Demand-Side Energy Management in the Smart Grid: Games and Prospects

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(ABSTRACT)

To mitigate the technical challenges faced by the next-generation smart power grid, in this thesis, novel frameworks are developed for optimizing energy management and trading between power companies and grid consumers, who own renewable energy generators and storage units. The proposed frameworks explicitly account for the effect on demand-side energy management of various consumer-centric grid factors such as the stochastic renewable energy forecast, as well as the varying future valuation of stored energy. In addition, a novel approach is proposed to enhance the resilience of consumer-centric energy trading scenarios by analyzing how a power company can encourage its consumers to store energy, in order to supply the grid's critical loads, in case of an emergency. The developed energy management mechanisms advance novel analytical tools from *game theory*, to capture the coupled actions and objectives of the grid actors and from the framework of *prospect theory* (PT), to capture the irrational behavior of consumers when faced with decision uncertainties. The studied PT and game-based solutions, obtained through analytical and algorithmic characterization, provide grid designers with key insights on the main drivers of each actor's energy management decision. The ensuing results primarily characterize the difference in trading decisions between rational and irrational consumers, and its impact on energy management. The outcomes of this thesis will therefore allow power companies to design consumer-centric energy management programs that support the sustainable and resilient development of the smart grid by continuously matching supply and demand, and providing emergency energy reserves for critical infrastructure.

(GENERAL AUDIENCE ABSTRACT)

The next-generation smart power grid is seen as a key enabler for effectively generating, delivering, and consuming electricity in a sustainable manner. Given the increasing demand for energy and the limited nature of various energy resources, the exchange of energy between producers and consumers must be optimally managed to pave the way towards the deployment of smart grid features on a larger scale. In particular, energy management schemes must deal with a complex and dynamic smart grid composed of utility companies, traditional power sources and loads, intermittent renewable energy generators, as well as new consumer-owned devices such as energy storage units and solar panels. In this thesis, we seek to model the different entities in the smart grid and the exchange of energy between them, in order to better understand the operation and effectiveness of various energy management schemes. In addition, this thesis accounts for the non-rational behavior of consumers in the energy trading process, when faced with various sources of uncertainty in the smart grid, including the intermittent renewable energy generation and the dynamic energy price. The results of this thesis will provide utility companies with key insights, crucial to the design of consumer-centric energy management programs, aimed towards matching electricity supply and demand, and insuring power supply to critical infrastructure.

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Chapter 1

Introduction

1.1 Motivation

The existing power grid is witnessing a noticeable structural transformation and is quickly evolving into a *smart grid* [1]. The smart grid is a power system that encompasses a number of intelligent components which can communicate for the purpose of generating, transmitting, and consuming energy in a more efficient, reliable, and resilient manner [2]. On the generation side, the smart grid tends to move away from the tradition central generation theme, and favors more the deployment of distributed energy resources (DERs), particularly renewable sources. In fact, this would result in less reliance on the inefficient central distribution network, from which stems 90% of all power outages and disturbances [2]. On the other hand, with the intermittent nature of renewable energy sources, a new set of challenges emerges, in relation to insuring that the grid's power quality is not affected by the fluctuating renewable power output. On the demand side, the smart grid must rely on responsible consumers, and their contribution to the overall energy management process, through smart and efficient consumption of energy. In fact, in the smart grid, each consumer will have a smart meter at its premises, which would provide bidirectional communication between the consumer and the power company [3]. Smart meters will enable consumers to better track their energy consumption and electricity bill by receiving real-time prices of consumption from the power company. This would also enable consumers to optimally utilize their renewable generation units and energy storage devices, in order to minimize their energy consumption cost. In addition, smart meters can provide bidirectional power metering, thus allowing the power company to accurately monitor the amount of electricity injected by the consumer into to grid, paving the way for more efficient energy management schemes and incentives [4].

1.2 Smart Grid Energy Management: Opportunities and Challenges

The deployment of smart and effective energy management schemes is one of the main foundations for an efficient, reliable, and resilient power grid [5]. Energy management pertains to the processes by which the smart grid's aggregate load, generation units, and energy exchange infrastructure are scheduled, in order to optimize the overall operational cost and efficiency. In this thesis, we mainly focus on energy management schemes that can enhance the overall energy usage at the customer's side of the grid.

In fact, one of the main challenges of the smart grid is to manage the consumer's consumption and to insure that it can be met with the existing energy generation units and transmission infrastructure. This task is made more challenging with the increased reliance on intermittent renewable energy sources, which would require a fair level of coordination and planning, to properly serve the load withing the grid [6]. Another major challenge that the smart grid must cope with is the growing market of electric vehicles (EVs) [7]. While showing great promise in curbing emissions and reducing transportation cost, the increasing number of electric vehicles represents a significant load increase, one which the current distribution grid might not be suitable to handle. In fact, the increasing number of EVs, could lead to a degradation of power quality, an increase in harmonics and voltage regulation issues, and might possibly damage utility and customer devices [8]. On the other hand, an EV can be used by customers as energy storage devices, which can be leveraged as emergency power, and managed to minimize the electricity bill. This further motivates the need for proper energy management schemes which seek to manage grid capacity and decrease consumption at high peak hours. The implementation of such schemes will be made possible by the ongoing deployment of bidirectional communication, between power companies and different grid consumers, which is a key feature of the smart grid [3].

In conclusion, energy management and planning schemes will be greatly involved in the evolution of the power grid, and are expected to play a major role at both generation and consumption sides. These schemes will be aimed at mitigating high peak consumption, managing the intermittent energy generators, and making the overall energy exchange process cheaper and more efficient. Next, we discuss some of the major relevant topics,

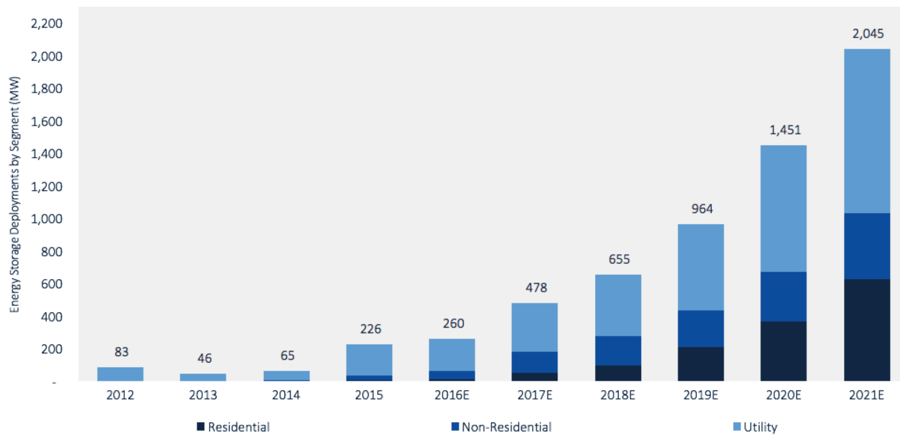

Figure 1.1: Energy storage market forecast [9]

related to energy planning and management in the smart grid.

1.2.1 Demand-Side Management: Consumers' Role in Energy Management

One of the main features of the smart grid is active consumer participation in demand-side management programs (DSM) [10–12]. Demand-side management programs are aimed towards varying or adjusting the energy consumption of participating customers, through financial incentives, in order to benefit the overall grid operation, by avoiding the formation of high peak hours and insuring that energy supply and demand match [12, 13]. Such programs would also benefit the power companies, by avoiding the huge investments needed to increase the capacity of the current power system. In fact, it is estimated that, the cost associated with delivering the top 100 hours of consumption is approximately 20 % of total electricity costs, since generation and transmission capacities are designed to meet peak load, which rarely occurs [14, 15].

A variety of the proposed DSM schemes fall under the umbrella of *demand response* (DR), and more particularly, time-based dynamic pricing programs [14, 15]. Using such programs, power companies can vary the pricing strategy, based on the optimal energy consumption it wishes the grid to have [16]. In fact, the energy load at peak times may be reduced by consumers shifting less essential loads to cheaper off-load consumption hours. Some of these shiftable loads can typically be battery storage units associated with EVs or small-scale renewable energy generators. In fact, recent forecasts predict noticeable growth in both residential and industrial energy storage markets in the next five years, as seen in Fig. 1.1 [9]. The effectiveness and success of DSM programs rely on the adequate design of pricing and incentive mechanisms, capable of decreasing the energy bill of participating consumers', while simultaneously fulfilling the power company's objectives (e.g. maximize profits, match supply and demand, and increase power quality) [14].

In summary, demand-side energy management faces many challenges that might hinder the sustainable development of the smart grid. In fact, DSM must cope with the ever increasing energy consumption, and insure that it is scheduled in a manner which can be supported by the available energy generation and transmission capacities. On the other hand, with the increased penetration of renewable energy generators, energy management schemes must be able to mitigate the intermittent output of these generators, and insure that the grid's consumption always matches the current level of generation.

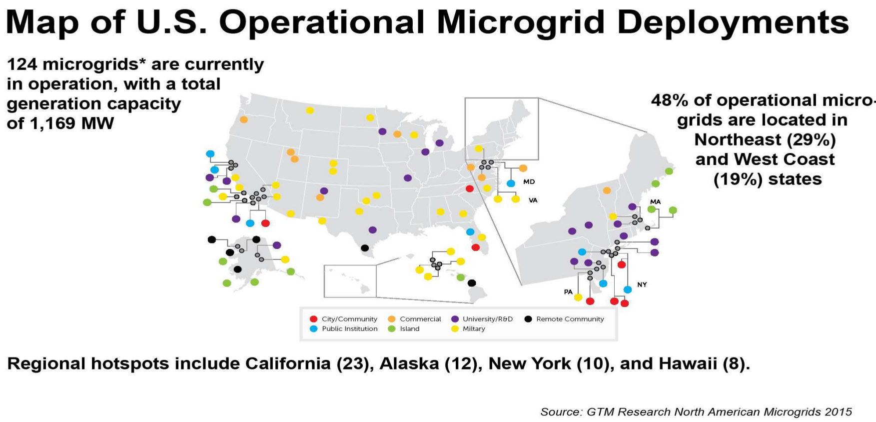

Figure 1.2: Currently deployed energy storage [17]

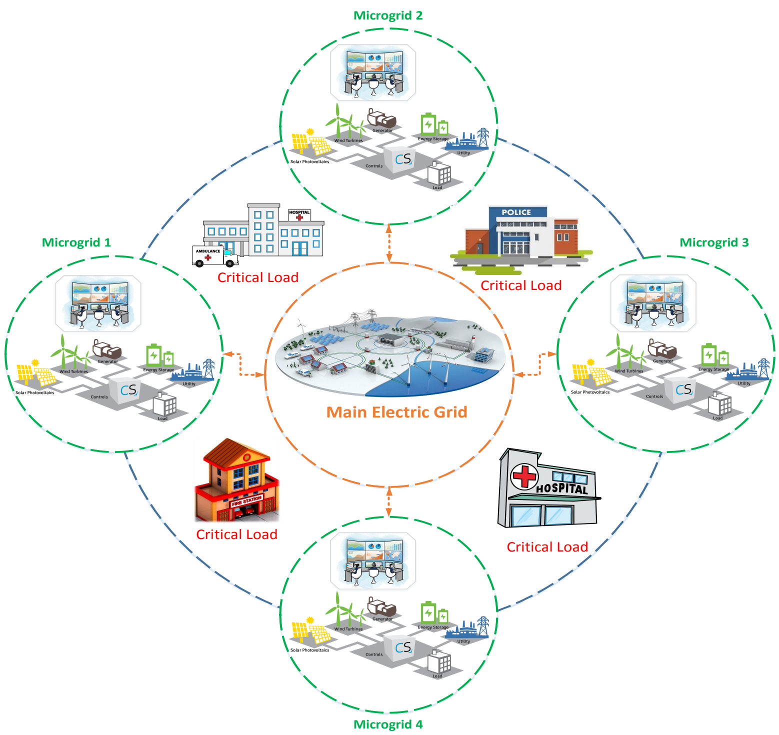

Figure 1.3: Structure of microgrids

1.2.2 Microgrid Energy Management

Given the technical challenges imposed by the growing needs of distributed generation, driven by the increase in energy consumption, the *microgrid* concept is growing into a versatile and dependable solution [18]. Microgrids are small-scale local versions of the centralized electricity system which can operate in either connected or isolated modes. A microgrid is designed such as the local load can mostly be met by the local energy generators or storage devices. A variety of recent works have attempted to tackle the main challenges of microgrid deployment in terms of control architecture [19–21], economic power dispatch [22, 23], and protection [24–26]. Microgrids bring forth numerous advantages to the power grid such as stability, flexibility, and reliability [27], and have already been extensively developed in the United States for communities, universities, office parks, and military bases, as seen in Fig. 1.2 [17]. In terms of reliability, the deployment of microgrids would decrease the susceptibility of the centralized grid to blackouts and failures, caused by natural disasters or cyber-physical attacks. In fact, distributed energy storage and generation units, similar to those associated with microgrids, have played a key part in preserving the operation of the grid's critical infrastructure during various recent emergency situations in the United States such as hurricanes Katrina and Rita [28].

In summary, the deployment of microgrids faces a number of challenges before it can be considered as a dependable solution on a larger scale. In fact, the first challenge lies in the design of the microgrid system itself, where the adequate generators, storage size, and suitable loads must be clustered together, for smooth and reliable grid operation. Given the new structure of microgrids, power dispatch and transmission must be optimized in a way to minimize cost and insure stability. On the other hand, given its reliance on renewable energy generators, insuring a constant supply to the microgrid's loads will be a challenging task, and would require a fair amount of planning and analysis.

