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COMPOSITE MODELS OF QUARKS AND LEPTONS

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(ABSTRACT)

We review the various constraints on composite models of quarks and leptons. Some dynamical mechanisms for chiral symmetry breaking in chiral preon models are discussed. We have constructed several "realistic candidate" chiral preon models satisfying complementarity between the Higgs and confining phases. The models predict three to four generations of ordinary quarks and leptons.

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INTRODUCTION

During the last thirty years, remarkable progress has been made in particle physics. A standard model of strong and electroweak interactions[1,2] based on a renomalizable quantum field theory and local $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance[3] has been developed. It describes successfully all our experimental information aobut the structure of matter down to distances of 10^{-16} cm (corresponding to an energy of 100 GeV). Three generations of quarks and leptons exist as fundamental constituents and their interactions are mediated by gluons, $W^{\pm}$ and Z bosons, and the photon. We illustrate in Table 1 the presently known or anticipated elementary particles and some of their basic properties. The strong and electroweak interactions are mediated through the exchange of gauge bosons G_μ^A ($A = 1,\ldots,8$), W_μ^I ($I = 1,\ldots,3$) and B_μ which are contained in the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The quantum numbers of the fermions and scalar under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ are shown in Table 2. Given the $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers of quark, lepton and Higgs field, their couplings to the gauge bosons G_μ^A, W_μ^I and B_μ are entirely determined up to three universal gauge coupling constants g_3, g_2, and g_1. On the contrary, the couplings between fermions and scalars are largely arbitrary. The most general gauge invariant Lagrangian of Yukawa couplings is given by

$$L_y = g^{(u)i}{}_j\, \bar{q}_{Li}\tilde{H}u_R^j + g^{(d)i}{}_j\, \bar{q}_{Li}Hd_R^j + g^{(e)i}{}_j\, \bar{\ell}_{Li}He_R^j + \text{h.c.} \qquad (1.1)$$

and involves 3 unconstrained complex 3x3 matrices of Yukawa couplings.

Table 1. Elementary Particles and Their Properties

particles	symbol	spin	charge	color	Mass(GeV)
Electron neutrino	ν_e	½	0	0	$<2 \times 10^{-8}$ [4]
Electron	e	½	-1	0	0.511×10^{-3}
Up quark	u	½	2/3	3	5×10^{-3}
Down quark	d	½	-1/3	3	9 x 1 0-3
Muon neutrino	ν_μ	½	0	0	$<0.25 \times 10^{-3}$ [5]
Muon	μ	½	-1	0	0.106
Charm quark	c	½	2/3	3	1.25
Strange quark	s	½	-1/3	3	0.175
Tau neutrino	ν_τ	½	0	0	<0.07 [5]
Tau	τ	½	-1	0	1.78
Top quark	t	½	2/3	3	>23?
Bottom quark	b	½	-1/3	3	4.5
Photon	γ	1	0	0	0
W boson	$W^{\pm}$	1	±1	0	81.8±1.5
Z boson	Z	1	0	0	92.6±1.7
Gluon	g	1	0	8	0
Higgs scalar	H	0	0	0	$7 \sim 1000$ [6,7]

Table 2. Fermion and Scalar Quantum Numbers in the Standard Model

particles	$SU(3)_C \times SU(2)_L \times U(1)_Y$
$q_L = \binom{u}{d}_L, \binom{c}{s}_L, \binom{t}{b}_L$	(3, 2, 1/3)
u_R, c_R, t_R	(3, 1, 4/3)
d_R, s_R, b_R	(3, 1, -2/3)
$\ell_L = \binom{\nu_e}{e}_L, \binom{\nu_\mu}{\mu}_L, \binom{\nu_\tau}{\tau}_L$	(1, 2, -1)
e_R, μ_R, τ_R	(1, 1, -2)
H	(1, 2, 1)

In the standard model, the effective potential of the Higgs field[8] H has a minimum which breaks the symmetry $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$:

$$\langle H\rangle = \frac{1}{\sqrt{2}} \binom{o}{v} . \tag{1.2}$$

This spontaneous symmetry breaking leads to masses for the $W^{\pm}$ and Z vector bosons:

$$M_W = \tfrac{1}{2}\, g_2 v \quad , \quad M_Z = \tfrac{1}{2}\, v\sqrt{g_1^2 + g_2^2} \; ,$$

$$G/\sqrt{2} = g_2^2 \,/\, 8M_W^2 = \tfrac{1}{2}\, v^2 \tag{1.3}$$

$$v = 246 \text{ GeV} .$$

Furthermore, one obtains from equations (1.1) and (1.2) mass matrices for u- and d-type quarks and for the charged leptons:

$$\begin{aligned} M^{(u)i}{}_j &= \frac{v}{\sqrt{2}}\, g^{(u)i}{}_j \\ M^{(d)i}{}_j &= \frac{v}{\sqrt{2}}\, g^{(d)i}{}_j \\ M^{(e)i}{}_j &= \frac{v}{\sqrt{2}}\, g^{(e)i}{}_j \end{aligned} \tag{1.4}$$

which yield the mass eignevalues $m_u, \ldots m_t$ and $m_e \ldots m_\tau$ (the masses of neutrinos are zero because of the absence of right-handed neutrinos) as well as the parameters θ_1, θ_2, θ_3 and δ of the Kobayashi-Maskawa matrix[9]. In the strong color $SU(3)_C$ gauge interaction there is a free parameter θ_{QCD} to describe strong violation of CP.

In summary, there are 19 parameters needed to specify the model. These are given in Table 3. The multiplet structure of quarks and leptons in the standard model is illustrated in Fig. 1.[10]

The structure of the matter sector of the standard model, as described above, gives rise to a number of important, theoretical questions which cannot be answered within this framework. First, there are questions provoked by the poor understanding of the Higgs sector. In particular, one asks for a possible dynamical origin of the scale of spontaneously symmetry breaking (SSB) of the weak interactions

$$\Lambda_{SSB} \equiv \left(\sqrt{2}\ G_F\right)^{-1/2} \simeq 250 \text{ GeV} \tag{1.5}$$

and for a mechanism which could determine the Higgs mass as well as the finite and rapidly increasing massses of quarks and leptons as shown in Table 1. Second, the question of the origin of the intriguing family and generation patterns of the quarks and leptons has to be answered.

In view of these questions, it is widely believed that there must be some new fundamental physics beyond the standard model. There are at least five approaches which have been proposed for describing the physics beyond the standard model. These are Grand Unified Theories,[11] Technicolor Theories,[12] Supersymmetric Theories,[13] Superstring Theories,[14] and Compositeness of Quarks and Leptons Theories.[15] The common feature of all of these approaches is the existence of a new fundamental underlying theory, valid at energies well above present energies, lead-

Table 3. Parameters Needed in the Standard Model

		No. of parameters
Gauge coupling constants	g_1, g_2, g_3	3
	θ_{QCD}	1
Masses	m_{qi} ; $i = u,c,t,d,s,b$	6
	$m_{\ell i}$; $i = e, \mu, \tau$	3
	M_H $(= (2\lambda)^{1/2}\, v)$	1
	M_W $(= \frac{1}{2} g_2 v)$	1
Mixing angles	θ_1, θ_2, θ_3, δ	4

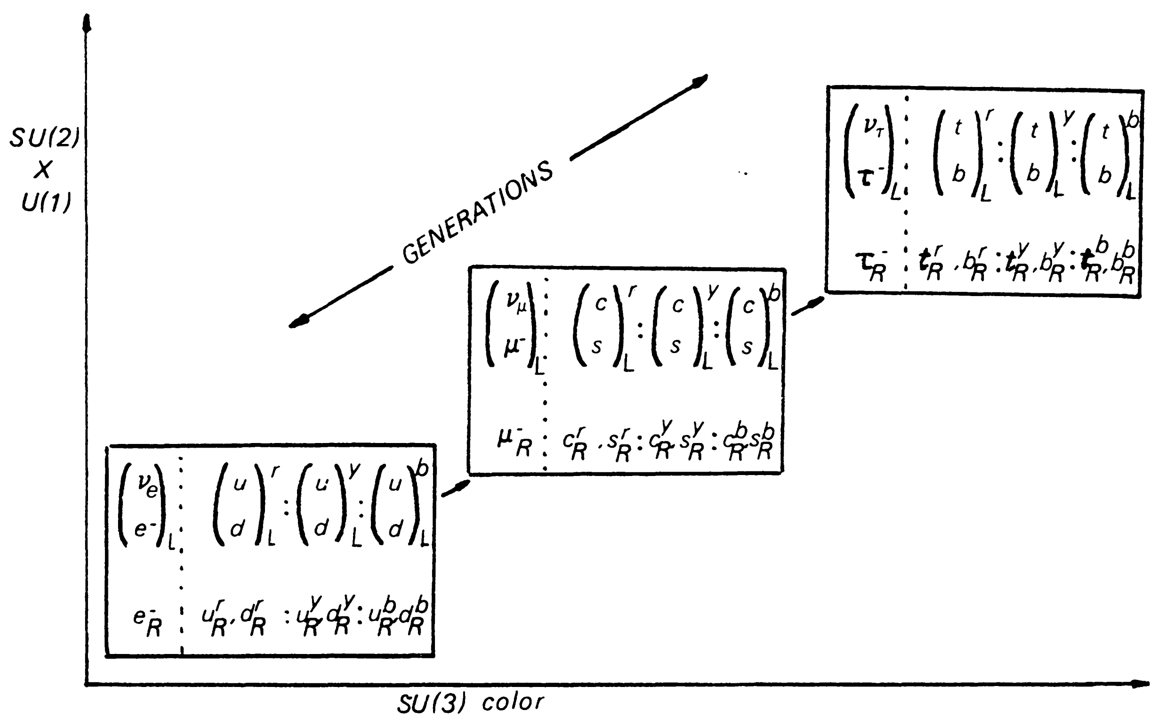

Fig. 1. Multiplet Structure in the Standard Model

ing to an effective low energy approximation which is consistent with the standard model. However, none of these approaches has produced a final theory. Each of these theories only explains some aspects of the question while other points are left unexplained. For example, the SO(10) GUT models [16] with an energy scale of about 10^{15} GeV unify only one generation of quarks and leptons into a single representation and the three interactions into one. But, the number of free parameters is increased beyond those of the standard model and no explanation is given of the generation puzzle.

In this thesis, we will discuss the current status of attempts to improve some of the unsatisfactory features of the standard model from the point of view of composite models of quarks and leptons without supersymmetry. The possiblity that quarks and leptons are bound states of more elemntary objects has been considered by many authors.[15] But the field is still very speculative because there is no experimental evidence whatsoever that quarks and leptons are composite.[17] Although various powerful theoretical constraints and interesting results have been obtained, so far no definite realistic model has been found. We will present the expectations as well as the difficulties in the composite model of quarks and leptons building.

The thesis is organized in the following way: In Chapter 2 we discuss the QCD type model for hadrons. We review the composite models of quarks and leptons in Chapter 3. We present some dynamical mechanisms for chiral symmetry breaking in chiral preon models in Chapter 4. In Chapter 5, two types of "realistic candidate" chiral preon models satisfying complementarity are given. Concluding remarks are given in Chapter 6.

Chapter 2

QCD TYPE MODEL FOR HADRONS

2.1 Preliminary

The $SU(3)_C$ gauge theory of quarks and gluons, known as quantum chromodynamics (QCD),[18] is the theory which describes the world of hadronic physics. The choice of this SU(3) color group is rather unique based on the requirements of the Pauli principle and group structure[19,20] on the constituent quark model of hadrons.

For n flavors of massless quarks, the corresponding symmetry is

$$SU(3)_C \times SU(n)_L \times SU(n)_R \times U(1)_L \times U(1)_R. \tag{2.1}$$

As pointed out by 't'Hooft,[21] the SU(2) instantons contained within $SU(3)_c$ break the two independent U(1) symmetries:

$$U(1)_L \times U(1)_R \xrightarrow{\text{instanton}} U(1)_{L+R} \times Z_{2n} \tag{2.2}$$

where the $U(1)_{L+R}$ symmetry is an unbroken vector symmetry or $U(1)_V$ which can be identified with the baryon charge and the discrete symmetry Z_{2n} comes from the breaking down of the $U(1)_{L-R}$ or $U(1)_A$ axial symmetry. The quantum numbers of the quarks under the symmetry $SU(3)_C \times SU(n)_L \times SU(n)_R \times U(1)_{L+R}$ are listed in Table 4.

It is well known that one assumption in QCD is that color singlet two-quark condensates $\langle q\,\bar{q}\rangle$ form where the condensates are

$$\langle q\,\bar{q}\rangle = (1\ ;\ n,\ \bar{n},\ 0) \tag{2.3}$$

Table 4. The Quantum Numbers of Quarks in QCD

	$SU(3)_C$	$SU(n)_L$	$SU(n)_R$	$U(1)_{L+R}$
q	3	n	1	$1/3$
$\bar{q}$	$\bar{3}$	1	$\bar{n}$	$-1/3$

under the symmetry $SU(3)_C \times SU(n)_L \times SU(n)_R \times U(1)_{L+R}$. These condensates with n equal vacuum expectation values (VEV's) break the chiral symmetry $SU(n)_L \times SU(n)_R$ down to the vector group $SU(n)_{L+R}$ while leaving the vector groups $SU(3)_C$ and $U(1)_{L+R}$ unbroken. The breaking of the chiral symmetry generates $n^2 - 1$ Nambu-Goldstone bosons.[22] All the quarks acquire masses because there is no chiral symmetry left to keep them massless and obviously, no massless fermionic states bind or all the hadrons are massive. However, if one examines the condensates (2.3) with care, one finds that the symmetry breaking pattern is not uniquely detemrined since the potential corresponding to the condensates is not known.[23] The symmetry $SU(3)_C \times SU(n)_L \times SU(n)_R \times U(1)_{L+R}$ could also break down to $SU(3)_C \times SU(n-1)_L \times SU(n-1)_R \times U(1)^2_{L+R}$ by the condensates. In this breaking, only one quark becomes massive while all the other quarks are still massless. However, the experiments rule out this possibility. For example, in the two-flavor case, the three observed pions are identified as three pseudo-Goldstone bosons. The masses of these bosons are zero at the chiral limit of quarks masses, i.e. $m_q = 0$ (cf. section 2.5). Thus, the three Goldstone bosons signal the symmetry breaking pattern $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$.

2.2 't Hooft Anomaly Matching Condition

In a classic paper[24] 't Hooft has proposed an important consistency condition to maintain unbroken chiral symmetry from the anomaly point of view, i.e. the massless composite particles must reproduce the same Adler-Bell-Jackiw (ABJ) anomalies[25] as are produced by the elementary fermions in the currents of all unbroken global chiral symmetries. This condition is a necessary condition for having massless composite fermions.[26] The reasons can be explained as follows.

