Optimal Resource Allocation Strategies to Protect Network-structured Systems

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Abstract

Protection of critical national infrastructure has received significant attention in the past recent years. As a result, researchers have developed methods to preserve and maintain critical infrastructure systems and minimize their vulnerability to disasters and disruptions. However, these models are often customized to meet the characteristics and functionality requirements for a particular system, and are computationally intensive and require simplifying assumptions. In this study, we first develop a tractable and relatively comprehensive model for optimizing maintenance planning of generic network-structured systems. We considered both linear and nonlinear objective functions for our problems. We then reformulate the model in order to enhance its computational effectiveness for large scale complex problems. The proposed modeling framework inherently captures the network topography, the stochastic nature of disruptions, and can be applied to network-structured systems for which performance is assessed based on network flow efficiency and mobility.

A hypothetical small-sized network is used to illustrate the developed models, and the proposed models are also applied to analyze a larger scale real network in order to assess their relative computational effectiveness and robustness. We selected the Istanbul highway network for the latter purpose because of its critical location and also because it has been considered in previous studies, which enables us to compare the effectiveness of our models with an existing model in the literature. We designed several test cases (considering single and multiple treatment types, and linear and nonlinear objectives), and solved them on the NEOS server using different available software. The results demonstrate that our models are capable of attaining optimal solutions within a very short time. Furthermore, the linear model is shown to yield a good approximation to the nonlinear model (it determined solutions within 0.3% of optimality, on average). Moreover, in both cases (our hypothetical illustrative example and the Istanbul highway network), the optimal policies obtained appear to favor the selection of fewer links and to apply a higher quality treatment to them.

To the memory of Azeem

Acknowledgements

In the Name of God, the Beneficient, the Merciful

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Chapter 1

Introduction

In recent years, and particularly after the 9/11 tragedy, the protection of critical national infrastructure has received significant attention. On a wide-scale, politicians and decision-makers have emphasized the importance and significance of protecting infrastructure systems such as electrical and power networks, water distribution systems, cyber infrastructure, and transportation networks against planned attacks, unplanned events, and natural disasters. Researchers, therefore, have tried to address two important issues: 1) what makes network systems vulnerable to disruptions and attacks, and 2) how can vulnerable systems be protected via appropriate pre-disaster and post disaster treatment plans. Scientists working on complex network systems have addressed the former challenge by looking into different types of network-structured configurations and their interactions that can make them vulnerable to failures and disruptions. On the other hand, Operations Research (OR) concepts and tools have been widely applied by managers and engineers to find optimal methods to maintain and repair systems and to minimize their vulnerability against disruptions and failures. In this study, we focus on the latter issue and propose a new maintenance optimization model that is simple and yet more generic as compared with existing models. We formulate the proposed maintenance optimization model for both single year and multi-year planning periods.

In the single year resource allocation problem (SRAP), the available maintenance budget is to be distributed across the elements of the network in order to maximize the performance of the network for one year. In the multi-year resource allocation problem (MRAP), the maintenance budget available in each year needs to be optimally allocated across different elements of the network for maximizing the network performance during the planning period.

This thesis mainly focuses on the SRAP model and is organized as follows. Chapter 2 provides a review of the current literature and existing studies concerning maintenance optimization of network-structured systems, and discusses the motivation of our research. Chapter 3 states the problem addressed herein and proposes mathematical models for the single and multi-year planning horizon versions of this problem. Chapter 4 then presents a suitable reformulation of the single year problem in order to enhance its solvability, along with a proposed algorithmic approach. Chapter 5 applies this developed procedure to a hypothetical illustrative network and a real large scale highway network and discusses the results obtained from the viewpoint of managerial insights. Finally, Chapter 6 provides conclusions and recommendations for future research.

Chapter 2

Literature Review

The maintenance and treatment of multi-component systems has been extensively studied in the past few decades. The main focus of these studies has been on parallel or series systems (Castanier et al. 2005; Barros et al. 2006; Zequeira and Berenguer, 2005; Satow and Osaki, 2003; Popova and Wilson, 1999; Nicolai and Dekker, 2008), and researchers have addressed various factors such as different types of dependencies between failure and maintenance of components (Dekker et al. 1996; Scarf and Deara, 1998; Scarf and Deara, 2003; Nicolai and Dekker, 2008). With the advancements in complex networks science, the research on network-based infrastructure systems has received significant attention in more recent years. Network science characterizes the interactions between the components of a system and provides a holistic view of the system that goes beyond a simple model of nodes and links (Barabasi, 2011). Researchers have used network science to understand the properties of social networks (Gonzalez et al. 2006), human mobility (Gonzalez et al. 2008), Web-space (Albert et al. 1999), and biological systems (Jeong et al. 2000; Ravasz et al. 2002), and to characterize vulnerability and failure impacts in infrastructure systems such as wireless communication systems (Buldyrev et al. 2010; Shao et al. 2011) and electrical networks (Rosas-Casals et al. 2007; Crucitti et al. 2004; Albert et al. 2004; Kinney et al. 2005).

Furthermore, several algorithms and methods have been developed to optimize maintenance planning for civil infrastructure facilities such as roadway networks, power grid networks, and water distribution systems in order to protect them against disasters and disruptions triggered by attacks or natural disasters. Among different infrastructure systems, roadways have received relatively more focus and attention. In the context of roadway networks, several studies have tried to plan maintenance activities for minimizing the vulnerability of the network due to a failure of bridges within the network. Liu and Frangopol (2005) used a genetic algorithm (GA) to allocate maintenance actions across bridges in the network with the objective of maximizing the time-dependent reliability of connectivity between specific origins and destinations and minimizing maintenance costs. They also considered user costs along with maintenance costs, as well as the cost of bridge failures (Liu and Frangopol, 2006), in addition to structural conditions and safety (Frangopol and Liu; 2007a). Later, Frangopol and Liu (2007b) used a two-stage stochastic dynamic program to solve a multi-objective optimization problem concerned with the maintenance planning of bridge networks. In the first stage, an optimal maintenance action that satisfies certain safety and service threshold requirements was identified for each individual bridge. In the second stage, the available limited annual budget was distributed across assets so that the optimal maintenance plans identified for each bridge can be implemented for a maximum number of bridges. Essahli and Madanat (2012) proposed an algorithm to optimize maintenance plans for bridge networks over a given planning horizon where the objective function captures the time-dependent connection reliabilities between origins and destinations within the network. However, due to the complexity of the problem, they only demonstrated the application of this optimization procedure for a five-bridge network and a one year planning horizon.

In addition to connectivity reliabilities that encapsulate the network topology, other factors such as traffic dynamics and interdependency of facilities and activities within the network have been addressed in maintenance scheduling problems. The concept of network topography dependency (NTD) was introduced by Papadakis and Kleindorfer (2005) for infrastructure systems, where they considered positive economic benefits if adjacent components are simultaneously grouped for maintenance. An undirected network was considered for this purpose and a maximum-flow-minimum-cut-based model was developed for deriving an exact and efficient solution algorithm.

Durango-Cohen and Sarutipand (2007) developed a quadratic programming model to optimize the maintenance policies for a transportation system, where they captured the interdependency of activities on the facilities within the system and attempted to coordinate the activities of adjacent facilities in order to minimize the total cost. A heuristic approach was used to accommodate the interdependency of facility activities within the proposed procedures.

Ng et al. (2009) formulated a mixed-integer bi-level program to minimize the cost of roadway maintenance activities and the system travel time over a long-term horizon. Traffic was simulated over the planning period and GA was used as a solution approach. The methodology was applied to a roadway having 24 links for a three year planning horizon. (The recorded time for the solution was over 2800 minutes using an Intel 3.00GHz Xeon(tm) CPU with 32-GB of RAM.)

In a few existing studies, researchers have formulated models to allocate pre-disaster (Peeta et al. 2010, Miller-Hooks et al. 2011) and post-disaster (Miller-Hooks et al. 2011) resources to roadways in the network in order to minimize the vulnerability of the system. Peeta et al. (2010) formulated the problem as a two-stage stochastic program, where investments in the first stage affect the failure probabilities of the corresponding links. They then reformulated the objective function as a non-increasing multi-linear function, and used a first-order Taylor series approximation in order to simplify the model to a knapsack problem, which was then used to derive a heuristic solution to the original problem. However, a drawback of this approach is that by disregarding the second and subsequent higher-order terms, the procedure does not capture the impact of simultaneous investments on more than one link.

Chen and Miller-Hooks (2011) developed an indicator for freight network resiliency based on the proportion of demands that are satisfied following a disastrous event by inherently capturing the operational and topographical attributes of the network. For each disaster (such as an earthquake, a terrorist attack, or a flood), a form of dependency in random link attributes (e.g., link capacity) was considered. They then developed a stochastic mixed-integer program to maximize resiliency while optimally scheduling post-event actions on links. A solution framework using Benders decomposition, column generation, and Monte Carlo simulation was proposed in order to reduce the computational effort by considering a tractable number of states for each disaster scenario. This study was improved by Miller-Hooks et al. (2011), who formulated a two-stage stochastic program to determine optimal preparedness actions (before a disaster) and recovery actions (after a disaster) on the network. A solution to this problem was obtained using the integer L-shaped method and Monte Carlo simulation.

The large size of networks, numerous facilities within the network, the number of maintenance options, and a multi-year planning horizon can significantly increase the complexity of such optimization problems. Researchers, therefore, have tried to use approximation techniques in order to reduce the accompanying computational effort. Several such strategies for resource/maintenance scheduling problems involve the use of approximate dynamic programming. Simão et al. (2009) developed an approximate dynamic programming approach for managing the fleet for a large-scale truck carrier. The problem considered a large number of factors such as driver attributes (location, domicile, capacity type, available time, days away from home, etc.) as well as attributes associated with trucks and loads. They were able to simulate the movements of 6000 drivers, and provided results that accurately matched with the historical performance of the company. Kuhn (2010) used approximate dynamic programming to select optimal maintenance policies for a heterogeneous set of facilities by approximating the value function of the Bellman equation. However, this work did not consider the connectivity, structure, and topography of the facilities in the network (the value function was simply taken as the summation of costs associated with each individual facility).

2.1 Motivation for the Present Study

The models developed for maintenance and treatment planning of infrastructure systems are customized based on the characteristics and functionality of the considered systems. Until today, research studies have not explicitly formulated a generic and comprehensive model for maintenance scheduling of network-structured systems. This model should ideally capture different factors such as the connection between network elements, interdependency of decisions on elements (for example, the benefits of grouping elements for maintenance), topography of the elements, network flow patterns, and factors such as stochasticity and uncertainty associated with, for example, disruptions and element functionality, etc. Moreover, current models are highly computationally intensive when considering large-scale networks, unless some oversimplifying assumptions are made. A computationally tractable model that captures the aforementioned details and provides acceptable solutions within reasonable times would be extremely useful to practitioners and engineers. In this study, we attempt to develop such a tractable and yet relatively comprehensive model for optimizing maintenance planning of generic network-structured systems. The model inherently captures the network topography as well as the stochastic nature of disruptions, and can be applied to network-structured systems for which performance is assessed based on network flow efficiency and mobility.

Chapter 3

Solution Methodology

In this chapter, we explain the problem that will be addressed in the present study and define the notation and parameters. We then provide the formulation for both the SRAP and MRAP models.