1.3 Outline of the Thesis

The main goal of this thesis is to develop new energy management and trading mechanisms between the power company and energy users, who typically own small-scale renewable generation units and storage devices. We seek to thoroughly study the typically coupled energy trading decisions of energy users through the various mathematical frameworks of game theory. In addition, we use the analytical frameworks of prospect theory, to account for the irrationality of energy users, when faced with various uncertainties in the smart grid such as: renewable energy outputs, real-time and dynamic pricing schemes, and incomplete information regarding opposing energy users in the grid.

The rest of this thesis is organized as follows.

- Chapter 2 reviews the current state-of-the-art on smart grid energy management, and discusses our contribution to this area.
- Chapter 3 introduces and discusses various frameworks from game and prospect theory, used in this thesis.
- Chapter 4 introduces a new framework for energy trading between consumers, which accounts for the intermittent nature of renewable energy generators on the demand side of the smart grid.
- Chapter 5 proposes a new hierarchical energy trading framework between smart grid consumers and a grid power company, while explicitly taking into account the uncertainty of the future energy price.

- Chapter 6 develops a new framework for analyzing the storage strategy of microgrid operators, when incentivized to store a portion of their generated excess of energy, in order to improve the central grid's resilience.
- Chapter 7 concludes the thesis and presents possible extensions to our current work.

Note: The notations used in the subsequent chapters will be specific to the chapter in which they are introduced.

Chapter 2

State of the Art and Contributions

In this chapter, we present an overview of works that addressed the various challenges in the smart grid, summarized in the previous chapter. We will then describe the main limitations of these works, and identify the areas that require further investigation, in order to guarantee the effectiveness of energy management schemes. Finally, we will discuss the contribution of this thesis in the study and analysis of energy management schemes, especially those where human decision makers are a key part of.

2.1 State of the Art

2.1.1 Demand Side Energy Management: Load Shifting, Storage Scheduling, and Energy Trading

To insure that energy consumption matches generation, and to avoid the costly peak consumption hours, power companies will devise DSM schemes, in order to better control the grid's energy consumption. Consumers will participate in DSM schemes, seeking to take advantage of the financial incentives offered by the power company and thus decrease their electricity bills. To that extent, a variety of works [29–40] proposed load shifting and scheduling models to thoroughly assess the effectiveness of DSM schemes.

First, the authors in [29–34, 41–46] proposed various frameworks that seek to flatten the load curve through DSM consumption scheduling. For example, the authors in [41] propose a DSM model which accurately predicts the consumption pattern of the consumers' water heaters, based on previous data, and ultimately propose a load shifting scheme for peak shaving. Similarly, the authors in [45] propose a queuing model for scheduling the consumers' air conditioner devices, based on consumption scheduling requirements. On the other hand, the work in [42] proposes a load management control strategy for optimizing the charging of electric vehicles, with the goal of insuring peak demand shaving and improving voltage regulation. Similarly, the works in [43] and [44] tackle the topic of multiple electric vehicles in a grid, seeking to choose the optimal time to start charging, in order minimize an individual cost which is mainly related to the total grid power. The authors in [29, 33, 47] propose a dynamic pricing scheme for day-ahead scheduling, while setting a price proportional to the peak to average load ratio. In [31] and [32], the authors propose a DSM scheme for effective scheduling of consumer-owned generation and storage units, in order to flatten the aggregate load consumption. In [30], the authors propose a heuristic-based evolutionary algorithm to for day-ahead DSM scheduling of a grid with residential, commercial, and industrial consumers. In [46], the authors utilize the framework of mechanism design from game theory to enable a power company to offer cost effective and attractive demand management contracts to consumers. The results from these various works show that the load curve can be significantly managed and flattened on a daily basis, through various centralized, as well as fully distributed DSM schemes.

Meanwhile, a number of works proposed scheduling schemes, for the purpose of matching grid consumption to the intermittent renewable energy supply [35–40]. For example, the authors in [37–39] propose a number of optimization algorithms for scheduling and operating a large renewable energy plant, which ultimately minimize the use of external energy sources and manage to serve all customers, at different delay rates. In addition, the author in [35] and [36], propose a game-theoretic framework to match the total grid demand to the forecasted output of a local wind farm, which has shown significant decrease in the supply/demand mismatch. The work in [40] studies the application of both a price-based and direct-load based DSM scheme, for matching supply and demand in the power network of the future.

A large body of DSM literature has adopted game-theoretic models and solutions [29, 31–36, 40, 43, 44, 47–52], to capture the fact that the actions of grid actors and consumers are coupled under the objective of managing the total grid load. In particular, a number of works [34, 48–50, 52] have used the framework of Stackelberg games in order to study the hierarchical interaction between the power company and the grid's consumers. This is due to the fact that, in a typical energy management setting, the power company acts first by announcing a certain incentive or pricing strategy, to which consumers respond by altering their consumption trend. For example, the work in [34] proposes a Stackelberg game approach to deal with demand response scheduling under load uncertainty based on real-time pricing in a residential grid. Similarly, the authors in [48, 50, 52] use a Stackelberg game approach between one power company and multiple users, which is aimed at smoothing the aggregated load in the system, while maximizing the revenues of all parties. The authors in [51], propose a multistage Stackelberg model between energy generators, energy retailers, and energy consumers, while seeking to maximize the retailers' procurement and pricing decisions. The authors in [49], propose a Stackelberg game approach between company and consumers, while studying the impact that a malicious attacker could have, through the manipulation of pricing data.

On the other hand, a number of recent works in DSM literature have studied the subject of irrational consumers in energy management scenarios, using the Nobel-winning framework of *prospect theory* (PT). For instance, the authors in [53, 54] introduce a storage management framework in which storage owners can choose to store or sell energy, while accounting for their subjective perceptions, through a prospect-theoretic framework. In [55], a prospect-theoretic framework is used to analyze a DSM scheme, where consumers seek to minimize the cost of their energy consumption by selecting the ideal time to start participating. To summarize, the existing works in DSM focus on flattening the load curve [35–40], as well as matching demand to the intermittent nature of large-scale renewable generation units [35–40]. However, most of these works fail to account for the irrational behavior of consumers, and the uncertain factors that lead to such behavior, which include consumer-owned renewables, and the stochastic nature of future energy prices associated with dynamic pricing.

2.1.2 Limitation of the Current DSM Literature

The main limitation of the DSM literature can be summarized as follows:

1- While the integration of utility scale renewable energy on the supply side of the grid has been well accounted for in DSM literature, the topic of small-scale renewable energy on the consumer side has been relatively under-explored. In fact, as most DSM schemes rely on day-ahead consumption scheduling, grid consumers might be forced to deviate from their declared consumption, given the unpredictable nature of their renewable energy output. This will severely impact the effectiveness of DSM programs, and can potentially require the power company to purchase additional energy in real-time, typically at higher costs, in order to compensate for such deviation.

2- The works in DSM literature also fail to account for the consumers' different valuations of the energy left in their storage. In fact, this valuation is based on the consumer's perceived future energy price, which is unknown given the variable nature of dynamic energy pricing. Changes in these valuations will undoubtedly affect a consumer's decision to either buy or sell energy, while participating in DSM schemes.

3- Stackelberg games have been extensively used to study the interaction between power company and consumers [33, 34, 48–52], given the fact the power company declares its pricing strategy first, before consumer react. All of these works make the assumption that consumers are fully rational players and will thus choose their strategy in accordance to classical game-theoretic analysis. However, the behavior of consumers, who are human players, can significantly deviate from the rational principles of classical game theory (CGT), when faced with the uncertainty of probabilistic outcomes.

4- While the authors in [53–55] account, to some extent, for the consumers' irrational behavior, the uncertainty from which this behavior stems from is usually associated with the probabilistic strategies of opponents. These works fail to account for various other sources of uncertainty on the consumer side of the grid, which include the intermittent output of their renewable energy generators, and the future price of any energy kept in their storage devices.

2.1.3 Microgrid Energy Management: Energy Trading, Operation Optimization, Reliability Enhancement

Recently, there has been an increasing interest in the study and deployment of microgrids (MGs), as an ideal solution for the growing need for distributed energy generation sources. In fact, there has been an abundant body of work dealing with microgid energy management [56–70], focused on the internal energy planning and scheduling of microgrids, as well as their energy trading with the power company and other microgrids.

First, a number of recent works have covered the topic of microgrid operation optimizing through an internal energy management process [56–62]. These works seek to make microgrids more autonomous and less dependent on outside energy sources. For instance, the authors in [56] and [60] propose an energy management system, for a microgrid with PhotoVoltaic (PV) installations and storage units, relying on solar energy and load forecasts. In addition, the authors in [61] develop a Monte Carlo simulation based framework to study a microgrid system, in order to assess the availability of wind energy and to approximate the needed storage capacity for a constant power supply. Similarly, the authors in [59] study the application and control of prime movers, such as gas engines, in a microgrid system, to compensate for the varying demand and intermittent output of renewable energy generators. In addition, the authors in [62] assess the introduction of a pitch angle and rotor speed controller to a wind power microgrid, in an attempt to maximize output power and stabilize the microgrid voltage during short circuit faults. The authors in [57], propose an energy management system for a droop-controlled microgrid, which varies generator output power in order to insure stable operation and minimize fuel consumption cost. Furthermore, the authors in [58], propose multiple control and power management strategies in a microgrid with multiple distributed generation units, in order to regulate voltage and compensate reactive power.

Second, the recent works in [63–70] have recently proposed frameworks for energy trading among different microgrids, as well as between microgrids and a central power company, to insure that any deficiency from local generation units is covered, and any excess is either stored or sold, to maximize the profits of various MG owners.

For example, the authors in [63] propose a neural network energy forecasting algorithm of a microgrid's renewable generators, in order to optimize the microgrid's scheduling and trading decisions. In [66], the authors propose a novel algorithm for forming coalitions between microgrids, in a cooperative game-theoretic framework, with the goal of optimizing power transfer and minimizing transmission losses. The works in [67] and [68] analyze the price competition between interconnected microgrid, using a game theoretic framework. Similarly, the authors in [64] propose a Stackelberg game model framework for energy trading between microgrids, where sellers move first by declaring the amount and price of energy they wish to trade, and buyers react by declaring their unit price bid to the sellers. The authors in [69] study the case where two neighboring microgrids, operating in island mode, can trade energy in a peer-to-peer fashion, in order to minimize the total energy generation and transmission cost. On the other hand, the authors in [65] utilize the weighting effect, from the framework of prospect theory, to account for the irrational behavior of each microgrid operator (MGO), originating from the uncertainty of the trading strategy of its opponents. In addition, the authors in [70], propose the implementation of a coalition control system, to provide efficient communication and control between a number of neighboring clustered microgrids.

More recently, in addition to providing energy management services, there has been considerable interest in using the storage abilities of microgrids to enhance the resilience of the smart grid against emergency events such as natural disasters or security breaches. The resilience of the grid reflects its ability to avoid the interruption of its critical services when faced with a large disturbance [71]. In this regard, various academic, industrial, and federal reports [72–74] have proposed leveraging the microgrids' storage capacity to mitigate the effect of loss of generation during emergencies by meeting the smart grid's most critical loads. Indeed, distributed storage and generation units, the integral constituents of MGs, have played an essential role in preserving the operation of hospitals, police stations, as well as fire fighting and rescue services centers in many recent emergency situations in the United States [74]. For instance, this has been the case during natural disasters such as hurricanes Katrina and Rita, and the wildfires which interrupted the transmission of electricity to parts of Utah in 1995 and 2003, as well as in the 2003 North American Northeast blackout [74]. In addition to the various reports [72–74] encouraging the use of MG storage to enhance grid resilience, a number of recent works [75, 76] has also investigated the issues related to power quality, that might arise, when a critical load is supplied by MG energy sources.

To summarize, the main body of work [56–70] focuses on producing an autonomous microgrid that relies on its internal generation and storage units to supply its load in an efficient and stable manner, while minimizing cost and power losses. While the topic of leveraging MG storage for increasing grid resilience has been supported by a number of technical reports [72–74], sufficient incentive schemes and models are lacking in the existing literature.

2.1.4 Limitations of the Current Microgrid Literature

Based on the conducted literature review, we can see that the current body of work related to microgrid energy management is lacking in multiple aspects:

1- There has been a lack of literature proposing energy trading scheme to incentivize microgrids to participate in covering the power grid's critical loads. Furthermore, models must be developed to assess incentive schemes, that encourage microgrid operators to store part of their energy excess for enhanced grid resilience. Such excess can, for example, be used in case of a power blackout, resulting from a natural disaster or malicious attack.

2- A number of recent works [64–69] have applied various game-theoretic models to study energy trading among MGOs. However, these works assume that, when choosing their optimal action, every operator has complete information regarding the system's architecture, the central power company, and other MGOs. However, in a real-life application, MGOs will rarely have complete information. Particularly, in a smart grid setting, given the intermittent output of renewable energy sources, an MGOs will find it challenging to predict its opponents' available energy output and storage level.

3- With the exception of the work in [65], the current literature on MGO energy trading assumes that all decision makers are rational. However, as shown by the studies under PT [77], human players (MGOs) will deviate significantly from the axioms of classical game theory, when faced with the uncertainty of outcomes. Such uncertainty could possibly stem from an MGO's incomplete information regarding its opponents' capabilities and energy generation/storage levels.

