STRESS CONCENTRATIONS IN UNDERCUT SPUR GEAR TEETH VIA THE FINITE ELEMENT METHOD

by

Jamshid Jalilvand

Thesis submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Mechanical Engineering

APPROVED:

R. G. Mitchiner, Chairman

H. H. Mabie C. E. Knight

November, 1983
Blacksburg, Virginia
STRESS CONCENTRATIONS IN UNDERCUT SPUR GEAR TEETH
VIA THE FINITE ELEMENT METHOD

by

Jamshid Jalilvand

(ABSTRACT)

An analysis of the influence of undercutting on the stress concentration factor for undercut gears using the finite element method is presented. The models used are in the shape of a whole gear with three teeth. The middle tooth is loaded assuming single-tooth contact. Thirty seven finite element models were used to compute stress concentrations in gear teeth. The results for non-undercut gears were compared with the Dolan and Broghamer results, and were not more than 9.5 percent different. The results are expressed in the form of a linear relationship giving the stress concentration factor at the root fillet as a function of the geometry of the tooth. It has been verified that this equation is an accurate formula for both undercut and non-undercut gears with nominal proportions.
ACKNOWLEDGEMENTS

The author wishes to thank Dr. R. G. Mitchiner for his advice and guidance during the course of this study. Appreciation is also extended to Dr. C. E. Knight and Dr. H. H. Macbie for serving on the Graduate Committee and providing advice and assistance.

A special thanks goes to my wife, , for her support, patience, and understanding during the course of this research project.

Finally, the author would like to thank his parents for their continuous support and encouragement.
**NOMENCLATURE**

- $a$: addendum.
- $b$: dedendum.
- $D$: overall width of Dolan and Broghamer models.
- $h$: height of load position above the "theoretical weakest section".
- $K$: combined tensile stress concentration factor.
- $K_c$: compressive bending stress concentration factor.
- $K_d$: concentration factor calculated using Dolan and Broghamer formula.
- $K_j$: concentration factor calculated using regression equation.
- $K_t$: tensile bending stress concentration factor.
- $L$: overall length of Dolan and Broghamer models.
- $N$: number of teeth.
- $N_1$: minimum number of teeth to avoid undercutting.
- $P$: diametral pitch.
- $q$: amount of radial undercutting.
- $R$: standard pitch radius.
- $R_b$: base radius.
- $R_c$: radius from center of gear to apex of parabola, $K$.
- $R_o$: addendum radius.
- $R_x$: radius from center of gear to point on involute at end of contact.
r  minimum radius of curvature of tooth fillet at intersection with root circle.

\( r_f \)  fillet radius on hob tip.

S_c  maximum compressive stress at fillet.

S_L  stress calculated at Lewis point.

S_R  normal stress.

S_t  maximum tensile stress at fillet.

t  thickness of tooth at Lewis' "theoretical weakest section."

W  tangential component of the external load.

W_R  radial component of the external load.

X_A, Y_A  coordinates of point A on the involute.

X_D, Y_D  coordinates of point D on addendum circle.

X_E, Y_E  coordinates of point E on the trochoid.

X_L, Y_L  coordinates of Lewis point.

X_R, Y_R  coordinates of point R on dedendum circle.

\( \alpha \)  construction angle.

\( \beta \)  construction angle.

\( \psi \)  construction angle.

\( \eta \)  construction angle.

\( \mu \)  construction angle.

\( \gamma \)  construction angle.

\( \theta \)  pressure angle.

\( \theta_l \)  angle between line of action and horizontal line.

\( \sigma \)  combined stress calculated at Lewis point.
\( \tau \) construction angle.

\( \xi \) construction angle.

\( \zeta \) construction angle.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>iv</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. LITERATURE REVIEW</td>
<td>3</td>
</tr>
<tr>
<td>III. THE GEOMETRY OF UNDERCUT GEAR TEETH</td>
<td>7</td>
</tr>
<tr>
<td>Introduction</td>
<td>7</td>
</tr>
<tr>
<td>Hob Tip Radius</td>
<td>7</td>
</tr>
<tr>
<td>Minimum Number of Teeth to Avoid Undercutting</td>
<td>9</td>
</tr>
<tr>
<td>Determination of the Amount of Undercutting</td>
<td>10</td>
</tr>
<tr>
<td>Loading the Tooth For the Largest Principal Stresses</td>
<td>13</td>
</tr>
<tr>
<td>Stress at the Lewis Point</td>
<td>17</td>
</tr>
<tr>
<td>IV. GEOMETRY MODELING AND ANALYSIS</td>
<td>20</td>
</tr>
<tr>
<td>Introduction</td>
<td>20</td>
</tr>
<tr>
<td>Tooth Profile Program</td>
<td>20</td>
</tr>
<tr>
<td>SUPERTAB</td>
<td>27</td>
</tr>
<tr>
<td>SUPERB</td>
<td>29</td>
</tr>
<tr>
<td>V. FINITE ELEMENT MODEL SIZE STUDY</td>
<td>30</td>
</tr>
<tr>
<td>Introduction</td>
<td>30</td>
</tr>
<tr>
<td>Dolan and Broghamer's Model</td>
<td>30</td>
</tr>
<tr>
<td>Boundary Condition</td>
<td>31</td>
</tr>
<tr>
<td>Mesh Size Study</td>
<td>35</td>
</tr>
<tr>
<td>Hole Size Study</td>
<td>43</td>
</tr>
<tr>
<td>Description of Models</td>
<td>46</td>
</tr>
<tr>
<td>VI. RESULTS</td>
<td>52</td>
</tr>
<tr>
<td>VII. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>66</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>75</td>
</tr>
</tbody>
</table>
Appendix

A. PROGRAM TO CALCULATE THE AMOUNT OF UNDERCUTTING . . 77
B. MAIN COMPUTER PROGRAM . . . . . . . . . . . . . . . . 79
VITA 96
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1. Comparison of Dolan and Broghamer's Results and Finite Element</td>
<td>34</td>
</tr>
<tr>
<td>Analysis Results</td>
<td></td>
</tr>
<tr>
<td>5.2. Comparison of Two Different Finite Element Models Against</td>
<td>39</td>
</tr>
<tr>
<td>Dolan and Broghamer's Results</td>
<td></td>
</tr>
<tr>
<td>5.3. Results of Mesh Size Study</td>
<td>41</td>
</tr>
<tr>
<td>5.4. Results of Hole Size Study</td>
<td>45</td>
</tr>
<tr>
<td>5.5. Dimensions of Gear Models</td>
<td>47</td>
</tr>
<tr>
<td>5.5. (Cont.)</td>
<td>48</td>
</tr>
<tr>
<td>6.1. Results</td>
<td>54</td>
</tr>
<tr>
<td>6.1. (Cont.)</td>
<td>55</td>
</tr>
<tr>
<td>7.1. Results of Regression Analysis</td>
<td>67</td>
</tr>
<tr>
<td>7.1. (Cont.)</td>
<td>68</td>
</tr>
<tr>
<td>7.2. Result of Testing Concentration Factor Formula</td>
<td>70</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1.</td>
<td>Enlarged hob tip</td>
<td>8</td>
</tr>
<tr>
<td>3.2.</td>
<td>Determination of the amount of undercutting</td>
<td>11</td>
</tr>
<tr>
<td>3.3.</td>
<td>Load at highest point for single-tooth contact</td>
<td>14</td>
</tr>
<tr>
<td>3.4.</td>
<td>Gear tooth model by Dolan and Broghamer</td>
<td>18</td>
</tr>
<tr>
<td>4.1.</td>
<td>Generation of tooth profile</td>
<td>21</td>
</tr>
<tr>
<td>4.2.</td>
<td>Generation of root trochoid</td>
<td>23</td>
</tr>
<tr>
<td>4.3.</td>
<td>Finite element model of a tooth</td>
<td>28</td>
</tr>
<tr>
<td>5.1.</td>
<td>Loading device used by Dolan and Broghamer</td>
<td>32</td>
</tr>
<tr>
<td>5.2.</td>
<td>Finite element model of Dolan and Broghamer model</td>
<td>33</td>
</tr>
<tr>
<td>5.3.</td>
<td>Three teeth finite element model</td>
<td>36</td>
</tr>
<tr>
<td>5.4.</td>
<td>Whole gear finite element model</td>
<td>37</td>
</tr>
<tr>
<td>5.5.</td>
<td>Tangential displacement of nodes on the boundaries</td>
<td>38</td>
</tr>
<tr>
<td>5.6.</td>
<td>Variation of maximum principal stress at the fillet versus the element length</td>
<td>42</td>
</tr>
<tr>
<td>5.7.</td>
<td>Variation of maximum principal stress at the fillet versus the ratio of pitch radius to hole radius</td>
<td>44</td>
</tr>
<tr>
<td>5.8.</td>
<td>Variation of tooth thickness at theoretical weakest section versus the amount of undercutting (b=1.2 in.)</td>
<td>49</td>
</tr>
<tr>
<td>5.9.</td>
<td>Variation of tooth thickness at theoretical weakest section versus the amount of undercutting (b=1.3 in.)</td>
<td>50</td>
</tr>
<tr>
<td>5.10.</td>
<td>Variation of tooth thickness at theoretical weakest section versus the amount of undercutting (b=1.4 in.)</td>
<td>51</td>
</tr>
</tbody>
</table>
6.1. Variation of maximum tensile stress at the fillet versus the amount of undercutting (b=1.2 in.) 57

6.2. Variation of maximum tensile stress at the fillet versus the amount of undercutting (b=1.3 in.) 58

6.3. Variation of maximum tensile stress at the fillet versus the amount of undercutting (b=1.4 in.) 59

6.4. Variation of theoretical combined tensile stress at the fillet versus the amount of undercutting (b=1.2 in.) . . . . . . . . . . . 60

6.5. Variation of theoretical combined tensile stress at the fillet versus the amount of undercutting (b=1.3 in.) . . . . . . . . . . . 61

6.6. Variation of theoretical combined tensile stress at the fillet versus the amount of undercutting (b=1.4 in.) . . . . . . . . . . . 62

6.7. Variation of stress concentration factor versus the amount of undercutting (b=1.2 in.) . . . 63

6.8. Variation of stress concentration factor versus the amount of undercutting (b=1.3 in.) . . . 64

6.9. Variation of stress concentration factor versus the amount of undercutting (b=1.4 in.) . . . 65

7.1. Variation of geometry factor, J, versus number of teeth . . . . . . . . . . . . . . . . . . . . 72

7.2. Variation of normal stress versus number of teeth 73
Chapter I
INTRODUCTION

When gear teeth are produced by hobbing teeth into the blank, if the number of teeth is less than a specific number, undercutting occurs because the cutting tool removes some portion of the tooth flank. As a result of undercutting, the tooth thickness at the theoretical weakest section would be thinner and the stress at the root fillet would be higher. But in many applications it's sometimes necessary to use undercut gears to achieve a specific ratio between gears. Therefore it is important to know how the stresses are affected by undercutting.

Many investigations have been made to relate stresses at the root fillet to the geometry of the tooth, but none of them includes the influence of undercutting. One of the earliest works in this area has been done by Wilfred Lewis [1]. Lewis inscribed a parabola within the tooth profile to represent a beam of uniform strength. The Lewis investigation still is the basis for most gear designs. Later in Section 3.5 it will be fully discussed how Lewis' formula has been modified.

---

1 Number in brackets refer to references at the end of the thesis.
In 1942 Dolan and Broghamer [2], using photoelastic techniques, obtained a real stress distribution at the gear tooth fillets. It should be noted that the Dolan and Broghamer's work, though it occurred 40 years ago, is still regarded as being the definitive work relating to stress concentrations in gear teeth. Further, the results of this work have been incorporated in the design standards of the American Gear Manufacturers Association (AGMA). All of the earlier investigations had been involved with non-undercut gears.

To relate stresses at the root fillet to the geometry of the tooth, using the finite element method, several gear tooth models were used in this study. These models covered both undercut and non-undercut gears, with a wide range of geometric dimensions. Chapter 3 fully covers the geometry of undercut gears. This chapter also includes the calculation of stresses at the root fillet and how the load was applied to cause the highest stress at the fillet.

SUPERTAB [3] and SUPERB [4] are two interactive computer programs used to generate tooth models and analyze them. Chapter 4 explains how the computer was used to create a finite element model and to perform the analysis.
Chapter II

LITERATURE REVIEW

Many earlier investigators attempted to relate the tensile stress at the root fillet to the geometry of the tooth. In 1893 Wilfred Lewis [1] used beam theory by inscribing a parabola within the tooth profile to represent a beam of uniform strength. Using photoelastic techniques, Baud and Timoshienko [5], obtained the stress distribution at the gear tooth fillets.

In 1942 Dolan and Broghamer [3] used photoelastic models of spur gear teeth to determine the maximum tensile stresses developed in the fillets of gear teeth. Their results were expressed in the form of a stress concentration factor, which was a function of the minimum radius of the fillets, the thickness of the tooth at the theoretical weakest section, the tooth pressure angle, and the height of the load. Dolan and Broghamer's models contained various standard gear teeth but did not include any undercut gears. Each model contained one full tooth and half of each of the adjacent teeth.

