STABILITY OF HEATED BOUNDARY LAYERS

by

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Chapter I

INTRODUCTION

The need for fuel efficient carriers has lead to studies in laminar flow control (Pfenninger et al.,1957; Bacon et al.,1959).It has been observed that considerable drag reduction and hence fuel economy could result if the flow over the body could be maintained laminar.A thorough understanding of the mechanics of laminar to turbulent flow is thus necessary. It has been shown by (Schubauer & Skramstad,1947)that during the initial stages,disturbances in the boundary layer are two dimensional and linear. These disturbances are called Tollmien - Schlichting or more appropriately Tollmien - Schubauer waves, which can grow or decay. During the process of amplification disturbances of a particular wavelength and frequency selectively grow inside the boundary layer (Tani,1969 ; Reshotko,1976 ; Morkovin ,1978).

Experimental studies show that initially the disturbances are two dimensional, later on these two-dimensional waves are observed to exhibit a three-dimensional behavior. At the next stage the disturbance amplitudes become large and modify the mean flow itself and nonlinear effects become important.

Linear stability theory, in which disturbances are considerd infinitesimal traveling harmonic waves, has been used to predict n factors, which are related to transition for a given problem (Liepmann,1945; Smith & Gamberoni,1956; van Ingen,1956; Jaffe, Okamura & Smith,1970; Mack,1975,1977). Initially all stability studies were confined to the case of the so-called "parallel boundary layer". Some of these are those conducted by Tollmien (1929), Schlichting(1935), Kurtz(1961), Kaplan(1964), Osborne(1967), Wazzan et al.(1968) and Mack(1969). For low Reynolds numbers these results differ by nearly 30% from the experimental results for the flow over the flat plate conducted by Schubauer & Skramstad (1947), Ross et al.(1970), Kachanov et al. (1975) and Strazisar and coworkers(1975,1977,1978). Experimental studies of boundary layer transition over a prolate spheroid were conducted by Meier and Kreplin(1980).

The parallel flow assumption was discarded by Bouthier(1972,1973), Nayfeh et al.(1974) and Gaster(1974) who incorporated the nonparallel effects on the stability of two-dimensional flows. Saric & Nayfeh (1975,1977) showed that the nonparallel effects are significant for low Reynolds numbers and unfavorable pressure gradients. The effects decrease as the Reynolds number increases or the pressure gradients become favorable.The minimum critical

Reynolds number predicted is quite close to that obtained experimentally.

Three-dimensional disturbances do form in two-dimensional as well as in three-dimensional flows (Klebanoff et al.,1962; Kovasznay et al.,1962; Kachanov et al.,1975).

A theoretical investigation of three-dimensional stability has been carried out by Gregory,Stuart & Walker(1955) who studied the three-dimensional temporal stability problem for a rotating disk flow. They reduced the three-dimensional temporal stability problem to a two-dimensional one through a transformation. Brown(1961) numerically solved the equations for a rotating disk flow. Nayfeh and Padhye(1980) calculated the three-dimensional neutral stability of two-dimensional as well as three-dimensional flows. Cebeci and Stewartson (1980) identified an absolute neutral curve called 'zarf' for a rotating disk flow. Sorokowski and Orszag (1977) using their computer code SALLY and the code of Kaups and Cebeci(1977) studied the temporal stability of parallel three-dimensional flows. Mack (1977) calculated the three-dimensional stability of supersonic two-dimensional and three-dimensional flows. Nayfeh and Padhye (1979) presented a relationship relating temporal and spatial stability for two-as well as three-dimensional flows. Mack (1978) calculated the stability of the Falkner-Skan-Cooke boundary layers and the effect of suction and cooling on the stability of three-dimensional boundary layers in supersonic flows (1979). Cebeci and Stewartson (1980) calculated n factors for a rotating disk flow. Malik, Wilkinson and Orszag (1981) studied the instability and calculated n factors for a rotating disk flow using the computer code SALLY.

Nayfeh (1980b,c) presented a nonparallel three-dimensional stability theory using the method of multiple scales and derived conditions for the direction of propagation. Cebeci and Stewartson (1980), for the case of parallel flows, arrived at the same conditions.Padhye and Nayfeh(1981) studied the nonparallel stability of three-dimensional flows and presented results for the X-21 wing.

The effects of heating/cooling on the stability of the boundary layer were investigated by Linke(1942) and Liepmann & Fila(1947). Diprima & Dunn (1965) quote unpublished results of McIntosh indicating large increases in the minimum critical Reynolds number for heated liquid boundary layers. Hauptmann (1968) also predicted strong stabilization in water for small wall heating.

Wazzan, Okamura & Smith (1968,1970a,b) and Wazzan, Keltner, Okamura, and Smith (1972) conducted extensive studies of the stability of heated and cooled water boundary layers.They showed that cooling destabilizes and heating stabilizes water boundary layers. Potter and Graber (1972) also obtained similar results for plane Poiseuille flow. Lowell and Reshotko (1974) considered temperature disturbances in their formulation but found no signifcant differences with the results of Wazzan et al. (1972) who did not consider temperature perturbations. An experimental study of uniformly and nonuniformly heated boundary layers was conducted by Strazisar, Reshotko and Prahl (1977) and Strazisar and Reshotko (1978), respectively.

Nayfeh and El-Hady (1980) considered the two-dimensional stability of heated boundary layers. They showed that nonsimilar effects must be taken into account for the case of nonuniformly heated boundary layers.

Experimental studies of boundary layer transition on a prolate spheroid were conducted by Meier and Kreplin (1980). Liepmann et al. (1982) and Liepmann and Nosenchuck (1982) used a dynamic heating concept to show the usefulness of heating in active laminar flow control.

A method for calclating the most dangerous frequency was proposed by Reed (1981) and Reed & Nayfeh (1982). Of all the different frequencies in the boundary layer one can calculate the frequency which might be responsible for triggering large growth rates and eventually transition.

The stability problem is formulated in Chapter II. The method of multiple scales is used to derive the three dimensional stability equations. The equations for the zeroth-order and the first-order problems are presented.The solvability condition is derived next. The solvability condition is then used to derive the wavenumber modulation equations which are then used to solve the Cauchy problem. The effect of heating a boundary layer is considered in Chapter III.Chapter IV covers the development of the boundary conditions and the method of solution of the eigenvalue problem. Next the method of calculating the most unstable disturbance and a modified definition of the most dangerous frequency are given. Results and discussion are presented in Chapter V. Conclusions are presented in Chapter VI.

Chapter II

PROBLEM FORMULATION

2.1 GOVERNING EQUATIONS

We consider the general three-dimensional Navier-Stokes equations with variable flow properties;that is

X-momentum equation:

$$\rho\{\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z\} = -\partial P/\partial x + \partial/\partial x(2\mu\partial U/\partial x + \lambda \mathrm{Div}\bar{V}) + \partial/\partial y(\mu\partial U/\partial y + \mu\partial V/\partial x) + \partial/\partial z(\mu\partial W/\partial x + \mu\partial U/\partial z) \qquad (2.1)$$

Y-momentum equation:

$$\rho\{\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z\}=-\partial P/\partial y + \partial/\partial x(\mu\partial V/\partial x + \mu\partial U/\partial y) + \partial/\partial y(2\mu\partial V/\partial y + \lambda\ \mathrm{Div}\bar{V}) + \partial/\partial z(\mu\partial V/\partial z + \mu\partial W/\partial y) \qquad (2.2)$$

Z-momentum equation:

$$\rho\{\partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z\} = -\partial P/\partial z + \partial/\partial x(\mu\partial W/\partial x + \mu\partial U/\partial z) + \partial/\partial y(\mu\partial V/\partial z + \mu\partial W/\partial y) + \partial/\partial z(2\mu\partial W/\partial z + \lambda \mathrm{Div}\bar{V}) \qquad (2.3)$$

Continuity equation:

$$\partial\rho/\partial t + \partial/\partial x(\rho U) + \partial/\partial y(\rho V) + \partial/\partial z(\rho W) = 0 \qquad (2.4)$$

Energy equation:

$$\rho Dh/Dt = DP/Dt + \partial/\partial x(\kappa\partial T/\partial x) + \partial/\partial y(\kappa\partial T/\partial y)\ \partial/\partial z(\kappa\partial T/\partial z) + \Phi \qquad (2.5)$$

where Φ is the dissipation function and it is defined as follows:

$$\Phi = \mu\{2(\partial U/\partial x)^2 + 2(\partial V/\partial y)^2 + 2(\partial W/\partial z)^2 + (\partial V/\partial x + \partial U/\partial y)^2 + (\partial W/\partial y + \partial V/\partial z)^2 + (\partial U/\partial z + \partial W/\partial x)^2\} + \lambda(\partial u/\partial x + \partial v/\partial y + \partial w/\partial z)^2 \quad (2.6)$$

where

ρ is the density

U,V,W are the x,y,z components of velocity $\bar{V}$

t is the time

κ is the coefficient of thermal conductivity

h is the enthalpy per unit of mass

μ is the coefficient of vicosity

λ is the second coefficient of viscosity

T is the temperature

P is the pressure

The fluid properties ρ,μ and κ are functions of temperature, which depend on the spatial coordinates.

Nondimensional variables are introduced using $L^* = (2\nu_e x/U_e)^{1/2}$ as the length scale, where x is the suface distance, the local edge velocity U_e as the velocity scale and the free stream values of viscosity, specific heat and thermal conductivity as the reference values for the fluid properties. The spatial variation in the fluid properties is evaluated through their dependence on temperature.

2.2 BASIC FLOW

The basic flow is assumed to be two-dimensional / axi-symmetric. It is governed by the following nondimensional set of equations:

X-momentum equation:

$$\rho\{U\partial U/\partial x + V\partial U/\partial y\} = \rho_e U_e dU_e/dx + (1/r^k)\partial/\partial y(r^k \mu \partial U/\partial y) \qquad (2.7)$$

Y-momentum equation:

$$\partial P/\partial y = 0 \qquad (2.8)$$

Continuity equation:

$$\partial(r^k \rho U)/\partial x + \partial(r^k \rho V)/\partial y = 0 \qquad (2.9)$$

Energy equation:

$$\rho(U\partial H/\partial x + V\partial H/\partial y) = (1/r^k)\partial/\partial y\{r^k[(\kappa/c_p)\partial H/\partial y + \mu(1-1/Pr)U\partial U/\partial y]\} \qquad (2.10)$$

where H is the total enthalpy

c_p is the specific heat at constant pressure

Pr is the Prandtl number

r is the radial distance from the axis

k = 0 for two-dimensional flow and k = 1 for axisymmetric flow,

the subscript e refers to conditions at the edge of the boundary layer.

2.3 STABILITY EQUATIONS

We restrict our attention to time-independent boundary layer flows that are slightly nonparallel ; that is , the transverse velocity is small compared with the streamwise component and all mean flow quantities are slowly varying functions of the streamwise position x and spanwise position z. Thus the basic flow is independent of time t and depends on x,y,z as follows:

$$U_s = U_s(x_1, y, z_1) \tag{2.11a}$$

$$V_s = \varepsilon \hat{V}_s(x_1, y, z_1) \qquad \hat{V}_s = O(1) \tag{2.11b}$$

$$W_s = W_s(x_1, y, z_1) \tag{2.11c}$$

$$P_s = P_s(x_1, z_1) \tag{2.11d}$$

$$T_s = T_s(x_1, y, z_1) \tag{2.11e}$$

where

$$x_1 = \varepsilon x \tag{2.12a}$$

$$z_1 = \varepsilon z \tag{2.12b}$$

$$\varepsilon = 1/R \tag{2.12c}$$

and $R = L^* U_e^* / \nu_e^*$. The quantities with an asterisk are dimensional and the subscript e refers to values at the edge of the boundary layer.

The effect of temperature perturbations on the stability results is negligible, Reshotko (1978) and Lowell & Reshotko (1974). Thus we consider the temperature profile obtained by solving the energy equation for the basic flow and do not consider the energy equation in deriving the disturbance equations.

Small disturbances are introduced and superimposed on the mean flow so that the total-flow quantities can be expressed as

$$Q = Q_s(x_1, y, z_1) + q(x_1, y, z_1, t) \tag{2.13}$$

where Q represents U,V,W and P, the subscript s refers to the basic state and the lower case u,v,w,p refer to the disturbance quantities.

Substituting the total flow variables in the Navier-Stokes equations, subtracting the basic-flow terms and linearizing the resulting equations, we obtain the disturbance equations.

2.4 DISTURBANCE EQUATIONS

X-momentum equation:

$$\rho_s\{\partial u/\partial t + U_s\partial u/\partial x + u\partial U_s/\partial x + \varepsilon V_s\partial u/\partial y + v\partial U_s/\partial y + W_s\partial u/\partial z + w\partial U_s/\partial z\} = -\partial p/\partial x + 1/R[\partial/\partial x\{2\mu_s\partial u/\partial x + \lambda_s(\partial u/\partial x + \partial v/\partial y + \partial w/\partial z)\} + \partial/\partial y\{\mu_s(\partial u/\partial y + \partial v/\partial x)\} + \partial/\partial z\{\mu_s(\partial w/\partial x + \partial u/\partial z)\}] \tag{2.14}$$

Y-momentum equation:

$$\rho_s\{\partial v/\partial t + U_s \partial v/\partial x + \varepsilon u \partial V_s/\partial x + \varepsilon V_s \partial v/\partial y + \varepsilon v \partial V_s/\partial y + W_s \partial v/\partial z + \varepsilon w \partial V_s/\partial z\} = -\partial p/\partial y + 1/R[\partial/\partial y\{2\mu_s \partial v/\partial y + \lambda_s(\partial u/\partial x + \partial v/\partial y + \partial w/\partial z)\} + \partial/\partial x\{\mu_s(\partial v/\partial x + \partial u/\partial y)\} + \partial/\partial z\{\mu_s(\partial v/\partial z + \partial w/\partial y)\}] \tag{2.15}$$

Z-momentum equation:

$$\rho_s\{\partial w/\partial t + U_s \partial w/\partial x + u \partial W_s/\partial x + \varepsilon V_s \partial w/\partial y + v \partial W_s/\partial y + W_s \partial w/\partial z + w \partial W_s/\partial z\} = -\partial p/\partial z + 1/R[\partial/\partial z\{2\mu_s \partial w/\partial z + \lambda_s(\partial u/\partial x + \partial v/\partial y + \partial w/\partial z)\} + \partial/\partial x\{\mu_s(\partial w/\partial x + \partial u/\partial z)\} + \partial/\partial y\{\mu_s(\partial v/\partial z + \partial w/\partial y)\}] \tag{2.16}$$

Continuity Equation:

$$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = o \tag{2.17}$$

2.5 BOUNDARY CONDITIONS

The no-slip and no-penetration boundary conditions demand the vanishing of the disturbance velocities at the wall; that is,

$$u = v = w = 0 \qquad \text{at } y = 0 \tag{2.18a}$$

Moreover, all disturbances decay away from the wall; that is,

$$u,\ v,\ w,\ p \rightarrow 0 \qquad \text{as } y \rightarrow \infty \tag{2.18b}$$

2.6 METHOD OF ANALYSIS

We seek a first-order uniform expansion for the disturbance quantities u,v,w,p in the traveling harmonic wave form

$$q(x,y,z,t) = [q_1(x_1,y,z_1) + \varepsilon q_2(x_1,y,z_1) + \ldots]\exp(i\theta) \tag{2.19}$$

where

$$\partial\theta/\partial x = \alpha(x_1,z_1) \tag{2.20a}$$

$$\partial\theta/\partial z = \beta(x_1,z_1) \tag{2.20b}$$

$$\partial\theta/\partial t = -\omega \tag{2.20c}$$

Here α is the wave number component in the x direction , β is the wave number component in the z direction , and ω is the nondimensional frequency. Using the chain rule, we have

$$\partial/\partial x = \alpha\,\partial/\partial\theta + \varepsilon\,\partial/\partial x_1 \tag{2.21a}$$

$$\partial/\partial z = \beta\,\partial/\partial\theta + \varepsilon\,\partial/\partial z_1 \tag{2.21b}$$

$$\partial/\partial t = -\omega\,\partial/\partial\theta + \varepsilon\,\partial/\partial t_1 \tag{2.21c}$$

For a spatial stability analysis, α and β are complex and ω is real; α_r and β_r then represent the wave-number components and $-\alpha_i$ and $-\beta_i$ represent the growth rates in the streamwise and spanwise directions, respectively. For a temporal stablity analysis, α and β are real and represent the wave-number compnents in the streamwise and spanwise directions and ω is complex whose real part represents the frequency and whose imaginary part represents the growth rate.

Substituting equations (2.19) - (2.21) into equations (2.14)-(2.18) and equating each of the coefficients of ε^0 and ε^1 on both sides, we have two sets of problems. These are called the zeroth-order and the first-order problems, respectively.

