.00627)	7.9280	0.455 (.01576)	0.028 (.00522)	7.9240
$\bar{N}$	0.596 (.01552)	0.054 (.00715)	3.5810	0.603 (.01548)	0.048 (.00676)	3.5705
NA	0.527 (.01580)	0.085 (.00882)	4.8975	0.549 (.01574)	0.076 (.00838)	4.9080
NL	0.606 (.01546)	0.093 (.00919)	3.0530	0.634 (.01524)	0.068 (.00796)	2.9920
J	0.569 (.01567)	0.031 (.00548)	4.3975	0.565 (.01569)	0.033 (.00565)	4.4600
AJ	0.489 (.01582)	0.019 (.00432)	7.2550	0.489 (.01582)	0.020 (.00443)	7.2435
JA	0.501 (.01582)	0.024 (.00484)	6.4860	0.492 (.01582)	0.022 (.00464)	6.6060
NJ	0.454 (.01575)	0.054 (.00715)	7.0250	0.471 (.01579)	0.044 (.00649)	7.0355
F	0.224 (.01319)	0.056 (.00727)	8.5105	0.224 (.01319)	0.051 (.00696)	8.5310
Kendall's W		0.596230 (.000)			0.605109 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 38 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.131	0.133	0.365	0.371
W	0.013	0.555	0.217	0.215
$\bar{N}$	0.052	0.344	0.299	0.305
NA	0.026	0.403	0.283	0.288
NL	0.053	0.255	0.340	0.352
J	0.020	0.388	0.292	0.300
AJ	0.010	0.553	0.218	0.219
JA	0.011	0.529	0.228	0.232
NJ	0.012	0.531	0.230	0.227
F	0.032	0.788	0.087	0.093

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 39

Parameters: n = 20 R^2 = 0.90 ρ^2 = 0.50 (k_1, k_2, k_3) = (4,4,4) m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.953218 (0.00501)	0.955223 (0.004313)
$Power_{NJ}$	0.981336 (0.00264)	0.981975 (0.003230)
$Power_F > Power_{NJ}$	0.0850	0.0700
$Power_{\overline{NJ}}$	0.985850 (0.00161)	0.980344 (0.003645)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000618	0.000679

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.893 (.00978)	0.107 (.00978)	2.0720	0.911 (.00901)	0.089 (.00901)	1.9555
W	0.950 (.00690)	0.050 (.00690)	7.9495	0.958 (.00635)	0.040 (.00620)	8.0805
$\bar{N}$	0.944 (.00727)	0.056 (.00727)	1.9945	0.958 (.00635)	0.042 (.00635)	1.9200
NA	0.906 (.00923)	0.082 (.00868)	9.0430	0.922 (.00848)	0.069 (.00802)	9.0455
NL	0.916 (.00878)	0.084 (.00878)	3.6565	0.926 (.00828)	0.074 (.00828)	3.6610
J	0.951 (.00683)	0.049 (.00683)	3.8565	0.962 (.00605)	0.038 (.00605)	3.8295
AJ	0.972 (.00522)	0.028 (.00522)	5.0935	0.983 (.00409)	0.017 (.00409)	5.1080
JA	0.946 (.00715)	0.034 (.00573)	7.0925	0.968 (.00557)	0.017 (.00409)	6.9660
NJ	0.946 (.00715)	0.051 (.00696)	6.3025	0.948 (.00702)	0.048 (.00676)	6.3385
F	0.941 (.00745)	0.056 (.00727)	7.9600	0.945 (.00721)	0.051 (.00696)	8.0955
Kendall's W		0.725840 (.000)			0.765228 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 39 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.714	0.004	0.142	0.140
W	0.198	0.300	0.248	0.254
$\bar{N}$	0.314	0.165	0.261	0.260
NA	0.189	0.266	0.274	0.271
NL	0.642	0.006	0.185	0.167
J	0.517	0.036	0.230	0.217
AJ	0.168	0.326	0.252	0.254
JA	0.116	0.381	0.253	0.250
NJ	0.137	0.332	0.274	0.257
F	0.130	0.508	0.180	0.182

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 40

Parameters: n = 20 R^2 = 0.90 $\rho^2 = 0.75$ $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.762011 (0.02317)	0.765334 (0.021423)
$Power_{NJ}$	0.925276 (0.00734)	0.927939 (0.006298)
$Power_F > Power_{NJ}$	0.0020	0.0030
$Power_{\overline{NJ}}$	0.931355 (0.00639)	0.922813 (0.008755)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000299	0.002257

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.904 (.00932)	0.096 (.00932)	1.8020	0.897 (.00962)	0.103 (.00962)	1.8595
W	0.953 (.00670)	0.044 (.00649)	8.4995	0.958 (.00635)	0.038 (.00605)	8.4435
$\bar{N}$	0.943 (.00734)	0.056 (.00727)	2.1715	0.936 (.00774)	0.063 (.00769)	2.2210
NA	0.922 (.00848)	0.075 (.00833)	7.8975	0.917 (.00873)	0.081 (.00863)	7.9015
NL	0.924 (.00838)	0.076 (.00838)	4.1210	0.916 (.00878)	0.084 (.00878)	4.1170
J	0.963 (.00597)	0.036 (.00589)	3.8080	0.964 (.00589)	0.034 (.00573)	3.8130
AJ	0.973 (.00513)	0.023 (.00474)	5.5275	0.975 (.00494)	0.022 (.00464)	5.4730
JA	0.967 (.00565)	0.027 (.00513)	5.7550	0.972 (.00522)	0.024 (.00484)	5.7125
NJ	0.952 (.00676)	0.045 (.00656)	6.4800	0.949 (.00696)	0.048 (.00676)	6.4750
F	0.906 (.00923)	0.038 (.00605)	8.9535	0.901 (.00945)	0.052 (.00702)	8.9840
Kendall's W	0.714063 (.000)			0.714354 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 40 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.723	0.002	0.146	0.129
W	0.326	0.131	0.274	0.269
$\bar{N}$	0.495	0.039	0.243	0.223
NA	0.383	0.072	0.284	0.261
NL	0.620	0.009	0.203	0.168
J	0.475	0.033	0.259	0.233
AJ	0.299	0.151	0.281	0.269
JA	0.289	0.136	0.293	0.282
NJ	0.298	0.136	0.290	0.276
F	0.132	0.432	0.227	0.209

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 41

Parameters: n = 20 R^2 = 0.75 ρ^2 = 0.50 $(k_1, k_2, k_3) = (4,2,6)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.835997 (0.01718)	0.493797 (0.024317)
$Power_{NJ}$	0.545168 (0.06271)	0.697731 (0.028751)
$Power_F > Power_{NJ}$	0.8670	0.0410
$Power_{\overline{NJ}}$	0.571180 (0.05718)	0.712192 (0.040815)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.004947	0.015095

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.914 (.00887)	0.086 (.00887)	1.9215	0.809 (.01244)	0.191 (.01244)	2.5080
W	0.950 (.00690)	0.032 (.00557)	5.7470	0.863 (.01088)	0.023 (.00474)	7.5140
$\bar{N}$	0.957 (.00642)	0.034 (.00573)	2.6920	0.906 (.00923)	0.053 (.00709)	2.8260
NA	0.446 (.01573)	0.078 (.00848)	8.7985	0.884 (.01013)	0.058 (.00740)	6.0435
NL	0.927 (.00823)	0.073 (.00823)	2.6015	0.859 (.01101)	0.139 (.01095)	3.2560
J	0.968 (.00557)	0.032 (.00557)	3.9395	0.855 (.01114)	0.133 (.01074)	4.4965
AJ	0.966 (.00573)	0.015 (.00385)	5.2740	0.871 (.01061)	0.028 (.00522)	5.9810
JA	0.455 (.01576)	0.018 (.00421)	9.2415	0.857 (.01108)	0.020 (.00443)	6.5115
NJ	0.715 (.01428)	0.049 (.00683)	7.3210	0.874 (.01050)	0.038 (.00605)	6.8490
F	0.918 (.00868)	0.055 (.00721)	7.4635	0.726 (.01411)	0.036 (.00589)	9.0145
Kendall's W		0.803209 (.000)			0.539330 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 41 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.463	0.002	0.486	0.049
W	0.057	0.542	0.287	0.114
$\bar{N}$	0.096	0.404	0.374	0.126
NA	0.025	0.589	0.147	0.239
NL	0.408	0.007	0.520	0.065
J	0.227	0.041	0.705	0.027
AJ	0.049	0.479	0.395	0.077
JA	0.009	0.719	0.133	0.139
NJ	0.017	0.650	0.195	0.138
F	0.070	0.565	0.299	0.066

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 42

Parameters: n = 20 R^2 = 0.90 ρ^2 = 0.25 (k_1, k_2, k_3) = (4,2,6) m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.997866 (0.00010)	0.971117 (0.002821)
$Power_{NJ}$	0.872804 (0.05373)	0.986056 (0.001893)
$Power_F > Power_{NJ}$	0.8120	0.1390
$Power_{\overline{NJ}}$	0.887162 (0.04525)	0.985571 (0.001983)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.004500	0.000839

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.906 (.00923)	0.094 (.00923)	2.4375	0.859 (.01101)	0.141 (.01101)	2.4020
W	0.963 (.00597)	0.037 (.00597)	6.3580	0.957 (.00642)	0.043 (.00642)	7.8345
$\bar{N}$	0.955 (.00656)	0.045 (.00656)	2.1065	0.954 (.00663)	0.046 (.00663)	1.9760
NA	0.623 (.01533)	0.071 (.00813)	9.1735	0.901 (.00945)	0.082 (.00868)	8.5175
NL	0.915 (.00882)	0.085 (.00882)	2.7830	0.877 (.01039)	0.123 (.01039)	3.5535
J	0.960 (.00620)	0.040 (.00620)	3.9960	0.902 (.00941)	0.098 (.00941)	4.2185
AJ	0.980 (.00443)	0.020 (.00443)	4.9525	0.977 (.00474)	0.023 (.00474)	4.8550
JA	0.635 (.01523)	0.021 (.00454)	9.1420	0.938 (.00763)	0.028 (.00522)	7.2875
NJ	0.885 (.01009)	0.042 (.00635)	6.9960	0.956 (.00649)	0.044 (.00649)	6.2085
F	0.953 (.00670)	0.047 (.00670)	7.0550	0.961 (.00613)	0.039 (.00613)	8.1470
Kendall's W	0.794920 (.000)			0.675568 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 42 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.477	0.004	0.453	0.066
W	0.034	0.738	0.118	0.110
$\dot{N}$	0.055	0.604	0.209	0.132
NA	0.030	0.666	0.092	0.212
NL	0.446	0.005	0.467	0.082
J	0.214	0.068	0.676	0.042
AJ	0.039	0.665	0.219	0.077
JA	0.005	0.803	0.078	0.114
NJ	0.023	0.741	0.108	0.128
F	0.055	0.705	0.152	0.088

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 43

Parameters: n = 20 R^2 = 0.90 ρ^2 = 0.90 $(k_1, k_2, k_3) = (6,2,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.978801 (0.00233)	0.818241 (0.018909)
$Power_{NJ}$	0.294895 (0.02752)	0.460615 (0.031290)
$Power_F > Power_{NJ}$	1.0000	0.9400
$Power_{\overline{NJ}}$	0.298929 (0.02734)	0.468354 (0.033959)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000087	0.005469

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.915 (.00882)	0.085 (.00882)	2.2940	0.897 (.00962)	0.103 (.00962)	1.9505
W	0.962 (.00605)	0.037 (.00597)	5.8900	0.950 (.00690)	0.041 (.00627)	5.9520
$\bar{N}$	0.962 (.00605)	0.038 (.00605)	2.3665	0.951 (.00683)	0.048 (.00676)	2.2985
NA	0.018 (.00421)	0.088 (.00896)	8.8820	0.222 (.01315)	0.100 (.00949)	8.7155
NL	0.916 (.00878)	0.084 (.00878)	2.4985	0.900 (.00949)	0.100 (.00949)	3.1375
J	0.973 (.00513)	0.027 (.00513)	3.9285	0.962 (.00605)	0.038 (.00605)	3.9215
AJ	0.976 (.00484)	0.024 (.00484)	4.8915	0.967 (.00565)	0.030 (.00540)	4.9040
JA	0.020 (.00443)	0.024 (.00484)	9.6555	0.188 (.01236)	0.034 (.00573)	9.4890
NJ	0.453 (.01575)	0.045 (.00656)	7.8205	0.637 (.01521)	0.058 (.00740)	7.4725
F	0.954 (.00663)	0.044 (.00649)	6.7730	0.908 (.00914)	0.060 (.00751)	7.1590
Kendall's W	0.853577 (.000)			0.812480 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 43 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.165	0.026	0.762	0.047
W	0.024	0.344	0.575	0.057
$\bar{N}$	0.045	0.246	0.649	0.060
NA	0.006	0.814	0.057	0.123
NL	0.141	0.042	0.768	0.049
J	0.081	0.083	0.814	0.022
AJ	0.035	0.309	0.622	0.034
JA	0.000	0.862	0.057	0.081
NJ	0.010	0.797	0.116	0.077
F	0.066	0.407	0.513	0.014

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 44

Parameters: n = 40 R^2 = 0.90 $\rho^2 = 0.50$ $(k_1, k_2, k_3) = (2,4,6)$ m = 500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.999995 (0.00000)	0.999994 (0.000000)
$Power_{NJ}$	0.999999 (0.00000)	0.999999 (0.000000)
$Power_F > Power_{NJ}$	0.0300	0.0420
$Power_{\overline{NJ}}$	0.999999 (0.00000)	0.999992 (0.000000)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000000	0.000000

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.932 (.01127)	0.068 (.01127)	2.6100	0.908 (.01294)	0.092 (.01294)	2.6080
W	0.950 (.00976)	0.050 (.00976)	7.0280	0.954 (.00938)	0.046 (.00938)	7.6390
$\bar{N}$	0.942 (.01046)	0.058 (.01046)	2.5150	0.952 (.00957)	0.048 (.00957)	2.2480
NA	0.944 (.01029)	0.056 (.01029)	9.7310	0.942 (.01046)	0.058 (.01046)	9.6470
NL	0.936 (.01096)	0.064 (.01096)	4.8400	0.920 (.01214)	0.080 (.01214)	4.8750
J	0.952 (.00957)	0.048 (.00957)	4.9030	0.918 (.01228)	0.082 (.01228)	4.9270
AJ	0.972 (.00739)	0.028 (.00739)	6.1140	0.984 (.00562)	0.016 (.00562)	5.8160
JA	0.976 (.00685)	0.024 (.00685)	6.3790	0.978 (.00657)	0.022 (.00657)	6.2700
NJ	0.944 (.01029)	0.056 (.01029)	7.3030	0.942 (.01046)	0.058 (.01046)	7.4060
F	0.948 (.00994)	0.052 (.00994)	3.5770	0.956 (.00918)	0.044 (.00918)	3.5640
Kendall's W		0.593434 (.000)			0.631435 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 44 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.962	0.000	0.020	0.018
W	0.748	0.010	0.110	0.132
$\bar{N}$	0.812	0.002	0.088	0.098
NA	0.656	0.022	0.122	0.200
NL	0.938	0.000	0.034	0.028
J	0.930	0.000	0.044	0.026
AJ	0.664	0.020	0.146	0.170
JA	0.594	0.036	0.148	0.222
NJ	0.638	0.018	0.148	0.196
F	0.482	0.060	0.182	0.276

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 45

Parameters: n=40 R^2 = 0.90 ρ^2=0.50 (k_1, k_2, k_3)=(6,2,4) m=500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	1.000000 (0.00000)	0.999995 (0.000000)
$Power_{NJ}$	0.931576 (0.02556)	0.997481 (0.000292)
$Power_F > Power_{NJ}$	0.9960	0.8380
$Power_{\overline{NJ}}$	0.936810 (0.02214)	0.997387 (0.000366)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000596	0.000061

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.904 (.01319)	0.096 (.01319)	3.1880	0.932 (.01127)	0.068 (.01127)	2.5950
W	0.946 (.01012)	0.054 (.01012)	4.9810	0.958 (.00898)	0.042 (.00898)	6.3030
$\bar{N}$	0.944 (.01029)	0.056 (.01029)	2.8870	0.954 (.00938)	0.046 (.00938)	2.4190
NA	0.668 (.02108)	0.086 (.01255)	9.1440	0.928 (.01157)	0.064 (.01096)	9.5000
NL	0.906 (.01306)	0.094 (.01306)	3.1820	0.936 (.01096)	0.064 (.01096)	2.8840
J	0.960 (.00877)	0.040 (.00877)	3.3070	0.962 (.00856)	0.038 (.00856)	3.5020
AJ	0.972 (.00739)	0.028 (.00739)	4.4130	0.984 (.00562)	0.016 (.00562)	4.5710
JA	0.690 (.02070)	0.032 (.00788)	9.2480	0.974 (.00712)	0.016 (.00562)	8.7650
NJ	0.918 (.01228)	0.056 (.01029)	7.5300	0.958 (.00898)	0.042 (.00898)	6.6600
F	0.934 (.01111)	0.066 (.01111)	7.1200	0.942 (.01046)	0.058 (.01046)	7.8010
Kendall's W		0.733267 (.000)			0.802168 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 45 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.614	0.000	0.360	0.026
W	0.248	0.160	0.484	0.108
$\bar{N}$	0.278	0.128	0.492	0.102
NA	0.078	0.458	0.208	0.256
NL	0.590	0.000	0.380	0.030
J	0.396	0.008	0.574	0.022
AJ	0.186	0.204	0.536	0.074
JA	0.052	0.508	0.204	0.236
NJ	0.078	0.458	0.244	0.220
F	0.200	0.222	0.506	0.072

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 46

Parameters: n = 40 R^2 = 0.90 ρ^2 = 0.75 $(k_1, k_2, k_3) = (4,2,6)$ m = 500

	H_1 vs H_2	H_1 vs H_3
$Power_F$	1.000000 (0.00000)	0.993914 (0.000118)
$Power_{NJ}$	0.957295 (0.00720)	0.999499 (1.7E-06)
$Power_F > Power_{NJ}$	1.0000	0.0000
$Power_{\overline{NJ}}$	0.959255 (0.00650)	0.999313 (5.1E-06)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000081	0.000002

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.936 (.01096)	0.064 (.01096)	2.7780	0.930 (.01142)	0.070 (.01142)	1.9070
W	0.956 (.00918)	0.044 (.00918)	4.8090	0.948 (.00994)	0.052 (.00994)	8.1520
$\bar{N}$	0.954 (.00938)	0.046 (.00938)	2.6420	0.960 (.00877)	0.040 (.00877)	1.7680
NA	0.762 (.01906)	0.062 (.01080)	9.2530	0.938 (.01080)	0.062 (.01080)	8.5950
NL	0.942 (.01046)	0.058 (.01046)	3.0340	0.936 (.01096)	0.064 (.01096)	5.5300
J	0.972 (.00739)	0.028 (.00739)	3.6570	0.950 (.00976)	0.050 (.00976)	3.5590
AJ	0.974 (.00712)	0.026 (.00712)	4.7330	0.974 (.00712)	0.026 (.00712)	4.8560
JA	0.774 (.01872)	0.028 (.00739)	9.3310	0.980 (.00627)	0.020 (.00627)	4.9990
NJ	0.954 (.00938)	0.042 (.00898)	7.8230	0.956 (.00918)	0.044 (.00918)	6.1280
F	0.946 (.01012)	0.054 (.01012)	6.9400	0.948 (.00994)	0.052 (.00994)	9.5060
Kendall's W		0.787329 (.000)			0.811429 (.000)	

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 46 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.818	0.000	0.182	0.000
W	0.642	0.000	0.350	0.008
$\bar{N}$	0.700	0.000	0.294	0.006
NA	0.190	0.188	0.148	0.474
NL	0.784	0.000	0.216	0.000
J	0.660	0.000	0.340	0.000
AJ	0.558	0.000	0.440	0.002
JA	0.136	0.252	0.166	0.446
NJ	0.276	0.112	0.304	0.308
F	0.508	0.000	0.490	0.002

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 47

Parameters: n = 20 R^2 = 0.70 $\rho^2 = 0.25$ $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.659586 (0.02455)	0.664004 (0.026183)
$Power_{NJ}$	0.692199 (0.04883)	0.693590 (0.045759)
$Power_F > Power_{NJ}$	0.2980	0.3200
$Power_{\overline{NJ}}$	0.758556 (0.03031)	0.719497 (0.058787)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.021574	0.029039

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.830 (.01188)	0.169 (.01186)	2.4010	0.822 (.01210)	0.176 (.01205)	2.4585
W	0.897 (.00962)	0.049 (.00683)	6.4200	0.887 (.01002)	0.036 (.00589)	6.5275
$\bar{N}$	0.899 (.00953)	0.062 (.00763)	3.1425	0.905 (.00928)	0.053 (.00709)	3.0760
NA	0.721 (.01419)	0.089 (.00901)	7.5870	0.695 (.01457)	0.090 (.00905)	7.6420
NL	0.860 (.01098)	0.137 (.01088)	2.9265	0.847 (.01139)	0.151 (.01133)	3.0175
J	0.877 (.01039)	0.115 (.01009)	4.2270	0.884 (.01013)	0.111 (.00994)	4.1770
AJ	0.890 (.00990)	0.034 (.00573)	5.8850	0.900 (.00949)	0.028 (.00522)	5.7165
JA	0.707 (.01440)	0.035 (.00581)	7.7885	0.687 (.01467)	0.026 (.00503)	7.8775
NJ	0.782 (.01306)	0.063 (.00769)	6.7170	0.770 (.01331)	0.064 (.00774)	6.6945
F	0.791 (.01286)	0.056 (.00727)	7.9055	0.803 (.01258)	0.061 (.00757)	7.8130
Kendall's W	0.531439 (.000)			0.530352 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 47 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.478	0.019	0.243	0.260
W	0.026	0.715	0.118	0.141
$\bar{N}$	0.057	0.603	0.154	0.186
NA	0.037	0.691	0.141	0.131
NL	0.420	0.026	0.269	0.285
J	0.299	0.160	0.274	0.267
AJ	0.031	0.701	0.122	0.146
JA	0.009	0.779	0.111	0.101
NJ	0.027	0.735	0.114	0.124
F	0.039	0.750	0.094	0.117

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 48

Parameters: n = 20 R^2 = 0.70 ρ^2 = 0.50 (k_1, k_2, k_3) = (4,4,4) m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.489874 (0.02145)	0.482599 (0.019873)
$Power_{NJ}$	0.646493 (0.03158)	0.643506 (0.032650)
$Power_F > Power_{NJ}$	0.1000	0.1060
$Power_{\overline{NJ}}$	0.701552 (0.02226)	0.671993 (0.047199)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.011548	0.021552

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.874 (.01050)	0.124 (.01043)	2.0010	0.870 (.01064)	0.128 (.01057)	1.9950
W	0.865 (.01081)	0.029 (.00531)	7.0070	0.819 (.01218)	0.036 (.00589)	7.1290
$\bar{N}$	0.883 (.01017)	0.045 (.00656)	3.1100	0.878 (.01035)	0.052 (.00702)	3.1065
NA	0.790 (.01289)	0.060 (.00751)	6.9030	0.776 (.01319)	0.076 (.00838)	6.9190
NL	0.901 (.00945)	0.094 (.00923)	2.8935	0.896 (.00966)	0.092 (.00914)	2.8505
J	0.927 (.00823)	0.052 (.00702)	3.9615	0.903 (.00936)	0.062 (.00763)	4.0140
AJ	0.837 (.01169)	0.021 (.00454)	6.4185	0.837 (.01169)	0.026 (.00503)	6.4025
JA	0.780 (.01311)	0.014 (.00372)	7.2345	0.757 (.01357)	0.022 (.00464)	7.2370
NJ	0.788 (.01293)	0.037 (.00597)	6.7595	0.791 (.01286)	0.056 (.00727)	6.7110
F	0.671 (.01487)	0.045 (.00656)	8.7115	0.679 (.01477)	0.054 (.00715)	8.6355
Kendall's W	0.601952 (.000)			0.601669 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 48 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.485	0.015	0.263	0.237
W	0.046	0.562	0.204	0.188
$\bar{N}$	0.101	0.409	0.259	0.231
NA	0.050	0.483	0.249	0.218
NL	0.407	0.030	0.293	0.270
J	0.264	0.119	0.334	0.283
AJ	0.051	0.583	0.186	0.180
JA	0.019	0.590	0.216	0.175
NJ	0.023	0.563	0.219	0.195
F	0.055	0.692	0.138	0.115

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 49

Parameters: n = 20 $R^2 = 0.70$ $\rho^2 = 0.75$ $(k_1, k_2, k_3) = (4,4,4)$ m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.261127 (0.00694)	0.260091 (0.006815)
$Power_{NJ}$	0.443709 (0.01872)	0.443418 (0.018369)
$Power_F > Power_{NJ}$	0.0160	0.0150
$Power_{\overline{NJ}}$	0.481531 (0.01624)	0.476259 (0.033498)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.003769	0.016178

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.809 (.01244)	0.132 (.01071)	2.0270	0.821 (.01213)	0.114 (.01006)	1.8900
W	0.625 (.01532)	0.032 (.00557)	7.4360	0.588 (.01557)	0.030 (.00540)	7.6155
$\bar{N}$	0.710 (.01436)	0.060 (.00751)	3.5780	0.724 (.01414)	0.055 (.00721)	3.5485
NA	0.641 (.01518)	0.090 (.00905)	5.6285	0.640 (.01519)	0.087 (.00892)	5.7495
NL	0.771 (.01329)	0.107 (.00978)	3.0200	0.779 (.01313)	0.095 (.00928)	2.9760
J	0.736 (.01395)	0.054 (.00715)	4.0400	0.764 (.01343)	0.043 (.00642)	3.9075
AJ	0.603 (.01548)	0.027 (.00513)	7.0545	0.617 (.01538)	0.022 (.00464)	6.9225
JA	0.600 (.01550)	0.029 (.00531)	6.5990	0.610 (.01543)	0.026 (.00503)	6.8385
NJ	0.590 (.01556)	0.056 (.00727)	6.8720	0.590 (.01556)	0.051 (.00696)	6.9210
F	0.373 (.01530)	0.054 (.00715)	8.7450	0.384 (.01539)	0.045 (.00656)	8.6310
Kendall's W	0.562652 (.000)			0.588004 (.000)		

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 49 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.294	0.043	0.332	0.331
W	0.053	0.514	0.210	0.223
$\bar{N}$	0.108	0.302	0.286	0.304
NA	0.066	0.379	0.281	0.274
NL	0.207	0.101	0.339	0.353
J	0.118	0.210	0.326	0.346
AJ	0.041	0.529	0.219	0.211
JA	0.037	0.503	0.229	0.231
NJ	0.044	0.489	0.231	0.236
F	0.056	0.715	0.113	0.116

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 50 (continued)

Testing H_2 *vs* H_3

Test	Reject Both	Reject Neither	Reject H_2	Reject H_3
N	0.069	0.204	0.346	0.381
W	0.007	0.691	0.146	0.156
$\bar{N}$	0.024	0.499	0.218	0.259
NA	0.024	0.561	0.187	0.228
NL	0.029	0.331	0.301	0.339
J	0.008	0.511	0.220	0.261
AJ	0.004	0.683	0.146	0.167
JA	0.005	0.659	0.158	0.178
NJ	0.012	0.669	0.145	0.174
F	0.033	0.817	0.072	0.078

Table IV.2.2 Results of Normal Deviate Experiments (continued)

Experiment: 50

Parameters: n = 20 R^2 = 0.70 ρ^2 = 0.90 (k_1, k_2, k_3) = (4,4,4) m = 1000

	H_1 vs H_2	H_1 vs H_3
$Power_F$	0.124813 (0.00086)	0.126036 (0.000898)
$Power_{NJ}$	0.222932 (0.00445)	0.226054 (0.004534)
$Power_F > Power_{NJ}$	0.0030	0.0040
$Power_{\overline{NJ}}$	0.238715 (0.00438)	0.245704 (0.010256)
$SSE(Power_{NJ}, Power_{\overline{NJ}})$	0.000558	0.005557

Test	$\hat{P}$	$\hat{\alpha}$	Avg. Rank	$\hat{P}$	$\hat{\alpha}$	Avg. Rank
N	0.584 (.01559)	0.109 (.00986)	2.0040	0.605 (.01547)	0.099 (.00945)	1.8865
W	0.338 (.01497)	0.035 (.00581)	7.6710	0.302 (.01453)	0.035 (.00581)	7.9435
$\bar{N}$	0.412 (.01557)	0.057 (.00734)	4.4900	0.447 (.01573)	0.047 (.00670)	4.4125
NA	0.381 (.01536)	0.087 (.00892)	4.7155	0.396 (.01547)	0.088 (.00896)	4.7140
NL	0.485 (.01581)	0.086 (.00887)	2.9515	0.515 (.01581)	0.080 (.00858)	2.9025
J	0.411 (.01557)	0.040 (.00620)	4.2370	0.449 (.01574)	0.034 (.00573)	4.2450
AJ	0.317 (.01472)	0.027 (.00513)	7.3710	0.353 (.01512)	0.023 (.00474)	7.3495
JA	0.340 (.01499)	0.029 (.00531)	6.5695	0.370 (.01528)	0.022 (.00464)	6.5430
NJ	0.306 (.01458)	0.056 (.00727)	6.9020	0.335 (.01493)	0.052 (.00702)	6.8290
F	0.134 (.01078)	0.057 (.00734)	8.0885	0.158 (.01154)	0.049 (.00683)	8.1745
Kendall's W		0.515612 (.000)			0.547394 (.000)	

Table IV.3.1 Mean Observed Power by $R^2 \times \rho^2$

(Equal k cases with n = 20)

N J
W AJ
$\bar{N}$ JA
NA NJ
NL F

R^2		ρ^2 0.25		0.50		0.75		0.90	
	0.50	0.7840	0.8220	0.7965	0.7740	0.6480	0.5010	0.3715	0.2345
		0.6280	0.6625	0.5025	0.5375	0.2855	0.3300	0.1120	0.1555
		0.7140	0.4710	0.6380	0.4935	0.4090	0.3330	0.1940	0.1655
		0.5170	0.5360	0.5410	0.5055	0.3785	0.3130	0.1916	0.1460
		0.8155	0.5405	0.7980	0.3785	0.5975	0.1745	0.2965	0.0685
	0.70	0.8260	0.8805	0.8720	0.9150	0.8150	0.7500	0.5945	0.4300
		0.8920	0.8950	0.8420	0.8370	0.6065	0.6100	0.3200	0.3550
		0.9020	0.6970	0.8805	0.7685	0.7170	0.6050	0.4295	0.3550
		0.7080	0.7760	0.7830	0.7895	0.6405	0.5900	0.3885	0.3205
R^2		0.8535	0.7970	0.8985	0.6750	0.7750	0.3785	0.5000	0.1460
	0.75	0.8818	0.9235	0.8595	0.9325	0.8605	0.8600	0.8058	0.7230
		0.9502	0.9652	0.9140	0.9300	0.7580	0.7795	0.6350	0.6730
		0.9428	0.8828	0.9245	0.8665	0.8410	0.7710	0.7428	0.6732
		0.8602	0.9072	0.8500	0.8880	0.7775	0.7560	0.6865	0.6468
		0.9000	0.9238	0.8995	0.8245	0.8560	0.5460	0.7440	0.4270
	0.90	0.8965	0.9470	0.9225	0.9642	0.9132	0.9628	0.8870	0.9040
		0.9600	0.9835	0.9565	0.9778	0.9562	0.9720	0.8610	0.8840
		0.9555	0.9100	0.9565	0.9655	0.9408	0.9688	0.9030	0.8900
		0.8675	0.9460	0.9280	0.9500	0.9332	0.9532	0.8680	0.8530
		0.9045	0.9495	0.9350	0.9450	0.9330	0.9278	0.8800	0.6285

Table IV.3.2 ANOVA of Analytic Power of NJ- and F-test

REPEATED MEASURE ANALYSIS ON ANALYTIC POWERS - F AND NJ

GENERAL LINEAR MODELS PROCEDURE

DEPENDENT VARIABLE: NJPOW

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	48	4.55391896	0.09487331	44.72
ERROR	35	0.07424462	0.00212127	PR > F
CORRECTED TOTAL	83	4.62816358		0.0001

R-SQUARE	C.V.	ROOT MSE	NJPOW MEAN
0.983958	5.9852	0.04605730	0.76952571

SOURCE	DF	TYPE I SS	F VALUE	PR > F
N	1	0.87600978	412.96	0.0001
R2	1	0.99481820	468.97	0.0001
P2	3	1.77033607	278.19	0.0001
K12	6	0.59630339	46.85	0.0001
N*R2	1	0.08250979	38.90	0.0001
N*P2	3	0.06928553	10.89	0.0001
N*K12	6	0.05301671	4.17	0.0029
R2*P2	3	0.07886497	12.39	0.0001
R2*K12	6	0.00499559	0.39	0.8788
P2*K12	18	0.02777894	0.73	0.7610

SOURCE	DF	TYPE III SS	F VALUE	PR > F
N	1	0.73953544	348.63	0.0001
R2	1	0.76262082	359.51	0.0001
P2	3	1.35584492	213.06	0.0001
K12	6	0.53914102	42.36	0.0001
N*R2	1	0.05362737	25.28	0.0001
N*P2	3	0.05125353	8.05	0.0003
N*K12	6	0.04564363	3.59	0.0071
R2*P2	3	0.07075796	11.12	0.0001
R2*K12	6	0.00421325	0.33	0.9160
P2*K12	18	0.02777894	0.73	0.7610

DEPENDENT VARIABLE: FPOW

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	48	5.18944017	0.10811334	70.71
ERROR	35	0.05351413	0.00152898	PR > F
CORRECTED TOTAL	83	5.24295430		0.0001

R-SQUARE	C.V.	ROOT MSE	FPOW MEAN
0.989793	4.8525	0.03910211	0.80581330

SOURCE	DF	TYPE I SS	F VALUE	PR > F
N	1	0.94471875	617.88	0.0001
R2	1	1.12215813	733.93	0.0001
P2	3	1.50889487	328.96	0.0001
K12	6	0.59666765	65.04	0.0001
N*R2	1	0.10283473	67.26	0.0001
N*P2	3	0.09839637	21.45	0.0001
N*K12	6	0.08442578	9.20	0.0001
R2*P2	3	0.11870441	25.88	0.0001
R2*K12	6	0.06015398	6.56	0.0001
P2*K12	18	0.55248551	20.07	0.0001

SOURCE	DF	TYPE III SS	F VALUE	PR > F
N	1	0.69178484	452.45	0.0001
R2	1	0.64640134	422.77	0.0001
P2	3	1.05814664	230.69	0.0001
K12	6	0.55217858	60.19	0.0001
N*R2	1	0.11706439	76.56	0.0001
N*P2	3	0.08118578	17.70	0.0001
N*K12	6	0.07242992	7.90	0.0001
R2*P2	3	0.08058190	17.57	0.0001
R2*K12	6	0.04935118	5.38	0.0005
P2*K12	18	0.55248551	20.07	0.0001

Table IV.3.2 ANOVA of Analytic Power of NJ- and F-test (continued)

UNIVARIATE TESTS OF HYPOTHESES FOR WITHIN SUBJECT EFFECTS

SOURCE: TEST

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
1	0.11903225	0.11903225	74.70	0.0001	.	.

SOURCE: TEST*N

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
1	0.00039837	0.00039837	0.25	0.6202	.	.

SOURCE: TEST*R2

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
1	0.00240060	0.00240060	1.51	0.2279	.	.

SOURCE: TEST*P2

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
3	0.01072928	0.00357643	2.24	0.1003	.	.

SOURCE: TEST*K12

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
6	1.00933209	0.16822201	105.57	0.0001	.	.

SOURCE: TEST*N*R2

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
1	0.00611296	0.00611296	3.84	0.0582	.	.

SOURCE: TEST*N*P2

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
3	0.02320526	0.00773509	4.85	0.0063	.	.

SOURCE: TEST*N*K12

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
6	0.04517247	0.00752875	4.72	0.0013	.	.

SOURCE: TEST*R2*P2

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
3	0.00081317	0.00027106	0.17	0.9159	.	.

SOURCE: TEST*R2*K12

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
6	0.03457636	0.00576273	3.62	0.0068	.	.

SOURCE: TEST*P2*K12

DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F	ADJ PR > F G - G	H - F
18	0.26157106	0.01453173	9.12	0.0001	.	.

SOURCE: ERROR(TEST)

DF	TYPE III SS	MEAN SQUARE
35	0.05577064	0.00159345

TESTS OF HYPOTHESES FOR BETWEEN SUBJECTS EFFECTS

SOURCE	DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F
N	1	1.43092192	1.43092192	695.70	0.0001
R2	1	1.40662155	1.40662155	683.89	0.0001
P2	3	2.40326228	0.80108743	389.48	0.0001
K12	6	0.08198752	0.01366459	6.64	0.0001
N*R2	1	0.16457880	0.16457880	80.02	0.0001
N*P2	3	0.10923405	0.03641135	17.70	0.0001
N*K12	6	0.07290107	0.01215018	5.91	0.0002
R2*P2	3	0.15052669	0.05017556	24.39	0.0001
R2*K12	6	0.01898807	0.00316468	1.54	0.1946
P2*K12	18	0.31869339	0.01770519	8.61	0.0001
ERROR	35	0.07198811	0.00205680		

Table IV.3.2 ANOVA of Analytic Power of NJ- and F-test (continued)

REPEATED MEASURE ANALYSIS ON ANALYTIC POWERS - F AND NJ

ANALYSIS OF VARIANCE OF CONTRAST VARIABLES

CONTRAST VARIABLE: TEST

SOURCE	DF	TYPE III SS	MEAN SQUARE	F VALUE	PR > F
MEAN	1	0.23806450	0.23806450	74.70	0.0001
N	1	0.00079673	0.00079673	0.25	0.6202
R2	1	0.00480121	0.00480121	1.51	0.2279
P2	3	0.02145856	0.00715285	2.24	0.1003
K12	6	2.01866417	0.33644403	105.57	0.0001
N*R2	1	0.01222592	0.01222592	3.84	0.0582
N*P2	3	0.04641051	0.01547017	4.85	0.0063
N*K12	6	0.09034494	0.01505749	4.72	0.0013
R2*P2	3	0.00162633	0.00054211	0.17	0.9159
R2*K12	6	0.06915272	0.01152545	3.62	0.0068
P2*K12	18	0.52314213	0.02906345	9.12	0.0001
ERROR	35	0.11154129	0.00318689		

MANOVA TEST CRITERIA FOR THE HYPOTHESIS OF NO TEST EFFECT

```
H = TYPE III SS&CP MATRIX FOR:  TEST
E = ERROR SS&CP MATRIX
P = DF OF RM EFFECT    =      1
Q = HYPOTHESIS DF      =      1
NE= DF OF E            =     35
S = MIN(P,Q)           =      1
M = .5(ABS(P-Q)-1)     =   -0.5
N = .5(NE-P)           =   17.0
```

```
WILKS' CRITERION        L = DET(E)/DET(H+E) =        0.31904875

        EXACT F = (1-L)/L*(NE+Q-P)/P
                            WITH P AND NE+Q-P DF

                F(1,35) =     74.70      PROB > F = 0.0001
```

```
PILLAI'S TRACE          V = TR(H*INV(H+E)) =         0.68095125

        F APPROXIMATION = (2N+S)/(2M+S+1) * V/(S-V)
                            WITH S(2M+S+1) AND S(2N+S) DF

                F(1,35) =     74.70      PROB > F = 0.0001
```

```
HOTELLING-LAWLEY TRACE = TR(E**-1*H) =               2.13431727

        F APPROXIMATION = (2S*N-S+2)*TR(E**-1*H)/(S*S*(2M+S+1))
                            WITH S(2M+S+1) AND 2S*N-S+2 DF

                F(1,35) =     74.70      PROB > F = 0.0001
```

```
ROY'S MAXIMUM ROOT CRITERION =                       2.13431727

        FIRST CANONICAL VARIABLE YIELDS AN F UPPER BOUND

                F(1,35) =     74.70      PROB > F = 0.0001
```

Table IV.3.3 Results of Non-Normal Deviate Experiments

Cox (N) Test: Cases Involving H_1 *vs* H_j, $j = 2,3$

$\hat{P}$ / $\hat{\alpha}$ / Avg. Rank		Distribution of Disturbance Term				
		$TN(0, \sigma_e^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_e^2)$	$N(0, \sigma_e^2)$
NN1:	(4,2)	0.8820	0.8760	0.8820	0.8900	0.8840
		0.1180	0.1220	0.1180	0.1100	0.1160
		2.3270	2.4610	2.3770	2.3250	2.2750
NN1:	(4,6)	0.7920	0.8080	0.8140	0.7940	0.8120
		0.2080	0.1900	0.1840	0.2060	0.1880
		2.7300	2.6400	2.6070	2.7260	2.6040
NN2:	(4,4)	0.8860	0.9220	0.9100	0.8660	0.9280
		0.1140	0.0780	0.0900	0.1340	0.0720
		2.3540	2.1120	2.2090	2.5030	2.0680
NN2:	(4,4)	0.8740	0.9020	0.9040	0.9000	0.9040
		0.1260	0.0980	0.0960	0.1000	0.0960
		2.2500	2.1520	2.1210	2.1600	2.0740
NN3:	(4,2)	0.9260	0.9280	0.9300	0.9080	0.9200
		0.0740	0.0720	0.0700	0.0920	0.0800
		2.1510	2.1300	2.0910	2.1990	2.1640
NN3:	(4,6)	0.8800	0.8800	0.8900	0.8760	0.8900
		0.1200	0.1200	0.1100	0.1240	0.1100
		1.9850	2.0310	1.8870	2.0480	1.8400
NN4:	(4,2)	0.9200	0.9140	0.9280	0.8980	0.9340
		0.0800	0.0840	0.0700	0.1020	0.0660
		2.0560	2.1310	2.0010	2.1940	1.9530
NN4:	(4,6)	0.8800	0.8780	0.8620	0.8540	0.8680
		0.1200	0.1180	0.1360	0.1440	0.1320
		1.9100	1.9240	2.0570	2.1340	1.9850
NN5:	(4,4)	0.8680	0.8880	0.8900	0.8620	0.8620
		0.1320	0.1120	0.1060	0.1360	0.1380
		2.0630	1.9690	1.9040	2.0640	2.1170
NN5:	(4,4)	0.8440	0.8500	0.8700	0.8560	0.8620
		0.1520	0.1360	0.1220	0.1440	0.1340
		2.1980	2.0960	2.0250	2.1810	2.0810
NN6:	(4,4)	0.8660	0.8580	0.8560	0.8280	0.8660
		0.1100	0.1220	0.1280	0.1300	0.1200
		1.8740	1.9980	2.0090	2.0020	1.9350
NN6:	(4,4)	0.8820	0.8740	0.8420	0.8900	0.8460
		0.1080	0.1120	0.1260	0.0800	0.1180
		1.8470	1.8730	1.9920	1.6310	1.9510
NN7:	(4,2)	0.9240	0.9060	0.9020	0.9240	0.9020
		0.0760	0.0880	0.0920	0.0760	0.0980
		1.7790	1.9570	2.0060	1.8800	1.8920
NN7:	(4,6)	0.6280	0.6760	0.6680	0.6520	0.6520
		0.2320	0.2220	0.2240	0.2460	0.2240
		2.8870	2.7760	2.8010	2.9970	2.7760
NN8:	(4,4)	0.9020	0.8900	0.8880	0.8760	0.8820
		0.0920	0.0840	0.0940	0.1120	0.1080
		1.6620	1.6610	1.6970	1.8390	1.8210
NN8:	(4,4)	0.8920	0.8980	0.9020	0.8860	0.9000
		0.0980	0.0720	0.0700	0.1020	0.0880
		1.7180	1.5590	1.5260	1.7590	1.6630

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

Cox (N) Test: H_2 *vs* H_3

Reject Both Reject Neither Reject H_2 Reject H_3		Distribution of Disturbance Term			
	$TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1: (2,6)	0.416	0.424	0.424	0.388	0.422
	0.000	0.010	0.010	0.004	0.006
	0.510	0.478	0.486	0.516	0.490
	0.074	0.088	0.080	0.092	0.082
NN2: (4,4)	0.560	0.574	0.574	0.592	0.576
	0.002	0.006	0.004	0.000	0.004
	0.214	0.218	0.224	0.200	0.218
	0.224	0.202	0.198	0.208	0.202
NN3: (2,6)	0.560	0.568	0.568	0.546	0.558
	0.002	0.002	0.002	0.000	0.000
	0.406	0.400	0.396	0.414	0.408
	0.032	0.030	0.034	0.040	0.034
NN4: (2,6)	0.544	0.564	0.600	0.528	0.588
	0.000	0.000	0.000	0.000	0.000
	0.444	0.420	0.390	0.458	0.404
	0.012	0.016	0.010	0.014	0.008
NN5: (4,4)	0.562	0.586	0.574	0.604	0.544
	0.002	0.010	0.010	0.004	0.012
	0.220	0.184	0.192	0.194	0.204
	0.216	0.220	0.224	0.198	0.240
NN6: (4,4)	0.430	0.538	0.510	0.454	0.472
	0.022	0.020	0.020	0.032	0.022
	0.302	0.246	0.240	0.220	0.254
	0.246	0.196	0.230	0.294	0.252
NN7: (2,6)	0.318	0.330	0.312	0.284	0.284
	0.002	0.002	0.002	0.002	0.002
	0.666	0.658	0.680	0.698	0.704
	0.014	0.010	0.006	0.016	0.010
NN8: (4,4)	0.472	0.574	0.498	0.532	0.472
	0.020	0.010	0.018	0.020	0.012
	0.220	0.198	0.238	0.232	0.260
	0.288	0.218	0.246	0.216	0.256

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

W-test: Cases Involving H_1 vs H_j, $j = 2,3$

$\hat{P}$ / $\hat{\alpha}$ / Avg. Rank		Distribution of Disturbance Term				
		$TN(0, \sigma_e^2)$	$t_{(3)}$	$\chi^2_{(3)}$	$\ln(0, \sigma_e^2)$	$N(0, \sigma_e^2)$
NN1:	(4,2)	0.9620	0.9440	0.9300	0.9440	0.9420
		0.0220	0.0260	0.0380	0.0340	0.0440
		5.6950	5.8870	5.9870	5.8340	5.7480
NN1:	(4,6)	0.8660	0.8600	0.8660	0.8460	0.8760
		0.0380	0.0380	0.0400	0.0500	0.0300
		8.1870	8.5930	8.5240	8.4060	8.3340
NN2:	(4,4)	0.9660	0.9620	0.9660	0.9540	0.9700
		0.0340	0.0360	0.0280	0.0460	0.0300
		7.4330	7.7810	7.6550	7.5920	7.5990
NN2:	(4,4)	0.9560	0.9540	0.9580	0.9640	0.9660
		0.0440	0.0420	0.0320	0.0360	0.0340
		8.8060	8.8990	8.9180	8.8750	8.9370
NN3:	(4,2)	0.9740	0.9620	0.9660	0.9460	0.9620
		0.0260	0.0360	0.0340	0.0520	0.0380
		6.5340	6.8520	6.6660	6.6330	6.5490
NN3:	(4,6)	0.9640	0.9280	0.9360	0.9600	0.9420
		0.0340	0.0600	0.0500	0.0360	0.0540
		9.1660	9.2120	9.2430	9.2050	9.1830
NN4:	(4,2)	0.9620	0.9540	0.9660	0.9500	0.9600
		0.0380	0.0360	0.0320	0.0500	0.0380
		6.2320	6.5260	6.3850	6.3600	6.2810
NN4:	(4,6)	0.9380	0.9320	0.9340	0.9160	0.9380
		0.0460	0.0320	0.0380	0.0360	0.0420
		8.9270	9.0410	8.9350	8.9180	8.8930
NN5:	(4,4)	0.9160	0.8740	0.8880	0.8780	0.8940
		0.0280	0.0400	0.0340	0.0520	0.0500
		7.1600	7.4880	7.4890	7.3560	7.2930
NN5:	(4,4)	0.8740	0.8300	0.8420	0.8740	0.8440
		0.0420	0.0580	0.0500	0.0340	0.0480
		8.6780	8.8540	8.8970	8.7760	8.7780
NN6:	(4,4)	0.7940	0.8360	0.8200	0.7440	0.7640
		0.0280	0.0220	0.0320	0.0400	0.0220
		7.8130	7.9340	7.8550	7.7710	7.6880
NN6:	(4,4)	0.7400	0.7580	0.7480	0.7220	0.7180
		0.0360	0.0380	0.0340	0.0360	0.0220
		8.7800	8.8740	8.8360	8.9030	8.6230
NN7:	(4,2)	0.9220	0.9180	0.9000	0.9040	0.9060
		0.0320	0.0420	0.0420	0.0220	0.0580
		5.0710	5.1860	5.2000	5.1030	5.0620
NN7:	(4,6)	0.2760	0.4780	0.4640	0.3620	0.3240
		0.0380	0.0280	0.0300	0.0260	0.0420
		7.8600	8.0280	7.9690	7.8430	7.9490
NN8:	(4,4)	0.8680	0.8680	0.8380	0.8500	0.8600
		0.0460	0.0380	0.0420	0.0460	0.0400
		8.4830	8.7030	8.5550	8.5500	8.4490
NN8:	(4,4)	0.8380	0.8760	0.8340	0.8420	0.8500
		0.0640	0.0320	0.0420	0.0420	0.0360
		8.7730	9.0440	8.8980	8.8400	8.8260

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

W-test: H_2 vs H_3

Reject Both Reject Neither Reject H_2 Reject H_3		$TN(0, \sigma_\epsilon^2)$	Distribution of Disturbance Term $t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(2,6)	0.012	0.008	0.016	0.010	0.016
		0.822	0.804	0.798	0.786	0.810
		0.102	0.106	0.102	0.114	0.094
		0.064	0.082	0.084	0.090	0.080
NN2:	(4,4)	0.054	0.060	0.054	0.046	0.050
		0.608	0.646	0.660	0.646	0.666
		0.180	0.150	0.138	0.150	0.142
		0.158	0.144	0.148	0.158	0.142
NN3:	(2,6)	0.096	0.114	0.108	0.106	0.114
		0.430	0.426	0.446	0.450	0.438
		0.328	0.324	0.308	0.320	0.304
		0.146	0.136	0.138	0.124	0.144
NN4:	(2,6)	0.180	0.192	0.206	0.216	0.200
		0.152	0.162	0.174	0.146	0.170
		0.546	0.546	0.516	0.540	0.524
		0.122	0.100	0.104	0.098	0.106
NN5:	(4,4)	0.092	0.108	0.118	0.118	0.098
		0.456	0.438	0.432	0.422	0.464
		0.226	0.194	0.208	0.222	0.206
		0.226	0.260	0.242	0.238	0.232
NN6:	(4,4)	0.084	0.172	0.158	0.136	0.100
		0.396	0.302	0.322	0.354	0.396
		0.290	0.276	0.270	0.226	0.262
		0.230	0.250	0.250	0.284	0.242
NN7:	(2,6)	0.058	0.094	0.106	0.072	0.096
		0.256	0.178	0.218	0.208	0.246
		0.620	0.684	0.630	0.648	0.610
		0.066	0.044	0.046	0.072	0.048
NN8:	(4,4)	0.128	0.262	0.188	0.232	0.150
		0.240	0.192	0.218	0.224	0.230
		0.304	0.258	0.300	0.274	0.296
		0.328	0.288	0.294	0.270	0.324

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

$\hat{N}$-test: Cases Involving H_1 *vs* H_j, $j = 2,3$

$\hat{P}$ / $\hat{\alpha}$ / Avg. Rank	Distribution of Disturbance Term: $TN(0, \sigma_\varepsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\varepsilon^2)$	$N(0, \sigma_\varepsilon^2)$
NN1: (4,2)	0.9680	0.9380	0.9320	0.9480	0.9500
	0.0240	0.0360	0.0460	0.0420	0.0460
	2.6370	2.5550	2.5640	2.5870	2.6960
NN1: (4,6)	0.9140	0.9060	0.9080	0.9100	0.9220
	0.0520	0.0460	0.0420	0.0540	0.0460
	3.4620	3.3090	3.2490	3.3660	3.4700
NN2: (4,4)	0.9520	0.9580	0.9620	0.9500	0.9660
	0.0480	0.0420	0.0340	0.0500	0.0340
	1.9630	2.0080	1.9570	1.9930	1.9370
NN2: (4,4)	0.9500	0.9560	0.9640	0.9640	0.9720
	0.0500	0.0440	0.0340	0.0360	0.0280
	2.2780	2.1820	2.2380	2.1940	2.2210
NN3: (4,2)	0.9700	0.9640	0.9640	0.9380	0.9540
	0.0300	0.0340	0.0360	0.0620	0.0460
	1.8890	1.9590	1.9560	2.1300	1.9950
NN3: (4,6)	0.9540	0.9360	0.9400	0.9560	0.9440
	0.0440	0.0600	0.0560	0.0420	0.0560
	2.2950	2.3030	2.3710	2.2180	2.3870
NN4: (4,2)	0.9600	0.9540	0.9600	0.9440	0.9560
	0.0400	0.0400	0.0380	0.0560	0.0420
	1.9470	1.9870	1.9590	2.0250	1.9720
NN4: (4,6)	0.9500	0.9320	0.9280	0.9240	0.9400
	0.0480	0.0500	0.0560	0.0580	0.0540
	2.2740	2.2900	2.3520	2.3260	2.2860
NN5: (4,4)	0.9320	0.9040	0.8960	0.9060	0.9120
	0.0480	0.0520	0.0540	0.0640	0.0540
	2.6320	2.5650	2.6230	2.6500	2.6600
NN5: (4,4)	0.9060	0.8760	0.8760	0.9100	0.9100
	0.0720	0.0620	0.0580	0.0600	0.0600
	3.2680	3.1170	3.1660	3.0760	3.2990
NN6: (4,4)	0.8700	0.8760	0.8600	0.7960	0.8320
	0.0480	0.0500	0.0700	0.0740	0.0660
	2.7960	2.6060	2.7370	3.0550	2.9220
NN6: (4,4)	0.8620	0.8560	0.8380	0.8180	0.8400
	0.0540	0.0620	0.0660	0.0460	0.0500
	3.2890	3.1050	3.1110	3.4080	3.2510
NN7: (4,2)	0.9460	0.9240	0.9100	0.9200	0.9140
	0.0320	0.0460	0.0420	0.0240	0.0660
	2.8490	2.6830	2.7000	2.7720	2.9450
NN7: (4,6)	0.4920	0.6040	0.6120	0.5420	0.5120
	0.0480	0.0480	0.0600	0.0520	0.0520
	4.3120	3.7520	3.8440	4.1260	4.2800
NN8: (4,4)	0.9180	0.9000	0.8920	0.9020	0.9040
	0.0560	0.0600	0.0660	0.0640	0.0660
	2.2980	2.4210	2.4890	2.3930	2.4100
NN8: (4,4)	0.9200	0.9120	0.9020	0.9000	0.9100
	0.0540	0.0400	0.0400	0.0620	0.0520
	2.3090	2.3580	2.4020	2.4280	2.3630

