

# ECONOMIC ANALYSIS OF THE VIRGINIA STEAM COAL MARKET

by

Martin L. Smith

Dissertation submitted to the Faculty of the  
Virginia Polytechnic Institute and State University  
in partial fulfillment of the requirements for the degree of  
Doctor of Philosophy  
in  
Mining and Minerals Engineering

APPROVED:

\_\_\_\_\_  
E. Topuz, Chairman

\_\_\_\_\_  
W. Hibbard

\_\_\_\_\_  
S. C. Sarin

\_\_\_\_\_  
J. R. Lucas

\_\_\_\_\_  
~~M. Karmis, Head~~

February, 1988  
Blacksburg, Virginia

# ECONOMIC ANALYSIS OF THE VIRGINIA STEAM COAL MARKET

by

Martin L. Smith

Committee Chairman: Dr. Ertugrul Topuz

Mining and Minerals Engineering

(ABSTRACT)

CSL 6/29/88

In recent years the Central Appalachian coal industry has suffered from a number of changes in the structure of the coal market. Foremost among these changes have been the collapse of the domestic steel industry and the passage of the Staggers Act. In the past high quality Central Appalachian coal was sold mainly as premium coking coal. This market failed, and continues to shrink. Regional coal producers are now looking to the rising demand for steam coal in a nation which is turning away from oil and nuclear power generation. With the possible passage of the Clean Air Act, low sulfur Central Appalachian coal may have a promising future, but only if its production can reach this new market.

Prior to 1980, regulated rail tariffs gave coal producers access to most consumers, while independent railroads competed for freight. Railroad deregulation greatly improved the position of the railroads, but weakened that of

regional coal producers. Central Appalachia and the southern coastal states were left with only two railroads, CSX and NS. These railroads now set their own rates and secretly negotiate contracts with shippers. Due to the configuration of ownership of these tracks, the vast majority of mines and utility plants were left with access to only one carrier. In this situation rail transportation has become the primary concern. Mines unable to originate shipments on the same railroad which is serving the utility plant find themselves locked out of that market unless their mine price is sufficiently low enough to compensate for the increased rail rate. Most of the coal burning plants in Virginia are located on CSX, while the vast majority of southwest Virginia's coal production is served by NS. With a higher mining cost than in eastern Kentucky and southern West Virginia, Virginia producers are having great difficulty in competing in the state's steam coal market. This dissertation is the first effort to quantitatively specify the regional steam coal market. This is done by identifying sources of data which are subsequently used to generate short term forecasts of mine price. These forecasts are used in the cost vector of a Virginia Coal Purchasing model which is solved to determine the optimal pattern of coal purchases for Virginia utilities. The structure of the model is based on a detailed market analysis which accounts for the influence of rail rates.

## ACKNOWLEDGEMENTS

I would like to thank my dissertation advisor, Dr. Ertugrul Topuz, for his patience and encouragement during the course of my research.

Special appreciation is extended to Dr. Walt Hibbard, who is the fountainhead of this research project both conceptually and financially through the auspices of the Virginia Mining and Minerals Resource Research Institute. Thanks are extended to Dr. Subash Sarin for assistance during the initial formulation of the optimization model, and to Dr. Robert Foutz for his generous advice in developing the Time Series models. Thanks are also extended to Dr. Michael Karmis and Dr. Richard Lucas for their review of this manuscript.

I extend much appreciation to Miss Carol White for her considerable help, and to my daughter, Amy Smith, for the many patient hours spent amusing herself while I worked.

Finally, much thanks to the many people who helped and encouraged me during my tenure, especially my parents, Raymond and Beatrice Smith, without whom none of this would have been possible.

# Table of Contents

Abstract .....	ii
Acknowledgements .....	iv
Table of Contents .....	v
List of Figures .....	ix
List of Tables .....	xi
Chapter 1: Introduction .....	1
1.1 Background .....	2
1.2 Study Objectives and Approach .....	16
Chapter 2: Liturature Review .....	20
2.1 Introduction .....	20
2.2 Survey of Relevant Energy Models .....	25
2.3 Applicability of Past Models .....	56
Chapter 3: Data Availability and Analysis .....	62
3.1 Introduction .....	62
3.2 Records of Individual Deliveries .....	63
3.2.1 Data Format .....	64
3.2.2 Source Identification.....	66
3.3 Source Specific Information .....	68
3.3.1 Coal Mining Directories .....	68
3.3.2 Keystone .....	70
3.3.3 MSHA Address/Employment Tapes .....	72
3.4 Railroad Specific Information .....	73

3.5 Utility Specific Information .....	76
3.6 Information on Existing Contracts .....	77
3.6.1 Coal Supply Contracts .....	77
3.6.2 Coal Haulage Contracts .....	79
 Chapter 4: Market Analysis .....	 80
4.1 Introduction .....	80
4.2 Regional Scope of Model .....	85
4.3 Supply .....	87
4.3.1 Long Term Contract Supply .....	89
4.3.2 Spot Market Supply .....	89
4.3.3 Determining Regional Division for Spot Supply .....	91
4.4 Factors Affecting Rail rates .....	108
4.5 Influence of Model Structure on Model Development .....	114
 Chapter 5: Forecasting of Mine Price to Virginia Utilities .....	 116
5.1 Introduction .....	116
5.2 Division of Forecast Regions .....	117
5.2.1 Long Term Market .....	117
5.2.2 Spot market .....	118
5.3 Forecasting Methodology .....	120
5.3.1 Inadequacies of Regression Modeling .....	121
5.3.2 Time Series models .....	131

5.4 Application of Time Series to Forecasting	
Mine Price .....	134
5.4.1 The Presence of Cross Sectional Data ....	134
5.4.2 Instability of The Market .....	144
5.4.3 Insufficient Data .....	149
5.4.4 Forecasting With Limited Data .....	151
5.4.5 Summary of The Forecasting Procedure ....	153
 Chapter 6: Development of a Virginia Utility Coal	
Purchasing Model .....	158
6.1 Model Assumptions and Approach .....	158
6.2 General Utility Supply Model .....	161
6.3 The Virginia Coal Purchasing Model .....	168
 Chapter 7: Model Input, Output, and Results .....	175
7.1 Model Input .....	175
7.1.1 Cost Vector .....	175
7.1.2 RHS Vector .....	179
7.2 Model Output .....	185
7.2.1 Sources in Basis .....	185
7.2.2 Nonbasis .....	188
7.2.3 Comparison of Actual vs. Optimal Purchases .....	194
7.2.4 Sensitivity Analysis .....	205
7.3 Parametric Analysis .....	212
7.3.1 Applications .....	212
7.3.2 Methodology .....	215

7.4 Supply and Demand Curves .....	217
7.4.1 Assumptions and Curves .....	218
7.4.2 Discussion of Results .....	225
Chapter 8: Conclusions and Recommendations .....	235
8.1 Perspective .....	235
8.2 Pioneering Aspects of the Study .....	237
8.3 The Steam Coal Market .....	239
8.4 Forecasting Mine Price .....	241
8.5 Modeling & Results .....	243
8.6 Recommendations .....	247
Bibliography .....	250
Appendix A: Virginia Coal Purchasing Model:	
LINDO formatted Input .....	256
Appendix B: Model Input and Sensitivity Analysis	
Output .....	269
Appendix C: Model Estimates and Forecasts for Contract	
Sources .....	282
Appendix D: Data Formats and Programs for Reading WVPSC	
HB966 and MSHA Work/Address Tapes .....	303
Vita .....	312

## List of Figures

Figure 1.1	Coal Counties of Central Appalachia and Rail Network .....	4
Figure 1.2	CSX Rail Network .....	5
Figure 1.3	NS Rail Network .....	6
Figure 1.4	Major Coal Burning Plants in Virginia .....	7
Figure 1.5a	Southwest Virginia Coalfield Rail System .....	8
Figure 1.5b	Southwest Virginia Coalfield .....	9
Figure 4.1	Time Trend of Quantity Delivered by State ...	84
Figure 4.2	Scatter Plot for Virginia to Chesterfield ..	102
Figure 4.3	Time Trend of Mine Price in Spot Market ....	106
Figure 4.4	Time Trend of Rail Rates for the Spot Market .....	107
Figure 4.5	Centered Rates and Quantities for Various Tonnages Shipped .....	111
Figure 4.6	Residual Rates and Quantities for Various Tonnages Shipped .....	113
Figure 5.1	Virginia Energy and Harman Mining to Virginia Power .....	130
Figure 7.1	Actual Demand Curve .....	228
Figure 7.2	Cumulative Supply (Actual): Kentucky Spot Sales .....	229
Figure 7.3	Cumulative Supply (Actual): West Virginia Spot	

	Sales .....	229
Figure 7.4	Cumulative Supply (Actual): Virginia Spot Sales .....	230
Figure 7.5	Marginal Cost (Supply) Curves: Virginia Spot Sales .....	231
Figure 7.6	Demand Curve: Virginia Spot Sales .....	231
Figure 7.7	Supply and Demand Curves: Kentucky Spot Sales .....	232
Figure 7.8	Supply and Demand Curves: West Virginia Spot Sales .....	232
Figure 7.9	Key for Combining Actual and Model Generated Curves .....	233
Figure 7.10	Combined Supply and Demand Curves .....	234
Figure C.1	Plot of Forecast and Actual Mine Price: Robinson Creek to Virginia Power .....	285
Figure C.2	Plot of Forecast and Actual Mine Price: Clinch- field to APCO .....	290
Figure C.3	Plot of Forecast and Actual Mine Price: GEX to Virginia Power .....	294
Figure C.4	Plot of Forecast and Actual Mine Price: Virginia Energy to Virginia Power .....	298
Figure C.5	Plot of Forecast and Actual Mine Price: Harman Mining to Virginia Power .....	302

# List of Tables

Table 1.1	Proportion of Virginia Coal Burned in Virginia Power Plants .....	14
Table 4.1	Counties Captive to a Single Carrier .....	93
Table 4.2	ANOVA of Mine Price Between Counties .....	94
Table 4.3	Comparison of Mean Rates .....	96
Table 4.4	ANOVA of Mine Price and Range Tests Between Freight Districts and Vepco Plants .....	98
Table 4.5	Pearson's Correlation Coefficients Between Rate and Mine Price .....	101
Table 4.6	Comparison of Rate and Mine Price Between Single Origin/Destination Pairs for the Period 6/85-12/86 .....	104
Table 5.1	OLS Results for Russell to APCO .....	128
Table 5.2	GLS Results for Russell to APCO .....	128
Table 5.3	OLS and GLS estimates of Spot Mine Price to Virginia Power .....	140
Table 5.4	Crosscorrelogram Between Btu and MinePrice ..	141
Table 7.1	Model Coding System .....	176
Table 7.2	Delivered Cost for Contract Sources .....	177
Table 7.3	Delivered Cost for Spot Sources .....	178
Table 7.4	Productivity Rating and Average Quality for Contract Sources .....	181
Table 7.5	Monthly Coal Consumption and Initial and Final Stockpiles .....	184
Table 7.6	Deliveries to Virginia Power .....	187

Table 7.7	Comparison of Lower 95% Confidence Interval With Entering Level Of Nonbasic Variables (all sources on contract i&L) .....	190
Table 7.8	Comparison of Lower 95% Confidence Interval With Entering Level Of Nonbasic Variables (all spot sources i&S) .....	191
Table 7.9	Delivered and Reduced Cost of Nonbasic Variables w/o Estimated Confidence Limits .....	193
Table 7.10a	Actual Deliveries (tons) on Contract to Virginia Power .....	199
Table 7.10b	Comparison of Deliveries From Virginia: Model Results vs. Actual .....	200
Table 7.10c	Comparison of Deliveries From West Virginia: Model Results vs. Actual .....	201
Table 7.10d	Comparison of Deliveries From Kentucky: Model Results vs. Actual .....	202
Table 7.11	Actual Spot Deliveries (Tons) to Virginia Power .....	204
Table 7.12	Sensitivity Analysis .....	211
Table C.1	Forecast and Model Estimate: Landmark to Virginia Power .....	283
Table C.2	Forecast and Model Estimate: Robinson to Virginia Power .....	284
Table C.3	Forecast and Model Estimate: Cobra Energy to Virginia Power .....	286
Table C.4	Forecast and Model Estimate: Clinchfield to APCO .....	289

Table C.5	Forecast and Model Estimate: Enterprise to Virginia Power .....	291
Table C.6	Forecast and Model Estimate: GEX to Virginia Power .....	292
Table C.7	Forecast and Model Estimate: Majestic to Virginia Power .....	295
Table C.8	Forecast and Model Estimate: Virginia Energy to Virginia Power .....	296
Table C.9	Forecast and Model Estimate: Wellmore to Virginia Power .....	299
Table C.10	Forecast and Model Estimate: Harman Mining to Virginia Power .....	300

# CHAPTER 1

## Introduction

Coal has long been the dominant economic force in southwest Virginia, the neighboring counties of southern West Virginia, and eastern Kentucky. In recent years the importance of coal in southwest Virginia has not diminished, yet there has been an ongoing deterioration of the competitive position of the state's coal industry. The specter of severe regional unemployment in a number of Virginia's coal producing counties has generated considerable interest in the nature of the regional coal market. This thesis is part of the effort to understand the structure and dynamics of the regional coal market: specifically, the steam coal market for Virginia utilities from mining sources in Central Appalachia. This market has been the subject of considerable qualitative analysis, but the following is the first development of a quantitative methodology for specifying and modeling the Virginia market for steam coal. This methodology includes:

- Data acquisition and analysis

- Statistical and economic specification of the market structure of steam coal deliveries
- A forecasting methodology for mine price
- Mathematical model development, implementation, and post optimality analysis

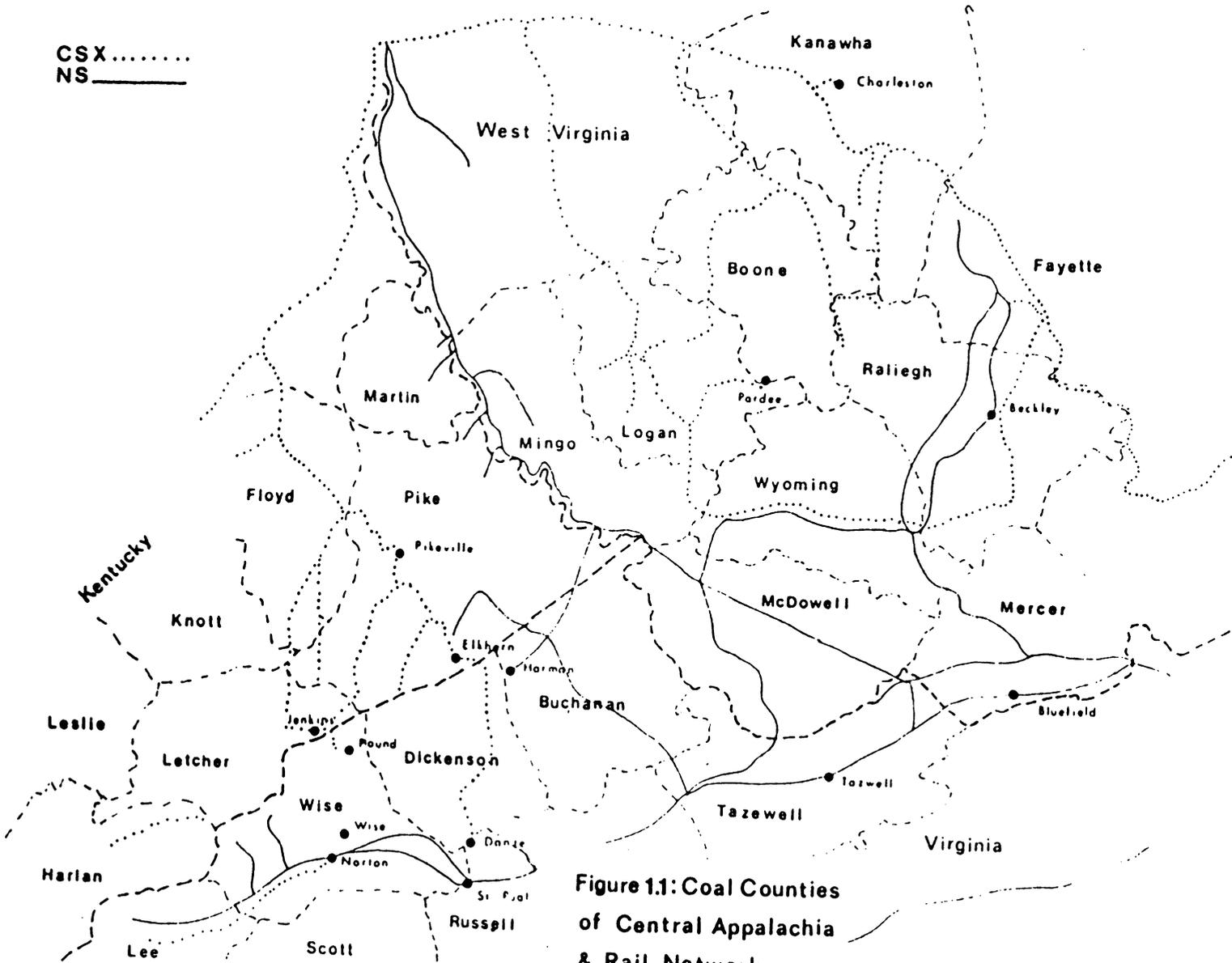
An introduction to the background of the Central Appalachian coal industry will be given in this chapter before entering into a predominately technical discussion of these four topics, and a market description of the results.

## 1.1 Background

The coal mining region referred to as Central Appalachia includes southwestern Virginia, southern West Virginia, and eastern Kentucky. Eastern Tennessee is also part of this region (coal districts 7 and 8), but is not a supplier of coal to Virginia Utilities. Figure 1.1 shows the coal mining counties of these three states including main rail lines. The entire region's rail network is served by two large corporations: Norfolk Southern (NS) and the Chessie System (CSX). More detailed maps for these two systems are given in Figures 1.2 and 1.3. Figure 1.4 shows the major Power Plants in Virginia which burn coal, including their connecting railroads. It should be noted that all Central

Appalachian coal reaches its domestic destinations by railroad, except for some transshipment via barge on the intercoastal waterway. Examination of Figures 1.1 - 1.4 shows that the two railroads serve certain key origins and destinations. Except for a few exceptions, all mines and utility plants are located on only one of the two carriers. Only four Virginia mines are served by both railroads; none of the coal consumers have the benefit of multiple rail access. Comparison of CSX and NS lines shows little access by CSX to Virginia's coal mines except for Dickenson county and a few mines in Wise county (see Figure 1.5). 371 out of 419 mines in Virginia are located on NS lines. Thus, any study of the competitive position of Virginia coal must account for the impact of NS's position regarding the delivery of coal and the setting of rail rates.

Central Appalachian coal producers are inextricably linked to their coal carriers. Because of this circumstance, this study gives greater emphasis to the role of the rail carrier than have previous coal supply models. The dependence of Virginia producers on rail transport is not a recent phenomenon. The relationship between the coal producers and railroads has been symbiotic, even though often adversarial. Large scale coal mining was not possible until railroads provided transportation to distant markets, while coal is the railroad companies' largest single source of revenue.



**Figure 1.1: Coal Counties  
of Central Appalachia  
& Rail Network**

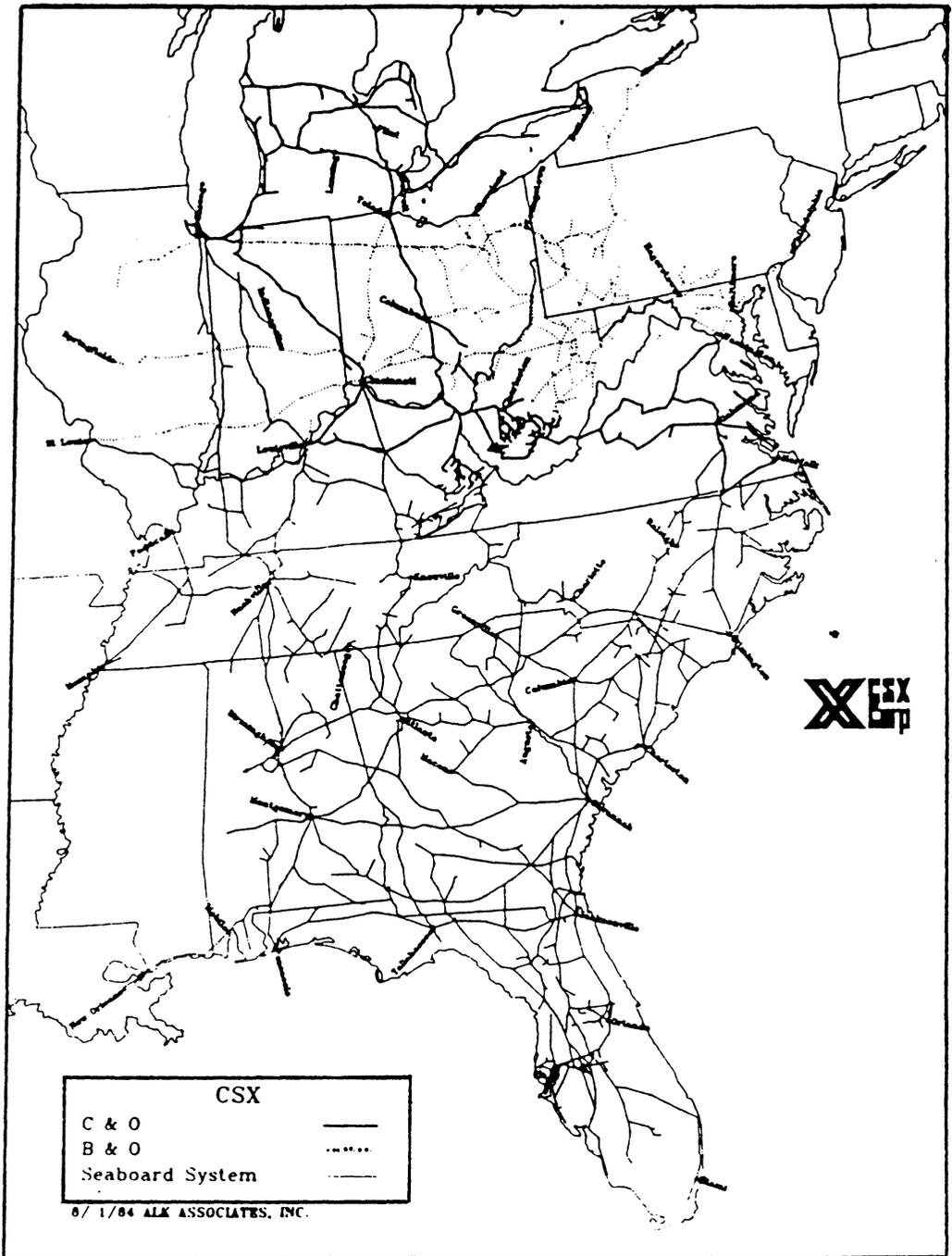


Figure 1.2: CSX Rail Network

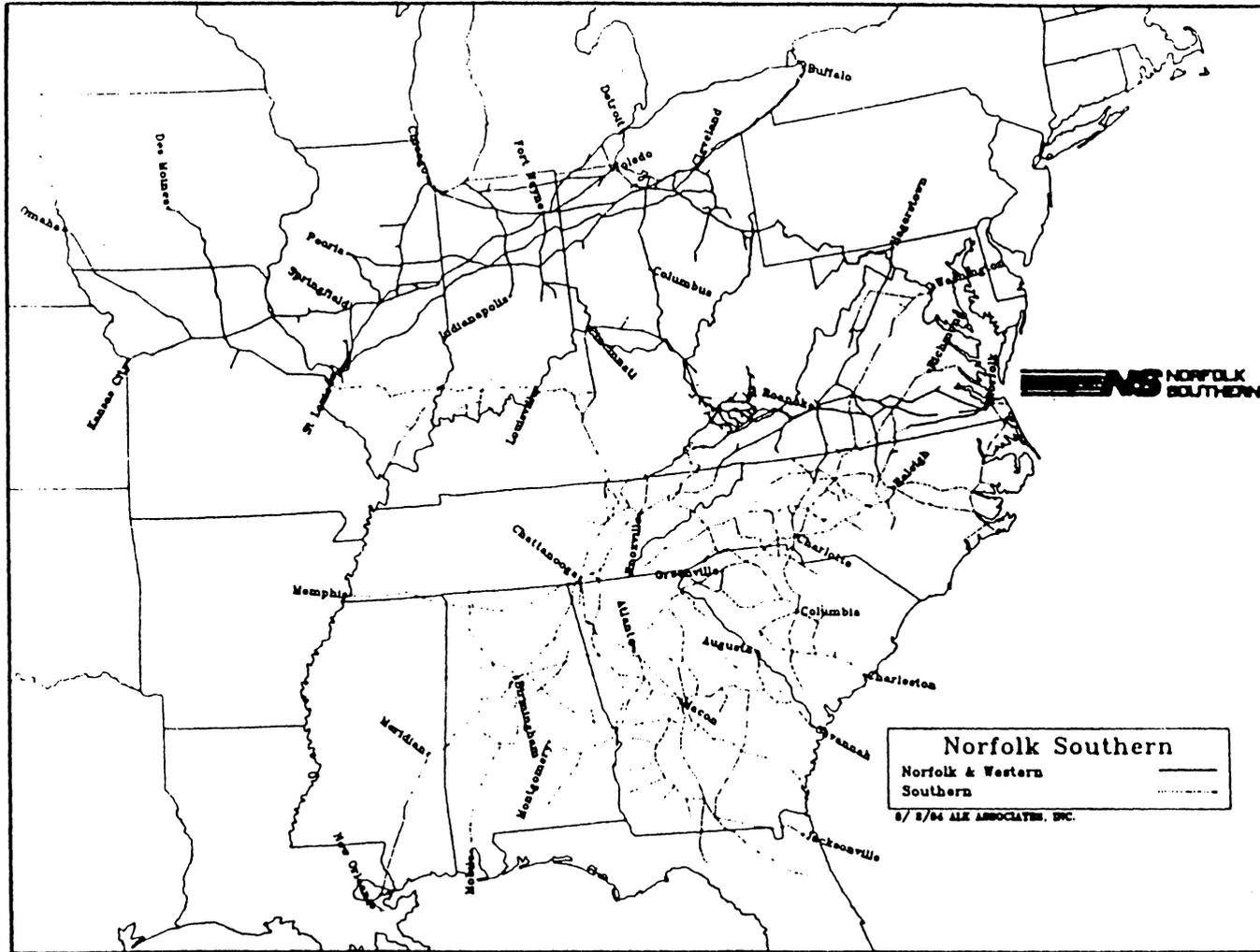


Figure 1.3: NS Rail Network

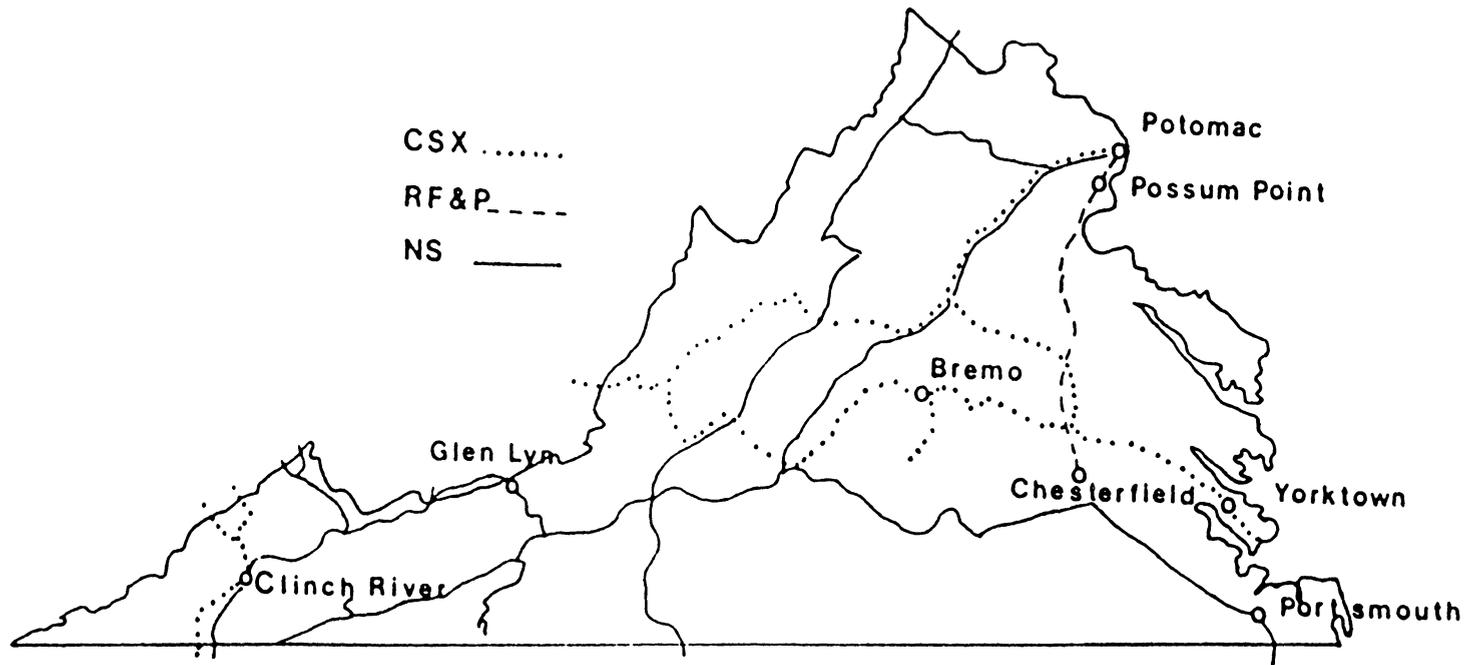


Figure 1.4: Major Coal Burning Plants in Virginia



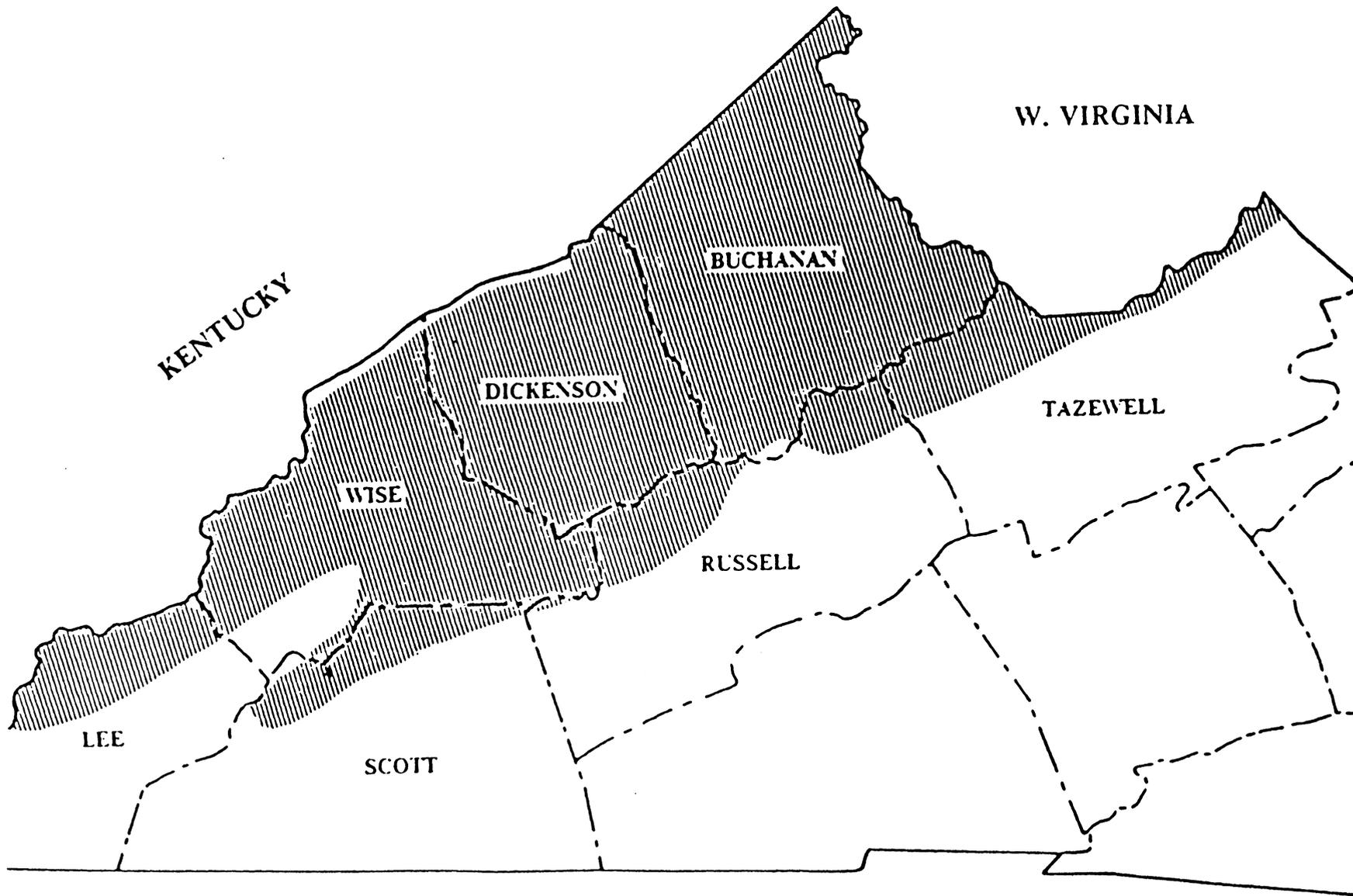


FIGURE 1.5b      SOUTHWEST VIRGINIA COALFIELD

Colonials first discovered coal in southwest Virginia in the mid 18th century in what are now Tazewell, Dickenson, Buchanan, and Wise counties. Large scale mining would not begin until 133 years later when N&W connected its New River branch with the Pocohontas mine in Tazewell. Access to rail connections was the determining factor in regional coal production. By 1892 Tazewell county was producing 697000 tons, while neighboring Russell, Lee, Wise, and Dickenson counties produced a total of 1045 tons without rail access. Tazewell's position soon changed with the advent of rail access in Wise county. In 1892 N&W's Clinch River branch and Louisville and Nashville (L&N) connected at Norton in Wise county. In this year Wise county production soared to 126216 tons. Wise county went on to dominate Virginia coal production for 40 years. This pattern has been followed by the other coal producing counties in Virginia. The following time table traces the parallel development of rail connections and coal mining in southwest Virginia (Hibbard, 1987a).

1883 - N&W's New River branch connects to Tazewell's Flat Top mining District.

1891 - N&W's Clinch River branch and L&N meet at Norton, Wise county.

- 1901 - The Interstate railroad connects L&N at Norton to N&W at Appalachia. This opened feeder lines for much of the county. C&O reached Wise county from Kentucky in 1903. With extensive rail connections Wise becomes the states largest producer by 1897.
- 1902 - The Carolina, Clinchfield, and Ohio (CC&O) connects N&W to mines in Dante and Wilder. This occurrence marked the beginning of large scale production in Russell county.
- 1907 - Norton is connected to St. Charles by the Virginia & Southwestern Railroad, allowing large scale production in Lee county to begin.
- 1911 - CC&O connects with L&N at St. Paul in Wise. With the CC&O running northward to Elkhorn, Kentucky, by 1915 Dickenson became a major coal producing county.
- 1918 - Production begins in Scott county.
- 1923 - N&W buys the Big Sandy & Cumberland (BS&C), a narrow gage railroad previously used to carry lumber out of Buchanan county. It was rebuilt to standard gage by 1931. After further development by N&W, Buchanan county became, and remains, Virginia's largest coal

producer.

By the time N&W opened up production in Buchanan county, seven railroads were serving southwest Virginia. After a series of mergers and takeovers, only two railroads were left by 1980: Norfolk Southern (N&W, Southern, and Interstate) and CSX (C&O, CC&O, L&N). For Virginia coal producers this development opened up more distant markets with shipments made on a single carrier. The major disadvantage of railroad consolidation was the resulting monopolistic position of the remaining two railroads. As mentioned earlier, very few mines, and no utilities, have a choice of originating or delivering carrier. Deregulation of the railroads occurred with the passage of the Staggers Act in 1980. Since then railroads have been permitted to set their own rates and negotiate long term confidential coal transportation contracts. Rail haulage on routes involving multiple carriers is usually prohibitively expensive compared to rates for a single carrier. Therefore, utilities will look for coal supply from mines located on the same railroad as that which serves the plant of destination. In addition to this, rail rates have shown greater responsiveness than mine price in situations where competition can be created. Currently, there is an over supply of coal. This, combined with intense competition among coal producers, has severely depressed coal prices. Utilities are

obliged to purchase coal at the lowest delivered price. Since rail rates show the greatest flexibility, utilities first negotiate with the railroad and then seek coal supplies from producers who can originate shipments on that carrier. If the rail carrier is unable to offer a competitive rate to a particular plant it is highly unlikely that any of the mines located on that railroad will be able to sell coal to that plant. This situation accounts for a large part of the dilemma that Virginia coal producers currently find themselves.

Virginia Power, the major utility in the state, has only one out of five coal burning plants located on NS, i.e., Portsmouth. Possum point is located on RF&P, which is primarily owned by CSX, but NS originating coal can find its way to Possum Point by being barged from NS's piers at Lambert's Point. Chesterfield is accessible with a short transfer on CSX tracks. In any case, outside of Portsmouth, increased transportation charges must be incurred in order to deliver coal originating on NS track to Virginia Power. Since the majority of Virginia coal originates on NS, most Virginia producers operate at a price disadvantage. Table 1.1 (Hibbard, 1987b) gives the proportion of Virginia coal burned by Virginia Power's plants receiving coal on NS. In fact, the main market for Virginia coal is in the south

Table 1.1 Proportion of Virginia Coal Burned  
in Virginia Power Plants  
(By Quarter & Percent)

	1985				1986				1987	
	1	2	3	4	1	2	3	4	1	2
Portsmouth	71	67	63	60	80	65	62	58	80	86
Chesterfield	28	25	25	14	18	25	10	10	24	26
Possum Point	0	0	0	0	0	0	0	3	18	18
Total	30	22	16	17	22	23	15	15	30	32

Data Source: Nat'l Coal Assoc., "Power Plant Deliveries",  
Monthly Reports, January 1985 - June 1987.

(North Carolina, South Carolina, and Georgia) where power plants are located on NS lines (Southern). Virginia plants purchase most of their coal from eastern Kentucky and southern West Virginia on CSX. This is not only because of lower rail rates on CSX. Virginia mine cost is significantly higher than in Kentucky and West Virginia. This resulted in declining market share for Virginia until 1987.

Many mines have ceased operations under the strains of oversupply and extreme competition. In 1983 741 mines were operating in Virginia. By 1986 this number had shrunk to 419 (Hibbard, 1987c). In response to competition Virginia producers have sought to decrease operating costs and increase productivity. This has been done by introducing longwalls, diesel haulage equipment, and more extensive use of contract miners. Contract miners decrease overhead and make it easier to turn production on or off depending on the market. A direct result of utilizing contract miners and increased mechanization has been increased unemployment. In 1978 Virginia employed 19000 coal miners. By 1986 this had dropped to 10240, in spite of increased production.

The economy of southwest Virginia is particularly dependent on coal mine employment. This is especially true of Buchanan, Wise, and Dickenson counties. With a regional job multiplier of one or two area jobs for each mining job, the

impact of decreases in mining employment are painfully obvious. Unemployment in the major coal producing counties mentioned above is very severe. It seems that improvement in employment in southwest Virginia is dependent on the competitiveness of the state's coal industry. Diversification of the regional economy has been a long recognized goal, but with little usable flat land, insufficient municipal services and highways, and locally controlled capital, the prospects are dim. This situation has resulted in a considerable awakening of interest on the part of the state legislature. In 1986 the Quillen Act was passed to encourage the purchase of Virginia coal. This act provides for a \$1/ton tax rebate to utilities who used more Virginia coal in 1987 than in 1985. The incentive increases to \$2/ton by 1988. The difficulty of putting together legislation of this sort is the lack of a means of quantifying the impact of this sort of policy on the competitive position of Virginia producers. This study is the first effort towards quantifying the regional market for steam coal.

## 1.2 Study Objectives and Approach

The primary objective of this study is to develop an analytic tool which will allow for a better understanding of the regional steam coal market to Virginia utilities. This

tool is in the form of a mathematical optimization model of the system of supply and demand of Central Appalachian coal producers competing for the Virginia Power and APCO market in Virginia. From the point of view of the utility company, development and use of a model derived along the guidelines described herein would be highly useful in determining an optimal coal purchasing strategy. The model could be used to plan coal purchases over the short range in order to decrease the cost of supply, and to increase competition between the two rail carriers. From the point of view of a state funded research organization, such as the Virginia Center for Coal and Energy Research (VCCER), it could be used to examine the impact on the market of various scenarios. For instance, the model could be used to project the possible increase of production of Virginia coal under the provisions of the Quillen Act at different tax rebate levels. The methodology used in determining model structure and input is at least as important as the model itself. The following chapters cover those topics central to the development of this model. This is a first time application of Time Series and Linear Programming, and it is unique in a number of ways as a result of the nature of the data and structure of the regional market. Chapter 2 reviews past energy models. Past studies which relate to or contrast with this effort in an informative manner are briefly reviewed. Chapter 3 discusses data availability. Different

sources of information used in this study are examined as to their usefulness and reliability. Data on coal deliveries are of particular interest as a rich source of information never before used in past modeling efforts. Chapter 4 makes use of the data discussed in Chapter 2 in order to conduct a market analysis of Central Appalachian steam coal sales to Virginia. Emphasis is given to the differences between the spot market and sources of supply on long term contract and the influence of different rail carriers and rail rates on mine price. This analysis is used to determine the market structure used in the optimization model. Chapter 5 establishes a methodology for short term forecasts of mine price using data on actual deliveries. These forecasts follow the market structure determined in the previous chapter. Different forecasting methodologies are given for the spot and long term markets. Forecasts of mine price are used with established rates to be used as elements in the objective function of the coal purchasing model. Chapter 6 presents the mathematical formulation of the Virginia Utility Coal Purchasing Model. Chapter 7 describes and discusses the input to and results of the model. A post optimality analysis is used to determine the sensitivity of model parameters. Parametric analysis is used to address broader questions and to generate supply and demand curves. Finally, in Chapter 8, conclusions are drawn from the results of the model, and recommendations for further

research are given.

## CHAPTER 2

### Literature Review

#### 2.1 Introduction

Since the oil embargo of 1973-1974, there have been numerous energy models developed to study the supply and demand mechanisms of fuels. The drastic rise in the price of oil was concurrent with a sharply increased demand for oil and gas in preference to steam coal for utility use. Given the renewed interest in domestic coal as an energy source, many studies and modeling efforts were sponsored to examine the potential for expansion of the coal industry, and the effects of increased government regulation.

Early attempts at modeling the coal market supply and demand will not be covered here except for those models which are of particular interest with regards to the methodology or structure of this dissertation.

The basic structure of coal models has remained the same.

This structure is integral to the problem itself and is used as a framework for this paper. The coal market is a three part system consisting of supply, demand, and transportation. In the following each of these three portions of the system will be examined in the light of previous efforts.

It should be noted that a review of past energy modeling efforts is mainly of academic interest to this study. Both the assumptions and goals of this model are quite divergent from those of past models. Comparisons between the model given in Chapter 6 and those included in the literature survey will be drawn in Section 2.3, but before considering previous models, several unusual features of the model used in this study should be pointed out.

Previous energy models have fallen into two distinct categories: econometric models and mathematical programs. Econometric models consist of a system of equations whose formulation is based on economic theory. These are estimated based on historic data using regression analysis (ordinary-least-squares, generalized-least-squares, or if simultaneous, two-stage-least-squares or a similar technique). Econometric models are useful when sufficient information is available and the system is simple or is

approached in broad fashion. A common sort of econometric model consists of supply and demand equations. Here supply and demand is modeled as a system of simultaneous equations (market equilibrium) based on variables such as fuel cost, GNP, etc. Several models of this type are discussed in the following section.

Mathematical programming models have been heavily relied upon to represent more complex energy models for which some objective is sought. In this case the system includes supply, transportation, and demand for which there is some objective. This objective may be to minimize cost of supply to the consumer on the demand side, or to maximize profits to the producer or carrier. Typically, such a system is represented by numerous equations which define the availability of supply, the transportation network and capacities, and demand. An objective function describes the costs and/or profits which are to be optimized.

Mathematical models have been the most popular means of modeling in coal market studies. They enable the researcher to represent much of the complexity inherent in the coal market without having the great mass of data required for estimation of an econometric model. Cost coefficients can be based on published figures or the results of other models. The same can be said of constants

representing levels of supply and demand. Even though these coefficients and constants may be uncertain it is assumed that the model's structure is sound, and that sensitivity analysis combined with common sense will account for any errors in the derivation of the model's coefficients. An unfortunate tendency in such models is to ignore current market conditions and reduce a complex economic structure to a series of engineering cost equations. A typical model will base the mine price of coal on the mining cost for a certain type of mine operating under different categories of geologic conditions, and will base rail transportation on simulated haulage costs or ICC tariff rates. This approach ignores the actual market price of coal and the competitive position of the railroad (or other carrier). As discussed in Chapter 4, mine price may have little in common with operating and capital costs, while rail rates are determined based on what the market will bear. This study uses mathematical programming as the framework for modeling, but derives the coefficients for mine price from econometric equations (see Chapter 5), and uses the actual rail rates as charged by the region's rail carriers during the study period. Thus, it is hoped that the model will more closely approximate actual market conditions.

Another important difference between this and past models

is its regional scope. Most of the early models were national in scope; they attempted to model all of the continental U.S.A. More recently, regional models have been developed. Three of these are discussed here: a Pennsylvania, a Central Appalachia, and a Florida supply model. The model in this analysis is restricted to demand in Virginia for steam coal. Supply has historically been from Central Appalachia. Therefore, a truly regional model is possible since the supply of coal to Virginia is a closed system which can be completely accounted for.

The data source used for this model breaks down delivered price into mine price (FOB) and transportation. Deliveries are classified as being purchased on the spot market or on long term contract. This data source has not previously been used and has some distinct advantages. The division of delivered price makes possible a model which accounts for the impact of rate setting policies and competition between railroads. Also, the spot and contract markets can be treated as two different markets, as they truly are.

The time frame of an energy model is typically long term. This introduces a number of complications including: estimation of reserves, expansion of mining capacity, and forecasts of demand for coal. Estimation of reserves for use in a model requires making a number of broad

assumptions. Anytime an assumption must be made the validity of a model becomes more doubtful. Future mining capacity is inextricably tied to demand and technological innovations. Technology cannot be predicted, while past long term forecasts of demand have been rendered severely inaccurate due to the unaccountability of shocks to the system such as: war, embargos, miners' strikes, and changes in technology. For a commodity as economically volatile as coal, forecasts are only accurate for the short term, i.e, over a period of time for which the well-informed forecaster can feel reasonably certain of market stability. The model used in this research is designed to mimic current market conditions. Forecasts are short term (six months) for the period spanning 6/86-12/86. This allows both the forecasts and the model to be validated against historical market behavior.

Even though this model shares little in common with past energy models a number of the more interesting efforts will be discussed to provide a historic perspective in coal market modeling.

## 2.2 Survey of Relevant Energy Models

Martin B. Zimmerman (1975,1978,1981) has addressed the

question of the effect on supply of the depletion of reserves. The crux of Zimmerman's argument is that econometric models based on historic time series data cannot capture the movement from cheaper to more costly reserves as the more attractive reserves are exhausted. The long-run cost of supply cannot be captured based on observations based on earlier, and probably more favorable, deposits. Zimmerman developed a method to include increasingly unfavorable conditions resulting from depletion into the cost estimate. This concept has had considerable influence upon many subsequent coal supply models. Previous efforts had assumed that the increase in cost due to depletion would be negligible due to the immense size of existing coal reserves.

The objective in all such studies is to develop a supply function (curve) in which depletion will play a relatively minor yet significant role. As Zimmerman points out, a large and complex number of factors contribute to the cost of mining coal. Unfortunately, only data on seam thickness and depth are generally available. Data on other factors are either proprietary or simply not kept. Zimmerman assumes that seam thickness is independent of all other factors related to production cost and lumps all of these into an unobservable factor,  $e$ , such that,

$$C = f(Th, Q) + e, \quad 2.1$$

the cost (C) is a function of seam thickness (Th), production (Q), and e, which can be treated as the error of estimation.

Data for seam thickness is available, but overall mine productivity (Q) must be found, again as a function of seam thickness. Zimmerman presents a productivity relationship,

$$q = A(Th)^g e \quad 2.2$$

where q is the output per unit shift and A and g are constants to be estimated. Q is based on the production unit, such as, a continuous miner in a drift mine. q is derived by dividing total mine output by the number of unit shifts. To capture the effect on production of mine size this can be rewritten as

$$Q = A(Th)^g S^b e \quad 2.3$$

where S is the number of production sections, and b is a constraint to be estimated

Given the number of production units, various classes of

expenditure are found as a function of production sections,  $S$ , using Ordinary Least Squares (OLS). Adding all cost functions (initial capital expenditure, present value of future capital expenditure, operating, supplies, labor) produces an expression for the total cost as a function of the number of producing sections. The annual capital charge is given by summing the regression equations for initial capital and NPV of future capital expenditures. This is done for each class of mine, for instance, deep vs. drift, and is converted to an annual capital charge for each mine type of interest.

The equations for total operating costs are similarly found based on the regressions for operating supplies and labor cost. Total annual cost is the sum of the above: annual operating costs plus capital costs. With total annual cost a function of  $S$ , one can substitute for  $S$  in the total annual cost equation; the solution to  $S$  is derived from the productivity equation. This yields an equation for total annual cost which is a function of annual output,  $Q$ , seam thickness, and  $e$ .

$$TC = a_0 + a_1 Q^{a_3} / a_2 (Th)^{g_e} \quad 2.4$$

There is a cost curve for each combination of seam thickness, sulfur category, and mining type. Dividing

this by output and differentiating with respect to  $Q$ , setting the result equal to zero and then solving for  $Q$  results in the output that minimizes average cost (AC):

$$AC = b/Th^k e \quad 2.5$$

where,  $b$  and  $k$  are constants.

This also represents the long run marginal cost of producing coal. Note that this is the curve for deep mining. Since  $e$  is unobservable, its distribution is assumed to be lognormal, its mean is assumed to be zero, and its variance is obtained from the estimation of the appropriate productivity equation. The distribution of tons of coal in the ground according to seam thickness was based on data for Pike county in eastern Kentucky. The distribution was found to be approximately lognormal. It is assumed that this county is representative of other coal producing-regions in the country. The distribution of tons of coal according to the cost of mining is lognormal. We can assume that the low cost portion of this distribution has been already mined out since low cost resources will be the first to be depleted. Thus, the actual distribution will be a truncated lognormal.

The desired end product is to define the "cumulative" cost

function, i.e., the supply curve. This is no more than the above truncated lognormal distribution integrated over the acceptable cost of production and multiplied by the total tonnage in that region that applies to the particular cost curve. This complex nonlinear function would result in considerable computational trouble if used as is. Zimmerman approximates the cumulative cost curves by step functions and then incorporates them into a Linear Program (LP) as constraints on production.

The Pennsylvania Coal Model (PCM) (Knight and Manula, 1977) is of particular interest here since this model was restricted to a single state. Thus, the model can afford to be highly disaggregate with supply and demand regions being defined on the county level. For each seam in each coal producing county a supply curve is defined based on available reserves and production costs for surface and underground mines. The supply curves consist of step functions. For each county the supply curve has four steps (one for each type of mine: old/new, surface/underground). The height of each curve is the production cost for that mine type, and the width is the annual production limit (figure 2.1).

Coal supply was limited to the available reserves within the state as given by Bureau of Mines (BOM) data.

Reserves for regions outside of Pennsylvania were assumed to be infinite. Production was determined for each mine type. Production from existing mines was assumed not to remain constant over their lifetime. A survey was taken to ascertain any expected changes in production, and this was projected into the model. The reserve and production data are used to assign annual and multiple year production limits by seam aggregated to the county level by coal type for each of the four mine types.

Production costs for deep mines were estimated by using detailed engineering cost programs. The computer simulator was used to determine mine design given depth and seam thickness. Based on this mine design, cost was estimated. A field survey of surface operations was conducted to arrive at the strip mine production cost.

Average costs weighted by state production figures were calculated for the five coals from out-of-state regions defined in the PCM with 1974 used as the base year. Deep or strip cost was multiplied by the respective production figure to derive total cost by state. State totals were then aggregated to get total cost by region. The regional cost was divided by the region's total production to yield average cost. Production is adjusted to account for cleaning losses.

Capital cost for opening new mines was calculated for deep and surface mines. For deep mines capital costs were approximated using a  $\log_e$  formulation which is not given in the report. For surface mines capital costs were obtained from some large coal company whose operations cover a wide range of strip ratios.

Shapiro and White (1980) relied heavily on Zimmerman's work. Six supply regions were defined and only two coal types were used: low (1) and high (2) sulfur coal. It was believed that such a simplification captures the competition between low and high sulfur coal in the utility market.

Nonlinear production cost functions were developed for each supply region. These are capable of accounting for the increase in marginal cost due to depletion as set forth by Zimmerman (2). Let,  $f_{ik}$  = total cost to extract  $r$  Btu's of type  $k$  coal from region  $i$ , let  $p_{ik}$  be the minimum cost of supplying those  $r$  Btu's. This is the marginal production cost, which is assumed to be lognormally distributed.

The total quantity,  $r(p)$ , which will be produced between a marginal extraction cost of 0 and  $p$  is,

$$r(p) = \int_0^p \phi(\rho) d\rho \quad 2.6$$

where  $\phi(\rho)$  is the density function expressing the marginal production at price  $p$ , and:

$$\phi(p) = R \cdot \text{lgn}(p; \mu, \sigma) \quad 2.7$$

where,  $R$  is the total reserve base, and  $\text{lgn}(p; \mu, \sigma)$  is the lognormal density function with parameters  $\mu$  and  $\sigma$ . Thus, integration of this density function from 0 to a marginal price  $p$  results in the cumulative extraction allowable under the economics of that marginal price.

$$\int_0^p \phi(\rho) d\rho = R \cdot N\{(\ln(p) - \mu) / \sigma\} = r(p) \quad 2.8$$

Once the cumulative extraction is defined, the total cost of producing  $r(p)$  tons can be found. This is equivalent to integrating up to a marginal price on the supply curve.

This approach of integrating lognormal density functions results in a highly nonlinear objective function. Shapiro and White avoid solving a large scale nonlinear program by using Decomposition theory. The model after decomposition is optimized as a sequence of smaller, interrelated Linear Programs.

The Coal Supply and Transportation Model (Science Applications Inc., 1983) generates piecewise linear supply curves based on step function supply curves. Each step represents the opening of a new class of mine. The length of each step represents the volume of coal available from a class of mine, while the height represents the projected price of coal from a mine in that class. Conversion to piecewise linear functions is accomplished by joining the midpoints of selected steps, with the exception of the first and last segments which are determined so as to ensure that f.o.b. mine price is a non-decreasing function of the level of production, and that the last segment has an extremely high slope to limit the supply of coal available from any one region or coal type.

