UNCERTAINTY AND THE EFFECTS OF A PAY-AS-YOU-GO SOCIAL SECURITY PROGRAM ON ECONOMIC GROWTH

by

Christopher McCoy

Dissertation submitted to the Graduate Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Economics

APPROVED:

Robert Mackay, Chairman

Melvin Hinich

Arthur Denzau

Daniel Orr

Carolyn Weaver

August, 1981
Blacksburg, Virginia

ACKNOWLEDGEMENTS

Thanks are due to the professors on my committee and to many other teachers for their direct and indirect aid in helping me complete this dissertation. I would like to mention especially Professor Robert Mackay, who initially aroused my interest in this topic and has provided many useful suggestions. Lastly, I thank my wife· , who over the past four years has sacrificed many evenings and has given me a great deal of inspiration in completing this work.

LIST OF FIGURES

LIST OF FIGURES (Continued)

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

During the past decade many economists have dealt with the subject of social security's impact on economic growth. To the writer's knowledge, no attempt was made to examine this topic in the setting of an uncertain economic future. This paper incorporates the alterations in individual savings behavior, caused by a simplified social security program, into stochastic economic growth models. This enables one to examine how a social security program affects uncertain future wages and interest rates. Two forms of uncertainty are examined, namely, production and labor force participation. This requires the integration of three economic fields of study: (1) the consumption-savings decision of individuals in a two period model, (2) the portfolio decisions of individuals and (3) stochastic economic growth.

The recent arguments on how social security impacts on individual savings behavior are first presented and evaluated in Chapter II. Feldstein's tax and benefit effects are presented and alternative theories by Munnell and Barro are introduced to counter them. Buchanan's reply and Feldstein's rebuttal follow and a conclusion is made that the negative tax and benefit effects are the two dominant forces influencing individual savings behavior.

In Chapter III individual consumption-savings decisions in a certainty setting are incorporated into a neo-classical growth model. A social security program is then imposed to show how equilibrium wages and interest rates change. In the latter part of Chapter III, production uncertainty is introduced and long-run stochastic equilibrium is derived. Social security is shown to raise stochastic interest rates and lower stochastic wages.

Additional questions concerning social security are analyzed within the production uncertainty in Chapter IV. They include: (1) determining the optimal amount of social security taxes and benefits for a particular generation, (2) examining the effects of a fully funded social security program on wages and interest rates, (3) studying the relationship between the rate of growth of labor force participation and private investment, (4) relaxing the assumption of identical wage rates and then determining how individuals' optimal amount of social security differs and (5) introducing technological growth into the model.

An alternative form of uncertainty, future labor participation, is examined in Chapter V to determine whether social security's impact on wages and interest rates varies, depending on the form of uncertainty. The results indicate that both forms of uncertainty yield the same result of lower wages and higher interest rates. The labor growth uncertainty model also shows that a social security program increases the variance of uncertain future wages.

CHAPTER II

SURVEY OF RECENT LITERATURE

This Chapter reviews and evaluates recent arguments concerning the effect of social security on private savings behavior. The conclusions in this section will determine how social security affects wages and interest rates in the latter chapters. There are three theories that are proposed by Feldstein, Munnell and Barro. All three theories assume that individuals live for two periods; they work the first period and retire sometime during the second. Feldstein's main argument is that social security reduces personal savings by taxing current wage income and promising to pay benefits during retirement. Munnell argues that Feldstein's negative tax effect does not exist and that the benefit effect is largely offset by the early retirement effect, caused by the social security program. In the latter part of this chapter, Barro's theory, that social security does not significantly reduce savings, but merely shifts the composition of savings between the older and younger generation, is analyzed. A conclusion is then drawn, which incorporates replies made by Feldstein and Buchanan to Barro's article.

II-1 Feldstein's Tax and Benefit Effects

Since Martin Feldstein's 1974 Journal of Political Economy article, "Social Security Induced Retirement and Aggregate Capital Accumulation," both academic and public interest in this program has risen sharply. Feldstein uses an extended life-cycle model to analyze the impact of social security on individual decisions concerning retirement and savings. He argues that social security reduces personal savings because people substitute private assets for expected retirement benefits. However, this program may have an offsetting positive savings effect because it tends to lengthen the period of retirement over which accumulated assets will be spread. He concludes that the net effect of social security will depend on the relative strength of these two forces.

Feldstein proposes that the program induces earlier retirement because a potential recipient, one who is age 65 or over, loses his social security benefits, if he earns more than a certain amount per month due to the earnings test. In order for one to receive his benefits, he is required to retire from regular employment. For an individual, who does not alter his retirement age, Feldstein maintains that social security benefits have an unambiguous effect of reducing savings. In figure 1 he illustrates the benefit and retirement effects on savings. In this diagram the horizontal axis measures income and consumption before 65; the vertical measures income and consumption after 65. This figure represents two different cases. The first case is where an individual would have retired at age 65 regardless of social security; the other is where he retires earlier because of the program. In the first case the individual's earnings are Y_{1A} in period 1 and zero in period 2. Given the opportunity to save and earn an interest of r, he will choose the pair of consumption levels denoted by the point I. In period 1, he consumes C_{1A} and saves $(Y_{1A} - C_{1A})$. Point B describes the individual's initial position under social security, assuming that the program pays a normal rate of return. His first-period income is reduced by the payroll tax to Y_{1B} and his second-period income is increased to B. In this case, social security does not alter the individual's budget line, thus the individual will not alter his consumption pattern. The reduction in disposable income plus the future retirement benefits cause savings to decline from $(Y_{1A} - C_{1A})$ to $(Y_{1B} - C_{1A})$, which is the amount of the payroll tax, $(Y_{1A} - Y_{1B})$. If social security promised a rate of return larger than r, the decrease in savings would be greater than the payroll tax.

In the second case the individual had planned to work after 65. In figure 1 point C is the individual's initial position, where he earns income Y_{1A} while he is young and income C when he is old. Equilibrium consumption is represented by point II. In the first period he consumes C_{1c} and saves $(Y_{1A} - C_{1c})$. When social security is introduced, causing him to retire completely at age 65, his initial position shifts from C to B, where first period disposable income is reduced by the payroll tax, and second period disposable income falls, because wage earnings are reduced to zero. With point B as his new initial position, the individual chooses the consumption pair at point I. The effect of social security is to change savings from $(Y_{1A} - C_{1c})$ to $(Y_{1B} - C_{1A})$. If $(Y_{1B} - C_{1A})$ is greater than $(Y_{1A} - C_{1c})$, then social security can increase savings.

Feldstein believes that the positive savings effect caused by earlier retirement is small, relative to the negative asset substitution

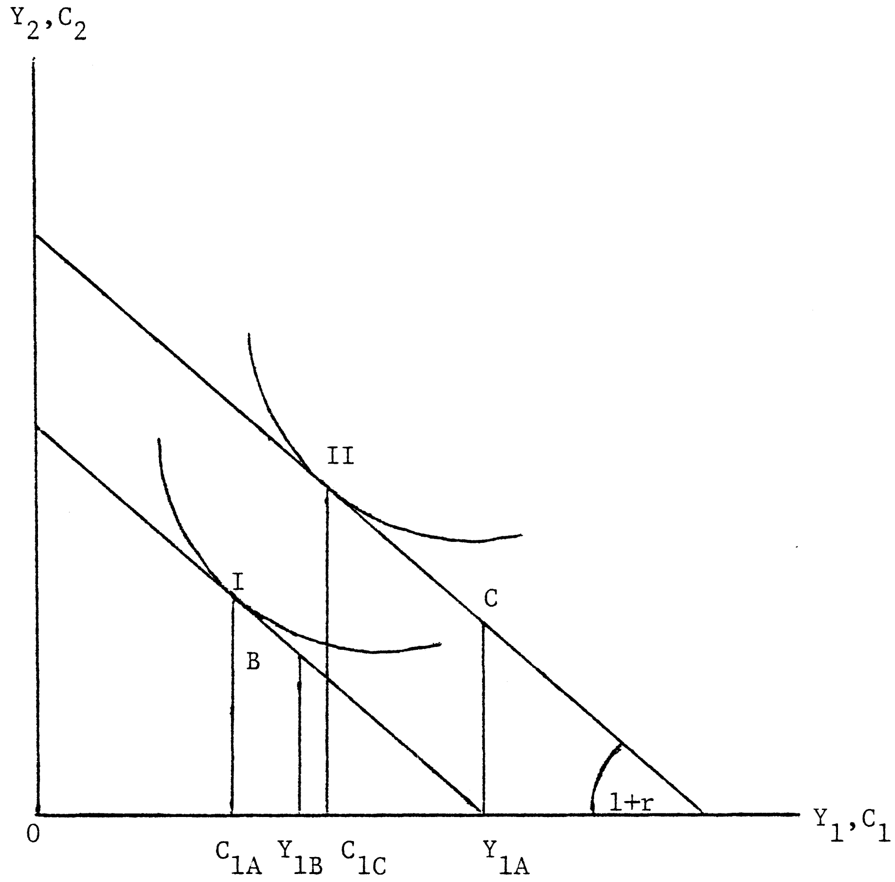

Figure 1

Optimal Consumption-Saving Decisions

and payroll tax effects. He concludes, that the net effect of social security on savings, is negative.

In the latter part of Feldstein's paper, he attempts to estimate the program's effects on personal savings. He concludes from his results, that the payroll tax and the anticipated benefits, combined to reduce savings in 1971 by $61 billion. This was approximately equal to the total personal savings for that year.

II-2 Munnell's Reply to Feldstein

In response to Feldstein's claim, that social security has reduced private savings by half, Alicia Munnell, in her article, "The Effect of Social Security on personal Savings," estimated, that the reduction in private savings for 1969 was only $3.6 billion, as compared to Feldstein's 1969 figure of $38.2 billion. Munnell used a more detailed approach to derive her estimates of savings reduction. She argues that Feldstein left out the unemployment rate in his estimate, which she believes would influence savings and is highly correlated with social security wealth. By leaving this data out, Munnell contends that Feldstein over-estimated the negative benefit effect on savings.

Munnell claims that there is no additional negative impact from the payroll tax. She argues that the program does not reduce disposable income in the aggregate; that transfers of income from workers to retirees would have only a very small effect on savings. She is saying that the reduction in savings, caused by the payroll tax, can be offset by an increase in savings by the retirees, who receive these benefits.

Munnell also points out that Feldstein does not include a variable for the retirement effect in his estimations. She argues that social security affects retirement in three ways. One effect is that retirement benefits moderate the reduction in income, that workers face when they retire. According to Munnell, this is a pure income effect that encourages older workers to choose leisure instead of work. The second way is that the earnings test makes it impossible for most workers to receive benefits without cutting back on work effort. The third result is that social security may contribute to the idea that sixty-five is the normal retirement age. She concludes that these three factors, which encourage an earlier retirement, almost entirely offset the decline in savings from social security benefits. Munnell claims that social security has caused only a small decrease in aggregate personal savings. However, she proposes, that in the future, due to the large benefit increase of the early 70's and the leveling off of participation of the aged in the labor market, the negative impact on savings in the future will be greater.

II-3 Barro's Bequest Effect

A totally different argument concerning the effects of social security on savings is presented by Robert Barro, in his article, "Are Government Bonds Net Wealth?". In this paper Barro builds a model based on intergenerational transfers. He concludes, that people will offset social security benefits and payroll taxes, by changing the amount they transfer to members of other generations, within their family.

Barro assumes a static economy where population and technology are constant. In Barro's world people live for two periods. In the first period individuals work and receive a wage of W, at the beginning of the period, for work done during that period. Members born in period t will allocate W between consumption while they are young, denoted by C_t^y, and savings, denoted by A_t^y, upon which interest is paid immediately. This means that if an individual wants to have savings of amount A_t^y at the beginning of period t+1, then it will cost him $(1-r)\ A_t^y$ at the beginning of period t. In the beginning of the second period when a member of generation t retires, he receives a bequest from his parents of amount A_{t-1}^0. With this bequest, plus the wealth he saved when he was young, the individual's total wealth is $A_{t-1}^0 + A_t^y$. He will allocate this wealth between consumption, C_t^0, to be used when he is old, and savings, to be left in the form of a bequest, A_t^0, on which he immediately receives interest. The budget constraint of a member of generation 1, who is old, is

$$\text{(II-1)} \quad A_1^y + A_0^0 = C_1^0 + (1-r)\ A_1^0.$$

The budget constraint of a member of generation 2 is

$$\text{(II-2)} \quad W + (1-r)A_1^0 = C_2^y + (1-r)C_2^0 + (1-r)^2 A_2^0.$$

Barro assumes that individuals derive utility from consumption when they are young, when they are old, and from the anticipated consumption levels of their offspring. For a member of generation 1 their utility is a function of consumption when they are young, when they are old and the utility of their children as shown in equation (II-3):

$$\text{(II-3)} \quad U_1 = U_1(C_1^y, C_1^0, U_2^*),$$

where

$$U_2^* = W + (1-r)A_1^0.$$

It is important to note that Barro assumes that utility depends on the endowment of their children, U_2^*, rather than on the gross bequest, A_1^0; and that transfers of resources to the younger generations can only be made in terms of unrestricted purchasing power.

When a member of generation 1 becomes old, he determines his allocation of resources, so as to maximize U_1, subject to the constraints below:

$$\text{(II-4)} \quad A_1^y + A_0^0 = C_1^0 + (1-r)A_1^0$$

and

$$\text{(II-5)} \quad W + (1-r)A_1^0 = C_2^y + (1-r)C_2^0 + (1-r)^2 A_2^0,$$

where A_1^y, A_1^0, A_0^0, A_2^0, C_1^0, C_2^0, C_2^y and $W > 0$.

The amount of the bequest, by a member of generation 1, takes into account the effect of A_1^0 on generation 2's resources, U_2^*, which is an argument in generation 1's utility function, U_1. He also notices the chain dependence of U_2 on U_3^* and U_3 on U_4^*. If an individual of generation 1 is going to leave a positive bequest to his children, then the budget constraint of this individual is

$$\text{(II-6)} \quad W + A_1^y + A_0^0 = C_1^0 + U_2^*.$$

Equation (II-6) can be called generation 1's social budget constraint. Writing the budget constraint this way shows that generation 1's opportunity set is increased if his offspring's wage increases.

The maximization problem facing an individual of generation 1, who is currently old, can be graphically shown in figure 2. Consumption is on the vertical axis and the anticipated consumption level of his offspring is located on the horizontal axis. It is assumed that the individual will leave a positive bequest, meaning that $U_2^* > W$. The individual maximizes his utility by choosing the optimal allocation, which in this graph is at point I. In this case he will leave a bequest of amount, $\hat{A}_1^0$, for his offspring. If the individual does not desire to leave a bequest, then he would be in a corner solution at W, because Barro assumes that the individual cannot leave a negative bequest.

Barro's next step is to impose a social security program, which gives amount S to the current older generation, that is financed by a lump sum tax on the younger generation. Barro makes the assumption that individuals know the program will eventually be terminated. Without altering Barro's model or results, it can be assumed, that the program is retired after two periods. This means that when generation 2 retires, each member will receive an amount S from generation 3, after which the program is aborted. The effect of social security on each member of

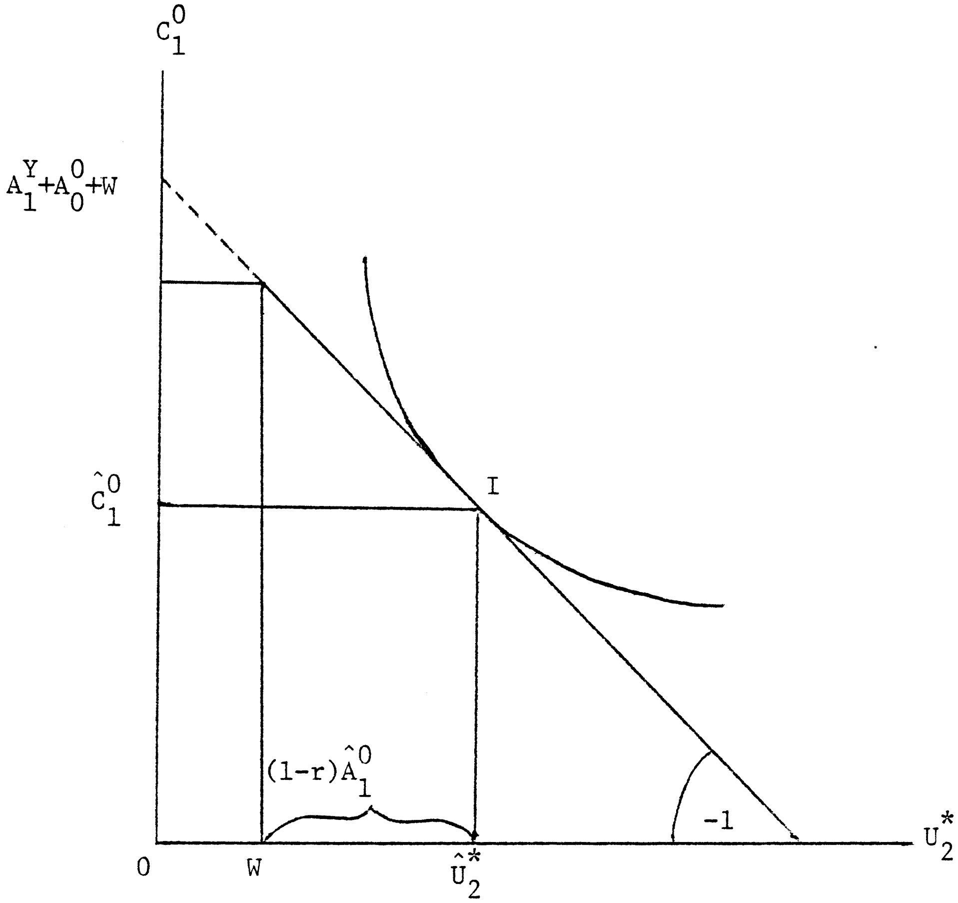

Figure 2

Optimal Bequest

generation 1 is to increase their individual wealth by amount S and to decrease the wealth of members of generation 2 by rS, since they pay taxes of amount S while they are young and receive benefits of amount S when they retire. The decrease in wealth experienced by a member of generation 3, in present value terms, is (1-r)S. The individual budget constraints for generations 1, 2 and 3 are written below:

$$\text{(II-7)}\quad A_1^y + A_0^0 + S = C_1^0 + (1-r)A_1^0,$$

$$\text{(II-8)}\quad W + (1-r)A_1^0 - rS = C_2^y + (1-r)C_2^0 + (1-r)^2A_2^0$$

and

$$\text{(II-9)}\quad (1-r)W + (1-r)^2A_2^0 - (1-r)S = C_3^y(1-r) + (1-r)^2C_3^0 + (1-r)^3A_3^0.$$

Members of generation 2 see the anticipated consumption levels of their children as

$$\text{(II-10)}\quad U_3^* = (1-r)W + (1-r)^2A_2^0 - (1-r)S.$$

An individual of generation 1, seeing the loss in income to his children of amount rS, and knowing that their children see the loss in income to their children, generation 3, of amount (1-r)S, view their children's consumption level as

$$\text{(II-11)}\quad U_2^* = W + (1-r)A_1^0 - S.$$

The social budget constraint for a member of generation 1 is

$$\text{(II-12)}\quad A_1^y + A_0^0 + W + S - rS - (1-r)S = C_1^0 + U_2^*.$$

The result is that the social budget constraint has not changed. Barro reasons that if the individual was going to leave a bequest before social security was introduced, then the introduction of the program, will cause members of generation 1, to save an additional amount, S, in order to offset the loss in social income of their children. For a member of generation 2, the loss in income when he is young is amount S, however, since he knows that he will now receive a bequest, that is larger by the amount (1+r)S, when he retires, he will not alter his consumption pattern. The result is that he will reduce savings while he is young by amount S. The total effect of social security on savings is to change the composition of savings. Older people will save more and younger people will save less, however, total savings will be the same as without the program. Barro concludes that social security does not affect net capital accumulation.