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

$\bar{N}$-test: H_2 *vs* H_3

Reject Both / Reject Neither / Reject H_2 / Reject H_3		Distribution of Disturbance Term				
		$TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(2,6)	0.018	0.018	0.030	0.020	0.028
		0.712	0.702	0.710	0.694	0.714
		0.192	0.178	0.158	0.186	0.154
		0.078	0.102	0.102	0.100	0.104
NN2:	(4,4)	0.094	0.098	0.094	0.096	0.092
		0.450	0.522	0.556	0.508	0.536
		0.250	0.190	0.182	0.198	0.188
		0.206	0.190	0.168	0.198	0.184
NN3:	(2,6)	0.172	0.174	0.172	0.166	0.188
		0.278	0.286	0.312	0.284	0.316
		0.414	0.412	0.390	0.430	0.370
		0.136	0.128	0.126	0.120	0.126
NN4:	(2,6)	0.284	0.286	0.300	0.298	0.292
		0.056	0.068	0.082	0.064	0.070
		0.580	0.572	0.542	0.570	0.560
		0.080	0.074	0.076	0.068	0.078
NN5:	(4,4)	0.178	0.204	0.176	0.202	0.174
		0.286	0.280	0.290	0.286	0.294
		0.266	0.230	0.252	0.258	0.246
		0.270	0.286	0.282	0.254	0.286
NN6:	(4,4)	0.158	0.310	0.266	0.232	0.226
		0.208	0.162	0.164	0.202	0.202
		0.364	0.272	0.296	0.254	0.298
		0.270	0.256	0.274	0.312	0.274
NN7:	(2,6)	0.110	0.152	0.150	0.118	0.128
		0.156	0.100	0.132	0.124	0.132
		0.674	0.706	0.678	0.698	0.696
		0.060	0.042	0.040	0.060	0.044
NN8:	(4,4)	0.306	0.424	0.358	0.386	0.314
		0.084	0.048	0.082	0.092	0.074
		0.278	0.270	0.284	0.274	0.310
		0.332	0.258	0.276	0.248	0.302

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

NA-test: Cases Involving H_1 vs H_j, $j = 2,3$

$\hat{P}$ / $\hat{\alpha}$ / Avg. Rank		$TN(0,\sigma^2_\epsilon)$	$t_{(3)}$	$\chi^2_{(3)}$	$\ln(0,\sigma^2_\epsilon)$	$N(0,\sigma^2_\epsilon)$
NN1:	(4,2)	0.4880	0.5220	0.4860	0.5220	0.4780
		0.0580	0.0600	0.1000	0.0640	0.0820
		8.7440	8.7980	8.8180	8.8120	8.7740
NN1:	(4,6)	0.8300	0.8460	0.8340	0.8140	0.8340
		0.0660	0.0860	0.0840	0.1040	0.0880
		6.5860	6.9390	6.9700	6.8390	6.7090
NN2:	(4,4)	0.8760	0.8880	0.8980	0.8540	0.9140
		0.0700	0.0720	0.0660	0.0840	0.0480
		9.2410	9.3020	9.2290	9.1910	9.2380
NN2:	(4,4)	0.8900	0.8780	0.8860	0.8980	0.8940
		0.0780	0.0800	0.0700	0.0580	0.0660
		8.7050	8.8090	8.7900	8.7150	8.8090
NN3:	(4,2)	0.5840	0.6000	0.5840	0.5920	0.5760
		0.0620	0.0680	0.0620	0.0820	0.0700
		9.1340	9.0990	9.1250	9.0350	9.0910
NN3:	(4,6)	0.9340	0.8900	0.9080	0.9340	0.9020
		0.0560	0.1060	0.0840	0.0640	0.0940
		7.6110	7.8720	7.7410	7.6700	7.6910
NN4:	(4,2)	0.3280	0.3560	0.3400	0.3260	0.3540
		0.0800	0.0720	0.0660	0.0940	0.0720
		8.9030	8.8840	8.9280	8.8560	8.9220
NN4:	(4,6)	0.9160	0.9060	0.9000	0.9100	0.9040
		0.0800	0.0780	0.0820	0.0680	0.0900
		6.7980	7.0810	6.8440	6.7600	6.7620
NN5:	(4,4)	0.8740	0.8560	0.8520	0.8500	0.8420
		0.0560	0.0780	0.0680	0.0720	0.0840
		7.4430	7.8670	7.8810	7.6810	7.5220
NN5:	(4,4)	0.8560	0.8240	0.8300	0.8460	0.8440
		0.0640	0.0880	0.0860	0.0860	0.0880
		6.9450	7.4410	7.3460	7.3090	7.0790
NN6:	(4,4)	0.8220	0.8340	0.8100	0.7560	0.7800
		0.0720	0.0620	0.0880	0.0880	0.0800
		6.1320	6.5870	6.5630	6.2380	6.1550
NN6:	(4,4)	0.8160	0.8420	0.8400	0.7700	0.8020
		0.0640	0.0640	0.0600	0.0780	0.0620
		5.7950	6.3010	6.1980	6.0150	5.7490
NN7:	(4,2)	0.0620	0.0860	0.0760	0.0560	0.0680
		0.0720	0.0760	0.0900	0.0500	0.0860
		8.7290	8.7230	8.7110	8.6870	8.7000
NN7:	(4,6)	0.5180	0.6200	0.6120	0.5680	0.5380
		0.0600	0.0660	0.0740	0.0680	0.0760
		3.6780	4.1540	4.1200	3.8910	3.8240
NN8:	(4,4)	0.8820	0.8660	0.8500	0.8580	0.8580
		0.0780	0.0740	0.0840	0.0860	0.0860
		6.0660	6.6750	6.3840	6.3780	6.1900
NN8:	(4,4)	0.8620	0.8760	0.8580	0.8640	0.8780
		0.0980	0.0680	0.0700	0.0780	0.0700
		6.0430	6.7180	6.3720	6.4320	6.1330

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

NA-test: H_2 *vs* H_3

Reject Both
Reject Neither
Reject H_2
Reject H_3

		Distribution of Disturbance Term				
		$TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(2,6)	0.024	0.020	0.020	0.016	0.024
		0.782	0.778	0.766	0.758	0.778
		0.070	0.050	0.048	0.070	0.048
		0.124	0.152	0.166	0.156	0.150
NN2:	(4,4)	0.046	0.046	0.034	0.042	0.040
		0.604	0.628	0.642	0.622	0.656
		0.192	0.152	0.156	0.172	0.146
		0.158	0.174	0.168	0.164	0.158
NN3:	(2,6)	0.036	0.044	0.034	0.028	0.036
		0.546	0.536	0.540	0.562	0.544
		0.136	0.136	0.132	0.140	0.128
		0.282	0.284	0.294	0.270	0.292
NN4:	(2,6)	0.020	0.032	0.032	0.032	0.028
		0.484	0.490	0.458	0.516	0.460
		0.106	0.120	0.120	0.096	0.118
		0.390	0.358	0.390	0.356	0.394
NN5:	(4,4)	0.084	0.102	0.102	0.100	0.090
		0.398	0.344	0.376	0.374	0.368
		0.260	0.256	0.254	0.268	0.262
		0.258	0.298	0.268	0.258	0.280
NN6:	(4,4)	0.102	0.212	0.202	0.154	0.138
		0.272	0.210	0.222	0.262	0.276
		0.348	0.294	0.290	0.258	0.282
		0.278	0.284	0.286	0.326	0.304
NN7:	(2,6)	0.006	0.010	0.004	0.016	0.008
		0.718	0.700	0.710	0.712	0.750
		0.044	0.036	0.042	0.056	0.034
		0.232	0.254	0.244	0.216	0.208
NN8:	(4,4)	0.216	0.340	0.264	0.300	0.218
		0.152	0.092	0.128	0.134	0.124
		0.302	0.282	0.312	0.288	0.322
		0.330	0.286	0.296	0.278	0.336

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

NL-test: Cases Involving H_1 *vs* H_j, $j = 2{,}3$

$\hat{P}$ / $\hat{\alpha}$ / Avg. Rank		$TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(4,2)	0.8940	0.8880	0.8880	0.8940	0.8940
		0.1060	0.1100	0.1120	0.1060	0.1060
		2.6500	2.7850	2.7560	2.7090	2.6070
NN1:	(4,6)	0.8440	0.8580	0.8440	0.8240	0.8440
		0.1560	0.1420	0.1520	0.1740	0.1540
		2.8700	2.9390	2.9970	3.0650	2.8450
NN2:	(4,4)	0.9040	0.9300	0.9240	0.8960	0.9280
		0.0960	0.0700	0.0760	0.1040	0.0720
		3.2520	3.1770	3.1630	3.2710	3.0870
NN2:	(4,4)	0.8920	0.9080	0.9260	0.9120	0.9180
		0.1080	0.0920	0.0740	0.0880	0.0820
		3.1760	3.2870	3.0230	3.1830	3.0170
NN3:	(4,2)	0.9360	0.9320	0.9320	0.9120	0.9280
		0.0640	0.0680	0.0680	0.0880	0.0720
		2.9600	3.0670	3.0140	3.0620	2.9860
NN3:	(4,6)	0.9060	0.8920	0.9020	0.8960	0.9000
		0.0940	0.1080	0.0980	0.1040	0.1000
		3.6530	4.1080	3.8610	3.9210	3.6570
NN4:	(4,2)	0.9220	0.9180	0.9340	0.9060	0.9380
		0.0780	0.0780	0.0640	0.0940	0.0620
		3.1420	3.2660	3.1160	3.2270	3.0530
NN4:	(4,6)	0.9040	0.9100	0.9080	0.8980	0.8960
		0.0940	0.0880	0.0880	0.0980	0.1020
		3.9330	4.3880	4.1200	4.2270	3.9810
NN5:	(4,4)	0.9040	0.8980	0.9060	0.9020	0.9040
		0.0920	0.1020	0.0840	0.0940	0.0940
		3.0660	3.3060	3.1850	3.0980	3.0680
NN5:	(4,4)	0.8720	0.8580	0.8840	0.9000	0.8900
		0.1200	0.1240	0.1060	0.0980	0.1040
		2.8530	3.1440	3.0100	2.8430	2.7490
NN6:	(4,4)	0.8680	0.8820	0.8640	0.8240	0.8600
		0.0800	0.0900	0.1020	0.1060	0.0900
		3.1020	3.4300	3.4820	3.3660	3.1730
NN6:	(4,4)	0.8780	0.8940	0.8600	0.8760	0.8540
		0.0780	0.0740	0.0900	0.0660	0.0800
		2.8060	3.1100	3.2080	2.8550	2.8780
NN7:	(4,2)	0.9260	0.9140	0.9040	0.9360	0.9100
		0.0680	0.0780	0.0880	0.0620	0.0880
		2.4010	2.6250	2.7030	2.4380	2.5090
NN7:	(4,6)	0.5900	0.6760	0.6620	0.6260	0.6040
		0.1940	0.1480	0.1640	0.1900	0.1820
		3.4070	3.2970	3.3640	3.4450	3.2820
NN8:	(4,4)	0.9000	0.8800	0.8740	0.8760	0.8720
		0.0760	0.0760	0.0820	0.0880	0.0920
		3.7630	4.5940	4.2020	4.2000	3.9330
NN8:	(4,4)	0.8800	0.8920	0.8840	0.8740	0.8980
		0.0920	0.0620	0.0660	0.0800	0.0640
		3.8210	4.4730	4.0650	4.1890	3.7600

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

NL-test: H_2 vs H_3

Reject Both Reject Neither Reject H_2 Reject H_3		$TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(2,6)	0.390	0.396	0.396	0.362	0.392
		0.004	0.016	0.010	0.004	0.010
		0.530	0.494	0.502	0.536	0.508
		0.076	0.094	0.092	0.098	0.090
NN2:	(4,4)	0.502	0.518	0.514	0.538	0.514
		0.006	0.006	0.008	0.008	0.008
		0.236	0.246	0.250	0.222	0.254
		0.256	0.230	0.228	0.232	0.224
NN3:	(2,6)	0.500	0.530	0.514	0.492	0.516
		0.002	0.004	0.002	0.000	0.004
		0.450	0.430	0.440	0.462	0.434
		0.048	0.036	0.044	0.046	0.046
NN4:	(2,6)	0.502	0.524	0.560	0.480	0.544
		0.000	0.000	0.000	0.000	0.000
		0.480	0.456	0.424	0.506	0.436
		0.018	0.020	0.016	0.014	0.020
NN5:	(4,4)	0.484	0.528	0.494	0.528	0.454
		0.010	0.014	0.020	0.008	0.024
		0.246	0.210	0.230	0.228	0.246
		0.260	0.248	0.256	0.236	0.276
NN6:	(4,4)	0.308	0.410	0.400	0.334	0.340
		0.058	0.044	0.058	0.060	0.060
		0.350	0.288	0.284	0.268	0.302
		0.284	0.258	0.258	0.338	0.298
NN7:	(2,6)	0.282	0.302	0.294	0.252	0.270
		0.004	0.004	0.006	0.004	0.004
		0.698	0.684	0.692	0.722	0.718
		0.016	0.010	0.008	0.022	0.008
NN8:	(4,4)	0.296	0.396	0.348	0.364	0.300
		0.068	0.040	0.060	0.068	0.060
		0.292	0.288	0.302	0.308	0.328
		0.344	0.276	0.290	0.260	0.312

Distribution of Disturbance Term

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

J-test: Cases Involving H_1 vs H_j, $j = 2,3$

$\hat{P}$ / $\hat{\alpha}$ / Avg. Rank		Distribution of Disturbance Term $TN(0, \sigma_e^2)$	$t_{(3)}$	$\chi^2_{(3)}$	$\ln(0, \sigma_e^2)$	$N(0, \sigma_e^2)$
NN1:	(4,2)	0.9580 0.0420 3.9660	0.9620 0.0340 3.8560	0.9480 0.0500 3.9290	0.9640 0.0360 3.8930	0.9420 0.0580 4.0240
NN1:	(4,6)	0.7460 0.2540 4.9240	0.7900 0.2040 4.6930	0.7880 0.2060 4.7330	0.7500 0.2440 4.8700	0.7720 0.2260 4.7770
NN2:	(4,4)	0.9320 0.0680 4.1050	0.9680 0.0320 3.8690	0.9540 0.0460 3.9770	0.9440 0.0560 3.9580	0.9620 0.0380 3.9930
NN2:	(4,4)	0.9120 0.0880 4.1990	0.9380 0.0620 4.0050	0.9440 0.0560 4.0400	0.9340 0.0660 4.0700	0.9440 0.0560 4.0800
NN3:	(4,2)	0.9800 0.0200 3.8960	0.9720 0.0260 3.8320	0.9700 0.0300 3.8830	0.9560 0.0440 3.9100	0.9600 0.0400 3.9610
NN3:	(4,6)	0.8680 0.1300 4.3740	0.8800 0.1200 4.1340	0.8960 0.1040 4.0970	0.8880 0.1120 4.1730	0.8760 0.1240 4.2670
NN4:	(4,2)	0.9780 0.0220 3.8650	0.9740 0.0220 3.7710	0.9740 0.0240 3.8570	0.9600 0.0400 3.8430	0.9720 0.0280 3.9210
NN4:	(4,6)	0.8880 0.1060 4.3440	0.8740 0.1180 4.2340	0.8620 0.1240 4.3530	0.8680 0.1260 4.3200	0.8720 0.1240 4.4200
NN5:	(4,4)	0.9380 0.0540 4.0670	0.9400 0.0460 3.9750	0.9320 0.0480 3.9550	0.9220 0.0640 4.0350	0.9340 0.0460 3.9550
NN5:	(4,4)	0.8760 0.1160 4.2320	0.8940 0.0820 3.9970	0.8980 0.0740 3.9570	0.8980 0.0960 4.1450	0.9060 0.0840 4.0140
NN6:	(4,4)	0.8920 0.0280 4.0960	0.8980 0.0340 3.9960	0.8940 0.0460 4.0180	0.8360 0.0500 4.0200	0.8640 0.0460 4.1720
NN6:	(4,4)	0.8700 0.0660 4.2060	0.8620 0.0720 4.1620	0.8580 0.0620 4.1020	0.8520 0.0520 4.1000	0.8500 0.0600 4.1470
NN7:	(4,2)	0.9620 0.0320 4.1470	0.9520 0.0340 4.1340	0.9500 0.0300 4.0620	0.9620 0.0280 4.1080	0.9540 0.0420 4.1240
NN7:	(4,6)	0.4620 0.2080 5.4260	0.5440 0.2080 5.4390	0.5440 0.1740 5.2230	0.4700 0.2260 5.4960	0.4660 0.2180 5.4080
NN8:	(4,4)	0.9160 0.0280 4.3130	0.9120 0.0180 3.9570	0.8880 0.0260 4.0610	0.8960 0.0340 4.0700	0.8920 0.0260 4.2260
NN8:	(4,4)	0.9060 0.0380 4.3110	0.9080 0.0320 4.0090	0.8820 0.0360 4.1460	0.9020 0.0460 4.1150	0.9040 0.0400 4.3150

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

J-test: H_2 vs H_3

Reject Both
Reject Neither
Reject H_2
Reject H_3

		Distribution of Disturbance Term				
		$TN(0, \sigma_\varepsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\varepsilon^2)$	$N(0, \sigma_\varepsilon^2)$
NN1:	(2,6)	0.178 0.096 0.684 0.042	0.144 0.130 0.684 0.042	0.160 0.094 0.694 0.052	0.166 0.110 0.684 0.040	0.172 0.118 0.666 0.044
NN2:	(4,4)	0.412 0.092 0.234 0.262	0.398 0.084 0.262 0.256	0.412 0.102 0.244 0.242	0.402 0.092 0.248 0.258	0.392 0.094 0.258 0.256
NN3:	(2,6)	0.306 0.014 0.650 0.030	0.322 0.026 0.628 0.024	0.306 0.032 0.640 0.022	0.288 0.014 0.670 0.028	0.314 0.028 0.630 0.028
NN4:	(2,6)	0.346 0.000 0.644 0.010	0.336 0.000 0.654 0.010	0.354 0.002 0.638 0.006	0.336 0.000 0.654 0.010	0.342 0.002 0.648 0.008
NN5:	(4,4)	0.342 0.052 0.306 0.300	0.374 0.066 0.254 0.306	0.334 0.106 0.270 0.290	0.390 0.056 0.260 0.294	0.338 0.086 0.272 0.304
NN6:	(4,4)	0.194 0.154 0.362 0.290	0.314 0.118 0.292 0.276	0.272 0.114 0.314 0.300	0.220 0.148 0.284 0.348	0.214 0.146 0.320 0.320
NN7:	(2,6)	0.146 0.012 0.840 0.002	0.170 0.014 0.810 0.006	0.178 0.010 0.808 0.004	0.168 0.012 0.810 0.010	0.166 0.006 0.824 0.004
NN8:	(4,4)	0.176 0.154 0.318 0.352	0.318 0.096 0.292 0.294	0.250 0.114 0.322 0.314	0.274 0.142 0.306 0.278	0.192 0.118 0.358 0.332

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

AJ-test: Cases Involving H_1 *vs* H_j, $j = 2,3$

$\hat{P}$ / $\hat{\alpha}$ / Avg. Rank	Distribution of Disturbance Term: $TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1: (4,2)	0.9700	0.9620	0.9460	0.9600	0.9660
	0.0120	0.0120	0.0260	0.0180	0.0220
	5.3280	5.1850	5.1920	5.2870	5.3060
NN1: (4,6)	0.9280	0.9120	0.9340	0.9120	0.9320
	0.0300	0.0280	0.0220	0.0400	0.0240
	5.0520	5.1210	5.0480	5.0340	5.1140
NN2: (4,4)	0.9760	0.9800	0.9780	0.9720	0.9840
	0.0240	0.0180	0.0180	0.0280	0.0160
	4.9130	4.9460	4.9680	4.9040	4.9860
NN2: (4,4)	0.9780	0.9840	0.9840	0.9860	0.9900
	0.0220	0.0140	0.0120	0.0140	0.0100
	4.8680	4.8060	4.8830	4.8790	4.8770
NN3: (4,2)	0.9860	0.9800	0.9820	0.9740	0.9840
	0.0140	0.0180	0.0180	0.0260	0.0160
	4.9460	4.8890	4.9190	4.8830	4.8760
NN3: (4,6)	0.9840	0.9640	0.9620	0.9740	0.9640
	0.0140	0.0280	0.0340	0.0240	0.0340
	4.9630	4.7910	5.0060	4.9540	5.0580
NN4: (4,2)	0.9800	0.9740	0.9780	0.9660	0.9800
	0.0200	0.0180	0.0200	0.0340	0.0180
	4.9440	4.8740	4.9570	4.9220	4.9550
NN4: (4,6)	0.9720	0.9420	0.9520	0.9300	0.9660
	0.0180	0.0340	0.0240	0.0360	0.0240
	5.3810	5.2240	5.2800	5.2930	5.3870
NN5: (4,4)	0.9200	0.9080	0.9080	0.8980	0.9320
	0.0220	0.0180	0.0260	0.0340	0.0180
	6.0760	5.8480	5.8560	5.9470	6.0310
NN5: (4,4)	0.9260	0.8800	0.8960	0.9220	0.9160
	0.0340	0.0320	0.0340	0.0240	0.0280
	5.7390	5.5740	5.6740	5.5460	5.7380
NN6: (4,4)	0.8280	0.8420	0.8280	0.7500	0.7700
	0.0100	0.0200	0.0320	0.0220	0.0180
	6.7180	6.3530	6.3550	6.4600	6.7150
NN6: (4,4)	0.8160	0.8240	0.8080	0.7800	0.7860
	0.0180	0.0260	0.0240	0.0220	0.0200
	6.5630	6.2630	6.2830	6.5000	6.5520
NN7: (4,2)	0.9300	0.9460	0.9220	0.9160	0.9360
	0.0320	0.0200	0.0200	0.0160	0.0360
	5.6370	5.4870	5.5260	5.6460	5.6160
NN7: (4,6)	0.3940	0.5460	0.5420	0.4460	0.4200
	0.0320	0.0320	0.0400	0.0260	0.0300
	6.7160	6.5160	6.6000	6.4390	6.6750
NN8: (4,4)	0.8900	0.9020	0.8700	0.8760	0.8860
	0.0260	0.0160	0.0180	0.0280	0.0160
	6.2980	5.8640	6.0680	6.0070	6.1840
NN8: (4,4)	0.8920	0.9040	0.8740	0.8780	0.8820
	0.0240	0.0140	0.0140	0.0260	0.0180
	6.1760	5.8190	6.0360	5.9630	6.1680

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

AJ-test: H_2 *vs* H_3

Reject Both Reject Neither Reject H_2 Reject H_3		$TN(0, \sigma_\epsilon^2)$	Distribution of Disturbance Term $t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(2,6)	0.006	0.008	0.012	0.014	0.008
		0.746	0.758	0.774	0.748	0.762
		0.188	0.184	0.162	0.180	0.168
		0.060	0.050	0.052	0.058	0.062
NN2:	(4,4)	0.052	0.052	0.058	0.056	0.052
		0.590	0.626	0.648	0.610	0.658
		0.196	0.172	0.150	0.170	0.140
		0.162	0.150	0.144	0.164	0.150
NN3:	(2,6)	0.092	0.116	0.092	0.098	0.104
		0.368	0.374	0.384	0.368	0.388
		0.450	0.430	0.454	0.462	0.436
		0.090	0.080	0.070	0.072	0.072
NN4:	(2,6)	0.216	0.214	0.230	0.218	0.212
		0.098	0.094	0.098	0.086	0.104
		0.638	0.652	0.638	0.660	0.654
		0.048	0.040	0.034	0.036	0.030
NN5:	(4,4)	0.090	0.104	0.106	0.104	0.088
		0.472	0.470	0.464	0.438	0.506
		0.210	0.196	0.190	0.204	0.188
		0.228	0.230	0.240	0.254	0.218
NN6:	(4,4)	0.062	0.146	0.134	0.120	0.100
		0.404	0.314	0.370	0.378	0.410
		0.298	0.288	0.262	0.220	0.254
		0.236	0.252	0.234	0.282	0.236
NN7:	(2,6)	0.092	0.118	0.126	0.094	0.108
		0.166	0.120	0.134	0.152	0.146
		0.714	0.750	0.718	0.720	0.716
		0.028	0.012	0.022	0.034	0.030
NN8:	(4,4)	0.118	0.254	0.172	0.224	0.130
		0.238	0.194	0.228	0.220	0.232
		0.304	0.262	0.300	0.282	0.316
		0.340	0.290	0.300	0.274	0.322

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

JA-test: Cases Involving H_1 *vs* H_j, $j = 2{,}3$

$\dot{P}$ $\dot{\alpha}$ Avg. Rank		Distribution of Disturbance Term $TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(4,2)	0.4940 0.0100 9.0140	0.5060 0.0220 9.0540	0.5000 0.0320 8.9730	0.5200 0.0160 9.0350	0.4820 0.0260 9.0230
NN1:	(4,6)	0.7900 0.0220 6.9580	0.8540 0.0220 6.8290	0.8580 0.0160 6.8750	0.8280 0.0300 6.7890	0.8260 0.0220 6.8850
NN2:	(4,4)	0.9060 0.0220 8.2240	0.9160 0.0160 8.2460	0.9220 0.0180 8.1800	0.8960 0.0240 8.1310	0.9220 0.0180 8.2290
NN2:	(4,4)	0.9300 0.0200 7.7180	0.9260 0.0140 7.8210	0.9220 0.0120 7.8750	0.9300 0.0120 7.8140	0.9260 0.0160 7.8730
NN3:	(4,2)	0.5940 0.0160 9.3120	0.6060 0.0200 9.3420	0.5740 0.0240 9.3640	0.5960 0.0380 9.2760	0.5660 0.0220 9.3130
NN3:	(4,6)	0.9720 0.0140 6.0610	0.9460 0.0360 6.1590	0.9480 0.0320 6.1080	0.9740 0.0220 6.1240	0.9560 0.0340 6.1720
NN4:	(4,2)	0.3380 0.0180 9.5220	0.3380 0.0160 9.4900	0.3340 0.0220 9.5590	0.3220 0.0340 9.4480	0.3320 0.0240 9.5740
NN4:	(4,6)	0.9720 0.0160 5.5110	0.9460 0.0220 5.2880	0.9480 0.0240 5.3230	0.9420 0.0240 5.4000	0.9600 0.0260 5.3960
NN5:	(4,4)	0.8900 0.0120 7.1420	0.8840 0.0260 7.0080	0.8600 0.0260 7.0470	0.8600 0.0360 7.0110	0.8600 0.0280 7.0560
NN5:	(4,4)	0.8500 0.0360 6.7820	0.8560 0.0300 6.6740	0.8600 0.0300 6.6330	0.8700 0.0380 6.8620	0.8580 0.0400 6.7350
NN6:	(4,4)	0.8320 0.0140 6.4660	0.8440 0.0180 6.4000	0.8260 0.0280 6.3570	0.7620 0.0280 6.3690	0.7720 0.0200 6.5810
NN6:	(4,4)	0.8020 0.0180 6.3300	0.8260 0.0320 6.1300	0.8180 0.0300 6.0310	0.7940 0.0220 6.1720	0.7900 0.0220 6.2850
NN7:	(4,2)	0.0600 0.0300 9.6030	0.0820 0.0280 9.5270	0.0920 0.0240 9.4610	0.0640 0.0220 9.6060	0.0900 0.0380 9.4910
NN7:	(4,6)	0.4100 0.0160 5.9350	0.5580 0.0280 6.0540	0.5500 0.0260 5.9450	0.4620 0.0200 5.9850	0.4320 0.0320 6.0450
NN8:	(4,4)	0.9020 0.0260 5.8460	0.9000 0.0180 5.3030	0.8740 0.0200 5.6030	0.8800 0.0260 5.6040	0.8860 0.0180 5.6800
NN8:	(4,4)	0.9000 0.0280 5.6920	0.9020 0.0160 5.4390	0.8780 0.0180 5.6530	0.8840 0.0240 5.5870	0.8820 0.0200 5.8400

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

JA-test: H_2 vs H_3

Reject Both Reject Neither Reject H_2 Reject H_3		Distribution of Disturbance Term $TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1 :	(2,6)	0.004	0.006	0.006	0.000	0.006
		0.874	0.878	0.850	0.874	0.876
		0.056	0.038	0.048	0.052	0.042
		0.066	0.078	0.096	0.074	0.076
NN2 :	(4,4)	0.026	0.016	0.014	0.022	0.016
		0.692	0.730	0.746	0.736	0.740
		0.146	0.126	0.118	0.118	0.122
		0.136	0.128	0.122	0.124	0.122
NN3 :	(2,6)	0.012	0.026	0.018	0.012	0.020
		0.684	0.666	0.668	0.660	0.670
		0.116	0.126	0.132	0.142	0.112
		0.188	0.182	0.182	0.186	0.198
NN4 :	(2,6)	0.010	0.016	0.016	0.016	0.016
		0.622	0.606	0.596	0.636	0.602
		0.108	0.124	0.116	0.094	0.112
		0.260	0.254	0.272	0.254	0.270
NN5 :	(4,4)	0.064	0.060	0.068	0.070	0.052
		0.502	0.450	0.478	0.474	0.492
		0.228	0.218	0.228	0.226	0.218
		0.206	0.272	0.226	0.230	0.238
NN6 :	(4,4)	0.052	0.126	0.144	0.116	0.088
		0.392	0.306	0.328	0.348	0.380
		0.314	0.306	0.258	0.226	0.266
		0.242	0.262	0.270	0.310	0.266
NN7 :	(2,6)	0.000	0.000	0.000	0.000	0.000
		0.828	0.824	0.818	0.792	0.820
		0.048	0.036	0.032	0.056	0.036
		0.124	0.140	0.150	0.152	0.144
NN8 :	(4,4)	0.130	0.260	0.186	0.222	0.142
		0.230	0.166	0.198	0.212	0.212
		0.302	0.288	0.314	0.290	0.322
		0.338	0.286	0.302	0.276	0.324

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

NJ-test: Cases Involving H_1 *vs* H_j, $j = 2,3$

$\dot{P}$ $\dot{\alpha}$ Avg. Rank		Distribution of Disturbance Term $TN(0, \sigma_\varepsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\varepsilon^2)$	$N(0, \sigma_\varepsilon^2)$
NN1:	(4,2)	0.7120	0.7300	0.7380	0.7520	0.7200
		0.0340	0.0380	0.0620	0.0440	0.0560
		7.1110	7.1100	7.1290	7.1280	7.1490
NN1:	(4,6)	0.8680	0.8960	0.8860	0.8660	0.8820
		0.0540	0.0440	0.0560	0.0640	0.0520
		6.2470	6.1010	6.1540	6.1150	6.2260
NN2:	(4,4)	0.9600	0.9500	0.9440	0.9340	0.9540
		0.0380	0.0400	0.0400	0.0580	0.0400
		6.4560	6.4560	6.4570	6.3800	6.5320
NN2:	(4,4)	0.9580	0.9500	0.9520	0.9460	0.9640
		0.0420	0.0400	0.0380	0.0420	0.0300
		6.1400	6.1610	6.2270	6.1870	6.1930
NN3:	(4,2)	0.9200	0.9120	0.9080	0.8900	0.9120
		0.0320	0.0440	0.0380	0.0540	0.0380
		7.1520	7.1070	7.1600	7.1330	7.1610
NN3:	(4,6)	0.9640	0.9220	0.9380	0.9540	0.9260
		0.0340	0.0700	0.0600	0.0440	0.0720
		6.4050	6.3460	6.4450	6.3740	6.4970
NN4:	(4,2)	0.8560	0.8640	0.8720	0.8480	0.8700
		0.0580	0.0500	0.0380	0.0620	0.0380
		7.6020	7.5130	7.5120	7.4920	7.5860
NN4:	(4,6)	0.9400	0.9380	0.9320	0.9180	0.9420
		0.0480	0.0460	0.0460	0.0480	0.0400
		6.8060	6.6610	6.7620	6.6980	6.8490
NN5:	(4,4)	0.8920	0.8800	0.8820	0.8740	0.8820
		0.0360	0.0440	0.0400	0.0580	0.0460
		6.6280	6.5100	6.5720	6.5650	6.6490
NN5:	(4,4)	0.8880	0.8440	0.8400	0.8860	0.8660
		0.0440	0.0620	0.0700	0.0500	0.0640
		6.1800	6.2270	6.3350	6.2370	6.3460
NN6:	(4,4)	0.8040	0.8340	0.8100	0.7360	0.7740
		0.0420	0.0380	0.0520	0.0460	0.0340
		6.8500	6.7100	6.6690	6.8090	6.7350
NN6:	(4,4)	0.7900	0.8400	0.8100	0.7840	0.7700
		0.0420	0.0300	0.0320	0.0460	0.0320
		6.5690	6.4710	6.4670	6.5550	6.6030
NN7:	(4,2)	0.2500	0.4040	0.3940	0.3260	0.3040
		0.0460	0.0560	0.0600	0.0320	0.0700
		7.7480	7.7950	7.7740	7.7270	7.7350
NN7:	(4,6)	0.3620	0.5300	0.5320	0.4280	0.3920
		0.0420	0.0460	0.0520	0.0420	0.0580
		6.6800	6.6540	6.7550	6.6190	6.6580
NN8:	(4,4)	0.8600	0.8700	0.8440	0.8520	0.8500
		0.0560	0.0480	0.0580	0.0580	0.0560
		6.9240	6.6300	6.7120	6.7370	6.8070
NN8:	(4,4)	0.8400	0.8840	0.8420	0.8440	0.8560
		0.0720	0.0340	0.0460	0.0580	0.0460
		6.9460	6.4840	6.7430	6.6470	6.7450

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

NJ-test: H_2 vs H_3

Reject Both / Reject Neither / Reject H_2 / Reject H_3		Distribution of Disturbance Term				
		$TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(2,6)	0.016	0.016	0.012	0.018	0.016
		0.818	0.842	0.818	0.828	0.834
		0.088	0.062	0.064	0.078	0.072
		0.078	0.080	0.106	0.076	0.078
NN2:	(4,4)	0.032	0.032	0.036	0.022	0.032
		0.638	0.692	0.718	0.694	0.720
		0.180	0.140	0.124	0.144	0.126
		0.150	0.136	0.122	0.140	0.122
NN3:	(2,6)	0.036	0.042	0.040	0.042	0.032
		0.578	0.572	0.586	0.564	0.590
		0.208	0.200	0.200	0.226	0.192
		0.178	0.186	0.174	0.168	0.186
NN4:	(2,6)	0.046	0.048	0.054	0.058	0.040
		0.440	0.426	0.436	0.448	0.452
		0.274	0.288	0.268	0.268	0.270
		0.240	0.238	0.242	0.226	0.238
NN5:	(4,4)	0.066	0.080	0.088	0.078	0.064
		0.462	0.436	0.444	0.434	0.470
		0.244	0.220	0.238	0.236	0.224
		0.228	0.264	0.230	0.252	0.242
NN6:	(4,4)	0.064	0.146	0.142	0.108	0.092
		0.394	0.294	0.328	0.350	0.396
		0.298	0.290	0.268	0.232	0.254
		0.244	0.270	0.262	0.310	0.258
NN7:	(2,6)	0.012	0.008	0.012	0.016	0.008
		0.744	0.686	0.684	0.700	0.704
		0.122	0.160	0.160	0.152	0.146
		0.122	0.146	0.144	0.132	0.142
NN8:	(4,4)	0.122	0.244	0.172	0.222	0.138
		0.250	0.176	0.206	0.218	0.234
		0.306	0.284	0.308	0.292	0.312
		0.322	0.296	0.314	0.268	0.316

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

F-test: Cases Involving H_1 *vs* H_j, $j = 2,3$

$\hat{P}$ $\hat{\alpha}$ Avg. Rank		Distribution of Disturbance Term $TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(4,2)	0.9320	0.9320	0.9060	0.9320	0.9360
		0.0480	0.0400	0.0600	0.0380	0.0480
		7.5280	7.3090	7.2750	7.3900	7.3980
NN1:	(4,6)	0.8480	0.8580	0.8680	0.8600	0.8760
		0.0600	0.0420	0.0460	0.0540	0.0480
		7.9840	7.8360	7.8430	7.7900	8.0360
NN2:	(4,4)	0.9760	0.9600	0.9480	0.9460	0.9540
		0.0240	0.0340	0.0440	0.0520	0.0460
		7.0590	7.1030	7.2050	7.0770	7.3310
NN2:	(4,4)	0.9480	0.9540	0.9640	0.9540	0.9640
		0.0520	0.0440	0.0280	0.0460	0.0360
		6.8600	6.8780	6.8850	6.9230	6.9190
NN3:	(4,2)	0.9560	0.9620	0.9620	0.9540	0.9580
		0.0440	0.0340	0.0380	0.0440	0.0420
		7.0260	6.7230	6.8220	6.7390	6.9040
NN3:	(4,6)	0.9320	0.9420	0.9400	0.9500	0.9580
		0.0600	0.0400	0.0400	0.0420	0.0360
		8.4870	8.0440	8.2410	8.3130	8.2480
NN4:	(4,2)	0.9440	0.9500	0.9380	0.9280	0.9420
		0.0560	0.0400	0.0580	0.0700	0.0540
		6.7870	6.5580	6.7260	6.6330	6.7830
NN4:	(4,6)	0.8640	0.8860	0.8640	0.8460	0.8580
		0.0600	0.0460	0.0520	0.0540	0.0400
		9.1160	8.8690	8.9740	8.9240	9.0410
NN5:	(4,4)	0.8200	0.8220	0.8200	0.8020	0.8080
		0.0420	0.0380	0.0400	0.0680	0.0380
		8.7230	8.4640	8.4880	8.5930	8.6490
NN5:	(4,4)	0.8060	0.8260	0.8140	0.8240	0.7740
		0.0500	0.0480	0.0500	0.0480	0.0600
		8.1250	7.8760	7.9570	8.0250	8.1810
NN6:	(4,4)	0.5720	0.6560	0.6260	0.5640	0.5560
		0.0520	0.0400	0.0480	0.0380	0.0360
		9.1530	8.9860	8.9550	8.9100	8.9240
NN6:	(4,4)	0.5500	0.6620	0.6500	0.5820	0.5200
		0.0400	0.0360	0.0360	0.0480	0.0460
		8.8150	8.7110	8.7720	8.8610	8.9590
NN7:	(4,2)	0.8680	0.8940	0.8720	0.8600	0.8800
		0.0620	0.0460	0.0440	0.0460	0.0540
		7.0360	6.8830	6.8570	7.0330	6.9260
NN7:	(4,6)	0.1780	0.2880	0.2920	0.2120	0.1960
		0.0460	0.0660	0.0640	0.0520	0.0460
		8.0990	8.3300	8.3790	8.1590	8.1030
NN8:	(4,4)	0.6200	0.7320	0.6880	0.6740	0.6520
		0.0560	0.0520	0.0460	0.0600	0.0560
		9.3470	9.1920	9.2290	9.2220	9.3000
NN8:	(4,4)	0.6620	0.7540	0.7040	0.7040	0.6780
		0.0440	0.0480	0.0340	0.0440	0.0380
		9.2110	9.0970	9.1590	9.0400	9.1870

Table IV.3.3 Results of Non-Normal Deviate Experiments (continued)

F-test: H_2 *vs* H_3

Reject Both Reject Neither Reject H_2 Reject H_3		Distribution of Disturbance Term $TN(0, \sigma_\epsilon^2)$	$t_{(3)}$	$\chi^2_{(2)}$	$\ln(0, \sigma_\epsilon^2)$	$N(0, \sigma_\epsilon^2)$
NN1:	(2,6)	0.022	0.022	0.024	0.024	0.024
		0.764	0.790	0.798	0.752	0.800
		0.136	0.114	0.104	0.146	0.104
		0.078	0.074	0.074	0.078	0.072
NN2:	(4,4)	0.056	0.058	0.052	0.068	0.058
		0.642	0.684	0.676	0.658	0.694
		0.170	0.130	0.130	0.136	0.124
		0.132	0.128	0.142	0.138	0.124
NN3:	(2,6)	0.136	0.132	0.126	0.126	0.112
		0.462	0.448	0.492	0.456	0.476
		0.330	0.342	0.316	0.346	0.338
		0.072	0.078	0.066	0.072	0.074
NN4:	(2,6)	0.218	0.218	0.224	0.210	0.198
		0.196	0.160	0.156	0.140	0.168
		0.558	0.598	0.602	0.614	0.612
		0.028	0.024	0.018	0.036	0.022
NN5:	(4,4)	0.082	0.094	0.102	0.096	0.096
		0.598	0.594	0.614	0.560	0.630
		0.152	0.150	0.142	0.158	0.142
		0.168	0.162	0.142	0.186	0.132
NN6:	(4,4)	0.048	0.094	0.086	0.072	0.058
		0.644	0.600	0.570	0.624	0.660
		0.164	0.166	0.170	0.120	0.152
		0.144	0.140	0.174	0.184	0.130
NN7:	(2,6)	0.106	0.126	0.132	0.102	0.112
		0.260	0.174	0.218	0.236	0.270
		0.620	0.692	0.646	0.654	0.608
		0.014	0.008	0.004	0.008	0.010
NN8:	(4,4)	0.078	0.112	0.098	0.124	0.088
		0.570	0.540	0.548	0.524	0.548
		0.166	0.188	0.194	0.168	0.188
		0.186	0.160	0.160	0.184	0.176

Table IV.3.4 Paired T-test on Power for Normal/Non-Normal Distributions

$(\hat{\mu}_t - \mu_N)$ t_{obs} p-value	Distribution of Disturbance Term $TN(0, \sigma^2_\varepsilon)$	 $t_{(3)}$	 $\chi^2_{(2)}$	 $\ln(0, \sigma^2_\varepsilon)$
Test:				
N	-0.004125	0.002250	0.001625	-0.009500
	-0.80	0.63	0.61	-1.57
	0.4387	0.5362	0.5531	0.1363
W	0.006250	0.013625	0.008750	-0.003750
	1.23	1.22	0.88	-0.80
	0.2363	0.2399	0.3920	0.4368
$\bar{N}$	0.007875	0.003625	-0.000375	-0.006875
	1.80	0.50	0.05	-1.84
	0.0926	0.6260	0.9605	0.0864
NA	0.004500	0.014000	0.006125	-0.003000
	0.80	1.91	1.01	-0.45
	0.4351	0.0748	0.3262	0.6613
NL	-0.001125	0.005750	0.003625	-0.005375
	-0.27	0.94	0.91	-1.08
	0.7911	0.3607	0.3753	0.2968
J	0.000875	0.012375	0.007000	-0.004250
	0.17	2.38	1.23	-1.31
	0.8673	0.0311	0.2386	0.2093
AJ	0.004750	0.009750	0.004375	-0.009625
	0.98	0.96	0.47	-2.48
	0.3433	0.3526	0.6436	0.0257
JA	0.013250	0.020750	0.009000	-0.001250
	1.96	3.27	1.86	-0.22
	0.0686	0.0051	0.0825	0.8298
NJ	0.000000	0.024000	0.016250	-0.001625
	0.00	2.13	1.50	-0.28
	1.0000	0.0505	0.1541	0.7863
F	-0.002125	0.035500	0.021625	0.005125
	-0.42	2.93	1.98	0.85
	0.6832	0.0104	0.0667	0.4079

V. An Empirical Study: Analysis of Food Spending Patterns

5.1 Introduction to the Study

This study of weekly expenditure data from the 1977-1978 National Food Consumption Survey (NFCS) is an example of an empirical situation in which non-nested hypothesis testing procedures can be employed for the purpose of selecting the most appropriate model. As previously stated, the underlying theory may provide the researcher with a set of alternative models, all of which, theoretically speaking, are candidates for the "true" (or at least most reasonable) model. Particularly in economic applications governed by the general Engel curve, the only structure dictated by the theory is the set of independent variables and limited restrictions on the equation's mathematical form. In this case, there are several functional forms of the general Engel curve which are feasible for modelling the relationship between household expenditure and such explanatory variables as income, household size and location. The various functional forms represent the mathematical form of a particular demand theory.

Depending on the functional form assumed, the 'goodness of fit' and estimated income elasticities (unitless measures of consumer sensitivity in food spending related to income changes) may be very different. Consequently, it is worthwhile for the researcher to investigate several alternative model specifications to see which theory is supported by the data. This investigation can be done by making comparisons on the basis of R^2, correlations between predicted and observed responses and correct signs of the estimated coefficients. More formally, hypothesis testing can be employed to judge whether or not one model has the ability to explain that which another model can, as well as "something extra," in terms of the behavior of the response variable. Therefore, the situation arises where the non-nested hypothesis testing procedures are useful tools to aid the researcher in the selection of the most appropriate model.

Specifically, an empirical study using actual household food expenditure data from the 1977-78 Nationwide Food Consumption Survey is presented here to discuss the role of non-nested hypothesis testing procedures in choosing the most appropriate functional form. For five alternative functional forms of the general Engel curve, the measures of fit in addition to the test results will be examined and employed in the choice of the best equation for modelling weekly household food expenditure.

The basic model layout for the study was taken from Salathe(1979) in which he examined alternative functional forms for modelling weekly food expenditure for a subset of data from the 1965 USDA Household Food Consumption Survey. His comparisons were based strictly on measures of fit, theoretical considerations (signs/magnitudes of estimated coefficients) and a goodness-of-fit measure based on the correlation between the observed and predicted responses. However, no formal hypothesis testing was used to aid in the determination of the most reasonable model specification. This aspect will be added to provide another dimension with which to select among alternative functional forms.

5.2 The General Engel Curve

For the study of weekly household food expenditure, the general Engel curve is of the form:

$$EXPEND = f(EDHM, EMPHM, SXHM, U1, U2, R1, R2, R4, S1, S3, S4, RAC, INC, HS, MEALS)$$

where

EXPEND = total weekly food expenditure,

EDHM = 1 if household manager not college educated;

0 otherwise (ow),

EMPHM = 1 if household manager unemployed; 0 ow,

SXHM = 1 if household manager female; 0 ow,

U1 = 1 if household located in central city; 0 ow,

U2 = 1 if household located in non-metropolitan area; 0 ow,

R1 = 1 if household located in Northeast; 0 ow,

R2 = 1 if household located in Midwest; 0 ow,

R4 = 1 if household located in West; 0 ow,

S1 = 1 if season spring (April-June, 1977); 0 ow,

S3 = 1 if season fall (October-December, 1977); 0 ow,

S4 = 1 if season winter (January-March, 1978); 0 ow,

RAC = 1 if household head non-white; 0 ow,

INC = annual household income (in dollars),

HS = household size (number of members),

MEALS = number of meals eaten from household food supply per week.

The food expenditures will be examined as a total amount spent per week by the household in addition to being subdivided into eight food categories. These can be broken down broadly as follows:

(1) Beverages (BEVEM_TO)

(2) Fats and oils (FATSM_TO)

(3) Fruits (FRUIM_TO)

(4) Grains (GRAIM_TO)

(5) Meat and meat alternates (MEATM_TO)

(6) Milk equivalents (MILKM_TO)

(7) Sugars and sweets (SUGAM_TO)

(8) Vegetables (VEGEM_TO)

For all categories (ALLM_TO), the response variable, EXPEND, measures in dollars the amount spent on foods bought for consumption in the home. In addition, a category for all other food expenditures was created: OTHERM_T. Hence, there will be ten dependent variables for which to determine the appropriate functional form of the Engel curve.

5.3 Alternative Functional Forms With Theoretical Comparisons

For this empirical study of weekly food expenditure, the (1) quadratic, (2) semi-log, (3) inverse, (4) double-log and (5) log-inverse functional specifications of the general Engel curve will be investigated. All models will be linear in the demographic variables with the indicated transformations on the exogenous variable, EXPEND, and the explanatory variables INC, HS and in particular cases only, MEALS. The other variables will be included in the model but are binary. The purpose of the demographic indicator variables is to model differences across various cross-sections of the population. In this framework, they will be used only in a linear additive manner. It may be reasonable to also use the demographic indicators as slope shifters on the continuous variables, such as INC and HS. However, if these are added to the model, the anticipated result would be strong collinearit among the regressors.

Prior to examining the results from estimating the five alternative models for the different food categories, consideration should be given to the structural differences and limitations of these functional forms. Table V.1 contains the formulations of the individual functional forms and some characteristics of each. Examination of these functional specifications yields some interesting comparisons.

The information provided by the computational formulas for marginal propensities to consume and income elasticities show how different the outcomes can be, depending on the choice of functional form. These two quantities are of particular interest to the economic researcher. The marginal propensity to consume (MPC) measures what part of the dollar bill you would spend, on a particular item, given the extra dollar just received. The income elasticity is a unitless measure of the consumer's sensitivity to income changes. It measures the change in expenditure for a given change in income, both on a percentage basis. Interestingly enough, a person's sensitivity to income changes can depend heavily on his position on the income scale. For making comparisons among elasticities, they are generally calculated at mean income and expenditure levels. Of the five functional forms considered, the double-log (4) formulation (also called log-linear) is unique in that its income elasticity is constant over the entire range of income/expenditure levels. If the double-log model proves to be the appropriate model, this constant elasticity has strong implications about the consumer's behavior or spending patterns.

The researcher is more concerned with understanding the consumer's spending patterns than in making predictions on the amount to be spent. Consequently, the correct signs on the estimated coefficients as well as the MPC and income elasticity (E_{INC}) are of utmost concern to the econometrician. Also, an elasticity measure for household size can be constructed in a similar manner and measures how sensitive the family manager's food expenditures are to the presence of an additional member in the household.

There are only three continuous regressor variables in this formulation: INC, HS and MEALS, all of which will have explanatory capability in terms of amount expended. However, they will not be treated in the same way. The MEALS variable is incorporated in the model as a separate regressor to explain or "pick up on" the variability in weekly food expenditure which relates to the number of meals actually eaten from the household food supply in a particular week. Therefore, meals eaten outside the home are not reflected in the expenditure relationships. When the appropriate functional form transformations are made on the regressor variables, they are applied to the INC and HS variables, but not generally to MEALS. In other words, MEALS can be considered a multi-level cross-sectional measure of variability from household to household. However, in cases involving the natural logarithm of the regressor variables, the transformation is also made on MEALS. (This stems from the linearization of a model with a specific error structure, such as the Cobb-Douglas production function.) Therefore, the five actual functional specifications, in terms of the continuous variables only, to be investigated in the current study are given in Table V.2.

Table V.1 Alternative Specifications of the Engel Function

Functional Form	Functional Specification	Marginal Propensity to Consume	Income Elasticity	Zero Observations?	Intercepts with y-Axis
(1) Quadratic	$E = \beta_0 + \beta_1 y + \beta_2 y^2$	$\beta_1 + 2\beta_2 y$	$\frac{(\beta_1 + 2\beta_2 y)y}{E}$	yes	can be negative
(2) Semi-log	$E = \beta_0 + \beta_1 \log(y)$	$\frac{\beta_1}{y}$	$\frac{\beta_1}{E}$	yes	positive
(3) Inverse	$E = \beta_0 + \beta_1 \frac{1}{y}$	$-\frac{\beta_1 E}{y^2}$	$-\frac{\beta_1}{y^2}$	yes	positive
(4) Double-log	$\log(E) = \beta_0 + \beta_1 \log(y)$	$\frac{\beta_1 E}{y}$	β_1	no	through origin
(5) Log-inverse	$\log(E) = \beta_0 + \beta_1 \frac{1}{y}$	$-\frac{\beta_1 E}{y^2}$	$-\frac{\beta_1}{y}$	no	positive

where $E = EXPEND$
$y = INC$

Table V.2 Functional Forms For the Empirical Study

Functional Form	Functional Specification	(for continuous variables only)
(1) Quadratic	$E = \beta_1 INC + \beta_2 INC^2 + \gamma_1 HS + \gamma_2 HS^2 + \theta_1 INCCHS + \alpha MEALS$	
(2) Semi-log	$E = \beta_1 \log(INC) + \gamma_1 \log(HS) + \alpha \log(MEALS)$	
(3) Inverse	$E = \beta_1 \frac{1}{INC} + \gamma_1 \frac{1}{HS} + \alpha MEALS$	
(4) Double-log	$\log(E) = \beta_1 \log(INC) + \gamma_1 \log(HS) + \alpha \log(MEALS)$	
(5) Log-inverse	$\log(E) = \beta_1 \frac{1}{INC} + \gamma_1 \frac{1}{HS} + \alpha MEALS$	

where $E = EXPEND$
$INCCHS = INC \times HS$

It is clear that the problem of dealing with zero observations on the dependent variable in the log models (4 and 5) must be handled in some satisfactory manner. Salathe (1979) circumvented the problem by replacing all zero expenditures with an arbitrarily small value of 0.01, or one cent. However, a large number of these one cent expenditures can result in estimated log models which reflect a distribution of responses which are skewed toward the negative tail. Therefore, the approach used here, which also has its pitfalls, is to drop all zero observations as if they were missing. Clearly, this procedure creates sample selection bias into the study, although the number of excluded observations is relatively small in the context of the data set. Consequently, the percentage of zero observations must be given due consideration when evaluating the resulting models.

5.4 Model Estimation

5.4.1 Estimation Considerations

When the data are a large sample from various cross-sections of a population, a variety of concerns arise. In the first place, there is the quantity of data involved. The data from the 1977-78 Nationwide Food Consumption Survey (NFCS 77-78) represent weekly observations taken from over 15,000 households located across the contiguous United States. For this empirical analysis, due to variable screening procedures, usable schedules for 9673 housekeeping households are employed. A housekeeping household, by definition, eats 10 or more meals at home from household food supplies in the survey week. Because of such a large amount of data, the degrees of freedom associated with the denominators of all tests of hypotheses of interest are exorbitant and the power of the tests is greatly increased. Consequently, a tug of war exists between the increased testing power and the correspondingly lowered model fitting due to the heterogeneity of the households. This issue must be taken into consideration throughout this discussion.

As a way to keep the influence of the large sample size from adversely clouding the results, the use of standardized p-values or tail-area probabilities are incorporated. Good (1982) proposed a method to obtain p-values associated with hypothesis tests as if they were based on a sample of 100 observations. Based on the relation that the Bayes factor for a specific tail-area probability is approximately inversely proportional to the square root of the sample size, Good proposed a reasonable adjustment. To have all the samples correspond to a set size of 100 observations, the standardized p-value is given to be the minimum of $(0.5, \sqrt{n/100}\, p)$. Good's adjustments constitute a more formal way of essentially shrinking the α, or type I error probability, for the test corresponding to the magnitude of the sample size. In particular, when the non-nested testing results are discussed in this chapter, both the observed and standardized p-values are presented.

The data for this study are based on information from households representing different sizes and income levels in addition to various demographic characteristics. Consequently, it is expected that there will be heterogeneous variances on the amount of food expenditure across different classes or cross-sections of households.

Often, the heteroskedasticity systematically reflects the behavior of one of the independent variables. Particularly, ine this type of expenditure model, it is common for the variance structure to be proportional to the income level:

$$Var(\varepsilon_i) \;=\; \sigma^2_{EXPEND_i} \;=\; \sigma^2\, INC_i^{\delta}\, e^{u_i}\,,$$

where $u_i \sim iidN(0,1)$. In order to estimate and test the significance of the parameter, δ (being different from zero), the Park-Glesjer approach is employed. (See Pindyck and Rubinfeld, 1981, pp. 150-152).

All the tests on the estimated δ parameters from each of the five functional specifications within each of the ten food groups were statistically significant. Therefore, 50 transformed models would need to be investigated, all of which now presented different dependent variables. With this in mind as well as the magnitudes of some of the $\hat{\delta}$ values, additional examination of the heteroskedasticity was necessary. Plots of the residuals from the initial models against the corresponding INC variable were produced. The various plots indicated that the problem was more one of outliers than of a systematic relationship between the disturbance terms and the income level. Consequently, this study employs the five alternative functional specifications as indicated, without any transformations to combat heterogeneity of variances.

5.4.2 Estimation Procedures

All model estimation was performed from the Maximum Likelihood (ML) approach which corresponds to OLS in the case of models which are linear in the parameters. Results from the initial estimations (in SAS output form) are provided in Appendix F, while summary information measuring fit and the estimated model characteristics are given in Table V.3. Examination of the initial models indicate that the quadratic and and double-log models tend to fit the data better. However, notice the extremely small coefficients of determination. Such small $R^{2\prime}$s are a common phenomena when dealing with such diverse cross-sections of a population (i.e., the households themselves).