2.2 Contributions and Publications

The main objective of this thesis is to develop new frameworks for studying and optimizing energy management and trading between the different actors in the smart grid (e.g. consumers, MGOs, power companies), in an effort to address the main challenges which could hamper the sustainable development of the future power grid. In fact, with the growing demand for energy, the limited capacity of the inefficient distribution grid coupled with the increased penetration of intermittent renewable energy generation, require novel solutions to efficiently manage the generation, transmission, and consumption of energy in the smart grid. In contrast to the existing body of literature, we propose day-ahead DSM frameworks, that can be used to analyze the energy trading decisions of consumers, while explicitly attempting to mitigate the real-time consumption deviation, from the day-ahead schedule, caused by the consumers' stochastic renewable outputs. Moreover, the proposed solutions also account for a consumer's varying stored energy valuations, caused by the uncertain future dynamic price of energy, and the impact of such valuations on the energy trading decisions. Furthermore, to enhance the grid's resilience while leveraging consumer owned storage units, this thesis develops a novel energy trading mechanism that can help incentivize microgrid operators to participate in a grid resilience enhancement scheme in an effort to protect the critical loads' power supply during natural disasters or malicious cyber-physical attacks. Within all of the developed frameworks, this thesis will introduce analytical techniques from *prospect theory*, to account for the irrational behavior of consumers and microgrid operators, when faced with uncertainties, stemming from renewable energy sources, energy price, and incomplete information regarding the capabilities of other entities in the grid. This, in turn, helps shed light on the impact of consumer behavior on the overall energy management and resilience of the grid. In particular, this thesis makes the following key contributions:

1- To optimize the process of demand-side energy management, we propose a novel framework for energy trading between *prosumers*, which are smart grid customers that can both consume and produce energy, which accounts for the intermittent nature of renewable energy generators on the demand side of the smart grid. In particular, we formulate a noncooperative game between multiple prosumers who must declare the amount of energy they wish to purchase or sell while maximizing their utility. The utility function captures the tradeoff between the profits gained from selling energy and the penalty induced by failure to meet the declared bid. In contrast to conventional game theory, we develop a prospect-theoretic framework that models the behavior of prosumers during energy trading. In particular, we account for each prosumer's perception of the probability of its possible profits from trading energy originating from the probabilistic outcomes of wind energy (probability weighting). We also account for the prosumer's valuation of its gains and losses with respect to its own preferences (framing effect). To solve this game, a best response algorithm is proposed that allows the system to reach a Nash equilibrium under both prospect theory and game theory. Simulation results show that the penalty factor has a more prominent effect on a prosumer under prospect theory analysis. In fact, the results show that a prosumer tends to sell less energy under prospect theory than under traditional game-theoretic analysis. The power company will thus need to revisit its pricing scheme or to acquire additional energy from external sources to account for this difference.

2- To capture the interactions between the power company and grid prosumers in a DSM scheme, we also propose a second framework for energy trading which explicitly accounts for the uncertainty of the future energy price. In particular, we formulate a single-leader-multiple-followers Stackelberg game, where the power company, seeking to maximize its profits, leads by declaring its pricing strategy, and prosumers react by choosing their optimal energy bid in a competing noncooperative manner. The prosumers' utility function accounts for the profits resulting from buying/selling energy at the current known price, as well as the uncertain future profits, originating from selling the energy in their storage. In contrast to conventional game theory, we develop a prospect-theoretic framework that models the behavior of prosumers when faced with the uncertainty of future profits, originating from their stored energy. In particular, we account for each prosumer's valuation of its gains and losses, compared to its own individual utility evaluation perspective, as captured via the PT framing effect, by a utility reference point. We show that under CGT the followers' noncooperative game admits a unique pure strategy Nash equilibrium. Moreover, under PT, we derive a set of conditions under which the pure strategy Nash equilibrium is proven to exist. In particular, we propose a number of algorithms that allow the prosumers and power company to reach an equilibrium in a distributed manner under both cases of CGT and PT. Simulation results show how the total grid load varies, with respect to the prosumers' reference point. In addition, simulation results highlight the impact of this variation on the power company's profits, which significantly decrease, when it fails to account for the prosumers' subjective perceptions under PT.

3- In order to fully exploit the role of microgrids in optimizing the central grid's resilience, we propose a new framework for analyzing the storage strategy of microgrid operators, when incentivized to store a portion of their generated excess of energy. In this regard, we formulate a noncooperative Bayesian game between multiple MGOs to account for the incomplete information of each MGO regarding the excess of energy of its opponents. In this game, each MGO must choose a portion of its MG's energy excess to store so as to maximize a utility function that captures the tradeoff between selling at the current market price and potentially selling in the future at a significantly higher emergency energy price. In contrast to conventional game theory, we develop a prospect-theoretic framework that models the behavior of MGOs when faced with the uncertainty of their opponents' stored energy, which stems from the presence of intermittent renewable energy sources. In particular, we account for each MGO's valuation of its gains and losses with respect to its own individual utility evaluation perspective, as captured via the PT framing effect. For this proposed game, we derive the closed-form expression for the Bayesian Nash equilibrium (BNE) for the classical game-theoretic scenario and interpret this equilibrium under different conditions. For the PT case, we propose a best response algorithm that allows the MGOs to reach a BNE in a decentralized fashion. Simulation results highlight the difference in MGO behavior between the fully rational case of classical game theory and the prospect-theoretic scenario. Indeed, for certain reference points, MGOs choose to store more energy under PT compared to CGT, while the case is reversed for other reference points where MGOs noticeably reduce their MGs' stored energy. In addition, the impact of the reference point is found to be more prominent as the emergency price increases. The power company must therefore quantify the subjective behavior of the MGOs before choosing the optimal emergency energy price, in order to meet the critical load at minimal cost.

2.2.1 Publications

The contributions of this thesis have resulted in the following publications:

1- G. El Rahi, W. Saad, A. Glass, N. B. Mandayam, and H. V. Poor, "Prospect theory for prosumer-centric energy trading in the smart grid", *In Proc. of the IEEE Power & Energy Society Innovative Smart Grid Technologies Conference (ISGT),* Minneapolis, Minnesota, USA, September 2016, (pp. 1-5).

2- G. El Rahi, S. Rasoul Etesami, W. Saad, N. B. Mandayam and H. V. Poor, "Managing Price Uncertainty in Prosumer-Centric Energy Trading: A Prospect-Theoretic Stackelberg Game Approach", Submitted to *IEEE Trans. on Smart Grid*, May 2017.

3- G. El Rahi, A. Sanjab, W. Saad, N. B. Mandayam and H. V. Poor, "Prospect theory for enhanced smart grid resilience using distributed energy storage", *In Proc. of the 54th Annual Allerton Conference on Communication, Control, and Computing* (Allerton), Monticello, IL, USA, September 2016, pp. 248-255.

In the next chapter, we will discuss in more details the different frameworks and solution concepts from game theory which are of relevance to smart grid problems addressed in this thesis. We will also discuss the various axioms of prospect theory which quantify the irrational behavior of human players, when faced with uncertain utility outcomes.

Chapter 3

Game and Prospect theory: Concepts and Techniques

3.1 Game Theory: Introduction and Examples

Game theory is a set of mathematical frameworks that enable the study of conflict and cooperation between intelligent decision makers [78]. It has been used in a variety of topics in economics, social sciences, biology, engineering, and more recently, smart grid [79–82]. A *game* represents a scenario where multiple individuals need to choose from a set of possible strategies, with each individual seeking to maximize its own payoff, which also depends on the strategy of other individuals. One classical example of a game is the popular "Prisoner's dilemma" [83]. In this scenario, two criminals are arrested and imprisoned. Each prisoner is placed in a separate cell with no means of communicating with the other. The prosecutor does not have sufficient evidence to convict the criminals on a principal charge, which sentences the felons to four years of jail. However, there is enough evidence to convict the prisoners of a minor crime, and sentenced to one year.

Each prisoner has the choice to either betray the other by testifying (T) that the other committed the major crime, or to stay quiet (Q). The prosecutor offers each criminal the following deal:

- If only one of the prisoners testifies, then that prisoner goes free and the other prisoner is jailed for 4 years.
- If both prisoners stay quiet, then they are both jailed for 2 years.
- If both prisoners testify, then they are both jailed for 3 years.

The game is represented in Table 3.1. One player acts as the row player and the other one plays as the column player, and both have the action options of testifying (T) or staying quiet (Q). It can be clearly seen that the actions of both players are coupled. In fact, the sentence served by each prisoner does not only depend on the prisoner's own action, but also on the opponent's choice. Regardless of the strategy of the other player, in the Prisoner's dilemma, each player has an incentive to always testify. It can be clearly seen that a prisoner testifying always results in less prison time for that prisoner. This would lead to both prisoners testifying, and thus both spending 3 years in jail. The result of (T,T) is known to be a *Nash equilibrium* in which no player can improve its utility by unilaterally changing its action. Although staying quiet, (Q,Q), will result in a better outcome for both players, greediness leads to an inefficient outcome.

Table 3.1: Prisoner's dilemma

	Testify	**Quiet**
Testify	(-3,-3)	(0,-4)
Quiet	(-4,0)	(-2,-2)

The Prisoner's dilemma illustrates the interaction between players and highlights the fact that interdependent decision-making does not always lead to the most efficient of outcomes. It is in fact a perfect of example of a noncooperative game, where cooperation between the different players is not possible. In what follows, we will discuss a number of frameworks from noncooperative game theory as well as their associated solution concepts.

3.2 Noncooperative Game Theory

Noncooperative game theory focuses on decision making scenarios in which cooperation or communication between the different involved players does not occur. The representation of the Prisoner's dilemma in Table 3.1 is referred to as a matrix game, and is suitable for representing games with two players with discrete action spaces. In games with continuous action spaces or large number of players, utility functions are usually defined to capture the dynamics of the game. We next define the different types of games used in this thesis.

3.2.1 Static Noncooperative Games with Complete Information

The concept of a static game is one in which decisions are made simultaneously, and only once. The concept of time has no effect in these types of games, contrary to dynamic games. Complete information is a term used to describe a game in which all players have complete information regarding the existence of every player, their possible actions, and their utility function. This type of game can be typically represented in strategic form as:

$$\Xi = \{\mathcal{N}, \{\mathcal{X}_n\}_{n\in\mathcal{N}}, \{U_n\}_{n\in\mathcal{N}}\}. \tag{3.1}$$

$\mathcal{N}$ is the set of all players, $\mathcal{X}_n$ is the action space of each player n, and U_n is the utility function that captures the gains achieved by player n. It is important to note that player n's utility function depends on both its chosen action x_n, as well as the vector of actions chosen by other players, $\boldsymbol{x}_{-n}$. In contrast to single-player optimization problems, where optimality has an unambiguous meaning, in multi-player decision making, optimality, in itself, is not a well-defined concept [84]. In fact, what is optimal for the overall system, might not be seen as the best outcome for each individual player. For example, in the Prisoner's dilemma, the outcome (Q , Q), which will result in a better payoff for both players, compared to the Nash equilibrium, is not seen as optimal by any player. In fact, each player would regret the choice of Q, given that T would have resulted in a better payoff. On the other hand, at the Nash equilibrium, such a scenario will not occur, given that it is a stable outcome. We next define the *pure strategy Nash equilibrium* (NE), as the widely agreed upon solution concept for this type of games.

Definition 1. *A strategy profile is said to constitute a* pure strategy Nash equilibrium *if,* *for each player $n \in \mathcal{N}$, we have:*

$$U_n(x_n^*, \boldsymbol{x}_{-n}^*) \geq U_n(x_n, \boldsymbol{x}_{-n}^*) \ \ \forall x_n \in \mathcal{X}_n. \tag{3.2}$$

In other words, at the Nash equilibrium, no player will have an incentive to unilaterally deviate from its current action, resulting in a stable solution of the game. In fact, given that other players do not choose to deviate, the player's utility function is already maximized. It is important to note that a pure strategy NE is not guaranteed to exist in every game. A game might also admit a mixed-strategy NE, which was not analyzed in this thesis. A mixed strategy is an assignment of a probability to each pure strategy which allows players to randomly select a pure strategy.

We next introduce *static noncooperative Bayesian games*, which map scenarios where completeness of information cannot be achieved.

3.2.2 Static Noncooperative Bayesian Games

A Bayesian game is a game in which players do not have complete information, regarding the utility function and action spaces of the other players. Instead, players have beliefs regarding their opponents, in the form of known probability distributions [85]. Formally, the strategic form of a Bayesian game is given by:

$$\Xi = \{\mathcal{N}, \{\mathcal{X}_n\}_{n\in\mathcal{N}}, \{\mathcal{T}_n\}_{n\in\mathcal{N}}, \{\mathcal{F}_n\}_{n\in\mathcal{N}}, \{U_n\}_{n\in\mathcal{N}}\}, \tag{3.3}$$

where $\mathcal{N}$ is the set of players, $\mathcal{X}_n$ is the action space of each player n, $\mathcal{T}_n$ is the set of possible types of each player, $\mathcal{F}_n$ is the set of beliefs of player n represented by the probability distributions of each of its opponents' type, and U_n is the utility function of player n. The term "type" represents a possible realization of the unknown or uncertain variable, which captures a player's incomplete information, regarding its opponents. In the smart grid, a variety of scenarios can arise where each player does not have complete information. For example, when making energy management decisions, it is unlikely that a consumer knows the storage energy level of other consumers, or the amount of energy generated by their solar panels. We next define the concept of *pure strategy Bayesian Nash equilibrium*, as the solution concept for this type of games [85].