Each segment  $j$  is defined by a range of quantities ( $CQ_j$ ,  $CQ_{j+1}$ ), an intercept ( $CY_j$ ), and a slope ( $CM_j$ ). Let,  $CT_i$  be the tons of coal produced from the region/coal type at any stage of the solution, and set  $j$  to an index pointing to the currently active segment of the supply curve for that region and coal type. The price of coal from region/coal type  $i$  will be,

$$CD_i = (Y_j + (CM_j * CT_i))$$

where,  $CD_i$  = Fob coal price from region/coal type  $i$ ,  
 $CY_j$  = intercept of the  $j^{\text{th}}$  supply curve segment,  
 $CM_j$  = slope of the  $j^{\text{th}}$  supply curve segment,  
 $CT_i$  = tons of coal produced in region  $i$ .

How cost and production figures were arrived at to develop the original step function is not described.

Soyster and Enscoe (1984) econometrically estimated a series of coal price equations in the form of a Cobbs-Douglas production function. Some of the dependent (exogenous) variables: annual production, total number of mines, total employment, average number of days worked per year, productivity, average price FOB mines (endogenous, dependent).

The equations were estimated for underground and surface mines for each supply region (generally, several states). For example, the surface relationship for Northern Appalachia is of this form:

$$S_{\text{price}} = b_0(S_{\text{prod}})^{b_1}(S_{\text{util}})^{b_2}(\text{year}/80)^{b_3} \quad 2.10$$

where  $S_{\text{price}}$  = price of surface coal,

$S_{\text{prod}}$  = surface productivity (tons/man day),  
 $S_{\text{util}}$  = surface utilization (days worked per  
year/365).

Note that geological parameters, such as seam thickness, were not considered. The data used was taken from EIA7 and EIA7A and FERC 423, which are Federal data bases recording basic statistics on coal deliveries to power utilities. All of these sources of information on delivered cost are only given as state averages. Not only does this limit disaggregation to the state level, but it also severely limits the number of observations, i.e., one per year, starting after the Arab oil embargo (1974). This problem is somewhat mitigated by including several states in each of the four supply regions. Productivity was found to be the most important variable. Equations for Appalachia showed the poorest fit.

The regional equations are decomposed into equations by state. This is done by comparing the regional estimate against historical data and adding a constant to the regional equation which is equal to the average difference between the estimated and actual results. This is equivalent to a vertical translation of the intercept to get from a regional equation to a state equation.

The coal supply equation by state is further disaggregated to account for different coal types. Given the proportion of each coal quality of interest in each region, a price premium is found based on the difference between Btu and sulfur levels from the average values.

The FERC 423 tapes are used to get the delivered price of coal by coal type, but to find FOB mine cost some estimate of transportation cost must be made. Delivered price for a particular year between fixed supply/demand pairs is estimated for spot and contract purchases using regression.

$$\text{Delivered Price} = b_0 + b_1(\text{Btu}) + b_2(\text{Sulfur}) \quad 2.11$$

where it is assumed that transportation cost is included in  $b_0$ . This provides the demand-side price of coal where  $b_0 = b_0' + \text{transportation cost}$ . Thus, the desired supply-side equation is:

$$\begin{aligned} \text{FOB mine cost} &= \text{Delivered Price} - \text{Transportation Cost} \\ &= b_0' + b_1(\text{BTU}) + b_2(\text{Sulfur}). \end{aligned} \quad 2.12$$

Yabroff and Dickenson (1980) developed a coal resource model to study the impact of increased coal mining

--particularly in the western U.S. Development of supply curves involved the following steps: developing an inventory of coal resources, estimation of cost factors for coal extraction, calculation of the extraction cost for the inventoried coal, efficiency of various coal extraction methods, and the added cost of state severance taxes.

Since the purpose of this model is long-term forecasting, extensive estimates of reserves have to be made using the BOM reserve categories of measured and inferred. Reserves are classified for each county by depth and seam thickness.

Next, the coal in each county was disaggregated by the extraction method. Surface, underground, and in-situ mining are the available technologies. Since in-situ mining is not practiced commercially it will not be covered in this report. When given by the BOM, the categories of surface and underground were used. The remaining tonnage was allocated to underground mining. Finally, the tonnage in each mining category was allocated to individual seams. For measured and indicated surface mineable seams it was assumed that 30% of the seams were 30 feet deep and 70% were 100 feet deep. For inferred seams, the depth is extended to 300 feet. This was

distributed at three depths: 10% at 50 feet, 30% at 100 feet, and 60% at 200 feet. Tonnage was uniformly distributed over three depth categories for deep mines.

Extraction costs were estimated for the coal seams inventoried above by using engineering cost equations developed by NUS (NUS corp., 1977). These equations give mining cost as a function of seam thickness, depth, production, and the type of mining equipment used for each type of extraction technology.

The coal mining technology for each combination of seam depth, seam thickness, and coal region was specified for input into SRI's version of the NUS cost models. For surface mining, Appalachian contour mining was assumed. For underground mining, drift configurations with continuous mining methods were assumed for seams up to 500 feet deep. Shaft mines were assumed for deeper mines. Continuous mining is assumed for seams less than seven feet thick except in Appalachia, which is left undefined. Longwall mining is assumed for seams thicker than seven feet.

Production is adjusted by a recovery factor which is dependent on seam thickness in surface mining. This is a linear relationship that varies from 0.8 for seams less

than or equal to four feet, to 0.98 for seams greater than 30 feet. For underground mining recovery factors of .43 and .45 were assumed for continuous and longwall mining.

A seam cost file was generated by using the mining cost models to compute the extraction cost for each thickness and depth category for each mine and technology configuration. A fixed size was assumed for various mine types: 5 MMT/yr for surface mines, and 1 MMT/yr for underground and contour mines, i.e., all Appalachian mines are fixed at 1 MMT/yr! A gross overestimate of typical mine size in this region. Indicated resources were assumed to cost twice as much to mine as measured resources. This is supposed to account for increased risk in investment.

As outlined above, the extraction cost was calculated for each seam. The state severance taxes and flue gas desulfurization were added, and the seams were then sorted by total cost by each region. By storing the quantities of coal available at each total cost by each extraction technology in each region, step function supply curves were developed for the marginal cost of extraction as a function of coal remaining for use in the model.

A Regional Coal Price Forecasting Model (CRA) was developed by CRA under contract with PEPCO. Particular emphasis was placed on supplies of coal from Central Appalachia, which is PEPCO's major supplier.

The analysis of supply is divided into three main components: surveying existing capacity, planned capacity additions, and whatever additional output can be produced by prices higher than the breakeven price of the typical new mine. Initial capacity is defined as 95% of the output of the prior year. Coal Age surveys of mine expansion are used to get a handle on planned expansion. The three total price functions (with corresponding price equations) are:

$$Y_{ti0} = .95(UC_{t-1} + SC_{t-1,i}), \quad P_{ti0} = .75(NMP_{ti}) \quad 2.13$$

which accounts for that portion of supply from old capacity;

$$Y_{ti1} = Y_{ti0} + (AUC_{ti} + ASC_{ti}), \quad P_{ti1} = NMP_{ti} \quad 2.14$$

increase in supply by increasing production to existing capacity;

$$Y_{tik} = Y_{ti1} + f_{ik}(e_{tik}UC_{ti} + e_{tik}US_{ti}),$$

$$P_{tik} = P_{ti1} + f_{ik}P_{ti1} \quad 2.15$$

where,  $k > 1$ , this is the supply possible when higher prices inspire new mine openings.

UC(SC) = existing underground (surface) capacity,

AUC(ASC) = additions to underground (surface) capacity,

y = total output,

f = the percentage increase in price above NMP (the required price for a new mine),

eu and es are the price elasticities of underground and surface output.

The subscripts are: time index (t), mining region(i), and price increments(k) on the supply curve.

A step-function supply curve was generated by using  $y_{ti0}$  for the first step.  $y_{ti1}$  is used as the second step. The k additional steps are increased capacity due to new mine openings. The step-function is linearized by having each step proceed from the midpoint of a segment on the supply curve to the midpoint of the following segment. Supplies are divided between low, medium, and high sulfur coal. Competition for low sulfur coal from non-utility customers is not accounted for. Also, the difference between spot and contract sales is not taken into account.

Modeling Long Term Production with the Argonne Coal Market Model (Dux, 1977): In the previous review those models using mathematical programming have all been LP's. LP's have the disadvantage of requiring many variables and additional constraints to account for supply and demand curves and cannot account for elasticities. The Argonne model uses Quadratic Programming. Such a model can have supply and demand revenues represented directly in the objective function since they are of quadratic form (squared terms).

The price for any given coal type is represented by the supply function,  $a_i + b_i x_i$ . Revenues on the sale of a quantity of coal ( $x_i$ ) will be:  $a_i x_i + b_i x_i^2$ . There is a supply curve for each combination of coal type and supply region. These are inserted in the objective function, as will be shown. When a step function is incorporated into a model each step is a separate variable in the objective function,  $a_{ijk} x_{ijk}$ , where there are  $n$  steps in the curve for region  $j$  producing coal type  $i$ ,  $a_{ijk}$  being the price of the  $k^{\text{th}}$  class of mine and  $x_{ijk}$  being its production. Each step/mine classification has an associated capacity constraint. Depletion of reserves can be accounted for in the successive time periods (assuming a time-dynamic model) by truncation of the leftmost portion of the previous time period's curve. In effect this is

equivalent to shifting to a new curve in which the less expensive reserves have been exhausted.

In Gordon's (1978) review of energy models he points out that the objective function should minimize the total resource cost of coal use. The original Argonne coal model was formulated to minimize the total market price. Market price can include economic rents, but in the resource minimization, formulation it is assumed that the low-cost suppliers can afford to cut prices to gain an increased share of the market.

The supply curve is the summation of the marginal cost curves. It is stated that the total resource cost at a particular quantity is:

$$C_i(x_i) = \int_0^{\bar{x}_i} \{dC_i/dx_i\} dx_i = \int_0^{\bar{x}_i} (a_i + b_i x_i) dx_i = a_i \bar{x}_i + b_i \bar{x}_i^2 / 2 \quad 2.16$$

where the marginal costs are  $dC_i/dx_i$ , and  $\bar{x}_i$  is the desired cumulative extraction.

Argonne derived its supply curves from the step functions developed by ICF in the PIE's coal analysis. A series of linear segments (i.e.,  $a_i + b_i x_i$ ) are fit to the step function.

Labys et. al. (1979) developed an econometric model of steam coal supply and demand. Two theoretical models were hypothesized. In one model, the demand for steam coal is assumed to be dependent upon total fuel requirements (TFR) and steam coal price (SCP). The utilities will minimize their cost. In the long run, the choice of fuel will depend on the price of fuel in competition with steam coal (IFP). Therefore, the demand for steam coal is:

$$SCD_t = f(TFR, SCP, IFP, SCD_{t-1}). \quad 2.17$$

An alternative model assumes that steam coal is the least preferred fuel. So steam coal demand satisfies only the remaining fuel requirements after supplies of oil, gas, nuclear, and hydroelectric have been used. This model will not be considered in this discussion.

Steam coal prices are a function of long-run average costs. Costs are assumed to be only a function of labor productivity (APL), alternate fuel prices (IFP), and a stock adjustment variable (VSTK). Utilities stockpile coal so as to minimize the cost of purchasing on the spot market during interruptions of contracted supply (strikes). Adjustment of stocks depends on the current inventory are defined as:

$$\Delta \text{STK} = \text{STK}_t - \text{STK}_{t-1} \quad 2.18$$

where  $\text{STK}_t = f(\text{SCD}, \text{STK}^*, \text{SCP}_t - \text{SCP}_{t-1}, \text{STK}_{t-1})$ ,  
 and  $\text{STK}^*$  is the maximum stock reserves considered  
 necessary in case of an interruption of supply.  
 Therefore, the equation for coal price is,

$$\text{SCP}_t = f(\text{SCP}_{t-1}, \text{APL}, \text{IFP}, \text{STK}). \quad 2.19$$

Two alternate hypotheses are used to describe steam coal production. The first assumes that the price of coal supply is not dependent on the type of mining method, although mine price obviously is. A partial adjustment formulation is used which includes short term ( $i$  years) and long term ( $j$  years) adjustment of price.

$$\text{SCQ} = f(\text{SCQ}_{t-1}, \text{SCP}_t, \text{SCP}_{t-i}, \text{SCP}_{t-j}) \quad 2.20$$

The alternate hypothesis implies that supply adjustment depends more closely on mining method and the type of production equipment. Surface mined coal supply is estimated as above, but uses the price of surface mined coal (where this data is found is not explained). Other relationships determine supply by mining method. Data acquisition would be much more difficult for this system

of equations.

Non-utility steam coal demand must be subtracted out of the total coal supply. The identity for the remaining coal is:

$$RCD = SCQ - SCD - BCD - VSTK - XCX, \quad 2.21$$

where  $BCD_t$  = demand for boiler coal (industrial use)  
 $= f(BCD_{t-1}, SCP/IFP)$ , and  
 $XCX$  = exports.

The equations for total coal supply, underground and surface mining shares of production, and residual coal demand are solved recursively using OLS (first total supply, then underground, and finally surface shares). The demand equations are solved as a simultaneous block of equations using 2SLS.

Labys and Yang (1980) developed a Quadratic Programming model for Central Appalachia. Supply and demand are driven by linear regression equations. Seven supply districts were defined including: S.W. Virginia, Virginia, E. Kentucky, and Tennessee. Underground and surface production are aggregated.

The theoretical formulation used was a "quasi-welfare function" as defined by Takayama (18). In Takayama and Judge's formulation, supply, demand, and transportation costs are accounted for in the framework of a quadratic program. Supply and demand are defined as being a linear function of price only.

$$\text{Supply} = Q_i = v_i - g_i P_i^q,$$

Which in terms of price,  $P_i$ , is:

$$P_i^q = n_i + o_i Q_i, \text{ for all supply regions } i. \quad 2.22$$

$$\text{Demand} = D_j = l_j - w_j P_j^d,$$

Which in terms of price,  $P_j$ , is:

$$P_j^d = a_j - b_j D_j, \text{ for all demand regions } j. \quad 2.23$$

As the authors admit, a single variable, price, is not sufficient to explain coal supply. Additional exogenous variables related to policies (regulation?) and productivities were somehow merged into the intercept term. This was done in order to permit the operation of the single commodity spatial equilibrium model which is as follows.

The regional quasi-welfare function is

$$z(Q_i, D_j) = \int_{\hat{D}_j}^{D_j} (a_j - b) dD_j - \int_{\hat{Q}_i}^{Q_i} (n_i + o_i Q_i) dQ_i$$

$$= K_i + K_j + a_j D_j - .5 b_j D_j^2 - n_i Q_i - .5 o_i Q_i^2, \text{ for all } i \text{ and } j$$

2.24

Dropping the constant terms, the quasi-welfare function over all supply and demand regions is,

$$\text{Max } Z = \sum_{j=1}^n a_j D_j - \sum_{i=1}^n n_i Q_i - .5 \sum_{j=1}^n b_j D_j^2 - .5 \sum_{i=1}^n o_i Q_i^2 - \sum_{i=1}^n \sum_{j=1}^n T_{ij} x_{ij}$$

2.25

$$\text{Subject to, } D_j \leq \sum_{i=1}^n x_{ij}, \text{ for all } j$$

2.26

$$Q_i \leq \sum_{j=1}^n x_{ij}, \text{ for all } i$$

2.27

where,  $x_{ij}$  is the quantity of coal shipped between regions  $i$  and  $j$  at a transportation rate  $T_{ij}$ .

$D_j$  is the demand,  $Q_i$  is the production, and  $a_j$ ,  $b_j$ ,  $o_i$ , and  $n_i$  are constants defining the slope and intercepts in the supply and demand curves.

Equation sets 2.26 and 2.27 are the constraints on demand not exceeding supply and shipments not exceeding production, respectively.

Shapiro and White (1980) incorporated an endogenous energy demand model with a supply and demand model. An aggregate

LP utility model is given for each demand region describing optimal capital investment and operating decisions for the region over the planning horizon. There is a separate utility model for each demand region. This is a short term model, such that, over a ten year planning horizon all plants are in place.

For each demand region  $j$  (e.g., New England) a separate LP is solved to minimize the cost of meeting energy demand. Thus, the objective function is to minimize the total discounted sum for the planning horizon (10 years) of total capital cost plus total fuel operating and maintenance costs. This is subject to the following constraints:

- (1) Generation capacity adjusted by an availability factor must be  $<$  additions to capacity over the planning horizon;
- (2) Added generating capacity for all electric generation over the planning horizon  $>$  the maximum expected rate of electricity demand times a unexplained coefficient which seems to account for available reserves;
- (3) Generation capacity must be sufficient to satisfy demand;

- (4) An equality to account for the rate of conversion between electricity and coal;
- (5) Industrial, metallurgical, and utility coal consumption equals demand for coal;
- (6) Sulfur emissions cannot exceed the percent set for region.

The exact formulation is given in the reference.

The Florida Statewide Conversion Study (Tobin, 1983) was undertaken to examine the feasibility of converting 14 of Florida's generating stations and 27 of its generating units from oil and gas to coal. Of particular interest was the means of coal transportation to handle this large increase in demand. Various supply regions and transportation modes were defined in the model in order to evaluate potential bottlenecks in transportation. Barge shipments were compared with rail transportation. Coal carried by rail was assumed to originate in Central Appalachia and Alabama, while barged coal could originate in the west via the Mississippi or from foreign sources.

This study is another example of externally defined coal demand. Demand was found for current coal use, future coal use without conversion, and an increased demand was found for the case of coal conversion. Current coal use was taken from historic data which broke coal use down into electric utility, industrial, residential and commercial, synfuels, and exports. Actual utility use of coal in Florida was determined. Future coal use was taken from the DRI coal model which forecast demand in the south-east region in 1990. Planned increases in coal-fired electricity generation were taken from a dispatch analysis conducted by Science Applications Inc. (SAI). Increases in coal demand resulting from conversion to coal of existing plants were also taken from SAI's dispatch analysis.

Hartman (1979) reviewed the current state-of-the-art in energy demand modeling in 1979, and made suggestions as to directions of further development. Hartman points out that in energy demand there are several interrelated decisions on the part of the energy user which tie together both the capital stock (utility type) and fuel. In the short run, capital stock is fixed, but over the long-run, size and characteristics of the capital stock are variable. The model used should be able to take this

into account. Modeling techniques are categorized as follows:

Treatment of the relationship of demand for energy and for its requisite capital in the short and long run:

- a) static equilibrium models,
- b) dynamic, partial adjustment models, or
- c) dynamic, multi equation models.

Treatment of fuel demand in relationship to other fuels and factors:

- a) single fuel demand models, or
- b) interfuel substitution models.

Static models focus on demand for a single fuel as a function of price, user's income, and other relevant variables such as climate. A static model assumes instantaneous changes in capital stock (increased utility capacity) to meet increased energy demand. Note that such a model does not differentiate between long and short-run elasticities. Nor does it consider the characteristics of the various utilities.

A partial adjustment model ties together changes in demand and the requisite capital stock. There is a short-run disequilibrium as utility capacity adjusts to changes in

demand. Thus, if the desired level of production is

$$q_t^* = q_t^*(p_t, y_t, w_t), \quad 2.28$$

and only a partial adjustment towards this desired level can be expected in the first time period, such that,

$$q_t - q_{t-1} = a(q_t^* - q_{t-1}), \quad 2.29$$

then the partial adjustment relation is

$$q_t = aq_t^* - (1-a)q_{t-1}. \quad 2.30$$

Dynamic multi-equation models explicitly recognize the different characteristics of short-run and long-run demand. Separate equations can be defined to deal with the different decisions related to energy use and capital stock. For instance,

$$q_t = u_t(p_t, y_t, w_t)k_t \quad 2.31$$

accounts for short-run fuel demand as a function of the desired level of demand, where  $u_t$  is the utilization rate of the fuel burning capital stock,  $k_t$ . The rate of utilization is a function of the price of the fuel of interest and of competing fuels,  $p_t$ , income level of user,  $y_t$ , and other relevant variables such as climate,  $w_t$ .

$$K_t = k_{t-1}(1-g) + k_t \quad 2.32$$

Here, the rate of utilization of capital stock,  $k_t$ , is defined by the rate of utilization of the previous year less retired capital stock plus additions to the capital stock which uses the fuel being analyzed.

$$k_t = f(p_t, cc_t, y_t, w_t) \quad 2.33$$

$k_t$  depends on the fuels of interest's desirability compared to alternate fuels. Thus, it is a function of the relative costs of all possible fuels,  $p_t$ , and the comparative characteristics of the capital required for the particular fuel,  $cc_t$ .

Interfuel substitution models deal with the competition from other fuels explicitly and with more detail. In these models demand is a function of the price, capital cost, and other non-cost related variables of all alternate fuels and their respective capital stock.

Hartman states that dynamic multi-equation demand models that account for fuel use and that take into consideration the fuels associated with the capital stock are the most useful. In this review econometric models are emphasized. Such models stress income, prices, and interfuel competition. They are weak due to their high level of

aggregation and their inability to explicitly model capital stock characteristics and non price policy variables. Optimization models are very useful for capturing technology and a disaggregate market, but inadequately account for price effects and interfuel competition. Thus, a desirable approach would be to combine econometric and optimization models, incorporating the advantages of both.

### 2.3 Applicability of Past Models

There exist many more energy models than those discussed in the previous section. Indeed, energy modeling is a field of endeavor unto itself. Only some of the more recent, interesting, and innovative models have been reviewed in order to give a feeling for the subject before giving an in-depth description of this research effort (see chapters 4,5, and 6). These models share some common features which are more or less applicable in this study. These features will be summarized according to their strengths and weaknesses, and how they compare with the approach used in this research effort.

**Objective Function:**

Usually the objective in past models has been to minimize the cost of coal supply to the consumer. This is effectively done by minimizing the total cost to all utilities. For most models, including the one used in this study, the objective function includes mining and transportation costs. A critical difference is that previous models have used mine cost as opposed to mine price, as determined by the market. Other approaches included the increased cost of resource depletion first taken into account by Zimmerman, or a more theoretical objective such as the Quasi-Welfare function (Judge, et al).

#### Regional Scope:

Most early models were in response to a perceived national energy shortage. Thus, these models were national or even international in scope. Mining costs were often based on data from the FERC 423 tapes which gave state averages for the cost of coal supply. Using this data, regional disaggregation could not extend further than individual states. For national models, states were often aggregated into supply regions (Central Appalachia, western states, Ohio Basin) and demand regions (New England, southeast, south central, etc.). In this manner the nation would be

roughly subdivided into energy sources and sinks with an interconnecting transportation network. Starting in the 1980s, research turned to regional models (three of these were covered in section 2.2). Regional models can be more disaggregate and more specific in modeling regional supply and demand than national models. In the PCM regional disaggregation extends down to the county level for supply. In the Florida Statewide Conversion Study, the emphasis was placed on describing demand from individual utilities and the transportation network needed to route the increased tonnage of coal that would be needed in case of the implementation of a planned statewide conversion from oil to coal. These regional models have shared the same structural characteristics as national models, but have been used to address specific questions of interest.

The regional models have been statewide on the supply side with the exception of the PCM. In contrast, the utility purchasing model described in this study first divides supply into two markets: spot and long-term contract. Long-term contracts are individually specified, while the spot market is divided by state (see Chapter 4).

Coal Supply:

Specification of supply of coal is beset by two problems: the multitude of individual sources (mines) and the lack of data on parameters related to mining costs. Because of this it is impractical, if not impossible, to accurately define individual mines by their cost and production. The approach used in coal supply models has been to generate supply curves which are either included directly in the model and solved as a Quadratic Program, as in the Argonne model, or are approximated as a piecewise polynomial. Step functions have been popular partially because they can be included directly in a Linear Program. Here, each step in the function represents the production available at or above the mining cost associated with that step. A different step function is generated for each supply region, coal type, and often in order to differentiate between surface and underground mines. Since mining costs are not known, reserves, production, and mining costs are estimated for each step. Mining cost is often based on engineering cost equation or computer simulations in which the parameters related to mining costs are varied to match the mining conditions and production at each step in the supply curve.

As suggested by Hartman, mining price is best estimated (in our case forecast) using econometric models which are

then included in a math programming model. Mining cost is not necessarily an accurate measure of the price of coal in the market place. In a situation of over supply the low-cost producers will set market price, while during a period of high demand, high-cost producers will determine market price while lower cost producers will increase their profits.

#### Transportation:

In long-term models the inclusion of the transportation network has played an important role. The concern has not been the competition (or lack thereof) between carriers, but rather the capacity of the network to deliver coal under conditions of increased coal demand. During the ICC regulation of rail rates, a great deal of emphasis was placed on estimating the cost of rail transportation as a function of the physical condition of the rail line, trackage design, and topography. This approach is not applicable to short-run models since the passage of the Staggers Act. In the short term, major increases in demand are not a concern and therefore capacity constraints can be ignored.

#### Demand:

In long-run models, demand for coal is based on long-run forecasts of energy demand, which is converted to demand for coal based on assumptions of conversions to coal-fired boilers and projected new coal burning capacity. Demand in a short-term model is based on utility projections of coal use for that period.

## CHAPTER 3

### Data Availability and Analysis

#### 3.1 Introduction

As should be true of any analytical study, the limiting factors are the availability and quality of the data. In a laboratory environment an experiment can be designed to achieve the clearest statistical evaluation. If the data are insufficient, more experiments can be run. Unfortunately, in many economic studies such as this analysis, the data are preexisting. Thus, the quality and extent of information is beyond the control of the researcher. It is the quality, extent, and quantity of data which dictates the scope of this analysis. While reading this thesis the reader is likely to ask himself why the extent of the model was not broader, or why certain details were not taken into consideration. In most cases the limitations are due to the data. A prime example of this is the manner in which this study predicts the mine price of steam coal. As discussed fully in Section 3.2.2, mine price is

forecast only as a function of coal quality and past price. This approach ignores the influence of the cost of mining. This is not simply because of the belief that mining costs are not significant in the current market, but mainly because of the inability to identify all the mines from which most deliveries of coal are made, due to the manner in which utilities have chosen to report the origin of their coal deliveries.

This chapter will only cover data sources which are used to forecast mine price or are used to obtain constants in the coal purchasing model. The constants in the model fall into three categories which relate to mine price, rail rates, and information specific to the utility, such as consumption of coal and monthly stockpile levels. A considerable body of data, such as railroad maps and mine directories, is also used in determining the structure of the model. All of these data sources are covered in the following sections.

### 3.2 Records of Individual Deliveries

Both Virginia and West Virginia require public utilities to report all purchases of coal. In West Virginia any utility which sells electricity within the state must

report all coal deliveries to all plants belonging to that utility. For instance, Virginia Power's Mountain Storm and Glen Lyn plants supply electricity to West Virginia customers, yet deliveries to plants solely in Virginia must also be reported. For West Virginia, this information is compiled by the West Virginia Public Service Commission (WVPSC), and published on computer tape as HB966. This tape records all deliveries to APCO and Virginia Power. Since these two utilities are the principle coal burning utilities in Virginia (with the past exception of PEPCO), HB966 is a sufficient source of information for deliveries of coal to Virginia power plants. The same information as that found in HB966 is compiled by Virginia's State Corporation Commission. This is available only in paper copy as report number FM-14. Since PEPCO is not regulated by Virginia (and therefore not included in FM-14) there is no advantage in using FM-14 in preference to HB966. In this study, only HB966 was used, primarily because of its availability on tape.

### 3.2.1 Data Format

As stated above, all information on coal deliveries to Virginia utilities (Virginia Power and APCO) is taken from a single source, WVPSC's HB966 tapes. Utilities are

required to report information on individual deliveries of fuel. In this study only rail shipments of coal from Central Appalachia to Virginia are considered. The variables used in this study taken from HB966 are: name of utility, plant, purchase type (spot vs. contract), coal mine type (surface vs. underground), state of origin, source\*, county of origin, quantity (tons), Btu/lb, sulfur content, ash, water content, rail cost, dumping cost, barge cost, truck cost, conveyer cost, handling cost, month of delivery, and name of coal sales company. The tape format and the program used to read the tape to a permanent SAS data set are given in the appendix.

Only rail carried deliveries are of interest in this model. The two railroads carry all coal traffic to Virginia utilities. Due to the distance between coal producing regions and this state's power plants, deliveries by truck are uneconomical. Conveyor belt costs refer to power plants which are sited by captive mines; this situation does not apply to any Virginia utilities with the exception of Carbo which was developed by APCO.

There has been some barging of coal by Virginia Power. 30,000 tons of intercoastal barge traffic was transhipped from Hampton Roads to Virginia Power's coastal plants. This traffic ceased after 1985 when Virginia Power had achieved its objective of lowering rail rates by introduc-

ing this competition. Virtually all coal shipments from the Central Appalachian states to eastern destinations are moved exclusively by rail. Another point to note is that Virginia utilities deliveries of coal originate almost exclusively from the three state region, districts 7 and 8. PEPCO receives some coal from mines located in western Maryland but is not included in this analysis. Therefore, HB966 provides the above information for all deliveries to Virginia utilities used in this study . These shipments are all being made by rail and originate in Virginia, southern West Virginia, or eastern Kentucky.

### 3.2.2 Source Identification

Mine price (FOB) under normal market conditions should be composed of fixed costs, variable costs, and profit. With this in mind, a forecast or model of mine price should include variables which control both operating and capital costs, and account for the expected profit margin. HB966 is used as the sole source of information in generating forecasts of mine price but is limited in information specific to the origin of the delivery. Only the basic coal quality characteristics and whether the source is a surface or underground mine are given. With only these variables given, it is most unlikely that any reasonably

accurate equation estimating the cost component of mine price can be found; more variables associated with mining costs are necessary. Section 3.3.3 summarizes the MSHA Address/Employment tapes. These tapes give information on every mine in the country on a quarterly basis. This information includes seam thickness, productivity, and other variables which could be used to estimate mining cost. Given sufficient time and money reasonable equations for estimating the mine price given in HB966 might be developed. Unfortunately, the origin of coal deliveries as given in HB966 is uncertain in the majority of cases. One would expect that utilities would be required to report the actual mine and company name, but this is generally not the case. Instead, utilities report the source as given to them by the coal vendor. The vendor gives the name of the source in a diverse number of ways. Either the company or mine name should be given, but instead what is typically given is a brand name or name of the coal sales company. Even when the mining company name is given, the source is often untraceable since many companies will have multiple mines active in the same county. Also, multiple sources might occur when coal is blended from more than one seam to meet a customer's needs. As a result of the frequent inability to identify the mine or mines from which a shipment of coal originates, the data on deliveries given in HB966 cannot be merged with data on the

individual mines, such as the MSHA tapes. Those data sources which contain information on the individual mines cannot be used to estimate mine price equations since only HB966 gives the price at which coal is being sold.

### 3.3 Source Specific Information

A number of sources of information are available on coal mines and their parent companies. Those which have proved useful in this study are described in this section; these are the directories of mines for Virginia and West Virginia, the annual report of the Kentucky Department of Mines and Minerals, Keystone, and the MSHA tapes mentioned in the previous section.

#### 3.3.1 Coal Mining Directories

Both the Virginia and West Virginia coal mining directories are produced in Blacksburg, Virginia. The VCCER publishes a directory of all Virginia mines permitted to produce coal. This in turn is based on the complete compilation of all mines under permit as recorded by the Virginia Division of Mines and Quarries. As noted in the 1986 directory, the 100 listed mines produced 62 percent

of Virginia's production, yet there were 681 active mines in the state for that year (1985). 1985 was a record year for production in the state. In periods of high demand, such as 1985, a large number of small producers become active, and then close down again during slumps in demand. A significant portion of production comes from mines producing under 100,000 tons. These readily open and close depending on the market. This could greatly complicate a model which attempts to account for the characteristics of individual mines. The variables contained in the Virginia directory are: company name, mine name(s), address, mine type (surface vs. underground), average yearly employment, man hours, production, productivity, seam height, seams, and railroad. Also included is information preparation plants as taken from Keystone (Nielson, 1985) and a list of longwalls as taken from Coal Mining, "Longwall Census". No information is available on prices, and only limited information is available on coal quality characteristics for some mines.

The West Virginia Directory & Buyer's Guide (Chisholm, 1986) is more elaborate in its organization, but contains the same information as the Virginia directory. Unlike the Virginia directory, longwalls and railroad are not listed, but the names of the managerial staff are listed. For the purposes of this study, the absence of the name of the

railroad is a serious drawback.

The last directory of mines for Kentucky was published in 1983. The only up-to-date and complete source of information on individual mines in Kentucky available is the Annual Report (Stanley, 1985) of the Kentucky Department of Mines and Minerals. This contains a directory of mines for each county. Mine listings in each county are subdivided into categories for combinations of mine type (underground, auger, or strip) and transportation (rail, water, or truck). The specific railroad is not given. The variables given are: company name, mine name(s), contact official, address, annual tonnage, men employed, days worked, seam, seam height, and accidents (fatal and nonfatal).

### 3.3.2 Keystone

McGraw-Hill annually publishes the Keystone Coal Industry Manual. This manual serves as the main national directory for the coal industry; it lists mining companies, mines, major coal consumers, coal associations, coal transportation companies, and information on major coal seams. The coal mine directory was the main source of information used out of Keystone, since it contains some information

which is not available in the state mining directories described above. Keystone lists mines alphabetically by state and mining company. The information given for each mine is not consistent in its detail, but is more complete than what is available in other directories for those mines which are listed. Keystone does not list all active mines. The variety of information which is given includes individual summaries of each mine describing the type of mine (drift, auger, etc.) and some details on the mining method and equipment. For strip mines the overburden ratio is given. Also given are reserves, coal sales agents, and principle applications (steam, industrial, or coke). When there is a preparation plant, the principle plant equipment, tonnage class, and coal analysis may be given. Some details of tipples may be given. Of major interest when available is the rail which ships the mine's production and the freight district. In its separate parts, Keystone is the most detailed source of data available, but the information described above is not listed for all mines. Unfortunately, Keystone is sketchy and incomplete, and therefore is only suitable as a supplementary source of information. The utility directory of steam power plants given in the manual is also useful. For each plant the delivering railroad at the plant, capacity, consumption, and any projected new capacity is given. However, better sources of data on utilities are available.

### 3.3.3 MSHA Address/Employment tapes

As stated in Section 3.2.2, information on individual mines cannot be merged with the data for mine price, i.e., HB966. Still, the MSHA tapes are of interest as the only complete documentation of all mines available on computer tape. Before discussing the content of these quarterly tapes, it should be noted that they are very expensive to read and lack adequate documentation. For this reason the JCL and SAS code used to read these tapes are included in the appendix.

The Address/Employment tapes record the operational status and accident reports of all U.S. mining companies. Of interest in this study are all active bituminous coal mines in Central Appalachia. The pertinent variables are: state, county, SIC code (for primary commodity mined), seam height, mine type (surface vs. underground), company name, mine name, and address. In a trailing record are given the following variables for each quarter of the year: number of employees (average for the quarter), number of man-hours, and tons of production.

If the MSHA tapes and HB966 could be matched by source these tapes would have two strong advantages over state directories and Keystone when estimating mine price. Not

only is the MSHA data immediately accessible by computer, but this information is also recorded quarterly, thus providing four times the data of an annual state directory.

### 3.4 Railroad Specific Information

This study is not heavily concerned with the transportation of coal in and of itself. The model is designed to determine the likely pattern of coal purchasing by Virginia utilities, rather than by what route deliveries of coal will be made. Railroads are important in terms of the rates they charge and what mines and power plants are captive to their lines. Because of this approach we are mainly interested in information which will identify rates, and the rail carrier at the mine or utility.

Rates can only be obtained from HB966, FM-14, or some other state public service commission report. These may be published or summarized elsewhere in paper copy as in Coal Week. Unlike the Federal Energy Regulatory Commissions reports (FERC 423 tapes), these state reports give the transportation and mine price (fob) components of delivered price. HB966 gives the total cost for rail transportation and the tonnage delivered, not the actual

rail rate, which can only be inferred from the transportation cost per ton.

The rail line going into the power plant is not difficult to obtain. In the case of this study, this was common knowledge, since there are not that many plants in Virginia. VCCER publications and Keystone also give this information. Identification of the rail line at the mine is far more difficult. For large mines with their own preparation plant and tipple there are a number of sources of information depending on the state and railroad. For Virginia the VCCER's coal mine directory and the VDMQ's database "TON.DBF" (available on diskette) list the railroad. Another good source of information is Norfolk Western's List of Stations And Sidings And Other Related Information (N&W, 1977) issued by the accounting department out of Roanoke. For each freight district this lists branch lines, the name of the station or siding, miles from the main line junction, and the distance to a major yard or destination, such as Bluefield or Norfolk. In the case of coal mines, the siding is given the name of the town or mine. With knowledge of the address of a mine or the nearest rail junction, this handbook can often be used to determine freight district and railroad (since, if it's not located on NS it must be located on CSX). It is much more difficult to determine the rail carrier for mines

outside of Virginia. Keystone sometimes gives the name of the freight district and railroad, but not always. The Kentucky Department of Mines and Minerals Annual Report only identifies mines which ship mainly by rail, not the name of the railroad or freight district. This is a major problem since both Kentucky and West Virginia have a number of counties which are serviced by both NS and CSX. Here identification is a matter of detective work using NW's list of stations and sidings and railroad map, which lists many of the mines located on NW sides within each district.

As shown above, mines located on railroad sidings with their own loading facilities can be identified by railroad and freight district, especially in Virginia. Even though almost all coal arrives at the plant by train much of that coal is still initially moved locally by truck. Many small operations are not located at a tipple and must truck their coal to a nearby tipple. For this reason individual identification of the railroad used by each mine is not practical. This is only done when counties are split between the two carriers (Pike county). Otherwise, if a county is essentially captive to one railroad, then all mines in that county are assigned that railroad.

### 3.5 Utility Specific Information

For each utility's power plants the following information is needed:

1. History of past deliveries - where deliveries came from, rail rate and mine price, quantity delivered, and coal quality analysis.
2. Coal consumption by month.
3. Stockpile levels by month including maximum and minimum levels allowable.
4. Boiler specifications - maximum (minimum) sulfur, ash, Btu and, water.

The history of past deliveries is recorded in HB966, which effectively also gives coal consumption by month. Monthly fuel consumption and stockpiles can be found in the National Coal Associations Steam Electric Market Analysis (NCA, 1986). This gives electricity production by state and source, fuel consumption and stockpiles at electric utility plants, and electricity generation by fuel type. Of primary interest are stockpile levels and coal consumption. Boiler specifications are needed in the model to

constrain the average monthly coal quality delivered to the plant to a level which meets those specifications. These boiler specifications are available from the bid sheets sent out by the utility to mines and coal sales companies. These specifications are then matched with the coal quality characteristics as given in HB966.

### 3.6 Information On Existing Contracts

The following sections describe what limited data is available to the public on coal supply and transportation contracts.

#### 3.6.1 Coal Supply Contracts

The model described in Chapter 5 relies on forecast of mine price. This approach is justifiable since actual coal purchasing contracts are not available. Even for the utility, fluctuations in contract mine price would have to be predicted, while mine price to competitor utility companies would only be available in HB966. If we accept the accuracy of forecast mine price for long term contracts, other details of the contract are still needed, specifically, the "quality as received" and "annual

tonnage under contract". Coal supply and transportation contracts are proprietary information and are not publicly available. For coal supply contracts one source of information is available from the publishers of Coal Outlook, their Guide to Coal Contracts (Zuercher, 1985). This guide is made up of selections from questionnaires sent to all utilities by the FERC. A complete tape is available, but is beyond the financial resources of this study. Each contract summary contains the following information: name and address of coal company under contract, county, mine type, seam name, leaseholder, date signed, expiration date, type of contract (e.g., base plus escalation), annual tonnage under contract, coal quality specifications, quality as received, mine price (fob), delivered price, and transportation (truck, rail, barge). This publication is of limited usefulness. Only a few contracts for each utility are given, the destination (plant) is not given, and the lag between new editions is too great to make it up-to-date. The last point presents a major drawback since the current practice is to renegotiate contracts annually. Still, the FERC tape itself could be very valuable if it wasn't so expensive.

### 3.6.2 Coal Haulage Contracts

If little information is available on coal supply contracts, even less is available on rail haulage contracts. Prior to 1980 and the passage of the Staggers Act, the Interstate Commerce Commission (ICC) set and published tariff rates. Currently, NS and CSX negotiate all rates by contract. These contracts are not made available, and many contain a clause such that divulgence of contract terms is grounds for cancellation of the contract. Rates can only be guessed at based on the transportation price given in HB966, while quantity rebates and gathering costs are totally unavailable. One source of information on rates is the Fielston Coal Transportation Manual (Lescroart, 1986). This also includes a few example contracts. Unfortunately, no rates are given for NS or CSX.

## CHAPTER 4

### Market Analysis

#### 4.1 Introduction

The following chapter deals with the market structure of steam coal sales to the major electric utilities currently in operation in Virginia. The objective of this analysis is to define the scope and level of detail necessary to produce a reasonably accurate model of competition for coal sales. This competition will be shown to exist on two levels: competition which exists between the two railroads (CSX and NS) which control all movement of coal in Central Appalachia , and the competition which exists between the coal sales companies acting in the interest of or owned by the mine operators. This analysis is based on statistics which are given a economic rationale. It should be understood that statistics cannot be any better than the available data.

## Structure of a Market Analysis:

In order to design a viable model a thorough understanding of the system is essential. This understanding can be obtained by a combination of personal experience, consultation with experts on the various facets of the system, and analysis of the system. The analysis includes:

1. Defining broad objectives and questions of interest.
2. Acquisition of relevant data.
3. Determination of the quality and applicability of the data.
4. Redefinition of narrower objectives that can be achieved given the availability of data and resources.
5. Detailed market analysis.

This discussion will focus on stage five. Stages one through three have already been covered in Chapters 1 and 3. From the view point of a modeler, the goal of a market analysis is to fix the assumptions inherent in the model structure and development. These assumptions are then validated based on statistics for the available data and are given an

economic rationale.

### Impact of Market Conditions on Virginia:

In the few years covered by this study a number of changes have occurred in the regional market for steam coal, which together have had a drastic impact on Virginia producers.

These changes were:

1. As will be covered in section 4.3.2 there was a collapse in the spot market in 1985. Following this collapse, the lower mine price made Virginia much less competitive and the tonnage purchased by state utilities has fallen drastically.
2. Virginia continues to be the high cost coal producer, followed by West Virginia. Kentucky with its higher proportion of surface mining enjoys a considerable price advantage over Virginia. Even though Virginia produces the highest quality coal, all three states surpass regional boiler specifications. With an excess supply of high quality coal available, premium coal from Virginia does not command a higher price.
3. Competition between the railroads has affected the

competitiveness of producers, resulting in an increased price advantage of Kentucky coal over Virginia coal. Rail traffic in Central Appalachia is provided by two rail carriers: NS and CSX. Rail rates are a major portion of final delivered price and, as will be shown, determine the competitiveness of coal at the destination. Almost all mines and most counties are captive to one of these two carriers. Eastern Kentucky is mostly captive to CSX (except for Pike and Martin counties) while Virginia is mostly captive to NS (except for Dickenson county). CSX has pursued a strategy of lowering its rates in order to increase its market share of coal tonnage shipped, resulting in Kentucky's price advantage over Virginia.

4. Prior to the market collapse in mid-1985 there was a strong increase in demand which drove up production to a record high. Since the drop in the spot price, there has been an oversupply of coal which has caused intense competition and continued to suppress the mine price.
5. Prior to the drop in the price of oil, conversion to coal-fired burners was scheduled for many boilers, but the continued low price of oil has caused many conversions to coal to be postponed.

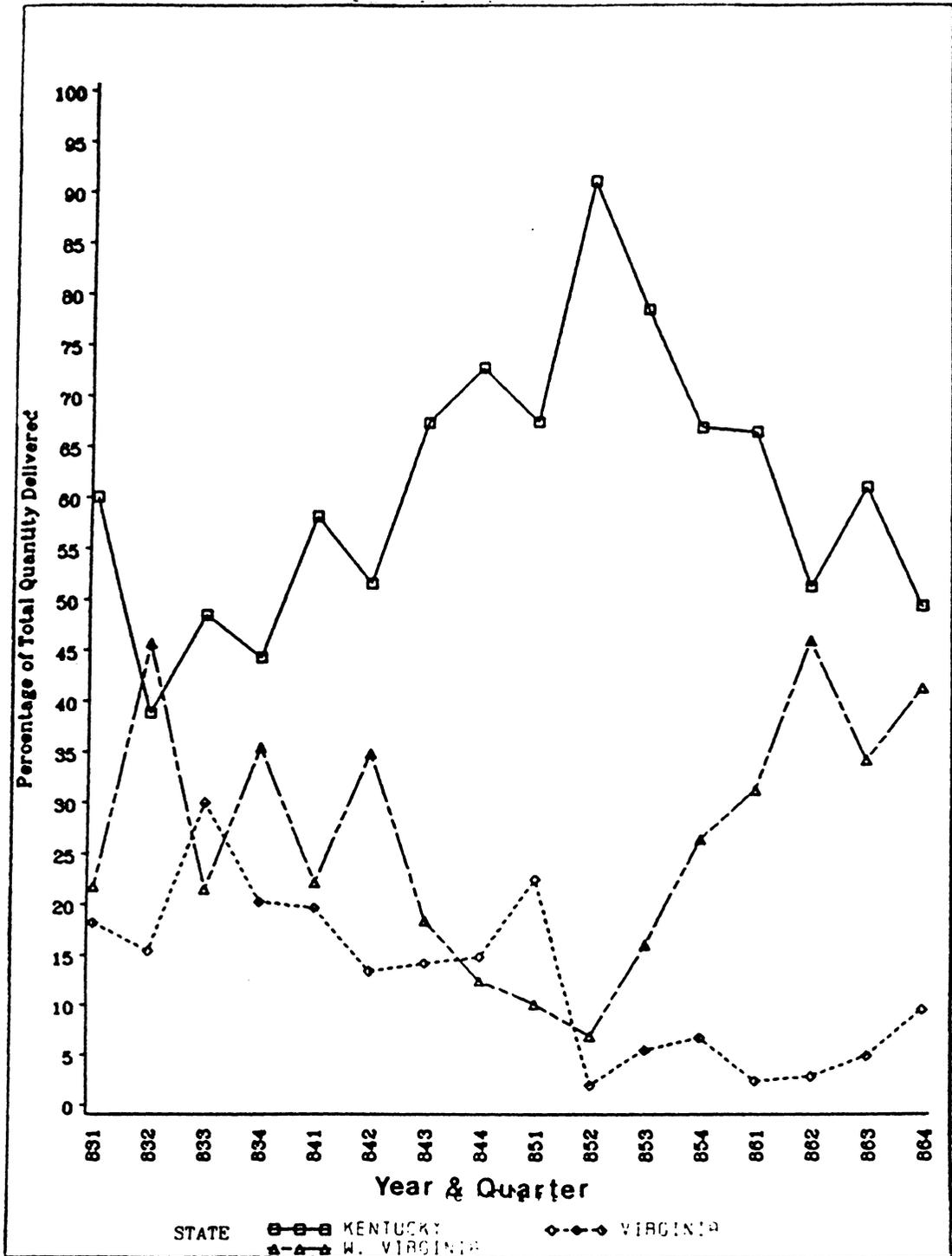


Figure 4.1

TIME TREND OF QUANTITY DELIVERED ON THE SPOT MARKET BY STATE  
 Deliveries to VEPCO From Kentucky, Virginia, and West Virginia, 83-86

As a result of the above developments Virginia is losing its share of the spot market. This is illustrated by Figure 4.1, which gives the percentage of spot coal purchased in each state by Virginia Power since 1983. Note that until 1985 around 20% of coal purchased came from Virginia, whereas after 1985 this figure has dropped to between 5 and 10%. Another development is that existing long term contracts are being renegotiated or bought out. There seems to be a shift to shorter term contracts which can be renegotiated annually.

#### 4.2 Regional Scope of Model

As seen in Chapter 2, previous energy models have in one way or another been national in scope. In some cases the emphasis has been regional, as in the Pennsylvania Coal Model, but none could be called a closed system. One of the unique features of this study is that it is specific in its scope regarding supply and demand. Demand is limited to Virginia utilities, while supply for these utilities comes almost entirely from Central Appalachia. Also, emphasis is given to the role played by the railroads.

## Utilities Included In Model:

This model is meant to mimic the market for steam coal as purchased by Virginia electric utilities. These utilities and their coal fired plants are:

VEPCO: Bremo, Chesterfield, Portsmouth, Possum Point, and Yorktown

APCO: Clinch River and Glen Lyn

PEPCO: Potomac River.

Unfortunately, PEPCO is not regulated by either the West Virginia Public Service Commission (WVPSC) or the Virginia Corporation Commission. Since these are the only two sources of information which break delivered price down into its FOB and transportation components, Potomac River cannot be adequately represented in this study. Thus, VEPCO and APCO plants represent the scope of demand for this model. However, this is not as grave a restraint as it might seem: PEPCO has sold its Virginia market to Virginia Power, choosing to concentrate on supplying the District of Columbia.

The demand nodes are the VEPCO and APCO plants as listed

above. The model will use historic consumption for these plants as given by the National Coal Association's publication, "Steam Electric Market Analysis". Since the model does not use forecast values for the demand, these quantities are the effective demands for coal at each plant which must be satisfied. The total consumption was found to correspond to those given by the data source used in this study.

The supply of coal to Virginia is limited to Central Appalachia not only because of specific interest in these three states, but also because these plants source all their coal needs from Virginia, eastern Kentucky, and southern West Virginia. As a result we have the happy circumstance of working with a closed system in terms of supply and demand. This is a circumstance which greatly limits the potential size of the model, while allowing for a more completely defined model.

#### 4.3 Supply

As shown above, Virginia utilities source all their shipments of coal from Central Appalachia. With a closed system, such as this, only the available supply of coal from these regions needs to be defined. Previous models have had long term time frames. This supposedly required defining resource

availability and using estimated reserves as a supply constraint. Since this model is short term (based on forecasts of the six month period 6/'86-12/'86), only the production capacities of the sources need be considered.

In the following analysis factors which influence mine price will be considered. Mine price can vary depending upon a number of interrelated parameters:

1. Due to the high cost of rail transportation one must consider both the carrier and the plant to which the delivery is bound; both the rate and the rate-setting policy may influence the mine price because of the highly competitive nature of the market.
2. The markets for spot sales vs. contracted coal vary to the extent that they need to be treated separately in this analysis.
3. Both coal quality and production costs can affect price; these influences must be considered as being specific to the source.

Sources have alternate definitions for the contract and spot markets. Those on long term contract are relatively simple to define and will be explained first. Defining spot market

sources is far more difficult and will be considered in some detail.

#### 4.3.1 Long Term Sources

Contract sources refer to specific existing contracts as of 5/86. Here the source is an existing mine or company that holds a contract with a utility to deliver a specified tonnage and quality of coal to that utility's plants. In this case production capacity is not a consideration, unless it can be determined that the contracted source is also involved in the spot market. The production capacity for contracted sources will be set at the contracted annual tonnage with that utility. This tonnage will be taken from Coal Outlook's "Guide to Coal Contracts" (Zuercher, 1985 edition). When the actual contracted tonnage is not available, the minimal tonnage will be taken as the minimum delivered in any month (as given in HB966) and the maximum as that source's production (as given in Keystone or that states directory of mines).

#### 4.3.2 Spot Market Supply

On the spot market, sources are defined as regions which for

various reasons can be said to be distinct. Overall, the entire Central Appalachian region competes in the same market due to the similarity in its coal quality and selling market, but for the purposes of the market, several regional distinctions are made. Primarily, the competition between the three states is of greatest interest. Therefore, the lowest and initial division is by state. Beyond this, further division must be made if there are distinct differences in the mine price on the spot market within the state. This nonhomogeneity within states may arise for two possible reasons. First, there may be an actual difference in the mine price due to location, coal quality, or mine cost. Secondly, there may be an influence arising from rail rates such that the mine operator feels that mine price must be adjusted for transportation charges in order to achieve a competitive delivered price to the utility. In summary, regional homogeneity and the influence of rail rates must both be considered before defining spot market regions. In the following analysis both matters will be resolved as well as possible given the available data. It should be remembered that the most elaborate statistical analysis and economic rationalizations can never be any better than the given data.

#### 4.3.3 Determining Regional Division of Spot Supply:

As long as mine price can be assumed to be independent of the rail rate, i.e., the mine operator does not consider or have knowledge of the final delivered cost when bidding, then it is only necessary to establish contiguous regions from which the spot mine price is essentially equivalent across all suppliers. Since it cannot be immediately assumed that rail rates will not influence mine price, it is necessary that the mine price be insulated from the effect of rail rates before testing the homogeneity of the spot market within any state.

Within any state the base rail rate will vary by the freight district in which the source is located and by the location of the plant. One way of removing the mine price from the rail rate would be to compare mine price across sources within the same freight district and delivering to the same plant. Counties which are captive to the same carrier are shown in Table 4.1. Note that in Wise, Virginia, there are only a couple of mines near Appalachia and Norton served by CSX. Otherwise, Wise is captive to NS. Thus, a good comparison of mine price could be made between Wise, Buchanan, and Tazewell counties. Likewise, in Kentucky, Harlan, Floyd, Letcher, and those sources in Pike known to be captive to CSX can be compared (there are no crossing or

duplication of lines in Pike). In West Virginia, McDowell, Wyoming, Mingo, and Mercer counties can be compared for NS, and Kanawha, Boone, Logan, Greene, and Raleigh are captive to CSX.

For a comparison of multiple means (in this case average mine price), with differing degrees of freedom, d.f., (number of deliveries), at multiple levels (between counties or states) a One-Way ANOVA (see Box, 1978) can be used to test the assumption that the mean spot price at all levels is equal. Here we test to see if the mean square between counties is significantly different from the mean square within counties by using an F test. In Table 4 the results of the between county ANOVA's for each state are given.

Table 4.2 gives the analysis for Virginia where only Buchanan and Wise had sufficient observations between 6/85 and 6/86 for comparison. Here the difference in spot price between the two counties is insignificant. For West Virginia all counties were compared; individual counties with too few observations to be considered individually were

Table 4.1: Counties Captive to a Single Carrier

Virginia:	Buchanan & Wise (NS) Dickenson (CSX)
Kentucky:	Harlan, Floyd, Letcher (CSX) Martin (NS) Pike (NS/CSX)
W. Virginia:	McDowell, Wyoming, Mingo, Mercer (NS) Kanawha, Logan, Boone, Greene, Raleigh (CSX)

grouped together (i.e., Greene, Fayette, and Raleigh). Mingo is captive to NS. There is little significant difference between the counties, especially when considering the badly unbalanced design (unequal degrees of freedom at the county level). Various range tests (pair-wise t-tests, Duncan, Tukey, and Bonferroni t-tests) indicate a possible difference between Mingo and the eastern grouping of counties. Note that this difference may be due to the difference in the carrier. In Kentucky there is little significant difference between counties. The range tests only point out a difference between Harlan and Floyd.