Let us consider the conserved current

$$J_\mu(x) = \bar{q}_{Li}(x)\, \gamma_\mu T^i_j\, q_{Lj}(x) \tag{2.4}$$

where T^i_j are the generators of the chiral symmetry of G at the elementary level and compute the three-point function

$$\begin{aligned}(2\pi)^4\delta^{(4)}(k_1+k_2+k_3)\Gamma_{\mu\nu\lambda}(k_1,k_2,k_3) &= \int d^4x_1 d^4x_2 d^4x_3 e^{i(k_1x_1+k_2x_2+k_3x_3)} \\ &\cdot \langle 0|T(J_\mu(x_1)\, J_\nu(x_2) J_\lambda(x_3))|0\rangle .\end{aligned} \tag{2.5}$$

Because of the ABJ triangular anomaly, $\Gamma_{\mu\nu\lambda}$ is not conserved. The anomaly divergence can be calculated perturbatively by evaluting the graphs shown in Fig. 2, from which we obtain

$$k_3^\lambda \Gamma_{\mu\nu\lambda}(k_1,k_2,k_3) = \frac{1}{\pi^2} A_{\mathrm{elementary}} \varepsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \tag{2.6}$$

where the anomaly coefficient $A_{\mathrm{elementary}}$ is equal to $\mathrm{Tr}(T^3)$. One can show that, at the symmetry point $k_1^2 = k_2^2 = k_3^2 = k^2$, $\Gamma_{\mu\nu\lambda}$ is singular and has the form

$$\begin{aligned}\Gamma_{\mu\nu\lambda} = \frac{A_{\mathrm{elementary}}}{k^2} \{&\varepsilon_{\mu\nu\alpha\beta}\, k_1^\alpha k_2^\beta k_{3\lambda} + \varepsilon_{\mu\nu\alpha\beta} k_3^\alpha k_1^\beta k_{2\lambda} \\ &+ \varepsilon_{\mu\nu\alpha\beta} k_2^\alpha k_3^\beta k_{1\lambda}\} + \text{non-singular terms.}\end{aligned} \tag{2.7}$$

$$\Gamma_{\mu\nu\lambda}(k_1,k_2k_3) = J_\mu(k_1)$$

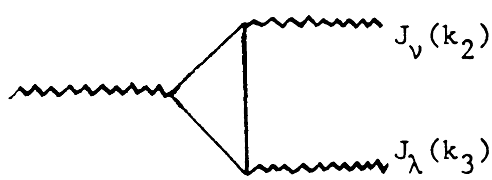

Fig. 2. Triangle Connected to the Chiral Anomaly

The pole at $k^2 = 0$ reveals the existence of massless physical particles which occur in the sum over intermediate states when we cut the three-point function. Since the elementary particles are confined, these physical states must be massless composite particles. Therefore, if G is unbroken, the elementary anomaly must be reproduced by the massless composite fermions, i.e.,

$$A_{\text{elementary}} = A_{\text{composite fermions}}. \tag{2.8}$$

This equation (2.8) is called the 't Hooft anomaly matching condition. However, the pole at $k^2 = 0$ can also be produced by a Goldstone boson coupling to J_μ which is shown in Fig. 3. In this case, the chiral symmetry G is spontaneously broken so that massless composite bosons--bound state Goldstone bosons exist.

The 't Hooft anomaly mathcing condition is an extremely powerful constraint on the pattern of chiral symmetry breaking. It implies that chiral symmetries which protect fermions from acquiring mass must be spontaneously broken unless it is possible to form physical fermions with the right quantum nubmers to match the anomalies of the elementary fermions. In his original paper,[24] 't Hooft used his condition (2.8) to argue that the chiral symmetry must be broken in QCD for more than two flavors. Let us reexamine the QCD model to see how credible the condition (2.8) is. For the two-flavor case, the quarks and all 3-quark color-singlet composites under the group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ are summarized in Table 5 where all the quarks and composites are left-handed (for the rest of this thesis we shall only work on the

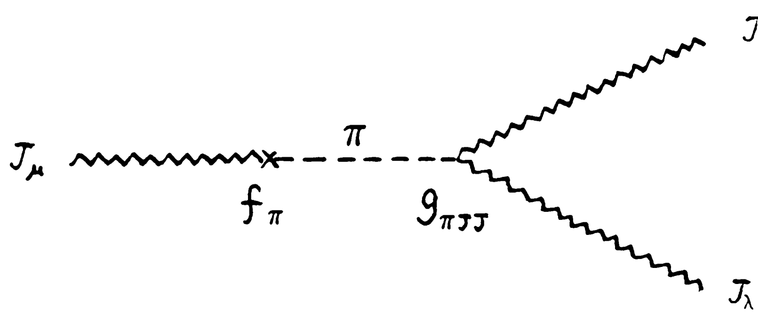

Fig. 3. Direct Coupling of Goldstone Boson to Current of Spontaneously Broken Symmetry.

Table 5. The Content of Quarks and Composites in Two-Flavor QCD Case

Quarks	$SU(3)_C$	$SU(2)_L$	$SU(2)_R$	$U(1)_{L+R}$	
q_1	3	2	1	1/3	
q_2	$\bar{3}$	1	2	-1/3	
Composites					Indices
$q_1q_1q_1$	1	4,2	1	1	ℓ_1^+,ℓ_1^-
$q_2q_2q_2$	1	1	4,2	-1	ℓ_2^+,ℓ_2^-
$q_1\bar{q}_2\bar{q}_2$	1	2	3,1	1	ℓ_3^+,ℓ_3^-
$\bar{q}_1\bar{q}_1q_2$	1	3,1	2	-1	ℓ_4^+,ℓ_4^-

left-handed elementary fermions (L) and left-handed composite fermions which are $(LLL)_M$ and $L(\bar{L}\bar{L})_A$ for 3-fermion composites[19] since $\bar{L} = R^+C$ where M, A, and C denote mixed symmetry, antisymmetrization, and charge-conjugation respectively) and the 't Hooft index ℓ is defined as the number of surviving left-handed massless composite fermions which is always a non-negative integer. The 't Hooft anomaly conditions are

For $[SU(2)_L]^2\ U(1)_{L+R}$ vertices: $1 = 10\ell_1^+ + \ell_1^- + 3\ell_3^+ + 3\ell_3^- - 8\ell_4^+$ (2.9a)

For $[SU(2)_R]^2\ U(1)_{L+R}$ vertices: $1 = 10\ell_2^+ + \ell_2^- + 3\ell_4^+ + 3\ell_4^- - 8\,\ell_3^+$ (2.9b)

For $[U(1)_{L+R}]^3$ vertices:

$$0 = 4(\ell_1^+ - \ell_2^+) + 2(\ell_1^- - \ell_2^-) + 6(\ell_3^+ - \ell_4^+) + 2(\ell_3^- - \ell_4^-) \qquad (2.9c)$$

It is easily seen that one of solutions of the three equations is $\ell_1^- = \ell_2^- = 1$ or $\ell_3^- = \ell_4^- = 1$ which corresponds to (1; 2,1,1) and (1; 1,2,-1) under the group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ with all other $\ell_i = 0$. Therefore, if chiral symmetry is not broken spontaneously, a massless nucleon doublet, i.e. a massless proton and neutron could emerge.

For the flavor $n > 3$ cases, we will first show that there is no solution of the 't Hooft anomaly matching conditions for $n = 3k$ ($k = 1,2,\ldots$) cases[27] and then, for any flavor n ($n > 2$) with the 't Hooft decoupling condition, which states that when one of the quarks is given a large mass, the remaining unbroken symmetry $SU(n-1)_L \times SU(n-1)_R$

$\times\ U(1)^2_{L+R}$ permits all composites containing this quark also to get a large mass. The latter result follows from the Appelquist-Carazone decoupling theorem[28] which states that the heavy fields effectively decouple and the low-momentum behavior of the theory is described by a renormalizable Lagrangian consisting of the massless fields only. The quarks and all 3-quarks composite fermions spectrum under the symmetry $SU(3)_C \times SU(n)_L \times SU(n)_R \times U(1)_{L+R}$ is shown in Table 6. There are five 't Hooft anomaly matching conditions which are:

$$[SU(n)_L]^3 : 3 = \frac{(n+3)(n+6)}{2}\ell_1^+ + \frac{(n-3)(n-6)}{2}\ell_1^- + (n^2-9)\ell_1^o + \\ + \frac{n(n+1)}{2}\ell_3^+ + \frac{n(n-1)}{2}\ell_3^- - (n+4)n\ell_4^+ - (n-4)n\ell_4^- \tag{2.10a}$$

$$[SU(n)_L]^3 :\ 3 = \frac{(n+3)(n+6)}{2}\ell_2^+ + \frac{(n+3)(n+6)}{2}\ell_2^- + (n^2-9)\ell_2^o \\ + \frac{n(n+1)}{2}\ell_4^+ + \frac{n(n-1)}{2}\ell_4^- - (n+4)n\ell_3^+ - (n-4)n\ell_3^- \tag{2.10b}$$

$$[SU(n)_L]^2U(1)_{L+R} :\ 1 = \frac{(n+2)(n+3)}{2}\ell_1^+ + \frac{(n-2)(n-3)}{2}\ell_1^- + (n^2-3)\ell_1^o \\ \frac{n(n+1)}{2}\ell_3^+ + \frac{n(n-1)}{2}\ell_3^- (n+2)n\ell_4^+ - (n-2)n\ell_4^- \tag{2.10c}$$

$$[SU(n)_R]^2U(1)_{L+R} :\ 1 = \frac{(n+2)(n+3)}{2}\ell_2^+ + \frac{(n-2)(n-3)}{2}\ell_2^- + n(n^2-3)\ell_2^o \\ \frac{n(n+1)}{2}\ell_4^+ + \frac{n(n-1)}{2}\ell_4^- - (n+2)n\ell_3^+ - (n-2)n\ell_3^- \tag{2.10d}$$

Table 6. The Content of Quarks and Composites in n-Flavor QCD Case

Quarks	$SU(3)_C$	$SU(n)_L$	$SU(n)_R$	$U(1)_{L+R}$	
q_1	3	$\square$	1	1/3	
q_2	$\bar{3}$	1	$\bar{\square}$	-1/3	
Composites					Indices
$q_1q_1q_1$	1	$\ydiagram{3}, \ydiagram{1,1,1}, \ydiagram{2,1}$	1	1	$\ell_1^+, \ell_1^-, \ell_1^o$
$q_2q_2q_2$	1	1	$\overline{\ydiagram{3}}, \overline{\ydiagram{1,1,1}}, \overline{\ydiagram{2,1}}$	-1	$\ell_2^+, \ell_2^-, \ell_2^o$
$q_1\bar{q}_2\bar{q}_2$	1	$\square$	$\ydiagram{2}, \ydiagram{1,1}$	1	ℓ_3^+, ℓ_3^-
$\bar{q}_1\bar{q}_1q_2$	1	$\overline{\ydiagram{2}}, \overline{\ydiagram{1,1}}$	$\bar{\square}$	-1	ℓ_4^+, ℓ_4^-

$$[U(1)_{L+R}]^3:\ 0 = \frac{n(n+1)(n+2)}{6}(\ell_1^+ + \ell_2^+) + \frac{n(n-1)(n-2)}{6}(\ell_1^- - \ell_2^-)$$

$$\frac{n(n^2-1)}{3}(\ell_1^o - \ell_2^o) + \frac{n^2(n+1)}{2}(\ell_3^+ - \ell_4^+) + \quad (2.10e)$$

$$\frac{n^2(n-1)}{2}(\ell_3^- - \ell_4^-) \ .$$

The above five equations can be reduced to

$$3 = \frac{(n+3)(n+6)}{2}\ell_1^+ + \frac{(n-3)(n-6)}{2}\ell_1^- + (n^2-9)\ell_1^o - \frac{n(n+7)}{2}\ell_3^+ - \frac{n(n-7)}{2}\ell_3^- \quad (2.11a)$$

and

$$1 = \frac{(n+2)(n+3)}{2}\ell_1^+ + \frac{(n-2)(n-3)}{2}\ell_1^- + (n^2-3)\ell_1^o - \frac{n(n+3)}{2}\ell_3^+ - \frac{n(n-3)}{2}\ell_3^- \quad (2.11b)$$

with $\ell_1^{\pm,o} = \ell_2^{\pm,o}$ and $\ell_3^{\pm} = \ell_4^{\pm}$. For n = 3k (k = 1,2,...), it is easy to see that there is no solution for Eqs. (2.11).[23] For the general n flavor case, with the 't Hooft decoupling condition which gives two more constraints.[24]

$$\ell_1^+ - \ell_3^+ + \ell_1^o = 0 \quad (2.11c)$$

and

$$\ell_1^- - \ell_3^- + \ell_1^o = 0 \ . \qquad (2.11d)$$

It is straightforward to show that there is no solution to all the Eqs. (2.11)[24,26]. However, the validity of using the Applequist-Carazone deocupling theorem has been criticized by some authors[27,29] because while it is clear that the composites containing the infinite mass quark must be also infinitely heavy, if this quark has a nonzero small mass, the composites containing this quark might have zero mass because of the strong color interaction. Actually, we have shown that even if one gives up the extra constraint of the 't Hooft decoupling condition, one can still draw the same conclusion with the generalized Pauli principle which states that the composite fermions must be totally antisymmetric under the color, flavor and spin symmetries by noting that $\ell_1^\pm$, $\ell_2^\pm$, ℓ_3^- and ℓ_4^- are zero. Therefore, the chiral symmetry must be broken in QCD in the $n > 2$ flavor case.

2.3 Mass Inequalities, the Large N Limit and Instanton Effects in Vector-like Theory

Weingarten, Witten, Nussinov (WWN)[30] and Vafa and Witten[31] have proved rigorously that, in QCD, the lightest pseudoscalar $q\bar{q}$ composite (the pion) cannot be heavier than the lightest baryon qqq composite. If quark confinement is assumed, according to the 't Hooft anomaly argument shown in the last section, there must be massless physical composite fermions for baryons coupling to the axial flavor currents if the chiral symmetry is not spontaneously broken. With WWN mass inequalities the pion must be a massless pseudoscalar and thus, it is a Goldstone boson. The chiral symmetry $SU(n)_L \times SU(n)_R \times U(1)_V$ must therefore be spontaneously broken, and mass inequalities can be invoked to be determined that the unbroken symmetry must be $SU(n)_V \times U(1)_V$. The above argument can be generalized to any vector-like gauge theory.

Coleman and Witten[32] have studied the behavior of QCD in the large N_c expansion limit, where N_c is the number of colors. Given confinement and some technical assumptions, it can be shown, without the WWN mass inequalities, that the chiral symmetry $SU(n)_L \times SU(n)_R \times U(1)_V$ is spontaneously broken down to $SU(n)_V \times U(1)_V$ in the limit $N_c \to \infty$.