As Jacobson [6] has pointed out, for reasons of size and cost, fully generated gears could not be made in a photoelastic material for each gear to be investigated. Dolan and
Broghamer applied the load to the flank of the tooth profile normally subjected to actual loads during the period of engagement. They found that the position of the actual maximum stress at the fillet obtained from the photoelastic analysis was located slightly closer to the gear center than the theoretical weakest section as defined by Lewis. However Dolan and Broghamer noted that Lewis' location for the theoretical weakest section was not greatly in error.

Using the ratio of the actual maximum stress in the fillet, as obtained from the photoelastic fringe pattern, to the calculated stress at the theoretical weakest section, the stress concentration factors were computed. Dolan and Broghamer used the Lewis beam theory to calculate the theoretical stresses at the fillet. As a result of these investigations, two equations were developed for values of the combined tensile stress concentration factor as follows:

For a 14.5 deg. pressure angle:

\[ K = 0.22 + (\frac{t}{R})^{0.2} \cdot (\frac{n}{t})^{0.4} \]

For a 20.0 deg. pressure angle:
In 1955, Jacobson [6] used two gear models, both having the same material elastic constants. The load was applied from one gear tooth to the other. Jacobson's results were in a good agreement with those of Dolan and Broghamer [7].

With the help of a modern digital computer it is possible to analyze gear tooth models via the finite element method. In 1973, Wilcox and Coleman [10] used finite element analysis to compute the gear tooth stresses. In their paper it is not mentioned what kind of elements or boundary condition have been used. They have compared their results with Dolan and Broghamer formula and concluded that the results are only a few percent different.

In 1972, Chabert, Dang Tran and Mathis [11] evaluated stresses and deflection of spur gear teeth under strain and concluded that their work permits to know the elastic deflection coefficient of the tooth with a high degree of accuracy. As to the value of the stress concentration factors, their result are substantially in agreement with Dolan and Broghamer photoelastic investigations.

All of the earlier investigations have been involved with non-undercut gears. As the result of literature review, it

\[ K = 0.18 + \left( \frac{t}{r} \right)^{0.15} \cdot \left( \frac{t}{h} \right)^{0.45} \]
should be pointed out that no treatment is made on slightly undercut teeth.
3.1 INTRODUCTION

This chapter presents the geometry of undercut gear teeth. Determination of the hob tip radius, the minimum number of teeth to avoid undercutting and the amount of undercutting is explained. The resulting finite element tooth models are based upon the geometry concepts presented herein.

3.2 HOB TIP RADIUS

Figure 3.1 illustrates an enlarged view of the hob tooth tip. As described by Mitchiner and Mabie [8], the distance \( \Delta \), which is the half of the rack tip land, may be expressed by:

\[
\Delta = \frac{\pi}{4p} - (b - r_f)\tan \phi - \frac{r_f}{\cos \phi}
\]  

(3.1)

When the tip is fully rounded ( \( \Delta = 0 \)), the hob tip radius is expressed as follows:
Fig. 3.1 Enlarged hob tip [8].
\[ r_f = \frac{1}{1 - \sin \phi} \left( \frac{\pi P}{4} \cos \phi - bs \sin \phi \right) \tag{3.2} \]

All the finite element models have the hob tip radius less than or equal to the amount of \( r_f \), calculated by using equation (3.2), to cover different size gear models with different hob tip radius.

### 3.3 MINIMUM NUMBER OF TEETH TO AVOID UNDERCUTTING

Undercutting occurs if the number of teeth cut in the blank is less than a specific number. This number of teeth has been expressed by Mitchiner, Mabie and Moosavi-Rad [9] as follows:

\[
N_1 = \frac{2P[b + r_f(s \sin \phi - 1)]}{\sin^2 \phi} \tag{3.3}
\]

Equation (3.3) has been used to determine the minimum number of teeth to avoid undercutting for different groups of models. Each group of gear models, with the same \( b, r_f \), and \( \phi \), contains two undercut gears with two or four teeth less than \( N_1 \). Two other gears of each group have teeth more than or equal to \( N_1 \) to avoid undercutting.
3.4 DETERMINATION OF THE AMOUNT OF UNDERCUTTING

It is important to calculate the amount of undercutting when a gear is undercut. This quantity, q, radial distance between the intersection point of the involute and trochoid and the base circle, has been determined by Mitchiner, Mabie, Moosavi-Rad, [9] as follows:

From Figure 3.2 it can be seen that:

\[ \cos \mu = \frac{R_b}{R_b + q} \]  \hspace{1cm} (3.4)

and

\[ \cos \psi = \frac{R - (b - r_f)}{R_b + q} \]  \hspace{1cm} (3.5)

from Spotts [12]

\[ R(\psi + \text{inv} \phi - \text{inv} \mu) = [R - (b - r_f)] \tan \psi + (b - r_f) \tan \phi \]  \hspace{1cm} (3.6)

Since

\[ R = \frac{N}{2p} \]
Fig. 3.2 Determination of the amount of undercutting [9].
then

\[ \frac{N}{2P} (\sin \varphi + \sin \psi) + (b - r_f) \tan \phi \]

\[ = (b - r_f) \tan \psi + \frac{N}{2P} \sin \phi \]  \hspace{1cm} (3.7)

by solving equations (3.4) and (3.5) for \( q \) and then equated,

\[ R_b \cos \psi = (R - b + r_f) \cos \varphi \]  \hspace{1cm} (3.8)

or

\[ \frac{N}{2P} \cos \phi \cos \psi = (\frac{N}{2P} - b + r_f) \cos \varphi \]  \hspace{1cm} (3.9)

The angle \( \varphi \) can be determined by solving simultaneously equations (3.7) and (3.9), and the amount of radial undercutting can be calculated from the following equation:

\[ q = \frac{N}{2P} \cos \phi \left( \frac{1}{\cos \varphi} - 1 \right) \]  \hspace{1cm} (3.10)
A computer program shown in Appendix A has been used to solve equations (3.7) and (3.9) simultaneously using the Newton-Raphson technique in double precision. This interactive program needs starting values for \( \mu \) and \( \psi \). It has been determined experimentally that .3° for \( \mu \) and .5° for \( \psi \) are good starting values.

3.5 LOADING THE TOOTH FOR THE LARGEST PRINCIPAL STRESSES

It was necessary to apply the force to a point of tooth profile that causes the largest stress in the root fillet. When the tooth is loaded by only one mating tooth. For the case of load sharing in a gear, as shown in Fig. 3.3, the point \( U \) is the highest intersection of the line of action and the centerline when only one tooth is in engagement [8].

Mitchiner and Mabie [8] have expressed the distance \( R_C \) between point \( U \) and center of gear as follow:

\[
R_{o2} = R_2 + a
\]  

(3.11)

\[
\tau = \sin^{-1} \left( \frac{R_2}{R_{o2}} \cos \phi \right)
\]  

(3.12)
Fig. 3.3 Load at highest point for single-tooth contact [8].
\[ R_x = \sqrt{Q^2 + R_1^2 - 2QR_1 \sin \phi} \]  

(3.13)

where

\[ Q = \frac{\cos(\tau + \phi)}{\sin \tau} R_2 \]  

(3.14)

Now

\[ K = \frac{\pi}{2N} - \text{inv} \left( \cos^{-1} \left( \frac{R_1}{R_x} \cos \phi \right) \right) + \text{inv} \phi \]  

(3.15)

\[ \xi = \frac{2\pi}{N} - K \]

\[ \zeta = \cos^{-1} \left[ \frac{R_x^2 + R_2^2 - (R_1 + R_2)^2}{2R_x R_2} \right] - \tau \]  

(3.16)
then

\[ R_c = \frac{R_x \sin(\xi)}{\sin(\xi + \xi)} \]  \hspace{1cm} (3.17)

\[ \phi_1 \] is the angle between the line of action and a horizontal line and is defined as follows [8]:

\[ \phi_L = \tan^{-1} \left( \frac{1 - \left( \frac{R}{R+a} \cos \phi \right)^2}{\frac{R}{R+a} \cos \phi} \right) - \frac{\pi}{2N} \]

\[ -\text{inv}[\tan^{-1} \left( \frac{1 - \left( \frac{R}{R+a} \cos \phi \right)^2}{\frac{R}{R+a} \cos \phi} \right)] + \text{inv} \phi \] \hspace{1cm} (3.18)

For all the models a unit force has been applied to the tooth along the line of action passing through point U. This force will cause the highest stress at the Lewis point in accordance with Mitchiner and Mabie [8].
3.6 **STRESS AT THE LEWIS POINT**

In order to calculate the stress concentration factor, the stresses at the Lewis point were calculated. Dolan and Broghamer [2] have computed the stress concentration factors as the ratio of the actual maximum stress at the fillet, as obtained from the experiments, to the calculated stress at the theoretical weakest section.

The theoretical weakest section is located between the two points of tangency of the tooth profile with a parabola inscribed in the tooth outline (see Fig. 3.4) with its apex at the intersection of the line of action of the applied force and the centerline of the tooth [2].

To locate the theoretical weakest section, a computer program has been written based on procedure expressed by Mitchiner and Mabie [8]. This program is a subroutine called POINT which will locate the Lewis point \((X_1, Y_1)\) and \(R_c\) (see Section 3.4). This subroutine is presented in Appendix B.

Neglecting the radial component, \(W_r\), of the applied load [2], the tensile or compressive bending (flexural) stresses, \(S_L\), was calculated by means of the Lewis equation:

\[
S_L = \frac{6wh}{I^2} \quad (3.19)
\]
Fig. 3.4 Gear tooth model by Dolan and Broghamer [3].
Where \( h \) is a distance between load position and the theoretical weakest section, or:

\[
h = R_c - Y_l
\]

A value for combined bending and normal stresses, \( \sigma \), at the tensile fillet was obtained by assuming that the stress developed by the radial component of load was equal to the load, \( W_R \), divided by the cross-sectional area of the tooth at the theoretical weakest section, and algebraically adding this value to the flexural stress \( S_L \) computed by equation (3.19) [2].

\[
S_R = \frac{w_R}{t} = \frac{w \tan \phi_L}{t} \tag{3.20}
\]

\[
\sigma = S_L - S_R = \frac{6wh}{t^2} - \frac{w}{t} \tan \phi_L \tag{3.21}
\]

All the models have been assumed to have a face width of unity.
Chapter IV
GEOMETRY MODELING AND ANALYSIS

4.1 INTRODUCTION

This chapter details the creation and analyses of the finite element models.

4.2 TOOTH PROFILE PROGRAM

It was necessary to use the equations of the trochoid and involute to define the profile of the tooth. Figure 4.1 shows a half tooth indicating its pitch and base circles. From this figure it can be seen that angle AOC (A is a point on the involute) is equal to:

\[ \angle AOC = \left( \frac{p}{2R} + \text{inv}\phi - \text{inv}\alpha \right) \]

and

\[ R_{\alpha} = R \frac{\cos\phi}{\cos\alpha} \]
Fig. 4.1 Generation of tooth profile.
As Fig. 4.1 shows, the coordinates of point A can be determined as follows:

\[ x_A = R \sin(AOC) \]

\[ x_A = R \frac{\cos \phi}{\cos \alpha} \cdot \sin \left[ \frac{P \cos \phi}{2R} + \text{inv} \phi - \text{inv} \alpha \right] \quad (4.1) \]

\[ y_A = R \frac{\cos \phi}{\cos \alpha} \cdot \cos \left[ \frac{P \cos \phi}{2R} + \text{inv} \phi - \text{inv} \alpha \right] \quad (4.2) \]

The coordinates of points on the trochid of a tooth have been defined parametrically by Mitchiner and Mabie [8] (see Fig. 4.2) as:
Fig. 4.2 Generation of root trochoid [8].
\[ x = (R - b + r_f) \sin(\beta + \theta) - R \cos(\beta + \theta) \]

\[ - \frac{r_f}{\sqrt{(b - r_f)^2 + R^2 \theta^2}} \left[ R \cos(\beta + \theta) + (b - r_f) \sin(\beta + \theta) \right] \quad (4.3) \]

\[ y = (R - b + r_f) \cos(\beta + \theta) + R \sin(\beta + \theta) \]

\[ + \frac{r_f}{\sqrt{(b - r_f)^2 + R^2 \theta^2}} \left[ R \sin(\beta + \theta) - (b - r_f) \cos(\beta + \theta) \right] \quad (4.4) \]

where

\[ \beta = \frac{\pi}{N} - \eta \quad (4.5) \]

\[ \eta = \frac{\Delta}{R} \quad (4.6) \]

and
\[ \Delta = \frac{\pi}{4p} - (b - r_f)\tan\phi - \frac{r_f}{\cos\phi} \quad (4.7) \]

It was also necessary to locate the point of tangency or the point of intersection of the involute tooth flank and the trochoidal tooth root (point I). For this purpose a computer program has been written, using the Newton-Rhapson method, to solve equations (4.1), (4.2), (4.3) and (4.4). This program (the main program) is presented in Appendix B.