2.7 THE ZEROTH ORDER PROBLEM

The zeroth-order problem is

$$L_1(u_0, v_0, w_0) = \rho_s\{ i\alpha u_0 + Dv_0 + i\beta w_0 \} + v_0 D\rho = 0 \qquad (2.22)$$

$$L_2(u_0, v_0, w_0, p_0) = \{ -i\Omega\rho_s + i\alpha\rho_s U_s + i\beta\rho_s W_s \} u_0 + v_0 DU_s + i\alpha p_0 - 1/R \{ i\alpha\mu_s(i\alpha r u_0 + sDv_0 + i\beta s w_0) - \beta\mu_s(\alpha w_0 + \beta u_0) + D\mu_s Du_0 + \mu_s D^2 u_0 + i\alpha(v_0 D\mu_s + \mu_s Dv_0)\} = 0 \qquad (2.23)$$

$$L_3(u_0, v_0, w_0, p_0) = \rho_s\{ i(-\Omega + \alpha U_s + \beta W_s) v_0 \} + Dp_0 - 1/R\{ i\alpha\mu_s(Du_0 + i\alpha v_0) + i\alpha s u_0 D\mu_s + i\alpha s\mu_s Du_0 + rD\mu_s Dv_0 + r\mu_s D^2 v_0 + i\beta s\, w_0 D\mu_s + i\beta s\mu_s Dw_0 + i\beta\mu_s(i\beta v_0 + Dw_0)\} = 0 \qquad (2.24)$$

and

$$L_4(u_0, v_0, w_0, p_0) = \rho_s\{ i(-\Omega + \alpha U_s + \beta W_s) w_0 + v_0 DW_s\} + i\beta p_0 - 1/R\{ i\alpha\mu_s(i\alpha w_0 + i\beta u_0) + D\mu_s Dw_0 + \mu_s D^2 w_0 + iD\mu_s \beta v_0 + i\beta\mu_s + i\beta\mu_s Dv_0 + i\beta\mu_s(i\alpha s u_0 + sDv_0 + i\beta r w_0)\} = 0 \qquad (2.25)$$

where the subscript 's' stands for basic-flow quantities and the subscript $_0$ stands for zeroth-order disturbance quantities and

$$s = \lambda_s / \mu_s \qquad (2.26a)$$

$$r = (2 + s) \tag{2.26b}$$

$$D = d/dy. \tag{2.26d}$$

Equations (2.22)-(2.26) constitute a system of four ordinary differential equations with variable coefficients, y is the independent variable, and x_1 and z_1 appear through the basic-flow quantities. The boundary conditions for the zeroth-order problem are

$$u_0 = v_0 = w_0 = 0 \qquad \text{at } y = 0 \tag{2.27a}$$

$$u_0 , v_0 , w_0 , p_0 \rightarrow 0 \qquad \text{as } y \rightarrow \infty \tag{2.27b}$$

The homogeneous sixth-order system of equations (2.22)-(2.25) together with the homogeneous boundary conditions (2.27) represent an eigenvalue problem. For a given basic flow and four of the six parameters $\alpha_r, \alpha_i, \beta_r, \beta_i, \omega_r$ and ω_i, we can calculate the other two as eigenvalues.

The four equations in (2.22)-(2.25) can be written as a set of six first-order equations which can then be numerically integrated over the region of interest. To this end, we define

$$Z_{10} = u_0, \quad Z_{20} = Du_0, \quad Z_{30} = v_0, \tag{2.28a}$$

$$Z_{40} = p_0, \quad Z_{50} = w_0, \quad Z_{60} = Dw_0 \tag{2.28b}$$

This enables us to write equations (2.22 - 2.25) and (2.27) as

$$D Z_{i0} - A_{ij} Z_{j0} = 0 \quad \text{for } i = 1,2,3...6 \tag{2.29}$$

$$Z_{10} = Z_{30} = Z_{50} = 0 \qquad \text{at } y = 0 \tag{2.30a}$$

$$Z_{n0} \rightarrow 0 \qquad \text{as } y \rightarrow \infty \tag{2.30b}$$

where A_{ij} is a 6x6 variable-coefficient matrix which is defined in Appendix A. The A_{ij} are also slowly varying functions of x_1 and z_1.

The solution of equations (2.29) and (2.30) can be written as

$$Z_{i0} = A\,(x_1, z_1, t_1)\zeta_i(x_1, y, z_1) \quad \text{for } i=1,2,\ldots,6 \tag{2.31}$$

where A is an unknown function at this level of approximation. It is determined by imposing the solvability condition at the next level of approximation (Nayfeh, 1980a).

2.8 THE FIRST-ORDER PROBLEM

Substituting equations (2.19) - (2.21) into equations (2.14)-(2.18) and equating the coefficients of ε on both sides, we obtain

$$L_1(u_1, v_1, w_1, p_1) = I_1 \tag{2.32}$$

$$L_2(u_1, v_1, w_1, p_1) = I_2 \tag{2.33}$$

$$L_3(u_1, v_1, w_1, p_1) = I_3 \tag{2.34}$$

$$L_4(u_1, v_1, w_1, p_1) = I_4 \tag{2.35}$$

$$u_1, v_1, w_1 = 0 \text{ at } y = 0 \tag{2.36a}$$

$$u_1, v_1, w_1, p_1 \rightarrow 0 \text{ as } y \rightarrow \infty \tag{2.36b}$$

where the operators L_1, L_2, L_3 and L_4 are defined in equations (2.22) -(2.25) and the inhomogeneous terms, I_1, I_2, I_3 and I_4 have contributions from the nonparallel effects of the basic flow and the variation of the eigenfunctions along the streamwise and spanwise directions; they are defined in Appendix B.

Equations (2.32)-(2.36) can be cast into a set of six first-order equations by defining

$$Z_{11} = u_1, \; Z_{21} = Du_1, \; Z_{31} = v_1, \tag{2.37a}$$

$$Z_{41} = p_1, \; Z_{51} = w_1, \; Z_{61} = Dw_1 \tag{2.37b}$$

The result is

$$DZ_{i1} - A_{ij}Z_{j1} = D_i \partial A/\partial t_1 + E_i \partial A/\partial x_1 + F_i \partial A/\partial z_1 + G_i \tag{2.38}$$

$$Z_{11} = Z_{31} = Z_{41} = 0 \qquad \text{at } y = 0 \tag{2.39a}$$

$$Z_{n1} \to 0 \qquad \text{as } y \to \infty \tag{2.39b}$$

where D_i, E_i, F_i, G_i are functions of the basic-flow quantities, the eigenfunctions of the zeroth-order problem, and the streamwise and spanwise derivatives of these quantities. They are defined in Appendix C.

Since the homogeneous part of the first-order problem has a nontrivial solution, the inhomogeneous first-order problem has a solution if the inhomogeneous parts are orthogonal to every solution of the adjoint homogeneous problem(Nayfeh, 1980a). This is the solvability or the consistency condition which must be satisfied by the first-order problem.

2.9 SOLVABILITY CONDITION

We use the concept of adjoint to arrive at the solvability condition for the first-order problem, which is solved after the solution to the zeroth-order problem has been obtained. The adjoint problem can be defined as follows: The zeroth-order homogeneous system is

$$\{ DZ_0 \} - [A] \{ Z_0 \} = 0 \tag{2.40}$$

$$Z_{10} = Z_{30} = Z_{50} = 0 \qquad \text{at } y = 0 \tag{2.41a}$$

$$Z_{n0} \to 0 \qquad \text{as } y \to \infty \tag{2.41b}$$

We multiply equation (2.40) from the left by $\{ W \}^T$ where $\{ W \}$ is the adjoint column vector and obtain

$$W^T DZ_0 - W^T A\, Z_0 = 0 \tag{2.42}$$

Integrating equation (2.42) by parts from $y = 0$ to $y = \infty$, we obtain

$$W^T Z_0 \Big|_0^\infty - \int_0^\infty (DW^T + W^T A)\, Z_0 \, dy = 0 \tag{2.43}$$

We now define the adjoint equation as

$$DW^T + W^T A = 0 \tag{2.44a}$$

or

$$DW + A^T W = 0 \tag{2.44b}$$

The boundary conditions for the adjoint system can be obtained as follows. Using equation (2.44a), we obtain from equation (2.43) that

$$W^T Z_0 \Big|_0^\infty = 0 \tag{2.45a}$$

or

$$[\, W_1 Z_{10} + W_2 Z_{20} + W_3 Z_{30} + \ldots + W_6 Z_{60} \,]_0^\infty = 0. \qquad (2.45b)$$

Since $Z_{n0} \to 0$ as $y \to \infty$, the terms evaluated at ∞ in equation (2.45) vanish if

$$W_n \to 0 \text{ as } y \to \infty \qquad (2.46)$$

Then using equation (2.41a), we find that equation (2.45a) becomes

$$[\, W_2 Z_{20} + W_4 Z_{40} + W_6 Z_{60} \,]_{y=0} = 0 \qquad (2.47)$$

We define the adjoint boundary conditions at y = 0 such that each of the coefficients of Z_{20}, Z_{40} and Z_{60} in equation (2.47) vanish independently; that is,

$$W_2 = W_4 = W_6 = 0 \qquad \text{at } y = 0 \qquad (2.48)$$

Therefore the adjoint problem is governed by equations (2.44) subject to the boundary conditions (2.46) and (2.48).

Having defined the adjoint problem, we return to the inhomogeneous problem. We multiply the matrix form of equations (2.38) from the left by W^T, integrate the result by parts from y = 0 to y = ∞, use the definition of the adjoint problem and equations (2.39), and obtain the solvability condition

$$\int_0^\infty W^T D \partial A/\partial t_1 dy + \int_0^\infty W^T E \partial A/\partial x_1 dy + \int_0^\infty W^T F \partial A/\partial z_1 dy + \int_0^\infty W^T G dy = 0. \qquad (2.49)$$

where D, E, F and G are column vectors whose components are the D_i, E_i, F_i and G_i.

2.10 AMPLITUDE-MODULATION EQUATION

Substituting equations (2.31) into equation (2.49) yields the following amplitude-modulation equations:

$$g_1 \partial A/\partial t_1 + g_2 \partial A/\partial x_1 + g_3 \partial A/\partial z_1 = \hat{h}_1 A \qquad (2.50)$$

or

$$\partial A/\partial t_1 + \omega_\alpha \partial A/\partial x_1 + \omega_\beta \partial A/\partial z_1 = h_1 A \qquad (2.51)$$

where

$$\omega_\alpha = g_2/g_1, \ \omega_\beta = g_3/g_1, \ h_1 = \hat{h}_1/g_1 \qquad (2.52)$$

and g_1, g_2, g_3 are defined in Appendix D. Equation (2.51) describes the modulation of the amplitude function A with x_1,z_1 and t_1.Here,ω_α,ω_β are the componens of the complex group velocity in the x and z directions ,respectively. The functions g_1,g_2 and g_3 are given in quadratures in terms of the basic flow,the eigenvalues and the eigenfunctions of the zeroth-order problem, and the eigenfunctions of the adjoint problem. The function $\hat{h}_1$ is given in quadratures in terms of the basic flow, the the adjoint eigenfunctions , the variation of the basic-flow quantities with the streamwise and spanwise directions, the nonparallel flow terms, and the variation of the eigenfunctions and eigenvalues of the zeroth-order problem in the streamwise and spanwise directions. It is defined in Appendix E.

2.11 WAVE NUMBER MODULATION EQUATIONS

The evaluation of h_1 demands the evaluation of the derivatives of the eigenfunctions of the zeroth-order problem. To this end we replace Z_i by ζ_i, the zeroth-order eigenfunction in equations (2.40) and (2.41), differentiate the resulting expressions with respect to x_1 and arrive at

$$D(\partial\zeta_i/\partial x_1) - A_{ij}(\partial\zeta_j/\partial x_1) = iE_i\partial\alpha/\partial x_1 + iF_i\partial\beta/\partial x_1 + H_{1i} \tag{2.53}$$

subject to the boundary conditions

$$\partial\zeta_1/\partial x_1 = \partial\zeta_3/\partial x_1 = \partial\zeta_5/\partial x_1 = 0 \text{ at } y = 0 \tag{2.54}$$

$$\partial\zeta_n/\partial x_1 \to 0 \text{ as } y \to \infty \tag{2.55}$$

Similarly differentiting equations (2.40) and (2.41) with respect to z_1 yields

$$D(\partial\zeta_i/\partial z_1) - A_{ij}(\partial\zeta_j/\partial z_1) = iE_i\partial\alpha/\partial z_1 + iF_i\partial\beta/\partial z_1 + H_{2i} \tag{2.56}$$

subject to the boundary conditions

$$\partial\zeta_1/\partial z_1 = \partial\zeta_3/\partial z_1 = \partial\zeta_5/\partial z_1 = 0 \text{ at } y = 0 \tag{2.57}$$

$$\partial\zeta_n/\partial z_1 \to 0 \text{ as } y \to \infty \tag{2.58}$$

where the E_i and F_i are given in Appendix C, whereas the H_{1i} and H_{2i} are given in Appendix F.

Since the homogeneous parts of (2.53) - (2.58) have nontrivial solutions, the inhomogeneous systems (2.53)-(2.55) and (2.56) - (2.58) have solutions if solvability conditions are satisfied. Application of these conditions yield

$$\omega_\alpha \, \partial\alpha/\partial x_1 + \omega_\beta \, \partial\beta/\partial x_1 = h_2 \tag{2.59}$$

and

$$\omega_\alpha \, \partial\alpha/\partial z_1 + \omega_\beta \, \partial\beta/\partial z_1 = h_3 \tag{2.60}$$

where h_2 and h_3 reflect the effects of nonparallelism in the streamwise and spanwise directions, respectively. They are given in Appendix E.

If the phase angle θ is assumed to be continuously differentiable, then it follows from equations (2.20) that

$$\partial^2\theta/\partial z\partial x = \varepsilon\partial\alpha/\partial z_1 = \partial^2\theta/\partial x\partial z = \varepsilon\partial\beta/\partial x_1$$

and hence

$$\partial\alpha/\partial z_1 = \partial\beta/\partial x_1 \tag{2.61}$$

Therefore equations (2.59) and (2.60) can be rewritten as

$$\omega_\alpha \, \partial\alpha/\partial x_1 + \omega_\beta \, \partial\alpha/\partial z_1 = h_2 \tag{2.62}$$

$$\omega_\alpha \, \partial\beta/\partial x_1 + \omega_\beta \, \partial\beta/\partial z_1 = h_3 \tag{2.63}$$

For this formulation to represent a physical problem, Nayfeh (1980b,c) showed that $\omega_\beta/\omega_\alpha$ must be a real quantity. Dividing equations (2.62) and (2.63) by ω_α, we have

$$\partial\alpha/\partial x_1 + (\omega_\beta/\omega_\alpha) \, \partial\alpha/\partial z_1 = h_2/\omega_\alpha \tag{2.64}$$

$$\partial\beta/\partial x_1 + (\omega_\beta/\omega_\alpha) \, \partial\beta/\partial z_1 = h_3/\omega_\alpha \tag{2.65}$$

These two equations represent a Cauchy problem in x_1 and z_1 .

A Cauchy problem needs initial data specified on an initial curve which is not a characteristic but intersects one of the characteristics. This initial data should satisfy the dispersion relationship and the condition $\omega_\beta/\omega_\alpha$ being real at a particular x and z location. In this work, the initial data is taken to be $\beta(x_1=a,z_1)=\beta_0(z_1)$ on $z_1 = T$.

Chapter III

HEATED BOUNDARY LAYERS

Heating in water and cooling in air significantly affect the stability characteristics of boundary layers. A study of heating effects is essential for evaluating the shear forces on the body which are needed in evaluating the performance of the vehicle as well as its proper control.

Qualitatitvely the effect of heating a liquid boundary layer on its stability can be explained as follows. For an inviscid,constant density flow, a velocity profile which has an inflection point is unstable. Considering viscosity to be variable, we reduce the boundary-layer equations for a flat plate to

$$d^2U/dy^2 = -(1/\mu)\ (d\mu/dy)(dU/dy) \quad \text{at the wall.} \tag{3.1}$$

For a heated flat plate $dT/dy < 0$ at the wall. Since μ decreases with increasing temperature $d\mu/dT < 0$. Hence $d\mu/dy > 0$ at the wall. Moreover since μ and dU/dy are greater than zero at the wall, it follows that $d^2U/dy^2 < 0$ at the wall for wall heating. On the other hand, for wall cooling of a liquid boundary layer, $d^2U/dy^2 > 0$ at the wall. Since $d^2U/dy^2 < 0$ as $y \rightarrow \infty$,it follows that if the curvature of the profile is positive at the wall there should be an inflection point in between, implying that heating stabilizes and cooling destabilizes a liquid boundary layer. The effect is just the opposite for gases.

Diprima and Dunn(1965) quoted unpublished results of McIntosh which indicate that for the two-dimensional Tollmien-Schubauer type of instabilty in a heated water boundary layer, the minimum critical Reynolds number increases ten times for a temperture difference of 50 ° C and an ambient temperature of 10°C. Hauptmann also predicted very strong stabilization in water for relatively small overheating. Wazzan, Okamura, and Smith (1968,1970a,b) and Wazzan, Keltner, Okamura and Smith (1972) conducted extensive studies of the stability of heated and cooled water boudary layers. Their results show that while cooling the wall destabilizes the flow, moderate heating stabilizes it. As the heating is increased , the minimum critical Reynolds number reaches a maximum with T_w and then decreases. They concluded that as T_w is increased above T_e , the velocity profile becomes more stable, whereas the variable viscosity terms tend to destabilize the flow. At moderate rates of heating, the effects of the mean velocity profiles on the minimum critical Reynolds number are dominant. As the heating increases, the destabilizing effect of the viscosity on the minimum critical Reynolds number increases whereas the stabilizing effect of heating due to the change in the velocity profiles begins to level off. The minimum critical Reynolds number then reaches a maximum at the critical value T_c of T_w. As T_w is increased beyond T_c, the destabilizing effects of the viscosity become dominant and the minimum critical Reynolds number decreases upon increasing T_w beyond the critical value. The results of Potter and Graber (1972) for the stability of Plane Poiseuille flow with heat transfer show similar results .