Since the $R^{2\prime}$s are not comparable across models with different transformations on the dependent variable, a direct measure of goodness of fit is the correlation between the observed and predicted responses, both in non-logarithmic and logarithmic forms: $\hat{\rho}(\hat{y}, y)$ and $\hat{\rho}(\widehat{\log y}, \log y)$. Of interest, too, are the magnitudes and signs associated with the elasticity measures. Clearly, food expenditures will be somewhat insensitive to income changes, at least at certain levels. So, small elasticity coefficients, E_{INC}, are expected and they should be positive based on the a priori questions regarding normal goods. For most categories of food and most models this hypothesis gains support. On the other hand, if consideration is also given to household size elasticity, E_{HS}, other possibilities exist. In general, household size elasticities are small and positive, except for the meat and vegetable categories. In particular, after a certain point of increasing the household size, economies of scale come into play and an actual decrease in per person food expenditure can be expected. The influence of such economies of scale is manifested in the elasticities observed for meats and vegetables. For comparison purposes, the magnitude of the positive elasticities for household size will generally be smaller than those for income.

From the initial results on the models, it appears that the real hypothesis testing situation will involve the quadratic and double-log models. In fact, they represent two "opposing" theories in terms of formulating patterns in food spending. Therefore, it will be interesting to see which one the data tend to support. On the basis of the measures of fit, there is no substantial reasoning for choosing one over the other in some of the commodity groups. The non-nested framework is very useful in such situations because it forces one model to be able to explain the performance of its

Table V.3 Initial Model Estimation: Summary

Commodity	FF	R^2	$\hat{\rho}(y, \hat{y})$	$\hat{\rho}(\ln y, \widehat{\ln y})$	MPC	Income Elasticity	Household Size Elasticity
	(1)	0.5394	0.7344	0.7791	5.7E-04	0.1562	0.0096
	(2)	0.5117	0.7157	0.5925	3.4E-04	0.0927	-0.0154
ALLM_TO	(3)	0.5243	0.7242	0.7121	7.1E-05	0.0196	1.8E-05
	(4)	0.6178	0.7360	0.7860	5.2E-04	0.1437	0.0051
	(5)	0.5853	0.7070	0.7651	1.1E-04	0.0294	6.6E-05
	(1)	0.0950	0.3083	0.3710	8.5E-05	0.3035	-0.5327
	(2)	0.0828	0.2886	0.2737	6.0E-05	0.2125	-0.3916
BEVEM_TO	(3)	0.0736	0.2713	0.3318	7.1E-06	0.0251	1.1E-05
	(4)	0.1671	0.2759	0.4088	6.6E-05	0.2340	0.1037
	(5)	0.1571	0.2565	0.3963	1.1E-05	0.0387	6.8E-05
	(1)	0.2621	0.5119	0.5575	-7.5E-06	-0.0646	0.0792
	(2)	0.2541	0.5042	0.4984	7.4E-06	0.0642	0.0274
FATSM_TO	(3)	0.2590	0.5089	0.5422	1.9E-06	0.0167	2.1E-05
	(4)	0.3165	0.5118	0.5626	1.2E-05	0.1055	-0.0032
	(5)	0.3003	0.4914	0.5480	2.5E-06	0.0213	7.1E-05

Table V.3 Initial Model Estimation: Summary(continued)

Commodity	FF	R^2	$\hat{\rho}(y, \hat{y})$	$\hat{\rho}(\ln y, \ln \hat{y})$	MPC	Income Elasticity	Household Size Elasticity
	(1)	0.1853	0.4305	0.4433	3.6E-05	0.1265	0.0019
	(2)	0.1755	0.4190	0.3852	2.3E-05	0.0813	-0.0460
FRUIM_TO	(3)	0.1799	0.4243	0.4179	7.0E-06	0.0248	7.2E-06
	(4)	0.2072	0.4291	0.4552	3.7E-05	0.1302	-0.0564
	(5)	0.1994	0.4192	0.4466	1.0E-05	0.0364	3.1E-05
	(1)	0.4351	0.6596	0.6871	1.7E-05	0.0369	0.3844
	(2)	0.4039	0.6363	0.5141	-8.9E-07	-0.0020	0.3124
GRAIM_TO	(3)	0.4238	0.6510	0.6675	1.5E-06	0.0033	2.4E-05
	(4)	0.4907	0.6614	0.7005	1.7E-05	0.0388	0.3162
	(5)	0.4774	0.6311	0.6910	2.7E-05	0.0061	8.7E-05
	(1)	0.4075	0.6384	0.6850	2.7E-04	0.1904	-0.0712
	(2)	0.3918	0.6268	0.5281	1.6E-04	0.1174	-0.0852
MEATM_TO	(3)	0.3949	0.6286	0.6114	3.3E-05	0.0238	1.9E-05
	(4)	0.4897	0.6411	0.6998	2.6E-04	0.1894	-0.0890
	(5)	0.4554	0.6088	0.6748	5.0E-05	0.0356	7.9E-05

Table V.3 Initial Model Estimation: Summary(continued)

Commodity	FF	R^2	$\hat{\rho}(y, \hat{y})$	$\hat{\rho}(\ln y, \ln \hat{y})$	MPC	Income Elasticity	Household Size Elasticity
	(1)	0.4258	0.6525	0.6458	3.5E-05	0.0733	0.3134
	(2)	0.3809	0.6190	0.4540	6.2E-06	0.0136	0.2813
MILKM_TO	(3)	0.4105	0.6408	0.5117	2.1E-06	0.0045	1.4E-05
	(4)	0.4477	0.6561	0.6691	2.5E-05	0.0548	0.2765
	(5)	0.4379	0.6371	0.6617	4.4E-06	0.0097	7.3E-05
	(1)	0.1335	0.3654	0.4110	5.4E-06	0.0521	0.3094
	(2)	0.1217	0.3501	0.3123	-6.1E-07	-0.0058	0.2173
SUGAM_TO	(3)	0.1291	0.3594	0.3615	-3.5E-07	-0.0034	1.5E-05
	(4)	0.2089	0.3640	0.4570	8.4E-07	0.0080	0.2084
	(5)	0.2037	0.3389	0.4513	-1.7E-10	-1.6E-06	6.7E-05
	(1)	0.2801	0.5292	0.5759	5.9E-05	0.1347	-0.1254
	(2)	0.2734	0.5229	0.5188	3.8E-05	0.0858	-0.1622
VEGEM_TO	(3)	0.2732	0.5228	0.5373	9.9E-06	0.0226	1.0E-05
	(4)	0.3473	0.5300	0.5893	6.1E-05	0.1383	-0.1622
	(5)	0.3203	0.5059	0.5660	1.5E-05	0.0336	5.1E-05

Table V.3 . Initial Model Estimation: Summary(continued)

Commodity	FF	R^2	$\hat{\rho}(y, \hat{y})$	$\hat{\rho}(\ln y, \ln \hat{y})$	MPC	Income Elasticity	Household Size Elasticity
	(1)	0.0762	0.2761	0.3216	3.4E-05	0.1908	-0.1479
	(2)	0.0773	0.2780	0.3076	2.4E-05	0.1338	-0.1802
OTHERM_T	(3)	0.0711	0.2667	0.2718	5.2E-06	0.0288	1.9E-05
	(4)	0.1171	0.2755	0.3422	3.2E-05	0.1791	-0.2906
	(5)	0.1056	0.2594	0.3249	8.1E-06	0.0447	3.5E-05

competitor in order to determined "valid." The objective, then, is to come up with the most appropriate functional specification for all commodities or a set of them for various commodities for modelling consumer behavior. Once again, it should be kept in mind that it is not unreasonable to assume that one of these five specifications under investigation is close enough to the true underlying relationship in the data to be considered valid.

5.5 The Application of Non-Nested Testing Procedures

In order to test among all five functional forms given, some extensions must be made to the linear regression model procedures discussed previously. In particular, for situations in which the two models under test are of the form:

$$H_1 : y_t = f(X_1) + u_{1t}$$

$$H_2 : h(y_t) = g(X_2) + u_{2t}.$$

As long as the function $h(y)$ is twice continuously differentiable and the transformation does not depend on any unknown parameters, the extension can be made in a straightforward manner.

5.5.1 Handling a Transformed Dependent Variable in the Non-Nested Setting

When testing between different error specifications, it is essential to have a procedure for testing models with transformed dependent variables. The AN procedures can be readily adapted to handle the situation. In particular, the extension of the J-test, called the P-test, is as easy to compute as the linear version of the J-test, and is based on the following artificial regression model

$$y_t - \hat{f}_t = \hat{F}_t \beta + \lambda (\hat{g}_t - \hat{h}_t(f_t)) + \varepsilon_t$$

where $\hat{F}$ is a matrix whose elements are the derivatives of $f(\beta)$ with respect to β, evaluated at $\hat{\beta}$. Therefore instead of fitting the observed y's on the AN model with a single estimate from the alternative model, the residuals from the maintained model are regressed on the difference in predicted responses from the two models. The corresponding t-test on λ yields the asymptotically valid N(0,1) P-test. A similar extended version of the JA-test is obtained in the context of the same artificially nested model but with the JA- appropriate estimator. In this empirical study, the $h(y_t)$ is $\log(y_t)$, which indeed fulfills the requirements of differentiability.

Although it has not been proved, it is reasonable to assume that the general behavior of the two procedures when transformed dependent variables are involved will echo cases discussed previously. Therefore, not only can the tests be easily performed, but their behavior is at least based on the same theoretical concept. There is another procedure proposed by Davidson and MacKinnon(1984) which handles the transformed dependent variable situation through the use of a double length regression model and a Lagrange Multiplier (LM) test. Mainly on the basis of the large data set involved here, this procedure will not be used in the current study.

5.5.2 Box-Cox Formulation for Transformed Dependent Variables

Under the more general Box-Cox regression setting, several of the functional forms presented here can be considered as nested in that framework: both the semilog and double-log models are nested in a more general framework with the dependent variable of the form y^{λ}, and $\lambda = 0,1$ are just two special cases. The same can said for the inverse and log-inverse models. However, in order to incorporate the Box-Cox likelihood ratio approach to testing hypotheses on λ, the more general model must be estimated to obtain an unrestricted maximized log-likelihood. Difficulties were encountered when estimation of unrestricted models was attempted.

Even in general models involving only a transformation on the dependent variable, maximum likelihood estimation would not converge to a solution. (Estimation was attempted using the

BoxCox procedure in SHAZAM.) To circumvent the problem, various subsamples of the original survey data in sizes of 500 and 1000 were employed. However, in these cases, too, the maximized unrestricted log-likelihood was not obtainable. (Subsampling was based on stratified sampling proportional to size from strata formed from the demographic indicator variables.)

What was estimable were the coefficients and an observed log-likelihood under set transformations. With these model estimates, a "simple versus simple" hypothesis test setting seems appropriate. However, since the degrees of freedom based on the number of restrictions associated with the large sample χ^2 procedure would cancel each other out, the test would be invalid. Consequently, the best that the Box-Cox formulation could do in this specific application was provide the log-likehoods for the functional forms as another measure of fit.

The purpose of the study was to select the most appropriate (valid) functional form not just the one that better fit the data. Consequently, the Box-Cox methods could not be used. Thus, one strength of the general non-nested framework has been highlighted. Estimation problems are minimal and all models can be tested under the same general approach even if there are transformed dependent variables involved. Also, under the Box-Cox regime, not all hypotheses concerning functional form are nested.

5.5.3 Application of the Tests

For this study, all of the asymptotically valid procedures were used where applicable: N, NA, NL, F, NJ, J and JA (with the AN procedures extended to the transformed cases). Small sample modifications were excluded. Since the quality of fit is quite low on the majority of the commodity groups and the squared canonical correlations between the various sets of regressors are quite large, low power would be expected from all the procedures, if their behavior reflected the small sample cases. However, on the basis of sample size alone and its influence on power, the two factors may wash each other out and result in reasonable behavior.

Therefore, attention is turned to the actual test results. The calculated test statistics for all ten commodity groups are provided in Table V.4, along with the p-values for rejecting the maintained model in the presence of a given alternative and the standardized p-values, as discussed in Section 5.4. Once again, it becomes clear that the real heart of testing the model validity involves the quadratic and double-log.

If consideration is given instead to the case of a linear versus a double-log model, another testing procedures could be applied to yield additional evidence than that based solely on the extended AN procedures. Godfrey and Wickens (1981) give the specific formulation for testing linear versus log-linear models derived from Andrew's approach(1971). (See also Bera and McAleer, 1983.) For models of the more general form:

$$H_1 : y_t = \sum_{k=1}^{K} \beta_k x_{tk} + \sum_{j=1}^{J} \alpha_j z_{tj} + u_t$$

$$H_2 : \ln y_t = \sum_{k=1}^{K} \beta_k \ln x_{tk} + \sum_{j=1}^{J} \alpha_j z_{tj} + v_t$$

Andrews proposes the use of a Taylor series expansion about λ^0 the maintained value of λ. Then an artificial variable is created and a t-test on the coefficient of that variable is the resulting test. The artificial variables are constructed as indicated:

$$H_1 : y_t = \sum_{k=1}^{K} \beta_k x_{tk} + \sum_{j=1}^{J} \alpha_j z_{tj} + (\lambda - \lambda^0)\bar{q}_{t1} + u_t$$

$$H_2 : \ln y_t = \sum_{k=1}^{K} \beta_k \ln x_{tk} + \sum_{j=1}^{J} \alpha_j z_{tj} + (\lambda - \lambda^0)\hat{q}_{t2} + v_t$$

with $\bar{q}_{t1} = f(\bar{y}_t) - \sum_{k=1}^{K} \bar{\beta}_k f(x_{tk})$ with $f(w) = w \ln w - w + 1$ and $\hat{q}_{t2} = g(\hat{y}_t) - \sum_{k=1}^{K} \hat{\beta}_k g(x_{tk})$ with $g(w) = 1/2(\ln w)^2$. The usual t-test is an exact procedure for testing the linear versus log-linear models. Although it is the quadratic which is of specific interest in this application, the information from this test can either support the decision rendered by the other procedures or it can bring to light concerns regarding the correct specification. (See Table V.5 for these results.)

Table V.4 Non-Nested Test Results for Weekly Food Expenditures

test statistic p-value standardized p-value		COMMODITY: ALLM_TO				
FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-2.855	-10.597	-6.981	-17.447	-300.46	-223.7
	.0043086	0	2.9E-12	0	0	0
	.0423762	0	2.9E-11	0	0	0
(1) VS (3)	-43.898	-137.22	4.369	3.397	603.494	-138.83
	0	0	1.2E-05	6.8E-04	0	0
	0	0	1.2E-04	.0066905	0	0
(1) VS (4)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(1) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(2) VS (3)	-17.745	-3.373	-20.542	2.875	-78.946	-385.78
	0	7.4E-04	0	0.004045	0	0
	0	.0073023	0	.0397829	0	0
(2) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(2) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(3) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(3) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(4) VS (5)	-27.285	-17.127	5.206	-26.887	74.900	-200.52
	0	0	1.9E-07	0	0	0
	0	0	1.9E-06	0	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: ALLM_TO

J-TEST		JA-TEST		NJ-TEST		F-TEST	
8.086	25.029	6.991	17.712	.	18.006	11.098	216.968
6.7E-16	0	2.9E-12	0	2.8E-12	0	2.3E-12	0
6.6E-15	0	2.9E-11	0	2.7E-11	0	2.2E-11	0
5.419	5.488	-4.369	-3.392	.	-3.395	6.736	175.621
6.1E-08	4.2E-08	1.3E-05	7.0E-04	1.2E-05	6.9E-04	2.8E-06	0
6.0E-07	4.1E-07	1.2E-04	.0068514	1.2E-04	.0067879	2.8E-05	0
6.839	1.636	-9.437	-1.054	.	.	.	.
8.5E-12	0.101872	0	0.291909	.	.	.	.
8.3E-11	0.5	0	0.5	.	.	.	.
5.107	-1.962	4.807	1.978	.	.	.	.
3.3E-07	.0497912	1.6E-06	.0479571	.	.	.	.
3.3E-06	0.489703	1.5E-05	0.471665	.	.	.	.
19.726	14.034	20.986	-2.900	.	-2.921	195.596	107.550
0	0	0	0.00374	0	.0034994	0	0
0	0	0	.0367838	0	.0344172	0	0
11.269	4.205	-11.928	-4.159	.	.	.	.
0	2.6E-05	0	3.2E-05	.	.	.	.
0	2.6E-04	0	3.2E-04	.	.	.	.
11.706	25.578	-20.408	-10.241	.	.	.	.
0	0	0	0	.	.	.	.
0	0	0	0	.	.	.	.
7.608	-1.555	-9.830	4.718	.	.	.	.
3.0E-14	0.119979	0	2.4E-06	.	.	.	.
3.0E-13	0.5	0	2.4E-05	.	.	.	.
3.169	5.846	-3.388	-14.330	.	.	.	.
.0015344	5.2E-09	7.1E-04	0	.	.	.	.
.0150912	5.1E-08	0.006952	0	.	.	.	.
-3.320	29.515	-5.208	27.927	.	28.585	19.265	294.500
9.0E-04	0	1.9E-07	0	1.9E-07	0	1.9E-12	0
.0088861	0	1.9E-06	0	1.8E-06	0	1.9E-11	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: BEVEM_TO

FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-0.855	-4.793	-0.308	-9.170	-1853.8	-662.56
	0.392683	1.6E-06	0.757829	0	0	0
	0.5	1.5E-05	0.5	0	0	0
(1) VS (3)	-5.701	-10.182	1.462	2.040	-285.78	-506.18
	1.2E-08	0	0.14372	.0413302	0	0
	1.1E-07	0	0.5	0.370638	0	0
(1) VS (4)	.	.	.	.	.	.
	:	:	:	:	:	:
(1) VS (5)	.	.	.	.	.	.
	:	:	:	:	:	:
(2) VS (3)	-2.161	-0.627	-0.976	3.496	-272.94	-1526.8
	.0307295	0.530666	0.328883	4.7E-04	0	0
	0.275573	0.5	0.5	.0042301	0	0
(2) VS (4)	.	.	.	.	.	.
		:	:	:	:	:
(2) VS (5)	.	.	.	.	.	.
	:	:	:	:	:	:
(3) VS (4)	.	.	.	.	.	.
		:	:	:	:	:
(3) VS (5)	.	.	.	.	.	.
	:	:	:	:	:	:
(4) VS (5)	-1.708	-1.276	2.841	1.734	-252.12	-495.16
	.0876819	0.202007	.0045019	0.082847	0	0
	0.5	0.5	.0403717	0.5	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: BEVEM_TO

J-TEST		JA-TEST		NJ-TEST		F-TEST	
-2.308	10.615	0.308	9.209	.	9.226	1.879	39.809
.0210244	0	0.75809	0	0.757942	0	.0804978	0
0.188541	0	0.5	0	0.5	0	0.5	0
-0.655	7.001	-1.460	-2.039	.	-2.040	2.715	102.067
0.512487	2.7E-12	0.144329	.0414827	0.144043	.0413379	.0186155	0
0.5	2.5E-11	0.5	0.372005	0.5	0.370707	0.166939	0
4.760	7.154	-4.308	-5.990	.	.	.	.
2.0E-06	9.2E-13	1.7E-05	2.2E-09	.	.	.	.
1.8E-05	8.2E-12	1.5E-04	2.0E-08	.	.	.	.
-0.663	9.264	-1.976	-5.178	.	.	.	.
0.50735	0	0.048189	2.3E-07	.	.	.	.
0.5	0	0.432145	2.1E-06	.	.	.	.
1.096	11.216	0.975	-3.495	.	-3.517	33.805	61.196
0.273112	0	0.32959	4.8E-04	0.326611	4.4E-04	0	0
0.5	0	0.5	.0042742	0.5	.0039417	0	0
4.130	1.715	-4.490	-3.637	.	.	.	.
3.7E-05	.0863838	7.2E-06	2.8E-04	.	.	.	.
3.3E-04	0.5	6.5E-05	.0024891	.	.	.	.
3.933	10.230	-3.580	-4.927	.	.	.	.
8.5E-05	0	3.5E-04	8.5E-07	.	.	.	.
7.6E-04	0	.0030995	7.6E-06	.	.	.	.
8.391	-1.994	-1.004	-0.131	.	.	.	.
5.6E-17	.0461858	0.315409	0.895779	.	.	.	.
5.0E-16	0.414182	0.5	0.5	.	.	.	.
5.187	0.402	-1.581	-5.643	.	.	.	.
2.2E-07	0.687695	0.113917	1.7E-08	.	.	.	.
2.0E-06	0.5	0.5	1.5E-07	.	.	.	.
-2.585	11.202	-2.839	-1.733	.	-1.748	15.313	47.779
.0097554	0	.0045369	.0831341	.0044453	.0805067	6.2E-10	0
.0874835	0	.0406855	0.5	.0398641	0.5	5.6E-09	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: FATSM_TO

FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-0.368	-0.923	-4.533	-6.345	-549.07	-424.92
	0.712718	0.356089	5.8E-06	2.2E-10	0	0
	0.5	0.5	5.6E-05	2.1E-09	0	0
(1) VS (3)	-12.858	-20.378	2.686	1.905	127.034	-279.87
	0	0	.0072418	0.056731	0	0
	0	0	.0700176	0.5	0	0
(1) VS (4)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(1) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(2) VS (3)	-2.299	-0.460	-6.507	-0.767	-171.16	-704.48
	.0214776	0.645666	7.7E-11	0.443285	0	0
	0.207656	0.5	7.4E-10	0.5	0	0
(2) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(2) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(3) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(3) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(4) VS (5)	-0.093	-1.023	2.694	7.801	-557.78	-386.12
	0.926116	0.306195	.0070622	6.1E-15	0	0
	0.5	0.5	.0682811	5.9E-14	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: FATSM_TO

J-TEST		JA-TEST		NJ-TEST		F-TEST	
5.524	10.467	4.533	6.354	.	6.384	5.310	44.324
3.4E-08	0	5.9E-06	2.2E-10	5.8E-06	1.8E-10	1.8E-05	0
3.3E-07	0	5.7E-05	2.1E-09	5.6E-05	1.7E-09	1.7E-04	0
3.042	0.895	-2.684	-1.904	.	-1.904	1.879	24.337
.0023567	0.37081	.0072876	.0569409	.0072807	.0569118	.0943686	2.9E-11
.0227853	0.5	.0704598	0.5	.0703934	0.5	0.5	2.8E-10
5.027	-0.835	0.413	0.112	.	.	.	.
5.1E-07	0.403739	0.679616	0.910826	.	.	.	.
4.9E-06	0.5	0.5	0.5	.	.	.	.
3.074	-4.889	2.972	2.111	.	.	.	.
.0021182	1.0E-06	.0029662	.0347988	.	.	.	.
.0204798	1.0E-05	.0286789	0.336453	.	.	.	.
9.954	6.224	10.596	0.766	.	0.766	41.271	20.703
0	5.1E-10	0	0.443696	0	0.443442	0	2.3E-13
0	4.9E-09	0	0.5	0	0.5	0	2.2E-12
7.131	4.148	-7.039	-4.489	.	.	.	.
1.1E-12	3.4E-05	2.1E-12	7.2E-06	.	.	.	.
1.0E-11	3.3E-04	2.0E-11	7.0E-05	.	.	.	.
7.289	13.923	-8.019	-8.313	.	.	.	.
3.4E-13	0	1.2E-15	8.3E-17	.	.	.	.
3.3E-12	0	1.2E-14	8.1E-16	.	.	.	.
5.402	0.260	1.355	3.335	.	.	.	.
6.8E-08	0.79487	0.17545	8.6E-04	.	.	.	.
6.5E-07	0.5	0.5	.0082793	.	.	.	.
0.443	4.538	0.258	-9.860	.	.	.	.
0.657776	5.8E-06	0.796413	0	.	.	.	.
0.5	5.6E-05	0.5	0	.	.	.	.
-1.612	15.575	-2.692	-7.820	.	-7.896	7.484	81.461
0.106996	0	.0071152	5.8E-15	.0070717	3.2E-15	5.3E-05	0
0.5	0	0.068793	5.6E-14	.0683725	3.1E-14	5.1E-04	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: FRUIM_TO

FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-1.588	-4.093	-4.037	0.508	-842.65	-499.02
	0.112384	4.3E-05	5.4E-05	0.611239	0	0
	0.5	4.1E-04	5.2E-04	0.5	0	0
(1) VS (3)	-19.354	-24.761	4.073	2.590	10.411	-347.36
	0	0	4.6E-05	.0096114	0	0
	0	0	4.4E-04	.0921691	0	0
(1) VS (4)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(1) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(2) VS (3)	-8.211	-1.271	-8.992	-1.388	-181.79	-1096.8
	2.2E-16	0.203694	0	0.165126	0	0
	2.1E-15	0.5	0	0.5	0	0
(2) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(2) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(3) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(3) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(4) VS (5)	-5.871	-1.912	-1.697	1.106	-177.97	-724.73
	4.3E-09	.0558453	.0897543	0.268721	0	0
	4.1E-08	0.5	0.5	0.5	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: FRUIM_TO

J-TEST		JA-TEST		NJ-TEST		F-TEST	
5.334	10.823	4.036	-0.508	.	-0.512	5.658	48.364
9.8E-08	0	5.5E-05	0.611466	5.4E-05	0.608746	7.1E-06	0
9.4E-07	0	5.3E-04	0.5	5.2E-04	0.5	6.8E-05	0
-4.062	4.570	-4.072	-2.589	.	-2.589	3.565	39.265
4.9E-05	4.9E-06	4.7E-05	.0096407	4.7E-05	.0096358	.0031901	0
4.7E-04	4.7E-05	4.5E-04	.0924502	4.5E-04	.0924033	0.030592	0
5.875	2.336	-5.094	2.991	.	.	.	.
4.4E-09	.0195127	3.6E-07	0.002788	.	.	.	.
4.2E-08	0.187118	3.4E-06	.0267361	.	.	.	.
4.393	1.649	2.527	1.162	.	.	.	.
1.1E-05	.0991819	.0115208	0.245266	.	.	.	.
1.1E-04	0.5	0.11048	0.5	.	.	.	.
8.035	5.786	9.024	1.387	.	1.388	30.031	13.344
1.0E-15	7.4E-09	0	0.165475	0	0.1653	0	1.1E-08
9.8E-15	7.1E-08	0	0.5	0	0.5	0	1.0E-07
7.978	-3.438	-7.101	2.985	.	.	.	.
1.7E-15	5.9E-04	1.3E-12	.0028433	.	.	.	.
1.6E-14	.0056447	1.3E-11	.0272656	.	.	.	.
8.553	8.890	-9.749	-1.721	.	.	.	.
0	0	0	.0852845	.	.	.	.
0	0	0	0.5	.	.	.	.
3.566	-0.089	-4.316	-0.426	.	.	.	.
3.6E-04	0.929084	1.6E-05	0.670118	.	.	.	.
.0034936	0.5	1.5E-04	0.5	.	.	.	.
1.633	-0.230	-4.123	-1.812	.	.	.	.
0.102503	0.818097	3.8E-05	.0700189	.	.	.	.
0.5	0.5	3.6E-04	0.5	.	.	.	.
1.108	9.681	1.695	-1.105	.	-1.111	1.150	31.288
0.267891	0	.0901093	0.269189	.0901168	0.266812	0.327521	0
0.5	0	0.5	0.5	0.5	0.5	0.5	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: GRAIN_TO

FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-1.445	-6.150	0.838	20.327	-397.68	-310.98
	0.14837	7.7E-10	0.402299	0	0	0
	0.5	7.6E-09	0.5	0	0	0
(1) VS (3)	-29.867	-77.279	-0.657	-9.233	346.057	-181.86
	0	0	0.511111	0	0	0
	0	0	0.5	0	0	0
(1) VS (4)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(1) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(2) VS (3)	-10.849	-1.954	-19.735	-8.645	-133.73	-475.61
	0	.0506582	0	0	0	0
	0	0.497844	0	0	0	0
(2) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(2) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(3) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(3) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(4) VS (5)	-6.061	-2.701	2.039	-7.321	-83.123	-397.93
	1.4E-09	.0069035	.0414183	2.5E-13	0	0
	1.3E-08	.0678439	0.407039	2.4E-12	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: GRAIM_TO

J-TEST		JA-TEST		NJ-TEST		F-TEST	
0.707	23.107	-0.837	20.758	.	20.874	2.600	182.733
0.479584	0	0.402613	0	0.402274	0	.0161343	0
0.5	0	0.5	0	0.5	0	0.15856	0
-1.543	12.271	0.656	9.266	.	9.267	0.611	98.126
0.122864	0	0.51184	0	0.511782	0	0.691658	0
0.5	0	0.5	0	0.5	0	0.5	0
2.169	4.836	-8.115	0.182	.	.	.	.
.0301071	1.3E-06	5.3E-16	0.855587	.	.	.	.
0.295877	1.3E-05	5.2E-15	0.5	.	.	.	.
-1.072	1.693	-3.594	-3.991	.	.	.	.
0.283747	.0904878	3.3E-04	6.6E-05	.	.	.	.
0.5	0.5	.0032162	6.5E-04	.	.	.	.
18.489	6.878	20.128	8.671	.	8.704	160.770	48.410
0	6.4E-12	0	0	0	0	0	0
0	6.3E-11	0	0	0	0	0	0
11.786	1.494	-14.005	-0.965	.	.	.	.
0	0.135208	0	0.334569	.	.	.	.
0	0.5	0	0.5	.	.	.	.
11.828	14.066	-17.195	-10.763	.	.	.	.
0	0	0	0	.	.	.	.
0	0	0	0	.	.	.	.
3.803	-2.320	-12.035	2.861	.	.	.	.
1.4E-04	.0203616	0	.0042322	.	.	.	.
.0014136	0.200104	0	.0415915	.	.	.	.
9.437	-4.351	-9.097	-8.590	.	.	.	.
0	1.4E-05	0	0	.	.	.	.
0	1.3E-04	0	0	.	.	.	.
-4.113	15.976	-2.054	7.335	.	7.415	5.778	89.464
3.9E-05	0	.0400026	2.4E-13	.0398779	1.3E-13	6.1E-04	0
3.9E-04	0	0.393126	2.4E-12	0.391901	1.3E-12	.0059706	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: MEATM_TO

FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-3.773	-8.546	-7.640	-11.980	-398.42	-293.55
	1.6E-04	0	2.2E-14	0	0	0
	.0015856	0	2.1E-13	0	0	0
(1) VS (3)	-31.020	-73.840	4.338	7.080	336.824	-192.24
	0	0	1.4E-05	1.4E-12	0	0
	0	0	1.4E-04	1.4E-11	0	0
(1) VS (4)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(1) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(2) VS (3)	-10.813	-2.679	-14.658	5.807	-122.22	-509.18
	0	.0073772	0	6.4E-09	0	0
	0	.0725368	0	6.2E-08	0	0
(2) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(2) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(3) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(3) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	
	.	.	.	.	.	.
(4) VS (5)	-17.339	-6.761	8.968	-14.931	-10.334	-421.56
	0	1.4E-11	0	0	0	0
	0	1.3E-10	0	0	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: MEATM_TO

J-TEST		JA-TEST		NJ-TEST		F-TEST	
7.850	17.149	7.656	12.060	.	12.166	10.892	107.507
4.6E-15	0	2.1E-14	0	2.1E-14	0	4.0E-12	0
4.5E-14	0	2.1E-13	0	2.0E-13	0	4.0E-11	0
6.184	-0.067	-4.338	-7.092	.	-7.101	8.497	124.258
6.5E-10	0.946583	1.5E-05	1.4E-12	1.4E-05	1.3E-12	4.9E-08	0
6.4E-09	0.5	1.4E-04	1.4E-11	1.4E-04	1.3E-11	4.8E-07	0
6.616	2.987	-8.987	-1.643	.	.	.	.
3.9E-11	.0028244	0	0.100416	.	.	.	.
3.8E-10	0.027771	0	0.5	.	.	.	.
5.817	5.416	3.774	5.339	.	.	.	.
6.2E-09	6.2E-08	1.6E-04	9.6E-08	.	.	.	.
6.1E-08	6.1E-07	.0015891	9.4E-07	.	.	.	.
14.754	11.386	14.810	-5.812	.	-5.839	103.935	87.058
0	0	0	6.4E-09	0	5.4E-09	0	0
0	0	0	6.3E-08	0	5.3E-08	0	0
8.001	8.003	-10.424	-7.752	.	.	.	.
1.4E-15	1.3E-15	0	9.9E-15	.	.	.	.
1.3E-14	1.3E-14	0	9.8E-14	.	.	.	.
8.276	22.713	-14.257	-8.177	.	.	.	.
1.4E-16	0	0	3.1E-16	.	.	.	.
1.4E-15	0	0	3.0E-15	.	.	.	.
9.103	1.429	-6.349	4.221	.	.	.	.
0	0.153037	2.3E-10	2.5E-05	.	.	.	.
0	0.5	2.2E-09	2.4E-04	.	.	.	.
-3.161	9.575	-3.983	-12.517	.	.	.	.
.0015771	0	6.9E-05	0	.	.	.	.
.0155075	0	6.7E-04	0	.	.	.	.
-2.146	27.029	-8.998	15.093	.	15.505	39.213	258.353
.0318978	0	0	0	0	0	0	0
0.313639	0	0	0	0	0	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: MILKM_TO

FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-1.860	-10.749	-1.790	-20.302	-427.41	-314.26
	.0628354	0	.0735173	0	0	0
	0.5	0	0.5	0	0	0
(1) VS (3)	-32.802	-90.098	-0.743	-10.462	338.590	-179.9
	0	0	0.457739	0	0	0
	0	0	0.5	0	0	0
(1) VS (4)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(1) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(2) VS (3)	-15.563	-2.176	-22.407	-6.652	-132.31	-508.37
	0	0.029588	0	2.9E-11	0	0
	0	0.290218	0	2.8E-10	0	0
(2) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(2) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(3) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(3) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(4) VS (5)	-9.984	-3.537	-1.872	-5.743	-83.256	-408.76
	0	4.0E-04	.0611601	9.3E-09	0	0
	0	.0039648	0.5	9.1E-08	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: MILKM_TO

J-TEST		JA-TEST		NJ-TEST		F-TEST	
2.787	27.416	1.788	20.735	.	21.070	1.671	253.440
.0053303	0	.0738075	0	0.07372	0	0.123664	0
.0522831	0	0.5	0	0.5	0	0.5	0
1.054	14.730	-0.742	10.513	.	10.513	0.267	128.321
0.291909	0	0.458105	0	0.458087	0	0.9314	0
0.5	0	0.5	0	0.5	0	0.5	0
2.070	5.135	-8.000	0.044	.	.	.	.
0.038479	2.9E-07	1.4E-15	0.964905	.	.	.	.
0.377428	2.8E-06	1.4E-14	0.5	.	.	.	.
1.096	2.873	-2.770	-4.048	.	.	.	.
0.273106	.0040749	.0056164	5.2E-05	.	.	.	.
0.5	.0399689	.0550894	5.1E-04	.	.	.	.
21.531	5.273	22.995	6.662	.	6.697	212.624	49.598
0	1.4E-07	0	2.8E-11	0	2.2E-11	0	0
0	1.3E-06	0	2.8E-10	0	2.2E-10	0	0
13.588	-3.245	-15.292	3.505	.	.	.	.
0	.0011785	0	4.6E-04	.	.	.	.
0	.0115598	0	.0044991	.	.	.	.
14.119	11.276	-19.157	-6.735	.	.	.	.
0	0	0	1.7E-11	.	.	.	.
0	0	0	1.7E-10	.	.	.	.
4.537	-3.335	-9.040	2.088	.	.	.	.
5.8E-06	8.6E-04	0	.0368241	.	.	.	.
5.7E-05	.0083984	0	0.361196	.	.	.	.
5.109	-4.828	-7.536	-5.138	.	.	.	.
3.3E-07	1.4E-06	5.3E-14	2.8E-07	.	.	.	.
3.2E-06	1.4E-05	5.2E-13	2.8E-06	.	.	.	.
-1.678	13.048	1.871	5.747	.	5.793	6.131	63.369
.0933796	0	.0613755	9.4E-09	.0611882	7.1E-09	3.7E-04	0
0.5	0	0.5	9.2E-08	0.5	7.0E-08	.0036069	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: SUGAM_TO

FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-0.280	-1.025	-0.528	-7.287	-912.22	-719.74
	0.779495	0.305368	0.597377	3.2E-13	0	0
	0.5	0.5	0.5	3.0E-12	0	0
(1) VS (3)	-6.296	-9.414	-1.126	-4.478	-224.56	-432.8
	3.1E-10	0	0.260209	7.6E-06	0	0
	2.9E-09	0	0.5	7.1E-05	0	0
(1) VS (4)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(1) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(2) VS (3)	-1.593	-0.284	-9.178	-2.833	-313.75	-1073.9
	0.111128	0.776102	0	.0046158	0	0
	0.5	0.5	0	.0434307	0	0
(2) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(2) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(3) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(3) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(4) VS (5)	-0.530	-1.168	2.789	-0.168	-730.33	-468.68
	0.596142	0.24283	.0052799	0.866482	0	0
	0.5	0.5	.0496786	0.5	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: SUGAM_TO

J-TEST		JA-TEST		NJ-TEST		F-TEST	
1.582	11.057	0.528	7.305	.	7.335	1.649	43.476
0.113685	0	0.597513	3.0E-13	0.597334	2.4E-13	0.129274	0
0.5	0	0.5	2.8E-12	0.5	2.3E-12	0.5	0
-0.281	6.337	1.125	4.478	.	4.478	0.465	23.737
0.778717	2.5E-10	0.26062	7.6E-06	0.260591	7.6E-06	0.802399	5.2E-11
0.5	2.3E-09	0.5	7.2E-05	0.5	7.2E-05	0.5	4.9E-10
2.413	4.548	-3.794	2.272	.	.	.	.
0.015842	5.5E-06	1.5E-04	.0231104	.	.	.	.
0.149058	5.2E-05	.0014041	0.217447	.	.	.	.
8.566	2.571	-3.903	2.832	.	.	.	.
0	.0101567	9.6E-05	.0046362	.	.	.	.
0	.0955652	9.0E-04	.0436226	.	.	.	.
8.566	2.571	9.213	2.832	.	2.836	36.052	10.989
0	.0101567	0	.0046362	0	.0045853	0	3.4E-07
0	.0955652	0	.0436226	0	.0431432	0	3.2E-06
5.651	-1.616	-7.100	2.957	.	.	.	.
1.6E-08	0.10613	1.3E-12	.0031147	.	.	.	.
1.5E-07	0.5	1.3E-11	.0293063	.	.	.	.
6.022	7.308	-7.013	-4.865	.	.	.	.
1.8E-09	2.9E-13	2.5E-12	1.2E-06	.	.	.	.
1.7E-08	2.8E-12	2.4E-11	1.1E-05	.	.	.	.
2.776	-0.459	-4.424	2.919	.	.	.	.
.0055148	0.646245	9.8E-06	.0035204	.	.	.	.
0.051889	0.5	9.2E-05	.0331238	.	.	.	.
3.284	-1.740	-3.897	-3.963	.	.	.	.
.0010274	.0818938	9.8E-05	7.5E-05	.	.	.	.
.0096671	0.5	9.2E-04	7.0E-04	.	.	.	.
-0.139	7.815	-2.788	0.168	.	0.169	2.873	22.248
0.889453	6.1E-15	.0053148	0.866587	.0038573	0.866097	.0348595	2.4E-14
0.5	5.7E-14	0.050007	0.5	.0362933	0.5	0.327995	2.3E-13

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: VEGEM_TO

FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-0.914	-2.157	-6.132	-4.950	-547.23	-348.52
	0.360952	0.031014	8.7E-10	7.4E-07	0	0
	0.5	0.303905	8.5E-09	7.3E-06	0	0
(1) VS (3)	-18.515	-28.629	4.050	5.887	110.810	-262.38
	0	0	5.1E-05	3.9E-09	0	0
	0	0	5.0E-04	3.9E-08	0	0
(1) VS (4)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(1) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(2) VS (3)	-5.383	-1.128	-10.779	1.613	-143.02	-755.86
	7.3E-08	0.259251	0	0.106788	0	0
	7.2E-07	0.5	0	0.5	0	0
(2) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(2) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(3) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(3) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(4) VS (5)	-4.816	-2.309	4.062	-0.675	-88.214	-593.44
	1.5E-06	.0209565	4.9E-05	0.499853	0	0
	1.4E-05	0.205352	4.8E-04	0.5	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: VEGEM_TO

J-TEST		JA-TEST		NJ-TEST		F-TEST	
7.358	11.175	6.137	4.952	.	4.982	9.036	48.149
2.0E-13	0	8.7E-10	7.5E-07	8.5E-10	6.4E-07	7.1E-10	0
2.0E-12	0	8.6E-09	7.3E-06	8.3E-09	6.3E-06	7.0E-09	0
4.264	-2.570	-4.049	-5.892	.	-5.893	3.638	55.041
2.0E-05	.0101848	5.2E-05	3.9E-09	5.2E-05	3.9E-09	.0027357	0
2.0E-04	.0998008	5.1E-04	3.9E-08	5.1E-04	3.9E-08	.0268066	0
7.496	-2.569	-4.126	-0.338	.	.	.	.
7.2E-14	.0102142	3.7E-05	0.735371	.	.	.	.
7.0E-13	0.100089	3.6E-04	0.5	.	.	.	.
4.396	-4.921	4.323	1.566	.	.	.	.
1.1E-05	8.8E-07	1.6E-05	0.117382	.	.	.	.
1.1E-04	8.6E-06	1.5E-04	0.5	.	.	.	.
10.458	10.881	10.836	-1.612	.	-1.614	46.783	47.448
0	0	0	0.106995	0	0.106599	0	0
0	0	0	0.5	0	0.5	0	0
6.594	3.915	-6.791	-4.647	.	.	.	.
4.5E-11	9.1E-05	1.2E-11	3.4E-06	.	.	.	.
4.4E-10	8.9E-04	1.2E-10	3.3E-05	.	.	.	.
7.254	18.994	-10.497	-8.444	.	.	.	.
4.4E-13	0	0	2.8E-17	.	.	.	.
4.3E-12	0	0	2.7E-16	.	.	.	.
7.058	-0.788	-1.566	2.375	.	.	.	.
1.8E-12	0.430716	0.117382	.0175685	.	.	.	.
1.8E-11	0.5	0.5	0.172153	.	.	.	.
-0.253	5.213	-0.670	-11.300	.	.	.	.
0.800274	1.9E-07	0.502874	0	.	.	.	.
0.5	1.9E-06	0.5	0	.	.	.	.
-2.228	20.305	-4.062	0.674	.	0.688	5.538	137.930
.0259036	0	4.9E-05	0.500328	4.9E-05	0.491253	8.5E-04	0
0.253829	0	4.8E-04	0.5	4.8E-04	0.5	.0083668	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: OTHERM_T						
FF	N-TEST		NA-TEST		NL-TEST	
(1) VS (2)	-0.552	-1.420	-5.907	1.866	-1551.7	-564.21
	0.580937	0.155708	3.5E-09	.0620116	0	0
	0.5	0.5	3.3E-08	0.5	0	0
(1) VS (3)	-7.572	-7.997	4.875	3.898	-199.39	-519.75
	3.7E-14	1.3E-15	1.1E-06	9.7E-05	0	0
	3.4E-13	1.2E-14	1.0E-05	9.1E-04	0	0
(1) VS (4)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(1) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(2) VS (3)	-0.233	-0.139	-0.522	-0.172	-249.5	-1928.5
	0.816083	0.889347	0.601466	0.863152	0	0
	0.5	0.5	0.5	0.5	0	0
(2) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(2) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(3) VS (4)	.	.	.	.	.	.
		.	.	.	.	.
		.	.	.	.	.
(3) VS (5)	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
(4) VS (5)	-2.188	-1.830	1.885	8.055	-536.93	-1062.9
	.0286835	0.067234	.0593848	8.0E-16	0	0
	0.268799	0.5	0.5	7.5E-15	0	0

Table V.4 Non-Nested Test Results for Weekly Food Expenditures (continued)

COMMODITY: OTHERM_T

J-TEST		JA-TEST		NJ-TEST		F-TEST	
4.977	2.341	5.912	-1.865	.	-1.867	5.911	8.466
6.6E-07	.0192545	3.5E-09	.0622148	3.5E-09	.0619483	3.6E-06	1.3E-05
6.2E-06	0.180438	3.3E-08	0.5	3.3E-08	0.5	3.4E-05	1.2E-04
4.601	1.521	-4.876	-3.898	.	-3.899	5.222	37.665
4.3E-06	0.128296	1.1E-06	9.8E-05	1.1E-06	9.8E-05	8.6E-05	4.2E-17
4.0E-05	0.5	1.0E-05	9.2E-04	1.0E-05	9.1E-04	8.1E-04	3.9E-16
5.027	-1.612	-4.942	2.313	.	.	.	.
5.1E-07	0.106998	7.9E-07	.0207457	.	.	.	.
4.8E-06	0.5	7.4E-06	0.194413	.	.	.	.
4.677	0.824	2.541	4.842	.	.	.	.
3.0E-06	0.409962	.0110707	1.3E-06	.	.	.	.
2.8E-05	0.5	0.103746	1.2E-05	.	.	.	.
0.297	7.862	0.522	0.172	.	0.172	5.909	25.675
0.766474	4.2E-15	0.601684	0.863441	0.601343	0.863315	5.1E-04	1.5E-16
0.5	4.0E-14	0.5	0.5	0.5	0.5	.0047332	1.4E-15
2.107	0.929	-3.422	-0.673	.	.	.	.
.0351459	0.352915	6.2E-04	0.500965	.	.	.	.
0.329361	0.5	0.005852	0.5	.	.	.	.
1.664	10.657	-2.267	-1.075	.	.	.	.
.0961482	0	.0234145	0.282404	.	.	.	.
0.5	0	0.219423	0.5	.	.	.	.
5.806	-0.643	-1.505	2.341	.	.	.	.
6.6E-09	0.520241	0.13236	.0192545	.	.	.	.
6.2E-08	0.5	0.5	0.180438	.	.	.	.
0.464	4.949	-2.271	-1.614	.	.	.	.
0.642659	7.6E-07	.0231711	0.106563	.	.	.	.
0.5	7.1E-06	0.217142	0.5	.	.	.	.
-0.343	10.932	-1.884	-8.077	.	-8.067	4.584	42.729
0.731607	0	0.059598	7.5E-16	.0594645	8.0E-16	.0032797	0
0.5	0	0.5	7.0E-15	0.5	7.5E-15	.0307351	0

Table V.5 Andrew's Test of Linear vs Log-Linear Models

	t-test on parameter λ	
Commodity	Linear Maintained	Log-Linear Maintained
$\hat{\lambda}$ p-value		
ALLM_TO	13.644 (0.001)	-2.078 (0.038)
BEVEM_TO	5.195 (0.001)	-6.565 (0.001)
FATSM_TO	7.254 (0.001)	-0.210 (0.834)
FRUIM_TO	7.498 (0.001)	0.732 (0.464)
GRAIM_TO	13.512 (0.001)	1.333 (0.183)
MEATM_TO	12.649 (0.001)	-3.651 (0.003)
MILKM_TO	13.516 (0.001)	1.460 (0.144)
SUGAM_TO	7.231 (0.001)	3.062 (0.002)
VEGEM_TO	9.042 (0.001)	0.369 (0.712)
OTHERM_T	6.220 (0.001)	0.337 (0.736)

5.6 Choice of Most Appropriate Model(s)

In this section, each commodity group will be discussed on an individual basis to determine the appropriate inference regarding the selection of an appropriate model. To start, the total weekly food expenditure model will be addresed.

In the case of all food expenditures the results from the non-nested procedures are far from clear cut. The only sure thing that can be stated is how out of line the Linearized Cox (NL) procedure's results are from the expected norm. However, based on the test between (1) and (4), this would indicate that the double-log form is the most appropriate model. Extra evidence in this favor is supplied by Andrews. Clearly, there are contradictory indicators throughout. In terms of the P-test, the double-log was maintained as valid against the inverse, whereas the extended JA-test was unable to detect it.

What is of particular interest here is the behavior of the semilog (2) model. It evidently is not as good as the other models in its basic properties, yet it rejects all the models tested against it. A non-significant result would be hoped for to indicate its inferiority. However, what is expected is that it is "so far off" from the "true" model that the true model would not reasonably pick up and explain the variability in this false model. Therefore, if this is indeed the scenario, the rejections in the presence of the semilog model are not unreasonable.

On the other hand, in the case of examining the expenditures only on beverages, the quadratic form could be considered the most appropriate model. In this case, the quadratic model did the best at retaining its validity in the presence of the other models. Although, the test of (1) versus (4) rejected in both instances, the quadratic was less likely to be rejected than the double-log. In terms of Andrews' procedure, the beverage commodity group is the only one in which the linear model at least held close ground on the double-log model. Also, the quadratic terms are significant in the initial model. This implies that a comparable version of Andrews' test for the quadratic versus the double-log form would yield even more conclusive indication of its validity.

For the fats and oils group of products, an unusual situation arises in which the inverse and double-log models are the frontrunners for being indicated as valid models. However, when these two models are tested against one another, the extended AN procedures split their vote across the two models. Since the P-test strongly rejected the inverse model when it was maintained, it would appear that the double-log provides the more appropriate model.

Turning to the expenditures on fruit, the oddity of having the inferior semilog model rejecting almost all of the viable alternatives was observed. In addition, if all the inferential evidence has beed compiled, there is only one possible model which could represent the valid functional form. Based on the information, although the double-log model is never maintained in the strict sense, it comes the closest to being maintained. Specifically, if the adjusted p-values as discussed by Good are utilized in this situation, the acceptable type I erorr for all the tests has been made smaller. Therefore, the power of these procedures has been reduced in a similar manner.

The grain commodity is fairly well behaved in terms of testing for model specification on the data set. When the quadratic and double-log are tested simultaneously, the P-test rejects the double-log model in favor of the quadratic model having more regressors whereas the extended JA-test will reject in favor of the double-log model with fewer regressors. Then in order to make a selection from these two, the strength with which they reject the alternative model is considered. When the magnitude of the procedures are examined, the double-log model earns the position of most appropriate functional form. In addition, the evidence drawn from the linear versus log-linear context also lends support to making the specific choice, paricularly since the INC^2 coefficient was deemed insignificant.

When considering the group of meat and meat alternates, the double-log model again appears to be the most appropriate functional specification, even though this result was arrived upon by a process of elimination. Problems still abound with the semilog model rejecting all alternatives. The cases of milk and milk equivalents, sugar and sweets, vegetables and other all lead to similar situations in which the double-log model comes out to be the most appropriate once the confusion has settled. This conclusion is drawn even though the double-log model was not capable of maintaining itself against all the other models in the non-nested setting. Also, there is nothing wrong with using supplemental information about the model to aid in the interpretation of the testing results in order to draw a sound inference concerning the models under consideration. Consequently, the most appropriate function form for the Engel Curve was selected for each commodity grouping based on the non-nested testing results.

5.7 Conclusions

Some interesting points have arisen in the context of this study. The first consideration, which was not necessarily a surprise, was to find how difficult it is to disentangle the results from various procedures, particularly when more than two models are being examined. This study evidenced such contrasting behavior several times over in the J and JA test procedures. Part of the difficulty stems from the apparent contradictions among the testing results. However, if the actual criteria used to evaluate the model specification is reviewed for each of the testing procedures, the contradictions can generally be sorted out and reexamined in a more helpful light. This illustration is a good example of the practical side of utilizing the non-nested testing procedures for determining the most appropriate functional specification.

Regarding the asymptotically valid tests, it seems rather obvious that the Linearized Cox (NL) test is not really worth utilizing in either the large or small sample cases. The remaining procedures, the Cox unadjusted test and the Atkinson AN-test appear to yield test statistics comparable to those of the AN and F procedures. Therefore, it seems safe to assume that the asymptotic properties of these procedures have been realized in this large sample setting.

In general, the weekly food expenditure study resulted in the selection of either the quadratic or double-log functional forms. Both of these models are direct realizations of competing economic theories for modelling consumer spending patterns. With regard to food expenditure data, in eight of the ten groups of commodities the double-log model was the preferred form of the general Engel curve. In other words, it was the functional form that the data best supported. Therefore, based on its estimation, information about the consumer's spending patterns, specifically income elasticity, is readily available to the economic researcher.

VI. Conclusions and Future Research

6.1 Summary

Throughout this discussion, much useful information has been compiled regarding the use of non-nested testing procedures to test functional form specification for linear regression models. First, extensive discussion of the approaches to testing hypotheses in the non-nested situation was presented. Through theoretical development and simulated work, comparisons of the ten more commonly used procedures for linear regression models were made. Incorporated in the comparison was a basic computational outline for generating the test statistics. In addition, a macro using PROC MATRIX of SAS was provided for this purpose.

Based on previous Monte Carlo work due to Godfrey and Pesaran, it was evident that the JA-test although exact under the maintained hypothesis was not the most powerful procedure by any standard. In particular, its very nature supports the tendency to select a model with fewer regressor variables. In order to take advantage of the test's strengths while improving its power, a modified version, the NJ-test, was proposed and its properties investigated. Although still a "conservative" testing procedure (i.e., unbiased under the maintained hypothesis), it provides larger power in cases where $k_1 > k_2$ and H_1 being the true model with only k_1 regressors. This result was confirmed theoretically as well as empirically by the simulation study. In addition, with a known, exact non-null distribution, power can be estimated for both this NJ-test and the Orthodox F-test prior to actually performing the tests. Since the NJ-test draws its error variance estimate from the comprehensive model approach, it serves as a compromise between the JA-test and the Orthodox F-test. In comparison to the F-test, it provides greater power as the collinearity between the competing models increases, particularly in cases where the quality of fit on the true model is high. Consequently, the modification to the JA-test is indeed simplistic but serves its purposes.

The main source of practical information was the Monte Carlo study. All ten testing procedures were included and their relative performance investigated. As an outgrowth of the investigation of the results which included cases where both models under test were invalid functional forms, some guidelines regarding the practical use of the $\check{N}$-, AJ-, F- and NJ-tests in situations with relatively small samples (20,40) were made. It was obvious that the asymptotically valid Cox procedures were too volatile in terms of significance levels to be useful in small sample applications. In addition, consideration of models with non-normal disturbances (both symmetric and skewed distributions) showed that all of the procedures were fairly robust to violations of assumptions of this type. For the model situations employed, only the presence of errors following the heavy-tailed t-distribution revealed any real reductions in power, although they were still not very dramatic.

In small sample applications, it is recommended to use the $\check{N}$ -test as a starting point. Then one or more of the conservative procedures - NJ, F, AJ - should be applied to see if there is evidence to support the inference of the $\check{N}$-test. In general the $\check{N}$-test has large power over a wider range of model conditions. Therefore, for a given situation (i.e., number of regressors in the models as well as the inference from the $\check{N}$-test) one of the three tests listed above should be more stringent in terms of rejecting the indicated hypothesis. Thus using the appropriate test forces the indicated model to supply even more evidence in its support. Following the guidelines set forth in Chapter IV should greatly improve the chances of drawing the correct conclusion from the non-nested tests.

Turning from the small-sample case to one with a large number of cross-sectional households providing observations, the study of food expenditure patterns provided a real data example in which non-nested procedures were used to determine the most appropriate functional form of the general Engel curve. The classical assumptions are not necessarily maintained (e.g., heteroscedastic variances). Just as in a consulting setting, the initial functional forms were investigated to see if there were problems with collinearity and/or heteroskedastic disturbances. Also, such a large sample of data provided an opportunity to employ the Cox procedures without small sample adjustments and thus provide a basis for comparisons with the other testing procedures. Briefly, the relation between the Box-Cox family of models and the more general non-nested models was drawn, even though estimation problems eliminated the Box-Cox procedures from practical use in this study.

Perhaps the most worthwhile part of the real data study was the chance it provided for interpreting the results of the testing procedures. In many cases, as in this one, the procedures can contradict each other. Therefore, how to utilize the testing results in order to draw the best inference regarding the validity of a model or set of models is very important. The key to making the best judgement about the results is having an understanding of the development of the tests and consequently their strengths and weaknesses.