Coordinates of the points on the root circle and those points on the addendum circle can be determined from Fig. 4.1 as follows:

\[ x_R = (R - b)\sin\theta \quad (4.8) \]

\[ y_R = (R - b)\cos\theta \quad (4.9) \]
This main computer program will calculate the coordinates of points on the root circle using equations (4.8) and (4.9) with $\theta$ ranging from $\pi/N$ to $\beta$ (see Fig. 4.1). Then the program will calculate the coordinates of points on the trochoid using equations (4.3) and (4.4) with $\theta$ starting from zero through the point of intersection (see Fig. 4.2). The program, using equations (4.1) and (4.2), will determine coordinates of points on the involute, from intersection point I until $a$ is equal to $\gamma$, where

$$
\gamma = \cos^{-1} \left( \frac{R \cos \phi}{R+a} \right)
$$

Equations 4.10 and 4.11 can be used to determine the coordinates of points on the addendum circle such that

$$
x_D = (R + a) \sin \theta
$$

$$
y_D = (R + b) \cos \theta
$$
4.3 SUPERTAB

SUPERTAB is an interactive program that frees designers and analysts from the routine demands of finite element model preparation and simplifies interpretation of analysis results via graphical displays [3]. SUPERTAB is a product of General Electric CAE International Inc.

After calculating coordinates of points on the tooth profile, these data have been recorded in a file for SUPERTAB. Using splines, lines, and arcs passing through points, a tooth model was created (see Fig. 4.3).

Then, using enhanced mesh generation, a mesh was generated. The advantage of enhanced mesh generation is that at user-specified points in the model, mesh density can be specified. It is necessary to have finer mesh around the root fillet and at the point where force is applied.

Then the nodes and elements were re-numbered to have minimum nodal band width. Nodes along the boundaries were fixed (see Section 5.2), and a force was applied to the closest
Fig. 4.3 Finite element model of a tooth.
node to the point of intersection of the line of action and the tooth profile (see Section 3.4).

Finally, a SUPERB file was written for the analysis of the finite element models.

4.4 SUPERB

SUPERB is a GE/CAE program which performs a finite element analysis. The finite element method (FEM) embodies the concept of physical structure represented by a model composed of a finite number of assembled sub-components or elements. Known loads are applied to this model, a system of equations is solved and results obtained [4].

In all of the gear models parabolic plane stress elements were used, with two nodal degrees of freedom per node, $U_x$ and $U_y$. It should be pointed out that calculated nodal stress is actually an average of the stresses located at the nearest Gauss point [4].

The results of the SUPERB analysis were recorded in a list file, listing averaged nodal stresses by descending maximum principal stresses, and a plot file, containing the stress contours and distorted geometry.
Chapter V

FINITE ELEMENT MODEL SIZE STUDY

5.1 INTRODUCTION

Several different finite element models have been investigated to study the effects of boundary conditions, mesh size, and hole size on solution convergence. On the basis of these studies, 37 finite element models were used to compute stress concentrations in gear teeth.

5.2 DOLAN AND BROGHAMER'S MODEL

The Dolan and Broghamer models were based on the shapes of spur gear teeth having a diametral pitch of two \(2\). Each model contained one full tooth and half of each of the two adjacent teeth. The overall dimensions \(L\) and \(D\) of the model as shown in Fig. 3.4 were 3.875 in. and 3.125 in., respectively. All of the gear tooth models were made from 0.25 in. thick polished plates of BT-61-893 clear bakelite.

For purposes of comparison the value of stress were divided by the applied load and multiplied by the thickness of the model to obtain equivalent stresses for a model of unit thickness and loaded with a unit load \(2\).

A finite element model was used to analyze a model used by Dolan and Broghamer and show the validity of modeling and
analysis techniques. The overall dimensions were the same as model 1 in the Dolan and Broghamer experiment ( \( \phi = 14.5^\circ \), a=0.5 in. b=.578 in., N=24, P=2 in., \( r_f = .132 \) in.). Figure 5.1 shows the loading device used by Dolan and Broghamer and Fig. 5.2 shows the finite element model with its boundary conditions and applied force.

A unit force was applied perpendicular to the centerline and 0.08 in. below the pitch point, identical to the Dolan and Broghamer applied load. Nodes at the boundaries were fixed, as is shown in Fig. 5.2, to have approximately the same boundary condition as Dolan and Broghamer's model.

The results of the finite element analysis and Dolan and Broghamer's results, listed in Table 5.1, are only 1.5 percent different.

5.3 **BOUNDARY CONDITION**

For the study of boundary conditions, a model with three full teeth was used. Figure 5.3 shows this model which had the diametral pitch of two and the same dimensions as model 1 in the Dolan and Broghamer experiment ( \( \phi = 14.5^\circ \), a=0.5 in. b=.578 in., N=24, \( r_f = .132 \) in.). The load was applied to the same position described in Section 5-1. Nodes on sides AB and CD were fixed only in the tangential direction, while the nodes on the hole side BD were fixed in both radial and
Fig. 5.1 Loading device used by Dolan and Broghamer [3].
Fig. 5.2 Finite element model of Dolan and Broghamer model 1.
**TABLE 5.1**

Comparison of Dolan and Broghamer's Results and Finite Element Analysis Results.

<table>
<thead>
<tr>
<th></th>
<th>$S_t$ (psi)</th>
<th>$S_c$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dolan and Broghamer's model</td>
<td>8.20</td>
<td>8.20</td>
</tr>
<tr>
<td>Finite element model</td>
<td>8.08</td>
<td>7.99</td>
</tr>
</tbody>
</table>
tangential directions. Then a whole gear model with the same dimensions was used (see Fig. 5.4). Since single-tooth contact was being studied, only three teeth were used in this model. All the nodes on the hole side were fixed in both the radial and tangential directions. This model had more realistic boundary conditions than either the previous model or the Dolan and Broghamer model.

Nodes on the sides AB and CD displaced in the tangential direction when a whole gear model was used. Because of the displacements of these nodes, and to get more reliable results, the whole gear models were used for all the finite element analyses.

The displacements of the nodes on the sides AB and CD for the whole gear model are shown in Fig. 5.5, and the stress results are listed in Table 5.2.

5.4 MESH SIZE STUDY

Mesh size has a very profound effect upon the accuracy of the results, of a finite element analysis. For more accurate results it is necessary to use a finer mesh around the area of loading and stress concentrations (see Fig. 4.3).

A group of gear models with the same dimensions as described in Section 5.3 were used. A unit force was applied as before. Each gear model had different size elements
Fig. 5.3 Three teeth finite element model.
Fig. 5.4 Whole gear finite element model.
Fig. 5.5 Tangential displacements of nodes on the boundaries.
### TABLE 5.2
Comparison of Two Different Finite Element Models Against Dolan and Broghamer's Results.

<table>
<thead>
<tr>
<th></th>
<th>$S_t$ (psi)</th>
<th>$S_c$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dolan and Broghamer's model</td>
<td>8.20</td>
<td>8.20</td>
</tr>
<tr>
<td>Three teeth model</td>
<td>8.24</td>
<td>7.90</td>
</tr>
<tr>
<td>Whole gear model</td>
<td>8.29</td>
<td>7.88</td>
</tr>
</tbody>
</table>
around the root fillet where the maximum tensile and compressive stresses occur. The results of this experiment are listed in Table 5.3 and plotted in Fig. 5.6. It is indicated that as the root element length gets smaller than .047 in., the stresses remain relatively unchanged, i.e. the finite element solution has converged.

Also it was noticed that using .05 in. length elements along the root fillet gives the closest agreement to Dolan and Broghamer results for the same model. Nodal stresses on the boundary close to theoretical weakest section changed by 9 percent from node to node when .05 in. length elements were used. It should be pointed out that calculated nodal stress is actually an average of the stresses located at the nearest Gauss points in the SUPERB output. Stresses of Gauss points approaching the boundary were increasing in such way that the stress at the boundary was not different more than five percent of the nodal stress at the same point.

As a result of this study, since there was only 1.4 percent different between Dolan and Broghamer result and the finite element result, for all the finite element models .05 in. length elements were used along the root fillet.

To apply the force as close as possible to the position which would cause the largest stress in the root fillet, it
TABLE 5.3
Results of Mesh Size Study

<table>
<thead>
<tr>
<th>Element length (in.)</th>
<th>$S_t$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.025</td>
<td>8.34</td>
</tr>
<tr>
<td>.032</td>
<td>8.31</td>
</tr>
<tr>
<td>.047</td>
<td>8.30</td>
</tr>
<tr>
<td>.059</td>
<td>8.04</td>
</tr>
<tr>
<td>.069</td>
<td>7.85</td>
</tr>
<tr>
<td>.083</td>
<td>7.74</td>
</tr>
</tbody>
</table>
Fig. 5.6 Variation of maximum principal stress at the fillet versus the element length.
was also necessary to have relatively fine mesh around the point where force was applied (refer to Section 3.4) Also, since stresses in the vicinity of the point of application of the load were not of interest, displacements of the load-ed node and adjacent nodes were neglected.

5.5 HOLE SIZE STUDY

The size of the hole in the gear models affected the results of experiments. For this study a group of gear models with the same dimensions as previous models (\( \phi =14.5^\circ \), a=0.5 in. b=.578 in., N=24, P=2 in., \( r_f = .132 \) in.), but with different size holes were analyzed. It was noted that as the ratio of pitch radius to hole radius decreases the maximum principal stress decreases as well. These result are shown in Fig. 5.7 and listed in Table 5.4. From Fig. 5.7 it can be noted that ratio of pitch radius to hole radius should not be less than 2 for consistent results.

As a result of this study, for all the finite element models the hole radius was chosen to be half of the pitch radius.
Fig. 5.7 Variation of maximum principal stress at the fillet versus the ratio of pitch radius to hole radius.
TABLE 5.4

Results of Hole Size Study

<table>
<thead>
<tr>
<th>Ratio of pitch radius to hole radius.</th>
<th>$S_t$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>6.91</td>
</tr>
<tr>
<td>1.5</td>
<td>8.04</td>
</tr>
<tr>
<td>2</td>
<td>8.30</td>
</tr>
<tr>
<td>3</td>
<td>8.10</td>
</tr>
<tr>
<td>4</td>
<td>8.17</td>
</tr>
<tr>
<td>5</td>
<td>8.23</td>
</tr>
<tr>
<td>6</td>
<td>8.10</td>
</tr>
</tbody>
</table>
5.6 DESCRIPTION OF MODELS

Thirty seven models were used to calculate stresses and determine stress concentration factor in spur gear teeth via the finite element method, based on previous experiments. These models, listed in Table 5.5, have been divided into three major groups. Gear models in group M have a dedendum of 1.2 in. For gear models in group N, the dedendum is equal to 1.3 in., but the dedendum of models in group J is equal to 1.4 in. Each group contains 12 models except group M, which has 13 models. Models in each group have been divided to three subgroups with different hob-tip radii, ranging from .10 in. to the maximum radius as calculated by equation (3.2) to cover different size gears. Each subgroup contains 4 models with different number of teeth. Number of teeth of the first two models in each subgroup are more than the minimum number of teeth to avoid undercutting. The rest of the models are undercut.

The amount of undercutting, q, has been determined by the method explained in Section 3.3. Figures 5.8, 5.9 and 5.10 show the tooth thicknesses at the theoretical weakest section vs. the amount of undercutting for different groups of models.
### TABLE 5.5

Dimensions of Gear Models

(\(\phi=20^\circ\) \(P=1.0\) in. \(a=1.0\) in.)

