Mechanism Design Theory for Service Contracts

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(ABSTRACT)

This paper presents a novel approach for designing and optimizing maintenance service contracts through the application of mechanism design theory. When offering a contract to its customer, the maintenance service provider seeks to specify contract terms – such as price, service features and incentives – that maximize the provider's profit, satisfy customer needs, allocate risks effectively and mitigate moral hazards. Optimal contract design has to account for asymmetric information and uncertainties associated with customer characteristics and behaviors. We illustrate our mechanism design approach by applying it to the contract design challenge of a gas turbine manufacturer, which also provides maintenance services for its aircraft engines. In our solution approach, we compute an optimal set of contracts. The entire set is presented to the customer and is designed such that the customer will accept one of the contract alternatives without negotiations. In addition to eliminating the costs and delays associated with negotiations, this approach also reveals the customer's private information to the service provider, which the provider can use to its benefit in maintenance management and future contract renewals. Furthermore, we design and incorporate win-win incentive mechanisms into the contracts, which reward the customer for actions that reduces maintenance costs. We present a deterministic and a stochastic mechanism design model, the latter accounting for uncertainties associated with customer actions, engine performance, and maintenance costs during the contract execution phase.

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My journey would not have been possible without the support of my family.

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1 Introduction

Maintenance service contracts describe the terms of an agreement between the maintenance service provider and the equipment operator for preventive maintenance and repair of the equipment [1]. The contract specifies contract duration, service scope, price, and the conditions under which the service provider is or is not responsible to provide maintenance services, along with other terms and conditions [2]. The contract may also include incentives that reward cost-saving behavior by the operator [3]. Original equipment manufacturers (OEMs) of complex and durable industrial products, such as production machines, mining equipment, and aircraft engines, typically provide the maintenance services for their products [4].

This paper focuses on service maintenance contracts for aircraft gas turbine engines, though the developed approach is generalizable and applicable to other products and industries. In the gas turbine industry, the main manufacturers and service providers are Rolls-Royce, General Electric and Pratt & Whitney. Competition is fierce, and companies are constantly looking for ways to gain a competitive edge [5]. Maintenance services represent a significant portion of the product's life cycle cost, and are thus a key consideration in a customer's aircraft engine purchasing decision. Similarly, maintenance services provide a significant revenue and profit stream for the OEMs. Consequentially, well-designed maintenance service contracts are central in ensuring competitiveness and profitability for the manufacturers.

Service contracts that cover scheduled maintenance and unexpected repairs can be regarded as a type of insurance that the provider sells to the equipment operator. As in the insurance industry, the contract provider aims to offer a profitable service product at a competitive price that helps the customer to manage costs and risks. Two types of problems make designing insurance contracts challenging: *adverse selection* and *moral hazard* [6].

Adverse selection refers to customers buying insurance only if they consider themselves as high risk and expect their costs to be greater than the insurance price. Adverse selection arises from asymmetric information about customers' characteristics, i.e., the contract provider has no access to the customers' private information and cannot adjust the insurance price accordingly.

Moral hazard refers to customer behavior that changes with insurance since risks and costs have been transferred to the contract provider. Asymmetric information is also the root cause of the moral hazard problem; however, here it is due to the contract provider's inability to monitor the customer's actions.

The following examples illustrate the concepts of asymmetric/private information, adverse selection and moral hazard. Gas turbine engines and their maintenance needs are affected by operating conditions (e.g., temperature, sand) and operator behavior (e.g., thrust, vertical acceleration). The customer has private information about these aspects at the time of contracting. A customer who foresees engine-deteriorating operating conditions may believe that they would benefit from purchasing a more comprehensive service contract. Opting for a more comprehensive service contract when expecting greater service needs and costs is adverse selection.

With insurance coverage, moral hazard becomes an issue if the customer engages in activities that increase engine deterioration and maintenance needs. For example, taxiing on one engine – a common behavior among operators – prevents the other engines from warming up completely and can be less than ideal for the cold engines during take-off. Moral hazards can result from indifference or carelessness, but can also be well-motivated, as in the given example, since taxiing on one engine saves fuel. If the contract provider cannot monitor this behavior, we again have a situation of asymmetric/private information. Moral hazard can also arise, if regulating or monitoring operator behavior is economically not feasible, not enforceable (though monitorable), or is legally prohibited.

The contract provider can reduce or eliminate the moral hazard and adverse selection problem through incentives that reward the customer for information sharing and cost-saving behavior. To mitigate moral hazard, the provider can offer price discounts to the customer for sharing pilot behavior and engine health monitoring (EHM) data, and make service coverage and copayments dependent on compliance with pre-defined targets. The shared information would also enable the provider to perform better and more cost-effective maintenance. Both parties can save costs – a win-win. Moreover, incentives can be used to mitigate adverse selection problems. A customer, who at the time of contracting prefers not to share EHM data despite significant incentives, reveals the private information that the engine is likely to be operated in harsher than expected conditions.

Additionally, well-designed maintenance service contracts account for uncertainties and risks that result from variations in engine characteristics, operating environments and the economy. In a competitive market place, the service provider benefits from identifying and quantifying risks associated with its service product and effectively allocating them through contracts. For example, an engine fuel consumption guarantee by the OEM is attractive to customers and has become an important marketing and sales instrument for aircraft engines. Fuel consumption can vary between engines of the same type due to manufacturing variance and is further affected by operating conditions and maintenance. Should the engine consume more fuel than guaranteed, the provider will pay for the difference. The provider's uncertainty and risk associated with the fuel guarantee is further amplified by changes in fuel price.

The above mentioned challenges in designing contracts can be addressed using *mechanism design theory* [2]. In mechanism design theory, a *principal* (contract provider) seeks to design a mechanism (contract) that will enable the principle to achieve its objective in a game (engine operations and maintenance actions) with an *agent* (customer). Mechanism design theory is also called reverse game theory, as the game outcome (actions, costs, and profit) is designed by the mechanism, as opposed to merely being the consequence of an existing game.

An important concept in mechanism design theory is the *revelation principle*, which states that a mechanism can be designed such that the agent – by making its optimal choice (here, among contract alternatives) – reveals its type. Type refers to the agent's characteristics, and in our case is the customer's private information on intended engine usage, service preferences and willingness to pay. In addition, if the agent's optimal choice also maximizes the principal's objective, here profit maximization, the mechanism is called *implementable*.

Another important concept in mechanism design theory is *incentive compatibility*. In an incentive compatible mechanism, the agent makes decisions in its own best interest, which are also in the principal's best interest. An initial conflict of interest can be overcome through incentives that align the agent's objectives and actions. In this research, we apply the revelation principle and generate incentive compatible contracts that implement the principal's objectives.

This paper is the first to apply mechanism design theory to the challenge of designing service maintenance contracts, and is among few papers that apply mechanism design theory to a real-world problem. Most papers on mechanism design theory use only small and fictitious example problems, while those that apply the theory merely use ideas, concepts, and select elements of the theory. This paper applies the key elements of mechanism design theory, i.e., the revelation principle, incentive compatibility and implementability, to a real-world problem. In addition to this contribution to mechanism design theory and its applications, this paper also contributes to the maintenance service literature by presenting a new contract design optimization approach.