II-4 Feldstein's Reply to Barro

In response to Barro's article, Feldstein questions two crucial assumptions that Barro makes. The first assumption is that transfers are made only in the form of unrestricted purchasing power left in a bequest. Feldstein argues that most families make intergenerational transfers in the form of support to their offspring's consumption, when they are children. An example of this is parental support for education. In this case, even if social security causes the older generation to make larger transfers of resources to their heirs, these transfers may not constitute a sizable increase in the older generation's bequests of unrestricted purchasing power.

Feldstein also questions Barro's assumption that the economy is static and that people believe that the social security program will eventually be terminated. These two assumptions are crucial to Barro's results. Feldstein claims that the post tax return on interest earnings, denoted by r_n, is likely to be less than the growth rate of the wage base, denoted by g, for the social security program. If r_n is less than or equal to g and people do not believe the program will be terminated in the future, then social security will create wealth in Barro's model. For example, assume that r_n equals g, then with the introduction of social security, the wealth of members of generation 1 increases by amount S. For a member of generation 2 the program taxes him by amount S when he is young and pays him back, in present value terms, amount $S(1+g)/(1+r_n)$. The individual's life time wealth is not altered by the program. For an individual of generation 3, social security, in present value terms, will tax him by amount $S(1+g)/(1+r_n)$ and pay him back $S(1+g)^2/(1+r)^2$, when he retires. The effect of social security on total social debt is

$$\text{(II-13)} \quad \int_0^{\infty} - Se^{(r_n-g)t} dt = - S(1-1) = 0.$$

Since there is no social debt created, but wealth of amount S was created for each member of generation 1, the social security program has created wealth. In this case, a member of generation 1 will increase both his consumption when he is old and the amount he will transfer to his children.

Figure 3 demonstrates how the increase of amount S in generation 1's wealth affects his behavior. Assuming that both goods are normal, he will increase, both his consumption from $\hat{C}_1^0$ to $\hat{\hat{C}}_1^0$ and his expenditures on bequest, from $(1-r)\hat{A}_1^0$ to $(1-r)\hat{\hat{A}}_1^0$. A member of generation 2 will increase his planned consumption in both periods and increase his bequest expenditures to his children, because he expects to receive a larger bequest, from his parents, of amount $(1-r)(\hat{\hat{A}}_1^0 - \hat{A}_1^0)$. The result is that savings of generation 1 will go up by amount $(1-r)(\hat{\hat{A}}_1^0 - \hat{A}_1^0)$; savings of generation 2 will go down by amount S, plus an additional amount, equal to the increase in generation 2's current consumption, caused by the larger bequest they expect to receive. The net effect on savings will be negative. If r_n is less than g, this would cause a further decrease in savings.

II-5 Buchanan's Reply to Barro

In another response to Barro's article, James Buchanan questions one of the main assumptions in Barro's model. This assumption is that people fully discount the payroll taxes in the future. If this is true, then this would mean that deficit spending does not affect the net sum of government and private savings in an economy. It would also mean that politicians would be indifferent to a pay-as-you-go and a fully funded social security program. In addition, it would mean that politicians and people would be indifferent toward taxation and deficit spending. This does not appear to be true.

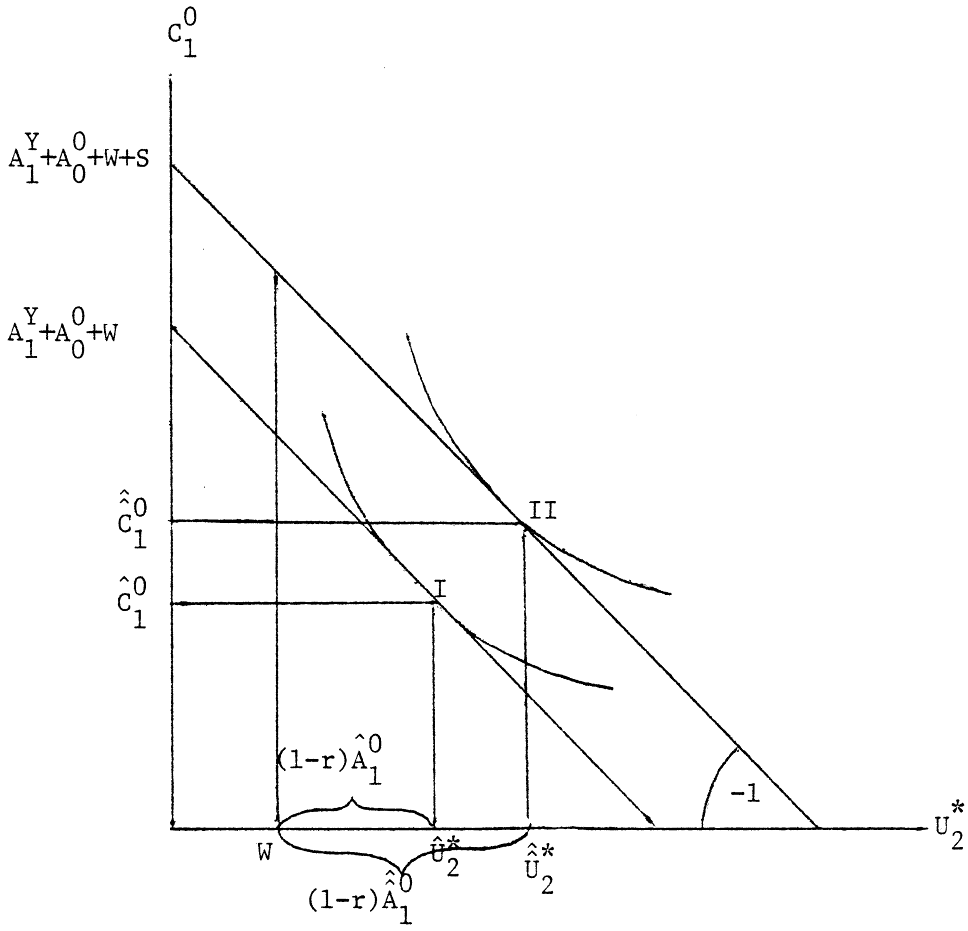

Figure 3

Social Security's Effect on Bequests

II-6 Conclusion

Barro's emphasis on bequests is an interesting extension of the life-cycle model, however, it does not seem likely that people would fully, or even substantially, offset the debt created by the social security program. In addition, Munnell's claim that the positive retirement effect of social security on savings offsets the negative benefit effect, loses some of its force once the program is fully mature. One of her arguments for early retirement is that social security has an income effect for older individuals. This has been true in the past because people who received benefits had paid very little into the program. This will not be true in the future. It is not likely that a mature social security program will create a substantial income effect for retirees in the future. Finally, Munnell's claim, that aggregate savings are unaffected by transfers of income from workers to retirees, is false in a mature social security program. Retirees do not view their benefits as transfers, rather, they have planned to receive these entitlements and have based their consumption plans on these expected benefits.

In the rest of this paper it is assumed that Barro's bequest effect and Munnell's retirement effect are negligible in a mature social security program. Future analysis will be limited to how Feldstein's benefit and payroll tax effects influence individual savings in both certainty and uncertainty settings. By incorporating Feldstein's effects on individual consumption-savings and portfolio decisions, it is now possible to examine the impact of a social security program on wages and interest rates.

CHAPTER III

DETERMINISTIC AND RANDOM GROWTH MODELS

In this section individual savings behavior is incorporated into single-sector, neo-classical growth models. The purpose of this chapter is to demonstrate how savings behavior affects long-run equilibrium capital-labor ratios and wage rates. Once these growth models are developed, a simplified pay-as-you-go social security program is imposed. The impact of the Feldstein benefit and tax effects on savings behavior is then followed through the model, to show how social security influences long-run equilibrium in both the deterministic and random growth models.

III-1 Deterministic Model

This section examines how a mature, pay-as-you-go social security program affects equilibrium wages and interest rates. In this model, which is based on Peter Diamond's 1965 article in American Economic Review, "National Debt In a Neoclassical Growth Model," it is assumed that the economy has an unchanging constant return to scale aggregate production function:

(III-1) $Y = F(K,L)$.

All decisions are made at the end of each period. These decisions include the allocation of wealth between current consumption and investment in equity for future consumption. The capital argument in the aggregate production function is equal to net savings plus the capital stock employed in the previous period. It is assumed that there is no depreciation and that labor grows at the rate of n percent each period. Individuals in this economy live for two periods; they work the first period and retire during the second period. Each person has a utility function, U, which is a function of consumption during the two periods of his life:

$$\text{(III-2)} \quad U = U(e^1_t, e^2_{t+1}),$$

where e^1_t is consumption in period t by an individual of the younger generation and e^2_{t+1} is consumption in period t+1 by a member of the older generation. It is assumed that there is no bequest motive. It is also assumed that all markets are competitive and that there is no uncertainty concerning the future.

By studying the behavior of a representative individual born in period t, it is possible to examine the market relationships in this economy. The individual works during period t. In return, he receives a wage, w_t, at the end of this period, which is equal to the marginal product of labor:

$$\text{(III-3)} \quad W_t = F_L(K_t, L_t).$$

He allocates this wage between current consumption and investment in equities for future consumption in order to maximize his utility. The rate of return on investment between t and t+1, denoted by r_{t+1}, is equal to the marginal product of capital. By investing in equities, the members of the younger generation make up the supply side of the capital market. In period t the individual consumes the difference between his wage and the amount he invests in the capital market:

$$\text{(III-4)} \quad e^1_t = W_t - S_t.$$

In period t+1 he consumes his savings plus the return on his investment:

$$\text{(III-5)} \quad e^2_{t+1} = S_t(1+r_{t+1}).$$

The capital demanders in this economy are firms, who wish to employ capital, for production during period t+1. These firms will demand capital up to the point where the marginal product of capital equals the interest rate:

$$\text{(III-6)} \quad r_{t+1} = F_K(K_{t+1},L_{t+1}).$$

The aggregate production function can be written as

$$\text{(III-7)} \quad F(K_t,L_t) = L_t f(k_t),$$

where

$$k_t = K_t/L_t.$$

This implies a relationship between the marginal product of labor and capital, which is known as the factor price frontier:

$$\text{(III-8)} \quad W = \phi(r).$$

Since the marginal product of capital is equal to the interest rate and the remainder of the output goes toward wages, (III-9) $W = f(k) - kf'(k)$,

an explicit relationship between wages and interest rates can be derived. This is brought about by expressing the capital-labor ratio as a function of the interest rate:

(III-10) $k = h(r)$;

differentiating k_t, with respect to r:

(III-11) $dk_t = h'dr$;

and then differentiating W, with respect to r, which results in equations (III-12) and (III-13):

(III-12) $$dW = f'(k)h'dr - kf''(k)h'dr - h'f'(k)dr$$
$$= -kf''(k)h'dr,$$

and

(III-13) $$dW/dr = -kf''(k)h' = \phi'(r).$$

Since $h' = dk/dr$ and $dr = f''(k)dk_t$, then $h' = 1/f''(k)$. Substituting this into equation (III-13) produces

(III-14) $$dW/dr = -k < 0, \text{ and } d^2W/dr^2 = -1/f''(k) > 0.$$

This negative relationship between equilibrium wages and interest rates, known as the factor price frontier, is represented by figure 4. Figure 4 demonstrates that, given any interest rate, the wage rate is determined for that period. In order to find the interest rate, it is necessary to explore the capital market.

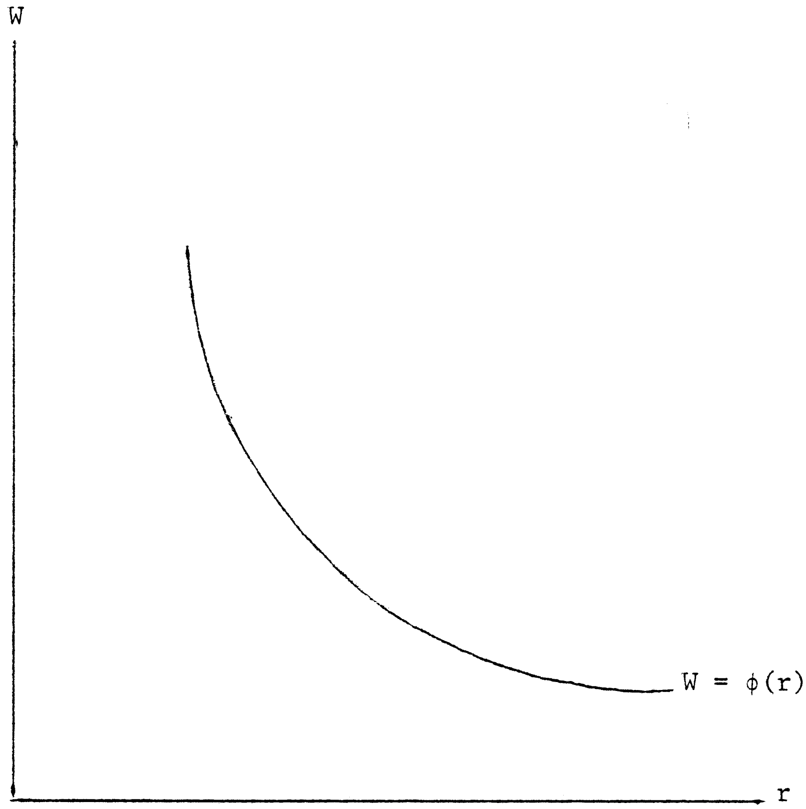

Figure 4

Factor-Price Frontier

The supply side of the capital market is the summation of savings from the younger generation. Their objective is to maximize life-time utility. The maximization problem is

$$\text{(III-15)} \quad \underset{e_t^1,\, e_{t+1}^2}{\text{Max}}\ U(e_t^1,\ e_{t+1}^2),$$

subject to

$$W_t = e_t^1 + e_{t+1}^2(1+r_{t+1}).$$

An individual's first period consumption can be expressed as, $W_t - s_t$, and his second period consumption as, $s_t(1+r_{t+1})$. The maximization problem for a representative individual can be rewritten as

$$\text{(III-16)} \quad \underset{s_t}{\text{Max}}\ U(W_t - s_t,\ s_t(1+r_{t+1})).$$

The first order condition with respect to s_t is

$$\text{(III-17)} \quad -U_1 + U_2(1+r_{t+1}) = 0,$$

where

$$U_1 = \partial U/\partial e_t \text{ and } U_2 = \partial U/\partial e_{t+1}.$$

Equation (III-17) can be rewritten as

$$\text{(III-18)} \quad U_1 = (1+r_{t+1})U_2.$$

Equation (III-18) shows the marginal rate of substitution between present and future consumption. Equations (III-16) and (III-18) demonstrate that the quantity saved is a function of the wage and the interest rate as shown below:

(III-19) $s_t = s(W_t, r_{t+1})$.

It is assumed that s_t is differentiable and that savings is a normal good, meaning, $0 < \partial s/\partial W < 1$. However, $\partial s/\partial r$ may be positive or negative.

To produce the supply schedule of capital, the individual savings functions are aggregated. This is shown in equation (III-20):

(III-20) $S_t = s_t L_t = L_t\ s(W_t, r_{t+1})$.

The demand curve for capital, which relates the capital stock in period t+1 to the interest rate, is the marginal product of capital as a function of the capital-labor ratio. This relationship is demonstrated in equation (III-21):

(III-21) $r_{t+1} = f'(K_{t+1}/L_{t+1})$,

where

$$\partial r_{t+1}/\partial K_{t+1} < 0 \text{ and } \partial^2 r_{t+1}/\partial K_{t+1}^2 > 0.$$

The equilibrium condition in the capital market is determined by equating demand and supply:

(III-22) $K_{t+1} = S_t$,

producing equation (III-23):

(III-23) $r_{t+1} = f'(S_t/L_{t+1}) = f'(s(W_t,r_{t+1})/(1+n))$.

Equation (III-21) implies that the demand curve is downward sloping, but the supply curve may be positively or negatively sloped. It is assumed that the supply curve always intersects the demand curve from above as shown in figures 5A and 5B. This assumption excludes the example shown in figure 5C. In this case an increase in the supply of capital causes the equilibrium quantity supplied and demanded to decline, and the equilibrium interest rate to rise.

Given that the case shown in figure 5C is ignored, a higher wage in period t results in an outward shift of the supply schedule, which causes an increase in equilibrium savings and a lower interest rate. By altering the wage levels in period t, the equilibrium interest rates in period t+1, can be traced out. This relationship is denoted as

(III-24) $r_{t+1} = \psi(W_t)$.

It is assumed that ψ is differentiable. The relationship between the current wage, W_t, and next period's interest rate, r_{t+1}, is determined by differentiating equation (III-23) by W_t. The results are shown in equation (III-25):

(III-25) $dr_{t+1}/dW_t = (f''\partial s/\partial W_t)/(1+n-f''\partial s/\partial r_{t+1}) < 0$.

Equation (III-25) implies that an increase in wages in period t causes individuals to save more at a given interest rate. This increase in savings results in a decline in the interest rate.