<table>
<thead>
<tr>
<th>MODEL</th>
<th>(b)</th>
<th>(r_f)</th>
<th>(P)</th>
<th>(a)</th>
<th>(N)</th>
<th>(t)</th>
<th>(h)</th>
<th>(q)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
</tr>
<tr>
<td>M1</td>
<td>1.20</td>
<td>0.10</td>
<td>22</td>
<td>1.9000</td>
<td>1.0526</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>1.20</td>
<td>0.10</td>
<td>20</td>
<td>1.8560</td>
<td>1.0618</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M3</td>
<td>1.20</td>
<td>0.10</td>
<td>18</td>
<td>1.8060</td>
<td>1.0724</td>
<td>0.27241E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M3a</td>
<td>1.20</td>
<td>0.10</td>
<td>17</td>
<td>1.7768</td>
<td>1.0783</td>
<td>0.13862E-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>1.20</td>
<td>0.10</td>
<td>16</td>
<td>1.7446</td>
<td>1.0848</td>
<td>0.33993E-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M5</td>
<td>1.20</td>
<td>0.30</td>
<td>20</td>
<td>1.8860</td>
<td>1.0105</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M6</td>
<td>1.20</td>
<td>0.30</td>
<td>18</td>
<td>1.8420</td>
<td>1.0239</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M7</td>
<td>1.20</td>
<td>0.30</td>
<td>15</td>
<td>1.7540</td>
<td>1.0485</td>
<td>0.76425E-04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M8</td>
<td>1.20</td>
<td>0.30</td>
<td>14</td>
<td>1.7178</td>
<td>1.0585</td>
<td>0.99729E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M9</td>
<td>1.20</td>
<td>0.40</td>
<td>18</td>
<td>1.8594</td>
<td>1.0001</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M10</td>
<td>1.20</td>
<td>0.40</td>
<td>16</td>
<td>1.8072</td>
<td>1.0173</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M11</td>
<td>1.20</td>
<td>0.40</td>
<td>14</td>
<td>1.7422</td>
<td>1.0382</td>
<td>0.23075E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>M12</td>
<td>1.20</td>
<td>0.40</td>
<td>12</td>
<td>1.6588</td>
<td>1.0642</td>
<td>0.16563E-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N1</td>
<td>1.30</td>
<td>0.10</td>
<td>22</td>
<td>1.8920</td>
<td>1.1263</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N2</td>
<td>1.30</td>
<td>0.10</td>
<td>21</td>
<td>1.8690</td>
<td>1.1302</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N3</td>
<td>1.30</td>
<td>0.10</td>
<td>20</td>
<td>1.8442</td>
<td>1.1343</td>
<td>0.10190E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N4</td>
<td>1.30</td>
<td>0.10</td>
<td>18</td>
<td>1.7874</td>
<td>1.1434</td>
<td>0.24844E-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N5</td>
<td>1.30</td>
<td>0.30</td>
<td>20</td>
<td>1.8760</td>
<td>1.0837</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N6</td>
<td>1.30</td>
<td>0.30</td>
<td>18</td>
<td>1.8248</td>
<td>1.0958</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N7</td>
<td>1.30</td>
<td>0.30</td>
<td>16</td>
<td>1.7630</td>
<td>1.1100</td>
<td>0.55721E-03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N8</td>
<td>1.30</td>
<td>0.30</td>
<td>14</td>
<td>1.6864</td>
<td>1.1271</td>
<td>0.45997E-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N9</td>
<td>1.30</td>
<td>0.35</td>
<td>20</td>
<td>1.8840</td>
<td>1.0712</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N10</td>
<td>1.30</td>
<td>0.35</td>
<td>18</td>
<td>1.8342</td>
<td>1.0840</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N11</td>
<td>1.30</td>
<td>0.35</td>
<td>16</td>
<td>1.7738</td>
<td>1.0990</td>
<td>0.28252E-04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N12</td>
<td>1.30</td>
<td>0.35</td>
<td>14</td>
<td>1.6990</td>
<td>1.1173</td>
<td>0.25031E-02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 5.5
(Cont.)

($\phi=20^\circ$, $P=1.0$ in., $a=1.0$ in.)

<table>
<thead>
<tr>
<th>MODEL</th>
<th>$b$</th>
<th>$r_f$</th>
<th>$N$</th>
<th>$t$</th>
<th>$h$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
<td>(in.)</td>
</tr>
<tr>
<td>J1</td>
<td>1.40</td>
<td>0.10</td>
<td>24</td>
<td>1.9264</td>
<td>1.1920</td>
<td>0.00</td>
</tr>
<tr>
<td>J2</td>
<td>1.40</td>
<td>0.10</td>
<td>23</td>
<td>1.9044</td>
<td>1.1948</td>
<td>0.00</td>
</tr>
<tr>
<td>J3</td>
<td>1.40</td>
<td>0.10</td>
<td>22</td>
<td>1.8808</td>
<td>1.1978</td>
<td>0.17995E-04</td>
</tr>
<tr>
<td>J4</td>
<td>1.40</td>
<td>0.10</td>
<td>20</td>
<td>1.8276</td>
<td>1.2043</td>
<td>0.17828E-02</td>
</tr>
<tr>
<td>J5</td>
<td>1.40</td>
<td>0.20</td>
<td>24</td>
<td>1.9386</td>
<td>1.1646</td>
<td>0.00</td>
</tr>
<tr>
<td>J6</td>
<td>1.40</td>
<td>0.20</td>
<td>22</td>
<td>1.8952</td>
<td>1.1716</td>
<td>0.00</td>
</tr>
<tr>
<td>J7</td>
<td>1.40</td>
<td>0.20</td>
<td>20</td>
<td>1.8446</td>
<td>1.1796</td>
<td>0.10190E-03</td>
</tr>
<tr>
<td>J8</td>
<td>1.40</td>
<td>0.20</td>
<td>18</td>
<td>1.7848</td>
<td>1.1887</td>
<td>0.24844E-02</td>
</tr>
<tr>
<td>J9</td>
<td>1.40</td>
<td>0.30</td>
<td>22</td>
<td>1.9096</td>
<td>1.1456</td>
<td>0.00</td>
</tr>
<tr>
<td>J10</td>
<td>1.40</td>
<td>0.30</td>
<td>20</td>
<td>1.8616</td>
<td>1.1548</td>
<td>0.00</td>
</tr>
<tr>
<td>J11</td>
<td>1.40</td>
<td>0.30</td>
<td>18</td>
<td>1.8044</td>
<td>1.1656</td>
<td>0.27241E-03</td>
</tr>
<tr>
<td>J12</td>
<td>1.40</td>
<td>0.30</td>
<td>16</td>
<td>1.7356</td>
<td>1.1782</td>
<td>0.33993E-02</td>
</tr>
</tbody>
</table>
Fig. 5.8 Variation of tooth thickness at theoretical weakest section versus the amount of undercutting (b=1.2 in.).
Fig. 5.9 Variation of tooth thickness at theoretical weakest section versus the amount of undercutting (b=1.3 in.).
DE DENDUM = 1.4 IN.

Fig. 5.10 Variation of tooth thickness at theoretical weakest section versus the amount of undercutting (b=1.4 in.).
Chapter VI
RESULTS

Following the generation of the 37 models, the maximum tensile and compressive stresses at the root fillets were obtained using the analysis program SUPERB [4]. Then simple beam stresses were calculated theoretically using Lewis' equations, (Eq. 3.19 for $S_L$ and Eq. 3.21 for $\sigma$):

$$S_L = \frac{6wh}{t^2}$$  \hspace{1cm} (3.19)

$$\sigma = \frac{6wh}{t^2} - \frac{W}{t} \tan \phi_L$$  \hspace{1cm} (3.21)

The compressive bending stress concentration factor was calculated as the ratio of $S_C$ to $S_L$:

$$K_c = \frac{S_C}{S_L}$$  \hspace{1cm} (6.1)
and tensile bending stress concentration factor as the ratio of $S_t$ to $S_L$:

$$K_t = \frac{S_t}{S_L} \quad (6.2)$$

The combined tensile stress concentration factor, ratio of $S_t$ to $\sigma$, was used for all comparisons against theoretical concentration factor using the Dolan and Broghammer equation for models with a $20^\circ$ pressure angle (Eq. 6.4).

$$K = \frac{S_t}{\sigma} \quad (6.3)$$

$$K_d = .18 + \left(\frac{L}{r}\right)^{15}\left(\frac{t}{h}\right)^{.45} \quad (6.4)$$

Table 6.1 presents the results of the analysis of the 37 finite element models. Using equation (6.4), $K_d$ has been calculated for those gears which are not undercut.
### TABLE 6.1

Results

<table>
<thead>
<tr>
<th>Model</th>
<th>$S_t$</th>
<th>$S_c$</th>
<th>$S_l$</th>
<th>$\theta$</th>
<th>$K_t$</th>
<th>$K_c$</th>
<th>$K$</th>
<th>$K_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>psi</td>
<td>psi</td>
<td>psi</td>
<td>psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>3.2577</td>
<td>3.9944</td>
<td>1.5984</td>
<td>2.0381</td>
<td>2.4990</td>
<td>2.3537</td>
<td>2.2085</td>
<td></td>
</tr>
<tr>
<td>M2</td>
<td>3.3759</td>
<td>4.0643</td>
<td>1.6865</td>
<td>1.4663</td>
<td>2.0017</td>
<td>2.4099</td>
<td>2.3023</td>
<td>2.1731</td>
</tr>
<tr>
<td>M3</td>
<td>3.4109</td>
<td>4.1234</td>
<td>1.7995</td>
<td>1.5724</td>
<td>1.8955</td>
<td>2.2914</td>
<td>2.1692</td>
<td></td>
</tr>
<tr>
<td>M3a</td>
<td>3.4218</td>
<td>4.2175</td>
<td>1.8689</td>
<td>1.6380</td>
<td>1.8309</td>
<td>2.2567</td>
<td>2.0890</td>
<td></td>
</tr>
<tr>
<td>M4</td>
<td>3.5901</td>
<td>4.2800</td>
<td>1.9498</td>
<td>1.7148</td>
<td>1.8413</td>
<td>2.1951</td>
<td>2.0936</td>
<td></td>
</tr>
<tr>
<td>M5</td>
<td>2.8189</td>
<td>3.4563</td>
<td>1.5540</td>
<td>1.3374</td>
<td>1.8140</td>
<td>2.2241</td>
<td>2.1077</td>
<td>1.9252</td>
</tr>
<tr>
<td>M6</td>
<td>2.8684</td>
<td>3.5692</td>
<td>1.6524</td>
<td>1.4298</td>
<td>1.7359</td>
<td>2.1600</td>
<td>2.0062</td>
<td>1.8897</td>
</tr>
<tr>
<td>M7</td>
<td>3.1292</td>
<td>3.8355</td>
<td>1.8652</td>
<td>1.6315</td>
<td>1.6777</td>
<td>2.0563</td>
<td>1.9180</td>
<td></td>
</tr>
<tr>
<td>M8</td>
<td>3.1908</td>
<td>3.8642</td>
<td>1.9639</td>
<td>1.7257</td>
<td>1.6247</td>
<td>1.9676</td>
<td>1.8490</td>
<td></td>
</tr>
<tr>
<td>M9</td>
<td>2.6817</td>
<td>3.3541</td>
<td>1.5829</td>
<td>1.3624</td>
<td>1.6942</td>
<td>2.1190</td>
<td>1.9684</td>
<td>1.8446</td>
</tr>
<tr>
<td>M10</td>
<td>2.7064</td>
<td>3.3770</td>
<td>1.7044</td>
<td>1.4773</td>
<td>1.5879</td>
<td>1.9813</td>
<td>1.8320</td>
<td>1.8038</td>
</tr>
<tr>
<td>M11</td>
<td>3.0044</td>
<td>3.6938</td>
<td>1.8727</td>
<td>1.6379</td>
<td>1.6043</td>
<td>1.9724</td>
<td>1.8343</td>
<td></td>
</tr>
<tr>
<td>M12</td>
<td>3.1759</td>
<td>3.8310</td>
<td>2.1220</td>
<td>1.8779</td>
<td>1.4967</td>
<td>1.8054</td>
<td>1.6912</td>
<td></td>
</tr>
<tr>
<td>N2</td>
<td>3.2847</td>
<td>4.0307</td>
<td>1.7722</td>
<td>1.5538</td>
<td>1.8353</td>
<td>2.2744</td>
<td>2.1140</td>
<td>2.1256</td>
</tr>
<tr>
<td>N3</td>
<td>3.3242</td>
<td>4.0643</td>
<td>1.8263</td>
<td>1.6046</td>
<td>1.8202</td>
<td>2.2254</td>
<td>2.0717</td>
<td></td>
</tr>
<tr>
<td>N4</td>
<td>3.4519</td>
<td>4.1901</td>
<td>1.9584</td>
<td>1.7291</td>
<td>1.7626</td>
<td>2.1396</td>
<td>1.9964</td>
<td></td>
</tr>
<tr>
<td>N5</td>
<td>2.8485</td>
<td>3.5076</td>
<td>1.6860</td>
<td>1.4681</td>
<td>1.6895</td>
<td>2.0804</td>
<td>1.9403</td>
<td>1.8653</td>
</tr>
<tr>
<td>N6</td>
<td>2.9518</td>
<td>3.6378</td>
<td>1.8008</td>
<td>1.5761</td>
<td>1.6392</td>
<td>2.0201</td>
<td>1.8729</td>
<td>1.8293</td>
</tr>
<tr>
<td>N7</td>
<td>3.1532</td>
<td>3.8509</td>
<td>1.9542</td>
<td>1.7214</td>
<td>1.6136</td>
<td>1.9706</td>
<td>1.8318</td>
<td></td>
</tr>
<tr>
<td>N8</td>
<td>3.3960</td>
<td>4.0894</td>
<td>2.1700</td>
<td>1.9274</td>
<td>1.5650</td>
<td>1.8845</td>
<td>1.7620</td>
<td></td>
</tr>
<tr>
<td>N9</td>
<td>2.7056</td>
<td>3.3618</td>
<td>1.6524</td>
<td>1.4354</td>
<td>1.6374</td>
<td>2.0345</td>
<td>1.8849</td>
<td>1.8396</td>
</tr>
<tr>
<td>N10</td>
<td>2.8419</td>
<td>3.4589</td>
<td>1.7633</td>
<td>1.5397</td>
<td>1.6117</td>
<td>1.9616</td>
<td>1.8457</td>
<td>1.8044</td>
</tr>
<tr>
<td>N11</td>
<td>3.0300</td>
<td>3.6979</td>
<td>1.9112</td>
<td>1.6799</td>
<td>1.5854</td>
<td>1.9349</td>
<td>1.8037</td>
<td></td>
</tr>
<tr>
<td>N12</td>
<td>3.3131</td>
<td>3.9809</td>
<td>2.1190</td>
<td>1.8783</td>
<td>1.5635</td>
<td>1.8787</td>
<td>1.7639</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 6.1
(Cont.)