The structure of this paper is as follows. Section 2 discusses the relevant literature on mechanism design theory and maintenance service contracts. Section 3 develops the model formulation and the general solution approach. Section 4 presents and solves an example problem motivated by a contract design challenge from the gas turbine industry. A stochastic extension and solution approach for the model is developed in Section 5. Future studies are discusses in Section 6. Finally, conclusions are discussed in Section 7.

2 Literature Review

2.1 Mechanism Design Theory

The foundational paper on mechanism design theory was written by Hurwicz [7] in 1960. However, the theory only became relevant for applications after the concept of incentive compatibility had been introduced by him in 1972 [8]. The theory further developed in the 1970s and 80s, most notably through contributions by Maskin [9] and Myerson [10]. Hurwicz, Maskin and Myerson received the 2007 Nobel Prize in Economics for their work.

In recent years, computer science has made important contributions to mechanism design theory, with a focus on computational complexities and algorithms to generate mechanisms [11-14]. Nisan and Ronen [11] introduced the algorithmic mechanism design approach for the design of Internet auctions. Since then, algorithmic mechanism design has been applied to balancing computational loads in distributed computer systems [15], online multi-unit auctions [12], wireless networking and web services [16], and real-time scheduling of computer processors [17].

Mechanism design theory had a major influence on contract theory [2]. In particular, it provided solutions to contract design problems with asymmetric information and the resulting adverse selection and moral hazard problems [2, 18-23]. Jullien [19] determined the conditions under which optimal contracts can be designed, despite adverse selection and asymmetric information problems. Moroni et al. [24] developed an approach to mitigate moral hazards in contract design for different utility functions of customer. Page [21] formulated a screening approach to identify incentive mechanisms that resolves the moral hazard problem. Building upon this work, Page [22] derived incentive compatible contract selection mechanisms for both moral hazard and adverse selection problems. Mukhopadhyay et al. [25] explored how offering a set of contracts, a so-called contract menu, to an agent can reveal the agent's private information and can help with optimal allocation of rewards. For a comprehensive literature review on mechanism design theory and contracts see Fudenberg and Tirole [26] and Chiappori and Salanié [27].

Papers that apply mechanism design theory to maintenance service contract design are rare. Notable exceptions are Gupta et al. [28] and Volker et al. [29]. Both papers studied contracts for public infrastructures construction and maintenance, such as for highways. Performance-based incentives were used to motivate agents to take a long-term interest in infrastructures performance while achieving low costs for the government agency. Incentive compatibility was analyzed, however, without consideration of the revelation principle. To the best of our knowledge, no paper has applied mechanism design theory with both revelation principle and incentive compatibility to the design of maintenance service contracts.

2.2 Maintenance Service Contracts

Maintenance service contracts have been studied without the explicit use of mechanism design theory. The main methods used and topics discussed in the maintenance contract literature are principal-agent models [3, 28, 30-37], game theory [24, 38, 39], incentive mechanisms [40-42], contract pricing [39, 43], contract negotiation

[4, 44], and applications in manufacturing [4, 39-41, 44-46] and aviation [47, 48], among others [49, 50].

Principal-agent models can analyze the interactions, objectives and decisions of a principal and an agent, where the principal seeks services from the agent in return for a payment [30, 34]. A typical example is the employer-employee or superior-subordinate relationship in organizations [51, 52]. To motivate the agent to act in the principal's interest, performance-based payments and other outcome-based incentives are typically used [53-60]. Through these incentives, the principal seeks to motivate the agent to provide a high level of effort, which however cannot be observed directly and is thus private information [31].

In the maintenance service literature, the customer typically takes on the role of the principal, since the customer pays the maintenance provider (agent) for its services and tries to incentivize service quality [28, 29, 35, 36, 61]. However, the perspective can also be reversed, as in our paper, where we consider the service provider to be the principal that offers and designs contracts and incentives such that the interests of the agent (customer) become aligned. Especially in papers where the maintenance contract is regarded as a type of insurance for the customer, this perspective is used; e.g., [31, 37].

Principal-agent models are based on game theory, which has been applied beyond the principal-agent model to analyze maintenance contracts. Murthy and Yeong [38] used a Bertrand-Stackelberg game formulation to derive optimal strategies for both the customer, who uses the equipment, and the maintenance service provider, who supplies the service parts. Murthy and Asgharizadeh [39] focused on optimal pricing strategies for maintenance contracts. These game-theoretic models, however, focus on actions after the contract is executed, but do not consider the contract design stage, and how to design incentives to align customer interests.

Incentives play a key role in maintenance service contracts. Taraki et al. [40] determined incentive mechanisms that lead the customer to choose a maintenance policy that maximizes the combined profits of manufacturer and customer. The paper was extended to account for manufacturing systems with multiple customers [41]. However, the concepts of moral hazard and adverse selection were not explicitly discussed.

Contract pricing and negotiations are well-studied topics in the maintenance service contract literature. Rinsaka and Sandoh [43] and Murthy and Asgharizadeh [39] developed models for pricing of service contracts with service level guarantees. Kumar et al. [4] and Jackson and Pascual [44] studied the negotiation process of service contracts, and identified modalities under which win-win situations for service provider and customer can be achieved. While most papers, such as Wang [35], focus on one service provider and one customer, Asgharizadeh and Murthy [36] considered a scenario with multiple customers serviced by one maintenance provider.

The majority of applications of maintenance contracts are in the domain of manufacturing [4, 39-41, 44-46]. Martin [45], for example, studied cost-effective contracts when maintenance is outsourced to service providers, and how these providers can then optimally manage maintenance. Ding et al. [46] developed a method for finding a set of maintenance contracts, which meets the required system availability while minimizing the contract provider's expected costs. However, none of the above listed papers considers information asymmetry between the contract provider and customer.

Beyond manufacturing, service contracts have been discussed in the context of urban green-space [49], healthcare [50], and aviation [47, 48]. In the aviation domain, Erickson et al. [47] surveyed industry practices of aircraft maintenance contracting in the defense and commercial sectors. Bowman and Schmee [48] used a simulation-based approach to determine optimal pricing in long-term engine maintenance contracts for a fleet of aircraft.

3 Model Formulation and Solution Approach

3.1 Model Formulation

In this section, we provide the details of the maintenance service contract design problem, introduce the notation, and present the mathematical formulation of the problem. The maintenance service provider seeks to offer a maintenance service contract to its customer and must decide which service features to include and how to price them. In accordance with mechanism design terminology, we refer to the service provider / contract designer as the *planner* and refer to the customer as the *agent*. The planner's goal is to maximize profit, while she also wants to ensure that the contract is accepted by the agent without negotiations. In addition, the planner seeks to include incentives in the contract to motivate cost-saving behavior by the agent. Lastly, the planner desires to obtain private information on expected engine usage from the agent during the contracting phase.

