Path Selection to Minimize Energy Consumption of an Electric Vehicle using Synthetic Speed Profiles and Predictive Terminal Energy

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Thesis submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Master of Science
In
Mechanical Engineering

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May 5, 2017
Blacksburg, VA

Keywords: eco-routing, battery electric vehicle, acceleration models, velocity profiles, environment, energy consumption
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Academic Abstract
Manufacturers of passenger vehicles are experiencing increased pressure from consumers and legislators due to the impact of transportation on the environment. Automotive manufacturers are responding by designing more sustainable forms of transportation through a variety of efforts, including increased vehicle efficiency and the electrification of vehicle powertrains (plug in hybrid electric vehicles (PHEV) and battery electric vehicles (BEV)). An additional method for reducing the environmental impact of personal transport is eco-routing, a methodology which selects routes on the basis of energy consumption.

Standard navigation systems offer route alternatives between a user clarified origin and destination when there are multiple paths available. These alternatives are commonly weighted on the basis of minimizing either total travel time (TTT) or trip distance. Eco-routing offers an alternative criterion – minimizing route energy consumption. Calculation of the energy consumption of a route necessitates the creation of a velocity profile which models how the route will be driven and a powertrain model which relates energy consumption to the constructed velocity profile. Existing research efforts related to both of these aspects typically require complex analysis and proprietary vehicle properties.

A new approach to weighting the energy consumption of different routes is presented within this paper. The process of synthesizing velocity profiles is an improvement upon simpler models while requiring fewer variables as compared to more complex models. A single input, the maximum acceleration, is required to tune driver aggressiveness throughout an entire route. Additionally, powertrain results are simplified through the application of a new parameter, predictive terminal energy. The parameter uses only glider properties as inputs, as compared to dedicated powertrain models which use proprietary vehicle information as inputs which are not readily available from manufacturers. Application of this research reduces computation time and increases the number of vehicles for which this analysis can be applied. An example routing scenario is presented, demonstrating the capability of the velocity synthesis and predictive terminal energy methodologies.
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General Audience Abstract
Research into environmental issues associated with greenhouse gas emissions (GHG) has placed increased pressure on a wide range of industries, transportation in particular. The studied impact of transportation on the environment is shaping legislative efforts and consumer expectations for more energy efficient vehicles. Vehicle manufactures are responding by designing more efficient vehicles such as plug in hybrid electric vehicles (PHEV) and battery electric vehicles (BEV). Beyond efforts into improving vehicle design, research is also being conducted into the efficient routing of vehicles.

Navigation systems often provide multiple options for traveling from a specified origin and destination. These systems typically report the trip distance and time enabling the traveler to make an informed decision of which route to select. Eco-routing seeks to add a new metric associated with each route option – the energy required to travel from the origin to the destination. Calculating the energy required to travel a given route involves estimating driver behavior and the powertrain response. Calculation of these two factors within existing research typically involves complicated analysis and a variety of vehicle parameters which are not easily accessible.

A new approach to modeling the driver behavior and route dynamics over a given route is presented in this thesis. The presented method for creating velocity profiles is notably less complex than existing research efforts. Additionally, calculation of the powertrain response, or the energy expended to traverse a given route, is explored. Eco-routing methods discussed in current research often require specific and proprietary information about vehicles to produce results. This thesis simplifies the process of estimating the energy required to complete a route by reducing the required information about passenger vehicles to solely publicly available information. An example routing scenario is presented which provides a demonstration of the discussed methods for approximating driver behavior and powertrain response.
Acknowledgements

I am appreciative of the Hybrid Electric Vehicle Team (HEVT) of Virginia Tech and all the team members I had the pleasure to work with. The team, participating in the Department of Energy and GM sponsored EcoCAR 3 competition, provided an opportunity to explore my passions related to hybrid and electric vehicles. I would not have pursued this research without the foundation I gained through participating with HEVT.

I would also like to thank my parents for providing the support which encourages taking risks and failing often. Finally, I would like to thank my advisor, Dr. Doug Nelson for sharing his patience, passion, and expertise. This research would not have been possible without Dr. Nelson’s generous time spent reviewing countless iterations of work.
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<td>Battery Electric Vehicle</td>
<td>(BEV)</td>
</tr>
<tr>
<td>Downloadable Dynamometer Database</td>
<td>(D3)</td>
</tr>
<tr>
<td>Greenhouse Gas</td>
<td>(GHG)</td>
</tr>
<tr>
<td>Green Vehicle Routing Problem</td>
<td>(GVRP)</td>
</tr>
<tr>
<td>Internal Combustion Engine Vehicle</td>
<td>(ICEV)</td>
</tr>
<tr>
<td>Inertial Specific Power</td>
<td>(ISP)</td>
</tr>
<tr>
<td>Plug-in Hybrid Electric Vehicle</td>
<td>(PHEV)</td>
</tr>
<tr>
<td>Root Mean Square</td>
<td>(RMS)</td>
</tr>
<tr>
<td>Total Travel Time</td>
<td>(TTT)</td>
</tr>
<tr>
<td>Urban Dynomometer Driving Schedule</td>
<td>(UDDS)</td>
</tr>
<tr>
<td>Vehicle Routing Problem</td>
<td>(VRP)</td>
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Introduction

The process of identifying the eco-route between a user clarified origin and destination involves several steps. First, a routing algorithm identifies potential paths, or route options, between the specified points. A velocity profile is then generated for each route option which captures the dynamics and constraints associated with a given path (speed limits, stops, etc.). Driver aggressiveness is incorporated into this portion of the eco-routing process as well. Once velocity profiles for each potential path are generated, a powertrain model is used to estimate the energy required to respond to the dynamics and constraints associated with each route option. A summary of this process can be seen in Figure A.1.

Figure A.1 Eco-routing process which identifies an optimal route on the basis of energy consumption in response to an input origin destination pair.

The objectives of this research focus on the portions contained within the red box above. This thesis is comprised of two journal papers which respectively focus on A, the derivation and validation of a velocity synthesis methodology, and B, the development of a new parameter to estimate route energy consumption. A chapter is dedicated to each journal paper with corresponding references and appendices. Additionally, a literature review is present at the beginning of each chapter to explore to the existing research associated with the content each journal paper. An aggregate literature review is thus excluded from this introduction to avoid repeating information. Finally, the thesis will finish with an overarching conclusion and supporting appendices containing information relevant to both journal papers. These appendices are not included within the individual chapters due to space limitations associated with journal submission. Any discussion of the supplemental information is present within the appendices themselves given that they are excluded from both journal papers.
1 Chapter One - Synthesis of speed versus time modes using variable acceleration rates for tractive energy analysis and route planning

1.1 Abstract

Eco-routing, the process of evaluating route options with the goal of selecting the route which minimizes energy consumption, represents great potential for reducing the environmental impact of personal transport. A new route synthesis technique is introduced which requires minimal route parameters to generate velocity versus time profiles. The synthesis methodology discussed is capable of generating full stop-to-stop velocity profiles with only distance, acceleration, cruise speed, and jerk rate as inputs. A synthesized alternative to the first 505 seconds of the UDDS certification cycle shows an overall tractive energy error of only -3.40%. Additionally, a constrained comparison methodology is introduced which captures tradeoffs between increased acceleration and decreased cruising speed. Further constrained comparisons are introduced which aid in the selection of appropriate distance horizons for on-board optimization of energy consumption. In an example presented, a distance horizon beyond 3.5 km is shown to be of little application.

Keywords: eco-routing, acceleration models, speed profiles, electric vehicle, tractive energy

1.2 Introduction

Rising environmental concerns have placed an increased emphasis on minimizing vehicle energy consumption. Significant progress has been made through recent engineering efforts to improve the efficiency of passenger transport, most notably through increasing vehicle electrification (Plug-in Hybrid Electric Vehicles [PHEV] and Battery Electric Vehicles [BEVI]). Efforts toward increasing vehicle fuel economy are not strictly limited between the energy source – battery or fuel – and the wheels; eco-routing is an example of research which reduces energy use from a focus outside of vehicle design. Eco-routing emphasizes selecting the most energy-efficient route (rather than the quickest or the shortest). Utilizing eco-routing requires no additional powertrain components and research efforts have cited fuel savings of 3.3% to 9.3% [1.1].

The primary focus of this research is to develop simple criterion for synthesizing velocity modes, or stop to stop velocity profiles. Specifically, the modes of interest feature a distance between two known stop locations. Synthesizing stop to stop behavior (initial and final speeds equal to zero) is especially important for predicting the energy consumption of urban trips, which often feature multiple route alternatives. Accurately predicting and comparing the energy consumption of different routes first requires a reasoned synthesis approach; poorly modeling the route dynamics may lead to selecting a less than optimal path. Reasoned construction of these velocity modes enables route analysis for the eventual use in an eco-routing application. The objectives of this paper are to analyze existing literature related to route synthesis and acceleration modeling, detail the development and calibration of a new acceleration model, and finally explore the accuracy of synthesized routes and evaluate route implications of different constrained velocity profiles. Related future work involving eco-routing will be discussed as well.
1.3 Existing Acceleration Models

A trivial outline of a given route velocity profile as a function of distance can be easily constructed through the knowledge of speed limits, stop sign locations, etc. However, this outline does not contain the accelerations and decelerations, which disproportionately contribute to fuel economy. A naturalistic driving study of over 100 vehicles noted that 20% of fuel was consumed during acceleration events despite these same events constituting only 6% of the distance traveled [1.2]. Deceleration events also heavily influence energy consumption figures for PHEV’s and BEV’s due to regenerative braking. Thus, incorporating the transitions between speeds into eco-routing calculations is important for accurately predicting energy consumption. This literature review serves to introduce the concept of acceleration models, introduce popular examples, and discuss model goals.

Several acceleration models exist with varying degrees of complexity. The simplest, the constant acceleration model, assumes a non-zero acceleration that does not change with respect to time. Constant acceleration models produce unrealistic velocity profiles due to the power and driving constraints. Two certification cycles – NEDC and Japan 10-15 Mode – utilize constant accelerations and decelerations; these cycles are being replaced with more natural driving profiles, the WLTP and JC08 respectively. Real world driving studies have shown that acceleration decreases as velocity increases, in an approximately linear fashion [1.3]. A linearly decreasing acceleration model produces a velocity profile which peaks to a cruising speed as acceleration reaches zero and more accurately captures real-world driving behavior (Equation 1-1).

\[ v(t) = v_{\text{max}} - (v_{\text{max}} - v_0)e^{-\beta t} \]  

Equation 1-1

where \( \beta \) = vehicle-specific constant.

Additional research supports the notion that a constantly decreasing acceleration model more accurately models real-world acceleration behavior, but suggests that the real world velocity shape is closer to that of an “S” and that a constantly decreasing model necessitates an unrealistically large initial acceleration [1.4]. Figure 1.1a and Figure 1.1b below compare the resulting velocity profiles of a typical linearly decreasing acceleration model and the more complicated polynomial model proposed by Akçelik and Biggs [1.4].

![Figure 1.1 Velocity profiles from linearly decreasing acceleration (a) and polynomial acceleration (b).](image)
The model proposed by Akçelik and Biggs is more involved, containing several tunable parameters within a detailed expression for acceleration.Rather than a linear formulation that decreases from a large maximum acceleration value, the polynomial acceleration model (Equation 1-2) begins with an initial acceleration of zero, gradually rises to a peak value, and tapers back to zero as the velocity approaches the cruise speed. Akçelik and Biggs contend this is a further improvement on the original constant acceleration model.

\[ a(t) = r a_m \theta^n (1 - \theta^m)^2 \]  

Equation 1-2

where \( a_m \) = maximum acceleration,
\( t_a \) = acceleration time,
\( \theta \) = time ratio \((t/t_a)\),
\( m, n \) = tunable constants,

and \( r = f(m, n) \), a third constant.

As shown above, the polynomial model is successful in synthesizing the “S” shaped behavior of real-world velocity profiles. The realistic peak acceleration value within the formulation is an additional model improvement; fuel consumption is directly related to acceleration and a velocity synthesis with unrealistic acceleration values could lead to selecting an incorrect route. Although the polynomial model more closely approximates the “S” behavior of real-world velocity profiles, the vehicle-specific constants of \( m \) and \( n \) are a drawback. Additional calibration of the polynomial model has been performed since the original formulation [1.5]. The goal of this paper is to produce a simpler acceleration model that still approximates the characteristics of real world driving.

Wang et al. analyzed both the linearly decreasing model and the polynomial models using driver data [1.6]. The driver data corroborated Long’s claim that the acceleration rates decreased as speed increased, although Wang found that the relationship between acceleration and velocity could not be assumed as linear. Wang agreed with Ackçelik’s suggestion of an “S” shaped velocity curve and found the polynomial model to be superior to the linearly decreasing model. However, although correct in shape, the polynomial model struggled to match the driver data as well. Wang proposes a separate polynomial model which is generated by applying a regression model directly to the driver data (Equation 1-3). One limitation of the regression model proposed by Wang is that the model is not tunable for more aggressive acceleration rates, relying solely on collected data. A model which can be adjusted based on driver aggressiveness is preferred for synthesis purposes.

\[ \sqrt{a(t)} = 1.381 - 0.011 * v(t) \]  

Equation 1-3

The models discussed vary in degrees of complexity and levels of accuracy in which they fit real world driver data. Several more acceleration models exist [1.7], [1.8] each featuring their own strengths and limitations. Additionally, several other papers approach route synthesis using vehicle parameters and control theory to minimize consumption; these methods are considered to be outside of the scope of this paper. A new acceleration model is desired which contains minimal tuning parameters while also satisfying the constraints discussed (zero acceleration at zero speed, “S” shape, etc). The paper will proceed by outlining a new acceleration model, model calibration, and constrained analysis between cases. Tractive energy implications from synthesized routes will be explored as well by utilizing a simple
glider model with standard vehicle dynamics expressions (Figure 1.2, Equation 1-4). Finally, the paper will conclude with a summary of results and outline appropriate future work.

![Free body diagram of a vehicle](image)

**Figure 1.2** Free body diagram of a vehicle.

\[ F_{tr} = F_i + F_{rr} + F_d + F_g \]  
Equation 1-4

Where \( F_{tr} \) = supplied tractive force,  
\( F_i \) = inertial force,  
\( F_{rr} \) = rolling resistance force,  
\( F_d \) = drag force,  
\( F_g \) = grade force.

1.4 Model Formulation

The main goal of this paper is to produce a velocity synthesis method which describes the vehicle trajectory between an initial and final speed separated by a known distance. The nature of synthesizing full stop to stop profiles, or modes, requires expanding beyond the acceleration models discussed in the previous section; a mode as discussed in this paper features an acceleration event, a cruise event (if necessary), and a deceleration event. Full modes are of particular interest because they contain a large array of driving events and can be employed to simplify complex driving profiles with few inputs (Section 1.3).

The first model devised to approximate stop to stop trajectory is very rudimentary. Acceleration behavior is modeled as 3 regions of constant acceleration: positive acceleration during launch, zero acceleration during cruise, and negative acceleration during deceleration. The piecewise acceleration model (Figure 1.3a) generates a trapezoid shaped velocity profile (Figure 1.3b), far from the “S” shape supported by Akçelik and Wang. The piecewise acceleration model (Equation 1-5 and Equation 1-6) will be referred to as the Trapezoid Model throughout the remainder of the paper. Note the presence of \( t \) and \( t_r \); \( t \) represents the global time of the mode while \( t_r \) represents the relative time within a particular piecewise event or “region”. For instance, if the vehicle stops cruising at 10 seconds and begins decelerating, one second later the global time is 11 seconds while the region time is 1 second.
\[
a(t) = \begin{cases} 
\alpha_{\text{max}} & t < t_a \\
0 & t_a \leq t \leq t_c \\
d_{\text{max}} & t > t_c 
\end{cases} 
\]

\[
v(t) = \begin{cases} 
\alpha_{\text{max}} t_r & t < t_a \\
v_c & t_a \leq t \leq t_c \\
v_c + d_{\text{max}} t_r & t > t_c 
\end{cases} 
\]

Equation 1-5

Equation 1-6

Where \( t_a \) = time required to accelerate to \( v_c \),
\( t_c \) = time at cruise completion,
\( t_r \) = time in a given piecewise region,
and \( v_c \) = cruise velocity.

