THE ECONOMIC DESIGN OF MULTIVARIATE

ACCEPTANCE SAMPLING PLANS

by

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CHAPTER I

INTRODUCTION AND HISTORICAL BACKGROUND

Man's concern for the quality of manufactured product is as old as civilization itself. The Egyptian construction inspector in the 15th century B.C. and the master craftsman in the early 19th century (8) were both ever-watchful to insure that each unit produced under his supervision was of the highest quality. To each of these people, and to each of the competent manufacturers in their interim, quality of product was a matter of personal pride as well as professional responsibility. The industrial revolution in the late 19th century brought rapid change to the industrial environment and, as a result, essentially removed the master craftsman from the forefront of American manufacture. The time-tested methods for assuring product quality were, however, retained and continued to be widely used. Many manufacturers felt that the only means to assess the overall quality of a lot was to inspect each unit of product and to judge it individually as being either acceptable or unacceptable. The theory that subsequently evolved was one which considered only a portion of a lot as the basis upon which the overall lot quality could be determined.

Statistical theory was first effectively applied to the control of product quality in the mid-1920's. Early research in the field was conducted at the Bell Telephone Laboratories by W. A. Shewhart, H. F. Dodge, and H. G. Romig (6, 33). These pioneers laid the foundation for the modern "control chart", set the pattern for subsequent applications of statistical methods to process control, and took the leadership in developing the application of statistical theory to sampling inspection. Shewhart, in his book (33), was primarily concerned with process control, using a "control chart" for the process mean to indicate the presence of assignable causes which may be contributing to the deterioration of product quality. Dodge and Romig (6) concentrated their efforts in the area of applying statistical theory to sampling inspection. Research continued at the Bell facility but industry was slow to accept these techniques.

The outbreak of World War II in 1939 caused the armed services to enter the commercial market as large consumers of American output and, in that capacity, the military had a considerable impact on existing quality standards and practices. Their adoption of scientifically designed sampling plans left those industrial firms which desired to remain government suppliers no alternative but to develop programs to prepare themselves for the implementation of statistical quality control methods.

The development of acceptance sampling procedures is only one of many ways in which statistical theory can be applied with respect to the control of product quality. The American Society for Quality Control (1) defines acceptance sampling as "sampling in which decisions are made to accept or to reject product; also, the science that deals with procedures by which decisions to accept or to reject product are based on the results of samples." This technique is concerned with determining whether the inspection lot as a whole conforms to a pre-specified quality standard, while examining only a randomly selected portion of the lot. Acceptance sampling plans are the scientifically designed procedures for the partial examination of an inspection lot and for specifying the acceptance/rejection criterion. These plans are broadly classified as either attributes or variables.

Attributes acceptance sampling plans attempt to statistically determine simply whether an inspection lot possesses, or does not possess, the specified quality characteristic to an acceptable degree. This determination is often made visually, or by using "go - no go" gauges, or by any other means which identifies the presence or absence of the attribute under consideration. For example, a manufacturer of cylindrical steel bars may wish to statistically determine whether or not the bars in an inspection lot of size L have the specified diameter. Deciding to implement an attributes acceptance sampling plan, he draws a randomly selected sample of size n_1 from the inspection lot, applies a "go - no go" gauge to each of the units, and records the number of bars which conform to the specified criteria and the number that do not. If the outcome of the inspection procedure shows, say, that two or less of the bars do not conform to the gauge, the manufacturer will accept the lot as having the specified diameter. If, on the other hand, three or more of the bars do not conform to the acceptance criteria, he will reject the lot for not possessing the desired attribute to an acceptable degree (in this case, too many bars having diameter outside the specification limits). The manufacturer has hence reached the statistical decision to either accept or to reject the inspection lot.

Variables sampling plans are fundamentally different from the attributes plans in that the former requires the quality characteristic under consideration to be physically measured on some continuous scale. These measurements can be made by using one of many types of micrometers, calipers, or other direct or indirect measuring devices, each of which may be specially designed for the particular task at hand. In this case, the sample mean or the sample range, or any other pertinent sample statistic is computed from the observed measurements and is used to assess the overall quality of the inspection lot. Owen (24) considers variables sampling plans which call for the acceptance of the inspection lot if both $\bar{X} - kS > LS$ and $\bar{X} + kS < US$, where LS and US are the lower and upper specification limits, respectively, and where $\bar{X}$ is the sample mean, S is the sample standard deviation, and k is an appropriate constant. Owen further notes that sampling plans based on the assumption that the characteristic being measured follows a normal distribution may save considerably on sample size over an attributes plan, pointing out that there is considerable justification for using, wherever applicable, variables acceptance sampling plans. In other research, Owen (23) offers the premise that variables acceptance sampling plans are not widely used except possibly in the area of life testing because of the uncertainty that the variate being measured is normally distributed, or the certainty that it is not. He concludes that it would be best, of course, to use sampling plans tailor-made to the particular distribution present, but if the sample size is not extremely small and the tail probabilities of the distribution are small then the effect of non-normality may not be severe.

Referring to the previous example, the manufacturer chooses here to implement a variables plan. He selects a random sample of size n_2 from the inspection lot (sample size n_2 for the variables plan is not necessarily equal to sample size n_1 for the attributes plan), measures the diameter of each of the bars, and then computes, say, the sample mean of these observations. Finally, the manufacturer assesses the overall quality of the inspection lot by comparing the computed value of the sample mean to statistically determined control limits. If the value of the sample mean falls between the control limits, he will accept the lot as possessing the specified diameter. Otherwise, he will reject the lot. The manufacturer has again reached a statistical decision to either accept or to reject the inspection lot. Thus, the decision alternatives which exist under the variables plan are the same as those which existed under the attributes plan.

The foremost concern of many manufacturers is, however, economic in nature. In this context, quality control plans might well be designed in a manner that attempts to minimize the total expected cost resulting from the implementation of the quality control plan. Widely referenced research on the economics of quality control was conducted by Duncan (7) and was primarily concerned with the economic design of $\bar{X}$ charts used to maintain the current control of a process. In this respect, he proposes a criterion that measures the approximate net income of a process when random shifts in the process mean may occur. Using elementary waiting line theory in his development, Duncan's solution provides the approximate optimal values of the sample size (n), the time between the taking of successive samples (h), and a constant of control limit dispersion (k). Goel, et. al. (13), present an iterative algorithm for the exact determination of these three parameters for Duncan's model.

Hald (14) points out that the full significance of a sampling inspection plan can only be developed on the basis of the prior distribution and the economic consequences of acceptance and rejection of the inspection lot. In his research, Hald derives the two basic equations for the determination, in the attributes case, of the sample size and the acceptance number by minimizing the given cost function without initially making any assumptions on the functional form of the prior distribution. He then investigates the solution for several reproducable distributions. Other authors (17, 27, 28) have investigated variations of this problem, all arriving at conclusions similar to those of Hald.

There are various costs which can be incurred by virtue of implementing an acceptance sampling plan. These costs are fundamentally divided into two categories:

a) the cost of reaching a decision to either accept or reject the inspection lot, and b) the cost associated with implementing that decision. Figure 1-1 shows these two costs areas schematically, and their components as developed in this research. In Chapter III, these cost components are developed in the framework of a mathematical model. These cost areas are characteristic of both attributes and variables cost-based quality control. It is, then, the minimum or near minimum cost quality control plan which is often the most attractive to management.

Again referring to the quality control situation described earlier in this chapter, the manufacturer now decides to consider a cost-based variables acceptance sampling plan. He must now mathematically model the situation, making various decisions as to the disposition of units found to be defective in sampling inspection as well as to the disposition of the entire inspection lot should the lot be rejected. His next task, and the one considered by some as being his most difficult, is to assign values to the various components of cost which are included in the cost model. Finally, the model must be optimized in order to determine the values of the decision variables (i.e. the sample size, lower control limit, and upper control limit) that specify the minimum cost quality control plan which he should subsequently implement. It

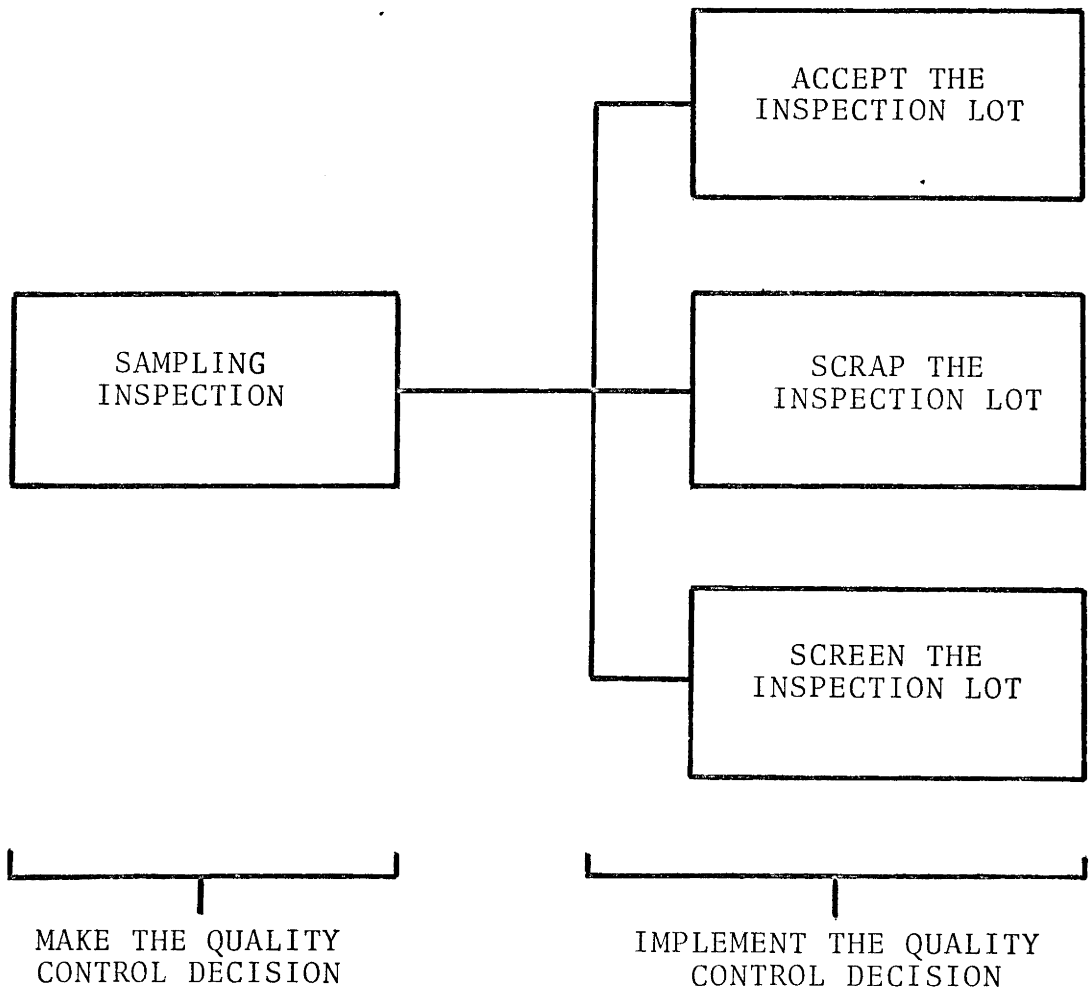

Figure 1-1. Schematic of the Major Cost Components.

is important to note that the acceptance sampling plan specified in this case is not necessarily, and is not usually, the same as that specified in the purely statistical case for variables acceptance sampling.

With the exception of recent research, cost-based as well as statistically computed acceptance sampling plans have been constructed upon the premise that an individual unit of product can concurrently possess only a single quality characteristic. Ghare and Torgersen (11) have conducted research concerning the statistical aspects of the problem, pointing out that the principal disadvantage of control by variables is that a separate control chart must be maintained for each quality characteristic being inspected. These authors developed the multicharacteristic (Q) chart that permits the monitoring of the central tendency of a number of continuous quality characteristics on one control chart. The Q Chart defines an accept/reject boundary and permits a plot over a time sequence but does not identify the one or more quality characteristics that may be out of control when an overall out-of-control condition is indicated on the chart. Montgomery and Klatt (21) have presented an economic model for the Hotelling T^2 control chart and have discussed a general methodology whereby the optimal test parameters for the control chart may be computed.

As opposed to multicharacteristic methods of process control, Schmidt and Bennett (30) have presented research concerning economic multiattribute acceptance sampling. These authors developed a mathematical model which considers the total expected cost of quality control per inspection lot as the criterion of optimality. The problem of interest was one in which several characteristics were controlled and the inspection lot was either accepted or scrapped. Results of that research indicated, for the examples cited, that the model and its optimum solutions are more sensitive to changes in the mean and variance of proportion defective per lot then to the shape of the distribution of proportion defective. Bennett and McCaslin (4) have also considered the multiattribute acceptance sampling problem. In their research, the inspection lot could be either accepted, scrapped, or screened. For the specific examples considered in that research, the authors have suggested that the cost model presented is, likewise, more sensitive to errors in the estimation of the parameters of the process distributions than to errors in identifying the distributional forms.

Many manufacturers are faced with the problem of simultaneously controlling several attributes and if the objective of the quality control effort in that case is economic in nature, then the research just cited (4, 30) may be applied very effectively. There are, however, products which have present in them several quality characteristics which are variables rather than attributes. This situation poses somewhat of a dilemma for the manufacturer, in that there are presently only a limited number of approaches which may be taken to the simultaneous control of several variables.

Multicharacteristic methods of control charting (11, 21) could be applied in the variables case, but the major weakness of these plans is that although they identify an out-of-control condition, they do not indicate the specific variable or variables which are causing this condition. Another approach might be to design an acceptance sampling plan for each of the individual variables from a purely statistical standpoint and then implement the resulting plans simultaneously. This procedure is a valid one, but the use of statistically determined acceptance sampling plans may result in more costly control of the variables than is really necessary. This approach to the simultaneous control of several variables often would not be attractive to management because of its adverse economic consequences.

The profit minded manufacturer realizes that the economic control of product quality may play an extremely important part in the financial success or failure of his firm. From an economic point of view, then, there exists the need to develop cost-based acceptance sampling plans for each of the variables under consideration, which when implemented in combination, result in the least total expected cost of controlling lot quality. The remainder of this research is concerned specifically with the design of economic multivariate acceptance sampling plans.

CHAPTER II

DESCRIPTION OF THE PROBLEM

A. The Problem

The lot-by-lot evaluation and subsequent attempt to control product quality is a problem familiar to many industrial managers. A system of acceptance sampling plans may be applied in an effort to cope with this problem and is, therefore, the approach to the control of product quality taken in this research. An acceptance sampling plan is often implemented in two phases. The first phase is to determine whether or not the inspection lot is of acceptable quality, i.e. to reach a quality control decision. The latter phase is concerned with the disposition of the lot in accordance with the overall quality control plan, i.e. implementing the quality control decision. The result of the first phase of the acceptance sampling procedure leads to one of three alternative decisions: accept the inspection lot, reject and scrap the lot, or reject and screen the lot.

The multivariate acceptance sampling system considered in this research is specified by three decision variables for each characteristic to be controlled: a sample size and a lower and upper control limit for the sample mean. The objective in this problem is to design the acceptance sampling plans such that when implemented in combination they provide for the control of product quality as economically as possible. The specific quality control problem of interest in this research is described in the remainder of this chapter.

Sampling inspection is conducted in the first phase of the quality control plan. Before discussing the sampling discipline, however, several aspects of the variables themselves must be considered. The type of product present here has one or more physically measurable characteristics, each being viewed as occurring independently of all the others. That is, there is no interaction among the variables with respect to their individual conformance to specifications. For instance, the diameter and the length of a cylindrical steel bar may be the two variables of interest. If the diameter of the bar is to be measured at both ends of the bar, each diameter is considered as a separate and independent variable and must be so designated.

The variable types are classified into three mutually exclusive categories. The first group is denoted as Variable Class A. In measuring or testing a product for this variable type, the product is destroyed and cannot be used for its intended purpose nor as an item for further sampling or screening inspection. Tensile or compressive testing in which the product must be shattered in order to obtain the desired measurement is one example of a variable type in this class.

The second category is Variable Class B. When inspecting this type of variable, the product is destroyed with respect to its intended purpose. The product can, however, be used as an item in subsequent inspection operations. For instance, a toothpaste tube must be ruptured in order to inspect for voids in the inner coating of the tube. This inspection operation renders the tube useless with regard to its intended purpose, but does not prohibit further inspection of such characteristics as the thread diameter at the neck of the tube.

The last of the variable classes is Variable Class C. This is a nondestructable category. Inspection for a variable type in this class leaves the product fit for its intended purpose as well as for further inspection operations. The variable types in Class C are referred to as "screenable" variables. A simple measuring operation for, say, length or diameter is representative of variable types in this class.

The sample statistic used in this problem to determine whether to accept or reject the inspection lot on a given variable is the sample mean of the observations taken on that variable. If the sample mean on a given variable conforms to the specified control limits for that variable the lot is accepted with respect to that variable. Otherwise, the lot is rejected on that variable.

The next aspect in this first phase of the quality control scheme is to specify the sampling inspection procedure. The model developed in this research is strictly dependent upon the order in which the variables being controlled are inspected. The model does not apply, and therefore must be altered, if any order of sampling other than that described as follows is used. The variables in Variable Class A (numbers $1,\ldots,k_1$) are inspected first, then those in Class B (numbers $k_1+1,\ldots,k_2$), and finally those in Variable Class C (numbers $k_2+1,\ldots,k_3$). The sample sizes specified by the acceptance sampling plans are arranged, within each of the variable classes, in ascending order. That is, $n_1 \leq n_2 \leq \ldots \leq n_{k_1}$, $n_{k_1+1} \leq n_{k_1+2} \leq \ldots \leq n_{k_2}$, $n_{k_2+1} \leq n_{k_2+2} \leq \ldots \leq n_{k_3}$.

The sampling discipline is as follows:

> Sample n_1 units on variable type 1. If the inspection lot is accepted on this variable, so note and go on to the next sampling inspection operation. If the lot is rejected on this

variable, reject and scrap the lot.

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Sample n_{k_1} units for variable type k_1. If the inspection lot is accepted on this variable, so note and go on to the next sampling inspection operation. If the lot is rejected on this variable, reject and scrap the lot.

Sample n_{k_1+1} units for variable k_1+1. If the inspection lot is accepted on this variable, so note and go on to the next sampling inspection operation. If the lot is rejected on this variable, reject and scrap the lot.

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Sample n_{k_2} units for variable k_2. If the lot is accepted on this variable, so note and go on to the next sampling inspection operation. If the lot is rejected on this variable, reject and scrap the lot.

Sample n_{k_2+1} units for variable k_2+1. If the lot is accepted on this variable, so note and go on to the next sampling inspection operation. If the lot is rejected on this variable, screen the lot on variable k_2+1, and go on to the next sampling inspection operation.

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Sample n_{k_3} units for variable k_3. If the lot is accepted on this variable, so note. If the lot is rejected on this variable, screen the lot on variable k_3. Inspection for all variables is completed.

The final aspect in the first phase of the acceptance sampling problem is to determine the costs which are incurred in order to reach a quality control decision, regardless of which decision is reached. The only cost incurred is that of performing the actual sampling inspection operations. This cost is specified on a per unit basis and is incurred for each variable inspected. This cost may vary from variable to variable but is maintained as a constant for all variables throughout the examples in this research.

As already indicated, a quality control decision is reached in the first phase of the acceptance sampling scheme. The remaining step is to implement that decision. Each of the three decision alternatives is now considered, characterizing the manner in which each is reached and the costs that are incurred.

The decision to accept the inspection lot is reached if, and only if, the sample mean of each and every one of the variables being controlled conforms to the control limits for that variable. When the lot is accepted only the units removed from the lot for the purpose of sampling inspection must be considered with regard to the accumulation of any replacement or repair costs. The units which have been destroyed during sampling inspection are discarded but may be subsequently replaced. These units which have not been destroyed but which have nevertheless been found to be defective are repaired and then returned to the inspection lot from which they were drawn.

The cost to replace a unit is assumed to be $C_{\rho 2}$. Replacement units are taken from an available supply and may be screened for all variables in Class C, depending upon management policy. If screening occurs, any defective units are repaired and the cost of performing the repair operations, if any, is also incurred. Any non-destroyed units remaining in the total sample which are found to be defective are repaired. In this case, as well as in the case of repairing defective replacement units, if multiple defects are found in a single unit, a cost to repair each defect is incurred.

Finally, the cost of accepting defectives in the lot must be considered. Since, in this decision alternative, the lot has been deemed acceptable on all variables, no defectives in the remainder of the lot will be detected and a cost is incurred by virtue of accepting defects in all variable classes.

If, at any point in the sampling inspection of Variable Class A or B, a sample mean does not conform to its control limits, the lot is rejected and scrapped. Sampling inspection is terminated and the lot and sample are immediately discarded. The cost to discard a unit is the cost $C_{\rho 1}$. This cost is incurred for every unit in the inspection lot.

The decision to reject and screen the inspection lot is reached if the lot is accepted on all variables in Classes A and B and subsequently rejected on one or more variables in Class C. The units destroyed during sampling inspection may be replaced, and replacement units may be screened, incurring the applicable costs. In addition, however, the costs of screening inspection and repairing observed defectives in the lot and the sample are incurred. The nondestroyed portion of the sample and lot are screened on only those Class C variables which were rejected during sampling inspection.

Since the lot is not scrapped in this case, a cost will be incurred by virtue of accepting some defectives. Defectives among the destructable variable types are not detected and a cost per defect for accepting these defectives is incurred. In addition, a cost for accepting defectives among all accepted Class C variables must be considered.

B. Method of Approach

The economic acceptance sampling problem just described is mathematically modeled in Chapter III of this research. The decision variables are the sample size and the lower and upper control limits for the mean of each characteristic under consideration. The model is then optimized using a conventional search technique, the pattern search (44), to find values of the decision variables that specify the acceptance sampling plans which, when implemented in combination, result in the minimum or near minimum total expected cost of controlling lot quality. Sensitivity analysis is performed for a specific example in an effort to determine the economic effect of errors in the estimation of the form of the distributions on the process means, provided that the mean and variance of these distributions have been determined with reasonable accuracy.

CHAPTER III

DEVELOPMENT OF THE COST MODEL

The purpose of this chapter is to present the development of a cost model consistent with the problem description. The mathematical model developed here expresses the total expected cost of quality control per inspection lot submitted for quality control as a function of the decision variables n_i, LCL_i, and UCL_i $(i = 1, \ldots, k_3)$, where n_i is the sample size and LCL_i and UCL_i are the lower and upper control limits for the sample mean for the ith variable, respectively. The sampling scheme in Chapter II must be used if the model is to be applied. The total expected cost of quality control per inspection lot, C_T, is

$$\begin{aligned} C_T = & \text{ E (cost of sampling inspection per inspection lot)} \\ & + \text{ E (cost of acceptance per inspection lot)} \\ & + \text{ E (cost of scrapping the lot per inspection lot)} \\ & + \text{ E (cost of screening the lot per inspection lot)} \end{aligned} \qquad [3\text{-}1]$$

For the purposes of the development of equation 3-1, this chapter is divided into eight major sections. The first three of these are concerned with the notation, assumptions and probability expressions, respectively. The next section presents a development of the cost expression associated with reaching a decision to either accept or to reject the inspection lot, i.e. the cost of sampling inspection. The succeeding three sections consider the costs of implementing the quality control decision, i.e. the cost of accepting the inspection lot, the cost of scrapping the lot, and the cost of screening the lot. Finally, an expression for the total expected cost of controlling the quality of an inspection lot is presented. Appendix A contains a schematic development of the cost model in "flow chart" form.

A. Notation

$1, \ldots, k_1$ = index representing variables in characteristic class A.

$k_1 + 1, \ldots, k_2$ = index representing variables in characteristic class B.

$k_2 + 1, \ldots, k_3$ = index representing variables in characteristic class C.

L = the number of items in an inspection lot.

n_i = sample size for the ith variable, $(i = 1, \ldots, k_3)$

LS_i = lower specification limit for the ith variable, $(i = 1, \ldots, k_3)$.

US_i = upper specification limit for the ith variable, $(i = 1, \ldots, k_3)$.

μ_i = mean product dimension for the ith variable in an inspection lot, (i = 1, ..., k_3).

$\underset{\sim}{\mu}$ = the vector (μ_1, μ_2, ..., μ_{k_3}).

$\bar{X}_i$ = sample mean for the ith variable, (i = 1, ..., k_3).

X_i = observed item dimension for the ith variable, (i = 1, ..., k_3).

σ_i^2 = variance of X_i for a given inspection lot, (i = 1, ..., k_3).

$f(\mu_i)$ = probability density function of μ_i, (i = 1, ..., k_3).

$g(\bar{x}_i | \mu_i)$ = conditional probability density function of $\bar{X}_i$ for samples drawn from an inspection lot with mean μ_i, (i = 1, ..., k_3).

$h(x_i | \mu_i)$ = conditional probability density function of X_i for units drawn from an inspection lot with a mean μ_i, (i = 1, ..., k_3).

LCL_i = lower control limit for the ith variable, (i = 1, ..., k_3).

UCL_i = upper control limit for the ith variable, (i = 1, ..., k_3).

C_{Ii} = the cost of performing the sampling inspection for the ith variable (i = 1, ..., k_3) on one unit of product.

$C_I(\underset{\sim}{\mu})$ = conditional expected cost of sampling inspection given the vector $\underset{\sim}{\mu}$.

C_{aLi} = cost which results from the undetected presence of the ith variable below the lower specification limit in an accepted or screened lot, $(i = 1, \ldots, k_3)$.

C_{aUi} = cost which results from the undetected presence of the ith variable above the upper specification limit in an accepted or screened lot, $(i = 1, \ldots, k_3)$.

$C_A(\text{accept}, \underset{\sim}{\mu})$ = conditional expected cost of accepting the inspection lot given acceptance and the vector $\underset{\sim}{\mu}$.

$C_A(\underset{\sim}{\mu})$ = conditional expected cost of accepting the inspection lot given the vector $\underset{\sim}{\mu}$.

$C_{\rho 1}$ = unit cost of scrapping an item.

$C_{\rho 2}$ = unit cost of replacement.

C_r = unit cost of materials handling.

$C_R(\text{scrap}, \underset{\sim}{\mu})$ = conditional expected cost of scrapping the inspection lot given rejection and the vector $\underset{\sim}{\mu}$.

$C_R(\underset{\sim}{\mu})$ = conditional expected cost of scrapping the inspection lot given the vector $\underset{\sim}{\mu}$.

C_{si} = unit cost of performing the screening inspection for the ith variable, $(i = k_2 + 1, \ldots, k_3)$.

$C_{\sigma Li}$ = cost of repairing one unit of product found to have the ith variable below the lower specification limit.

$C_{\sigma Ui}$ = cost of repairing one unit of product found to have the ith variable above the upper specification limit.

$C_S(\underset{\sim}{\mu})$ = conditional expected cost of screening the inspection lot given the vector $\underset{\sim}{\mu}$.

C_T = total expected cost of quality control per inspection lot submitted for control.

N^* = total number of items removed from the inspection lot for the purpose of sampling inspection.

$$= \max \left[\sum_{i=1}^{k_1} n_i + n_{k_2}, \sum_{i=1}^{k_1} n_i + n_{k_3} \right]$$

δ_0 = the relationship between the largest sample size among the variables in class C and the largest sample size among the variables in class B.

$$= \begin{cases} 1, & n_{k_3} > n_{k_2} \\ 0, & n_{k_3} \le n_{k_2} \end{cases}$$

δ_{1i} = the relationship between the sample size for the ith screenable variable and the largest sample size among the variables in class B.

$$= \begin{cases} 1, & n_i > n_{k_2} \\ 0, & n_i \le n_{k_2} \end{cases} \qquad i = k_2 + 1, \ldots, k_3$$

N_{1i} = the minimum between the ith screenable sample size minus the (i -1)st sample size and the ith sample size minus the largest sample size among the variables in class B. The variable δ_{1i}, above is used in conjunction with N_{1i} in order to maintain the latter as a nonnegative quantity.

$= \min[n_i - n_{i-1}, n_i - n_{k_2}], \; i = k_2 + 1, \ldots, k_3$

N_{2i} = the maximum between the sample size for the ith screenable variable minus the largest sample size among the variables in class B and zero.

$= \max[n_i - n_{k_2}, 0], \; i = k_2 + 1, \ldots, k_3$

δ_2 = indicates management's decision with respect to the replacement policy.

$= \begin{cases} 1, & \text{units destroyed during sampling inspection are replaced in accepted lots.} \\ 0, & \text{units destroyed during sampling inspection are not replaced in accepted lots.} \end{cases}$

δ_3 = indicates management's decision with respect to the screening of replacement units.

$= \begin{cases} 1, & \text{units are screened for all class C variables} \\ 0, & \text{units are not screened} \end{cases}$

B. <u>Assumptions</u>

1. The random variables $\mu_1, \ldots, \mu_{k_3}$ are independently distributed.
2. The random variables $\bar{X}_1, \ldots, \bar{X}_{k_3}$ are independently distributed.
3. The random variables $X_1, \ldots, X_{k_3}$ are independently

distributed.

4. Errors, if any, which occur in sampling or screening inspection are assumed to be negligible.
5. A specific defect type can occur only once in a unit of product.
6. The sampling inspection of a unit of product for a characteristic falling in class A completely destroys the unit and further sampling inspection of that unit cannot be performed.
7. The sampling inspection of a unit for a characteristic falling in class B renders the unit unfit for its intended use but does not prohibit further sampling inspection of that unit.
8. The sampling or screening inspection of a unit for a characteristic in class C is nondestructive and lot rejection results in screening of the lot, given acceptance on variables in classes A and B.
9. The number of units sampled for the ith variable, if inspection on that variable takes place, is without exception, n_i. $(i = 1, \ldots, k_3)$.
10. If, at the completion of sampling inspection for a given destructable characteristic type, the decision is reached to reject the lot, the entire inspection lot is immediately scrapped and further sampling inspection is not performed.

11. Sampling inspection for the ith destructable characteristic type must be completed and the lot must be accepted on that defect type before sampling is begun for the $(i+1)^{st}$ variable type. $(i = 1, \ldots, k_2)$.
12. A unit that has been rendered unfit for its intended use during sampling inspection is scrapped, but may or may not be replaced, if the inspection lot is accepted or screened.
13. In an inspection lot that has not been scrapped, a unit that has not been destroyed during sampling inspection but has been found defective during subsequent sampling or screening inspection on a nondestructable defect type is assumed to be repaired.
14. A unit that has been accepted, or repaired, but has multiple defects will incur the cumulative cost of accepting, or repairing, each characteristic type.
15. If the inspection lot is accepted on all destructable characteristic types and the policy is to replace all units destroyed in sampling inspection, replacement units may be screened on all nondestructable characteristic types and defectives are repaired accordingly.
16. When screening for multiple characteristic types, the inspection of any one unit is carried out to completion even though a defect in the unit may be detected prior

to the inspection of all characteristic types on that unit.

17. An inspection lot which is accepted on all destructable characteristic types cannot be scrapped. If the inspection lot is subsequently rejected on one or more screenable characteristic types, the remainder of the inspection lot is screened for those defect types only.

18. Sample sizes are arranged, within a given characteristic class, in ascending order.

C. Frequently Used Probability Expressions

Several probability expressions appear often during the development of the cost model. This section presents mathematical statements of these expressions and the corresponding notation that will be used throughout the remainder of this chapter.

The measured value of the ith variable is either within or not within the specification limits for that variable. In the former case the unit is termed as possessing the ith quality characteristic within acceptable limits and termed as being defective in the latter case. The conditional probability that the measured dimension of the ith characteristic ($i = 1, \ldots, k_3$) is within the specification limits for the ith characteristic given μ_i is

$$P[G_i | \mu_i] = \int_{LS_i}^{US_i} h(x_i | \mu_i)\, dx_i \qquad [3\text{-}2]$$

In the event that an individual unit is defective on the ith characteristic ($i = 1, \ldots, k_3$), it is of particular interest whether the measured dimension of that variable falls below the lower specification limit or above the upper specification limit. The conditional probability that an individual unit is observed to be defective on the ith variable and that the measured value of that dimension falls below the ith lower specification limit given μ_i is

$$P[D_{Li} | \mu_i] = \int_{-\infty}^{LS_i} h(x_i | \mu_i)\, dx_i \qquad [3\text{-}3]$$

Similarly, the conditional probability that an individual unit is observed to be defective on the ith variable and that the measured value of that dimension falls above the ith upper specification limit given μ_i is

$$P[D_{Ui} | \mu_i] = \int_{US_i}^{\infty} h(x_i | \mu_i)\, dx_i \qquad [3\text{-}4]$$

Another probability expression used repeatedly is the conditional probability that the inspection lot is accepted on the ith variable given μ_i, i.e. that the computed value of the sample mean of the n_i observed measurements on the ith variable is within the control limits for that variable. This probability statement is given in Equation 3-5 below.

$$P[A_i \mid \mu_i] = \int_{LCL_i}^{UCL_i} g(\bar{x}_i \mid \mu_i)\, d\bar{x}_i \qquad [3\text{-}5]$$

Equations 3-2 through 3-5 have given the basic probability statements used most frequently throughout the model development. Other probability expressions are required in the development of the model but all are related to these four and will not be presented here.

D. Cost of Sampling Inspection

Determining whether to accept or to reject the inspection lot is the first step in the practical application of a quality control plan for acceptance sampling. The sampling discipline described in Chapter II and presented schematically in Appendix A is the primary support for the development of this cost expression.

In order to perform the sampling inspection for the ith variable ($i = 2, \ldots, k_2$), the inspection lot must have been accepted on the previous $i - 1$ variables, with inspection on the first variable being performed without exception. There exists, then, a conditional relationship between the ith inspection operation and those which preceed it. That is,

$$P(\text{Accept on variables } 1, \ldots, i\text{-}1 \mid \mu_1, \ldots, \mu_{i-1})$$

$$= \prod_{j=1}^{i-1} P[A_j \mid \mu_j] \qquad [3\text{-}6]$$

The expression in Equation 3-6 is required for defect types in Variable Classes A and B, $i = 1, \ldots, k_2$.

Since the inspection lot cannot be rejected and scrapped on Variable Class C variables, the sampling inspection for these variables is conditioned only on having accepted the inspection lot on variables 1 through k_2, inclusive and

$$P(\text{Accept on variables } 1, \ldots, k_2 \mid \mu_1, \ldots, \mu_{k_2})$$

$$= \prod_{j=1}^{k_2} P[A_j \mid \mu_j] \qquad [3\text{-}7]$$

The cost of performing the ith sampling inspection operation ($i = 1, \ldots, k_3$) given that it is performed is, without exception, expressed as $n_i\ C_{Ii}$. This cost is the unconditional sampling inspection cost for the ith variable. The sampling procedure is, however, a conditional procedure.