The planner is uncertain about the agent's expected engine usage and engine operating conditions. However, the planner can envision all possible scenarios and can estimate the likelihood of each scenario for a given agent. As customary in mechanism design theory, we model this uncertainty by considering multiple agent types $\theta \in \Theta$, with the probability p_θ that the agent is indeed of type θ.

A maintenance service contract is a collection of service features q_i and is offered at a price of t to the agent. For a given agent type θ, the planner seeks to find the optimal contract features $Q_\theta^* = \left\{q_1^\theta, ..., q_i^\theta, \ldots q_m^\theta\right\}$ and the optimal price t_θ^*. A service feature is included in an agent's contract if $q_i^\theta = 1$, and zero otherwise. An example of a service feature is life limited parts (LLP) coverage, under which the planner replaces certain engine parts at regular intervals at no additional cost to the agent.

Once the contract is signed and the engine is in operation, the agent can take a number of actions that affect engine performance and maintenance. Actions can lead to increases or decreases in maintenance costs. For example, taxiing on one engine before take-offs increases cost, while sharing EHM data can help to reduce costs. The set of actions taken by an agent of type θ is denoted by $A_\theta = \left\{a_1^\theta, ..., a_j^\theta, \ldots a_n^\theta\right\}$, with $a_j^\theta \in \{0,1\}$ indicating whether or not the agent takes the action with index *j*.

To motivate cost-saving actions by the agent, the planner can include incentives I_θ as part of the contract. The planner seeks to determine the set of optimal incentives $I_\theta^* = \left\{i_1^\theta, .., i_k^\theta, \ldots i_o^\theta\right\}$, with $i_k^\theta \in \{0,1\}$, for a given agent type θ.

The attractiveness of a contract for planner and agent is represented by value functions V_p and V_θ, respectively. The value functions depend on price, contract features, incentives and the agent's actions, and are thus $V_P(t_\theta, Q_\theta, I_\theta, A_\theta)$ and $V_\theta(t_\theta, Q_\theta, I_\theta, A_\theta)$. We assume these functions to be continuous and monotonic in price t_θ. Both players have a reservation value v_θ and v_P that is the minimum value a contract has to provide from them to participate. Fig. 1 illustrates both players' value functions (here, linear in price t_θ), their reservation values, and the resulting feasible range for the contract price. Table 1 summarizes the notation.

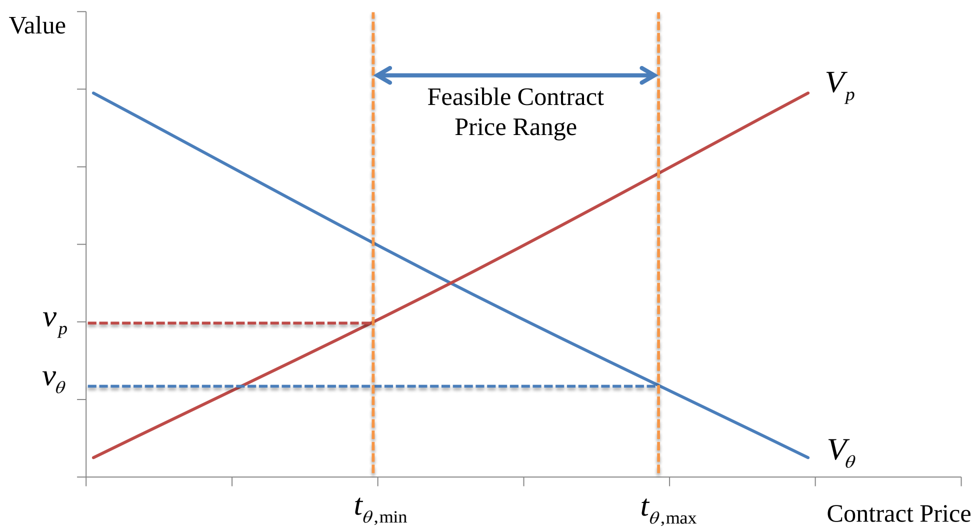

Fig. 1: Value functions, reservations values and feasible contract price range

Table 1: Overview of notation

θ	Agent type	t_θ	Contract price
p_θ	Probability of agent type θ	V_p	Value function of planner
Q_θ	Set of contract features	V_θ	Value function of agent
I_θ	Set of incentives	v_θ	Reservation value of agent
A_θ	Set of agent actions	v_p	Reservation value of planner

The contracting game can be analyzed by dividing it into two stages. In the first stage, the planner designs a contract for each agent type that maximizes her value function V_p. The planner chooses service features Q_θ, incentives I_θ, and price t_θ for each agent type θ. The entire set of contracts is then offered to the agent.

In the second stage, the agent chooses among the multiple contracts offered. In choosing the contract, the agent considers his optimal actions A_θ that he will take for each contract, and then picks the contract that maximizes his value V_θ. The agent's contract choice reveals private information (here, on future actions A_θ) to the planner. In mechanism decision theory, the first stage is referred to as a screening game, while the second stage is called a signaling game.

The two-stage game can be represented as an optimization problem:

$$\max_{t_\theta, Q_\theta, I_\theta} \sum_{\theta \in \Theta} p_\theta \cdot \mathrm{V}_p(t_\theta, Q_\theta, I_\theta, A_\theta) \tag{1}$$

subject to

$$V_\theta(t_\theta, Q_\theta, I_\theta, A_\theta) \geq v_\theta \quad \forall \theta \tag{2}$$

$$V_p(t_\theta, Q_\theta, I_\theta, A_\theta) \geq v_{\mathrm{p}} \quad \forall \theta \tag{3}$$

$$V_\theta(t_\theta, Q_\theta, I_\theta, A_\theta) > V_\theta(t_{\theta^-}, Q_{\theta^-}, I_{\theta^-}, A_{\theta^-}) \quad \forall \theta^- \in \Theta \setminus \theta \tag{4}$$

$$\text{with } Q_\theta \in B^m,\ I_\theta \in B^o, A_\theta \in B^n,\ t_\theta \in R^+. \tag{5}$$

The objective function (1) represents the planner's objective of maximizing her expected profit by choosing service features Q_θ, incentives I_θ, and contract price t_θ for each agent type
θ. The probability p_θ is the planner's prior belief that the agent is of type θ.

Constraints (2) and (3) ensure that the contracts meet the reservation values of the players. The constraints put upper and lower bounds on price t_θ, as illustrated in Fig. 1.

Constraint (4) represents the optimization problem of the agent in the second stage of the game. The agent will be offered $|\Theta|$ contracts, and will choose the one contract that maximizes his value. The set of contracts offered by the planner to the agent is designed such that the agent reveals his type by choosing a specific contract. Moreover, incentives are chosen by the planner such that the agent's actions maximize the planner's objective function. In mechanism design terminology, the planner's contract design and the agent's contract choice represent a Bayesian Nash equilibrium, which satisfies the incentive compatibility constraint and the revelation principle.