Wazzan et al.(1968,1972) formulated the two-dimensional stability problem by taking into account the viscosity and temperature variations of the basic flow but ignoring the temperature and viscosity disurbances. The result is a fourth-order modified Orr-Sommerfeld equation. Lowell and Reshotko (1974), on the other hand, included the temperature perturbations, and hence included the energy equation. Their sixth-order system yields results very close to the results of the fourth-order system of Wazzan et al.(1968,1972). Also the neutral stability characteristics and growth rates calculated by Lowell and Reshotko (1974) " are sufficiently close (as compared to Wazzan et al.) so that there is no important quantitative difference between the two" (Reshotko, 1978). Heating dramatically decreases the amplification rates as well as the range of frequencies and wave numbers undergoing amplification. All these imply a more stable boundary layer. Thermal disturbances and hence perturbations in the viscosity and other fluid properties hardly affect the stability characteristics. This is attributed to the fact that the thermal bounday layer is extremely thin due to the large Prandtl number of water. Thus these disturbances are of importance only in a region which is much smaller than the veloctiy boundary layer .

Experimental studies of nonuniformly heated boundary layers were conducted by Strazisar and Reshotko (1978). They cosidered step changes in temperature and a power law temperature variation along a flat plate. The temperature variation is of the form

$$T_w - T_e = A x^N \tag{3.2}$$

where T is temperature, x is the distance along the plate, N is an exponent which may be positive or negative, the subscript w denotes the wall, the subscript e denotes the edge of the boundary layer, and A is a constant. Their results show that at $x= x_r$ as N increases the instability increases whereas as N decreases the instability decreases; that is, $N < 0$ results in growtn rates that are lower than those for $N = 0$ or $N >0$.

These results were explained by Nayfeh and El-Hady(1980) for the case of two-dimensional stability. They showed that it is essential to consider the basic flow as nonsimilar in the case of nonuniform wall heating. Their results agree qualitatively with those of Strazisar and Reshotko (1978).

We formulate the three-dimensional stability problem without considering the temperature fluctuations and hence all fluid property disturbances are ignored. The mean velocity and temperature profiles are evaluated using the Transition Analysis Program System (TAPS)(Gentry, 1976 ;Gentry and Wazzan, 1976). The fluid properties are evaluated using the formulae given by Lowell and Reshotko (1974). We evaluate the most dangerous frequency responsible for the possible trigerring of transition for a flat plate and an axisymmetric body. The effect of heating is also considered and the effect of the exponent N is evaluated for three different boundary layers.

Chapter IV

METHOD OF SOLUTION

4.1 GOVERNING EQUATIONS

The zeroth-order problem given by equations (2.29) and (2.30) can be written as

$$DZ = AZ \tag{4.1}$$

$$Z_1 = Z_3 = Z_5 = 0 \qquad \text{at } y = 0 \tag{4.2}$$

$$Z_1,\ Z_3,\ Z_5 \rightarrow 0 \qquad \text{as } y \rightarrow \infty \tag{4.3}$$

Equations (4.1)-(4.3) constitute an eigenvalue problem. For a given mean flow the dispersion relationship is

$$\omega = \omega(\alpha, \beta, R) \tag{4.4}$$

The eigenvalues are determined, in general, numerically by integrating the system of equations (4.1) in the transverse direction and imposing the boundary conditions (4.2) and (4.3). The boundary conditions at y=0 present no difficulties, but the boundary conditions (4.3) are reformulated in the numerical procedure that we use.

4.2 APPLICATION OF BOUNDARY CONDITIONS AT INFINITY

Instead of applying the boundary conditions (4.3) numerically, which is very expensive and not so accurate, we determine an analytic solution to equations (4.1) outside the boundary layer, apply these boundary conditions, and replace them with three other conditions at a finite value of y just outside the boundary layer. We reformulate the boundary conditions for both the zeroth-order problem and its adjoint following Ragab and Nayfeh (1981).

The set of equations (4.1) is a variable-coefficient system inside the boundary layer because of the variation of the basic-flow quantities. Outside the boundary layer all the basic-flow quantities are constant, and equations (4.1) reduce to the following set of differential equations with constant coefficients:

$$DZ = CZ \tag{4.5}$$

where C is given in Appendix G. The eigenvalues of C are given by

$$\lambda_1 = \{\alpha^2 + \beta^2\}^{1/2} \tag{4.6}$$

$$\lambda_2 = \{\alpha^2 + \beta^2 + i\rho_e R(\alpha U_e - \omega)/\mu_e\}^{1/2} \tag{4.7}$$

$$\lambda_3 = \lambda_2 \tag{4.8}$$

$$\lambda_4 = -\lambda_1 \tag{4.9}$$

$$\lambda_5 = -\lambda_2 \tag{4.10}$$

$$\lambda_6 = \lambda_5 \tag{4.11}$$

The general solution of equations (4.5) is obtained by superposing six linearly independent solutions as

$$\{Z\} = [Y]\ \{b\} \tag{4.12}$$

where the elements b_i of $\{b\}$ are given by

$$b_i = c_i \exp(\lambda_i y) \tag{4.13}$$

The matrix Y depends on y because the constant-coefficient matrix has repeated eigenvalues. To develop a matrix that is independent of y, we reduce the matrix C to a Jordan canonical form through the similarity transformation

$$J = P^{-1}CP \tag{4.14}$$

where the columns of the matrix P are the generalized eigenvectors of C. The columns are arranged so that the generalized eigenvectors corresponding to the eigenvalues having positive real parts appear in the first three columns. The matrix P is defined in Appendix H. Then we let

$$Z = P\xi \tag{4.15}$$

in equation (4.5), use equation (4.14), and obtain

$$\xi' = J\xi \tag{4.16}$$

Since the first three rows in J correspond to the eigenvalues with positive real parts, the boundary conditions (4.3) demand that

$$\xi_1 = \xi_2 = \xi_3 = 0 \quad \text{as } y \to \infty \tag{4.17}$$

But it follows from equation (4.15) that

$$\xi = P^{-1}\ Z \tag{4.18}$$

hence equation (4.17) implies that

$$Tz = \{ 0 \} \qquad \text{at } y=y_{max} \tag{4.19}$$

where T consists of the first three rows of P^{-1} and $y_{max} > \delta$, where δ is the boundary-layer thickness. The disadvantage of using T is the evaluation of P^{-1} at each iteration. To avoid this we use the adjoint problem to develop a matrix T that needs no inversion.

Multiplying equation (4.1) by Z^{*T} where Z^* is the adjoint of Z, and integrating the result by parts from y = 0 to y = y_{max} to transfer the derivatives from Z to Z^*, we obtain

$$Z^{*T}Z\Big|_0^{y_{max}} - \int_0^{y_{max}} (DZ^{*T} + Z^{*T}A)\, dy = 0 \tag{4.20}$$

The adjoint equations are defined as

$$DZ^{*T} + Z^{*T}A = 0 \tag{4.21}$$

or

$$DZ^* + A^TZ^* = 0 \tag{4.22}$$

Then equation (4.20) reduces to

$$Z^{*T}Z|_{ymax} - Z^{*T}Z|_0 = 0 \tag{4.23}$$

We choose the adjoint boundary conditions such that both the terms in (4.23) vanish independently ;that is,

$$Z^{*T}Z = 0 \text{ at } y = y_{max} \tag{4.24}$$

and

$$Z_1^*Z_1 + Z_2^*Z_2 + Z_3^*Z_3 + \ldots . Z_6^*Z_6 = 0 \text{ at } y = 0 \tag{4.25}$$

Using equations (4.2), we reduce equations (4.25) to

$$Z_2^*Z_2 + Z_4^*Z_4 + Z_6^*Z_6 = 0 \text{ at } y = 0 \tag{4.26}$$

Then the boundary conditions for the adjoint problem at $y=0$ are obtained by setting each of the coefficients of $Z_2(0), Z_4(0)$ and $Z_6(0)$ equal to zero. The result is

$$Z_2^* = Z_4^* = Z_6^* = 0 \text{ at } y = 0 \tag{4.27}$$

Substituting the transformation (4.15) into equation (4.24) yields

$$Z^{*T}P\xi = 0 \quad \text{at } y = y_{max} \tag{4.28}$$

Letting

$$\xi^* = P^T Z^* \tag{4.29}$$

we rewrite equation (4.27) as

$$\xi^{*T}\xi = 0 \quad \text{at } y = y_{max} \tag{4.30}$$

Using equation (4.17) in equation (4.30) yields

$$\xi_4^*\xi_4 + \xi_5^*\xi_5 + \xi_6^*\xi_6 = 0 \text{ at } y = y_{max} \tag{4.31}$$

The boundary conditions for the adjoint problem are obtained by setting each of the coefficients of ξ_4, ξ_5, ξ_6 in equation (4.31) equal to zero. The result is

$$\xi_4^* = \xi_5^* = \xi_6^* = 0 \text{ at } y = y_{max} \tag{4.32}$$

Thus the adjoint boundary conditions at $y = y_{max}$ are

$$T^* Z^* = 0 \qquad \text{at } y = y_{max} \tag{4.33}$$

where T^* consists of the last three rows of P^T.

We now consider the adjoint of the adjoint problem for $y > y_{max}$. Then, equation (4.22) reduces to

$$DZ^* + C^T Z^* = 0 \tag{4.34}$$

We can reduce $C^* = -C^T$ to a Jordan canonical form by using the similarity transformation

$$J^* = P^{*-1}C^*P^* \tag{4.35}$$

where P^* is a matrix whose columns are the generalized eigenvectors of C^*, rearranged so that the first three columns correspond to the three eigenvalues of C^* having positive real parts. Using the transformation

$$Z^* = P^*\xi^* \tag{4.36}$$

we rewrite equation (4.34) as

$$D\xi^* = J^*\xi^* \tag{4.37}$$

Since the first three rows of J^* correspond to the eigenvalues with positive real parts, then for decaying solutions we have

$$\xi_1^* = \xi_2^* = \xi_3^* = 0 \quad \text{at } y = y_{max}. \tag{4.38}$$

Using arguments similar to the preceding arguments, we arrive at the following boundary conditions at $y = y_{max}$ for the original problem:

$$TZ = 0 \qquad \text{at } y = y_{max}, \tag{4.39}$$

where T consists of the last three rows of P^{*T}. The matrix P^* is defined in Appendix I.

4.3 NUMERICAL PROCEDURE FOR THE BOUNDARY VALUE PROBLEM

To solve the two-point boundary value problem given by equations (4.1),(4.2) and (4.39), we use the code SUPORT developed by Scott and Watts (1975,1977). The integration procedure consists of the superposition of a set of linearly independent solutions coupled with an orthonormalization procedure that ensures the linear independence of the individual solution vectors.

The boundary-value problem is converted into an initial value problem. The boundary conditions at $y=y_{max}$ are known. These three conditions eliminate three of the six linearly independent solutions. The remaining three solutions are then integrated through the boundary layer. Linear combinations of these three solutions do not satisfy,in general, the boundary conditions at y=0 unless the parameters ω, α, β and R are chosen so that the dispersion relation (4.4) is satisfied. A simple Newton-Raphson procedure is used to iterate on the eigenvalues (two of the parameters $\omega_r, \omega_i, \alpha_r, \alpha_i, \beta_r, \beta_i$ and R given the other parameters.) to satisfy the boundary conditions at $y = 0$.

4.4 METHOD OF DETERMINING THE MOST UNSTABLE DISTURBANCE

Determination of the most unstable disturbance is done by solving the Cauchy problem defined by the wave number modulation equations (2.64) and (2.65). Equations (2.64) and (2.65) govern the variation of the wave numbers in the streamwise and spanwise directions. Their characteristics are given by

$$dx_1/ds = 1 \tag{4.40}$$

$$dz_1/ds = \omega_\beta/\omega_\alpha \tag{4.41}$$

Along these characteristics, equations (2.64) and (2.65) become

$$d\alpha/ds = h_2/\omega_\alpha \tag{4.42}$$

$$d\beta/ds = h_3/\omega_\alpha \tag{4.43}$$

Equations (4.42) and (4.43) show that if the wave number modulation equations were homogeneous, the wave numbers would be constant along each characteristic obtained by integrating equations (4.40) and (4.41). This would be the case for a "parallel boundary layer". Since the boundary layer is not "parallel" and all flow quantities are slowly varying functions of the streamwise and spanwise positions, the resulting wave number modulation equations are inhomogeneous, in general, and need to be numerically integrated with the dispersion relation (4.4) as a constraint subject to initial conditions on a given curve which intersects a characteristic curve. The initial conditions are given in the form

$$x_1(a,\tau) = a \tag{4.44}$$

$$z_1(a,\tau) = \tau \tag{4.45}$$

$$\beta(x_1 = a,\tau) = \beta_0(z_1) \tag{4.46}$$

The general solution of the characteristic equations (4.40) and (4.41) can be written as

$$x_1 = s + c_1 \tag{4.47}$$

$$z_1 = \int_a^s (\omega_\beta/\omega_\alpha)\, ds + c_2 \tag{4.48}$$

where c_1 and c_2 are constants. Applying the initial conditions (4.44) and (4.45) and the choice s=a at the initial curve, we obtain $x_1 = s$ and $c_2 = \tau$. Hence,

$$x_1 = s \tag{4.49}$$

$$z_1 = \int_a^s (\omega_\beta/\omega_\alpha)\, ds + \tau \tag{4.50}$$

To proceed further, we need to determine the partial derivatives of α and β and hence the ζ_i with respect to x_1 and z_1 along the characteristics. It follows from equation (4.43) and the initial condition (4.46) that

$$\beta = \int_a^s h_3/\omega_\alpha\, dt + \beta_0(\tau) \tag{4.51}$$

Moreover, it follows from equation (4.50) that

$$\tau = z_1 - \int_a^s (\omega_\beta/\omega_\alpha)\, dt \tag{4.52}$$

Since

$$\partial\beta/\partial x_1 = (\partial\beta/\partial s)\, \partial s/\partial x_1 + (\partial\beta/\partial\tau)\, \partial\tau/\partial x_1 \tag{4.53}$$

and

$$\partial s/\partial x_1 = 1\ ,\ \partial\tau/\partial x_1 = -\omega_\beta/\omega_\alpha \tag{4.54}$$

it follows from equations (4.52) and (2.61) that

$$\partial\alpha/\partial z_1 = \partial\beta/\partial x_1 = h_3/\omega_\alpha - \omega_\beta/\omega_\alpha\, \beta_0'(\tau) \tag{4.55}$$

Then, it follows form equation (2.65) that

$$\partial\beta/\partial z_1 = \beta_0'(\tau) \tag{4.56}$$

Substituting equation (4.55) into equation (2.64) yields

$$\partial\alpha/\partial x_1 = h_2/\omega_\alpha - (\omega_\beta/\omega^2_\alpha)\, h_3 + (\omega_\beta/\ \omega_\alpha)^2\ \beta'_0(\tau) \tag{4.57}$$

These partial derivatives can now be used to solve equations (2.53) - (2.58) and hence determine $\partial\zeta_i/\partial x_1$ and $\partial\zeta_i/\partial z_1$, which in turn are used to determine h_1. Then, equation (2.51) can be rewritten along the characteristic as,

$$dA/ds = (h_1/\omega_\alpha)\ A \tag{4.58}$$

To determine the most unstable disturbance, we determine the dimensional frequency ω^* and the real part of the dimensional spanwise wave number β_r^* such that the n factor is a maximum. To this end, for a given ω^*, we select a β_r^* and let $\beta_0(\tau) = \beta_r^*$ in equation (4.46) and numerically integrate equations (4.42),(4.43),(4.49),(4.50) and (4.58). The initial values for β_i, α_r and α_i are determined by solving the zeroth-order eigenvalue problem and imposing the condition that $\omega_\beta/\omega_\alpha$ must be real. Then, s is incrementd by Δs and new values for x_1, z_1, α and β are calculated from

$$x_1 = s + \Delta s \tag{4.59}$$

$$z_1 = (\omega_\beta/\omega_\alpha)\Delta s + \tau \tag{4.60}$$

$$\alpha = \alpha_{old} + (h_2/\omega_\alpha)\Delta s \tag{4.61}$$

$$\beta = \beta_{old} + (h_3/\omega_\alpha)\Delta s \tag{4.62}$$

The zeroth-order eigenvalue problem is solved at the new location to check whether the predicted values of α and β satisfy the eigenvalue problem and the condition that $\omega_\beta/\omega_\alpha$ must be real. If these conditions are not satisfied, the step size Δs is reduced until they are satisfied. Then, the n factor is calculated as

$$n = -\int_{\alpha}^{s} \{\alpha_i + (\omega_\beta/\omega_\alpha)\beta_i - \mathrm{real}(h_1/R\omega_\alpha)\}\, ds \qquad (4.63)$$

The process is repeated for other values of β^*_r and ω^* to determine the values that maximize n or that make n exceed a critical value in the shortest distance.

Chapter V

RESULTS AND DISCUSSION

5.1 FLOW OVER A FLAT PLATE

Numerical results were obtained for the flow over a flat plate. The boundary layer-equations were solved using the Transition Analysis Program System (TAPS), see Gentry (1976). It uses the Keller box technique.