**Figure 1.3** Acceleration (a) and velocity (b) profiles for the Trapezoid Model.

Vehicle simulations using the Trapezoid Model highlight a problem with the constant acceleration formulation echoed by Lee - as the speed increases, the power demand from overcoming the vehicle inertia continues to rise unrealistically. Large inertial power events at high speeds are especially problematic when considering the strong relationship between drag power and speed (\( \propto V^3 \)). The confluence of large inertial power demand as well as large drag power demand requires adjusting the constant acceleration behavior of the Trapezoid Model. Acceleration should taper as speed rises to reflect the limitations of real world driving (powertrain constraints, for instance). However, as Akçelik notes, a linearly decreasing acceleration model requires unrealistic initial accelerations whereas real world driving starts with an acceleration of zero. A modification to the Trapezoid Model is needed which enforces zero initial acceleration and an intermediate maximum acceleration that tapers off as speed rises. In real world driving, power demand rises with acceleration and tapers off with acceleration to a smaller cruise value [1.3]. The Trapezoid model is adjusted with the goals of better approximating these acceleration and power behaviors.

Rather than deriving the speed transition model using a piecewise acceleration with constant terms, the model is altered by instead employing variable acceleration rates. Variable acceleration is implemented through a piecewise jerk constraint, where jerk represents the change of acceleration over time, similar to efforts explored in [1.9] and [1.10]. The acceleration and velocity profiles are modified accordingly; Equation 1-7, Equation 1-8, and Equation 1-9 contain the jerk, acceleration, and velocity expressions during acceleration while Figure 1.4a, Figure 1.4b, and Figure 1.4c display these quantities versus time.
Similar to the designation of the Trapezoid Model, the piecewise jerk model will be referred to as the Hill Model.

\[
j(t) = \begin{cases} 
  j_{\text{max}} & t < t_1, \\
  0 & t_1 \leq t \leq t_2, \\
  j_{\text{min}} & t > t_3 
\end{cases} 
\]  
\text{Equation 1-7}

\[
a(t) = \begin{cases} 
  j_{\text{max}}t_r & t < t_1, \\
  a_{\text{max}} & t_1 \leq t \leq t_2, \\
  a_{\text{max}} + j_{\text{min}}t_r & t > t_3 
\end{cases} 
\]  
\text{Equation 1-8}

\[
v(t) = \begin{cases} 
  j_{\text{max}}t_r^2/2 & t < t_1, \\
  a_{\text{max}}t_r & t_1 \leq t \leq t_2, \\
  j_{\text{min}}t_r^2/2 + a_{\text{max}}t_r & t > t_3 
\end{cases} 
\]  
\text{Equation 1-9}

Where $t_1$ = time at the conclusion of Region 1, $t_2$ = time at the conclusion of Region 2, $t_3$ = time at the conclusion of Region 3, $j_{\text{max}}$ = the maximum jerk rate, $j_{\text{min}}$ = the minimum jerk rate.

**Figure 1.4** Jerk (a) and acceleration (b) profiles for the Hill Model and the resulting velocity profile (c).
Enforcing a constant, piecewise jerk constraint solves several problems. The acceleration profile begins and finishes at zero and the resulting velocity profile achieves the desired “S” shape. The full Hill Model (Figure 1.5) contains 7 regions which each exhibit different portions of the piecewise jerk expression. Region 4 represents the time spent cruising, while regions 5, 6, and 7 describe the vehicle deceleration. Regions 5, 6, and 7 are assumed to be symmetrical to regions 1, 2, and 3; the acceleration begins at zero, jerks to a maximum deceleration (region 5), maintains a constant value (region 6), and jerks back to zero (region 7).

![Figure 1.5 Full Hill Model (velocity-time) with regions annotated.](image)

Generating a full mode requires a prescribed maximum acceleration, deceleration, positive jerk, negative jerk, initial velocity, final velocity, cruise velocity, and distance. The time of a mode synthesis is a function of the input parameters and not explicitly prescribed. Two simplifying relationships are employed to reduce the overall number of inputs for the Hill model. These include: equating the positive and negative maximum acceleration values (as is often observed in real world driving), and relating the positive and negative jerk rates to the maximum acceleration values. It is reasoned that more aggressive drivers who accelerate at higher rates also jerk between acceleration rates quickly (sharp pedal adjustments). A tuned constant of 0.5 effectively relates the maximum acceleration for a given mode to the jerk value used to transition between accelerations (Equation 1-10). By equating acceleration extremes and relating them to jerk rates, a single parameter can be used to drive the acceleration intensity of the entire mode. For instance, if a maximum acceleration value of 1.5 m/s² is prescribed, the maximum deceleration will be -1.5 m/s² and the jerk values applied will be 0.75 m/s³ and -0.75 m/s³. Part of these assumptions will be adjusted in the calibration section.

\[ \alpha = \frac{j_{max}}{a_{max}} \]  

**Equation 1-10**

Where \( \alpha \) = tuned constant,

\[ a_{max} = \text{maximum prescribed acceleration}, \]

and \( j_{max} = \text{maximum jerk}. \)

Finally, it is worth noting that the time to complete the mode is not an input, but instead a response to the previously stated mode parameters including distance. A model formulation with constrained time and responsive distance is possible. However, the time constrained model was not pursued given the
nature of eco-routing; information along a route is inherently distance based (location of speed limits, for instance). Synthesizing profiles using distance as an input is thus chosen. A brief description of each region as a function of the inputs above follows with relevant equations present within Appendix A.

Region 1 begins with zero acceleration and maximum jerk, continuing until the prescribed maximum acceleration develops. The time spent during region 1 is simply the time required for the jerk rate to produce the maximum acceleration. Final acceleration, velocity, and distance values for a given region are used as the initial values for each subsequent region to maintain continuity. Jerk is the only model parameter which is not continuous. Region 2 begins with an initial velocity equivalent to the velocity developed during the jerk event and features constant acceleration (jerk equals zero). The duration of region 2 is determined by the velocity difference necessary to produce the cruising speed. Regions 1 and 3 last for known amounts of time and thus result in known velocity differences; region 2 lasts the necessary amount of time to force the prescribed cruise speed to develop. Finally, region 3 begins with the maximum prescribed acceleration and the negative jerk rate. The region continues until the velocity reaches the cruise speed simultaneously with the acceleration reaching zero. Regions 5, 6, and 7 represent the transition from the cruise speed to the final speed and are calculated in a similar manner. Time and distance contributions for regions 1-3 and 5-7 are easily determined analytically. Region 4, the time spent at the cruising speed with zero acceleration, lasts as long as necessary to make up the difference in distance between the prescribed value for the overall mode and the contributions from the six other regions.

It is important to note a few constraints present within the synthesis methodology described above. The maximum and minimum acceleration values as well as the cruise velocity are assumed to develop. Certain input combinations can violate this assumption and cause synthesis errors. For instance, a cruise velocity smaller than the speed differences from regions 1 and 3 prevents region 2 from developing. A distance input smaller than the distance traveled in regions 1-3 and 5-7 causes model discontinuities as well. 0 contains analytical expressions for each region and the conditions which ensure these governing assumptions are not violated. Additionally, route dynamics and powertrain limitations are not enforced. Accelerations are modeled blind of the impact of grade and without consideration for powertrain performance. A scenario involving a steep grade and a limited powertrain may be insufficient in meeting accelerations prescribed by the velocity synthesis. A related study performed by Rakha et al. [1.11] includes grade and vehicle limitations in a similar velocity synthesis. The analysis within this paper is intentionally vehicle agnostic and complications originating from the inclusion of grade information are not explored. It is assumed that drivers select similar levels of acceleration and aggression regardless of vehicle selection and the acceleration rates within this paper do not infringe upon powertrain constraints. If average grade information is known, it could be incorporated into the velocity synthesis method by adjusting the acceleration rate. For instance, a speed transition with a maximum acceleration of 2.5 m/s² could also represent the tractive power demand associated with a lower acceleration rate occurring on a mild grade.

The Hill Model serves as a modification of the original Trapezoid model, which exhibits unrealistic power demands due to sustained peak accelerations at high speeds. An important comparison evaluates the two route synthesis models to understand if the jerk constraint and tapering of the acceleration correctly alleviates the large power demand issue. Inertial Specific Power (ISP), a route specific parameter that does not depend on vehicle properties, is used for this evaluation. ISP is an instantaneous parameter which is calculated by multiplying the acceleration and velocity at a given instant (Equation 1-11).
\[ ISP(t) = v(t) \times a(t) \]  

Equation 1-11

Where \( ISP(t) \) = Inertial Specific Power at time \( t \), \( \left( \frac{m^2}{s^3} \right) \), \( \left( \frac{W}{kg} \right) \),

\[ v(t) = \text{velocity at time } t, \left( \frac{m}{s} \right), \]

and \( a(t) = \text{acceleration at time } t, \left( \frac{m}{s^2} \right). \)

ISP is used to estimate the normalized inertial power per unit mass required by a vehicle to traverse a given route. The quantity is absent of a constant term (rolling resistance) or a term relating to velocity squared (drag) making its utility limited. The parameter is not to be confused with Vehicle Specific Power, or VSP, a quantity which does capture the additional power demands from rolling resistance, aerodynamic drag, and grade. VSP is not preferred given the large amount of vehicle-specific parameters necessary for calculation; evaluating the ISP only requires the route speed profile. Although VSP is a more illustrative parameter, for some purposes ISP is still effective at illustrating the power demand related to overcoming the inertial force, which is most prominent during acceleration events. More involved power calculations require vehicle parameters which exit the scope of route comparison. A major goal of this paper is to introduce a method of route synthesis and analysis blind of vehicle properties. Figure 1.6a and Figure 1.6b below shows constructed velocity profiles for both the Trapezoid and Hill models, as well as the resulting ISP profiles. Note that the distance, time, cruise speed, and average accelerations are constant between cases.

![Figure 1.6 Comparisons of Hill & Trapezoid velocity profiles and ISP profiles.](image)

The ISP profiles are not as initially expected. Adjusting the Trapezoid model by tapering the acceleration as speed rises (Hill model) successfully causes the ISP to peak occur at an intermediate speed, but the magnitude of this Hill peak ISP exceeds the Trapezoid peak of 37.5 m²/s³ versus 35.4 m²/s³. Although the Hill formulation has the correct shape, the main flaw of the Trapezoid model – unrealistically high power demand – remains unsolved. Further adjustment is required to expand upon tailoring the “S” shape to agree with the power behavior of real-world driving, tuning which is not performed in [1.9] and [1.10].

**1.5 Model Calibration**

Calibration of the model is performed by adjusting the Hill model input parameters against profiles that are known to represent real-world driving. An alternate way to understand the relative power demand of a route is to analyze the operating points on an acceleration versus velocity plot. Plotting the quantities
against one another effectively shows the behavior of acceleration as speed increases and allows for visual comparison between different driving profiles. The US06 certification cycle is considered to be one of the most aggressive in terms of vehicle power demand and provides an example of aggressive driving to compare against. Figure 1.7 shows the acceleration behavior of a single 60 mph cruise mode synthesized using the Hill Model as a function of velocity against the operating points of the entire multi-mode US06 drive cycle.

![Figure 1.7 Hill Model and US06 acceleration cloud comparison.](image)

Inspection of Figure 1.7 highlights the main issue with the synthesis approach – unreasonably high accelerations persist at increasing speeds despite the presence of region 3 within the Hill Model which tapers accelerations. The high accelerations at high speeds are largely to blame for the high ISP values previously discussed within the Hill model. Acceleration must taper as speed increases, and this tapering effect must begin much earlier within the speed transition. In other words, region 3 must last longer relative to regions 1 and 2. The formulation for region 3 (0) shows that the time spent tapering the acceleration is a function of the negative jerk rate and the positive acceleration. Increasing the time spent in region 3 – without increasing the time spent in region 1 – requires reducing the magnitude of the negative jerk rate relative to the positive jerk rate. Note that a prior assumption equated the positive and negative jerk rates; moving forward, this assumption will be modified.

A single parameter is desired to tune the Hill model velocity profile, simplifying the calibration process. Investigating a wide range of model inputs revealed that the most influential relationship on the mode shape is the relationship between the positive and negative jerk rate. A large positive jerk rate and a small negative jerk rate results in the desired inflation of region 3 and region 5 time. Rather than individually adjusting each jerk value, a new constant is created for tuning purposes (Equation 1-12).

\[ n = -j_{\text{max}}/j_{\text{min}} \]  

Equation 1-12

Where \( n \) = tuning constant.

The effect of \( n \) on the Hill profile can be best illustrated by evaluating the same US06 acceleration cloud with modes generated from different tuning constants (Figure 1.8). Decreasing the value of \( n \) results in a
significant impact on the velocity profile. An \( n \) value of 0.3 most accurately warps the previous “S” shape to fit with additional urban drive cycles (1.12).

![Figure 1.8](image)

**Figure 1.8** Accel-velocity of Hill Model overlaid on US06 acceleration cloud.

Note that the calibration procedure serves only to appropriately warp the template “S” mode profile. The same synthesis procedure with identical bounding speeds \( (V_i, V_c, V_f) \) can produce significantly different modes using acceleration as an input. As stated before, increasing the acceleration increases the mode deceleration and the magnitude of the jerk values. Adjusting the acceleration of a mode can be thought of as adjusting the driver aggressiveness but also reducing the time and distance required to achieve a cruise speed. The overall impact of calibrating \( n \) to 0.3 can be seen in Figure 1.9a and Figure 1.9b below, which contains velocity and acceleration profiles for the original Hill Model and the Calibrated Hill Model.

![Figure 1.9](image)

**Figure 1.9** Velocity (a) and acceleration (b) profiles for the original Hill Model and the Calibrated Hill Model.

The impact of the Hill calibration using \( n \) on the acceleration tapering can be most effectively understood by analyzing the ISP plots discussed earlier. Although the velocity profile helps show the shape of a given mode, it is difficult to grasp the implications a given acceleration may have on the inertial power a vehicle must overcome by looking at velocity alone. The plot in Figure 1.10 shows the ISP versus time graph from earlier with a trace for the Calibrated Hill Model added alongside the original Hill Model and Trapezoid
Model. Each mode shown is constrained to the same distance and time (average speed) for fair comparison. Maximum accelerations of 1.33 m/s², 1.48 m/s², and 1.50 m/s² are prescribed for the three traces. Note the symmetry of the ISP plot for each trace – this behavior results from accelerating and decelerating in an identical manner.

**Figure 1.10** ISP versus time for all three mode formulations.

Figure 1.10 confirms that the calibration process successfully alleviated the peak ISP issue present in the original Hill model. The largest ISP value is a reduction in magnitude from the Trapezoid model and occurs significantly earlier and at a lower speed during the acceleration. Further calibration is possible by comparing against real-world trajectories in addition to the drive cycles evaluated in Appendix B. However, the Calibrated Hill Model presented is considered sufficiently refined for the route analysis which follows.

### 1.6 Drive Cycle Approximation

A further understanding of how effectively the Calibrated Hill Model approximates real world driving can be found by synthesizing a route consisting of multiple modes with different cruise speeds and acceleration intensities. Comparisons between route parameters (average acceleration, RMS speed, etc.) as well as vehicle-specific parameters (energy at the wheels) help illustrate situations where the Calibrated Hill Model accurately approximates real world driving and where it does not. Synthesizing a real-world driving route first requires selecting a reference route to be simplified. The 505 drive cycle, or the first 505 seconds of the Urban Dynamometer Driving Schedule certification cycle, is chosen as an urban route to approximate given the number of complete stops and full acceleration events. Additionally, using a certification cycle facilitates energy consumption comparisons between simulation results and publicly available vehicle test results.

The 505 cycle starts with the vehicle at rest and contains five separate modes from stop to stop. These cycle modes will be referred to as hills, from Hill 1 to Hill 5. Each hill is approximated by using the Calibrated Hill Model discussed in the previous section. Each hill synthesis is constrained to match the average speeds of the respective hills contained in the 505. Peak acceleration values for the modes are selected by using the peak acceleration values present in the corresponding 505 hills. Note that for hill 4 a slightly higher acceleration was required to satisfy the average speed constraint. Table 1.1 summarizes
the defining characteristics of each 505 hill and each synthesized mode. The table rows describing the total cycle include idle times. Figure 1.11 shows the 505 cycle and the synthesized approximation, which was constructed by combining each of the modes with the appropriate cycle idle times.

