Friction of Extensible Strips: an Extended Shear Lag Model with Experimental Evaluation

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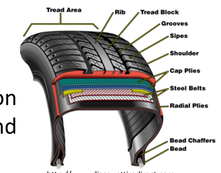
Motivation

- Friction plays an important role in our daily life
- Tires, shoes, wearable sensors, etc.
- Material stiffness varies widely depending on their application, material properties and geometry
- National Operating Committee on Standards for Athletic Equipment (NOCSAE) DOC (ND) 019 – 10m15a
- Standard Test Method and Performance Specification for Newly Manufactured Football Players Hand Coverings
- Requirement: Static Coefficient of Friction ≤ 4.5
- But the effect of stiffness on COF is not considered



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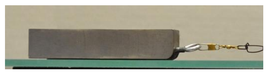
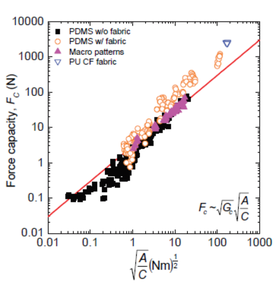


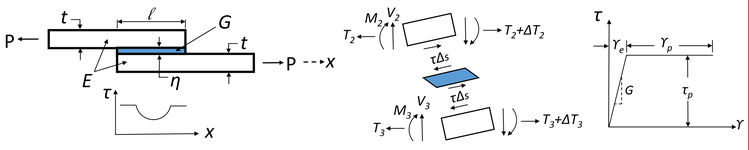
Background

- A scaling law has been developed by Dr. Crosby's group to relate the shear adhesion strength to the stiffness

$$F_c \sim \sqrt{G_c}\sqrt{\frac{A}{C}}$$



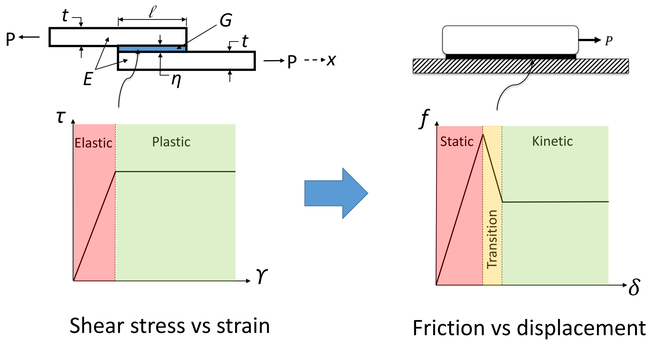
Bartlett, Michael D., et al. *Advanced Materials* 24.8 (2012): 1078-1083.



- Shear Lag model was developed by Hart-Smith for analysis of Adhesive-Bonded Single-Lap Joints experiencing adhesive's plasticity

Hart-Smith, L., Adhesive-bonded single-lap joints. 1973: NASA Technical Report

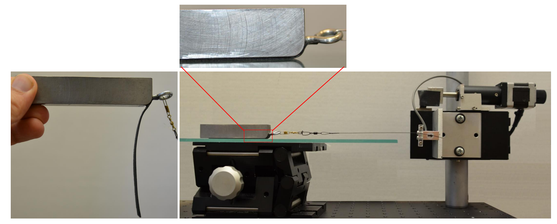
Analogy between Elastic-Plastic stress and Static-Kinetic friction:

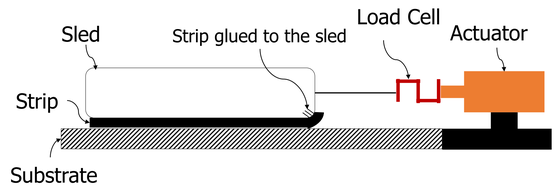


Shear stress vs strain

Friction vs displacement

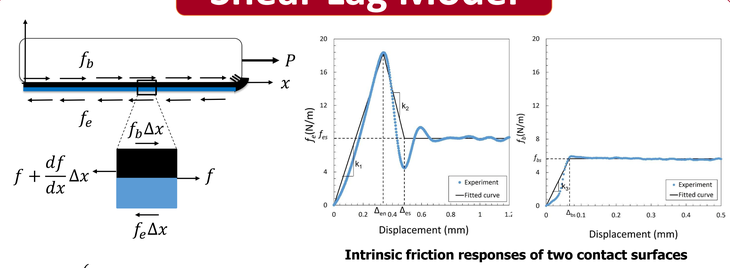
Experiments





- A custom built friction setup is used to measure the friction vs displacement of the strips
- A translational actuator pulls a steel sled (mass 190g, length 100mm, and width 10mm), the nose of which secures the end of the extensible strip
- Different strip materials and substrates were used, e.g. elastomer-coated fabric, braided elastic strip, glass, and tape
- The intrinsic friction force was obtained by rigidly adhering the strips to the sled

