AN EMPIRICAL COMPARISON OF AUTOREGRESSIVE AND RATIONAL MODELS OF PRICE EXPECTATIONS

by

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CHAPTER I

INTRODUCTION

Few economists would argue with the notion that expectations play a crucial role in determining observed economic behavior. In general terms, one may define expectations as those ex ante beliefs held by individuals about the future development of some economic variable. A dynamic economic model that does not treat the formation and effect of expectations in some manner is clearly incomplete. Current research in the field of macroeconomics and the concomitant analysis of government policy indicates an increasing awareness by economists of expectations' importance. One area of macroeconomic analysis has explicitly made expectations the cornerstone of its theoretical foundation. Indeed, the growing list of published works, both theoretical and empirical, dealing with the measurement of expectations and their influence serves to underscore not only the significance of the subject but also the diversity of opinion.

The foremost obstacle facing the empirical researcher is the fact that expectations are not directly observable.[1] To overcome this difficulty one may use current data as a proxy for the expected future value. This approach fails because, as Jacob Mincer has noted, "The use

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[1]An exception to this is the use of survey data on expectations of consumers. For an excellent introduction to the methods of collection and results using such data, see George Katona, "Theory of Expectations," in Human Behavior in Economic Affairs: Essays in Honor of George Katona, ed. by Strumpel et al. (San Francisco: Jossey-Bass, 1972).

of current rather than anticipated values is equivalent to a hypothesis that expectations are largely based on current magnitudes and do not differ from them in any systematic way."[2] The issue, therefore, is to provide an empirically refutable hypothesis of the mechanism by which expectations are believed to be formed.

The purpose of this introductory chapter is to discuss the informational basis of two hypotheses of the expectations mechanism that have achieved wide popularity. One theory, commonly referred to as autoregressive expectations, has been used extensively in empirical research since it was first introduced by Irving Fisher. The distinguishing characteristic of this general class of model is that expectations are assumed to be formed through an autogressive scheme whereby only the past values of the forecasted variable are used. The other approach, known as rational expectations, asserts that "expectations about a variable are . . . rational if they depend, in the proper way, on the same things that economic theory says actually determine that variable."[3] Examination of these alternative approaches is relegated to Chapters II and III.

Prior to examining these theories, it will be useful to consider the possible information sets which may provide the basis for price or

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[2]Jacob Mincer, "Models of Adaptive Forecasting," in Economic Forecasts and Expectations, ed. by Jacob Mincer (New York: National Bureau of Economic Research, 1969); cited in E. L. Feige and D. K. Pearce, "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Measures of Monetary and Fiscal Policy?" Journal of Political Economy (June, 1976), 499-522.

[3]Thomas Sargent and Neil Wallace, "Rational Expectations and the Dynamics of Hyperinflation," International Economic Review, 14 (June, 1973), 328.

inflation expectations formation. Following Hicks,[4] I shall classify information by three types: Type I information is considered non-economic and includes such things as politics, individuals' physical and psychological well-being, the weather, etc. Although recent economic analyses of the political process and its influence on observed economic activity and the increasing sophistication of consumer survey techniques have begun to provide valuable insights into the workings of such elements, their effect on expectations is not well known. As such, the components of this information set are generally regarded as stochastic and yielding no systematically predictable data.

The second information set, referred to as Type II, that individuals may exploit contains only the past price information. Assuming that only Type II data are available, individuals would form their expectations of future prices based solely on past values of prices. As mentioned above, some extrapolative scheme may be utilized in which past price experience is used to forecast the future path of prices. Type III information is comprised of the structural economic data produced by the economy's dynamic behavior. Individuals forming expectations based on this information are, in the sense of Muth, making "predictions of the relevant economic theory."[5] These rational expectations are derived from the individual's perceived theoretical structure of the economy and

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[4]John Hicks, Value and Capital (2nd ed.; Oxford: Clarendon Press, 1974), pp. 204-205.

[5]John Muth, "Rational Expectations and the Theory of Price Movements," Econometrica, 24 (July, 1961), 316. See, also, Sargent and Wallace, "Rational Expectations."

the observed interrelationships between prices and other economic variables in the model. The way in which one theoretically models the economy will, therefore, determine the information used in forming expectations.

The preceding discussion suggests that only Types II and III information will be systematically used by individuals to calculate predictions of future prices.[6] But, given Type II information, why would individuals engage in employing additional resources to gather the information found under the Type III heading? Assuming that the individuals act in a rational, cost-minimizing manner, it is fairly straightforward to demonstrate why information other than Type II may be used.

Let the cost of misestimating future prices be specified by the following simple quadratic loss function:[7]

$$C = k \cdot \{P_t - E_{t-1}(P_t)(1)\}^2 \quad (1)$$

where C is the non-negative cost of an incorrect price prediction, k is a constant, P_t represents the actual price level in period t, and $E_{t-1}(P_t)$ is the price level forecast made in period t-1 for period t. When $E_{t-1}(P_t) = P_t$, price expectations are correct and the cost of misestimation becomes zero. The individual's optimal behavior is, obviously, to reduce the squared forecast error on the right-hand side of equation (1).

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[6]This is not to say that Type I information has no influence on future prices or expectations. Rather, the position taken here is that events considered under this heading are unpredictable and are viewed as stochastic shocks.

[7]This derivation is found in Feige and Pearce, "Innovations," 502.

Letting the squared forecast error be a function of the different information sets available to the individual, i.e.,

$$\{P_t - E_{t-1}(P_t)(1)\} = f(I_1, I_2, I_3, \ldots, I_n)^2 \tag{2}$$

$$f'_{I_i} < 0, \; f''_{I_i} > 0$$

and further assuming that each type of information $I_i (i=1,2,3,\ldots,n)$ can only be obtained at some non-negative cost,[8] the total cost of an incorrect price forecast may be shown by

$$TC = k \cdot f(I_1, I_2, I_3, \ldots, I_n) + \sum_{i=1}^{n} c_i I_i \; . \tag{3}$$

Setting the derivative of equation (3) with respect to the i^{th} type of information equal to zero yields the cost minimizing solution,

$$c_i = -k \cdot f'_i \; ; \qquad (i=1,2,3,\ldots,n) \; . \tag{4}$$

Equation (4) suggests that the individual's cost minimizing action, i.e., the reduction of costs imposed by incorrect price forecasts, is to acquire information of various types until the per unit cost of information equals the reduction in the cost of a forecast error. Thus, depending on the costs of acquiring different information sets relative to the costs of a wrong price prediction, it is possible that expectations will be formed using (1) only Type II information, (2) only Type III information, or (3) some combination of both sets.

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[8] The literature concerning the economics of information is adamant on this point. For a good introduction, see Armen Alchian, "Information Costs, Pricing, and Resource Unemployment," in Microeconomic Foundations of Employment and Inflation Theory, ed. by Phelps et al. (New York: W. W. Norton and Company, 1970).

The next two chapters are devoted to examining two widely used methods of empirically determining the appropriate technique by which expectations can be accurately measured. As stated earlier, these approaches can be classified as a class of autoregressive models and those in which expectations are assumed to be formed rationally by exploiting the structural information obtained from the relevant theory of the economy; it is to these models that we now turn.

CHAPTER II

AUTOREGRESSIVE MODELING OF EXPECTATIONS

The most popular method of empirically measuring unobservable price or inflationary expectations has been to represent expectations by a distributed lag model of some form. Analogous to weak-form tests,[1] this procedure uses only past price data and implicitly assumes that past prices reflect all the necessary information in the system. If this assumption is true, then the best ex ante prediction (in the sense of minimizing the forecast error) of future prices is one that fully exploits the informational content of the historical price series.

It is generally agreed that Irving Fisher was the first to introduce this technique of modeling price expectations.[2] After much detailed analysis of the relationship between the value of money, i.e., its purchasing power, and interest rates, Fisher postulated that in a world of perfect foresight and instantaneous adjustment, the nominal rate of interest would be determined by the real rate of interest plus the expected appreciation or depreciation of money. In other words, Fisher maintained that, assuming the real rate constant, the nominal rate of interest would be predominantly influenced by anticipated movements in the general price level.

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[1]For an excellent discussion of the various efficient markets tests, see Eugene Fama, "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance, 25 (May, 1970), 383-417.

[2]Irving Fisher, The Theory of Interest (New York: Kelly and Millman, Inc., 1930).

Fisher empirically tested this hypothesized relationship between the nominal interest rate and expected price movements through simple correlation analysis using data for the United States and Great Britain. Though Fisher clearly recognized that such tests did not represent a complete theory of interest rate determination, he argued that the effects of price expectations were pervasive and so important in the process governing the actions of financial markets to warrant this special consideration.[3] His empirical approximation of the expected rate of change of prices was formed by positing a distributed lag of past price changes. The technique employed by Fisher constrained the weights of the lagged distribution to (1) decline in a linear fashion, eventually reaching zero at the far tail of the distribution, and (2) sum to unity.[4]

Seeking to maximize the correlation between nominal interest rates and his measure of price expectations, Fisher tested several lengths of lag periods and found that the correlation increased markedly as the lag was extended back through time:

> But when we make the much more reasonable supposition that price changes do not exhaust their effect in a single year but manifest their influence with diminishing intensity, over long periods which vary in length with the conditions, we find a very significant relationship.[5]

For both Great Britain and the United States, Fisher found the appropriate lag scheme to be quite long. He writes that

> For Great Britain in 1898-1924, the highest value of r(+0.980) is reached when effects of price changes are assumed to be

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[3]Ibid., esp. chs. II and XIX. [4]Ibid., p. 420.

[5]Ibid., p. 423.

spread over 28 years or for a weighted average of 9.3 years, while for the United States the highest r(+0.857) is for a distribution of the influence due to price changes over 20 years or a weighted average of 7.3 years.[6]

Though surprising at first, Fisher regarded these results to be quite indicative of the way in which expectations of future prices are formed. Indeed, in the following passage Fisher suggests that past prices may well reflect certain non-price information in the economy:

> A little thought should convince the reader that the effects of bumper wheat crops, revolutionary discoveries and inventions, Japanese earthquakes, Mississippi floods, and similar events project their influence upon price and interest rates over many future years even after the original event has been forgotten.[7]

The preceding discussion reveals why Irving Fisher is generally regarded as the first writer to explicitly provide an empirically tested model of price expectations formation. Contrary to this popular belief is the position taken by John Rutledge.[8] Rutledge suggests that Fisher's aim was not to model price or inflationary expectations and analyze their influence on the behavior of nominal interest rates but to investigate "the total relation between inflation and interest, which . . . is based on many effects additional to those of expected inflation."[9] To close the discussion of Fisher's contribution to the field of expectations modeling, we shall briefly address this claim by Rutledge.

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[6]Ibid. [7]Ibid., p. 428 (italics added).

[8]John Rutledge, A Monetarist Model of Inflationary Expectations (Lexington, MA: Lexington Books, 1974). See, also, his "Irving Fisher and Autoregressive Expectations," American Economic Review, 67 (February, 1977), 200-205.

[9]Rutledge, Monetarist Model, p. 21.

Early in The Theory of Interest it is readily apparent that Fisher regarded the effect of expected price movements to be paramount in explaining observed variations in the nominal market interest rates. He reveals in Chapter II, "Money Interest and Real Interest," that the crucial link between price movements and resultant nominal interest rate fluctuations to be the purchasing power of an individual's money stock. Effects of an appreciation or depreciation in the purchasing power of money, caused by movements in the general price level, "on the money rate of interest will be different according to whether or not that change is foreseen."[10] The importance ascribed to foresight is a recurring theme throughout The Theory of Interest. Indeed, the sole purpose of Fisher's empirical work in Chapter XIX, "The Relation of Interest to Money and Prices," is to specifically examine the impact of price expectations on interest rates. This is clearly illustrated by his own words:

> How is it possible for a borrower or lender to foresee variations in the general price level with the resultant increase or decrease in the buying power of money? Most people . . . think instinctively of money as constant and incapable of appreciation or depreciation. Yet it may be true that they do take account . . . of a change in the buying power of money, under the guise of a change in the level of prices in general. . . . If inflation is going on, they will scent rising prices ahead and so rising money profits . . . and their willingness to borrow will itself tend to raise interest.[11]

And lest there be any doubt that Fisher was not focusing his attention on the influence of expectations, he continues,

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[10]Fisher, Theory of Interest, p. 37 (italics in original).

[11]Ibid., p. 400 (italics added).

> And today especially, foresight is clearer and more prevalent than ever before. The business man makes a definite effort to look ahead not only to his own particular business but as to general business conditions, including the trend of prices.
>
> Evidence that an expected change in the price level does have an effect on the money rate of interest may be obtained from several sources.[12]

In the opinion of this author, the passages quoted above prove to be more than adequate in repudiating Rutledge's position. That Fisher sought to explicitly measure price expectations with his distributed lag model can hardly, in light of the preceding evidence, be questioned.

A major innovation in the measurement of expectations is found in Phillip Cagan's classic piece examining the economic effects of hyperinflation in seven European countries.[13] Based on Milton Friedman's "Restatement" of the quantity theory, Cagan hypothesized that observed fluctuations in real money balances could be in large part explained by the public's expectation of future inflation rates. Assuming that desired real cash balances always equal actual real cash balances, Cagan tested a simple money demand model of the form

$$\log_e(M/P) = -aE - g + e_t , \qquad (5)$$

where M represents an end-of-month index of money in circulation, P is an end-of-month index of the price level, a and g are constants, and e_t is an error term. The variable E denotes the expected rate of inflation which is assumed to depend on actual inflation rates from the past. The way in which past inflation rates influence current expectations differs

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[12] Ibid. (italics added).

[13] Phillip Cagan, "The Monetary Dynamics of Hyperinflation," in Studies in the Quantity Theory of Money, ed. by Milton Friedman (Chicago: University of Chicago Press, 1956).

from Fisher's analysis in that "the expected rate of change in prices is revised per period of time in proportion to the difference between the actual rate of change in prices and the rate of change that was expected."[14]

This latter assumption is formalized by the expression:

$$(dE/dt)_t = b(C_t - E_t) \; ; \quad b > 0 \tag{6}$$

where C_t is the actual rate of inflation in time t, and E_t is defined as above. The term b is assumed by Cagan to be constant and represents the "coefficient of expectation," i.e., b measures the adjustment speed of inflationary expectations to actual rates where smaller valued bs indicate relatively slower adjustment.[15] Solving equation (6) for E_t, Cagan posited that expected inflation is approximated by a distributed lag of past rates with the lag weights constrained to fall off exponentially (compare to Fisher). This suggests that the expected rate of inflation may be written as

$$E_t = (1 - b) \sum_{i=0}^{w} b^i C_{t-1} \; ; \quad 0 \leq b \leq 1 \; . \tag{7}$$

Letting L, the lag operator, be defined such that $L^i X_t = X_{t-i}$, equation (7) may be written in the form

$$E_t = (1 - b)\{ \sum_{i=0}^{w} (bL)^i \} \; C_t \; . \tag{8}$$

or, simplifying,

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[14]Ibid., p. 37. [15]Ibid.

$$E_t = \{(1 - b)/(1 - bL)\} C_t \ . \tag{8}$$

Equation (8) indicates that the weight placed on past inflation values is a function of the coefficient of expectations term b.[16] The value of b determines the "steepness" of the lag distribution, i.e., the weights attached to past values of inflation. Higher values of b imply a more rapid adjustment of expected to actual inflation rates and a relatively greater discounting of inflation rates observed beyond the recent past. In other words, the greater the b values, the more weight is attached to recent inflationary experiences relative to those of the past.[17]

An obvious shortcoming of Cagan's method is that it requires a priori construction of the geometrically declining weights. As such, one must first hypothesize some initial value for the expectations coefficient b, calculate the appropriate weighting pattern, and then test these lag patterns until the best fitting lag scheme is empirically determined. Though a necessarily tedious process, Cagan's success in measuring the impact of expected inflation on the demand for real cash balances during hyperinflation prompted Milton Friedman to state that such expectations models "can be carried over to other variables as well

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[16]Although equation (8) in the text is not an exact duplicate of Cagan's, it is equivalent. For similar versions as that presented in the text, see Sargent and Wallace, "Rational Expectations"; and Zvi Griliches, "Distributed Lags: A Survey," Econometrica, 35, no 1 (1967), 18-21.

[17]Both Fisher and Cagan hypothesized that the weights assigned to inflation rates of the distant past would have little or no influence on current expectations. However, whereas Fisher assumed the weights to eventually reach zero, Cagan's formulation requires that the weights approach zero asymptotically.

and so is likely to be important in fields other than money."[18] Indeed, this approach was used by Friedman to measure expected income and its relative importance in explaining consumption.[19]

A more general approach to estimating the distributed lag weights is found in Shirley Almon's article, "The Distributed Lag Between Capital Appropriations and Expenditures."[20] The primary feature of this approach is that instead of estimating n lag coefficients b_0, b_1, b_2, . . ., b_n, these terms are approximated by fitting a polynomial lag distribution to the past values of the variable forecasted. Once the degree of polynomial and the number of lags have been specified, the weights obtained by ordinary least squares are thus constrained to satisfy the degree of polynomial.[21] This procedure is quite common in empirical research dealing with the formation of expectations.

The works of Box and Jenkins have provided yet another procedure to analyze time-series data in which the predictive information contained in the past price series is efficiently utilized.[22] This approach is

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[18]Milton Friedman, "The Quantity Theory of Money: A Restatement," in Studies in the Quantity Theory of Money, ed. by M. Friedman (Chicago: University of Chicago Press, 1956), p. 19.

[19]Milton Friedman, A Theory of the Consumption Function (New York: National Bureau of Economic Research, 1957).

[20]Shirley Almon, "The Distributed Lag Between Capital Appropriations and Expenditures," Econometrica, 33 (January, 1965), 178-196.

[21]In other words, the weights are constrained to lie on the chosen polynomial curve.

[22]G. E. P. Box and G. M. Jenkins, Time Series Analysis, Forecasting and Control (San Francisco: Holden-Day, 1970).

premised on the basic assumption that the observed price series is actually a realization of some underlying stochastic process. This means that the value of a given series chosen at discrete time intervals, say P_1, P_2, P_3, . . ., P_T, is in fact derived from a set of jointly distributed random variables. In other words, it is assumed that there exists a probability distribution $p(P_1,P_2,P_3,\ldots,P_T)$ which assigns to every possible combination of values for $(P_1,P_2,P_3,\ldots,P_T)$ some probability p. Thus, by knowing the density function $p_1,p_2,p_3,\ldots p_T(P_1,P_2,P_3,\ldots,P_T)$ statements of the probable outcomes for P_{T+1} can be made.[23]

The stochastic process generating the observed price series may be represented by the weighted average of past price observations and the current period disturbance, or

$$P_t = \phi_1 P_{t-1} + \phi P_{t-2} + \phi P_{t-3} + \ldots + \phi P_{t-p} + e_t , \qquad (9)$$

where $\phi_i(i=1,2,3,\ldots,p)$ is the weight assigned to past price observations, P_{t-i} represents past prices, and e_t is the current period disturbance term. The process may also be described by the weighted sum of past random shocks. Formally, this is illustrated by

$$P_t = e_t + \Theta_1 e_{t-1} + \Theta_2 e_{t-2} + \Theta_3 e_{t-3} + \ldots + \Theta_q e_{t-q} , \qquad (10)$$

where $e_{t-i}(i=1,2,3,\ldots,q)$ denotes the current and past random shocks, and $\Theta_i(i=1,2,3,\ldots,q)$ are weights assigned to the past disturbance terms.

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[23]R. S. Pindyck and D. L. Rubinfeld, Econometric Models and Economic Forecasts (New York: McGraw-Hill, 1976), pp. 454-455.

A prior condition for the estimation of either equations (9) or (10) is that the series exhibit stationarity--that any point value of the series be invariant with respect to time.[24] This requires that the sum of the respective weights converge, i.e.,

$$\sum_{i=1}^{p} \phi_i < \infty \tag{11}$$

and

$$\sum_{i=0}^{q} \Theta_i < \infty \ .^{25} \tag{12}$$

To circumvent the estimation of equations (8) and (9) separately, Box and Jenkins utilize a general class of models known as autoregressive-integrated-moving-average (ARIMA) models. The ARIMA model exploits the informational content of both equations (8) and (9) and may be written in the general form of

$$\phi(B)\Delta^d P_t = \Theta(B)e_t \ , \tag{13}$$

with O(B) representing the autoregressive component (equation (8)), $\Theta(B)$ the moving average component (equation (9)), Δ^d is the degree of differencing needed to achieve stationarity of the raw data series, (B) is the lag operator where $B^i X_t = X_{t-i}$, and P_t and e_t are defined as before. The appropriate order of the ARIMA model (the "memory" of equations (9) and (10)) is determined by means of the sample autocorrelation function as the sample autocorrelations are compared to their

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[24]C. R. Nelson, Applied Time Series Analysis (San Francisco: Holden-Day, 1973), pp. 19-23.

