Investigating the Performance of Process-observation-error-estimator and Robust Estimators in Surplus Production Model: A Simulation Study

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Abstract

This study investigated the performance of the three estimators of surplus production model including process-observation-error-estimator with normal distribution (POE_N), observation-error-estimator with normal distribution (OE_N), and process-error-estimator with normal distribution (PE_N). Estimators with fat-tailed distributions including Student's t distribution and Cauchy distribution were also proposed and their performances were compared with the estimators with normal distribution. This study used Bayesian method, revised Metropolis Hastings within Gibbs sampling algorithm (MHGS) that was previously used to solve POE_N (Millar and Meyer, 2000), developed the MHGS for the other estimators, and developed the methodologies which enabled all the estimators to deal with data containing multiple indices based on catch-per-unit-effort (CPUE). Simulation study was conducted based on parameter estimation from two example fisheries: the Atlantic weakfish *(Cynoscion regalis*) and the black sea bass (*Centropristis striata*) southern stock.

Our results indicated that POE_N is the estimator with best performance among all six estimators with regard to both accuracy and precision for most of the cases. POE_N is also the robust estimator to outliers, atypical values, and autocorrelated errors. OE_N is the second best estimator. PE_N is often imprecise. The estimators with fat-tailed distribution usually result in some estimates more biased than the estimators with normal distribution. The performance of POE_N and OE_N can be improved by fitting multiple indices. Our study suggested that POE_N be used for population dynamic models in future stock assessment. Multiple indices from valid surveys should be incorporated into stock assessment models. OE_N can be considered when multiple indices are available.

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1. Research needs

The surplus production model (Ricker 1975), also known as the biomass dynamic model (Hilborn and Walters 1992), is an age-aggregated model in fish stock assessment. Compared to more complex models such as the delay difference model (Hilborn and Walters 1992) and age structured models, the surplus production model has undemanding data needs and simple forms, but is able to provide valid parameter estimation that can capture basic fishery characteristics. Under certain situations, the surplus production model is superior to the more complicated models (Ludwig and Walters 1985, Prager 1994), so it remains as a fundamental method and is widely used in biology and fisheries stock assessments.

To solve the surplus production model, there are historically three estimators classified based on the assumptions of error structure: process-error-estimator (PE), observation-error-estimator (OE), process-observation-error-estimator (POE). Selection of an appropriate estimator to fit the biomass dynamic model to the observed data is very important with regard to the reliability of parameter estimation (Punt 1988, 1992, de Valpine and Hastings 2003). The POE considers both process error, which shows the fluctuations of the population dynamics, and observation error, which represents the measurement error of the records. The PE limits the random effects to the fluctuation of population change, whereas the OE only takes the measurement error into consideration. The importance of process error and observation error varies according to different fish populations and different fisheries. The results of parameter estimation sometimes differ dramatically because of the choice of different estimators (Polacheck et al. 1993), which make the choice of the reliable estimator important to population assessment of fish species. The aim of my research was to determine the performance of the three estimators and to select the best estimator that can perform well under a variety of situations.

Besides the original estimators, new estimators with new assumptions on the error distribution were worth further examination. Ordinarily, the errors in the surplus production model are assumed to have multiplicative error with a lognormal distribution. Fat-tailed distributions were studied to model the errors and were found to lead to a more stable estimation (Chen and Fournier 1999). Outliers, which are caused by data deficiency, and atypical values, which come from the abnormal population change, commonly occur in fishery data (Prager 2002). Commonly used fat-tailed distributions include Student's t distribution and Cauchy distribution. Autocorrelated errors and errors that follow mixed distributions are also common in fishery data (Beamish 1995, Beamish et al. 1999). Compared to the normal distribution, the fat-tailed distribution has a heavier tail which exhibit tolerance to noises. In that, estimators with the multiplicative error following a log-fat-tailed distribution has the potential to be more resistant to abnormal values, such as outliers, atypical values, and autocorrelated errors. The robustness of the estimators to these situations needed further consideration.

In present, estimators adopted by fishery stock assessment using population dynamic models are usually under the assumption that the error follows a log-normal distribution. From previous study, POE had the best performance but the most computation complexity among the three estimators (de Valpine and Hastings 2000). OE had better performance than PE (Polacheck et al. 1993). Among the three estimators, OE is currently widely adopted by fishery biologist and fishery managers when they encounter population dynamic problems. One reason is that it can be solved by just basic statistical methods which can be realized using tools as simple as Excel spread sheet, while POE requires much more difficult statistical methods and software. The other reason is that OE has relatively better performance than PE, which also requires simple statistical computations. Because of the improvement of computer performance, people started to take less concern on computation problems. Also with the possibility of compiling statistical methods into easy operating software, a new discussion of how to choose estimators for reality use was needed, rebalancing the estimators' complexity and performance.

In a summery, the previous work on estimators of surplus production model lacked a systematic framework for comparing estimators under different situations. Therefore, a more comprehensive study with different data and estimator assumptions needed to be developed. New methodologies on more complicated estimators needed to be developed and popularized if necessary.

2. Background information

2.1 Introduction of surplus production models and different estimators

The surplus production model as one of the age-aggregated models is widely used to analyze fishery population dynamics, especially when age data are not available or are unable to be precisely obtained. The surplus production model only requires time series of catch data and effort data. It incorporates the overall effects of growth, mortality, and recruitment into a single production function--the surplus production.

The model is composed of the process equation and the observation equation. In the deterministic process equation, the total biomass dynamics are described as

$$B_{t+1} = B_t + g(B_t) - C_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

where B_t is the biomass of the fish population at the start of year t, C_t is the biomass of catch during year t, $g(B_t)$ is the surplus production of year t, which demonstrates a density dependent population growth. There are several forms of surplus production based on fisheries with different characteristics. The most common ones include Schaefer form (Schaefer 1954), Fox form (Fox 1970), and Pella-Tomlison form (Tomlison 1969). All of them incorporate the two important parameters: the intrinsic growth rate and the carrying capacity. They have different structures so that the most productive population size that result in the fastest population growth varies based on different forms. By using Schaefer form, it is assumed that the fastest population growth happen when the population is half of the carrying capacity. By using the Fox form and the Pella-Tomlison form, with their different asymmetry shapes, the fastest growth is assumed to happen either when the population size is smaller or larger than half of the carrying capacity. In reality, the different forms chosen to be used should be based on the computational complexity and the characteristics of the individual fish population (Prager 2002, Maunder 2003). Also, all of the variables representing biomass in equation (1) can be changed to population abundance depending on the unit with which the fishery data are recorded.

The deterministic observation equation, which is typically written as

$$I_t = qB_t, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

where I_t is the index of relative abundance, which are observable values to calibrate population size. The catch-per-unit-effort (CPUE) is most often used as the index of relative abundance in stock assessment models. The observation equation in surplus production model assumes a linear, proportional relationship between the index and the abundance. The slope of the proportional relationship, q, is known as the catch-ability coefficient.

The suggested error assumptions for process and observation equations include additive normal distribution and multiplicative log-normal distribution (Quinn and Deriso 1999). The three estimators that are commonly used to fit the surplus production model to data are based on the assumptions of whether the process error or the observation error is incorporated into the process equation or observation equation respectively. The process-error-estimator (PE) only considers the fluctuation on the process equation and leaves the observation equation to be deterministic. The observation-error-estimator (OE) only adds random effects to the observation equation and considers the population dynamic to be deterministic. The process-observation-error-estimator (POE) assumes that error exists in both equations (Quinn and Deriso 1999).

Historically, different statistical methods have been used to solve the three estimators. For OE, the maximum likelihood method has been commonly used to solve the estimator (Polacheck 1993). The likelihood function is based on errors of CPUE. The least square method has also been used on occasion (de Valpine and Hastings 2002). For PE, as the observation equation is deterministic, B_t in the process equation can be all represented by I_t to diminish the number of equations. Both the ordinary least square method and the maximum likelihood method have been used to solve PE (Polacheck, 1993, de Valpine and Hastings 2002).

For POE, more complex statistical methods are needed. The Kalman filter method (Kalman 1960) has been applied to state space models with process equation and observation equation to be linear and with error normally distributed, such as the catch at length model (Sullivan 1992), the catch at age model (Schnute 1994) and the delay difference biomass model (Kimura et al. 1996). The extended Kalman filter method has been used to fit non-linear state space models only when additional information is available, such as an arbitrary ratio of the two variances (Pella 1993). Another modification to the Kalman filter is the numerically integrated state-space (NISS) method (de Valpine and Hastings 2002) which can be implemented on any nonlinear, non-Gaussian models.

The penalized likelihood method (Schnute 1994, Richards and Schnute 1998) is an alternative to the Kalman filter. It is applicable without linear or Gaussian assumptions, but it may result in some bias in the estimation of fixed effects (Lin and Breslow 1996).

Bayesian analysis has also been used. The Markov Chain Monte Carlo approach with a Gibbs sampling was used in non-normal and nonlinear state-space modeling (Carlin et al. 1992). Then a modification was made to use Metropolis-Hastings within Gibbs sampling (Millar and Meyer 1999, 2000). Compared to the other methods, the Bayesian method has the advantage of providing an estimation of parameter distribution instead of just the point parameter value, although it is at a cost of increased computing complexity.

2.2 Review of the research on the performance of different estimators

Historically, there are several studies comparing the performances of the three estimators using both example fisheries and simulation studies (Polacheck et al. 1993, de Valpine 2003). Polacheck et al. (1993) compared PE and OE with normal distribution in the surplus production model. They assumed that both the observation error and process error were multiplicative and log-normally distributed. The result showed that the OE usually provides a lower estimate of maximum sustainable yield and is less biased and more precise. Even for the simulated data with only process error, the OE can give better parameter estimation than PE. In their comparison, the least square method was used to solve for the PE, and the maximum likelihood method was used to solve for the OE.

de Valpine (2003) compared three estimators using the Beverton-Holt model and the Ricker model (Quinn and Deriso 1999). The observation error and process error were both assumed to be multiplicative log-normally distributed. The simulated data he used were all with the existence of both process and observation errors, but changes were made on the relative magnitude of the two errors. Among the three estimators, POE with a thorough consideration of errors was shown to have the lowest estimation bias and variance in most of the simulated cases. The exceptions were for the Ricker model with a large stable equilibrium value, the PE performed best with the simulated data of large process error and small observation error, and the OE performed the best with the simulated data of large observation error and small process error. In his comparison, he used the least square method to solve both PE and OE, and used the NISS method to solve POE.

2.3 Review of the research on outliers/atypical values

Large errors are commonly associated with fisheries data, which result from many sources. Outliers are data with large error or even mistakes that shift the observed value from the true value greatly. Atypical values are actually true values but result from abnormal environmental or human effects. In many cases, the outliers and atypical values are left behind when fitting model to data, which will certainly cause bias in parameter estimation. One way to deal with outliers and atypical values is to identify them after fitting the model, remove them, (interpolate new values), and to refit the model again on the new data set without outliers or atypical values (Chen and Paloheimo 1995, Prager 2002). To distinguish the outliers or atypical values from ordinary data is not easy. Another way to eliminate the influence of abnormal errors is to build estimation methods that can resist those unique values. Studies have been conducted to use robust objective functions for the estimators (Chen et al. 1994, Shertzer and Prager 2002) or robust error distributions (Chen and Fournier 1999, Chen et al. 2000) with the estimators so that a stable estimation could be achieved.

In solving von Bertalanffy growth models, fat-tailed distributions, which exhibit heavier tails than the normal distribution have been shown to be more resistant to outliers or atypical values (Chen and Fournier 1999). They also are less sensitive to prior mis-specification in the Bayesian estimation than the normal distribution when OE was used for catch at length model (Chen el al. 2000). Student's t distribution, Cauchy distribution and modified normal distribution have all been used as fat-tailed distribution (Chen and Fournier 1999, Chen el al. 2000). However, the assumption of fat-tailed distributed errors has its disadvantages when outliers or atypical values are few (Chen and Fournier 1999). There has been discussion about the mathematical considerations of the modified normal distribution (Mclane 1967). Studies are moving towards development of good estimators that are not only less sensitive to outliers and atypical values, but can also have good performances in the absence of outliers and atypical values.

2.4 Review of research on autocorrelated data and mixed error data

Climate or environmental effects can have strong autocorrelated long term variations that influence the recruitment, growth rate, and natural mortality of fish populations. Sometimes, these external effects can also exhibit sudden changes in means or variations which may lead to a mixed population effects. For example the temperature fluctuation changes from strong to mild in May in the northern hemisphere; because in summer the temperature usually stays high while in spring it often fluctuates in a wider range. In stock assessment models, people use autocorrelated error (Beamish et al. 1999) and mixed error to model these phenomena.

For the observation error, there are also nonstationary situations. Autocorrelated fluctuations can happen if the errors show self dependence caused by the fishing gear, fishing people, fishing location, etc. The change of error distribution can result from the change of fishing facilities and fishing environment.

The autocorrelated error assumptions in the population dynamic equation have been widely used in recruitment analysis (Walters and Parma 1996, Katsukawa and Matsuda 2003) and population growth models (Jiao et al. 2008, 2009) by both real population analysis and simulation studies. First order autocorrelations are assumed in all of the cases. The effects of mixed error in the population dynamics have not been studied yet.

In the observation equation, the autocorrelated error was once assumed by Ishimura et al. (2005). First degree autocorrelations were used in the model. Prager (2002) simulated observation error composed of three mixed distributions with the coefficients of variation of 2%, 10% and 25% respectively.

2.5 Research goals and objectives

My research aimed to give a further comparison of these three estimators and to explore their performance with fat-tailed distributions. To make the study convincing, my work was based on two fish populations along Atlantic coast, Atlantic weakfish (*Cynoscion regalis*) and the black sea bass (*Centropristis striata*) southern stock, the different structures of which can provide a more general background to compare the three estimators. In my simulation study, I conducted several scenarios to test the performance of different estimators under different situations. The assumptions on error distributions were discussed together with the estimators. The goal of this work was to compare the performance of the estimators in surplus production model in order to decide the best estimator in surplus production model with good parameter estimation and moderate complexity, and to find the robust estimator that was resistant to different kinds of error in fishery data. Based on the goal, I accomplished six objectives.

More specifically, I investigated 1) the performance of PE, OE, and POE in surplus production model by comparing the accuracy and precision of their parameter estimations; 2) the performance of PE, OE, and POE by looking at their responses to outliers or atypical values; 3) the performance of PE, OE, and POE with fat-tailed distributions; 4) the performance of PE, OE, and POE with fat-tailed distribution when outliers or atypical values exist; 5) the performance of PE, OE, and POE when residuals are autocorrelated or following a mixture of distributions; 6) the performance of PE, OE, and POE under a different number of CPUEs, i.e., more measurement errors.

3. Contributions from my research

There are some limitations in previous studies on the comparisons of the three estimators. My research corrected these limitations and further explored the performances of the three estimators in surplus production model.

First, we managed to use the same statistical methods, Metropolis Hastings within Gibbs sampling, to solve the three estimators. Previous studies did not use a consistent statistical method to solve different estimators, so it was possible that the parameter estimation results contained combined effects of both the statistical methods used and the characteristic of different estimators. In previous study, Polacheck et.al. (1993) compared the OE and PE, but they chose to solve the estimators using the maximum likelihood method and the ordinary least square method respectively. de Valpine and Hastings (2002) compared the three estimators, but they chose to use the ordinary least square method to solve OE and PE, and the NISS method to solve POE. He intended to compare the statistical methods at the same time and concluded that the NISS method gave the best estimation. In my opinion, his result was a mixture of both the performance of estimators and statistical methods together, and the effects of the estimator and the estimation method must interfere with each other. In this research, I used the unified statistical method so that all focus was on the performance comparison of the three estimators. The result of our study was thus more solid than previous studies.

Second, our results of the estimators of the best performance were more convincing, because we conducted simulations by generating data from different “true” models with different assumptions on error structure which avoided the preordainment in the previous study. de Valpine and Hastings (2002) simulated data of three process error and observation error scenarios: large standard deviation for both, and large for one but small for the other. It was obvious that all of their scenarios were based on the process observation error assumption so that it was not surprising that POE behaved the best. A solid analysis should simulate data with only observation error or only process error or both and then run the three estimators based on different simulated data to get a fair comparison. To verify the universality of the robust estimators, this research conducted data with only observation error, only process error and both process error and observation error to test performance of the three estimators.

Third, my study had the first comprehensive conclusions on the performance of the POE in surplus production models compared with the other estimators. Previously there were only studies on the process-observation-error-estimator in the surplus production model (Millar and Meyer 2000) and other more complex models, such as the delay difference model (Meyer and Millar 1998) and age-structured models (de Valpine 2004). Polacheck et.al. (1993) only compared the two estimators with only single error existed in surplus production models. de Valpine and Hastings (2002) compared the three estimators and considered POE the best, but the models he chose to operate on were the Beverton-Holt model and the Ricker model, which were simple population growth models. My research provided a better idea of the performance of estimators for surplus production model and was quite valuable and useful in current fisheries stock assessment.

Fourth, this was the first research studying the applicability of estimators with fat-tailed distribution in the surplus production model. Studies with fat-tailed distribution incorporated in the estimators have been used in stock recruitment model, individual growth model, and catch at age models with Bayesian methods (Chen and Paloheimob 1998, Chen and Fournier 1999, Chen el al. 2000). No evaluations have been found in surplus production models. This study discussed whether the performance of the three estimators with fat-tailed distribution was better than those with the normal distribution in the surplus production model, and whether the estimators with fat-tailed distribution could have good performance in both situations with or without the existence of outliers and atypical values. Our study on estimators with a fat-tailed distribution provided a beneficial test of their applicability before applying them in the surplus production model for real fishery problems.

Fifth, this was the first study to compare the performance of the three estimators with fat-tailed distributions. Studies have been focused on building the fat-tailed distribution into the parameter estimation methods (Chen and Paloheimo 1998, Chen and Fournier 1999, Chen et al. 2000). The modification of error distribution assumption can have possibilities to change the performance of different estimators. Our simulation study provided a picture of how the estimators performed with fat-tailed distribution and how the modification of the distribution could influence the choice of estimators among different complexities.

Sixth, our studies also discussed the performances of the three estimators dealing with data of autocorrelated error and mixed error. Previous studies were all limited in analyzing recruitment models and population growth models with autocorrelated errors. This study tested surplus production models to fit data with nonstationary process and observation errors.

Seventh, my study investigated the influence of the number of the measurement errors, i.e., one measurement error from one CPUE index and three or five from more than one CPUE indices. The influence of the measurement errors increased in this situation which reasonably lead this study to investigate that whether the performance of the estimators could be improved and whether OE could outcompete POE when multiple observation errors exist.

4. Methodology

There are several ways to fit a surplus production model to observed data, which can be found in section 2.2 about the background information. When fitting a model to data, an estimator with specific error structure should be chosen, error distributions should be assumed, and the statistical methodology should be selected to solve the estimator. Choices of fat-tailed distributions and statistical methods in my study are listed in section 4.2. PE, OE, and POE estimators are illustrated in section 4.3 with the combination of multiplicative errors following lognormal distribution and log fat-tailed distributions. The statistical methods are explained in section 4.4 and 4.5. The estimating process used to analyze the example fisheries and used in simulation studies are introduced in detail in sections 4.6 and 4.7.

4.1 Surplus production model and its characteristics

The surplus production model (Ricker 1975) in a deterministic form can be written as:

$$\begin{cases} B_{t+1} = B_t + g(B_t) - C_t, \\ I_t = qB_t. \end{cases} \quad \text{......} \quad (3)$$

In the process equation, Schaefer growth form was used to model the surplus production in my study (Schaefer 1954). The Schaefer form is a quadratic polynomial function of the population biomass that is relatively simple, and it can utilize parameters with biological interests to reflect the fish population status. It has the following form:

$$g(B) = rB(1 - \frac{B}{K}) \quad \text{......} \quad (4)$$

in which, K is the carrying capacity that represents the equilibrium population size in the original natural environment without human influence. We can also interpret it as the largest population size that the environment can support at a saturation state. Once the population size is larger than K, the population will decrease to carrying capacity because of lack of food supply or other resources to support the population. *r* represents the intrinsic growth rate. A fish population that has a larger value of intrinsic growth rate can exhibit a faster increase of abundance or biomass due to the self-growth or reproduction of the population. Because the Schaefer form follows a quadratic polynomial function, it has a symmetrical curve (Figure 1). When the population biomass changes from zero to the carrying capacity, the surplus production can reach a maximum value when the biomass stays at K/2.

Besides parameters in the numerical model, some biological reference points are also very important values for fishery management. These quantities of management interests include maximum sustainable yield (MSY), the total biomass at maximum sustainable yield B_{MSY}, and the fishing mortality at maximum sustainable yield F_{MSY}. The MSY is defined as the maximum yield we can exploit while the stock remains at a sustainable state. In Schaefer model, MSY can be achieved at the same time the surplus production reaches its max value. So MSY equals $\frac{rK}{4}$, with B_{MSY} equals $\frac{K}{2}$ (Figure 1). F_{MSY}, which is the entire fishery harvest divided by the population biomass when maximum sustainable yield is reached, equals $\frac{r}{2}$. We can also define relative fishing mortality rate at the latest year of fishery to be F_T / F_{MSY}, and relative stock biomass after the latest year B_T / B_{MSY}. F_T and B_T are the fishing mortality and fish population biomass respectively at the latest year of the recorded time series T. If F_T / F_{MSY} is larger than 1, we can tell the fishery is currently under overfishing. If B_T / B_{MSY} is larger than 1, we can tell the fishery has been over fished.

To benefit the statistical methodology used to solve the model, I used a revised form of the surplus production model by replacing B by P (relative population biomass) which equals B/K. The deterministic Surplus production model with a Schaefer form changed to

$$\begin{cases} P_{t+1} = P_t + rP_t(1-P_t) - C_t / K, \\ I_t = qKP_t. \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)'$$

In this way, the values for relative population biomass P, carrying capacity K, and other parameters can be calculated from the statistical model. B, B_{MSY}, and other biological reference points is then calculated based on these parameter values. I use equation (1)' instead of (1) to explain the error assumption and Bayesian method in later sections.

4.2 Probability density function under consideration in this study

To add random effects on process error equation and observation error equation, the multiplicative errors were considered other than additive error. The process equation with multiplicative error can be written as

$$P_{t+1} = [P_t + rP_t(1-P_t) - C_t / K] \times e^{\varepsilon_{1,t}},$$

where $\varepsilon_{1,t}$ represent the process error of year t. The observation equation with multiplicative error can be written as

$$I_t = (qKP_t) \times e^{\varepsilon_{2,t}},$$

where $\varepsilon_{2,t}$ represent the observation error of year t.

I use ε below to represent all errors including $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, t=1,…,T. The distribution of ε is usually assumed to be normal with a probability density function as:

$$p(\varepsilon)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(\varepsilon-\mu)^2}{2\sigma^2}},\varepsilon\in R$$

The error has mean value, μ, and variance σ^2. We write $\varepsilon \sim N(\mu,\sigma^2)$. When we assume the error of observation equation or process equation follows the lognormal distribution, we consider both μ and σ as parameters to be estimated. Since our study is a simulation study focusing on the performance of different estimators, we assume $\mu = 0$.

In my research, I explored the multiplicative error with fat-tailed distribution as well. Compared to normal distribution, the fat-tailed distribution exhibits wider kurtosis and thicker tails like what is illustrated as red curve compared to the blue curve that represents the normal distribution (Figure 2). The scientific definition of fat-tailed distribution is a distribution when

$\Pr[X > x] \sim x^{-\alpha}$, as $x \to \infty$, $\alpha > 0$.

Student’s t distribution has been incorporated into fisheries research previously (Chen et al. 2000). Other than the Student’s t distribution, I also experimented on the Cauchy distribution, another choice of fat-tailed distribution. The three estimators to solve the surplus production model, PE, OE and POE, are combined with different assumptions on the error distribution, the normal, Student’s t, and the Cauchy distributions and are shown in section 4.3.

Student's t-distribution is a classic fat-tailed distribution that is most commonly used. It has the density function as

$$f(t) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\,\Gamma(\frac{\nu}{2})}(1+\frac{t^2}{\nu})^{-(\nu+1)/2}, t \in (-\infty,+\infty). \qquad (7)$$

ν, known as degree of freedom, is a real number that ranges from 0 to $+\infty$ and Γ is the Gamma function that has the following form

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt .$$

When $\nu > 2$, the expected value for the t distribution is 0, and the variance is $\frac{\nu}{\nu-2}$.

When $1 < \nu \leq 2$, the expected value for the t distribution is 0, and the variance is $+\infty$. For the t distribution with $0 < \nu \leq 1$, there is no mean or variance. When we assume the error in observation equation or process equation follows log-Student's t distribution, we consider ν as a parameter to be estimated.

Using the log-Student's t distribution as the error distribution in the surplus production model has limitation. The shape of Student's t distribution has limitations. As we can tell from the Figure 23, the smaller ν is, the more spread out the distribution is. And the Student's t distribution will converge to the standard normal distribution as ν increases to positive infinity. The most towering shape the Student's t distribution can get is as standard normal distribution (Figure 23). The peak of the Student's t distribution can never exceed the peak of the standard normal distribution, which means that it fails to mimic any distribution of $N(0,\sigma^2)$ where σ is far smaller than 1.

The **Cauchy distribution** is a continuous distribution with the probability density function:

$$f(x;x_0,\gamma)=\frac{1}{\pi\gamma[1+(\frac{x-x_0}{\gamma})^2]}=\frac{1}{\pi}[\frac{\gamma}{(x-x_0)^2+\gamma^2}]\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

where x_0 is the location parameter indicating the peak of the distribution, and γ is the scale parameter that specifies the half-width at half-maximum (HWHM). The larger γ is, the more spread out the distribution other than concentrate. The Cauchy distribution has no mean and no variance. When we assume the error of observation equation or the process equation follows the log-Cauchy distribution, we consider both x_0 and γ as parameters to be estimated. Since my study is a simulation study focused on the performance of different estimators, I assumed $x_0=0$.

4.3 Estimators under consideration in my study

The process-error-estimator (PE) assumes the process error structure. The observation equation is considered precise, and the error lies in the population dynamics equation. So for the process-error-estimator with multiplicative error, we have

$$\begin{cases} P_{t+1}=(P_t+g(P_t)-C_t)e^{\varepsilon_{1,t}} \\ I_t=qKP_t \end{cases}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

The error $\varepsilon_{1,t}$ can follow a normal distribution or fat-tailed distributions; and $g(P_t)$ follows the Schaefer form.

Assume $\varepsilon_{1,t}\sim N(0,\sigma_1^2)$. The likelihood function of P_t,given P_{t-1} using log-normal distribution assumption is:

$$f(P_t \mid P_{t-1}) = \frac{1}{\sqrt{2\pi}\sigma_1} \exp\{\frac{-(\log P_t - \log \hat{P}_t)^2}{2\sigma_1^2}\}$$

where $\hat{P}_t = P_t + g(P_{t-1}) - C_t$. $\varepsilon_{1,t} = \log P_t - \log \hat{P}_t$ represents the process error of year t.

Assume $\varepsilon_{1,t} \sim t(\upsilon_1)$. The likelihood function changes to

$$f(P_t \mid P_{t-1}) = \frac{\Gamma(\frac{\nu_1 + 1}{2})}{\sqrt{\nu_1 \pi}\Gamma(\frac{\nu_1}{2})} (1 + \frac{(\log P_t - \log \hat{P}_t)^2}{\nu_1})^{-(\nu_1+1)/2}$$

Assume $\varepsilon_{1,t} \sim Cauchy(0, \gamma_1)$. The likelihood function changes to

$$f(P_t \mid P_{t-1}) = \frac{1}{\pi\gamma_1[1 + (\frac{\log P_t - \log \hat{P}_t}{\gamma_1})^2]} = \frac{1}{\pi}[\frac{\gamma_1}{(\log P_t - \log \hat{P}_t)^2 + \gamma_1{}^2}]$$

$\hat{P}_t$ is a function of I_t, parameters q, and parameters $P_{initial}, K$, r from the deterministic process equation. The parameters to be estimated from the likelihood function are q, $P_{initial}, K$ and r. In addition to those parameters, PE with normal distribution (PE_N) also has parameter σ_1. PE with Student's t distribution (PE_T) also has parameter ν_1. PE under Cauchy distribution (PE_C) also has parameter γ_1.

Observation-error-estimator (OE) adopts the observation error structure assumption. The population dynamics equation is assumed to be deterministic, and the error occurs in the proportional relationship between the CPUE and the population biomass. It is summarized in the following equations:

$$\begin{cases} P_{t+1} = P_t + g(P_t) - C_t / K \\ I_t = (qKP_t)e^{\varepsilon_{2,t}} \end{cases}$$

The error $\varepsilon_{2,t}$ can follow a normal distribution or fat-tailed distributions, and $g(P_t)$ follows the Schaefer form.

Assume $\varepsilon_{2,t} \sim N(0, \sigma_2^2)$. The likelihood functions of I_t given $\hat{I}_t$ for observation-error-estimator with log-normal assumption is

$$f(I_t \mid \hat{I}_t) = \frac{1}{\sqrt{2\pi\sigma_2}} \exp\{\frac{-(\log I_t - \log \hat{I}_t)^2}{2\sigma_2^2}\}$$

where $\hat{I}_t = P_t \times K \times q$. Define $\varepsilon_{2,t} = \log I_t - \log \hat{I}_t$ which represent the observation error for year t.

Assume $\varepsilon_{2,t} \sim t(\upsilon_2)$. The likelihood function changes to

$$f(I_t \mid \hat{I}_t) = \frac{\Gamma(\frac{\nu_2 + 1}{2})}{\sqrt{\nu_2 \pi}\Gamma(\frac{\nu_2}{2})} (1 + \frac{(\log I_t - \log \hat{I}_t)^2}{\nu_2})^{-(\nu_2+1)/2}$$

To use log Cauchy distribution instead of log-normal distribution, the likelihood function changes to

$$f(I_t \mid \hat{I}_t) = \frac{1}{\pi\gamma_2[1 + (\frac{\log I_t - \log \hat{I}_t}{\gamma_2})^2]} = \frac{1}{\pi}[\frac{\gamma_2}{(\log I_t - \log \hat{I}_t)^2 + \gamma_2^2}]$$

Where $\hat{I}_t$ is a function of P_t, parameters q and $P_{initial}$, and K, and r are obtained from the deterministic process equation. The parameters to be estimated from the likelihood function are q, $P_{initial}$, K and r. In addition to those parameters, OE with the normal distribution (OE_N) also has the parameter σ_2 . OE with Student's t distribution (OE_T) also has parameter ν_2. OE with Cauchy distribution (OE_C) also has the parameter γ_2.

Process-observation-error-estimator (POE) incorporates both process error and observation error into the surplus production model:

$$\begin{cases} P_{t+1} = (P_t + g(P_t) - C_t / K)e^{\varepsilon_{1,t}} \\ I_t = (qKP_t)e^{\varepsilon_{2,t}} \end{cases}$$

To solve the process-observation-error-estimator, we need to maximize the likelihood function based on the distribution assumption of the error, which is the product of the likelihood of process-error-estimator and the likelihood of observation-error-estimator. The parameters to be estimated from the likelihood function are q , $P_{initial}, K$ and r . In addition to those parameters, POE with normal distribution (POE_N) also has parameters σ_1, σ_2 . POE with Student's t distribution (POE_T) also has parameters ν_1, ν_2. POE with Cauchy distribution (POE_C) also has parameters γ_1, γ_2.

4.4 Statistical algorithms to solve the three estimators---Metropolis Hastings within Gibbs sampling method

According to Gibbs sampling, with the parameters $\theta_1, \theta_2, \cdots \theta_n$ and the observations $x_1, x_2, \cdots x_n$, the joint posterior distribution $f(\theta_1, \theta_2, \cdots \theta_n \mid x_1, x_2, \cdots x_n)$ can be achieved from generating the data starting from the initial vector ($\theta_1^0, \theta_2^0, \cdots \theta_n^0$) and then doing iterations following a Gibbs sampling procedure:

simulate $\theta_1^{(m)} \sim f(\theta_1 \mid \theta_2^{m-1}, \theta_3^{m-1}, \cdots \theta_n^{m-1}, x_1, x_2, \cdots, x_n)$,

simulate $\theta_2^{(m)} \sim f(\theta_2 \mid \theta_1^{m}, \theta_3^{m-1}, \cdots \theta_n^{m-1}, x_1, x_2, \cdots, x_n)$,

$$\vdots \qquad \text{..................(10)}$$

$$\text{simulate} \quad \theta_n^{(m)} \sim f(\theta_n \mid \theta_1^m, \theta_2^m, \cdots \theta_{n-1}^m, x_1, x_2, \cdots, x_n),$$

where $f(\theta_i \mid \theta_1^{m-1}, \theta_2^{m-1}, \cdots, \theta_{i-1}^{m-1}, \theta_{i+1}^{m-1} \cdots \theta_n^{m-1}, x_1, x_2, \cdots, x_n)$ is the full conditional distribution of θ_i, given the parameter value of the other parameters that are generated from the previous iteration.

To add Metropolis Hastings process into the Gibbs sampling procedure, each θ_i^m value sampled from the full conditional distribution will be judged by Metropolis-Hastings algorithm to decide whether it will update the previous value from iteration m-1, θ_i^{m-1}.

After getting the new sample θ_i^m, calculate

$$\alpha_{m-1}^m = \min\{1, \frac{f(\theta_i^m \mid \theta_1^m, \cdots, \theta_{i-1}^m, \theta_{i+1}^{m-1}, \cdots \theta_n^{m-1})}{f(\theta_i^{m-1} \mid \theta_1^m, \cdots, \theta_{i-1}^m, \theta_{i+1}^{m-1}, \cdots \theta_n^{m-1})}\}.$$

Generate a random variable u from the uniform distribution in [0, 1]. Then

1) if $u < \alpha_{m-1}^m$, accept the θ_i^m value, then move to the next step.
2) if $u >= \alpha_{m-1}^m$, don't upgrade θ_i, just keep $\theta_i^m = \theta_i^{m-1}$.

A burning period of data from the beginning of the iterations was excluded to avoid the unstableness caused by the position of initial values. The number of iterations was set large enough for the distribution of the generated data to converge to the joint posterior distribution. A thinning interval was used to diminish autocorrelation effect from the Markov Chain. Heidelberger & Welch (1983) convergence test was used to make sure the final convergence of the Markov Chain after the chosen iteration. After the qualified iterations, the joint posterior distribution $f(\theta_1, \theta_2, \cdots \theta_n \mid x_1, x_2, \cdots x_n)$ was achieved from the data pool of $(\theta_1, \theta_2, \cdots \theta_n)$. The Bayesian estimation of the parameters was derived from the mean or median value of the parameter value from the whole data set throughout all the m iterations.

Below I illustrate the full conditional distribution for all parameters of the three surplus production model estimators. Based on the full conditional distribution, we can follow the Metropolis Hastings within Gibbs sampling algorithm to find the posterior distribution of the parameters.

Process-error-estimator

The surplus production model of process-error-estimator with Bayesian form can be written as

$$\log(P_1) \mid a, \sigma_1^2 = a + \varepsilon_{1,1}$$

$$\log(P_{t+1}) \mid P_t, K, r, \sigma_1^2 = \log[P_t + rP_t(1 - P_t) - C_t / K] + \varepsilon_{1,t}$$

$$I_t \mid P_t, K, q = q \times P_t \times K$$
$$t = 1, 2, \cdots\cdots,$$

Where a is a parameter that can reflect the depletion, which is the population biomass divided by the carrying capacity, at the beginning of the time series of data studied. To present it in a formula, if $E(\varepsilon_{1,1}) = 0$, $a = E[\log(B_1) - \log(K)] = E[\log(\frac{B_1}{K})] = E[\log P_1]$.

