THE USE OF DEFENSIVE INTERVALS IN CORPORATE FAILURE PREDICTION AND AUDITORS' GOING CONCERN EVALUATIONS

by

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CHAPTER 1

INTRODUCTION

Going concern is one of the most fundamental and firmly established concepts generally accepted in discussions of financial accounting and policy formation. The going concern postulate may be simply described:

> In the absence of evidence to the contrary, the entity is viewed as remaining in operation indefinitely.[1]

Accounting Principles Board Statement No. 4 provides the rationale for this basic feature of accounting:

> Because of the relative permanence of enterprises, financial accounting is formulated basically for going concerns. Past experience indicates that continuation of operations is highly probable for most enterprises although continuation cannot be known with certainty.[2]

In practice, the application of the evidence gathering process vis-à-vis enterprise continuity has often been deficient. When liquidation seems imminent, firms are no longer treated as going concerns and traditional historical cost accounting treatments are modified to encompass a realizable value standard. In the absence of such clear evidence of impending liquidation, however, one automatically defaults to the going concern presumption.

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[1] Committee on Concepts and Standards Underlying Corporate Financial Statements, Accounting and Reporting Standards for Corporate Financial Statements, (American Accounting Association, 1957), p. 2.

[2] Accounting Principles Board, Basic Concepts and Accounting Principles Underlying Financial Statements of Business Enterprises--APB Statement No. 4, (AICPA, 1970), p. 45.

Fremgen implies that such an approach is misdirected:

> Perhaps the definition cited . . . should be altered to read that "the entity is viewed as remaining in operation indefinitely" in recognition of evidence to that effect, not "in the absence of evidence to the contrary."[3]

Thus, enterprise continuity should be viewed as a conclusion or expectation based upon evidence, not as a universally applied presumption.

Sterling advances a similar argument and concludes:

> A strong case can be made that the accounting reports ought to show something about the likelihood of the firms continuing instead of the reports being prepared under the assumption that it [sic] will continue. If one accepts this as a proper function of accounting, then the status of the going concern concept ought to be changed from an assumption to a prediction.[4]

From this perspective, going concern is interpreted as an expectation which is proposed in an environment of future uncertainty.

It is in this arena of the unknown future that the auditor faces a major challenge. In an attempt to evaluate the overall fairness of financial statement presentations, the auditor must be cognizant of the impact of the uncertain future events. Of primary relevance in this regard is the auditor's evaluation of the capacity of the firm to continue operations as a going concern. This evaluation is largely a matter of auditor judgement based upon the application of analytical techniques to a myriad of evidential matter.

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[3]James Fremgen, "The Going Concern Assumption: A Critical Appraisal," Accounting Review, (October, 1968), p. 650.

[4]Robert R. Sterling, "The Going Concern: An Examination," Accounting Review, (July, 1968), p. 494.

Bankruptcy prediction models have been offered in the literature as one important source of input to such an analytical review process and serve as the impetus for this research project. In reviewing the present and potential impact of empirical research in accounting, Kaplan identifies the study of bankruptcy prediction as offering ". . . the most immediate relevance to practicing auditors":

> . . . auditors get into much of their troubles when they give an unqualified opinion to a firm which shortly thereafter goes bankrupt. Analytic techniques which help an auditor decide when a firm is approaching default or insolvency would seem to offer significant benefits in reducing legal and insurance expenses.[5]

In Auditing Research Monograph No. 1, Carmichael develops a classification scheme identifying the basic factors which suggest going concern problems (i.e., imperiled continuing operations). One major class of going concern problem indicators relates to the enterprise's difficulty in meeting obligations. Liquidity deficiency (defined by Carmichael as a working capital deficit position) is proposed as an important component of this group of factors signalling financial distress.[6] This basic notion of liquidity will serve as a primary focus of this research.

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[5]Robert S. Kaplan, "The Information Content of Financial Accounting Numbers: A Survey of Empirical Evidence," in The Impact of Accounting Research on Practice and Disclosure, A. Rashad Abdel-khalik and Thomas F. Keller, eds., (North Carolina: Duke Univ. Press, 1978), p. 162.

[6]Douglas R. Carmichael, The Auditor's Reporting Obligation: The Meaning and Implementation of the Fourth Standard of Reporting--Auditing Research Monograph No. 1, (AICPA, 1972), p. 94.

Liquidity Versus Solvency

On conceptual grounds, one may properly contrast liquidity (nearness to cash) with solvency (ability to pay debts as they fall due).[7] Friedland describes liquid assets as those ". . . that may be readily exchanged for cash at their equilibrium value."[8] Van Horne identifies the two basic dimensions involved in such a view of liquidity:

1) the time required to convert the asset into money, and
2) the certainty of the price realized.[9]

The nature of these dimensions may be exemplified by considering the relative liquidity of receivables and inventory. Receivables are generally regarded as more liquid than inventory because they are only one step (collection) away from cash, whereas inventory is two steps away (sale and collection). The nearness of receivables to cash is given greater weight in this case since the relative length of the two steps is disproportionate. However, depending upon expectations concerning the actual resources to be realized from conversion, initial indications of liquidity based solely upon nearness to cash may have to be revised. In this second dimension, readily marketable inventories may be viewed as more liquid than aged receivables that

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[7] Paul M. Van Arsdell, Corporation Finance: Policy, Planning, Administration, (New York: Ronald Press, 1968), pp. 266-269.

[8] Seymour Friedland, Principles of Financial Management: Corporate Finance, Investments, and Macrofinance, (Massachusetts: Winthrop Publishers, 1978), p. 14.

[9] James C. Van Horne, Financial Management and Policy, 5th ed., (New Jersey: Prentice-Hall, 1980), p. 717.

will be collectible only in part. Thus, liquidity involves the speed of convertibility to cash as well as the expectation of the amount realized upon conversion.

Solvency may also be viewed in two different contexts. In the equity sense, solvency concerns the ability to pay debts as they come due. In the bankruptcy (legal) sense, solvency applies to situations in which enterprise asset values exceed existing obligations. Thus, it is possible for the firm to be insolvent in the equity sense without being insolvent from the strict legal perspective.

The literature has generally approached liquidity as a means to an end. Holdings of cash and near-cash assets are deemed instrumental in assuring that debts can be met as they fall due. Those firms with strong liquid positions are seen as better able to meet maturing obligations than their less liquid competitors. Financial ratios designed as liquidity indicators can thus provide an indirect measure of the firm's solvency. The focus of this research is on such liquidity indicators.

Evolution of Ratio Analysis

The development of ratios as an aid to financial statement analysis is a relatively recent phenomenon, predicated in large part on the development of financial institutions as major forces in the economy.[10] Beginning in the 1890's, a movement arose among commercial

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[10] James O. Horrigan, "A Short History of Financial Ratio Analysis," Accounting Review, (April, 1968), p. 284.

banks to demand financial statements from clients so that the credit granting process could be better managed.[11] Credit analysis was primarily concerned with the "ability to pay" and financial statements were sought to provide information on evaluation of credit risks. The literature of this time however was practically void of any concrete analytical procedure suggestions.[12]

Within the next decade, certain quantitative measurements (ratios) based upon financial statement data were being suggested. The first proposed comparison (intended to be used in the credit granting function) was between current assets (referred to as "quick assets" at the time) and current liabilities:

> Credit is usually extended on the strength of quick assets, and many good judges feel that the ratio of quick assets to liabilities should be about $2\frac{1}{2}$ to 1.[13]

The pervasive adoption of this one measurement (which came to be known as the current ratio) could hardly have been anticipated. Perhaps excluding earnings per share, no other ratio has achieved such impact:

> For many years [the] "two for one" current ratio was the alpha and omega of balance sheet analysis; even today the businessmen are legion who believe this single ratio to be the one infallible guide to balance sheet interpretation.[14]

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[11] Roy A. Foulke, Practical Financial Statement Analysis, (New York: McGraw Hill, 1968), pp. 13-19.

[12] John N. Meyer, Financial Statement Analysis, (New Jersey: Prentice-Hall, 1969), pp. 5-6.

[13] William M. Rosendale, "Credit Department Methods," Bankers Magazine, (February, 1908), p. 187.

[14] Foulke, Statement Analysis, p. 178.

Credit analysts eventually came to question the proficiency of the current ratio as the sole indicator of the short run debt paying ability of the firm. Concern centered around the liquidity of pre-paid expenses and especially inventories, due primarily to the uncertain time span involved in achieving realization in cash. As a result, a new supplemental ratio was proposed to compare the more liquid (quick) assets and current liabilities. This comparison is referred to as the acid-test or quick ratio.

Since their introduction, the current and quick ratios have served (despite criticism) as important indicators of an enterprise's ability to meet maturing obligations. These two measures, it is argued, provide relative yardsticks for evaluating the overall liquidity position of the firm.

Refinement of Liquidity Measures

One of the major limitations of these two ratios lies in their basically static nature. Walter concludes that the traditional approach to solvency determination which emphasizes the availability at a given point in time of current assets to meet current liabilities is deficient:

> The appropriate topic . . . appears, rather, to be whether prevailing cash inflows (plus cash resources) cover existing cash outflows by a sufficient margin to protect against possible reductions in inflows or increments in outflows.[15]

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[15] James E. Walter, "Determination of Technical Solvency," Journal of Business, (January, 1957), p. 43.

In a similar manner, Lemke argues:

> Liquidity depends primarily on cash flows, and to some extent on cash and near-cash holdings. However, little indication of the pattern of future cash flows can be gleaned from balances of working capital items on a particular date.[16]

The implication then is that a study of liquidity should emphasize a dynamic, rather than static process--with particular emphasis on cash flow characteristics. Information concerning cash inflows and outflows, their trends, timing and general future expectations, is desirable for such an analysis. Unfortunately, such information is not usually available to the outside analyst. As a result, suggested refinements in liquidity measures which incorporate cash flow data have tended to stagnate at the conceptual stage, with little hope of application in practice given the current state of external financial reporting by enterprises.

One notable suggestion for improvement in the area of evaluating a firm's short run debt paying ability comes from Sorter and Benston who propose the defensive interval measure as a superior alternative to the traditional current ratio.[17]

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[16] Kenneth W. Lemke, "The Evaluation of Liquidity: An Analytical Study," Journal of Accounting Research, (Spring, 1970), p. 61.

[17] George H. Sorter and George Benston, "Appraising the Defensive Position of a Firm: The Interval Measure," Accounting Review, (October, 1960), pp. 633-640; and Sidney Davidson, George H. Sorter, and Hemu Kalle, "Measuring the Defensive Position of a Firm," Financial Analysts Journal, (January-February, 1964), pp. 23-29.

The primary ratio offered for consideration is:

(I) Basic Defensive Interval $= \dfrac{\text{Defensive Assets}}{\text{Projected Daily Operating Expenditures}}$

(where defensive assets include cash, marketable securities, and net receivables; and projected expenditures include operating expenses adjusted for items not requiring the use of defensive assets). This ratio attempts to provide a measure of the number of days that the stock of defensive assets could satiate a firm's operating requirements without further inflows to that stock.

Rather than just one ratio, Sorter and Benston propose an expandable set of measures that consider alternative environmental assumptions including:[18]

a. continued availability of credit
(adjusted to negate window dressing effects)

(II) Adjusted Interval Measure $= \dfrac{\text{Defensive Assets - Actual Liabilities + Average Liabilities}}{\text{Projected Daily Operating Expenditures}}$

b. cessation of availability of credit

(III) No Credit Interval $= \dfrac{\text{Defensive Assets - Actual Liabilities}}{\text{Projected Daily Operating Expenditures}}$

c. doubtful turnover of receivables

(IV) Cash Interval $= \dfrac{\text{Cash + Marketable Securities}}{\text{Projected Daily Operating Expenditures}}$

d. partial sales decline

(V) Reduced Sales Interval $= \dfrac{\text{Defensive Assets}}{\text{Projected Daily Operating Expenditures Less Anticipated Daily Receipts}}$

e. cost behavior changes

(VI) Reduced Operations Interval $= \dfrac{\text{Defensive Assets}}{\text{Projected Daily Operating Expenditures Less Savings on Expenditures Due to Reduced Operations}}$

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[18] *Ibid*.

Theoretical Support For Defensive Intervals

Conceptually, these interval measures offer a number of important improvements over the current ratio which has served for so long as the primary liquidity position indicator. First, the ratio descriptions are consistent with the motives underlying liquidity preference.[19] Under the relevant economic theory, firms maintain a stock of highly liquid assets to serve as a shield or buffer against contingencies, especially unexpected alterations in future cash flow patterns. Such a defensive stock will enable the firm to meet these contingencies without significantly altering its normal operations or financing requirements. It is reasonable then to investigate a relationship (i.e., defensive interval measure) in which this stock of defensive assets is compared to expenditure requirements that are necessary to protect the continued operations of a firm in the face of a decline or discontinuance of inflows to the stock.[20]

Second, as previously illustrated by ratios (II) through (VI) on page 9, the defensive interval measure is a flexible tool which may be extended to encompass a variety of environmental factor combinations. Depending upon the analyst's needs, new manifestations of the basic defensive interval can be derived by incorporating relevant assumptions for a particular enterprise in a given environmental

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[19] John Maynard Keynes, The General Theory of Employment, Interest and Money, (New York: Harcourt, Brace and World, 1936), chaps. 13, 15.

[20] Van Arsdell, Corporation Finance, pp. 934-935; and Raymond P. Kent, Corporate Financial Management, (Illinois: Richard D. Irwin, 1969), p. 138.

setting. In this way, the interval measure may be tailored to fit a specific purpose in the analysis of liquidity.

Third, with respect to debt paying ability, it may be argued that defensive assets provide a more effective measure of the firm's present stock from which short term debts will have to be paid. In contrast, the asset stock measure in the current ratio includes inventory and certain prepaid items which are not now (and may never be) in a liquid form to meet maturing obligations. In the event of significant financial distress in which a partial or complete curtailment of normal revenues occurs, inventory realization is hindered and the current ratio will overstate the immediately available stock of debt paying assets. The interval measure specifically excludes inventories from the defensive asset stock and thus provides a reasonable liquidity indicator in the face of such limitations on sales levels.

Fourth, the current ratio is highly susceptible to "window dressing" effects which result in ratio changes without any underlying changes in debt paying ability. One manifestation of window dressing may be defined as:

> . . . the distortion of the amount of current liabilities at [the] balance sheet date due to the purposeful timing of exchanges of assets and liabilities.[21]

For example, assume the simple case of a firm with $200,000 cash and $100,000 of accounts payable, representing the only current assets

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[21]Sorter and Benston, "The Interval Measure," p. 637.

and liabilities. By scheduling a liability payment of $50,000 at the close of the accounting period, both the current and quick ratios would reflect a 50% increase (from 2:1 to 3:1) without a corresponding increase in the real ability to pay future obligations. Even if one discounts the widespread use of such a voluntary subterfuge, the deficiency of the current and quick ratios still exists since the basic window dressing effect can also arise unintentionally. The specific timing of certain transactions (e.g., repayment of customary business loans) which result in liability amounts at year end which are not representative of normal levels will yield the same distortion, no matter how coincidental. Sorter and Benston demonstrated that the adjusted interval measure (II) is immune from this type of misrepresentation (either voluntary or accidental).[22] Even if the average amount of liabilities is unknown and one must revert to the basic interval measure (I), the advantage over the more traditional liquidity measures is still evident:

> In general, most of the events over which the firm has complete control will decrease the unadjusted interval measure but increase the current or quick ratio. Management thus could worsen its unadjusted interval but would be hard put to better it.[23]

Fifth, consistent with the earlier comments concerning the need for consideration of dynamic processes in liquidity measurement, the defensive interval represents a step in the right direction. Although

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[22] *Ibid.*, pp. 637-638.

[23] *Ibid.*, p. 638.

the defensive asset stock is a static concept, the incorporation of projected expenditures into the interval calculation adds a dynamic dimension lacking from the traditional liquidity measures. The result is a measure which draws attention to the important connection between liquidity (expressed in terms of a stock of defensive assets) and the need for liquidity (expressed in terms of future expenditure requirements).

Sixth, the defensive interval results in a measure expressed in a natural dimension (time) which is readily interpretable. In contrast, the current and quick ratios exhibit no such dimension to give them operational meaning. As a result, it is extremely difficult to interpret the size and direction of changes in these ratios in a uniform manner.

Finally, because the interval measure is expressed in a meaningful form, it is possible to specifically consider the impact of projected changes in a firm's liquidity position. By assigning some sort of opportunity cost interest rate concept involved in the holding of the defensive stock, one could ascribe a cost (or benefit) to a given proposal for altering the defensive strength of the firm. Such an approach would thus provide the liquidity management function with a basis for evaluating the change in advance.

Having thus identified the primary factors which support the relative superiority of the defensive interval measure, it remains to be seen whether such a measure does in fact impart information which is different from the more traditional liquidity ratios. By definition, the defensive interval is calculated using financial items and a natural time dimension which distinguish it from the current and quick ratios. But such theoretical differentiations in themselves do nothing to answer the question of the supposed increased utility of the interval measure. Such a question is inherently empirical in nature.

Scientific Inquiry -- The Need For Empirical Verification

In philosophical discussions, five primary methods of attaining and validating knowledge may be identified: authoritarianism (reliance on the testimony of others); mysticism (reliance on a super-sensuous faculty of instinct and intuition); pragmatism (meaning of a concept is tied to its practical consequences); rationalism (meaning is derived by abstract reasoning from a set of universal principles); and empiricism (meaning is derived from perceptual experience).[24] The modern tradition of the scientific approach concentrates on the last two of these logical theory methods and they therefore deserve further discussion.

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[24]William P. Montague, The Ways of Knowing, (London: George Allen and Unwin Ltd., 1925), chaps. 1 through 5.

Rationalism, as an historic tradition of philosophic thought, had its beginnings with Socrates and Plato and reached its zenith with the writings of Descartes, Spinoza and Leibniz. Such an approach emphasizes reason as the supreme source of genuine knowledge. Using the methods of mathematics as a model, the rationalist method begins with a set of self-evident truths (axioms) from which inescapable conclusions are determined through a series of deduced propositions. The importance of this approach in attaining truth has been stated by Descartes:

> In the subjects we propose to investigate, our inquiries should be directed, not to what others have thought, nor to what we ourselves conjecture, but to what we can clearly and perspicuously behold and with certainty deduce; for knowledge is not won in any other way.[25]

Thus, reasoning is viewed as the ultimate source of knowledge.

A problem with such a philosophical position lies not so much in the process of deduction (which may be quite sound), but rather in the character of the original postulates chosen. Even if the chain of deduction cannot be faulted, the process can often be criticized because so much reliance is placed on the initial premises (which may not in fact be truly self-evident). As an example of the difficulties which may result, consider the case of the astronomical description of planetary motion. Aristotle had adopted as a basic premise that the world was created by a perfect God and deduced that the planets

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[25] René Descartes, Rules For the Direction of the Mind, in The Philosophical Works of Descartes, translated by Elizabeth S. Haldane and G. R. T. Ross, (London: Cambridge University Press, 1911), p. 5.

must therefore move in a perfect orbit (i.e., a circle). This notion persisted for centuries, with astronomers constantly arguing over which particular system of epicycles was more accurate. When Kepler finally observed (correctly) that planetary movement followed elliptical instead of circular patterns, his conclusions were rejected because they were not derived deductively.[26] There is a danger, then, in operating on the basis of "a priori truths" which conflict with reality. The empirical approach can often be applied to determine if such a conflict exists.

Empiricism, espoused by such writers as Bacon, Locke, Berkeley, Hume, and Peirce, adopts the position that all knowledge is based upon experience and sense perception, and the general concepts which can be derived from them. As Locke has stated:

> Our observation, employed either about external sensible objects, or about the internal operations of our minds, perceived and reflected on by ourselves, is that which supplies our understandings with all the materials of thinking. These two are the fountains of knowledge, from whence all the ideas we have, or can naturally have, do spring.[27]

Thus, knowledge stems from either sensation or reflection.

Inherent in this empirical approach is the application of inductive reasoning--the process by which one leaps from the known particular to the unknown generality. This process allows one to

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[26] Frederick Vivian, Thinking Philosophically, (New York: Basic Books, 1969), pp. 57-58.

[27] John Locke, An Essay Concerning Human Understanding, in The English Philosophers From Bacon to Mill, Edwin A. Burtt, ed., (New York: Random House, 1939), p. 248.

extend the boundaries of knowledge beyond mere original sense perception. Implied in this inductive process are the methods of hypothesis, observation, and experiment:

> We establish clear meaning by testing an idea in use--by tracing out its concrete applications and consequences rather than by intuitive inspection or abstract definition. . . . The ultimate test of truth is that it is verified by facts open to inspection and admitted to be such by all qualified observers.[28]

This, in essence, is the perspective adopted by the scientific method.

Scientific inquiry encompasses both rational and empirical aspects in its methodology. A basic scientific cycle may be described:

DATA---INDUCTION---HYPOTHESIS---DEDUCTION---
PREDICTION---VERIFICATION---DATA[29]

From an initial sensory input an explanatory hypothesis is formed through induction. From this hypothesis a certain prediction is deduced. Observations are then made (e.g., experiments are performed) to determine if the facts support or refute the prediction. In such a methodology, empirical verification plays a paramount role:

> . . . it is particularly the empiricist who comes into his own in scientific activity. For however rationalistic some of the steps in scientific method may be . . . it is a process that must both begin and end in sensation. If sense data are lacking at either the beginning or the end of the process, we do not have science.[30]

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[28] Melvin Rader, The Enduring Questions: Main Problems of Philosophy, (New York: Holt, Rinehart and Winston, 1969), p. 131.

[29] Vivian, Thinking Philosophically, p. 65.

[30] Hunter Mead, Types and Problems of Philosophy, (New York: Henry Holt and Company, 1959), p. 186.

The thrust of this research is empirical in nature, and thus represents part of the theory verification stage of the scientific process. The defensive interval measure has been proposed as an improvement over the more traditional liquidity ratios. Verification of this assertion first requires identification of some criterion upon which to base evaluation. One such standard discussed in the literature for evaluating alternatives is predictive ability.[31] Such a standard emphasizes the capacity to suggest outcomes which can then be empirically tested:

> According to this criterion, alternative accounting measurements are evaluated in terms of their ability to predict events of interest to decision-makers. The measure with the greatest predictive power with respect to a given event is considered to be the "best" method for that particular purpose.[32]

The "event of interest" selected in this research is bankruptcy. Predictability of business failure will thus serve as the evaluative criterion. Defensive interval measures will be appraised in the context of their contribution to improvement of failure prediction.

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[31]C. West Churchman, Prediction and Optimal Decision, (New Jersey: Prentice-Hall, 1961)

[32]William H. Beaver, John W. Kennelly, and William M. Voss, "Predictive Ability as a Criterion for the Evaluation of Accounting Data," Accounting Review, (October, 1968), p. 675.

Overview of the Remaining Chapters

Chapter two provides a basic literature review. Emphasis is placed here on previous bankruptcy prediction studies and the specific models which are employed.

Chapter three presents the results of a preliminary empirical analysis performed to gain insight into general characteristics of defensive intervals relative to other liquidity measures.

Chapter four describes the research methodology employed in this study, including a description of the data collection process. Attention is also directed to certain improvements over previous methodologies.

Chapter five reports the results of the bankruptcy prediction study. Included here are the outcomes of the specific test procedures.

Chapter six extends the analysis to the area of auditor opinions. The models developed in Chapter five are used to obtain failure predictions which are then compared with auditors' evaluations of going concern problems.

Chapter seven briefly summarizes all results and offers certain recommendations.

CHAPTER 2

LITERATURE REVIEW

The primary focus of this research is the potential contribution which defensive interval measures can make to the prediction of business failure. This is essentially an empirical question and therefore the review which follows is limited to a discussion of past empirical research which has specifically addressed failure prediction. Particular emphasis will be placed on detailing those studies whose methodologies have significant implications applicable to this research.

Prior studies concentrating on the role of financial ratios in such prediction may be classified into two broad groups based upon the fundamental perspective employed. The first group of studies emphasizes the predictive power of individual ratios and may be categorized under the rubric "univariate." The second group is characterized by the simultaneous consideration of ratios and may be classified as "multivariate" in approach.

Univariate Failure Prediction

A number of early works appear in this area. In general, these studies were marred by certain limitations and statistical deficiencies which reduce the strength and generalizability of their conclusions. In spite of their shortcomings, however, they did lay the groundwork for future studies of the utility of ratios in evaluating distress.[33]

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[33]James O. Horrigan, "A Short History of Financial Ratio Analysis," Accounting Review, (April, 1968), pp. 284-294.

They are included here, without a detailed review of methodology, for the sake of historical completeness.

Ramser and Foster[34] investigated thirty-three ratios of 173 firms whose securities were registered in Illinois during the period 1920 to 1927. Results indicated that failed and less successful firms exhibited lower ratios than the more successful firms, although this conclusion did not span the entire set of ratios selected for study.

FitzPatrick[35] investigated thirteen ratios of nineteen pairs of failed and nonfailed firms for the period 1920 to 1929. Results showed consistent differences in ratio trends for up to three years prior to failure, with net income/net worth and net worth/debt providing the best indications of financial distress.

Smith and Winakor[36] investigated twenty-one ratios of 183 failed firms for the period 1923 to 1931. Analysis was made of the trends of the means of the ratios up to ten years prior to failure. Results indicated a basic deterioration of average ratio values with the rate

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[34] J. R. Ramser and Louis O. Foster, A Demonstration of Ratio Analysis, Bulletin No. 40, (Illinois: University of Illinois, Bureau of Business Research, 1931).

[35] Paul J. FitzPatrick, Symptoms of Industrial Failures, (Washington, D.C.: Catholic University of America Press, 1931); and "A Comparison of Ratios of Successful Industrial Enterprises With Those of Failed Firms," Certified Public Accountant, (October, November and December, 1932), pp. 598-605, 656-662, 727-731, resp.

[36] Raymond F. Smith and Arthur H. Winakor, Changes in the Financial Structure of Unsuccessful Industrial Corporations, Bulletin No. 51, (Illinois: University of Illinois, Bureau of Business Research, 1935); an earlier study evaluated twenty-nine firms with similar results-- A Test Analysis of Unsuccessful Industrial Companies, Bulletin No. 31, (Illinois: University of Illinois, Bureau of Business Research, 1930).

of decline increasing rapidly as failure approached. Net working capital/total assets was found to be the most consistent failure indicator.

Merwin[37] investigated an unspecified number of ratios for 200 small manufacturing corporations "failing" (i.e., ceasing to file income tax returns) during the period 1926 to 1936. Comparison was made of the mean industry ratios for "continuing" and "discontinuing" firms with deterioration of ratio trends being exhibited up to six years prior to failure. Three ratios were found to be very sensitive indicators of failure: net working capital/total assets; net worth/debt; and the current ratio.

Studies by Beaver

These early studies did not attempt prediction per se, but rather were concerned with describing the magnitude and trend of ratios exhibited by failed firms. Beaver[38] was the first to actually apply a relatively rigorous statistical methodology to the prediction of corporate failure by financial ratios.

A rather broad definition of failure was adopted for his study with several critical events assumed to be indicative of distress, including such situations as bankruptcy, bond default, nonpayment of

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[37] Charles L. Merwin, Financing Small Corporations in Five Manufacturing Industries: 1926-36, (New York: National Bureau of Economic Research, 1942).

[38] William H. Beaver, "Financial Ratios as Predictors of Failure," (unpublished Ph.D. dissertation, University of Chicago, 1965); and "Financial Ratios as Predictors of Failure," Empirical Research in Accounting: Selected Studies, 1966, Journal of Accounting Research, pp. 71-111ff.

preferred stock dividends, and overdrawing bank accounts. Seventy-nine firms exhibiting one or more of these characteristics during the period 1954 to 1964 were identified. A matched sample of seventy-nine nonfailed firms was then selected, with industry and asset size used as the matching criteria. Thirty ratios were computed for this combined set of firms for the five year period prior to failure.

For each year preceeding failure, means of the ratios for the two groups of firms were calculated and compared. The resulting profile analyses support the results of the earlier studies cited in this review. Ratio trends for the failed firms were found to deteriorate (relative to the nonfailed firms) over the five year period, with differences between the two trend lines increasing as failure approached. Since such a profile analysis approach concentrates only on mean values of the two groups, the impact of other distributional characteristics is ignored. As a result, such a technique is only partially descriptive in nature and does not necessarily lend insight into the underlying predictive power of the ratios.