[25]Pindyck and Rubinfeld, Econometric Models.

standard errors.[26] A further check for the overall appropriateness of the specified model is provided by the Q-statistic,[27] which is approximated by a χ^2 distribution with (T - p - q) degrees of freedom. Once the appropriate ARIMA specification for the price series is obtained, it is then possible to make ex ante forecasts x periods ahead.

In this chapter, a brief survey of various autoregressive approaches to modeling expectations has been offered. As the discussion illustrates, a feature common to all these approaches is the utilization of only past price data in the measurement of expectations. Numerous studies have employed the different techniques described here in explaining a wide variety of economic behavior. Because a complete listing is beyond the scope of this chapter, the reader is referred to the illustrative works of Gibson, Feldstein and Eckstein, Feldstein and Summers, Roll, Goldfeld, Frenkel, Modigliani and Sutch, Turnovsky, and Lahiri.[28] Interestingly,

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[26]Ibid., pp. 453-473.

[27]The Q-statistic is defined by $Q = T \sum_{j=1}^{k} r_j^2$ where T is the size of the sample period, k is the total lag for the autocorrelation of the sample, and r_j is the estimate of the theoretical autocorrelation coefficient ρ_j of lag j.

[28]W. E. Gibson, "Price Expectations Effects on Interest Rates," Journal of Finance (March, 1970), 19-34 and "Interest Rates and Inflationary Expectations: New Evidence," American Economic Review (December, 1972), 854-865; M. Feldstein and O. Eckstein, "The Fundamental Determinants of the Interest Rate," Review of Economics and Statistics, 52 (November, 1970), 363-375; M. Feldstein and M. Summers, "Inflation, Tax Rules, and the Long-Term Interest Rate," Brookings Papers on Economic Activity (no. 1, 1978), 61-110; R. Roll, "Interest Rates on Monetary Assets and Commodity Price Index Changes," Journal of Finance, 27 (May, 1972), 251-278; S. Goldfeld, "The Case of Missing Money," Brookings Papers on Economic Activity (no. 3, 1976), 683-740; J. Frenkel, "The Forward Exchange Rate, Expectations, and the Demand for Money: The German Hyperinflation," American Economic Review, 67 (September, 1977), 653-670; F. Modigliani and

each study listed here makes no effort to examine alternative expectations formation specifications. Such an alternative approach to the autoregressive method, known as the theory of rational expectations, is the topic of the next chapter.

R. Sutch, "Debt Management and the Term Structure of Interest Rates: An Empirical Analysis of Recent Experience," Journal of Political Economy, 75 (August, 1976), 569-589; S. Turnovsky, "Empirical Evidence on the Formation of Price Expectations," Journal of the American Statistical Association, 65 (December, 1970), 1441-1454; K. Lahiri, "Inflationary Expectations: Their Formation and Interest Rate Effects," American Economic Review, 66 (March, 1976), 124-131.

CHAPTER III

RATIONAL EXPECTATIONS

The purpose of this chapter is to examine the characteristics pertaining to the theory of rational expectations. An essential feature of this modeling approach, as opposed to the ad hoc nature of autoregressive models discussed in the preceding chapter, is that expectations are viewed to be essentially predictions of the model of the economy conditional on information available at the time when expectations are formed. Originally described by John Muth,[1] individuals' subjective probability distribution of possible outcomes are assumed to be identical to the objective probability distributions that actually occur. In other words, individuals act as if they know the underlying processes that will generate the outcome in question. If the true model of the economy implies that this variable or that policy will have a predictable influence on the development or evolution of the variable being forecasted, then individuals forming expectations "rationally" will attempt to fully exploit the informational content of these hypothesized structural relationships. Thus, rational expectations are essentially mathematical expectations of the predicted variable conditional on the information of all the pertinent variables and relationships in the model known at the time of prediction.

Though the concept of rational expectations is by no means new, it was not until the appearance of Muth's pioneering contribution and

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[1]Muth, "Price Movements," 316.

later extensions by Robert E. Lucas, Jr. (discussed below) that the relevance of this approach was recognized. Especially in the area of policy analysis, more specifically monetary policy, recent investigations have suggested that under the assumptions of rational expectations counter-cyclical policies are doomed to fail because the public has already taken full account of the authority's behavior in forming its (the public's) expectations of the future. The monetary authority, therefore, can influence real economic activity only when it behaves in an unpredictable manner. As Robert Barro has demonstrated, such unsystematic behavior may serve only to impart increased instability into the economic system.

To illustrate how such policy implications are derived and at the same time illustrate the workings of the theory of rational expectations, it is useful to examine the application of rational expectations to the natural rate hypothesis. Originally coined by Milton Friendman in his 1967 American Economic Association Presidential address, the natural rate hypothesis asserts that observed trade-offs between inflation and unemployment are only temporary at best and caused by unanticipated movements in the price level.[2] As Friedman was to later state the notion, "only suprises matter."[3] When inflation is fully anticipated (which may occur when the public "catches on" to the actual underlying process generating observed inflation), the trade-off

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[2]Milton Friedman, "The Role of Monetary Policy," <u>American Economic Review</u>, 58 (March, 1968), 11.

[3]Milton Friedman, "Nobel Lecture: Inflation and Unemployment," <u>Journal of Political Economy</u>, 85 (June, 1977), 456.

disappears and employment settles at its "natural" rate. In this situation, the Phillips curve becomes vertical at the natural rate and the exploitable policy trade-off between inflation and unemployment vanishes from the policy maker's choice set.

In his article "Econometric Testing of the Natural Rate Hypothesis," Robert Lucas explicitly models the natural rate hypothesis under the assumption of rational expectations.[4] Lucas begins his analysis by positing an aggregate supply function of the form

$$y_t = a(P_t - P_t^*) , \qquad (14)$$

where y_t is the log of real output in period t, P_t is the log of the price level, and P_t^* represents the log of expected prices for the next (t+1) period. The term a is a parameter with the a prior restriction that it be greater than zero. This restriction is based on the notion that "the initial or first-period response of P_t^* to a change in P_t is less than proportional."[5] Given some unanticipated change in the price level P_t, there will be a change in the level of output until that time when $P_t = P_t^*$ denoting a condition of stable prices. In other words, the preceding discussion suggests that the aggregate supply curve will be upward sloping.

The aggregate demand curve in Lucas's model is given by

$$y_t + P_t = x_t , \qquad (15)$$

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[4]Robert E. Lucas, Jr., "Econometric Testing of the Natural Rate Hypothesis," in The Econometrics of Price Determination, ed. by O. Eckstein (Washington: Board of Governors of the Federal Reserve System, 1972), pp. 50-59.

[5]Ibid., p. 52.

where x_t is defined by Lucas as the log of nominal GNP and interpreted as an exogenous shift parameter accounting for variations in demand, possibly the result of some governmental policy action. Such a policy, as defined by Lucas, is "a rule giving the current value of x_t as a function of the state of the system."[6] A policy rule, therefore, determines the way in which authorities manipulate (or attempt to) current values of the shift parameter x_t. If the authorities follow such a rule, then rational expectations asserts that individuals know its "form" and will use the information contained therein to forecast future output levels. To illustrate this, let

$$x_t = r_1 x_{t-1} + r_2 x_{t-2} + e_t \tag{16}$$

be the policy rule chosen by the authorities where e_t is a random error term with mean zero and constant variance.

To close this simple model, it is necessary to specify the process by which P^*_t is determined. Under the rational expectations hypothesis, the expectation formed in the current period of the price next period may be written as

$$P^*_t = E_t(P_{t+1}) + n_t \ , \tag{17}$$

where E_t is the mathematical expectations of P_{t+1} formed in time t conditional on information available up to the end of time t, and n_t is another error term with the same properties of e_t but assumed to be independently distributed from e_t. Equation (17) suggests that rational

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[6]Ibid., p. 56.

expectations of P_{t+1} deviate from the actual value only by some white noise process.[7]

Given the system of equations specified by (14) through (17), it is possible to solve for the reduced form output equation. Substituting equation (14) into (15) and solving for P_t yields

$$P_t = (1/1+a)x_t + (a/1+a)P^*_t \,. \tag{18}$$

To find the expected value of P_{t+1} formed in t, take the E_t operator through equation (18) to give

$$E_t(P_{t+1}) = (1/1+a)E_t(x_{t+1}) + (a/1+a)P^*_{t+1} \,, \tag{19}$$

since $E_t(P^*_{t+1}) = P^*_{t+1}$. Given the appearance of P^*_t in equation (18) and P^*_{t+1} in equation (19), it is not possible to solve these equations simultaneously. Rather, it is necessary to use one method of dealing with difference equations known as the underdetermined coefficients technique.[8]

This approach requires that a trial solution for the reduced form equation of P_t be hypothesized a priori. Following Lucas, the trial

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[7]Benjamin Friedman argues that conditional expectations such as equation (18) require such large amounts of information as to render them useless. He notes that missing from rational expectations models "is a clear outline of the way in which economic agents derive the knowledge which they use to formulate expectations. . . ." For a fuller discussion on this and related topics, see his "Optimal Expectations and the Extreme Information Assumptions of 'Rational Expectations' Macromodels" (Harvard University, mimeographed, May, 1978).

[8]See, Alpha Chiang, Fundamental Methods of Mathematical Economics (2nd ed.; New York: McGraw-Hill, 1974), pp. 541-543. The procedure in the text is found in, Warren Weber, "Notes on Rational Expectations" (Virginia Polytechnic Institute and State University, mimeo, undated).

solution of P_t is posited to be

$$P_t = \Pi_1 x_t + \Pi_2 x_{t-1} + \Pi_3 n_t \ . \tag{20}$$

Assuming equation (17) to be the "true" representation of the expectations formation process, equation (20) may be written in the form

$$P_t^* = E_t(P_{t+1}) + n_t = \Pi_1 E_t(x_{t-1}) + \Pi_2 x_t + n_t \tag{21}$$

since $E_t(x_t) = x_t$ and, under the assumption that n_t is a white noise process, $E_t(n_{t+1}) = 0$.

Determination of the term $E_t(x_{t+1})$ is accomplished by following the same procedure used to generate equation (19) only now applied to equation (16). Remembering that e_t is assumed to be white noise, taking the expected value operator E_t through equation (16) yields

$$E_t(x_{t+1}) = r_1 x_t + r_2 x_{t-1} \ . \tag{22}$$

Equation (22) obtains because $E_t(x_t) = x_t$ and $E_t(x_{t-1}) = x_{t-1}$ since both x_t and x_{t-1} are, by assumption, known at the end of period t.

The solution for P_t^* is now easily found by substituting equation (22) into equation (21) to get

$$P_t^* = \Pi_1(r_1 x_t + r_2 x_{t-1}) + \Pi_2 x_t + n_t \ . \tag{23}$$

Since the solutions for P_t (equation (20)) and P_t^* (equation (23)) must satisfy equation (18), it is necessary to substitute equations (20) and (23) into equation (18) in order to evaluate the restrictions on the parameters Π_1, Π_2, and Π_3 found in equation (20). These substitutions give

$$\Pi_1 x_t + \Pi_2 x_{t-1} + \Pi_3 n_t = (a/1+a)\{(r_1\Pi_1 + \Pi_2)x_t + \Pi_1 r_2 x_{t-1} + n_t\} + (1/1+a)x_t \quad (24)$$

which, upon evaulating the coefficients Π_1, Π_2, and Π_3, reveals that

$$\Pi_1 = a(r_1\Pi_1 + \Pi_2)/(1+a) + (1/1+a)$$

$$\Pi_2 = a(\Pi_1 r_2)/(1+a)$$

and

$$\Pi_3 = a/(1+a) \ . \quad (25)$$

Simultaneously solving equations(s) (25) for Π_1 and Π_2 yields the linear restrictions,[9]

$$\Pi_1 = (1+a)/(r_2 a^2) + (1+a)\ \{1+a(1-r)\}$$

and

$$\Pi_2 = (r_2 a)/(1+a)\{1 + a(1-r) + r_2 a^2\} \ . \quad (26)$$

These restrictions are substituted into equation (20) and, using the information provided by equation (15), the reduced form output solution is found to be

$$y_t = a(1+a)(1-r_1)/(1+a)\{(1+a(1-r_1) + (r_2 a^2)\}x_t - r_2 a/(1+a)\{1+a(1-r_1) + (r_2 a^2)\}x_{t-1} - (a/1-a)n_t \ . \quad (27)$$

Wrong

The output equation (27) suggests several important implications for testing the natural rate hypothesis under the assumption of rational

[9]The solution technique uses Cramer's Rule.

expectations. Most "non-natural" tests of equations like (27) have taken the long-run measure of a once-and-for-all demand shift to be the sum of the coefficients for the variables x_t and x_{t-1}.[10] As Lucas notes, this approach is equivalent to "the linear restriction that the sum of the coefficients be zero."[11] Empirical testing of equation (27), however, implies that no such restrictions exist under the assumption of rational expectations. The "correct" test is clearly one that first estimates the policy rule given by equation (16) to obtain the true values of the parameters r_1 and r_2. This may be done by estimating equation (16) over a period in which the policy rule is temporally stable, i.e., the underlying structure of (16) remains unchanged over the time period being estimated.[12]

Once these estimates of r_1 and r_2 are obtained, they serve to place testable restrictions on the coefficients of x_t and x_{t-1} found in equation (27). In other words, equation (27) is tested with and without thse restrictions whereupon standard statistical tests for loss of explanatory power due to restrictions (a F-test on the sum of squared residuals) are utilized. If imposition of the restrictions implied by the rational expectations assumptions leads to a significant loss of explanatory power, then rejection of the rational expectations-

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[10]Thomas Sargent, "Testing for Neutrality and Rationality," in A Prescription for Monetary Policy: Proceedings from a Seminar Series (Minneapolis: Federal Reserve Bank, December, 1976), p. 67.

[11]Lucas, "Econometric Testing," 57.

[12]Tests of equation (27) have been attempted in various forms by Lucas. See his "Some International Evidence on Output-Inflation Trade-Offs," American Economic Review (June, 1973), 326-334.

natural rate hypothesis is implied. Clearly, such estimation procedures provide a more robust test of the natural rate hypothesis relative to those assuming some a priori restrictions based on an ad hoc estimation of the reduced form output equation.

The second major implication derived from equation (27), and one that is implicit in the preceding discussion, deals with the effect of government policy on real output.[13] If demand is perfectly stable, such that the policy parameters in equation (16) are zero ($r_1 = r_2 = 0$, $\sigma^2 = 0$), then equation (27) reduces to[14]

$$y_t = -(a/1-a)n_t \ , \tag{28}$$

which implies that real output will vary only due to the forecast error n_t. In the same way, if policy follows a martingale without trend, implying that $r_1 = 1$, $r_2 = 0$, and σ^2 arbitrary, then again real output would vary unsystematically as in equation (28).[15] The implication of equation (27), therefore, is that although expansionary policy in the current period may lead to an increase in real output, "if one views a policy as a rule describing a response to given values of state variables ('indicators'), this recommendation is not a policy."[16] Thus, only unsystematic behavior on the part of the authority will influence

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[13]Lucas, "Econometric Testing," 57.

[14]This equation is derived from the reduced form price and output equations. See, Weber, "Notes on Rational Expectations."

[15]Note that under these circumstances, prices follow a random walk. See, ibid.

[16]Lucas, "Econometric Testing," 56.

real output--all other behavior by the monetary policy maker is known and incorporated into the public's expectations formation mechanism.

The effects of policy, particularly monetary policy, has become a focal point for much of the rational expectations literature. Robert Barro's theoretical work,[17] basically an extension of Lucas's earlier contributions, has dealt with the role of monetary policy under different informational advantages of the monetary authority vis-à-vis the public. Assuming that prices and quantities are determined by competitive market-clearing relationships between geographically dispersed markets, and that some types of information--e.g., prices in other markets--are received with a lag, Barro investigates the effects of policy when (1) the monetary authority lacks superior information, and (2) the authority possesses superior information about the economy. The essence of Barro's findings is that if and only if the monetary authority has superior information about the economy, and, more importantly, its own money rule, can it control the money supply in such a way as to effect counter-cyclical policies designed to guide the development of real output. When the authority does not possess such informational advantages, the optimal behavior in the sense of reducing the variance of real output is to reduce the variance of the money supply's rate of growth, e.g., to follow a constant gorwth money rule similar to that suggested by Milton Friedman.

Barro has recently attempted to test these theoretical implications by examining the effects of the predicted and unpredicted components of

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[17] Robert Barro, "Rational Expectations and the Role of Monetary Policy," Journal of Monetary Economics, 2, no. 1 (1978), 1-33.

the money supply growth rate on output, unemployment, and the price level for the United States.[18] Using annual data from 1941 to 1975, Barro estimates what amounts to a money rule similar to equation (16),[19] and takes the residuals (actual minus predicted) from this equation as a representation of the unanticipated component of money growth. Implicitly assuming that the monetary authority does not possess superior information (i.e., the public knows the money rule that is being followed), Barro finds that unanticipated money growth relative to actual and predicted money growth to have substantial explanatory power for variations in real output, unemployment, and prices. With regard to unemployment, Barro states that "statistical tests confirmed the underlying hypothesis that actual money growth was irrelevant for unemployment given values of unanticipated money growth."[20]

The works of Thomas Sargent and Neil Wallace have theoretically demonstrated the ineffectiveness of counter-cyclical monetary policy when expectations are formed rationally. In their paper, "Rational Expectations and the Theory of Economic Policy," Sargent and Wallace show

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[18]Robert Barro, "Unanticipated Money Growth and Unemployment in the United States," American Economic Review, 67 (March, 1977), 101-115; "Unanticipated Money, Output, and the Price Level in the United States," Journal of Political Economy, 86 (August, 1978), 549-580.

[19]See, Barro, "Unanticipated Money Growth," 103. The actual equation Barro estimates is $DM_t = a_0 + b_0DM_{t-1} + b_1DM_{t-2} + b_2FEDV + b_3UN_{t-1} + e_t$, where DM_{t-1} represents the growth rate of the money stock (M1), FEDV is a measure of federal government expenditures relative to "normal" levels, and UN is a cyclical variable, defined by the unemployment rate, and derived as $UN \equiv \log(U/1-U)$.

[20]Barro, "Unanticipated Money Growth," 114.

that estimation of standard reduced form macroeconomic equations without reference to the policy parameters in the monetary rule may seriously jeopardize the resulting conclusion that a constant money growth rule is suboptimal to a money rule with feedback. They note that "the parameters of estimated reduced forms like (1) in part reflect the policy responses in operation during the periods over which they are estimated."[21] Thus, failure to account for changes in the parameters of the money rule, or any of the policy equations in the model for that matter, vitiates the standard implicit assumption underlying most estimations of macromodels that the estimated reduced form parameters are invariant over the sample period with respect to changes in governmental policy actions.

The question of parameter invariance has begun to receive much attention in the rational expectations literature. Directed mainly at existing Keynesian macromodels, it is argued that such specifications ignore the restrictions on the reduced form parameters that the theoretical structure of the model would suggest and in "general assume that all other equations remain unchanged when an equation describing a policy variable is changed."[22] Thus it has been suggested that an important consideration in explaining the recent breakdown of such models is that there might have occurred significant changes in the structural

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[21] Thomas Sargent and Neil Wallace, "Rational Expectations and the Theory of Economic Policy," Journal of Monetary Economics (no. 2, 1976), 173. Equation (1) referred to in the quotation is given by $y_t = a + a_1 y_{t-1} + a_2 m_t + u_t$ where y is some variable which the authority is interested in controlling and m_t is a policy variable.

[22] Robert Lucas and Thomas Sargent, "After Keynesian Economics" (mimeographed, 1978), p. 22.

equations specifying government policy.[23] Indeed, the empirical investigation by Muench et al. reveals a great deal of structural instability in the reduced form coefficients of the FRB-MIT model and the Michigan Quarterly model over the recent past.[24] In testing the St. Louis equation, it was found that it, too, exhibited significant structural change in the estimated coefficients over the sample period 1953.I-1977.IV.[25] The common denominator of these macromodels is a general disregard for changes in policy variables which may alter reduced form parameters in a crucial way. Only when policy regime breaks are explicitly considered in the estimation procedure can the estimates of reduced form equations be regarded as accurately portraying the true interrelationships within the economy.