Our mechanism is also implementable, i.e., achieves the planner's goal of profit maximization. Maskin [9] derived the conditions under which a mechanism is implementable; however, his theorems only apply to games with three or more agents. Since our game has only two players, we adopted the subgame perfect implementation approach developed by Moore and Repullo [62]. The authors showed that the revelation principle and incentive compatibility are necessary, but not sufficient, conditions for a mechanism to be implementable. But, if the game outcome is subgame perfect, the mechanism is implementable. Since our game is formulated as a sequential game, a solution is guaranteed to be subgame perfect and thus implementable. Lastly, a revelation mechanism may have equilibria that are not agent type-revealing. By using an inequality (>) instead of ($\geq$) in constraint (4), we ensure that by choosing a contracts, the agent reveals his type. Such mechanisms are also called truthfully implementable [26].

3.2 Solution Approach

Our model is a mixed-integer optimization problem. Contract features, incentives and agent actions are binary variables; the contract price is a real variable. We used the optimization software LINGO to obtain the optimal solution to our numeric example problem, which we introduce in the following section. LINGO uses an integer solver, which generates constraint cuts to eliminate non-integer solutions of the linear program relaxation. We also used LINGO to solve the stochastic problem extension presented in Section 5. For stochastic optimizations, LINGO uses the Benders decomposition method.

To speed up the computations, we modify constraint (4) to

$$V_\theta(t_\theta, Q_\theta, I_\theta, A_\theta) - \varepsilon \geq V_\theta(t_{\theta^-}, Q_{\theta^-}, I_{\theta^-}, A_{\theta^-}) \quad \forall \theta^- \in \Theta \setminus \theta. \qquad (6)$$

Constant $\varepsilon > 0$ takes on a small real value near zero. This modification enables us to change the inequality (>) to ($\geq$), which is computationally less expensive. The constant ε needs to be sufficiently small, in particular, smaller than the difference between any two values V_θ, i.e.,

$$\varepsilon < \min_{\theta, \theta^-}[\left| V_\theta(t_\theta, Q_\theta, I_\theta, A_\theta) - V_\theta(t_{\theta^-}, Q_{\theta^-}, I_{\theta^-}, A_{\theta^-}) \right|] \qquad (7)$$

In practice, ε is typically chosen to be one or more magnitudes smaller than the other variables [63]. Optimization results would show if ε had not be sufficiently small. In our numerical examples, relevant variables values are all greater than 1, and we chose $\varepsilon = 0.1$.

4 Example Problem

In this section, we provide an example problem to illustrate the application of the contract design model. We begin by specifying and introducing additional model variables and a vector notation, which are used in the ensuing numerical example.

4.1 Additional Model Variables and Vector Formulation

In this example, the planner can offer up to three contract service features to the agent. We use a vector notation to specify this and other variables. For contract features the vector is $\mathbf{q}_{\boldsymbol{\theta}} = [q_1, q_2, q_3]^T$. We assume that the agent knows how much each contract feature is worth to him, while the planner does not know this private information of the agent. However, the planner can probabilistically (via agent type) estimate the agent's valuation. The planner considers three possible agent types $\theta_1, \theta_2, \theta_3$. We denote the value of the contract features to the agent by vector $\mathbf{v}_{\mathbf{Q}(\boldsymbol{\theta})}$. Each contract feature comes with a cost to the planner, denoted by vector $\mathbf{c}_{\mathbf{Q}(\boldsymbol{\theta})}$.

The agent type also specifies which actions $\mathbf{a}_{\boldsymbol{\theta}} = [a_1, a_2, a_3]^T$ the agent will take. Each action a_j is associated with a service contract features q_j. The inclusion of a service feature results in predictable agent behavior. The action variable is binary, and $a_j = 1$ indicates that the agent is compliant, i.e., follows the planner's recommendations and takes cost-savings measures. Action $a_j = 0$ indicates non-compliance resulting in higher cost for the planner. The cost of an agent's action to the planner is denoted by vector $\mathbf{c}_{\bar{\mathbf{A}}(\boldsymbol{\theta})}$, and the value to the agent is denoted by vector $\mathbf{v}_{\bar{\mathbf{A}}(\boldsymbol{\theta})}$. We do not directly use the action variable in the formulation of our example problem, but instead include it in the cost vector $\mathbf{c}_{\bar{\mathbf{A}}(\boldsymbol{\theta})}$ and value vector $\mathbf{v}_{\bar{\mathbf{A}}(\boldsymbol{\theta})}$.

Due to the agent's actions, the cost $\mathbf{c}_{\mathbf{Q}(\boldsymbol{\theta})}$ and the value $\mathbf{v}_{\mathbf{Q}(\boldsymbol{\theta})}$ of the contract features change. The net cost to the planner of providing the service contract features is $\bar{\mathbf{c}}_{\mathbf{Q}(\boldsymbol{\theta})} := \mathbf{c}_{\mathbf{Q}(\boldsymbol{\theta})} + \mathbf{c}_{\bar{\mathbf{A}}(\boldsymbol{\theta})}$, while the net value to the agent is $\bar{\mathbf{v}}_{\mathbf{Q}(\boldsymbol{\theta})} := \mathbf{v}_{\mathbf{Q}(\boldsymbol{\theta})} + \mathbf{v}_{\bar{\mathbf{A}}(\boldsymbol{\theta})}$. These cost and value vectors and the implicit agent actions are based on a contract without incentives.

With incentives, the agent can be motivated to change from non-compliant to compliant actions. The planner chooses whether to include incentives $\mathbf{i}_{\boldsymbol{\theta}} = [i_1, i_2, i_3]^T$. The planner's choice depends on whether providing an incentive is worth the cost. For the planner, incentives have the cost $\mathbf{c}_{\mathbf{I}(\boldsymbol{\theta})}$. In return, the planner receives the benefit of a reduction in costs associated with the now compliant actions, which change from $\mathbf{c}_{\bar{\mathbf{A}}(\boldsymbol{\theta})}$ to $\mathbf{c}_{\mathbf{A}(\boldsymbol{\theta})}$. The net cost of offering an incentive is denoted by $\bar{\mathbf{c}}_{\mathbf{I}(\boldsymbol{\theta})} := \mathbf{c}_{\mathbf{I}(\boldsymbol{\theta})} + \mathbf{c}_{\mathbf{A}(\boldsymbol{\theta})} - \mathbf{c}_{\bar{\mathbf{A}}(\boldsymbol{\theta})}$. Similarly, the agent evaluates whether he wants to accept the incentive. The value $\mathbf{v}_{\bar{\mathbf{A}}(\boldsymbol{\theta})}$ of the action before incentives now changes to the action value with incentives $\mathbf{v}_{\mathbf{A}(\boldsymbol{\theta})}$. The net value of accepting a contract with incentives is $\bar{\mathbf{v}}_{\mathbf{I(\theta)}} := \mathbf{v}_{\mathbf{I(\theta)}} + \mathbf{v}_{\mathbf{A(\theta)}} - \mathbf{v}_{\bar{\mathbf{A}}\mathbf{(\theta)}}$. Table 2 provides an overview of the newly introduced notation.