6.2 Topics for Further Research

Much information has been obtained regarding the use of the non-nested testing procedures in general. Examination of the type of models under consideration has focused on the number of regressors (equal or unequal), the quality of fit, sample size and collinearity between competing models. A limited set of experimental runs employing non-normal disturbances showed much promise for the testing procedures to remain inferentiably sound. However, in the context of econometric applications which involve time series and/or cross-sectional data, there are other violations of the classical assumptions on the disturbances which arise. Often, the time series data have serial correlated erorrs which warrant corrective action in the estimation method to improve the quality of the resulting parameter estimates. As was noted in the demand study (Chapter V), cross-sectional data (household budget data) often violate the homogeneity of variance assumption.

There is no reason to suppose that such violations in the classical assumptions imply that the model is misspecified, or has an incorrect functional form. Consequently, though, since the model will be estimated using a more consistent procedure, the tests should be based on these "corrected" models. Thornton (1985) gives an example of test results both before and after estimation adjustments to correct for autocorrelated disturbance terms were made. However, there are many unanswered questions about when and how to make such adjustments for autocorrelated/heteroscedastic disturbances in the context of testing for correct functional form under the non-nested setting. It is important that one type of model misspecification not be masked by the detection and/or correction for another. (See Kennedy(1985) for an overview of the different types of model misspecification.) In other words, it is essential for the presence of heteroskedastic disturbance terms, which are not taken into account by the estimation technique, not to indicate a misspecified model by these procedures when indeed it is the appropriate functional form.

The results of the Monte Carlo study clearly indicate that the small-sample modifications to the various testing procedures are quite worthwhile in terms of improving the inferential ability of the test. Specifically, the $\bar{N}$ (and W, to some degree,) adjustment to the Cox test and the AJ adjustment to the J-test led to substantial improvement over the associated "parent" test in terms of power to make the correct inference. Although the JA-test is exact under the maintained hypothesis, it too was improved upon by utilizing the NJ modification, particularly in situations where the JA-test was lacking in power. Therefore, it seems that further developments into making such adjustments could prove useful.

Since the AN procedures are easier to compute than the Cox based procedures, even when considering small sample adjustments, investigation should center around the gereral AN familiy of procedures discussed by Pesaran(1982b). The properties of this family can be discussed in general terms so that small sample adjustments could be proposed and investigated for any of the family of procedures in general. Therefore, on the basis of this information, it could be determined which of the family of procedures are best in terms of making the correct inference for a given pair of models. Recall this family of procedures as defined in equation (2.48) where the consistent linear estimator $R\underline{y}$ was employed in the artificially nested model (2.41). In this context and given a particular adjustment, the choice of the R matrix can be made to achieve the "maximum" power. Consequently, small sample modifications to the AN procedures could prove useful in real data applications.

There are an abundance of specific areas of interest into which further work can be applied. From the econometric framework, these special cases would concern particular model structures such as qualitative choice models (using Logit/Tobit analysis), systems of nonlinear simultaneous equations and seemingly unrelated systems of equations (SUR). Situations such as these abound and are all of great importance to the researcher.

However, one broad topic where further research is warranted is the overlap between Box-Cox formulations and the general non-nested family of models. It was evident from the study in Chapter V that there are inherent problems with the maximum likelihood estimation in the Box-Cox model when the sample was large, and these patterns were not as severe in the non-nested setting. However, this empirical study does not provide any information about their relative behavior in small-samples. To fully examine their performance under the more common cross-over models which would be those with the log of the response as the dependent variable in the specified model, an extensive Monte-Carlo would be required and would provide an interesting project for future work.

In conclusion, this research has endeavored to provide meaningful information and practical guidelines for using non-nested testing procedures to test model specification under multiple alternatives. The information presented here goes a long way toward educating the researcher on the appropriate use of the non-nested methodology for linear regression models. However, there will always be additional factors which warrant investigation. Additional research in this area should pay dividends.

Bibliography

Aguirre-Torres, V., and Gallant, A. R.(1983). "The Null and Non-Null Asymptotic Distribution of the Cox Test for Multivariate Nonlinear Regression: Alternatives and a New Distribution-Free Cox Test," Journal of Econometrics, 21, 5-33.

Andrews, D. F.(1971). "A Note on the Selection of Data Transformations," Biometrika, 58, 247-254.

Atkinson, A. C.(1969). "A Test for Discriminating Between Models," Biometrika, 56(2), 337-347.

Atkinson, A. C.(1970). "A Method for Discriminating Between Models," Journal of the Royal Statistical Society, Series B, 32, 323-345.

Bera, A. K., and McAleer, M.(1983). "Some Exact Tests for Model Specification," Review of Economics and Statistics, 65(2), 351-354.

Box, G. E. P., and Cox, D. R.(1964). "An Analysis of Transformations," Journal of the Royal Statistical Society, Series B, 26, 211-243.

Cox, D. R.(1960). "Tests of Separate Families of Hypotheses," Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, University of California, Berkeley, 105-123.

Cox, D. R.(1962). "Further Results on Tests of Separate Families of Hypotheses," Journal of the Royal Statistical Society, Series B, 24, 406-424.

Das Gupta, S. and M. D. Perlman(1974). "Power of the Noncentral F-test: Effect of Additional Variates on Hotelling's T -Test," Journal of the American Statistical Association, 69 (345), 174-180.

Davidson, R., and MacKinnon, J. G.(1981a). "Several Tests for Model Specification in the Presence of Alternative Hypotheses," Econometrica, 49, 781-793.

Davidson, R., and MacKinnon, J. G.(1981b). "On a Simple Procedure for Testing Non-Nested Regression Models," Economics Letters, 5, 45-48.

Davidson, R., and MacKinnon, J. G.(1982). "Some Non-Nested Tests and the Relations Among Them," Review of Economic Studies, 49, 551-565.

Davidson, R., and MacKinnon, J. G.(1983a). "Testing the Specification of Multivariate Models in the Presence of Alternative Hypotheses," Journal of Econometrics, 23, 301-313.

Davidson, R., and MacKinnon, J. G.(1983b). "Small Sample Properties of Alternative Forms of the Lagrange Multiplier Test," Economics Letters, 12, 269-275.

Davidson, R., and MacKinnon, J. G.(1984). "Model Specification Tests Based on Artificial Linear Regressions," International Economic Review, 25(2), 485-502.

Epps, T. W., Singleton, K. J., and Pulley, L. B.(1982). "A Test of Separate Families of Distributions Based on the Empirical Moment Generating Function," Biometrika, 69, 391-399.

Fisher, G. R., and McAleer, M.(1979). "On the Interpretation of the Cox Test in Econometrics," Economics Letters, 4, 145-150.

Fisher, G. R., and McAleer, M.(1981). "Alternative Procedures and Associated Tests of Significance for Nonnested Hypotheses," Journal of Econometrics, 16, 103-119.

Fisher, G. R.(1983). "Tests for Two Separate Regressions," Journal of Econometrics, 21, 117-132.

Godfrey, L. G., and Pesaran, M. H.(1982). "Small Sample Adjustments for the J-Test," Discussion Paper No. 944, Harvard Institute for Economic Research, Harvard University, Cambridge.

Godfrey, L.G., and Wickens, M. R.(1982). "Tests of Misspecification Using Locally Equivalent Alternative Models," in G. C. Chow and P. Corsi, eds., Evaluating the Reliability of Macro-Economic Models, (Wiley, New York), 71-99.

Godfrey, L. G., and Pesaran, M. H.(1983). "Tests of Non-Nested Regression Models - Small Sample Adjustments and Monte-Carlo Experiments," Journal of Econometrics, 21, 133-154.

Good, I. J.(1982). "Standardized Tail-Area Probabilities,(C140)" Journal of Statistical Computation and Simulation, 16, 1 , 65-66.

Graybill, F. A., and G. A. Milliken(1969). "Quadratic Forms and Idempotent Matrices With Random Elements," The Annals of Mathematical Statistics, 40, 1430-38.

Judge, G. G., Griffiths, W. E., Hill, R. C., Lutkepohl, H., and Lee, T. C.(1985). The Theory and Practice of Econometrics, 2nd ed. (Wiley, New York), 881-895.

Kendall, M. G. and Babington-Smith, B.(1939). "The Problem of m Rankings," Annals of Mathematical Statistics, 10, 495-513.

Kennedy, P.(1985). "Violating Assumption One: Specification Errors," A Guide to Econometrics, 2nd ed. (The MIT Press, Cambridge), 67-89.

Loh, W. Y.(1985). "A New Method for Testing Separate Families of Hypotheses," Journal of the American Statistical Society, 80(390), 362-368.

MacKinnon, J. G.(1983a). "Model Specification Tests Against Non-Nested Alternatives," Econometric Review, 2(1), 85-110.

MacKinnon, J. G.(1983b). "Model Specification Tests Against Non-Nested Alternatives: Reply," Econometric Review, 2(1), 151-158.

McAleer, M.(1981). "A Small Sample Test for Non-Nested Regression Models," Economics Letters, 7, 335-338.

McAleer, M., and Fisher, G.(1982). "Testing Separate Regression Models Subject to Specification Error," Journal of Econometrics, 19(1), 125-145.

McAleer, M., Fisher, G., and Volker, P.(1982). "Separate Misspecified Regressions and the U.S. Long-run Demand for Money Function," The Review of Economics and Statistics, LXIV, 572-583.

McAleer, M.(1983). "Exact Tests of a Model Against Nonnested Alternatives," Biometrika, 70(1), 285-292.

McAleer, M., and Bera, A.(1983). "Model Specification Tests Against Non-Nested Alternatives: Comment," Econometric Review, 2(1), 121-130.

Milliken, G. A., and Graybill, F. A.(1970). "Extensions of the General Linear Hypothesis Model," Journal of the American Statistical Association, 65, 797-807.

Pereira, B. de B.(1977b). "A Note on the Consistency and the Finite Sample Comparisons of Some Tests of Separate Families of Hypotheses," Biometrika, 64, 109-113.

Pesaran, M. H.(1974). "On the General Problem of Model Selection," Review of Economic Studies, 41, 153-171.

Pesaran, M. H. and Deaton, A. S.(1978). "Testing Nonnested Nonlinear Regression Models," Econometrica, 46, 677-694.

Pesaran, M. H.(1981). "Pitfalls of Testing Non-Nested Hypotheses By the Lagrange Multiplier Method," Journal of Econometrics, 17, 323-331.

Pesaran, M. H.(1982a). "Comparison of Local Power of Alternative Tests of Nonnested Regression Models," Econometrica, 50, 1287-1306.

Pesaran, M. H.(1982b). "On the Comprehensive Method of Testing Non-Nested Regression Models," Journal of Econometrics, 18, 263-274.

Pesaran, M. H.(1982c). "A Critique of the Proposed Tests of the Natural Rate-Rational Expectations Hypothesis," The Economics Journal, 92, 529-554.

Pesaran, M. H.(1983). "Model Specification Tests Against Non-Nested Alternatives: Comment," Econometric Review,2(1), 145-149.

Pindyck, R. S., and Rubinfeld, D. L.(1981). Econometric Models and Economic Forecasts, 2nd ed. (McGraw-Hill Book Company, New York), 150-152.

Salathe, L.(1979). "A Comparison of Alternative Functional Forms for Estimating Household Engel Curves," Unpublished paper.

White, H.(1982). "Regularity Conditions for Cox's Test of Non-Nested Hypotheses," Journal of Econometrics, 19, 301-318.

Additional Reading

Non-Nested Hypothesis Testing Applications

Capps, Jr., O., and Kramer, R. A.(1985). "Analysis of Food Stamp Participation Using Qualitative Choice Models" American Journal of Agricultural Economics, 67(1), 49-59.

Davis, E. P.(1984). "The Consumption Function in Macroeconomic Models: A Comparative Study," Applied Economics, 16, 799-838.

Deaton, A. S.(1978). "Specification and Testing in Applied Demand Analysis," Economics Journal, 88(351), 524-536.

Deaton, A.(1982). "Model Selection Procedures, or Does the Consumption Function Exist?," in G. C. Chow and P. Corsi, eds., Evaluating the Reliability of Macro-Economic Models, (Wiley, New York), 43-69.

Godfrey, L. G.(1984). "On the Uses of Misspecification Checks and Tests of Non-Nested Hypotheses in Empirical Econometrics," Economics Journal (Supplement), 94, 69-81.

Gregory, A. W., and McAleer, M.(1983). "Testing Non-Nested Specifications of Money Demand for Canada," Canadian Journal of Economics, 16(4), 593-602.

Green, R. D., Pope, R. D., and Phipps, T. T.(1981). "Discriminating Among Alternative Habit Formation Schemes in Single-Equation Demand Models," Applied Economics, 13(3), 399-409.

Pope, R., Green, R., and Eales, J.(1979). "Testing for Homogeneity and Habit Formation in a Flexible Demand Specification of U. S. Meat Consumption," Working Paper #79-8, University of California, Davis, Dept. of Agricultural Economics.

Rossi, P. E.(1985). "Comparison of Alternative Functional Forms in Production," Journal of Econometrics, 30, 345-361.

Singleton, K. J.(1985). "Testing Specifications of Economic Agents' Intertemporal Optimum Problems in the Presence of Alternative Models," Journal of Econometrics, 30, 391-413.

Thornton, D. L.(1985). "The Appropriate Interest Rate and Scale Variable in Money Demand: Results From Non-Nested Tests," Applied Economics, 17, 735-744.

More General Functional Form Applications

Bender,B., Gronberg, T. J., and Hwang,H. S.(1980). "Choice of Functional Form and the Demand for Air Quality," Review of Economics and Statistics, 62(4), 638-643.

Benus, J., Kmenta, J., and Shapiro, H.(1976). "The Dynamics of Household Budget Allocation to Food Expenditures," The Review of Economics and Statistics, 58(2), 129-138.

Blaylock, J., and Green, R.(1980). "Analysis of Flexible Engel Functions" Agricultural Economic Research, 32(4), 12-20.

Chang, H.(1977). "Functional Forms and the Demand for Meat in the United States," Review of Economics and Statistics, 59, 355-359.

Chang, H.(1980). "Functional Forms and the Demand for Meat in the United States: A Comment," Review of Economics and Statistics, 62(1), 148-150.

Gemmill, G.(1980). "Using the Box-Cox Form for Estimating Demand: A Comment," Review of Economics and Statistics, 62(1), 147-148.

Gerking, S. D., and Boyes, W. J.(1980). "The Role of Functional Form in Estimating Elasticities of Housing Expenditures," Southern Economics Journal, 47(2), 287-302.

Ghosh, S., Gilbert, C. L., and Hallett, A. J. H.(1983). "Tests of Functional Form in Dynamic Econometric Models: Some Empirical Evidence," Empirical Economics, 8(2), 63-69.

Hassan, Z. A., and Johnson, S. R.(1979). "The Demand for Meat in Canada: An Application of the Transformation of the Variables," Canadian Journal of Agricultural Economics, 27(3), 1-12.

Huang, C. J., and Grawe, O. R.(1980). "Functional Forms and the Demand for Meat in the United States: A Comment," Review of Economics and Statistics, 62(1), 144-146.

Huang, C. L.(1979). "Estimating U. S. Demand for Meat With a Flexible Functional Form," Southern Journal of Agricultural Economics, 11(2), 17-20.

McGowan, I.(1983). "Choice of Functional Form in Economics," British Review of Economic Issues, 5(12), 32-45.

Savin, N. E., and White, K. J.(1978). "Estimation and Testing for Functional Form and Autocorrelation," Journal of Econometrics, 8, 1-12.

Smith, M. A., and Maddala, G. S.(1983). "Multiple Model Testing for Non-Nested Heteroskedastic Censored Regression Models," Journal of Econometrics, 21, 71-81.

Zarembka, P.(1968). "Functional Forms in the Demand for Money," Journal of the American Statistical Association, 63, 502-511.

Ziemer, R. F., Musser, W. N., and Hill, R. C.(1980). "Recreation Demand Equations: Functional Form and Consumer Surplus," American Journal of Agricultural Economics, 62(1), 136-141.

Appendix A: The Work of Graybill and Milliken

For the development of the distributional properties of the JA- and NJ- tests, it is necessary to employ the work of Graybill and Milliken (1969) regarding quadratic forms, $\underset{\sim}{y}'A\underset{\sim}{y}$, in which the matrix A contains random elements. In particular, Theorems 3.1 and 3.2 from their 1969 paper are directly applicable. Therefore, these theorems as well as the proof of the first, are presented here.

Theorem 3.1 (Graybill and Milliken, 1969, p. 1431)

Let the $n \times 1$ random vector $\underset{\sim}{y}$ be such that $y \sim N(\underset{\sim}{\mu}, I)$. Let K be any non-zero $r \times n$ matrix of constants of rank $k < n$; let L be any non-zero $n \times n$ matrix of constants such that the rows of L are in the orthogonal complement of the row space of K. Let A be an $n \times n$ matrix with elements a_{ij} where $a_{ij} = f_{ij}(K\underset{\sim}{y})$, and where $f_{ij}(\cdot)$ is a Borel function of the random vector $K\underset{\sim}{y}$. The random variable $w = \underset{\sim}{y}'A\underset{\sim}{y}$ is distributed as a noncentral chi-square if the following four conditions hold with probability one.

(1) $A = L'AL$;

(2) A is idempotent;

(3) $tr(A) = m$; m is a constant positive integer;

(4) $\underset{\sim}{\mu}'A\underset{\sim}{\mu} = \lambda$; λ is a constant.

PROOF: Define the random variable $\underset{\sim}{\mu}$ by

$$\underset{\sim}{\mu} = \begin{bmatrix} K \\ L \end{bmatrix} \underset{\sim}{y} = \begin{bmatrix} K\underset{\sim}{y} \\ L\underset{\sim}{y} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}.$$

Then $\underset{\sim}{\mu}_1 \sim N(K\underset{\sim}{\mu}, KK')$, $\underset{\sim}{\mu}_2 \sim N(L\underset{\sim}{\mu}, LL')$ and $\underset{\sim}{\mu}_1$ is independent of $\underset{\sim}{\mu}_2$ since $LK' = 0$ (ie. the rows of L are in the orthogonal complement of the row space of K). From condition (1) we obtain $w = \underset{\sim}{y}'A\underset{\sim}{y} = \underset{\sim}{y}'L'AL\underset{\sim}{y} = \underset{\sim}{\mu}_2'A\underset{\sim}{\mu}_2$. Since A depends only on the random vector $\underset{\sim}{\mu}_1$ and since $\underset{\sim}{\mu}_1$ and $\underset{\sim}{\mu}_2$ are independent, the distribution of the conditional random variable $w \mid \underset{\sim}{\mu}_1 = \underset{\sim}{\mu}_1{}^*$ is by Lemma

2.4[1] non-central chi-square with m degrees of freedom if conditions (2), (3) and (4) hold. But this distribution is the same for every allowable value of μ_1*, hence the marginal distribution of w is non-central chi-square with m degrees of freedom.

Theorem 3.2 (Graybill and Milliken, 1969, p. 1432)

Let $\underset{\sim}{y}$, K and L be defined as in Theorem 3.1. Let the elements of the $n \times n$ matrices A and B be Borel functions of the vector $K\underset{\sim}{y}$. The two random variable w_1 and w_2, where $w_1 = \underset{\sim}{y}'A\underset{\sim}{y}$ and $w_2 = \underset{\sim}{y}'B\underset{\sim}{y}$, are independent if the following nine conditions hold with probability one:

(1) $L'AL = A$;

(2) $L'BL = B$;

(3) $A = A^2$;

(4) $B = B^2$;

(5) $tr(A) = m_1$;

(6) $tr(B) = m_2$;

(7) $\underset{\sim}{\mu}'A\underset{\sim}{\mu} = \lambda_1$;

(8) $\underset{\sim}{\mu}'B\underset{\sim}{\mu} = \lambda_2$;

(9) $AB = 0$;

where m_1, m_2 are constant positive integers, λ_1 and λ_2 are constants.

---

[1] Lemma 2.4. Let $\underset{\sim}{y} \sim N(\underset{\sim}{\mu}, V)$, V is the $n \times n$ of rank k, and let A be an $n \times n$ matrix with constant elements. The quadratic form $\underset{\sim}{y}'A\underset{\sim}{y}$ is distributed as a non-central chi-square variable with m degrees of freedom if and only if $H'AH$ is idempotent and $tr(H'AH) = m$ where H is any $n \times k$ matrix of rank k such that $V = HH$. (The non-centrality parameter is $\frac{1}{2}\underset{\sim}{\mu}'A\underset{\sim}{\mu}$).

Appendix B: Model Characteristics Under the Monte-Carlo Design

B.1 Variance-Covariance Structure for X_1 and X_2

Under the Monte Carlo design, the true and alternative models are generated using N(0,1) deviates as the regressor variables in the manner discussed in Section 4.2.3. This construction, proposed by Godfrey and Pesaran (1982,1983), provides a method of controlling the amount of collinearity between the models. The regressors within the true (and alternative) model are generated identically and independently and those in the alternative model are generated so that the canonical correlation between the sets of regressors in H_1 and that in the alternative hypothesis (H_2 or H_3) can be controlled to be a particular value, say ρ^2.

In particular, for a model H_1 with k_1 regressors and a model H_2 with k_2 regressors ($k_0 = 0$, no overlapping variables or exact collinearities between models), the following variance-covariance matrices are obtained:

$$\Sigma_{11} = V(X_1) = \frac{1}{n}X'_1X_1 = I_{k_1}$$

$$\Sigma_{22} = V(X_2) = \frac{1}{n}X'_2X_2 = \begin{cases} \left[\begin{array}{c|c} \frac{1}{1-\rho^2}I_{k_1} & 0_{k_1 \times (k_2 - k_1)} \\ 0_{(k_2 - k_1)\times k_1} & I_{k_2 - k_1} \end{array}\right] & \text{if } k_2 > k_1 \\ \frac{1}{1-\rho^2}I_{k^2} & \text{if } k_2 \leq k_1 \end{cases}$$

with the covariance structure between the two sets of regressors given by:

$$\Sigma_{12} = C(X_1, X_2) = \frac{1}{n}X'_1X_2 = \begin{cases} \left[\dfrac{\rho}{(1-\rho^2)^{1/2}}I_{k_1} \mid 0_{k_1 \times (k_2 - k_1)}\right] & \text{if } k_2 > k_1 \\ \begin{bmatrix} \dfrac{\rho}{(1-\rho^2)^{1/2}}I_{k_2} \\ 0_{(k-k_2)\times k_2} \end{bmatrix} & \text{if } k_2 \le k_1 \end{cases}$$

and $\Sigma_{21} = \Sigma_{12}'$.

Consequently, using the structure on the regressor variables from each of the models, the squared canonical correlations between the sets of variables are the solutions to:

$$|\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} - \lambda\Sigma_{11}| = 0.$$

Making use of the above variance-covariance matrices,

$$\left| \begin{bmatrix} \rho^2 I_{k_s} & | & 0_{s\times(k_1-s)} \\ 0_{(k_1-s)\times s} & | & 0_{k_1-s} \end{bmatrix} - \lambda I_{k_1} \right| = 0.$$

where $s \equiv \min(k_1, k_2)$. From this formulation, it is clear that $\lambda = \rho^2$ are s of the k_1 solutions, with the remaining $(k_1 - s)$ solutions being 0. Therefore, by generating models as indicated in Section 4.2.3, equations (4.1)-(4.7), the collinearity structure between models can be controlled through the parameter ρ^2.

B.2 The Noncentrality Parameters for the NJ- and F-tests

In the derivations below, the variance-covariance form of the models will be used in the derivation of the noncentrality parameters. In place of X'_iX_j, Σ_{ij} for $i,j = 1,2$ will be used throughout this discussion.

B.2.1 Derivation of $\bar{\lambda}_{NJ} = \lambda_{NJ}|_{\underline{y} = E_2(\underline{y})}$

Recall the following expression for λ_{NJ}:

$$\lambda_{NJ} = \frac{1}{2\sigma_2^2}\frac{(\underline{\beta}'_2X'_2P_1P_2M_1X_2\underline{\beta}_2)^2}{\underline{\beta}'_{22}P_1P_2M_1P_2P_1X_2\underline{\beta}_2} = \frac{1}{2\sigma_2^2}\frac{A^2}{B}$$

where $A = \underline{\beta}'_2X'_2P_1X_2\underline{\beta}_2 - \underline{\beta}'_2X'_2P_1P_2P_1X_2\underline{\beta}_2$

and $B = \underline{\beta}'_2X'_2P_1P_2P_1X_2\underline{\beta}_2 - \underline{\beta}'_2X'_2P_1P_2P_1P_2P_1X_2\underline{\beta}_2$.

For simplification, the case in which $k_1 = k_2$ will be considered first. Under the equal k case ($k = k_1 = k_2$),

$$A = \underline{\beta}'_2[\frac{\rho}{(1-\rho^2)^{1/2}}I_k I_k^{-1}\frac{\rho}{(1-\rho^2)^{1/2}}I_k]\underline{\beta}_2$$
$$- \underline{\beta}'_2[\frac{\rho}{(1-\rho^2)^{1/2}}I_k I_k^{-1}\frac{\rho}{(1-\rho^2)^{1/2}}I_k(\frac{1}{1-\rho^2}I_k)^{-1}\frac{\rho}{(1-\rho^2)^{1/2}}I_k I_k^{-1}\frac{\rho}{(1-\rho^2)^{1/2}}I_k]\underline{\beta}_2$$

$$A = \underline{\beta}'_2[\frac{\rho^2}{1-\rho^2}I_k]\underline{\beta}'_2 - \underline{\beta}'_2[\frac{\rho^4}{1-\rho^2}I_k]\underline{\beta}_2 = \rho^2\sum_{j=1}^{k}\beta_{2j}^2$$

Similarly for the equal k case,

$$B = \underline{\beta}'_2X'_2P_1P_2P_1X_2\underline{\beta}_2 - \underline{\beta}'_2X'_2P_1P_2P_1P_2P_1X_2\underline{\beta}_2,$$

and using the derivations for A:

$$B = \underline{\beta}'_2[\frac{\rho^4}{1-\rho^2}I_k]\underline{\beta}_2 - \underline{\beta}'_2\{C_k I_k^{-1} \& C_k[\frac{1}{1-\rho^2}I_k]^{-1}C_k I_k^{-1}C_k[\frac{1}{1-\rho^2}I_k]^{-1}C_k I_k^{-1}C_k\}\underline{\beta}_2$$

where $C_k = \frac{\rho}{(1-\rho^2)^{1/2}}I_k$. Since all the matrices of interest are scalar multiples of the identity, again the result simplifies nicely to:

$$B = \underline{\beta}'_2[\frac{\rho^4 - \rho^6}{1 - \rho^2} I_k]\underline{\beta}_2 = \rho^4 \sum_{j=1}^{k} \beta_{2j}^2 .$$

Therefore, combining A and B together, in the equal k case,

$$\bar{\lambda}_{NJ} = \frac{1}{2\sigma_2^2} \frac{A^2}{B} = \frac{\sum_{j=1}^{k} \beta_{2j}^2}{2\sigma_2^2} .$$

The only difference in the formulation of $\bar{\lambda}_{NJ}$ when $k_1 \neq k_2$, is the form of Σ_{12} substituted in for X'_1X_2. Based on the expression for Σ_{12} given in B.1, two cases result. If $k_2 > k_1$, then only the $k_1 = s = \min(k_1, k_2)$ β_{2j}'s associated with the non-orthogonal pieces of the X'_1X_2 matrix are included in the sum. Similarly, if $k_2 < k_1$, then all $k_2 = s$ β_{2j}'s are included in the summation.

Therefore, in general,

$$\bar{\lambda}_{NJ} = \frac{\sum_{j=1}^{s} \beta_{2j}^2}{2\sigma_2^2}$$

where s is defined as indicated above.

B.2.2 Derivation of λ_F

The general form of the noncentrality parameter for the Orthodox F-test is:

$$\lambda_F = \frac{1}{2\sigma_2^2} \underline{\beta}'_2 X'_2 M_1 X_2 \underline{\beta}_2 = \frac{1}{2\sigma_2^2} \underline{\beta}'_2 [X'_2X_2 - X'_2X_1(X'_1X_1)^{-1}X'_1X_2]\underline{\beta}_2 .$$

Once again, to see the exact form of the noncentrality parameter, two cases are considered. First, assume that $k_2 \leq k_1$ and the appropriate formulation of Σ_{12}. In this case,

$$\lambda_F = \frac{1}{2\sigma_2^2}\underline{\beta}'_2\{\frac{1}{1-\rho^2}I_{k_2} - [\frac{\rho}{(1-\rho^2)^{1/2}}I_{k_2}|0_{k_2 \times (k_1 - k_2)}]\, I_{k_1}^{-1}[\frac{\rho}{(1-\rho^2)^{1/2}}I_{k_2}|0_{k_2 \times (k_1 - k_2)}]'\}\underline{\beta}_2$$

which simplifies to

$$\lambda_F = \frac{1}{2\sigma_2^2}\underline{\beta}'_2[\frac{1}{1-\rho^2}I_{k_2} - \frac{\rho^2}{1-\rho^2}I_{k_2}]\underline{\beta}_2 = \frac{1}{2\sigma_2^2}\sum_{j=1}^{k_2}\beta_{2j}^2 .$$

Then, for the case where $k_2 > k_1$, the formulation remains the same since the extra regressors in H_2 (i.e., $k_2 - k_1$) are independent of any of the k_1 regressors in H_1, as is reflected in both Σ_{22} and Σ_{12} for this case. In other words, by making the proper substitutions, the noncentrality parameter is given by:

$$\lambda_F = \frac{1}{2\sigma_2^2}\underline{\beta}'_2 \left[\begin{array}{c|c} \frac{1}{1-\rho^2}I_{k_1} & 0_{(k_2 - k_1)\times k_1} \\ 0_{k_1 \times (k_2 - k_1)} & I_{k_2 - k_1} \end{array}\right] - \left[\begin{array}{c|c} \frac{\rho^2}{1-\rho^2}I_{k_1} & 0_{(k_2 - k_1)\times k_1} \\ 0_{k_1 \times (k_2 - k_1)} & 0_{k_2 - k_1} \end{array}\right] \underline{\beta}_2$$

$$\lambda_F = \frac{1}{2\sigma_2^2}\underline{\beta}'_2\, I_{k_2}\, \underline{\beta}_2 .$$

It should be quite clear that the value of λ_F is based on the sum of all the β_{2j}'s, and not only on the s of the regressors which are correlated with one another as in the case of the NJ-test.

Appendix C: Example of Simulated Results for n = 40, 500 vs 1000 Replications

The purpose of this example is to illustrate that using only 500 replications within an experimental run of the Monte Carlo instead of 1000 does not reduce the accuracy and strength of the findings enough to warrant the extra cost of using 1000. This example is based on Experiment 27 of the normal distribution runs. For this case, $n = 40$, $R^2 = 0.90$, $\rho^2 = 0.50$ and $(k_1, k_2, k_3) = (2,4,6)$. Below are the observed power, significance level and average p-value rankings for comparison. Different seeds were used in each as the random start to generate the two samples.

	m=1000		H_1 vs H_2	m=500		
Test	$\dot{P}$	$\dot{\alpha}$	Avg. Rank	$\dot{P}$	$\dot{\alpha}$	Avg. Rank
N	0.939 (.00757)	0.061 (.00649)	2.5245	0.918 (.01228)	0.082 (.01228)	2.7450
W	0.956 (.00649)	0.044 (.00649)	7.3300	0.948 (.00994)	0.052 (.00944)	7.1450
$\tilde{N}$	0.958 (.00635)	0.042 (.00635)	2.3605	0.932 (.01127)	0.068 (.01127)	2.6230
NA	0.948 (.00702)	0.052 (.00702)	9.776	0.940 (.01063)	0.060 (.01063)	9.7280
NL	0.949 (.00696)	0.051 (.00696)	4.725	0.926 (.01172)	0.074 (.01172)	4.7950
J	0.965 (.00581)	0.035 (.00581)	4.8865	0.954 (.00938)	0.046 (.00938)	4.8370
AJ	0.985 (.00385)	0.015 (.00385)	6.116	0.978 (.00657)	0.022 (.00657)	6.3730
JA	0.980 (.00443)	0.020 (.00443)	6.468	0.978 (.00657)	0.022 (.00657)	6.3730
NJ	0.952 (.00676)	0.048 (.00676)	7.369	0.946 (.01012)	0.054 (.01012)	7.2840
F	0.966 (.00573)	0.034 (.00573)	3.4455	0.958 (.00898)	0.042 (.00898)	3.4540

Kendall's W 0.64164 0.586124

Appendix C: Example of Simulated Results for n = 40, 500 vs 1000 Replications(continued)

	m=1000		H_1 *vs* H_3	m=500		
Test	$\dot{P}$	$\dot{\alpha}$	Avg. Rank	$\dot{P}$	$\dot{\alpha}$	Avg. Rank
N	0.915 (.00882)	0.085 (.00882)	2.488	0.920 (.01214)	0.080 (.01214)	2.5060
W	0.940 (.00751)	0.060 (.00751)	7.7488	0.944 (.01029)	0.056 (.01029)	7.8150
$\tilde{N}$	0.938 (.00763)	0.062 (.00763)	2.273	0.942 (.01046)	0.058 (.01046)	2.3090
NA	0.933 (.00791)	0.067 (.00791)	9.647	0.940 (.01063)	0.060 (.01063)	9.6870
NL	0.921 (.00853)	0.079 (.00853)	4.741	0.928 (.01157)	0.072 (.01157)	4.9280
J	0.932 (.00796)	0.068 (.00796)	4.8755	0.942 (.01046)	0.058 (.01046)	4.7800
AJ	0.973 (.00513)	0.027 (.00513)	5.9095	0.974 (.00712)	0.026 (.00712)	5.8080
JA	0.967 (.00565)	0.033 (.00565)	6.1895	0.974 (.00712)	0.974 (.00712)	6.2310
NJ	0.940 (.00751)	0.060 (.00751)	7.4875	0.944 (.01029)	0.056 (.01029)	7.4740
F	0.944 (.00727)	0.056 (.00727)	3.6405	0.968 (.00788)	0.032 (.00788)	3.4620
Kendall's W		0.64704			0.656564	

It is clear that any differences between these two sets of runs are a result of the different generated samples and not primarily of the number of replications. Although this is but one example, it supports the choice of using only 500 replications in experiments involving samples of size 40.

Appendix D Non-Nested Macro and Monte Carlo Programs

Non-Nested Macro

```
//BO###NN JOB acct#,NNMACRO,TIME=1,REGION=1024K
/*LONGKEY #####
//STEP01 EXEC SAS
//SYSIN DD *
OPTIONS LS=80 NODATE;
PROC MATRIX;
 FETCH Y DATA=YDAT; FETCH X1 DATA=X1DAT; FETCH X2 DATA=X2DAT;
  N=NROW(Y); K1=NCOL(X1); K2=NCOL(X2);   IN=I(N);  R2=J(1,2);
* CREATE AND INITIALIZE MATRICES FOR TEST STAT VALUES;
 C=J(1,2);   W=J(1,2); N0=J(1,2);
JJ=J(1,2);  JA=J(1,2);  F=J(1,2); NJ=J(1,2);
NA=J(1,2);  NL=J(1,2); AJ=J(1,2);          IN=I(N);
X1P=X1';              X2P=X2';
X1PX1=X1'*X1;          X2PX2=X2'*X2;
XI1=SOLVE(X1PX1,X1P); XI2=SOLVE(X2PX2,X2P);
A1=X1*XI1;            A2=X2*XI2;
M1=IN-A1;             M2=IN-A2;
B1=XI1*Y;             B2=XI2*Y;
TRM12=TRACE(M1*M2);
TRA12=TRACE(A1*A2);  TRA122=TRACE(A1*A2*A1*A2);
 TRB12=K2 - TRA122 - ( K2 - TRA12 )**2 #/ (N-K1);
 TRB21=K1 - TRA122 - ( K1 - TRA12 )**2 #/ (N-K2);
```

```
SSY=Y'*Y - (Y(+,))**2#/N;
* OLS AND MLE ON SEPARATE MODELS;  R22=J(3,1,1);
YH1=X1*B1;                YH2=X2*B2;
E1=Y-YH1;                 E2=Y-YH2;
SSR1=E1'*E1;              SSR2=E2'*E2;
S1LS=SSR1#/(N-K1);        S2LS=SSR2#/(N-K2);
S1ML=SSR1#/N;             S2ML=SSR2#/N;
R2(,1)=1 - SSR1#/SSY;     R2(,2)=1 - SSR2#/SSY;
B1M2=B1'*X1'*M2*X1*B1;  B1MM2=B1'*X1'*M2*M1*M2*X1*B1;
B2M1=B2'*X2'*M1*X2*B2;  B2MM1=B2'*X2'*M1*M2*M1*X2*B2;
E21=M2*X1*B1;  E12=M1*X2*B2;
E211=M1*E21;   E122=M2*E12;
* COX TEST;
O21=(E21'*E21 + S1LS*TRM12) #/ (N-K2);
O12=(E12'*E12 + S2LS*TRM12) #/ (N-K1);
S12ML=S2ML+B2M1#/N;  S12LS=S12ML*N#/(N-K1);
S21ML=S1ML+B1M2#/N;  S21LS=S21ML*N#/(N-K2);
 C12N=(N#/2)*LOG(S2ML#/S21ML);
 C21N=(N#/2)*LOG(S1ML#/S12ML);
V12=SQRT(S1ML*B1MM2#/S21ML**2);
V21=SQRT(S2ML*B2MM1#/S12ML**2);
C(,1)=C12N#/V12;
C(,2)=C21N#/V21;
*W TEST;
W(,1)=(N-K2)*(S2LS-O21) #/SQRT(2*S1LS**2*TRB12 + 4*S1LS*E211'*E211);
W(,2)=(N-K1)*(S1LS-O12) #/SQRT(2*S2LS**2*TRB21 + 4*S2LS*E122'*E122);
*N0 TEST;
T012=0.5*(N-K2)*LOG(S2LS#/O21);  T021=0.5*(N-K1)*LOG(S1LS#/O12);
```

```
V012=(S1LS#/O21**2)*(E211'*E211 +0.5*S1LS*TRB12);
V021=(S2LS#/O12**2)*(E122'*E122 +0.5*S2LS*TRB21);
N0(,1)=T012#/SQRT(V012);    N0(,2)=T021#/SQRT(V021);
*ATKINSON'S TEST;
AD12=SQRT(S1ML*Y'*A1*A2*M1*A2*A1*Y);
AD21=SQRT(S2ML*Y'*A2*A1*M2*A1*A2*Y);
NA(,1)=-(Y'*M1*A2*A1*Y)#/AD12;
NA(,2)=-(Y'*M2*A1*A2*Y)#/AD21;
*LINEARIZED COX TEST---NL;
NL(,1)=.5*Y'*(A2-A1*A2*A1)*Y #/AD12;
NL(,2)=.5*Y'*(A1-A2*A1*A2)*Y #/AD21;
*J TEST;
X12=X1||YH2;  X12P=X12'; X12PX12=X12'*X12; XI12=SOLVE(X12PX12,X12P);
SSRJ1=Y'*(IN-X12*XI12)*Y;
    SJ1=SSRJ1#/(N-K1-1);
JJ(,1)=(B2'*X2'*M1*Y)#/SQRT(SJ1*B2M1);
X21=X2||YH1;  X21P=X21'; X21PX21=X21'*X21; XI21=SOLVE(X21PX21,X21P);
SSRJ2=Y'*(IN-X21*XI21)*Y;
    SJ2=SSRJ2#/(N-K2-1);
JJ(,2)=(B1'*X1'*M2*Y)#/SQRT(SJ2*B1M2);
*ADJUSTED J-TEST:::: AJ;
P12=(K2-TRA12)#/(N-K1); AY12=YH2-P12*E1;
P21=(K1-TRA12)#/(N-K2); AY21=YH1-P21*E2;
****CALCULATION OF SIG HAT FOR THE ADJUSTED J-TEST;
A12=X1||AY12; A21=X2||AY21;
A12P=A12'; A12PA12=A12'*A12; AI12=SOLVE(A12PA12,A12P);
A21P=A21'; A21PA21=A21'*A21; AI21=SOLVE(A21PA21,A21P);
SA12=Y'*(IN-A12*AI12)*Y #/ (N-K1-1);
```

```
SA21=Y'*(IN-A21*AI21)*Y #/ (N-K2-1);
AJ(,1)=E1'*AY12#/SQRT(SA12*AY12'*M1*AY12);
AJ(,2)=E2'*AY21#/SQRT(SA21*AY21'*M2*AY21);
*JA TEST;
 YH12=A2*A1*Y; YH21=A1*A2*Y;
N12=(Y'*M1*YH12)#/SQRT(YH12'*M1*YH12);
N21=(Y'*M2*YH21)#/SQRT(YH21'*M2*YH21);
*SIGS FOR JA-TEST;
XJ12=X1||YH12;  XJ21=X2||YH21;
XJ12P=XJ12'; XJ12P12=XJ12'*XJ12; XIJ12=SOLVE(XJ12P12,XJ12P);
XJ21P=XJ21'; XJ21P21=XJ21'*XJ21; XIJ21=SOLVE(XJ21P21,XJ21P);
SJA12=Y'*(IN-XJ12*XIJ12)*Y#/(N-K1-1);
SJA21=Y'*(IN-XJ21*XIJ21)*Y#/(N-K2-1);
JA(,1)=N12#/SQRT(SJA12);
JA(,2)=N21#/SQRT(SJA21);
*CLASSICAL F-TEST;
X12=X1||X2; X12P=X12'; X12PX12=X12'*X12; XI12=SOLVE(X12PX12,X12P);
 M12=IN-X12*XI12; SIG12=(Y'*M12*Y)#/(N-K1-K2);
 SSREG12=Y'*(IN-M12)*Y;
F(,1) =(SSREG12-B1'*X1'*Y)#/(SIG12*K2);
F(,2) =(SSREG12-B2'*X2'*Y)#/(SIG12*K1);
*NEW JA TEST;
NJ(,1)=N12#/SQRT(SIG12);
NJ(,2)=N21#/SQRT(SIG12);
*;
*CALCULATE P-VALUES ;
CP12= ( 1- PROBNORM(ABS(C(,1))))*2;
CP21= ( 1- PROBNORM(ABS(C(,2))))*2;
```

```
WP12= ( 1- PROBNORM(ABS(W(,1))))*2;
WP21= ( 1- PROBNORM(ABS(W(,2))))*2;
N0P12= ( 1- PROBNORM(ABS(N0(,1))))*2;
N0P21= ( 1- PROBNORM(ABS(N0(,2))))*2;
NAP12= ( 1- PROBNORM(ABS(NA(,1))))*2;
NAP21= ( 1- PROBNORM(ABS(NA(,2))))*2;
NLP12= ( 1- PROBNORM(ABS(NL(,1))))*2;
NLP21= ( 1- PROBNORM(ABS(NL(,2))))*2;
 JP12= ( 1 -PROBT(ABS(JJ(,1)),N-K1-1))*2;
 JP21= ( 1 -PROBT(ABS(JJ(,2)),N-K2-1))*2;
AJP12= ( 1 -PROBT(ABS(AJ(,1)),N-K1-1))*2;
AJP21= ( 1 -PROBT(ABS(AJ(,2)),N-K2-1))*2;
JAP12= ( 1 -PROBT(ABS(JA(,1)),N-K1-1))*2;
JAP21= ( 1 -PROBT(ABS(JA(,2)),N-K2-1))*2;
NJP12= ( 1 -PROBT(ABS(NJ(,1)),N-K1-K2))*2;
NJP21= ( 1 -PROBT(ABS(NJ(,2)),N-K2-K1))*2;
FP12=1 - PROBF( F(,1),K1,N-K1-K2);
FP21=1 - PROBF( F(,2),K2,N-K1-K2);
IF (CP12 LE 0.05) THEN DO;
   IF (CP21 LE 0.05) THEN CCODE=11;  ELSE CCODE=10; END;
 ELSE IF (CP21 LE 0.05) THEN CCODE=01; ELSE CCODE=00; END;
IF (WP12 LE 0.05) THEN DO;
   IF (WP21 LE 0.05) THEN WCODE=11;  ELSE WCODE=10; END;
 ELSE IF (WP21 LE 0.05) THEN WCODE=01; ELSE WCODE=00; END;
IF (N0P12 LE 0.05) THEN DO;
   IF (N0P21 LE 0.05) THEN N0CODE=11;  ELSE N0CODE=10; END;
 ELSE IF (N0P21 LE 0.05) THEN N0CODE=01; ELSE N0CODE=00; END;
IF (NAP12 LE 0.05) THEN DO;
```

```
   IF (NAP21 LE 0.05) THEN NACODE=11;  ELSE NACODE=10; END;
 ELSE IF (NAP21 LE 0.05) THEN NACODE=01; ELSE NACODE=00; END;
IF (NLP12 LE 0.05) THEN DO;
   IF (NLP21 LE 0.05) THEN NLCODE=11;  ELSE NLCODE=10; END;
 ELSE IF (NLP21 LE 0.05) THEN NLCODE=01; ELSE NLCODE=00; END;
IF (JP12 LE 0.05) THEN DO;
   IF (JP21 LE 0.05) THEN JCODE=11;  ELSE JCODE=10; END;
 ELSE IF (JP21 LE 0.05) THEN JCODE=01; ELSE JCODE=00; END;
IF (AJP12 LE 0.05) THEN DO;
   IF (AJP21 LE 0.05) THEN AJCODE=11;  ELSE AJCODE=10; END;
 ELSE IF (AJP21 LE 0.05) THEN AJCODE=01; ELSE AJCODE=00; END;
IF (JAP12 LE 0.05) THEN DO;
   IF (JAP21 LE 0.05) THEN JACODE=11;  ELSE JACODE=10; END;
 ELSE IF (JAP21 LE 0.05) THEN JACODE=01; ELSE JACODE=00; END;
IF (JAP12 LE 0.05) THEN DO;
   IF (JAP21 LE 0.05) THEN JACODE=11;  ELSE JACODE=10; END;
 ELSE IF (JAP21 LE 0.05) THEN JACODE=01; ELSE JACODE=00; END;
IF (NJP12 LE 0.05) THEN DO;
   IF (NJP21 LE 0.05) THEN NJCODE=11;  ELSE NJCODE=10; END;
 ELSE IF (NJP21 LE 0.05) THEN NJCODE=01; ELSE NJCODE=00; END;
IF (FP12 LE 0.05) THEN DO;
   IF (FP21 LE 0.05) THEN FCODE=11;  ELSE FCODE=10; END;
 ELSE IF (FP21 LE 0.05) THEN FCODE=01; ELSE FCODE=00; END;
PRINT CCODE WCODE N0CODE NACODE NLCODE JCODE AJCODE JACODE NJCODE
   FCODE;
RETURN;
```