<table>
<thead>
<tr>
<th>Model</th>
<th>St</th>
<th>Sc</th>
<th>S1</th>
<th>σ</th>
<th>Kt</th>
<th>Kc</th>
<th>K</th>
<th>Kd</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>psi</td>
<td>psi</td>
<td>psi</td>
<td>psi</td>
<td>psi</td>
<td>psi</td>
<td>psi</td>
<td>psi</td>
</tr>
<tr>
<td>J1</td>
<td>3.1606</td>
<td>3.9003</td>
<td>1.7617</td>
<td>1.5512</td>
<td>1.7941</td>
<td>2.2139</td>
<td>2.0375</td>
<td>2.1143</td>
</tr>
<tr>
<td>J2</td>
<td>3.2316</td>
<td>3.9602</td>
<td>1.8061</td>
<td>1.5927</td>
<td>1.7893</td>
<td>2.1927</td>
<td>2.0290</td>
<td>2.0990</td>
</tr>
<tr>
<td>J3</td>
<td>3.2495</td>
<td>3.9962</td>
<td>1.8557</td>
<td>1.6391</td>
<td>1.7511</td>
<td>2.1535</td>
<td>1.9825</td>
<td></td>
</tr>
<tr>
<td>J4</td>
<td>3.3784</td>
<td>4.1159</td>
<td>1.9744</td>
<td>1.7507</td>
<td>1.7111</td>
<td>2.0846</td>
<td>1.9297</td>
<td></td>
</tr>
<tr>
<td>J5</td>
<td>2.9262</td>
<td>3.5944</td>
<td>1.6995</td>
<td>1.4904</td>
<td>1.7218</td>
<td>2.1150</td>
<td>1.9634</td>
<td>1.9483</td>
</tr>
<tr>
<td>J6</td>
<td>3.0057</td>
<td>3.7008</td>
<td>1.7874</td>
<td>1.6816</td>
<td>2.0705</td>
<td></td>
<td>1.9114</td>
<td>1.9198</td>
</tr>
<tr>
<td>J7</td>
<td>3.1458</td>
<td>3.8350</td>
<td>1.8982</td>
<td>1.6766</td>
<td>1.6573</td>
<td>2.0203</td>
<td>1.8763</td>
<td></td>
</tr>
<tr>
<td>J8</td>
<td>3.2920</td>
<td>4.0158</td>
<td>2.0422</td>
<td>1.9825</td>
<td></td>
<td>1.9664</td>
<td>1.8163</td>
<td></td>
</tr>
<tr>
<td>J9</td>
<td>2.7960</td>
<td>3.4647</td>
<td>1.7214</td>
<td>1.5081</td>
<td>1.6243</td>
<td>2.0127</td>
<td>1.8540</td>
<td>1.8413</td>
</tr>
<tr>
<td>J10</td>
<td>2.9235</td>
<td>3.5894</td>
<td>1.8248</td>
<td>1.6052</td>
<td>1.6021</td>
<td>1.9670</td>
<td>1.8213</td>
<td>1.8101</td>
</tr>
<tr>
<td>J11</td>
<td>3.1057</td>
<td>3.7181</td>
<td>1.9589</td>
<td>1.7317</td>
<td>1.5854</td>
<td>1.9306</td>
<td>1.7934</td>
<td></td>
</tr>
<tr>
<td>J12</td>
<td>3.2942</td>
<td>3.9681</td>
<td>2.1400</td>
<td>1.9036</td>
<td>1.5393</td>
<td>1.8543</td>
<td>1.7305</td>
<td></td>
</tr>
</tbody>
</table>
Figures 6.1, 6.2 and 6.3 show the maximum tensile stresses at the fillet, $S_t$, versus the amount of undercutting for different groups of models. Figures 6.4, 6.5 and 6.6 show the theoretical combined tensile stresses at fillet, $\sigma$, versus the amount of undercutting. Figures 6.7, 6.8 and 6.9 show the stress concentration factors via finite element method versus the amount of undercutting.
Fig. 6.1 Variation of maximum tensile stress at the fillet versus the amount of undercutting (b=1.2 in.).
Fig. 6.2 Variation of maximum tensile stress at the fillet versus the amount of undercutting (b=1.3 in.).
Fig. 6.3 Variation of maximum tensile stress at the fillet versus the amount of undercutting (b=1.4 in.).
Fig. 6.4 Variation of theoretical combined tensile stress at the fillet versus the amount of undercutting (b=1.2 in.).
Fig. 6.5 Variation of theoretical combined tensile stress at the fillet versus the amount of undercutting (b=1.3 in.).
Fig. 6.6 Variation of theoretical combined tensile stress at the fillet versus the amount of undercutting ($b=1.4$ in.).
**DEDENDDUM = 1.2 IN.**

--- HOB TIP RADIUS = .10 IN.  X--- HOB TIP RADIUS = .30 IN.

--- HOB TIP RADIUS = .40 IN.

Fig. 6.7 Variation of stress concentration factor versus the amount of undercutting (b=1.2 in.).
Fig. 6.8 Variation of stress concentration factor versus the amount of undercutting (b=1.3 in.).
Fig. 6.9 Variation of stress concentration factor versus the amount of undercutting (b=1.4 in.).
Chapter VII
CONCLUSIONS AND RECOMMENDATIONS

Using the results obtained from the finite element analyses, a multiple linear regression analysis was performed. The result was a linear equation in terms of the hob tip radius, $r_f$, the tooth thickness at theoretical weakest section, $t$, and the height of load position above the theoretical weakest section, $h$.

\[
K_J = (-1.253234)r_f + (.675457)t + (-1.772398)h + (2.976752)
\]

Using equation 7.1, stress concentration factors for all previous models (listed in Tables 5.5 and 6.1) were calculated. These results are listed in Table 7.1 beside those stress concentration factors obtained from the finite element analyses. Percentage differences between these two results are listed, which is not more than 3.5 percent and in most cases is less than 1.5 percent.

To verify equation 7.1 an undercut gear model was used ($\phi=20.0^\circ$, $a=1.0$ in., $b=1.25$ in., $N=16$, $P=1$ in., $r_f=0.25$ in.), which was not among those models previously analyzed. Com-
### TABLE 7.1

Results of Regression Analysis

<table>
<thead>
<tr>
<th>MODEL</th>
<th>$t$ (in.)</th>
<th>$h$ (in.)</th>
<th>$K$</th>
<th>$K_j$</th>
<th>%ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>1.9000</td>
<td>1.0526</td>
<td>2.3537</td>
<td>2.2692</td>
<td>3.5897</td>
</tr>
<tr>
<td>M2</td>
<td>1.8560</td>
<td>1.0618</td>
<td>2.3023</td>
<td>2.2231</td>
<td>3.4392</td>
</tr>
<tr>
<td>M3</td>
<td>1.8060</td>
<td>1.0724</td>
<td>2.1692</td>
<td>2.1706</td>
<td>0.0624</td>
</tr>
<tr>
<td>M3a</td>
<td>1.7768</td>
<td>1.0783</td>
<td>2.0890</td>
<td>2.1404</td>
<td>2.4602</td>
</tr>
<tr>
<td>M4</td>
<td>1.7446</td>
<td>1.0848</td>
<td>2.0936</td>
<td>2.1071</td>
<td>0.6466</td>
</tr>
<tr>
<td>M5</td>
<td>1.8860</td>
<td>1.0105</td>
<td>2.1077</td>
<td>2.0837</td>
<td>1.1415</td>
</tr>
<tr>
<td>M6</td>
<td>1.8420</td>
<td>1.0239</td>
<td>2.0062</td>
<td>2.0302</td>
<td>1.1993</td>
</tr>
<tr>
<td>M7</td>
<td>1.7540</td>
<td>1.0485</td>
<td>1.9180</td>
<td>1.9272</td>
<td>0.4789</td>
</tr>
<tr>
<td>M8</td>
<td>1.7178</td>
<td>1.0585</td>
<td>1.8490</td>
<td>1.8850</td>
<td>1.9475</td>
</tr>
<tr>
<td>M9</td>
<td>1.8594</td>
<td>1.0001</td>
<td>1.9684</td>
<td>1.9588</td>
<td>0.4845</td>
</tr>
<tr>
<td>M10</td>
<td>1.8072</td>
<td>1.0173</td>
<td>1.8320</td>
<td>1.8931</td>
<td>3.3348</td>
</tr>
<tr>
<td>M11</td>
<td>1.7422</td>
<td>1.0382</td>
<td>1.8343</td>
<td>1.8121</td>
<td>1.2083</td>
</tr>
<tr>
<td>M12</td>
<td>1.6588</td>
<td>1.0642</td>
<td>1.6912</td>
<td>1.7097</td>
<td>1.0953</td>
</tr>
<tr>
<td>N1</td>
<td>1.8920</td>
<td>1.1263</td>
<td>2.1200</td>
<td>2.1331</td>
<td>0.6220</td>
</tr>
<tr>
<td>N2</td>
<td>1.8690</td>
<td>1.1302</td>
<td>2.1140</td>
<td>2.1107</td>
<td>0.1554</td>
</tr>
<tr>
<td>N3</td>
<td>1.8442</td>
<td>1.1343</td>
<td>2.0717</td>
<td>2.0867</td>
<td>0.7244</td>
</tr>
<tr>
<td>N4</td>
<td>1.7874</td>
<td>1.1434</td>
<td>1.9964</td>
<td>2.0322</td>
<td>1.7945</td>
</tr>
<tr>
<td>N5</td>
<td>1.8760</td>
<td>1.0837</td>
<td>1.9403</td>
<td>1.9472</td>
<td>0.3571</td>
</tr>
<tr>
<td>N6</td>
<td>1.8248</td>
<td>1.0958</td>
<td>1.8729</td>
<td>1.8912</td>
<td>0.9777</td>
</tr>
<tr>
<td>N7</td>
<td>1.7630</td>
<td>1.1100</td>
<td>1.8318</td>
<td>1.8243</td>
<td>0.4102</td>
</tr>
<tr>
<td>N8</td>
<td>1.6864</td>
<td>1.1271</td>
<td>1.7620</td>
<td>1.7422</td>
<td>1.1213</td>
</tr>
<tr>
<td>N9</td>
<td>1.8840</td>
<td>1.0712</td>
<td>1.8849</td>
<td>1.9121</td>
<td>1.4419</td>
</tr>
<tr>
<td>N10</td>
<td>1.8342</td>
<td>1.0840</td>
<td>1.8457</td>
<td>1.8558</td>
<td>0.5426</td>
</tr>
<tr>
<td>N11</td>
<td>1.7738</td>
<td>1.0990</td>
<td>1.8037</td>
<td>1.7884</td>
<td>0.8482</td>
</tr>
<tr>
<td>N12</td>
<td>1.6990</td>
<td>1.1173</td>
<td>1.7639</td>
<td>1.7054</td>
<td>3.3143</td>
</tr>
</tbody>
</table>
\textbf{TABLE 7.1}

(Cont.)