3.1 Problem Statement

Consider a network $G(N, K)$, where the model described below can represent any network-structured system such as an electrical network, a water distribution system, a communication network, or a roadway infrastructure. Latora and Marchiori (2001) introduced network efficiency as a function of the inverse of the shortest path as follows:

$$H(G) = \frac{1}{|N|(|N|-1)} \sum_{\{i,j\} \in OD} \frac{1}{d_{ij}}\,, \tag{1}$$

where $H(G)$ is a measure of how efficiently information is exchanged within the network, OD is the set of origin-destinations, and d_{ij} is the minimum path resistivity between node pair $\{i, j\} \in OD$. However, the measure does not account for f_{ij}, the amount of data, information, or load, etc., transferred between each node pair $\{i, j\} \in OD$. Taking f_{ij} into account, and dropping the constant multiplier $\frac{1}{|N|(|N|-1)}$, we will consider the following objective function:

$$\text{Maximize} \sum_{\{i,j\} \in OD} \frac{f_{ij}}{d_{ij}}, \tag{2a}$$

where d_{ij} represents the shortest path between i and j. In a probabilistic framework where each link has an associated probability density function of attenuated effective transmissivity (e.g., length), one could let d_{ij} represent either the expected shortest path length between i and j or the shortest expected path length between i and j. Note that these two measures are different in that the first represents the expected value of the minimum of path length random variables corresponding to paths between i and j, whereas the second represents the minimum of these expected path lengths. In this study, we use the second of these two measures because it is more amenable to an optimization modeling approach. On the other hand, computing the expected shortest path length, in general, requires determining the probability density function of the minimum of a set of dependent path length random variables corresponding to paths between i and j (or an enumeration of exponential potential discretized states of the system with simultaneous probabilistic link length attenuations), which is typically an onerous task. Furthermore, each maintenance action, a, will reduce the failure probability of link k by r_{ka} (assuming a two-state (generalized Bernoulli) link length random variable), where this reduction could possibly be independent of the links and their failure probabilities, i.e., $r_{ka} \equiv r_a, \forall k$. As an alternative, we will also consider the following objective function that seeks to minimize the total flow multiplied by the shortest path distance-related cost (or inefficiency):

$$\text{Minimize} \sum_{\{i,j\} \in OD} f_{ij} d_{ij}\,. \tag{2b}$$

The goal is then to develop suitable models and algorithms for determining a set of optimal maintenance actions for the network links in order to optimize either (2a) or (2b), and to compare the resulting policies, given budgetary and other physical constraints.

3.2 Notation

3.2.1 Sets and Parameters

$G(N, K)$: Network G with set of nodes N and set of links K.

OD: Set of origin-destination pairs $\{i, j\}$ of interest.

$A : \{a_1, a_2, ..., a_m\}$: Set of possible maintenance actions on the links.

B: Total available budget.

p_k: Disruption probability of link k before treatment.

c_{ka}: Cost of treating link k with maintenance action a.

r_{ka}: Reduction in disruption probability if link k receives maintenance type a ($r_{ka} \leq p_k, \forall a \in A, k \in K$).

l_k: Resistivity (length, travel time, etc.) of link k if undisrupted.

u_k: Resistivity of link k when disrupted ($u_k > l_k, \forall k \in K$).

q_{ij}: Path between node pair $\{i, j\} \in OD$.

Q_{ij}: Set of viable paths between node pair $\{i, j\} \in OD$ ($|Q_{ij}| \geq 1$ and is assumed to be relatively small $\forall \{i, j\} \in OD$).

U_q: Upper bound on the expected resistivity of path q (where the resistivity of a path is given by the sum of the resistivities of links in the path) for any set of disruption probabilities that do not exceed the untreated values ($p_k, \forall k \in q$). This can be computed as follows: $\sum_{k \in q} [(1 - p_k)l_k + p_k u_k]$, $\quad \forall q \in Q_{ij}, \{i, j\} \in OD$.

f_{ij}: Amount of flow, data, etc., to be transferred between node pair $\{i, j\} \in OD$.

3.2.2 Decision Variables

Principal decision variables:

x_{ka}: Binary variable that equals 1 if link k receives treatment type a, and is 0 otherwise.

Auxiliary decision variables:

y_k: Disruption probability of link k after treatment.

d_{ij}: Shortest expected path resistivity between node pair $\{i, j\} \in OD$ following link treatments.

λ_q^{ij}: Binary variable that equals 1 if path $q \in Q_{ij}$ computes d_{ij}, and is 0 otherwise.

3.3 Problem Formulation

In this section, we provide mathematical formulations for both single year and multi-year planning models.

3.3.1 Single Year Planning Model

In the single year planning model, we seek to optimally allocate the maintenance budget B, available for one year across different links in order to maintain the links in the network such that the network objective stated in the form of (2a) or (2b) is optimized. The model formulation for the single year resource allocation problem (SRAP) is given below.

$$\textbf{SRAP:} \quad \text{Maximize} \quad \sum_{\{i,j\} \in OD} \frac{f_{ij}}{d_{ij}}, \tag{3}$$

subject to:

$$\sum_{a \in A} x_{ka} \leq 1, \quad \forall k \in K, \tag{4}$$

$$\sum_{k \in K} \sum_{a \in A} c_{ka} x_{ka} \leq B, \tag{5}$$

$$y_k = p_k - \sum_{a \in A} r_{ka} x_{ka}, \quad \forall k \in K, \tag{6}$$

$$d_{ij} \leq \sum_{k \in q} \left[(1-y_k)l_k + y_k u_k\right], \quad \forall q \in Q_{ij}, \{i,j\} \in OD, \tag{7}$$

$$d_{ij} \geq \sum_{k \in q} \left[(1-y_k)l_k + y_k u_k\right] - (1-\lambda_q^{ij})U_q, \quad \forall q \in Q_{ij}, \{i,j\} \in OD, \tag{8}$$

$$\sum_{q \in Q_{ij}} \lambda_q^{ij} = 1, \quad \forall \{i,j\} \in OD, \tag{9}$$

$$x_{ka} \in \{0,1\}, \quad \forall k \in K,\, a \in A, \tag{10}$$

$$\lambda_q^{ij} \in \{0,1\}, \quad \forall q \in Q_{ij}, \{i,j\} \in OD. \tag{11}$$

In this formulation, the objective function (3) seeks to maximize the network efficiency. Constraint (4) requires each link to receive at most one treatment action; Constraint (5) enforces the budgetary restrictions; Constraint (6) defines the relationship between the disruption probability of a link before treatment and after treatment; and Constraints (7)-(9) enforce the relationship that

$$d_{ij} = \min_{q \in Q_{ij}} \left\{ \sum_{k \in q} \left[(1-y_k)l_k + y_k u_k\right] \right\}, \quad \forall \{i,j\} \in OD, \tag{12}$$

where, for each $\{i,j\} \in OD$, (7) represents (12) as a set of less-than-or-equal-to inequalities, while (8) and (9), together with (7), require equality to hold for some (and therefore the minimal) expected path resistivity value. Note that for any $q \in Q_{ij}$, $\{i,j\} \in OD$, we have from (6) that

$$\sum_{k \in q} \left[(1-y_k)l_k + y_k u_k\right] = \sum_{k \in q} \left[l_k + y_k(u_k - l_k)\right] \leq \sum_{k \in q} \left[l_k + p_k(u_k - l_k)\right] = U_q, \tag{13}$$

and so (8) is redundant whenever $\lambda_q^{ij} = 0$. Finally, Constraints (10) and (11) represent binary restrictions on the $x-$ and $\lambda-$ variables, respectively. Actually, due to the nature of the objective function, it is evident that (8), (9), and (11) are sufficient to ensure (12), and the model remains equivalent by omitting (7). However, we shall retain (7) for the sake of clarity and investigate its computational effects in our numerical study.

3.3.2 Multi-Year Planning Model

In this section, we shall describe a multi-year extension for Model SRAP where an optimal set of policies need to be selected over a finite planning horizon in order to maximize the total discounted network efficiency. Let $t = 1, ..., T$ index the years encompassing the planning horizon of total duration T. Let $\mathbf{x}_t \equiv (x_{kat}, k \in K, a \in A)$ for $t = 1, ..., T$ be the decision policy that is to be applied at the beginning of year t, where for each link k and $t \in \{1, ..., T\}$, x_{kat} equals 1 if link k receives treatment a at the beginning of year t, and is 0 otherwise. The overall treatment schedule for year t is restricted by a budget B_t, i.e., $\mathbf{x}_t \in X_t \equiv \left\{\mathbf{x}_t \in \{0,1\}^{|K||A|} : \sum_{a \in A} x_{kat} \leq 1, \forall k \in K; \sum_{k \in K} \sum_{a \in A} c_{kat} x_{kat} \leq B_t\right\}$, $\forall t = 1, ..., T$, where c_{kat} is the cost of treating link k with maintenance action type a in year t. Let y_{kt} denote the resulting disruption probability of link k during year t based on the applied treatment and the prior disruption probability at the beginning of the year, where p_k is the initial disruption probability of link k at the beginning of the planning horizon, i.e., $y_{k0} \equiv p_k, \forall k$. For each year $t \in \{1, ..., T\}$, and for any link $k \in K$, we assume that if link k receives treatment $a \in A$, then its resulting disruption probability during year t, y_{kt}, is reduced by r_{ka} from its prior value $y_{k(t-1)}$, but if it receives no treatment (i.e., $\sum_a x_{kat} = 0$), then y_{kt} increases by some amount ξ_{kt}, where ξ_{kt} is a randomly-generated parameter value with, for example, a beta distribution (we set $\xi_{k1} \equiv 0, \forall k$, assuming that the first year's case is accommodated within the p_k-values). Furthermore, we assume that with no treatment over the planning horizon, we have that $p_k + \sum_{t=1}^{T} \xi_{kt} \leq 1$, so that $y_{kt} \leq 1$, $\forall k, t$. Moreover, we assume that $p_k - T\max_{a \in A}\{r_{ka}\} \geq 0$, $\forall k \in K$, which implies that $y_{kt} \geq 0$, $\forall k, t$. Let f_{ijt} denote the amount of flow, traffic, etc., which is to be transferred between node pair $\{i, j\} \in OD$ during year t. As defined before, let l_k be the resistivity of link k when the link is operating, and let u_k denote the resistivity of link k when the link is disrupted. Let Q_{ijt} denote the set of viable paths (indexed by $q \in Q_{ijt}$) between node pair $\{i, j\} \in OD$ during year t. Note that, for each $\{i, j\} \in OD$, we permit Q_{ijt} to vary over time based on the anticipated dynamics of the network. (In a similar vein, to accommodate potential shifts in population centers, we could define OD_t as the set of origin-destination pairs $\{i, j\}$ to be considered during year t with obvious modifications in the model proposed below.) Similar to U_q as derived by (13), let U_{qt}

be an upper bound on the expected resistivity of path q during year t for any set of disruption probabilities that are less than or equal to the values $p'_{kt} \equiv p_k + \sum_{t'=1}^{t} \xi_{kt'}, \forall k \in K$, which is computed as $U_{qt} \equiv \sum_{k\in q} \left[(1 - p'_{kt})l_k + p'_{kt}u_k\right], \forall q \in Q_{ijt}, \{i, j\} \in OD, t = 1, ..., T$. Let d_{ijt} be the shortest expected path resistivity between node pair $\{i, j\} \in OD$ during year t, and let λ_{qt}^{ij} denote a binary variable that equals 1 if path $q \in Q_{ijt}$ represents d_{ijt} during year t, and is 0 otherwise. For conceptual purposes, let $\mathbf{s}_t \equiv \mathbf{y}_{t-1} \equiv (y_{k,t-1}, k \in K)$ represent the input state of the system in year t with each link having a particular disruption probability, where the initial state $\mathbf{s}_1 \equiv \mathbf{y}_0$ for $t = 1$ corresponds to the disruption probabilities $p_k, \forall k \in K$. For each $t \in \{1, ..., T\}$, given $\mathbf{s}_t$ and $\mathbf{x}_t$, let $\sigma(\mathbf{s}_t, \mathbf{x}_t)$ be the transition function that dictates the modified disruption probabilities $\mathbf{y}_t$ given by (18) below, which hence contributes toward determining the system efficiency $u(\mathbf{s}_t, \mathbf{x}_t)$ during year t according to (2), as well as defines the input state of the system during the next year, i.e., $\mathbf{s}_{t+1} \equiv \mathbf{y}_t \equiv \sigma(\mathbf{s}_t, \mathbf{x}_t)$ (see Figure 1). Let $V_t(\mathbf{s}_t)$ represent the value function of the network at year t given state $\mathbf{s}_t$, and let α represent the discount rate factor.

