PART 1

THE CALIBRATION AND USE OF A SPECTROPHOTOMETER WITH THE PHENOLDISULFONIC ACID METHOD OF NITRATE ANALYSIS

PART 2

THE PRESENTATION OF NEW CONCEPTS ON FRICTION IN CONDUITS IN A FORM SUITABLE FOR DESIGN CRITERIA

by

Maurice Andrew Person

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APPROVED:

Director of Graduate Studies

Dean of Engineering

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Head of Department

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PART 1

THE CALIBRATION AND USE OF A SPECTROPHOTOMETER WITH THE PHENOLDISULFONIC ACID METHOD OF NITRATE ANALYSIS

I. INTRODUCTION

The statistical and medical evidence linking potable water supplies high in nitrate content with methemoglobinemia in infants has led to increased interest in the nitrate content of water supplies. The high nitrate content waters used in preparing infants formulae have been indicated as the source (12).

Infants suffering from the disease methemoglobinemia are frequently referred to as "blue babies". The disease is characterized by a grayish- or brownish-blue coloration, first appearing on the lips, then spreading to the fingers and toes, and may eventually cover the entire body. This discoloration may be caused by nitrates being converted to nitrites in the more alkaline stomachs of infants. The nitrites enter the blood where they react with the blood causing a decrease in the oxygen transporting capabilities. The bluish color is indicative of a lack of oxygen (12).

A report by the Committee on Water Supply, Engineering Section, American Public Health Association, based on information revealed by a questionnaire sent to the 48 states, Alaska, and Hawaii, revealed important information regarding the occurrence of methemoglobinemia and the related water supply nitrate content (12). It revealed that with few exceptions, cases of methemoglobinemia in the United States occurred in the North central states where the nitrate content of ground water supplies was high. The incidence of the disease increased with increased nitrate content above 10 parts per million as nitrogen.

Nitrate content in high concentrations occur in industrial wastes evolved in the preparation of trinitrotoluene and nitroglycerine (10), in wells, particularly in the North central states (12), and in industrial and domestic sewage in an advanced state of decomposition (9).

Because of the occasional occurrence of high nitrate concentration and its association with methemoglobinemia in infants, a reliable, accurate method of analysis of nitrate concentration is required. Such an analysis may be employed in an attempt to avoid the use of waters high in nitrate content for infants and to prevent chance contamination of currently potable water supplies with nitrates.

Two methods, a phenoldisulfonic acid method and a reduction method, for nitrate analysis are listed in Standard Methods for the Examination of Water, Sewage, and Industrial Wastes (1). The phenoldisulfonic acid method of nitrate analysis is a method by which a yellow color, its intensity in proportion to the nitrate content in a sample, is formed by a reaction between phenoldisulfonic acid and the residue formed when the sample is evaporated to dryness; the solution then diluted and made alkaline. The reduction method requires a conversion of nitrates to ammonia and a determination of the ammonia formed as a measure of nitrate concentration.

Chlorides, nitrites, colored ions, and materials which physically modify the color system interfere with the results obtained in using the phenoldisulfonic acid method. Ammonia, nitrites, organic nitrogen, incomplete conversion of nitrates to amonia, and incomplete recovery of the ammonia present are interferences to the reduction method (1).

Both the phenoldisulfonic acid method and the reduction method have serious limitations. In many cases, the phenoldisulfonic acid method may prove more advantageous because of the shorter length of time required in its application and the simpler laboratory procedures involved.

Since the color formed in the phenoldisulfonic acid method of nitrate analysis is a function of the nitrate content in a sample, it is necessary to prepare a series of color standards of known nitrate content to compare with samples of unknown nitrate content. Since it is difficult to distinguish accurately, by visual means, changes in color intensity of a given wavelength, the spectrophotometer, is the instrument of choice for this type of analysis. Using a spectrophotometer, concentrations of colored solutions can be compared rapidly and monochromatically.

The objective of this investigation was to utilize spectrophotometric methods to refine and improve the phenoldisulfonic acid method of nitrate analysis for water supplies, thereby contributing to the protection of the health of the public.

II. SURVEY OF LITERATURE

A series of four papers published in 1909, 1910, and 1911; the first two by Chanot and Pratt (4) (5) and the third and fourth with Redfield (6) (7), offer an excellent analysis of the phenoldisulfonic acid method of nitrate determinations. The first paper (4) confirmed the fact that the reagent involved was 1-2-4-disulfonic acid with small amounts of the para acid. The composition of the yellow compound was reported in the second paper (5) to be an alkali salt of nitrophenoldisulfonic acid. The third paper (6) presented the chief errors which may affect the results as obtained by the phenoldisulfonic method. The chief sources of error reported were the instability and indefinite composition of the sulfonic acid reagent with chlorides, carbonates, organic matter, and nitrites reported as interferences to test results. The fourth paper (7) describes a method of preparing a stable sulfonic acid reagent free of the mono and tri acids. The absence of monosulfonic acid makes reliable results and the comparison with permanent standards possible. Methods of removal of carbonate interference by neutralization, of chloride interference by addition of silver nitrate, and of nitrite interference by boiling with hydrogen peroxide are offered.

Much research has been done in studying the phenoldisulfonic acid method of nitrate analysis with particular emphasis placed on interfering agents encountered. The current procedure for the determination of nitrates in water, as presented in Standard Methods for the Examination of Water, Sewage, and Industrial Wastes (1) recommends, initially, decolorization of highly colored samples with an aluminum hydroxide suspension, precipitation of chlorides with silver sulfate, and neutralization to approximately pH 7. This is followed by evaporation to dryness, addition of phenoldisulfonic acid, dilution, and alkalization.

Standard Methods (1) offers a photometric method for determining color concentrations in samples in addition to a visual comparison method. In employing the Beckman model B spectrophotometer, use is made of Beer's law, as presented in Clinical Analyses With the Beckman B and DU Spectrophotometers (3) which states that "the amount of light obsorbed is in direct proportion to the amount of the absorbing substance". Beckman also presents a description of the light source including wave lengths available for use with and without filters, a method for the elimination of interferences, and operating principles for instrumental analysis. The analysis involves a comparison of monochromatic light absorption by a sample with standards of known concentration.

III. METHODS AND MATERIALS

The phenoldisulfonic acid method of nitrate analysis procedure listed in Standard Methods for the Examination of Water, Sewage, and Industrial Wastes (1) was used in preparing color standards. The only differences were in the concentration of nitrate originally employed and the dilutions.

Color Standard Preparations

0-1 part per million (ppm) Nitrate Nitrogen Range

For preparing color standards of known nitrate content, a stock nitrate solution was made by dissolving 3.6080 grams of reagent grade potassium nitrate, molecular weight 101.10, in a liter of doubly distilled water. This solution contained 500 ppm nitrate as nitrogen. One hundred mls. was taken and diluted to one liter, resulting in a solution containing 50 ppm nitrate nitrogen. Evaporation to dryness of 100 mls. of the diluted stock solution on a steam bath yielded a nitrate residue in the evaporating dish. Two mls. of reagent grade phenoldisulfonic acid was added to the residue with intimate contact facilitated by rubbing with a glass rod. This solution was diluted with 20 mls. of doubly distilled water. A 1:1 dilution of ammonium hydroxide was added until the maximum yellow color was developed, after which the solution was diluted with doubly distilled water to one liter. The colored solution then contained five ppm nitrate nitrogen.

Dilutions, of 0.01 ppm, 0.05 ppm, and thence in increments of 0.05 ppm to one ppm nitrate nitrogen, of the colored solution were made in 50 ml. Nessler tubes. A blank, containing no added nitrate, was prepared by evaporating 50 mls. of doubly distilled water, adding phenoldisulfonic acid, with dilution and alkalization. The blank was considered to allow 100 per cent light transmission. The per cent light transmission through the colored standards could then be read as that not absorbed by the colored, alkaline, nitrate salt.

0-10 ppm Nitrate Nitrogen Range

Color standards for the nitrate nitrogen range of from one to ten ppm were prepared. The dissolving of 3.6080 grams of reagent grade potassium nitrate, molecular weight 101.10, in one liter of doubly distilled water resulted in a 500 ppm nitrate nitrogen solution. One hundred ml. of the 500 ppm solution were evaporated to dryness on a steam bath. The addition of two ml. phenoldisulfonic acid, dilution with 20 ml. doubly distilled water, followed by alkalization to develop maximum color, and further dilution to one liter resulted in a 50 ppm nitrate nitrogen colored solution. One to ten ml. of this solution, in increments of 0.5 ml., when diluted to 50 ml., provided a series of color standards for from one to ten ppm nitrate nitrogen, in increments of 0.5 ppm. A blank solution was prepared as in the lower nitrate nitrogen range investigated.

THE BECKMAN MODEL B SPECTROPHOTOMETER

The Beckman model B spectrophotometer uses a Fery prism (2) to disperse the light from an incandescent lamp into its different wave lengths. The light is transmitted to the prism and from the prism to a sensitive phototube by a system of mirrors. The phototube measures the light incident upon it. If a colored sample is placed in the light path prior to its reaching the phototube, the amount of light absorbed by or transmitted through the sample may be measured.

Colors are manifested by selective transmission of certain wavelengths and different colors represent selective transmission of light of different dominant wave lengths (3). For colorimetric analysis, a wave length is usually selected which affords maximum absorption (minimum transmission) through the colored solution.

The wave length of light incident upon the phototube may be controlled by an external adjustment on the spectrophotometer, which moves the Fery prism by translation and rotation to select monochromatic light in wave lengths of the visible spectrum (320 to 1000 millimicrons). The wave length used, instead of representing purely monochromatic light, represents a small band of light of wave lengths centered about that selected. The width of this band may be controlled by adjustment of a slit in the path of the dispersed light.

Taras (11) reports that nitrate nitrogen contents of from 0.01 to two ppm can be determined accurately using monochromatic light at a wave length of 410 millimicrons, and that conformity to Beer's law extends to 470 and 480 millimicrons, hence accurate determination of nitrate nitrogen contents up to 12 ppm is possible.

The Calibration Curves

0-1 ppm Nitrate Nitrogen Range

The model B spectrophotometer may be used in any of four sensitivities, increasing in precision from a setting of from one to four. The increased sensitivity is associated with a decreased slit width. The sensitivity selected for use in the tests reported was two. This permitted the use of a slit width of approximately 0.2 millimeters for the series of tests.

The spectrophotometer was equipped with four, one centimeter, pyrex, sample cells, which could be moved, consecutively, by a carriage mechanism, into the path of the monochromatic light prior to its reaching the phototube. The blank, prepared with no added nitrate, was inserted into the first of the four cells. The other three were filled with the same color standard. A slit width adjustment to approximately 0.2 millimeter permitted a reading of 100 per cent transmission through the blank. Moving the three colored sample cells consecutively into the monochromatic light path permitted readings of light transmittance through these samples to be made relative to 100 per cent transmission through the blank. The three readings obtained for the same colored sample permitted the detection of errors due to sample or cell surface contamination.

The wave length of minimum transmission for the yellow color was found to be 405 millimicrons. Only a slight difference in per cent transmission was detectable at 410 millimicrons. To obtain the minimum transmission wave length, per cent transmission readings were made through a one ppm nitrate nitrogen color standard, the upper limit of the range being considered, for various wave lengths of monochromatic light. Per cent transmission values were found to increase for wave lengths above and below 405 millimicrons.

Readings of per cent transmittance were made for each colored standard in each of 21 series of standards prepared. Including the blank, there were 22 color standard solutions in each series for a total of 462 color standard solutions.

Chaney (3) states that absorbance varies directly with concentration and that absorbance is the negative logarithm of the transmittance. Mathematically, these points may be indicated by

$$A = f\ (C)$$

and

$$A = \log_{10}\frac{1}{T}$$

where A is absorbance, C is concentration, and T is transmittance.

It can therefore be seen that concentration is directly proportional to the logarithm of the transmittance.

$$C = f\ (\log_{10} T)$$

Since the intensity or concentration of color varies directly with the nitrate concentration, a curve relating nitrate concentration and per cent light transmittance would take the form of a straight line if plotted on semi-logarithmic paper and could be represented by the following general formula.

$$T = mC + b$$

where

$T = \log_{10}$ per cent transmission

m = slope

C = concentration

b = the T intercept

The method of least squares (8) was used in obtaining an algebraic expression for the curve best fitting the data obtained. Computations employed in obtaining the calibration curve are shown in Appendix A. To obtain a measure of the accuracy of the calibration curve relationship, a correlation coefficient and the standard deviation for the data were computed. The correlation coefficient calculations are shown in Appendix B and the standard deviation computations in Appendix C.

