Friction of Extensible Strips: an Extended Shear Lag Model with Experimental Evaluation

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Introduction

Friction of elastomeric materials is a topic of great practical importance, with many applications, such as tires, rubber seals, conveyer belts, etc. In many cases, the stiffness of the elastomer in a direction parallel to the friction force is low, giving rise to a significant deformation and resulting to a deviation from intrinsic frictional behavior of the elastomer. Furthermore, in some applications, a backing or support layer may be applied to or embedded within the elastomer layer, depending on the application and the properties needed on the two surfaces, leading to a change in the effective stiffness of the elastomers. These circumstances could also be applicable to the other flexible systems, where there is a significant deformation due to the low stiffness of the material. Although the frictional properties of soft materials have been well studied in the past decades [1-4], the effect of effective stiffness on the intrinsic frictional response of these materials is not well understood from either an experimental or theoretical basis.

In the classical modeling of friction of polymers, it has been shown that there is a correlation between adhesion and friction in elastomers [5]. A few papers investigated the effect of backing layer on the adhesion of elastomeric materials, experimentally. For example, Bartlett et al. [6] investigated the effect of backing stiffness on the shear adhesion strength of elastomers and found a scaling law between adhesive force capacity, projected area of contact, and compliance of the fabric-backed adhesive in shear loading. They showed that by decreasing the compliance of the backing fabric, the shear adhesion strength of the elastomeric adhesives increases.

Several papers theoretically modelled the intrinsic frictional behavior of the polymers using a correlation between viscoelastic properties, adhesion, and friction [3, 7, 8]. However, to the best of authors’ knowledge, there is no theoretical model that directly predicts the effect of backing stiffness on the frictional response of elastomers. Friction between elastomers and a substrate has some analogy with adhesive joints, e.g. lap shear joints, where there is a load transfer mechanism between two members via intrinsic frictional response for the former and the shear stresses in the adhesive for the latter. The shear lag model introduced by Volkersen [9] is widely used in the analysis and design of adhesive joints. This model has been extended for analysis of elastic-plastic adhesives [10], fiber-matrix interaction in composite materials [11], and interfacial friction stress at frictionally bonded fiber-matrix interface in single fiber push-out tests [12].

In this paper, the effect of backing layer stiffness on the friction of confined elastomeric strips was investigated, both experimentally and theoretically. The elastomer strips were confined by a sled of fixed mass resting on the top of the strip. The friction force required to move the sled with respect to the base was measured as a function of displacement. It has been found that, by decreasing the stiffness of the backing, the peak corresponding to the static friction force decreased dramatically. For a softer backing, there was no observable static peak, whereas there was a slope change in the force-displacement curve at the point where slippage initiated. The local displacement of several points along the length of the strip was measured by image analysis, revealing that for the more flexible backing elastomer, there was a combination of no-slip, transition, and slip zones along the length of the elastomeric strip. The onset of slipping of the entire sample was found to occur when the slip zone was fully developed along the length of the elastomer. A new theoretical model based on the classic extension of the classic shear lag model to elastic-plastic interlayers, was proposed to predict the friction response as a function of backing stiffness. Using the intrinsic frictional response of an elastomer, obtained from rigidly adhering an elastomer-coated strip to a steel sled and pulling it across a glass substrate, the behavior of elastomers with different backing stiffness was predicted using the analytical model developed. Three regions were defined along the length of the elastomeric strip, namely no-slip, transition, and slip zones. Using a longitudinal force equilibrium condition, the governing equations corresponding to each region were derived. Boundary conditions between each region were satisfied based on continuity of both the displacement and stress. The resulting solution was used to predict force-displacement curves for confined elastomers supported by a flexible backing. The results obtained from the shear lag analysis were compared with experimental results and shown to be in good agreement.

Experiments

The friction-displacement measurements were performed using a custom-built friction tester (Figure 1): a translational actuator (PI,M-229.26S, Karlsruhe, Germany) pulled a steel sled (mass 190g, length 100mm, and width 10mm), the nose of which secured the end of the elastomer-coated strip (Jo-Ann Fabric and Crafts, product number:13421664). The total friction force was measured using a load cell with a full range of 5.9N (OMEGA, LCAE-600G, Stamford, USA). The sample was glued just on the front edge of the sled such that there was a relative motion between sample and sled due to sample stretching. A camera (Nikon D7000) was also used to capture the localized displacement of several points marked along the length of the samples. The experiments were conducted at a displacement rate of 2.5 mm/sec, and the results represent the means of triplicate experiments.