Table 4.2: ANOVA of Mine Price Between Counties

<u>State</u>	<u>Counties</u>	<u>F value</u>	<u>Prob&gt;F</u>
Va.	Buch. vs Wise	0.93	.34
	W.Va. Boone, East <sup>1</sup> ,		
	Logan, Mingo	1.98	.12
Ky.	Floyd, Harl,		
	<u>Letch, Pike</u>	<u>2.11</u>	<u>.10</u>
State	Ky, Va, WVa	3.66	.03
	1. Fayette, Raleigh, and Greene		
	(PROC GLM, SAS Users Guide: Statistics)		

Considering the lack of significant difference between counties within states, it might be asked if the same analysis would point out significant differences in the mine price on the spot market between the three states. The ANOVA indicates a definite difference in the spot market mine price between states.

In conclusion, the spot market is homogeneous within each of the three states as long as we can eliminate the influence of the carriers and rail rates. There is a significant difference in mine price between the three states. The next question to be addressed is how the carrier and rates influence mine price.

#### The Relationship Between Rail Carrier and Mine Price:

It was shown above that the spot price of coal is homogeneous within a state, especially when on the same carrier. Due to the scarcity of data there is little opportunity to compare mean spot price between counties in the same state who are also captive to different carriers. Virginia is essentially captive to NS. In West Virginia, very little has been shipped to VEPCO from NS dominated counties except from Mingo during the current market period, 6/85-6/86. In Kentucky only Pike and Martin counties have a strong NS presence. A good comparison would then be between sources

on NS vs. CSX in Pike.

First, compare the mean price as given in Table 4.3 using a t-test.

Table 4.3: Comparison of Mean Rail Cost

	<u>N</u>	<u>MEAN</u>	<u>STD</u>
CSX	54	25.61	1.84
NS	25	25.99	0.64

$$t = |x_1 - x_2| / s_p \sqrt{1/n_1 + 1/n_2}$$

where,

$$s_p = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

This turns out to have  $t=1.0$ , strongly indicating no difference in mine price between NS and CSX in Pike. We might assume from this that the choice of railroad does not influence mine price, but what about the influence of rail rates on the mine price? This question will be examined in the next section.

## The Influence of Rail Rates on Mine Price:

Specific origin/destination pairs can be compared for differences in mine price. If we make the somewhat broad assumption that rail rates are consistent when a delivery originates from the same freight district on the same carrier, and is going to the same plant, then it may be possible to compare mine prices to see if there is a significant difference in the price bid to different plants from the same origin. To make this comparison, the mean mine price was compared for origins within the same freight district with differing destinations.

Table 4.4:

ANOVA Of Mine Price & Range Tests Between Freight  
Districts and VEPCO Plants

<u>Origin</u>	<u>Destination</u>	<u>F value</u>	<u>Prob&gt;F</u>
Buch/Wise	22, 27	47.06	.0001
	A		
	B		
Kanawha	22, 21, 28	9.47	.0002
	A		
	B B		
Floyd, Ky	21, 22, 28, 32	9.20	.0001
	A A A		
	B		
Pike(CSX)	27, 28, 22, 32, 21	3.09	.024
	A A A A		
	B B B B		
Pike(NS)	28, 22, 27	25.61	.0001
	A A		
	B C		

21=Bremo, 22=Chesterfield, 27=Portsmouth, 28=Possum Point,  
32=Yorktown

A,B,C are groupings of significantly similar rail costs for  
the given destinations.

Table 4.4 summarizes the results from five ANOVA's and Bonferroni t tests of the differences between means, one set for each freight district. The results of the t tests are given following each ANOVA. Plants to which the mine price does not significantly differ are categorized using the same letter. For instance, shipments from Pike on NS are categorized for Portsmouth, Chesterfield, and Possum Point. In this case mine price to all are significantly different except for Possum Point and Portsmouth.

There is a strongly significant difference in mine price depending on the plant. This would be even more apparent if only the mine price of those plants with significantly different rail rates were compared. Thus, it is reasonable to conclude that rates are taken into account when bidding and that this is done in order to obtain a competitive delivered price.

Since mine operators will vary a bid in order to attain a competitive delivered price, one might ask if rail carriers likewise adjust rates in consideration of mine price in order to increase tonnage.

To address this question the following can be examined:

1. The correlation between mine price and rates within the

same origin/destination pair. If the rail carrier manipulates rates in view of maintaining a competitive delivered price, then there should be a negative correlation between mine price and rail rates for the same origin/destination pair.

2. The correlation between rates and mine price across different destinations for the same origin. This will give us another measure of the impact of rail rates on the mine price. If mine operators adjust their bid depending on the rate this correlation should be negative.
3. Comparisons of the variation in mine price and rates. If rail rates are essentially a fixed base rate for a specific origin/destination pair while mine price fluctuates due to intense competition then higher variance in the mine price can be expected and vice versa.

Table 4.5 gives the correlations needed to consider points 1 and 2 above. Not all combinations of origin and destination are accounted for because of insufficient data. Table 4.6 gives additional information relative to point 3.

Table 4.5  
Pearson's Correlation Coefficients  
Between Rail Cost & Mine Price

Source	Plant					Overall
	Bremo	Chest	Port	Possum	York	
Pike(CSX)	---	.40	.40	-.02	---	-.20
Pike(NS)	-.12	.36	.62	.26	-.85	-.40
Kanawha	-.12	.10	---	-.15	---	-.01
Floyd	.42	.26	---	.11	.37	.20
Buch/Wise	---	-.93	-.02	---	---	-.33

(PROC CORR, SAS User's Guide: Basics)

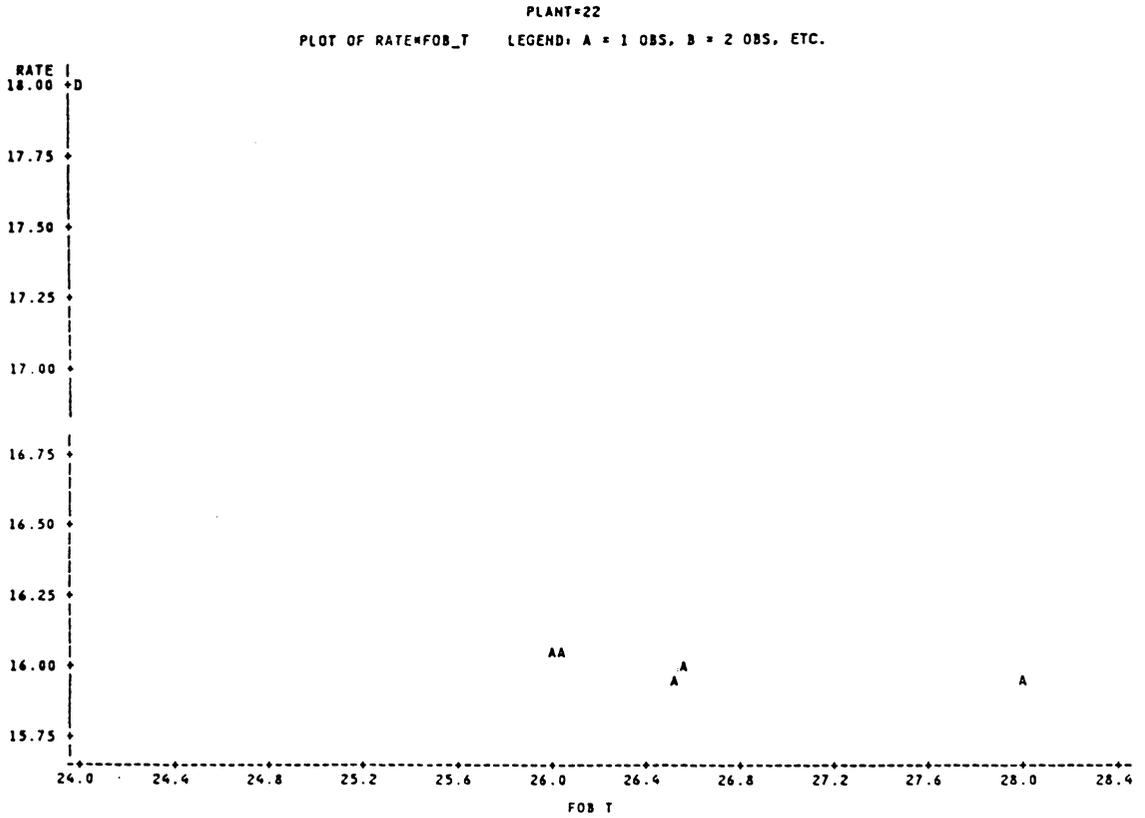


Figure 4.2 Scatter Plot For Virginia To Chesterfield

The correlations given in Table 4.5 should be viewed with some caution. In some cases there were too few observations to generate the coefficient. These were left blank in the table, but more observations are needed. Let's consider the overall trends and individually look at the exceptions. The correlation between mine price and rail cost for specific origin/destination pairs is on the whole inconclusive. In some cases there is a weak positive correlation, but on the whole there is no strong relationship between rail cost and mine price. There are two exceptions: Virginia to Chesterfield and Pike county to Yorktown. Both have strong negative correlations, which is what we would look for if rates were being set in consideration of the mine price. To look at these two cases in greater detail the scatter plots of the rates vs. mine price can be examined. The scatter plot for shipments from Virginia to Chesterfield is given in Figure 4.2. The symbol "D" at a rate of \$18.00 represents four deliveries made to Chesterfield. Note that all other deliveries were made at about a \$16.00 rate regardless of the mine price. This one set of expensive deliveries is responsible for the strong negative correlation, and therefore should not be considered a valid correlation. Now, note the overall correlation for shipments from a freight district to all destinations. Here the overall trend is negative, which supports the ANOVA results. These results

are weak and cannot stand on their own for making conclusions.

Table 4.6 gives a more detailed breakdown of the data used in Table 4.5. Upon examination of these statistics one can see two trends:

1. The highest rail cost is usually associated with the lowest mine price and vice-versa. This is particularly true when there is a significant spread in the rail cost.
2. Variance is lower for rail cost than for mine price. Exceptions to this are due to sharp changes in the rates in a particular month rather than fluctuation between individual deliveries in the same month. As an illustration of this consider the case of Pike county shipping on CSX to Portsmouth. In this case both the standard deviation and the range of the rail costs exceeds the same for the mine price. Figures 4.3 and 4.4 are time plots of the mine price and rail cost for this origin destination pair. The mine price fluctuates more broadly than the rail cost except for a rate reduction in late 1985. It is this sort of variation in rail cost which at time seems to show the variation in the rate being greater than that of the mine price.

Table 4.6. Comparison of Rail Cost & Mine Price Between Single Origin Destination Pairs For the Period 6/85-12/86 (FOB is mine price, Rate and FOB in \$/ton)

<u>Pike (NW)</u>								
Plant	N	Mean & Rank		Signif. Group	Std.		Range	
		Rail Cost	Rank FOB		Rate	FOB	Rate	FOB
Chester.	39	17.03(2)	26.43(3)		0.71	2.95	3.56	10.52
Possum.	15	19.49(3)	25.54(1)		0.52	2.64	3.44	8.01
Ports.	48	14.94(1)	25.84(2)		1.64	1.65	5.03	10.23 *
<u>Pike (CSX)</u>								
Plant	N	Mean & Rank		Signif. Group	Std.		Range	
		Rail Cost	Rank FOB		Rate	FOB	Rate	FOB
Ports.	13	13.98(1)	26.87(5)	A	2.07	1.09	6.09	3.98 *
Possum	6	16.39(4)	25.72(4)	A B	0.85	1.12	2.17	3.15
Chest.	24	16.19(3)	25.41(3)	A B	1.10	1.20	3.37	3.26
York.	7	18.30(5)	24.73(2)	B	3.66	3.85	10.18	10.87 *
Bremo	4	15.72(2)	24.08(1)	B	1.69	.2	2.94	.39
<u>Kanawha District</u>								
Plant	N	Mean & Rank		Signif. Group	Std.		Range	
		Rail Cost	Rank FOB		Rate	FOB	Rate	FOB
Chest.	48	14.87(2)	24.68(3)	A	.76	.87	2.54	2.85
Bremo	32	14.13(1)	24.68(2)	B	.73	.67	2.56	3.00
Possum	25	15.60(3)	24.63(1)	B	.47	1.69	1.75	4.78
<u>Floyd, Ky.</u>								
Plant	N	Mean & Rank		Signif. Group	Std.		Range	
		Rail Cost	Rank FOB		Rate	FOB	Rate	FOB
York.	10	16.04(2)	26.23(4)	A	.15	.50	0.32	2.00
Chest.	29	15.57(1)	24.70(3)	B	.98	.88	2.96	3.00 *
Bremo	7	16.18(4)	24.57(2)	B	1.25	1.11	2.65	3.24
Possum	22	16.12(3)	24.43(1)	B	.63	1.07	1.59	3.70
<u>Buchanan &amp; Wise</u>								
Plant	N	Mean & Rank		Signif. Group	Std.		Range	
		Rail Cost	Rank FOB		Rate	FOB	Rate	FOB
Port.	29	13.28(1)	26.23(3)		.66	1.36	1.97	4.60
York.	3	16.27(2)	26.00(2)		.25	.87	.43	1.50
Chest.	9	16.88(3)	25.45(1)		1.05	1.50	2.03	4.00

\* Rates for this O/D pair were cut during this period.

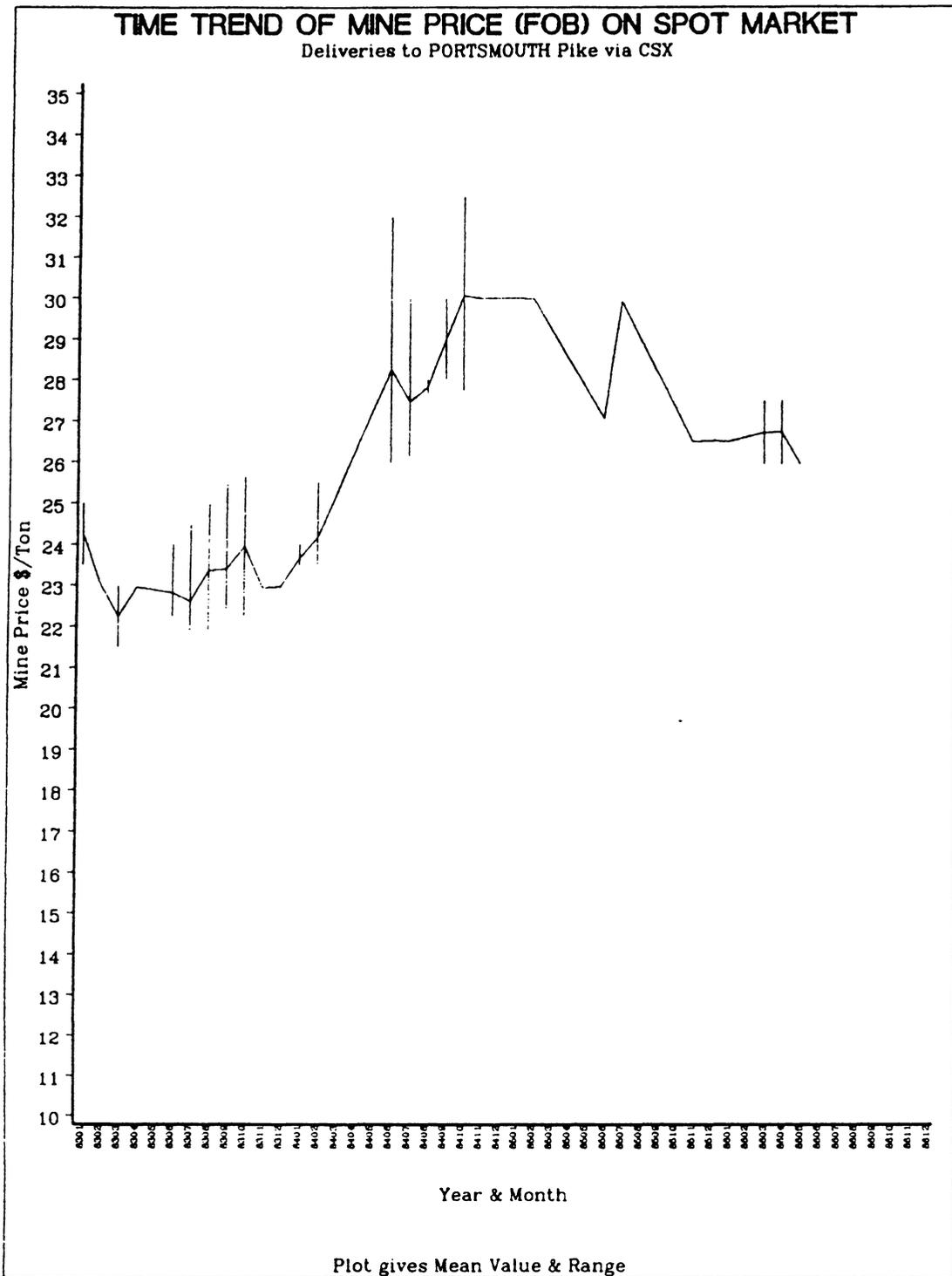


Figure 4.3

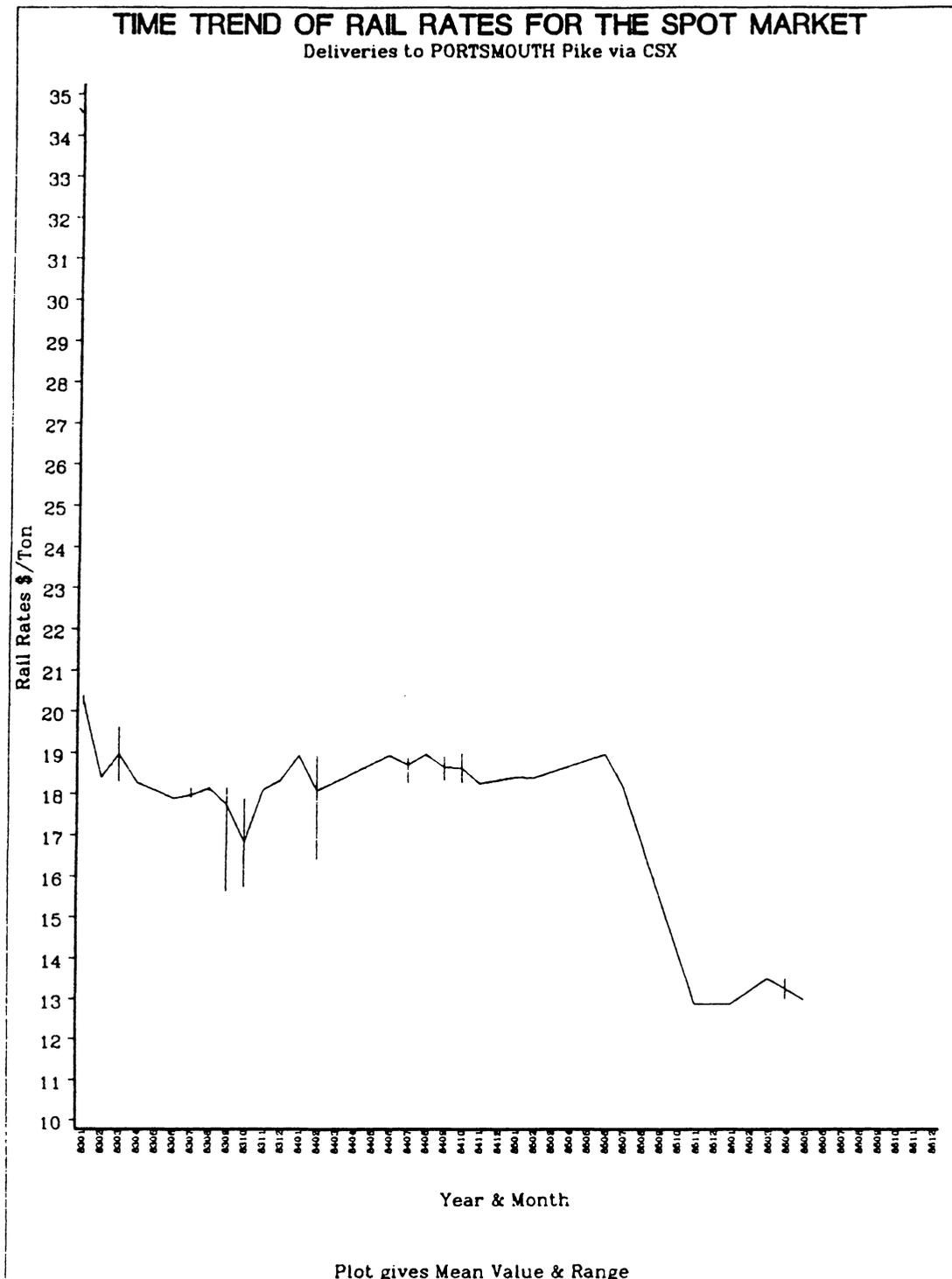


Figure 4.4

These results suggest that mine operators lower their bid to plants with higher rates in order to achieve a competitive bid price and take advantage of lower rates. In comparison with mine price, rates are relatively constant at any one time period and only vary sharply over time. This indicates that the rail carriers do not openly manipulate rates in consideration of mine price. As a consequence, mine operators bear the burden of competition and are highly constrained by rail rates.

#### 4.4 Factors Affecting Rail Rates

Electric utilities purchase coal using a number of criteria, the most important of which are: boiler specifications (coal quality), diversification of supply (regional, contract vs. spot, union vs. nonunion), use of a preferred supplier list, and delivered price. The primary consideration is delivered price. Boiler specifications are easily met with coal from this region. The strategy used in determining price coefficients for the objective function of the model (see chapter 6) is to forecast the mine price and add to this the rail cost. This direct use of rail cost rests on the assumption that rail cost is not influenced by factors relating to the individual shipment (such as shipment size) and that other factors will be fixed by the structure of the model. In this

section this assumption will be justified.

Rates are a function of:

1. The distance hauled - This is roughly fixed for any origin/destination pair and need not be considered further.
2. Gathering costs - This does vary considerably depending upon the region, carrier, and mine. For instance, in Virginia NS gathers together car loads of coal coming from a number of sources into a gathering yard, except for those cars shipped from a prep plant as unit trains. The rates are therefore based on single cars. CSX makes more use of unit train rates. The facilities at the mine site or tipple also are accounted for in the rate. Determination of gathering costs is not possible with the available data.
3. Unloading facilities - The speed and efficiency with which trains can be unloaded is accounted for in the rates. This is fixed for any specific plant.
4. Handling costs - This is included in HB966 as a separate charge.

5. Quantity shipped. - There is no question that quantity incentives are included in rail transportation contracts to encourage the utility to increase the tonnage shipped on that particular line. Unfortunately, the form of this incentive is an end of year rebate or surcharge depending on whether the total tonnage was over or under the base rate volume. Because of the rebate system, the influence of the quantity shipped does not show up in the available data. The data was examined using regression to see if a case could be made for quantity having an influence on price or if different rates were charged for single cars than for unit trains. The significance of quantity on rates was highly erratic for different freight districts. Usually the regressor for quantity was insignificant. In some cases it was significant and positive in sign. An example of this is found in deliveries to Virginia Power from CSX's Kanawha district. The plot of centered rail costs vs. the quantity shipped is given in Figure 4.5. Centering is the subtraction of the mean from individual observations. This places all observations at the same level, aiding in visual examination of the variation in the rates (see Myers, 1986). The plot includes the curve for a cubic regression of the rate as a function of the quantity. This provides a best fitting curve. Note that the observations fall into categories of 7000 tons (unit trains) with a mass of single car (less than a unit

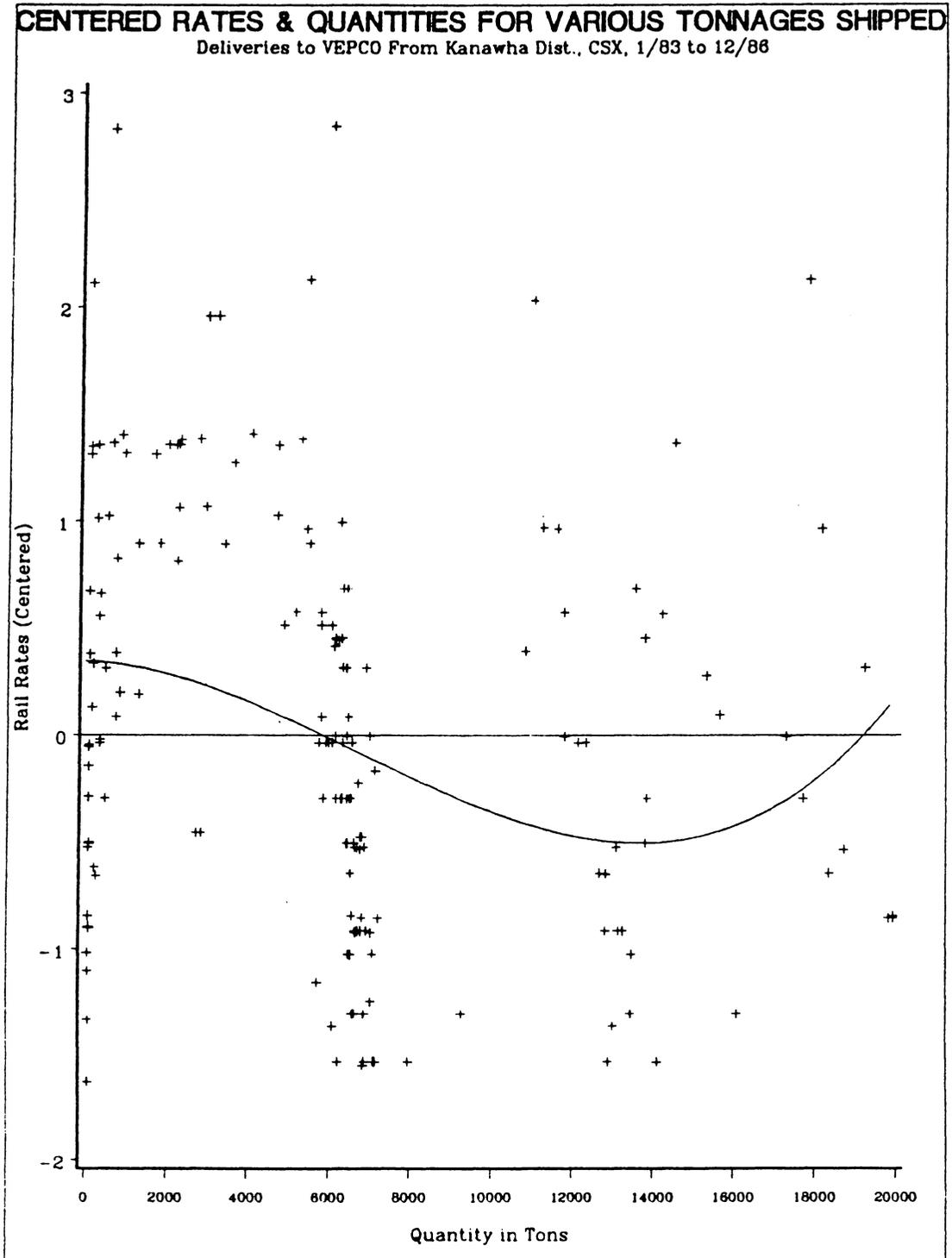


Figure 4.5

train) shipments. Careful examination of these deliveries showed that unit train and multi-unit train deliveries were during a period of peak demand when the price was low, not that unit trains necessarily would have lower rates. Figure 4.6 gives the same plot for deliveries made by NS from Buchanan and Wise to Virginia Power. This shows absolutely no relationship between quantity and rates.

6. Competition for lower delivered price -- This probably exists, and is best accounted for by using actual rail rates.

Rates and mine price have already been shown to be positively correlated. This may largely be a coincidence of the analysis being for the current market during which rates and mine price have both been falling. Regression analysis with a time component shows this to be the case.

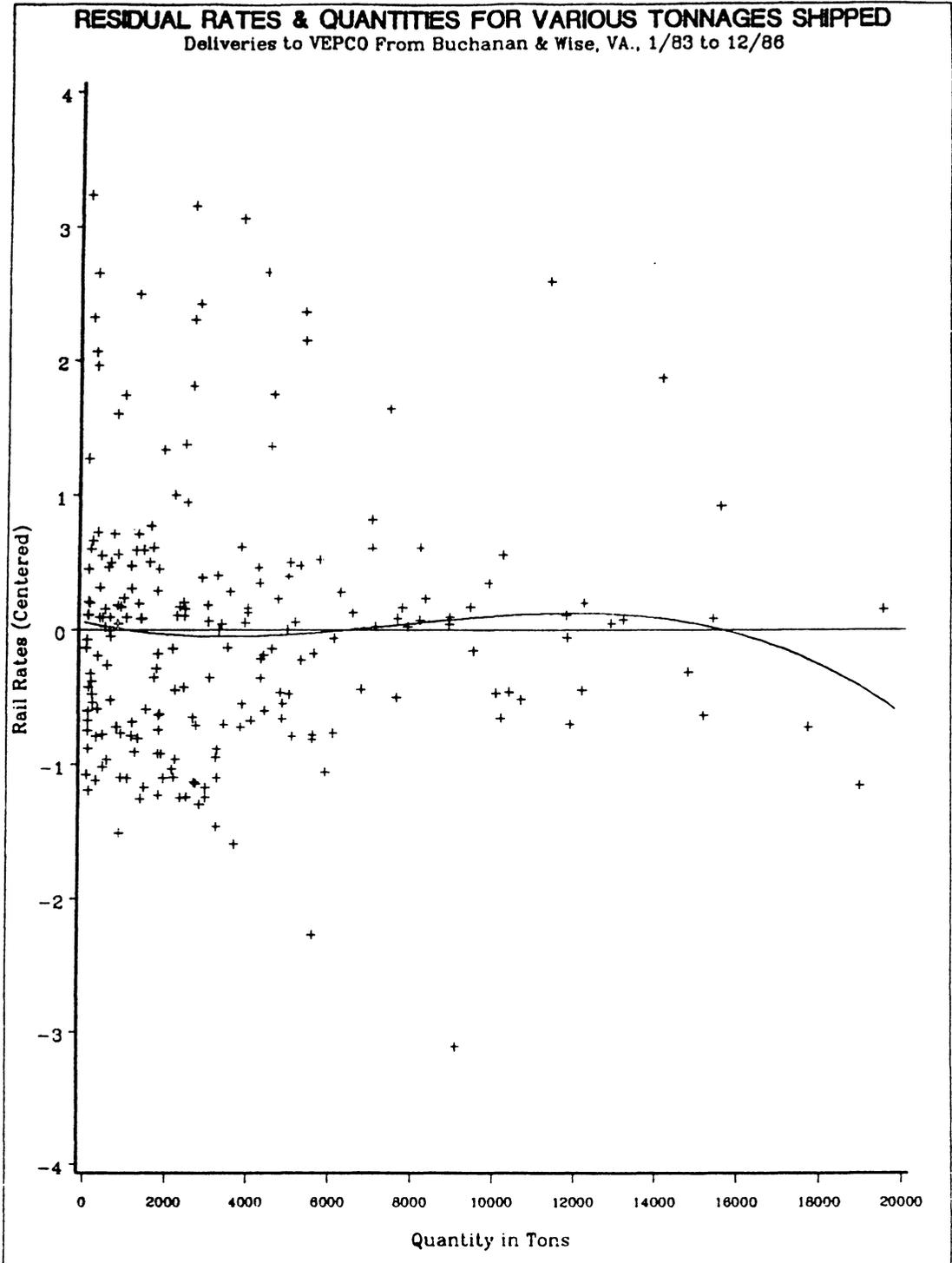


Figure 4.6

Quantity incentives are not immediately reflected in reported costs and therefore are not apparent in the available data.

#### 4.5 Influence of Market Structure on Model Development

The coal purchasing model that is described in Chapter 6 can be used to mimic the market characteristics that have been described in this chapter. From the point of view of the utility, the steps involved in obtaining coal supply follow a pattern which can be represented in a mathematical model. The utility first determines demand for power generation in the upcoming year. We will use the actual usage of coal, but the utility forecasts could be used if this model was to be used for a future time period. It used to be that coal supply was first negotiated before arranging for transportation to the plants, but since 1980 and the Staggers Act, acquisition of transportation contracts with NS and CSX have become more dominant concerns. Rail rates are first negotiated, and following this, only those mines which are on the lines of the contracted railroads are given the opportunity to bid. The utility negotiates coal supply with the mining companies (or their proxies) with the objective of obtaining the lowest total delivered price. This procedure can be represented in a mathematical framework (see Chapter 6) and

solved to minimize total delivered cost to the utility. By considering delivered cost in its two separate components, different strategies aimed at increasing competition between the carriers and the mining companies can be explored, thus lowering the cost of supply.

## CHAPTER 5

# Forecasting of Mine Price to Virginia Utilities

### 5.1 Introduction:

The likely purpose for a utility coal purchasing model is to determine an optimal future coal supply policy. This requires some estimates as to what the mine price of steam coal will be in the near future, as well as estimates of the upper and lower limits of the possible deviation of the estimated mine price. Thus, a forecast of the mine price along with confidence intervals is needed. The forecast expected value is used to generate the initial most likely scenario after which the confidence limits are used in a post-optimality sensitivity analysis.

In this chapter the full scope of obtaining forecasts of mine price of steam coal will be examined. This includes a review of the division of forecast regions (i.e., supply regions), methodology, applications and modifications of classical forecasting methods, as well as the final

results.

## 5.2 Division of Forecast Regions:

As was explained in detail in Chapter 3, the market for steam coal can initially be decomposed into existing long-term contracts and the spot market. Beyond this the state in which the shipment originates and the rail rate to the destination can be used to separate mine price on the spot market.

### 5.2.1 Long Term Market

Most utilities try to ensure stable coal supply and fuel costs by contracting coal supply with various coal sales and mining companies. To do so a long term contract is established between the utility and the coal supplier. A contract will typically extend from one to five years. Earlier contracts extended for longer time periods, but the current trend is towards contracts which can be renewed annually. These contracts specify coal quality as received, annual tonnage, and price f.o.b. mine. The price is usually a base price plus cost escalation.

In the case of contracted coal supply the source of the coal is obvious, and, as a result, there should be a

different forecast of mine price for each active contract. The source might be a single mine or local mining company, or it might be a coal sales company supplying coal from a number of different sources. In either case, coal price quality and annual tonnage is specified by contract and can be expected to remain relatively stable.

### 5.2.2 Spot Market

As described in Chapter 4, the spot market is drastically different from the long term contract market in its behavior and structure. The spot market is characterized by unstable prices and a large number of suppliers.

The effect of competition and demand are most immediately felt in spot sales. A contract offers security to both the coal supplier and the utility, but when making a bid many other competitors may have to be taken in to consideration both as to their mine price and their likely rail cost. As a result of this, intense competition occurs during a period of over-supply, as is currently the case. Conversely, when there is a sharp increase in demand, the spot price of coal will rapidly increase. This was the case from 1984 through the beginning of 1985. Mines almost

invariably operated below capacity and can readily increase production. Also, many small contract miners go into production when the spot price picks up, thus meeting the lag between supply and demand. When demand by utilities falls, production capacity is high, resulting in sharp oversupply. It is costly to cut back on production and coal operators must continue to sell even if by doing so only their operating costs can be met. As a result, production cannot adjust itself to demand rapidly enough to avoid severe reductions in market price and the elimination of many small and more costly operations from the market. These market realities have to be taken into account when attempting to forecast mine price on the spot market. Again, they are:

1. Large numbers of suppliers.
2. Extreme instability of the market price.
3. Cut-throat competition during periods of oversupply resulting in fewer low-cost operators remaining in the market.
4. Diversity as to individual sources of supply.

Each of these characteristics must be accounted for when

developing a methodology to forecast mine price in this market. In determining the division of forecast regions for the spot market, point four determines the degree of resolution which is possible. Unlike long term contract sales, an individual mine rarely has a sufficiently long period of continuous sales to a utility to allow for forecasts for individual sources. Instead, we must look for groupings of sources for which the level and behavior of mine price has been relatively uniform. As discussed in chapter four, the most reasonably divisional breakdown is by state followed by the rail carrier and destination. Thus, for each of the three states, separate forecasts will be made for both of the carriers. In those cases where the difference in rail rates between different destinations is sufficient to influence mine price, a separate forecast will be made.

### 5.3 Forecasting Methodology:

Classical forecasting methodology using either Regression or Time Series models are accepted econometric tools. Detailed discussion of the models used in this study are available elsewhere as listed in the references. The data on coal deliveries is not directly suitable to either Regression or Time Series, particularly in the case of

spot sales. Thus, a series of techniques not addressed in previous studies were necessary to forecast mine price using this complex and unwieldy data source. Prior to discussion of adaptation of Time Series to the data, the inadequacies of simple regression models will be reviewed and basic Time Series models will be summarized.

### 5.3.1 Inadequacies of Regression Modeling

Initially, price functions were to be built based both on variables related to mining cost, such as productivity and seam height, as well as data on the price paid for coal deliveries. After considerable fruitless effort, it was conceded that reasonably accurate cost functions could not be built given the available data nor estimated using Regression. The main reasons for this problem will be outlined in the following section.

#### Insufficient Explanatory Power of Data:

As discussed in Chapter 2, the nature of the data dictates that mine specific data, which could be used to explain the cost component of mine price, is not mergible with data on actual coal deliveries. As long as mining cost

does not account for the difference in price between competing mines this is not a problem. In this case we assume that the price of coal is entirely market driven and therefore mine operators sell for whatever price they can get: a not unreasonable assumption in the current spot market. In either case, the specification error must be considered. This is the case when variables which are relevant to mine price are omitted from a Regression equation.

An obvious consequence of leaving out relevant variables is a reduction in the explanatory power of the equation, i.e., lower  $R^2$ . Since it must be assumed that there is a rational explanation behind the price of coal (?), then given the correct variables a functional form could be found that yields a high  $R^2$ . Unfortunately this has not been the case. Regression based only on data from HB966 typically yields very low  $R^2$ . This lack-of-fit is one reason for abandoning the attempt to generate price functions, but not only was  $R^2$  low, the coefficients for those significant variable included in the model were inconsistent in both sign and magnitude. One possible explanation for this is what is termed specification error.

Let's assume that the true Regression equation for mine

price is a linear function of Btu and Productivity, representing the market and production cost components of mine price respectively.

$$FOB_i = \beta_2 BTU_i + \beta_3 PROD_i + \varepsilon_i \quad (5.1)$$

Instead of using 5.1 an equation is fit which assumes that operating costs are not significant, i.e.,

$$FOB_i = \beta_2^* BTU_i + \varepsilon_i^* \quad (5.2)$$

For this univariate equation the Ordinary Least Squares (OLS) slope estimate is,

$$\hat{\beta}_2^* = \frac{\sum x_{2i} FOB_i}{\sum x_{2i}^2} \quad (5.3)$$

Substitution of the actual estimator of  $FOB_i$  as given by 5.1 into 5.3 will yield,

$$\hat{\beta}_2^* = \beta_2 + \frac{\beta_{3i} \sum x_{2i} x_{3i}}{\sum x_{2i}^2} = \beta_2 + \frac{COV(x_2, x_3)}{VAR(x_2)} \quad (5.4)$$

Since the  $E(\varepsilon_i) = 0$ .

As a result, when the  $COV(x_2, x_3) = 0$  the slope parameter  $\hat{\beta}_2^*$

will be biased in the direction of this covariance and the estimate will be inconsistent (see Myers, 1986).

### Colliniarity:

Some of the quality characteristics exhibit dependence among themselves; in particular, Btu, Ash, Water, and Sulfur. This collinear relationship results in several problems which renders OLS estimates of questionable validity.

- Parameter estimates become biased upwards.
- Parameter estimates are unstable and highly dependent on the data set used.

Due to colliniarity, regression equations often have parameters with wrong sign, misleading tests of significance, and are not generally applicable given other data sets. As a result, cost functions containing terms for Btu and sulfur estimated using OLS can have a positive coefficient for sulfur, yet irrespective of the data we know that there is no price premium paid for sulfur. What might be happening is that a premium is paid for high Btu coal and since sulfur does burn and thus raise the Btu value (colliniarity), the positive coefficient for sulfur may be due to sulfur acting as a proxy for Btu.

### Consequences of Time Dependent Data:

Much of this study is based on the fact that mine price varies over time in a predictable manner. This implies that there exists a correlation between past values and the current value, and that once this serial correlation has been quantified it can be used to predict future values. Even though serial correlation can be used to advantage it is a major stumbling block when using OLS to estimate a regression.

OLS estimation provides the "Best Linear Unbiased Estimate" (BLUE) when residuals don't exhibit trends and are uncorrelated.

$$E(\varepsilon) = 0 \quad \text{and} \quad \text{Cov}(\varepsilon) = E(\varepsilon\varepsilon') = \sigma^2 I \quad (5.5)$$

The assumption is that the population value of the error term is zero (no trend) and that there is no serial correlation such that the Variance-Covariance matrix used in the OLS parameter estimator  $X(X'X)^{-1}X'Y$  is the identity matrix, i.e., constant variance over time (for an excellent discussion of OLS see, Draper and Smith, 1966). Times series data often is serially correlated and has trends over time. When this is the case the variance-covariance

matrix is no longer the identity matrix assumed for OLS. Rather, the variances on the diagonal can be changing over time while the covariances on the off diagonals can be nonzero. In this situation the actual residual series is not independent but autocorrelated over time. For instance, consider first order serial correlation in which the current error term,  $\varepsilon_t$ , is a function of the past error,  $\varepsilon_{t-1}$ , as well as pure error,  $u_t$ .

$$\varepsilon_t = \rho_{t,t-1}\varepsilon_{t-1} + u_t \quad \text{for all } t, \quad |\rho| < 1 \quad (5.6)$$

As a result,  $E(\varepsilon\varepsilon') = \sigma^2\Omega$  where,  $\Omega$  is the matrix of correlation coefficients between the residuals and  $u_t$  in this first order autoregressive process is  $N(0, \sigma^2)$  and independent over time.

Serial correlation results in a loss of efficiency of the OLS regression estimators and the failure of the usual tests of significance. Also, least squares estimators do not exhibit minimum variance.

BLUE estimates can be obtained by using Generalized Least Squares (GLS) instead of OLS. In this case an estimate of the actual variance-covariance matrix is used so that the GLS estimator becomes,  $X(X'\Omega X)^{-1}X'Y$  (see, Pindyck, 1981). Here the major obstacle is to determine the degree (number

of lags) of the autocorrelation and then obtain estimates of the terms in  $\Omega$ .

As an example of the improvement in efficiency as well as fit when using GLS instead of OLS for serially correlated data, consider the OLS results in Table 5.1, and the same data estimated using GLS in an autoregressive model in Table 5.2.

#### Nonstationary Market Price:

Stationarity is essential when modeling a time series. A stationary process is one which remains in equilibrium about a constant mean level. This includes the process mean, variance, and covariance. With a stationary process it can be assumed that a market has not changed markedly over time any of the parameters used in the model. For instance, we would expect to see the Btu level fluctuating about the same mean level during any subset of the entire time period of the data set. Likewise, we would not see any trend in price over time. When this is true, the process is stationary. Therefore, a model estimate for any subset time period should have the same values and significance levels regardless of the time frame of the subset, i.e., the estimation will be consistent and

Table 5.1: OLS Results for Russel to APCO

ORDINARY LEAST SQUARES ESTIMATES				
SSE		136.2113	DFE	151
MSE		0.9020617	ROOT MSE	0.9497693
SBC		467.1769	AIC	442.6257
REG RSQ		0.2270	TOTAL RSQ	0.2270
DURBIN-WATSON		0.5707		

VARIABLE	DF	B VALUE	STD ERROR	T RATIO	APPROX PROB
INTERCPT	1	38.5518088	20.3251755	1.897	0.0598
DEL_C	1	0.5315670	0.2352715	2.259	0.0253
QUANTITY	1	-.0000044385	.00000298074	-1.489	0.1386
SULFUR	1	-3.1499351	1.4370202	-2.192	0.0299
BTU	1	0.0006944001	0.001295074	0.536	0.5926
ASH	1	-0.2197181	0.1926799	-1.140	0.2560
WATER	1	0.1357078	0.2606362	0.521	0.6034
SURF	1	-0.2390234	0.1680026	-1.423	0.1569

Table 5.2: GLS Results for Russel to APCO

A U T O R E G P R O C E D U R E				
PRELIMINARY MSE=		0.3788788		
ESTIMATES OF THE AUTOREGRESSIVE PARAMETERS				
LAG	COEFFICIENT	STD ERROR	T RATIO	
1	-0.45323519	0.08103378	-5.593164	
2	-0.20738215	0.08754353	-2.368903	
3	-0.16781544	0.08103378	-2.070932	

YULE-WALKER ESTIMATES				
SSE		27.20732	DFE	148
MSE		0.1838333	ROOT MSE	0.4287578
SBC		227.2408	AIC	193.4829
REG RSQ		0.0689	TOTAL RSQ	0.8456

VARIABLE	DF	B VALUE	STD ERROR	T RATIO	APPROX PROB
INTERCPT	1	25.7511654	9.95761550	2.586	0.0107
DEL_C	1	-0.1860196	0.14676494	-1.267	0.2070
QUANTITY	1	-.0000013613	.00000111025	-1.226	0.2221
SULFUR	1	0.3922376	0.66730350	0.588	0.5576
BTU	1	0.001236974	0.0006304626	1.962	0.0516
ASH	1	0.1555591	0.09293285	1.674	0.0963
WATER	1	0.1908785	0.13012904	1.467	0.1445
SURF	1	-0.0212217	0.06252740	-0.339	0.7348

stable. This is a very valuable property since it allows confirmation of the validity of the model and the application of the model to other data sets. The implications for forecasting are obvious. Conversely, when the data is nonstationary the applicability of any cost function based on that data is dubious. The same can be said of any forecast.

As an example of a nonstationary series consider the time plot of mine price for Virginia Energy and Harman Mining to Virginia Power in Figure 5.1. Here there has been an overall trend towards increasing price. Such a trend can be accounted for in a forecast as will be discussed later., but this trend renders any general cost function estimated using OLS invalid. The ramifications of having a nonstationary time series are that:

- OLS estimates are inefficient and tests of significance are invalid.
- Historic data can not be replicated. Estimated coefficients will not refer to any actual time frame.
- The time trend will not be accountable in cost functions or forecasts.
- Modeling and estimation becomes much more complex.

Time series modeling can be used to incorporate the



movement of serially dependent data into a model while using GLS to account for the covariance structure of the residuals. As an example consider, the OLS and autoregressive models estimated as shown in table 5.1 and 5.2.

### 5.3.2 Time Series Models

For the reasons given previously, Least Squares (LS) is inadequate for estimation as are simple regression models when applied to time series data which is serially correlated and/or nonstationary. Time series models will often provide a superior fit and the best forecast. Thus, prior to discussion of the application of time series models to this study, a brief review of classical time series models follows.

#### Autoregressive Models:

Consider a model for a time series  $y_t$  which is only a function of  $p$  of its own past values, i.e., autoregressive of order  $p$  (for the following see, Box, 1976).

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_{t-p} y_{t-p} + \varepsilon_t$$

$$(1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) y_t = \varepsilon_t$$

$$\phi(B)y_t = \varepsilon_t \quad (5.7)$$

Here,  $Y_t$  is considered to be stationary such that after centering  $y_t = Y_t - \mu$  is the deviation of a stationary stochastic process from its origin,  $\mu$ , and  $\varepsilon_t$  is white noise, at least in part due to unmeasured influences.  $B$  is the back shift operator, such that,  $B^j y_t = y_{t-j}$ .

Thus, using 5.7, mine price can be described as a function of its own past values such that mine price fluctuates over time in some identifiable pattern.

#### Moving Average Models:

A Moving Average process of order  $q$  expresses  $y_t$  linearly as a finite number,  $q$ , of previous errors.

$$\begin{aligned} y_t &= \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \\ &= (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q) \varepsilon_t \\ &= \theta(B) \varepsilon_t, \text{ where } \theta < 1 \end{aligned} \quad (5.8)$$

Here the fluctuation of the process about its mean value (under the assumption of stationarity) is described as a moving average of the errors about that mean. Unlike a strict moving average the sum of the  $q$  terms need not sum

to unity, nor are the individual coefficients,  $\theta$ , restricted to positive values.

#### Nonstationary Homogeneity:

Mine price of steam coal is typically nonstationary. Time series modeling will not work for nonstationary data, but in most cases where a series is not severely nonstationary it can be transformed into a stationary series by differencing. Such a series is referred to as being homogeneous nonstationary.

A necessary but not sufficient condition for stationarity is that the mean of the process,  $\mu$ , is constant over time. Thus it can be shown that the sum of the autoregressive parameter must sum to less than unity,

$$\phi_1 + \phi_2 + \dots + \phi_p < 1 \quad (5.9)$$

Otherwise, the process is drifting away from any specific  $\mu$ . If this is the case then the series may be restored to stationarity by differencing  $d$  times. For example, first order differencing of the series  $y_t$ ,

$$\tilde{y}_t = \phi y_{t-1} + \varepsilon_t \quad (5.10)$$

where,  $\tilde{Y}_t = Y_t - Y_{t-1}$

Differencing will act to homogenize the level of  $y_t$  throughout the series or to flatten dominating trends. A mixed AR(p) MA(q) model in which the time series has been differenced  $d$  times is classified as being an ARIMA(p,d,q) model. In this study mine price never required differencing more than once.

#### General ARIMA Models:

The characteristics of both an autoregressive model of order  $p$ , AR(p), and a moving average of order  $q$ , MA(q), can be combined additively after differencing  $d$  times to form a ARIMA(p,d,q) model.

$$\begin{aligned}
 Y_t &= \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q} \\
 \phi(B)Y_t &= \theta(B)\varepsilon_t \qquad (5.11)
 \end{aligned}$$

Here it is assumed that the time series is only a function of the lagged dependent variable and the error process over time, but in the case of mine price other independent variables help determine the price of coal. These independent variables can be treated as independent time series and used as further input in the ARIMA model.

If the input variables can be assumed to be independent of each other (i.e., not collinear), and to only influence the value of the dependent series in the current time period (no lagged independent series parameters in the model) then a model which combines regression and time series can be formulated. The regression part of the model can be thought of as performing its usual role in prediction while the ARIMA portion of the model helps to explain the residual variation as a time series leaving an error term,  $\eta$ , which is pure error. An example of this would be if the Btu value was included in the model, such that,

$$y_t = \beta_0 + \beta_1 \text{BTU}_t + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \eta_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q} \quad (5.12)$$

Outside of the complexities of the spot market this model is often used. In cases where there is a lag between the input series and its impact on the mine price ( $y_t$ ) model formulation becomes far more complex requiring the use of Transfer Functions, but such lagged independent variables are rarely significant.

#### 5.4 Application of Time Series to Forecasting Mine Price:

As discussed in section 4.3.1 simple regression modeling of the data given by HB966 on deliveries of steam coal is inadequate for either cost functions or forecasting. In some ways ARIMA modeling seems to be the solution to some of these problems. Section 5.3.2 summarized ARIMA model as commonly used. Unfortunately, some practical limitations to ARIMA modeling exist when applied to this data set. An ideal data set would:

- be stationary
- be serially correlated
- have no missing observations
- have one observation per time period
- be univariate
- and would have a sufficiently long and continuous time series running up to the last observation prior to the first forecast period.

Unfortunately, in a number of cases the above conditions are violated. This section will consider the nature of these problems will be discussed as well as the means used to deal with them.

#### 5.4.1 The Presence of Cross Sectional Data

##### Nature of the Problem:

In HB966, deliveries are reported on a monthly basis. As a result, there are typically a number of deliveries made from any one source during a month. Time series modeling calls for one observation per time period. Since the minimum time period is one month, a way must be found to deal with multiple observations within time periods, i.e., cross sectional data. The problems arising from the incorporation of time series and cross sectional data (respectively, serial correlation and heteroscedasticity) have long been a topic of research in regression modeling, but have not been resolved in the more recent field of Time Series analysis.

##### Solution Approach:

The severity of this problem is much greater in the spot market than for deliveries on contract. As explained in chapter 4, the spot market is statewide in scope and therefore a single forecast is derived from a multitude of producers while contract forecasts are often based on deliveries made from a specific mine. Naturally, on the spot market the majority of the data will be cross

sectional. In order to use time series analysis the raw data must meet the conditions set forth at the beginning of this section. In this section we address the problem of reducing the cross sectional component of the raw data set to a single observation per month. One possible way of doing this would be to use the average mine price for each month. This might follow a screening out of all deliveries with mine prices greater than twice the standard error. Cross sectional averaging and screening has two potential drawbacks:

1. It must be assumed that the movement of the average value is representative of the market as a whole. If there were sufficient cross sectional data in each month to allow the distribution of mine price in each month to be defined, then the equivalence of this distribution could be compared across time. If these monthly distributions remained uniform then it would be reasonable to use an average value since all else would remain equal. In fact, the calculated distribution could be used to determine the actual confidence band. Unfortunately, there is not sufficient cross sectional data to determine the monthly distribution of mine price. With so few observations in a typical month, a consistent distribution cannot be expected.

2. After averaging much of the predictive ability of input time series (such as coal quality characteristics) are lost, eliminating their usefulness as independent regression variables. This relates to point 1 where the distribution of the data is lost by averaging. In this case it is the multivariate distribution between the mine price and the independent variables which is lost. Specifically, we lose the covariances (or equivalently, the cross correlations) between the price and its predictor variables (Btu, Sulfur, ect.).

As an illustration of this problem, consider the OLS and autoregressive estimates given in Table 5.3 and the crosscorrelogram between Btu and FOB mine price in Table 5.4. In the OLS estimate of the regression equation based on the raw data (both cross sectional and time series data) Btu is significant in its predictive power. The Durban Watson (DW) statistic indicated that this data set was serially correlated indicating a need for lagged dependent variables (endogenous) in the regression equation and estimation with GLS. The Autoregression shows Btu as continuing to be significant even in the presence of lagged values of the mine price and using Yule-Walker equations for estimation. Now consider the cross correlations given in Table 5.4 for the average monthly values of mine price and Btu. Using twice the standard

Table 5.3

## OLS ESTIMATE OF SP MINE PRICE FOR VA

## ANALYSIS OF VARIANCE

SOURCE	DF	SUM OF SQUARES	MEAN SQUARE	F VALUE	PROB>F
MODEL	8	509.38147	63.67268392	6.749	0.0001
ERROR	385	3632.13189	9.43410880		
C TOTAL	393	4141.51336			
ROOT MSE		3.071499	R-SQUARE	0.1230	
DEP MEAN		27.29426	ADJ R-SQ	0.1048	
C.V.		11.25328			

## PARAMETER ESTIMATES

VARIABLE	DF	PARAMETER ESTIMATE	STANDARD ERROR	T FOR H0: PARAMETER=0	PROB >  T
INTERCEP	1	-8.12687538	12.14541821	-0.669	0.5038
DEL_C	1	-0.001598733	0.000534809	-2.989	0.0030
QUANTITY	1	-0.000012817	0.000030389	-0.422	0.6734
SULFUR	1	-0.09887740	0.51483502	-0.192	0.8478
BTU	1	0.002520885	0.000782218	3.223	0.0014
ASH	1	0.17026260	0.15824543	1.076	0.2826
WATER	1	0.45375434	0.17348472	2.616	0.0093
SURF	1	-0.85681617	0.33464572	-2.560	0.0108
INC	1	-1.30434353	0.60235098	-2.165	0.0310

## A U T O R E G P R O C E D U R E

## YULE-WALKER ESTIMATES

SSE	929.3547	DFE	376
MSE	2.471688	ROOT MSE	1.57216
SBC	1565.728	AIC	1494.154
REG RSQ	0.2597	TOTAL RSQ	0.7756

VARIABLE	DF	B VALUE	STD ERROR	T RATIO	APPROX PROB
INTERCPT	1	3.45293732	6.15292559	0.561	0.5750
DEL_C	1	-.0000782013	0.0003105833	-0.252	0.8013
QUANTITY	1	0.00005143	0.0000147067	3.497	0.0005
SULFUR	1	0.22387942	0.24990489	0.896	0.3709
BTU	1	0.00168862	0.000386293	4.371	0.0001
ASH	1	0.11730757	0.07628506	1.538	0.1250
WATER	1	0.27481993	0.08809642	3.120	0.0020
SURF	1	-0.50816940	0.17326992	-2.933	0.0036
INC	1	-5.74835280	0.99507355	-5.777	0.0001



error as significance bounds (the dashed lines) we can see that there is no longer any significant correlation between price and Btu after averaging.