To actually evaluate this predictive ability, Beaver first performed a dichotomous classification test. Such a technique involves arraying for each year a given ratio's values (and the corresponding failure or nonfailure status) and then selecting a cutoff point which will maximize the distinction between the two groups of firms (i.e., minimize the number of incorrect predictions based upon the decision rule selected). This procedure was repeated with each ratio and the resulting percentages of misclassification served as the criteria for selecting the best predictors over the five year period: cash flow/total debt; net income/total assets; total debt/total assets; working capital/total assets; and the current ratio.

To partially overcome the criticism of this test as being entirely ex post, Beaver then adopted a calibration sample approach. The original data were randomly divided into two groups and optimal cutoff points were derived for each ratio in each subsample. Cutoff points from one subsample were then used to classify the firms in the other subsample. Results indicated a small but persistent increase in the misclassification errors over those obtained when the cutoff points were applied to the firms in the subsample from which the points were originally derived.

Beaver recognized other limitations of the classification test procedure. Since the decision rule for categorization involves a dichotomous choice between failure and nonfailure, information concerning the magnitude of the particular ratio is lost. Such information can be very important in evaluating the strength of the resulting prediction. Secondly, the cutoff values derived from such a test are not generalizable to an actual decision making environment since no explicit consideration is given to either: 1) the relative difference in "costs" of the two types of misclassification errors possible; or 2) the discrepancy between the odds of failure in the population relative to the percentage of failed firms in the sample drawn for study.

To overcome these last deficiencies, Beaver broadened his view and adopted the perspective of ratios as assessments of the likelihood of failure or nonfailure. Such an approach incorporates Bayesian inference into the analysis and allows for the explicit consideration of the cost of misclassification and prior odds of failure, as well as the actual magnitude of the ratios under study. Basically, this analysis involves revision of the prior probability of failure on the basis of observed ratio values, resulting in a decision rule expressed in terms of the likelihood estimates. A Bayesian approach was adopted for the current research and specifics are described in greater detail in Appendix 1.

Beaver later performed two additional studies on his 158 firm sample.[39] Utilizing essentially the same analyses described above, he investigated certain a priori arguments in the literature concerning the relative predictability of liquid and nonliquid asset ratios. Fourteen variables were selected for study and results indicated that, in general, nonliquid asset measures outperformed the liquid asset measures in both the short and long run. Evidence also showed that the two most frequently advocated liquid asset measures (based on current and quick assets) were outperformed by less popular measures (based on cash and working capital). Such a

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[39] William H. Beaver, "Alternative Accounting Measures as Predictors of Failure," Accounting Review, (January, 1968), pp. 113-122; and "Market Prices, Financial Ratios, and the Prediction of Failure," Journal of Accounting Research, (Autumn, 1968), pp. 179-192.

result points up the critical need for empirical verification of a priori beliefs. In the second study, the association between the trends of certain financial ratios and market rates of returns was investigated. Results indicated that stock prices are adjusted to reflect a deteriorating solvency position and such changes behave as if investors impound the ratio data in their assessments.

Other Univariate Studies

As a potential aid to discriminating between high and low risk firms, Wilcox has suggested a simple conceptual model based upon a variant of the gambler's ruin probabilistic process.[40] Using Markov chain concepts, a single predictive criterion is developed which incorporates both profitability and liquidity characteristics, with an emphasis on cash flow. In a later study, empirical evidence was actually gathered to evaluate the model.[41] Fifty-two firms that had filed for bankruptcy during 1949 to 1971 were analyzed. Results of a paired analysis indicated improved predictability over Beaver's best ratios, although the improvement appears to be only nominal.

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[40] Jarrod W. Wilcox, "A Gambler's Ruin Prediction of Business Failure Using Accounting Data," Sloan Management Review, (Spring, 1971), pp. 1-10; and "A Simple Theory of Financial Ratios as Predictors of Failure," Journal of Accounting Research, (Autumn, 1971), pp. 389-395.

[41] Jarrod W. Wilcox, "A Prediction of Business Failure Using Accounting Data," Empirical Research in Accounting: Selected Studies, 1973, Journal of Accounting Research, pp. 163-179ff.

Tamari[42] investigated ten ratios of twenty-eight Israeli industrial companies during the period 1956 to 1960. Sixteen of these firms had actually been declared bankrupt and the remaining twelve had been given substantial debt relief and were "virtually bankrupt." Similar to the earlier univariate studies, Tamari generally found deteriorating ratio trends up to five years prior to failure, with failed firm ratios being lower than nonfailed industrial firms taken as a whole. Tamari also suggested an "index of risk" composed of six ratios measuring financial soundness, each ratio being weighted according to a consensus of subjective considerations supplied by a broad group of financial analysts, commercial bankers, and credit rating institutions.[43] This risk index, with its emphasis on incorporating information from a variety of ratios, represents a connecting link between the univariate research approach and the multivariate approach which follows.

Criticism of the Univariate Approach

Univariate analysis in the area of failure prediction is subject to a number of limitations. First, results are often ambiguous since different ratios may yield conflicting indications of a firm's condition. One is then left with the task of attempting to determine which ratio indicator takes precedence.

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[42]Meir Tamari, "Financial Ratios as a Means of Forecasting Bankruptcy," Management International Review, (Vol. 4, 1966), pp. 15-21.

[43]For a more complete description of the construction of this index and empirical studies performed to test its validity as a predictor of risk, see Meir Tamari, Financial Ratios: Analysis and Prediction, (London: Paul Elek Ltd., 1978).

Secondly, insignificance of a particular ratio when considered in isolation does not imply lack of predictive ability since the variable may materially aid explanatory power when considered in a multi-ratio setting. For example, consider the case exhibited in Illustration 1A. Two groups of observations (encompassed by the ellipses G_1 and G_2) are graphed in bivariate space. When considering the variables in isolation, X would be deemed significant since there is a difference between the group means when measured on this dimension (i.e., a location shift is evident from point a to b). However, on the Y dimension, there is no difference between the group means (point c) and Y would appear insignificant as a distinguishing characteristic of the two groups. By considering the variables simultaneously (i.e., considering the relationships between the two variables), it would be possible to construct a differentiating line (L_1) which would result in a minimal overlap of the groups. Thus, the information provided by variable Y <u>is</u> important and the conclusion reached on a univariate basis is erroneous.

This case also illustrates that by considering multiratio information, increased power over the univariate approach might be attained. In Illustration 1B, a line (L_2) is constructed halfway between the ellipse centroids to differentiate between the two groups on the basis of variable X only. Those observations to the left of the line would be categorized as members of G_1 and those to the right as members of G_2 . The resulting misclassified observations are represented by the shaded areas. Note that the area of misclassification which

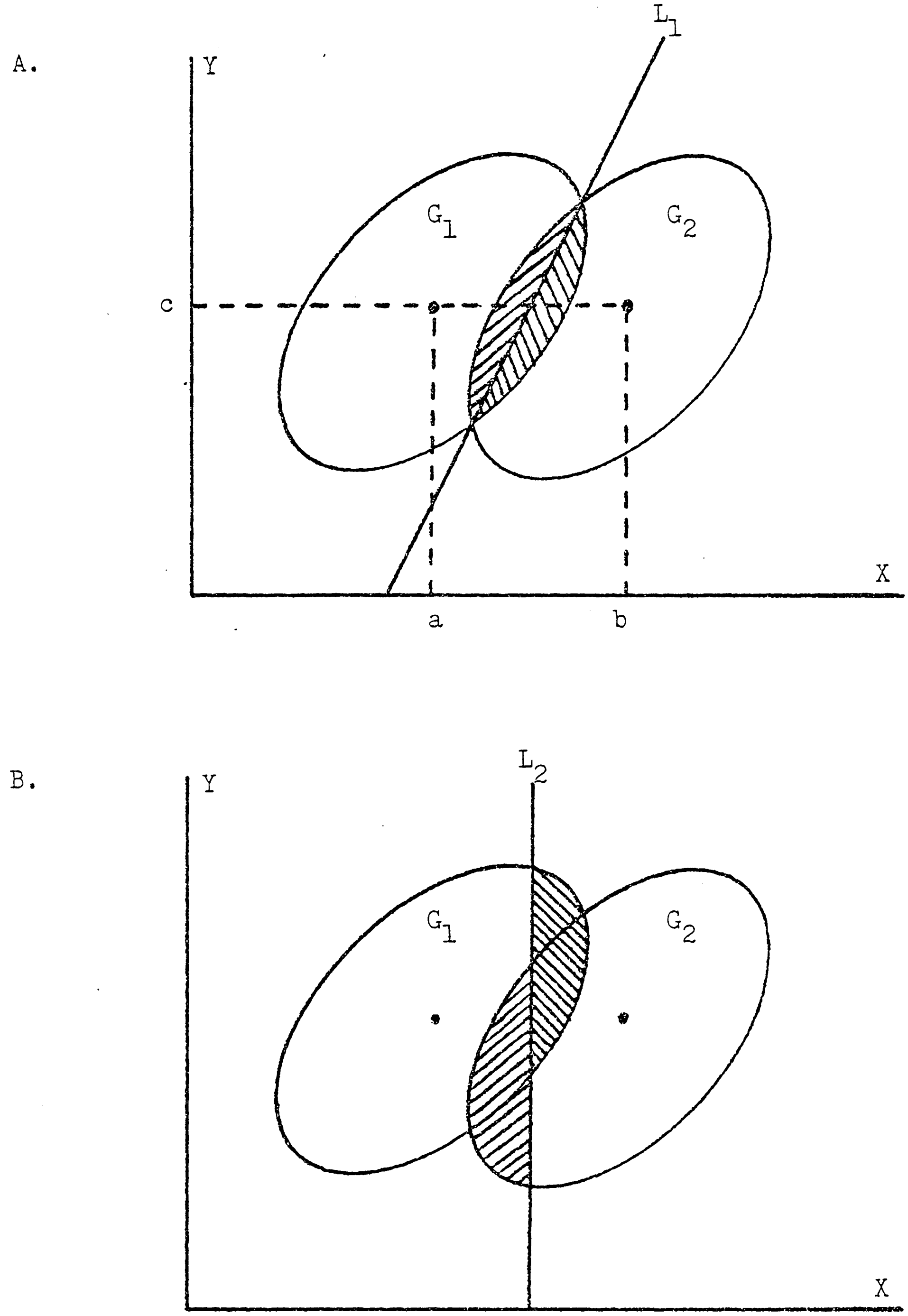

ILLUSTRATION 1

MULTIVARIATE AND UNIVARIATE GROUP DISTINCTION results from this univariate approach is greater than that in Illustration 1A in which the overlap has been minimized by a multivariate approach. The univariate analysis would thus lead to a sub-optimal result in this case.

Finally, it is highly unlikely that the nature of the financial condition of a firm can be captured in a single descriptive measure. It seems reasonable then to consider simultaneously an expanded set of measures which can offer information concerning different dimensions of a firm's status.

Multivariate Failure Prediction

Studies by Altman

The first major empirical work adapting a multivariate approach to failure prediction came from Edward Altman. Since he has published numerous works in this area (with essentially the same statistical methodology), only the more significant ones will be reviewed here.

Altman's seminal research appeared in 1968.[44] He investigated thirty-three manufacturers filing for bankruptcy during the period 1946 to 1965, and a matched sample (based upon similar asset size and industry) of thirty-three nonfailed firms. A group of twenty-two ratios originally selected for study was reduced through various statistical and ad hoc methods to a final set of five which offered

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[44] Edward I. Altman, "Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy," Journal of Finance, (September, 1968), pp. 589-609; see also, Corporate Bankruptcy in America, (Massachusetts: D.C. Heath and Company, 1971).

the best predictive power: working capital/total assets; retained earnings/total assets; earnings before interest and taxes/total assets; market value of equity/book value of debt; and sales/total assets.

The basic technique employed in his research was multiple discriminant analysis (hereinafter referred to as MDA). This is a statistical tool designed to classify an observation into discrete a priori categories based upon its characteristics. In failure studies, the output of this technique is a discriminant function which transforms independent variables (financial ratios) into a composite score which can then be used to classify individual observations (firms) into one of two dichotomous groups (failed or non-failed). The lower the resulting composite score, the greater is the predicted failure potential. Specific details of this MDA approach will be described in Chapter 4.

Altman's final discriminant model contained the five ratios listed above. Applied to financial statement information drawn one year prior to bankruptcy for the sixty-six firm sample, an overall correct classification rate of 95% was achieved. Applying the model to information drawn for the next preceeding year resulted in a deterioration of predictive power. The overall correct classification rate dropped to 83%.

Recognizing that the power of his model could be subject to search bias (because of the way the original variable set was reduced) and therefore nongeneralizable to the population, Altman performed a series of additional tests. First, a holdout sample approach was adopted in which model estimates were derived from a subsample of the original data and the remainder of the sample was then classified using this model. Five replications, based upon five different methods of drawing the holdout sample, were performed with an average correct classification rate of 93.5%, which is only slightly below that attained in the original test. Second, a new sample of twenty-five bankrupt firms was identified and the model was applied to ratios one year prior to bankruptcy. A correct classification rate of 96% was attained. Third, a random sample of sixty-six manufacturing firms with net losses in either 1958 or 1961 were subjected to the model and a correct classification rate of 79% resulted. Altman's analysis indicates that this drop in predictive ability was due in large part to the concentration of firms around the cutoff point.

Altman then investigated the long range predictive ability of his model. Returning to the original thirty-three firm failed sample, predictions were made for each of the five years preceeding bankruptcy. Correct classification rates resulted of 94%, 72%, 48%, 29% and 36% for the respective years. Thus, the model performed well as a forecaster up to two years prior to failure, but as the lead time increased its accuracy quickly deteriorated. A univariate trend analysis on the model's components provided a potential reason for these results by indicating that the ratios exhibited a declining course with the most significant change occurring between the second and third years preceeding failure.

Results of this basic MDA modeling approach were used by Altman in a number of subsequent studies including prediction of railroad bankruptcy,[45] failure implications for commercial loan evaluation,[46] and prediction of security dealer failures.[47] Evidence in these studies indicates the broad applicability of the MDA technique as an explanatory and predictive tool.

In 1977, Altman presented a revised failure classification model incorporating a number of statistical improvements over his earlier work.[48] An investigation was performed on fifty-three bankrupt firms (including retailers as well as manufacturers, in contrast to the earlier model development) and a matched group (on the basis of industry) of fifty-five nonfailed firms. The bankrupt firms came from the period 1962 to 1975, with 94% failing in the last seven years, and the average asset size exceeded those firms included in the original study by a factor of four times.

An original variable set of twenty-seven ratios was reduced

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[45] Edward I. Altman, "Railroad Bankruptcy Propensity," Journal of Finance, (May, 1971), pp. 333-346; and "Predicting Railroad Bankruptcies in America," Bell Journal of Economics and Management Science, (Spring, 1973), pp. 184-211.

[46] Edward I. Altman, "Corporate Bankruptcy Prediction and Its Implications for Commercial Loan Evaluation," Journal of Commercial Bank Lending, (December, 1970), pp. 8-22; and Edward I. Altman, Michel Margaine, Michel Schlosser, and Pierre Vernimmen, "Financial and Statistical Analysis for Commercial Loan Evaluation: A French Experience," Journal of Financial and Quantitative Analysis, (March, 1974), pp. 195-211.

[47] Edward I. Altman and Bettina Loris, "A Financial Early Warning System for Over-the-Counter Broker-Dealers," Journal of Finance, (September, 1976), pp. 1201-1217.

[48] Edward I. Altman, Robert G. Haldeman, and P. Narayanan, "ZETA Analysis: A New Model to Identify Bankruptcy Risk of Corporations," Journal of Banking and Finance, (June, 1977), pp. 29-54.

(using six different tests as criteria for the actual variables to be eliminated) to a final profile of seven: earnings before interest and taxes/total assets; standard error of estimate of earnings before interest and taxes/total assets; earnings before interest and taxes/ total interest payments; retained earnings/total assets; current ratio; market value of equity/total capital; and total assets. These measures were designed to capture the corporate characteristics of return on assets, stability of earnings, debt service, cumulative profitability, liquidity, and size, respectively. Several components of this list are distinguished from the earlier model in their incorporation of normalized measures and trend components.

The MDA that was performed included specific consideration of unequal dispersion matrices, unequal prior probabilities of group membership, and unequal costs of misclassification. The importance of these statistical refinements will be discussed in Chapter 4. Results based on the original sample, as well as a series of replications designed for validation, indicated overall correct classification ranging from 96% one year prior to failure to 70% up to five years prior to failure. The accuracy of failed firm classification was almost consistently less than that of nonfailed firms in all but the first year prior to failure, with the linear model generally outperforming the quadratic variant. Comparison with Altman's original model and certain naive strategies indicated the ZETA model's relative superiority.

Studies by Deakin

Edward Deakin has replicated the methodology of both the Beaver and Altman original research studies.[49] Thirty-two failed firms (including bankrupts, insolvents, and those liquidated for the benefit of creditors) were identified during the period 1964 to 1970 and matched (on the basis of asset size and industry) with thirty-two nonfailed firms. Results of a dichotomous classification test generally confirmed Beaver's observations. Departing from the paired design used by Beaver and Altman, Deakin then selected a random sample of thirty-two nonfailed firms and performed a discriminant analysis on fourteen of Beaver's ratios. Results of this MDA indicated high predictive ability up to three years in advance, with error rates increasing markedly as the lead time was expanded to four and five years. Evidence also indicated the relative superiority of the multivariate approach, since the MDA consistently outperformed the best predictors in a univariate setting.

In a second study, Deakin incorporated several methodological refinements in the application of MDA.[50] A number of these had been previously identified and discussed in the literature (e.g., a priori odds of group membership, quadratic classification rules), but Deakin appears to be the first to have actually given explicit consideration

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[49] Edward B. Deakin, "A Discriminant Analysis of Predictors of Business Failure," Journal of Accounting Research, (Spring, 1972), pp. 167-179.

[50] Edward B. Deakin, "Business Failure Prediction: An Empirical Analysis," Working Paper 76-13, University of Texas at Austin, Bureau of Business Research, (November, 1975).

to them in the model development. (Additional details will be given in Chapter 4.) Deakin concluded that MDA models can predict failure with a relatively high degree of accuracy but tempered his statement by noting that nonfailure misclassification errors remain a persistent problem.

Other Multivariate Studies

Numerous multivariate studies of failure prediction have appeared since Altman's pioneering work in this area "shed the light." The more significant of these will be briefly reviewed.

Daniel[51] essentially adopted the Altman MDA methodology in his research. A distinguishing feature of his work was the more refined method of selecting variables for inclusion in the model. Employing factor analysis, stepwise regression, and correlation analysis, a function was developed which exhibited good predictive power in failure identification.

Blum[52] investigated a sample of 115 firms that had failed (i.e., entered bankruptcy or reduced debts through creditor agreement) during the period 1954 to 1968 and a matched group of nonfailed firms. Unlike previous studies, this matching was performed on the basis of industry, number of employees, and sales volume. His work is also

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[51]Troy E. Daniel, "Discriminant Analysis for the Prediction of Business Failures," (unpublished Ph.D. dissertation, University of Alabama, 1968).

[52]Marc Blum, "The Failing Company Doctrine," (unpublished Ph.D. dissertation, Columbia University, 1969); and "Failing Company Discriminant Analysis," Journal of Accounting Research, (Spring, 1974), pp. 1-25.

distinguished from previous research in its explicit consideration of ratio trends and variance as predictors. Results of applying a twelve variable MDA model (containing six variability indicators) were similar to those attained by both Beaver and Altman.

Edmister[53] extended the MDA approach to the area of small business failure prediction. Twenty-one failed firms whose Small Business Administration loans had been written off as losses were identified along with twenty-one nonloss borrowers. An original set of nineteen ratios was reduced through stepwise regression techniques to a seven variable profile. Considerable emphasis was placed on trends and industry averages in the ratio development. Results indicated the general efficacy of the model.

Pinches and Trieschmann[54] applied the MDA approach to insurance company insolvency. Failed firms were defined as those that had involuntarily entered liquidation, receivership, conservatorship or rehabilitation during the period 1966 to 1971. Twenty-six property-liability firms were identified as exhibiting these criteria and a

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[53]Robert O. Edmister, "Financial Ratios as Discriminant Predictors of Small Business Failure," (unpublished Ph.D. dissertation, Ohio State University, 1970); "Financial Ratios and Credit Scoring for Small Business Loans," Journal of Commercial Bank Lending, (September, 1971), pp. 10-23; and "An Empirical Test of Financial Ratio Analysis for Small Business Failure Prediction," Journal of Financial and Quantitative Analysis, (March, 1972), pp. 1477-1497.

[54]George E. Pinches and James S. Trieschmann, "A Multivariate Model for Predicting Financially Distressed P-L Insurers," Journal of Risk and Insurance, (September, 1973), pp. 327-338; "The Efficiency of Alternative Models for Solvency Surveillance in the Insurance Industry," Journal of Risk and Insurance, (December, 1974), pp. 563-577; and "Discriminant Analysis, Classification Results, and Financially Distressed P-L Insurers," Journal of Risk and Insurance, (June, 1977), pp. 289-298.

matched sample of solvent firms was obtained. From an original seventy variable set, a final six ratio profile was selected and a correct classification rate of 94% was noted. In their second study, an empirical comparison was made between univariate and multivariate prediction approaches, based upon their original sample of firms. Results indicated the superiority of the MDA technique. In their third study, attention was directed to consideration of certain refinements in the MDA approach (e.g., quadratic analysis and jack-knife validation).

The last two studies in this section specifically illustrate application of the Beaver suggestion to use predictive ability in evaluating alternative accounting measures. Elam[55] investigated the effects of including capitalized lease data in financial ratios. Forty-eight pairs of failed and nonfailed firms during the period 1966 to 1972 were evaluated on the basis of twenty-eight ratios, using both a univariate and multivariate approach for analysis. Results indicated that the addition of capitalized long term lease information did not significantly improve prediction of failure.

Ketz[56] investigated the question of whether or not ratios derived from general price-level statements improve predictability

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[55] Rick Elam, "The Effect of Lease Data on the Predictive Ability of Financial Ratios," Accounting Review, (January, 1975), pp. 25-43.

[56] J. Edward Ketz, "A Comparison of the Predictability of Business Failure by the Financial Ratios of General Price Level Statements With Those of Historical Cost Statements," (unpublished Ph.D. dissertation, Virginia Polytechnic Institute and State University, 1977); and "The Effect of General Price-Level Adjustments on the Predictive Ability of Financial Ratios," Journal of Accounting Research, Supplement 1978, pp. 273-284ff.

over ratios derived from historical cost statements. Seventy-five failed firms (supplied by Dun and Bradstreet) during the period 1970 to 1975 and 100 randomly drawn nonfailed firms were evaluated on the basis of both an original and reduced variable set. Results generally indicated only a slight improvement by considering price-level information. The methodology which Ketz employed in his research incorporated many of the refinements suggested by Altman and Deakin. Also, it appears that this study was the first to adopt a Bayesian inference approach explicitly in a multivariate failure prediction setting.

Summary

Prior research has generally illustrated the utility of financial ratios in business failure prediction. Application of the predictive ability criterion to the evaluation of alternative accounting measures has been demonstrated to be a viable methodological approach. Specifically, the use of multiple discriminant analysis (MDA) has been shown to be a powerful tool, offering distinct advantages over the univariate approach. Therefore, in the current research, defensive interval measures will be evaluated in terms of their predictive ability in the area of business failure, using MDA as the technique of analysis.

CHAPTER 3

PRELIMINARY EMPIRICAL ANALYSIS

In Chapter 1, the conceptual arguments favoring the superiority of defensive interval measures were identified. It remains to be seen, however, whether such measures do in fact impart information different from the more traditional liquidity ratios. The results of certain empirical investigations performed to explore this question are reported in this chapter.

Selection of Liquidity Ratios

A review of the literature on financial statement analysis indicates that three measures have generally served as the basis for liquidity evaluation: current ratio; quick ratio; and working capital ratio. These measures serve as the benchmarks against which the defensive intervals are compared. Table 1 indicates the components used in the ratio calculations.

Further comment is required concerning the defensive interval computations. Recall from Chapter 1 that the defensive interval attempts to provide a measure of the number of days that the stock of defensive assets could satiate a firm's operating requirements without further inflows to that stock. Defensive assets include cash, short-term marketable securities and net receivables. Cash and near-cash assets represent the primary stock from which unexpected fund requirements can be met without adversely affecting normal production or seeking additional financing. Inventories and prepayments are specifically excluded based on the assumptions

TABLE 1

RATIO CALCULATIONS FOR LIQUIDITY ANALYSIS

Ratio		Calculation
(CR) Current Ratio	=	$\frac{\text{Current Assets}}{\text{Current Liabilities}}$
(QAR) Quick Ratio	=	$\frac{\text{Cash + Marketable Securities + Net Receivables}}{\text{Current Liabilities}}$
(WCR) Working Capital Ratio	=	$\frac{\text{Current Assets - Current Liabilities}}{\text{Total Assets}}$
(DIB1, DIB2) *Basic Defensive Interval	=	$\frac{\text{Cash + Marketable Securities + Net Receivables}}{\text{Projected Daily Operating Expenditures}}$
(DIC1, DIC2) *Cash Defensive Interval	=	$\frac{\text{Cash + Marketable Securities}}{\text{Projected Daily Operating Expenditures}}$
(DCR1, DCR2) *No Credit Defensive Interval	=	$\frac{\text{Cash + Marketable Securities + Net Receivables - Current Liabilities}}{\text{Projected Daily Operating Expenditures}}$

*Two different bases were used to estimate projected daily operating expenditures for each of the three defensive intervals. Those measures denoted by a "1" (e.g. DIB1) are based on income statement data. Those measures denoted by a "2" (e.g. DIB2) are based on statement of changes in financial position data.

that: 1) continuation of normal operations requires maintenance of certain levels of these assets; 2) prepayments will not generally undergo cash conversion and are therefore not part of the available defensive buffer; and 3) conversion of inventories requires continuance of sales and a delay in revenue generation is one of the situations which gives rise to the need for a defensive stock of assets. The defensive stock is not reduced by the amount of short-term obligations under the assumption that in the normal course of business a rollover effect will occur in which payment is offset by the incurrence of additional liabilities. Thus, there will be no net effect requiring employment of the defensive stock.

The basic defensive interval (DIB1 and DIB2) adopts the broadest view of the defensive stock. Two other more restrictive cases have been suggested. First, if serious question as the timing or eventual collectability of receivables is raised, they may be excluded and a cash interval calculated (DIC1 and DIC2). The resulting measure indicates the number of days the firm could meet operating requirements without relying on receivable collections or additional revenues. Secondly, if the ability to obtain additional short-term financing is impaired, a no credit interval may be calculated (DCR1 and DCR2). Such a measure indicates the number of days that operating requirements could be met from the defensive stock after current financing is paid off.

Calculation of the denominator of the interval measures poses some significant problems. Specific information about the operating costs which require use of defensive assets could be obtained from a cash budget for the coming period. However, since this information is usually not available to the outside analyst, another source of data must be found. Sorter and Benston suggested the income statement as this potential source.[57] Under the assumption that a firm will in the short run continue its operations as in the past, data from one year is used to project the next year's expenditures. Operationally, this surrogate approach involves adjusting income statement "expense" information to "expenditure" estimates by removing the effects of items not requiring the use of defensive assets. More specifically, operating expenses (e.g., cost of goods sold, and selling, general, and administrative expenses) are adjusted for such non-cash expenses as depreciation, amortization, and depletion. The defensive interval measures that were calculated using this income statement approach will be hereinafter identified by inserting a "1" following the interval abbreviation (e.g., DIB1, DIC1, and DCR1).

As an alternative to the income statement approach, the statement of changes in financial position is suggested as a source for estimating projected expenditures. A major component of this statement presents a measure of the financial resources provided from operations of the firm. While several funds concepts can be employed in measuring these financial resources provided, by far the more common approach used is working capital.[58] Although a cash equivalent

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[57] Sorter and Benston, "The Interval Measure," p. 635.

[58] George Dick and Richard Rikert, eds., Accounting Trends and Techniques, (New York: American Institute of Certified Public Accountants, 1977), p. 332. A survey of 600 industrial and merchandising companies indicated that only 36 firms opted for a cash or cash equivalent approach in measuring changes in financial position.

approach would be more closely alligned to the actual definition of defensive assets, use of a working capital fund concept is appropriate as a basis for estimating expenditure requirements in the short run. The relevant section of the statement of changes in financial position may be represented as follows:

Revenues -Expenses +Extraordinary Items* +Depreciation & Amortizations* +Deferred Taxes* +Other Funds From Operations	=	Total Funds From Operations After Extraordinary Items

Rearranging terms, the resulting relationship is emphasized:

Expenses -Deferred Taxes* -Depreciation & Amortizations*	=	Revenues +Other Funds From Operations -Total Funds From Operations After Extraordinary Items +Extraordinary Items*

(*Note that the signs of these items will be reversed if the original effect was to either increase income or reduce expenses.)