The planner's objective is to maximize price minus the cost for providing service, minus the cost of providing incentives. For the agent, the objective is to maximize the value he receives from the contract features and incentives, minus the price of the contract. We assume both agents to have linear and risk neutral value and cost preferences. The formulation of the optimization problem is:

$$\max_{t_\theta, \mathbf{q_\theta}, \mathbf{i_\theta}} \sum_{\theta \in \Theta} p_\theta \cdot (t_\theta - \bar{\mathbf{c}}_{\mathbf{Q(\theta)}} \mathbf{q_\theta} - \bar{\mathbf{c}}_{\mathbf{I(\theta)}} \mathbf{i_\theta}) \tag{8}$$

subject to

$$\bar{\mathbf{v}}_{\mathbf{Q(\theta)}} \mathbf{q_\theta} + \bar{\mathbf{v}}_{\mathbf{I(\theta)}} \mathbf{i_\theta} - t_\theta \geq v_\theta \quad \forall \theta \tag{9}$$

$$t_\theta - \bar{\mathbf{c}}_{\mathbf{Q(\theta)}} \mathbf{q_\theta} - \bar{\mathbf{c}}_{\mathbf{I(\theta)}} \mathbf{i_\theta} \geq v_{\mathrm{p}} \quad \forall \theta \tag{10}$$

$$\bar{\mathbf{v}}_{\mathbf{Q(\theta)}} \mathbf{q_\theta} + \bar{\mathbf{v}}_{\mathbf{I(\theta)}} \mathbf{i_\theta} - t_\theta > \bar{\mathbf{v}}_{\mathbf{Q(\theta)}} \mathbf{q_{\theta^-}} + \bar{\mathbf{v}}_{\mathbf{I(\theta)}} \mathbf{i_{\theta^-}} - t_{\theta^-} \quad \forall \theta^- \in \Theta \setminus \theta \tag{11}$$

$$\text{with } \mathbf{q_\theta} \in \mathbf{B^m},\ \mathbf{i_\theta} \in \mathbf{B^o},\ t_\theta \in R^+ \tag{12}$$

Table 2: Overview of vector notation

$\mathbf{q_\theta}$	Contract features	$\mathbf{i_\theta}$	Incentives
$\mathbf{c}_{\mathbf{Q(\theta)}}$	Contract feature costs	$\mathbf{v}_{\mathbf{Q(\theta)}}$	Contract feature values
$\mathbf{c}_{\mathbf{I(\theta)}}$	Incentive costs	$\mathbf{v}_{\mathbf{I(\theta)}}$	Incentive values
$\mathbf{c}_{\bar{\mathbf{A}}\mathbf{(\theta)}}$	Original action costs	$\mathbf{v}_{\bar{\mathbf{A}}\mathbf{(\theta)}}$	Original action values
$\mathbf{c}_{\mathbf{A(\theta)}}$	Modified action costs	$\mathbf{v}_{\mathbf{A(\theta)}}$	Modified action values
$\bar{\mathbf{c}}_{\mathbf{Q(\theta)}}$	Net contract feature costs	$\bar{\mathbf{v}}_{\mathbf{Q(\theta)}}$	Net contract feature values
$\bar{\mathbf{c}}_{\mathbf{I(\theta)}}$	Net incentive costs	$\bar{\mathbf{v}}_{\mathbf{I(\theta)}}$	Net incentive values

4.2 Example and Numerical Analysis

We consider three contract features that are typical options in service maintenance contracts in the gas turbine industry: 1) mission care agreement (MCA), which covers all maintenance except for damage directly caused by the agent, 2) fuel consumption guarantee, for which the planner pays for any fuel consumed above the guaranteed level, and 3) life limited parts (LLP) coverage, for which the planner replaces these parts at regular time intervals and bears the cost.

The planner is unsure about the agent type, but knows that the agent falls within one of three categories: 1) a carrier, who operates in cold regions and has low-skilled maintenance staff, 2) a carrier, who operates in sandy conditions (e.g., in the Middle East) and has basic maintenance facilities, or 3) a carrier, who operates in humid conditions (e.g., in Southeast Asia) and has state-of-the-art maintenance facilities. Furthermore, agent type 2 and 3 are sensitive to fuel costs, and they are willing to take actions to improve fuel efficiency, even at their own cost. Agent type 1 has a tight flight schedule with high utilization of aircrafts and is therefore interested in replacing parts before they reach the end of life in order to avoid unscheduled maintenance. The probability distribution of the agent type is shown in Table 3.

Table 3: Probability distribution of agent types

Agent type	θ_1	θ_2	θ_3
Probability	.5	.25	.25

In contracts with MCA, the agent may exhibit behavior that leads to additional maintenance, since the planner bears the maintenance cost. The planner can incentivize the agent to reduce unnecessary maintenance by offering to perform non-covered maintenance services at no cost, if MCA maintenance costs are below a certain threshold.

In contracts with a fuel consumption guarantee, the agent may not perform maintenance on his own terms to reduce fuel consumption. The planner can offer to perform free maintenance in return for engine-friendly operator behavior. To receive this incentive, the agent has to agree to share EHM data on a regular basis, so the planner can confirm operator compliance and can pro-actively perform fuel-saving maintenance.

In contracts with LLP coverage, the agent may request replacement of parts that still have life remaining when maintenance on other parts of the engine is performed. Each maintenance event results in aircraft downtime and the agent benefits from combining multiple maintenance activities into one. As an incentive, the planner can offer to bear some of the downtime cost, if it is economically advantageous for the planner to perform separate maintenance activities. Table 4 summarizes contract features, agent actions, and incentives for each agent type.

Table 4: Summary of contract features, agent actions, and incentives

Agent type	1) Operates in cold region, low skilled maintenance staff, tight flight schedule	2) Operates in sandy conditions, basic maintenance facility, moderately tight flight schedule, sensitive about fuel price	3) Operates in humid conditions, state-of-the-art maintenance facility, sensitive about fuel price
Contract feature	1) MCA	2) Fuel consumption guarantee	3) LLP coverage
Non-compliant agent action	Requests unnecessary maintenance	No actions to improve fuel consumption	Request replacement of parts with life remaining
Incentive	Planner pays for additional maintenance caused by agent, if maintenance requests/ cost are low.	Planner offers free maintenance to improve fuel efficiency, if agent shares data and exhibits good operating behavior.	Planner covers share of agent's downtime cost to avoid replacement of parts with remaining life.

We provide exemplary data for our numerical analysis. Table 5 shows the data for the agent, and Table 6 for the planner. The data was chosen in accordance with the agents' and planner's preferences and characteristics. For example, the numbers in the contract feature value vector $\mathbf{v_Q(\theta)}$ in Table 5 reflect that agent type 2 and 3 value the fuel consumption guarantee more than type 1. It also shows that agent type 1, and to a lesser extent agent type 2, value the MCA and LLP coverage more than agent type 3, since agent type 3 has insufficient maintenance capabilities. We further assume a reservation value of $v_\theta = 5$ for the agent, and $v_P = 3$ for the planner.