End of Non-Nested Macro

Normal Deviate Case: Simulation Program

```
//BO###ND JOB acct#,NORMAL,TIME=11,REGION=3072K
/**TIME=11 FOR N=20,TIME=23 FOR N=40
/*LONGKEY #####
/*PRIORITY IDLE
/*JOBPARM LINES=5
//STEP1 EXEC FORTVC
//FORT.SYSIN DD *
C
C *** This program illustrates calling a FORTRAN Function from SAS.
C
      INTEGER FUNCTION MATSUB( NARG, ARGS )
      INTEGER*4 NARG
      INTEGER*4 ARGS( 1 )
      INTEGER*4 MIN, MAX, ROW, COL, ILOC, OLOC, NTOTAL
C     IARRAY is an input  array passed from SAS to FORTRAN.
C     OARRAY is an output array generated by FORTRAN and returned to SAS.
      REAL*8 IARRAY( 1 ), OARRAY( 1 )
C     The following Declarations are used in the implementation of
C     the IMSL Subroutine GGNML:
C        XX     is a single precision vector used to contain the values
C               generated by GGNML.  These values are then assigned to
C               output matrix OARRAY.
```

```
C          NOTE:  SAS programs expect passed arrays to be declared
C               as REAL*8 variables.  Although this program
C               links in the IMSL Double Precision library,
C               Subroutine GGNML returns Single Precision values.
C      DSEED  is a double precision number used as the seed for the
C          random number generator.
      REAL*4 XX(10000)
      DOUBLE PRECISION DSEED
      DATA DSEED/40687.D0/
C** TEST TO ENSURE THAT ONLY ONE ARGUMENT IS PASSED TO THIS PROCEDURE
      IF( NARG.NE.1 ) THEN
       MATSUB = 5
       RETURN
      ENDIF
C
C** TEST TO ENSURE THAT THE ONE ARGUMENT IS A MATRIX
C    (i.e. the input value is at least a 1 X 1 array)
      CALL ARG( ARGS(1), ROW, COL, ILOC, IARRAY)
      MIN = MIN0( ROW, COL)
      IF(MIN.LT.1) THEN
       MATSUB = 6
       RETURN
      ENDIF
C
C** DEFINE THE OUTPUT MATRIX
C
C-- Routine SETUP defines the output matrix and has the form:
C
```

```
C    CALL SETUP(IRES,NROWS,NCOLMS)
C
C       where  IRES   is a result number -- can use 1.
C              NROWS  is the number of rows in the output matrix.
C              NCOLMS is the number of columns in the output matrix.
C
C --  Use the following with ZRPOLY:
C        MAX  =  MAX0( ROW,COL)
C        CALL SETUP(1,MAX-1,2)
C --  Use the following with GGNML:
       CALL SETUP(1,ROW,COL)
C
C --  Subroutine ARG is used to get the dimensions and location of
C     the matrices according to their symbol table number IARG(I):
C
     CALL ARG( 1, ROW,COL, OLOC, OARRAY )
     IF( ROW.EQ. 0 .OR. COL.EQ.0 ) THEN
       MATSUB =  1
       RETURN
     ENDIF
C
C --- Call the desired IMSL Subroutine:
C
C -   GNML  is a Gaussian (Normal) random deviate generator:
C          XX is used as a temporary 'array' for storing  the
C             generated random deviates which are then placed
C             in array OARRAY which is passed from FORTRAN to SAS
C             (indexing into OARRAY starts at location OLOC).
```

```
    NTOTAL = ROW * COL
    CALL GGNML( DSEED, NTOTAL,  XX )
    IJ = 0
    DO 1000 I=1,ROW
    DO 1000 J=1,COL
      IJ = IJ + 1
1000   OARRAY(OLOC+IJ-1) = XX(IJ)
    RETURN
    END
/*
//* STEP0002 EXEC PGM=IEWL,PARM='MAP,LIST'
//STEP0002 EXEC PGM=IEWL
//SYSPRINT  DD SYSOUT=A
//SYSUT1    DD UNIT=SYSDA,SPACE=(TRK,(40,40))
//SYSLIB    DD DSN=SYS2.SAS.SUBLIB,DISP=SHR
//          DD DSN=SYS2.SAS.LIBRARY,DISP=SHR
//          DD DSN=SYS2.PLIBASE,DISP=SHR
//          DD DSN=SYS2.R3.VFORTLIB,DISP=SHR
//          DD DSN=VPI.IMSL.DP,DISP=SHR
//* In the lines which follow:
//* The SETSSI statement describes the characteristics of the input
//*     function.  The values in positions 3 and 4 specify the number
//*     of arguments passed to the function;  these should be equal.
//*     If all arguments are numeric, the last four digists are zero.
//*     For additional information regarding this statement, see:
//*     Technical Report:  P-139.  SAS Programmers Guide Version 5.
//* The NAME statement specifies the name used to call the function
//*     from within the SAS program.  The R designates that any previous
```

```
//*      function having this name will be replaced.
//SYSLIN    DD DSN=&&LOADSET,DISP=(OLD,DELETE,DELETE)
//         DD *
 INCLUDE SYSLIB(MATMAIN)
 ENTRY MATMAIN
 SETSSI BF110000
 NAME XXXXXX(R)
/*
//* IN THE
//SYSLMOD   DD DSN=&LIBRARY,DISP=(NEW,PASS,DELETE),UNIT=SYSDA,
//             SPACE=(CYL,(10,20,20),,CONTIG)
//STEP0003 EXEC SAS
//SYSIN DD *
OPTIONS  NODATE LS=80;
PROC MATRIX;   TITLE 'MONTE-CARLO FOR NORMAL DEVIATE CASE';
TITLE3 'EXPT ##';
  ************SET SIMULATION CONTROL VARIABLES*******************;
NITER=####;  N= ##; K1= #; K2= #;  K3= #; R21=0.##; P21=0.##;
*PRINT R21 P21 N NITER K1 K2 K3;
  PARMTRS=N||R21||P21||K1||K2||K3||NITER;   PRINT PARMTRS;
  ****************************************************************;
  *********** SET CRITICAL VALUES FOR TESTS ********************;
  ******* NN=1-4 FOR N=20, NN=5-8 FOR N=40  ********************;
  ******* 1,5 FOR 246, 2,6 FOR 426, 3,7 FOR 624, AND 4,8 FOR 444**;
NN=#;
  ******* SET UP CONSTANT VALUES AND CALCULATE MODEL CONTROLS ****;
IN=I(N);            VAR1=K1*(1-R21)#/R21;  LB=SQRT(P21#/(1-P21));
*PRINT VAR1 LB;
```

```
   BETA1=J(K1,1,1); BETA2=J(K2,1,1); BETA3=J(K3,1,1); Y=J(N,1,1);
******** SET UP CHECKING VECTORS FOR TEST RESULTS ***************;
R0=0 0;   R1=0 1; R2=1 0; R3=1 1;
*********CREATE AND INITIALIZE THE P-VALUE AND RANK MATRICES;
PV12=J(NITER,10,0); PV13=J(NITER,10,0);
RK12=J(NITER,10,0); RK13=J(NITER,10,0);
********** INITIALIZE COUNTERS FOR POWER AND TYPE 1 ERROR PROBABILITIES;
CPC12=0;  CEC12=0;   CPC13=0; CEC13=0;
CPW12=0;  CEW12=0;   CPW13=0; CEW13=0;
CPN12=0; CEN12=0;    CPN13=0; CEN13=0;  CPNJ12=0; CENJ12=0;
CPNA12=0; CENA12=0; CPNA13=0; CENA13=0;
CPNL12=0; CENL12=0; CPNL13=0; CENL13=0;
CPJ12=0; CEJ12=0;    CPJ13=0; CEJ13=0;  CPNJ13=0; CENJ13=0;
CPAJ12=0; CEAJ12=0; CPAJ13=0; CEAJ13=0;
CPJA12=0; CEJA12=0;  CPJA13=0; CEJA13=0;
CPF12=0; CEF12=0;    CPF13=0; CEF13=0;
*************VECTORS TO HOLD NCP VALUES: E FOR ESTIMATED;
NCPT=J(NITER,2,0);
NCPF=J(NITER,2,0);
******VECTORS TO HOLD POWER VALUES;
    PT=J(NITER,2,0); PF2=J(NITER,2,0);
 *****************************;
********INITIALIZE CRITICAL VALUE VECTORS;
*  F (J AND JA TOO) CRITICAL VALUES ARE ONLY APPROX FOR N=40;
VC95=J(1,6,1.9600);
VJ95=  2.110 2.110 2.131 2.131 2.160 2.160 /
    2.131 2.131 2.110 2.110 2.160 2.160 /
    2.160 2.160 2.110 2.110 2.131 2.131 /
```

```
     2.131 2.131 2.131 2.131 2.131 2.131 /
     2.042 2.042 2.042 2.042 2.042 2.042 /
     2.042 2.042 2.042 2.042 2.042 2.042 /
     2.042 2.042 2.042 2.042 2.042 2.042 /
     2.042 2.042 2.042 2.042 2.042 2.042;
VF95= 3.11 3.00 3.74 3.22 3.89 3.48 /
     3.74 3.22 3.11 3.00 3.48 3.89 /
     3.89 3.48 3.00 3.11 3.22 3.74 /
     3.26 3.26 3.26 3.26 3.26 3.26 /
     2.69 2.42 3.32 2.42 3.32 2.69 /
     3.32 2.42 2.69 2.42 2.69 3.32 /
     3.32 2.69 2.42 2.69 2.42 3.32 /
     2.69 2.69 2.69 2.69 2.69 2.69;
VNJ95= 2.145 2.179 2.145 2.228 2.179 2.228 /
     2.145 2.228 2.145 2.179 2.228 2.179 /
     2.179 2.228 2.179 2.145 2.228 2.145 /
     2.179 2.179 2.179 2.179 2.179 2.179 /
     2.042 2.042 2.042 2.042 2.042 2.042 /
     2.042 2.042 2.042 2.042 2.042 2.042 /
     2.042 2.042 2.042 2.042 2.042 2.042 /
     2.042 2.042 2.042 2.042 2.042 2.042;
****** CREATE AND INITIALIZE MATRICES FOR TEST STAT VALUES;
 C=J(NITER,6,0);     W=J(NITER,6,0);  N0=J(NITER,6,0);
JJ=J(NITER,6,0);     JA=J(NITER,6,0);  F=J(NITER,6,0); NJ=J(NITER,6,0);
NA=J(NITER,6,0);     NL=J(NITER,6,0);  AJ=J(NITER,6,0);
******* CREATE AND INITIALIZE COUNTER VECTORS FOR # OF SIG TEST STATS;
 CC95=J(1,6,0);   CW95=J(1,6,0);  CN95=J(1,6,0);
```

```
CJ95=J(1,6,0);   CJA95=J(1,6,0);  CF95=J(1,6,0);  CNJ95=J(1,6,0);
CNA95=J(1,6,0);   CNL95=J(1,6,0); CAJ95=J(1,6,0);
*********R2 COUNTER INITIALIZATIONS;
      SR2=J(3,1,0); SUSR2=J(3,1,0);
********SETUP FOR KENDALL'S COEF OF CONCORDANCE --ADJUST FOR TIES;
   TIECK=J(1,10,1);  SUMTIE12=0;  SUMTIE13=0;
*****INITIALIZE 2X2 COUNT MATRIX FOR TESTS OF MODEL 2 VS 3;
    CNT23=J(10,4,0);
*;
****** GENERATE MATRICES TO BE SENT TO IMSL FOR RANDOM NORMAL DEVIATES;
  X1H=J(N,K1,1); X2H=J(N,K2,1); X3H=J(N,K3,1); ERRH=J(N,1,1);
***************BEGINNING OF ITERATIVE LOOP*******************;
DO M=1 TO NITER;
***************************************************************;
********** GENERATE X VALUES AND ERROR TERMS AND Y***********;
    X1=XXXXXX(X1H); X2=XXXXXX(X2H); X3=XXXXXX(X3H);
  IF K2>=K1 THEN X2(,1:K1) = LB*X1 +X2(,1:K1);
   ELSE  X2= LB* X1(,1:K2) + X2;
  IF K3>=K1 THEN X3(,1:K1) = LB*X1 +X3(,1:K1);
   ELSE  X3= LB* X1(,1:K3) + X3;
 ERR=XXXXXX(ERRH);  ERR=SQRT(VAR1)*ERR;
   Y=X1*BETA1 +ERR;
************COMPUTE NECESSARY MODEL ESTIMATION PIECES*********;
X1P=X1';            X2P=X2';            X3P=X3';
X1PX1=X1'*X1;       X2PX2=X2'*X2;       X3PX3=X3'*X3;
XI1=SOLVE(X1PX1,X1P); XI2=SOLVE(X2PX2,X2P);  XI3=SOLVE(X3PX3,X3P);
A1=X1*XI1;          A2=X2*XI2;          A3=X3*XI3;
M1=IN-A1;           M2=IN-A2;           M3=IN-A3;
```

```
B1=XI1*Y;        B2=XI2*Y;        B3=XI3*Y;
TRM12=TRACE(M1*M2);
TRM13=TRACE(M1*M3);
TRM23=TRACE(M2*M3);
TRA12=TRACE(A1*A2);  TRA122=TRACE(A1*A2*A1*A2);
TRA13=TRACE(A1*A3);  TRA132=TRACE(A1*A3*A1*A3);
TRA23=TRACE(A2*A3);  TRA232=TRACE(A2*A3*A2*A3);
 TRB12=K2 - TRA122 - ( K2 - TRA12 )**2 #/ (N-K1);
 TRB21=K1 - TRA122 - ( K1 - TRA12 )**2 #/ (N-K2);
 TRB13=K3 - TRA132 - ( K3 - TRA13 )**2 #/ (N-K1);
 TRB31=K1 - TRA132 - ( K1 - TRA13 )**2 #/ (N-K3);
 TRB23=K3 - TRA232 - ( K3 - TRA23 )**2 #/ (N-K2);
 TRB32=K2 - TRA232 - ( K2 - TRA23 )**2 #/ (N-K3);
  SSY=Y'*Y - (Y(+,))**2#/N;
* OLS AND MLE ON SEPARATE MODELS;  R22=J(3,1,1);
YH1=X1*B1;           YH2=X2*B2;
E1=Y-YH1;            E2=Y-YH2;
SSR1=E1'*E1;         SSR2=E2'*E2;
S1LS=SSR1#/(N-K1);   S2LS=SSR2#/(N-K2);
S1ML=SSR1#/N;        S2ML=SSR2#/N;
R22(1,)=1 - SSR1#/SSY;   R22(2,)=1 - SSR2#/SSY;
YH3=X3*B3;
E3=Y-YH3;
SSR3=E3'*E3;
S3LS=SSR3#/(N-K3);
S3ML=SSR3#/N;
R22(3,)=1 - SSR3#/SSY;
```

```
*******************************************************************;
****************COUNT UPDATES FOR R2;
 DO I=1 TO 3;  SR2(I,)=SR2(I,) + R22(I,);
   SUSR2(I,)=SUSR2(I,)+ R22(I,)**2; END;
************************* E(YHJ) UNDER HI *********************;
 YH12=A2*A1*Y; YH21=A1*A2*Y; YH13=A3*A1*Y; YH31=A1*A3*Y;
 YH23=A3*A2*Y; YH32=A2*A3*Y;
*******************************************************************;
E21=M2*YH1;  E12=M1*YH2;
E31=M3*YH1;  E13=M1*YH3;
E32=M3*YH2;  E23=M2*YH3;
E211=M1*E21;   E122=M2*E12;
E311=M1*E31;   E133=M3*E13;
E322=M2*E32;   E233=M3*E23;
 B1M2=E21'*E21; B1MM2=E211'*E211;
 B1M3=E31'*E31; B1MM3=E311'*E311;
 B3M2=E23'*E23; B3MM2=E233'*E233;
 B3M1=E13'*E13; B3MM1=E311'*E311;
 B2M1=E12'*E12; B2MM1=E122'*E122;
 B2M3=E32'*E32; B2MM3=E322'*E322;
*******************************************************;
********CALCULATION OF TEST STATISTICS*****************;
*******************************************************;
***************** COX TEST****************************;
*******************************************************;
O21=(B1M2 + S1LS*TRM12) #/ (N-K2);
O31=(B1M3 + S1LS*TRM13) #/ (N-K3);
O32=(B2M3 + S2LS*TRM23) #/ (N-K3);
```

```
O12=(B2M1 + S2LS*TRM12) #/ (N-K1);
O13=(B3M1 + S3LS*TRM13) #/ (N-K1);
O23=(B3M2 + S3LS*TRM23) #/ (N-K2);
S12ML=S2ML+B2M1#/N;  S12LS=S12ML*N#/(N-K1);
S13ML=S3ML+B3M1#/N;  S13LS=S13ML*N#/(N-K1);
S23ML=S3ML+B3M2#/N;  S23LS=S23ML*N#/(N-K2);
S21ML=S1ML+B1M2#/N;  S21LS=S21ML*N#/(N-K2);
S31ML=S1ML+B1M3#/N;  S31LS=S31ML*N#/(N-K3);
S32ML=S2ML+B2M3#/N;  S32LS=S32ML*N#/(N-K3);
 C12N=(N#/2)*LOG(S2ML#/S21ML);
 C13N=(N#/2)*LOG(S3ML#/S31ML);
 C23N=(N#/2)*LOG(S3ML#/S32ML);
 C21N=(N#/2)*LOG(S1ML#/S12ML);
 C31N=(N#/2)*LOG(S1ML#/S13ML);
 C32N=(N#/2)*LOG(S2ML#/S23ML);
V12=SQRT(S1ML*B1MM2#/S21ML**2);
V13=SQRT(S1ML*B1MM3#/S31ML**2);
V23=SQRT(S2ML*B2MM3#/S32ML**2);
V21=SQRT(S2ML*B2MM1#/S12ML**2);
V31=SQRT(S3ML*B3MM1#/S13ML**2);
V32=SQRT(S3ML*B3MM2#/S23ML**2);
C(M,1)=C12N#/V12;
C(M,2)=C13N#/V13;
C(M,4)=C23N#/V23;
C(M,3)=C21N#/V21;
C(M,5)=C31N#/V31;
C(M,6)=C32N#/V32;
  **********COMPARE TO CRITICAL VALUES;
```

```
CH95=ABS(C(M,))>=VC95;
CC95=CC95+CH95;
* POWER AND TYPE 1 ERROR COUNTS;
CH9513=CH95(,1 3); CH9525=CH95(,2 5);
IF ALL(CH9513 = R1) THEN CPC12=CPC12+1;
ELSE IF ALL(CH9513=R2) OR ALL(CH9513=R3) THEN CEC12=CEC12+1;
IF ALL(CH9525 = R1) THEN CPC13=CPC13+1;
ELSE IF ALL(CH9525=R2) OR ALL(CH9525=R3) THEN CEC13=CEC13+1;
*COMPARE AND COUNT FOR 2 VS 3;
IF (CH95(,4 6)=R3) THEN CNT23(1,1)=CNT23(1,1)+1;
ELSE IF (CH95(,4 6)=R0) THEN CNT23(1,2)=CNT23(1,2)+1;
ELSE IF (CH95(,4 6)=R1) THEN CNT23(1,3)=CNT23(1,3)+1;
ELSE CNT23(1,4)=CNT23(1,4)+1;
******************W TEST***************************************;
**************************************************************;
W(M,1)=(N-K2)*(S2LS-O21) #/SQRT(2*S1LS**2*TRB12 + 4*S1LS*B1MM2);
W(M,2)=(N-K3)*(S3LS-O31) #/SQRT(2*S1LS**2*TRB13 + 4*S1LS*B1MM3);
W(M,4)=(N-K3)*(S3LS-O32) #/SQRT(2*S2LS**2*TRB23 + 4*S2LS*B2MM3);
W(M,3)=(N-K1)*(S1LS-O12) #/SQRT(2*S2LS**2*TRB21 + 4*S2LS*B2MM1);
W(M,5)=(N-K1)*(S1LS-O13) #/SQRT(2*S3LS**2*TRB31 + 4*S3LS*B3MM1);
W(M,6)=(N-K2)*(S2LS-O23) #/SQRT(2*S3LS**2*TRB32 + 4*S3LS*B3MM2);
*************COMPARE TO CRITICAL VALUES;
WH95=ABS(W(M,))>=VC95;
CW95=CW95+WH95;
* POWER COUNTS;
IF ALL(WH95(,1 3) = R1) THEN CPW12=CPW12+1;
ELSE IF ALL(WH95(,1 3)=R2) OR ALL(WH95(,1 3)=R3)
THEN CEW12=CEW12+1;
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```
IF ALL(WH95(,2 5) = R1) THEN CPW13=CPW13+1;
ELSE IF ALL(WH95(,2 5)=R2) OR ALL(WH95(,2 5)=R3)
 THEN CEW13=CEW13+1;
*COMPARE AND COUNT FOR 2 VS 3;
IF (WH95(,4 6)=R3) THEN CNT23(2,1)=CNT23(2,1)+1;
ELSE IF (WH95(,4 6)=R0) THEN CNT23(2,2)=CNT23(2,2)+1;
ELSE IF (WH95(,4 6)=R1) THEN CNT23(2,3)=CNT23(2,3)+1;
ELSE CNT23(2,4)=CNT23(2,4)+1;
******************N0 TEST**********************************;
**********************************************************;
T012=0.5*(N-K2)*LOG(S2LS#/O21);  T021=0.5*(N-K1)*LOG(S1LS#/O12);
T013=0.5*(N-K3)*LOG(S3LS#/O31);  T031=0.5*(N-K1)*LOG(S1LS#/O13);
T023=0.5*(N-K3)*LOG(S3LS#/O32);  T032=0.5*(N-K2)*LOG(S2LS#/O23);
V012=(S1LS#/O21**2)*(B1MM2 +0.5*S1LS*TRB12);
V013=(S1LS#/O31**2)*(B1MM3 +0.5*S1LS*TRB13);
V023=(S2LS#/O32**2)*(B2MM3 +0.5*S2LS*TRB23);
V021=(S2LS#/O12**2)*(B2MM1 +0.5*S2LS*TRB21);
V031=(S3LS#/O13**2)*(B3MM1 +0.5*S3LS*TRB31);
V032=(S3LS#/O23**2)*(B3MM2 +0.5*S3LS*TRB32);
N0(M,1)=T012#/SQRT(V012);    N0(M,3)=T021#/SQRT(V021);
N0(M,2)=T013#/SQRT(V013);    N0(M,5)=T031#/SQRT(V031);
N0(M,4)=T023#/SQRT(V023);    N0(M,6)=T032#/SQRT(V032);
 *****COMPARE TO CRITICAL VALUES;
NH95=ABS(N0(M,))>=VC95;
CN95=CN95+NH95;
*POWER COUNTS;
IF ALL(NH95(,1 3)=R1) THEN CPN12=CPN12+1;
ELSE IF ALL(NH95(,1 3)=R2) OR ALL(NH95(,1 3)=R3)
```

```
 THEN CEN12=CEN12+1;
IF ALL(NH95(,2 5)=R1) THEN CPN13=CPN13+1;
ELSE IF ALL(NH95(,2 5)=R2) OR ALL(NH95(,2 5)=R3)
 THEN CEN13=CEN13+1;
*COMPARE AND COUNT FOR 2 VS 3;
IF (NH95(,4 6)=R3) THEN CNT23(3,1)=CNT23(3,1)+1;
ELSE IF (NH95(,4 6)=R0) THEN CNT23(3,2)=CNT23(3,2)+1;
ELSE IF (NH95(,4 6)=R1) THEN CNT23(3,3)=CNT23(3,3)+1;
ELSE CNT23(3,4)=CNT23(3,4)+1;
*************** *ATKINSON'S TEST************************;
*******************************************************;
AD12=SQRT(S1ML*YH12'*M1*YH12);
AD21=SQRT(S2ML*YH21'*M2*YH21);
AD13=SQRT(S1ML*YH13'*M1*YH13);
AD31=SQRT(S3ML*YH31'*M3*YH31);
AD23=SQRT(S2ML*YH23'*M2*YH23);
AD32=SQRT(S3ML*YH32'*M3*YH32);
NA(M,1)=-(E1'*YH12)#/AD12;
NA(M,3)=-(E2'*YH21)#/AD21;
NA(M,2)=-(E1'*YH13)#/AD13;
NA(M,5)=-(E3'*YH31)#/AD31;
NA(M,4)=-(E2'*YH23)#/AD23;
NA(M,6)=-(E3'*YH32)#/AD32;
 *COMPARE TO CRITICAL VALUES;
 NAH95=ABS(NA(M,))> =VC95;
 CNA95=CNA95+NAH95;
 *POWER COUNTS;
 IF ALL(NAH95(,1 3)=R1) THEN CPNA12=CPNA12+1;
```

```
ELSE IF ALL(NAH95(,1 3)=R2) OR ALL(NAH95(,1 3)=R3)
 THEN CENA12=CENA12+1;
IF ALL(NAH95(,2 5)=R1) THEN CPNA13=CPNA13+1;
 ELSE IF ALL(NAH95(,2 5)=R2) OR ALL(NAH95(,2 5)=R3)
 THEN CENA13=CENA13+1;
 *COMPARE AND COUNT FOR 2 VS 3;
 IF (NAH95(,4 6)=R3) THEN CNT23(4,1)=CNT23(4,1)+1;
 ELSE IF (NAH95(,4 6)=R0) THEN CNT23(4,2)=CNT23(4,2)+1;
 ELSE IF (NAH95(,4 6)=R1) THEN CNT23(4,3)=CNT23(4,3)+1;
 ELSE CNT23(4,4)=CNT23(4,4)+1;
***************LINEARIZED COX TEST---NL**********************;
**************************************************************;
NL(M,1)=0.5*(YH2'*YH2 - YH12'*YH12) #/AD12;
NL(M,3)=0.5*(YH1'*YH1 - YH21'*YH21) #/AD21;
NL(M,2)=0.5*(YH3'*YH3 - YH13'*YH13) #/AD13;
NL(M,5)=0.5*(YH1'*YH1 - YH31'*YH31) #/AD31;
NL(M,4)=0.5*(YH3'*YH3 - YH23'*YH23) #/AD23;
NL(M,6)=0.5*(YH2'*YH2 - YH32'*YH32) #/AD32;
 *COMPARE TO CRITICAL VALUES;
 NLH95=ABS(NL(M,))>=VC95;
 CNL95=CNL95+NLH95;
 *POWER COUNTS;
 IF ALL(NLH95(,1 3)=R1) THEN CPNL12=CPNL12+1;
 ELSE IF ALL(NLH95(,1 3)=R2) OR ALL(NLH95(,1 3)=R3)
  THEN CENL12=CENL12+1;
 IF ALL(NLH95(,2 5)=R1) THEN CPNL13=CPNL13+1;
 ELSE IF ALL(NLH95(,2 5)=R2) OR ALL(NLH95(,2 5)=R3)
  THEN CENL13=CENL13+1;
```

```
*COMPARE AND COUNT FOR 2 VS 3;
IF (NLH95(,4 6)=R3) THEN CNT23(5,1)=CNT23(5,1)+1;
 ELSE IF (NLH95(,4 6)=R0) THEN CNT23(5,2)=CNT23(5,2)+1;
  ELSE IF (NLH95(,4 6)=R1) THEN CNT23(5,3)=CNT23(5,3)+1;
   ELSE CNT23(5,4)=CNT23(5,4)+1;
*******************J TEST************************************;
*************************************************************;
X12=X1||YH2;  X12P=X12′; X12PX12=X12′*X12; XI12=SOLVE(X12PX12,X12P);
SSRJ1=Y′*(IN-X12*XI12)*Y;
   SJ1=SSRJ1#/(N-K1-1);
JJ(M,1)=(B2′*X2′*M1*Y)#/SQRT(SJ1*B2M1);
X21=X2||YH1;  X21P=X21′; X21PX21=X21′*X21; XI21=SOLVE(X21PX21,X21P);
SSRJ2=Y′*(IN-X21*XI21)*Y;
   SJ2=SSRJ2#/(N-K2-1);
JJ(M,3)=(B1′*X1′*M2*Y)#/SQRT(SJ2*B1M2);
X13=X1||YH3;  X13P=X13′; X13PX13=X13′*X13; XI13=SOLVE(X13PX13,X13P);
SSRJ1=Y′*(IN-X13*XI13)*Y;
   SJ1=SSRJ1#/(N-K1-1);
JJ(M,2)=(B3′*X3′*M1*Y)#/SQRT(SJ1*B3M1);
X31=X3||YH1;  X31P=X31′; X31PX31=X31′*X31; XI31=SOLVE(X31PX31,X31P);
SSRJ3=Y′*(IN-X31*XI31)*Y;
   SJ3=SSRJ3#/(N-K3-1);
JJ(M,5)=(B1′*X1′*M3*Y)#/SQRT(SJ3*B1M3);
X23=X2||YH3;  X23P=X23′; X23PX23=X23′*X23; XI23=SOLVE(X23PX23,X23P);
SSRJ2=Y′*(IN-X23*XI23)*Y;
   SJ2=SSRJ2#/(N-K2-1);
JJ(M,4)=(B3′*X3′*M2*Y)#/SQRT(SJ2*B3M2);
```

```
X32=X3||YH2;  X32P=X32'; X32PX32=X32'*X32; XI32=SOLVE(X32PX32,X32P);
SSRJ3=Y'*(IN-X32*XI32)*Y;
   SJ3=SSRJ3#/(N-K3-1);
JJ(M,6)=(B2'*X2'*M3*Y)#/SQRT(SJ3*B2M3);
    *COMPARE TO CRITICAL VALUES;
    JH95=JJ(M,)>=VJ95(NN,);
    CJ95=CJ95+JH95;
    *COUNT FOR POWER AND TYPE1 ERROR;
    IF ALL(JH95(,1 3)=R1) THEN CPJ12=CPJ12+1;
     ELSE IF ALL(JH95(,1 3)=R2) OR ALL(JH95(,1 3)=R3)
                      THEN CEJ12=CEJ12+1;
    IF ALL(JH95(,2 5)=R1) THEN CPJ13=CPJ13+1;
     ELSE IF ALL(JH95(,2 5)=R2) OR ALL(JH95(,2 5)=R3)
                    THEN CEJ13=CEJ13+1;
    *COMPARE AND COUNT FOR 2 VS 3;
    IF (JH95(,4 6)=R3) THEN CNT23(6,1)=CNT23(6,1)+1;
     ELSE IF (JH95(,4 6)=R0) THEN CNT23(6,2)=CNT23(6,2)+1;
      ELSE IF (JH95(,4 6)=R1) THEN CNT23(6,3)=CNT23(6,3)+1;
       ELSE CNT23(6,4)=CNT23(6,4)+1;
****************ADJUSTED J-TEST:::: AJ********************;
*******************************************************;
P12=(K2-TRA12)#/(N-K1); AY12=YH2-P12*E1;
P21=(K1-TRA12)#/(N-K2); AY21=YH1-P21*E2;
P13=(K3-TRA13)#/(N-K1); AY13=YH3-P13*E1;
P31=(K1-TRA13)#/(N-K3); AY31=YH1-P31*E3;
P23=(K3-TRA23)#/(N-K2); AY23=YH3-P23*E2;
P32=(K2-TRA23)#/(N-K3); AY32=YH2-P32*E3;
****CALCULATION OF SIG HAT FOR THE ADJUSTED J-TEST;
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A12=X1||AY12; A21=X2||AY21; A13=X1||AY13; A31=X3||AY31;
A23=X2||AY23; A32=X3||AY32;
A12P=A12'; A12PA12=A12'*A12; AI12=SOLVE(A12PA12,A12P);
A21P=A21'; A21PA21=A21'*A21; AI21=SOLVE(A21PA21,A21P);
A13P=A13'; A13PA13=A13'*A13; AI13=SOLVE(A13PA13,A13P);
A31P=A31'; A31PA31=A31'*A31; AI31=SOLVE(A31PA31,A31P);
A23P=A23'; A23PA23=A23'*A23; AI23=SOLVE(A23PA23,A23P);
A32P=A32'; A32PA32=A32'*A32; AI32=SOLVE(A32PA32,A32P);
SA12=Y'*(IN-A12*AI12)*Y #/ (N-K1-1);
SA21=Y'*(IN-A21*AI21)*Y #/ (N-K2-1);
SA13=Y'*(IN-A13*AI13)*Y #/ (N-K1-1);
SA31=Y'*(IN-A31*AI31)*Y #/ (N-K3-1);
SA23=Y'*(IN-A23*AI23)*Y #/ (N-K2-1);
SA32=Y'*(IN-A32*AI32)*Y #/ (N-K3-1);
AJ(M,1)=E1'*AY12#/SQRT(SA12*AY12'*M1*AY12);
AJ(M,3)=E2'*AY21#/SQRT(SA21*AY21'*M2*AY21);
AJ(M,2)=E1'*AY13#/SQRT(SA13*AY13'*M1*AY13);
AJ(M,5)=E3'*AY31#/SQRT(SA31*AY31'*M3*AY31);
AJ(M,4)=E2'*AY23#/SQRT(SA23*AY23'*M2*AY23);
AJ(M,6)=E3'*AY32#/SQRT(SA32*AY32'*M3*AY32);
    *COMPARE TO CRITICAL VALUES;
    AJH95=AJ(M,)>=VJ95(NN,);
    CAJ95=CAJ95+AJH95;
    *COUNT FOR POWER AND TYPE1 ERROR;
    IF ALL(AJH95(,1 3)=R1) THEN CPAJ12=CPAJ12+1;
     ELSE IF ALL(AJH95(,1 3)=R2) OR ALL(AJH95(,1 3)=R3)
                 THEN CEAJ12=CEAJ12+1;
   IF ALL(AJH95(,2 5)=R1) THEN CPAJ13=CPAJ13+1;
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```
    ELSE IF ALL(AJH95(,2 5) = R2) OR ALL(AJH95(,2 5) = R3)
                    THEN CEAJ13 = CEAJ13 + 1;
    *COMPARE AND COUNT FOR 2 VS 3;
   IF (AJH95(,4 6) = R3) THEN CNT23(7,1) = CNT23(7,1) + 1;
    ELSE IF (AJH95(,4 6) = R0) THEN CNT23(7,2) = CNT23(7,2) + 1;
      ELSE IF (AJH95(,4 6) = R1) THEN CNT23(7,3) = CNT23(7,3) + 1;
       ELSE CNT23(7,4) = CNT23(7,4) + 1;
********************JA TEST*******************************;
**********************************************************;
N12 = (Y'*M1*YH12)#/SQRT(YH12'*M1*YH12);
N13 = (Y'*M1*YH13)#/SQRT(YH13'*M1*YH13);
N32 = (Y'*M3*YH32)#/SQRT(YH32'*M3*YH32);
N23 = (Y'*M2*YH23)#/SQRT(YH23'*M2*YH23);
N21 = (Y'*M2*YH21)#/SQRT(YH21'*M2*YH21);
N31 = (Y'*M3*YH31)#/SQRT(YH31'*M3*YH31);
*SIGS FOR JA-TEST;
XJ12 = X1||YH12;  XJ21 = X2||YH21;
XJ13 = X1||YH13;  XJ31 = X3||YH31;
XJ23 = X2||YH23;  XJ32 = X3||YH32;
XJ12P = XJ12'; XJ12P12 = XJ12'*XJ12; XIJ12 = SOLVE(XJ12P12,XJ12P);
XJ21P = XJ21'; XJ21P21 = XJ21'*XJ21; XIJ21 = SOLVE(XJ21P21,XJ21P);
XJ13P = XJ13'; XJ13P13 = XJ13'*XJ13; XIJ13 = SOLVE(XJ13P13,XJ13P);
XJ31P = XJ31'; XJ31P31 = XJ31'*XJ31; XIJ31 = SOLVE(XJ31P31,XJ31P);
XJ23P = XJ23'; XJ23P23 = XJ23'*XJ23; XIJ23 = SOLVE(XJ23P23,XJ23P);
XJ32P = XJ32'; XJ32P32 = XJ32'*XJ32; XIJ32 = SOLVE(XJ32P32,XJ32P);
SJA12 = Y'*(IN-XJ12*XIJ12)*Y#/(N-K1-1);
SJA13 = Y'*(IN-XJ13*XIJ13)*Y#/(N-K1-1);
SJA23 = Y'*(IN-XJ23*XIJ23)*Y#/(N-K2-1);
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SJA32=Y'*(IN-XJ32*XIJ32)*Y#/(N-K3-1);
SJA21=Y'*(IN-XJ21*XIJ21)*Y#/(N-K2-1);
SJA31=Y'*(IN-XJ31*XIJ31)*Y#/(N-K3-1);
JA(M,1)=N12#/SQRT(SJA12);
JA(M,2)=N13#/SQRT(SJA13);
JA(M,3)=N21#/SQRT(SJA21);
JA(M,4)=N23#/SQRT(SJA23);
JA(M,5)=N31#/SQRT(SJA31);
JA(M,6)=N32#/SQRT(SJA32);
    *COMPARE TO CRITICAL VALUES;
    JAH95=JA(M,)>=VJ95(NN,);
    CJA95=CJA95+JAH95;
    *COUNT FOR POWER AND TYPE1 ERROR;
    IF ALL(JAH95(,1 3)=R1) THEN CPJA12=CPJA12+1;
     ELSE IF ALL(JAH95(,1 3)=R2) OR ALL(JAH95(,1 3)=R3)
                      THEN CEJA12=CEJA12+1;
    IF ALL(JAH95(,2 5)=R1) THEN CPJA13=CPJA13+1;
     ELSE IF ALL(JAH95(,2 5)=R2) OR ALL(JAH95(,2 5)=R3)
                    THEN CEJA13=CEJA13+1;
    *COMPARE AND COUNT FOR 2 VS 3;
    IF (JAH95(,4 6)=R3) THEN CNT23(8,1)=CNT23(8,1)+1;
     ELSE IF (JAH95(,4 6)=R0) THEN CNT23(8,2)=CNT23(8,2)+1;
      ELSE IF (JAH95(,4 6)=R1) THEN CNT23(8,3)=CNT23(8,3)+1;
       ELSE CNT23(8,4)=CNT23(8,4)+1;
********************CLASSICAL F-TEST**********************;
***********************************************************;
X12=X1||X2; X12P=X12'; X12PX12=X12'*X12; XI12=SOLVE(X12PX12,X12P);
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```
X13=X1||X3; X13P=X13'; X13PX13=X13'*X13; XI13=SOLVE(X13PX13,X13P);
X23=X2||X3; X23P=X23'; X23PX23=X23'*X23; XI23=SOLVE(X23PX23,X23P);
 M12=IN-X12*XI12; SIG12=(Y'*M12*Y)#/(N-K1-K2);
 M13=IN-X13*XI13; SIG13=(Y'*M13*Y)#/(N-K1-K3);
 M23=IN-X23*XI23; SIG23=(Y'*M23*Y)#/(N-K2-K3);
 SSREG12=Y'*(IN-M12)*Y;
 SSREG13=Y'*(IN-M13)*Y;
 SSREG23=Y'*(IN-M23)*Y;
F(M,1) =(SSREG12-B1'*X1'*Y)#/(SIG12*K2);
F(M,3) =(SSREG12-B2'*X2'*Y)#/(SIG12*K1);
F(M,2) =(SSREG13-B1'*X1'*Y)#/(SIG13*K3);
F(M,5) =(SSREG13-B3'*X3'*Y)#/(SIG13*K1);
F(M,4) =(SSREG23-B2'*X2'*Y)#/(SIG23*K3);
F(M,6) =(SSREG23-B3'*X3'*Y)#/(SIG23*K2);
**********************************************************;
****COMPUTE TRUE NCP'S FOR F-TEST;
      NCPF(M,1)=BETA1'*X1'*M2*X1*BETA1#/(2*VAR1);
      NCPF(M,2)=BETA1'*X1'*M3*X1*BETA1#/(2*VAR1);
**********************************************************;
    *COMPARE TO CRITICAL VALUES;
     FH95=F(M,)>=VF95(NN,);
    CF95=CF95+FH95;
    *COUNT FOR POWER AND TYPE 1 ERROR;
     IF ALL(FH95(,1 3)=R1) THEN CPF12=CPF12+1;
     ELSE IF ALL(FH95(,1 3)=R2) OR ALL(FH95(,1 3)=R3)
    THEN CEF12=CEF12+1;
    IF ALL(FH95(,2 5)=R1) THEN CPF13=CPF13+1;
    ELSE IF ALL(FH95(,2 5)=R2) OR ALL(FH95(,2 5)=R3) THEN CEF13=CEF13+1;
```

```
*COMPARE AND COUNT FOR 2 VS 3;
 IF (FH95(,4 6)=R3) THEN CNT23(10,1)=CNT23(10,1)+1;
  ELSE IF (FH95(,4 6)=R0) THEN CNT23(10,2)=CNT23(10,2)+1;
   ELSE IF (FH95(,4 6)=R1) THEN CNT23(10,3)=CNT23(10,3)+1;
    ELSE CNT23(10,4)=CNT23(10,4)+1;
*******************NEW JA TEST ****************************;
**********************************************************;
NJ(M,1)=N12#/SQRT(SIG12);
NJ(M,2)=N13#/SQRT(SIG13);
NJ(M,3)=N21#/SQRT(SIG12);
NJ(M,4)=N23#/SQRT(SIG23);
NJ(M,5)=N31#/SQRT(SIG13);
NJ(M,6)=N32#/SQRT(SIG23);
***************************************************;
****COMPUTE NCP'S FOR NJ TEST;
 NCPT(M,1)=(BETA1'*X1'*M2*A1*A2*Y)##2#/
      (Y'*A2*A1*M2*A1*A2*Y#2#VAR1);
 NCPT(M,2)=(BETA1'*X1'*M3*A1*A3*Y)##2#/
      (Y'*A3*A1*M3*A1*A3*Y#2#VAR1);
 *****NCPT EVALUATED AT E1 (Y)  *********;
ENCPT(M,1)=(BETA1'*X1'*M2*A1*A2*X1*BETA1)##2#/
      (BETA1'*X1'*A2*A1*M2*A1*A2*X1*BETA1#2#VAR1);
ENCPT(M,2)=(BETA1'*X1'*M3*A1*A3*Y)##2#/
      (BETA1'*X1'*A3*A1*M3*A1*A3*X1*BETA1#2#VAR1);
******************************************************;
***********************************************************;
******************************************************;
  *COMPARE TO CRITICAL VALUES;
```

```
    NJH95=ABS(NJ(M,))>=VNJ95(NN,);
    CNJ95=CNJ95+NJH95;
    *POWER AND TYPE 1 ERROR COUNTS;
     IF ALL(NJH95(,1 3)=R1) THEN CPNJ12=CPNJ12+1;
      ELSE IF ALL(NJH95(,1 3)=R2) OR ALL(NJH95(,1 3)=R3) THEN
     CENJ12=CENJ12+1;
    IF ALL(NJH95(,2 5)=R1) THEN CPNJ13=CPNJ13+1;
     ELSE IF ALL(NJH95(,2 5)=R2) OR ALL(NJH95(,2 5)=R3) THEN
    CENJ13=CENJ13+1;
    *COMPARE AND COUNT FOR 2 VS 3;
    IF (NJH95(,4 6)=R3) THEN CNT23(9,1)=CNT23(9,1)+1;
     ELSE IF (NJH95(,4 6)=R0) THEN CNT23(9,2)=CNT23(9,2)+1;
       ELSE IF (NJH95(,4 6)=R1) THEN CNT23(9,3)=CNT23(9,3)+1;
      ELSE CNT23(9,4)=CNT23(9,4)+1;
*****************************************************;
*CALCULATE P-VALUES ASSOCIATED WITH REJECTING FALSE MODEL;
*****************************************************;
IF CH95(,1)=0 THEN PV12(M,1)=(1-PROBNORM(ABS(C(M,3))))*2;
  ELSE PV12(M,1)=1;
IF WH95(,1)=0 THEN PV12(M,2)=(1-PROBNORM(ABS(W(M,3))))*2;
  ELSE PV12(M,2)=1;
IF NH95(,1)=0 THEN PV12(M,3)=(1-PROBNORM(ABS(N0(M,3))))*2;
  ELSE PV12(M,3)=1;
IF NAH95(,1)=0 THEN PV12(M,4)=(1-PROBNORM(ABS(NA(M,3))))*2;
  ELSE PV12(M,4)=1;
IF NLH95(,1)=0 THEN PV12(M,5)=(1-PROBNORM(ABS(NL(M,3))))*2;
  ELSE PV12(M,5)=1;
IF JH95(,1)  =0 THEN PV12(M,6)=(1-PROBT(ABS(JJ(M,3)),N-K1-1))*2;
```

```
 ELSE PV12(M,6) = 1;
IF AJH95(,1) = 0 THEN PV12(M,7) = (1-PROBT(ABS(AJ(M,3)),N-K1-1))*2;
 ELSE PV12(M,7) = 1;
IF JAH95(,1) = 0 THEN PV12(M,8) = (1-PROBT(ABS(JA(M,3)),N-K1-1))*2;
 ELSE PV12(M,8) = 1;
IF NJH95(,1) = 0 THEN PV12(M,9) = (1-PROBT(ABS(NJ(M,3)),N-K1-K2))*2;
 ELSE PV12(M,9) = 1;
IF FH95(,1) = 0 THEN PV12(M,10) = 1-PROBF(F(M,3),K2,N-K1-K2);
 ELSE PV12(M,10) = 1;
*************************************************************;
   *RANK P VALUES WITHIN EACH ITERATION (ROW);
    RK12(M,) = RANKTIE(PV12(M,));
   *COUNT TIES FOR ADJUSTMENT ON KENDALL'S C.C. ;
    TIES12 = (PV12(M,) = TIECK);
    NTIES12 = TIES12(,+); SUMTIE12=SUMTIE12+(NTIES12**3-NTIES12)#/12;
**************************************************************;
*CALCULATE P-VALUES ASSOCIATED WITH REJECTING FALSE MODEL--13;
IF CH95(,2) = 0 THEN PV13(M,1) = (1-PROBNORM(ABS(C(M,5))))*2;
 ELSE PV13(M,1) = 1;
IF WH95(,2) = 0 THEN PV13(M,2) = (1-PROBNORM(ABS(W(M,5))))*2;
 ELSE PV13(M,2) = 1;
IF NH95(,2) = 0 THEN PV13(M,3) = (1-PROBNORM(ABS(N0(M,5))))*2;
 ELSE PV13(M,3) = 1;
IF NAH95(,2) = 0 THEN PV13(M,4) = (1-PROBNORM(ABS(NA(M,5))))*2;
 ELSE PV13(M,4) = 1;
IF NLH95(,2) = 0 THEN PV13(M,5) = (1-PROBNORM(ABS(NL(M,5))))*2;
 ELSE PV13(M,5) = 1;
IF JH95(,2)  = 0 THEN PV13(M,6) = (1-PROBT(ABS(JJ(M,5)),N-K1-1))*2;
```

```
 ELSE PV13(M,6)=1;
IF AJH95(,2)=0 THEN PV13(M,7)=(1-PROBT(ABS(AJ(M,5)),N-K1-1))*2;
 ELSE PV13(M,7)=1;
IF JAH95(,2)=0 THEN PV13(M,8)=(1-PROBT(ABS(JA(M,5)),N-K1-1))*2;
 ELSE PV13(M,8)=1;
IF NJH95(,2)=0 THEN PV13(M,9)=(1-PROBT(ABS(NJ(M,5)),N-K1-K3))*2;
 ELSE PV13(M,9)=1;
IF FH95(,2)=0 THEN PV13(M,10)=1-PROBF(F(M,5),K2,N-K1-K3);
 ELSE PV13(M,10)=1;
**********RANK P VALUES WITHIN EACH ITERATION (ROW);
    RK13(M,)=RANKTIE(PV13(M,));
    *COUNT TIES FOR ADJUSTMENT ON KENDALL'S C.C. ;
    TIES13 = (PV13(M,)=TIECK);
    NTIES13 = TIES13(,+); SUMTIE13=SUMTIE13+(NTIES13**3-NTIES13)#/12;
*******************************************************;
************************************************************;
*****TAKE CARE OF POWER COMPUTATIONS*************************;
************************************************************;
**FOR F: 2 VS 1; R=K1; DF2=N-K1-K2; CV=VF95(NN,3); L=NCPF(M,1);
 PF2(M,1)=1-FPROB(CV,R,DF2,L);
**FOR F: 3 VS 1; R=K1; DF2=N-K1-K3; CV=VF95(NN,5); L=NCPF(M,2);
 PF2(M,2)=1-FPROB(CV,R,DF2,L);
  ****POWER COMPUTATIONS FOR NJ TEST****;
**FOR T: 2 VS 1; R= 1; DF2=N-K1-K2; CV=VNJ95(NN,3)##2; L=NCPT(M,1);
 PT(M,1)=1-FPROB(CV,R,DF2,L);
**FOR T: 3 VS 1; R= 1; DF2=N-K1-K3; CV=VNJ95(NN,5)##2; L=NCPT(M,2);
 PT(M,2)=1-FPROB(CV,R,DF2,L);
************NJ POWER BASED ON EXPECTED VALUE OF Y UNDER H1 *****;
```

```
**FOR T: 2 VS 1;  R= 1; DF2=N-K1-K2; CV=VNJ95(NN,3)##2; L=ENCPT(M,1);
EPT(M,1)=1-FPROB(CV,R,DF2,L);
**FOR T: 3 VS 1;  R= 1; DF2=N-K1-K3; CV=VNJ95(NN,5)##2; L=ENCPT(M,2);
EPT(M,2)=1-FPROB(CV,R,DF2,L);
********POWER SUMS AND SS CALCULATIONS FOLLOWING THE LOOP;
***********************************************************;
*END OF ITERATIVE LOOP;
END;
***********************************************************;
************POWER COMPARISONS AND AVG POWER CALCS*********;
          SUMPF2=PF2(+,); SUMPT=PT(+,);
          AVGPF2=SUMPF2#/NITER; AVGPT=SUMPT#/NITER;
          SSF2=(PF2#PF2)(+,); SST=(PT#PT)(+,);
    STDERPF2=(SSF2-SUMPF2#SUMPF2#/NITER)#/(NITER-1);
    STDERPT=(SST-SUMPT#SUMPT#/NITER)#/(NITER-1);
           SUMEPT=EPT(+,);
           AVGEPT=SUMEPT#/NITER;
           SSET=(EPT#EPT)(+,);
     STDEREPT=(SSET-SUMEPT#SUMEPT#/NITER)#/(NITER-1);
HF=AVGPF2'||STDERPF2';  HT=AVGPT'||STDERPT';  HET=AVGEPT'||STDEREPT';
**********COMPUTE A SSE FOR DEVIATIONS BETWEEN POWER T AND E(POWERT)
        ITERATION BY ITERATION  **********************************;
DIFFPT=PT-EPT;  SSDIFFPT=(DIFFPT#DIFFPT)(+,);
 AVG2DIFF=SSDIFFPT#/NITER;
 DIFFT=SSDIFFPT//AVG2DIFF;
*********ITERATION BY ITERATION COMPARISON OF POWER********;
  COMPF2T=(PF2 > PT);
```

```
AVGCOF2T=( COMPF2T(+,) )#/NITER;
POWRCOMP=HF//HT//AVGCOF2T//HET//DIFFT;   PRINT POWRCOMP;
***************************************************************;
***************************************************************;
****CREATE VECTORS FOR STORING POWER,STND ERR, IERROR, STND ERR;
***************************************************************;
C12=J(1,4,0); W12=J(1,4,0); N12=J(1,4,0); NA12=J(1,4,0); NL12=J(1,4,0);
C13=J(1,4,0); W13=J(1,4,0); N13=J(1,4,0); NA13=J(1,4,0); NL13=J(1,4,0);
J12=J(1,4,0); AJ12=J(1,4,0); JA12=J(1,4,0); NJ12=J(1,4,0);
J13=J(1,4,0); AJ13=J(1,4,0); JA13=J(1,4,0); NJ13=J(1,4,0);
F12=J(1,4,0);
F13=J(1,4,0);
*NEED TO COMPUTE POWER AND TYPE 1 ERROR PROBABILITIES;
C12(,1)=CPC12#/NITER; C12(,3)=CEC12#/NITER;
C13(,1)=CPC13#/NITER; C13(,3)=CEC13#/NITER;
C12(,2)=SQRT((CPC12-CPC12**2#/NITER)#/(NITER*(NITER-1)));
C12(,4)=SQRT((CEC12-CEC12**2#/NITER)#/(NITER*(NITER-1)));
C13(,2)=SQRT((CPC13-CPC13**2#/NITER)#/(NITER*(NITER-1)));
C13(,4)=SQRT((CEC13-CEC13**2#/NITER)#/(NITER*(NITER-1)));
W12(,1)=CPW12#/NITER; W12(,3)=CEW12#/NITER;
W13(,1)=CPW13#/NITER; W13(,3)=CEW13#/NITER;
W12(,2)=SQRT((CPW12-CPW12**2#/NITER)#/(NITER*(NITER-1)));
W12(,4)=SQRT((CEW12-CEW12**2#/NITER)#/(NITER*(NITER-1)));
W13(,2)=SQRT((CPW13-CPW13**2#/NITER)#/(NITER*(NITER-1)));
W13(,4)=SQRT((CEW13-CEW13**2#/NITER)#/(NITER*(NITER-1)));
N12(,1)=CPN12#/NITER; N12(,3)=CEN12#/NITER;
N13(,1)=CPN13#/NITER; N13(,3)=CEN13#/NITER;
N12(,2)=SQRT((CPN12-CPN12**2#/NITER)#/(NITER*(NITER-1)));
```

```
N12(,4) = SQRT((CEN12-CEN12**2#/NITER)#/(NITER*(NITER-1)));
N13(,2) = SQRT((CPN13-CPN13**2#/NITER)#/(NITER*(NITER-1)));
N13(,4) = SQRT((CEN13-CEN13**2#/NITER)#/(NITER*(NITER-1)));
NA12(,1) = CPNA12#/NITER;  NA12(,3) = CENA12#/NITER;
NA13(,1) = CPNA13#/NITER;  NA13(,3) = CENA13#/NITER;
NA12(,2) = SQRT((CPNA12-CPNA12**2#/NITER)#/(NITER*(NITER-1)));
NA12(,4) = SQRT((CENA12-CENA12**2#/NITER)#/(NITER*(NITER-1)));
NA13(,2) = SQRT((CPNA13-CPNA13**2#/NITER)#/(NITER*(NITER-1)));
NA13(,4) = SQRT((CENA13-CENA13**2#/NITER)#/(NITER*(NITER-1)));
NL12(,1) = CPNL12#/NITER;  NL12(,3) = CENL12#/NITER;
NL13(,1) = CPNL13#/NITER;  NL13(,3) = CENL13#/NITER;
NL12(,2) = SQRT((CPNL12-CPNL12**2#/NITER)#/(NITER*(NITER-1)));
NL12(,4) = SQRT((CENL12-CENL12**2#/NITER)#/(NITER*(NITER-1)));
NL13(,2) = SQRT((CPNL13-CPNL13**2#/NITER)#/(NITER*(NITER-1)));
NL13(,4) = SQRT((CENL13-CENL13**2#/NITER)#/(NITER*(NITER-1)));
J12(,1) = CPJ12#/NITER;  J12(,3) = CEJ12#/NITER;
J13(,1) = CPJ13#/NITER;  J13(,3) = CEJ13#/NITER;
J12(,2) = SQRT((CPJ12-CPJ12**2#/NITER)#/(NITER*(NITER-1)));
J12(,4) = SQRT((CEJ12-CEJ12**2#/NITER)#/(NITER*(NITER-1)));
J13(,2) = SQRT((CPJ13-CPJ13**2#/NITER)#/(NITER*(NITER-1)));
J13(,4) = SQRT((CEJ13-CEJ13**2#/NITER)#/(NITER*(NITER-1)));
AJ12(,1) = CPAJ12#/NITER;    AJ12(,3) = CEAJ12#/NITER;
AJ13(,1) = CPAJ13#/NITER;    AJ13(,3) = CEAJ13#/NITER;
AJ12(,2) = SQRT((CPAJ12-CPAJ12**2#/NITER)#/(NITER*(NITER-1)));
AJ12(,4) = SQRT((CEAJ12-CEAJ12**2#/NITER)#/(NITER*(NITER-1)));
AJ13(,2) = SQRT((CPAJ13-CPAJ13**2#/NITER)#/(NITER*(NITER-1)));
AJ13(,4) = SQRT((CEAJ13-CEAJ13**2#/NITER)#/(NITER*(NITER-1)));
JA12(,1) = CPJA12#/NITER;    JA12(,3) = CEJA12#/NITER;
```

```
JA13(,1)=CPJA13#/NITER;    JA13(,3)=CEJA13#/NITER;
JA12(,2)=SQRT((CPJA12-CPJA12**2#/NITER)#/(NITER*(NITER-1)));
JA12(,4)=SQRT((CEJA12-CEJA12**2#/NITER)#/(NITER*(NITER-1)));
JA13(,2)=SQRT((CPJA13-CPJA13**2#/NITER)#/(NITER*(NITER-1)));
JA13(,4)=SQRT((CEJA13-CEJA13**2#/NITER)#/(NITER*(NITER-1)));
NJ12(,1)=CPNJ12#/NITER;    NJ12(,3)=CENJ12#/NITER;
NJ13(,1)=CPNJ13#/NITER;    NJ13(,3)=CENJ13#/NITER;
NJ12(,2)=SQRT((CPNJ12-CPNJ12**2#/NITER)#/(NITER*(NITER-1)));
NJ12(,4)=SQRT((CENJ12-CENJ12**2#/NITER)#/(NITER*(NITER-1)));
NJ13(,2)=SQRT((CPNJ13-CPNJ13**2#/NITER)#/(NITER*(NITER-1)));
NJ13(,4)=SQRT((CENJ13-CENJ13**2#/NITER)#/(NITER*(NITER-1)));
F12(,1)=CPF12#/NITER;      F12(,3)=CEF12#/NITER;
F13(,1)=CPF13#/NITER;      F13(,3)=CEF13#/NITER;
F12(,2)=SQRT((CPF12-CPF12**2#/NITER)#/(NITER*(NITER-1)));
F12(,4)=SQRT((CEF12-CEF12**2#/NITER)#/(NITER*(NITER-1)));
F13(,2)=SQRT((CPF13-CPF13**2#/NITER)#/(NITER*(NITER-1)));
F13(,4)=SQRT((CEF13-CEF13**2#/NITER)#/(NITER*(NITER-1)));
*PRINT C12 W12 N12 NA12 NL12 J12 AJ12 JA12 NJ12 F12 ;
*PRINT C13 W13 N13 NA13 NL13 J13 AJ13 JA13 NJ13 F13 ;
TESTS12=C12//W12//N12//NA12//NL12//J12//AJ12//JA12//NJ12//F12 ;
TESTS13=C13//W13//N13//NA13//NL13//J13//AJ13//JA13//NJ13//F13 ;
***HOLD TEST RESULTS TO COMBINE WITH AVG RANKS FURTHER DOWN;
************************************************************;
***********************************************************;
*****CALCULATE MEAN AND STND ERROR OF R2 FOR`ALL 3 MODELS;
***********************************************************;
   MR2=SR2#/NITER; SER2=J(3,1,0);
   DO I=1 TO 3;
```

```
SER2(I,)=SQRT((SUSR2(I,)-SR2(I,)**2#/NITER)#/(NITER*(NITER-1)));
END; R2INFO=MR2||SER2; PRINT R2INFO;
***********************************************************;
*******CALCULATE KENDALL'S COEFFICIENT OF CONCORDANCE--KW12;
SUMRK12=RK12(+,); RBAR=J(1,10,(NITER*6));
RKB=SUMRK12-RBAR;
WSUM12=(RKB#RKB)(,+);
KW12=12*WSUM12#/(990*NITER**2 - NITER*SUMTIE12);
***CALCULATE THE AVERAGE RANKING OF EACH TEST FOR THIS RUN;
AVRANK12=SUMRK12#/NITER;
TESTS12=TESTS12||AVRANK12`; PRINT TESTS12;
***CALCULATE THE P-VALUE ASSOCIATED WITH CORRES S FOR KENDALL'S W;
S12=NITER*9*KW12;
SIGW12=1-PROBCHI(S12,9);
*PRINT SUMRK12 AVRANK12; KENDAL12=WSUM12||KW12||S12||SIGW12;
PRINT KENDAL12;
************************************************************;
******CALCULATE KENDALL'S COEFFICIENT OF CONCORDANCE--KW13;
************************************************************;
SUMRK13=RK13(+,); RBAR=J(1,10,(NITER*6));
RKB=SUMRK13-RBAR;
WSUM13=(RKB#RKB)(,+);
KW13=12*WSUM13#/(990*NITER**2 - NITER*SUMTIE13);
***CALCULATE THE AVERAGE RANKING OF EACH TEST FOR THIS RUN;
AVRANK13=SUMRK13#/NITER;
TESTS13=TESTS13||AVRANK13`; PRINT TESTS13;
***CALCULATE THE P-VALUE ASSOCIATED WITH CORRES S FOR KENDALL'S W;
S13=NITER*9*KW13;
```

```
    SIGW13 = 1-PROBCHI(S13,9);
*PRINT SUMRK13 AVRANK13 ;  KENDAL13 = WSUM13||KW13||S13||SIGW13;
PRINT KENDAL13;
************************************************************;
   ***CALCULATE AND PRINT % FOR MODEL 2 VS 3;
PCNT23 = CNT23#/NITER;
  PRINT PCNT23;
***********************************************************;
*********************** THE END *************************;
***********************************************************;
/*
//
```