$\varepsilon_{1,t}, t = 1, 2, \cdots$ are the random effects in process equations. Here I use f to present the density function. All of the likelihood function f on the right part of the equation can be found in section 5.3. $\pi(\theta)$ is the inferior distribution of parameter θ.

$$f(K \mid r, a, \sigma_1^2, q, \sigma_1^2, I_1, ..., I_t) \propto \pi(K) f(\frac{I_1}{qK} \mid a, \sigma_1^2) \prod_{t=2}^{T} (\frac{I_t}{qK} \mid \frac{I_{t-1}}{qK}, K, r, \sigma_1^2)$$

$$f(r \mid K, a, q, \sigma_1^2, I_1, ..., I_T) \propto \pi(r) \prod_{t=2}^{T} f(\frac{I_t}{qK} \mid \frac{I_{t-1}}{qK}, K, r, \sigma_1^2)$$

$$f(a \mid K, r, q, \sigma^2, I_1, ..., I_T) \propto \pi(a) f(\frac{I_1}{qK} \mid a, \sigma_1^2)$$

$$f(\sigma_1^2 \mid K, r, a, q, I_1, ..., I_T) \propto \pi(\sigma^2) f(\frac{I_1}{qK} \mid K, \sigma_1^2) \prod_{t=2}^{T} f(\frac{I_t}{qK} \mid \frac{I_{t-1}}{qK}, K, r, \sigma_1^2)$$

$$f(q \mid K, r, a, \sigma_1^2, I_1, ..., I_T) \propto \pi(q) f(\frac{I_1}{qK} \mid a, \sigma_1^2) \prod_{t=2}^{T} f(\frac{I_t}{qK} \mid \frac{I_{t-1}}{qK}, K, r, \sigma_1^2)$$

Observation-error-estimator

The surplus production model of observation-error-estimator with Bayesian form can be written as

$$\log(P_1) \mid a = a$$

$$\log(P_{t+1}) \mid P_t, K, r = \log[P_t + rP_t(1 - P_t) - C_t / K]$$

$$\log(I_t) \mid P_t, K, q, \sigma_2^2 = \log(q) + \log(P_t) + \log(K) + \varepsilon_{2,t}$$
$$t = 1, 2, \cdots\cdots,$$

Where a is logarithm of the depletion value at the beginning of the time series data studied. To present it as a formula, $a = \log(\frac{B_1}{K})$. $v_t, t = 1, 2, \cdots\cdots$ are the random effects in observation equations. Here we use f to present the density function. All of the likelihood function f on the right part of the equation can be found in section 5.3. $\pi(\theta)$ is the inferior distribution of parameter θ.

$$f(K \mid r, a, \sigma_2^2, q, I_1, \cdots, I_T) \propto \pi(K) \prod_{t=1}^{T} f(I_t \mid K, a, q, \sigma_2^2)$$

$$f(r \mid K, a, \sigma_2^2, q, I_1, \cdots, I_T) \propto \pi(r) \prod_{t=2}^{T} f(I_t \mid K, a, q, \sigma_2^2)$$

$$f(a \mid K, r, \sigma_2^2, q, I_1, \cdots, I_T) \propto \pi(a) \prod_{t=1}^{T} f(I_t \mid K, a, q, \sigma_2^2)$$

$$f(q \mid K, r, a, \sigma_2^2, I_1, \cdots, I_T) \propto \pi(q_i) \prod_{t=1}^{T} f(I_t \mid K, a, q, \sigma_2^2)$$

$$f(\sigma_2^2 \mid K, r, a, q, I_1, \cdots, I_T) \propto \pi(\sigma^2) \prod_{t=1}^{T} f(I_t \mid K, a, q, \sigma_2^2)$$

Process-observation-error-estimator

The surplus production model of process-observation-error-estimator with Bayesian form can be written as

$$\log(P_1) \mid a, \sigma_1^2 = a + \varepsilon_{1,1}$$

$$\log(P_{t+1}) \mid P_t, K, r, \sigma_1^2 = \log[P_t + rP_t(1 - P_t) - C_t / K] + \varepsilon_{1,t}$$

$$\log(I_t) \mid P_t, K, q, \sigma_2^2 = \log(q) + \log(P_t) + \log(K) + \varepsilon_{2,t}$$
$$t = 1, 2, \cdots\cdots,$$

Where a is the mean difference between the biomass at the starting time and the carrying capacity. $\varepsilon_{1,t}, \varepsilon_{2,t}, t = 1, 2, \cdots\cdots$ are the random effects in process and observation equations. Here we assume the errors are following the normal distribution. The forms following fat-tailed distributions can be derived by only modifying the distribution function.

Using the Metropolis-Hastings within-Gibbs samplings in my study, the full conditional distribution of each parameter are given below:

$$f(P_1, K, r, a, \sigma_1^2, q, \sigma_2^2, P_2, \cdots, P_T, I_1, \cdots, I_T)$$
$$\propto f(P_1 \mid a, \sigma_1^2) f(I_1 \mid P_1, K, q, \sigma_2^2) f(P_2 \mid P_1, K, r, \sigma_1^2)$$

$$f(P_t \mid K, r, a, \sigma_1^2, q, \sigma_2^2, P_1, \cdots, P_{t-1}, P_{t+1}, \cdots P_T, I_1, \cdots I_T)$$
$$\propto f(P_t \mid P_{t-1}, K, r, \sigma_1^2) f(I_t \mid P_t, K, q, \sigma_2^2) f(P_{t+1} \mid P_t, r, \sigma_1^2), t = 2, 3, \cdots$$

$$f(K \mid r, a, \sigma_1^2, q, \sigma_2^2, P_1, \cdots, P_T, I_1, \cdots, I_T) \propto \pi(K) f(P_1 \mid a, \sigma_1^2) \prod_{t=2}^{T} f(P_t \mid P_{t-1}, K, r, \sigma_1^2)$$

$$f(r \mid K, a, \sigma_1^2, q, \sigma_2^2, P_1, \cdots, P_T, I_1, \cdots, I_T) \propto \pi(r) \prod_{t=2}^{T} f(P_t \mid P_{t-1}, K, r, \sigma_1^2)$$

$$f(a \mid K, r, \sigma_1^2, q, \sigma_2^2, P_1, \cdots, P_T, I_1, \cdots, I_T) \propto \pi(a) f(P_1 \mid a, \sigma_1^2)$$

$$f(\sigma_1^2 \mid K, r, a, q, \sigma_2^2, P_1, \cdots, P_T, I_1, \cdots, I_T) \propto \pi(\sigma_1^2) f(P_1 \mid a, \sigma_1^2) \prod_{t=2}^{T} f(P_t \mid P_{t-1}, K, r, \sigma_1^2)$$

$$f(q \mid K, r, a, \sigma_1^2, \sigma_2^2, P_1, \cdots, P_T, I_1, \cdots, I_T) \propto \pi(q) \prod_{t=1}^{T} f(I_t \mid P_t, K, q, \sigma_2^2)$$

$$f(\sigma_2^2 \mid K, r, a, \sigma_1^2, q, P_1, \cdots, P_T, I_1, \cdots, I_T) \propto \pi(\sigma_2^2) \prod_{t=1}^{T} f(I_t \mid P_t, K, q, \sigma_2^2)$$

where $\propto$ means the left hand distribution has the same shape as the right hand formula, and $\pi(\cdot)$ represent the prior distribution .

Cauchy and Student's t distribution just change $\sigma_1,\ \sigma_2$ to $\gamma_1,\ \gamma_2$ or $\tau_1,\ \tau_2$ and the likelihood function will use the one for Cauchy and Student's t distribution respectively.

With the normal distribution assumption, there are totally six parameters in surplus production model $K, r, a, \sigma_1^2, \sigma_2^2, q$. The parameters changed to K, r, a, t_1, t_2, q for t distribution and $K, r, a, \gamma_1^2, \gamma_2^2, q$ for Cauchy distribution.

The estimation results varied corresponding to different choices of priors (Millar and Meyer 2000, Chen et al. 2000). Estimators with the fat-tailed distributions are found to be less sensitive to different choices of priors than those with normal distributions (Chen et al. 2000). In my study, I will use non-informative priors. The reason to choose the non-informative prior in the real fishery analysis is discussed in section 5.1. Non-informative priors are also used in the simulation study because my focus is on comparing the performance of different estimators with normal or fat-tailed distributions. Using informative prior will confound with the choice of estimators.

To give a reference to use informative priors, Millar and Meyer (2000) chose log-normal distribution to be the prior distribution for K, r and a, an inverse gamma distribution for σ_1^2 and σ_2^2, and a uniform distribution for $\log q$. For t distribution, the extra parameter ν, which is the degree of freedom, can be assumed to follow a normal distribution (Chen et al. 2000). The scale parameter γ in Cauchy distribution is considered with the same scale of σ in the normal distribution (Chen et al. 2000). So compared to σ^2, γ can be obtained by the root value of γ^2 which is assumed to follow an inverse gamma distribution.

4.5 Statistical algorithms change to deal with multiple indices based on CPUEs

For multiple indices based on CPUEs, the full conditional distributions for all parameters are changed by incorporating the likelihood functions of the additional CPUEs. The full conditional distributions for the three estimators using multiple indices based on CPUEs are listed below with a form using normal distribution. For Cauchy and Student's t distribution, the parameters σ_1, σ_2 need to be changed to γ_1, γ_2, or τ_1, τ_2 respectively. Function f is the relative likelihood function for the three estimations with the normal distribution, Cauchy distribution, or Student's t distribution given in section 4.2 and 4.3.

Process-error-estimator

The full conditional distributions for five parameters when PE is used are

$$f(K \mid r, a, \sigma_1^2, q_1,...q_M, I_{11},..., I_{,TM}) \propto \pi(K)\prod_{i=1}^{M} f(\frac{I_{1i}}{q_i} \mid K, a, \sigma_1^2)\prod_{t=2}^{T}\prod_{i=1}^{M} f(\frac{I_{ti}}{q_i} \mid \frac{I_{t-1,i}}{q_i}, K, r, \sigma_1^2)$$

$$f(r \mid K, a, \sigma_1^2 q_1,...q_M, I_{11},..., I_{TM}) \propto \pi(r)\prod_{t=2}^{T}\prod_{i=2}^{M} f(\frac{I_{ti}}{q_i} \mid \frac{I_{t-1,i}}{q_i}, K, r, \sigma_1^2)$$

$$f(a \mid K, r, \sigma_1^2, q_1,...q_M, I_{11},..., I_{TM}) \propto \pi(a)\prod_{i=1}^{M} f(\frac{I_{1i}}{q_i} \mid K, a, \sigma_1^2)$$

$$f(\sigma_1^2 \mid K, r, a, q_i, I_{1i},..., I_{Ti}) \propto \pi(\sigma_1^2) f(\frac{I_{1i}}{q_i} \mid K, a, \sigma_1^2)\prod_{t=2}^{T} f(\frac{I_{ti}}{q_i} \mid \frac{I_{t-1,i}}{q_i}, K, r, \sigma_1^2)$$

$$f(q_i \mid K, r, a, \sigma_1^2, I_{1i},..., I_{Ti}) \propto \pi(q_i) f(\frac{I_{1i}}{q_i} \mid K, a, \sigma_1^2)\prod_{t=2}^{T} f(\frac{I_{ti}}{q_i} \mid \frac{I_{t-1,i}}{q_i}, K, r, \sigma_1^2)$$

T is the length of the time series and M is the total number of multiple indices based on CPUEs.

Observation-error-estimator

The full conditional distributions for five parameters when OE is used are

$$f(K \mid r, a, \sigma_1^2,...\sigma_M^2, q_1,...q_M, I_{11}, \cdots, I_{TM}) \propto \pi(K)\prod_{t=1}^{T}\prod_{i=1}^{M} f(I_{ti} \mid K, a, q_i, \sigma_i^2)$$

$$f(r \mid K, a, \sigma_1^2,...\sigma_M^2, q_1,...q_M, I_{11}, \cdots, I_{TM}) \propto \pi(r)\prod_{t=2}^{T}\prod_{i=1}^{M} f(I_{ti} \mid K, a, q_i, \sigma_i^2))$$

$$f(a \mid K, r, \sigma_1^2,...\sigma_M^2, q_1,...q_M, I_{11}, \cdots, I_{TM}) \propto \pi(a)\prod_{t=1}^{T}\prod_{i=1}^{M} f(I_{ti} \mid K, a, q_i, \sigma_i^2)$$

$$f(q_i \mid K, r, a, \sigma_1^2,...\sigma_M^2, I_{1M}, \cdots, I_{TM}) \propto \pi(q_i)\prod_{t=1}^{T} f(I_{ti} \mid K, a, q_i, \sigma_i^2), i = 1,..., M$$

$$f(\sigma_i^2 \mid K, r, a, q_1,...q_M, I_{1M}, \cdots, I_{TM}) \propto \pi(\sigma_i^2)\prod_{t=1}^{T} f(I_{ti} \mid K, a, q_i, \sigma_i^2)$$

T is the length of the time series and M is the total number of multiple indices based on CPUEs.

Process-observation-error-estimator

The full conditional distributions for seven parameters when POE is used are

$$f(P_1 \mid K, r, a, \sigma_1^2, q_1,...q_M, \sigma_{2,1}^2,..., \sigma_{2,M}^2, P_2, \cdots, P_T, I_{11}, I_{12},..., I_{1M}, I_{21},..., I_{2M}, \cdots, I_{T1},..., I_{TM})$$
$$\propto f(P_1 \mid a, \sigma_1^2) \prod_{i=1}^{M} f(I_{1i} \mid P_1, K, q_i, \sigma_{2i}^2) f(P_2 \mid P_1, K, r, \sigma_1^2)$$

$$f(P_t \mid K, r, a, \sigma_1^2, q_1,...q_M, \sigma_{2,1}^2,..., \sigma_{2,M}^2, P_1, \cdots, P_{t-1}, P_{t+1}, \cdots P_T, I_{11}, I_{12},..., I_{1M}, I_{21},..., I_{2M}, \cdots, I_{T1},..., I_{TM})$$
$$\propto f(P_t \mid P_{t-1}, K, r, \sigma_1^2) \prod_{i=1}^{8} f(I_{ti} \mid P_t, K, q_i, \sigma_{2i}^2) f(P_{t+1} \mid P_t, K, r, \sigma_1^2), t = 2,3, \cdots$$

$$f(K \mid r, a, \sigma_1^2, q_1,...q_M, \sigma_{2,1}^2,..., \sigma_{2,M}^2, P_1, \cdots, P_T, I_{11},..., I_{TM}) \propto \pi(K) f(P_1 \mid a, \sigma_1^2) \prod_{t=2}^{T} f(P_t \mid P_{t-1}, K, r, \sigma_1^2)$$

$$f(r \mid K, a, \sigma_1^2, q_1,...q_M, \sigma_{2,1}^2,..., \sigma_{2,M}^2, P_1, \cdots, P_T, I_{11},..., I_{TM}) \propto \pi(r) \prod_{t=2}^{T} f(P_t \mid P_{t-1}, K, r, \sigma_1^2)$$

$$f(a \mid K, r, \sigma_1^2, q_1,...q_M, \sigma_{2,1}^2,..., \sigma_{2,M}^2, P_1, \cdots, P_T, I_{11}, \cdots, I_{TM}) \propto \pi(a) f(P_1 \mid a, \sigma_1^2)$$

$$f(\sigma_1^2 \mid K, r, a, q_1,...q_M, \sigma_{2,1}^2,..., \sigma_{2,M}^2, P_1, \cdots, P_T, I_{11}, \cdots, I_{TM}) \propto \pi(\sigma_1^2) f(P_1 \mid a, \sigma_1^2) \prod_{t=2}^{T} f(P_t \mid P_{t-1}, K, r, \sigma_1^2)$$

$$f(q_i \mid K, r, a, \sigma_1^2, \sigma_{2,1}^2,..., \sigma_{2,M}^2, P_1, \cdots, P_T, I_{11}, \cdots, I_{TM}) \propto \pi(q_i) \prod_{t=1}^{T} f(I_{ti} \mid P_t, K, q_i, \sigma_{2i}^2), i = 1,..., M$$

$$f(\sigma_{2i}^2 \mid K, r, a, \sigma_1^2, q_1,...q_M, P_1, \cdots, P_T, I_{11}, \cdots, I_{T8}) \propto \pi(\sigma_{2i}^2) \prod_{t=1}^{T} f(I_{ti} \mid P_t, K, q_i, \sigma_{2i}^2), i = 1,..., M$$

T is the length of the time series and M is the total number of multiple indices based on CPUEs.

4.6 Example fisheries used in this study

Atlantic weakfish and black sea bass are used as two example fisheries. The simulation study introduced in the next section is based on the results from the example fisheries analysis. The two example fisheries are very different species in respect to their habitat and life history (Table 1). Fisheries with various biological characteristics and catch histories are chosen to be example fisheries. The results of parameter estimation and the biological reference point estimation are also different shown in section 5.1. Simulation studies based on two fisheries can improve the reliability and generalization of the final judgments on the three estimators.

4.6.1 Atlantic weakfish

Atlantic weakfish *(Cynoscion regalis*) mainly range from North Carolina to New England. Some of them have also been found in Florida. The Atlantic weakfish inhibit primarily the estuarine and inshore oceanic areas. They migrate seasonally. In autumn, they travel from northern area to Carolina and Virginia waters where they stay over winter. In spring, they travel back to north and spawn there. Delaware Bay and Chesapeake Bay are the main spawning and nursery areas. They are gonochoristic unisexual. Ninety percent mature at age 1. The maximum age determined using otoliths is 17 years. The weakfish technical committee considers 12 years to be $T_{\max}$, which is used as the eldest age group in the age structured model. The Atlantic weakfish can grow as long as 1 meters and weigh as much as 7.75 kg. Studies have shown various results about the diet of Atlantic weakfish. Juveniles in Delaware Bay mainly live on mysid shrimp, while in Chesapeake Bay the bay anchovy make up a large percentage of the weakfish diet. Older Atlantic weakfish prefer large prey such as Atlantic menhaden or other clupeid. The natural annual mortality rate of Atlantic weakfish is considered to be 0.25. The Fisheries Management Plan of Atlantic States Marine Fisheries Commission considers weakfish to be a unit stock by genetic analysis. So the weakfish population assessment targets all the Atlantic weakfish along Atlantic coast (ASMFS 2006).

The data used in population dynamics model to assess the Atlantic weakfish population include catch data and catch-per-unit-effort (CPUE) data. All the data were from the 48th northeast regional stock assessment workshop (48th SAW) assessment report (Northeast Fisheries Science Center 2009). The Atlantic weakfish catch data contain commercial landings from 1950 to 2007 and recreational landings from 1981 to 2007. According to Figure 3, the commercial catch data increased a little from 1950 to 1955, decreased till 1967, and then increased dramatically by 10 times to 1979. Both commercial and recreational landings follow a decreasing trend from 1980 to 2003, and both landings reached their historical minimum value at the last year during available data (ASMFS 2006).

CPUE plots (Figure 4) used standardized CPUE by dividing original CPUE by the mean value among years. I only use standardized CPUE to plot Figures so that the CPUE collected from different surveys are visibly comparable. The original CPUE records are used in the population assessment model later so that we can get catch-ability coefficient estimation respectively for each survey. There are eight surveys available to provide CPUE data. The Marine Recreational Fisheries Statistics Survey (MRFSS) provides fishery dependent catch-per-unit-effort indices. The Northeast Fisheries Science Center (NEFSC) and the Southeast Area Monitoring and Assessment Program (SEAMAP) conducted the federally operated coastal wide fishery independent surveys (trawl net surveys). The other five are all state operated fishery independent surveys that operated in Rhode Island (RI), New Jersey (NJ), Delaware Bay (DE), Maryland (MD), and North Carolina (NC). The first four are trawl net surveys and the last one is a gill net survey (ASMFS 2006). According to the CPUE plots shown by (Figure 4), the CPUE trend varies dramatically in different surveys. For example during 1980-1990, CPUEs are at a low level from surveys of NEFSC, RI, and DE, at a high level from surveys of MRFSS, and vary a lot from survey of MD. During the years around 2000, CPUEs are at a low level from surveys of RI and MD, at a high level from surveys of MRFSS, DE, NC, and vary a lot from surveys of NEFSC, SEAMAP and NJ. In this case, the reliability of these surveys should be examined and possible combinations of CPUEs need to be selected to achieve reasonable parameter estimation for Atlantic weakfish population.

4.6.2 Black sea bass

The black sea bass (*Centropristis striata*) inhabit large ranges from Florida to Maine (SEDAR 2006). For the purposes of this study, I focus on the southern stock distinguished from Florida to North Carolina. They mainly feed on benthic organisms such as rock crabs, hermit crabs, squids, and razor clams. According to the diet preference, they can be found in offshore waters up to a depth of 130 meters as well as the inshore waters. They are hermaphroditic. The age at which individuals switch from female to male is variable. Most of the fish change sex before age 6. Females are sexually mature by age 2, while mature males don't occur until age 4. Black sea bass spawn from January through July along the southeastern U.S. coast. Occasionally, spawning can also be observed in fall (October and November). However, the greatest percentage of females spawning in condition occurs during March through May. Females reach a maximum length of 38 centimeters. Males can attain maximum length of over 60 centimeters. 20 years is known as the male longevity, because all older individuals are males. So this is also used as longevity for the whole species. People commonly accept the natural mortality rate as 0.3 year. Two populations exist along the Atlantic coast separated by Cape Hatteras in North Carolina. The one that was studied in my research ranges from Cape Hatteras south to Florida (SEDAR 2006).

All the black sea bass data were from the 2006 report of black sea bass population assessment (SEDAR 2006). The available catch data for black sea bass that are used in my research include the commercial landings from 1973 to 2003 and the recreational landings from 1978 to 2003 and are plotted in Figure 5. The commercial catch increased dramatically after 1977, fluctuated a lot during the period from 1980 to 1990 and has decreased since then. The recreational catch was always lower than commercial catch, but followed the same pattern since recording began in 1978 (SEDAR 2006).

There are five surveys on CPUE data that are used in my research. The head boat data are from surveys on the log book data in units of tons per angler-trip. The other four are all fishery independent surveys. The index from hook and line gear is in units of number of fish per collection-hour, and the indices from trap gear are in units of number of fish per trap-hour. The CPUEs plotted in Figure 6 are also standardized CPUEs, which are the original values divided by the mean value (SEDAR 2006). Compared to the CPUE plots of Atlantic weakfish, most of the lines representing CPUEs from different surveys in Figure 6 are of similar pattern. So the five CPUEs are more consistent than the CPUEs of Atlantic weakfish fisheries discussed earlier.

Comparing the two example fisheries, they differ in several aspects. First, they are fish species from different genus. One of them, Atlantic weakfish, mainly come from colder waters in the North Atlantic, from North Carolina to Maine, and the other, black sea bass, come from south Atlantic, from Cape Hatteras to Florida. Black sea bass inhabit in waters deeper than Atlantic weakfish due to its diet preference. The black sea bass have a much longer spawning span than Atlantic Weakfish, which improves their recruitment rate. Black sea bass also show 5 percent natural mortality higher than Atlantic weakfish.

The biggest difference is that Atlantic weakfish are gonochoristic fish with females and males to be separated individuals, whereas, black sea bass are hermaphroditic and are born as female, and then change sex to male which could happen as late as age 6. Accordingly, the longevity, length and weight of black sea bass are much different from the Atlantic weakfish. I listed the main differences between these two species in Table 1. Considering all of these differences about biological traits and life history status, the two fisheries are qualified to be selected as two example fishery to be used in the simulation study.

4.7 Simulation Study

The simulation study is based on the parameter estimation from the two example fisheries. For objective 1 and objective 3, I simulated data using two sets of parameters, i.e., from both the Atlantic weakfish and from black sea bass. For objectives 2, 4, 5, and 6, I only conducted the simulation based on the parameters from the Atlantic weakfish. Figure 7 explained the main procedure of each simulation study. The flow chart of Figure 8 explained how the data were generated based on assumptions of the “true models”, and then how the data were fitted using different estimators to compare the performances of the estimators.

Objective 1: Investigate performance of the three estimators in surplus production model

Objective 1 was to compare the performance of the PE, OE, and POE estimators with normal distribution. I designed three groups of simulated data with three choices of error structures: observation error only, process error only, and process observation error together. Process error and observation error were both assumed to be multiplicative and follow a log-normal distribution. Each of the three groups of data was fitted using the three estimators one by one, which resulted in nine groups of estimated results. Table 2 shows all the nine scenarios from I.1 to III.3 conducted for this objective. The scenarios designated by the same Roman numerals fit the same generated data, whereas, the Arabian numerals 1 to 6 represent different estimators including different distribution assumptions. For future reference, this numbering rule applies to all scenarios from I.1 to XIV. 4, but scenarios from XV.1 to XV. 4 are different. XV.1 and XV.2 both fit data with three CPUEs, whereas XV.3 and XV.4 fit data with five CPUEs. I conducted all these scenarios twice based on example fisheries of both the Atlantic weakfish and the black sea bass, which guaranteed that the performance of the estimators we compared were not limited to specific fisheries.

Indices designed to calibrate the accuracy and precision of the estimation results were used to evaluate performance of the estimators. These indices are usually composed of different calculations of error items. The most often used indices include the mean of errors (ME, $\frac{1}{Iteration}\sum_{i=1}^{Iteration}(\hat{r}_i - r_{true})$), the mean of squared errors (MSE, $\frac{1}{Iteration}\sum_{i=1}^{Iteration}(\hat{r}_i - r_{true})^2$), the mean of proportional errors (MPE $\sum_{i=1}^{Iteration}\frac{\hat{r}_i - r_{true}}{r_{true}}$), the mean of squared proportional errors (MSPE, $\sum_{i=1}^{Iteration}(\frac{\hat{r}_i - r_{true}}{r_{ruet}})^2$). Among these four indices, the first and third indices reflect the bias of the estimation, which represents the accuracy of the estimators. The second and fourth indices reflect the precision of the estimators. If we change r_{true} to $\bar{r} = \frac{1}{iteration}\sum_{i=1}^{Iteration}\hat{r}_i$ in the second and fourth formula, the new indices reflect the variance of the estimated value. Some stable indices were suggested to be used by previous studies to test the estimation results, which provide evaluation from a more stable perspective than the methods of comparing the average values (Prager 2002). I revised the indices listed above to be the stable indices so that the stable indices were comparable with the previous indices. Instead of using the average value, I used the median value of error calculations among the simulation runs. The stable indices are median of errors (MDE, $median_{i=1}^{Iteration} \hat{r}_i - r_{true}$), median of proportional errors (MDPE, with formula $median_{i=1}^{Iteration} \frac{\hat{r}_i - r_{true}}{r_{true}}$), median of squared errors (MDSE, $median_{i=1}^{Iteration} (\hat{r}_i - r_{true})^2$), median of squared proportional error (MDSPE, $median_{i=1}^{Iteration} (\frac{\hat{r}_i - r_{true}}{r_{true}})^2$). Indices built on other parameters were derived by replacing the population intrinsic growth rate r with the parameter of interest. Indices built on certain biological reference points were also considered in my study.

Objective 2: Investigate the response of the three estimators to outliers/atypical values

Objective 2 was to compare the performance of PE, OE, and POE estimators with normal distribution when they fit data with outliers or atypical values. There are several ways to design the outliers or atypical values in the data to be simulated. In this research, the outliers or atypical values were considered as errors that have an abnormally large variance or biased mean value compared to ordinary errors.

In this section, I generated data with both process and observation errors. The errors are multiplicative, following a log-normal distribution, with the outliers and atypical values either in process error or in observation error. Based on the reality of fishery data, eight groups of data were generated as explained in the following paragraph. Each group of data was fitted using two estimators: process-observation-error-estimator with normal distribution (POE_N) and observation-error-estimator with normal distribution (OE_N). Sixteen scenarios, from scenario IV.1 to XI.2, were tested in this research, which can be found in Table 3.

In this study, I assumed that all simulated data were with process observation error which is different from what I did in objective 1. This is because firstly, in most of the cases, process error and observation error both exist in reality. Second, from the study on objective 1 I found that process-error-estimator did not perform well compared to the other two estimators. Third, observation-error-estimator and process-observation-error-estimator were the two estimators that are widely used when people do fishery stock assessment using a population dynamics model. So I generated data based on assumption of process observation error but fitted the data using both the observation-error-estimator and process-observation-error-estimator here.

In scenarios IV.1 and IV.2, I assumed that the outliers and atypical values were in the observation error. Their logarithm values had a larger variance compared to the ordinary observation errors, and were randomly distributed along the time series. The outliers or atypical values of this kind can be explained by all possible aspects that lead to increased scale or variation of observation error; for example, mistaken records, bad weather or current influence, random change of fishing gears, ships or observers, etc. In scenario IV.1 I fitted the data using POE_N and in scenario IV.2 I fitted the data using OE_N.

In scenarios V.1 and V.2, I assumed that the outliers and atypical values were in the observation error. The logarithm values of them had a larger variance compared to ordinary observation errors, but instead of being randomly spread along the time period, the outliers and atypical values were located at fixed period of time. The outliers and atypical values of this kind can be explained by all possible reasons that lead to increased scale or vibration of observation error in some particular years; for example, change of fishing gears in some continuous years, change of ships or observers in some continuous years, etc. In scenario V.1 I fitted the data using POE_N and in scenario V.2 I fitted the data using OE_N.

In scenarios VI.1 and VI.2, I assumed that the outliers and atypical values were in the observation error. Their logarithm values had a biased mean value, and they are randomly distributed along the time series. The outliers or atypical values of this kind can be explained by mistaken records, bad weather or current influence, random change of fishing gear, ships or observers, etc., all of which constantly drive the error towards one direction. In scenario VI.1 I fitted the data using POE_N and in scenario VI.2 I fitted the data using OE_N.

In scenarios VII.1 and VII.2, I assumed that the outliers and atypical values were in the observation error and their logarithm value had a biased mean value compared to ordinary observation errors. Instead of randomly spread along the time period, the outliers and atypical values were located at fixed period of time. The outliers or atypical values of this kind can be explained by mistaken records, bad weather or current influence, change of fishing gears, ships or observers, and etc, all of which last for some time and constantly drive the error towards one direction. In scenario VII.1 I fitted the data using POE_N and in scenario VII.2 I fitted the data using OE_N.

In scenarios VIII.1 and VIII.2, I assumed the outliers and atypical values were in the process error, their logarithm values had a large variance compared to ordinary process errors, and the outliers and atypical values were randomly distributed along time. The outliers and atypical values of this kind can be explained by abnormal population variation due to random environmental change, random predation-prey relationship change, random human effects, etc. In scenario VIII.1 I fitted the data using POE_N and in scenario VIII.2 I fitted the data using OE_N.

In scenarios IX.1 and IX.2, I assumed the outliers and atypical values were in process error. Their logarithm values had a large variance compared to ordinary process errors, but instead of randomly spread along time, the outliers and atypical values were located at fixed period of time. The outliers or atypical values of this kind can be explained by abnormal population variation due to environmental change, period predation-prey relationship change, human effects, and etc, all of which last for some period of time. In scenario IX.1 I fitted the data using POE_N and in scenario IX.2 I fitted the data using OE_N.

In scenarios X.1 and X.2, I assumed the outliers and atypical values were in the process error, their logarithm had a biased mean compared to ordinary process errors, and the outliers and atypical values were randomly distributed along time. The outliers or atypical values of this kind can be explained by irregular tendency of population change due to random environmental change, random period predation-prey relationship change, random human effects, etc. In scenario X.1 I fitted the data using POE_N and in scenario X.2 I fitted the data using OE_N.

In scenarios XI.1 and XI.2, I assumed the outliers and atypical values were in the process error, their logarithm values had a biased mean compared to ordinary process errors, but instead of randomly spread along time, the outliers and atypical values were located at fixed period of time. The outliers or atypical values of this kind can be explained by irregular tendency of population change due to environmental change, predator-prey relationship change, human effects, etc., all of which last for some period of time. In scenarioXI.1 I fitted the data using POE_N and in scenario XI.2 I fitted the data using OE_N.

The percentage of outliers and atypical values in the data, the variance of outliers and atypical values, the biased means, and the specific time period that outliers and atypical values existed were chosen based on example fisheries. To compare the estimation results of different estimators, indices in objective 1 were also used in this objective.

Objective 3: Investigate performance of the three estimators with fat-tailed (Cauchy) distribution

Objective 3 was to compare the performance of PE, OE, and POE estimators with fat-tailed distribution. I used the same groups of data generated for objective 1, which included data with process error only, data with observation error only, and data with both process error and observation error. Errors were multiplicative errors following log-normal distribution. Each group of data was fitted using the process error with fat-tailed distribution, observation error with fat-tailed distribution, process observation error with fat-tailed distribution respectively. Table 4 showed all the nine scenarios from I.4 to III.6 that I conducted for this objective. Scenarios with same Roman numerals were tested on the same generated data. By fitting the same data, the performances of estimators with fat-tailed distribution are comparable with the performance of estimators with normal distribution. I also conducted all these scenarios twice based on the example fisheries of both Atlantic weakfish and black sea bass.

To compare the estimation results, indices listed in objective 1 were also used in this objective.

Objective 4: Investigate response of the three estimators with fat-tailed distribution to outliers/atypical values

Objective 4 was to compare the performance of PE, OE, and POE estimators with fat-tailed distribution when they fit data with outliers or atypical values. I used the same groups of data as in objective 2. The data were generated with both process error and observation error, and outliers and atypical values were in either process error or observation error with either an abnormal large variance or biased mean, either randomly distributed along time or at a fixed location. Scenarios IV.3 to XI.4 can be found in Tables 5. The performance of POE estimator with fat-tailed distribution and OE estimator with fat-tailed distribution were tested. PE estimator with fat-tailed distribution was not tested because its performance, shown in objective 3, was not as good as the other two estimators with fat-tailed distribution. By fitting the same data with objective 1, the performance of estimators with fat-tailed distribution is comparable with the performance of estimators with normal distribution when they fit data with outliers or atypical values. All results were illustrated in section 6.

To compare the estimation results, indices listed in objective 1 were also used in this objective.

Objective 5: Investigate the response of three estimators to data with autocorrelated errors

Objective 5 was to compare the performance of estimators with normal distribution and the performance of estimators with fat-tailed distribution, when they fit data with errors that are autocorrelated. Data generated in this objective was assumed to have both process and observation errors. The reason for this assumption can be found in objective 2. Three conditions were tested including data with only process error autocorrelated, data with only observation error autocorrelated, and data with both errors autocorrelated.

The autocorrelated process error was assumed first order autocorrelation, which was usually used to model the delay effects of population change. The process error of current year is related to the process error of last year by following

$$P_{t+1} = (P_t + rP_t(1-P_t) - C/K)e^{\varepsilon_{1,t}}$$
$$\varepsilon_{1,t} = a_1 + b_1\varepsilon_{1,t-1} + \xi_{1,t},$$

where $\xi_{1,t} \sim N(0,\rho_1^2), t = 1,2,...,T$. a_1, b_1, ρ_1 are constant values. Considering the mean value, we have

$E(\varepsilon_{1,t}) = a_1 + b_1 \times E(\varepsilon_{1,t-1}) + E(\xi_{1,t}) = a_1 + b_1 \times E(\varepsilon_{1,t-1}) = a_1$.

Because the ordinary process error was usually assumed unbiased, a_1 was required to be zero. The autocorrelation equation is change to

$$\varepsilon_{1,t} = b_1\varepsilon_{1,t-1} + \xi_{1,t}.$$

The variance of process error can be calculated as

$$\sigma_1^2 = Var(\varepsilon_{1,t}) = E(\varepsilon_{1,t}^2) - [E(\varepsilon_{1,t})]^2 = E(\varepsilon_{1,t}^2) = \frac{\rho_1^2}{1-b_1^2}.$$

The value of b_1, ρ_1 should be properly defined so that the variance of autocorrelated process error can be equal to the variance of independent process error in objective 1 and 3. By having process error to remain the same variance, the estimation results from this objective were to be compared to results from objective 1 and 3.

Similar to the autocorrelated process error, I assumed the autocorrelated observation error with first degree autocorrelation. The observation error of the current year is related to the observation error of previous year by following

$$I_t = (q \times P_t \times K)e^{\varepsilon_{2,t}}$$
$$\varepsilon_{2,t} = a_2 + b_2\varepsilon_{2,t-1} + \xi_{2,t},$$

where $\xi_{2,t} \sim N(0, \rho_2^2), t = 1,2,..., T$. a_2 are defined to be zero. b_2 and ρ_2 are to properly defined so that the variance $\sigma_2^2 = \dfrac{\rho_2^2}{1-b_2^2}$ was the same with the variance of independent observation error used in objective 1 and 3.