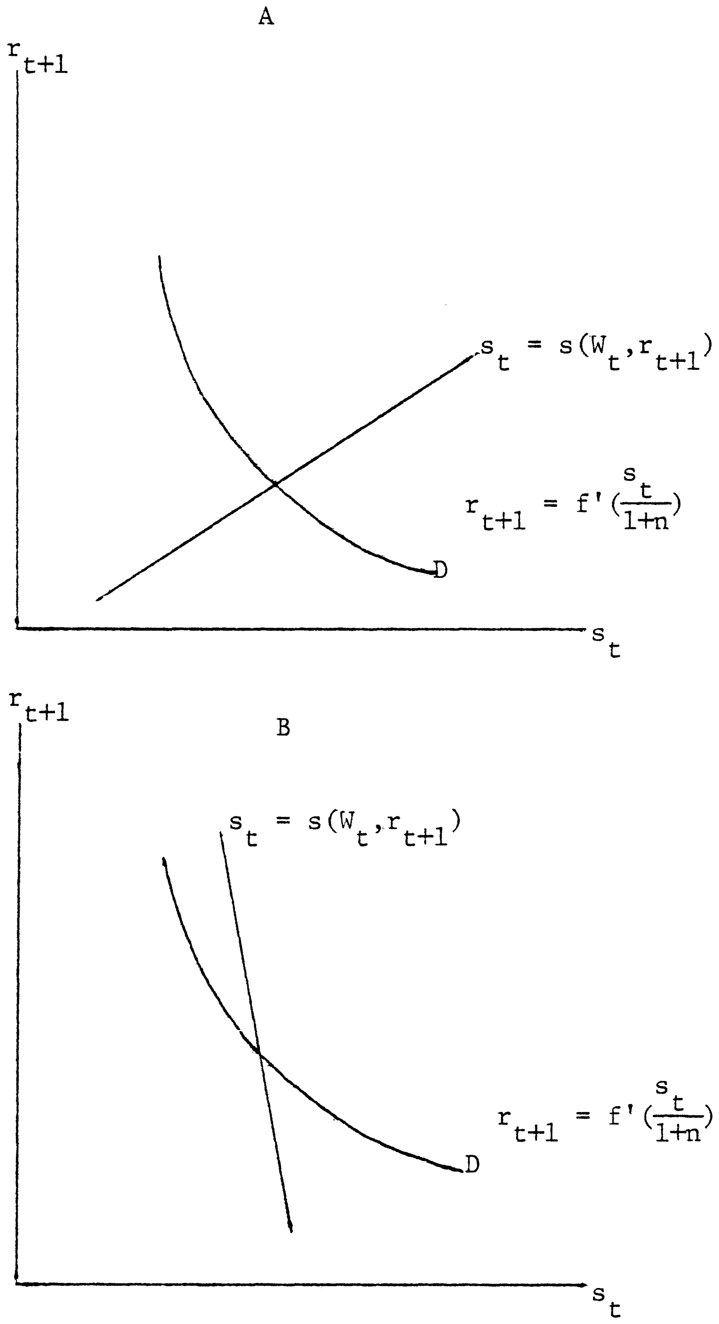

Figure 5

Capital Market Supply and Demand Curves

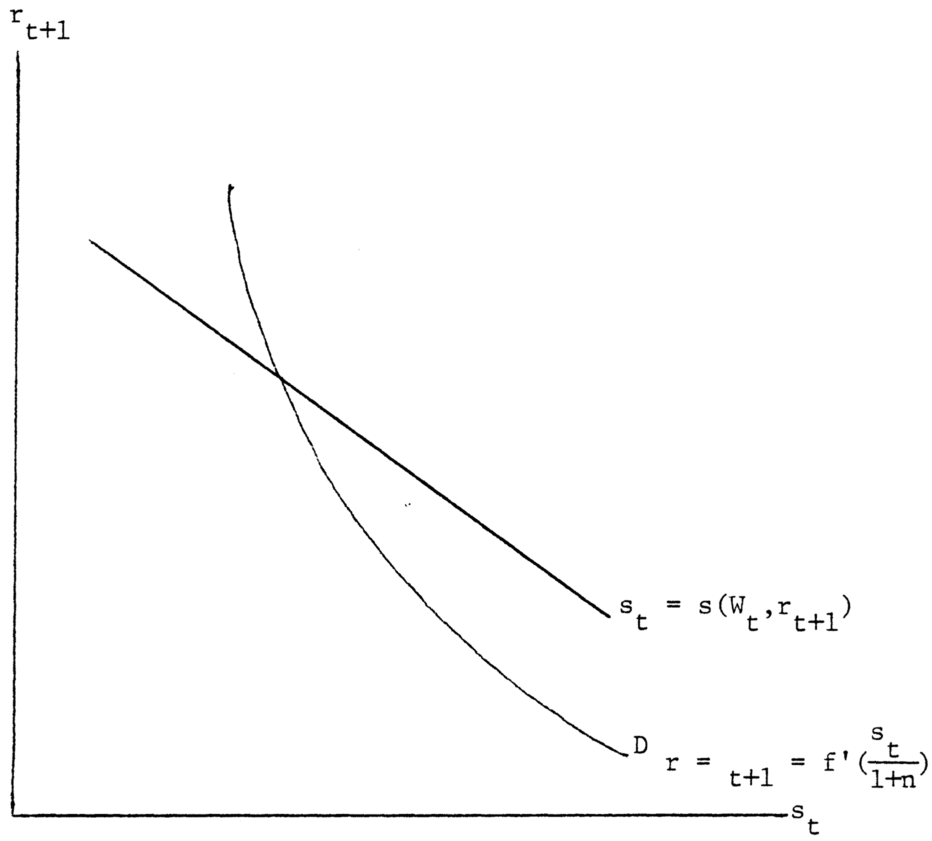

Figure 5C

Perverse Capital Market Supply Curve

The economy is represented in figure 6. This diagram contains both the factor price frontier shown in figure 4, and the wage-interest rate relationship in equation (III-22). A necessary condition for the economy to have a stable equilibrium is that the ϕ function intersects the ψ function from below. Given this assumption, the economy can start at any point on the factor-price frontier and a path can be traced toward long-term equilibrium. For example, if the economy is at point I, the interest rate is r_1. The wage rate, W_1, is determined by the factor-price relationship. Once W_1 is known, the interest rate for the next period is determined by the ψ function. At point E the economy is in long-run equilibrium where the wage rate, W^*, and the interest rate, r^*, remain constant over time. The capital-labor ratio will also remain constant.

III-2 The Effect of Social Security on Wages and Interest Rates

It is now possible to introduce a simplified social security program into the economy to examine how it affects long-run equilibrium wages and interest rates. In this model it is assumed that the social security tax rate, τ, is constant over time. Current benefits will depend on the tax rate, the current wage and population growth. In order to find the program's effect on savings, it is necessary to investigate the supply side of the capital market, where individuals make their consumption-savings decisions. The representative individual earns a wage of W_t which is taxed at the rate of τ in the first period of his life. He allocates his net income between current consumption in period t and

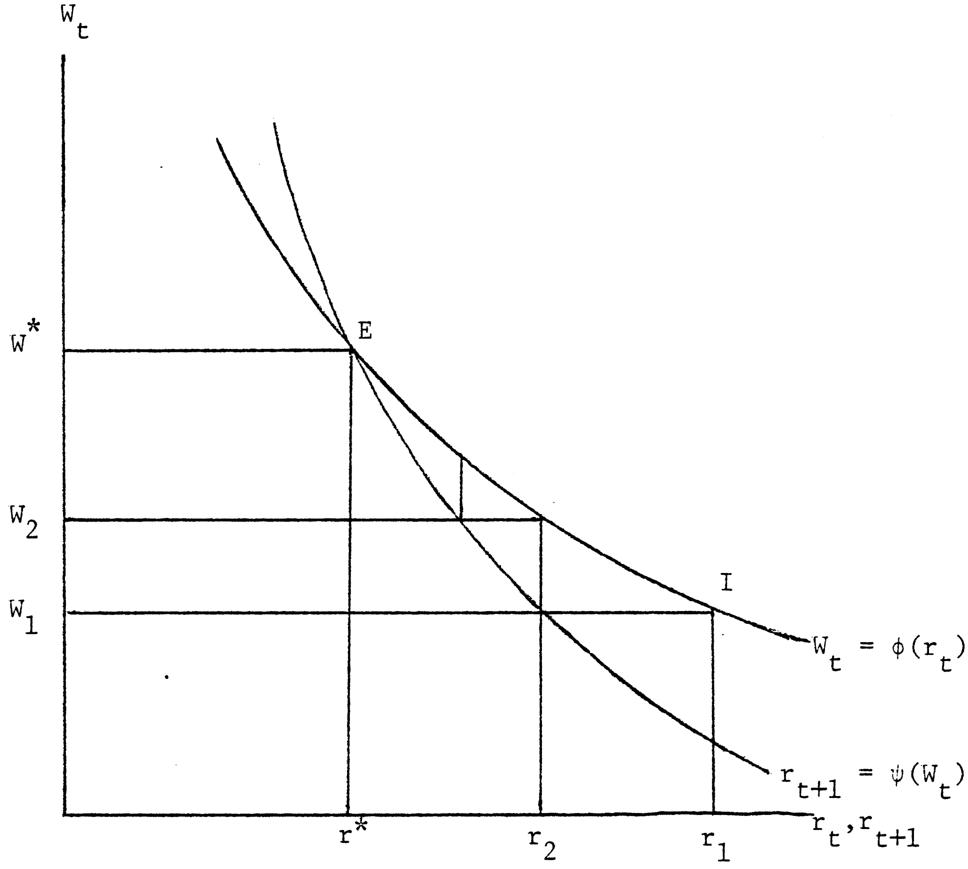

Figure 6

Long-run Equilibrium

savings for future consumption. In period t+1 the individual consumes his social security benefits plus his savings. In formal terms the optimization problem faced by a representative individual is

$$\text{(III-26)} \quad \underset{e_t^1, e_{t+1}^2}{\text{Max}}\ U(e_t^1, e_{t+1}^2),$$

subject to

$$W_t + SS_{t+1}(1+r_{t+1}) = \tau\ W_t + e_t^1 + e_{t+1}^2/1+r_{t+1}.$$

The term, SS_{t+1}, is the social security benefit an individual receives in the second period of his life. It is assumed that current social security tax payments are equal to current benefits as shown below:

$$\text{(III-27)} \quad SS_{t+1} = \tau\ W_{t+1}(1+n).$$

$\tau\ W_t$ is the payroll tax he pays during his working life. The maximization problem can be rewritten as

$$\text{(III-28)} \quad \underset{S_t}{U}(W_t(1-\tau) - S_t,\ S_t(1+r_{t+1}) + \tau\ W_{t+1}(1+n)),$$

where

$$e_t = W_t(1-\tau) - S_t \text{ and } e_{t+1} = S_t(1+r_{t+1}) + \tau\ W_{t+1}(1+n).$$

The first order condition, with respect to savings, is

$$\text{(III-29)} \quad -U_1 + U_2(1+r_{t+1}) = 0.$$

In order to find out how social security affects savings, equation (III-29) is totally differentiated with respect to S_t and τ. The result is represented in equation (III-30):

$$\text{(III-30)} \quad dS_t/d\tau = (-U_{11}W_t - U_{22}(1+r_{t+1})W_{t+1})/(U_{11}+U_{22}(1+r_{t+1})^2) < 0.$$

The first term in the numerator shows that social security decreases disposable income, causing the individual to reallocate some of his intended savings toward current consumption. This term is known as the Feldstein payroll tax effect. The second part of the numerator demonstrates that social security increases second period income, causing a reduction in the need to save, which leads to a further reduction in savings. This second term is Feldstein's benefit effect.

Equation (III-30) implies that social security reduces the amount individuals save at a given wage rate. This means that the supply of capital is reduced, causing the ψ function to shift out as shown in figure 7 and equation (III-31):

$$\text{(III-31)} \quad dr_{t=1}/d\tau = (f''\partial s/\partial\tau)/(1+n-f''\partial S/\partial r_{t+1}) > 0.$$

The long term result of this social security program is to decrease the equilibrium capital-labor ratio, which leads to an increase in interest rates from r* to r**, and a decrease in wages from W* to W**. A simplified example of social security's effects on wages and interest rates is presented in the model below.

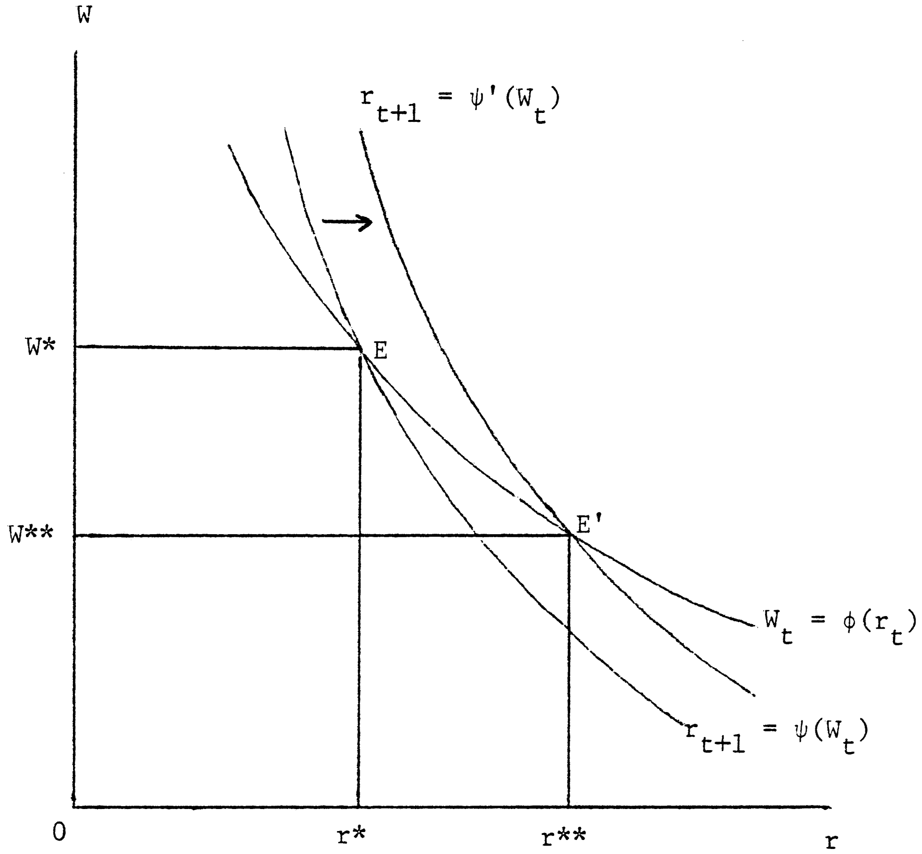

Figure 7

Social Security's Effect on Long-run Equilibrium

An Example

Consider an economy with Cobb-Douglas production and utility functions. The utility function can be expressed as

(III-32) $U(e_t, e_{t+1}) \equiv \gamma \log e_t + (1-\gamma) \log e_{t+1}$.

The maximization problem for a representative individual is

(III-33) $\underset{S_t}{\text{Max}}\ \gamma \log (W_t - S_t) + (1-\gamma)\log(S_t(1+r_{t+1}))$.

The saving function derived from equation (iii-32) is

(III-34) $S_t = (1-\gamma)W_t$.

Given that the production function is

(III-35) $y_t = A(k)^{\alpha}$,

the ψ function becomes

(III-36) $r_{t+1} = \alpha A((1-\gamma)W_t/(1+n))^{\alpha-1}$.

The factor price relationship is

(III-37) $W_t = (1-\alpha)\, \alpha^{\frac{\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} r_t^{\frac{\alpha}{\alpha-1}}$.

The long-run equilibrium interest rate, r*, is determined by combining equations (III-36) and (III-37) which becomes

(III-38) $r_{t+1} = \alpha^{1-\alpha}(1-\gamma)^{\alpha-1}(1-\alpha)(1+n)^{1-\alpha}r_t^{\alpha}$, and then setting r_t equal to r_{t+1}, producing

(III-39) $r^* = \alpha(1+n)/(1-\gamma)(1-\alpha)$.

The long-run equilibrium wage rate is

(III-40) $W^* = A^{\frac{1}{1-\alpha}} ((1+n)/(1-\gamma)(1-\alpha))^{\frac{\alpha}{\alpha-1}}$.

By introducing a social security program into this economy the maximization problem is altered to

(III-41) $\text{Max}\ \gamma \log(W_t(1-\tau)-S_t) + (1-\gamma) \log (S_t(1+r_{t+1}) + \tau W_{t+1}(1+n))$.

The savings function derived from equation (III-41) is

(III-42) $S'_t = (1-\gamma)(1-\tau)W_t/(1+\tau(1+n)(1-\gamma))$.

By comparing equation (III-42) with equation (III-34), it is evident that social security reduces the amount people save at a given wage level. The ψ function in this case is

(III-43) $r_{t+1} = \alpha A((1-\gamma)(1-\tau)W_t/(1+\tau(1+n)(1-\gamma))(1+n))^{\alpha-1}$.

Equation (III-43) shows that an increase in the social security tax rate causes the ψ function to shift out. This was previously demonstrated in Figure 7 for the general case.

The long-run equilibrium interest rate, r^{**}, is

(III-44) $r^{**} = \alpha(1+n)(1+\tau(1+n)(1-\gamma))/(1-\alpha)(1-\gamma)(1-\tau)$.

The long-run equilibrium wage rate is

$$(\text{III-45})\quad W^{**} = A^{\frac{1}{1-\alpha}}((1+n)(1+\tau(1+n)(1-\gamma))/(1-\alpha)(1-\gamma)(1-\tau))^{\frac{\alpha}{\alpha-1}}.$$

Comparing equations (III-44) and (III-45) with (III-39) and (III-40) indicates that social security causes the long-run equilibrium interest rate to rise and the wage rate to fall as shown in figure 8.

III-3 Production Uncertainty

In this section technological uncertainty is introduced into the basic framework. Then the existence and properties of long-run stochastic equilibrium are derived and examined in order to learn how social security affects this equilibrium. A definition of long-run stochastic equilibrium is given below.

Definition: Long-run stochastic equilibrium, or steady-state, is defined as a closed interval of capital-labor ratios, in which the economy's capital-labor ratio enters and is incapable of escaping, regardless of the starting point of the economy.

In this model interest rates and wages are influenced, but are not totally determined, by the amount individuals decide to invest in firms, in order to provide income for their retirement years. After describing the existence and properties of long-run equilibrium, a pay-as-you-go social security program is introduced into the economy to examine how equilibrium is altered. In this economy, which has an infinite life, social security shifts the steady-state capital-labor ratio distribution to a lower level, causing wages to be lower and interest rates to be higher.

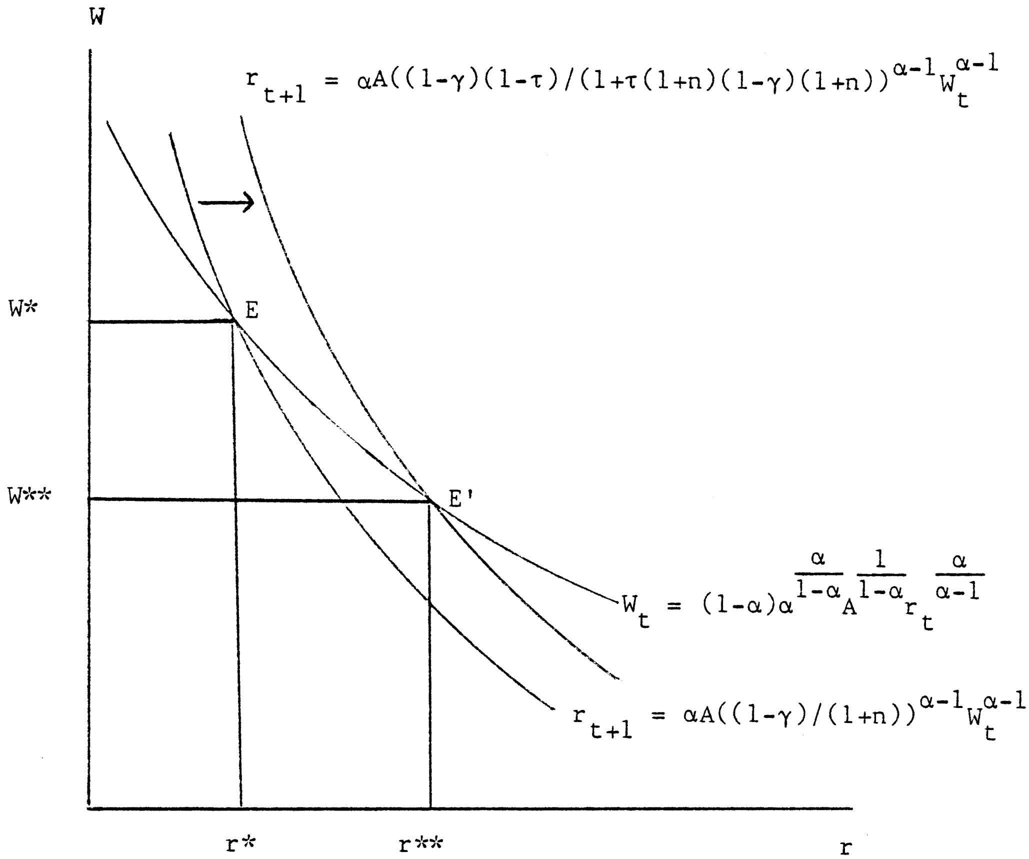

Figure 8

Social Security's Effect on Long-run
Equilibrium in a Cobb-Douglas
Production and Utility Function Model

As in the certainty model, individuals are assumed to be identical and live for two periods. In the first period they work and receive a wage. They retire in the second period and then die at the end of that period. All consumption and investment decisions are made at the end of each period. In addition, it is assumed that people can perceive n possible states that may occur next period and can attach probabilities to the likelihood of each possible state.