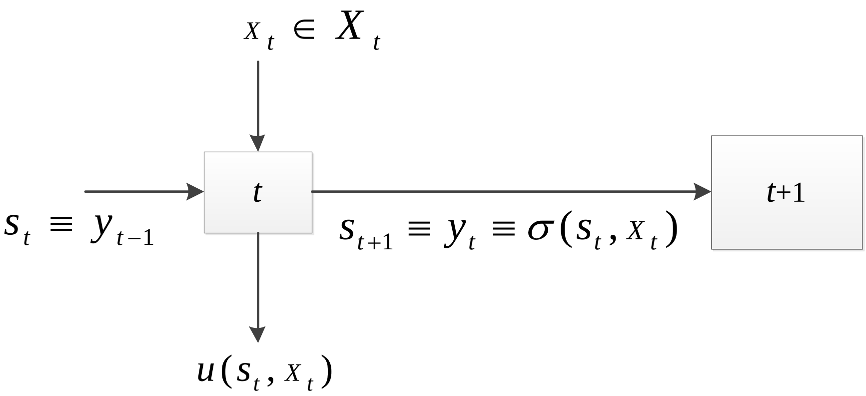

Figure 1: Stage Diagram for Model MRAP

In the multi-year resource allocation problem (MRAP), the goal therefore is to determine maintenance policies for each year $t = 1, ..., T$, such that the overall discounted system value is optimized over the planning horizon. We can formulate this planning model using the following Bellman's equation:

$$V_t(\mathbf{s}_t) = \max_{\mathbf{x}_t \in X_t} \left\{u(\mathbf{s}_t, \mathbf{x}_t) + \alpha V_{t+1}\left[\sigma(\mathbf{s}_t, \mathbf{x}_t)\right]\right\}, \forall t = 1, ..., T, \quad (14)$$

where $V_{T+1}(.) \equiv 0$. The exponential enumeration required by (14) entails significant computational effort and complexity. As such, (14) simply represents a conceptual framework. In order to derive a more tractable model, we formulate an alternative mathematical program, denoted MRAP, as follows:

$$\textbf{MRAP:} \quad \text{Maximize} \quad \sum_{t=1}^{T} \alpha^{t-1} \sum_{\{i,j\} \in OD} \frac{f_{ijt}}{d_{ijt}}, \tag{15}$$

subject to:

$$\sum_{a \in A} x_{kat} \leq 1, \quad \forall k \in K, t = 1, ..., T, \tag{16}$$

$$\sum_{k \in K} \sum_{a \in A} c_{kat} x_{kat} \leq B_t, \quad \forall t = 1, ..., T, \tag{17}$$

$$y_{kt} = y_{k(t-1)} + (1 - \sum_{a \in A} x_{kat}) \xi_{kt} - \sum_{a \in A} r_{ka} x_{kat}, \quad \forall k \in K, t = 1, ..., T, \tag{18}$$

$$d_{ijt} \leq \sum_{k \in q} \left[(1 - y_{kt}) l_k + y_{kt} u_k \right], \quad \forall q \in Q_{ijt}, \{i, j\} \in OD, t = 1, ..., T, \tag{19}$$

$$d_{ijt} \geq \sum_{k \in q} \left[(1 - y_{kt}) l_k + y_{kt} u_k \right] - (1 - \lambda_{qt}^{ij}) U_{qt}, \quad \forall q \in Q_{ijt}, \{i, j\} \in OD, t = 1, ..., T, \tag{20}$$

$$\sum_{q \in Q_{ijt}} \lambda_{qt}^{ij} = 1, \quad \forall \{i, j\} \in OD, t = 1, ..., T, \tag{21}$$

$$x_{kat} \in \{0, 1\}, \quad \forall k \in K, \, a \in A, t = 1, ..., T, \tag{22}$$

$$\lambda_{qt}^{ij} \in \{0, 1\}, \quad \forall q \in Q_{ijt}, \{i, j\} \in OD, t = 1, ..., T. \tag{23}$$

Here, the objective function (15) seeks to maximize the overall discounted efficiency of the network over the planning horizon T. Constraints (16)-(18) are similar to (4)-(6), respectively. Note that (18) implies that $0 \leq y_{kt} \leq 1$ since $y_{kt} \geq p_k - T\max_{a\in A}\{r_{ka}\} \geq 0$ and $y_{kt} \leq p'_{kt} \leq p_k + \sum_{t'=1}^{T} \xi_{kt'} \leq 1, \forall k \in K, t = 1, ..., T$. Constraints (19)-(23) are similar to (7)-(11), respectively. Observe that Model SRAP is a special case of Model MRAP for $T \equiv 1$. Furthermore, similar to Problem SRAP, due to the nature of the objective function (15) and based on (20), (21), and (23), the Model MRAP remains equivalent upon omitting Constraint (19).

Chapter 4

Reformulation and Algorithm for the Single Year Planning Model

In this chapter, we present an improved formulation for Problem SRAP, the single year planning model described in Chapter 3, and design solution algorithms for the problem with both objectives (2a) and (2b).

To begin with, we shall first reformulate Problem SRAP, in an equivalent manner, whereby the λ_q^{ij}-variables can be relaxed to take on *continuous* values in [0,1] and yet automatically be binary-valued at optimality. The main observation to accomplish this is that (7) and (8) can be rewritten as follows:

$$d_{ij} = \sum_{q \in Q_{ij}} \lambda_q^{ij} \left\{ \sum_{k \in q} [(1 - y_k) l_k + y_k u_k] \right\}, \forall \{i, j\} \in OD. \tag{24}$$

Note that for given y-values, (24) writes d_{ij} as a convex combination of the quantities $\left\{ \sum_{k \in q} [(1 - y_k) l_k + y_k u_k] \right\}$ for $q \in Q_{ij}$, where the λ_q^{ij}-variables satisfy (9) plus nonnegativity restrictions. Hence, by virtue of the objective function preferring smaller d_{ij}-values (using either objectives (2a) or (2b)), we will automatically set $\lambda_q^{ij} = 1$ for the q-index that achieves the minimum value in (12), and zero otherwise, for each $\{i, j\} \in OD$. Thus, (24) evaluates the d_{ij}-variables precisely as in (12). However, this occurs at the expense of creating a nonlinear function in (24), which we shall resolve next.

A second observation is that Problem SRAP is essentially defined in the primary $\mathbf{x}$-variable space and can be stated as follows:

$$\text{Maximize } \{F(\mathbf{x}) : \mathbf{x} \in X\} \ , \tag{25}$$

where

$$X \equiv \left\{ \mathbf{x} \in \{0,1\}^{|K||A|} : (4) \, \text{and} \, (5) \, \text{hold} \right\} \ , \tag{26}$$

and where $F(\mathbf{x})$ is evaluated via (3) (likewise for (2b)) for any $\mathbf{x} \in X$ by first computing the y_k-variables from (6), and then determining the d_{ij}-variables using (12). In fact, by eliminating the y_k-variables from Problem SRAP using (6), and noting that we can rewrite (24) as

$$\begin{aligned} d_{ij} &= \sum_{q \in Q_{ij}} \lambda_q^{ij} \left\{ \sum_{k \in q} [(1-p_k)l_k + p_k u_k] - \sum_{k \in q} (u_k - l_k) \left[\sum_{a \in A} r_{ka} x_{ka} \right] \right\} \\ &= \sum_{q \in Q_{ij}} \left\{ U_q \lambda_q^{ij} - \sum_{k \in q} (u_k - l_k) \lambda_q^{ij} \left[\sum_{a \in A} r_{ka} x_{ka} \right] \right\} , \end{aligned}$$

we get that SRAP can be equivalently restated as follows:

$$\text{Maximize } \sum_{\{i,j\} \in OD} \frac{f_{ij}}{d_{ij}}$$

subject to

$$(4), \ (5), \ (9)$$

$$d_{ij} = \sum_{q \in Q_{ij}} U_q \lambda_q^{ij} - \sum_{q \in Q_{ij}} \sum_{k \in q} \sum_{a \in A} (u_k - l_k) r_{ka} \lambda_q^{ij} x_{ka}, \quad \forall \{i,j\} \in OD, \tag{27}$$

$$\lambda_q^{ij} \geq 0, \, \forall q \in Q_{ij}, \{i,j\} \in OD, \ \text{and} \ x_{ka} \in \{0,1\} \, , \forall k \in K, \, a \in A. \tag{28}$$

As a next manipulation, we shall linearize the constraints in (27) using the special-structured

Reformulation-Linearization Technique (RLT) of Sherali et al. (1998) by substituting

$$\xi_{qka}^{ij} = \lambda_q^{ij} x_{ka}, \forall q \in Q_{ij}, k \in q, a \in A, \text{for each } \{i,j\} \in OD, \tag{29}$$

and constructing the following GUB (generalized upper bounding)-based product constraints:

$$\left[1 - \sum_{a \in A} x_{ka}\right] * \lambda_q^{ij} \geq 0 \Rightarrow \lambda_q^{ij} - \sum_{a \in A} \xi_{qka}^{ij} \geq 0, \quad \forall q \in Q_{ij},\, k \in q,\, \{i,j\} \in OD, \tag{30}$$

$$\left[1 - \sum_{a \in A} x_{ka}\right] * (1 - \lambda_q^{ij}) \geq 0 \Rightarrow \lambda_q^{ij} + \sum_{a \in A} (x_{ka} - \xi_{qka}^{ij}) \leq 1,$$
$$\forall q \in Q_{ij}, k \in q, \{i,j\} \in OD, \tag{31}$$

$$(1 - \lambda_q^{ij}) * x_{ka} \geq 0 \Rightarrow x_{ka} - \xi_{qka}^{ij} \geq 0, \quad \forall q \in Q_{ij}, k \in q, a \in A, \{i,j\} \in OD, \tag{32}$$

$$\lambda_q^{ij} * x_{ka} \geq 0 \Rightarrow \xi_{qka}^{ij} \geq 0, \quad \forall q \in Q_{ij}, k \in q, a \in A, \{i,j\} \in OD. \tag{33}$$

Note that (30) and (31) together imply (4) (as seen by summing these inequalities), and so (4) can be dropped from the model in the presence of (30)-(33). As per Sherali et al. (1998), it can be readily verified that for binary values of the x_{ka}-variables, the linear inequalities (30)-(33) imply that the nonlinear product relationship (29) holds true. Accordingly, we can rewrite Problem SRAP as follows:

$$\text{Maximize} \sum_{\{i,j\} \in OD} \frac{f_{ij}}{d_{ij}}$$

subject to

(5), (9), (28), (30)-(33), and

$$d_{ij} = \sum_{q \in Q_{ij}} U_q \lambda_q^{ij} - \sum_{q \in Q_{ij}} \sum_{k \in q} \sum_{a \in A} (u_k - l_k) r_{ka} \xi_{qka}^{ij}, \quad \forall \{i,j\} \in OD, \tag{34}$$

Proposition 1 (*Additional valid inequalities*): For any $\{i,j\} \in OD$,
let $K_{ij} = \{k \in K : k \in q \text{ for all } q \in Q_{ij}\}$. Then we can include the following valid inequalities in the model based on the variables defined in (29) for such a link:

$$\sum_{q \in Q_{ij}} \xi_{qka}^{ij} = x_{ka}, \quad \forall k \in K_{ij},\, a \in A, \text{ for each } \{i,j\} \in OD. \tag{35}$$

Proof: Inequality (35) is validated by multiplying (9) by $x_{ka}, \forall a \in A$, for the given $k \in K_{ij}$, $\{i,j\} \in OD$, and then linearizing the resulting equation by using the substitution (29), where all the ξ_{qka}^{ij}-variables involved in (35) are defined within (29) by virtue of the hypothesis of the proposition. □

As a further modeling expedient, we next derive a result that can be applied to *a priori* set certain ξ_{qka}^{ij}-variables to zero. Toward this end, first note from (12) and (13) that

$$d_{ij} \leq \min_{q \in Q_{ij}} U_q \equiv U_{ij}, \text{ say}, \forall \{i,j\} \in OD. \tag{36}$$

Likewise, we can derive a lower bound L_{ij} for each d_{ij} by observing that, if $y_{kmin} \equiv p_k - \max_{a \in A} \{r_{ka}\}, \forall k \in K$, then we have,

$$\begin{aligned}\sum_{k \in q} [(1 - y_k) l_k + y_k u_k] &= \sum_{k \in q} [l_k + y_k (u_k - l_k)] \geq \sum_{k \in q} [l_k + y_{kmin} (u_k - l_k)] \\ &= \sum_{k \in q} [(1 - y_{kmin}) l_k + y_{kmin} u_k] \equiv L_q, \text{say}.\end{aligned} \tag{37}$$

Hence, we can assert from (12) that

$$d_{ij} \geq \min_{q \in Q_{ij}} \{L_q\} \equiv L_{ij}, \text{ say}. \tag{38}$$

Proposition 2: Suppose that for some $q \in Q_{ij}$ for an $\{i,j\} \in OD$, we have that $[L_q, U_q] \cap [L_{ij}, U_{ij}] = \phi$. Then, we can set

$$\lambda_q^{ij} = 0 \quad \text{along with} \quad \xi_{qka}^{ij} = 0, \quad \forall k \in q, a \in A. \tag{39}$$