Definition 2. *A strategy profile is said to constitute a* pure strategy Bayesian Nash equilibrium *if, for each player $n \in \mathcal{N}$, we have:*

$$\mathbb{E}_{\boldsymbol{T}_{-n}}\left[U_n(x_n^*, \boldsymbol{x}_{-n}^*, \boldsymbol{T}_{-n})\right] \geq \mathbb{E}_{\boldsymbol{T}_{-n}}\left[U_n(x_n, \boldsymbol{x}_{-n}^*, \boldsymbol{T}_{-n})\right], \forall x_n \in \mathcal{X}_n, \tag{3.4}$$

In other words, given its beliefs $f_n(\boldsymbol{T}_{-n})$ of an opponent's type $\boldsymbol{T}_{-n}$, no player can increase its expected utility by unilaterally deviating from its current strategy.

Next, we define the framework of Stackelberg games in which, a hierarchy of play exists between different players.

3.2.3 Stackelberg Games

A Stackelberg game is a game in which players referred to as *leaders* move first. Another set of players known as followers, after observing the leaders' actions, choose their own actions [82]. This game model is highly popular in economics situations in which a dominant leader firm moves first (e.g., chooses price, quantity, market location), and the follower firms react. Hierarchical models are common in smart grid energy management studies [34,48–50]. Typically, smart grid consumers are the followers that react to the utility company's strategy, which might include the choice of price, incentive, or scheduling scheme. We next define the *Stackelberg equilibrium* as game theoretic solution for this type of games [86]. The following definition notation is that of a single-leader-multiple-follower Stackelberg game.

Definition 3. *A strategy profile ($\boldsymbol{x}_n^*, y^*$) is said to be a* Stackelberg equilibrium (SE) *if it satisfies the following conditions:*

$$U_n(x_n^*, \boldsymbol{x}_{-n}^*, y^*) \geq U_n(x_n, \boldsymbol{x}_{-n}^*, y^*) \ \forall n \in \mathcal{N}, \tag{3.5}$$

$$\min_{\boldsymbol{x}_n^*} U_{leader}(\boldsymbol{x}_n^*, y^*) = \max_{y} \min_{\boldsymbol{x}_n^*} U_{leader}(\boldsymbol{x}_n^*, y), \tag{3.6}$$

where y is the leader's action and $\boldsymbol{x}_n$ is the followers' action vector.

3.3 Prospect Theory

One of the main drawbacks of classical game theory is the assumption that all players are rational, decision-makers [87]. For example, when faced with uncertain and probabilistic outcomes, a rational decision-maker would chose the action which maximize the expected value of its utility function. However, in practice, empirical studies [77] have shown that real-world decision makers, may deviate significantly from the rational axioms, when faced with the uncertainty of outcomes. The most prominent of studies was that done by Kahneman and Tversky within the context of *prospect theory* , which won the 2002 Nobel prize in economic sciences. We next describe one of the experiments conducted by Kahneman and Tversky, in their seminal work back in 1979:

A number of individuals were given two choices:

- Option A: Enter a lottery with 80% chance of winning 4000 dollars.
- Option B: Take 3000 dollars.

Taking the expected value of both options, we get $U(A) = 3200$ and $U(B) = 3000$. The expected value of option A clearly exceeds that of option B, and should rationally be preferred. However, the experiment showed that 83% of the individuals actually preferred option B. Further experiments also highlighted the effect of how the options are perceived by different individuals. The next example highlights the difference in results between two equivalent experiments which were described to individuals in a different manner. In the first experiment, each subject is given 1000 dollars and then asked to choose between two options:

- Option A: Enter a lottery with 50% chance of winning an additional 1000 dollars.
- Option B: Take an additional 500 dollars.

84% of subjects chose option B. In the second experiment, each subject is given 2000 dollars and then asked to choose between two options:

- Option A: Enter a lottery with 50% chance of losing 1000 dollars.
- Option B: Lose 500 dollars.

69% of subjects choose option A. It can clearly be seen that option A from both experiments is the same. They both result in a gain of 1000 dollars with a 50% chance, or gain of 2000 dollars with a 50% chance. However, in the second experiment, option A is preferred, in contrast to the first experiment where option B was preferred instead. This highlights the importance of how the problem is described to the individuals, and how they perceive their different options.

According to prospect theory, when assessing an option with probabilistic outcomes, a non-rational individual will subjectively weight the probability of each outcome. In addition, the value of each outcome is also *framed* differently by each individual. The different methods by which a rational (expected utility theory) and irrational individual (prospect theory) respectively assess the expected payoff of an option "*A*", with K probabilistic outcomes, is given by:

$$\text{Expected Utility Theory: } U(A) = \sum_{k=1}^{K} p_k U_{A,k}, \tag{3.7}$$

$$\text{Prospect Theory: } U(A) = \sum_{k=1}^{K} W(p_k) V(U_{A,k}), \tag{3.8}$$

where $U_{A,k}$ is the value of each outcome k, and p_k is the probability of k occurring. In addition, $W(P)$ is the function by which a non-rational individual weights the probability of each outcome. $V(U_{A,k})$ is the function by which a non-rational individual valuates

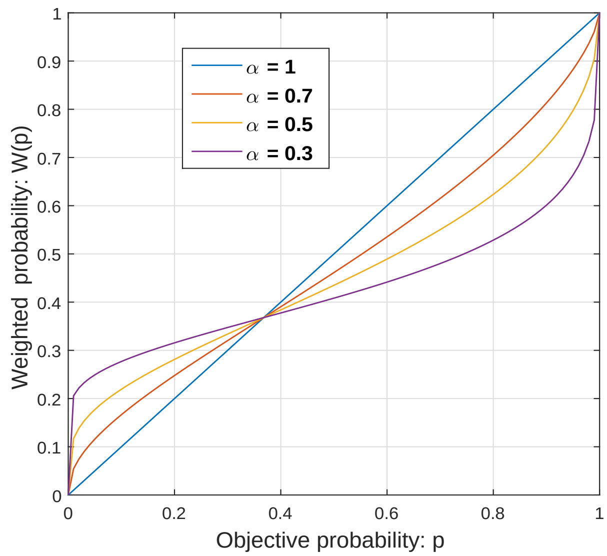

Figure 3.1: Probability weighting function

each outcome. In what follows, we will take a closer look at both of these functions, and describe the axioms that they quantify.

3.3.1 Probability Weighting Effect

The first important PT notion is the so-called weighting effect. In particular, it is observed that in real-life decision-making, people tend to subjectively weight the probability of uncertain outcomes. The weighting effect is mapped by the Prelec function [88] given by:

$$W(p_k) = \exp(-(-\ln(p_k))^{\alpha}), \text{with } 0 < \alpha \leq 1. \tag{3.9}$$

As seen in Figure 3.1, for $\alpha = 1$, the weighted probabilities are equal to the objective probabilities, and the individual is rational. The main axioms of the probability weighting effect are:

- Individuals tend to overweight low-probability events in judgment, and underweight highly probable outcomes. This phenomenon can be used to explain the demand for insurance, in which an individual overweights the very small probability with which an accident might occurs. It can also be used to explain the tendency of individual to buy lottery tickets where the probability of winning is greatly overweighted .

- Individuals tend to greatly alter extreme probabilities. This can be clearly seen in Fig. 3.1. One of the many experiments used to illustrate this phenomenon, is the Russian roulette experiment. The study showed that an individual is willing to pay a greater amount of money to remove the last bullet from a gun in a game of Russian roulette, compared to removing the fourth bullet. Even though both removals reduce the risk by the same percentage of one-sixth, eliminating the risk completely seems more important than to simply reduce it to 50%. This response is perfectly normal from an emotional perspective, but not from a rational point of view.

3.3.2 Utility Framing Effect

The second PT notion is referred to as the framing effect. In particular, it is observed that each individual perceives a utility as either a loss or a gain, after comparing it to its individual reference point. The reference point is typically different for each individual and originates from its past experiences and future aspirations of profits. Furthermore, individuals tend to evaluate losses in a very different manner compared to gains. The

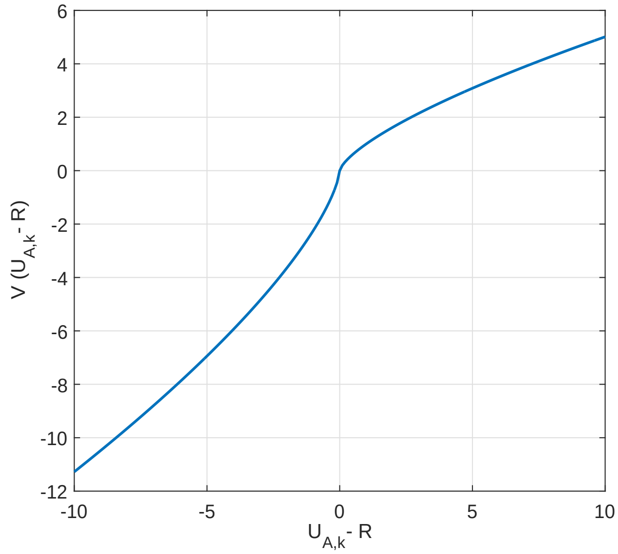

Figure 3.2: Probability framing function

utility framing function, Figure 3.2, which maps an individual's subjective perception of utility is given by [89]:

$$V(U) = \begin{cases} (U_{A,k} - R)^{\beta^+} & \text{if } U_{A,k} > R, \\ -\lambda (R - U_{A,k})^{\beta^-} & \text{if } U_{A,k} < R, \end{cases} \tag{3.10}$$

where $0 < \beta^- \leq 1, 0 < \beta^+ \leq 1$ and $\lambda \geq 1$.

The main axioms of utility framing are:

- Individuals perceive utility according to changes in value with respect to a reference point rather than an absolute value.
- Individuals assign a higher value to differences between small gains or losses close to the reference point in comparison to those further away. This is effect is referred to as diminishing sensitivity, an is captured by the coefficients β^+ and β^-.

- Individuals feel greater aggravation for losing a sum of money than satisfaction associated with gaining the same amount of money. This phenomenon is referred to as loss aversion and is captured by the aversion coefficient λ.

In this chapter, we have introduced a number of framework from game theory and prospect theory, which will be used to study the irrational behavior of human decision makers in the smart grid. In fact, the coupled decisions of these players will deviate from the axioms of expected utility theory, when faced with the uncertainty of their payoffs, stemming from various factors such as intermittent renewable energy, random future price of energy, and incomplete information regarding other entities in the grid.

Chapter 4

Prosumer-Centric Energy Trading: The Effect of Wind Generation Uncertainty

In this chapter, the problem of energy trading between smart grid prosumers that can simultaneously consume and produce energy is studied. The problem is formulated as a noncooperative game between prosumers whose goal is to meet their energy demands at minimum cost by optimally utilizing their storage units and renewable (wind) energy sources. In this game, each prosumer will declare the amount of energy that will be sold or bought to maximize a utility function that captures the tradeoff between the profits gained from selling energy and the penalty incurred for failing to meet the declared amount, due to the stochastic nature of wind energy. The proposed game explicitly accounts for each prosumer's subjective perceptions using the framework of prospect theory. In particular, a prosumer's perception of the probability of its possible profits from trading energy is captured via the weighting effect. In addition, the prosumer's valuation of its gains and losses with respect to its own preferences is captured via the so-called framing effect. To find the equilibrium of this game, a best response algorithm is proposed. Simulation results show the difference in prosumer behavior using traditional game-theoretic and prospect-theoretic analysis. In particular, the results show that probability weighting increases the sensitivity of the prosumers to penalties. Moreover, under PT, a prosumer tends to sell less energy compared to a conventional game-theoretic scenario.

4.1 Introduction

With the increased penetration of renewable energy generators and storage devices at the demand side of the power grid, there is a growing need for adequate modeling and analysis of prosumers in DSM models, which account for their consumption, generation, and storage capabilities [90]. Most of the existing models rely on day-ahead scheduling, where prosumers will declare the amount of energy they plan to buy/sell at the start of the day. However, with the stochastic energy output prediction available on a day-ahead basis, adjustments are needed to the current DSM schemes, to avoid real-time consumer deviation from the declared amount of energy. In the model introduced in this chapter, we propose the inclusion of a penalty factor, which penalizes consumers from any sort of deviation. On the other hand, with the addition of the penalty factor, a prosumer is unsure of the payoff it would receive, when its choice of energy amount to trade is made. Such uncertainty of outcomes will result in consumers making irrational energy trading choices, which deviate from the rationality assumption in previous DSM analysis.

To capture this uncertainty, in this chapter, we formulate a PT-based energy trading model which accounts for the irrational behavior of prosumers, when maximizing an uncertain utility function, that accounts for energy trading profits and bid-deviation penalties.

4.2 System Model and Problem Formulation

Consider the set $\mathcal{N}$ of N grid prosumers each of which owns a storage unit and a wind turbine. Each prosumer $n \in \mathcal{N}$, has a known load profile L_n that must be satisfied and cannot be changed even if the declared energy is not enough to cover it. Each prosumer has an initial stored energy Q_n available in a storage device, originating from an excess of energy at a previous time. In our model, the power company requires its prosumers to declare the amount of energy that they will be buying or selling at the start of the day. This is analogous to the day-ahead scheduling models commonly used in the DSM literature such as in [35] and [33]. Consequently, we let b_n be the amount declared by prosumer n where $b_n > 0$ implies an amount of energy that will be bought and $b_n < 0$ will represent the amount of energy that will be sold. $b_n = 0$ indicates that no amount of energy is traded. At the start of the day, the prosumers do not know for certain the amount of wind energy that will be available.

Here, we assume that the wind energy day-ahead prediction follows a Gaussian distribution of known variance σ_n and mean M_n as shown in [35]. We let W_n be a random variable that represents the amount of wind energy produced. The domain of W_n will be discretized into K outcomes $W_{n,k}$ with probability $p_{n,k}$ each. The outcomes are sorted in an increasing order where $W_{n,1}$ refers to the smallest possible amount of wind energy and

$W_{n,K}$ is the largest amount.