We first consider the unheated case. Table 1 shows the results for a free stream velocity of 13.58 m/sec corresponding to a free stream Reynolds No. per foot (RPF) of 4.5 x 10^6, at a free stream temperature of 23.9°c. In each case ω^* is kept a constant. To calculate the most dangerous frequency we proceed as follows. For flows over a flat plate we assume that transition corresponds to n = 9. Keeping this in mind the most dangerous frequency should be defined as the one which gives n=9 in the shortest possible distance along the plate.

Based on this condition, Table 1 shows that the most dangerous frequency is $F \approx 24.9 \times 10^{-6}$ corresponding to a dimensional frequency $\omega^* \approx 5000$ Hz. R_x in all the tables is the Reynolds number based on the surface distance x and is defined as $R_x = U_e^* \, x^*/\nu^*$.

TABLE 1

n-Factor vs Frequency

U_e = 1358.2 cm/sec, T_e = 534.69°R, RPF = 4.5 x 10^6
DT = 0.0, ρ_e = 0.9965 g/cc; μ_e =0.0091676 cp

ω^*	F x 10^{-6}	n	R_x x 10^{-6}
2500	12.46	18.50	9.28
2500	12.46	9.00	5.25
4500	22.42	9.00	3.23
4900	24.44	9.00	3.14
4950	24.68	9.00	3.13
5000	24.93	9.00	3.13
5100	25.43	9.00	3.15
5200	25.93	9.00	3.15

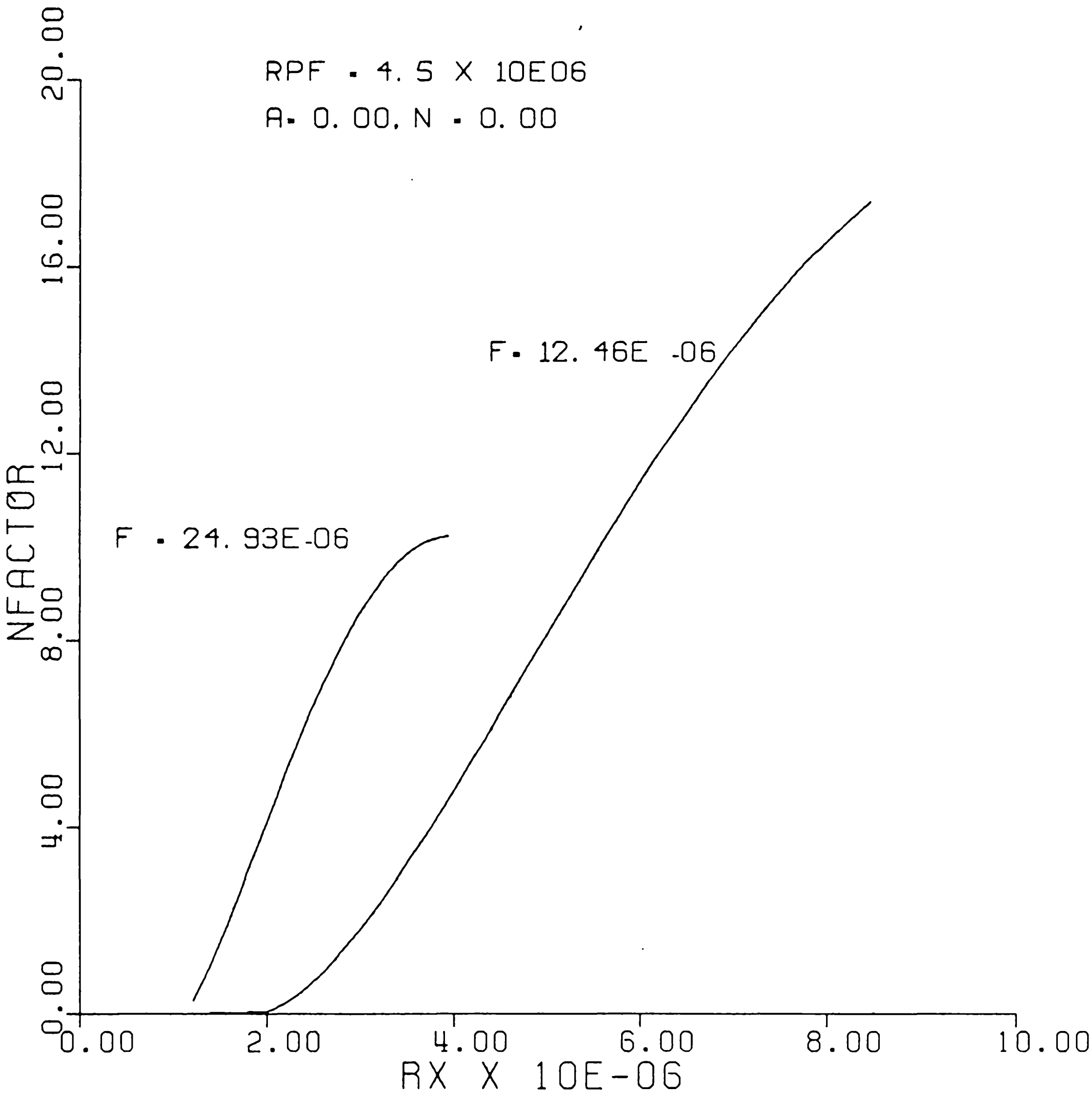

Figure 1: n Factor vs distance.

It is interesting to note that $F = 12.46 \times 10^{-6}$ gives n factors greater than 20 if the integration is carried out along a much larger distance on the plate. Had we based the definition of the most dangerous frequency on the maximum value of n, irrespective of how far the diturbance travels to reach this value, the results would have been erroneous. Since transition on a flat plate is assumed to occur when n reaches 9, there is no point in performing any calculations once this critical value is reached for any frequency. Consequently the modified definition of the most critical frequency should be used.

These results compare very well with those of Saric(1983) who found that $F = 25 \times 10^{-6}$ is the most dangerous frequency for a flat plate. A sample plot of the variation of the n factor with distance is shown in Fig.1.

5.1.1 Effect of Three-Dimensional Disturbances

Three-dimensional disturbances were considered with different dimensional wavelengths in the spanwise direction. In all the cases considered, Table 2 shows that the resulting n factors are equal to or less than those obtained for two-dimensional disturbances. Introduction of three-dimensional disturbances reduces the wave number as well as the

TABLE 2

Effect of Three-dimensional disturbances

U_e = 1358.2 cm/sec, T_e = 534.69°R, RPF = 4.5 x 10^6
DT = 0.0, ρ_e = 0.9965 g/cc, μ_e =0.0091676 cp

ω^*	F x 10^{-6}	β_r^*	n	R_x x 10^{-6}
4900	24.44	1.0e-04	9.35	3.26
4900	24.44	3.0e+00	9.16	3.26
4900	24.44	5.0e+00	8.76	3.26
5000	24.93	1.0e-04	9.35	3.26
5000	24.93	1.0e-01	9.35	3.26
5000	24.93	3.0e-01	9.35	3.26
5000	24.93	5.0e-01	9.35	3.26
5000	24.93	1.0e+01	6.16	3.12
5000	24.93	1.5e+01	3.09	2.46

growth rates.The stronger the three dimensionality is, the lower are the growth rates as shown in Fig2.

5.2 HEATING EFFECTS

We consider the effect of wall heating for the flow past a flat plate where the variation of wall overheat with distance is shown in Fig.3. Since wall overheat is a function of the streamwise position,we use nonsimilar boundary layer solutions to solve for the mean- flow quantities. This was shown to be a requirement by Nayfeh and El-Hady (1980). The mean flow is obtained from the TAPS code .

The effect of power-law wall heating is shown in Table 3 for $T_w(x_r) - T_e = 10^0R$ at $x_r = 10.7$ cm; and A = 16.87 and N = 0.5 in $T_w = T_e + Ax^N$.

The effect of wall overheat is apparent and large reductions in the n factors occur as shown in Fig.4.

5.2.1 Effect of Three-Dimensional Disturbances

Table 4 shows the calculated n factors for three- dimensional disturbances for a RPF of 4.5×10^6 corresponding to a free stream velocity $U_e = 13.582$ m/sec and $F = 24.44 \times 10^{-6}$. Also shown are the results of the unheated case. It is clear from the data that three-dimensional disturbances result in lower growth rates for both unheated and heated

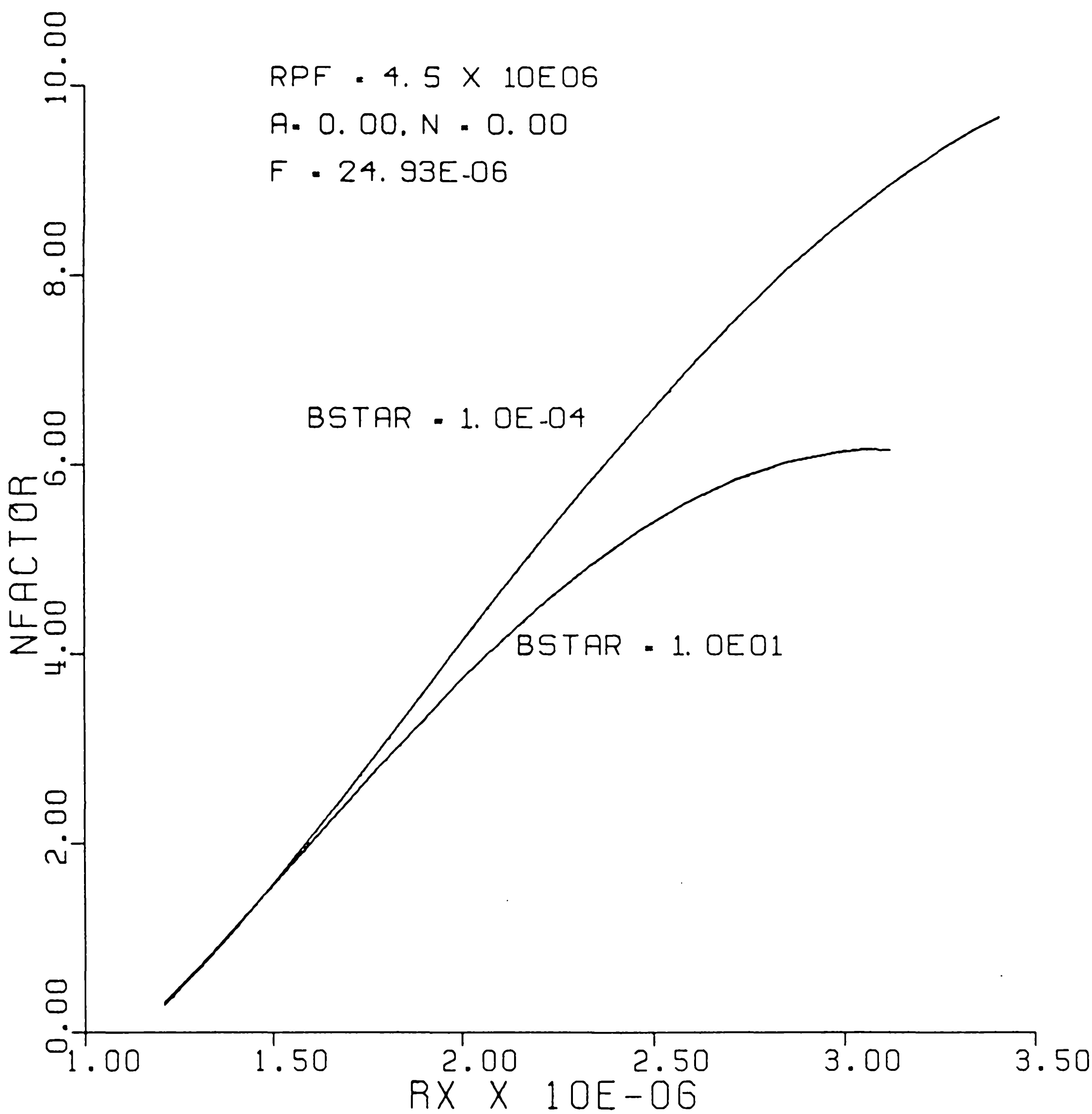

Figure 2: Effect of Three-Dimensional Disturbances.

boundary layers over a flat plate, irrespective of the wall overheat. The introduction of three dimensionality actually lowers the streamwise wave numbers α_r and the streamwise growth rates $-\alpha_i$.

5.2.2 Effect of Exponent N in Power Law Heating

Wall temperature was varied as $T_w = T_e + Ax^N$ keeping $\Delta T = T_w - T_e = 10^0R$ at x=10.71 cm for all N. The temperature variation along the length of the plate is shown in Fig.3. The exponent N was varied between -1.0 and 1.0. The results are presented in Tables 5 and 6 for $F = 24.44 \times 10^{-6}$ and $F = 19.95 \times 10^{-6}$, respectively. The free stream velocity is 13.582 m/s (RPF $=4.5 \times 10^6$) and the free stream temperature is 534.69⁰R.

The effect of increasing the exponent N is to decrease the n factors. This result, which is shown in Fig. 5, appears to contradict the results of Strazisar and Reshotko (1978) and Nayfeh and El-Hady(1980) who found that decreasing N is stabilizing.

If we compare the growth rates at x_r = 10.71 cm, shown in Table 7, we see that they agree with the conclusions of Strazisar and Reshotko (1978) and Nayfeh and El-Hady (1980).

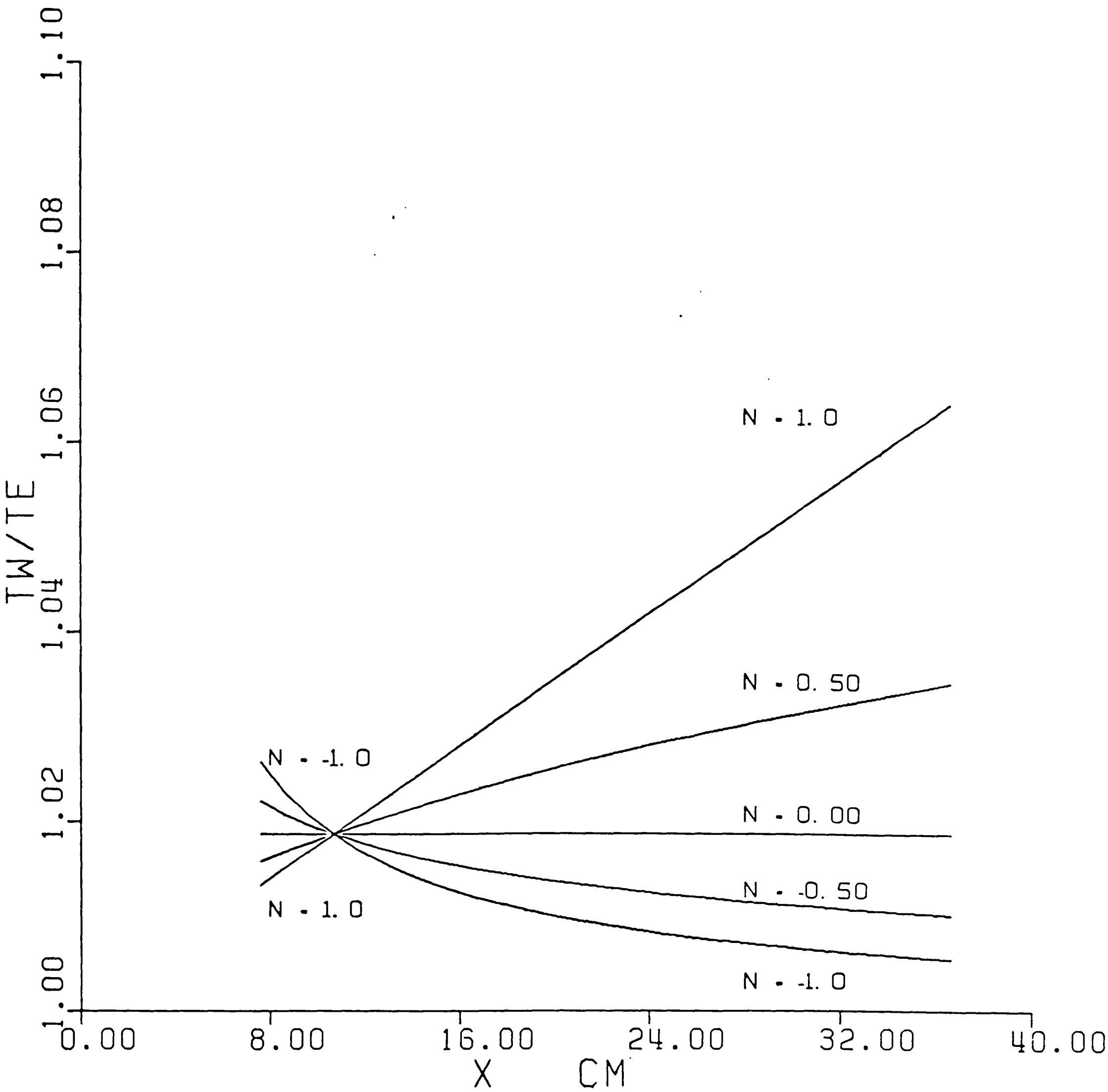

Figure 3: Temperature Variation over the Flat Plate

TABLE 3

Heating Effects

U_e = 1358.2 cm/sec, T_e = 534.69°R, RPF = 4.5×10^6
ρ_e = 0.9965 g/cc, μ_e = 0.0091676 cp

ω^*	$F \times 10^{-06}$	n	$R_x \times 10^{-6}$	
4000	19.95	8.75	3.40	No heating
4000	19.95	3.65	4.01	Heating
4900	24.44	9.35	3.26	No heating
4900	24.44	3.63	3.04	Heating