In this objective, three groups of data were generated, each of which was fitted using POE_N, OE_N, POE estimator with fat-tailed distribution, OE estimator with fat-tailed distribution. The 12 scenarios can be found listed in Table 6.

Objective 6: Investigate the performance of the three estimators benefiting from incorporating multiple indices based on CPUEs

Objectives 1 to 5 used estimators to fit data containing catch records and one CPUE. Objective 6 was to investigate the performance of estimators when they fit data containing catch records and multiple CPUE records, and how they can benefit from using more CPUE information. Three groups of data were simulated with both process and observation error. Data was generated with catch records and one CPUE, three CPUEs, and five CPUEs for each group respectively. Each group of data was fitted using POE_N and OE_N respectively. Six scenarios can be found in Table 7. Only estimators with normal distribution were considered because firstly estimators with fat-tailed distribution didn't perform as well as estimators with normal distribution in objective 1 and 3. Second, using estimators with the same distribution will allow the objective focus on the effect of incorporating multiple indices based on CPUEs.

5. Results

5.1 The example fishery

5.1.1 Atlantic weakfish

To get the parameter estimation from the Atlantic weakfish population, I used both POE_N and POE_C to fit the data. After comparing the estimation results of using different combinations from CPUEs data together with catch data, I finally chose to use the three CPUEs from MRFSS, RI, and DE, the three of which were also recommended by the 2006 Atlantic weakfish stock assessment (NEFSC 2006). There were two choices for the length of the time series. Considering the start of the CPUEs records at 1966, I chose the time series from 1966 to 2007 and fitted the data using POE_N and POE_C (weakfish analyses 1 and 2 respectively). To account for the validation of the catch data records from the year incorporating recreational catch data at 1981, I chose the time series from 1981 to 2007 and fitted the data using POE_N and POE_C (weakfish analyses 3 and 4 respectively).

Non-informative priors were used for all of the parameters in the model. The priors were set to be uniform distribution with lower and upper boundaries wide enough so that they would not influence the estimation. The boundaries were also limited to meet the biological facts and statistical definition. The parameters include population intrinsic growth rate r, carrying capacity K, catch-ability coefficient q, logarithm of the depletion at the starting year of the time series a, the estimated relative biomass P (here, P equals population biomass divided by carrying capacity) at each year, and corresponding parameters in either normal or fat-tailed distributions. The priors' distributions for all parameters can be found in Table 8. There were two reasons to choose non-informative priors instead of informative prior. First, the weakfish technical committee rejected all previous stock assessments including the ones using dynamic models at 2006, because none of them can effectively explain the current situation for weakfish. So, the old estimated parameters' values found in previous weakfish population analysis were not valid enough to be incorporated into prior solicitation. Second, my purpose of this study is to investigate the estimators proposed based on this example fishery analysis. To use non-informative prior can exclude the possible influences caused by the priors so that we are sure that all of the differences in the parameter estimates were because of the estimators selected.

For the selection of candidates of robust distributions or fat-tailed distributions, I used Cauchy distribution instead of Student's t distribution in the three estimators. Cauchy distribution can present the distribution of both process error and observation error more accurately than Student's t distribution in the surplus production model proposed. The parameter estimation of the standard deviation in normal distribution (here the standard deviation of $\log(P)$) by POE_N is approximately 0.2 for process error and 0.6 (here the standard deviation of $\log(I)$) for observation error (Table 8). The student's t distribution has its limitation in that it cannot mimic normal distributions with standard deviations smaller than 1. If Student's t distribution was assumed as the distribution of process error or observation error, which is far too flat to reflect the true distribution, it would result in inaccurate parameter estimations of r, K and others because of its inability to distinguish high likelihood values and capture the exact parameter value that maximize the likelihood (Figure 23). Further usability of Student's t distribution was discussed in section 9.

To interpret the estimation results, the posterior distributions of parameters of interest were shown in Figures 9 to 12. The posterior mean of the parameter estimation was shown in Table 8. There are ten parameters for POE_N, including population intrinsic growth rate r, carrying capacity K, the mean value of logarithm of depletion at the first year a, standard deviation of process error σ_1, catch-ability coefficient q_i (i=1,2,3), and standard deviation of observation error $\sigma_{2,i}$ (i=1,2,3). The relative population biomass P_t, which equals the biomass of the year t divided by the carrying capacity K, are estimated. For POE_C, the scale parameter for process error γ_1 and the scale parameter for observation error $\gamma_{2,i}$ (i=1,2,3) are estimated instead of the standard deviation in normal distribution when POE_N was used. For the population intrinsic growth rate r, the estimated value is larger when POE_C was used than when POE_N was used (Figure 9). For the carrying capacity K, estimated values were smaller when fitting the short time series than when fitting the long time series (Figure 9). For a, the estimated value is larger when fitting short time series than when fitting long time series, which met with the P_t estimation that the relative population is greater at year 1981 than at year 1966 (Figure 13). For q_i (i=1,2,3), the estimated values were larger when POE_C was used than when POE_N was used. The estimated q_i values were larger when fitting short time series than when fitting long time series, which implied the possibility that the catch-ability were lower during 1966 to 1981 (Figure 10). Figure 14 showed the estimated distribution for process error and the observation error including normal distributions from analyses 1 and 2 and Cauchy distributions from analyses 3 and 4. For the biomass estimation at the first year B_0 (year 1966 for Atlantic weakfish analyses 1 and 3, year 1981 for Atlantic weakfish analyses 2 and 4), POE_C always resulted in smaller estimation than POE_N (Figure 11).

To interpret the estimation of parameters of management interest, POE_C resulted in smaller value for B_T / B_{MSY}, and larger value for F_T / F_{MSY} (T=2007). According to the estimation by POE_N, the Atlantic weakfish population is not over fished with B_T / B_{MSY} estimated greater than one, but it is not the case according to the estimation by POE_C. All analyses showed that over fishing is not happening with F_T / F_{MSY} estimated smaller than 1 (Figure 12). This probability resulted from the low catch level around 2007 (Figure 3). Although I used the weakfish example here, I do not recommend the results here for management purposes directly. Other factors such as relative abundance indices used and models such as age-structured models need to be considered as part of a comprehensive stock assessment for management purposes.

Figure 13 showed the estimated relative biomass P_t and population biomass B_t from 1966 to 2007 together with their 90% confidence interval. Atlantic weakfish was showed to have experienced a strong variation during 1981-1991, increased greatly from the lowest value at 1991 to highest value at 1996 due to the sudden drop of commercial and recreational catch, kept its highest population biomass during 1996-2001, and then went back to similar population level as in 1966.

The true parameter values used in simulation study were chose based on the parameter estimations from the four weakfish analyses. I adopt the population intrinsic growth rate to be 5.7 based on the estimation by POE_N, carrying capacity to be 80000, a to be -4.5 as the mean log transformed relative biomass at year 1981, standard deviation of process error 0.1, standard deviation of observation error 0.63. However in the simulation study, I chose the standard deviation of the process error to be 0.1, which is lower than it is estimated, because the estimated value around 0.2 is very high according to the rule of coefficient of variation (CV). From the process function:

$$\log(P_t) = \log(\hat{P}_t) + \varepsilon_{1,t}, \varepsilon_{2,t} \sim N(0, \sigma_1^2),$$

the coefficient of variation of process error is derived

$$CV_1 = \frac{\sigma_1}{E(|\log(\hat{P}_t)|)},$$

where $E(|\log(\hat{P}_t)|) = \frac{1}{T}\sum_{t=1}^{T} |\log(\hat{P}_t)|$. In this way, the CVs are calculated to be 0.4554 and 0.4985 for Atlantic weakfish analyses 1 and 2 respectively (Table 11). I did a test simulation on these values and the population dynamics turned out to fluctuate greatly. The population biomass with process error was as high as 1.8 times the original values. Considering the dramatically changed population during 1980-1990 (Figure 13), I assumed that the process error for Atlantic weakfish were with outliers and atypical values of large variance. To avoid losing ordinary signals in the process equation, the standard deviation was assumed to be 0.1 in the simulation study which corresponding to a CV value of $\log(P)$ equals to 0.2139 (Table 13). In order to choose one set of reasonable values for the catch-ability coefficient and observation error standard deviation from three surveys, I also calculated the CV for observation error. From the observation function:

$$\log(I_{i,t}) = \log(\hat{I}_{i,t}) + \varepsilon_{2,i,t}, \varepsilon_{2,i,t} \sim N(0, \sigma_{2,i}^2),$$

the coefficient of variation of observation error is derived

$$CV_2 = \frac{\sigma_{2,i}}{E_t(|\log(I_{i,t})|)},$$

where $E_t(|\log(I_{i,t})|) = \frac{1}{T}\sum_{t=1}^{T} |\log(I_{i,t})|$.

I adopt the catch-ability coefficient and the standard deviation estimated for Delaware Bay as true value for simulation study, which is 0.002 and 0.63 respectively. Table 11 also showed that CPUE from New Jersey resulted in the largest CV value in both weakfish analyses 1 and 2, which is almost 1.7 times of the CVs from MRFSS and DE. In the standardized CPUE plot, the CPUEs from NJ showed different pattern from CPUEs from MRFSS and DE (Figure 6). So, I also considered that CPUEs from NJ had the outliers or atypical values of large variance. The CV values for process error of Atlantic weakfish and for CPUE from NJ were used as a reference when I simulated outliers and atypical values. The details were shown in section 5.2.2.

5.1.2 Black sea bass

I used catch data and all five CPUEs to conduct the black sea bass population analysis. I also used both POE_N and POE_C as estimators for surplus production model. The Cauchy distribution was used instead of Student's t distribution because the deficiency of Student's t distribution explained in section 5.1.1. In the black sea bass analysis 1, I used POE_N to fit data ranged from 1974-2007. In the black sea bass analysis 2, I used POE_N to fit data ranged from 1978-2007. In the black sea bass analysis 3, I used POE_C to fit data ranged from 1974-2007. In the black sea bass analysis 4, I used POE_C to fit data ranged from 1978-2007. Two time series were used as that the CPUE records are not available until 1974 and the recreational catch data were not available until 1978. Design was the same as in Atlantic weakfish analysis.

Non-informative priors were used for black sea bass analysis. The prior distribution and the estimated value for parameters were shown in Table 9. I used posterior mean of parameters to be the estimated values, which was shown in Figures 16 to 19.

For the population growth rate r, the estimated values were larger when POE_N was used than when POE_C was used (Figure 16). For carrying capacity K, the estimated values were smaller when POE_N was used than when POE_C was used (Figure 16). For a, the estimated values when fitting short time series were larger than when fitting long time series, which met with the results that the relative population was larger at year 1978 than at year 1974 (Figure 19). For q_i (i=1,…,5), the estimated values were larger when POE_N was used than when POE_C was used. Figure 21 showed the estimated distribution for process error and observation error including normal distribution from analyses 1 and 2 and Cauchy distributions from analyses 3 and 4. For the biomass estimation at the first year B_0 (year 1978 for black sea bass analyses 1 and 3, year 1978 for black sea bass analyses 2 and 4), POE_C always resulted in larger estimation than POE_N (Figure 18).

To interpret the estimation of parameters of management interest, POE_C resulted in larger value for B_T / B_{MSY}, and larger value for F_T / F_{MSY} (T=2003). All the four analyses showed that the black sea bass population is over fished with B_T / B_{MSY} estimated smaller than one (Figure 19). The over fishing of black sea bass is happening with F_T / F_{MSY} estimated larger than one (Figure 19).

Figure 20 showed the estimated values of relative biomass and population biomass from 1974 to 2003 and their 90% confidence intervals. It showed that black sea bass population fluctuated during year 1974-1986, decreased slowly from 1986 to the lowest value at 1995, increased a little bit till 1998, and kept decreasing till 2003. The decrease of population from 1986 was probably resulted from the increased catch at previous years, and the recovery of population in 1998 was probably due to the decreased harvest since 1995 (Figure 5). The same with Atlantic weakfish analysis, the results of black sea bass population assessment in this study are not recommended for management purposes directly.

The true parameter values used in simulation study were chosen based on the parameter estimations from four black sea bass analyses. I adopt the population intrinsic growth rate to be 0.6 based on the estimation by POE_N, carrying capacity to be 6500 based on the estimation by POE_N, a to be -2.5 as the mean log transformed relative biomass at year 1978, the standard deviation of process error to be 0. I chose to use the catch-ability coefficient and the standard deviation estimated from head boat survey as the true value for simulation study, which is 0.0016 and 0.07 respectively, because firstly the head boat survey covered the longest time period among five surveys from 1974 to 2003. Second, the estimated CV of the observation error from the head boat survey was as low as 0.05 which was distinct from the CV of 0.1399 estimated for observation error of Atlantic weakfish (Table 11). As the CV values for process error of Atlantic weakfish and black sea bass are similar, to be 0.2139 and 0.2358 respectively (Tables 11 and 13), the data generated for simulation study based on both Atlantic weakfish and black sea bass exhibited different balance between process error and observation error. All the true values for simulation study based on black sea bass were shown in Table 12. I notified that the observation error from survey of FL snapper trap had the CV value of 0.2536, which was far larger than CVs for observation error from other surveys (Table 11). According to the standardized CPUE plot (Figure 6), the CPUE from SEAMAP FL snapper trap fluctuating greatly during 1980-1987 was most abnormal compared to CPUEs from other surveys. I assumed the CPUE from FL snapper trap survey had outliers and atypical values. The CV value estimated for observation error from FL snapper trap survey was used as a reference when I simulated outliers and atypical values in the simulation study. The details were shown in section 5.2.2.

Table 10 listed the proportional difference of parameter values estimated by POE_N and POE_C for both Atlantic weakfish and black sea bass. The proportional difference of parameter values were calculated from

$$\text{proportional difference}=\frac{|X^1_{POE\_N}-X^2_{POE\_N}|}{(X^1_{POE\_N}+X^2_{POE\_N})/2}\, or \,\frac{|X^3_{POE\_C}-X^4_{POE\_C}|}{(X^3_{POE\_C}+X^4_{POE\_C})/2}.$$

X_{POE_N}, X_{POE_C} denote the parameter values estimated by POE_N and POE_C respectively. Numbers in the top right corner denote the analyses 1 to 4 for Atlantic weakfish and black sea bass. This proportional difference index indicated that how different the estimation is for the same estimator fitting data of two different time series. Table 10 showed that the stability of POE_N and POE_C fitting two time series were different for Atlantic weakfish and black sea bass. POE_C resulted in more stable estimation for Atlantic weakfish, with the proportional difference indices smaller, compared to the estimation results when POE_N was used, whereas POE_N resulted in more stable estimation for black sea bass analysis. There was no conclusion on robust estimators based on real fishery analysis.

5.2 Simulation study

5.2.1 Performance of the three estimators to log-normally distributed errors

Two groups of data were simulated based on the Atlantic weakfish and the black sea bass respectively. Data based on Atlantic weakfish fishery were simulated from 1981-2007. Data based on black sea bass fishery were simulated from 1978-2003. All simulated data contained the total catch and one CPUE for the study of objectives one to five, but one, three, and five CPUES for the study of objective 6.

For data based on the Atlantic weakfish, three error types, including data with both process and observation errors (data set I), data with only observation error (data set II), and data with only process error (data set III), were simulated based on the "true" values listed in Table 14. For data based on black sea bass, only two error types, including data with both process and observation errors, and data with only process error, were simulated. Data with only observation error were not simulated for black sea bass because the population biomass were decreased to zero in the last eight years of the time series when true values listed in Table 14 were used. All of the simulated data are fitted using POE_N (estimator 1), OE_N (estimator 2), PE_N (estimator 3), POE_C (estimator 4), OE_C (estimator 5), and PE_C (estimator 6). In all, there were eighteen scenarios for Atlantic weakfish and twelve scenarios for black sea bass. Each scenario was tested using one hundred simulations. Mean of proportional error (MPE) and mean of squared proportional error (MSPE) were used as indices to evaluate the accuracy of the estimation. Median of proportional error (MDPE) and median of squared proportional error (MDSPE) were used as indices to evaluate the precision of estimation. r, K, MSY, and B/B_{MSY} were selected as parameters to present the estimation results. The values of six indices based on the three parameters were shown in Tables 15 and 16 (Table 15 based on Atlantic weakfish; Table 16 based on black sea bass). Box-plots of proportional error and squared proportional error for the estimation of three parameters can be found in Figures 24 to 27 (Figures 24, 26 and 27 based on Atlantic weakfish; Figures 25, 28 based on black sea bass). The bottom and top of the box in the box plot represent the 25th and 75th percentile respectively; the band near the middle of the box represents the median value; and the red stars represent outliers. The outliers of the estimation results are defined of values either larger than $q_3 + 1.5 \times (q_3 - q_1)$ or smaller than $q_1 - 1.5 \times (q_3 - q_1)$, where q_1 and q_3 are the 25th and 75th percentiles respectively.

Figure 24 and 25 showed that when POE with log-normally distributed errors was the true model in generating data, POE_N always performed the best in assessing r, K, MSY, and B/B_{MSY} concerning estimation accuracy and precision. POE_N estimation had the smallest MPEs and MDPEs of r, K, MSY, and B/B_{MSY} among the six estimators, with the values always close to 0 (Tables 15 and 16). MSPEs and MDSPEs of r, K, MSY, and B/B_{MSY} when POE_N was used among the smallest sometimes close to those values when POE_C or OE_N were used (Tables 15 and 16). Among estimators with Cauchy distribution, POE_C performed the best most of the times. Estimators with Cauchy distribution had more stable estimation for r compared to estimators with normal distribution, but they resulted in large bias and variance when estimating K, MSY, and B/B_{MSY}. PE_N and PE_C resulted in estimations more biased and less precise compared to the other estimators with the same distribution.

Figure 26 showed that when OE with log-normally distributed errors was the true model in generating data, POE_N and OE_N performed better than the other estimators in assessing r, K, MSY, and B/B_{MSY} most of times. The MPEs and MDPEs of r, K, MSY, and B/B_{MSY} when POE_N and OE_N were used were very similar to each other, but always smaller than values estimated by the other estimators (Tables 15 and 16). POE_N gave smaller values for five out of six indices than OE_N. The MSPEs and MDSPEs of K, MSY, and B/B_{MSY} when POE_N and OE_N were used were always smaller than when other estimators were used (Tables 15 and 16). As for MSPEs and MDSPEs of r, OE_C provided similar values with POE_N and OE_N, and with one value smaller. Among estimators with Cauchy distribution, OE_C performed the best concerning eight out of twelve indices, with POE_C better for the other four indices (Tables 15 and 16). Estimators with Cauchy distribution resulted in more stable estimation for r compared to estimators with normal distribution, but they resulted in large bias and variance when estimating K, MSY, and B/B_{MSY}. PE_N and PE_C resulted in estimations that are more biased and less precise compared to the other estimators with the same distribution. The values of indices when the true model was OE were smaller than these when the true model is POE.

Figures 27 and 28 showed that when PE was the true model in generating data, POE_N performed better than the other estimators in assessing r, K, MSY, and B/B_{MSY} most of times. The MPEs and MDPEs of r, K, MSY, and B/B_{MSY} when POE_N was used are usually the smallest, though sometimes those values when OE_N was used were close (Table 15). The MSPEs and MDSPEs of r, K, MSY, and B/B_{MSY} by POE_N are among the smallest, though sometimes those values when POE_C, OE_C, OE_N were used are close (Tables 15 and 16). Among estimators with Cauchy distribution, POE_C performed usually the best (Tables 15 and 16). Estimators with Cauchy distribution has more stable estimation for r compared to estimators with normal distribution, but they results in large bias and variance when estimating K, MSY, and B/B_{MSY}. The performances of PE_N and PE_C were improved compared to the situation when the true model is POE or OE, but it was still not as good as POE_N. The values of indices when the true model is PE are smaller than the values when the true model is POE or OE.

In general, POE_N had good performances no matter the true model is POE, OE, or PE. The performances of OE_N and PE_N were getting closer to POE_N when they were used to fit data generated from their own model, but were still not as good as POE_N most of the times. Estimators with Cauchy distribution provided more stable estimation for r, but the estimation for K, MSY, and B/B_{MSY} are far away from 0 and with poor precision.

5.2.2 Performance of the estimators to outliers and atypical values

Eight data types (from IV to XI) with both process and observation errors were simulated. The data was based on Atlantic weakfish and was from 1981-2007. Each types of simulated data was fitted using four estimators including POE_N (estimator 1), OE_N (estimator 2), POE_C (estimator 3), and OE_C (estimator 4). PE_N and PE_C were not tested because the performances of them were shown not as good as the other four estimators in section 5.2.1. There were in all 32 scenarios and each of them was performed 100 simulations. Results were compared using the same indices used in section 5.2.2 (Table 17, Figure 29-36).

For data set IV, I assumed that the outliers and atypical values were of large variance and were in observation error. The standard deviation of outliers and atypical values were set to be 1.0 compared to 0.63 which is the standard deviation of ordinary observation error. The CV of outliers and atypical values is 0.2213 compared to 0.1394 which is the CV of ordinary observation error (Table 13). This distribution was created based on the values estimated for NJ CPUE in weakfish analysis. The CV value of outliers and atypical values were set to be the same scale as CV value for NJ CPUE, which is around 0.24 (Table 11). The number of outliers and atypical values were set to be 5 (18% of the total length) and randomly distributed along the time series. Figure 29 showed that, POE_N and OE_N performed better than the other estimators in assessing r, K, MSY, and B/B_{MSY}. POE_N and OE_N had similar performance most of the times but OE_N has more biased estimation for K and MSY (Table 17). Among estimators with Cauchy distribution, POE_C performed much better than OE_C but not as well as POE_N and OE_N most of the time. To point out is that although the MPE and MDPE of

r by POE_C and OE_C is biased from zeros, the MSPE and MDPE by POE_C and OE_C are smaller than by POE_N and OE_N because of small variance of the estimated r by estimators with Cauchy distribution. Estimators with Cauchy distribution gave more stable estimation for r compared to estimators with normal distribution, but they resulted in large bias and variance when estimating K, MSY, and B/B_{MSY}. Comparing the results with the ones fitting POE data, the values of indices when outliers and atypical values existed are more biased and imprecise than the values when POE without outliers and atypical values are the true model.

For data set V, I assumed that the outliers and atypical values were of large variance and appeared as observation error at fix period. Different from data set IV, all the observation errors of large variance were set to be the first five CPUE records considering that poor fishery techniques in the early years most probably resulted in errors of large variance. Figure 30 showed that, POE_N and OE_N performed better than the other estimators in assessing r, K, MSY, and B/B_{MSY} most of times. POE_N and OE_N had similar performance most of the times expect OE_N had a larger variance for r (Table 17). Among estimators with Cauchy distribution, POE_C performed better than OE_C. Although biased than POE_N and OE_N, the estimation of POE_C and OE_C had smaller MDSPE value of r because of their smaller variance. Estimators with Cauchy distribution also resulted in large bias and variance when estimating K, MSY, and B/B_{MSY}. Comparing the results with the ones fitting POE data, the values of indices when outliers and atypical values existed are more biased and imprecise than the values when POE without outliers and atypical values are the true model. Comparing the results with the ones fitting data with outliers and atypical values of large variance randomly distributed among observation error, the values of indices when outliers and atypical values at fixed time period is more biased and imprecise. The influence of outliers and atypical values of large variance to the estimation results is larger when the outliers and atypical values continuously lasted for a time while.

For data set VI, I assumed that the outliers and atypical values were of biased mean value and were in observation error. The biased mean of outliers and atypical values were set to be 0.3, which satisfied that the mean value of the exponential outliers and atypical values were at the same scale of the value for outliers and atypical values of large variance (Table 13). The mean value of exponential outliers and atypical values of large variance is 1.6487 while the mean value of exponential outliers and atypical values of biased mean is 1.6462 (Table 13). It meant that the ordinary observation error can resulted in the expectation of recorded CPUE value 1.2195 times the true CPUE value, while outliers and atypical values would result in the expectation of recorded CPUE 1.6 times the true value (Table 13). Figure 28 showed that, POE_N and OE_N performed better than the other estimators in assessing r , K , MSY, and B/B_{MSY} most of times. OE_N performed the best in estimating r . POE_C provided a more stable estimation of r than POE_N but is more biased and less stable in estimating K , MSY, and B/B_{MSY} (Figure 31). Comparing the results with the ones fitting POE data, the values of indices when outliers and atypical values existed were more biased and imprecise than the values when the data were generated without outliers and atypical values.

For data set VII, I assumed that the outliers and atypical values were of biased mean value and appeared as observation error at fixed time period. Different from data set V, all the observation errors of biased mean were set to be the first five CPUE records considering that poor fishery techniques in the early years most probably resulted in errors of biased mean. Figure 32 showed that, POE_N and OE_N performed better than the other estimators in assessing r, K, MSY, and B/B_{MSY} most of times. POE_N and OE_N had similar performance most of the times expect that OE_N had more biased estimation for K and MSY (Figure 32). Among estimators with Cauchy distribution, POE_C performed better than OE_C. Although POE_C resulted in much smaller MSPE and MDSPE of r, it resulted in higher MSPE and MDSPE of K, MSY, and B/B_{MSY}, and it resulted in MPEs and MDPEs of K, MSY, and B/B_{MSY} further away from 0. POE_N and OE_N resulted in MPE and MDPE of K, MSY, and B/B_{MSY} closer to 0 and resulted in MSPEs and MDSPEs of K, MSY, and B/B_{MSY} much smaller. Comparing the results with the ones fitting POE data, the values of indices when outliers and atypical values existed are more biased and imprecise than the values when POE without outliers and atypical values were used as the true model in the simulation. Comparing the results with the ones fitting data with outliers and atypical values of biased mean randomly distributed among observation error, the performance of estimators fitting data with outliers and atypical values at fixed time period have more biased estimation of r and B/B_{MSY}.

For data set VIII, I assumed that the outliers and atypical values were of large variance and were in process error. The standard deviation of outliers and atypical values were set to be 0.2 compared to 0.1 which is the standard deviation of ordinary process error. The CV of outliers and atypical values is 0.4277 compared to 0.2139 which is the CV of ordinary process error (Table 13). This distribution was created based on the values estimated in weakfish analysis. The CV of process error is 0.4554 from weakfish analysis 1 and 0.4985 from weakfish analysis 2 (Table 11). The number of outliers and atypical values were set to be 5 (18% of the total length) and randomly distributed along the time series. Figure 33 showed that, POE_N performed the best among estimators most of times. Although POE_C gave smallest value of MPEs, MDPEs, MSPEs and MDSPEs of r, it resulted greater biased in estimating K, MSY, and B/B_{MSY} with MPEs and MDPEs further away from 0, and it resulted in MSPE and MDSPEs of K, MSY, and B/B_{MSY} much larger.

For data set IX, I assumed that the outliers and atypical values were of large variance and appeared as process error at fix period. Different from data set IV, all the observation errors of large variance were set to be the first five CPUE records considering that poor fishery techniques in the early years most probably resulted in errors of large variance. Figure 34 showed that, POE_N and OE_N performed better than the other estimators in assessing r, K, MSY, and B/B_{MSY} most of times. POE_N and OE_N had similar performance most of the times expect OE_N had a larger variance for r (Table 17). Among estimators with Cauchy distribution, POE_C performed better than OE_C. Although biased than POE_N and OE_N, the estimation of POE_C and OE_C had smaller MDSPE value of r because of their smaller variance. Estimators with Cauchy distribution also resulted in large bias and variance when estimating K, MSY, and B/B_{MSY}. Comparing the results with the ones fitting POE data, the values of indices when outliers and atypical values existed are more biased and imprecise than the values when POE without outliers and atypical values are the true model. Comparing the results with the ones fitting data with outliers and atypical values of large variance randomly distributed among observation error, the values of indices when outliers and atypical values at fixed time period is more biased and imprecise. The influence of outliers and atypical values of large variance to the estimation results was larger when the outliers and atypical values continuously lasted for a period of time.

For data set X, I assumed that the outliers and atypical values were of biased mean value and were in observation error. The biased mean of outliers and atypical values were set to be 0.3, which satisfied that the mean value of the exponential outliers and atypical values were at the same scale of the value for outliers and atypical values of large variance (Table 13). The mean value of exponential outliers and atypical values of large variance is 1.6487 while the mean value of exponential outliers and atypical values of biased mean is 1.6462 (Table 13). It meant that the ordinary observation error can resulted in the expectation of recorded CPUE value 1.2195 times the true CPUE value, while outliers and atypical values would result in the expectation of recorded CPUE 1.6 times the true value (Table 13). Figure 33 showed that, POE_N and OE_N performed better than the other estimators in assessing r, K, MSY, and B/B_{MSY} most of times. OE_N performed the best in assessing r. POE_C provided a more stable estimation of r than POE_N but result in more biased and less stable estimation of K, MSY, and B/B_{MSY} (Figure 33). Comparing the results with the ones fitting POE data, the values of indices when outliers and atypical values existed indicated that the results were more biased and imprecise than the values when POE without outliers and atypical values were used as the true model.

For data set VI, I assumed that the outliers and atypical values were of biased mean value and appeared as observation error at fix period. Different from data set V, all the observation errors of biased mean were set to be in the first five CPUE records.

Figure 34 showed that, POE_N and OE_N performed better than the other estimators in assessing r, K, MSY, and B/B_{MSY} most of times. POE_N and OE_N result in similar performance most of the times expect OE_N had a more biased estimation for K (Figure 34). Among estimators with Cauchy distribution, POE_C performed better than OE_C. Although POE_C resulted in much smaller MSPE and MDSPE of r, it resulted in higher MSPE and MDSPE of K, MSY, and B/B_{MSY}, and it resulted in MPEs and MDPEs of K, MSY, and B/B_{MSY} further away from 0. POE_N and OE_N resulted in MPE and MDPE of K, MSY, and B/B_{MSY} closer to 0 and resulted in MSPEs and MDSPEs of K, MSY, and B/B_{MSY} much smaller. Comparing the results with the ones fitting POE data, the values of indices when outliers and atypical values existed were more biased and imprecise than the values when POE without outliers and atypical values were used as the true model. Comparing the results with the ones fitting data with outliers and atypical values of biased mean randomly distributed among observation error, the performance of estimators fitting data with outliers and atypical values at fixed time period was similar but more biased estimation of r and B/B_{MSY} were observed.

5.2.3 Performance of the estimators to autocorrelated errors

Three data types (from XII to XIV) with both process error and observation error were simulated. The data was based on Atlantic weakfish and was from 1981-2007. Each types of simulated data was fitted using four estimators including POE_N (estimator 1), OE_N (estimator 2), POE_C (estimator 3), and OE_C (estimator 4). PE_N and PE_C were not tested because the performances of them were shown not as good as the other four estimators in section 5.2.1. There were in all 9 scenarios and each of them was performed 100 simulations. Results were compared using the same indices given in section 5.2.2 (Table 18, Figure 35-37).

For data set XII, I assumed that the process error was ordinary but observation error was with first order auto-correlation. The auto-correlation coefficient of observation error b_2 was set to be 0.6. The standard deviation of the random error ρ_2 was 0.48 so that the variance of the autocorrelated observation error remained to equal to σ_2^2. Figure 35 showed that POE_N and OE_N performed better in assessing r, K and B/B_{MSY} most of times. Although POE_C resulted in the smallest MSPE and MDSPE of r, it also resulted in MPE and MDPE of K and B/B_{MSY} further away from 0, and the MSPE and MDSPE of K much larger than the other estimators.

For data set XIII, I assumed that the observation error was ordinary but process error was with first order auto-correlation. The auto-correlation coefficient of process error b_1 was set to be 0.6. The standard deviation of the random error ρ_1 was 0.08 so that the variance of the autocorrelated observation error remained σ_1^2. Figure 36 showed that POE_N and OE_N performed better in assessing r, K, MSY, and B/B_{MSY} most of times. Although POE_C and OE_C resulted in smaller MSPE and MDSPE of r and MPE and MDPE of r closer to zero, it also resulted in MPE and MDPE of K, MSY, and B/B_{MSY} further away from 0, and the MSPE and MDSPE of K and MSY much larger than the other estimators.

For data set XIV, I assumed that both observation error and process error were with first order auto-correlation. The values of auto-correlation coefficient and the standard deviation were set according to data set XII and XIII (Table 12). Figure 37 showed that POE_N and OE_N performed better in assessing r, K, MSY, and B/B_{MSY}

most of the times. Although POE_C and OE_C resulted in smaller MSPE and MDSPE of r and MPE and MDPE of r closer to zero, it also resulted in MPE and MDPE of K, MSY, and B/B_{MSY} further away from 0, and the MSPE and MDSPE of K and MSY much larger than the other estimators. Comparing to estimation in fitting data XII and XIII, the parameter estimation was worse in fitting data XIV.

In general, POE_N and OE_N performed better than the other estimators in assessing r, K, MSY, and B/B_{MSY} most of times. POE_C and OE_C had good performance in assessing r, but tended to result in more biased K and B/B_{MSY} than when POE_N and OE_N were used.

5.2.4 Performance of the estimators fitting data with multiple indices based on CPUEs

Three groups of data with both process error and observation error were simulated. Data from group I was used directly as the data with one CPUE. The other two groups contained three CPUEs and five CPUEs respectively. The catch-ability coefficients and standard deviations for three CPUEs and five CPUEs were shown in Table 14. The data was generated from 1981-2007. Each groups of simulated data was fitted using two estimators including POE_N (estimator 1), OE_N (estimator 2). PE_N were not tested because the performances of them were shown not as good as the other four estimators in section 5.2.1. There were in all 4 scenarios and each of them was performed 100 simulations. Results were compared using the same indices given in section 5.2.1 (Table 19, Figure 38).

Figure 38 showed that POE usually performed better than OE in assessing r, K, MSY, and B/B_{MSY} no matter fitting data with one CPUE, three CPUEs, or five CPUEs.

Estimators fitting data with multiple indices based on CPUEs performed usually better than the same estimators fitting data with less numbers of CPUEs. Although there were exception that POE_N and OE_N fitting data with 3 CPUEs resulted in MPE and MDPE of K and MSY further away from 0 than them fitting data with one CPUE, but they also resulted in smaller MSPE and MDSPE of K at the same time.

6. Discussion

Study based on only real fishery analysis is hard to investigate the robustness and appropriateness of the estimators of interest. In this study, the estimates of the parameters when POE_N was used were very close to these when POE_C were used (Table 8, Figures 9 and 10 for weakfish; Table 9, Figures 16 and 17 for black sea bass). The estimated biomass and relative biomass from POE_N and POE_C were at the same scale and of the same trend (Table 8, Figures 12 and 13 for weakfish; Table 9, Figures 19 and 20 for black sea bass), but the differences for the parameters of management interest were still substantial. POE_C was less sensitive to the length of the time series in the example fishery of Atlantic weakfish, but POE_N was more robust to the length of the time series in the example fishery of black sea bass. To search for robust estimators by comparing the stability of the parameter estimates to the number of CPUEs and length of the time series is not enough. Other reasons such as the non-stationary dynamics of the population (e.g., change of the distributions of population growth rate or carrying capacity) can influence the estimation results other than the estimators.

Through the simulation study, POE_N was found to be the estimators with best performance among three estimators with normal distribution and three estimators with Cauchy distribution (Figures 24 to 27) among my proposed scenarios. OE_N was the second best estimators and was comparable to POE_N only when the true model generating the data was OE. POE_N was found to be the robust estimator when the data were with outliers and atypical values and when the data was with autocorrelated residuals; OE_N was the second best robust estimators under these situations. The performance of OE_N was comparable to POE_N in assessing K, MSY, and B_T / B_{MSY} for some cases, but it resulted in more biased and imprecise r in the simulation study. Estimators with Cauchy distribution provided a stable estimation of r, but they tended to estimate smaller r, and larger K and MSY, and smaller B_T / B_{MSY} than their true values. Estimator with Cauchy distribution also resulted in less stable estimation for K, MSY, and B_T / B_{MSY} compared to estimators with normal distribution.