Proof: Note that if $[L_q, U_q] \cap [L_{ij}, U_{ij}] = \phi$, then path $q \in Q_{ij}$ will never determine d_{ij} via (12), and so we can set $\lambda_q^{ij} = 0$, which then from (29) (or equivalently from (30) and (33)) implies that $\xi_{qka}^{ij} = 0$, $\forall k \in q$, $a \in A$. □

Henceforth, we shall assume that the problem reduction implied by Proposition 2 has been conducted by preprocessing the data to eliminate all such paths $q \in Q_{ij}, \forall \{i,j\} \in OD$. Finally, defining

$$\delta_{ij} \equiv \frac{1}{d_{ij}}, \forall \{i,j\} \in OD, \tag{40}$$

we derive our proposed reformulation of Problem SRAP (denoted as SRAP*) as follows, where we have included the implied bound on the d_{ij}- and δ_{ij}-variables as given by (36) and (38) for the sake of algorithmic convenience:

$$\textbf{SRAP*}: \quad \text{Maximize} \quad \sum_{\{i,j\} \in OD} f_{ij}\delta_{ij}$$

subject to

$$\sum_{k \in K} \sum_{a \in A} c_{ka} x_{ka} \leq B,$$

$$\sum_{q \in Q_{ij}} \lambda_q^{ij} = 1, \quad \forall \{i,j\} \in OD,$$

$$d_{ij} = \sum_{q \in Q_{ij}} U_q \lambda_q^{ij} - \sum_{q \in Q_{ij}} \sum_{k \in q} \sum_{a \in A} (u_k - l_k) r_{ka} \xi_{qka}^{ij}, \quad \forall \{i,j\} \in OD,$$

$$\lambda_q^{ij} - \sum_{a \in A} \xi_{qka}^{ij} \geq 0, \quad \forall q \in Q_{ij},\ k \in q,\ \{i,j\} \in OD,$$

$$\lambda_q^{ij} + \sum_{a \in A} (x_{ka} - \xi_{qka}^{ij}) \leq 1, \forall q \in Q_{ij}, k \in q, \{i,j\} \in OD,$$

$$x_{ka} - \xi^{ij}_{qka} \geq 0, \quad \forall q \in Q_{ij}, k \in q, a \in A, \{i,j\} \in OD,$$

$$\sum_{q \in Q_{ij}} \xi^{ij}_{qka} = x_{ka}, \quad \forall k \in K_{ij},\ a \in A, \text{ for each } \{i,j\} \in OD,$$

$$d_{ij}\delta_{ij} = 1, \forall \{i,j\} \in OD, \tag{41}$$

$$L_{ij} \leq d_{ij} \leq U_{ij} \quad \text{and} \quad \frac{1}{U_{ij}} \leq \delta_{ij} \leq \frac{1}{L_{ij}}, \forall \{i,j\} \in OD, \tag{42}$$

$$\xi^{ij}_{qka} \geq 0, \quad \forall q \in Q_{ij}, k \in q, a \in A, \{i,j\} \in OD,$$

$$\lambda^{ij}_q \geq 0, \forall q \in Q_{ij}, \{i,j\} \in OD, \text{ and } x_{ka} \in \{0,1\}, \forall k \in K,\ a \in A.$$

Note that Problem SRAP* is a bilinear 0-1 mixed-integer programming problem, which is linear except for the bilinear constraint (41). This problem can be solved using the RLT-based branch-and-bound algorithm discussed in Sherali and Alameddine (1992) by partitioning on the x_{ka}- and d_{ij}-variables, or just on the x_{ka}-variables by using bound-factor products composed via (42) to generate a relaxation for SRAP*. More specifically, the latter relaxation is given by the linear programming (LP) problem obtained by relaxing $x_{ka} \in \{0,1\}$ to $x_{ka} \geq 0, \forall k \in K, a \in A$, and by replacing the bilinear constraint (41) with the following bound-factor product constraints, noting that $d_{ij}\delta_{ij} = 1$ from (41), and that the objective function prompts the inclusion of only such constraints that impose lower bounds on the δ_{ij}-variables:

$$(U_{ij} - d_{ij})(\delta_{ij} - \frac{1}{U_{ij}}) \geq 0 \Rightarrow \frac{d_{ij}}{U_{ij}} + U_{ij}\delta_{ij} \geq 2, \tag{43}$$

$$(d_{ij} - L_{ij})(\frac{1}{L_{ij}} - \delta_{ij}) \geq 0 \Rightarrow \frac{d_{ij}}{L_{ij}} + L_{ij}\delta_{ij} \geq 2. \tag{44}$$

Alternatively, we can use the commercial software BARON to solve Problem SRAP* directly.

We adopt this option since the most recent version, BARON 10.3, is itself fashioned to construct and use RLT constraints of the type (43) and (44), and we will compare the relative efficacy of BARON in solving SRAP versus SRAP* in our computational study.

Remark 1: Note that for the linear objective (2b), the corresponding reformulated problem, denoted SRAP*(2b), is a linear 0-1 mixed-integer program (MIP) given as follows:

SRAP*(2b):

$$\text{Minimize} \quad \left\{ \sum_{\{i,j\}\in OD} f_{ij} d_{ij} : (5), (9), (28), (30)-(33), (34), \text{ and } (35) \right\}.$$

This problem can be solved directly using CPLEX, and in our numerical experiments, we shall likewise compare the relative effectiveness of CPLEX 12.2 in solving SRAP(2b) (which is Model SRAP given by (4)-(11) along with objective (2b)) versus SRAP*(2b), as well as assess the contributions of the valid inequalities (35) of Proposition 1. □

Problems SRAP and MRAP are large-scale optimization problems with solution spaces of the order of $m^{|K|}$ and $(m^{|K|})^T$, respectively. The main complexity issues are as follows:

- The solution space is extremely large. Developing a search heuristic to cleverly sample across the solution space and rapidly reach a good quality solution can reduce the computational complexity.
- Calculation of *U(G)* can be computationally impossible in some cases particularly if $|Q_{ij}|$ is not small. For example, the *U(G)*-value for the network depends on $2^{|K|}$ scenario realizations, which is computationally intractable. Nonetheless, the foregoing model formulations implicitly represent the combinatorial aspects of the problem, and are amenable to both exact and heuristic methods using partial enumeration techniques or Branch-and-Bound (B&B) and relax-and-fix type of heuristic procedures (Sherali et al. 1999).

Chapter 5

Computational Results

In this chapter, we demonstrate the application of our models to an illustrative example as well as to a real large scale highway network. We analyze the computational results, discuss our findings, and compare the efficiency of our models with those in previous studies.

5.1 Illustrative Example

In this section, we present a preliminary application of our proposed approach using an illustrative example. Figure 2 depicts a hypothetical network having five nodes and eight links. This network could represent any infrastructure such as a water distribution system, a roadway system, or a communication network. Tables 1 and 2 provide the information for the links and the treatment costs as well as the routes.

Table 1: Link and Treatment Characteristics

				c_{ka}			r_{ka}		
Link	l_k	u_k	p_k	a_1	a_2	a_3	a_1	a_2	a_3
1	100	150	0.5	100	150	250	0.15	0.25	0.45
2	70	100	0.45	70	85	110	0.15	0.25	0.45
3	65	120	0.55	110	170	220	0.15	0.25	0.45
4	150	270	0.65	250	380	480	0.15	0.25	0.45
5	55	90	0.75	200	300	400	0.15	0.25	0.45
6	120	150	0.6	80	120	150	0.15	0.25	0.45
7	75	100	0.45	45	65	85	0.15	0.25	0.45
8	45	120	0.45	150	200	400	0.15	0.25	0.45

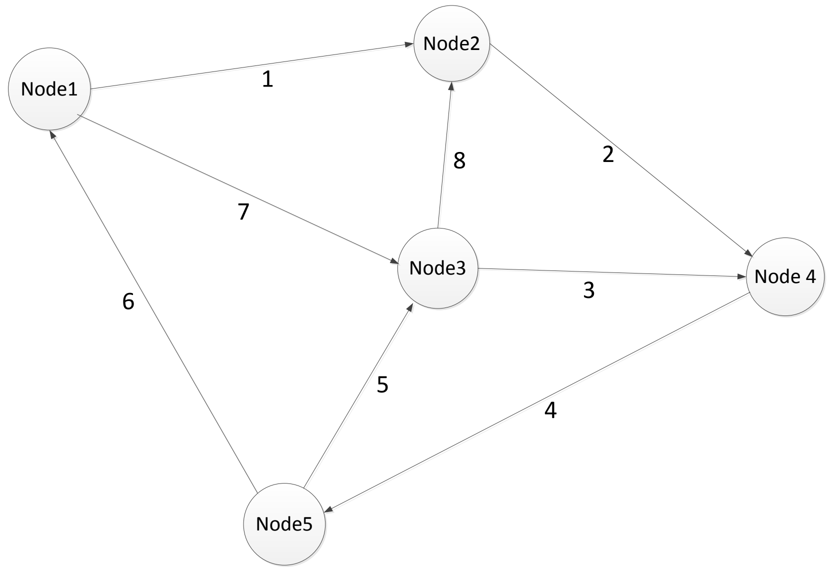

Figure 2: Illustrative Network

Table 2: OD Path Information

Node i	Node j	Q_{ij}sets	Links in Path $q \in Q_{ij}$	f_{ij}
1	4	Q_{14}	$\{1, 2\}$	1000
			$\{3, 7\}$	
			$\{7, 8, 2\}$	
2	1	Q_{21}	$\{2, 4, 6\}$	300
3	4	Q_{34}	$\{3\}$	250
			$\{8, 2\}$	
5	3	Q_{53}	$\{5\}$	100
			$\{6, 7\}$	

We solved the maintenance optimization problem for the network shown in Figure 2 for different levels of the available budget with Models SRAP(2b) and SRAP*(2b). The details of the results obtained are provided in Appendix A. In general, as shown in Figure 3 and as expected, the objective value decreases with an increase in the available budget. This is simply because more available budget will help improve the reliabilities of the links and ultimately reduce the expected minimum path resistivity between different origins and destinations. The results also show that, regardless of the available budget, the optimal solutions tend to favor the idea of "selecting fewer links to apply high quality treatments" rather than “improving more links with lower treatment quality”. However, it is evident that the optimal solutions are sensitive to the values of treatment costs (c_{ka}) and treatment qualities (r_{ka}) provided in Table 1. A significant difference between the costs of different maintenance classes would possibly change the aforementioned tendency of optimal solutions. There is also a negligible difference between the CPU times required to solve Models SRAP(2b) and SRAP*(2b), but this difference could be more significant for larger sized problems.