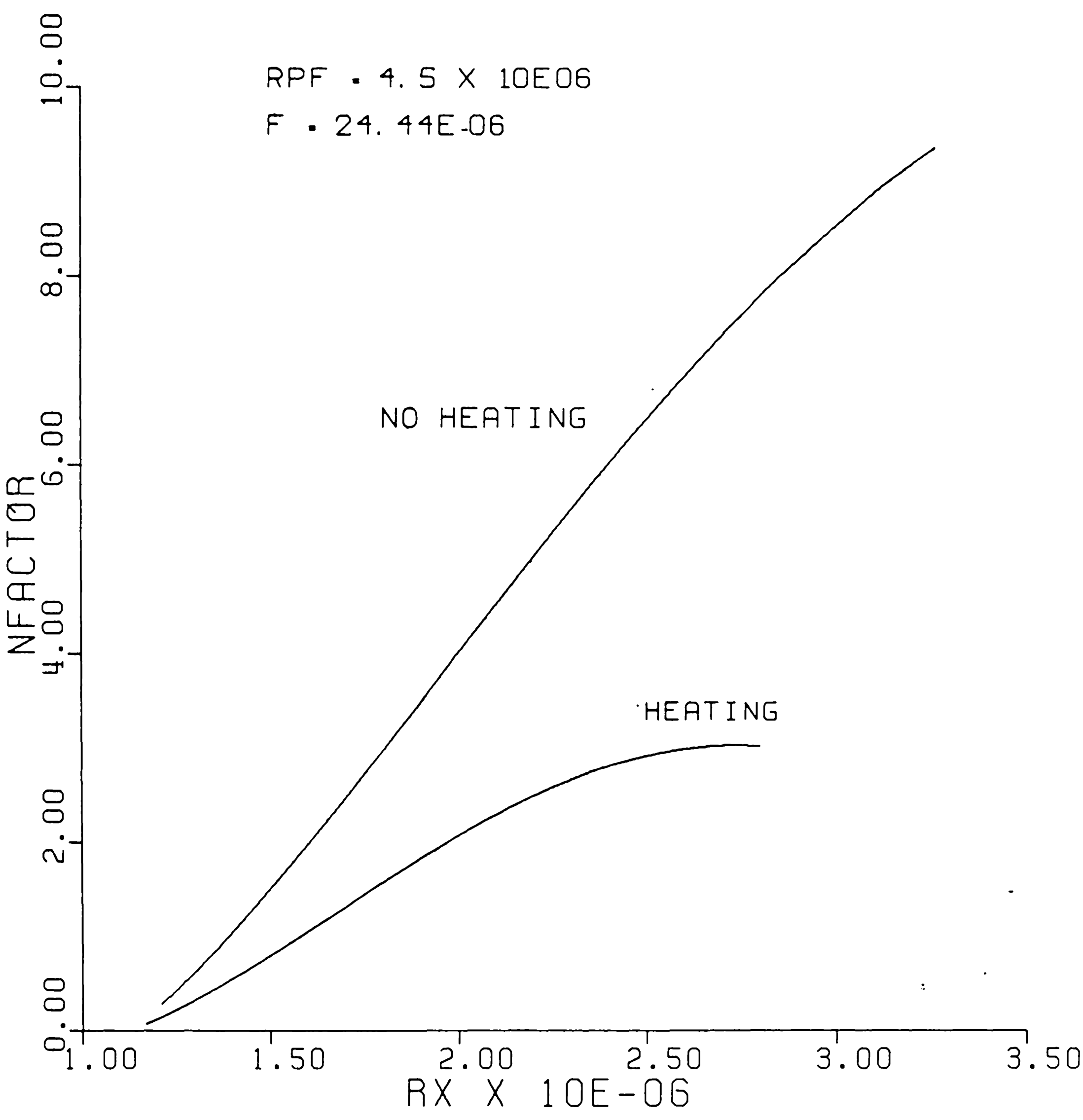

Figure 4: Effect of Wall Heating. $DT=Ax^N$, $A=28.46$, $N=1.0$

TABLE 4

Effect of heating on Three Dimensional Disturbances

U_e = 1358.2 cm/sec, T_e = 534.69°R, RPF = 4.5 x 10^6

F = 24.44 x 10^{-06}, ρ_e = 0.9965 g/cc, μ_e =0.0091676 cp

β_r^*	T_w	n	$R_x \times 10^{-6}$
1.0e-04	534.69	9.36	3.26
3.0e+00	534.69	9.16	3.26
5.0e+00	534.69	9.01	3.40
1.0e-04	A=10.0,N=0.0	4.40	3.48
1.0e+00	A=10.0,N=0.0	4.37	3.40
1.0e-04	A=16.87,N=0.5	3.63	3.04
1.0e+00	A=16.87,N=0.5	3.61	3.12
3.0e+00	A=16.87,N=0.5	3.45	2.98
5.0e+00	A=16.87.N=0.5	3.12	2.98
1.0e-04	A=5.93,N=-0.5	5.52	3.83
1.0e+00	A=5.93,N=-0.5	5.47	3.83
1.0e-04	A=3.52,N=-1.0	6.89	4.18
1.0e+00	A=3.52,N=-1.0	6.84	4.18
1.0e-04	A=28.46,N=1.0	3.02	2.73
1.0e+00	A=28.46,N=1.0	3.01	2.73

TABLE 5

Effect of Exponent N

U_e = 1358.2 cm/sec, T_e = 534.69°R, RPF = 4.5 x 10^6

F = 24.44 x 10^{-06}, ρ_e = 0.9965 g/cc, μ_e =0.0091676 cp

β_r^*	T_w	n	R_x x 10^{-6}
1.0e-04	534.69	9.36	3.26
1.0e-04	A=3.52,N=-1.0	6.89	4.18
1.0e-04	A=5.93,N=-0.5	5.52	3.83
1.0e-04	A=10.00,N=0.0	4.40	3.48
1.0e-04	A=16.87,N=0.5	3.63	3.04
1.0e-04	A=28.46,N=1.0	3.02	2.73

TABLE 6

Effect of Exponent N

U_e = 1358.2 cm/sec, T_e = 534.69°R, RPF = 4.5 x 10^6

F = 19.95 x 10^{-06}, ρ_e = 0.9965 g/cc, μ_e =0.0091676 cp

β_r^*	T_w	n	$R_x \times 10^{-6}$
1.0e+00	A=3.52,N=-1.0	8.34	4.62
1.0e+00	A=5.93,N=-0.5	6.94	4.49
1.0e+00	A=10.00,N=0.0	5.41	4.50
1.0e+00	A=16.87,N=0.5	3.63	4.01
1.0e+00	A=28.46,N=1.0	2.37	3.21

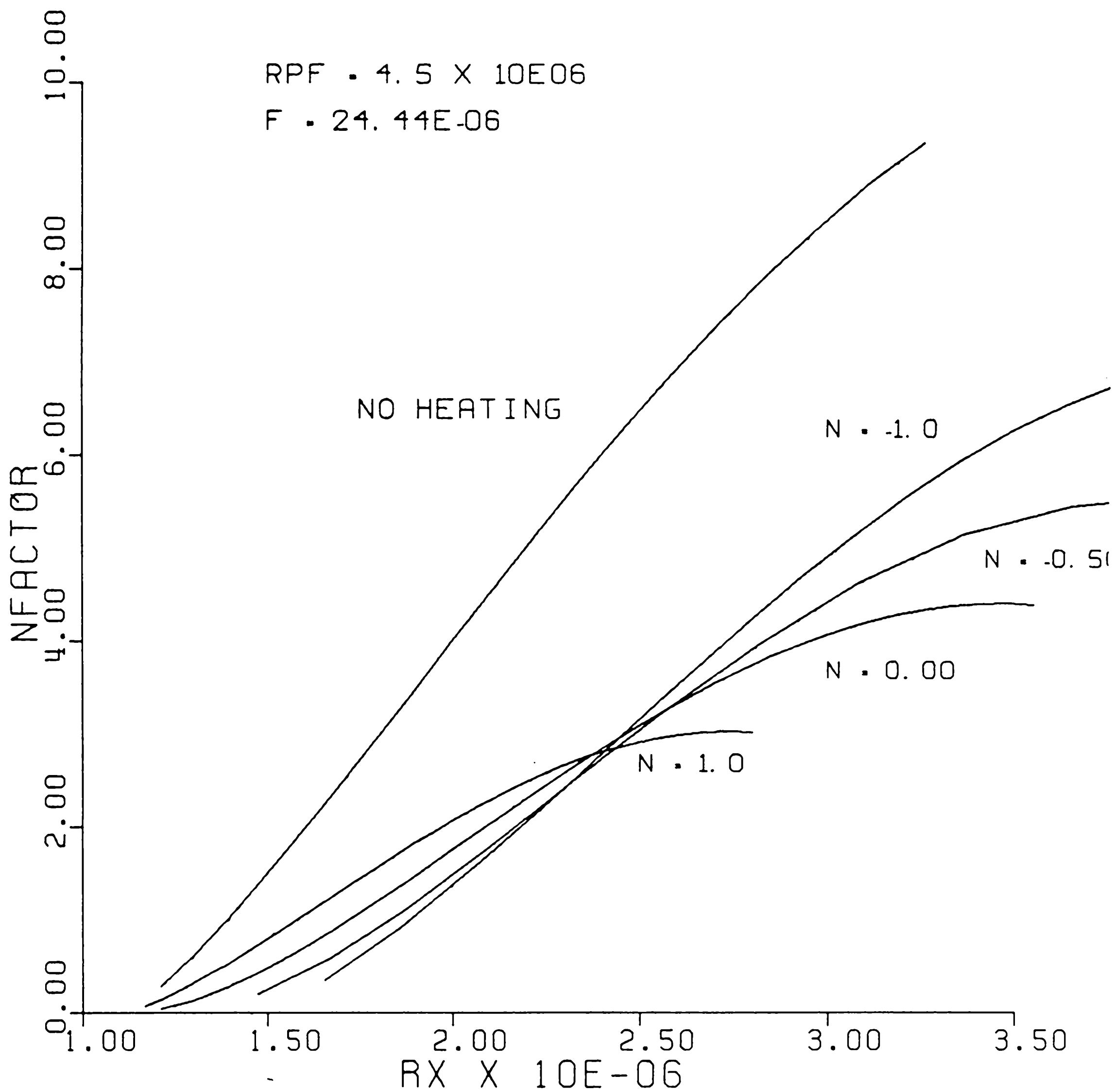

Figure 5: Effect of Exponent N on n Factors.

TABLE 7

Effect of the Exponent N on growth rates

Ue = 1358.2 cm/sec, Te = 534.69°R, RPF = 4.5×10^6
F = 19.95×10^{-06}, ρ_e = 0.9965 g/cc, μ_e =0.0091676 cp

x(cm)	N	α_i
10.71	N=-1.0	-2.51e-04
10.71	N=-0.5	-0.54e-03
10.71	N=0.5	-0.15e-02
10.71	N=1.0	-0.17e-02

They investigated the effect of the exponent N on the growth rates at $x = x_r$. At $x = x_r$, decreasing N is stabilizing because the temperature variation in Fig.3 shows that wall overheat increases for all $x < x_r$ as the exponent N is decreased. However for $x > x_r$ the opposite is true and wall overheat increases for increasing N.

It should be emphasized that, if x_r is greater than or equal to or even slightly less than the streamwise position of Branch II of the neutral stability curve, decreasing N will appear to be stabilizing. This is the reason why Nayfeh and El-Hady (1980) after calculating n factors have arrived at the conclusion that decreasing N is stabilizing. As a test we changed x_r to 25.0 cm for the case of $\omega^* =$ 4900, for which the branch II point occurs at $x = 27.5$ cm and calculated the effect of the exponent N on the n factors. At this value of x_r, decreasing N appears to be stabilizing as shown in Table 8, which agrees with the conclusions of Strazisar & Reshotko (1978) and Nayfeh & El-Hady(1980). The conclusion that decreasing or increasing N is stabilizing actually depends on the location of the reference point x_r and is thus not universal.

TABLE 8

Effect of Exponent N for x_r = 25cm.

U_e = 1358.2 cm/sec, Te = 534.69°R, RPF = 4.5 x 10^6

F = 24.44 x 10^{-06}, ρ_e = 0.9965g/cc, μ_e = 0.0091676cp

β_r^*	T_w	n	$R_x \times 10^{-6}$
1.0e-04	534.69	9.36	3.26
1.0e-04	A=8.21,N=-1.0	completely stable	
1.0e-04	A=9.06,N=-0.5	3.04	3.69
1.0e-04	A=10.00,N=0.0	4.40	3.48
1.0e-04	A=11.04,N=0.5	5.56	3.26
1.0e-04	A=12.19,N=1.0	6.48	3.26

5.3 FALKNER-SKAN FLOWS

For the pressure gradient parameter of -0.025, corresponding to a flow over a wedge having a vertex angle of 4.5 degrees, we calculated the n factors for the case of two dimensional stability. The results are shown in Table 9 and Fig.6. Note that $F = 12.5 \times 10^{-6}$ produces a larger n factor but $F = 27.4 \times 10^{-6}$ reaches n=9 much earlier. According to our modified definition, this is the most dangerous frequency. As a check the n factors for three-dimensional disturbances were calculated for $F = 28.17 \times 10^{-6}$. The result is the same as that for the flat plate; that is, three-dimensional disturbances yield lower n factors than two-dimensional disturbances.

The effect of heating on Falkner - Skan boundary layers is the same as that for the flow over a flat plate. Three-dimensional disturbances in heated boundary layers result in n factors equal to or less than those produced by two-dimensional waves. For this value of x_r, increasing the exponent N appears to be stabilizing. The results are shown in Table 10 and Fig. 7.

TABLE 9

Falkner - Skan Profiles

U_e = 1358.2 cm/sec, T_e = 534.69°R, β=-0.025, RPF = 4.5×10^6
DT = 0.0, ρ_e = 0.9965 g/cc, μ_e = 0.0091676 cp

ω^*	$F \times 10^{-06}$	n	$R_x \times 10^{-6}$
2500	12.46	9.0	4.44
4800	23.90	9.0	2.67
5000	24.93	9.0	2.66
5500	27.42	9.0	2.64
5650	28.17	9.0	2.66
6000	29.92	9.0	2.83

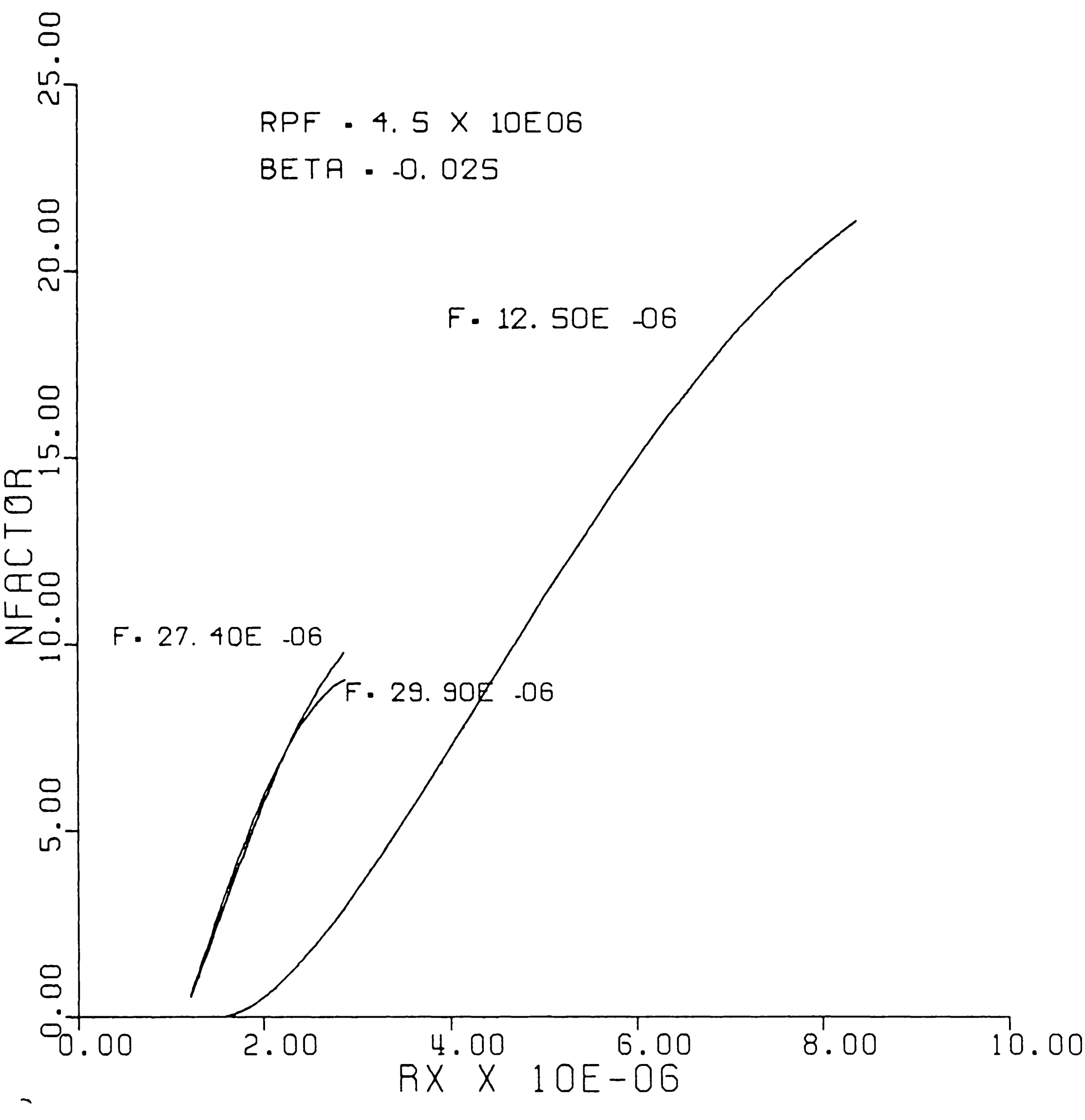

Figure 6: n Factor vs distance

5.4 AXISYMMETRIC BOUNDARY LAYERS

We consider the axisymmetric flow past the prolate spheroid shown in Fig. 8. The expressions for the inviscid edge-velocity components past this body are given by

$$u/U = 2/(\alpha - \alpha_0)[1 - (x-a)\cos\theta_s/(a^2(x-a)^2/a^4 + y^2/b^4)] \tag{5.1}$$

$$v/U = 2/(\alpha - \alpha_0)[1 - (x-a)\sin\theta_s/(a^2(x-a)^2/a^4 + y^2/b^4)] \tag{5.2}$$

and

$\cos\theta_s = x/[x^2 + y^2/(b^4/a^4)]^{1/2}$

$\alpha_0 = 2(1-e^2)[1/2 \log((1+e)/(1-e)) - e]/e^3$

$e = [1 - b^2/a^2]^{1/2}$

where

θ_s is the angle between the x axis and the surface normal

e is the eccentricity

a is the semi-major axis

b is the semi-minor axis

The prolate spheroid under study is 6 feet long and 1 foot thick at the center. The free stream velocity is 6.036 m/sec and the Reynolds number per foot (RPF) based on the free stream velocity is 2.0×10^6.