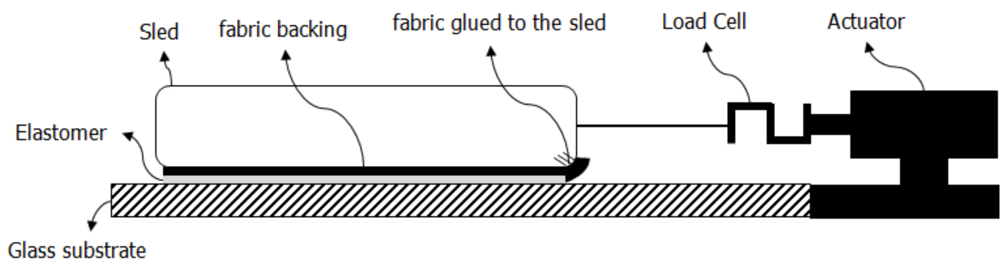


Figure 1. Schematic of experimental setup

Theoretical model

The shear lag model is used to predict the effect of backing stiffness on the friction of elastomer. The model's development was inspired by the related solution by Hart-Smith [10] for analysis of adhesively bonded lap joints considering adhesive plasticity. The intrinsic friction response of the elastomer with rigid backing is analogous to the elastic-plastic response of the adhesive in the lap joints shear loading. In particular, static friction force would be analogous to the linear elastic stress while the kinetic friction is analogous to the yielding and plastic stress. However, in the friction versus displacement response of the elastomer, there was a drop from static peak to the kinetic friction that is considered as a transition region, whereas in the adhesive layer with perfectly plastic response there is a plateau after yielding in the stress-strain curve.

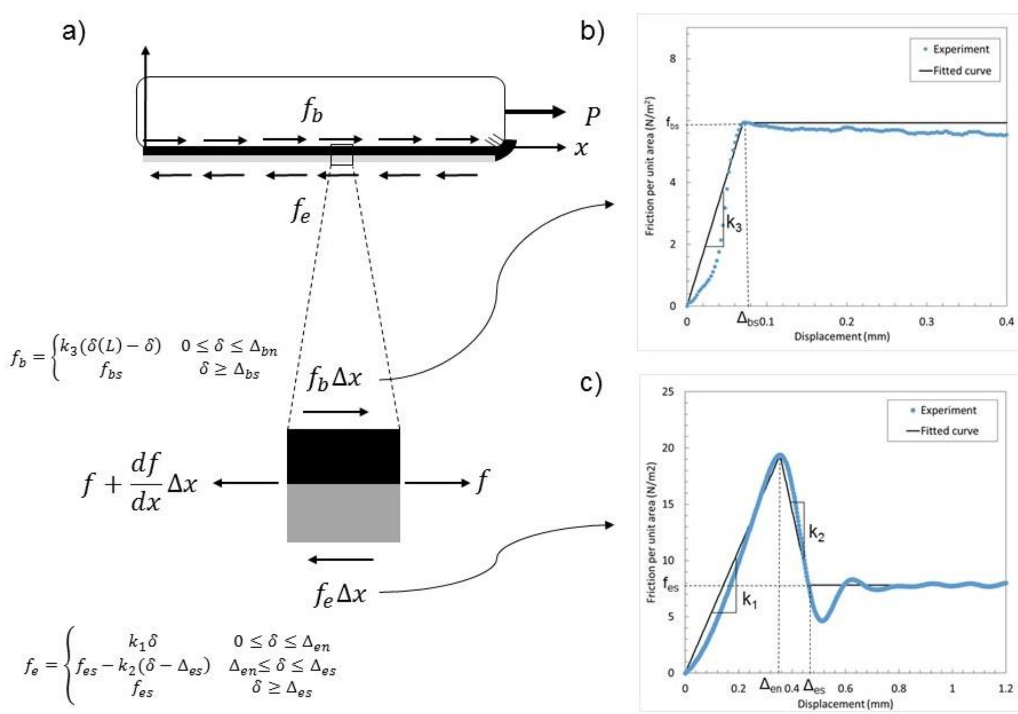
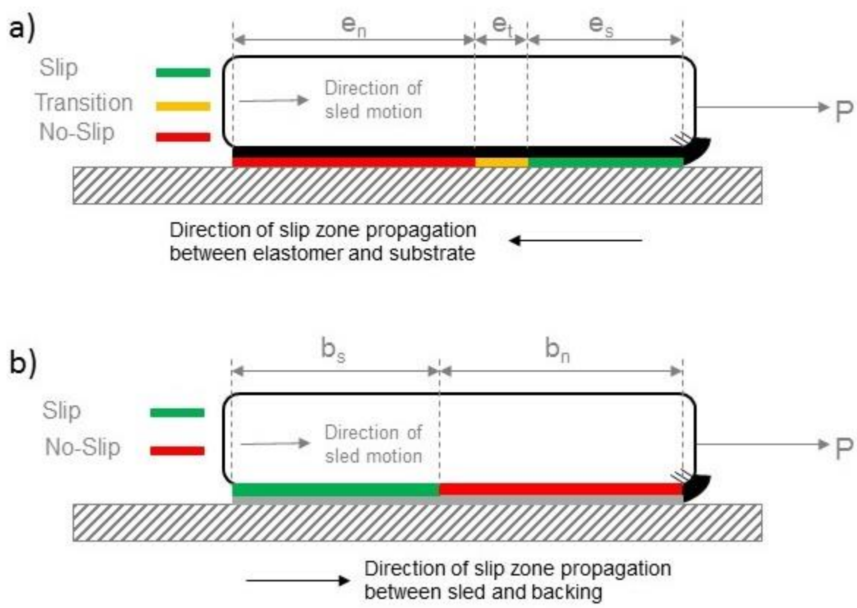