Non-Normal Deviate Case: Simulation Program

```
//BO###NND JOB acct#,NONNORM,TIME = 15,REGION = 3072K
/*LONGKEY #####
/*PRIORITY IDLE
/*JOBPARM LINES = 5
//STEP1 EXEC FORTVC
//FORT.SYSIN DD *
C
C *** This program illustrates calling a FORTRAN Function from SAS.
C
      INTEGER FUNCTION MATSUB( NARG, ARGS )
      INTEGER*4 NARG
```

```
      INTEGER*4 ARGS( 1 )
      INTEGER*4 MIN, MAX, ROW, COL, ILOC, OLOC, NTOTAL
C     IARRAY is an input  array passed from SAS to FORTRAN.
C     OARRAY is an output array generated by FORTRAN and returned to SAS.
      REAL*8 IARRAY( 1 ), OARRAY( 1 )
C     The following Declarations are used in the implementation of
C     the IMSL Subroutine GGNML:
C        XX     is a single precision vector used to contain the values
C               generated by GGNML.  These values are then assigned to
C               output matrix OARRAY.
C               NOTE:  SAS programs expect passed arrays to be declared
C                      as REAL*8 variables.  Although this program
C                      links in the IMSL Double Precision library,
C                      Subroutine GGNML returns Single Precision values.
C        DSEED  is a double precision number used as the seed for the
C               random number generator.
      REAL*4 XX(10000)
      DOUBLE PRECISION DSEED
      DATA DSEED/28217.D0/
C** TEST TO ENSURE THAT ONLY ONE ARGUMENT IS PASSED TO THIS PROCEDURE
      IF( NARG.NE.1 ) THEN
        MATSUB = 5
        RETURN
      ENDIF
C
C** TEST TO ENSURE THAT THE ONE ARGUMENT IS A MATRIX
C     (i.e. the input value is at least a 1 X 1 array)
      CALL ARG( ARGS(1), ROW, COL, ILOC, IARRAY)
```

```
      MIN=MIN0( ROW, COL)
      IF(MIN.LT.1) THEN
        MATSUB= 6
        RETURN
      ENDIF
C
C** DEFINE THE OUTPUT MATRIX
C
C-- Routine SETUP defines the output matrix and has the form:
C
C     CALL SETUP(IRES,NROWS,NCOLMS)
C
C         where  IRES   is a result number -- can use 1.
C                NROWS  is the number of rows in the output matrix.
C                NCOLMS is the number of columns in the output matrix.
C
C -- Use the following with ZRPOLY:
C        MAX = MAX0( ROW,COL)
C        CALL SETUP(1,MAX-1,2)
C -- Use the following with GGNML:
        CALL SETUP(1,ROW,COL)
C
C -- Subroutine ARG is used to get the dimensions and location of
C     the matrices according to their symbol table number IARG(I):
C
      CALL ARG( 1, ROW,COL, OLOC, OARRAY )
      IF( ROW.EQ. 0 .OR. COL.EQ.0 ) THEN
        MATSUB= 1
```

```
      RETURN
    ENDIF
C
C --- Call the desired IMSL Subroutine:
C
C -  GNML  is a Gaussian (Normal) random deviate generator:
C        XX is used as a temporary 'array' for storing  the
C          generated random deviates which are then placed
C          in array OARRAY which is passed from FORTRAN to SAS
C          (indexing into OARRAY starts at location OLOC).
    NTOTAL = ROW * COL
    CALL GGNML( DSEED, NTOTAL,  XX )
    IJ = 0
    DO 1000 I=1,ROW
    DO 1000 J=1,COL
      IJ = IJ + 1
 1000   OARRAY(OLOC+IJ-1) = XX(IJ)
    RETURN
    END
/*
//* STEP0002 EXEC PGM=IEWL,PARM='MAP,LIST'
//STEP0002 EXEC PGM=IEWL
//SYSPRINT  DD SYSOUT=A
//SYSUT1    DD UNIT=SYSDA,SPACE=(TRK,(40,40))
//SYSLIB    DD DSN=SYS2.SAS.SUBLIB,DISP=SHR
//          DD DSN=SYS2.SAS.LIBRARY,DISP=SHR
//          DD DSN=SYS2.PLIBASE,DISP=SHR
//          DD DSN=SYS2.R3.VFORTLIB,DISP=SHR
```

```
//         DD DSN=VPI.IMSL.DP,DISP=SHR
//* In the lines which follow:
//* The SETSSI statement describes the characteristics of the input
//*     function. The values in positions 3 and 4 specify the number
//*     of arguments passed to the function; these should be equal.
//*     If all arguments are numeric, the last four digists are zero.
//*     For additional information regarding this statement, see:
//*     Technical Report: P-139. SAS Programmers Guide Version 5.
//* The NAME statement specifies the name used to call the function
//*     from within the SAS program. The R designates that any previous
//*     function having this name will be replaced.
//SYSLIN    DD DSN=&&LOADSET,DISP=(OLD,DELETE,DELETE)
//          DD *
 INCLUDE SYSLIB(MATMAIN)
 ENTRY MATMAIN
 SETSSI BF110000
 NAME XXXXXX(R)
/*
//* IN THE
//SYSLMOD   DD DSN=&LIBRARY,DISP=(NEW,PASS,DELETE),UNIT=SYSDA,
//             SPACE=(CYL,(10,20,20),,CONTIG)
//STEP3 EXEC FORTVC
//FORT.SYSIN DD *
C
C *** This program illustrates calling a FORTRAN Function from SAS.
C
      INTEGER FUNCTION MATSUB( NARG, ARGS )
      INTEGER*4 NARG
```

```
      INTEGER*4 ARGS( 1 )
      INTEGER*4 MIN, MAX, ROW, COL, ILOC, OLOC, NTOTAL
C     IARRAY is an input  array passed from SAS to FORTRAN.
C     OARRAY is an output array generated by FORTRAN and returned to SAS.
      REAL*8 IARRAY( 1 ), OARRAY( 1 ), WORK2( 1 ), WORK3( 1 )
      REAL*4 TN(20), XX(20), CHI(20), T(20)
      DOUBLE PRECISION DSEED
      DATA DSEED/39441.D0/
C** TEST TO ENSURE THAT ONLY ONE ARGUMENT IS PASSED TO THIS PROCEDURE
      IF( NARG.NE.1 ) THEN
        MATSUB  =  5
        RETURN
      ENDIF
C
C** TEST TO ENSURE THAT THE ONE ARGUMENT IS A MATRIX
C     (i.e. the input value is at least a 1 X 1 array)
      CALL ARG( ARGS(1), ROW, COL, ILOC, IARRAY)
      MIN=MIN0( ROW, COL)
      IF(MIN.LT.1) THEN
        MATSUB=  6
        RETURN
      ENDIF
C
          CALL SETUP(1,ROW,COL)
      CALL ARG( 1, ROW,COL, OLOC, OARRAY )
      IF( ROW.EQ. 0 .OR. COL.EQ.0 ) THEN
        MATSUB=  1
        RETURN
```

```
      ENDIF
C
C --- Call the desired IMSL Subroutine:
C
      NTOTAL = ROW / 4
      CALL GGNML( DSEED, NTOTAL,  XX )
 1499 DO 1500 K=1,NTOTAL
      HOLD=GGNQF(DSEED)
      IF(ABS(HOLD).GE.(1.6449)) GOTO 1499
      TN(K)=HOLD
 1500 CONTINUE
      DO 1620 K=1,NTOTAL
      CALL GGCHS(DSEED,2,WORK2,CHI(K))
      CALL GGCHS(DSEED,3,WORK3,CHI3H)
      T(K)=XX(K)/SQRT(CHI3H/3)
 1620 CONTINUE
      DO 1000 I=1,NTOTAL
      OARRAY(OLOC+I-1) = TN(I)
      OARRAY(OLOC+NTOTAL+I-1) = T(I)
      OARRAY(OLOC+2*NTOTAL+I-1) = XX(I)
      OARRAY(OLOC+3*NTOTAL+I-1) = CHI(I)
 1000 CONTINUE
      RETURN
      END
/*
//* STEP0004 EXEC PGM=IEWL,PARM='MAP,LIST'
//STEP0004 EXEC PGM=IEWL
//SYSPRINT  DD SYSOUT=A
```

```
//SYSUT1    DD UNIT=SYSDA,SPACE=(TRK,(40,40))
//SYSLIB    DD DSN=SYS2.SAS.SUBLIB,DISP=SHR
//          DD DSN=SYS2.SAS.LIBRARY,DISP=SHR
//          DD DSN=SYS2.PLIBASE,DISP=SHR
//          DD DSN=SYS2.R3.VFORTLIB,DISP=SHR
//          DD DSN=VPI.IMSL.DP,DISP=SHR
//SYSLIN    DD DSN=&&LOADSET,DISP=(OLD,DELETE,DELETE)
//          DD *
 INCLUDE SYSLIB(MATMAIN)
 ENTRY MATMAIN
 SETSSI BF110000
 NAME GENERR(R)
/*
//* IN THE
//SYSLMOD   DD DSN=&LIBRARY,DISP=(MOD,PASS,DELETE),UNIT=SYSDA,
//             SPACE=(CYL,(10,20,20),,CONTIG)
//STEP0005 EXEC SAS
//SYSIN DD *
OPTIONS  NODATE LS=80;
PROC MATRIX;    TITLE 'MONTE-CARLO FOR NONNORMAL DEVIATE CASE';
TITLE3 'EXPT #';
  ************SET SIMULATION CONTROL VARIABLES*******************;
NITER= 500;  N= 20; K1= #; K2= #;  K3= #; R21=0.##; P21=0.##;
  PARMTRS=N||R21||P21||K1||K2||K3||NITER;   PRINT PARMTRS;
  **************************************************************;
  *********** SET CRITICAL VALUES FOR TESTS ********************;
  ******* NN=1-4 FOR N=20              *********************;
```

```
*******            2   FOR 426   AND           4   FOR 444**;
NN=#;
 ******* SET UP CONSTANT VALUES AND CALCULATE MODEL CONTROLS ****;
IN=I(N);             VAR1=K1*(1-R21)#/R21;  LB=SQRT(P21#/(1-P21));
     ONE=J(N,1,1);
  ****TO COMPUTE LOG-NORMAL DEVIATES--G0 AND G1;
    G02=LOG(0.5 + 0.5#SQRT(1 + 4#VAR1));
    G1=EXP(0.5#G02);   G0=SQRT(G02);
  ***************;
   BETA1=J(K1,1,1); BETA1M=J(K1,5,1);    Y=J(N,5,1);
******** SET UP CHECKING VECTORS FOR TEST RESULTS ***************;
R0=0 0;  R1=0 1; R2=1 0; R3=1 1;
*********CREATE AND INITIALIZE THE P-VALUE AND RANK MATRICES;
PV12=J(NITER*5,10,0); PV13=J(NITER*5,10,0);
RK12=J(NITER*5,10,0); RK13=J(NITER*5,10,0);
********** INITIALIZE COUNTERS FOR POWER AND TYPE 1 ERROR PROBABILITIES;
CPC12=J(5,1,0);  CEC12=J(5,1,0); CPC13=J(5,1,0); CEC13=J(5,1,0);
CPW12=J(5,1,0);  CEW12=J(5,1,0);   CPW13=J(5,1,0); CEW13=J(5,1,0);
CPN12=J(5,1,0); CEN12=J(5,1,0);    CPN13=J(5,1,0); CEN13=J(5,1,0);
CPNJ12=J(5,1,0); CENJ12=J(5,1,0);
CPNA12=J(5,1,0); CENA12=J(5,1,0); CPNA13=J(5,1,0); CENA13=J(5,1,0);
CPNL12=J(5,1,0); CENL12=J(5,1,0); CPNL13=J(5,1,0); CENL13=J(5,1,0);
CPJ12=J(5,1,0); CEJ12=J(5,1,0);    CPJ13=J(5,1,0); CEJ13=J(5,1,0);
CPNJ13=J(5,1,0); CENJ13=J(5,1,0);
CPAJ12=J(5,1,0); CEAJ12=J(5,1,0); CPAJ13=J(5,1,0); CEAJ13=J(5,1,0);
CPJA12=J(5,1,0); CEJA12=J(5,1,0);  CPJA13=J(5,1,0); CEJA13=J(5,1,0);
CPF12=J(5,1,0); CEF12=J(5,1,0);    CPF13=J(5,1,0); CEF13=J(5,1,0);
********INITIALIZE CRITICAL VALUE VECTORS;
```

```
* F (J AND JA TOO) CRITICAL VALUES ARE ONLY APPROX FOR N=40;
VC95=J(5,6,1.9600);
VJ95= 2.110 2.110 2.131 2.131 2.160 2.160 /
    2.131 2.131 2.110 2.110 2.160 2.160 /
    2.160 2.160 2.110 2.110 2.131 2.131 /
    2.131 2.131 2.131 2.131 2.131 2.131 /
    2.042 2.042 2.042 2.042 2.042 2.042 /
    2.042 2.042 2.042 2.042 2.042 2.042 /
    2.042 2.042 2.042 2.042 2.042 2.042 /
    2.042 2.042 2.042 2.042 2.042 2.042;
VF95= 3.11 3.00 3.74 3.22 3.89 3.48 /
    3.74 3.22 3.11 3.00 3.48 3.89 /
    3.89 3.48 3.00 3.11 3.22 3.74 /
    3.26 3.26 3.26 3.26 3.26 3.26 /
    2.69 2.42 3.32 2.42 3.32 2.69 /
    3.32 2.42 2.69 2.42 2.69 3.32 /
    3.32 2.69 2.42 2.69 2.42 3.32 /
    2.69 2.69 2.69 2.69 2.69 2.69;
VNJ95= 2.145 2.179 2.145 2.228 2.179 2.228 /
    2.145 2.228 2.145 2.179 2.228 2.179 /
    2.179 2.228 2.179 2.145 2.228 2.145 /
    2.179 2.179 2.179 2.179 2.179 2.179 /
    2.042 2.042 2.042 2.042 2.042 2.042 /
    2.042 2.042 2.042 2.042 2.042 2.042 /
    2.042 2.042 2.042 2.042 2.042 2.042 /
    2.042 2.042 2.042 2.042 2.042 2.042;
*****CREATE MATRIX 5 * 6 CRITICAL VALUES FOR THE F AND NJ TESTS;
VF95M=VF95(NN,) // VF95(NN,) // VF95(NN,) // VF95(NN,) // VF95(NN,);
```

```
VNJ95M=VNJ95(NN,) // VNJ95(NN,) // VNJ95(NN,) // VNJ95(NN,) //VNJ95(NN,);
***************CREATE JH95, AJH95 AND JAH95 FOR FILLING IN ROW BY ROW;
     JH95=J(5,6,0);   AJH95=J(5,6,0);   JAH95=J(5,6,0);
  ******************************************************;
****** CREATE AND INITIALIZE MATRICES FOR TEST STAT VALUES;
 C=J(NITER*5,6,0);      W=J(NITER*5,6,0);  N0=J(NITER*5,6,0);
JJ=J(NITER*5,6,0);     JA=J(NITER*5,6,0);   F=J(NITER*5,6,0); NJ=J(NITER*5,6,0);
NA=J(NITER*5,6,0);     NL=J(NITER*5,6,0);  AJ=J(NITER*5,6,0);
******* CREATE AND INITIALIZE COUNTER VECTORS FOR # OF SIG TEST STATS;
 CC95=J(5,6,0);    CW95=J(5,6,0);  CN95=J(5,6,0);
 CJ95=J(5,6,0);   CJA95=J(5,6,0);  CF95=J(5,6,0);  CNJ95=J(5,6,0);
CNA95=J(5,6,0);   CNL95=J(5,6,0); CAJ95=J(5,6,0);
*********R2 COUNTER INITIALIZATIONS;
     SR2=J(3,5,0); SUSR2=J(3,5,0);
********SETUP FOR KENDALL'S COEF OF CONCORDANCE --ADJUST FOR TIES;
       TIES12=J(5,10,0);     TIES13=J(5,10,0);
      NTIES12=J(5,1,0);       NTIES13=J(5,1,0);
   TIECK=J(1,10,1);  SUMTIE12=J(5,1,0);  SUMTIE13=J(5,1,0);
*****INITIALIZE 2X2 COUNT MATRIX FOR TESTS OF MODEL 2 VS 3;
    CNT23=J(50,4,0);
*;
****** GENERATE MATRICES TO BE SENT TO IMSL FOR RANDOM NORMAL DEVIATES;
  X1H=J(N,K1,1); X2H=J(N,K2,1); X3H=J(N,K3,1); ERRH=J(N*4,1,1);
***************BEGINNING OF ITERATIVE LOOP******************;
DO M=1 TO NITER;    LOC=(M-1)*5+1;
***LOC FOR START POSITION TO PLACE APPROP. DEVIATES IN TEST STAT LISTS**;
***********************************************************;
********** GENERATE X VALUES AND ERROR TERMS AND Y***********;
```

```
X1=XXXXXX(X1H); X2=XXXXXX(X2H); X3=XXXXXX(X3H);
IF K2>=K1 THEN X2(,1:K1) = LB*X1 +X2(,1:K1);
 ELSE X2= LB* X1(,1:K2) + X2;
IF K3>=K1 THEN X3(,1:K1) = LB*X1 +X3(,1:K1);
 ELSE X3= LB* X1(,1:K3) + X3;
ERR=GENERR(ERRH);    SIGMA1=SQRT(VAR1);
   ERR1=ERR(1:N,1)#SIGMA1#/SQRT(0.6230336); * 1--TRUNCATED NORMALS;
   ERR2=ERR(N+1:2*N,1)#SQRT(VAR1#/3);     * 2--STUDENT T-S;
   ERR3=EXP(ERR(2*N+1:3*N,1)#G0)-ONE#G1;       * 3--LOGNORMALS;
      TWO=J(N,1,2);
   ERR4=(ERR(3*N+1:4*N,1)-TWO)#SIGMA1#/2;  * 4--CHI-SQUARES;
   ERR5=ERR(2*N+1:3*N,1)#SIGMA1;            * 5--NORMALS;
TRUEY=X1*BETA1;
 Y(,1)=TRUEY + ERR1;  Y(,2)=TRUEY + ERR2;
 Y(,3)=TRUEY + ERR3;  Y(,4)=TRUEY + ERR4;
 Y(,5)=TRUEY + ERR5;
************COMPUTE NECESSARY MODEL ESTIMATION PIECES*********;
X1P=X1';           X2P=X2';           X3P=X3';
X1PX1=X1'*X1;      X2PX2=X2'*X2;      X3PX3=X3'*X3;
XI1=SOLVE(X1PX1,X1P); XI2=SOLVE(X2PX2,X2P); XI3=SOLVE(X3PX3,X3P);
A1=X1*XI1;         A2=X2*XI2;         A3=X3*XI3;
M1=IN-A1;          M2=IN-A2;          M3=IN-A3;
B1=XI1*Y;          B2=XI2*Y;          B3=XI3*Y;
TRM12=TRACE(M1*M2);
TRM13=TRACE(M1*M3);
TRM23=TRACE(M2*M3);
TRA12=TRACE(A1*A2);  TRA122=TRACE(A1*A2*A1*A2);
```

```
TRA13=TRACE(A1*A3);  TRA132=TRACE(A1*A3*A1*A3);
TRA23=TRACE(A2*A3);  TRA232=TRACE(A2*A3*A2*A3);
 TRB12=K2 - TRA122 - ( K2 - TRA12 )**2 #/ (N-K1);
 TRB21=K1 - TRA122 - ( K1 - TRA12 )**2 #/ (N-K2);
 TRB13=K3 - TRA132 - ( K3 - TRA13 )**2 #/ (N-K1);
 TRB31=K1 - TRA132 - ( K1 - TRA13 )**2 #/ (N-K3);
 TRB23=K3 - TRA232 - ( K3 - TRA23 )**2 #/ (N-K2);
 TRB32=K2 - TRA232 - ( K2 - TRA23 )**2 #/ (N-K3);
    YSUM=DIAG(Y(+,));
  SSY=Y'*Y - YSUM##2#/N;
* OLS AND MLE ON SEPARATE MODELS;  R22=J(3,5,1);  I5=I(5);
YH1=X1*B1;              YH2=X2*B2;
E1=Y-YH1;               E2=Y-YH2;
SSR1=E1'*E1;            SSR2=E2'*E2;
S1LS=SSR1#/(N-K1);       S2LS=SSR2#/(N-K2);
S1ML=SSR1#/N;            S2ML=SSR2#/N;
R22(1,)=VECDIAG(I5 - SSR1#/SSY)';  R22(2,)=VECDIAG(I5 - SSR2#/SSY)';
YH3=X3*B3;
E3=Y-YH3;
SSR3=E3'*E3;
S3LS=SSR3#/(N-K3);
S3ML=SSR3#/N;
R22(3,)=VECDIAG(I5 - SSR3#/SSY)';
*****************************************************************;
****************COUNT UPDATES FOR R2;
 DO I=1 TO 3;  SR2(I,)=SR2(I,) + R22(I,);
   SUSR2(I,)=SUSR2(I,)+ R22(I,)##2;  END;
************************ E(YHJ) UNDER HI ********************;
```

```
YH12=A2*A1*Y; YH21=A1*A2*Y;  YH13=A3*A1*Y;  YH31=A1*A3*Y;
YH23=A3*A2*Y; YH32=A2*A3*Y;
***************************************************************;
E21=M2*YH1;  E12=M1*YH2;
E31=M3*YH1;  E13=M1*YH3;
E32=M3*YH2;  E23=M2*YH3;
E211=M1*E21;   E122=M2*E12;
E311=M1*E31;   E133=M3*E13;
E322=M2*E32;   E233=M3*E23;
B1M2=E21'*E21;  B1MM2=E211'*E211;
B1M3=E31'*E31;  B1MM3=E311'*E311;
B3M2=E23'*E23;  B3MM2=E233'*E233;
B3M1=E13'*E13;  B3MM1=E311'*E311;
B2M1=E12'*E12;  B2MM1=E122'*E122;
B2M3=E32'*E32;  B2MM3=E322'*E322;
******************************************************;
********CALCULATION OF TEST STATISTICS*****************;
******************************************************;
***************** COX TEST*****************************;
******************************************************;
O21=(B1M2 + S1LS#TRM12) #/ (N-K2);
O31=(B1M3 + S1LS#TRM13) #/ (N-K3);
O32=(B2M3 + S2LS#TRM23) #/ (N-K3);
O12=(B2M1 + S2LS#TRM12) #/ (N-K1);
O13=(B3M1 + S3LS#TRM13) #/ (N-K1);
O23=(B3M2 + S3LS#TRM23) #/ (N-K2);
S12ML=S2ML+B2M1#/N;  S12LS=S12ML#N#/(N-K1);
S13ML=S3ML+B3M1#/N;  S13LS=S13ML#N#/(N-K1);
```

```
S23ML = S3ML + B3M2#/N;  S23LS = S23ML#N#/(N-K2);
S21ML = S1ML + B1M2#/N;  S21LS = S21ML#N#/(N-K2);
S31ML = S1ML + B1M3#/N;  S31LS = S31ML#N#/(N-K3);
S32ML = S2ML + B2M3#/N;  S32LS = S32ML#N#/(N-K3);
 C12N = LOG(VECDIAG(S2ML#/S21ML))#(N#/2);
 C13N = LOG(VECDIAG(S3ML#/S31ML))#(N#/2);
 C23N = LOG(VECDIAG(S3ML#/S32ML))#(N#/2);
 C21N = LOG(VECDIAG(S1ML#/S12ML))#(N#/2);
 C31N = LOG(VECDIAG(S1ML#/S13ML))#(N#/2);
 C32N = LOG(VECDIAG(S2ML#/S23ML))#(N#/2);
V12 = (VECDIAG(S1ML#B1MM2#/S21ML##2))##0.5;
V13 = (VECDIAG(S1ML#B1MM3#/S31ML##2))##0.5;
V23 = (VECDIAG(S2ML#B2MM3#/S32ML##2))##0.5;
V21 = (VECDIAG(S2ML#B2MM1#/S12ML##2))##0.5;
V31 = (VECDIAG(S3ML#B3MM1#/S13ML##2))##0.5;
V32 = (VECDIAG(S3ML#B3MM2#/S23ML##2))##0.5;
C(LOC:LOC + 4,1) = C12N#/V12;
C(LOC:LOC + 4,2) = C13N#/V13;
C(LOC:LOC + 4,4) = C23N#/V23;
C(LOC:LOC + 4,3) = C21N#/V21;
C(LOC:LOC + 4,5) = C31N#/V31;
C(LOC:LOC + 4,6) = C32N#/V32;
  **********COMPARE TO CRITICAL VALUES;
    CH95 = ABS(C(LOC:LOC + 4,)) > = VC95;
    CC95 = CC95 + CH95;
    * POWER AND TYPE 1 ERROR COUNTS;
    CH9513 = CH95(,1 3); CH9525 = CH95(,2 5);
DO J = 1 TO 5;
```

```
IF ALL(CH9513(J,) = R1) THEN CPC12(J,)=CPC12(J,)+1;
   ELSE IF ALL(CH9513(J,)=R2) OR ALL(CH9513(J,)=R3)
     THEN CEC12(J,)=CEC12(J,)+1;
IF ALL(CH9525(J,) = R1) THEN CPC13(J,)=CPC13(J,)+1;
   ELSE IF ALL(CH9525(J,)=R2) OR ALL(CH9525(J,)=R3)
     THEN CEC13(J,)=CEC13(J,)+1;
  *COMPARE AND COUNT FOR 2 VS 3;
IF (CH95(J,4 6)=R3) THEN CNT23(J,1)=CNT23(J,1)+1;
  ELSE IF (CH95(J,4 6)=R0) THEN CNT23(J,2)=CNT23(J,2)+1;
  ELSE IF (CH95(J,4 6)=R1) THEN CNT23(J,3)=CNT23(J,3)+1;
  ELSE CNT23(J,4)=CNT23(J,4)+1;
END;
*****************W TEST***************************************;
**************************************************************;
W(LOC:LOC+4,1)=VECDIAG(S2LS-O21)#(N-K2)
   #/(VECDIAG(S1LS##2#2#TRB12 + S1LS#B1MM2#4))##0.5;
W(LOC:LOC+4,2)=VECDIAG(S3LS-O31)#(N-K3)
   #/(VECDIAG(S1LS##2#2#TRB13 + S1LS#B1MM3#4))##0.5;
W(LOC:LOC+4,4)=VECDIAG(S3LS-O32)#(N-K3)
   #/(VECDIAG(S2LS##2#2#TRB23 + S2LS#B2MM3#4))##0.5;
W(LOC:LOC+4,3)=VECDIAG(S1LS-O12)#(N-K1)
   #/(VECDIAG(S2LS##2#2#TRB21 + S2LS#B2MM1#4))##0.5;
W(LOC:LOC+4,5)=VECDIAG(S1LS-O13)#(N-K1)
   #/(VECDIAG(S3LS##2#2#TRB31 + S3LS#B3MM1#4))##0.5;
W(LOC:LOC+4,6)=VECDIAG(S2LS-O23)#(N-K2)
   #/(VECDIAG(S3LS##2#2#TRB32 + S3LS#B3MM2#4))##0.5;
    *************COMPARE TO CRITICAL VALUES;
     WH95=ABS(W(LOC:LOC+4,))>=VC95;
```

```
    CW95=CW95+WH95;
    * POWER COUNTS;
DO J=1 TO 5;
    IF ALL(WH95(J,1 3) = R1) THEN CPW12(J,)=CPW12(J,)+1;
    ELSE IF ALL(WH95(J,1 3)=R2) OR ALL(WH95(J,1 3)=R3)
      THEN CEW12(J,)=CEW12(J,)+1;
    IF ALL(WH95(J,2 5) = R1) THEN CPW13(J,)=CPW13(J,)+1;
    ELSE IF ALL(WH95(J,2 5)=R2) OR ALL(WH95(J,2 5)=R3)
      THEN CEW13(J,)=CEW13(J,)+1;
    *COMPARE AND COUNT FOR 2 VS 3;
    IF (WH95(J,4 6)=R3) THEN CNT23(5+J,1)=CNT23(5+J,1)+1;
    ELSE IF (WH95(J,4 6)=R0) THEN CNT23(5+J,2)=CNT23(5+J,2)+1;
    ELSE IF (WH95(J,4 6)=R1) THEN CNT23(5+J,3)=CNT23(5+J,3)+1;
    ELSE CNT23(5+J,4)=CNT23(5+J,4)+1;
END;
******************N0 TEST***********************************;
***********************************************************;
  T012=LOG(VECDIAG(S2LS#/O21))#0.5#(N-K2);
  T021=LOG(VECDIAG(S1LS#/O12))#0.5#(N-K1);
  T013=LOG(VECDIAG(S3LS#/O31))#0.5#(N-K3);
  T031=LOG(VECDIAG(S1LS#/O13))#0.5#(N-K1);
  T023=LOG(VECDIAG(S3LS#/O32))#0.5#(N-K3);
  T032=LOG(VECDIAG(S2LS#/O23))#0.5#(N-K2);
V012=VECDIAG((S1LS#/O21##2)#(B1MM2 +S1LS#TRB12#0.5));
V013=VECDIAG((S1LS#/O31##2)#(B1MM3 +S1LS#TRB13#0.5));
V023=VECDIAG((S2LS#/O32##2)#(B2MM3 +S2LS#TRB23#0.5));
V021=VECDIAG((S2LS#/O12##2)#(B2MM1 +S2LS#TRB21#0.5));
V031=VECDIAG((S3LS#/O13##2)#(B3MM1 +S3LS#TRB31#0.5));
```

```
V032=VECDIAG((S3LS#/O23##2)#(B3MM2  +S3LS#TRB32#0.5));
N0(LOC:LOC+4,1)=T012#/V012##0.5;  N0(LOC:LOC+4,3)=T021#/V021##0.5;
N0(LOC:LOC+4,2)=T013#/V013##0.5;  N0(LOC:LOC+4,5)=T031#/V031##0.5;
N0(LOC:LOC+4,4)=T023#/V023##0.5;  N0(LOC:LOC+4,6)=T032#/V032##0.5;
     *****COMPARE TO CRITICAL VALUES;
    NH95=ABS(N0(LOC:LOC+4,))>=VC95;
    CN95=CN95+NH95;
    *POWER COUNTS;
DO J=1 TO 5;
    IF ALL(NH95(J,1 3)=R1) THEN CPN12(J,)=CPN12(J,)+1;
    ELSE IF ALL(NH95(J,1 3)=R2) OR ALL(NH95(J,1 3)=R3)
     THEN CEN12(J,)=CEN12(J,)+1;
    IF ALL(NH95(J,2 5)=R1) THEN CPN13(J,)=CPN13(J,)+1;
    ELSE IF ALL(NH95(J,2 5)=R2) OR ALL(NH95(J,2 5)=R3)
     THEN CEN13(J,)=CEN13(J,)+1;
    *COMPARE AND COUNT FOR 2 VS 3;
    IF (NH95(J,4 6)=R3) THEN CNT23(10+J,1)=CNT23(10+J,1)+1;
    ELSE IF (NH95(J,4 6)=R0) THEN CNT23(10+J,2)=CNT23(10+J,2)+1;
    ELSE IF (NH95(J,4 6)=R1) THEN CNT23(10+J,3)=CNT23(10+J,3)+1;
    ELSE CNT23(10+J,4)=CNT23(10+J,4)+1;
END;
*************** *ATKINSON'S TEST************************;
*******************************************************;
 ATD12=VECDIAG(YH12'*M1'*YH12);
 ATD21=VECDIAG(YH21'*M2'*YH21);
 ATD13=VECDIAG(YH13'*M1'*YH13);
 ATD31=VECDIAG(YH31'*M3'*YH31);
 ATD23=VECDIAG(YH23'*M2'*YH23);
```

```
ATD32=VECDIAG(YH32'*M3'*YH32);
AD12=(VECDIAG(S1ML)#ATD12)##0.5;
AD21=(VECDIAG(S2ML)#ATD21)##0.5;
AD13=(VECDIAG(S1ML)#ATD13)##0.5;
AD31=(VECDIAG(S3ML)#ATD31)##0.5;
AD23=(VECDIAG(S2ML)#ATD23)##0.5;
AD32=(VECDIAG(S3ML)#ATD32)##0.5;
NA(LOC:LOC+4,1)=-VECDIAG(E1'*YH12)#/AD12;
NA(LOC:LOC+4,3)=-VECDIAG(E2'*YH21)#/AD21;
NA(LOC:LOC+4,2)=-VECDIAG(E1'*YH13)#/AD13;
NA(LOC:LOC+4,5)=-VECDIAG(E3'*YH31)#/AD31;
NA(LOC:LOC+4,4)=-VECDIAG(E2'*YH23)#/AD23;
NA(LOC:LOC+4,6)=-VECDIAG(E3'*YH32)#/AD32;
    *COMPARE TO CRITICAL VALUES;
    NAH95=ABS(NA(LOC:LOC+4,))>=VC95;
    CNA95=CNA95+NAH95;
    *POWER COUNTS;
DO J=1 TO 5;
    IF ALL(NAH95(J,1 3)=R1) THEN CPNA12(J,)=CPNA12(J,)+1;
    ELSE IF ALL(NAH95(J,1 3)=R2) OR ALL(NAH95(J,1 3)=R3)
     THEN CENA12(J,)=CENA12(J,)+1;
   IF ALL(NAH95(J,2 5)=R1) THEN CPNA13(J,)=CPNA13(J,)+1;
    ELSE IF ALL(NAH95(J,2 5)=R2) OR ALL(NAH95(J,2 5)=R3)
     THEN CENA13(J,)=CENA13(J,)+1;
    *COMPARE AND COUNT FOR 2 VS 3;
    IF (NAH95(J,4 6)=R3) THEN CNT23(15+J,1)=CNT23(15+J,1)+1;
    ELSE IF (NAH95(J,4 6)=R0) THEN CNT23(15+J,2)=CNT23(15+J,2)+1;
     ELSE IF (NAH95(J,4 6)=R1) THEN CNT23(15+J,3)=CNT23(15+J,3)+1;
```

```
    ELSE CNT23(15+J,4)=CNT23(15+J,4)+1;
END;
***************LINEARIZED COX TEST---NL*********************;
************************************************************;
NL(LOC:LOC+4,1)=VECDIAG(YH2`*YH2 - YH12`*YH12)#0.5#/AD12;
NL(LOC:LOC+4,3)=VECDIAG(YH1`*YH1 - YH21`*YH21)#0.5#/AD21;
NL(LOC:LOC+4,2)=VECDIAG(YH3`*YH3 - YH13`*YH13)#0.5#/AD13;
NL(LOC:LOC+4,5)=VECDIAG(YH1`*YH1 - YH31`*YH31)#0.5#/AD31;
NL(LOC:LOC+4,4)=VECDIAG(YH3`*YH3 - YH23`*YH23)#0.5#/AD23;
NL(LOC:LOC+4,6)=VECDIAG(YH2`*YH2 - YH32`*YH32)#0.5#/AD32;
  *COMPARE TO CRITICAL VALUES;
  NLH95=ABS(NL(LOC:LOC+4,))>=VC95;
  CNL95=CNL95+NLH95;
  *POWER COUNTS;
DO J=1 TO 5;
   IF ALL(NLH95(J,1 3)=R1) THEN CPNL12(J,)=CPNL12(J,)+1;
   ELSE IF ALL(NLH95(J,1 3)=R2) OR ALL(NLH95(J,1 3)=R3)
    THEN CENL12(J,)=CENL12(J,)+1;
   IF ALL(NLH95(J,2 5)=R1) THEN CPNL13(J,)=CPNL13(J,)+1;
   ELSE IF ALL(NLH95(J,2 5)=R2) OR ALL(NLH95(J,2 5)=R3)
     THEN CENL13(J,)=CENL13(J,)+1;
   *COMPARE AND COUNT FOR 2 VS 3;
   IF (NLH95(J,4 6)=R3) THEN CNT23(20+J,1)=CNT23(20+J,1)+1;
    ELSE IF (NLH95(J,4 6)=R0) THEN CNT23(20+J,2)=CNT23(20+J,2)+1;
     ELSE IF (NLH95(J,4 6)=R1) THEN CNT23(20+J,3)=CNT23(20+J,3)+1;
      ELSE CNT23(20+J,4)=CNT23(20+J,4)+1;
END;
************************************************************;
```

```
***** MATRIX CALCULATIONS FOR F-TEST,NJ-TEST **************;
N12=VECDIAG(Y'*M1*YH12)#/ATD12##0.5;    *NUM FOR JA AND NJ TESTS;
N13=VECDIAG(Y'*M1*YH13)#/ATD13##0.5;
N32=VECDIAG(Y'*M3*YH32)#/ATD32##0.5;
N23=VECDIAG(Y'*M2*YH23)#/ATD23##0.5;
N21=VECDIAG(Y'*M2*YH21)#/ATD21##0.5;
N31=VECDIAG(Y'*M3*YH31)#/ATD31##0.5;
P12=(K2-TRA12)#/(N-K1); AY12=YH2-E1#P12; *AJ TEST ADJUSTED Y2HATS;
P21=(K1-TRA12)#/(N-K2); AY21=YH1-E2#P21;
P13=(K3-TRA13)#/(N-K1); AY13=YH3-E1#P13;
P31=(K1-TRA13)#/(N-K3); AY31=YH1-E3#P31;
P23=(K3-TRA23)#/(N-K2); AY23=YH3-E2#P23;
P32=(K2-TRA23)#/(N-K3); AY32=YH2-E3#P32;
********************CLASSICAL F-TEST***********************;
**********************************************************;
X12=X1||X2; X12P=X12'; X12PX12=X12'*X12; XI12=SOLVE(X12PX12,X12P);
X13=X1||X3; X13P=X13'; X13PX13=X13'*X13; XI13=SOLVE(X13PX13,X13P);
X23=X2||X3; X23P=X23'; X23PX23=X23'*X23; XI23=SOLVE(X23PX23,X23P);
 M12=IN-X12*XI12; SIG12=(Y'*M12*Y)#/(N-K1-K2);
 M13=IN-X13*XI13; SIG13=(Y'*M13*Y)#/(N-K1-K3);
 M23=IN-X23*XI23; SIG23=(Y'*M23*Y)#/(N-K2-K3);
 SSREG12=Y'*(IN-M12)*Y;
 SSREG13=Y'*(IN-M13)*Y;
 SSREG23=Y'*(IN-M23)*Y;
F(LOC:LOC+4,1) =VECDIAG((SSREG12-B1'*X1'*Y)#/(SIG12#K2));
F(LOC:LOC+4,3) =VECDIAG((SSREG12-B2'*X2'*Y)#/(SIG12#K1));
F(LOC:LOC+4,2) =VECDIAG((SSREG13-B1'*X1'*Y)#/(SIG13#K3));
F(LOC:LOC+4,5) =VECDIAG((SSREG13-B3'*X3'*Y)#/(SIG13#K1));
```

```
F(LOC:LOC+4,4) =VECDIAG((SSREG23-B2'*X2'*Y)#/(SIG23#K3));
F(LOC:LOC+4,6) =VECDIAG((SSREG23-B3'*X3'*Y)#/(SIG23#K2));
*************************************************************;
*************************************************************;
    *COMPARE TO CRITICAL VALUES;
     FH95=F(LOC:LOC+4,)>=VF95M;
    CF95=CF95+FH95;
    *COUNT FOR POWER AND TYPE 1 ERROR;
DO J=1 TO 5;
     IF ALL(FH95(J,1 3)=R1) THEN CPF12(J,)=CPF12(J,)+1;
     ELSE IF ALL(FH95(J,1 3)=R2) OR ALL(FH95(J,1 3)=R3)
    THEN CEF12(J,)=CEF12(J,)+1;
    IF ALL(FH95(J,2 5)=R1) THEN CPF13(J,)=CPF13(J,)+1;
    ELSE IF ALL(FH95(J,2 5)=R2) OR ALL(FH95(J,2 5)=R3)
      THEN CEF13(J,)=CEF13(J,)+1;
    *COMPARE AND COUNT FOR 2 VS 3;
     IF (FH95(J,4 6)=R3) THEN CNT23(45+J,1)=CNT23(45+J,1)+1;
      ELSE IF (FH95(J,4 6)=R0) THEN CNT23(45+J,2)=CNT23(45+J,2)+1;
       ELSE IF (FH95(J,4 6)=R1) THEN CNT23(45+J,3)=CNT23(45+J,3)+1;
        ELSE CNT23(45+J,4)=CNT23(45+J,4)+1;
END;
*******************NEW JA TEST ******************************;
**************************************************************;
NJ(LOC:LOC+4,1)=N12#/(VECDIAG(SIG12))##0.5;
NJ(LOC:LOC+4,2)=N13#/(VECDIAG(SIG13))##0.5;
NJ(LOC:LOC+4,3)=N21#/(VECDIAG(SIG12))##0.5;
NJ(LOC:LOC+4,4)=N23#/(VECDIAG(SIG23))##0.5;
NJ(LOC:LOC+4,5)=N31#/(VECDIAG(SIG13))##0.5;
```

```
NJ(LOC:LOC+4,6)=N32#/(VECDIAG(SIG23))##0.5;
*****************************************************;
******************************************************;
    *COMPARE TO CRITICAL VALUES;
    NJH95=ABS(NJ(LOC:LOC+4,))>=VNJ95M;
    CNJ95=CNJ95+NJH95;
    *POWER AND TYPE 1 ERROR COUNTS;
DO J=1 TO 5;
     IF ALL(NJH95(J,1 3)=R1) THEN CPNJ12(J,)=CPNJ12(J,)+1;
      ELSE IF ALL(NJH95(J,1 3)=R2) OR ALL(NJH95(J,1 3)=R3) THEN
     CENJ12(J,)=CENJ12(J,)+1;
    IF ALL(NJH95(J,2 5)=R1) THEN CPNJ13(J,)=CPNJ13(J,)+1;
     ELSE IF ALL(NJH95(J,2 5)=R2) OR ALL(NJH95(J,2 5)=R3) THEN
    CENJ13(J,)=CENJ13(J,)+1;
    *COMPARE AND COUNT FOR 2 VS 3;
    IF (NJH95(J,4 6)=R3) THEN CNT23(40+J,1)=CNT23(40+J,1)+1;
     ELSE IF (NJH95(J,4 6)=R0) THEN CNT23(40+J,2)=CNT23(40+J,2)+1;
      ELSE IF (NJH95(J,4 6)=R1) THEN CNT23(40+J,3)=CNT23(40+J,3)+1;
     ELSE CNT23(40+J,4)=CNT23(40+J,4)+1;
 END;
************************************************************;
** DUE TO RECONSTRUCTING REGRESSOR VECTORS TO INCLUDE VARIOUS
** Y HATS IN THE J- AJ- AND JA- TESTS -- THEY ARE COMPUTED
** FOR EACH Y DISTRIBUTIONS SEPARATELY   .... LOOP FOR K=1 TO5;
******************J TEST***********************************;
************************************************************;
```

```
    DO K = 1 TO 5;
X12=X1||YH2(,K);  X12P=X12'; X12PX12=X12'*X12; XI12=SOLVE(X12PX12,X12P);
SSRJ1=Y(,K)'*(IN-X12*XI12)*Y(,K);
    SJ1=SSRJ1#/(N-K1-1);
JJ(LOC+K-1,1)=(B2(,K)'*X2'*M1*Y(,K))#/SQRT(SJ1*B2M1(K,K));
X21=X2||YH1(,K);  X21P=X21'; X21PX21=X21'*X21; XI21=SOLVE(X21PX21,X21P);
SSRJ2=Y(,K)'*(IN-X21*XI21)*Y(,K);
    SJ2=SSRJ2#/(N-K2-1);
JJ(LOC+K-1,3)=(B1(,K)'*X1'*M2*Y(,K))#/SQRT(SJ2*B1M2(K,K));
X13=X1||YH3;  X13P=X13'; X13PX13=X13'*X13; XI13=SOLVE(X13PX13,X13P);
SSRJ1=Y(,K)'*(IN-X13*XI13)*Y(,K);
    SJ1=SSRJ1#/(N-K1-1);
JJ(LOC+K-1,2)=(B3(,K)'*X3'*M1*Y(,K))#/SQRT(SJ1*B3M1(K,K));
X31=X3||YH1(,K);  X31P=X31'; X31PX31=X31'*X31; XI31=SOLVE(X31PX31,X31P);
SSRJ3=Y(,K)'*(IN-X31*XI31)*Y(,K);
    SJ3=SSRJ3#/(N-K3-1);
JJ(LOC+K-1,5)=(B1(,K)'*X1'*M3*Y(,K))#/SQRT(SJ3*B1M3(K,K));
X23=X2||YH3(,K);  X23P=X23'; X23PX23=X23'*X23; XI23=SOLVE(X23PX23,X23P);
SSRJ2=Y(,K)'*(IN-X23*XI23)*Y(,K);
    SJ2=SSRJ2#/(N-K2-1);
JJ(LOC+K-1,4)=(B3(,K)'*X3'*M2*Y(,K))#/SQRT(SJ2*B3M2(K,K));
X32=X3||YH2(,K);  X32P=X32'; X32PX32=X32'*X32; XI32=SOLVE(X32PX32,X32P);
SSRJ3=Y(,K)'*(IN-X32*XI32)*Y(,K);
    SJ3=SSRJ3#/(N-K3-1);
JJ(LOC+K-1,6)=(B2(,K)'*X2'*M3*Y(,K))#/SQRT(SJ3*B2M3(K,K));
     *COMPARE TO CRITICAL VALUES;
     JH95(K,)=JJ(LOC+K-1,)>=VJ95(NN,);
*    CJ95=CJ95+JH95;
```

```
*COUNT FOR POWER AND TYPE1 ERROR;
IF ALL(JH95(K,1 3)=R1) THEN CPJ12(K,)=CPJ12(K,)+1;
 ELSE IF ALL(JH95(K,1 3)=R2) OR ALL(JH95(K,1 3)=R3)
          THEN CEJ12(K,)=CEJ12(K,)+1;
IF ALL(JH95(K,2 5)=R1) THEN CPJ13(K,)=CPJ13(K,)+1;
 ELSE IF ALL(JH95(K,2 5)=R2) OR ALL(JH95(K,2 5)=R3)
          THEN CEJ13(K,)=CEJ13(K,)+1;
*COMPARE AND COUNT FOR 2 VS 3;
IF (JH95(K,4 6)=R3) THEN CNT23(25+K,1)=CNT23(25+K,1)+1;
 ELSE IF (JH95(K,4 6)=R0) THEN CNT23(25+K,2)=CNT23(25+K,2)+1;
  ELSE IF (JH95(K,4 6)=R1) THEN CNT23(25+K,3)=CNT23(25+K,3)+1;
   ELSE CNT23(25+K,4)=CNT23(25+K,4)+1;
*****************ADJUSTED J-TEST:::: AJ*********************;
***********************************************************;
****CALCULATION OF SIG HAT FOR THE ADJUSTED J-TEST;
A12=X1||AY12(,K); A21=X2||AY21(,K); A13=X1||AY13(,K); A31=X3||AY31(,K);
A23=X2||AY23(,K); A32=X3||AY32(,K);
A12P=A12'; A12PA12=A12'*A12; AI12=SOLVE(A12PA12,A12P);
A21P=A21'; A21PA21=A21'*A21; AI21=SOLVE(A21PA21,A21P);
A13P=A13'; A13PA13=A13'*A13; AI13=SOLVE(A13PA13,A13P);
A31P=A31'; A31PA31=A31'*A31; AI31=SOLVE(A31PA31,A31P);
A23P=A23'; A23PA23=A23'*A23; AI23=SOLVE(A23PA23,A23P);
A32P=A32'; A32PA32=A32'*A32; AI32=SOLVE(A32PA32,A32P);
SA12=Y(,K)'*(IN-A12*AI12)*Y(,K) #/ (N-K1-1);
SA21=Y(,K)'*(IN-A21*AI21)*Y(,K) #/ (N-K2-1);
SA13=Y(,K)'*(IN-A13*AI13)*Y(,K) #/ (N-K1-1);
SA31=Y(,K)'*(IN-A31*AI31)*Y(,K) #/ (N-K3-1);
SA23=Y(,K)'*(IN-A23*AI23)*Y(,K) #/ (N-K2-1);
```

```
SA32=Y(,K)'*(IN-A32*AI32)*Y(,K) #/ (N-K3-1);
AJ(LOC+K-1,1)=E1(,K)'*AY12(,K)#/SQRT(SA12*AY12(,K)'*M1*AY12(,K));
AJ(LOC+K-1,3)=E2(,K)'*AY21(,K)#/SQRT(SA21*AY21(,K)'*M2*AY21(,K));
AJ(LOC+K-1,2)=E1(,K)'*AY13(,K)#/SQRT(SA13*AY13(,K)'*M1*AY13(,K));
AJ(LOC+K-1,5)=E3(,K)'*AY31(,K)#/SQRT(SA31*AY31(,K)'*M3*AY31(,K));
AJ(LOC+K-1,4)=E2(,K)'*AY23(,K)#/SQRT(SA23*AY23(,K)'*M2*AY23(,K));
AJ(LOC+K-1,6)=E3(,K)'*AY32(,K)#/SQRT(SA32*AY32(,K)'*M3*AY32(,K));
    *COMPARE TO CRITICAL VALUES;
    AJH95(K,)=AJ(LOC+K-1,)>=VJ95(NN,);
  *  CAJ95=CAJ95+AJH95;
    *COUNT FOR POWER AND TYPE1 ERROR;
    IF ALL(AJH95(K,1 3)=R1) THEN CPAJ12(K,)=CPAJ12(K,)+1;
     ELSE IF ALL(AJH95(K,1 3)=R2) OR ALL(AJH95(K,1 3)=R3)
                THEN CEAJ12(K,)=CEAJ12(K,)+1;
   IF ALL(AJH95(K,2 5)=R1) THEN CPAJ13(K,)=CPAJ13(K,)+1;
    ELSE IF ALL(AJH95(K,2 5)=R2) OR ALL(AJH95(K,2 5)=R3)
                THEN CEAJ13(K,)=CEAJ13(K,)+1;
    *COMPARE AND COUNT FOR 2 VS 3;
   IF (AJH95(K,4 6)=R3) THEN CNT23(30+K,1)=CNT23(30+K,1)+1;
    ELSE IF (AJH95(K,4 6)=R0) THEN CNT23(30+K,2)=CNT23(30+K,2)+1;
     ELSE IF (AJH95(K,4 6)=R1) THEN CNT23(30+K,3)=CNT23(30+K,3)+1;
      ELSE CNT23(30+K,4)=CNT23(30+K,4)+1;
*******************JA TEST*****************************;
*****************************************************;
*SIGS FOR JA-TEST;
XJ12=X1||YH12(,K);  XJ21=X2||YH21(,K);
XJ13=X1||YH13(,K);  XJ31=X3||YH31(,K);
XJ23=X2||YH23(,K);  XJ32=X3||YH32(,K);
```

```
XJ12P=XJ12′; XJ12P12=XJ12′*XJ12; XIJ12=SOLVE(XJ12P12,XJ12P);
XJ21P=XJ21′; XJ21P21=XJ21′*XJ21; XIJ21=SOLVE(XJ21P21,XJ21P);
XJ13P=XJ13′; XJ13P13=XJ13′*XJ13; XIJ13=SOLVE(XJ13P13,XJ13P);
XJ31P=XJ31′; XJ31P31=XJ31′*XJ31; XIJ31=SOLVE(XJ31P31,XJ31P);
XJ23P=XJ23′; XJ23P23=XJ23′*XJ23; XIJ23=SOLVE(XJ23P23,XJ23P);
XJ32P=XJ32′; XJ32P32=XJ32′*XJ32; XIJ32=SOLVE(XJ32P32,XJ32P);
SJA12=Y(,K)′*(IN-XJ12*XIJ12)*Y(,K)#/(N-K1-1);
SJA13=Y(,K)′*(IN-XJ13*XIJ13)*Y(,K)#/(N-K1-1);
SJA23=Y(,K)′*(IN-XJ23*XIJ23)*Y(,K)#/(N-K2-1);
SJA32=Y(,K)′*(IN-XJ32*XIJ32)*Y(,K)#/(N-K3-1);
SJA21=Y(,K)′*(IN-XJ21*XIJ21)*Y(,K)#/(N-K2-1);
SJA31=Y(,K)′*(IN-XJ31*XIJ31)*Y(,K)#/(N-K3-1);
JA(LOC+K-1,1)=N12(K,)#/SQRT(SJA12);
JA(LOC+K-1,2)=N13(K,)#/SQRT(SJA13);
JA(LOC+K-1,3)=N21(K,)#/SQRT(SJA21);
JA(LOC+K-1,4)=N23(K,)#/SQRT(SJA23);
JA(LOC+K-1,5)=N31(K,)#/SQRT(SJA31);
JA(LOC+K-1,6)=N32(K,)#/SQRT(SJA32);
   *COMPARE TO CRITICAL VALUES;
   JAH95(K,)=JA(LOC+K-1,)>=VJ95(NN,);
 *  CJA95=CJA95+JAH95;
   *COUNT FOR POWER AND TYPE1 ERROR;
   IF ALL(JAH95(K,1 3)=R1) THEN CPJA12(K,)=CPJA12(K,)+1;
    ELSE IF ALL(JAH95(K,1 3)=R2) OR ALL(JAH95(K,1 3)=R3)
                  THEN CEJA12(K,)=CEJA12(K,)+1;
   IF ALL(JAH95(K,2 5)=R1) THEN CPJA13(K,)=CPJA13(K,)+1;
    ELSE IF ALL(JAH95(K,2 5)=R2) OR ALL(JAH95(K,2 5)=R3)
                 THEN CEJA13(K,)=CEJA13(K,)+1;
```

```
*COMPARE AND COUNT FOR 2 VS 3;
IF (JAH95(K,4 6)=R3) THEN CNT23(35+K,1)=CNT23(35+K,1)+1;
 ELSE IF (JAH95(K,4 6)=R0) THEN CNT23(35+K,2)=CNT23(35+K,2)+1;
  ELSE IF (JAH95(K,4 6)=R1) THEN CNT23(35+K,3)=CNT23(35+K,3)+1;
   ELSE CNT23(35+K,4)=CNT23(35+K,4)+1;
*****************************************************;
 END;    **** END OF K  FROM 1 TO 5 LOOP;
*****************************************************;
*CALCULATE P-VALUES ASSOCIATED WITH RELECTING FALSE MODEL;
*****************************************************;
 DO L = 1 TO 5;
***********************;
IF CH95(L,1)=0 THEN PV12(LOC+L-1,1)=(1-PROBNORM(ABS(C(LOC+L-1,3))))*2;
 ELSE PV12(LOC+L-1,1)=1;
IF WH95(L,1)=0 THEN PV12(LOC+L-1,2)=(1-PROBNORM(ABS(W(LOC+L-1,3))))*2;
 ELSE PV12(LOC+L-1,2)=1;
IF NH95(L,1)=0 THEN PV12(LOC+L-1,3)=(1-PROBNORM(ABS(N0(LOC+L-1,3))))*2;
 ELSE PV12(LOC+L-1,3)=1;
IF NAH95(L,1)=0 THEN PV12(LOC+L-1,4)=(1-PROBNORM(ABS(NA(LOC+L-1,3))))*2;
 ELSE PV12(LOC+L-1,4)=1;
IF NLH95(L,1)=0 THEN PV12(LOC+L-1,5)=(1-PROBNORM(ABS(NL(LOC+L-1,3))))*2;
 ELSE PV12(LOC+L-1,5)=1;
IF JH95(L,1)  =0 THEN PV12(LOC+L-1,6)=(1-PROBT(ABS(JJ(LOC+L-1,3)),N-K1-1))*2;
 ELSE PV12(LOC+L-1,6)=1;
IF AJH95(L,1)=0 THEN PV12(LOC+L-1,7)=(1-PROBT(ABS(AJ(LOC+L-1,3)),N-K1-1))*2;
 ELSE PV12(LOC+L-1,7)=1;
IF JAH95(L,1)=0 THEN PV12(LOC+L-1,8)=(1-PROBT(ABS(JA(LOC+L-1,3)),N-K1-1))*2;
```

```
 ELSE PV12(LOC+L-1,8)=1;
IF NJH95(L,1)=0 THEN PV12(LOC+L-1,9)=(1-PROBT(ABS(NJ(LOC+L-1,3)),N-K1-K2))*2;
 ELSE PV12(LOC+L-1,9)=1;
IF FH95(L,1)=0 THEN PV12(LOC+L-1,10)=1-PROBF(F(LOC+L-1,3),K2,N-K1-K2);
 ELSE PV12(LOC+L-1,10)=1;
*********************************************************;
   *RANK P VALUES WITHIN EACH ITERATION (ROW);
    RK12(LOC+L-1,)=RANKTIE(PV12(LOC+L-1,));
   *COUNT TIES FOR ADJUSTMENT ON KENDALL'S C.C. ;
    TIES12(L,) = (PV12(LOC+L-1,)=TIECK);
NTIES12(L,) = TIES12(L,+); SUMTIE12(L,)=SUMTIE12(L,)
+(NTIES12(L,)##3-NTIES12(L,))#/12;
************************************************************;
*CALCULATE P-VALUES ASSOCIATED WITH REJECTING FALSE MODEL--13;
IF CH95(L,2)=0 THEN PV13(LOC+L-1,1)=(1-PROBNORM(ABS(C(LOC+L-1,5))))*2;
 ELSE PV13(LOC+L-1,1)=1;
IF WH95(L,2)=0 THEN PV13(LOC+L-1,2)=(1-PROBNORM(ABS(W(LOC+L-1,5))))*2;
 ELSE PV13(LOC+L-1,2)=1;
IF NH95(L,2)=0 THEN PV13(LOC+L-1,3)=(1-PROBNORM(ABS(N0(LOC+L-1,5))))*2;
 ELSE PV13(LOC+L-1,3)=1;
IF NAH95(L,2)=0 THEN PV13(LOC+L-1,4)=(1-PROBNORM(ABS(NA(LOC+L-1,5))))*2;
 ELSE PV13(LOC+L-1,4)=1;
IF NLH95(L,2)=0 THEN PV13(LOC+L-1,5)=(1-PROBNORM(ABS(NL(LOC+L-1,5))))*2;
 ELSE PV13(LOC+L-1,5)=1;
IF JH95(L,2) =0 THEN PV13(LOC+L-1,6)=(1-PROBT(ABS(JJ(LOC+L-1,5)),N-K1-1))*2;
 ELSE PV13(LOC+L-1,6)=1;
IF AJH95(L,2)=0 THEN PV13(LOC+L-1,7)=(1-PROBT(ABS(AJ(LOC+L-1,5)),N-K1-1))*2;
```

```
  ELSE PV13(LOC+L-1,7)=1;
IF JAH95(L,2)=0 THEN PV13(LOC+L-1,8)=(1-PROBT(ABS(JA(LOC+L-1,5)),N-K1-1))*2;
  ELSE PV13(LOC+L-1,8)=1;
IF NJH95(L,2)=0 THEN PV13(LOC+L-1,9)=(1-PROBT(ABS(NJ(LOC+L-1,5)),N-K1-K3))*2;
  ELSE PV13(LOC+L-1,9)=1;
IF FH95(L,2)=0 THEN PV13(LOC+L-1,10)=1-PROBF(F(LOC+L-1,5),K2,N-K1-K3);
  ELSE PV13(LOC+L-1,10)=1;
**********RANK P VALUES WITHIN EACH ITERATION (ROW);
    RK13(LOC+L-1,)=RANKTIE(PV13(LOC+L-1,));
    *COUNT TIES FOR ADJUSTMENT ON KENDALL'S C.C. ;
    TIES13(L,) = (PV13(LOC+L-1,)=TIECK);
NTIES13(L,) = TIES13(L,+); SUMTIE13(L,)=SUMTIE13(L,)
+(NTIES13(L,)##3-NTIES13(L,))#/12;
**********
   END;  END;  *** END OF P-VALUE AND RANK LOOPS;
*****************************************************************;
********POWER SUMS AND SS CALCULATIONS FOLLOWING THE LOOP;
***********************************************************;
*END OF ITERATIVE LOOP;
END;
***********************************************************;
*****************************************************************;
****CREATE VECTORS FOR STORING POWER,STND ERR, IERROR, STND ERR;
*****************************************************************;
C12=J(5,4,0); W12=J(5,4,0); N12=J(5,4,0); NA12=J(5,4,0); NL12=J(5,4,0);
C13=J(5,4,0); W13=J(5,4,0); N13=J(5,4,0); NA13=J(5,4,0); NL13=J(5,4,0);
J12=J(5,4,0); AJ12=J(5,4,0); JA12=J(5,4,0); NJ12=J(5,4,0);
J13=J(5,4,0); AJ13=J(5,4,0); JA13=J(5,4,0); NJ13=J(5,4,0);
```

```
F12=J(5,4,0);
F13=J(5,4,0);
*NEED TO COMPUTE POWER AND TYPE 1 ERROR PROBABILITIES;
C12(,1)=CPC12#/NITER; C12(,3)=CEC12#/NITER;
C13(,1)=CPC13#/NITER; C13(,3)=CEC13#/NITER;
C12(,2)=SQRT((CPC12-CPC12##2#/NITER)#/(NITER#(NITER-1)));
C12(,4)=SQRT((CEC12-CEC12##2#/NITER)#/(NITER#(NITER-1)));
C13(,2)=SQRT((CPC13-CPC13##2#/NITER)#/(NITER#(NITER-1)));
C13(,4)=SQRT((CEC13-CEC13##2#/NITER)#/(NITER#(NITER-1)));
W12(,1)=CPW12#/NITER; W12(,3)=CEW12#/NITER;
W13(,1)=CPW13#/NITER; W13(,3)=CEW13#/NITER;
W12(,2)=SQRT((CPW12-CPW12##2#/NITER)#/(NITER#(NITER-1)));
W12(,4)=SQRT((CEW12-CEW12##2#/NITER)#/(NITER#(NITER-1)));
W13(,2)=SQRT((CPW13-CPW13##2#/NITER)#/(NITER#(NITER-1)));
W13(,4)=SQRT((CEW13-CEW13##2#/NITER)#/(NITER#(NITER-1)));
N12(,1)=CPN12#/NITER; N12(,3)=CEN12#/NITER;
N13(,1)=CPN13#/NITER; N13(,3)=CEN13#/NITER;
N12(,2)=SQRT((CPN12-CPN12##2#/NITER)#/(NITER#(NITER-1)));
N12(,4)=SQRT((CEN12-CEN12##2#/NITER)#/(NITER#(NITER-1)));
N13(,2)=SQRT((CPN13-CPN13##2#/NITER)#/(NITER#(NITER-1)));
N13(,4)=SQRT((CEN13-CEN13##2#/NITER)#/(NITER#(NITER-1)));
NA12(,1)=CPNA12#/NITER;  NA12(,3)=CENA12#/NITER;
NA13(,1)=CPNA13#/NITER;  NA13(,3)=CENA13#/NITER;
NA12(,2)=SQRT((CPNA12-CPNA12##2#/NITER)#/(NITER#(NITER-1)));
NA12(,4)=SQRT((CENA12-CENA12##2#/NITER)#/(NITER#(NITER-1)));
NA13(,2)=SQRT((CPNA13-CPNA13##2#/NITER)#/(NITER#(NITER-1)));
NA13(,4)=SQRT((CENA13-CENA13##2#/NITER)#/(NITER#(NITER-1)));
NL12(,1)=CPNL12#/NITER;  NL12(,3)=CENL12#/NITER;
```

```
NL13(,1)=CPNL13#/NITER;  NL13(,3)=CENL13#/NITER;
NL12(,2)=SQRT((CPNL12-CPNL12##2#/NITER)#/(NITER#(NITER-1)));
NL12(,4)=SQRT((CENL12-CENL12##2#/NITER)#/(NITER#(NITER-1)));
NL13(,2)=SQRT((CPNL13-CPNL13##2#/NITER)#/(NITER#(NITER-1)));
NL13(,4)=SQRT((CENL13-CENL13##2#/NITER)#/(NITER#(NITER-1)));
J12(,1)=CPJ12#/NITER;  J12(,3)=CEJ12#/NITER;
J13(,1)=CPJ13#/NITER;  J13(,3)=CEJ13#/NITER;
J12(,2)=SQRT((CPJ12-CPJ12##2#/NITER)#/(NITER#(NITER-1)));
J12(,4)=SQRT((CEJ12-CEJ12##2#/NITER)#/(NITER#(NITER-1)));
J13(,2)=SQRT((CPJ13-CPJ13##2#/NITER)#/(NITER#(NITER-1)));
J13(,4)=SQRT((CEJ13-CEJ13##2#/NITER)#/(NITER#(NITER-1)));
AJ12(,1)=CPAJ12#/NITER;    AJ12(,3)=CEAJ12#/NITER;
AJ13(,1)=CPAJ13#/NITER;    AJ13(,3)=CEAJ13#/NITER;
AJ12(,2)=SQRT((CPAJ12-CPAJ12##2#/NITER)#/(NITER#(NITER-1)));
AJ12(,4)=SQRT((CEAJ12-CEAJ12##2#/NITER)#/(NITER#(NITER-1)));
AJ13(,2)=SQRT((CPAJ13-CPAJ13##2#/NITER)#/(NITER#(NITER-1)));
AJ13(,4)=SQRT((CEAJ13-CEAJ13##2#/NITER)#/(NITER#(NITER-1)));
JA12(,1)=CPJA12#/NITER;    JA12(,3)=CEJA12#/NITER;
JA13(,1)=CPJA13#/NITER;    JA13(,3)=CEJA13#/NITER;
JA12(,2)=SQRT((CPJA12-CPJA12##2#/NITER)#/(NITER#(NITER-1)));
JA12(,4)=SQRT((CEJA12-CEJA12##2#/NITER)#/(NITER#(NITER-1)));
JA13(,2)=SQRT((CPJA13-CPJA13##2#/NITER)#/(NITER#(NITER-1)));
JA13(,4)=SQRT((CEJA13-CEJA13##2#/NITER)#/(NITER#(NITER-1)));
NJ12(,1)=CPNJ12#/NITER;    NJ12(,3)=CENJ12#/NITER;
NJ13(,1)=CPNJ13#/NITER;    NJ13(,3)=CENJ13#/NITER;
NJ12(,2)=SQRT((CPNJ12-CPNJ12##2#/NITER)#/(NITER#(NITER-1)));
NJ12(,4)=SQRT((CENJ12-CENJ12##2#/NITER)#/(NITER#(NITER-1)));
NJ13(,2)=SQRT((CPNJ13-CPNJ13##2#/NITER)#/(NITER#(NITER-1)));
```

```
NJ13(,4)=SQRT((CENJ13-CENJ13##2#/NITER)#/(NITER#(NITER-1)));
F12(,1)=CPF12#/NITER;      F12(,3)=CEF12#/NITER;
F13(,1)=CPF13#/NITER;      F13(,3)=CEF13#/NITER;
F12(,2)=SQRT((CPF12-CPF12##2#/NITER)#/(NITER#(NITER-1)));
F12(,4)=SQRT((CEF12-CEF12##2#/NITER)#/(NITER#(NITER-1)));
F13(,2)=SQRT((CPF13-CPF13##2#/NITER)#/(NITER#(NITER-1)));
F13(,4)=SQRT((CEF13-CEF13##2#/NITER)#/(NITER#(NITER-1)));
*PRINT C12 W12 N12 NA12 NL12 J12 AJ12 JA12 NJ12 F12 ;
*PRINT C13 W13 N13 NA13 NL13 J13 AJ13 JA13 NJ13 F13 ;
TESTS12=C12//W12//N12//NA12//NL12//J12//AJ12//JA12//NJ12//F12 ;
TESTS13=C13//W13//N13//NA13//NL13//J13//AJ13//JA13//NJ13//F13 ;
***HOLD TEST RESULTS TO COMBINE WITH AVG RANKS FURTHER DOWN;
***********************************************************;
*********************************************************;
*****CALCULATE MEAN AND STND ERROR OF R2 FOR ALL 3 MODELS;
*********************************************************;
     MR2=SR2#/NITER; SER2=J(3,5,0);
    DO I=1 TO 3;
SER2(I,)=((SUSR2(I,)-SR2(I,)##2#/NITER)#/(NITER*(NITER-1)))##0.5;
    END;  R2INFO=MR2||SER2;   PRINT R2INFO;
***********************************************************;
************************************************************;
******CALCULATE KENDALL'S COEFFICIENT OF CONCORDANCE--KW12 AND 13;
************************************************************;
SUMRK12=J(5,10,0);  SUMRK13=J(5,10,0);   RBAR=J(5,10,(NITER*6));
  ONE5=J(5,1,1);
   **OBTAIN 5 INDIVIDUAL SUMS OF RANKINGS;
DO M = 1 TO NITER;   LOC=(M-1)*5 + 1;
```

```
  SUMRK12=SUMRK12+RK12(LOC:LOC+4,);
  SUMRK13=SUMRK13+RK13(LOC:LOC+4,);
 END;
   RKB12=SUMRK12-RBAR;          RKB13=SUMRK13-RBAR;
   WSUM12=(RKB12##2)(,+);   WSUM13=(RKB13##2)(,+);
  KW12=WSUM12#12#/(ONE5#990#NITER**2 - SUMTIE12#NITER);
  KW13=WSUM13#12#/(ONE5#990#NITER**2 - SUMTIE13#NITER);
***CALCULATE THE AVERAGE RANKING OF EACH TEST FOR THIS RUN;
    AVRANK12=SUMRK12#/NITER;
    AVRANK13=SUMRK13#/NITER;
****STACK RANKINGS BY TESTS AND DISTRIBUTIONS WITHIN EACH TEST TO
  ****** MATCH WITH OTHER TEST STAT INFO ************************;
ARK12=AVRANK12(,1)//AVRANK12(,2)//AVRANK12(,3)//AVRANK12(,4)//
    AVRANK12(,5)//AVRANK12(,6)//AVRANK12(,7)//AVRANK12(,8)//
    AVRANK12(,9)//AVRANK12(,10);
ARK13=AVRANK13(,1)//AVRANK13(,2)//AVRANK13(,3)//AVRANK13(,4)//
    AVRANK13(,5)//AVRANK13(,6)//AVRANK13(,7)//AVRANK13(,8)//
    AVRANK13(,9)//AVRANK13(,10);
***** COMBINE ALL TEST STAT DATA TOGETHER FOR 1 VS 2 AND 3 ****;
TESTS12=TESTS12||ARK12;     PRINT TESTS12;
TESTS13=TESTS13||ARK13;     PRINT TESTS13;
***CALCULATE THE P-VALUE ASSOCIATED WITH CORRES S FOR KENDALL'S W;
   S12=KW12#NITER#9;   S13=KW13#NITER#9;
  SIGW12=J(5,1,0);   SIGW13=J(5,1,0);
 DO J = 1 TO 5;
    SIGW12(J,)=1-PROBCHI(S12(J,),9);
    SIGW13(J,)=1-PROBCHI(S13(J,),9);
  END;
```

```
     KENDAL12=KW12||S12||SIGW12;
     KENDAL13=KW13||S13||SIGW13;
PRINT KENDAL12 KENDAL13;
*************************************************************;
   ***CALCULATE AND PRINT % FOR MODEL 2 VS 3;
PCNT23=CNT23#/NITER;
  PRINT PCNT23;
***********************************************************;
*********************** THE END *************************;
***********************************************************;
/*
//
```