<u>0-10 ppm Nitrate Nitrogen Range</u>

The same general procedure was used in obtaining the 0-10 ppm concentration-per cent transmission curve as in the 0-1 ppm curve. A wave length of 465 millimicrons was selected since maximum meter deflections on the spectrophotometer would be on the upper two-thirds of the scale, where better accuracy can be expected (2). A sensitivity of two and a slit width of approximately 0.2 millimeters was employed.

Twelve sets of colored standards were prepared for the nitrate nitrogen range from 0-10 ppm. Each set contained 19 colored standards for nitrate concentrations in the range from one to ten ppm in increments of 0.5 ppm. Two hundred, forty individual per cent transmittance readings were made for 12 sets of colored standards.

The method of least squares was employed to determine the line fitting the data obtained. The logarithms of the per cent transmission readings were obtained and used in this calculation. A correlation coefficient and standard deviation was computed as a measure of the reliability of the relationship developed.

IV. RESULTS

0-1 ppm Nitrate Nitrogen Range

Twenty-one sets of per cent transmission readings were made on the 21 sets of colored standards prepared. These readings are presented in Table 1. They were obtained using the Beckman model B spectrophotometer using 405 millimicron monochromatic light, a sensitivity of two, and a slit width of approximately 0.2 millimeters.

The line of best fit for the relation between per cent transmittance and nitrate nitrogen obtained from these data is presented as Figure 1.

0-10 ppm Nitrate Nitrogen Range

Per cent light transmission readings for 12 sets of nitrate color standards prepared were determined. These readings are provided as Table 2. The readings were made using 410 millimicron monochromatic light with a sensitivity setting of two and a slit width of approximately 0.2 millimeters.

The line of best fit for the relation between per cent transmittance and nitrate nitrogen obtained from these data is presented as Figure 2.

TABLE 1

PER CENT LIGHT TRANSMISSION FOR NITRATE CONCENTRATION INDICATED

ppm NO_3-N	Run No. 1	Run No. 2	Run No. 3	Run No. 4	Run No. 5
0	100	100	100	100	100
0.01	97	98.2	98.1	97.1	99
0.05	92	92.8	81.5	81	93
0.1	89	89.5	89.6	89	90
0.15	85	85	85	84	86
0.2	81.9	82.1	81.3	81.1	83.5
0.25	78	78.1	78	78	78
0.3	73	73	73	73	71.5
0.35	71	71.3	71	71	72
0.4	68	68	67	67.5	67.5
0.45	64.5	64	64	64	64
0.5	61	60.8	61	60.1	61
0.55	57.5	57	57	57	57.5
0.6	55	55	54.1	54.4	55.5
0.65	51.8	52	51	51	52
0.7	50	50	50	50	51
0.75	47	47.8	47	47	48
0.8	45.3	45.5	45	45	47
0.85	43	43.2	42.5	43	44.5
0.9	41	41	41	40.8	41.5
0.95	37.5	39	38	38	38.5
1.0	36.5	37.3	37	37	38

TABLE 1

PER CENT LIGHT TRANSMISSION FOR NITRATE CONCENTRATION INDICATED

ppm NO_3-N	Run No. 6	Run No. 7	Run No. 8	Run No. 9
0	100	100	100	100
0.01	99	97	99	99
0.05	92.5	91	94	95
0.1	90.5	91	91	90
0.15	85	84	87	86
0.2	83	83	81	83
0.25	77.5	78	77	78
0.3	70.5	75	73	75
0.35	72.5	71	70	71
0.4	68	67.8	67	68
0.45	63	64	64	64.5
0.5	61	61	61	62.5
0.55	57	59	59	59
0.6	55	56	56.8	56
0.65	51.5	53	53	54
0.7	52	49	50.5	51
0.75	48	49	48	48
0.8	47	46	45	46
0.85	44	44	43	44
0.9	41.5	42.5	40	42
0.95	39.5	40	38	40
1.0	38	37.5	38	39

TABLE 1

PER CENT LIGHT TRANSMISSION FOR NITRATE CONCENTRATION INDICATED

ppm NO_3-N	Run No. 10	Run No. 11	Run No. 12	Run No. 13
0	100	100	100	100
0.01	99	98	99	99.3
0.05	95	96	95.4	95.5
0.1	90	91	90	90.2
0.15	86	86.5	86	86
0.2	82	82	82.2	82
0.25	78	79	77.9	78
0.3	74	73.5	74.3	74
0.35	71	71	71	70.4
0.4	67.5	68.5	67	67
0.45	64	64	63.4	63.7
0.5	61	61	61	61
0.55	58	60	58	58
0.6	55	56	55	55
0.65	53	52	52.5	52
0.7	50	51	50	50
0.75	47.5	47.5	47.5	47.2
0.8	46	45.5	45.5	45
0.85	43.5	44	43	43
0.9	41	42	41	41
0.95	39.5	40	39	39
1.0	37.5	38	37.5	37

TABLE 1

PER CENT LIGHT TRANSMISSION FOR NITRATE CONCENTRATION INDICATED

ppm NO_3-N	Run No. 14	Run No. 15	Run No. 16	Run No. 17
0	100	100	100	100
0.01	99.5	99	99.5	99
0.05	96	95.5	96	96
0.1	90	90	90	90
0.15	86	86	84	86
0.2	82	82	82	83
0.25	78	78	78	78
0.3	75	74	75	75
0.35	71.5	71	72	71.5
0.4	67	67	67	67.5
0.45	64	63.2	63.8	64
0.5	62	61.2	62	62
0.55	58.5	57.8	58.5	58
0.6	55	55	55	55.2
0.65	53	52.5	53	53.2
0.7	50	49.5	50	50.2
0.75	48	47	48	48
0.8	46	45	46	46
0.85	43.5	43	43.2	43
0.9	40.2	40.8	41	42
0.95	39.5	38.8	40.2	39.2
1.0	38.5	37	37.5	37.5

TABLE 1

PER CENT LIGHT TRANSMISSION FOR NITRATE CONCENTRATION INDICATED

ppm NO_3-N	Run No. 18	Run No. 19	Run No. 20	Run No. 21
0	100	100	100	100
0.01	99.2	99	99	99
0.05	96	95.5	95	95.7
0.1	90	90	90	90
0.15	84.8	86	85.2	86
0.2	83	82	83.2	82.5
0.25	78	78	78	78
0.3	75	75	74.2	74.8
0.35	72	71	71	71
0.4	67	67.5	67	67.5
0.45	64	64	64	64
0.5	62	62	62	61.5
0.55	58	59	58.5	58
0.6	55.5	56	55.5	55.5
0.65	53	53	53	53
0.7	50.2	51	50.2	50
0.75	48	48.3	48	48
0.8	46	46.3	46	46
0.85	43.5	44.3	43.5	43.8
0.9	41.5	42	42	41.5
0.95	39.5	39.2	40	40
1.0	38	38	38	37.5

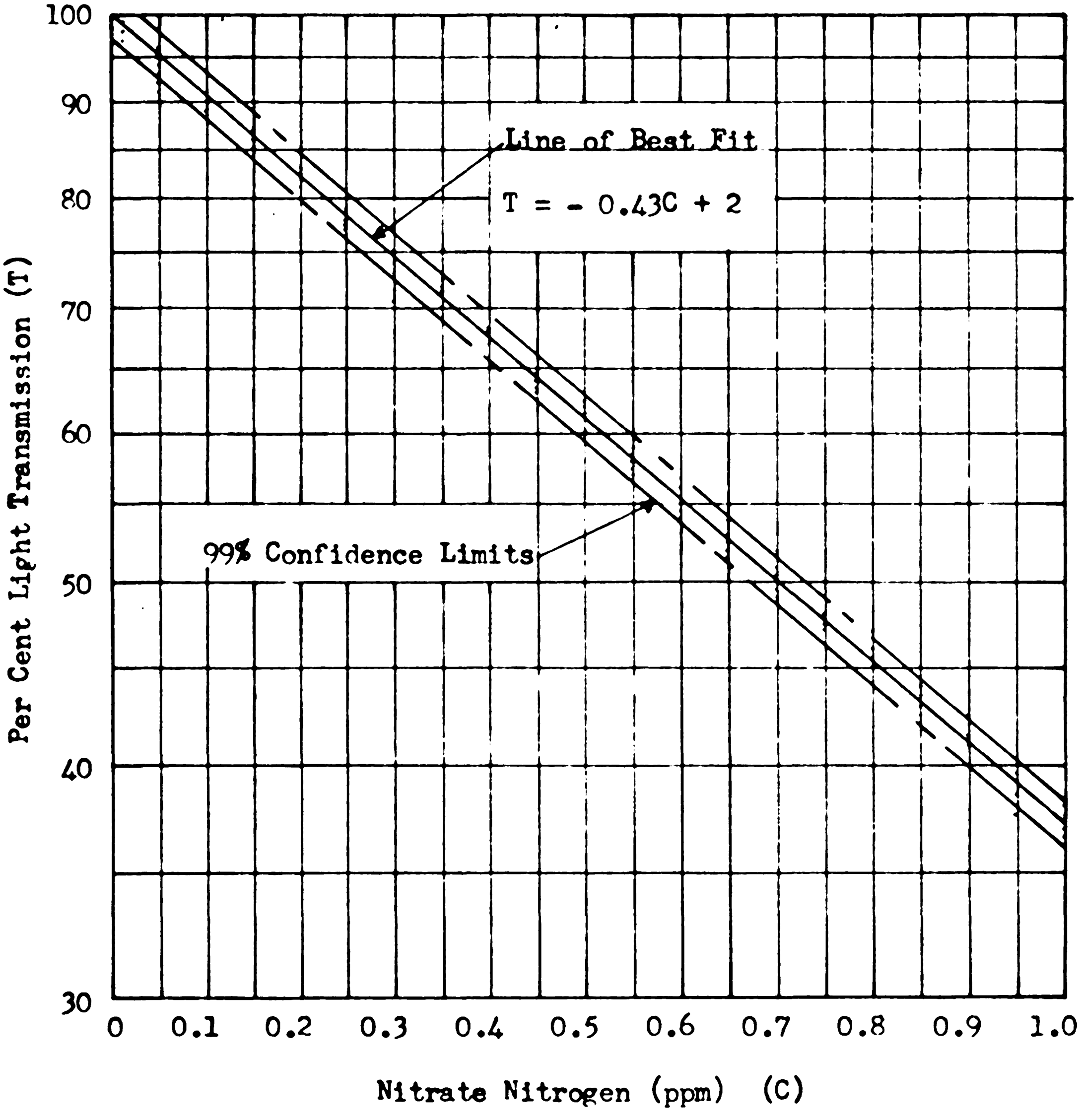

Figure 1

Nitrate Content-Per Cent Light Transmission for

405 Millimicron Monochromatic Light

TABLE 2

PER CENT LIGHT TRANSMISSION FOR NITRATE CONCENTRATION INDICATED

ppm NO_3-N	Run No. 1	Run No. 2	Run No. 3	Run No. 4
0	100	100	100	100
1	91.7	91.3	92	92.1
1.5	87.3	87.2	87.4	87.5
2	82.3	83	83.1	83.5
2.5	80.3	80	81	81
3	77	76.3	77	77
3.5	73	72.7	73.9	73.7
4	70	70	71	71.2
4.5	67.5	66.6	67.8	67.8
5	64.1	63.9	64.9	65.1
5.5	61.9	61.2	62.7	62.2
6	59.1	58.5	59.1	59.3
6.5	56.1	55.5	56.5	56.5
7	54.1	53.1	54.7	55
7.5	52	50.8	52.2	52.6
8	49.7	49	50	50
8.5	47.3	47	48	48.2
9	44.9	45	46	46.2
9.5	42.9	42.8	43.7	44
10	41.2	41.2	42.5	42.8

TABLE 2

PER CENT LIGHT TRANSMISSION FOR NITRATE CONCENTRATION INDICATED

ppm NO_3-N	Run No. 5	Run No. 6	Run No. 7	Run No. 8
0	100	100	100	100
1	92.2	93.2	92.3	91.9
1.5	87.1	88	87.9	87.3
2	83.9	84.9	83.6	83.1
2.5	81.1	81.5	81	81
3	76.5	78.3	77.1	77.1
3.5	73.5	74	74.1	73.7
4	71.5	72.4	71.6	71.2
4.5	67.8	69	68	67.7
5	65.3	66.1	65.2	64.9
5.5	63.2	63.9	63	62.8
6	60	60.7	60	60
6.5	57.3	58.2	57.5	57.1
7	55.8	56.2	55.3	55.1
7.5	52.9	53.5	53	52.6
8	50.8	52	50.3	50.4
8.5	49	49.3	49	48.3
9	47	47.9	46.7	46.4
9.5	44.9	45.6	44.5	44.2
10	43.1	44.2	43.1	42.6