Due to the random nature of wind energy generation, a prosumer may have an excess of energy $(W_{n,k} + Q_n > L_n)$ in which case the excess could be sold to the power company for a monetary profit or, instead, a deficiency in which case additional energy must be purchased from the power company to satisfy the load. The price of selling or buying one unit of energy is related to the total energy consumption declared by all the prosumers. In our pricing model, the prosumer is billed based on the amount that is declared. If a prosumer deviates from its declared amount, it will incur a penalty. This is similar to the recent work in [91] that analyzes wind energy bidding. The unit energy price for buying/selling energy is denoted by ρ and is given by:

$$\rho\left(b_n, \boldsymbol{b}_{-n}\right) = \rho_{\text{base}} + \alpha \sum_{n \in \mathcal{N}} b_n, \tag{4.1}$$

where ρ_{base} and α are design parameters set by the power company. α is a normalization coefficient. Clearly, increasing α will cause the action of each prosumer to have a more prominent effect on the unit price of energy which, in turn, will change the total price for the rest of the prosumers of the grid. In (1), $\boldsymbol{b}_{-n}$ is a vector that represents the amount of energy declared by all the prosumers in the set $\mathcal{N} \setminus \{n\}$.

We let $F(x)$ be the penalty function used by the power company whenever a prosumer deviates from the declared amount. For example, consider a prosumer that declares an amount $-b_n$ to be sold. Certain outcomes of the wind energy, will result in a sufficient excess to meet the declared bid. On the other hands, under other wind conditions, the prosumer will be unable to meet its declared bid and will be forced to deviate and then penalized. A similar situation may occur when the prosumer declares an amount to be bought. Even though any penalty function can be accommodated in our model, for tractability, we use the following function:

$$F(x_{b_n,k}) = \begin{cases} \tau x_{b_n,k}, & \text{if } x_{b_n,k} < 0, \\ 0 & \text{otherwise,} \end{cases} \tag{4.2}$$

where $x_{b_n,k} = Q_n + W_{n,k} + b_n - L_n$. $x_{n,k}$ represents the amount of deviation from the declared energy bid for each outcome of the wind energy. In addition, a positive deviation induces no penalty. This is a reasonable assumption since each prosumer owns a storage unit whose capacity is large enough to store the excess of energy. τ is the monetary penalty of a deviation of one unit of energy set by the power company.

The set of possible values of b_n for each prosumer n is given by

$\mathcal{B}_n = \{b_n \in \mathbb{Z} : B_{n,\text{min}} \leq b_n \leq B_{n,\text{max}}\}$. $-B_{n,\text{min}}$ represents the least possible deficiency or maximum possible excess of energy that prosumer n might encounter for a given W_n, Q_n and L_n. $B_{n,\text{max}}$ represents the maximum possible deficiency or least possible excess in energy. $B_{n,\text{min}}$ occurs at the largest possible value of wind energy and is given by:

$$B_{n,\text{min}} = -\left[W_{n,K} + Q_n - L_n\right]. \tag{4.3}$$

On the other hand, $B_{n,\text{max}}$ occurs at the lowest possible value of wind energy and is given by:

$$B_{n,\text{max}} = -\left[W_{n,1} + Q_n - L_n\right]. \tag{4.4}$$

For a chosen energy bid b_n, each outcome of wind energy will produce a certain monetary profit depending on whether it is sufficient to meet the bid or not. The total expected monetary profit is captured by the following utility function:

$$U_n\left(b_n, \boldsymbol{b}_{-n}\right) = \sum_{k=1}^{K} p_{n,k} E_{n,k}\left(b_n, \boldsymbol{b_{-n}}, W_{n,k}\right), \tag{4.5}$$

where $E_{n,k}$ is the utility induced by each wind energy outcome $W_{n,k}$ for action b_n and action vector $\boldsymbol{b}_{-n}$ and is given by:

$$E_{n,k}\left(b_n, \boldsymbol{b}_{-n}, W_{n,k}\right) = -\rho\left(b_n, \boldsymbol{b}_{-n}\right) b_n + F\left(Q_n + W_{n,k} + b_n - L_n\right). \tag{4.6}$$

When $b_n = B_{n,\text{max}}$, the prosumer can meet the bid for all possible wind outcomes. In fact, $B_{n,\text{max}}$ represents the maximum possible deficiency or least possible excess. Thus $E_{n,k}$ has one possible value for $b_n = B_{n,\text{max}}$. This follows from the fact that every wind outcome $W_{n,k}$ that is sufficient for the prosumer to meet its bid will result in the same $E_{n,k}$ as seen in (4.6). On the other hand, each $W_{n,k}$ that does not meet the bid, will incur a different penalty and thus a different $E_{n,k}$ as seen in (4.2). As b_n decreases, the total purchasing cost will decrease (revenue will increase), however, the bid can no longer be met for additional wind outcomes. In addition, the penalty paid for the wind outcomes that do not meet the load increases as b_n decreases as seen from (4.2). For $b_n = B_{n,\text{min}}$, the purchasing cost will be the lowest (revenue will be at its maximum), however the prosumer can only meet its bid for only one of the wind energy outcomes ($W_{n,K}$) and will be penalized for all others. In our energy trading problem, the goal of each prosumer is to declare the optimal amount of energy that would maximize its expected utility. In addition, when choosing its energy bid, the prosumer must account for the actions of all other prosumers since they affect its expected utility via the price of energy. This motivates the use of game theory [82] in our analysis. However, due to the uncertainty of wind energy, the perception about an optimal bid will differ between prosumers based on each individual's subjective evaluation of the expected utility. This motivates the use of prospect theory [77] to account for the irrational behavior of the prosumer when declaring its energy bid in face of wind generation uncertainty.

4.3 Energy Trading as a Noncooperative Game

We formulate a static noncooperative game between the different prosumers in the set $\mathcal{N}$. In this game, each prosumer seeks to maximize its utility by choosing the optimal declared energy. Since the decisions on the amount of energy to sell or buy are coupled, as captured by (4.6) then, a game-theoretic approach is apropos. The strategic form of our game is: $\Xi = \{\mathcal{N}, \{\mathcal{B}_n\}_{n\in\mathcal{N}}, \{U_n\}_{n\in\mathcal{N}}\}$. $\mathcal{N}$ is the set of all prosumers, $\mathcal{B}_n$ is the action space of each player n, and U_n is the utility of player n that captures the benefits of selling or buying energy along with the penalty of deviating from the declared amount of energy. Under a classical game-theoretic model, the utility achieved by prosumer n for choosing a certain action b_n is given in (4.6).

4.3.1 Utility under Prospect Theory

In a classical noncooperative game, a player evaluates an objective expected utility. However, in real-life, individuals will subjectively perceive their utility when faced with probabilistic outcomes [77]. The wind generation in our energy trading model will induce multiple probabilistic outcomes for each different action. In fact, a subjective prosumer, will alter the probabilities of its possible profits from trading energy. In addition, a subjective prosumer will also perceive the possible profits, in terms of gains and losses, based on its own preferences [77]. This motivates the application of prospect theory in order to account for the prosumer's subjectivity while choosing the optimal energy bid. PT is a widely used tool in the understanding of human behavior when faced with uncertainty of alternatives. In our analysis, we will inspect the effect of two key notions from prospect theory: utility framing and probability weighting.

Utility Framing

In PT, the notion of utility framing pertains to the fact that, in the real world, humans will perceive the values of a utility in terms of gains and losses based on their own *reference point* R_n. A utility is perceived as a gain if it is larger than the reference point, while it is considered a loss if it is below the reference point. The choice of R_n is different between every prosumer and it reflects personal expectations of revenue from energy trading. For example, a revenue of $\$50$ originating from a given energy trade, will be perceived differently by a high-income prosumer as opposed to a low-income prosumer. In fact, a high-income prosumer probably has a high reference point and will consider the $\$50$ as a loss, while the low-income producer will consider it as a gain. Consequently, to take into account utility framing, the function in (5) must be rewritten as:

$$U_{f,n}\left(b_n, \boldsymbol{b}_{-n}\right) = \sum_{k=1}^{K} p_{n,k} V\left(\, E_{n,k}\left(b_n, \boldsymbol{b}_{-n}, W_{n,k}\right) - R_n\right), \tag{4.7}$$

where R_n is the reference point of prosumer n. $V(X)$ is the value framing function concave in gains and convex in losses with a larger slope for losses than for gains. In fact, PT states that the aggravation that an individual feels for losing a sum of money is greater than the satisfaction associated with gaining the same amount [77], which explains the shape of the value function that is typically given by:

$$V(\boldsymbol{X}) = \begin{cases} X^{\gamma} & \text{if } X > 0, \\ (-\lambda)(-X)^{\beta} & \text{if } X < 0, \end{cases} \tag{4.8}$$

where $0 < \beta, \gamma \leq 1$ and $\lambda \geq 1$.

Probability Weighting

In our energy trading problem, the stochastic wind energy will induce multiple probabilistic utility outcomes for each action. This motivates the application of another important notion of PT: *the probability weighting effect*. In [77], it has been observed that, in real-life decisions, individuals tend to subjectively weight the probability of uncertain outcomes. In fact, a PT prosumer will overweight low probabilities and underweight high probabilities. For example, an energy trade that leads to a given profit with probability 0.1 will be perceived differently by a PT prosumer compared to a rational, objective prosumer. While an objective prosumer does not alter the probability, a PT prosumer will overweight a probability of 0.1 such that $w(0.1) > 0.1$, where $w(0.1)$ is the weighted probability. We assume that players weight the probabilities according to the commonly used Prelec function given by:

$$w(p) = \exp(-(-\ln p)^{\zeta}), \tag{4.9}$$

with the distortion coefficient $\zeta \in [0, 1]$. Note that, under the PT weighting effect, a prosumer weights the probability of each unique utility outcome D_n. In other words, for a given action b_n, the occurrences of $W_{n,k}$ that produce the same utility $E_{n,k}$ will be considered as one event. As explained in Section II, all wind energy outcomes that meet the energy bid will result in the same utility $E_{n,k}$ and will be combined together into one D_n. The probability of such a joint outcome is denoted by $\pi_{n,i}$ and is equal to the sum of the probabilities p_{n_k} of the wind energy occurrences that produce the same utility. On the other hand, the energy outcomes that fail to meet the energy bid will result in a different D_n. Each player's utility under probability weighting is then given by:

$$U_{w,n}(b_n, \boldsymbol{b}_{-n}) = \sum_{i=1}^{M_{b_n}} w(\pi_{n,i}) D_n\left(b_n, \boldsymbol{b}_{-n}, W_{n,i}\right), \tag{4.10}$$

where M_{b_n} is the total number of unique outcomes D_n for each action b_n. Given each player's action set and its expected utility for tradtional game theory and PT analysis, next, we define the concept of a pure strategy Nash equilibrium, as the game-theoretic solution under both EUT and PT:

Definition 4. *A strategy profile is said to constitute a* pure strategy Nash equilibrium *if, for each player $n \in \mathcal{N}$, we have:*

$$U_n(b_n^*, \boldsymbol{b}_{-n}^*) \geq U_n(b_n, \boldsymbol{b}_{-n}^*) \forall b_n \in \mathcal{B}_{\mathcal{N}} \tag{4.11}$$

In other words, a strategy profile is a Nash equilibrium in our problem if no prosumer can improve its expected utility by unilaterally changing its energy bid.

In order to compute the Nash equilibrium of the proposed energy trading game in a practical manner, we use the best response approach illustrated in Table 4.1. In fact, finding the closed-form expression for the pure strategy Nash equilibrium or proving its existence is a challenging task for our general game due to the discrete strategy action space [82]. However, it is known that, when it converges, the best response algorithm will always reach a Nash equilibrium [82]. The bidirectional communication between the power company and the prosumers, provided by the smart meters, is the key feature allowing implementation of this algorithm. The power company first communicates to the prosumers the design parameters for energy price (ρ_{base} and α) and penalty (τ). The prosumers then submit their initial energy bids and the utility company will update the energy price. At the start of each iteration of the best response algorithm, the prosumers sequentially pick the energy bid perceived as optimal with respect to the action of all other prosumers. Each prosumer knows the total sum of the bids of others prosumers through the price of energy which is updated after each submitted bid. The algorithm will keep iterating until a Nash equilibrium is reached. At the Nash equilibrium, no prosumer can increase its expected utility by changing the amount of energy that it has declared. Even though an analytical proof of existence/convergence was not possible, as observed from our simulations, the best response always converged to an equilibrium.

Table 4.1: Proposed noncooperative game approach

Best Response Dynamics: Each prosumer $n \in \mathcal{N}$ selects an initial strategy b_n^{init} **repeat, sequentially** For each iteration j 1) The action of all prosumers $i \in \mathcal{N} \setminus n$ is fixed. 2) Each consumer n chooses its best response strategy $r_n(\boldsymbol{b}_{-n})$. 3) Repeat $\forall n \in \mathcal{N}$. **conversion** 4) The algorithm converges when $b_n^j = b_n^{j+1}$ of all players n. 5) If $\forall j, b_n^j \neq b_n^{j+1}$, then there exists no pure strategy Nash equilibrium.