2.4. Chiral Symmetry Breaking Induced by One-Gluon Exchange

At the beginning of this chapter, we have pointed out that the choice of VEV's for the quark concensates $\langle q\bar{q} \rangle$, which break the flavor symmetry, are not uniquely determined. Here, we would like to show that equal VEV's for all n flavors are preferred by assuming that single gluon exchange is dominant in the theory; it then follows that the chiral symmetry of n flavors $SU(n)_L \times SU(n)_R$ is broken down to the diagonal $SU(n)_{L+R}$. We will use the formalism of Cornwall, Jackiw and Tomboulis[33] which allows one to write an effective potential for the two-body condensate and construct the ordered vacuum in the presence of an external field, and then see if the order in this vacuum survives when the field is turned off. We will study the effective potential for the condensate $\langle q\bar{q} \rangle$ which should indicate which vacuum for the condensate $\langle q\bar{q} \rangle$ is preferable.

To evaluate the effective potential, we will first write the partition function for a theory of massless fermions with a non-local source

K which induces chiral symmetry breaking:

$$Z[K(x,y)] = \exp[-W(K)] = \int \exp[-\int (\bar{q}\, \not{\partial}\, q - \bar{q}(x)K(x,y)q(y)] \tag{2.11}$$

where q and $\bar{q}$ are the chiral fermion fields. The nonlocal two-body operator for the fermionic field is

$$\langle q(x)_a \bar{q}(y)_b \rangle_K = \int \frac{d^4k}{(2\pi)^4} \exp[-ik(x-y)] \; S_{ab}(k) \tag{2.12}$$

with

$$S_{ab}(k) = [\frac{i}{\not{k} + \Sigma(k^2)}]_{ab} . \tag{2.13}$$

The non-local source K(x,y) is chosen so that the operator $S_{ab}(k)$ corresponds to the physical propagator of the theory with $\Sigma_{ab}(k^2)$ denoting the dynamically induced fermionic mass matrix element of flavors a,b. The effective potential Γ has the following form

$$\Gamma = - \mathrm{Tr}\,\ell n S^{-1} + \mathrm{Tr}(S^{-1} - \not{\partial})S - (\text{Fig. 4}) \tag{2.14}$$

where the one-gluon exchange is assumed to be the dominant contribution and Fig. 4 presents the two-particle irreducible vacuum diagram. One can calculate Γ by using the equation (2.14) and Fig. 4, which gives

$$\Gamma = \frac{1}{4\pi^2} \int_0^\infty dk\, k^3\, \mathrm{Tr}[-\ell n[k^2 + \Sigma^2(k^2)] + \frac{2\Sigma^2(k^2)}{k^2 + \Sigma^2(k^2)}]$$

$$- \frac{g^2}{8\pi^2} \int_o^\infty \int_o^\infty dkdp \frac{k^3p^3}{\max(k^2,p^2)} \mathrm{Tr} \left[\frac{\Sigma(k^2)\Sigma(p^2)}{(k^2+\Sigma^2(k^2))(p^2+\Sigma^2(p^2))}\right] \tag{2.15}$$

$$\equiv \mathrm{Tr}F(\Sigma^2)$$

where

$$F(\Sigma^2) = \frac{1}{4\pi^2} \int_o^\infty dk\ k^3[-\ell n[k^2+\Sigma^2(k^2)] + \frac{2\Sigma^2(k^2)}{k^2+\Sigma^2(k^2)}$$

$$- \frac{g^2}{8\pi^2} \int_o^\infty \int_o^\infty dkdp \frac{k^3p^3}{\mathrm{Max}(k^2,p^2)} \left[\frac{\Sigma(k^2)\Sigma(p^2)}{[k^2+\Sigma^2(k^2)][p^2+\Sigma^2(p^2)]}\right] . \tag{2.16}$$

If we denote the eigenvalues of Σ^2 by m_i^2, $i = 1,\ldots n$, then

$$\Gamma = \sum_i F(m_i^2) \tag{2.17}$$

since the eigenvalues are independent variables. To minimize this sum (2.17), we must minimize each term. Each eigenvalue must be at the minimum F, and thus the eigenvalues are either zero (no chiral symmetry breaking) or are all equal and non-zero (the chiral symmetry $SU(n)_L$ x

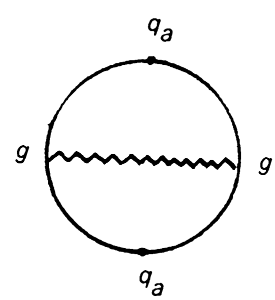

Fig. 4. Diagram contributing to the effective potential Γ up to order g^2. The blobs on the solid lines dennote the fermionic propagator with a dynamically induced mass Σ. The wiggly line denotes the gluon propagator.

$SU(n)_R$ breaks down to the diagonal $SU(n)_{L+R}$). The latter case happens when the gauge coupling g becomes quite strong. This is because the quadratic term Σ^2 of the effective potential may become negative and the chiral symmetric vacuum is unstable. Therefore, the breaking of chiral symmetry to the diagonal $SU(n)_{L+R}$ is favored by the one-gluon exchange approximation.

Several remarks follow:[34]

1. The condensate (2.14) is non-local in the time and thus the solution is not stationary.
2. The effective potential is not bound from below.
3. The condensate (2.14) is produced only if the gauge coupling constant $\alpha = \frac{g^2}{4\pi}$ is large enough, i.e. of the order of unity. But the validity of the perturbation theory used to deduce the required result is then in question.

However, despite these objections, we believe that this method gives some indication for the existence of chirally unstable vacua at the one-loop approximation in a vector-like theory.

2.5 Chiral Symmetry Breaking Induced by Instantons

It is well known that the axial $U(1)_A$ chiral symmetry in QCD is spontaneously broken by the non-perturbative effects--instantons[21]--in a manner such that the associated Goldstone excitation is coupled to a gauge conserved current and hence there is no need for a light pseudoscalar particle to appear in the physical spectrum. It has been suggested that instanton dynamics may be responsible for the breaking of the $SU(n)_L \times SU(n)_R$ chiral symmetry[35] in addition to the $U(1)_A$ chiral symmetry.

We shall use the original papers of 't Hooft[21]; Caldi[36]; Callan, Dashen and Gross[35]; Carlitz and Creamer[37] and the lecture notes of Peskin[34] to discuss how the chiral symmetry breaking in QCD is realized with the instanton mechanism.

With the same notation of the last section, the partition function for the theory is

$$\begin{aligned} Z(k) &= \sum_n \exp[-\int K\Sigma]\, Z_n(\Sigma) \\ &= \exp[-\int K\Sigma]\, Z_o(\Sigma)\left(1+\sum_{n\neq 0} A_n(\Sigma)\right) \end{aligned} \tag{2.18}$$

where n is the Pontryagin index and $Z_o(\Sigma)$ is the value given by perturbation theory. In the dilute instanton gas approximation[35], one has

$$\left(1+\sum_{n\neq 0} A_n(\Sigma)\right) \approx \exp\left[A_1(\Sigma)+A_{-1}(\Sigma)\right] \tag{2.19}$$

where $A_1(\Sigma)$ is called the one-instanton amplitude and $A_{-1}(\Sigma) = (A_1(\Sigma))^*$. Using Eqs. (2.18) and (2.19) and the definition of effective potential

$$\Gamma = \ln Z[K] - \int K\Sigma , \tag{2.20}$$

we find

$$\Gamma(\Sigma) = (\text{kinematic terms}) - \left(A_1(\Sigma) + A_{-1}(\Sigma)\right) \qquad (2.21)$$

where the gluon effect has been ignored. It can be shown that[21,34]

$$A_1(\Sigma) \sim \int d^4x \int d\rho\, A(g^2) \exp[-8\pi^2/g^2]\, I(\Sigma) \exp[-2/3C(r)\ln(\rho^2\mu_o^2)] \qquad (2.22)$$

where C(r) is the index of the representation r, μ_o is the normalization subtraction point, and

$$\begin{aligned} I(\Sigma) &\to B \cdot \Sigma \quad (\Sigma \to 0) \\ &\to \exp[2/3\; C(r)\ln\Sigma^2\rho^2] \quad (\Sigma\to\infty) \end{aligned} \qquad (2.23)$$

where B is a numerical constant.[21] Thus, we have

$$A_1(\Sigma) \geqslant 0. \qquad (2.23a)$$

The equality sign in (2.23a) is the case when the chiral symmetry is unbroken which corresponds to $\Sigma = 0$. So, the strong growth of $A_1(\Sigma)$ generates an instability toward the chiral symmetry breaking. Using the fermion action

$$\int [\bar{q}\, \not{D}\, q + \bar{q}_i \Sigma_{ij} q_j] \qquad (i,j = 1,\ldots n)\,, \qquad (2.24)$$

one finds

$$A_1(\Sigma) \sim \prod_{i=1}^{n} \Sigma_{ii} \, . \qquad (2.25)$$

If the chiral symmetry is broken, we have

$$< \prod_{i=1}^{n} q_i\bar{q}_i > \neq 0 \qquad (2.26)$$

from Eq. (2.25). To have maximum $A_1(\Sigma)$, we expect

$$< q_i\bar{q}_i > = < q_j\bar{q}_j > \neq 0 \qquad (i,j = 1,\ldots n). \qquad (2.27)$$

Thus, the chiral symmetry of QCD with n massless quarks breaks down to the diagonal $SU(n)_{L+R} \times U(1)_{L+R}$. However, it has been questioned how reliable Eq. (2.27) is because it has not been calculated directly yet.

2.6 Summary

From the arguments of the last few sections, we can see that nature seems to prefer the symmetry breaking pattern $SU(n)_L \times SU(n)_R \to SU(n)_{L+R}$ in QCD. This pattern corresponds to the maximum unbroken global symmetry (MUGS)[24] which allows all fermions to acquire mass.

The non-vanishing quark condensates also explain the finite pseudo-Goldstone boson masses. For example, three zero mass Goldstone bosons (pions) come from the chiral symmetry breaking $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$ in two-flavor QCD and one can then show that the pion masses satisfy the relation[38]

$$m_\pi^2 \, f_\pi^2 = m_q \, \langle o | \, q \, \bar{q} \, | o \rangle \tag{2.28}$$

where m_π is the mass of pion, f_π is its decay constant, and m_q is the (Higgs-induced) quark mass. The scheme of spontaneous symmetry breaking, involving the pseudoscalar mesons as Goldstone bosons, implies that the vacuum expectation value of condensates does not vanish at the chiral limit ($m_q \to 0$), and one has

$$\langle o | \, q \, \bar{q} \, | o \rangle \sim \Lambda_{QCD}^3 \approx (200 \text{ MeV})^3 \tag{2.29}$$

and thus

$$f_\pi \sim \Lambda_{QCD} \tag{2.30}$$

where Λ_{QCD} is the QCD mass scale parameter. From Eqs. (2.28), (2.29), and (2.30), one has

$$m_\pi^2 \sim m_q \, \Lambda_{QCD} \, . \tag{2.31}$$

Therefore the pion masses vanish in the chiral limit. We note that the masses of composite baryons are of order Λ_{QCD} , i.e.

$$M_{hadron} \sim \Lambda_{QCD} \, . \tag{2.32}$$

Chapter 3

REVIEW OF COMPOSITE MODELS OF QUARKS AND LEPTONS

3.1 Preliminary

The idea that quarks and leptons may have a further substructure was first speculated by Pati and Salam[39] in 1974. Since then, a large number of physicists[40] have been interested in constructing composite models of quarks and leptons. As we showed in Tables 1 and 2 and Fig. 1, quarks and leptons are different objects, but there are several similarities. The differences are:

1. Quarks have color and fractional electric charge while leptons are colorless and integrally charged.
2. Quarks are confined while leptons are observable in the free state.

The similarities are:

1. quarks and leptons are spin $1/2$ fermions and their charges are quantized in a related way (e.g. $e_e = 3e_d$);
2. all three families of quarks and leptons have the same quantum nubmers;
3. the triangle anomalies and SU(2) Witten global anomalies[41] of quarks and leptons are connected. In the standard model, both quarks and leptons separately have non-vanishing triangular anomalies on $[SU(2)_L]^2\ U(1)_Y$ and $[U(1)_Y]^3$ and non-vanishing SU(2) Witten global anomaly (because there are three doublets for quarks and one doublet for leptons in a single family) but that together the quarks and leptons of a single family are free of these perturbative and non-perturbative anomalies. Without both quarks and leptons, the stan-

dard model would not be renormalizable and mathematically consistent.

The similarities and linkages between quarks and leptons may hint that the quarks and leptons may have a common origin and be composites of more elementary objects called "preons."

3.2 Experimental Constraints on Preon Models

In analogy with the QCD quark model in which the quarks are confined by the color group $SU(3)_C$ at the energy scale of Λ_{QCD}, if quarks and leptons are composites of more fundamental constitutents or preons, they must be bound together by a new force (metacolor force) which should be confining and become strong at a scale Λ_{MC} which is also a measure of the typical size r of composite states. It is clear that Λ_{MC} must be large since there are no strong indications that quarks and leptons are other than point particles from the experiments so far. In this section, we will review some experimental bounds on Λ_{MC}.

1. Anomalous magnetic moment of muon[42]

The precise measurements of the anomalous magnetic moment of the muon provide a constraint on the substructure of leptons. The difference between the observed and calculated (standard model) values of the muon anomalous magnetic moment value has an upper bound[43] of

$$\delta a \sim 3 \times 10^{-8} . \tag{3.1}$$

This small difference could come from the lepton's non-point-like substructure. The effective interaction has the form (cf. Fig. 5):

$$\delta L_{eff} = ef^2\left(\frac{m_\mu}{\Lambda_{MC}^2}\right) \bar{\mu}_L \sigma^{\mu\nu} F_{\mu\nu} \mu_R \tag{3.2}$$

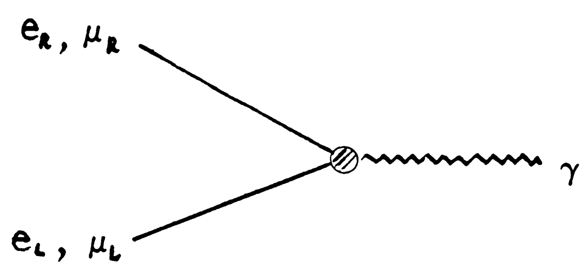

Fig. 5 Anomalous magnetic moments

where e is the electromagnetic charge of the muon, f is the model-dependent form factor, and m_μ is the mass of the muon. Since the non-vanishing small bound (3.1) requires the breaking of the global chiral symmetry in the light fermion system, the muon mass term must appear in (3.2) and Λ_{MC}^{-2} is needed to give the correct physical dimensions. The Lorentz invariance requires the form $\bar{\mu}_L \sigma^{\mu\nu} F_{\mu\nu} \mu_R$ in Eq. (3.2). We therefore find

$$\delta a = f \left(\frac{m_\mu}{\Lambda_{MC}}\right)^2 . \tag{3.3}$$

The Eqs. (3.1) and (3.3) give the bound

$$\Lambda_{MC} > f \cdot 500 \text{ GeV} . \tag{3.4a}$$

If we assume that f is of order unity, we have

$$\Lambda_{MC} > 500 \text{ GeV} . \tag{3.4b}$$

Hence, the bound on Λ_{MC} depends on the factor f.