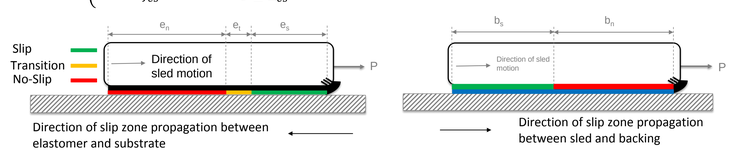
Shear Lag Model



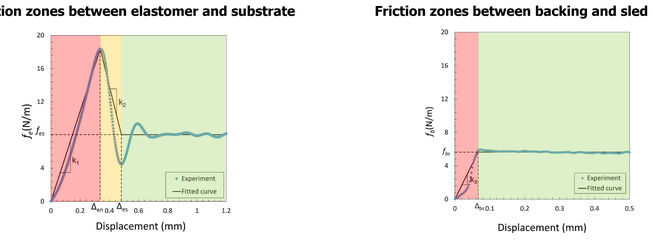
Intrinsic friction responses of two contact surfaces

$$f_e = \begin{cases} k_1\delta & 0 \le \delta \le \Delta_{en} \\ f_{es} - k_2(\delta - \Delta_{es}) & \Delta_{en} \le \delta \le \Delta_{es} \\ f_{es} & \delta \ge \Delta_{es} \end{cases}$$

$$f_b = \begin{cases} k_3(\delta(L) - \delta) & 0 \le \delta \le \Delta_{bn} \\ f_{bs} & \delta \ge \Delta_{bs} \end{cases}$$



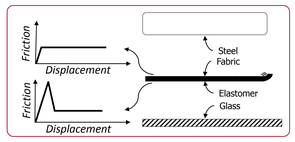
Friction zones between elastomer and substrate | **Friction zones between backing and sled**

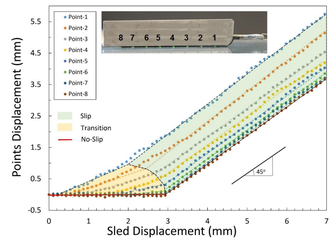


$$\frac{\partial^2\delta}{\partial x^2} - \frac{f_e - f_b}{EA} = 0$$

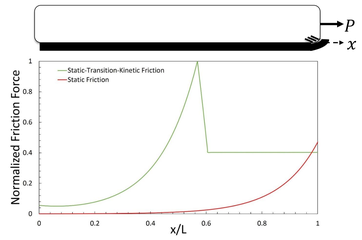
- The governing differential equations are obtained from a differential element at contact interfaces and equilibrium condition, for each region
- Boundary conditions are satisfied by continuity of the displacement and force at the boundaries of each region
- The displacement field and therefore the friction force can be obtained by solving the corresponding differential equations

Results

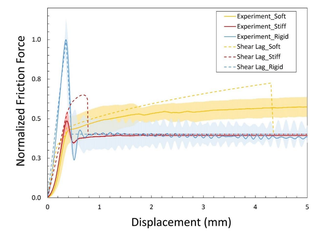




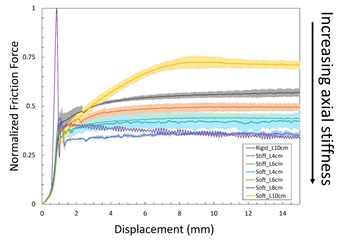
Displacement of different points and development of slippage zone across the length of a strip. The least-square fits (dash lines) confirm the formation of transition and slip zones



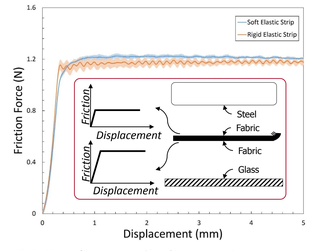
Normalized friction force per unit length across the length of the strip for two cases obtained from shear lag model



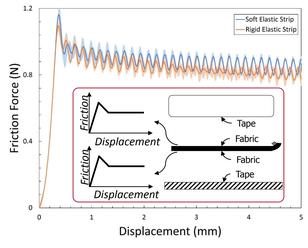
Comparison of friction force versus displacement response for strips with three different backing stiffnesses (shear lag model and experiment). Clouds' widths correspond to two standard deviations



Comparison of friction force versus displacement response for strips with different lengths and backing stiffnesses. Clouds' width correspond to two standard deviations



Friction force vs displacement response of an elastic strip with two fabric sides in contact with steel on the top and glass on the bottom



Friction force vs displacement response of an elastic strip with two fabric sides in contact with tape on both sides (symmetric)

Conclusion

- Effective stiffness has a profound effect on both static and kinetic friction of extensible strips
- There are three distinct regions along the length of the strip, namely no-slip, transition, and slip zones
- The static friction decreases with effective stiffness whereas the kinetic friction increases by decreasing the effective stiffness
- An extended shear lag model is developed to predict the frictional response of extensible strips
- The analysis resembles that obtained when shear lag theory is applied to lap shear joints experiencing adhesive layer plasticity

Acknowledgement

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