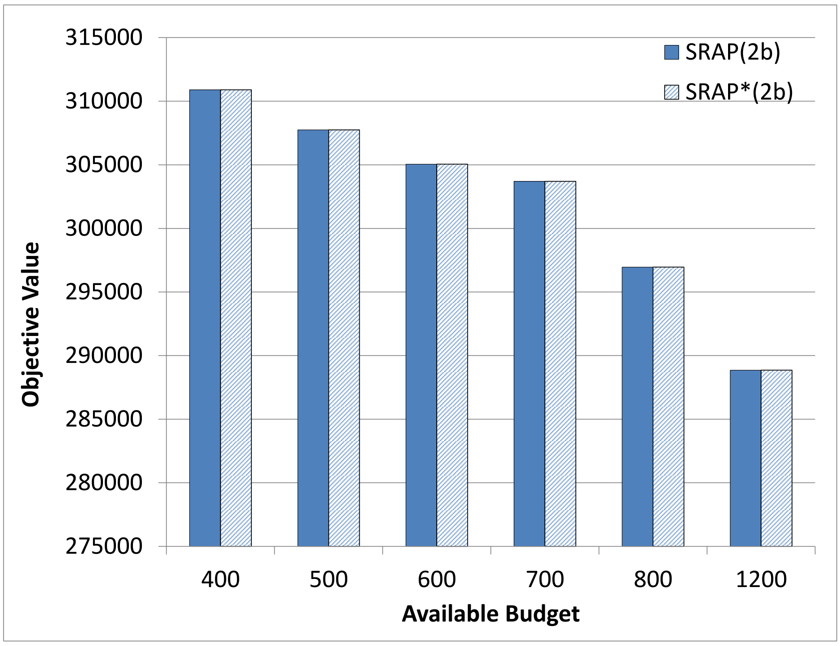

Figure 3: Objective Value - Available Budget Relationship

5.2 Application to a Large Scale Istanbul Highway Network

In this section, we demonstrate the application of our model to a larger real network. We selected the Istanbul highway system for our case study (see Figure 4). Our motivation for doing so is as follows:

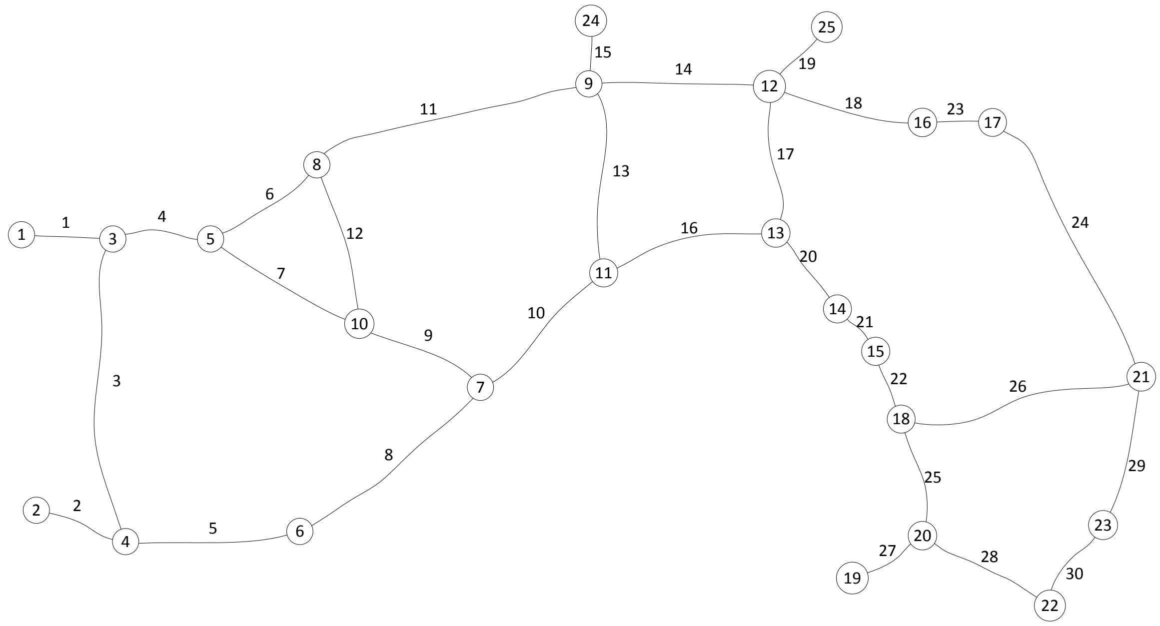

Figure 4: Istanbul Highway Network

- The Istanbul highway is a critical network that connects the Asian side of Istanbul to its European side.
- The highway network is located in an earthquake-prone region and thus is subject to disruptions and failures, particularly from bridge collapses.

This network has also been used in previous studies (Peeta et al. (2010)), and hence serves as a useful benchmarking case study for our models. For comparison purposes, we chose the same network characteristics (failure probabilities, treatment costs, etc.) as were used

Table 3: Network Characteristics

Link	l_k	u_k	C_k	p_k
1	2.46	4.46	80	0.2
2	2.2	4.2	80	0.2
3	8	10	320	0.2
4	2.56	4.56	260	0.3
5	4.57	6.56	160	0.2
6	3.44	5.44	420	0.4
7	4.19	6.19	160	0.2
8	5.6	7.6	620	0.4
9	3.71	5.17	120	0.2
10	4.44	6.44	340	0.3
11	7.11	9.11	940	0.45
12	4.03	6.03	160	0.2
13	5.02	7.02	620	0.4
14	4.55	6.55	1180	0.5
15	1.36	3.36	40	0.2
16	4.26	6.26	940	0.45
17	3.64	5.6	300	0.3
18	4.19	6.19	520	0.4
19	1.98	3.98	40	0.2
20	2.45	4.45	800	0.45
21	1.8	3.8	40	0.2
22	1.97	3.97	160	0.3
23	1.61	3.61	40	0.2
24	8.09	10.09	620	0.4
25	2.87	4.87	260	0.3
26	6.35	8.35	780	0.4
27	2.27	4.27	800	0.45
28	3.91	5.91	120	0.2
29	4.11	6.11	220	0.3
30	2.27	4.27	500	0.4

in Peeta et al. (2010), which are provided in Tables 3 and 4. It is important to note that, in both studies, the optimization models tend to optimize maintenance planning. However, our objective function is different from that in Peeta et al., where the latter adopts a simple linear function of costs.

For our analysis, we made the following assumptions as well as changes to the network characteristics in Peeta et al. (2010):

Table 4: Path Information

OD pair	Q_{ij}sets	Links in Path $q \in Q_{ij}$	f_{ij}
$\{14, 20\}$	$Q_{14,20}$	$\{21, 22, 25\}$	31
		$\{21, 22, 26, 29, 30, 28\}$	
		$\{20, 17, 18, 23, 24, 26, 25\}$	
		$\{20, 17, 18, 23, 24, 29, 30, 28\}$	
$\{14, 7\}$	$Q_{14,7}$	$\{20, 16, 10\}$	31
		$\{20, 17, 14, 13, 10\}$	
		$\{20, 17, 14, 11, 12, 9\}$	
		$\{20, 16, 13, 11, 12, 9\}$	
		$\{20, 17, 14, 11, 6, 7, 9\}$	
		$\{20, 16, 11, 13, 6, 7, 9\}$	
$\{12, 8\}$	$Q_{12,8}$	$\{17, 20, 21, 22\}$	28
		$\{14, 16, 13, 20, 21, 22\}$	
		$\{18, 23, 24, 26\}$	
		$\{18, 23, 24, 29, 30, 28, 25\}$	
$\{9, 7\}$	$Q_{9,7}$	$\{13, 10\}$	19
		$\{11, 12, 9\}$	
		$\{14, 17, 16, 10\}$	
		$\{11, 6, 7, 9\}$	
$\{4, 8\}$	$Q_{4,8}$	$\{3, 4, 6\}$	35
		$\{5, 8, 9, 12\}$	
		$\{3, 4, 7, 12\}$	
		$\{5, 8, 9, 7, 6\}$	
		$\{5, 8, 10, 13, 11\}$	
		$\{5, 8, 10, 16, 17, 14, 11\}$	
$\{17, 19\}$	$Q_{17,19}$	$\{24, 26, 25, 27\}$	30
		$\{24, 29, 30, 28, 27\}$	
$\{15, 22\}$	$Q_{15,22}$	$\{22, 25, 28\}$	30
		$\{22, 26, 29, 30\}$	

- The lengths of links represent resistivity (l_k) in our problem. We selected reasonable disrupted resistivity values (u_k) for the links based on their l_k- values.
- The link survivability, p_e, was converted to disruption probability, i.e., $p_k = 1 - p_e, \forall k$.
- There is only one maintenance type, i.e., $A = \{a_1\}$, as considered in Peeta et al. We later consider three maintenance types for further analysis of our proposed models..
- c_e is the cost to treat a link, which restores its failure probability back to 0, i.e., $r_{ka} = p_k, \forall k$.

Table 5: Test Runs for a Single Treatment Type

	SRAP		SRAP*		Budget		
	a	b	a	b	B1	B2	B3
TRa1	√				√		
TRa2	√					√	
TRa3	√						√
TRb1		√			√		
TRb2		√				√	
TRb3		√					√
TR*a1			√		√		
TR*a2			√			√	
TR*a3			√				√
TR*b1				√	√		
TR*b2				√		√	
TR*b3				√			√

- The M- values used in Peeta et al. (2010) represent the relative criticality of an OD pair in terms of an imposed penalty cost whenever no path exists between that pair. Therefore, in our model, M can be taken to represent the amount of flow (which is used as a priority weight) between an OD pair. In other words, $f_{ij} = M_{ij}, \forall \{i, j\} \in OD$.
- We considered three budget levels: B1, B2, and B3, corresponding to 10%, 20%, and 30% of the total budget needed to invest in all links ($11,640), respectively.

We designed several test runs (TRs) considering different combinations of our models (SRAP and SRAP*), objective functions (given by (2a) and (2b)), and budget levels. Table 5 describes these test runs, where, for example, "TR*a2" represents the test run for Model SRAP*(2a) with the second budget level.

We solved each TR with several suitable solvers available on the NEOS server, including BARON, AlphaECP, DICOPT, SBB, and CoinCbc. The results obtained by implementing these solvers within the different test runs are presented in Appendix B. Figure 5 depicts the results for different TRs at a glance, where the treated links are highlighted (in green).

We found that for the TRs with Objective (2a), the resulting decisions were very similar to those reported in Peeta et al. (2010). The advantage of our model, however, to that of Peeta et al. is that the later requires approximations and Monte Carlo simulation for simplification

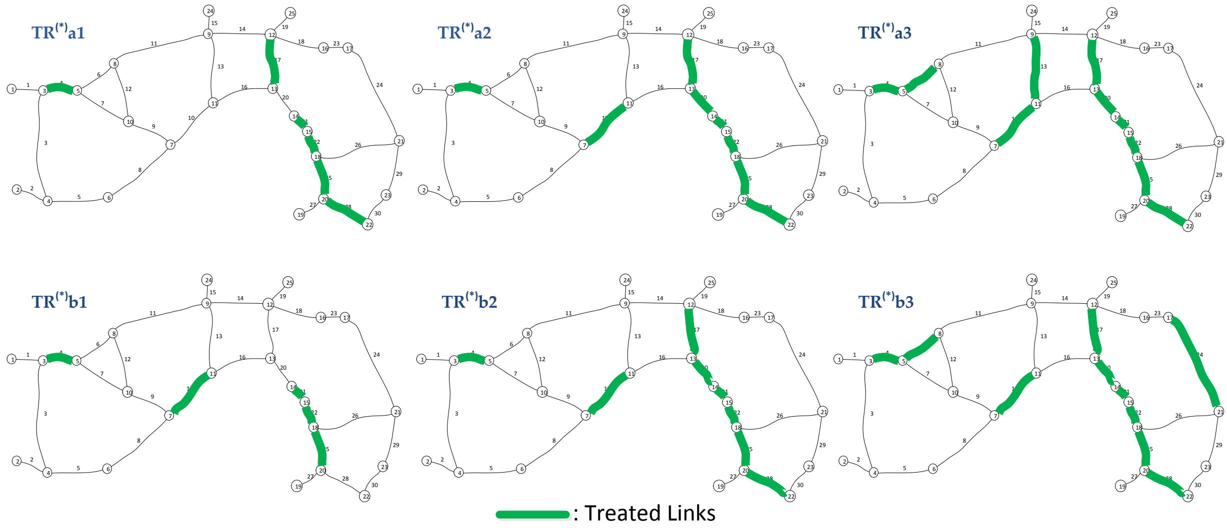

Figure 5: Results for the Single Treatment Case

purposes. Moreover, the linear Objective (2b) is novel to the present study.

We further increased the computational complexity of the problem in order to assess the full scope of our models by considering three maintenance types for the network. The maintenance types and their characteristics are shown in Table 6. Again, we designed different test runs (see Table 7) and solved them with several suitable solvers on the NEOS server. The detailed results obtained are tabulated in Appendix B. Figure 6 depicts the results for different TRs at a glance, where the treated links are highlighted in different colors for different treatment types.

In the following section, we summarize our findings for both the single and multiple treatment type runs.