TABLE 2

PER CENT LIGHT TRANSMISSION FOR NITRATE CONCENTRATION INDICATED

ppm NO_3-N	Run No. 9	Run No. 10	Run No. 11	Run No. 12
0	100	100	100	100
1	92.1	92.9	92	91.8
1.5	87.5	88	87.3	87.7
2	83.9	84.8	83.9	84.2
2.5	81	82.1	80.3	80.9
3	77	78.6	77.1	77.3
3.5	73.3	75	74.1	74.2
4	71	72.8	70.2	71
4.5	67.3	69.3	67	67.7
5	64.8	66.9	64.6	65.4
5.5	62.1	64.3	61.7	61.8
6	59.3	61.8	58.7	59.3
6.5	56.9	59	56.3	57.3
7	54.9	57	54.1	55
7.5	52.2	55	52	52.3
8	49.9	52.7	50	50.4
8.5	48	50.3	47.8	48.2
9	46	48.3	45.5	46.2
9.5	43.9	46.2	43.7	44.6
10	42.5	45	42.2	42.8

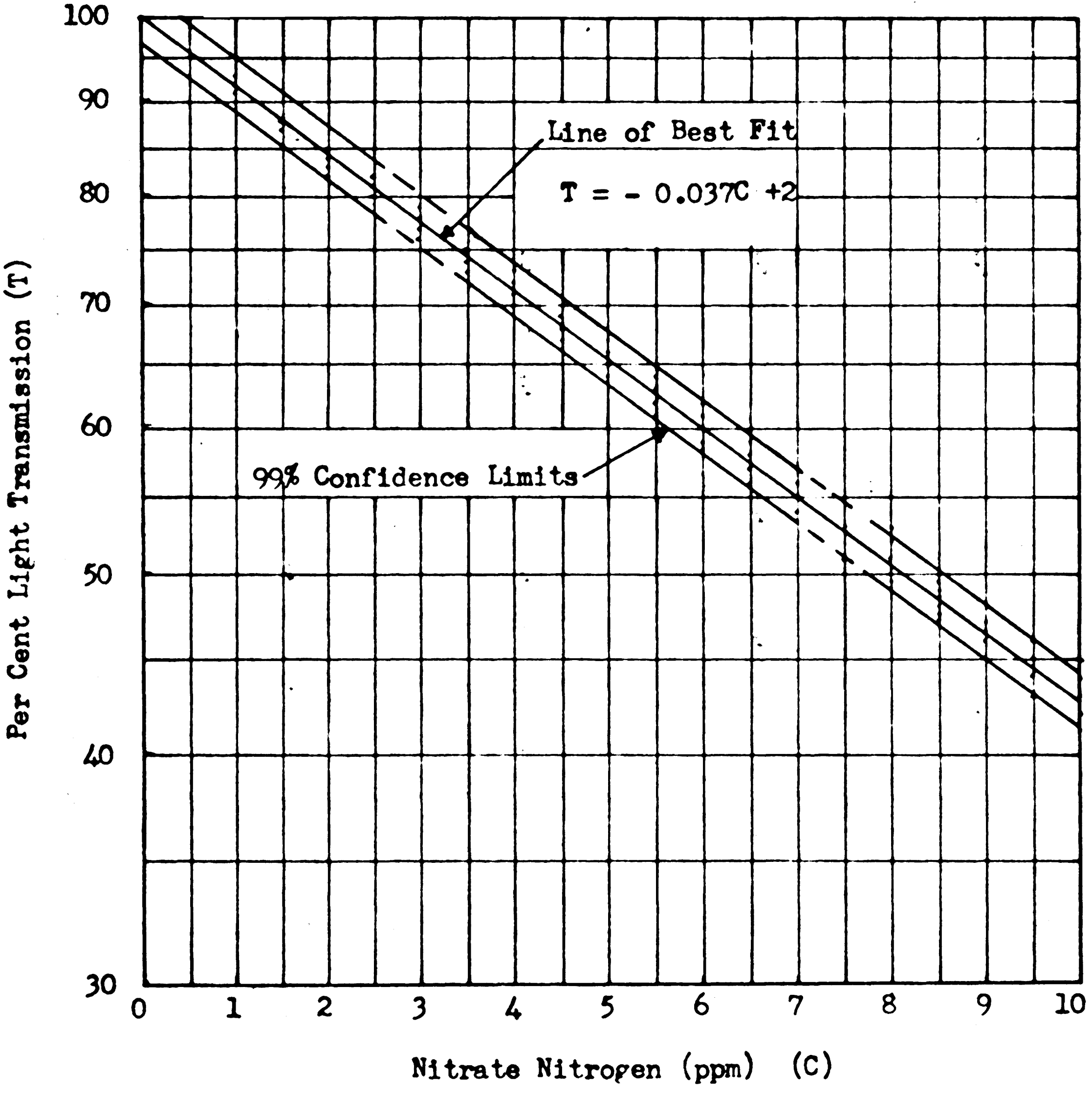

Figure 2

Nitrate Content-Per Cent Light Transmission For

465 Millimicron Monochromatic Light

V. GENERAL DISCUSSION

Practical employment of the relationship between nitrate content and per cent light transmission, using the phenoldisulfonic acid method of nitrate analysis, may be made. After determining the per cent light transmission through a sample of unknown nitrate content, the selection of the related nitrate concentration from the graphical relationship provided is a routine procedure.

In the 0-1 ppm nitrate nitrogen-logarithm, base 10, per cent light transmission relationship, the standard deviation for the $\log_{10}$ per cent transmission reading was computed in Appendix C to be 0.00498. This represents a nitrate nitrogen range of 0.012 ppm. The 99 per cent confidence range for $\log_{10}$ per cent light transmittance was 0.013 which represents a nitrate nitrogen range of 0.03 ppm. This indicates that the worst error to be expected 99 per cent of the time, in using the calibration curve, is an error of 0.03 ppm nitrate nitrogen.

The standard deviation in $\log_{10}$ per cent transmittance for the 0-10 ppm nitrate nitrogen range was computed in Appendix C to be 0.00556. This represents a nitrate nitrogen range of .15 ppm. The 99 per cent confidence value computed was 0.0145, representing a 0.4 ppm nitrate nitrogen range. This indicates that the worst error to be expected 99 per cent of the time, in using the calibration curve, is 0.4 ppm nitrate nitrogen.

For the visual comparison method with the phenoldisulfonic acid method of nitrate analysis, a series of color standards are prepared for visual comparison with the color developed in a sample of unknown nitrate content. The color standards recommended by Standard Methods are for 0.02, 0.06, 0.10, 0.14, 0.20, 0.30, 0.40, 0.70, 1.20, 2.00, 3.00, 4.00, and 6.00 ppm nitrate nitrogen. Since color intensities for nitrate contents up to one half the differences in nitrate contents in two adjacent standards would be read as the nearest standard, the accuracy of measurement would depend on the nitrate range between standards.

In the 0-1 ppm nitrate range, precision to be expected would, therefore be 0.04 at 0.1 ppm, 0.2 at 0.4 ppm, and 0.4 at 0.7 ppm. These values suffer by comparison with the 0.03 ppm precision to be expected using a spectrophotometer with the calibration curve derived.

In the 0-10 ppm nitrate nitrogen range, for visual comparison, a precision of 0.65 ppm at 1.2 ppm, 1 ppm at 3 ppm, and 1.5 ppm at 4 ppm is the best that can be expected. Again these values are much worse than the 0.4 ppm precision to be expected using a spectrophotometer and the calibration curve derived.

The relationship developed between nitrate content and per cent light transmittance can be used in water analyses to study nitrate contents in determining whether a water supply is safe for use by infants. Its use in the analysis of waters suspected of being sufficiently high in nitrate to have been the cause of an infant's contracting methemoglobinemia may help in determining an accurate harmful concentration of nitrate in water. In the performance of the preceding studies, the relationships provided as a refinement and improvement to the phenoldisulfonic acid method of nitrate analysis may aid in the elimination of methemoglobinemia in infants caused by high nitrate content water supplies.

VI. CONCLUSIONS

The number of per cent light transmission readings utilized in preparing the relationships between per cent light transmittance and nitrate content compensates for extreme, high or low, readings and provides data for a good relationship, calibration curve.

Using the calibration curves provided, no color standards need be made when the phenoldisulfonic acid method of nitrate analysis is employed. The per cent light transmitted through a sample of unknown nitrate content may be used with the calibration curve to determine the related nitrate content.

VII. SUMMARY

Calibration curves and mathematical relationships between nitrate content and per cent light transmission have been derived. The per cent light transmission was measured through samples with which the phenoldisulfonic acid method of nitrate analysis was employed to develop the characteristic yellow color.

Relationships were established for the nitrate nitrogen ranges of from 0 to 1 ppm and from 0 to 10 ppm using 405 and 465 millimicron monochromatic light respectively. In the 0-1 ppm relationship, an accuracy of 0.03 ppm nitrate nitrogen may be expected 99 per cent of the time. In the 0-10 ppm relationship, an accuracy of 0.4 ppm may be expected 99 per cent of the time.

VIII. BIBLIOGRAPHY

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PART 2

THE PRESENTATION OF NEW CONCEPTS ON FRICTION IN CONDUITS IN A FORM SUITABLE FOR DESIGN CRITERIA

I. INTRODUCTION

In recent publications regarding rough conduit flow, Dr. Henry M. Morris uses roughness index and relative roughness spacing parameters in his derivation of expressions for conduit roughness factor, for three different types of flow based on roughness conditions of conduit interiors. The publications reporting this approach are Dr. Morris' doctoral thesis entitled A New Concept of Flow in Rough Conduits[1] and his American Society of Civil Engineers transactions publication entitled Flow in Rough Conduits[2].

The roughness index is defined by (λ/h), the ratio of the roughness element spacing to the radial height of the projections. The relative roughness spacing is defined as (r_o/λ), the ratio of radius to spacing of the roughness elements.

In defining the three types of flow considered, the fact that a vortex was formed at the crest of roughness elements projecting into the flowing liquid which spread and dissipated radially and in a downstream direction was employed. If the vortex is formed and completely dissipated before the next roughness element is encountered so that smooth flow conditions exist prior to the encountering of the

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1. Morris, Henry M., Jr., A New Concept of Flow in Rough Conduits, A Thesis, University of Minnesota, 1950.

2. Morris, Henry M., Jr., Flow in Rough Conduits, Transactions, American Society of Civil Engineers, Paper No. 2745, 1955.

next roughness element, the flow is termed "Isolated Roughness Flow". If the next roughness element is encountered prior to dissipation of the vortex so that the new vortex is formed while the effects of the preceding vortex are still evident, the flow is called "Wake Interference Flow". "Skimming Flow" or "Quasi-Smooth Flow" exists when the roughness elements are so close together that the flow essentially skims the crests of the elements. Regions of dead water containing stable vortices occur between the roughness elements. The roughness elements and the stable vortices form a pseudo-wall over which conditions of flow are similar to smooth conduit flow.

Dr. Morris has also denoted the boundary between wake interference flow and isolated roughness flow by equating the resistance functions for the two types of flow.

In order that the most advantage may be derived from the principles formulated by Dr. Morris, the material presented should be in a form that is easily usable in computing the desired factor or ratio usable as design criteria. The presentation in such an easily used form is the purpose of this study.

II. WAKE INTERFERENCE FLOW

To present Dr. Morris' formulated friction factor relationships for various roughnesses, it was decided to employ graphical means of relating conduit Reynolds numbers and friction factors, as used in the Darcy equation, for various relative roughness spacing values. These relationships were computed from the following three known factors.

(1) Dr. Morris' equation for the resistance function in wake interference flow

$$k_a = \frac{1}{\sqrt{f}} - 2 \log_{10} \frac{r_o}{\lambda} = 1.75 + \frac{\sqrt{2}}{2} \left(\frac{c\ \lambda}{r_o}\right) \left(2.5 - \psi\right)$$

k_a = resistance function for axial flow

f = friction factor employed in the Darcy equation

r_o/λ = relative roughness spacing

r_o = pipe racius measured to the roughness element crests

λ = spacing of the wall roughness elements in the direction of flow

c = coefficient such that the distance $y = c\ \lambda$ locates the break between the core and wall velocity distributions

y = radial distance from the roughness crests to any point in the cross section

ψ = slope of the curve v/v_s versus $\log_e (y/\lambda)$ in the wall region

v = velocity of flow at any point in the cross section

v_s = shear velocity

Note: For two dimensional flow, a factor b is used instead of r_o, representing the distance from the roughness crests to the channel center.