Figure 2. a) A differential element of an elastomer-coated fabric in contact with a substrate b) friction force between sled and backing fabric c) intrinsic friction response of elastomer with rigid backing

Figure 2(a) shows the geometry and free body diagram of a differential element of an elastomer-coated fabric in contact with a substrate. Using the longitudinal force equilibrium of the differential element dx and the stress-strain relation of the elastomer-coated fabric, the governing differential equation can be written as:

$$\frac{d^2\delta}{dx^2} + \frac{f_e - f_b}{Et} = 0 \tag{1}$$

where δ, f_e, f_b, and Et are the displacement, intrinsic friction force per unit area between elastomer with rigid backing and substrate, intrinsic friction force per unit area between sled and fabric backing, and the effective stiffness of the elastomer-coated fabric, respectively. Figure 2(b) and (c) depict the f_e and f_b friction force curves together with corresponding equations that will be used in the model as the intrinsic properties of the contact surfaces. The corresponding constants for f_e and f_b can be obtained from the rigid backing elastomer friction experiment and by measuring friction between sled and fabric, respectively.



Based on the intrinsic behavior of the elastomer and the axial compliance of the backing, three regions are defined along the length of elastomer in contact with substrate, namely no-slip, transition, and slip zones, while two regions are considered between sled and backing, namely no-slip and slip zones. However, the initiation of slippage between the backing and sled results in relative movement between the backing and elastomer, making the slip zone more dominant compared to the no-slip zone, in contact between backing and elastomer. Therefore, for the sake of simplicity, after initiation of the slip between backing and sled, the corresponding friction force is considered to be constant up to the slippage of the entire sample. Figure 3 demonstrates a schematic image of the mentioned regions along the length of the elastomer-coated fabric. By using the general solution for Equation (1) and the continuity of displacement and stress and the balance of the forces as the boundary conditions, the displacement and friction force can be numerically solved.