Empirical tests of parameter invariance across monetary regime changes has thus far been restricted to studies involving tests of the rational expectations-natural rate hypothesis. Using the concpet of observational equivalence to derive two models that appear to be econometrically identical yet have dramatically different policy implications associated with their respective coefficients,[26] Salih

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[23]Sargent "Neutrality and Rationality," 76-83; see, also, ibid.

[24]T. Muench, et al., "Tests for Structural Change and Prediction Intervals for the Reduced Forms of Two Structural Models of the U.S.: The FRB-MIT and Michigan Quarterly Models," Annals of Economic and Social Measurement, 3, no. 3 (1974), 491-519.

[25]R. W. Hafer, "A Note on the Temporal Stability of the St. Louis Equation" (Virginia Polytechnic Institute and State Univ., mimeo, 1978).

[26]Thomas Sargent, "The Observational Equivalence of Natural and Un-natural Rate Theories of Macroeconomics," Journal of Political Economy, 84 (June, 1976), 631-640.

Neftci and Neftci and Sargent have tested the relationship between the money supply and output for the United States over the post-war period.[27] Finding a monetary policy break in 1962.II and 1964, respectively, each study concludes that the relationship between real output and unexpected changes in the money supply were invariant with respect to policy regime changes, thus supporting the rational expectations version of the natural rate hypothesis. The implication of these results, though still tentative, is that one should explicitly account for regime changes if robust tests of reduced form coefficients are to be made. Only when such regime changes are incorporated into the theoretical-empirical analysis of macromodels can one compare the relative efficiency of various monetary policy prescriptions.

In the preceding survey, it was attempted to focus attention on some of the main contributions to and applications of the rational expectations hypothesis. In addition, this was not a critical review of rational expectations but was expository in nature. This is not to suggest that the theory is error free, as the insightful article by Robert Shiller suggests.[28] In the next chapter, the theoretical derivation of the models which form the basis for future empirical estimattion are presented.

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[27]See, for example, S. Neftci, "Unanticipated Money Growth and the Stability of Economic Relationships" (George Washington University, mimeo, 1978); S. Neftci and T. Sargent, "A Little Bit of Evidence on the Natural Rate Hypothesis," Journal of Monetary Economics, no. 4 (1978), 315-319.

[28]Robert Shiller, "Rational Expectations and the Dynamic Structure of Macroeconomic Models: A Critical Review," Journal of Monetary Economics, 4 (January, 1978), 1-44.

CHAPTER IV

DERIVATION OF EXPECTATIONS MODELS

Chapters II and III postulated two alternative methods of measuring expectations. The objective of this chapter is to present two empirically testable models which will form the basis for the empirical work found in subsequent chapters.

The discussion begins with a specification from the general class of autoregressive models of expectations. More general than one which a priori assigns some lag pattern to past observations, the approach suggested here allows the weighting scheme and length of the lag pattern to be determined empirically. Such an autoregressive model is given by the specification

$$\dot{P}^e_t = \alpha + \sum_{i=1}^{n} \beta_i \dot{P}_{t-1} + e_t \qquad (29)$$

where $\dot{P}^e_t$ is the expected (predicted) rate of inflation for time t, $\dot{P}_{t-1}$ (i=1,2,3,...,n) represents actual past inflation rates, α is a constant, β_i(i=1,2,3,...,n) is the empirically determined weight assigned to the n lagged inflation rates, and e_t is a random error term.

Compared to other autoregressive models of expectations, the approach suggested by equation (29) is clearly the least restrictive. Whereas the Fisher and Cagan methods restricted the β_i lag weights to decline in some predetermined fashion, this is not the case for equation (29). The Almon procedure requires the β_is of equation (29) to fit some a priori polynomial function with the further possibility for estimation with or without various combinations of endpoint restrictions.[1] While the estimation and comparison of estimates obtained from these various autoregressive models would be interesting, such is not the stated purpose of the present snalysis. Rather, the predictive abilities of this naive autoregressive specification will be compared to those of the rational expectations model (discussed below). If such a simple model of expectations provides empirically superior predictions relative to the informationally intensive rational expectations model, then a definite statement may be made concerning the current debate among macroeconomists concerning the mechanism by which expectations are formed.

The expectations mechanism described by equation (29) is representative of an ad hoc approach because it is not founded in any theoretical structure. The general acceptance of such models is based firmly in their empirical successes. It should be noted, however, that disappointingly few comparative empirical investigations of models like the autoregressive one presented above and other theoretically based specifications have appeared in the expectations literature.[2]

In recent years there has been a growing dissatisfaction with the ad hoc expectations models such as equation (29). Following Muth, one group of theorists argues against the autoregressive approach primarily

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[1]Almon, "Distributed Lag."

[2]Possible exceptions include, Turnovsky, "Formation of Price Expectations"; E. L. Feige and D. K. Pearce, "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Measures of Monetary and Fiscal Policy?: Journal of Political Economy (June, 1976), 499-522.

because of its disregard for the underlying theoretical interrelationships among various economic variables in the economy. In short, the approach known as rational expectations is founded in theory and based on the assumption that information is scarce and that it is not wasted by rational economic individuals. Thus, the theory of rational expectations provides a theoretical-empirical counterpart to models like equation (29). In what follows the rational expectations model's reduced form price equation is presented which, as will be seen in Chapter VI wherein a modified version of the reduced form equation is tested, provides an alternative to equation (29).

The theoretical model used to describe the economy and the one from which the reduced form price equation is derived is basically a variant of the aggregate supply-aggregate demand model found in many macroeconomics texts. The specification used here follows the analysis of Sargent and Wallace with slight modifications supplied by Bennett McCallum.[3] To begin with, an aggregate supply schedule is specified. Following Lucas, the schedule is of the form

$$y_t = a_0 + a_1(P_t - E_{t-1}P_t) + a_2 y_{t-1} + u_{1t} \quad (a_1 > 0,\ 0 < a_2 < 1) \qquad (30)$$

where y_t measures the logarithm of current real income, P_t is the log of the price level in time t, and u_{1t} is a random error term with zero mean. The term $E_{t-1}(P_t)$ represents the rationally formed price expectation of

[3]Thomas Sargent and Neil Wallace, "'Rational Expectations,' the Optimal Monetary Instrument, and the Optimal Money Supply Rule," Journal of Political Economy (April, 1975), 241-254; B. T. McCallum, "Dating, Discounting, and the Robustness of the Lucas-Sargent Proposition," Journal of Monetary Economics (January, 1978).

the current price level made in time t-1 and based on information available to the individual at the end of the period. Robert E. Lucas has empirically tested such an aggregate supply schedule and found it to explain movements in real income quite well.[4] The a priori assumption on the parameter a_1 incorporates the conditions which underlie the natural rate hypothesis discussed in Chapter III: only unanticipated variations in the price level ($P_t \neq E_{t-1}P_t$) induce changes in the path of output. The additional assumption concerning the sign of a_2 is a stability condition.

The aggregate demand or "IS" curve of the economy is given by

$$y_t = b_0 + b_1\{r_t - E_{t-1}(P_{t+1} - P_t)\} + b_2 Z_t + u_{2t} \quad (b_1 < 0) \tag{31}$$

where r_t is the nominal rate of interest, Z_t is a vector of real exogenous variables which for analytical ease shall be omitted in the subsequent analysis, and u_{2t} is another error term with zero mean and assumed to be independently distributed from u_{1t}. The form of equation (31) is premised on the notion that real investment depends on the real rate of interest; thus, the hypothesized sign for the parameter b_1.[5]

The "LM" curve of the economy may be written as

$$m_t = P_t + c_1 y_t + c_2 r_t + u_{3t} \; ; \; (c_1 > 0, \; c_2 < 0) \; . \tag{32}$$

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[4]Robert Lucas, "Some International Evidence on Output-Inflation Trade-Offs," American Economic Review (June, 1973), 326-334.

[5]S. Turnovsky, "On the Role of Inflationary Expectations in a Short-Run Macro-Economic Model," Economic Journal (June, 1974), 317-337.

In this equation m_t denotes the log of the current money stock, P_t is the current price level (in logs), y_t and r_t are defined as before, and u_{3t} is yet another error term with the properties identical to u_{1t} and u_{2t}. As equation (32) suggests, the demand for real money balances will vary directly with real income (y_t) and inversely with the nominal rate of interest (r_t).

To complete the rational expectations model it is necessary to specify some policy rule which governs the money supply process. It is assumed that monetary policy is determined by a rule of the type

$$m_t = d_0 + d_1 m_{t-1} + d_2 y_{t-1} + d_3 G_{t-1} + u_{4t} \tag{33}$$

where u_{4t} is a fourth error term orthogonal to u_{1t}, u_{2t}, and u_{3t}, d_1, d_2, and d_3 are parameters (they may, as in Neftci, be thought of as one-sided lag distributions), and m_{t-1} and y_{t-1} are defined as before, except that now they are lagged. The variable G_{t-1} captures the federal government's revenue motive for money creation (this is discussed in greater detail in the next chapter). The money rule specified by equation (33) indicates that the monetary authority can control the money supply only up to the white noise disturbance term u_{4t}. This disturbance term may be thought of as representing those exogenous forces operating upon the money stock which are beyond the authority's control or it may represent the unsystematic behavior of the monetary authority itself.[7] In addition, it is assumed that past values of money and output are used by the policy maker in determining its money supply policy for the future.

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[7]See, ibid., and McCallum, "Dating, Discounting, and Robustness."

Utilizing the solution technique developed by Lucas,[8] one may solve the structural equations (30) through (33) for the reduced form solutions of the model. Following McCallum and Weber,[9] the model's linearity properties are utilized and reduced form solutions are hypothesized in which the endogenous variables of the system--y_t, P_t, and r_t--are given as functions of the exogenous variables and the stochastic error terms. Thus,

$$y_t = \Pi_{10} + \Pi_{11}m_{t-1} + \Pi_{12}y_{t-1} + \Pi_{13}G_{t-1} + \Pi_{14}u_{1t} + \Pi_{15}u_{2t} + \Pi_{16}u_{3t} + \Pi_{17}u_{4t} \quad , \qquad (34)$$

$$P_t = \Pi_{20} + \Pi_{21}m_{t-1} + \Pi_{22}y_{t-1} + \Pi_{23}G_{t-1} + \Pi_{24}u_{1t} + \Pi_{25}u_{2t} + \Pi_{26}u_{3t} + \Pi_{27}u_{4t} \quad , \qquad (35)$$

and

$$r_t = \Pi_{30} + \Pi_{31}m_{t-1} + \Pi_{32}y_{t-1} + \Pi_{33}G_{t-1} + \Pi_{34}u_{1t} + \Pi_{35}u_{2t} + \Pi_{36}u_{3t} + \Pi_{37}u_{4t} \quad . \qquad (36)$$

In order to evaluate the $\Pi_{10}, \ldots, \Pi_{37}$ parameters of equations (34) through (36) in terms of the model's structural parameters, equations (34) and (35) were substituted into the aggregate supply schedule given by equation (30). First, however, one must specify how

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[8]Lucas, "Econometric Testing."

[9]See, Warren Weber, "Some Implications of the Elimination of Intertemporal Arbitrage Opportunities for Interest Rate Changes" (Virginia Polytechnic Institute and State University, mimeo, 1978); and McCallum, "Dating, Discounting, and Robustness."

$E_{t-1}(P_t)$ is derived. Using the assumption of rational expectations, from (35)

$$E_{t-1}(P_t) = \Pi_{20} + \Pi_{21}m_{t-1} + \Pi_{22}y_{t-1} + \Pi_{23}G_{t-1} \ . \tag{37}$$

Substituting equations (34), (35), and (37) into (30) yields

$$\begin{aligned}\Pi_{10} &+ \Pi_{11}m_{t-1} + \Pi_{12}y_{t-1} + \Pi_{13}G_{t-1} + \Pi_{14}u_{1t} + \Pi_{15}u_{2t} + \Pi_{16}u_{3t} \\ +\Pi_{17}u_{4t} &= a_0 + a_1\{\Pi_{20} + \Pi_{21}m_{t-1} + \Pi_{22}y_{t-1} + \Pi_{23}G_{t-1} + \Pi_{24}u_{1t} \\ &\quad + \Pi_{25}u_{2t} + \Pi_{26}u_{3t} + \Pi_{27}u_{4t} - \Pi_{20} - \Pi_{21}m_{t-1} - \Pi_{22}y_{t-1} \\ &\quad - \Pi_{23}G_{t-1}\} + a_2y_{t-1} + u_{1t} \ .\end{aligned} \tag{38}$$

From equation (38) obtain the restrictions

$$\begin{aligned}\Pi_{10} &= a_0 \ , \\ \Pi_{11} &= 0 \ , \\ \Pi_{12} &= a_2 \ , \\ \Pi_{13} &= 0 \ , \\ \Pi_{14} &= a_1\Pi_{24} + 1 \ , \\ \Pi_{15} &= a_1\Pi_{25} \ , \\ \Pi_{16} &= a_1\Pi_{26} \ , \text{ and} \\ \Pi_{17} &= a_1\Pi_{27} \ .\end{aligned} \tag{39}$$

Before deriving the restrictions from similar substitutions into equations (31) and (32), it is necessary to note that, from equations (34) and (35),

$$E_{t-1}(P_t) = \Pi_{20} + \Pi_{21}d_0 + \Pi_{22}\Pi_{10} + (\Pi_{21}d_1 + \Pi_{22}\Pi_{11})m_{t-1}$$

$$+ (\Pi_{21}d_2 + \Pi_{22}\Pi_{12})y_{t-1} + (\Pi_{21}d_3 + \Pi_{22}\Pi_{13})G_{t-1} . \qquad (40)$$

Substituting equations (33) through (37) and (40) into equations (31) and (32) yields additional restrictions: from (31)

$$\Pi_{10} = b_0 + b_1(\Pi_{30} - \Pi_{21}d_0 - \Pi_{22}\Pi_{10}) ,$$

$$\Pi_{11} = b_1(\Pi_{31} - \Pi_{21}d_1 - \Pi_{22}\Pi_{11} + \Pi_{21}) ,$$

$$\Pi_{12} = b_1(\Pi_{32} - \Pi_{21}d_2 - \Pi_{22}\Pi_{12} + \Pi_{22}) ,$$

$$\Pi_{13} = b_1(\Pi_{33} - \Pi_{21}d_3 - \Pi_{22}\Pi_{13} + \Pi_{23}) ,$$

$$\Pi_{14} = b_1\Pi_{34} ,$$

$$\Pi_{15} = b_1\Pi_{35} + 1 ,$$

$$\Pi_{16} = b_1\Pi_{36} , \text{ and}$$

$$\Pi_{17} = b_1\Pi_{37} . \qquad (41)$$

From equation (32), one obtains the restrictions

$$d_0 = \Pi_{20} + c_1\Pi_{10} + c_2\Pi_{30} ,$$

$$d_1 = \Pi_{21} + c_1\Pi_{11} + c_2\Pi_{31} ,$$

$$d_2 = \Pi_{22} + c_1\Pi_{12} + c_2\Pi_{32} ,$$

$$d_3 = \Pi_{23} + c_1\Pi_{13} + c_2\Pi_{33} ,$$

$$1 = \Pi_{27} + c_1\Pi_{17} + c_2\Pi_{37} \,,$$

$$0 = \Pi_{24} + c_1\Pi_{14} + c_2\Pi_{34} \,,$$

$$0 = \Pi_{25} + c_1\Pi_{15} + c_2\Pi_{35} \,, \text{ and}$$

$$0 = \Pi_{26} + c_1\Pi_{16} + c_2\Pi_{36} \,. \tag{42}$$

The linear restrictions derived from equations (39), (41), and (42) may be solved simultaneously for the parameters $\Pi_{10},\ldots,\Pi_{37}$ in order that they may be evaluated in terms of the model's structural parameters. Since the focus of this study is on the reduced form price equation, only the reduced form parameters $\Pi_{20},\ldots,\Pi_{27}$ found in equation (35) will be reported and discussed. The explicit reduced form price equation is

$$P_t = \Pi_{20} + \Pi_{21}m_{t-1} + \Pi_{22}y_{t-1} + \Pi_{23}G_{t-1} + e_t \tag{43}$$

where

$$\Pi_{20} = h^{-1}\{b_1(d_0 + a_0c_1 - c_2\Pi_{21}\Pi_{10} + a_1\Pi c_1\Pi_{20}) + c_2(b_0 - a_0 + a_1\Pi_{20})\}$$

$$\Pi_{21} = h^{-1}\{b_1(d_1 + c_2\Pi_{21} - c_2\Pi_{21}d_1 - c_2\Pi_{22}\Pi_{11}) + a_1\Pi_{21}(c_2 + b_1c_1)\}$$

$$\Pi_{22} = h^{-1}\{b_1(d_2 - a_2c_1 + c_2\Pi_{22} - c_2\Pi_{21}d_2 + c_2\Pi_{22}\Pi_{12}) + c_2(a_1\Pi_{22} - a_2) + a_1b_1c_1\Pi_{22}\}$$

$$\Pi_{23} = h^{-1}\{b_1(d_d + c_2\Pi_{23} - c_2\Pi_{21}d_3 - c_2\Pi_{22}\Pi_{13}) + a_1\Pi_{23}(c_2 + b_1c_1)\}$$

$$e_t = c_2 + b_1 c_1\{u_{1t} + c_2 u_{2t} + b_1(u_{4t} - u_{3t})\}h^{-1} \text{ , and}$$

$$h^{-1} = 1/\left(a_1 c_2 + b_1(a_2 c_1 + 1)\right)$$

The reduced form price solution of the model is of the standard form found in most discussion of rational expectations concerned with more than the output solution alone.[10] Careful examination of the parameters in equation (43) reveals that the monetary authority's choice of policy parameters in the money rule will directly influence the time path of the price level. This is because the policy parameters d_0, d_1, and d_3 are "imbedded" in the reduced form price solution's parameters. Thus, if the monetary authority adopts a new rule governing the time path of the money supply, we would expect the parameters in equation (43) to be affected in a systematic, empirically testable way.

The preceding discussion of autoregressive and rational expectations approaches to modeling the expectations mechanism suggests that one possible method of determining the appropriate model of the expectations formation process is to estimate each specification across different monetary policy regimes. Once a regime break is found, the appropriate test for rationality is then to determine whether or not the estimated coefficients of equations (29) and (43) remain invariant to the break in policy.

The rational expectations model (and the variant tested in Chapter VI) indicates that the estimated relationship should not be stable across regimes. Contrary to the usual rational expectations-

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[10]See, for example, Weber, "Interest Rate Changes."

natural rate discussion concerning output-money relationships wherein the reduced form output coefficients are being tested for invariance with respect to the money rule break point, the goal here is to find the reduced form price parameters exhibiting instability with respect to changes in the monetary regime. In other words, one would expect to find, if the model is rational in the sense of Sargent,[11] that if the money rule parameters are not stable over the sample period, then the parameters of the reduced form price equation should likewise exhibit testable indications of structural instability during the period.[12]

It was noted in Chapter I that an implicit assumption underlying the autoregressive approach to measuring expectations is that past price or inflation data may already incorporate and reflect pertinent information from the structural interrelationships within the economy that may affect the time path of prices. Assuming that the money supply process is an essential force in determining the observed price level, a change in this process, i.e., a change in the money rule, should also be reflected in the estimated coefficients of equation (29). If the autoregressive model is "rational," then it would also be expected to exhibit parameter instability across monetary policy regime changes.[13]

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[11]Sargent, "Neutrality and Rationality."

[12]For articles dealing with the testing of the neutrality proposition regarding the natural rate hypothesis, see, especially, Neftci and Sargent, "Natural Rate Hypothesis"; Neftci, "Unanticiapted Money Growth"; and Sargent, "Neutrality and Rationality."

[13]To see this, suppose current prices (or inflation) are a function of the money supply in the preceding period: $P_t = f(M_{t-1})$. In such a situation, it is clear that past prices are also functions of past money

In this chapter two alternative models of expectations were presented. In general, most investigations examine the rate-of-change of the price level. In the subsequent chapters, this approach will be adopted and the inflation rate, not the general price level, will be dealt with. Even so, it should be emphasized that the empirical implications of the models examined in this chapter, especially the rational expectations model, remain unchanged. The essence of the ensuing empirical work is in testing models that can be accepted as a basic statement of the alternative expectations mechanisms suggested above. Any deviation from the confines of the theoretical boundaries set forth in this chapter is based solely on the empirical evidence obtained from several experiments.