(2) The relationship between the resistance function and the wall Reynolds number for wake interference flow furnished in the publications by Dr. Morris.

(3) The equation for the wall Reynolds number

$$R_w = \left(R_c \sqrt{f}\right) / \left(r_o / \lambda\right)$$

R_w = wall Reynolds number

R_c = conduit Reynolds number

f, r_o, and λ are as defined previously

Related wall Reynolds numbers and resistance functions were selected for the range from departure from smooth pipe relationship to the reaching of a constant value of resistance function. Using the first two terms equated in the resistance function formula, it was possible to compute friction factors for varying values of the resistance function and relative roughness spacing. The corresponding conduit Reynolds numbers were computed from the wall Reynolds number equation, having computed f, using the same relative roughness spacing values, and the wall Reynolds numbers related to the resistance functions used in computing f.

Resistance function-wall Reynolds number relationships for wake interference flow, caused by sharp edged strip roughnesses, corrugation strip roughnesses, and uniform spot roughnesses, are provided by Dr. Morris. Since these relationships differ, it was necessary to compute related friction factors and conduit Reynolds numbers for the three types of roughnesses. These computations, for varying relative roughness spacings, are presented as Table 3, for sharp edged strip roughnesses; Table 4, for corrugation strip roughnesses; and Table 5, for uniform spot roughnesses. The relationships are presented in graphical form as Figures 3, 4, and 5 for sharp edged strip, corrugation strip, and uniform spot roughnesses respectively.

Example of Wake Interference Flow Employing the Friction Factor Reynolds Number Relationship

A 24 in. circular corrugated metal pipe, 500 ft. long, has crests 3/4 in. apart. Water at 60° F. flows through the pipe at 3 ft. per second. The pipe flows full. What head loss can be expected?

(1) Relative roughness spacing

$$r_o/\lambda = \frac{12 \text{ in.}}{.75 \text{ in.}} = 16$$

(2) Reynolds number

$$R_c = \frac{VD}{\nu} = \left(\frac{24 \text{ in.}}{12 \text{ in./ft.}}\right)\left(\frac{3 \text{ ft./sec.}}{.000012 \text{ ft.}^2\text{/sec.}}\right) = 5 \times 10^5$$

V = velocity in ft. per sec.

D = diameter in ft.

ν = kinematic viscosity in ft.^{2}/sec.

(3) Friction factor from figure 4 for a relative roughness spacing of 16 and a Reynolds number of 5×10^5.

$$f = .04$$

(4) Head loss, using the Darcy equation

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = .04 \times \frac{500 \text{ ft./sec.} \times 12 \text{ in./ft.}}{24 \text{ in.}} \times \frac{3 \text{ ft./sec.}^2}{2 \times 32.2 \text{ ft./sec.}^2}$$

h_f = $\underline{\underline{1.40 \text{ ft.}}}$ Answer

TABLE 3

Related Values of Friction Factor and Reynolds Number for Wake Interference Flow, Sharp Edged Strip Roughness.

		$r_o/\lambda = 1$		$r_o/\lambda = 2$		$r_o/\lambda = 3$	
k_a	R_w	f	R_c	f	R_c	f	R_c
3.32	118	.0907	3.92×10^2	.0651	9.26×10^2	.0546	1.52×10^3
3.33	126	.0902	4.20×10^2	.0647	9.90×10^2	.0543	1.62×10^3
3.30	142	.0918	4.69×10^2	.0657	1.11×10^3	.0551	1.82×10^3
3.20	164	.0977	5.25×10^2	.0693	1.25×10^3	.0578	2.05×10^3
2.94	200	.1157	5.88×10^2	.0798	1.42×10^3	.0657	2.34×10^3
2.80	250	.1275	7.00×10^2	.0865	1.70×10^3	.0708	2.82×10^3
2.60	300	.1479	7.80×10^2	.0977	1.92×10^3	.0789	3.20×10^3
2.34	500	.1827	1.17×10^3	.1157	2.94×10^3	.0918	4.95×10^3
2.13	1000	.2204	2.13×10^3	.1342	5.46×10^3	.1047	9.27×10^3
2.00	1900	.2500	3.80×10^3	.1479	9.88×10^3	.1141	1.69×10^4
1.88	5000	.2829	9.40×10^3	.1626	2.48×10^4	.1240	4.26×10^4
1.83	10000	.2985	1.83×10^4	.1639	4.86×10^4	.1285	8.37×10^4
1.79	20000	.3121	3.58×10^4	.1751	9.56×10^4	.1342	1.64×10^5
1.76	50000	.3299	8.80×10^4	.1795	2.36×10^5	.1351	4.08×10^5
1.75	100000	.3264	1.75×10^5	.1811	4.70×10^5	.1362	8.13×10^5

$r_o/\lambda = 5$		$r_o/\lambda = 10$		$r_o/\lambda = 15$	
f	R_c	f	R_c	f	R_c
.0449	2.78×10^3	.0353	6.28×10^3	.0310	1.00×10^4
.0447	2.98×10^3	.0351	6.74×10^3	.0310	1.08×10^4
.0453	3.34×10^3	.0356	7.52×10^3	.0313	1.20×10^4
.0473	3.77×10^3	.0370	8.53×10^3	.0324	1.37×10^4
.0531	4.34×10^3	.0410	9.88×10^3	.0357	1.59×10^4
.0567	5.25×10^3	.0434	1.20×10^4	.0376	1.93×10^4
.0625	6.00×10^3	.0472	1.38×10^4	.0409	2.23×10^4
.0715	9.35×10^3	.0530	2.17×10^4	.0454	3.52×10^4
.0803	1.76×10^4	.0586	4.13×10^4	.0496	6.73×10^4
.0865	3.23×10^4	.0625	7.60×10^4	.0528	1.24×10^5
.0930	4.92×10^4	.0664	1.90×10^5	.0558	3.17×10^5
.0959	1.61×10^5	.0681	3.83×10^5	.0570	6.27×10^5
.0983	3.19×10^5	.0696	7.58×10^5	.0582	1.24×10^6
.1002	7.90×10^5	.0709	1.88×10^6	.0590	3.08×10^6
.1008	1.57×10^6	.0710	3.75×10^6	.0594	6.16×10^6

TABLE 3

Related Values of Friction Factor and Reynolds Number for Wake Interference Flow, Sharp Edged Strip Roughness.

r_o/λ = 30		r_o/λ = 60		r_o/λ = 125		r_o/λ = 250	
f	R_c	f	R_c	f	R_c	f	R_c
.0255	2.22×10^4	.0212	4.86×10^4	.0177	1.11×10^5	.0152	2.40×10^5
.0254	2.37×10^4	.0211	5.20×10^4	.0177	1.19×10^5	.0151	2.56×10^5
.0256	2.66×10^4	.0213	5.73×10^4	.0178	1.33×10^5	.0152	2.87×10^5
.0265	3.02×10^4	.0219	6.64×10^4	.0183	1.52×10^5	.0156	3.28×10^5
.0289	3.53×10^4	.0238	7.78×10^4	.0197	1.79×10^5	.0167	3.87×10^5
.0304	4.31×10^4	.0248	9.52×10^4	.0204	2.19×10^5	.0173	4.75×10^5
.0326	4.99×10^4	.0264	1.11×10^5	.0217	2.55×10^5	.0182	5.55×10^5
.0358	7.92×10^4	.0288	1.77×10^5	.0234	4.08×10^5	.0196	8.93×10^5
.0389	1.52×10^5	.0310	3.41×10^5	.0250	7.90×10^5	.0208	1.73×10^6
.0410	2.82×10^5	.0325	6.32×10^5	.0260	1.47×10^6	.0216	3.23×10^6
.0430	7.23×10^5	.0338	1.63×10^6	.0271	3.80×10^6	.0224	8.35×10^6
.0440	1.43×10^6	.0345	3.23×10^6	.0275	7.53×10^6	.0227	1.66×10^7
.0445	2.84×10^6	.0350	6.40×10^6	.0279	1.50×10^7	.0230	3.30×10^7
.0452	7.05×10^6	.0355	1.59×10^7	.0282	3.72×10^7	.0232	8.20×10^7
.0455	1.41×10^7	.0356	3.18×10^7	.0283	7.43×10^7	.0233	1.84×10^8

r_o/λ = 500		r_o/λ = 1000		r_o/λ = 5000	
f	R_c	f	R_c	f	R_c
.0131	5.15×10^5	.0115	1.10×10^6	.0095	3.36×10^6
.0131	5.50×10^5	.0115	1.18×10^6	.0094	3.87×10^6
.0132	6.19×10^5	.0116	1.32×10^6	.0095	4.34×10^6
.0135	7.06×10^5	.0118	1.51×10^6	.0097	4.98×10^6
.0144	8.34×10^5	.0125	1.79×10^6	.0102	5.91×10^6
.0149	1.03×10^6	.0129	2.20×10^6	.0105	7.28×10^6
.0156	1.20×10^6	.0135	2.58×10^6	.0110	8.56×10^6
.0167	1.94×10^6	.0144	4.17×10^6	.0116	1.39×10^7
.0176	3.77×10^6	.0151	8.13×10^6	.0121	2.73×10^7
.0182	7.03×10^6	.0156	1.52×10^7	.0125	5.08×10^7
.0189	1.82×10^7	.0161	3.90×10^7	.0128	1.31×10^8
.0191	3.62×10^7	.0163	7.83×10^7		
.0193	7.19×10^7	.0165	1.56×10^8		
.0195	1.79×10^8				

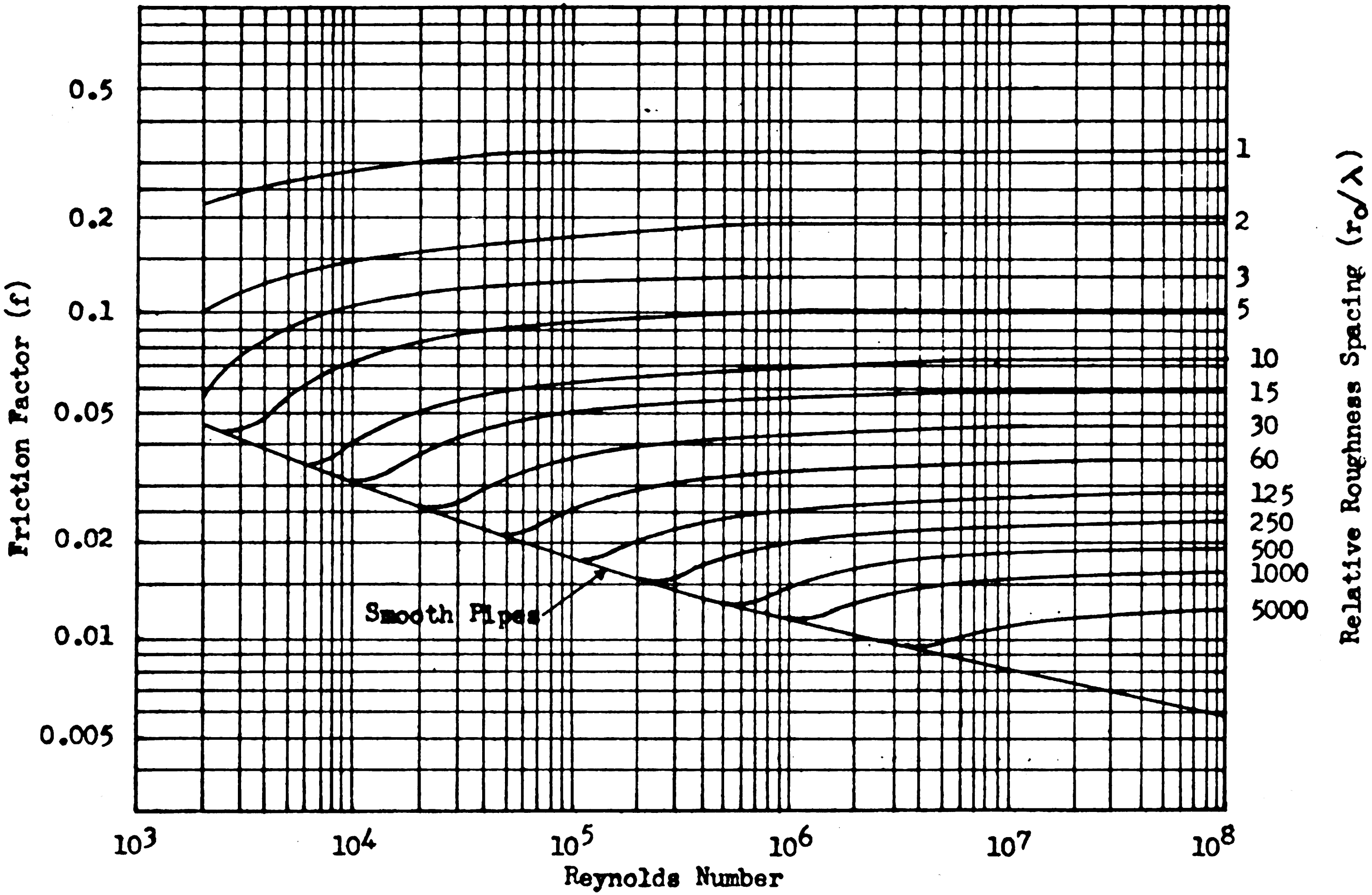

Figure 3. Friction Factor for Wake Interference Flow, Sharp Edged Strip Roughness

TABLE 4

Related Values of Friction Factor and Reynolds Number for Wake Interference Flow, Corrugation Strip Roughness.