Apparently, much predictive power is lost when cross sectional averaging is used for deliveries which vary widely in price and other price influencing characteristics. It seems that the application of cross sectional averaging is dependent upon the number of observations and the variance of the mine price and independent variables. With this in mind, consider the pertinent features of the spot and contract markets.

#### Spot Market:

1. Many deliveries come from different sources.
2. The number of deliveries and sources depend on the condition of the market. During a bullish market, existing mines increase production and shipments. Also, small contract miners come into production to meet the lag in supply and demand. During a period of oversupply production is cut back, and contract miners drop out of the market.
3. As a result of the diversity in the number of suppliers

there is increased variability in coal quality and mine specific factors which influence cost.

#### Contract Market:

1. By definition a contract source consists of one mine or sales company shipping from a single county for a specific price.
2. Long term contracts are one means of providing secure sales and supply for the miner and utility. For contract sources a continuous stream of deliveries can be expected with only a few shipments per month.
3. Since the coal comes from a single mine or group of mines working in similar seams, the variation in coal quality and mining costs should be low.

Based on the differences in the nature of the data between the two markets two different approaches are justified. For contract sales the fluctuation in price and quality as well as the number of deliveries will be small. Because of this, use of the average value is justifiable. After averaging the mine price and input series a combined Time Series/Regression model can be fit to the data using equation 5.12. Note that this model does not account for a

time lag between the value of the input series and its effect on the mine price. It is assumed that the influence of lagged independent variables (e.g.  $Btu_{t-1}$ ) is not significant. This assumption agrees with past experience as well as common sense. The inclusion of lagged input series parameters necessitates the use of Transfer Functions in the model, a topic not covered herein.

For spot sales, straight use of average values is not justifiable given the high variance about the mean monthly value. Certainly, before going to a time series analysis one should account for this cross-sectional variation. In the next section a combined trend/regression model will be given which will be estimated using the raw data (5.13). The multiple regression portion of the model will be used as a filter to remove that part of the variance in the data which is cross-sectional in its nature, and can only be accounted for by considering individual deliveries. This will be discussed in more detail in sections 5.4.2 and 5.4.5.

#### 5.4.2 Instability of the Market

This topic is discussed from a more analytic perspective in sections 5.3.1 and 5.3.2 under the heading of

nonstationarity. In this section the instability of the market price will be covered as well as the means used to account for price instability as a trend in time with a long cycle.

#### Nature of the Problem:

Section 5.2 and Chapter 4 discuss the phenomenon of instability of the spot market. Sources on contract also experience trends. Typically, there is a gradual increase in price over time since most contracts include a price escalation clause. This sort of continuous linear trend can be easily forecast. When the price fluctuates wildly in a short period of time or has a sharp dominating feature the accuracy of forecasts is questionable.

In the 1983-1986 time frame this was the situation. The sharp bell shaped trend in spot mine price dominates any forecast, pulling projected mine price well below any reasonable level.

Fitting a trend model to the raw data does not represent a problem in itself. Regardless of the complexity of the time trend it can be fit using either a Fourier Transform or a high order polynomial. The real problem is to identify the nature of the trend so that overfitting does not damage the true time series that should be found in

the residuals. Let's consider this confusing topic in greater detail before continuing.

There are two alternate assumptions about the spot market that can be made prior to generating a model that will be used to forecast mine price:

1. If the current market is compared with the bull market that existed prior to about 6/85, it can be argued that these two markets are so different that they should not be included in the same forecast. Before the market collapsed there was a supply side economy existed in which the miner could seek to make substantial profits. In the current market the mine operator is often lucky to be able to cover production costs. The current market is demand driven. Since the market has changed it might make better sense to base forecasts only on data from the latter half of 1985 on.
2. The high market level reached in the first half of 1985 was due to influences which occur repeatedly over time, and which are periodic or cyclic themselves. One likely reason for the increase in demand by utilities in that period was the threatened strike by the mine workers. In preparation for this the utilities may have increase stockpile levels. Also, an increased demand for power

generation and high oil prices resulted in increased purchases of coal. The economy, oil prices, and demand for coal are all tied together, while mine strikes are predictable shocks to the system. Thus, mine price is a component which is long term and cyclic in nature. The periodicity of this "macro" cycle is not apparent in the relatively short time period used in this study, and since a full cycle is not available it cannot be quantified in the model. Instead this periodicity causes the data to be highly nonstationary. Hidden by this dominating trend is the short term variation in price that we wish to forecast. This "micro" time series may be seasonal (responding to peak power demand periods of the year) and/or fluctuation of price due to competition between mine operators attempting to find a price which will maximize profits. This is the time series that is of greatest interest. Once an accurate trend has been identified the residual series can be modeled as a time series and added to the trend to produce a forecast.

Unfortunately, it would be difficult if not impossible to prove which of the above two scenarios is the most accurate. A long term cycle could be looked for in another data set tracing mine price back many years, but the number of major changes in the market would make

explanation very difficult. In particular consider the passage of the Staggers Act in 1980. Deregulation of the rail carriers and the subsequent abandonment of ICC rail tariff rates in the early 1980s has completely changed the coal market in Central Appalachia. A model based on data prior to the Staggers Act would have to be structurally and conceptually different from the current model. As noted in the literature review of chapter 2, this has been the case with other models. For a short term model the first scenario would probably be the safest, but the second scenario is probably closer to the truth. The strategy used in this study is to base spot market forecasts on data for the current market when sufficient data are available and seem to indicate market equilibrium. The methodology used is covered in section 5.4.4. Otherwise, a simple trend model is identified using a modified Fourier Transform.

The model used to form the majority of spot market forecasts is a combined multiple regression/trend model which is estimated using the raw data set.

$$\begin{aligned}
 \text{FOB}_i = & \beta_0 + \beta_1 \sin\left(\frac{2\pi}{a_1} \text{TIME}_i + \gamma_1\right) + \beta_2 \cos\left(\frac{2\pi}{a_2} \text{TIME}_i + \gamma_2\right) + \beta_3 x_{1i} + \dots \\
 & + \beta_{n+2} x_{ni} + \varepsilon_i \qquad (5.13)
 \end{aligned}$$

where,  $\text{FOB}_i$  is the mine price (f.o.b.) of observation  $i$ ,

$\text{TIME}_i$  is the time period corresponding to the year and month numbered in ascending order (i.e.,  $1, 2, \dots, T$ ),  
 $x_{1i}, \dots, x_{ni}$  are the regressor variables such as Btu and Ash,  
 $\alpha, \beta,$  and  $\gamma$  are the parameters to be estimated,  
 $\varepsilon_i$  is the error term which we hope contains the short term time series.

This model is nonlinear and is estimated using the Gauss-Newton gradient search technique as described in the SAS version 5 "User's Guide: Statistics" under PROC NLIN. Equation 5.13 can be thought of as a filter prior to the initial time series which removes that variance in the data which is due to the macro time trend and other significant observation specific variance such as coal quality. The residual series from this model can then be averaged by month to produce a univariate time series which can be modeled as ARIMA. The specific methodology used to produce the forecast price and its confidence intervals will be covered in section 5.4.5.

### 5.4.3 Insufficient Data

The SAS statistics package is being used in this study.

From the Econometric package procedure ARIMA is being used to model the mine price of coal over a 42 month period, 1/83 to 6/86, and then to forecast price into the next six months. In terms of the available degrees of freedom the limitations and complications of doing this are as follows:

1. PROC ARIMA requires a minimum of 30 observations to fit the basic model. To this must be added six additional degrees of freedom to forecast the six month period. Typically, at least an uninterrupted time span of three years is need to forecast an ARIMA model.
2. Gaps in shipments often occur. Time series analysis requires an unbroken series uninterrupted by missing values. When there are missing values for only a few observations, interpolation is used to supply the needed values.
3. Some sources may have supplied considerable coal in the past, but haven't made any deliveries lately. If the last shipment occurred only a few months prior to 6/86 then the length of the forecast can be extended. Otherwise, it is assumed that the contract has expired and the operator is not selling on the spot market.

4. In the situation where a Source is currently shipping coal on contract but has not shipped for a long enough time period to allow the estimation of an ARIMA model, non-optimal forecasting methods can be used as discussed in section 5.4.4.

Thus, 42 observations are often not available, and for some of the more complex multivariate models are barely adequate. In some cases insufficient data is available to generate ARIMA forecasts. When this happens forecasts can be made using less desirable extrapolation procedures.

#### 5.4.4 Forecasting With Limited Data

Often, the available data is not suited to ARIMA modeling. Either the series is too short in length or exhibits heavy trends and therefore lacks the necessary property of stationarity.

When there is inadequate data for producing a Time Series forecast using an ARIMA model, other forecasting models are available which can yield good forecasts, but are non-optimal in a statistical sense of not minimizing the sum-of-squares error.

### Non-optimal Forecasting Procedures:

The following discussion is taken from SAS's FORECAST procedure in the version 5 "SAS/ETS User's Guide" from which the "STEPAR" method was used in this study.

The procedure used here to forecast is based on assumptions similar to those used in developing spot market forecasts. A time trend is used in conjunction with an autoregressive model. The time trend is a polynomial of degree three or less with autoregressive terms being added which meet a minimum significance level. Using the same notation as for 5.13 we have,

$$FOB_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 + e_t \quad (5.14)$$

where,  $t=TIME$ , the current month and  $\beta$  are parameters which are estimated using OLS.

The residual series  $e_t$  from the OLS estimate of 5.13 are fit as an autoregressive model,

$$e_t = a_0 + a_1 e_{t-1} + a_2 e_{t-2} + \dots + a_p e_{t-p} + \varepsilon_t \quad (5.15)$$

where,  $\varepsilon_t$  should be white noise.

This procedure is particularly useful for contract sources which have been supplying a utility only recently.

Use of Actual Values (or, when all else fails, punt):

The 1985 market collapse was followed by drastic losses in sales in most supply regions. Some regions continued to ship coal to Virginia Power, but only sporadically. A good example of this trend occurred in mines located on CSX in Dickenson county. With so little information forecasting is not justified. In cases such as this the actual mine price experienced at or near that date will be used in the model.

#### 5.4.5 Summary of the Forecasting Procedure

The methodology used in generating forecasts of mine price has been loosely covered in this chapter. In this section the forecasting procedure will be reviewed step-by-step to help resolve any confusion the reader may be experiencing.

Spot Market Procedure:

To forecast mine price and the confidence intervals of forecasts for the spot market the following procedure is followed.

1. Fit the regression/trend model (5.13) to the raw data set.
2. Forecast any independent regressors for the period of interest using the non-optimal forecasting procedures described in section 5.4.4.
3. Using the estimated parameters of 5.13 and the forecast values from step 2 use forecast mine price for TIME = T, T+1, ..., T+6.
4. Generate the confidence intervals for the results of step 3 as follows.

$$\hat{F}OB_{T+1} - t_{1-\alpha} s_f \leq FOB_{t+1} \leq \hat{F}OB_{T+1} + t_{1-\alpha} s_f \quad (5.16)$$

where,  $\hat{F}OB_{T+1}$  is the forecast mine price from step 3,  
 $t_{1-\alpha}$  is the percentile of a t-distribution at  
 T-2 degrees of freedom,  
 $s_f$  is the standard deviation of the forecast  
 which is calculated as follows.

For a trend model without any regression components find,

$$s_f^2 = s^2 \left[ 1 + \frac{1}{T} + (x_{T+1} - \bar{x})^2 / \sum (\bar{x}_t - \bar{x})^2 \right] \quad (5.17)$$

where,  $\bar{x}$  is the univariate input series,

$\bar{x}_t$  is the average value for month  $t$ ,

$$s^2 = \frac{1}{T-2} \sum (\overline{FOB}_t - \hat{FOB}_t)^2 \quad (5.18)$$

For a combined trend/regression model find,

$$s_f^2 = s^2 \tilde{X} (X'X)^{-1} \tilde{X}' \quad (5.19)$$

where,  $X$  is a  $T \times k$  matrix of independent variable observations averaged by month,

$\tilde{X}$  is the vector of forecast independent values.

5. Generate a time series of monthly averages of the residuals from 5.13. Model this as a univariate ARIMA using equation 5.11.
6. Add the forecast and confidence intervals from steps 5 and 6 to obtain the final forecast that will be used in the coal purchasing model described in chapter 6.

### Long Term Contract Procedure:

For the following it is assumed that the data lends itself to ARIMA modeling, i.e., it is either stationary or homogeneous (can be made stationary via differencing once or twice); that the series can be reduced to one observation per time period without unacceptable loss of information; that the series is of sufficient length to generate a six month forecast; and that serial correlation is present.

1. Generate the time series data set from the raw data by:
  - a. Averaging together all deliveries from the same month.
  - b. Using interpolation to replace missing values.
  - c. Replacing outliers with interpolated values.
2. Identify the required degree of differencing (if any) needed to produce a stationary time series.
3. Examine the Autocorrelation function (ACF) and the Partial Autocorrelation Function (PACF) for initial estimates of the degrees of autoregressive and moving average terms,  $p$  and  $q$ , respectively.

4. Fit an ARIMA/regression model to the series generated in step 1 using a model of degree  $p$  and  $q$  as estimated in step 3 including any significant input series as determined in previous analysis.
  
5. Examine the ACF and PACF of the residuals of the model fit in step 4. The residual series should be white noise and therefore have no significant terms in the ACF or PACF. If the residuals are not white, modify the model as indicated by any significant terms in the ACF and PACF and return to step 4. Otherwise, continue to step 6.
  
6. Using the model estimated in step 5, forecast the output time series (mine price) and the confidence interval.

If the data for a particular source is not suited to either of the above two procedures then the methods covered in section 5.4.4 can be used.

## CHAPTER 6

# Development of a Virginia Utility Coal Purchasing Model

### 6.1 Model Assumptions and Approach

General model assumptions and regional scope has been covered in Chapters 4 and 5. In this section specific assumptions in the formulation of the model will be covered before giving the mathematical formulation and description.

The model is developed from the perspective of the utility. One of the objectives of a utility is to minimize the cost of coal supply under constraints related to: state and corporate policy, boiler specifications, transportation, existing coal supply contracts, stockpile levels, and the availability of supply. A number of assumptions have had to be made, particularly in defining the availability of coal supply and forecasting the cost coefficients in the objective function.

Assumptions related to supply encompass the definition of supply regions, production capacities of the individual sources of supply, and the range of production of a source which must be purchased to meet contract requirements between utilities and mines under long term contract obligations. Coal supply for Virginia utilities is assumed to be sourced entirely from Central Appalachia. This is an accurate assumption except for PEPCO's Potomac River plant which also receives coal from Maryland and northern West Virginia. For the reasons given in Chapter 3, PEPCO plants will not be included in this study. Thus, this assumption holds true. Sources of supply are initially separated into two categories: the spot market and mines on long term contract. Several assumptions are made depending on how a coal source is categorized. The spot market in Central Appalachia is assumed to be initially broken down by state. Beyond this, states are subdivided by destinations with sufficiently different rail rates so as to result in a lowering of the mine price. Mine price is lowered in order to attain a competitive delivered price as explained in Chapter 4. Those mines on long term contract are individually specified. For contracted sources, assumptions have to be made as to their mining capacity and the contracted annual tonnage. Monthly mining capacities are projected from daily production capacities as taken from mine permits. Annual contracted tonnage is specified in the contract which is

proprietary information. Examination of the discrepancies between actual delivered tonnage and contracted tonnage for past contracts with APCO and Virginia Power has shown that the actual tonnage delivered and contracted annual tonnages vary only slightly in all but a few cases. Therefore, the actual delivered tonnage for 1986 is used as a baseline to constrain the delivered tonnage to a range which hopefully encompasses the actual tonnage under contract. Spot sources are unconstrained as to their mining capacities. This assumption is justifiable considering the vast difference in demand by Virginia utilities when compared to the production of Central Appalachia under the current conditions of over supply. This model follows Virginia Power's current policy of buying between 40 and 60 percent of their needs on long term contract. Other simplifying assumptions are made about the transportation of coal. Transportation to Virginia Power and APCO plants in Virginia is limited to rail movement. This reflects the inescapable dominance of CSX and NS. Virginia Power has used barging of coal as a tool to increase competition between the carriers, but barging was only used for a short while. It is a possible factor for imports of Colombian coal, but is unlikely to play a significant role for domestic coal. Much coal is initially carried from the mine by truck, but this is only used as a means of transportation to the tipple or prep plant. Trucking is not a feasible method of transportation except

for short distances. Thus, it is assumed that all transportation of coal to the utility plants is by rail. Many previous coal supply models have included the rail network and have specified capacity constraints for main haulage lines in the network. This model does not include a network components or constrain the movement of coal. Since the model is short term and demand is limited to a few specific plants. There is no concern as to the ability of the existing rail network to deliver sufficient coal to meet demand.

All plants include large coal stockpiles as an integral part of consumption planning and purchasing policy. Stockpiles are depleted or increased in order to lower the cost of supply and ensure no interruption of electrical generation. Stockpiles have an engineering limit as to their maximum size. Minimum stockpile level is assumed to be consistent with Virginia Powers stated policy of having at least one months supply on hand.

## 6.2 General Utility Supply Model

Before presentation of the steam coal supply model for Virginia utilities a supply model of general application will be presented.

Notation:

$x_{ijt}$  = the quantity of coal (tons) delivered to plant  $j$  in month  $t$  from source  $i$

$S_{jt}$  = the stockpile level at plant  $j$  at the end of time  $t$

$\mathcal{Q}$  is the grouping of contiguous months used in defining the fluctuation of coal deliveries for all  $i \in L_k$

$T$  = the final time period in the model, such that  $t=1, \dots, T$

$I_k$  = the set of all sources making deliveries to utility  $k$

$L_k$  = the set of all mines on long term contract with utility  $k$ , such that  $i \in L_k$

$I_j$  = the set of all  $i$  historically supplying  $j$

$J_i$  = the set of all  $j$  historically supplied by  $i$

$D_{jt}$  = the demand at plant  $j$  in time  $t$

$x_{ijt}^*$  is the actual coal tonnage delivered in month  $t$ , which is used to tie contract deliveries at  $t=1$  and  $t=T$  into

the actual deliveries in the months preceding and following the model

$q_i$  = the minimum semi-annual tonnage purchased by utility  $k$  on long term contract with source  $i$

$Q_{it}$  = mine capacity for month  $t$  for all  $i \in L_k$

$p_i$  and  $P_i$  are the minimum and maximum percent change in delivered tonnage from the preceding  $\ell$  months for all  $i \in L_k$

$F_k^{\min}$  and  $F_k^{\max}$  are the percentage of total deliveries purchased on contract reflecting the policy of utility  $k$

$Ash_j, Btu_j, Sul_j^{\min}, Sul_j^{\max}$  are the coal quality specifications at plant  $j$  for ash, Btu, and sulfur

$\overline{Ash}_i, \overline{Btu}_i, \overline{Sul}_i$  are the average coal quality characteristics for coal source  $i$

General Case:

The following is a utility steam coal supply model of

general application. Here there are  $K$  utilities who are independently minimizing their total cost of coal deliveries over a time frame of months  $t=1, \dots, T$ . The multiple objective function is considered in the form of a min{max} problem such that,

given  $K$  objectives,  $z_k$ ,

$$\text{Minimize } z = \text{maximum}\{z_1, z_2, \dots, z_K\}$$

where each of the  $K$  objectives is of the form

$$z_k = \sum_{i \in I_k} \sum_{j \in k} \sum_{t=1}^T c_{ijt} x_{ijt}$$

This can be made linear by including the  $K$  objectives in the model as  $K$  constraints  $\leq$  an artificial decision variable  $z$ , and then minimizing  $z$ . Thus, the full model is of the form,

Minimize  $Z = z$

Subject to,

$$\sum_{i \in L_k} \sum_{j \in K} \sum_{t=1}^T c_{ij} x_{ij} t \leq z, \quad \text{for all } k \quad (6.1)$$

$$\sum_{j \in J_i} \sum_{t=0}^{t+l-1} x_{ij} t > 0, \quad \text{for all } i \in L_k, k, t=0, \dots, T-l+1 \quad (6.2)$$

$$\sum_{j \in J_i} \sum_{t=l}^{l+1} x_{ij} t \geq p_i \sum_{j \in J_i} \sum_{t=1}^{l-1} (x_{ij} t + x_{ij}^*), \quad \text{for all } i \in L_k, k \quad (6.3)$$

$$\sum_{j \in J_i} \sum_{t=l}^{l+1} x_{ij} t \leq p_i \sum_{j \in J_i} \sum_{t=1}^{l-1} (x_{ij} t + x_{ij}^*), \quad \text{for all } i \in L_k, k \quad (6.4)$$

$$\sum_{j \in J_i} \sum_{t=0}^{t+l-1} x_{ij} t \geq p_i \sum_{j \in J_i} \sum_{t=l}^{t-1} x_{ij} t, \quad \text{for all } i \in L_k, k$$

and  $(l+1) < t < (T-l)$  (6.5)

$$\sum_{j \in J_i} \sum_{t=0}^{t+l-1} x_{ij} t \leq p_i \sum_{j \in J_i} \sum_{t=l}^{t-1} x_{ij} t, \quad \text{for all } i \in L_k, k$$

and  $(l+1) < t < (T-l)$  (6.6)

$$\sum_{j \in J_i} \sum_{t=T-l+2}^T (x_{ij} t + x_{ij, T+1}^*) \geq p_i \sum_{i \in J_i} \sum_{t=T-l+1}^{T-l} x_{ij} t, \quad \text{for all } i \in L_k, k \quad (6.7)$$

$$\sum_{j \in J_i} \sum_{t=T-l+2}^T (x_{ij} t + x_{ij, T+1}^*) \leq p_i \sum_{i \in J_i} \sum_{t=T-l+1}^{T-l} x_{ij} t, \quad \text{for all } i \in L_k, k \quad (6.8)$$

$$\sum_{j \in I_i} \sum_{t=1}^T x_{ijt} \geq q_i, \quad \text{for all } i \in L_k, k \quad (6.9)$$

$$\sum_{i \in I_k} \sum_{t=1}^T x_{ijt} \geq F_k^{\min} \sum_{i \in I_k} \sum_{t=1}^T x_{ijt}, \quad \text{for all } k \quad (6.10)$$

$$\sum_{i \in I_k} \sum_{t=1}^T x_{ijt} \leq F_k^{\max} \sum_{i \in I_k} \sum_{t=1}^T x_{ijt}, \quad \text{for all } k \quad (6.11)$$

$$\sum_{i \in I_j} x_{ijt} + S_{j,t-1} - D_{jt} = S_{jt}, \quad \text{for all } j, t \quad (6.12)$$

$$\sum_j x_{ijt} \leq Q_{it}, \quad \text{for all } i \in L, t \quad (6.13)$$

$$\overline{\sum_{i \in I_j} A s_i x_{ijt}} \leq A s_j \sum_{i \in I_j} x_{ijt}, \quad \text{for all } j, t \quad (6.14)$$

$$\overline{\sum_{i \in I_j} B t_u x_{ijt}} \leq B t_u \sum_{i \in I_j} x_{ijt}, \quad \text{for all } j, t \quad (6.15)$$

$$\overline{\sum_{i \in I_j} S u_l x_{ijt}} \geq S u_l^{\min} \sum_{i \in I_j} x_{ijt}, \quad \text{for all } j, t \quad (6.16)$$

$$\overline{\sum_{i \in I_j} S u_l x_{ijt}} \leq S u_l^{\max} \sum_{i \in I_j} x_{ijt}, \quad \text{for all } j, t \quad (6.17)$$

$$D_{j,t+1} \leq S_{jt} \leq S_j^{\max} \quad \text{for all } j, t < T$$

$$z, S_{jt}, x_{ijt} \geq 0 \quad \text{for all } i, j, t$$

Constraints 6.2 -6.8 define the allowable fluctuation in delivered tonnage for a source on contract with utility  $k$  (i.e.,  $i \in L_k$ ). Here we consider the change in tonnage between consecutive groupings of months of length  $\ell$ . Thus, in constraint 6.2, if  $\ell=2$  then there must be some delivery of coal during any two consecutive months. This controls the length of gaps in shipments. The following 6 constraint sets constrain the allowable percent change in delivered tonnage from the tonnage in previous grouping of  $\ell$  months.  $P_i$  is the maximum percent increase, while  $p_i$  is the maximum percent decrease. The model is tied to the actual tonnage for the month preceding the beginning of the model by constraints 6.3 and 6.4. Constraints 6.7 and 6.8 do the same for the month following the end of the model for cases where the model is a historical study.

Utilities are required to meet their contractual obligations to purchase at least a minimum tonnage for all contract sources by 6.9. Also, utility policy as to the percent split between contract and spot purchases is defined in 6.10 and 6.11. Monthly plant stockpiles,  $S_{jt}$ , are decision variables in the model. The material balance in constraint 6.12 controls the current month's stockpile size.  $S_{jt}$  must be at least large enough to meet demand in the following month while being no larger than the maximum stockpile size. A constraint on mine capacity (6.13) is included in the

general case, but would rarely be included in a case study. Monthly mine capacities are published but it would be difficult to define all demand for a particular source. Coal quality requirements at the plant are defined in 6.14 through 6.17. This set of constraints only includes Btu, ash, and sulfur since these are the only quality characteristics reported in the data. Here the weighted sum of coal quality from all sources making deliveries in any month must meet the given boiler specifications.

### 6.3 The Virginia Coal Purchasing Model

A formulation for a utility coal purchasing model based on the economic structure of the Central Appalachian market, the assumptions made in this study, and the available data as given in Chapter 3 is as follows. Additional notation not covered in the general case is as follows.

$I$  = the set of all mines

$L$  = the set of all mines on long term contract, such that  $i \in L$

$S$  = the set of all spot market regions, such that  $i \in S$

$V$  = the set of all Virginia Power plants, such that

$j \in V$

$A$  = the set of all APCO plants, such that  $j \in A$

$K$  = the union of  $V$  and  $A$ , the set of all Virginia plants

$S_{j0}, S_{j6}$  are the actual stockpile levels in June and December of 1987 used as initial and final control levels.

$x_{ij0}^*$  is the actual initial coal delivery in June, 1987

$P_i, p_i$  the maximum and minimum percentage change, respectively, in deliveries between two adjacent months found during the study period.

$$\text{Min } Z = \sum_{i \in L} \sum_j \sum_t^T c_{ij t} x_{ij t} + \sum_{i \in S} \sum_j \sum_t^T c_{ij t} x_{ij t} \quad (6.17)$$

Subject To

$$\sum_{j \in I_j} x_{ij t} \leq p_i \sum_{j \in I_j} x_{ij 0}^* \quad \text{for all } i \in L \quad (6.18)$$

$$\sum_{j \in I_j} x_{ij t} \geq p_i \sum_{j \in I_j} x_{ij 0}^* \quad \text{for all } i \in L \quad (6.19)$$

$$\sum_{j \in I_j} x_{ij t} \leq p_i \sum_{j \in I_j} x_{ij, t-1} \quad \text{for all } t > 1, i \in L \quad (6.20)$$

$$\sum_{j \in I_j} x_{ij t} \geq p_i \sum_{j \in I_j} x_{ij, t-1} \quad \text{for all } t > 1, i \in L \quad (6.21)$$

$$\sum_{i \in J_i} x_{ij t} + S_{j, t-1} - D_{j t} = S_{j t} \quad \text{for all } j, t \quad (6.22)$$

$$\sum_{i \in L} \sum_{j \in V} \sum_t^T x_{ij t} \geq 0.4 \sum_{i \in I} \sum_{j \in V} \sum_t^T x_{ij t} \quad (6.23)$$

$$\sum_{i \in L} \sum_{j \in V} \sum_t^T x_{ij t} \leq 0.6 \sum_{i \in I} \sum_{j \in V} \sum_t^T x_{ij t} \quad (6.24)$$

$$\sum_{j \in k} \sum_t^T x_{ij t} \geq q_i \quad \text{for all } i \in L, t \text{ and } k \in \{A, V\} \quad (6.25)$$

$$D_{j,t+1} \leq S_{jt} \leq 2D_{j,t+1} \quad \text{for all } j, t < 6$$

$$S_{jt} \text{ and } x_{ijt} \geq 0 \quad \text{for all } i, j, t$$

The objective function as given by 6.17 is to minimize the total delivered cost of coal supply. This is a time dynamic model which optimizes for deliveries during  $t=1,2,\dots,T$ . The supply of coal is divided into two categories: purchases on long term contract ( $i \in L$ ), and on the spot market ( $i \in S$ ). The delivered cost,  $c_{ijt}$ , is composed of the estimated mine price,  $M_{ijt}$ , (see Chapter 5), and the current estimated rail rate,  $R_{ijt}$ . In this case there is only a single objective. This is possible since neither of the two utilities shared any sources of supply for this time frame. Since most of the values of  $M_{ijt}$  are estimated as time series, then  $c_{ijt}$  is stochastic in nature, and has upper and lower confidence limits which should be taken into account in an ensuing sensitivity analysis (see Chapter 7).

Constraints 6.18 - 6.21 control the fluctuation in the quantity delivered from all long term sources.  $P_i$  is the maximum percentage increase or decrease between any two adjacent months. This percentage is based on the past fluctuation for that source and can exceed 100%, i.e., a value of  $P_i > 1$ . Constraints 6.18 and 6.19 tie the model in with the production levels found in the month prior to the

beginning of the study period (June), while 6.20 and 6.21 control fluctuation in deliveries during the time frame of the model. Previous model runs which lacked these constraints yielded an optimal solution in which long term contract requirements were satisfied by making large block purchases and buying nothing in higher price months. This is not a practice followed in reality by the utilities. These constraints effectively supplanted capacity constraints.

Constraints 6.22 balances supply and demand where supply includes the decision variable  $S_{j,t-1}$ , which is the stockpile level at the end of the prior month.  $S_{j0}$  is input directly as the actual stockpile level in the month preceding the study period (July).  $D_{jt}$  is the actual consumption (tons) of coal or that utilities projected consumption for  $j \in k$ . Stockpile levels for months  $t=1, \dots, 5$  are found as material balances when summing across all  $t$  for 6.22. The total materials balance is found by summing across all  $t$ . Stockpiles are given as end-of-the-month levels. Therefore, to tie the model in with reality the initial stockpile,  $S_{j0}$ , is used for June and the final stockpile,  $S_{j6}$ , is used for December of 1987.

Utilities maintain a policy of purchasing a percentage of their total coal needs on long term contract. For APCO's

two Virginia plants all coal comes from Clinchfield and Wellmore on contract, so only these sources need be defined. Virginia Power's policy is to buy between 40 and 60 percent of supply from contracted sources. This policy is included in the model by equations 6.23 and 6.24.

Total deliveries from mines on contract is constrained to a minimum delivery level encompassing the actual annual contracted tonnage by constraints 6.25, where  $q_i$  is the lower limit.

Decision variable  $S_{jt}$  is bounded by the maximum and minimum stockpile levels. These follow Virginia Power's policy of maintaining between one and two months demand on hand in the stockpile in case of any interruption in supply. The decision variables,  $S_{jt}$  and  $x_{ijt}$ , are constrained to be nonnegative.

In initial runs of the model an estimate of the monthly production capacity was made for each contract source. All deliveries from that source,  $x_{ijt}$ , were constrained by the estimated capacity,  $Q_{it}$ , such that

$$\sum_j x_{ijt} \leq Q_{it} \quad \text{for all } i, t$$

Production capacity was allowed to increase on the short run

as a monthly percentage increase of the initial capacity estimate:  $Q_{i,t+1} \leq (1+P)Q_{it}$  with  $Q_{it}$  being a given value for  $t=1$  and being a variable to be determined in the model for  $t>1$ . This constraint set allows short term increase in mine production under conditions of increased demand. In the presence of constraints on the maximum allowable percentage change in production (see equations 6.18 - 6.21) capacity constraints were not found to be binding in the majority of cases, and therefore were dropped.

Coal quality (Btu, ash, and sulfur) must meet boiler specifications of the utilities. Quality constraints have been used in initial formulations of the model, but were not found to be binding. In, fact the vast majority of Central Appalachian coal exceeds the requirements of the plants. Since coal blending and cleaning takes place at the plant to produce a more uniform feed, any coal source is of acceptable quality.

## Chapter 7

### Model Input, Output, and Results

#### 7.1 Model Input

The linear model described in the previous chapter requires input for the objective function coefficients and the right-hand-side vector (RHS). The cost vector of the objective consists of the estimated or actual delivered cost per ton of coal. The RHS values depend upon the constraint class as described in Chapter 6. This section gives tabulated values for input to the model.

##### 7.1.1 Cost Vector

The coding system used in the model is given in Table 7.1. For each source,  $i$ , all possible destinations, i.e., plants,  $j$ , are listed. The time period,  $t$ , and the railroad,  $r$ , are also specified.

Table 7.1: Coding System

<u>j</u>	<u>Plant</u>	<u>r</u>	<u>Railroad</u>	<u>t</u>	<u>Month/Year</u>
1	Chesterfield	1	CSX	1	7/86
2	Portsmouth	2	NS	2	8/86
3	Possum Point			3	9/86
4	Yorktown			4	10/86
5	Bremo Bluff			5	11/86
6	Clinchfield			6	12/86
7	Glen Lyn				

Table 7.2 gives the breakdown for the delivered price,  $c_{ijt}$ , for individual mining sources on contract. Table 7.3 does the same for the spot market. Complete input listings are given in Tables B.1 and B.2 of Appendix B. Values in these tables are either forecast, actual, or inferred. Inferred values are based on actual deliveries where there was not a sufficient number of deliveries on which a forecast could be based. Inferred values are typically based on one or two deliveries to a plant, and are indicated by being enclosed in parentheses. Actual deliveries can be recognized by the absence of any forecast confidence interval.  $M_{ijt}$  is the mine price. This is given as the predicted value, PRED, and the upper and lower 95% confidence interval.  $R_{ijt}$  is the rail rate, which is actual or inferred, and follows the same sequence as given for the plants,  $j$ .

Table 7.2 Delivered Cost for Contract Sources (\$/ton)

i	j*	t	r	M <sub>ijt</sub>			R <sub>ijt</sub>			C <sub>ijt</sub>		
				PRED	L95	U95	PRED	L95	U95	PRED	L95	U95
Cobra	1	2/3/4	1	1	29.40	29.10	29.69	17.98/13.18(20.00)	47.38/42.58(49.40)	44.63/42.22(49.69)	45.22/42.87(49.69)	
		2/3/4	2	2	29.35	29.04	29.67	17.98/13.18(20.00)	47.33/42.53(49.67)	47.02/42.22(49.04)	47.65/42.85(49.67)	
		2/3/4	3	3	29.31	28.98	29.64	17.98/13.18/20.00	47.29/42.49/49.31	46.96/42.16/48.98	47.62/42.82/49.69	
		2/3/4	4	4	29.24	28.89	29.59	(17.98)13.20(20.00)	(47.22)42.44(49.24)	(46.87)42.09(48.89)	(47.57)42.79(49.59)	
		2/3/4	5	5	29.20	28.83	29.56	(17.98)13.23(20.00)	(47.18)42.43(49.20)	(46.81)42.06(48.83)	(47.54)42.79(49.56)	
		2/3/4	6	6	29.16	28.77	29.55	17.98/13.23(20.00)	47.14/42.39(49.16)	46.75/42.00(48.77)	47.53/42.78(49.55)	
Harman Mining	2	2/3/4	1	1	33.08	32.29	33.88	(17.98)13.08	(51.06)46.16	(50.27)45.37	(51.76)46.96	
		2/3/4	2	2	31.13	32.33	33.94	17.98/13.18	49.11/44.31	50.31/45.51	51.92/47.12	
		2/3/4	3	3	33.20	32.39	34.01	17.98/13.20	51.18/46.40	50.37/45.59	51.99/47.21	
		2/3/4	4	4	33.26	32.44	34.09	17.98(13.20)	51.24(46.46)	50.42(45.64)	52.07(47.29)	
		2/3/4	5	5	33.31	32.48	34.15	17.98(13.20)	51.29(46.51)	50.46(45.68)	52.13(47.35)	
		2/3/4	6	6	33.25	32.41	34.10	17.98(13.20)	51.23(46.46)	50.39(45.61)	52.08(47.30)	
Va. Energy	3	2/3/4	1	1	32.27	31.27	32.85	15.54/13.09	47.81/45.36	47.23/44.78	48.39/45.94	
		2/3/4	2	2	32.36	31.36	32.96	17.98/13.09	50.34/45.45	49.74/44.85	50.94/46.05	
		2/3/4	3	3	32.43	31.43	33.04	17.98/13.18	50.42/45.61	49.82/45.01	51.03/46.22	
		2/3/4	4	4	32.50	31.50	33.11	(17.98)13.21	(50.48)45.71	(49.48)45.10	(51.09)46.32	
		2/3/4	5	5	32.56	31.56	33.16	17.98/13.23	50.54/45.79	49.93/45.18	51.16/46.41	
		2/3/4	6	6	32.63	32.63	33.25	17.98/13.23	50.61/45.86	49.99/45.24	51.23/46.48	
GEX	4	2/3/4	1	1	30.51	30.26	30.76	15.54/13.17	46.05/43.68	45.80/43.43	46.30/43.93	
		2/3/4	2	2	30.61	30.34	30.88	(17.98)13.18	(48.59)43.79	(48.32)43.52	(48.86)44.06	
		2/3/4	3	3	30.72	30.44	30.99	17.98/13.18	48.70/43.90	48.42/43.62	48.97/44.17	
		2/3/4	4	4	30.83	30.56	31.11	17.98(13.18)	48.81(44.01)	48.54(43.74)	49.09(44.29)	
		2/3/4	5	5	30.84	30.56	31.12	17.98(13.18)	48.82(44.02)	48.54(43.74)	49.10(44.30)	
		2/3/4	6	6	30.92	30.64	31.20	17.98(13.18)	48.90(44.10)	48.62(43.82)	49.18(44.38)	

i=source j=plant/destination t=month M<sub>ijt</sub>=mine price(fob) R<sub>ijt</sub>=rail rate C<sub>ijt</sub>=delivered cost (\*\*)\*\*=inferred

\* Entries for R<sub>ijt</sub> and C<sub>ijt</sub> correspond to the same sequence as given for j, i.e., j<sub>1</sub>/j<sub>2</sub>/... For detailed key see Table 7.1.

Table 7.3 Delivered Cost for Spot Sources (\$/ton)

i	j*	t	M <sub>ij</sub> t			R <sub>ij</sub> t			C <sub>ij</sub> t		
			L95	U95	L95	U95	L95	U95	L95	U95	
											PRED
Virginia	4&5**	1 2	(22.80)		(17.30)		(41.10)				
17	4&5	2	22.80		17.30		41.10				
	4&5	3	22.80		17.30		41.10				
	4&5	4	22.77		17.30		41.07				
	4&5	5	22.70		17.30		41.00				
	4&5	6	(22.70)		(17.30)		(41.00)				
Virginia	2	1 2	24.00		16.50		40.50				
18	2	2	24.00		16.50		40.50				
	2	3	(24.00)		(16.50)		(40.50)				
	2	4	(24.00)		(16.50)		(40.50)				
	2	5	(24.00)		(16.50)		(40.50)				
	2	6	(24.00)		(16.50)		(40.50)				
Virginia	2/3	1 1	24.90 22.83 22.87		(17.98/13.18)		(42.88/38.08)		(40.81/36.01)	(40.95/36.15)	
19	2/3	2	24.69 22.13 27.25		(17.98/13.18)		(42.67/37.87)		(40.11/35.31)	(45.23/40.43)	
	2/3	3	24.46 21.31 27.62		17.98/13.18		42.44/37.64		39.29/34.49	45.60/40.80	
	2/3	4	24.23 20.38 28.07		17.98/13.20		42.21/37.43		38.36/33.58	46.05/41.27	
	2/3	5	23.98 18.36 28.60		(17.98)13.20		(41.96)37.18		(36.34)31.56	(46.58)41.80	
	2/3	6	23.73 18.24 29.21		(17.98/13.20)		(41.71/36.93)		(36.22/31.42)	(47.19/42.41)	
Kentucky	3	1 2	24.33 22.15 26.51		(13.18)		(37.51)		(35.33)	(39.69)	
20	3	2	23.66 21.44 25.87		(13.18)		(36.84)		(34.62)	(39.05)	
	3	3	23.25 21.02 25.48		13.18		36.43		34.20	38.66	
	3	4	23.08 20.77 25.40		13.21		36.29		33.98	38.61	
	3	5	22.75 20.56 25.41		13.19		35.93		33.77	38.60	
	3	6	22.21 20.41 25.37		(13.19)		(35.40)		(33.60)	(38.56)	

i=source j=plant/destination t=month M<sub>ij</sub>t=mine price(fob) R<sub>ij</sub>t=rail rate C<sub>ij</sub>t=delivered cost (\*\*)\*\*=inferred

\* Entries for R<sub>ij</sub>t and C<sub>ij</sub>t correspond to the same sequence as given for j, i.e., j<sub>1</sub>/j<sub>2</sub>/... For detailed key see Table 7.1.

\*\* A single estimate is made for this combination of destinations.

### 7.1.2 RHS Vector

The RHS vector is the right side of the system of equations  $Ax=b$ , i.e., the vector  $b$ ;  $x$  is the matrix of decision variables,  $x_{ijt}$ ; and  $A$  is the array of technological coefficients. As per the model formulation of Chapter 6,  $x_{ijt}$  represents the tonnage of coal purchased from a given source  $i$ , shipped to a plant  $j$ , in month  $t$ . The RHS can be broken down into three categories: availability, demand, and stockpiles. The input for this vector is described in this section.

#### Availability:

Resource availability is typically represented in a supply or transportation model as a fixed amount of product or other quantity, such that total shipments cannot exceed this limit. Here availability will be broadly interpreted as any limiting bound on the supply of coal.

Two initial model formulations included constraints on production capacity at the mine and coal quality. Both sets of constraints were found not to be binding on the feasible region of the model and were therefore dropped from the final formulation. As a point of interest these values are included in Table 7.4. This table includes the

maximum allowable percentage change in shipments between adjacent months for a contract source,  $P_i$ , and the minimum total quantity delivered during the course of the model,  $q_i$ . These are defined in equation sets 6.2-6.5 and 6.9, respectively. The coal quality characteristics are weighted averages (by tonnage of delivery).

#### Demand and Stockpiles:

Constraint set 6.6 is a material balance between incoming deliveries, the stockpile, and consumption. The inputs are the monthly demand and the initial and final stockpile levels. These are given in Table 7.5 as the actual tonnage consumed at the plant during that month.

Table 7.4 Productivity Ratings and Average Quality for Contact Sources

l	t	$\overline{\text{BTU}}_{lt}$ (%)	$\overline{\text{ASH}}_{lt}$ (%)	$\overline{\text{SULFUR}}_{lt}$ (%)	Total '86 Tonnage	$q_{lk}$ (tons)	$P_l$ (factor)
Cobra Energy 1	1	13264	7.6	1.0	212991	95846	2
	2	12160	11.0	0.83			
	3	12689	9.9	1.03			
	4	12695	10.5	0.8			
	5	12686	9.8	0.7			
	6	12540	9.7	1.03			
Harman Mining 2	1	13344	7.42	1.03	228964	103034	4.5
	2	13354	6.96	1.12			
	3	13388	6.76	1.17			
	4	13553	5.93	1.2			
	5	13406	6.61	1.1			
	6	13614	6.27	0.96			
Va. Energy 3	1	13204	7.49	1.24	280269	126121	1.72
	2	13062	8.07	1.27			
	3	13380	7.05	1.27			
	4	13013	8.47	1.3			
	5	13198	7.74	1.06			
	6	12923	8.2	1.42			
GEX 4	1	13045	6.59	1.17	188826	84972	1.95
	2	13053	7.12	1.31			
	3	13100	6.95	1.4			
	4	13123	6.9	1.32			
	5	13142	6.36	1.14			
	6	12898	6.63	1.17			
Robinson 5	1	13234	6.8	1.3	155646	70041	2.34
	2	13031	7.37	1.4			
	3	13165	6.32	1.1			
	4	13261	6.17	1.2			
	5	13081	6.6	1.5			
	6	13117	6.9	1.2			
Majestic 6	1	12805	8.16	1.0	250118	112553	1.5
	2	13175	7.79	0.8			
	3	13290	6.89	0.8			
	4	13292	7.19	0.8			
	5	13055	8.03	1.0			
	6	12838	9.0	1.1			

l=source t=month columns 3,4,&5 are weighted average coal quality  
 $q_{lk}$ =minimum contract purchase  $P_l$ =maximum monthly production increase

Table 7.4 cont. Productivity Ratings and Average Quality for Contact Sources

$l$	$t$	$\overline{\text{BTU}}_{lt}$ (%) $_{lt}$	$\overline{\text{ASH}}_{lt}$ (%) $_{lt}$	$\overline{\text{SULFUR}}_{lt}$ (%) $_{lt}$	Total '86 Tonnage	$q_{lk}$ (tons)	$P_l$ (factor)
Landmark 7	1	13350	6.15	1.2	235042	105769	1.5
	2	13529	5.44	1.3			
	3	13475	5.55	1.4			
	4	13565	5.45	1.4			
	5	13395	5.0	1.2			
	6	13437	5.74	1.1			
Race Fork 8	1				11883	5347	1.71
	2	13856	5.9	0.9			
	3	13683	7.1	0.9			
	4						
	5						
	6						
Price 9	1	13144	5.0	0.8	176418	79388	4
	2	13128	5.75	0.8			
	3	13000	8.0	1.5			
	4	12913	6.65	0.8			
	5	13066	6.3	0.9			
	6	13194	5.8	0.9			
Mountain Top 10	1	12468	9.8	0.8	230398	103679	1.58
	2	12727	9.5	0.8			
	3	12770	8.5	0.8			
	4	12625	9.7	1.0			
	5	12716	9.2	0.7			
	6	12483	9.5	0.7			
Hobet 11	1				146491		2
	2	12405	9.0	0.7			
	3	12540	9.5	1.2			
	4	12604	9.3	0.8			
	5	12631	9.4	0.7			
	6	12670	9.0	0.9			

$l$ =source  $t$ =month columns 3,4,&5 are weighted average coal quality  
 $q_{lk}$ =minimum contract purchase  $P_l$ =maximum monthly production increase

Table 7.4 cont. Productivity Ratings and Average Quality for Contact Sources

I	t	$\overline{\text{BTU}}_{it}$ (%)	$\overline{\text{ASH}}_{it}$ (%)	$\overline{\text{SULFUR}}_{it}$ (%)	Total '86 Tonnage	$q_{ik}$ (tons)	$P_i$ (factor)
Calvin Branch 12	1	12822	9.5	0.9	517116	232702	2
	2	12885	10.3	0.7			
	3	12772	9.3	0.7			
	4	12712	9.4	0.7			
	5	12668	9.8	0.7			
	6	12539	10.0	0.75			
Enterprise 13	1	13092	7.5	1.4	285030	128264	2
	2	12920	7.4	1.4			
	3	13153	6.6	1.4			
	4	13153	6.7	1.3			
	5	13139	6.6	1.3			
	6	13088	7.4	1.6			
Red River 14	1	13243	8.2	0.8	105116	47302	1.66
	2	13066	7.4	1.0			
	3	13299	8.13	0.89			
	4	13403	6.1	0.8			
	5						
	6	13000	6.8	1.5			
Clinchfield 15	1	12565	12.7	0.8			2
	2	12613	12.4	0.7			
	3	12683	11.5	0.8			
	4	12871	12.2	0.8			
	5	13013	11.0	0.8			
	6	12886	11.5	0.7			
Wellmore 16	1	11963	14.8	0.9			2
	2	12002	14.7	0.9			
	3	11946	14.3	0.9			
	4	12073	13.2	0.9			
	5	11949	14.0	0.9			
	6	11706	13.5	0.9			

I=source t=month columns 3,4,&5 are weighted average coal quality  
 $q_{ik}$ =minimum contract purchase  $P_i$ =maximum monthly production increase

Table 7.5 Monthly Coal Consumption  
and Initial and Final Stockpiles

J	t	$D_{J,t}$	$S_{J,t}$	J	t	$D_{J,t}$	$S_{J,t}$
1	0		85829	2	0		458926
	1	62268			1	274342	
	2	44648			2	245768	
	3	21327			3	120656	
	4	19498			4	119739	
	5	95038			5	120155	
	6	54813	103453		6	263496	582435
3	0		119514	4	0		68753
	1	85151			1	72212	
	2	75214			2	57068	
	3	83028			3	19819	
	4	51261			4	63222	
	5	35763			5	50504	
	6	39293	122563		6	58057	144534
5	0		96848	6	0		162622
	1	48940			1	101879	
	2	29414			2	20127	
	3	33459			3	53993	
	4	36311			4	53562	
	5	28300			5	76673	
	6	32653	145906		6	97122	214070
7	0		145193				
	1	74802					
	2	54389					
	3	72847					
	4	47509					
	5	50372					
	6	41468	175210				

J=plant t=month  $D_{J,t}$ =demand (tons)  $S_J$ =stockpile (tons)

## 7.2 Model Output

The optimal solution to the model described in Chapter 6 and given the input of Tables 7.2 - 7.5 is broken down into those decision variables which are active (basic), and those which are nonactive, i.e., equal to zero (nonbasic). First, the breakdown between the basic and nonbasic solution will be given and compared to the actual deliveries experienced during the study period. This is followed by a sensitivity analysis of the decision variables, which will be expanded in section 7.3 to include the RHS and objective function.

### 7.2.1 Sources in Basis

Table 7.6 gives the optimal monthly tonnages for those variables in the basis. This is broken down by state, contract, and spot. For Virginia Power, the spot market accounts for 1104497 tons, about 42% of total purchases. This approaches the model minimum of 40%. One assumes that if constraints on minimum contract tonnage had not been so high, a larger percentage of spot sales would have been realized. This possibility will be examined later in section 7.3.

Other points of interest, which will be examined more fully later, are the exclusion of Virginia from the spot market and the lack of spot sales. The model prefers to buy large quantities when the market price is low. It should be noted that sources on contract were constrained not to exceed a percentage change between months. This acted to spread out sales more evenly than on the spot market.



### 7.2.2 Nonbasis

In the final optimal solution many variables are not in the basic solution. These are either sources which were too expensive, or particular months in which the price for coal was unacceptably high. These nonbasic variables are given in Tables 7.7 - 7.9. Nonbasic variables which used a forecast value for  $C_{ijt}$  are included in Tables 7.7 and 7.8, which are for contract and spot sources respectively. Following the delivered price are columns for the Reduced Cost, and the lower 95% confidence limit.

The Reduced Cost is a measure of the rate at which the objective function solution deteriorates as  $x_{ijt}$  is increased from zero. The Reduced Cost is the amount by which the price,  $C_{ijt}$ , must be reduced before  $x_{ijt}$  will enter the basis. This is the same as the dual price. Column four gives the price level at which  $x_{ijt}$  will enter the basis. However, this figure does not give the volume of sales that will accompany such a reduction in price. This requires parametric analysis of the price coefficient in the objective function. Comparison of this value with the lower confidence limit gives a better feeling for the likelihood of a nonbasic variable entering the basis. Since the  $c_{ijt}$  are in many cases forecast with considerable error, the confidence interval should be

examined as well as the expected value used as input in the model. In Tables 7.7 and 7.8 column six is the difference (labeled as the CRITICAL DIFFERENCE) between the lower confidence limit and the price at which  $x_{ijt}$  is expected to enter the basis. Where this difference is negative the price at which the variable enters the basis is within the 95% confidence interval. In this case there is a probability, less than 95%, that  $x_{ijt}$  will actually be nonzero. This probability is a function of the percentage of the difference between  $C_{ijt}$  and  $L95$  which the Reduced Cost accounts for and also the probability distribution of the confidence interval. When the difference between column four and column five is positive, the value at which  $x_{ijt}$  will enter the basis lies outside of the confidence limit. In this case we can be reasonably certain that this source of coal will not be economically attractive to the utility.

A number of sources were input in the model as projections based on a few actual deliveries. Since no forecast could be made, no confidence intervals are available. These sources and their Reduced Costs are given in table 7.9.