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such that $P_{t-1} = f(M_{t-i-1})$, $i=1,2,3,\ldots,n$. If this is true, then the autoregressive equation given by (29) may be written in the form $P_t = f(P_{t-1}, P_{t-2}, \ldots, P_{t-n}, M_{t-n-1})$ where the influence (or weight) on prices (and money) beyond some point, say, k ($k < n$), becomes insignificantly small. Thus, in estimating equation (29), the lag distribution is truncated at some point k, disregarding prices and money data beyond this empirically determined point.

Note, however, that this procedure does not remove the notion that past prices (or inflation) are in fact influenced by past money. As such, even though money data are not explicitly incorporated into the "reduced form" equation given by (29), its influence _is_ reflected in past prices. Thus, changes in the money supply process should induce changes in the process by which expectations are formed. For this reason, the autoregressive model represented by equation (29) should exhibit parameter instability across money rule breaks.

CHAPTER V

ESTIMATION OF THE MONEY RULE

In this chapter, we shall examine the use of monetary regime breaks as one possible avenue in detecting model rationality. This test, suggested by Sargent,[1] is based on the idea that perceived changes in the underlying monetary rule may directly influence the time path of inflation and, therefore, the public's expectations formation process. In essence, the methodology proposed here is a variant of standard structural stability tests of estimated models common to most time series analyses. Investigations into the impacts of changes in the money rule (or, put another way, changes in the monetary standard)[2] have increasingly begun to receive consideration by macroeconomic theorists. The argument underlying this approach is that reduced form equations derived from structural models may not, when estimated outright, present an accurate portrayal of the hypothesized relationship unless regime changes in the policy equations are tested for and incorporated explicitly into the analysis. As noted in the discussion of the reduced form price equation derived from the rational expectations model, changes in the estimated parameters of the hypothesized money rule place testable restrictions on the coefficients of that equation.

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[1]Sargent, "Neutrality and Rationality."

[2]Benjamin Klein, "Our New Monetary Standard: The Measurement and Effects of Price Uncertainty in the United States, 1880-1973," Economic Inquiry (December, 1975), 461-484.

At this time, there are only a few studies addressing this problem. Two recent papers by Neftci and Neftci and Sargent specifically test for breaks in the estimated money rule and apply these results to the rational expectations-natural rate hypothesis.[3] The fundamental premise to these studies is that the rational expectations version of the natural rate hypothesis, i.e., that only unexpected changes in money growth affect variations in real economic variables such as output or unemployment, should exhibit parameter invariance across different money rules. Their tentative empirical results indicate that the rational expectations specification of the natural rate hypothesis does exhibit the desired invariance property across changing monetary regimes.

The next section of this chapter presents a brief overview of previous estimations of the money rule and tests for detecting possible changes in regime. The second section presents the money rule equation tested in this analysis. As noted previously in Chapter IV, the form of the money rule is important in determining the actual inflation equation to be tested under the heading of rational expectations. The third section reports the test for regime change in the estimated money rule and some thoughts on the appropriate use of the empirical results provided by such estimations.

Previous Tests of the Money Rule

Empirical tests of the money rule have been based primarily on two alternative specifications. In the works of Neftci and Neftci

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[3]Neftci, "Unanticipated Money Growth"; and Neftci and Sargent, "Natural Rate Hypothesis."

and Sargent,[4] a money rule equation of the following form is utilized:

$$M_t = \sum_{i=1}^{k} a_i M_{t-i} + \sum_{i=1}^{k} b_i y_{t-i} + u_t \qquad (44)$$

where M_t is the log of the narrowly defined money stock (M1), y_{t-i} (i=1,2,3,...,k) is the log of real GNP, and u_t is a serially uncorrelated random error term. In each study, the original data were first regressed on a constant, exponential trend and seasonal dummy variables to remove obvious deterministic components from each time series. The residuals from these regressions, approximating linearly regular or indeterministic stochastic processes, were then used as data in estimating equation (44). Unfortunately, the actual empirical form of equation (44) is not reported in either study. Although this omission is disturbing, it is the reported break point test results which are of current interest.

The Neftci study attempts to determine the break point in the money rule (44) by employing the econometric technique developed by Richard Quandt.[5] This procedure assumes that over the entire sample period, two alternative relationships exist to explain the variation in the dependent variable. Given these two relationships

$$y_i = a_1 + b_1 x_i + u_1 \quad ; \quad i=1,2,3,\ldots,t \qquad (45a)$$

and

$$y_i = a_2 + b_2 x_j + u_2 \quad ; \quad j=t+1,t+2,\ldots,T \qquad (45b)$$

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[4]Ibid.

[5]Richard Quandt, "The Estimation of the Parameters of a Linear Regression Obeying Two eaprate Regimes," Journal of the American Statistical Association (December, 1958), 873-880.

where u_1 and u_2 are independently distributed error terms with zero mean and standard deviations σ_1 and σ_2, respectively, the probelem is to detect that point t where the above relationships hold. In other words, one attempts to empirically ascertain the break point t at which equation (45a) explains the realtionship between x and y over the subsample 1 to t and from t+1 to T, equation (45b) provides the best specification.

Following Quandt, the likelihoods for each sample estimates are

$$(1/\sqrt{2\Pi}\sigma_1)^t \exp - 1/2\sigma_1^2 \sum_{i=1}^{t} (y_i - b_1x_i - a_1)^2 , \tag{46a}$$

and

$$(1/\sqrt{2\Pi}\sigma_2)^{T-t} \exp - 1/2\sigma_2^2 \sum_{j=t+1}^{T} (y_j - b_2x_j - a_2)^2 . \tag{46b}$$

The likelihood of the entire sample (1 to T) is, therefore,

$$(1/\sqrt{2\Pi}\sigma_1)^t (1/\sqrt{2\Pi}\sigma_2)^{T-t} \exp - 1/2\sigma_1^2 \sum_{i=1}^{t} (y_i - b_1x_i - a_1)^2$$
$$- 1/2\sigma_2^2 \sum_{j=t+1}^{T} (y_j - b_2x_j - a_2)^2 . \tag{47}$$

Taking the log of equation (47), one obtains the log likelihood function

$$L = -T\log\sqrt{2\Pi} - t\log \sigma_1 - (T-t)\log\sigma_2 - (1/2\sigma_1^2 \sum_{i=1}^{t} (y_i - b_1x_i - a_1)^2$$
$$-(1/2\sigma_2^2) \sum_{j=t+1}^{T} (y_j - b_2x_j - a_2)^2 \tag{48}$$

Setting the partial derivative of equation (48) with respect to a_1, b_1, a_2, and b_2 equal to zero yields the log of the maximum likelihood function for a given sample of size T:

$$L(t) = -T\log\sqrt{2\Pi} - t\log\sigma_1^2 - (T-t)\log\sigma_2^2 - T/2 , \tag{49}$$

which is a direct function of t. To implement this test, an arbitrary t is chosen whereupon successive iterations are made until the maximum value of L(t) is found.

Using quarterly data from 1947 to 1974 for the United States, Neftci reports that the money rule break point occurred in the fourth quarter of 1962.[6] Examination of the reported values of the likelihood function (the L(t)s of equation (49)) for the estimated money rule over the period 1962.IV to 1964.IV casts some doubt on this conclusion. During this period, the values of L(t) range from 1189.0 to 1184.5. The value for 1962.IV (1189.0) is a maximum for the entire period and, therefore, may represent the "most likely" break point candidate. However, the occurrence of L(t) values over the 1962.IV to 1964.IV period in reasonably close proximity to that of 1962.IV urges caution in concluding that the fourth quarter of 1962 is the break point.

In an earlier study, Neftci and Sargent test for break points in the money rule described by equation (44).[7] Applying the same quasi-filtering technique found in Neftci to quarterly, postwar time series data for real GNP ($1958), M1, and M2, Neftci and Sargent attempt to find possible break points in the money rule by using the

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[6]There is a discrepancy in Neftci's work. Though he reports in the text that the break point is found for 1962.II, the data indicates that the actual point of regime change is 1962.IV. It should also be noted that in Neftci and Sargent, "Natural Rate Hypothesis," the break point for the estimated money rule was found to be 1964.I. No reason, save a different estimation technique, is offered by Neftci for these differing results.

[7]Ibid.

Chow test. Splitting the sample period at 1964.I, the null hypothesis that the money rule remained stable across the period was rejected for the M1 series: the calculated F-statistic is $F_{78}^{13} = 2.54$, which exceeds the 1 percent critical value of $F_{70}^{12} = 2.45$. On the other hand, applying the same test procedure to the money rule which uses the M2 series, the calculated F-statistic does not exceed the critical F at the 5 percent level of significance. Although the test results for the M2 series indicate no structural break at 1964.I, the M1 test results do suggest a possible breakdown in the stability of equation (44) sometime during the mid-1960s.[8]

Robert Barro's recent empirical works employ a different specification of the money rule than that used by Neftci and Neftci and Sargent.[9] Basically, Barro's money rule equation includes, in addition to lagged values of money growth, the lagged unemployment rate, and a measure of federal government expenditures relative to an empirically derived "normal" level. Thus, the money rule tested by Barro appears as

$$\dot{M}_t = a_0 + b_1\dot{M}_{t-1} + b_2\dot{M}_{t-2} + b_3 FEDV_t + b_4 U_{t-1} + e_t \tag{50}$$

where $\dot{M}$ is the growth rate of the narrowly defined money stock (M1), FEDV is a measure of government expenditures relative to normal, and U is the unemployment rate. The unemployment rate is utilized to capture possible

[8] To assess the similarities (or differences) of the various statistical techniques discussed in the text, an interesting experiment would be to apply each method to a single money rule equation and compare the break point results. Such a test, though beyond the scope of the present study, is planned for the future.

[9] Barro, "Unanticipated Money Growth," and "Money, Output, and the Price Level."

counter-cyclical behavior on the part of the monetary authority. In addition, the actual measure of the unemployment variable is entered as the logarithm of the ratio (U/1-U) which restricts the unemployment rate to the (0,1) interval.[10]

Barro employs the variable FEDV on the premise that it reflects an important aspect of the federal government's revenue motive. The essence of Barro's argument is that government expenditures are assumed to be financed through a combination of taxation and money creation. On the tax side, revenues are generated through an institutional framework which administers the collection of taxes and is maintained by capital expenditures which perpetuate its tax-raising ability. "The amount of capital invested in the tax-raising 'industry,'" Barro argues, "will depend on 'long-run' value of such variables as government expenditures and national income."[11] If the government attempts to "minimize the total costs associated with raising revenue,"[12] deviation of expenditures from their "normal" level will not lead to an expansion of the "tax-raising industry" in the short run, but to greater utilization of the relatively less costly avenue of financing expenditures through money creation.

Barro's use of the variable FEDV may be questioned on two points. The first is that his measure of "normal" expenditures may be largely conditioned on the sample period. Consider, for example, Barro's sample period of 1941-1975 which includes World War II, the Korean and the Vietnam wars. Estimating the value of government expenditures relative

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[10]Barro, "Unanticipated Money Growth," 107. [11]Ibid., 102.

[12]Ibid.; this assertion is open to question.

to some "normal" value which is obtained from the same period may introduce a significant bias in the estimated values for FEDV. Failure to account for the possible upward bias in the measure during war years may greatly influence resulting empirical estimates of equation (50). In a forthcoming article, Small demonstrates that, using zero-one dummy variables for the war years, a significant degree of the explanatory power of Barro's money rule (Barro reports the estimated R^2 to be 0.90) may be due to the implicit assumption that war and non-war years come from the same regression population.[13]

The second criticism concerns the inability of Barro's FEDV measure to capture the possible increase in the accumulated "capital" of the tax-raising apparatus over the postwar period. If this institution of the government's revenue raising operation has itself grown over the period, assuming that deviations of expenditures relative to normal would be met equally in all periods by monetary expansion is debatable. Failure to incorporate such a secular growth in the "tax-raising industry" may impart an additional bias to the estimated values of money growth when the variable FEDV is utilized in equation (50).

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[13]D. H. Small, "A Comment on Barro's 'Unanticipated Money Growth and Unemployment in the United States'," American Economic Review (forthcoming). It is questionable whether or not Small's comment is in fact appropriate. This is because Small estimates his equations using Cochrane-Orcutt and Barro uses ordinary least-squares. Because of this , Small's dependent variable (the unemployment rate) is not the same as Barro's due to the adjustment for first-order serial correlation. See any econometrics text for a description of the adjustment procedure that is implemented by the Cochrane-Orcutt procedure.

In the next section, the money rule estimated in this study is presented.

The Money Rule

The money rule equation which is employed in this study is a variant of equation (33) found in Chapter IV. The specification used here approximates Barro's model discussed above. Instead of using a measure of federal expenditures relative to a normal level, the value of the federal government surplus relative to nominal GNP is entered into the money rule in an attempt to capture the revenue motive via monetary expansion. A specification similar to this has been successfully used by J. E. Tanner and J. Trapani in an investigation concerning the impacts of unanticipated money growth on stock prices.[14] In addition to this variable, and in keeping with the spirit of the theoretical model outlined in Chapter IV, lagged real output measured by real GNP ($1972) is entered along with several lagged money growth rates. The empirical form of the money rule equation is, therefore,

$$\dot{M}_t = a_0 + b_1\dot{M}_{t-1} + b_2\dot{M}_{t-s} + b_3\dot{M}_{t-3} + b_4\dot{M}_{t-4} + b_5\dot{M}_{t-5}$$

$$+ b_6(Sur/GNP)_{t-1} + b_7(Sur/GNP)_{t-2} + b_8\dot{y}_{t-1} + e_t \qquad (51)$$

where $\dot{M}_{t-1}$(i=1,2,3,4,5) measures the lagged growth rate of the narrowly defined money stock (M1), (Sur/GNP) is the level of the federal government

[14] J. E. Tanner and J. Trapani, "Can the Quantity Theory be Used to Predict Stock Prices?" Southern Economic Journal (October, 1977), 261-270.

surplus relative to nominal GNP, $\dot{y}_{t-1}$ is the lagged growth rate of real output (GNP in \$1972), and e_t is a random error term.[15]

Equation (51) was estimated using seasonally adjusted quarterly data for the United States over the period 1949.I-1977.IV. As such, all growth rates are viewed as quarterly rates-of-change. Using ordinary least-squares, the estimated money rule is, with absolute values of the t-statistics in parentheses,

$$\dot{M}_t = \underset{(1.68)}{0.002} + \underset{(7.06)}{0.690}\,\dot{M}_{t-1} - \underset{(0.71)}{0.084}\,\dot{M}_{t-2} + \underset{(0.57)}{0.067}\,\dot{M}_{t-3}$$

$$- \underset{(1.34)}{0.152}\,\dot{M}_{t-4} + \underset{(2.35)}{0.219}\,\dot{M}_{t-5} - \underset{(3.27)}{0.159}\,(Sur/GNP)_{t-1}$$

$$+ \underset{(2.24)}{0.110}\,(Sur/GNP)_{t-2} + \underset{(2.04)}{0.090}\,\dot{y}_{t-1} \qquad (52)$$

$$R^2 = 0.594 \quad ; \quad D.W. = 1.94 \quad ; \quad \hat{\sigma} = 0.004 \quad ; \quad F^{8}_{102} = 18.64$$

where $\hat{\sigma}$ is the standard error of the regression and D.W. is the Durbin-Watson statistic.[16]

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[15]Note that the variable (Sur/GNP) is entered in level form. This specification is based solely on empirical experiments with various forms wherein the present derivation proved empirically superior relative to those in which (Sur/GNP) was entered in log or growth rate forms. It was also tried using actual government expenditures in place of the surplus variable. In all cases, the empirical results were statistically inferior to those reported in the text.

[16]Several experiments were tried wherein the independent variables' lags were extended back through time. Such lengthening on the lag structures, relative to the result reported in the text, did not significantly increase the explanatory power of the equation.

Turning to the estimated coefficients of the money rule, the results are somewhat surprising. Consider first the coefficients on the lagged money growth variables. The estimated b_i values for the $\dot{M}_{t-1}$ and $\dot{M}_{t-5}$, 0.690 and 0.219, respectively, are the only lagged money growth coefficients that attain statistical significance at any acceptable level. Both are significant at the 5 percent level. The sign pattern of the lagged money coefficients suggests something of a dampened oscillatory effect over time. In addition, the summed impact of the lagged money variables is 0.738, which is similar to the estimate obtained by Tanner and Trapani.[17]

A priori it was thought that the sign on the variables (Sur/GNP) would be negative. Intuitively one would expect that, given the arguments offered above, when the government surplus relative to GNP increases, this would reduce the role of money creation in servicing expenditures and maximizing government revenues. Although the estimated coefficient on the $(Sur/GNP)_{t-2}$ variable is of the "wrong" sign, the realtive magnitudes of the two (Sur/GNP) variables are such that the sum two-period effect on the money growth rate is in fact negative; this is seen by comparing the $(Sur/GNP)_{t-1}$ coefficient of -0.159 to that for $(Sur/GNP)_{t-2}$, +0.110. Thus, over two periods, the sum coefficient for the variable (Sur/GNP) is -0.049 which, although quite small, is in accord with a priori reasonsing. In addition, the estimated coefficients

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[17]Tanner and Trapani, "Qnatity Theory," 265. Using a lag structure on past money growth of eight periods, Tanner and Trapani report the cumulative impact of past money to be 0.830.

for both (Sur/GNP) variables are statistically significant at the 5 percent levels.

Lastly, consider the coefficient on lagged real output. The estimated value of 0.090 (significant at the 5 percent level) implies that an increase in the growth rate of real output last period may induce the monetary authority to expand the current money stock. This finding tentatively suggests that the monetary authority may react positively to the expanding needs of trade in the economy.

The usefulness of estimating the money rule is twofold. The primary reason for estimating the money rule equation is to find possible break points across the sample period. As discussed in Chapter IV and in the first section of this chapter, break points in the money rule may be used to test the rationality of the alternative expectations equations studied here. Such tests are found in the following chapter. Another reason to estimate the money rule, and one which is beginning to receive attention by the profession (although it is somewhat outside the present purpose of this study) is to obtain a time-series representing the unanticipated component of observed money growth rates. Since equation (52) is actually a forecasting model, the residuals from this equation may be used as a measure of unanticipated money growth. In Appendix A this series will be used in a methodological issue concerning the use of actual and unanticipated money growth series in certain empirical studies.

This section presented the empirical estimates of the money rule selected for use in this study. In the next section, this specification is tested for regime change over the 1950.II-1977.IV sample period.

Break Point Test of the Money Rule

To test for a possible regime change in the money rule, it has been decided to split the sample at 1964.I. One might argue that this arbitrary approach is perhaps too insensitive to the possibility of multiple breaks over the sample period and that a more rigorous search methodology should be utilized. While this may be true if the current purpose was to examine the capabilities of competing methods for detecting regime change, the present cause is well served by merely examining the stability of the estimated money rule and, using this information, the stability of the expectations models across money rule regime changes.

The test used to determine whether or not 1964.I represents a possible break point in the money rule is provided by Gregory Chow.[18] As the Chow test is designed to examine two possible regimes (i.e., one break point), the implementation of the procedure in this study is demonstrated as follows. Consider the basic estimated model to be

$$y = xB + u \quad , \tag{53}$$

where y is a $(T \times 1)$ column vector, x is a $(T \times k)$ matrix, and B is a $(k \times 1)$ vector or parameters. The term u represents a random error term with zero mean. Estimation of equation (53) yields the sum of squared residuals

$$(e'e)_T = y_T'y_T - b'x_T'y_T \quad , \tag{54}$$

[18] Gregory Chow, "Tests for Equality Between Two Sets of Coefficients in Two Linear Regressions," Econometrica (July, 1960), 591-605.

where y_T is a (T × 1) vector. In most standard applications of the Chow test, one would attempt to test whether or not this relationship (equation (53)) holds over the entire sample. If a break point is hypothesized, the test procedure is to estimate equation (53) over the n sample observations prior to the hypothesized break point to obtain the subsample sum of squared residuals denoted by

$$(e'e)_n = y_n'y_n - b'x_n'x_n \quad , \tag{55}$$

where y_n is an (n × 1) vector. Equation (53) is then reestimated over the post-break sample m(m=n+1,...,T) to obtain the second subsample sum of squared residuals,

$$(e'e)_m = y_m'y_m - b'x_m'x_m \quad . \tag{56}$$

With the information provided by equations (54), (55), and (56), form the F-statistic,

$$F = \frac{(e'e)_T - (e'e)_n - (e'e)_m/k}{(e'e)_n + (e'e)_m/T-2k} \tag{57}$$

where $T \equiv m + n$. Computing the F-statistic given by (57) permits one to test for acceptance at some specified statistical level of significance of the null hypothesis of no regime change. A significant F-statistic would suggest that the subperiod regression results come from different populations and should not be combined to estimate one overall relationship.