Table 5: Values of contract features, incentives, and actions

Agent type	*Contract feature value* $\mathbf{v_Q(\theta)}$	*Incentive value* $\mathbf{v_I(\theta)}$	*Original action value* $\mathbf{v_{\bar{A}}(\theta)}$	*Modified action value* $\mathbf{v_A(\theta)}$
θ_1	[70, 40, 32]	[7, 3, 2]	[9, 6, 2]	[5, 5, 4]
θ_2	[55, 60, 29]	[5, 5, 2]	[12, 7, 3]	[3, 5, 5]
θ_3	[30, 60, 25]	[3, 6, 2]	[7, 10, 4]	[7, 6, 4]

Table 6: Costs of contract features, incentives, and actions

Agent type	*Contract feature cost* $\mathbf{c_Q(\theta)}$	*Incentive value* $\mathbf{c_I(\theta)}$	*Original action cost* $\mathbf{c_{\bar{A}}(\theta)}$	*Modified action cost* $\mathbf{c_A(\theta)}$
θ_1	[50, 40, 30]	[4, 5, 2]	[10, 5, 3]	[6, 3, 8]
θ_2	[50, 40, 30]	[4, 5, 2]	[8, 5, 4]	[7, 2, 6]
θ_3	[50, 40, 30]	[4, 5, 2]	[5, 6, 4]	[5, 4, 4]

We apply the data to the optimization problem (6-10) to find the optimal set of three contracts that the planner should offer to the agent. Table 7 summarizes the results. For agent type 1, the optimal contract includes MCA, fuel guarantee and LLP coverage and should be offered at a price of 157. It is optimal to offer an incentive for MCA, but not for the other service features. For agent type 2, the optimal contract offers MCA and the fuel guarantee at price 131, with incentive only for the fuel guarantee. For agent type 3, only the fuel consumption guarantee should be offered at a price of 65, without any incentives. Given the probability distribution of agent types, the expected profit is 20.75. Per design, the optimal contracts are unique for each agent type and the agent will prefer the contract designed for him over the other contracts. Consequentially, the agent will reveal its type by making his optimal choice.

Table 7: Optimal contract design for each agent type

Agent type	*Optimal contract set*	*Optimal incentive set*	*Price*	*Planner's profit*
θ_1	[1, 1, 1]	[1, 0, 0]	157	19
θ_2	[1, 1, 0]	[0, 1, 0]	131	26
θ_3	[0, 1, 0]	[0, 0, 0]	65	19

5 Stochastic Model Extension

In the previous example, we assumed that the actions of the agent, engine performance and resulting costs and values are deterministic. In the real world, however, these variables come with uncertainty and thus need to be modeled probabilistically. Only after the contract is signed, and the engine is in operation, will agent actions and engine performance become known.

We can differentiate four contract phases, which are shown in Table 2. In the first phase, the contract offer phase, the planner designs a contract that maximizes her expected profit under uncertainty. This time, the uncertainty is not only about agent type, but also related to agent actions and engine performance. As mentioned before, this phase is also called a screening game, since the design of the optimal contract set will screen for the agent type.

In the second phase, the contract acceptance phase, the agent chooses among contract alternatives, of which at least one meets his reservation value and is acceptable. By choosing his optimal contract, the agent signals his type to the planner. This phase is therefore also called a signaling game. In his decision of which contract alternative to choose, the agent consider the uncertainties associated with receiving an incentives, since engine performance is only probabilistically correlated with his actions.

In the third phase, the uncertainties of agent actions and engine performance will be resolved. The agent decides whether he wants to take actions in response to incentives offered by the contract. Compliant actions increase the probability, but do not guarantee the receipt on the incentive. The engine performance and resulting cost is another factor.

Our model considers these three stages, but one could consider a fourth stage of contract renewal. The duration of contracts for gas turbine engines are typically around ten years in the industry. After the contract expires, a new contract can be negotiated. The information discovered during contract acceptance (agent type) and contract execution (engine performance) will inform the contract renewal. The new contract can be designed in the same fashion as before, starting with a contract offer by the planner and a contract acceptance by the agent.

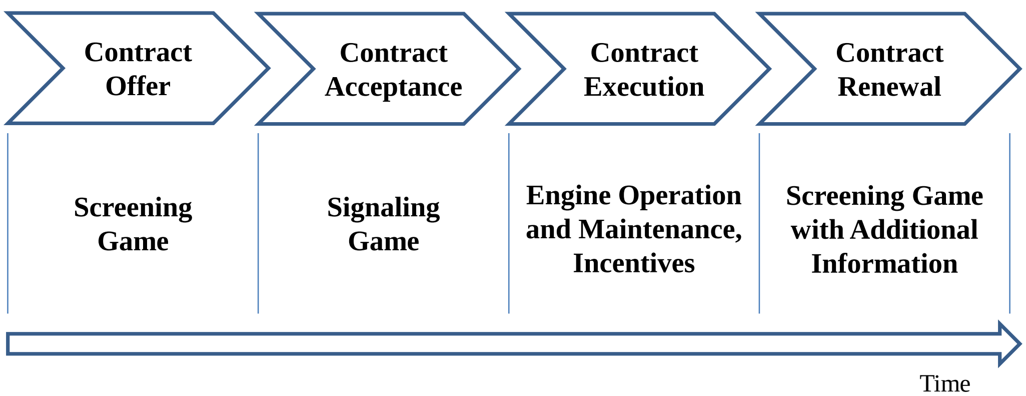

Fig. 2. Contract phases

5.1 Stochastic Formulation

The general form of a stochastic two-stage linear program [64] that extends our deterministic model is

$$\text{Max } \mathrm{f}\,(x) = cx + E[h(x,\omega)] \quad (13)$$
$$\text{s.t. } x \in \mathbf{X} \quad (14)$$

where **X** is a convex polyhedral set, and where

$$h(x,\omega) = \text{Max } gy \quad (15)$$
$$\text{s.t. } Wy = \omega - Tx, \quad (16)$$
$$y \geq 0\ . \quad (17)$$

In the first stage of the stochastic optimization (13-14), the decision variable x is determined. The objective function (13) depends on *x*, but also on random variable ω. In our example, *x* refers to the contract features, and ω refers to the engine performance, agent's actions and resulting maintenance cost. In the second stage, the planner tries to find an optimal *y*, which in our case is the incentive mechanism.

Based on this general form of the stochastic two-stage optimization problem, we derive the specific formulation for our example problem. We modify the deterministic variable notation to account for the fact that variables are now probability distributions. Except for $\mathbf{v}_{\mathbf{Q(\theta)}}$ and $\mathbf{c}_{\mathbf{Q(\theta)}}$, which remain deterministic, we add ω to the subscript of all vectors from our original notation.