Using Equations 3-6 and 3-7 to impose the appropriate conditions on the cost components of inspection, the conditional expected cost of sampling inspection given $\underset{\sim}{\mu}$, denoted by $C_I(\underset{\sim}{\mu})$, is formulated as

$$C_I(\underset{\sim}{\mu}) = C_{I1}\, n_1 + \sum_{i=2}^{k_2} C_{Ii}\, n_i \prod_{j=1}^{i-1} P[A_j \mid \mu_j]$$

$$+ \sum_{i=k_2+1}^{k_3} C_{Ii}\, n_i \prod_{j=1}^{k_2} P[A_j \mid \mu_j] \qquad [3\text{-}8]$$

E. Cost of Accepting the Inspection Lot

A decision to accept the inspection lot is one of the three possible outcomes of the sampling inspection procedure. The components of cost included in this section are incurred solely as the result of implementing this decision alternative and are conditioned accordingly. This section is presented in four parts, each containing a distinct component of the cost of accepting the inspection lot. Note that accepting the inspection lot implies that the sample mean, $\bar{X}_i$, falls within the specified limits, LCL_i and UCL_i, for all i, where

$$P(\text{Accept on all variables}|\underset{\sim}{\mu}) = \prod_{i=1}^{k_3} P[A_i|\mu_i] \qquad [3\text{-}9]$$

i. <u>Cost of units destroyed or rejected in sampling inspection</u>

The nature of the variables included in this model may cause some units to be destroyed during sampling inspection. A total of

$$\sum_{i=1}^{k_1} n_i + n_{k_2} \qquad [3\text{-}10]$$

units are destroyed during sampling inspection of an accepted or screened inspection lot. A cost $C_{\rho 1}$, the per unit cost to scrap an item, and the per unit materials handling cost, C_r, are incurred for each of these units. The conditional expected cost of the units destroyed during sampling inspection given lot acceptance and the vector $\underset{\sim}{\mu}$ is

$$C_{A1_1} = [C_{\rho 1} + C_r] \, [\sum_{i=1}^{k_1} n_i + n_{k_2}] \qquad [3\text{-}11]$$

If the sample size n_i, for some $i = k_2+1, \ldots, k_3$, is greater than the sample size n_{k_2}, there exists units in the total sample which have not been destroyed during sampling inspection. Some of these units may be rejected during the completion of the sampling procedure for nondestructable variables. For the purposes of developing this component of cost, the reader should recall that

$$\delta_{1i} = \begin{cases} 1, & n_i > n_{k_2} \\ 0, & n_i \le n_{k_2} \end{cases}, \quad i = k_2+1, \ldots, k_3 \qquad [3\text{-}12]$$

and,

$$N_{1i} = \text{Min}\ [n_i - n_{i-1}, n_i - n_{k_2}],\ i = k_2+1, \ldots, k_3 \qquad [3\text{-}13]$$

Those units rejected but not destroyed during sampling inspection likewise incur the materials handling cost, but by assumption, the cost of repairing the observed defectives will also be incurred. Hence,

$$N_{2i} = \text{Max}[n_i - n_{k_2}, 0]\ ,\ i = k_2+1, \ldots, k_3 \qquad [3\text{-}14]$$

The conditional expected cost of rejecting units during the sampling inspection and repairing the observed defectives, given lot acceptance and the vector $\underset{\sim}{\mu}$ is

$$C_{A1_2} = C_r \sum_{i=k_2+1}^{k_3} \delta_{1i} N_{1i} \left(1 - \prod_{j=i}^{k_3} P[G_j \mid \mu_j]\right)$$

$$+ \sum_{i=k_2+1}^{k_3} N_{2i} \{C_{\sigma Ui} P[D_{Ui} \mid \mu_i] + C_{\sigma Li} P[D_{Li} \mid \mu_i]\}$$

[3-15]

Let C_{A1} (Accept, $\underset{\sim}{\mu}$) denote the conditional expected cost of units destroyed or rejected in sampling inspection given lot acceptance and the vector $\underset{\sim}{\mu}$. This cost is given as

$$C_{A1} (\text{Accept}, \underset{\sim}{\mu}) = C_{A1_1} + C_{A1_2} \qquad [3\text{-}16]$$

ii. Cost of replacing units destroyed in sampling inspection

A manufacturing firm may elect to replace all units destroyed in sampling inspection, thereby maintaining a constant "outgoing" inspection lot size. If this policy is in effect several additional costs are incurred. In order to indicate the presence or absence of this policy, the following notation is introduced.

$$\delta_2 = \begin{cases} 1, \text{ Replace units destroyed in sampling inspection} \\ 0, \text{ Do not replace units destroyed in sampling inspection} \end{cases}$$

[3-17]

The cost to physically replace a destroyed unit will simply be the cost, $C_{\rho 2}$. As before, the number of units that must be replaced if the policy is in effect are those which were rendered unfit for their intended use during sampling inspection. This number of units is given by Equation 3-10. The conditional expected cost of replacing these units given lot acceptance and the vector $\underset{\sim}{\mu}$ is

$$C_{A2_1} = C_{\rho 2}\left[\sum_{i=1}^{k_1} n_i + n_{k_2}\right] \qquad [3\text{-}18]$$

In addition, all replacement units may be screened for Class C variables, in which case, $\delta_3=1$; $\delta_3=0$, otherwise. The conditional expected cost of performing the screening inspection of replacement units given lot acceptance and the vector $\underset{\sim}{\mu}$ is

$$C_{A2_2} = \delta_3 \sum_{i=k_2+1}^{k_3} \left[C_{si}\left(\sum_{j=1}^{k_1} n_j + n_{k_2}\right)\right] \qquad [3\text{-}19]$$

The conditional expected cost associated with repairing the observed defectives among the replacement units given lot acceptance and the vector $\underset{\sim}{\mu}$ is

$$C_{A2_3} = \delta_3 \left[\sum_{i=1}^{k_1} n_i + n_{k_2} \right] \sum_{i=k_2+1}^{k_3} (C_{\sigma Ui} \; P[D_{Ui} | \mu_i]$$

$$+ \; C_{\sigma Li} \; P[D_{Li} | \mu_i]) \qquad [3\text{-}20]$$

Since the replacement units cannot be inspected for variables in Classes A and B without again being destroyed, defectives among these variables will pass undetected. Hence, the conditional expected cost of accepting these undetected defectives among replacement units given lot acceptance and the vector $\underset{\sim}{\mu}$ is

$$C_{A2_4} = \left[\sum_{i=1}^{k_1} n_i + n_{k_2} \right] \sum_{i=1}^{\delta_3 k_2 + (1 - \delta_3) k_3} \{C_{aUi}$$

$$P[D_{Ui} | \mu_i] + C_{aLi} \; P[D_{Li} | \mu_i]\} \qquad [3\text{-}21]$$

Let C_{A2} (Accept, $\underset{\sim}{\mu}$) denote the total conditional expected cost of implementing a replacement policy given lot acceptance and the vector $\underset{\sim}{\mu}$. This cost is given as

$$C_{A2} \text{ (Accept, } \underset{\sim}{\mu}) = \delta_2 [C_{A2_1} + C_{A2_2} + C_{A2_3} + C_{A2_4}]$$

$$[3\text{-}22]$$

iii. <u>Cost of accepting defective units in the uninspected portion of the inspection lot</u>

The implementation of a sampling procedure has as its objective an attempt to control the quality of an inspection lot without subjecting the entire lot to inspection. In many cases it is impossible to totally inspect the entire lot without destroying it. In some cases, the optimal sampling plans will not be ones which call for 100 percent inspection of the lot on all variables. This situation gives rise to the component of cost presented here, the cost of accepting defectives among the non-inspected portion of the inspection lot when the lot is accepted.

The first step in the development of this component of cost is to determine the number of units in an accepted inspection lot which have not been inspected at all. Hence,

$$N^* = \text{Max}\left[\sum_{i=1}^{k_1} n_i + n_{k_2}, \sum_{i=1}^{k_1} n_i + n_{k_3}\right] \qquad [3\text{-}23]$$

Equation 3-23 specified the total number of units which are removed from the inspection lot for the purpose of sampling. In an inspection lot of size L there are, then L - N* units remaining in the lot which have not been inspected. Once again when considering accepted defectives, the concern is not only with the number of defectives but also with whether the units are defective below the lower specification limit or defective above the upper specification limit. The conditional expected cost of accepting defectives among the uninspected portion of the inspection lot given lot acceptance and the vector $\underset{\sim}{\mu}$ is

$$C_{A3}\ (\text{Accept},\ \underset{\sim}{\mu}) = (L - N^*) \sum_{i=1}^{k_3} \{C_{aUi}\ P[D_{Ui}|\mu_i] + C_{aLi}\ P[D_{Li}|\mu_i]\} \qquad [3\text{-}24]$$

iv. <u>Cost of accepting defective units among the nondestroyed, uninspected portion of the total sample</u>

Again referring to the sampling discipline, recall that if $n_{k_2} < n_{k_3}$ there are $n_{k_3} - n_{k_2}$ nondestroyed units remaining among the total number of units removed from the inspection lot for the purpose of sampling. A cost associated with accepting defectives among these units is now considered.

Since the inspection lot is accepted on all variable types, no screening inspection will be performed and if $n_{k_2} < n_{k_2+1}$ there are $n_{k_2+1} - n_{k_2}$ nondestroyed units which have not been inspected for variables 1, ..., k_2; $n_{k_2+2} - n_{k_2+1}$ nondestroyed units which have not been inspected for variables 1, ..., k_2+1; etc., up through the interval $n_{k_3} - n_{k_3-1}$. The preceding analysis can be conducted regardless of the interval in which the sample size n_{k_2} falls. The conditional expected cost of accepting these defectives given lot acceptance and the vector $\underset{\sim}{\mu}$ is

$$C_{A4}\ (\text{Accept}, \underset{\sim}{\mu}) = \sum_{i=k_2+1}^{k_3} \delta_{1i}\ N_{1i}\ \Big(\sum_{j=1}^{i-1} C_{aUj}\ P[D_{Uj}|\mu_j] + C_{aLj}\ P[D_{Lj}|\mu_j]\Big) \qquad [3\text{-}25]$$

v. <u>Total expected cost of accepting the inspection lot given the vector $\underset{\sim}{\mu}$</u>

From Equations 3-16, 3-22, 3-24, 3-25, the total conditional expected cost of accepting the inspection lot given lot acceptance and the vector $\underset{\sim}{\mu}$, C_A (Accept, $\underset{\sim}{\mu}$), is

$$C_A\ (\text{Accept}, \underset{\sim}{\mu}) = C_{A1}\ (\text{Accept}, \underset{\sim}{\mu}) + C_{A2}\ (\text{Accept}, \underset{\sim}{\mu}) + C_{A3}\ (\text{Accept}, \underset{\sim}{\mu}) + C_{A4}\ (\text{Accept}, \underset{\sim}{\mu}) \qquad [3\text{-}26]$$

Taking the expectation of Equation 3-27 with respect to accepting the inspection lot on all variables, the total expected cost of accepting the inspection lot given the vector $\underset{\sim}{\mu}$ is

$$C_A(\underset{\sim}{\mu}) = \prod_{i=1}^{k_3} C_A\ (\text{Accept}, \underset{\sim}{\mu})\ P[A_i|\mu_i] \qquad [3\text{-}27]$$

F. Cost of Scrapping the Inspection Lot

Another of the mutually exclusive outcomes of the sampling inspection procedure is to reject and scrap the inspection lot. The inspection lot is rejected and scrapped if, for any $i = 1, \ldots, k_2$; $\bar{X}_i$ falls outside the control limits LCL_i and UCL_i, where

$$P(\text{Accept on variables } 1, \ldots, k_2 | \mu_1, \ldots, \mu_{k_2})$$

$$= \prod_{i=1}^{k_2} P[A_i | \mu_i] \qquad [3\text{-}28]$$

The conditional probability of scrapping the inspection lot is the complement of Equation 3-28.

The only costs associated with this event are the unit scrapping cost and the unit materials handling cost. These costs are associated with each of the units in the total inspection lot of size L. The conditional expected cost of scrapping the inspection lot given the vector $\underset{\sim}{\mu}$ is

$$C_R(\underset{\sim}{\mu}) = L(C_r + C_{\rho 1}) \ \{1 - \prod_{i=1}^{k_2} P[A_i | \mu_i]\} \qquad [3\text{-}29]$$

G. Cost of Screening the Inspection Lot

The last of the mutually exclusive outcomes of the sampling inspection procedure is to screen the inspection lot. The cost expression developed in this section along with the cost expressions developed in Sections E and F of this chapter complete the collectively exhaustive set of outcomes of the sampling procedure.

The inspection lot is rejected and screened if, and only if, the following events occur:

a. the inspection lot is accepted on all destructable variables (Class A and B); and

b. the inspection lot is rejected on one or more screenable variables.

The conditional probability that the inspection lot is accepted on all variables in Classes A and B was previously given in Equation 3-7. The probability of rejecting the inspection lot on variables in Class C is considered as each of the individual cost expressions is presented.

i. <u>Cost of screening inspection</u>

The screening inspection procedure is conducted only for the nondestructable variables (Class C). Furthermore, screening is conducted only for those variables in which $\bar{X}_i < LCL_i$ or $\bar{X}_i > UCL_i$, $i = k_2+1, \ldots, k_3$, during sampling inspection.

Consider, first, the units in the inspection lot which have not been selected for the purpose of sampling inspection. This number is $L - N^*$. These units will be screened for variable i if, and only if, the above relationship holds for i, $i = k_2+1, \ldots, k_3$. The conditional expected cost of screening inspection on the L - N* units given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S1_1} = (L - N^*) \sum_{i=k_2+1}^{k_3} C_{si} \left(1 - P[A_i \mid \mu_i]\right) \qquad [3\text{-}30]$$

If n_{k_3} is greater than n_{k_2} there are $n_{k_3} - n_{k_2}$ nondestroyed units remaining among the total number of units which had previously been removed from the inspection lot for the purpose of sampling inspection. Let

$$\delta_0 = \begin{cases} 1, & n_{k_3} > n_{k_2} \\ 0, & n_{k_3} \leq n_{k_2} \end{cases} \qquad [3\text{-}31]$$

Applying the analysis of Section E, Part iv, here the conditional expected cost of screening the nondestroyed portion of the total sample given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S1_2} = \delta_0 \sum_{i=k_2+1}^{k_3} C_{si} [\delta_{1i} (n_{k_3} - n_i) + (1 - \delta_{1i})$$

$$(n_{k_3} - n_{k_2})](1 - P[A_i \mid \mu_i]) \qquad [3\text{-}32]$$

Let C_{S1} (Screen, $\underset{\sim}{\mu}$) denote the total conditional expected cost of screening inspection given acceptance on variables in Classes A and B. Then

$$C_{S1}\ (\text{Screen},\ \underset{\sim}{\mu}) = C_{S1_1} + C_{S1_2} \qquad [3\text{-}33]$$

ii. <u>Cost of items destroyed or rejected in sampling and screening inspection</u>

The cost expression presented here is developed in four components: the cost of units destroyed in sampling inspection; the cost of units rejected in sampling inspection; the cost of units rejected in the screening inspection of the sample; the cost of units rejected in the screening inspection of the remainder of the inspection lot. These four cost expressions are denoted as C_{S2_1}, C_{S2_2}, C_{S2_3}, and C_{S2_4}, respectively.

The number of units destroyed during sampling inspection was previously given in Equation 3-10. Since each of these destroyed units cannot be used for its intended purpose, the conditional expected cost incurred due to the destruction of these units given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S2_1} = (C_r + C_{\rho 1}) \left(\sum_{i=1}^{k_1} n_i + n_{k_2} \right) \left(1 - \prod_{i=k_2+1}^{k_3} P[A_i | \mu_i]\right)$$

[3-34]

Although some units may have been destroyed during the sampling inspection for variables in Classes A and B, an additional number of units which have not been destroyed, if any such units remain in the total sample, may subsequently be rejected on one or more variables in Class C. The cost of units rejected during sampling inspection given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S2_2} = C_r \left[\sum_{i=k_2+1}^{k_3} \delta_{1i} N_{1i} \left\{1 - \prod_{j=i}^{k_3} P[G_j | \mu_j]\right\}\right]$$

$$\left(1 - \prod_{t=k_2+1}^{k_3} P[A_t | \mu_t]\right) \quad [3\text{-}35]$$

There is also a cost associated with repairing the observed defectives among these units. Conditioned upon the acceptance of variables in Classes A and B and the vector $\underset{\sim}{\mu}$, this expected conditional cost is

$$C_{S2_3} = \sum_{i=k_2+1}^{k_3} N_{2i} \quad (C_{\sigma Ui} \; P[D_{Ui}|\mu_i]$$

$$+ C_{\sigma Li} \; P[D_{Li}|\mu_i]) \; (1 - \sum_{j=k_2+1}^{k_3} P[A_j|\mu_j])$$

[3-36]

The next cost to be considered is the conditional expected cost associated with the units remaining in the total sample which have not been destroyed but which have been screened for one or more Class C variables. The conditional expected cost of units rejected during the screening inspection of the nondestroyed portion of the total sample given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S2_4} = C_r \sum_{i=k_2+1}^{k_3} \delta_{1i} \; (n_{k_3} - n_i) \; (1 - P[G_i|\mu_i])$$

$$(1 - P[A_i|\mu_i]) \qquad [3\text{-}37]$$

These units, too, are repaired in a manner similar to all other rejected units. The conditional expected cost of repairing the units rejected during the screening inspection of the sample given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S2_5} \doteq \sum_{i=k_2+1}^{k_3} [\delta_{1i} (n_{k_3} - n_i) + (1 - \delta_{1i}) (n_{k_3} - n_{k_2})] (C_{\sigma Ui} P[D_{Ui}|\mu_i] + C_{\sigma Li} P[D_{Li}|\mu_i]) (1 - P[A_i|\mu_i]) \qquad [3\text{-}38]$$

The last portion of the cost development here is the conditional cost of units rejected during the screening inspection of the remainder of the inspection lot and the conditional cost of repairing the observed defectives among these units. Recall that the number of units remaining in the inspection lot is L - N*. The materials handling cost, C_r, is associated with each of these units found to be defective. Note that in order for a unit to be screened on variable i, the inspection lot must have rejected on i ($i = k_2+1, \ldots, k_3$). The conditional expected cost of units rejected during the screening inspection of the L - N* units remaining in the inspection lot given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S2_6} = C_r \sum_{i=k_2+1}^{k_3} (L - N^*) \; (1 - P[G_i | \mu_i])$$

$$(1 - P[A_i | \mu_i]) \qquad [3\text{-}39]$$

The cost incurred in order to repair the defectives among these units must finally be considered. Recall that whether the observed defect is below the lower specification limit or above the upper specification limit is of importance. The conditional expected cost of repairing these defects given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S2_7} = (L - N^*) \sum_{i=k_2+1}^{k_3} (C_{\sigma Ui} \; P[D_{Ui} | \mu_i]$$

$$+ C_{\sigma Li} \; P[D_{Li} | \mu_i]) \; (1 - P[A_i | \mu_i]) \qquad [3\text{-}40]$$

The conditional total expected cost of items destroyed or rejected during sampling and screening inspection given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S2} \; (\text{Screen}, \underset{\sim}{\mu}) = C_{S2_1} + C_{S2_2} + C_{S2_3} + C_{S2_4}$$

$$+ C_{S2_5} + C_{S2_6} + C_{S2_7} \qquad [3\text{-}41]$$

iii. Cost of replacing units destroyed in sampling inspection

The cost of replacing destroyed units is incurred only if management policy is to take this corrective action. As before, δ_2 is the variable in the model which controls this action. The costs associated with replacing these destroyed units were previously discussed in Part ii of Section E of this chapter. The mathematical development of the cost expression here is identical to that presented in Equations 3-18 through 3-21, inclusive. This cost expression is summarized in Equation 3-22. The component of cost required here is obtained simply by properly conditioning Equation 3-22. Hence, the conditional expected cost of replacing units destroyed in sampling inspection given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S3}\ (\text{Screen},\ \underset{\sim}{\mu}) = \delta_2\ (C_{A2_1} + C_{A2_2} + C_{A2_3} + C_{A2_4})\left(1 - \prod_{i=k_2+1}^{k_3} P[A_i \mid \mu_i]\right) \qquad [3\text{-}42]$$

iv. Cost of accepting defectives among the uninspected portion of the inspection lot

The conditional cost of accepting defectives among the L - N* units in the inspection lot will be developed in two parts. The conditional cost of accepting defectives in Classes A and B ($i = 1, \ldots, k_2$) is considered first, then the cost of accepting defectives on variable i ($i = k_2+1, \ldots, k_3$) is presented under the condition that the lot was screened, but not for variable i.

The conditional expected cost of accepting defectives in Variable Class A and B given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S4_1} = (L - N^*) \sum_{i=1}^{k_2} (C_{aUi} \; P[D_{Ui} | \mu_i] + C_{aLi} \; P[D_{Li} | \mu_i]) \; (1 - \prod_{j=k_2+1}^{k_3} P[A_j | \underset{\sim}{\mu}]) \quad [3\text{-}43]$$

The cost of accepting defective units with respect to variable i given that the lot was accepted on variable i, $i = k_2+1, \ldots, k_3$, while being rejected on some other i, $i = k_2+1, \ldots, k_3$, is given in Equation 3-44. This expected cost, conditioned upon acceptance of variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S4_2} = (L - N^*) \sum_{i=k_2+1}^{k_3} (C_{aUi} \; P[D_{Ui}|\mu_i]$$

$$+ C_{aLi} \; P[D_{Li}|\mu_i]) \; P[A_i|\mu_i]$$

$$\cdot \left(1 - \prod_{\substack{j=k_2+1 \\ j \neq i}}^{k_3} P[A_j|\mu_j]\right) \qquad [3\text{-}44]$$

The conditional expected cost of accepting defectives in the uninspected portion of the inspection lot given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S4} \; (\text{Screen}, \underset{\sim}{\mu}) = C_{S4_1} + C_{S4_2} \qquad [3\text{-}45]$$

v. <u>Cost of accepting defectives among the nondestroyed, uninspected portion of the sample</u>

This component of cost is similar to the one developed in Section E, Part iv, of this chapter but the approach here will be somewhat different. In order to incur the cost of accepting defectives in the total sample, n_{k_3} must be greater than n_{k_2}; i.e. the total sample must not have been destroyed during the sampling inspection procedure.

For the purpose of developing this cost expression, recall the notation in Equation 3-31. The conditional expected cost incurred by virtue of accepting defectives in Classes A and B among the $n_{k_3} - n_{k_2}$ nondestroyed units, if any such units exist, given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S5_1} = \delta_0 \sum_{i=k_2+1}^{k_2} \{\delta_{1i} N_{1i} \sum_{j=1}^{k_2} (C_{aUj} \; P[D_{Uj}|\mu_j] + C_{aLj} \; P[D_{Lj}|\mu_j])\} \; (1 - \prod_{t=k_2+1}^{k_3} P[A_t|\mu_t]) \qquad [3\text{-}46]$$

Finally, the cost of accepting defectives in Class C must be considered. In order for variable i not to be detected as defective, the lot must have been accepted on variable i, $i = k_2+1, \ldots, k_3$, during sampling inspection while, at the same time, being rejected on some other i, $i = k_2+1, \ldots, k_3$, during sampling inspection. The conditional expected cost in this case given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_{S5_2} = \delta_0 \sum_{i=k_2+1}^{k_3} [\delta_{1i} (n_{k_3} - n_i) + (1 - \delta_{1i})$$

$$(n_{k_3} - n_{k_2})] \; (C_{aUi} \; P[D_{Ui} | \mu_i] + C_{aLi} \; P[D_{Li} | \mu_i])$$

$$\cdot \; (P[A_i | \mu_i])(1 - \prod_{\substack{j=k_2+1 \\ j \neq i}}^{k_3} P[A_j | \mu_j]) \qquad [3\text{-}47]$$

Equations 3-46 and 3-47 give the total conditional expected cost of accepting defectives among the nondestroyed, uninspected units in the total sample given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$. Thus

$$C_{S5} \text{ (Screen, } \underset{\sim}{\mu}) = C_{S5_1} + C_{S5_2} \qquad [3\text{-}48]$$

vi. <u>Total cost of screening the inspection lot</u>

The total conditional expected cost of rejecting and screening the inspection lot given acceptance on variables in Classes A and B and the vector $\underset{\sim}{\mu}$ is

$$C_S\ (\text{Screen}, \underset{\sim}{\mu}) = C_{S1}\ (\text{Screen}, \underset{\sim}{\mu}) + C_{S2}\ (\text{Screen}, \underset{\sim}{\mu})$$

$$+ C_{S3}\ (\text{Screen}, \underset{\sim}{\mu}) + C_{S4}\ (\text{Screen}, \underset{\sim}{\mu})$$

$$+ C_{S5}\ (\text{Screen}, \underset{\sim}{\mu}) \qquad [3\text{-}49]$$

Taking the expectation of Equation 3-49 with respect to acceptance of the lot on variables in Classes A and B, the total conditional expected cost of rejecting and screening the inspection lot given the vector $\underset{\sim}{\mu}$ is

$$C_S\ (\underset{\sim}{\mu}) = C_S\ (\text{Screen}, \underset{\sim}{\mu})\ (\prod_{i=1}^{k_2} P[A_i|\mu_i]) \qquad [3\text{-}50]$$

H. Total Expected Cost of Controlling the Quality of the Inspection Lot

The total expected cost of controlling the quality of an inspection lot was first expressed in Equation 3-1. Each component of that cost expression, conditioned upon the vector $\underset{\sim}{\mu}$, has been developed in Sections D through G, inclusive, of this chapter. Recall once again that the total cost of controlling lot quality consists of the cost of reaching a quality control decision and the cost of implementing that decision. Taking the expectation of the sum of Equations 3-8, 3-27, 3-29, and 3-49 the total expected cost of controlling the quality of an inspection lot is

$$C_T = \prod_{i=1}^{k_3} \int_{-\infty}^{\infty} [C_I(\underset{\sim}{\mu}) + C_A(\underset{\sim}{\mu}) + C_R(\underset{\sim}{\mu}) + C_S(\underset{\sim}{\mu})]$$

$$f(\mu_i)\, d\mu_i \qquad [3\text{-}51]$$

CHAPTER IV

RESULTS

A. Introduction

The development of a mathematical model, which characterizes an economic system of acceptance sampling plans for the simultaneous control of several variables, may be useful in analyzing the cost effectiveness of existing quality control systems. The model's real importance to the practicing quality control engineer, however, lies in its ability to be used to determine the minimum cost quality control system in a variety of situations. The next section in this chapter of the thesis contains three examples to which the model presented herein has been applied. The examples are considered for several reasons. First, the acceptance sampling plans computed on an individual basis may not be the most economic set of multivariate plans, and this aspect of the problem is considered. Second, a computational procedure is suggested for determining an optimum set of acceptance sampling plans and is illustrated for each of the examples. Finally, the dependency of the model upon the cost parameters specified as part of the input data is demonstrated.

The last section in this chapter deals with the sensitivity of the model with regard to the forms of the probability density functions of the random variables representing the lot mean for each of the characteristics being controlled. Sensitivity analysis may be important since identifying the distributions of lot means often presents a significant problem when using a model of this type. Practically speaking, the shapes of these distributions are never known and are sometimes difficult to estimate. These distributions are, nevertheless, part of the model and errors in estimating their shapes may be critical. The sensitivity analysis is conducted for a specific example and although no general conclusions should be drawn from the results, the method of analysis may be extended to any problem of this nature.

B. Applied Examples

This section of the chapter contains three examples to which the model has been applied. The examples are now presented individually, identifying the parameters and an optimum multivariate quality control system for each. A method which can be used to compute the optimum system of acceptance sampling plans is also suggested.

i. Alternative optimization approaches

In order to identify the minimum cost system of acceptance sampling plans, the model must be optimized with respect to the decision variables n_i, LCL_i, and UCL_i ($i = 1, \ldots, k_3$). Due to the complexity of the model, no attempt was made to apply classical optimization techniques in this regard. The pattern search was selected to compute an indicated optimum solution for the model. Validation of this approach is presented in Appendix F.

One method of systematically applying the pattern search to the model may be briefly summarized as follows:

> Consider variable 1, disregarding the presence of the other variables; starting from a point of acceptance without sampling, optimize the model in order to obtain an optimum acceptance sampling plan for variable 1.
>
> Consider variable 2, disregarding the presence of the other variables; starting from a point of acceptance without sampling, optimize the model in order to obtain an optimum acceptance sampling plan for variable 2.
>
> .
>
> .
>
> .
>
> Consider variable k_3, disregarding the presence of the other variables; starting from a point of acceptance

without sampling, optimize the model in order to obtain an optimum acceptance sampling plan for variable k_3.

Evaluate the model for these individually computed acceptance sampling plans in order to obtain the actual cost of multivariate quality control. Starting from this point, optimize all variables together in the model to obtain an optimum multivariate set of acceptance sampling plans.

Another systematic approach to the optimization of the model for a specific problem is executed in two steps. This method is:

Select starting points of acceptance without sampling.

Optimize the model in order to obtain the optimum multivariate set of acceptance sampling plans.

For the examples herein, the first approach was found to be computationally more efficient than the second and was, therefore, the one selected for use in this thesis. The results of using each of the approaches with respect to the computation of the optimum multivariate set of acceptance sampling plans for Example 1 are summarized in Appendix D.

ii. Example 1

The acceptance sampling problem of interest in this example is one in which four variables are to be simultaneously controlled. Two of the variables are in class A, one in class B, and one in class C, i.e. $k_1 = 2$, $k_2 = 3$, and $k_3 = 4$. The random variables μ_i, $\bar{X}_i$, and X_i ($i = 1, 2, 3, 4$) are all normally distributed. The parameters of the variables considered in this example are given in Table 4-1. The inspection lot size is 10000 units and the replacement option is in effect, $\delta_2 = 1$. The replacement units are screened, $\delta_3 = 1$. The cost parameters for the variables are provided in Table 4-2. The acceptance costs for each of the variables in this example are given in Table 4-3.

The principal probability statements used in the derivation of the expected cost model are as follows:

a. P (the measured value of the ith variable falls within the specification limits)

$$= \int_{-\infty}^{\infty} \int_{LS_i}^{US_i} h(x_i|\mu_i)\ f(\mu_i)\ dx_i\ d\mu_i \qquad i = 1, 2, 3, 4 \qquad [4\text{-}1]$$

TABLE 4-1. VARIABLES PARAMETERS - ALL EXAMPLES					
VARIABLE NUMBER	VARIABLE TYPE	PROCESS MEAN	STANDARD DEVIATION	LOWER SPECIFICATION LIMIT	UPPER SPECIFICATION LIMIT
1	A	20.00	0.50	18.40	21.60
2	A	50.00	1.40	45.80	54.20
3	B	5.00	0.80	3.00	7.00
4	C	15.00	1.00	12.30	17.70

TABLE 4-2. COST PARAMETERS - ALL EXAMPLES

VARIABLE NUMBER	SAMPLING INSPECT. C_{Ii}	SCREENING INSPECT. C_{si}	REPAIR $C_{\sigma Li}$	REPAIR $C_{\sigma Ui}$	SCRAP $C_{\rho 1}$	REPLACE $C_{\rho 2}$	MATERIALS HANDLING C_r
1	$0.005	———	———	———			
2	$0.005	———	———	———	$2.00	$2.00	$0.00
3	$0.005	———	———	———			
4	$0.005	$0.005	$4.75	$4.75			

TABLE 4-3. ACCEPTANCE COSTS - SPECIFIC EXAMPLES

EXAMPLE NUMBER	VARIABLE NUMBER	C_{aLi}	C_{aUi}
1	1	\$55.85	\$55.85
	2	34.75	34.75
	3	12.00	12.00
	4	5.00	5.00
2	1	55.85	55.85
	2	5.00	5.00
	3	12.00	12.00
	4	5.00	5.00
3	1	20.00	20.00
	2	5.00	5.00
	3	12.00	12.00
	4	5.00	5.00

b. P (accept on the ith variable)

$$= \int_{-\infty}^{\infty} \int_{LCL_i}^{UCL_i} g(\bar{x}_i | \mu_i)\, f(\mu_i)\, d\bar{x}_i\, d\mu_i$$

$$i = 1, 2, 3, 4 \qquad [4\text{-}2]$$

c. P (measured value of the ith variable falls within the specification limits for a lot accepted on i)

$$= \int_{-\infty}^{\infty} \int_{LCL_i}^{UCL_i} \int_{LS_i}^{US_i} h(x_i | \mu_i)\, g(\bar{x}_i | \mu_i)\, f(\mu_i)\, dx_i\, d\bar{x}_i\, d\mu_i$$

$$i = 1, 2, 3, 4 \qquad [4\text{-}3]$$

Since each of the probability statements in Equations 4-1, 4-2, and 4-3 are formed by combinations of normal density functions, these three expressions can be simplified. The mathematical development forming the basis for this simplification is presented in Appendix B of the thesis. Each of the expressions in Equations 4-1 and 4-2 reduces to a single integral and is evaluated, when required, by applying the approximation to the normal distribution found in Appendix C. Unlike the first two, the simplified form of Equation 4-3 remains a double integral and cannot be evaluated by a single approximation. A numerical integration technique in conjunction with the approximating relationship for the normal distribution function is used to evaluate this probability statement.

The problem now is to compute the optimum multivariate set of acceptance sampling plans for the simultaneous control of the four variables. Each of the variables is first considered as occurring individually, disregarding the presence of the others. In these four initial problems, the pattern search [44] is applied to the model in order to determine an optimal acceptance sampling plan for each of the variables. These results are summarized in Table 4-4.

The total expected costs of quality control per inspection lot, C_T, in Table 4-4 represent the cost of controlling the ith variable, disregarding the presence of the other variables. Since the variables being controlled appear together rather than individually, the total expected cost of controlling the overall quality of the inspection lot on all variables remains unknown at this point. This overall total expected cost is generally not the sum of the individual costs, C_T, in Table 4-4 and, in this sense, these costs are somewhat misleading. The total expected cost per inspection lot that is actually incurred as the result of simultaneously implementing the

TABLE 4-4. SINGLY OPTIMIZED ACCEPTANCE SAMPLING PLANS - EXAMPLE 1

VAR NUMBER	POINT	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
1	START	0	19.50	20.50	$0.00	$6367.62	$ 0.00	$ 0.00	$6367.62
	OPT	30	19.25	20.75	0.15	3879.90	1189.01	0.00	5069.06
2	START	0	48.50	51.50	0.00	6147.27	0.00	0.00	6147.27
	OPT	32	47.94	52.06	0.16	3905.91	1274.14	0.00	5180.21
3	START	0	3.50	6.50	0.00	5772.03	0.00	0.00	5772.03
	OPT	22	3.70	6.30	0.11	4573.38	861.95	0.00	5435.44
4	START	0	12.50	17.50	0.00	1639.23	0.00	0.00	1639.23
	OPT	36	14.40	15.60	0.18	303.00	0.00	1292.69	1595.87

optimum individually computed acceptance sampling plans as evaluated in the multivariate model is the point labeled "Start" in Table 4-5.

From this point, the pattern search is applied to the model in order to compute the optimum multivariate set of acceptance sampling plans. The optimum multivariate set of plans in this case indicates that sampling for each of the variables should be conducted. As shown in Table 4-5 a reduction in the total expected cost of quality control per inspection lot can be realized by implementing the optimum multivariate acceptance sampling plans rather than by simply implementing the optimum individual plans.

iii. Example 2

The second example is identical to the first in all respects except one. In this case, the acceptance costs have been adjusted as shown in the Table 4-3. The method of approaching optimization of the problem in this example is the same as was used in the first example.

The individually computed acceptance sampling plans for this example are presented in Table 4-6. This set of plans indicates that sampling for all of the variables except variable 2 should be conducted. The optimum multivariate set of acceptance sampling plans is summarized in Table 4-7. The optimum plans in this case, although not

TABLE 4-5. MULTIVARIATE OPTIMIZATION OF THE MODEL - EXAMPLE 1

POINT	VAR NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
START	1	30	19.25	20.75	\$0.55	\$6344.28	\$3146.50	\$6016.18	\$15507.51
	2	32	47.94	52.06					
	3	22	3.70	6.30					
	4	36	14.40	15.60					
OPT.	1	36	19.42	20.58	1.13	4127.35	6846.35	3653.21	14628.04
	2	34	48.34	51.66					
	3	28	3.98	6.02					
	4	214	14.37	15.63					

TABLE 4-6. SINGLY OPTIMIZED ACCEPTANCE SAMPLING PLANS - EXAMPLE 2

VAR NUMBER	POINT	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
1	START	0	19.50	20.50	$0.00	$6367.62	$ 0.00	$ 0.00	$6367.62
	OPT	30	19.25	20.75	0.15	3879.90	1189.01	0.00	5069.06
2	START	0	46.50	53.50	0.00	884.50	0.00	0.00	884.50
	OPT	0	46.50	53.50	0.00	884.50	0.00	0.00	884.50
3	START	0	3.50	6.50	0.00	5772.03	0.00	0.00	5772.03
	OPT	22	3.70	6.30	0.11	4573.38	861.95	0.00	5435.44
4	START	0	12.50	17.50	0.00	1639.23	0.00	0.00	1639.23
	OPT	36	14.40	15.60	0.18	303.00	0.00	1292.69	1595.87

TABLE 4-7. MULTIVARIATE OPTIMIZATION OF THE MODEL - EXAMPLE 2									
POINT	VAR NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
START	1	30	19.25	20.75	$0.42	$5140.86	$1999.77	$5102.09	$12243.14
	2	0	46.50	53.50					
	3	22	3.70	6.30					
	4	36	14.40	15.60					
OPT	1	35	19.38	20.62	1.00	4020.60	3867.01	4019.56	11908.17
	2	0	45.74	54.26					
	3	26	3.90	6.10					
	4	176	14.41	15.61					

the same as the individually computed plans, also indicate that sampling should be conducted for all variables except variable 2. Again, as shown in Table 4-7, reduction in the total expected cost of quality control per inspection lot can be realized by implementing the optimum multivariate set of acceptance sampling plans rather than simply using the optimum individual plans.

iv. Example 3

The third example is also identical to the first example in all respects except one. Here the acceptance costs have been adjusted as shown in Table 4-3. The method of approaching the optimization of the problem in this case is the same as in the first and second examples.