		$r_o/\lambda = 1$		$r_o/\lambda = 2$		$r_o/\lambda = 3$	
k_a	R_w	f	R_c	f	R_c	f	R_c
3.34	120	.0896	4.01×10^2	.0644	9.43×10^2	.0541	1.55×10^3
3.40	150	.0865	5.10×10^2	.0625	1.20×10^3	.0526	1.96×10^3
3.44	260	.0845	8.94×10^2	.0613	2.10×10^3	.0516	3.43×10^3
3.42	300	.0855	1.03×10^3	.0619	2.42×10^3	.0521	3.94×10^3
3.34	500	.0896	1.67×10^3	.0644	3.94×10^3	.0541	6.45×10^3
3.15	1000	.1008	3.15×10^3	.0711	7.50×10^3	.0592	1.23×10^4
2.81	3000	.1267	8.43×10^3	.0860	2.04×10^4	.0704	3.39×10^4
2.47	8000	.1639	1.98×10^4	.1061	4.92×10^4	.0850	8.23×10^4
2.38	10000	.1766	2.38×10^4	.1126	5.96×10^4	.0896	1.00×10^5
2.18	20000	.2104	4.36×10^4	.1294	1.11×10^5	.1014	1.88×10^5
2.01	40000	.2475	8.04×10^4	.1486	2.08×10^5	.1134	3.56×10^5
1.90	70000	.2770	1.33×10^5	.1600	3.50×10^5	.1223	6.01×10^5
1.84	101000	.2954	1.86×10^5	.1679	4.92×10^5	.1275	8.49×10^5
1.78	200000	.3156	3.56×10^5	.1766	9.52×10^5	.1332	1.64×10^6
1.75	350000	.3265	6.13×10^5	.1811	1.65×10^6	.1362	2.85×10^6

$r_o/\lambda = 5$		$r_o/\lambda = 10$		$r_o/\lambda = 15$	
f	R_c	f	R_c	f	R_c
.0445	2.84×10^3	.0350	6.41×10^3	.0309	1.03×10^4
.0434	3.60×10^3	.0343	8.10×10^3	.0302	1.30×10^4
.0427	6.29×10^3	.0338	1.42×10^4	.0298	2.26×10^4
.0431	7.23×10^3	.0340	1.63×10^4	.0300	2.60×10^4
.0445	1.18×10^4	.0350	2.67×10^4	.0309	4.28×10^4
.0483	2.27×10^4	.0377	5.15×10^4	.0330	8.26×10^4
.0564	6.32×10^4	.0431	1.44×10^5	.0375	2.32×10^5
.0668	1.55×10^4	.0500	3.58×10^5	.0430	5.78×10^5
.0700	1.89×10^5	.0520	4.38×10^5	.0446	7.10×10^5
.0780	3.58×10^5	.0572	8.36×10^5	.0486	1.36×10^6
.0860	6.82×10^5	.0622	1.60×10^6	.0527	2.61×10^6
.0918	1.16×10^6	.0658	2.73×10^6	.0553	4.47×10^6
.0952	1.64×10^6	.0678	3.88×10^6	.0570	6.35×10^6
.0989	3.18×10^6	.0700	7.56×10^6	.0586	1.24×10^7
.1008	5.51×10^6	.0710	1.31×10^7	.0595	2.16×10^7

TABLE 4

Related Values of Friction Factor and Reynolds Number for Wake Interference Flow, Corrugation Strip Roughness.

$r_o/\lambda = 30$		$r_o/\lambda = 60$		$r_o/\lambda = 125$		$r_o/\lambda = 250$	
f	R_c	f	R_c	f	R_c	f	R_c
.0254	$2.26x10^4$	.0210	$4.97x10^4$	.0176	$1.13x10^5$	.0151	$2.44x10^5$
.0249	$2.85x10^4$	.0207	$6.26x10^4$	.0174	$1.42x10^5$	.0149	$3.07x10^5$
.0245	$4.97x10^4$	.0204	$1.09x10^5$	.0172	$2.48x10^5$	.0147	$5.36x10^5$
.0247	$5.73x10^4$	.0205	$1.26x10^5$	.0172	$2.85x10^5$	.0148	$6.16x10^5$
.0254	$9.43x10^4$	.0210	$2.06x10^5$	.0176	$4.71x10^5$	.0151	$1.02x10^6$
.0270	$1.83x10^5$	.0223	$4.02x10^5$	.0186	$9.18x10^5$	.0158	$2.00x10^6$
.0303	$5.17x10^5$	.0247	$1.15x10^6$	.0204	$2.62x10^6$	.0173	$5.70x10^6$
.0342	$1.30x10^6$	.0275	$2.89x10^6$	.0225	$6.66x10^6$	.0189	$1.55x10^7$
.0354	$1.60x10^6$	.0284	$3.56x10^6$	.0231	$8.22x10^6$	.0194	$1.80x10^7$
.0381	$3.08x10^6$	.0305	$6.88x10^6$	.0246	$1.60x10^7$	.0205	$3.49x10^7$
.0409	$5.93x10^6$	.0324	$1.33x10^7$	.0260	$3.10x10^7$	.0215	$6.80x10^7$
.0427	$1.02x10^7$	.0336	$2.29x10^7$	.0270	$5.33x10^7$	.0223	$1.17x10^8$
.0438	$1.45x10^7$	.0344	$3.27x10^7$	.0275	$7.61x10^7$		
.0449	$2.84x10^7$	.0352	$6.40x10^7$	.0280	$1.50x10^8$		
.0455	$4.93x10^7$	.0355	$1.11x10^8$				

$r_o/\lambda = 500$		$r_o/\lambda = 1000$		$r_o/\lambda = 5000$	
f	R_c	f	R_c	f	R_c
.0131	$5.24 x 10^5$	.0115	$1.12 x 10^6$	.0095	$3.71 x 10^6$
.0129	$6.60 x 10^5$	.0114	$1.41 x 10^6$	.0093	$4.66 x 10^6$
.0128	$1.15 x 10^6$	.0112	$2.46 x 10^6$	.0093	$8.11 x 10^6$
.0128	$1.32 x 10^6$	.0113	$2.83 x 10^6$	.0093	$9.34 x 10^6$
.0131	$2.19 x 10^6$	.0115	$4.67 x 10^6$	.0095	$1.54 x 10^7$
.0137	$4.28 x 10^6$	.0120	$9.15 x 10^6$	.0098	$3.14 x 10^7$
.0148	$1.23 x 10^7$	.0129	$2.64 x 10^7$	.0105	$8.79 x 10^7$
.0161	$3.15 x 10^7$	.0140	$6.78 x 10^7$	.0113	$2.26 x 10^8$
.0165	$3.89 x 10^7$	.0142	$8.38 x 10^7$		
.0174	$7.58 x 10^7$	.0150	$1.64 x 10^8$		
.0182	$1.48 x 10^8$				

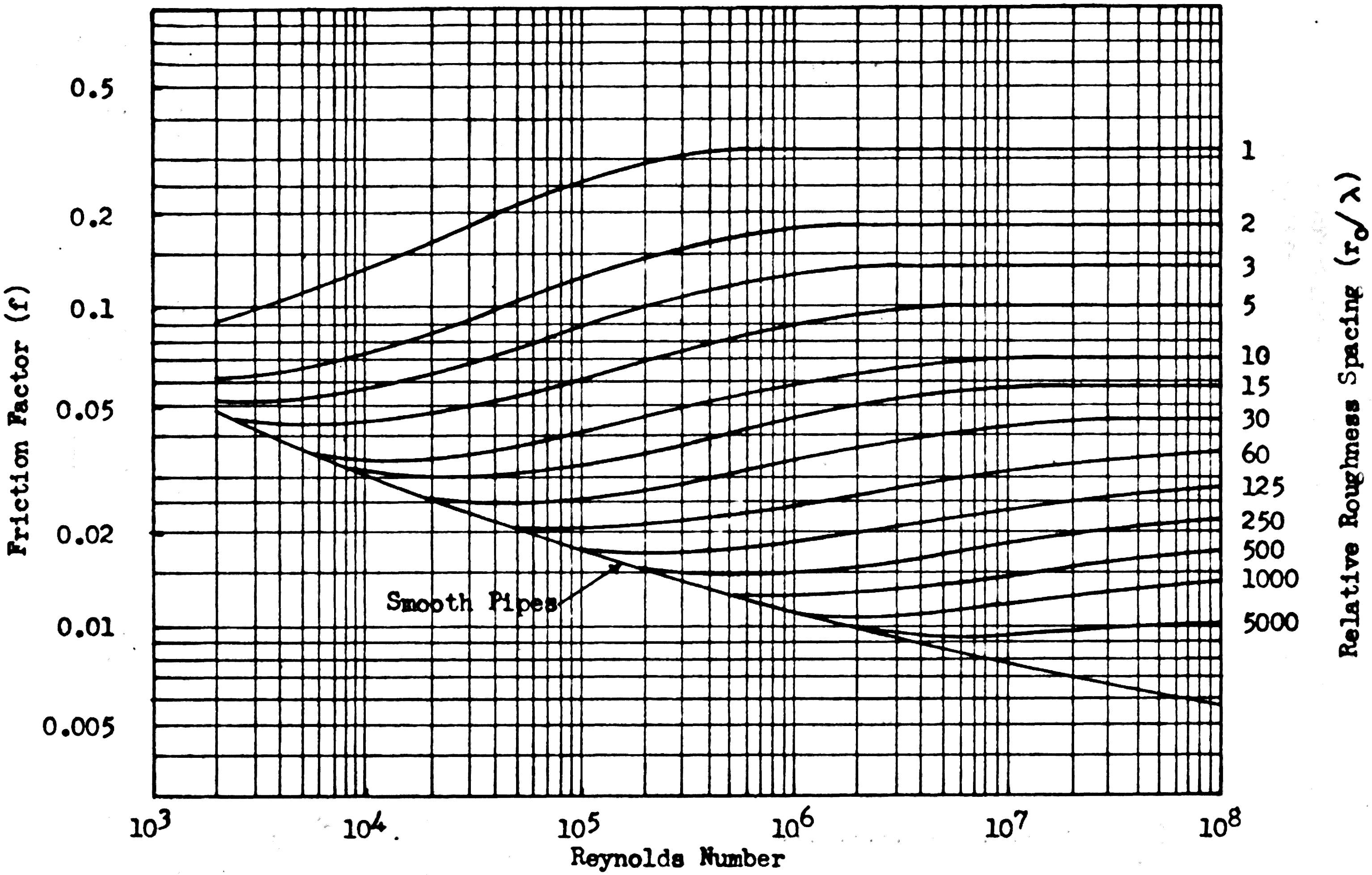

Figure 4. Friction Factor for Wake Interference Flow, Corrugation Strip Roughnesses

TABLE 5

Related Values of Friction Factor and Reynolds Number for Wake Interference Flow, Uniform Spot Roughness.

