PREDICTION OF CUBIC-FOOT VOLUME OF LOBLOLLY PINE TO ANY TOP DIAMETER LIMIT AND TO ANY POINT ON TREE BOLE

by

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CHAPTER ONE : INTRODUCTION

Total tree volumes are usually estimated using volume equations. These equations often predict tree volumes from diameter at breasr height (dbh) and either total or merchantable height. However, foresters are more interested in estimating merchantable volumes, that is, the content of tree boles from stump height to some fixed top diameter or to a point on the bole specified by either height to that point or a proportion of total height. Hence it is useful to have volume equations which estimate individual tree volumes to any specified standard of utilization expressed as stump height, top diameter, or section height.

Merchantable volume equations developed in the past often used dbh and total height as independent variables. When equations for different merchantable volumes are fitted independently, they often have the undesirable characteristic of producing volume lines that cross illogically within the range of the data. Consequently, inconsistent estimates are produced for different merchantable volumes of a single stem.

To eliminate this problem, volume equations to various top limits can be constrained to differ by a constant amount, so that all volume lines are parallel. This method assumes that top volume remains the same despite changes in tree size. Actually, for a specified small top diameter, top volume varies little from small to large trees. As the top limit increases, the change in top volume is more evident. Volume lines are not really parallel to one another; merchantable volume lines should approach but not cross the total volume line as tree size increases.

Scope of Study

Recent studies have attempted to define the mathematical relationship between dbh, total height, top diameter limit, and merchantable and total volumes. In the present study, different models employed in the past were reviewed; suitable models were used and some others were modified to ensure that predicted volumes to various top limits are logically related. All of the models chosen or modified were compared to determine the "best" models for predicting merchantable volumes to any desired top diameter limit and to any specified height or proportion of total height. In determining the "best" model in each case, both bias and precision were evaluated.

Literature Review

Standard merchantable volume tables list volumes for different combinations of dbh and height classes, each table representing a fixed stump height and top diameter(s). Specifications of merchantability may vary according to product and local cutting regulations; however, many tables do not reflect current utilization practices. Various techniques have been developed to overcome these difficulties. Hummel *et al.* (1951)

presented volume tables to a 3-inch top diameter and also expressed volumes to a 6- and 9-inch top as percentages of volume to a 3-inch top. The ratios between merchantable volumes to different top limits were assumed to be constant whereas they actually vary with tree size. Volume tables published by the British Columbia Forest Service (Browne 1962) also included conversion factors that could be applied to an estimate of total tree volume to obtain merchantable volumes to three degrees of utilization.

Schumacher and Hall (1933) suggested that their standard logarithmic volume equations could be modified to estimate merchantable volume to a fixed diameter top by substracting constants from each of the variables before converting the data into logarithmic form. Spurr (1952) criticized this method as being not fully valid since the corrections did not take into account the relationship between top volume, dbh, and height. He developed instead an equation that predicted merchantable volume to a 4-inch top diameter, based on the Australian total volume equation.

$$v_{4''} = b_1 + b_2D^2 + b_3H + b_4D^2H - b_5\left[\frac{H}{D-4}\right]$$

where $v_{4''}$ = merchantable volume to a 4-inch top limit,

H = total height,

D = dbh, and

b_i = regression coefficient to be estimated.

Beck (1963) in his study of yellow poplar stated that the merchantable volume equations using fixed diameter limits were related by an additive constant. In a study of loblolly pine volumes, Bailey and Clutter (1970) constrained the volume equations to various top limits to differ by a constant amount.

Honer (1964) predicted the ratio of merchantable to total volume from

$$v/V = b_1 + b_2X_i + b_3X_i^2$$

where V = total volume,

v = volume to height h or top diameter inside bark (ib) d,

X_1 = h/H,

X_2 = $(d/D_{ib})^2$, and

D_{ib} = dbh ib.

The volume ratio equation based on top diameter was adjusted later (Honer 1967) to reflect varying stump heights.

$$v'/V = b_1 + b_2X_3 + b_3X_3^2$$

where v' = merchantable volume to top diameter ib d,

X_3 = $(d/D_{ib})^2(H+h_1)/H$, and

h_1 = stump height.

Burkhart (1977) established a nonlinear volume ratio model of the form

$$v'/V' = 1 + b_1(d^{b_2}/D^{b_3})$$

where V' = total volume above stump.

Merchantable volumes (v') can be predicted by applying these ratios to total volume above the stump. Total volume was estimated with the combined variable equation.

Stem profile greatly affects the merchantable volumes, and studies on tree taper have produced many taper equations that, when integrated, can be used to predict merchantable volumes to various heights of the tree. Behre (1927) proposed a hyperbolic equation that related tree diameter (in percentage of dbh) to distance from tip to measuring point (in percentage of total height). Matte (1949) found that the stem profile above breast height could be described by the equation

$$y = x\sqrt{b_1 + b_2x + b_3x^2}$$

where $y = d/D_{ib}$,

$x = p/(H-4.5)$, and

p = distance from tip to measuring point.

Fries (1965) and Fries and Matérn (1965) proposed a multivariate approach to construct tree taper curves. Kozak and Smith (1966), after having analyzed the multivariate techniques, concluded that the simpler methods were best. A simple taper equation that predicted diameter at any given height from the ground was suggested by Munro (1966).

$$d = D\sqrt{b_1 + b_2h/(H-4.5)}$$

Later he introduced another taper equation (Munro 1968) :

$$d^2/D^2 = b_1 + b_2(h/H) + b_3(h/H)^2$$

Addition of terms up to a fifth-degree polynomial in (h/H) improved the prediction of the taper equation. Kozak et al. (1969) conditioned Munro's (1968) equation so that predicted diameter equaled zero at the tree tip and obtained a new form

$$d^2/D^2 = b_1(h/H - 1) + b_2(h^2/H^2 - 1)$$

Bruce et al. (1968) derived an estimating equation expressing ratio of squared diameter ib to squared dbh as a function of dbh, total height, and the 3/2, 3rd, 32nd, and 40th powers of relative height. Bennett and Swindle (1972) used taper equations consisting of third-degree polynomials. A nonlinear taper equation was developed by Ormerod (1971)

$$d = D_k \left[\frac{H-h}{H-k}\right]^{b_1}$$

where D_k is diameter measured at height k.

Bruce (1972) described a set of equations that permit flexible transformation of Behre's equation (1927) from one top diameter to another. In a series of papers, Demaerschalk (1971, 1972, 1973) developed a compatible taper-volume system that, by integration of the taper equation, produced identical estimation of total volumes to those given by existing volume equations. Compatible taper equations were derived from the combined variable, logarithmic, Honer's volume equations and from the volume over basal area equation.

Goulding and Murray (1976) suggested a compatible taper equation of the form

$$d^2 = \frac{V}{0.00545415\ H} f(p/H)$$

The results obtained showed that at the 95% level f(p/H) was a fifth-degree polynomial in (p/H) such that the regression coefficients satisfied

$$\sum_{i=0}^{5} \frac{b_i}{i+1} = 1$$

Tree form is complex and this makes it difficult to describe stem profiles by a single model. Ormerod (1973) modified his previous nonlinear taper equation (1971) and produced a step function that provided a better description of the bole by dividing it into sections.

$$d = (D_i - C_i)\left[\frac{H_i - h}{H_i - k_i}\right]^{b_i} + C_i$$

where H_i = height to top of section i,

C_i = section diameter intercept,

D_i = section diameter measured at height k_i, and

b_i = fitted exponent on the closed interval $\left[h=H_{i-1},\ h=H_i\right]$.

Three submodels describing three sections of tree bole were joined to form a segmented polynomial regression model by Max and Burkhart (1976). This model was found to be superior to the simple quadratic model presented by Kozak et al. (1969) in describing tree taper.

Demaerschalk and Kozak (1977) developed a system comprising two conditioned taper equations. The system was so complicated that one regression coefficient of the bottom model had to be determined for each tree by an iteration procedure. Furthermore, volumes involving the top section of the tree are obtained through numerical integration. The complexity of the system is the main drawback that makes it impractical.

Merchantable length is the entire length of the bole from the stump to a predetermined top diameter limit. A taper equation, upon transformation, can predict changes in merchantable length as top diameter varies. Gideon and Faurot (1977) utilized a modified logistic growth function which estimated the length directly rather than a geometric expression of stem form.

CHAPTER TWO : PROCEDURES AND METHODS

Data

Data from trees felled on plots in loblolly pine plantations and natural stands were used in this study. Sample tree data from plantations were obtained in the Virginia Piedmont and Coastal Plain and the Coastal Plain of Delaware, Maryland, and North Carolina. Natural-stand sample trees were from the Virginia Piedmont and Coastal Plain and the Coastal Plain of North Carolina.