4.4 Simulation Results and Analysis

For simulating the proposed model, we consider a smart grid with $N = 6$ active consumers unless stated otherwise. As mentioned in [92], the average daily energy consumption per household is 11.2 kWh. Here, we will arbitrarily choose each L_n within the range $[8, 16]$ kWh. Each prosumer owns a storage device of maximum capacity 20 kWh [93] and, thus, we arbitrarily choose Q_n within the range $[2, 6]$. All prosumers own two 2.5 kW wind turbines with daily wind energy generation mean of M_n within the range $[10, 16]$ as in [94]. In addition, the base price of energy is chosen to be $\rho_{base} = 10$ cents/kWh [95], and $\alpha = 0.3$. The penalty factor is chosen to be $\tau = 15$ unless stated otherwise. We simulate the system for three scenarios: traditional game theory, PT with framing effects, and PT with weighting.

In Figs. 4.1 and 4.2, we consider the case of PT with probability weighting. The effect of weighting is more prominent as ζ decreases. Fig. 4.1 shows the probability of certain utility outcomes before and after weighting. It highlights the fact that the very unlikely outcomes experience the largest increase (from 0.01 to 0.11). In our model, these probabilities are associated with the small outcomes of wind energy $W_{n,k}$ and thus the highest penalties. In traditional game theory, the weight of these outcomes on the total expected utility is negligible due to their very small probabilities. However, in the case of prospect theory, these outcomes have a more prominent weight. As fig. 4.2 shows, prosumers which follow PT analysis will avoid the high penalties and will tend to sell less energy. In fact, the net energy sales decrease, from 10 kWh for the conventional game-theoretic case (which corresponds to $\zeta = 1$) to 3 kWh for the extreme case of PT weighting in the six prosumers case. As the amount of energy sold decreases, the power company will now need to either modify its pricing or to acquire additional energy from an external source to account for this difference, in order to serve the demand of the rest of the grid.

In Fig. 4.3, we consider the case with framing effects and we show the impact of a varying reference point on the total net amount of energy sold. The studied range of the reference point lies between the smallest and largest possible profits that a prosumer can obtain. In fact, a prosumer cannot logically expect a profit that is not possible to achieve. Fig. 4.3 shows that for a low reference point of -50, the total energy sold is the same under traditional game theory and PT. In fact, for a very low reference point, most utility outcomes are considered as gains and are framed similarly. This will thus reduce the impact of prosumer behavior and the framing effect. As the reference point increases from

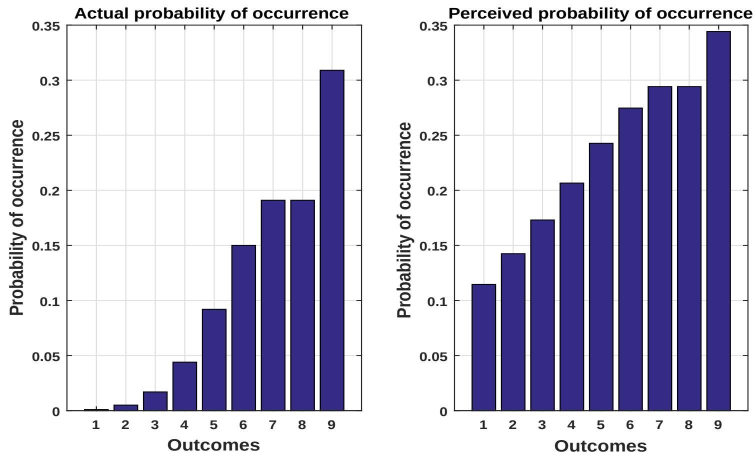

Figure 4.1: Effect of probability weighting

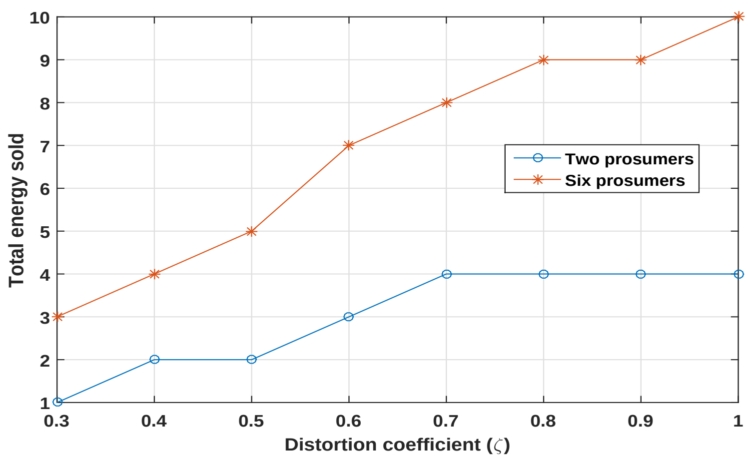

Figure 4.2: Total energy sold by prosumers for different coefficients ζ

-50 to -10 in our example, the total net amount of energy sold decreases from 12 kWh to 7 kWh. Certain possible profits related to the outcomes of wind energy that do not satisfy the bid are now smaller than the reference point and are considered as losses. The value of these losses are amplified since losses loom larger than gains. The prosumer will then increase its bid b_n (decrease the amount sold), since higher values of b_n will result in a lower penalty and thus higher values of profits. These profits are still above the reference point and are still considered as gains. The total expected utility is now larger for higher

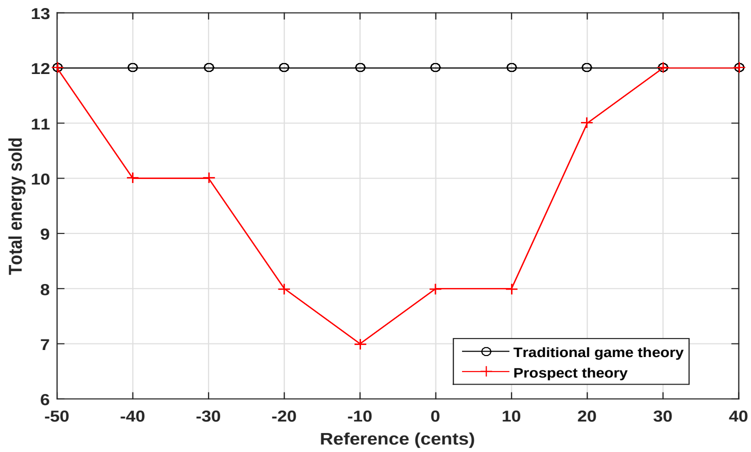

Figure 4.3: Total energy sold by prosumers as a fucntion of the reference point

values of b_n. Moreover, from Figure 4, we can see that, as the reference point increases from -10 to 30, the total amount of net energy sold increase from 7 to 12 kWh. Here profits related to the outcomes of wind that satisfy the bid are now smaller than the reference point as well. All possible profits related to the current b_n are now perceived as losses. The prosumer will then increase the amount sold since, as previously mentioned, lower values of b_n will generate a higher profit (lower cost) for outcomes of wind energy that cover the bid. These outcomes are still above the reference point and are considered as gains which results in a higher total expected utility for lower values of b_n. At a reference point of 40, PT and traditional game theory produce the same result. In fact, at a very high reference point, most of the utility outcomes are considered as losses and are framed similarly and the effect of framing is not prominent.

Fig. 4.4 shows the effect of changing the penalty factor τ on traditional game theory and PT. Here, we set: $R = 0, \beta = 1, \gamma = 0.7, \lambda = 5, \zeta = 0.6$. For very low penalty factors, the effect of weighting is minimal and the total energy sold under it is close to that of traditional game theory. However, under the effect of utility framing, the total amount

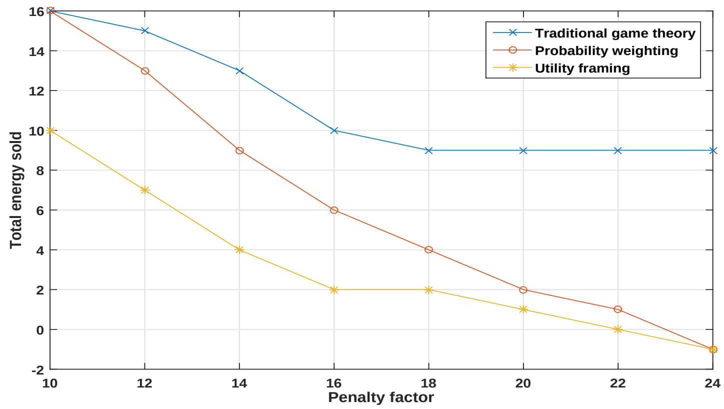

Figure 4.4: Total energy Sold by prosumers for different penalty factors τ

sold is lower for small penalty factors. In fact, weighting only affects the probability of each outcome of the utility and not the outcomes themselves; and for low penalty factors, the utility of low wind occurrences is not very low and thus increasing their probability will not greatly affect the total expected utility value. However, framing impacts the utility outcomes directly, and for a reference point of 0, small losses will be amplified and will be seen as higher losses which explains the decrease in total sold energy. Another important observation is that for traditional game-theoretic analysis, the effect of the increasing penalty factor is not as prominent as the other two cases. In other words, under PT, prosumers are more affected by the penalty factor since both PT effects amplify possible losses received for low wind occurrences. Thus, the power company, has better control on total amount sold or bought by prosumers who are more subjective in perceiving their expected utility. In addition, when choosing the penalty factor, the power company must account for prosumer subjectivity in order to keep the total net energy sold in its desirable range.

4.5 Conclusion

In this chapter, we have proposed a new framework for energy trading between grid prosumers that own renewable energy and storage units. We have formulated the problem as a noncooperative game between prosumers that must declare their energy trading bids in a day-ahead fashion. Subsequently, we have used novel concepts from prospect theory to model the behavior of prosumers when choosing their optimal bidding strategy for energy trading in face of wind generation uncertainty. In particular, we have analyzed the impact of the prospect-theoretic notions of weighting and framing, on the overall energy trading process. Simulation results have shown how behavioral considerations can impact the overall outcome of energy trading, particularly, due to the intermittency of wind energy generation.

Chapter 5

Price Uncertainty in Prosumer-Centric Energy Trading: A Stackelberg Game Approach

In this chapter, the problem of energy trading between smart grid prosumers, that can simultaneously consume and produce energy, and a grid power company is studied. The problem is formulated as a single-leader, multiple-follower Stackelberg game between the power company and multiple prosumers. In this game, the power company acts as a leader who determines the pricing strategy that maximizes its profits, while the prosumers act as followers who react by choosing the amount of energy to buy or sell so as to optimize their current and future profits. The proposed game accounts for each prosumer's subjective decision when faced with the uncertainty of profits, induced by the random future price.

In particular, the framing effect, from the framework of prospect theory, is used to account for each prosumer's valuation of its gains and losses with respect to an individual utility reference point. The reference point changes between prosumers and stems from their past experiences and future aspirations of profits. The followers' noncooperative game is shown to admit a unique pure strategy Nash equilibrium under classical game theory which is obtained using a fully distributed algorithm. The results are extended to account for the case of PT using algorithmic solutions in order to acheive an NE under certain conditions. Simulation results show that the total grid load varies significantly with the prosumers' reference point and their loss-aversion level. In addition, it is shown that the power company's profits considerably decreases when it fails to account for the prosumer's subjective perceptions under PT.

5.1 Introduction

The participation of prosumers in the overall grid energy management process has drawn recent interest from power companies [96, 97]. As mentioned in the previous chapter, it is essential for DSM models to account for the ability of prosumers to generate and store energy. Another key component to consider in prosumer DSM energy trading, is the variable and uncertain future energy price. In fact, given the storage capability of prosumers, one must accurately account for their valuation of the unsold energy in their storage, which might significantly impact their decision to buy or sell energy. In addition, given that DSM schemes are proposed by the power company, its role as decision maker must be accounted for when modeling the prosumer energy trading process. In fact, typically, a power company would first declare its DSM pricing strategy, to which the prosumers would adapt their energy exchange to.

In this context, in this chapter, we formulate a PT-based hierarchical Stackelberg model for energy trading in the smart grid, between the power company and prosumers attempting to maximize their utility function, which accounts for the current energy sale profits, as well as the random future value of the energy left in storage.

5.2 System Model

Consider the set $\mathcal{N}$ of N grid prosumers. Each prosumer $n \in \mathcal{N}$ owns an energy storage unit of capacity $Q_{\text{max},n}$, and a solar PV panel which produces a daily amount of energy W_n. Each prosumer has a known load profile L_n that must be satisfied and an initial stored energy Q_n available in a storage device, originating from an excess of energy at a previous time. In our model, the power company requires prosumers to declare the amount of energy that they will be buying or selling at the start of the as done in day-ahead scheduling models used in DSM literature such as in [35] and [33]. We let x_n be the amount of energy declared by prosumer n, where $x_n > 0$ implies an amount of energy that will be bought and $x_n < 0$ will represent the amount of energy that will be sold. $x_n = 0$ indicates that energy is traded.

The price of selling or buying one unit of energy is related to the total energy declared by all the prosumers. In our pricing model, each prosumer is billed based on the amount that is declared. We assume that the prosumers are truthful and have no incentive to deviate, given the possible penalties that will be incurred. The unit energy price for buying/selling energy is denoted by ρ and is given by:

$$\rho\left(x_n, \boldsymbol{x}_{-n}\right) = \rho_{\text{base}} + \alpha \sum_{n \in \mathcal{N}} x_n, \tag{5.1}$$

where ρ_{base} and α are design parameters set by the power company. For simplicity, we assume that α is fixed and positive, and that the company only varies ρ_{base} to control the amount of energy bough/sold by the prosumers. In (1), $\boldsymbol{x}_{-n}$ is a vector that represents the amount of energy declared by all the prosumers in the set $\mathcal{N} \setminus \{n\}$. The price of unit of energy ρ is regulated and must be within a range $[\rho_{\text{min}}, \rho_{\text{max}}]$.