Results and Discussion

Figure 3 shows the position and displacement of eight points equally spaced through the length of the strip as a function of sled displacement. The displacement of each point was obtained from image analysis of the friction experiment. The first point was near the leading edge of the sled whereas the last point was at the end, and the others were in between, as indicated in inset in Figure 3. The first point moved simultaneously with the sled, while the others had a delay in movement due to the propagation of slippage zone up to the end, as shown in Figure 3. The onset of total

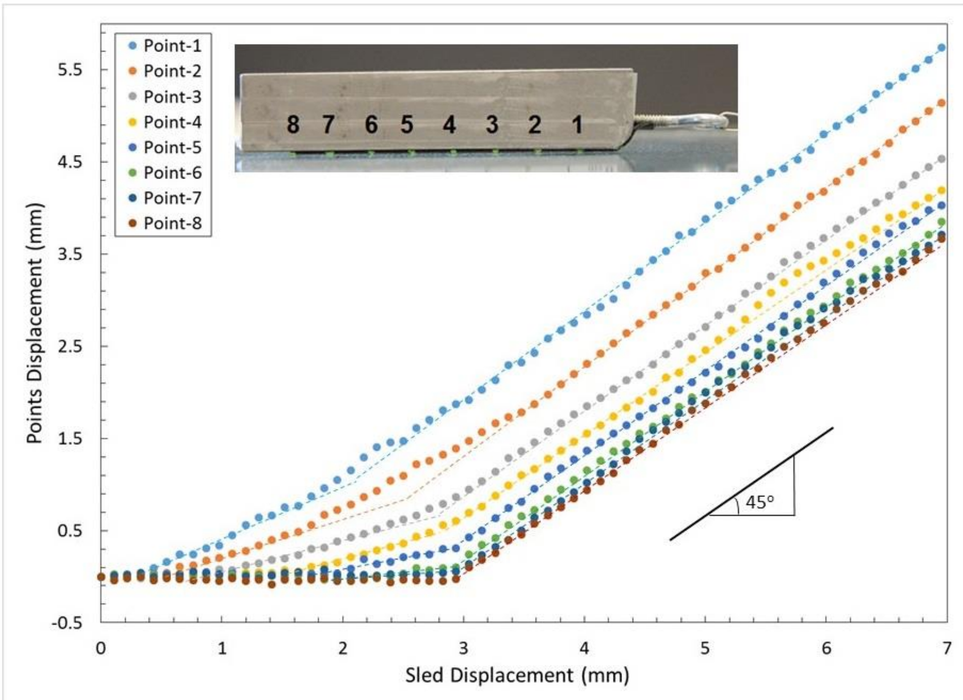


Figure 3. Displacement of different points and development of slippage zone across the length of strip. The least-square fits (dash lines) confirm the formation of transition and slip zones

slippage occurred when the last point started to move, (i.e. the moment at which point-8 had a rise in displacement as a function of sled displacement) and occurred after 3mm of sled displacement. The progressive development of the slippage zone between elastomer and substrate along the length is also obvious in the displacements of the points from the beginning to the end of the sample. Furthermore, there is a change in the slope of the curves, which was determined from the intersection of least-square fits of the two regions for each point. This change in slope corresponds to the transition region between no-slip and slippage zones, as indicated in Figure 3.

Figure 4 demonstrates the force-displacement response of strips with three different backing stiffnesses and comparisons with the theoretical model predictions. By decreasing the stiffness of the backing, the peak corresponding to the static friction force decreases dramatically up to a point where there is no peak corresponding to the static friction force. Therefore, the strip with rigid backing had the highest static friction peak whereas one with lowest stiffness experienced no static