The optimum acceptance sampling plans computed for each variable, when the presence of the others is disregarded, is presented in Table 4-8. In this example, the individually computed plans indicate that two variables, variables 1 and 2, should be accepted without inspection. The optimum multivariate set of acceptance sampling plans in Table 4-9, however, indicates that inspection for variable 1 should be performed while variable 2 remains to be accepted without inspection. A savings in the total expected cost of quality control per inspection lot can likewise be realized in this example

TABLE 4-8. SINGLY OPTIMIZED ACCEPTANCE SAMPLING PLANS - EXAMPLE 3

VAR NUMBER	POINT	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
1	START	0	19.25	20.75	$0.00	$2280.26	$ 0.00	$ 0.00	$2280.26
	OPT	0	19.25	20.75	0.00	2280.26	0.00	0.00	2280.26
2	START	0	46.50	53.50	0.00	884.50	0.00	0.00	884.50
	OPT	0	46.50	53.50	0.00	884.50	0.00	0.00	884.50
3	START	0	3.50	6.50	0.00	5772.03	0.00	0.00	5772.03
	OPT	22	3.70	6.30	0.11	4573.38	861.95	0.00	5435.44
4	START	0	12.50	17.50	0.00	1639.23	0.00	0.00	1639.23
	OPT	36	14.40	15.60	0.18	303.00	0.00	1292.69	1595.87

TABLE 4-9. MULTIVARIATE OPTIMIZATION OF THE MODEL - EXAMPLE 3

POINT	VAR NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
START	1	0	19.25	20.75	$0.28	$4490.11	$ 862.00	$4634.88	$9987.27
	2	0	46.50	53.50					
	3	22	3.70	6.30					
	4	36	14.40	15.60					
OPT	1	17	19.06	20.94	0.95	4032.78	1895.61	3846.76	9776.10
	2	0	46.56	53.44					
	3	26	3.86	6.14					
	4	164	14.37	15.63					

by implementing the optimum multivariate set of acceptance sampling plans rather than by simply using the optimum individually computed plans.

v. Comparison of the examples

The method of optimization, the parameters of the system, and the components of cost, with the exception of one category, are identical for all the examples and, in each case, as one might expect, the optimum set of multivariate acceptance sampling plans changes because of the change in this one cost category. The dependency of the model upon cost parameters is clearly demonstrated. A reduction in the total expected cost of quality control per inspection lot may be realized by implementing the optimum multivariate set of acceptance sampling plans rather than the optimum individually computed plans. There seems, then, to be an economic incentive to use the multivariate model rather than several single variable models, at least for the examples considered here.

C. Sensitivity Analysis

In order to illustrate the importance of sensitivity analysis, consider the following example. A manufacturer is confronted with the problem of designing an economic acceptance sampling system for the simultaneous control of several variables. The accounting department has provided him with the necessary cost parameters and past information on the system has been used to accurately identify the distributions of the sample mean and the individual product dimensions of each of the variables. Data on the relative frequency with which values of the sample mean have occurred has been used to compute estimates of the mean and variance of the distributions of μ_i for each of the variables; but the shapes of these distributions remain unidentified. Since errors may be made in estimating the shapes of the distributions of μ_i, the manufacturer's main concern is that the total expected cost of quality control will be significantly greater in this case than could be realized if these distributions were known. In conducting the sensitivity analysis, the manufacturer assigns various shapes to the process distributions and then computes the optimum set of acceptance sampling plans and the total expected cost of quality control per inspection lot which would be incurred in that particular quality control system. He then computes the total expected cost error incurred by virtue of implementing these plans in a quality control system in which the shapes of the distributions of μ_i are other than the assumed shapes. In this way, the relative importance or unimportance of accurately identifying the shapes of the distributions of μ_i for this particular problem may be determined.

The analysis here is presented in order to demonstrate a method which can be used to determine the sensitivity of the model to errors in estimating the shapes of the distributions of μ_i when the model is applied to a specific problem. In this section, previously defined notation is extended such that

$f(\mu_i)$ = true distribution of μ_i, i = 1, 2, 3, 4.

C_T = the total expected cost of quality control per inspection lot incurred by virtue of implementing the optimum set of acceptance sampling plans based upon $f(\mu_i)$; and

$$C_T = C_I + C_A + C_R + C_S$$

$\hat{f}(\mu_i)$ = assumed distribution of μ_i, i = 1, 2, 3, 4.

$\hat{C}_T$ = the total expected cost of quality control per inspection lot incurred by virtue of implementing the optimum set of acceptance sampling plans based upon $\hat{f}(\mu_i)$ when $f(\mu_i)$ is the true process distribution.

$$\hat{C}_T = \hat{C}_I + \hat{C}_A + \hat{C}_R + \hat{C}_S$$

The problem of interest in this analysis is that given in Example 1, with parameters in Tables 4-1, 2-4, and 4-3.

Since any data which may be collected on the distributions of μ_i is usually in discrete form, the distribution of μ_i (i = 1, 2, 3, 4) in this analysis was allowed to assume the shape of seven different discrete relative frequency functions. Using the various discrete distributions should not adversely effect the validity of the analysis since increasing the number of points in the discrete function increases the accuracy with which it approximates a continuous counterpart, although the continuous density functions for μ_i, i = 1, 2, ..., k_3), may also be used in the model where appropriate. Each of the relative frequency distributions is designed such that it has seven points, one at its desired mean value and three symmetrically placed to each side of the mean. The points are equally spaced about the mean and their positions determined such that the variance of the discrete approximates the variance of the normal within $\pm$ 0.003, with the mean of the discrete equal to the corresponding normal. The parameters for these discrete distributions, as well as for the normal, are contained in Appendix E.

The sensitivity analysis is conducted in two steps. First, let the distribution $f(\mu_i)$, for i = 1, 2, 3, 4 be the discrete relative frequency function (b) in Figure 4-1. The model must then be optimized in order to obtain the optimum multivariate set of acceptance sampling plans and the total expected cost of quality control per inspection lot, C_T, incurred by virtue of implementing these plans. The results of this step of the analysis are summarized in Table 4-10.

Second, assume the distribution of μ_i is normal, denoted $\hat{f}(\mu_i)$. The optimum system of acceptance sampling plans obtained for this assumption is given as "Normal (a)" in Table 4-10, and is, of course, the same set obtained in Example 1 of the chapter. The total expected cost of quality control per inspection lot, $\hat{C}_T$, incurred by virtue of implementing this assumed optimum set of sampling plans when the true distribution of μ_i, $f(\mu_i)$, is the discrete function b given in Table 4-11. The total expected cost of quality control, $\hat{C}_T$, will be in error since the true distributions of the lot means are other than assumed normal.

The final aspect of this step of the sensitivity analysis is to determine the cost error incurred by virtue of making the assumption that $\hat{f}(\mu_i)$ is normal when, in fact, $f(\mu_i)$ is actually the relative frequency function b.

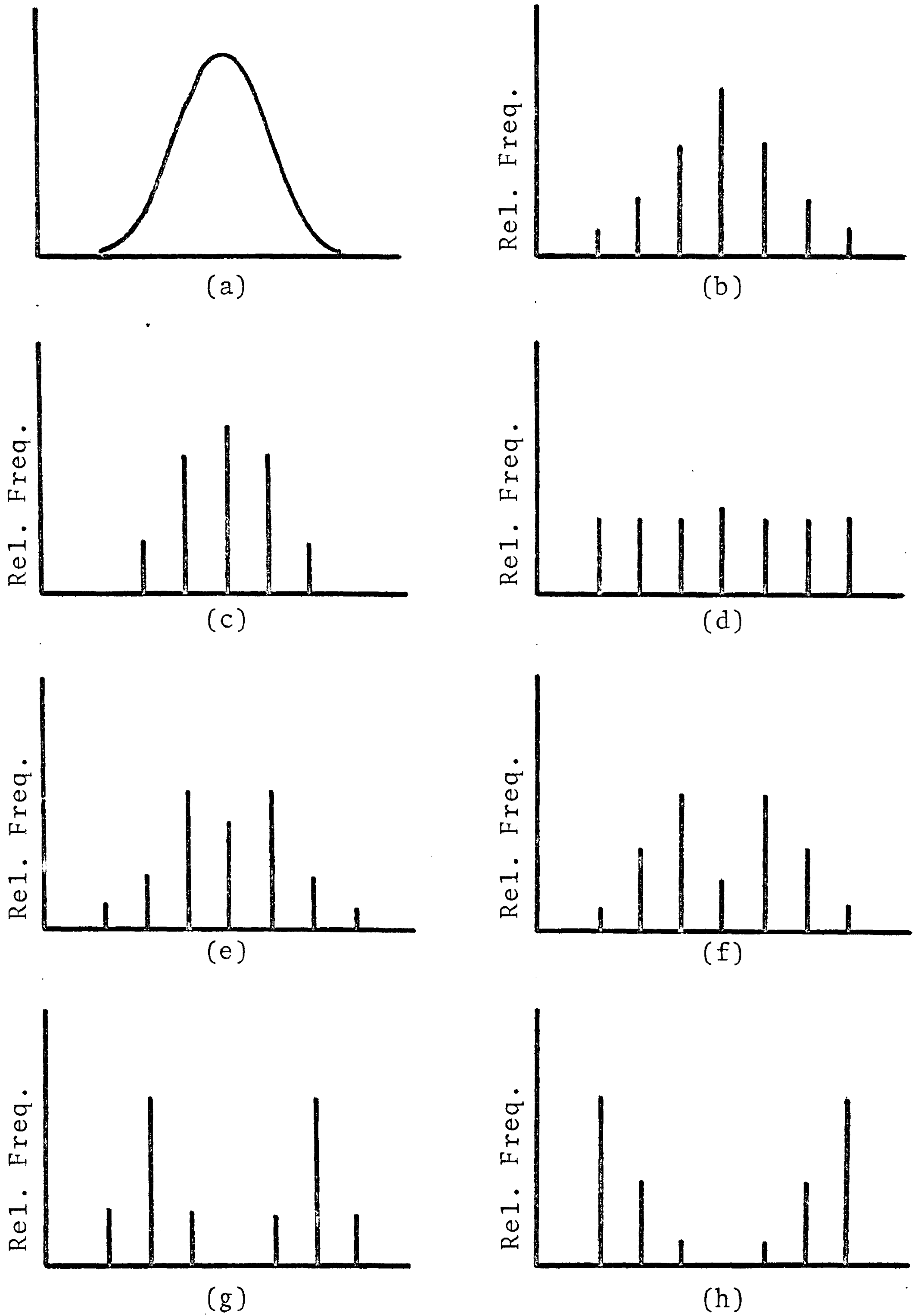

Figure 4-1. Sensitivity Analysis - Distributions of μ_i

TABLE 4-10. SENSITIVITY ANALYSIS - $f(\mu_i)$ KNOWN

$f(\mu_i)$	VAR. NO.	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
NORMAL (a)	1	36	19.4200	20.5789	$1.13	$4127.35	$6846.35	$3653.21	$14628.04
	2	34	48.3430	51.6508					
	3	28	3.9824	6.0174					
	4	214	14.3730	15.6335					
DISCRETE (b)	1	36	19.3819	20.6179	1.09	5340.61	6464.01	2978.26	14783.97
	2	36	48.2375	51.7653					
	3	35	3.9699	6.0300					
	4	184	14.2907	15.7143					

TABLE 4-10. SENSITIVITY ANALYSIS - $f(\mu_i)$ KNOWN (CONTINUED)

$f(\mu_i)$	VAR NO.	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
DISCRETE (c)	1	46	19.4736	20.5261	$0.80	$1458.42	$9558.24	$4005.99	$15023.45
	2	44	48.5089	51.4914					
	3	37	4.1124	5.8874					
	4	109	14.6760	15.3737					
DISCRETE (d)	1	9	19.3224	20.6775	0.82	3884.84	6896.35	5599.86	16381.87
	2	51	48.6275	51.3725					
	3	0	——	——					
	4	166	14.4210	15.5749					

TABLE 4-10. SENSITIVITY ANALYSIS - $f(\mu_i)$ KNOWN (CONTINUED)

$f(\mu_i)$	VAR NO.	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
DISCRETE (e)	1	38	19.3933	20.6058	$0.86	$5514.62	$6337.79	$2984.16	$14837.43
	2	38	48.2659	51.7331					
	3	38	3.9775	6.0212					
	4	105	14.2239	15.7840					
DISCRETE (f)	1	48	19.4097	20.5908	1.09	5329.76	5833.25	4193.93	15358.03
	2	46	48.3032	51.6952					
	3	42	4.0262	5.9737					
	4	136	14.3115	15.6943					

TABLE 4-10. SENSITIVITY ANALYSIS - $f(\mu_i)$ KNOWN (CONTINUED)									
$f(\mu_i)$	VAR NO.	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
DISCRETE (g)	1	0	—	—	\$1.17	\$3069.44	\$0.00	\$12991.30	\$16061.91
	2	0	—	—					
	3	0	—	—					
	4	234	14.4332	15.5535					
DISCRETE (h)	1	0	—	—	1.63	5903.29	0.00	9834.96	15739.88
	2	0	—	—					
	3	0	—	—					
	4	326	14.2980	15.6996					

TABLE 4-11. SENSITIVITY ANALYSIS - $\hat{f}(\mu_i)$ ASSUMED NORMAL

$f(\mu_i)$	$\hat{C}_I$	$\hat{C}_A$	$\hat{C}_R$	$\hat{C}_S$	$\hat{C}_T$	PERCENT ERROR
NORMAL (a)	$1.13	$4127.35	$6846.35	$3656.21	$14628.04	0.00
DISCRETE (b)	1.10	4545.28	7407.87	2872.99	14827.24	0.29
DISCRETE (c)	1.05	2739.27	8153.85	4321.63	15215.80	1.28
DISCRETE (d)	1.15	4158.82	6522.23	5859.17	16541.37	0.97
DISCRETE (e)	1.12	4884.37	7027.39	2959.33	14872.21	0.23
DISCRETE (f)	1.18	5144.40	6142.97	4113.61	15402.16	0.29
DISCRETE (g)	1.34	2858.62	3463.61	9938.24	16261.81	1.24
DISCRETE (h)	1.47	4337.26	1511.14	10466.50	16316.37	3.66

The error for this particular example is then computed as

$$\text{Percent Error} = \frac{\hat{C}_T - C_T}{C_T} \text{ X } 100$$

$$= \frac{\$14{,}827.24 - \$14{,}783.97}{\$14{,}783.97} \text{ X } 100$$

$$= 0.29\%$$

The preceding analysis was conducted with $f(\mu_i)$ assuming the shape of each of the remaining six discrete distributions in Figure 4-1, while $\hat{f}(\mu_i)$ was assumed normal in all cases. The results of the analysis are summarized in Tables 4-10 and 4-11. In most of the cases considered, the cost error incurred by virtue of inaccurately identifying the shapes of the distributions of μ_i was less than one percent. The largest cost error was incurred in the case of the U-shaped discrete relative frequency function and even in this case was a relatively small 3.66 percent. As might be expected, when the shape of $f(\mu_i)$ departs markedly from that of the assumed normal, the magnitude of the cost error incurred tends to increase.

The results of this research seems to concur with those obtained by other authors who have considered cost-based quality control systems. Bennett and McCaslin (4), as well as Schmidt and Bennett (30), have suggested that their respective cost models for the simultaneous control of several attributes are more sensitive to errors in the estimation of the parameters of the lot mean distributions than to errors in identifying the distributional forms. Pfanzagl (27) has reached similar conclusions for the single attribute cost model.

The model presented herein has been applied to a single example in which an economic system of acceptance sampling plans is to be developed for the simultaneous control of four variables. Various discrete shapes of the lot mean distributions were analyzed with respect to the cost errors incurred by virtue of inaccurately identifying these shapes. Provided that the mean and variance of the distributions of μ_i of interest can be accurately specified, the model seems to be robust with respect to reasonable assumptions for the shapes of these distributions, at least for the example considered.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FURTHER RESEARCH

A. Summary and Conclusions

The model presented in this thesis forms a basis for developing a set of acceptance sampling plans for the economic, simultaneous control of several variables. The decision variables in the model are the sample size and the lower and upper control limits for the sample mean of each of the variables being controlled. The sample mean for a variable is the criterion of lot acceptability for that variable. The optimum or near optimum quality control system is specified by using an iterative search technique, the pattern search, to identify the values of the decision variables such that the total expected cost of multivariate quality control per inspection lot is minimized. The model assumes that the variables being controlled are independently distributed, but does not restrict the number of variables that can be controlled simultaneously. The model also assumes that errors, if any, which occur during sampling or screening inspection are negligible.

Three categories of variables are considered in the model. The first class, denoted Class A, is a destructable category and inspection of a unit for a variable of this type destroys the product for its intended use as well as prohibits further sampling or screening inspection of that unit. Variable Class B is also a destructable category but inspection of a unit for a variable of this type simply destroys the product for its intended use. Variable Class C is a nondestructable or "screenable" category. Inspection of a unit for a variable of this type does not impair the product in any way. The order of inspection is specified as Class A first, then B, and finally C. The order within a category is determined by the sample sizes of the optimum acceptance sampling plans, inspection being conducted first for the variable in a given class having the smallest sample size, then the variable having the next largest sample size, and so on. The model is developed under the assumption that this specific sampling discipline is incorporated and if any other order of sampling is to be used then the model must be modified accordingly.

If the lot is rejected on a variable in Classes A or B during the sampling inspection procedure, the lot is scrapped and inspection immediately terminated. If the lot is accepted on all variables in Classes A and B and subsequently rejected on one or more variables in Class C, the lot is screened for only those variables rejected during the sampling inspection process. The model assumes that defective units in an accepted or screened lot are repaired, while units destroyed during sampling inspection may or may not be replaced, depending upon management policy.

The use of the model is demonstrated through the presentation of several applied examples. Optimization of these problems is achieved by first considering each of the variables separately and then using the individual optima as the starting point in the optimization of the multivariate model. This approach to determining the optimum set of acceptance sampling plans seems to be computationally more efficient than considering all of the variables together in the multivariate model initially.

As might be expected, economic benefits may be derived from multivariate computation of the acceptance sampling plans used to control product quality, depending, of course, on the specific problem of interest. In some of the examples cited, the optimal acceptance sampling plans indicated that one or more of the characteristics under consideration should be accepted without sampling. The results presented in this research lead the author to believe, based upon the examples considered, that the model is more sensitive to the parameters of the distributions of the lot mean of the variables than to the shapes of these distributions.

B. Recommendations for Further Research

The results of this research indicate several areas worthy of further study. Analytic solutions for the optimum quality control system for the model presented herein, and for other cost-based models, would remove the necessity to rely upon conventional search techniques in this respect. In addition, a more exact sensitivity analysis could be performed based upon these analytic solutions.

In many products, both variables and attributes are present and require control. A model which forms the basis for developing an economic quality control system which incorporates both of these types of characteristics would be beneficial. Similarly, a model may also be considered which not only provides for the control of the quality of the outgoing product but also takes into account the control of the process producing the product.

Consider a problem in which several screenable variables are to be simultaneously controlled. In some instances, the quality of one or more of the variables has deteriorated to the extent that the lot should be scrapped rather than rectified for those variables. A multicharacteristic model which is designed with two sets of control limits for each of the variables may be effectively applied in this case. As an example of this situation, let the lower control limits for the sample mean of the ith variable be denoted as C_{LSi} and C_{LRi} and the upper control limits be denoted as C_{USi} and C_{URi}, such that $C_{LSi} \leq C_{LSi} \leq C_{USi} \leq C_{URi}$. If the value of the sample mean for the ith variable, $\bar{X}_i$, is such that $C_{LSi} \leq \bar{X}_i \leq C_{USi}$ then the lot is accepted on variable i. If $C_{LRi} \leq \bar{X}_i < C_{LSi}$ or $C_{USi} < \bar{X}_i \leq C_{URi}$, the lot is screened for variable i. If $\bar{X}_i < C_{LRi}$ or $\bar{X}_i > C_{URi}$, then the lot is scrapped. A similar quality control system may be designed for the attributes case by considering dual acceptance numbers, rather than dual control limits.

If one knows initially which characteristics should economically not be controlled, the optimum acceptance sampling plans for these variables can automatically be specified as acceptable without sampling and these variables given no further consideration. In general, the expected cost of accepting items defective with respect to a given variable must be balanced against the expected cost of controlling that variable. At some point, the expected cost of control may overshadow the expected acceptance cost to the extent that the variable should be given no further consideration. An analytic technique for identifying the variable or variables which should not be controlled would reduce the scope of the quality control problem to only those variables which can and should be controlled.

In some quality control systems, the characteristics are inspected in a predetermined order as in the case of the model developed here. It would be useful to consider the economic aspects of inspecting the variables in Class C first, then Class B, then Class A, and so on. Although analysis for the model presented herein was not conducted in this respect, information concerning the model's dependency upon the sampling discipline would be helpful. Considering these topics, and more like them, the area of cost-based quality control systems is one in which much of the development and refinement is yet to come.

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APPENDIX A

LOGICAL DEVELOPMENT OF THE MODEL

INSPECTION 1

Draw n_1 units from the lot;

Inspect n_1 for Variable Type 1.

COST: $(C_{I1})(n_1)$

DECISION BLOCK 1 (A)

RESULT: $LCL_1 \leq \bar{X}_1 \leq UCL_1$

DECISION: Accept lot on Variable Type 1.

ACTION: Continue inspection.

DECISION BLOCK 1 (B)

RESULT: $\bar{X}_1 < LCL_1$ or $\bar{X}_1 > UCL_1$

DECISION: Reject lot on Variable Type 1.

ACTION: Reject and scrap lot; stop inspection.

COST: $(L)(C_r + C_{\rho 1})$

INSPECTION 2

Draw n_2 new units from the lot;

Inspect n_2 for Variable Type 2.

COST: $(C_{I2})(n_2)$

1

1

DECISION BLOCK 2 (A)

RESULT: $LCL_2 \leq \bar{X}_2 \leq UCL_2$

DECISION: Accept lot on Variable Type 2.

ACTION: Continue inspection.

DECISION BLOCK 2 (B)

RESULT: $\bar{X}_2 < LCL_2$ or

$\bar{X}_2 > UCL_2$

DECISION: Reject lot on Variable Type 2.

ACTION: Reject and scrap lot; stop inspection.

COST: $(L)(C_r + C_{\rho 1})$

INSPECTION k_1

Draw n_{k_1} new units from the lot.

Inspect n_{k_1} for Variable Type k_1.

COST: $(C_{Ik_1})(n_{k_1})$

2

2

DECISION BLOCK k_1 (A)

RESULT: $LCL_{k_1} \le \bar{X}_{k_1} \le UCL_{k_1}$

DECISION: Accept lot on Variable Type k_1.

ACTION: Continue inspection.

DECISION BLOCK k_1 (B)

RESULT: $\bar{X}_{k_1} < LCL_{k_1}$ or

$\bar{X}_{k_1} > UCL_{k_1}$

DECISION: Reject lot on Variable Type k_1.

ACTION: Reject and scrap lot; stop inspection.

COST: $(L)(C_r + C_{\rho 1})$

INSPECTION $k_1 + 1$

Draw n_{k_1+1} new units from the lot;

Inspect n_{k_1+1} for Variable Type k_1+1.

COST: $(C_{Ik_1+1})(n_{k_1+1})$

3

4

3

DECISION BLOCK k_1+1 (A)

RESULT: $LCL_{k_1+1} \leq \bar{X}_{k_1+1} \leq UCL_{k_1+1}$

DECISION: Accept lot on Variable Type k_1+1.

ACTION: Continue inspection.

4

DECISION BLOCK k_1+1 (B)

RESULT: $\bar{X}_{k_1+1} < LCL_{k_1+1}$ or $\bar{X}_{k_1+1} > UCL_{k_1+1}$

DECISION: Reject lot on Variable Type k_1+1.

ACTION: Reject and scrap lot; stop inspection.

COST: $(L)(C_r + C_{\rho 1})$

INSPECTION k_1+2

Draw $(n_{k_1+2} - n_{k_1+1})$ new units from the lot.

Inspect n_{k_1+2} for Variable Type k_1+2.

COST: $(C_{Ik_1+2})(n_{K_1+2})$

5

6

5

DECISION BLOCK k_1+2 (A)

RESULT: $LCL_{k_1+2} \leq \bar{X}_{k_1+2} \leq UCL_{k_1+2}$

DECISION: Accept lot on Variable Type k_1+2.

ACTION: Continue inspection.

6

DECISION BLOCK k_1+2 (B)

RESULT: $\bar{X}_{k_1+2} < LCL_{k_1+2}$ or $\bar{X}_{k_1+2} > UCL_{k_1+2}$

DECISION: Reject lot on Variable Type k_1+2

ACTION: Reject and scrap lot; stop inspection.

COST: $(L)(C_r + C_{\rho 1})$

INSPECTION k_2

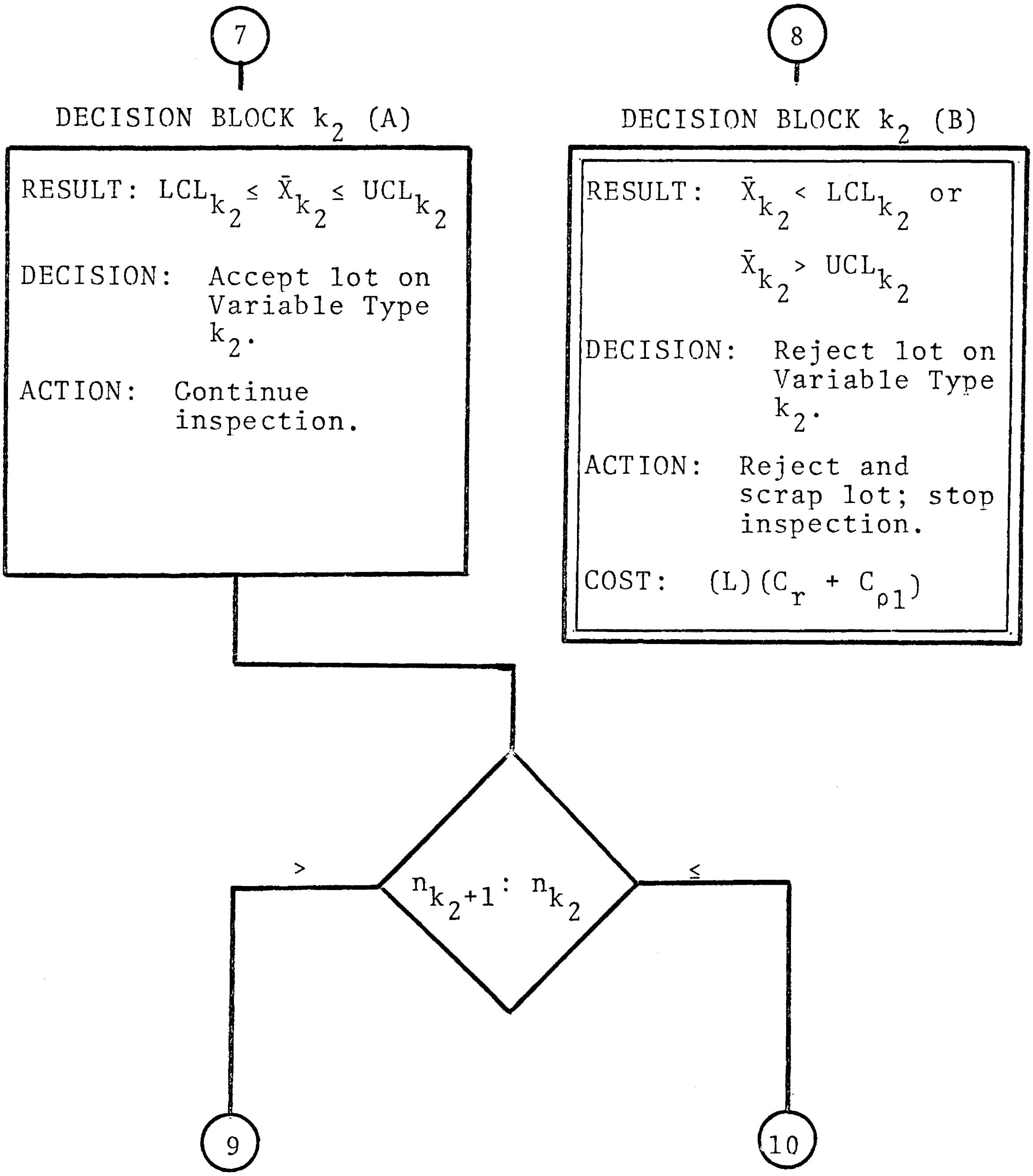
Draw $(n_{k_2} - n_{k_2-1})$ new units from the lot.

Inspect n_{k_2} for Variable Type k_2.

COST: $(C_{Ik_2})(n_{k_2})$

7

8


7
8
DECISION BLOCK k_2 (A)
RESULT: $LCL_{k_2} \le \bar{X}_{k_2} \le UCL_{k_2}$
DECISION: Accept lot on Variable Type k_2.
ACTION: Continue inspection.
DECISION BLOCK k_2 (B)
RESULT: $\bar{X}_{k_2} < LCL_{k_2}$ or $\bar{X}_{k_2} > UCL_{k_2}$
DECISION: Reject lot on Variable Type k_2.
ACTION: Reject and scrap lot; stop inspection.
COST: $(L)(C_r + C_{\rho 1})$
$n_{k_2+1} : n_{k_2}$
$>$
$\le$
9
10

9

INSPECTION k_2+1

Draw $(n_{k_2+1} - n_{k_2})$ new units from the lot.

Inspect n_{k_2+1} for Variable Type k_2+1

COST: $(C_{Ik_2+1})(n_{k_2+1})$

10

INSPECTION k_2+1

Inspect n_{k_2+1} for Variable Type k_2+1.

COST: $(C_{Ik_2+1})(n_{k_2+1})$

DECISION BLOCK k_2+1 (A)

RESULT: $LCL_{k_2+1} \leq \bar{X}_{k_2+1} \leq UCL_{k_2+1}$

DECISION: Accept lot on Variable Type k_2+1.

ACTION: Continue inspection.

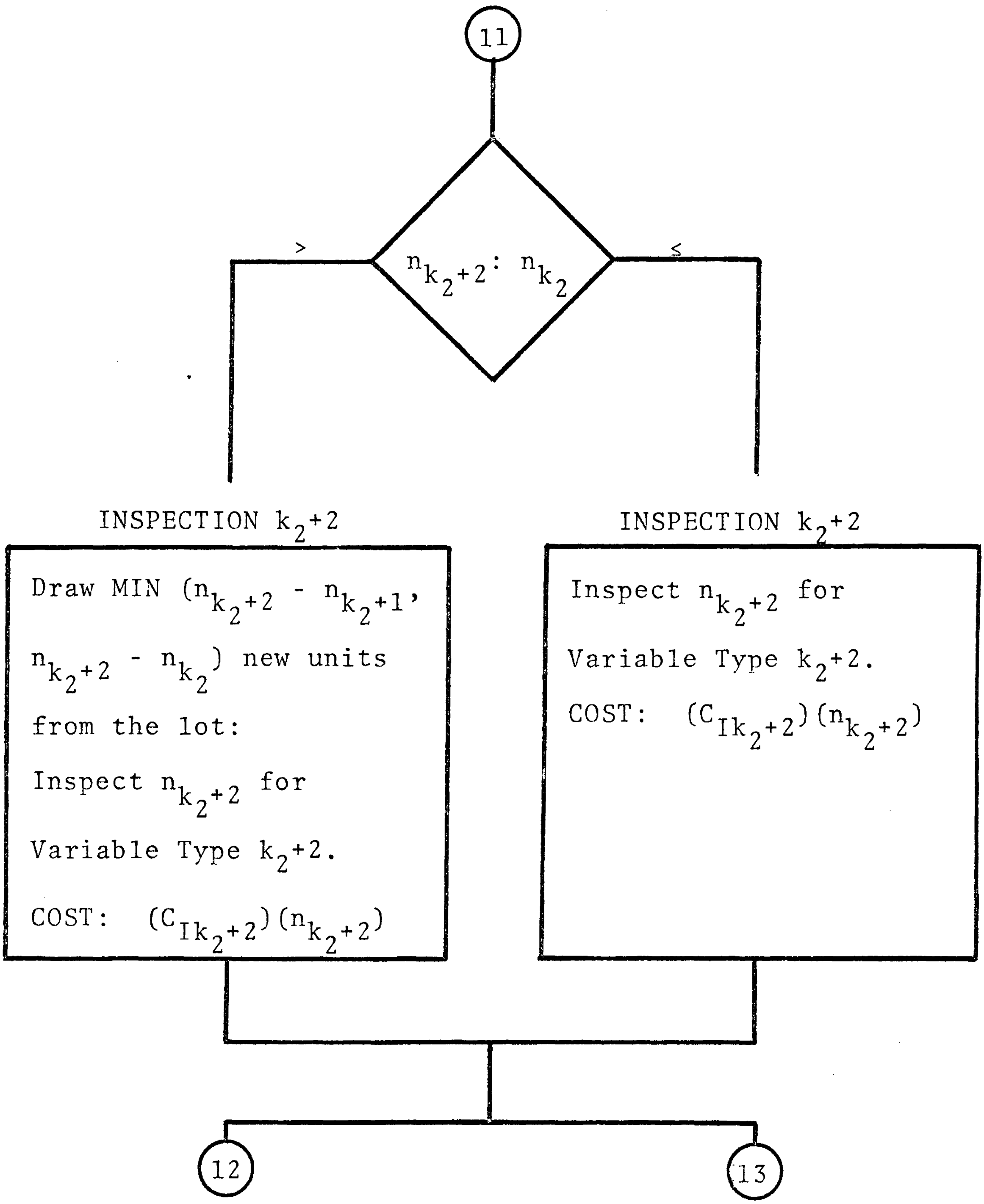
DECISION BLOCK k_2+1 (B)

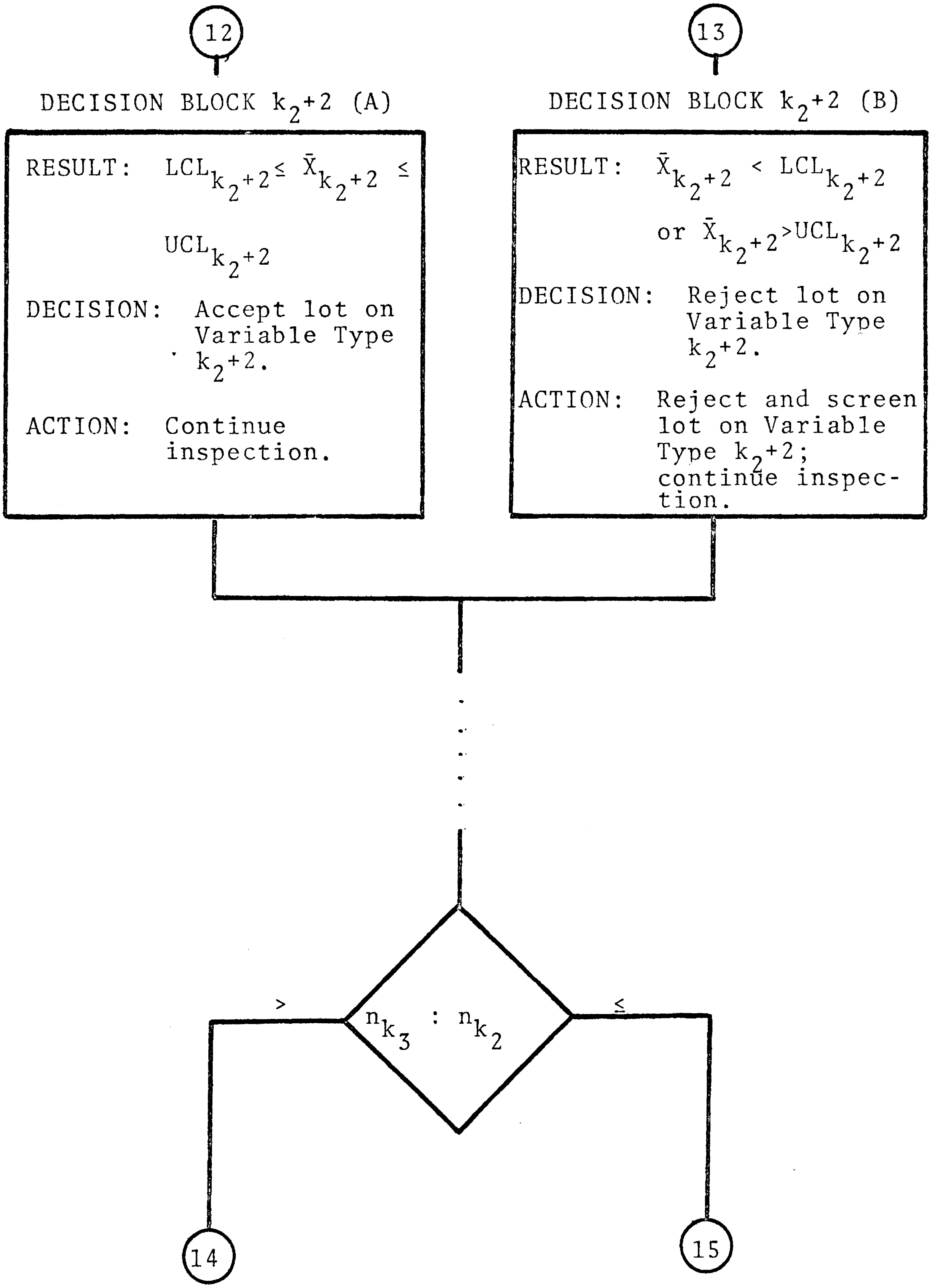
RESULT: $\bar{X}_{k_2+1} < LCL_{k_2+1}$ or $\bar{X}_{k_2+1} > UCL_{k_2+1}$

DECISION: Reject lot on Variable Type k_2+1

ACTION: Reject and screen lot on Variable Type k_2+1; Continue inspection.