The left side of this last equation indicates an adjustment of "expenses" to "fund expenditures" by allowing for those items not requiring the outlay of fund resources. Operationally, this calculation is to be performed based on information from the statement of changes in financial position as indicated by the right side of the equation. Since the resulting measure is intended to be an estimate of future expenditures based upon a projection of past data, the effect of any extraordinary (i.e., nonrecurring) items must be removed. The last equation cited accomplishes this. The defensive interval measures which were calculated using this funds statement approach will be hereinafter identified by inserting a "2" following the interval abbreviation (e.g. DIB2, DIC2, and DCR2).

Selection of Firms For Analysis

Having specified the set of liquidity measures of interest, a universe of firms was then identified for study. As indicated by Table 2, the New York (NYSE) and American (AMEX) Stock Exchanges dominate the organized capital markets in this country along a number of dimensions. Of the 6,703 securities listed for trading on registered stock exchanges in 1978, 95% are traded in these two markets, accounting for 99.6% of the value of all stocks and bonds listed.[59]

In view of this significant concentration of wealth, the current research will be limited to firms listed on the NYSE or AMEX. In order to maintain a reasonable degree of homogeneity among the firms selected and to assure availability of the data necessary for the ratio calculations, the universe is further restricted to nonregulated companies. Thus, transportation and insurance firms, utilities, and financial institutions are specifically excluded from consideration.

The primary data source for this section is the COMPUSTAT service for 1979.[60] This service provides magnetic tape libraries of financial, statistical, and market information covering over 2,000 industrial companies. From a listing of COMPUSTAT-covered firms as of December, 1978, a random sample of 608 companies was selected to meet the criteria specified above (i.e., nonregulated

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[59] information compiled from <u>45th Annual Report of the Securities and Exchange Commission</u>, (Washington, D.C.: U.S. Government Printing Office, 1979).

[60] <u>Industrial COMPUSTAT Manual</u>, Standard and Poor's Compustat Services, Inc., 1979.

TABLE 2

SECURITIES EXCHANGES: DESCRIPTIVE STATISTICS FOR THE YEAR 1978*
(all data in billions)

	Market Value of Stocks & Bonds Listed	Market Value of Stocks Listed	Market Value of All Equity Securities Traded	Share Volume
New York Stock Exchange	$1287.63	$822.7	$210.55	7.50
American Stock Exchange	43.13	39.2	18.94	1.01
All Registered Stock Exchanges	1335.36	864.8	268.51	9.36

*Source: SEC Annual Report, 1979

firms traded on the NYSE or AMEX).[61] These firms form the data base for the rest of the analyses performed in this chapter.

Statement of the Research Question

A general working hypothesis for investigation in this chapter may be formulated to describe the relationships of interest:

> H_o: There is no difference between the refined measure (defensive interval) and traditional measures (current, quick, and working capital ratios) as indicators of liquidity position.

A number of analyses will be performed in order to evaluate this assertion.

First, ratios were calculated for each firm for each of the years 1971 to 1978 so that some perspective of the relative numbers involved could be gained. Table 3 reports by year and by ratio the resulting first (Q_1), second (i.e., median), and third (Q_3) quartiles. Due to the diverse construction of these ratios and the resulting lack of consistent dimensionality, location tests comparing defensive intervals with the traditional ratios cannot be applied and interpreted meaningfully. Therefore, other approaches must be adopted.

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[61] see Appendix 3 for a description of the random sampling approach employed and a discussion of the steps undertaken for data validation.

TABLE 3

QUARTILES OF LIQUIDITY RATIOS, 1971-1978

	CR	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
'71: Q3	3.17	1.66	.44	129.34	123.75	41.84	40.62	41.98	42.34
Median	2.31	1.20	.31	93.22	89.03	19.86	18.81	14.48	13.70
Q1	1.72	.86	.18	67.06	61.19	11.26	10.11	-10.12	-11.85
'72: Q3	3.07	1.62	.43	127.25	117.40	37.84	36.48	38.43	35.47
Median	2.32	1.18	.32	90.74	85.63	18.46	17.08	12.47	12.08
Q1	1.73	.89	.18	67.32	61.18	9.55	8.98	-7.88	-9.97
'73: Q3	2.88	1.49	.42	127.14	114.11	37.33	33.73	34.31	31.86
Median	2.19	1.10	.32	88.05	82.52	15.25	13.56	6.97	6.36
Q1	1.65	.82	.18	64.79	59.69	8.20	7.43	-14.52	-15.44
'74: Q3	2.81	1.39	.43	116.34	106.03	29.31	25.86	25.26	24.13
Median	2.18	1.05	.31	83.55	77.30	13.51	12.28	3.16	3.05
Q1	1.61	.77	.17	58.80	55.64	7.63	7.21	-19.41	-19.25
'75: Q3	3.11	1.60	.45	117.11	109.40	37.22	34.73	36.39	36.55
Median	2.25	1.12	.32	84.51	78.63	18.85	16.86	8.58	8.25
Q1	1.68	.84	.18	60.30	56.27	9.18	8.13	-12.08	-12.35
'76: Q3	3.01	1.57	.44	120.18	109.91	39.83	35.77	34.50	33.50
Median	2.27	1.15	.32	80.47	74.16	18.94	17.54	9.64	8.66
Q1	1.72	.86	.19	59.75	55.48	8.74	8.23	-10.19	-10.91
'77: Q3	2.98	1.53	.43	118.62	109.11	39.56	34.61	35.11	35.37
Median	2.20	1.13	.32	83.67	77.67	16.38	15.12	9.50	9.07
Q1	1.67	.86	.18	57.80	53.97	7.59	7.12	-12.49	-12.56
'78: Q3	2.79	1.49	.42	125.32	117.84	34.90	34.44	35.27	33.71
Median	2.15	1.12	.30	83.47	77.36	15.09	14.47	9.05	8.71
Q1	1.65	.84	.18	61.46	56.53	6.82	6.66	-13.35	-14.00

Cross-Sectional Analysis

Spearman Correlation

One possible perspective is to employ the concept of independence and investigate the degree of association exhibited between a pair of variables. Spearman[62] has introduced a distribution-free test statistic to measure this mutual relationship, expressed here in its most convenient computational form:

$$r_s = 1 - \frac{6 \sum^{n} (R_1 - R_2)}{n\,(n^2 - 1)}$$

where R_1 and R_2 represent the sample ranks of the paired variables. This Spearman Rank Correlation Coefficient is actually the classic sample correlation coefficient applied to the within sample rankings of the observations.[63]

To test the significance of the resulting statistic, the following hypotheses are formulated:

$$H_o: \rho = 0$$
$$H_1: \rho \neq 0$$

where ρ (rho) represents the correlation parameter of the bivariate distribution. The null hypothesis implies that the two variables are not correlated in the population and the test statistic differs from zero only by chance. For large samples, significance may be

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[62] C. Spearman, "The Proof and Measurement of Association Between Things," American Journal of Psychology, (Vol. 15, 1904), pp. 72-101.

[63] Sidney Siegel, Nonparametric Statistics For the Behavioral Sciences, (New York: McGraw Hill Book Co., 1956), pp. 203-204.

tested by reference to the Student's t distribution with the critical value:[64]

$$t_{(n-2)} = r_s \sqrt{\frac{n-2}{1-r_s}}$$

Spearman Rank Correlation Coefficients were computed for each possible pair of liquidity ratio measures, based on all information available for the specified firms.

The Statistical Analysis System (SAS) package was utilized to perform these calculations and test the significance of the resulting coefficients.[65] Using the large sample approximation described above, tests of significance for a sample of coefficients were independently performed to validate the alpha levels reported by SAS.[66]

Results for all liquidity ratio pairs for 1978 are summarized in Table 4. The correlation coefficients and corresponding alpha levels attained are reported for each two-variable set. These alpha levels correspond to the probability of making a Type I error (i.e., incorrectly rejecting a true null hypothesis) and thus represent the lowest possible significance levels which would still lead to rejection of the null hypothesis of independence.

Just a glance at the data indicates the extremely small alpha

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[64] Maurice G. Kendall, Rank Correlation Methods, (London: Charles Griffin and Co., Ltd., 1948), pp. 47-48.

[65] Jane T. Helwig and Kathryn A. Council, eds., SAS User's Guide, 1979 Edition, (North Carolina: SAS Institute Inc., 1979); Version 79.3B of SAS was employed.

[66] critical values obtained from F. James Rohlf and Robert R. Sokal, Statistical Tables, (San Francisco: W.H. Freeman and Co., 1969), pp. 160-161.

TABLE 4

SPEARMAN CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1978

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.7686 (.0001)	.8542 (.0001)	.1729 (.0001)	.2125 (.0001)	.1803 (.0001)	.2419 (.0001)	.6997 (.0001)	.7229 (.0001)
QAR		.5758 (.0001)	.5243 (.0001)	.5339 (.0001)	.4898 (.0001)	.5141 (.0001)	.9599 (.0001)	.9372 (.0001)
WCR			.1195 (.0032)	.1741 (.0002)	.0372 (.3617)	.1045 (.0244)	.5094 (.0001)	.5273 (.0001)
DIB1				.9765 (.0001)	.6793 (.0001)	.6483 (.0001)	.5600 (.0001)	.5453 (.0001)
DIB2					.6557 (.0001)	.6685 (.0001)	.5549 (.0001)	.5520 (.0001)
DIC1						.9857 (.0001)	.5254 (.0001)	.5356 (.0001)
DIC2							.5396 (.0001)	.5362 (.0001)
DCR1								.9777 (.0001)

levels resulting from the significance tests. Thus, the probability of observing the actual coefficients under H_0 is almost negligible and one may conclude that there is a significant association among each of the variable pairs. Upon closer inspection some exceptions to this generalization are found. Certain of the pairings made with WCR (especially DIC1) yield higher alpha levels. The associated coefficients are the smallest in the table and thus less distinguishable from the zero value hypothesized. Thus, there is more Type I risk involved in these cases.

Kendall Correlation

Numerous other statistics are available to test for the significance of association between variates. Perhaps the most popular competitor to Spearman's estimate is Kendall's tau. In contrast to r_s, which attempts to measure the correlation coefficient for the underlying bivariate population (i.e., a distributional parameter), Kendall's statistic estimates a probability parameter (tau):[67]

$$\tau = 2\,P\,\{\,(X_1 - X_2)\,(Y_1 - Y_2) > 0\,\} - 1$$

If X and Y are truly independent, $P\,\{\,(X_1 - X_2)\,(Y_1 - Y_2) > 0\,\} = \frac{1}{2}$ and the value of τ (tau) is zero. Calculation of the Kendall estimate involves identifying concordant and discordant pairs of observations:[68]

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[67] Miles Hollander and Douglas A. Wolfe, Nonparametric Statistical Methods, (New York: John Wiley and Sons, 1973), pp. 185ff.

[68] Ibid., p. 189.

(X_i, X_j) and (Y_i, Y_j) are defined as concordant pairs if

1) $X_i > X_j$ and $Y_i > Y_j$ or
2) $X_i < X_j$ and $Y_i < Y_j$, thus resulting in
$(X_i - X_j)(Y_i - Y_j) > 0$;

(X_i, X_j) and (Y_i, Y_j) are defined as discordant pairs if

1) $X_i < X_j$ and $Y_i > Y_j$ or
2) $X_i > X_j$ and $Y_i < Y_j$, thus resulting in
$(X_i - X_j)(Y_i - Y_j) < 0$.

The actual parameter estimate is provided by:[69]

$$\hat{\tau} = \frac{S}{\frac{n}{2}(n - 1)}$$

where S = the number of concordant pairs less the number of discordant pairs. Basically then, this approach measures association in terms of the agreement among orderings of all possible pairs of observations within the samples.

To test the significance of the resulting statistic, the following hypotheses are formulated:

$$H_0: \tau = 0$$

$$H_1: \tau \neq 0$$

The null hypothesis implies that the X and Y variables are independent and the test statistic differs from zero only by chance. For large samples, significance can be tested by reference to

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[69] James V. Bradley, Distribution-Free Statistical Tests, (Englewood Cliffs, New Jersey: Prentice-Hall, 1968), pp. 284-287.

standard normal tables with the critical value:[70]

$$z = \frac{\hat{\tau}}{\sqrt{\frac{2(2n + 5)}{9n(n - 1)}}}$$

The numerical values of r_s and $\hat{\tau}$ are not directly comparable measures of correlation since they are based on different underlying scales. Sokal and Rohlf point out:[71]

> If the two variables . . . are actually independent, then the numerical value of Spearman's coefficient of rank correlation is highly correlated with that of Kendall's coefficient . . . . If the true correlation is not zero, then one would expect the two coefficients to be sensitive to different kinds of departures from independence. It is not possible, at present, to give any suggestions as to when each coefficient would best be used.

Since the Spearman significance tests indicated that the true correlation was generally not zero, the analysis in this section was extended to include calculation of the Kendall statistics and performance of significance tests on them. SAS was employed for these procedures.

Results for all liquidity ratio pairs for 1978 are reported in Table 5. Note in general the very small alpha levels attained, thus indicating again the lack of independence between the sets of variables, this time as measured by $\hat{\tau}$.

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[70] Robert M. Thorndike, Correlational Procedures For Research, (New York: Gardner Press, 1978), p. 90.

[71] Robert R. Sokal and F. James Rohlf, Biometry, (San Francisco: W.H. Freeman and Co., 1969), p. 540.

TABLE 5

KENDALL CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1978

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.5767 (.0001)	.6745 (.0001)	.1200 (.0000)	.1457 (.0000)	.1209 (.0000)	.1613 (.0000)	.5159 (.0001)	.5370 (.0001)
QAR		.4072 (.0001)	.3755 (.0001)	.3830 (.0001)	.3419 (.0001)	.3588 (.0001)	.8321 (.0001)	.8145 (.0001)
WCR			.0844 (.0019)	.1225 (.0001)	.0245 (.3682)	.0700 (.0242)	.3606 (.0001)	.3744 (.0001)
DIB1				.9454 (.0001)	.4991 (.0001)	.4733 (.0001)	.4217 (.0001)	.4100 (.0001)
DIB2					.4795 (.0001)	.4903 (.0001)	.4214 (.0001)	.4167 (.0001)
DIC1						.9653 (.0001)	.3789 (.0001)	.3862 (.0001)
DIC2							.3908 (.0001)	.3869 (.0001)
DCR1								.9706 (.0001)

To provide additional evidence on the reliability of the statistics measuring the association between liquidity ratios, the cross-sectional analysis was extended to encompass all years from 1971 to 1978, inclusive. Spearman and Kendall correlations are summarized in Tables 6 and 7, respectively. Only those estimates which would not lead to rejection of the null hypotheses of independence have been denoted by a " * ". More complete tabulations for all possible variable pairs for the individual years 1971 to 1977 are provided in Appendix 2.

Discussion of Results

Complete independence is a very strict criterion to apply in evaluating the difference between the refined and traditional liquidity measures. Given the large sample size and the common elements in the various ratio calculations, the rejection of the null hypotheses with the resulting conclusion that the variables are associated is not surprising. These results should not be construed as compelling evidence supporting the working hypothesis of this chapter. Complete independence represents a sufficient but not necessary condition to imply a difference between the defensive intervals and traditional liquidity ratios.

The small alpha levels attained in the significance tests allow the confident use of the corresponding correlation measures as descriptive indices. This conclusion stems from the nature of the sampling design which specified selection of a large random sample of firms. If higher significance levels had resulted, one would be

TABLE 6

SPEARMAN CORRELATION SUMMARY STATISTICS*

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
'71 CR	.7920	.8084	.1036*	.0925*	.1469	.1442	.7147	.6912
'72	.7579	.8092	.1024*	.0885*	.1448	.1330	.6974	.6931
'73	.7173	.8011	.1205	.1091*	.1486	.1307	.6569	.6483
'74	.7153	.8342	.0946*	.1176*	.0945*	.1357	.6842	.6933
'75	.7666	.8363	.1937	.1920	.1883	.2046	.7011	.6846
'76	.7780	.8350	.1666	.1912	.1849	.2077	.7155	.6934
'77	.7770	.8432	.1621	.1918	.1689	.2114	.7070	.7049
'78	.7686	.8542	.1729	.2125	.1803	.2419	.6997	.7229
'71 QAR		.5610	.4222	.4253	.4069	.4011	.9541	.9227
'72		.5188	.4553	.4368	.4249	.3992	.9637	.9483
'73		.4525	.5026	.4884	.4717	.4307	.9661	.9497
'74		.4945	.4629	.4601	.3911	.3864	.9652	.9415
'75		.5592	.5310	.5393	.4810	.4802	.9656	.9445
'76		.5712	.5078	.5317	.4863	.5022	.9631	.9409
'77		.5781	.4999	.5330	.4575	.4954	.9611	.9480
'78		.5758	.5243	.5339	.4898	.5141	.9599	.9372
'71 WCR			.0285*	.0753*	-.0298*	.0256*	.5058	.5281
'72			.0248*	.0510*	-.0205*	.0221*	.4783	.5092
'73			.0218*	.0624*	-.0731*	-.0334*	.3984	.4324
'74			.0208*	.0855*	-.0738*	.0154*	.4618	.4978
'75			.1080	.1434	.0338*	.0877*	.5027	.5113
'76			.0969*	.1572	.0293*	.0772*	.5265	.5310
'77			.1064	.1445	.0314*	.0730*	.5185	.5112
'78			.1195	.1741	.0372*	.1045*	.5094	.5273

* indicates null hypothesis is not rejected at $\alpha = .01$

TABLE 7

KENDALL CORRELATION SUMMARY STATISTICS*

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
'71 CR	.6034	.6340	.0739	.0686*	.1019	.1018	.5392	.5264
'72	.5696	.6374	.0733	.0637*	.1004	.0934	.5178	.5172
'73	.5347	.6300	.0849	.0778*	.0980	.0859	.4854	.4830
'74	.5343	.6626	.0671*	.0841	.0639*	.0948	.5070	.5182
'75	.5771	.6562	.1355	.1336	.1278	.1392	.5203	.5095
'76	.5861	.6571	.1180	.1333	.1286	.1422	.5286	.5184
'77	.5847	.6617	.1142	.1342	.1168	.1454	.5233	.5240
'78	.5767	.6745	.1200	.1457	.1209	.1613	.5159	.5370
'71 QAR		.4025	.3013	.3050	.2860	.2830	.8281	.8026
'72		.3695	.3261	.3099	.2984	.2777	.8434	.8288
'73		.3201	.3612	.3503	.3325	.3008	.8505	.8315
'74		.3499	.3317	.3303	.2743	.2723	.8466	.8306
'75		.3983	.3830	.3903	.3372	.3361	.8503	.8298
'76		.4038	.3672	.3829	.3418	.3508	.8418	.8251
'77		.4067	.3577	.3809	.3204	.3454	.8360	.8265
'78		.4072	.3755	.3830	.3419	.3588	.8321	.8145
'71 WCR			.0222*	.0541*	-.0203*	.0181*	.3668	.3840
'72			.0196*	.0370*	-.0135*	.0160*	.3419	.3637
'73			.0189*	.0463*	-.0512*	-.0250*	.2825	.3060
'74			.0173*	.0618*	-.0492*	.0117*	.3287	.3543
'75			.0764	.1033	.0231*	.0620*	.3586	.3661
'76			.0701	.1122	.0213*	.0522*	.3742	.3787
'77			.0745	.1019	.0207*	.0487*	.3665	.3615
'78			.0844	.1225	.0245*	.0700*	.3606	.3744

* indicates null hypothesis is not rejected at $\alpha = .01$

less likely to accept the correlation measures as indicative of the true degree of relationship in the population. Adopting this perspective, one now has a basis for comparing the relative correlation (i.e., association) between different sets of variables and thus a means for offering some insight into the research question posed. The discussion which follows is based on general impressions from both the Spearman and Kendall summaries as presented in Tables 6 and 7. The observations noted are equally applicable to either set of test results.

First, adopting the current ratio (CR) as the standard for comparison, note the rather high correlation with the other two traditional liquidity ratios (QAR and WCR). All of the defensive interval correlations with CR, regardless of either the defensive stock definition or the approach adopted for estimating projected expenditures, are less than those between either CR and QAR, or CR and WCR. Thus, taken as groups, the other traditional ratios are more closely associated with the current ratio than are the defensive intervals.

Moving from a group approach to specific comparisons, the basic defensive intervals (DIB1 and DIB2) and the cash intervals (DIC1 and DIC2) are only slightly correlated with the current ratio. In fact, in a number of cases, the null hypothesis could not be rejected and one is lead to infer (at $\alpha = .01$) that there is independence between the specified variables. In contrast, the no credit intervals(DCR1 and DCR2) indicate substantially more correlation with the current ratio than exhibited by the other defensive intervals.

When the quick ratio (QAR) is adopted as the standard for comparison, somewhat similar results are obtained. The basic and cash interval measures still display rather low correlations, but large increases in the no credit interval correlations with QAR are apparent.

Finally, when the working capital ratio (WCR) is applied as the benchmark, the most dramatic differences arise. The basic and cash intervals manifest as a group the smallest correlations in the entire table of coefficients. In most of these cases (especially DIC1 and DIC2) the null hypothesis could not be rejected at $\alpha = .01$. Even at much higher significance levels, rejection still would not result and independence is therefore strongly implied. The no credit intervals exhibit their lowest correlations in the table when compared with WCR.

To summarize, several cases in which liquidity variables were compared have indicated independence. Thus, if one were to use these liquidity position indicators (i.e., ratios) as a basis for ordering firms, there would be no statistical association between the resulting rankings. In the more numerous cases, significant associations were found. Relying on the low alpha levels attained and the resulting confidence in the parameter estimates, one may conclude by inspection that in general the correlations between the defensive intervals and traditional ratios are rather small. Thus, liquidity rankings of firms in these cases would result in statistically significant associations, but the magnitude of these would in fact be minimal.

The evidence thus far tends to support rejection of the working hypothesis and thus suggests that there is indeed a difference between the traditional and refined measures as liquidity indicators. To further verify this conclusion, two supplemental analyses were performed.

Time-Series Analysis

Correlation Over Time

In this section the cross-sectional approach, in which a firm's liquidity ratios are compared at a given point in time, is dropped in favor of an analysis over time. This will allow for consideration of liquidity indicators in a dynamic setting.

A 300 firm random subsample of the original data set was selected for investigation, and the nine liquidity measures of interest were calculated for each firm for each year 1971 to 1978. This data was then analyzed by computing and comparing for each firm the year to year changes in the liquidity ratios. An example of the scheme that was adopted follows, in which the current ratio (CR) and one basic defensive interval (DIB1) are compared for a sample firm:

Year(t)	CR_t	$DIB1_t$	Sign of $(CR_t - CR_{t-1})$	Sign of $(DIB1_t - DIB1_{t-1})$
1971	x	y		
1972	x	y	+	-
⋮	⋮	⋮	⋮	⋮
1978	x	y	-	+

Note that the magnitude of the changes is ignored and only the direction of change is considered.

The tabulations of the comparisons of the year to year changes were then combined to represent performance across the entire sample. Results are reported in Table 8. These figures indicate, for each possible liquidity ratio comparison, the percentage of cases in the entire sample in which the year to year changes agreed in signs. If the traditional ratios and defensive interval measures provide the same information over time as to the changing liquidity status of the firm, one would expect very large percentages in this table of comparisons. Results thus indicate a significant number of times that the traditional and refined indicators move in opposite directions.

Stability of the Measures

A second supplemental approach was undertaken to provide additional evidence about the relative movement of the liquidity measures over time. Using the same 300 firm data base as in the previous section, coefficients of variation were investigated.

The sample estimate for the population coefficient of variation is provided by:[72]

$$CV = \frac{s}{\bar{X}}$$

where s is the sample standard deviation. The measure of variability in the data (s) is thus expressed as a fraction of the mean of the data ($\bar{X}$). Since both statistics are based on the same unit of

[72]George W. Snedecor and William G. Cochran, Statistical Methods, (Ames, Iowa: Iowa State University Press, 1967), p. 62.

TABLE 8

PERCENTAGE OF CASES IN WHICH YEAR TO YEAR LIQUIDITY CHANGES AGREED IN DIRECTION

	Comparison Measures: QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
Standard:								
CR	82.92	86.52	53.12	53.83	57.47	57.76	79.85	80.18
QAR		79.70	64.40	64.32	65.34	66.20	90.94	91.10
WCR			58.73	57.96	61.63	61.94	79.39	79.39

measurement, the resulting estimate of relative variation (CV) is independent of the original measurement base employed. Use of the CV thus allows for the direct comparison of liquidity ratio variations which have been derived from alternative liquidity measurement approaches.

Coefficients of variation were calculated for each firm for each of the nine liquidity measures during the 1971 to 1978 time period. For each firm, comparisons of magnitude were then made between the CV of each traditional ratio (in turn) and the CV of each defensive interval measure. Tabulations for all individual firms are combined and reported in Table 9. These figures represent the percentage of firms for which the CV of the comparison measure was less than the CV of the standard measure. Results indicate that relative to the traditional liquidity ratios, the basic defensive intervals exhibit more stability over time. However, the cash and no credit intervals appear quite unstable relative to the standard measures.

Summary

Evidence provided in this chapter generally supports rejection of the working hypothesis previously stated. Results of the cross-sectional analyses indicated rather small correlations between the traditional and refined liquidity measures. In several cases, statistical independence was even implied. Results of the time-series analyses corroborated this evidence by indicating that the various liquidity measures provide different indications of the changes in

TABLE 9

PERCENTAGE OF FIRMS FOR WHICH THE COEFFICIENT OF VARIATION OF THE COMPARISON WAS LESS THAN THAT OF THE STANDARD

Standard:	Comparison Measures: QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	18.16	52.16	51.87	53.60	7.20	7.78	3.75	3.17
QAR		65.42	68.30	66.86	11.82	12.68	6.63	5.76
WCR			50.14	49.86	12.97	13.54	6.34	6.34

liquidity status over time.

These conclusions provide the justification for incorporating the defensive interval measures as liquidity indicators in the variable sets investigated in the remainder of this research. Attention will now be directed to describing the particular methodology to be employed in evaluating the potential contribution of these interval measures to business failure prediction.

CHAPTER 4

METHODOLOGY

This chapter will describe specifics of the methodology to be used to evaluate the contribution of defensive interval measures to business failure prediction. In order to state the problem investigated more precisely, the following null hypothesis is offered:

> H_o: The ability to discriminate between failed and nonfailed firms does not improve if one incorporates the use of defensive interval measures as liquidity indicators

Multiple discriminant analysis models will be built to test this hypothesis. The following discussion will serve to explain discriminant analysis and its underlying assumptions. In addition, a description of the approaches to be taken in specifying various components of the models will be given.

Discriminant Analysis and the Theory of Classification

The basic problem of classification, as viewed from the perspective of multiple discriminant analysis, involves assignment of an observation of unknown origin to one of two or more categories on the basis of measurements taken on that observation. In the context of business failure prediction, this process entails classification of firms into one of two dichotomous groups (failed or nonfailed) based on observed firm characteristics (financial ratios).

The Fisher Design

Two basic derivations of discriminant analysis have appeared in the literature. One approach, first suggested by Fisher,[73] defined discrimination in terms of achieving the maximum separation of the two underlying populations. Basically, this method involves the transformation of a set of individual variables to a single weighted composite score which will then have maximum utility in distinguishing between the two groups.

Consider a set of p variables X_1, X_2 . . . X_p to be used as the basis for determining differences between the groups. The desired discriminant function is a linear combination of the variables of the form:[74]

$$z = \lambda_1 X_1 + \lambda_2 X_2 + \cdot \cdot \cdot \cdot + \lambda_p X_p$$

The task then becomes one of appropriately determining the weighting coefficients (λ's) so that maximum differentiation between the groups occurs. The criterion suggested by Fisher to measure the degree of group separation was the between-groups variation relative

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[73] Ronald A. Fisher, "The Use of Multiple Measurements in Taxonomic Problems," Annals of Eugenics, Vol. 7, 1936, pp. 179-188; and "The Statistical Utilization of Multiple Measurements," Annals of Eugenics, Vol. 8, 1938, pp. 376-386.

[74] the following development is from Paul G. Hoel, Introduction to Mathematical Statistics, (New York: John Wiley and Sons, 1971), pp. 181-184ff; and, John E. Overall and C. James Klett, Applied Multivariate Analysis, (New York: McGraw Hill Book Co., 1972), pp. 244-247ff.

to the within-group variation:[75]

$$G = \frac{(\bar{z}_1 - \bar{z}_2)^2}{\sum_i^2 \sum_j^n (z_{ij} - \bar{z}_i)^2}$$

where $\bar{z}_i$ represents the z mean of the respective groups and z_{ij} represents the z value of the j^{th} individual in the i^{th} group. Values of λ are chosen so as to maximize the criterion function G.