<table>
<thead>
<tr>
<th>MODEL</th>
<th>t</th>
<th>h</th>
<th>K</th>
<th>K_j</th>
<th>%ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(in.)</td>
<td>(in.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J1</td>
<td>1.9264</td>
<td>1.1920</td>
<td>2.0375</td>
<td>2.0399</td>
<td>0.1183</td>
</tr>
<tr>
<td>J2</td>
<td>1.9044</td>
<td>1.1948</td>
<td>2.0290</td>
<td>2.0201</td>
<td>0.4386</td>
</tr>
<tr>
<td>J3</td>
<td>1.8808</td>
<td>1.1978</td>
<td>1.9825</td>
<td>1.9988</td>
<td>0.8252</td>
</tr>
<tr>
<td>J4</td>
<td>1.8276</td>
<td>1.2043</td>
<td>1.9297</td>
<td>1.9514</td>
<td>1.1220</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J5</td>
<td>1.9386</td>
<td>1.1646</td>
<td>1.9634</td>
<td>1.9714</td>
<td>0.4098</td>
</tr>
<tr>
<td>J6</td>
<td>1.8952</td>
<td>1.1716</td>
<td>1.9114</td>
<td>1.9297</td>
<td>0.9561</td>
</tr>
<tr>
<td>J7</td>
<td>1.8446</td>
<td>1.1796</td>
<td>1.8763</td>
<td>1.8813</td>
<td>0.2684</td>
</tr>
<tr>
<td>J8</td>
<td>1.7848</td>
<td>1.1887</td>
<td>1.8163</td>
<td>1.8248</td>
<td>0.4700</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J9</td>
<td>1.9096</td>
<td>1.1456</td>
<td>1.8540</td>
<td>1.8602</td>
<td>0.3337</td>
</tr>
<tr>
<td>J10</td>
<td>1.8616</td>
<td>1.1548</td>
<td>1.8213</td>
<td>1.8114</td>
<td>0.5392</td>
</tr>
<tr>
<td>J11</td>
<td>1.8044</td>
<td>1.1656</td>
<td>1.7934</td>
<td>1.7537</td>
<td>2.2176</td>
</tr>
<tr>
<td>J12</td>
<td>1.7356</td>
<td>1.1782</td>
<td>1.7305</td>
<td>1.6849</td>
<td>2.6377</td>
</tr>
</tbody>
</table>
paring results for this model, a 0.27 percent difference was noted between the finite element analysis concentration factor and the factor determined by using equation 7.1. These results are listed in Table 7.2. It is concluded that equation 7.1 is an accurate formula for both undercut and non-undercut gears.

Shigley and Mitchell [13] show that normal stress corresponding to the total load, \( W \), acting at the highest point of single-tooth contact and including the effects of stress concentration factor may be determined from:

\[
\sigma = \frac{w_p}{F} \frac{t}{J}
\]

(7.2)

where the geometry factor, \( J \), can be obtained from:

\[
J = \frac{Y}{K}
\]

(7.3)

where

\[
Y = \frac{1}{\cos \phi_L (1.5 \cos \phi \left( \frac{\phi_L}{x} - \tan \frac{\phi_L}{t} \right))}
\]

(7.4)
<table>
<thead>
<tr>
<th>MODEL</th>
<th>( t )</th>
<th>( h )</th>
<th>( K )</th>
<th>( K_j )</th>
<th>( %ERROR )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEST</td>
<td>1.7644</td>
<td>1.0973</td>
<td>1.9052</td>
<td>1.9104</td>
<td>0.2737</td>
</tr>
</tbody>
</table>

TABLE 7.2

Result of Testing Concentration Factor Formula
Using equations 7.3 and 7.2 for a gear model which had a diametral pitch of 1.0 in., addendum of 1.0 in., dedendum of 1.3 in., hob tip radius of 0.3 in. and a cutting pressure angle of $20^\circ$, the geometry factor, $J$, and the maximum normal stress were calculated for various numbers of teeth. Stress concentration factors used in equation 7.3 are calculated by using the regression equation (Eq. 7.1). These results are plotted in Figures 7.1 and 7.2 indicating that as the number of teeth decreases, stress at the root fillet for those slightly undercut gears increases as the same ratio for non-undercut gears.

Finally, the results of this investigation were expressed in form of a linear relationship giving the stress concentration factor at the root fillet as a function of the geometry of the tooth.

For further studies it is recommended that two gear teeth models in contact be studied instead of applying a single load to a tooth. This makes it possible to study load-sharing between gear teeth as well as stresses. It should be noted that Equation 7.1 is valid only for $20^\circ$
P = 1 IN. ADDENDUM = 1 IN. DEEDENDUM = 1.3 IN. HOB TIP RADIUS = .3 IN. PHI = 20 DEG.

+...USING REGRESSION EQUATION (EQ. 7.1)
*...USING DOLAN AND BROGHAMER EQUATION

**FIG. 7.1 VARIATION OF GEOMETRY FACTOR, J, VERSUS NUMBER OF TEETH.**
P = 1 IN. ADDENDUM = 1 IN. DEEDENDUM = 1.3 IN. HOBB TIP RADIUS = .3 IN. PHI = 20 DEG.

...USING REGRESSION EQUATION (EQ. 7.1)
...USING DOLAN AND BROGHAMER EQUATION

FIG. 7.2 VARIATION OF NORMAL STRESS VERSUS NUMBER OF TEETH.
pressure angle, but with further study a relation can be found for all common cutting pressure angles.
REFERENCES


2 Dolan, T. J., and Broghamer, E. L., A Photoelastic Study of Stresses in Gear Tooth Fillets, University of Illinois, Engineering Experiment Station, Bulletin No. 335, 1942.


11 Chabert, G., Dang Tran, T., and Mathis, R., An Evaluation of Stresses and Deflection of Spur Gear Teeth Under Strain,


Appendix A

PROGRAM TO CALCULATE THE AMOUNT OF UNDERCUTTING

IMPLICIT REAL*8 (A-H,O,Z)
REAL*8 N,MU
P=1.
PHID=20.
10 TYPE *, ' P=1. PHI=20. DEG. ENTER B, RF, N'
ACCEPT *,B,RF,N
20 TYPE *, ' ENTER STARTING VALUES FOR MU AND PSI'
ACCEPT *,MU,PSI
IF(MU.GT.1.0R.PSI.GT.1.) STOP
C
30 F1=N*DCOS(PSI)*DCOS(PHI)-(N-(B-RF)*2.)*DCOS(MU)
C
F2=-N*(PSI+MU-DTAN(PSI)-DTAN(MU))*.5
C+N*(PHI-DTAN(PHI)*.5-(B-RF)*DTAN(PSI)
C+(B-RF)*DTAN(PHI)
C
F1MU=(N-(B-RF)*2.)*DSIN(MU)
C
F1PSI=-N*DSIN(PSI)*DCOS(PHI)
C
F2MU=N*DTAN(MU)**2/2.
C
F2PSI=N*DTAN(PSI)**2/2.-B+RF-(B-RF)*DTAN(PSI)**2
C
DET=F1MU*F2PSI-F2MU*F1PSI
C
DELMU=(-F1*F2PSI+F2*F1PSI)/DET
C
DELPSI=(-F2*F1MU+F1*F2MU)/DET
C
40 TYPE 40,MU,PSI,DELMU,DELPSI
FORMAT(4E15.5)
C
IF MU OR PSI ARE NEGETIVE ASK FOR STARTING VALUE
C
IF (MU.LT.0.0R.PSI.LT.0.0) GO TO 20
C
C
IF DELMU AND DELPSI ARE LESS THAN 1.E-10 STOP
C
C
MU=MU+DELMU
PSI=DELPSI+PSI
GO TO 30

\[
Q = \frac{N}{2 \cdot P} \cdot \text{DCOS} \left( \text{PHI} \right) \cdot \left( \frac{1}{\text{DCOS} \left( \text{MU} \right)} - 1 \right)
\]

TYPE *, /5X, 'Q=', Q
GO TO 10
END
Appendix B

MAIN COMPUTER PROGRAM

DIMENSION NOEN(4), JJX(4), X(3000), Y(3000), NEND(4)
DIMENSION NST(4), NPTS(9,15)
DIMENSION NM(8)
CHARACTER*10 FINAM
CHARACTER*4 EXT, DAT
CHARACTER*15 FNPNT, FNLIN, FNEDG, FNDAT
REAL N
COMMON/TOOTH/N, P, A, D, RHF, THKNSS, PHI, R
DATA PI/3.1415927/, IYES/IHY/, EXT/'.TTH'/, DAT/'.INF'/

322 TYPE 11
11 FORMAT( ' ENTER DIAMETRAL PITCH, ADDENDUM, DEDENDUM' , / ,
C ' HOB TIP RADIUS, NUMBER OF TEETH' , / )
ACCEPT *, P, A, D, RHF, N

TYPE 80
80 FORMAT( ' IS THIS TO BE A STANDARD GEAR Y OR N> ?', ' )
ACCEPT 81, IRPLY
81 FORMAT(A1)
12 FORMAT( ' ENTER CUTTING PRESSURE ANGLE IN DEGREES ' )
ACCEPT *, PHIDEG
GO TO 84

TYPE 85
85 FORMAT( ' ON CUTTING PITCH CIRCLE ENTER', / ,
C ' PITCH RADIUS ', F15.9, ' FOR STD GEARS' , / ,
C ' CUTTING PRESSURE ANGLE IN DEGREES' , / ,
C ' TOOTH THICKNESS ', F15.9, ' FOR STD GEARS' , / )
ACCEPT *, R, PHIDEG, THKNSS

84 PHI=PHIDEG*PI/180.

TYPE 14, P, A, D, RHF, N, R, PHIDEG, THKNSS
14 FORMAT( ' INPUT DATA : ' , / ,
C ' PITCH ', F15.7, / ,
C ' ADD. FRACTION ', F15.7, / ,
C ' DED. FRACTION ', F15.7, / ,
C ' HOB TIP RAD ', F15.7, / ,
C ' NO. OF TEETH ', F15.7, / ,
C ' CTG. PITCH R ', F15.7, / ,
C ' CTG. PR. ANG. ', F15.7, / ,
C ' TOOTH THKNSS ', F15.7, / ,
C ' ENTER THE NO OF INTERVALS PER CURVE24 MAX-4 MIN>' )
ACCEPT *, XINT
INTERV=IFIX(XINT+.5)
SPACN=.01*((A+D)/P)
SPMIN=.95*SPACN
SPMAX=1.05*SPACN
X(1)=0.
Y(1)=0.

LOCATE INTERSECTION/TANGENCY POINT

CALL INTERSECT(RI,XI,YI,ALPHAI,THETAI,ITER,0,IER)

IF(IER.NE.0) TYPE 320
FORMAT(' CANNOT LOCATE TROCH/INVOLUTE INTERSECTION')
IF(IER.EQ.99) TYPE 328
FORMAT(' LACK OF CONVERGENCE IN 2000 ITERATIONS')
IF(IER.EQ.1) TYPE 329
FORMAT(' POINT OF INTERSECTION NOT LOCATED PAUSIBLY')
IF(IER.EQ.1) GO TO 322
IF(IER.EQ.0) TYPE 331, XI,YI,RI,ITER
FORMAT(' INTERSECTION POINT LOCATED AT
*(',F12.8,',' ,F12.8,' )'/,
1 ' AT RADIUS OF ',F12.8,'/
2 ' POINT LOCATED IN ',IS,' ITERATIONS.')

LOCATE LEWIS POINT

CALL POINT(RLEWIS,XLEWIS,YLEWIS,TOUT,ITER,1,IER,RC)

IF(IER.NE.0) TYPE 420
FORMAT(' CANNOT LOCATE LEWIS POINT')
IF(IER.EQ.99) TYPE 428
FORMAT(' LACK OF CONVERGENCE IN 2000 ITERATIONS')
IF(IER.EQ.1) TYPE 429
FORMAT(' POINT OF INTERSECTION NOT LOCATED PLAUSIBLY')
IF(IER.EQ.1) GO TO 322
IF(IER.EQ.0) TYPE 430, XLEWIS,YLEWIS,RLEWIS,TOUT,ITER,RC
FORMAT('LEWIS POINT LOCATED AT (' ,F12.8,',' ,F12.8,' )'/,
1 ' ,AT RADIUS OF ',F12.8,' AND THETA OF ',F12.8,'/
2 ' LEWIS POINT LOCATED IN ',IS,' ITERATIONS.',
3 ' RC=',E12.5)

ANGLE OF LINE OF ACTION FOR SINGLE TOOTH LOADING.