11


11
n_{k_2+2} : n_{k_2}
>
≤
INSPECTION k_2+2
Draw MIN (n_{k_2+2} - n_{k_2+1}, n_{k_2+2} - n_{k_2}) new units from the lot:
Inspect n_{k_2+2} for Variable Type k_2+2.
COST: $(C_{Ik_2+2})(n_{k_2+2})$
INSPECTION k_2+2
Inspect n_{k_2+2} for Variable Type k_2+2.
COST: $(C_{Ik_2+2})(n_{k_2+2})$
12
13


12
13
DECISION BLOCK k_2+2 (A)
RESULT: $LCL_{k_2+2} \leq \bar{X}_{k_2+2} \leq UCL_{k_2+2}$
DECISION: Accept lot on Variable Type k_2+2.
ACTION: Continue inspection.
DECISION BLOCK k_2+2 (B)
RESULT: $\bar{X}_{k_2+2} < LCL_{k_2+2}$ or $\bar{X}_{k_2+2} > UCL_{k_2+2}$
DECISION: Reject lot on Variable Type k_2+2.
ACTION: Reject and screen lot on Variable Type k_2+2; continue inspection.
$n_{k_3} : n_{k_2}$
>
≤
14
15

14

INSPECTION k_3

Draw MIN ($n_{k_3} - n_{k_3-1}$, $n_{k_3} - n_{k_2}$) new units from the lot.

Inspect n_{k_3} for Variable Type k_3.

COST: $(C_{Ik_3})(n_{k_3})$

15

INSPECTION k_3

Inspect n_{k_3} for Variable Type k_3.

COST: $(C_{Ik_3})(n_{k_3})$

DECISION BLOCK k_3 (A)

RESULT: $LCL_{k_3} \leq \bar{X}_{k_3} \leq UCL_{k_3}$

DECISION: Accept lot on Variable Type k_3.

ACTION: Inspection completed.

DECISION BLOCK k_3 (B)

RESULT: $\bar{X}_{k_3} < LCL_{k_3}$ or

$X_{k_3} > UCL_{k_3}$

DECISION: Reject lot on Variable Type k_3.

ACTION: Reject and screen lot on Variable Type k_3; Inspection completed.

16

16

POST-INSPECTION ALTERNATIVE 1

Lot accepted on <u>all</u> variable types.

17

POST-INSPECTION ALTERNATIVE 2

Lot accepted on all destructable variable types $(1,\ldots,k_2)$ and rejected on one or more non-destructable types $(k_2+1,\ldots,k_3)$.

28

17

Cost of units destroyed or rejected in sampling inspection.

19

Cost of accepting defectives in the non-inspected nondestroyed portion of the sample.

26

Cost of accepting defectives in the non-inspected portion of the lot.

$$N^* = \text{MAX}\left(\sum_{i=1}^{k_1} n_i + n_{k_2},\ \sum_{i=1}^{k_1} n_i + n_{k_3}\right)$$

$(L - N^*)$ $[C_{aL1}$ P(unit defective on Variable 1 below lower spec) + C_{aUi} P(unit defective on variable 1 above upper spec)]

18

18

(L - N*) $[C_{aL2}$ P(unit defective on variable 2 below lower spec) + C_{aU2} P(unit defective on variable 2 above upper spec)]

.
.
.
.
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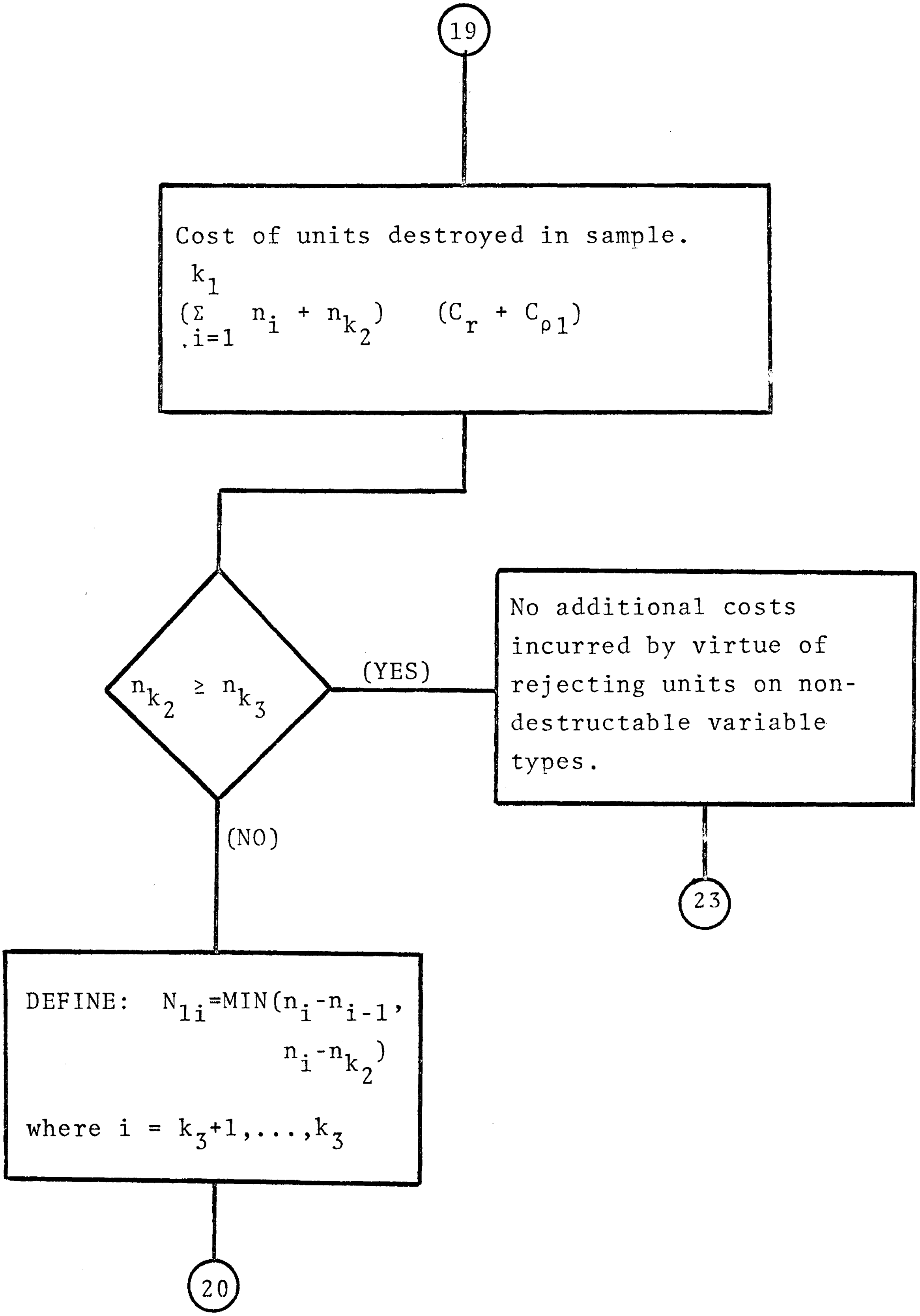
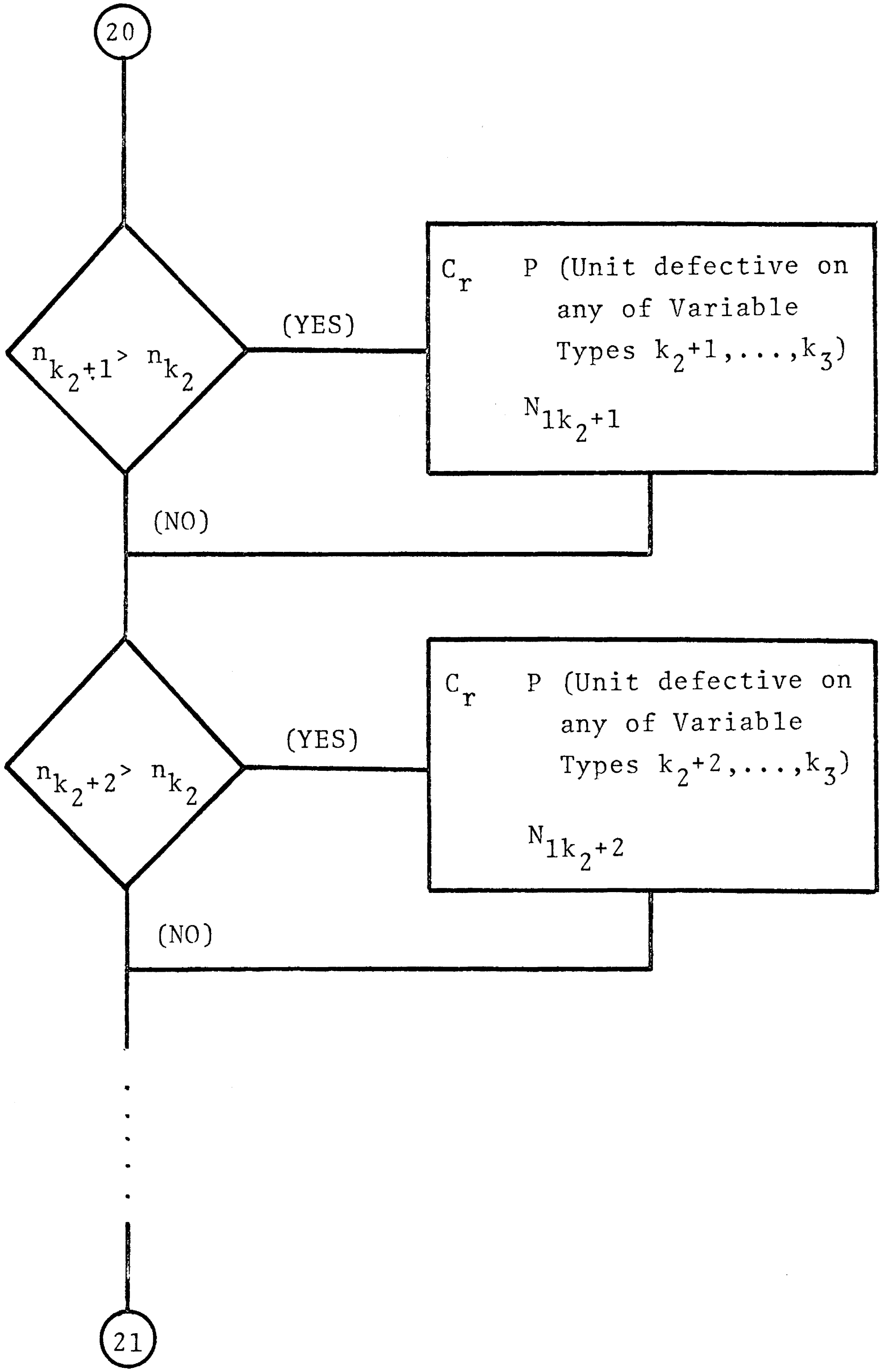
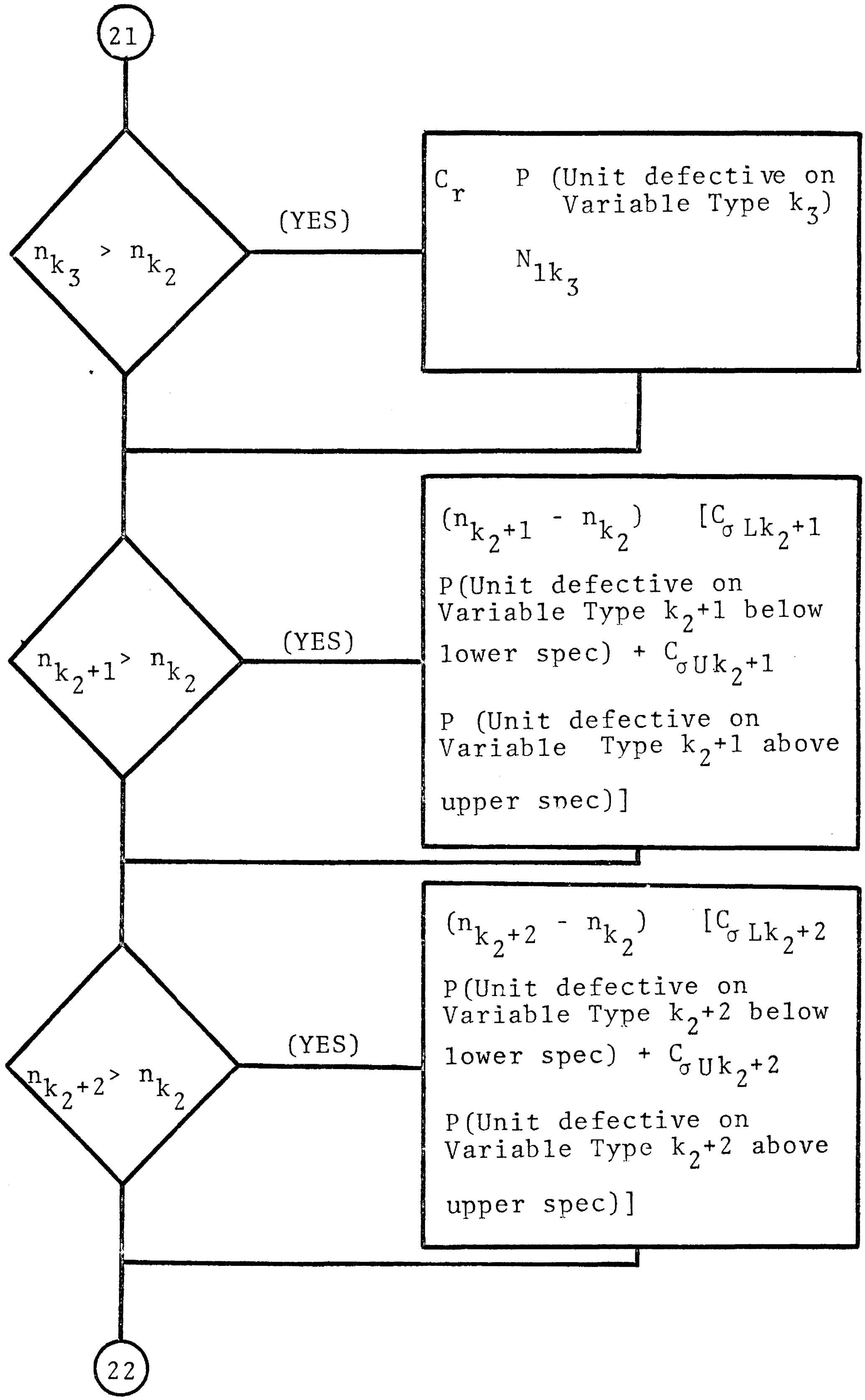
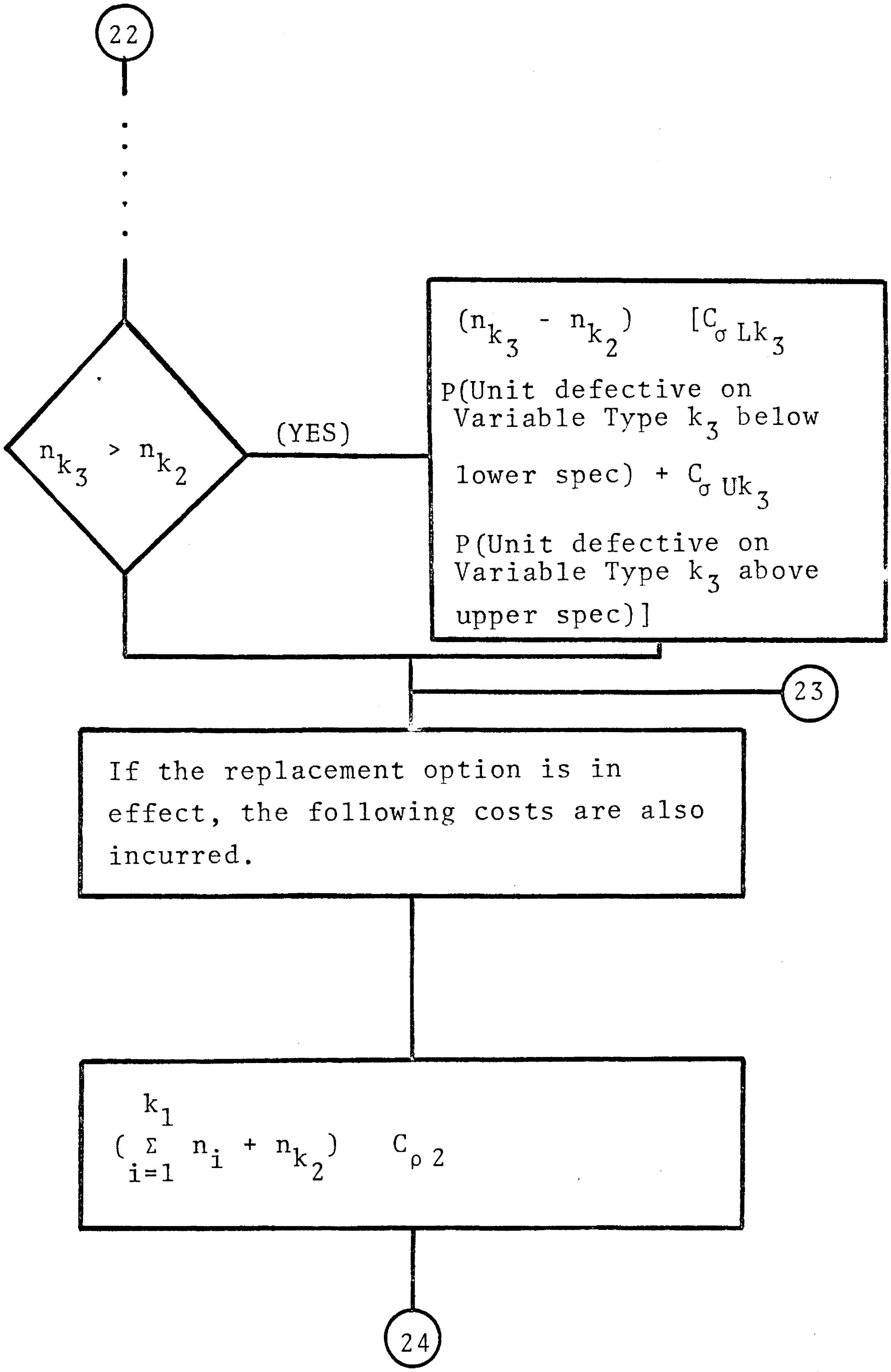
(L - N*) $[C_{aLk_3}$ P(unit defective on variable k_3 below lower spec) + C_{aUk_3} P(unit defective on variable k_3 above upper spec)]


19
Cost of units destroyed in sample.
$(\sum_{i=1}^{k_1} n_i + n_{k_2}) \quad (C_r + C_{\rho 1})$
$n_{k_2} \geq n_{k_3}$
(YES)
No additional costs incurred by virtue of rejecting units on non-destructable variable types.
23
(NO)
DEFINE: N_{1i}=MIN(n_i-n_{i-1}, n_i-n_{k_2})
where i = k_3+1,...,k_3
20


20
$n_{k_2+1} > n_{k_2}$
(YES)
C_r P (Unit defective on any of Variable Types $k_2+1,\ldots,k_3$)
N_{1k_2+1}
(NO)
$n_{k_2+2} > n_{k_2}$
(YES)
C_r P (Unit defective on any of Variable Types $k_2+2,\ldots,k_3$)
N_{1k_2+2}
(NO)
21


21
$n_{k_3} > n_{k_2}$
(YES)
C_r P (Unit defective on Variable Type k_3)
N_{1k_3}
$n_{k_2+1} > n_{k_2}$
(YES)
$(n_{k_2+1} - n_{k_2})$ [$C_\sigma Lk_2+1$
P(Unit defective on Variable Type k_2+1 below lower spec) + $C_\sigma Uk_2+1$
P (Unit defective on Variable Type k_2+1 above upper spec)]
$n_{k_2+2} > n_{k_2}$
(YES)
$(n_{k_2+2} - n_{k_2})$ [$C_\sigma Lk_2+2$
P(Unit defective on Variable Type k_2+2 below lower spec) + $C_\sigma Uk_2+2$
P(Unit defective on Variable Type k_2+2 above upper spec)]
22


22
$n_{k_3} > n_{k_2}$
(YES)
$(n_{k_3} - n_{k_2})$ [$C_\sigma Lk_3$
P(Unit defective on Variable Type k_3 below lower spec) + $C_\sigma Uk_3$
P(Unit defective on Variable Type k_3 above upper spec)]
23
If the replacement option is in effect, the following costs are also incurred.
$(\sum_{i=1}^{k_1} n_i + n_{k_2})$ $C_{\rho 2}$
24

24

$$\left(\sum_{i=1}^{k_1} n_i + n_{k_2}\right) C_{sk_2+1}$$

$$\vdots$$

$$\left(\sum_{i=1}^{k_1} n_i + n_{k_2}\right) \quad C_{sk_3}$$

$$\left(\sum_{i=1}^{k_1} n_i + n_{k_2}\right) \quad [C_{\sigma Lk_2+1}$$

P(Unit defective on Variable Type k_2+1 below lower spec) + $C_{\sigma Uk_2+1}$

P(Unit defective on Variable Type k_2+1 above upper spec)]

25

25

$(\sum_{i=1}^{k_1} n_i + n_{k_2})$ $[C_\sigma L_{k_2+2}$

P(Unit defective on Variable Type k_2+2 below lower spec) + $C_\sigma U_{k_2+2}$

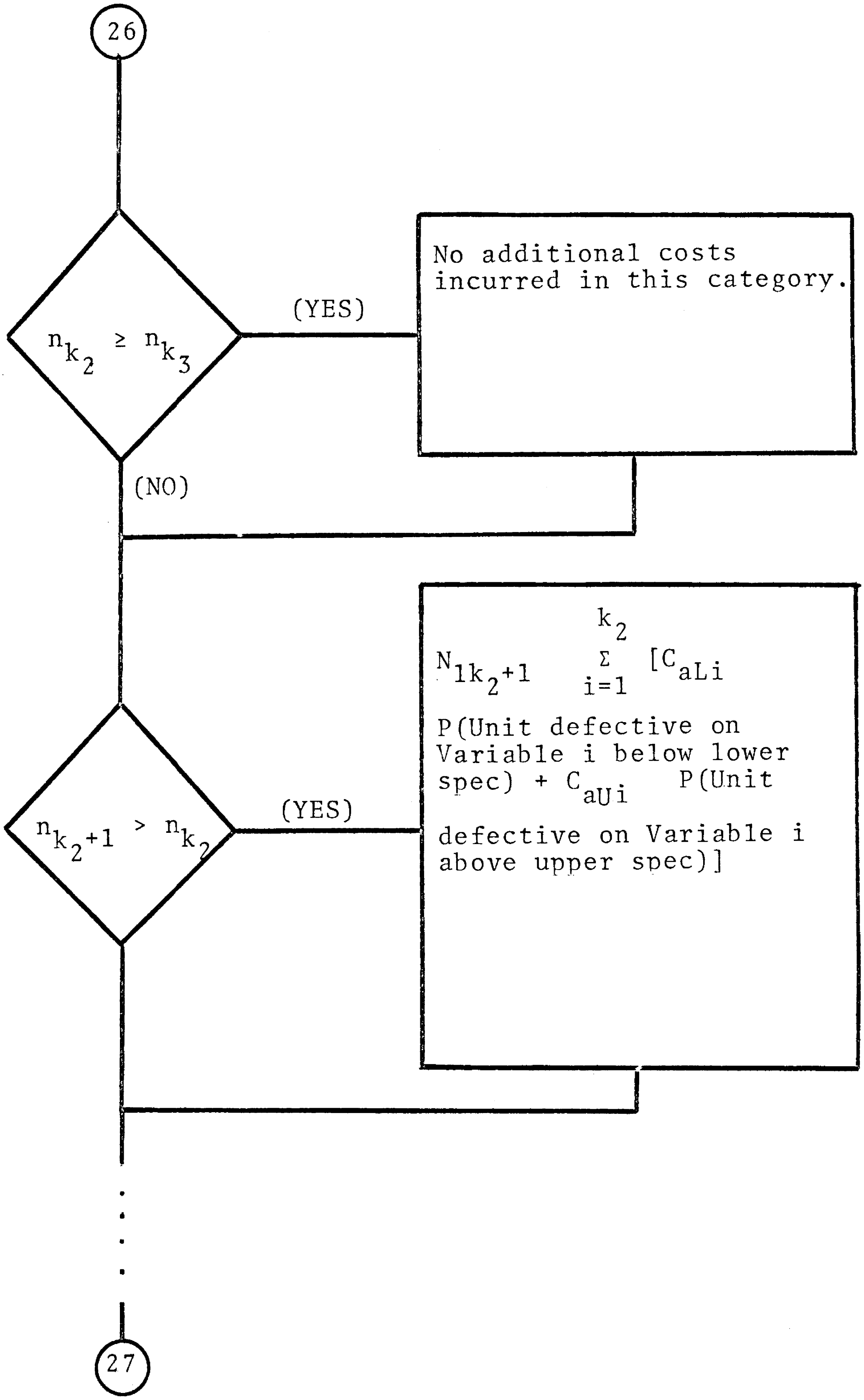
P(Unit defective on Variable Type k_2+2 above upper spec)]

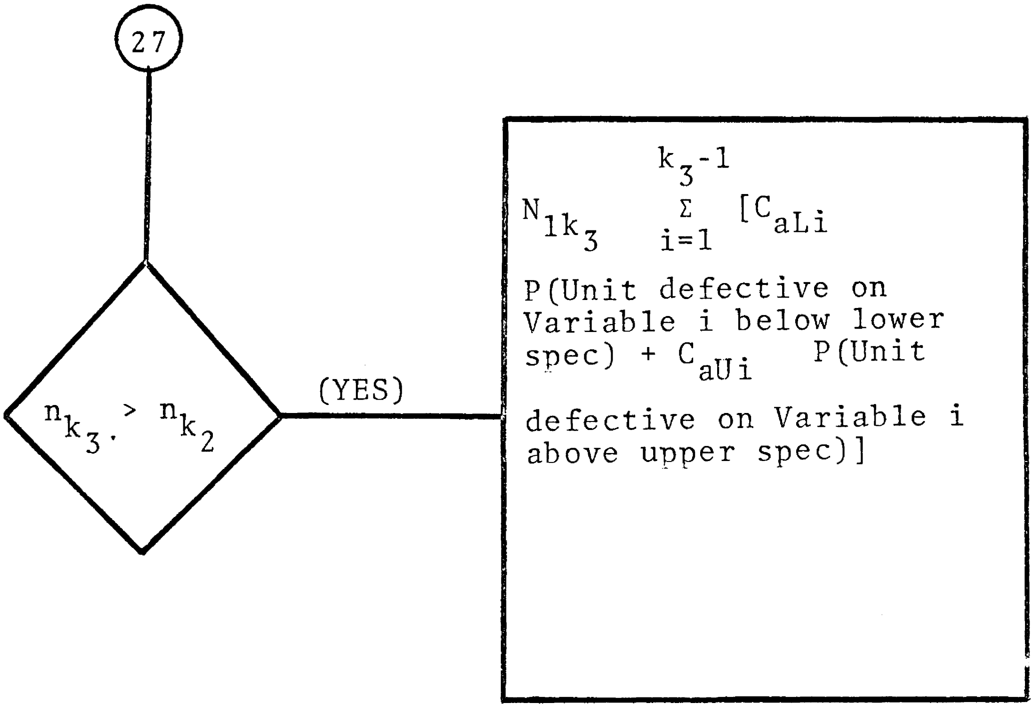
.
.
.
.
.

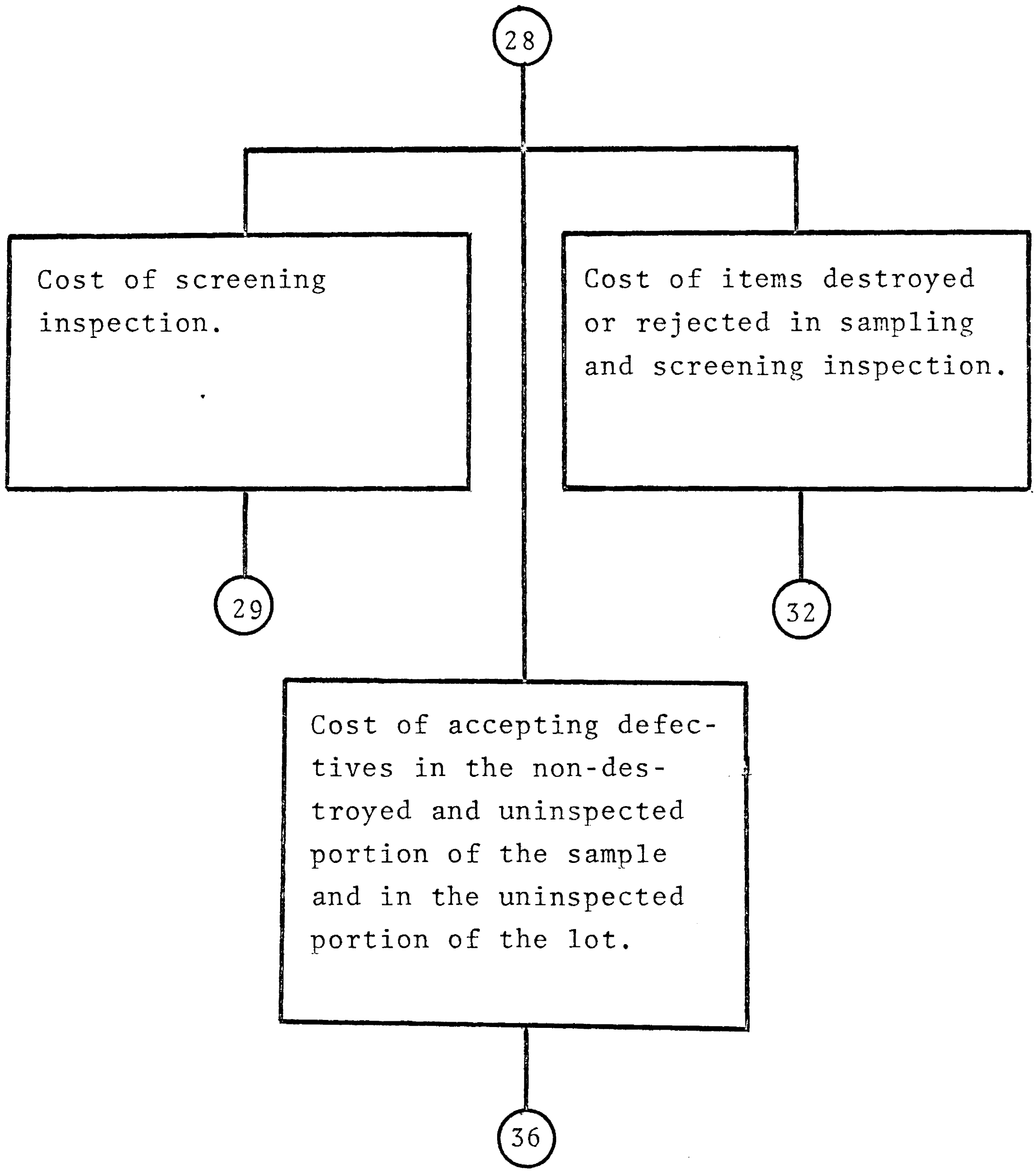
$(\sum_{i=1}^{k_1} n_i + n_{k_3})$ $[C_\sigma L_{k_3}$

P(Unit defective on Variable Type k_3 below lower spec) + $C_\sigma U_{k_3}$

P(Unit defective on Variable Type k_3 above upper spec)]


26
$n_{k_2} \geq n_{k_3}$
(YES)
No additional costs incurred in this category.
(NO)
$n_{k_2+1} > n_{k_2}$
(YES)
$N_{1k_2+1} \sum_{i=1}^{k_2} [C_{aLi}$
P(Unit defective on Variable i below lower spec) + C_{aUi} P(Unit
defective on Variable i above upper spec)]
27


27
$n_{k_3} > n_{k_2}$
(YES)
$N_{1k_3} \sum_{i=1}^{k_3-1} [C_{aLi}$
P(Unit defective on Variable i below lower spec) + C_{aUi} P(Unit defective on Variable i above upper spec)]


28
Cost of screening inspection.
Cost of items destroyed or rejected in sampling and screening inspection.
29
32
Cost of accepting defectives in the non-destroyed and uninspected portion of the sample and in the uninspected portion of the lot.
36

29

(L - N*) $[C_{sk_2+1}$

P(Lot rejected on Variable Type k_2+1)]