The simulation also found that estimators tend to perform better when there are multiple indices based on CPUEs available. POE_N and OE_N resulted in much better results when there are multiple indices based on CPUEs in the simulation. Parameters estimated by POE_N and OE_N fitting 3 CPUEs or 5 CPUEs were less biased and more stable than fitting one CPUE. The performance of OE_N fitting 3 CPUEs and 5 CPUEs was comparable to or sometimes better than POE_N fitting 1 CPUE. Using multiple indices based on CPUEs could allow decreasing of the model complexity by using OE_N instead of POE_N while maintaining good performance for stock assessment.

The performances of the estimators with fat-tailed distribution (Cauchy here) were not as good as expected. They always resulted in a negative biased r and positive biased K which is partly because of the inverse correlation between r and K. The negative bias of B_T / B_{MSY} is heavily because of the biased estimation of K. One possible reason was that the fat-tailed distribution was more flat than normal distribution that it had trouble to distinguish the high likelihood value of the “true” parameter value. Because of the multiplicative error assumption, when the population biomass was generated, its distribution tended to have a long right-side tail with higher percentage of high values. The CPUE was generated based on the population biomass following a long right-side tailed lognormal distribution and were applied multiplicative observation error again. In this way, the shape of the distribution of CPUE was further skewed after adding the noise from two kinds of multiplicative errors. The whole population size was very easily to be overestimated because of the heavy right-side tail of the distribution of CPUE data. This also explained the positive bias of K estimated by the estimators with normal distribution in some scenarios. Because the fat-tailed distribution was less sensible to the high likelihood value of the true value, estimators with fat-tailed distribution was more likely to have higher estimated population biomass and carrying capacity K. Other possible reasons for the unexpected performance of estimators with fat-tailed distribution were due to my operation of the program. The estimators with fat-tailed distributions needed longer iterations for Markov chains to converge to the posterior distribution. I used 100,000 iterations with thinning interval of 5 for all of the estimators with normal or Cauchy distributions. Statistical convergence test was used for the first of 100 simulations for each scenario in objective 1 and 3. Posterior distributions estimated by all estimators pasted the convergence test within the proposed iterations, and the same iteration number and thinning interval was used for all simulations. Among the proportional error and squared proportional error from some simulation, we observed some extremely large abnormal values. It might be resulted from the unconverged Markov chains. These large abnormal proportional errors and squared proportional errors were mostly observed when estimators with fat-tailed distribution were used. Although the iteration I used in my simulation study is twice of the number used in Meyer and Miller (2000), there was still chance that the posterior distribution was not able to converge within the number of iterations. A built-in convergence test is suggested when the MCMC is used so that the convergence of Markov chains was guaranteed.

In this research, I chose Cauchy distribution instead of Student's t distribution to be the fat-tailed distribution. Student's t distribution has its limitation in that it can only be flatter than the standard normal distribution and it has to be symmetrical besides zero. Figure 23 showed that as the degree of freedom increased, the shape of Student's t distribution grow higher till merged to the standard normal distribution. The estimated distributions for both process error and observation error of Atlantic weakfish and black sea bass using estimators with normal distribution were with standard deviation far smaller from 1. So Student's t distribution could not fit the shape of the exact distribution of observation error and process error. I tried the process-observation-error-estimator with Student's t distribution to assess the Atlantic weakfish population. It failed to capture the true value of r with the posterior distribution of r close to a uniform distribution (results not shown here). Although the Student's t distribution was not used in this research, it can still be applied to other fishery population. In order to satisfy the usefulness of the Student's t distribution i.e., the standard deviation is larger than or equal to 1, the $mean(|\log(I)|)$ or $mean(|\log(P)|)$ value should at least equal to or higher than 3, calculated by an average CV of around 30%. So, it should be useful when $\log(B)$ instead of $\log(P)$ is used in the process equation. Further exploration of the Student's t distribution as a robust distribution in modeling process error and observation error in the surplus production model may be needed.

Parameters in the model themselves were strongly correlated to each other caused by the fixed model form. In the surplus production model, r and K are strongly correlated (Figure 15 and Figure 22). Estimators, which resulted in a negative biased r estimates, always resulted in positive biased K estimates and negative biased B_T / B_{MSY}. The correlation coefficients marked in the legend of Figures 15 and 22 were not that large partly because the relationship between r and K was non-linear which can be found from the process function. Correlations also existed among other parameters. So it was suggested that the joint posterior distribution of all parameters instead of marginal distribution for each single parameter, be used for the projection study or risk analysis for a real fishery analysis.

From my study, the performance of the three estimators with normal distribution and the three estimators with Cauchy distribution was evaluated using both example fisheries and a simulation study. This evaluation helped us to choose appropriate estimators in future fishery population dynamics and stock assessment. POE estimator was strongly suggested to be used for surplus production model because it performs better than OE estimator and PE estimator no matter process error exist or not, it was also robust to outliers, atypical values, and autocorrelated errors. Multiple indices based on CPUEs which help improve the parameter estimation and performance of the estimators was suggested to be used if available in population dynamic analysis. OE estimators could be considered when multiple indices based on CPUEs are available which had the similar performance with POE estimators with only one CPUE.

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7 List of Tables

Table 1: The characteristics of Atlantic weakfish and black sea bass

	Atlantic weakfish	Black sea bass (the southern stock)	
Order, Family	Perciformes, Sciaenidae	Perciformes, Sciaenidae	
Genius	*Cynoscion*	*Centropristis*	
Range	North Carolina to New England	Florida to North Carolina	
Habitat	estuarine and inshore oceanic areas	inshore and offshore waters	
Diet	mysid shrimp (young) bay anchovy (young) Atlantic menhaden or other clupeid fish (adult fish)	bottom-feeding invertebrates including crabs, squids, clams	
Spawn Time	late May through the summer	January till November	
Natural Mortality	0.25	0.3	
Sex	gonochoristism	hermaphroditism (potengetic)	
		female ----→	male
Maturity	90% mature at age	2	4
Size	1 m	0.3 m	0.6 m
Longevity	12 years	8	20

Table 2: Scenarios to investigate performances of estimators with normal distribution

	Data Generated:	Normal POE	Normal OE	Normal PE
Estimators Used	POE with Normal distribution assumption	I.1	II.1	III.1
	OE with Normal distribution assumption	I.2	II.2	III.2
	PE with Normal distribution assumption	I.3	II.3	III.3

Table 3: Scenarios to investigate the performance of estimators with normal distribution to outliers and atypical values

	Data Generated:	POE: PE~N(0,0.1), OE~N(0,0.63) Outliers in OE of Large Variance ~ N(0,1) Random Distributed	 Fixed first 5 year
Estimator used	POE with Normal distribution assumption	IV.1	V.1
	OE with Normal distribution assumption	IV.2	V.2
	Data Generated:	POE: PE~N(0,0.1), OE~N(0,0.63) Outliers in OE of Biased Mean ~ N(0.3, 0.6) Random Distributed	 Fixed first 5 year
Estimator used	POE with Normal distribution assumption	VI.1	VII.1
	OE with Normal distribution assumption	VI.2	VII.2
	Data Generated:	POE: PE~N(0,0.1), OE~N(0,0.63) Outliers in PE of Large Variance ~ N(0,0.2) Random Distributed	 Fixed middle 5 year
Estimator used	POE with Normal distribution assumption	VIII.1	IX.1
	OE with Normal distribution assumption	VIII.2	IX.2
	Data Generated:	POE: PE~N(0,0.1), OE~N(0,0.63) Outliers in PE of Biased Mean ~ N(0.1, 0.1) Random Distributed	 Fixed middle 5 year
Estimator used	POE with Normal distribution assumption	X.1	XI.1
	OE with Normal distribution assumption	X.2	XI.2

Table 4: Scenarios to investigate the performance of estimators with fat-tailed distribution

	Data Generated:	Normal POE	Normal OE	Normal PE
Estimators Used	POE under Cauchy distribution assumption	I.4	II.4	III.4
	OE under Cauchy distribution assumption	I.5	II.5	III.5
	PE under Cauchy distribution assumption	I.6	II.6	III.6

Table 5: Scenarios to investigate the performance of estimators with fat-tailed distribution to outliers and atypical values

	Data Generated:	Normal POE, PE~N(0,0.1), OE~N(0,0.63) Outliers in OE of Large Variance ~ N(0,1)	
		Random Distributed	Fixed first 5 year
Estimator used	POE with Cauchy distribution assumption	IV.3	V.3
	OE with Cauchy distribution assumption	IV.4	V.4
	Data Generated:	Normal POE, PE~N(0,0.1), OE~N(0,0.63) Outliers in OE of Biased Mean ~ N(0.3, 0.63)	
		Random Distributed	Fixed first 5 year
Estimator used	POE with Cauchy distribution assumption	VI.3	VII.3
	OE with Cauchy distribution assumption	VI.4	VII.4
	Data Generated:	Normal POE, PE~N(0,0.1), OE~N(0,0.63) Outliers in PE of Large Variance ~ N(0,0.2)	
		Random Distributed	Fixed middle 5 year
Estimator used	POE with Cauchy distribution assumption	VIII.3	IX.3
	OE with Cauchy distribution assumption	VIII.4	IX.4
	Data Generated:	Normal POE, PE~N(0,0.1), OE~N(0,0.63) Outliers in PE of Biased Mean ~ N(0.1, 0.1)	
		Random Distributed	Fixed middle 5 year
Estimator used	POE with Cauchy distribution assumption	X.3	XI.3
	OE with Cauchy distribution assumption	X.4	XI.4

Table 6: Scenarios to investigate the performance of estimators to autocorrelated errors

	Data Generated:	Normal distributed POE OE autocorrelated	 PE autocorrelated	 OE and PE both autocorrelated
Estimators Used	POE with Normal distribution assumption	XII.1	XIII.1	XIV.1
	OE with Normal distribution assumption	XII.2	XIII.2	XIV.2
	POE with Cauchy distribution assumption	XII.3	XIII.3	XIV.3
	POE with Cauchy distribution assumption	XII.4	XIII.4	XIV.4

Table 7: Scenarios to investigate the performance of estimators fitting data with multiple indices based on CPUEs

	Data Generated:	Normal distributed POE 1 CPUE	 3 CPUE	 5 CPUE
Estimators Used	POE with Normal distribution assumption	I.1	XV.1	XV.3
	OE with Normal distribution assumption	I.2	XV.2	XV.4

Table 8 Results for Atlantic weakfish analysis

Parameters and their priors	Analysis 1 POE_N 1966-2007	Analysis 2 POE_N 1981-2007	Analysis 3 POE_C 1966-2007	Analysis 4 POE_C 1981-2007
$r \sim U(0,2)$				
	0.5735	0.5896	0.6943	0.6334
$K \sim U(0,10^7)$				
	8.5519e+004	7.7387e+004	8.1588E+04	7.8823E+04
$a \sim U(-10,0)$				
	-0.5261	-0.4158	-0.8459	-0.4619
$\sigma_1 \sim U(0,4)$				
	0.1733	0.2331		
$\gamma_1 \sim U(0,4)$				
			0.2162	0.2112
$q \sim U(0,1)$				
	2.1370E-06	2.5575E-06	2.9968E-06	3.2341E-06
	1.1290E-03	1.3605E-03	1.4239E-03	1.5387E-03
	1.8070E-03	2.2974E-03	2.9774E-03	3.3382E-03
$\sigma_2 \sim U(0,4)$				
	0.3369	0.2907		
	0.9727	0.9677		
	0.6365	0.6321		
$\gamma_2 \sim U(0,4)$				
			0.0778	0.0786
			0.6877	0.6871
			0.3512	0.3466
$P \sim U(0,1)$				
1966	0.6261		0.5471	
1967	0.6984		0.5370	
1968	0.7458		0.5720	
1969	0.7855		0.6995	
1970	0.8048		0.7306	
1971	0.8078		0.7456	
1972	0.7731		0.6411	
1973	0.7313		0.6171	
1974	0.6870		0.5799	
1975	0.7230		0.6669	
1976	0.7274		0.6767	
1977	0.7181		0.6671	
1978	0.7230		0.6628	
1979	0.7106		0.6397	

1980	0.6779		0.6850	
1981	0.6534	0.7007	0.6917	0.6936
1982	0.5503	0.5289	0.3979	0.4094
1983	0.6068	0.6120	0.6355	0.6487
1984	0.5823	0.5585	0.5042	0.5159
1985	0.6327	0.6275	0.6135	0.6199
1986	0.7011	0.7343	0.8408	0.8448
1987	0.6164	0.6300	0.6432	0.6444
1988	0.5222	0.4957	0.4063	0.4146
1989	0.4467	0.4058	0.3341	0.3368
1990	0.4249	0.3685	0.2751	0.2785
1991	0.4747	0.4216	0.3541	0.3536
1992	0.5005	0.4287	0.2952	0.2993
1993	0.6027	0.5444	0.4560	0.4579
1994	0.7323	0.7078	0.6983	0.7038
1995	0.8154	0.7921	0.7493	0.7558
1996	0.8702	0.8633	0.8738	0.8777
1997	0.8697	0.8633	0.9091	0.9093
1998	0.8531	0.8431	0.8868	0.8882
1999	0.8512	0.8434	0.8734	0.8761
2000	0.8530	0.8507	0.8902	0.8871
2001	0.7833	0.7440	0.6230	0.6246
2002	0.7516	0.7036	0.6035	0.6017
2003	0.6652	0.5703	0.5079	0.5125
2004	0.7296	0.6769	0.5661	0.5652
2005	0.7184	0.6609	0.5301	0.5313
2006	0.6956	0.6220	0.4719	0.4704
2007	0.6827	0.5843	0.4360	0.4280

Table 9 Results for black sea bass analysis

Parameters and their priors	Analysis 1 POE_N 1974-2003	Analysis 2 POE_N 1978-2003	Analysis 3 POE_C 1974-2003	Analysis 4 POE_C 1978-2003
$r \sim U(0,2)$				
	0.6047	0.5980	0.4403	0.4081
$K \sim U(0,10^7)$				
	6.5138E+03	6.5170E+03	9.9842E+03	9.3613E+03
$a \sim U(-10,0)$				
	-0.3495	-0.2291	-0.3824	-0.3146
$\sigma_1 \sim U(0,4)$				
	0.1953	0.2018		
$\gamma_1 \sim U(0,4)$				
			0.1276	0.1360
$q \sim U(0,1)$				
	2.1939E-03	2.1614E-03	1.4955E-03	1.4348E-03
	2.5472E-03	2.5153E-03	1.6377E-03	1.5817E-03
	1.6124E-03	1.5840E-03	1.0992E-03	1.0466E-03
	5.4804E-03	5.4178E-03	3.6783E-03	3.5041E-03
	1.6446E-03	1.6215E-03	1.1474E-03	1.0909E-03
$\sigma_2 \sim U(0,4)$				
	0.3995	0.4044		
	0.5927	0.6011		
	0.1589	0.1599		
	0.3184	0.3225		
	0.0736	0.0605		
$\gamma_2 \sim U(0,4)$				
			0.3187	0.3206
			0.3993	0.4089
			0.1068	0.1113
			0.2150	0.2200
			0.0497	0.0428
$P \sim U(0,1)$				
1974	0.7356		0.7994	
1975	0.9062		0.8573	
1976	0.6884		0.7468	
1977	0.7571		0.8356	
1978	0.8779	0.8886	0.9266	0.9230
1979	0.7679	0.7727	0.8469	0.8612
1980	0.7962	0.8069	0.8876	0.9045
1981	0.8138	0.8228	0.9115	0.9236
1982	0.8258	0.8352	0.9128	0.9284
1983	0.8001	0.8165	0.9003	0.9197

1984	0.7350	0.7462	0.8106	0.8309
1985	0.7514	0.7605	0.8609	0.8716
1986	0.7170	0.7337	0.7937	0.8167
1987	0.7201	0.7406	0.7897	0.8161
1988	0.6479	0.6606	0.6997	0.7212
1989	0.4599	0.4619	0.4983	0.5104
1990	0.4367	0.4474	0.4656	0.4834
1991	0.3533	0.3571	0.3775	0.3885
1992	0.3259	0.3340	0.3522	0.3646
1993	0.2559	0.2603	0.2746	0.2837
1994	0.2278	0.2306	0.2510	0.2573
1995	0.1801	0.1785	0.2184	0.2198
1996	0.1924	0.1920	0.2227	0.2252
1997	0.2274	0.2313	0.2599	0.2657
1998	0.2401	0.2433	0.2781	0.2835
1999	0.2725	0.2788	0.3178	0.3248
2000	0.2188	0.2197	0.2450	0.2499
2001	0.2005	0.2006	0.2235	0.2278
2002	0.1714	0.1723	0.1919	0.1961
2003	0.1663	0.1678	0.1839	0.1876

Table 10 Performance of POE_N and POE_C in real fishery analysis

Parameters		Weakfish POE_N	Weakfish POE_C	Black sea bass POE_N	Black sea bass POE_C
r					
		0.0277	0.0917	0.0111	0.0759
K					
		0.0998	0.0345	0.0005	0.0644
σ_1					
		0.2943		0.0327	
γ_1					
			0.0234		0.0637
q					
		0.1791	0.0762	0.0149	0.0414
		0.1860	0.0775	0.0126	0.0348
		0.2390	0.1143	0.0178	0.0490
				0.0115	0.0485
				0.0141	0.0505
σ_2					
		0.1472		0.0122	
		0.0052		0.0141	
		0.0069		0.0063	
				0.0128	
				0.1954	
γ_2					
			0.0102		0.3197
			0.0009		0.4041
			0.0132		0.1091
					0.2175
					0.0463
P					
	1978			0.0121	0.0039
	1979			0.0062	0.0167
	1980			0.0133	0.0189
	1981	0.0699	0.0027	0.0110	0.0132
	1982	0.0397	0.0285	0.0113	0.0169
	1983	0.0085	0.0206	0.0203	0.0213
	1984	0.0417	0.0229	0.0151	0.0247
	1985	0.0083	0.0104	0.0120	0.0124
	1986	0.0463	0.0047	0.0230	0.0286
	1987	0.0218	0.0019	0.0281	0.0329
	1988	0.0521	0.0202	0.0194	0.0303
	1989	0.0960	0.0080	0.0043	0.0240
	1990	0.1422	0.0123	0.0242	0.0375
	1991	0.1185	0.0014	0.0107	0.0287
	1992	0.1545	0.0138	0.0245	0.0346
	1993	0.1016	0.0042	0.0170	0.0326
	1994	0.0340	0.0078	0.0122	0.0248
	1995	0.0290	0.0086	0.0089	0.0064

1996	0.0080	0.0045	0.0021	0.0112
1997	0.0074	0.0002	0.0170	0.0221
1998	0.0118	0.0016	0.0132	0.0192
1999	0.0092	0.0031	0.0229	0.0218
2000	0.0027	0.0035	0.0041	0.0198
2001	0.0515	0.0026	0.0005	0.0191
2002	0.0660	0.0030	0.0052	0.0216
2003	0.1536	0.0090	0.0090	0.0199
2004	0.0749	0.0016		
2005	0.0834	0.0023		
2006	0.1117	0.0032		
2007	0.1553	0.0185		

Table 11 Coefficient of variation (CV) of observation error from real fisheries analysis

	σ_1	CV_1		σ_2	CV_2
Weakfish analysis 1	0.1733	0.4554	MRFSS	0.3368	0.1498
			NJ	0.9727	0.2478
			DE	0.6365	0.1409
Weakfish analysis 2	0.2331	0.4985	MRFSS	0.2907	0.1293
			NJ	0.9677	0.2465
			DE	0.6321	0.1399
Blackseabass analysis 1	0.1953	0.2358	Hook & Line	0.3995	0.1762
			FL Snapper Trap	0.5927	0.2536
			Blackfish Trap	0.1589	0.0793
			Chevron Trap	0.3184	0.1547
			Headboat	0.0736	0.0500
Blackseabass analysis 2	0.2018	0.2236	Hook & Line	0.4044	0.1783
			FL Snapper Trap	0.6011	0.2572
			Blackfish Trap	0.1599	0.0798
			Chevron Trap	0.3225	0.1566
			Headboat	0.0605	0.0411

$$CV_1 = \frac{\sigma_1}{mean(\log(P))}$$

$$CV_2 = \frac{\sigma_2}{mean(\log(I))}$$

Table 12 True values for objectives 1-6

	Weakfish	Black sea bass
r	0.57	0.6
K	80000	6500
a	-0.45	-0.25
σ_1	0.1	0.2
q	0.002	0.0016
	0.0005	Na
	0.001	Na
	0.005	Na
	0.01	Na
σ_2	0.63	0.07
	0.4	Na
	0.5	Na
	0.6	Na
	0.7	Na
Outliers and atypical values in observation error	$\mu_{2,OA} = 0.3$ $\sigma_{2,OA} = 1.0$	Na
Outliers and atypical values in process error	$\mu_{1,OA} = 0.1$ $\sigma_{1,OA} = 0.2$	Na
Autocorrelation in observation error	$b_2 = 0.6$ $\rho_2 = 0.48$	Na
Autocorrelation in process error	$b_1 = 0.6$ $\rho_1 = 0.08$	Na

Table 13 Reference values for outliers and atypical values

	$Mean(e^{\varepsilon})$	CV_1 or CV_2
$\varepsilon_1 \sim N(0, \sigma_1^2)$	1.0050	0.2139
$\varepsilon_1 \sim N(0, \sigma_{1,OA}^2)$	1.1331	0.4277
$\varepsilon_1 \sim N(\mu_{1,OA}, \sigma_1^2)$	1.1107	0.2139
$\varepsilon_2 \sim N(0, \sigma_2^2)$	1.2195	0.1394
$\varepsilon_2 \sim N(0, \sigma_{2,OA}^2)$	1.6487	0.2213
$\varepsilon_2 \sim N(\mu_{2,OA}, \sigma_2^2)$	1.6462	0.1394

Table 14 True values and CVs for simulated multiple indices based on CPUEs

	q	σ_2	CV_2
1 CPUE, 3 CPUEs, 5 CPUEs	0.002	0.63	0.1416
3 CPUEs, 5 CPUEs	0.0005	0.4	0.1277
3 CPUEs	0.001	0.5	0.1307
3 CPUEs	0.005	0.6	0.1104
3 CPUEs, 5 CPUEs	0.01	0.7	0.1142

$$CV_2 = \frac{\sigma_2}{mean(\log(I))}$$

Table 15 Results for objective 1 and 3 --- simulation based on Atlantic weakfish

MPE of r= $Mean(\frac{\hat{r}_i - r_{true}}{r_{true}})$

MPE of K= $Mean(\frac{\hat{K}_i - K_{true}}{K_{true}})$

MPE of B/Bmsy= $Mean(\frac{\hat{d}_i - d_{i,true}}{d_{i,true}})$, d=Bt/Bmsy

MSPE of r= $Mean[(\frac{\hat{r}_i - r_{true}}{r_{true}})^2]$

MSPE of K= $Mean[(\frac{\hat{K}_i - K_{true}}{K_{true}})^2]$

MSPE of B/Bmsy= $Mean[(\frac{\hat{d}_i - d_{i,true}}{d_{i,true}})^2]$, d=Bt/Bmsy

MDPE of r= $Median(\frac{\hat{r}_i - r_{true}}{r_{true}})$

MDPE of K= $Median(\frac{\hat{K}_i - K_{true}}{K_{true}})$

MDPE of B/Bmsy= $Median(\frac{\hat{d}_i - d_{i,true}}{d_{i,true}})$, d=Bt/Bmsy

MDSPE of r= $Median[(\frac{\hat{r}_i - r_{true}}{r_{true}})^2]$

MDSPE of K= $Median[(\frac{\hat{K}_i - K_{true}}{K_{true}})^2]$

MDSPE of B/Bmsy= $Median[(\frac{\hat{d}_i - d_{i,true}}{d_{i,true}})^2]$, d=Bt/Bm

senarios	MPE r	MPE K	MPE MSY	MPE B/Bmsy	MSPE r	MSPE K	MSPE MSY	MSPE B/Bmsy
I.1	0.0256	0.0138	0.0588	-0.0684	0.0129	0.0378	0.0991	0.007
I.2	0.1375	0.0347	0.1999	-0.0693	0.0173	0.042	0.1347	0.0076
I.3	0.1969	0.0803	0.3178	-0.1216	0.016	0.0521	0.1691	0.0067
I.4	-0.0617	2.773	2.615	-0.2272	0.0016	5.2358	5.4313	0.0143
I.5	-0.0853	4.6569	4.2478	-0.2442	0.001	7.8306	7.6204	0.0147
I.6	-0.1217	9.2952	8.2133	-0.2802	0.0011	35.8111	32.9296	0.0135
II.1	-0.0004	0.0006	0.0243	-0.0772	0.0279	0.0314	0.1148	0.0003
II.2	0.0222	0.0036	0.043	-0.0782	0.017	0.0281	0.0843	0.0004
II.3	0.1803	0.0639	0.284	-0.1295	0.0285	0.0402	0.1682	0.0004
II.4	-0.0617	2.092	1.9669	-0.2295	0.0016	4.1685	4.245	0.0064
II.5	-0.0147	0.9104	0.9226	-0.2479	0.0022	1.0661	1.2476	0.0052
II.6	-0.128	3.9617	3.5103	-0.2816	0.0107	5.1741	6.011	0.006
III.1	0.0572	0.0034	0.0753	-0.0942	0.0105	0.032	0.0803	0.007
III.2	0.1324	0.0706	0.235	-0.0963	0.0156	0.0525	0.1452	0.0072
III.3	0.1377	0.0106	0.1674	-0.1468	0.0157	0.0304	0.0997	0.0068
III.4	-0.068	2.4142	2.2469	-0.2495	0.0018	3.6348	3.7301	0.012
III.5	-0.0918	3.2506	2.9007	-0.2678	0.0007	4.1608	3.8734	0.0131
III.6	-0.0774	2.6949	2.5623	-0.3012	0.0045	9.7726	10.0269	0.0108
senarios	MDPE r	MDPE K	MDPE MSY	MDPE B/Bmsy	MDSPE r	MDSPE K	MDSPE MSY	MDSPE B/Bmsy
I.1	0.0243	0.005	0.0494	-0.07	0.0057	0.0124	0.0387	0.0062
I.2	0.1342	0.0111	0.1681	-0.0679	0.018	0.011	0.0519	0.0058
I.3	0.194	0.0518	0.2469	-0.1235	0.0376	0.0175	0.061	0.0153
I.4	-0.0748	2.9148	2.6021	-0.2237	0.0056	8.4969	6.7716	0.05
I.5	-0.0915	4.8119	4.2462	-0.2313	0.0084	23.1546	18.0306	0.0535
I.6	-0.1281	7.5	6.3893	-0.278	0.0164	56.3556	40.9075	0.0773
II.1	0.0019	0.005	0.0099	-0.0767	0.0111	0.0124	0.0652	0.0059
II.2	0.0239	0.0103	0.0383	-0.0761	0.0087	0.0107	0.0506	0.0058
II.3	0.1794	0.0518	0.2393	-0.1268	0.0322	0.0175	0.0763	0.0161
II.4	-0.0771	2.034	1.7916	-0.2312	0.0059	4.1388	3.2114	0.0535
II.5	-0.0136	0.409	0.3866	-0.2367	0.0018	0.1765	0.1587	0.056
II.6	-0.1377	4.2821	3.434	-0.2774	0.019	18.3365	11.7925	0.0769
III.1	0.0535	0.0053	0.0575	-0.0947	0.0065	0.012	0.0409	0.0093
III.2	0.1284	0.0288	0.1691	-0.0945	0.0165	0.0303	0.0702	0.0093
III.3	0.1343	0.0215	0.1465	-0.1503	0.018	0.011	0.0504	0.0226
III.4	-0.0693	2.3508	2.0801	-0.2346	0.0048	5.5409	4.3394	0.055
III.5	-0.0936	3.4484	3.0288	-0.2464	0.0088	11.8918	9.1746	0.0607
III.6	-0.0716	1.4163	1.2715	-0.2807	0.0051	2.0266	1.6355	0.0788

Table 16 Results for objective 1 and 3 --- simulation based on black sea bass

senarios	MPE r	MPE K	MPE MSY	MPE B/Bmsy	MSPE r	MSPE K	MSPE MSY	MSPE B/Bmsy
I.1	0.0256	0.0006	0.0006	-0.0549	0.0129	0.0314	0.01	0.0243
I.2	0.1375	0.021	0.021	-0.0558	0.0173	0.0342	0.0102	0.1342
I.3	0.1969	0.0639	0.0639	-0.1084	0.016	0.0402	0.0091	0.194
I.4	-0.0617	2.6108	2.6108	-0.2104	0.0016	4.2611	0.0138	-0.0771
I.5	-0.0853	4.5026	4.5026	-0.229	0.001	6.9445	0.0125	-0.0926
I.6	-0.1217	8.8571	8.8571	-0.2638	0.0011	28.3862	0.0124	-0.1298
III.1	0.0572	0.0034	0.0034	-0.0654	0.0105	0.032	0.0186	0.0535
III.2	0.1324	0.0706	0.0706	-0.0664	0.0156	0.0525	0.0187	0.1284
III.3	0.1377	0.0106	0.0106	-0.1183	0.0157	0.0304	0.0169	0.1343
III.4	-0.068	2.4142	2.4142	-0.2186	0.0018	3.6348	0.021	-0.0693
III.5	-0.0918	3.2506	3.2506	-0.2375	0.0007	4.1608	0.0186	-0.0936
III.6	-0.0774	2.6949	2.6949	-0.2714	0.0045	9.7726	0.0187	-0.0716
senarios	MDPE r	MDPE K	MDPE MSY	MDPE B/Bmsy	MDSPE r	MDSPE K	MDSPE MSY	MDSPE B/Bmsy
I.1	0.0256	0.0006	0.0006	-0.0549	0.0129	0.0314	0.01	0.0243
I.2	0.1375	0.021	0.021	-0.0558	0.0173	0.0342	0.0102	0.1342
I.3	0.1969	0.0639	0.0639	-0.1084	0.016	0.0402	0.0091	0.194
I.4	-0.0617	2.6108	2.6108	-0.2104	0.0016	4.2611	0.0138	-0.0771
I.5	-0.0853	4.5026	4.5026	-0.229	0.001	6.9445	0.0125	-0.0926
I.6	-0.1217	8.8571	8.8571	-0.2638	0.0011	28.3862	0.0124	-0.1298
III.1	0.0572	0.0034	0.0034	-0.0654	0.0105	0.032	0.0186	0.0535
III.2	0.1324	0.0706	0.0706	-0.0664	0.0156	0.0525	0.0187	0.1284
III.3	0.1377	0.0106	0.0106	-0.1183	0.0157	0.0304	0.0169	0.1343
III.4	-0.068	2.4142	2.4142	-0.2186	0.0018	3.6348	0.021	-0.0693
III.5	-0.0918	3.2506	3.2506	-0.2375	0.0007	4.1608	0.0186	-0.0936
III.6	-0.0774	2.6949	2.6949	-0.2714	0.0045	9.7726	0.0187	-0.0716

Table 17 Results for objective 2 and 4 --- simulation based on Atlantic weakfish

senarios	MPE r	MPE K	MPE MSY	MPE B/Bmsy	MSPE r	MSPE K	MSPE MSY	MSPE B/Bmsy
IV.1	0.0034	0.5867	0.7233	-0.0730	0.0573	1.0944	1.893	0.0107
IV.2	0.0053	2.9515	3.1585	-0.0735	0.1004	0.4868	3.0161	0.0102
IV.3	-0.1256	14.0099	12.6311	-0.2388	0.0038	111.5082	108.4649	0.0125
IV.4	-0.1185	24.343	22.1109	-0.2366	0.0144	61.9104	88.3077	0.0289
V.1	-0.0858	2.4349	2.8593	-0.0958	0.0749	17.7201	29.2932	0.0133
V.2	0.0334	2.3980	4.4593	-0.0957	0.3303	28.0193	95.5335	0.0134
V.3	-0.1479	19.6328	17.0775	-0.2379	0.0022	143.2332	125.2923	0.0145
V.4	-0.1518	30.1079	26.4873	-0.2349	0.0041	385.7871	355.9916	0.0281
VI.1	-0.0376	1.5401	1.9018	-0.0738	0.0691	7.9867	14.4979	0.0095
VI.2	-0.0396	33.779	32.9985	-0.0762	0.0128	79.6155	123.6097	0.0099
VI.3	-0.1367	13.1467	11.7205	-0.2499	0.0035	95.5134	91.0232	0.0131
VI.4	-0.1377	23.2092	22.8581	-0.2483	0.0286	433.6112	605.6389	0.0225
VII.1	-0.1576	1.5041	1.5652	-0.1123	0.07	4.4017	7.8595	0.0087
VII.2	-0.1575	3.2678	2.7916	-0.1118	0.0629	1.3532	3.1743	0.0114
VII.3	-0.1570	12.7086	11.0595	-0.2899	0.0042	94.3859	85.3417	0.012
VII.4	-0.1566	12.8809	12.3079	-0.2893	0.0206	213.0193	231.5217	0.025
VIII.1	0.2721	-0.1133	0.1711	-0.0259	0.0835	0.1050	0.3699	0.1324
VIII.2	0.2733	0.1874	0.6591	-0.0252	0.1097	1.2209	3.0943	0.1339
VIII.3	-0.1951	1.0463	0.7320	-0.0890	0.0342	0.5280	0.8789	0.1199
VIII.4	-0.1805	0.8179	0.7869	-0.0818	0.1846	1.0941	3.0045	0.1541
IX.1	0.1097	0.0607	0.2331	-0.0949	0.0717	0.1075	0.3815	0.0154
IX.2	0.1112	0.0575	0.2746	-0.0921	0.1697	0.1431	0.6885	0.0285
IX.3	-0.2980	1.0327	0.5157	-0.1453	0.0323	0.5202	0.7634	0.0159
IX.4	-0.2957	0.7736	0.4972	-0.1443	0.0836	1.7831	2.5631	0.0195
X.1	0.2436	-0.1035	0.1429	-0.0771	0.0588	0.0319	0.1719	0.0164
X.2	0.2464	-0.1231	0.2045	-0.0804	0.1811	0.1648	0.7519	0.0129
X.3	-0.2360	0.8681	0.5002	-0.1435	0.0315	0.3499	0.6189	0.016
X.4	-0.2278	2.0242	1.5901	-0.1427	0.0465	4.8024	5.5263	0.0196
XI.1	0.0606	0.1444	0.2642	-0.0898	0.0663	0.0986	0.3648	0.0023
XI.2	0.0608	-0.0079	0.1719	-0.0952	0.0743	0.5082	1.0878	0.0056
XI.3	-0.2468	0.7708	0.4088	-0.1547	0.0391	0.2781	0.5712	0.0028
XI.4	-0.1862	0.7754	0.7646	-0.1539	0.1421	1.8027	3.4828	0.0166

senarios	MDPE r	MDPE K	MDPE MSY	MDPE B/Bmsy	MDSPE r	MDSPE K	MDSPE MSY	MDSPE B/Bmsy
IV.1	0.0151	0.1830	0.2129	-0.0801	0.0377	0.0441	0.1924	0.0095
IV.2	0.0209	3.2278	3.4447	-0.0785	0.0661	10.4191	11.8672	0.0089
IV.3	-0.1270	10.5905	9.2279	-0.2473	0.0161	112.1689	85.164	0.0612
IV.4	-0.1157	24.0318	21.4715	-0.2316	0.0146	577.5251	461.0252	0.0564
V.1	-0.0353	0.3261	0.3407	-0.1095	0.0362	0.1067	0.3415	0.0136
V.2	0.0595	-0.0822	0.0785	-0.1097	0.2654	0.3003	0.7595	0.0137
V.3	-0.1459	19.1629	16.1733	-0.2474	0.0213	367.2173	261.5757	0.0612
V.4	-0.1524	30.3798	25.6125	-0.2432	0.0232	922.9454	656.0353	0.0591
VI.1	-0.0410	0.2764	0.2502	-0.0670	0.0523	0.0774	0.2935	0.0066
VI.2	-0.0410	35.9448	34.7545	-0.0742	0.0076	1292.029	1207.991	0.0065
VI.3	-0.1362	10.7936	9.1889	-0.2509	0.0185	116.5018	84.4351	0.0630
VI.4	-0.1362	19.0071	16.2894	-0.2470	0.0236	361.3032	265.3722	0.0610
VII.1	-0.1638	0.5463	0.2834	-0.1138	0.0460	0.2985	0.3861	0.0138
VII.2	-0.1634	3.7698	2.9171	-0.1029	0.0416	14.2115	8.5102	0.0121
VII.3	-0.1572	10.3269	8.5741	-0.2706	0.0247	106.6499	73.5202	0.0732
VII.4	-0.1572	7.4088	6.1330	-0.2699	0.0247	54.9034	37.6264	0.0729
VIII.1	0.3746	-0.1808	-0.0046	-0.1067	0.1404	0.0913	0.1370	0.0238
VIII.2	0.3908	-0.2205	-0.1816	-0.1073	0.1639	0.2432	0.3092	0.0240
VIII.3	-0.1943	0.9102	0.5171	-0.1576	0.0378	0.8285	0.2676	0.0405
VIII.4	-0.1704	0.6220	0.2921	-0.1480	0.1020	0.3869	0.5817	0.0437
IX.1	0.1069	-0.0049	0.0963	-0.1170	0.0456	0.0231	0.1345	0.0162
IX.2	0.1069	-0.0181	0.0834	-0.1006	0.1188	0.0340	0.2793	0.0178
IX.3	-0.3070	0.8936	0.2184	-0.1616	0.0943	0.7985	0.1347	0.0279
IX.4	-0.3128	0.5662	-0.2058	-0.1550	0.1091	0.3206	0.4409	0.0267
X.1	0.2399	-0.139	0.0675	-0.0804	0.0575	0.022	0.0771	0.0112
X.2	0.2399	-0.2038	-0.0228	-0.0776	0.1267	0.0785	0.2695	0.0086
X.3	-0.2323	0.7545	0.3013	-0.1467	0.0540	0.5693	0.1213	0.0236
X.4	-0.2323	1.6709	0.7382	-0.1494	0.0540	2.792	0.5451	0.0240
XI.1	0.0576	0.0838	0.1174	-0.0919	0.0422	0.0241	0.1298	0.0085
XI.2	0.0576	-0.1294	-0.2288	-0.0919	0.0469	0.1681	0.3658	0.0085
XI.3	-0.2444	0.6680	0.1844	-0.1552	0.0640	0.4462	0.0991	0.0241
XI.4	-0.1845	0.5600	0.0160	-0.1551	0.1079	0.3136	0.5495	0.0241