Firms are wholly equity financed, with each firm's production plan, contingent on only one possible state that may occur next period. This means that if a particular state does not occur the value of a firm becomes zero. These firms issue equity shares during the current period that pay off the following period, only if one particular state occurs, meaning, that all securities are Arrow-Debreu securities. There are n types of firms, one type for each possible state, with many firms in each state. For a given potential state, all firms can be aggregated into a single constant returns to scale production function of the form

$$\text{(III-46)} \quad Y_j = F_j(K_j,\ L_{t+1},\ A_j), \quad \text{for} \quad j=1,\ldots,n,$$

where

$Y_j \equiv$ total potential output of all state j firms,

$$A_j \equiv \begin{cases} A_j & \text{if state } j \text{ occurs} \\ 0 & \text{if state } j \text{ does not occur,} \end{cases}$$

$K_j \equiv$ total capital stock invested in all state j firms,

$L_{t+1} \equiv$ labor contribution in period t+1. It is assumed that labor grows at a constant rate of n% per period, and that the labor market always clears.

The random event, A_j, describes the potential technological state that may occur the following period. It is assumed that the A_j's are independent of the economy and that the probability distribution of the A_j's is constant over time. It is also assumed that the A_j's are bounded above and below by positive finite values. This means that there exist numbers, $0 < \theta < \psi < \infty$, such that, $A_j \in [\theta, \psi]$, for $j=1,\ldots,n$.

Equation (III-46) can be divided by L_{t+1}, to describe potential output during period t+1 in per-capita terms:

(III-47) $Y_j/L_{t+1} = F_j(K_j/L_{t+1}, 1, A_j) = f_j(k_j, A_j)$.

The following Inada conditions are assumed:

(III-48) $\lim_{k \to 0} f'(k, A_j) = \infty$, and $\lim_{k \to \infty} f'(k, A_j) = 0$

In addition, all markets are assumed to be competitive. Given these assumptions, the rates of return on equities and the wage rate for a given state j are

(III-49) $r_j = f'_j(k_j, A_j)$,

and

(III-50) $w_j = f_j(k_j, A_j) - k_j f'_j(k_j, A_j)$,

where $\partial w_j/\partial A_j > 0$.

It is assumed that if $w_j > w_k$ for a given $k_j = k_k$, then $w_j > w_k$ for all positive values where $k_j = k_k$. This means that the wage functions in (III-50) do not cross each other at different capital-labor ratio levels.

Capital demanders in this economy are firms who wish to employ capital for production during period t+1. The demand curve for capital, in a particular state contingent market, is the horizontal summation of the demand schedules, of all firms, specializing in that particular state contingent production plan. It is assumed that all firms within a particular state have identical production functions and that they demand capital up to the point where the marginal product of capital is equal to the interest rate. This is shown by equation (III-49). The slope of the demand curve, for any of the n states, is determined by taking the first derivative of equation (III-49) with respect to K_j:

$$\text{(III-51)} \quad \partial r_j / \partial K_j = f''_j(K_j/L_{t+1}, A_j) \frac{1}{L_{t+1}} < 0.$$

Equation (III-51) indicates that the demand curve is downward sloping for each of the n state contingent capital markets.

In order to derive the supply side of the n capital markets some definitions are necessary.

DEFINITIONS:

P_j ≡ value of all state j firms or the amount invested in all state j firms at time t.

X^i_j ≡ proportion of the value of all state j firms demanded by individual i at time t.

Π_j ≡ probability that state j will occur.

As in the certainty model, members of the younger generation make up the supply side of the capital market. At the end of each period these individuals have the opportunity to allocate their wage income between current consumption, e_t, and investment for future consumption, e_{t+1}. It is assumed that the utility functions are concave and additively separable. The maximization problem for a representative individual at the end of period t is

$$\text{(III-52)} \quad \max_{e_t, X_j} U(e_t) + \sum_{j=1}^{n} \pi_j V(X_j P_j (1+r_j)),$$

where

$$e_t = W_t - \sum_{j=1}^{n} X_j P_j,$$

and

$$E(e_{t+1}) = \sum_{j=1}^{n} \Pi_j \left(X_j P_j (1 + r_j)\right).$$

It is assumed that

$$\text{(III-53)} \quad \lim_{e_t \to 0} U'(e_t) = \lim_{e_{t+1} \to 0} V'(e_{t+1}) = \infty,$$

and

$$\text{(III-54)} \quad \lim_{e_t \to \infty} U'(e_t) = \lim_{e_{t+1} \to \infty} V'(e_{t+1}) = 0.$$

The objective function can now be rewritten in the form:

$$\text{(III-55)} \quad \max_{X_j} U(W_t - \sum_{j=1}^{n} X_j P_j) + \sum_{j=1}^{n} \Pi_j V(X_j P_j (1+r_j)).$$

The first-order conditions for equation (III-55) are

$$(\text{III-56}) \quad - U'P_j + \Pi_j V'P_j(1+r_j) = 0, \text{ for } j=1,\ldots,n.$$

The equations in (III-56) can be rearranged to show the marginal rates of substitution between current consumption and expected future consumption in the n possible states:

$$(\text{III-57}) \quad \frac{U'}{V'} = \Pi_j(1+r_j), \quad \text{for} \quad j = 1,\ldots,n.$$

Equations (III-55) and (III-57) imply that the X_j's are a function of the aggregate value of all firms in that particular state, the current wage, the rate of return on all n state securities, and the probabilities of the occurrence of each state:

$$(\text{III-58}) \quad X_j = X_j(P_j, W_t, \Pi_1, \ldots, \Pi_n, r_1, \ldots, r_n).$$

The supply schedule of capital for a particular state is the sum of the L_t individual investment functions, where in equilibrium

$$(\text{III-59}) \quad \sum_{i=1}^{L_t} X^i_j(P_j, W_t, \Pi_1, \ldots, \Pi_n, r_1, \ldots, r_n)P_j = P_j, \quad \text{for } j=1,\ldots,n.$$

Since individuals are identical, the amount of capital supplied by a person can be expressed as P_j/L_t. By combining the demand and the supply schedules for each state and by equating P_j and K_j, the equilibrium conditions in the capital markets are established. These conditions relate the potential rate of return of a particular equity to the current wage rate, the present and future labor supply, the probability of various states occurring, and their rates of returns on equity:

(III-60) $r_j = f'(P_j\ (W_t, L_t, \Pi_1, \ldots, \Pi_n, r_1, \ldots, r_n)/L_{t+1}, A_j)$.

Equation (III-60) is similar to the ψ function in the certainty case, although there are now n of these functions.

Given the assumptions made above, the capital demand curves are downward sloping, but the supply curves may have a positive or negative slope. The assumption is made, that the supply curves are either positively or more negatively sloped than the demand curves, as represented in Figure 9, A and B.

Assuming that consumption in both periods are normal goods, an increase in W_t shifts the supply curves out, in either 9A or 9B causing the capital-labor ratio in period t+1 to increase. This can be demonstrated mathematically by rewriting the individual's maximization problem as

(III-61) $$\underset{P_j/L_t}{\text{Max}}\ U(W_t - \sum_{j=1}^{n} P_j/L_t) + \sum_{j=1}^{n} \Pi_j\ V(P_j/L_t(1+r_j)),$$

and then setting the first order conditions equal to zero.

(III-62) $- U' + \Pi_j V'\ (1+r_j) = 0$, for $j=1,\ldots,n$.

Next, totally differentiating the equations in (III-62) by the P_j's and W_t to get:

(III-63) $\partial P_j/\partial W_t = U''/(U'' + [\Pi_j(1+r_j)]^2 V'') > 0$, for $j=1,\ldots,n$.

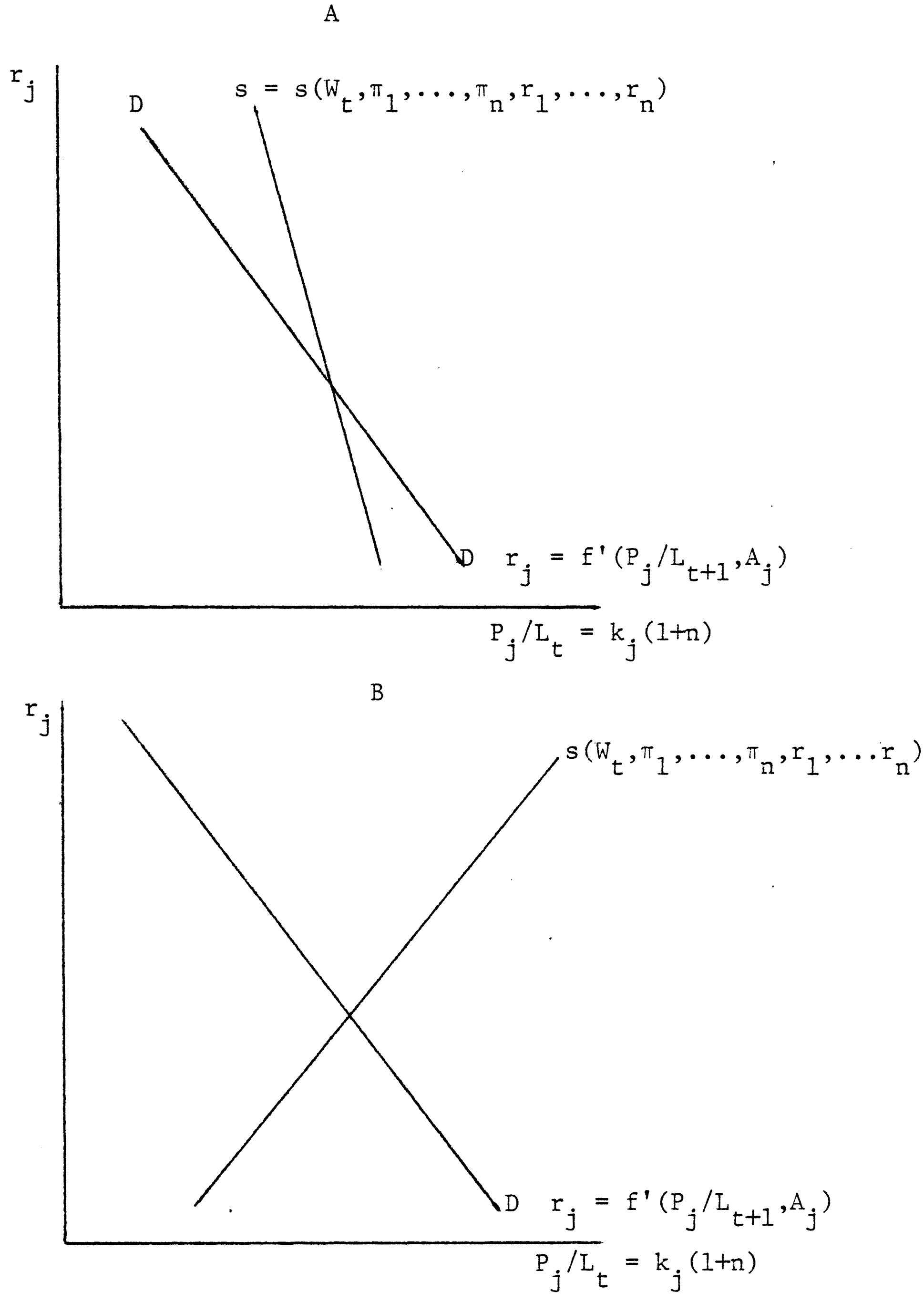

Figure 9

Capital Market Supply and Demand Curves

The equations in (III-63) imply that an increase in current wages shifts the supply curves out in each capital market.

The interest rate equilibrium condition in equation (III-60) can be rewritten to express the equilibrium capital-labor ratio for each state:

$$\text{(III-64)} \quad \sum_{j=1}^{n} P_j/L_{t+1} = H(W_t, r_1(W_t),\ldots,r_n(W_t), \Pi_1,\ldots,\Pi_n) = h(W_t),$$

where $P_j/L_{t+1} = h_j(W_t)$.

The capital-labor equilibrium condition in equation (III-64) represents the results, from the interaction of the supply and demand functions in each state contingent capital market. Once the current wage is known, the potential capital-labor ratios and interest rates can be determined. This is because the probability distribution of potential states, the A_j's, and the rate of growth of the labor force is assumed to be constant. The actual capital-labor ratio which occurs in period t+1 depends on W_t and the actual A_j, which occurs in period t+1. This is represented in equation (III-65):

$$\text{(III-65)} \quad k_{t+1} = h(W_t, A_{t+1}).$$

The current wage rate can be represented by a single stochastic difference equation:

$$\text{(III-66)} \quad W_{t+1} = f(h(W_t,A_{t+1}), A_{t+1}) - h(W_t,A_{t+1})f'(h(W_t,A_{t+1}), A_{t+1}).$$

Equation (III-66) is the stochastic difference equation driving the economy. It is now possible to examine how the economy approaches a steady-state. A formal definition of long-run stochastic equilibrium is stated below.

DEFINITION: The interval $[\underline{k}, \overline{k}]$ is said to be a stable interval for the stochastic process:

$k_{t+1} = h(W_t, A_{t+1})$ if for all $k \in [\underline{k}, \overline{k}]$, the smallest P_j/L_{t+1} in $h(f(k, \theta) - k\, f'(k, \theta)) \geq \underline{k}$ and the largest P_j/L_{t+1} in $h(f(k, \psi) - k\, f'(k, \psi)) \leq \overline{k}$,

where θ and ψ are the lower and upper bounds on the Aj's.

In order to derive the steady-state distribution that is bounded below by a $\underline{k}$ greater than zero, showing that the economy does not disappear, and bounded above by a $\overline{k}$ less than infinity, the following two theorems are stated and proved.

Theorem 1:

The steady-state capital-labor ratio is bounded above by a value less than infinity.

Proof:

From the Inada condition, $\lim_{k\to\infty} f'_j(k, A_j) = 0$, there exists a P^*_t/L_{t+1}, large enough, such that, $f(P^*_t/L_{t+1}, A_j) - P^*_t/L_{t+1} f(P^*/L_{t+1}, A_j) = W^* < P^*_t/L_{t+1}$, for all A_j. This implies that there exists a capital-labor ratio large enough, that it cannot be sustained. Define $\overline{\overline{P}}_t/L_{t+1}$ as the maximum P_j/L_{t+1} from $h(W_t)$, where W_t starts at W^*, and let $A_j = \psi$, for all t. If t grows large enough, $\overline{\overline{P}}_t/L_{t+1}$ will eventually equal $\overline{k}$. Once $\overline{k}$ is reached it will remain constant for $t=1,\ldots,\infty$. This $\overline{k}$ exists since $h(W_t)$ is positive and continuous. If $\overline{\overline{P}}_{t+1}/L_{t+2}$ is greater than $\overline{k}$, then $\overline{\overline{P}}_{t+2}/L_{t+3}$ will be less than $\overline{\overline{P}}_{t+1}/L_{t+2}$, and as t grows large enough $\overline{\overline{P}}_t/L_{t+1}$ will eventually equal $\overline{k}$, which is the least upper-bound of steady-state.

End of Proof

Theorem 1 is demonstrated graphically in Figure 10. In this diagram the capital-labor ratios are on the horizontal axis and the wage rates are on the vertical axis. By letting A_j equal ψ, and the investment function equal the maximum P_j/L_{t+1} from $h(W_t)$, it is unnecessary to examine any of the other possible states or investment functions, in order to find the upper bound on the steady-state equilibrium. If the economy starts at a capital-labor ratio greater than $\overline{k}$, it must eventually approach $\overline{k}$.

The following theorem proves that there is a positive steady state lower bound. Lemma 1 will be useful in proving the next theorem.

Lemma 1:

As $W_t \rightarrow 0$, the smallest $P_{j,t}/L_{t+1}$ from $h(W_t)$ will have the property that $h(f(P_{j,t}/L_{t+1}, \theta) - P_{j,t}/L_{t+1}f'(\ )) > W_t$.

<u>Proof</u>:

From the Inada condition, $\lim_{k\rightarrow 0} f'_j(k, A_j) = \infty$, for all A_j, there exists a $P^{**}_t/L_{t+1} > 0$, such that, $f(P^{**}_t/L_{t+1}, \theta) - P^{**}_t/L_{t+1}f'(\ )$ $\geqslant P^{**}_t/L_{t+1}$. For any P_t/L_{t+1}, $0 < P_t/L_{t+1} < P^{**}_t/L_{t+1}$,

$$\lim_{P_j/L_{t+1}\rightarrow 0} ([f(P_t/L_{t+1}, \theta) - P_t/L_{t+1}\ f'(P_t/L_{t+1}, \theta)] \div P_t/L_{t+1}) \rightarrow \infty$$

End of Proof

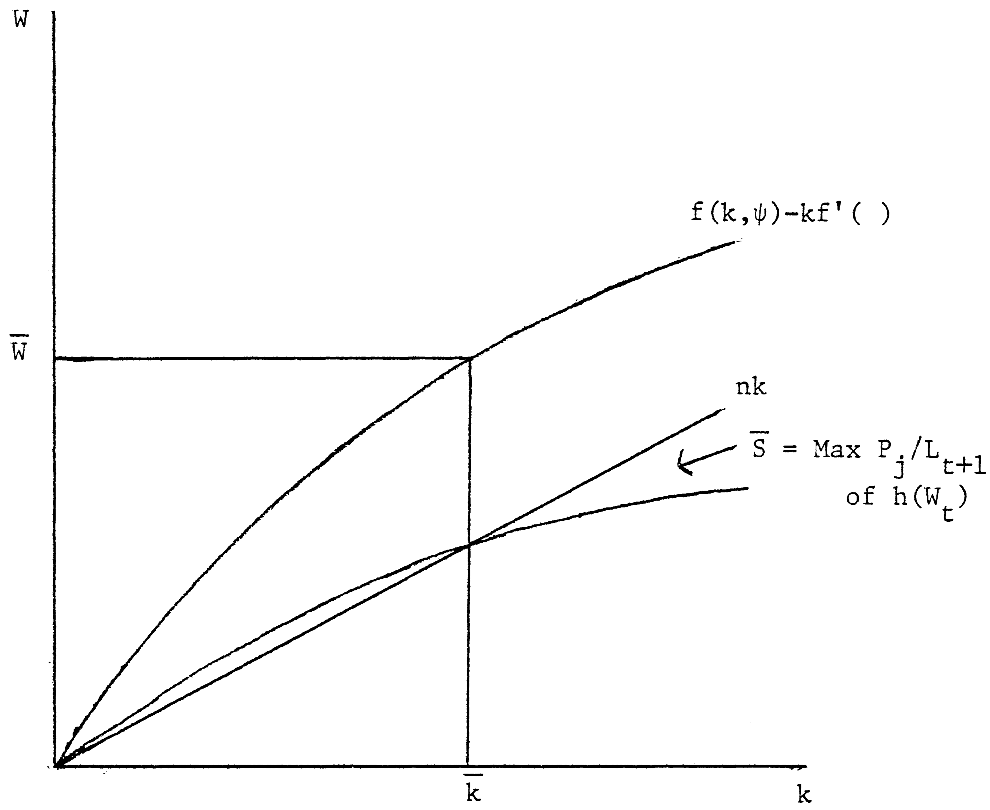

Figure 10

Upper Bound on Steady-State Wages
and Capital-Labor Ratios

Theorem II:

The steady-state capital-labor ratio is bounded below by a positive value.