(L - N*) $[C_{sk_2+2}$

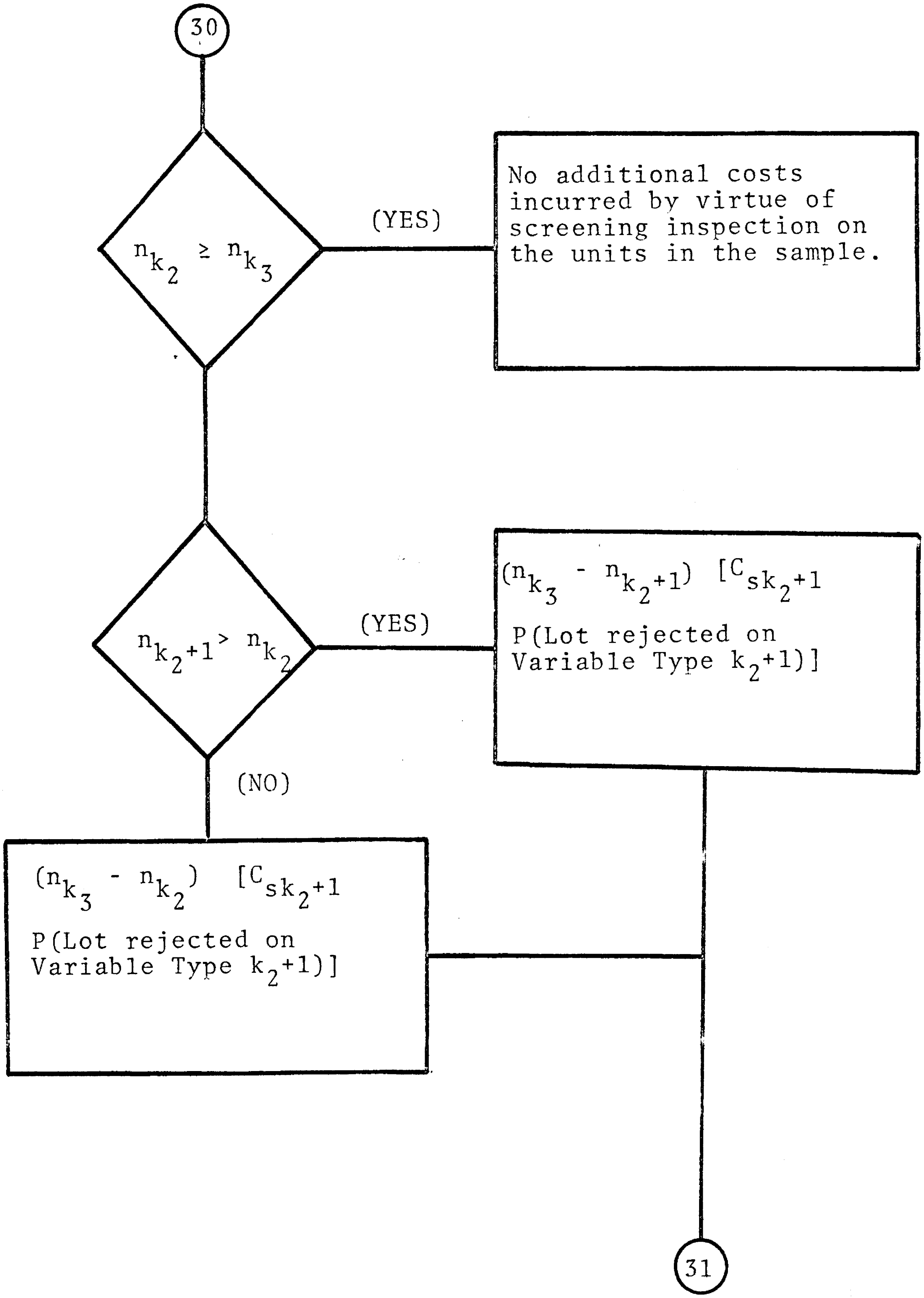
P(Lot rejected on Variable Type k_2+2)]

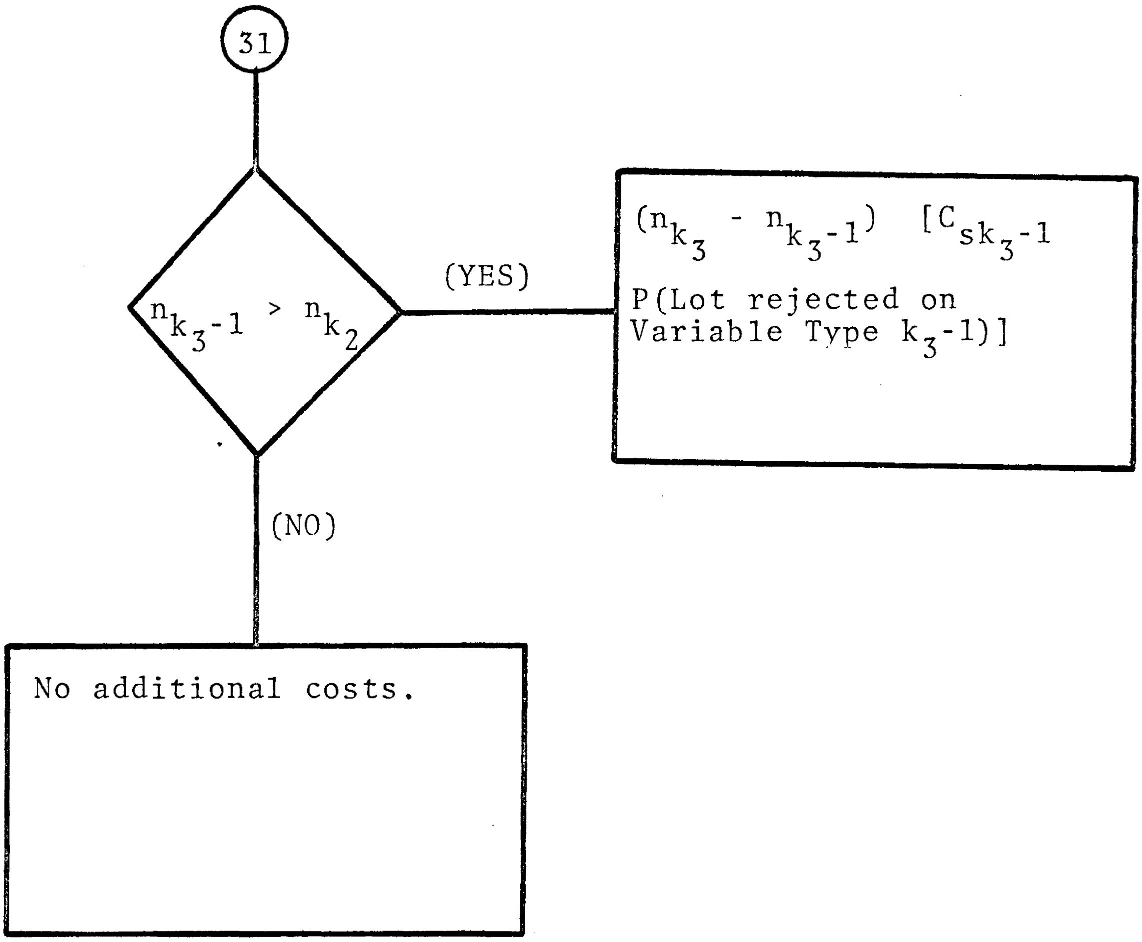
.
.
.
.
.

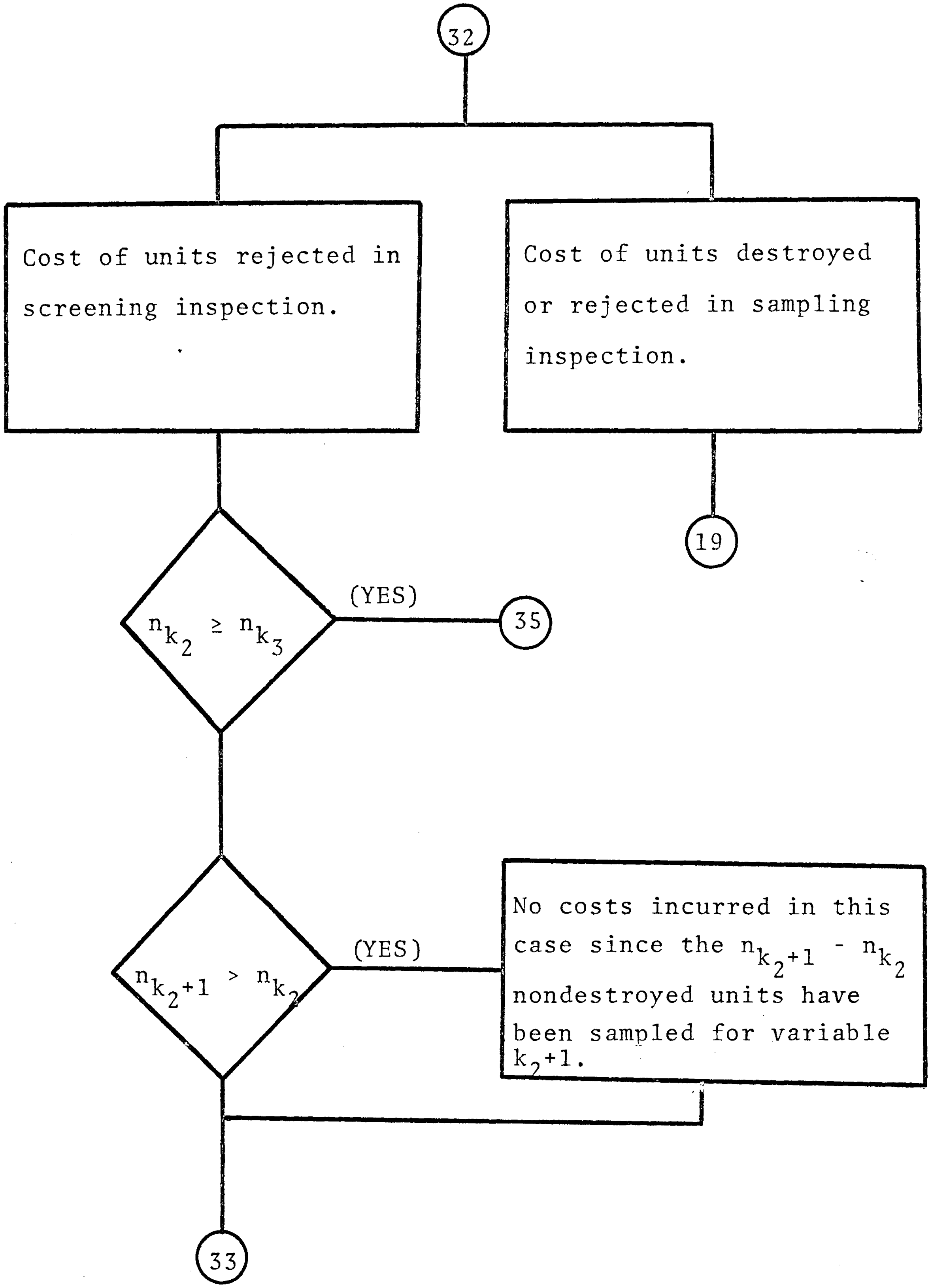
(L - N*) $[C_{sk_3}$

P(Lot rejected on Variable Type k_3)]

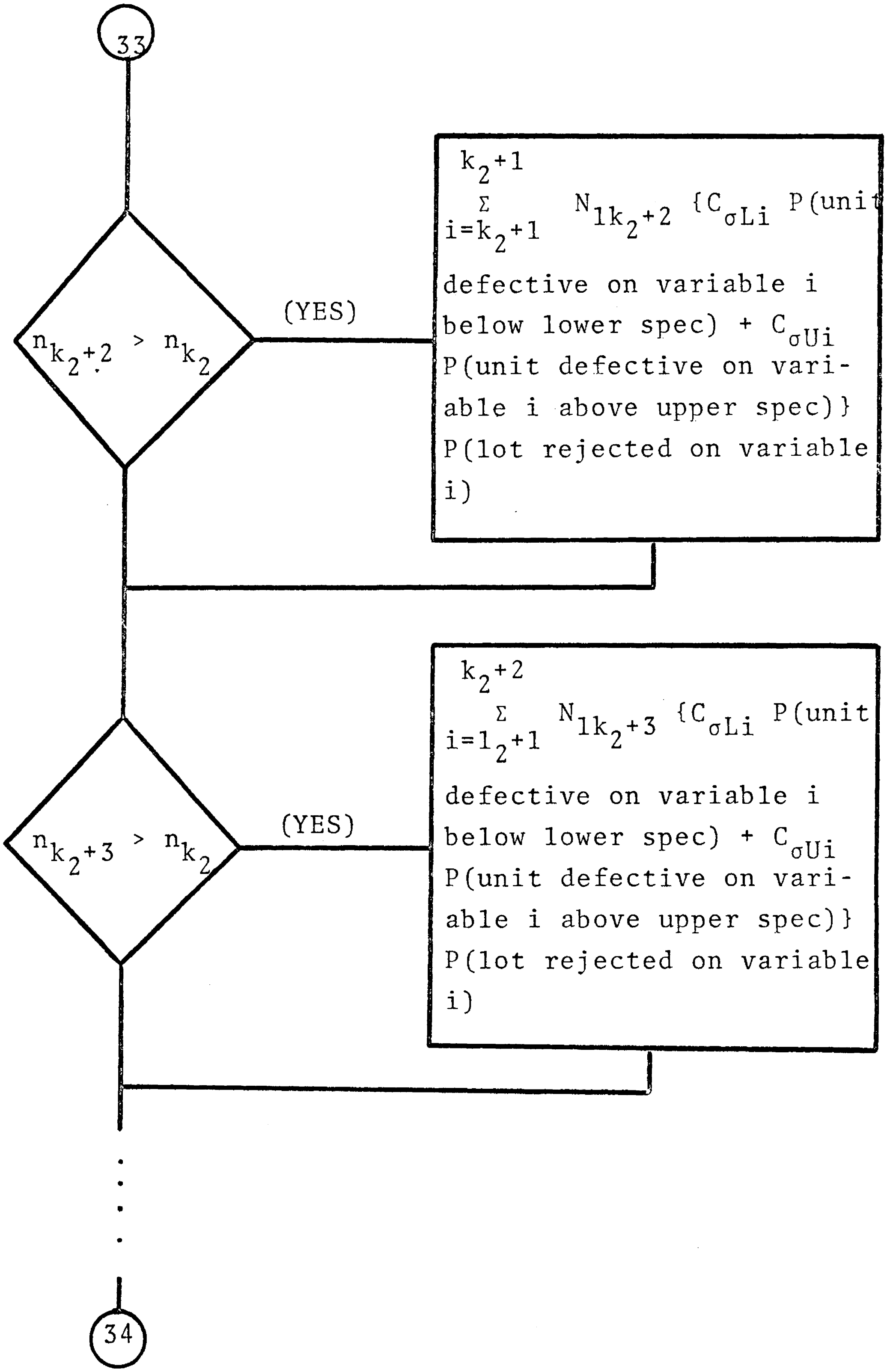
30

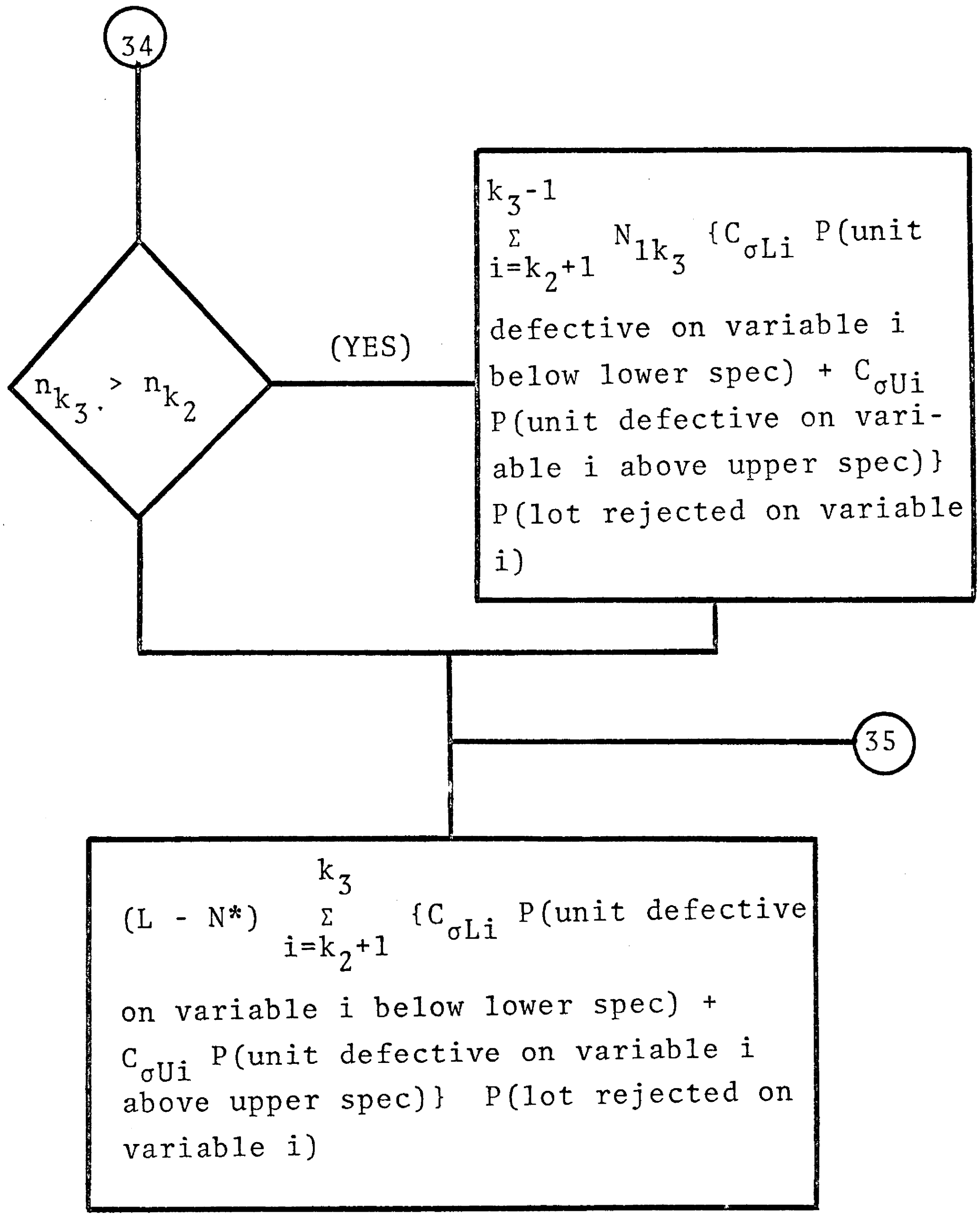

30
$n_{k_2} \geq n_{k_3}$
(YES)
No additional costs incurred by virtue of screening inspection on the units in the sample.
$n_{k_2+1} > n_{k_2}$
(YES)
$(n_{k_3} - n_{k_2+1})$ [C_{sk_2+1}
P(Lot rejected on Variable Type k_2+1)]
(NO)
$(n_{k_3} - n_{k_2})$ [C_{sk_2+1}
P(Lot rejected on Variable Type k_2+1)]
31

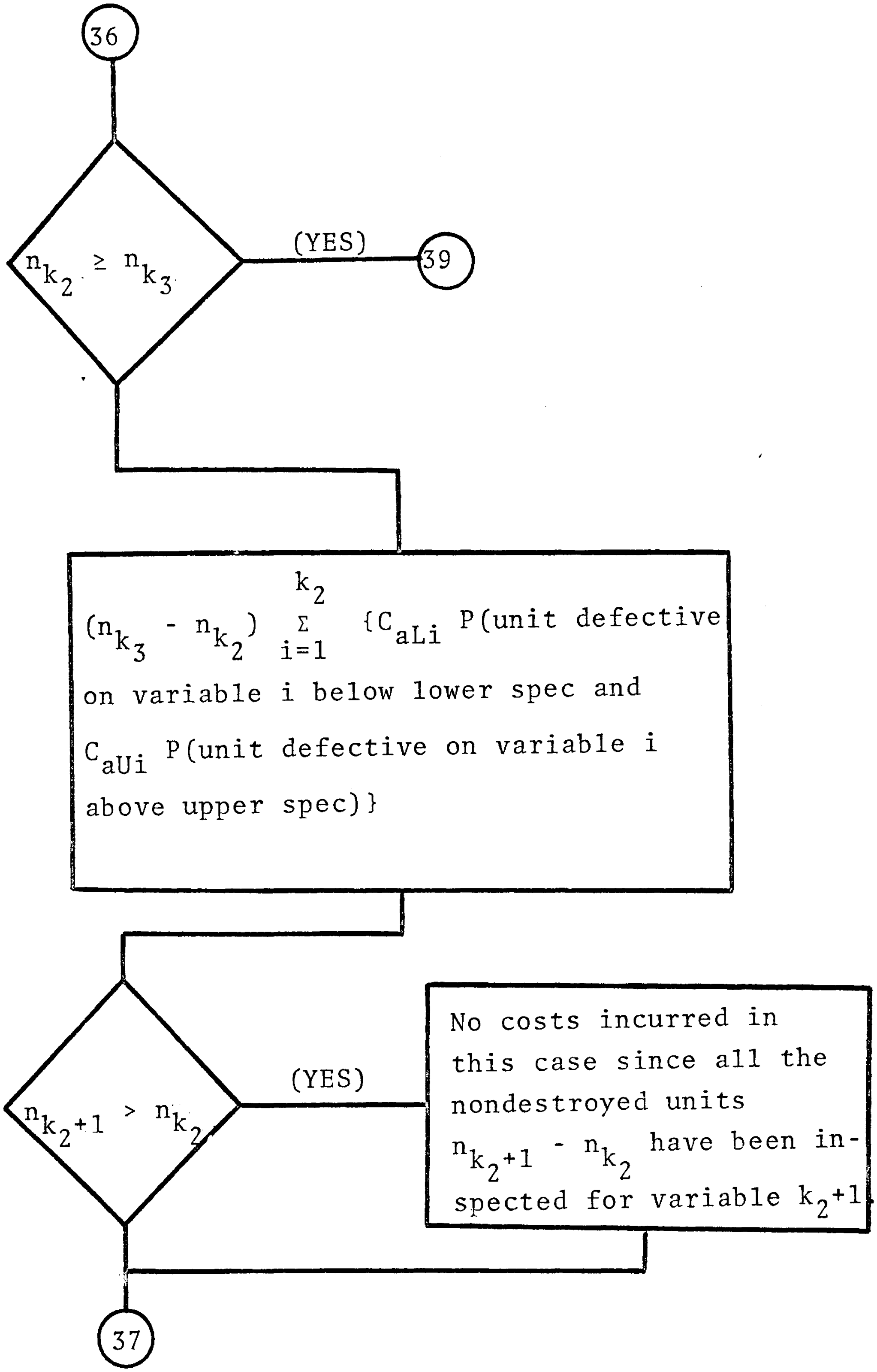

31
$n_{k_3-1} > n_{k_2}$
(YES)
$(n_{k_3} - n_{k_3-1})$ [C_{sk_3-1}
P(Lot rejected on
Variable Type k_3-1)]
(NO)
No additional costs.

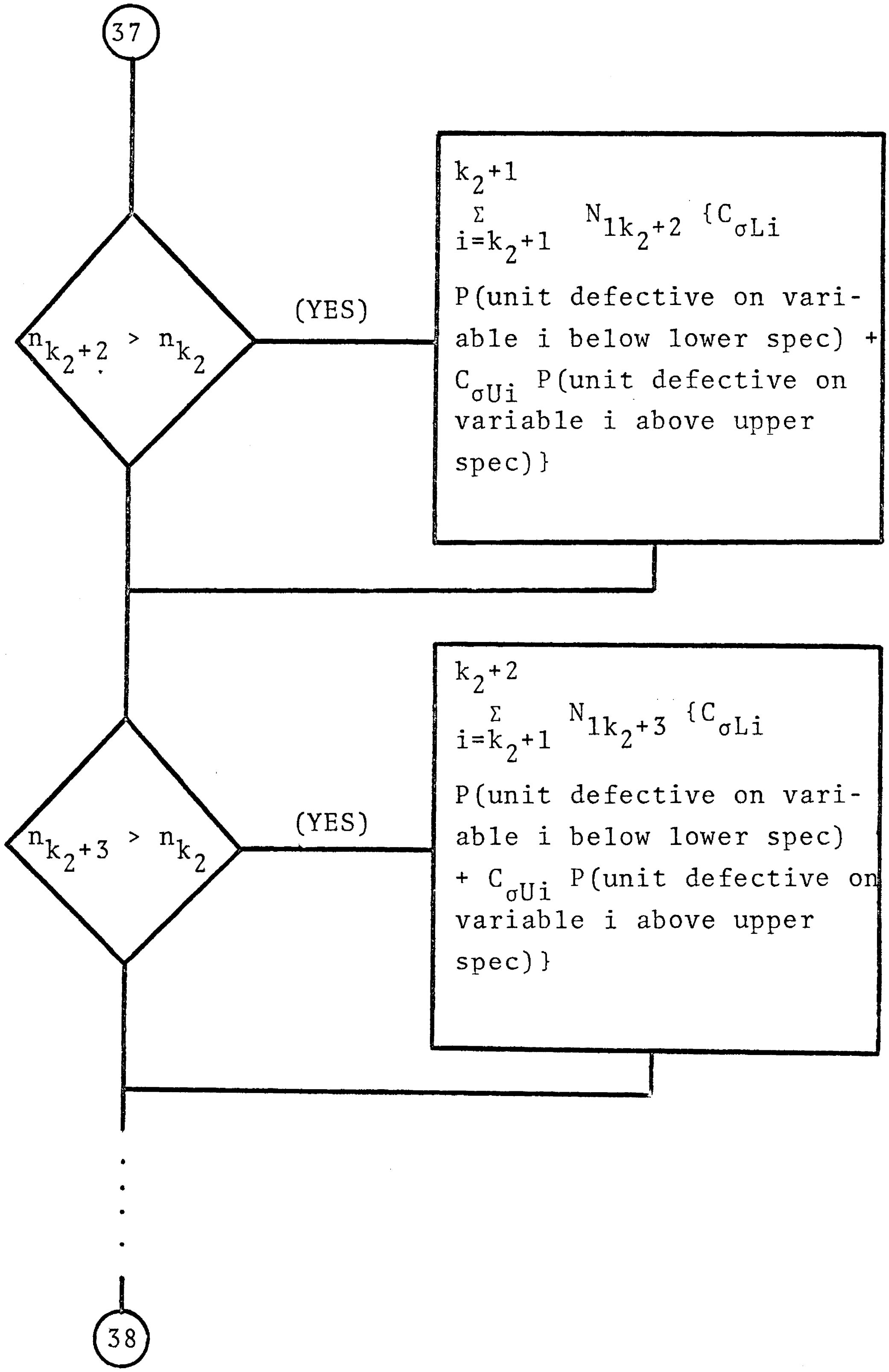


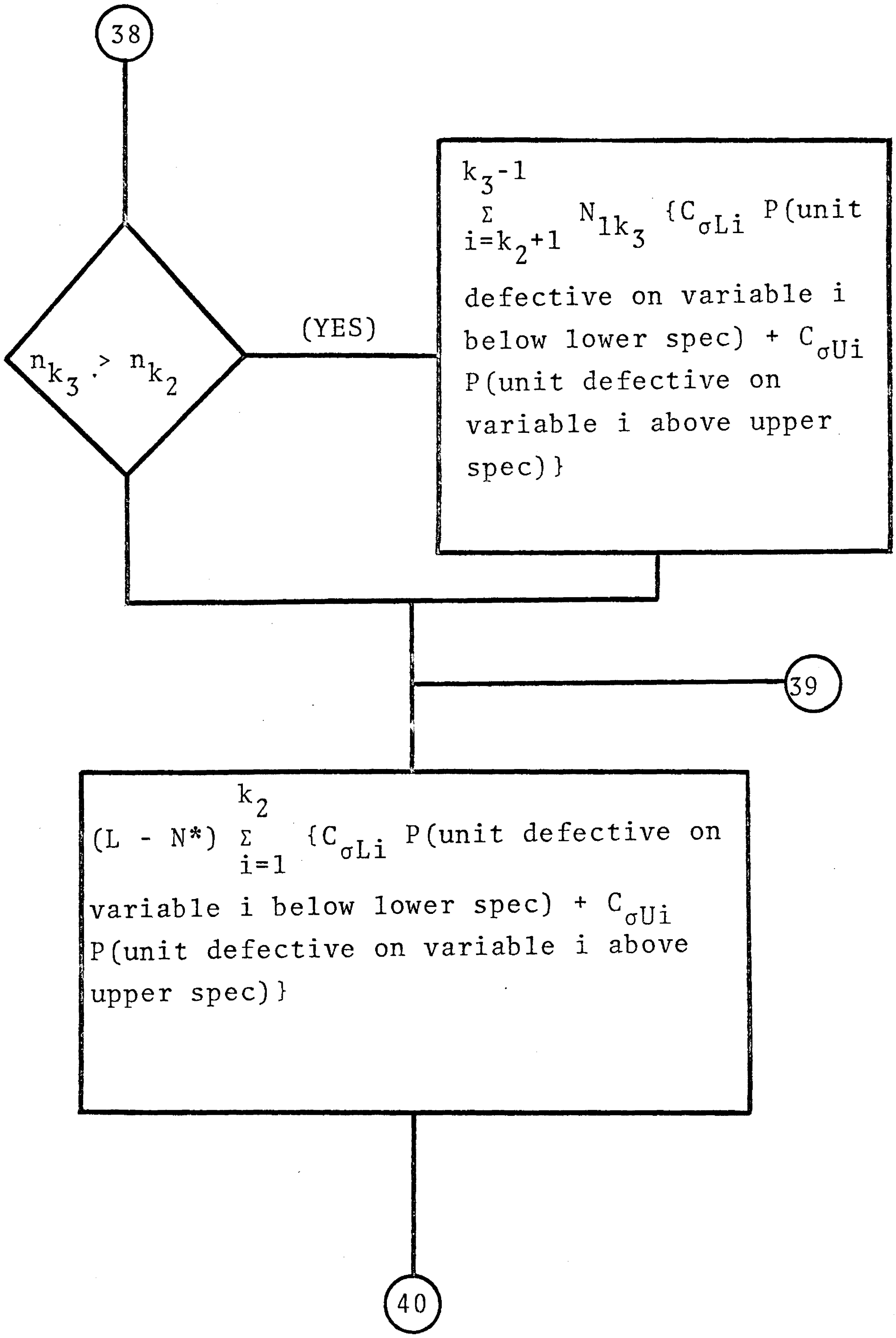
32
Cost of units rejected in screening inspection.
Cost of units destroyed or rejected in sampling inspection.
19
$n_{k_2} \geq n_{k_3}$
(YES)
35
$n_{k_2+1} > n_{k_2}$
(YES)
No costs incurred in this case since the $n_{k_2+1} - n_{k_2}$ nondestroyed units have been sampled for variable k_2+1.
33


33
$n_{k_2+2} > n_{k_2}$
(YES)
$\sum_{i=k_2+1}^{k_2+1} N_{1k_2+2}$ {$C_{\sigma Li}$ P(unit defective on variable i below lower spec) + $C_{\sigma Ui}$ P(unit defective on variable i above upper spec)} P(lot rejected on variable i)
$n_{k_2+3} > n_{k_2}$
(YES)
$\sum_{i=1_2+1}^{k_2+2} N_{1k_2+3}$ {$C_{\sigma Li}$ P(unit defective on variable i below lower spec) + $C_{\sigma Ui}$ P(unit defective on variable i above upper spec)} P(lot rejected on variable i)
34


34
$n_{k_3.} > n_{k_2}$
(YES)
$\sum_{i=k_2+1}^{k_3-1} N_{1k_3} \{C_{\sigma Li}$ P(unit defective on variable i below lower spec) + $C_{\sigma Ui}$ P(unit defective on variable i above upper spec)} P(lot rejected on variable i)
35
$(L - N^*) \sum_{i=k_2+1}^{k_3} \{C_{\sigma Li}$ P(unit defective on variable i below lower spec) + $C_{\sigma Ui}$ P(unit defective on variable i above upper spec)} P(lot rejected on variable i)


36
$n_{k_2} \geq n_{k_3}$
(YES)
39
$(n_{k_3} - n_{k_2}) \sum_{i=1}^{k_2} \{C_{aLi}$ P(unit defective on variable i below lower spec and C_{aUi} P(unit defective on variable i above upper spec)}
$n_{k_2+1} > n_{k_2}$
(YES)
No costs incurred in this case since all the nondestroyed units $n_{k_2+1} - n_{k_2}$ have been inspected for variable k_2+1.
37


37
$n_{k_2+2} > n_{k_2}$
(YES)
$\sum_{i=k_2+1}^{k_2+1} N_{1k_2+2} \{C_{\sigma Li}$ P(unit defective on variable i below lower spec) + $C_{\sigma Ui}$ P(unit defective on variable i above upper spec)}
$n_{k_2+3} > n_{k_2}$
(YES)
$\sum_{i=k_2+1}^{k_2+2} N_{1k_2+3} \{C_{\sigma Li}$ P(unit defective on variable i below lower spec) + $C_{\sigma Ui}$ P(unit defective on variable i above upper spec)}
38


38
$n_{k_3} > n_{k_2}$
(YES)
$\sum_{i=k_2+1}^{k_3-1} N_{1k_3} \{C_{\sigma Li}$ P(unit defective on variable i below lower spec) + $C_{\sigma Ui}$ P(unit defective on variable i above upper spec)}
39
$(L - N^*) \sum_{i=1}^{k_2} \{C_{\sigma Li}$ P(unit defective on variable i below lower spec) + $C_{\sigma Ui}$ P(unit defective on variable i above upper spec)}
40

40

$(L - N^*) \sum_{i=k_2+1}^{k_3}$ $\{C_{\sigma Li}$ P(unit defective on variable i below lower spec) + $C_{\sigma Ui}$ P(unit defective on variable i above upper spec)$\}$

APPENDIX B

MATHEMATICAL SIMPLIFICATION OF THE MULTIPLE INTEGRALS

1. $$K = \int_{-\infty}^{\infty} \int_{LCL}^{UCL} \int_{LS}^{US} h(x|\mu)\ g(\bar{x}|\mu)\ f(\mu)\ dx\ d\bar{x}\ d\mu$$

[B-1]

where

$$h(x|\mu) = \frac{1}{\sigma\sqrt{2\pi}}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad [B-2]$$

$$g(\bar{x}|\mu) = \frac{\sqrt{n}}{\sigma\sqrt{2\pi}}\ e^{-\frac{(\bar{x}-\mu)^2}{2\sigma^2/n}} \qquad [B-3]$$

$$f(\mu) = \frac{\sqrt{L'}}{\sigma\sqrt{2\pi}}\ e^{-\frac{(\mu-\mu_o)^2}{2\sigma^2/L'}} \qquad [B-4]$$

Combining terms results in:

$$K = \int_{-\infty}^{\infty} \int_{LCL}^{UCL} \int_{LS}^{US} \frac{\sqrt{L'n}}{2\pi\,\sigma^3\sqrt{2\pi}}\ e^{-\frac{n(\bar{x}-\mu)^2 + L'(\mu-\mu_o)^2}{2\sigma^2}}\ e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ dx\ d\bar{x}\ d\mu \qquad [B-5]$$

Collecting terms, completing the square of μ in the exponent, and interchanging the order of integration yields:

$$K = \int_{LCL}^{UCL} \int_{LS}^{US} \frac{\sqrt{nL'/(n+L'+1)}}{2\pi\sigma^2} e^{-\frac{\frac{n}{n+L'+1}(\bar{x}-x)^2}{2\sigma^2}}$$

$$e^{-\frac{\frac{L'}{L'+n+1}(x-\mu_o)^2 + \frac{nL'}{n+L'+1}(\bar{x}-\mu_o)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} \frac{\sqrt{n+L'+1}}{\sigma\sqrt{2\pi}} e^{-\frac{\left(\mu - \frac{n\bar{x}+L'\mu_o+x}{n+L'+1}\right)^2}{2\sigma^2/L'+n+1}} d\mu \; dx \; d\bar{x}$$

[B-6]

Noting that μ is integrated over its entire range, the innermost integral reduces to unity.

Collecting terms and completing the square with respect to $\bar{x}$ gives:

$$K = \int_{LCL}^{UCL} \frac{\sqrt{nL'/(n+L')}}{\sigma\sqrt{2\pi}} e^{-\frac{\frac{nL'}{n+L'}(\bar{x}-\mu_o)^2}{2\sigma^2}}$$

$$\int_{LS}^{US} \frac{\sqrt{(n+L')/(n+L'+1)}}{\sigma\sqrt{2\pi}} e^{-\frac{\frac{n+L'}{n+L'+1}\left(x - \frac{n\bar{x}-L'\mu_o}{n+L'}\right)^2}{2\sigma^2}}$$

$$dx\ d\bar{x} \qquad [B-7]$$

2. $$G = \int_{-\infty}^{\infty} \int_{LCL}^{UCL} g(\bar{x}|\mu) f(\mu)\ d\bar{x}\ d\mu \qquad [B-8]$$

where $g(\bar{x}|\mu)$ and $f(\mu)$ are given by Equations B-3 and B-4, respectively.