**Table 1.1** 505 Hills and corresponding synthesized modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Time</th>
<th>Distance</th>
<th>Average Speed</th>
<th>RMS Velocity</th>
<th>Peak Acceleration</th>
<th>Average + Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hill 1, cycle</td>
<td>105</td>
<td>0.67</td>
<td>23.1</td>
<td>23.97</td>
<td>1.34</td>
<td>0.39</td>
</tr>
<tr>
<td>Hill 1, synthesis</td>
<td>105</td>
<td>0.67</td>
<td>23.1</td>
<td>23.97</td>
<td>1.34</td>
<td>0.90</td>
</tr>
<tr>
<td>Hill 2, cycle</td>
<td>170</td>
<td>1.96</td>
<td>41.5</td>
<td>43.93</td>
<td>1.48</td>
<td>0.48</td>
</tr>
<tr>
<td>Hill 2, synthesis</td>
<td>170</td>
<td>1.96</td>
<td>41.5</td>
<td>42.88</td>
<td>1.48</td>
<td>1.15</td>
</tr>
<tr>
<td>Hill 3, cycle</td>
<td>51</td>
<td>0.37</td>
<td>26.0</td>
<td>27.79</td>
<td>1.48</td>
<td>0.61</td>
</tr>
<tr>
<td>Hill 3, synthesis</td>
<td>51</td>
<td>0.37</td>
<td>26.0</td>
<td>28.50</td>
<td>1.48</td>
<td>1.09</td>
</tr>
<tr>
<td>Hill 4, cycle</td>
<td>27</td>
<td>0.14</td>
<td>18.8</td>
<td>20.55</td>
<td>1.48</td>
<td>1.04</td>
</tr>
<tr>
<td>Hill 4, synthesis</td>
<td>27</td>
<td>0.14</td>
<td>18.8</td>
<td>21.18</td>
<td>1.60</td>
<td>1.14</td>
</tr>
<tr>
<td>Hill 5, cycle</td>
<td>57</td>
<td>0.45</td>
<td>28.3</td>
<td>29.64</td>
<td>1.48</td>
<td>0.72</td>
</tr>
<tr>
<td>Hill 5, synthesis</td>
<td>57</td>
<td>0.45</td>
<td>28.3</td>
<td>30.63</td>
<td>1.48</td>
<td>1.13</td>
</tr>
<tr>
<td>Total Cycle</td>
<td>505</td>
<td>3.59</td>
<td>25.60</td>
<td>31.36</td>
<td>1.48</td>
<td>0.54</td>
</tr>
<tr>
<td>Total Synthesis</td>
<td>505</td>
<td>3.59</td>
<td>25.60</td>
<td>31.06</td>
<td>1.60</td>
<td>1.09</td>
</tr>
</tbody>
</table>

**Figure 1.11** Velocity and acceleration profiles for the original and calibrated Hill models.

Each pair of modes occur over the same distance and time (average speed) and this characteristic is true over the entire cycle as well. However, the synthesized modes consistently feature noticeably larger average positive accelerations. The average positive acceleration for the entire synthesized route is nearly double that of the 505 – 1.09 m/s² versus 0.54 m/s². A major reason for this discrepancy is the amount of noise in the 505 cycle. Incremental accelerations at high speeds bias the overall average down while the accelerations present in the synthesized route only occur during large speed transitions. (Events with zero acceleration are not included in the average value). Comparing the acceleration clouds of the cycle and syntheses shows that the major accelerations are similar in magnitude and taper nearly identically (Figure...
1.12). However, the small accelerations at high speeds contribute to differences in energy consumption estimations and cannot be discounted.

![Acceleration clouds of the 505 drive cycle and the synthesized approximation.](image)

**Figure 1.12** Acceleration clouds of the 505 drive cycle and the synthesized approximation.

Additional disparities are present between the velocity profiles of the two routes. The first two hills of the 505 reach noticeably higher speeds and a trace difference of nearly 30 mph occurs during the second hill. Given the discrepancies between the 505 and the synthesized route in both speed and acceleration, it remains unclear whether the modal simplifications are an accurate approach toward estimating route energy consumption.

The accuracy of the simplified route is best understood by comparing energy consumption results which are obtained by utilizing a scalable powertrain model [1.12]. The powertrain model chosen provides energy consumption as a function of distance. A 2013 Nissan Leaf is selected as the simulation vehicle due to the extensive validation data that is available [1.13]. Additionally, physical validation of the 2013 Leaf specifically is present within [1.12] as well. Energy results present within the scope of this paper are restricted to the tractive output at the wheels. As a result, the glider properties are the only vehicle-specific quantities required for the analysis and conclusions of this paper; more exhaustive energy values pertaining to the Calibrated Hill Model which include results from the vehicle powertrain can be found in [1.14]. Such analysis is intentionally avoided within the context of this paper to determine what conclusions can be made with the fewest number of inputs; a route evaluation strategy that requires complex details about vehicle operation and control strategy is inherently limited in application. Ideally, eco-routing solutions will only require basic vehicle glider properties which are easily obtained from manufacturers or emissions testing dynamometer data.

Aggregate wheel energy results are encouraging. The synthesized route estimates the energy at the wheels over the full cycle with an error of -3.4%. Table 1.2 contains the wheel energy results broken down by hill and energy components. Additionally, Figure 1.13a and Figure 1.13b. show the components of the energy at the wheels as a function of time for both routes. The components of energy tabulated and plotted can be found from Equation 1-13 to Equation 1-17. Note that the 505th second of the UDDS contains a deceleration event. An additional second is added to the fifth hill to allow the inertial energy to reach zero.
\[ E_{r0} = \int m g c_{rr0} dx \]  
Equation 1-13

\[ E_{r1} = \int m g c_{rr1} v(x) dx \]  
Equation 1-14

\[ E_d = \int \rho c_d A_f v(x)^2 / 2 \cdot dx \]  
Equation 1-15

\[ E_i = \int m_i a(x) dx \]  
Equation 1-16

\[ E_w = E_{r0} + E_{r1} + E_d + E_i \]  
Equation 1-17

Where  
\( E_{r0} \) = constant rolling resistance energy,  
\( E_{r1} \) = linearly varying rolling resistance energy,  
\( E_d \) = drag energy,  
\( E_i \) = inertial energy,  
\( E_w \) = total energy at the wheels,  
\( c_{rr0} \) = constant rolling resistance coefficient,  
\( c_{rr1} \) = linear rolling resistance coefficient,  
\( m_i \) = inertial mass,  
\( \rho \) = density of air,  
\( c_d \) = vehicle drag coefficient,  
and  
\( A_f \) = frontal area.

**Table 1.2** Energy results for original and synthesized 505 drive cycle.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Time (seconds)</th>
<th>Rolling Resistance</th>
<th>Drag (kJ)</th>
<th>Propel (kJ)</th>
<th>Brake (kJ)</th>
<th>Net, Wheels</th>
<th>Error (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Er0, constant</td>
<td>Er1, linear</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>0</td>
<td>153.7</td>
<td>14.2</td>
<td>57.6</td>
<td>443.3</td>
<td>-219.2</td>
<td>224.1</td>
</tr>
<tr>
<td>Hill 1, cycle</td>
<td>105</td>
<td>153.8</td>
<td>14.2</td>
<td>55.9</td>
<td>315.7</td>
<td>-91.8</td>
<td>223.9</td>
</tr>
<tr>
<td>Hill 1, synthesis</td>
<td>105</td>
<td>447.7</td>
<td>76.5</td>
<td>581.7</td>
<td>1462.7</td>
<td>-358.4</td>
<td>1104.3</td>
</tr>
<tr>
<td>Hill 2, cycle</td>
<td>170</td>
<td>447.8</td>
<td>72.9</td>
<td>508.0</td>
<td>1308.9</td>
<td>-280.3</td>
<td>1028.7</td>
</tr>
<tr>
<td>Hill 2, synthesis</td>
<td>170</td>
<td>84.0</td>
<td>9.4</td>
<td>47.0</td>
<td>310.4</td>
<td>-170.2</td>
<td>140.2</td>
</tr>
<tr>
<td>Hill 3, cycle</td>
<td>51</td>
<td>84.1</td>
<td>9.9</td>
<td>52.0</td>
<td>323.0</td>
<td>-177.2</td>
<td>145.9</td>
</tr>
<tr>
<td>Hill 3, synthesis</td>
<td>51</td>
<td>32.1</td>
<td>2.8</td>
<td>11.2</td>
<td>162.7</td>
<td>-117.6</td>
<td>45.1</td>
</tr>
<tr>
<td>Hill 4, cycle</td>
<td>27</td>
<td>32.2</td>
<td>3.0</td>
<td>12.8</td>
<td>194.4</td>
<td>-146.5</td>
<td>47.9</td>
</tr>
<tr>
<td>Hill 4, synthesis</td>
<td>27</td>
<td>102.3</td>
<td>11.9</td>
<td>61.1</td>
<td>333.3</td>
<td>-159.6</td>
<td>173.7</td>
</tr>
<tr>
<td>Hill 5, cycle</td>
<td>58</td>
<td>102.4</td>
<td>12.7</td>
<td>70.0</td>
<td>376.7</td>
<td>-191.6</td>
<td>185.2</td>
</tr>
<tr>
<td>Hill 5, synthesis</td>
<td>58</td>
<td>819.9</td>
<td>114.8</td>
<td>758.7</td>
<td>2712.4</td>
<td>-1025.1</td>
<td>1687.3</td>
</tr>
<tr>
<td>Total Cycle</td>
<td>506</td>
<td>820.3</td>
<td>112.7</td>
<td>698.6</td>
<td>2518.8</td>
<td>-887.3</td>
<td>1631.6</td>
</tr>
<tr>
<td>Total Synthesis</td>
<td>506</td>
<td>819.9</td>
<td>114.8</td>
<td>758.7</td>
<td>2712.4</td>
<td>-1025.1</td>
<td>1687.3</td>
</tr>
</tbody>
</table>
Wheel energy results between the cycle and synthesis differ most prevalently over Hill 2. The velocity profile of Hill 2 proves to be too erratic for the Calibrated Hill Model to approximate with a high degree of accuracy. Large fluctuations in speed are not captured by a single synthesized mode which contains a notably lower cruising speed. The root mean square (RMS) velocity – which can be used to predict drag energy – for the synthesized hill is 42.88 mph as compared to 43.98 mph for the cycle hill. Similar RMS differences are present between the routes for hills 3, 4, and 5. However, incremental differences in RMS values disproportionately impact drag estimation as speed rises given the behavior of squared differences. The RMS error between Hill 2 representations causes the drag component of the energy at the wheels for the synthesized route to be calculated 73.72 kJ below the same value for the cycle route. This drag error almost entirely represents the overall Hill 2 wheel energy error of -75.59 kJ and the majority of the overall route wheel energy error of -57.42 kJ. Hill 2 of the 505 serves as an example route portion where the Calibrated Hill Model struggles using a single simple mode. However, the model limitations caused by sporadic driving are considered a worthwhile tradeoff considering the limited number of inputs required for synthesis and the overall accuracy of the full cycle synthesis at approximating the energy at the wheels. The Hill method can have more complex speed variations, but these complexities would require more input parameters.

## 1.7 Constrained Analysis

The prior section synthesized a full route by splicing several modes together, each generated by the Calibrated Hill Model under average speed constraints. The Calibrated Hill Model can synthesize a mode with minimal inputs – initial, cruise, and final velocities in addition to the acceleration intensity. Tuning the acceleration intensity and cruise velocity simultaneously enables synthesizing modes with very different characteristics which satisfy the identical route constraints. The focus of this section seeks to analyze different route options for traversing between the same two stops, a common feature of urban routes (stoplight to stoplight for instance). Fully constraining stop to stop behavior with a common average speed (fixed distance, fixed time) across cases better captures the consequences of acceleration. The impact of high accelerations is well studied within the context of aggregate datasets [1.13] or unconstrained accelerations [1.15]. However, there is little research related to the impact of acceleration under strict route constraints. Properly constraining both distance and time captures trade-offs that other studies do not consider; for instance, vehicles that are slow to accelerate must cruise at a higher speed to traverse a given section of roadway compared to more aggressive alternatives. Additionally, constraining
the average speed is a requirement for fairly comparing route alternatives; synthesis options with larger average speeds are expected to consume more energy.

Figure 1.14 shows an example of a route situation that would be known ahead of time, with possible synthesis choices (generated using the Calibrated Hill Model). The three different synthesis options complete the route in the same amount of time (Figure 1.15) and consist of different peak accelerations. Note the differences in cruise speed for each route option; trajectories with higher accelerations exert more power during launch, but also cruise at a lower speed. Common eco-driving advice suggests limiting acceleration intensities to a minimum and delicately transitioning between speeds to avoid large power events. Such advice does not capture the penalty of a higher cruising speed present within this constrained analysis.

![Figure 1.14 Velocity profiles of stop to stop example with route synthesis options containing different levels of acceleration.](image1)

![Figure 1.15 Distance versus time for the three constrained cases with varying acceleration intensities.](image2)

Top speeds of 72.4 kph, 53.1 kph, and 49.9 kph are reached respectively across the three routes. The mode with a peak acceleration of 1.5 m/s^2 represents an edge case, where the cruise speed develops for a single timestep. The prescribed distance over the mode is equal to the distance contributions of Regions 1-3 and 5-7, preventing the necessity of Region 4. This minimum modal distance, which cannot
be further reduced without adjusting other parameters will be referred to as the critical distance. Equation 1-20 in Appendix A contains the distance constraint which will produce the critical distance when minimized.

Interestingly, the route with the smallest acceleration experienced the largest ISP event due to the higher speeds achieved (Figure 1.16a). A more comprehensive understanding of the energy requirements associated with each synthesize can be found by analyzing the energy at the wheels. Figure 1.16b contains the aggregate wheel energy results for each of the three route options. The same 2013 Nissan Leaf glider parameters are used within this analysis.

![Figure 1.16 ISP and wheel energy results for the three constrained stop to stop synthesis options as a function of time.](image)

The net acceleration in a stop to stop event is zero which causes inertial impacts at the wheel to cancel (Figure 1.16a). As a result, final wheel energy results from the constrained analysis (Figure 1.16b, end of route) describe the net road load required to complete a given synthesis. A more accurate predictor for synthesis tractive energy consumption is the mode RMS velocity rather than the synthesis peak acceleration. In the comparison shown in Figure 1.16a and Figure 1.16b, the RMS velocities for the 1.5 m/s\(^2\) trace, 2.5 m/s\(^2\) trace, and 3.5 m/s\(^2\) trace are 47.0 kph, 43.8 kph, and 43.0 kph respectively.

The results from the energy analysis confirm the conclusion suggested by the ISP values – the synthesis with the smallest acceleration required the largest energy to complete as measured at the wheels while the synthesis with the largest acceleration required the smallest. Due to the higher cruise speed needed to fulfill the average speed constraint, the 1.5 m/s\(^2\) acceleration case has a higher RMS speed, more energy lost to drag, and a higher net road load. Specifically, 110.4 kJ are required at the wheel for the 2013 Leaf to complete the 1.5 m/s\(^2\) mode as compared to 93.4 kJ for the 3.5 m/s\(^2\) mode, a difference of 18%. This analysis shows that urban routing constraints may result in larger inertial power events during lower acceleration modes. This, a simple rule such as always seeking to minimize acceleration to optimize energy consumption cannot be claimed. [1.14] discusses this constrained analysis further, analyzing results at the battery terminals including powertrain losses and comparing the results against the conclusions made using wheel energy values.

1.8 Cruise Time Analysis for Understanding Distance Horizons

An alternative method for understanding the energy requirements of a synthesized mode involves analyzing the tractive energy road load requirements for speed transitions (acceleration and deceleration)
separate from the tractive energy requirements for cruising. For simplicity, the energy consumption during cruising will be referred to as the “cruise consumption” while the energy consumption during speed transitions will be referred to as the “transition consumption”. The total distance normalized energy consumption (Wh/km) for a full mode is simply a weighted average between the transition consumption and the cruise consumption. A mode with no cruising time – for instance, the critical distance example in Section 1.7 – will have an energy consumption value equal to the transition consumption. On the other hand, a mode with a long cruising time will have an energy consumption value close to the cruising road load.