Proof:

Given a $\underline{\underline{W}}_t$ where $\underline{\underline{W}}_t > 0$, define the minimum P_j/L_{t+1} of $h(\underline{\underline{W}}_t)$ as $\underline{\underline{P}}_t/L_{t+1}$. Assume $A_j = \theta$ over all t. As t grows large enough $\underline{\underline{P}}_t/L_{t+1}$ will eventually equal $\underline{k}$. Once $\underline{k}$ is reached it will remain for $t=1,\ldots,\infty$. If $\underline{\underline{P}}_t/L_{t+1}$ is less than $\underline{k}$, then from Lemma 1, $\underline{\underline{P}}_{t+1}/L_{t+2}$ will be greater than $\underline{\underline{P}}_t/L_{t+1}$; as t grows large enough, $\underline{\underline{P}}_t/L_{t+1}$ will eventually equal $\underline{k}$, which is the greatest lower bound of steady-state.

End of Proof

Theorem II is demonstrated in Figure 11. By letting A_j equal θ and by letting the investment function equal the minimum P_j/L_{t+1} from $h(W_t)$, it is unnecessary to examine any of the other possible states or investment functions, in order to find the lower bound on steady-state. If the economy starts at a capital-labor ratio less than $\underline{k}$, it must eventually approach $\underline{k}$.

Theorem I and II prove that if the economy starts with a capital-labor ratio greater than zero but less than infinity, it will eventually approach steady-state where $k \in [\underline{k},\overline{k}]$. Once the capital-labor ratio is within this interval, it will fluctuate due to the random technological variable, A_j, but it will remain within the lower and upper bounds. Figure 12 represents this stochastic equilibrium.

A simplified example of an economy in steady-state is given below.

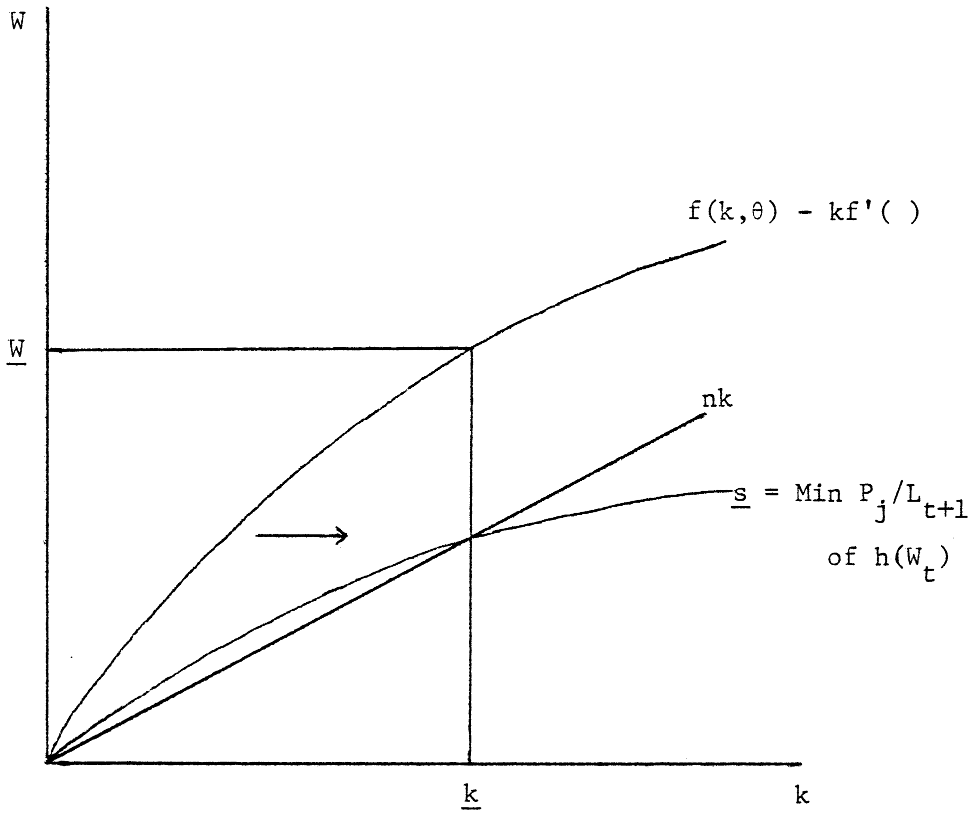

Figure 11

Lower Bound on Steady-State Wages
and Capital-Labor Ratios

An Example

As in section 1, consider an economy with Cobb-Douglas production and utility functions. The maximization problem can be expressed as

(III-67) $\text{Max}\ \ \gamma \log e_t + \Pi_1(1-\gamma) \log e_1 + \ldots + \Pi_n(1-\gamma) \log e_n,$
subject to

$$W_t = e_t + e_1/1+r_1 + \ldots + e_n/1+r_n.$$

The investment functions derived from equation (III-67) are

(III-68) $\sum_{j=1}^{n} P_j/L_{t+1} = \sum_{j=1}^{n} \Pi_j(1-\gamma)W_t/1+n.$

Equation (III-68) is the specific form of equation (III-64). The actual capital-labor ratio for period t+1 will depend upon which state becomes A_{t+1}:

(III-69) $k_{t+1} = \Pi_{t+1}(1-\gamma)W_t/1+n.$

Equation (III-69) is the specific form of equation (III-64).

Assuming a production function of the form:

(III-70) $y_t = A_t k_t^{\alpha},$

the capital market equilibrium condition in the n capital markets can be represented by equation (III-71):

(III-71) $r_j = \alpha A_j\, \Pi_j(1-\gamma)W_t/(1+n)^{\alpha-1}, \quad \text{for } j=1, \ldots, n.$

The wage in period t+1 will depend on equation (III-68) and the particular A_j that becomes A_{t+1}:

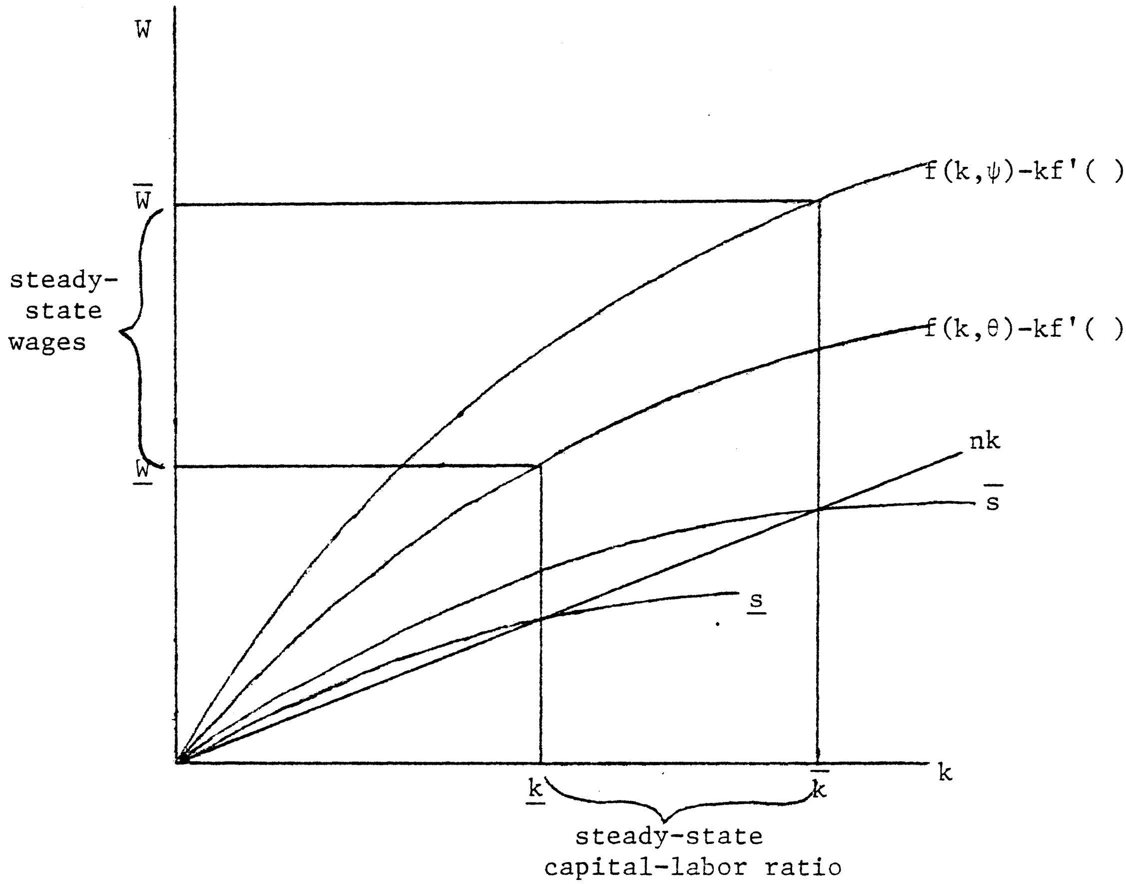

Figure 12

Long-run Equilibrium Wages and Capital-Labor Ratios

$$\text{(III-72)} \quad W_{t+1} = A_{t+1}(1-\alpha)\Pi^{\alpha}_{t+1} \; (1-\gamma)^{\alpha}(1+n)^{-\alpha}\, W^{\alpha}_{t} \, .$$

Equation (III-72) is the stochastic difference equation that is driving the economy.

In order to determine the steady-state lower bound, let A_j equal θ and $\underline{\Pi}$ equal the minimum Π_j for all t. The steady-state lower bound on wages and capital-labor ratios satisfies the conditions in equations (III-73) and (III-74):

$$\text{(III-73)} \quad \underline{W} = \lim_{t\to\infty} W_t = \underline{\Pi}^{\frac{\alpha}{1-\alpha}}\, \theta^{\frac{1}{1-\alpha}}(1-\alpha)^{\frac{1}{1-\alpha}}(1-\gamma)^{\frac{\alpha}{1-\alpha}}(1+n)^{\frac{-\alpha}{1-\alpha}},$$

and

$$\text{(III-74)} \quad \underline{k} = \underline{\Pi}(1-\gamma)\underline{W}/(1+n) = (\underline{\Pi}\theta(1-\alpha)(1-\gamma)/(1+n))^{\frac{1}{1-\alpha}}.$$

The steady-state upper bound is determined by letting A_j equal ψ and $\overline{\Pi}$ equal the maximum Π_j for all t. The steady-state upper bounds on wages and capital-labor ratios satisfies the conditions in equations (III-75) and (III-76):

$$\text{(III-75)} \quad \overline{W} = \lim_{t\to\infty} W_t = \overline{\Pi}^{\frac{\alpha}{1-\alpha}}\, \psi^{\frac{1}{1-\alpha}}\, (1-\alpha)^{\frac{1}{1-\alpha}}\, (1-\gamma)^{\frac{\alpha}{1-\alpha}}\, (1+n)^{\frac{-\alpha}{1-\alpha}},$$

and

$$\text{(III-76)} \quad \overline{k} = \frac{\overline{\Pi}(1-\gamma)\overline{W}}{1+n} = (\overline{\Pi}\, \psi(1-\alpha)(1-\gamma)/(1+n))^{\frac{1}{1-\alpha}}.$$

III-4 Effects of Social Security on Long-run Stochastic Equilibrium

With the introduction of a "Pay-As-You-Go" social security program, members of the younger generation experience two additional factors. During the working years social security taxes reduce disposable income, and when retirement arrives social security pays benefits. Equation (III-76) represents the revised maximization problem for an individual of the younger generation:

$$\text{(III-77)} \quad \underset{P_j/L_t}{\text{Max}} \; U(W_t(1-\tau_t) - \sum_{j=1}^{n} P_j/L_t) + \sum_{j=1}^{n} \Pi_j V[(P_j/L_t)(1+r_j)+\tau_{t+1}W_j(1+n)],$$

where:

τ_t is the social security tax in period t.

τ_{t+1} is the social security tax in period t+1.

It is assumed that $\tau_t = \tau_{t+1}$, and that these tax rates are constant over time. The first-order conditions from equation (77) are

$$\text{(III-78)} \quad - U' + \Pi_j \; V'(1+r_j) = 0, \text{ for } j=1,\ldots,n.$$

Totally differentiating the equations in (III-78) with respect to the P_j's and τ_t, gives the results:

$$\text{(III-79)} \quad dP_j/d\tau_t = -U''/(U'' + [\Pi_j(1+r_j)]^2V'') < 0, \text{ for } j=1,\ldots,n.$$

The equations in (III-79) imply that a reduction in disposable income, caused by the social security tax, decreases the amount invested in each of the n capital markets.

In order to examine the effect of social security benefits on investments, the equations in (III-78) are totally differentiated with respect to P_j and τ_{t+1}:

$$\text{(III-80)} \quad dP_j/d\tau_{t+1} = \frac{-\Pi_j(1+r_j)W_j(1+n)V"}{U" + [\Pi_j(1+r_j)]^2V"} < 0, \text{ for } j=1,\ldots,n.$$

The equations in (III-80) imply that social security benefits, by guaranteeing that a proportion of the future generation's wage will be transferred to the retired population, decreases the amount people invest in each of the n capital markets at a given wage.

The negative tax and benefit effects are represented diagrammatically by figures 13A or 13B. The two negative effects cause the supply curves to shift in because individuals desire to invest less in each of the capital markets.

With the introduction of social security into the economy, the equilibrium condition in the capital markets represented by equation (III-64), is altered. Assuming that the social security tax rate is constant over time, the new equation, representing the capital market equilibrium condition is

$$\text{(III-81)} \quad P_{j,t}/L_{t+1} = h_j^s(W_t),$$

or

$$\text{(III-82)} \quad \Sigma P_{j,t}/L_{t+1} = h^s(W_t), \text{ for } j=1,\ldots n.$$

In this case, $h(W_t) > h^s(W_t)$, for all $P_{j,t}/L_{t+1}$. This indicates that for a given W_t, k_{t+1} and W_{t+1} are less than they would be in the absence of a

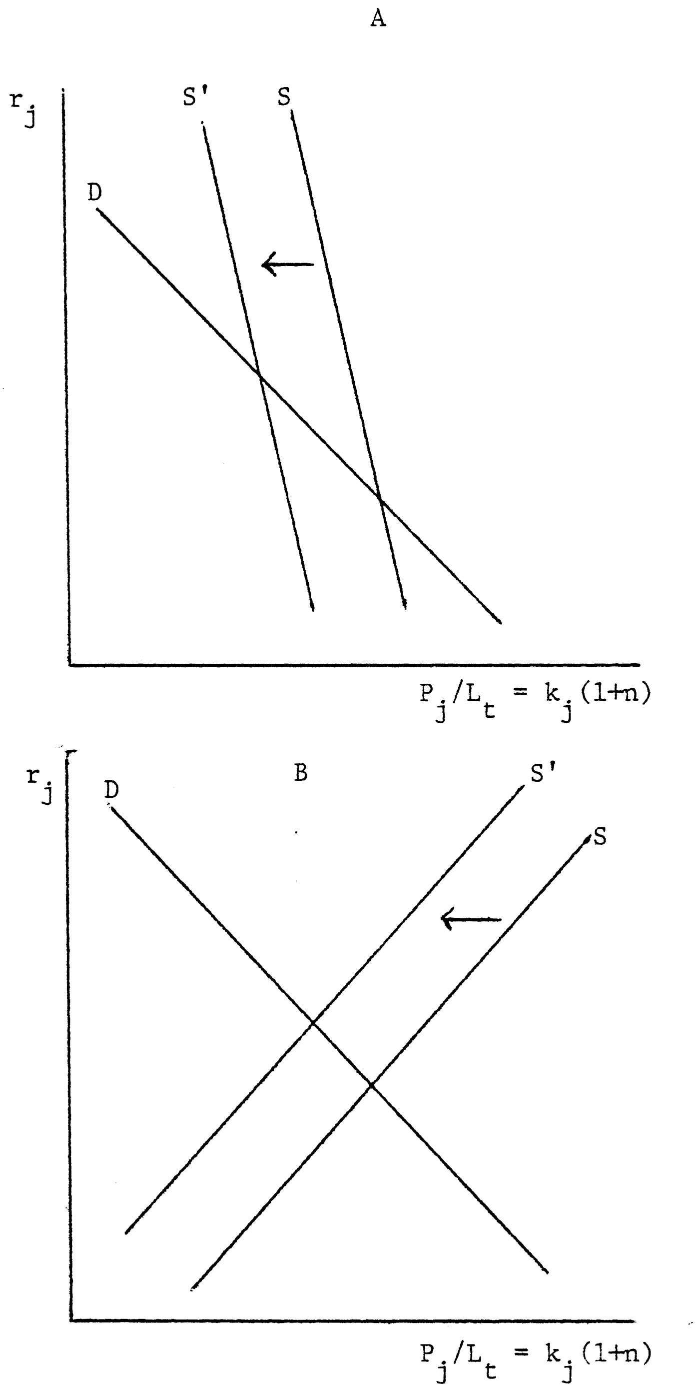

Figure 13

Social Security's Effect on Capital Markets social security program. As people invest less in each state contingent asset, the steady-state capital-labor ratios decline. This also causes steady-state wages to decline. The new least upper bound on capital-labor ratios can be defined as $\overline{k}^s$ where $\overline{k}^s < \overline{k}$. The new greatest lower bound can be defined as $\underline{k}_s$ where $\underline{k}_s < \underline{k}$. Figure 14 represents the change in the steady-state distribution. The result of a social security program is to shift the steady-state distribution of capital-labor ratios to the left and the distribution of wages down, causing future generations to have less income.