Combining terms in the exponent,

$$G = \int_{-\infty}^{\infty} \int_{LCL}^{UCL} \frac{\sqrt{nL'}}{2\pi\sigma^2} e^{-\frac{n(\bar{x}-\mu)^2 + L'(\mu-\mu_o)^2}{2\sigma^2}}\ d\bar{x}\ d\mu$$

$$[B-9]$$

Collecting terms, completing the square of μ in the exponent, and interchanging the order of integration yields:

$$G = \int_{LCL}^{UCL} \cdot \frac{\sqrt{nL'/(n+L')}}{\sigma\sqrt{2\pi}} e^{-\frac{\frac{nL'}{n+L'}(\bar{x}-\mu_\circ)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} \frac{\sqrt{n+L'}}{\sigma\sqrt{2\pi}} e^{-\frac{(n+L')\left(\mu - \frac{n\bar{x}+L'\mu_\circ}{n+L'}\right)}{2\sigma^2}} d\mu \; d\bar{x}$$

[B-10]

Integrating with respect to μ is unity, which further reduces the integral to:

$$G = \int_{LCL}^{UCL} \frac{\sqrt{nL'/(n+L')}}{\sigma\sqrt{2\pi}} e^{-\frac{\frac{nL'}{n+L'}(\bar{x}-\mu_\circ)^2}{2\sigma^2}} d\bar{x} \qquad [B\text{-}11]$$

3. $$H = \int_{-\infty}^{\infty} \int_{LS}^{US} h(x|\mu)\, f(\mu)\, dx\, d\mu \qquad [B\text{-}12]$$

where $h(x|\mu)$ and $f(\mu)$ are given by Equations B-2 and B-4, respectively.

Combining terms

$$H = \int_{\infty}^{\infty} \int_{LS}^{US} \frac{\sqrt{L'}}{2\pi\sigma^2} e^{-\frac{(x-\mu)^2 + L'(\mu-\mu_o)^2}{2\sigma^2}} dx\, d\mu \qquad [B\text{-}13]$$

Collecting terms, completing the square of μ in the exponent, and interchanging the order of integration:

$$H = \int_{LS}^{US} \frac{\sqrt{L'/(L'+1)}}{\sigma\sqrt{2\pi}} e^{-\frac{\frac{L'}{L'+1}(x-\mu_o)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{\infty} \frac{\sqrt{L'+1}}{\sigma\sqrt{2\pi}} e^{-\frac{(L'+1)\left(\mu-\frac{x+L'\,{}_o}{L'+1}\right)^2}{2\sigma^2}} d\mu\, dx \qquad [B\text{-}14]$$

Integrating over μ, which reduces to unity, the resulting integral becomes

$$H = \int_{LS}^{US} \frac{\sqrt{L'/(L'+1)}}{\sigma\sqrt{2\pi}} e^{-\frac{\frac{L'}{L'+1}(x-\mu_o)^2}{2\sigma^2}} dx \qquad [B-15]$$

APPENDIX C

APPROXIMATION TO THE NORMAL INTEGRAL[1]

Function: $\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$

Range: $0 < x < \infty$

Approximation:

$$\Phi^*(x) = 1 - \frac{1}{(1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5)^8}$$

$a_1 = 0.14112821$ $a_4 = -0.00039446$

$a_2 = 0.08864027$ $a_5 = 0.00328975$

$a_3 = 0.02743349$

[1]Taken from Approximations for Digital Computers by Cecil Hastings, Jr., Princeton, N.J.: Princeton University Press (1955).

APPENDIX D

OPTIMIZATION OF THE COST MODEL

This appendix to the research has been included in order to provide support and justification for using the individually optimum acceptance sampling plans as the starting point in the optimization of the multivariate model. The optimization times footnoted in Tables D-1, D-2, and D-3 are execution times for the IBM 370/155 digital computer located at the Virginia Polytechnic Institute and State University Computing Center.

Table D-1 shows the acceptance sampling plans, considered individually, with each optimized from an initial starting point of acceptance without sampling. Table D-2 then shows the optimization of the multivariate model starting from the optimum points in Table D-1.

Table D-3 shows the optimization of the multivariate model with starting points for all variables as acceptance without sampling. It is important to note that although the optimum total expected costs of acceptance sampling in Tables D-2 and D-3 are not significantly different, a reduction of some 36 percent in the total execution time to reach these points can be realized by virtue of optimizing the cost model as outlined in Tables D-1 and D-2. This approach was, therefore, applied in all cases through this research.

TABLE D-1. SINGLY OPTIMIZED ACCEPTANCE SAMPLING PLANS *									
VAR NUMBER	POINT	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
1	START	0	19.50	20.50	$0.00	$6367.62	$ 0.00	$ 0.00	$6367.62
	OPT	30	19.25	20.75	0.15	3879.90	1189.01	0.00	5069.06
2	START	0	48.50	51.50	0.00	6147.27	0.00	0.00	6147.27
	OPT	32	47.94	52.06	0.16	3905.91	1274.14	0.00	5180.21
3	START	0	3.50	6.50	0.00	5772.03	0.00	0.00	5772.03
	OPT	22	3.70	6.30	0.11	4573.38	861.95	0.00	5435.44
4	START	0	12.50	17.50	0.00	1639.23	0.00	0.00	1639.23
	OPT	36	14.40	15.60	0.18	303.00	0.00	1292.69	1595.87

* 1 Minute, 46.56 Seconds

TABLE D-2. MULTIVARIATE OPTIMIZATION OF THE MODEL*

POINT	VAR NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
START	1	30	19.25	20.75	\$0.55	\$6344.28	\$3146.50	\$6016.18	\$15507.51
	2	32	47.94	52.06					
	3	22	3.70	6.30					
	4	36	14.40	15.60					
OPT	1	36	19.42	20.58	1.13	4127.35	6846.35	3653.21	14628.04
	2	34	48.34	51.66					
	3	28	3.98	6.02					
	4	214	14.37	15.63					

* 13 Minutes, 2.50 Seconds

TABLE D-3. MULTIVARIATE OPTIMIZATION OF THE MODEL*

POINT	VAR NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
START	1	0	19.50	20.50	$0.00	$19926.16	$ 0.00	$ 0.00	$19926.16
	2	0	48.50	51.50					
	3	0	3.50	6.50					
	4	0	12.50	17.50					
OPT	1	36	19.43	20.57	0.94	4014.48	6982.11	3633.52	14631.05
	2	34	48.36	51.63					
	3	28	3.97	6.02					
	4	158	14.38	15.62					

*23 Minutes, 9.54 Seconds

APPENDIX E

DISTRIBUTIONAL FORMS - $f(\mu_i)$

The distributional forms and parameters specified herein are used in the sensitivity analysis of the cost model presented in this research.

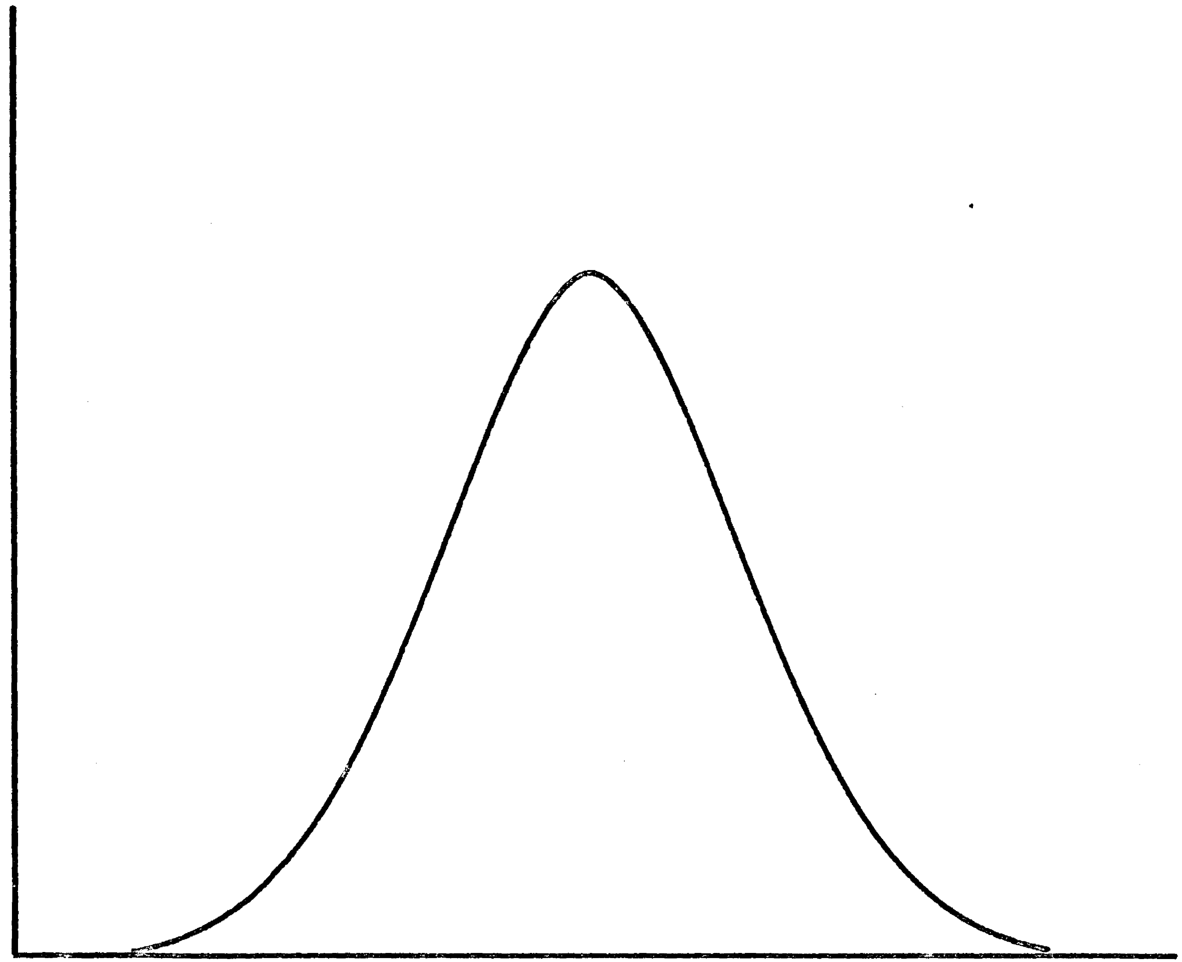


FIGURE E-1. NORMAL DISTRIBUTION A

TABLE E-1. PARAMETERS OF NORMAL DISTRIBUTION A

VARIABLE NUMBER	DESIRED MEAN	STANDARD DEVIATION
1	20.00	0.39
2	50.00	1.08
3	5.00	0.62
4	15.00	0.77

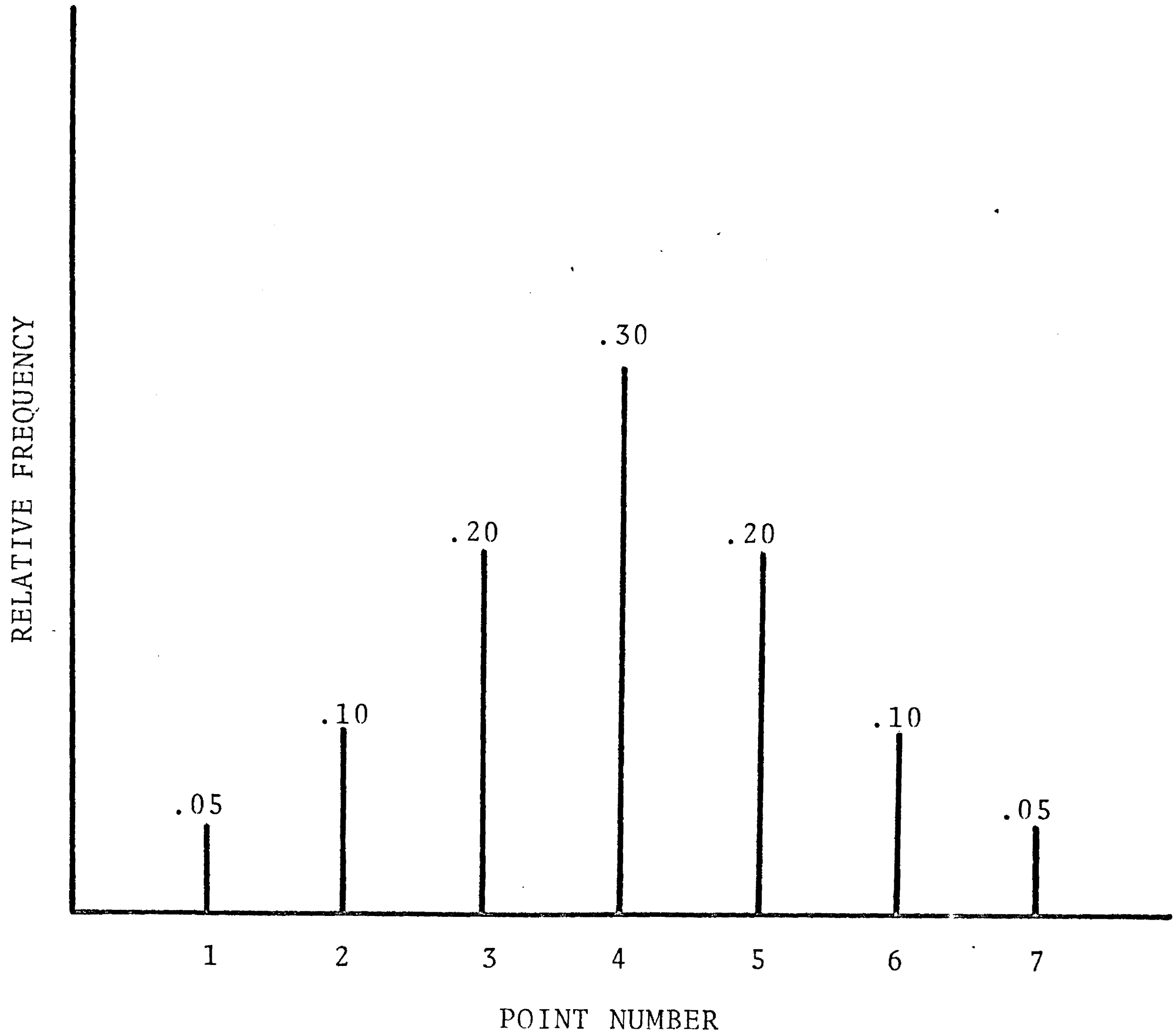


FIGURE E-2. DISCRETE DISTRIBUTION B

TABLE E-2. POINT VALUES - DISCRETE DISTRIBUTION B							
VARIABLE NUMBER	POINT NUMBER						
	1	2	3	4	5	6	7
1	19.205	19.470	19.735	20.000	20.265	20.530	20.795
2	47.756	48.504	49.252	50.000	50.748	51.496	52.244
3	3.722	4.148	4.574	5.000	5.426	5.852	6.278
4	13.398	13.932	14.466	15.000	15.534	16.068	16.602

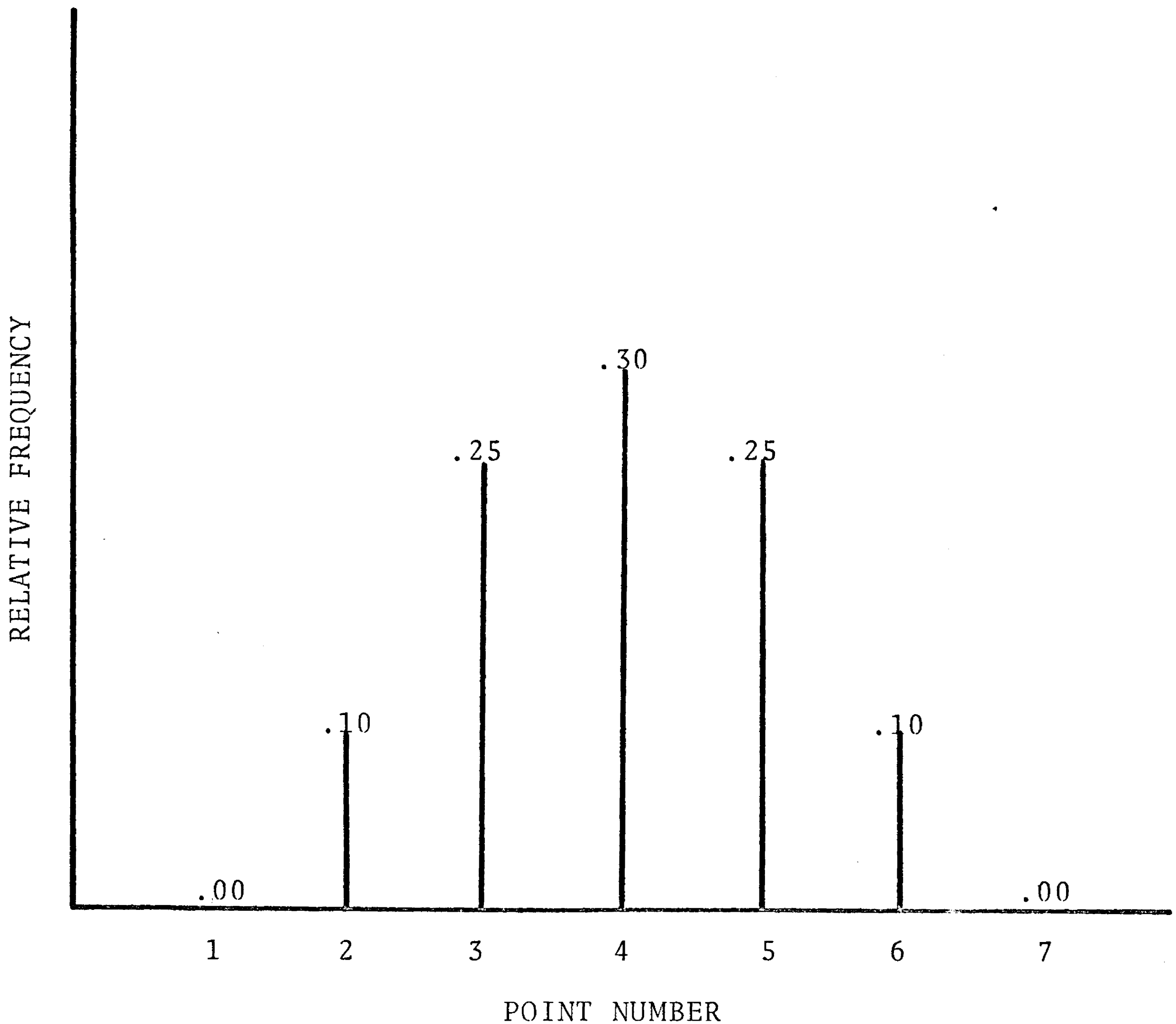


FIGURE E-3. DISCRETE DISTRIBUTION C

TABLE E-3. POINT VALUES - DISCRETE DISTRIBUTION C

VARIABLE NUMBER	POINT NUMBER						
	1	2	3	4	5	6	7
1	18.989	19.326	19.663	20.000	20.337	20.674	21.011
2	47.150	48.100	49.050	50.000	50.950	51.900	52.850
3	3.374	3.916	4.458	5.000	5.542	6.084	6.626
4	12.966	13.644	14.322	15.000	15.678	16.356	17.034

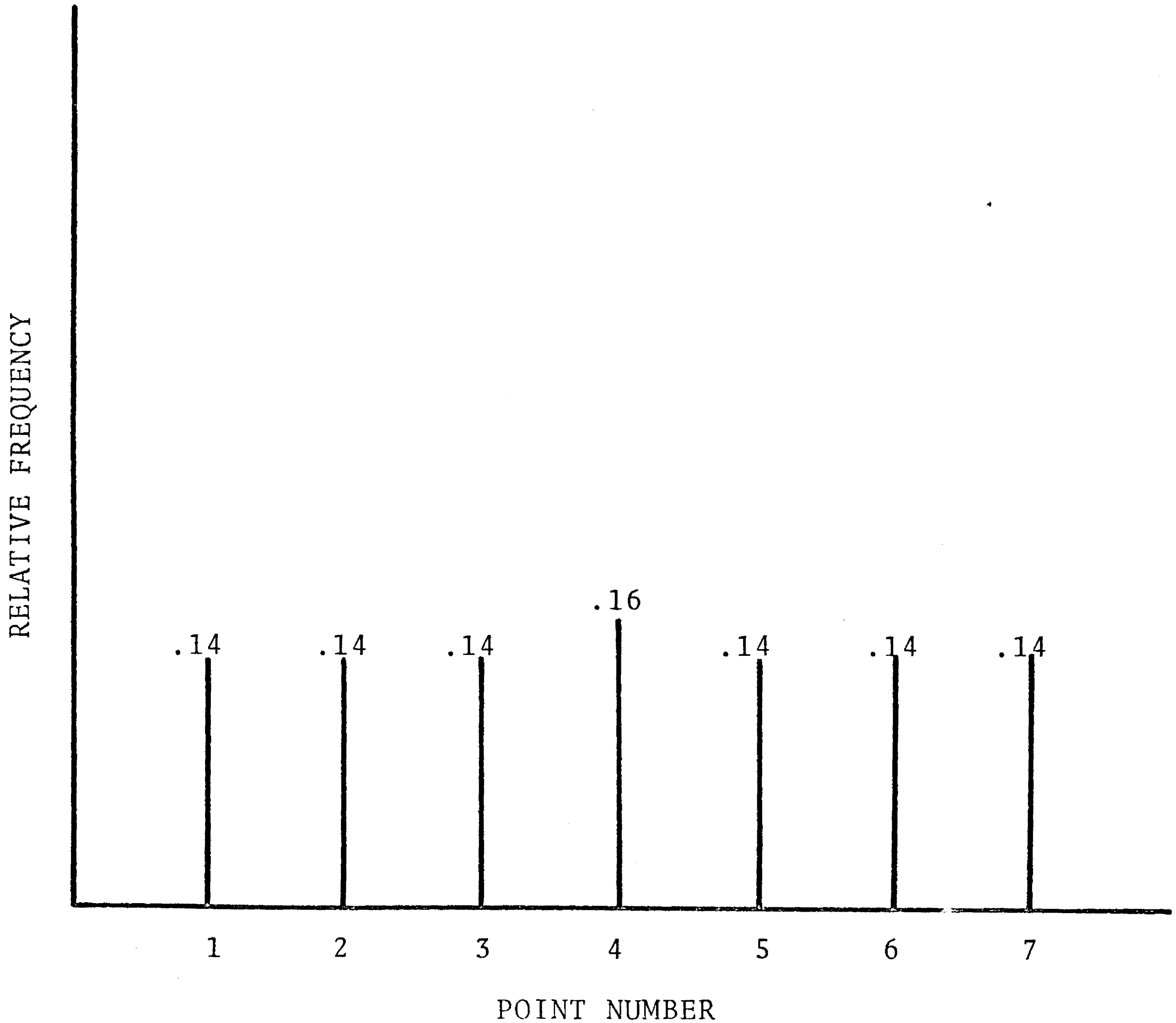


FIGURE E-4. DISCRETE DISTRIBUTION D

TABLE E-4. POINT VALUES - DISCRETE DISTRIBUTION D							
VARIABLE NUMBER	POINT NUMBER						
	1	2	3	4	5	6	7
1	19.418	19.612	19.806	20.000	20.194	20.388	20.582
2	48.356	48.904	49.452	50.000	50.548	51.096	51.644
3	4.064	4.376	4.688	5.000	5.312	5.624	5.936
4	13.827	14.218	14.609	15.000	15.391	15.782	16.173

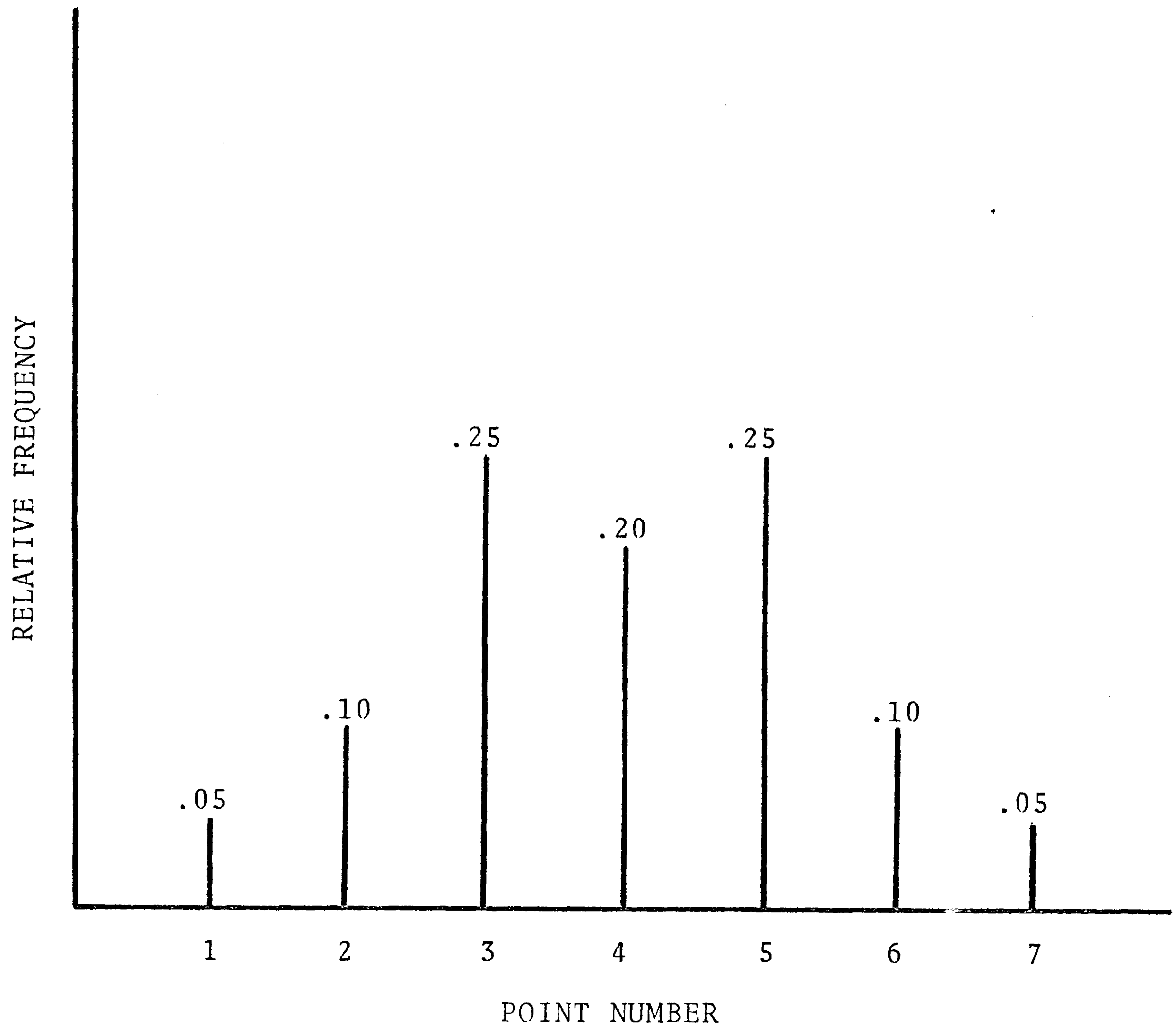


FIGURE E-5. DISCRETE DISTRIBUTION E

TABLE E-5. POINT VALUES - DISCRETE DISTRIBUTION E

VARIABLE NUMBER	POINT NUMBER						
	1	2	3	4	5	6	7
1	19.223	19.482	19.741	20.000	20.259	20.518	20.777
2	47.807	48.538	49.269	50.000	50.731	51.462	52.193
3	3.749	4.166	4.583	5.000	5.417	5.834	6.251
4	13.437	13.958	14.479	15.000	15.521	16.042	16.563

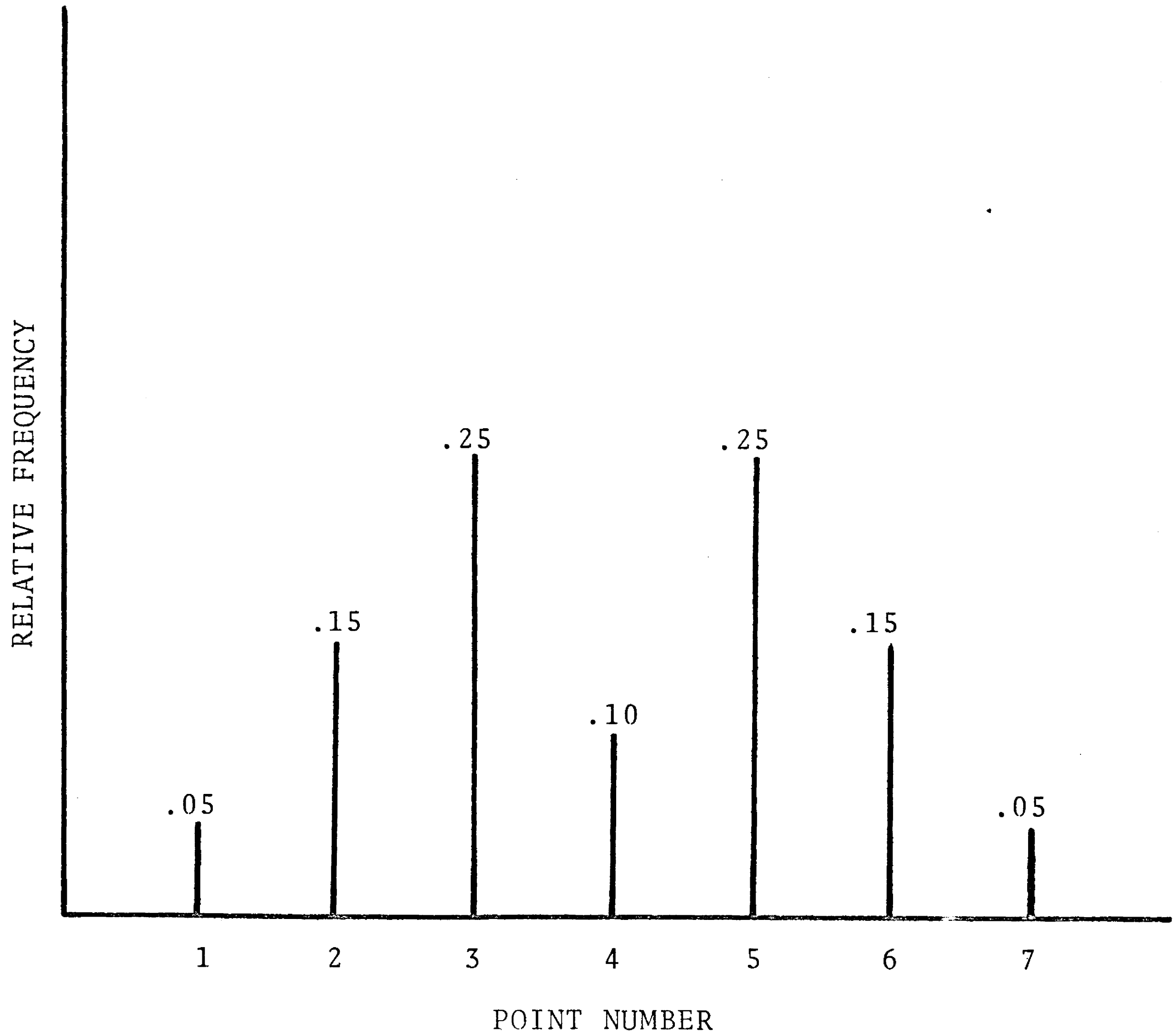


FIGURE E-6. DISCRETE DISTRIBUTION F

TABLE E-6. POINT VALUES - DISCRETE DISTRIBUTION F							
VARIABLE NUMBER	POINT NUMBER						
	1	2	3	4	5	6	7
1	19.286	19.524	19.762	20.000	20.238	20.476	20.714
2	47.984	48.656	49.328	50.000	50.672	51.344	52.016
3	3.851	4.234	4.617	5.000	5.383	5.766	6.149
4	13.560	14.040	14.520	15.000	15.480	15.960	16.440

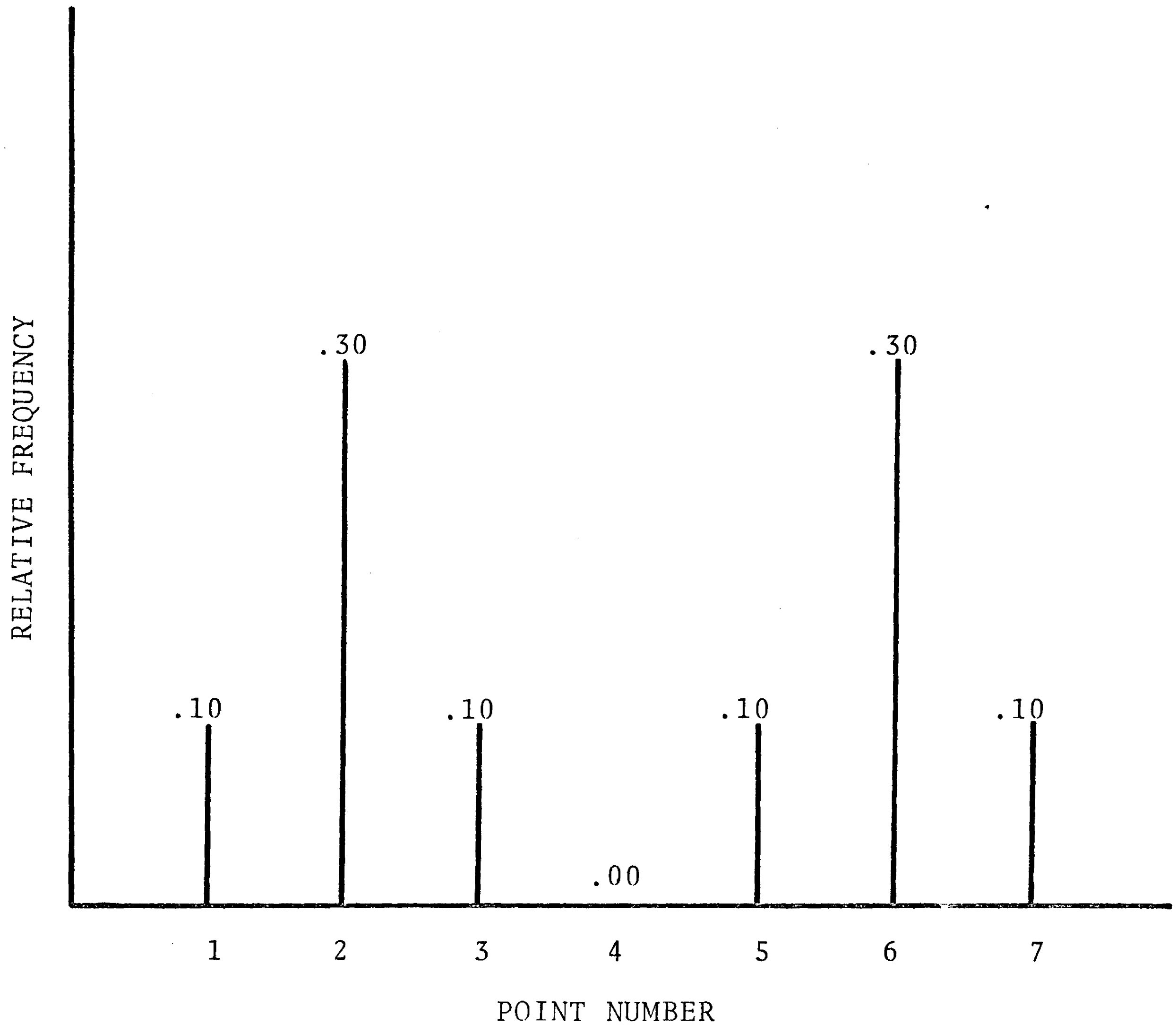


FIGURE E-7. DISCRETE DISTRIBUTION G

TABLE E-7. POINT VALUES - DISCRETE DISTRIBUTION G							
VARIABLE NUMBER	POINT NUMBER						
	1	2	3	4	5	6	7
1	19.451	19.634	19.817	20.000	20.183	20.366	20.549
2	48.449	48.966	49.483	50.000	50.517	51.034	51.551
3	4.115	4.410	4.705	5.000	5.295	5.590	5.885
4	13.893	14.262	14.631	15.000	15.369	15.738	16.107

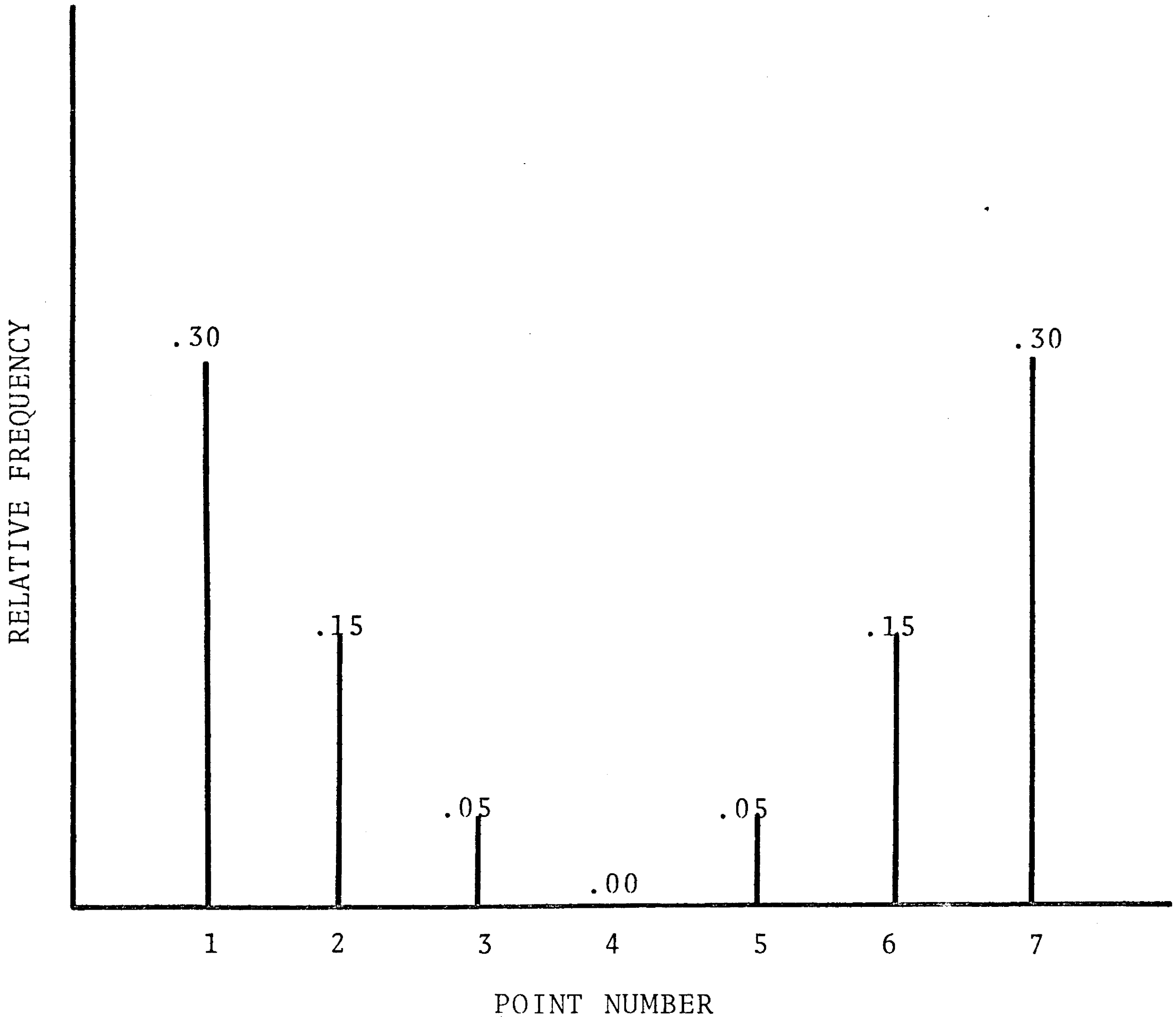


FIGURE E-8. DISCRETE DISTRIBUTION H

TABLE E-8. POINT VALUES - DISCRETE DISTRIBUTION H							
VARIABLE NUMBER	POINT NUMBER						
	1	2	3	4	5	6	7
1	19.553	19.702	19.851	20.000	20.149	20.298	20.447
2	48.743	49.162	49.581	50.000	50.419	50.838	51.257
3	4.283	4.522	4.761	5.000	5.239	5.478	5.717
4	14.103	14.402	14.701	15.000	15.299	15.598	15.897

APPENDIX F

VALIDATION OF THE INDICATED OPTIMUM SOLUTION

Optimization of the examples in Chapter IV was achieved by systematically applying the pattern search to the model. In this appendix, the search procedure is initiated at several different sets of starting points. Tables F-1 through F-5 give the results of these procedures. Variation in the starting points for the multivariate search seems

a. to produce no significant change in the indicated optimum value of the objective function; and

b. to produce little change in the indicated optimum acceptance sampling plans for the destructable variables (numbers 1, 2, and 3).