These trees were felled and cut into 4-foot sections. Stump heights were not measured but all were about 0.5 foot and a constant height of 0.5 foot was assumed. Tree dbh and total height were measured, together with diameters inside bark (ib) and outside bark (ob) at the stump and at 4-foot intervals up the stem to an approximate 2-inch top diameter ob. Individual tree volumes were determined using Smalian's formula for each 4-foot and then summing the volumes of the appropriate sections. The top was assumed to be a cone and the stump a cylinder for purposes of computing their volumes.

There were 427 sample trees varying from 3 to 12 inches in dbh and from 15 to 90 feet in total height from plantations, and 209 sample trees ranging from 5 to 14 inches in dbh and from 20 to 90 feet in total height from natural stands. Twenty-five per cent of the sample trees were selected at random from each 1-inch dbh class. These trees, which constitute an independent data set, were considered representative of the original data and were withheld for testing purposes. The remaining trees for fitting of the models are referred to as the sample data.

Notation

The following notation will be used in the subsequent models.

a_i, b_i = regression coefficients to be estimated from sample data,

D = dbh in inches,

d = top diameter (ib or ob) in inches at height h,

H = total height in feet,

h = height above the ground to top diameter d, or height to limit of utilization,

h_1 = lower limit of integration with respect to h, or stump height (0.5 foot in this study),

h_2 = upper limit of integration with respect to h, either height above the ground to the limit of utilization or computed height to top diameter d,

$K = \pi / \left[4(144)\right] = 0.00545415$,

p = distance from the tip to top diameter d or to limit of utilization, $p = H-h$ and $p_2 = H-h_2$,

p_1 = distance from the tip to the stump, $p_1 = H-h_1$,

V = total cubic-foot volume above the ground (ib or ob),

V' = total cubic-foot volume above stump (ib or ob),

v = cubic-foot volume from the ground to some top diameter limit or height,

v' = merchantable cubic-foot volume (above stump) to some top diameter or height limit,

$x = h/H$, $x_1 = h_1/H$, and $x_2 = h_2/H$,

$z = p/H$, $z_1 = p_1/H$, and $z_2 = p_2/H$.

Models

Models employed in the comparison include some used in the past as well as other modified models. All of these models should provide logically-related estimation of merchantable volumes to various top diameter or height limits. Independent variables measured on standing trees were restricted to dbh and total height. Limits of utilization include stump height and either top diameter or height to the top bolt.

Models were classified into two categories : (1) Category a -- models that predict merchantable volume to any point on the tree bole specified by either distance from tip or height above the ground, and (2) Category b -- models that give merchantable volumes to predetermined top diameter limits. Merchantable volumes may be predicted using two different approaches. Integration of taper equations is a logical way to obtain estimates of merchantable volumes or volumes of certain segments of the bole, the limits of integration being either limits of utilization or heights to the desired top and bottom. On the other hand, volume ratio models provide estimation of the ratios of merchantable to total volume. Merchantable volumes are obtained in turn as products of these ratios and total volume; the latter is estimated using a total volume equation. After evaluating a number of total volume models, Burkhart (1977) concluded that the simple combined variable equation ($V = b_1' + b_2'D^2H$) performed as well as, if not better than any of the alternatives compared. Thus this equation is employed in the present study to provide estimates of total stem volumes.

Volume Ratio Models

Model 1a (Honer).

Honer (1967) expressed the ratio $R=v/V$ as a function of (h/H)

$$R = b_0 + b_1 x + b_2 x^2$$

It is necessary to condition the above equation so that R is 1 when h equals H. This results in

$$b_0 + b_1 + b_2 = 1$$

Substituting this constraint for the coefficients in the original equation and factoring similar terms yields the final model

$$\text{(1a)} \qquad (R-1) = b_1(x-1) + b_2(x^2-1)$$

Volume to height h is given by

$$v = RV$$

where total volume is estimated from $V = b_1' + b_2'D^2H$.

Model 1b (Honer).

The ratio of merchantable volume (above stump) to total volume, $R = v'/V$, is predicted from top diameter limit, dbh, total height, and stump height.

$$R = b_1 + b_2X + b_3X^2 \tag{1b}$$

where $X = (d^2/D^2)(1 + h_1/H)$.

Merchantable volume to top diameter d is estimated from $v' = RV$, $(V = b_1' + b_2'D^2H)$.

Model 2b (Burkhart).

Burkhart (1977) used the following nonlinear model to relate $R' = v'/V'$ to top diameter and dbh :

$$R' = 1 + b_1(d^{b_2}/D^{b_3}) \tag{2b}$$

Model 2a (Modified Burkhart).

A similar nonlinear model was developed to predict R' from p and H, instead of d and D.

$$R' = 1 + b_1(p^{b_2}/H^{b_3}) \tag{2a}$$

Model 3a (Modified Burkhart weighted by $1/p^2$).

The weights $1/p^2$ were applied to model 2a because of heterogeneity of variance and with the hope that more reasonable predicted volumes near the tip could be obtained.

$$(R'-1)/p = b_1p^{b_2}/H^{b_3} \tag{3a}$$

Model 3b (Burkhart weighted by $1/d^2$).

Weights of $1/d^2$ were applied with the purpose of improving estimated values for smaller top diameters.

$$(R'-1)/d = b_1 d^{b_2}/D^{b_3} \qquad (3b)$$

In models 2 and 3, the volume ratio R' may be used to convert total volumes above stump (estimated from $V' = b_3' + b_4' D^2 H$) to merchantable volumes to various points on the bole specified by either height or top diameter limits.

Model 4a (Sixth-degree polynomial volume ratio model).

Munro (1968) found that his taper equation had good results when (d^2/D^2) was fitted by a fifth-degree polynomial in (p/H). Goulding and Murray (1976) employed a fifth-degree polynomial in (p/H) in their compatible taper equation. Integration of both models yields volume as a function of terms up to a sixth-degree polynomial in either (h/H) or (p/H).

Based on these findings the author attempted to express $R = v/V$ in terms of a sixth-degree polynomial in (p/H), that is:

$$R = \sum_{i=0}^{6} (b_i z^i)$$

The equation was conditioned so that R=1 when p=0 and R=0 when p=H.

$$\sum_{i=0}^{6} b_i = 0 \quad \text{and} \quad b_0 = 1$$

This volume ratio model becomes

$$(4a) \qquad (R+z-1) = b_2(z^2-z) + b_3(z^3-z) + b_4(z^4-z) + b_5(z^5-z) + b_6(z^6-z)$$

Model 4b (Sixth-degree polynomial volume ratio model).

A second sixth-degree polynomial volume ratio model was developed, using (d/D) instead of (p/H).

$$R = \sum_{i=0}^{6} (b_i X^i)$$

where $X = d/D$.

As before, the equation should be conditioned in order that volume estimates are logically related. The constraint that R equal 1 when d is 0 results in $b_0 = 1$. The final model is as follows

$$(4b) \qquad (R-1) = b_1X + b_2X^2 + b_3X^3 + b_4X^4 + b_5X^5 + b_6X^6$$

Again, volume to height h or top diameter d is given by $v = RV$, where $V = b_1' + b_2'D^2H$.

Integration of Taper Equations

In this approach, a taper equation is fitted on sample data and then integrated to provide volume of any desired section of the bole.

Model 5 (Kozak *et al.*).

The quadratic model proposed by Kozak *et al.* (1969) was employed to describe tree taper.

$$d^2/D^2 = b_1(x-1) + b_2(x^2-1) \tag{5}$$

Merchantable volume to a given point is obtained by integration of the equation from the stump to that point.

$$v = KD^2H \left[b_2x^3/3 + b_1x^2/2 - (b_1+b_2)x \right]_{x_1}^{x_2} \tag{5'}$$

If the top limit is specified by height h_2 from the ground, $x_2 = h_2/H$. In case of a top diameter limit, x_2 may be computed from equation (5)

$$x_2 = \frac{1}{2b_2} \left[-b_1 \pm \sqrt{b_1^2 + 4b_2(b_1 + b_2 + d^2/D^2)} \right], \quad 0 \leq x_2 \leq 1.$$

<u>Model 6</u> (Ormerod).

The simple nonlinear taper equation by Ormerod (1971) was considered.

$$d = D_k \left[\frac{p}{H-k} \right]^{b_1}$$

Height k in the original equation was standardized at 4.5 feet. This model is conditioned so that $d = D_k$ when $p = H-4.5$. Therefore D_k is dbh if the taper equation is for diameter outside bark. In case of diameter inside bark, D_k becomes dbh inside bark, which can be estimated from dbh outside bark using a linear equation. The final model is

(6) $$d = D_{4.5}\left[\frac{p}{H-4.5}\right]^{b_1}$$

where $D_{4.5} = D = \text{dbh}$, if d is diameter ob,

$= D_{ib} = \text{dbh ib}$, if d is diameter ib, and

$D_{ib} = b_5' + b_6'D$, b_5' and b_6' are regression coefficients from $dib = b_5' + b_6'\ dob$.