This discrimination technique may be described graphically for the bivariate case as in Illustration 2.[76] The elliptical contour lines represent a locus of points of equal density for the respective groups G_1 and G_2. The intersection of these contours defines a straight line (L) from which a perpendicular line through the origin is constructed (Z). If the individual points in bivariate space are then projected onto line Z, the two frequency distributions $f(z_1)$ and $f(z_2)$ result, with $\bar{z}_1$ and $\bar{z}_2$ representing the means of the respective distributions. An individual's composite discriminant score represents its location along this line. The intersection point between lines L and Z, z' , divides the discriminant space into two regions denoting the probable group membership. Those individuals whose z scores fall to the left of z' would be classified as G_1 members, and those whose z scores fall to the right of z' would

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[75]Hoel, Mathematical Statistics, p. 183.

[76]adapted from W.W. Cooley and P.R. Lohnes, Multivariate Procedures for the Behavioral Sciences, (New York: John Wiley and Sons, 1962), pp. 116-117.

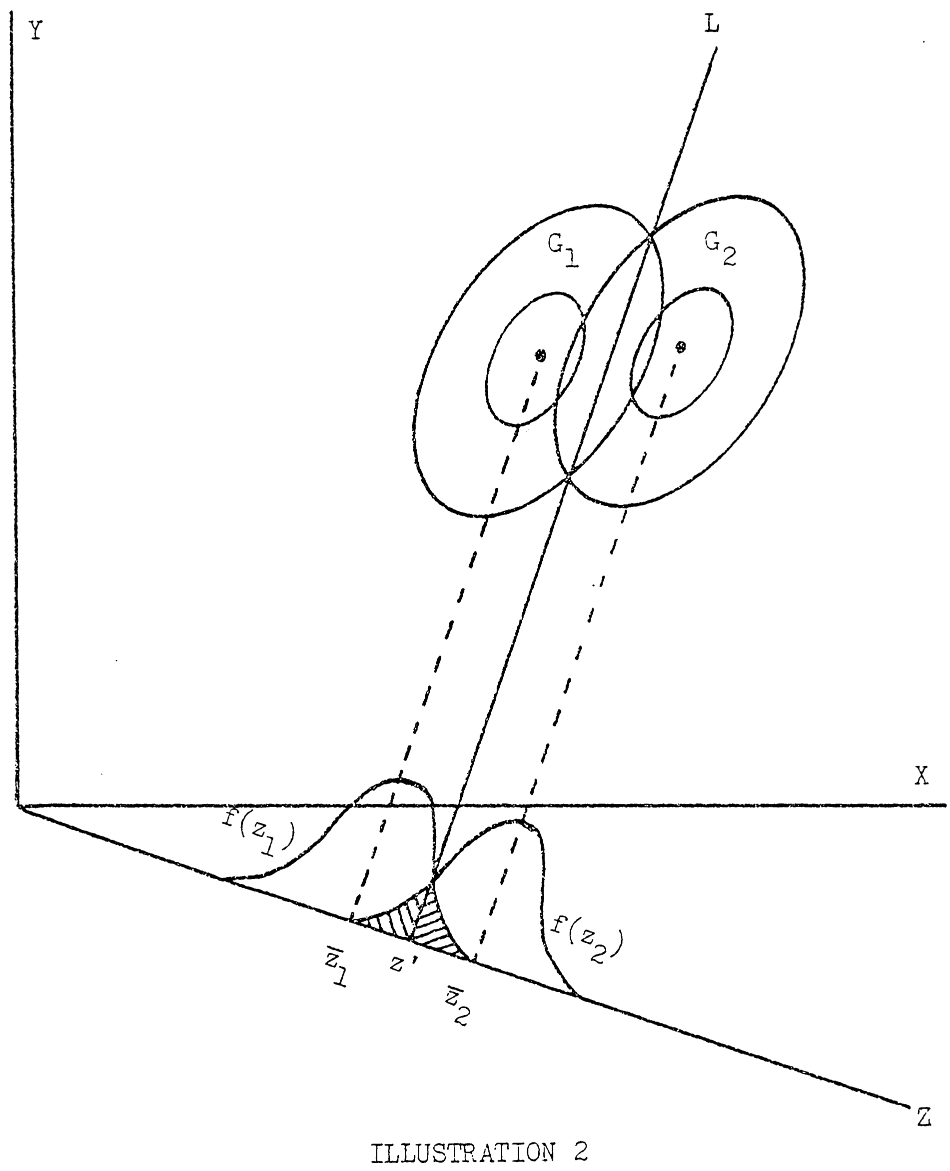

ILLUSTRATION 2

THE FISHER APPROACH TO GROUP DISCRIMINATION be classified as G_2 members. Point z' is chosen so that the minimum overlap of the group distributions occurs. This overlap, denoted by the shaded area, represents the misclassification zone that would result from the selected discriminant function and assignment rule. Thus, in this approach to discrimination, misclassification is minimized by maximizing group separation.

The Neyman-Pearson Design

A second approach, proposed by Neyman and Pearson[77] and implemented by Welch,[78] concentrates on developing an assignment rule that will minimize the total probability of misclassification. Kendall[79] has demonstrated that while the two basic approaches to discriminant analysis possess different mathematical derivations, they are essentially equivalent. Since the second method (used in this research) allows for explicit consideration of a number of refinements over the original Fisher design, it will now be discussed at some length.

To describe the specific components of this method more precisely, consider the following mathematical expansion.[80] Let π_0 and π_1 represent two populations whose multivariate frequency distributions

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[77] J. Neyman and E.S. Pearson, "On the Problem of the Most Efficient Tests of Statistical Hypotheses," Philosophical Transactions of the Royal Society of London, Series A, Vol.231, 1933, pp. 289-337.

[78] B.L. Welch, "Note on Discriminant Functions," Biometrika, Vol. 31, 1939, pp. 218-220.

[79] Maurice Kendall, Multivariate Analysis, (London: Charles Griffin and Co. Ltd., 1975), pp. 147-148.

[80] the development which follows was adapted from Theodore W. Anderson, An Introduction to Multivariate Statistical Analysis, (New York: John Wiley and Sons, 1958), pp. 127-128ff.

(densities) are denoted by $f_o(x)$ and $f_1(x)$, respectively. Further, let the prior probability that an observation comes from π_o be q_o, and from π_1 be q_1. Let R represent the entire p-dimensional space which is divided in some manner into two mutually exclusive regions R_o and R_1. If an observation falls in R_o it would be assigned to π_o and similarly, an observation in R_1 would be deemed to come from π_1.

The conditional probability of correctly categorizing an observation that is actually a member of π_o is given by:

$$P(0|0, R) = \int_{R_o} f_o(x)\, dx$$

where $dx = dx_1 , dx_2 \ldots . dx_p$. The probability of misclassifying this observation is then:

$$P(1|0, R) = \int_{R_1} f_o(x)\, dx$$

Similarly, the conditional probability of correctly categorizing an observation that is actually a member of π_1 is given by:

$$P(1|1, R) = \int_{R_1} f_1(x)\, dx$$

The probability of misclassifying this observation is then:

$$P(0|1, R) = \int_{R_o} f_1(x)\, dx$$

Incorporating the prior odds of group membership, one may denote $q_oP(0|0,R)$ and $q_oP(1|0,R)$ as the respective probabilities of drawing an observation from π_o and then correctly or incorrectly classifying it. In the same manner, $q_1P(1|1,R)$ and $q_1P(0|1,R)$ represent the respective probabilities of drawing an observation from π_1 and then correctly or incorrectly classifying it. The total probability of misclassification may then be expressed as in (I) and restated in more convenient form as in (II):[81]

$$\text{(I)} \qquad q_oP(1|0,R) + q_1P(0|1,R)$$

$$= q_o \int_{R_1} f_o(x)dx + q_1 \int_{R_o} f_1(x)dx$$

$$= q_o[\, 1 - \int_{R_o} f_o(x)dx \,] + q_1 \int_{R_o} f_1(x)dx$$

$$\text{(II)} \qquad = q_o + \int_{R_o} [\, q_1f_1(x) - q_of_o(x) \,]\, dx$$

Minimization of this last quantity is accomplished by choosing R_o such that $[q_1f_1(x) - q_of_o(x)] < 0$ for all points in R_o. The following decision rule for classification results:[82]

$$\text{(III)} \qquad \text{If } \frac{f_o(x)}{f_1(x)} > \frac{q_1}{q_o} \text{ assign } X \text{ to } \pi_o,$$

otherwise assign X to π_1.

To this point in the presentation, the Fisher approach has been improved upon by incorporating unequal prior odds into the specification of the assignment rule. Further refinements are possible.

Two basic types of errors of classification are possible in discriminant analysis: 1) classifying a π_o individual as a member of π_1; and 2) classifying a π_1 individual as a member of π_o. Since

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[81] Peter A. Lachenbruch, Discriminant Analysis, (New York: Hafner Press, 1975), p. 10.

[82] Ibid., p. 11.

these two errors may not be equally undesirable, simple minimization of the probability of misclassification (I) is not sufficient. What is needed is an assignment rule which will consider the relative unacceptability of the two errors.

Let $C(1|0)$ (>0) and $C(0|1)$ (>0) represent the respective costs of the two types of classification errors previously described. The total cost of misclassification can then be expressed as:[83]

$$\text{(IV)} \qquad C(1|0)q_oP(1|0,R) + C(0|1)q_1P(0|1,R)$$

Following the same progression as that from (I) to (III), minimization of this last expression is accomplished by selecting the following decision rule:[84]

$$\text{(V)} \qquad \text{If } \frac{f_o(x)}{f_1(x)} > \frac{C(0|1)q_1}{C(1|0)q_o} \quad \text{assign X to } \pi_o,$$

otherwise assign X to π_1.

This approach therefore adopts as its goal the minimization of the expected cost of misclassification rather than just the minimization of the total probability of error.

A special case of the general approach previously outlined occurs when π_o and π_1 are multivariate normal with respective mean vectors μ_o and μ_1 (measured over p variables) and common covariance matrix Σ.[85] The population densities in this case may be denoted by

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[83]Anderson, Statistical Analysis, p. 127.

[84]Lachenbruch, Discriminant Analysis, pp. 14-15.

[85]this approach was first used by A. Wald, "On a Statistical Problem Arising in the Classification of an Individual Into One of Two Groups," Annals of Mathematical Statistics, Vol.15, 1944, pp. 145-162.

the expression:[86]

$$f_i(x) = (2\pi)^{-\frac{1}{2}p} |\Sigma|^{-\frac{1}{2}} \exp[-\tfrac{1}{2}(X-\mu_i)' \Sigma^{-1}(X-\mu_i)]$$

The ratio of densities thus becomes:[87]

$$\frac{f_o(x)}{f_1(x)} = \exp\{-\tfrac{1}{2}[(X-\mu_o)' \Sigma^{-1} (X-\mu_o) - (X-\mu_1)' \Sigma^{-1} (X-\mu_1)]\}$$

Expressed in logarithm form and incorporating both prior odds of group membership and different costs of misclassification, the assignment rule becomes:[88]

(VI) If $X'\Sigma^{-1}(\mu_o-\mu_1)-\frac{1}{2}(\mu_o+\mu_1)'\Sigma^{-1}(\mu_o-\mu_1) > \ln k$,

assign X to π_o, otherwise assign X to π_1.

From the earlier discussion,[89] k in this decision rule is denoted by:

$$\frac{C(0|1)\ q_1}{C(1|0)\ q_o}$$

The sample analogue to (VI) is simply:

(VII) $X'\ S^{-1}\ (\overline{X}_o-\overline{X}_1) - \frac{1}{2}(\overline{X}_o+\overline{X}_1)'\ S^{-1}\ (\overline{X}_o-\overline{X}_1) > \ln k$

where $\overline{X}_i$ i=1,2 and S are the sample estimates for the parameters μ_i and Σ, respectively.

The discussion to this point has presumed equal covariances in the two groups and the consequent application of linear discriminant

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[86] John P. Van de Geer, Introduction to Multivariate Analysis for the Social Sciences, (San Francisco: W.H. Freeman and Co., 1971), p. 79.

[87] Anderson, Statistical Analysis, p. 133.

[88] Ibid., p. 134.

[89] see the derivation of expression (V).

analysis. If this assumption cannot be met, quadratic discrimination is suggested.[90] Assignment rules for the sample analogue are similar to those expressed in (VI) except that a quadratic function results in which separate group dispersion matrices are considered instead of one common matrix:[91]

$$\text{(VIII)} \quad \text{If } X'(S_0^{-1} - S_1^{-1})X - 2(\overline{X}_0' S_0^{-1} - \overline{X}_1' S_1^{-1})X + \overline{X}_0' S_0^{-1} \overline{X}_0 - \overline{X}_1' S_1^{-1} \overline{X}_1 < \ln |S_0 \cdot S_1^{-1}| - 2 \ln k ,$$

assign X to π_0 , otherwise assign X to π_1.

The actual classification procedure applied in this research employs a Bayesian inference approach which incorporates the prior odds and costs of misclassification refinements already discussed. Specifics of this methodology are presented in Appendix 1.

Experimental Design

The previous discussion of classification theory has identified a number of critical issues which must be addressed in the application of discriminant analysis. These issues will now be explained in terms of their specific application to this research on business failure prediction.

Identification of Group Membership

A primary assumption of discriminant analysis is the existence of discrete, identifiable, and mutually exclusive groups. As

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[90] Robert A. Eisenbeis and Robert V. Avery, Discriminant Analysis and Classification Procedures, (Lexington, Mass.: D.C. Heath and Co., 1972), p. 16.

[91] Ibid.

explained in Chapter 3, the universe of firms selected for study consists of nonregulated companies traded on the New York (NYSE) and American (AMEX) Stock Exchanges. The failed group is to be composed of those firms that have filed a petition for bankruptcy under either Chapter X or XI of the Chandler Act during the period 1972 through 1978.

Identification of the elements of this group is a laborious undertaking since no list of firms meeting these criteria is available. Previous studies have generally overcome this data problem by reducing the scope of firms studied or arbitrarily adjusting the definition of failure to conform to the characteristics of those companies in available failure listings. It was deemed important in this study to maintain a strict failure definition in order to avoid an arbitrary grouping scheme and thus assure the discreteness of the groups under study.

The actual search process to be employed in identifying the failed firm group is described in Illustration 3. Potential failed firm group members are identified in Step #1 from three basic sources:[92] F & S Index of Corporate Change; Wall Street Journal Index; and Dun and Bradstreet. Step #2 is accomplished primarily by reference to bankruptcy reports in news articles from various business publications.

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[92] Funk and Scott Index of Corporate Change, Annual Editions 1971 through 1979, (Cleveland: Predicasts, Inc.); Wall Street Journal Index, Annual Editions 1971 through 1979, (New York: Dow Jones and Co., Inc.); and "List of Large Business Failures: 1971 to 1978," prepared by Business Economics Division, (New York: Dun and Bradstreet, Inc.).

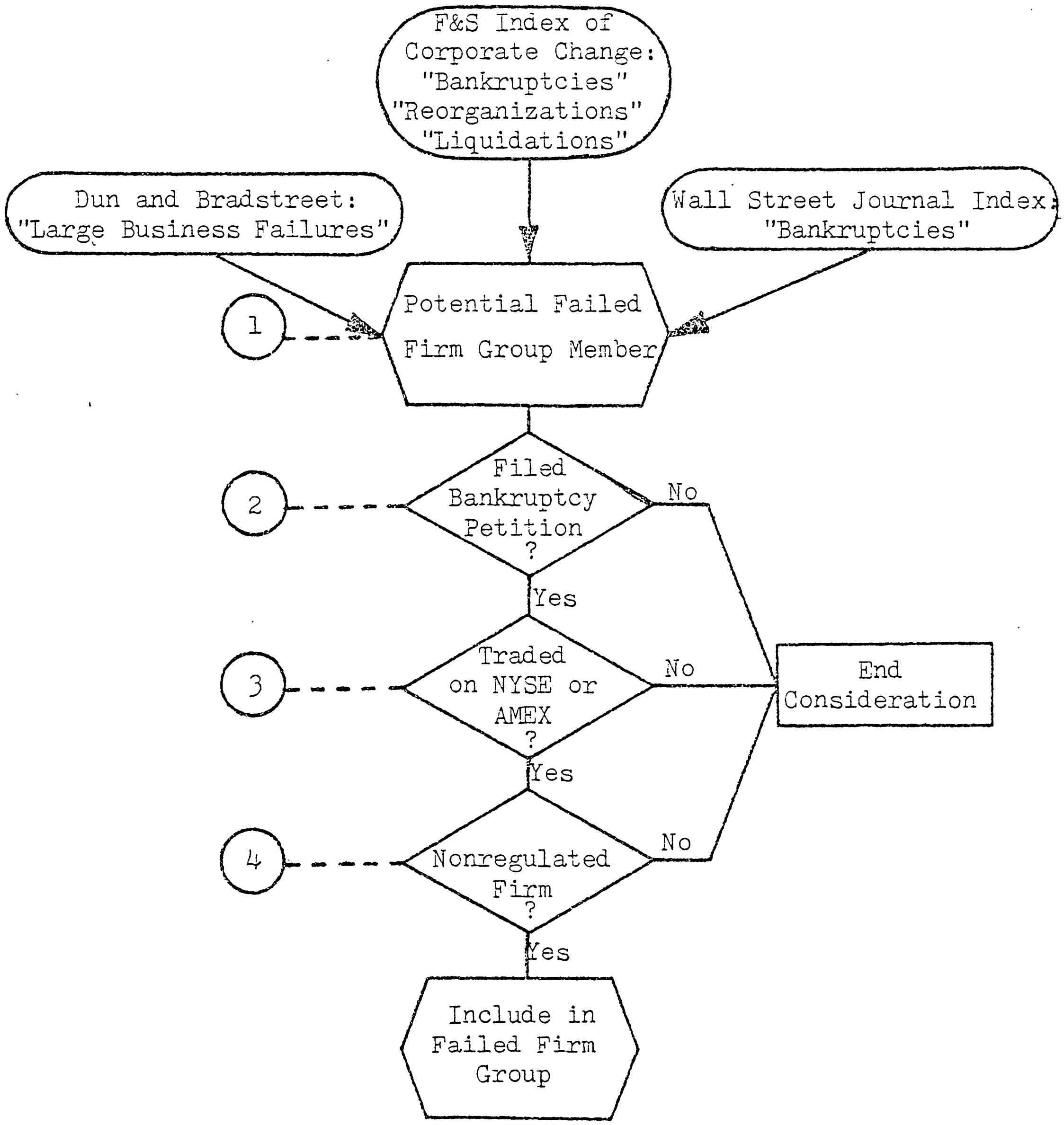

ILLUSTRATION 3

SEARCH PROCESS FOR IDENTIFICATION OF FAILED GROUP MEMBERSHIP

Step #3 information is derived from exchange listings in Moody's Industrial Manual, corroborated by the Daily Stock Price Record.[93] Step #4 is achieved by reference to Standard and Poor's Register and the F & S Index of Corporations and Industries.[94] This last step involves exclusion of firms with Standard Industrial Classification (SIC) code numbers in the 4000 or 6000 range. These two ranges include financial institutions and transportation, utility, and insurance companies.

The nonfailed firm group is identified as those nonregulated NYSE and AMEX companies that did not file for bankruptcy during the period 1972 through 1978. Because of the year to year fluctuations in firms traded on these exchanges, it is necessary to select a reference point to be used as the basis for establishing the nonfailed firm population. December, 1978 was selected as this point of reference.

Rejection of Paired Sample Design

Most of the research reviewed in Chapter 2 utilized a paired sample design in which failed firms were matched with an equal number of nonfailed firms. Given an identified group of failed firms, similar nonfailed firms were then selected, usually on the

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[93] Moody's Industrial Manual, Annual Editions 1971 through 1979, (New York: Moody's Investors Service, Inc.); ISL Daily Stock Price Index, 1971 and 1972, and ISL Daily Stock Price Record, 1973 to 1978, (New York: Investment Statistics Laboratory).

[94] Standard and Poor's Register of Corporations, Directors and Executives, (New York: Standard and Poor's Corporation); and Funk and Scott Index of Corporations and Industries, Annual Editions 1971 through 1979, (Cleveland: Predicasts, Inc.).

basis of industry or asset size. This approach can be criticized from a number of standpoints.

First, as Deakin has noted, such pairing is a violation of the discriminant analysis assumption that the group members are drawn at random from their respective populations.[95] Secondly, there is no assurance that the particular matching criteria selected are effective or even appropriate in the circumstances.[96] Finally, the application of the paired sample design has generally been implemented by choosing a nonfailed firm group constrained by the size of the failed firm group. There is no valid reason for such a restriction on the size of the nonfailed group. Based on these arguments, the paired design will be rejected in this study in favor of a random selection of firms from the nonfailed population.

Selection of Ratios For Study

A third assumption of discriminant analysis is that the individual observations of each group can be described by measurements made on a common set of variables. Prior research is characterized by investigation of a multitude of financial ratios as firm descriptors. Since the potential set of financial ratios is seemingly boundless, it was decided in advance to limit the variables included in this study.

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[95] Deakin, "A Discriminant Analysis," p. 172

[96] John Neter, "Discussion of Financial Ratios as Predictors of Failure," Empirical Research in Accounting: Selected Studies, 1966, Journal of Accounting Research, p. 114.

Those ratios found to be the best predictors in Altman's and Beaver's original research will be selected for investigation. Thus, ratios which have already demonstrated utility in failure prediction in both univariate and multivariate studies will be chosen as the starting point for further analysis. Besides limiting the variable sets to a manageable size, this approach sets a very stringent standard in evaluating the potential contribution of defensive interval measures.

Multivariate Normality

A fourth assumption of discriminant analysis is that the variables selected have a multivariate normal distribution in the respective populations. Deakin[97] and Mecimore[98] have demonstrated that many financial ratios are not distributed normally in the univariate case. However, these results are by no means conclusive since other studies have tended to support normality.[99] It should also be noted that such univariate analyses provide only a preliminary step in evaluating multivariate observations since marginal normality does

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[97] Edward B. Deakin, "Distributions of Financial Accounting Ratios: Some Empirical Evidence," Accounting Review, (January, 1976), pp. 90-96.

[98] Charles D. Mecimore, "Some Empirical Distributions of Financial Ratios," Management Accounting, (September, 1968), pp. 13-16.

[99] see for example, James O. Horrigan, "Some Empirical Bases of Financial Ratio Analysis," Accounting Review, (July, 1965), pp. 558-568; and R.G. Bird and A.J. McHugh, "Financial Ratios--An Empirical Study," Journal of Business Finance and Accounting, (Spring, 1977), pp. 29-45.

not necessarily imply joint normality.[100]

Little is currently known about the robustness of discriminant analysis techniques when faced with departures from normality. Most of the statistical research performed in this area has concentrated on either specified types of nonnormal distributions or on situations in which normality is violated in known ways.[101]

Particular attention has been paid in the literature to the classification of discrete variables. Relevant research has appeared in this area by Revo,[102] Moore,[103] and Gilbert.[104] All of these cited studies evaluated the linear discriminant function relative to alternative classification procedures. Results generally indicated effective performance of the linear function over a wide range of discrete data types. Gilbert concluded that the loss in predictive accuracy from use of the linear rule ". . . for classification as

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[100]R. Gnanadesikan, Methods for Statistical Data Analysis of Multivariate Observations, (New York: John Wiley and Sons, 1977) pp. 162-163.

[101]Robert A. Eisenbeis, "Pitfalls in the Application of Discriminant Analysis in Business, Finance, and Economics," Journal of Finance, (June, 1977), pp. 876-877.

[102]Lawrence T. Revo, "On Classifying With Certain Types of Ordered Qualitative Variates: An Evaluation of Several Procedures," North Carolina Institute of Statistics Mimeo Series, #708, 1970.

[103]D.H. Moore, "Evaluation of Five Discrimination Procedures for Binary Variables," Journal of the American Statistical Association, Vol. 68, 1973, pp. 399-404.

[104]Ethel S. Gilbert, "On Discrimination Using Qualitative Variables," Journal of the American Statistical Association, Vol. 63, 1968, pp. 1399-1412.

opposed to any other procedure is too small to be of much importance."[105]

Much less work has been done in evaluating robustness of the discriminant function when applied to continuous distributions. This is due in large part to the paucity of identified multivariate continuous distributions.[106] Lachenbruch, Sneeringer and Revo[107] investigated linear and quadratic discriminant functions in the context of three specific nonnormal multivariate distributions. In contrast to the earlier studies cited here, results indicated rather severe sensitivity to nonnormality. Also interesting was their finding that transformations of the data, while achieving greater normality, did little to improve classification results.

In the current research, discriminant analysis will play a crucial role in investigating the utility of defensive interval measures. Since this technique relies on the assumption of multivariate normality, and in view of the previous discussion, some further justification for its use is perhaps necessary.

In general, concern over normality should not be carried to such an extreme that analytical approaches are discarded merely because one of the underlying assumptions is not strictly met. What is important is the severity of the assumption violations and

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[105] Ibid., p. 1410

[106] Lachenbruch, Discriminant Analysis, p. 55.

[107] Peter A. Lachenbruch, Cheryl Sneeringer and Lawrence T. Revo, "Robustness of the Linear and Quadratic Discriminant Function to Certain Types of Non-Normality," Communications in Statistics, Vol. 1, 1973, pp. 39-56.

their consequent impact on the validity of the results. Suggestions have been made in the literature that departures from multivariate normality may not have serious adverse effects on the application of MDA.

In this regard, Marriott[108] has discussed the generalization of the Central Limit Theorem to the multivariate case and the resultant importance to robustness considerations. With particular reference to discriminant analysis, Cooley and Lohnes have suggested that:[109]

> . . . by the Central Limit Theorem linear functions of variates are more likely to be normal than are the component variates, [and] multiple-discriminant scores may satisfy the important assumption of a multivariate normal distribution better than the original test scores.

Thus, even if there is some concern about the actual distributions of the underlying populations, the reduction in dimensionality allows for increased confidence in the classification results.

Equality of Dispersion Matrices

A fifth assumption, applicable to linear discriminant analysis, is that the covariance matrices are equal across the groups. When such a property holds, the linear transformation from the original to reduced space preserves the relative Euclidean distances of the

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[108] F.H.C. Marriott, The Interpretation of Multiple Observations, (New York: Academic Press, 1974), pp. 14-15.

[109] Cooley and Lohnes, Multivariate Procedures, p. 116.

observations. However, if this assumption is not met, a "warping" of these observation positions occurs which may seriously affect both group mean significance tests and classification rules.[110] Quadratic discriminant analysis, which drops the homogeneity of dispersion assumption, should be used in such a case.[111] Therefore, in the current research a test of the equality of the group covariance matrices will be performed. On the basis of the test result, the appropriate discriminant analysis technique (i.e., linear or quadratic) will be selected.

A Priori Odds and Misclassification Costs

Two refinements in the application of discriminant analysis have been previously identified: 1) consideration of a priori probabilities of group membership; and 2) consideration of unequal costs of misclassification errors. Much of the research cited in Chapter 2 has ignored these notions and therefore defaulted to a situation of equal costs and equal priors. Joy and Tollefson[112] and Eisenbeis[113] have demonstrated the misleading implications which can arise as a consequence of this default, especially when the priors are asymmetrical. Since neither of these equalities is descriptive of the business failure environment, explicit consideration of these refinements will be made in this study.

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[110] Eisenbeis, "Pitfalls," p. 878.

[111] Eisenbeis and Avery, Discriminant Analysis, pp. 37-38.

[112] O. Maurice Joy and John O. Tollefson, "On the Financial Applications of Discriminant Analysis," Journal of Financial and Quantitative Analysis, (December, 1975), pp. 733-735ff.

[113] Eisenbeis, "Pitfalls," pp. 889-891.

Proper specification of the a priori probabilities of group membership poses some problems. If two populations are pooled and a random sample from this combined set is drawn, it can be shown that use of the resulting group subsample proportions as a priori probability estimates should yield relatively unbiased misclassification errors.[114] However, as previously described, group members in this study are to be determined under a different type of sampling scheme. Thus, another approach to designation of prior odds must be found.

In order to obtain a sufficiently large group of failed firms for study, it will be necessary to pool observations from the selected time period 1972 through 1978. The addition of this time dimension leads to a question of the proper approach to prior odds estimation. Possibilities include: 1) reference to relative population sizes in a given year; 2) calculation of an average relative frequency over the time period; or 3) modification to an atemporal estimate.[115] This last perspective will be adopted for this research and the specific prior odds selection will be explained in the next chapter.