These results suggest that the model seems to be robust with respect to the screenable variable's sampling plan.

TABLE F-1. MULTIVARIATE OPTIMIZATION OF THE MODEL

POINT	VAR. NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
START	1	0	19.50	20.50	$0.00	$19926.16	$ 0.00	$ 0.00	$19926.16
	2	0	48.50	51.50					
	3	0	3.50	6.50					
	4	0	12.50	17.50					
OPT	1	36	19.43	20.57	0.94	4014.48	6982.11	3633.52	14631.05
	2	34	48.36	51.63					
	3	28	3.97	6.02					
	4	158	14.38	15.62					

TABLE F-2. MULTIVARIATE OPTIMIZATION OF THE MODEL

POINT	VAR. NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
START	1	30	19.25	20.75	$0.55	$6344.28	$3146.50	$6016.18	$15507.51
	2	32	47.94	52.06					
	3	22	3.70	6.30					
	4	36	14.40	15.60					
OPT	1	36	19.42	20.58	1.13	4127.35	6846.35	3653.21	14628.04
	2	34	48.34	51.66					
	3	28	3.98	6.02					
	4	214	14.37	15.63					

TABLE F-3. MULTIVARIATE OPTIMIZATION OF THE MODEL

POINT	VAR. NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
START	1	75	19.95	20.05	$ .46	$ .07	$19995.23	$ 1.55	$19997.31
	2	150	49.95	50.05					
	3	250	4.95	5.05					
	4	750	14.95	15.05					
OPT	1	35	19.41	20.59	1.92	3999.16	6876.39	3750.73	14629.20
	2	34	48.35	51.65					
	3	28	3.99	6.01					
	4	456	14.39	15.61					

TABLE F-4. MULTIVARIATE OPTIMIZATION OF THE MODEL

POINT	VAR. NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
	1	25	19.5	20.5					
	2	50	48.5	51.5					
START	3	50	3.7	6.3	$1.42	$3264.70	$7473.47	$4234.83	$14974.42
	4	300	14.5	15.5					
	1	35	19.41	20.59					
	2	34	48.35	51.65					
OPT	3	29	3.99	6.01	.91	4019.94	6866.84	3740.48	14628.17
	4	146	14.41	15.64					

TABLE F-5. MULTIVARIATE OPTIMIZATION OF THE MODEL									
POINT	VAR. NUMBER	SAMPLE SIZE	LOWER CONTROL LIMIT	UPPER CONTROL LIMIT	C_I	C_A	C_R	C_S	C_T
START	1	75	18.5	22.5	$2.30	$17732.79	$1909.40	$ 0.00	$19644.49
	2	150	45.5	52.5					
	3	250	1.7	9.3					
	4	10	10.5	19.5					
OPT	1	38	19.44	20.61	.46	7894.63	6752.76	0.00	14647.85
	2	35	48.34	51.58					
	3	25	3.95	6.03					
	4	9	12.17	17.75					

APPENDIX G

COMPUTER PROGRAM FOR THE MULTIVARIATE COST MODEL

A. Notation

Variable	Definition
K1	k_2
K2	$k_2 - k_1$
K3	$k_3 - k_2$
KDESTR	$k_1 + k_2$
KTOTAL	$k_1 + k_2 + k_3$
L	L
N(I)	n_i
DMEAN(I)	Desired value of μ_i
STD(I)	δ_i
SPEC(I, 1)	LS_i
SPEC(I, 2)	US_i
CLIMIT(I, 1)	LCL_i
CLIMIT(I, 2)	UCL_i
UCR	C_r
UCR01	$C_{\rho 1}$
UCR02	$C_{\rho 2}$
UCAL(I)	C_{aLi}

Variable	Definition
UCAU(I)	C_{aUi}
UCRLOW(I)	$C_{\sigma Li}$
UCRUP(I)	$C_{\sigma Ui}$
UCI(I)	C_{Ii}
UCSCR(I)	C_{si}
CI	C_I
CA1	C_{A1}
CA2	C_{A2}
CA3	C_{A3}
CA4	C_{A4}
CR	C_R
CS1	C_{S1}
CS2	C_{S2}
CS3	C_{S3}
CS4	C_{S4}
NSTAR	N^*
N1STAR(I)	N_{1i}
N2STAR(I)	N_{2i}
DELTA1(I)	δ_{1i}
DELTA2	δ_2
DELTA3	δ_3

NPROB = number of problems.

$$\text{IPRINT} = \begin{cases} 0, & \text{all iterations printed} \\ 1, & \text{only improved iterations printed} \end{cases}$$

MX = number of decision variables

NS = number of changes of step size

BASE(I) = initial value of decision variable i

XMIN(I) = minimum value of decision variable i

XMAX(I) = maximum value of decision variable i

DELT(I) = initial value of the step size for decision variable i.

DELTM(I) = minimum value of the step size for decision variable

DELTU(I) = maximum value of the step size for decision variable

NOTE:

Decision Variable Number	Corresponds To
1	n_1
2	LCL_1
3	UCL_1
4	n_2
5	LCL_2
6	UCL_2
.	.
.	.
.	.

B. Major Program Limitation

The total number of decision variables must be less than or equal to 45; i.e. the total number of variables to be controlled must be less than or equal to 15.

C. Input Cards

Card	VARIABLE NAME(s)	FORMAT
1	NPROB	1X, I5
2	IPRINT	1X, I5
3	K1, K2, K3, L	4I10
4	DELTA2, DELTA3	2F10.4
5	N(1), DMEAN(1), STD(1), CLIMIT(1, 1), CLIMIT(1, 2), SPEC(1, 1), SPEC(1, 2) (Same as above for any additional variables)	I10, 6F10.4
6	UCI(1), UCAC(1), UCAL(1), UCRUP(1), UCRLOW(1), UCSCR(1) (Same as above for any additional variables)	6F10.4
7	UCR, UCR01, UCR02	3F10.4
8	MX, NS	1X, 2I5
9	BASE(1), XMIN(1), XMAX(1), DELT(1), DELTM(1), DELTU(1)	1X, 6F8.4

Card	VARIABLE NAME(s)	FORMAT
10	BASE(2), XMIN(2), XMAX(2)	1X, 6F8.4
	DELT(2), DELTM(2), DELTU(2)	
11	BASE(3), XMIN(3), XMAX(3)	1X, 6F8.4
	DELT(3), DELTM(3), DELTU(3)	
	(Above three card sequence repeats for additional variables)	