Meier and Kreplin (1980) observed "growth of instabilities" at x/2a = 0.14 and "large increases in local wall

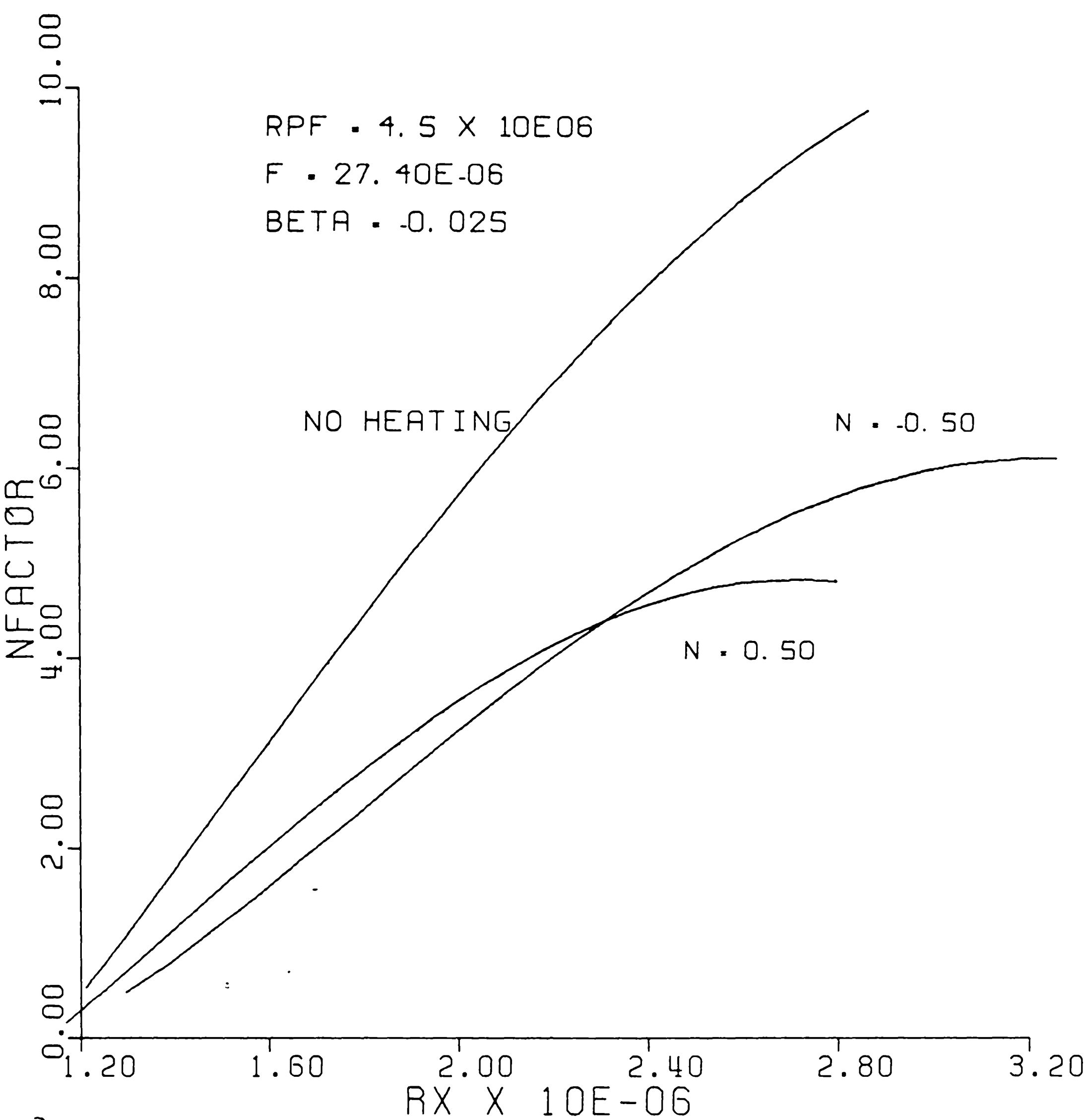

Figure 7: Effect of Exponent N on n Factors.

TABLE 10

Effect of exponent N

Falkner Skan Profiles

Ue = 1358.2 cm/sec, Te = 534.69°R, RPF = 4.5 x 10^6
ρ_e = 0.9965 g/cc, μ_e =0.0091676 cp

F x 10^{-06}	T_w	n	R_x x 10^{-6}
27.43	A=5.93,N=-0.5	6.11	3.22
27.43	A=16.87,N=0.5	4.83	2.73

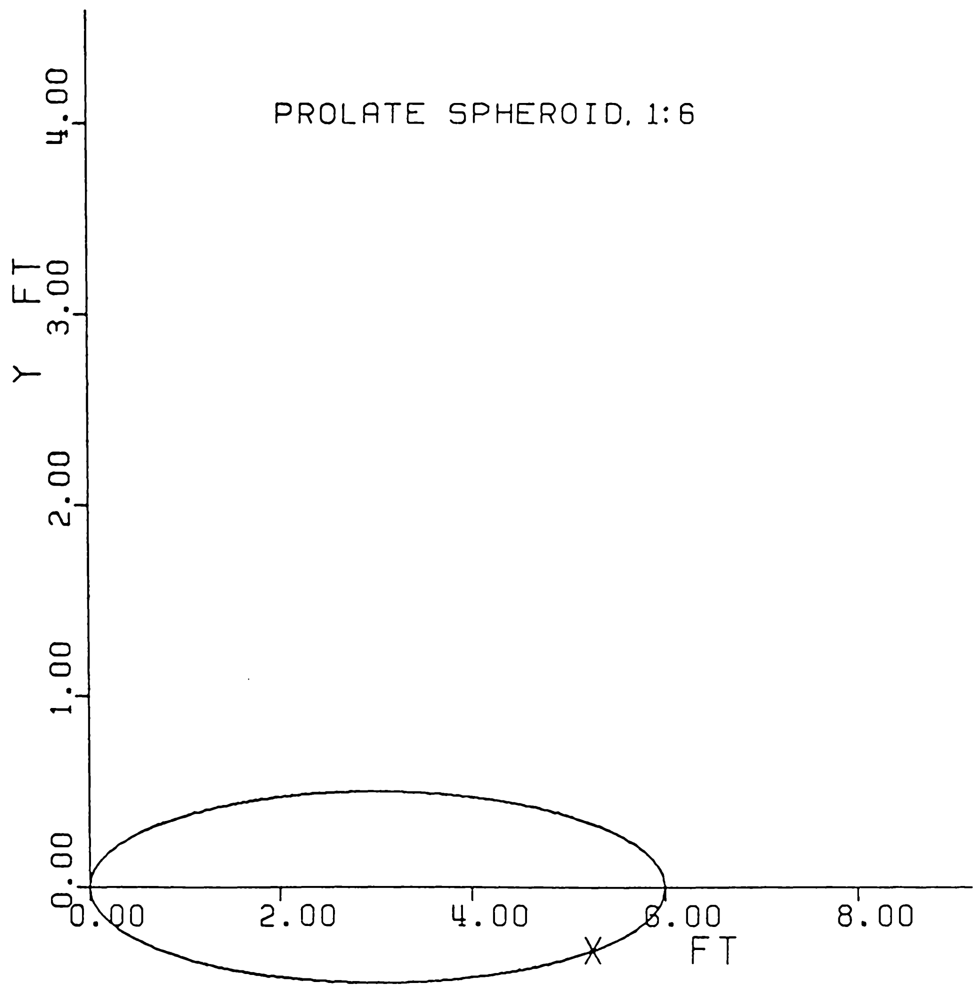

Figure 8: Axisymmetric Body.

shear stress" at $0.4 < x/2a < 0.5$ in their experimental investigation of flow past a 1 : 6 prolate spheroid. Their smaller value corresponds to higher free stream velocities. Their maximum RPF value is 1.219×10^6.

We calculated the n factors for a RPF of 1.219×10^6 for a dimensional frequency of 200 Hz. The neutral point is obtained at $x/2a = 0.15$ and $n = 9$ occurs at $x/2a = 0.36$. For a dimensional frequency of 800 Hz. and a RPF of 2.0×10^6 the disturbances become unstable at $x/2a = 0.14$ and $n = 9$ occurs at $x/2a = 0.38$.

The modified definition of the most dangerous frequency is employed and is found to be in the range (820 - 900) Hz. The results are shown in Tables 11 and 12 and a sample plot is shown in Fig 9.

5.4.1 Effect of Three-Dimensional Disturbances

We calculated the n factors for three-dimensional disturbances at the frequency $\omega^* = 875$ Hz. For this axisymmetric flow, three-dimensional disturbances result in slightly lower values of n. It seems that three-dimensional disturbances are not critical for axisymmetric problems either. The results are shown in Table 13. and a sample plot is shown in Fig.10.

TABLE 11

Axisymmetric Flow

Prolate Spheroid 1 : 6

$\dot{U}_{\infty}$ = 6.036 m/sec, T_e = 534.69°R, RPF = 2.0 x 10^6

ρ_e = 0.9965 g/cc, μ_e = 0.0091676 cp

ω^*	n_{max}	x(cm)	x/2a
600	no growth	--	--
750	no growth	--	--
800	11.30	92.78	0.507
803	11.23	91.41	0.499
806	11.18	91.41	0.499
807	11.16	91.41	0.499
809	11.12	91.41	0.499
812	11.06	90.04	0.492
818	10.94	88.70	0.485
825	10.75	88.70	0.485
837	10.54	86.07	0.471
850	10.33	84.74	0.463
875	9.93	80.83	0.442
900	9.52	77.06	0.421
950	8.78	72.12	0.394
1030	6.17	65.33	0.357

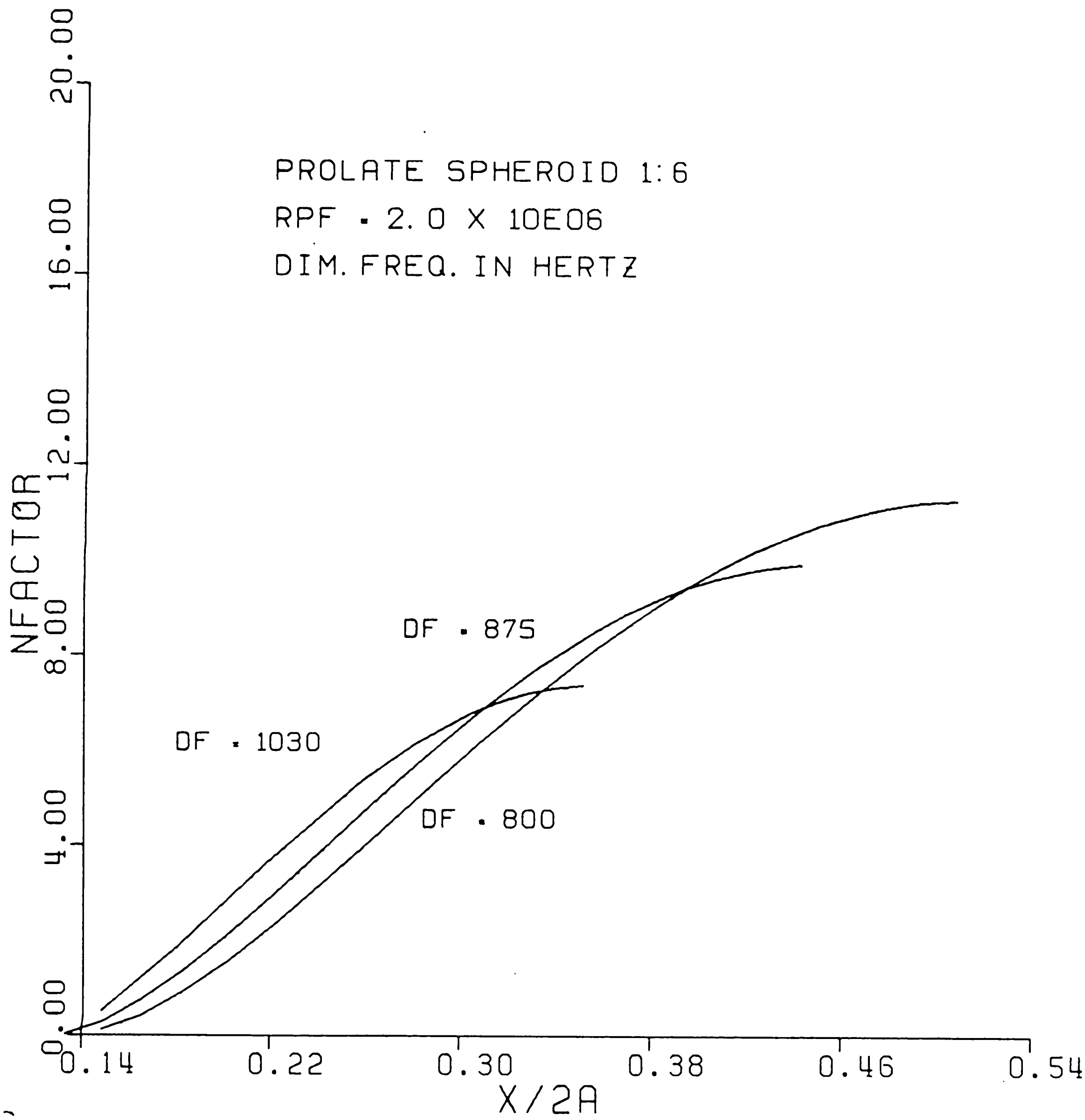

Figure 9: Evaluation of the Critical Frequency.