2. $\mu \to e\gamma$ decay

In the standard model this process is entirely forbidden, due to separate conservation of electron and muon number. The current upper limit on the branching ratio (BR) for the process $\mu \to e\gamma$ (cf. Fig. 6) is $< 4.9 \times 10^{-11}$ [44] which may come from the compositeness of leptons. For describing this possible small effect, we may use the effective coupling[45]

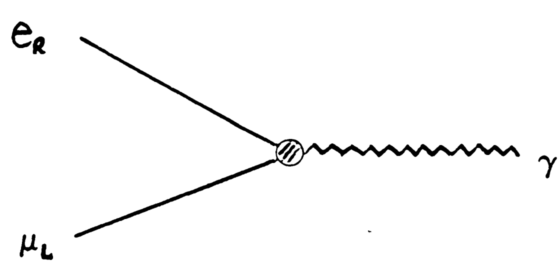

Fig. 6. $\mu \to e\gamma$ decay

$$\delta L_{eff} = e \frac{m_\mu}{\Lambda^2_{MC}} \bar{\mu}_L \sigma^{\mu\nu} F_{\mu\nu} e_R \tag{3.5}$$

where the structure in Eq. (3.5) is the same as in Eq. (3.2). We find

$$\Lambda_{MC} \approx \left[96\ \pi^3 \alpha / G^2 BR(\mu \to e\gamma)\right]^{1/4} > 240 \text{ TeV} . \tag{3.6}$$

Here, we have assumed that the model-dependent form factor for the differnt families (e and μ) is order of one, which may be a gross overestimate since μ and e belong to different families.[47]

3. K_L - K_S mass difference

The K_L - K_S mass difference comes from the mixing between s and d quarks which can be calculated by means of the box graph (cf. Fig. 7a) in the standard model. If quarks are composites, it may have an effective form (cf. Fig. 7b):

$$\delta L_{eff} = \frac{\varepsilon}{\Lambda^2_{MC}} \bar{s}_L \gamma^\mu d_L \bar{s}_L \gamma_\mu d_L . \tag{3.7}$$

The factor ε is model dependent. If ε is of order 1, the observed K_L - K_S mass difference gives[46]

$$\Lambda_{MC} > 1000 \text{ TeV} . \tag{3.8}$$

However, the processes could be completely eliminated since it is possible that the flavor-changing operators are forbidden by symmetries.

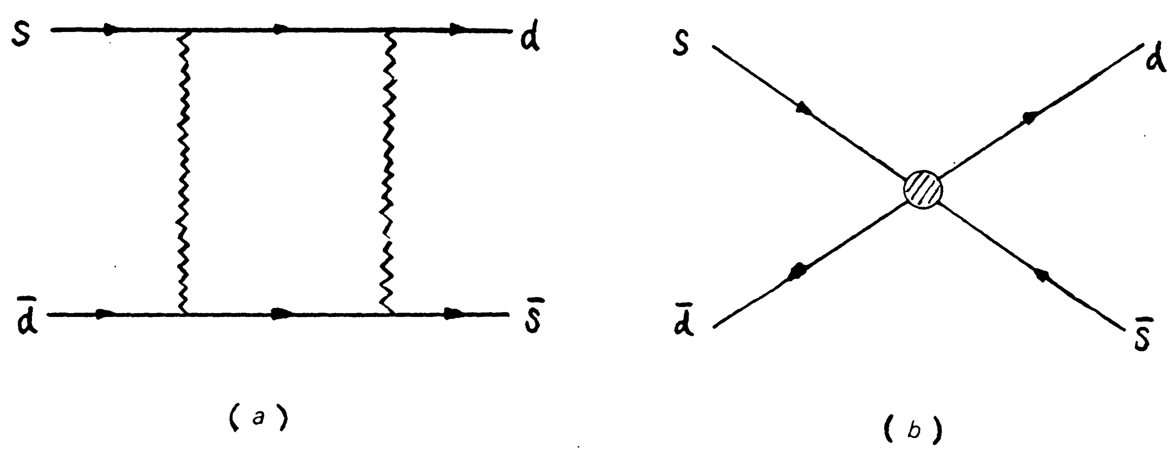

Fig. 7. $K_L - K_S$ mixing: (a) the box graph of the standard model; (b) the composites effect.

3.3 Dynamical constraints on preon models

The discussions of the previous section tell us that $\Lambda_{MC} > 0.5$ TeV and $r_{\ell,q} < 2 \times 10^{-17}$ cm if quarks and leptons have a substructure. Unlike the quark model in which the masses of composite baryons are of the order of the QCD scale Λ_{QCD}, the masses of composite quarks and leptons are much less than the metacolor scale Λ_{MC}:

$$m_q \;,\; m_\ell << \Lambda_{MC}. \tag{3.12}$$

That is to say, in preon models, the composites must be approximately massless with respect to their compositeness scale Λ_{MC}. Therefore, a certain symmetry must exist to keep quarks and leptons massless. Without introducing supersymmetry, the simplest symmetry which protects fermions from acquiring mass is a chiral symmetry. If one starts with global chiral symmetry G on the preon level, one expects that some unbroken chiral symmetry G^F must remain in the composite level. The 't Hooft triangular anomaly matching condition (2.8), presented in Chapter 2 for the existence of massless baryons, is valid here for the massless composite quarks and leptons (on the metacolor scale). We replace Eq. (2.8) by

$$A_{\text{preons}} = A_{\text{composite fermions}} \tag{3.13}$$

where A_{preons} and $A_{\text{composite fermions}}$ are the sum of anomalies for the preons and for the composites, respectively.

Let us now consider the general group $G_{MC} \times G_{MF}$ for a preon model where G_{MC} is a gauged metacolor group with the metacolor scale Λ_{MC} and G_{MF} is a global metaflavor group. For massless preons,

G_{MF} is a chiral symmetry. Quarks and leptons are the bound states of preons which are singlet under the metacolor group G_{MC}, similar to the way baryons are singlets under the color group $SU(3)_C$. Assuming that the unbroken global chiral symmetry is G^F_{MF} which is a subgroup of G_{MF}, the masses of some composites are zero under the protection of this chiral group. The triangular anomalies of G^F_{MF} for the preons and the composites must match each other in accordance with the 't Hooft triangular anomaly matching condition (3.13).

Since the dynamics of the preon constituents inside quarks and leptons is described by the metacolor gauged group G_{MC}, like QCD, the gauge interaction must be asymptotically free so that the coupling constant of the theory is assumed to become strong at the scale Λ_{MC} and follow an appropriate renormalization group curve. Moreover, the gauge sector must be anomaly-free to have renormalizability of the theory. In four-dimensional space time, only triangular anomalies and Witten SU(2) global anomalies exist. Thus, the theory requires freedom from both triangular anomalies and Witten SU(2) global anomalies. Witten has shown that for any SU(2) gauge theory the number of fermion zero modes N_o must be an even number in order for the theory to be free of the global anomaly where $N_o = \sum_{R_i} T(R_i)$ and $T(R_i)$ is the index of SU(2) in the SU(2) representation, R_i, defined by

$$\mathrm{Tr}T_aT_b\big|_{R_i} = \mathrm{Tr}(R_i)\delta_{ab} \qquad (3.14)$$

with $T_a(a = 1,2,3)$ the generators of SU(2). In a recent paper,[48] we have shown that for a gauge theory with gauge group $G \supset SU(2)$ and $\pi_4(G) = 0$ (where π_4 is the four dimensional homotopy group), one has the relation

$$Q_3(R) = Q_2(R) \pmod{2} \tag{3.15}$$

where $Q_n(R)$ is the nth order Dynkin index normalized to $Q_n(\square) = 1$ and $Q_3(R)$ and $Q_2(R)$ are the triangular anomaly coefficient A and the number of fermion zero modes N_0 for representation R, respectively. This relation is called the even-odd rule. Thus, the triangular anomaly-free condition guarantees the absence of the Witten SU(2) anomaly. Since all the simple compact Lie groups except group Sp(N)(Sp(1) = SU(2)) have the property $\pi_4(G) = 0$,[49] we conclude that for preon models in four dimensions we only need consider the triangular anomaly if the metacolor group G_{MC} is one of the simple compact Lie groups (except Sp(N)). Following the argument of 't Hooft, the Witten SU(2) global anomaly of metaflavor symmetry must be absent for both preons and composites by adding spectator fermions. Therefore, if the number of zero modes is even (odd) for preons, it must be also true for the composites. With the even-odd rule (3.15), the 't Hooft triangular anomaly matching condition (3.13) guarantees this requirement. So there is no new constraint for the massless composites in addition to the 't Hooft triangular anomaly matching condition by considering the Witten SU(2) non-perturbative anomaly.

In Chapter 2, we have shown that in vector-like gauge theory, such as QCD, the global chiral symmetry is spontaneously broken down to the maximal non-chiral subgroup and there are no massless composites in this type of model. Therefore, for the composite model of quarks and leptons without supersymmetry, the metacolor group G_{MC} must be chiral. It is interesting to note that, unlike vector-like gauge theory, the WWN type result that the lowest mass composite boson is less than the lowest mass composite fermion, does not apply in the chiral gauge theories. The reason for this is that, in chiral gauge theory, the Euclidean functional integral for chiral fermions in the presence of a background gauge field has an ambiguity in the form of a phase exp $(i n \pi/2)$ per fermion representation. The chiral gauge theory requires that the preons are in complex representation under the metacolor group G_{MC}. Since only the simple groups SU(N)(N > 3), SO(4K + 2)(K > 2) and E_6 have complex representations,[50] any realistic composite model must therefore be constructed on the basis of one of these three groups. However, progress in preon model building has been slow because very little is known about the behavior of these kinds of chiral gauge theories beyond perturbation theory and there is lack of experimental guidance beyond the standard model. Nevertheless, some progress has been made and we next report on the work in which we have been involved.

3.4 Some Chiral Preon Models

In order to understand the dynamical constraints and problems in chiral preon model building, in this section we study three well-known models. These three models will be also discussed in the next chapter with tumbling complementarity.

1. Georgi chiral preon model with SU(N)(N>4) metacolor[51,52]

The Georgi preon model has (N-4) left-handed Weyl preons in the fundamental ($\square$) representation (F) and one in the two-rank antisymmetric-bar ($\bar{\boxminus}$) representation (A) of the metacolor $SU(N)_{MC}$ group. The model has a global $SU(N-4)_F \times U(1)_{F1} \times U(1)_{F2}$ chiral symmetry. Due to instanton effects, the symmetry breaks down to SU(N-

$4)_F$ x $U(1)_F$ where the charge of $U(1)_F$ is the linear combination of $U(1)_{F1}$ and $U(1)_{F2}$ charges because of the anomaly-free condition for $[SU(N)_{MC}]^2 U(1)_F$. The spectrum of the theory, which is summarized in Table 7, consists of composites bound by the strong $SU(N)_{MC}$ force and none of the metacolor $SU(N)_{MC}$ and metaflavor $SU(N-4)_F$ x $U(1)_F$ is spontaneously broken. The 't Hooft anomaly mathcing conditions are

$$[SU(N-4)_F]^3 : \quad N = N\ell_1 + (N-8)\ell_2 \tag{3.16a}$$

$$[SU(N-4)_F]^2 U(1)_F : \quad N(N-2) = (N-2)N\ell_1 + (N-6)N\ell_2 \tag{3.16b}$$

$$\begin{aligned} [U(1)_F]^3 : \quad N(N-4)(N-2)^3 - \frac{N(N-1)(N-4)^3}{2} &= \frac{(N-4)(N-3)N^3}{2}\ell_1 \\ &+ \frac{(N-4)(N-5)N^3}{2}\ell_2 . \end{aligned} \tag{3.16c}$$

The unique solution of these three equations is $\ell_1 = 1$ and $\ell_2 = 0$. Thus, the massless composite is

$$(1 \; ; \Box\!\Box , N) \tag{3.17}$$

under the symmetry $SU(N)_{MC}$ x $SU(N-4)_F$ x $U(1)_F$. The quantum numbers of the massless fermions in (3.17) cannot be identified as the quantum numbers of ordinary quarks and leptons for any N. So, this model is a toy model.

Table 7. Particle Content of the Georgi Chiral Preon Model

preons	$SU(N)_{MC}$	$SU(N-4)_F$	$U(1)_F$	
F	$\square$	$\square$	N-2	
A	$\overline{\begin{smallmatrix}\square\\ \square\end{smallmatrix}}$	1	-(N-4)	
Composites				Indices
FFA	1	$\square\square, \begin{smallmatrix}\square\\ \square\end{smallmatrix}$	N	ℓ_1, ℓ_2

2. Bars-Yankielowicz chiral preon model with SU(N) metacolor[53,54]

The Bars-Yankielowicz chiral preon model is a chiral SU(N) metacolor gauge theory with three left-handed Weyl preons (P_1, P_2, P_3) in $(N+M+4)F$, $\overline{S}$, and $M\overline{F}$ representations $(M > 0)$ under $SU(N)_{MC}$, respectively, where $F(\overline{F})$ stands for the fundamental representation $\square(\overline{\square})$ and $\overline{S}$ the two-rank symmetry-bar representation $\overline{\square\square}$. This model respects a global metaflavor symmetry G_{MF} where

$$G_{MF} = SU(N+M+4)_F \times SU(M)_F \times U(1)_{F1} \times U(1)_{F2} \ . \tag{3.18}$$

By assuming that the metacolor $SU(N)_{MC}$ and metaflavor G_{MF} are unbroken, we write the particle content in Table 8. For the composite fermion $P_2\overline{P}_3\overline{P}_3$, the singlet under the $SU(N)_{MC}$ implies that $\overline{P}_3\overline{P}_3$ is in the symmetric representation. Since the total spin for $\overline{P}_3\overline{P}_3$ must be zero, which gives an antisymmetric spin contribution to the wave function, the meta-Pauli principle requires that under the symmetry $SU(M)_F$, $\overline{P}_3\overline{P}_3$ must also be symmetric. Therefore, the index ℓ_5 is zero. There are eight 't Hooft anomaly matching conditions, i.e. $[SU(N+M+4)_F]^3$, $[SU(N+M+4)_F]^2 U(1)_{F1}$, $[SU(N+M+4)_F]^2$, $U(1)_{F2}$, $[SU(M)_F]^3$, $[SU(M)_F]^2 U(1)_{F1}$, $[SU(M)_F]^2 U(1)_{F2}$, $[U(1)_{F1}]^3$ and $[U(1)_{F2}]^3$. The unique solution of these equations is $\ell_2 = \ell_3 = \ell_4 = 1$ and $\ell_1 = \ell_5 = 0$. Thus, the massless fermions are

$$(1; \begin{smallmatrix}\square\\ \square\end{smallmatrix}, 1, N+M, 0), \ (1 ; 1, \square\square, (N+M+2), 2N)$$

$$\text{and } (1 ; \overline{\square}, \overline{\square}, -(N+M+2), -N) \tag{3.19}$$

Table 8. Particle Content of the Bars-Yankielowicz Chiral Preon Model

Preon	$SU(N)_{MC}$	$SU(N+M+4)_F$	$SU(M)_F$	$U(1)_{F1}$	$U(1)_{F2}$	
P_1	$\square$	$\square$	1	(N+M+2)	−M	
P_2	$\overline{\square\square}$	1	1	−(N+M+4)	2M	
P_3	$\overline{\square}$	1	$\overline{\square}$	−(N+M+4)	M−N	
Composite						Indices
$P_1P_1P_2$	1	$\square\square$, $\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	N+M	0	ℓ_1, ℓ_2
$\overline{P}_1\overline{P}_2P_3$	1	$\overline{\square}$	$\overline{\square}$	−(N+M+2)	−N	ℓ_3
$P_2\overline{P}_3\overline{P}_3$	1	1	$\square\square$, $\begin{smallmatrix}\square\\\square\end{smallmatrix}$	(N+M+2)	2N	ℓ_4, ℓ_5

under the symmetry $SU(N)_{MC} \times SU(N+M+4)_F \times SU(M)_F \times U(1)_{F1} \times U(1)_{F2}$. If we take $N = 4$ and $M = 8$, the unbroken global symmetry is $G_F = SU(16)_F \times SU(8)_F \times U(1)_{F1} \times U(1)_{F2}$. At the level of $SO(10) \times Sp(8)$, the massless composite fermions in (3.19) are

$$(120,1),\ (1,36) \text{ and } (\overline{16},8) \tag{3.20}$$

where $\overline{16}$ is the spinor representation of SO(10). The fermions in (3.20) have no triangular and SU(2) Witten global anomalies. Since 120 and 36 are real representations of SO(10) and Sp(8), respectively, the first two terms in (3.20) become massive, leaving $(\overline{16},8)$ as massless fermions which can be identified as eight families of ordinary quarks and leptons. However, this model does not satisfy complementarity which will be shown in the next chapter.