		$r_o/\lambda = 1$		$r_o/\lambda = 2$		$r_o/\lambda = 3$		$r_o/\lambda = 5$		$r_o/\lambda = 10$		$r_o/\lambda = 15$	
k_a	R_w	f	R_c	f	R_c	f	R_c	f	R_c	f	R_c	f	R_c
2.01	30	-	-	-	-	-	-	-	-	-	-	.0526	1.96×10^{3}
2.13	40	-	-	-	-	-	-	-	-	.0586	1.65×10^{3}	.0498	2.69×10^{3}
2.17	50	-	-	-	-	-	-	-	-	.0575	2.09×10^{3}	.0489	3.39×10^{3}
2.20	60	-	-	-	-	-	-	-	-	.0567	2.52×10^{3}	.0483	4.09×10^{3}
2.19	70	-	-	-	-	-	-	-	-	.0570	2.93×10^{3}	.0485	4.77×10^{3}
2.18	80	-	-	-	-	-	-	-	-	.0572	3.34×10^{3}	.0488	5.43×10^{3}
2.16	90	-	-	-	-	-	-	-	-	.0578	3.74×10^{3}	.0492	6.09×10^{3}
2.14	100	-	-	-	-	-	-	.0798	1.77×10^{3}	.0583	4.14×10^{3}	.0496	6.74×10^{3}
1.99	150	-	-	-	-	-	-	.0870	2.54×10^{3}	.0628	5.99×10^{3}	.0531	9.77×10^{3}
1.88	200	-	-	-	-	.1240	1.70×10^{3}	.0930	3.28×10^{3}	.0664	7.76×10^{3}	.0559	1.27×10^{4}
1.78	300	-	-	-	-	.1332	2.47×10^{3}	.0989	4.77×10^{3}	.0700	1.13×10^{4}	.0586	1.86×10^{4}
1.75	400	.3265	7.00×10^{2}	.1811	1.88×10^{3}	.1362	3.25×10^{3}	.1008	6.30×10^{3}	.0711	1.50×10^{4}	.0595	2.46×10^{4}

TABLE 5

Related Values of Friction Factor and Reynolds Number for Wake Interference Flow, Uniform Spot Roughness.

$r_o/\lambda = 30$		$r_o/\lambda = 60$		$r_o/\lambda = 125$		$r_o/\lambda = 250$	
f	R_c	f	R_c	f	R_c	f	R_c
.0406	$4.46x10^3$	.0322	$1.00x10^4$	.0260	$2.33x10^4$	.0216	$5.11x10^4$
.0388	$6.09x10^3$	.0309	$1.37x10^4$	.0250	$3.16x10^4$	.0208	$6.93x10^4$
.0381	$7.68x10^3$	.0305	$1.72x10^4$	.0247	$3.98x10^4$	.0206	$8.71x10^4$
.0377	$9.27x10^3$	.0301	$2.07x10^4$	.0245	$4.79x10^4$	.0204	$1.05x10^5$
.0379	$1.08x10^4$	.0302	$2.42x10^4$	.0246	$5.58x10^4$	.0205	$1.22x10^5$
.0380	$1.23x10^4$	.0303	$2.76x10^4$	.0246	$6.37x10^4$	.0205	$1.40x10^5$
.0383	$1.38x10^4$	.0306	$3.09x10^4$	.0248	$7.14x10^4$	.0206	$1.57x10^5$
.0386	$1.53x10^4$	.0308	$3.42x10^4$	.0250	$7.91x10^4$	.0208	$1.74x10^5$
.0410	$2.22x10^4$	.0325	$4.99x10^4$	.0262	$1.16x10^5$	.0217	$2.55x10^5$
.0428	$2.90x10^4$	.0338	$6.53x10^4$	.0271	$1.52x10^5$	.0224	$3.34x10^5$
.0447	$4.26x10^4$	.0351	$9.61x10^4$	.0281	$2.24x10^5$	.0231	$4.94x10^5$
.0453	$5.64x10^4$	.0355	$1.27x10^5$	.0284	$2.97x10^5$	.0233	$6.55x10^5$

$r_o/\lambda = 500$		$r_o/\lambda = 1000$		$r_o/\lambda = 5000$	
f	R_c	f	R_c	f	R_c
.0182	$1.11 x 10^5$	.0156	$2.40 x 10^5$	.0125	$8.06 x 10^5$
.0176	$1.51 x 10^5$	.0151	$3.25 x 10^5$	.0121	$1.09 x 10^6$
.0175	$1.89 x 10^5$	.0150	$4.09 x 10^5$	.0120	$1.37 x 10^6$
.0173	$2.28 x 10^5$	.0149	$4.92 x 10^5$	.0119	$1.65 x 10^6$
.0174	$2.66 x 10^5$	.0149	$5.73 x 10^5$	.0120	$1.92 x 10^6$
.0174	$3.03 x 10^5$	.0149	$6.54 x 10^5$	.0120	$2.19 x 10^6$
.0175	$3.40 x 10^5$	.0150	$7.34 x 10^5$	.0120	$2.46 x 10^6$
.0176	$3.77 x 10^5$	.0151	$8.14 x 10^5$	.0121	$2.73 x 10^6$
.0183	$5.54 x 10^5$	.0157	$1.20 x 10^6$	.0125	$4.02 x 10^6$
.0189	$7.28 x 10^5$	.0161	$1.58 x 10^6$	.0128	$5.30 x 10^6$
.0194	$1.08 x 10^6$	.0165	$2.33 x 10^6$	.0131	$7.86 x 10^6$
.0196	$1.43 x 10^6$	.0166	$3.10 x 10^6$	.0132	$1.04 x 10^7$

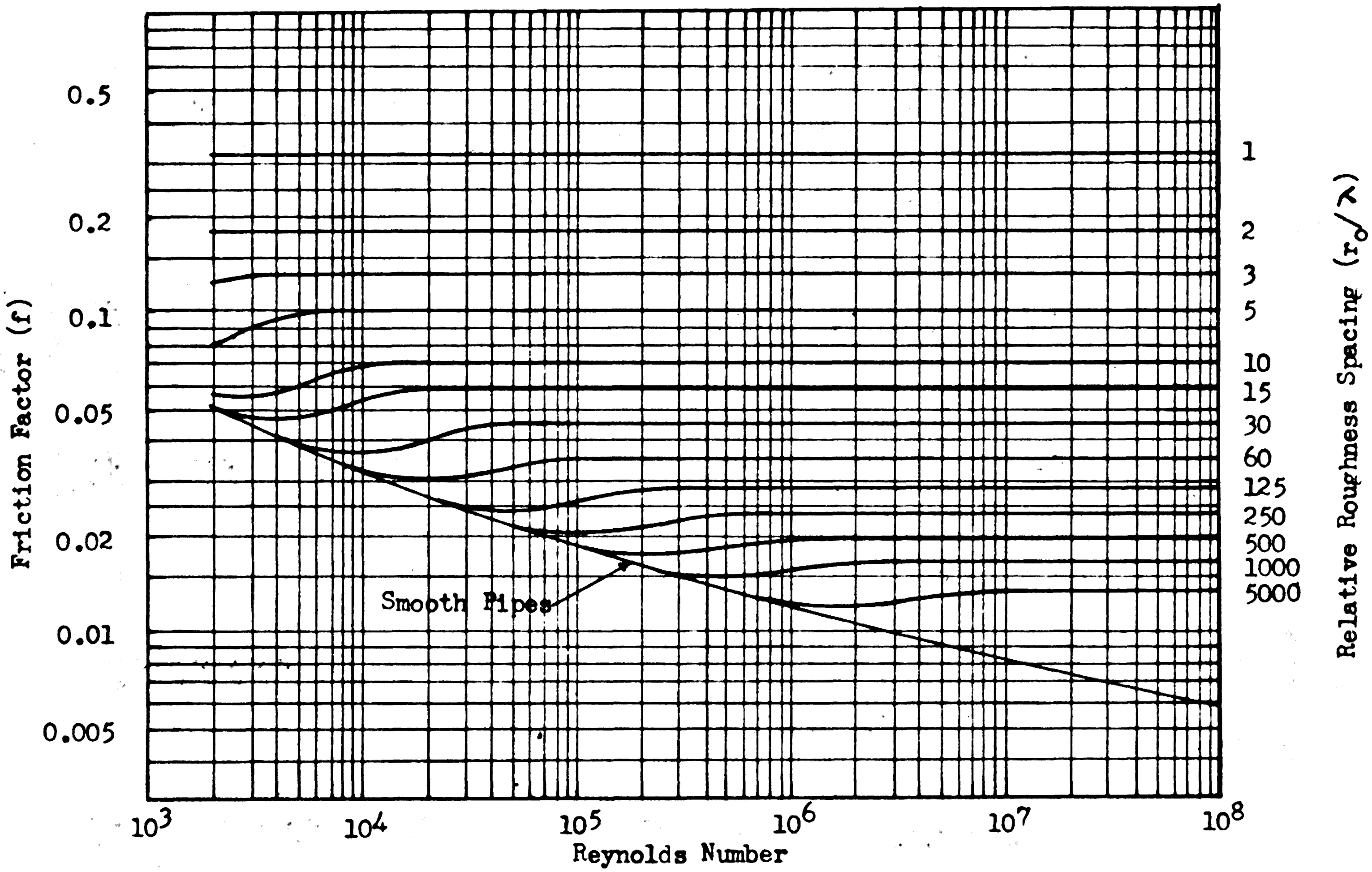

Figure 5. Friction Factor for Wake Interference Flow, Uniform Spot Roughnesses

III. ISOLATED ROUGHNESS FLOW

Dr. Morris' characterization of isolated roughness flow is of an independent development and dissipation of form turbulence behind each element and by friction drag turbulence along the wall between the elements.

To consider the concept of isolated roughness, the possibility of other roughness conditions being superimposed was eliminated by considering the wall between roughness elements as smooth. The following equation, then, equates the conduit friction factor for isolated roughness flow conditions.

$$f = f_s \left[1 + \frac{67.2\ C_d}{\lambda/h} \left(1 - \frac{ns}{P} \right) \right]$$

f = conduit friction factor

f_s = smooth conduit friction factor

C_d = drag coefficient for roughness element

λ/h = roughness index

λ = spacing of the wall roughness elements in the direction of flow

h = radial height of the wall roughness elements

n = number of individual roughness elements in a periphery

s = clear peripheral spacing between roughness elements

P = wetted perimeter

Since a relationship between Reynolds number and smooth conduit friction factor was available, it was decided that the plotting of curves relating Reynolds number and conduit friction factor for various values of a variable $(1 - ns/P)/(\lambda/h)$ could be accomplished. This variable was designated as ϕ and the aforementioned curves were plotted as Figure 6, using the drag coefficient for a commonly encountered roughness element, spheres. Values of ϕ were tabulated for varying values of ns/P and λ/h and are presented for convenient usage in Table 6. When roughness elements other than spheres are encountered, it is necessary to modify values of ϕ by multiplication with a factor listed in Table 7 for various roughness element shapes.

The values for drag coefficients used in Table 7 were selected by Dr. Morris in the light of present information on drag coefficients for submerged bodies in unconfined flow and in order to obtain reasonable correlation between observed and computed friction factors.

Table 8 contains the computed values of friction factors, for use in the Darcy equation, for different values of Reynolds number and ϕ used in the plotting of Figure 6.

Since λ/h values of unity and above are associated with skimming flow or wake interference flow, λ/h values in the neighborhood of unity were not used in the tabulation of ϕ values. The lowest value of λ/h used in the tabulation was three, which resulted in a maximum ϕ value of .333. The maximum correction to be multiplied to ϕ in Table 7 is 3.8, indicating that the maximum value of ϕ expected to be encountered would be .333 x 3.8 or 1.267. This appears to be a conservitively high value, since a λ/h value of three is probably not high enough to produce isolated roughness flow.

Analytic Example of Isolated Roughness Flow Employing the Friction Factor-Reynolds Number Curve

A 24 in. cast iron water pipe flows full at a velocity of 10 ft. per second. Joints are riveted so that rivet heads extend 1 in. into the flow. There are 50 rivets at each joint having a head diameter of 1 in. The joints are 5 ft. apart, the pipe is 1/4 in. thick, and the temperature of the flowing water is 60° F. What is the head loss in 1000 ft.?

(1) Determine λ/h

$$\frac{\lambda}{h} = \frac{5 \text{ ft.} \times 12 \text{ in./ft.}}{1 \text{ in.}} = 60$$

determine ns/P

P = wetted perimeter

$P = (\text{O.D.} - 2t) = (24 - 1/2) = 73.83$ in.

O. D. = outside diameter

t = pipe thickness

$n = 50$

$s = (P - 50d)/50 = (73.83 - 50\times1)/50 = .48$ in.

d = rivet head diameter

$ns/P = (50 \times .48)/73.83 = .325$

From Table 6 for $ns/P = .325$ and $\lambda/h = 60$

$\phi = .012$

(2) Since rivet head shapes are approximately spherical segments, the ϕ value must be multiplied by a correction factor of 0.75 from Table 7.