Table 18 Results for objective 5 --- simulation based on Atlantic weakfish

senarios	MPE r	MPE K	MPE MSY	MPE B/Bmsy	MSPE r	MSPE K	MSPE MSY	MSPE B/Bmsy
XII.1	0.168	-0.1104	0.0635	-0.0317	0.0457	0.0214	0.1226	0.0533
XII.2	0.1703	-0.0476	0.1432	-0.0227	0.1084	0.0107	0.1763	0.088
XII.3	-0.2005	0.5658	0.3154	-0.0924	0.0308	0.1947	0.4126	0.0486
XII.4	-0.2044	0.4471	0.3992	-0.089	0.1121	0.8149	1.7743	0.0737
XIII.1	0.2177	-0.112	0.111	-0.0532	0.0338	0.0469	0.168	0.0726
XIII.2	0.2206	-0.1373	0.1049	-0.0534	0.061	0.0795	0.2915	0.0717
XIII.3	0.1323	15.7837	18.6369	-0.2146	0.0123	47.6724	89.0833	0.0619
XIII.4	0.1299	18.6966	23.0291	-0.2183	0.0279	153.1262	310.7878	0.0474
XIV.1	0.2365	-0.1617	0.0655	-0.0777	0.0387	0.0364	0.1516	0.0501
XIV.2	0.2707	-0.1599	0.1447	-0.0843	0.2818	0.0344	0.4471	0.0456
XIV.3	-0.0623	16.0754	15.9105	-0.2311	0.0149	71.7707	94.5617	0.0531
XIV.4	-0.0556	24.1129	25.2145	-0.2563	0.0379	219.1789	364.6736	0.2085
senarios	MDPE r	MDPE K	MDPE MSY	MDPE B/Bmsy	MDSPE r	MDSPE K	MDSPE MSY	MDSPE B/Bmsy
XII.1	0.1836	-0.1305	0.0304	-0.0789	0.0411	0.0194	0.0542	0.0173
XII.2	0.1943	-0.0555	0.1454	-0.0620	0.0704	0.0051	0.1027	0.0213
XII.3	-0.1964	0.5125	0.2013	-0.1303	0.0396	0.2626	0.1289	0.0258
XII.4	-0.1957	0.3487	0.0399	-0.1345	0.0765	0.3207	0.4874	0.0293
XIII.1	0.2521	-0.188	0.0352	-0.0756	0.0648	0.0458	0.0697	0.0161
XIII.2	0.2668	-0.2363	-0.0092	-0.0722	0.0918	0.0765	0.1276	0.0164
XIII.3	0.1272	16.5682	18.9344	-0.2482	0.0169	274.5069	358.5116	0.0621
XIII.4	0.1224	16.7941	18.8104	-0.2382	0.0187	282.0425	353.8302	0.0593
XIV.1	0.2885	-0.2103	0.0237	-0.1136	0.0833	0.0563	0.0678	0.0229
XIV.2	0.4012	-0.2072	0.1277	-0.1342	0.2671	0.0542	0.2469	0.0223
XIV.3	-0.0644	17.172	16.2381	-0.2490	0.0059	294.8766	263.6783	0.0640
XIV.4	-0.0560	23.3679	21.9949	-0.4000	0.0124	546.1193	483.8898	0.2059

Table 19 Results for objective 6--- simulation based on Atlantic weakfish

senarios	MPE r	MPE K	MPE MSY	MPE B/Bmsy	MSPE r	MSPE K	MSPE MSY	MSPE B/Bmsy
I.1	0.0256	0.0138	0.0588	-0.0684	0.0129	0.0378	0.0991	0.007
I.2	0.1375	0.0347	0.1999	-0.0693	0.0173	0.042	0.1347	0.0076
XV.1	0.0165	-0.0154	0.0042	-0.0558	0.0035	0.0044	0.0148	0.0092
XV.2	0.0201	-0.0425	-0.0118	-0.0687	0.0088	0.0288	0.0659	0.0087
XV.3	0.0148	0.0265	0.0426	-0.063	0.0004	0.0043	0.0066	0.008
XV.4	0.0296	0.0531	0.0876	-0.0529	0.0015	0.0171	0.0277	0.0082
senarios	MDPE r	MDPE K	MDPE MSY	MDPE B/Bmsy	MDSPE r	MDSPE K	MDSPE MSY	MDSPE B/Bmsy
I.1	0.0243	0.005	0.0494	-0.07	0.0077	0.0154	0.0443	0.0067
I.2	0.1342	0.0111	0.1681	-0.0679	0.018	0.0159	0.0599	0.0069
XV.1	0.014	-0.0214	0.0018	-0.0453	0.0026	0.0036	0.0084	0.0046
XV.2	0.016	-0.0796	-0.0505	-0.0569	0.0064	0.0128	0.0234	0.0052
XV.3	0.0177	-0.0069	0.0127	-0.0608	0.0004	0.0005	0.0014	0.0055
XV.4	0.0355	-0.0137	0.0251	-0.0508	0.0016	0.0021	0.0056	0.0049

8 List of Figures

Figure 1: Surplus production curve when Schaefer model is used

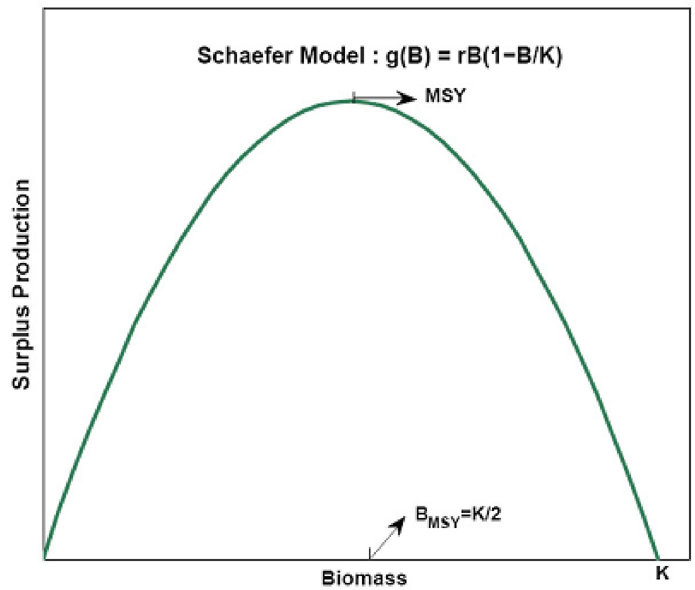


Figure 2: Comparison between fat-tailed distribution (blue curve) and Normal distribution (red curve)

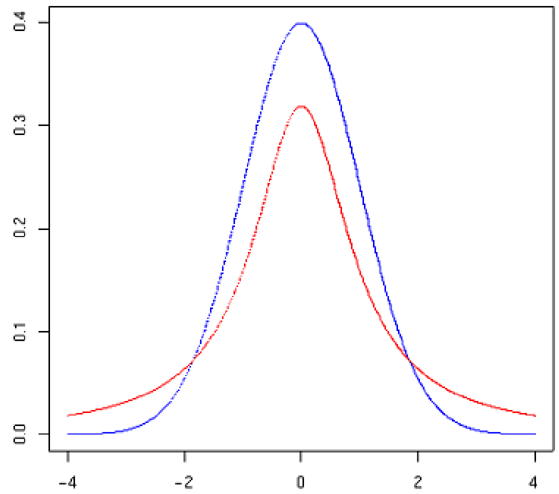


Figure 3: Commercial and recreational landings of Atlantic weakfish. The commercial and recreational landing data were from Northeast Fisheries Science Center (2009).

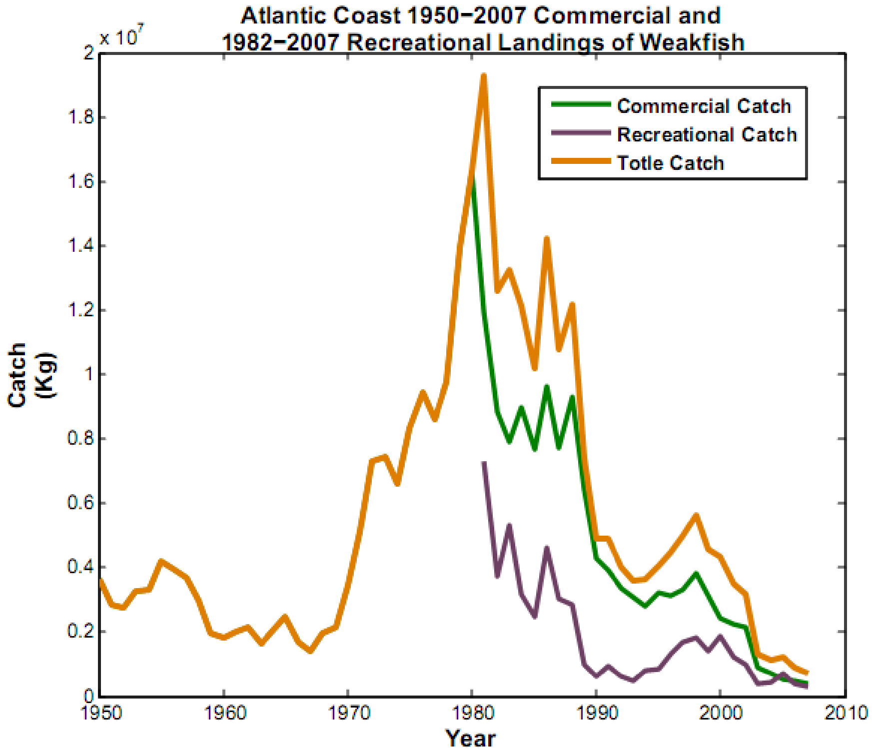


Figure 4: Standardized CPUE from 8 surveys for Atlantic weakfish. The catch-per-unit-effort data were from Northeast Fisheries Science Center (2009).

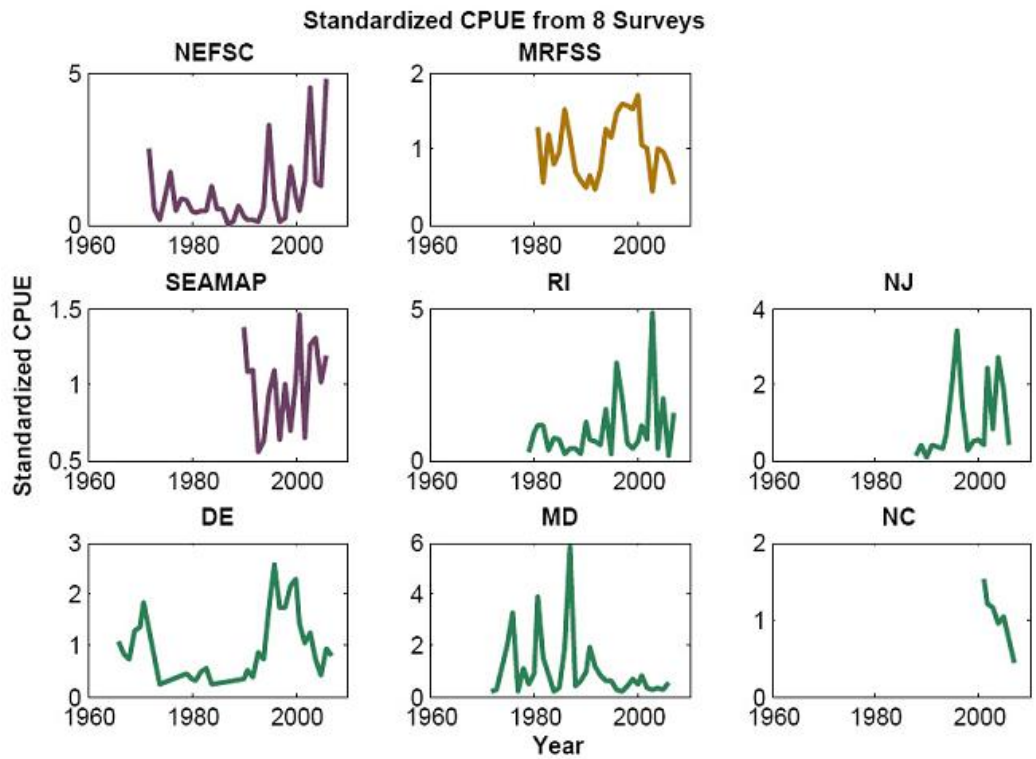


Figure 5: Commercial and recreational landings of black sea bass. The black sea bass commercial and recreational landings data were from the 2006 report of black sea bass population assessment (SEDAR 2006).

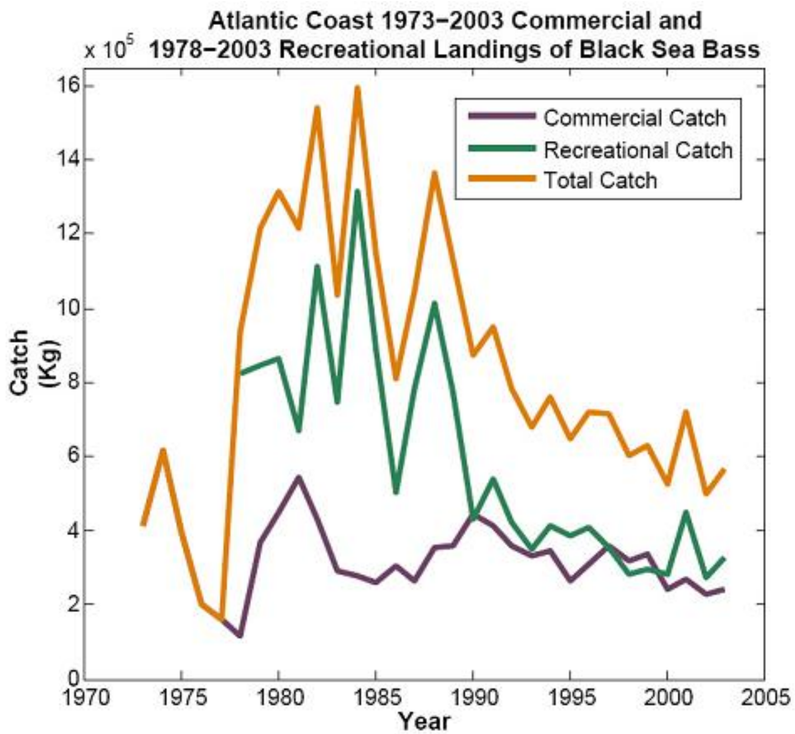


Figure 6: Standardized CPUE from 5 surveys for black sea bass. The black sea bass catch-per-unit-effort data were from the 2006 report of black sea bass population assessment (SEDAR 2006).

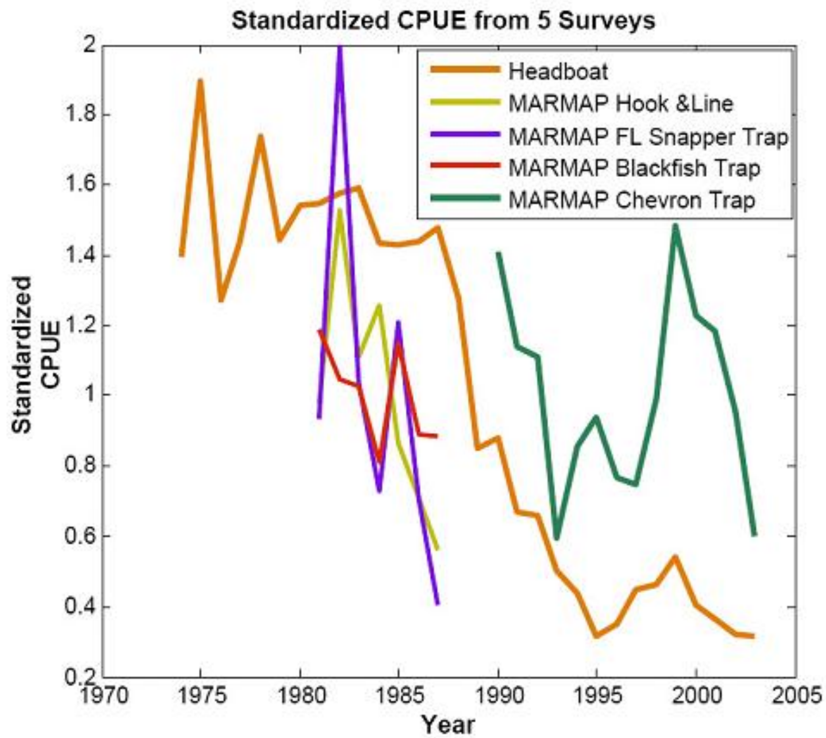


Figure 7: Procedure of the simulation study

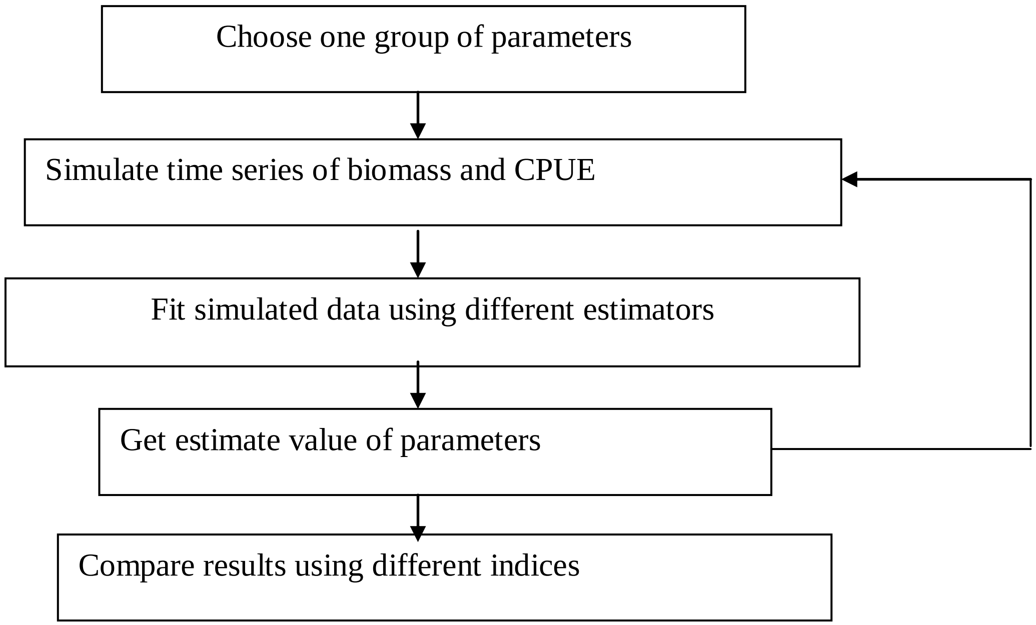


Figure 8: Scenarios of data simulation and estimation models

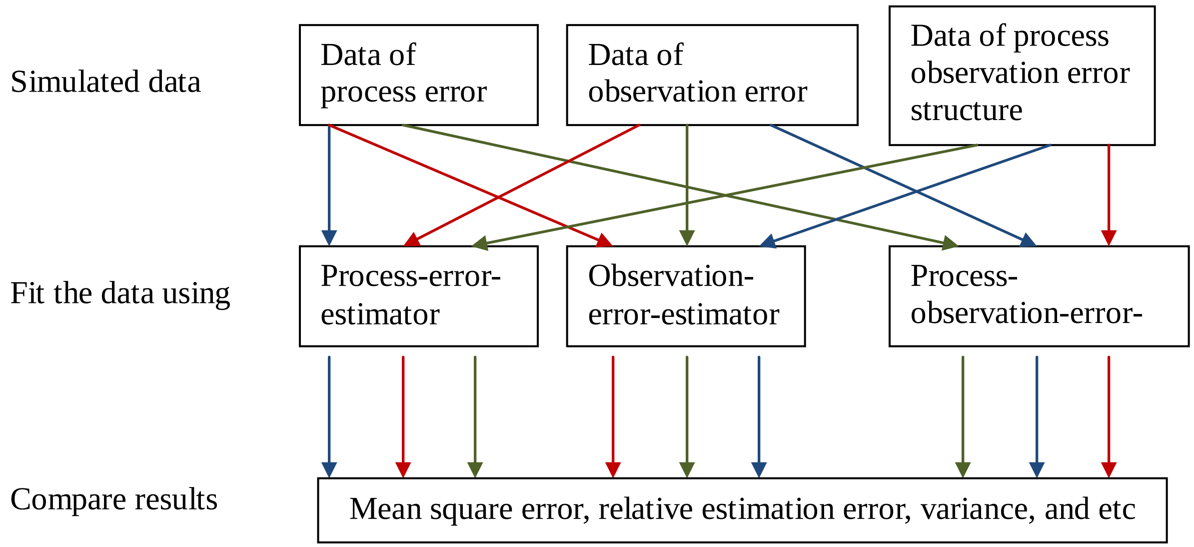


Figure 9: Atlantic weakfish parameters' posterior distributions: r, K, a, σ_1.For all figures on Weakfish, red line represent the results for process-observation-error-estimator with normal distribution (POE_N) fit data from 1966-2007 (Atlantic weakfish analysis 1); pink line represent the results for POE_N fit data from 1981-2007 (Atlantic weakfish analysis 2); blue line represent the results for process-observation-error-estimator with Cauchy distribution (POE_C) fit data from 1966-2007 (Atlantic weakfish analysis 3); light blue line represnet the results for POE_C fit the data from 1981-2007 (Atlantic weakfish analysis 4). The dashed lines represent the posterior mean of the parameter estimation. The mean values are listed in the legend.

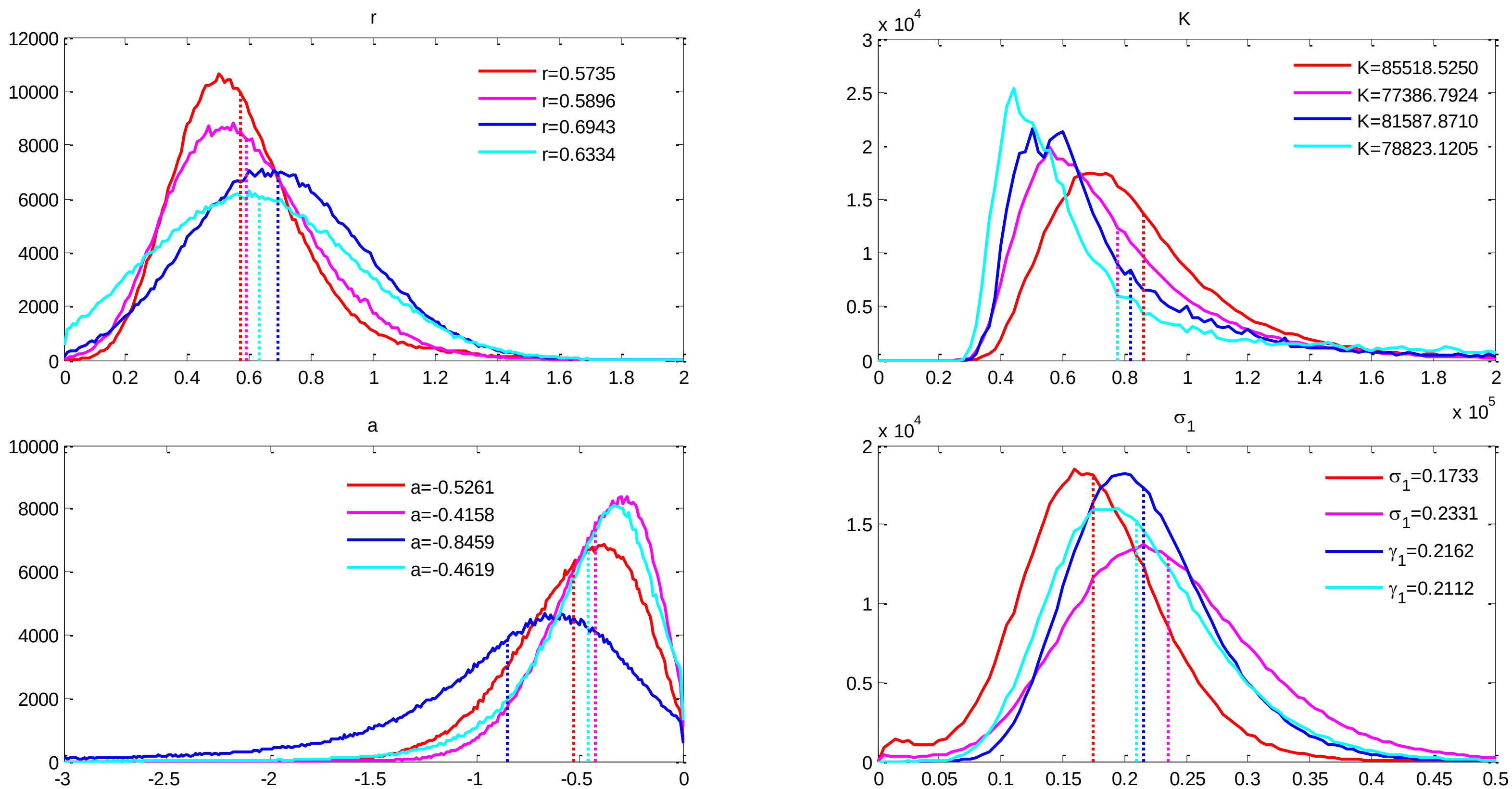


Figure 10: Atlantic weakfish parameters' posterior distributions: q, σ_2, γ_2. The dashed lines represent the posterior mean of the parameter estimation. The mean values are listed in the legend. Three rows list the parameters values for three CPUE

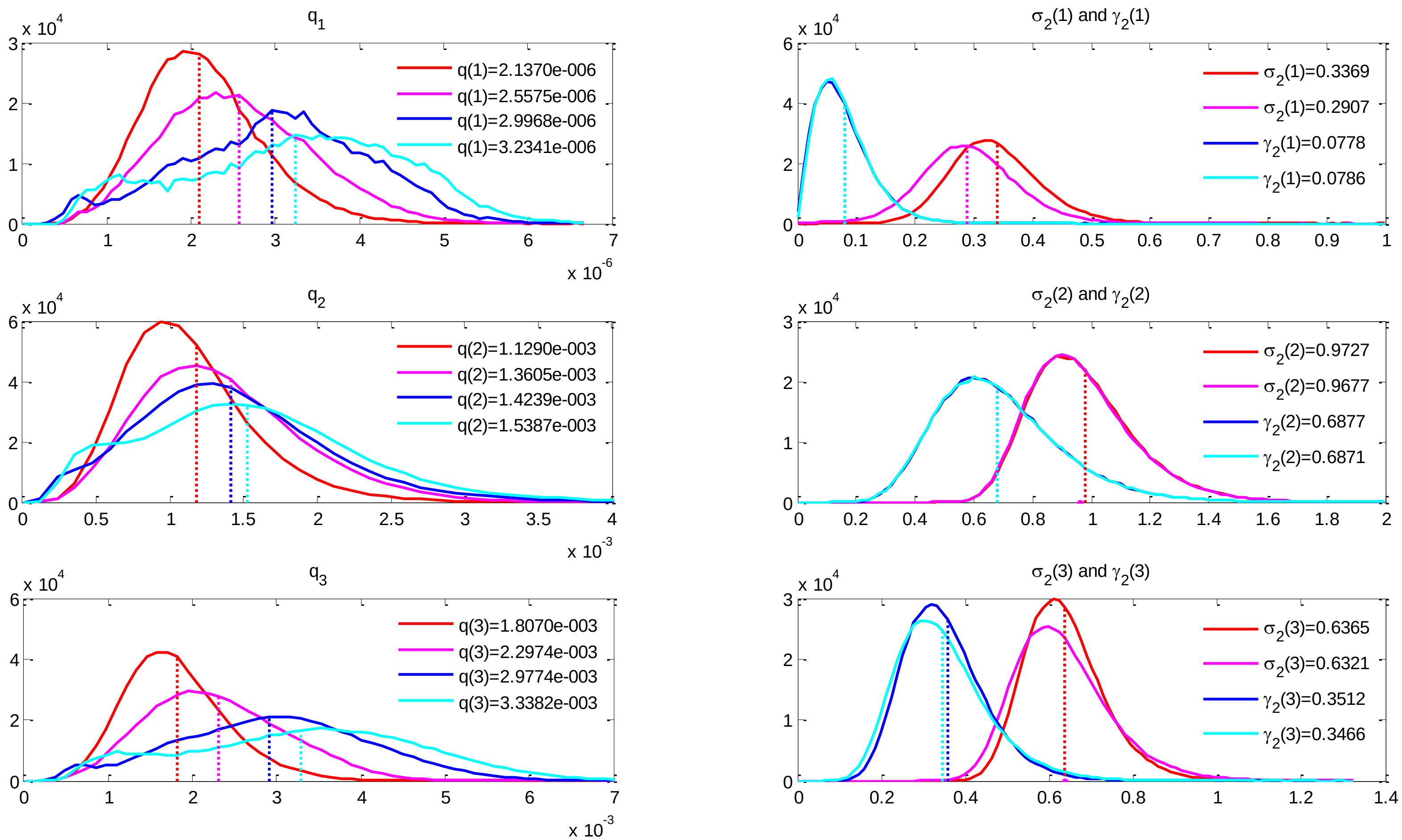


Figure 11: Atlantic weakfish parameters' posterior distributions: B_0, B_T. The dashed lines represent the posterior mean of the parameter estimation. The mean values are listed in the legend.

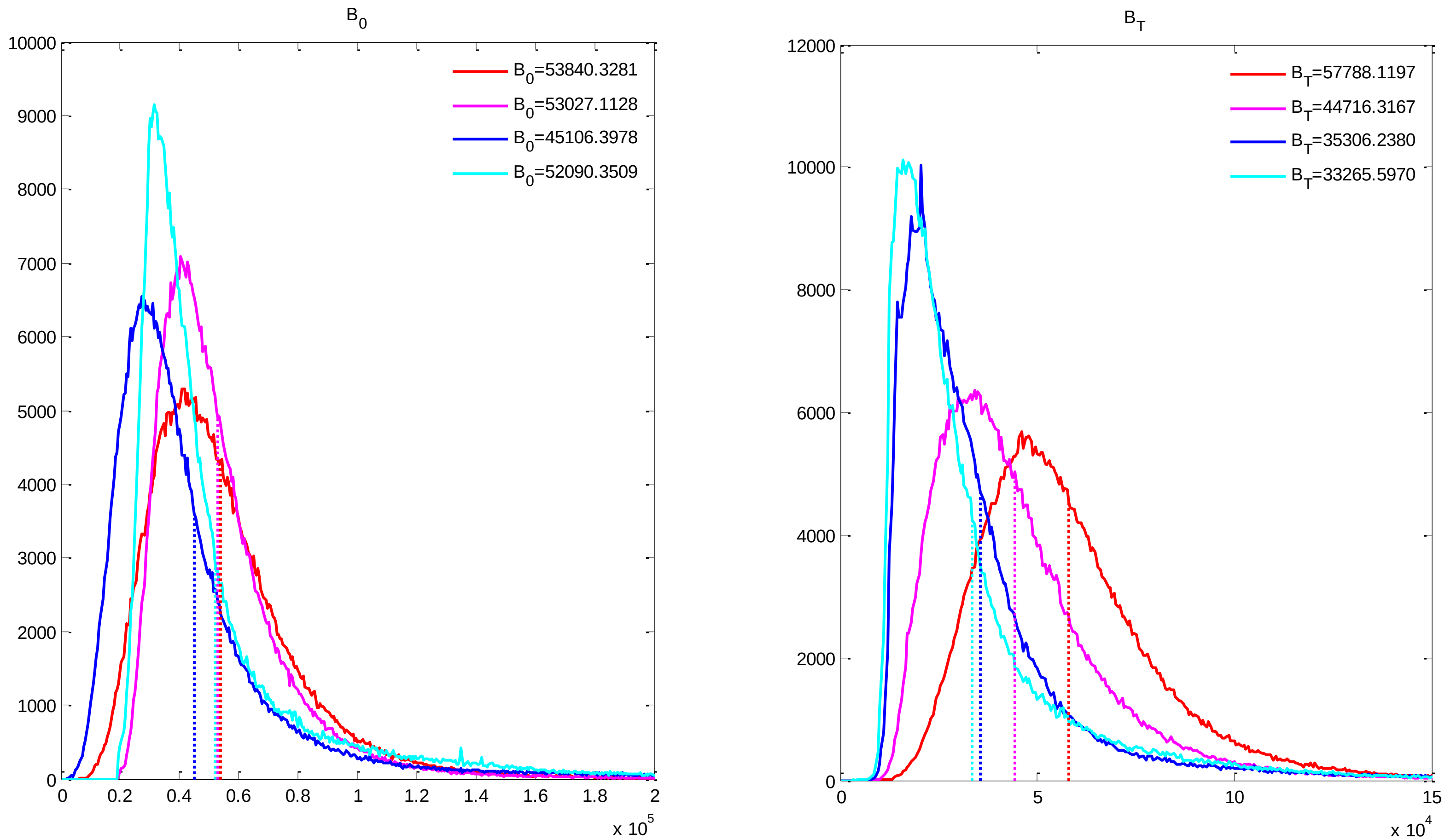


Figure 12: Atlantic weakfish values of management interest: B_T / B_{MSY} and F_T / F_{MSY}. The dashed lines represent the posterior mean of the parameter estimation. The mean values are listed in the legend.