Integration of the taper equation provides merchantable volume

(6') $$v = \frac{KD_{4.5}^2}{(2b_1+1)(H-4.5)^{2b_1}}\left[p^{2b_1+1}\right]_{p_2}^{p_1}$$

If merchantable volume to a top diameter d is desired, p_2 needs be calculated from the taper equation.

$$p_2 = (H-4.5)(d/D_{4.5})^{1/b_1}\ ,\ 0 \leqslant p_2 \leqslant H\ .$$

Model 7a (Demaerschalk).

Demaerschalk (1973) derived different sets of functions from five volume equations. Among these compatible taper equations, the one derived from the combined variable equation was chosen in order to be consistent with other models evaluated in this study.

(7a) $$\frac{d^2}{D^2} = \frac{b_1'(b_1+1)}{KD^2H}z^{b_1} + \frac{b_2'(b_2+1)}{K}z^{b_2}$$

where b_1 and b_2 are "free" parameters to be estimated,

b_1' and b_2' are regression coefficients from the combined variable function $V = b_1' + b_2'D^2H$.

Since there is no easy way to compute z_2 for a given top diameter limit, this taper equation can give only merchantable volumes to various heights when integrated.

(7a')
$$v = \left[b_1' z^{b_1+1} + b_2' D^2 H \, z^{b_2+1} \right]_{z_2}^{z_1}$$

Model 8 (Modified Demaerschalk).

Demaerschalk's above taper equation was modified to contain only one "free" parameter, thus circumventing the difficulty of computing z_2 (relative distance from tip to top diameter limit) in the original model.

(8)
$$\frac{d^2}{D^2} = \left[\frac{b_1+1}{K}\right] \left[\frac{b_1'}{D^2H} + b_2'\right] z^{b_1}$$

Merchantable volume is again derived from the taper equation by integration.

(8')
$$v = (b_1' + b_2'D^2H) \left[z^{b_1+1} \right]_{z_2}^{z_1}$$

In case of merchantable volume to a top diameter d, z_2 is given from

$$z_2 = \left[\frac{Kd^2H}{(b_1+1)(b_1'+b_2'D^2H)} \right]^{1/b_1}, \quad 0 \leqslant z_2 \leqslant 1 \ .$$

Model 9a (Goulding-Murray).

Goulding and Murray (1976) proposed a polynomial compatible taper equation of the form

$$d^2 = \frac{V}{KH} \sum_{i=1}^{n} (b_i z^i)$$

where the coefficients of the polynomial satisfy

$$\sum_{i=1}^{n} \frac{b_i}{i+1} = 1$$

Their study showed that a fifth-degree polynomial was significant at the 95% level. Imposing the above condition on this equation results in

$$(d^2KH/V - 2z) = b_1(3z^2-2z) + b_2(4z^3-2z) + b_3(5z^4-2z) + b_4(6z^5-2z) \tag{9a}$$

Merchantable volume is then

$$v = EV \left[(1-b_1-b_2-b_3-b_4)z^2 + b_1 z^3 + b_2 z^4 + b_3 z^5 + b_4 z^6\right]_{z_2}^{z_1} \tag{9a'}$$

where EV is estimated total volume from $EV = b_1' + b_2'D^2H$.

Model 10 (Modified Goulding-Murray).

Goulding-Murray's compatible polynomial taper equation was simplified to become a quadratic model so that z_2 may be easily computed for a given top diameter d.

$$d^2 = \frac{V}{KH} (b_{01} z + b_{02} z^2)$$

In order that the taper equation is compatible, the condition $b_{01}/2 + b_{02}/3 = 1$ must be satisfied. The final model is

$$(10) \qquad (d^2KH/V - 2z) = b_1(3z^2-2z)$$

Merchantable volume is obtained by integration of equation (10).

$$(10') \qquad v = EV \left[(1-b_1)z^2 + b_1 z^3 \right]_{z_2}^{z_1}$$

where $EV = b_1' + b_2'D^2H$,

$z_2 = (H-h_2)/H$, for top height limit, or

$$z_2 = \frac{1}{3b_1}\left[(b_1-1) \pm \sqrt{(b_1-1)^2 + 3b_1 d^2KH/V} \right], \; 0 \leq z_2 \leq 1,$$

for top diameter limit.

Model 11 (Max-Burkhart).

The segmented taper equation developed by Max and Burkhart (1976) was used. As seen in Fig. 1, the tree bole is divided into three sections, each of which is fitted with a different submodel of the form of Kozak et al.'s taper equation (model 5). These models are then grafted together using the segmented polynomial regression technique described by Fuller (1969), Gallant and Fuller (1973), and Gallant (1974).

$$(11) \qquad d^2/D^2 = b_1(x-1) + b_2(x^2-1) + b_3(a_1-x)^2I_1 + b_4(a_2-x)^2I_2$$

where a_1 and a_2 are two join points to be estimated,

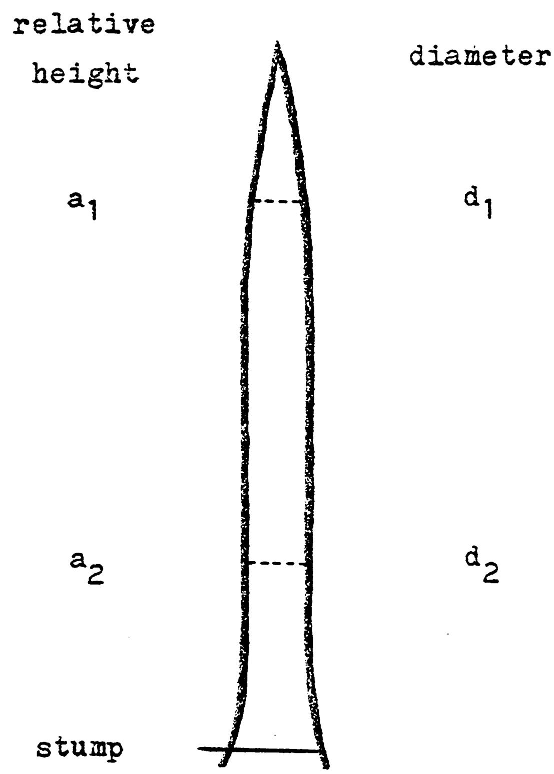

Fig. 1. Three sections of tree bole.

$$I_i = 1, \quad x \le a_i, \qquad i=1,2$$
$$\quad = 0, \quad x > a_i .$$

The diameters at the join points can be obtained from equation (11).

$$x = a_1, \quad d_1 = D\sqrt{b_1(a_1-1) + b_2(a_1^2-1)}$$

$$x = a_2, \quad d_2 = D\sqrt{b_1(a_2-1) + b_2(a_2^2-1) + b_3(a_1-a_2)^2}$$

Integration of the taper equation gives merchantable volume

$$(11') \qquad v = KD^2H\left[\frac{b_2}{3}x^3 + \frac{b_1}{2}x^2 - (b_1+b_2)x - \frac{b_3}{3}(a_1-x)^3 I_1 - \frac{b_4}{3}(a_2-x)^3 I_2\right]_{x_1}^{x_2}$$

If merchantable volume to top diameter d is desired, x_2 must be computed from an equation of the form

$$Ax_2^2 + Bx_2 + C = 0$$

where $A = b_2 + I_1'b_3 + I_2'b_4$,

$B = b_1 - 2I_1'a_1b_3 - 2I_2'a_2b_4$,

$C = -b_1 - b_2 - d^2/D^2 + I_1'a_1^2b_3 + I_2'a_2^2b_4$,

$$I_i' = 1, \quad d \ge d_i, \qquad i=1,2$$
$$\quad = 0, \quad d < d_i .$$

$$x_2 = \frac{1}{2A}\left(-B \pm \sqrt{B^2-4AC}\right), \quad 0 \le x_2 \le 1 .$$

<u>Model</u> <u>12</u> (Segmented polynomial model).

Similar to Max and Burkhart's taper equation, this quadratic-quadratic-quadratic model consists of three submodels grafted at two join points by the same segmented polynomial regression technique. Each section of the bole is described by modified Goulding and Murray taper equation (model 10) instead of by the Kozak <u>et</u> <u>al</u>. equation as in the previous model. Gallant and Fuller (1973) developed a method by which simple polynomial models can be written directly in reparameterized form. In Appendix A this rather complex model is derived by the same method to ensure that the correct model is obtained. The model, written in reparameterized form, is

$$(12) \qquad (d^2KH/V - 2z) = b_1(3z^2-2z) + b_2(z-a_1)^2I_1 + b_3(z-a_2)^2I_2$$

where $I_i = 1$ if $z \geq a_i$, $\quad i=1,2$
$\qquad = 0$ if $z < a_i$.

a_1 and a_2 are two join points to be fitted from the sample data.