$.75 \times .012 = .009$

(3) Reynolds number

$$N_r = \frac{DV}{\nu} = \frac{23.5 \text{ in.} \times 10 \text{ ft./sec.}}{12 \text{ in./ft.} \times .000012 \text{ ft.}^2\text{/sec.}} = 1.63 \times 10^6$$

From figure 6 for Reynolds number = 1.63×10^6 and $\phi = .009$

$f = .0142$

(4) Head loss from Darcy equation

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

$$h_f = .0142 \times \frac{1000 \text{ ft.} \times 12 \text{ in./ft.}}{23.5 \text{ in.}} \times \frac{(10 \text{ ft./sec.})^2}{64.4 \text{ ft./sec.}^2}$$

$h_f = \underline{\underline{11.24 \text{ ft.}}}$ Answer

TABLE 6

Values of ϕ for Indicated Values of λ/h and ns/P

ns/P	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
λ/h										
3	.333	.300	.267	.233	.200	.167	.133	.100	.067	.033
5	.200	.180	.160	.140	.120	.100	.080	.060	.040	.020
10	.100	.090	.080	.070	.060	.050	.040	.030	.020	.010
15	.067	.060	.053	.047	.040	.031	.027	.020	.013	.007
30	.033	.030	.027	.023	.020	.017	.013	.010	.007	.003
60	.017	.015	.014	.012	.010	.009	.007	.005	.004	.002
125	.008	.007	.006	.006	.005	.004	.003	.002	.002	.001
250	.004	.004	.003	.003	.002	.002	.002	.001	.001	.0005
500	.002	.002	.001	.001	.001	.001	.001	.0005	.0005	.0002

TABLE 7

Drag Coefficients and Corrections to ϕ Values From Table 6 For Different Roughness Element Shapes

Type of Roughness Element	Drag Coefficient	Corrections To From Table 6
Spheres	.500	1.00
Spherical Segments	.375	.75
Cones	.450	.90
Short Angles	1.600	3.20
Long Angles	1.900	3.80
Sand Grains	.500	1.00

Example of use of correction

If a ϕ value of .030 is obtained from Table 6 and the roughness elements in the conduit being studied are in the shape of cones, the ϕ value of .030 must be multiplied by a correction of .9 to obtain a corrected ϕ value of .027 to use with Figure 6.

TABLE 8

Conduit Friction Factors in Isolated Roughness Flow

ϕ	0	.02	.05	.1	.18	.3	.5	.8	1.2
N_r									
3.5×10^3	.0414	.0692	.1110	.1810	.292	.459	.737	1.15	1.71
5×10^3	.0371	.0620	.0994	.162	.261	.411	.660	1.03	1.53
1×10^4	.0308	.0515	.0825	.134	.217	.341	.548	.859	1.27
2×10^4	.0258	.0471	.0691	.112	.182	.286	.459	.719	1.07
5×10^4	.0209	.0349	.0560	.0911	.147	.232	.372	.583	.864
1×10^5	.0180	.0301	.0482	.0785	.127	.199	.320	.502	.744
2×10^5	.0156	.0261	.0418	.0680	.110	.173	.278	.435	.645
5×10^5	.0131	.0219	.0351	.0571	.0923	.145	.233	.365	.541
1×10^6	.0117	.0196	.0314	.0510	.0825	.130	.208	.326	.483
2×10^6	.0103	.0172	.0276	.0449	.0726	.114	.183	.287	.426
5×10^6	.009	.0150	.0241	.0392	.0634	.0997	.160	.251	.372
1×10^7	.0081	.0135	.0217	.0353	.0571	.0897	.144	.226	.335
1×10^8	.0059	.0099	.0158	.0257	.0416	.0654	.105	.164	.244

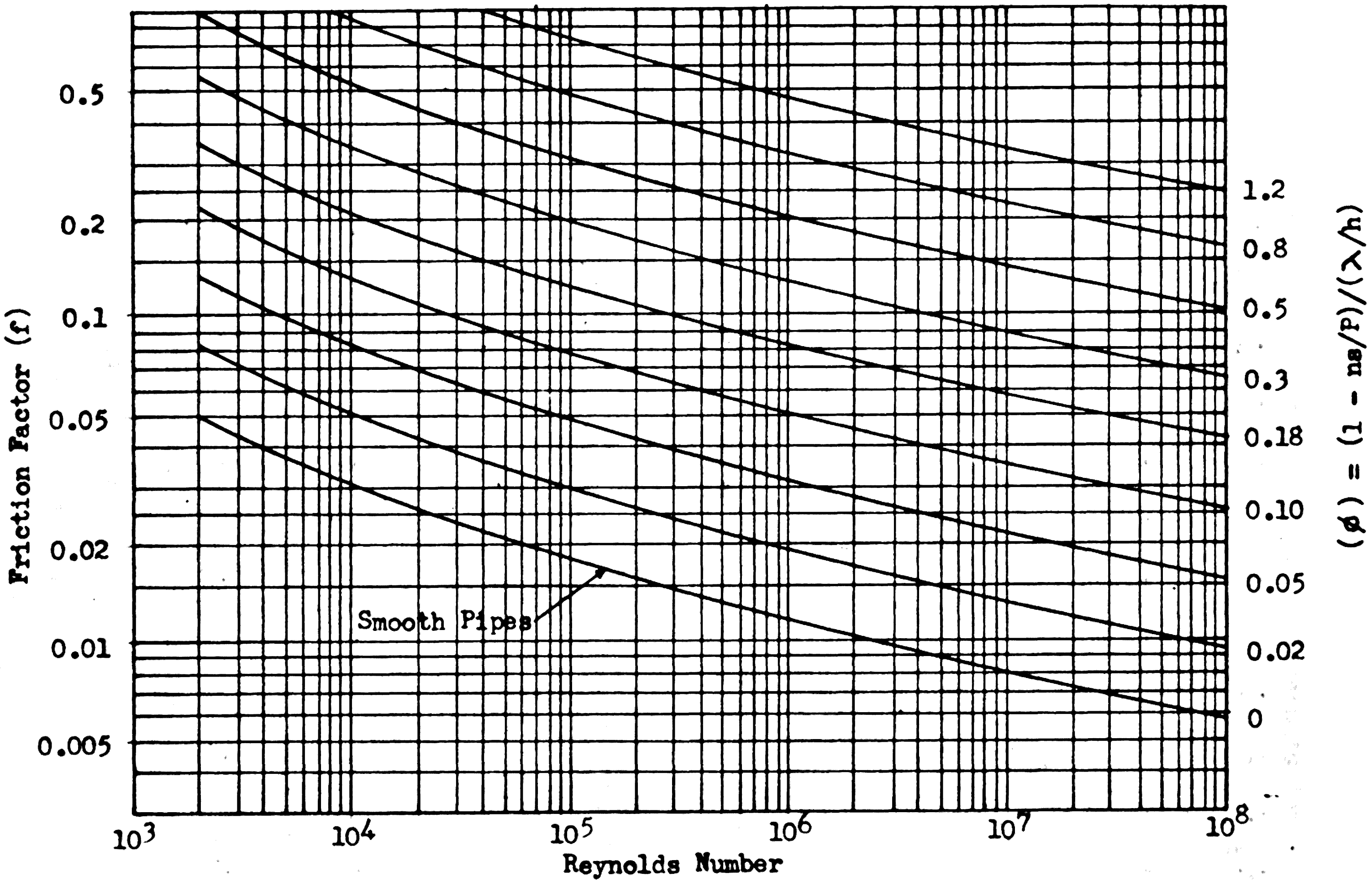

Figure 6. Friction Factor for Isolated Roughness Flow

IV. QUASI-SMOOTH FLOW

Dr. Morris' equation for friction factor in conduits for quasi-smooth flow indicates that vortex generation is the predominant source of turbulence.

$$f = f_s + \frac{\left(\frac{c_w\, v_w}{V}\right)^3}{\frac{\lambda}{j,h}}$$

in which

f = friction factor,

f_s = friction factor for smooth conduit,

c_w = a coefficient which when multiplied by v_w, is equal to the velocity of the vortex perimeter,

V = average velocity of flow,

λ = spacing of wall roughness elements in the direction of flow,

h = radial height of wall roughness projections,

j = groove width.

The roughness index term λ/h or λ/j to be used would be the larger value, or the smaller value between h and j.

As in the case of isolated roughness flow, it was decided to plot values of Reynolds number and friction factor for different values of a variable. The variable selected was χ representing $(c_w\, v_w/V)^3/(\lambda/h)$. Dr. Morris indicates the possible order of magnitude of c_w and v_w/V might approximate 1/2, and lists some experimental work where skimming flow was encountered, in which values of c_w of approximately 1/2 and values of v_w/V of 2/3 were obtained.

The boundary between quasi-smooth and wake interference flow conditions, according to Dr. Morris appears to be where the groove width j becomes significantly larger than the depth h.

Values of $\left(\frac{c_w\ v_w}{V}\right)^3$ for values of c_w and v_w/V in .1 in. increments from 0 to 1 are presented in Table 9. Values of χ for different $(c_w\ v_w/V)^3$ and (λ/h) values are presented in Table 10. Table 11 contains values of conduit friction factors for various Reynolds numbers and values of λ. The information in this table was obtained by adding a constant χ value to smooth pipe friction factors for a series of Reynolds numbers. This information was plotted as Figure 7.

<u>Example Showing Use of the Reynolds Number-Friction Factor Curves</u>

A 12 in., inside diameter, concrete pipe flows full of water. The surface of the pipe has roughness elements which cause quasi-smooth flow. Aggregate spacing is such that $\lambda/h = 1$. Pipe of this type has a c_w value of approximately .5 and a v_w/V ratio of .7. The velocity of flow is 10 ft. per sec. and the water temperature is 60°F. What is the friction factor for this pipe?

(1) From Table 9, since $c_w = .5$ and $v_w/V = .7$,

$(c_w\ v_w/V)^3 = .0429$

(2) From Table 10, since $\lambda/h = 1$ and $(c_w\ v_w/V)^3 = .0429$,

$\chi = .043$

(3) Reynolds number

$$N_r = \frac{DV}{\nu} = \frac{12\text{ in.} \quad 10\text{ ft./sec.}}{12\text{ in./ft.} \quad .000012\text{ ft.}^2\text{/sec.}}$$

$N_r = 8.33 \times 10^5$

(4) From Figure 7, since $\chi = .043$ and $N_r = 8.33 \times 10^5$

$f = \underline{\underline{.057}}$ = Answer.

TABLE 9

Computed Values of $\left(\frac{c_w\ v_w}{V}\right)^3$

c_w	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1
$\frac{v_w}{V}$											
0	0	0	0	0	0	0	0	0	0	0	0
.1	0	1×10^{-6}	8×10^{-6}	2.7×10^{-5}	6.4×10^{-5}	1.25×10^{-4}	2.16×10^{-4}	3.43×10^{-4}	5.12×10^{-4}	7.29×10^{-4}	1×10^{-3}
.2	0	8×10^{-6}	6.4×10^{-5}	2.16×10^{-4}	5.12×10^{-4}	1×10^{-3}	1.73×10^{-3}	2.74×10^{-3}	4.10×10^{-3}	5.83×10^{-3}	8×10^{-3}
.3	0	1.7×10^{-5}	2.16×10^{-4}	7.29×10^{-4}	1.73×10^{-3}	3.38×10^{-3}	5.83×10^{-3}	9.26×10^{-2}	1.38×10^{-2}	1.97×10^{-2}	2.7×10^{-2}
.4	0	6.4×10^{-5}	5.12×10^{-4}	1.73×10^{-3}	4.10×10^{-3}	8×10^{-3}	1.38×10^{-2}	2.20×10^{-2}	3.28×10^{-2}	4.67×10^{-2}	6.4×10^{-2}
.5	0	1.25×10^{-4}	1×10^{-3}	3.38×10^{-3}	8×10^{-3}	1.56×10^{-2}	2.7×10^{-2}	4.29×10^{-2}	6.4×10^{-2}	9.11×10^{-2}	.125
.6	0	2.16×10^{-4}	1.73×10^{-3}	5.83×10^{-3}	1.38×10^{-2}	2.7×10^{-2}	4.67×10^{-2}	7.41×10^{-2}	.111	.157	.216
.7	0	3.43×10^{-4}	2.74×10^{-3}	9.26×10^{-3}	2.20×10^{-2}	4.29×10^{-2}	7.41×10^{-2}	.118	.176	.250	.343
.8	0	5.12×10^{-4}	4.10×10^{-3}	1.38×10^{-2}	3.28×10^{-2}	6.4×10^{-2}	.111	.176	.262	.373	.512
.9	0	7.29×10^{-4}	5.83×10^{-3}	1.97×10^{-2}	4.67×10^{-2}	9.11×10^{-2}	.157	.250	.373	.531	.729
1	0	1×10^{-3}	8×10^{-3}	2.7×10^{-2}	6.4×10^{-2}	.125	.216	.343	.512	.729	1