TEM=ATAN(SQRT(1.-((R*COS(PHI)/(R+A))**2)/(R*COS(PHI)/
C(R+A))
PHIL=TEMP-PI/(2.*N)-XINV(TEMP)+XINV(PHI)
PHILDEG=PHIL*180./PI
TYPE *, 'PHIL=', PHILDEG

STRESS AT LEWIS POINT

WLOAD=l.*COS(PHIL)
HEIGHT=RC-YLEWIS
TYPE *, 'HEIGHT=', HEIGHT
THICKNS=2.*XLEWIS
TYPE *, 'THICKNESS=', THICKNS
STRESS=6.*WLOAD*HEIGHT/THICKNS**2
SIGMA=STRESS-WLOAD*TAN(PHIL)/THICKNS

DEVELOP ROOT CIRCLE

DELTA=((PI/P-THKNSS)/2.)-(D/P-RHF/P)*TAN(PHI)-RHF/(P*COS(PHI))
ETA=DELTA/R
BETA=PI/N-ETA
XE=(R-D/P)*SIN(BETA)
YE=(R-D/P)*COS(BETA)
XS=(R-D/P)*SIN(PI/N)
YS=(R-D/P)*COS(PI/N)
XL=SQRT((XS-XE)**2+(YS-YE)**2)
BETA=PI/N-ATAN(XE/YE)
LL=IFIX(XL/SPACNG)
IF(LL.EQ.O) LL=l
IF(LL.GT.INTERVL) LL=INTERVL
DEL=23./24.*BETA/FLOAT(LL)
TH=PI/N+DEL
DO 120 I=2,3000
TH=TH-DEL
X(I)=(R-D/P)*SIN(TH)
Y(I)=(R-D/P)*COS(TH)
IF(X(I).LE.XE) GO TO 121
CONTINUE
121 X(I)=XE
Y(I)=YE
IDEDTROCH=I
NN=I+1

C
DEL=.01
TH=-DEL+1.E-7

C

TROCHOID
SPMINN=SPMIN
SPMAXX=SPMAX
NNN=NN
TH=-DEL+1.E-7
NTROCHPT=0
DO 60 I=NNN,3000
NTROCHPT=NTROCHPT+1
CONTINUE
TH=TH+DEL

C
C
CALL TROCH(X(I),Y(I),TH)
XX=X(I)-X(I-1)
YY=Y(I)-Y(I-1)
SP=SQR(T(XX**2+YY**2))
IF(SP.GE.SPMINN.AND.SP.LE.SPMAXX) GO TO 63
TH=TH-DEL
IF(SP.GT.SPMAXX) DEL=.99*DEL
IF(SP.LT.SPMINN) DEL=1.01*DEL
GO TO 61

C
63
RM=SQR(T(X(I)**2+Y(I)**2))
IF(RM.GE.RI.AND.NTROCHPT.LT.INTERVL) GO TO 70
IF(RM.LT.RI) GO TO 60
SPMAXX=SPMAXX*1.1
SPMINN=SPMAXX*.9
GO TO 65
CONTINUE
70
X(I)=XI
Y(I)=YI
ITROCHINV=I
NN=I+1
DEL=.01
ALPHA=ALPHAI
RO=R+A/P
C
C
INVOLUTE
C

SPMAXX=SPMAX
SPMINN=SPMIN
NNN=NN
NINVPT=0
ALPHA=ALPHAI-DEL
DO 10 I=NNN,3000
NINVPT=NINVPT+1
CONTINUE
ALPHA=ALPHA+DEL
CALL INVOL(X(I),Y(I),ALPHA)
XX=X(I)-X(I-1)
YY=Y(I)-Y(I-1)
SP=SQR(T(XX**2+YY**2))
IF(SP.GE.SPMINN.AND.SP.LE.SPMAXX) GO TO 163
ALPHA=ALPHA-DEL
IF(SP.GT.SPMAXX) DEL=.99*DEL
IF(SP.LT.SPMINN) DEL=1.01*DEL
GO TO 161

C
163 RM=SQRT(X(I)**2+Y(I)**2)
IF(RM.GE.RO.AND.NINVPT.LT.INTERVL) GO TO 20
IF(RM.LT.RO) GO TO 10
SPMAXX=SPMAXX*1.1
SPMINN=SPMAXX*.9
GO TO 170

C
10 CONTINUE

C
20 NN=I-1
ININVADD=I-1
NNX=NN

C
DEVELOP ADDENDUM CIRCLE
C
PHIA=ACOS(R/(R+A/P)*COS(PHI))
TO=(R+A/P)*(THKNSS/(2.*R)+XINV(PHI)-XINV(PHIA))
BETA=TO/(R+A/P)
LL=IFIX(TO/SPACNG)
IF(LL.EQ.0) LL=1
IF(LL.GT.INTERVL) LL=INTERVL
DEL=1.001*BETA/FLOAT(LL)
TH=BETA+DEL
DO 270 I=NN,3000
TH=TH-DEL
X(I)=(R+A/P)*SIN(TH)
Y(I)=(R+A/P)*COS(TH)
IF(X(I).LE.O) GO TO 271

C
270 CONTINUE

C
271 X(I)=0.
Y(I)=R+A/P
NN=I
IEND=I

C
STRESS CONCENTRATION FACTOR
C
RDOLAN=RHF+(D-RHF)**2/(R+D-RHF)
DOLANK=.18+(THICKNS/RDOLAN)**.15*(THICKNS/HEIGHT)**.45

C
TYPE 273

C
273 FORMAT( ' DO YOU WANT A SCREEN DWG OF THE TOOTH',
C ' HALFWAY?', 'Y OR N> ')
ACCEPT 81, IRPLY
IF(IRPLY.NE.IYES) GO TO 900

CALL INITT(1)
CALL BINITT
CALL DLIMY(0.,RO)
CALL DLIMX(0.,RO)
X(1)=FLOAT(NN-1)
Y(1)=FLOAT(NN-1)
CALL CHECK(X,Y)
CALL DSPLAY(X,Y)
CALL TINPUT(IOP)

OUTPUT TO SDRC GRAPHICS FILES

900 TYPE 901
901 FORMAT('DO YOU WANT OUTPUT TO SDRC-TYPE FILES Y OR N> ? ')
ACCEPT 902, IRPLY
902 FORMAT(A1)
IF(IRPLY.NE.IYES) GO TO 1000
TYPE 906
906 FORMAT(' ENTER THE NAME OF THE FILE ' ' TO BE WRITTEN - ')
ACCEPT 907, FINAM
907 FORMAT(A)
FNPNT=FINAM//EXT
FNDAT=FINAM//DAT
TYPE 908, FNPNT
908 FORMAT(1X,A,' CONTAINS THE POINT, CIRCLE, SPLINE, LINE DATA')
OPEN(UNIT=21,NAME=FNPNT,TYPE='UNKNOWN')
OPEN(UNIT=22,NAME=FNDAT,TYPE='UNKNOWN')

TYPE 1300
1300 FORMAT(' ENTER RADIUS OF HOLE IN BLANK')
ACCEPT *, HRADIUS
TYPE 2000
2000 FORMAT(' ENTER NUMBER OF TEETH IN FE MODEL')
ACCEPT *, XTEETH
LTEETH=IFIX(XTEETH+.5)
LLTH=(LTEETH/2)*2
IF(LLTH.EQ.LTEETH) LTEETH=LTEETH+1

GENERATE 2ND HALF OF TOOTH IN X AND Y

LEND=2*IEND-3
LL=IEND
DO 2010 I=IEND+1,LEND
LL=LL-1
85

\[ X(I) = -X(LL) \]
\[ Y(I) = Y(LL) \]

C

\[ \text{POINT FILE} \]

WRITE (21, 903)

\[ \text{FORMAT}(' -1',/',', ' 25') \]

C

IPTNUM = 1
ZETA = 2. * PI / N
XI = FLOAT((LTEETH / 2)) * ZETA + ZETA

C

DO 2100 I = 1, LTEETH
XI = XI - ZETA
DO 2200 J = 2, LEND
IPTNUM = IPTNUM + 1
XNEW = Y(J) * SIN(XI) + X(J) * COS(XI)
YNEW = Y(J) * COS(XI) - X(J) * SIN(XI)
WRITE (21, 2210) IPTNUM, XNEW, YNEW

\[ \text{FORMAT}((110, 9X, '0', 19X, '8', 2E13.5, 10X, 'O.O') \]

IF (I .NE. 1 .AND. J .NE. 2) GO TO 2200
X6 = 0.0
Y6 = -SQRT(XNEW**2 + YNEW**2)

2200 CONTINUE

XI = FLOAT((LTEETH / 2)) * ZETA + PI / N
XNEW = (R - D / P) * SIN(-XI)
YNEW = (R - D / P) * COS(-XI)
IPTNUM = IPTNUM + 1
WRITE (21, 2210) IPTNUM, XNEW, YNEW
WRITE (21, 909)

\[ \text{FORMAT}(' -1') \]

C

\[ \text{SPLINE FILE} \]

WRITE (21, 1200)

\[ \text{FORMAT}(' -1', '/', ', ' 28') \]
NM(1) = (IDEDTROCH - 2) + 1
NM(2) = (ITROCHINV - IDEDTROCH) + 1
NM(3) = (INVADD - ITROCHINV) + 1
NM(4) = (IEND - INVADD) + 1
NPTS(1, 1) = 2
NPTS(2, 1) = IDEDTROCH
NPTS(3, 1) = ITROCHINV
NPTS(4, 1) = INVADD
NPTS(5, 1) = IEND
NPTS(6, 1) = IEND + NM(4) - 1
NPTS(7, 1) = IEND + NM(4) + NM(3) - 2
NPTS(8,1)=IEND+NM(4)+NM(3)+NM(2)-3  
NPTS(9,1)=IEND+NM(4)+NM(3)+NM(2)+NM(1)-4  
NM(5)=NM(4)  
NM(6)=NM(3)  
NM(7)=NM(2)  
NM(8)=NM(1)  
DO 3000 I=2,15  
   DO 3001 J=1,9  
      NPTS(J,I)=NPTS(J,I-1)+2*IEND-4  
   3001 CONTINUE  
3000 CONTINUE  
C  
DO 3010 J=1,LTEETH  
   DO 3020 I=1,8  
      LABEL=(J-1)*8+I  
      WRITE(21,3030) LABEL,NM(I)  
   3020 CONTINUE  
3010 CONTINUE  
C  
POINT FILE FOR HOLE and LINE OF ACTION  
C  
WRITE(21,903)  
XI=FLOAT((LTEETH/2))*ZETA+PI/N  
X1=0.  
Y1=HRADIUS  
X2=HRADIUS*SIN(XI)  
Y2=HRADIUS*COS(XI)  
X3=HRADIUS*SIN(-XI)  
Y3=HRADIUS*COS(-XI)  
X4=0.0  
Y4=RC  
X5=2.0*COS(PHIL)  
Y5=2.0*SIN(PHIL)+RC  
X7=0.0  
Y7=-Y1  
N1=NPTS(9,LTEETH)+1  
N2=N1+1  
N3=N2+1  
N4=N3+1  
N5=N4+1  
N6=N5+1  
N7=N6+1  
WRITE(21,2210) N1,X3,Y3  
WRITE(21,2210) N2,X1,Y1  
WRITE(21,2210) N3,X2,Y2  
WRITE(21,2210) N4,X4,Y4
WRITE(21,2210) N5,X5,Y5
WRITE(21,2210) N6,X6,Y6
WRITE(21,2210) N7,X7,Y7
WRITE(21,909)

C
C CIRCLE FILE FOR HOLE
C
WRITE(21,1400)
1400 FORMAT(' -1',/,' 27',/,' 9X,' '1',9X,' 8',9X, 'C')
WRITE(21,1410) N1,N2,N3
1410 FORMAT(3I10)
WRITE(21,1401)
1401 FORMAT(9X,'2',9X,'1',9X,'8',9X,'1')
WRITE(21,1410) N3,N7,N1
WRITE(21,1402)
1402 FORMAT(9X,'3',9X,'1',9X,'8',9X,'1')
WRITE(21,1411) N6,NPTS(9,LTEETH)
1411 FORMAT(9X,'2',2I10)
WRITE(21,909)

C
C LINE FILE
C
WRITE(21,913)
913 FORMAT(' -1',/,' 26')
WRITE(21,914) NPTS(9,LTEETH),N1
914 FORMAT(9X,'1',9X,'8',9X,'1',2I10)
WRITE(21,915) N3
915 FORMAT(9X,'2',9X,'8',9X,'1',9X,'2',I10)
WRITE(21,916) N4,N5
916 FORMAT(9X,'3',9X,'8',9X,'1',2I10)
WRITE(21,909)

C
C DATA FILE
C
WRITE(22,930) P,A,D,RHF,N,R,PHIDEG,THKNSS,XLEWIS,
1 YLEWIS,RC,PHILDEG,STRESS,SIGMA,DOLANK,SPACNG
930 FORMAT(' INPUT DATA : ' ,//,
1 ' PITCH ',F15.8,/, 2 ' ADD. FRACTION ',F15.8,/, 3 ' DED. FRACTION ',F15.8,/, 4 ' HOB TIP RAD ',F15.8,/, 5 ' NO. OF TEETH ',F15.8,/, 6 ' CTG. PITCH R ',F15.8,/, 7 ' CTG. PR. ANG. ',F15.8,/, 8 ' TOOTH THKNSS ',F15.8,/, 9 ' X LEWIS POINT ',F15.8,/, 1 ' Y LEWIS POINT ',F15.8,/, 2 ' POINT K REDIUS ',F15.8,/, 3 ' PHI(L) ',F15.8,/)
FUNCTION XINV(X)
XINV = TAN(X) - X
RETURN
END

SUBROUTINE INTERSECT(RI,X,Y,AOUT,TOUT,ITER,IDBUG,IER)

THIS ROUTINE WILL LOCATE THE POINT OF TANGENCY OR
THE POINT OF INTERSECTION OF THE INVOLUTE TOOTH
FLANK AND THE TROCHOIDAL TOOTH ROOT FOR STANDARD OR
NON-STANDARD EXTERNAL INVOLUTE GEAR TEETH