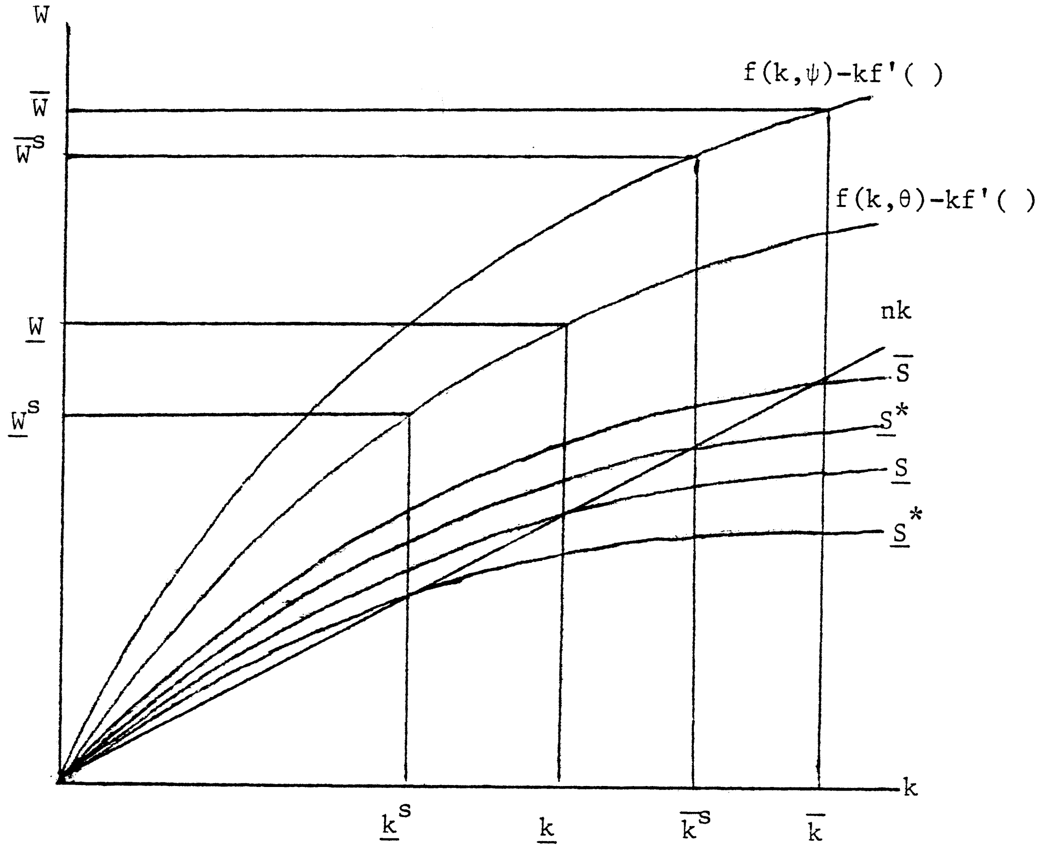

Figure 14

Social Security's Effect on Long-run Equilibrium

CHAPTER IV

EXTENSIONS TO THE PRODUCTION UNCERTAINTY MODEL

In this chapter extensions are added to the random growth model in order to examine how the model handles the following five economic questions. These questions include: determining the optimal amount of social security and how the level changes when individual wages differ, the relationship between the growth rate of the labor force and private investment, the impact of a fully funded social security program as compared to a pay-as-you-go program, and the introduction of technological growth.

IV-1 Optimal Social Security--Planner's Problem

In this section the assumption of a single social security tax rate for all states is relaxed, so that, specific tax rates can be set for each state. The tax rate for a specific state is defined as τ_j and tax rates are assumed to be constant over time. The expected social security benefits for state j are

$$\text{(IV-1)} \quad SS_j = \tau_j(1+n)W_j.$$

The maximization problem for a representative individual is

$$\text{(IV-2)} \quad \text{Max } U(W_t(1 - \sum_{j=1}^{n} \tau_j) - \sum_{j=1}^{n} P_j/L_t) +$$

$$\sum_{j=1}^{n} \Pi_j V(P_j/L_t(1+r_j) + \tau_j W_j(1+n)).$$

The optimal social security tax in period t and the benefits in period t+1 are determined by maximizing equation (IV-2) with respect to the τ_j's, for all n states.

In determining the optimal τ_j, the planner, unlike an individual who has negligible impact on the capital market must consider how the tax rates influence future wages and interest rates. As social security taxes and future benefits increase, individuals decrease the amount they privately invest in state contingent securities. This results in an expected increase in interest rates and a decrease in wage rates, for each of the n possible states. The increase in the interest rates is demonstrated by differentiating equation (IV-3), with respect to P_j, to get equation (IV-4):

$$\text{(IV-3)} \quad r_j = f'(P_j/L_{t+1}, A_j),$$

and

$$\text{(IV-4)} \quad \partial r_j/\partial P_j = f''(P_j/L_{t+1}, A_j)\ 1/L_{t+1} < 0.$$

The decrease in expected wage rates is determined by differentiating equation (IV-5), with respect to P_j, to get equation (IV-6):

$$\text{(IV-5)} \quad W_j = f(P_j/L_{t+1}, A_j) - f'(\ )\ P_j/L_{t+1},$$

and

$$\text{(IV-6)} \quad \partial W_j/\partial P_j = -f''(\ )\ P_j/(L_{t+1})^2 > 0.$$

Optimal social security is determined by taking the n first order conditions of equation (IV-2), with respect to the τ_j's, to get

$$\text{(IV-7)} \quad -U'W_t + \Pi_j\, V'[W_j(1+n) + (P_j/L_t)\ \partial r_j/\partial P_j\ \partial P_j/\partial \tau_j + \tau_j(1+n)\ \partial W_j/\partial P_j\ \partial P_j/\partial \tau_j] = 0, \quad \text{for } j=1,\ldots,n.$$

The equations in (IV-7) demonstrate that optimal social security is attained, when the marginal disutilities from both social security taxes on current consumption, $-U'W_t$, and the reduction in the amount of expected social security benefits, caused by lower expected wages, $\Pi_j V'\tau_j(1+n)\partial W_j/\partial P_j\ \partial P_j/\partial \tau j$, are equal to the marginal utility derived by higher expected social security benefits from higher taxes, $\Pi_j V'W_j(1+n)$, and higher private asset returns, resulting from higher interest rates, $\Pi_j V'P_j/L_t\ \partial r_j/\partial P_j\ \partial P_j/\partial \tau_j$. It is possible that at the optimum level of social security, individuals experience greater utility than they would without the program because a social security program may offer an alternative trade-off between present and expected future consumption as compared to the private market. An increase in the potential rates of return on private assets can allow individuals to increase their second period consumption even though their disposable income has declined.

Equation (IV-7) also shows that a decision maker, who is in charge of setting the tax rates on a pay-as-you-go social security program, has the capability of altering potential wages and interest rates. If the planner desires future lower interest rates and higher wages, he can lower the current tax rates and decrease expected future benefits.

IV-2 A Fully funded Social Security Program

In this section a fully funded social security program is analyzed in order to see how its effects on individual investment behavior differ from that of a pay-as-you-go program. A fully funded program is defined, as current benefits being equal to last period's tax revenues invested in the state that occured, multiplied by the current rate of returns on investments. This is represented by equation (IV-8):

$$\text{(IV-8)} \quad SS_j = \tau_j(W_t)(1+r_j).$$

It is assumed that social security tax revenues are invested in private securities. The maximization problem for a representative individual is

$$\text{(IV-9)} \quad \underset{P_j/L_t}{\text{Max}}\; U(W_t(1-\sum_{j=1}^{n}\tau_j) - \sum_{j=1}^{n}\frac{P_j}{L_t}) + \sum_{j=1}^{n}\Pi_j V(\frac{P_j}{L_t}(1+r_j) + \sum_{j=1}^{n}\tau_j W_t(1+r_j)).$$

The first order conditions for an individual who is maximizing his utility are

$$\text{(IV-10)} \quad -U' + \Pi_j\, V'(1+r_j) = 0, \text{ for } j=1,\ldots,n.$$

The impact of a fully funded social security program on private investment can be determined by totally differentiating the equations in (IV-11) with respect to the P_j's and τ_j's. The results are

$$\text{(IV-12)} \quad dP_j/d\tau_j = -(U''W_t + \Pi_j^2 V''(1+r_j)^2 W_t)/(U''/L_t+(\Pi_j^2 V''/L_t)(1+r_j)^2) < 0,$$

$$\text{for } j = 1,\ldots,n.$$

The equations in (IV-13) demonstrate that a fully funded social security program causes a decline in private investments, similar to that of a pay-as-you-go program.

The optimal tax rate can be determined by differentiating the objective function in (IV-9), with respect to the τ_j's, and setting the first order conditions equal to zero. This is shown in equation (IV-13):

$$\text{(IV-13)} \quad - U'W_t + \pi_j V'W_t(1+r_j) +$$

$$\Pi_j V' \frac{P_t}{L_t} \frac{\partial r_j}{\partial P_j} \frac{\partial P_j}{\partial \tau_j} + \Pi_j V' \tau_j W_t + \frac{\partial r_j}{\partial P_j} \frac{\partial P_j}{\partial \tau_j} = 0,$$

$$\text{for } j = 1,\ldots,n.$$

In the case of a fully funded social security program, the reduction in private investments is offset by public investment of the tax revenues. These public investments yield the same rate of return as private assets. This means that individuals view the expected social security retirement benefits as a perfect substitute for private investment. As long as current public investment in state securities does not exceed what private investment would have been in the absense of the program, the net effect on total investment, which includes both private and public, is zero. A fully funded social security program merely substitutes private investment for public investment. If public investment does not exceed what private investment would have been without the program, expected interest rates and wages remain unchanged. This allows the equations in (IV-13) to be simplified to produce (IV-14):

$$\text{(IV-14)} \quad - U'W_t + \pi_j V'W_t(1+r_j) = 0, \quad \text{for } j=1,\ldots,n.$$

The fully funded program has no effect on capital accumulation. It merely substitutes public investments for private investments. The utility of individuals is unchanged, unless public investment of tax revenues in state securities exceed what private investment would have been, in the absence of the program. In this case a fully funded program can alter expected wages and interest rates.

IV-3 Relationship Between Labor Growth and Private Investments

In this section the relatinship, between the growth in labor force participation, n, and private investment, is analyzed in order to determine how n affects capital investments. Differentiating wages and interest rates, with respect to n, it is shown that an increase in next period's labor force has the effect of raising expected interest rates and decreasing expected wages for each particular state. This is shown in the equations in (IV-15) and (IV-16):

$$\text{(IV-15)}\quad \partial r_j/\partial n = f''(\ ) - P_j/L_t(1+n)^2 > 0, \qquad \text{for } j=1,\ldots,n,$$

and

$$\text{(IV-16)}\quad \partial W_j/\partial n = f''(\ )\, P_j^2/L_t^2(1+n)^3 < 0, \qquad \text{for } j=1,\ldots,n.$$

As the labor force grows, the proportion of capital allocated to a unit of labor declines. This reduces the expected marginal product of labor and increases the marginal product of capital.

An increasing labor force leads to two opposing effects on expected retirement benefits. This is demonstrated, in equation (IV-17), by differentiating expected social security benefits, with respect to n:

$$\text{(IV-17)}\quad \partial SS_j/\partial n = \tau_j(W_j + (1+n)\,\partial W_j/\partial n), \qquad \text{for } j=1,\ldots,n.$$

As the expected labor force increases, more people will be contributing to the social security program. This tends to raise the expected benefits for future retirees. On the other hand, an increase in n lowers future wages, as demonstrated by the equations in (IV-16). This leads to a reduction in SS_j. If the elasticity of demand for labor is greater than one, then the equations in (IV-17) are positive. If it is less than one, then the equations are negative, indicating that an increase in the labor force leads to a decrease in the total wage bill. This would cause a decrease in expected social security benefits.

The representative individual's maximization problem is given in equation (IV-18):

$$\text{(IV-18)}\quad \underset{P_j/L_t}{\text{Max}}\; U(W_t(1 - \sum_{j=1}^{n} \tau_j) - \sum_{j=1}^{n} P_j/L_t)$$

$$+ \sum_{j=1}^{n} \pi_j V(P_j/L_t(1+r_j) + SS_j).$$

The first order conditions are

$$\text{(IV-19)}\quad - U' + \pi_j V'(1+r_j) = 0, \qquad \text{for } j=1,\ldots,n.$$

The impact, of an increase in the expected labor force next period, on private investment, can be determined by totally differentiating the first order conditions in (IV-19), with respect to the P_j's and n. The results are

$$\text{(IV-20)}\quad -[U''/L_t + \pi_j^2 (1+r_j)^2 V''/L_t]dP_j =$$

$$[\pi_j^2 V''(1+r_j)][P_j/L_t\, \partial r_j/\partial n + \partial SS_j/\partial n]dn$$

$$+ \pi_j V'\, \partial r_j/\partial n\, dn, \qquad \text{for } j=1,\ldots,n,$$

or

$$\text{(IV-21)}\quad dP_j/dn = -\{\pi_j^2\, V''(1+r_j)][P_jL_t\ \partial r_j/\partial n + \partial SS_j/\partial n] + \pi_j V'\ \partial r_j/\partial n$$

$$[U''/L_t + \pi_j^2(1+r_j)^2\ V''/L_t],$$

$$\text{for } j=1,\ldots,n.$$

If the elasticity of demand for labor is greater than one, then the equations in (IV-21) have two negative income effects arising from increases in expected social security benefits and expected returns on private investments. These two factors would tend to reduce the amount of private investments. However, there would be a positive substitution effect, arising from a higher rate of return earned on private assets. The net result on current investment, of an increase in the future labor force is ambiguous, although in the presence of a pay-as-you-go social security program, there is an additional income effect, arising from higher future labor participation. If the elasticity of demand for labor is less than one, then the equations in (IV-21) have only one negative income effect, arising from an increase in expected returns on private investments. There would be a positive income effect arising from a decrease in expected social security benefits along with the positive substitution effect. The net result on current investment, of an increase in future labor force participation, is ambiguous.

IV-4 Optimal Social Security When Wages Differ

In this Section the assumption that all individuals receive the same wage, is relaxed in order to examine whether people's preferences for social security taxes are influenced by their level of income, when a public pension program is progressive.

When individuals receive the same wage, the maximization problem for each individual is

$$\text{(IV-22)} \quad \text{Max } U(W_t(1 - \sum_{j=1}^{n} \tau_j) - \sum_{j=1}^{n} P_j/L_t + \sum_{j=1}^{n} \pi_j V(P_j/L_t(1+r_j) + SS_j).$$

The optimal level of taxes is determined by differentiating equation (IV-22), with respect to the τ_j's. The results are

$$\text{(IV-23)} \quad -U'W_t + \pi_j V' \, \partial SS_j/\partial\tau_j + \pi_j V'[P_j/L_t \, \partial r_j/\partial P_j \, \partial P_j/\partial\tau_j + \partial SS_j/\partial P_j \, \partial P_j/\partial\tau_j] = 0, \text{ for } j=1,\ldots,n.$$

The results in (IV-23) are the same as those of a single planner, who is trying to optimize the present generation's utility. The assumption of a single wage rate is now relaxed. Individuals are divided into two levels of labor skills. In the lower paid group, individuals earn W_t. The other group, which possesses greater labor skills, earns $W_t(1+q)$. In other words, the marginal product of labor of the latter is q percent higher than the former. Those who earn W_t are assumed to compose Z percent of the labor force; the others represent the remaining labor force. These percentages are assumed to remain constant over time.

In this setting the social security tax rates are the same, but expected benefits are equal for both groups. This means that lower paid wage earners receive a higher rate of return on their expected social security benefits, than the higher paid group. These expected benefits are

$$\text{(IV-24)} \quad SS^*_j = \tau_j[ZW_j + (1-Z)(1+q)W_j](1+n) = \tau_j Q_j,$$

$$\text{where } Q_j = [ZW_j + (1-Z)(1+q)W_j](1+n),$$

$$\text{for } j=1,\ldots,n.$$

The expected rates of return for individuals who currently earn W_t are, Q_j/W_t, for $j=1,\ldots,n$; the rates for those of the higher paid group are, $Q_j/W_t(1+q)$, for $j=1,\ldots,n$. As q rises the rates of returns diverge between the two groups.

The maximization problem is

$$\text{(IV-25)} \quad \underset{P_j/L_t}{\text{Max}}\ U(W_t(1+q)(1 - \sum_{j=1}^{n} \tau_j) - \sum_{j=1}^{n} P_j/L_t) +$$

$$\sum_{j=1}^{n} \pi_j V(P_j/L_t(1+r_j) + SS^*_j).$$

For those who currently earn W_t, q equals zero. The optimum level of social security tax rates for each group is determined by differentiating equation (IV-25), with respect to the τ_j's, and setting these first order conditions equal to zero. The results are

$$\text{(IV-26)} \quad - U'W_t(1+q) + \pi_j V'Q_j +$$

$$\pi_j V'[P_j/L_t\ \partial r_j/\partial P_j\ \partial P_j/\partial \tau_j + \partial SS^*/\partial P_j\ \partial P_j/\partial \tau_j] = 0,$$

$$\text{for } j=1,\ldots,n.$$

In order to examine how the optimal tax rates for the two groups change as q rises, the first order conditions in (IV-26) are totally differentiated, with respect to the τ_j's and q. The results are

$$\text{(IV-27)} \quad d\tau_j/dq = \{U''W_t{}^2(1+q)(1 - \sum_{j=1}^{n} \tau_j) + U'W_t - \pi_j^2 V''Q_j \tau_j(1-Z)W_j(1+n) - \pi_j V'(1-Z)W_j(1+n)\} \div \{U''W_t^2 + \pi_j^2 V''[ZW_j + (1-Z)(1+q)W_j]\},$$

for j=1,...,n.

By defining the divisors in (IV-27) by M_j, the four effects in each equation can be separated into four terms. These are

$$\text{(IV-28)} \quad (U''W_t{}^2(1+q)(1-\Sigma\tau_j))/M_j > 0,$$

$$\text{(IV-29)} \quad U'W_t/M_j < 0,$$

$$\text{(IV-30)} \quad -(\pi_j^2 V''Q_j T_j(1-Z)W_j(1+n))/M_j \gtrless 0,$$

and

$$\text{(IV-31)} \quad -(\pi_j V'(1-Z)W_j(1+n))/M_j > 0, \text{ for } j=1,\ldots,n.$$

For individuals who earn W_t, the income and substitution effects in (IV-28) and (IV-29) are equal to zero, because their current wages have not changed. If the demand elasticity for labor is greater than one, then there is a negative income effect, caused by higher expected social security benefits. This is shown by the equations in (IV-30). These equations indicate that these individuals would desire a lower social security tax rate if q increases. The reason for this is that expected benefits have increased for the given tax rate, causing the marginal utility of expected benefits to decline, relative to the marginal disutility of current consumption, that is foregone, due to the tax. If the elasticity of demand for labor is less than one, then the income effects in (IV-30) are positive. This would lead individuals who earn W_t to desire a higher tax and benefit level. In addition, there is a positive substitution effect shown in (IV-31), caused by having a higher level of expected social security benefits per tax rate. This causes the lower paid workers to desire higher social security tax rates. From the equations in (IV-32) and (IV-33), it is shown that if the elasticity of demand for labor is less than one, then an increase in q will lead individuals, who earn W_t, to desire higher social security tax rates and benefits. If the elasticity of the demand for labor is greater than one, then the results are ambiguous because of the opposing income and substitution effects.

The income and substitution effects in (IV-30) and (IV-31), for the higher paid individuals, are the same as for the lower paid group. In addition, there are other income and substitution effects as shown in the equations in (IV-28) and (IV-29). These two effects are the result of earning a higher current wage. The positive income effect in (IV-28) causes these individuals to desire higher tax rates and benefits because they have more current income. The negative substitution effect in (IV-29) causes them to desire a lower tax rate because they are paying out more social security tax revenues, per tax rate. The combined effects of (IV-28) through (IV-31) lead to an unambiguous answer, as to whether these individuals desire a higher or lower tax rate.

The net effect on the level of desired tax rates and benefits caused by allowing incomes to differ, is ambiguous due to opposing income and substitution effects. It cannot be shown in this model whether one group will desire higher or lower tax rates compared to the other wage group. However, if the elasticity of demand for labor is less than one, then the lower paid group will desire a larger public pension program.