3. Okamoto-Marshak chiral preon model with E_6 metacolor[55]

Okamoto and Marshak have presented a chiral preon model based on the E_6 metacolor gauge group with a single preon in the 27 fundamental representation. The metaflavor G_{MF} is $SU(N)_F$ ($N < 22$ is required to have an asymptotically-free theory). The preon and composite representations are in

$$P = (27\ ;\ \square) \tag{3.21}$$

and

$$PPP = \left(1\ ;\ \square\square\square + \begin{array}{ll}\square & \square \\ \square & \end{array} + \begin{array}{l}\square \\ \square \\ \square\end{array}\right) \tag{3.22}$$

respectively, under the symmetry $E_6 \times SU(N)_F$. It is easy to show that there is no solution of the 't Hooft anomaly condition for $SU(N)_F$ when the meta-Pauli principle is imposed. Thus, the group G_{MF} must break spontaneously down to a subgroup G^F_{MF}. In particular, they considered the case G_{MF}^{F} = SU(16) x SU(N-16) (for the realistic model N must be larger than 16); the U(1) symmetry that would appear in the breakdown $G_{MF} \rightarrow G_{MF}^{F}$ is assumed to be dynamically broken by the metacolor force. They found that there is no solution to the 't Hooft anomaly matching condition for N = 17, 19, 20, and 21 with the meta-Pauli principle and the Bars-index[54] condition, i.e. $\ell_2 < 1$. For the case N = 18, after G_{MF} is spontaneously broken down to $G_{MF}^{F} = SU(16)_F \times SU(2)_F$, the preon representations are in

$$P_1 \; (27 \; ; \; 16, 1)$$

and (3.23)

$$P_2 = (27 \; ; \; 1,2)$$

under the symmetry E_6 x SU(16) x SU(2). The particle content is summarized in Table 9 (with the meta-Pauli principle). The 't Hooft anomaly matching condition comes from the three SU(16) currents and we have

$$247\ell_1 + 40\ell_2^{+} + 24\ell_2^{-} + 3\ell_3^{+} + \ell_3^{-} = 27 \; . \tag{3.24}$$

Because of the requirement $\ell_i < 1$, the unique solution is $\ell_3^{+} = \ell_2^{-} = 1$ and all other indices $\ell_i = 0$. The massless composites are

$$(1 \; ; \; \begin{smallmatrix}\square\\ \square\end{smallmatrix} \; , \; 2) \tag{3.25a}$$

and

$$(1 \; ; \; \square \; , \; 3). \tag{3.25b}$$

The composite (3.25b) can be identified as the three generations of quarks and leptons at the gauged SU(10) level while the composite (3.25a) becomes massive (120 is a real representaiton under the SO(10) group) according to the survival hypothesis.[51] Thus, one is left with just three generations of ordinary quarks and leptons and no exotic fermions. However, this model also does not satisfy complementarity.

Table 9. Particle Content of the Okamoto-Marshak Chiral Preon Model

preons	E_6	SU(16)	SU(2)	
P_1	27	$\square$	1	
P_2	27	1	2	
Composites				**Indices**
$P_1P_1P_1$	1	$\begin{smallmatrix}\square\square\\\square\end{smallmatrix}$	1	ℓ_1
$P_1P_1P_2$	1	$\square\square$, $\begin{smallmatrix}\square\\\square\end{smallmatrix}$	2	ℓ_2^+, ℓ_2^-
$P_1P_2P_2$	1	$\square$	3,1	ℓ_3^+, ℓ_3^-
$P_2P_2P_2$	1	1	2	ℓ_4

In summary, we have studied three chiral preon models with the 't Hooft anomaly matching condition. The Georgi-type model is a toy model because the massless composite fermions (1 ; $\square\square$, N) in (3.17) under the symmetry $SU(N)_{MC}$ x $SU(N-4)_F$ x $U(1)_F$ cannot be identified as ordinary quarks and leptons. The Bars-Yankielosicz model with N = 4 and M = 8 predicts eight families of ordinary quarks and leptons at the gauged SO(10) level without exotic fermions. The model has a gauged Sp(8) family symmetry. But as we will show in the next chapter, the assumption of unbroken metaflavor does not hold with tumbling complementarity. The Okamoto-Marshak model is a quite attractive model because only one preon representation is used and it predicts three ordinary quarks and leptons with an SU(2) family group without exotic fermions by assuming a special unbroken global symmetry. Unfortunately, it does not satisfy complementarity.

Chapter 4

SOME DYNAMICAL MECHANISMS FOR CHIRAL SYMMETRY BREAKING IN CHIRAL PREON MODELS

4.1 Preliminary

Although the 't Hooft anomaly mathcing condition is a very strong constraint in chiral preon models, it is not sufficient to unqiuely determine the spectrum of massless composites. Several important questions were left unanswered in the last chapter, which can be summarized as follows:

1. The 't Hooft anomaly matching condition is only applicable when the metacolor force preserves some global chiral symmetries. If the chiral symmetries are broken there is no need for massless composite fermions even though the consistency conditions have solutions.
2. For a given G_{MF}, how does one choose the correct unbroken global metaflavor G_{MF}^{F} symmetry among many possibilities?
3. How does one select the likely solution among many 't Hooft anomaly matching conditions?

Thus additional dynamical arguments are necessary to obtain a less ambiguous result. In this chapter, we would like to study some dynamical mechanisms for the chiral symmetry breaking in analogy with the discussions of the vector-like QCD model in Chapter 2 which may provide some hints to solve the above problems. Unfortunately, the WWN mass inequalities do not have their counterparts in chiral gauge theories, as we mentioned in section 3.3, and it has not yet been proved possible to predict unambiguously the behavior of a chiral gauge theory in the $N \to \infty$ limit.[56] Thus, we will only discuss the one-gauge boson exchange and instanton approaches first mentioned in Sections 4.2 and 4.3, respectively.

4.2. Tumbling and Complementarity

The phenomenon of tumbling, resulting from a gauge group breaking itself by forming fermion condensates in a nontrivial representation, is the consequence of a rule suggested by Dimopoulos, Raby and Susskind (DRS).[57] The theoretical justification is based on the single gauge boson exchange approximation. Assuming that the chiral fermions are $\psi_{\alpha i}$, where $\alpha = 1,2$ is a spin index and i refers to metacolor and metaflavor, the effective potential Γ can be constructed by means of the most general scalar source term:

$$\int K^{ij}(y,x) \; [\varepsilon^{\alpha\beta}\psi_{\alpha i}(x)\psi_{\beta j}(y) + \text{h.c.}] \; . \qquad (4.1)$$

By following the discussion of section 2.4 and with a more general class of propagators:[34]

$$\bar{\psi}\, S^{-1}\psi \to \psi_i\, \partial\!\!\!/ \psi_i + {}^1\!/\!{}_2 \; [\; \Sigma^{ij}\varepsilon^{\alpha\beta}\psi_{\alpha i}\psi_{\beta j} + \text{h.c.}] \; , \qquad (4.2)$$

where a pair of indices (i,j) of Σ corresponds to a pair of chiral fermions in the representations r_i and r_j of the gauge group, we have the effective potential:

$$\Gamma = \frac{1}{4\pi^2} \int_0^\infty dkk^3 \mathrm{Tr}\{ -\ell n[k^2+\Sigma_{ij}(k^2)\Sigma_{ij}^*(k^2)] + \frac{2\Sigma_{ij}(k^2)\Sigma_{ij}^*(k^2)}{k^2+\Sigma_{ij}(k^2)\Sigma_{ij}^*(k^2)} \}$$

$$- \frac{3g^2}{64\pi^4} \left(C_2(r_i)+C_2(r_j)-C_2(R) \right) \int_0^\infty \int dkdp \frac{k^3 p^3}{\max(k^2,p^2)} \tag{4.3}$$

$$\cdot \mathrm{Tr} \left[\frac{\Sigma_{ij}(p^2)\Sigma_{ij}(k^2)}{\left(k^2+\Sigma_{ij}(k^2)\Sigma_{ij}^*(k^2)\right)\left(p^2+\Sigma_{ij}(p^2)\Sigma_{ij}^*(p^2)\right)} \right]$$

where R is an irreducible representaion of the product $r_i \times r_j$ and $C_2(\xi)$ is the second-order Casimir operator of the representation ξ. An instability for this potential (4.3) will appear for sufficiently strong coupling and for any representation R in $r_i \times r_j$ such that

$$A \equiv C_2(r_i) + C_2(r_j) - C_2(R) > 0 \tag{4.4}$$

The value of Γ depends not only on A and the running coupling g, but also on the eigenvalues of Σ in flavor space like the case of QCD in section 2.6. Therefore, the fermion condensate is in the channel which minimizes the Eq. (4.3) and preserves the maximum unbroken global symmetry (MUGS)[34,57] through equal eigenvalues of Σ. This channel is called the most attractive channel (MAC). The condensate corresponding to the MAC in the Higgs-like sector breaks the symmetry spontaneously.

To appreciate the idea, let us study the Georgi preon model in section 3.4. It is easy to show that the MAC of this model is

$$F \cdot A = (\square\,;\,\square\,,\,(N-2)) \times (\overline{\square\square}\,;\,1,\,-(N-4)) \to (\overline{\square}\,;\,\square\,,\,2)\,. \qquad (4.5)$$

By means of this condensate, the symmetry $SU(N)_{MC} \times SU(N-4)_F \times U(1)_F$ breaks down to $SU(4)_{MC} \times SU(N-4)'_F \times U(1)'_F$ to have MUGS where $SU(N-4)'_F$ is the diagonal subgroup of $SU(N-4)_F$ and the $SU(N-4)_{MC}$ subgroup of $SU(N)_{MC}$, and $U(1)'_F$ is a linear combinaton of $U(1)_F$ and the $U(1)_{MC}$ subgroup of $SU(N)_{MC}$, namely $U(1)'_F = U(1)_F - \tfrac{1}{2}\, U(1)_{MC}$. The preons F and A in Table 7 then branch into:

	$SU(4)_{MC}$	$\times\ SU(N-4)'_F$	$\times\ U(1)'_F$		
	1	$\square\square$	N		
F →	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	N	*	
	$\square$	$\square$	$(N+4)/4$	*	(4.6)
A →	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	1	0	*	
	1	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$-N$	*	
	$\overline{\square}$	$\overline{\square}$	$-(N+4)/2$	*	

The fermions with asterisk in (4.6), which are real or vector-like representations under the symmetry $SU(4)_{MC} \times SU(N-4)'_F \times U(1)'_F$, become massive,[51] leaving

$$(1\,;\,\square\square,\,N) \qquad (4.7)$$

under the symmetry $SU(4)_{MC} \times SU(N-4)'_F \times U(1)'_F$ as the massless fermions.

It is remarkable that the massless fermions (4.7) found in the Higgs phase in which the gauge symmetry is broken spontaneously by the Higgs-like scalar are the same as the massless composite fermions (3.17) of the metacolor confining phase in which none of the metacolor or metaflavor symmetries is spontaneously broken and the composites are singlet under the metacolor group. It seems that there is no effect on the spectrum of massless composites whether or not the system actually tumbles. DRS[52] noted this and pointed out that this result is not an accident and is expected from a theorem in lattice gauge theory for the SU(N) gauge group given by Fradkin and Shenker.[59] The theorem states that in a SU(N) gauge theory with the scalar in the fundamental representation, the Higgs and confining phases are the same. The phase diagram is shown in Fig. 8. The reason for this theorem is because the fundamental Higgs fields break the center of the SU(N) gauge group completely. Based on this theorem, a principle of complementarity is conjectured,[52] which states that in general the spectra of massless states are identical in the Higgs phase (with the scalar in the fundamental representation) and the confining phase of the SU(N) gauge theory.