The model, using our vector notation, becomes:

<u>First Stage Problem:</u>

$$\max_{t_\theta,\mathbf{q_\theta}}\ f(t_\theta,\mathbf{q_\theta}) = \sum\nolimits_{\theta\in\Theta} p_\theta \cdot (t_\theta - \mathbf{c}_{\mathbf{Q(\theta)}}\mathbf{q_\theta}) - E[h(\mathbf{q_\theta},\omega)] \quad (18)$$

subject to

$$\mathbf{v}_{\mathbf{Q(\theta)}}\mathbf{q_\theta} - t_\theta \geq v_\theta \ \ \forall\theta \quad (19)$$
$$t_\theta - \mathbf{c}_{\mathbf{Q(\theta)}}\mathbf{q_\theta} \geq v_{\mathrm{p}} \ \ \forall\theta \quad (20)$$
$$\text{with } \mathbf{q_\theta} \in \mathbf{B^m},\ t_\theta \in R^{+} \quad (21)$$

<u>Second Stage Problem:</u>

$$h(\mathbf{q_\theta},\omega) = \min_{\mathbf{i_\theta}}\ \sum\nolimits_{\theta\in\Theta} p_\theta \cdot (\overline{\mathbf{c}}_{\mathbf{I(\theta},\omega)}\mathbf{i_\theta}) \quad (22)$$

subject to

$$\overline{\mathbf{v}}_{\mathbf{Q(\theta},\omega)}\mathbf{q_\theta} + \overline{\mathbf{v}}_{\mathbf{I(\theta},\omega)}\mathbf{i_\theta} - t_\theta \geq v_\theta \ \ \forall\theta \quad (23)$$
$$t_\theta - \overline{\mathbf{c}}_{\mathbf{Q(\theta},\omega)}\mathbf{q_\theta} - \overline{\mathbf{c}}_{\mathbf{I(\theta},\omega)}\mathbf{i_\theta} \geq v_{\mathrm{p}} \ \ \forall\theta \quad (24)$$
$$\overline{\mathbf{v}}_{\mathbf{Q(\theta},\omega)}\mathbf{q_\theta} + \overline{\mathbf{v}}_{\mathbf{I(\theta},\omega)}\mathbf{i_\theta} - t_\theta > \overline{\mathbf{v}}_{\mathbf{Q(\theta},\omega)}\mathbf{q_{\theta^-}} + \overline{\mathbf{v}}_{\mathbf{I(\theta},\omega)}\mathbf{i_{\theta^-}} - t_{\theta^-} \ \ \forall\theta^- \in \Theta\setminus\theta \quad (25)$$
$$\text{with } \mathbf{i_\theta} \in \mathbf{B^o} \quad (26)$$

The goal of the two-stage stochastic program is to find the first stage decisions and recourse decisions to maximize the planner's expected profit. The first stage decision variables are t_θ, $\mathbf{q_\theta}$. The value of h is determined by the recourse variable $\mathbf{i_\theta}$, which becomes a decision variable in the second stage.

Solving the two-stage stochastic program requires an iterative two-step procedure. In step 1, we initialize the iteration algorithm by solving (18-21) with $\mathbf{i_\theta} = \mathbf{0}$ and $E[h(\mathbf{q_\theta}, \omega)] = 0$. The optimal solutions for t_θ and $\mathbf{q_\theta}$ of step 1 are then applied to the optimization problem (22-26). In this second step, new values for $\mathbf{i_\theta}$ and $E[h(\mathbf{q_\theta}, \omega)]$ are determined. These value are then applied again to the optimization problem (18-21) of step 1.The algorithm continues in this fashion until the solution converges and optimal values for t_θ, $\mathbf{q_\theta}$ and $\mathbf{i_\theta}$ are found.

The main difficulty of a stochastic integer problem lies in the non-convex and discontinuous recourse objective function. To address this challenge, one can relax the integer constraint of decision variables and apply stochastic optimization techniques without affecting the structure of the problem. Laporte and Louveaus [65] presented the L-shaped method for stochastic integer programs inspired by Benders decomposition. Caroe and Tind [66] proposed a cutting-plane approach to mixed 0-1 stochastic integer programs. Sakalauskas [67] applied Monte-Carlo estimator approach to nonlinear stochastic programming.

5.2 Example and Numerical Analysis

We provide a modified version of the example presented above. The agent type characteristics are summarized in Table 8. In this example, we consider two possible agent types with equal probability (Table 9). The value and cost vectors of agent and planner are provided in Table 10. The table shows that agent type 1 values the fuel consumption guarantee more than agent type 2 due to his fuel price sensitivity. Agent type 2, on the other hand, values the LLP coverage more, since he does not have good maintenance operations.

Table 8: Summary of contract features, incentives, and the agent actions

Agent type	1) Operates in cold region, state-of-the-art maintenance facility, tight flight schedule, sensitive about fuel price	2) Operates in humid region, low skilled maintenance staff
Contract feature	1) Fuel consumption guarantee	2) LLP coverage
Non-compliant agent action	Less likely to perform maintenance to increase the fuel efficiency	More likely to request replacement of parts with life remaining
Incentive	Planner pays for agent's maintenance cost, if EHM data shows agent's effort to save fuel	Planner covers share of agent's downtime cost to avoid replacement of parts with remaining life.

Table 9: Probability distribution of agent types

Agent type	θ_1	θ_2
Probability	.5	.5

Table 10: Values and costs of contract features

Agent type	*Contract feature value* $\mathbf{v_Q(\theta)}$	*Contract feature cost* $\mathbf{c_Q(\theta)}$
θ_1	[30, 15]	[20, 10]
θ_2	[20, 35]	[15, 15]

Table 11 provides the probability distributions of the values and costs given original and modified actions. The original and modified actions have the same values and costs, but the probability of the agent taking the original (non-compliant) action decreases as the planner provides incentives. For example, the probability of agent type 1 taking the non-complaint action 1 is 0.5, but the probability decreases to 0.25 with incentive. We assume that the valuation by both agents is equal, i.e., costs and values take on the same numeric values.

The value the agent receives from an incentive is shown in Table 12. Table 13 shows the corresponding cost of the incentive for the planner. The tables reflect that while the different agent types value the incentives differently, the cost for providing the incentive is the same for the planner in both cases.

Table 11: Probability distribution of values and costs

Variable	*Original action 1*	*(Prob.)*	*Original action 2*	*(Prob.)*	*Variable*	*Modified action 1*	*(Prob.)*	*Modified action 2*	*(Prob.)*
$\mathbf{v_{\bar{A}}(\theta_1), c_{\bar{A}}(\theta_1)}$	10	(.5)	5	(.2)	$\mathbf{v_A(\theta_1), c_A(\theta_1)}$	10	(.25)	5	(.1)
	0	(.5)	0	(.8)		0	(.75)	0	(.9)
$\mathbf{v_{\bar{A}}(\theta_2), c_{\bar{A}}(\theta_2)}$	3	(.33)	12	(.6)	$\mathbf{v_A(\theta_2), c_A(\theta_2)}$	3	(.17)	12	(.4)
	0	(.67)	0	(.4)		0	(.83)	0	(.6)