IV-5 Impact of Technological Growth on Optimal Savings

In this section a technological growth trend is incorporated into the production function in order to determine whether it would change the level of investments. It is assumed that growth is in the form of labor enhancing technology and that the rate of growth is defined as λ per period. Given these assumptions, the effective labor force in period t, E_t, is defined as the number of laborers, multiplied by the growth of labor enhancing technology, over the time period from 0 to t. This is defined in equation (IV-32):

(IV-32) $E_t = L_t\, e^{\lambda t}$.

The expected wages and interest rates are

(IV-33) $r_{j,t} = f'(P_{j,t}/L_t\, e^{\lambda t}, A_j)$,

for $j=1,\ldots,n$,

and

(IV-34) $W_{j,t} = e^{\lambda t}[f(\ )-(P_{j,t}/L_t\, e^{\lambda t})f'(\ )]$,

for $j=1,\ldots,n$.

The effect of an increase in λ on wages and interest rates is determined by differentiating the equations in (IV-33) and (IV-34) by λ to get

(IV-35) $\partial r_j/\partial\lambda = -f''(\ )\ P_{j,t}t/L_t e^{\lambda t} > 0,$

for $j=1,\ldots,n$

and

(IV-36) $\partial W_j/\partial\lambda = e^{\lambda t}[tW_j + f''(\ )(P_{j,t}/L_t\ e^{\lambda t})],$

for $j=1,\ldots,n.$

The equations in (IV-35) show that an increase in labor enhancing technology will cause interest rates to rise, because each unit of capital is now working with a larger amount of effective labor. The equations in (IV-36) show that an increase in λ causes the wage rate, per unit of effective labor, to decline and also causes the number of effective labor units to increase. If the elasticity of demand for effective labor, ε_E, is greater than one, then the net effect of an increase in λ is to increase the wage received by an individual. If ε_E is less than one, then an increase in λ cause wages to fall.

Expected social security benefits are

(IV-37) $SS_j = \tau\ e^{\lambda t}\ W_j(1+n),$

for $j=1,\ldots,n.$

If ε_E is greater than one, then an increase in λ causes expected wages to rise, which in turn causes expected social security benefits to increase. This is demonstrated in the equations in (IV-38):

(IV-38) $\partial SS_j/\partial\lambda = \tau_j(1+n)\ \partial W_j/\partial\lambda > 0,$ for $j=1,\ldots,n.$

If ε_E is less than one, then

(IV-39) $\partial SS_j/\partial\lambda = \tau_j(1+n)\ \partial W_j/\partial\lambda < 0$, for $j=1,\ldots,n$.

The maximization problem for a representative individual is

(IV-40) $$\underset{P_j/L_t}{\text{Max}}\ U(W_t(1-\Sigma\tau_j) - \Sigma\ P_j/L_t + \Sigma\ \pi_j\ V(P_j/L_t(1+r_j) + SS_j),$$

for $j=1,\ldots,n$.

The first order conditions are

(IV-41) $$-U' + \pi_j\ V'(1+r_j) = 0,$$

for $j=1,\ldots,n$.

The effect of an increase in λ on investment is determined by totally differentiating the equations in (IV-42) by the P_j's and λ. The results are

(IV-42) $$[dP_j/d\lambda] = \frac{U''\partial W_t/\partial\lambda - \pi_j^2 V''[\partial r_j/\partial\lambda + \partial SS_j/\partial\lambda] + \pi_j V'[P_j/L_t\partial r_j/\partial\lambda]}{U'' + \pi_j^2\ V''(1+r_j)^2},$$

for $j=1,\ldots,n$.

Each equation in (IV-43) has three income effects and one substitution effect. The first term in the numerator shows that if ε_E is greater than one, then an increase in λ leads to individuals wanting to invest part of their increase in current income, in private assets. If ε_E is less than one, then the income effect is negative. The second term is a negative income effect, caused by an increase in the rates of return on expected private assets. This causes individuals to invest less in private assets. The third term in the numerator is an income effect. If ε_E is greater than one, then an increase in λ causes expected social security benefits to increase. This causes the need to invest in private assets to decline. If ε_E is less than one, then the income effect is positive. The fourth term is a positive substitution effect caused by the higher expected rate of return on private investments.

Due to the opposing income and substitution effects, the results of an increase in λ, on private investment, is ambiguous, however, if ε_E is greater than one, then there is an additional negative income effect with a pay-as-you-go social security program, present in this model. This would tend to cause individuals to invest less in private securities when λ increased than they would in the absence of this retirement program.

CHAPTER V

LABOR GROWTH UNCERTAINTY IN A NEO-CLASSICAL GROWTH MODEL

This chapter examines an alternative form of uncertainty in order to find out if the implications of social security's effects on steady-state capital-labor ratios and wages differ, depending on the type of uncertainty. In this section production technology is viewed as certain, whereas, next period's labor force is uncertain. After long-run, stochastic equilibrium is derived, a pay-as-you-go social security program is introduced into the economy. The results are similar to those found in the production uncertainty model. It is shown that social security reduces both current and long-run capital-labor ratios and wage rates. In addition, it is shown that the social security program increases the uncertainty of future wage rates.

V-1 The Model

The technological assumptions are the same as those stated in the certainty case in Chapter IV. In addition, people's expectations concerning the distribution of next period's wage and interest rates are assumed to be correct. All firms are assumed to be identical and to issue equity shares in period t that offer a distribution of potential future rate of returns in period t+1. These firms can be aggregated into a single, constant returns to scale production function of the form

$$\text{(V-1)} \quad Y_{t+1} = F(K_{t+1}, L_t(1+\tilde{n})),$$

where

> $L_t(1+\tilde{n}) = L_{t+1}$, and $\tilde{n}$ is bounded above and below by positive finite values, $\tilde{n} \,\varepsilon\, [\underline{n}, \bar{n}]$. The probability distribution of the $\tilde{n}$'s is identical over time.

Equation (V-1) can be divided by L_{t+1} in order to derive per-capita potential output:

(V-2) $Y_{t+1}/L_{t+1} = F(K_{t+1}/L_{t+1}, 1) = f(K_{t+1}/L_{t+1})$.

The following Inada conditions are assumed:

(V-3) $\lim_{k\to 0} f'(k) = \infty$, and $\lim_{k\to\infty} f'(k) = 0$.

The rate of return on equity and the wage rate in period t+1 are

(V-4) $\tilde{r}_{t+1} = f'(K_{t+1}/L_t(1+\tilde{n}))$,

and

(V-5) $\tilde{W}_{t+1} = f(K_{t+1}/L_t(1+\tilde{n})) - (K_{t+1}/L_t(1+\tilde{n}))f'(\ )$.

Equations (V-4) and (V-5) show that the distributions on interest rates and wages are dependent on the distribution of $\tilde{n}$. If the random variable, $\tilde{n}$, is large in period t+1, then the return on equity will be higher and the wage rate will be lower, than if the value had been smaller. This is illustrated by differentiating equations (V-4) and (V-5) by n to get:

$$\text{(V-6)} \quad \partial \tilde{r}_{t+1}/\partial n = -f''(\ )\, K_{t+1}/L_t(1+\tilde{n})^2 > 0,$$

and

$$\text{(V-7)} \quad \partial \tilde{W}_{t+1}/\partial n = f''(\ )\, K_{t+1}^2/L_t^2(1+\tilde{n})^3 < 0.$$

Firms are assumed to demand capital up to the point where the expected marginal product of capital equals the expected interest rate. This equilibrium relationship is represented in equation (V-8):

$$\text{(V-8)} \quad \int_{\underline{n}}^{\overline{n}} r_{t+1}(n)\, \Pi(n)dn = \int_{\underline{n}}^{\overline{n}} f'(K_{t+1}/L_t(1+n))\Pi(n)dn.$$

The slope of the demand curve is determined by differentiating equation (V-8) by K_{t+1} to get

$$\text{(V-9)} \quad \frac{\partial \int_{\underline{n}}^{\overline{n}} r_{t+1}(n)\Pi(n)dn}{\partial K_{t+1}} = \int_{\underline{n}}^{\overline{n}} f''(K_{t+1}/L_t(1+n))\, \frac{1}{L_t(1+n)}\, \Pi(n)dn < 0.$$

Equation (V-9) indicates that the demand curve for capital is negatively sloped.

The supply side of the capital market is composed of the savings functions of the younger generation. The objective function for a representative individual is

$$\text{(V-10)} \quad \max_{S_t} U(W_t - S_t) + \int_{\underline{n}}^{\overline{n}} V(S_t(1+r(n))\Pi(n)dn,$$

where

S_t is per-capita savings of an individual in period t.

The first order condition with respect to S_t is

$$(\text{V-11}) \quad -U' + \int_{\underline{n}}^{\overline{n}} V'(1+r(n))\Pi(n)dn = 0.$$

Equations (V-10) and (V-11) imply that per-capita savings, in period t, are a function of the current wage rate and the probability distribution, of interest rates in period t+1. This is represented in equation (V-12):

$$(\text{V-12}) \quad S_t = S_t(W_t, \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn)$$

By combining the demand and supply schedules and equating S_tL_t and K_{t+1}, the equilibrium condition in the capital market is attained. This condition relates the distribution of next period's interest rates to current wage rate:

$$(\text{V-13}) \quad \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn = \int_{\underline{n}}^{\overline{n}} f'(S(W_t, \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn)/(1+n))\Pi(n)dn.$$

Equation (V-13) is similar to the capital market equilibrium condition in Chapter III-1, where $r_{t+1} = \psi(W_t)$. Once the current wage rate is known, then the expected interest rate distribution for period t+1 can be determined. The relationship between W_t and the probability distribution of next period's interest rates, is determined by differentiating equation (V-13) with respect to W_t, to get:

$$\text{(V-14)}\quad \frac{d\int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn}{dW_t} = \int_{\underline{n}}^{\overline{n}} \left(\frac{f''\ \partial S_t/\partial W_t}{1+n-f''\ \partial S_t/\partial r(n)}\right)\Pi(n)dn < 0.$$

Equation (V-14) indicates that an increase in current wages causes the distribution of interest rates, for period t+1, to decline. In addition, the probability distribution of wage rates in period t+1 shift upward, as current wages increase. The actual wage and interest rate, in period t+1, depends on the actual capital labor ratio that occurs in period t+1. The capital labor ratio is

$$\text{(V-15)}\quad K_{t+1}/L_{t+1} = S_t(W_t, \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn)/L_t(1+n_{t+1}).$$

The wage rate is

$$\text{(V-16)}\quad W_{t+1} = f(S_t(W_t, \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn)/L_t(1+n_{t+1}))$$

$$- (S_t(W_t, \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn)/L_t(1+n_{t+1}))f'(\ ).$$

Equation (V-16) is the single, stochastic difference equation driving the economy. If the current wage rate increases, then the probability distribution of next period's wages increase. If the value of n is large in period t+1, then W_{t+1} will be lower than if n had been small.

With equation (V-16) it is now possible to examine how this economy approaches long-run equilibrium. A formal definition of steady-state is first defined.

DEFINITION: The interval $[\underline{k}, \overline{k}]$ is said to be a stable interval for the stochastic process:

$$k_{t+1} = S_t(W_t, \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn)/L_t(1+n_{t+1}) \text{ if for all } k \in [\underline{k}, \overline{k}],$$

$$S_t(f(k,\overline{n}) - kf'(k,\overline{n}), \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn)/(1+n) \geq$$

$$\underline{k}, \text{ and } S_t(f(k,\underline{n}) - kf'(k,\underline{n}), \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn)/(1+n) \leq \overline{k}.$$

In order to derive the steady-state distribution which is bounded below by a $\underline{k}$ greater than zero, and bounded above by a $\overline{k}$ less than infinity, it is necessary to prove the following two theorems.

Theorem III:

The steady-state capital-labor ratio is bounded above by a value less than infinity.

Proof:

From the Inada condition,

$$\lim_{k \to \infty} f'(k) = 0,$$

there exists a $S_t^*/L_t(1+n)$, large enough, such that $f(S_t/L_t(1+n)) - S_t/L_t(1+n)f'(\ ) = W^* < S_t^*/L_t(1+n)$, for all $n \in [\underline{n}, \overline{n}]$. This implies that there exists a capital-labor ratio, large enough, that it cannot be sustained over time. Let n equal $\underline{n}$ for all t. If t grows large enough, then $S_t(\ldots)/L_t(1+\underline{n})$ will eventually approach $\overline{k}$. If $S_t(\ldots)/L_t(1+\underline{n}) > \overline{k}$, then $S_{t+1}(\ldots)/L_{t+1}(1+\underline{n})$ will eventually equal $\overline{k}$, which is the least upper bound of the steady-state distribution.

End of Proof

Theorem III is demonstrated graphically in Figure 15. By letting n equal $\underline{n}$, it is unnecessary to examine any of the other possible values of n, in order to find the upper bound on steady-state wages and capital-labor ratios. If the economy's capital-labor ratio is greater than $\overline{k}$, then it will be unable to maintain that level.

The following theorem proves that there is a positive steady-state lower bound. A lemma similar to Lemma 1 is first proved in order to help prove the Theorem.

Lemma 2:

As $W_t \to 0$, $S_t(W_t, \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn)$ will have the property that $f(S_t(\ldots)/L_{t+1}) - S(\ldots)/L_{t+1}\, f'(\ldots) > W_t$.

<u>Proof:</u>

From the Inada condition,

$$\lim_{k \to 0} f'(k) = \infty,$$

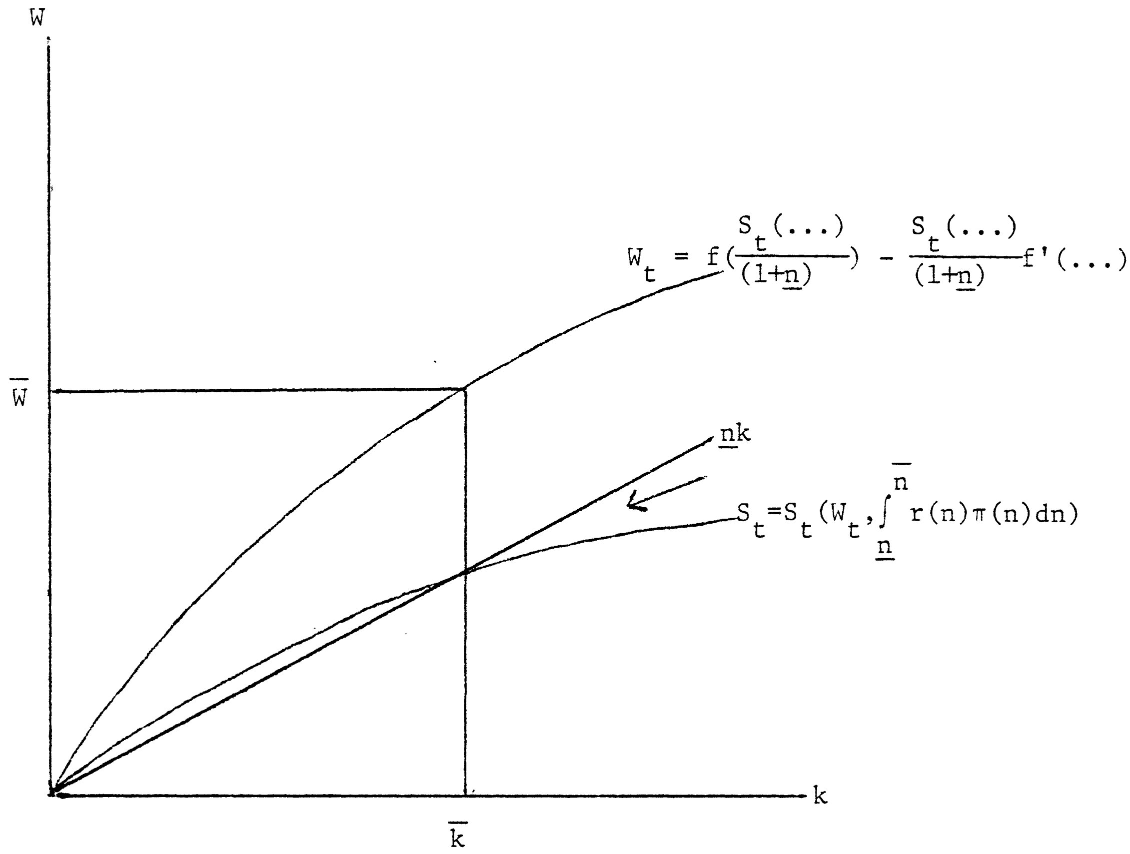

Figure 15

Upper Bound on Steady-State Wages and Capital-Labor Ratios there exists a $S_t^{**}/L_{t+1} > 0$, such that, $f(S_t^{**}/L_{t+1}) - S_t^{**}/L_{t+1}f'(\ldots) \geq S_t^{**}/L_{t+1}$. For any S_t/L_{t+1}, where $0 < S_t/L_{t+1} < S_t^{**}/L_{t+1}$,

$$\lim_{S_t/L_{t+1} \to 0} [(f(S_t/L_{t+1}) - S_t/L_{t+1}f'(\ldots)) \div S_t/L_{t+1}] = \infty.$$

End of Proof

Theorem IV:

The steady-state capital-labor ratio is bounded below by a positive value.

Proof:

From Lemma 2, there exists a $S_t^{**} > 0$, small enough, such that, $f(S_t^{**}/(1+n)) - S_t^{**}/(1+n)f'(\ldots) \geq S_t^{**}/(1+n)$, for all $n \in [\underline{n},\bar{n}]$. Let n equal $\bar{n}$, for all t. As t grows large enough, $S_t/(1+\bar{n})$ will eventually approach $\underline{k}$, which is the greatest lower bound of the steady-state distribution, of capital labor ratios.

End of Proof

Theorem IV is demonstrated graphically in Figure 16. By letting n equal $\bar{n}$, it is unnecessary to examine any of the other possible values of n in order to determine the steady-state lower bound. If the economy starts at a capital labor ratio below $\underline{k}$, then it will eventually equal or exceed $\underline{k}$.