Even though the Fradkin and Shenker theorem is only proved for SU(N) gauge group, people have conjectured a similar "complementarity" theorem for the SO(4N+2)$(N > 2)$[58,60] and E_6[61] gauge theories. Thus, Gérard, Okamoto and Marshak[61] showed that the princple of complementarity is valid in E_6 gauge theory with the scalar in the fundamental 27 representation of E_6. For SO(10) metacolor gauge theory, which is the

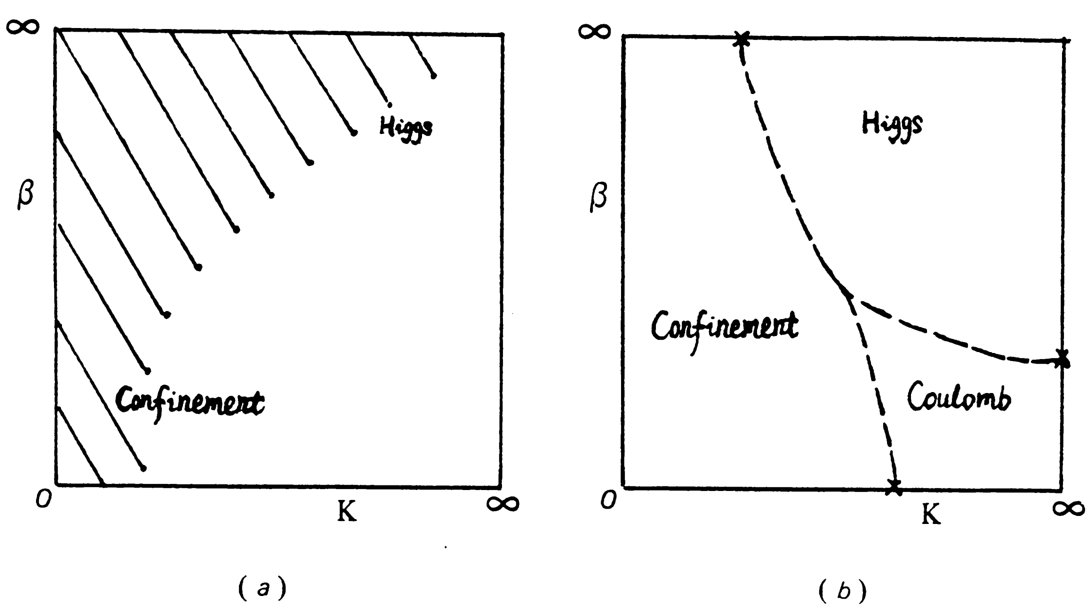

Fig. 8. The phase diagram for SU(N) gauge group: (a) Higgs fields in the fundamental representation (the Higgs and confining regions belong to the same phase); (b) Higgs fields in adjoint representation (there is a phase boundary between the Higgs and confining regions) where the dimensionless coupling constants β and K are related to the gauge coupling constant g and to the Higgs length R which represents the strength of the Higgs fields through the relations $K = 1/g^2$ and $\beta = R^2$ so that the Higgs and confining regions are in the large and small parts of the β and K regions, respectively.

smallest SO(4N+2)(N $\geq$ 2) group, King[60] has argued that the usual complementarity principle may not hold because the fundamental (10) representation only breaks the center Z_4 of SO(10) to Z_2, thereby implying the existence of two phases and the breakdwon of complementarity. King therefore suggested that a Higgs scalar in an attractive channel (AC) belonging to the spinorial 16 representation can initiate tumbling consistent with complementarity (since it completely breaks Z_4).

Complementarity implies a one-to-one correspondence between the surviving massless fermions in the "tumbling" Higgs phase and the massless composite fermions remaining from 't Hooft anomaly matching in the confining phase. Tumbling is supposed to be triggered by the two-fermion (scalar) condensate acting as the Higgs boson in the metacolor symmetry breaking $G_{MC} \to G'_{MC}$. In the process of tumbling, the global chiral symmetry G_{MF} is also broken down to a subgroup G'_{MF} by the condensate, and those fermions under the symmetry $G'_{MC} \times G'_{MF}$ which particpated in the condensate or which can form mass terms become massive. Tumbling is repeated until we reach $G^F_{MC} \times G^F_{MF}$ in which the surviving massless fermions are all G^F_{MC} singlets (or no massless fermion survives) and the symmetry G^F_{MF} is MUGS. The condensates are in the fundamental representation of SU(N) or E_6 metacolor and supposedly in the spinoral representation of SO(10) metacolor.

Let us use this complementarity principle to study the models 2 and 3 in section 3.4 to see how complementarity works.

For the Bars-Yankielowicz chiral preon model, the MAC is

$$P_1P_2 = (\square\ ; \square,\ 1,\ (N+M+2),\ -M) \times (\overline{\square\square};\ 1,\ 1,\ -(N+M+4),\ 2M) \rightarrow (\overline{\square}\ ;\ \square,\ 1,\ -2,\ M). \qquad (4.8)$$

With this condensate, the symmetry $SU(N)_{MC} \times SU(N+M+4)_F \times SU(M)_F \times U(1)_{F1} \times U(1)_{F2}$ breaks down to $SU(N)'_F \times SU(M+4)_F \times SU(M)_F \times U(1)'_{F1} \times U(1)'_{F2}$ which is the MUGS. The assumption in section 3.4 that the global symmetry $SU(N+M+4)_F \times SU(M)_F \times U(1)_{F1} \times U(1)_{F2}$ is unbroken is not correct. Further details on this model will be given in the next chapter.

For the Okamoto-Marshak $E_6 \times SU(18)_F$ chiral preon model,[55,61] in the Higgs phase, the metacolor part of the MAC condensate transforms like the symmetric $\overline{27}$, i.e.

$$27 \times 27 \rightarrow (\overline{27})_S\ . \qquad (4.9)$$

By the meta-Pauli principle, the metaflavor part of the condensate is $\square\square$ under $SU(18)_F$. The MAC condensate is thus

$$(\overline{27}\ ; \square\square) \qquad (4.10)$$

under the symmetry $E_6 \times SU(18)_F$. To obtain the MUGS, we use the branching rules of E_6 into one of its maximal little groups $SU(2)_{MC} \times SU(6)_{MC}$:

$$\overline{27} \rightarrow (1\ ;\ \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}) + (\square\ , \square) \qquad (4.11)$$

and $SU(18)_F$ into $SU(6)_F \times SU(3)_F$:

$$\square \rightarrow (\square\ , \square)$$

and

$$\square\square \rightarrow (\square\square, \square\square) + (\begin{smallmatrix}\square\\\square\end{smallmatrix}, \begin{smallmatrix}\square\\\square\end{smallmatrix}). \qquad (4.12)$$

The MUGS can be identified as $SU(6)_F \times SU(2)_F$ instead of the symmetry $SU(16)_F \times SU(2)_F$ given in section 3.4. The fermions (27; $\square$) under $E_6 \times SU(18)_F$ branch as follows;

$$(2 ; 1, 2+1) , (2 ; 35, 2+1) , (1 ; 20, 2+1)$$

and (4.13)

$$(1 ; 70, 2+1)$$

under $SU(2)_{MC} \times SU(6)'_F \times SU(2)_F$ where $SU(6)'$ is the diagonal subgroup of $SU(6)_{MC}$ and $SU(6)_F$ and $SU(2)_F$ is the subgroup of $SU(3)_F$. Since 35 and 20 are real representations of $SU(6)'_F$, the fermions corresponding to the first three terms of (4.13) acquire mass, leaving only

$$(1 ; 70, 2+1) \tag{4.14}$$

as massless fermions. So, there are three copies of (1 ; 70) under $SU(2)_{MC} \times SU(6)'_F$. The family group is SU(2). It is easy to check that in the confining phase, there is a complementarity solution corresponding to the massless fermions in (4.14) for the 't Hooft anomaly matching condition on $[SU(6)]^3$. However, this model is not realistic becasue the 70 of SU(6) does not contain the correct quantum numbers of ordinary quarks and leptons.

One consequence of complementarity is that the 't Hooft indices for composite fermions are bounded in magnitude by 1, i.e.

$$\ell_i \leq 1 .$$

This condition was first conjectured by Bars[50] on the basis of family symmetry considerations without complmentarity. With it, Okamoto and Marshak[62] argue that when $\ell_i = n(n>1)$, one has n-fold degeneracies with massless fermions in the Higgs phase, which imply the existence of a "new" SU(n) global symmetry that is not a subgroup of the original global metaflavor symmetry. By considering the MUGS principle, some "family symmetries" such as U(1) and SU(2), contained in metaflavor symmetry, prevent degeneracies of surviving massless fermions in the Higgs phase, requiring the condition (4.15). Thus, a preon model solution of the generation problem satisfying complementarity will require the natural emergence of a "family" group. This complementarity result is reflected in the Okamoto-Marshak maodel where the $SU(2)_F$ family group comes out naturally by satisfying MUGS in the Higgs phase; this restricts the 't Hooft index $\ell \leq 1$.

The 't Hooft index constraint makes preon model building harder since the metaflavor group must contain the family group in addition to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ group. On the other hand, we have no idea whether the family group will be gauged or not and how it breaks.

4.3 Instanton Effects in Chiral Gauge Theory

The behavior of instantons in chiral gauge theory is more speculative and complicated than in vector-like gauge theory. Peskin[34] has generalized the Eqs. (2.24) and (2.25) to

$$\langle \prod_{i=1}^{m} \psi_i \rangle \neq 0 \qquad (4.16a)$$

and

$$\langle \psi_i \psi_j \rangle \neq 0 \qquad (4.16b)$$

respectively, where $\psi_{i,j}(i,j = 1,\ldots,m)$ are the chiral fermion fields of the theory and m is the total number of the fermion zero modes which is always even for a triangular anomaly-free theory according to a recent paper of Geng, Marshak, Okubo and Zhao.[48]

We will use the Eqs. (4.16a) and (4.16b) to study the three models in section 3.4.

For the Georgi preon model, there are (N-4) fermion zero modes for F preons and (N-2) fermion zero modes for A preons. The equations corresponding to (4.16a) and (4.16b) are:

$$\langle (F \cdot A)^{N-4} \cdot (A \cdot A) \rangle \neq 0 \, , \tag{4.17}$$

$$\langle F \cdot A \rangle \neq 0 \, , \tag{4.18}$$

and

$$\langle A \cdot A \rangle \neq 0 \tag{4.19}$$

where F • A and A • A are in the representations($\overline{\Box}$; $\Box$, 2) and ($\overline{\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}}$; 1, -2(N-4)) respectively to guarantee the invariance of $(F\cdot A)^{N-4} \cdot (A\cdot A)$ in Eq. (4.17) under the symmetry $SU(N)_{MC} \times SU(N-4)_F \times U(1)_F$. The channel (4.18) is just the MAC in Eq. (4.5) which gives the unbroken metacolor symmetry $SU(4)_{MC}$ and the MUGS $SU(N-4)^1_F \times U(1)^1_F$ shown in the last section. The channel $\langle AA \rightarrow ($ $\overline{\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}}$; 1, -2(N-4)$)\rangle$ is an AC which also breaks the symmetry $SU(N)_{MC} \times SU(N-4)_F \times U(1)_F$ down to $SU(4)_{MC} \times SU(N-4)^{'}_F \times U(1)^{'}_F$ because the representation $\overline{\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}}$ is singlet under $SU(4)_{MC}$. Thus, the MUGS obtained with the condensates (4.18) and (4.19) agrees with the result of using MAC (4.18) alone.

For the Bars-Yankielowicz model, let us only study the simple M=0 case. There are (N+4) fermion zero modes for P_1 and (N+2) fermion zero modes for P_2. One therefore, has

$$\langle (P_1 \cdot P_2)^{N+2} (P_1 P_2) \rangle \neq 0 \tag{4.20}$$

with

$$\langle P_1 \cdot P_2 \rightarrow (\overline{\Box} ; \Box , -2) \rangle \neq 0 \tag{4.21}$$

and

$$< P_1 \cdot P_2 \rightarrow (\boxminus ; \boxminus , -2(N+4)) > \neq 0 \ . \qquad (4.22)$$

For the condensate (4.21) which is MAC, the MUGS is $SU(N)_F \times SU(4)_F \times U(1)_F$ (the details of this model are given in the next chapter). However, for the condensate (4.22) which is AC, the MUGS is $SU(n-4)_F \times SU(4)_F \times SU(4)_F \times U(1)_F$ which is a subgroup of $SU(N)_F \times SU(4)_F \times U(1)_F$. Therefore, the symmetry breaking pattern favored by instanton effects of the Peskin version does not agree with that of the one-gauge boson exchange approximation for this model. In our point of view the reason is that the Eq. (4.16b) may not be valid for arbitrary ψ_i and ψ_j because Eq. (4.16a) does not guarantee Eq. (4.16b). The vector-like QCD theory escapes this criticsm since the condensates in Eq. (2.12) are equivalent for any flavor i. On the contrary, the chiral preon models with more than one preon representation (for the one preon representation case, cf the Okamoto-Marshak model below) generally do not have this structure (the Georgi chiral preon mdoel is an accident in our view because of the special structure of the groups). To solve this problem, one must be able to calculate the value of $< \psi_i \psi_j >$ in (4.16b) directly. Unfortunately, this is very difficult to carry out.

For the Okamoto-Marshak $E_6 \times SU(18)_F$ model, there are 108 fermion zero modes corresponding to the preon P = (27;18) under $E_6 \times SU(18)_F$. We therefore have

$$< 27^{108} > \neq 0 \qquad (4.23)$$

with

$$< 27 \cdot 27 \rightarrow \overline{27}_S > \neq 0 \ . \qquad (4.24)$$

The condensate (4.24) is the MAC shown in Eq. (4.9). Thus, the symmetry breaking pattern in the section 4.2 follows.

In summary, although the instanton mechanism gives another way to determine symmetry breaking patterns, it has serious limitations in chiral preon models primarily because two-preon condensates cannot give rise to metacolor singlets as in vector-like models.

Chapter 5

REALISTIC CANDIDATE PREON MODELS WITH COMPLEMENTARITY

5.1 Preliminary

We now discuss some interesting results obtained for SU(N) preon models satisfying complementarity. Until now, the principle of complementarity has produced many toy models but all to few "realistic candidate" chiral preon mdoels. A "realistic candidate" preon model of quarks and leptons should have the following properties:

1. it should predict at least three families of massless (composite) fermions on the metacolor scale Λ_{MC};
2. the quantum numbers of each family should correspond to those of the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$;
3. the number of (composite) exotic fermions should be consistent with asymptotic freedom in the (composite) metaflavor sector.

After a "realistic candidate" model is found, one must explain the finite and rapidly increasing masses of the three families of quarks and leptons. In this chapter, we would like to present some "realistic candidate" chiral preon models[63,64] satisfying the principle of complementarity between the Higgs and the confining phases.

5.2 High Composite Scale Preon Models

In this class of models, the gauged subgroup g_{MF} of the metaflavor symmetry G_{MF} is the grand unification (GUT) group such as SO(10) or SU(5) with the GUT energy scale, Λ_{GUT} so that the confining scale Λ_{MC} is higher than Λ_{GUT}.

The models are obtained by placing the preon in the fundamental representations $F^i(\square)$ and $\bar{F}^i(\bar{\square})$ and the two-index symmetric-bar representation $\bar{S}_{ij}(\overline{\square\square})$ under the metacolor $SU(N)_{MC}$ group. The numbers of $\square$, $\bar{\square}$ and $\overline{\square\square}$, denoted by $N_{\square}$, $N_{\bar{\square}}$ and $N_{\overline{\square\square}}$ respectively, are determined by the anomaly-free condition of $SU(N)_{MC}$ which can be written as

$$N_{\square} - N_{\bar{\square}} - (N+4)N_{\overline{\square\square}} = 0 \; . \tag{5.1}$$

Case 1

Taking $N_{\overline{\square\square}} = 1$ and $N_{\bar{\square}} = M$ we have $N_{\square} = N+M+4$ from Eq. (5.1). The symmetry group for the preons is $SU(N)_{MC} \times SU(N+M+4)_F \times SU(M)_F \times U(1)_{F1} \times U(1)_{F2}$ with three preon representations $(\square ; \square, 1, (N+M+2), -M)$, $(\overline{\square\square} ; 1, 1, -(N+M+4), 2M)$ and $(\bar{\square} ; 1, \square, -(N+M+4), (M-N))$ where the charges of $U(1)_{F1}$ and $U(1)_{F2}$ are determined by the anomaly-free conditions of $[SU(N)_{MC}]^2\, U(1)_{F1}$ and $[SU(N)_{MC}]^2 U(1)_{F2}$ respectively. This is exactly the Bars-Yankelowicz model in sections 3.4. The simplest version of this class of chiral preon models is the M = 0 case.[63] In this version, the symmetry is $SU(N)_{MC} \times SU(N+4)_F \times U(1)_F$ with two preon representations.