To test the null hypothesis of no structural change across the postulated 1964.I break point, the money rule equation (51) was reestimated for the period 1950.II to 1964.I and 1964.II to 1977.IV. The regression results for these two subperiods, along with the estimates for the entire sample, are presented in Table 1. Absolute values of the t-statistics are given in parentheses. SSR represents the sum of squared residuals which are used in the Chow test.

Prior to examining the Chow test results, it may be useful to examine the subperiod money rule estimates. Perhaps the most striking result is the lack of any persistent pattern in the lagged money growth variables during the period 1950.II-1964.I. Only the estimated coefficient on the $\dot{M}_{t-1}$ variable is statistically significant (1 percent level). The estimated values of the other lagged money growth rates are less than their respective stnadard errors. The lagged (Sur/GNP) variables, however, maintain their sign pattern and statistical significance during this subperiod. Lagged real output, again positively signed and with an estimated coefficient almost identical to that from the entire period regression, becomes statistically insignificant.

For the 1964.II-1977.IV period, the sign pattern of all estimated coefficients is identical to that of the entire period estimation. The estimated $\dot{M}_{t-1}$ and $\dot{M}_{t-5}$ coefficients are again statistically significant at the 1 percent and 5 percent levels, respectively. The summed impact of the lagged money growth variables is 0.591 compared to 0.738 for the entire period. The marked decline in the summed effect for the subperiod relative to the entire sample remains to be explained. Notice also that, except for the estimated $(Sur/GNP)_{t-1}$ coefficient which is very close to being statistically significant at the 5 percent level (the

Table 1

Money Rule Regression Results[a]

Equation Tested: $\dot{M}_t = a_0 + b_1\dot{M}_{t-1} + b_2\dot{M}_{t-2} + b_3\dot{M}_{t-3} + b_4\dot{M}_{t-4} + b_5\dot{M}_{t-5} + b_6(Sur/GNP)_{t-1} + b_7(Sur/GNP)_{t-2} + b_8\dot{y}_{t-1}$

	Sample Period		
Variables	1950.II-1977.IV	1950.II-1964.I	1964.II-1977.IV
a_0	0.002 (1.68)	0.003 (2.71)**	0.004 (1.38
$\dot{M}_{t-1}$	0.690 (7.06)**	0.612 (4.26)**	0.547 (3.94)**
$\dot{M}_{t-2}$	-0.084 (0.71)	0.017 (0.10)	-0.207 (1.29)
$\dot{M}_{t-3}$	0.067 (0.57)	-0.142 (0.87)	0.172 (1.09)
$\dot{M}_{t-4}$	-0.153 (1.34)	-0.104 (0.66)	-0.255 (1.62)
$\dot{M}_{t-5}$	0.219 (2.35)*	-0.021 (0.16)	0.334 (2.40)*
$(Sur/GNP)_{t-1}$	-0.159 (3.27)**	-0.140 (2.46)*	-0.168 (1.99)
$(Sur/GNP)_{t-2}$	0.110 (2.24)*	0.134 (2.44)*	0.093 (1.05)
$\dot{y}_{t-1}$	0.090 (2.04)*	0.091 (1.64)	0.119 (1.71)
$\bar{R}^2$	0.594	0.522	0.442
D-W	1.94	2.00	1.89
SEE	0.004	0.004	0.004
SSR	0.00194	0.00072	0.00091

[a]Absolute values of t-statistics appear in parentheses. ** and * denote statistical significance at 1 and 5 percent levels, respectively. $\bar{R}^2$ is the coefficient of determination, D-W is the Durbin-Watson statistic, SEE is the standard error of the estimate, and SSR is the sum of squared residuals.

critical value is 2.00), estimates of both the $(Sur/GNP)_{t-2}$ and $\dot{y}_{t-1}$ variables are not statistically significant at any acceptable levels.

Using the information provided in Table 1, one may form the F-statistic given by equation (57). The F-statistic used to test for a regime change in the estimated money rule at 1964.I is calculated by

$$F = \frac{(0.00194 - 0.00072 - 0.00091)/9}{(0.00072 + 0.00091)/111 - 18} = 1.98$$

which is distributed as F^{9}_{93}. The critical values for F^{9}_{100} are 1.97 at the 5 percent level and 2.59 for the 1 percent level of significance. The calculated F value above exceeds the 5 percent critical value and thus permits rejection of the null hypothesis of no regime change in the money rule over the sample period. With this result, 1964.I does, therefore, represent a possible break point in the money rule. The usefulness of this finding, alluded to in earlier discussions, is that it may be employed in determining the rationality of the expectations models tested in this study. In the next chapter, such rationality tests are described and implemented.[19]

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[19]Previous investigations using the break point approach, most notably Neftci, "Unanticipated Money Growth," and Neftci and Sargent, "Money, Output, and Price Level," have implicitly assumed that there is no regime change when calculating the unexpected money series. If a break point is found, then using the entire period regression results to obtain values for unexpected money is clearly incorrect. Failure to account for changes in the underlying relationships across regime changes will produce inaccurate estimates of the unexpected money-output relationships. Such an approach is not only econometrically incorrect, but it also assumes irrational behavior on the part of individuals in the economy. In Appendix A, some empirical evidence on this point as it applies to the present analysis is presented.

CHAPTER VI

RATIONALITY TESTS USING MONEY RULE BREAK POINTS

The purpose of this chapter is to present the empirical representations of the expectations models suggested in Chapter IV and the results of the break point test for rationality. In Chapter IV, two alternative approaches to measuring inflationary expectations were offered. One is relatively simple autoregressive model wherein expectations of inflation are based solely on past inflation rates. Such forecasting mechanisms are known in a different context as weak-form tests.[1] The second approach posits that forecasters exploit the pertinent information available in the economy in predicting future inflation. These information requirements are specified by the theory which the forecaster believes truly represents the structural relationships within the economy. Referred to as the rational expctations model of inflation expectations, it was shown in Chapter IV that the specification of the money rule determines the variables used in forecasting inflation.

Obviously these two alternative models do not encompass the entire spectrum of possible specifications of the process by which expectations of inflation are formed. The assumptions of each are clearly quite restrictive. In an attempt to relax these specific informational requirements, a third alternative model, more general than those previously mentioned, is also tested in this and future chapters. This

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[1]For an excellent discussion of such tests, see, Eugene Fama, "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance (May, 1970), 383-417.

specification, which shall be referred to as the general model of expectations, incorporates the informational content of the autoregressive and the rational expectations models. Though admittedly ad hoc in its derivation, it does allow the forecaster to use an information set which is more general than the rival specifications.

The first section of this chapter presents the expectations models to be estimated. The second section contains the empirical estimates of each model for the sample period 1951.II to 1977.IV. In addition, a statistical comparison of the various models is made. Finally, the results of testing for rationality via the break point approach are reported in the third section.

Alternative Models of Expectations

It was suggested in Chapter IV that the simple autoregressive model is perhaps one of the least restrictive (informationally and computationally) approaches to measuring expectations. The general form of the autoregressive model, which is rewritten here for convenience, is

$$\dot{P}_t = a_0 + \sum_{i=1}^{n} b_i \dot{P}_{t-1} + u_t \quad , \tag{58}$$

where $\dot{P}_t$ represents the rate of change of prices, $\dot{P}_{t-1}$ (i=1,2,...,n) is past inflation rates, and u_t is an error term. The number of lagged inflation rates cannot be specified a priori and, therefore, it is determined empirically. It should also be noted that estimation of equation (58) does not necessitate the a priori specification of the lagged weight structure. Relative to other autoregressive methods, which may require the lag weights to decline geometrically or some a priori assumption that the lag structure fit a polynomial of degree n, the approach offered by equation (58) is the most general.

As an alternative to equation (58), it was demonstrated in Chapter IV that a reduced form equation for inflation could be derived from a structural model of the economy. Such a model (a variant of (43)) incorporates the assumption of rational expectations and includes only the exogenous variables of the postulated economic system as explanatory variables in forecasting future inflation rates. For expositional convenience, the theoretical model was kept to a manageable size. The key outcome of that theoretical discussion is that the form of the hypothesized money rule, both in variables and lag structure, would determine the final specification of the inflation equation to be empirically tested. Extending the lag structure of the money rule affects equally the lag structure of the inflation model and, most importantly, does <u>not</u> affect the conclusion that changes in the structure of the money rule should impart testable instability in the structure of the rational expectations inflation equation. In other words, the rational expectations inflation model should exhibit structural instability across money rule break points.

Chapter V provides the empirical form of the money rule used in this study. Given the preceding discussion, the empirical representation of the rational expectations inflation model becomes

$$\dot{P}_t = a_0 + b_1\dot{M}_{t-1} + b_2\dot{M}_{t-2} + b_3\dot{M}_{t-3} + b_4\dot{M}_{t-4} + b_5\dot{M}_{t-5} + b_6(\text{Sur/GNP})_{t-1}$$

$$+ b_7(\text{Sur/GNP})_{t-2} + b_8\dot{y}_{t-1} + e_t \quad , \qquad (59)$$

where $\dot{P}_t$ is the current period rate of inflation, $\dot{M}_{t-i}$ (i=1,...,5) represents lagged money growth rates, $(Sur/GNP)_{t-i}$ (i=1,2) is the federal government surplus scaled by nominal GNP, $\dot{y}_{t-1}$ measures last period's growth rate of real output, and e_t is a random error term. The theoretical discussion in Chapter IV indicates that, if the model is rational, equation (59) should display structural instability across money rule break points. This proposition is tested in the third section of this chapter.

To conclude this section yet another expectations model is suggested which incorporates the informational content of those previously discussed. This model, though admittedly ad hoc, is founded on the presumption that individuals attempting to make optimal forecasts (in the sense of minimizing the forecast error) may not disregard certain information provided by the historical evolution of the economy. Referred to as the general model of expectations, it may be written in the form

$$\dot{P}_t = a_0 + b_1\dot{M}_{t-1} + b_2\dot{M}_{t-2} + b_3\dot{M}_{t-3} + b_4\dot{M}_{t-4} + b_5\dot{M}_{t-5}$$
$$+ b_6(Sur/GNP)_{t-1} + b_7(Sur/GNP)_{t-2} + b_8\dot{y}_{t-1}$$
$$+ \sum_{i=1}^{n} b_i\dot{P}_{t-i} + u_t \quad . \tag{60}$$

As with equation (58), the appropriate number of lagged inflation rates is determined empirically by estimating equation (58). Intuitively, one may argue that expected signs on the estimated coefficients, especially b_1 through b_8, should conform with those hypothesized for equation (59). However, since equation (60) is an ad hoc approach <u>suggested</u> by the alternative models herein, such a priori convictions must be tempered.

The following test reports the empirical estimates of equations (58), (59), and (60). These estimates form the basis of the break-point test for rationality found in the third section. Since the dependent variable $\dot{P}_t$ is the same in all estimated equations, the relative explanatory power of the alternative models (based on the coefficient of determination adjusted for degress of freedom) may be compared. In addition, estimating the general model suggests testing the relevance of the variables in the rational expectations model once lagged inflation rates are given. This test may also be applied to lagged inflation given the variables in the rational expectations model. We now turn to these various tests.

Regression Results

Utilizing seasonally adjusted quarterly data on the narrowly defined money stock (M1), the GNP deflator (1972=100), real GNP ($1972), and the federal government surplus, equations (58), (59), and (60) were estimated over the postwar period 1951.II through 1977.IV (107 observations). Except for the (Sur/GNP) variables, all others are expressed as quarterly rates-of-change.

The regression results are reported in Table 2. Looking at equation (58) results first, the autoregressive model, the empirically determined lag structure was found to be four lags. Increasing the number of lagged terms did not improve the explanatory power of the equation when adjusted for degrees of freedom. The rapidity with which the persistence effects decline over time is somewhat surprising. Indeed, these results suggest that the adjustment response is virtually completed within a single year.

Table 2

Regression Results for Expectations Models, 1951.II-1977.IV[a]

	Model Tested		
Variables	Autoregressive (58)	Rational Expectations (59)	General (60)
Constant	0.002 (2.46)***	0.003 (2.19)**	0.001 (0.66)
$\dot{M}_{t-1}$		0.220 (1.90)*	0.103 (1.03)
$\dot{M}_{t-2}$		0.027 (0.19)	-0.009 (0.08)
$\dot{M}_{t-3}$		0.279 (2.03)**	0.212 (1.83)*
$\dot{M}_{t-4}$		0.111 (0.82)	-0.003 (0.02)
$\dot{M}_{t-5}$		0.111 (0.99)	-0.001 (0.01)
$(Sur/GNP)_{t-1}$		-0.037 (0.55)	-0.008 (0.11)
$(Sur/GNP)_{t-2}$		-0.011 (0.17)	-0.049 (0.79)
$\dot{y}_{t-1}$		-0.160 (2.93)***	-0.067 (1.37)
$\dot{P}_{t-1}$	0.401 (4.27)***		0.288 (2.59)**
$\dot{P}_{t-2}$	0.206 (2.07)**		0.187 (1.81)*
$\dot{P}_{t-3}$	0.221 (2.35)**		0.220 (2.34)**
$\dot{P}_{t-4}$	-0.068 (0.77)		-0.099 (1.11)
$\bar{R}^2$	0.517	0.362	0.568
D-W	1.55	1.09	1.70
$\hat{\sigma}$	0.004	0.005	0.004

[a]Absolute values of t-statistics shown in parentheses. All other definitions are the same as those described in Table 1. ***, **, and * denote statistical significance at the 1, 5, and 10 percent levels, respectively.

Equation (58) indicates that over 50 percent of the variation in $\dot{P}$ is explained by the simple autoregressive model. The reported $\bar{R}^2$ is statistically significant at the 1 percent level. The Durbin-Watson indicates a moderate degree of serial correlation in the residuals. It is well known, however, that this test may be questioned when lagged dependent variables are used as explanatory variables.[2]

The regression results for the rational expectations model, equation (59), are interesting on several points. Comparing the $\bar{R}^2$s of the autoregressive and rational expectations models, there is a marked decline in the explanatory power: from 0.517 for the autoregressive to 0.362 for the rational expectations model. Although the reported $\bar{R}^2$ for equation (59) is significant at the 1 percent level, only three estimated coefficients achieve acceptable statistical significance: $\dot{M}_{t-1}$ and and $\dot{M}_{t-3}$ at the 5 percent level, and $\dot{y}_{t-1}$ at the 1 percent level. The estimated $(Sur/GNP)_{t-1}$ and $(Sur/GNP)_{t-2}$ coefficients are not statistically different from zero and have signs different than those hypothesized a priori.

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[2] J. Durbin, "Testing for Serial Correlation in Least Squares Regression When Some of the Regressors Are Lagged Dependent Variables," <u>Econometrica</u> (May, 1970), 410-421. The Durbin-h statistic is calculated as

$$h = (1 - 1/2d)\ \sqrt{T/1 - T \cdot \hat{V}(\beta_1)}$$

where d is the Durbin-d statistic, T is the sample size, and $\hat{V}(\beta_1)$ is the estimate of variance of $\hat{\beta}_1$. The calculated h-statistic for the autoregressive model is 9.91. Since the h-statistic is tested as a standard normal deviate, the h calculated here exceeds 1.645 which does not allow rejection of the hypothesis of zero autocorrelation at the 5 percent level of significance. But, as the present purpose is to forecast inflation and the presence of serial correlation in the residuals affects the estimated standard errors of the independent variables and not the R^2, it is debatable whether or not the presence of serial correlation is of great concern.

In addition, equation (59) indicates a significant degree of serial correlation in the residuals. Based on standard statistical criteria, the rational expectations model of inflationary expectations is inferior to the simple autoregressive model.

The empirical results from estimating the general model of expectations suggest that it may be statistically superior to the alternative models examined in this chapter. The adjusted R^2 far exceeds that of the rational expectations model and is slightly better than that of the autoregressive approach: 0.568 versus 0.362 and 0.517, respectively. Interestingly enough, only the estimated $\dot{P}_{t-1}$ and $\dot{P}_{t-3}$ coefficients (0.288 and 0.220) achieve statistical significance at the 5 percent level. The estimates of $\dot{M}_{t-3}$ and $\dot{P}_{t-2}$ are statistically significant at the 10 percent level. Of the remaining eight variables, only three--$\dot{M}_{t-1}$, $\dot{y}_{t-1}$, and $\dot{P}_{t-4}$--have estimated coefficients that exceed their respective standard errors. Even so, it should again be emphasized that for the purpose of forecasting, the main criteria in selecting the "best" equation is the minimization of the standard error of the estimated equation (adjusted for degrees of freedom). On this basis, the general model of expectations, one which incorporates past inflation rates <u>and</u> structural economic information suggested by theory, is the better forecasting equation.

Before turning to the break-point test for rationality found in the next section, it is useful to statistically ascertain whether or not the general model is in fact "relevant." Using the standard

F-test for exclusion of variables, one can test whether or not adding lagged inflation rates to the rational expectations model and, conversely, adding the structural information provided by the rational expectations model to past inflation data significantly increases the explanatory power, i.e., significantly reduces the sum of squared residuals.

Consider first the test for the irrelevance of the structural information given lagged inflation rates. This is equivalent to testing the null hypothesis that the estimated coefficient b_1 through b_8 in equation (60) are simultaneously zero. This test is implemented by first regressing $\dot{P}_t$ on an array of explanatory variables that includes both the structural information set and the past history of inflation. The actual test examines the impact on the sum of squared residuals when the former is deleted.[3] This procedure yielded the statistic $F^8_{94} = 2.09$ which exceeds the 5 percent critical value of $F^8_{100} = 2.03$. Thus the hypothesis that the structural information set as a whole is irrelevant to the determination of the rate of inflation, given lagged inflation rates, is not accepted at the 5 percent level of significance. The opposite test for the irrelevance of lagged inflation given the set of structural variables produces the F-statistic $F^4_{98} = 12.5$ with the 1 percent critical value being $F^4_{100} = 3.51$. The importance of the past values of inflation is thus confirmed by this test. These results, combined withthe previous comparison of the $\overline{R}^2$s, provide

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[3]See, J. Kmenta, Elements of Econometrics (New York: Macmillan Publishing Company, Inc., 1971), pp. 370-372.

support to the contention that the general model of inflation expectations may be statistically the more appropriate way to measure expectations of future inflation relative to the autoregressive and rational expectations models. Utilizing the information provided by the empirical results reported in this section, we now turn to the break-point test for rationality.

Break-Point Test for Rationality

Once a regime change in the money rule is found, the Chow test described in Chapter V may be applied to the autoregressive, rational expectations, and general models of inflationary expectations. Rationality, in the sense of Sargent,[4] of each alternative model will be determined by model instability across the money rule break point. From the earlier discussion of the autoregressive model,[5] we would expect to find, if this approach is rational, a regime change occurring at that point in time where a break in the money rule was discovered. Similarly, the rational expectations and general models of expectations should also exhibit structural instability in the estimated coefficients across different monetary regimes. Failure to satisfy this test criterion at acceptable statistical levels of significance provides one bit of evidence about the relative efficacy of each approach in modeling the process of inflation expectations during the postwar period.

Chapter V presented the break point test of the estimated money rule. Using the Chow test procedure, it was established that a possible

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[4]Sargent, "Neutrality and Rationality." [5]See fn. 13, Ch. IV.

break in the structure of the money rule occurred in 1964.I. Statistically significant at the 5 percent level, this test result suggests that the two subperiods (1951.II-1964.I and 1964.II-1977.IV) do not come from the same regression population. This information--the time of the break--can be used to test for rationality of the alternative inflation expectations models. Applying the same Chow test procedure to each expectations equation, the calculated F-statistic will reveal whether or not the respective equation remained stable across changes in the monetary regime. As mentioned earlier, instability of the expectations equations (rejection of the null hypothesis of no change) is the present criterion for detecting model rationality.

The calculated F-statistics and appropriate critical values for the Chow test are reported in Table 3. In all cases, the null hypothesis of no structural change is rejected at the 1 percent level of significance. Based on the criterion of structural variance across different money rules, model rationality is, therefore, not rejected for each of the alternative expectations models.