The diameters at the join points are derived from the taper equation when z equals in turn a_1 and a_2.

$$d_1 = \sqrt{V/KH\left[3b_1a_1^2 + 2a_1(1-b_1)\right]}$$

and $$d_2 = \sqrt{V/KH\left[3b_1a_2^2 + 2a_2(1-b_1) + b_2(a_2-a_1)^2\right]}$$

Integration of equation (12) provides merchantable volume to some top limit

$$(12') \quad v = V\left[b_1z^3 + (1-b_1)z^2 + b_2(z-a_1)^3I_1/3 + b_3(z-a_2)^3I_2/3\right]_{z_2}^{z_1}$$

In case of a top diameter limit d, z_2 is a solution of the following equation

$$Az_2^2 + Bz_2 + C = 0$$

where $A = 3b_1 + I_1'b_2 + I_2'b_3$,

$B = 2(1 - b_1 - I_1'a_1b_2 - I_2'a_2b_3)$,

$C = I_1'a_1^2b_2 + I_2'a_2^2b_3 - d^2KH/V$,

$$I_i' = \begin{cases} 1, & d \geq d_i, \\ 0, & d < d_i. \end{cases} \qquad i=1,2$$

$$z_2 = \frac{1}{2A}(-B \pm \sqrt{B^2 - 4AC}), \quad 0 \leq z_2 \leq 1 .$$

Testing of Accuracy and Precision

All of the models were evaluated to determine the "best" choice for predicting merchantable volume to various top diameter limits and the "best" alternative for predicting merchantable volume to various heights of the tree. In addition, the taper equations were compared to determine which models best described tree taper.

Merchantable volumes outside bark were computed at ten-percent intervals of total height. These actual volumes were compared to predicted volumes from each of the models. Predicted diameters outside bark at these points on tree boles were also compared to actual diameters. Identical procedures were carried out for volumes and diameters inside bark.

Let D_i and D_i' be the differences of the two volumes and two diameters, respectively.

$$D_i = V_{i\ \text{actual}} - V_{i\ \text{predicted}}$$

and

$$D_i' = d_{i\ \text{actual}} - d_{i\ \text{predicted}}$$

where $i = 1,2,\ldots,N,$

N = number of pairs of volumes or diameters compared for each model,

V_i = the ith merchantable volume ob or ib,

d_i = the ith diameter ob or ib.

The following three criteria were employed to evaluate the models in terms of merchantable volumes.

Bias :

$$\overline{D} = \frac{1}{N}\sum_{i=1}^{N} D_i$$

Mean absolute deviation :

$$\overline{|D|} = \frac{1}{N}\sum_{i=1}^{N} |D_i|$$

Standard error of the differences :

$$s_D = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (D_i - \bar{D})^2} = \sqrt{\frac{1}{N-1} \left[\sum_{i=1}^{N} D_i^2 - (\sum_{i=1}^{N} D_i)^2/N \right]}$$

The same criteria were applied to the comparison of taper equations with the substitution of D_i' for D_i in the above formulae.

The first part of the model comparison was to determine how well each of the models fits the <u>sample data</u>. The second step was to compare how well the models predict for the <u>population</u> which is represented by the independent data already withheld for this purpose. Consequently, a total of four data sets were used in the comparison : (1) plantations sample data, (2) natural stands sample data, (3) plantations independent data, and (4) natural stands independent data.

For each data set, biases of all models were computed and compared to one another. A rank was then assigned to each model; rank number one corresponded to the "best" model which had the lowest absolute value of bias. The same procedures were repeated for the other two criteria -- mean absolute deviation and standard error of the differences. The lower these two values, the better the model and thus the lower the ranks. A total of the three ranks for each model demonstrates how well it behaves as compared with other models.

CHAPTER THREE : RESULTS AND DISCUSSION

Regression techniques were employed to fit the outside and inside bark models to the sample data from plantations and from natural stands. Coefficients for predicting total volumes above the ground and above stump are presented in Tables 1 and 2, respectively. Table 3 shows the coefficients in the simple linear equations that estimate diameter at breast height inside bark from diameter at breast height outside bark. Parameter estimates for the volume ratio and taper equation models are displayed in Tables 4 and 5.

Values of bias, mean absolute deviation, and standard error of the differences, as well as their ranks for each model, may be found in Appendix B. Tables 6.1, 6.2, and 6.3 summarize these results by showing the overall ranks of the models. Model 2a (Modified Burkhart) is ranked "best" among twelve models predicting merchantable volumes to various heights whereas model 12 (segmented polynomial model) proves to be the "best" among ten models to estimate volumes to different top diameter limits. The comparison of seven taper equations shows that model 11 (Max-Burkhart) "best" predicts tree diameters at various points on the bole.

In general, the results are similar for different data sets. The ranks of the models do not differ much from plantations to natural stands for both sample data and independent data sets. On the other hand, the relative ranks of models predicting

Table 1. Regression coefficients for estimating total volumes above the ground outside and inside bark of loblolly pine trees from different sources $(V = b_1' + b_2'D^2H)$.

Source	b_1'	b_2'
Plantation-grown trees		
Outside bark	0.397655	0.002387
Inside bark	0.149600	0.001887
Naturally-grown trees		
Outside bark	0.576771	0.002510
Inside bark	-0.246556	0.002140

Table 2. Coefficients for estimating total volumes above stump outside and inside bark ($V' = b_3' + b_4'D^2H$)

Source	b_3'	b_4'
Plantation-grown trees		
Outside bark	0.326874	0.002330
Inside bark	0.106759	0.001844
Naturally-grown trees		
Outside bark	0.437955	0.002463
Inside bark	-0.331073	0.002104

Table 3. Parameter estimates for predicting diameter at breast height inside bark from diameter at breast height outside bark (dib = $b_5' + b_6'$ dob).

Source	b_5'	b_6'
Plantations	0.150013	0.837482
Natural stands	0.211908	0.853269

Table 4.1. Coefficients of outside bark models fitted to sample data from plantations.

Model	b_1	b_2	b_3	b_4	b_5 (a_1)	b_6 (a_2)
1a	2.012202	-1.114996				
1b	1.044419	-0.651680	-0.037775			
2a	-0.721225	2.466685	2.389190			
2b	-0.350339	2.997422	2.598475			
3a	-0.690652	1.567574	2.470999			
3b	-0.312503	2.142370	2.669479			
4a		-0.394007	0.510228	-5.225757	7.860563	-3.764329
4b	0.065846	-1.176716	5.521988	-11.927296	9.060787	-2.231897
5	-2.951714	1.474172				
6	0.782063					
7a	0.023094	2.359977				
8	1.961174					
9a	-7.847354	21.576563	-23.716711	9.358150		
10	0.971623					
11	-3.511893	1.644598	-1.151284	49.684312	0.803555	0.120703
12	0.573201	-0.422506	149.109975		0.262901	0.902561

Table 4.2. Coefficients of inside bark models fitted to sample data from plantations.

Model	b_1	b_2	b_3	b_4	b_5 (a_1)	b_6 (a_2)
1a	2.062945	-1.132486				
1b	1.031473	-0.684694	-0.395270			
2a	-0.736785	2.392042	2.313887			
2b	-0.856533	3.258128	3.041093			
3a	-0.720564	1.517269	2.423779			
3b	-0.916980	2.403970	3.202241			
4a		-0.382811	0.400642	-5.404655	8.195029	-3.822529
4b	0.895249	-10.018726	39.523158	-72.057779	56.289604	-15.576732
5	-1.822844	0.813285				
6	0.714332					
7a	-0.260226	1.887507				
8	1.714812					
9a	-7.693594	21.842831	-24.249739	9.514920		
10	0.756240					
11	-2.566028	1.193330	-1.204713	40.396194	0.707480	0.109560
12	0.445060	-1.384433	141.048335		0.527785	0.903034

Table 5.1. Coefficients of outside bark models fitted to sample data from natural stands.