Table 7.7

COMPARISON OF LOWER 95% C.I. WITH ENTERING LEVEL OF NONBASIC VARIABLES  
(all sources on contract, IEL)

VARIABLE	$C_{ijt}$	REDUCED COST	L95	ENTERS BASIS AT	CRITICAL DIFFERENCE
X126	49.66	1.72	46.75	47.94	1.19
X141	47.17	1.29	49.69	45.88	-3.80
X142	42.42	6.61	49.04	35.81	-13.22
X143	49.19	0.91	48.98	48.28	-0.69
X144	47.13	0.88	48.89	46.25	-2.63
X145	42.38	0.88	48.83	41.50	-7.32
X146	49.15	1.12	48.77	48.03	-0.73
X221	51.05	0.10	50.27	50.95	0.68
X225	51.17	0.02	50.46	51.14	0.68
X226	46.39	1.74	50.39	44.65	-5.73
X233	51.28	0.02	45.59	51.26	5.67
X322	45.35	0.08	49.74	45.26	-4.47
X326	45.60	1.72	49.99	43.88	-6.10
X331	50.47	2.34	44.78	48.12	3.34
X424	43.78	0.01	48.54	43.76	-4.77
X425	48.69	0.04	48.54	48.64	0.10
X426	43.89	1.77	48.62	42.12	-6.49
X431	48.80	2.42	43.43	46.37	2.94
X541	49.17	0.20	48.68	48.97	0.29
X542	48.51	5.06	48.68	43.45	-5.22
X543	49.17	0.32	48.68	48.84	0.16
X544	48.65	0.55	48.68	48.09	-0.58
X545	49.17	0.44	48.68	48.72	0.04
X546	48.77	0.95	48.68	47.82	-0.85
X626	43.34	1.72	47.19	41.62	-5.56
X631	48.09	2.44	43.02	45.65	2.63
X633	48.05	3.00	42.67	45.05	2.38
X634	43.30	0.01	42.58	43.29	0.71
X723	45.68	0.37	45.11	45.31	0.20
X724	46.41	0.40	45.01	46.01	1.00
X725	45.68	0.34	45.01	45.34	0.33
X726	46.41	2.62	44.71	43.79	-0.91
X742	46.41	5.00	45.74	41.41	-4.32
X1145	42.72	1.20	40.81	37.59	-3.21
X1146	43.41	1.29	40.11	33.00	-7.10
X1342	44.09	4.29	43.34	39.79	-3.54
X1345	43.98	0.30	43.68	43.67	-0.00
X1346	44.22	0.98	43.68	43.23	-0.44
X1351	43.98	2.05	43.45	41.93	-1.51
X1353	44.32	0.30	43.58	44.01	0.43

Table 7.8

COMPARISON OF LOWER 95% C.I. WITH ENTERING LEVEL OF NONBASIC VARIABLES  
(all spot sources, IES)

VARIABLE	$C_{ijt}$	REDUCED COST	L95	ENTERS BASIS AT	CRITICAL DIFFERENCE
X1923	42.66	4.83	39.29	37.83	-1.45
X1924	37.86	4.59	38.36	33.27	-5.08
X1925	42.43	4.34	36.34	38.09	1.75
X1926	37.63	4.58	36.22	33.05	-3.16
X1931	42.20	4.27	36.01	37.92	1.91
X1932	37.42	4.07	35.31	33.35	-1.95
X1933	41.95	4.84	34.49	37.11	2.62
X1934	37.17	4.59	33.58	32.58	-0.99
X1935	41.70	4.31	31.56	37.39	5.83
X1936	36.92	3.82	31.42	33.10	1.68
X2031	37.50	4.71	35.33	32.79	-2.53
X2032	36.83	4.04	34.62	32.79	-1.82
X2033	36.42	3.63	34.20	32.79	-1.40
X2034	36.28	3.45	33.98	32.83	-1.14
X2035	35.92	3.06	33.77	32.86	-0.90
X2036	35.39	2.29	33.60	33.10	-0.49
X2111	37.63	0.43	33.08	37.20	4.12
X2112	37.59	1.36	32.92	36.22	3.30
X2113	38.32	0.42	33.50	37.89	4.39
X2114	38.28	1.13	34.07	37.14	3.07
X2115	38.57	1.72	34.08	36.84	2.76
X2116	37.68	2.06	34.38	35.62	1.24
X2122	37.86	0.08	32.03	37.77	5.74
X2123	39.34	0.47	32.23	38.87	6.64
X2124	38.07	1.02	32.65	37.05	4.40
X2125	38.89	1.93	32.98	36.96	3.98
X2126	38.63	3.69	33.04	34.93	1.89
X2143	40.40	0.19	33.05	40.21	7.16
X2144	39.40	1.65	34.42	37.74	3.32
X2145	40.64	2.78	34.97	37.85	2.88
X2146	39.54	1.80	33.77	37.73	3.96
X2151	41.53	0.41	33.73	41.11	7.38
X2154	39.82	0.38	33.42	39.43	6.01
X2155	40.55	0.98	33.43	39.57	6.14
X2156	40.51	0.81	33.73	39.69	5.96
X2221	39.31	1.72	43.02	37.59	-5.42
X2222	39.31	5.85	42.90	33.46	-9.43
X2223	43.44	5.77	42.80	37.67	-5.12
X2224	39.25	5.67	42.69	33.57	-9.11
X2225	43.37	5.58	42.57	37.79	-4.77
X2226	39.17	6.97	42.46	32.19	-10.26
X2231	43.28	6.52	38.82	36.76	-2.05
X2232	38.51	6.45	38.70	32.06	-6.63
X2233	43.19	6.38	38.60	36.81	-1.78
X2234	38.44	5.68	37.92	32.76	-5.15
X2235	43.10	5.58	37.83	37.52	-0.30
X2236	38.35	5.25	37.71	33.10	-4.60

Table 7.8 cont.

COMPARISON OF LOWER 95% C.I. WITH ENTERING LEVEL OF NONBASIC VARIABLES  
(all spot sources, IES)

VARIABLE	$C_{ij}$	REDUCED COST	L95	ENTERS BASIS AT	CRITICAL DIFFERENCE
X2411	38.07	0.86	36.75	37.20	0.45
X2412	38.58	1.68	37.27	36.90	-0.36
X2413	38.80	0.16	37.35	38.63	1.28
X2415	38.82	0.11	37.20	38.71	1.51
X2421	39.08	0.99	37.26	38.09	0.83
X2422	39.04	1.22	37.21	37.81	0.60
X2423	39.34	1.44	37.31	37.90	0.59
X2424	38.91	1.45	37.26	37.46	0.20
X2425	39.06	1.68	37.47	37.38	-0.08
X2426	39.50	3.40	37.68	36.10	-1.57
X2451	39.02	0.94	37.48	38.08	0.60
X2452	39.29	1.25	37.51	38.03	0.52
X2453	39.61	0.70	37.61	38.90	1.29
X2454	39.10	0.49	37.70	38.61	0.91
X2455	39.52	0.60	37.79	38.92	1.13

Table 7.9

DELIVERED & REDUCED COSTS OF NONBASIC VARIABLES W/O ESTIMATED  
CONFIDENCE LIMITS (all 1)

VARIABLE	$C_{ijt}$	REDUCED COST	VARIABLE	$C_{ijt}$	REDUCED COST
X1013	45.16	8.29	X1751	40.07	1.38
X1021	43.75	0.24	X1752	40.07	1.39
X1024	43.12	1.72	X1753	40.00	1.25
X1025	43.54	1.72	X1754	40.00	1.29
X1026	44.23	3.39	X1755	40.00	1.25
X1041	42.91	0.20	X1756	40.00	0.83
X1042	43.33	4.95	X1821	40.00	1.90
X1043	44.02	9.64	X1822	40.00	1.90
X1044	42.91	1.27	X1823	40.00	1.90
X1045	43.33	1.27	X1824	40.00	1.90
X1046	44.02	1.46	X1825	40.00	1.90
X1123	43.41	1.70	X1826	40.00	3.37
X1124	42.33	1.63	X2341	39.339	1.01
X1125	42.66	1.65	X2342	39.509	6.18
X1126	43.41	3.22	X2344	38.859	0.10
X1141	42.37	0.01	X2345	38.969	0.21
X1142	42.72	5.02	X2521	42.389	4.79
X1143	43.41	1.34	X2522	42.389	4.79
X1741	40.10	1.77	X2523	43.500	5.89
X1742	40.10	1.22	X2524	42.469	4.85
X1743	40.10	6.74	X2525	41.449	3.83
X1744	40.10	2.13			
X1745	40.10	1.29			
X1746	40.10	1.45			

### 7.2.3 Comparison of Actual vs. Optimal Purchases

In any comparison between model results and the system it attempts to evaluate, the degree of variation between reality and the model can be expected to be directly proportional to the complexity of the system. This is especially true for optimization models of economic structures, such as the steam coal market. Not only are there numerous imponderables, but it must also be noted that model results are based on foreknowledge of the price of coal and corresponding optimal decisions are based only on the total cost of supply. Also, the forecasts of mine price assumes that the future is like the past. These factors alone make for a drastic departure from the fuel purchasing procedure of a utility. The coal purchasing agent has only a general idea as to what the delivered cost\* from any source will be in the near future.

This is especially true on the spot market. The idea of using forecasts of mine price is one of the distinguishing features of this model, rather than common practice among utilities. Also, state owned utilities do not think entirely in terms of minimizing delivered cost. Among many constraints which are unquantifiable for lack of information are political demands which do not figure in the cost of coal. With this in mind consider the record of monthly totals of the actual deliveries from sources

corresponding to those used in the model. These are given in Tables 7.10 and 7.11. Compare the actual pattern of deliveries in Tables 7.10 -7.11 with the model results summarized in Table 7.6. Spot deliveries exhibit the most notable differences. In the model, Virginia is entirely cut out of the spot market. This is not surprising, considering that Virginia is the high cost producer. What is surprising is the dominance of West Virginia over Kentucky, the opposite of what actually happened. This occurrence seems to be due largely to the very low mine price in December for NS shipments to Chesterfield (see X<sub>2526</sub> in Table 7.6). In fact, most deliveries from West Virginia occurred in December. In reality, Virginia Power did not buy from this extremely low cost source. It is not possible from the available information to analyze Virginia Power's purchases. It must be remembered that the model acts with foreknowledge. It has information on all coal prices for the entire six month period, and optimizes for this entire time frame. Virginia Power does not have foreknowledge of spot prices, and may miss some opportunities as a result.

\* Henceforth coal purchasing shall be referred to from the point of view of the utility. Therefore we will consider the problem of minimizing delivered cost, rather than price.

Another interesting comparison is between the total

tonnages shipped on the two railroads. The split in West Virginia is probably skewed due to X<sub>2526</sub> which ships on NS. Disregarding this source, the total tonnage closely approximates what that state actually shipped. In reality, NS carried only 48248 tons out of 382959. In Kentucky, CSX completely captures the market in the model, which is not far from what actually happened with only 48283 out of 665004 tons being carried by NS.

Tables 7.10b - 7.10d give a more detailed comparison between model results and actual deliveries. These tables are organized by state, market, and decision variable. Sources in Virginia on contract follow a pattern common to all contract sources. Since contract coal is more expensive than that available in the spot market, the optimal solution will typically purchase as little as allowable and still meet contracted minimum tonnages.

Those sources in the model which produced as much or more than it actually did indicate a source which is particularly competitive. In such a case the utility is buying coal from this source in preference to spot coal. This phenomenon occurs only in Virginia. Specifically, Cobra Energy, Harman Mining, and Red River all ship more to Virginia Power than the required minimum. This does not occur in the other states. In spite of Virginia's

dismal position in the spot market, model results indicate a competitive edge for these contract sources. Outside of this, Virginia follows the same pattern in the model results as the other two states. Individual monthly deliveries typically don't compare with actual monthly totals. The optimal strategy seems to be to purchase large quantities in months in which the price is low. This results in more spotty purchasing than what Virginia Power actually does. Another situation occurs in West Virginia: the model's solution has Virginia Power satisfying contract requirements by shipping to that plant with the lowest delivered price and shipping nothing to the other plants. Instead, these plants which are receiving less contract coal are freed to purchase lower cost spot coal. This is the case for Mountain Top shipping to Possum Point and Hobet shipping to Chesterfield and Possum Point. Mountain Top ships a very large quantity (31318 tons) to Chesterfield ( $\times_{1023}$ ) in the low cost month of September. This allows Virginia Power to meet contract obligations while avoiding purchases of high cost coal from Mountain Top bound for Possum Point. The same strategy is used in the case of purchases from Hobet Mining. purchases from Hobet Mining. It is much less expensive to ship Hobet coal to Bremo Bluff than to ship it to Chesterfield or Possum Point. In reality Virginia Power sent 33705 tons to Chesterfield and 45774 tons to

Possum Point. Instead of this the model sends twice the actual tonnage delivered to Bremono Bluff while sending none to the other plants. This satisfies the contract at the lowest possible cost, since lower cost spot coal can be purchased for Chesterfield and Possum Point.

Table 7.10a

ACTUAL DELIVERIES (TONS) ON CONTRACT TO VIRGINIA POWER		WEST VIRGINIA		KENTUCKY	
VARIABLE	TONS	VARIABLE	TONS	VARIABLE	TONS
X121	12905	X1011	12934	X421	21713
X122	2205	X1012	19461	X423	14945
X123	2140	X1013	25936	X424	10304
X126	4394	X1014	13498	X425	17236
X131	7349	X1022	6408	X426	13984
X132	9830	X1024	6790	X431	335
X133	11546	X1025	13117	X432	10377
X134	23075	X1026	13099	X433	5317
X135	7645	X1041	4291	X546	12251
X136	11145	X1042	1391	X551	11269
X222	7593	X1043	96	X552	18139
X223	20151	X1112	6607	X553	7475
X224	17928	X1113	13542	X554	17478
X225	8283	X1114	13458	X555	13453
X226	14188	X1115	98	X621	20189
X231	6959	X1124	6621	X623	5127
X232	24405	X1125	20008	X624	5716
X233	105	X1126	19145	X625	15245
X321	14280	X1145	6986	X626	3144
X322	6054	X1221	36036	X631	2106
X323	581	X1222	75083	X632	16125
X325	14896	X1223	75665	X633	18853
X326	3426	X1224	44477	X634	18036
X331	5498	X1225	88323	X635	3698
X332	16813	X1226	59714	X636	15917
X333	23127	CONTRACT	582784	X721	6173
X334	31636			X722	18927
X335	3465			X723	25130
X336	15245			X724	25971
X832	5622			X726	19970
X833	6261			X741	6668
X1431	15030			X745	19612
X1432	6106			X746	6846
X1433	10163			X941	6360
X1434	4579			X942	25460
X1436	3999			X943	190
CONTRACT	378627			X944	25883
				X945	13095
				X946	19882
				X1341	99
				X1345	6696
				X1351	21819
				X1352	17504
				X1353	15736
				X1354	31744
				X1355	17878
				X1356	39473
				CONTRACT	669548

Table 7.10b: COMPARISON OF DELIVERIES FROM VIRGINIA  
MODEL RESULTS vs. ACTUAL

VARIABLE	MODEL	ACTUAL	VARIABLE	MODEL	ACTUAL
<b>Cobra Energy to Chesterfield</b>			<b>Virginia Energy to Chesterfield</b>		
X121	0	12905	X321	43628	14280
X122	6114	2205	X322	0	6054
X123	4852	2140	X323	0	581
X124	1223	0	X324	11436	0
X126	0	4394	X325	6633	14896
<b>Subtotal</b>	<b>12189</b>	<b>21644</b>	X326	0	3426
<b>Cobra Energy to Portsmouth</b>			<b>Subtotal</b>	<b>61697</b>	<b>39237</b>
X131	15374	7349	<b>Virginia Energy to Portsmouth</b>		
X132	1572	9830	X331	0	5498
X133	0	11546	X332	33997	16813
X134	8481	23075	X333	19718	23127
X135	19409	7645	X334	0	31636
X136	38818	11145	X335	0	3465
<b>Subtotal</b>	<b>83656</b>	<b>70590</b>	X336	10707	15245
<b>Total</b>	<b>95845</b>	<b>92234</b>	<b>Subtotal</b>	<b>64423</b>	<b>95784</b>
<b>Harman Mining to Chesterfield</b>			<b>Total</b>	<b>126120</b>	<b>135021</b>
X222	0	7593	<b>Race Fork to Portsmouth</b>		
X223	14545	20151	X831	158	0
X224	3054	17928	X832	270	5622
X225	0	8283	X833	462	6261
X226	0	14188	X834	790	0
<b>Subtotal</b>	<b>17600</b>	<b>68143</b>	X835	1352	0
<b>Harman Mining to Portsmouth</b>			X836	2312	0
X231	15392	6959	<b>Total</b>	<b>5346</b>	<b>11883</b>
X232	69265	24405	<b>Red River to Portsmouth</b>		
X233	0	105	X1431	10243	15030
X235	641	0	X1432	6145	6106
X236	134	0	X1433	3687	10163
<b>Subtotal</b>	<b>85433</b>	<b>31469</b>	X1434	5027	4579
<b>Total</b>	<b>103033</b>	<b>99612</b>	X1435	8345	0
			X1436	13853	3999
			<b>Total</b>	<b>47301</b>	<b>39877</b>
			<b>STATE</b>		
			<b>TOTAL</b>	<b>377649</b>	<b>378627</b>
			<b>(no spot sales in model results)</b>		

Table 7.10c: COMPARISON OF DELIVERIES FROM WEST VIRGINIA  
MODEL RESULTS vs. ACTUAL

VARIABLE	MODEL CONTRACT	ACTUAL	VARIABLE	MODEL SPOT	ACTUAL
Mountain Top to Breomo Bluff			W.Va. to Portsmouth via CSX		
X1011	12545	12934	X2341		9949
X1012	4246	19461	X2342		7889
X1013	0	25936	X2343	83118	22153
X1014	19730	13498	X2344		31282
X1015	12430		X2345		8988
X1016	7831		X2346	45524	3749
Subtotal	56784	71829	Total	128642	84010
Mountain Top to Chesterfield			W.Va. to Breomo Bluff via CSX		
X1022	15575	6408	X2411	0	21437
X1023	31318	0	X2412	0	30441
X1024	0	6790	X2413	0	21428
X1025	0	13117	X2414	152966	16324
X1026	0	13099	X2416	39544	0
Subtotal	46894	39414	Total	192510	89630
Mountain Top to Possum Point			W.Va. to Chesterfield via CSX		
X1041	0	4291	X2421	0	26105
X1042	0	1391	X2422	0	34849
X1043	0	96	X2423	0	25931
Subtotal	0	5778	X2424	0	13405
Total	103678	117021	X2425	0	27448
Hobet to Breomo Bluff			X2426	0	19932
X1111	26712	0	Total	0	147670
X1112	20236	6607	W.Va. to Yorktown via CSX		
X1113	10118	13542	X2451	0	8245
X1114	5059	13458	X2452	0	5156
X1115	2529	98	X2456	16046	34849
X1116	1264	0	Total	16046	48250
Subtotal	65920	33705	W.Va. to Chesterfield via NS		
Hobet to Chesterfield			X2522	0	25931
X1124	0	6621	X2523	0	13405
X1125	0	20008	X2524	0	27448
X1126	0	19145	X2525	0	19932
Subtotal	0	45774	X2526	506581	8245
Hobet to Possum Point			Total	506581	94961
X1145	0	6986	Calvin Branch to Chesterfield		
Total	65920	86465	X1221	17439	36036
			X1222	18963	75083
			X1223	37926	75665
			X1224	75853	44477
			X1225	151706	88323
			X1226	75853	59714
			Total	377742	379298

Table 7.10d: COMPARISON OF DELIVERIES FROM KENTUCKY  
MODEL RESULTS vs. ACTUAL

VARIABLE	MODEL	ACTUAL	VARIABLE	MODEL	ACTUAL
CONTRACT			CONTRACT		
GEX to Chesterfield			Landmark to Chesterfield		
X421	26777	21713	X721	17530	6173
X423	11696	14945	X722	11920	18927
X424	0	10304	X723	0	25130
X425	0	17236	X724	0	25971
X426	0	13984	X726	0	19970
Subtotal	38473	78182	Subtotal	29450	96171
GEX to Portsmouth			Landmark to Possum Point		
X431	0	335	X741	0	6668
X432	22935	10377	X743	15112	0
X433	0	5317	X744	22668	0
X434	5965	0	X745	15414	19612
X435	5965	0	X746	23122	6846
X436	11632	0	Subtotal	76318	33126
Subtotal	46498	16029	Total	105768	129297
Total	84971	94211			
Robinson Creek to Possum Point			Price to Possum Point		
X546	0	12251	X941	4669	6360
			X942	1167	25460
Robinson Creek to Yorktown			X943	865	190
X551	5281	11269	X944	3461	25883
X552	2270	18139	X945	13845	13095
X553	2889	7475	X946	55380	19882
X554	6760	17478	Total	79387	90870
X555	15820	13453			
X556	37019	0	Enterprise to Possum Point		
Subtotal	70040	67814	X1341	10225	99
Total	70040	80065	X1343	7528	0
			X1344	10456	0
			X1345	0	6696
			Subtotal	28209	6795
Majestic to Chesterfield			Enterprise to Yorktown		
X621	39968	20189	X1351	0	21819
X623	18481	5127	X1352	5112	17504
X624	12567	5716	X1353	0	15736
X625	1436	15245	X1354	4600	31744
X626	0	3144	X1355	30113	17878
Subtotal	72453	49421	X1356	60227	39473
			Subtot.	100054	144154
Majestic to Portsmouth			Total	128263	150949
X631	0	2106			
X632	27178	16125			
X633	0	18853			
X634	0	18036			
X635	7109	3698			
X636	5811	15917			
Subtotal	40099	74735			
Total	112552	124156			

Table 7.10d cont.: COMPARISON OF DELIVERIES FROM KENTUCKY  
MODEL RESULTS vs. ACTUAL

VARIABLE	MODEL SPOT	ACTUAL	VARIABLE	MODEL SPOT	ACTUAL
Ky. to Portsmouth via CSX			Ky. to Possum Point via CSX		
X2033	0	21268	X2141	45633	30081
X2034	0	27099	X2142	38470	28189
<u>X2035</u>	<u>0</u>	<u>182</u>	X2143	0	25835
Total	0	48549	X2144	0	19307
Ky. to Bremo Bluff via CSX			X2145	0	7433
X2111	0	39503	<u>X2146</u>	<u>0</u>	<u>6543</u>
X2112	0	15249	Total	84103	117388
X2113	0	20091	Ky. to Yorktown via CSX		
<u>X2114</u>	<u>0</u>	<u>2991</u>	X2151	0	12773
Total	0	77834	X2152	35759	0
Ky. to Chesterfield via CSX			X2153	36273	0
X2121	104579	31930	X2154	0	6543
X2122	0	104012	<u>X2155</u>	<u>0</u>	<u>6611</u>
X2123	0	110696	Total	72033	25927
X2124	0	63746	Ky. to Chesterfield via NS		
X2125	0	36539	X2223	0	25679
<u>X2126</u>	<u>0</u>	<u>100</u>	<u>X2224</u>	<u>0</u>	<u>4584</u>
Total	104579	347023	Total	0	30263
			Ky. to Portsmouth via NS		
			X2233	0	5759
			X2234	0	8130
			<u>X2235</u>	<u>0</u>	<u>4131</u>
			Total	0	18020



### 7.2.3 Sensitivity Analysis

Sensitivity analysis is used to determine the sensitivity of model results to relatively small changes in the various elements of the model. These elements include objective function coefficients, technological coefficients, the RHS elements, and the possible addition of new constraints or variables. These will be jointly referred to as model parameters. When the input for a model is uncertain, or if elements of the model may be changed in the future, sensitivity analysis of these uncertain or potentially changing elements becomes an integral part of model analysis. The discussion presented in this section will use much of the modeling terminology presented previously. The basic sensitivity analysis presented in this section presents a range analysis. This is the range in which changes in the parameter will not result in a change in the basis, i.e., those source/destination combinations ( $x_{ijt}$ ) which are in the optimal solution at a positive value. This effectively gives some idea of the sensitivity of the model to changes in specific parameters. As the value of the parameter changes beyond the allowable increase or decrease there will be a resulting change in the basis. For example, the major concern in the coal purchasing model is the sensitivity of the volume and pattern of coal purchases to

the delivered cost. As  $C_{ijt}$  decreases past the allowable decrease, the basis will change such that  $x_{ijt}$  will either enter the basis, if it was initially nonbasic, or will decrease in value as purchases are shifted to some other source or as  $x_{ijt}$  leaves the basic solution. The same sort of analysis can be done for the RHS of constraints. Here the validity of the basis for a range of resource availability is considered. Also the sensitivity of the model to the addition of new constraints or decision variables can be examined. A sensitivity analysis only gives information on the sensitivity of the basic solution with respect to the elements of the model or in changes to the model. It does not give information on how the basis will change; what will enter the basis, will leave, or how the objective solution will change. These sort of questions are addressed using Parametric Analysis (section 7.3).

The sensitivity analysis for the decision variables coefficients,  $c_{ijt}$ , is presented in Table 7.12. This gives the coefficient and the allowable increase and decrease before there will be a change in the basic solution. For many of the variables the basis is insensitive to change in  $c_{ijt}$ . In this case the range is given as INFINITY. This is the result of one of two possible situations, depending if the variable is basic or

nonbasic in the optimal solution. If the variable is nonbasic then the allowable increase in the cost will be infinity, since increased cost will only make the variable an even more unattractive candidate for entering the basis. For variables in the basis and members of the contract market the allowable decrease will be infinity when production has reached the allowable maximum.

The sensitivity analysis of the decision variables falls into three categories: contract, spot, and division by rail carrier. Contract sources are in most cases very sensitive to changes in price. This is not surprising since there were numerous optimal solutions involving this class of variables. First, the price fluctuation is low when compared to spot sources. As a result, small price changes will cause a shift in purchases between different months. Purchasing patterns will only change between months since the utility is committed to buying a minimum tonnage during the time span of the model. Since contract sources are significantly higher priced than sources in the spot market, Virginia Power purchases the minimum that it is constrained to. Therefore, the differences in tonnage between sources on contract will experience little if any change, leaving total deliveries to change between months for the same source.

Sensitivity analysis of the spot market presents a more interesting case. The patterns of spot sales have already been discussed in section 7.2.3. Here the rigidity of that supply pattern can be considered.

Virginia spot sources are variables X1741 to X1826 for shipments out of Dickenson County on CSX, and X1921 to X1936 for shipments from the remainder of the state on NS. For all of these sources the allowable increase is infinite. Since none of them are in the basic solution increasing cost will not affect the basis. As mentioned earlier, Virginia is not in the spot market in the model. The ALLOWABLE DECREASE is the cost reduction necessary before Virginia Power will begin purchasing Virginia coal on the spot market. It is interesting to note the difference in price reduction between sources captive to CSX vs. NS. For Dickenson county (CSX), a decrease in price around two to three dollars will allow entry into the market. For other Virginia sources captive to NS, this figure is in excess of four to five dollars. This cannot be explained by the difference in rail rates. Actually, the delivered price of coal going to Possum Point on NS is by far the cheapest coal available in Virginia. There must be other complicating factors within the model/market which are resulting in a disinclination to purchase Virginia coal. It seems that demand by Possum

Point is entirely satisfied by contract sources. This eliminates inexpensive spot coal. In reality, spot shipments on NS to Possum Point accounted for 76% of all of Virginia Power's deliveries from Virginia on the spot market.

Sources in Martin and Pike counties in Kentucky which are served by NS do not participate in the spot market to Chesterfield and Portsmouth. The Chesterfield market is controlled by CSX out of Kentucky and West Virginia. A very large price reduction would be necessary before NS sources could enter the market for these two plants.

In the current model, Kentucky is shut out of Bremo Bluff, but the range analysis shows that a price reduction of between one and two dollars would result in Kentucky entering the spot market to Bremo. West Virginia currently controls Bremo, but it is sensitive to increases in price. Sources on CSX in Kentucky could readily compete against West Virginia given a reduction in price. West Virginia does not participate in the market for shipments to Chesterfield from Kentucky on CSX. In this case a price reduction of between one to two dollars by CSX or more likely, mines on CSX, would result in the entry of West Virginia producers into this market. In summary, outside of Virginia producers there are plenty of possibilities in the pattern of deliveries on the spot

market. With the exception of the results for Virginia, the model's results are not that dissimilar from reality when the sensitivity of these results are taken into account. One must also remember that the  $c_{ijt}$  were forecast with considerable error and that the confidence limits should be considered before drawing any further conclusions from this model.

The sensitivity of the optimal solution to changes in stockpile levels are given in Table 7.13. Only one example table is given. the remainder is presented as Table B.3 in Appendix B. Here the optimal levels of the stockpiles are given along with the range analysis. It is interesting to note that the solution is, in most cases, highly sensitive. Changes of a fraction of a ton result in a change in the basis.

Table 7.12 SENSITIVITY ANALYSIS

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
	Cobra Energy to Chesterfield		
X121	47.37	INFINITY	-0.000006
X122	47.32	-0.000020	0.002039
X123	47.28	0.002451	0.277879
X124	47.21	0.017928	0.000026
X125	47.17	INFINITY	-0.000021
X126	47.13	INFINITY	1.726511
	Cobra Energy to Portsmouth		
X131	42.57	-0.000006	0.502695
X132	42.52	0.002039	-0.000020
X133	42.48	INFINITY	0.002451
X134	42.43	0.000026	0.017928
X135	42.42	-0.000021	0.542629
X136	42.38	0.310074	17.677872
	Cobra Energy to Possum Point		
X141	49.39	INFINITY	1.290003
X142	49.66	INFINITY	6.610010
X143	49.30	INFINITY	0.912455
X144	49.23	INFINITY	0.885416
X145	49.19	INFINITY	0.885401
X146	49.15	INFINITY	1.126490
	Harman Mining to Cesterfield		
X221	51.05	INFINITY	0.100001
X222	49.10	INFINITY	-0.000020
X223	51.17	0.022410	2.197881
X224	51.23	0.000026	1.903945
X225	51.28	INFINITY	0.029963
X226	51.22	INFINITY	1.746501
	Harman Mining to Portsmouth		
X231	46.15	0.100001	1.414690
X232	44.30	-0.000020	18.083344
X233	46.39	INFINITY	0.022410
X234	46.45	INFINITY	0.000026
X235	46.50	0.029963	1.752487
X236	46.45	1.746501	1.198415
	Virginia Energy to Chesterfield		
X321	47.80	2.349980	INFINITY
X322	50.33	INFINITY	0.089986
X323	50.41	INFINITY	0.007573
X324	50.47	0.008064	0.025015
X325	50.53	0.000020	0.043130
X326	50.60	INFINITY	1.726491

### 7.3 Parametric Analysis

Sensitivity analysis considers changes in the basis over a small range. It cannot be used to consider what will happen to the optimal solution as a coefficient or vector changes over a broad range. For this reason sensitivity analysis cannot be used to check the validity of assumptions. This model is based on a series of assumptions. Some of these are of uncertain validity. This section will give the background of Parametric Analysis, a methodology by which parameters of the model can be changed over a broad range in order to check the validity of assumptions.

#### 7.3.1 Applications

Most Linear Programming packages enable the user to continuously permute a parameter from its value in the optimal solution to some user defined maximum. Typically, as the parameter value changes the new solution, or some subset of the solution, will be output for each change in the basis. In this model the elements of the RHS and cost vector are permuted. The methodology of parametric analysis is described in the next section. The reasons for doing so are as follows.

Considerable uncertainty is associated with the estimates of mine price. In the model results given in the previous sections of this chapter, the delivered cost used in the cost vector (objective function) uses the expected value of the forecasted mine price. This expected value's uncertainty is partially quantified by its 95% confidence interval. One possible application of parametric analysis is to permute the expected value of an element in the cost vector over the range of its confidence interval. As an example, consider deliveries from X2526. Due to the surprisingly low cost coefficient for this source (\$36.13/ton) Chesterfield purchases an enormous quantity of spot coal from this source in this month (506582 tons). No coal is bought from this source in previous months. This is because of a sharp drop in the forecast of the expected value. In the previous month the delivered price was \$41.45/ton. Thus, we can be justifiably suspicious that the forecast for this month is severely underestimated. To check this, parametric analysis can be performed on the cost vector for this element only. Specifically,  $c_{ijt}$  can be permuted from its current level up to the upper confidence value. Unfortunately,  $c_{2526}$  is based on the actual mine price, since there was insufficient data on which to base a forecast. Another candidate for this sort of analysis are the sources shipping to Portsmouth,

captive to NS in Virginia. These sources are currently out of the spot market, but what if the delivered cost was actually at the lower limit (see L95 in Table 7.3 for X1931 - X1936). Here the difference between the predicted price and its lower confidence interval is as great as \$5.62 in the fifth month, well in excess of the required decrease for that source to enter the basis. The question is, how many tons will be purchased from that source at that price? This question will be answered in section 7.4. This application will yield the activity level for a source or subset of sources as the price changes over a given interval. This is equivalent to generating a demand curve for a subset of the cost vector.

Parametric Analysis can also be used to permute the RHS vector from its value in the optimal solution to a user specified maximum. As the RHS side is varied, the impact of this change on the objective value and on the dual price for the associated constraint can be examined.

Let's consider a constraint on production in which the RHS defines minimum production, i.e.,  $\sum a_i x_i \geq b_i$ . As  $b_i$  is permuted from its lower value (increasing from zero), this constraint will remain nonbinding until the permuted value of the RHS exceeds the optimal activity level. Beyond this point the new constraint is binding in the optimal solution, and has a nonzero (in this case negative) dual

price. This dual price can be thought of as the marginal cost incurred as a result of forcing supply beyond the level found to be optimal in the original model. As the RHS continues to increase, the basis will change at discrete points and will have an increasing dual price/marginal cost. Thus, as the supply increases from this source the objective function will deteriorate and the marginal cost will increase. When these changes are plotted as marginal cost vs. production, the result is a supply curve (see section 7.4). Other applications of parametric analysis are possible. Another interesting possibility would be to push down the tonnage requirements from contract sources while simultaneously increasing the price from low cost spot sources. This technique could be used to simulate classical supply/demand relationships under conditions of increasing demand. For the objectives of this study, such an analysis is best left for later studies.

### 7.3.2 Methodology

In the previous section, parametric analysis was introduced as a means of studying the behavior of the model while permuting elements of the cost and/or RHS vector over broad ranges. This section will briefly

introduce the methodology used in parametric analysis. Only changes in the RHS and cost vector will be considered.

#### Parametric Analysis of the Cost Vector:

Assume that the model has already been solved for an optimal objective function value,  $Z$ , and decision variable vector,  $x$ . The objective function is of the form,  $Z = \sum_j^n c_j x_j$ . We are interested in permuting the cost vector,  $c$ , in a direction  $\alpha$ , and by a magnitude  $\theta > 0$ . Thus, we are looking for a series of new optimal solutions to a modified objective function,

$$Z(\theta) = \sum_j^n (c_j + \alpha_j \theta) x_j \quad 7.1$$

where  $\alpha_j$  is the relative rate of change of the coefficients. Increasing  $\theta$  from zero will change the coefficients at these relative rates, but for the purposes of this study all  $\alpha_j$  will be integer valued (0,1) to enable us to examine subsets of the cost vector.

#### Parametric Analysis of the RHS:

Here elements of the RHS,  $b_i$ , are permuted as for the cost

vector. The  $b_i$  are replaced by  $(b_i + \alpha_i \theta)$ , for all  $i=1, \dots, m$  constraints. Again,  $\alpha$  will be integer in value allowing for the examination of specific constraints in the model. Specifically, for the generation of supply curves of the spot market in the three states. In this case the problem becomes,

$$\begin{aligned} & \text{Minimize } Z(\theta) = cx \\ & \text{subject to, } \sum_j^n a_{ij} x_j \geq b_i + \alpha_i \theta, \text{ for } i=1, \dots, m \quad 7.2 \\ & \qquad \qquad \qquad x_j \geq 0 \end{aligned}$$

It is of interest to trace the changes in the resulting optimal objectives,  $Z^*(\theta)$ , and decision variables,  $x_j$ .

#### 7.4 Supply and Demand Curves

The use and methodology of parametric programming was discussed in section 7.3. Here the actual results of the application of parametric programming will be presented in the form of supply and demand curves. Both the implications and assumptions of these curves will be considered.

#### 7.4.1 Assumptions and Curves

The linear program presented in Chapter 6 quantifies the supply/demand relationship between Central Appalachian producers and Virginia utilities for the last half of 1986. The optimal solution to this model yields a point solution. For this solution, there is only one demand level,  $x_{ij,t}$ , for a given price,  $c_{ij,t}$ . Permutation of the price level in the optimal solution yields a set of new basic solutions. In this way the initial optima can be manipulated in order to define supply and demand curves.

#### Actual Demand:

Actual spot sales from the three states were plotted in Figure 7.1 in the form of demand curves. The horizontal axis is the total tonnage purchased at or below the given price, i.e., the cumulative quantity which can be sold at that price. Spot sales are given as totals for each state, regardless of the rail carrier. Since there is only one purchaser, Virginia Power, in the three state market for coal, there can only be one demand curve. This single demand curve for Virginia Power is defined by the summation of the three state curves, and is included in Figure 7.1.

When using Figure 7.1 as a demand curve, some assumptions must be made. With the total demand from Virginia Power shared between the three states, the shape of the demand curve for quantities beyond actual total demand must be inferred. This is unfortunate since the most interesting question is that of finding out which price level will be necessary to capture an increased portion of the market. The use of the curve for Virginia requires that its slope is assumed to decline at the same rate. Since these curves are based on sales over a six month period, it must be assumed that the overall price level during that period was fairly stationary. Some final assumptions are that the spot market is purely competitive, that a preferred buyers list is not used, or even if it is, that bids are still judged simply by delivered price, and that price is not influenced by characteristics such as coal quality, which might vary between bidders, but which in reality is controlled by the quality specifications released to the bidders by the utility. The results of the market analysis of Chapter 4 support these assumptions.

If the actual Virginia Power demand curve is considered in its three state curve components, the following observations can be made.

There is little that can be said about Virginia's

cumulative demand curve due to its limited range. It should be pointed out that Virginia coal is not particularly overpriced when compared to West Virginia or Kentucky. Possibly, the problem is the lack of low priced coal, which is available in the other states. Both Kentucky's and West Virginia's curves are very similar. This is indicative of the strong competition between the two states. The first 100000 tons is available for between 39.50 and 43.50 \$/ton. Over this range the curve drops off steeply, reflecting Virginia Power's limited interest in high priced coal. The critical price seems to be about \$39.00/ton. Enormous gains in sales can be made below this price. Marginal gains in sales are made for prices below \$38.00/ton. In reality, this probably reflects the lowest cost producers. The full demand curve for spot sales to Virginia Power is the one that any Central Appalachian coal producer should refer to. This quantifies the actual demand for the region during this period. The area of particular interest is the flat sloped portion of the curve over the price range of about 38 to 39 \$/ton. In this region large gains in sales can be made. These are the low cost producers found in West Virginia and Kentucky, a portion of the market Virginia producers are having great difficulty competing with. One problem with using actual deliveries to establish a demand curve is that there are many sales made at different times

over a six month period. A true demand curve should be based on sales coming from a single market closing. Since this is not the case there are probably a number of coal sales made at excessively high or low prices which effectively exist outside of the true market. Some of the high cost deliveries may have been the result of preference or rush buying while some of the low cost sales might have been sold below cost to alleviate cash flow problems. Because of this, the flatter portion of the actual demand curve, less its extreme tails, is probably closer to the true demand.

#### Actual Supply:

By reversing the procedure used to generate the previously described actual demand curves (plotting the cumulative deliveries from the lowest to highest price paid) actual supply curves can be generated. These curves are given in Figures 7.2 - 7.4 for each state. Unlike the actual demand curve, separate supply curves can be made for each state. This is because the curves quantify sales from a particular supply region rather than demand from a single customer. Here an assumption must be made that the three states can be represented as significantly different spot sources. This was determined to be the case in Chapter 4. Evaluation of these curves will be left for the final

section of this chapter.

#### Derived Demand:

Demand curves based on actual sales are limited by the availability of data. This is illustrated by the curve for Virginia. An alternative is to use the model to generate demand curves. In this case there is no limitation on the range of the price, and the quantity can extend up to the maximum demand. The only assumptions are those inherent in the model. A major advantage of using the optimization model to generate demand curves is the ability to individually specify the demand from a specific source or subset of producers. In this way, demand curves were generated for the spot market in each state.

Figures 7.5 and 7.6 are the supply and demand curves, respectively, for Virginia spot sales. These were generated from the optimization model using parametric analysis. To generate the demand curve, the coefficients for Virginia spot sources ( $i=17, \dots, 19$ ) decreased by \$.20 intervals ( $\alpha=-1, \theta=.20$ ). As the price decreases by \$1.40/ton, Virginia coal enters the basis. Two vertical price scales are given for these curves. The Price Shift is the price change experienced by all spot sources in the state. This does not directly relate to a \$/ton value,

since there are a number of sources within each state having differing prices. At a price level, where Virginia sources are in the basis, the weighted average price which is associated with the Price Shift is given. For a given Price Shift, demand from each source is weighted by its percentage of total demand for the state. As a result, the weighted price is not evenly scaled. Rather, it serves as a reference.

In the model Virginia does not enter the spot market until the delivered price is reduced by \$1.40/ton. At this point Dickenson county begins shipping to Yorktown in 12/86 (via CSX). Virginia tonnage does increase significantly until the price is reduced by \$2.40/ton, and sales are also made to Possum Pint at \$38.60/ton. By the time that the spot price has gone down to \$37.84, Virginia will account for nearly 500,000 tons of Virginia Power purchases. There is no increase in sales beyond this point until the price is cut by \$4.20/ton, at which time Virginia captures the entire spot market.

#### Marginal Cost of Supply:

The supply curve is generated by including in the model a constraint which forces a minimum production from a

source, in this case all spot sources in a given state. The RHS is permuted upwards from zero. As the permuted RHS exceeds the optimal production in the original solution a marginal cost is incurred. This is the additional price resulting from forcing the utility to purchase coal from this source while less expensive sources are available. Since no marginal cost is incurred at a sales volume less than the optima, that portion of the supply curve below market equilibrium is not available by manipulating the model.

Figure 7.5 is the model generated supply curve for Virginia. The two vertical axes are the marginal cost and weighted average price/ton. Here the assumption is that this gives the quantity that Virginia sources are willing or able to produce at this price. Not until the price increases from the current average (an unweighted average of \$40.48/ton) by \$1.77/ton, does production begin to increase.

Figures 7.7 and 7.8 give the combined supply and demand curves for Kentucky and West Virginia as generated from the model. Note that these two states are part of the optimal basis. Because of this, the supply and demand curves intersect. Such is not the case for Virginia. Both states have fairly linear demand functions. For

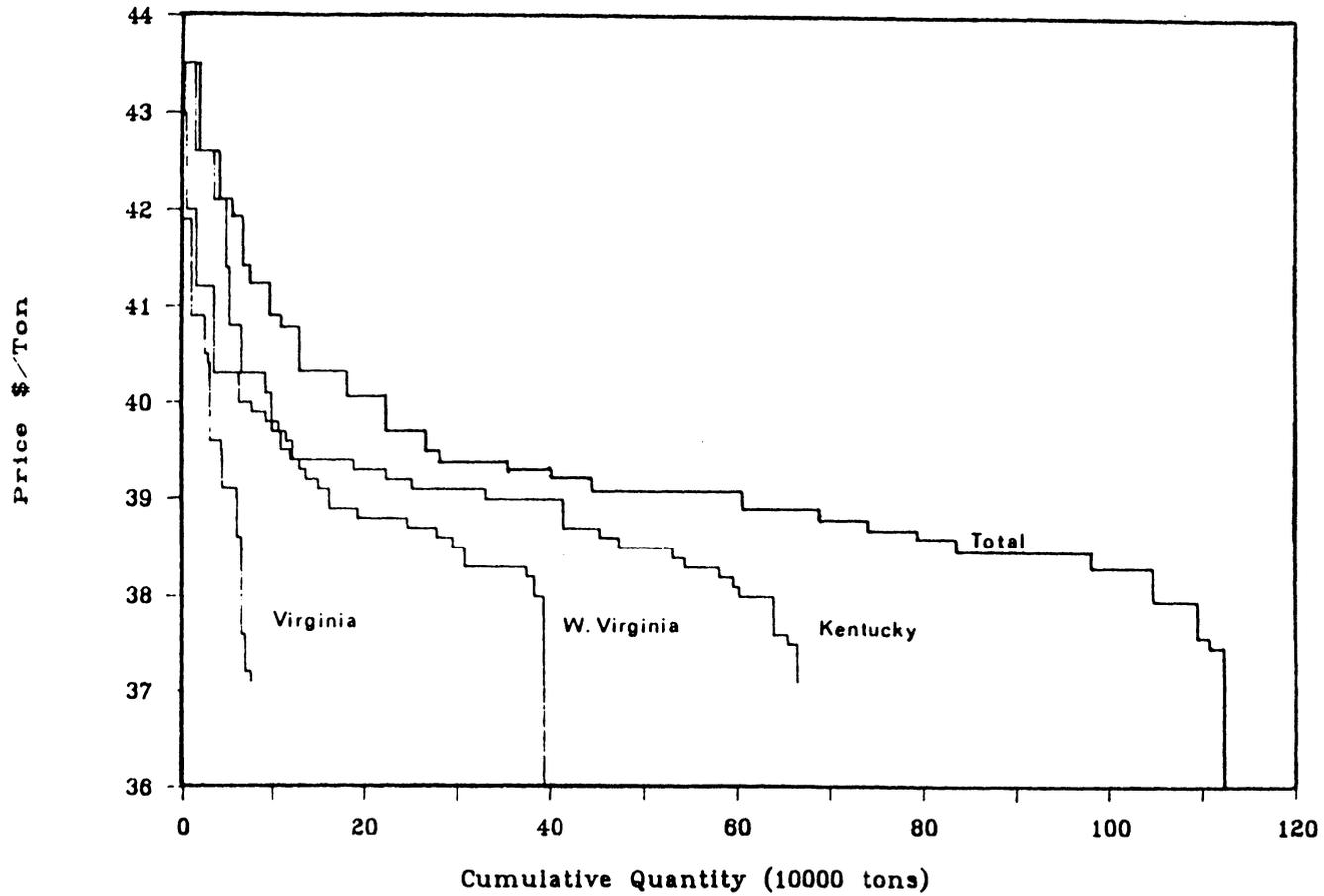
Kentucky, increased sales of about 100,000 tons can be obtained if prices are decreased very slightly from the equilibrium point. Note that the equilibrium point was determined in the initial optimal solution. Large gains in sales occur at price reductions of 1 and 3 dollars. At a shift of \$3/ton, Kentucky captures the spot market. In this model West Virginia already dominates most of the spot market. Thus, smaller price reductions are necessary to capture the remainder. Note that West Virginia can increase its overall price by as much as a dollar before suffering a serious loss in market share.

#### 7.4.2 Discussion of Results

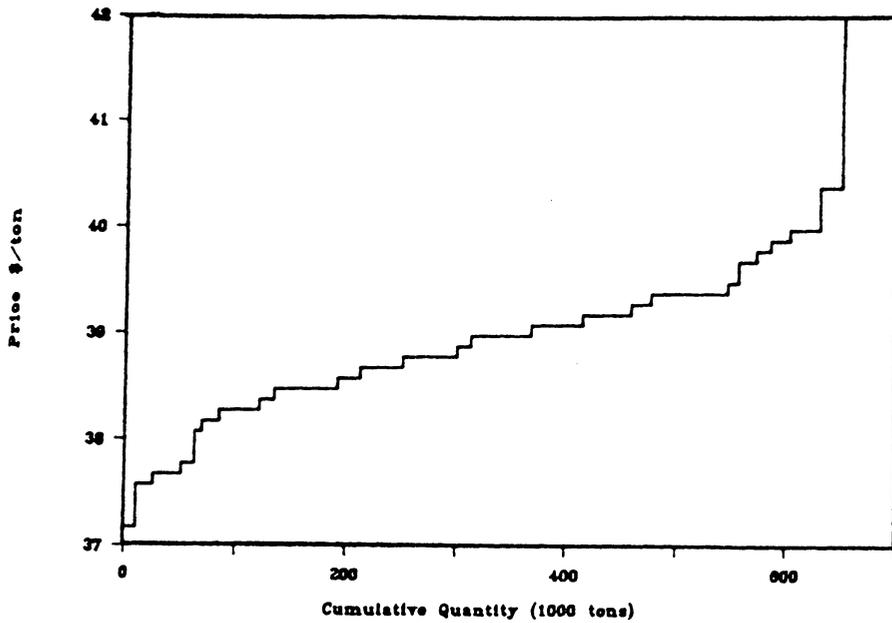
Supply and demand curves are used to concisely quantify markets. This is what has been attempted in this section, but there are several complications. For curves generated from data on actual deliveries, a number of assumptions had to be made and the range of the supply curves is limited to actual total sales volume. The demand curve is for all spot sales to Virginia Power rather than individual states. Model generated curves retain the assumptions used in the model itself. Also, model generated supply curves are incomplete showing only the upper portion of the curve. If the assumptions made for

both the model and the actual demand and supply curves are correct then the two sets of curves should complement each other. This would allow model generated and actual sales curves to be combined, yielding a complete specification of the spot market. There is one critical difference between the actual market and the model. The model finds an optimal solution for obtaining coal supply at the minimum cost, while in reality Virginia Power does not simply base coal purchases on minimizing total cost. Even if this were their stated policy, it would not be possible without prior knowledge of future coal prices. Therefore, actual sales will never match model results since the model will generate curves at a lower price level, but curves generated using Parametric Analysis do not presume any specific price level. Rather, the price levels in the optimal solution are permuted. This is why the model generated curves plot the price "shift" vs. quantity instead a specific price. Thus, the two sets of curves can be combined under the assumptions already made. Let's review two critical assumptions. For the actual supply curves, the top of the curve is the maximum price paid for coal from that state by Virginia Power. If this curve is the correct specification for the supply curve then the demand curve should intersect this supply curve at this point. Thus, the actual supply curve is only the lower portion of the full supply curve, which lies below the

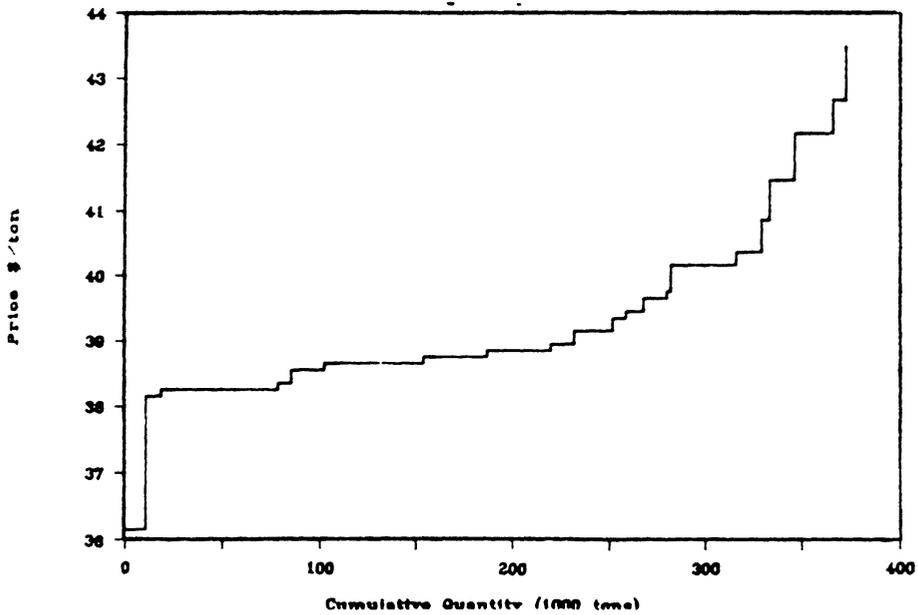
intersection of the supply and demand curves. The optimal solution to the model can be thought of as the market equilibrium. The model generated supply curve is based on the marginal cost incurred as a consequence of moving past market equilibrium along the supply curve. Thus the model generated supply curve is the top portion of the full supply curve lying above the intersection of the supply and demand curves. With this in mind the two curves can be thought of as two ends of the same supply curve intersected by that state's demand curve, which is obtained from the model. This situation is illustrated by Figure 7.9. The main question is where is the true location of the intersection of the generated demand and joint supply curves. Assuming the correctness of the upper tail of the actual supply curve, Figure 7.10 presents the complete supply and demand curves as set forth in Figure 7.9.