The constrained analysis in Section 1.7 contains an edge case where the length of the lowest acceleration mode is the synthesis critical distance. This section seeks to perform a similar comparison between acceleration intensities, but with distances larger than the critical distance. It is hypothesized that the distance normalized tractive energy of a full mode will approach the distance normalized tractive energy of maintaining the cruise speed as the synthesis distance increases. In other words, the tractive energy consumption across varying levels of driver aggressiveness all approach a common value as the stop to stop distance increases. An understanding of the relationship between mode distance and driver aggressiveness influence has eco-routing application; if driver aggressiveness has little impact on energy consumption for long modes, there is little purpose in finely detailing velocity profiles for long stop-to-stop distances. More practically, results from this analysis could be used to tune an appropriate “distance horizon” for on-board energy optimization strategies.

The analysis within this section utilizes slightly different constraints than those imposed in 1.7. Prior comparison between synthesized modes required fixing both the mode distance and time (average speed); synthesized modes within this section will be constrained only to the same distance. Any differences in average speed across each set of modes is expected to minimize as the synthesized distance rises. The acceleration rates used for synthesis comparisons in 1.7 are used in this analysis as well (1.5 m/s², 2.5 m/s², 3.5 m/s²). Cruise speeds of 50 kph, 70 kph, and 90 kph are selected to characterize route segments from urban driving to highway driving. Note that the relaxation of the average speed constraint permits each synthesized mode in each comparison to fully reach the cruising speed. Figure 1.17 below shows an example of three modes with varying levels of aggressiveness synthesized over a common distance. Traveling the 0.58km synthesis distance required travel times of 42.2 seconds, 35.7 seconds, and 32.9 seconds for the 1.5 m/s², 2.5 m/s², and 3.5 m/s² routes respectively.

Figure 1.17 Constrained example for cruise consumption analysis.
Synthesis comparisons for a given cruise speed start by generating modes over the critical distance dictated by the mode with the smallest peak acceleration (the 1.5 m/s² mode in Figure 1.14). The distance over which all mode cases are synthesized is then increased iteratively. Figure 1.18 shows energy consumption results for three levels of driver aggression for the 90 kph cruise case. Note that the first collection of points at 0.58 km are the results for the synthesized routes shown in Figure 1.17. Each additional set of points at larger distances are the results of a similarly constrained set of modes.

Figure 1.18 Net energy consumption results across different levels of driver aggressiveness.

As expected, increasing the synthesis distance for mode comparisons results in the net energy consumption of each mode approaching the cruise energy consumption. Although the shape of the curves is trivial, the results are still meaningful; analyzing the error in energy consumption results versus distance helps illustrate the threshold at which emphasizing acceleration behavior is no longer meaningful. The energy consumption of route portions involving sufficiently large distances between stops can thus be assumed to be the cruise consumption. An alternative representation of the cruise analysis can be seen below in Figure 1.19 and Table 1.3, which show the amount of distance necessary to converge to the cruise consumption for different cruise speeds. The modes evaluated against the cruise speed are synthesized with an acceleration of 1.5 m/s². Note that modes synthesized with higher accelerations would converge at quicker rate; Figure 1.18 shows that increasing the acceleration rate increases the impact synthesis distance has on minimizing energy consumption error.

Figure 1.19 The effect of synthesis distance on the error between cruise consumption and modal energy consumption for different cruise speeds.
Table 1.3 Distances required for discrete error percentages between modal energy consumption and cruise consumption

<table>
<thead>
<tr>
<th>Error</th>
<th>50 kph</th>
<th>70 kph</th>
<th>90 kph</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0%</td>
<td>0.55</td>
<td>1.42</td>
<td>2.79</td>
</tr>
<tr>
<td>4.0%</td>
<td>0.68</td>
<td>1.78</td>
<td>3.49</td>
</tr>
<tr>
<td>3.0%</td>
<td>0.91</td>
<td>2.37</td>
<td>4.65</td>
</tr>
<tr>
<td>2.0%</td>
<td>1.37</td>
<td>3.56</td>
<td>6.97</td>
</tr>
<tr>
<td>1.0%</td>
<td>2.73</td>
<td>7.12</td>
<td>13.94</td>
</tr>
<tr>
<td>0.5%</td>
<td>5.46</td>
<td>14.25</td>
<td>27.88</td>
</tr>
</tbody>
</table>

On-board vehicle optimization seeks to analyze the route ahead and make control strategy adjustments to minimize energy consumption. Typical implementation of control optimization requires a defined “horizon” to look ahead and take into consideration. The contents of Table 1.3 can be used to determine an appropriate distance horizon. Note that the percentages shown represent the error in energy consumption between the full contents of a mode against only the cruise consumption for a given cruise speed; comparing between two levels of aggressiveness – full mode versus full mode – would result in even smaller error percentages. As an example, see Figure 1.20, which is an altered representation of the 90 kph data present in Figure 1.18. The error calculated in this case shows the difference in modal energy consumption between the higher acceleration cases and the 1.5 m/s².

Figure 1.20 The effect of synthesis distance on the modal energy consumption error of higher acceleration modes (2.5 m/s²; 3.5 m/s²) versus the energy consumption of a lower acceleration mode (1.5 m/s²).

As the impact of driver aggressive on energy consumption minimizes, so does the opportunity for on-board optimization to produce meaningful results. For instance, in a stop-to-stop urban routing scenario with a cruise speed of 90 kph, there is little reason for a distance horizon beyond 3.5 km. While the energy consumption error between a full mode and the cruise consumption is greater than 5% (Figure 1.19), the maximum error between full modes is 2% (Figure 1.20). Additional analysis evaluating the powertrain energy consumption can be found in [1.14]. The paper finds an energy difference of only 2.8% at the battery terminals between the 505 cycle and the synthesized approximation using the method discussed within this paper.
1.9 Conclusions & Future Work

Transitions between speeds along a route are important to capture accurately for meaningful energy consumption results. Several acceleration models exist, each with different tradeoffs of complexity and accuracy. A new acceleration model is presented which synthesizes full speed transitions with minimal inputs. Although calibration of the original Hill Model is performed, there remains additional room for improvement by calibrating against large databases of real world driving. However, the Calibrated Hill Model introduced is considered to be a reliable synthesis technique to facilitate the analysis performed within this paper. Additionally, the ability to easily tune the Calibrated Hill Model with a single input for higher levels of driver aggressiveness is a strong feature. A stated limitation is the exclusion of grade information as an input to the velocity modeling process; incorporating such information would require selecting vehicle parameters which this study seeks to avoid.

Wheel energy results between a real route and a synthesized approximation are encouraging. The Calibrated Hill Model estimated the wheel energy over the entire route with an error of only -3.4%. However, additional analysis involving the vehicle powertrain is necessary to fully understand powertrain energy consumption results and how they relate to the energy at the wheels. The route synthesis of the 505 drive cycle displayed an important limitation of the Calibrated Hill Model – sporadic driving. Further tuning or generation of more complex modes of route synthesis methodology could be used to better capture more complex routes at the expense of more input requirements. Potential solutions include splicing modes that do not start and stop at zero as well as separately modeling the acceleration and deceleration behaviors of a given mode (which are currently considered symmetric).

Finally, a new methodology for understanding the impacts of acceleration is presented. Constraining both the distance traveled and the time allotted for multiple route syntheses helps better capture tradeoffs between cruise speed and acceleration intensity. Additional synthesis comparison is performed to evaluate the length of meaningful distance horizons given the cruise speed of a link portion. Wheel energy results are presented for both comparisons, but further research is required to understand energy consumption from the powertrain beyond at the wheels. [1.14] continues the analysis introduced within this paper and discusses powertrain specific results for a Nissan 2013 Leaf.
1.10 References


1.11 Appendix A - Hill Formulation & Constraints

Table 1.4 Equations for distance, velocity, acceleration, and jerk as functions of the region time.

<table>
<thead>
<tr>
<th>Region</th>
<th>Region</th>
<th>x(t)</th>
<th>v(t)</th>
<th>a(t)</th>
<th>j(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$j_{\max} * t_{r}^2/6 + v_0 * t_r$</td>
<td>$j_{\max} * t_{r}^2/2 + v_0$</td>
<td>$j_{\max} * t_r$</td>
<td>$j_{\max}$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$a_{\max} * t_r^2/2 + v_1 * t_r + x_1$</td>
<td>$a_{\max} * t_r + v_1$</td>
<td>$a_{\max}$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$j_{\min} * t_{r}^3/6 + a_{\max} * t_r^2/2 + v_2 * t_r + x_2$</td>
<td>$j_{\min} * t_{r}^2/2 + a_{\max} * t_r + v_2$</td>
<td>$j_{\min} * t_r + a_{\max}$</td>
<td>$j_{\min}$</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>$v_c * t_r + x_3$</td>
<td>$v_c + v_3$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>$j_{\min} * t_{r}^3/6 + v_c * t_r + x_4$</td>
<td>$j_{\min} * t_{r}^2/2 + v_4$</td>
<td>$j_{\min} * t_r$</td>
<td>$j_{\min}$</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>$a_{\min} * t_r^2/2 + v_5 * t_r + x_5$</td>
<td>$a_{\min} * t_r + v_5$</td>
<td>$a_{\min}$</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>$j_{\max} * t_{r}^3/6 + a_{\min} * t_r^2/2 + v_6 * t_r + x_6$</td>
<td>$j_{\max} * t_{r}^2/2 + v_6$</td>
<td>$j_{\max} * t_r + a_{\min}$</td>
<td>$j_{\max}$</td>
</tr>
</tbody>
</table>

Table 1.5 Equations for the distance, velocity, and time differences during each region

<table>
<thead>
<tr>
<th>Region</th>
<th>Region</th>
<th>$\Delta x_n$</th>
<th>$\Delta v_n$</th>
<th>$\Delta t_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$j_{\max} * \Delta t_1^2/6 + v_0 * \Delta t_1$</td>
<td>$j_{\max} * \Delta t_1^2/2$</td>
<td>$a_{\max}/j_{\max}$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$a_{\max} * \Delta t_2^2/2 + v_1 * \Delta t_2$</td>
<td>$a_{\max} * \Delta t_2$</td>
<td>$v_c - \Delta v_1 - \Delta v_3 - v_0 \over a_{\max}$</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$j_{\min} * \Delta t_3^2/6 + a_{\max} * \Delta t_3^2/2 + v_2 * \Delta t_3$</td>
<td>$j_{\min} * \Delta t_3^2/2 + a_{\max} * \Delta t_3$</td>
<td>$-a_{\max}/j_{\min}$</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>$v_c * \Delta t_4$</td>
<td>0</td>
<td>$X - \sum_{i=1}^{n} x_n \over \sum_{i=1}^{n} x_n$</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>$j_{\min} * \Delta t_5^2/6 + v_c * \Delta t_5$</td>
<td>$j_{\min} * \Delta t_5^2/2$</td>
<td>$a_{\min}/j_{\min}$</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>$a_{\min} * \Delta t_6^2/2 + v_5 * \Delta t_6$</td>
<td>$a_{\min} * \Delta t_6$</td>
<td>$v_f - v_c - \Delta v_5 - \Delta v_7 \over a_{\min}$</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>$j_{\max} * \Delta t_7^2/6 + a_{\min} * \Delta t_7^2/2 + v_6 * \Delta t_7$</td>
<td>$j_{\max} * \Delta t_7^2/2 + a_{\min} * \Delta t_7$</td>
<td>$-a_{\min}/j_{\max}$</td>
</tr>
</tbody>
</table>

Equations 18 - 20. Model constraints

\[ v_c - v_1 \geq \frac{a_{\max}^2}{2 * j_{\max}} - \frac{a_{\max}^2}{2 * j_{\min}} \]  
\[ v_c - v_f \geq \frac{a_{\min}^2}{j_{\max}} \]  
\[ X \geq \sum_{i=1}^{n} x_n + \sum_{i=5}^{n} x_n \]
1.12 Appendix B - Hill Calibration Against Common Drive Cycles

Figure 1.21 Single modes from the Hill Model and HWFET acceleration cloud comparison.

Figure 1.22 Hill Model and 505 Hill 1 acceleration cloud comparison.

Figure 1.23 Hill Model and UDDS acceleration cloud comparison.
2 Chapter Two – Path selection using predictive terminal energy for electric vehicle eco-routing

2.1 Abstract

Eco-routing is a popular research field that seeks to identify the route option which minimizes vehicle energy consumption rather than time or distance. Existing eco-routing methods typically require detailed vehicle properties to reach route selection conclusions. An alternative method is introduced which estimates the energy consumption of an electric vehicle by adjusting the tractive energy at the wheels. A new parameter, predictive terminal energy, is introduced which requires significantly fewer vehicle properties. The parameter is calibrated against a validated electric vehicle model for a variety of velocity and acceleration ranges. An error of 4.4% is calculated between the predictive terminal energy and the battery terminal energy for the 505 drive cycle. Finally, the performance of the predictive terminal energy is investigated in an eco-routing scenario where the eco-route must be determined from three route alternatives. In most cases, the error between the predictive terminal energy and the battery terminal energy is within 5%. Finally, parameter limitations and factors which influence higher error values, such as stops and grade, are discussed.

Keywords: eco-routing, powertrain model, electric vehicle, tractive energy, predictive tractive energy

2.2 Introduction

Increasing research is being conducted to reduce the environmental impact of personal transport. One promising research area is eco-routing, a strategy for determining the most energy-efficient route rather than the shortest in distance or time. Successful selection of the optimal route option requires accurately capturing the vehicle dynamics and powertrain response. This paper serves as a continuation of prior research into vehicle trajectory synthesis [2.1] by incorporating powertrain modeling results. Several eco-routing strategies exist, but these often require significant powertrain and route properties that are not readily available; a major goal of this analysis is to develop a methodology for correctly selecting the optimal path using as little vehicle and route information as possible. The objectives of this paper are to discuss existing research related to eco-routing and energy consumption estimation, introduce the powertrain model utilized, analyze the energy consumption of vehicle speed profiles, and finally conclude with a discussion of results and future work to be explored.

2.3 Literature Review

Eco-routing is a relatively new research field, but stems from the much larger topic of the Vehicle Routing Problem (VRP) which has been explored extensively. The VRP can be thought of as the traveling salesman problem [2.2], but with a fleet of vehicles. VRPs have many applications, especially involving the transportation of goods to a large amount of consumers (postal routes, for instance). A typical VRP solution emphasizes optimizing the aggregate distance traveled by the fleet. Calculation of the VRP involves the construction of networks featuring nodes and edges, where each edge is associated with a certain penalty or cost. Common penalties associated with edges include travel time and edge distance [2.3]. The solution to the VRP is the routing strategy that minimizes the aggregate penalty incurred by a
fleet while still satisfying the fleet demand. VRP are classified as non-deterministic polynomial-time hard, or NP-hard, requiring significant computational time to obtain solutions [2.4].

Existing literature for solving VRPs is vast and several variations are widely explored which include additional variables such as time windows [2.5], traffic information [2.6], compartments of goods [2.7], etc. One promising variant in particular is the Green Vehicle Routing Problem (GVRP). GVRPs utilize similar analysis methods as VRPs (nodes and edges), but seek to minimize the fleet energy consumption rather than the fleet distance or fleet time. GVRPs introduce an added difficulty as approximating the energy consumption of route edges requires significantly more information about both the route dynamics and the vehicles within the fleet. For instance, a GVRP must be capable of differentiating the energy consumption penalties associated with a low efficiency vehicle on a route involving high accelerations as compared to a high efficiency vehicle in more favorable conditions. Several studies involving GVRPs exist, but these approaches usually feature limited analysis related to both vehicle dynamics and route kinematics [2.8]. For example, [2.9] assigns edge penalties by multiplying the vehicle weight by the edge distance. This approach is an improvement over VRPs with no emphasis on energy minimization, but more analysis is needed to accurately capture the energy penalties associated with routes (energy lost due to drag, grade, etc.).