Identification of the costs of errors in classification is also a difficult problem. Although these cost considerations have been discussed widely in the statistical literature, very few attempts at explicit incorporation in an empirical model building context have been made. There are a few notable exceptions.

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[114] Eisenbeis and Avery, Discriminant Analysis, p. 52.

[115] Altman, Haldeman and Narayanan, "ZETA Analysis," pp. 43-44.

Altman[116] has identified certain cost components from the standpoint of the commercial bank loan function. Costs of accepting a loan that subsequently defaults (C_1) and costs of rejecting a loan that would have been paid off (C_2) were specified as follows:

$$C_1 = 1 - \frac{LLR}{GLL} \quad \text{and} \quad C_2 = r - i ,$$

where LLR and GLL represent the amount of loan loss recovered and the gross loan loss, respectively; and r and i represent the effective interest rate on the loan and the effective bank opportunity cost, respectively. Investigation of the loan loss recovery experience of a sample of banks during the period 1971 through 1975 indicated 70% as an estimate for C_1.[117] The estimate of C_2 was arbitrarily chosen as 2%.[118]

Ketz[119] specified the cost components from the standpoint of the outside investor. It was assumed that investment in a firm presumed to be nonfailed, but that eventually failed, would lead to a complete loss of the invested funds. Thus, the cost of this type of error would be 100%. Failure to invest in a company that is mistakenly presumed to be a failure would lead to the opportunity loss

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[116] estimates were derived in Edward I. Altman, "Some Estimates of the Cost of Lending Errors for Commercial Banks," Journal of Commercial Bank Lending, (October, 1977), pp. 51-58, and then incorporated into the model in Altman, "ZETA Analysis," pp. 44-46ff.

[117] Altman, "Cost of Lending Errors," pp. 55-56.

[118] Altman, et. al., "ZETA Analysis," p. 46.

[119] Ketz, "A Comparison," p. 82 and p. 140.

of the incremental return over a risk-free investment. The cost of this error was assumed to be 10%.

Actual specification of misclassification costs is thus dependent upon the choice of the decision maker of interest. In this research, the auditor of financial statements has been chosen as the focal point. Therefore, the cost of errors in classifying firms should be estimated from the auditor's standpoint.

Assigning actual values to each of the two types of errors appears to be an insurmountable task. However, as the classification rule (V) on page 74 indicates, only a measure of the relative costs of the errors is necessary. But even this situation poses some significant problems. Failure by the auditor to identify firms that soon go bankrupt can be seen as a potentially very costly error since perturbed investors may institute numerous law suits as a result. On the other hand, identifying a nonfailed firm as a potential bankrupt may result in the loss of future audit fees as disgruntled clients seek other auditors. More importantly, the failure signal transmitted by the auditor in this case may in fact become a self-fulfilling prophecy, leading to the eventual downfall of the firm. The cost of this error must certainly be high.

Different auditors may form very different assessments of the undesirability of making these two types of errors. Thus, it would be unwise to presume one cost scheme in the analysis. Since not even the relative costs of misclassification can be established with any degree of confidence or consistency, it was decided in this study to construct various models incorporating relative costs from a range of values. The specific cost selections will be described in the next chapter.

Time Factor Considerations

With regard to the actual selection of ratio values, most previous research has concentrated on the year before failure. Operationally, this has involved taking ratio information from the most recent financial statement prior to failure. In practice this may result in selecting measurements of ratios which preceed actual bankruptcy filing by only a few days. In other cases, since there is a time lag between the financial statement date and the actual issuance date, it is possible that the availability of the ratio information would actually follow bankruptcy filing.

If the objective is to be able to evaluate failure potential substantially before the fact, it is clearly improper to make ratio measurements from data so close to the actual filing date. Therefore, in the current study, ratios will be drawn from the financial statements closest to fourteen months prior to the date of filing for bankruptcy. This will effectively limit consideration to statements dated between eight and twenty months prior to bankruptcy. Assuming approximately a two month lag between statement date and issuance, this design will insure that no ratios are calculated from information first available closer than six months prior to filing.

Discriminant Function Evaluation

In this final section of the methodology review, a general description will be given of the means to be employed to evaluate the discriminant models built. Specifics of the actual test procedures applied will be given in the next chapter.

Lachenbruch[120] has suggested three separate avenues to discriminant function assessment: 1) tests of between-group differences; 2) tests of the sufficiency of a subset of variables; and 3) estimation of error rates. Each of these approaches will be employed in this study, with particular emphasis being given to the error rate evaluations.

If a discriminant function is to have any real power to distinguish between groups, then the groups must be different with respect to the predictor variables used to build the function. Therefore, a multivariate test of the group means will be performed to determine if the groups actually differ along the ratio dimensions selected.

A second evaluation involves determining if all of the ratios included in the function are necessary for discrimination. This is essentially a model reduction approach in which the discriminatory sufficiency of a subset of variables is investigated. In this study, since the original variable sets to be selected are so small, model reduction will not be a major objective. Nevertheless, the impact of including defensive interval measures in the original variable sets will be evaluated from this perspective.

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[120] Lachenbruch, Discriminant Analysis, p. 25.

Assessing Error Rates

Although the initial avenue of investigation described in this section is a necessary first step to evaluate the discriminant function, results are not sufficient to indicate that the classification rule adopted will be effective. Even if group means are found to be different, there may still be significant overlap of the groups. Thus, a classification rule may in fact accomplish the goal of minimization of errors, but the size of the error rate may be very large.[121] Therefore, performance evaluation of the discriminant function should concentrate on the associated error rate and not just on group mean differences.

The actual error rate of the sample discriminant function can be calculated with confidence when population parameters are known. However, in the typical case, parameter estimates which are subject to sampling errors are involved and the misclassification rate must be estimated. This poses some significant statistical problems since the sampling distributions of the classification rules are quite complex.[122] Numerous alternatives have been suggested in the literature to estimate classification errors.[123] So far, no one best method has surfaced for discriminant functions developed from parameter estimates. Several of the more common sample-based approaches will now be briefly described.

[121] Maurice G. Kendall and Alan Stuart, The Advanced Theory of Statistics, Vol. 3, (London: Charles Griffin and Co. Ltd., 1966), p. 322.

[122] Eisenbeis and Avery, Discriminant Analysis, pp. 21-22.

[123] for an extensive bibliography on this topic see G.T. Toussaint, "Bibliography on Estimation of Misclassification," IEEE Transactions on Information Theory, 1974.

The Original Sample (or Resubstitution) method was first proposed by Smith.[124] In this approach, observations used to build the discriminant model are categorized on the basis of the classification rules developed. An "apparent error rate" results which represents the proportion of the original observations misclassified by the sample discriminant. This approach has often been criticized in the literature as being a biased estimator which understates the probability of misclassification.[125] However, it appears that this criticism applies only to small or moderate size samples. Fukunaga and Kessell[126] have demonstrated empirically that as the sample size increases, this bias (although still present) is significantly reduced.

Kendall has explained the phenomena at work which can account for this empirical result:[127]

> There are, in practice, two sources of error in this kind of empirical determination of the misclassification error. First of all, we do not know the parent parameters, and our discriminant is based on estimates from the sample. On the average we might expect that for this reason our estimate of error will be greater than the true value. On the other hand, our empirical estimate is derived from data to which it has been fitted. Consequently the empirical estimate will, on the average, be less than it would have been had the discriminator been applied to a new sample. . . .

[124] C.A.B. Smith, "Some Examples of Discrimination," Annals of Eugenics, Vol. 18, 1947, pp. 272-283.

[125] see for example Ronald E. Frank, William F. Massy and Donald G. Morrison, "Bias in Multiple Discriminant Analysis," Journal of Marketing Research, (August, 1965), pp. 250-258.

[126] Keinosuke Fukunaga and David L. Kessell, "Estimation of Classification Error," IEEE Transactions on Computers, (December, 1971), pp. 1521-1527.

[127] Kendall, Multivariate Analysis, p. 160.

Apparently, as the sample size is increased, these over and under estimation effects are diluted, resulting in a convergence on the true error rate.

The Holdout (or Split-Sample) method involves a partitioning of the original sample into two or more parts. One subsample is employed to develop the discriminant function which is then used to classify the members of the other subsample(s). Error rate estimations are made on the basis of the subsample classifications.

Although widely used in practice, this method suffers from several drawbacks:[128] 1) large samples are required that may not be available; 2) the discriminant function evaluated (based on the subsample) is not the same as the function actually used in classification (based on the complete sample), and thus the error rate estimates are not generalizable to the performance of the final classification rules developed;[129] 3) estimates of error rates will likely vary based upon the size of the holdout sample selected; and 4) the data are not economically used, since a sample must be drawn just to provide error estimates that is often larger than necessary to provide an effective discriminant function. Empirical results also indicate that this method is not clearly superior to the other available approaches.[130]

[128] Peter A. Lachenbruch and M. Ray Mickey, "Estimation of Error Rates in Discriminant Analysis," Technometrics, (February, 1968), pp. 2-3.

[129] G.J. McLachlan, "Confidence Intervals for the Conditional Probability of Misallocation in Discriminant Analysis," Biometrics, (March, 1975), p. 162

[130] Robert A. Eisenbeis, "Problems in Applying Discriminant Analysis in Credit Scoring Models," Journal of Banking and Finance, (October, 1978), p. 215.

The Leaving-One-Out method, originally proposed by Lachenbruch,[131] combines features of the previous two approaches. In this method, the discriminant function is developed on the basis of all but one observation, and that omitted observation is then classified. The process is replicated, holding out a different observation each time, and misclassifications are tallied. The result is an almost unbiased estimate of the probability of misclassification, for which confidence intervals may be constructed.[132]

It has been demonstrated empirically that both the Smith and Lachenbruch methods of error estimation perform well when extended to quadratic discriminant analysis.[133] Evidence seems to indicate preference for the Leaving-One-Out method when small samples are involved. However, as Lachenbruch suggests;[134]

> When large samples are available, it makes no sense to try the fancier estimates, as the bias has already been reduced to a very low level.

Since large samples will be obtained in this research, the Smith method of error estimation will therefore be utilized.

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[131] Peter A. Lachenbruch, "Estimation of Error Rates in Discriminant Analysis," (unpublished Ph.D. dissertation, University of California at Los Angeles, 1965).

[132] Peter A. Lachenbruch, "An Almost Unbiased Method of Obtaining Confidence Intervals for the Probability of Misclassification in Discriminant Analysis," Biometrics, (December, 1967), pp. 639-645.

[133] Fukunaga and Kessel, "Classification Error," pp. 1521-1527.

[134] Lachenbruch, Discriminant Analysis, p. 35.

CHAPTER 5

RESULTS OF THE DISCRIMINANT ANALYSES

The previous chapter has outlined in some detail the methodology to be employed in evaluating the contribution of defensive interval measures to business failure prediction. This chapter will report the results of the ex post study undertaken.

Identification of Group Membership

The failed firm search process previously described in Chapter 4 (page 78) led to identification of sixty-one AMEX and NYSE companies which failed during the period 1972 to 1978, inclusive. These firms are listed alphabetically in Table 10, along with their respective Standard Industrial Classification code, stock exchange affiliation, date of bankruptcy petition filing, and financial statement date drawn for analysis.

Evidence of actual petition filings could only be found for fifty-nine of these companies. Two other firms (Servotronics and Western Orbis) had publicly reported the intention to institute bankruptcy proceedings and investigation of 10-K reports filed with the Securities and Exchange Commission indicated that board of director approval had been given for such action. Surrogate filing dates were chosen for these two firms which correspond to the date that the American Stock Exchange halted trading in their securities. These two substitute dates have been denoted by a " * " in Table 10.

Except for Sequoyah Industries, all of the identified bankruptcy petitions were originally filed under Chapter XI. The

Table 10

FAILED FIRM GROUP MEMBERS

Company Name	SIC No.	Exchange	Date of Filing	F.S. Date Drawn
Aerodex Inc.	3720	AMEX	6/09/76	12/31/74
Alan Wood Steel	3310	AMEX	6/13/77	12/31/75
Allied Supermarkets Inc.	5411	NYSE	11/07/78	6/25/77
American Book-Stratford Press Inc.	2731	AMEX	6/05/73	12/31/71
American Girl Fashions Inc.	3140	AMEX	2/05/75	11/24/73
American Kitchen Foods Inc.	2000	AMEX	10/17/75	8/02/74
American Training Services	8200	AMEX	5/17/76	2/28/75
Ancorp National Services Inc.	5812	NYSE	3/20/73	12/31/71
Arlan's Department Stores Inc.	5331	NYSE	5/14/73	1/29/72
Armac Enterprises Inc.	3940	AMEX	5/21/76	12/31/74
Associated Food Stores Inc.	5411	AMEX	6/16/75	7/27/74
Bohack Corp.	5411	AMEX	7/31/74	1/27/73
Botany Industries Inc.	2300	AMEX	4/25/72	7/31/71
Bowmar Instrument Corp.	3662	AMEX	2/10/75	9/30/73
Coit International Inc.	5949	AMEX	8/28/75	5/31/74
Commodore Corp.	2450	AMEX	8/09/74	6/30/73
Commonwealth Oil Refining Co.	2911	NYSE	3/02/78	12/31/76
DCA Development Corp.	3250	AMEX	2/06/73	12/31/71
Drew National Corp.	5712	AMEX	8/12/75	8/31/74
Dynamics Corp. of America	3662	NYSE	8/02/72	12/31/70
Electrospace Corp.	3662	AMEX	4/29/74	12/31/72
Ernst (E.C.), Inc.	1700	AMEX	12/04/78	3/31/78
Esgro Inc.	5063	AMEX	3/19/73	4/30/72
FAS International Inc.	8200	NYSE	2/08/72	9/30/70
Federal's Inc.	5311	NYSE	8/16/72	7/31/71
Fishman (M.H.) Co.	5331	AMEX	12/27/74	12/31/73
Food Fair Inc.	5411	NYSE	10/02/78	7/30/77
Frigitemp Corp.	3580	AMEX	3/20/78	12/31/76
Giant Stores Corp.	5331	AMEX	8/17/73	1/29/72
Goodway Inc.	2731	AMEX	10/04/73	2/29/72
Grant (W.T.) Co.	5311	NYSE	10/03/75	1/30/75

TABLE 10
(Continued)
FAILED FIRM GROUP MEMBERS

Company Name	SIC No.	Exchange	Date of Filing	F.S. Date Drawn
Gruen Industries Inc.	3870	AMEX	4/15/77	3/31/76
Hartfield-Zodys Inc.	5331	AMEX	11/15/74	2/01/74
Harvard Industries	3679	AMEX	11/20/72	9/30/71
Interstate Stores Inc.	5311	NYSE	5/22/74	1/28/73
Leader International Industries	9997	AMEX	11/30/73	12/31/72
Mammoth Mart Inc.	5331	AMEX	6/14/74	2/03/73
Mangel Stores Corp.	5211	AMEX	3/04/74	2/03/73
Maule Industries Inc.	3270	AMEX	7/23/76	12/31/74
Old Town Corp.	3950	AMEX	6/05/73	12/31/71
Paterson Parchment Paper Co.	2600	AMEX	12/23/74	12/31/73
Penn Fruit Co.	5411	NYSE	9/04/75	8/31/74
Permaneer Corporation	2400	AMEX	6/29/76	11/01/75
Plaza Group Inc.	5961	AMEX	12/10/74	12/31/73
Potter Instrument Co.	3573	AMEX	4/21/75	6/30/74
RAI Inc.	2300	AMEX	2/16/73	4/30/72
R.E.D.M. Corp.	3610	AMEX	3/20/72	12/31/70
Rosenau Brothers Inc.	2300	AMEX	2/27/74	12/31/72
Scottex Corp.	2200	AMEX	1/16/74	12/31/72
Sequoyah Industries Inc.	2270	AMEX	1/24/74	4/28/73
Servotronics	3679	AMEX	4/23/76*	12/31/74
Sitkin Smelting & Refining Inc.	3341	AMEX	3/14/78	6/30/77
Stelber Industries Inc.	3750	AMEX	3/12/76	6/30/75
Stratton Group Ltd.	5331	AMEX	9/30/73	12/30/72
Supronics Corp.	2844	AMEX	10/26/76	8/31/75
Tennessee Forging Steel	3310	AMEX	12/05/77	6/30/76
UDS Inc.	5211	AMEX	11/30/76	1/31/76
Volume Merchandise Inc.	5331	AMEX	7/23/73	1/31/72
Western Orbis	2450	AMEX	9/19/74*	6/30/73
Willcox and Gibbs Inc.	5080	AMEX	11/19/76	12/31/75
Winston Mills Inc.	2200	AMEX	5/22/78	1/01/77

voluntary proceedings under this section of the Chandler Act are generally less restrictive than those under Chapter X, allowing for continued management control and a scaling-down of unsecured creditor claims.[135] Sequoyah originally filed under Chapter X and Interstate Stores filed under Chapter XI but proceedings were later transferred to Chapter X coverage. The "date of filing" reported in Table 10 represents the date the original bankruptcy petition was initiated.

With regard to stock exchange affiliation, fifty of the sixty-one firms (82%) were traded on the AMEX. More than sixty-one NYSE and AMEX companies were originally identified in the search process as potential failed firm group members. Further investigation revealed that some of these firms had been delisted from their respective stock exchanges several years before bankruptcy petitions were actually filed, and their securities were no longer being traded. Since one of the variable sets investigated in this study incorporates market value of stock information, these few firms had to be excluded from consideration. The rationale for this decision is that the actual filing for bankruptcy was ancillary to failure signals conveyed years earlier.

As described in Chapter 3, the nonfailed group was randomly selected from a listing of NYSE and AMEX firms as of December, 1978. This group is composed of 608 companies.

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[135]For a more complete description, see Altman, Corporate Bankruptcy in America, pp. 6-12.

Selection of Ratios For Study

Two primary sets of ratios were chosen for investigation in this study. Variable set #1 contains those ratios found by Beaver to be the best failure predictors in a univariate setting.[136] Variable set #2 contains the components of Altman's multivariate prediction model.[137] Ratio definitions for these two groups are given in Table 11. These ratio sets serve as the benchmarks for evaluating defensive interval contributions to failure prediction. Discriminant models were developed for each of these variable groups and results were then compared with the performance attained by adding defensive interval measures.

Ratio calculations for the failed firm group were derived from financial statements closest to fourteen months prior to the actual date of bankruptcy filing.[138] The market value of equity securities was determined by reference to stock prices on the day closest to the financial statement date selected.

The respective financial statement dates of the failed firms are reported in the last column of Table 10. Analysis of the time lags between statement and filing dates indicated a rather even distribution over the range of 8 to 20 months, with a mean of 14 and median of 13.6 months.

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[136] Beaver, "Predictors of Failure," 1966.

[137] Altman, "Prediction of Corporate Bankruptcy," 1968.

[138] information was compiled from Moody's Industrial Manual, Annual Reports, and 10-K's filed with the Securities and Exchange Commission.

TABLE 11

ORIGINAL RATIO SETS INVESTIGATED

Variable Set #1 -- Beaver's Best Predictors:		
CF/TOTD	=	$\frac{\text{Cash Flow}}{\text{Total Debt}}$
NI/TOTA	=	$\frac{\text{Net Income}}{\text{Total Assets}}$
LIAB/TOTA	=	$\frac{\text{Current + Long Term Liabilities}}{\text{Total Assets}}$
WC/TOTA	=	$\frac{\text{Current Assets - Current Liabilities}}{\text{Total Assets}}$
CA/CL	=	$\frac{\text{Current Assets}}{\text{Current Liabilities}}$
Variable Set #2 -- Altman's Best Predictors:		
WC/TOTA	=	$\frac{\text{Current Assets - Current Liabilities}}{\text{Total Assets}}$
RE/TOTA	=	$\frac{\text{Retained Earnings}}{\text{Total Assets}}$
EBIT/TOTA	=	$\frac{\text{Earnings Before Interest and Taxes}}{\text{Total Assets}}$
MVE/BVD	=	$\frac{\text{Market Value of Equity}}{\text{Book Value of Debt}}$
SAL/TOTA	=	$\frac{\text{Sales}}{\text{Total Assets}}$

Since the failed firms were selected over a range of years, changing economy-wide influences may affect a particular ratio's values. To alleviate the confounding effects of these environmental factors, ratios of the nonfailed firms were calculated from comparable statement dates. This was accomplished by partitioning the nonfailed group into seven time-based subsamples whose sizes were derived from the relative frequencies of the financial statement dates drawn for the failed group. Results of this partitioning scheme are described in Table 12.

Ratio calculations for these nonfailed group subsamples were performed by utilizing the Financial Analysis System.[139] This is a software package which is designed to access financial data on the COMPUSTAT Industrial Tapes.

A Priori Odds and Costs of Misclassification

No failure rate estimates are available which specifically apply to the universe of firms identified in this research. Dun and Bradstreet, Inc.[140] has been collecting comprehensive business failure statistics for over 100 years and recent estimates indicate an average failure rate of between .4% and .5% per year, although individual yearly rates vary. However, these statistics are based upon a business enterprise population in excess of 2.6 million companies.

[139] Carl E. Ferguson, Jr., and Warren G. Glimpse, FAS User's Guide, Management System Developments, Inc., May, 1978. Version 4.0 of the Financial Analysis System was employed.

[140] Business Failure Record, Business Economics Division, (New York: Dun and Bradstreet, Inc., 1976).

TABLE 12

PARTITIONING THE NONFAILED GROUP IN ORDER TO DRAW COMPARABLE TIME-BASED RATIOS

Distribution of Failed Firms' Financial Statement Dates Selected For Study	Number of Failed Firms	Size of Nonfailed Partitions
1970		
1971	10	100
1972	11	110
1973	12	120
1974	11	109
1975	7	70
1976	5	49
1977	5	50
1978		
	$\Sigma = 61$	$\Sigma = 608$

Table 13 reports the distribution by year of the failed firms selected in this study. Similar to the findings of other research in the area of bankruptcy, there is not a stationary yearly failure rate. Selection of the frequency of failure in one year as the a priori probability estimate would therefore yield inconsistent results, dependent upon the actual year arbitrarily chosen. Altman has suggested that the a prior probability of failure should be viewed in an atemporal sense without regard to a given year's reported statistics. Although precision is impossible, he estimates the true a priori odds to be somewhere in the range of 2% to 5%.[141]

Analysis of the December, 1978 listing of COMPUSTAT firms meeting the exchange and industry requirements previously specified revealed a total nonfailed population of 1813 firms. Viewing the 1972 to 1978 period as one time unit and assuming a relatively constant size of the nonfailed population, the universe of firms is determined to be made up of 1874 companies (i.e., 1813 nonfailed and 61 failed firms). The failure frequency observed in this universe is therefore 3.3%.

In future analyses, the probability of group membership in the failed population will be assumed to be 3.3% and conversely, the probability of nonfailure status will be valued at 96.7%. This scheme perhaps overestimates the true failure probability and is offered only as an approximate measurement approach. However, in

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[141] Altman, et. al., "ZETA Analysis," p. 44.

TABLE 13

DISTRIBUTION OF FAILURES BY YEAR

Year of Failure	No. of Failed Firms
1972	6
1973	12
1974	14
1975	9
1976	10
1977	3
1978	7
Total	61

view of Altman's previous "guesstimates," 3.3% appears reasonable. At the very least, this approach is an improvement over merely assuming equal a priori odds of group membership.

As previously discussed in Chapter 4, estimation of the costs of misclassification of firms from the auditor's standpoint cannot easily be ascertained with any degree of confidence in even a relative sense. Therefore, the investigations which were conducted presumed a range of relative costs.

Recall from the discriminant analysis discussion in Chapter 4 that the assignment rule incorporates the relative costs of making two different types of errors: 1) classifying a nonfailed firm as a failed firm $\{ C(1|0)\ (> 0) \}$; and 2) classifying a failed firm as a nonfailed firm $\{ C(0|1)\ (> 0) \}$. The ratios of these two costs chosen for analysis included:

$$\frac{C(1|0)}{C(0|1)} = \text{a) } 3\ ; \quad \text{b) } 1\ ; \quad \text{c) } 1/3\ ; \quad \text{d) } 1/30\ .$$

These relationships represent the alternative situations in which: a) it is three time worse (i.e., more costly) to misclassify nonfailed firms; b) the costs of misclassifying nonfailed and failed firms are equal; c) it is three times worse to misclassify failed firms; and d) it is thirty times worse to misclassify failed firms. This last cost relation approximates the implicit cost considerations to which Altman defaulted in his early prediction studies.

Tests of Between-Group Differences

Before the actual discriminant models were built, several preliminary analyses were conducted. The first of these involved testing for group differences with respect to the ratios selected for study.

A multivariate test of the equality of group means can be performed in order to investigate the following hypotheses:[142]

$$H_0: \quad \mu_0 = \mu_1 \qquad \text{when } \Sigma \text{ is known}$$

$$H_1: \quad \mu_0 \neq \mu_1$$

where μ_0 represents the vector of means of the nonfailed firm ratios,

μ_1 represents the vector of means of the failed firm ratios, and

Σ represents the true variance-covariance matrix of the failed firm ratios.

Since the failed firm group members comprise a population and not a sample, μ_1 and Σ are known. Only μ_0 must be estimated ($\overline{X}_0$) by drawing a random sample from the nonfailed firm population.

Significance may be tested by reference to Hotelling's T^2 distribution with the critical value:[143]

$$T^2_{(p\,,\,\infty)}(\alpha) \quad = \quad n\,D^2$$

where D^2 represents the Mahalanobis distance defined as $(\overline{X}_0 - \mu_1)'\,\Sigma^{-1}\,(\overline{X}_0 - \mu_1)$.

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[142] Clyde Y. Kramer, A First Course in Methods of Multivariate Analysis, (Blacksburg, Virginia: By the Author, 1972) pp. 36-40.

[143] Ibid., p. 37 and p. 62.

Both the Beaver and Altman variable sets were evaluated using this procedure. Individual components used to develop the test statistics are reported in Tables 14 and 15, along with selected critical values from Hotelling's T^2 distribution.[144] The null hypothesis is strongly rejected in both cases with alpha levels attained < .0001 . Thus, the failed and nonfailed groups do differ with respect to these ratio groups.

Simultaneous Confidence Intervals

A second type of analysis was then performed which is actually an extension of the test procedure just described. A combined variable set was created which included Beaver's and Altman's best failure predictors as well as the six defensive interval measures previously described in Chapter 3. A test of the equality of group means was then performed on this large ratio set. Results indicated a D^2 value of 278.84 . The test statistic of 169531.96 was significant at $\alpha = < .0001$, implying strong rejection of the hypothesis of equality of group means.

This last test procedure, while rejecting the null hypothesis, does not necessarily imply that all mean components of the combined ratio set are different. Thus, one could not presume that the groups actually differ with respect to the defensive interval measures. The appropriateness of adding these ratios to the variable sets to be used in subsequent discriminant analyses would thus be subject

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[144] critical values obtained from D.R. Jensen and R.B. Howe, "Upper Percentage Points of Hotelling's T^2 Distribution," March, 1968, Appendix A in Kramer, Multivariate Analysis.