Model	b_1	b_2	b_3	b_4	b_5 (a_1)	b_6 (a_2)
1a	1.963013	-1.050972				
1b	1.096218	-0.881685	0.093655			
2a	-0.550269	2.268871	2.133744			
2b	-0.423985	2.721949	2.447156			
3a	-0.525456	1.346947	2.195245			
3b	-0.502613	2.110716	2.890919			
4a		-0.419301	-0.073852	-4.436686	8.096890	-4.185694
4b	-0.310797	1.144112	1.301775	-9.597877	9.273504	-2.568687
5	-2.644718	1.245303				
6	0.686081					
7a	18.836281	1.328388				
8	1.898565					
9a	-11.231219	30.685269	-34.139192	13.562324		
10	0.847583					
11	-2.460230	0.975057	-0.704371	131.484230	0.802554	0.086667
12	0.388314	-0.592892	226.512284		0.304622	0.907050

Table 5.2. Coefficients of inside bark models fitted to sample data from natural stands.

Model	b_1	b_2	b_3	b_4	b_5 (a_1)	b_6 (a_2)
1a	2.014661	-1.066311				
1b	1.016628	-0.440418	-0.824273			
2a	-0.650676	2.200890	2.099639			
2b	-1.545017	3.696713	3.708773			
3a	-0.625364	1.306444	2.188251			
3b	-1.602608	2.793788	3.810633			
4a		-0.335124	-0.696014	-3.326899	7.020436	-3.678812
4b	0.964471	-11.592252	47.957411	-88.767111	69.935624	-19.496783
5	-1.643459	0.654330				
6	0.633201					
7a	-1.103280	1.513583				
8	1.584941					
9a	-9.537819	27.540025	-31.399483	12.517565		
10	0.594789					
11	-4.341248	2.119558	-2.368180	87.482034	0.797441	0.088320
12	0.240303	-1.361376	250.256777		0.508715	0.915870

volumes to top height limits are different from those of models estimating volumes to various top diameters. If the model is in the form of a taper equation, merchantable volume to a given top diameter is obtained by first computing height to that point from the same taper equation and then using this value as one of the two limits of integration. Since additional bias is gained through estimation of height from top diameter, the difference between actual and predicted volumes is generally greater for volumes to top diameter limits than for volumes to given heights. A closer look at the models appears to be necessary in order that one can understand why these models behave as they do.

Model 1 (Honer) is a volume ratio model and model 5 (Kozak et al.) is basically a taper equation. However, both are simple quadratic models and neither performs well in estimating merchantable volumes in comparison with other models. Model 5 also fails to accurately estimate tree diameters. The results indicate that a simple quadratic equation apparently cannot adequately describe tree taper or predict volumes.

Model 7 (Demaerschalk) and model 8 (Modified Demaerschalk) are nonlinear compatible taper equations. Neither ranked well in predicting either volumes or diameters. Simplification of model 7 by omitting one "free" parameter results in model 8, which does not perform as well for estimation of merchantable volumes as does model 7. On the other hand, model 8 is the "worst" model among eight taper equations evaluated to describe

Table 6.1. Overall ranks of the models to predict merchantable volumes to various heights

Model	Rank total outside bark					Rank total inside bark					Rank sum	Overall rank
	(1)	(2)	(3)	(4)	Total	(1)	(2)	(3)	(4)	Total		
1a	33	36	33	32	134	34	36	28	23	121	255	12
2a	4	5	3	6	18	6	11	6	12	35	53	1
3a	8	7	6	6	27	11	17	11	16	55	82	2
4a	11	6	16	16	49	9	8	12	14	43	92	3
5	34	33	34	34	135	34	32	25	26	117	252	11
6	17	23	18	20	78	22	24	25	11	82	160	7
7a	26	23	13	12	74	29	23	34	34	120	194	8
8	23	18	31	31	103	26	23	32	33	114	217	10
9a	13	16	16	15	60	10	11	8	13	42	102	4
10	24	19	28	28	99	23	22	26	27	98	197	9
11	26	24	20	21	91	18	8	13	9	48	139	6
12	15	24	16	13	68	12	19	14	16	61	129	5

(1) = Plantations sample data,

(2) = Plantations independent data,

(3) = Natural stands sample data,

(4) = Natural stands independent data,

Rank total = sum of ranks over the three criteria for each data set,

Rank sum = sum of rank totals from different data sets, ob and ib,

Overall ranks = final ranks of the models based on values of their rank sums.

Table 6.2. Overall ranks of the models to predict merchantable volumes to various top diameters.

Model	Rank total outside bark					Rank total inside bark					Rank sum	Overall rank
	(1)	(2)	(3)	(4)	Total	(1)	(2)	(3)	(4)	Total		
1b	19	23	21	22	85	21	21	19	21	82	167	7
2b	5	9	18	13	45	8	12	12	13	45	90	4
3b	9	17	5	5	26	9	17	10	15	51	77	3
4b	12	3	10	10	35	10	10	6	10	36	71	2
5	30	30	30	30	120	30	29	25	25	109	229	10
6	12	12	12	15	51	17	18	18	8	61	112	6
8	25	22	27	27	101	27	22	30	30	109	210	9
10	22	21	24	23	90	24	20	25	26	95	185	8
11	23	19	12	14	68	12	8	12	7	39	107	5
12	8	9	6	6	29	7	8	8	10	33	62	1

(1) = Plantations sample data,

(2) = Plantations independent data,

(3) = Natural stands sample data,

(4) = Natural stands independent data,

Rank total = sum of ranks over the three criteria for each data set,

Rank sum = sum of rank totals from different data sets, ob and ib,

Overall ranks = final ranks of the models based on values of their rank sums.

Table 6.3. Overall ranks of the models to predict tree diameters.

Model	Rank total outside bark					Rank total inside bark					Rank sum	Overall rank
	(1)	(2)	(3)	(4)	Total	(1)	(2)	(3)	(4)	Total		
5	20	17	14	15	66	12	13	15	15	55	121	6
6	6	7	6	7	26	12	15	12	11	50	76	2
7	19	20	11	11	61	20	20	22	22	84	145	8
8	14	12	24	20	70	10	7	16	16	49	119	5
9	16	18	13	13	60	20	21	12	12	65	125	7
10	11	11	15	14	51	15	12	15	15	57	108	4
11	10	11	11	12	44	5	5	5	5	20	64	1
12	12	12	14	16	54	14	15	11	12	52	106	3

(1) = Plantations sample data,

(2) = Plantations independent data,

(3) = Natural stands sample data,

(4) = Natural stands independent data,

Rank total = sum of ranks over the three criteria for each data set,

Rank sum = sum of rank totals from different data sets, ob and ib,

Overall ranks = final ranks of the models based on values of their rank sums.

tree taper.

Model 9 (Goulding-Murray) and model 10 (Modified Goulding-Murray) are also compatible taper equations but in the form of a polynomial. Model 9, consisting of a higher degree polynomial, predicts very well merchantable volumes but predicts poorly tree diameters (ranked seventh among eight taper equations).

The fact that the two complex, original compatible taper equations introduced by Demaerschalk (1973) and Goulding and Murray (1976) do not give good results in predicting tree diameters raises a question as to whether or not this type of taper equation should be employed if the sole purpose is to describe tree taper. This study shows that a complex compatible taper equation predicts merchantable volumes well but works poorly on estimating stem diameters.

Model 6 (Ormerod) proves to be a good nonlinear taper equation (ranked second). A closer look at the ranks reveals that this model predicts diameters outside bark better than diameters inside bark. In case of diameter inside bark, dbh ib in the model has to be estimated from dbh ob, causing some loss in both accuracy and precision. Integration of the taper equation, however, does not provide very good estimates of merchantable volumes.

Model 11 (Max-Burkhart) is the "best" model to estimate tree diameters at various points on the bole. The superior predictive ability of this taper equation is due mostly to its increased flexibility. The use of three quadratic submodels to describe stem taper of three sections of the bole helps account for butt swell often underestimated by a single taper equation no matter how complicated it is. The drawback of this model is that it is not compatible. Despite its excellent performance as a taper equation, integration of model 11 over the entire bole does not give total volume and due to this inconsistency the prediction of merchantable volumes to various top limits is not as good as may be expected.

Model 12 is a segmented polynomial taper equation consisting of three submodels, each in the form of model 10 which is itself a compatible quadratic taper equation. Integrating model 12 over the entire bole, $z_1 = 1$ (ground) and $z_2 = 0$ (tree tip), estimates total tree volume

$$V = EV\left[1 + b_2(1-a_1)^3/3 + b_3(1-a_2)^3/3\right]$$

or $$V = (EV)(C)$$

where EV is estimated total volume from $EV = b_1' + b_2'D^2H$.

Table 7 shows that C is very close to 1 for both volumes outside and inside bark of trees from plantations and from natural stands. The relationship between regression coefficients suggests that model 12 may be considered as a compatible taper equation. This is possibly the main difference between models 11 and 12, even though they are both segmented taper equations. Model 12 consistently provides better volume estimates than does

Table 7. Values of C from equation (13) for volumes outside bark and inside bark of loblolly pines from different sources

	Plantations	Natural stands
Outside bark	0.9896	0.9942
Inside bark	0.9943	0.9959

model 11 when integrated. In fact, it was ranked first in predicting merchantable volumes to top diameter limits.