This paper will focuses on a specific type of the GVRP – eco-routing. Eco-routing reduces the scope of the GVRP from a fleet of vehicles to a single vehicle. Additionally, eco-routing generally consists of a single origin and destination (rather than a collection of customers which must be included in the route). This reduction in scope changes many of the intended applications; while GVRP solutions may be of interest to transportation fleet operators or municipalities, eco-routing seeks to provide the optimal route for a single user (an individual trying to find the optimal commute to work, for instance). However, the intended purpose of selecting the optimal route on an energy basis rather than on a distance or time basis is common between eco-routing and GVRP uses.

Early eco-routing research is encouraging. A study by [2.10] showed that the energy consumption of the eco-route is 9% lower than the consumption of the time-optimized route. The same study cited a 9% increase in the travel time. These results are typical; the eco-route often requires a time tradeoff with the quickest route alternative. An additional study by [2.11] corroborates this finding. Their results showed an average savings of 21% on the eco-route compared to the time-optimized route at the average penalty of a time inflation of 10%.

Existing eco-routing approaches can be segregated into two main categories: macroscopic and microscopic. Macroscopic models are typically non-iterative and use overall route properties to estimate energy consumption. For instance, [2.12] calculates the energy consumption of a given route by using average acceleration and velocity values as inputs into a simplified vehicle model. Macroscopic models require a very limited amount of inputs and computation time, but their simplicity comes at the expense of accuracy. [2.13] analyzes both microscopic and macroscopic approaches and concludes that macroscopic models can produce inaccurate conclusions by failing to take instantaneous effects into consideration.

Microscopic models differ by analyzing routes with a greater scrutiny. Instead of using aggregate parameters that describe route sections (or entire routes), microscopic models capture time and distance varying parameters such as vehicle speed, acceleration, grade, etc. These more detailed route characteristics are then fed into more comprehensive vehicle models which capture transient effects such
as the impact a red light may have on an urban route. Tradeoffs are associated with microscopic models as well; significantly more information is required for both the route characteristics and the vehicle properties. For instance, rather than simply using speed limit data, microscopic models must synthesize speed transitions and incorporate grade data. Additional vehicle properties which describe the powertrain efficiency – such as engine maps – are also required in most microscopic analyses. An alternative method to avoid extensive powertrain analysis is to employ regression relationships relating instantaneous route conditions to energy consumption. [2.14] presents an expansion of earlier research, outlining a method for estimating fuel consumption of 60 different vehicles based on only route information. Finally, additional research minimizes the energy consumption by optimizing the velocity profile of a given route rather than differentiating between the energy consumption of potential route options [2.15]. The velocity synthesis procedure introduced in Section 2.5 is considered to reasonably approximate realistic driving behavior; thus, offline eco-driving is not evaluated within this paper.

Figure 2.1 Microscopic eco-routing process for an origin destination pair with three potential route alternatives.

Figure 2.1 describes the process necessary to evaluate the energy consumption of a given route using microscopic analysis. An origin-destination pair (1) is input into a routing model which produces a number of route alternatives (2). Three route alternatives are shown above, but the number of potential route options could vary depending on location. A velocity profile is synthesized for each route option with driver aggressiveness as an optional tuning parameter (3). Each route synthesis, along with grade information if available, is then fed into a powertrain model which estimates the energy consumption over each route option (4). The route which requires the least amount of energy is then output as a suggestion to the user (5). The red outline included in Figure 2.1 describes the scope of the work in this paper. Route options and grade data is assumed to be known data, and its acquisition will not be discussed. However, the utilization of this information will be described. One factor that is not included is the effect of traffic on route synthesis. Several models exist which analyze historical traffic data and use this information as an input to the eco-routing process [2.6], [2.16], [2.17]. Traffic effects are not yet considered in this work due to the difficulty in obtaining historical traffic data reliably, as this information is often proprietary and comes with a cost. However, knowledge of traffic information does increase eco-routing accuracy and would be treated as input into the route synthesis process.
Note that Figure 2.1 does not incorporate grade data into the velocity synthesis model present within column A. It is assumed that the acceleration rates modeled will be met regardless of grade or powertrain limitations, an assumption which may be violated in certain scenarios such as an insufficient powertrain attempting to accelerate while on a steep grade. A similar eco-routing study by Rakha et al. [2.18] presents similar research which includes grade and vehicle limitations in their velocity synthesis method. In this study, powertrain constraints and the impact of grade are not explored until column B.

The paper will proceed by first describing the powertrain model utilized for producing energy consumption results in Section 2.4. Section 2.5 follows by outlining the synthesis procedure for constructing velocity profiles from route data. Validation of synthesized routes compared to actual routes will be presented in Section 2.6. Section 2.7 explores the relationship between the tractive energy and the energy at the terminals of the battery electric test vehicle, discussing the necessity of powertrain simulations. An eco-routing example is evaluated in Section 2.8 and the paper will end with Section 2.9 which summarizes the results and describing future work.

### 2.4 Powertrain Model

Prior research efforts outlined in [2.1] primarily focus on the tractive energy consumption of routes at the wheels. A major goal of this paper is to expand the route selection process by also incorporating vehicle powertrain response. Although tractive energy consumption is an important metric, the main goal of eco-routing is to minimize the powertrain energy consumption. A powertrain model is required to expand the tractive energy calculations and calculate more meaningful results.

Vehicle models fit within two main categories – backward facing and forward facing. A backward facing model accepts the route as an input, assumes the vehicle will perfectly match the route speed, and calculates the energy required starting from the wheels and moving backwards through the powertrain. The assumption that no route trace misses will occur is limiting in application; non-iterative backwards facing models are generally not capable of enforcing powertrain constraints such as maximum power or speed. A forward facing model permits error between the vehicle trajectory and desired trajectory rather than assuming a vehicle will match the input route. However, the presence of error between the driven and desired trajectory is also a modeling limitation – especially for eco-routing purposes where the fuel consumption of trajectories is desired. A backward facing model is considered appropriate for this work due to the realistic trajectories simulated (no powertrain constraints broken) and the importance of accurately capturing route energy consumption (no trajectory error). The scalable model introduced in [2.19] is employed for this purpose. Note that the selected powertrain model is capable of estimating powertrain consumption for battery electric vehicles (BEV), plug-in hybrid electric vehicles (PHEV), and internal combustion engine vehicles (ICEV). The BEV model is emphasized throughout this paper, given the simplicity of the powertrain and the significant validation data available for the vehicle chosen – a 2013 Nissan Leaf [2.20]. Additionally, prior eco-routing research suggests that BEVs stand to benefit more from eco-routing than traditional vehicles; (7) found that eco-routing reduced energy consumption by 12.5% for BEVs as compared to 8.4% for ICEVs while (5) found improvements of 2.8% for ICEVs and 8.5% for BEVs. A simplified overview of the BEV powertrain model described by [2.19] follows with modeling equations described afterward.
Figure 2.2 Calculation sequence of backward facing BEV powertrain model from the wheels to the battery.

Figure 2.2 outlines the backward facing BEV model, which calculates the powertrain response necessary to satisfy the energy at the wheels. Green arrows represent the flow of energy while the numbers represent the order in which variables are calculated. Tractive power at the wheels (1) is a function of the current route characteristics and vehicle properties. Knowledge of the tractive power required at the wheels is needed for calculating the final drive loss (2). Output motor power can be calculated once the downstream power requirements and losses are known (3). Additionally, motor losses (4) can be calculated with an understanding of the motor output conditions. Information regarding motor output power and motor losses are used to inform the battery component model of the necessary output power (5) to meet the tractive power at the wheels. An accessory load (6) is enforced to capture non-propulsive power demand. Finally, a battery loss model is employed to calculate the internal battery losses (7) that result from satisfying the overall vehicle power demand.

Figure 2.3 Free body diagram of a vehicle glider.
\[ F_{tr} = F_i + F_{rr} + F_d + F_g \]  \hspace{1cm} \text{Equation 2-1}

Where \( F_{tr} \) = supplied tractive force,
\( F_i \) = inertial force,
\( F_{rr} \) = rolling resistance force,
\( F_d \) = drag force,
\( F_g \) = grade force.

The tractive force at the wheel is calculated from the vehicle glider model (Figure 2.3) and serves as an input for the powertrain model (Equation 2-1). Driveline speed and torque is calculated using vehicle speed, tractive force, and wheel radius (Equation 2-2 and Equation 2-3). Equation 2-4 describes the final drive output, which is simply the product of the axle speed and torque.

\[ \omega_a = \frac{V}{r} \]  \hspace{1cm} \text{Equation 2-2}

\[ T_a = F_{tr} * r \]  \hspace{1cm} \text{Equation 2-3}

\[ P_{FD,\text{out}} = \omega_a * T_a = F_{tr} * V \]  \hspace{1cm} \text{Equation 2-4}

Where \( \omega_a \) = wheel axle speed,
\( T_a \) = wheel axle torque,
\( V \) = vehicle velocity,
\( F_{tr} \) = tractive force,
\( r \) = wheel radius,
and \( P_{FD,\text{out}} \) = final drive output power, tractive power.

The BEV model assumes a fixed gear ratio between the driveline and wheels. The fixed gear amplifies the axle speed (Equation 2-5) and lowers the axle torque (Equation 2-6) back to the motor. Figure 2.4 shows the final drive quantities with relation to the larger powertrain model.

\[ \omega_s = FD * \omega_a \]  \hspace{1cm} \text{Equation 2-5}

\[ T_s = \frac{T_a}{FD} \]  \hspace{1cm} \text{Equation 2-6}

Where \( \omega_s \) = motor shaft speed,
\( FD \) = motor final drive ratio,
and \( T_s \) = motor shaft torque.
The torque which must be overcome to spin the axle is assumed to be a fixed fraction of the maximum motor torque (Equation 2-7). This assumption is applied for scaling purposes – a larger motor is almost always associated with a larger final drive. Loss due to the final drive can be easily calculated using the shaft speed and driveline loss (Equation 2-8). Finally, the input power supplied by the motor can be solved by either performing a power balance on the final drive or multiplying the shaft speed and torque (Equation 2-9).

\[
T_{FD,\text{loss}} = 0.012 \times T_{\text{max}} \quad \text{Equation 2-7}
\]

\[
P_{FD,\text{loss}} = T_{FD,\text{loss}} \times \omega_s \quad \text{Equation 2-8}
\]

\[
P_{M,\text{out}} = P_{FD,\text{in}} = P_{FD,\text{out}} + P_{FD,\text{loss}} = T_s \times \omega_s \quad \text{Equation 2-9}
\]

Where \( T_{FD,\text{loss}} \) = final drive torque loss on motor input side,
\( T_{\text{max}} \) = maximum electric motor torque,
\( P_{FD,\text{loss}} \) = final drive power loss,
\( P_{M,\text{out}} \) = electric motor output power,
\( P_{FD,\text{in}} \) = final drive input power.

A similar component model is utilized for the electric motor system; downstream factors determine the output power, losses are calculated using physical parameters, and the input power is the sum of the output power and losses (Equation 2-9 and Equation 2-10). The main difference between the electric motor model and the final drive model is the calculation of losses (Equation 2-11). Figure 2.5 displays the motor parameters within the context of the overall powertrain. Tuned constants represent different losses as a function of the motor output torque and output speed. Specifically, \( k_c \) scales the copper loss, \( k_l \) scales the hysteresis loss, \( k_w \) scales the windage loss, and \( C \) represents a constant loss associated with inverter/motor operation.
Figure 2.5 Motor model with model parameters labelled.

\[ P_{M,\text{in}} = P_{M,\text{out}} + P_{M,\text{loss}} \]  
\[ P_{M,\text{loss}} = k_c T_s^2 + k_i \omega_s + k_w \omega_s^3 + C \]

Where \( P_{M,\text{in}} \) = electric motor input power, 
\( P_{M,\text{loss}} \) = electric motor power loss, 
\( k_c \) = copper loss coefficient, 
\( k_i \) = hysteresis loss coefficient, 
\( k_w \) = windage loss coefficient, 
\( C \) = constant loss coefficient.

Note that the motor loss coefficients are not easily found and represent an approximation of the motor losses for different operating points. [2.21] proposes a scaling methodology for adjusting loss values from a motor with known loss coefficients. The loss expression described by Equation 2-12 is employed to model the motor losses from the selected vehicle. Further discussion regarding the application of the scaled loss equation can be found in [2.19] as well.

\[ P_{M,\text{loss,\text{scaled}}} = \left[ \frac{T_{\text{max}} \omega_{\text{base}}}{T_{\text{ref}} \omega_{\text{ref}}} \right] \left[ k_c T_s \left( \frac{T_{\text{ref}}}{T_{\text{max}}} \right)^2 + k_i \omega_s \left( \frac{\omega_{\text{ref}}}{\omega_{\text{base}}} \right) + k_w \omega_s^3 \left( \frac{\omega_{\text{ref}}}{\omega_{\text{base}}} \right)^3 + C \right] \]

Where \( P_{M,\text{loss,\text{scaled}}} \) = scaled motor loss, 
\( \omega_{\text{base}} \) = base speed of the motor being approximated, 
\( T_{\text{ref}} \) = max torque of the reference motor, 
\( \omega_{\text{ref}} \) = base speed of the reference motor.

Calculation of the battery power is now possible with knowledge of the input power demand from the motor. In addition to powering the electric motor, the battery must also supply power to satisfy a specified accessory load (Equation 2-13). The accessory load demand represents the power required for non-propulsive energy use – lights, passenger comfort, radio, etc. Accessory demand is assumed constant irrespective of the route dynamics, making the parameter mostly inconsequential to route selection except for assigning an energy penalty to long idling events.
\[ P_{B,\text{out}} = P_{M,in} + P_{\text{Acc}} \]  \hspace{1cm} \text{Equation 2-13}

Where \( P_{B,\text{out}} \) = electrical power out of the battery, and \( P_{\text{Acc}} \) = accessory power load.

\[ R_{\text{int}} = R_0 \frac{V_{\text{OC}}^2}{E_{\text{cap}}} \]  \hspace{1cm} \text{Equation 2-14}

\[ V_T = V_{\text{OC}} - I_B R_{\text{int}} \]  \hspace{1cm} \text{Equation 2-15}

\[ P_{B,\text{loss}} = I_B R_{\text{int}}^2 \]  \hspace{1cm} \text{Equation 2-16}

\[ I_B = \frac{V_{\text{OC}} - \sqrt{V_{\text{OC}}^2 - 4 R_{\text{int}} P_{B,\text{out}}}}{2 \times R_{\text{int}}} \]  \hspace{1cm} \text{Equation 2-17}

\[ P_{B,\text{int}} = P_{B,\text{loss}} + P_{B,\text{out}} \]  \hspace{1cm} \text{Equation 2-18}

\[ E_{B,\text{int}} = \int P_{B,\text{int}} \, dt \]  \hspace{1cm} \text{Equation 2-19}
Where $R_{\text{int}}$ = battery internal resistance,
$R_0$ = a scaling constant,
$V_{OC}$ = battery open circuit voltage,
$E_{\text{cap}}$ = battery energy capacity,
$V_T$ = battery terminal voltage,
$I_B$ = battery current,
$P_{B,\text{loss}}$ = battery power loss,
$P_{B,\text{int}}$ = internal battery power,
and $E_{B,\text{int}}$ = internal battery energy.

The powertrain overview within this section assumed a positive tractive force at the wheels, resulting in energy flow from the battery to the wheels. BEV models are capable of energy flowing in the opposite direction through regenerative braking; kinetic brake energy is siphoned from the wheels, through the final drive, converted to electricity using the electric motor as a generator, and power is delivered to the battery terminals for recharging. All of the loss models and power flow balances also work for negative power flow, including cases where the power changes sign through the powertrain. Refer to [2.19] for regenerative braking logic and assumptions for more detail. [2.19] also discusses maximum torque, power, and minimum speeds constraints associated with regenerative braking.

### 2.5 Velocity Modes and Route Synthesis

The powertrain model described in the previous section requires a velocity profile as an input. Thus, generation of velocity profiles is required for each route option. A goal of velocity synthesis is to produce profiles that closely match real world driving and the dynamics of velocity transitions. [2.16] presents a route synthesis technique that accurately models speed transitions for accelerations, but the model is not easily tunable to adjust acceleration intensity. [2.1] presents the synthesis technique utilized within this paper, which was primarily chosen due to the low number of input parameters required and the ease of tuning for driver aggressiveness. A short summary of the synthesis technique from [2.1] follows.