TABLE 14

COMPONENTS OF THE BEAVER RATIO SET GROUP MEANS TEST

Ratio	$\bar{X}_o$	μ_1	$d = \bar{X}_o - \mu_1$
CF/TOTD	.2455288	-.0418771	.2874059
NI/TOTA	.0554233	-.0890590	.1444823
LIAB/TOTA	.4886616	.7980902	-.3094286
WC/TOTA	.3049997	.1321951	.1728046
CA/CL	2.4407930	1.4422787	.9985143

True Covariance Matrix of Failed Firm Group

	CF/TOTD	NI/TOTA	LIAB/TOTA	WC/TOTA	CA/CL
CF/TOTD	.016034	.022325	-.021684	.021673	.015405
NI/TOTA	.022325	.043110	-.045080	.052498	.042179
LIAB/TOTA	-.021684	-.045080	.071328	-.070416	-.088633
WC/TOTA	.021673	.052498	-.070416	.101444	.141903
CA/CL	.015405	.042179	-.088633	.141903	.345639

Inverse of True Covariance Matrix

	CF/TOTD	NI/TOTA	LIAB/TOTA	WC/TOTA	CA/CL
CF/TOTD	321.255	-273.537	1.003	109.449	-25.615
NI/TOTA	-273.537	369.202	40.083	-171.970	48.018
LIAB/TOTA	1.003	40.083	58.195	12.845	4.714
WC/TOTA	109.449	-171.970	12.845	134.473	-35.807
CA/CL	-25.615	48.018	4.714	-35.807	14.084

Test Statistic and Critical Values

$D^2 = 22.366509$ $\qquad$ $T^2_{(5,\infty)}(\alpha=.05) = 11.070$

$nD^2 = 13598.837$ $\qquad$ $T^2_{(5,\infty)}(\alpha=.01) = 15.086$

TABLE 15

COMPONENTS OF THE ALTMAN RATIO SET GROUP MEANS TEST

Ratio	$\overline{X}_o$	μ_1	$d = \overline{X}_o - \mu_1$
WC/TOTA	.3049997	.1321951	.1728046
RE/TOTA	.2920446	-.0328410	.3248856
EBIT/TOTA	.1335583	-.0539475	.1875058
MVE/BVD	16.5040620	.7675531	15.7365090
SAL/TOTA	1.5407640	2.0713689	-.5306049

True Covariance Matrix of Failed Firm Group

	WC/TOTA	RE/TOTA	EBIT/TOTA	MVE/BVD	SAL/TOTA
WC/TOTA	.101444	.089059	.026970	.036142	-.020075
RE/TOTA	.089059	.123899	.028525	.058195	-.029901
EBIT/TOTA	.026970	.028525	.019290	.007236	.015784
MVE/BVD	.036142	.058195	.007236	1.453990	-.070199
SAL/TOTA	-.020075	-.029901	.015784	-.070199	2.152340

Inverse of True Covariance Matrix

	WC/TOTA	RE/TOTA	EBIT/TOTA	MVE/BVD	SAL/TOTA
WC/TOTA	29.3363	-17.5949	-15.1362	.0572	.1420
RE/TOTA	-17.5949	23.2177	-9.7466	-.4330	.2158
EBIT/TOTA	-15.1362	-9.7466	88.0567	.2840	-.9131
MVE/BVD	.0572	-.4330	.2840	.7030	.0154
SAL/TOTA	.1420	.2158	-.9131	.0154	.4761

Test Statistic and Critical Values

$D^2 = 173.8865$	$T^2_{(5,\infty)}(\alpha=.05) = 11.070$
$nD^2 = 105722.95$	$T^2_{(5,\infty)}(\alpha=.01) = 15.086$

to question.

Kramer[145] has suggested employing simultaneous confidence intervals in order to determine which of the variables actually contribute to the rejection of the null hypothesis (i.e., to determine which of the ratios actually differ between the groups). A general expression for the desired 1 - α confidence intervals is given by:[146]

$$c\,\bar{X}_o' - \sqrt{\frac{1}{n}\,c\,\Sigma\,c'}\cdot\left[T_{(p\,,\,\infty)}(\alpha)\right] \leq c\,\mu_o \leq c\,\bar{X}_o' + \sqrt{\frac{1}{n}\,c\,\Sigma\,c'}\cdot\left[T_{(p\,,\,\infty)}(\alpha)\right]$$

where c represents a row vector of zeros, except for a 1 positioned to pick up the particular ratio for which the confidence interval is being constructed, and

$T_{(p\,,\,\infty)}(\alpha)$ is the positive square root of $T^2_{(p\,,\,\infty)}(\alpha)$.

The last three columns of Table 16 report the estimates of μ_o and the lower and upper limits of the 99% confidence intervals. The first column of this table contains the means of the failed firm ratios (μ_1). For a given ratio, if the failed firm mean falls within the confidence interval of the nonfailed firm mean, that variable did not aid rejection of the null hypothesis. Since none of the μ_1 values are contained within the respective confidence interval ranges, all of the listed variables contributed to rejection and one may conclude that the failed and nonfailed groups differ with respect to each of the ratios. These results provide the statistical justification for adding the defensive interval measures to the two basic variable sets

[145] Kramer, Multivariate Analysis, pp. 56-62.

[146] Ibid., p. 62.

TABLE 16

MEANS OF THE FAILED FIRM RATIOS AND CONFIDENCE INTERVALS FOR THE MEANS OF THE NONFAILED GROUP RATIOS

Ratio	μ_1	$\overline{X}_o$	$CI_o^{(LL)}$	$CI_o^{(UL)}$
CF/TOTD	-.0419	.2455	.2171	.2739
NI/TOTA	-.0891	.0554	.0089	.1020
LIAB/TOTA	.7981	.4887	.4288	.5486
WC/TOTA	.1322	.3050	.2336	.3764
CA/CL	1.4423	2.4408	2.3089	2.5726
RE/TOTA	-.0328	.2920	.2131	.3710
EBIT/TOTA	-.0539	.1336	.1024	.1647
MVE/BVD	.7676	16.5041	16.2336	16.7745
SAL/TOTA	2.0714	1.5408	1.2118	1.8698
DIB1	67.0299	108.7745	96.5822	120.9668
DIB2	65.2932	99.2975	87.5759	111.0190
DIC1	8.7196	33.9190	32.2994	35.5387
DIC2	8.5361	30.6294	29.0606	32.1982
DCR1	-56.4263	11.0383	-2.2492	24.3259
DCR2	-54.1152	12.2830	.1420	24.4239

in the discriminant analyses which were performed.

Equality of Dispersion Matrices

A third preliminary step was undertaken in order to select the appropriate MDA technique (i.e., linear or quadratic). Choice of the proper modeling approach depends upon the equality of the underlying group dispersion matrices. To investigate the existence of this equality, the following hypotheses are formulated:

$$H_0: \quad \Sigma_0 = \Sigma_1$$

$$H_1: \quad \Sigma_0 \neq \Sigma_1$$

where Σ_i, $i=0,1$ represents the true covariance matrix of the non-failed and failed firm group ratios, respectively.

To test the homogeneity of the within group covariance matrices for two groups, the following test criterion is developed:[147]

$$V = \frac{\prod_{i=0}^{1} |\text{Within SS Matrix}_i|^{\frac{1}{2}n_i}}{|\text{Pooled SS Matrix}|^{\frac{1}{2}N}}$$

where n_i and N represent the number of observations in the i^{th} group and the total number of observations, respectively. Kendall and Stuart have suggested that under the null hypothesis the following expression is distributed approximately as χ^2 with degrees of freedom equal to $\frac{1}{2}\,p(p+1)$ for the two group case:[148]

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[147] Kendall and Stuart, Theory of Statistics, p. 266.

[148] Ibid., p. 282.

$$- 2 \text{ rho } \ln \left[\frac{N^{\frac{1}{2}pN} \cdot V}{\prod_{i=0}^{1} n_i^{\frac{1}{2}pn_i}} \right]$$

where $\text{rho} = 1 - \left[\sum_{0}^{1} \left(\frac{1}{n_i - 1} - \frac{1}{N-2} \right) \right] \cdot \frac{2p^2 + 3p - 1}{6(p+1)}$

Tests were separately performed for the Beaver ratio set, the Altman ratio set, and the combined ratio set which included all of the defensive interval measures. Resulting chi-square test statistics for these three variable sets were 592.87 (df=15), 879.25 (df=15), and 2675.78 (df=120), respectively. Critical values of the χ^2 distribution at α = .01 are 30.578 (for 15 degrees of freedom) and 158.95 (for 120 degrees of freedom).[149] The null hypothesis of homogeneity of group dispersion matrices was strongly rejected (α = < .0001) in all cases. Because of these results, quadratic rather than linear discriminant analyses were performed on the various ratio sets.

A number of other variable sets, to be described later, were also evaluated using this test procedure. Results in all cases studied indicated strong rejection of the null hypothesis of equal dispersion matrices. Therefore quadratic MDA was employed throughout this research.

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[149]critical values obtained from G.B. Beus and D.R. Jensen, "Upper Percentage Points of the Bonferroni Chi-Square Statistics," September, 1967, Appendix D in Kramer, Multivariate Analysis.

Quadratic Discriminant Analysis

The basic hypothesis investigated in this section is:

> H_o: The ability to discriminate between failed and nonfailed firms does not improve if one incorporates the use of defensive interval measures as liquidity indicators

The Beaver and Altman "best predictor" variable sets have been previously identified as the two standards against which the ratio sets incorporating defensive interval measures are to be compared. Therefore, the first series of discriminant analyses were performed to establish the classification effectiveness of the separate Beaver and Altman standards. The Statistical Analysis System (SAS) was employed to develop quadratic MDA classification rules incorporating the a priori odds and costs of misclassification previously described.

Classification results for each of the two standard variable sets are reported in Table 17. The elements of this table are arranged for each of the four possible alternative cost of misclassification assumptions according to the scheme which follows:

		PREDICTED STATUS:	
		Nonfailed	Failed
ACTUAL STATUS:	Nonfailed		
	Failed		

Each cell in this arrangement represents, per actual status, the number of firms classified as nonfailed or failed. The northeast and southwest cells thus represent the nonfailed and failed firm misclassifications, respectively.

TABLE 17

QUADRATIC MDA CLASSIFICATION RESULTS:
BEAVER AND ALTMAN RATIO SETS

	3x Worse Misclass. N-F Firm		Equal Cost of Misclass.		3x Worse Misclass. F Firm		30x Worse Misclass. F Firm	
Variable Set #1: Beaver	598	10	593	15	583	25	497	111
	34	27	29	32	24	37	7	54
Variable Set #2: Altman	582	26	556	52	462	146	223	385
	19	42	9	52	4	57	1	60

Table 18 converts these misclassification cells to relative proportions based upon the total number of observations in each actual status group. In addition, this table reports the overall error rates calculated by the Smith method described in Chapter 4.

As one moves from left to right in Tables 17 and 18, the misclassification of failed firms is presumed to be more and more costly. Note that for both variable sets the number of misclassified failed firms and the resulting proportion of failed firms improperly categorized (reported in the southwest corner cells) drops as the cost of making this type of error increases. However, as the northeast cells in these tables reflect, this results in an increasing number of nonfailed firms being misclassified. Thus, there is a tradeoff involved in the classification rules developed. To correctly categorize more failed firms, assignment regions must be altered, but this consequently results in more nonfailed firm misclassifications. Converse statements can be made from the standpoint of correctly classifying nonfailed firms, as one moves from right to left in these tables.

Based on total error rates, the Beaver variable set outranks the Altman ratios, regardless of the relative costs of misclassification assumed. Beaver's ratios also consistently provide better categorizations of the nonfailed firms. However, the Altman set consistently outperforms the Beaver set with regard to correctly classifying failed firms.

TABLE 18

QUADRATIC MDA ERROR RATES:
BEAVER AND ALTMAN RATIO SETS

	3x Worse Misclass. N-F Firm		Equal Cost of Misclass.		3x Worse Misclass. F Firm		30x Worse Misclass. F Firm	
Variable Set #1: Beaver		.0164		.0247		.0411		.1826
	.5574		.4754		.3934		.1148	
Total Error Rate≈	6.58%		6.58%		7.32%		17.64%	
Variable Set #2: Altman		.0428		.0855		.2401		.6332
	.3115		.1475		.0656		.0164	
Total Error Rate≈	6.73%		9.12%		22.42%		57.70%	

Modification of the Original Variable Sets

Once the classification results and error rates of the standard variable sets had been established, attention was directed to appraising the effects of incorporating the defensive interval measures. Each of the six defensive intervals was added, one at a time, to the original Beaver "best predictor" set. Then, groups of two defensive intervals were added. Since there are two alternative measurement approaches to the calculation of the projected daily operating expenditures, these groups of two were selected so that measurement bases were not mixed. Finally, groups of three defensive intervals were added, again avoiding mixtures of the measurement bases.

Quadratic MDA models were built for each of the resulting variable sets, under each of the four costs of misclassification assumptions. A priori odds of group membership were also incorporated into the models. Classification results are reported in Tables 19, 20, and 21. Misclassification error rates for nonfailed and failed firms (as well as overall error rates) for these variable sets are reported in Tables 22, 23, and 24. Performance results of the original Beaver ratio group are reproduced at the top of each table in order to aid comparisons.

This entire variable group modification process was then replicated for the original Altman "best predictor" set. Classification results are reported in Tables 25, 26 and 27. Individual and total error rates are summarized in Tables 28, 29, and 30.

TABLE 19

QUADRATIC MDA CLASSIFICATION RESULTS:
BEAVER RATIO SET PLUS DEFENSIVE INTERVALS ADDED ONE AT A TIME

Variable Set	3x Worse Misclass. N-F Firm		Equal Cost of Misclass.		3x Worse Misclass. F Firm		30x Worse Misclass. F Firm	
STANDARD: 5 Beaver Ratios	598	10	593	15	583	25	497	111
	34	27	29	32	24	37	7	54
5 Beaver Ratios + DIB1	598	10	591	17	582	26	486	122
	30	31	27	34	21	40	5	56
5 Beaver Ratios + DIB2	598	10	593	15	583	25	492	116
	30	31	29	32	22	39	5	56
5 Beaver Ratios + DIC1	591	17	583	25	545	63	454	154
	26	35	17	44	11	50	3	58
5 Beaver Ratios + DIC2	591	17	585	23	548	60	461	147
	27	34	17	44	13	48	3	58
5 Beaver Ratios + DCR1	599	9	595	13	582	26	492	116
	33	28	27	34	21	40	3	58
5 Beaver Ratios + DCR2	598	10	594	14	583	25	498	110
	34	27	29	32	22	39	4	57

TABLE 20

QUADRATIC MDA CLASSIFICATION RESULTS:
BEAVER RATIO SET PLUS DEFENSIVE INTERVALS ADDED TWO AT A TIME

Variable Set	3x Worse Misclass. N-F Firm		Equal Cost of Misclass.		3x Worse Misclass. F Firm		30x Worse Misclass. F Firm	
STANDARD: 5 Beaver Ratios	598	10	593	15	583	25	497	111
	34	27	29	32	24	37	7	54
5 Beaver Ratios + DIB1 & DIC1	588	20	574	34	534	74	454	154
	22	39	16	45	11	50	1	60
5 Beaver Ratios + DIB2 & DIC2	590	18	583	25	542	66	469	139
	25	36	17	44	10	51	3	58
5 Beaver Ratios + DIB1 & DCR1	596	12	588	20	571	37	476	132
	31	30	26	35	18	43	5	56
5 Beaver Ratios + DIB2 & DCR2	595	13	587	21	577	31	495	113
	31	30	27	34	18	43	6	55
5 Beaver Ratios + DIC1 & DCR1	590	18	573	35	538	70	450	158
	23	38	17	44	9	52	2	59
5 Beaver Ratios + DIC2 & DCR2	592	16	581	27	552	56	471	137
	24	37	18	43	12	49	3	58

TABLE 21

QUADRATIC MDA CLASSIFICATION RESULTS:
BEAVER RATIO SET PLUS DEFENSIVE INTERVALS ADDED THREE AT A TIME

Variable Set	3x Worse Misclass. N-F Firm	Equal Cost of Misclass.	3x Worse Misclass. F Firm	30x Worse Misclass. F Firm
STANDARD: 5 Beaver Ratios	598 | 10 34 | 27	593 | 15 29 | 32	583 | 25 24 | 37	497 | 111 7 | 54
5 Beaver Ratios + DIB1 & DIC1 & DCR1	589 | 19 21 | 40	561 | 47 13 | 48	531 | 77 10 | 51	450 | 158 2 | 59
5 Beaver Ratios + DIB2 & DIC2 & DCR2	589 | 19 23 | 38	579 | 29 18 | 43	545 | 63 11 | 50	478 | 130 4 | 57

TABLE 22

QUADRATIC MDA ERROR RATES:
BEAVER RATIO SET PLUS DEFENSIVE INTERVALS ADDED ONE AT A TIME

Variable Set	3x Worse Misclass. N-F Firm	Equal Cost of Misclass.	3x Worse Misclass. F Firm	30x Worse Misclass. F Firm
STANDARD: 5 Beaver Ratios	.0164	.0247	.0411	.1826
	.5574	.4754	.3934	.1148
Total Error Rate≈	6.58%	6.58%	7.32%	17.64%
5 Beaver Ratios + DIB1	.0164	.0280	.0428	.2007
	.4918	.4426	.3443	.0820
Total Error Rate≈	5.98%	6.58%	7.03%	18.98%
5 Beaver Ratios + DIB2	.0164	.0247	.0411	.1908
	.4918	.4754	.3607	.0820
Total Error Rate≈	5.98%	6.58%	7.03%	18.09%
5 Beaver Ratios + DIC1	.0280	.0411	.1036	.2533
	.4262	.2787	.1803	.0492
Total Error Rate≈	6.43%	6.28%	11.06%	23.47%
5 Beaver Ratios + DIC2	.0280	.0378	.0987	.2418
	.4426	.2787	.2131	.0492
Total Error Rate≈	6.58%	5.98%	10.91%	22.42%
5 Beaver Ratios + DCR1	.0148	.0214	.0428	.1908
	.5410	.4426	.3443	.0492
Total Error Rate≈	6.28%	5.98%	7.03%	17.79%
5 Beaver Ratios + DCR2	.0164	.0230	.0411	.1809
	.5574	.4754	.3607	.0656
Total Error Rate≈	6.58%	6.43%	7.03%	17.04%

TABLE 23

QUADRATIC MDA ERROR RATES:
BEAVER RATIO SET PLUS DEFENSIVE INTERVALS ADDED TWO AT A TIME

Variable Set	3x Worse Misclass. N-F Firm	Equal Cost of Misclass.	3x Worse Misclass. F Firm	30x Worse Misclass. F Firm
STANDARD:	.0164	.0247	.0411	.1826
5 Beaver Ratios	.5574	.4754	.3934	.1148
Total Error Rate≈	6.58%	6.58%	7.32%	17.64%
5 Beaver Ratios	.0329	.0559	.1217	.2533
+ DIB1 & DIC1	.3607	.2623	.1803	.0164
Total Error Rate≈	6.28%	7.47%	12.71%	23.17%
5 Beaver Ratios	.0296	.0411	.1086	.2286
+ DIB2 & DIC2	.4098	.2787	.1639	.0492
Total Error Rate≈	6.43%	6.28%	11.36%	21.23%
5 Beaver Ratios	.0197	.0329	.0609	.2171
+ DIB1 & DCR1	.5082	.4262	.2951	.0820
Total Error Rate≈	6.43%	6.88%	8.22%	20.48%
5 Beaver Ratios	.0214	.0345	.0510	.1859
+ DIB2 & DCR2	.5082	.4426	.2951	.0984
Total Error Rate≈	6.58%	7.17%	7.32%	17.79%
5 Beaver Ratios	.0296	.0576	.1151	.2599
+ DIC1 & DCR1	.3770	.2787	.1475	.0328
Total Error Rate≈	6.13%	7.77%	11.81%	23.92%
5 Beaver Ratios	.0263	.0444	.0921	.2253
+ DIC2 & DCR2	.3934	.2951	.1967	.0492
Total Error Rate≈	5.98%	6.73%	10.16%	20.93%

TABLE 24

QUADRATIC MDA ERROR RATES:
BEAVER RATIO SET PLUS DEFENSIVE INTERVALS ADDED THREE AT A TIME

Variable Set	3x Worse Misclass. N-F Firm	Equal Cost of Misclass.	3x Worse Misclass. F Firm	30x Worse Misclass. F Firm
STANDARD:	.0164	.0247	.0411	.1826
5 Beaver Ratios	.5574	.4754	.3934	.1148
Total Error Rate≈	6.58%	6.58%	7.32%	17.64%
5 Beaver Ratios	.0313	.0773	.1266	.2599
+ DIB1 & DIC1 & DCR1	.3443	.2131	.1639	.0328
Total Error Rate≈	5.98%	8.97%	13.00%	23.92%
5 Beaver Ratios	.0313	.0477	.1036	.2138
+ DIB2 & DIC2 & DCR2	.3770	.2951	.1803	.0656
Total Error Rate≈	6.28%	7.03%	11.06%	20.03%

TABLE 25

QUADRATIC MDA CLASSIFICATION RESULTS:
ALTMAN RATIO SET PLUS DEFENSIVE INTERVALS ADDED ONE AT A TIME

Variable Set	3x Worse Misclass. N-F Firm		Equal Cost of Misclass.		3x Worse Misclass. F Firm		30x Worse Misclass. F Firm	
STANDARD: 5 Altman Ratios	582	26	556	52	462	146	223	385
	19	42	9	52	4	57	1	60
5 Altman Ratios + DIB1	564	44	501	107	357	251	224	384
	13	48	6	55	5	56	3	58
5 Altman Ratios + DIB2	577	31	523	85	393	215	232	376
	14	47	7	54	5	56	3	58
5 Altman Ratios + DIC1	503	105	408	200	347	261	275	333
	6	55	3	58	2	59	2	59
5 Altman Ratios + DIC2	515	93	416	192	348	260	279	329
	6	55	4	57	2	59	2	59
5 Altman Ratios + DCR1	575	33	533	75	447	161	273	335
	13	48	7	54	3	58	1	60
5 Altman Ratios + DCR2	578	30	546	62	479	129	294	314
	12	49	4	57	3	58	1	60

TABLE 26

QUADRATIC MDA CLASSIFICATION RESULTS:
ALTMAN RATIO SET PLUS DEFENSIVE INTERVALS ADDED TWO AT A TIME

Variable Set	3x Worse Misclass. N-F Firm		Equal Cost of Misclass.		3x Worse Misclass. F Firm		30x Worse Misclass. F Firm	
STANDARD: 5 Altman Ratios	582	26	556	52	462	146	223	385
	19	42	9	52	4	57	1	60
5 Altman Ratios + DIB1 & DIC1	463	145	389	219	340	268	276	332
	7	54	5	56	3	58	2	59
5 Altman Ratios + DIB2 & DIC2	501	107	412	196	358	250	286	322
	8	53	5	56	2	59	2	59
5 Altman Ratios + DIB1 & DCR1	553	55	488	120	381	227	254	354
	8	53	5	56	3	58	3	58
5 Altman Ratios + DIB2 & DCR2	567	41	531	77	451	157	290	318
	9	52	5	56	5	56	3	58
5 Altman Ratios + DIC1 & DCR1	498	110	414	194	364	244	282	326
	5	56	3	58	2	59	1	60
5 Altman Ratios + DIC2 & DCR2	531	77	458	150	389	219	294	314
	5	56	4	57	1	60	1	60

TABLE 27

QUADRATIC MDA CLASSIFICATION RESULTS:
ALTMAN RATIO SET PLUS DEFENSIVE INTERVALS ADDED THREE AT A TIME

Variable Set	3x Worse Misclass. N-F Firm		Equal Cost of Misclass.		3x Worse Misclass. F Firm		30x Worse Misclass. F Firm	
STANDARD: 5 Altman Ratios	582	26	556	52	462	146	223	385
	19	42	9	52	4	57	1	60
5 Altman Ratios + DIB1 & DIC1 & DCR1	456	152	403	205	366	242	290	318
	6	55	4	57	4	57	1	60
5 Altman Ratios + DIB2 & DIC2 & DCR2	528	80	455	153	396	212	312	296
	7	54	4	57	1	60	1	60

TABLE 28

QUADRATIC MDA ERROR RATES:
ALTMAN RATIO SET PLUS DEFENSIVE INTERVALS ADDED ONE AT A TIME

Variable Set	3x Worse Misclass. N-F Firm	Equal Cost of Misclass.	3x Worse Misclass. F Firm	30x Worse Misclass. F Firm
STANDARD: 5 Altman Ratios	.0428 .3115	.0855 .1475	.2401 .0656	.6332 .0164
Total Error Rate≈	6.73%	9.12%	22.42%	57.70%
5 Altman Ratios + DIB1	.0724 .2131	.1760 .0984	.4128 .0820	.6316 .0492
Total Error Rate≈	8.52%	16.89%	38.27%	57.85%
5 Altman Ratios + DIB2	.0510 .2295	.1398 .1148	.3536 .0820	.6184 .0492
Total Error Rate≈	6.73%	13.75%	32.29%	56.65%
5 Altman Ratios + DIC1	.1727 .0984	.3289 .0492	.4293 .0328	.5477 .0328
	16.59%	30.34%	39.31%	50.07%
5 Altman Ratios + DIC2	.1530 .0984	.3158 .0656	.4276 .0328	.5411 .0328
	14.80%	29.30%	39.16%	49.48%
5 Altman Ratios + DCR1	.0543 .2131	.1234 .1148	.2648 .0492	.5510 .0164
	6.88%	12.26%	24.51%	50.22%
5 Altman Ratios + DCR2	.0493 .1967	.1020 .0656	.2122 .0492	.5164 .0164
	6.28%	9.87%	19.73%	47.09%

TABLE 29

QUADRATIC MDA ERROR RATES:
ALTMAN RATIO SET PLUS DEFENSIVE INTERVALS ADDED TWO AT A TIME

Variable Set	3x Worse Misclass. N-F Firm	Equal Cost of Misclass.	3x Worse Misclass. F Firm	30x Worse Misclass. F Firm
STANDARD: 5 Altman Ratios	.0428 .3115	.0855 .1475	.2401 .0656	.6332 .0164
Total Error Rate≈	6.73%	9.12%	22.42%	57.70%
5 Altman Ratios + DIB1 & DIC1	.2385 .1148	.3602 .0820	.4408 .0492	.5461 .0328
Total Error Rate≈	22.72%	33.48%	40.51%	50.07%
5 Altman Ratios + DIB2 & DIC2	.1760 .1311	.3224 .0820	.4112 .0328	.5296 .0328
Total Error Rate≈	17.19%	30.04%	37.67%	48.43%
5 Altman Ratios + DIB1 & DCR1	.0905 .1311	.1974 .0820	.3734 .0492	.5822 .0492
Total Error Rate≈	9.42%	18.68%	34.38%	53.36%
5 Altman Ratios + DIB2 & DCR2	.0674 .1475	.1266 .0820	.2582 .0820	.5230 .0492
Total Error Rate≈	7.47%	12.26%	24.22%	47.98%
5 Altman Ratios + DIC1 & DCR1	.1809 .0820	.3191 .0492	.4013 .0328	.5362 .0164
Total Error Rate≈	17.19%	29.45%	36.77%	48.88%
5 Altman Ratios + DIC2 & DCR2	.1266 .0820	.2467 .0656	.3602 .0164	.5164 .0164
Total Error Rate≈	12.26%	23.02%	32.88%	47.09%

TABLE 30

QUADRATIC MDA ERROR RATES:
ALTMAN RATIO SET PLUS DEFENSIVE INTERVALS ADDED THREE AT A TIME

Variable Set	3x Worse Misclass. N-F Firm	Equal Cost of Misclass.	3x Worse Misclass. F Firm	30x Worse Misclass. F Firm
STANDARD: 5 Altman Ratios	.0428	.0855	.2401	.6332
	.3115	.1475	.0656	.0164
Total Error Rate≈	6.73%	9.12%	22.42%	57.70%
5 Altman Ratios + DIB1 & DIC1 & DCR1	.2500	.3372	.3980	.5230
	.0984	.0656	.0656	.0164
Total Error Rate≈	23.62%	31.24%	36.77%	47.68%
5 Altman Ratios + DIB2 & DIC2 & DCR2	.1316	.2516	.3487	.4868
	.1148	.0656	.0164	.0164
Total Error Rate≈	13.00%	23.47%	31.84%	44.39%

Evaluating the Discriminant Functions

Several alternative perspectives can be adopted to evaluate this mass of data, and thus appraise the contribution which defensive intervals make to failure prediction: 1) overall error rates; 2) errors in misclassifying nonfailed firms; 3) errors in misclassifying failed firms; and 4) expected cost of misclassification. Evaluations based on the first three of these criteria can be made directly from Tables 19 through 30. The fourth criterion requires additional calculations.

In Chapter 4, the total cost of misclassification was established as:

$$C(1|0)\ q_o\ P(1|0) + C(0|1)\ q_1\ P(0|1).$$

It is this total cost which the discriminant functions seek to minimize. The a priori odds $\{q_o$ and $q_1\}$ have been previously identified as .967 and .033. The costs of misclassification $\{C(1|0)$ and $C(0|1)\}$, although not valued individually, have been expressed in relative terms including 3:1, 1:1, 1:3, and 1:30 . The probabilities of misclassifying nonfailed firms $\{P(1|0)\}$ and misclassifying failed firms $\{P(0|1)\}$ are found in the northeast and southwest cells, respectively, of the error rate tables.