TABLE 12

Axisymmetric Flow

Prolate Spheroid 1 : 6

U_∞ = 6.036 m/sec, T_e = 534.69°R, RPF = 2.0×10^6

ρ_e = 0.9965 g/cc, μ_e = 0.0091676 cp

ω^*	n	x(cm)	x/2a
600	no growth	--	--
750	no growth	--	--
790	9.0	69.79	0.382
800	9.0	69.46	0.381
818	9.0	68.97	0.377
837	9.0	68.91	0.377
850	9.0	68.61	0.375
862	9.0	68.43	0.374
870	9.0	68.40	0.374
875	9.0	68.36	0.374
880	9.0	68.40	0.374
887	9.0	68.43	0.374
900	9.0	68.77	0.376
950	8.78	72.12	0.394
1030	6.17	65.33	0.357

TABLE 13

Axisymmetric Flow

Prolate Spheroid 1 : 6

U_∞ = 6.036 m/sec, Te = 534.69°R, RPF = 2.0 x 10^6

ρ_e = 0.9965 g/cc, μ_e = 0.0091676 cp

ω^*	T_w	n_{max}	x(cm)	x/2a
875	534.69	9.93	80.83	0.44
875	A=7.5,N=-1.0	8.65	85.15	0.466
875	A=8.6,N=-0.5	5.75	75.07	0.411
875	A=10.0,N=0.0	3.17	68.45	0.374
875	A=11.55,N=0.5	0.70	55.15	0.302
875	A=13.33,N=1.0	no growth		

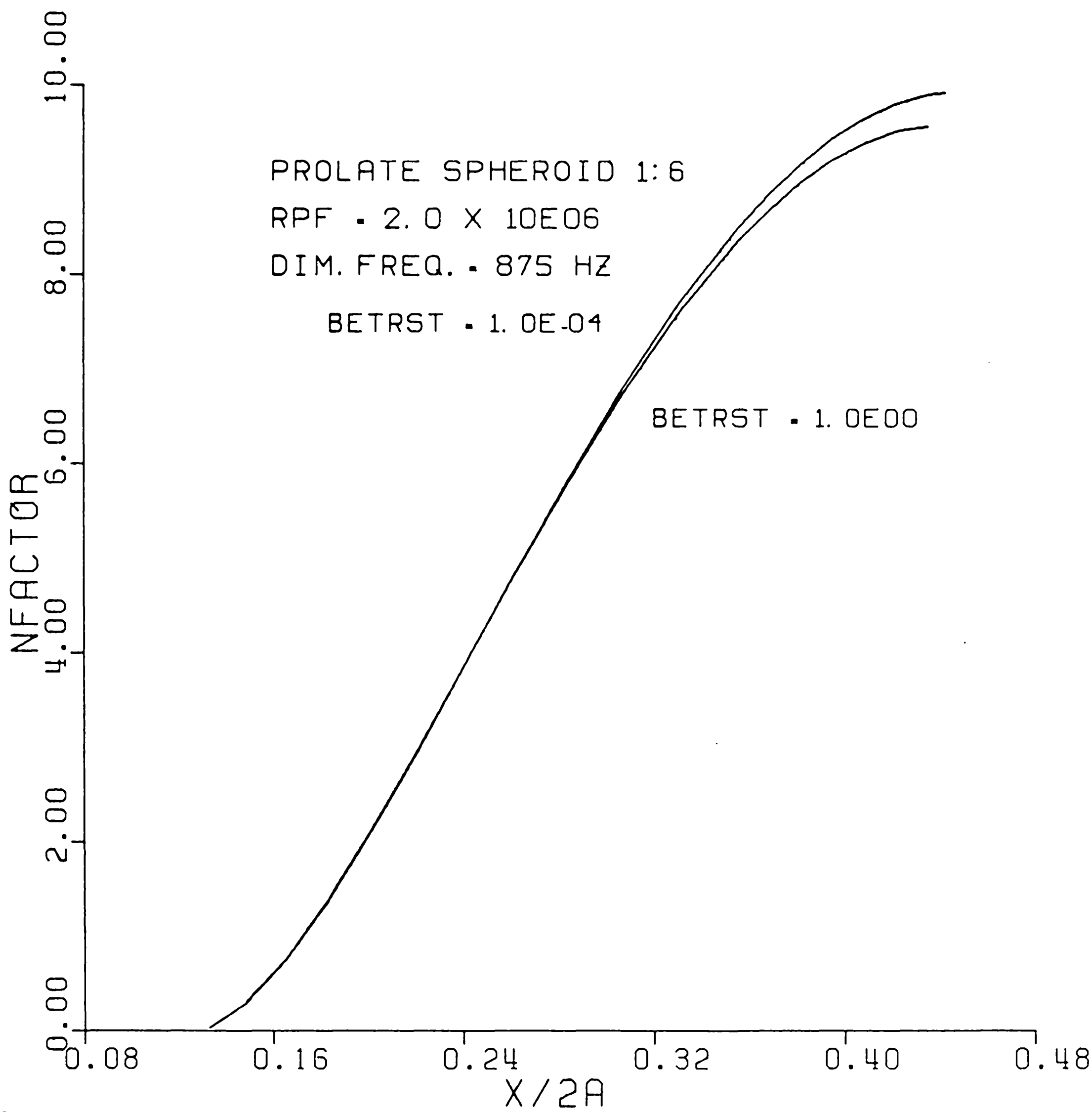

Figure 10: Effect of three-dimensional disturbances.

5.4.2 Effect of Wall Overheat

The power-law heating equation was used to arrive at the variable wall overheat. The results are presented in Table 14 and Fig.11. Heating dramatically reduces the n factors . The tabulated n factors are the maximum values, and the disturbances are stable further downstream. For this particular pattern of heating, increasing N appears to be stabilizing.

Calculations were also performed for three-dimensional disturbances for the heated boundary layer case with variable wall overheat. We show the results in Table 15 and a sample plot in Fig.12. Three dimensional-disturbances appear to be less critical resulting in lower growth rates than those produced by two-dimensional disturbances.

TABLE 14

Heating And Three Dimensional Effects

Prolate Spheroid 1:6

$U_\infty = 6.036$ m/sec, $T_e = 534.69^0R$, RPF = 2.0×10^6

$\omega^* = 875, \rho_e = 0.9965$ g/cc, $\mu_e = 0.0091676$ cp

β_r^*	T_w	n	x/2a
1.0e-04	A=0.00,N=0.00	9.93	0.442
1.0e-04	A=8.66,N=-0.5	5.75	0.411
1.0e-02	A=8.66,N=-0.5	5.75	0.411
1.0e-01	A=8.66,N=-0.5	5.75	0.411
1.0e+00	A=8.66,N=-0.5	5.49	0.416

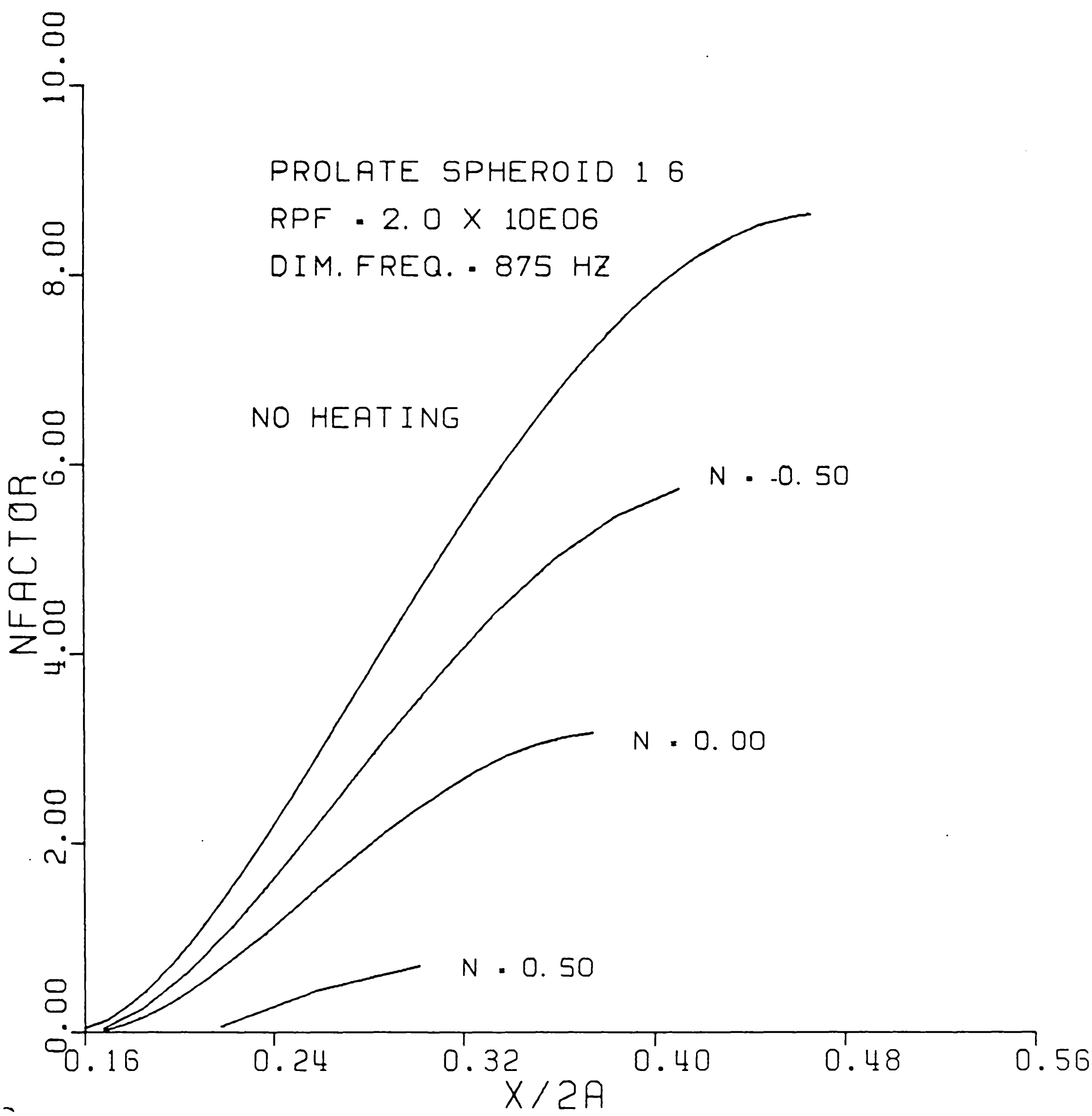

Figure 11: Effect of the Exponent N on n Factors.

TABLE 15

Three Dimensional Effects

Prolate Spheroid 1:6

U_∞ = 6.036 m/sec, T_e = 534.69°R, RPF = 2.0 x 10^6

ω^* = 875, ρ_e = 0.9965 g/cc, μ_e = 0.0091676 cp

β_r^*	n	x/2a
1.0e-04	9.93	0.442
1.0e-02	9.93	0.442
1.0e-01	9.92	0.442
1.0e+00	9.57	0.435

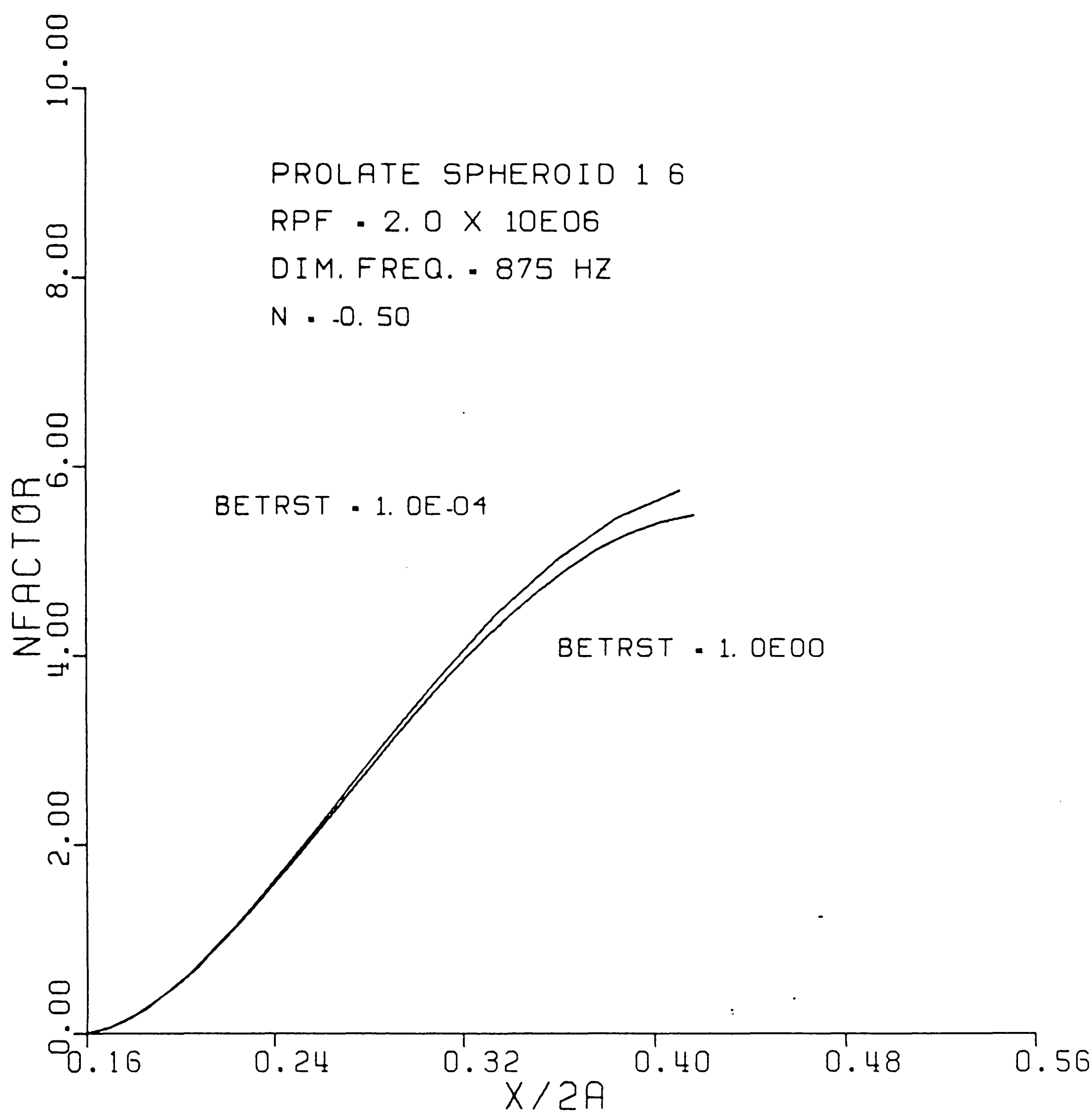

Figure 12: Effect of three-dimensional disturbances.

Chapter VI

CONCLUSIONS

1. We have analyzed the three-dimensional stability of two-dimensional and axisymmetric boundary-layer flows. Numerical results are presented for the case of heated water boundary layers.

2. A modified definition for determining the critical frequency has been suggested. The critical frequency is the one that gives $n = n_{tr}$, where n_{tr} corresponds to transition, in the shortest possible distance on the body. This definition has been used to obtain the most dangerous frequency for the case of the Blasius boundary layer, the Falkner-Skan boundary layer, and the axisymmetric boundary layer over a prolate spheroid.

3. Three-dimensional disturbances have been studied. The results show that two-dimensional waves are more critical and yield higher n factors than three-dimensional waves for all heated and unheated boundary layers irrespective of the pressure gradient and the wall geometry.

4. The effect of heating a water boundary layer on the n factors has been evaluated. Moreover the effect of variable wall overheat has been studied.The results show that the stability strongly depends on the actual heat distribution.

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Appendix A

COMPONENTS OF MATRIX A

All $A_{ij} = 0$ except the following:

$A_{12} = 1.0$

$A_{21} = R/\mu_s[-i\omega\rho_s + i\alpha U_s\rho_s] + (\alpha^2+\beta^2)$

$A_{22} = -D\mu_s/\mu_s$

$A_{23} = R/\mu_s[\ \rho_s DU_s - i\alpha D\mu_s/R + i\alpha\mu_s(s+1)D\rho_s\ /R\rho_s]$

$A_{24} = i\alpha R/\mu_s$

$A_{31} = -i\alpha$

$A_{33} = -D\rho_s/\rho_s$

$A_{35} = -i\beta$

$A_{41} = [-2i\alpha D\mu_s + ir\alpha\mu_s(D\rho_s/\rho_s)]/R$

$A_{42} = -i\alpha\mu_s/R$

$A_{43} = -i\rho_s(-\omega+\alpha U_s) - [\mu_s(\alpha^2+\beta^2 + rD(D\rho_s/\rho_s)) + r(D\mu_s D\rho_s/\rho_s - \mu_s(D\rho_s\ /\rho_s)^2]/R$

$A_{45} = i\beta(r\mu_s D\rho_s/\rho_s - 2D\mu_s)/R$

$A_{46} = -i\beta\mu_s/R$

$$A_{56} = 1.0$$

$$A_{63} = -i\beta(D\mu_s/\mu_s - (s+1)D\rho_s/\rho_s)$$

$$A_{64} = i\beta R/\mu_s$$

$$A_{65} = i\rho_s R(-\omega+\alpha U_s)/\mu_s + \alpha^2 + \beta^2$$

$$A_{66} = -D\mu_s/\mu_s$$

Appendix B

$I_1 = -\rho_s[\partial u_0/\partial x_1 + \partial w_0/\partial z_1] - u_0\partial\rho_s/\partial x_1 - w_0\partial\rho_s/\partial z_1$

$I_2 = -\rho_s[\partial u_0/\partial t_1 + U_s\partial u_0/\partial x_1 + V_s\partial u_0/\partial y + u_0\partial U_s/\partial x_1 + W_s\partial u_0/\partial z_1 + w_0\partial U_s/\partial z_1] - \partial p_0/\partial x_1 + i/R\{ 2\mu_s i\alpha\partial u_0/\partial x_1 + 2\mu_s iu_0\partial\alpha/\partial x_1 + 2i\alpha u_0\partial\mu_s/\partial x_1 + 2i\alpha\mu_s\partial u_0/\partial x_1 + i\alpha\lambda_s\partial u_0/\partial x_1 + i\alpha u_0\partial\lambda_s/\partial x_1 + i\alpha\lambda_s\partial u_0/\partial x_1 + i\lambda_s u_0\partial\alpha/\partial x_1 + \lambda_s\partial^2 v_0/\partial x_1\partial y + \partial v_0/\partial y\ \partial\lambda_s/\partial x_1 + i\beta w_0\ \partial\lambda_s/\partial x_1 + i\lambda_s\beta\partial w_0/\partial x_1 + i\alpha\lambda_s\partial w_0/\partial z_1 + iw_0\ \lambda_s\partial\beta/\partial x_1 + \partial\mu_s/\partial y\ \partial v_0/\partial x_1 + \mu_s\partial^2 v_0/\partial y\ \partial x_1 + i\alpha w_0\partial\mu_s/\partial z_1 + i\alpha\mu_s\partial w_0/\partial z_1 + i\beta\mu_s\ \partial w_0/\partial x_1 + i\mu_s w_0\partial\alpha/\partial z_1 + i\beta u_0\partial\mu_s/\partial z_1 + 2i\beta\mu_s\partial u_0/\partial z_1 + i\mu_s u_0\partial\beta/\partial z_1\}$