TABLE 10

Values of χ

$\left(\frac{c_w v_w}{V}\right)^3$	0	.005	.010	.015	.020	.025	.030	.035	.040	.045	.050
λ/h											
.2	0	.0250	.0500	.0750	.1000	.1250	.1500	.1750	.2000	.2250	.2500
.4	0	.0125	.0250	.0375	.0500	.0625	.0750	.0875	.1000	.1125	.1250
.6	0	.0083	.0167	.0250	.0333	.0417	.0500	.0583	.0666	.0750	.0833
.8	0	.0063	.0125	.0188	.0250	.0313	.0375	.0438	.0500	.0563	.0625
1.0	0	.0050	.0100	.0150	.0200	.0250	.0300	.0350	.0400	.0450	.0500
1.2	0	.0042	.0083	.0125	.0167	.0208	.0250	.0292	.0333	.0375	.0417
1.4	0	.0036	.0071	.0107	.0143	.0179	.0214	.0250	.0286	.0321	.0357

TABLE 11

Conduit Friction Factors For Quasi-Smooth Flow

χ	0	.003	.009	.018	.035	.055	.080	.12	.18	.25
N_r										
3.5×10^3	.0414	.0434	.0504	.0594	.0764	.0964	.1214	.1614	.2214	.2914
5×10^3	.0371	.0401	.0461	.0551	.0721	.0921	.1171	.1571	.2171	.2871
1×10^4	.0308	.0338	.0398	.0488	.0658	.0858	.1108	.1508	.2108	.2808
2×10^4	.0258	.0288	.0348	.0438	.0608	.0808	.1058	.1458	.2058	.2758
5×10^4	.0209	.0239	.0299	.0389	.0559	.0759	.1009	.1409	.2009	.2709
1×10^5	.0180	.0210	.0270	.0360	.0530	.0730	.0980	.1380	.1980	.2680
2×10^5	.0156	.0186	.245	.0336	.0506	.0706	.0956	.1356	.1956	.2656
5×10^5	.0131	.0161	.0221	.0311	.0481	.0681	.0931	.1331	.1931	.2631
1×10^6	.0117	.0147	.0207	.0297	.0467	.0667	.0917	.1317	.1917	.2617
2×10^6	.0103	.0133	.0193	.0283	.0453	.0653	.0903	.1303	.1903	.2603
5×10^6	.0090	.0120	.0180	.0270	.0440	.0640	.0890	.1290	.1890	.2590
1×10^7	.0081	.0111	.0171	.0261	.0430	.0631	.0881	.1281	.1881	.2581
1×10^8	.0059	.0089	.0149	.0239	.0409	.0609	.0859	.1259	.1859	.2559

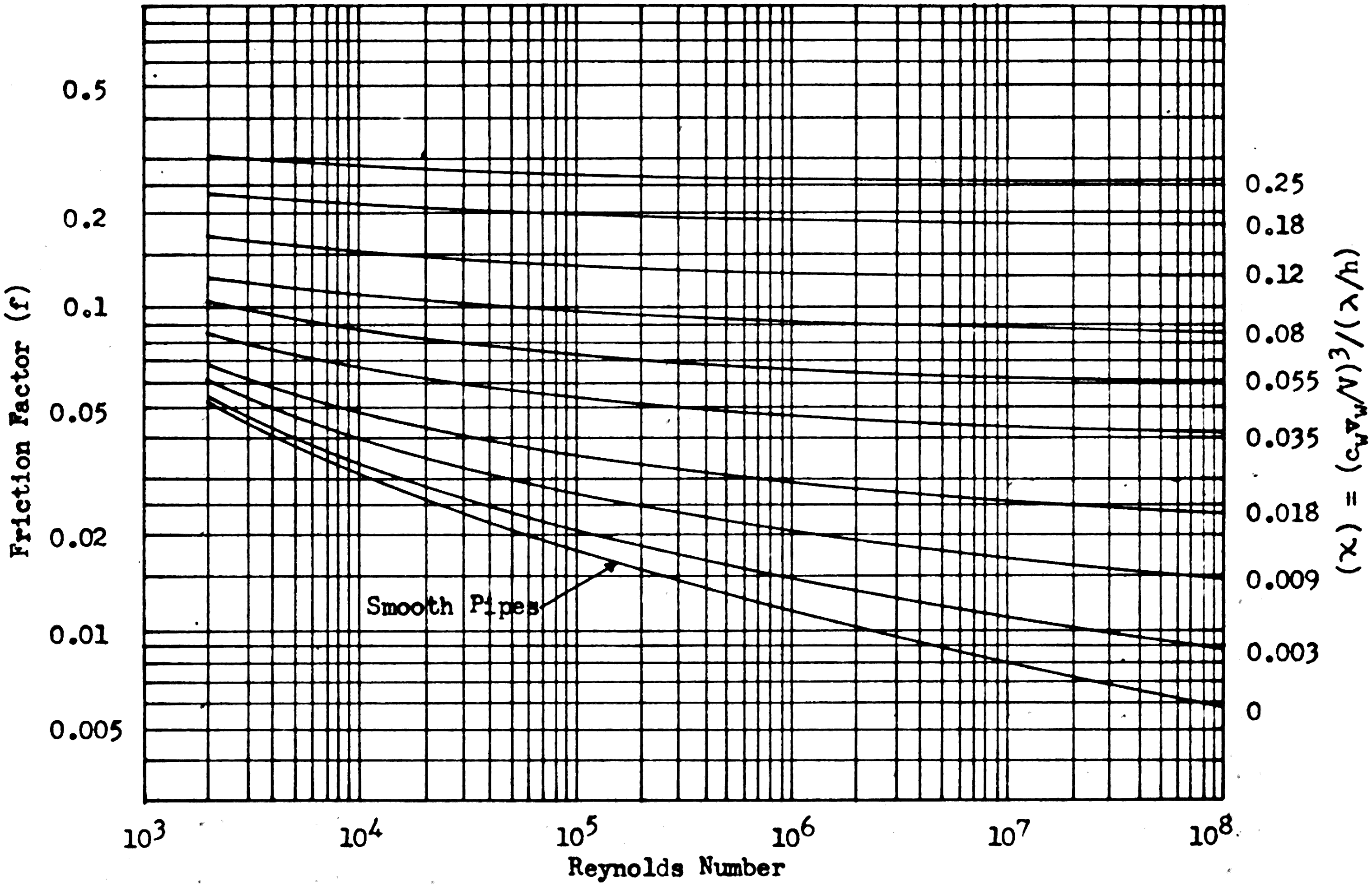

Figure 7. Friction Factor for Quasi-Smooth (Skimming) Flow

V. CONCLUSIONS

The Reynolds number friction factor curves for wake interference, isolated roughness, and quasi-smooth flow conditions make possible easier computation of the desired factor in the relationship.

The curves graphically illustrate Dr. Morris' concepts on conduit flow and could lead to further adoption and use of these concepts in the field of hydraulics.

VI. SUMMARY

Friction factor-Reynolds number curves have been developed for Dr. Morris' concepts of wake interference, isolated roughness, and quasi-smooth flow conditions. Tabulated data to aid in obtaining the variables, dependent or conduit surface roughness, which relate Reynolds numbers with friction factors are provided.

Analytical examples illustrating the use of the graphical Reynolds number-friction factor relationships are provided for each of the three flow conditions.

VII. ACKNOWLEDGMENTS

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APPENDICES

APPENDIX A

Equation for the Curve of Best Fit by the Method of Least Squares

$T = \log_{10}$ per cent transmittance

C = nitrate nitrogen content in ppm

n = number of readings

m = slope

b = the "T" intercept

Normal Equations

$$(1) \quad \Sigma T = m \Sigma C + nb$$

$$(2) \quad \Sigma TC = m \Sigma C^2 + b \Sigma C$$

From the data in Table 1 for the 0-1 ppm nitrate nitrogen range

$\Sigma T = 829.364$

$\Sigma C = 220.71$

$\Sigma TC = 377.027$

$\Sigma C^2 = 150.675$

$n = 462$

Substituting numerical values in the normal equations

$$(1) \quad 829.364 = 220.71m + 462b$$

$$(2) \quad 377.027 = 150.675m + 220.71b$$

Solving the two equations simultaneously

$b = 2$

$m = -0.43$

Equation for curve of best fit

$$T = -0.43C + 2$$

From the data in Table 2 for the 0-10 ppm nitrate nitrogen range

$\Sigma T = 433.285$

$\Sigma C = 1254$

$\Sigma TC = 2187.779$

$\Sigma C^2 = 8607$

$n = 240$

Substituting numerical values in the normal equations

(1) $433.285 = 1254m + 240b$

(2) $2187.779 = 8607m + 1254b$

Solving the two equations simultaneously

$b = 2$

$m = -0.037$

Equation for curve of best fit

$T = -0.037C + 2$

APPENDIX B

Calculation of Correlation Coefficient Between Nitrate Content and Logarithm, base 10, Per Cent Light Transmittance

The equation for the correlation coefficient

$$r = \frac{\Sigma TC - \frac{(\Sigma T)(\Sigma C)}{n}}{\sqrt{\left[\Sigma C^2 - \frac{(\Sigma C)^2}{n}\right]\left[\Sigma T^2 - \frac{(\Sigma T)^2}{n}\right]}}$$

For the range 0-1 ppm nitrate nitrogen

$\Sigma T = 829.364$

$\Sigma T^2 = 1496.988$

$\Sigma C = 220.71$

$\Sigma C^2 = 150.675$

$\Sigma TC = 377.027$

$n = 462$

Substituting numerical values in the correlation coefficient equation

$$r = \frac{377.027 - \frac{(829.364)(220.71)}{462}}{\sqrt{\left[150.675 - \frac{(220.71)^2}{462}\right]\left[1496.988 - \frac{(829.364)^2}{462}\right]}}$$

$r = -0.9993$

For the range 0-10 ppm nitrate nitrogen

$$\Sigma T = 433.285$$

$$\Sigma T^2 = 785.061$$

$$\Sigma C = 1254$$

$$\Sigma C^2 = 8607$$

$$\Sigma TC = 2187.779$$

$$n = 240$$

Substituting numerical values in the correlation coefficient equation

$$r = \frac{2187.779 - \frac{(433.285)(1254)}{240}}{\sqrt{\left[8607 - \frac{(1254)^2}{240}\right]\left[785.061 - \frac{(433.285)^2}{240}\right]}}$$

$$r = -0.9987$$

APPENDIX C

Standard Deviation Calculations

(σ_r) = Standard deviation

r = Correlation coefficient

The standard deviation equation

$$(\sigma_r)^2 = (1 - r^2) \left(\frac{\Sigma(T^2) - \frac{(\Sigma T)^2}{n}}{n - 2} \right)$$

For the 0-1 ppm nitrate nitrogen range

$r = -0.9993$

$\Sigma T^2 = 1496.988$

$\Sigma T = 829.364$

$n = 462$

Substituting numerical values

$$(\sigma_r)^2 = \left(1 - (-0.9993)^2\right) \left(\frac{1496.988 - \frac{(829.364)^2}{462}}{460} \right)$$

$$(\sigma_r)^2 = 0.0000248$$

$$\sigma r = 0.00498$$

A standard deviation of 0.00498 in $\log_{10}$ per cent transmittance represents a nitrate nitrogen content range of 0.012 ppm.

For 99 per cent confidence with 460 degrees of freedom

"t" (statistical tables) = 2.59

Confidence limit = .00498 × 2.59

= .0129 per cent light transmittance

Representing 0.03 ppm nitrate nitrogen

For the 0-10 ppm nitrate nitrogen range

$r = -0.9987$

$\Sigma (T^2) = 785.061$

$\Sigma T = 433.285$

$n = 240$

Substituting numerical values in the equation for standard deviation

$$(\sigma_r)^2 = \left(1 - (-0.9987)^2\right)\left(\frac{785.061 - \frac{(433.285)^2}{240}}{238}\right)$$

$$(\sigma_r)^2 = 0.00003094$$

$$\sigma_r = 0.00556$$

A standard deviation of 0.00556 in $\log_{10}$ per cent transmittance represents a nitrate nitrogen content range of 0.15 ppm.

For 99 per cent confidence with 258 degrees of freedom

"t" (statistical tables) = 2.60

Confidence limit = 0.00556 × 2.60

= 0.0145 per cent light transmittance

Representing 0.4 ppm nitrate nitrogen