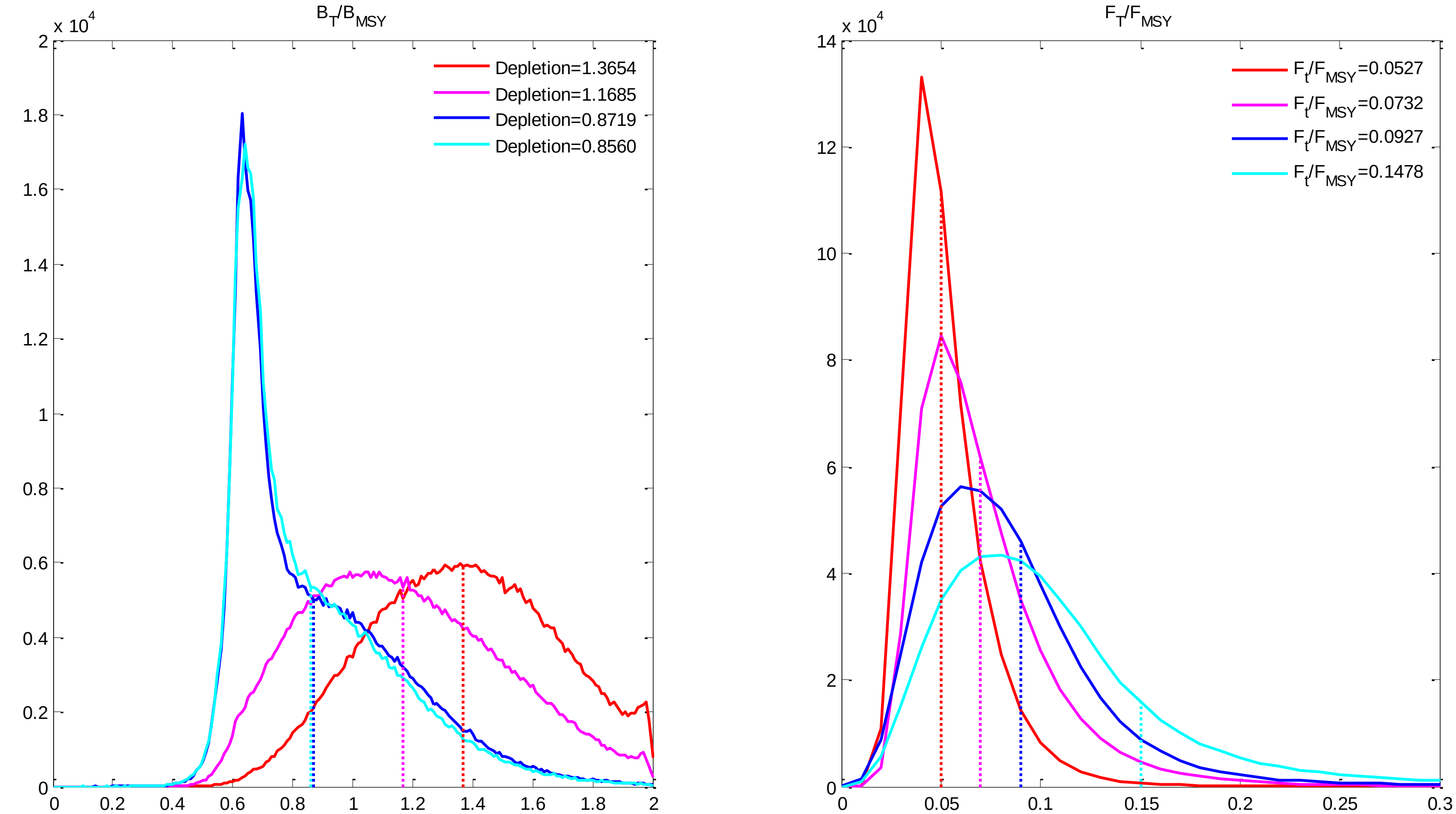


Figure 13: Atlantic weakfish population biomass estimation from 1966 to 2007. Solid lines represented posterior mean of the relative population biomass or population biomass estimation. Dashed lines are values corresponding to 5% (below solid lines) and 95 % (above solid lines) in the cumulative density function of posterior distribution.

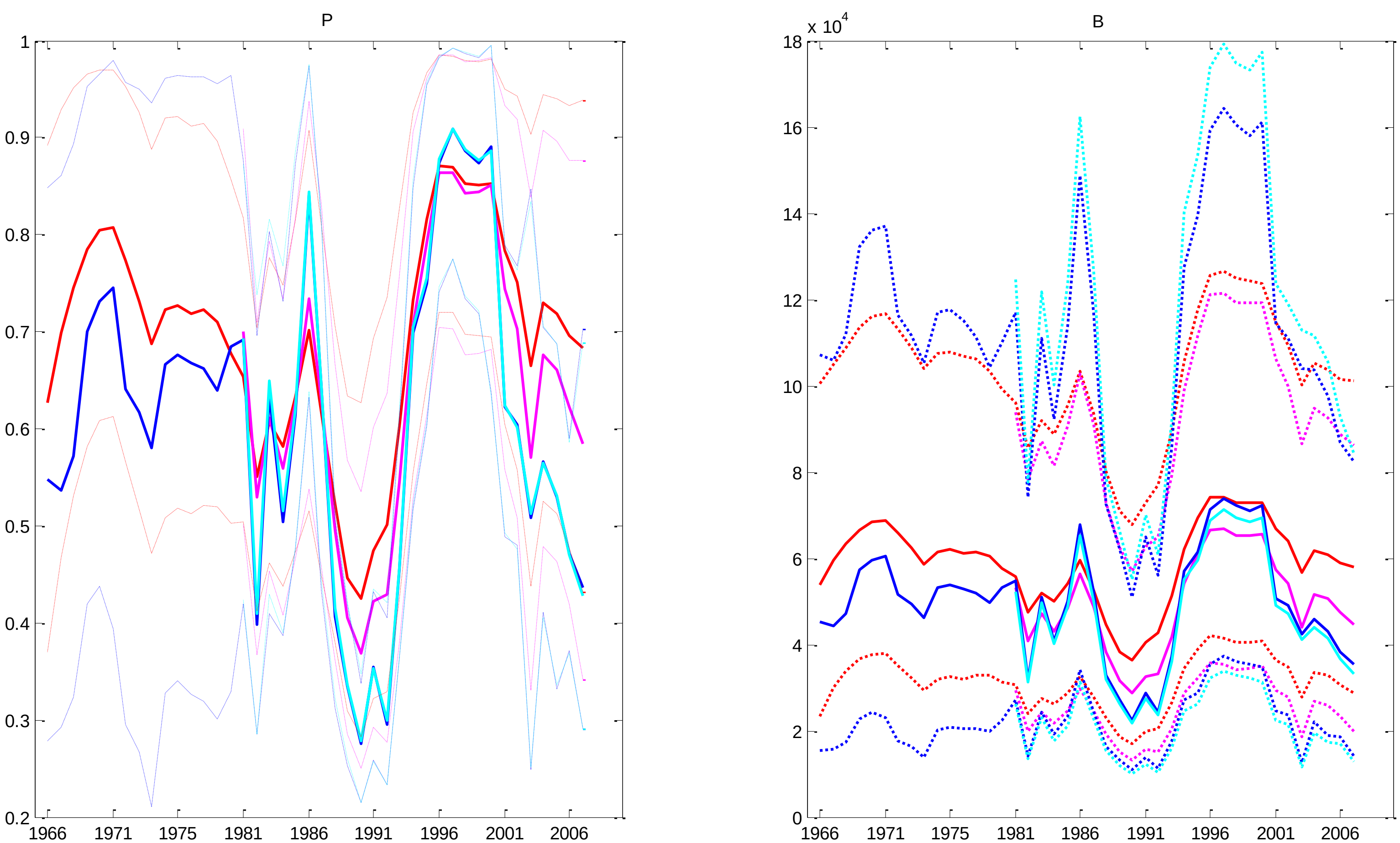


Figure 14: Error distributions of Atlantic weakfish estimated by POE_N and POE_C. Left column is the estimated process error distribution for Atlantic weakfish. Right column is the estimated observation error distribution for Atlantic weakfish. The values of parameters $\sigma_1, \sigma_2, \gamma_1, \gamma_2$ used are mean values of their posterior distributions.

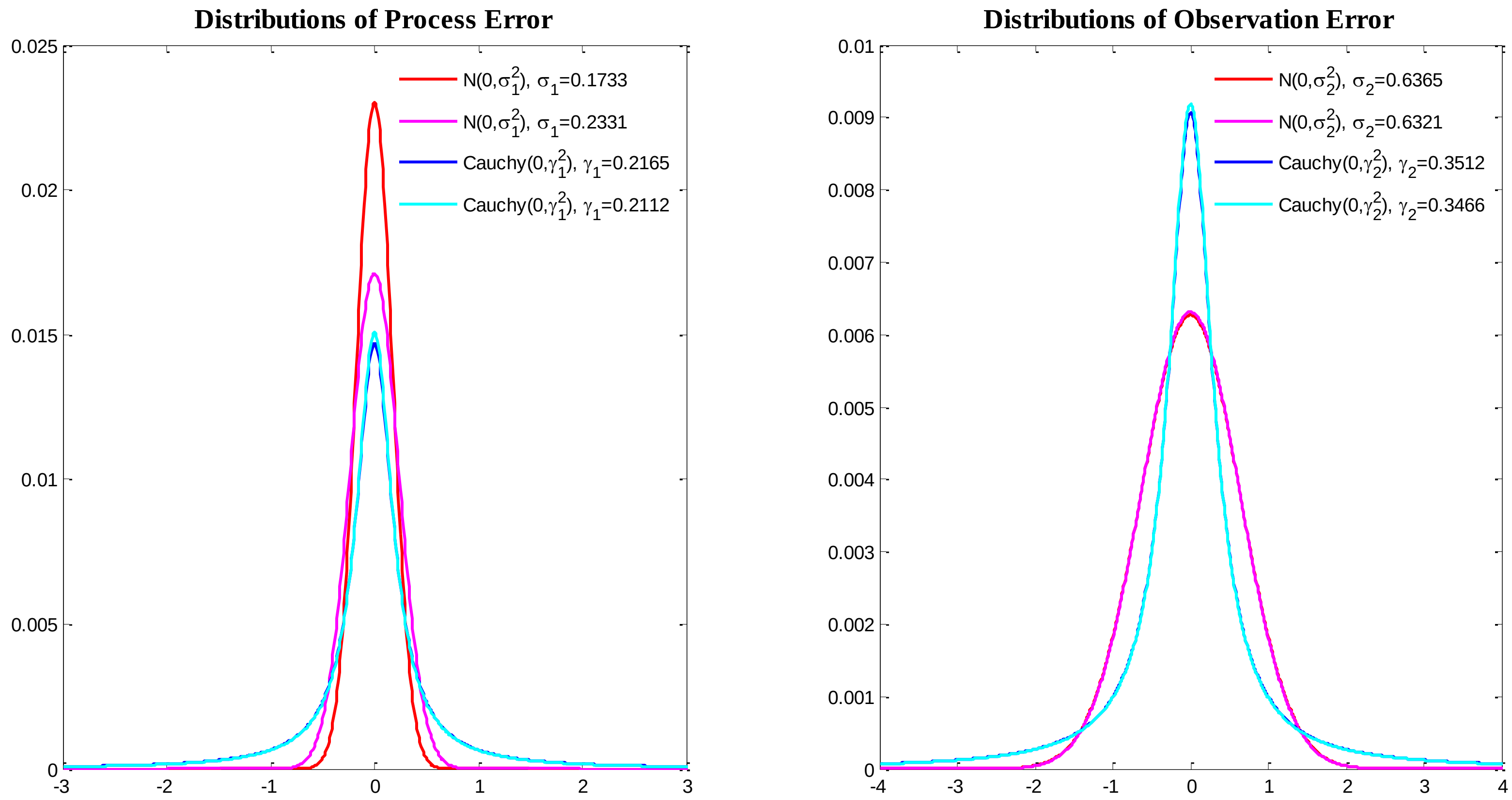


Figure 15: Atlantic weakfish joint posterior distribution of r and K

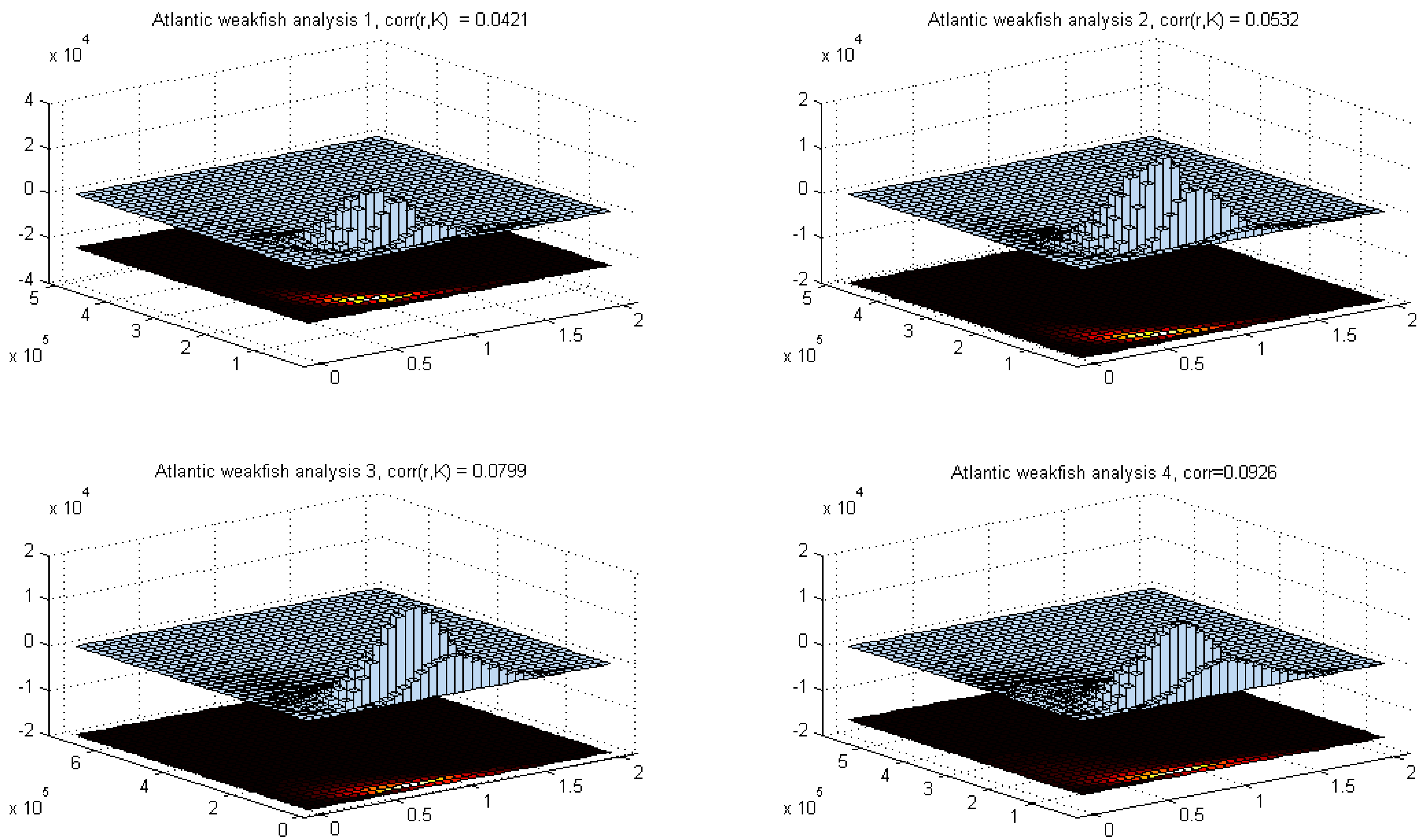


Figure 16: Black sea bass parameter estimation: r, K, a, σ_1 . For all figures on black sea bass, red line represent the results for POE_N fit data from 1974-2003 (black sea bass analysis 1); pink line represent the results for POE_N fit data from 1978-2003 (black sea bass analysis 2); blue line represent the results for POE_C fit data from 1974-2003 (black sea bass analysis 3); light blue line represent the results for POE_C fit the data from 1978-2003 (black sea bass analysis 4). The dashed lines represent the posterior mean of the parameter estimation. The mean values are listed in the legend.

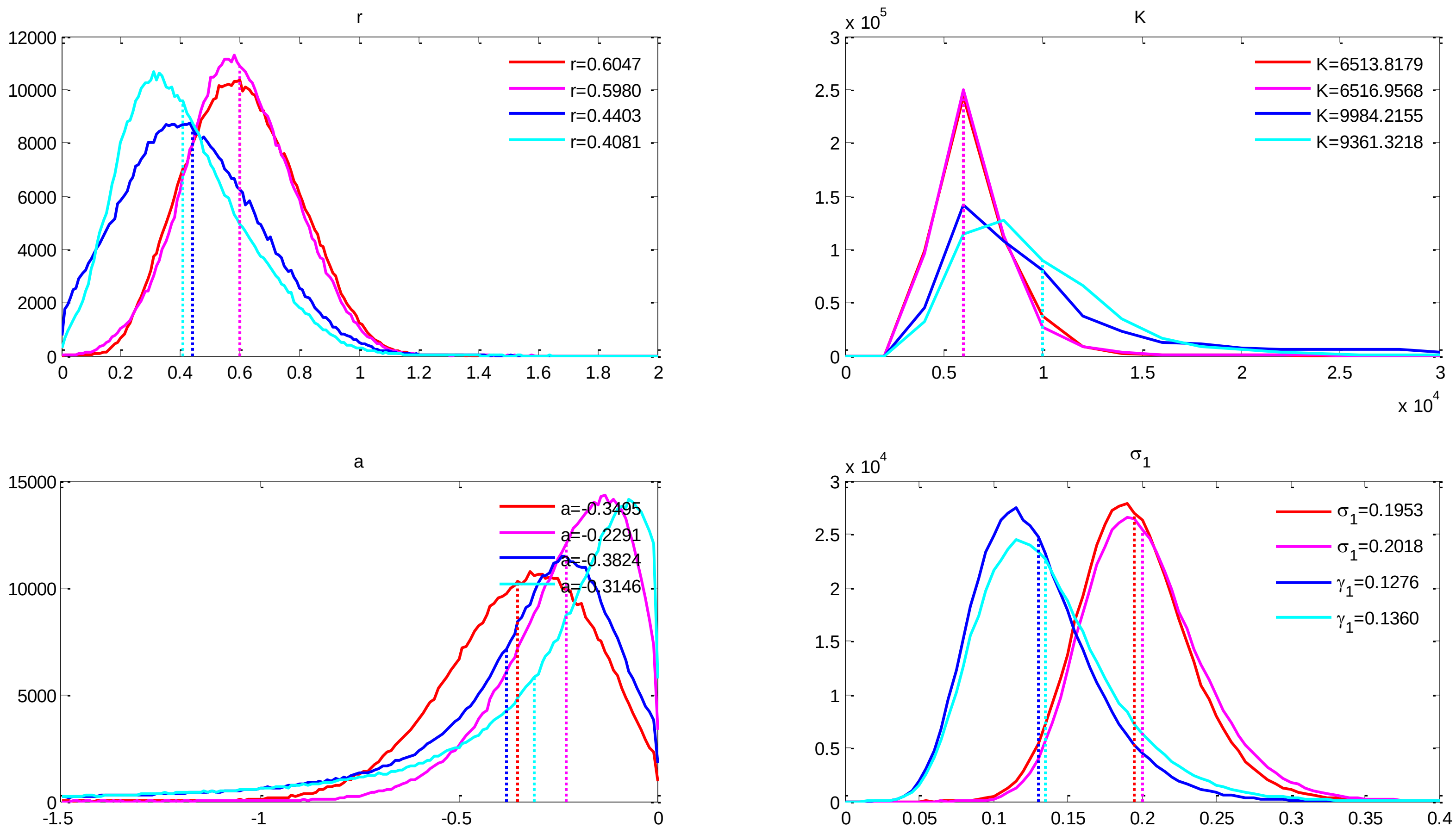


Figure 17: Black sea bass parameter estimation: q, σ_2, γ_2. The dashed lines represent the posterior mean of the parameter estimation. The mean values are listed in the legend.

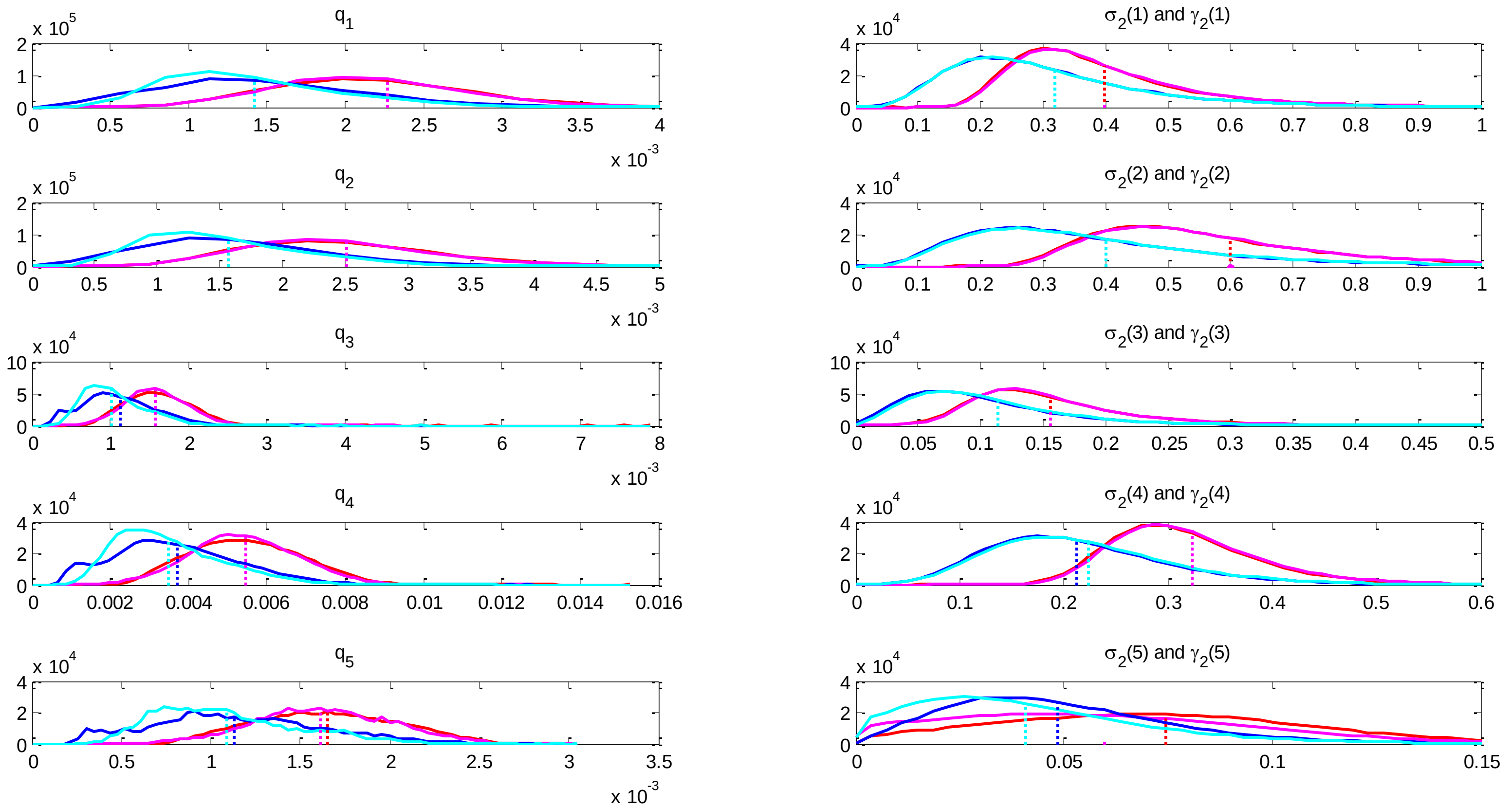
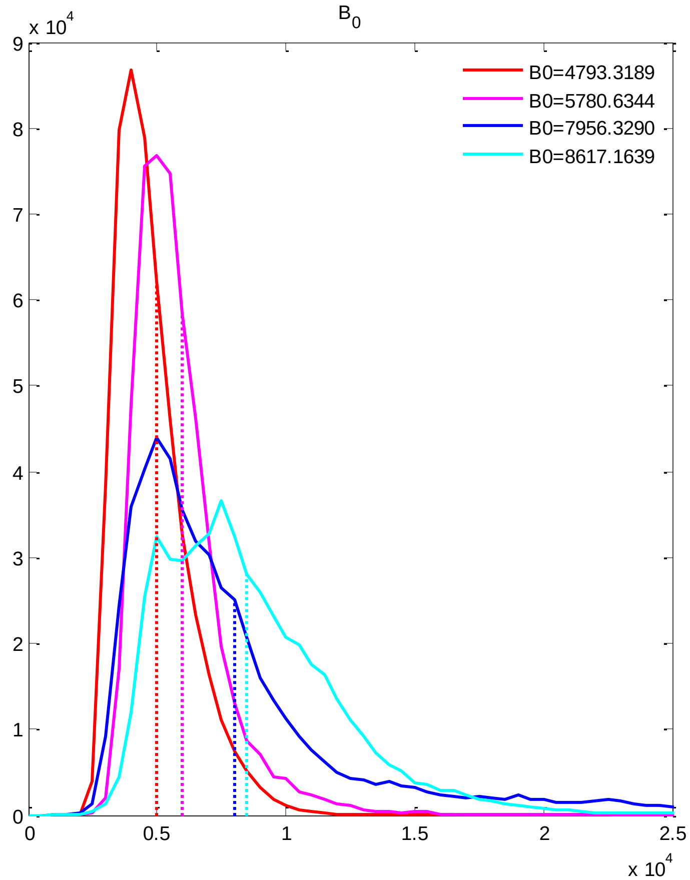


Figure 18: Black sea bass parameter estimation: B_0, B_T. The dashed lines represent the posterior mean of the parameter estimation. The mean values are listed in the legend.



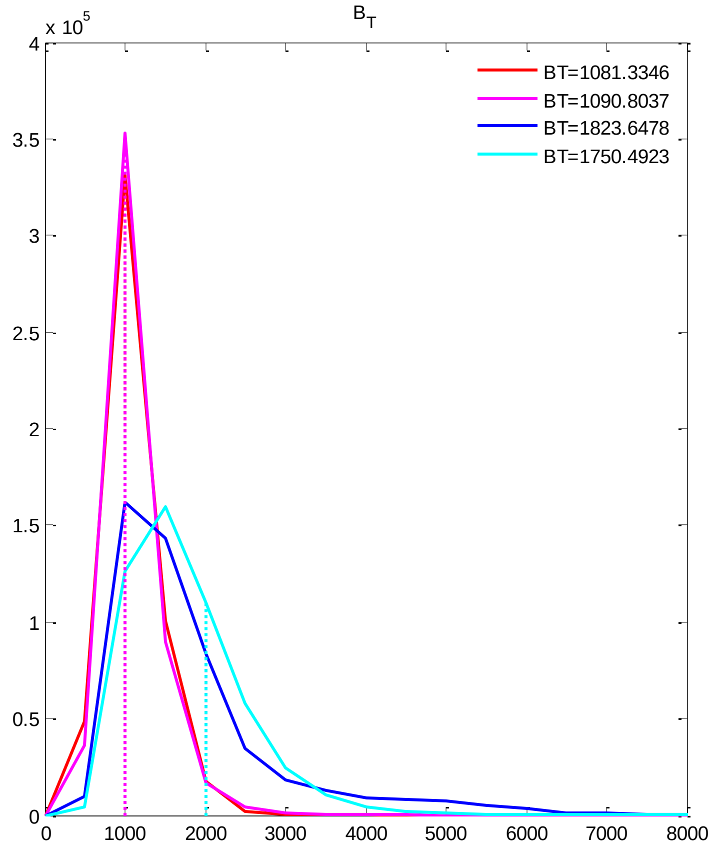


Figure 19: Black sea bass values of management interest: B_T / B_{MSY} and F_T / F_{MSY}. The dashed lines represent the posterior mean of the parameter estimation. The mean values are listed in the legend.

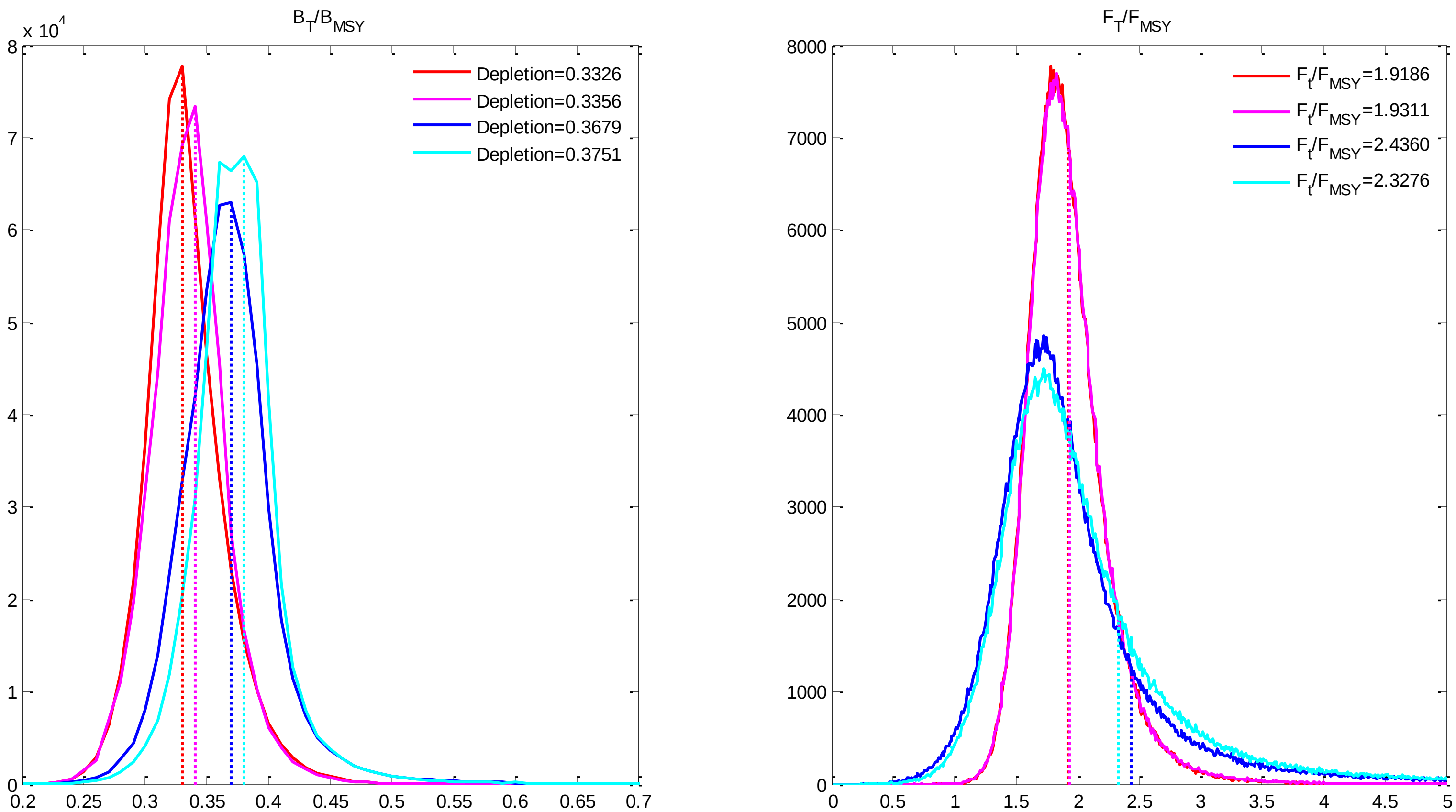


Figure 20: Black sea bass estimated relative population biomass and population biomass from 1974 to 2003. Solid lines represented posterior mean of the relative population biomass or population biomass estimation. Dashed lines are values corresponding to 5% (below solid lines) and 95 % (above solid lines) in the cumulative density function of posterior distribution.

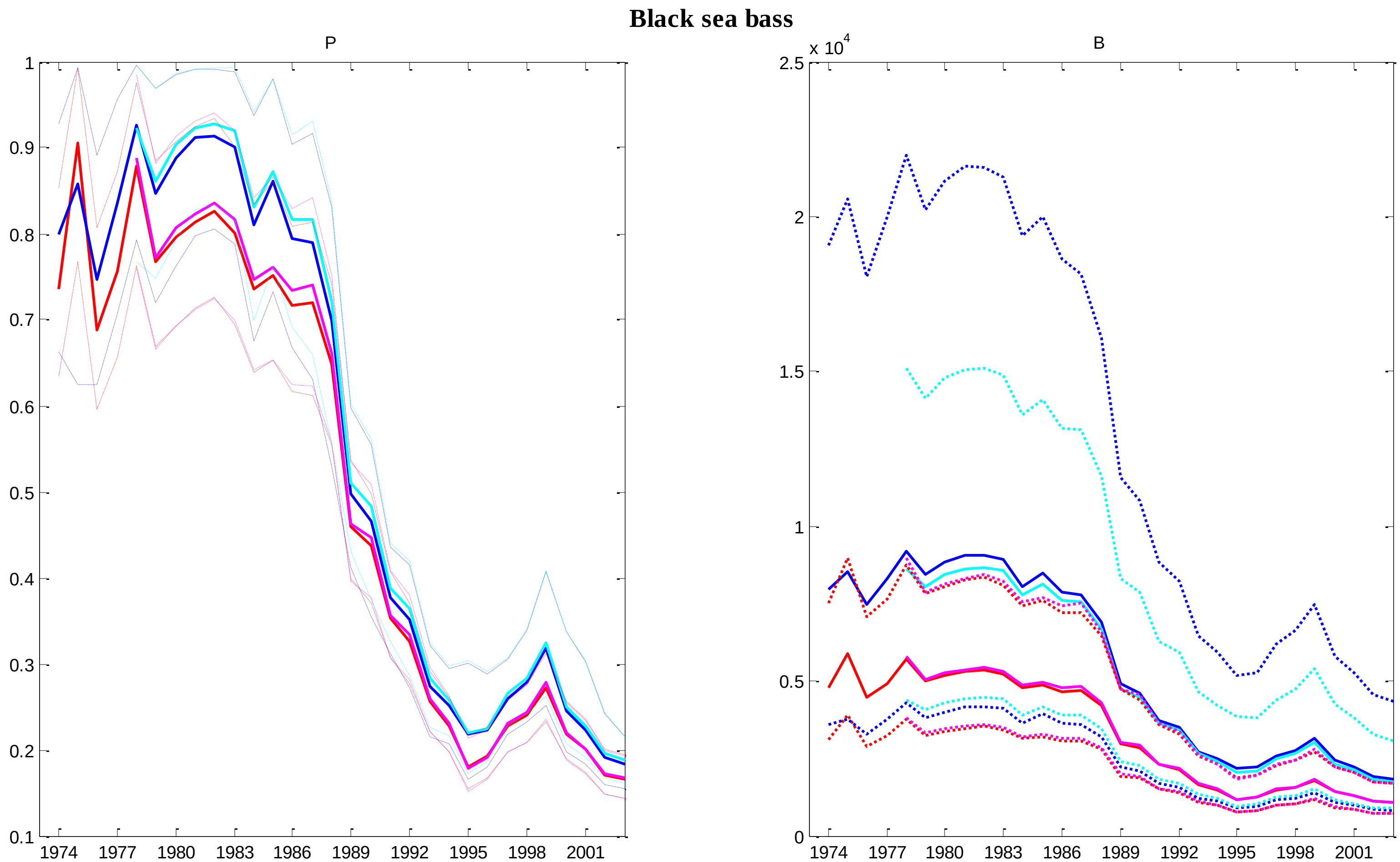


Figure 21: Error distributions of black sea bass estimated by POE_N and POE_C. Left column is the estimated process error distribution for black sea bass. Right column is the estimated observation error distribution for black sea bass. The values of parameters $\sigma_1, \sigma_2, \gamma_1, \gamma_2$ used are mean values of their posterior distributions.