The future price of energy is perceived to be unknown by the prosumers, given the uncertainty related to future solar energy generation and the pricing strategy of the power company. The future price of energy is thus modeled by a random variable ρ_f. For simplicity, we assume that ρ_f follows a uniform distribution $[\rho_{\text{min}}, \rho_{\text{max}}]$. However, most of our analysis can be extended to the case in which ρ_f follows more general distributions.

The set of possible values of x_n for each prosumer n is
$\mathcal{X}_n = \{x_n \in \mathbb{R} : x_{n,\text{min}} \leq x_n \leq x_{n,\text{max}}\}$. $x_{n,\text{min}} = -W_n - Q_n + L_n$ is a prosumer's maximum sold/minimum bought energy. $x_{n,\text{max}} = -W_n - Q_n + L_n + Q_{\text{max},n}$, is the maximum energy that prosumer n can purchase. For a chosen energy bid x_n, the prosumer's utility function will be:

$$C_n\left(x_n, \boldsymbol{x}_{-n}, \rho_{\text{base}}\right) \;=\; \Big(\rho_{\text{base}} + \alpha(x_n + \sum_{m\in\mathcal{N}\setminus n} x_m)\Big)x_n + \left(W_n + Q_n - L_n + x_n\right)\rho_f. \tag{5.2}$$

In (5.2), the first term represents the revenue/cost of prosumer n at the current time, while the second term represents the future monetary value associated with unsold energy. The prosumers' actions are coupled through the energy price and they will thus be competing to maximize their respective revenues. On the other hand, the power company will purchase (sell) the energy bough (sold) by the prosumers in the energy market at the current market clearing price ρ_{mar}. Given the current market price, the power company's utility function is given by:

$$U_{pc}(\boldsymbol{x}, \rho_{\text{base}}) = \left(\rho_{\text{base}} + \alpha\sum_{n\in\mathcal{N}} x_n\right)\sum_{n\in\mathcal{N}} x_n - \rho_{\text{mar}}\sum_{n\in\mathcal{N}} x_n. \tag{5.3}$$

The power company's revenues are clearly affected by the prosumers and their energy bids. On the other hand, since the prosumers react to the power company's choice of ρ_{base}, the prosumers' utility is directly affected by the power company's action. We thus model the energy trading problem as a hierarchical Stackelberg game [82] with the power company acting as leader, and the prosumers acting as followers.

5.2.1 Stackelberg Game Formulation

We formulate a single-leader, multiple-follower Stackelberg game [82], between the power company and the prosumers. The power company (leader), will act first by choosing ρ_{base} to maximize its profits. The prosumers, having received the power company's pricing strategy, will engage in a noncooperative game. In fact, the final price of energy is proportional to the grid's total load, to which each prosumer contributes. We first formulate the prosumers' problem under CGT as follows:

$$\begin{aligned} \max_{x_n} U_n^{\text{CGT}}(\boldsymbol{x}, \rho_{\text{base}}) :=& \mathbb{E}_{\rho_f}[C_n(x_n, \boldsymbol{x}_{-n}, \rho_{\text{base}})] \\ &\text{s.t } x_n \in [x_{\min,n}, x_{\max,n}]. \end{aligned} \tag{5.4}$$

In (5.4), prosumer n attempts to maximize its expected profits, given the actions of other prosumers and the power company. The previous formulation assumes all prosumers to be rational expected utility maximizers. Moreover, the power company's problem will be:

$$\begin{aligned} &\max_{\rho_{\text{base}}} U_{pc}(\boldsymbol{x}, \rho_{\text{base}}), \\ &\text{s.t } \rho_{\text{base}} \in [\rho_{\min}, \rho_{\max}]. \end{aligned} \tag{5.5}$$

To solve this game, one suitable concept is that of a Stackelberg equilibrium (SE) as the game-theoretic solution of our model.

Definition 5. *A strategy profile ($\boldsymbol{x}^*, \rho_{\text{base}}^*$) is a* Stackelberg equilibrium *if it satisfies the* *following conditions:*

$$
\begin{aligned}
&U_n^{\mathrm{CGT}}(x_n^*, \boldsymbol{x}_{-n}^*, \rho_{\text{base}}^*) \geq U_n^{\mathrm{CGT}}(x_n, \boldsymbol{x}_{-n}^*, \rho_{\text{base}}^*) \quad \forall n \in \mathcal{N}, \\
&\min_{\boldsymbol{x}^*} U_{pc}(\boldsymbol{x}^*, \rho_{\text{base}}^*) = \max_{\rho_{\text{base}}} \min_{\boldsymbol{x}^*} U_{pc}(\boldsymbol{x}^*, \rho_{\text{base}}),
\end{aligned}
\tag{5.6}
$$

where $\boldsymbol{x}_n^*$ is the solution to problem (5.4) for all prosumers in $\mathcal{N}$, and ρ_{base}^* is the solution to problem (5.5).

Remark 1. *Note that in Definition 5, in the case where the followers' problem admits a unique solution $\boldsymbol{x}^*$, the second condition in (5.6) reduces to $U_{pc}(\boldsymbol{x}^*, \rho_{\text{base}}^*) = \max_{\rho_{\text{base}}} U_{pc}(\boldsymbol{x}^*, \rho_{\text{base}})$.*

Next, we will analyze the SE under CGT.

5.3 Game Solution under CGT

The analysis under CGT assumes that all prosumers are expected utility maximizers. Thus, we seek to find a solution that solves both problems (5.4) and (5.5), while satisfying (5.6). First, we start by solving the follower's problem while assuming the the leader's action is fixed to ρ_{base}. We now introduce the following notations:

$$
\begin{aligned}
&\theta := \rho_{\text{base}} - \frac{\rho_{\max} + \rho_{\min}}{2}, \quad \bar{x}_{-n} := \sum_{k \neq n} x_k, \\
&\delta_n := (W_n + Q_n - L_n)\frac{\rho_{\max} + \rho_{\min}}{2}, \quad n \in \mathcal{N}.
\end{aligned}
$$

Here, the expected utility of prosumer $n \in \mathcal{N}$ will be:

$$U_n^{\text{CGT}}(x_n, \bar{x}_{-n}, \rho_{\text{base}}) = -\alpha x_n^2 - (\theta + \alpha \bar{x}_{-n})x_n + \delta_n. \tag{5.7}$$

Next, we denote by x_n^r the best response of player n, which is the solution of problem (5.4), given that all the other players choose a specific strategy profile $\boldsymbol{x}_{-n}$. The following theorem explicitly characterizes the best response of each prosumer n.

Theorem 1. *The best response of player n is given by:*

$$x_n^r(\bar{x}_{-n}) = \begin{cases} -\frac{\theta}{2\alpha} - \frac{\bar{x}_{-n}}{2} & \textit{if } -\frac{\theta}{2\alpha} - \frac{\bar{x}_{-n}}{2} \in [x_{n,\min}, x_{n,\max}], \\ x_{n,\min} & \textit{if } -\frac{\theta}{2\alpha} - \frac{\bar{x}_{-n}}{2} \leq x_{n,\min}, \\ x_{n,\max} & \text{else.} \end{cases} \tag{5.8}$$

Proof. See Appendix A.1. □

In fact, one can rewrite the best responses of all the players in Theorem 1 in a combined single matrix form. We define $\boldsymbol{A}$ to be an $n \times n$ matrix with all entries equal to -$\frac{1}{2}$ except the diagonal entries which are 0, i.e., $A_{ij} = -\frac{1}{2}$ if $j \neq i$, and $A_{ij} = 0$, otherwise. Let $\boldsymbol{a} = -\frac{\theta}{2\alpha}\mathbf{1}$ where $\mathbf{1}$ is the vector of all 1's. Then, we can rewrite (5.8) for all players as

$$\boldsymbol{x}^r = \Pi_\Omega\big[\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}\big], \tag{5.9}$$

where $\Pi_\Omega[\cdot]$ is the projection operator on the n dimensional cube $\Omega := \prod_{n \in \mathcal{N}} [x_{n,\min}, x_{n,\max}]$

in $\mathbb{R}^n$. Our analysis will later use this closed-form representation of the best response dynamics.

5.3.1 Existence and Uniqueness of the Followers' NE under CGT

One key question with regard to the prosumers' game is whether such a game admits a pure-strategy NE. This is important as it allows us to stabilize the demand market in an equilibrium where each prosumer is satisfied with its payoff, as shown next.

Theorem 2. *The prosumers' game admits a unique pure-strategy NE.*

Proof. See Appendix A.2. □

In fact, one can characterize the unique pure strategy NE of the prosumers' game in more detail. As we saw earlier, the best response of the followers can be written as (5.9). Since at equilibrium every player must play its best response, therefore an action profile $\boldsymbol{x}^*$ is an equilibrium if and only if we have $\boldsymbol{x}^* = \Pi_\Omega[\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}^*]$, which means $\boldsymbol{x}^* = \text{argmin}\|\boldsymbol{z} - (\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}^*)\|^2, \boldsymbol{z} \in \Omega$. Since the former is a convex optimization problem, we can write its dual as

$$\max\ \mathcal{D}(\boldsymbol{\mu}, \boldsymbol{\nu}) := -\frac{1}{4}\|\boldsymbol{\mu} - \boldsymbol{\nu}\|^2 + (\boldsymbol{\mu} - \boldsymbol{\nu})'(\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}^*) - \boldsymbol{\mu}'\mathbf{1}$$

$$\boldsymbol{\mu}, \boldsymbol{\nu} \geq 0,$$

where $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)$ and $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_n)$ are the dual variables corresponding to the constraints $z_n \leq x_{n,\max}$ and $z_n \geq x_{n,\min}$, respectively. We denote the optimal solution

Algorithm 1 The relaxation learning algorithm

Given that at time step $t = 1, 2, \ldots$ players have requested $(x_1(t), \ldots, x_n(t))$ units of energy, at the next time step player $n \in \mathcal{N}$ requests $x_n(t+1)$ energy units given by

$$x_n(t+1) = (1 - \frac{1}{\sqrt{t}})x_n(t) + \frac{1}{\sqrt{t}}x_n^r(t),$$

where $x_n^r(t)$ denotes the best response of player n, given the actions of all other players $\boldsymbol{x}_{-n}(t)$ at time step t.

of the dual by $(\boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$. Since, we already know that $\boldsymbol{x}^*$ is the optimal solution of the primal, due to the strong duality the values of the primal and dual must be the same, i.e., $\mathcal{D}(\boldsymbol{\mu}^*, \boldsymbol{\nu}^*) = \|(I - \boldsymbol{A})\boldsymbol{x}^* - \boldsymbol{a}\|^2$. Moreover, the KKT conditions must hold at the optimal solution [98], which together with the earlier equation provide the following system of $3n$ equations with $3n$ variables $(\boldsymbol{x}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$ which characterizes the equilibrium point $\boldsymbol{x}_n^*$ using dual variables:

$$\begin{aligned}
&\mathcal{D}(\boldsymbol{\mu}^*, \boldsymbol{\nu}^*) = \|(I - \boldsymbol{A})\boldsymbol{x}^* - \boldsymbol{a}\|^2, \\
&\mu_n^*(x_n^* - x_{\max,n}) = 0, \\
&\nu_n^*(x_n^* - x_{\min,n}) = 0, \ \ \forall n \in \mathcal{N}.
\end{aligned} \tag{5.10}$$

Solving these equations can be done to derive the unique pure strategy NE of the followers' game.

5.3.2 Distributed Learning of the Followers NE

Next, we propose a distributed learning algorithm which converges in a polynomial rate to the unique pure-strategy NE of the prosumers' game as formally stated in Algorithm 1.

At each stage of Algorithm 1, prosumer n selects its next action as a convex combination of its current action and its best response at that stage. One of the main advantages of Algorithm 1 is that it can be implemented in a completely distributed manner as each prosumer needs only to know its own actions and best response function, and does not require any information about others' actions. Moreover, the prosumers do not need to keep track of their actions history which is the case in many other learning algorithms. Note that Algorithm 1 can be viewed as a special case of more general algorithms known as *relaxation* algorithms [99]. The implementation of this algorithm is made possible by the bidirectional communication between the power company and the prosumers, provided by the smart meters. In fact, at each iteration, the prosumer would send the power company its current strategy, and would receive the updated energy price.

Next, we consider the following definition which will be handy in proving our main convergence result in this section:

Definition 6. *Given an n players game with utility functions $\{u_n(\cdot)\}_{n\in\mathcal{N}}$ and any two action profiles $\boldsymbol{x}$ and $\boldsymbol{y}$, the* Nikaido-Isoda *function associated with this game is given by* $\Psi(\boldsymbol{x},\boldsymbol{y}) := \sum_{n\in\mathcal{N}}[u_n(y_n,x_{-n}) - u_n(x_n,x_{-n})]$.

The Nikaido-Isoda function measures the social income due to selfish deviation of individuals. This function admits several key properties. As an example we always have $\Psi(\boldsymbol{x},\boldsymbol{x}) = 0, \forall \boldsymbol{x}$. Moreover, given a fixed action profile $\boldsymbol{x}$, $\Psi(\boldsymbol{x},\boldsymbol{y})$ is maximized when y_n, equals the best response of player n with respect to $\boldsymbol{x}_{-n}$. In particular, for such a best response action profile $\boldsymbol{y}$, $\Psi(\boldsymbol{x},\boldsymbol{y}) = 0$ if and only if $\boldsymbol{x}$ is a pure strategy NE of the game. See [99,100] for other properties of this function. Using the Nikaido-Isoda function associated to the prosumers' game we can prove the following theorem:

Theorem 3. *If every prosumer updates its energy request bid based on Algorithm 1, then their action profiles will jointly converge to an pure strategy NE. After t steps the joint actions will be an ϵ-NE where $\epsilon = O(t^{-\frac{1}{4}})$ (i.e., the convergence rate to an NE is $O(t^{-\frac{1}{4}})$).*

Proof. See Appendix A.4. □

As it has been shown in Appendix A.2, the prosumers' game is a *concave game* [101], which is known to admit a distributed leaning algorithm for obtaining its NE points (see, e.g., [101, Theorem 10]). However, in general obtaining distributed learning algorithms with provably fast convergence rates to NE points in concave games is a challenging task. Therefore, one of the main advantages of Theorem 3 that it establishes a polynomial convergence rate for the relaxation Algorithm 1 leveraging rich structure of the prosumers' utility functions.