**Figure 7.1**  
**Actual Demand Curve**  
 Cumulative Deliveries to Virginia Power



**Figure 7.2: Cumulative Supply (Actual)**  
Kentucky Spot Sales



**Figure 7.3: Cumulative Supply (Actual)**  
West Virginia Spot Sales

# Cumulative Supply (Actual)

Virginia Spot Sales

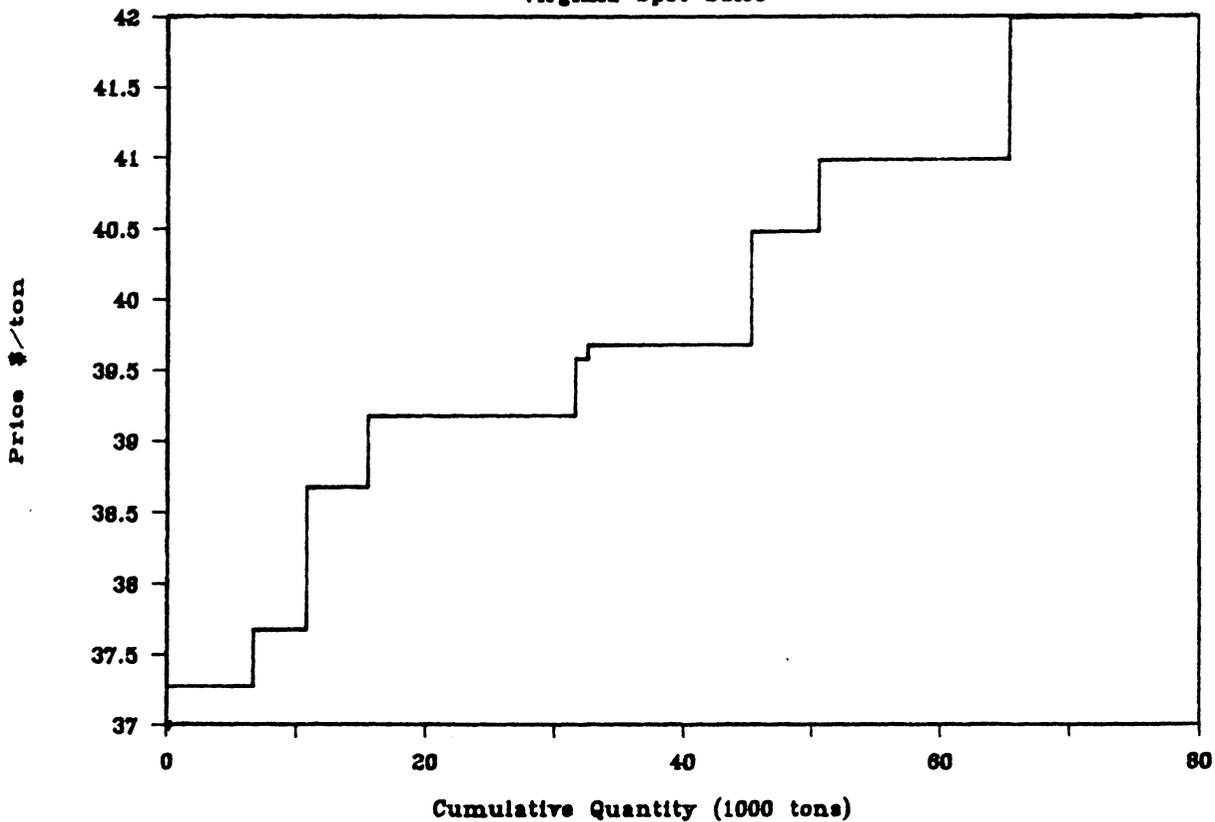


Figure 7.4

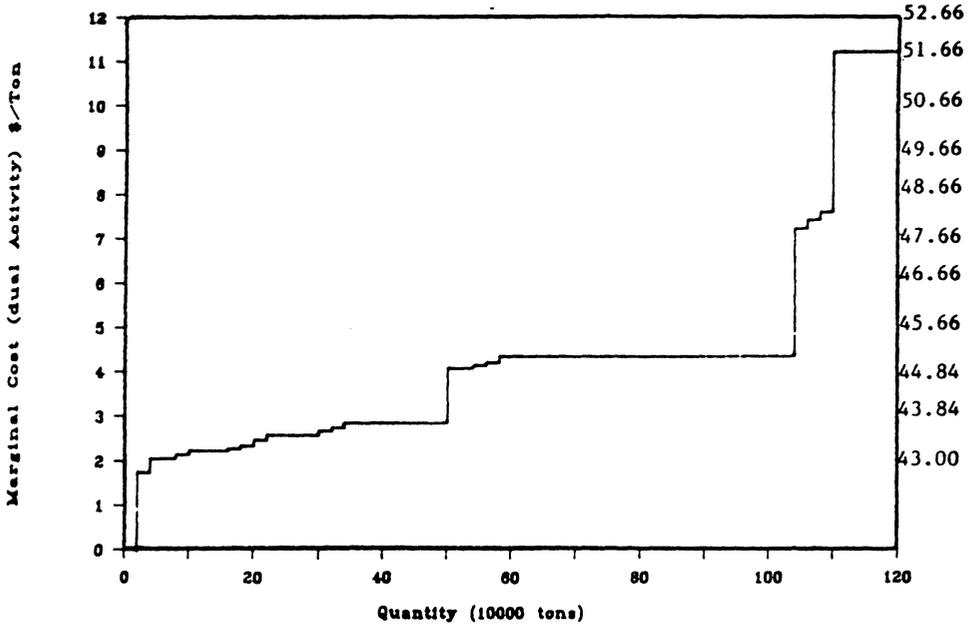


Figure 7.5 Marginal Cost (Supply) Curve  
Virginia Spot Sales

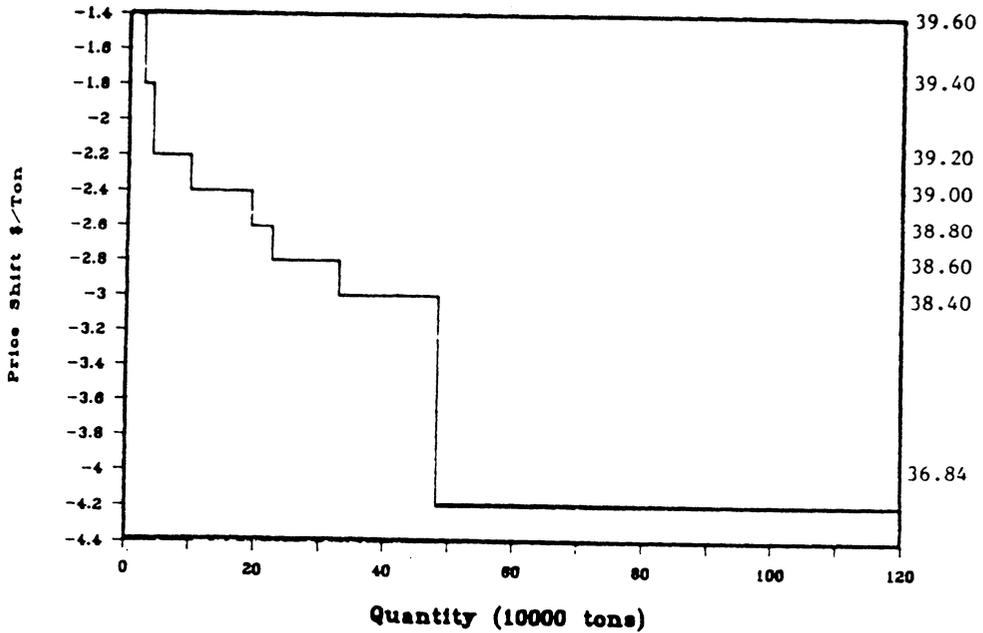
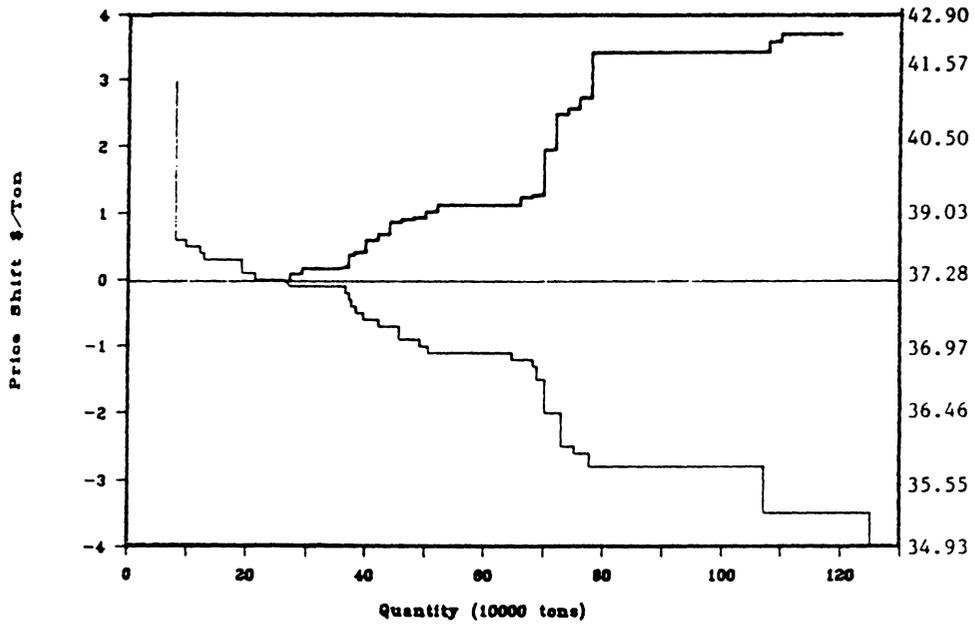
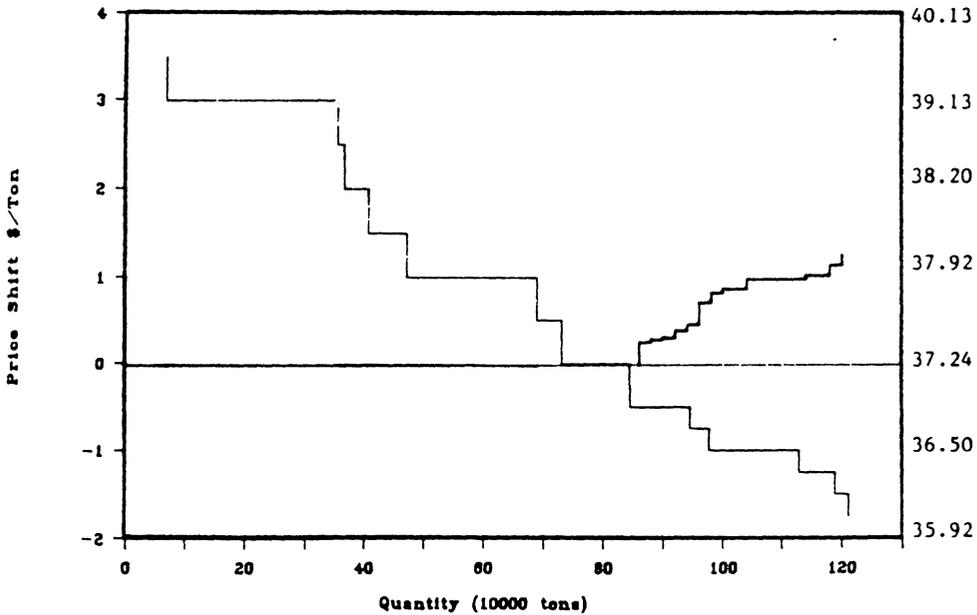


Figure 7.6 Demand Curve  
Virginia Spot Sales



**Figure 7.7: Supply & Demand Curves  
Kentucky Spot Sales**



**Figure 7.8 Supply & Demand Curves  
West Virginia Spot Sales**

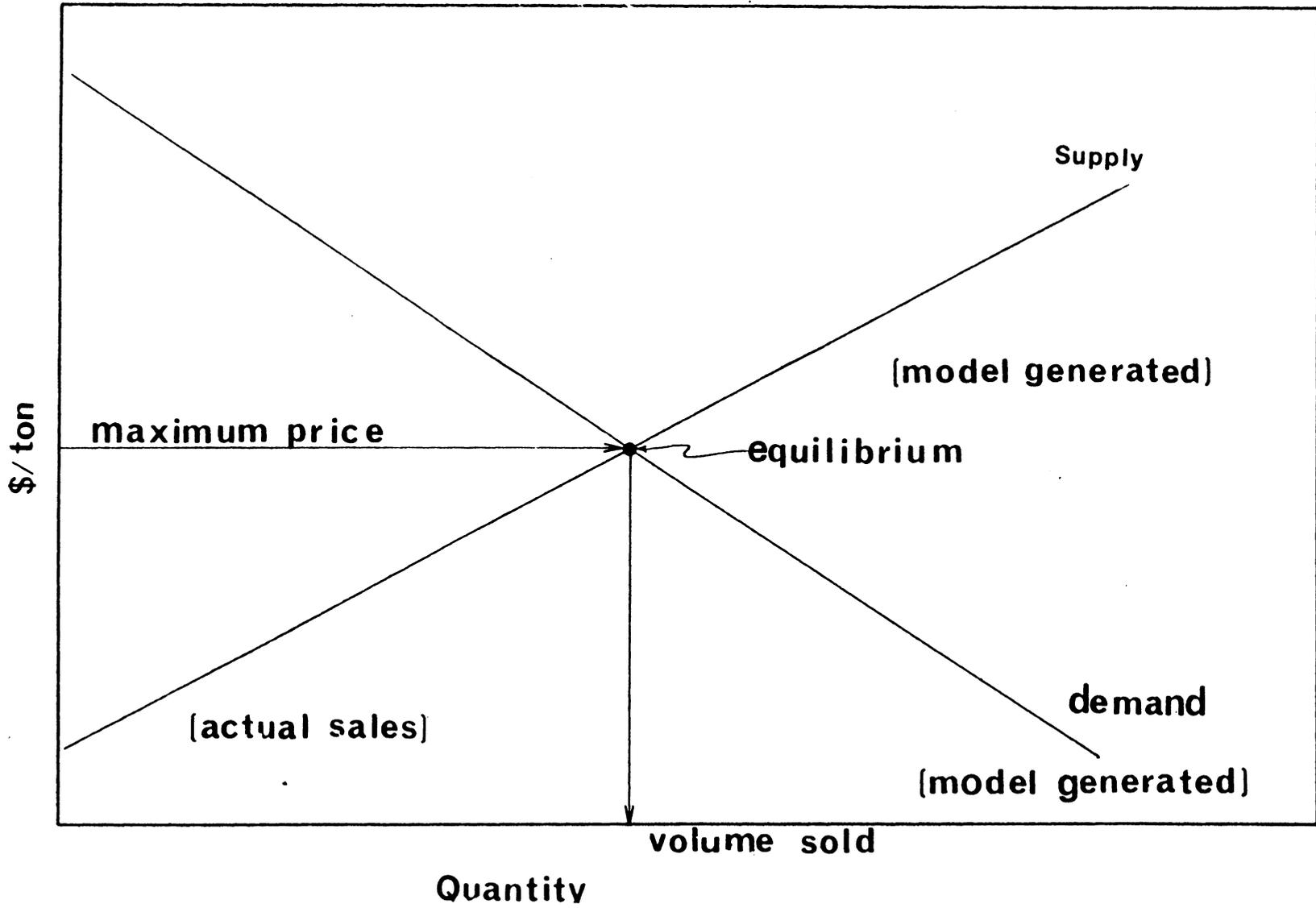


Figure 7.9: Key For Combining Actual & Model Generated Curves

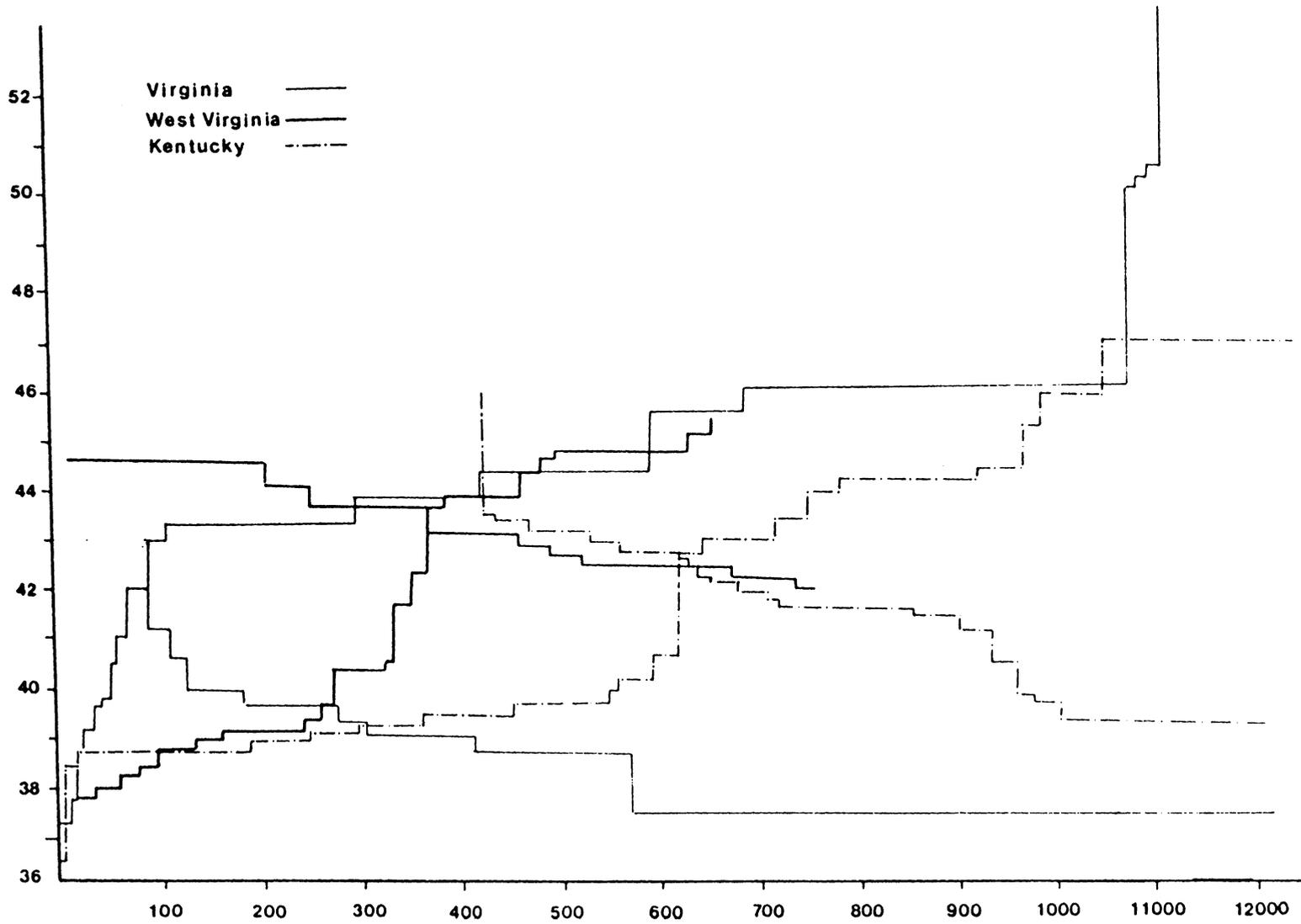


Figure 7.10: Combined Supply & Demand Curves

## CHAPTER 8

### Conclusions and Recommendations

#### 8.1 Perspective

Virginia's coal fields began large scale production with the advent of railroad transportation. The relationship between railroads and coal producers has remained essentially the same over the past hundred years of production: railroads rely on coal transportation for the bulk of their income, and coal producers are tied to rail transport as the only means of reaching distant markets. Early in this century, a number of regional railroad companies competed in southwest Virginia. These included L&N, CC&O, Southern, Virginian, C&O, N&W, and others. By 1980 this situation had changed dramatically. Mergers and takeovers resulted in two large corporations, CSX and NS. These two large railroad companies were left in positions of regional monopolies, so that very few coal producers or consumers had access to both carriers. With the passage in 1980 of the Staggers Act, railroad tariff regulation changed: railroads could

now set their own rates without public disclosure. Prior to deregulation, rates were fixed and a known quantity; utilities could seek the least cost supplier of coal, knowing in advance what their rail cost would be. This situation is now reversed. Today, utilities seek coal transportation contracts with the railroads, and then look for coal supply from mines able to deliver on that railroad. As a result, the majority of mines are effectively captive to their railroad. In the case of the majority of Virginia producers, this situation has had dire consequences.

Virginia's mining cost is higher than than that of eastern Kentucky and West Virginia. In addition, Virginia producers have limited access to the state's utilities. The majority of mines are located on NS, while only one of Virginia Power's coal burning power plants is located on NS. The remaining plants are located on CSX or RF&P (majority owned by CSX). Depending on the distance from NS tracks, Virginia producers work with a price disadvantage when trying to sell to Virginia Power. Due to the configuration of NS tracks, Virginia producers have a difficult time selling coal in the east and north. Most of Virginia's production goes to customers in Georgia and the Carolinas on Southern's tracks (NS).

Virginia's position as the high cost producer in Central Appalachia has resulted in loss of market share during this

period of overproduction and depressed prices. Loss of sales has caused severe unemployment in the coal producing counties of southwest Virginia. There seems to be little prospect of diversification of the economies of this part of the state. Since coal is likely to remain the major economic element of these counties, there has been much interest in improving the position of Virginia's coal producers. To this end, the state legislatures have considered various actions. In Virginia, there has been the passage of the Quillen Act, which provides for a tax rebate to buyers of Virginia coal. In West Virginia, legislation that would open trackage rights throughout the state is being examined. When considering the impact of this sort of legislation, what is lacking is a means of quantifying the regional coal market structure. This dissertation is the first time a methodology has been developed to quantify the Central Appalachian steam coal market.

## 8.2 Pioneering Aspects of Study

Past coal models concentrated on the national and international markets for coal. This included all end uses, i.e., steam generation, synfuels, industrial, chemicals, export, and steel. This study is regional in scope; this is the

first model of Virginia coal use. Major simplification has been achieved by recognizing the limited or diminishing importance of non-utility consumption in the state. Instead of defining markets by end use (steam vs. coke), markets are specified as they exist in reality. The major division is between producers on long term contract and those selling on the spot market. This critical difference has not been recognized in past models. Indeed, it is not possible to do so without using data on individual deliveries. A model with the greatest possible detail would individually specify mines. This would be hopelessly complex. By recognizing in the model the nature of the spot market, this complexity is reduced to a very manageable level. Spot sources were defined by state of origin and by destination following the results of a detailed market analysis.

Previous models used cost estimating functions to generate supply curves, typically for each coal type and classification of mine. The basic assumption was that mine price was largely based on mining cost, or that mine price could be statistically estimated for entire geographic regions based on variables which influence mining cost. Past studies, including this effort, showed this approach to be ineffective in Central Appalachia. Instead, this model adopts a new approach. Time series models are used to generate short-term forecasts of mine "price" based on data for

actual deliveries. By concentrating on mine price rather than cost estimates, the realities of market position and competition can be accounted for. By breaking delivered price down into its transportation and mining components, the impact of rail rates and railroad competition (and local monopoly) is taken into account. The inclusion of railroads in a post-Staggers Act environment is critically important.

Models of most consumption regions, such as Pennsylvania, must account for imports of coal from regions external to the model. For Virginia and the southern coastal states virtually all coal supply is sourced from Central Appalachia. This situation allows for a model with a closed system of supply.

### 8.3 The Steam Coal Market

Steam coal experienced increases in demand and price from 1983 until the middle of 1985. In mid '85 the market collapsed. This was most apparent in the highly competitive spot market. Virginia, as the high cost producer, lost much of its share of the spot market to Virginia Power. Virginia Power took advantage of overproduction to renegotiate or buy out old contracts, while buying less expensive spot coal from West Virginia and Kentucky.

Comparison of spot market sales and mines on contract showed that spot and contract sources had to be treated as separate markets. Mine price for contract sources tended to be higher than price on the spot market. This can be attributed to the oversupply of coal in this period. Contract mine price showed greater stationarity over time, with the price changing gradually until sharp reductions occurred as a result of renegotiations in late 1986. Mine price in the spot market has been far less stable. Deliveries from individual mines are far less frequent and less predictable as to quantity and price than for contract sources. A market analysis of spot deliveries had the following conclusions:

- Mine price has been declining but steady since late 1985,
- Price is significantly different between the three states with Virginia and Kentucky being the high and low price sources, respectively,
- Within a state, mine price varies inversely with rail rates when there is a significant difference between rail rates to differing destinations,
- Access to various markets depends upon the identity of the

carrier.

In consideration of these points it was determined that producers react to their expectation of the level of rail rates and how rates will affect competition. Recognizing that utilities look for the lowest delivered price, coal producers will lower their bid to compensate for higher rail rates in order to have a competitive delivered price.

The railroads have also been lowering their rates during this period of oversupply. CSX has pursued a policy of capturing a larger share of the coal freight market. They have been largely successful in this effort . In reaction to CSX competition, NS's rates have also dropped, but not before Virginia producers on NS lost their share of the spot market to Kentucky and West Virginia producers on CSX tracks.

#### 8.4 Forecasting

Short term forecasts of mine price were made using data on deliveries of coal compiled by the West Virginia Public Service Commission (HB966). The original intention was to combine this information on mine price and coal quality with data on production and employment related data published by

MSHA. The MSHA tapes could not be merged with HB966 since the specific origin of a delivery was usually not cross identifiable. Time series modeling was found to be the best means of modeling mine price for reasons other than the lack of descriptive variables. The nonstationary nature of mine price, collinearity, and lack of data placed severe limits on the application of regression in modeling mine price, especially in the spot market.

Time series, particularly ARIMA models, are well suited to contract data, but not to the spot market. Time series models require autocorrelation of the dependent variable (price) over time, stationarity, one observation per time period (month), and a sufficiently long series with no missing observations (deliveries every month). Deliveries from sources on contract were found to be well suited to these restrictions with some minor adjustments, but the spot market violates most of the requirements of a time series. Mine price from sources on the spot market was found to be highly nonstationary with a broad range of prices and multiple deliveries in any one month. The exception to this is for those spot regions which experienced gaps in shipments; for instance, the interruption of the Virginia spot market following the market collapse in 1985. A methodology was developed for forecasting spot mine price. The variation in mine price explained by cross-sectional data

(deliveries occurring in the same time period) and long term trends were incorporated into the model using a combined time trend/regression model. Short term fluctuation in mine price was accounted for by modeling the residual series of the trend/regression model as an ARIMA. In all cases the division of forecast regions was based on the results of the market analysis.

## 8.5 Modeling

Using forecasts of mine price and actual rail rates, a linear program was used to minimize the total cost of coal deliveries to APCO and Virginia Power plants in Virginia. A single objective function was sufficient to optimize deliveries for both utilities since neither share any of the same sources. The cost coefficients in the objective function were based on forecasts of mine price which had an associated confidence interval. This confidence interval was used in conjunction with a sensitivity analysis to identify sources for which the model would be particularly sensitive.

Parametric analysis was used to examine how the optimal solution would change as key parameters changed over a broad range. This technique was used to generate supply and

demand curves for the spot market in each state. These curves were based on the assumption of an ideal free market economy and optimal purchasing activities on the part of Virginia Power. Demand curves were generated by permuting the cost vector, and recording the change in demand as the price level from the region of interest changed. Supply curves were based on the marginal cost associated with constraining the purchasing activity of the utility to a level exceeding that found in the optimal solution. Since no marginal cost (dual activity) will be incurred until the RHS of this purchasing constraint passes the optimal solution, and since the optimal solution is equivalent to equilibrium point, then the lower half of the supply curve cannot be generated. Under the assumptions of a free market and a single clearing price, actual cumulative supply during the modeling period was used as the lower portion of the supply curve.

Model results show that West Virginia and Kentucky have a strong advantage over Virginia in the spot market. In both the model and reality, Virginia is cut out of the spot market. Given a drop in delivered price of between \$1-2/ton, sources on CSX in Virginia (Dickenson county) can expect to reenter the spot market. Virginia will start to be competitive with a price reduction of \$1.40/ton, and will began to rapidly increase sales volume to Virginia Power

when price reductions exceed \$2.20/ton. This is the situation for sources on CSX. The majority of the state's producers located on NS have dimmer prospects. The model is very insensitive to changes in delivered price for NS sources. It seems that a price reduction (in the model) of as much as \$4 or more might be necessary before NS sources in Virginia's spot market will enter the basis. In the contract market, Virginia's position is much better. Virginia producers on contract are the only contract sources in the three states who are able to compete with spot purchases. With renegotiations of contracts in late 1986 and early 1987, it is likely that long term contracts with Virginia suppliers will become even more attractive, especially from mines able to deliver on CSX or to Yorktown. Overall, NS was shown to be in a weak position in Virginia and Kentucky, but showed great potential for increasing its tonnage out of West Virginia as a result of very low spot mine price from that region in the end of 1986. This resulted in the main discrepancy between model results and actual sales. The model optimized purchasing for the entire six month period. With foreknowledge of the availability of very low cost coal in West Virginia in January 1986, the model purchased an enormous amount from this source. This made West Virginia the largest producer of coal for Virginia Power. In reality, it is unlikely that Virginia Power had sufficient advance notice of this sharp drop in price and

was not in a position to take advantage of it so late in the year.

In summary, the model indicates that Virginia Power does not appear to follow a strict policy of minimizing cost when procuring supply. Following an optimal coal purchasing policy would have the following results: Virginia would lose what little share of the spot market it currently holds, and NS would lose most of its tonnage in Virginia and Kentucky to CSX, but markedly improve its position in West Virginia as long as very low cost coal remained available. A new development in 1987 has been the passage of the Quillen Act. It is not yet known the effect of this act will be since the data for 1987 is just now becoming available, but the preliminary results of the dollar tax rebate seem to coincide with what the model suggests for the latter half of 1986. There has been an increase in contract shipments from Virginia sources; the majority of these have been on CSX sources in Wise and Dickenson counties. Model results support the belief that the competitive position of the coal producers is tied to that of their railroad.

## 8.7 Recommendations

The following are suggestions for extensions or improvements of this study which were not expedited either for lack of time or resources.

- Extension of model to cover entire market region:

This study restricted steam coal consumption to Virginia. From the point of view of Virginia consumers this model is adequate since Virginia receives all its coal from Central Appalachia. From the point of view of the producers the model is incomplete since Virginia is only a part of the market for Central Appalachia. Extension of the model to the north would be inadvisable. This would greatly complicate the model since supply from outside of Central Appalachia would have to be accounted for. The main market for Central Appalachia is in the southern coastal states. A model including this market would retain the same structure as the current model, supply from Central Appalachia being delivered by the same two railroad companies. A new data source similar to HB966 would have to be found in order to make this extension.

- Including intercoastal barging of coal:

The current model considers only rail transportation. Intercoastal barging was used for a limited time by

Virginia Power from NS docks on Lamberts Point to their Possum Point plant, which is located on CSX. This was done to encourage competition between the railroads. As a result of this, CSX sharply reduced its rates to Possum Point, and Virginia Power stopped barging coal. In light of this happening it would be instructive to allow for the barging of coal to all plants located on waterways. This would open up cross carrier traffic to Possum Point, Yorktown, and Portsmouth. If this model was extended to Florida, barging would have to be included.

- Optimal spot market forecasting procedure: Develop a statistical methodology capable of using both cross-sectional as well as time series data in forecasts. This is essential for modeling data, such as coal deliveries, which occur at irregular intervals and as a result often have multiple occurrences in the time period reported.
- Quillen Act: Rerun the model using actual data for 1987 in order to study the impact of the Quillen Act on sales of Virginia coal to Virginia Power.
- Clean Air Act: Virginia coal quality is the highest in Central Appalachia. Passage of the Clean Air Act might result in increased usage of Virginia coal. The model

could be used to study this by including coal quality constraints. This scenario would be particularly interesting in an enlarged model including the southeastern states.

## BIBLIOGRAPHY

- Bazaraa, M.S. and Jarvis, J.J. (1977), Linear Programming and Network Flows. Wiley.
- Box, G. and Hunter, J.S. (1978), Statistics for Experimenters. Wiley.
- Box and Jenkins (1976), Time Series Analysis: Forecasting and Control. Wiley.
- Chisolm, R.H., editor (1986), "1986 West Virginia Directory and Buyer's Guide,"Coal Mining Directories.
- Draper, N.R. and Smith, H. (1966), Applied Regression Analysis. Wiley.
- Dux, Christopher, et. al. (1977), Modeling Long Term Coal Production with the Argonne Coal Market Model. Argonne National Laboratories, CONF-771005-7.
- Gordon, R.L. (1977), Economic Analysis of Coal Supply: An Assessment of Existing Studies. EPRI.

- Hartman, Raymond S.(1979), "Frontiers in Energy Demand Modeling", Annual Review of Energy, Vol.4, pp.433-466.
- Hibbard, W., and Prelaz, L. (1986), "1986 Virginia Coal Mine Directory", VCCER, October.
- Hibbard, W. (1987), "An Abridged History of the Southwest Virginia Coal Industry", VCCER, July.
- Hibbard, W. (1987), "A Study of the Virginia Coal Industry", Materials and Society, Vol. 11, No. 1, pp.85-129.
- Hibbard, W. (1987), "Energy Scout", VCCER, Vol. 7, No. 2, March.
- Hibbard, W. (1987), "Impact of Virginia Utility Coal Incentive Act", VCCER.
- Knight, C.B. and Manula, Charles (1977), "The Pennnsylvania Coal Model", Executive Summary, NTIS, PB 283 966.
- Labys, W.C. and Yang, C.W. (1980), "A Quadratic Programming Model of the Applachian Steam Coal Market", Energy Economics, April.

Labys, W.C., Yang, C.W., and Liebenthal, A.M. (1979), "An Econometric Simulation Model of the U.S. Market for Steam Coal", Energy Economics, January.

Lescroart, J.E., editor (1986), 1986-1987 Fieldston Coal Transportation Manual. Fieldston Company Inc.

Myers, R.H. (1986), Classical and Modern Regression with Applications. Wiley.

National Coal Association (1986), Electric Utility Coal Stockpiles.

National Coal Association (1986), Steam Electric Plant Factors.

Nielsen, G.F., editor (1985), 1985 Keystone Coal Industry Manual. McGraw-Hill.

Norfolk and Western (1977), "List of Stations and Sidings and Other Related Information", Accounting Department.

NUS Corporation (1977), "Underground Mines" and "Surface Mines", Vols. 1 and 2 in Coal Mining Cost Models. Prepared for EPRI, EPRI EA-437.

- Pindyck, R.S. and Rubinfeld, D.L. (1981), Econometric Models and Economic Forecasts. McGraw-Hill.
- SAS Institute Inc. (1984), SAS/ETS User's Guide: Version 5 Edition. SAS Institute Inc., Cary, N.C.
- SAS Institute Inc. (1985), SAS User's Guide: Basics, Version 5 Edition. SAS Institute Inc., Cary, N.C.
- SAS Institute Inc. (1985), SAS User's Guide: Statistics, Version 5 Edition. SAS Institute Inc., Cary, N.C.
- Schrage, L. (1986), Linear, Integer, and Quadratic Programming with LINDO. Scientific Press.
- Science Applications Inc. (1983), Coal Supply and Transportation Model: Model Description and Data Documentation.
- Shapiro, J.F. and White, D. (1980), "Decomposition and Integration of Coal Supply and Demand Models", MIT Energy Lab., Working paper, MIT-EL 80-029WP.
- Soyster, A.L. and Ensore, E.E. (1984), "A Short-term, Econometrically Based Coal Supply Model", Materials and Society, Vol. 8, No. 3, pp.567-587.

- Stanley, W. (1985), Annual Report. Kentucky Department of Mines and Minerals, Lexington, Kentucky.
- Tobin, R.L. (1983), The Florida Statewide Coal Conversion Study: Coal Supply and Transportation Analysis. Argonne National Laboratories.
- Yarbroff, I.W. and Dickenson, E.M. (1980), Coal Resources Model: An Impact Analysis Tool. Final Report, SRI International.
- Zimmerman, M.B. (1978), "Estimating a Policy Model of U.S. Coal Supply", Materials and Society, Vol.2, p. 67.
- Zimmerman, M.B. (1975), Long-Run Mineral Supply: The Case of Coal in the U.S. PhD Dissertation, Dartmouth College.
- Zimmerman, M.B. (1975), "Modeling Depletion in a Mineral Industry: The Case of Coal", Bell Journal of Economics, Vol. 8, pp.41-65.
- Zimmerman, M.B. (1975), The Supply of Coal in the Long-Run: The Case of Eastern Deep Coal. MIT Press.
- Zimmerman, M.B. (1981), The U.S. Coal Industry: The Economics of Policy Choice. MIT Press.

Zuercher, R. and Buist, H., editors (1985), Guide to Coal Contracts. Coal Outlook.

## APPENDIX A

### Virginia Coal Purchasing Model: LINDO Format Input

MIN 47.38X121+42.58X131+49.40X141+47.33X122+42.53X132+49.67X142+  
47.29X123 +42.49X133+49.31X143+47.22X124+42.44X134+49.24X144+47.18X125+  
42.43X135 +49.20X145+47.14X126+42.39X136+49.16X146+51.06X221+46.16X231+  
49.11X222 +44.31X232+51.18X223+46.40X233+51.24X224+46.46X234+51.29X225+  
46.51X235 +51.23X226+46.46X236+47.81X321+45.36X331+50.34X322+45.45X332+  
50.42X323 +45.61X333+50.48X324+45.71X334+50.54X325+45.79X335+50.61X326+  
45.86X336 +46.05X421+43.68X431+48.59X422+43.79X432+48.70X423+43.90X433+  
48.81X424 +44.01X434+48.82X425+44.02X435+48.90X426+44.10X436+49.18X541+  
48.52X551 +49.18X542+48.66X552+49.18X543+48.78X553+49.18X544+48.89X554+  
49.18X545 +49.00X555+49.18X546+49.18X556+45.81X621+43.45X631+48.20X622+  
43.40X632 +45.15X623+43.35X633+48.10X624+43.33X634+48.06X625+43.31X635+  
48.02X626 +43.27X636+45.69X721+46.42X741+45.69X722+46.42X742+45.69X723+  
46.42X743 +45.69X724+46.42X744+45.69X725+46.48X745+46.44X726+46.44X746+  
42.49X831+42.49X832+42.49X833+42.49X834+42.49X835+42.49X836+46.23X941+  
46.23X942+46.25X943+45.96X944+45.70X945+45.70X946+43.85X1011+44.48X1021  
+45.17X1041+43.96X1012+44.35X1022+45.04X1042+43.75X1013+34.14X1023  
+44.89X1043+43.13X1014+43.55X1024+44.24X1044+42.92X1015+43.34X1025  
+44.03X1045  
+42.92X1016+43.34X1026+44.03X1046+39.28X1111+39.67X1121+40.42X1141  
+39.28X1112+39.67X1122+40.42X1142+42.28X1113+42.67X1123+43.42X1143  
+42.34X1114+42.67X1124+43.42X1144+42.38X1115+42.73X1125+43.42X1145  
+42.48X1116+42.73X1126+43.42X1146+38.13X1221+38.13X1222+37.93X1223  
+37.41X1224+37.09X1225+37.09X1226+43.99X1341+44.10X1351+43.99X1342  
+44.23X1352+43.99X1343+44.23X1353+43.99X1344+44.26X1354+44.33X1345  
+44.29X1355+44.33X1346+44.29X1356+42.64X1431+42.64X1432+42.64X1433  
+42.14X1434+42.14X1435+42.14X1436+44.84X1561+44.94X1562+45.08X1563  
+45.11X1564+45.08X1565+45.06X1566+48.85X1671+47.38X1672+46.27X1673  
+46.07X1674+45.94X1675+41.10X1676+41.10X1741+41.10X1751+41.07X1742  
+41.00X1752+41.00X1743+41.10X1753+41.10X1744+41.10X1754+41.00X1745  
+41.00X1755+41.00X1746+41.00X1756+40.50X1821+40.50X1822+40.50X1823  
+40.50X1824+40.50X1825+40.50X1826+42.88X1921+38.08X1931+42.67X1922  
+37.87X1932+42.44X1923+37.64X1933+42.21X1924+37.43X1934+41.96X1925  
+37.18X1935+41.71X1926+36.93X1936+37.51X2031+36.84X2032+36.43X2033  
+36.29X2034+35.93X2035+35.40X2036+37.64X2111+37.60X2121+38.33X2141

+38.29X2151+38.58X2112+37.69X2122+33.33X2142+37.87X2152+39.35X2113  
 +38.08X2123+38.90X2143+38.64X2153+40.06X2114+38.64X2124+40.41X2144  
 +39.41X2154+40.65X2115+39.55X2125+41.54X2145+40.00X2155+41.17X2116  
 +39.83X2126+40.56X2146+40.52X2156+39.32X2221+39.32X2231+43.45X2222  
 +39.25X2232+43.38X2223+39.18X2233+43.29X2224+38.52X2234+43.20X2225  
 +38.45X2235+43.11X2226+38.36X2236+39.34X2341+39.51X2342+38.71X2343  
 +38.86X2344+38.97X2345+38.75X2346+38.08X2411+38.59X2421+38.81X2451  
 +38.89X2412+38.83X2422+39.13X2452+39.09X2413+39.05X2423+39.35X2453  
 +38.89X2412+38.83X2422+39.13X2452+39.09X2413+39.05X2423+39.35X2453  
 +38.92X2414+39.07X2424+39.51X2454+39.03X2415+39.30X2425+39.62X2455  
 +39.11X2416+39.53X2426+39.70X2456+42.39X2521+42.39X2522+43.50X2523  
 +42.47X2524+41.45X2525+40.45X2526

## SUBJECT TO

$$X121+X131+X141 \leq 61496$$

$$X122+X132+X142-2X121-2X131-2X141 \leq 0$$

$$X123+X133+X143-2X122-2X132-2X142 \leq 0$$

$$X124+X134+X144-2X123-2X133-2X143 \leq 0$$

$$X125+X135+X145-2X124-2X134-2X144 \leq 0$$

$$X126+X136+X146-2X125-2X135-2X145 \leq 0$$

$$X221+X231 \leq 34996$$

$$X222+X232-4.5X221-4.5X231 \leq 0$$

$$X223+X233-4.5X222-4.5X232 \leq 0$$

$$X224+X234-4.5X223-4.5X233 \leq 0$$

$$X225+X235-4.5X224-4.5X234 \leq 0$$

$$X226+X236-4.5X225-4.5X235 \leq 0$$

$$X321+X331 \leq 43628$$

$$X322+X332-1.72X321-1.72X331 \leq 0$$

$$X323+X333-1.72X322-1.72X332 \leq 0$$

$$X324+X334-1.72X323-1.72X333 \leq 0$$

$$X325+X335-1.72X324-1.72X334 \leq 0$$

$$X326+X336-1.72X325-1.72X335 \leq 0$$

$$X421+X431 \leq 26777$$

$$X422+X432-1.95X421-1.95X431 \leq 0$$

$X423+X433-1.95X422-1.95X432 \leq 0$   
 $X424+X434-1.95X423-1.95X433 \leq 0$   
 $X425+X435-1.95X424-1.95X434 \leq 0$   
 $X426+X436-1.95X425-1.95X435 \leq 0$   
 $X541+X551 \leq 24564$   
 $X542+X552-2.34X541-2.34X551 \leq 0$   
 $X543+X553-2.34X542-2.34X552 \leq 0$   
 $X544+X554-2.34X543-2.34X553 \leq 0$   
 $X545+X555-2.34X544-2.34X554 \leq 0$   
 $X546+X556-2.34X545-2.34X555 \leq 0$   
 $X621+X631 \leq 41937$   
 $X622+X632-1.5X621-1.5X631 \leq 0$   
 $X623+X633-1.5X622-1.5X632 \leq 0$   
 $X624+X634-1.5X623-1.5X633 \leq 0$   
 $X625+X635-1.5X624-1.5X634 \leq 0$   
 $X626+X636-1.5X625-1.5X635 \leq 0$   
 $X721+X741 \leq 38670$   
 $X722+X742-1.5X721-1.5X741 \leq 0$   
 $X723+X743-1.5X722-1.5X742 \leq 0$   
 $X724+X744-1.5X723-1.5X743 \leq 0$   
 $X725+X745-1.5X724-1.5X744 \leq 0$   
 $X726+X746-1.5X725-1.5X745 \leq 0$   
 $X832-1.71X831 \leq 0$   
 $X833-1.71X832 \leq 0$   
 $X834-1.71X833 \leq 0$   
 $X835-1.71X834 \leq 0$   
 $X836-1.71X835 \leq 0$   
 $X941 \leq 28150$   
 $X942-4X941 \leq 0$   
 $X943-4X942 \leq 0$   
 $X944-4X943 \leq 0$   
 $X945-4X944 \leq 0$   
 $X946-4X945 \leq 0$   
 $X1011+X1021+X1041 \leq 28245$

$X1012+X1022+X1042-1.58X1011-1.58X1021-1.58X1041\leq 0$   
 $X1013+X1023+X1043-1.58X1012-1.58X1022-1.58X1042\leq 0$   
 $X1014+X1024+X1044-1.58X1013-1.58X1023-1.58X1043\leq 0$   
 $X1015+X1025+X1045-1.58X1014-1.58X1024-1.58X1044\leq 0$   
 $X1016+X1026+X1046-1.58X1015-1.58X1025-1.58X1045\leq 0$   
 $X1111+X1121+X1141\leq 26712$   
 $X1112+X1122+X1142-2X1111-2X1121-2X1141\leq 0$   
 $X1113+X1123+X1143-2X1112-2X1122-2X1142\leq 0$   
 $X1114+X1124+X1144-2X1113-2X1123-2X1143\leq 0$   
 $X1115+X1125+X1145-2X1114-2X1124-2X1144\leq 0$   
 $X1116+X1126+X1146-2X1115-2X1125-2X1145\leq 0$   
 $X1221\leq 69754$   
 $X1222-2X1221\leq 0$   
 $X1223-2X1222\leq 0$   
 $X1224-2X1223\leq 0$   
 $X1225-2X1224\leq 0$   
 $X1226-2X1225\leq 0$   
 $X1341\leq 40900$   
 $X1342+X1352-2X1341-2X1351\leq 0$   
 $X1343+X1353-2X1342-2X1352\leq 0$   
 $X1344+X1354-2X1343-2X1353\leq 0$   
 $X1345+X1355-2X1344-2X1354\leq 0$   
 $X1346+X1356-2X1345-2X1355\leq 0$   
 $X1431\leq 34142$   
 $X1432-1.66X1431\leq 0$   
 $X1433-1.66X1432\leq 0$   
 $X1434-1.66X1433\leq 0$   
 $X1435-1.66X1434\leq 0$   
 $X1436-1.66X1435\leq 0$   
 $X1562-2X1561\leq 0$   
 $X1563-2X1562\leq 0$   
 $X1564-2X1563\leq 0$   
 $X1565-2X1564\leq 0$   
 $X1566-2X1565\leq 0$

$$X1672 - 2X1671 \leq 0$$

$$X1673 - 2X1672 \leq 0$$

$$X1674 - 2X1673 \leq 0$$

$$X1675 - 2X1674 \leq 0$$

$$X1676 - 2X1675 \leq 0$$

$$X121 + X131 + X141 \geq 15374$$

$$X122 + X132 + X142 - .5X121 - .5X131 - .5X141 \geq 0$$

$$X123 + X133 + X143 - .5X122 - .5X132 - .5X142 \geq 0$$

$$X124 + X134 + X144 - .5X123 - .5X133 - .5X143 \geq 0$$

$$X125 + X135 + X145 - .5X124 - .5X134 - .5X144 \geq 0$$

$$X126 + X136 + X146 - .5X125 - .5X135 - .5X145 \geq 0$$

$$X221 + X231 \geq 3675$$

$$X222 + X232 - .21X221 - .21X231 \geq 0$$

$$X223 + X233 - .21X222 - .21X232 \geq 0$$

$$X224 + X234 - .21X223 - .21X233 \geq 0$$

$$X225 + X235 - .21X224 - .21X234 \geq 0$$

$$X226 + X236 - .21X225 - .21X235 \geq 0$$

$$X321 + X331 \geq 17820$$

$$X322 + X332 - .58X321 - .58X331 \geq 0$$

$$X323 + X333 - .58X322 - .58X332 \geq 0$$

$$X324 + X334 - .58X323 - .58X333 \geq 0$$

$$X325 + X335 - .58X324 - .58X334 \geq 0$$

$$X326 + X336 - .58X325 - .58X335 \geq 0$$

$$X422 + X432 \geq 7000$$

$$X422 + X432 - .51X421 - .51X431 \geq 0$$

$$X423 + X433 - .51X422 - .51X432 \geq 0$$

$$X424 + X434 - .51X423 - .51X433 \geq 0$$

$$X425 + X435 - .51X424 - .51X434 \geq 0$$

$$X426 + X436 - .51X425 - .51X435 \geq 0$$

$$X541 + X551 \geq 5281$$

$$X542 + X552 - .43X541 - .43X551 \geq 0$$

$$X543 + X553 - .43X542 - .43X552 \geq 0$$

$$X544 + X554 - .43X543 - .43X553 \geq 0$$

$$X545 + X555 - .43X544 - .43X554 \geq 0$$

$$X546+X556-.43X545-.43X555>=0$$

$$X621+X631>=19011$$

$$X622+X632-.68X621-.68X631>=0$$

$$X623+X633-.68X622-.68X632>=0$$

$$X624+X634-.68X623-.68X633>=0$$

$$X625+X635-.68X624-.68X634>=0$$

$$X626+X636-.68X625-.68X635>=0$$

$$X721+X741>=17530$$

$$X722+X742-.68X721-.68X741>=0$$

$$X723+X743-.68X722-.68X742>=0$$

$$X724+X744-.68X723-.68X743>=0$$

$$X725+X745-.68X724-.68X744>=0$$

$$X726+X746-.68X725-.68X745>=0$$

$$X832-.58X831>=0$$

$$X833-.58X832>=0$$

$$X834-.58X833>=0$$

$$X835-.58X834>=0$$

$$X836-.58X835>=0$$

$$X941>=4669$$

$$X942-.25X941 >=0$$

$$X943-.25X942 >=0$$

$$X944-.25X943 >=0$$

$$X945-.25X944 >=0$$

$$X946-.25X945 >=0$$

$$X1011+X1021+X1041>=11863$$

$$X1012+X1022+X1042-.63X1011-.63X1021-.63X1041>=0$$

$$X1013+X1023+X1043-.63X1012-.63X1022-.63X1042>=0$$

$$X1014+X1024+X1044-.63X1013-.63X1023-.63X1043>=0$$

$$X1015+X1025+X1045-.63X1014-.63X1024-.63X1044>=0$$

$$X1016+X1026+X1046-.63X1015-.63X1025-.63X1045>=0$$

$$X1111+X1121+X1141>=6544$$

$$X1112+X1122+X1142-.5X1111-.5X1121-.5X1141>=0$$

$$X1113+X1123+X1143-.5X1112-.5X1122-.5X1142>=0$$

$$X1114+X1124+X1144-.5X1113-.5X1123-.5X1143>=0$$

$$X1115+X1125+X1145-.5X1114-.5X1124-.5X1144 \geq 0$$

$$X1116+X1126+X1146-.5X1115-.5X1125-.5X1145 \geq 0$$

$$X1221 \geq 17439$$

$$X1222-.5X1221 \geq 0$$

$$X1223-.5X1222 \geq 0$$

$$X1224-.5X1223 \geq 0$$

$$X1225-.5X1224 \geq 0$$

$$X1226-.5X1225 \geq 0$$

$$X1341 \geq 10225$$

$$X1342+X1352-.5X1341-.5X1351 \geq 0$$

$$X1343+X1353-.5X1342-.5X1352 \geq 0$$

$$X1344+X1354-.5X1343-.5X1353 \geq 0$$

$$X1345+X1355-.5X1344-.5X1354 \geq 0$$

$$X1346+X1356-.5X1345-.5X1355 \geq 0$$

$$X1431 \geq 10243$$

$$X1432-.6X1431 \geq 0$$

$$X1433-.6X1432 \geq 0$$

$$X1434-.6X1433 \geq 0$$

$$X1435-.6X1434 \geq 0$$

$$X1436-.6X1435 \geq 0$$

$$X1562-.5X1561 \geq 0$$

$$X1563-.5X1562 \geq 0$$

$$X1564-.5X1563 \geq 0$$

$$X1565-.5X1564 \geq 0$$

$$X1566-.5X1565 \geq 0$$

$$X1672-.5X1671 \geq 0$$

$$X1673-.5X1672 \geq 0$$

$$X1674-.5X1673 \geq 0$$

$$X1675-.5X1674 \geq 0$$

$$X1676-.5X1675 \geq 0$$

$$+.6X121+.6X131+.6X141+.6X122+.6X132+.6X142+.6X123$$

$$+.6X133+.6X143+.6X124+.6X134+.6X144$$

$$+.6X125+.6X135+.6X145+.6X126+.6X136+.6X146+.6X221$$

$$+.6X231+.6X222+.6X232+.6X223+.6X233$$

+.6x224+.6x234+.6x225+.6x235+.6x226+.6x236+.6x321  
 +.6x331+.6x322+.6x332+.6x323+.6x333  
 +.6x324+.6x334+.6x325+.6x335+.6x326+.6x336+.6x421  
 +.6x431+.6x422+.6x432+.6x423+.6x433  
 +.6x424+.6x434+.6x425+.6x435+.6x426+.6x436+.6x541  
 +.6x551+.6x542+.6x552+.6x543+.6x553  
 +.6x544+.6x554+.6x545+.6x555+.6x546+.6x556+.6x621  
 +.6x631+.6x622+.6x632+.6x623+.6x633  
 +.6x624+.6x634+.6x625+.6x635+.6x626+.6x636+.6x721  
 +.6x741+.6x722+.6x742+.6x723+.6x743  
 +.6x724+.6x744+.6x725+.6x745+.6x726+.6x746+.6x831  
 +.6x832+.6x833+.6x834+.6x835+.6x836  
 +.6x941+.6x942+.6x943+.6x944+.6x945+.6x946+.6x1011  
 +.6x1021+.6x1041+.6x1012+.6x1022+.6x1042  
 +.6x1013+.6x1023+.6x1043+.6x1014+.6x1024+.6x1044  
 +.6x1015+.6x1025+.6x1045+.6x1016+.6x1026+.6x1046  
 +.6x1111+.6x1121+.6x1141+.6x1112+.6x1122+.6x1142  
 +.6x1113+.6x1123+.6x1143+.6x1114+.6x1124+.6x1144  
 +.6x1115+.6x1125+.6x1145+.6x1116+.6x1126+.6x1146  
 +.6x1221+.6x1222+.6x1223+.6x1224+.6x1225+.6x1226  
 +.6x1341+.6x1351+.6x1342+.6x1352+.6x1343+.6x1353  
 +.6x1344+.6x1354+.6x1345+.6x1355+.6x1346+.6x1356  
 +.6x1431+.6x1432+.6x1433+.6x1434+.6x1435+.6x1436  
 -.4x1741-.4x1751-.4x1742-.4x1752-.4x1743-.4x1753  
 -.4x1744-.4x1754-.4x1745-.4x1755-.4x1746-.4x1756  
 -.4x1821-.4x1822-.4x1823-.4x1824-.4x1825-.4x1826  
 -.4x1921-.4x1931-.4x1922-.4x1932-.4x1923-.4x1933  
 -.4x1924-.4x1934-.4x1925-.4x1935-.4x1926-.4x1936  
 -.4x2031-.4x2032-.4x2033-.4x2034-.4x2035-.4x2036  
 -.4x2111-.4x2121-.4x2141-.4x2151-.4x2112-.4x2122  
 -.4x2142-.4x2152-.4x2113-.4x2123-.4x2143-.4x2153  
 -.4x2114-.4x2124-.4x2144-.4x2154-.4x2115-.4x2125  
 -.4x2145-.4x2155-.4x2116-.4x2126-.4x2146-.4x2156  
 -.4x2221-.4x2231-.4x2222-.4x2232-.4x2223-.4x2233

-.4x2224-.4x2234-.4x2225-.4x2235-.4x2226-.4x2236  
 -.4x2341-.4x2342-.4x2343-.4x2344-.4x2345-.4x2346  
 -.4x2411-.4x2421-.4x2451-.4x2412-.4x2422-.4x2452  
 -.4x2413-.4x2423-.4x2453-.4x2414-.4x2424-.4x2454  
 -.4x2415-.4x2425-.4x2455-.4x2416-.4x2426-.4x2456  
 -.4X2521-.4X2522-.4X2523-.4X2524-.4X2525-.4X2526>=0  
 +.4X121+.4X131+.4X141+.4X122+.4X132+.4X142+.4X123  
 +.4X133+.4X143+.4X124+.4X134+.4X144  
 +.4x125+.4x135+.4x145+.4x126+.4x136+.4x146+.4x221  
 +.4x231+.4x222+.4x232+.4x223+.4x233  
 +.4x224+.4x234+.4x225+.4x235+.4x226+.4x236+.4x321  
 +.4x331+.4x322+.4x332+.4x323+.4x333  
 +.4x324+.4x334+.4x325+.4x335+.4x326+.4x336+.4x421  
 +.4x431+.4x422+.4x432+.4x423+.4x433  
 +.4x424+.4x434+.4x425+.4x435+.4x426+.4x436+.4x541  
 +.4x551+.4x542+.4x552+.4x543+.4x553  
 +.4x544+.4x554+.4x545+.4x555+.4x546+.4x556+.4x621  
 +.4x631+.4x622+.4x632+.4x623+.4x633  
 +.4x624+.4x634+.4x625+.4x635+.4x626+.4x636+.4x721  
 +.4x741+.4x722+.4x742+.4x723+.4x743  
 +.4x724+.4x744+.4x725+.4x745+.4x726+.4x746+.4x831  
 +.4x832+.4x833+.4x834+.4x835+.4x836  
 +.4x941+.4x942+.4x943+.4x944+.4x945+.4x946+.4x1011  
 +.4x1021+.4x1041+.4x1012+.4x1022+.4x1042  
 +.4x1013+.4x1023+.4x1043+.4x1014+.4x1024+.4x1044  
 +.4x1015+.4x1025+.4x1045+.4x1016+.4x1026+.4x1046  
 +.4x1111+.4x1121+.4x1141+.4x1112+.4x1122+.4x1142  
 +.4x1113+.4x1123+.4x1143+.4x1114+.4x1124+.4x1144  
 +.4x1115+.4x1125+.4x1145+.4x1116+.4x1126+.4x1146  
 +.4x1221+.4x1222+.4x1223+.4x1224+.4x1225+.4x1226  
 +.4x1341+.4x1351+.4x1342+.4x1352+.4x1343+.4x1353  
 +.4x1344+.4x1354+.4x1345+.4x1355+.4x1346+.4x1356  
 +.4x1431+.4x1432+.4x1433+.4x1434+.4x1435+.4x1436  
 -.6x1741-.6x1751-.6x1742-.6x1752-.6x1743-.6x1753