We start by writing down the two preon representations:

	$SU(N)_{MC}$	$SU(N+4)_F$	$U(1)_F$		
$F^{i\alpha}$	$\square$	$\square$	$(N+2)$	$i = 1,\ldots N$	(5.2)
$\bar{S}_{ij}$	$\overline{\square\square}$	1	$-(N+4)$	$\alpha = 1,\ldots(N+4)$.	

In the Higgs phase, we have: MAC: $\Phi^{\alpha}_{j} \equiv F^{i\alpha}\overline{S}_{ij} = (\Box\,;\,\Box\,,\,(N+2)) \times (\overline{\Box\Box}\,;\,1,\,-(N+4)) \to (\overline{\Box}\,;\,\Box\,,\,-2)$. Since Φ^{α}_{j} is in the fundamental representation, we may proceed[52]; the choice for the Higgs scalar to obtain the MUGS is:

$$\Phi^{\alpha}_{j} = \langle F^{i\alpha}\, \overline{S}_{ij} \rangle = \Lambda\, \delta^{\alpha}_{j+4} \quad (\Lambda \text{ a constant}) \tag{5.3}$$

Now:

$$SU(N+4)_F \to SU(N)_F \times SU(4)_F \times U(1)'_F \tag{5.4}$$

with

$$\Box \to \left\{ \begin{array}{ccc} \Box & 1 & 4 \\ 1 & \Box & -N \end{array} \right. \tag{5.5}$$

and hence we get:

$$SU(N)_{MC} \times SU(N+4)_F \times U(1)_F \to SU(N)'_F \times SU(4)_F \times U(1)''_F \tag{5.6}$$

where $SU(N)'_F$ is the diagonal subgroup of $SU(N)_{MC}$ and $SU(N)_F$ and $U(1)''_F$ is a linear combination of $U(1)'_F$ and $U(1)_F$, namely $U(1)''_F = U(1)'_F + 2U(1)_F$, which gives zero charge to MAC. Under $SU(N)'_F \times SU(4)_F \times U(1)''_F$, the preon representations decompose into:

$$\begin{array}{cccc} F^{i\alpha} \to & \Box\Box & 1 & 2(N+4) \\ & \begin{smallmatrix}\Box\\\Box\end{smallmatrix} & 1 & 2(N+4) \\ & \Box & \Box & N+4 \\ \overline{S}_{ij} \to & \overline{\Box\Box} & 1 & -2(N+4) \end{array} \quad . \tag{5.7}$$

It immediately follows that the massless fermions are:

$$\left(\begin{smallmatrix}\square\\\square\end{smallmatrix},\ 1,\ 2(N+4)\right)\ ;\ \left(\square,\ \square,\ N+4\right) \tag{5.8}$$

It is seen from eq. (5.8) that the family group SU(4) has emerged in a natural way in the Higgs picture but we must now show that the massless fermions in the Higgs picture [four $\square$'s and one $\begin{smallmatrix}\square\\\square\end{smallmatrix}$ of SU(N)] match the massless (composite) fermions in the confining picture.

To prove that complementarity works, we write down the representations for the preons and all 3-preon composites (there are no five-and seven-preon composites) under the group $SU(N)_{MC} \times SU(N)_F \times SU(4)_F \times U(1)''_F$ in Table 10. The 't Hooft anomaly matching equations (five in number) are:

$$[SU(N)_F]^3:\ N = 4\ell_1 + (N+4)\ell_2 + (N-4)\ell'_2 \tag{5.9a}$$

$$\begin{aligned}[SU(N)_F]^2U(1)''_F:\ 2(N+4)N &= 4(N+4\ \ell_1 + 2(N+4)(N+2)\ell_2 \\ &+ 2(N+4)(N-2)\ell'_2\end{aligned} \tag{5.9b}$$

$$[SU(4)_F]^3:\ N = N\ell_1 + 8\ell_3 \tag{5.9c}$$

$$[SU(4)_F]^2U(1)''_F:\ (N+4)N = (N+4)N\ell_1 \tag{5.9d}$$

Table 10. Particle Content in Confining Phase of Case 1

preons		$SU(N)_{MC}$	$SU(N)_F$	$SU(4)_F$	$U(1)_F''$	
	P_1'	$\square$	$\square$	1	2(N+4)	
$F^{i\alpha} \rightarrow$						
	P_1''	$\square$	1	$\square$	N+4	
$\overline{S}_{ij} \rightarrow$	P_2	$\overline{\square\square}$	1	1	-2(N+4)	
composites						Indices
$P_1'P_1''P_2$		1	$\square$	$\square$	N+4	ℓ_1
$P_1'P_1'P_2$		1	$\square\square + \begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	2(N+4)	ℓ_2 , ℓ_2'
$P_1''P_1''P_2$		1	1	$\square\square + \begin{smallmatrix}\square\\\square\end{smallmatrix}$	0	ℓ_3 , ℓ_3'

$$[U(1)''_F]^3:\ N^2[2(N+4)]^3 + 4N(N+4)^3 - \frac{N(N+1)}{2}[2(N+4)]^3 =$$

$$4N(N+4)^3\ell_1 + \frac{N(N+1)}{2}[2(N+4)]^3\,\ell_2 + \tag{5.9e}$$

$$\frac{N(N-1)}{2}[2(N+4)]^3\ell_2'$$

It is easily seen that the only solution of all these equations is $\ell_1 = \ell_2' = 1$ with all other $\ell_i = 0$, in precise agreement with the Higgs picture. We therefore conclude that a complementarity solution of the $SU(N)_{MC} \times SU(N+4)_F \times U(1)_F$ model exists and that the massless (composite) fermions are ($\begin{smallmatrix}\square\\\square\end{smallmatrix}$, 1, 2(N+4)) and ($\square$, 4, (N+4)) under $SU(N)_F \times SU(4)_F \times U(1)_F$.

We next show that the choice of N = 16 (Model I) give rise to 4 families of massless composites that can be identified with quarks and leptons (without exotics) at the gauged SO(10) level while the choice of N = 15 (Model II) predicts 3 families of quarks and leptons with no exotics at the gauged SU(5) level. Model I is straightforward; the choice N = 16 in Eq. (5.8) implies 4(16) and 1(120) at the gagued g_F = SO(10) subgroup level ($U(1)_F$ is broken down after gauging). Since the representation 120 is real, this single family of exotic fermions should acquire a large mass.[51] Model II, corresponding to the choice N = 15, requires more care. The massless (composite) fermions satisfying complementarity then become (Eq. (5.8)):

$$(\begin{smallmatrix}\square\\\square\end{smallmatrix},\ 1,\ 38) \text{ and } (\square,\ \square,\ 19) \text{ under } SU(15)_F \times SU(4)_F \times U(1)_F\,. \tag{5.10}$$

We can now take the global chiral symmetry $G_F = SU(15) \times U(1)$ and gauge its subgroup $g_F = SU(5)$ so that the preons transform according to the $\bar{5} + 10$ representation of SU(5). From Eq. (5.10), we have

$$\begin{aligned} \square &\to \bar{5} + 10 \\ \begin{smallmatrix}\square\\ \square\end{smallmatrix} &\to 5 + \overline{10} + \overline{45} + 45. \end{aligned} \tag{5.11}$$

and, taking Georgi's survival hypothesis into account,[51] we end up with 3 families of $(\bar{5} + 10)$ of SU(5) without exotics. It is interesting to note that one of the four families of $\bar{5} + 10$ in the representation of SU(15) is eliminated by the mirror family $5 + \overline{10}$ in the representation of SU(15).

We now return to the general Bars-Yankelowicz group. It can be shown that complementarity holds in the general case and that the surviving massless (composite) fermions are:

$$\begin{aligned} &(\,\square\,,\ \square\,,\ 1,\ N(M+1) + (M+2)^2,\ M(N+M+4))\ , \\ &(\,\begin{smallmatrix}\square\\ \square\end{smallmatrix}\,,\ 1,\ 1,\ (M+2)(N+M+4),\ 2M(M+4))\ \text{and} \\ &(\,\overline{\square}\,,\ 1,\ \overline{\square},\ -(M+2)(N+M+4),\ -\ (M-N)(M+4)) \end{aligned} \tag{5.12}$$

under $SU(N)_F \times SU(M+4)_F \times SU(M)_F \times U(1)_F \times U(1)_F$. Eq. (5.12) reduces to Eq. (5.8) for $M = 0$. If one now takes $N = 16$ (15) and gauges $SU(N)_F$ at the SO(10) [SU(5)] levels, taking account of the survival hypothesis,[51] Eq. (5.12) yields the $M = 0$ result for arbitrary M.

We have thus identified two "realistic candidate" models that uniquely satisfy complementarity within the framework of the entire

Bars-Yankelosicz class of chiral preon models. The common family group SU(4) that naturally emerges makes a distinction between "SO(10)" and "SU(5)" quarks and leptons both as regards the number of predicted families and whether a right-handed neutrino exists or not.

Case 2

In this case, we take $N_{\overline{\square\square}} = 2$ and $N_{\overline{\square}} = 0$ and therefore, $N_{\square} = 2(N+4)$ from Eq. (5.1). The global symmetry of this case is $SU(2(N+4))_F \times SU(2)_F \times U(1)_F$. Thus, the chiral preon model is based on the symmetry $SU(N)_{MC} \times SU(2(N+4)_F \times SU(2)_F \times U(1)_F$ with two preon representations

$$\begin{aligned} P_1 &= \left(\square\ ;\ \square,\ \ 1,\ \frac{(N+2)}{2}\right) \quad \text{and} \\ P_2 &= \left(\overline{\square\square};\ \ 1,\ \square,\ -\frac{(N+4)}{2}\right) \end{aligned} \tag{5.13}$$

where the $U(1)_F$ charges are determined by the anomaly-free condition on $[SU(N)_{MC}]^2U(1)_F$. To obtain a "realistic candidate" model, let us study the N = 4 case (Model III) in which the symmetry is $SU(4)_{MC} \times SU(16)_F \times SU(2)_F \times U(1)_F$ with the two preon representations[64]

$$\begin{aligned} P_1 &= (4;\ 16,\ 1,\ 3) \quad \text{and} \\ P_2 &= (\overline{10};\ 1,\ 2,\ -4)\ . \end{aligned} \tag{5.14}$$

The Higgs and confining phases are:

(i) Higgs Phase

The first MAC binds P_1 and P_2 into a condensate in the representation:

$$\Phi = (\overline{4};\ 16,\ 2,-1) \tag{5.15}$$

under $SU(4)_{MC} \times SU(16)_F \times SU(2)_F \times U(1)_F$. Now $SU(4)_{MC}$ breaks down to $SU(2)_{MC} \times SU(2)'_{MC} \times U(1)_{MC}$ so that:

$$4 \to (2,1,-1) + (1,2,1) \tag{5.16}$$

$$\overline{10} \to (3,1,-2) + (2,2,0) + (1,3,2) \tag{5.17}$$

and $SU(16)_F$ breaks down to $\to SU(15)_F \times U(1)'_F$ so that

$$16 \to (15,1) + 1,-15). \tag{5.18}$$

Thus, the symmetry $SU(4)_{MC} \times SU(16)_F \times SU(2)_F \times U(1)_F$ breaks down to $SU(2)_{MC} \times SU(15)_F \times SU(2)'_F \times U(1)_{F1} \times U(1)_{F2}$ via the condensate (5.15) where $SU(2)'_F$ is the diagonal subgroup of $SU(2)_F$ and $SU(2)'_{MC}$, and $U(1)_{F1}$ and $U(1)_{F2}$ are linear combinations of $U(1)_{MC}$ and $+\ U(1)'_F$ or $U(1)_F$ respectively; that is, $U(1)_{F1} = 15\ U(1)_{MC} + U(1)'_F$ and $U(1)_{F2} = U(1)_{MC} + U(1)_F$, which give zero charges to MAC. Under $SU(2)_{MC} \times SU(15)_F \times SU(2)'_F \times U(1)_{F1} \times U(1)_{F2}$, the decompostiions of the preon representations are shown in Table 11. The fermions with asterisk in Table 11 become massive, leaving the others as massless fermions.

The next MAC condensate is given by

$$(3;1,2,-30),-6)\text{x}(2;1,3,0,-4) \to \Phi' = (2;1,2,-30,-10) \tag{5.19}$$

which breaks the symmetry $SU(2)_{MC} \times SU(15)_F \times SU(2)'_F \times U(1)_{F1} \times U(1)_{F2}$ down to $SU(15)_F \times SU(2)^f_F \times U(1)^f_F$, where $SU(2)^f_F$ is the diagonal subgroup of $SU(2)_{MC}$ and $SU(2)'_F$, and $U(1)^f_F$ is a linear combinaton of $U(1)_{F1}$ and $U(1)_{F2}$, i.e. $U(1)^f_F = U(1)_{F1} - 3U(1)_{F2}$. The massless fermions in Table 11 then decompose into:

Table 11. Higgs Phase of Case 2

	$SU(2)_{MC}$	$SU(15)_F$	$SU(2)'_F$	$U(1)_{F1}$	$U(1)_{F2}$
	2	15	1	16	4
	2	1	1	0	4*
$P_1 \rightarrow$	1	15	2	-14	2
	1	1	2	-30	2*
	1	1	2	30	-2*
	1	1	4	30	-2
$P_2 \rightarrow$	2	1	1	0	-4*
	2	1	3	0	-4
	3	1	2	-30	-6

$$(15,2,4),(15,2,-20),(1,4,36),(1,2,-12),(1,4,-12),(1,2,12) \text{ and } (1,4,12) \tag{5.20}$$

under $SU(15)_F \times SU(2)_F^f \times U(1)_F^f$. The fermions corresponding to the last four terms in (5.20) become massive, leaving

$$(15,2,4),(15,2,-20) \text{ and } (1,4,36) \tag{5.21}$$

as the massless fermions.