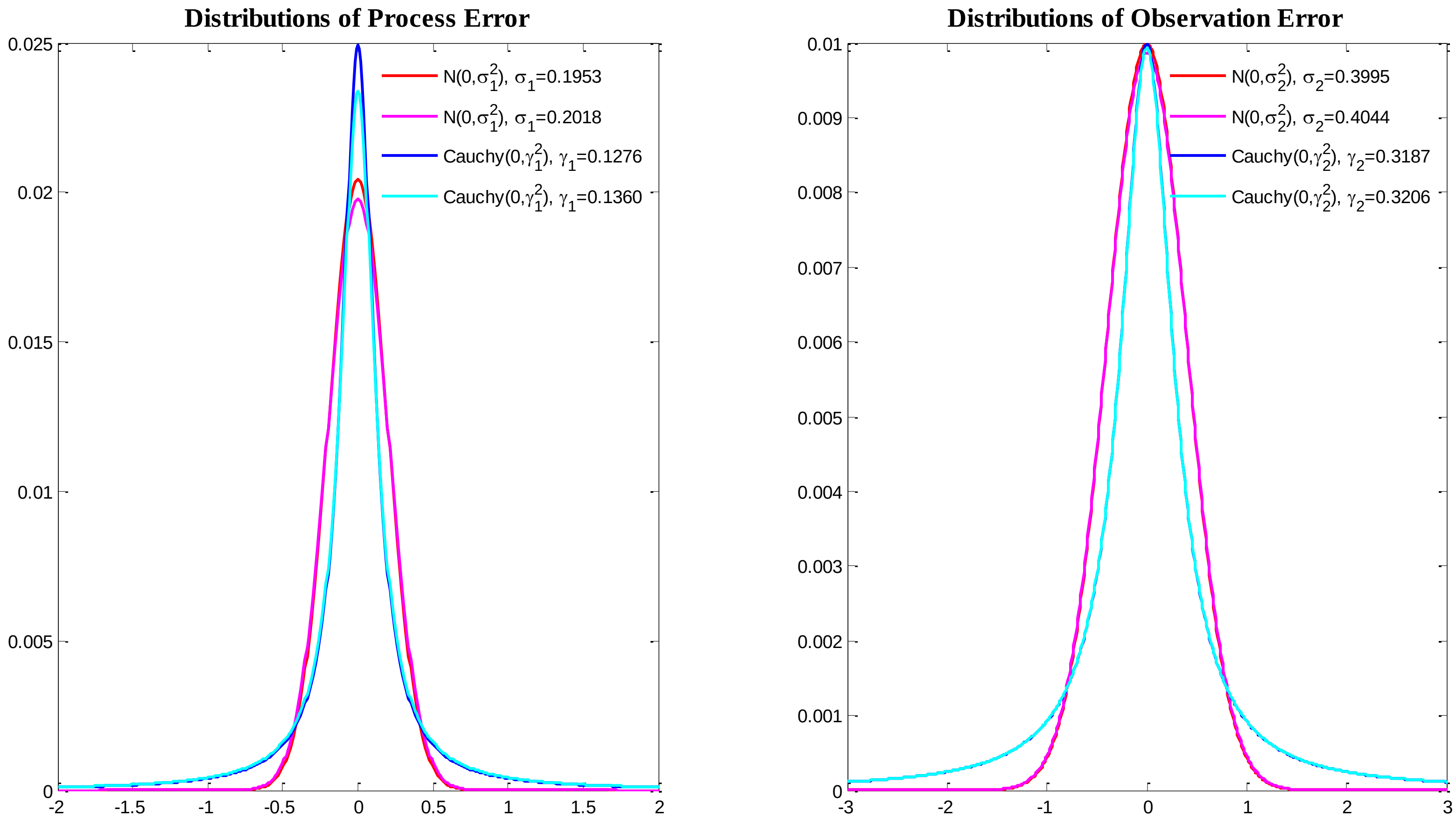


Figure 22: Black sea bass joint posterior distribution of r and K

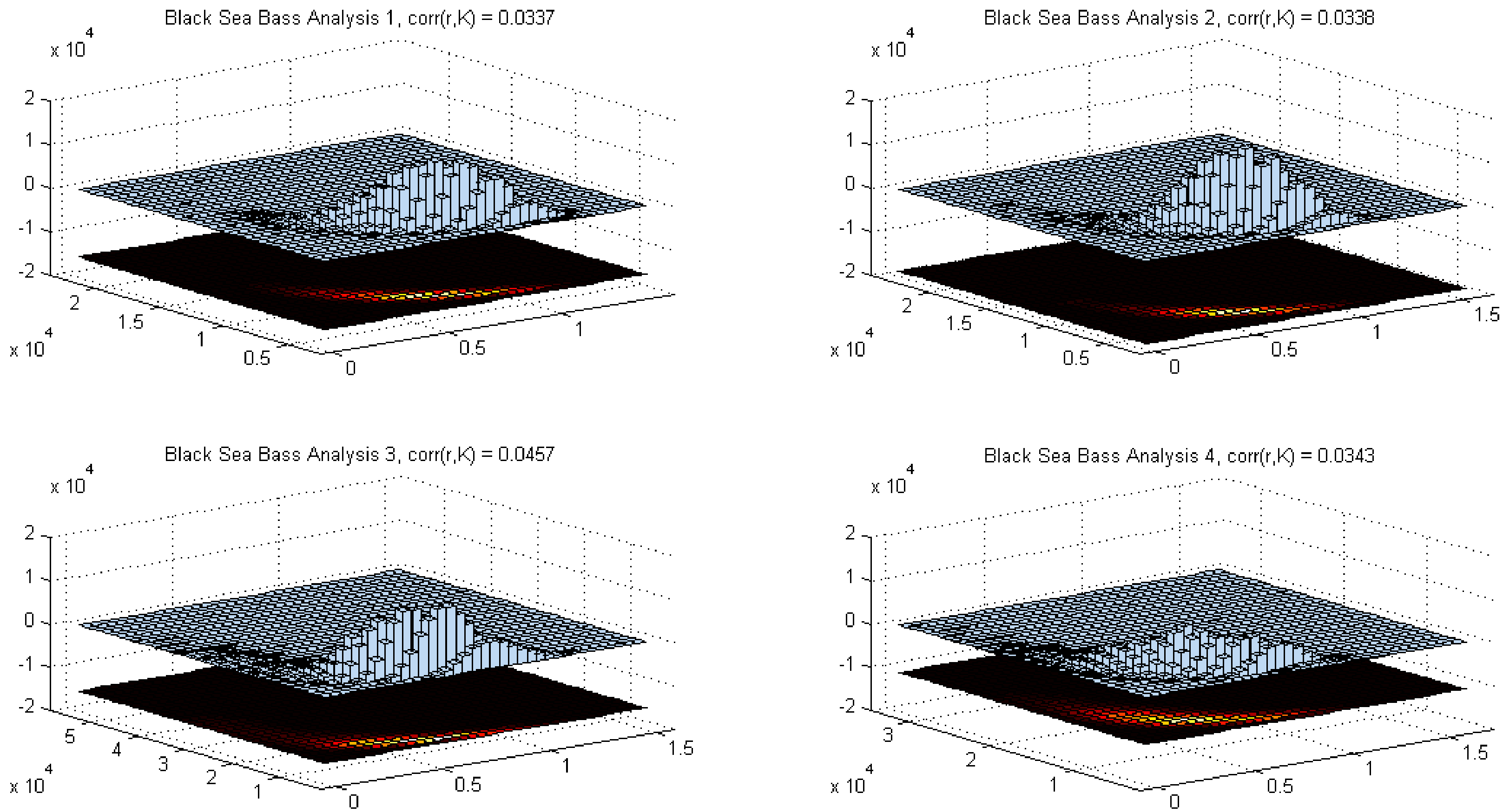
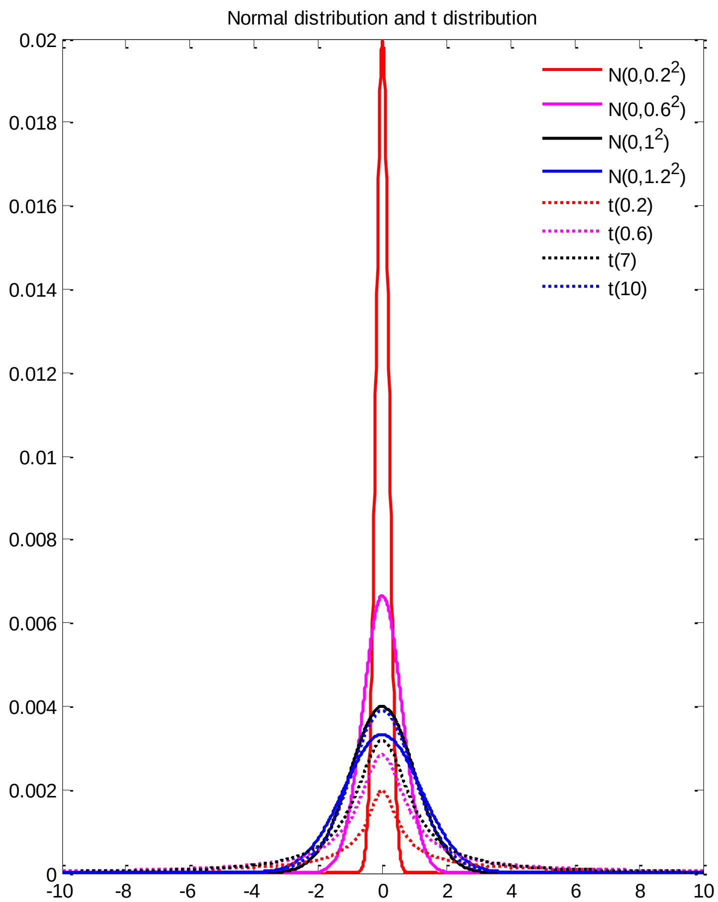


Figure 23: Normal distribution, Student's t distribution, and Cauchy distribution



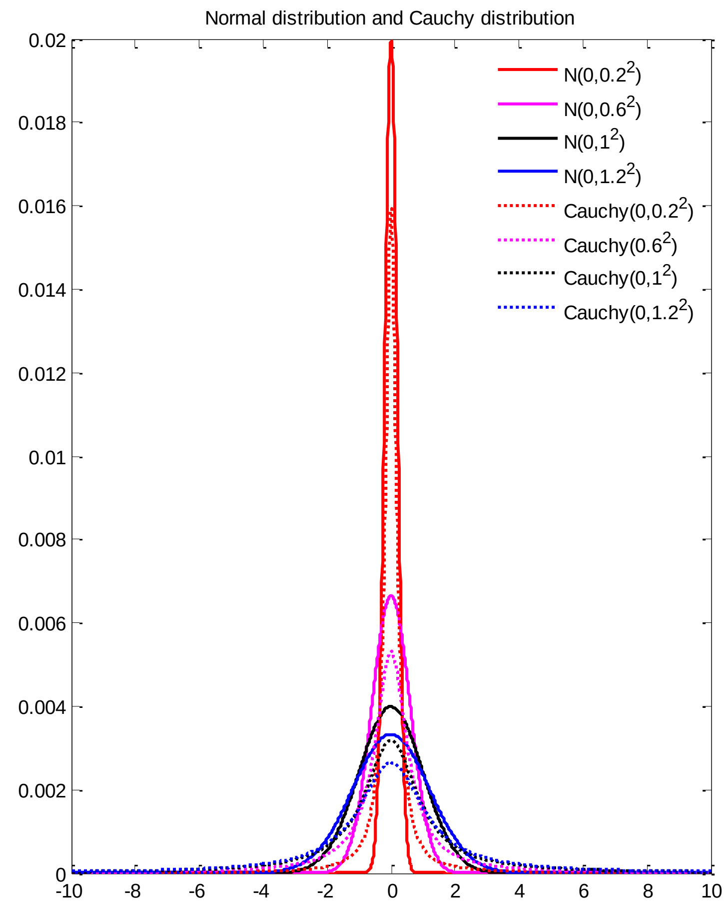


Figure 24: Objective 1 and 3, performance of estimators when OE and PE both existed based on Atlantic weakfish population

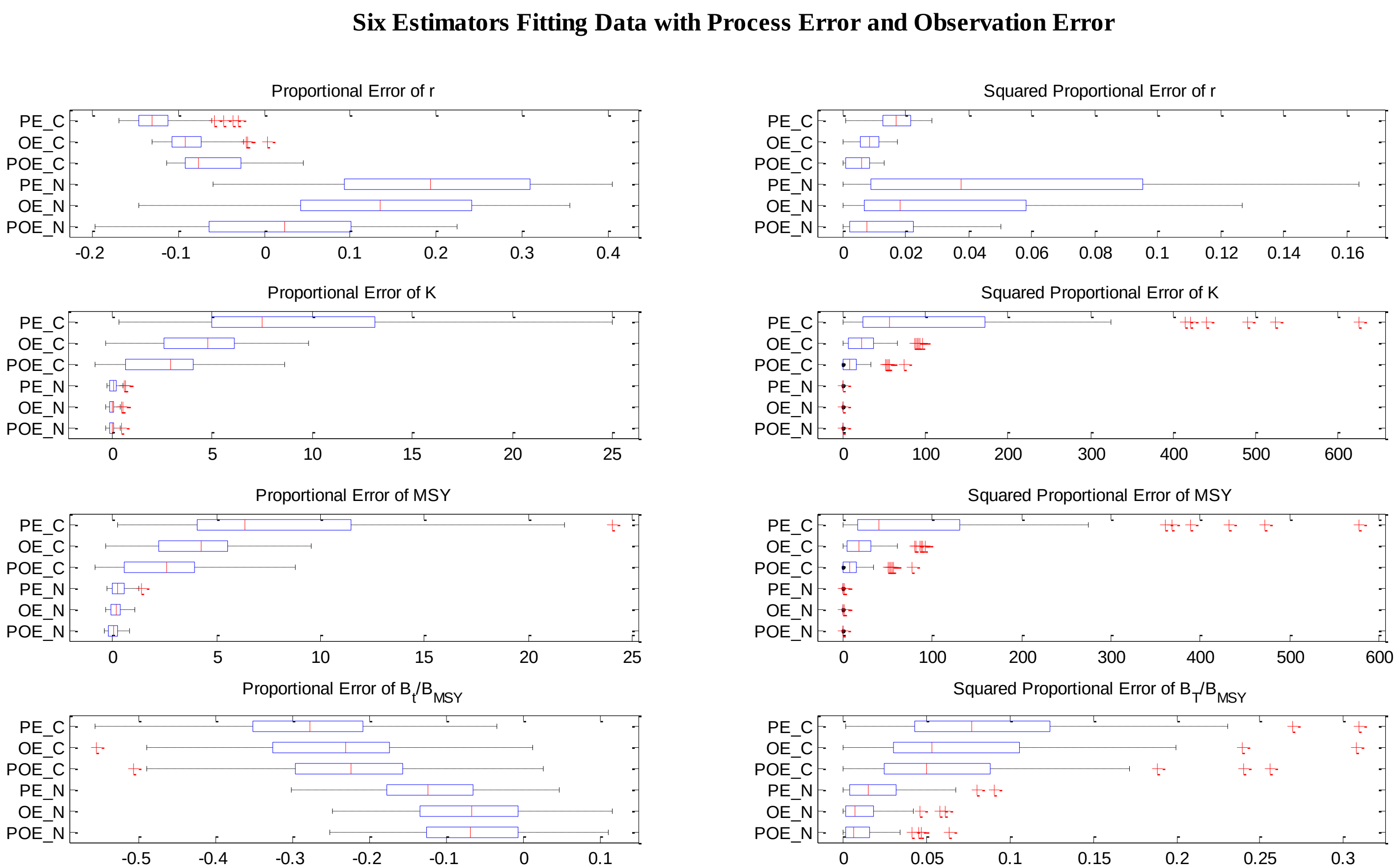


Figure 25: Objective 1 and 3, performance of estimators when OE and PE both existed based on black sea bass population

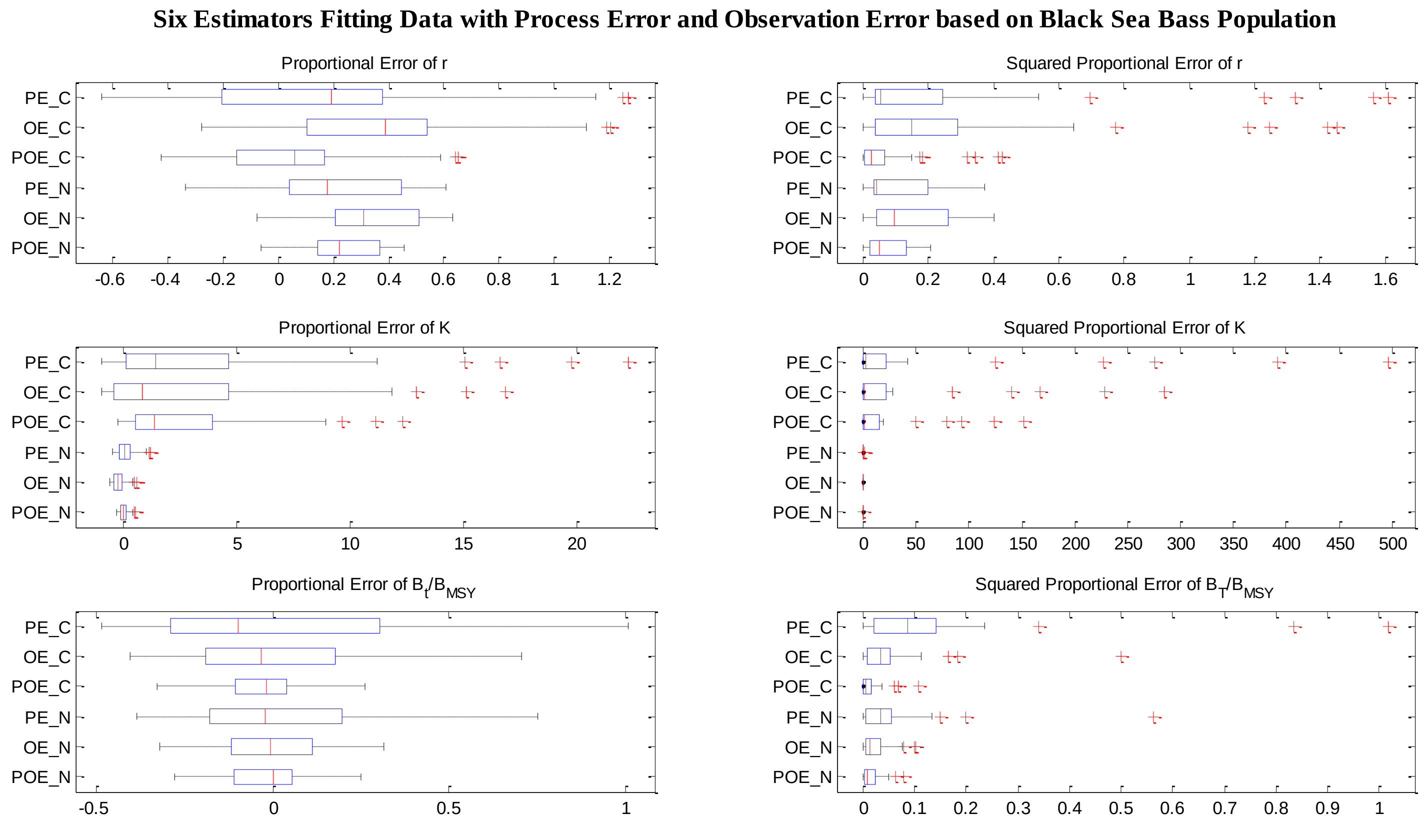


Figure 26: Objective 1 and 3, performance of estimators when only OE existed based on Atlantic weakfish population

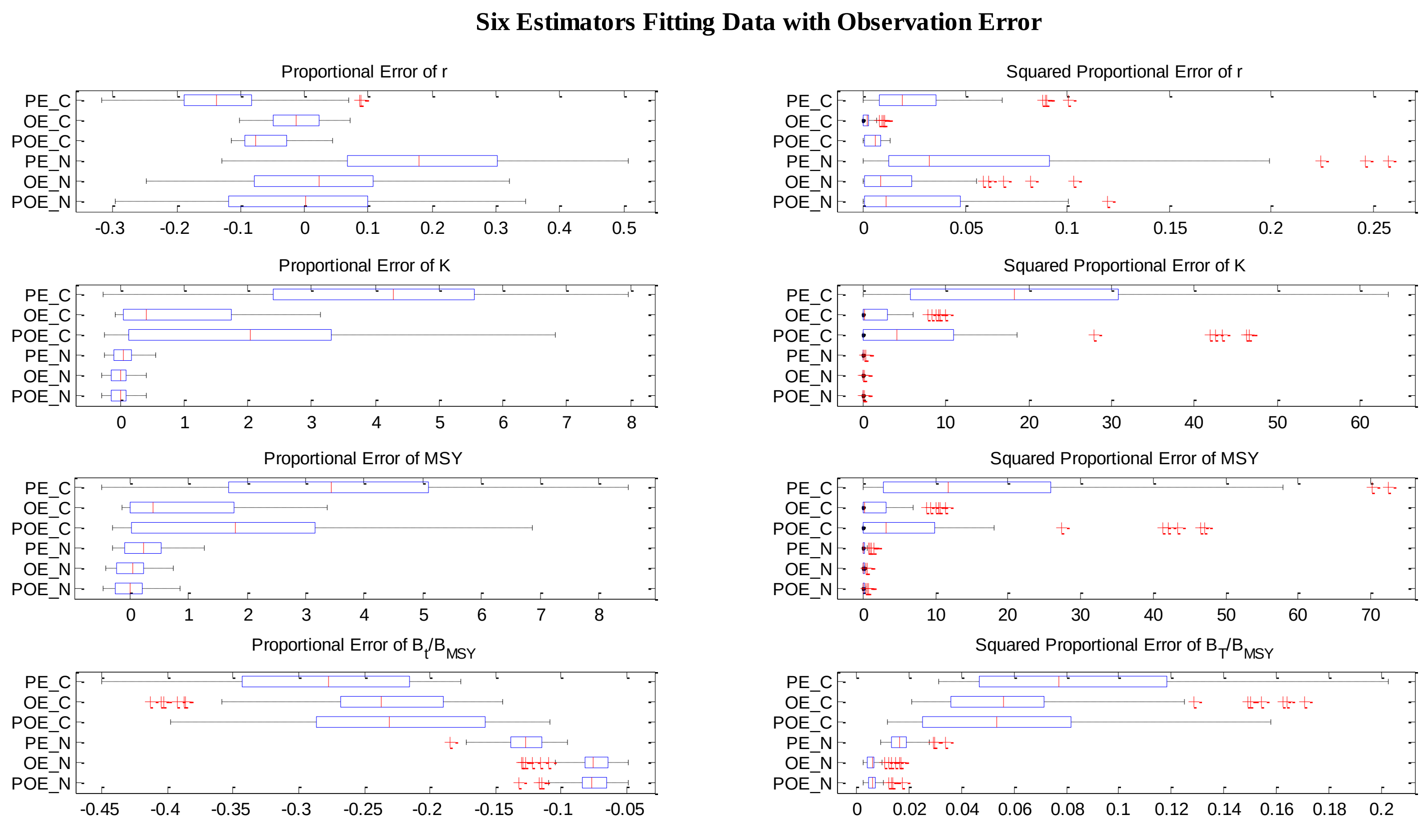


Figure 27: Objective 1 and 3, performance of estimators when only PE existed based on Atlantic weakfish population

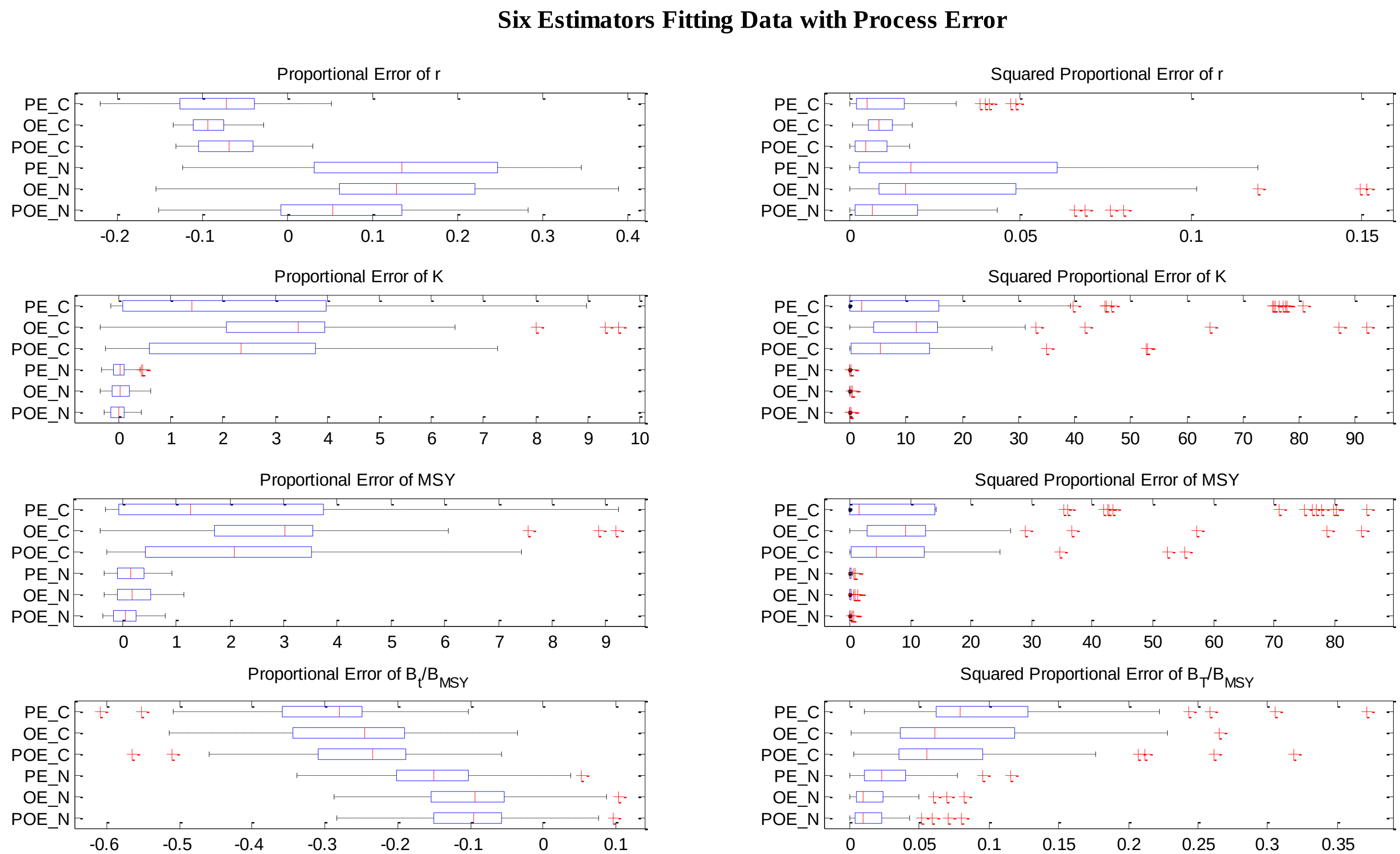


Figure 28: Objective 1 and 3, performance of estimators when only PE existed based on black sea bass population

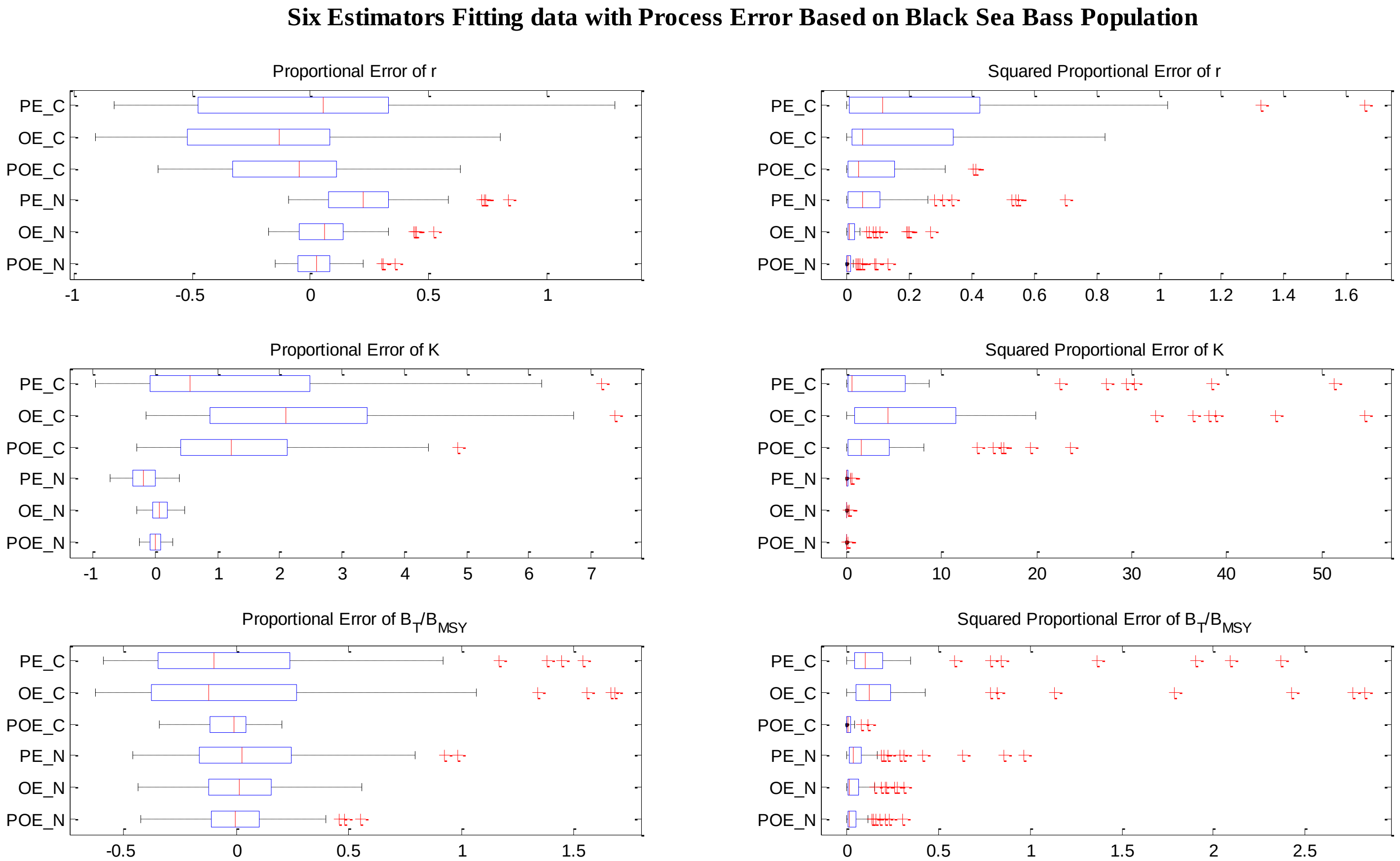


Figure 29: Objective 2 and 4, performances of estimators to OA of large variance in observation error

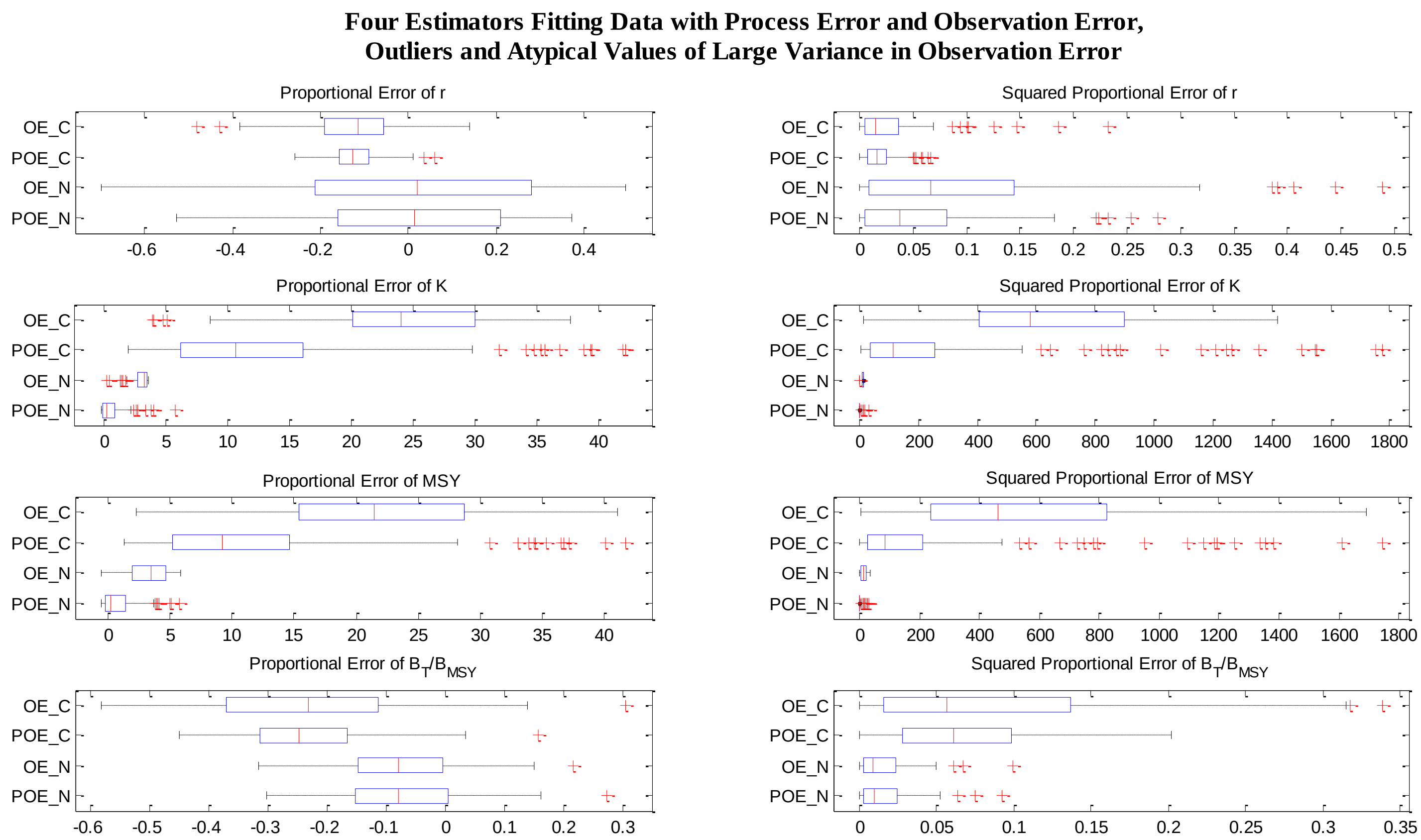


Figure 30: Objective 2 and 4, performances of estimators to OA of large variance in observation error at fixed time

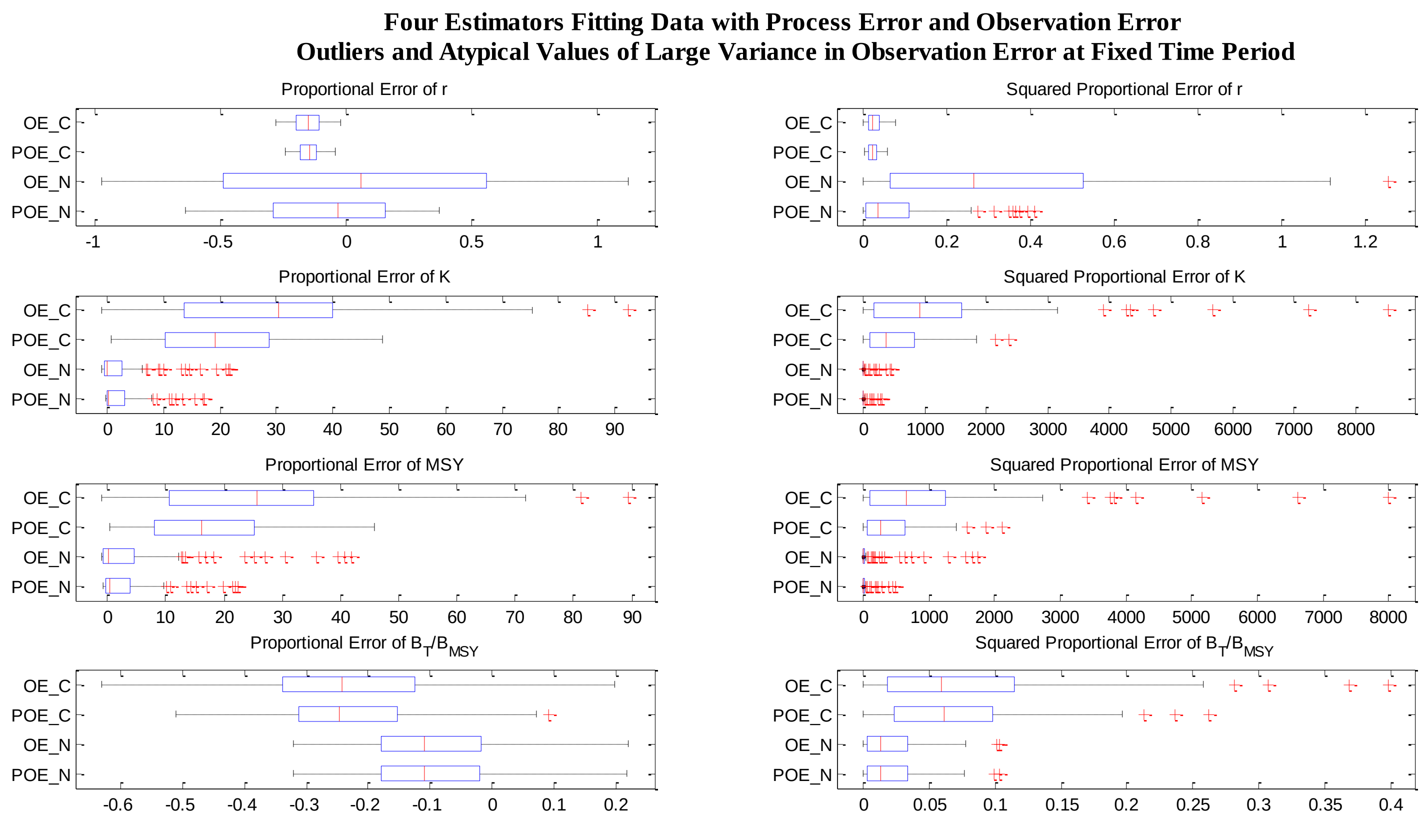


Figure 31: Objective 2 and 4, performances of estimators to biased OA in observation error