$-.6x1744-.6x1754-.6x1745-.6x1755-.6x1746-.6x1756$   
 $-.6x1821-.6x1822-.6x1823-.6x1824-.6x1825-.6x1826$   
 $-.6x1921-.6x1931-.6x1922-.6x1932-.6x1923-.6x1933$   
 $-.6x1924-.6x1934-.6x1925-.6x1935-.6x1926-.6x1936$   
 $-.6x2031-.6x2032-.6x2033-.6x2034-.6x2035-.6x2036$   
 $-.6x2111-.6x2121-.6x2141-.6x2151-.6x2112-.6x2122$   
 $-.6x2142-.6x2152-.6x2113-.6x2123-.6x2143-.6x2153$   
 $-.6x2114-.6x2124-.6x2144-.6x2154-.6x2115-.6x2125$   
 $-.6x2145-.6x2155-.6x2116-.6x2126-.6x2146-.6x2156$   
 $-.6x2221-.6x2231-.6x2222-.6x2232-.6x2223-.6x2233$   
 $-.6x2224-.6x2234-.6x2225-.6x2235-.6x2226-.6x2236$   
 $-.6x2341-.6x2342-.6x2343-.6x2344-.6x2345-.6x2346$   
 $-.6x2411-.6x2421-.6x2451-.6x2412-.6x2422-.6x2452$   
 $-.6x2413-.6x2423-.6x2453-.6x2414-.6x2424-.6x2454$   
 $-.6x2415-.6x2425-.6x2455-.6x2416-.6x2426-.6x2456$   
 $-.6x2521-.6x2522-.6x2523-.6x2524-.6x2525-.6x2526 \leq 0$

$$X1011+X1111+X2111+X2411-S11=-23561$$

$$X1012+X1112+X2112+X2412+S11-S12=44648$$

$$X1013+X1113+X2113+X2413+S12-S13=21327$$

$$X1014+X1114+X2114+X2414+S13-S14=19498$$

$$X1015+X1115+X2115+X2415+S14-S15=95038$$

$$X1016+X1116+X2116+X2416+S15=158266$$

$$X121+X221+X321+X421+X621+X721+X1021+X1121+X1221+X1821+X1921+X2121$$

$$+X2221+X2421+X2521-S21=-184584$$

$$X122+X222+X322+X422+X622+X722+X1022+X1122+X1222+X1822+X1922+X2122$$

$$+x2222+x2422+x2522+s21-s22=245768$$

$$X123+X223+X323+X423+X623+X723+X1023+X1123+X1223+X1823+X1923+X2123$$

$$+x2223+x2423+x2523+s22-s23=120656$$

$$X124+X224+X324+X424+X624+X724+X1024+X1124+X1224+X1824+X1924+X2124$$

$$+x2224+x2424+x2524+s23-s24=119739$$

$$X125+X225+X325+X425+X625+X725+X1025+X1125+X1225+X1825+X1925+X2125$$

$$+x2225+x2425+x2525+s24-s25=120155$$

$$X126+X226+X326+X426+X626+X726+X1026+X1126+X1226+X1826+X1926+X2126$$

$$+X2226+X2426+X2526+S25=845931$$

$X131+X231+X331+X431+X631+X831+X1431+X1931+X2031+X2231-S31=-34363$   
 $X132+X232+X332+X432+X632+X832+X1432+X1932+X2032+X2232+S31-S32=75214$   
 $X133+X233+X333+X433+X633+X833+X1433+X1933+X2033+X2233+S32-S33=83028$   
 $X134+X234+X334+X434+X634+X834+X1434+X1934+X2034+X2234+S33-S34=51261$   
 $X135+X235+X335+X435+X635+X835+X1435+X1935+X2035+X2235+S34-S35=35763$   
 $X136+X236+X336+X436+X636+X836+X1436+X1936+X2036+X2236+S35=161856$   
 $X141+X541+X741+X941+X1041+X1141+X1341+X1741+X2141+X2341-S41=3459$   
 $X142+X542+X742+X942+X1042+X1142+X1342+X1742+X2142+X2342+S41-S42=57068$   
 $X143+X543+X743+X943+X1043+X1143+X1343+X1743+X2143+X2343+S42-S43=19819$   
 $X144+X544+X744+X944+X1044+X1144+X1344+X1744+X2144+X2344+S43-S44=63222$   
 $X145+X545+X745+X945+X1045+X1145+X1345+X1745+X2145+X2345+S44-S45=50504$   
 $X146+X546+X746+X946+X1046+X1146+X1346+X1746+X2146+X2346+S45=202591$   
 $X551+X1351+X1751+X2151+X2451-S51=-47908$   
 $X552+X1352+X1752+X2152+X2452+S51-S52=29414$   
 $X553+X1353+X1753+X2153+X2453+S52-S53=33459$   
 $X554+X1354+X1754+X2154+X2454+S53-S54=36311$   
 $X555+X1355+X1755+X2155+X2455+S54-S55=28300$   
 $X556+X1356+X1756+X2156+X2456+S55=178599$   
 $X1561-S61=-60743$   
 $X1562+S61-S62=60743$   
 $X1563+S62-S63=53993$   
 $X1564+S63-S64=53562$   
 $X1565+S64-S65=76673$   
 $X1566+S65=311192$   
 $X1671-S71=-70391$   
 $X1672+S71-S72=54389$   
 $X1673+S72-S73=72847$   
 $X1674+S73-S74=47509$   
 $X1675+S74-S75=50372$   
 $X1676+S75=216678$   
 $X121+X131+X141+X122+X132+X142+X123+X133+X143+X124+X134+X144$   
 $+X125+X135+X145+X126+X136+X146 \geq 95846$   
 $X221+X231+X222+X232+X223+X233+X224+X234+X225+X235+X226+X236 \geq 103034$   
 $X321+X331+X322+X332+X323+X333+X324+X334+X325+X335+X326+X336 \geq 126121$

$X_{421}+X_{431}+X_{422}+X_{432}+X_{423}+X_{433}+X_{424}+X_{434}+X_{425}+X_{435}+X_{426}+X_{436} >=84972$   
 $X_{541}+X_{551}+X_{542}+X_{552}+X_{543}+X_{553}+X_{544}+X_{554}+X_{545}+X_{555}+X_{546}+X_{556} >=70041$   
 $X_{621}+X_{631}+X_{622}+X_{632}+X_{623}+X_{633}+X_{624}+X_{634}+X_{625}+X_{635}+X_{626}+X_{636} >=112553$   
 $X_{721}+X_{741}+X_{722}+X_{742}+X_{723}+X_{743}+X_{724}+X_{744}+X_{725}+X_{745}+X_{726}+X_{746} >=105769$   
 $X_{831}+X_{832}+X_{833}+X_{834}+X_{835}+X_{836} >=5347$   
 $X_{941}+X_{942}+X_{943}+X_{944}+X_{945}+X_{946} >=79388$   
 $X_{1011}+X_{1021}+X_{1041}+X_{1012}+X_{1022}+X_{1042}+X_{1013}+X_{1023}+X_{1043}+X_{1014}+X_{1024}$   
 $+X_{1044}+X_{1015}+X_{1025}+X_{1045}+X_{1016}+X_{1026}+X_{1046} >=103679$   
 $X_{1111}+X_{1121}+X_{1141}+X_{1112}+X_{1122}+X_{1142}+X_{1113}+X_{1123}+X_{1143}+X_{1114}+X_{1124}$   
 $+X_{1144}+X_{1115}+X_{1125}+X_{1145}+X_{1116}+X_{1126}+X_{1146} >=65921$   
 $X_{1221}+X_{1222}+X_{1223}+X_{1224}+X_{1225}+X_{1226} >=232702$   
 $X_{1341}+X_{1351}+X_{1342}+X_{1352}+X_{1343}+X_{1353}+X_{1344}+X_{1354}+X_{1345}+X_{1355}+X_{1346}$   
 $+X_{1356} >=128264$   
 $X_{1431}+X_{1432}+X_{1433}+X_{1434}+X_{1435}+X_{1436} >=47302$   
 $X_{1561}+X_{1562}+X_{1563}+X_{1564}+X_{1565}+X_{1566} >=172181$   
 $X_{1671}+X_{1672}+X_{1673}+X_{1674}+X_{1675}+X_{1676} >=58937$

$s_{11} <= 89296$	$s_{24} <= 240310$	$s_{42} <= 39638$	$s_{55} <= 65306$	$S_{73} <= 142526$
$s_{11} >= 44648$	$s_{24} >= 120155$	$s_{42} >= 19819$	$s_{55} >= 32653$	$s_{73} >= 47509$
$s_{12} <= 42654$	$s_{25} <= 526992$	$s_{43} <= 126444$	$S_{61} <= 182229$	$S_{74} <= 151116$
$s_{12} >= 21327$	$s_{25} >= 263496$	$s_{43} >= 63222$	$S_{61} >= 60743$	$s_{74} >= 50372$
$s_{13} <= 38996$	$s_{31} <= 150428$	$s_{44} <= 101008$	$S_{62} <= 161979$	$S_{75} <= 124404$
$s_{13} >= 19498$	$s_{31} >= 75214$	$s_{44} >= 50504$	$s_{62} >= 53993$	$s_{75} >= 41468$
$s_{14} <= 190076$	$s_{32} <= 166056$	$s_{45} <= 116114$	$S_{63} <= 160686$	
$s_{14} >= 95038$	$s_{32} >= 83028$	$s_{45} >= 58057$	$s_{63} >= 53562$	
$s_{15} <= 109626$	$s_{33} <= 102522$	$s_{51} <= 58828$	$S_{64} <= 230019$	
$s_{15} >= 54813$	$s_{33} >= 51261$	$s_{51} >= 29414$	$s_{64} >= 76673$	
$s_{21} <= 491536$	$s_{34} <= 71526$	$s_{52} <= 66918$	$S_{65} <= 291366$	
$s_{21} >= 245768$	$s_{34} >= 35763$	$s_{52} >= 33459$	$s_{65} >= 97122$	
$s_{22} <= 241312$	$s_{35} <= 78586$	$s_{53} <= 72622$	$S_{71} <= 163167$	
$s_{22} >= 120656$	$s_{35} >= 39293$	$s_{53} >= 36311$	$s_{71} >= 54389$	
$s_{23} <= 239478$	$s_{41} <= 114136$	$s_{54} <= 56600$	$S_{72} <= 218541$	
$s_{23} >= 119739$	$s_{41} >= 57068$	$s_{54} >= 28300$	$s_{72} >= 72847$	

## APPENDIX B

### Model Input and Sensitivity Analysis Output

TABLE B.1 (Table 7.2 cont.): Delivered Cost for Contract Sources (\$/ton)

i	j*	t r	M <sub>ijt</sub>			R <sub>ijt</sub>			C <sub>ijt</sub>		
			PRED	L95	U95	PRED	L95	U95	PRED	L95	U95
Robinson	4/5	1	33.20	32.90	33.51	(15.42)/15.32	(49.18)/48.52	(48.32)/48.22	(48.93)/48.83		
5	4/5	2	33.34	32.98	33.89	(15.42)/15.32	(49.18)/48.86	(48.40)/48.30	(49.11)/49.01		
	4/5	3	33.46	33.09	33.84	(15.42)/15.32	(49.18)/48.78	(48.41)/48.41	(49.26)/49.16		
	4/5	4	33.57	33.19	33.96	(15.42)/15.32	(49.18)/48.89	(48.61)/48.51	(49.38)/49.28		
	4/5	5	33.68	33.29	34.07	(15.42)/15.32	(49.18)/49.00	(48.71)/48.61	(49.49)/49.39		
	4/5	6	33.76	33.36	34.15	15.42/(15.32)	49.18/(49.18)	48.78/(48.61)	49.57/(49.39)		
Majestic	2/3	1 1	30.27	29.84	30.71	15.45/13.18	45.81/43.45	45.38/43.02	46.25/43.89		
6	2/3	2	30.22	29.64	30.80	(17.98)/13.18	(48.20)/43.40	(47.62)/42.82	(48.78)/43.98		
	2/3	3	30.17	29.49	30.84	17.98/13.18	45.15/43.35	47.47/42.67	48.82/44.02		
	2/3	4	30.12	29.37	30.86	17.98/13.21	48.10/43.33	47.35/42.58	48.84/44.07		
	2/3	5	30.08	29.28	30.87	17.98/13.21	48.06/43.31	47.26/42.51	48.85/44.10		
	2/3	6	30.04	29.21	30.88	17.98/13.23	48.02/43.27	47.19/42.44	48.86/44.11		
Landmark	2/4	1 2	31.06	30.51	31.61	14.63/15.36	45.69/46.42	45.14/45.87	46.24/46.87		
7	2/4	2	31.06	30.38	31.73	14.63(15.36)	45.69(46.42)	45.01(45.74)	46.36(47.09)		
	2/4	3	31.06	30.28	31.84	14.63(15.36)	45.69(46.42)	45.11(45.74)	46.47(47.20)		
	2/4	4	31.06	30.18	31.93	14.63(15.36)	45.69(46.42)	45.01(45.74)	46.56(47.29)		
	2/4	5	31.06	30.10	32.01	(14.63)/15.42	(45.69)/46.48	(45.01)/45.52	(46.64)/47.43		
	2/4	6	31.06	30.02	32.09	14.63/15.42	46.44/46.44	44.71/47.51	46.78/47.51		
Race Fork	3	1 2	(29.31)			(13.18)	(42.49)				
8	3	2	29.31			13.18	42.49				
	3	3	29.31			13.18	42.49				
	3	4	(29.31)			(13.18)	(42.49)				
	3	5	(29.31)			(13.18)	(42.49)				
	3	6	(29.31)			(13.18)	(42.49)				

i=source j=plant/destination t=month M<sub>ijt</sub>=mine price(fob) R<sub>ijt</sub>=rail rate C<sub>ijt</sub>=delivered cost (\*\*)\*\*=inferred  
 \* Entries for R<sub>ijt</sub> and C<sub>ijt</sub> correspond to the same sequence as given for j, i.e., j<sub>1</sub>/j<sub>2</sub>/... For detailed key see Table 7.1.

Table B.1 (Table 7.2 cont.): Delivered Cost for Contract Sources (\$/ton)

i	j*	t r	M <sub>ijt</sub>		R <sub>ijt</sub>		C <sub>ijt</sub>	
			PRED	L95	U95	PRED	L95	U95
Price	4	1 2	30.87		15.36		46.23	
9	4	2	30.87		15.36		46.23	
	4	3	30.89		15.36		46.25	
	4	4	30.58		15.38		45.96	
	4	5	30.28		15.42		45.70	
	4	6	30.28		15.42		45.70	
Mountain Top	1/2/4	1 2	30.34		13.51(14.14)14.83		43.85(44.48)45.17	
10	1/2/4	2	30.21		13.75/14.14/14.83		43.96/44.35/45.04	
	1/2/4	3	30.00		13.75(14.14)14.88		43.75(34.14)44.88	
	1/2/4	4	29.35		13.78/14.20(14.88)		43.13/43.55(44.24)	
	1/2/4	5	29.14		(13.78)14.20(14.88)(42.92)		43.34(44.03)	
	1/2/4	6	29.14		(13.78)14.20(14.88)(42.92)		43.34(44.03)	
Hobet	1/2/4	1 2	25.53		13.75(14.14/14.88)		39.28(39.67/40.42)	
11	1/2/4	2	25.53		13.75(14.14/14.88)		39.28(39.67/40.42)	
	1/2/4	3	26.53		13.75(14.14/14.88)		42.28(42.67/43.42)	
	1/2/4	4	26.53		13.81/14.14(14.88)		42.34/42.67(43.42)	
	1/2/4	5	26.53		13.85/14.20/14.88		42.38/42.73/43.42	
	1/2/4	6	26.53		(13.85)14.20(14.88)(42.48)		42.73(43.42)	
Calvin Branch	2	1 2	23.99		14.14		36.13	
12	2	2	23.99		14.14		36.13	
	2	3	23.99		13.84		37.93	
	2	4	23.64		13.77		37.41	
	2	5	23.29		13.80		37.09	
	2	6	23.29		13.80		37.09	

i=source j=plant/destination t=month M<sub>ijt</sub>=mine price(fob) R<sub>ijt</sub>=rail rate C<sub>ijt</sub>=delivered Cost (\*\*)\*\*=inferred  
 \* Entries for R<sub>ijt</sub> and C<sub>ijt</sub> correspond to the same sequence as given for j, i.e., j<sub>1</sub>/j<sub>2</sub>/... For detailed key see Table 7.1.

TABLE B.1 (Table 7.2 cont.): Delivered Cost for Contract Sources (\$/ton)

i	j	t	M <sub>ijt</sub>			R <sub>ijt</sub>	C <sub>ijt</sub>		
			PRED	L95	U95		PRED	L95	U95
Enterprise 13	4/5	1	28.91	28.28	29.56	15.08/15.19	43.99/44.10	43.34/43.45	44.64/44.75
	4/5	2	28.91	28.28	29.56	(15.08)15.32	(43.99)44.23	(43.34)43.58	(44.64)44.88
	4/5	3	28.91	28.28	29.56	(15.08)15.32	(43.99)44.23	(43.34)43.58	(44.64)44.88
	4/5	4	28.91	28.28	29.56	(15.08)15.35	(43.99)44.26	(43.34)43.61	(44.64)44.91
	4/5	5	28.91	28.28	29.56	15.42/15.38	44.33/44.29	43.68/43.64	44.98/44.94
	4/5	6	28.91	28.28	29.56	(15.42)15.38	(44.33)44.29	(43.68)43.64	(44.98)/44.94
Red River 14	3	1	29.46			13.18	42.64		
	3	2	29.46			13.18	42.64		
	3	3	29.46			13.18	42.64		
	3	4	28.91			13.23	42.14		
	3	5	(28.91)			(13.23)	(42.14)		
	3	6	28.91			13.23	42.14		
Clinchfield 15	3	1	43.35	40.74	45.95	1.49	44.84	42.23	47.44
	3	2	43.44	40.16	46.71	1.50	44.94	41.66	48.21
	3	3	43.50	39.89	47.11	1.58	45.08	41.47	48.69
	3	4	43.55	39.76	47.35	1.56	45.11	41.32	48.91
	3	5	43.59	39.70	47.49	1.49	45.08	41.19	48.96
	3	6	43.62	39.67	47.58	1.44	45.06	41.11	49.02
Wellmore 16	4	1	42.20	40.75	43.66	6.05	48.85	46.60	49.71
	4	2	41.32	39.54	43.09	6.06	47.38	45.60	49.15
	4	3	40.19	38.29	42.09	6.08	46.27	44.37	48.17
	4	4	40.51	38.54	42.49	6.10	46.61	44.64	48.59
	4	5	39.97	37.97	41.97	6.10	46.07	44.07	48.07
	4	6	39.84	37.82	41.87	6.10	45.94	43.92	47.97

i=source j=plant/destination t=month M<sub>ijt</sub>=mine price(fob) R<sub>ijt</sub>=rail rate C<sub>ijt</sub>=delivered Cost (\*\*)\*\*=inferred  
 \* Entries for R<sub>ijt</sub> and C<sub>ijt</sub> correspond to the same sequence as given for j, i.e., j<sub>1</sub>/j<sub>2</sub>/... For detailed key see Table 7.1.

TABLE B.2 (Table 7.3 cont.): Delivered Price for Spot Sources (\$/ton)

i	j	t	r	M <sub>ijt</sub>			R <sub>ijt</sub>			C <sub>ijt</sub>		
				PRED	L95	U95	PRED	L95	U95	PRED	L95	U95
Kentucky 21	1/2/4/5	1	2	22.97	14.41	29.00	14.67/14.63/15.36/15.32	37.64/37.60/38.33/38.29	33.08/33.04/33.77/33.73	43.67/43.63/44.36/44.32		
				22.55	16.87	28.33	16.03/15.14/15.67(15.32)	38.58/37.69/33.33(37.87)	32.92/32.03/32.56(32.19)	44.36/43.47/44.00(43.65)		
				23.32	17.47	29.63	16.03/14.76/15.58(15.32)	39.35/38.08/38.90(38.64)	33.50/32.23/33.05(32.79)	45.66/44.39/45.21(44.95)		
				24.03	18.04	30.30	16.03/14.81/16.38/15.38	40.06/38.64/40.41/39.41	34.07/32.65/34.42/33.42	46.33/44.91/46.68/45.68		
				24.62	18.05	30.99	(16.03)14.83/16.92/15.38	(40.65)39.55/41.54/40.00	(34.08)32.98/34.97/33.43	(47.02)45.92/47.91/46.37		
				25.14	18.35	34.34	(16.03)14.89/15.42(15.38)	(41.17)39.83/40.56(40.52)	(34.38)33.04/33.77(33.73)	(50.37)49.03/49.76(49.72)		
Kentucky 22	2/3	1	1	25.54	25.04	26.03	(17.98/13.78)	(39.32/39.32)	(43.02/38.82)	(44.01/39.81)		
				2	25.47	24.92	26.03	(17.98/13.78)	(43.45/39.25)	(42.90/38.70)	(44.01/13.78)	
				3	25.40	24.82	25.98	17.98/13.78	43.38/39.18	42.80/38.60	43.96/39.76	
				4	25.31	24.71	25.91	17.98/13.21	43.29/38.52	42.69/37.92	43.89/39.12	
				5	25.22	24.59	25.85	(17.98)13.23	(43.20)38.45	(42.57)37.83	(43.83)39.08	
				6	25.13	24.48	25.79	(17.98/13.23)	(43.11/38.36)	(42.46/37.71)	(43.77/39.02)	
W. Virginia 23	4	1	2	23.98			15.38	39.34				
				2	23.47			16.04	39.51			
				3	22.98			15.73	38.71			
				4	23.28			15.58	38.86			
				5	23.36			15.61	38.97			
				6	22.97			15.78	38.75			
W. Virginia 24	1/2/5	1	2	24.45	23.12	25.77	13.63/14.14/14.36	38.08/38.59/38.81	36.75/37.26/37.48	39.40/39.91/40.13		
				2	24.76	23.14	26.37	14.13/14.07/14.37	38.89/38.83/39.13	37.27/37.21/37.51	40.50/40.44/40.74	
				3	24.98	23.24	26.72	14.11/14.07(14.37)	39.09/39.05(39.35)	37.35/37.31(37.61)	40.83/40.79(41.09)	
				4	25.14	23.33	26.84	13.78/13.95(14.37)	38.92/39.07(39.51)	37.11/37.26(37.70)	40.72/40.87(41.31)	
				5	25.25	23.42	27.08	(13.78)14.05(14.37)	(39.03)39.30(39.62)	(37.20)37.47(37.79)	(40.86)41.13(41.45)	
				6	25.33	23.48	27.18	(13.78)14.20(14.37)	(39.11)39.53(39.70)	(37.26)37.68(37.85)	(40.96)41.38(41.55)	
W. Virginia 25	2	1	1	(28.25)			(14.14)	(42.39)				
				2	28.25			14.14	42.39			
				3	25.51			17.99	43.50			
				4	26.38			16.09	42.47			
				5	27.25			14.20	41.45			
				6	22.90			13.23	36.13			

i=source j=plant/destination t=month M<sub>ijt</sub>=mine price(fob) R<sub>ijt</sub>=rail rate C<sub>ijt</sub>=delivered price (\*\*)\*\*=inferred  
 \* Entries for R<sub>ijt</sub> and C<sub>ijt</sub> correspond to the same sequence as given for j, i.e., j<sub>1</sub>/j<sub>2</sub>/... For detailed key see Table 7.1.

TABLE B.3 (Table 7.12 cont.): SENSITIVITY ANALYSIS

VARIABLE	OBJ COEFFICIENT RANGES		ALLOWABLE DECREASE
	CURRENT COEF	ALLOWABLE INCREASE	
Virginia Energy to Possum Point			
X331	45.35	INFINITY	2.349980
X332	45.44	0.002713	0.008415
X333	45.60	0.004677	0.021323
X334	45.70	INFINITY	0.009999
X335	45.78	INFINITY	0.000020
X336	45.85	0.003985	0.001285
GEX to Chesterfield			
X421	46.04	2.429998	INFINITY
X422	48.58	INFINITY	-0.000010
X423	48.69	0.009143	0.002948
X424	48.80	INFINITY	0.019995
X425	48.81	INFINITY	0.049978
X426	48.89	INFINITY	1.776495
GEX to Portsmouth			
X431	43.67	INFINITY	2.429998
X432	43.78	-0.000010	0.001503
X433	43.89	INFINITY	0.002462
X434	44.00	0.017928	0.005780
X435	44.01	0.002505	0.007771
X436	44.09	0.001285	0.003985
Robinson Creek to Possum Point			
X541	49.17	INFINITY	0.200002
X542	49.17	INFINITY	5.060002
X543	49.17	INFINITY	0.329991
X544	49.17	INFINITY	0.559972
X545	49.17	INFINITY	0.449971
X546	49.17	INFINITY	0.950002
Robinson Creek to Yorktown			
X551	48.51	0.200002	1.448308
X552	48.65	5.060002	1.110707
X553	48.77	0.329991	0.483047
X554	48.88	0.559972	0.458398
X555	49.00	0.449971	0.787229
X556	49.17	0.458398	16.338867
Majesti9988 Chesterfield			
X621	45.80	0.106051	25.915543
X622	48.19	INFINITY	0.000029
X623	45.14	3.002439	0.155089
X624	48.09	0.010023	0.228072
X625	48.05	-0.000021	0.000020
X626	48.01	INFINITY	1.726515

TABLE B.3 (Table 7.12 cont.): SENSITIVITY ANALYSIS

VARIABLE	OBJ COEFFICIENT RANGES		ALLOWABLE DECREASE
	CURRENT COEF	ALLOWABLE INCREASE	
Majestic to Portsmouth			
X631	43.44	INFINITY	2.440001
X632	43.39	0.000029	4.111541
X633	43.34	INFINITY	3.002439
X634	43.32	INFINITY	0.010023
X635	43.30	0.000020	-0.000021
X636	43.26	1.726515	0.493237
Landmark to Chesterfield			
X721	45.68	0.000015	0.661758
X722	45.68	5.000015	0.393906
X723	45.68	INFINITY	0.377548
X724	45.68	INFINITY	0.404556
X725	45.68	INFINITY	0.344559
X726	46.43	INFINITY	2.620010
Landmark to Possum Point			
X741	46.41	INFINITY	0.000015
X742	46.41	INFINITY	5.000015
X743	46.41	0.049951	0.017352
X744	46.41	0.033301	0.020597
X745	46.47	0.068898	0.030289
X746	46.43	0.008777	0.033301
Race Fork to Portsmouth			
X831	42.48	INFINITY	0.159052
X832	42.48	3.051360	0.265927
X833	42.48	1.784422	0.356870
X834	42.48	1.043522	0.427245
X835	42.48	0.544409	0.601587
X836	42.48	0.268119	22.047592
Price to Portsmouth			
X941	46.22	INFINITY	2.413543
X942	46.22	INFINITY	5.930846
X943	46.25	164.120911	0.577641
X944	45.95	12.130469	0.407503
X945	45.69	1.630013	0.101885
X946	45.69	0.077627	9.255919
Mountain Top to Bremo Bluff			
X1011	43.84	0.200012	4.577253
X1012	43.95	0.430008	-0.000013
X1013	43.75	INFINITY	8.292419
X1014	43.12	1.279984	4.783873
X1015	42.91	1.279984	2.653867
X1016	42.91	1.469986	1.290911

TABLE B.3 (Table 7.12 cont.): SENSITIVITY ANALYSIS

VARIABLE	OBJ COEFFICIENT RANGES		
	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
Mountain Top to Chesterfield			
X1021	44.47	INFINITY	0.240005
X1022	44.34	-0.000013	0.430008
X1023	34.13	3.950689	8.662107
X1024	43.54	INFINITY	1.724564
X1025	43.33	INFINITY	1.724564
X1026	43.33	INFINITY	3.399994
Mountain Top to Possum Point			
X1041	45.16	INFINITY	0.200012
X1042	45.03	INFINITY	4.959991
X1043	44.88	INFINITY	9.642426
X1044	44.23	INFINITY	1.279984
X1045	44.02	INFINITY	1.279984
X1046	44.02	INFINITY	1.469986
Hobet to Bremo Bluff			
X1111	39.27	-0.000013	INFINITY
X1112	39.27	0.000002	1.237482
X1113	42.27	1.349993	2.474964
X1114	42.33	1.249987	1.617495
X1115	42.37	1.209993	1.197847
X1116	42.47	1.299974	0.683346
Hobet to Chesterfield			
X1121	39.66	INFINITY	-0.000013
X1122	39.66	INFINITY	0.000002
X1123	42.66	INFINITY	1.707552
X1124	42.66	INFINITY	1.634569
X1125	42.72	INFINITY	1.654573
X1126	42.72	INFINITY	3.229982
Hobet to Possum Point			
X1141	40.41	INFINITY	0.019991
X1142	40.41	INFINITY	5.020006
X1143	43.41	INFINITY	1.349993
X1144	43.41	INFINITY	1.249987
X1145	43.41	INFINITY	1.209993
X1146	43.41	INFINITY	1.299974
Calvin Branch to Chesterfield			
X1221	38.12	INFINITY	0.529999
X1222	38.12	0.088196	0.028432
X1223	37.92	0.044098	0.014216
X1224	37.40	0.022049	0.007108
X1225	37.08	0.011025	0.003554
X1226	37.08	0.022049	0.007108

TABLE B.3 (Table 7.12 cont.): SENSITIVITY ANALYSIS

VARIABLE	OBJ COEFFICIENT RANGES		ALLOWABLE DECREASE
	CURRENT COEF	ALLOWABLE INCREASE	
Enterprise to Possum Point			
X1341	43.98	INFINITY	1.484014
X1342	43.98	INFINITY	4.299993
X1343	43.98	0.309987	0.402870
X1344	43.98	0.309972	0.389999
X1345	44.32	INFINITY	0.309972
X1346	44.32	INFINITY	0.989995
Enterprise to Yorktown			
X1351	44.09	INFINITY	2.054006
X1352	44.22	4.299993	1.456014
X1353	44.22	INFINITY	0.309987
X1354	44.25	0.389999	0.309972
X1355	44.28	0.309972	1.177532
X1356	44.28	0.655012	9.194976
Red River to Portsmouth			
X1431	42.63	INFINITY	1.337798
X1432	42.63	INFINITY	1.092086
X1433	42.63	INFINITY	0.682555
X1434	42.13	3.696418	0.180462
X1435	42.13	0.480029	0.404396
X1436	42.13	0.252368	17.997208
Clinchfield to Clinch River			
X1561	44.83	0.103569	INFINITY
X1562	44.93	0.595001	0.103569
X1563	45.07	1.190002	0.007141
X1564	45.10	0.049988	0.043330
X1565	45.07	0.024994	0.044994
X1566	45.05	0.012497	0.297501
Wellmore to Glen Lyn			
X1671	48.84	INFINITY	1.470016
X1672	47.37	1.470016	2.399986
X1673	46.26	1.599991	0.199997
X1674	46.06	0.199997	3.356674
X1675	45.93	10.070023	4.840004
X1676	41.09	4.840004	INFINITY
VA Spot to Possum Point via CSX			
X1741	41.09	INFINITY	2.770004
X1742	41.06	INFINITY	7.740005
X1743	41.00	INFINITY	2.290009
X1744	41.09	INFINITY	2.349991
X1745	41.00	INFINITY	2.250000
X1746	41.00	INFINITY	2.250000

TABLE B.3 (Table 7.12 cont.): SENSITIVITY ANALYSIS

VARIABLE	OBJ COEFFICIENT RANGES		ALLOWABLE DECREASE
	CURRENT COEF	ALLOWABLE INCREASE	
	VA Spot to Yorktown via CSX		
X1751	41.09	INFINITY	3.229996
X1752	41.00	INFINITY	3.130005
X1753	41.09	INFINITY	2.459991
X1754	41.09	INFINITY	2.080002
X1755	41.00	INFINITY	1.980011
X1756	41.00	INFINITY	1.300003
	VA Spot to Chesterfield via CSX		
X1821	40.50	INFINITY	2.900009
X1822	40.50	INFINITY	2.900009
X1823	40.50	INFINITY	2.897568
X1824	40.50	INFINITY	2.884583
X1825	40.50	INFINITY	2.884583
X1826	40.50	INFINITY	4.370010
	VA Spot to Chesterfield via NS		
X1921	42.87	INFINITY	5.279999
X1922	42.66	INFINITY	5.070007
X1923	42.43	INFINITY	4.837555
X1924	42.20	INFINITY	4.594574
X1925	41.95	INFINITY	4.344574
X1926	41.70	INFINITY	5.580002
	VA Spot to Possum Point via NS		
X1931	38.07	INFINITY	5.279999
X1932	37.86	INFINITY	5.070007
X1933	37.63	INFINITY	4.840012
X1934	37.42	INFINITY	4.594589
X1935	37.17	INFINITY	4.314590
X1936	36.92	INFINITY	3.823502
	KY Spot to Portsmouth CSX		
X2031	37.50	INFINITY	4.710007
X2032	36.83	INFINITY	4.040009
X2033	36.42	INFINITY	3.630005
X2034	36.28	INFINITY	3.454590
X2035	35.92	INFINITY	3.064590
X2036	35.39	INFINITY	2.293503
	KY to Bremo Bluff via CSX		
X2111	37.63	INFINITY	0.430008
X2112	38.57	INFINITY	1.369995
X2113	39.34	INFINITY	0.429993
X2114	40.05	INFINITY	1.139999
X2115	40.64	INFINITY	1.729996
X2116	41.16	INFINITY	2.060013

TABLE B.3 (Table 7.12 cont.): SENSITIVITY ANALYSIS

VARIABLE	OBJ COEFFICIENT RANGES		ALLOWABLE DECREASE
	CURRENT COEF	ALLOWABLE INCREASE	
	KY Spot to Chesterfield via CSX		
X2121	37.59	-17.230759	0.000015
X2122	37.68	INFINITY	0.089996
X2123	38.07	INFINITY	0.477554
X2124	38.63	INFINITY	1.024582
X2125	39.54	INFINITY	1.934570
X2126	39.82	INFINITY	3.699997
	KY Spot to Possum Point via CSX		
X2141	38.32	0.000015	5.000000
X2142	33.32	4.299993	INFINITY
X2143	38.89	INFINITY	0.190002
X2144	40.40	INFINITY	1.659988
X2145	41.53	INFINITY	2.789993
X2146	40.55	INFINITY	1.809998
	KY Spot to Yorktown via CSX		
X2151	38.28	INFINITY	0.419998
X2152	37.86	0.419998	0.200002
X2153	38.63	0.309987	0.329991
X2154	39.40	INFINITY	0.389999
X2155	40.00	INFINITY	0.980011
X2156	40.51	INFINITY	0.819992
	KY Spot to Chesterfield via NS		
X2221	39.31	INFINITY	1.720001
X2222	43.44	INFINITY	5.850006
X2223	43.37	INFINITY	5.777557
X2224	43.28	INFINITY	5.674576
X2225	43.19	INFINITY	5.584579
X2226	43.10	INFINITY	6.979996
	KY Spot to Portsmouth via NS		
X2231	39.31	INFINITY	6.520004
X2232	39.25	INFINITY	6.450012
X2233	39.17	INFINITY	6.380005
X2234	38.51	INFINITY	5.684586
X2235	38.44	INFINITY	5.584595
	W.VA. to Portsmouth via CSX		
X2341	39.33	INFINITY	1.010010
X2342	39.50	INFINITY	6.180008
X2343	38.70	0.017352	0.049951
X2344	38.85	INFINITY	0.109985
X2345	38.96	INFINITY	0.219986
X2346	38.75	0.049951	0.017352

TABLE B.3 (Table 7.12 cont.): SENSITIVITY ANALYSIS

VARIABLE	OBJ COEFFICIENT RANGES		ALLOWABLE DECREASE
	CURRENT COEF	ALLOWABLE INCREASE	
	W.VA. Spot to Bremo Bluff via CSX		
X2411	38.07	INFINITY	0.869995
X2412	38.88	INFINITY	1.680008
X2413	39.08	INFINITY	0.169998
X2414	38.91	0.110001	1.209993
X2415	39.02	INFINITY	0.110001
X2416	39.10	0.683346	0.189987
	W.VA. Spot to Chesterfield via CSX		
X2421	38.58	INFINITY	0.990005
X2422	38.82	INFINITY	1.229996
X2423	39.04	INFINITY	1.447556
X2424	39.06	INFINITY	1.454575
X2425	39.29	INFINITY	1.684570
X2426	39.52	INFINITY	3.400009
	W.VA. Spot to Yorktown via CSX		
X2451	38.80	INFINITY	0.940002
X2452	39.12	INFINITY	1.259995
X2453	39.34	INFINITY	0.709991
X2454	39.50	INFINITY	0.490005
X2455	39.61	INFINITY	0.600006
X2456	39.69	0.819992	0.458398
	W.VA. Spot to Chesterfield via NS		
X2521	42.38	INFINITY	4.790009
X2522	42.38	INFINITY	4.790009
X2523	43.50	INFINITY	5.897568
X2524	42.46	INFINITY	4.854568
X2525	41.44	INFINITY	3.834579
X2526	36.12	0.007108	0.022049

TABLE B.4 (Table 7.13): Sensitivity Analysis - Stockpiles

VARIABLE	OBJ COEFFICIENT RANGES		ALLOWABLE DECREASE
	ACTIVITY	ALLOWABLE INCREASE	
Bremo Bluff			
S11	62818	-0.000013	0.430008
S12	42654	1.710007	INFINITY
S13	31445	1.349993	0.169998
S14	189703	0.110001	1.209993
S15	109626	0.189987	INFINITY
Chesterfield			
S21	434506	0.001895	-0.000013
S22	241312	0.002441	INFINITY
S23	239478	0.012985	INFINITY
S24	223874	0.010758	0.005271
S25	263496	INFINITY	1.485428
Portsmouth			
S31	75530	-0.000006	0.159052
S32	161681	0.005127	0.004677
S33	102522	0.035416	INFINITY
S34	71526	0.029999	INFINITY
S35	78586	0.241089	INFINITY
Possum Point			
S41	57068	INFINITY	5.000000
S42	39638	5.380005	INFINITY
S43	126444	0.040009	INFINITY
S44	99808	0.033301	0.020597
S45	78564	0.027887	0.012260
Yorktown			
S51	53184	0.200002	0.419998
S52	66918	0.770004	INFINITY
S53	72622	0.379990	INFINITY
S54	47672	0.600006	0.309972
S55	65306	0.680008	INFINITY
Clinchfield			
S61	168729	0.103569	INFINITY
S62	161979	0.198334	INFINITY
S63	134982	0.170000	0.007141
S64	125198	0.198334	0.008331
S65	136080	0.297501	0.012497
Glen Lyn			
S71	89339	INFINITY	1.470016
S72	72847	INFINITY	1.599991
S73	47509	INFINITY	0.199997
S74	50372	INFINITY	3.356674
S75	72226	INFINITY	4.840004

## APPENDIX C

### Model Estimates and Forecast for Contract Sources

## ARIMA ANALYSIS FOR LANDMARK/PIKE KY. TO VEPCO

## ARIMA: CONDITIONAL LEAST SQUARES ESTIMATION

PARAMETER	ESTIMATE	APPROX. STD ERROR	T RATIO	LAG
MA1,1	0.284884	0.182643	1.56	1

VARIANCE ESTIMATE = 0.0784004  
 STD ERROR ESTIMATE = 0.280001  
 AIC = 9.44893\*  
 SBC = 10.8162\*  
 NUMBER OF RESIDUALS= 29  
 \* DOES NOT INCLUDE LOG DETERMINANT

MODEL FOR VARIABLE FOB\_T  
 NO MEAN TERM IN THIS MODEL.  
 PERIODS OF DIFFERENCING= 1.

MOVING AVERAGE FACTORS  
 FACTOR 1

1-.284884B\*(1)  
 ARIMA ANALYSIS FOR LANDMARK/PIKE KY. TO VEPCO

## FORECASTS FOR VARIABLE FOB\_T

OBS	FORECAST	STD ERROR	LOWER 95%	UPPER 95%	ACTUAL	RESIDUAL
2	29.0000	0.2800	28.4512	29.5488	29.0000	0.0000
3	29.0000	0.2800	28.4512	29.5488	29.0000	0.0000
4	29.0000	0.2800	28.4512	29.5488	29.1200	0.1200
5	29.0858	0.2800	28.5370	29.6346	29.3100	0.2242
6	29.2461	0.2800	28.6973	29.7949	29.3100	0.0639
7	29.2918	0.2800	28.7430	29.8406	28.4800	-0.8118
8	28.7113	0.2800	28.1625	29.2601	29.6400	0.9287
9	29.3754	0.2800	28.8266	29.9242	29.6400	0.2646
10	29.5646	0.2800	29.0158	30.1134	30.0600	0.4954
11	29.9189	0.2800	29.3701	30.4677	30.1300	0.2111
12	30.0699	0.2800	29.5211	30.6186	30.1300	0.0601
13	30.1129	0.2800	29.5641	30.6617	30.3200	0.2071
14	30.2610	0.2800	29.7122	30.8098	30.5100	0.2490
15	30.4391	0.2800	29.8903	30.9879	30.5100	0.0709
16	30.4898	0.2800	29.9410	31.0386	30.4900	0.0002
17	30.4899	0.2800	29.9412	31.0387	30.4700	-0.0199
18	30.4757	0.2800	29.9269	31.0245	30.4700	-0.0057
19	30.4716	0.2800	29.9228	31.0204	30.5600	0.0884
20	30.5348	0.2800	29.9860	31.0836	30.6500	0.1152
21	30.6172	0.2800	30.0684	31.1660	30.6500	0.0328
22	30.6407	0.2800	30.0919	31.1894	30.7300	0.0893
23	30.7045	0.2800	30.1558	31.2533	30.8000	0.0955
24	30.7728	0.2800	30.2240	31.3216	30.8000	0.0272
25	30.7923	0.2800	30.2435	31.3410	30.7200	-0.0723
26	30.7406	0.2800	30.1918	31.2894	30.7100	-0.0306
27	30.7187	0.2800	30.1699	31.2675	30.7100	-0.0087
28	30.7125	0.2800	30.1637	31.2613	30.8500	0.1375
29	30.8108	0.2800	30.2620	31.3596	31.0400	0.2292
30	30.9747	0.2800	30.4259	31.5235	31.0900	0.1153

-----FORECAST BEGINS-----

31	31.0572	0.2800	30.5084	31.6059
32	31.0572	0.3442	30.3825	31.7318
33	31.0572	0.3982	30.2766	31.8377
34	31.0572	0.4457	30.1835	31.9308
35	31.0572	0.4886	30.0994	32.0149
36	31.0572	0.5281	30.0221	32.0922

## ESTIMATES FROM FORECAST FOR ENTERPRISE/ROBINSON

OBS	_TYPE_	TIME	FOB_T
1	N	23	23
2	SIGMA	23	0.14201
3	CONSTANT	23	31.71926
4	LINEAR	23	0.06757213
5	AR1	23	0.6318446
6	AR2	23	.
7	AR3	23	.
8	AR4	23	.
9	AR5	23	-0.237451
10	AR6	23	.
11	AR7	23	.
12	AR8	23	.
13	AR9	23	.
14	AR10	23	.

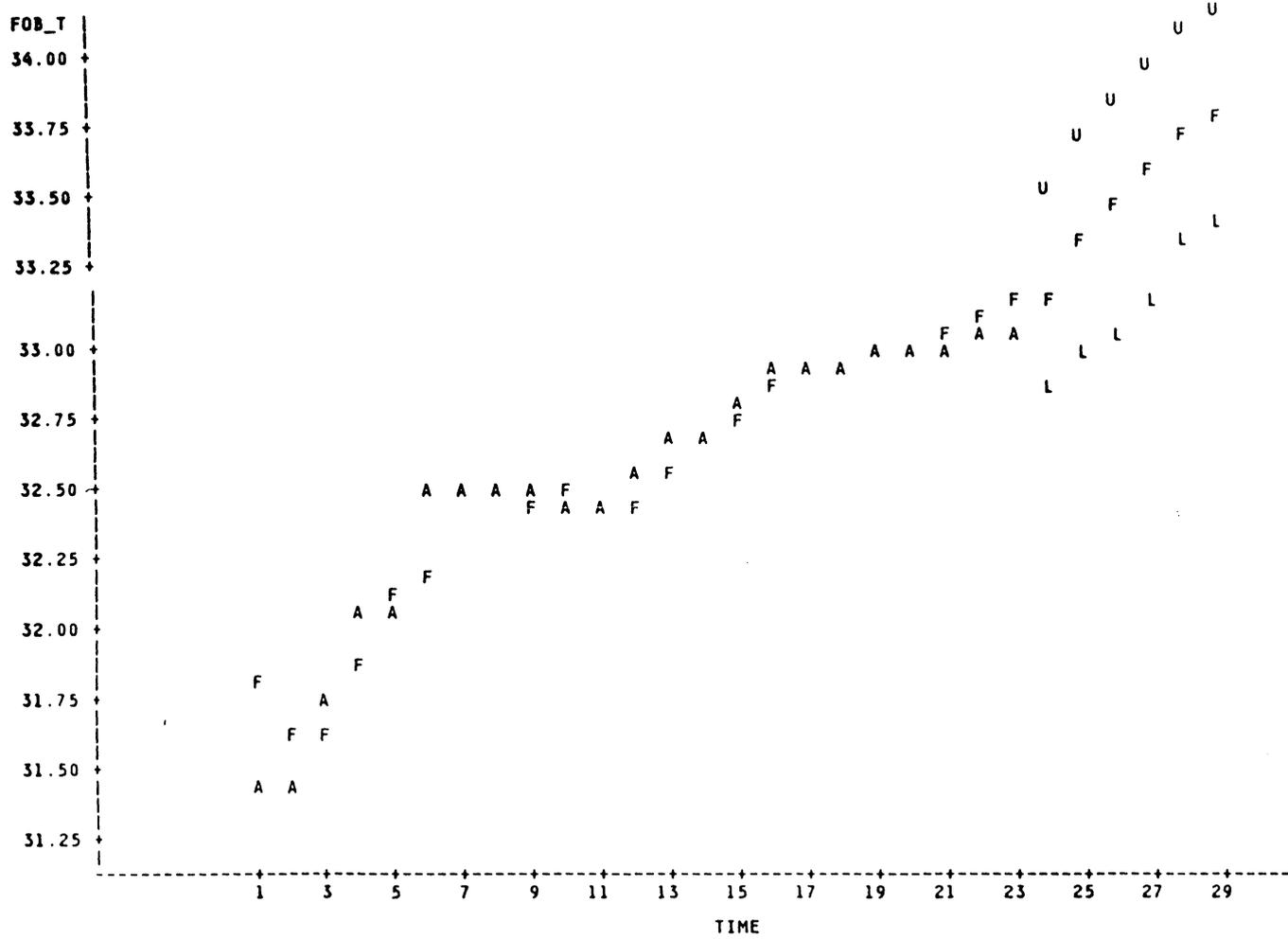
## THE OUTPUT FORECASTED FROM PROC FORECAST FOR ENTERPRISE/ROB

OBS	TIME	_TYPE_	_LEAD_	FOB_T
1	1	ACTUAL	0	31.4190
2	1	FORECAST	0	31.7868
3	2	ACTUAL	0	31.4200
4	2	FORECAST	0	31.6220
5	3	ACTUAL	0	31.7350
6	3	FORECAST	0	31.6475
7	4	ACTUAL	0	32.0510
8	4	FORECAST	0	31.8714
9	5	ACTUAL	0	32.0510
10	5	FORECAST	0	32.0960
11	6	ACTUAL	0	32.4790
12	6	FORECAST	0	32.2082
13	7	ACTUAL	0	32.4800
14	7	FORECAST	0	32.5193
15	8	ACTUAL	0	32.4900
16	8	FORECAST	0	32.4860
17	9	ACTUAL	0	32.4740
18	9	FORECAST	0	32.4582
19	10	ACTUAL	0	32.4610
20	10	FORECAST	0	32.4891
21	11	ACTUAL	0	32.4600
22	11	FORECAST	0	32.4201
23	12	ACTUAL	0	32.5750
24	12	FORECAST	0	32.4602
25	13	ACTUAL	0	32.6730
26	13	FORECAST	0	32.5714
27	14	ACTUAL	0	32.6900
28	14	FORECAST	0	32.6780
29	15	ACTUAL	0	32.8430
30	15	FORECAST	0	32.7328
31	16	ACTUAL	0	32.9260
32	16	FORECAST	0	32.8706

## ROBINSON(CONT.)

OBS	TIME	TYPE	LEAD	FOB_T
33	17	ACTUAL	0	32.9200
34	17	FORECAST	0	32.9367
35	18	ACTUAL	0	32.9530
36	18	FORECAST	0	32.9505
37	19	ACTUAL	0	32.9820
38	19	FORECAST	0	33.0083
39	20	ACTUAL	0	33.0160
40	20	FORECAST	0	33.0312
41	21	ACTUAL	0	32.9960
42	21	FORECAST	0	33.0739
43	22	ACTUAL	0	33.0490
44	22	FORECAST	0	33.1036
45	23	ACTUAL	0	33.0500
46	23	FORECAST	0	33.1702
47	24	FORECAST	1	33.2048
48	24	L95	1	32.9018
49	24	U95	1	33.5079
50	25	FORECAST	2	33.3355
51	25	L95	2	32.9824
52	25	U95	2	33.6887
53	26	FORECAST	3	33.4638
54	26	L95	3	33.0908
55	26	U95	3	33.8368
THE OUTPUT FORECASTED FROM PROC FORECAST FOR ENTERPRISE/ROB				
OBS	TIME	_TYPE_	_LEAD_	FOB_T
56	27	FORECAST	4	33.5731
57	27	L95	4	33.1907
58	27	U95	4	33.9556
59	28	FORECAST	5	33.6829
60	28	L95	5	33.2949
61	28	U95	5	34.0710
62	29	FORECAST	6	33.7565
63	29	L95	6	33.3633
64	29	U95	6	34.1496

PLOT OF FORECAST FOR ENTERPRISE/ROBINSON  
 PLOT OF FOB\_T\*TIME    SYMBOL IS VALUE OF \_TYPE\_



## ESTIMATES FROM FORECAST FOR COBRA

OBS	_TYPE_	TIME	MFOB
1	N	28	28
2	SIGMA	28	0.1280155
3	CONSTANT	28	27.72234
4	LINEAR	28	0.1339445
5	QUAD	28	-0.00271292
6	AR1	28	0.2837283
7	AR2	28	.
8	AR3	28	.
9	AR4	28	.
10	AR5	28	.
11	AR6	28	.
12	AR7	28	.
13	AR8	28	.
14	AR9	28	.
15	AR10	28	.
16	AR11	28	-0.273216
17	AR12	28	.

SAS

OBS	TIME	_TYPE_	_LEAD_	MFOB
1	1	ACTUAL	0	27.9021
2	1	FORECAST	0	27.8536
3	2	ACTUAL	0	27.8739
4	2	FORECAST	0	27.9931
5	3	ACTUAL	0	28.0416
6	3	FORECAST	0	28.0698
7	4	ACTUAL	0	28.1428
8	4	FORECAST	0	28.1982
9	5	ACTUAL	0	28.2644
10	5	FORECAST	0	28.3038
11	6	ACTUAL	0	28.4775
12	6	FORECAST	0	28.4114
13	7	ACTUAL	0	28.4802
14	7	FORECAST	0	28.5410
15	8	ACTUAL	0	28.7102
16	8	FORECAST	0	28.6070
17	9	ACTUAL	0	28.7712
18	9	FORECAST	0	28.7336
19	10	ACTUAL	0	28.7711
20	10	FORECAST	0	28.8084
21	11	ACTUAL	0	28.9200
22	11	FORECAST	0	28.8620
23	12	ACTUAL	0	29.0578
24	12	FORECAST	0	28.9407
25	13	ACTUAL	0	29.5107
26	13	FORECAST	0	29.0677
27	14	ACTUAL	0	29.0504
28	14	FORECAST	0	29.2252
29	15	ACTUAL	0	29.0792
30	15	FORECAST	0	29.1364
31	16	ACTUAL	0	29.0406
32	16	FORECAST	0	29.1754
33	17	ACTUAL	0	29.1151
34	17	FORECAST	0	29.1650
35	18	ACTUAL	0	29.1720
36	18	FORECAST	0	29.2387
37	19	ACTUAL	0	29.1720
38	19	FORECAST	0	29.2400
39	20	ACTUAL	0	29.2187
40	20	FORECAST	0	29.2659
41	21	ACTUAL	0	29.3475
42	21	FORECAST	0	29.3165
43	22	ACTUAL	0	29.3339
44	22	FORECAST	0	29.3442
45	23	ACTUAL	0	29.3095
46	23	FORECAST	0	29.3292
47	24	ACTUAL	0	29.2575
48	24	FORECAST	0	29.2197
49	25	ACTUAL	0	29.2391
50	25	FORECAST	0	29.3464
51	26	ACTUAL	0	29.3810
52	26	FORECAST	0	29.3437
53	27	ACTUAL	0	29.5202
54	27	FORECAST	0	29.3996
55	28	ACTUAL	0	29.5194

## COBRA ENERGY (CONT.)

OBS	TIME	_TYPE_	_LEAD_	MFOB
56	28	FORECAST	0	29.4184
57	29	FORECAST	1	29.3969
58	29	L95	1	29.1030
59	29	U95	1	29.6908
60	30	FORECAST	2	29.3511
61	30	L95	2	29.0367
62	30	U95	2	29.6655
63	31	FORECAST	3	29.3089
64	31	L95	3	28.9795
65	31	U95	3	29.6382
66	32	FORECAST	4	29.2399
67	32	L95	4	28.8938
68	32	U95	4	29.5859
69	33	FORECAST	5	29.1969
70	33	L95	5	28.8317
71	33	U95	5	29.5620
72	34	FORECAST	6	29.1588
73	34	L95	6	28.7720
74	34	U95	6	29.5455

## CLINCHFIELD

PARAMETER ESTIMATE STD ERROR T RATIO LAG  
 AR1,1 0.763271 0.0968657 7.88 1

VARIANCE ESTIMATE = 1.76741  
 STD ERROR ESTIMATE = 1.32944  
 AIC = 144.972  
 SBC = 146.71  
 NUMBER OF RESIDUALS = 42

DATA HAVE BEEN CENTERED.  
 NO MEAN TERM IN THIS MODEL.

AUTOREGRESSIVE FACTORS  
 FACTOR 1  
 1-.763271B\*\*(1)  
 SAS

MODEL FOR VARIABLE MFOB  
 DATA HAVE BEEN CENTERED.  
 NO MEAN TERM IN THIS MODEL.