Table 6: Treatment Characteristics

	r_{ka}			c_{ka}		
k	a_1	a_2	a_3	a_1	a_2	a_3
1	0.02	0.07	0.20	8	40	80
2	0.02	0.07	0.20	8	40	80
3	0.02	0.07	0.20	32	160	320
4	0.03	0.11	0.30	26	130	260
5	0.02	0.07	0.20	16	80	160
6	0.04	0.14	0.40	42	210	420
7	0.02	0.07	0.20	16	80	160
8	0.04	0.14	0.40	62	310	620
9	0.02	0.07	0.20	12	60	120
10	0.03	0.11	0.30	34	170	340
11	0.05	0.16	0.45	94	470	940
12	0.02	0.07	0.20	16	80	160
13	0.04	0.14	0.40	62	310	620
14	0.05	0.18	0.50	118	590	1180
15	0.02	0.07	0.20	4	20	40
16	0.05	0.16	0.45	94	470	940
17	0.03	0.11	0.30	30	150	300
18	0.04	0.14	0.40	52	260	520
19	0.02	0.07	0.20	4	20	40
20	0.05	0.16	0.45	80	400	800
21	0.02	0.07	0.20	4	20	40
22	0.03	0.11	0.30	16	80	160
23	0.02	0.07	0.20	4	20	40
24	0.04	0.14	0.40	62	310	620
25	0.03	0.11	0.30	26	130	260
26	0.04	0.14	0.40	78	390	780
27	0.05	0.16	0.45	80	400	800
28	0.02	0.07	0.20	12	60	120
29	0.03	0.11	0.30	22	110	220
30	0.04	0.14	0.40	50	250	500

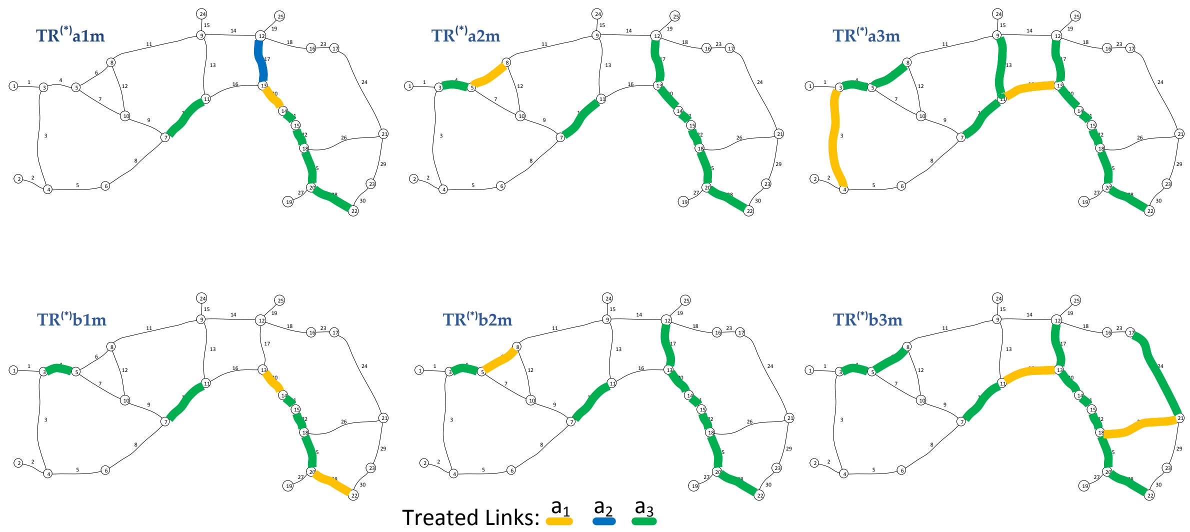

Figure 6: Results for the Multiple Treatment Case

Table 7: Test Runs for Multiple (Three) Treatment Types

	SRAP		SRAP*		Budget		
	a	b	a	b	B1	B2	B3
TRa1m	√				√		
TRa2m	√					√	
TRa3m	√						√
TRb1m		√			√		
TRb2m		√				√	
TRb3m		√					√
TR*a1m			√		√		
TR*a2m			√			√	
TR*a3m			√				√
TR*b1m				√	√		
TR*b2m				√		√	
TR*b3m				√			√

5.3 Summary of Findings

In this section, we summarize our findings based on the analysis of the results for the single and multiple treatment cases. These findings are as follows:

1. For Objective (2a), in general, the solvers were able to attain a better solution with Model SRAP* as compared with using the original un-manipulated model SRAP. This was more evident for TRs that consider three maintenance types, for which the problem then becomes more complex. Table 8 displays the optimality gap for each TR using different solvers. The optimal solution value for each test run was taken as the best objective value found for the same instance involving the given objective function choice and budget level over all runs of SRAP and SRAP* using the different solvers. These (assumed) optimal values are displayed in the second column of Table 8. The stated optimality gaps in this table are defined with respect to these optimal values. For example, the optimality gaps for TRa1m, TRa2m, and TRa3m using the solver SBB were 1.9% and 1.8%, and 2.5%, respectively, whereas these gaps were all zero using SRAP*. Likewise, for TRa1, TRa2, and TRa3, the optimality gaps for SBB were 0.2%, 0.5%, and 2.5%, respectively, but this solver again attained optimal solutions for all runs using SRAP*. Although the optimality gaps resulting from using Model SRAP with different solvers are within 2.5%, yet, this speaks to the robustness and value of using Model SRAP*, where the improved structure and the additional inequalities afforded by this model evidently enables solvers to optimize the test instances more effectively. For the simpler linear objective function (2b), however, all test runs using either SRAP or SRAP* found optimal solutions.

Table 8: Optimality Gaps Using Different Solvers

TR	Optimal Value	Optimality Gap (%)			
		AlphaECP	BARON	Dicopt	SBB
TRa1m	18.4734	0.0	0.0	0.5	1.9
TRa2m	19.035	0.0	0.0	0.9	1.8
TRa3m	19.3356	0.0	0.0	1.1	2.5
TR*a1m	18.4734	0.0	0.0	0.0	0.0
TR*a2m	19.035	0.0	0.0	0.0	0.0
TR*a3m	19.3356	0.0	0.0	0.0	0.0
TRa1	18.3982	0.0	0.0	0.4	0.2
TRa2	19.0228	0.0	0.0	1.0	0.5
TRa3	19.3073	0.0	0.0	0.6	2.5
TR*a1	18.3982	0.0	0.0	0.0	0.0
TR*a2	19.0228	0.0	0.0	0.0	0.0
TR*a3	19.3073	0.0	0.0	0.0	0.0

2. The optimal solution for the nonlinear model with Objective (2a) can be approximated by the results obtained using the linear model with Objective (2b). Let $x^*(2a)$ represent the optimal solution found for a TR with Objective (2a), and let $Z^{SRAP(2a)}(x^*(2a))$ be its associated optimal objective value. Similarly, let $x^*(2b)$ represent the optimal solution found for the corresponding TR with Objective (2b). For almost all cases, we found that:

$$Z^{SRAP(2a)}(x^*(2b)) \approx Z^{SRAP(2a)}(x^*(2a)). \tag{45}$$

Table 9 presents the percentage optimality gap when using $Z^{SRAP(2a)}(x^*(2b))$ to estimate $Z^{SRAP(2a)}(x^*(2a))$. This implies that the solution obtained by the relatively more readily solvable linear model (SRAP(2b)) can be used to approximate an optimal solution for SRAP(2a) with an acceptable optimality gap (0.3% on average).

Table 9: Optimality Gap in Estimating the Solution to SRAP(2a) Using SRAP(2b)

	TR*a1	TR*a2	TR*a3	TR*a1m	TR*a2m	TR*a3m
$Z^{SRAP(2a)}(x^*(2a))$	18.3982	19.0228	19.3073	18.4735	19.0350	19.335
$Z^{SRAP(2a)}(x^*(2b))$	18.3218	19.0228	19.2018	18.3767	19.0350	19.228
Optimality Gap (%)	0.42	0.00	0.55	0.52	0.00	0.55

3. Generally, the optimal solutions tend to choose a path between an OD pair that has a minimum number of links and then select the corresponding links for treatment. This is because a path having a minimum number of links in our test cases is usually the shortest path between the OD pair. Moreover, better treatments can be applied to a smaller number of links (in multiple treatment cases), and consequently this results in relatively superior quality solutions.

4. The network efficiency (Objective (2a)) increases with an additional allocation of the treatment budget, as expected, but with diminishing marginal returns, i.e., at some point, the extra budgetary allocations do not result in significant efficiency improvements (see Figure 7). Another observation is that the network efficiency does not improve significantly in cases where several treatment types are considered compared to cases with a single treatment. This might be because of the specific r_{ka}- and c_{ka}- values in the data, which do not justify the selection of maintenance types 1 and 2. Hence, for the Istanbul network, similar to our small illustrative network, optimal solutions tend to favor policies that select fewer links and apply higher quality treatments.

5. The CPU time for all TRs were in the order of milli- and deci- seconds. For TRs with three maintenance types and with Model SRAP*, the execution time was in the order of 0.01-0.02 seconds. For TRs using the original non-reformulated model (SRAP), the execution time was between 0.001-0.008 seconds, although this model resulted in sub-optimal solutions using some solvers as noted above.

Hence, overall, we recommend Model SRAP* for implementation, with the possible use of the linear objective (2b) as an approximation to (2a) for solving relatively large-scale complex problem instances. We anticipate similar results for the multiple period models MRAP and MRAP*, but we relegate the detailed study of these models to future research.

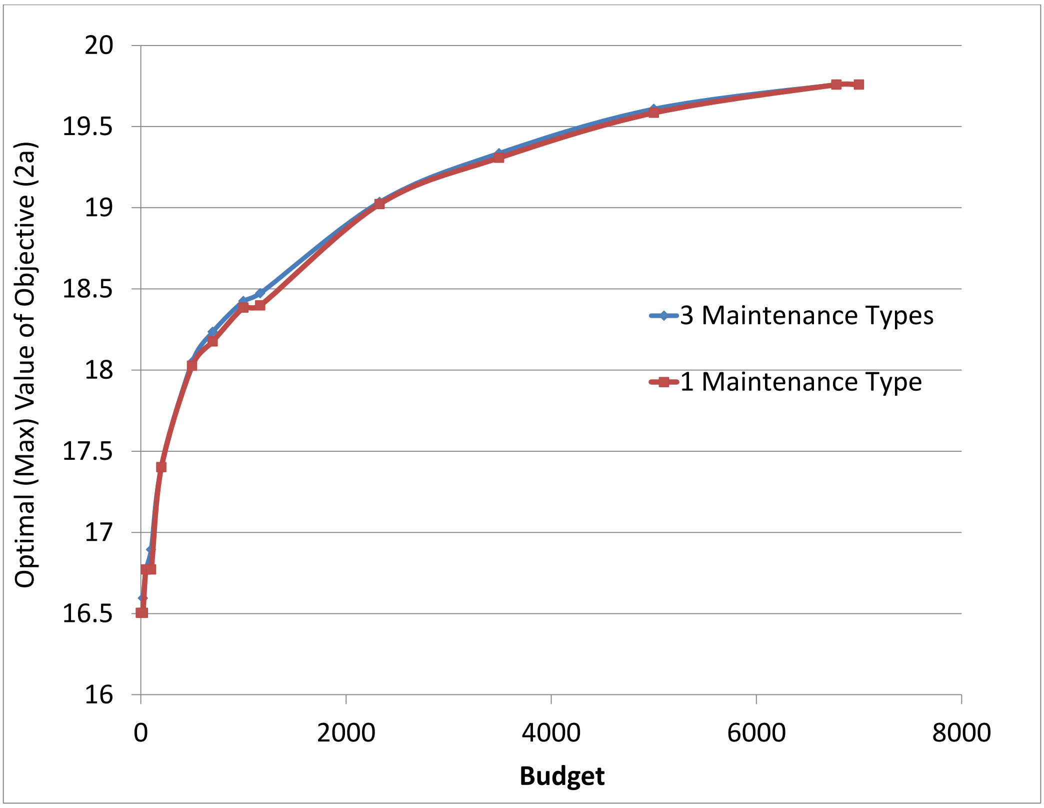

Figure 7: Budgetary Impact on Network Performance

Chapter 6

Discussion

Protection of critical national infrastructure has become a serious issue, particularly after the 9/11 attacks. Moreover, frequent worldwide natural disasters such as the Japan earthquake, hurricane Katrina, and hurricane Sandy in the past few years have made researchers more interested in developing models to select optimal maintenance policies in order to minimize the consequences of disasters on infrastructure systems. As a result, many models have been developed for vulnerability analysis, and for pre- and post-disaster management of roadways, water distribution systems, power networks, and communication systems. However, in reality, infrastructure systems are large-scale networks and existing models often entail simplifying assumptions (which sometimes ignore certain salient properties of the infrastructure system) in order to provide optimal solutions within a reasonable computational time-frame.