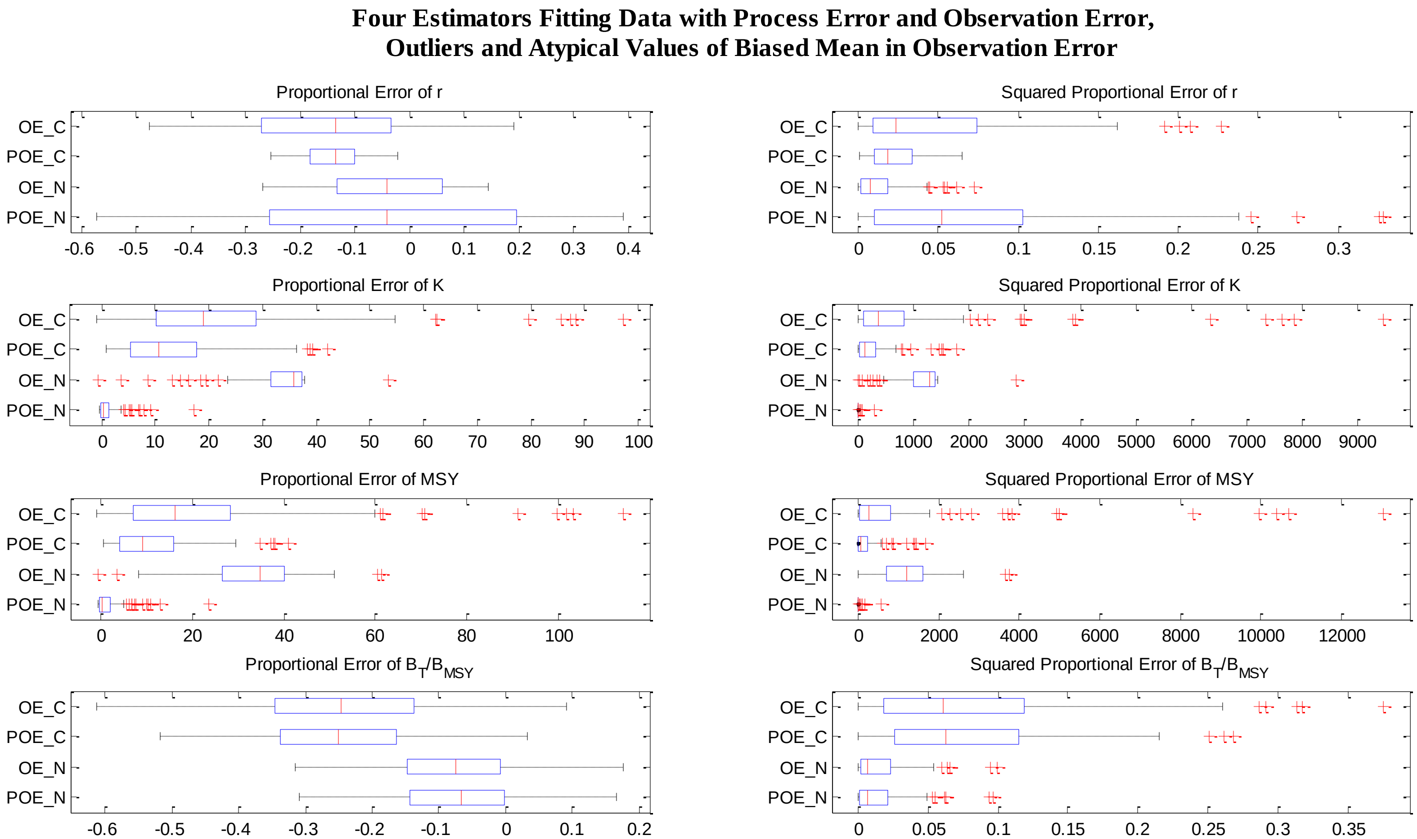


Figure 32: Objective 2 and 4, performances of estimators to biased OA in observation error at fixed time

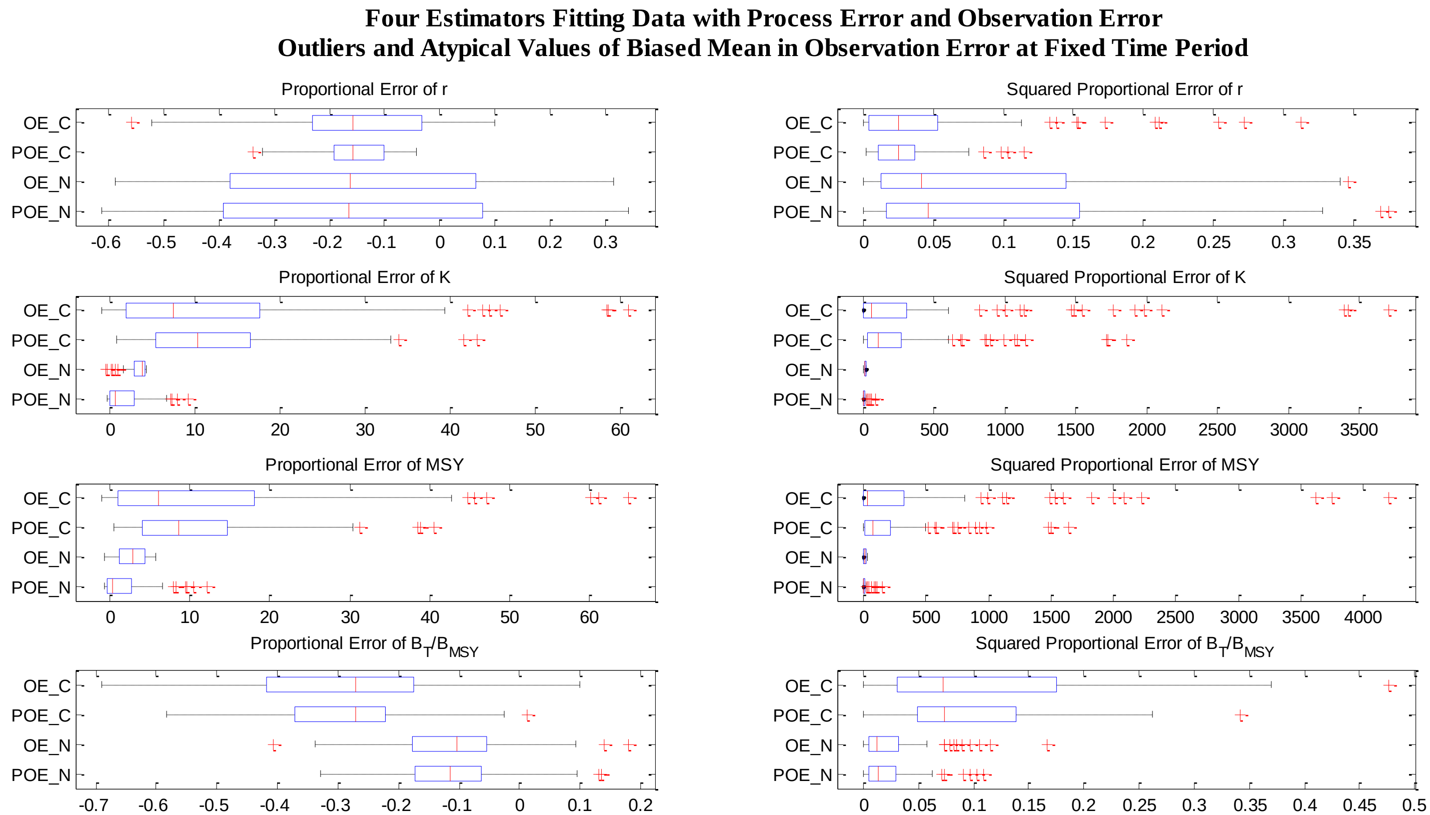


Figure 33: Objective 2 and 4, performances of estimators to OA of large variance in process error

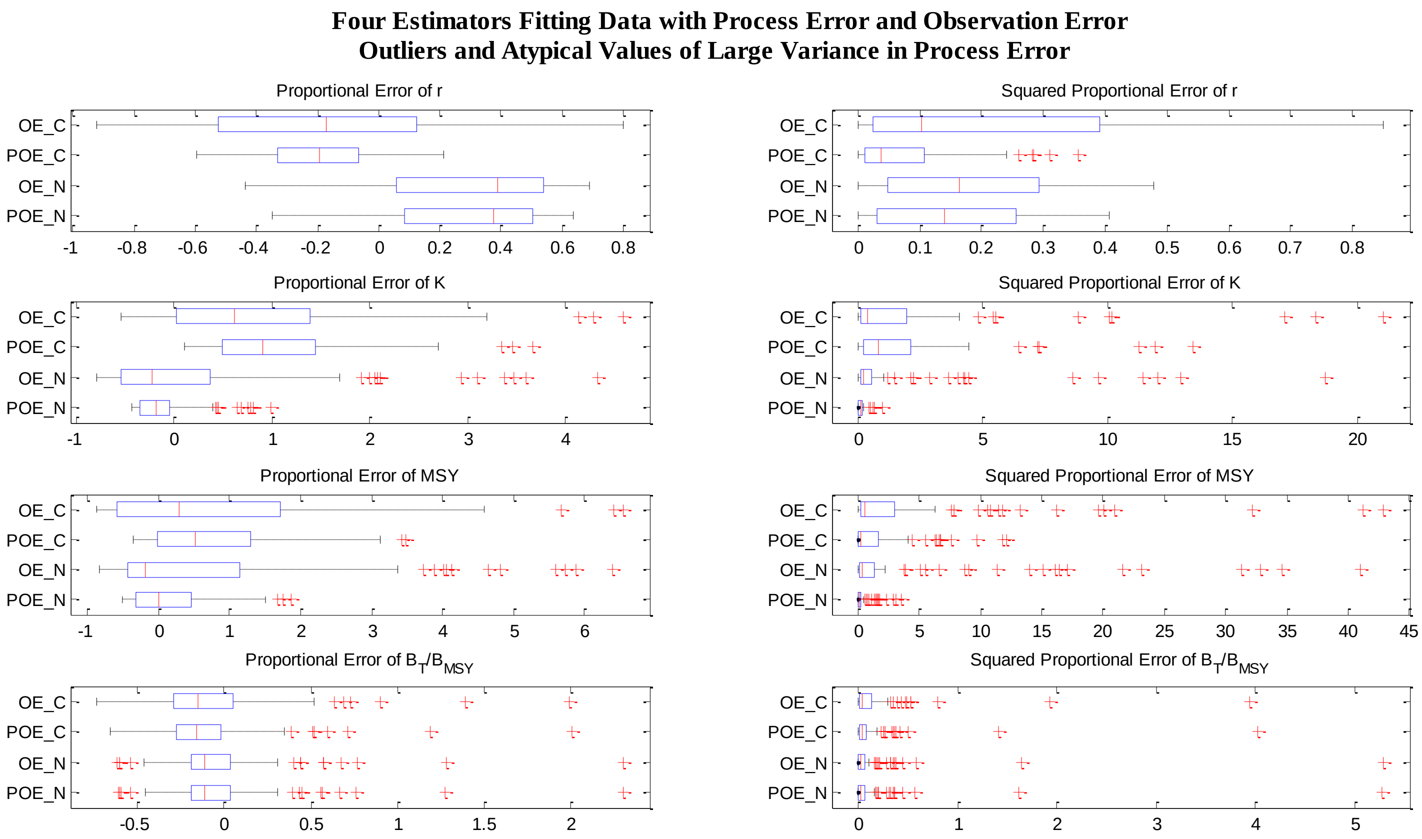


Figure 34: Objective 2 and 4, performances of estimators to OA of large variance in process error at fixed time

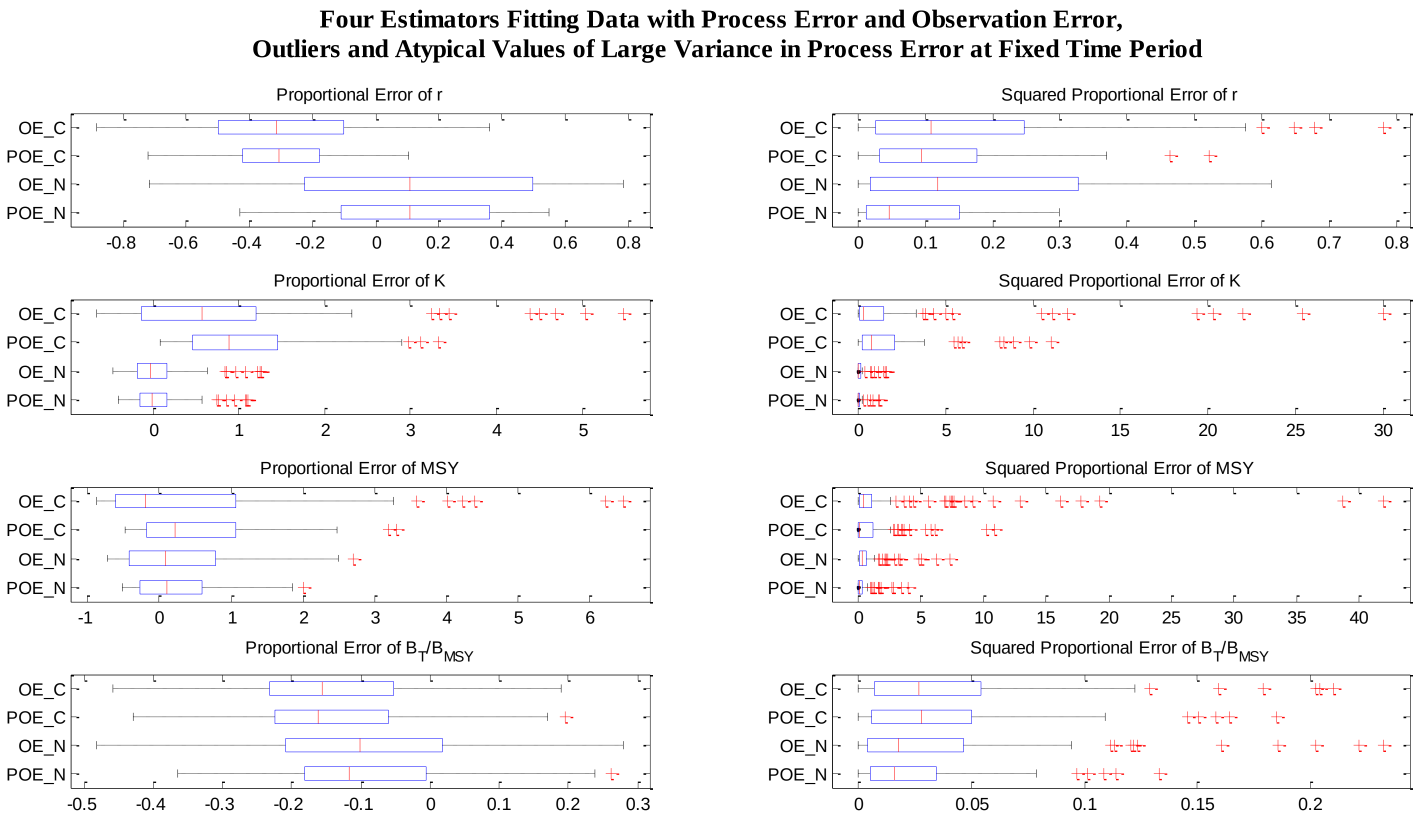


Figure 35: Objective 2 and 4, performances of estimators to biased OA in process error

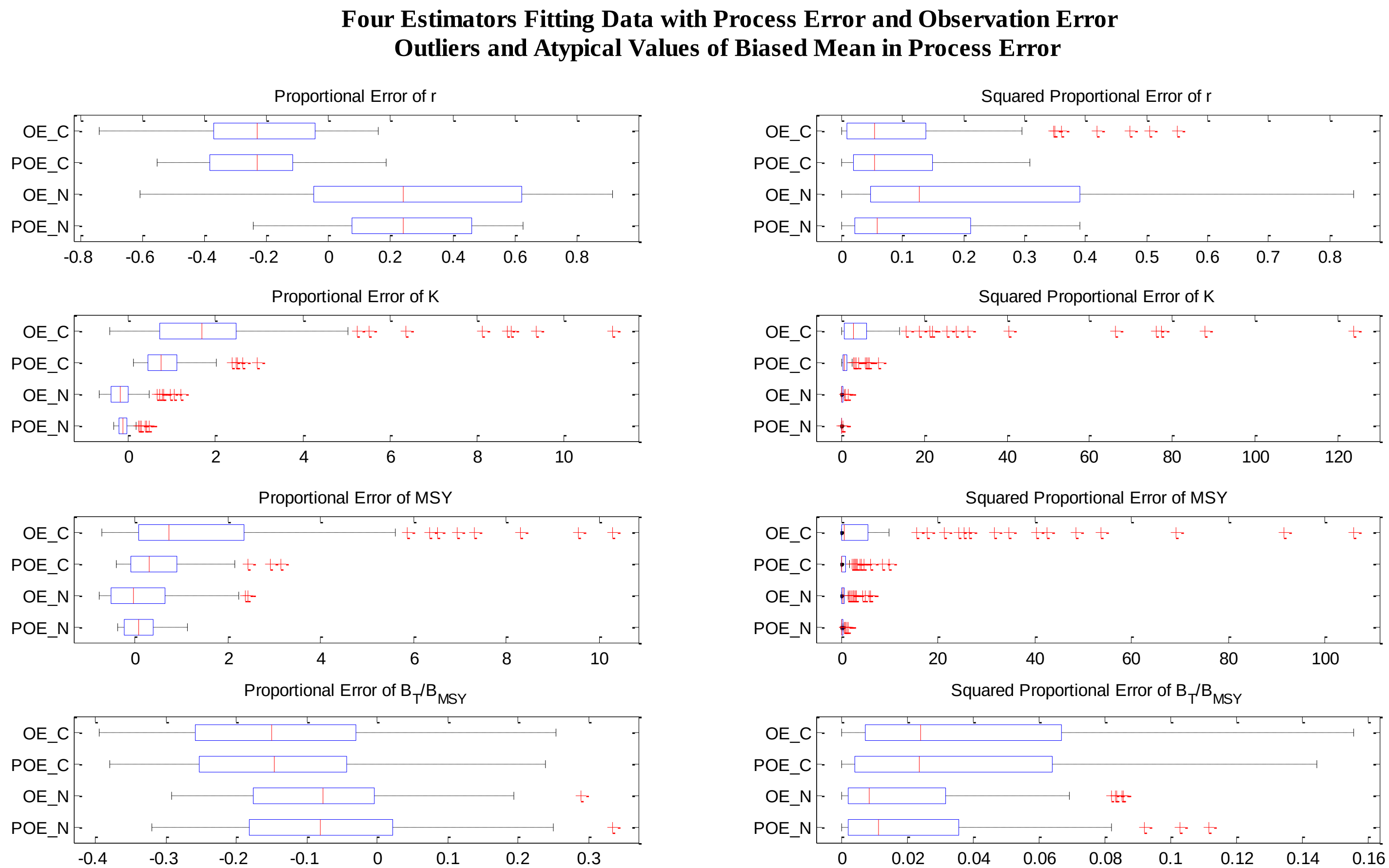


Figure 36: Objective 2 and 4, performances of estimators to biased OA in process error at fixed time

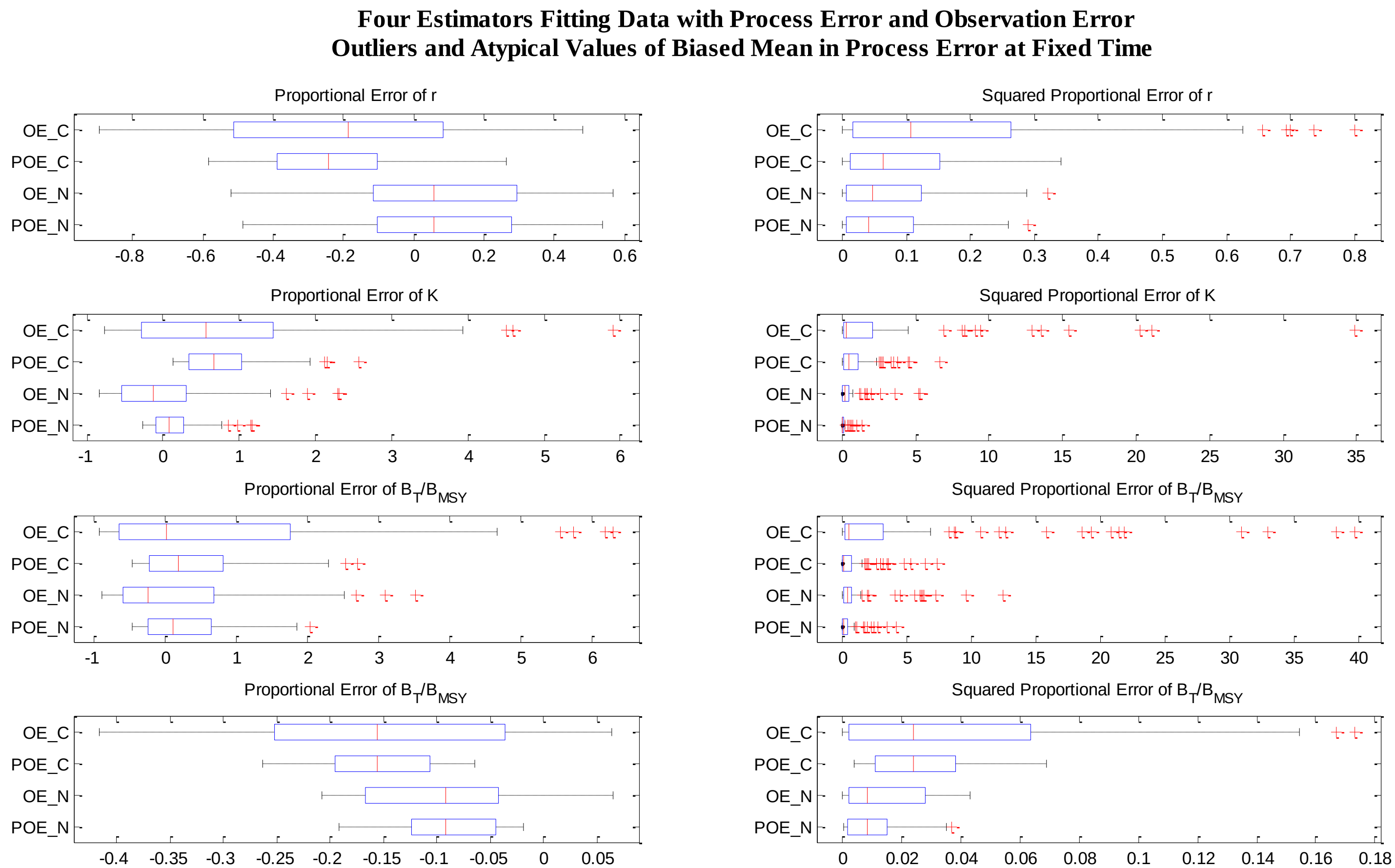


Figure 37: Objective 5, performances of estimators to autocorrelated observation error

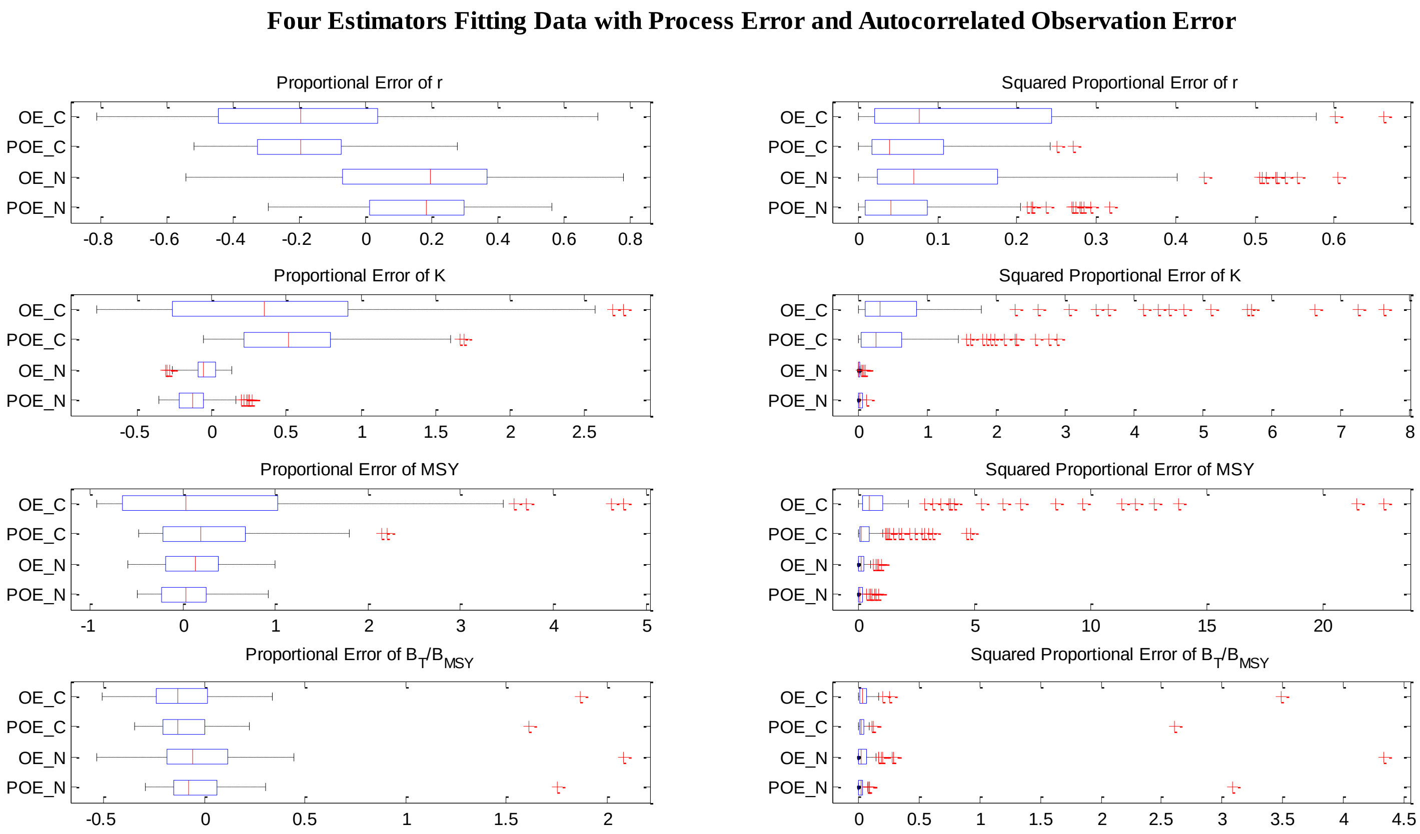


Figure 38: Objective 5, performances of estimators to autocorrelated process error

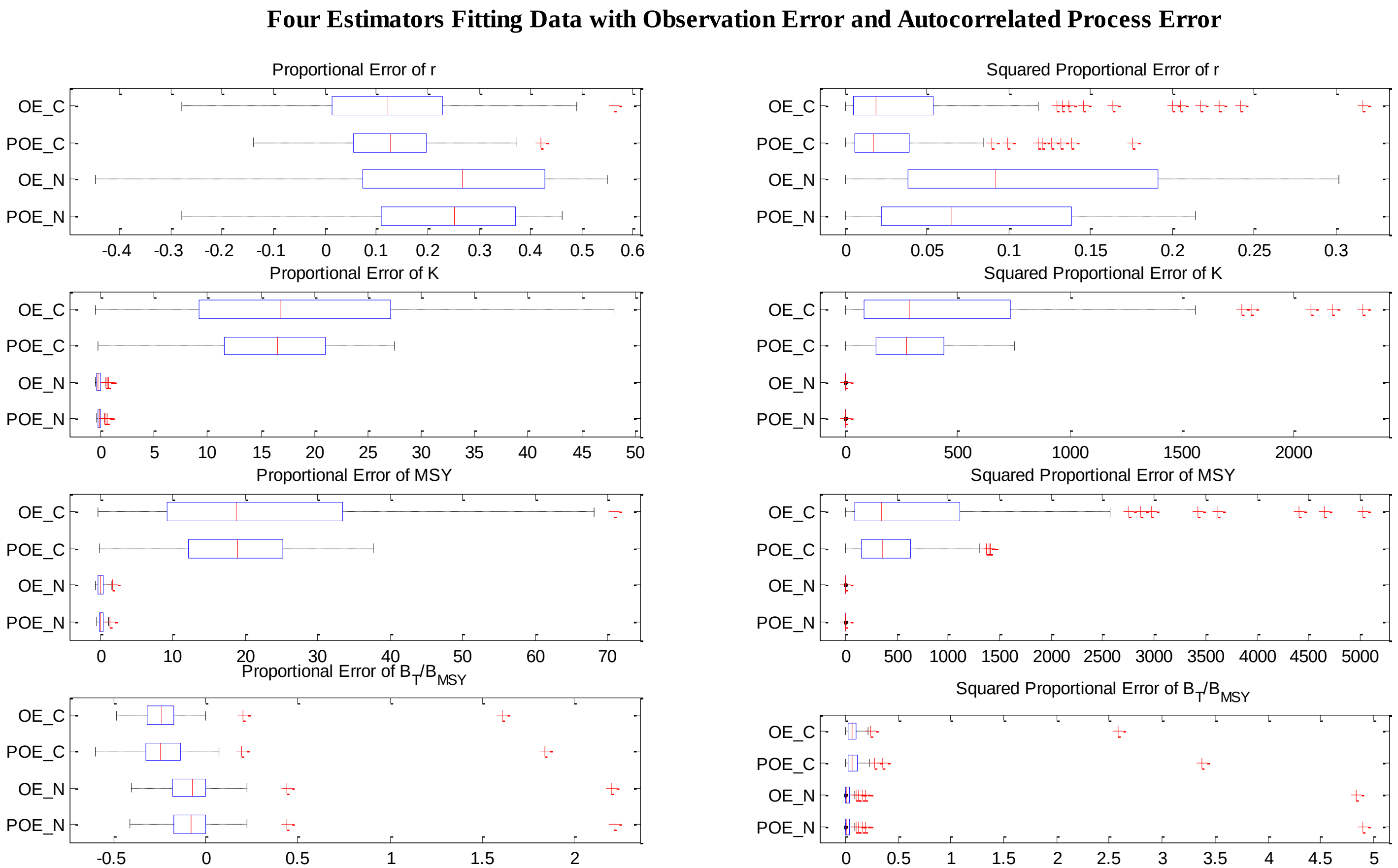


Figure 39: Objective 5, performances of estimators to autocorrelated errors

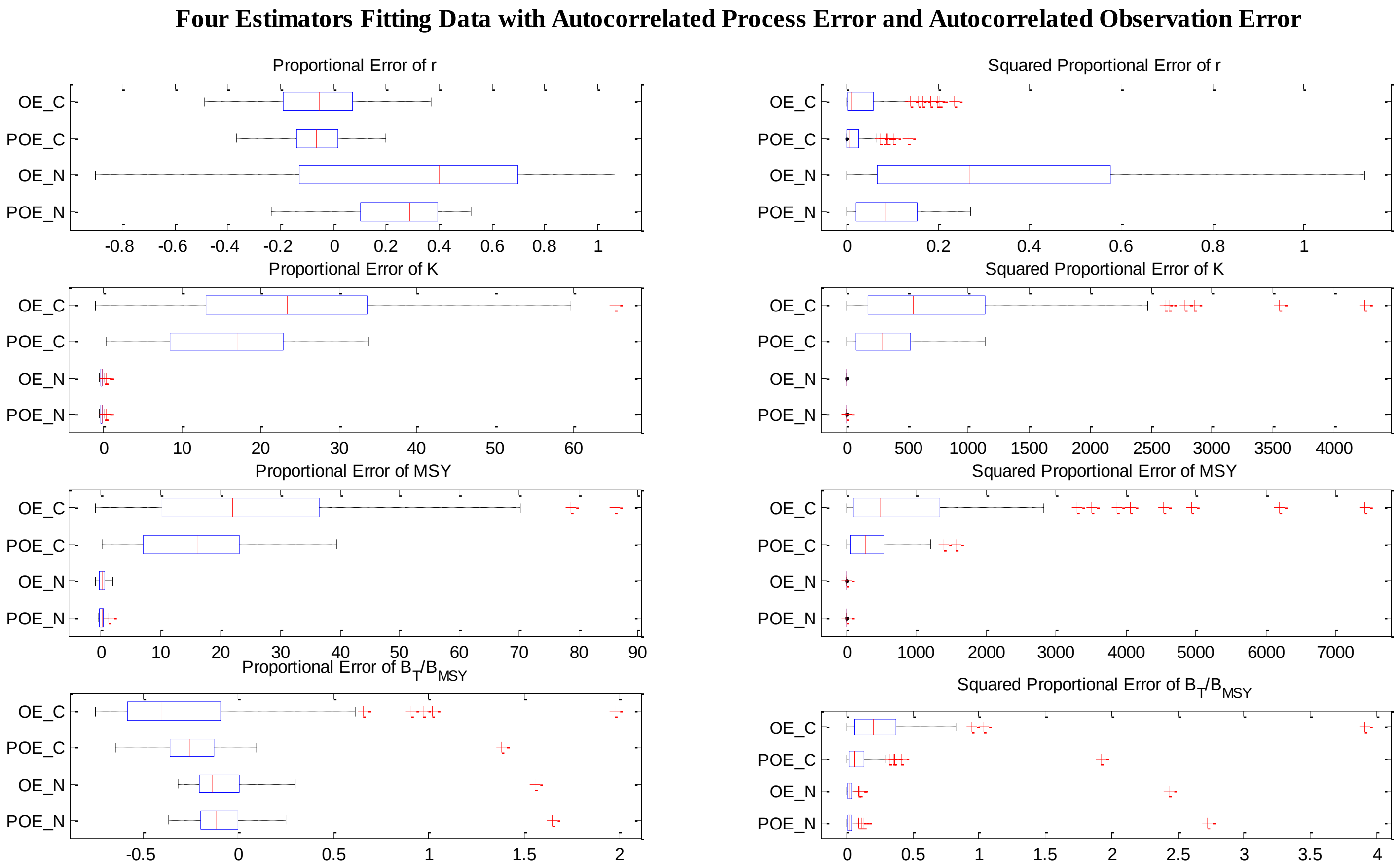


Figure 40: Objective 6, performances of POE_N and OE_N fitting data with multiple indices based on CPUEs