ARGUMENTS:
RI RADIUS OF INTERSECTION/TANGENCY POINT
X,Y COORDINATES OF THE POINT OF INTERSECTION/TANGENCY
AOUT ALPHA OF THE INVOLUTE INTERSECTION
TOUT THETA OF THE TROCHOID INTERSECTION
ITER ITERATION COUNT TO SOLUTION
IDBUG 0 FOR NO OUTPUT OF SUBROUTINE
1 FOR OUTPUT AT EACH ITERATION
IER 0 FOR PLAUSIBLE OUTPUT
1 FOR RI IN AN UNREASONABLE RANGE
99 FOR LACK OF CONVERGENCE

SUBROUTINES CALLED:
DERIV, INVOL, TROCH

IMPLICIT REAL*8 (A-H, O-Z)
REAL N
COMMON/TOOTH/N,P,A,D,RHF,THKNSS,PHI,R
REAL*4 X,Y,AOUT,TOUT,XI1S,Y11S,XT1S,YT1S,F1AS,F1TS,
C F2AS,F2TS,RI,P,D,RHF,THKNSS,PHI,R,A,RO,RMIN
IER=0
ITER=0
FACTOR=1.D0
EPS=1.D-4
APPROXIMATE ALPHA AND THETA TO BEGIN ITERATION AND SET THE HISTORY VARIABLES

ALPHA= .3DO*(1.DO-DEXP(-(DBLE(N)-20.DO)/25.DO))
ALPHA= .70
ALPHAP=ALPHA
THETA= .3DO*(DEXP(-(DBLE(N)-20.DO)/45.DO))
THETAP=THETA

BEGIN LOOP

CONTINUE

THIS LOOP LOCATES THE POINT OF TANGENCY OR INTERSECTION BETWEEN THE INVOLUTE AND THE TROCHOID

ITER=ITER+1

TEST ITERATION COUNTER FOR LACK OF CONVERGENCE

IF(ITER.GT.2000) GO TO 900

SET THE FACTORS TO DAMPEN OSCILLATIONS

IF(ITER.GE.10) FACTOR=.10DO
IF(ITER.GE.50) FACTOR=.01DO
IF(ITER.GE.200) FACTOR=.001DO

GET THE DERIVATIVES OF Xi-XT (F1) AND Yi-YT (F2) SINGLE PRECISION

CALL DERIV(SNGL(ALPHA),SNGL(THETA),F1AS,F1TS,F2AS,
          F2TS)
F1A=DBLE(F1AS)
F1T=DBLE(F1TS)
F2A=DBLE(F2AS)
F2T=DBLE(F2TS)

FORM THE JACOBIAN

DJAC=F1A*F2T-F2A*F1T

GET THE CURRENT LOCATIONS ON THE TROCHOID AND INVOLUTE

CALL INVOL(X1S,Y1S,SNGL(ALPHA))
CALL TROCH(XT1S,YT1S,SNGL(THETA))
X1=DBLE(X1S)
Y1=DBLE(Y1S)
XT1=DBLE(XT1S)
YT1=DBLE(YT1S)
DTEMP1=(Y11-YT1)*F1T-(X11-XT1)*F2T
DTEMP2=(X11-XT1)*F2A-(Y11-YT1)*F1A

COMPUTE THE NEWTON-RAPHSON CORRECTIONS

DELA=DTEMP1/DJAC*FACTOR
DELTH=DTEMP2/DJAC*FACTOR
ALPHA=ALPHA+DELA
THETA=THETA+DELTH

COMPUTE ERRORS AND COMPARE WITH ERROR CRITERION

EPSA=ABS((ALPHA-ALPHAP)/ALPHA)
EPST=ABS((THETA-THETAP)/THETA)

IF BOTH ERRORS SMALL THEN EXIT

IF(EPSI.LT.EPS.AND.EPSA.LT.EPS) GO TO 3

IF DEBUG SWITCH .NE. 0 WRITE ITERATION RESULTS

IF(IDBUG.NE.0) TYPE 2, ITER, ALPHA, THETA
FORMAT(' INTERSECT *** ITER=' ,I5,' ALPHA=' ,D15.8,
' THETA=' ,D15.8)
ALPHAP=ALPHA
THETAP=THETA

DON'T LET ALPHA OR THETA GO NEGATIVE

IF(ALPHA.LT.0.) ALPHA=1.D-10
IF(THETA.LT.0.) THETA=1.D-10

START LOOP AGAIN

GO TO 1

EXIT LOOP

ALPHAS=SNGL(ALPHA)
CALL INVOL(X,Y,ALPHAS)
AOUT=SNGL(ALPHA)
TOUT=SNGL(THETA)
RI=SQRT(X*X+Y*Y)
RO=R+A/P
RMIN=R-D/P
IER=1
IF(RI.GE.RMIN.AND.RI.LE.RO) IER=0
RETURN
LOOP FOR CONVERGENCE FAILURE

IER = 99
RETURN
END

SUBROUTINE TROCH(XTR,YTR,TH)

COMPUTE THE LOCATION OF A POINT ON THE TROCHOID (XTR,YTR) GIVEN THE PARAMETER TH (THETA)

REAL N
COMMON/TOOTH/N,P,A,D,RHF,THKNSS,PHI,R

TROCHOID

DELTA=((PI/P-THKNSS)/2.)-(D/P-RHF/P)*TAN(PHI)-RHF/(P*COS(PHI))
ETA=DELTA/R
BETA=PI/N-ETA
XTR=-TH*R*COS(BETA+TH)+(R-D*P**(-1)+P**(-1)*RHF)*
COS(BETA+TH)-P**(-1)*RHF*(TH**(-1)*R**(-1)*P**(-1)*RHF)*
CD-RHF)*(TH**(-2)*R**(-2)*P**(-2)*(D-RHF)**2+1.)*(-.5)
C*SIN(BETA+TH)+(TH**(-2)*R**(-2)*P**(-2)*(D-RHF)**2+C+1.)*(-.5)*COS(BETA+TH)
YTR=TH*R*SIN(BETA+TH)+(R-D*P**(-1)+P**(-1)*RHF)*
CCOS(BETA+TH)+P**(-1)*RHF*(-TH**(-1)*R**(-1)*P**(-1)
C)**(D-RHF)*(TH**(-2)*R**(-2)*P**(-2)*(D-RHF)**2+1.)*(-.5)
C**2+1.)*(-.5)*SIN(BETA+TH)
RETURN
END

SUBROUTINE INVOL(XINV,YINV,ALPHA)

COMPUTE THE LOCATION OF A POINT ON THE INVOLUTE (XINV,YINV) GIVEN THE PARAMETER ALPHA

REAL N
COMMON/TOOTH/N,P,A,D,RHF,THKNSS,PHI,R

INVOLUTE

XINV=R*COS(PHI)/COS(ALPHA)*SIN(((THKNSS/2.)/R)
C+TAN(PHI)-PHI-TAN(ALPHA)+ALPHA)
YINV=R*COS(PHI)/COS(ALPHA)*COS(((THKNSS/2.)/R)
C+TAN(PHI)-PHI-TAN(ALPHA)+ALPHA)
RETURN
END
SUBROUTINE DERIV(ALPHA, THETA, FLA, FLT, F2A, F2T)

FORM F1 AS XINV-XTROCH
F2 AS YINV-YTROCH
TAKE DERIVATIVES W/R ALPHA -- FLA and F2A
THETA -- FLT and F2T

REAL N
COMMON/TOOTH/N, P, A, D, RHF, THKNSS, PHI, R
ALDEL=ALPHA+.001
THDEL=THETA+.001
CALL INVOL(XI1, YI1, ALPHA)
CALL INVOL(XI2, YI2, ALDEL)
CALL TROCH(XT1, YT1, THETA)
CALL TROCH(XT2, YT2, THDEL)
FLA=(XI2-XI1)*1000.
FLT=-(XT2-XT1)*1000.
F2A=(YI2-YI1)*1000.
F2T=-(YT2-YT1)*1000.
RETURN
END

SUBROUTINE POINT(RI, X, Y, TOUT, ITER, IDBUG, IER, RC)

THIS ROUTINE WILL LOCATE THE POINT OF HIGHEST
TENSILE STRESS ON THE ROOT FOR STANDARD OR
NON-STANDARD EXTERNAL INVOLUTE GEAR TEETH

ARGUMENTS:
RI  RADIUS OF POINT
X, Y  COORDINATES OF THE POINT
TOUT  THETA OF THE TROCHOID

POINT
ITER  ITERATION COUNT TO SOLUTION
IDBUG  0 FOR NO OUTPUT OF SUBROUTINE
        1 FOR OUTPUT AT EACH ITERATION
IER  0 FOR PLAUSIBLE OUTPUT
        1 FOR RI IN AN UNREASONABLE RANGE
        99 FOR LACK OF CONVERGENCE
RC  RADIUS OF K POINT FOR SINGLE-TOOTH LOADING

SUBROUTINES CALLED:
DERIV, INVOL, TROCH

IMPLICIT REAL*8 (A-H,O-Z)
REAL N
COMMON/TOOTH/N,P,A,D,RHF,THKNSS,PHI,R
REAL*4 X,Y,TOUT,RI,P,D,RHF,THKNSS,PHI,R,A,RO,RMIN
IER=0
ITER=0
DEL=.002
FACTOR=1.0D0
EPS=1.0D-4

C APPROXIMATE THETA TO BEGIN ITERATION
   AND SET THE HISTORY VARIABLE

THETA=.2D0*(DEXP(-(DBLE(N)/40.D0)))+.1D-8
THETAP=THETA

C BEGIN LOOP

1 CONTINUE

C THIS LOOP LOCATES THE POINT OF TANGENCY OR INTERSECTION
   BETWEEN THE INVOLUTE AND THE TROCHOID

ITER=ITER+1

C TEST ITERATION COUNTER FOR LACK OF CONVERGENCE

IF(ITER.GT.200) GO TO 900

C SET THE FACTORS TO DAMPEN OSCILLATIONS

IF(ITER.GE.10) FACTOR=.10D0
IF(ITER.GE.50) FACTOR=.01D0
IF(ITER.GE.200) FACTOR=.001D0

C GET THE DERIVATIVE
   SINGLE PRECISION

CALL DERIV2(THETA,FTHETA,FPRIME,RC)
THETAP=THETA
THETA=THETA-FTHETA/FPRIME

C COMPUTE ERROR AND COMPARE WITH ERROR CRITERION

EPST=ABS((THETA-THETAP)/THETA)

C IF ERROR SMALL THEN EXIT

IF(EPST.LT.EPS) GO TO 3

C IF DEBUG SWITCH .NE. 0 WRITE ITERATION RESULTS
IF,IDBUG.NE.0) TYPE 2, ITER,THETA
FORMAT(' POINT *** ITER=',I5,' THETA=',D15.8)

DON'T LET THETA GO NEGATIVE
IF(THETA.LT.0.) THETA=1.D-10

START LOOP AGAIN
GO TO 1

EXIT LOOP

CONTINUE
THETAS=SNGL(THETA)
CALL TROCH(X,Y,THETAS)
TOUT=THETAS
RI=SQR((X*X+Y*Y)
RO=R+A/P
RMIN=R-D/P
IER=1
IF(RI.GE.RMIN.AND.RI.LE.RO) IER=0
RETURN

LOOP FOR CONVERGENCE FAILURE

900 IER=99
RETURN
END

SUBROUTINE DERIV2(THETA,FTHETA,FPRIME,RC)

RC RADIUS OF POINT K

REAL K,N,MJE,MJE2
COMMON/TOOTH/N,P,A,D,RHF,THKNSS,PHI,R
PI=3.1415927
DEL=.01
THDEL=THETA+DEL
DELT=((PI/P-THKNSS)/2.)-(D/P-RHF/P)*TAN(PHI)-RHF/(P*COS(PHI))
ETA=DELT/R
BETA=PI/N-ETA
MJE=-(1.+(D-RHF)/(R*THETA)*TAN(BETA+THETA))/
* ((D-RHF)/(R*THETA)-TAN(BETA+THETA))
MJE2=-(1.+(D-RHF)/(R*THDEL)*TAN(BETA+THDEL))/
* ((D-RHF)/(R*THDEL)-TAN(BETA+THDEL))
R1=R
R2=R1*1.
RO2=R2+A
TAU=ASIN(R2*COS(PHI)/RO2)
Q=COS(TAU+PHI)*R2/SIN(TAU)
RX=SQRT(Q**2+R1**2-2.*Q*R1*SIN(PHI))
TEMP=ACOS(R1*COS(PHI)/RX)
K=PI/(2.*N)-XINV(TEMP)+XINV(PHI)
PSI=(2.*PI/N)-K
ETHA=ACOS((RX**2+RO2**2-((R1+R2)**2))/(2.*RX*RO2))-TAU
RC=RX*SIN(ETHA)/SIN(ETHA+PSI)
CALL TROCH(XT,YT,THETA)
CALL TROCH(XT2,YT2,THDEL)
FTHETA=MJE+2.*(RC-YT)/XT
FTHETA2=MJE2+2.*(RC-YT2)/XT2
FPRIME=(FTHETA2-FTHETA)/DEL
RETURN
END
The vita has been removed from the scanned document.