![Velocity, acceleration, and jerk profiles](image)

**Figure 2.7** Velocity (a), acceleration, and jerk (b) profiles of a synthesized mode.

Figure 2.7a shows a typical output of the synthesis method described in [2.1]. The velocity profile shown is what will be referred to as a mode, or a combination of an acceleration event, a cruise event, and a deceleration event. Modes are of particular interest because they fully define a stop-to-stop event within...
a route – a common feature of urban routes. Constructing the velocity profile shown requires specifying only the mode distance, maximum acceleration, and velocity thresholds (initial – usually zero, cruise, and final – usually zero). Tuning the maximum acceleration allows for easily adjusting the mode dynamics for varying levels of driver aggressiveness; increasing the acceleration value also increase the magnitude of the deceleration. Figure 2.7b shows the acceleration profile which produces the shape of the velocity mode. Jerk rates, which represent the change in acceleration with respect to time, are used to control the acceleration. Acceleration starts at zero, increases to a maximum, and tapers as the speed rises. This behavior closely resembles that of real-world acceleration profiles with respect to rising speed [2.22]. Deceleration is modeled in a symmetric manner to minimize synthesis input parameters, and is also representative of typical driver behavior.

As stated in the previous section, velocity profiles are used as inputs to the powertrain model to estimate energy consumption. The powertrain model then calculates the energy consumption for each distance step or time step. Plotting this data effectively illustrates the impact of route properties on powertrain energy consumption. Figure 2.8 shows a sample synthesized mode with the response of the 2013 Nissan Leaf powertrain. Note that time or distance varying consumption information is exclusive to microscopic modeling. Macroscopic approaches estimate a single value to describe entire route sections, which prevents the generation of similar figures with respect to time.

![Figure 2.8 Synthesized 1.5 m/s², 50 kph mode with the energy response of the 2013 Nissan Leaf powertrain.](image)

Both the tractive energy at the wheels and the electric energy coming out of the battery terminals are shown; the difference between these values represents the powertrain losses which must be overcome to propel the vehicle over the velocity profile. The three regions of the plot effectively show the impact of acceleration on energy consumption. Energy is rapidly depleted from the battery during the initial acceleration, after which the vehicle begins to cruise. Cruising results in a linear rise in energy consumption versus time with a slope equal to the value of the power required to overcome the vehicle road load. Finally, energy is returned to the batteries during the deceleration event due to the effects of regenerative braking. A plot of the energy consumption of an ICEV would look similar, except for a larger
powertrain loss and a small amount of energy consumption during the deceleration event instead of regenerative braking.

Real world routes are often comprised of more than just stop-to-stop profiles; speed transitions, such as a change in the roadway speed limit, are also common in routing scenarios. The selected synthesis technique is capable of constructing transitions between nonzero speeds as well. Velocity transition profiles can be easily constructed with a similarly limited number of inputs. Figure 2.9 shows an example of acceleration and deceleration transitions; the initial velocity, final velocity, and acceleration intensity are the only quantities required to define the speed transition shown.

![Velocity and Acceleration Example](image)

**Figure 2.9** Example of a synthesized velocity transition between nonzero speeds.

The ability to synthesize stop-to-stop modes and speed transitions allows for constructing more extensive, complicated routes. Section 2.6 follows with synthesis and model validation.

### 2.6 Synthesis and Powertrain Validation

Selecting the route which requires the least amount of energy requires both accurately modeling energy consumption and the velocity profile being simulated. Inaccuracy in either the powertrain model or the synthesis procedure introduces error which can result in a sub-optimal route option being selected as the eco-route. This section seeks to validate both the powertrain model and the approximation of real world driving behavior into synthesis profiles to provide confidence in the simulation results of the route approximations.

The powertrain model is first validated against accurate test data. Reference Nissan Leaf 2013 data is obtained through Argonne National Laboratory’s Downloadable Dynamometer Database (D3) [2.20]. D3 data contains relevant parameters such as state of charge and axle torque in response to chassis dynamometer drive cycle tests. Powertrain validation is performed by simulating the same velocity traces in the model and comparing results. Figure 2.10 shows the state of charge of the 2013 Leaf tested by ANL as well as the state of charge of the powertrain model, both in response to the speed and acceleration shown. The test route – number 61402070 as defined within the database - is particularly useful; velocity values start from zero and accelerate in 10 mph increments to 80 mph. A wide range of steady state velocity values and transient accelerations is ideal because areas of poor model correlation can be easily identified.
After calibrating the powertrain loss model, the SOC error between the model and measured data ranges from -0.11% to 0.19%, both well within the typical SOC reporting increment of 0.5% found in most electric vehicles. The magnitude of the calculated error is encouraging, especially considering the approximately 9 minute duration of the 6.2 mile route. Additional validation can be easily obtained by comparing the energy consumption in Wh/mi at different cruising speeds. Additional data offered by ANL describes the energy consumption of a 2013 Leaf at different cruising speeds (10 mph to 80 mph) and different grades (0% and 6%) [2.20]. Analyzing the cruise consumption values in Figure 2.11 more effectively shows where the powertrain model is accurate and where it is not. Understanding the impact of grade information on model accuracy is also important, as hilly terrain often represents energy consumption penalties.
While Figure 2.10 is effective for showing the consistency of the model SOC and the validation SOC over an entire route, Figure 2.11 clearly illustrates under which conditions the model data is not accurate. Specifically, the powertrain model is the most inaccurate at very low speeds and low grades - 53.7% energy consumption error at 10 mph and 0% grade. This large error quickly subsides to 18% error at 20 mph and 9% error at 30 mph (both cases at 0% grade). Fortunately, the low steady speeds at which the model accuracy suffers do not comprise significant portions of most routes. The model is thus considered a reliable tool for estimating and differentiating the energy consumption between routes as shown by the validation data. Additional validation information can be found in [2.19]. Finally, note that internal battery energy is not validated directly. Internal battery losses are assumed to be minimal in comparison to the energy consumption routes require. Results minimizing terminal energy, such as the determination of the eco-route, are assumed to minimize internal battery energy as well.

The velocity synthesis technique requires validation as well. A calibrated powertrain model has little utility if the input velocity profile does not accurately represent real world driving. For instance, a poorly tuned velocity synthesis with very low accelerations will under-predict energy consumption. Large discrepancies between real world driving and the selected velocity synthesis model could result in the incorrect path being selected. Synthesis validation is performed by comparing the energy consumption of a certification cycle against a synthesized version of the same route. [2.1] validates the velocity synthesis method at the wheels but does not compare powertrain results between a realistic velocity profile and a synthesized profile at the battery terminals.

Energy consumption is calculated for both cases using the powertrain model introduced in Section 2.4. The first 505 seconds of the Urban Dynamometer Driving Schedule (UDDS), or the “505” is selected for comparison route given its urban characteristics and wide availability. An approximate route is synthesized by constructing each of the 5 stop-to-stop portions of the 505 separately. Each synthesized “hill” is constrained in both distance and time (fixed average speed) with the corresponding hills in the 505 to ensure a fair comparison. Figure 2.12 shows the 505 cycle with driver behavior, the synthesized approximation, and state of charge results from the powertrain model.

![Figure 2.12](image)

**Figure 2.12** Velocity and SOC profiles for the 505 cycle and the synthesized alternative.
The error between the model SOC traces ranges from -0.39% to 0.32%. Similar to the D3 comparison earlier in this section, the error calculated is smaller than the typical SOC reporting increment of 0.5%. Additionally, the SOC error between the model and the ANL dynamometer data is nearly indistinguishable, further validating the powertrain model. SOC results from the powertrain model for the cycle and synthesis differ most prevalently over hill 2, which contains both the maximum and minimum SOC errors. The velocity profile of hill 2 proves to be difficult to match with a single synthesized mode, largely due to the interrupted acceleration and lack of a consistent cruising speed. Note that hills 3, 4, and 5 better fit the shape of a mode and produce little SOC error. Fortunately, the velocity behavior of the later hills better match uninhibited real world driving. The erratic nature of hills 1 and 2 are likely results of traffic and roadway interruptions which are not currently considered to be an input of this analysis. Further analysis regarding the validation and calibration of the utilized velocity model can be found in [2.1].

In summary, shortcomings of both the powertrain model and the velocity synthesis procedure are found. However, the conditions which cause poor powertrain agreement (10 mph cruise speeds) and poor synthesis accuracy (erratic driving) are not critical. Typical cruise speeds far exceed 10 mph and the effects of traffic are outside of the scope of this paper. The powertrain model and velocity synthesis are considered to be sufficiently validated for differentiating the energy consumption between routes.

### 2.7 Ttractive Energy and Terminal Energy Evaluation

Estimating the energy consumption of multiple route options can now be performed with a validated synthesis model for generating reasonable velocity profiles and a validated powertrain model which translates the velocity profiles into energy results. As per Figure 2.9, a full velocity synthesis is generated for the speed limits and speed transitions of each route and the energy consumption results are calculated. Total travel time (TTT) is calculated for each route in response to synthesis inputs (acceleration intensity) and route constraints (speed limits, stops). The eco-route is the path which requires the least amount of energy, rather than the shortest amount of distance or time. One downside of this methodology is the significant amount of vehicle-specific parameters required for energy consumption results at the battery. Table 2.1 below shows the variables required for calculating both the ttractive energy and the terminal energy (assuming the use of the scaled motor model explained in Section 2.4). Note that 7 extra variables are needed, some of which are difficult to obtain easily from the manufacturer. The large number of vehicle-specific parameters required for the determination of terminal energy represents a significant eco-routing limitation for real world implementation. While many of the parameters within the terminal energy column are not commonly cited, vehicle mass, drag coefficient, and frontal area are frequently public information. An alternate method for estimating the energy penalty of a given route which requires fewer model inputs is desired.

**Table 2.1 Vehicle-specific parameters required for calculation**

<table>
<thead>
<tr>
<th>Ttractive Energy</th>
<th>Terminal Energy</th>
<th>Required Vehicle Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>$m$</td>
<td>Test mass</td>
</tr>
<tr>
<td>$Cd$</td>
<td>$Cd$</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>$A_f$</td>
<td>$A_f$</td>
<td>frontal area</td>
</tr>
<tr>
<td>$Crr$</td>
<td>$Crr$</td>
<td>rolling resistance coefficient</td>
</tr>
<tr>
<td>-</td>
<td>$r$</td>
<td>wheel radius</td>
</tr>
<tr>
<td>-</td>
<td>$FD$</td>
<td>final drive ratio</td>
</tr>
</tbody>
</table>
Table 2.1 Vehicle-specific parameters required for calculation

| - | $T_{max}$ | maximum motor torque |
| - | $\omega_{max}$ | maximum motor speed |
| - | $C$ | constant motor loss |
| - | $V_{oc}$ | battery open circuit voltage |
| - | $E_{cap}$ | battery energy capacity |

This section seeks to revisit and explore the comparison between the tractive and terminal energy values discussed in the prior section. As shown in Figure 2.8, there are strong similarities between the energy at the wheels and the energy at the terminals. Increases in route power demand cause both the tractive energy and the battery energy to rise. Additionally, braking events which cause regenerative effects at the terminals reduce the aggregate energy at the wheels and terminals. Analyzing the tractive energy appears to be a strong predictor for the terminal energy; an eco-driving study by Mensing et al. [2.23] showed that the most energy efficient cycle also featured the least amount of tractive energy. However, there is some difference between the tractive energy and the terminal energy due to the powertrain losses described in Section 2.4. Figure 2.13 shows the energy at the wheels versus the energy at the terminals for the route traversed in Figure 2.8. Traces are shown indicating the mode portions which correspond with the plotted operating points.

![Figure 2.13 Tractive energy versus terminal energy in response to the velocity profile plotted in Figure 2.8.](image)

Although the relationship between the tractive energy and terminal energy changes throughout the traversed mode, there is a strong linear relationship within each region. The slope of the trace at a given point in Figure 2.13 represents the relative energy at the wheels compared to the energy at the terminals, or the powertrain efficiency. Note that the powertrain efficiency is reversed for regenerative braking, where the tractive energy is the powertrain input and the terminal energy is the powertrain output. In this case, the reciprocal of the plotted slope represents the powertrain efficiency. Vehicle acceleration and cruise cause the terminal and tractive energy to rise with two distinct slopes (or powertrain efficiencies), while the vehicle deceleration causes both energy values to reduce with a third slope value. The endpoint of the entire trace represents the energy consumption required for the entire mode. A line between the initial and final energy values (the dashed line in Figure 2.13) represents the net energy consumption with a slope of the net powertrain efficiency. Knowledge of the different - but relatively constant - powertrain efficiencies for each portion of a mode has eco-routing implications; establishing a
simplified relationship between the wheels and the terminals allows for consumption prediction using only the vehicle parameters from the tractive energy column in Table 2.1. Although [2.23] showed that the eco-route also featured the smallest tractive energy of all route alternatives, this is not always the case. Further analysis using the distinct regions in Figure 2.13 is explored below to increase the accuracy of selecting the eco-route using tractive energy to predict terminal energy.

The shape of the curve in Figure 2.13 is common across all modes. However, the powertrain efficiency varies depending on the mode parameters. Acceleration intensity and cruise speed both influence the acceleration and deceleration efficiencies while the cruise efficiency is solely a function of the cruise speed. Selecting powertrain efficiencies for acceleration, cruise, and deceleration which are representative of a wide array of driving profiles is thus necessary for accurately predicting the terminal energy for a given route portion. Powertrain efficiencies for a matrix of modes generated using three acceleration levels (1.5 m/s², 2.5 m/s², and 3.5 m/s²), eight speeds (40 kph to 110 kph), and cruise times of five seconds are shown below. Greater detail can be found in Appendix A, which contains the powertrain efficiencies reported by mode component (acceleration, cruise, and deceleration).

![Figure 2.14](image)

**Figure 2.14** Powertrain efficiencies for a variety of synthesized modes.

Scatter points in Figure 2.14 appear linear, but further inspection reveals that the powertrain efficiency varies widely from 40% to 70%. Thus, relying solely on a single correction factor for predicting the terminal energy from the tractive energy is unreliable (as is shown by the large disparity between tractive and terminal energy in Figure 2.8). The efficiency plots broken down by mode components in Appendix A are much more encouraging: acceleration efficiency ranges from 84.0% to 90.1%, and deceleration efficiency ranges from 71.4% to 84.3%. However, the powertrain efficiency during cruising varies a large amount, from 31.3% at 10 kph to 85.7% at 120 kph (Figure 2.22, Appendix A). Constant factors of 86% and 74% are selected to best represent the powertrain efficiency during acceleration and deceleration while a cruise efficiency as a linear function of speed is employed to approximate the powertrain efficiency during cruise. These factors are then used to calculate a new parameter, the predictive terminal energy. Equation 2-20 and Equation 2-21 represent expression for the standard and predictive terminal energy. Equation 2-22 describes the conditional powertrain coefficient which is triggered by the vehicle state. The conditional powertrain coefficient represents the estimated terminal power required to produce the current tractive power. Estimating the power out of the terminals for propel events (positive tractive power) requires dividing by the powertrain efficiency while estimating the regen energy into the terminals requires
multiplying by the powertrain efficiency. In summary, the powertrain model terminal energy requires an exhaustive amount of vehicle properties and is distinctly separate from the predictive terminal energy, which requires far fewer vehicle parameters and is calculated as a function of the tractive energy. The synthesis and powertrain response in Figure 2.8 is shown again in Figure 2.15, with an additional trace plotted representing the newly defined predictive terminal energy.

\[
E_w = \int P_{tr}(t) \, dt \\
E_{wp} = \int P_{tr}(t) \cdot \eta_p(t) \, dt \\
\beta(t) = \begin{cases} 
1/0.86 & a(t) > 0 \quad \text{(accel)} \\
1/(0.0045 \cdot v(t) + 0.3819) & a(t) = 0 \quad \text{(cruise)} \\
0.81 & a(t) < 0 \quad \text{(decel)}
\end{cases}
\]

Where \( E_w = P_{FD,out} \) = tractive energy, \( E_{wp} \) = predictive terminal energy, \( \beta(t) \) = conditional powertrain coefficient, \( a(t) \) = vehicle acceleration, and \( v(t) \) = vehicle speed.