These components were combined to derive measures of the expected cost of misclassification for each discriminant function. Results for each variable set are presented in Tables 31 and 32. Individual expectations in these tables cannot meaningfully be interpreted in an absolute sense since the choice of actual costs was arbitrary. However, these cost selections were consistently applied within each

TABLE 31

EXPECTED COSTS OF MISCLASSIFICATION:
BEAVER RATIO SET PLUS MODIFICATIONS

Variable Set	3x Worse Misclass. N-F Firm	Equal Cost of Misclass.	3x Worse Misclass. F Firm	30x Worse Misclass. F Firm
5 Beaver Ratios	.06445	.03822	.07527	.28044
+ DIB1	.06248	.04044	.07250	.26848
+ DIB2	.06248	.03822	.07233	.25888
+ DIC1	.09427	.04823	.11672	.28998
+ DIC2	.09476	.04503	.11492	.27883
+ DCR1	.05930	.03404	.07250	.22936
+ DCR2	.06445	.03657	.07233	.23451
+ DIB1 & DIC1	.10656	.06209	.13428	.26046
+ DIB2 & DIC2	.09843	.04823	.12009	.26602
+ DIB1 & DCR1	.07257	.04470	.08563	.28439
+ DIB2 & DCR2	.07752	.04674	.07603	.26888
+ DIC1 & DCR1	.09745	.06423	.12492	.28162
+ DIC2 & DCR2	.08834	.05192	.10704	.26282
+ DIB1 & DIC1 & DCR1	.10141	.08137	.13755	.28162
+ DIB2 & DIC2 & DCR2	.10239	.05512	.11672	.26643

TABLE 32

EXPECTED COSTS OF MISCLASSIFICATION:
ALTMAN RATIO SET PLUS MODIFICATIONS

Variable Set	3x Worse Misclass. N-F Firm	Equal Cost of Misclass.	3x Worse Misclass. F Firm	30x Worse Misclass. F Firm
5 Altman Ratios	.13389	.08736	.23880	.62896
+ DIB1	.21708	.17367	.40780	.65693
+ DIB2	.15530	.13905	.35037	.64413
+ DIC1	.50551	.32051	.41937	.56079
+ DIC2	.44818	.30929	.41772	.55439
+ DCR1	.16441	.12314	.26128	.54923
+ DCR2	.14936	.10091	.21026	.51567
+ DIB1 & DIC1	.69748	.35185	.43200	.55924
+ DIB2 & DIC2	.51609	.31519	.40182	.54323
+ DIB1 & DCR1	.26729	.19394	.36663	.60901
+ DIB2 & DCR2	.20056	.12526	.25783	.55159
+ DIC1 & DCR1	.52888	.31100	.39221	.53487
+ DIC2 & DCR2	.37087	.24127	.35087	.51567
+ DIB1 & DIC1 & DCR1	.73045	.32905	.39196	.52207
+ DIB2 & DIC2 & DCR2	.38640	.24602	.33972	.48696

column, and therefore expectations may be compared in a relative sense. Comparisons can only be made properly within, and not across, columns.

Summary of Results

Performance of the discriminant functions for each variable set which included defensive intervals was compared against the original Beaver and Altman ratio set results. These comparisons were made on the basis of all four of the criteria listed in the last subsection. Findings are summarized in Tables 33 and 34. The "*" notations indicate the cases in which reductions were achieved (relative to the standard ratio sets) in overall error rates, individual error rates, or total expected cost of misclassification.

Since two different standards were selected as benchmarks (namely, Beaver's and Altman's "best predictor" sets), across-the-board generalizations are difficult to make. However, a number of patterns are evident.

First, evaluations are heavily dependent upon the actual cost of misclassification structure selected for analysis, as well as the specific ratio set selected as the standard. Second, different conclusions can be reached depending upon the specific evaluation criterion selected, or at least the strength of those conclusions may vary. Third, improvements in classificatory ability are generally achieved by adding only one defensive interval. Little overall enhancement is attained by expanding consideration to several defensive measures. A likely reason for this result is that the basic advantage afforded by the dynamic nature of the defensive

TABLE 33

SUMMARY OF DISCRIMINANT FUNCTION EVALUATIONS:
BEAVER RATIO SET VERSUS DEFENSIVE INTERVAL MODIFICATIONS

		+ DIB1	+ DIB2	+ DIC1	+ DIC2	+ DCR1	+ DCR2	+ DIB1 & DIC1	+ DIB2 & DIC2	+ DIB1 & DCR1	+ DIB2 & DCR2	+ DIC1 & DCR1	+ DIC2 & DCR2	+ DIB1 & DIC1 & DCR1	+ DIB2 & DIC2 & DCR2
Total Error Rate	A:	*	*	*	E	*	E	*	*	*	E	*	*	*	*
	B:	E	E	*	*	*	*		*						
	C:	*	*			*	*				E				
	D:						*								
Misclassification of Nonfailed Firms	A:	E	E			*	E								
	B:		E			*	*								
	C:		E				E								
	D:						*								
Misclassification of Failed Firms	A:	*	*	*	*	*	E	*	*	*	*	*	*	*	*
	B:	*	E	*	*	*	E	*	*	*	*	*	*	*	*
	C:	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	D:	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Total Expected Cost of Misclassification	A:	*	*			*	E								
	B:		E			*	*								
	C:	*	*			*	*								
	D:	*	*		*	*	*	*	*		*		*		*

KEY:
- A: 3 times worse to misclassify nonfailed firms
- B: Equal costs of misclassifying nonfailed and failed firms
- C: 3 times worse to misclassify failed firms
- D: 30 times worse to misclassify failed firms

* indicates a reduction relative to the Beaver Standard
E indicates the Beaver Standard is exactly equalled

TABLE 34

SUMMARY OF DISCRIMINANT FUNCTION EVALUATIONS:
ALTMAN RATIO SET VERSUS DEFENSIVE INTERVAL MODIFICATIONS

		+ DIB1	+ DIB2	+ DIC1	+ DIC2	+ DCR1	+ DCR2	+ DIB1 & DIC1	+ DIB2 & DIC2	+ DIB1 & DCR1	+ DIB2 & DCR2	+ DIC1 & DCR1	+ DIC2 & DCR2	+ DIB1 & DIC1 & DCR1	+ DIB2 & DIC2 & DCR2
Total Error Rate	A:		E				*								
	B:														
	C:						*								
	D:		*	*	*	*	*	*	*	*	*	*	*	*	*
Misclassification of Nonfailed Firms	A:														
	B:														
	C:						*								
	D:	*	*	*	*	*	*	*	*	*	*	*	*	*	*
Misclassification of Failed Firms	A:	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	B:	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	C:			*	*	*	*	*	*	*		*	*	E	*
	D:					E	E					E	E	E	E
Total Expected Cost of Misclassification	A:														
	B:														
	C:														
	D:			*	*	*	*	*	*	*	*	*	*	*	*

KEY:

A: 3 times worse to misclassify nonfailed firms
B: Equal costs of misclassifying nonfailed and failed firms
C: 3 times worse to misclassify failed firms
D: 30 times worse to misclassify failed firms

* indicates a reduction relative to the Altman Standard
E indicates the Altman Standard is exactly equalled

intervals is captured by considering a single refined liquidity ratio. Fourth, there appears to be a slight advantage of the funds statement based interval measures over their income statement based counterparts.

Perhaps the most striking feature evident in Tables 33 and 34 is the improvement noted in the classification of the failed firms. In the vast majority of cases, addition of defensive intervals resulted in a reduction of the failed firms misclassified. The improvement was often substantial. Since correct identification of failed firms is perhaps the crux of the test, these results are encouraging. Evidence thus suggests that, in general, incorporating defensive interval measures in the analysis does improve discriminatory ability, although this conclusion does not extend to each and every case studied.

CHAPTER 6

MULTIPLE DISCRIMINANT ANALYSIS AS AN AID TO GOING CONCERN EVALUATION

Results presented in the previous chapter have illustrated the utility of financial ratios in discriminating between failed and nonfailed firms. The contribution of defensive interval measures to this discrimination process has also been demonstrated. Attention will now be directed toward a comparison of MDA model predictions and going concern evaluations as reported in auditor opinions on financial statement presentations.

Selection of "Best Predictor" Ratio Sets

The first task performed was the identification of appropriate standards against which auditor opinions could be compared. The performance of thirty different variable sets and their resulting quadratic MDA models has been described in Chapter 5. Results generally indicated that the addition of defensive interval measures to either the Beaver or Altman original ratio sets significantly improved the classification of failed firms.

Since the analyses presented in this chapter concentrate on failed and "failing" firms, a natural inclination would be to choose as the "best" model that variable combination which resulted in the lowest misclassification of failed firms. However, as previously described, a tradeoff exists in which improvement in the classification of failed firms is accomplished at the expense of increased misclassification of nonfailed firms. Therefore, one would expect this approach to seriously bias auditor vs. model comparisons in favor of discriminant analysis.

A second possible approach would be to select that variable set which results in the lowest overall misclassification percentage. However, different mixes of failed and nonfailed misclassifications can result in the same overall error rate. Since the relative costs of these two types of misclassification may not be equal, concentration on total error rates without consideration of the mix of individual errors would be inappropriate.

A third approach, the expected cost of misclassification, specifically considers the relative mix of individual error rates as well as their individual costs.[150] This alternative procedure was adopted.[151] Comparisons of the total cost of misclassification were made for each possible variable set. Results indicated that the addition of the funds statement based no credit interval measure (DCR2) resulted in a total cost expectation equal to or less than that resulting from the addition of any other single defensive interval or group of intervals. These expected costs were also less than those attained from the discriminant models based on either the original Beaver or Altman ratio sets. Two basic standards were thus selected for subsequent comparison purposes: 1) Beaver ratio set plus DCR2; and 2) Altman ratio set plus DCR2.

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[150]see page 131 for a description of this approach. Tables 31 and 32 report the calculated costs of misclassification for each ratio set's corresponding discriminant model.

[151]all analyses performed in this and later sections assumed the moderately asymmetrical case in which it is three times worse (i.e., more costly) to incorrectly classify failed firms than it is to incorrectly classify nonfailed firms.

Analysis of Failed Firms

A random sample of thirty companies was selected from the failed firm population identified in Chapter 5. Auditor opinions relating to the financial statements for these firms were then evaluated.

Fifteen of these firms received unqualified opinions and seven firms received qualified opinions for reasons other than going concern problems. These twenty-two companies are listed in Table 35. This table reports the date of bankruptcy petition filing,[152] the financial statement date drawn for analysis, the associated auditor opinion date, and classification results of the two quadratic discriminant models selected as the standards for comparison. The modified Beaver ratio set correctly identified thirteen of these companies as failed firms (denoted by "F" in the table), and the modified Altman ratio set correctly classified all twenty-two firms.

Auditors disclaimed opinions on financial statements for three companies and issued qualified opinions relating to going concern problems for five companies. These eight firms are listed in Table 36 along with the relevant dates and discriminant model classifications. Both the modified Beaver and Altman ratio sets led to correct categorizations of all eight firms.

Results for this thirty firm sample are summarized in percentage terms in Table 37. Only 26.7% of the firms known ex post to have filed for bankruptcy were identified by the auditor as having going concern problems approximately one year before the fact. In contrast, discriminant models based upon the modified Beaver and Altman ratio

[152] recall from Chapter 5 that a surrogate filing date was selected for Servotronics.

TABLE 35

AUDITOR VERSUS MDA CLASSIFICATIONS:
FAILED FIRMS WITHOUT AUDITOR-IDENTIFIED GOING CONCERN PROBLEMS

	File Date	F.S. Date	Opinion Date	Beaver + DCR2	Altman + DCR2
Unqualified Opinions Issued:					
Alan Wood Steel	6/13/77	12/31/75	1/20/76	F	F
Armac Enterprises	5/21/76	12/31/74	3/25/75	F	F
Bohack Corp.	7/31/74	1/27/73	4/27/73	N-F	F
Commodore Corp.	8/09/74	6/30/73	8/03/73	N-F	F
Electrospace Corp.	4/29/74	12/31/72	4/18/73	N-F	F
Federal's Inc.	8/16/72	7/31/71	9/27/71	F	F
Gruen Industries	4/15/77	3/31/76	7/16/76	F	F
Mammoth Mart	6/14/74	2/03/73	4/09/73	N-F	F
Maule Industries	7/23/76	12/31/74	3/27/75	N-F	F
Old Town Corp.	6/05/73	12/31/71	3/25/72	N-F	F
Penn Fruit Co.	9/04/75	8/31/74	11/22/74	N-F	F
Scottex Corp.	1/16/74	12/31/72	3/30/73	F	F
Servotronics	4/23/76*	12/31/74	4/25/75	F	F
Sitkin Smelting & Ref.	3/14/78	6/30/77	9/07/77	F	F
Supronics Corp.	10/26/76	8/31/75	11/26/75	F	F
"Subject To" or "Except For" Opinions Issued Without Regard to Going Concern Problems:					
Frigitemp Corp.	3/20/78	12/31/76	5/10/77	N-F	F
Harvard Industries	11/20/72	9/30/71	1/05/72	F	F
Paterson Parchment	12/23/74	12/31/73	3/29/74	F	F
Plaza Group Inc.	12/10/74	12/31/73	3/22/74	F	F
RAI Inc.	2/16/73	4/30/72	8/04/72	F	F
Stratton Group Ltd.	9/30/73	12/30/72	3/07/73	N-F	F
Willcox and Gibbs	11/19/76	12/31/75	5/07/76	F	F

TABLE 36

AUDITOR VERSUS MDA CLASSIFICATIONS:
FAILED FIRMS WITH AUDITOR-IDENTIFIED GOING CONCERN PROBLEMS

	File Date	F.S. Date	Opinion Date	Beaver + DCR2	Altman + DCR2
Opinions Disclaimed Because of Going Concern Problems:					
Botany Industries	4/25/72	7/31/71	10/18/71	F	F
Tennessee Forging Steel	12/05/77	6/30/76	10/15/76	F	F
Goodway Inc.	10/04/73	2/29/72	12/11/72	F	F
"Subject To" Opinions Issued Relating to Going Concern Problems:					
Allied Supermarkets	11/07/78	6/25/77	8/10/77	F	F
American Training Serv.	5/17/76	2/28/75	5/28/75	F	F
Arlan's Dept. Stores	5/14/73	1/29/72	4/14/72	F	F
Leader Int'l Industries	11/30/73	12/31/72	3/24/73	F	F
Sequoyah Industries	1/24/74	4/28/73	9/21/73	F	F

TABLE 37

PERCENTAGES OF COMPANIES IN FAILED FIRM SAMPLE IDENTIFIED AS HAVING GOING CONCERN PROBLEMS

	Auditor Opinion	Beaver Based Model	Altman Based Model
Table 35 Firms (n = 22)	0%	59.1%	100%
Table 36 Firms (n = 8)	100%	100%	100%
Total Firms (n = 30)	26.7%	70.0%	100%

ratio sets classified 70% and 100%, respectively, of the companies as failed firms. Thus, for firms that actually filed bankruptcy petitions, the selected models provided signals of going concern problems much more frequently than auditor opinions. Altman and McGough[153] and Deakin[154] have noted similar results.

Since the auditor's opinion is not a prediction of bankruptcy per se, but rather a comment on the fairness of presentation of financial statements, this inquiry was perhaps biased in favor of discriminant analysis which was actually utilized as a failure prediction technique. To temper the results of this section, a further investigation was performed.

Analysis of "Failing" Firms

In this section, companies that had been identified by their auditors as having going concern problems are studied. The data source utilized for selection of these firms is the Disclosure Journal.[155] Consideration was restricted to nonregulated companies covered by COMPUSTAT that filed 10-K's with the Securities and

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[153] Edward I. Altman and Thomas P. McGough, "Evaluation of a Company as a Going Concern," Journal of Accountancy, (December, 1974), pp. 50-57.

[154] Edward B. Deakin, "Business Failure Prediction: An Empirical Analysis," in Financial Crises: Institutions and Markets in a Fragile Environment, Edward I. Altman and Arnold W. Sametz, eds., (New York: John Wiley and Sons, 1977), pp. 72-87.

[155] Disclosure Journal: Index of Corporate Events, Annual Volumes 2 and 3, (Silver Spring, Maryland: Disclosure, Inc., 1974 and 1975).

Exchange Commission during 1974 which contained qualified opinions or disclaimers citing going concern problems. Over-the-Counter as well as NYSE and AMEX firms had to be included in order to obtain a reasonable sample size. Twenty-one companies meeting the above criteria were identified, and thus comprise the group of "failing" firms studied.

Histories of these companies were then investigated for the inclusive period 1971 to 1978 and it was discovered that thirteen firms had filed petitions for bankruptcy. Financial statements were selected for each firm corresponding to the latest yearend in which the associated auditor opinion date preceeded the filing date by at least one month.[156] In addition, the next two previous years' financial statements were drawn for analysis.

Table 38 lists these thirteen "failing" firms with the set of three financial statement and auditor opinion dates selected. Auditor opinions are denoted by "E" if a disclaimer or qualified opinion relating to going concern problems was issued. The designation "OK" indicates that either an unqualified opinion or qualified opinion not citing significant going concern problems was issued. Classification results for the two quadratic MDA models selected as standards are reported in the last two columns. Classifications as failed or nonfailed are designated by "F" or "N-F", respectively.

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[156]the one exception to this scheme involved Western Orbis. Since a surrogate filing date was selected as described in Chapter 5, the only restriction imposed was that the latest financial statement drawn preceed this date.

TABLE 38

AUDITOR VERSUS MDA CLASSIFICATIONS:
"FAILING" FIRMS THAT DID FILE FOR BANKRUPTCY

	File Date	F.S. Date	Opinion Date	Opinion	Beaver + DCR2	Altman + DCR2
Albee Homes	3/04/75	6/30/72	11/30/72	E	F	F
		6/30/73	4/22/74	E	F	F
		6/30/74	12/10/74	E	F	F
Assoc. Food Stores	6/16/75	7/29/72	10/06/72	OK	N-F	F
		7/28/73	12/20/73	OK	F	F
		7/27/74	11/03/74	E	F	F
Career Academy	12/13/74	1/31/72	3/24/72	OK	F	F
		1/31/73	5/18/73	OK	F	F
		1/31/74	6/07/74	E	F	F
Commodore Corp.	8/09/74	6/30/71	8/01/71	OK	F	F
		6/30/72	7/31/72	OK	N-F	F
		6/30/73	8/03/73	OK	N-F	F
Eon Corp.	1/12/73	2/28/70	6/27/70	OK	F	N-F
		2/28/71	5/14/71	OK	F	F
		2/29/72	5/19/72	OK	F	F
Interstate Stores	5/22/74	1/31/71	4/19/71	OK	N-F	F
		1/30/72	4/17/72	OK	N-F	F
		1/28/73	5/01/73	E	F	F
Mangel Stores	3/04/74	1/30/71	4/14/71	OK	N-F	F
		1/29/72	4/14/72	OK	N-F	F
		2/03/73	4/24/73	OK	N-F	F
NRG Inc.	12/02/77	12/31/74	3/27/75	E	N-F	N-F
		12/31/75	4/20/76	E	F	F
		12/31/76	4/20/77	E	F	N-F
Optics Technology	9/05/74	4/30/71	5/27/71	OK	N-F	F
		4/30/72	6/09/72	OK	F	F
		4/30/73	7/31/73	E	F	F
Potter Instrument	4/21/75	6/30/72	8/31/72	OK	F	F
		6/30/73	10/22/73	OK	N-F	F
		6/30/74	1/24/75	E	F	F
Stelber Industries	3/12/76	6/30/73	9/25/73	OK	N-F	F
		6/30/74	9/27/74	E	F	F
		6/30/75	9/27/75	E	F	F
Volume Merchandise	7/23/73	1/31/70	4/29/70	OK	N-F	N-F
		1/31/71	5/07/71	OK	N-F	F
		1/31/72	5/02/72	OK	N-F	F
Western Orbis	9/19/74*	6/30/72	8/30/72	OK	N-F	F
		6/30/73	9/19/73	OK	N-F	F
		6/30/74	10/31/74	E	F	F

For the thirteen selected first prior yearends, the auditor opinions indicated going concern problems in nine cases. This compares with ten and twelve cases categorized as failures by the modified Beaver and Altman ratio set discriminant models, respectively. For the selected second and third prior yearends, the auditor opinions indicated going concern problems in three and two cases, respectively. Discriminant models based on the modified Beaver ratio set indicated failures in seven and five cases for these last two prior yearends. The modified Altman ratio based models indicated failures in thirteen and ten cases for these respective yearends. Thus, the Beaver and Altman based models (both incorporating the no credit defensive interval measure) consistently outperformed the auditor opinion classifications.

Results are summarized in percentage terms in Table 39. Correct failure classifications generally exhibit a deteriorating trend as the time lag between the evaluation date and the date of filing increases.[157] Nevertheless, the quadratic discriminant model based on the Altman ratio set plus DCR2 correctly classified 76.9% of these firms from information drawn up to three years prior to filing for bankruptcy.

Eight of the twenty-one firms studied in this section did not file for bankruptcy so there was no standard time reference point to use in identifying prior yearends as in the previous discussion.

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[157]the one anomaly to this generalization occurred as a result of the Altman based model misclassification of NRG Inc. for the 1976 statement year.

TABLE 39

PERCENTAGES OF CORRECT CLASSIFICATIONS
OF "FAILING" FIRMS THAT DID FILE FOR BANKRUPTCY

	Auditor Opinion	Beaver Based Model	Altman Based Model
First Prior Yearend	69.2%	76.9%	92.3%
Second Prior Yearend	23.1%	53.8%	100%
Third Prior Yearend	15.4%	38.5%	76.9%

Therefore, all available financial statements and associated auditor opinions during the inclusive period 1971 to 1975 were evaluated. Attention was directed in the analysis to a determination of the relative timing of going concern problem indicators.

Results are reported in Table 40. All notations retain the same meanings as described for Table 38. Compared to the auditor evaluations, the discriminant model based on the modified Beaver ratio set indicated going concern problems earlier in seven (87.5%) of the eight cases. In the remaining case the auditor opinion identified going concern difficulties prior to this model. The Altman based model indicated going concern problems earlier than the auditor opinions in six (75%) of the eight cases. The auditor opinion and model identified problems in the same year in one case, and in the last case the opinion signalled difficulties earlier than this model.

Implications For Auditor Going Concern Evaluations

The primary function of financial statements is to provide information for economic decision making.[158] Auditors are charged with the task of expressing an opinion on the "fairness" of those

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[158] Study Group on the Objectives of Financial Statements, Objectives of Financial Statements, (New York: American Institute of Certified Public Accountants, 1973), p. 13

TABLE 40

AUDITOR VERSUS MDA CLASSIFICATIONS: "FAILING" FIRMS THAT DID NOT FILE FOR BANKRUPTCY

	F.S. Date	Opinion Date	Opinion	Beaver + DCR2	Altman + DCR2
Aries Corp.	6/30/71	6/13/72	OK	F	F
	6/30/72	12/13/72	E	F	F
	6/30/73	12/28/73	E	F	F
	6/30/74	9/30/74	E	F	F
Energy Conversion Devices	6/30/71	9/28/71	E	N-F	N-F
	6/30/72	9/25/72	E	N-F	N-F
	6/30/73	9/27/73	E	N-F	N-F
	6/30/74	9/25/74	E	F	F
	6/30/75	9/18/75	E	F	F
Grant Advertising Int'l	12/31/71	4/10/72	OK	F	F
	12/31/72	7/27/73	E	F	F
	12/31/73	5/31/74	E	F	F
	12/31/74	3/31/75	E	F	F
Lindal Cedar Homes	12/31/71	Unaudited	?	N-F	N-F
	12/31/72	3/14/73	OK	N-F	F
	12/31/73	4/08/74	OK	F	F
	12/31/74	4/11/75	E	F	F
	12/31/75	3/19/76	E	F	F
Newport Pharmaceuticals	2/28/71	3/16/71	OK	N-F	N-F
	4/30/72	7/13/72	OK	F	N-F
	4/30/73	6/25/73	E	F	F
	4/30/74	7/03/74	E	F	F
	4/30/75	6/09/75	E	F	N-F
Stanwick Corp	4/30/71	7/27/71	OK	F	F
	4/30/72	8/10/72	E	F	F
	4/30/73	7/25/73	E	F	F
	4/30/74	7/22/74	E	F	F
	4/30/75	7/18/75	OK	N-F	F
Telex Corp.	3/31/71	6/04/71	OK	N-F	N-F
	3/31/72	6/02/72	OK	N-F	N-F
	3/31/73	8/03/73	OK	F	F
	3/31/74	7/11/74	E	F	F
	3/31/75	6/06/75	E	F	F
Westbury Fashions	12/31/71	3/03/72	OK	N-F	N-F
	12/31/72	3/20/73	OK	F	F
	12/31/73	4/04/74	E	F	F
	12/31/74	3/06/75	E	F	F

statements:[159]

> A major part of the auditor's present role is to evaluate whether the information presented by the company adequately portrays its financial position and earnings <u>and the related uncertainties surrounding their measurement</u>. (Italics mine.)

One of the most significant uncertainties of concern to the auditor is the entity's ability to continue in existence. Imperiled continuing operations have serious implications for the adequacy and appropriateness of the financial statement presentations. Thus, consideration of a firm's going concern status weighs heavily in the auditor's discharge of his reporting obligation. William Casey, former Chairman of the Securities and Exchange Commission, has expressed the importance of this aspect of the auditor evaluation function:[160]

> Auditors sometimes find themselves so dubious about a company's viability as a going concern that they find themselves unable to give an opinion as to the overall fairness of the financial statements, which rest after all on the implicit assumption that there is a going business here which can reasonably be expected to continue operating for an indefinite period in the future.
>
> We think it imperative that such prime candidates for bankruptcy or reorganization proceedings be spotted at the earliest possible moment so that investors may guide themselves accordingly.

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[159] <u>Commission on Auditors' Responsibilities: Report, Conclusions, and Recommendations</u>, (New York: AICPA, 1978), p. 28.

[160] William J. Casey, "Investor Relations and Corporate Credibility," paper presented at the October 3, 1972 meeting of the National Investor Relations Institute, Washington, D.C.

Although continued existence is generally presumed, the auditor must remain receptive to evidence which raises a question about the entity's ability to remain a going concern.[161] The discriminant analysis models developed in this research are offered as one possible source of this evidence.

The ability of ratio sets incorporating defensive interval measures to identify failing firms successfully has been demonstrated. As results in the last section have shown, this identification often anticipated subsequent auditor recognition of going concern problems. If Casey's "imperative" is accepted, these models should be seriously considered in going concern evaluations, since they may offer some early signals of imperiled continuing operations.

The final evaluation of a company's going concern status must rest of course on the professional judgement and expertise of the auditor. No simple mathematical model based on six financial ratio measures can hope to capture the nature and essence of a firm's future solvency condition completely. In addition, numerous mitigating factors may exist which tend to refute the problem indicators. It is up to the auditor to identify and evaluate both failure signals and mitigating circumstances in any meaningful determination of going concern status.

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[161] Auditing Standards Board, The Auditor's Considerations When a Question Arises About an Entity's Continued Existence, Statement on Auditing Standards No. 34, (New York: American Institute of Certified Public Accountants, 1981).

CHAPTER 7

SUMMARY AND SUGGESTIONS FOR FUTURE RESEARCH

Defensive interval measures, first introduced by Sorter and Benston in 1960, have been largely ignored in the theoretical and applied literature. As discussed in Chapter 1, these measures offer a number of important conceptual improvements over the more traditional liquidity position indicators. By emphasizing projected daily operating expenditures, defensive intervals incorporate a dynamic element that overcomes much of the criticism of static liquidity measures. Also, these intervals are expressed in a natural dimension (time) which results in an operational meaning that allows for easy interpretation.

Although the theoretical superiority of defensive intervals can be supported, such arguments ignore the empirical question of whether or not these measures actually impart information that is different from the more traditional liquidity ratios. Chapter 3 describes the preliminary investigations undertaken to explore this question. Spearman and Kendall correlation tests of the cross-sectional degree of association between liquidity variables were performed. In the majority of cases, significant associations between the traditional and defensive ratios were found, although the actual correlation estimates were generally quite small. In a number of other cases, statistical independence was established. These results were corroborated by time-series analyses which demonstrated that the traditional and refined liquidity indicators often move in opposite directions over time.

The literature review of bankruptcy studies in Chapter 2 indicates the important role that liquidity variables play in discriminating between failed and nonfailed firms. In view of the alleged superiority of the defensive intervals, it was postulated that consideration of these refined liquidity measures might improve discriminatory ability. The primary purpose of this dissertation was therefore to investigate the contribution that defensive intervals make to business failure prediction.

Discriminant analysis was the basic technique employed to evaluate this contribution. Using the ratio sets found to be good predictors by Beaver in his univariate research and Altman in his multivariate research as a starting point, discriminant models were constructed that incorporated various combinations of defensive interval measures. A number of refinements over the typical application of discriminant analysis were considered in this model development.

First, a priori odds of group membership and relative costs of the two types of misclassifications were included in the construction of the discriminant models. Results generally indicated the sensitivity of the model classifications to alternative cost specifications. Therefore, some of the conclusions reached in prior bankruptcy research that ignored these considerations may be suspect.