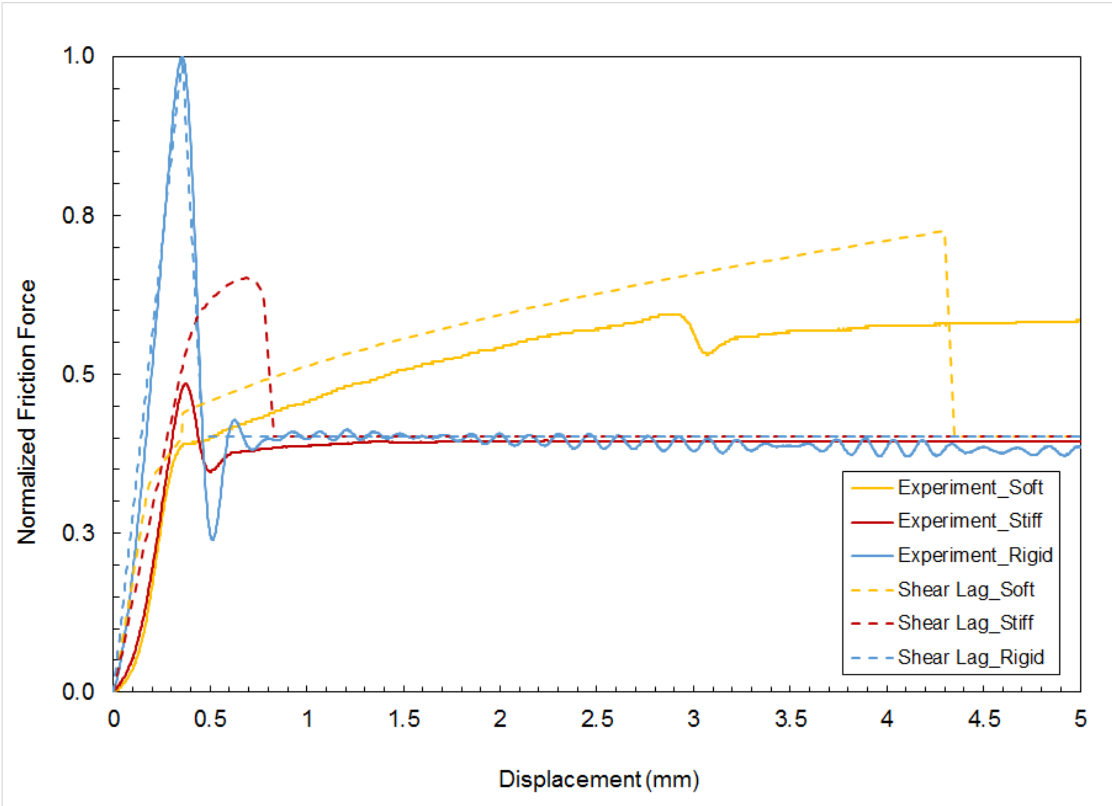


Figure 4. Force versus displacement curve of strips with three different backing stiffness

friction peak. This result is consistent with the experiment performed by Bartlett et al. [6] for the effect of backing stiffness on the shear adhesion strength of elastomers, while there is a correlation between adhesion and friction [5]. In their experiment, they used carbon fiber fabric as a backing of elastomer adhesives to decrease the compliance of the adhesives in the shear loading direction and found a scaling law between the force capacity and compliance of the adhesives in several order of magnitudes loading range. In the friction experiment, the elastomer with flexible backing experienced higher variation of deformation from the leading edge to the end, resulting to the combination of static and kinetic friction across the length of the strip and softening the overall static peak due to averaging the local static and kinetic friction forces. On the other hand, in the rigid backing elastomer, there was no significant change in the deformation through the length due to the rigidity of the backing and all the contact points moved together resulting to either static or kinetic friction across the length of the sample. This resulted in a sharp peak at static friction and a drop to kinetic friction after slippage. The shear lag model was able to predict this behavior very accurately. It predicted the initiation of slippage, where there is a slope change in the curve, as well as the point at which the slippage zone fully developed, where there is a small drop in curve for soft backing.

Conclusion

We found that the effective stiffness has a profound effect on the frictional response of the elastomers. In particular, by decreasing the effective stiffness, the peak corresponding to the static friction force decreased dramatically. For the more flexible backing, there was no observable peak corresponding to the static friction and the friction force increased with a small drop associated to the full development of the slippage zone. The local displacements of the several points along the elastomer length were analyzed using image processing and the development of the local slippage zone was observed. Furthermore, a new application of the classic shear lag model was proposed to predict the effect of effective stiffness on the friction force of elastomer-coated strip. Three regions were defined along the length of elastomer, namely no-slip, transition, and slip zones. The results obtained from the shear lag analysis were compared with experimental results and shown to be in good agreement. Interestingly, the analysis resembles that obtained when shear lag theory is applied to lap shear joints experiencing adhesive layer plasticity at the end.

Acknowledgments

The authors would like to thank the Provost's Office and College of Engineering for partial support of AM, the Biomedical Engineering and Mechanics (BEAM) Department for use of equipment, and the Macromolecules and Interfaces Institute at Virginia Tech for fostering interdisciplinary research in polymer science.

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