Some precision in estimating diameters is sacrificed in order that the taper equation is compatible; this characteristic is shared by model 12 with other compatible taper equations. However, the presence of three submodels certainly helps to better describe stem taper. Model 12 was ranked only after models 11 (Max-Burkhart) and 6 (Ormerod) in predicting diameters.

All volume ratio models considered (2, 3, and 4), except model 1, yielded very good results when predicting merchantable volumes, probably because they are fitted directly from volume data whereas the taper equations have to be integrated to provide volume estimates. Models 2 and 3 predict R', ratio of merchantable volume to total volume above stump. It is assumed that stump heights do not vary much and may be considered constant. Model 2a (Modified Burkhart) was ranked first among other models to predict merchantable volumes to various height limits. Model 3a which is model 2a weighted by $1/p^2$ was ranked second. This suggests that weighting of model 2a does not help in the attempt to improve the prediction of merchantable volumes. In estimating volumes to top diameters, model 3b (model 2b weighted by $1/d^2$) was ranked third among ten models. The weighting in this case really improves volume estimates for smaller top diameters and the overall result is the superior predictive ability of model 3b as compared with that of the original model 2b (Burkhart).

Other weights were also applied, $1/p^4$ and $1/p^6$ to model 2a,

$1/d^4$ and $1/d^6$ to model 2b. The comparison of these models showed that models 2a and 3b still provide better results. Models 2 and 3 consist only of three coefficients and still fit the data well owing to the flexibility of the nonlinear model form. Model 4, on the other hand, is a sixth-degree polynomial volume ratio model comprising five parameters (4a) or six parameters (4b) to be estimated from the sample data. This model estimates the volume ratios which are used to convert total stem volumes into volumes to various top limits, all volumes are above the ground. Merchantable volume from stump height to some top limit is calculated by substraction. Model 4a and 4b give consistently good results, they were ranked third and second, respectively. The main advantage model 4 has over models 2 and 3 is that the former is flexible enough to deal with various stump heights.

The above comparison of all models demonstrates that one model may perform well in one case and may not be as good in another. Various models should be employed to attain different objectives. If a single model is desired, it should be a good taper equation that, when integrated, predicts well merchantable volumes to either top height or top diameter limits. The best-ranked models for various objectives are shown in Table 8.

The two sets of volume ratio models, (2a, 3b) and (4a, 4b), yield about equally good volume estimates. These two sets of models are recommended for predicting merchantable volumes to various heights and/or top diameters. The choice between them will depend largely upon the data. If stump heights are approx-

Table 8. Best-ranked models for various objectives.

Objectives	Best-ranked models	
Volumes to various heights	2a	Modified Burkhart
	4a	6th-degree polynomial model
	9a	Goulding-Murray
Volumes to top diameters	12	Segmented taper equation
	4b	6th-degree polynomial model
	3b	Burkhart weighted by $1/d^2$
Diameters	11	Max-Burkhart
	6	Ormerod
	12	Segmented taper equation
One model for various purposes	12	Segmented taper equation

imately the same and may be assumed constant, models 2a and 3b with fewer parameters to be estimated are more appropriate. On the other hand, models 4a and 4b are more flexible in case there are considerable variations among stump heights. If the sole objective is to describe tree taper, model 11 proves to be the "best" model. In another case, model 12 is the "all around" taper equation that provides consistently good estimation of both diameters and volumes.

CHAPTER FOUR : APPLICATIONS OF THE RECOMMENDED MODELS

Model 12 (segmented taper equation) is able to predict volume of any segment of the stem when integrated. The limits of integration are relative distances from the tip to the two ends of the segment of interest. These end values may be computed from the same taper equation if one or both are specified by stem diameters.

Models 2a and 3b (Modified Burkhart and Burkhart weighted by $1/d^2$) convert total volume above the stump into merchantable volumes to various heights or to diameter limits. The volume of a segment between two predetermined diameters or heights or a combination of both may then be computed by substraction.

Models 4a and 4b (sixth-degree polynomial volume ratio models) predict volumes above the ground to top heights or diameters. The volume of any section of the stem may also be obtained by substraction. In particular, merchantable volumes are given by substracting stump volumes, which are estimated from model 4a, from volumes to top limits.

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APPENDIX A

REPARAMETERIZATION FOR MODEL 12

The compatible quadratic taper equation (Modified Goulding-Murray) is simple enough and still has some nice properties. This model can be written as

$$y_i = b_1(3z^2 - 2z) + e_i$$
$$= -2b_1 z + 3b_1 z^2 + e_i$$

where $y = d^2KH/V - 2z$,

$z = (H-h)/H$.

The segmented polynomial regression model can be expressed as

$$y_i = f(z_i) + e_i$$

where $f(z)$ is a sequence of three grafted submodels,

$$\begin{aligned} f(z) = f_1(z) &= -2b_{11}z + 3b_{11}z^2 \ , \ 0 \leq z \leq a_1 \ , \\ = f_2(z) &= b_{02} + b_{12}z + b_{22}z^2 \ , \ a_1 < z \leq a_2 \ , \\ = f_3(z) &= b_{03} + b_{13}z + b_{23}z^2 \ , \ a_2 < z \leq 1 \ . \end{aligned}$$

These submodels are then grafted together at the join points a_1 and a_2 by imposing some restrictions on the model. These constraints demand that f be continuous and have continuous first partial derivatives in z over $[0,1]$.

The restriction that f be continuous at a_1 requires that

(1) $$-2b_{11}a_1 + 3b_{11}a_1^2 = b_{02} + b_{12}a_1 + b_{22}a_1^2$$

The restriction that f have a continuous first partial derivative with respect to z at a_1 requires that

$$-2b_{11} + 6b_{11}a_1 = b_{12} + 2b_{22}a_1 \tag{2}$$

Equation (1) and (2) give

$$-3b_{11}a_1^2 = b_{02} - b_{22}a_1^2$$

(3) or

$$b_{02} = (b_{22} - 3b_{11})a_1^2$$

Equation (2) gives

$$b_{12} = -2b_{11} - 2(b_{22} - 3b_{11})a_1 \tag{4}$$

Substitution of (3) and (4) for b_{02} and b_{12} in $f_2(z)$ results in

$$\begin{aligned} f_2(z) &= b_{02} + b_{12}z + b_{22}z^2 \\ &= (b_{22} - 3b_{11})a_1^2 - 2b_{11}z - 2(b_{22} - 3b_{11})a_1 z \\ &\quad + (b_{22} - 3b_{11})z^2 + 3b_{11}z^2 \\ &= -2b_{11}z + 3b_{11}z^2 + (b_{22} - 3b_{11})(a_1^2 - 2a_1 z + z^2) \end{aligned}$$

$$f_2(z) = f_1(z) + (b_{22} - 3b_{11})(z - a_1)^2 \tag{5}$$

Identical computations are carried out when the restrictions that f be continuous at a_2 and have continuous first partial derivative at a_2 are imposed.

$$f_3(z) = f_2(z) + (b_{23} - b_{22})(z - a_2)^2 \tag{6}$$

Combining equation (5) and (6) provides

$$f_3(z) = f_1(z) + (b_{22} - 3b_{11})(z - a_1)^2 + (b_{23} - b_{22})(z - a_2)^2$$

Let $b_1 = b_{11}$,

$b_2 = b_{22} - 3b_{11}$,

$b_3 = b_{23} - b_{22}$,

$$I_i = \begin{cases} 1 & \text{if } z \geq a_i , \\ 0 & \text{if } z < a_i . \end{cases} \qquad i=1,2$$