**Figure 2.15** Powertrain response to synthesized 50 kph mode with adjusted tractive energy metric.

As shown above, the application of the conditional powertrain coefficient effectively relates the tractive energy to the terminal energy as compared to the unadjusted tractive energy. Producing the predictive terminal energy requires only the route dynamics and minimal vehicle properties. Modes with low or high cruising speeds are more prone to error. Figure 2.16 shows an edge case for a 110 kph mode with a predictive error of 13.8\% for estimating terminal energy consumption. The high speed of the mode results in an overprediction for each of the three mode portions (acceleration, cruise, and deceleration). Despite the weakness in accuracy caused by the high speed, the predictive terminal energy still outperforms the tractive energy in estimating the terminal energy for the edge case shown. The imperfection in terminal energy prediction is considered a worthwhile tradeoff to avoid the requirement of vehicle parameters which are often not publicly available as well as test data required for powertrain validation.
Figure 2.16 Energy results for a 110 kph mode with a peak prescribed acceleration of 2.5 m/s^2.

Performance of the predictive terminal energy metric has been evaluated on a mode basis to this point. A more robust validation is achieved by analyzing a synthesis containing a variety of cruise speeds and cruise times. The 505 approximation is again selected as a test route given the existing synthesis approximation and the availability of results to compare against. Selection of the 505 permits comparisons between the predictive terminal energy to the energy at the terminals of both the powertrain model and the physical vehicle tested by ANL. The latter comparison, between predictive tractive energy and physical data, captures the full scope of simplifications and assumptions applied throughout this eco-routing methodology.

Figure 2.17 505 cycle and synthesized approximation with accompanying tractive and terminal energy traces.
Table 2.2 Powertrain and predictive energy results for the 505 cycle and synthesis

<table>
<thead>
<tr>
<th>Trace</th>
<th>Cumulative Route Energy (kJ)</th>
<th>Route Energy Error (kJ)</th>
<th>Route Energy Error (%)</th>
<th>Synthesis Energy Difference (kJ)</th>
<th>Synthesis Energy Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>505 Cycle Powertrain Model Terminal Energy</td>
<td>2562</td>
<td>0</td>
<td>0.0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>505 Synthesis Powertrain Model Terminal Energy</td>
<td>2489</td>
<td>-73</td>
<td>-2.8</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>505 Synthesis Predictive Terminal Energy</td>
<td>2595</td>
<td>N/A</td>
<td>N/A</td>
<td>106</td>
<td>4.4</td>
</tr>
<tr>
<td>505 Synthesis Wheel Energy</td>
<td>1631</td>
<td>N/A</td>
<td>N/A</td>
<td>-858</td>
<td>-34.5</td>
</tr>
</tbody>
</table>

Figure 2.17 shows tractive and terminal energy consumption results alongside the 505 cycle and the 505 synthesized approximation. Unaltered tractive energy for the 505 synthesis is represented by the black line, predictive terminal energy for the 505 synthesis by the red line, powertrain model terminal energy for the 505 synthesis by the purple line, and powertrain model terminal energy for the 505 cycle by the orange dashed line. Table 2.2 summarizes the results in Figure 2.17 by comparing the powertrain model terminal energy results for the 505 cycle and the 505 synthesis approximation, which describes the accuracy of the simplified velocity profile versus the original profile. The table also compares the synthesis predictive terminal energy and synthesis wheel energy to the synthesis powertrain model energy, illustrating the improvement in accuracy by applying the conditional powertrain coefficient to the tractive energy (Equation 2-21 and Equation 2-22).

The -73 kJ, -2.8% error between the terminal energy for the cycle (2562 kJ) versus the terminal energy for the synthesis (2489 kJ) represents the penalty in accuracy due to approximating the route as a series of simplified modes. The additional 4.4% error between the terminal energy for the synthesis (2489 kJ) and the predictive terminal energy for the synthesis (2595 kJ) represents the penalty in accuracy due to using conditional powertrain efficiency in lieu of the full powertrain model. The calculated error associated with the synthesis approximation and the predictive tractive energy are considered an acceptable tradeoff given the limited number of publicly available parameters necessary for calculations and the significant improvement on the tractive energy compared to the battery terminal energy. Understanding these errors requires recalling the purpose of approximating the route and powertrain; eco-routing seeks to determine the least energy intensive route option rather than predicting energy consumption exactly. The error between the powertrain model terminal energy results for the 505 cycle and 505 synthesis as well as the difference between 505 synthesis powertrain model terminal energy and predictive terminal energy are considered small enough to select the correct route out of multiple alternative a high percentage of the time. The following section contains an example routing scenario with an application of the predictive terminal energy method.

2.8 Eco-routing Example

The validation presented in the Section 2.6 primarily focuses on the accuracy of the predictive terminal
energy in isolated cases – single modes or a single route. Although the metric closely follows the 505 example, additional verification is needed to confirm the ability to accurately select the eco-route. This section seeks to apply the predictive terminal energy methodology to an eco-routing example originally explored in [2.24]. A routing scenario describing three routes in the Blacksburg, Virginia area are shown in Figure 2.18. The routes vary considerably in their characteristics: route 1 is 5.6 km in length and predominately urban; route 2 is 9.1 km in length and mostly highway driving; route 3 is 5.6km in length and a mix of highway and urban driving. However, each of the three routes alternatives shown feature an estimated travel time of 10 minutes by a popular routing platform. A driver using a standard navigation system would be tempted to use any of the routes shown without prior knowledge of the roadway. Such a situation is an ideal application of eco-routing; when multiple route alternatives are similar in time, there is little penalty for selecting the most energy efficient route. In this example, roadway data including speed limits and grade are considered known variables and inputs to the problems (Figure 2.1). Additionally, the effects of traffic are outside of the scope of this paper and thus not included.

![Figure 2.18](image) Routes used for eco-routing example.

There are 15, 11, and 7 potential stops present within route 1, route 2, and route 3 respectively. Stops are distinguished as either conditional (a stoplight which could be green or red) or mandatory (a stop sign or a traffic circle). In one case, on Route 2, a stoplight is treated as a mandatory stop given its placement on an off-ramp. A stop time of 0 seconds is assumed for mandatory stops which describes a vehicle stopping to obey a stop sign at an empty intersection and proceeding. A stop time of 25 seconds is assumed for stops at stoplights. The prediction of traffic signals (and signal timing) is considered outside of the scope of this analysis similar to the effects of congestion. Instead of attempting to predict which signals will cause stops, the edge cases of each route will be explored where all traffic signals are assumed red and then compared to the opposite edge cases where all traffic signals are assumed green. Stop times directly
impact the TTT, but only slightly increase energy consumption due to the selection of a BEV as the simulated powertrain. Energy consumption results for the same study using an ICEV as the simulated vehicle would be more impacted by stop time due to the larger energy consumption during idling as compared to a BEV.

The process necessary to determine the eco-route is as follows: roadway data for all route options is gathered, synthesized routes characterizing the route dynamics (such as speed limits and stops) are generated, and finally powertrain results are calculated using the synthesized routes. To investigate the significance of grade, the described methodology is performed for the routes with roadway grade considered and without roadway grade considered. Table 2.3 (without considering grade) and Table 2.4 (considering grade) contain aggregate terminal energy and predictive terminal energy for each of the route options. Note that routes which features no conditional stops are identified with NCS and routes which feature all conditional stops are identified with ACS. The route which requires the least amount of energy is highlighted in green for each case. Refer to Appendix B for the synthesized velocity profiles which produce the energy results. Plots showing the velocity synthesis for all traffic signals red and all traffic signals green are included for each route. Additionally, terminal energy traces are included within the Appendix B plots for simulations with and without grade data.

**Table 2.3** Route synthesis characteristics and powertrain results without considering grade data

<table>
<thead>
<tr>
<th>Route</th>
<th>Synthesis Time (s)</th>
<th>Average Speed (kph)</th>
<th>Top Speed (kph)</th>
<th>Stops</th>
<th>Tractive Energy (kJ)</th>
<th>Predictive Terminal (kJ)</th>
<th>Terminal Energy (kJ)</th>
<th>Predictive Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route 1 NCS</td>
<td>448.8</td>
<td>45.2</td>
<td>64.4</td>
<td>3</td>
<td>1361</td>
<td>2252</td>
<td>2158</td>
<td>4.3%</td>
</tr>
<tr>
<td>Route 2 NCS</td>
<td>489.9</td>
<td>64.5</td>
<td>104.6</td>
<td>3</td>
<td>3189</td>
<td>4460</td>
<td>4326</td>
<td>3.1%</td>
</tr>
<tr>
<td>Route 3 NCS</td>
<td><strong>441.3</strong></td>
<td><strong>44.3</strong></td>
<td><strong>104.6</strong></td>
<td>5</td>
<td>1592</td>
<td>2504</td>
<td>2441</td>
<td>2.6%</td>
</tr>
<tr>
<td>Route 1 ACS</td>
<td>784.2</td>
<td>25.8</td>
<td>64.4</td>
<td>15</td>
<td>1258</td>
<td><strong>2401</strong></td>
<td>2478</td>
<td>-3.1%</td>
</tr>
<tr>
<td>Route 2 ACS</td>
<td>757.5</td>
<td>41.7</td>
<td>104.6</td>
<td>11</td>
<td>2969</td>
<td>4600</td>
<td>4490</td>
<td>2.4%</td>
</tr>
<tr>
<td>Route 3 ACS</td>
<td><strong>508.9</strong></td>
<td><strong>38.0</strong></td>
<td><strong>104.6</strong></td>
<td>7</td>
<td>1553</td>
<td>2476</td>
<td><strong>2451</strong></td>
<td>1.0%</td>
</tr>
</tbody>
</table>

**Table 2.4** Route synthesis characteristics and powertrain results considering grade data

<table>
<thead>
<tr>
<th>Route</th>
<th>Time (s)</th>
<th>Average Speed (kph)</th>
<th>Top Speed (kph)</th>
<th>Stops</th>
<th>Tractive Energy (kJ)</th>
<th>Predictive Terminal (kJ)</th>
<th>Terminal Energy (kJ)</th>
<th>Predictive Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route 1 NCS</td>
<td>448.8</td>
<td>45.2</td>
<td>64.4</td>
<td>3</td>
<td>1317</td>
<td><strong>2112</strong></td>
<td>2146</td>
<td>-1.6%</td>
</tr>
<tr>
<td>Route 2 NCS</td>
<td>489.9</td>
<td>64.5</td>
<td>104.6</td>
<td>3</td>
<td>3479</td>
<td>4789</td>
<td>4639</td>
<td>3.2%</td>
</tr>
<tr>
<td>Route 3 NCS</td>
<td><strong>441.3</strong></td>
<td><strong>44.3</strong></td>
<td><strong>104.6</strong></td>
<td>5</td>
<td>1778</td>
<td>2837</td>
<td>2640</td>
<td>7.4%</td>
</tr>
<tr>
<td>Route 1 ACS</td>
<td>784.2</td>
<td>25.8</td>
<td>64.4</td>
<td>15</td>
<td>1200</td>
<td><strong>2113</strong></td>
<td>2450</td>
<td>-13.7%</td>
</tr>
<tr>
<td>Route 2 ACS</td>
<td>757.5</td>
<td>41.7</td>
<td>104.6</td>
<td>11</td>
<td>3263</td>
<td>4866</td>
<td>4800</td>
<td>1.4%</td>
</tr>
<tr>
<td>Route 3 ACS</td>
<td><strong>508.9</strong></td>
<td><strong>38.0</strong></td>
<td><strong>104.6</strong></td>
<td>7</td>
<td>1720</td>
<td>2775</td>
<td>2648</td>
<td>4.8%</td>
</tr>
</tbody>
</table>

Table 2.3 and Table 2.4 illustrate the impact of vehicle stops and roadway grade on terminal energy.
consumption as well as the accuracy of the predictive terminal energy. In Table 2.3, Route 1 is the eco-route when no conditional stops are present within the synthesis. However, the inclusion of all conditional stops causes Route 3 to become the eco-route. The eco-route switches due to the larger number of conditional stops present within Route 1 (12 stops) compared to Route 3 (2 stops). Although the minimum terminal energy adjusts from Route 1 to Route 3, the predictive terminal energy parameter identifies Route 1 as the eco-route for the ACS group. Figure 2.19a and Figure 2.19b show the cumulative powertrain model terminal energy, predictive terminal energy, and wheel energy for Route 1 NCS and Route 1 ACS. Note that a real world comparison between the three route alternatives would most likely involve energy consumption values which are between the edge cases of all conditional stops and no conditional stops. Energy consumption values of each route alternative can be approximated between these edge cases with knowledge of the route traffic signal green times. An understanding of the green times for the traffic signals present within each route would enable estimation of where typical energy consumption values lie between each pair of edge cases. For instance, if the green times for a given route’s traffic signals suggests lights will be green 70% of the time, a more realistic approximation of the route energy consumption would be calculated by weighting the no conditional stops consumption by 70% and the all conditional stops consumption by 30%.

![Figure 2.19](image)

**Figure 2.19** Powertrain model terminal energy, predictive tractive energy, and wheel energy for Route 1 NCS (a) and Route 1 ACS (b).

Figure 2.19a and Figure 2.19b illustrate the accuracy of the predictive terminal energy in response to route dynamics such as stops and cruise speeds. In Figure 2.19a, extended cruise events cause the predictive terminal energy to diverge from the powertrain model terminal energy due to slight error in the regression relationship used for the powertrain coefficient during cruise events (Equation 2-22, Figure 2.22). Figure 2.19b shows that the stops in Route 1 ACS improve the reliability of the predictive terminal energy as compared to Route 1 NCS. Despite the increase in accuracy, the predictive terminal energy identifies Route 1 as the eco-route when Route 3 is identified as the eco-route by the powertrain model terminal energy. This is largely due to slight difference in energy consumption of 27 (kJ) calculated between Route 1 ACS and Route 3 ACS using the powertrain model. A real world eco-routing application involving such a minor energy difference would likely incorporate the TTT (Figure 2.20) when selecting an appropriate route. The impact of stops on the predictive terminal energy error extends to Route 2 and Route 3 as well; error for Route 2 NCS drops from 3.1% to 2.4% when all stops are considered while the error for Route 3 NCS drops from 2.6% to 1.0%. 

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Figure 2.20 Variations in synthesized estimated TTT between the six route options evaluated.

Table 2.4, which considers the effects of grade, shows that Route 1 is the eco-route when both all conditional stops are considered and no conditional stops are considered. The difference in the determination of eco-route between Table 2.3 and Table 2.4 when all conditional stops are considered suggests there is a relationship between the impact factors of grade and vehicle stops on terminal energy consumption. In addition to influencing battery terminal energy consumption, the presence of grade data as an input impacts the predictive energy consumption as well. The magnitude of the predictive error generally rises with the inclusion of grade data. This result is expected, as the powertrain coefficient is calibrated against velocity profiles without grade data considered. Additional calibration using instantaneous grade as an input to powertrain efficiency estimation would increase model accuracy while still not requiring vehicle parameters. In all cases expect for Route 1 ACS with grade considered, the predictive terminal energy reliably estimates the powertrain model energy consumption within 5% while only requiring basic vehicle properties.

Finally, the eco-routing and predictive terminal energy results must be understood in context. Route 1 is the eco-route when no stops are considered regardless of grade. However, when all stops are considered, the determination of the eco-route is split between Route 1 and Route 3 depending on whether grade is utilized as model input. In a real world application, this discrepancy is insignificant compared to the large difference in TTT (Figure 2.20).

2.9 Conclusions & Future Work

Eco-routing is the process of determining which route option requires the least amount of energy as compared to the shortest route in time or distance. Calculation of the energy requirements for each route option evaluated requires two main steps: the generation of a velocity profile from the origin to the destination, and the calculation of the energy required to propel the selected vehicle in accordance to the generated velocity profile. Existing eco-routing efforts vary in the generation of route velocity profiles, but typically involve detailed powertrain models which require many proprietary vehicle properties. The large number of required vehicle parameters represents a significant limitation to existing eco-routing applications due to the limited number of vehicles for which the sufficient amount of vehicle information necessary are known.
An alternative method of analyzing the tractive energy requirements of an electric vehicle is investigated in an effort to reduce the number of vehicle properties necessary to produce route energy consumption results from 11 parameters to 4. Energy consumption results are evaluated using a new parameter – predictive terminal energy – which is a function of the tractive energy requirements to complete a route. The predictive terminal energy parameter is calibrated against a validated electric vehicle powertrain model over a variety of velocity profiles to enable application for a variety of route types (urban, highway, etc.). An error of 4.4% between the predictive terminal energy and the battery terminal energy for an example BEV is calculated for the 505 drive cycle, a considerable improvement upon the -34.5% difference between the unaltered tractive energy and the battery terminal energy for the same velocity profile. Additionally, a validated velocity synthesis method is introduced which further simplifies the number of eco-routing input parameters required for results.