Second, tests of the equality of the group dispersion matrices were performed in order to select the appropriate form of statistical analysis. Results indicated in all cases that quadratic discriminant analysis should be employed. This contrasts with the linear form of analysis generally applied in previous bankruptcy research.

Third, the paired sample design was rejected as being in violation of the discriminant analysis assumption that group members be drawn at random from their respective populations. Nonfailed firms in this research were randomly selected from the population of interest. In contrast to most previous studies, this selection was not constrained by either the size of the failed firm group or the characteristics of its members. In addition, a comprehensive search process was undertaken to assure that all failed firms meeting the specified criteria were identified for analysis. This process is viewed as an improvement over the "hit or miss" failure identifications common to other studies.

Fourth, a Bayesian approach was adopted in which the total expected cost of misclassification was used to evaluate the models. This approach specifically incorporates consideration of the two types of misclassification errors possible, as well as their relative costs and probabilities of occurrence. Past research has generally concentrated on total misclassification error rates as the criterion for evaluating discriminatory ability. Evidence has illustrated that as classification regions are altered to improve categorizations of one group, increased misclassifications of the other group result. Because of this tradeoff effect, it is important to extend the analysis beyond total misclassifications to a consideration of the mix of individual error rates. The Bayesian approach accomplishes this extension.

Chapter 5 reports the classification results and error rates of the various quadratic discriminant models that were developed. Since large sample sizes were obtained, the Smith method of error estimation was used. Evidence indicates that incorporating defensive interval measures in the analysis does indeed improve discriminatory ability. Most striking was the improvement noted in the correct classification of failed firms. In general, this improvement is achieved by adding only one defensive interval. Little overall enhancement was attained by expanding consideration to sets of defensive measures. In addition, there appears to be a slight advantage of the funds statement based interval measures over their income statement based counterparts. However, the marginal improvement does not seem to warrant the additional time and expense required to accumulate the funds statement information.

In view of the encouraging results reported in Chapter 5, the analysis was extended to a comparison of model predictions and going concern evaluations as reported in auditor opinions on financial statement presentations. These investigations are described in Chapter 6. Evaluation of a subsample of the failed firm population indicated that the selected quadratic MDA models provided advance signals of going concern problems much more frequently than the auditor opinions.

An independent sample was then selected which contained companies that had been identified by their auditors as having going concern problems. For those firms that actually filed for bankruptcy, the discriminant models consistently outperformed the auditor opinions in terms of correct classification of going concern status. This advantage extended up to three years prior to the actual filing date.

For those firms that did not file for bankruptcy, the models generally indicated going concern problems earlier than the auditor opinions.

Discriminant models which incorporate defensive interval measures can provide some important input to the auditor's going concern review. As demonstrated, these models often provide early signals of imperiled continuing operations and thus may offer the auditor an alternative perspective to consider. The final determination of going concern must, however, rest upon the professional judgement of the auditor. Mitigating factors may exist which refute certain problem indicators and these must be considered before a meaningful going concern evaluation can be completed.

In this dissertation, the evaluation of defensive interval contributions to failure prediction has concentrated on improvements over those classifications provided by the original Beaver and Altman reduced ratio sets. Although these two groups of ratios served as rather stringent standards, future research could be directed toward an evaluation of alternative variable combinations, perhaps including ratio trend information. In this way, additional evidence could be gathered to reaffirm the conclusion of this study that incorporating defensive intervals improves discriminatory ability.

Dramatic increases in interest rates have occurred in recent years and the resulting high opportunity costs should have a major impact upon the nature and size of firms' liquid asset holdings. Since defensive stocks are composed of these cash and near-cash assets, it would be interesting to investigate the performance of the defensive interval measures in the context of such a high interest environment.

Extension of the sample design to more recent periods may be hampered by the effects of the Bankruptcy Reform Act of 1978 which became effective October, 1979. Increased petition filings under this less restrictive Act will perhaps blur the distinction between true "failed" and "nonfailed" firms. Therefore, new criteria for failure may have to be developed in future studies.

Results in this research have illustrated the sensitivity of model results to different presumed expected costs of misclassification. Additional work needs to be performed in an attempt to ascertain relative costs which are relevant to the auditor in practice. Quadratic MDA models have demonstrated discriminatory ability over a range of cost specifications. Pinning down relevant costs would be a first step towards actual implementation of this research as an aid to the auditor's review process.

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APPENDIX 1

BAYESIAN DISCRIMINANT ANALYSIS

The discriminant models and classification procedures developed in this dissertation are based upon a Bayesian inference approach. The purpose of this appendix is to briefly describe specifics of the methodology in the context of the two group case.[162]

For i = 0 or 1, let q_i be the a priori (or prior) probability that an individual comes from population π_i. Further, let $q_o + q_1 = 1$. A vector of observations taken on an individual is denoted by x. The following relationship derives from probability theory:

$$P(\pi_i, x) = P(x|\pi_i)\ q_i = P(\pi_i|x)\ P(x)$$

where $P(\pi_i, x)$ represents the probability of the joint occurrence of π_i and x, $P(x|\pi_i)$ is the conditional probability that x is observed given that the individual comes from π_i, and $P(\pi_i|x)$ is the conditional probability that the individual comes from π_i given x. This last expression is referred to as the a posteriori (or posterior) probability. P(x) represents the unconditional probability of x and may be calculated by:

$$P(x) = \Sigma\ P(x|\pi_i)\ q_i$$

Bayes' Theorem asserts that:

$$P(\pi_i|x) = \frac{P(\pi_i, x)}{P(x)}$$

Incorporating the previous definitions, this equation may be expressed

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[162]the exposition which follows was adapted from Van de Geer, Multivariate Analysis, pp. 258-263, and Afifi and Azen, Statistical Analysis, pp. 237-241.

for the two group case as:

$$P(\pi_i|x) = \frac{P(x|\pi_i)\ q_i}{P(x|\pi_o)\ q_o + P(x|\pi_1)\ q_1}$$

Assuming multivariate normal distributions and substituting the density functions $f_o(x)$ and $f_1(x)$ for the conditional probabilities $P(x|\pi_i)$, one obtains:

$$P(\pi_i|x) = \frac{f_i(x)\ q_i}{f_o(x)\ q_o + f_1(x)\ q_1}$$

The Bayes approach essentially represents the application of a logical learning process. Before any measurements are taken on an individual, prior odds of group membership (q_i) in a given population (π_i) would form the basis for classification. After measurements are taken (i.e., observation vector x is obtained), judgement is revised to reflect new probabilities of group membership (posterior odds). Bayes' Theorem provides the framework for accomplishing this revision.

The classification rules resulting from the Bayesian inference approach may be simply stated:

If $P(\pi_o|x) > P(\pi_1|x)$, assign to π_o;
otherwise assign to π_1.

Thus, given the observation vector x, if the posterior probability of group membership in π_o exceeds that of π_1, the individual is presumed to have come from π_o. Such a procedure yields results equivalent to the classification rules developed in Chapter 4 which were designed to minimize the total probability of misclassification. This can be demonstrated by expressing the density based posterior odds specifications in ratio form and then comparing the results with the classification rules expressed in equation III, p. 73, which are in likelihood ratio format.

A refinement of this approach incorporates the relative costs of the two types of misclassification errors possible. A minor adjustment is required as discussed in Chapter 4. Classification rules stemming from this refinement are presented in equation IV, p. 74. These rules minimize the expected cost of misclassification rather than the probability of misclassification.

Although this Bayesian approach yields classification rules equivalent to the conventional discriminant analysis, an important by-product results: conditional probabilities that an observation is a member of π_i given a particular vector of measurements. This allows the researcher or analyst to establish and apply threshold values as the minimum acceptable posterior probability for classification in a particular group.

The Statistical Analysis System's quadratic DISCRIM procedure was utilized to develop the posterior odds calculations.[163] The SAS program defines the conditional probabilities of interest as:

$$P(\pi_i | x) = \frac{\exp\left[\frac{1}{2} D_i^2(x)\right]}{\Sigma \exp\left[\frac{1}{2} D_i^2(x)\right]} \qquad \text{for } i = 0 \text{ or } 1$$

where $D_i^2(x) = (X-\overline{X}_i)' \, S_i^{-1} \, (X-\overline{X}_i) + \ln |S_i| - 2 \ln (\text{Prior}_i)$. In this

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[163] SAS User's Guide, pp. 183-190.

last expression, S_i represents the within covariance matrix of group i, and Prior_i represents (the cost of misclassifying a member of π_i) times (q_i). Firms were then classified as members of the group for which the posterior probability $\{ P(\pi_i | x) \}$ was largest.

APPENDIX 2

SPEARMAN AND KENDALL CORRELATIONS
AND SIGNIFICANCE LEVELS: 1971-1977

TABLE 41

SPEARMAN CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1971

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.7920 (.0001)	.8084 (.0001)	.1036 (.0115)	.0925 (.0573)	.1469 (.0003)	.1442 (.0030)	.7147 (.0001)	.6912 (.0001)
QAR		.5610 (.0001)	.4222 (.0001)	.4253 (.0001)	.4069 (.0001)	.4011 (.0001)	.9541 (.0001)	.9227 (.0001)
WCR			.0285 (.4880)	.0753 (.1220)	-.0298 (.4686)	.0256 (.5990)	.5058 (.0001)	.5281 (.0001)
DIB1				.9826 (.0001)	.6590 (.0001)	.6321 (.0001)	.5042 (.0001)	.4865 (.0001)
DIB2					.6228 (.0001)	.6387 (.0001)	.4977 (.0001)	.4981 (.0001)
DIC1						.9831 (.0001)	.4497 (.0001)	.4227 (.0001)
DIC2							.4339 (.0001)	.4341 (.0001)
DCR1								.9726 (.0001)

TABLE 42

SPEARMAN CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1972

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.7579 (.0001)	.8092 (.0001)	.1024 (.0118)	.0885 (.0616)	.1448 (.0004)	.1330 (.0049)	.6974 (.0001)	.6931 (.0001)
QAR		.5188 (.0001)	.4553 (.0001)	.4368 (.0001)	.4249 (.0001)	.3992 (.0001)	.9637 (.0001)	.9483 (.0001)
WCR			.0248 (.5425)	.0510 (.2816)	-.0205 (.6142)	.0221 (.6417)	.4783 (.0001)	.5092 (.0001)
DIB1				.9803 (.0001)	.6705 (.0001)	.6205 (.0001)	.5163 (.0001)	.4919 (.0001)
DIB2					.6239 (.0001)	.6328 (.0001)	.4926 (.0001)	.5063 (.0001)
DIC1						.9850 (.0001)	.4558 (.0001)	.4196 (.0001)
DIC2							.4196 (.0001)	.4310 (.0001)
DCR1								.9862 (.0001)

TABLE 43

SPEARMAN CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1973

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.7173 (.0001)	.8011 (.0001)	.1205 (.0028)	.1091 (.0193)	.1486 (.0002)	.1307 (.0050)	.6569 (.0001)	.6483 (.0001)
QAR		.4525 (.0001)	.5026 (.0001)	.4884 (.0001)	.4717 (.0001)	.4307 (.0001)	.9661 (.0001)	.9497 (.0001)
WCR			.0218 (.5916)	.0624 (.1814)	-.0731 (.0711)	-.0334 (.4747)	.3984 (.0001)	.4324 (.0001)
DIB1				.9832 (.0001)	.6574 (.0001)	.6075 (.0001)	.5320 (.0001)	.5120 (.0001)
DIB2					.6070 (.0001)	.6146 (.0001)	.5095 (.0001)	.5240 (.0001)
DIC1						.9859 (.0001)	.4905 (.0001)	.4475 (.0001)
DIC2							.4443 (.0001)	.4586 (.0001)
DCR1								.9869 (.0001)

TABLE 44

SPEARMAN CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1974

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.7153 (.0001)	.8342 (.0001)	.0946 (.0191)	.1176 (.0106)	.0945 (.0193)	.1357 (.0032)	.6842 (.0001)	.6933 (.0001)
QAR		.4945 (.0001)	.4629 (.0001)	.4601 (.0001)	.3911 (.0001)	.3864 (.0001)	.9652 (.0001)	.9415 (.0001)
WCR			.0208 (.6073)	.0855 (.0638)	-.0738 (.0679)	.0154 (.7392)	.4618 (.0001)	.4978 (.0001)
DIB1				.9711 (.0001)	.6101 (.0001)	.5517 (.0001)	.4203 (.0001)	.4076 (.0001)
DIB2					.5462 (.0001)	.5663 (.0001)	.4151 (.0001)	.4437 (.0001)
DIC1						.9734 (.0001)	.3588 (.0001)	.3488 (.0001)
DIC2							.3522 (.0001)	.3798 (.0001)
DCR1								.9748 (.0001)

TABLE 45

SPEARMAN CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1975

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.7666 (.0001)	.8363 (.0001)	.1937 (.0001)	.1920 (.0001)	.1883 (.0001)	.2046 (.0001)	.7011 (.0001)	.6846 (.0001)
QAR		.5592 (.0001)	.5310 (.0001)	.5393 (.0001)	.4810 (.0001)	.4802 (.0001)	.9656 (.0001)	.9445 (.0001)
WCR			.1080 (.0072)	.1434 (.0015)	.0338 (.4022)	.0877 (.0530)	.5027 (.0001)	.5113 (.0001)
DIB1				.9589 (.0001)	.6775 (.0001)	.6241 (.0001)	.5592 (.0001)	.5463 (.0001)
DIB2					.6236 (.0001)	.6539 (.0001)	.5566 (.0001)	.5655 (.0001)
DIC1						.9655 (.0001)	.4981 (.0001)	.4743 (.0001)
DIC2							.4817 (.0001)	.4906 (.0001)
DCR1								.9671 (.0001)

TABLE 46

SPEARMAN CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1976

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.7780 (.0001)	.8350 (.0001)	.1666 (.0001)	.1912 (.0001)	.1849 (.0001)	.2077 (.0001)	.7115 (.0001)	.6934 (.0001)
QAR		.5712 (.0001)	.5078 (.0001)	.5317 (.0001)	.4863 (.0001)	.5022 (.0001)	.9631 (.0001)	.9409 (.0001)
WCR			.0969 (.0160)	.1572 (.0005)	.0293 (.4679)	.0772 (.0908)	.5265 (.0001)	.5310 (.0001)
DIB1				.9830 (.0001)	.7319 (.0001)	.7005 (.0001)	.5603 (.0001)	.5728 (.0001)
DIB2					.6915 (.0001)	.7035 (.0001)	.5812 (.0001)	.5734 (.0001)
DIC1						.9870 (.0001)	.5298 (.0001)	.5361 (.0001)
DIC2							.5447 (.0001)	.5346 (.0001)
DCR1								.9876 (.0001)

TABLE 47

SPEARMAN CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1977

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.7770 (.0001)	.8432 (.0001)	.1621 (.0001)	.1918 (.0001)	.1689 (.0001)	.2114 (.0001)	.7070 (.0001)	.7049 (.0001)
QAR		.5781 (.0001)	.4999 (.0001)	.5330 (.0001)	.4575 (.0001)	.4954 (.0001)	.9611 (.0001)	.9480 (.0001)
WCR			.1064 (.0082)	.1445 (.0016)	.0314 (.4373)	.0730 (.1125)	.5185 (.0001)	.5112 (.0001)
DIB1				.9842 (.0001)	.7136 (.0001)	.6759 (.0001)	.5451 (.0001)	.5684 (.0001)
DIB2					.6710 (.0001)	.6830 (.0001)	.5732 (.0001)	.5561 (.0001)
DIC1						.9866 (.0001)	.4992 (.0001)	.5310 (.0001)
DIC2							.5350 (.0001)	.5177 (.0001)
DCR1								.9870 (.0001)

TABLE 48

KENDALL CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1971

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.6034 (.0001)	.6340 (.0001)	.0739 (.0070)	.0686 (.0351)	.1019 (.0002)	.1018 (.0018)	.5392 (.0001)	.5264 (.0001)
QAR		.4025 (.0001)	.3013 (.0001)	.3050 (.0001)	.2860 (.0001)	.2830 (.0001)	.8281 (.0001)	.8026 (.0001)
WCR			.0222 (.4175)	.0541 (.0965)	-.0203 (.4600)	.0181 (.5790)	.3668 (.0001)	.3840 (.0001)
DIB1				.9493 (.0001)	.4811 (.0001)	.4580 (.0001)	.3833 (.0001)	.3710 (.0001)
DIB2					.4497 (.0001)	.4618 (.0001)	.3804 (.0001)	.3802 (.0001)
DIC1						.9626 (.0001)	.3241 (.0001)	.3064 (.0001)
DIC2							.3150 (.0001)	.3154 (.0001)
DCR1								.9677 (.0001)

TABLE 49

KENDALL CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1972

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.5696 (.0001)	.6374 (.0001)	.0733 (.0070)	.0637 (.0443)	.1004 (.0002)	.0934 (.0032)	.5178 (.0001)	.5172 (.0001)
QAR		.3695 (.0001)	.3261 (.0001)	.3099 (.0001)	.2984 (.0001)	.2777 (.0001)	.8434 (.0001)	.8288 (.0001)
WCR			.0196 (.4703)	.0370 (.2431)	-.0135 (.6205)	.0160 (.6141)	.3419 (.0001)	.3637 (.0001)
DIB1				.9413 (.0001)	.4951 (.0001)	.4531 (.0001)	.3953 (.0001)	.3770 (.0001)
DIB2					.4567 (.0001)	.4633 (.0001)	.3779 (.0001)	.3866 (.0001)
DIC1						.9646 (.0001)	.3318 (.0001)	.3043 (.0001)
DIC2							.3049 (.0001)	.3122 (.0001)
DCR1								.9773 (.0001)

TABLE 50

KENDALL CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1973

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.5347 (.0001)	.6300 (.0001)	.0849 (.0017)	.0778 (.0126)	.0980 (.0003)	.0859 (.0059)	.4854 (.0001)	.4830 (.0001)
QAR		.3201 (.0001)	.3612 (.0001)	.3503 (.0001)	.3325 (.0001)	.3008 (.0001)	.8505 (.0001)	.8315 (.0001)
WCR			.0189 (.4853)	.0463 (.1382)	-.0512 (.0583)	-.0250 (.4230)	.2825 (.0001)	.3060 (.0001)
DIB1				.9474 (.0001)	.4834 (.0001)	.4435 (.0001)	.3943 (.0001)	.3781 (.0001)
DIB2					.4435 (.0001)	.4478 (.0001)	.3759 (.0001)	.3858 (.0001)
DIC1						.9683 (.0001)	.3553 (.0001)	.3214 (.0001)
DIC2							.3184 (.0001)	.3284 (.0001)
DCR1								.9790 (.0001)

TABLE 51

KENDALL CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1974

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.5343 (.0001)	.6626 (.0001)	.0671 (.0129)	.0841 (.0064)	.0639 (.0180)	.0948 (.0021)	.5070 (.0001)	.5182 (.0001)
QAR		.3499 (.0001)	.3317 (.0001)	.3303 (.0001)	.2743 (.0001)	.2723 (.0001)	.8466 (.0001)	.8306 (.0001)
WCR			.0173 (.5212)	.0618 (.0449)	-.0492 (.0686)	.0117 (.7041)	.3287 (.0001)	.3543 (.0001)
DIB1				.9396 (.0001)	.4424 (.0001)	.3978 (.0001)	.3162 (.0001)	.3061 (.0001)
DIB2					.3944 (.0001)	.4065 (.0001)	.3130 (.0001)	.3312 (.0001)
DIC1						.9591 (.0001)	.2580 (.0001)	.2512 (.0001)
DIC2							.2542 (.0001)	.2721 (.0001)
DCR1								.9715 (.0001)

TABLE 52

KENDALL CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1975

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.5771 (.0001)	.6562 (.0001)	.1355 (.0000)	.1336 (.0000)	.1278 (.0000)	.1392 (.0000)	.5203 (.0001)	.5095 (.0001)
QAR		.3983 (.0001)	.3830 (.0001)	.3903 (.0001)	.3372 (.0001)	.3361 (.0001)	.8503 (.0001)	.8298 (.0001)
WCR			.0764 (.0045)	.1033 (.0007)	.0231 (.3908)	.0620 (.0409)	.3586 (.0001)	.3661 (.0001)
DIB1				.9284 (.0001)	.4980 (.0001)	.4596 (.0001)	.4226 (.0001)	.4137 (.0001)
DIB2					.4584 (.0001)	.4791 (.0001)	.4245 (.0001)	.4292 (.0001)
DIC1						.9496 (.0001)	.3614 (.0001)	.3428 (.0001)
DIC2							.3502 (.0001)	.3548 (.0001)
DCR1								.9629 (.0001)

TABLE 53

KENDALL CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1976

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.5861 (.0001)	.6571 (.0001)	.1180 (.0000)	.1333 (.0000)	.1286 (.0000)	.1422 (.0000)	.5286 (.0001)	.5184 (.0001)
QAR		.4038 (.0001)	.3672 (.0001)	.3829 (.0001)	.3418 (.0001)	.3508 (.0001)	.8418 (.0001)	.8251 (.0001)
WCR			.0701 (.0091)	.1122 (.0002)	.0213 (.4292)	.0522 (.0873)	.3742 (.0001)	.3787 (.0001)
DIB1				.9460 (.0001)	.5486 (.0001)	.5198 (.0001)	.4266 (.0001)	.4315 (.0001)
DIB2					.5151 (.0001)	.5235 (.0001)	.4379 (.0001)	.4329 (.0001)
DIC1						.9680 (.0001)	.3854 (.0001)	.3855 (.0001)
DIC2							.3917 (.0001)	.3852 (.0001)
DCR1								.9776 (.0001)

TABLE 54

KENDALL CORRELATIONS AND SIGNIFICANCE LEVELS FOR 1977

	QAR	WCR	DIB1	DIB2	DIC1	DIC2	DCR1	DCR2
CR	.5847 (.0001)	.6617 (.0001)	.1142 (.0000)	.1342 (.0000)	.1168 (.0000)	.1454 (.0000)	.5233 (.0001)	.5240 (.0001)
QAR		.4067 (.0001)	.3577 (.0001)	.3809 (.0001)	.3204 (.0001)	.3454 (.0001)	.8360 (.0001)	.8265 (.0001)
WCR			.0745 (.0057)	.1019 (.0009)	.0207 (.4415)	.0487 (.1128)	.3665 (.0001)	.3615 (.0001)
DIB1				.9498 (.0001)	.5268 (.0001)	.4958 (.0001)	.4102 (.0001)	.4234 (.0001)
DIB2					.4937 (.0001)	.5008 (.0001)	.4313 (.0001)	.4172 (.0001)
DIC1						.9705 (.0001)	.3588 (.0001)	.3793 (.0001)
DIC2							.3838 (.0001)	.3708 (.0001)
DCR1								.9771 (.0001)

APPENDIX 3

RANDOM SAMPLING DESIGN
AND DATA VALIDATION

In this appendix, the random sampling approach taken to identify the nonfailed firms will be detailed. Also, a description will be given of the steps undertaken to validate the data collected.

A sample of nonfailed firms was drawn in order to provide a data base for the preliminary empirical investigations reported in Chapter 3. This sample also serves as the nonfailed firm group in the discriminant analyses described in Chapters 4 and 5.

Appendix A of the 1979 Industrial COMPUSTAT Manual contains an alphabetical listing of the companies included in the Primary, Supplementary, and Tertiary Industrial Files. The total COMPUSTAT coverage of industrial companies, determined by merging these three files, consists of 2,491 firms as of December 15, 1978. This listing was partitioned into 100 sections in order to facilitate the drawing and application of random numbers for sampling purposes. Since 2,491 could not be divided evenly by 100, it was necessary to deviate slightly from equal size sections. The partitioning was constructed so that ninety-one of the sections contained twenty-five companies each, and the remaining nine sections contained twenty-four companies each.

Utilizing "A Table of 14,000 Random Units," 610 two-digit numbers were selected.[164] The sections corresponding to these random numbers represent the groupings of companies from which the nonfailed firm members were drawn. For each identified section, another

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[164] Samuel M. Selby, ed., CRC Standard Mathematical Tables, (Cleveland: Chemical Rubber Co., 1967), pp. 564-568.

two-digit random number was then selected. The following scheme was used to accomplish the actual firm identifications:

if random number is in this range:	01 - 04	05 - 08	. . . . .	93 - 96	97 - 00
then choose firm number:	1	2	. . . . .	24	25

If the firms thus identified met the specified criteria (i.e. nonregulated companies traded on either the New York or American Stock Exchange that did not file for bankruptcy during 1972 through 1978), they became members of the nonfailed group investigated. If one or more of the criteria were not met, a replacement random number was selected to identify an alternative firm. This process was replicated for each of the original 610 random numbers selected, resulting in a sample of 610 companies.

Later investigation revealed that one of these companies had reported an intention to file a petition for bankruptcy and was therefore not properly a member of the nonfailed population. Another company did not have sufficient financial statement data available for the year selected for analysis. These two companies were excluded, resulting in a final nonfailed sample containing 608 firms.

In Chapter 5, a partitioning scheme was described in which the 608 member sample was divided into seven subsamples in order to identify the specific financial statement dates to analyze in the MDA model developments. Six firms within each of the seven partitions were selected for data validation.

Individual components of the ratio calculations were output from the COMPUSTAT tapes for each of these forty-two companies. Verification data was obtained from 10-K reports filed with the Securities and Exchange Commission. A comparison of 10-K and COMPUSTAT information for the selected data items revealed no discrepancies. The effect of any inconsistencies not discovered by this validation process are presumed to be mitigated by the large number of firms selected for analysis.

A different approach was adopted for collection and validation of the failed firm group ratios. Since so few of the failed firms had information on either the Current or Research COMPUSTAT tapes recent enough to be of use, it was necessary to select an alternative data source. Moody's Industrial Manual offers one of the most comprehensive compilations of financial statement data, however, statement of changes in financial position information is not generally reported by this source. In order to avoid problems of cross-validation among a number of information sources, it was decided to draw all necessary financial statement data directly from 10-K reports. Ratios for the sixty-one failed firms were calculated from this S.E.C. derived information.

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THE USE OF DEFENSIVE INTERVALS IN
CORPORATE FAILURE PREDICTION AND
AUDITORS' GOING CONCERN EVALUATIONS

by

Jon Robert Carpenter

(ABSTRACT)

Defensive interval measures, first introduced by Sorter and Benston in 1960, have been largely ignored in the theoretical and applied literature. In this dissertation, the conceptual superiority of these ratios is explored and empirical investigations are undertaken to determine if these measures actually impart information different from the more traditional liquidity position indicators. Correlation tests of the cross-sectional degree of association between liquidity variables were performed. Significant associations between the traditional and defensive ratios were generally found, although the actual parameter estimates were usually quite small. In a number of other cases, statistical independence was established. These results were corroborated by time-series analyses.

A literature review of bankruptcy studies indicates the important role that liquidity variables play in discriminating between failed and nonfailed firms. In view of the alleged superiority of the defensive intervals, it was postulated that consideration of these refined liquidity measures might improve discriminatory ability. The primary purpose of this dissertation was therefore to investigate the contribution that defensive intervals make to business failure

prediction.

Multiple discriminant analysis (MDA) was the basic technique employed to evaluate this contribution. Using ratio sets found to be good predictors in prior research as a starting point, discriminant models were constructed that incorporated various combinations of defensive interval measures. A number of refinements over the typical application of MDA were considered in this model development: a priori odds of group membership were identified; a range of relative costs of misclassification errors was considered; tests of the equality of group dispersion matrices were performed in order to select the appropriate form of statistical analysis; the paired sample design was rejected; and a Bayesian inference approach was adopted to evaluate the models.

Various quadratic MDA models were developed and evaluated. Evidence indicates that incorporating defensive interval measures in the analysis does indeed improve discriminatory ability. Most striking was the improvement noted in the correct classification of failed firms.

The analysis was extended to a comparison of model predictions and going concern evaluations as reported in auditor opinions on financial statement presentations. Evaluation of a subsample of the failed firm population indicated that the selected quadratic models provided advance signals of going concern problems much more frequently than the auditor opinions.

An independent sample was drawn containing companies that had been identified by their auditors as having going concern problems.

For those firms that actually filed for bankruptcy, the discriminant models consistently outperformed the auditor opinions in terms of correct classification of going concern status. This advantage extended up to three years prior to the actual filing date. For those firms that did not file for bankruptcy, the models generally indicated going concern problems earlier than the auditor opinions.

Discriminant models which incorporate defensive interval measures can provide some important input to the auditor's going concern review. As demonstrated in this dissertation, these models often provide early signals of imperiled continuing operations and thus may offer the auditor a valuable alternative perspective to consider in going concern evaluations.