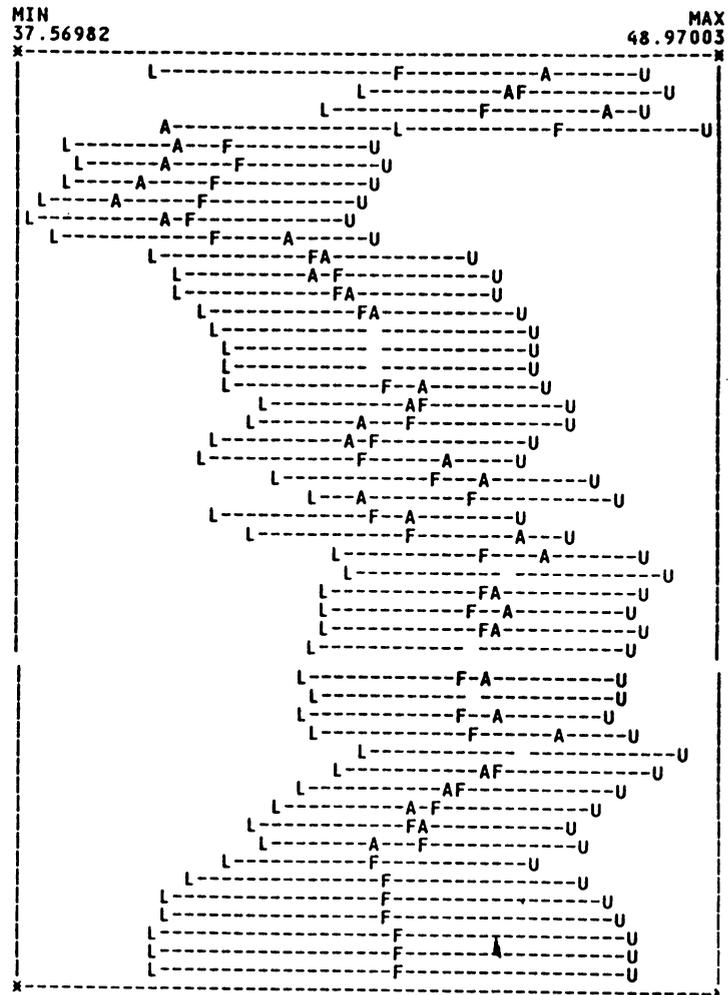
AUTOREGRESSIVE FACTORS  
 FACTOR 1  
 1-.763271B\*\*(1)  
 SAS

## FORECASTS FOR VARIABLE MFOB

OBS	FORECAST	STD ERROR	LOWER 95%	UPPER 95%	ACTUAL	RESIDUAL
1	43.7179	2.0577	39.6848	47.7509	46.2001	2.4822
2	45.6125	1.3294	43.0068	48.2181	45.4797	-0.1328
3	45.0626	1.3294	42.4570	47.6683	47.1852	2.1226
4	46.3644	1.3294	43.7587	48.9700	39.8001	-6.5643
5	40.7275	1.3294	38.1219	43.3332	40.0649	-0.6626
6	40.9297	1.3294	38.3240	43.5353	39.7576	-1.1721
7	40.6951	1.3294	38.0895	43.3008	39.4422	-1.2529
8	40.4544	1.3294	37.8487	43.0600	39.0768	-1.3776
9	40.1755	1.3294	37.5698	42.7811	39.7393	-0.4362
10	40.6811	1.3294	38.0755	43.2868	41.8026	1.1215
11	42.2560	1.3294	39.6503	44.8616	42.3823	0.1263
12	42.6985	1.3294	40.0928	45.3041	42.3440	-0.3545
13	42.6692	1.3294	40.0636	45.2749	42.8442	0.1750
14	43.0510	1.3294	40.4454	45.6567	43.1857	0.1347
15	43.3117	1.3294	40.7060	45.9173	43.2557	-0.0560
16	43.3651	1.3294	40.7595	45.9708	43.2149	-0.1502
17	43.3340	1.3294	40.7283	45.9396	43.3579	0.0239
18	43.4431	1.3294	40.8375	46.0488	44.0862	0.6431
19	43.9990	1.3294	41.3934	46.6047	43.8638	-0.1352
20	43.8293	1.3294	41.2236	46.4349	43.1559	-0.6734
21	43.2889	1.3294	40.6833	45.8946	42.9355	-0.3534
22	43.1207	1.3294	40.5151	45.7264	44.4729	1.3522
23	44.2942	1.3294	41.6885	46.8998	45.1670	0.8728
24	44.8239	1.3294	42.2183	47.4296	43.0146	-1.8093
25	43.1811	1.3294	40.5754	45.7867	43.9637	0.7826
26	43.9055	1.3294	41.2999	46.5112	45.6374	1.7319
27	45.1830	1.3294	42.5773	47.7886	46.0723	0.8893
28	45.5149	1.3294	42.9093	48.1206	45.5184	0.0035
29	45.0922	1.3294	42.4865	47.6978	45.3975	0.3053
30	44.9999	1.3294	42.3942	47.6055	45.4669	0.4670
31	45.0529	1.3294	42.4472	47.6585	45.2430	0.1901
32	44.8820	1.3294	42.2763	47.4876	44.9365	0.0545
33	44.6480	1.3294	42.0424	47.2537	45.1381	0.4901
34	44.8019	1.3294	42.1962	47.4075	44.8671	0.0652
35	44.5950	1.3294	41.9894	47.2007	45.3339	0.7389
36	44.9513	1.3294	42.3457	47.5570	46.2720	1.3207
37	45.6674	1.3294	43.0617	48.2730	45.8137	0.1463
38	45.3176	1.3294	42.7119	47.9232	45.0058	-0.3118
39	44.7009	1.3294	42.0953	47.3066	44.3978	-0.3031
40	44.2368	1.3294	41.6312	46.8425	43.8520	-0.3848
41	43.8202	1.3294	41.2146	46.4259	44.1717	0.3515
42	44.0643	1.3294	41.4586	46.6699	43.2334	-0.8309
-----FORECAST BEGINS-----						
43	43.3481	1.3294	40.7424	45.9537		
44	43.4356	1.6724	40.1577	46.7136		
45	43.5024	1.8431	39.8901	47.1148		
46	43.5534	1.9356	39.7598	47.3471		
47	43.5924	1.9875	39.6970	47.4877		
48	43.6221	2.0171	39.6687	47.5755		
49	43.6447	2.0447	39.6687	47.6117		

CLINCHFIELD (CONT.)

TIME	FORECAST FOR MFOB	MFOB	LOWER 95% CONFIDENCE LIMIT	UPPER 95% CONFIDENCE LIMIT
1	43.72	46.20	39.68	47.75
2	45.61	45.48	43.01	48.22
3	45.06	47.19	42.46	47.67
4	46.36	39.80	43.76	48.97
5	40.73	40.06	38.12	43.33
6	40.93	39.76	38.32	43.54
7	40.70	39.44	38.09	43.30
8	40.45	39.08	37.85	43.06
9	40.18	39.74	37.57	42.78
10	40.68	41.80	38.08	43.29
11	42.26	42.38	39.65	44.86
12	42.70	42.34	40.09	45.30
13	42.67	42.84	40.06	45.27
14	43.05	43.19	40.45	45.66
15	43.31	43.26	40.71	45.92
16	43.37	43.21	40.76	45.97
17	43.33	43.36	40.73	45.94
18	43.44	44.09	40.84	46.05
19	44.00	43.86	41.39	46.60
20	43.83	43.16	41.22	46.43
21	43.29	42.94	40.68	45.89
22	43.12	44.47	40.52	45.73
23	44.29	45.17	41.69	46.90
24	44.82	43.01	42.22	47.43
25	43.18	43.96	40.58	45.79
26	43.91	45.64	41.30	46.51
27	45.18	46.07	42.58	47.79
28	45.51	45.52	42.91	48.12
29	45.09	45.40	42.49	47.70
30	45.00	45.47	42.39	47.61
31	45.05	45.24	42.45	47.66
32	44.88	44.94	42.28	47.49
33	44.65	45.14	42.04	47.25
34	44.80	44.87	42.20	47.41
35	44.60	45.33	41.99	47.20
36	44.95	46.27	42.35	47.56
37	45.67	45.81	43.06	48.27
38	45.32	45.01	42.71	47.92
39	44.70	44.40	42.10	47.31
40	44.24	43.85	41.63	46.84
41	43.82	44.17	41.21	46.43
42	44.06	43.23	41.46	46.67
43	43.35	.	40.74	45.95
44	43.44	.	40.16	46.71
45	43.50	.	39.89	47.11
46	43.55	.	39.76	47.35
47	43.59	.	39.70	47.49
48	43.62	.	39.67	47.58
49	43.64	.	39.66	47.63



FORECAST OF ENTERPRISE/PIKE/LT/ TO VEPCO

OBS	_TYPE_	TIME	MFOB
1	N	6	6
2	SIGMA	6	0.3059412
3	CONSTANT	6	28.91
4	AR1	6	.
5	AR2	6	.
6	AR3	6	.

SAS

OBS	TIME	_TYPE_	_LEAD_	MFOB
1	1	ACTUAL	0	28.2900
2	1	FORECAST	0	28.9100
3	2	ACTUAL	0	28.9900
4	2	FORECAST	0	28.9100
5	3	ACTUAL	0	28.9900
6	3	FORECAST	0	28.9100
7	4	ACTUAL	0	29.0500
8	4	FORECAST	0	28.9100
9	5	ACTUAL	0	29.0700
10	5	FORECAST	0	28.9100
11	6	ACTUAL	0	29.0700
12	6	FORECAST	0	28.9100
13	7	FORECAST	1	28.9100
14	7	L95	1	28.2623
15	7	U95	1	29.5577
16	8	FORECAST	2	28.9100
17	8	L95	2	28.2623
18	8	U95	2	29.5577
19	9	FORECAST	3	28.9100
20	9	L95	3	28.2623
21	9	U95	3	29.5577
22	10	FORECAST	4	28.9100
23	10	L95	4	28.2623
24	10	U95	4	29.5577
25	11	FORECAST	5	28.9100
26	11	L95	5	28.2623
27	11	U95	5	29.5577
28	12	FORECAST	6	28.9100
29	12	L95	6	28.2623
30	12	U95	6	29.5577

## ESTIMATES FROM FORECAST FOR GEX/PIKE

OBS	_TYPE_	TIME	FOB_T
1	N	25	25
2	SIGMA	25	0.1180783
3	CONSTANT	25	29.10821
4	LINEAR	25	0.05791
5	AR1	25	0.3993971
6	AR2	25	.
7	AR3	25	.
8	AR4	25	.
9	AR5	25	.
10	AR6	25	.
11	AR7	25	-0.291362
12	AR8	25	.
13	AR9	25	.
14	AR10	25	.
15	AR11	25	.
16	AR12	25	-0.263559

## THE OUTPUT FORECASTED FROM PROC FORECAST FOR GEX/PIKE

OBS	TIME	_TYPE_	_LEAD_	FOB_T
1	1	ACTUAL	0	29.0100
2	1	FORECAST	0	29.1661
3	2	ACTUAL	0	29.0000
4	2	FORECAST	0	29.1617
5	3	ACTUAL	0	29.1370
6	3	FORECAST	0	29.1925
7	4	ACTUAL	0	29.2710
8	4	FORECAST	0	29.2820
9	5	ACTUAL	0	29.2600
10	5	FORECAST	0	29.3703
11	6	ACTUAL	0	29.4950
12	6	FORECAST	0	29.4006
13	7	ACTUAL	0	29.6420
14	7	FORECAST	0	29.5293
15	8	ACTUAL	0	29.6420
16	8	FORECAST	0	29.6683
17	9	ACTUAL	0	29.8000
18	9	FORECAST	0	29.7228
19	10	ACTUAL	0	29.9600
20	10	FORECAST	0	29.7977
21	11	ACTUAL	0	29.6400
22	11	FORECAST	0	29.8742
23	12	ACTUAL	0	30.0000
24	12	FORECAST	0	29.8012
25	13	ACTUAL	0	29.9420
26	13	FORECAST	0	29.9694
27	14	ACTUAL	0	30.0260
28	14	FORECAST	0	29.9729
29	15	ACTUAL	0	30.1120
30	15	FORECAST	0	30.0373
31	16	ACTUAL	0	30.1150

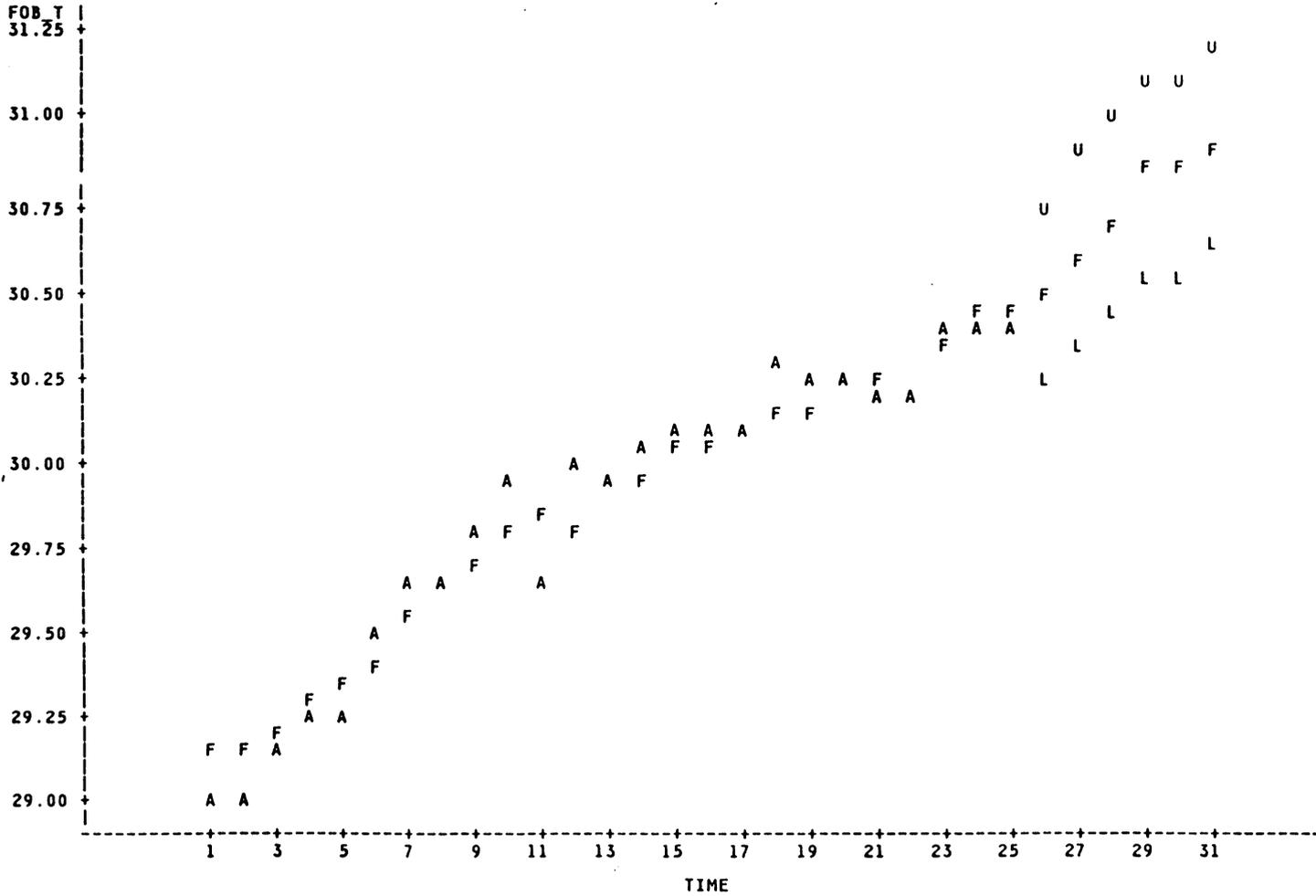
## GEX(CONT.)

OBS	TIME	TYPE	LEAD	FOB_T
32	16	FORECAST	0	30.0572
33	17	ACTUAL	0	30.1010
34	17	FORECAST	0	30.0816
35	18	ACTUAL	0	30.2820
36	18	FORECAST	0	30.1742
37	19	ACTUAL	0	30.2620
38	19	FORECAST	0	30.1698
39	20	ACTUAL	0	30.2300
40	20	FORECAST	0	30.2456
41	21	ACTUAL	0	30.2000
42	21	FORECAST	0	30.2336
43	22	ACTUAL	0	30.2010
44	22	FORECAST	0	30.2213
45	23	ACTUAL	0	30.4100
46	23	FORECAST	0	30.3721
47	24	ACTUAL	0	30.3770
48	24	FORECAST	0	30.4317
49	25	ACTUAL	0	30.4110
50	25	FORECAST	0	30.4480
51	26	FORECAST	1	30.5122
52	26	L95	1	30.2618
53	26	U95	1	30.7625
54	27	FORECAST	2	30.6062
55	27	L95	2	30.3372

THE OUTPUT FORECASTED FROM PROC FORECAST FOR GEX/PIKE

OBS	TIME	_TYPE_	_LEAD_	FOB_T
56	27	U95	2	30.8751
57	28	FORECAST	3	30.7186
58	28	L95	3	30.4449
59	28	U95	3	30.9922
60	29	FORECAST	4	30.8338
61	29	L95	4	30.5574
62	29	U95	4	31.1101
63	30	FORECAST	5	30.8381
64	30	L95	5	30.5592
65	30	U95	5	31.1170
66	31	FORECAST	6	30.9216
67	31	L95	6	30.6402
68	31	U95	6	31.2031

PLOT OF FORECAST FOR GEX/PIKE  
 PLOT OF FOB\_T\*TIME SYMBOL IS VALUE OF \_TYPE\_



ARIMA ANALYSIS FOR MAJESTIC/PIKE KY. TO VEPCO

ARIMA: CONDITIONAL LEAST SQUARES ESTIMATION

PARAMETER ESTIMATE APPROX. STD ERROR T RATIO LAG  
 AR1,1 0.897762 0.0964541 9.31 1

VARIANCE ESTIMATE = 0.0485233  
 STD ERROR ESTIMATE = 0.22028  
 AIC = -4.65205\*  
 SBC = -3.25086\*  
 NUMBER OF RESIDUALS= 30

MODEL FOR VARIABLE FOB\_T  
 DATA HAVE BEEN CENTERED.  
 NO MEAN TERM IN THIS MODEL.

AUTOREGRESSIVE FACTORS  
 FACTOR 1

1-.897762B\*\*(1)

ARIMA ANALYSIS FOR MAJESTIC/PIKE KY. TO VEPCO

MODEL FOR VARIABLE FOB\_T  
 DATA HAVE BEEN CENTERED.  
 NO MEAN TERM IN THIS MODEL.

OBS	FORECAST	STD ERROR	LOWER 95%	UPPER 95%	ACTUAL	RESIDUAL
1	29.7113	0.2203	29.2796	30.1431	29.0000	-0.7113
2	29.0727	0.2203	28.6410	29.5045	29.0000	-0.0727
3	29.0727	0.2203	28.6410	29.5045	29.0000	-0.0727
4	29.0727	0.2203	28.6410	29.5045	29.0000	-0.0727
5	29.0727	0.2203	28.6410	29.5045	28.7710	-0.3017
6	28.8671	0.2203	28.4354	29.2989	29.4000	0.5329
7	29.4318	0.2203	29.0001	29.8636	29.2550	-0.1768
8	29.3017	0.2203	28.8699	29.7334	29.3790	0.0773
9	29.4130	0.2203	28.9812	29.8447	29.3800	-0.0330
10	29.4139	0.2203	28.9821	29.8456	29.6100	0.1961
11	29.6204	0.2203	29.1886	30.0521	29.8480	0.2276
12	29.8340	0.2203	29.4023	30.2658	29.8500	0.0160
13	29.8358	0.2203	29.4041	30.2676	29.9050	0.0692
14	29.8852	0.2203	29.4535	30.3169	30.2150	0.3298
15	30.1635	0.2203	29.7318	30.5952	30.2150	0.0515
16	30.1635	0.2203	29.7318	30.5952	30.2160	0.0525
17	30.1644	0.2203	29.7327	30.5961	30.1080	-0.0564
18	30.0674	0.2203	29.6357	30.4992	30.0850	0.0176
19	30.0468	0.2203	29.6151	30.4785	30.0440	-0.0028
20	30.0100	0.2203	29.5782	30.4417	29.7010	-0.3090
21	29.7021	0.2203	29.2703	30.1338	29.7010	-0.0011
22	29.7021	0.2203	29.2703	30.1338	29.7820	0.0799
23	29.7748	0.2203	29.3430	30.2065	29.8650	0.0902
24	29.8493	0.2203	29.4175	30.2810	29.8690	0.0197
25	29.8529	0.2203	29.4211	30.2846	29.8690	0.0161
26	29.8529	0.2203	29.4211	30.2846	29.7900	-0.0629
27	29.7820	0.2203	29.3502	30.2137	29.8380	0.0560
28	29.8250	0.2203	29.3933	30.2568	30.0150	0.1900
29	29.9840	0.2203	29.5522	30.4157	30.2890	0.3050
30	30.2299	0.2203	29.7982	30.6617	30.3400	0.1101
-----FORECAST BEGINS-----						
31	30.2757	0.2203	29.8440	30.7075		
32	30.2180	0.2960	29.6378	30.7982		
33	30.1662	0.3452	29.4897	30.8428		
34	30.1197	0.3802	29.3745	30.8649		
35	30.0780	0.4062	29.2817	30.8742		
36	30.0405	0.4261	29.2054	30.8754		

## ESTIMATES FROM FORECAST FOR VA. ENERGY

OBS	_TYPE_	TIME	FOB_T
1	N	29	29
2	SIGMA	29	0.2748904
3	CONSTANT	29	30.35441
4	LINEAR	29	0.06497044
5	AR1	29	0.2775931
6	AR2	29	.
7	AR3	29	.
8	AR4	29	.
9	AR5	29	.
10	AR6	29	.
11	AR7	29	.
12	AR8	29	.
13	AR9	29	.
14	AR10	29	.
15	AR11	29	.
16	AR12	29	.

## THE OUTPUT FORECASTED FROM PROC FORECAST

OBS	TIME	_TYPE_	_LEAD_	FOB_T
1	1	ACTUAL	0	29.2800
2	1	FORECAST	0	30.4194
3	2	ACTUAL	0	30.5000
4	2	FORECAST	0	30.1681
5	3	ACTUAL	0	30.5000
6	3	FORECAST	0	30.5537
7	4	ACTUAL	0	30.6400
8	4	FORECAST	0	30.6006
9	5	ACTUAL	0	30.7400
10	5	FORECAST	0	30.6864
11	6	ACTUAL	0	30.7500
12	6	FORECAST	0	30.7611
13	7	ACTUAL	0	30.9100
14	7	FORECAST	0	30.8108
15	8	ACTUAL	0	30.9900
16	8	FORECAST	0	30.9022
17	9	ACTUAL	0	30.7500
18	9	FORECAST	0	30.9713
19	10	ACTUAL	0	31.2700
20	10	FORECAST	0	30.9516
21	11	ACTUAL	0	31.3400
22	11	FORECAST	0	31.1429
23	12	ACTUAL	0	31.3400
24	12	FORECAST	0	31.2093
25	13	ACTUAL	0	31.4600
26	13	FORECAST	0	31.2562
27	14	ACTUAL	0	31.5900
28	14	FORECAST	0	31.3364
29	15	ACTUAL	0	31.5900
30	15	FORECAST	0	31.4195
31	16	ACTUAL	0	31.5900

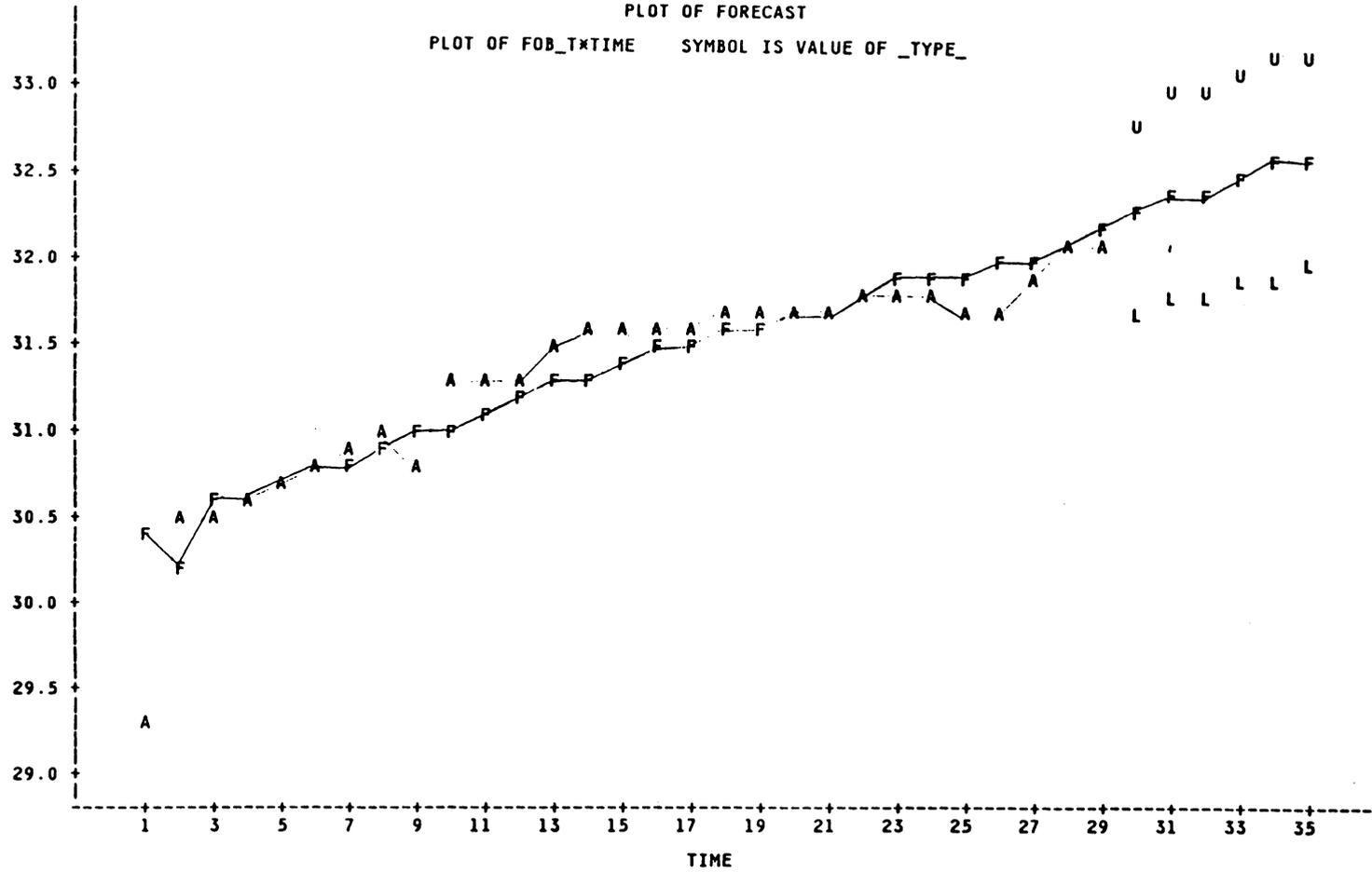
## VA. ENERGY (CONT.)

THE OUTPUT FORECASTED FROM PROC FORECAST				
OBS	TIME	TYPE	LEAD	FOB_T
32	16	FORECAST	0	31.4664
33	17	ACTUAL	0	31.5800
34	17	FORECAST	0	31.5133
35	18	ACTUAL	0	31.6500
36	18	FORECAST	0	31.5575
37	19	ACTUAL	0	31.7200
38	19	FORECAST	0	31.6239
39	20	ACTUAL	0	31.6800
40	20	FORECAST	0	31.6902
41	21	ACTUAL	0	31.7100
42	21	FORECAST	0	31.7261
43	22	ACTUAL	0	31.8000
44	22	FORECAST	0	31.7813
45	23	ACTUAL	0	31.8000
46	23	FORECAST	0	31.8532
47	24	ACTUAL	0	31.7700
48	24	FORECAST	0	31.9002
49	25	ACTUAL	0	31.7100
50	25	FORECAST	0	31.9388
51	26	ACTUAL	0	31.7400
52	26	FORECAST	0	31.9691
53	27	ACTUAL	0	31.9000
54	27	FORECAST	0	32.0243
55	28	ACTUAL	0	32.1200
THE OUTPUT FORECASTED FROM PROC FORECAST				
OBS	TIME	_TYPE_	_LEAD_	FOB_T
56	28	FORECAST	0	32.1157
57	29	ACTUAL	0	32.1200
58	29	FORECAST	0	32.2237
59	30	FORECAST	1	32.2706
60	30	L95	1	31.6940
61	30	U95	1	32.8472
62	31	FORECAST	2	32.3594
63	31	L95	2	31.7600
64	31	U95	2	32.9587
65	32	FORECAST	3	32.4309
66	32	L95	3	31.8262
67	32	U95	3	33.0357
68	33	FORECAST	4	32.4977
69	33	L95	4	31.8888
70	33	U95	4	33.1067
71	34	FORECAST	5	32.5632
72	34	L95	5	31.9499
73	34	U95	5	33.1765
74	35	FORECAST	6	32.6283
75	35	L95	6	32.0105
76	35	U95	6	33.2462

# VIRGINIA ENERGY

PLOT OF FORECAST

PLOT OF FOB\_T\*TIME SYMBOL IS VALUE OF \_TYPE\_



## ARIMA ANALYSIS FOR WELLMORE

PARAMETER	ESTIMATE	APPROX. STD ERROR	T RATIO	LAG	VARIABLE	SHFT
AR1,1	0.698395	0.14545	4.80	1	FOB_T	0
AR1,2	-0.374125	0.165684	-2.26	5	FOB_T	0
NUM1	3.26153	2.37991	1.37	0	SULFUR	13
NUM1,1	2.09245	2.43538	0.86	1	SULFUR	13

VARIANCE ESTIMATE = 0.549055  
 STD ERROR ESTIMATE = 0.740982  
 AIC = 64.1054\*  
 SBC = 69.2887\*

NUMBER OF RESIDUALS= 27  
 \* DOES NOT INCLUDE LOG DETERMINANT

MODEL FOR VARIABLE FOB\_T  
 DATA HAVE BEEN CENTERED.  
 NO MEAN TERM IN THIS MODEL.

AUTOREGRESSIVE FACTORS

FACTOR 1

1-.698395B\*\*(1)+0.374125B\*\*(5)

INPUT NUMBER 1 IS SULFUR  
 WITH A SHIFT OF 13  
 PERIODS OF DIFFERENCING=1.  
 THE NUMERATOR FACTORS ARE

FACTOR 1

3.26153-2.09245B\*\*(1)

FULL MODEL WITH NOISE AR(1,5) AND INPUT SERIES (B,R,S)=(13,0,1)

## FORECASTS FOR VARIABLE FOB\_T

OBS	FORECAST	STD ERROR	LOWER 95%	UPPER 95%	ACTUAL	RESIDUAL
16	41.0667	0.7410	39.6144	42.5190	40.6560	-0.4107
17	40.7916	0.7410	39.3393	42.2439	42.2470	1.4554
18	41.9062	0.7410	40.4539	43.3585	41.3890	-0.5172
19	41.2779	0.7410	39.8256	42.7302	41.5460	0.2681
20	41.3043	0.7410	39.8520	42.7566	42.7840	1.4797
21	42.7473	0.7410	41.2950	44.1996	43.4610	0.7137
22	41.8659	0.7410	40.4136	43.3182	40.7930	-1.0729
23	40.9617	0.7410	39.5094	42.4140	40.8490	-0.1127
24	40.8312	0.7410	39.3789	42.2835	40.7050	-0.1262
25	39.8896	0.7410	38.4373	41.3419	39.5630	-0.3266
26	39.4884	0.7410	38.0361	40.9407	39.6460	0.1576
27	39.7086	0.7410	38.2563	41.1609	40.6670	0.9584
28	41.1801	0.7410	39.7278	42.6324	40.7020	-0.4781
29	41.2892	0.7410	39.8369	42.7415	40.5920	-0.6972
30	40.3156	0.7410	38.8633	41.7679	39.9270	-0.3886
31	41.6864	0.7410	40.2341	43.1387	41.1140	-0.5724
32	40.7636	0.7410	39.3113	42.2158	40.0860	-0.6776
33	40.9220	0.7410	39.4698	42.3743	40.4650	-0.4570
34	40.5348	0.7410	39.0825	41.9871	40.2900	-0.2448
35	40.6474	0.7410	39.1951	42.0997	39.7600	-0.8874
36	40.5607	0.7410	39.1084	42.0130	40.2150	-0.3457
37	40.8789	0.7410	39.4266	42.3312	40.4810	-0.3979
38	40.5858	0.7410	39.1335	42.0381	40.9710	0.3852
39	41.4888	0.7410	40.0365	42.9411	42.1930	0.7042
40	42.4418	0.7410	40.9895	43.8941	43.3440	0.9022
41	42.9639	0.7410	41.5116	44.4162	42.2920	-0.6719
42	41.7973	0.7410	40.3450	43.2496	42.2180	0.4207

-----FORECAST BEGINS-----

43	42.2044	0.7410	40.7521	43.6567
44	41.3158	0.9038	39.5444	43.0872
45	40.1867	0.9734	38.2789	42.0945
46	40.5078	1.0056	38.5369	42.4787
47	39.9665	1.0209	37.9655	41.9674
48	39.8436	1.0325	37.8199	41.8672

## ESTIMATES FROM FORECAST FOR HARMAN MINING

OBS	_TYPE_	TIME	FOB_T
1	N	23	23
2	SIGMA	23	0.3735385
3	CONSTANT	23	32.25731
4	LINEAR	23	0.03497036
5	AR1	23	.
6	AR2	23	.
7	AR3	23	.
8	AR4	23	.
9	AR5	23	.
10	AR6	23	.
11	AR7	23	.
12	AR8	23	-0.433906
13	AR9	23	.
14	AR10	23	.
15	AR11	23	.

## THE OUTPUT FORECASTED FROM PROC FORECAST

OBS	TIME	_TYPE_	_LEAD_	FOB_T
1	1	ACTUAL	0	32.0000
2	1	FORECAST	0	32.2923
3	2	ACTUAL	0	32.0000
4	2	FORECAST	0	32.3273
5	3	ACTUAL	0	32.1700
6	3	FORECAST	0	32.3622
7	4	ACTUAL	0	32.5200
8	4	FORECAST	0	32.3972
9	5	ACTUAL	0	32.5200
10	5	FORECAST	0	32.4322
11	6	ACTUAL	0	32.5500
12	6	FORECAST	0	32.4671
13	7	ACTUAL	0	32.9300
14	7	FORECAST	0	32.5021
15	8	ACTUAL	0	32.8800
16	8	FORECAST	0	32.5371
17	9	ACTUAL	0	32.9300
18	9	FORECAST	0	32.6989
19	10	ACTUAL	0	32.9300
20	10	FORECAST	0	32.7490
21	11	ACTUAL	0	32.9300
22	11	FORECAST	0	32.7254
23	12	ACTUAL	0	32.8000
24	12	FORECAST	0	32.6237
25	13	ACTUAL	0	32.6600
26	13	FORECAST	0	32.6738
27	14	ACTUAL	0	32.6700
28	14	FORECAST	0	32.7109
29	15	ACTUAL	0	31.2700
30	15	FORECAST	0	32.5962
31	16	ACTUAL	0	32.8500
32	16	FORECAST	0	32.6680

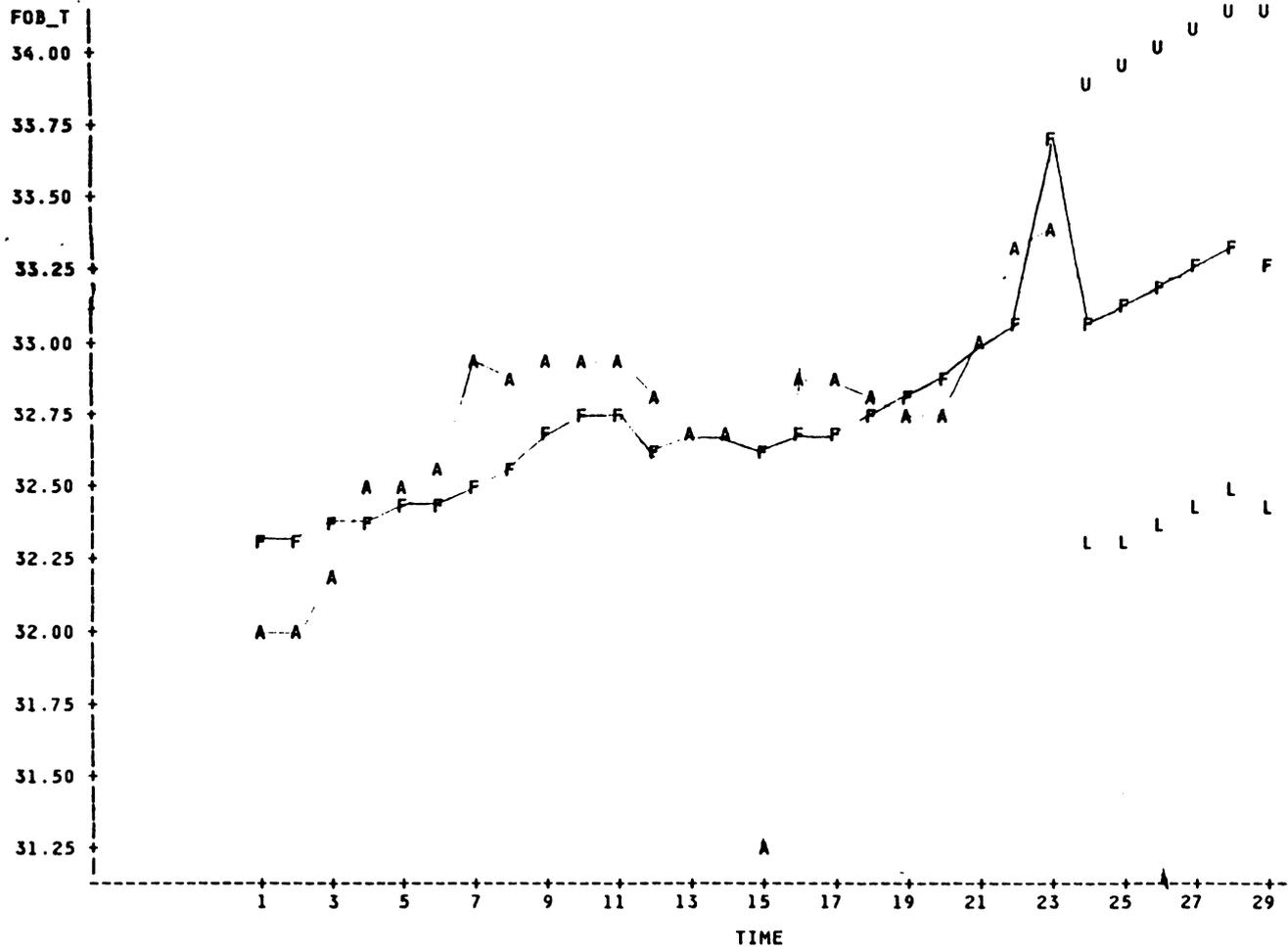
HARMAN MINING (CONT.)				
OBS	TIME	TYPE	LEAD	FOB_T
33	17	ACTUAL	0	32.8500
34	17	FORECAST	0	32.6965
35	18	ACTUAL	0	32.8100
36	18	FORECAST	0	32.7466
37	19	ACTUAL	0	32.7800
38	19	FORECAST	0	32.7968
39	20	ACTUAL	0	32.7800
40	20	FORECAST	0	32.9033
41	21	ACTUAL	0	33.0300
42	21	FORECAST	0	33.0142
43	22	ACTUAL	0	33.3300
44	22	FORECAST	0	33.0600
45	23	ACTUAL	0	33.3800
46	23	FORECAST	0	33.7176
47	24	FORECAST	1	33.0822
48	24	L95	1	32.2850
49	24	U95	1	33.8794
50	25	FORECAST	2	33.1324
51	25	L95	2	32.3269
52	25	U95	2	33.9378
53	26	FORECAST	3	33.1999
54	26	L95	3	32.3855
55	26	U95	3	34.0142

THE OUTPUT FORECASTED FROM PROC FORECAST

OBS	TIME	_TYPE_	_LEAD_	FOB_T
56	27	FORECAST	4	33.2630
57	27	L95	4	32.4393
58	27	U95	4	34.0867
59	28	FORECAST	5	33.3132
60	28	L95	5	32.4796
61	28	U95	5	34.1468
62	29	FORECAST	6	33.2548
63	29	L95	6	32.4108
64	29	U95	6	34.0989

HARMAN MINING

PLOT OF FOB\_T\*TIME SYMBOL IS VALUE OF \_TYPE\_



## APPENDIX D

### Data Formats & Programs for Reading HB966 & Work Address/Employment Tapes

HSAC-MSHA  
DOCUMENTATION

SYSTEM - CONTRACTOR  
RD - ADDRESS-EMPLOYMENT  
DATA ELEMENTS - 2

02/17/83

<u>CHARACTER POSITIONS</u>	<u>DATA ELEMENTS</u>	<u>PICTURE</u>	<u>DESCRIPTION</u>
1-3	Contractor	Pic X(3)	MSHA Contractor code assigned to an independent contractor.
4-10	Mine ID	Pic 9(7)	Constant value of zeros.
11-12	Filler	Pic 99	
13-16	Inspection Office	Pic 9(4)	Constant value of 9998.
17-18	State Code	Pic 99	Constant value of 98.
19-21	County Code	Pic 999	Constant value of 998.
22-26	SIC	Pic 9(5)	Constant value of 99998.
27	Canvass or Class	Pic 9	Constant value of 9.
28-29	Mine Type	Pic 99	Constant value of 14.
30	Status Code	Pic X	Code for status of operations of contractor (active to permanently closed.) Coal = Alpha A through H, Metal/Nonmetal = Numeric - 1, 2, and 3.
31-36	Status Date	Pic X(6)	Date of latest add or change of status. YMMDD.
37-40	Seam Height	Pic 9(4)	Constant value of zeros.

<u>CHARACTER POSITIONS</u>	<u>DATA ELEMENTS</u>	<u>PICTURE</u>	<u>DESCRIPTION</u>
41-42	Education & Training District	Pic 99	Constant value of zeros.
43	Surface/Underground Indicator	Pic X	Constant value of spaces.
44-46	Travel Area	Pic X(3)	Constant value of spaces.
47	Mailing Control	Pic 9	Provides for suppression of mailouts. 0 = no suppression; 1 = suppress selected mailouts.
48-77	Company Name	Pic X(30)	Company owning or having primary responsibility for this contractor code.
78-107	Mine or Plant Name	Pic X(30)	Constant value of ALL MINING OPERATIONS.
108-137	Street or PO Box Number	Pic X(30)	Mailing address for this contractor.
138-150	City	Pic X(13)	City to which mail is sent for this contractor.
151-152	State Abbreviation	Pic XX	State abbreviation for mailing purposes.
153-157	Zip Code	Pic 9(5)	Zip Code for mailing purposes.
158-181	County Name	Pic X(24)	Constant value of VARIOUS COUNTIES.

<u>CHARACTER POSITIONS</u>	<u>DATA ELEMENTS</u>	<u>PICTURE</u>	<u>DESCRIPTION</u>
			The next two items represent information supplied quarterly by the contractor on Form 7000-2. They do not accurately reflect actual accidents/illnesses reported. Occurs 4 times - once for each report quarter.
182 (Occurrence 1)	Injury Flag	Pic 9	Contractor statement indicating whether there were reportable injuries or illnesses during this report quarter. 1 if yes; 2 if no.
183-185 (Occurrence 1)	Injury Count	Pic 9(3)	Number of reportable accidents and illnesses given on employment form.
198-199	Filler	Pic XX	
200-201	Update Addition Year	Pic 99	Year address information was added to file.
202-204	Update Addition Number	Pic 999	Update cycle number address information was added to file.
205-206	Update Change Year	Pic 99	Year of latest change to address information.
207-209	Update Change Number	Pic 999	Update cycle number of latest change to address information.
210	Number of Subunit Operations	Pic 9	Number of subunit operations for the contractor (employment trailer count). Value can vary from zero to nine.

SAC-MSHA  
DOCUMENTATION

SYSTEM - CONTRACTOR  
RD - ADDRESS-EMPLOYMENT  
DATA ELEMENTS - 5

02/17/83

Following the address portion of the record, there may be from 0 to 9 employment "trailers" each containing 4 quarters of employment data for that operations subunit. See character position 210.

Employment trailer length - 130.

<u>CHARACTER POSITIONS</u>	<u>DATA ELEMENTS</u>	<u>PICTURE</u>	<u>DESCRIPTION</u>
			Information obtained from Form 7000-2.
211-212	Operations Subunit	Pic 99	Subunit operations code.  Next four elements are repeated four times representing four calendar quarters.
213-221	Document Number	Pic 9(9)	Number assigned to the document upon receipt in mailroom of HSAC and stamped on form.
222-226	Number of Employees	Pic 9(5)	Average number of persons working during quarter in this operations subunit. Item 1 (2).
227-234	Number of Employee-Hours	Pic 9(8)	Total employee-hours worked during the quarter in this operations subunit. Item 1 (3).
235-244	Tons of Production	Pic 9(10)	Production of clean coal (short tons) during quarter. Item 1 (4).

The Address/Employment tape is a sequential file. The key is the Contractor Code and the Mine-ID (positions 1-10) of each record.

JCL & SAS CODE FOR READING MSHA ADDRESS/EMPLOYMENT TAPES AND GENERATING  
AND PRINTING A SAS DATA SET

```
//boxid JOB acctnt,name ,REGION=1024K,PRTY=IDLE
/*ROUTE PRINT VPIVM1.HTIMS
/*ROUTE PUNCH VPIVM1.HTIMS
/*JOBPARM LINES=4,CARDS=100,ACCTPG
//STEP1 EXEC SAS
//WRK44 DD DSN=WORK844,
// UNIT=TAPE,
// VOL=SER=MSHA44,
// DISP=(OLD,KEEP),
// DCB=(RECFM=U,BLKSIZE=800,DEN=3),
// LABEL=(1,NL)
//WKDISK DD DSN=A32555.WORK844,
// UNIT=SYSDA,
// SPACE=(6160,(50,20),RLSE),
// DISP=(NEW,CATLG,DELETE)
//SYSIN DD *
* SAS DATA SET GENERATION AND SORTING ROUTINES;
DATA WKDISK.WORK844;
  INFILE WRK44;
  INPUT MINEID 4-10 STATEID 17-18 COUNTYID 19-21 SIC 22-26
        TYPE 28-29 STATUS $ 30-30 DATE 31-36 SEAMH 37-40 MINE $ 43-43
        COMPANY $ 48-77 SOURCE $ 78-107 ADR $ 108-137 CITY $ 138-150
        STATE $ 151-152 ZIP 153-157 COUNTY $ 158-181 SUOP 210 @;
IF SUOP=0 THEN INPUT;
IF SUOP=1 THEN INPUT NM11 222-226 NMH11 227-234 TP11 235-244
                    NM12 254-258 NMH12 259-266 TP12 267-276
                    NM13 286-290 NMH13 291-298 TP13 299-308
                    NM14 318-322 NMH14 323-330 TP14 331-340;

IF SUOP=2 THEN INPUT NM11 222-226 NMH11 227-234 TP11 235-244
                    NM12 254-258 NMH12 259-266 TP12 267-276
                    NM13 286-290 NMH13 291-298 TP13 299-308
                    NM14 318-322 NMH14 323-330 TP14 331-340
                    NM21 352-356 NMH21 357-364 TP21 365-374
                    NM22 384-388 NMH22 389-396 TP22 397-406
                    NM23 416-420 NMH23 421-428 TP23 429-438
                    NM24 448-452 NMH24 453-460 TP24 461-470;

IF SUOP=3 THEN INPUT NM11 222-226 NMH11 227-234 TP11 235-244
                    NM12 254-258 NMH12 259-266 TP12 267-276
                    NM13 286-290 NMH13 291-298 TP13 299-308
                    NM14 318-322 NMH14 323-330 TP14 331-340
                    NM21 352-356 NMH21 357-364 TP21 365-374
                    NM22 384-388 NMH22 389-396 TP22 397-406
                    NM23 416-420 NMH23 421-428 TP23 429-438
                    NM24 448-452 NMH24 453-460 TP24 461-470
                    NM31 482-486 NMH31 487-494 TP31 495-504
                    NM32 514-518 NMH32 519-526 TP32 527-536
```

NM33 546-550 NMH33 551-558 TP33 559-568  
 NM34 578-582 NMH34 583-590 TP34 591-600;

IF SUOP=4 THEN INPUT NM11 222-226 NMH11 227-234 TP11 235-244  
 NM12 254-258 NMH12 259-266 TP12 267-276  
 NM13 286-290 NMH13 291-298 TP13 299-308  
 NM14 318-322 NMH14 323-330 TP14 331-340  
 NM21 352-356 NMH21 357-364 TP21 365-374  
 NM22 384-388 NMH22 389-396 TP22 397-406  
 NM23 416-420 NMH23 421-428 TP23 429-438  
 NM24 448-452 NMH24 453-460 TP24 461-470  
 NM31 482-486 NMH31 487-494 TP31 495-504  
 NM32 514-518 NMH32 519-526 TP32 527-536  
 NM33 546-550 NMH33 551-558 TP33 559-568  
 NM34 578-582 NMH34 583-590 TP34 591-600  
 NM41 612-616 NMH41 617-624 TP41 625-634  
 NM42 644-648 NMH42 649-656 TP42 657-666  
 NM43 676-680 NMH43 681-688 TP43 689-698  
 NM44 708-712 NMH44 713-720 TP44 721-730;

OPTIONS NODATE NONUMBER NOLABEL NOCENTER;  
 IF STATEID = 21 OR STATEID = 54 OR STATEID = 51;  
 IF SIC = 12110;  
 IF STATUS = 'A';  
 IF TYPE = 11 OR TYPE = 12;

DATA STATE0 (DROP = NM11 NMH11 TP11  
 NM12 NMH12 TP12  
 NM13 NMH13 TP13  
 NM14 NMH14 TP14  
 NM21 NMH21 TP21  
 NM22 NMH22 TP22  
 NM23 NMH23 TP23  
 NM24 NMH24 TP24  
 NM31 NMH31 TP31  
 NM32 NMH32 TP32  
 NM33 NMH33 TP33  
 NM34 NMH34 TP34  
 NM41 NMH41 TP41  
 NM42 NMH42 TP42  
 NM43 NMH43 TP43  
 NM44 NMH44 TP44);

SET WKDISK.WORK844;  
 IF SUOP=0;  
 IF \_N\_ >=50 THEN STOP;  
 \*IF STATE='WV';  
 \*IF COUNTY='KANA' OR COUNTY='BOON' OR COUNTY='LOGA';  
 PROC PRINT DATA=STATE0;  
 VAR MINEID STATEID COUNTYID SIC TYPE STATUS DATE SEAMH MINE COMPANY  
 SOURCE ADR CITY STATE ZIP COUNTY SUOP;  
 DATA STATE1 (DROP = NM21 NMH21 TP21

```

NM22 NMH22 TP22
NM23 NMH23 TP23
NM24 NMH24 TP24
NM31 NMH31 TP31
NM32 NMH32 TP32
NM33 NMH33 TP33
NM34 NMH34 TP34
NM41 NMH41 TP41
NM42 NMH42 TP42
NM43 NMH43 TP43
NM44 NMH44 TP44);

```

```

SET WKDISK.WORK844;
IF SUOP=1;
IF _N_ >=50 THEN STOP;
*IF STATE='WV';
*IF COUNTY='KANA' OR COUNTY='BOON' OR COUNTY='LOGA';
PROC PRINT DATA=STATE1;
  VAR MINEID STATEID COUNTYID SIC TYPE STATUS DATE SEAMH MINE COMPANY
  SOURCE ADR CITY STATE ZIP COUNTY SUOP;
PROC PRINT DATA=STATE1;
  VAR NM11 NMH11 TP11 NM12 NMH12 TP12 NM13 NMH13 TP13 NM14 NMH14 TP14;

```

```

DATA STATE2 (DROP = NM31 NMH31 TP31
                NM32 NMH32 TP32
                NM33 NMH33 TP33
                NM34 NMH34 TP34
                NM41 NMH41 TP41
                NM42 NMH42 TP42
                NM43 NMH43 TP43
                NM44 NMH44 TP44);

```

```

SET WKDISK.WORK844;
IF SUOP=2;
IF _N_ >=50 THEN STOP;
*IF STATE='WV';
*IF COUNTY='KANA' OR COUNTY='BOON' OR COUNTY='LOGA';
PROC PRINT DATA=STATE2;
  VAR MINEID STATEID COUNTYID SIC TYPE STATUS DATE SEAMH MINE COMPANY
  SOURCE ADR CITY STATE ZIP COUNTY SUOP;
PROC PRINT DATA=STATE2;
  VAR NM11 NMH11 TP11 NM12 NMH12 TP12 NM13 NMH13 TP13 NM14 NMH14 TP14;
PROC PRINT DATA=STATE2;
  VAR NM21 NMH21 TP21 NM22 NMH22 TP22 NM23 NMH23 TP23 NM24 NMH24 TP24;
DATA STATE3 (DROP = NM41 NMH41 TP41
                NM42 NMH42 TP42
                NM43 NMH43 TP43
                NM44 NMH44 TP44);

```

```

SET WKDISK.WORK844;
IF SUOP=3;
IF _N_ >=50 THEN STOP;
*IF STATE='WV';
*IF COUNTY='KANA' OR COUNTY='BOON' OR COUNTY='LOGA';

```

```

PROC PRINT DATA=STATE3;
  VAR MINEID STATEID COUNTYID SIC TYPE STATUS DATE SEAMH MINE COMPANY
  SOURCE ADR CITY STATE ZIP COUNTY SUOP;
PROC PRINT DATA=STATE3;
  VAR NM11 NMH11 TP11 NM12 NMH12 TP12 NM13 NMH13 TP13 NM14 NMH14 TP14;
PROC PRINT DATA=STATE3;
  VAR NM21 NMH21 TP21 NM22 NMH22 TP22 NM23 NMH23 TP23 NM24 NMH24 TP24;
PROC PRINT;
  VAR NM31 NMH31 TP31 NM32 NMH32 TP32 NM33 NMH33 TP33 NM34 NMH34 TP34;
DATA STATE4;
SET WKDISK.WORK844;
IF SUOP=4;
IF _N_ >=50 THEN STOP;
*IF STATE='WV';
*IF COUNTY='KANA' OR COUNTY='BOON' OR COUNTY='LOGA';
PROC PRINT DATA=STATE4;
  VAR MINEID STATEID COUNTYID SIC TYPE STATUS DATE SEAMH MINE COMPANY
  SOURCE ADR CITY STATE ZIP COUNTY SUOP;
PROC PRINT DATA=STATE4;
  VAR NM11 NMH11 TP11 NM12 NMH12 TP12 NM13 NMH13 TP13 NM14 NMH14 TP14;
PROC PRINT DATA=STATE4;
  VAR NM21 NMH21 TP21 NM22 NMH22 TP22 NM23 NMH23 TP23 NM24 NMH24 TP24;
PROC PRINT DATA=STATE4;
  VAR NM31 NMH31 TP31 NM32 NMH32 TP32 NM33 NMH33 TP33 NM34 NMH34 TP34;
PROC PRINT DATA=STATE4;
  VAR NM41 NMH31 TP41 NM42 NMH42 TP42 NM43 NMH43 TP43 NM44 NMH44 TP44;
/*
//

```

**The vita has been removed from  
the scanned document**