In this research, we have developed a relatively simple, generic, and robust model that provides optimal maintenance policies in order to protect critical infrastructure systems against stochastic disruptions triggered by attacks or natural disasters. Our model has a simple structure and is defined as a one-stage mixed-integer problem that inherently captures the configurational properties of the network. The model is generic because it can be applied to any infrastructure system that has a network structure (transportation, power, water, and communication). We enhanced our basic model with additional inequalities to restructure the model representation so that it could be solved more effectively and robustly. We applied our model to a real large scale network (Istanbul highway systems) to assess its robustness. The resulting model was able to attain optimal solutions within a very short time without the need for simplifying assumptions and approximation techniques that have been used in previous studies. We also found that, in general, optimal policies prefer to choose fewer links and to apply higher quality treatments rather than implement lower quality treatments on several links.

Another contribution of this research is that our model with a suitable linear objective function can approximate optimal solutions for the main nonlinear model with an acceptable accuracy. This means that the model with the linear objective can be used instead of the nonlinear model when solving very large scale problems in order to further reduce computational effort.

Future studies can further extend the technical and managerial aspects of this research. With respect to the technical aspect, a reformulated version of model MRAP can be developed and its robustness and tractability on large scale network applications can be assessed. The models can also be applied to other infrastructure systems such as power and communication networks to evaluate their efficacy in addressing a wider scope of applications.

From a managerial perspective, characterizing optimal policies for different types of systems and deriving general protection guidelines based on optimization approaches would be a very interesting topic for future investigation. For example, it is of interest to explore whether the "choose few links and apply higher quality treatments" is a policy that generically applies to different types of networks of various structures and sizes.

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Appendices

Appendix A

Results for the Illustrative Network

Tables 10 through 15 detail the preliminary results for both the SRAP(2b) and SRAP*(2b) models. In these tables, the following variable values are shown in the stated format:

$$d_{ij} = [d_{14}, d_{21}, d_{34}, d_{53}]$$

$$\lambda_q^{ij} = [\lambda_q^{14}, \lambda_q^{14}, \lambda_q^{14}, \lambda_q^{21}, \lambda_q^{34}, \lambda_q^{34}, \lambda_q^{53}, \lambda_q^{53}]$$

$$y_k = [y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8]$$

Table 10: Results for SRAP (2b) and SRAP*(2b) with B=1200

	SRAP(2b)			SRAP*(2b)		
	x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3
1	0	0	0	0	0	0
2	0	0	1	0	0	1
3	0	0	1	0	0	1
4	0	0	1	0	0	1
5	0	0	0	0	0	0
6	0	0	1	0	0	1
7	0	0	1	0	0	1
8	0	0	0	0	0	0
Obj. Value	288850			288850		
d_{ij}	[145.5, 368.5, 70.5, 81.25]			[145.5, 368.5, 70.5, 81.25]		
y_k	[0.5, 0, 0.1, 0.2, 0.75, 0.15, 0, 0.45]			-		
λ_q^{ij}	[0, 1, 0, 1, 1, 0, 1, 0]			[0, 1, 0, 1, 1, 0, 1, 0]		
B	1200			1200		

Table 11: Results for SRAP (2b) and SRAP*(2b) with B=800

	SRAP(2b)			SRAP*(2b)		
	x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	1	0	0	1
4	0	0	1	0	0	1
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	1	0	0	1
8	0	0	0	0	0	0
Obj. Value	296950			296950		
d_{ij}	[145.5, 395.5, 70.5, 81.25]			[145.5, 395.5, 70.5, 81.25]		
y_k	[0.5, 0.45, 0.1, 0.2, 0.75, 0.6, 0, 0.45]			-		
λ_q^{ij}	[0, 1, 0, 1, 1, 0, 1, 0]			[0, 1, 0, 1, 1, 0, 1, 0]		
B	800			800		

Table 12: Results for SRAP (2b) and SRAP*(2b) with B=700

	SRAP(2b)			SRAP*(2b)		
	x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3
1	0	0	0	0	0	0
2	0	0	1	0	0	1
3	0	0	1	0	0	1
4	1	0	0	1	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	1	0	0	1
8	0	0	0	0	0	0
Obj. Value	303700			303700		
d_{ij}	[145.5, 418, 70.5, 81.25]			[145.5, 418, 70.5, 81.25]		
y_k	[0.5, 0, 0.1, 0.5, 0.75,0.6, 0, 0.45]			-		
λ_q^{ij}	[0, 1, 0, 1, 1, 0, 1, 0]			[0, 1, 0, 1, 1, 0, 1, 0]		
B	700			700		

Table 13: Results for SRAP (2b) and SRAP*(2b) with B=600

	SRAP(2b)			SRAP*(2b)		
	x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3
1	0	0	0	0	0	0
2	0	0	1	0	0	1
3	0	0	1	0	0	1
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	1	0	0	1
7	0	0	1	0	0	1
8	0	0	0	0	0	0
Obj. Value	305050			305050		
d_{ij}	[145.5, 422.5, 70.5, 81.25]			[145.5, 422.5, 70.5, 81.25]		
y_k	[0.5, 0, 0.1, 0.65, 0.75, 0.15, 0, 0.45]			-		
λ_q^{ij}	[0, 1, 0, 1, 1, 0, 1, 0]			[0, 1, 0, 1, 1, 0, 1, 0]		
B	600			600		

Table 14: Results for SRAP (2b) and SRAP*(2b) with B=500

	SRAP(2b)			SRAP*(2b)		
	x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3
1	0	0	0	0	0	0
2	0	0	1	0	0	1
3	0	0	1	0	0	1
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	1	0	0	1	0	0
7	0	0	1	0	0	1
8	0	0	0	0	0	0
Obj. Value	307750			307750		
d_{ij}	[145.5, 431.5, 70.5, 81.25]			[145.5, 431.5, 70.5, 81.25]		
y_k	[0.5, 0, 0.1, 0.65, 0.75, 0.45, 0, 0.45]			-		
λ_q^{ij}	[0, 1, 0, 1, 1, 0, 1, 0]			[0, 1, 0, 1, 1, 0, 1, 0]		
B	500			500		

Table 15: Results for SRAP (2b) and SRAP*(2b) with B=400

	SRAP(2b)			SRAP*(2b)		
	x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3
1	0	0	0	0	0	0
2	0	1	0	0	1	0
3	0	0	1	0	0	1
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	0	0	0	0	0	0
7	0	0	1	0	0	1
8	0	0	0	0	0	0
Obj. Value	310900			310900		
d_{ij}	[145.5, 442, 70.5, 81.25]			[145.5, 442, 70.5, 81.25]		
y_k	[0.5, 0.2, 0.1, 0.65, 0.75, 0.6, 0, 0.45]			-		
λ_q^{ij}	[0, 1, 0, 1, 1, 0, 1, 0]			[0, 1, 0, 1, 1, 0, 1, 0]		
B	400			400		

Appendix B

Results for the Istanbul Highway Network

The results for the single and multiple treatment test runs are presented in different tables given below, where the d_{ij}- variable values are displayed in the following format:

$$d_{ij} = [d_{14,20}, d_{14,7}, d_{12,18}, d_{9,7}, d_{4,8}, d_{17,19}, d_{15,22}].$$

Table 16: Results for TRa1

	x_{ka}			
Link	AlphaECP	BARON	DICOPT	SBB
1				
2				
3				
4	1	1	1	
5				
6				
7				
8				
9				
10			1	1
11				
12				
13				
14				
15				
16				
17	1	1		1
18				
19				
20				
21	1	1	1	1
22	1	1	1	1
23				
24				
25	1	1	1	1
26				
27				
28	1	1		
29				
30				
Obj. Value	18.3982	18.3982	18.3218	18.3692
Used Budget	1140	1140	1060	1100
d_{ij}(AlphaECP)	[6.6, 13.6, 10.8, 10.9, 15.2, 22.1, 8.8]			
d_{ij}(BARON)	[6.6, 13.6, 10.8, 10.9, 15.2, 22.1, 8.8]			
d_{ij}(DICOPT)	[6.6, 13.0, 11.3, 10.3, 15.2, 22.1, 9.2]			
d_{ij}(SBB)	[6.6, 13.0, 10.8, 10.3, 15.8, 22.1, 9.2]			
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i, j\} \in OD$			

Table 17: Results for TRa2

	x_{ka}			
Link	AlphaECP	BARON	DICOPT	SBB
1				
2				
3				
4	1	1	1	
5				
6			1	
7				
8				
9				1
10	1	1	1	1
11				
12				1
13				
14				
15				
16				
17	1	1		1
18				
19				
20	1	1	1	1
21	1	1	1	1
22	1	1	1	1
23				
24				
25	1	1	1	1
26				
27				
28	1	1		1
29				
30				
Obj. Value	19.0228	19.0228	18.841	18.9354
Used Budget	2280	2280	2280	2300
d_{ij}(AlphaECP)	[6.6, 12.1, 9.9, 10.3, 15.2, 22.1, 8.8]			
d_{ij}(BARON)	[6.6, 12.1, 9.9, 10.3, 15.2, 22.1, 8.8]			
d_{ij}(DICOPT)	[6.6, 12.1, 10.4, 10.3, 14.4, 22.1, 9.2]			
d_{ij}(SBB)	[6.6, 12.1, 9.9, 10.3, 15.8, 22.1, 8.8]			
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i, j\} \in OD$			

Table 18: Results for TRa3

	x_{ka}			
Link	AlphaECP	BARON	DICOPT	SBB
1				
2				
3				
4	1	1	1	
5				
6	1	1	1	1
7				1
8				
9		1		1
10	1	1	1	1
11				
12				1
13	1	1		1
14				
15				
16			1	
17	1	1		1
18				
19				
20	1	1	1	1
21	1	1	1	
22	1	1	1	1
23				
24				
25	1	1	1	1
26				
27				
28	1	1	1	1
29				
30				
Obj. Value	19.3073	19.3073	19.1986	18.8341
Used Budget	3320	3440	3340	3460
d_{ij}(AlphaECP)	[6.6, 12.1, 9.9, 9.5, 14.4, 22.1, 8.8]			
d_{ij}(BARON)	[6.6, 12.1, 9.9, 9.5, 14.4, 22.1, 8.8]			
d_{ij}(DICOPT)	[6.6, 11.2, 10.4, 10.3, 14.4, 22.1, 8.8]			
d_{ij}(SBB)	[7.0, 12.1, 10.3, 9.5, 15.0, 22.1, 8.8]			
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i, j\} \in OD$			

Table 19: Results for TR*a1

	x_{ka}			
Link	AlphaECP	BARON	DICOPT	SBB
1				
2				
3				
4		1		1
5				
6				
7				
8				
9				
10	1		1	
11				
12				
13				
14				
15				
16				
17		1		1
18				
19				
20				
21	1	1	1	1
22	1	1	1	1
23				
24				
25	1	1	1	1
26				
27				
28	1	1	1	1
29				
30				
Obj. Value	18.3842	18.3982	18.3842	18.3982
Used Budget	920	1140	920	1140
d_{ij}(AlphaECP)	[6.6, 13.0, 11.3, 10.3, 15.8, 22.1, 8.8]			
d_{ij}(BARON)	[6.6, 13.6, 10.8, 10.9, 15.2, 22.1, 8.8]			
d_{ij}(DICOPT)	[6.6, 13.0, 11.3, 10.3, 15.8, 22.1, 8.8]			
d_{ij}(SBB)	[6.6, 13.6, 10.8, 10.9, 15.2, 22.1, 8.8]			
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i,j\} \in OD$			

Table 20: Results for TR*a2

	x_{ka}			
Link	AlphaECP	BARON	DICOPT	SBB
1				
2				
3				
4	1	1	1	1
5				
6				
7				
8				
9				
10	1	1	1	1
11				
12				
13				
14				
15				
16				
17	1	1	1	1
18				
19				
20	1	1	1	1
21	1	1	1	1
22	1	1	1	1
23				
24				
25	1	1	1	1
26				
27				
28	1	1	1	1
29				
30				
Obj. Value	19.0228	19.0228	19.0228	19.0228
Used Budget	2280	2280	2280	2280
d_{ij}(AlphaECP)	[6.6, 12.1, 9.9, 10.3, 15.2, 22.1, 8.8]			
d_{ij}(BARON)	[6.6, 12.1, 9.9, 10.3, 15.2, 22.1, 8.8]			
d_{ij}(DICOPT)	[6.6, 12.1, 9.9, 10.3, 15.2, 22.1, 8.8]			
d_{ij}(SBB)	[6.6, 12.1, 9.9, 10.3, 15.2, 22.1, 8.8]			
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i, j\} \in OD$			