(ii) Confining Phase

We write down the representations for the preons and relevant composites under the group $SU(4)_{MC} \times SU(15)_F \times SU(2)_F'' \times U(1)_F'$ where $U(1)_F'' = U(1)_F' - 3U(1)_F$ in Table 12. The 't Hooft anomaly matching equations are:

$$\begin{aligned}
&[SU(15)_F]^3: 4 = 2\ell_1 + 2\ell_2 + 4\ell_3 + \ldots \\
&[SU(15)_F]^2\, U(1)_F'' : -32 = -40\ell_1 + 8\ell_2 + 160\ell_3 + \ldots \\
&[SU(2)_F]^2 U(1)_F'' : 120 = -30\ell_1 + 60\ell_2 + 60\ell_3 + 36\ell_4 + 30\ell_5 + \ldots \\
&[U(1)_F''] : -51456 = -24\text{x}10^4\ell_1 + 1,920\ell_2 + 3,840\ell_3 + 93,312\ell_4 \\
&\qquad + 186,624\ell 1_5 + \ldots
\end{aligned} \tag{5.22}$$

Evidently, $\ell_1 = 1$, $\ell_2 = 1$, $\ell_5 = 1$ and all other $\ell_i = 0$ is a solution of (5.22). Consequently, complementarity holds in this model. The massless fermions in (5.20) can be identified as four families of quarks and leptons together with four heavy neutrinos at the SU(5)GUT level. Since $SU(2)_F \times U(1)_F$ essentially serves as the family group, the four families of quarks and leptons are really (2+2) families, with the bifurcation resulting from the "spurion-like" two-preon (q_2q_3) and four-

Table 12. Confining Phase of Case 2.

Preons		$SU(4)_{MC}$	$SU(15)_F$	$SU(2)_F$	$U(1)''_F$	
$P_1 \rightarrow$	q_1	4	15	1	-8	
	q_2	4	1	1	-24	
$P_2 \rightarrow$	q_3	$\overline{10}$	1	2	12	
Composites						**Indices**
$q_1q_2q_3$		1	15	2	-20	ℓ_1
$q_1q_2q_3^3$		1	15	2,4	4	ℓ_2,ℓ_3
$\bar{q}_2^2q_3\bar{q}_3^2$		1	1	2,4	36	ℓ_4,ℓ_5

preon $(q_2 q_3^3)$ quasi-scalars in the final fermion composites.

Finally, it should be noted that similar models based on the $SU(3)_{MC}$ or $SU(5)_{MC}$ confining groups predict three or five families of quarks and leptons respectively although one more preon representation has be to be introduced in comparison with the $SU(4)_{MC}$ confining group of Model III.

5.3 A Low Composite Scale Preon Model[64]

In the last section, we described several models with the composite scale Λ_{MC} larger than the GUT scale Λ_{GUT}. This makes the composite model phenomenologically difficult to test. In order to have more likely checks of compositeness it would be desirable to identify a "realistic candidate" model with $\Lambda_{MC} \ll \Lambda_{GUT}$.[65] A low composite scale can be achieved if the gauged subgroup g_{MF} of the global metaflavor group G_{MF} could be the observed low energy $SU(3)_C \times SU(2)_L \times U(1)_Y$ group of the standard model. In this section, we present the first "realistic candidate" chiral preon model with low composite scale that satisfies complementarity.

The low composite scale model follows directly from the $SU(4)_{MC} \times SU(16)_F \times SU(2)_F \times U(1)_F$ chiral preon model in the last section. Replacing $SU(16)_F$ by $SU(4)_{CF} \times SU(2)_{LF} \times SU(2)_{RF} (G_{422})$ in this model, the preon group becomes $SU(4)_{MC} \times G_{422} \times SU(2)_F \times U(1)_{F1} \times U(1)_{F2}$ with the preons given by:

$$p_1 = (4;\ 4,\ 2,\ 1;\ 1,\ 1,\ 1),$$
$$p_2 = (4;\ 4,\ 1,\ 2;\ 1,\ -1,\ 2) \text{ and} \qquad (5.23)$$
$$p_3 = (\overline{10};\ 1,\ 1,\ 1;\ 2,\ 0,\ -2)$$

where the two U(1) charges are determined by the anomaly-free condition on $[SU(4)_{MC}]^2 \cdot U(1)_{Fi}$. Consider now the:

(i) Higgs Phase

The first MAC binds P_2 and P_3 into a condensate in the representation:

$$\phi = (\overline{4};\ \overline{4},\ 1,\ 2;\ 2,\ -1,\ 0) \qquad (5.24)$$

under $SU(4)_{MC} \times G_{422} \times SU(2)_F \times U(1)_{F1} \times U(1)_{F2}$. By using the branching rule (5.16), (5.17) and

$$SU(4)_{CF} \rightarrow SU(3)_{CF} \times U(1)_{C4}$$
$$4 \rightarrow (3,-1) + (1,+3) \qquad (5.25)$$
$$\overline{4} \rightarrow (\overline{3},1) + (1,\ -3),$$

and $SU(2) \rightarrow U(1)_2$ with

$$2 \rightarrow (+1)+(-1), \qquad (5.26)$$

the symmetry $SU(4)_{MC} \times G_{422} \times SU(2)_F \times U(1)_{F1} \times U(1)_{F2}$ breaks down to $SU(2)_{MC} \times SU(3)_{CF} \times SU(2)_{LF} \times U(1)_Y \times SU(2)'_F \times U(1)'_{F1} \times U(1)_{F2}$, where $U(1)_Y = -1/3\ U(1)_{C4} + U(1)_{R2}$, $SU(2)'_F$ is the diagonal subgroup of $SU(2)_F$ and $SU(2)'_{MC}$ and $U(1)'_{F1} = U(1)_{MC} + U(1)_{F1}$.

The preons P_1, P_2 and P_3 in (5.23) then branch into:

	$SU(2)_{MC}$	x $SU(3)_{CF}$	x $SU(2)_{LF}$	x $U(1)_Y$	x $SU(2)'_F$	x $U(1)'_{F1}$	x $U(1)_{F2}$	
$P_1 \rightarrow$	2	3	2	1/3	1	2	1	
	2	1	2	-1	1	2	1	
	1	3	2	1/3	2	0	1	
	1	1	2	-1	2	0	1	
$P_2 \rightarrow$	2	$\bar{3}$	1	2/3	1	0	2	
	2	$\bar{3}$	1	-4/3	1	0	2	
	2	1	1	2	1	0	2	
	2	1	1	0	1	0	2*	
	1	$\bar{3}$	1	2/3	2	-2	-2	(5.27)
	1	$\bar{3}$	1	-4/3	2	-2	-2	
	1	1	1	2	2	-2	-2	
	1	1	1	0	2	2	-2*	
$P_3 \rightarrow$	1	1	1	0	2	2	-2*	
	1	1	1	0	4	2	-2	
	2	1	1	0	1	0	-2*	
	2	1	1	0	3	0	-2	
	3	1	1	0	2	-2	-2	

The fermions with asterisk become massive and all other fermions are still massless.

The next MAC condensate is:

$$(3;1,1,0,2,-2,-2)\mathrm{x}(2;1,1,0;3,0,-2) \rightarrow \Phi' = (2;1,1,0;2,-2,-4) \qquad (5.28)$$

The symmetry $SU(2)_{MC}$ x $SU(3)_{CF}$ x $SU(2)_{LF}$ x $U(1)_Y$ x $SU(2)'_F$ x $U(1)'_{F1}$ x $U(1)_{F2}$ breaks down to $SU(3)_{CF}$ x $SU(2)_{LF}$ x $U(1)_Y$ x $SU(2)''_F$ x $U(1)''_F$ where $SU(2)''_F$ is the diagonal subgroup of $SU(2)_{MC}$ and $SU(2)'_F$ and $U(1)''_F$ = $2U(1)'_{F1}$ - $U(1)_{F2}$. The massless fermions under $SU(3)_{CF}$ x $SU(2)_{LF}$ x $U(1)_Y$ x $SU(2)''_F$ x $U(1)''_F$ are given in Table 13.

Table 13. Massless Fermions in Higgs Phase (low composite scale model)

$SU(3)_{CF}$	$\times\ SU(2)_{LF}$	$\times\ U(1)_Y$	$\times\ SU(2)''_F$	$\times\ U(1)''_F$
3	2	1/3	2	3
3	2	1/3	2	-1
1	2	-1	2	3
1	2	-1	2	-1
$\bar{3}$	1	2/3	2	-2
$\bar{3}$	1	2/3	2	-6
$\bar{3}$	1	-4/3	2	-2
$\bar{3}$	1	-4/3	2	-6
1	1	2	2	-2
1	1	2	2	-6
1	1	0	4	6

(ii) Confining Phase

We write down the representations for the preons and composites under the group $SU(4)_{MC} \times SU(3)_{CF} \times SU(2)_{LF} \times U(1)_Y \times SU(2)_F \times U(1)_F$, where $U(1)_Y = -1/3\ U(1)_{C4} + U(1)_{L2}$ and $U(1)_F = 2U(1)_{F1} - U(1)_{F2}$ in Table 14.

These are nine 't Hooft anomaly matching constraints, i.e. $[SU(3)_{CF}]^3$, $[SU(3)_{CF}]^2U(1)_Y$, $[SU(3)_{CF}]^2\ U(1)_F$, $[SU(2)_{LF}]^2U(1)_Y]^3$, $[SU(2)_F]^2U(1)_Y$, $[SU(2)_F]^2U(1)_F$ and $[U(1)_F]^3$. By solving all of these equations, we find a complementarity solution which corresponds to the massless fermions in Table 13, that is, $\ell_i = 1$, $i = 1,2,\ldots,10$, $\ell'_{11} = 1$ and all others $\ell_i = 0$. When the $SU(3)_{CF} \times SU(2)_{LF} \times U(1)_Y$ gauge interactions are turned on, the first ten terms of massless fermions in Table 13 comprise four generations of ordinary quarks and leptons and the single last term in Table 13 represents four neutrinos which become heavy. The symmetry $SU(2)_F \times U(1)_F$ serve as the family group and there are no exotic fermions.

It is not surprising that our low composite scale model bears a strong resemblance to the high composite scale model discussed in the last section (case 2), since the metaflavor symmetry $SU(4)_{CF} \times SU(2)_{LF} \times SU(2)_{RF}$ is a subgroup of $SU(16)_{MF}$. But the point is that we can start with Eq. (5.23) and relying upon tumbling complementarity, we can identify a "realistic candidate" preon model with the low-dimensional confining group $SU(4)_{MC}$ that predicts four generations of quarks and leptons at the gauge group level $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the standard model. Moreover, there is the curious twist that the residual global flavor group $SU(2) \times U(1)$ has the same structure as the electroweak part of the gauged composite standard group.

Table 14. Confining Phase (low composite scale model)

Preons		$SU(4)_{MC}$	x $SU(3)_{CF}$	x $SU(2)_{LF}$	x $U(1)_Y$	x $SU(2)_F$	x $U(1)_F$	
$p_1 \rightarrow$	q_1	4	3	2	1/3	1	1	
	q_2	4	1	2	-1	1	1	
$p_2 \rightarrow$	q_3	4	$\bar{3}$	1	2/3	1	-4	
	q_4	4	$\bar{3}$	1	-4/3	1	-4	
	q_5	4	1	1	2	1	-4	
$p_3 \rightarrow$	q_6	4	1	1	0	1	-4	
	q_7	$\overline{10}$	1	1	0	2	2	
Composites								Indices
$q_1 q_6 q_7$		1	3	2	1/3	2	-1	ℓ_1
$q_2 q_6 q_7$		1	1	2	-1	2	-1	ℓ_2
$q_3 q_6 q_7$		1	$\bar{3}$	1	2/3	2	-6	ℓ_3
$q_4 q_6 q_7$		1	$\bar{3}$	1	-4/3	2	-6	ℓ_4
$q_5 q_6 q_7$		1	1	1	2	2	-6	ℓ_5
$q_1 q_6 q_7^3$		1	3	2	1/3	2,4	3	ℓ_6, ℓ_6'
$q_2 q_6 q_7^3$		1	1	2	-1	2,4	3	ℓ_7, ℓ_7'
$q_3 q_6 q_7^3$		1	3	1	2/3	2,4	-2	ℓ_8, ℓ_8'
$q_4 q_6 q_7^3$		1	$\bar{3}$	1	-4/3	2,4	-2	ℓ_9, ℓ_9'
$q_5 q_6 q_7^3$		1	1	1	2	2,4	-2	ℓ_{10}, ℓ_{10}'
$\bar{q}_6^2 q_7 \bar{q}_7^2$		1	1	1	0	2,4	6	ℓ_{11}, ℓ_{11}'

Chapter 6

CONCLUDING REMARKS

In the last chapter, we have presented several "realistic candidate" chiral preon models satisfying complementarity between the Higgs and confining phases. These models have common gauge group structures in that the preon representations of the models all come from the fundamental representations $\square$ and $\bar{\square}$ and the two-index symmetric-bar representation $\overline{\square\square}$. The different choices of $N_{\square}$, $N_{\bar{\square}}$ and $N_{\overline{\square\square}}$, which give different global metaflavor symmetries, distinguish the models. These models are rather unique for the following reasons.

Among non-supersymmetry chiral preon models, the simplest is one with a simple group and a single complex irreducible anomaly-free representation R_{MC}. Then the metacolor group must be SO(4N+2)(N $\geq$ 2) or E_6 since they are the only anomaly-free groups with complex representations. But for SO(4N+2)(N $\geq$ 2) groups,[58] th spinoral representations are the smallest complex representations. Using the spinoral representation, the asymptotic freedom condition in the metacolor sector restricts the number of metaflavors to $N_f < 11N \times 2^{4-2n}$. For 3-preon models, the representation R_{MC} is further restricted by the singlet condition $R_{MC} \times R_{MC} \times R_{MC} \supset 1$. These conditions together make it impossible to construct realistic SO(4N+2)(N $\geq$ 2) preon models. For the E_6 metacolor group, Gerard, Okamoto and Marshak[61] have shown that it is impossible to have a realistic model with a single preon represntation satisfying complementarity.

If we allow two or more preon representations, we can construct quarks and leptons out of preons confined by a SU(N) metacolor group.

If we limit ourselves to no more than two preon representations, it is easily seen that the Georgi-type models and our "realistic candidate" high composite scale models are the simplest models satisfying complementarity. Since the massless composite fermions of the Georgi-type $SU(N)_{MC} \times SU(N-4)_F \times U(1)_F$ models satisfying complementarity are ($\square\!\square$, N) under the unbroken global symmetry $SU(N-4)_F \times U(1)_F$ as shown in chapters 3 and 4; these massless composites cannot be identified as quarks and leptons for any N. Our high composite scale "realistic candidate" preon models are the only simplest chiral preon mdels satisfying the principle of complementarity between the Higgs and confining phases. Our "realistic candidate" preon models, both of high composite scale and of low compsite scale, have certain features in common. They predict four and three families of ordinary quarks and leptons while the family groups emerge naturally.

In this thesis, we have not touched on other crucial problems in preon model building. For example, we must explain the finite and rapid increasing masses of the observed three families of quarks and leptons (all small compared to Λ_{MC}). To try to use our "realistic candidate" models to generate finite masses with a reasonable family pattern is a much more difficult step since we are not able to calculate dynamically any of the relevant condensates--even in toy models.[66] Moreover, the problems of which subgroup of global metaflavor to gauge and how to break the family group are still unanswered. Clearly, some new ideas are needed before we can claim to have a truly realistic composite model of quarks and leptons.

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