Appendix E: ANOVA Results for Equal k Cases

This section presents the analyses for cases involving samples of size 20 as well as an equal number of regressors in the competing models. Although the assumptions for the analysis have been violated strictly speaking, the results indicate the effects of R^2 and ρ^2 on both the observed power and significance level of the testing procedures. In both cases, the two-way ANOVA for each procedure is presented. When the effects are significant, further information is given by Duncan's Multiple Range Test on the appropriate means.

E.1. Observed Significance Level

Cox (N):

DEPENDENT VARIABLE: CA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.04723897	0.00314926	10.99
ERROR	24	0.00687700	0.00028654	PR > F
CORRECTED TOTAL	39	0.05411598		0.0001

R-SQUARE	C.V.	ROOT MSE	CA MEAN
0.872921	14.4341	0.01692754	0.11727500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.02438093	28.36	0.0001
P2	3	0.01200145	13.96	0.0001
R2*P2	9	0.01085659	4.21	0.0023

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.02014613	23.44	0.0001
P2	3	0.01254664	14.60	0.0001
R2*P2	9	0.01085659	4.21	0.0023

W-test:

DEPENDENT VARIABLE: WA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.00157822	0.00010521	2.28
ERROR	24	0.00110875	0.00004620	PR > F
CORRECTED TOTAL	39	0.00268698		0.0349

R-SQUARE	C.V.	ROOT MSE	WA MEAN
0.587361	18.3824	0.00679690	0.03697500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.00126306	9.11	0.0003
P2	3	0.00004801	0.35	0.7920
R2*P2	9	0.00026715	0.64	0.7501

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.00114510	8.26	0.0006
P2	3	0.00006260	0.45	0.7185
R2*P2	9	0.00026715	0.64	0.7501

$\tilde{N}$-test:

DEPENDENT VARIABLE: NTA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.00176610	0.00011774	2.72
ERROR	24	0.00103750	0.00004323	PR > F
CORRECTED TOTAL	39	0.00280360		0.0139

R-SQUARE	C.V.	ROOT MSE	NTA MEAN
0.629940	12.7916	0.00657489	0.05140000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.00048914	3.77	0.0238
P2	3	0.00038800	2.99	0.0509
R2*P2	9	0.00088896	2.28	0.0514

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.00054168	4.18	0.0163
P2	3	0.00025161	1.94	0.1501
R2*P2	9	0.00088896	2.28	0.0514

NA-test:

DEPENDENT VARIABLE: NAA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.00272572	0.00018171	1.73
ERROR	24	0.00252125	0.00010505	PR > F
CORRECTED TOTAL	39	0.00524698		0.1121

R-SQUARE	C.V.	ROOT MSE	NAA MEAN
0.519485	13.4906	0.01024949	0.07597500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.00060377	1.92	0.1540
P2	3	0.00014415	0.46	0.7146
R2*P2	9	0.00197781	2.09	0.0721

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.00036481	1.16	0.3464
P2	3	0.00013703	0.43	0.7301
R2*P2	9	0.00197781	2.09	0.0721

NL-test:

DEPENDENT VARIABLE: NLA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.02955523	0.00197035	10.83
ERROR	24	0.00436775	0.00018199	PR > F
CORRECTED TOTAL	39	0.03392298		0.0001

R-SQUARE	C.V.	ROOT MSE	NLA MEAN
0.871245	14.0855	0.01349035	0.09577500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.01477393	27.06	0.0001
P2	3	0.00912531	16.71	0.0001
R2*P2	9	0.00565598	3.45	0.0073

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.01199366	21.97	0.0001
P2	3	0.01001103	18.34	0.0001
R2*P2	9	0.00565598	3.45	0.0073

J-test:

DEPENDENT VARIABLE: JA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.02802165	0.00186811	20.89
ERROR	24	0.00214625	0.00008943	PR > F
CORRECTED TOTAL	39	0.03016790		0.0001

R-SQUARE	C.V.	ROOT MSE	JA MEAN
0.928856	16.8717	0.00945659	0.05605000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.00859948	32.05	0.0001
P2	3	0.01566939	58.41	0.0001
R2*P2	9	0.00375278	4.66	0.0012

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.00779228	29.05	0.0001
P2	3	0.01618153	60.32	0.0001
R2*P2	9	0.00375278	4.66	0.0012

AJ-test:

DEPENDENT VARIABLE: AJA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.00034427	0.00002295	0.75
ERROR	24	0.00073050	0.00003044	PR > F
CORRECTED TOTAL	39	0.00107478		0.7106

R-SQUARE	C.V.	ROOT MSE	AJA MEAN
0.320323	22.9160	0.00551702	0.02407500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.00005607	0.61	0.6126
P2	3	0.00002652	0.29	0.8319
R2*P2	9	0.00026169	0.96	0.4986

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.00006696	0.73	0.5424
P2	3	0.00002314	0.25	0.8581
R2*P2	9	0.00026169	0.96	0.4986

JA-test:

DEPENDENT VARIABLE: JAA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.00031797	0.00002120	0.60
ERROR	24	0.00085200	0.00003550	PR > F
CORRECTED TOTAL	39	0.00116998		0.8487

R-SQUARE	C.V.	ROOT MSE	JAA MEAN
0.271779	23.1611	0.00595819	0.02572500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.00003164	0.30	0.8271
P2	3	0.00002455	0.23	0.8742
R2*P2	9	0.00026179	0.82	0.6043

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.00002275	0.21	0.8860
P2	3	0.00001832	0.17	0.9142
R2*P2	9	0.00026179	0.82	0.6043

NJ-test:

DEPENDENT VARIABLE: NJA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.00074590	0.00004973	0.97
ERROR	24	0.00123450	0.00005144	PR > F
CORRECTED TOTAL	39	0.00198040		0.5142

R-SQUARE	C.V.	ROOT MSE	NJA MEAN
0.376641	14.2584	0.00717199	0.05030000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.00031682	2.05	0.1332
P2	3	0.00001553	0.10	0.9589
R2*P2	9	0.00041355	0.89	0.5455

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.00028104	1.82	0.1702
P2	3	0.00001989	0.13	0.9420
R2*P2	9	0.00041355	0.89	0.5455

F-test:

DEPENDENT VARIABLE: FA

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.00061210	0.00004081	0.79
ERROR	24	0.00124350	0.00005181	PR > F
CORRECTED TOTAL	39	0.00185560		0.6791

R-SQUARE	C.V.	ROOT MSE	FA MEAN
0.329866	14.2819	0.00719809	0.05040000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.00012631	0.81	0.4994
P2	3	0.00023367	1.50	0.2390
R2*P2	9	0.00025212	0.54	0.8304

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.00011718	0.75	0.5309
P2	3	0.00024579	1.58	0.2199
R2*P2	9	0.00025212	0.54	0.8304

Duncan's Multiple Range Test on Mean Observed Significance Levels:

```
DUNCAN'S MULTIPLE RANGE TEST FOR VARIABLE: CA
NOTE: THIS TEST CONTROLS THE TYPE I COMPARISONWISE ERROR RATE,
      NOT THE EXPERIMENTWISE ERROR RATE

              ALPHA=0.05  DF=24  MSE=2.9E-04

        WARNING: CELL SIZES ARE NOT EQUAL.
                 HARMONIC MEAN OF CELL SIZES=9.6

        NUMBER OF MEANS          2          3         4
        CRITICAL RANGE  0.0159386  0.016737 0.0172928

 MEANS WITH THE SAME LETTER ARE NOT SIGNIFICANTLY DIFFERENT.

        DUNCAN   GROUPING            MEAN        N  R2

                     A            0.157000       8  0.5

                     B            0.131375       8  0.7

                     C            0.109417      12  0.75

                     D            0.089250      12  0.9
```

Cox (N)-test

```
DUNCAN'S MULTIPLE RANGE TEST FOR VARIABLE: WA
NOTE: THIS TEST CONTROLS THE TYPE I COMPARISONWISE ERROR RATE,
      NOT THE EXPERIMENTWISE ERROR RATE

              ALPHA=0.05  DF=24  MSE=4.6E-05

        WARNING: CELL SIZES ARE NOT EQUAL.
                 HARMONIC MEAN OF CELL SIZES=9.6

        NUMBER OF MEANS          2          3         4
        CRITICAL RANGE  .00639659 0.0067204 .00694356

 MEANS WITH THE SAME LETTER ARE NOT SIGNIFICANTLY DIFFERENT.

        DUNCAN   GROUPING            MEAN        N  R2

                     A            0.041917      12  0.9
                     A
                     A            0.040000      12  0.75
                     A
                     A            0.035250       8  0.7

                     B            0.026750       8  0.5
```

W-test

```
DUNCAN'S MULTIPLE RANGE TEST FOR VARIABLE: NTA
NOTE: THIS TEST CONTROLS THE TYPE I COMPARISONWISE ERROR RATE,
      NOT THE EXPERIMENTWISE ERROR RATE

              ALPHA=0.05  DF=24  MSE=4.3E-05

        WARNING: CELL SIZES ARE NOT EQUAL.
                 HARMONIC MEAN OF CELL SIZES=9.6

        NUMBER OF MEANS          2          3         4
        CRITICAL RANGE  .00618765 .00650088 .00671676

 MEANS WITH THE SAME LETTER ARE NOT SIGNIFICANTLY DIFFERENT.

        DUNCAN   GROUPING            MEAN        N  R2

                     A            0.053875       8  0.7
                     A
                     A            0.053417      12  0.75
                     A
                     A            0.052333      12  0.9

                     B            0.044500       8  0.5
```

$\tilde{N}$-test

```
DUNCAN'S MULTIPLE RANGE TEST FOR VARIABLE: NAA
NOTE: THIS TEST CONTROLS THE TYPE I COMPARISONWISE ERROR RATE,
      NOT THE EXPERIMENTWISE ERROR RATE

              ALPHA=0.05  DF=24  MSE=1.1E-04

        WARNING: CELL SIZES ARE NOT EQUAL.
                 HARMONIC MEAN OF CELL SIZES=9.6

        NUMBER OF MEANS          2          3          4
        CRITICAL RANGE   .00964584 0.0101341 0.0104706

 MEANS WITH THE SAME LETTER ARE NOT SIGNIFICANTLY DIFFERENT.

        DUNCAN    GROUPING             MEAN        N   R2

                        A           0.083375       8   0.7
                        A
                B       A           0.075250       8   0.5
                B       A
                B       A           0.075083      12   0.75
                B
                B                   0.072417      12   0.9
```

NA-test

```
DUNCAN'S MULTIPLE RANGE TEST FOR VARIABLE: NLA
NOTE: THIS TEST CONTROLS THE TYPE I COMPARISONWISE ERROR RATE,
      NOT THE EXPERIMENTWISE ERROR RATE

              ALPHA=0.05  DF=24  MSE=1.8E-04

        WARNING: CELL SIZES ARE NOT EQUAL.
                 HARMONIC MEAN OF CELL SIZES=9.6

        NUMBER OF MEANS          2          3          4
        CRITICAL RANGE   0.0126958 0.0133385 0.0137814

 MEANS WITH THE SAME LETTER ARE NOT SIGNIFICANTLY DIFFERENT.

        DUNCAN    GROUPING             MEAN        N   R2

                        A           0.127375       8   0.5

                        B           0.105250       8   0.7

                        C           0.090167      12   0.75

                        D           0.074000      12   0.9
```

NL-test

```
DUNCAN'S MULTIPLE RANGE TEST FOR VARIABLE: JA
NOTE: THIS TEST CONTROLS THE TYPE I COMPARISONWISE ERROR RATE,
      NOT THE EXPERIMENTWISE ERROR RATE

              ALPHA=0.05  DF=24  MSE=8.9E-05

        WARNING: CELL SIZES ARE NOT EQUAL.
                 HARMONIC MEAN OF CELL SIZES=9.6

        NUMBER OF MEANS          2          3          4
        CRITICAL RANGE   .00889963 .00935015 .00966064

 MEANS WITH THE SAME LETTER ARE NOT SIGNIFICANTLY DIFFERENT.

        DUNCAN    GROUPING             MEAN        N   R2

                        A           0.078625       8   0.5

                        B           0.063875       8   0.7

                        C           0.054000      12   0.75

                        D           0.037833      12   0.9
```

J-test

```
DUNCAN'S MULTIPLE RANGE TEST FOR VARIABLE: CA
NOTE: THIS TEST CONTROLS THE TYPE I COMPARISONWISE ERROR RATE,
      NOT THE EXPERIMENTWISE ERROR RATE

                ALPHA=0.05  DF=24  MSE=2.9E-04

       NUMBER OF MEANS           2          3          4
       CRITICAL RANGE   0.0156087  0.0163989  0.0169434

MEANS WITH THE SAME LETTER ARE NOT SIGNIFICANTLY DIFFERENT.

       DUNCAN   GROUPING            MEAN      N   P2

                    A           0.144000     10   0.25

                    B           0.118200     10   0.5
                    B
                    B           0.111400     10   0.75

                    C           0.095500     10   0.9
```

Cox (N)-test

```
DUNCAN'S MULTIPLE RANGE TEST FOR VARIABLE: NTA
NOTE: THIS TEST CONTROLS THE TYPE I COMPARISONWISE ERROR RATE,
      NOT THE EXPERIMENTWISE ERROR RATE

                ALPHA=0.05  DF=24  MSE=4.3E-05

       NUMBER OF MEANS           2          3          4
       CRITICAL RANGE   .00606264  .00636954  .00658105

MEANS WITH THE SAME LETTER ARE NOT SIGNIFICANTLY DIFFERENT.

       DUNCAN   GROUPING            MEAN      N   P2

                    A           0.056300     10   0.75
                    A
               B    A           0.051300     10   0.9
               B    A
               B    A           0.050100     10   0.25
               B
               B                0.047900     10   0.5
```

Ñ-test

```
DUNCAN'S MULTIPLE RANGE TEST FOR VARIABLE: JA
NOTE: THIS TEST CONTROLS THE TYPE I COMPARISONWISE ERROR RATE,
      NOT THE EXPERIMENTWISE ERROR RATE

                ALPHA=0.05  DF=24  MSE=8.9E-05

       NUMBER OF MEANS           2          3          4
       CRITICAL RANGE   .00871983  .00916124  .00946545

MEANS WITH THE SAME LETTER ARE NOT SIGNIFICANTLY DIFFERENT.

       DUNCAN   GROUPING            MEAN      N   P2

                    A           0.088600     10   0.25

                    B           0.055400     10   0.5

                    C           0.045400     10   0.75

                    D           0.034800     10   0.9
```

J-test

E.2. Observed Power

Cox(N)-test:

DEPENDENT VARIABLE: CP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	0.69032412	0.04602161	26.44
ERROR	24	0.04177625	0.00174068	PR > F
CORRECTED TOTAL	39	0.73210038		0.0001

R-SQUARE	C.V.	ROOT MSE	CP MEAN
0.942956	5.1326	0.04172142	0.81287500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.34966417	66.96	0.0001
P2	3	0.18759316	35.92	0.0001
R2*P2	9	0.15306680	9.77	0.0001

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.32796659	62.80	0.0001
P2	3	0.22366267	42.83	0.0001
R2*P2	9	0.15306680	9.77	0.0001

W-test:

DEPENDENT VARIABLE: WP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	2.57651637	0.17176776	25.18
ERROR	24	0.16374400	0.00682267	PR > F
CORRECTED TOTAL	39	2.74026038		0.0001

R-SQUARE	C.V.	ROOT MSE	WP MEAN
0.940245	11.2552	0.08259944	0.73387500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	1.60787767	78.56	0.0001
P2	3	0.76408812	37.33	0.0001
R2*P2	9	0.20455059	3.33	0.0089

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	1.51832404	74.18	0.0001
P2	3	0.78129032	38.17	0.0001
R2*P2	9	0.20455059	3.33	0.0089

$\tilde{N}$-test:

DEPENDENT VARIABLE: NTP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	1.80073052	0.12004870	34.06
ERROR	24	0.08459125	0.00352464	PR > F
CORRECTED TOTAL	39	1.88532178		0.0001

R-SQUARE	C.V.	ROOT MSE	NTP MEAN
0.955132	7.5805	0.05936864	0.78317500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	1.09319019	103.39	0.0001
P2	3	0.49991101	47.28	0.0001
R2*P2	9	0.20762932	6.55	0.0001

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	1.05041046	99.34	0.0001
P2	3	0.54687931	51.72	0.0001
R2*P2	9	0.20762932	6.55	0.0001

NA-test:

DEPENDENT VARIABLE: NAP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	1.74509490	0.11633966	24.99
ERROR	24	0.11172950	0.00465540	PR > F
CORRECTED TOTAL	39	1.85682440		0.0001

R-SQUARE	C.V.	ROOT MSE	NAP MEAN
0.939828	9.5254	0.06823046	0.71630000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	1.33353294	95.48	0.0001
P2	3	0.29522727	21.14	0.0001
R2*P2	9	0.11633469	2.78	0.0221

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	1.24137597	88.88	0.0001
P2	3	0.31075021	22.25	0.0001
R2*P2	9	0.11633469	2.78	0.0221

NL-test:

DEPENDENT VARIABLE: NLP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	1.05184727	0.07012315	25.14
ERROR	24	0.06695150	0.00278965	PR > F
CORRECTED TOTAL	39	1.11879878		0.0001

R-SQUARE	C.V.	ROOT MSE	NLP MEAN
0.940158	6.5617	0.05281710	0.80492500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.44657657	53.36	0.0001
P2	3	0.39982782	47.78	0.0001
R2*P2	9	0.20544289	8.18	0.0001

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.41777872	49.92	0.0001
P2	3	0.44689725	53.40	0.0001
R2*P2	9	0.20544289	8.18	0.0001

J-test:

DEPENDENT VARIABLE: JP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	1.54371237	0.10291416	24.22
ERROR	24	0.10198800	0.00424950	PR > F
CORRECTED TOTAL	39	1.64570038		0.0001

R-SQUARE	C.V.	ROOT MSE	JP MEAN
0.938028	8.0992	0.06518819	0.80487500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	0.70157404	55.03	0.0001
P2	3	0.57291136	44.94	0.0001
R2*P2	9	0.26922698	7.04	0.0001

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	0.66314814	52.02	0.0001
P2	3	0.63439964	49.76	0.0001
R2*P2	9	0.26922698	7.04	0.0001

AJ-test:

DEPENDENT VARIABLE: AJP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	2.42994897	0.16199660	28.09
ERROR	24	0.13842600	0.00576775	PR > F
CORRECTED TOTAL	39	2.56837498		0.0001

R-SQUARE	C.V.	ROOT MSE	AJP MEAN
0.946104	10.0487	0.07594578	0.75577500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	1.52861393	88.34	0.0001
P2	3	0.70814815	40.93	0.0001
R2*P2	9	0.19318689	3.72	0.0048

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	1.44124571	83.29	0.0001
P2	3	0.73289632	42.36	0.0001
R2*P2	9	0.19318689	3.72	0.0048

JA-test:

DEPENDENT VARIABLE: JAP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	2.22027952	0.14801863	23.01
ERROR	24	0.15435925	0.00643164	PR > F
CORRECTED TOTAL	39	2.37463878		0.0001

R-SQUARE	C.V.	ROOT MSE	JAP MEAN
0.934997	11.2113	0.08019748	0.71532500

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	1.77399623	91.94	0.0001
P2	3	0.33863890	17.55	0.0001
R2*P2	9	0.10764440	1.86	0.1086

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	1.64864897	85.44	0.0001
P2	3	0.34481124	17.87	0.0001
R2*P2	9	0.10764440	1.86	0.1086

NJ-test:

DEPENDENT VARIABLE: NJP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	2.30598465	0.15373231	25.59
ERROR	24	0.14419375	0.00600807	PR > F
CORRECTED TOTAL	39	2.45017840		0.0001

R-SQUARE	C.V.	ROOT MSE	NJP MEAN
0.941150	10.8151	0.07751176	0.71670000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	1.64572727	91.31	0.0001
P2	3	0.52891383	29.34	0.0001
R2*P2	9	0.13134355	2.43	0.0400

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	1.54384381	85.65	0.0001
P2	3	0.54203603	30.07	0.0001
R2*P2	9	0.13134355	2.43	0.0400

F-test:

DEPENDENT VARIABLE: FP

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE
MODEL	15	3.39813290	0.22654219	31.55
ERROR	24	0.17234950	0.00718123	PR > F
CORRECTED TOTAL	39	3.57048240		0.0001

R-SQUARE	C.V.	ROOT MSE	FP MEAN
0.951729	13.5004	0.08474213	0.62770000

SOURCE	DF	TYPE I SS	F VALUE	PR > F
R2	3	1.88144861	87.33	0.0001
P2	3	1.34469320	62.42	0.0001
R2*P2	9	0.17199109	2.66	0.0268

SOURCE	DF	TYPE III SS	F VALUE	PR > F
R2	3	1.64879940	76.53	0.0001
P2	3	1.27674432	59.26	0.0001
R2*P2	9	0.17199109	2.66	0.0268

Appendix F: Initial Models Estimation: Weekly Food Expenditures

This appendix contains the SAS Proc Reg output for initial model estimation for each individual food categories. It is clear from the values of the variance inflation factors, VIF's, that collinearity within the models is not severe enough to merit a biased estimation technique.

F.1. All Food

QUADRATIC MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	3738314.62	207684.15	628.097	0.0001
ERROR	9654	3192155.56	330.65626		
C TOTAL	9672	6930470.18			

ROOT MSE	18.18396	R-SQUARE	0.5394
DEP MEAN	45.62719	ADJ R-SQ	0.5385
C.V.	39.85333		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	7.61262286	1.19383255	6.377	0.0001	0
EDHM1	1	1.19814436	0.43656577	2.744	0.0061	1.13171555
EMPSHM2	1	-1.43548859	0.40726555	-3.525	0.0004	1.16380591
SXHM	1	-3.00161183	0.75741061	-3.963	0.0001	1.11766659
U1	1	1.11137996	0.48542088	2.290	0.0221	1.45972231
U2	1	-1.85108045	0.45874271	-4.035	0.0001	1.39854626
R1	1	6.49047916	0.50358664	12.889	0.0001	1.37233489
R2	1	0.29505085	0.50076760	0.589	0.5557	1.35197229
R4	1	0.37402918	0.57054920	0.656	0.5121	1.31857810
S1	1	-1.03845883	0.54081633	-1.920	0.0549	1.54140657
S3	1	-0.31005818	0.52459624	-0.591	0.5545	1.58876131
S4	1	0.23853778	0.52741481	0.452	0.6511	1.57489750
RAC	1	-0.11237341	0.57573419	-0.195	0.8453	1.24897442
INC	1	0.000379795	0.000101356	3.747	0.0002	17.49835719
HS	1	0.88761883	0.49414754	1.796	0.0725	19.66931410
MEALS	1	0.57447549	0.01544368	37.198	0.0001	7.28964657
INC2	1	-4.27771E-09	3.20398E-09	-1.335	0.1819	16.06221486
HS2	1	-0.32973107	0.04429018	-7.445	0.0001	10.71073230
INCCHS	1	0.000098457	0.000016398	6.004	0.0001	10.94444688

SEMILOG MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3546389.64	236425.98	674.678	0.0001
ERROR	9657	3384080.54	350.42772		
C TOTAL	9672	6930470.18			

ROOT MSE	18.71971	R-SQUARE	0.5117
DEP MEAN	45.62719	ADJ R-SQ	0.5110
C.V.	41.02754		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-108.13408	4.09144066	-26.429	0.0001	0
EDHM1	1	0.58992125	0.44830720	1.316	0.1882	1.12607585
EMPSHM2	1	-1.71016339	0.42063229	-4.066	0.0001	1.17140892
SXHM	1	-6.94630325	0.78217866	-8.881	0.0001	1.12470785
U1	1	1.22815816	0.49939595	2.459	0.0139	1.45781249
U2	1	-2.05010305	0.47211821	-4.342	0.0001	1.39771394
R1	1	6.80263893	0.51884562	13.111	0.0001	1.37456827
R2	1	0.98366775	0.51486697	1.911	0.0561	1.34853960
R4	1	0.34794211	0.58769480	0.592	0.5538	1.32008425
S1	1	-1.18717470	0.55646475	-2.133	0.0329	1.53982451
S3	1	-0.75078209	0.53944655	-1.392	0.1640	1.57721563
S4	1	-0.20285975	0.54218072	-0.374	0.7083	1.57041375
RAC	1	-0.007930182	0.58982736	-0.013	0.9893	1.25690864
LINC	1	4.22818365	0.32051829	13.192	0.0001	1.59454446
LHS	1	-0.70412578	0.94454990	-0.745	0.4560	8.10670451
LMEALS	1	31.25111989	0.90065831	34.698	0.0001	7.73194199

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3633665.99	242244.40	709.582	0.0001
ERROR	9657	3296804.19	341.39010		
C TOTAL	9672	6930470.18			

ROOT MSE	18.47674	R-SQUARE	0.5243
DEP MEAN	45.62719	ADJ R-SQ	0.5236
C.V.	40.49503		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	26.50235359	1.42821946	18.556	0.0001	0
EDHM1	1	-0.20461767	0.43439614	-0.471	0.6376	1.08526467
EMPSHM2	1	-2.19992708	0.40229522	-5.468	0.0001	1.09986854
SXHM	1	-3.58931319	0.78439574	-4.576	0.0001	1.16183626
U1	1	0.55627772	0.49195980	1.131	0.2582	1.45217314
U2	1	-2.50116876	0.46403974	-5.390	0.0001	1.38603654
R1	1	6.94922544	0.51194377	13.574	0.0001	1.37366890
R2	1	0.98483519	0.50761692	1.940	0.0524	1.34552986
R4	1	0.64318147	0.57983059	1.109	0.2673	1.31900892
S1	1	-0.98421853	0.54925165	-1.792	0.0732	1.53976538
S3	1	-0.39514265	0.53244214	-0.742	0.4580	1.57719943
S4	1	0.30673366	0.53508409	0.573	0.5665	1.57006475
RAC	1	-1.70550257	0.57632302	-2.959	0.0031	1.21218046
INVINC	1	-11262.56860	1455.40493	-7.738	0.0001	1.36396878
INVHS	1	-10.14495931	1.17645373	-8.623	0.0001	3.81331206
MEALS	1	0.31165755	0.009322028	34.887	0.0001	2.57247598

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2185.83952	145.72263	1040.714	0.0001
ERROR	9657	1352.19097	0.14002185		
C TOTAL	9672	3538.03049			

ROOT MSE	0.3741949	R-SQUARE	0.6178
DEP MEAN	3.651411	ADJ R-SQ	0.6172
C.V.	10.24795		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-0.39752530	0.08178524	-4.861	0.0001
EDHM1	1	0.001460917	0.008961369	0.163	0.8705
EMPSHM2	1	-0.03442762	0.008408164	-4.095	0.0001
SXHM	1	-0.09478938	0.01563524	-6.063	0.0001
U1	1	0.03045153	0.009982600	3.050	0.0023
U2	1	-0.03960751	0.009437536	-4.197	0.0001
R1	1	0.13661152	0.01037139	13.172	0.0001
R2	1	0.008606571	0.01029186	0.836	0.4030
R4	1	0.01254638	0.01174764	1.068	0.2856
S1	1	-0.04620660	0.01112337	-4.154	0.0001
S3	1	-0.02047090	0.01078319	-1.898	0.0577
S4	1	-0.005437072	0.01083784	-0.502	0.6159
RAC	1	-0.004897130	0.01179027	-0.415	0.6779
LINC	1	0.14366889	0.006406952	22.424	0.0001
LHS	1	0.005087778	0.01888094	0.269	0.7876
LMEALS	1	0.72416867	0.01800357	40.224	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2070.88688	138.05913	908.730	0.0001
ERROR	9657	1467.14361	0.15192540		
C TOTAL	9672	3538.03049			

ROOT MSE	0.3897761	R-SQUARE	0.5853
DEP MEAN	3.651411	ADJ R-SQ	0.5847
C.V.	10.67467		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	3.75222244	0.03012900	124.539	0.0001
EDHM1	1	-0.01265720	0.009163802	-1.381	0.1672
EMPSHM2	1	-0.02772012	0.008486617	-3.266	0.0011
SXHM	1	-0.08520810	0.01654722	-5.149	0.0001
U1	1	0.01961793	0.01037814	1.890	0.0587
U2	1	-0.04400833	0.009789148	-4.496	0.0001
R1	1	0.14741714	0.01079971	13.650	0.0001
R2	1	0.02271840	0.01070843	2.122	0.0339
R4	1	0.02758294	0.01223181	2.255	0.0242
S1	1	-0.04719782	0.01158631	-4.074	0.0001
S3	1	-0.02154254	0.01123213	-1.918	0.0551
S4	1	-0.001020084	0.01128786	-0.090	0.9280
RAC	1	-0.03608083	0.01215782	-2.968	0.0030
INVINC	1	-369.27846	30.70248858	-12.028	0.0001
INVHS	1	-0.83054480	0.02481787	-33.466	0.0001
MEALS	1	0.007387972	0.000196653	37.569	0.0001

F.2. Beverages

QUADRATIC MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	18287.00329	1015.94463	46.882	0.0001
ERROR	8023	174157.11	21.70723078		
C TOTAL	8041	192444.12			

ROOT MSE	4.659102	R-SQUARE	0.0950
DEP MEAN	3.747659	ADJ R-SQ	0.0930
C.V.	124.3203		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	2.84124162	0.35257193	8.059	0.0001	0
EDHM1	1	-0.13374574	0.12028728	-1.112	0.2662	1.12392550
EMPSHM2	1	-0.38853104	0.11255875	-3.452	0.0006	1.14133790
SXHM	1	-2.23278174	0.22852989	-9.770	0.0001	1.11340682
U1	1	-0.11410586	0.13580497	-0.840	0.4008	1.44576257
U2	1	-0.41451210	0.12820133	-3.233	0.0012	1.36239827
R1	1	0.56113862	0.14043551	3.996	0.0001	1.39174836
R2	1	0.22454060	0.14046644	1.599	0.1100	1.37760254
R4	1	0.38862025	0.16106120	2.413	0.0158	1.32492960
S1	1	0.07744726	0.15110656	0.513	0.6083	1.55411833
S3	1	-0.46519903	0.14721197	-3.160	0.0016	1.56841165
S4	1	-0.25083528	0.14820458	-1.692	0.0906	1.56225881
RAC	1	-0.14856120	0.16402734	-0.906	0.3651	1.24707275
INC	1	0.000074967	0.000028861	2.597	0.0094	17.80468274
HS	1	-0.30067563	0.13851241	-2.171	0.0300	19.52838635
MEALS	1	0.06492677	0.004153351	15.632	0.0001	6.68189159
INC2	1	4.54337E-10	8.84118E-10	0.514	0.6073	16.14473795
HS2	1	-0.04948454	0.01251407	-3.954	0.0001	11.08458319
INCCHS	1	-4.90604E-07	.00000453115	-0.108	0.9138	10.93446112

SEMILOG MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	15941.93325	1062.79555	48.328	0.0001
ERROR	8026	176502.18	21.99130109		
C TOTAL	8041	192444.12			

ROOT MSE	4.689488	R-SQUARE	0.0828
DEP MEAN	3.747659	ADJ R-SQ	0.0811
C.V.	125.1312		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-11.74264780	1.13423631	-10.353	0.0001	0
EDHM1	1	-0.23062735	0.12072347	-1.910	0.0561	1.11746776
EMPSHM2	1	-0.38332983	0.11362856	-3.374	0.0007	1.14811203
SXHM	1	-2.36374974	0.23050421	-10.255	0.0001	1.11809593
U1	1	-0.12407955	0.13658449	-0.908	0.3637	1.44351710
U2	1	-0.43629990	0.12896066	-3.383	0.0007	1.36077718
R1	1	0.59098665	0.14140779	4.179	0.0001	1.39285855
R2	1	0.25931488	0.14114829	1.837	0.0662	1.37384096
R4	1	0.39151052	0.16218104	2.414	0.0158	1.32606423
S1	1	0.06037481	0.15199942	0.397	0.6912	1.53224982
S3	1	-0.53169916	0.14798507	-3.593	0.0003	1.56445508
S4	1	-0.31431387	0.14895028	-2.110	0.0349	1.55763557
RAC	1	-0.16425851	0.16417355	-1.001	0.3171	1.23315937
LINC	1	0.79656284	0.08886867	8.963	0.0001	1.48515070
LHS	1	-1.46761772	0.25702955	-5.710	0.0001	7.34381183
LMEALS	1	3.11608537	0.24467520	12.736	0.0001	7.05637833

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	14156.98164	943.79878	42.487	0.0001
ERROR	8026	178287.15	22.21369725		
C TOTAL	8041	192444.12			

ROOT MSE	4.713141	R-SQUARE	0.0736
DEP MEAN	3.747639	ADJ R-SQ	0.0718
C.V.	125.7623		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	4.94269350	0.40891271	12.087	0.0001	0
EDHM1	1	-0.37224663	0.11909733	-3.126	0.0018	1.07667783
EMPSHM2	1	-0.44661494	0.11100456	-4.023	0.0001	1.08472835
SXHM	1	-2.21056889	0.23574022	-9.377	0.0001	1.15776069
U1	1	-0.20722178	0.13697281	-1.513	0.1304	1.43720260
U2	1	-0.50189760	0.12912885	-3.887	0.0001	1.35066972
R1	1	0.66093897	0.14202916	4.654	0.0001	1.39105871
R2	1	0.33357709	0.14166976	2.355	0.0186	1.36935691
R4	1	0.49542776	0.16283747	3.042	0.0024	1.32543670
S1	1	0.06568196	0.15275169	0.430	0.6672	1.53196145
S3	1	-0.52141605	0.14872450	-3.506	0.0005	1.56430844
S4	1	-0.27889729	0.14966118	-1.864	0.0624	1.55679576
RAC	1	-0.47550851	0.16322474	-2.913	0.0036	1.20674313
INVINC	1	-1251.79782	421.32565	-2.971	0.0030	1.29024528
INVHS	1	-0.57266774	0.34339053	-1.668	0.0954	2.83996008
MEALS	1	0.03250239	0.002549483	12.749	0.0001	2.46031112

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	1706.82064	113.78804	107.353	0.0001
ERROR	8026	8507.11971	1.05994514		
C TOTAL	8041	10213.94034			

ROOT MSE	1.029536	R-SQUARE	0.1671
DEP MEAN	0.7478031	ADJ R-SQ	0.1656
C.V.	137.6748		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-2.94658629	0.24901171	-11.833	0.0001
EDHM1	1	-0.05182611	0.02650379	-1.955	0.0506
EMPSHM2	1	-0.11354822	0.02494616	-4.552	0.0001
SXHM	1	-0.65504932	0.05060519	-12.944	0.0001
U1	1	0.004086979	0.02998594	0.136	0.8916
U2	1	-0.10726036	0.02831219	-3.788	0.0002
R1	1	0.20162965	0.03104485	6.495	0.0001
R2	1	0.07176476	0.03098788	2.316	0.0206
R4	1	0.11626639	0.03560544	3.265	0.0011
S1	1	-0.09278850	0.03337015	-2.781	0.0054
S3	1	-0.15631036	0.03248883	-4.811	0.0001
S4	1	-0.09825077	0.03270074	-3.005	0.0027
RAC	1	-0.05644177	0.03604288	-1.566	0.1174
LINC	1	0.23399074	0.01951034	11.993	0.0001
LHS	1	0.10366665	0.05642860	1.837	0.0662
LMEALS	1	0.55083215	0.05371631	10.254	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	1604.14556	106.94304	99.692	0.0001
ERROR	8026	8609.79478	1.07273795		
C TOTAL	8041	10213.94034			

ROOT MSE	1.035731	R-SQUARE	0.1571
DEP MEAN	0.7478031	ADJ R-SQ	0.1555
C.V.	138.5031		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	1.65003272	0.08986012	18.362	0.0001
EDHM1	1	-0.09139378	0.02617209	-3.515	0.0004
EMPSHM2	1	-0.13811976	0.02439367	-5.662	0.0001
SXHM	1	-0.67423469	0.05180481	-13.015	0.0001
U1	1	-0.01579651	0.03010030	-0.525	0.5997
U2	1	-0.12573750	0.02837656	-4.431	0.0001
R1	1	0.22120176	0.03121145	7.087	0.0001
R2	1	0.09744051	0.03113247	3.130	0.0018
R4	1	0.13832193	0.03578415	3.865	0.0001
S1	1	-0.09022888	0.03356777	-2.688	0.0072
S3	1	-0.15251034	0.03268277	-4.666	0.0001
S4	1	-0.08794900	0.03288861	-2.674	0.0075
RAC	1	-0.11512684	0.03586926	-3.210	0.0013
INVINC	1	-514.63722	92.58791422	-5.558	0.0001
INVHS	1	-0.90781440	0.07546137	-12.030	0.0001
MEALS	1	0.006156410	0.000560259	10.989	0.0001

F.3. Fats and Oils

QUADRATIC MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	3270.52557	181.69586	184.067	0.0001
ERROR	9329	9208.81340	0.98711688		
C TOTAL	9347	12479.33897			

ROOT MSE	0.9935376	R-SQUARE	0.2621
DEP MEAN	1.467334	ADJ R-SQ	0.2607
C.V.	67.71038		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	0.10306512	0.06718598	1.534	0.1250	0
EDHM1	1	0.12780281	0.02419929	5.281	0.0001	1.12849349
EMPSHM2	1	-0.02009304	0.02259299	-0.889	0.3738	1.16021845
SXHM	1	0.03720184	0.04310400	0.863	0.3881	1.11075658
U1	1	0.04552283	0.02696143	1.688	0.0914	1.45194984
U2	1	0.04916710	0.02542717	1.934	0.0532	1.39099260
R1	1	0.13342298	0.02794946	4.774	0.0001	1.37472176
R2	1	0.02172535	0.02782119	0.781	0.4349	1.36014433
R4	1	-0.02035142	0.03170983	-0.642	0.5210	1.31981128
S1	1	-0.13078312	0.03020931	-4.329	0.0001	1.53402969
S3	1	-0.05813278	0.02908340	-1.999	0.0457	1.58196312
S4	1	-0.04135561	0.02925092	-1.414	0.1574	1.57648006
RAC	1	0.03658995	0.03252474	1.125	0.2606	1.24770269
INC	1	0.000010086	.00000564226	1.788	0.0739	17.51365230
HS	1	0.09064406	0.02748066	3.298	0.0010	19.55628360
MEALS	1	0.01604934	0.000854625	18.779	0.0001	7.16559963
INC2	1	-8.07901E-11	1.77577E-10	-0.455	0.6491	16.09960537
HS2	1	-0.01192305	0.002449189	-4.868	0.0001	10.63716934
INCCHS	1	.00000155034	9.10667E-07	1.702	0.0887	10.93546029

SEMILOG MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3171.03350	211.40223	211.940	0.0001
ERROR	9332	9308.30547	0.99746094		
C TOTAL	9347	12479.33897			

ROOT MSE	0.9987297	R-SQUARE	0.2541
DEP MEAN	1.467334	ADJ R-SQ	0.2529
C.V.	68.06423		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-2.98544125	0.22288857	-13.394	0.0001	0
EDHM1	1	0.11105324	0.02427034	4.576	0.0001	1.12335838
EMPSHM2	1	-0.02897189	0.02278197	-1.272	0.2035	1.16747471
SXHM	1	-0.08093047	0.04344697	-1.863	0.0625	1.11690289
U1	1	0.05018944	0.02707955	1.853	0.0639	1.44950989
U2	1	0.04324632	0.02555392	1.692	0.0906	1.39032458
R1	1	0.14014721	0.02811544	4.985	0.0001	1.37667253
R2	1	0.03810212	0.02793268	1.364	0.1726	1.35684867
R4	1	-0.02228019	0.03189520	-0.699	0.4849	1.32143960
S1	1	-0.13582081	0.03035239	-4.475	0.0001	1.53253584
S3	1	-0.06986038	0.02920382	-2.392	0.0168	1.57854832
S4	1	-0.05427245	0.02936149	-1.848	0.0646	1.57194911
RAC	1	0.04308699	0.03252239	1.325	0.1853	1.23458562
LINC	1	0.09428304	0.01745134	5.403	0.0001	1.56856065
LHS	1	0.04021453	0.05127534	0.784	0.4329	7.94836240
LMEALS	1	0.90956043	0.04892212	18.592	0.0001	7.57784631

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3231.78945	215.45263	217.420	0.0001
ERROR	9332	9247.54952	0.99095044		
C TOTAL	9347	12479.33897			

ROOT MSE	0.9954649	R-SQUARE	0.2590
DEP MEAN	1.467334	ADJ R-SQ	0.2578
C.V.	67.84173		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	0.75325765	0.07892580	9.544	0.0001	0
EDHM1	1	0.09712324	0.02375901	4.088	0.0001	1.08559570
EMPSHM2	1	-0.03544291	0.02203340	-1.609	0.1077	1.09918852
SXHM	1	0.01481510	0.04398109	0.337	0.7362	1.15194699
U1	1	0.03365157	0.02693578	1.249	0.2116	1.44358178
U2	1	0.03539265	0.02536502	1.395	0.1629	1.37884616
R1	1	0.14282151	0.02801084	5.099	0.0001	1.37542504
R2	1	0.03611728	0.02780677	1.299	0.1940	1.35347877
R4	1	-0.01527597	0.03177094	-0.481	0.6307	1.31977795
S1	1	-0.12994690	0.03025296	-4.295	0.0001	1.53251415
S3	1	-0.06051426	0.02910846	-2.079	0.0377	1.57856000
S4	1	-0.04042566	0.02926329	-1.381	0.1672	1.57171067
RAC	1	0.005759986	0.03208733	0.180	0.8575	1.20967076
INVINC	1	-311.50353	82.15161089	-3.792	0.0002	1.35505104
INVHS	1	-0.38247897	0.06514113	-5.872	0.0001	2.97745611
MEALS	1	0.01487967	0.000510527	29.146	0.0001	2.54715112

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2266.28257	151.08550	288.129	0.0001
ERROR	9332	4893.39592	0.52436733		
C TOTAL	9347	7159.67849			

ROOT MSE	0.7241321	R-SQUARE	0.3165
DEP MEAN	0.06618731	ADJ R-SQ	0.3154
C.V.	1094.065		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-3.98655424	0.16160607	-24.668	0.0001
EDHM1	1	0.06355830	0.01759729	3.612	0.0003
EMPSHM2	1	-0.02058405	0.01651814	-1.246	0.2127
SXHM	1	0.04672252	0.03150281	1.483	0.1381
U1	1	0.007403425	0.01963411	0.377	0.7061
U2	1	0.01312915	0.01852795	0.709	0.4786
R1	1	0.09585896	0.02038519	4.702	0.0001
R2	1	0.02364662	0.02025268	1.168	0.2430
R4	1	-0.02187484	0.02312571	-0.946	0.3442
S1	1	-0.13236722	0.02200710	-6.015	0.0001
S3	1	-0.04874585	0.02117432	-2.302	0.0214
S4	1	-0.03372245	0.02128864	-1.584	0.1132
RAC	1	0.04748428	0.02358047	2.014	0.0441
LINC	1	0.10546870	0.01265315	8.335	0.0001
LHS	1	-0.003245369	0.03717735	-0.087	0.9304
LMEALS	1	0.77453846	0.03547114	21.836	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2150.16237	143.34416	267.029	0.0001
ERROR	9332	5009.51613	0.53681056		
C TOTAL	9347	7159.67849			