By combining figures 15 and 16 into figure 17, a steady-state distribution of capital-labor ratios and wage rates is depicted. This figure shows that, if the economy starts with a positive capital-labor

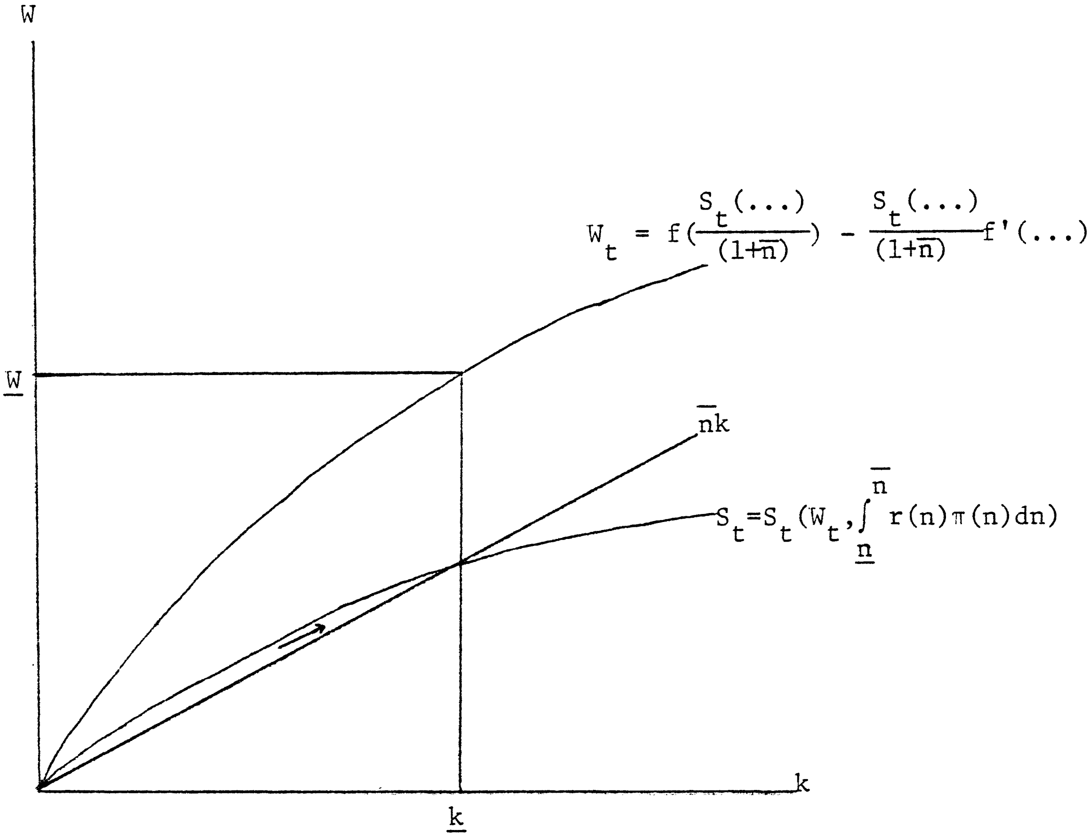

Figure 16

Lower Bound on Steady-State Wages and Capital-Labor Ratios ratio, then it will eventually enter long-run stochastic equilibrium. The economy will then remain within this equilibrium distribution.

A simplified example of this model is presented below.

An Example

Consider the discrete case where there are two possible values of n, namely, n_1 and n_2, and that Π_1 and Π_2 are their respective probabilities. Assuming a Cobb-Douglas production and utility function, the objective function is

$$\text{(V-17)}\quad \text{Max } \gamma\log(W_t - S_t) + \Pi_1(1-\gamma)\log(S_t(1+r_1)) + \Pi_2(1-\gamma)\log(S_t(1+r_2)),$$

where

$$r_1 = \alpha A(S_t/(1+n_1))^{\alpha-1} \text{ and } r_2 = \alpha A(S_t/(1+n_2))^{\alpha-1} .$$

The investment function derived from equation (V-17) is

$$\text{(V-18)}\quad S_t = W_t(1-\gamma).$$

In this case the savings function is independent of the expected interest rate distribution. Equation (V-18) is the specific form of the general savings function in equation (V-12).

The equilibrium condition is the capital market is

$$\text{(V-19)}\quad \Pi_1 r_1 + \Pi_2 r_2 = \Pi_1 \alpha A\left(\frac{W_t(1-\gamma)}{1+n_1}\right)^{\alpha-1} + \Pi_2 \alpha A\left(\frac{W_t(1-\gamma)}{1+n_2}\right)^{\alpha-1}.$$

Equation (V-19) is the specific form of the general capital-market equilibrium condition in equation (V-13). The wage in period t+1 will

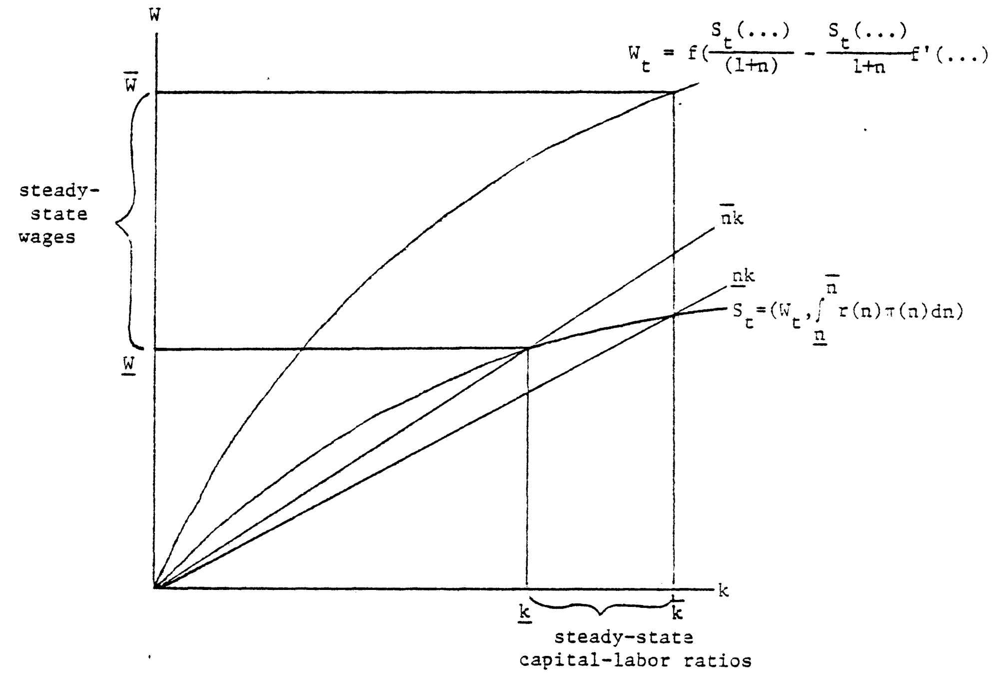

Figure 17

Long-run Equilibrium Wages and
Capital-Labor Ratios

depend on the savings function in equation (V-18) and the particular, n that occurs in period t+1. This is expressed as

$$\text{(V-20)} \quad W_{t+1} = (1-\alpha)A(W_t(1-\gamma)/n_{t+1})^{\alpha} \; .$$

Equation (V-20) is the stochastic equation driving the economy.

In order to derive the steady-state, upper and lower bounds, on capital-labor ratios and wage rates, assume that n_1 is less than n_2. The steady-state lower bound on wages and capital-labor ratios is

$$\text{(V-21)} \quad \underline{W} = \lim_{t\to\infty} W_t = (1-\gamma)^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} (1-\gamma)^{\frac{\alpha}{1-\alpha}} n_2^{\frac{-\alpha}{1-\alpha}},$$

and

$$\text{(V-22)} \quad \underline{k} = (1-\gamma)\underline{W}/(1+n_2).$$

The steady-state upper bounds are

$$\text{(V-23)} \quad \underline{W} = \lim_{t\to\infty} W_t = (1-\gamma)^{\frac{1}{1-\alpha}} A^{\frac{1}{1-\alpha}} (1-\gamma)^{\frac{\alpha}{1-\alpha}} n_1^{\frac{-\alpha}{1-\alpha}},$$

and

$$\text{(V-24)} \quad \overline{k} = (1-\gamma)\overline{W}/1+n_1.$$

V-2 Impact of Social Security on Long-run Stochastic Equilibrium

Social security's impact on the steady-state distribution of capital-labor ratios is a result of altering the savings behavior of individuals. The objective function of an individual when social security is present in the model is

$$(V\text{-}25)\quad \text{Max } U(W_t(1-\tau)-S_t) + \int_{\underline{n}}^{\overline{n}} V(S_t(1+r(n)) + \tau\, W_{t+1}(n)(1+n))\Pi(n)dn$$

Equation (V-25) indicates that expected benefits are a function of the tax rate, the probability distribution of next period's wages, and the probability distribution of n. The first order condition with respect to S_t is

$$(V\text{-}26)\quad -U' + \int_{\underline{n}}^{\overline{n}} V'(1+r(n))\Pi(n)dn = 0$$

To determine how social security affects individual savings behavior, equation (V-26) is totally differentiated with respect to S_t and τ. This becomes

$$(V\text{-}27)\quad dS_t/dT = \frac{-U''W_t - \int_{\underline{n}}^{\overline{n}} V''(1+r(n))W_{t+1}(n)(1+n)\Pi(n)dn}{U'' + \int_{\underline{n}}^{\overline{n}} V''(1+r(n))^2\Pi(n)dn} < 0$$

Equation (V-27) indicates that there are two negative income effects caused by the presence of a social security program. First, the tax on current income leads individuals to save less. This is shown by the first term in the numerator of equation (V-27). The second term demonstrates that promised benefits reduce the need to save. The net result is that individuals allocate a smaller proportion of their gross wage income toward savings.

In the presence of a social security program, individual savings is a function of the current wage, the distribution of expected interest rates, and the expected distribution of social security benefits. This is shown in equation (V-28):

$$\text{(V-28)} \quad S_t^* = S_t^*(W_t, \int_{\underline{n}}^{\overline{n}} r(n)\Pi(n)dn, \int_{\underline{n}}^{\overline{n}} \tau\, W(n)(1+n)\Pi n)dn.$$

Since individuals save proportionately less of their gross wage income, capital-labor ratios are lower. This causes wages to be lower and interest rates to be higher.

Figure 18 depicts the impact of social security on the steady-state distribution of capital-labor ratios and wage rates. The results are similar to those stated in the production uncertainty case. A pay-as-you-go public pension program causes the long-run equilibrium capital-labor ratios and wage rates to be lower. In addition, due to diminishing marginal returns to labor, as the capital-labor ratio increases, the social security program causes the variance of long-run wages to increase. This is demonstrated by the wider gap between the least upper and the greatest lower bound, when social security is present in the model. Figure 17 shows that $(\overline{W}^* - \underline{W}^*) > (\overline{W}-\underline{W})$.

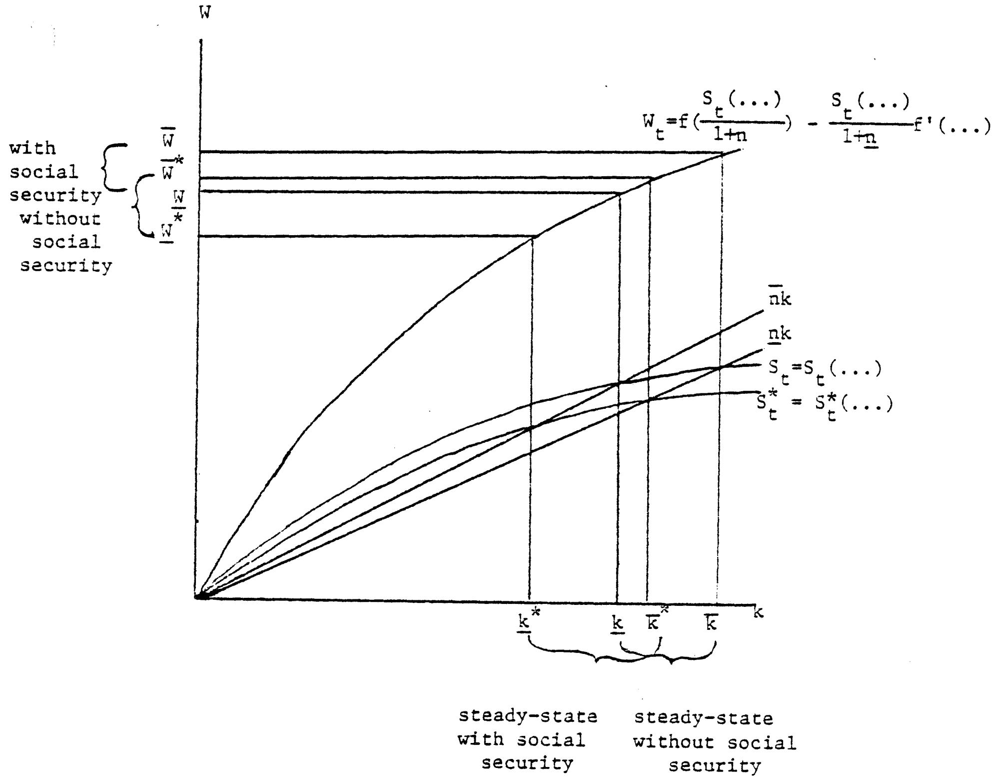

Figure 18

Social Security's Effect on
Long-run Equilibrium

CHAPTER VI

CONCLUSION

The deterministic and random growth models have shown that the introduction of these simplified pay-as-you-go social security programs depress individual savings. This leads to lower capital-labor ratios which in turn reduce wages and raise interest rates. The uncertainty models demonstrate that social security shifts the long-run distribution of capital-labor ratios and wages downward and shifts the interest rate distribution upward. The labor uncertainty model also demonstrates that social security by shifting the distribution of capital-labor ratios downward, increases the uncertainty of next period's wage rates. These results are due to the pay-as-you-go nature of a social security program where the negative tax and benefit effects are not offset by public investments.

One of the problems that needs to be addressed in these models is the sequential nature of investment behavior. This requires extending an individual's life beyond two periods. Other limitations in these models include the strong assumptions imposed on the social security program and the aggregation problems that concern growth models in general.

The models in this paper are greatly simplified to allow manipulations to be made, however, it is the writer's belief that the uncertainty models are useful and can be applied to other economic questions. These subjects include government debt, the changing behavior of labor participation on savings, and the changes in tax laws that affect productivity and individual savings behavior. These problems along with others can be easily incorporated into these random growth models in order to examine their implications.

BIBLIOGRAPHY

K. J. Arrow, _Aspects of the Theory of Risk-Bearing_, Helsinki 1965.

R. J. Barro, "Are Government Bonds Net Wealth?" _Journal of Political Economy_, November/December 1974, 82, 1095-1117.

W. Brock and L. Mirman, "Optimal Economic Growth and Uncertainty: The Discounted Case," _J. Econ. Theory_, 1972, 4, 479-513.

R. Brumberg and F. Modigliani, "Utility Analysis and the Consumption Function: An Interpretation of Cross Section Data." _Post Keynesian Economics_, K. Kurihara, ed., New Brunswick 1954.

J. M. Buchanan, "Barro on the Ricardian Equivalence Theorem," _Journal of Political Economy_, 1976, 84, no. 2, 337-41.

D. Cass, "Optimal Growth in an Aggregative Model of Capital Accumulation," _Rev. Econ. Stud._, 1965, 32, 233-240.

P. A. Diamond, "National Debt in a Neoclassical Growth Model," _Amer. Econ. Rev._, December 1965, 55, 1126-50.

E. F. Fama and M. H. Miller, _The Theory of Finance_, Hinsdale 1972.

M. S. Feldstein, "Perceived Wealth in Bonds and Social Security," _Journal of Political Economy_, 1976, 84, no. 2, 331-36.

__________, "Social Security, Induced Retirement and Aggregate Capital Accumulation," _Journal of Political Economy_, September/October 1974, 82, 905-25.

__________, "Social Security and Private Saving: International Evidence in an Extended Life-Cycle Model," _The Economics of Public Services_, Martin Feldstein and Robert Inman, eds., London 1977, 174-205.

I. Fisher, _The Theory of Interest_, New York 1930.

F. H. Hahn, "Savings and Uncertainty," _Rev. of Econ. Stud._, 1970, 37, 21-4.

J. Hirshleifer, "Investment Decision under Uncertainty: Choice Theoretic Approaches," _Quart. J. Econ._, 1965, 79, 509-36.

S. C. Hu, "On the Dynamic Behavior of the Consumer and the Optimal Provision of Social Security," _Rev. Econ. Stud._, October 1978, 45, 437-45.

D. Levhari and T. N. Srinivasan, "Optimal Savings Under Uncertainty," Technical Report no. 8, Institute for Mathematical Studies in the Social Sciences, Stanford University, December 1967.

J. B. Long, Jr., "Consumption-Investment Decisions and Equilibrium in the Securities Market," Studies in the Theory of Capital Markets, Michael C. Jensen, ed., New York 1972, 146-222.

D. Mayers, "Nonmarketable Assets and Capital Market Equilibrium Under Uncertainty," Studies in the Theory of Capital Markets, Michael C. Jensen, ed., New York 1972, 223-48.

L. J. Mirman, "On the Existence of Steady State Measures for One Sector Growth Models with Uncertain Technology," Int. Econ. Rev., 1972, 31, no. 2, 271-86.

__________, "Uncertainty and Optimal Consumption Decisions," Econometrica January 1971, 39, no. 1, 179-85.

J. A. Mirrlees, "Optimum Accumulation under Uncertainty: The Case of Stationary Returns to Investment," Allocation Under Uncertainty: Equilibrium and Optimality, Jacques H. Dreze, ed., New York 1974, Ch. 3.

J. Mosin, The Economic Efficiency of Financial Markets, Lexington 1977.

A. Munnell, The Future of Social Security, Brookings Institution 1977.

E. S. Phelps, "The Accumulation of Risky Capital: A Sequential Utility Analysis," Econometrica, 1962, 30, no. 4, 729-43.

__________, "The Golden Rule of Accumulation: A Fable for Growthmen," Am. Econ. Rev., Sept. 1961, 51, 638-43.

P. A. Samuelson, "An Exact Consumption Loan Model of Interest With or Without the Social Contrivance of Money," Journal of Political Economy, Dec. 1958, 66, 467-82.

__________, "Lifetime Portfolio Selection by Dynamic Stochastic Programming," Review of Economics and Statistics, 1969, 51, 204-11.

__________, "Optimal Social Security in a Life Cycle Growth Model," Int. Econ. Rev., Oct. 1975, 16, 539-44.

__________, "The Pure Theory of Public Expenditures," Rev. Econ. and Statis., November 1954.

A. Sandmo, "Capital Risk, Consumption and Portfolio Choice," Econometrica 1969, 37, 586-99.

A. Sandmo, "The Effect of Uncertainty on Saving Decisions," Rev. Econ. Studies, 1970, 37, 353-60.

J. Tobin, "Liquidity Preference as Behavior Towards Risk," Rev. Econ. Stud., Feb. 1958, 25, no. 67, 65-86.

__________, "The Theory of Portfolio Selection," The Theory of Interest Rates, F. H. Hahn and F.P.R. Brechling, eds., London 1965, Ch. 1.

The vita has been removed from the scanned document

UNCERTAINTY AND THE EFFECTS OF A PAY-AS-YOU-GO SOCIAL SECURITY PROGRAM ON ECONOMIC GROWTH

by

Christopher McCoy

(ABSTRACT)

This paper examines the implications of a simplified social security program on future wages and interest rates in the context of uncertainty. Individual consumption-savings and portfolio decisions are integrated into stochastic growth models to find how a social security program alters intermediate and long-run equilibrium factor payments. Two forms of uncertainty are used, namely, production uncertainty and labor force participation.

In the first part of this paper recent arguments over how social security affects individual savings are presented and evaluated. Conclusions are made that Feldstein's tax and benefit effects represent the dominant influences on personal savings behavior. A neo-classical growth model is then built, that integrates the two period consumption-savings decisions of individuals, and long-run equilibrium wages and interest rates are derived. A social security program is then introduced, causing wages to fall and interest rates to rise. Production uncertainty is then added to the model to find how social security impacts on factor payments via individual consumption and portfolio decisions.

Certain questions regarding a social security program are then examined within the production uncertainty model. They include:

determining the optimal amount of social security; examining the implications of a fully funded program; studying the relationship between future labor force participation and private investment; examining if optimal social security varies, depending on the individual's wage income and introducing technological growth to see how it effects optimal savings.

An alternative form of uncertainty, labor force participation, is then substituted into the model to see if the implications of social security differ, depending on the form of uncertainty. The results are similar to those found in the production uncertainty model. In addition, it is shown that social security tends to increase the variance of future stochastic wages.