The final model written in reparameterized form is

$$y = b_1(3z^2 - 2z) + b_2 I_1(z - a_1)^2 + b_3 I_2(z - a_2)^2$$

APPENDIX B

Table B1.1. Ranks of models that predict merchantable volumes outside bark and inside bark to various heights -- Plantations sample data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
1a	0.3861	12	0.4352	12	0.4748	9	33
2a	0.0002	1	0.2352	1	0.3768	2	4
3a	-0.0037	2	0.2389	3	0.3809	3	8
4a	-0.0133	3	0.2387	2	0.3885	6	11
5	-0.2423	11	0.3690	11	0.6609	12	34
6	0.1345	10	0.2645	6	0.3741	1	17
7a	-0.1133	7	0.2820	9	0.4767	10	26
8	-0.1184	9	0.2673	7	0.4274	7	23
9a	0.0443	4	0.2434	4	0.3845	5	13
10	-0.1182	8	0.2676	8	0.4276	8	24
11	-0.0560	5	0.2977	10	0.5321	11	26
12	0.0813	6	0.2508	5	0.3840	4	15
			INSIDE BARK				
1a	0.1874	12	0.2667	12	0.3774	10	34
2a	0.0026	2	0.2019	3	0.3540	1	6
3a	-0.0024	1	0.2057	5	0.3582	5	11
4a	-0.0129	4	0.2003	1	0.3579	4	9
5	-0.1204	11	0.2364	11	0.4322	12	34
6	0.0591	7	0.2187	9	0.3700	6	22
7a	-0.0643	8	0.2189	10	0.3911	11	29
8	-0.0715	10	0.2165	8	0.3759	8	26
9a	0.0282	5	0.2011	2	0.3549	3	10
10	-0.0659	9	0.2157	7	0.3750	7	23
11	0.0086	3	0.2074	6	0.3766	9	18
12	0.0547	6	0.2050	4	0.3544	2	12

Table B1.2. Ranks of models that predict merchantable volumes outside and inside bark to various heights -- Plantations independent data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
1a	0.3768	12	0.4074	12	0.4647	12	36
2a	0.0084	3	0.2116	1	0.3268	1	5
3a	0.0036	1	0.2129	3	0.3283	3	7
4a	0.0052	2	0.2123	2	0.3269	2	6
5	-0.1887	11	0.3075	11	0.4523	11	33
6	0.1335	10	0.2441	9	0.3290	4	23
7a	-0.0830	6	0.2336	8	0.3585	9	23
8	-0.0943	8	0.2262	5	0.3360	5	18
9a	0.0594	5	0.2218	4	0.3371	7	16
10	-0.0942	7	0.2262	6	0.3362	6	19
11	-0.0136	4	0.2460	10	0.3650	10	24
12	0.0944	9	0.2314	7	0.3439	8	24
			INSIDE BARK				
1a	0.1684	12	0.2295	12	0.3721	12	36
2a	-0.0101	1	0.1801	4	0.3289	6	11
3a	-0.0154	3	0.1856	6	0.3305	8	17
4a	-0.0172	4	0.1774	2	0.3217	2	8
5	-0.1103	11	0.2110	11	0.3371	10	32
6	0.0336	6	0.1923	7	0.3643	11	24
7a	-0.0628	8	0.1934	8	0.3295	7	23
8	-0.0726	10	0.1962	10	0.3249	3	23
9a	0.0215	5	0.1748	1	0.3273	5	11
10	-0.0673	9	0.1944	9	0.3252	4	22
11	0.0108	2	0.1821	5	0.3160	1	8
12	0.0484	7	0.1775	3	0.3316	9	19

Table B1.3. Ranks of models that predict merchantable volumes outside and inside bark to various heights -- Natural stands sample data.

Model	Bias Value	Bias Rank	Mean absolute deviation Value	Mean absolute deviation Rank	Standard error of the differences Value	Standard error of the differences Rank	Rank Total
			OUTSIDE BARK				
1a	0.8245	12	0.9265	12	0.9260	9	33
2a	0.0027	1	0.5416	1	0.8035	1	3
3a	-0.0117	2	0.5470	2	0.8086	2	6
4a	-0.0700	6	0.5736	4	0.8384	6	16
5	-0.5335	11	0.8119	11	1.1702	12	34
6	0.2597	8	0.6104	7	0.8141	3	18
7a	-0.0493	5	0.5640	3	0.8249	5	13
8	-0.4505	10	0.7443	10	1.0042	11	31
9a	-0.0199	3	0.5828	6	0.8563	7	16
10	-0.3830	9	0.7021	9	0.9676	10	28
11	0.0249	4	0.6267	8	0.9053	8	20
12	0.1300	7	0.5743	5	0.8235	4	16
			INSIDE BARK				
1a	0.3576	11	0.5591	10	0.7258	7	28
2a	0.0135	2	0.4698	1	0.7027	3	6
3a	-0.0038	1	0.4790	5	0.7094	5	11
4a	-0.0499	4	0.4770	4	0.7038	4	12
5	-0.2573	9	0.5442	8	0.7424	8	25
6	0.1227	6	0.5209	7	0.8318	12	25
7a	-0.3950	12	0.6064	12	0.7664	10	34
8	-0.2622	10	0.5665	11	0.7799	11	32
9a	0.0415	3	0.4755	3	0.7019	2	8
10	-0.2168	8	0.5470	9	0.7649	9	26
11	0.1225	5	0.4745	2	0.7142	6	13
12	0.1227	7	0.4795	6	0.6979	1	14

Table B1.4. Ranks of models that predict merchantable volumes outside and inside bark to various heights -- Natural stands independent data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
1a	0.8205	12	0.9458	12	0.8726	8	32
2a	0.0118	3	0.5922	1	0.8306	2	6
3a	-0.0028	1	0.5988	2	0.8385	3	6
4a	-0.0563	6	0.6067	4	0.8469	6	16
5	-0.5022	11	0.8177	11	1.1777	12	34
6	0.2701	8	0.6820	8	0.8446	4	20
7a	-0.0378	4	0.6019	3	0.8456	5	12
8	-0.4297	10	0.7494	10	0.9957	11	31
9a	-0.0069	2	0.6111	6	0.8487	7	15
10	-0.3636	9	0.7109	9	0.9631	10	28
11	0.0456	5	0.6780	7	0.9394	9	21
12	0.1402	7	0.6104	5	0.8238	1	13
			INSIDE BARK				
1a	0.3721	12	0.6205	10	0.7343	1	23
2a	0.0332	3	0.5489	3	0.7766	6	12
3a	0.0159	1	0.5577	7	0.7866	8	16
4a	-0.0266	2	0.5509	5	0.7865	7	14
5	-0.2336	9	0.5866	8	0.8099	9	26
6	0.1313	5	0.5400	2	0.7580	4	11
7a	-0.3685	11	0.6401	12	0.8555	11	34
8	-0.2347	10	0.6237	11	0.8690	12	33
9a	0.0631	4	0.5503	4	0.7698	5	13
10	-0.1903	8	0.6082	9	0.8506	10	27
11	0.1389	6	0.5288	1	0.7345	2	9
12	0.1427	7	0.5514	6	0.7560	3	16

Table B2.1. Ranks of models estimating merchantable volumes outside and inside bark to top diameters -- Plantations sample data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
1b	-0.1555	7	0.4126	6	0.6545	6	19
2b	0.0322	1	0.3598	1	0.5599	3	5
3b	0.0339	2	0.3649	3	0.5699	4	9
4b	-0.0345	3	0.3692	4	0.6031	5	12
5	-0.3740	10	0.5550	10	0.9768	10	30
6	0.1168	6	0.3723	5	0.5577	1	12
8	-0.2095	9	0.4236	8	0.6637	8	25
10	-0.2073	8	0.4233	7	0.6636	7	22
11	-0.0989	5	0.4287	9	0.7557	9	23
12	0.0429	4	0.3644	2	0.5589	2	8
			INSIDE BARK				
1b	-0.1074	7	0.3530	7	0.5844	7	21
2b	0.0158	3	0.3218	3	0.5393	2	8
3b	0.0064	2	0.3232	4	0.5421	3	9
4b	-0.0250	4	0.3206	2	0.5528	4	10
5	-0.2005	10	0.3803	10	0.6755	10	30
6	0.0356	6	0.3421	6	0.5543	5	17
8	-0.1364	9	0.3575	9	0.5949	9	27
10	-0.1204	8	0.3535	8	0.5907	8	24
11	-0.0049	1	0.3261	5	0.5773	6	12
12	0.0340	5	0.3177	1	0.5362	1	7

Table B2.2. Ranks of models estimating merchantable volumes outside and inside bark to top diameters -- Plantations independent data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
1b	-0.1383	7	0.3591	9	0.5188	7	23
2b	0.0350	2	0.3254	3	0.5137	4	9
3b	0.0356	3	0.3359	5	0.5340	9	17
4b	-0.0149	1	0.3136	1	0.4734	1	3
5	-0.3082	10	0.4686	10	0.6955	10	30
6	0.1093	6	0.3329	4	0.4792	2	12
8	-0.1836	9	0.3590	8	0.5170	5	22
10	-0.1816	8	0.3586	7	0.5174	6	21
11	-0.0527	5	0.3585	6	0.5248	8	19
12	0.0429	4	0.3180	2	0.4908	3	9
			INSIDE BARK				
1b	-0.1206	7	0.3203	8	0.5031	6	21
2b	-0.0073	1	0.2858	3	0.5074	8	12
3b	-0.0154	5	0.2888	5	0.5064	7	17
4b	-0.0375	6	0.2839	2	0.4820	2	10
5	-0.1924	10	0.3412	10	0.5294	9	29
6	-0.0081	2	0.3043	6	0.5350	10	18
8	-0.1435	9	0.3205	9	0.5014	4	22
10	-0.1287	8	0.3148	7	0.5014	5	20
11	-0.0103	3	0.2866	4	0.4815	1	8
12	0.0137	4	0.2730	1	0.4950	3	8