D. Program Listing

```
C
C*** PATTERN SEARCH
C
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      DIMENSION BASE(45),XMIN(45),XMAX(45),DELT(45),RF(45),TB(45)
      DIMENSION DELTM(45),DELTU(45),PB(45)
      COMMON/OPTU/ BEST(45), YMAX
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      COMMON/NTRL/SAMPLE, DELTA2
      COMMON/BLOK/INTEXX(15,3)
      COMMON/BLOKL/RF
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/CREF/CIREF,CACREF,CRJREF,CSCREF
C
C*** READ NUMBER OF PROBLEMS
C
      READ(5,9090) NPROB
      WRITE(6,9090)NPROB
C
C*** READ PRINT CONTROL VARIABLE (IF = 1, WILL PRINT EACH FUNCTIONAL
C*** EVALUATION; IF = 0, WILL PRINT ONLY INPROVED EVALUATIONS
C
      READ(5,9090) IPRINT
      WRITE(6,9090)IPRINT
      DO 200 IJ=1,NPROB
      IND=0
      CALL SUBSYS(IND,YY)
```

```
      IND=1
C
C*** READ THE NUMBER OF DECISION VARIABLES AND THE MAXIMUM NUMBER OF
C*** STEPS
C
      READ(5,9000) MX,NS
      WRITE(6,9000)MX,NS
      NITER=0
      DO 30 I=1,MX
C
C*** READ STARTING VALUES OF DECISION VARIABLES, MINIMUM AND MAXIMUM
C*** VALUE THE DECISION VARIABLE CAN ASSUME, THE STARTING STEP SIZE
C*** AND THE MAXIMUM AND MINIMUM STEP SIZE
C
      READ(5,9010) BASE(I),XMIN(I),XMAX(I),DELT(I),DELTM(I),DELTU(I)
      WRITE(6,9010)BASE(I),XMIN(I),XMAX(I),DELT(I),DELTM(I),DELTU(I)
      TB(I)=BASE(I)
      RF(I)=BASE(I)
      PB(I)=BASE(I)
   30 CONTINUE
C
C*** PROCEED WITH SEARCH
C
  310 IC=1
      YMAX=-10.**20
      JJ=1
   50 II=1
      DO 60 I=1,MX
   60 RF(I)=TB(I)
      CALL SUBSYS(IND,YY)
      YY=-YY
      IF(IPRINT.LE.0) GO TO 230
```

```
      WRITE(6,9000) IC
      WRITE(6,9020) YY,(RF(L2),L2=1,MX)
  230 IF(YY.LE.YMAX) GO TO 70
      IF(IPRINT.GT.0) GO TO 240
      WRITE(6,9000) IC
      WRITE(6,9020) YY,(RF(L2),L2=1,MX)
      WRITE(6,99) CIREF,CACREF,CRJREF,CSCREF
  240 NITER=0
      DO 280 I=1,MX
  280 BEST(I)=RF(I)
      YMAX=YY
      IF(JJ.NE.1) II=2
   70 DO 90 I=1,MX
      RF(I)=TB(I)+DELT(I)
      IF(RF(I).GT.XMAX(I)) RF(I)=XMAX(I)
      CALL SUBSYS(IND,YY)
      YY=-YY
      IF(IPRINT .LE.0) GO TO 250
      WRITE(6,9000) IC
      WRITE(6,9020) YY,(RF(L2),L2=1,MX)
  250 IF(YY.GT.YMAX) GO TO 80
      RF(I)=TB(I)-DELT(I)
      IF(RF(I).LT.XMIN(I)) RF(I)=XMIN(I)
      CALL SUBSYS(IND,YY)
      YY=-YY
      IF(IPRINT.LE.0) GO TO 260
      WRITE(6,9000) IC
      WRITE(6,9020) YY,(RF(L2),L2=1,MX)
  260 IF(YY.GT.YMAX) GO TO 80
      RF(I)=TB(I)
      GO TO 90
   80 YMAX=YY
```

```
      BEST(I)=RF(I)
      IF(IPRINT.GT.0) GO TO 270
      WRITE(6,9000) IC
      WRITE(6,9020) YY,(RF(L2),L2=1,MX)
      WRITE(6,99) CIREF,CACREF,CRJREF,CSCREF
   99 FORMAT(4F14.2)
  270 NITER=0
      II=2
   90 CONTINUE
      GO TO (120,100),II
  100 JJ=2
      DO 110 I=1,MX
      PB(I)=BASE(I)
      BASE(I)=RF(I)
      TB(I)=2.*BASE(I)-PB(I)
      IF(TB(I).LT.XMIN(I)) TB(I)=XMIN(I)
      IF(TB(I).GT.XMAX(I)) TB(I)=XMAX(I)
  110 CONTINUE
      GO TO 50
  120 IF(IC.GT.NS) GO TO 180
      IC=IC+1
      DO 170 I=1,MX
      DELT(I)=DELT(I)/2.
      IF(DELT(I).LT.DELTM(I)) GO TO 210
      TB(I)=BASE(I)
      PB(I)=BASE(I)
      RF(I)=BASE(I)
  170 CONTINUE
      GO TO 70
  210 NITER=NITER+1
      IF(NITER.GT.1) GO TO 180
      DO 220 I=1,MX
```

```
      DELT(I)=DELTU(I)
      BASE(I)=TB(I)
  220 RF(I)=BASE(I)
      GO TO 50
  180 WRITE(6,9030)
      IC=IC-1
      WRITE(6,9080) YMAX
      WRITE(6,9020) (BEST(I),I=1,MX)
      WRITE(6,9100) IC
      CALL OUTPT2
      DO 300 I=1,MX
  300 RF(I)=BEST(I)
      NND=1
  200 CONTINUE
      CALL EXIT
 9000 FORMAT(1X,8I5)
 9010 FORMAT(1X,8F8.4)
 9020 FORMAT(1X,7(3X,F14.4))
 9030 FORMAT(1H0,'FINAL SOLUTION',//)
 9040 FORMAT(1H0,'ACCELERATION STEP TO ',//)
 9080 FORMAT(1X,'OBJECTIVE FUNCTION = ',E14.7,/,1X,'DECISION VARIABLES '
     1,//)
 9090 FORMAT(1X,I5)
 9100 FORMAT(1X,'ITERATIONS = ',I5)
 9200 FORMAT(1X,'PERCENT SAVINGS = ',E14.7)
 9210 FORMAT(1X,'MODEL INVALID FOR INPUT DATA')
      END
```

```
      SUBROUTINE SUBSYS(IND,CT)
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      DIMENSION RF(45)
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      COMMON/NTRL/SAMPLE, DELTA2
      COMMON/BLOK/INTEXX(15,3)
      COMMON/BLCKL/RF
      COMMON/REF/IREFPT
      COMMON/HIST/NPTS(6), FREQ(6,10), SPREAD(6,10)
      COMMON/CREF/CIREF,CACREF,CRJREF,CSCREF
      COMMON/STEP/ KONTRC
      INTEGER DELTA1
      CTREF = 0.00
      CIREF = 0.00
      CACREF = 0.00
      CRJREF = 0.00
      CSCREF = 0.00
      IF (IND .EQ. 0) CALL INPUT
      IF (IND .EQ. 0) CALL OUTPT1
      IF (IND .EQ. 0) CALL LOAD
      IF (IND .EQ. 0) RETURN
C
C*** CHECK TO SEE IF CONTROL LIMITS HAVE CROSSED
C
      DO 20 I = 1,KTOTAL
      K = (I - 1) * 3 + 2
```

```
      KK= (I - 1) * 3 + 3
      IF (RF(KK) .LE. RF(K)) GO TO 10
      GO TO 20
   10 CT = 10. ** 30
      RETURN
   20 CONTINUE
      IF (K1 .LE. 1 .AND. K2 .LE. 1 .AND. K3 .LE. 1) GO TO 30
      CALL ORDER
      GO TO 70
   30 KVAR = 3 * KTOTAL
C
C*** STORE SAMPLE SIZES AND CONTROL LIMITS AS PASSED FROM SEARCH
C*** ROUTINE
C
      J = 0
      DO 40 I = 1,KVAR,3
      J = J + 1
      N(J) = RF(I)
   40 CONTINUE
      J = 0
      DO 50 I = 2,KVAR,3
      J = J + 1
      CLIMIT(J,1) = RF(I)
   50 CONTINUE
      J = 0
      DO 60 I = 3,KVAR,3
      J = J + 1
      CLIMIT(J,2) = RF(I)
   60 CONTINUE
C
C*** INITIALIZE ALL COMPONENTS OF COST
C
```

```
   70 CONTINUE
      CI = 0.00
      CA1 = 0.00
      CA2 = 0.00
      CA3 = 0.00
      CA4 = 0.00
      CR = 0.00
      CS1 = 0.00
      CS2 = 0.00
      CS3 = 0.00
      CS4 = 0.00
C
C*** CALL COMPUTATIONAL ROUTINES
C
      CALL APROX1
      IF (IREFPT .EQ. 1) CALL INSPCT
      IF (IREFPT .EQ. 1) CALL SCRAP
      IF (IREFPT .EQ. 1) GO TO 100
      CALL APROX2
      CALL NUMER
      CALL INSPCT
      CALL ACCPT1
      IF (DELTA2 .EQ. 1.00) CALL ACCPT2
      CALL ACCPT3
      IF (KDESTR .NE. 0) GO TO 80
      CALL ACCPT4
      GO TO 90
   80 IF (N(KDESTR) .LT. N(KTOTAL)) CALL ACCPT4
   90 IF (KDESTR .NE. 0) CALL SCRAP
      IF (KONTRC .EQ. 1) GO TO 906
      IF (K3 .NE. 0) CALL SCREN1
      IF (K3 .NE. 0) CALL SCREN2
```

```
      IF (K3 .NE. 0 .AND. KDESTR .NE. 0 .AND. DELTA2 .EQ. 1.00) CALL
     1   SCREN3
      IF (K3 .NE. 0) CALL SCREN4
  906 CS = CS1 + CS2 + CS3 + CS4
      CT = CI + CA1 + CA2 + CA3 + CA4 + CR + CS
  100 IF (IREFPT .EQ. 1) CT = CI + CR
      CIREF = CI
      CACREF = CA1 + CA2 + CA3 + CA4
      CRJREF = CR
      CSCREF = CS1 + CS2 + CS3 + CS4
      RETURN
      END
```

```
      SUBROUTINE INPUT
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      COMMON/NTRL/SAMPLE, DELTA2
      COMMON/HIST/NPTS(6), FREQ(6,10), SPREAD(6,10)
      INTEGER DELTA1
C
C*** INPUT THE NUMBER OF DEFECTS IN EACH DEFECT CLASS AND THE LOT SIZE
C
      READ(5,10) K1,K2,K3,L
   10 FORMAT(4I10)
C
C*** INPUT CONTROL PARAMETERS
C
      READ(5,20) DELTA2,DELTA3
   20 FORMAT(2F10.4)
C
C*** COMPUTE TOTAL NUMBER OF DESTRUCTABLE DEFECT TYPES AND THE TOTAL
C*** NUMBER OF DEFECT TYPES
C
      KDESTR = K1 + K2
      KTOTAL = K1 + K2 + K3
      L1 = KDESTR + 1
C
C*** INPUT THE INITIAL SAMPLE SIZES AND CONTROL LIMITS, AND THE DESIRED
C*** PROCESS MEANS AND STANDARD DEVIATIONS FOR EACH DEFECT TYPE
```

```
C
      DO 40 I = 1,KTOTAL
      READ(5,30) N(I), DMEAN(I), STD(I), (CLIMIT(I,J), J = 1,2), (SPEC(I
     %,J), J = 1,2)
   30 FORMAT(I5,6F10.4)
   40 CONTINUE
C
C*** INPUT COST PARAMETERS
C
      DO 60 I = 1,KTOTAL
      READ(5,50) UCI(I), UCAU(I), UCAL(I), UCRUP(I), UCRLOW(I), UCSCR(I)
   50 FORMAT(6F10.4)
   60 CONTINUE
      READ (5,70) UCR, UCR01,UCR02
   70 FORMAT(3F10.4)
      RETURN
      END
```

```
      SUBROUTINE OUTPT1
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/NTRL/SAMPLE, DELTA2
      COMMON/HIST/NPTS(6), FREQ(6,10), SPREAD(6,10)
C
C*** GO TO NEW PAGE
C
      WRITE(6,10)
   10 FORMAT(1H1)
C
C*** OUTPUT TITLE
C
      WRITE(6,20)
   20 FORMAT(33X,'MASTERS THESIS',///,20X,'A MULTIVARIABLE QUALITY CONTR
     1OL COST MODEL',///)
C
C*** OUTPUT PROGRAM CONTROL PARAMETERS
C
      WRITE(6,30)
   30 FORMAT(1X,'      CONTROL PARAMETERS',//)
      IF (DELTA2 .EQ. 1.) WRITE(6,40)
   40 FORMAT(15X,'REPLACEMENT OPTION IS IN EFFECT',///)
      IF (DELTA2 .EQ. 0.) WRITE(6,50)
   50 FORMAT(15X,'REPLACEMENT OPTION NOT IN EFFECT',///)
      IF (DELTA3 .EQ. 1.) WRITE(6,60)
   60 FORMAT(15X,'REPLACEMENT UNITS SCREENED FOR ALL VARIABLES',///)
      IF (DELTA3 .EQ. 0.) WRITE(6,70)
   70 FORMAT(15X,'REPLACEMENT UNITS NOT SCREENED',///)
C
```

```
C*** OUTPUT NUMBER OF DEFECT TYPES IN EACH DEFECT CLASS
C
      WRITE(6,80) K1, K2, K3
   80 FORMAT(1X,'      NUMBER OF DEFECT TYPES',//,15X,'DEFECT CLASS A = 
     1',I3,/,15X,'DEFECT CLASS B = ',I3,/,15X,'DEFECT CLASS C = ',I3,///
     1)
C
C*** OUTPUT LOT SIZE
C
      WRITE(6,90) L
   90 FORMAT(1X,'      LOT SIZE AND INITIAL SAMPLE SIZES',//,15X,'LOT SI
     1ZE = ',I10,//,15X,'DEFECT NO.',10X,'SAMPLE SIZE',/)
C
C*** OUTPUT INITIAL SAMPLE SIZES FOR EACH DEFECT TYPE
C
      DO 110 I = 1,KTOTAL
      WRITE(6,100) I, N(I)
  100 FORMAT(21X,I2,4X,'........',2X,I6,/)
  110 CONTINUE
C
C*** OUTPUT DEFECT DIMENSIONAL PARAMETERS
C
      WRITE(6,120)
  120 FORMAT(//,1X,'      DEFECT DIMENSIONAL PARAMETERS',//,28X,'DESIRED
     1',3X,'PROCESS',8X,'LOWER',11X,'UPPER',/,28X,'PROCESS',3X,'STANDARD
     1',3X,'SPECIFICATION',3X,'SPECIFICATION',/,15X,'DEFECT NO.',5X,'MEA
     1N',4X,'DEVIATION',6X,'LIMIT',11X,'LIMIT',/)
      DO 140 I = 1,KTOTAL
      WRITE(6,130) I, DMEAN(I), STD(I), (SPEC(I,J), J = 1,2)
  130 FORMAT(15X,I10,F10.4,2X,F10.4,3X,F10.4,6X,F10.4,/)
  140 CONTINUE
C
```

```
C*** OUTPUT INITIAL VALUES OF CONTROL LIMITS FOR EACH DEFECT TYPE
C
      WRITE(6,150)
  150 FORMAT(///,7X,   'INITIAL VALUES OF LOWER AND UPPER CONTROL LIMITS
     1',//,15X,'DEFECT NO.',5X,'LOWER',5X,'UPPER',/)
      DO 170 I = 1,KTOTAL
      WRITE(6,160) I, (CLIMIT(I,J), J = 1,2)
  160 FORMAT(15X,I10,2F10.4,/)
  170 CONTINUE
C
C*** OUTPUT COST PARAMETERS
C
      WRITE(6,180) UCRO1, UCRO2, UCR
  180 FORMAT(///,7X,'COST PARAMETERS',//,15X,'UNIT COST OF SCRAPPING  = 
     1',F11.4,//,15X,'UNIT COST OF REPLACEMENT = ',F10.4,//,15X,'UNIT CO
     2ST OF MATERIALS HANDLING = ',F10.4,///)
      WRITE(6,190)
  190 FORMAT(15X,'UNIT COST OF INSPECTION',//,20X,'DEFECT NO.',10X,'SAMP
     1LING',10X,'SCREENING',/)
      DO 210 I = 1,KTOTAL
      WRITE(6,200) I, UCI(I), UCSCR(I)
  200 FORMAT(26X,I2,10X,F10.4,9X,F10.4,/)
  210 CONTINUE
      WRITE(6,220)
  220 FORMAT(///,15X,'UNIT COST OF ACCEPTING DEFECTIVES OUTSIDE SPECIFIC
     1ATION LIMITS',//,20X,'DEFECT NO.',10X,' < LOWER',10X,'  > UPPER',/
     2)
      DO 240 I = 1,KTOTAL
      WRITE(6,230) I, UCAL(I), UCAU(I)
  230 FORMAT(26X,I2,10X,F10.4,9X,F10.4,/)
  240 CONTINUE
      WRITE(6,250)
```

```
250 FORMAT(15X,'UNIT COST OF REPAIRING DEFECTIVES OUTSIDE SPECIFICATIO
   1N LIMITS',//,20X,'DEFECT NO.',10X,' < LOWER',10X,'  > UPPER',/)
    DO 270 I = 1,KTOTAL
    WRITE(6,260) I, UCRLOW(I), UCRUP(I)
260 FORMAT(26X,I2,10X,F10.4,9X,F10.4,/)
270 CONTINUE
    WRITE(6,10)
    RETURN
    END
```

```
      SUBROUTINE LOAD
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/HIST/NPTS(6), FREQ(6,10), SPREAD(6,10)
      COMMON/BLK9/DUMVAL(15,30)
C
C*** STORE PARAMETER VALUES IN TEMPORARY STORAGE
C
      DO 10 I = 1,KTOTAL
      DUMVAL(I,1) = UCAU(I)
      DUMVAL(I,2) = UCAL(I)
      DUMVAL(I,3) = UCRUP(I)
      DUMVAL(I,4) = UCRLOW(I)
      DUMVAL(I,5) = SPEC(I,1)
      DUMVAL(I,6) = SPEC(I,2)
      DUMVAL(I,7) = UCI(I)
      DUMVAL(I,8) = UCSCR(I)
      DUMVAL(I,9) = DMEAN(I)
      DUMVAL(I,10) = STD(I)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ORDER
      DIMENSION RF(45)
      DIMENSION ADUMP(15), ID(15)
      COMMON/BLOKL/RF
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK9/DUMVAL(15,30)
      COMMON/HIST/NPTS(6), FREQ(6,10), SPREAD(6,10)
      DO 10 I = 1,KTOTAL
      K = (3 * I) - 2
      ADUMP(I) = RF(K)
   10 CONTINUE
      IF (K1 .EQ. 0) GO TO 60
      IF (K1 .GT. 1) GO TO 20
      ID(1) = 1
      GO TO 60
C
C*** PUT SAMPLE SIZES OF DEFECT CLASS A INTO ASCENDING ORDER
C
   20 DO 50 I = 1,K1
      BEST = 10.**30
      DO 40 J = 1,K1
      IF (ADUMP(J) .GT. 10.**20) GO TO 40
      IF (ADUMP(J) .LE. BEST) GO TO 30
      GO TO 40
   30 BEST = ADUMP(J)
      ISAVE = J
   40 CONTINUE
      ID(I) = ISAVE
      ADUMP(ISAVE) = 10.**30
```

```
   50 CONTINUE
   60 IF (K2 .EQ. 0) GO TO 110
      IF (K2 .GT. 1) GO TO 70
      INEXT = K1 + 1
      ID(INEXT) = INEXT
      GO TO 110
   70 INEXT = K1 + 1
      KEND = INEXT + K2 - 1
C
C*** PUT SAMPLE SIZES OF DEFECT CLASS B INTO ASCENDING ORDER
C
      DO 100 I = INEXT,KEND
      BEST = 10.**30
      DO 90 J = INEXT,KEND
      IF (ADUMP(J) .GT. 10.**20) GO TO 90
      IF (ADUMP(J) .LE. BEST) GO TO 80
      GO TO 90
   80 BEST = ADUMP(J)
      ISAVE = J
   90 CONTINUE
      ID(I) = ISAVE
      ADUMP(ISAVE) = 10.**30
  100 CONTINUE
  110 IF (K3 .EQ. 0) GO TO 160
      IF (K3 .GT. 1) GO TO 120
      ILAST = K1 + K2 + 1
      ID(ILAST) = ILAST
      GO TO 160
  120 ILAST = K1 + K2 + 1
      IEND = ILAST + K3 - 1
C
C*** PUT SAMPLE SIZES OF DEFECT CLASS C INTO ASCENDING ORDER
```

```
C
      DO 150 I = ILAST,IEND
      BEST = 10.**30
      DO 140 J = ILAST,IEND
      IF (ADUMP(J) .GT. 10.**20) GO TO 140
      IF (ADUMP(J) .LE. BEST) GO TO 130
      GO TO 140
  130 BEST = ADUMP(J)
      ISAVE = J
  140 CONTINUE
      ID(I) = ISAVE
      ADUMP(ISAVE) = 10.**30
  150 CONTINUE
C
C*** ARRANGE OTHER PARAMETERS ACCORDING TO ORDER OF SAMPLE SIZES
C*** WITHIN EACH DEFECT CLASS
C
  160 DO 170 I = 1,KTOTAL
      KORD = ID(I)
      KREF1 = (KORD - 1) * 3 + 1
      KREF2 = (KORD - 1) * 3 + 2
      KREF3 = (KORD - 1) * 3 + 3
      N(I) = RF(KREF1)
      CLIMIT(I,1) = RF(KREF2)
      CLIMIT(I,2) = RF(KREF3)
      UCAU(I) = DUMVAL(KORD,1)
      UCAL(I) = DUMVAL(KORD,2)
      UCRUP(I) = DUMVAL(KORD,3)
      UCRLOW(I) = DUMVAL(KORD,4)
      SPEC(I,1) = DUMVAL(KORD,5)
      SPEC(I,2) = DUMVAL(KORD,6)
      UCI(I) = DUMVAL(KORD,7)
```

```
      UCSCR(I) = DUMVAL(KORD,8)
      DMEAN(I) = DUMVAL(KORD,9)
      STD(I) = DUMVAL(KORD,10)
  170 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INSPCT
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      INTEGER DELTA1
C
C*** INITIALIZE COST OF INSPECTION TO ZERO
C
      CI = 0.00
C
C*** CHECK FOR ABSENCE OF DESTRUCTABLE DEFECT TYPES
C
      IF (KDESTR .EQ. 0) GO TO 40
C
C*** COMPUTE FIRST COMPONENT OF INSPECTION COST
C
      CI = UCI(1) * FLOAT( N(1) )
      IF (KDESTR .EQ. 1) GO TO 20
C
C*** COMPUTE ADDITIONAL COMPONENTS OF INSPECTION COST
C
      DO 10 I = 2,KDESTR
      K = I - 1
      CI = CI + UCI(I) * FLOAT( N(I) ) * INPROD(K)
   10 CONTINUE
C
C*** CHECK FOR ABSENCE OF NON-DESTRUCTABLE DEFECT TYPES
```

```
C
   20 IF (K3 .EQ. 0) RETURN
C
C*** COMPUTE FINAL COMPONENTS OF INSPECTION COST
C
      DO 30 I = L1,KTOTAL
      CI = CI + UCI(I) * FLOAT( N(I) ) * INPROD(KDESTR)
   30 CONTINUE
      RETURN
C
C*** IF NO DESTRUCTABLE DEFECT TYPES ARE PRESENT, INSPECTION COST IS
C*** SIMPLY THE UNIT COSTS OF INSPECTION TIMES THE SAMPLE SIZES
C
   40 DO 50 I = 1,K3
      CI = CI + UCI(I) * FLOAT( N(I) )
   50 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ACCPT1
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      COMMON/BLOC/NSUM
      INTEGER DELTA1
C
C*** INITIALIZE COST OF UNITS DESTROYED OR REJECTED IN SAMPLING
C*** INSPECTION
C
      CA1 = 0.00
      NSUM = 0
C
C*** CHECK FOR ABSENCE OF DESTRUCTABLE DEFECT TYPES
C
      IF (KDESTR .EQ. 0) GO TO 40
C
C*** CHECK FOR ABSENCE OF DESTRUCTABLE DEFECT TYPES IN DEFECT CLASS A
C
      IF (K1 .EQ. 0) GO TO 20
C
C*** COMPUTE NUMBER OF UNITS DESTROYED BY VIRTUE OF SAMPLING FOR DEFECT
C*** CLASS A DEFECT TYPES
C
      DO 10 I = 1,K1
      NSUM = NSUM + N(I)
   10 CONTINUE
```

```
C
C*** CHECK FOR ABSENCE OF DEFECT TYPES IN DEFECT CLASS B
C
   20 IF (K2 .EQ. 0) GO TO 30
C
C*** CONTINUE TO COMPUTE TOTAL NUMBER OF UNITS DESTROYED IN SAMPLING
C*** INSPECTION
C
      NSUM = NSUM + N(KDESTR)
C
C*** COMPUTE COST ASSOCIATED WITH UNITS DESTROYED IN SAMPLING INSPECTION
C
   30 CA1 = CA1 + (UCR + UCRO1) * FLOAT(NSUM) * INPROD (KTOTAL)
C
C*** CHECK TO SEE IF THERE ARE ANY UNITS REMAINING IN THE TOTAL SAMPLE
C*** WHICH HAVE NOT BEEN DESTROYED IN SAMPLING INSPECTION
C
      IF (K3 .EQ. 0 .OR. N(KDESTR) .GE. N(KTOTAL)) RETURN
      GO TO 70
C
C*** COMPUTE NUMBER OF UNITS REJECTED IN SAMPLING INSPECTION
C
   40 DO 60 I = 1,KTOTAL
      DELTA1(I) = 1
      IF (I .EQ. 1) N1STAR(1) = N(1)
      K = I - 1
      IF (I .NE. 1) N1STAR(I) = N(I) - N(K)
      N2STAR(I) = N(I)
   60 CONTINUE
      GO TO 90
   70 DO 80 I = L1,KTOTAL
      N2STAR(I) = 0
```

```
      K = I - 1
      DELTA1(I) = 1
      IF(N(I) .LE. N(KDESTR)) DELTA1(I) = 0
      IF(N(I) .LE. N(KDESTR)) GO TO 80
      N1STAR(I) = N(I) - N(KDESTR)
      IF (N1STAR(I) .GT. (N(I) - N(K))) N1STAR(I) = N(I) - N(K)
      N2STAR(I) = N(I) - N(KDESTR)
   80 CONTINUE
C
C*** COMPUTE COST ASSOCIATED WITH UNITS REJECTED IN SAMPLING INSPECTION
C
   90 DO 130 I = L1,KTOTAL
      IF (DELTA1(I) .EQ. 0) GO TO 130
      DUM1 = 1.00
      DUM2 = 1.00
      DO 100 J = I,KTOTAL
      DUM1 = DUM1 * INTGRL(J)
      DUM2 = DUM2 * NUMINT(J,2)
  100 CONTINUE
      DUM3 = (DUM1 - DUM2) * N1STAR(I)
      IF (I .EQ. 1) GO TO 110
      K = I - 1
      CA1 = CA1 + UCR * DUM3 * INPROD(K)
      GO TO 120
  110 CA1 = CA1 + UCR * DUM3
C
C*** COMPUTE COST OF REPAIRING UNITS REJECTED IN SAMPLING INSPECTION
C
  120 IF (N2STAR(I) .LE. 0) GO TO 130
      CA1 = CA1 + (UCRUP(I) * NUMINT(I,3) + UCRLOW(I) * NUMINT(I,1)) *
     1 (INPROD(KTOTAL) / INTGRL(I)) * FLOAT(N2STAR(I))
  130 CONTINUE
```

```
      RETURN                                                             MAS  690
      END                                                                MAS  691
```

```
      SUBROUTINE ACCPT2
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      COMMON/BLOC/NSUM
      INTEGER DELTA1
C
C*** COMPUTE PER UNIT COST OF ACTUALLY REPLACING DESTROYED UNITS
C
      CA2 = UCRO2 * INPROD(KTOTAL)
      DUM1 = 0.00
C
C*** COMPUTE PER UNIT COST OF ACCEPTING DEFECTS IN CLASSES A AND B
C*** AMONG REPLACEMENT UNITS
      KKEND = DELTA3*FLOAT(KTOTAL)+(1.0-DELTA3)*FLOAT(KDESTR)
      DO 10 I = 1,KKEND
      DUM1 = DUM1 + (UCAU(I) * NUMINT(I,3) + UCAL(I) * NUMINT(I,1)) * (I
     %NPROD(KDESTR) / INTGRL(I))
   10 CONTINUE
C
C*** CHECK FOR ABSENCE OF CLASS C DEFECT TYPES
C
      IF (K3 .EQ. 0 .OR. DELTA3 .EQ. 0.00) GO TO 30
C
C*** COMPLETE REMOVAL OF CONDITIONALITY
C
      DO 20 I = L1,KTOTAL
```

```
      DUM1 = DUM1 * INTGRL(I)
   20 CONTINUE
   30 CA2 = CA2 + DUM1
C
C*** CHECK FOR ABSENCE OF CLASS C DEFECT TYPES
C
      IF (K3 .EQ. 0 .OR. DELTA3 .EQ. 0.00) GO TO 50
      DUM2 = 0.00
C
C*** COMPUTE PER UNIT COST OF SCREENING REPLACEMENT UNITS FOR CLASS C
C*** DEFECT TYPES AND REPAIRING DEFECTIVES ACCORDINGLY
C
      DO 40 I = L1,KTOTAL
      DUM2 = DUM2 + UCSCR(I)
      CA2 = CA2 + (UCRUP(I) * NUMINT(I,3) + UCRLOW(I) * NUMINT(I,1)) * (
     %INPROD(KTOTAL) / INTGRL(I))
   40 CONTINUE
C
C*** COMPUTE UNCONDITIONAL PER UNIT COST OF SCREENING REPLACEMENT UNITS
C
      CA2 = CA2 + DUM2 * INPROD(KTOTAL)
C
C*** COMPUTE TOTAL COST FOR REPLACING DESTROYED UNITS WHEN THE
C*** INSPECTION LOT IS ACCPTED
C
   50 CA2 = CA2 * FLOAT(NSUM)
      RETURN
      END
```

```
      SUBROUTINE ACCPT3
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      INTEGER DELTA1
C
C*** CHECK FOR ABSENCE OF DESTRUCTABLE DEFECT TYPES
C
   10 IF (KDESTR .NE.0) GO TO 20
C
C*** COMPUTE THE TOTAL NUMBER OF UNITS REMOVED FROM THE INSPECTION LOT
C*** FOR THE PURPOSE OF SAMPLING INSPECTION
C
      NSTAR = N(KTOTAL)
      GO TO 60
   20 IDUM = 0
      IF (K1 .EQ. 0) GO TO 40
      DO 30 I = 1,K1
      IDUM = IDUM + N(I)
   30 CONTINUE
      NSTAR = IDUM
      IF (K2 .EQ. 0) GO TO 50
   40 NSTAR = N(KDESTR) + IDUM
   50 IF (K3 .EQ. 0) GO TO 60
      NDUM = N(KTOTAL) + IDUM
      IF (NSTAR .LT. NDUM) NSTAR = NDUM
C
```

```
   60 SUM = 0.00
C
C*** COMPUTE THE PER UNIT COST OF ACCEPTING DEFECTIVES AMONG THE
C*** NON-INSPECTED REMAINDER OF THE INSPECTION LOT
C
      DO 70 I = 1,KTOTAL
      SUM = SUM + (UCAU(I) * NUMINT(I,3) + UCAL(I) * NUMINT(I,1)) * (INP
     %ROD(KTOTAL) / INTGRL(I))
   70 CONTINUE
C
C*** COMPUTE THE TOTAL COST FOR ALL UNITS
C
      CA3 = (FLOAT(L) - FLOAT(NSTAR)) * SUM
      RETURN
      END
```

```
      SUBROUTINE ACCPT4
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), OMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      INTEGER DELTA1
C
C*** INITIALIZE COST OF ACCEPTING DEFECTIVES AMONG THE NON-DESTROYED
C*** NON-INSPECTED REMAINDER OF THE SAMPLE
C
      CA4 = 0.00
C
C*** CONSIDER ONLY CLASS C DEFECT TYPES
C
      DO 20 I = L1,KTOTAL
C
C*** CHECK TO SEE IF THE SAMPLE SIZE OF THE DEFECT UNDER CONSIDERATION
C*** IS GREATER THAN THE LARGEST DEFECT CLASS B SAMPLE SIZE
C
      IF (DELTA1(I) .EQ. 0) GO TO 20
      K = I - 1
C
C*** COMPUTE ABOVE COST FOR ALL DEFECT TYPES PRIOR TO THE DEFECT TYPE
C*** UNDER CONSIDERATION
C
      DO 10 J = 1,K
      CA4 = CA4 + ((FLOAT(N1STAR(I)) * (UCAU(J) * NUMINT(J,3) + UCAL(J)
     % * NUMINT(J,1))) * (INPROD(KTOTAL) / INTGRL(J)))
```

```
   10 CONTINUE                                                          MAS  831
   20 CONTINUE                                                          MAS  832
      RETURN                                                            MAS  833
      END                                                               MAS  834
```

```
      SUBROUTINE SCRAP
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
C
C*** COMPUTE THE COST OF SCRAPPING THE LOT
C
      CR = FLOAT (L) *(UCR + UCRO1) * (1.00 - INPROD(KDESTR))
      RETURN
      END
```

```
      SUBROUTINE SCREN1
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      COMMON/BLOK/INTEXX(15,3)
      INTEGER DELTA1
      DUM = 0.00
C
C*** COMPUTE PER UNIT COST OF SCREENING INSPECTION
C
      DO 10 I = L1,KTOTAL
      DUM = DUM + UCSCR(I) * (1.00 - INTGRL(I))
   10 CONTINUE
C
C*** COMPUTE TOTAL COST OF SCREENING REMAINDER OF INSPECTION LOT
C
      CS1 = (FLOAT(L) - FLOAT(NSTAR)) * DUM
C
C*** CHECK FOR ABSENCE OF DESTRUCTABLE DEFECT TYPES
C
      IF (KDESTR .EQ. 0) GO TO 20
C
C*** REMOVE CONDITIONALITY
C
      CS1 = CS1 * INPROD(KDESTR)
C
C*** CHECK FOR ABSENCE OF CLASS B DEFECT TYPES
```

```
C
      IF (K2 .EQ. 0) GO TO 20
C
C*** CHECK TO SEE IF THE LARGEST SAMPLE SIZE OF A CLASS   DEFECT TYPE
C*** IS GREATER THAN OR EQUAL TO THE LARGEST SAMPLE SIZE OF A CLASS C
C*** DEFECT TYPE
C
      IF (N(KDESTR) .GE. N(KTOTAL)) RETURN
   20 ADUM = 0.00
      DUM = 0.00
      IF (K2 .NE. 0) DUM = FLOAT(N(KDESTR))
C
C*** COMPUTE COST OF SCREENING NON-DESTROYED UNITS REMAINING IN SAMPLE
C
      DO 30 I = L1,KTOTAL
      ADUM = ADUM + UCSCR(I) * (FLOAT( DELTA1(I) ) * (FLOAT( N(KTOTAL) )
     1  - FLOAT( N(I) )) + (1.00 - FLOAT( DELTA1(I) )) * (FLOAT( N(KTOTA
     1L) ) - DUM)) * (1.00 - INTGRL(I))
   30 CONTINUE
C
C*** CHECK FOR ABSENCE OF DESTRUCTABLE DEFECT TYPES
C
      IF (KDESTR .EQ. 0) GO TO 40
C
C*** REMOVE CONDITIONALITY
C
      CS1 = CS1 + ADUM * INPROD(KDESTR)
      RETURN
   40 CS1 = CS1 + ADUM
      RETURN
      END
```

```
      SUBROUTINE SCREN2
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      COMMON/NTRL/SAMPLE, DELTA2
      COMMON/BLOK/INTEXX(15,3)
      COMMON/BLOM/DUM
      COMMON/BLOC/NSUM
      INTEGER DELTA1
      CS2 = 0.00
      DUM = 1.00
      DO 10 I = L1,KTOTAL
      DUM = DUM * INTGRL(I)
   10 CONTINUE
      IF (KDESTR .EQ. 0) GO TO 20
C
C*** COMPUTE COST OF UNITS DESTROYED IN SAMPLING INSPECTION
C
      CS2 = FLOAT(NSUM) * (UCR + UCRO1) * INPROD(KDESTR) * (1.00 - DUM)
      IF (K2 .EQ. 0) GO TO 20
      IF (N(KDESTR) .GE. N(KTOTAL)) GO TO 190
   20 SUM = 0.00
      DO 60 I = L1,KTOTAL
      IF (DELTA1(I) .EQ. 0) GO TO 70
      DUM1 = 1.00
      DUM2 = 1.00
      DO 30 J = I,KTOTAL
```

```
      DUM1 = DUM1 * INTEXX(J,2)
      DUM2 = DUM2 * NUMINT(J,2)
   30 CONTINUE
      IF (I .EQ. L1) GO TO 50
      KK = I - 1
      DO 40 K = L1 , KK
      DUM2 = DUM2 * INTGRL(K)
   40 CONTINUE
C
C*** COMPUTE COST OF UNITS REJECTED IN SAMPLING INSPECTION
C
   50 SUM = SUM + (UCR * FLOAT(N1STAR(I)) * (1.00 - DUM1 - DUM + DUM2))
   60 CONTINUE
      IF (KDESTR .EQ. 0) GO TO 70
      CS2 = CS2 + SUM * INPROD(KDESTR)
      GO TO 80
   70 CS2 = CS2 + SUM
   80 SUM = 0.00
      DO 100 I = L1,KTOTAL
      DUMNUM = (1.00 - INTEXX(I,2) - INTGRL(I) + NUMINT(I,2))
      IF (DELTA1(I) .EQ. 0) GO TO 90
C
C*** COMPUTE COST OF UNITS REJECTED IN SCREENING INSPECTION OF THE
C*** NON-DESTROYED PORTION OF THE SAMPLE
C
      SUM = SUM + (UCR * (FLOAT(N(KTOTAL)) - FLOAT(N(I))) * DUMNUM)
C
C*** COMPUTE COST OF UNITS REJECTED IN SCREENING INSPECTION OF THE
C*** REMAINDER OF THE INSPECTION LOT
C
   90 SUM = SUM + UCR * (FLOAT(L) - FLOAT(NSTAR)) * DUMNUM
  100 CONTINUE
```

```
      IF (KDESTR .EQ. 0) GO TO 110
      CS2 = CS2 + SUM * INPROD(KDESTR)
      GO TO 120
  110 CS2 = CS2 + SUM
  120 SUM = 0.00
      DO 140 I = L1,KTOTAL
      IF (N2STAR(I) .LE. 0) GO TO 140
      DUM3 = 1.00
      DO 130 J = L1,KTOTAL
      IF (J .EQ. I) GO TO 130
      DUM3 = DUM3 * INTGRL(J)
  130 CONTINUE
C
C*** COMPUTE COST OF REPAIRING NON-DESTROYED UNITS REJECTED IN
C*** SAMPLING INSPECTION OF SAMPLE
C
      SUM = SUM + N2STAR(I) * (UCRUP(I) * (INTEXX(I,3) - DUM3 * NUMINT(I
     1 ,3)) + UCRLOW(I) * (INTEXX(I,1) - DUM3 * NUMINT(I,1)))
  140 CONTINUE
      IF (KDESTR .EQ. 0) GO TO 150
      CS2 = CS2 + SUM * INPROD(KDESTR)
      GO TO 160
  150 CS2 = CS2 + SUM
  160 SUM = 0.00
      DO 170 I = L1,KTOTAL
      DUM4 = 0.00
      IF (K2 .NE. 0) DUM4 = FLOAT(N(KDESTR))
C
C*** COMPUTE COST OF REPAIRING NON-DESTROYED UNITS REJECTED IN
C*** SCREENING INSPECTION OF SAMPLE
C
      SUM = SUM + ((FLOAT(DELTA1(I)) * (FLOAT(N(KTOTAL)) - FLOAT(N(I))))
```

```
     1 + ((1.00 - FLOAT(DELTA1(I))) * FLOAT(N(KTOTAL)) - DUM4)) *(UCRUP(
     2I) * (INTEXX(I,3) - NUMINT(I,3)) + UCRLOW(I) * (INTEXX(I,1) - NUMI
     3NT(I,1)))
  170 CONTINUE
      IF (KDESTR .EQ. 0) GO TO 180
      CS2 = CS2 + SUM * INPROD(KDESTR)
      GO TO 190
  180 CS2 = CS2 + SUM
  190 SUM = 0.00
      DO 200 I = L1,KTOTAL
C
C*** COMPUTE COST OF REPAIRING UNITS REJECTED IN SCREENING INSPECTION
C*** OF THE REMAINDER OF THE LOT
C
      SUM = SUM + UCRUP(I) * (INTEXX(I,3) - NUMINT(I,3)) + UCRLOW(I) * (
     1 INTEXX(I,1) - NUMINT(I,1))
  200 CONTINUE
      IF (KDESTR .EQ. 0) GO TO 210
      CS2 = CS2 + SUM * (FLOAT(L) - FLOAT(NSTAR)) * INPROD(KDESTR)
      RETURN
  210 CS2 = CS2 + SUM * (FLOAT(L) - FLOAT(NSTAR))
      RETURN
      END
```

```
      SUBROUTINE SCREN3
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      COMMON/NTRL/SAMPLE, DELTA2
      COMMON/BLOC/NSUM
      COMMON/BLOM/DUM
      COMMON/BLOK/INTEXX(15,3)
      INTEGER DELTA1
      SUM = 0.00
      DO 10 I = L1,KTOTAL
      SUM = SUM + UCSCR(I) * DELTA3
   10 CONTINUE
      SUM = SUM + UCRO2
C
C*** COMPUTE COST OF SCREENING REPLACEMENT UNITS FOR UNITS DESTROYED
C*** IN SAMPLING INSPECTION
C
      CS3 = SUM * FLOAT(NSUM) * INPROD(KDESTR) * (1.00 - DUM)
      SUM = 0.00
      DO 30 I = L1,KTOTAL
      DUM1 = 1.00
      DO 20 J = L1,KTOTAL
      IF (J .EQ. I) GO TO 20
      DUM1 = DUM1 * INTGRL(J)
   20 CONTINUE
C
```

```
C*** COMPUTE COST OF REPAIRING DEFECTIVES FOUND AMONG REPLACEMENT
C*** UNITS
C
      SUM = SUM + UCRUP(I) * (INTEXX(I,3) - (NUMINT(I,3) * DUM1)) + UCRL
     1 OW(I) * (INTEXX(I,1) - NUMINT(I,1) * DUM1)
   30 CONTINUE
      CS3 = CS3 + SUM * FLOAT(NSUM) * INPROD(KDESTR)
C
C*** COMPUTE COST OF ACCEPTING DESTRUCTABLE DEFECT TYPES AMONG
C*** REPLACEMENT UNITS
      KKEND = DELTA3*FLOAT(KTOTAL)+(1.0-DELTA3)*FLOAT(KDESTR)
      SUM = 0.00
      DO 40 I = 1,KKEND
      SUM = SUM + (UCAU(I) * NUMINT(I,3) + UCAL(I) * NUMINT(I,1)) * INPR
     1 OD(KDESTR) / INTGRL(I)
   40 CONTINUE
      CS3 = CS3 + SUM * (1.00 - DUM) * FLOAT(NSUM)
      RETURN
      END
```

```
      SUBROUTINE SCREN4
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK3/UCR,UCRHO,UCI(15),UCAU(15),UCAL(15),UCRUP(15),UCRLOW(1
     15),UCSCR(15),UCRO1,UCRO2,DELTA3
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/COST/CI,CA1,CA2,CA3,CA4,CR,CS1,CS2,CS3,CS4
      COMMON/PASS/N1STAR(15), N2STAR(15), DELTA1(15), NSTAR
      COMMON/NTRL/SAMPLE, DELTA2
      COMMON/BLOC/NSUM
      COMMON/BLOK/INTEXX(15,3)
      COMMON/BLOM/DUM
      INTEGER DELTA1
      CS4 = 0.00
      IF (KDESTR .EQ. 0) GO TO 20
      SUM = 0.00
      DO 10 I = 1,KDESTR
      SUM = SUM + (UCAU(I) * NUMINT(I,3) + UCAL(I) * NUMINT(I,1)) * (INP
     1 ROD(KDESTR) / INTGRL(I))
   10 CONTINUE
C
C*** COMPUTE COST OF ACCEPTING DESTRUCTABLE DEFECT TYPES AMONG THE
C*** REMAINDER OF THE INSPECTION LOT
C
      CS4 = CS4 + SUM * (FLOAT(L) - FLOAT(NSTAR)) * (1.00 - DUM)
   20 SUM = 0.00
      IF (K3 .EQ. 1) GO TO 50
      DO 30 I = L1,KTOTAL
      SUM = SUM + (UCAU(I) * NUMINT(I,3) + UCAL(I) * NUMINT(I,1)) * INTG
     1 RL(I) * (1.00 - (DUM / INTGRL(I)))
   30 CONTINUE
```

```
      IF (KDESTR .EQ. 0) GO TO 40
C
C*** COMPUTE COST OF ACCEPTING SCREENABLE DEFECT TYPES AMONG THE
C*** REMAINDER OF THE INSPECTION LOT
C
      CS4 = CS4 + SUM * INPROD (KDESTR) * (FLOAT(L) - FLOAT(NSTAR))
      GO TO 50
   40 CS4 = CS4 + SUM * (FLOAT(L) - FLOAT(NSTAR))
   50 IF (K2 .EQ. 0) GO TO 60
      IF (N(K2) .GE. N(KTOTAL)) RETURN
   60 IF (KDESTR .EQ. 0) GO TO 90
      DO 80 I = L1,KTOTAL
      IF (DELTA1(I) .EQ. 0) GO TO 80
      DO 70 J = 1,KDESTR
C
C*** COMPUTE THE COST OF ACCEPTING DESTRUCTABLE DEFECT TYPES AMONG
C*** THE NON-DESTROYED, NON-INSPECTED PORTION OF THE SAMPLE
C
      CS4 = CS4 + (FLOAT(N1STAR(I)) * (UCAU(J) * NUMINT(J,3) + UCAL(J) *
     1 NUMINT(J,1))) * (INPROD(KDESTR) / INTGRL(J)) * (1.00 - DUM)
   70 CONTINUE
   80 CONTINUE
   90 SUM = 0.00
      IF (K3 .EQ. 1) RETURN
      DO 100 I = L1,KTOTAL
      DUM5 = 0.00
      IF (K2 .NE. 0) DUM5 = FLOAT(N(KDESTR))
      SUM = SUM + (UCAU(I) * NUMINT(I,3) + UCAL(I) * NUMINT(I,1)) * INTG
     1 RL(I) * (1.00 - (DUM / INTGRL(I))) * (DELTA1(I) *(FLOAT(N(KTOTAL)
     2 ) - FLOAT(N(I))) + (1.00 - DELTA1(I)) * (FLOAT(N(KTOTAL)) - DUM5)
     3 )
  100 CONTINUE
```

```
      IF (KDESTR .EQ. 0) GO TO 110                                      MAS 1146
C                                                                       MAS 1147
C*** COMPUTE THE COST OF ACCEPTING SCREENABLE DEFECT TYPES AMONG         MAS 1148
C*** THE NON-DESTROYED, NON-INSPECTED PORTION OF THE SAMPLE              MAS 1149
C                                                                       MAS 1150
      CS4 = CS4 + SUM * INPROD(KDESTR)                                  MAS 1151
      RETURN                                                            MAS 1152
  110 CS4 = CS4 + SUM                                                   MAS 1153
      RETURN                                                            MAS 1154
      END                                                               MAS 1155
```

```
      SUBROUTINE APROX1
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      DIMENSION A(5), X(15,2)
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/REF/IREFPT
      COMMON/STEP/ KONTRC
      REAL INT(2)
      IREFPT = 0
C
C***INITIALIZE APPROXIMATING CONSTANTS
C
      A(1) = 0.14112821
      A(2) = 0.08864027
      A(3) = 0.0274339
      A(4) = -0.00039446
      A(5) = 0.00328975
C
C
C***COMPUTE APPROXIMATING INTEGRAL FOR ALL VARIABLES
C
      DO 30 K = 1,KTOTAL
      IF (N(K).NE. 0) GO TO 21
      INTGRL(K) = 1.00
      GO TO 25
   21 CONTINUE
C
C***COMPUTE TRANSFORMATION FOR LOWER AND UPPER LIMITS OF INTEGRATION
C
      R = FLOAT(L) / 6000.0000
      DO 20 I = 1,2
```

```
      X(K,I) = (SQRT(FLOAT(N(K)) * R         / (R         + FLOAT(N(K))))
     % * (CLIMIT(K,I) - DMEAN(K)))/ (STD(K) * SQRT(2.000))
      SUM = 0.0
      DO 10 J = 1,5
      SUM = SUM + A(J) * ABS( X(K,I) ) ** J
   10 CONTINUE
      DENOM =(1.0 + SUM) ** 8
      PHISTR = 1.0 - (1.0 / DENOM)
C
C***COMPUTE APPROXIMATE INTEGRAL FROM MINUS (-) INFINITY TO LIMITS OF
C***INTEGRATION
C
      IF (X(K,I) .EQ. 0.00) GO TO 11
      GO TO 12
   11 INT(I) = 0.500
      GO TO 20
   12 INT(I) = 0.500 + (( X(K,I) / ABS(X(K,I))) * 0.500 * PHISTR)
   20 CONTINUE
C
C***COMPUTE APPROXIMATE INTEGRAL BETWEEN LIMITS OF INTEGRATION
C
      INTGRL(K) = INT(2) - INT(1)
      IF (INTGRL(K) .EQ. 0.00) IREFPT = 1
C
C***COMPUTE PRODUCTS OF SUCCESSIVE INTEGRALS
C
   25 IF (K .EQ. 1) INPROD(K) = INTGRL(K)
      KDUM = K - 1
      IF (K .GT. 1) INPROD(K) = INPROD(KDUM) * INTGRL(K)
   30 CONTINUE
      IF (K3 .EQ. 0) RETURN
      KONTRC = 1
```

```
      DO 906 I = L1,KTOTAL
      IF (INTGRL(I) .LT. 0.9990) KONTRC = 0
  906 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE APROX2
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
C
      DIMENSION A(5), X(15,2)
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/BLOK/INTEXX(15,3)
      REAL INT(2)
C***INITIALIZE APPROXIMATING CONSTANTS
C
      A(1) = 0.14112821
      A(2) = 0.08864027
      A(3) = 0.0274339
      A(4) = -0.00039446
      A(5) = 0.00328975
C
C
C***COMPUTE APPROXIMATING INTEGRAL FOR ALL VARIABLES
C
      DO 30 K = 1,KTOTAL
C
C***COMPUTE TRANSFORMATION FOR LOWER AND UPPER LIMITS OF INTEGRATION
C
      R = FLOAT(L) / 6000.0000
      DO 20 I = 1,2
      X(K,I) = SQRT(R        / (R        + 1.00)) * (SPEC(K,I) - DMEAN(K
     1 )) / (STD(K) * SQRT(2.000))
      SUM = 0.0
      DO 10 J = 1,5
      SUM = SUM + A(J) * ABS( X(K,I) ) ** J
   10 CONTINUE
```

```
      DENOM =(1.0 + SUM) ** 8
      PHISTR = 1.0 - (1.0 / DENOM)
C
C***COMPUTE APPROXIMATE INTEGRAL FROM MINUS (-) INFINITY TO LIMITS OF
C***INTEGRATION
C
      INT(I) = 0.50 + ((X(K,I) / ABS( X(K,I) )) * 0.50 * PHISTR)
   20 CONTINUE
C
C***COMPUTE APPROXIMATE INTEGRAL BETWEEN LIMITS OF INTEGRATION
C
      INTEXX(K,1) = INT(1)
      INTEXX(K,2) = INT(2) - INT(1)
      INTEXX(K,3) = 1.00 - INT(2)
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE NUMER
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/KEEP/KDUMMY
      COMMON/LAST/JWAY
      COMMON/BLOK/INTEXX(15,3)
      TOL = 0.001
      ITFIN = 10
      DO 30 JWAY = 1,3
   10 DO 20 I = 1,KTOTAL
      IF(N(I) .EQ. 0) GO TO 15
      KDUMMY = I
      AL = CLIMIT(I,1)
      BL = CLIMIT(I,2)
      NUMINT(I,JWAY) = GAUSS(AL,BL,TOL,ITFIN)
      GO TO 20
   15 NUMINT(I,JWAY) = INTEXX(I,JWAY)
   20 CONTINUE
   30 CONTINUE
      RETURN
      END
```

```
      FUNCTION GAUSS(AL,BL,TOL,ITFIN)
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
      DOUBLE PRECISION V, R
C
C*** GAUSSIAN QUADRATURE ... 11 POINTS PER SUBINTERVAL
C*** INTEGRATES FDUM FROM AL TO BL
C
      DIMENSION V(11), R(11)
      COMMON/KEEP/KDUMMY
      COMMON/LAST/JWAY
      DATA V,R /-.978228658146057D0, -.887062599768095D0,
     1 -.730152005574049D0,-.519096129206812D0,
     2 -.269543155952345D0, 0.0D0, .269543155952345D0,
     3 .519096129206812D0, .730152005574049D0,
     4 .887062599768095D0, .978228658146057D0,
     5 .055668567116174D0, .125580369464905D0,
     6 .186290210927734D0, .233193764591990D0,
     7 .262804544510247D0, .272925086777901D0,
     8 .262804544510247D0, .233193764591990D0,
     9 .186290210927734D0, .125580369464905D0, .055668567116174D0 /
C
      NINT = 1
      IT = 0
   10 IT = IT + 1
      DEL = (BL - AL) / (FLOAT(NINT))
      SS = 0.0
      DO 30 I = 1,NINT
      B = AL + DEL * (FLOAT(I))
      A = B - DEL
      S = 0.0
      DO 20 J = 1,11
      X1= ( (B+A) + (B-A)*V(J) ) * 0.5
```

```
      F = FDUM(X1)                                                      MAS 1329
      S = S + R(J) * F                                                  MAS 1330
   20 CONTINUE                                                          MAS 1331
      S = (B-A) * S * 0.5                                               MAS 1332
      SS = SS + S                                                       MAS 1333
   30 CONTINUE                                                          MAS 1334
      IF (IT .GT. 1) GO TO 50                                           MAS 1335
   40 SAVE = SS                                                         MAS 1336
      NINT = 2 * NINT                                                   MAS 1337
      GO TO 10                                                          MAS 1338
   50 IF ( ABS(SAVE) - TOL ) 60,60,70                                   MAS 1339
   60 X1= ABS(SAVE - SS)                                                MAS 1340
      GO TO 80                                                          MAS 1341
   70 X1= ABS( (SAVE - SS) / SAVE )                                     MAS 1342
   80 IF (X1.GT. TOL .AND. IT .LE. ITFIN) GO TO 40                      MAS 1343
      IF (X1.GT. TOL) GO TO 100                                         MAS 1344
      GAUSS = SS                                                        MAS 1345
   90 RETURN                                                            MAS 1346
  100 WRITE(6,110) IT, ITFIN, X1, SS                                    MAS 1347
      CALL EXIT                                                         MAS 1348
  110 FORMAT(1H0, 'GAUSS   IT,ITFIN,DIFF,FINAL ',2I4,2E20.8)            MAS 1349
      END                                                               MAS 1350
```

```
      FUNCTION AMULT(ALIM)
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT
C
      DIMENSION A(5), X(15,2)
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)
      COMMON/BLK4/INTGRL(15), NUMINT(15,3), INPROD(15)
      COMMON/KEEP/KDUMMY
      COMMON/LAST/JWAY
      REAL INT(2)
C***INITIALIZE APPROXIMATING CONSTANTS
C
      A(1) = 0.14112821
      A(2) = 0.08864027
      A(3) = 0.0274339
      A(4) = -0.00039446
      A(5) = 0.00328975
C
C***COMPUTE TRANSFORMATION FOR LOWER AND UPPER LIMITS OF INTEGRATION
C
      K = KDUMMY
      R = FLOAT(L) / 6000.0000
      DO 20 I = 1,2
      X(K,I) = SQRT((FLOAT(N(K)) + R       ) / (FLOAT(N(K)) + R       +
     1 1.000)) * (SPEC(K,I) - ((FLOAT(N(K)) * ALIM + R        * DMEAN(K)
     2 ) / (FLOAT(N(K)) + R       ))) / (STD(K) * SQRT(2.000))
      SUM = 0.0
      DO 10 J = 1,5
      SUM = SUM + A(J) * ABS( X(K,I) ) ** J
   10 CONTINUE
      DENOM =(1.0 + SUM) ** 8
      PHISTR = 1.0 - (1.0 / DENOM)
```

```
C
C***COMPUTE APPROXIMATE INTEGRAL FROM MINUS (-) INFINITY TO LIMITS OF
C***INTEGRATION
C
      INT(I) = 0.50 + ((X(K,I) / ABS( X(K,I) )) * 0.50 * PHISTR)
   20 CONTINUE
C
C***COMPUTE APPROXIMATE INTEGRAL BETWEEN LIMITS OF INTEGRATION
C
      IF (JWAY .EQ. 1) AMULT = INT(1)
      IF (JWAY .EQ. 2) AMULT = INT(2) - INT(1)
      IF (JWAY .EQ. 3) AMULT = 1.000 - INT(2)
      RETURN
      END
```

```
      FUNCTION FDUM(ALIM)                                               MAS 1397
      REAL*8 INPROD,INTEXX,INTGRL,NUMINT                                MAS 1398
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1                             MAS 1399
      COMMON/BLK2/CLIMIT(15,2), L, N(15), DMEAN(15), SPEC(15,2), STD(15)MAS 1400
      COMMON/KEEP/KDUMMY                                                MAS 1401
      COMMON/LAST/JWAY                                                  MAS 1402
      I = KDUMMY                                                        MAS 1403
      R = FLOAT(L) / 6000.0000                                          MAS 1404
      BMULT = SQRT((FLOAT(N(I)) * R       ) / (FLOAT(N(I)) + R       )) MAS 1405
     1 / (STD(I) * SQRT(6.2832))                                        MAS 1406
      CMULT = ((FLOAT(N(I)) * R       ) / (FLOAT(N(I)) + R       )) *   MAS 1407
     1 ((ALIM - DMEAN(I)) ** 2) / (2.000 * STD(I) **2)                  MAS 1408
      CMULT = - CMULT                                                   MAS 1409
      DMULT = BMULT * EXP(CMULT)                                        MAS 1410
      FDUM = DMULT * AMULT(ALIM)                                        MAS 1411
      RETURN                                                            MAS 1412
      END                                                               MAS 1413
```

```
      SUBROUTINE OUTPT2
      COMMON/BLK1/K1,K2,K3,KDESTR,KTOTAL,L1
      COMMON/OPTU/ BEST(45), YMAX
      WRITE(6,10)
   10 FORMAT(1H1,1X,'OPTIMUM MULTIVARIATE ACCEPTANCE SAMPLING PARAMETERS
     1',///,43X,'LOWER',12X,'UPPER',/,26X,'SAMPLE',10X,'CONTROL',10X,
     2 'CONTROL',/,7X,'DEFECT NO.',11X,'SIZE',12X,'LIMIT',12X,'LIMIT',/
     3 /)
      DO 30 I = 1,KTOTAL
      J = (I - 1) * 3 + 1
      J1= (I - 1) * 3 + 2
      J2= (I - 1) * 3 + 3
      NSAMPL = BEST(J)
      WRITE(6,20) I, NSAMPL, BEST(J1), BEST(J2)
   20 FORMAT(7X,I6,12X,I6,10X,F7.4,10X,F7.4,/)
   30 CONTINUE
      TECOST = - YMAX
      WRITE(6,40) TECOST
   40 FORMAT(///,7X,'TOTAL EXPECTED COST = $',F14.2)
      WRITE(6,50)
   50 FORMAT(1H1)
      RETURN
      END
```

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THE ECONOMIC DESIGN OF MULTIVARIATE
ACCEPTANCE SAMPLING PLANS

by

Stephen Clay Chapman

(ABSTRACT)

A total expected cost model for multivariate acceptance sampling is developed. The components of cost included in the model are presented in two phases: the cost of making the quality control decision (sampling inspection) and the cost of implementing the quality control decision (accept the inspection lot, scrap the lot, or screen the lot). Several variables are to be controlled within their given specification limits, where the sample mean for each of the variables in the criterion by which lot acceptance for that variable is determined. The decision variables are the sample size and the lower and upper control limits for each of the characteristics subjected to the control. The pattern search is used to determine the values of the decision variables which minimize the total expected cost of quality control per inspection lot submitted for control.

The lot mean, sample mean, and individual unit measurements for each of the quality characteristics are considered to be random variables. Sensitivity analysis

is performed to determine the robustness of the model to changes in the form of the distributions on the lot means given that the desired mean and variance of these distributions has been accurately estimated.