$I_3 = -\rho_s[\partial v_0/\partial t_1 + U_s\partial v_0/\partial x_1 + V_s\partial v_0/\partial y + v_0\partial V_s/\partial y + W_s\partial v_0/\partial z_1] + 1/R\{i\alpha v_0\partial\mu_s/\partial x_1 + i\alpha\mu_s\partial v_0/\partial x_1 + i\mu_s v_0\partial\alpha/\partial x_1 + i\alpha\partial v_0/\partial x_1 + \mu_s\partial^2 u_0/\partial x_1\partial y + \lambda_s\partial^2 u_0/\partial x_1\partial y + \partial\lambda_s/\partial y\ \partial u_0/\partial x_1 + \partial\lambda_s/\partial y\ \partial w_0/\partial z_1 + \lambda_s\ \partial^2 w_0/\partial y\partial z_1 + i\mu_s v_0\partial\beta/\partial z_1 + i\beta v_0\partial\mu_s/\partial z_1\ i\beta\mu_s\partial v_0/\partial z_1 + i\mu_s\beta\partial v_0/\partial z_1 + \partial\mu_s/\partial z_1\ \partial w_0/\partial y + \mu_s\partial^2 w_0/\partial y\partial z_1$

$I_4 = -\rho_s[\partial w_0/\partial t_1 + U_s\partial w_0/\partial x_1 + u_0\partial W_s/\partial x_1 + V_s\partial w_0/\partial y + W_s\partial w_0/\partial z_1 + w_0\partial W_s/\partial z_1] - \partial p_0/\partial z_1 - 1/R\{-w_0(i\alpha\partial\mu_s/\partial x_1 + i\mu_s\partial\alpha/\partial x_1 + 2i\beta\partial\mu_s/\partial z_1 + i\beta\partial\lambda_s/\partial z_1 + 2i\mu_s\partial\beta/\partial z_1\ i\lambda_s\partial\beta/\partial z_1) - 2i\alpha\mu_s\partial w_0/\partial x_1 + u_0(-i\mu_s\ \partial\beta/\partial x_1 - i\alpha\partial\lambda_s/\partial z_1 - i\beta\partial\mu_s/\partial x_1 - i\lambda_s\ \partial\alpha/\partial z_1) + \partial u_0/\partial x_1(-i\beta\mu_s - i\beta\lambda_s) + \partial u_0/\partial z_1\ (-i\alpha\mu_s - i\alpha\lambda_s) + \partial^2 v_0/\partial y\partial z_1(-\mu_s-\lambda_s) + \partial v_0/\partial z_1(-\partial\mu_s/\partial y) + \partial w_0/\partial z_1(-2i\mu_s\beta-2i\mu_s -2i\beta\lambda_s) + \partial v_0/\partial y(-\partial\lambda_s/\partial z_1)\}$

Appendix C

$D_1 = E_1 = F_1 = G_1 = 0$

$D_2 = \rho_s R\zeta_{01}/\mu_s$

$E_2 = R(\rho_s U_s \zeta_{01} + \zeta_{04})/\mu_s$

$F_2 = \rho_s R W_s \zeta_{01}/\mu_s$

$G_2 = \rho_s R\{\partial U_s/\partial x_1 \zeta_{01} + \partial U_s/\partial z_1 \zeta_{05} + U_s \partial\zeta_{01}/\partial x_1 + V_s \partial\zeta_{01}/\partial y + W_s \partial\zeta_{01}/\partial z + 1/\rho_s \partial\zeta_{04}/\partial x_1\}/\mu_s$

$D_3 = 0$

$E_3 = -\zeta_{01}$

$F_3 = -\zeta_{05}$

$G_3 = -\{\partial\zeta_{01}/\partial x_1 + \zeta_{01}/\rho_s \partial\rho_s/\partial x_1 + \zeta_{05}/\rho_s \partial\rho_s/\partial z_1 + \partial\zeta_{05}/\partial z_1\}$

$D_4 = -\rho_s \zeta_{03}$

$E_4 = -\rho_s U_s \zeta_{03}$

$F_4 = -\rho_s W_s \zeta_{03}$

$G_4 = -\rho_s\{U_s \partial\zeta_{03}/\partial x_1 + V_s \partial\zeta_{03}/\partial y + \zeta_{03}\partial V_s/\partial y + W_s \partial\zeta_{03}/\partial z_1\}$

$D_5 = E_5 = F_5 = G_5 = 0$

$D_6 = \rho_s R\zeta_{05}/\mu_s$

$E_6 = \rho_s R U_s \zeta_{05}/\mu_s$

$F_6 = R/\mu_s\{\rho_s W_s \zeta_{05} + \zeta_{04}\}$

$G_6 = \rho_s R/\mu_s\{U_s \partial\zeta_{05}/\partial x_1 + V_s \partial\zeta_{05}/\partial y + W_s \partial\zeta_{05}/\partial z_1 + \zeta_{01}\partial W_s/\partial x_1 + \zeta_{05}\partial W_s/\partial z_1 + 1/\rho_s \partial\zeta_{04}/\partial z_1\}$

Appendix D

$$g_1 = \int_0^\infty \{ \rho_s R/\mu_s [W_{12}\zeta_{01} + W_{16}\zeta_{05}] - \rho_s \zeta_{03} W_{14} \} \, dy$$

$$g_2 = \int_0^\infty \{ \rho_s R U_s/\mu_s [W_{12}(\zeta_{01} + \zeta_{04}/U_s) + W_{16}\zeta_{05}] - W_{13}\zeta_{01} - \rho_s U_s \zeta_{03} W_{14} \} \, dy$$

$$g_3 = \int_0^\infty \{ \rho_s R W_s/\mu_s [W_{12}\zeta_{01} + W_{16}\zeta_{05}] - W_{13}\zeta_{05} - \rho_s W_s \zeta_{03} W_{14} + R W_{16}\zeta_{04}/\mu_s \} \, dy$$

Appendix E

$$\hat{h}_1 = \int_0^\infty \{ \rho_s R/\mu_s [\zeta_{01} \partial U_s/\partial x_1 + \zeta_{05} \partial U_s /\partial z_1 + U_s \partial\zeta_{01}/\partial x_1 + V_s \partial\zeta_{01}/\partial y + W_s \partial\zeta_{01}/\partial z_1 + 1/\rho_s \, \partial\zeta_{04}/\partial x_1] W_{12} - W_{13} [\partial\zeta_{01}/\partial x_1 + \zeta_{01}/\rho_s \, \partial\rho_s/\partial x_1 + \zeta_{05}/\rho_s \, \partial\rho_s/\partial z_1 + \partial\zeta_{05}/\partial z_1] - \rho_s W_{14} [U_s \partial\zeta_{03}/\partial x_1 + V_s \partial\zeta_{03} /\partial y + W_s \partial\zeta_{03}/\partial z_1 + \zeta_{03} \partial V_s/\partial y] + \rho_s R W_{16} /\mu_s [U_s \partial\zeta_{05}/\partial x_1 + V_s \partial\zeta_{05}/\partial y + W_s \partial\zeta_{05}/\partial z_1 + \zeta_{01} \partial W_s/\partial x_1 + \zeta_{05} \partial W_s/\partial z_1 + 1/\rho_s \, \partial\zeta_{04}/\partial z_1 \} \, dy$$

$$h_2 = \int_0^\infty \{ [(-\omega + \alpha U_s + \beta W_s)(iR\rho_s/\mu_s \, \partial\rho_s/\partial x_1 - iR\rho_s/\mu_s^2 \, \partial\mu_s/\partial x_1) + iR\rho_s/\mu_s \, (\alpha \, \partial U_s/\partial x_1 + \beta \partial W_s/\partial x_1)] \zeta_{01} W_{12} + [\partial\mu_s/\partial x_1 (-\rho_s RDU_s/\mu_s^2) + \rho_s R/\mu_s \, \partial(DU_s)/\partial x_1 + RDU_s /\mu_s \, \partial\rho_s/\partial x_1] \zeta_{03} W_{12} - [iR\alpha/\mu_s^2 \, \partial\mu_s/\partial x_1] \zeta_{04} W_{12} + [-1/\rho_s \, \partial(D\rho_s)/\partial x_1 + D\rho_s/\rho_s^2 \, \partial\rho_s/\partial x_1] \zeta_{03} W_{13} + [-i\partial\rho_s/\partial x_1 (-\omega + \alpha U_s + \beta W_s) - i\rho_s (\alpha \partial U_s/\partial x_1 + \beta \partial W_s/\partial x_1)] \zeta_{03} W_{14} + [\partial\rho_s/\partial x_1 \, R/\mu_s \, DW_s] \zeta_{03} W_{16} + [iR/\mu_s \, \partial\rho_s/\partial x_1 (-\omega + \alpha U_s + \beta W_s) \zeta_{05} W_{16} + [-\rho_s \, RDW_s/\mu_s^2 \zeta_{03} W_{16} - iR\beta\zeta_{04} W_{16} + iR\rho_s/\mu_s \, (\omega - \alpha U_s - \beta W_s) \zeta_{05} W_{16}] \partial\mu_s/\partial x_1 + [\rho_s R /\mu_s \, \partial(DW_s)/\partial x_1] \zeta_{03} W_{16} + [iR\rho_s/\mu_s (\alpha \, \partial U_s/\partial x_1 + \beta \partial W_s/\partial x_1)] \zeta_{05} W_{16} \}/g_1 \, dy$$

Replace x_1 by z_1 in the expression for h_2 to obtain h_3.

Appendix F

$H_{11} = 0$

$H_{12} = \zeta_{01}\{(-\omega + \alpha U_s + \beta W_s)[iR\rho_s/\mu_s \; \partial\rho_s/\partial x_1 - iR\rho_s/\mu_s^2 \; \partial\omega_s/\partial x_1] + iR\rho_s/\mu_s \; \alpha\partial U_s/\partial x_1 + iR\rho_s\beta/\mu_s \; \partial W_s/\partial x_1 \} + \zeta_{02}\{ RDU_s/\mu_s \; \partial\rho_s/\partial x_1 + R\rho_s/\mu_s \; \partial(D \; U_s)/\partial x_1 - R\rho_s DU_s/\mu_s^2 \; \partial\mu_s/\partial x_1\} + \zeta_{04}\{ -iR\alpha/\mu_s^2 \; \partial\mu_s/\partial x_1\}$

$H_{13} = \zeta_{03}\{-1/\rho_s \; \partial(D\rho_s)/\partial x_1 + D\rho_s/\rho_s^2 \; \partial\rho_s/\partial x_1\}$

$H_{14} = \zeta_{03}\{-i\partial\rho_s/\partial x_1(-\omega+\alpha U_s+\beta W_s) - i\rho_s \; (\alpha\partial U_s/\partial x_1 + \beta\partial W_s/\partial x_1)\}$

$H_{15} = 0$

$H_{16} = \zeta_{03}\{R/\mu_s \; DW_s\partial\rho_s/\partial x_1 + R\rho_s/\mu_s \; \partial(DW_s)/\partial x_1 - R\rho_s DW_s\partial\mu_s/\partial x_1\} + \zeta_{04}\{-iR\beta/\mu_s^2 \; \partial\mu_s/\partial x_1\} + \{iR/\mu_s \; \partial\rho_s/\partial x_1(-\omega+\alpha U_s+\beta W_s) - iR\rho_s/\mu_s^2 \; \partial\mu_s/\partial x_1(-\omega+\alpha U_s+\beta W_s) + i\rho_s R/\mu_s\alpha\partial U_s/\partial x_1 \; iR\rho_s/\mu_s\beta\partial W_s/\partial x_1\}\zeta_{05}$

Replace x_1 by z_1 in the above expressions to obtain H_{2i}

Appendix G

COMPONENTS OF MATRIX C

All $C_{ij} = 0$ except the following.

$C_{12} = 1.0$

$C_{21} = R\{-i\omega\rho_e + i\alpha\rho_e U_e\}/\mu_e + \alpha^2 + \beta^2$

$C_{24} = i\alpha R/\mu_e$

$C_{31} = -i\alpha$

$C_{35} = -i\beta$

$C_{42} = -i\alpha\mu_e/R$

$C_{43} = -i\rho_e(-\omega+\alpha U_e) - \mu_e(\alpha^2+\beta^2)/R$

$C_{46} = -i\beta\mu_e/R$

$C_{56} = 1.0$

$C_{64} = i\beta R/\mu_e$

$C_{65} = i\rho_e R/\mu_e\ (-\omega+\alpha U_e) + \alpha^2 + \beta^2$

Appendix H

COMPONENTS OF MATRIX P

$$\lambda_1 = -\{\alpha^2 + \beta^2\}^{1/2}$$

$$\lambda_2 = -\{\alpha^2 + \beta^2 + i\rho_e R(\alpha U_e - \omega)/\mu_e\}^{1/2}$$

$$P_{11} = P_{13} = P_{14} = P_{15} = P_{53} = P_{56} = 1$$

$$P_{42} = P_{43} = P_{45} = P_{46} = 0$$

$$P_{12} = P_{23} = P_{63} = \lambda_2$$

$$P_{16} = P_{25} = P_{66} = -\lambda_2$$

$$P_{21} = \lambda_1$$

$$P_{24} = \lambda_1$$

$$P_{22} = P_{26} = \lambda_2{}^2$$

$$P_{31} = P_{34} = \lambda_1/i\alpha$$

$$P_{32} = -2i\alpha$$

$$P_{33} = -i(\alpha + \beta)/\lambda_2$$

$$P_{35} = i\alpha(1 - \lambda_2)/\lambda_2$$

$$P_{36} = -i(\alpha - \beta/\lambda_2)$$

$$P_{41} = \rho_e(\omega - \alpha U_e)/\alpha$$

$$P_{42} = P_{43} = P_{45} = P_{46} = 0$$

$$P_{44} = \rho_e(\omega/\alpha - U_e)$$

$$P_{51} = P_{54} = \beta/\alpha$$

$$P_{52} = - P_{55} = \alpha\lambda_2/\beta$$

$$P_{61} = - P_{64} = \beta\lambda_1/\alpha$$

$$P_{62} = P_{65} = \alpha\lambda_2{}^2/\beta$$

Appendix I

COMPONENTS OF MATRIX P^*

$\lambda_1 = -\{\alpha^2 + \beta^2\}^{1/2}$

$\lambda_2 = -\{\alpha^2 + \beta^2 + i\rho_e R(\alpha U_e - \omega)/\mu_e)\}^{1/2}$

$P^*_{11} = P^*_{14} = P^*_{51} = P^*_{54} = P^*_{65} = 0$

$P^*_{31} = P^*_{34} = P^*_{36} = P^*_{43} = 1$

$P^*_{12} = i\alpha - \lambda_2\beta/\alpha$

$P^*_{13} = i\alpha\mu_e/R - \lambda_2{}^2$

$P^*_{15} = (\alpha^2 - \lambda_2{}^2)\beta/(\alpha\lambda_2)$

$P^*_{16} = (\lambda_2{}^2 - i\alpha)/\lambda_2$

$P^*_{21} = P_{24} = i\alpha/\lambda_2{}^2$

$P^*_{22} = - P_{25} = \beta/\alpha$

$P^*_{23} = P_{32} = \lambda_2$

$P^*_{33} = \lambda_2\mu_e/R$

$P^*_{35} = i\beta$

$P^*_{41} = \lambda_1/\lambda_2{}^2$

$P^*_{42} = R/\mu_e$

$P^*_{44} = -\lambda_1 R/\lambda_2{}^2\mu_e$

$P^*_{45} = -iR\beta/(\mu_e\lambda_2)$

$P^*_{46} = -R/(\mu_e\lambda_2)$

$P^*_{52} = (\lambda_2{}^2 + i\beta\lambda_2 - \beta^2)/i\beta$

$P^*_{53} = (\lambda_2{}^2\mu_e(1+i\alpha R/\mu_e) - \beta^2\mu_e)/i\beta R$

$P^*_{55} = \beta^2/\lambda_2$

$P^*_{56} = (-\lambda_2{}^2(1 + i\alpha) + \beta^2)/(i\beta\lambda_2)$

$P^*_{61} = P_{64} = i\beta/\lambda_2{}^2$

$P^*_{62} = -(i\beta + \lambda_2)/i\beta$

$P^*_{63} = (-\mu_e\lambda_2(1 + i\alpha R/\mu_e))/i\beta R$

$P^*_{66} = -(1 + i\alpha)/i\beta$

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STABILITY OF HEATED BOUNDARY LAYERS

WAQAR ASRAR

(ABSTRACT)

A three-dimensional linear stability analysis is presented for two-dimensional boundary layer flows. The method of multiple scales is used to derive the amplitude and the wave number modulation equations, which take into account the nonparallelism of the basic flow. The zeroth-order eigenvalue problem is numerically integrated to calculate the quasi-parallel growth rates which are then integrated together with the nonparallel growth rates along the characteristics of the wave number modulation equations to evaluate the n-factors. The n-factors are used to determine the most dangerous frequency.

The most critical frequency is defined to be the one that yields the n-factor corresponding to transition in the shortest possible distance. This definition is used to evaluate the critical frequency for the Blasius boundary layer, a wedge flow and an axisymmetric boundary layer.

The effect of three-dimensional disturbances is evaluated and found to be less critical than two-dimensional disturbances regardless of the pressure gradient, the temperature distribution of the wall and the wall geometry.

The effect of heating the boundary layer is evaluated for the Blasius, Falkner-Skan and axisymmetric boundary layers. In all the cases considered, heating substantially reduces the n-factors. Results are compared with those of

Strazisar & Reshotko (1978) and Nayfeh & El-Hady (1980).