The test results presented in this chapter should not be viewed as conclusive evidence for or against any one of the models. Indeed, on the basis of the Chow test reported in this section, each model may be considered "rational" in the sense of Sargent. However, it is not possible at this time to make any definitive statement regarding the relative efficacy of one approach vis-à-vis its rivals in measuring expectations. Chapters VII and VIII offer further tests for rationality and predictive accuracy that will provide a sounder foundation upon which the relative abilities of the alternative models can be compared.

Table 3

Chow-Test Results for Structural Stability[a]
(1951.II-1964.I, 1964.II-1977.IV)

Model	Calculated F	Critical F
Autoregressive (58)	$F^{5}_{97} = 13.01$	$F^{5}_{100} = 3.20$ (1%)
		$= 2.30$ (5%)
Rational Expectations (59)	$F^{9}_{80} = 5.26$	$F^{9}_{80} = 2.64$ (1%)
		$= 1.99$ (5%)
General (60)	$F^{13}_{81} = 4.37$	$F^{13}_{80} = 2.41$ (1%)
		$= 1.88$ (5%)

[a]The F-statistic is calculated by

$$F^{k}_{T-2k} = \frac{SSR_1 - (SSR_2 + SSR_3)/k}{(SSR_2 + SSR_3)/T - 2k}$$

where

SSR_1 = sum of squared residuals from entire period regression,

SSR_2 = sum of squared residuals from first subperiod regression,

SSR_3 = sum of squared residuals from second subperiod regression,

k = number of variables, and

T = number of observations.

CHAPTER VII

ASSESSING THE FORECASTING ACCURACY OF THE AUTOREGRESSIVE, RATIONAL EXPECTATIONS, AND GENERAL MODELS

The tests presented in this chapter are based on the assumption that economically rational individuals will seek out and employ that predictive mechanism which minimizes the forecast error. It was demonstrated in Chapter I that if there exists some non-negative cost associated with misestimating future rates of inflation, then the optimizing forecaster will attempt to minimize the expected value of the loss function $E[\dot{P}_t - E_{t-1}(\dot{P}_t)]$. Using the out of sample forecasts generated by each of the models estimated in Chapter VI, it is possible to examine the respective forecasting ability of the alternative specifications. To this end, two standard statistical criteria for measuring predictive accuracy have been employed: the root-mean-squared error (RMSE) and the Theil-U coefficient. In the next section, the test procedure and the test statistics are described, and the empirical results are presented. The second section offers some concluding remarks.

Tests of Post-Sample Forecasting Accuracy

One way to measure a model's ability to predict future movements in some variable is to compare its post-sample extrapolations to actual realized events. This may be accomplished by estimating the model and using the parameter estimates to form predictions beyond the estimating sample. Comparing the RMSE and Theil-U statistics of these forecasts for each competing model provides evidence concerning the relative accuracy of each model's predictions.

Prior to estimating the post-sample forecasts for each model, the temporal stability of the forecasting models, as discussed in Chapter VI, must be considered. In the preceding chapter it was found that all three models were not structurally stable across the entire sample period 1951.II-1977.IV. Applying the Chow test, it was found that 1964.I represents a break point common to all three models. Since the parameter estimates from the two subperiods 1951.II-1964.I and 1964.II-1977.IV apparently belong to different regression populations, the subperiods will be utilized in the following analysis.

Table 4 reports the 1951.II-1964.I subperiod regression results for each forecasting model. The most striking result is the general breakdown in the models' ability to explain movements of inflation during the period. This is indicated by the low $\bar{R}^2$ values for each model. Relative to its competitors, the rational expectations model appears to be the "best" specification even though the $\bar{R}^2$ is only 0.127. Overall, the results for the first period are disappointing. Many of the estimated coefficients are not statistically significant at any reasonable level. The signs of the parameter estimates differ in many cases from those hypothesized a priori. These findings do, however, agree with those reported by Turnovsky.[1] He, too, found a significant lack of explanatory power of various inflation equations over a similar period (1954-1964). Indeed, the results presented in Table 4

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[1]Turnovsky, "Formation of Price Expectations."

Table 4

Regression Results for Expectations Models,[a] 1951.II-1964.I

Variables	Autoregressive	Rational Expectations	General
		Model Tested	
Constant	0.005 (5.27)***	0.005 (5.10)***	0.006 (3.95)***
$\dot{M}_{t-1}$		-0.030 (0.23)	-0.030 (0.21)
$\dot{M}_{t-2}$		-0.356 (2.35)**	-0.296 (1.83)*
$\dot{M}_{t-3}$		0.308 (2.05)	0.250 (1.53)
$\dot{M}_{t-4}$		0.072 (0.49)	0.068 (0.40)
$\dot{M}_{t-5}$		-0.223 (1.83)*	-0.170 (1.27)
$(Sur/GNP)_{t-1}$		-0.083 (1.18)	-0.114 (1.20
$(Sur/GNP)_{t-2}$		0.090 (1.48)	0.153 (1.61)
$\dot{y}_{t-1}$		0.114 (1.96)*	0.122 (1.97)*
$\dot{P}_{t-1}$	-0.043 (0.41)		-0.012 (0.09)
$\dot{P}_{t-2}$	-0.086 (0.83)		-0.119 (0.90)
$\dot{P}_{t-3}$	0.191 (2.13)**		0.011 (0.09)
$\dot{P}_{t-4}$	-0.095 (1.08)		-0.124 (1.19)
$\bar{R}^2$	0.029	0.127	0.096
D-W	1.89	1.92	1.45
$\hat{\sigma}$	0.0040	0.0035	0.0035

[a]Absolute values of t-statistics shown in parentheses; ***, **, and * denote statistical significance at, respectively, 1, 5, and 10 percent levels. $\bar{R}^2$ is the coefficient of determination adjusted for degrees of freedom; D-W is the Durbin-Waston statistic, and $\hat{\sigma}$ is the standard error of the estimate.

suggest that over the 1951.II-1964.I period, inflation may have been relatively constant. If so, this may explain the lack of any significant empirical relationship between the explanatory variables and the actual time path of inflation rates.

Using the parameter estimates generated by each model, post sample forecasts of the inflation rate over the period 1964.II-1967.I (12 quarters) are generated. The actual rate of inflation and the models' forecasts for this period are found in Table 5. An examination of the actual and forecasted values of inflation rates reveals a relatively poor performance by each model. This is especially true for the 1966.I-1966.IV period where all models substantially underpredict the actual rate of inflation.[2] In addition, the rational expectations and general models predict a large decline in the rate of inflation for 1965.I when in fact the actual rate increased nearly four times its 1964.IV value.[3]

To statistically assess the post sample predictive accuracy of the various models, the RMSE and Theil-U for each model has been calculated using the actual and predicted values of inflation over the 1964.II-1967.I period. Minimization of the RMSE is considered to be one criterion for predictive accuracy when comparing alternative forecasts. The RMSE measures the absolute deviation of forecasted values from the actual observed values. The RMSE statistic is calculated by

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[2]Over this period, the average error for the autoregressive, rational expectations, and general models are, respectively, -0.483, -0.544, and -0.551.

[3]The forecast errors for the rational expectations and general models for 1965.I are, respectively, 0.824 and 0.751, whereas the error for the autoregressive model is 0.367.

Table 5

Post-Sample Forecasts, 1964.II-1967.I

Year and Quarter	Actual	Autoregressive	Rational Expectations	General
1964.II	0.336	0.455	0.520	0.626
1964.III	0.552	0.535	0.608	0.678
1964.IV	0.220	0.412	0.207	0.210
1965.I	0.821	0.454	-0.003	0.070
1965.II	0.506	0.494	0.658	0.692
1965.III	0.668	0.371	0.679	0.656
1965.IV	0.494	0.538	0.419	0.596
1966.I	0.988	0.414	0.254	0.254
1966.II	1.188	0.469	0.267	0.322
1966.III	0.586	0.369	0.382	0.329
1966.IV	0.911	0.489	0.594	0.566
1967.I	0.588	0.518	0.750	0.691

NOTE: Regression run on sample 1951.II-1964.I and forecasted 12 quarters ahead.

$$RMSE = \frac{1}{T} \sqrt{\sum_{t=1}^{T} (\dot{P}_t^e - P_t^a)^2} \qquad (61)$$

where $\dot{P}_t^e$ represents the predicted inflation rate for time t, $\dot{P}_t^e$ is the actual rate, and T denotes the number of periods or size of the sample. Applying the RMSE criterion to the post sample forecast error series generated by each model thus permits one evaluation of the relative accuracy of their respective forecasts. The model producing the smallest (i.e., closest to zero) RMSE may be judged as providing the more accurate inflation forecast.

The second measure of predictive accuracy is the Theil-U coefficient. It is calculated as

$$U = \sqrt{\frac{\sum_{t=1}^{T} (\dot{P}_t^e - \dot{P}_t^e)^2}{\sum_{t=1}^{T} (\dot{P}_t^a)^2}} \quad . \qquad (62)$$

The variables $\dot{P}_t^e$ and $\dot{P}_t^a$ are defined as before. This measure is useful in that it may be thought of as indicating the "seriousness" of the forecast error.[4] As the measure clearly indicates, when U = 0, all forecasts are "perfect," i.e., $\dot{P}_t^e = \dot{P}_t^a$ for all t = 1,2,3,...,T. For the three models examined in this study, it is possible to compare respective U-values and, based on the relative magnitudes of the estimates, make some statement concerning the predictive capabilities of each model vis-à-vis the others. Thus, the model with the lower U-value may be though of as a more accurate forecasting specification.

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[4]H. Theil, Applied Economic Forecasting (2nd ed.; Amsterdam: North-Holland Publishing Company, 1966), p. 26.

The RMSE and Theil-U statistics for each model's post-1964.I forecasts are reported in Table 6. Comparing these statistics across models indicates that the simple autoregressive model provides the relatively more accurate post-sample inflation forecasts. Although the calculated RMSE values are relatively close for each alternative model, the Theil-U statistic for the autoregressive model is substantially less than either the rational expectations or the general models.[5]

Given the regression results reported in Table 4, this finding is somewhat surprising. Note, however, that the autoregressive model is essentially predicting the mean inflation value over the sample period 1951.II-1964.I. This is indicated by the low $\bar{R}^2$ and highly significant constant term. This being the case, post sample forecasts derived from this specification essentially yield forecasts so influenced. In essence, then, that model which basically forecasts the mean from the regression sample (e.g., the autoregressive model) appears to provide relatively more accurate post sample forecasts of the inflation rate. On the basis of the statistics reported in Table 6, the autoregressive model is judged as providing the more accurate inflation forecasts over the post sample period 1964.II-1967.I.

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[5]Ibid. Theil suggests a test for the significance of the difference between the alternative U-values. This test, which utilizes the estimated standard errors for the U^2 values, is based on the assumption that n pairs of predicted ($\dot{P}^e_t$) and actual ($\dot{P}^a_t$) changes in the inflation rate are essentially random drawings from a bivariate normal population. In a sense, this procedure is similar to testing the null hypothesis that the different model's forecasts are as good or as bad as constant change extrapolations. In the present analysis, there is no reliable method to test the significance between the U-values of the three models since each has nonconstant change predictions. For a similar problem to the one faced here, see F. Zahn and W. R. Hosek, "Impact of Trade Credit on the Velocity of Money and the Market Rate of Interest," Southern Economic Journal (October, 1973), 202-209.

Table 6

Root-Mean-Squared Errors and Theil-U Coefficients for Inflation Models: 1964.II-1967.I Extrapolations[a]

Model	RMSE	Theil-U
Autoregressive	0.0969	0.4750
Rational Expectations	0.1264	0.6195
General	0.1255	0.6154

[a]Based on Table 5.

To analyze the post-sample forecasting abilities of the models during the 1964.II-1977.IV subperiod, the competing inflation specifications were reestimated. Because the data set used in this study ends in 1977.IV, and in that we desire to make post-sample forecasts, the inflation models were reestimated over the period 1964.II-1974.IV and post-sample (1975.I-1977.IV) forecasts were generated. The 1964.II-1974.IV regression results are presented in Table 7. Examining the results from the 1964.II-1974.IV period reported in Table 7, the most interesting outcome is the relatively close explanatory power of the autoregressive and general models. The $\bar{R}^2$ for the simple autoregressive model is 0.744, whereas the same measure for the general model is 0.795. Since both of these estimates far exceed that for the rational expectations model ($\bar{R}^2$ = 0.486), this suggests that each would be chosen (on the basis of historical performance) over the rational expectations model by the optimizing forecaster. However, since we are concerned here with each model's ability to forecast inflation, the $\bar{R}^2$ criterion is not sufficient in choosing one model over the others.

Using the parameter estimates reported in Table 7, post-sample extrapolations for each model over the period 1975.I-1977.IV (12 quarters) have been generated. The actual inflation rate for this period, as well as each model's forecasts, are found in Table 8. Relative to the autoregressive and general models, the rational expectations model again exhibits a poor forecasting ability in the 1975.I-1977.IV post-sample period. Although the autoregressive model produces substantial forecasting errors in the first few quarters, its forecasts after 1975.I

Table 7

Regressions Results for Expectations Models,[a] 1964.II-1974.IV

Dependent Variable: $\dot{P}_t$	Model Tested		
Variables	Autoregressive	Rational Expectations	General
Constant	0.0002 (0.16)	0.0045 (1.22)	0.0001 (0.04)
$\dot{M}_{t-1}$		0.130 (0.63)	-0.092 (0.59)
$\dot{M}_{t-2}$		0.214 (0.98)	0.219 (1.35
$\dot{M}_{t-3}$		0.252 (0.98)	0.025 (0.13)
$\dot{M}_{t-4}$		0.040 (0.16)	-0.185 (0.94)
$\dot{M}_{t-5}$		0.172 (0.89)	0.064 (0.39)
$(Sur/GNP)_{t-1}$		0.033 (0.19)	0.109 (0.88)
$(Sur/GNP)_{t-2}$		-0.045 (0.23)	-0.090 (0.61)
$\dot{y}_{t-1}$		-0.402 (4.02)***	-0.015 (0.16)
$\dot{P}_{t-1}$	0.573 (3.53)***		0.534 (2.73)***
$\dot{P}_{t-2}$	0.273 (1.53)		0.272 (1.31)
$\dot{P}_{t-3}$	0.255 (1.37)		0.162 (0.64)
$\dot{P}_{t-4}$	-0.035 (0.20)		0.109 (0.52)
$\bar{R}^2$	0.744	0.486	0.795
D-W	1.99	1.18	1.99
$\hat{\sigma}$	0.0033	0.0050	0.0034

[a]See footnote to Table 4 for a definition of terms.

Table 8

Actual and Forecasted Rates of Inflation: 1975.I-1977.IV[a]

Year and Quarter	Actual	Autoregressive	Rational Expectations	General
1975.I	2.565	3.094	1.910	2.758
1975.II	1.396	2.909	2.311	2.849
1975.III	1.764	2.185	0.576	1.668
1975.IV	1.519	1.965	0.382	2.172
1976.I	0.998	1.641	1.206	1.955
1976.II	1.208	1.410	0.503	1.135
1976.III	1.116	1.313	0.991	1.274
1976.IV	1.326	1.193	1.148	1.418
1977.I	1.291	1.360	1.384	1.132
1977.II	1.720	1.366	0.899	1.388
1977.III	1.183	1.659	1.282	1.708
1977.IV	1.422	1.452	1.300	1.323

[a]Based on regerssion equations in Table 7.

are generally on target. This is also the case for the general model even though the magnitude of its post-sample forecast errors are generally smaller in magnitude.

To assess the overall predictive abilities of the autoregressive, rational expectations, and general models, the post-1974 forecasts of each are used to calculate the RMSE and Theil-U statistics. The results of these calculations are presented in Table 9. As an examination of Table 9 will confirm, the autoregressive model provides the lowest RMSE of the three specifications. Its value (0.1627) is below that of the rational expectations model (0.1908) and the general specification (0.1657). Using the RMSE as a basis of comparison, the findings in Table 9 suggest that over the 1964.II-1977.IV subperiod, the autoregressive model may have produced forecasts that were superior to those of the rational expectations and general models.

With regard to the Theil-U statistics presented in Table 9, the preceding notion is supported further. Compared to its rival specifications, the autoregressive approach to forecasting inflation again appears to be the superior approach. Its Theil-U value of 0.3733 is less than that of the rational expectations model (0.4378) and the general specification (0.3803). Although the closeness of the RMSE and Theil-U statistics found in Table 9 invites some caution,[6] by the present criterion (minimization of the RMSE and Theil-U), the autoregressive model represents the more accurate forecasting model during the second subperiod.

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[6]See Zahn and Hosek, "Impact of Trade Credit."

Table 9

Root-Mean-Sqaured Errors and Theil-U Coefficients
for Inflation Models: 1975.I-1977.IV
Extrapolations[a]

Model	RMSE	Theil-U
Autoregressive	0.1627	0.3733
Rational Expectations	0.1908	0.4378
General	0.1657	0.3803

[a]Based on Table 8

Concluding Remarks

The findings of this chapter may be briefly summarized. Based on the belief that economically rational individuals will seek out and employ that predictive model which minimizes the forecast error, this chapter offers on possible approach to find such a model. The approach used here was to generate post-sample forecasts of inflation from each alternative model and compare their respective RMSE and Theil-U statistics. Such calculations for the two subperiods suggested in Chapter VI reveal that the simple autoregressive model provides relatively more accurate post-sample forecasts than either the rational expectations or general approaches. Indeed, these results strongly suggest that, once past inflation rates are known, additional economic information such as past money growth rates, the federal government's fiscal activities, and past growth rates of real output may be irrelevant in obtaining accurate forecasts of inflation.[7]

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[7]Such findings are similar to those of Feige and Pearce, "Economically Rational Expectations."

CHAPTER VIII

AN ECONOMIC APPLICATION OF INFLATION PREDICTIONS

It has been suggested that a powerful test for rationality is one that explicitly incorporates forecasts obtained from alternative expectations models into certain economic contexts.[1] Such an approach is premised on the belief that individuals in the economy will exploit that model which provides relatively better information about the actual inflation process. To illustrate how this approach is applied in the present analysis, the effects of the various measures of inflation expectations on nominal interest rates in the United States will be estimated for the period 1951.II-1977.IV. Using the well-known Fisher equation, the expected inflation series generated by each alternative expectations model examined in this study is utilized. Determination of the model that more accurately measures the actual process by which individuals form their inflationary expectations can be achieved by the statistical criterion of goodness-of-fit, since each regression equation employs the same dependent variable. In other words, if the expected inflation series from, say, the autoregressive model minimizes the standard error of the estimating equation (adjusted for degrees of freedom) relative to the rational expectations or general models in explaining movements in nominal interest rates, then it may be considered a better empirical representation of the

[1]Sargent, "Neutrality and Rationality."

expectations formation mechanism. In the next section, the set-up of the test, the data used, and the regression results are described.

The Effects of Expected Inflation on Nominal Interest Rates

The effect of expected inflation on nominal interest rates has been examined by economists for many years. Since the appearance of Irving Fisher's explanation of the Gibson Paradox, the problem has been argued on both theoretical and, more recently, empirical grounds.[2] Perhaps the most widely used technique in the various empirical studies has been to assume that expectations of future inflation are based solely on past inflation experiences. Thus, various hypotheses to explain the autoregressive formation of expectations--weighted, adaptive, and extrapolative--have been tested in this context.

Some researchers have also employed directly observed expectations data to address this question. Such data, compiled by Joseph A. Livingston, have been substituted for the various sutoregressive measures of expected inflation.[3] One such study concludes that support of the

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[2]See, for example, A. Alchian and R. Kessel, "Effects of Inflation," Journal of Political Economy (December, 1962), 521-537; R. A. Mundell, "Inflation and Real Interest," Journal of Political Economy (June, 1963), 280-283; W. P. Yohe and D. Karnosky, "Interest Rates and Price Level Changes," Federal Reserve Bank of St. Louis Review (December, 1969), 19-36; W. E. Gibson, "Price Expectations Effects on Interest Rates," Journal of Finance (March, 1970), 19-34, and "Interest Rates and Inflationary Expectations: New Evidence," American Economic Review (December, 1972), 854-865; D. H. Pyle, "Observed Price Expectations and Interest Rates," Review of Economics and Statistics (August, 1972), 275-280; and K. Lahiri, "Inflationary Expectations: Their Formation and Interest Rate Effects," American Economic Review (March, 1976), 124-131.

[3]See, for example, Gibson, "Price Expectations Effects," and Pyle, "Observed Price Expectations."