Table B2.3. Ranks of models estimating merchantable volumes outside and inside bark to top diameters -- Natural stands sample data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
1b	-0.3806	7	1.1277	7	1.6077	7	21
2b	0.1304	6	0.9253	6	1.3617	6	18
3b	0.0025	1	0.9005	2	1.3142	2	5
4b	-0.0713	3	0.9106	3	1.3288	4	10
5	-0.8751	10	1.2938	10	1.9192	10	30
6	0.1082	5	0.9150	4	1.3200	3	12
8	-0.7502	9	1.2273	9	1.7240	9	27
10	-0.6626	8	1.1707	8	1.6580	8	24
11	-0.0599	2	0.9240	5	1.3609	5	12
12	0.1081	4	0.8704	1	1.2654	1	6
			INSIDE BARK				
1b	-0.1771	7	0.8606	6	1.2354	6	19
2b	0.0339	4	0.8218	4	1.2017	4	12
3b	0.0193	2	0.8232	5	1.2011	3	10
4b	-0.0156	1	0.8147	3	1.1899	2	6
5	-0.4616	9	0.9618	9	1.3372	7	25
6	0.0208	3	0.9068	7	1.3658	8	18
8	-0.4811	10	0.9840	10	1.4092	10	30
10	-0.3921	8	0.9491	8	1.3728	9	25
11	0.1311	5	0.8056	2	1.2211	5	12
12	0.1409	6	0.8028	1	1.1822	1	8

Table B2.4. Ranks of models estimating merchantable volumes outside and inside bark to top diameters -- Natural stands independent data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
1b	-0.4006	7	1.1522	7	1.5991	8	22
2b	0.0964	6	0.9580	4	1.2853	3	13
3b	-0.0243	1	0.9328	2	1.2681	2	5
4b	-0.0816	3	0.9562	3	1.3058	4	10
5	-0.8600	10	1.3274	10	1.8745	10	30
6	0.0914	5	0.9907	5	1.8085	5	15
8	-0.7498	9	1.2541	9	1.6591	9	27
10	-0.6650	8	1.2013	8	1.5970	7	23
11	-0.0654	2	1.0109	6	1.3827	6	14
12	0.0872	4	0.9111	1	1.2219	1	6
			INSIDE BARK				
1b	-0.1660	7	0.9389	7	1.2885	7	21
2b	0.0445	4	0.9261	5	1.2325	4	13
3b	0.0308	3	0.9281	6	1.2357	6	15
4b	-0.0044	1	0.9135	4	1.2332	5	10
5	-0.4436	9	1.0123	8	1.3793	8	25
6	0.0157	2	0.9088	3	1.2232	3	8
8	-0.4559	10	1.0628	10	1.4780	10	30
10	-0.3694	8	1.0325	9	1.4362	9	26
11	0.1390	5	0.8737	1	1.1889	1	7
12	0.1490	6	0.9030	2	1.2101	2	10

Table B3.1. Ranks of models that give diameters outside and inside bark -- Plantations sample data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
5	0.0479	6	0.3306	7	0.4557	7	20
6	-0.0301	4	0.2126	1	0.2995	1	6
7a	-0.0279	3	0.4007	8	0.5396	8	19
8	0.0134	2	0.3114	6	0.4232	6	14
9a	-0.1076	8	0.2888	4	0.3870	4	16
10	0.0117	1	0.3098	5	0.4222	5	11
11	-0.0388	5	0.2405	2	0.3330	3	10
12	-0.0811	7	0.2589	3	0.3318	2	12
			INSIDE BARK				
5	-0.0451	5	0.2433	3	0.3412	4	12
6	-0.0958	8	0.2351	2	0.2978	2	12
7a	-0.0441	4	0.2990	8	0.4039	8	20
8	-0.0278	1	0.2452	4	0.3415	5	10
9a	-0.0906	7	0.2572	7	0.3438	6	20
10	-0.0393	2	0.2527	6	0.3439	7	15
11	-0.0420	3	0.2073	1	0.2854	1	5
12	-0.0905	6	0.2494	5	0.3210	3	14

Table B3.2. Ranks of models that give diameters outside and inside bark -- Plantations independent data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
5	0.0221	3	0.2991	7	0.4158	7	17
6	-0.0614	5	0.2147	1	0.2841	1	7
7a	-0.0519	4	0.3598	8	0.4911	8	20
8	-0.0092	1	0.2730	5	0.3803	6	12
9a	-0.1296	8	0.2882	6	0.3737	4	18
10	-0.0108	2	0.2715	4	0.3792	5	11
11	-0.0643	6	0.2471	2	0.3248	3	11
12	-0.1033	7	0.2587	3	0.3205	2	12
			INSIDE BARK				
5	-0.0753	5	0.2313	3	0.3128	5	13
6	-0.1307	8	0.2386	5	0.2789	2	15
7a	-0.0730	4	0.2726	8	0.3681	8	20
8	-0.0563	1	0.2263	2	0.3096	4	7
9a	-0.1188	7	0.2565	7	0.3280	7	21
10	-0.0678	2	0.2339	4	0.3129	6	12
11	-0.0722	3	0.2107	1	0.2730	1	5
12	-0.1187	6	0.2492	6	0.3087	3	15

Table B3.3. Ranks of models that give diameters outside and inside bark -- Natural stands sample data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
5	0.0234	1	0.4824	6	0.6439	7	14
6	-0.0732	4	0.2912	1	0.3911	1	6
7a	-0.0787	7	0.2996	2	0.3932	2	11
8	0.0863	8	0.5527	8	0.7065	8	24
9a	-0.0404	3	0.3876	5	0.5426	5	13
10	0.0325	2	0.4849	7	0.6383	6	15
11	-0.0761	5	0.3053	3	0.4030	3	11
12	-0.0783	6	0.3125	4	0.4156	4	14
			INSIDE BARK				
5	-0.0684	5	0.3736	5	0.4930	5	15
6	-0.1161	7	0.3167	3	0.4007	2	12
7a	-0.1489	8	0.3947	8	0.4971	6	22
8	0.0137	1	0.3821	7	0.5167	8	16
9a	-0.0558	4	0.3331	4	0.4573	4	12
10	-0.0206	2	0.3756	6	0.5027	7	15
11	-0.0312	3	0.2428	1	0.3300	1	5
12	-0.0816	6	0.3048	2	0.4137	3	11

Table B3.4. Ranks of models that give diameters outside and inside bark -- Natural stands independent data.

Model	Bias		Mean absolute deviation		Standard error of the differences		Rank Total
	Value	Rank	Value	Rank	Value	Rank	
			OUTSIDE BARK				
5	0.0023	1	0.4854	7	0.6389	7	15
6	-0.0929	5	0.3135	1	0.4178	1	7
7a	-0.0995	7	0.3248	2	0.4235	2	11
8	0.0632	4	0.5428	8	0.6910	8	20
9a	-0.0627	3	0.4019	5	0.5566	5	13
10	0.0097	2	0.4838	6	0.6278	6	14
11	-0.0965	6	0.3305	3	0.4406	3	12
12	-0.1003	8	0.3322	4	0.4432	4	16
			INSIDE BARK				
5	-0.0720	5	0.3918	5	0.5093	5	15
6	-0.1197	7	0.3370	2	0.4270	2	11
7a	-0.1526	8	0.4103	8	0.5112	6	22
8	0.0103	1	0.3970	7	0.5277	8	16
9a	-0.0587	4	0.3589	4	0.4901	4	12
10	-0.0237	2	0.3957	6	0.5195	7	15
11	-0.0351	3	0.2767	1	0.3722	1	5
12	-0.0843	6	0.3378	3	0.4577	3	12

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PREDICTION OF CUBIC-FOOT VOLUME OF LOBLOLLY PINE

TO ANY TOP DIAMETER LIMIT AND TO ANY POINT ON TREE BOLE

by

Quang Van Cao

(ABSTRACT)

This study considers the problem of estimating merchantable volume to some specified top diameter or height limit. The models were separated into two categories. Volume ratio models give the ratios of merchantable to total volume. Taper equations when integrated provide volume estimates of any segment of the bole. Data from plantations and natural stands of loblolly pine were used to compare the models for ability to predict merchantable volumes. Additional evaluations were made among the taper equations to determine the one that "best" describes stem taper. Results showed that different models should be used for different objectives.