ROOT MSE	0.7326736	R-SQUARE	0.3003
DEP MEAN	0.06618731	ADJ R-SQ	0.2992
C.V.	1106.97		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	0.01130286	0.05809029	0.195	0.8457
EDHM1	1	0.06053834	0.01748691	3.462	0.0005
EMPSHM2	1	-0.000607066	0.01621684	-0.037	0.9701
SXHM	1	0.05264477	0.03237059	1.626	0.1039
U1	1	-0.000172491	0.01982504	-0.009	0.9931
U2	1	0.01449173	0.01866895	0.776	0.4376
R1	1	0.10458838	0.02061630	5.073	0.0001
R2	1	0.03575996	0.02046610	1.747	0.0806
R4	1	-0.007316654	0.02338377	-0.313	0.7544
S1	1	-0.13478141	0.02226652	-6.053	0.0001
S3	1	-0.05216837	0.02142416	-2.435	0.0149
S4	1	-0.03215377	0.02153812	-1.493	0.1355
RAC	1	0.02475270	0.02361664	1.048	0.2946
INVINC	1	-269.74337	60.46452431	-4.461	0.0001
INVHS	1	-0.90397856	0.04794462	-18.855	0.0001
MEALS	1	0.007292690	0.000375753	19.408	0.0001

F.4. Fruits

<u>QUADRATIC MODEL</u>

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	15426.27531	857.01538	115.974	0.0001
ERROR	9177	67815.69445	7.38974550		
C TOTAL	9195	83241.96976			

ROOT MSE	2.718409	R-SQUARE	0.1853
DEP MEAN	3.576274	ADJ R-SQ	0.1837
C.V.	76.0123		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	1.53887267	0.18634401	8.258	0.0001	0
EDHM1	1	-0.52972080	0.06629052	-7.991	0.0001	1.12726171
EMPSHM2	1	0.12962976	0.06241343	2.077	0.0378	1.16199780
SXHM	1	0.27223913	0.11958289	2.277	0.0228	1.10881906
U1	1	0.09897976	0.07423407	1.333	0.1825	1.45487847
U2	1	-0.07732663	0.07037547	-1.099	0.2719	1.39053321
R1	1	0.81901897	0.07707121	10.627	0.0001	1.38798137
R2	1	0.16218744	0.07693110	2.108	0.0350	1.36681711
R4	1	0.96713769	0.08709450	11.104	0.0001	1.33305740
S1	1	-0.41476624	0.08313276	-4.989	0.0001	1.53863224
S3	1	-0.55045015	0.08039769	-6.847	0.0001	1.58222331
S4	1	-0.51824950	0.08076047	-6.417	0.0001	1.57716896
RAC	1	-0.01589039	0.08910050	-0.178	0.8585	1.25178760
INC	1	.00000427465	0.000015609	0.274	0.7842	17.64745560
HS	1	-0.09444185	0.07586843	-1.245	0.2132	19.54520555
MEALS	1	0.03352153	0.002355443	14.232	0.0001	7.17513998
INC2	1	-3.51845E-10	4.92579E-10	-0.714	0.4751	16.32896969
HS2	1	-0.01233538	0.006804614	-1.813	0.0699	10.69321196
INCCHS	1	0.000013397	.00000252673	5.302	0.0001	11.15756816

<u>SEMILOG MODEL</u>

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	14607.80077	973.85338	130.255	0.0001
ERROR	9180	68634.16899	7.47648900		
C TOTAL	9195	83241.96976			

ROOT MSE	2.734317	R-SQUARE	0.1755
DEP MEAN	3.576274	ADJ R-SQ	0.1741
C.V.	76.45713		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-6.22719256	0.61832407	-10.071	0.0001	0
EDHM1	1	-0.55855111	0.06655509	-8.392	0.0001	1.12309440
EMPSHM2	1	0.12341376	0.06296138	1.960	0.0500	1.16877115
SXHM	1	0.01799715	0.12063501	0.149	0.8814	1.11533919
U1	1	0.11253730	0.07461579	1.508	0.1315	1.45282542
U2	1	-0.08821234	0.07077402	-1.246	0.2127	1.39001084
R1	1	0.83296295	0.07757403	10.738	0.0001	1.38983688
R2	1	0.20778722	0.07728521	2.689	0.0072	1.36342469
R4	1	0.95308729	0.08764324	10.875	0.0001	1.33424611
S1	1	-0.41564356	0.08357596	-4.973	0.0001	1.53703908
S3	1	-0.57385620	0.08077991	-7.104	0.0001	1.57877116
S4	1	-0.54366311	0.08112451	-6.702	0.0001	1.57295602
RAC	1	-0.01040250	0.08918071	-0.117	0.9071	1.23949270
LINC	1	0.29081157	0.04842409	6.006	0.0001	1.58183633
LHS	1	-0.16433395	0.14171836	-1.160	0.2462	8.00535876
LMEALS	1	1.93747933	0.13505216	14.346	0.0001	7.61643216

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	14978.51705	998.56780	134.286	0.0001
ERROR	9180	68263.45272	7.43610596		
C TOTAL	9195	83241.96976			

ROOT MSE	2.726922	R-SQUARE	0.1799
DEP MEAN	3.576274	ADJ R-SQ	0.1786
C.V.	76.25037		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	2.18904255	0.21781186	10.050	0.0001	0
EDHM1	1	-0.59732495	0.06520130	-9.161	0.0001	1.08372317
EMPSHM2	1	0.10357378	0.06087925	1.701	0.0889	1.09868125
SXHM	1	0.24572526	0.12211867	2.012	0.0442	1.14914932
U1	1	0.07004997	0.07426821	0.943	0.3456	1.44713815
U2	1	-0.11152707	0.07029505	-1.587	0.1126	1.37862885
R1	1	0.82950518	0.07735807	10.723	0.0001	1.38961478
R2	1	0.19801229	0.07702626	2.571	0.0102	1.36165812
R4	1	0.96209524	0.08737872	11.011	0.0001	1.33340662
S1	1	-0.40505623	0.08334505	-4.860	0.0001	1.53685776
S3	1	-0.55354360	0.08056485	-6.871	0.0001	1.57890406
S4	1	-0.51551664	0.08090347	-6.372	0.0001	1.57289165
RAC	1	-0.10237342	0.08803040	-1.163	0.2449	1.21428214
INVINC	1	-1129.57159	226.90742	-4.978	0.0001	1.36164242
INVHS	1	-0.33013131	0.17955244	-1.839	0.0660	2.99352277
MEALS	1	0.03138681	0.001409944	22.261	0.0001	2.55489491

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	1373.26493	91.55099565	159.960	0.0001
ERROR	9180	5254.05485	0.57233713		
C TOTAL	9195	6627.31978			

ROOT MSE	0.7565297	R-SQUARE	0.2072
DEP MEAN	0.959613	ADJ R-SQ	0.2059
C.V.	78.83696		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-2.44017485	0.17107764	-14.264	0.0001
EDHM1	1	-0.17587505	0.01841443	-9.551	0.0001
EMPSHM2	1	0.06514984	0.01742013	3.740	0.0002
SXHM	1	0.12348675	0.03337726	3.700	0.0002
U1	1	0.05729016	0.02064466	2.775	0.0055
U2	1	-0.01893693	0.01958173	-0.967	0.3335
R1	1	0.28477684	0.02146315	13.268	0.0001
R2	1	0.11597989	0.02138324	5.424	0.0001
R4	1	0.29690588	0.02424909	12.244	0.0001
S1	1	-0.12413701	0.02312376	-5.368	0.0001
S3	1	-0.12604284	0.02235015	-5.639	0.0001
S4	1	-0.10323387	0.02244550	-4.599	0.0001
RAC	1	0.001169166	0.02467448	0.047	0.9622
LINC	1	0.13017703	0.01339796	9.716	0.0001
LHS	1	-0.05640520	0.03921058	-1.439	0.1503
LMEALS	1	0.55191117	0.03736617	14.770	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	1321.52526	88.10168381	152.432	0.0001
ERROR	9180	5305.79452	0.57797326		
C TOTAL	9195	6627.31978			

ROOT MSE	0.7602455	R-SQUARE	0.1994
DEP MEAN	0.959613	ADJ R-SQ	0.1981
C.V.	79.22418		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	0.71999505	0.06072431	11.857	0.0001
EDHM1	1	-0.18781141	0.01817763	-10.332	0.0001
EMPSHM2	1	0.06691197	0.01697268	3.942	0.0001
SXHM	1	0.15043134	0.03404577	4.419	0.0001
U1	1	0.04740371	0.02070542	2.289	0.0221
U2	1	-0.02324890	0.01959717	-1.186	0.2355
R1	1	0.28792349	0.02156685	13.350	0.0001
R2	1	0.12177198	0.02147434	5.671	0.0001
R4	1	0.30429180	0.02436053	12.491	0.0001
S1	1	-0.12334249	0.02323597	-5.308	0.0001
S3	1	-0.12555030	0.02246088	-5.590	0.0001
S4	1	-0.09917605	0.02255528	-4.397	0.0001
RAC	1	-0.02439265	0.02454222	-0.994	0.3203
INVINC	1	-463.78934	63.26008851	-7.331	0.0001
INVHS	1	-0.40030351	0.05005787	-7.997	0.0001
MEALS	1	0.006390081	0.000393082	16.256	0.0001

F.5. Grains

QUADRATIC MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	81130.40873	4507.24493	412.416	0.0001
ERROR	9639	105343.54	10.92888674		
C TOTAL	9657	186473.95			

ROOT MSE	3.305887	R-SQUARE	0.4351
DEP MEAN	5.659228	ADJ R-SQ	0.4340
C.V.	58.41586		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	0.47939017	0.21740819	2.205	0.0275	0
EDHM1	1	0.11383259	0.07945930	1.433	0.1520	1.13109845
EMPSHM2	1	-0.28091918	0.07412381	-3.790	0.0002	1.16406634
SXHM	1	-0.45536080	0.13805517	-3.298	0.0010	1.11734408
U1	1	0.01087763	0.08833606	0.123	0.9020	1.45904081
U2	1	-0.31276058	0.08343151	-3.749	0.0002	1.39765294
R1	1	1.36486620	0.09160607	14.899	0.0001	1.37244402
R2	1	0.50381435	0.09111581	5.529	0.0001	1.35235781
R4	1	0.04663896	0.10383444	0.449	0.6533	1.31826179
S1	1	-0.13233073	0.09840616	-1.345	0.1787	1.54050675
S3	1	0.13888261	0.09542896	1.455	0.1456	1.58017756
S4	1	0.48698920	0.09593623	5.076	0.0001	1.57450163
RAC	1	-0.60210776	0.10494854	-5.737	0.0001	1.25037123
INC	1	-0.000058911	0.000018454	-3.192	0.0014	17.51994049
HS	1	0.85272386	0.08990916	9.484	0.0001	19.66083822
MEALS	1	0.05669006	0.002807938	20.189	0.0001	7.27595237
INC2	1	4.72016E-10	5.82919E-10	0.810	0.4181	16.06353226
HS2	1	-0.06571607	0.008059344	-8.154	0.0001	10.72045561
INCCHS	1	0.000021232	.00000298379	7.116	0.0001	10.95126054

SEMILOG MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	75317.87866	5021.19191	435.553	0.0001
ERROR	9642	111156.07	11.52832082		
C TOTAL	9657	186473.95			

ROOT MSE	3.395338	R-SQUARE	0.4039
DEP MEAN	5.659228	ADJ R-SQ	0.4030
C.V.	59.99649		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-7.30911903	0.74301730	-9.837	0.0001	0
EDHM1	1	0.07367045	0.08140275	0.905	0.3655	1.12537947
EMPSHM2	1	-0.29730573	0.07637101	-3.893	0.0001	1.17146446
SXHM	1	-1.00850081	0.14225938	-7.090	0.0001	1.12442663
U1	1	0.02045417	0.09066705	0.226	0.8215	1.45713658
U2	1	-0.33903111	0.08566352	-3.958	0.0001	1.39682127
R1	1	1.41651917	0.09416176	15.043	0.0001	1.37469103
R2	1	0.61021924	0.09346386	6.529	0.0001	1.34896711
R4	1	0.05055749	0.10670538	0.474	0.6357	1.31977929
S1	1	-0.14886368	0.10101681	-1.474	0.1406	1.53892052
S3	1	0.09182077	0.09790112	0.938	0.3483	1.57663318
S4	1	0.44279062	0.09839107	4.500	0.0001	1.56999767
RAC	1	-0.58382946	0.10728275	-5.442	0.0001	1.23867062
LINC	1	-0.01116432	0.05822913	-0.192	0.8480	1.59591904
LHS	1	1.76807491	0.17142362	10.314	0.0001	8.08664763
LMEALS	1	3.10005637	0.16343252	18.968	0.0001	7.70790013

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	79019.42940	5267.96196	472.699	0.0001
ERROR	9642	107454.52	11.14442218		
C TOTAL	9657	186473.95			

ROOT MSE	3.338326	R-SQUARE	0.4238
DEP MEAN	5.659228	ADJ R-SQ	0.4229
C.V.	58.98988		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	2.43071433	0.25832147	9.410	0.0001	0
EDHM1	1	0.03575518	0.07858484	0.455	0.6491	1.08494301
EMPSHM2	1	-0.38377362	0.07276245	-5.274	0.0001	1.10000630
SXHM	1	-0.48196478	0.14209737	-3.392	0.0007	1.16083909
U1	1	-0.04921319	0.08897394	-0.553	0.5802	1.45156147
U2	1	-0.38915623	0.08387153	-4.640	0.0001	1.38511756
R1	1	1.38471975	0.09255009	14.962	0.0001	1.37378298
R2	1	0.54514685	0.09178825	5.939	0.0001	1.34585004
R4	1	0.01835915	0.10486864	0.175	0.8610	1.31864680
S1	1	-0.10195081	0.09932057	-1.026	0.3047	1.53891905
S3	1	0.16953299	0.09625794	1.761	0.0782	1.57665605
S4	1	0.52911277	0.09672941	5.470	0.0001	1.56968752
RAC	1	-0.64954428	0.10440483	-6.221	0.0001	1.21351671
INVINC	1	-233.18969	263.31811	-0.886	0.3759	1.36456954
INVMS	1	-1.69842269	0.21300627	-7.974	0.0001	3.01057272
MEALS	1	0.07505451	0.001684754	44.549	0.0001	2.56865710

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3146.98350	209.79890	619.283	0.0001
ERROR	9642	3266.48781	0.33877700		
C TOTAL	9657	6413.47132			

ROOT MSE	0.5820455	R-SQUARE	0.4907
DEP MEAN	1.442548	ADJ R-SQ	0.4899
C.V.	40.34843		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-1.68222438	0.12737167	-13.207	0.0001
EDHM1	1	-0.003228478	0.01395446	-0.231	0.8170
EMPSHM2	1	-0.03770624	0.01309189	-2.880	0.0040
SXHM	1	-0.14360494	0.02438337	-5.889	0.0001
U1	1	0.01169479	0.01554259	0.752	0.4518
U2	1	-0.03509028	0.01468486	-2.390	0.0169
R1	1	0.22715691	0.01614167	14.073	0.0001
R2	1	0.08465539	0.01602203	5.284	0.0001
R4	1	-0.02239494	0.01829196	-1.224	0.2209
S1	1	-0.03422824	0.01731680	-1.977	0.0481
S3	1	0.03425541	0.01678269	2.041	0.0413
S4	1	0.06990071	0.01686668	4.144	0.0001
RAC	1	-0.11444718	0.01839094	-6.223	0.0001
LINC	1	0.03879361	0.009981924	3.886	0.0001
LMS	1	0.31619906	0.02938624	10.760	0.0001
LMEALS	1	0.65872444	0.02801640	23.512	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3062.06573	204.13772	587.305	0.0001
ERROR	9642	3351.40558	0.34758407		
C TOTAL	9657	6413.47132			

ROOT MSE	0.5895626	R-SQUARE	0.4774
DEP MEAN	1.442548	ADJ R-SQ	0.4766
C.V.	40.86953		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	1.50447786	0.04562067	32.978	0.0001
EDHM1	1	-0.001551588	0.01387842	-0.112	0.9110
EMPSHM2	1	-0.02567085	0.01285016	-1.998	0.0458
SXHM	1	-0.12934927	0.02509500	-5.154	0.0001
U1	1	0.005283391	0.01571318	0.336	0.7367
U2	1	-0.03513837	0.01481207	-2.372	0.0177
R1	1	0.23109888	0.01634476	14.139	0.0001
R2	1	0.08993554	0.01621020	5.548	0.0001
R4	1	-0.01663942	0.01852025	-0.898	0.3690
S1	1	-0.03359960	0.01754063	-1.916	0.0555
S3	1	0.03646658	0.01699956	2.145	0.0320
S4	1	0.07634901	0.01708282	4.469	0.0001
RAC	1	-0.11892283	0.01843834	-6.450	0.0001
INVINC	1	-76.63698416	46.50309829	-1.648	0.0994
INVMS	1	-1.08883660	0.03761781	-28.945	0.0001
MEALS	1	0.009002880	0.000297535	30.258	0.0001

F.6. Meat and Meat Alternates

QUADRATIC MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	600829.57	33379.42030	368.666	0.0001
ERROR	9649	873631.67	90.54116129		
C TOTAL	9667	1474461.23			

ROOT MSE	9.515312	R-SQUARE	0.4075
DEP MEAN	17.50795	ADJ R-SQ	0.4064
C.V.	54.34853		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	0.67959083	0.62520823	1.087	0.2771	0
EDHM1	1	1.80334519	0.22851755	7.891	0.0001	1.13143876
EMPSHM2	1	-0.82611277	0.21320273	-3.875	0.0001	1.16406973
SXHM	1	-1.56052389	0.39684505	-3.932	0.0001	1.11750165
U1	1	1.20636015	0.25407349	4.748	0.0001	1.45926955
U2	1	-0.97920211	0.24008180	-4.079	0.0001	1.39813095
R1	1	2.14862368	0.26359593	8.151	0.0001	1.37214259
R2	1	-0.89335887	0.26207831	-3.409	0.0007	1.35212183
R4	1	-1.14285455	0.29862670	-3.827	0.0001	1.31839717
S1	1	0.48976409	0.28305817	1.730	0.0836	1.54134082
S3	1	1.03509903	0.27458725	3.770	0.0002	1.58056758
S4	1	1.02161340	0.27604084	3.701	0.0002	1.57483438
RAC	1	3.55636915	0.30131322	11.804	0.0001	1.24921261
INC	1	0.000277150	0.000053047	5.225	0.0001	17.49286385
HS	1	-0.03374823	0.25860781	-0.130	0.8962	19.66353090
MEALS	1	0.23989091	0.008081726	29.683	0.0001	7.28603205
INC2	1	-2.56487E-09	1.67677E-09	-1.530	0.1261	16.05562191
HS2	1	-0.10041120	0.02318023	-4.332	0.0001	10.71168609
INCCHS	1	0.000017572	.00000858314	2.047	0.0407	10.94624061

SEMILOG MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	577697.07	38513.13805	414.522	0.0001
ERROR	9652	896764.16	92.90967260		
C TOTAL	9667	1474461.23			

ROOT MSE	9.638966	R-SQUARE	0.3918
DEP MEAN	17.50795	ADJ R-SQ	0.3909
C.V.	55.0548		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-49.95754015	2.10755765	-23.704	0.0001	0
EDHM1	1	1.52933745	0.23090466	6.623	0.0001	1.12575131
EMPSHM2	1	-0.97392914	0.21666155	-4.495	0.0001	1.17149994
SXHM	1	-3.15788708	0.40328808	-7.830	0.0001	1.12466232
U1	1	1.26669519	0.25720763	4.925	0.0001	1.45736944
U2	1	-1.04999165	0.24313148	-4.319	0.0001	1.39732333
R1	1	2.26281270	0.26724110	8.467	0.0001	1.37440107
R2	1	-0.64199124	0.26514790	-2.421	0.0155	1.34869940
R4	1	-1.14223522	0.30267878	-3.774	0.0002	1.31989105
S1	1	0.42294833	0.28659106	1.476	0.1400	1.53977655
S3	1	0.85069098	0.27784183	3.062	0.0022	1.57700369
S4	1	0.83792351	0.27923005	3.001	0.0027	1.57035433
RAC	1	3.62867613	0.30375497	11.946	0.0001	1.23717718
LINC	1	2.05615979	0.16509253	12.455	0.0001	1.59457164
LHS	1	-1.49139173	0.48645814	-3.066	0.0022	8.10292568
LMEALS	1	13.12971067	0.46397304	28.298	0.0001	7.72907393

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	582254.95	38816.99658	419.927	0.0001
ERROR	9652	892206.28	92.43745152		
C TOTAL	9667	1474461.23			

ROOT MSE	9.61444	R-SQUARE	0.3949
DEP MEAN	17.50795	ADJ R-SQ	0.3940
C.V.	54.91472		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	9.26595354	0.74374191	12.459	0.0001	0
EDHM1	1	1.17182906	0.22611692	5.182	0.0001	1.08506600
EMPSHM2	1	-1.14568390	0.20940287	-5.471	0.0001	1.09990915
SXHM	1	-1.91160912	0.40872207	-4.677	0.0001	1.16107562
U1	1	0.99426076	0.25606867	3.883	0.0001	1.45177957
U2	1	-1.22982531	0.24149445	-5.093	0.0001	1.38561242
R1	1	2.36132208	0.26646870	8.862	0.0001	1.37344844
R2	1	-0.59309886	0.26417760	-2.245	0.0248	1.34568602
R4	1	-0.97495750	0.30178700	-3.231	0.0012	1.31882797
S1	1	0.48785659	0.28585843	1.707	0.0879	1.33974005
S3	1	0.97186486	0.27714026	3.507	0.0005	1.57706528
S4	1	1.02688580	0.27849162	3.687	0.0002	1.57003946
RAC	1	2.85359898	0.29992971	9.514	0.0001	1.21237523
INVINC	1	-5242.38940	757.45211	-6.921	0.0001	1.36386048
INVHS	1	-4.13551251	0.61246899	-6.752	0.0001	3.01267720
MEALS	1	0.19658753	0.004851242	40.523	0.0001	2.57150173

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2698.96979	179.93132	617.545	0.0001
ERROR	9652	2812.26157	0.29136568		
C TOTAL	9667	5511.23136			

ROOT MSE	0.539783	R-SQUARE	0.4897
DEP MEAN	2.618292	ADJ R-SQ	0.4889
C.V.	20.61584		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-2.35482621	0.11802342	-19.952	0.0001
EDHM1	1	0.10409748	0.01293068	8.050	0.0001
EMPSHM2	1	-0.04800192	0.01213307	-3.956	0.0001
SXHM	1	-0.13294384	0.02258417	-5.887	0.0001
U1	1	0.05392372	0.01440365	3.744	0.0002
U2	1	-0.06092138	0.01361539	-4.474	0.0001
R1	1	0.10172969	0.01496553	6.798	0.0001
R2	1	-0.05752649	0.01484831	-3.874	0.0001
R4	1	-0.07015331	0.01695004	-4.139	0.0001
S1	1	0.002490827	0.01604913	0.155	0.8767
S3	1	0.03601229	0.01555917	2.315	0.0207
S4	1	0.05779564	0.01563691	3.696	0.0002
RAC	1	0.21490361	0.01701031	12.634	0.0001
LINC	1	0.18939497	0.009245185	20.486	0.0001
LHS	1	-0.08895191	0.02724178	-3.265	0.0011
LMEALS	1	0.85890383	0.02598253	33.057	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2509.68757	167.31250	538.023	0.0001
ERROR	9652	3001.54380	0.31097636		
C TOTAL	9667	5511.23136			

ROOT MSE	0.5576525	R-SQUARE	0.4554
DEP MEAN	2.618292	ADJ R-SQ	0.4545
C.V.	21.29833		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	2.76173465	0.04313819	64.021	0.0001
EDHM1	1	0.08577653	0.01311513	6.540	0.0001
EMPSHM2	1	-0.03698626	0.01214569	-3.045	0.0023
SXHM	1	-0.13609044	0.02370652	-5.741	0.0001
U1	1	0.04162314	0.01485192	2.803	0.0051
U2	1	-0.06537633	0.01400706	-4.667	0.0001
R1	1	0.11804935	0.01545560	7.638	0.0001
R2	1	-0.03625243	0.01532271	-2.366	0.0180
R4	1	-0.04802309	0.01750412	-2.744	0.0061
S1	1	-0.000658892	0.01658024	-0.040	0.9683
S3	1	0.03201556	0.01607457	1.992	0.0464
S4	1	0.06068512	0.01615295	3.757	0.0002
RAC	1	0.17293166	0.01739639	9.941	0.0001
INVINC	1	-447.29156	43.93340701	-10.181	0.0001
INVHS	1	-0.99228046	0.03552416	-27.933	0.0001
MEALS	1	0.007147971	0.000281380	25.403	0.0001

F.7. Milk Equivalents

QUADRATIC MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	83323.87772	4629.10432	395.530	0.0001
ERROR	9602	112377.52	11.70355311		
C TOTAL	9620	195701.39			

ROOT MSE	3.421046	R-SQUARE	0.4258
DEP MEAN	5.75983	ADJ R-SQ	0.4247
C.V.	59.39491		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	1.85804244	0.22592794	8.224	0.0001	0
EDHM1	1	-0.39988711	0.08229361	-4.859	0.0001	1.13070862
EMPSHM2	1	-0.12002920	0.07682397	-1.562	0.1182	1.16376573
SXHM	1	-0.54705068	0.14349553	-3.812	0.0001	1.11634871
U1	1	-0.13377099	0.09149495	-1.462	0.1438	1.45584041
U2	1	-0.15608724	0.08646982	-1.805	0.0711	1.39518920
R1	1	1.09378831	0.09494275	11.521	0.0001	1.37298670
R2	1	0.30554738	0.09445966	3.235	0.0012	1.35282574
R4	1	0.79744535	0.10761355	7.410	0.0001	1.31924995
S1	1	-0.31773838	0.10204977	-3.114	0.0019	1.54191808
S3	1	-0.01101348	0.09896356	-0.111	0.9114	1.58158572
S4	1	0.02903864	0.09950693	0.292	0.7704	1.57559180
RAC	1	-1.39833058	0.10903999	-12.824	0.0001	1.24537215
INC	1	-0.000065461	0.000019130	-3.422	0.0006	17.48785789
HS	1	0.32096601	0.09327370	3.441	0.0006	19.66086840
MEALS	1	0.05892022	0.002912083	20.233	0.0001	7.28014141
INC2	1	3.41722E-10	6.03981E-10	0.566	0.5716	16.05742945
HS2	1	-0.01641366	0.008346522	-1.967	0.0493	10.70084062
INCCHS	1	0.000030126	.00000309502	9.734	0.0001	10.96314578

SEMILOG MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	74550.11243	4970.00750	394.027	0.0001
ERROR	9605	121151.28	12.61335578		
C TOTAL	9620	195701.39			

ROOT MSE	3.551529	R-SQUARE	0.3809
DEP MEAN	5.75983	ADJ R-SQ	0.3800
C.V.	61.66031		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-7.58847506	0.77902325	-9.741	0.0001	0
EDHM1	1	-0.44851608	0.08522407	-5.263	0.0001	1.12520110
EMPSHM2	1	-0.13589156	0.08000903	-1.698	0.0855	1.17121647
SXHM	1	-1.22296318	0.14943226	-8.184	0.0001	1.12530815
U1	1	-0.11610259	0.09492685	-1.223	0.2213	1.45400747
U2	1	-0.18601193	0.08973723	-2.073	0.0382	1.39423650
R1	1	1.14733764	0.09864334	11.631	0.0001	1.37519835
R2	1	0.44463130	0.09793651	4.540	0.0001	1.34935055
R4	1	0.79978519	0.11178193	7.155	0.0001	1.32075874
S1	1	-0.32501242	0.10588796	-3.069	0.0022	1.54034309
S3	1	-0.05024677	0.10262581	-0.490	0.6244	1.57812854
S4	1	-0.007221012	0.10315833	-0.070	0.9442	1.57120469
RAC	1	-1.38407036	0.11263040	-12.289	0.0001	1.23289442
LINC	1	0.07810533	0.06097879	1.281	0.2003	1.38454749
LHS	1	1.62049452	0.17977495	9.014	0.0001	8.07171009
LMEALS	1	3.21584546	0.17144804	18.757	0.0001	7.69952231

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	80335.97239	5355.73149	445.903	0.0001
ERROR	9605	115365.42	12.01097578		
C TOTAL	9620	195701.39			

ROOT MSE	3.465685	R-SQUARE	0.4105
DEP MEAN	5.75983	ADJ R-SQ	0.4096
C.V.	60.16993		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	2.64797246	0.26921311	9.836	0.0001	0
EDHM1	1	-0.52064073	0.08165345	-6.376	0.0001	1.08469337
EMPSHM2	1	-0.27322354	0.07565778	-3.611	0.0003	1.09981284
SXHM	1	-0.61819387	0.14820327	-4.171	0.0001	1.16032089
U1	1	-0.20761976	0.09244864	-2.246	0.0247	1.44830571
U2	1	-0.25794769	0.08721220	-2.958	0.0031	1.38292280
R1	1	1.12389021	0.09622235	11.680	0.0001	1.37414986
R2	1	0.37801472	0.09545852	3.960	0.0001	1.34622386
R4	1	0.77260043	0.10903332	7.086	0.0001	1.51962674
S1	1	-0.27237107	0.10332663	-2.636	0.0084	1.54028541
S3	1	0.03852143	0.10014418	0.385	0.7005	1.57809455
S4	1	0.09132108	0.10065605	0.907	0.3643	1.57092712
RAC	1	-1.50634896	0.10881889	-13.843	0.0001	1.20858044
INVINC	1	-326.31685	273.88687	-1.191	0.2335	1.35577735
INVHS	1	-1.03630803	0.22179133	-4.672	0.0001	3.00168240
MEALS	1	0.08098881	0.001750879	46.256	0.0001	2.564389

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2927.22827	195.14855	519.098	0.0001
ERROR	9605	3610.88198	0.37593774		
C TOTAL	9620	6538.11025			

ROOT MSE	0.6131376	R-SQUARE	0.4477
DEP MEAN	1.45586	ADJ R-SQ	0.4469
C.V.	42.11516		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-1.60819772	0.13449095	-11.958	0.0001
EDHM1	1	-0.10275231	0.01471313	-6.984	0.0001
EMPSHM2	1	-0.03416597	0.01381280	-2.474	0.0134
SXHM	1	-0.12678244	0.02579806	-4.914	0.0001
U1	1	-0.02029277	0.01638787	-1.238	0.2156
U2	1	-0.03560862	0.01549228	-2.298	0.0216
R1	1	0.18864068	0.01702983	11.077	0.0001
R2	1	0.05879427	0.01690781	3.477	0.0005
R4	1	0.12805401	0.01929809	6.636	0.0001
S1	1	-0.07223552	0.01828055	-3.951	0.0001
S3	1	0.003477212	0.01771737	0.196	0.8444
S4	1	0.006261140	0.01780930	0.352	0.7252
RAC	1	-0.29562948	0.01944457	-15.204	0.0001
LINC	1	0.05484338	0.01052741	5.210	0.0001
LHS	1	0.27652511	0.03103643	8.910	0.0001
LMEALS	1	0.64379343	0.02959887	21.751	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2862.78481	190.85232	498.768	0.0001
ERROR	9605	3675.32545	0.38264711		
C TOTAL	9620	6538.11025			

ROOT MSE	0.6185848	R-SQUARE	0.4379
DEP MEAN	1.45586	ADJ R-SQ	0.4370
C.V.	42.48931		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	1.52514298	0.04805143	31.740	0.0001
EDHM1	1	-0.10716552	0.01457420	-7.353	0.0001
EMPSHM2	1	-0.03077826	0.01350404	-2.279	0.0227
SXHM	1	-0.09979520	0.02645257	-3.773	0.0002
U1	1	-0.02935292	0.01650101	-1.779	0.0753
U2	1	-0.03936672	0.01556637	-2.529	0.0115
R1	1	0.19277292	0.01717458	11.224	0.0001
R2	1	0.06324576	0.01703826	3.712	0.0002
R4	1	0.13380414	0.01946119	6.875	0.0001
S1	1	-0.07014840	0.01844261	-3.804	0.0001
S3	1	0.007383221	0.01787458	0.413	0.6796
S4	1	0.01439961	0.01796596	0.801	0.4229
RAC	1	-0.30821012	0.01942291	-15.868	0.0001
INVINC	1	-121.97860	48.88563808	-2.495	0.0126
INVHS	1	-0.91633643	0.03958719	-23.147	0.0001
MEALS	1	0.009573408	0.000312512	30.634	0.0001

F.8. Sugars and Sweets

QUADRATIC MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	3467.33764	192.62987	75.653	0.0001
ERROR	8834	22499.20259	2.54688732		
C TOTAL	8852	25966.54023			

ROOT MSE	1.595897	R-SQUARE	0.1335
DEP MEAN	1.331326	ADJ R-SQ	0.1318
C.V.	119.8727		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	0.27360317	0.11269239	2.428	0.0152	0
EDHM1	1	0.01560746	0.04003609	0.390	0.6967	1.12742671
EMPSHM2	1	-0.01210650	0.03723219	-0.325	0.7451	1.15434338
SXHM	1	-0.09820075	0.07388316	-1.329	0.1838	1.10745238
U1	1	-0.15907534	0.04467300	-3.561	0.0004	1.45554414
U2	1	0.11426407	0.04188594	2.728	0.0064	1.39117649
R1	1	0.03605179	0.04610570	0.782	0.4345	1.36680038
R2	1	0.09855843	0.04578799	2.152	0.0314	1.35513797
R4	1	0.08962554	0.05264490	1.702	0.0887	1.31063710
S1	1	0.16483111	0.05011837	3.289	0.0010	1.53603658
S3	1	0.15209941	0.04803080	3.167	0.0015	1.59241553
S4	1	0.12788027	0.04828774	2.648	0.0081	1.58592250
RAC	1	-0.30387306	0.05317776	-5.714	0.0001	1.25721011
INC	1	-.0000071277	.00000935976	-0.762	0.4464	17.63723378
HS	1	0.08323467	0.04500145	1.850	0.0644	19.46384392
MEALS	1	0.01225643	0.001383290	8.860	0.0001	6.97098957
INC2	1	-3.83681E-10	2.94184E-10	-1.304	0.1922	16.20593890
HS2	1	-0.006841960	0.004000904	-1.710	0.0873	10.82769353
INCCHS	1	.00000721184	.00000149519	4.823	0.0001	11.10546535

SEMILOG MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3160.62687	210.70846	81.647	0.0001
ERROR	8837	22805.91336	2.58073027		
C TOTAL	8852	25966.54023			

ROOT MSE	1.606465	R-SQUARE	0.1217
DEP MEAN	1.331326	ADJ R-SQ	0.1202
C.V.	120.6665		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-1.71409788	0.36866430	-4.649	0.0001	0
EDHM1	1	0.009263375	0.04018072	0.231	0.8177	1.12069541
EMPSHM2	1	-0.02276072	0.03759355	-0.605	0.5449	1.16142640
SXHM	1	-0.24683651	0.07456576	-3.310	0.0009	1.11321780
U1	1	-0.15523182	0.04493627	-3.454	0.0006	1.45343719
U2	1	0.10607667	0.04214806	2.517	0.0119	1.39017016
R1	1	0.04785914	0.04644639	1.030	0.3028	1.36888443
R2	1	0.12788182	0.04602745	2.778	0.0055	1.35139232
R4	1	0.09019176	0.05302761	1.701	0.0890	1.31232375
S1	1	0.16611346	0.05042345	3.294	0.0010	1.53440509
S3	1	0.14772295	0.04828680	3.059	0.0022	1.58833010
S4	1	0.12177756	0.04853379	2.509	0.0121	1.58111621
RAC	1	-0.30965514	0.05323713	-5.817	0.0001	1.24349537
LINC	1	-0.007724012	0.02898941	-0.266	0.7899	1.57350436
LHS	1	0.28934832	0.08449823	3.424	0.0006	7.69926187
LMEALS	1	0.75251057	0.08084885	9.308	0.0001	7.35071960

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3352.36853	223.49124	87.334	0.0001
ERROR	8837	22614.17170	2.55903267		
C TOTAL	8852	25966.54023			

ROOT MSE	1.599698	R-SQUARE	0.1291
DEP MEAN	1.331326	ADJ R-SQ	0.1276
C.V.	120.1582		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	0.46093276	0.13205988	3.490	0.0005	0
EDHM1	1	-0.002003967	0.03930317	-0.051	0.9593	1.08136929
EMPSHM2	1	-0.04408559	0.03634250	-1.213	0.2252	1.09461476
SXHM	1	-0.10999944	0.07549190	-1.457	0.1451	1.15071768
U1	1	-0.17199100	0.04465739	-3.851	0.0001	1.44762379
U2	1	0.09397283	0.04178556	2.249	0.0245	1.37794558
R1	1	0.04354032	0.04623382	0.942	0.3464	1.36788372
R2	1	0.11351934	0.04577142	2.480	0.0132	1.34773065
R4	1	0.08632107	0.05277522	1.636	0.1020	1.31088299
S1	1	0.17483257	0.05020277	3.483	0.0005	1.53390018
S3	1	0.16359287	0.04807953	3.403	0.0007	1.58807551
S4	1	0.14042671	0.04832254	2.906	0.0037	1.58067132
RAC	1	-0.33432165	0.05243196	-6.376	0.0001	1.21639311
INVINC	1	57.22735216	135.45853	0.422	0.6727	1.35720066
INVHS	1	-0.24658756	0.10989841	-2.244	0.0249	2.93417085
MEALS	1	0.01678725	0.000831815	20.181	0.0001	2.50391380

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2809.15452	187.27697	155.532	0.0001
ERROR	8837	10640.67525	1.20410493		
C TOTAL	8852	13449.82977			

ROOT MSE	1.097317	R-SQUARE	0.2089
DEP MEAN	-0.308564	ADJ R-SQ	0.2075
C.V.	-355.621		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-3.50291815	0.25182100	-13.910	0.0001
EDHM1	1	-0.03323457	0.02744597	-1.211	0.2260
EMPSHM2	1	-0.01666665	0.02567877	-0.649	0.5163
SXHM	1	-0.10408371	0.05093312	-2.044	0.0410
U1	1	-0.12959277	0.03069431	-4.222	0.0001
U2	1	0.11234786	0.02878978	3.902	0.0001
R1	1	-0.08202991	0.03172582	-2.586	0.0097
R2	1	0.02670560	0.03143966	0.849	0.3957
R4	1	0.01473990	0.03622120	0.407	0.6841
S1	1	0.03210048	0.03444240	0.932	0.3514
S3	1	0.13797393	0.03298294	4.183	0.0001
S4	1	0.09598416	0.03315165	2.895	0.0038
RAC	1	-0.18221895	0.03636432	-5.011	0.0001
LINC	1	0.008035737	0.01980160	0.406	0.6849
LHS	1	0.20842102	0.05771763	3.611	0.0003
LMEALS	1	0.76679845	0.05522487	13.885	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	2739.21882	182.61459	150.670	0.0001
ERROR	8837	10710.61095	1.21201889		
C TOTAL	8852	13449.82977			

ROOT MSE	1.100917	R-SQUARE	0.2037
DEP MEAN	-0.308564	ADJ R-SQ	0.2023
C.V.	-356.788		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-0.52745075	0.09088405	-5.804	0.0001
EDHM1	1	-0.02858402	0.02704857	-1.057	0.2906
EMPSHM2	1	-0.001457749	0.02501103	-0.058	0.9535
SXHM	1	-0.05922101	0.05195378	-1.140	0.2544
U1	1	-0.13535755	0.03073337	-4.404	0.0001
U2	1	0.11351635	0.02875696	3.947	0.0001
R1	1	-0.08118844	0.03181827	-2.552	0.0107
R2	1	0.02693314	0.03150006	0.855	0.3926
R4	1	0.01983094	0.03632009	0.546	0.5851
S1	1	0.03231211	0.03454972	0.935	0.3497
S3	1	0.14117205	0.03308850	4.266	0.0001
S4	1	0.10267469	0.03325573	3.087	0.0020
RAC	1	-0.18949572	0.03608385	-5.252	0.0001
INVINC	1	21.04952171	93.22501682	0.226	0.8214
INVHS	1	-0.85907316	0.07563245	-11.359	0.0001
MEALS	1	0.01072313	0.000571907	18.750	0.0001

F.9. Vegetables

QUADRATIC MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	39646.21994	2202.56777	207.139	0.0001
ERROR	9583	101898.89	10.63329774		
C TOTAL	9601	141545.11			

ROOT MSE	3.260874	R-SQUARE	0.2801
DEP MEAN	5.540241	ADJ R-SQ	0.2787
C.V.	58.85798		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	1.65029389	0.21695437	7.607	0.0001	0
EDHM1	1	-0.10233424	0.07853102	-1.303	0.1925	1.13192686
EMPSHM2	1	-0.02210224	0.07332063	-0.301	0.7631	1.16378149
SXHM	1	0.48287632	0.13854370	3.485	0.0005	1.11100451
U1	1	0.18978274	0.08739513	2.172	0.0299	1.45762719
U2	1	-0.01313993	0.08250732	-0.159	0.8735	1.39720263
R1	1	0.15953774	0.09059640	1.761	0.0783	1.37353148
R2	1	-0.17942756	0.09013442	-1.991	0.0465	1.35292949
R4	1	-0.47544235	0.10274953	-4.627	0.0001	1.31924070
S1	1	-0.68574334	0.09739239	-7.041	0.0001	1.54055416
S3	1	-0.76810098	0.09440262	-8.136	0.0001	1.58092936
S4	1	-0.83367557	0.09490427	-8.784	0.0001	1.57514362
RAC	1	-0.38615701	0.10373842	-3.722	0.0002	1.24942524
INC	1	0.000075084	0.000018269	4.110	0.0001	17.55315828
HS	1	-0.09310035	0.08891738	-1.047	0.2951	19.61296737
MEALS	1	0.06894690	0.002776124	24.836	0.0001	7.24648607
INC2	1	-9.55101E-10	5.76209E-10	-1.658	0.0974	16.08675850
HS2	1	-0.02881357	0.007958500	-3.620	0.0003	10.70219530
INCCHS	1	.00000273331	.00000294732	0.927	0.3537	10.92728768

SEMILOG MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	38691.93097	2579.46206	240.408	0.0001
ERROR	9586	102853.18	10.72952026		
C TOTAL	9601	141545.11			

ROOT MSE	3.275595	R-SQUARE	0.2734
DEP MEAN	5.540241	ADJ R-SQ	0.2722
C.V.	59.12369		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-12.48807613	0.72273335	-17.279	0.0001	0
EDHM1	1	-0.16957792	0.07871388	-2.154	0.0312	1.12700575
EMPSHM2	1	-0.06666553	0.07388323	-0.902	0.3669	1.17111220
SXHM	1	0.07829235	0.13957210	0.561	0.5748	1.11748776
U1	1	0.21169379	0.08772638	2.413	0.0158	1.45552656
U2	1	-0.02816519	0.08285832	-0.340	0.7339	1.39647864
R1	1	0.18838681	0.09108125	2.068	0.0386	1.37582253
R2	1	-0.12526412	0.09043138	-1.385	0.1660	1.34964577
R4	1	-0.47894634	0.10327319	-4.638	0.0001	1.32077004
S1	1	-0.70644154	0.09778194	-7.225	0.0001	1.53897613
S3	1	-0.81551119	0.09472520	-8.609	0.0001	1.57747704
S4	1	-0.88176108	0.09519968	-9.262	0.0001	1.57075181
RAC	1	-0.35900426	0.10371576	-3.461	0.0005	1.25767961
LINC	1	0.47543477	0.05641948	8.427	0.0001	1.59311360
LHS	1	-0.89886514	0.16640766	-5.402	0.0001	8.08189173
LMEALS	1	3.92239145	0.15899489	24.670	0.0001	7.70130504

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	38670.84866	2578.05658	240.228	0.0001
ERROR	9586	102874.26	10.73171934		
C TOTAL	9601	141545.11			

ROOT MSE	3.27593	R-SQUARE	0.2732
DEP MEAN	5.540241	ADJ R-SQ	0.2721
C.V.	59.12975		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	3.53517069	0.25466878	13.881	0.0001	0
EDHM1	1	-0.21729439	0.07728475	-2.812	0.0049	1.08623088
EMPSHM2	1	-0.04421513	0.07165485	-0.617	0.5372	1.10130817
SXHM	1	0.43108852	0.14167686	3.043	0.0024	1.15120953
U1	1	0.15610215	0.08756013	1.783	0.0747	1.44971782
U2	1	-0.04894486	0.08251643	-0.593	0.5531	1.38469426
R1	1	0.19651423	0.09107224	2.158	0.0310	1.37326822
R2	1	-0.12558219	0.09035569	-1.390	0.1646	1.34711143
R4	1	-0.44062801	0.10324183	-4.268	0.0001	1.31967711
S1	1	-0.69396756	0.09779444	-7.096	0.0001	1.53905452
S3	1	-0.79633573	0.09472994	-8.406	0.0001	1.57731152
S4	1	-0.85021738	0.09518654	-8.932	0.0001	1.56999551
RAC	1	-0.53360196	0.10271464	-5.195	0.0001	1.21365253
INVINC	1	-1578.19426	262.26417	-6.018	0.0001	1.36998741
INVMS	1	-0.72858362	0.20999222	-3.470	0.0005	2.98523954
MEALS	1	0.05286990	0.001656541	31.916	0.0001	2.55654221

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	1825.64559	121.70971	340.070	0.0001
ERROR	9586	3430.79214	0.35789611		
C TOTAL	9601	5256.43773			

ROOT MSE	0.5982442	R-SQUARE	0.3475
DEP MEAN	1.477003	ADJ R-SQ	0.3463
C.V.	40.50393		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-2.70086899	0.13199772	-20.461	0.0001
EDHM1	1	-0.05388904	0.01437605	-3.749	0.0002
EMPSHM2	1	0.004813735	0.01349380	0.357	0.7213
SXHM	1	0.17402380	0.02549100	6.827	0.0001
U1	1	0.04231972	0.01602207	2.641	0.0083
U2	1	-0.02986578	0.01513298	-1.974	0.0485
R1	1	0.05263949	0.01663479	3.164	0.0016
R2	1	-0.02944651	0.01651610	-1.783	0.0746
R4	1	-0.06781582	0.01886149	-3.595	0.0003
S1	1	-0.12569021	0.01785858	-7.038	0.0001
S3	1	-0.11255607	0.01730031	-6.506	0.0001
S4	1	-0.13872667	0.01738697	-7.979	0.0001
RAC	1	-0.06735444	0.01894232	-3.556	0.0004
LINC	1	0.13833997	0.01030427	13.425	0.0001
LMS	1	-0.16216719	0.03039217	-5.336	0.0001
LMEALS	1	0.78102016	0.02903832	26.896	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	1683.71471	112.24765	301.173	0.0001
ERROR	9586	3572.72302	0.37270217		
C TOTAL	9601	5256.43773			

ROOT MSE	0.6104934	R-SQUARE	0.3203
DEP MEAN	1.477003	ADJ R-SQ	0.3193
C.V.	41.33326		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	1.39848156	0.04745937	29.467	0.0001
EDHM1	1	-0.06061017	0.01440257	-4.208	0.0001
EMPSHM2	1	0.02357363	0.01335340	1.765	0.0775
SXHM	1	0.19064119	0.02640251	7.221	0.0001
U1	1	0.03439559	0.01631747	2.108	0.0351
U2	1	-0.02966907	0.01537753	-1.929	0.0537
R1	1	0.06137006	0.01697197	3.616	0.0003
R2	1	-0.01689472	0.01683844	-1.003	0.3157
R4	1	-0.05264309	0.01923971	-2.736	0.0062
S1	1	-0.12794541	0.01822470	-7.020	0.0001
S3	1	-0.11648217	0.01765361	-6.598	0.0001
S4	1	-0.13828588	0.01773870	-7.796	0.0001
RAC	1	-0.09577629	0.01914162	-5.004	0.0001
INVINC	1	-423.25440	48.87483148	-8.660	0.0001
INVMS	1	-0.65819224	0.03913357	-16.818	0.0001
MEALS	1	0.006783996	0.000308709	21.975	0.0001

F.10. Other Items

QUADRATIC MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	18	3867.33661	214.85203	40.176	0.0001
ERROR	8763	46862.25269	5.34774081		
C TOTAL	8781	50729.58930			

ROOT MSE	2.312518	R-SQUARE	0.0762
DEP MEAN	2.310006	ADJ R-SQ	0.0743
C.V.	100.1088		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	0.71400008	0.16530043	4.319	0.0001	0
EDHM1	1	0.48145570	0.05828978	8.260	0.0001	1.12738133
EMPSHM2	1	0.04696628	0.05421566	0.866	0.3864	1.15318578
SXHM	1	0.02211982	0.10753775	0.206	0.8370	1.09972633
U1	1	-0.11817629	0.06501378	-1.818	0.0691	1.43631925
U2	1	-0.09622878	0.06066699	-1.586	0.1127	1.38856163
R1	1	-0.05985955	0.06671850	-0.897	0.3696	1.37828652
R2	1	-0.18283823	0.06652056	-2.749	0.0060	1.35356010
R4	1	-0.21790539	0.07724718	-2.821	0.0048	1.30779068
S1	1	0.06119091	0.07291277	0.839	0.4014	1.53489264
S3	1	0.07745784	0.06987418	1.109	0.2677	1.58902854
S4	1	0.13542781	0.07014987	1.931	0.0536	1.58381269
RAC	1	-0.70632617	0.08003808	-8.825	0.0001	1.21570486
INC	1	0.000041034	0.000013597	3.018	0.0026	17.74972116
HS	1	-0.03642136	0.06772062	-0.538	0.5907	20.49084958
MEALS	1	0.02022571	0.002038516	9.922	0.0001	7.09580666
INC2	1	-1.27612E-10	4.26988E-10	-0.299	0.7650	16.37673493
HS2	1	-0.01053936	0.006224032	-1.693	0.0904	11.54759214
INCCHS	1	-.0000011058	.00000219165	-0.505	0.6139	11.07579170

SEMILOG MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3920.96334	261.39756	48.953	0.0001
ERROR	8766	46808.62596	5.33979506		
C TOTAL	8781	50729.58930			

ROOT MSE	2.310799	R-SQUARE	0.0773
DEP MEAN	2.310006	ADJ R-SQ	0.0757
C.V.	100.0344		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	-4.86179453	0.53578453	-9.074	0.0001	0
EDHM1	1	0.44482438	0.05806624	7.661	0.0001	1.12041577
EMPSHM2	1	0.02611392	0.05436916	0.480	0.6309	1.16059658
SXHM	1	-0.09081920	0.10761088	-0.844	0.3987	1.10286151
U1	1	-0.11772181	0.06491898	-1.813	0.0698	1.45446817
U2	1	-0.10671362	0.06061486	-1.761	0.0784	1.38823955
R1	1	-0.05243911	0.06671333	-0.786	0.4319	1.38012405
R2	1	-0.16886542	0.06638098	-2.544	0.0110	1.34989202
R4	1	-0.22222107	0.07723175	-2.877	0.0040	1.30921408
S1	1	0.05741226	0.07282796	0.788	0.4305	1.53360337
S3	1	0.06269390	0.06974470	0.899	0.3687	1.58550133
S4	1	0.12036856	0.07000585	1.719	0.0856	1.57966393
RAC	1	-0.70271631	0.07953070	-8.836	0.0001	1.20212699
LINC	1	0.30901792	0.04202822	7.353	0.0001	1.57983032
LHS	1	-0.41626422	0.12221781	-3.406	0.0007	7.90220525
LMEALS	1	1.19421291	0.11635469	10.264	0.0001	7.50428817

INVERSE MODEL

DEP VARIABLE: EXPEND

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	3604.88557	240.32570	44.705	0.0001
ERROR	8766	47124.70373	5.37585030		
C TOTAL	8781	50729.58930			

ROOT MSE	2.318588	R-SQUARE	0.0711
DEP MEAN	2.310006	ADJ R-SQ	0.0695
C.V.	100.3715		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|	VARIANCE INFLATION
INTERCEP	1	1.94207736	0.19247000	10.090	0.0001	0
EDHM1	1	0.40267031	0.05722371	7.037	0.0001	1.08083932
EMPSHM2	1	0.02227824	0.05294681	0.421	0.6739	1.09408850
SXHM	1	-0.04639243	0.10951607	-0.424	0.6719	1.13459694
U1	1	-0.13921512	0.06499879	-2.142	0.0322	1.42835271
U2	1	-0.11863171	0.06057197	-1.959	0.0502	1.37697765
R1	1	-0.03182361	0.06687826	-0.476	0.6342	1.37765386
R2	1	-0.14362483	0.06650718	-2.160	0.0308	1.34594116
R4	1	-0.18752192	0.07740566	-2.423	0.0154	1.30629612
S1	1	0.05765186	0.07307884	0.789	0.4302	1.53383045
S3	1	0.06080655	0.06998473	0.869	0.3850	1.58572548
S4	1	0.12699333	0.07024744	1.808	0.0707	1.57991725
RAC	1	-0.78243704	0.07913589	-9.887	0.0001	1.18223817
INVINC	1	-852.25572	206.23635	-4.132	0.0001	1.37520064
INVMS	1	-0.57128376	0.15765550	-3.624	0.0003	3.00418192
MEALS	1	0.01171632	0.001226782	9.551	0.0001	2.55608312

DOUBLE-LOG MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	1335.80881	89.05392075	77.497	0.0001
ERROR	8766	10073.20026	1.14912164		
C TOTAL	8781	11409.00907			

ROOT MSE	1.071971	R-SQUARE	0.1171
DEP MEAN	0.3544733	ADJ R-SQ	0.1156
C.V.	302.4123		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	-3.89261973	0.24854839	-15.661	0.0001
EDHM1	1	0.29721590	0.02695670	11.034	0.0001
EMPSHM2	1	0.05762137	0.02521237	2.285	0.0223
SXHM	1	0.03963341	0.04992027	0.794	0.4273
U1	1	-0.04680785	0.03011566	-1.554	0.1202
U2	1	-0.05587842	0.02811900	-1.987	0.0469
R1	1	-0.13935286	0.03094806	-4.503	0.0001
R2	1	-0.19483929	0.03079388	-6.327	0.0001
R4	1	-0.25344391	0.03582751	-7.074	0.0001
S1	1	-0.05120019	0.03378461	-1.515	0.1297
S3	1	0.03867139	0.03235430	1.195	0.2320
S4	1	0.09435389	0.03247545	2.905	0.0037
RAC	1	-0.38710130	0.03689399	-10.492	0.0001
LINC	1	0.17912315	0.01949673	9.187	0.0001
LMS	1	-0.29059158	0.05669637	-5.125	0.0001
LMEALS	1	0.70978397	0.05397649	13.150	0.0001

LOG-INVERSE MODEL

DEP VARIABLE: LEXP

ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	15	1204.48241	80.29882727	68.979	0.0001
ERROR	8766	10204.52666	1.16410297		
C TOTAL	8781	11409.00907			

ROOT MSE	1.078936	R-SQUARE	0.1056
DEP MEAN	0.3544733	ADJ R-SQ	0.1040
C.V.	304.3772		

PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB > |T|
INTERCEP	1	0.23414590	0.08956435	2.614	0.0090
EDHM1	1	0.28134886	0.02662859	10.566	0.0001
EMPSHM2	1	0.07134677	0.02463836	2.896	0.0038
SXHM	1	0.04808985	0.05096241	0.944	0.3454
U1	1	-0.05511673	0.03024666	-1.822	0.0685
U2	1	-0.05661774	0.02818667	-2.009	0.0446
R1	1	-0.12855864	0.03112125	-4.131	0.0001
R2	1	-0.17848189	0.03094858	-5.767	0.0001
R4	1	-0.23382189	0.03602009	-6.491	0.0001
S1	1	-0.05382493	0.03400664	-1.583	0.1135
S3	1	0.03266913	0.03256682	1.003	0.3158
S4	1	0.09335606	0.03268908	2.856	0.0043
RAC	1	-0.42187684	0.03682524	-11.456	0.0001
INVINC	1	-572.41129	95.97040743	-5.964	0.0001
INVMS	1	-0.44510131	0.07336370	-6.067	0.0001
MEALS	1	0.005058571	0.000570836	8.862	0.0001

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