Fisher effect is not dependent on the use of distributed lag schemes to measure expected inflation.[4]

It is the intention of this author to empirically compare the relative efficacy of the various measures of expected inflation (derived from the models used in this study) in explaining variations in the nominal interest rate. This is achieved by estimating the equation

$$i_t = a_0 + a_1\dot{P}_1^e + u_t \qquad (63)$$

where i_t is the nominal interest rate and $\dot{P}_t^e$ is the expected rate of inflation. Briefly, equation (63) is based on the notion that, assuming the real rate of interest (empirically measured by the constant term a_0) to be constant, the nominal rate (i_t) will vary with the expected rate of inflation. Thus, if the real rate is constant, the Fisher hypothesis that the nominal rate fully adjusts to expected inflation would be confirmed by the coefficient a_1 taking on an estimated value of unity. On the other hand, $\hat{a}_1 = 0$ implies that the nominal rate is not affected (there is no adjustment) by expected inflation.

To empirically estimate the relationship between the nominal interest rate and the alternative measures of expected inflation, two commonly used maturity horizons have been chosen for the interest rate: the four to six month commerical paper rate and the Aaa seasoned bond yield. The $\dot{P}_t^e$ measures are obtained from estimates of the autoregressive, rational expectations, and general models. The criterion for selecting the best fitting equation, i.e., the "best" approach to measuring inflationary

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[4]See, Pyle, "Observed Price Expectations."

expectations, is the coefficient of determination corrected for degrees of freedom ($\bar{R}^2$). Since this procedure is valid only when a common dependent variable is used, comparison of the $\bar{R}^2$s can be done only for each separate interest rate series.

Estimates of equation (63) for the two market rates are presented in Table 10. In every case, the estimated value of a_1 is statistically significant at the 1 percent level suggesting a strong positive association between nominal interest rates and each expected inflation series during the 1951.II-1977.IV period. The adjustment effect of market rates to expected inflation appears to be fairly rapid: the average $\hat{a}_1$ coefficient for the four to six month paper rate is 0.811 and, for the Aaa bond rate, 0.798. According to the Fisher hypothesis discussed above, a coefficient of $\hat{a}_1 = 1$ would indicate perfect adjustment of the interest rate to expected inflation. Among the estimated coefficients found in Table 10, however, the largest value for a_1 is 0.844 which, based on a t-test, is significantly different from unity at the 5 percent level (1 = 1.92). These results are not, however, consistent with those of previous estimates (see Gibson, Pyle, and Lahiri).[5] In these works, the estimated coefficient a_1 was generally found to be in much closer proximity to unity.

One possible explanation for this divergence of findings is the different time periods studied. This research focuses on the period 1951.II-1977.IV, whereas the earlier works examined the relationship during the 1952-1970 era. Given the virulent nature of inflation since

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[5]Gibson, "Price Expectations Effects," Pyle, "Observed Price Expectations," and Lahiri, "Inflationary Expectations."

Table 10

Interest Rates and Expected Rates of Inflation,
Regression Results[a]
(1951.II-1977.IV)

Equation tested: $i_t = a_0 + a_1\dot{P}_t^e$

Model	Interest Rate	a_0	a_1	$\bar{R}^2$	$\hat{\sigma}$
Autoregressive	4-6 month paper	0.017* (5.76)	0.816* (10.87)	0.525	0.0138
	Aaa bond	0.026* (9.56)	0.753* (10.62)	0.513	0.0131
Rational Expectations	4-6 month paper	0.016* (4.57)	0.832* (8.90)	0.425	0.0152
	Aaa bond	0.023* (7.54)	0.844* (10.43)	0.500	0.0132
General	4-6 month paper	0.018* (6.79)	0.787* (11.87)	0.569	0.0132
	Aaa bond	0.025* (11.69)	0.795* (14.62)	0.667	0.0180

[a]"*" denotes significance at 1 percent level (one-tailed), $\bar{R}^2$ is the coefficient of determiniation corrected for degrees of freedom, and $\hat{\sigma}$ is standard error of the estimate.

1970, it is quite possible that the previously estimated relationship has since broken down.[6]

Examination of the reported $\bar{R}^2$s for each equation reveals that the general model of expectations may be the preferred approach. For both the four to six month paper rate and the Aaa bond yield, the general model's $\bar{R}^2$, 0.569 and 0.667, respectively, is the largest. Interestingly, the simple autoregressive model yields the second highest $\bar{R}^2$ values for the entire period estimates. In addition to the $\bar{R}^2$ estimates, the general and autoregressive models maintain their relative superiority if one examines the calculated t-statistics for each equation. Thus, on the basis of the evidence presented in Table 10, it would appear that a rational individual would do well to employ the general model to forecast inflation.

To be consistent with the findings of Chapter VI that there exists a break in the structure of the estimated inflation equations at 1964.I, it may be useful to examine subperiod estimates of equation (63). Doing so allows me to test the notion that temporal changes in inflationary expectations may have substantial impacts on the hypothesized relationship between market interest rates and expected inflation. Such a reestimation is in the spirit of previous research wherein a general break in equation (63) was discovered around 1960.[7]

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[6] This may be especially true regarding the assumption of a constant real rate of interest.

[7] See, Gibson, "Price Expectations Effects," and Lahiri, "Inflationary Expectations."

Subperiod estimates of equation (63) for 1951.II-1964.I and 1964.II-1977.IV are found in Tables 11 and 12, respectively.[8] Comparison of these subperiod results with those from the entire period suggests that the relationship embodied in equation (63) is indeed highly sensitive to the estimation period. Turning first to the 1951.II-1964.I findings in Table 11, it appears that an overall breakdown in the hypothesized relationship occurs. None of the estimated a_1 values achieve statistical significance at any reasonable level and, in the case of the autoregressive model, the coefficient for a_1 takes on a negative value. Based on the underlying theory, these subperiod results imply that the market rates did not adjust to expected inflation. Although difficult to explain, my results for this period are quite similar to those reported by others.[9]

The increase in the estimated values of a_1 and the $\bar{R}^2$s for the 1964.II-1977.IV period relative to the previous period is indeed dramatic. Consider, first, the regression results for the four to six month commerical paper rate. The largest $\hat{a}_1$ value, 0.687, is obtained from the autoregressive model. This value implies that a

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[8]To substantiate the use of subperiod estimates, a Chow test for structural stability was employed. These test results are as follows:

Model	Calculated F-statistics (F^2_{103}): 4-6 Month Paper Rate	Aaa Bond Rate
Autoregressive	59.78	79.57
Rational Expectations	41.77	65.69
General	48.78	37.79

In all cases, the calculated F-statistics far exceed the critical F-value at the 1 percent level of significance (Critical F^2_{100} = 4.89). Thus, we reject the null hypothesis of no structural change for each specification.

[9]Gibson, "Price Expectations Effects," and Lahiri, "Inflationary Expectations."

Table 11

Interest Rates and Expected Rates of Inflation, Regression Results[a]
(1951.II-1964.I)

Equation tested: $i_t = a_0 + a_1\dot{P}^e_t$

Model	Interest Rate	a_0	a_1	$\bar{R}^2$	$\hat{\sigma}$
Autoregressive	4-6 month paper	0.030* (6.66)	-0.058 (0.24)	0.001	0.0080
	Aaa bond	0.037* (10.78)	-0.038 (0.21)	0.001	0.0061
Rational Expectations	4-6 month paper	0.026* (8.91)	0.152 (1.03)	0.021	0.0079
	Aaa bond	0.034* (15.33)	0.110 (0.98)	0.019	0.0060
General	4-6 month paper	0.026* (9.69)	0.127 (0.93)	0.017	0.0079
	Aaa bond	0.034* (16.39)	0.119 (1.15)	0.026	0.0060

[a]"*" denotes significance at 1 percent level (one-tailed), $\bar{R}^2$ is the coefficient of determination corrected for degrees of freedom, and $\hat{\sigma}$ is standard error of the estimated.

Table 12

Interest Rates and Expected Rates of Inflation,
Regression Results[a]
(1964.II-1977.IV)

Equation tested: $i_t = a_0 + a_1\dot{P}^e_t$

Model	Interest Rate	a_0	a_1	$\bar{R}^2$	$\hat{\sigma}$
Autoregressive	4-6 month paper	0.026* (6.20)	0.687* (8.71)	0.581	0.0107
	Aaa bond	0.039* (10.68)	0.553* (7.81)	0.526	0.0096
Rational Expectations	4-6 month paper	0.028* (4.20)	0.634* (4.83)	0.293	0.0139
	Aaa bond	0.035* (6.92)	0.640* (6.43)	0.428	0.0106
General	4-6 month paper	0.027* (6.77)	0.654* (8.55)	0.572	0.0108
	Aaa bond	0.041* (11.27)	0.519* (7.45)	0.502	0.0098

[a]"*" denotes significance at 1 percent level (one-tailed), $\bar{R}^2$ is the coefficient of determination corrected for degrees of freedom, and $\hat{\sigma}$ is standard error of the estimate.

1 percent increase in the expected rate of inflation will induce a 0.687 percent increase in the nominal rate of interest. Although this adjustment factor is significantly less than unity, relative to the 1951.II-1964.I findings it indicates a much more rapid adjustment of the paper rate to expected inflation during more recent times. Also, the $\bar{R}^2$ for the autoregressive model is the largest obtained from any of the alternative models tested.

The findings for the Aaa bond rate also suggest that the autoregressive model may be superior to its rivals over this latter period. The $\bar{R}^2$ for the autoregressive model is 0.526 compared to 0.502 and 0.428 for the general and rational expectations models, respectively. An interesting result from this subperiod is the general size differential between the a_1 estimates for the commercial paper rate and the Aaa bond rate. Others have found that the impact of expected inflation decreases with a lengthening of the term to maturity. Comparing the average a_1 estimates for the paper rate (0.658) to those for the bond rate (0.571) supports such a view.

The subperiod estimates strongly suggest a significant structural change in the empirical relationship between nominal interest rates and expected inflation rates around the time 1964.I. Recalling that in Chapter VI the Chow test indicated a break in the structure of the inflation equations, and that the equations performed significantly better over the second subperiod, these two findings suggest that changes in economic behavior (e.g., adjustment of market rates) may have been greatly influenced by the rate at which expectations were being altered. In other words, if the prevalence of inflation during the 1964.II-1977.IV period was such that relatively more rapid adjustment in behavior was necessary to maintain certain profit opportunities, then the observed difference in estimated a_1 coefficients between the two subperiods is not as surprising as one might think.

Although previous research concerning the effects of expected inflation on various market rates of interest have generally tended to support the Fisher hypothesis, my results do not. The largest adjustment coefficient estimated in this chapter is 0.844 which, based on a t-test, is significnatly different from unity. This value, however, was obtained from an entire period regression. Subsequent analysis for structural change in the interest rate-expected inflation relationship revealed that a break at 1964.I is present. In light of this, the largest a_1 coefficient, found using the autoregressive model for the 1964.II-1977.IV period, is 0.687. Again this value is significantly different from unity.

This lack of perfect adjustment in the nominal rates to expected inflation may be the outcome of several possible forces at work in the economy. One, already alluded to above, is that individuals may not have the ability to adjust their inflation expectations at rates fast enough to keep abreast of the actual inflation process. This lack of response may explain the less than perfect adjustment in market rates. Another possible consideration concerns the maintained assumption that the real rate of interest remains virtually unchanging over the estimation period. Irrespective of the model used to generate inflation expectations and period tested, the findings reported here strongly suggest that inflationary expectations may have exerted a significant influence on the real rate over time. This is implied by the estimated a_1 values of less than unity. Clearly, the results of the present analysis suggest the need for further research.

To conclude this chapter, the present issue at hand will be addressed. Based on the hypothesis that individuals acting rationally would exploit that model which provides inflation predictions which minimize the loss of a decision made in regard to some economic context, the purpose of this chapter was to provide such an economic framework from which a superior expectations model might be chosen. To this end, the much examined Fisher hypothesis was employed. Estimating equation (63) over the 1951.II-1977.IV period, along with what appears to be the statistically appropriate subperiods of 1951.II-1964.I and 1964.II-1977.IV, the evidence points to the autoregressive model as providing the "best" approach to measuring inflation expectations. Especially when one considers the evidence suggesting that perhaps only the second subperiod may be used for analysis, the autoregressive model yields the largest estimates of the $\bar{R}^2$, and, therefore, the superior approach to modeling expectations.

CHAPTER IX

SUMMARY

In this study, several empirical tests were presented to measure the relative abilities of alternative models in capturing the unobservable process by which individuals may form expectations of future inflation. These models differ in that each is based on the assumption that individuals may exploit different information sets available at the time when expectations of the future are formed. As noted in Chapter I, individuals will employ information of various types in an attempt to minimize the disparity between ex ante forecasts and actual outcomes.

Three econometric representations of the inflation expectations process have been tested: an autoregressive model in which the information set used by the individual is assumed to consist of only past inflation data; a rational expectations model in which the structural economic relationships in the economy (excluding past inflation) make up the available information set; and a general model which utilizes both structural economic data and past inflation rates in generating predictions of future inflation rates. Although these models do not encompass the entire spectrum of available specifications, they are representative examples of alternative avenues to measuring expectations.

The competing models were each subjected to tests for rationality and forecasting efficacy. The rationality tests consisted of the breakpoint test and one which uses inflation predictions from each model in an economic context. To determine which model provides relatively more accurate inflation forecasts, post-sample Theil-U and root-mean-squared error (RMSE) statistics were compared for each model. To conclude this study, it is useful to review these various test results.

The breakpoint test for rationality is found in Chapter VI. Rationality, in the sense of Sargent, of each model is determined by testing instability across money rule regime changes. In essence, this test asserts that if there are changes in underlying structural relationships within the economy (for example, a change in the money supply process), then we would expect rational economic individuals to adjust their expectations formation process accordingly. Indeed, the breakpoint test results indicate exactly this: using the money rule break point (1964.I) as a reference point, it was found that each expectations model exhibited a regime change at this point. On the basis of this test, the autoregressive, rational expectations, and general models of inflationary expectations appear to be rational.

It was felt that comparing the relative accuracy of each model in predicting future (i.e., post-sample) inflation would provide statistical evidence toward determining which model may better represent the inflation expectations process. Using standard measures of forecasting accuracy (the Theil-U and RMSE), it was found in Chapter VII that the autoregressive model provides subperiod inflation rate forecasts for each of the subperiods analyzed. Indeed, the autoregressive model's statistical performance far exceeded that of the rational expectations model and was generally better than the general model.

To further analyze the rationality of the models' predictions, the forecasts generated by each expectations specification were incorporated into the well-known Fisher equation. This approach provides a more economic orientation to the problem of choosing between alternative expectations models and is not without precedent (see, for example, John Rutledge). If the expected (predicted) inflation series from, say, the autoregressive model minimizes the standard error of the estimating equation relative to the rational expectations or general models in explaining movements in market interest rates, then it may be considered a better representation of the expectations process and one which rational economic individuals appear to be utilizing. Given the break-point test results discussed previously, the relationship between nominal interest rates and expected inflation was examined for the two subperiods 1951.II-1964.I and 1964.II-1977.IV. Due to the lack of any reliable results for the first period, we shall focus on the second subperiod findings.

The evidence from this latter period points to the autoregressive model as providing the "best" approach to modeling inflationary expectations. Based on the $\bar{R}^2$ criterion, this specification better explains movements in both the four to six month commercial paper rate and the Aaa seasoned bond rate during the 1964.II-1977.IV period. From this test, the autoregressive model would be chosen by rational economic individuals, because it would minimize the forecast errors associated with predicting movements in nominal interest rates.

From the various tests performed in this study, it would appear that a rational economic individual attempting to obtain optimal (error minimizing) forecasts of future inflation would select the autoregressive over the rational expectations and general approaches. In three out of the four tests presented, the autoregressive model performed as well if not better than its competition. For the purposes of forecasting inflation, the simple autoregressive model, which utilizes only past inflation information, outperforms the relatively more information-intensive rational expectations and general models.

Expectations play a crucial role in determining economic behavior. If one consider expectations to be informed predictions of the future, then finding the superior forecasting model may also provide information concerning the unobservable process by which individuals form expectations. In this study, an attempt has been made to provide an empirical format by which the more "appropriate" model of the unobservable inflation expectations process can be ascertained. The results suggest that such a model is one that uses only past inflation data. Clearly, there is a need for additional research in this area, especially in two directions. The first concerns the further investigation into models derived from some structural model of the economy. Models of that nature are useful in that they allow the researcher to more fully investigate the impacts of certain structural changes on the expectations process. The second direction of future research is in the way of applying more advanced econometric techniques to the problem at hand. Separately, and in combination, such investigations can only enhance our incomplete knowledge of the inflationary expectations process.

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APPENDIX A

A NOTE ON THE MEASUREMENT AND EFFECTS OF UNANTICIPATED MONEY GROWTH

Recently the rational expectations literature has devoted a significant amount of attention to analyzing the differential impacts of actual and unanticipated money growth on real economic variables. Utilizing the concept of observational equivalence, Neftci and Neftci and Sargent have sought to empirically estimate the effects of unanticipated money on real output during post-war years for the United States.[1] In a recent paper, Robert Barro has extended his previous research to include an analysis of unexpected money growth's impact on the price level.[2] Briefly, Barro's empirical results suggest that once the effects of unanticipated money are accounted for, actual money growth is irelevant to the determination of the price level.

The discussion in this Appendix should be viewed as a methodological aside to the text. As mentioned in the text, the procedure has been to estimate a money rule and then to use the residuals (actual minus predicted) form the equation as a measure of unexpected money. Although this procedure is acceptable within certain bounds, failure to explicitly account for regime breaks in the money rule may impart bias in the unexpected money growth series. The previous research in this area, especially those cited above, do not explicitly state whether or not such adjustments for the break are made. In the tests performed here, it is shown that once the regime break is taken into account, unexpected

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[1]Neftci, "Unanticipated Money Growth," and Neftci and Sargent, "Natural Rate Hypothesis."

[2]Barro, "Money, Output, and Price Level."

money growth may be irrelevant to the determination of future inflation when actual money or past inflation is given.

Quite simply, the test involves the estimation of the money rule presented in Chapter V and the construction of two alternative unexpected money growth series. One series is found by estimating the residuals of the money rule over the entire estimation period. This series implicitly assumes that there is no regime change in the money rule. It is this series which appears to be the one used in previous studies. Clearly the opportunity for biased estimates of unexpected money growth is substantial in this situation. The second series is obtained by calculating the money rule residuals over the two subperiods so delineated by a regime change. In the text, it is reported that, based on a Chow test, 1964.I appears to be a breakpoint in the money rule. Therefore, the second unexpected money series comes from each subperiod as would be consitent with the finding of different monetary regimes.

To demonstrate the importance of this regime change recognition, the money rule for the period 1950.II to 1977.IV has been estimated, and the alternative unexpected money growth series has been generated in accordance with the preceding discussion. After several trial estimations, the appropriate lag structure for the unexpected money series was established. In the estimated inflation equations, unanticipated money growth was substituted for actual money in keeping with the spirit of previous work.[3] Using the regime break unadjusted series, the regression results for the entire period are (t-statistics in parentheses)

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[3]Weber, "Notes on Sargent's 'Observational Equivalence'."

$$\dot{P}_t = \underset{(12.03)}{0.039} + \underset{(1.86)}{0.250}\ UM_{t-1} + \underset{(1.88)}{0.252}\ UM_{t-2} + \underset{(3.20)}{0.426}\ UM_{t-3}$$

$$+ \underset{(3.28)}{0.446}\ UM_{t-4} - \underset{(0.28)}{0.084}\ (Sur/GNP)_{5-1} - \underset{(1.24)}{0.353}\ (Sur/GNP)_{t-2}$$

$$- \underset{(3.46)}{0.214}\ \dot{y}_{t-1} \qquad \bar{R}^2 = 0.208$$

where UM_{t-1} (i=1,...,4) represents the unexpected money growth variable. Estimating the equation with the regime break adjusted series gives

$$\dot{P}_t = \underset{(10.21)}{0.009} + \underset{(0.38)}{0.060}\ UM'_{t-1} + \underset{(0.26)}{0.042}\ UM'_{t-2} + \underset{(1.56)}{0.0249}\ UM'_{t-3}$$

$$+ \underset{(1.38)}{0.222}\ UM'_{t-4} - \underset{(0.74)}{0.061}\ (Sur/GNP)_{t-1} - \underset{(0.65)}{0.050}\ (Sur/GNP)_{t-2}$$

$$- \underset{(1.85)}{0.121}\ \dot{y}_{t-1} \qquad \bar{R}^2 = 0.048$$

Comparing these results by the $\bar{R}^2$ criterion suggests that failure to account for regime changes in deriving the unexpected money growth series may seriously bias the results. Indeed, the marked decline in the degree of explanatory power form the first to the second equation is quite striking. Though tentative, the results suggest that unexpected money growth, when properly measured, may have little explanatory power in regard to inflation. Indeed, these results are quite at odds with the results of Barro.