Table 12: Probability distribution of agent's incentive value

Variable	*Incentive 1*	*(Prob.)*		*(Prob.)*	*Incentive 2*	*(Prob.)*		*(Prob.)*
$\mathbf{v_I(\theta_1)}$	2	(.5)	0	.5	2	(.75)	0	(.25)
$\mathbf{v_I(\theta_2)}$	3	(.5)	0	.5	3	(.33)	0	(.67)

Table 13: Probability distribution of planner's incentive cost

Variable	*Incentive 1*		*Incentive 2*	
		(Prob.)		*(Prob.)*
$\mathbf{c_I(\theta_1)}$	2	(.5)	6	(.25)
	0	(.5)	0	(.75)
$\mathbf{c_I(\theta_2)}$	4	(.25)	8	(.125)
	0	(.75)	0	(.825)

We compute the optimal contract set using LINGO, as described in Section 3.2. The results are summarized in Table 14. The optimal contract set for agent type 1 includes the fuel consumption guarantee, but no incentives. The contract is offered at a price of 34.9 and results in an expected profit of 9.9 to the planner. The optimal contract set of type 2 includes both fuel guaranteed and LLP coverage, and incentives associated with LLP coverage are provide. The contract is offered at a price of 62.2 and results in an expected profit of 24.9. The overall expected profit of the contract set, given equal probability of both agent types, is 17.4.

Table 14: Optimal contract design for each agent type

Agent type	*Optimal contract set*	*Optimal incentive set*	*Price*	*Planner's expected profit*
θ_1	[1, 0]	[0, 0]	34.9	9.9
θ_2	[1, 1]	[0, 1]	62.2	24.9

The profit that the planner realizes in the end depends on the realization of the random variables. In particular, the agent may decide not to take a compliant action even when an incentive is offered. If the agent takes non-compliant actions, despite incentives, the planner's profits drops to 3.9 and 18.1 for agent type 1 and 2, respectively, compared to the expected profit of 9.9 and 24.9. If on the other hand, the agent takes the compliant actions, the planners profit will rise to 15.9 and 33 for agent type 1 and 2, respectively.

Furthermore, receipt of the incentive for the agent is uncertain since it depends on engine performance. Compliant actions increase the chance of receiving an incentive, but the outcome is probabilistic. The relationship between the agent's action, engine performance, and probability of receiving an incentive is shown in Figure 3. For the agent's action, we assumed in this graph that actions are not just binary (compliant, non-compliant), but can be continuous, which can be interpreted as a degree of compliance. The figure shows how the probability of receiving an incentive by the agent increases with a higher degree of action compliance and better engine performance.

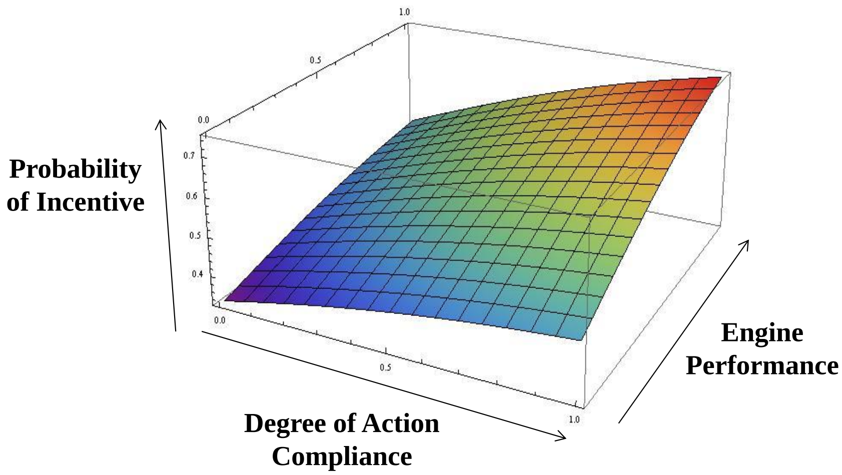

Fig. 3. Relationship between agent's actions, engine performance and incentive

6 Future Research

This paper demonstrated a novel approach to optimize the service contract design. However our research raises additional questions to be considered and answered in the future research.

The first question is how to capture dynamic behavior of the agent type. The service contract duration normally lasts more than 10 years. As the contract duration prolongs, the chance of the agent changing type (due to changes in his business strategy or environment) increases. Our service contract design mechanism identifies the agent type only once in the beginning of the contract and regards it unchanged during the contract term. To mitigate uncertainties causes by the dynamic agent type during the contract duration, ideas from stochastic game theory could be applied [68]. However the study by Shapley [68] demonstrated in game theoretical perspective. There are challenges to apply his study in a mechanism design framework.

The second question is how to improve our algorithm in the computational perspective. The biggest challenge in computation is the incentive compatibility constraint of our formulation. If n possible agent types are considered in the problem, then $n(n-1)$ incentive compatibility constraints are needed in the formulation. As the number of the possible agent type increases, our problem becomes more expensive in computation. We propose the stratified sampling technique to improve the computation efficiency [69]. Our problem inherits a very well defined structure of strata, agent types. The sample space is divided into subgroups of each agent type, and is used in stratified sampling. This technique guarantees improvement in computation.

7 Conclusion

We applied mechanism design theory to the problem of designing service maintenance contracts. In particular, we discussed maintenance service contracts in the gas turbine industry and provided real word based examples.

We formulated the maintenance service contract problem as a two-stage sequential optimization problem using mechanism design principles, and presented a modeling and solution approach for a deterministic and a stochastic case. We first analyzed the deterministic case, where customer actions and future outcomes are predictable, followed by a stochastic extension, which accounted for uncertainties in these variables. In both cases, an optimal contract set is computed, which is offered in its entirety to the customer. We provided a numerical example for the deterministic and the stochastic case that illustrated our modeling and solution approach.

The mathematical formulation and resulting solutions satisfy the key concepts of mechanism design theory, which are the revelation principle, incentive compatibility and implementability. Offering a specifically designed menu of contract alternatives ensures that in choosing its most preferred contract, the customer reveals its private information (revelation principle). This information can be used by the contract provider to better plan for and manage maintenance efforts. The information can also be used in future contract negotiations and helps to mitigate the adverse selection problem.

The contracts include incentives that align the interests of contract provider and customer and thereby minimizes the problems associated with moral hazards (incentive compatibility). As the analysis of the stochastic model showed, these incentives mitigate, but do not fully eliminate (as in the deterministic case), the moral hazard problem. The actions by the customer are in response to future outcomes, which are uncertain at the time of contracting. Similarly, compliant actions by the customer merely increase, but do not guarantee, the chance of receiving an incentive. Maintenance events and cost, which are the basis for determining the incentives, are favorably influenced by compliant customer actions, but a degree of uncertainty remains. Still, incentive compatible contracts create win-wins for customer and contract provider, i.e., the expected profit increase for the contracts provider and the expected cost savings for the customer are positive.

Lastly, the optimal contract set meets the provider's objective of profit maximization and ensures that negotiations can be avoided (implementability). The objective function of the optimization problem maximizes the expected profit over the various possible agent characteristics (types). In the constraints, a minimum reservation value condition for the customer ensures that at least one contract in the contract set is acceptable.

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