Table 21: Results for TR*a3

	x_{ka}			
Link	AlphaECP	BARON	DICOPT	SBB
1		1		
2		1		
3				
4	1	1	1	1
5				
6	1	1	1	1
7				
8				
9				
10	1	1	1	1
11				
12				
13	1	1	1	1
14				
15				
16				
17	1	1	1	1
18				
19				
20	1	1	1	1
21	1	1	1	1
22	1	1	1	1
23				
24				
25	1	1	1	1
26				
27				
28	1	1	1	1
29				
30				
Obj. Value	19.3073	19.3073	19.3073	19.3073
Used Budget	3320	3480	3320	3320
d_{ij}(AlphaECP)	[6.6, 12.1, 9.9, 9.5, 14.4, 22.1, 8.8]			
d_{ij}(BARON)	[6.6, 12.1, 9.9, 9.5, 14.4, 22.1, 8.8]			
d_{ij}(DICOPT)	[6.6, 12.1, 9.9, 9.5, 14.4, 22.1, 8.8]			
d_{ij}(SBB)	[6.6, 12.1, 9.9, 9.5, 14.4, 22.1, 8.8]			
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i, j\} \in OD$			

Table 22: Results for TRb1 & TR*b1

	x_{ka}		
Link	CoinCbc	Gurobi	CPLEX
1			
2			
3			
4	1	1	1
5			
6			
7			
8			
9			
10	1	1	1
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
21	1	1	1
22	1	1	1
23			
24			
25	1	1	1
26			
27			
28			
29			
30			
Obj. Value	2588.874	2588.874	2588.874
Used Budget	1060	1060	1060
d_{ij}(CoinCbc)	[6.6, 13.0, 11.3, 10.3, 15.2, 22.1, 9.2]		
d_{ij}(Gurobi)	[6.6, 13.0, 11.3, 10.3, 15.2, 22.1, 9.2]		
d_{ij}(CPLEX)	[6.6, 13.0, 11.3, 10.3, 15.2, 22.1, 9.2]		
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i,j\} \in OD$		

Table 23: Results for TRb2 & TR*b2

	x_{ka}		
Link	CoinCbc	Gurobi	CPLEX
1			
2			
3			
4	1	1	1
5			
6			
7			
8			
9			
10	1	1	1
11			
12			
13			
14			
15			
16			
17	1	1	1
18			
19			
20	1	1	1
21	1	1	1
22	1	1	1
23			
24			
25	1	1	1
26			
27			
28	1	1	1
29			
30			
Obj. Value	2507.31	2507.31	2507.31
Used Budget	2280	2280	2280
d_{ij}(CoinCbc)	[6.6, 12.1, 9.9, 10.3, 15.2, 22.1, 8.8]		
d_{ij}(Gurobi)	[6.6, 12.1, 9.9, 10.3, 15.2, 22.1, 8.8]		
d_{ij}(CPLEX)	[6.6, 12.1, 9.9, 10.3, 15.2, 22.1, 8.8]		
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i, j\} \in OD$		

Table 24: Results for TRb3 & TR*b3

	x_{ka}		
Link	CoinCbc	Gurobi	CPLEX
1			
2			
3			
4	1	1	1
5			
6	1	1	1
7			
8			
9			
10	1	1	1
11			
12			
13			
14			
15			
16			
17	1	1	1
18			
19			
20	1	1	1
21	1	1	1
22	1	1	1
23			
24	1	1	1
25	1	1	1
26			
27			
28	1	1	1
29			
30			
Obj. Value	2455.31	2455.31	2455.31
Used Budget	3320	3320	3320
d_{ij}(CoinCbc)	[6.6, 12.1, 9.9, 10.3, 14.4, 21.3, 8.8]		
d_{ij}(Gurobi)	[6.6, 12.1, 9.9, 10.3, 14.4, 21.3, 8.8]		
d_{ij}(CPLEX)	[6.6, 12.1, 9.9, 10.3, 14.4, 21.3, 8.8]		
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i, j\} \in OD$		

Table 25: Results for TRa1m

Solvers	AlphaECP			BARON			DICOPT			SBB		
	x_{ka}			x_{ka}			x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
1												
2												
3												
4	1								1			
5												
6	1											
7												
8												
9												
10			1			1			1			
11												
12												
13	1											
14												
15												
16												
17	1				1							1
18												
19												
20	1			1			1			1		
21			1			1			1		1	
22			1			1			1			1
23												
24												
25			1			1			1			1
26												
27												
28			1			1	1					1
29												1
30												
Obj. Value	18.4722			18.4734			18.3767			18.1137		
Used Budget	1160			1150			1152			1160		
d_{ij}(AlphaECP)	[6.6, 12.9, 11.2, 10.2, 15.7, 22.1, 8.8]											
d_{ij}(BARON)	[6.6, 12.9, 11.0, 10.3, 15.8, 22.1, 8.8]											
d_{ij}(DICOPT)	[6.6, 12.9, 11.2, 10.3, 15.2, 22.1, 9.1]											
d_{ij}(SBB)	[6.9, 13.5, 10.9, 10.9, 15.8, 22.1, 8.8]											
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall\{i,j\} \in OD$											

Table 26: Results for TRa2m

Solvers	AlphaECP			BARON			DICOPT			SBB		
	x_{ka}			x_{ka}			x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
1												
2												
3												
4			1			1			1			
5										1		
6	1			1					1			
7												
8												
9												1
10			1			1			1			1
11												
12												1
13												
14												
15												
16												
17			1			1	1					1
18												
19												
20			1			1			1			1
21			1			1			1		1	
22			1			1			1			1
23												
24												
25			1			1			1			1
26												
27												
28			1			1	1					1
29										1		
30												
Obj. Value	19.035			19.035			18.8706			18.6865		
Used Budget	2322			2322			2322			2318		
d_{ij}(AlphaECP)	[6.6, 12.1, 9.9, 10.3, 15.1, 22.1, 8.8]											
d_{ij}(BARON)	[6.6, 12.1, 9.9, 10.3, 15.1, 22.1, 8.8]											
d_{ij}(DICOPT)	[6.6, 12.1, 10.4, 10.3, 14.4, 22.1, 9.1]											
d_{ij}(SBB)	[6.9, 12.1, 10.1, 10.3, 15.8, 22.1, 8.8]											
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall\{i,j\} \in OD$											

Table 27: Results for TRa3m

Solvers	AlphaECP			BARON			DICOPT			SBB		
	x_{ka}			x_{ka}			x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
1												
2												
3				1				1				
4			1			1			1			
5												
6			1			1			1			1
7												1
8												
9												1
10			1			1			1			1
11												
12												1
13			1			1	1					1
14												
15												
16	1			1					1			
17			1			1	1					1
18												
19												
20			1			1			1			1
21			1			1			1	1		
22			1			1			1			1
23												
24												
25			1			1			1			1
26	1											
27												
28			1			1	1					1
29										1		
30												
Obj. Value	19.3338			19.3356			19.1167			18.87		
Used Budget	3492			3446			3484			3486		
d_{ij}(AlphaECP)	[6.6, 12.0, 9.9, 9.5, 14.4, 22.0, 8.8]											
d_{ij}(BARON)	[6.6, 12.0, 9.9, 9.5, 14.4, 22.1, 8.8]											
d_{ij}(DICOPT)	[6.6, 11.2, 10.4, 10.2, 14.3, 22.1, 9.1]											
d_{ij}(SBB)	[7.0, 12.1, 10.2, 9.5, 15.0, 22.1, 8.8]											
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i,j\} \in OD$											

Table 28: Results for TRb1m & TR*b1m

Solvers	CoinCbc			Gurobi			CPLEX		
	x_{ka}			x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
1									
2									
3									
4			1			1			1
5									
6									
7									
8									
9									
10			1			1			1
11									
12									
13									
14									
15									
16									
17									
18									
19									
20	1			1			1		
21			1			1			1
22			1			1			1
23				1					
24									
25			1			1			1
26									
27									
28	1			1			1		
29									
30									
Obj. Value	2581.774			2581.774			2581.774		
Used Budget	1152			1156			1152		
d_{ij}(all solvers)	[6.6, 12.9, 11.2, 10.3, 15.2, 22.1, 9.1]								
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall\{i,j\} \in OD$								

Table 29: Results for TRb2m & TRb*2m

Solvers	CoinCbc			Gurobi			CPLEX		
	x_{ka}			x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
1									
2									
3									
4			1			1			1
5									
6	1			1			1		
7									
8									
9									
10			1			1			1
11									
12									
13									
14									
15									
16									
17			1			1			1
18									
19									
20			1			1			1
21			1			1			1
22			1			1			1
23									
24									
25			1			1			1
26									
27									
28			1			1			1
29									
30									
Obj. Value	2504.51			2504.51			2504.51		
Used Budget	2322			2322			2322		
d_{ij}(all solvers)	[6.6, 12.1, 9.9, 10.3, 15.1, 22.1, 8.8]								
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i,j\} \in OD$								

Table 30: Results for TRb3m & TRb*3m

Solvers	CoinCbc			Gurobi			CPLEX		
	x_{ka}			x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
1									
2									
3									
4			1			1			1
5									
6			1			1			1
7									
8									
9									
10			1			1			1
11									
12									
13									
14									
15									
16	1			1			1		
17			1			1			1
18									
19									
20			1			1			1
21			1			1			1
22			1			1			1
23									
24			1			1			1
25			1			1			1
26	1			1			1		
27									
28			1			1			1
29									
30									
Obj. Value	2449.81			2449.81			2449.81		
Used Budget	3492			3492			3492		
d_{ij}(all solvers)	[6.6, 12.0, 9.9, 10.3, 14.4, 21.2, 8.8]								
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i, j\} \in OD$								

Table 31: Results for TR*a1m

Solvers	AlphaECP			BARON			DICOPT			SBB		
	x_{ka}			x_{ka}			x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
1												
2												
3												
4												
5												
6												
7												
8												
9				1								
10			1			1			1			1
11												
12												
13												
14												
15												
16												
17		1			1			1			1	
18												
19												
20	1			1			1			1		
21			1			1			1			1
22			1			1			1			1
23												
24												
25			1			1			1			1
26												
27												
28			1			1			1			1
29												
30												
Obj. Value	18.4734			18.4734			18.4734			18.4734		
Used Budget	1150			1162			1150			1150		
d_{ij}(all solvers)	[6.6, 12.9, 11.0, 10.3, 15.8, 22.1, 8.8]											
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall\{i,j\} \in OD$											

Table 32: Results for TR*a2m

Solvers	AlphaECP			BARON			DICOPT			SBB		
	x_{ka}			x_{ka}			x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
1												
2												
3												
4			1			1			1			1
5												
6	1			1					1			1
7												
8												
9												
10			1			1			1			1
11												
12												
13												
14												
15												
16												
17			1			1	1			1		
18												
19												
20			1			1			1			1
21			1			1			1			1
22			1			1			1			1
23												
24												
25			1			1			1			1
26												
27												
28			1			1	1			1		
29												
30												
Obj. Value	19.035			19.035			19.035			19.035		
Used Budget	2322			2322			2322			2322		
d_{ij}(all solvers)	[6.6, 12.1, 9.9, 10.3, 15.1, 22.1, 8.8]											
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i,j\} \in OD$											

Table 33: Results for TR*a3m

Solvers	AlphaECP			BARON			DICOPT			SBB		
	x_{ka}			x_{ka}			x_{ka}			x_{ka}		
Link	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
1												
2												
3	1			1			1			1		
4			1			1			1			1
5												
6			1			1			1			1
7												
8												
9												
10			1			1			1			1
11												
12												
13			1			1			1			1
14												
15												
16	1			1			1			1		
17			1			1			1			1
18												
19												
20			1			1			1			1
21			1			1			1			1
22			1			1			1			1
23												
24												
25			1			1			1			1
26												
27												
28			1			1			1			1
29												
30												
Obj. Value	19.3356			19.3356			19.3356			19.3356		
Used Budget	3446			3446			3446			3446		
d_{ij}(all solvers)	[6.6, 12.0, 9.9, 9.5, 14.4, 22.1, 8.8]											
λ_q^{ij}(all solvers)	$\lambda_1^{ij} = 1, \forall \{i,j\} \in OD$											