The velocity synthesis and predictive terminal energy concepts introduced are also applied to an eco-routing scenario involving three different route options. Edge cases considering all traffic signals to be green and all traffic signals to be red are evaluated to understand the impact of stops on route selection and predictive terminal energy accuracy. Battery terminal energy and predictive terminal energy results are compared with typical error values of less than 5%. The routing example also highlights limitations of the predictive terminal energy, namely grade and frequent vehicle stops. Additional calibration of the predictive terminal energy considering instantaneous grade would improve the parameter accuracy and reliability. As presented, the error associated with the metric is considered a worthwhile tradeoff in order to significantly reduce the requisite vehicle properties for analysis.
2.10 References


2.11 Appendix A - Acceleration, cruise, and deceleration powertrain efficiency for matrix of test modes

![Powertrain Efficiency vs. Mode Cruise Speed (kph)](image)

**Figure 2.21** Variation of powertrain efficiency during mode acceleration for a matrix of mode cruise speeds and accelerations / decelerations.

![Powertrain Efficiency vs. Mode Cruise Speed (kph)](image)

**Figure 2.22** Variation of powertrain efficiency at various cruise speeds.

![Powertrain Efficiency vs. Mode Cruise Speed (kph)](image)

**Figure 2.23** Variation of powertrain efficiency during mode deceleration for a matrix of mode cruise speeds and accelerations / decelerations.
2.12 Appendix B – Velocity and grade profiles for eco-routing example

Figure 2.24 Speed profile of the first route alternative. Both edge cases are considered with all traffic signals assumed red (a) and green (b).

Figure 2.25 Speed profile of the first route alternative. Both edge cases are considered with all traffic signals assumed red (a) and green (b).

Figure 2.26 Speed profile of the first route alternative. Both edge cases are considered with all traffic signals assumed red (a) and green (b).
3 Research Conclusions

Eco-routing is a promising research area which enables reduced energy consumption of a vehicle without changing any of its physical components. The process of determining which route requires the least amount of energy contains several steps after establishing an origin destination pair. This research focused on two major steps: velocity profile generation and energy consumption calculation. In both cases, existing literature offered thorough analysis, but often using complex analysis. The objectives of this thesis, to simplify existing velocity synthesis and powertrain calculation methods, are considered satisfied.

Chapter One introduces a methodology for generating routes with a minimal number of inputs. Using a single input, $a_{max}$, the driver aggressiveness of a route can be altered. Adjustment of $a_{max}$ influences both the intensity of speed transitions as well as the shape of the velocity profile. The velocity synthesis model, calibrated against representative certification cycles, approximates the 505 drive cycle with only a -3.3% error at the wheels. Powertrain analysis in Chapter Two for the same route comparison (505 cycle versus synthesized alternative) results in a -2.8% energy error at the battery terminals. The analysis shows that by accurately modeling acceleration and capturing the average speed of stop to stop driving, accurate energy consumption results can be calculated for more complex real world driver behavior. The velocity synthesis analysis in Chapter One also discusses constrained analysis and distance horizon calculations. One example shows there is little utility in a distance horizon beyond 3.5 km for a 90 kph speed. The conclusions reached regarding distance horizons are expanded upon in Appendix II which explores the impact of driver aggressiveness on route energy consumption. Results from the driver aggressiveness study suggest that speed transitions have a decreasing impact on net energy consumption as route distances increase.

Chapter Two expands the velocity synthesis methodology by analyzing results using a powertrain model for a representative BEV. The velocity synthesis method and the BEV model used are both validated within the chapter. Comparisons between the tractive energy and the powertrain battery terminal energy reveal opportunity for simplification. A new parameter, predictive terminal energy, is introduced which attempts to approximate the powertrain battery terminal energy while only requiring glider properties of simulated vehicles. In an eco-routing example comparing three route alternatives in the Blacksburg, Virginia area, the predictive terminal energy estimates the powertrain battery terminal energy with less than 5% difference in almost every case. Grade data and stops are found to influence the accuracy of the predictive terminal energy parameter. Future work is necessary to expand this analysis to include grade as an input into the conditional powertrain coefficient and to consider additional vehicles as well (PHEVs, BEVs).
### 4 Supplemental Appendices

#### 4.1 Appendix AA – Powertrain model input parameters

![Table of parameters](image)

**Figure 4.1** Input parameters used within the scalable powertrain model developed by [1.11].

Figure 4.1 shows a summary of the input parameters used to simulate a Nissan 2013 Leaf in the scalable powertrain model described by [1.11]. Inputs shaded in blue represent model inputs associated with the selected vehicle, inputs shaded in brown represent optional scaling factors, and inputs shaded in green represent additional inputs not necessarily associated with specific vehicles.

---

<table>
<thead>
<tr>
<th>Vehicle Glider Parameters</th>
<th>Battery Parameters</th>
<th>Regen Parameters</th>
<th>Ref. Motor Parameters</th>
<th>Scaled Motor Parameters</th>
<th>Driveline Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013 Nissan Leaf</td>
<td>AESC Li-ion</td>
<td></td>
<td>UQM PP125 PM</td>
<td>Leaf Motor</td>
<td></td>
</tr>
<tr>
<td>Mass</td>
<td>Type</td>
<td>Fraction</td>
<td></td>
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<tr>
<td>1498 [kg]</td>
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</tr>
<tr>
<td>3302 [lb]</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{\text{mass}} )</td>
<td>( E_{\text{cap}} )</td>
<td></td>
<td></td>
<td></td>
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</tr>
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<td>1.00 [---]</td>
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<td>( k_{\text{cap}} )</td>
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<td>1.00 [---]</td>
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<td>( V_{\text{oc}} )</td>
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<tr>
<td>1498 [kg]</td>
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<tr>
<td>3302 [lb]</td>
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<td>31.9 [lbf]</td>
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<td>EPA B</td>
<td>( R_{\text{int}} )</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0.116 [lbf/mph]</td>
<td>0.11 [( \Omega )]</td>
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<tr>
<td>0.0178 [lbf/mph²]</td>
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</tr>
<tr>
<td>( c_{\text{r0}} )</td>
<td>( SOC_{i} )</td>
<td></td>
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</tr>
<tr>
<td>0.0097 [---]</td>
<td>60.00 [%]</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>( c_{\text{r1}} )</td>
<td>( SOC_{\text{min}} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0001 [1/(m/s)]</td>
<td>20 [%]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C_{\text{dA}} )</td>
<td>( SOC_{\text{max}} )</td>
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<td></td>
</tr>
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<td>30 [%]</td>
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<td>( n_{\text{charger}} )</td>
<td></td>
<td></td>
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<td>1.00 [---]</td>
<td>87 [%]</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4.1** shows a summary of the input parameters used to simulate a Nissan 2013 Leaf in the scalable powertrain model described by [1.11]. Inputs shaded in blue represent model inputs associated with the selected vehicle, inputs shaded in brown represent optional scaling factors, and inputs shaded in green represent additional inputs not necessarily associated with specific vehicles.
4.2 Appendix BB – Terminal energy, tractive energy, and predictive tractive energy for a 2013 Nissan Leaf at various cruise speeds.

Figure 4.2 Comparison between the predictive terminal energy consumption, powertrain model terminal energy consumption, and tractive energy consumption for different cruise speeds.

Figure 4.2 compares the cruise consumption of a Nissan Leaf at the wheels and at the battery terminals. An additional trace shows the predicted terminal energy (Equation 2-21) which is calculated as a function of the energy at the wheels using a speed dependent powertrain efficiency factor. The data shown is an extension of the data shown in Figure 2.22. Results from this figure show that the predictive terminal energy will under-predict battery terminal energy consumption at low speeds and over predict at high speeds. The parameter is very accurate between 35 kph and 55 kph – speeds which constitute large portions of urban driving.
4.3 Appendix CC – Impact of $a_{\text{max}}$ on velocity profile shape, tractive power, tractive energy, and internal battery energy during acceleration for a 2013 Nissan Leaf.

Figure 4.3 Impact of $a_{\text{max}}$ on speed profile shape and required tractive power during acceleration from 0 kph to 50 kph.

Figure 4.4 Impact of $a_{\text{max}}$ on speed profile shape and required tractive power during acceleration from 0 kph to 70 kph.

Figure 4.5 Impact of $a_{\text{max}}$ on speed profile shape and required tractive power during acceleration from 0 kph to 100 kph.
Figure 4.6 Internal battery energy and tractive wheel energy results for different acceleration rates from 0 kph to 50 kph.

Figure 4.7 Internal battery energy and tractive wheel energy results for different acceleration rates from 0 kph to 70 kph.

Figure 4.8 Internal battery energy and tractive wheel energy results for different acceleration rates from 0 kph to 100 kph.
4.4 Appendix DD – Impact of $a_{\text{max}}$ on velocity profile shape, tractive power, tractive energy, and internal battery energy during deceleration for a 2013 Nissan Leaf.

**Figure 4.9** Impact of $a_{\text{max}}$ on speed profile shape and required tractive power during deceleration from 50 kph to 0 kph.

**Figure 4.10** Impact of $a_{\text{max}}$ on speed profile shape and required tractive power during deceleration from 70 kph to 0 kph.

**Figure 4.11** Impact of $a_{\text{max}}$ on speed profile shape and required tractive power during deceleration from 100 kph to 0 kph.
Figure 4.12 Impact of $a_{\text{max}}$ on the tractive energy and terminal energy during a deceleration from 50 kph to 0 kph.

Figure 4.13 Impact of $a_{\text{max}}$ on the tractive energy and terminal energy during a deceleration from 70 kph to 0 kph.

Figure 4.14 Impact of $a_{\text{max}}$ on the tractive energy and terminal energy during a deceleration from 100 kph to 0 kph.
4.5 Appendix EE – Impact of $n$ on velocity profiles for varying cruise speeds with a maximum acceleration of 1.5 m/s$^2$

![Graph showing impact of $n$ on velocity profiles.](image)

**Figure 4.15** Effect of mode tuning constant on inertial specific power (ISP) during acceleration to 50 kph.

![Graph showing impact of $n$ on velocity profiles.](image)

**Figure 4.16** Effect of mode tuning constant on inertial specific power (ISP) during acceleration to 75 kph.

![Graph showing impact of $n$ on velocity profiles.](image)

**Figure 4.17** Effect of mode tuning constant on inertial specific power (ISP) during acceleration to 100 kph.
4.6 Appendix FF – Impact of $a_{max}$ on the motor limitations of a 2013 Nissan Leaf

Figure 4.18 Motor operating points for a constrained set of low speed modes with varying acceleration levels.

Figure 4.19 Motor operating points for a constrained set of medium speed modes with varying acceleration levels.

Figure 4.20 Motor operating points for a constrained set of high speed modes with varying acceleration levels.
4.7 Appendix GG – Tractive energy vs terminal energy for low, medium, and high speed modes

**Figure 4.21** Relationship between the tractive and terminal energy for a constrained set of low speed modes with varying acceleration levels.

**Figure 4.22** Relationship between the tractive and terminal energy for a constrained set of medium speed modes with varying acceleration levels.

**Figure 4.23** Relationship between the tractive and terminal energy for a constrained set of high speed modes with varying acceleration levels.
4.8 Appendix HH – Tractive, terminal, and predictive energy for each hill of the 505

Energy results for each hill of the 505 drive cycle. Chapter Two analyzes the 505 as a whole, but does not discuss the hill results individually. The predictive terminal energy struggles most on Hill 1 and Hill 2 due to the variability in driving speed which the synthesis struggles to approximate. Hills 3, 4, and 5, are better approximated by the Calibrated Hill Model due to the lack of local acceleration events. The velocity profiles illustrate areas of strength and weakness for the synthesis model.

Figure 4.24 Terminal, predictive, and tractive energy for Hill 1 of the 505 certification cycle.

Figure 4.25 Terminal, predictive, and tractive energy for Hill 2 of the 505 certification cycle.
Figure 4.26 Terminal, predictive, and tractive energy for Hill 3 of the 505 certification cycle.

Figure 4.27 Terminal, predictive, and tractive energy for Hill 4 of the 505 certification cycle.

Figure 4.28 Terminal, predictive, and tractive energy for Hill 5 of the 505 certification cycle.
4.9 Appendix II – Synthesized 505 with varying levels of driver aggressiveness for constrained and unconstrained comparisons

Figure 4.29 The 505 cycle approximated using the Calibrated Hill Model with three levels of maximum acceleration with an average speed constraint enforced for each hill.

Figure 4.29 contains three different velocity profiles which each approximate the 505 drive cycle with varying levels of intensity ($a_{max} = 1.5\text{m/s}^2, 2.5\text{m/s}^2, 3.5\text{m/s}^2$). The more aggressive velocity profiles cruise at lower speeds in order to satisfy the average speed constraint, which is enforced for each hill. Final energy consumption values of 2560 kJ, 2568 kJ, and 2570 kJ are calculated for the $1.5\text{m/s}^2$, $2.5\text{m/s}^2$, and $3.5\text{m/s}^2$ cases respectively. The largest difference of 0.4%, is calculated between the least and most aggressive cases, is within the noise of the powertrain model accuracy. The results from this analysis corroborate the conclusions from Section 1.8 – as the distances of a given route increase, acceleration has a diminishing effect on energy consumption. In all three cases, the net energy consumption of each hill is primarily impacted by the cruise speed rather than the intensity of speed transitions.

A similar comparison is explored in Figure 4.30 which relaxes the average speed constraint. Instead, each velocity synthesis shares a common cruise speed for each mode. Velocity profiles with higher levels of driver aggression are permitted to complete the route in a shorter amount of time, which more closely mimics the motivations of aggressive driving in real world conditions. A larger energy consumption difference between cases is calculated for the unconstrained comparison: 2601 kJ, 2650 kJ, and 2680 kJ for the $1.5\text{m/s}^2$, $2.5\text{m/s}^2$, and $3.5\text{m/s}^2$ cases respectively. The difference between the least aggressive and most aggressive cases in the unconstrained comparison is 3% - an order of magnitude larger than the difference calculated for the constrained comparison. Table 4.1 summarizes the energy consumption results for the constrained and unconstrained comparisons.
Figure 4.30 The 505 cycle approximated using the Calibrated Hill Model with three levels of maximum acceleration with no time constraints enforced.

Table 4.1 Energy consumption results for synthesized approximations of the 505 drive cycle with varying levels of driver aggressiveness.

<table>
<thead>
<tr>
<th>Distance Constraint</th>
<th>Maximum Acceleration</th>
<th>Time (s)</th>
<th>Tractive Energy (kJ)</th>
<th>Terminal Energy (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enforced</td>
<td>1.5 m/s²</td>
<td>505</td>
<td>1623</td>
<td>2561</td>
</tr>
<tr>
<td>Enforced</td>
<td>2.5 m/s²</td>
<td>505</td>
<td>1572</td>
<td>2568</td>
</tr>
<tr>
<td>Enforced</td>
<td>3.5 m/s²</td>
<td>505</td>
<td>1551</td>
<td>2570</td>
</tr>
<tr>
<td>Unenforced</td>
<td>1.5 m/s²</td>
<td>505</td>
<td>1623</td>
<td>2601</td>
</tr>
<tr>
<td>Unenforced</td>
<td>2.5 m/s²</td>
<td>486</td>
<td>1643</td>
<td>2651</td>
</tr>
<tr>
<td>Unenforced</td>
<td>3.5 m/s²</td>
<td>476</td>
<td>1652</td>
<td>2681</td>
</tr>
</tbody>
</table>

| 505 Drive Cycle     | 1.5 m/s²             | 505      | 1687                 | 2562                 |