To further examine the influence of regime adusted unexpected money growth on inflation, the irrelevance of unexpected money, once actual money and lagged inflation rates are known, has been tested. The procedure used here is identical to that employed in Chapter VI. Briefly, testing the hypothesis that lagged UM_{t-i}s are irrelevant given lagged actual money growth rates yields an F-statistic of $F^{4}_{94} = 1.92$, critical F at the 5 percent level $F^{4}_{100} = 2.46$. Thus, the hypothesis that, given actual money, unexpected money growth is relevant toward explaining the inflationary process can be rejected. The reverse test, the null hypothesis that actual money is irrelevant given unexpected money, is soundly rejected: the calculated F-statistic is $F^{5}_{94} = 10.77$, which far exceeds the 1 percent critical value of $F^{5}_{100} = 3.51$. Together, these test results strongly support the contention that once actual money growth is known, unexpected money growth is irrelevant in determining the path of inflation.[4]

Finally, the hypothesis that unexpected money is irrelevant, given lagged inflation, is tested. The F value for this test is $F^{7}_{96} = 1.50$, which is less than the 5 percent critical value of $F^{7}_{100} = 2.10$. Performing the reverse test (testing for the irrelevance of lagged inflation given unexpected money) yields an F value of $F^{4}_{95} = 26.74$, which far exceeds the critical value of $F^{4}_{100} = 3.51$ at the 1 percent level of significance. These results provide further evidence regarding the role of unexpected money in explaining inflation.

To summarize, the purpose of this Appendix was to suggest a possible methodological error in previous works examining the effect of unexpected money growth on certain economic variables. This error, it has been argued, stems from the possible disregard for observed regime changes in

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[4]The test results for the irrelevance of lagged actual money growth yield an F-statistic of $F^{5}_{94} = 2.08$, which exceeds the critical value of $F^{5}_{100} = 2.03$, 5 percent level of significance. Thus, the hypothesis that, given lagged inflation, actual money growth is irrelevant to the determination of inflation can be rejected.

the money rule when calculating the unexpected money series. As tentative regression results and subsequent rests for loss of explanatory power reveal, such a disregard severly biases the estimates when unadjusted unexpected money series are used. Clearly, further research into this area in warranted.

APPENDIX B

DATA COLLECTION

Time	Money Supply (Ml)	($1972) Real GNP (y)	Surplus	(1972=100) GNP Deflator (P)	GNP	4-6 Month Paper	Aaa Bond Rate
1949.I	111.20	490.90	0.8	52.98	260.10	1.560	2.707
1949.II	111.37	488.90	-2.9	52.48	256.60	1.560	2.707
1949.III	111.03	493.40	-3.9	52.41	258.60	1.457	2.630
1949.IV	111.03	489.20	-3.9	52.43	256.50	1.363	2.597
1950.I	112.03	511.50	-4.8	52.28	267.40	1.310	2.577
1950.II	113.67	525.10	7.6	52.73	276.90	1.310	2.610
1950.III	114.93	542.40	16.4	54.30	294.50	1.460	2.633
1950.IV	115.93	554.60	17.1	55.16	305.90	1.710	2.670
1951.I	117.13	562.40	18.0	56.88	319.90	1.953	2.700
1951.II	118.20	573.20	8.2	57.17	327.70	2.193	2.900
1951.III	119.70	584.70	0.1	57.19	334.40	2.253	2.887
1951.IV	121.90	585.70	01.3	57.79	338.50	2.257	2.953
1952.I	123.50	591.30	0.1	57.69	341.10	2.380	2.957
1952.II	124.53	592.10	-3.8	57.64	341.30	2.323	2.933
1952.III	125.80	598.30	-7.6	58.00	347.00	2.310	2.947
1952.IV	127.07	612.50	-3.7	58.64	359.20	2.310	2.987
1953.I	127.57	622.20	=4.5	58.73	365.40	2.327	3.070
1953.II	128.43	626.20	-6.2	58.90	369.80	2.620	3.323
1953.III	128.63	622.40	-5.7	59.09	367.80	2.747	3.270
1953.IV	128.73	616.40	-11.7	58.82	362.60	2.370	3.133
1954.I	129.10	608.10	-10.5	59.53	362.00	2.037	2.957
1954.II	129.40	605.60	-6.6	59.74	361.80	1.633	2.877
1954.III	130.63	614.40	-5.0	59.60	366.20	1.363	2.883
1954.IV	131.97	626.10	-1.8	59.90	375.00	1.310	2.887
1955.I	133.50	641.10	1.3	60.44	387.50	1.613	2.960
1955.II	134.30	650.80	4.0	60176	395.40	1.967	3.033
1955.III	134.87	660.30	5.0	61.18	404.00	2.327	3.100
1955.IV	135.10	667.00	6.0	61.50	410.20	2.833	3.117

Time	Money Supply (M1)	($1972) Real GNP (y)	Surplus	(1972=100) GNP Deflator (P)	GNP	4-6 Month Paper	Aaa Bond Rate
1956.I	135.57	664.10	6.3	62.02	411.90	3.000	3.097
1956.II	135.93	667.50	5.5	62.53	417.40	3.263	3.260
1956.III	135.97	667.90	4.9	63.24	422.40	3.350	3.423
1956.IV	136.60	675.70	6.0	63.77	430.90	3.630	3.677
1957.I	136.87	680.40	4.3	64.51	438.90	3.630	3.700
1957.II	136.93	680.90	2.5	64.77	441.00	3.683	3.773
1957.III	136.97	685.60	2.6	65.37	448.20	3.953	4.070
1957.IV	136.23	676.70	-1.5	65.44	442.80	3.993	3.997
1958.I	136.07	663.40	-8.1	65.69	435.80	2.817	3.607
1958.II	1937.63	668.20	-12.4	65.83	439.90	1.717	3.580
1958.III	139.00	684.40	-10.8	66.20	453.10	2.130	3.870
1958.IV	140.70	702.10	-9.8	66.42	466.30	3.213	4.093
1959.I	142.60	710.70	-4.2	66.98	476.00	3.303	4.103
1959.II	143.80	726.30	0.8	67.45	489.90	3.603	4.353
1959.III	144.53	718.60	-1.0	67.70	486.50	4.193	4.473
1959.IV	143.63	726.20	-0.6	67.96	493.50	4.760	4.570
1960.I	143.00	740.70	7.1	68.40	506.60	4.687	4.553
1960.II	142.77	738.90	5.6	68.55	506.50	4.073	4.453
1960.III	143.93	735.70	1.5	68.81	506.20	3.373	4.313
1960.IV	144.23	731.90	-0.6	68.94	504.60	3.270	4.320
1961.I	144.83	736.60	-4.9	68.84	507.10	3.013	4.270
1961.II	146.03	749.00	-4.5	69.19	518.20	2.860	4.283
1961.III	146.87	758.70	-3.8	69.49	527.20	2.897	4.437
1961.IV	148.30	776.90	-1.9	69.60	540.70	3.057	4.410
1962.I	149.17	788.10	-5.0	70.17	553.00	3.243	4.410
1962.II	149.83	798.30	-4.6	70.41	562.10	3.203	4.297
1962.III	149.53	804.30	-2.6	70.60	567.80	3.333	4.337
1962.IV	150.43	805.80	-3.2	71.02	572.30	3.263	4.257

Time	Money Supply (Ml)	($1972) Real GNP (y)	Surplus	(1972=100) GNP Deflator (P)	GNP	4-6 Month Paper	Aaa Bond Rate
1963.I	151.83	813.50	-2.4	71.32	580.20	3.310	4.197
1963.II	153.33	823.70	1.8	71.37	587.90	3.317	4.220
1963.III	154.80	838.80	1.2	71.59	600.50	3.697	4.287
1963.IV	156.40	846.90	2.1	72.08	610.40	3.907	4.333
1964.I	157.33	861.10	-2.5	72.28	622.40	3.950	4.370
1964.II	158.83	872.00	-6.3	72.52	632.40	3.933	4.407
1964.III	161.43	880.50	-2.7	72.92	642.10	3.910	4.410
1964.IV	163.40	883.90	-0.6	73.08	646.00	4.063	4.430
1965.I	164.50	903.00	4.4	73.69	665.40	4.300	4.420
1965.II	165.73	916.40	4.7	74.06	678.70	4.380	4.443
1965.III	167.63	932.30	-3.1	74.56	695.10	4.380	4.497
1965.IV	170.53	952.00	-1.1	74.93	713.30	4.470	4.613
1966.I	173.33	969.60	1.4	75.67	733.70	4.970	4.813
1966.II	175.43	976.30	3.0	76.58	747.60	5.427	5.003
1966.III	175.23	985.40	-1.2	77.02	759.00	5.790	5.320
1966.IV	175.50	992.80	-4.1	77.73	771.70	6.000	5.383
1967.I	177.17	994.40	-11.6	78.19	777.50	5.450	5.120
1967.II	179.63	1001.30	-12.5	78.48	785.80	4.717	5.263
1967.III	183.87	1013.60	-13.1	79.23	803.10	4.973	5.617
1967.IV	186.70	1021.50	-12.3	80.15	818.70	5.303	6.027
1968.I	189.17	1031.40	-9.8	81.18	837.30	5.580	6.127
1968.II	192.67	1049.40	-11.2	82.12	861.80	6.080	6.253
1968.III	196.63	1061.80	-3.9	82.88	880.00	5.963	6.077
1968.IV	200.80	1064.70	-1.1	84.03	894.70	5.963	6.243
1969.I	204.27	1074.80	9.1	84.95	913.00	6.657	6.700
1969.II	206.00	1079.60	11.7	86.05	929.00	7.540	6.887
1969.III	207.20	1083.40	5.1	87.40	946.90	8.487	7.063
1969.IV	208.73	1077.50	3.4	88.47	953.30	8.620	7.467

Time	Money Supply (M1)	($1972) Real GNP (y)	Surplus	(1972=100) GNP Deflator (P)	GNP	4-6 Month Paper	Aaa Bond Rate
1970.I	210.53	1073.60	-4.5	89.91	964.20	8.553	7.893
1970.II	213.03	1074.10	-14.1	90.91	976.50	8.167	8.140
1970.III	215.67	1082.00	-15.4	91.74	992.60	7.837	8.220
1970.IV	218.77	1071.40	-20.5	92.99	996.30	6.293	7.907
1971.I	222.63	1095.30	-18.5	94.40	1034.00	4.590	7.217
1971.II	227.83	1103.30	-23.8	95.73	1056.20	5.040	7.473
1971.III	231.63	1111.00	-23.4	96.53	1072.40	5.743	7.557
1971.IV	233.27	1120.50	-22.5	97.38	1091.20	5.067	7.300
1972.I	237.80	1141.20	-13.4	98.76	1127.00	4.060	7.233
1972.II	242.17	1163.00	-20.0	99.46	1156.70	4.577	7.277
1972.III	247.20	1178.00	-10.8	100.29	1181.40	4.933	7.207
1972.IV	252.83	1202.20	-24.9	101.14	1219.40	5.333	7.137
1973.I	258.03	1229.80	-10.9	102.89	1265.30	6.283	7.220
1973.II	261.63	1231.10	-7.4	104.65	1288.40	7.467	7.307
1973.III	265.00	1236.30	-4.8	106.57	1317.50	9.873	7.587
1973.IV	268.43	1242.60	-4.6	109.05	1355.10	8.980	7.650
1974.I	273.23	1230.20	-5.3	111.28	1369.00	8.303	7.897
1974.II	276.47	1224.50	-7.9	114.30	1400.10	10.457	8.363
1974.III	278.87	1216.90	-8.0	117.52	1430.10	11.533	8.987
1974.IV	282.00	1199.70	-25.5	121.06	1452.40	9.050	9.017
1975.I	283.50	1169.80	-48.5	124.21	1453.00	6.563	8.707
1975.II	287.87	1188.20	-99.2	125.96	1496.60	5.920	8.873
1975.III	292.30	1220.70	-65.5	128.20	1564.90	6.667	8.913
1975.IV	294.27	1229.80	-67.6	130.16	1600.70	6.120	8.810
1976.I	297.70	1256.00	-60.3	131.46	1651.20	5.290	8.557
1976.II	302.77	1271.50	-46.2	133.06	1691.90	5.570	8.533
1976.III	305.57	1283.70	-53.5	134.56	1727.30	5.530	8.463
1976.IV	311.10	1287.40	-55.9	136.35	1755.40	4.990	8.183

Time	Money Supply (M1)	($1972) Real GNP (y)	Surplus	(1972=100) GNP Deflator (P)	GNP	4-6 Month Paper	Aaa Bond Rate
1977.I	316.50	1311.00	-38.8	138.12	1810.80	4.810	8.033
1977.II	322.90	1330.70	-40.3	140.52	1869.90	5.237	8.013
1977.III	329.43	1347.40	-56.4	142.19	1915.90	5.807	7.946
1977.IV	335.33	1360.20	-58.6	144.23	1961.80	6.593	8.103

SOURCE: Money Supply, Real GNP, GNP Deflator, GNP, 4-6 Month Ppaer Rate, and Aaa Bond Yield supplied by Research Department, Federal Reserve Bank of St. Louis. Surplus: Survey of Current Business, various issues.

Actual and Predicted Inflation: 1951.II-1977.IV Quarterly Rates of Change

	Model			
Time	Autoregressive	Rational Expectations	General	Actual
1951.II	0.68	0.59	0.67	0.51
1951.III	0.21	0.64	0.40	0.00
1951.IV	0.87	0.71	0.80	1.05
1952.I	0.23	0.20	0.00	-0.19
1952.II	0.35	0.10	0.01	-0.01
1952.III	0.69	0.56	0.73	0.62
1952.IV	0.32	0.62	0.63	1.11
1953.I	0.38	0.20	0.27	0.14
1953.II	0.50	0.39	0.42	0.28
1953.III	0.60	0.63	0.63	0.34
1953.IV	0.36	0.12	0.01	-0.46
1954.I	0.51	0.47	0.51	1.19
1954.II	0.50	0.26	0.28	0.36
1954.III	0.24	0.11	-0.01	-0.23
1954.IV	0.72	0.55	0.61	0.49
1955.I	0.43	0.33	0.30	0.91
1955.II	0.31	0.54	0.53	0.52
1955.III	0.49	0.51	0.55	0.70
1955.IV	0.53	0.64	0.68	0.51
1956.I	0.40	0.48	0.47	0.85
1956.II	0.48	0.29	0.36	0.82
1956.III	0.40	0.40	0.42	1.13
1956.IV	0.47	0.45	0.47	0.83
1957.I	0.42	0.66	0.56	1.15
1957.II	0.49	0.39	0.42	0.40
1957.III	0.41	0.56	0.43	0.93
1957.IV	0.54	0.64	0.62	0.01

	Model			
Time	Autoregressive	Rational Expectations	General	Actual
1958.I	0.36	0.36	0.26	0.39
1958.II	0.59	0.55	0.60	0.22
1958.III	0.36	0.48	0.41	0.56
1958.IV	0.50	0.19	0.25	0.32
1959.I	0.42	0.82	0.70	0.84
1959.II	0.50	0.46	0.41	0.71
1959.III	0.38	0.33	0.29	0.37
1959.IV	0.53	0.37	0.38	0.38
1960.I	0.48	0.53	0.52	0.64
1960.II	0.42	0.74	0.68	0.22
1960.III	0.45	0.32	0.40	0.38
1960.IV	0.53	0.26	0.39	0.20
1961.I	0.42	0.23	0.25	-0.15
1961.II	0.51	0.88	0.90	0.50
1961.III	0.47	0.65	0.70	0.44
1961.IV	0.37	0.27	0.33	0.16
1962.I	0.54	0.73	0.78	0.82
1962.II	0.46	0.48	0.56	0.35
1962.III	0.38	0.56	0.51	0.26
1962.IV	0.58	0.51	0.54	0.60
1963.I	0.41	0.54	0.51	0.42
1963.II	0.42	0.18	0.23	0.01
1963.III	0.53	0.28	0.32	0.30
1963.IV	0.48	0.71	0.74	0.67
1964.I	0.39	0.46	0.51	0.28
1964.II	0.60	0.54	0.47	0.34
1964.III	0.58	0.73	0.40	0.55
1964.IV	0.59	0.97	0.55	0.22

	Model			
Time	Autoregressive	Rational Expectations	General	Actual
1965.I	0.50	1.12	0.60	0.82
1965.II	0.84	0.53	0.85	0.51
1965.III	0.67	0.70	0.64	0.67
1965.IV	0.86	0.58	0.66	0.49
1966.I	0.63	0.57	0.76	0.99
1966.II	0.98	0.72	1.10	1.19
1966.III	1.15	1.16	1.22	0.59
1966.IV	0.93	0.98	0.88	0.91
1967.I	0.96	0.89	0.66	0.59
1967.II	0.69	0.97	0.39	0.37
1967.III	0.65	0.80	0.60	0.96
1967.IV	0.86	0.88	0.91	1.15
1968.I	1.11	1.33	1.13	1.28
1968.II	1.37	1.40	1.33	1.15
1968.III	1.25	1.27	1.24	0.92
1968.IV	1.06	1.45	1.31	1.38
1969.I	1.26	1.66	1.48	1.08
1969.II	1.14	1.48	1.55	1.29
1969.III	1.34	1.46	1.51	1.56
1969.IV	1.41	1.28	1.30	1.22
1970.I	1.34	1.41	1.28	1.50
1970.II	1.46	1.17	1.34	1.22
1970.III	1.24	1.07	1.22	0.90
1970.IV	1.09	1.03	1.20	1.36
1971.I	1.21	1.71	1.12	1.51
1971.II	1.39	0.89	1.38	1.40
1971.III	1.49	1.53	1.52	0.83
1971.IV	1.06	1.68	1.30	0.89

		Money		
Time	Autoregressive	Rational Expectations	General	Actual
1972.I	0.94	1.57	0.95	1.40
1972.II	1.19	1.29	1.07	0.71
1972.III	0.97	1.14	0.99	0.83
1972.IV	0.98	1.32	1.25	1.13
1973.I	1.00	1.22	1.04	1.42
1973.II	1.37	1.38	1.47	1.70
1973.III	1.63	1.91	1.64	1.81
1973.IV	1.75	1.69	1.74	2.31
1974.I	2.08	1.51	1.99	2.02
1974.II	1.97	1.88	2.11	2.71
1974.III	2.40	1.63	2.47	2.74
1974.IV	2.43	1.64	2.52	2.97
1975.I	2.74	1.89	2.61	2.56
1975.II	2.42	2.21	2.26	1.40
1975.III	1.65	1.03	1.50	1.76
1975.IV	1.54	1.06	1.72	1.00
1976.I	1.33	1.39	1.26	1.00
1976.II	1.22	1.04	1.17	1.21
1976.III	1.15	1.36	1.10	1.11
1976.IV	1.07	1.25	0.95	1.33
1977.I	1.32	1.58	1.14	1.29
1977.II	1.29	1.27	1.32	1.72
1977.III	1.60	1.31	1.57	1.18
1977.IV	1.31	1.54	1.39	1.42

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AN EMPIRICAL COMPARISON OF AUTOREGRESSIVE AND RATIONAL MODELS OF PRICE EXPECTATIONS

by

Rik W. Hafer

(ABSTRACT)

This dissertation presents several empirical tests to measure the relative abilities of alternative models in capturing the unobservable process by which economic individuals may form expectations of future inflation. Three empirical representations of the inflation expectations process are tested: an autoregressive model which uses only past inflation data; a rational expectations model which utilizes the structural economic relationships in the economy (excluding past inflation); and a general model which exploits both the information sets just described.

These competing approaches are each subjected to tests for rationality and predictive accuracy. The rationality tests employed in this study are the breakpoint test suggested by Sargent and the incorporation of each model's inflation predictions into an analysis of the Fisher equation. To gauge the predictive accuracy of each model, post-sample extrapolations were generated and compared by means of the root-mean-squared error and Theil inequality coefficient.

The outcome of these various tests porvides support to the contention that, for an individual attempting to obtain optimal

(error minimizing) forecasts of future inflation would select the relatively simple autoregressive model over the rational expectations or general approaches. In three out of four tests presented, the autoregressive model performed as well if not better than its more informational intensive competitors.