5.3.3 Finding the Stackelberg Nash Equilibrium under CGT

While Algorithm 1 achieves the unique pure strategy of the prosumers' game under CGT, our final goal is to obtain the Stackelberg equilibrium of the entire market. For this purpose, we leverage Algorithm 1 using one of the following methods to construct the SE of the entire market under CGT:

Method 1

The Stackelberg equilibrium of the game can be found by solving the following non-linear optimization problem. Let $\boldsymbol{x}^*(\rho_{\text{base}})$ be the unique NE obtained by the followers when the power company's action is ρ_{base}. Note that $\boldsymbol{x}^*(\rho_{\text{base}})$ is a well-defined continuous function of ρ_{base}. First, the power company solves the following optimization problem a priori to find its unique optimal action ρ^*_{base} and announces it to the prosumers.The problem is defined as:

$$\begin{aligned}&\max_{\rho_{\text{base}}} U_{\text{pc}}(\boldsymbol{x}^*(\rho_{\text{base}}), \rho_{\text{base}})\\ &s.t. \quad \boldsymbol{x}^*(\rho_{\text{base}}) = \Pi_{\Omega}[\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}^*(\rho_{\text{base}})].\end{aligned} \tag{5.11}$$

We next present a second method, which does not require the power company to solve the non-linear optimization problem in (5.11). In addition, the second method allows the players to reach the SE quickly and efficiently in $1/\epsilon^5$ steps and will be mainly used in our simulation results in Section V.

Method 2

Given any small $\epsilon > 0$ for which the power company and the prosumers want to find their ϵ-SE with precision ϵ (i.e., no one can gain more than ϵ by deviating), the company partitions its action interval and sequentially announces prices $\rho_{\text{base}} = \epsilon, k = 1, \ldots, \lfloor \frac{1}{\epsilon} \rfloor$. For each such price ρ_{base}, prosumers obtain their ϵ-equilibrium in no more than $\frac{1}{\epsilon^4}$ steps, and the company must repeat this process at most $\frac{1}{\epsilon}$ steps and choose the action that maximized

$$U_n^{\text{PT}}(x_n, \bar{x}_{-n}, \rho_{\text{base}}) = \begin{cases} \frac{(c\rho_{\text{max}} + d - R_n)^{\beta^+ + 1} - (c\rho_{\text{min}} + d - R_n)^{\beta^+ + 1}}{c(\beta^+ + 1)\rho_{\text{d}}}, & \text{if } R_n < \rho_{\text{min}} c + d, \\ \frac{(c\rho_{\text{max}} + d - R_n)^{\beta^+ + 1}}{c(\beta^+ + 1)\rho_{\text{d}}} - \lambda_n \frac{(-c\rho_{\text{min}} - d + R_n)^{\beta^- + 1}}{c(\beta^- + 1)\rho_{\text{d}}}, & \text{if } \rho_{\text{min}} c + d < R_n < \rho_{\text{max}} c + d, \\ \frac{\lambda_n(-c\rho_{\text{max}} - d + R_n)^{\beta^- + 1} - \lambda_n(-c\rho_{\text{min}} - d + R_n)^{\beta^- + 1}}{c(\beta^- + 1)\rho_{\text{d}}}, & \text{if } \rho_{\text{max}} c + d < R_n. \end{cases} \tag{5.12}$$

its utility. The running time in this case will be $\frac{1}{\epsilon^5}$ to find an ϵ-SE.

Our analysis thus far assumed that all prosumers are fully rational and their behavior can thus be modeled using CGT. However, this assumption might not hold , given that prosumer are humans that can have different subjective valuations on their uncertain energy trading payoffs. Next, we extend our result using PT [77] to model the behavior of prosumers when faced with the unknown future price of energy and thus the actual value of the stored energy.

5.4 Prospect Theoretic Analysis

In a classical noncooperative game, a player evaluates an objective expected utility. However, in practice, individuals tend to subjectively perceive their utility when faced with uncertainty [77]. In our model, a prosumer's uncertainty originates from the unknown future energy price and power company pricing strategy. Consequently, we will analyze the effect of the key notion of *utility framing* from PT. Utility framing states that a utility is considered a gain if it is larger than the reference point, while it is perceived as a loss if it is smaller than that reference point. This reference captures a prosumer's anticipated profits and originates from past energy trading transactions and future aspirations of profits, which can differ in between different prosumers [89]. Let R_n be the reference point of a given prosumer n. Thus, to capture such subjective perceptions, we use PT framing [89] to redefine the utility function:

$$V\left(C_n\left(\boldsymbol{x}, \rho_{\text{base}}\right)\right) = \begin{cases} \left(C_n(\boldsymbol{x}, \rho_{\text{base}}) - R_n\right)^{\beta^+} & \text{if } C_n(\boldsymbol{x}, \rho_{\text{base}}) > R_n, \\ -\lambda_n \left(R_n - C_n(\boldsymbol{x}, \rho_{\text{base}}\right)^{\beta^-} & \text{if } C_n(\boldsymbol{x}, \rho_{\text{base}}) < R_n, \end{cases} \tag{5.13}$$

where $0 < \beta^- \leq 1, 0 < \beta^+ \leq 1$ and $\lambda \geq 1$.

$V(\cdot)$ is a framing value function, concave in gains and convex in losses with a larger slope for losses than for gains [89]. Experimental studies have shown that the aggravation experienced by an individual for losing a certain amount of money is larger than the satisfaction felt for gaining the same amount [77], which justifies the introduction of the loss multiplier λ_n. In addition, the framing principle states that an individual's sensitivity to marginal change in its utility diminishes as we move further away from the reference point, which explains the introduction of the exponents β^+ and β^-.

The expected utility function of prosumer n, under prospect theory, for a given action profile $\boldsymbol{x}$, is given by (5.12) where $c := W_n + Q_n + x_n - L_n$, $d := -\left(\rho_{\text{base}} + \alpha(x_n + \bar{x}_{-n})\right) x_n$, and $\rho_{\text{d}} := \rho_{\max} - \rho_{\min}$.

5.4.1 Existence and Uniqueness of the NE under PT

To study the existence of the followers' NE under PT, we analyze the concavity of the utility function in (5.12). The concavity of the PT utility function provides a sufficient condition to conclude the existence of at least one pure-strategy NE [101, Theorem 1]. Here, we note that prosumer n's expected utility function can take multiple forms over the product action space Ω, depending on the conditions in (5.12). It is thus challenging to prove that the utility function is concave, which makes it extremely difficult to analyze the existence and uniqueness of the followers' NE. Thus, we inspect a number of conditions under which the PT utility function is concave. Here, for simplicity and to provide more closed-form solutions, we disregard the diminishing sensitivity effect and thus set $\beta^+ = \beta^- = 1$. However, proving uniqueness remains challenging even under these conditions. The following theorem provides sufficient conditions under which the prosumers' game under PT admits a pure strategy NE.

Theorem 4. *In either of the following cases, the prosumers' game under PT admits at least one pure strategy NE:*

- *Case 1:* $\Delta_1 > 0,$ and $x_{r1} < x_{n,\text{min}}, x_{n,\text{max}} < x_{r2}$.
- *Case 2:* $\Delta_2 < 0,$ or $x_{n,\text{max}} < x_{r3},$ or $x_{r4} < x_{n,\text{min}}$.
- *Case 3:* $(\Delta_2 > 0, x_{r3} < x_{n,\text{min}}, x_{n,\text{max}} < x_{r4})$, and $(\Delta_1 < 0,$ or $x_{n,\text{max}} < x_{r1}$ *or* $x_{r2} < x_{n,\text{min}})$ and $\left(x_{\text{max},n} < 1 - \frac{b_1}{a_1}\right)$,

where

$$
\begin{aligned}
&k_n := W_n + Q_n - L_n,\\
&\Delta_1 := (\rho_{\text{min}} - \rho_{\text{base}} - \alpha\bar{x}_{-n})^2 + 4\alpha(k_n\rho_{\text{min}} - R_n),\\
&\Delta_2 := (\rho_{\text{max}} - \rho_{\text{base}} - \alpha\bar{x}_{-n})^2 + 4\alpha(k_n\rho_{\text{max}} - R_n),\\
&x_{r1,r2} := \frac{\pm\sqrt{\Delta_1} + (\rho_{\text{min}} - \rho_{\text{base}} - \alpha\bar{x}_{-n})}{2\alpha},\\
&x_{r3,r4} := \frac{\pm\sqrt{\Delta_2} + (\rho_{\text{max}} - \rho_{\text{base}} - \alpha\bar{x}_{-n})}{2\alpha},\\
&m_1 := 64(W_n + Q_n - L_n),\ a_1 = 48\alpha^2(1-\lambda_n),\\
&b := (176\alpha^2 k_n + 32\alpha(\rho_{\text{base}} - \rho_{\text{max}} + \alpha\bar{x}_{-n}))(1-\lambda_n).
\end{aligned}
$$

Proof. See Appendix A.5. □

As an immediate corollary of Theorem 4, under any of the above conditions one can again obtain the SE of the entire market for PT prosumers using the same procedure used under CGT (i.e., using Algorithm 1 in the prosumers' side together with either of the methods in Subsection 5.3.3). This is simply because under any of the conditions in Theorem 4, the prosumers' game again becomes a concave game which is sufficient for the convergence of Algorithm 1. However, we note that under PT, we do not necessarily have the same Algorithm convergence rates.

Finally, whenever the concavity of the game cannot be guaranteed, we propose a sequential best response algorithm, that build on our previous work in [102]. This is a special case of Algorithm 1, where $x_n(t+1) = x_n^r(t)$, and where players update their strategy sequentially instead of simultaneously. An analytical proof of existence/convergence is challenging, given that no proof for the game's concavity could be derived, as previously

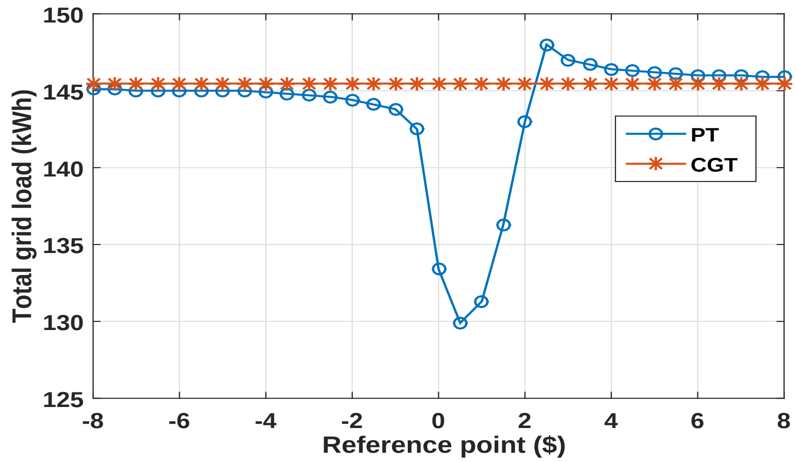

Figure 5.1: Total grid load under expected utility theory and prospect theory.

discussed. However, when it converges, this algorithm is guaranteed to reach an NE. In fact, as observed from our simulations in Section V, the algorithm always converged and found a pure strategy NE, for all simulated scenarios.

5.5 Simulation Results and Analysis

For our simulations, we consider a smart grid with $N = 9$ prosumers, unless otherwise stated, each of which having a load L_n arbitrarily chosen within the range $[10, 30]$ kWh. In addition, the storage capacity $Q_{\text{max},n}$ is set to 25 kWh and $\alpha = 1/N$. β^+ and β^- are taken to be both equal to 0.88 and $\lambda = 2.25$, unless stated otherwise [89]. We set, $\rho_{\text{base}} = \$0.04$ and $R_n = \$1$, unless stated otherwise. When the leader's action is not fixed, Method 2 from Section III-C was used to find the SE.

Fig. 6.2 compares the effect of different prosumer reference points on the total energy sold or bought for both CGT and PT, while fixing the power company's action. For CGT, a prosumer's reference point is naturally irrelevant. For the PT case, for a reference point below $-\$2$, the prosumers' action profile is not significantly affected compared to CGT, since most potential payoffs of the action profile are still viewed as gains, above the reference point. As the reference point increases from $-\$2$ to $\$0.5$, the total energy consumed will decrease from around 145 kWh to 130 kWh, since some of the potential payoffs of the current action profile will start to be perceived as losses, as they cross the reference point. Given that losses have a larger weight under PT compared to CGT, the expected utility of the current strategy profile will significantly decrease thus causing the followers to exhibit a risk-averse behavior. In fact, as some of the potential future profits are perceived as losses, a prosumer will sell more energy at the current time slot. As the reference point increases from $\$0.5$ to $\$2$, the present profits are now perceived as losses, and prosumers will start exhibiting risk-seeking behavior. In fact, each prosumer will consider the present profit as insignificant and will thus store more energy in the hope of selling it in the future at higher prices. Finally, as the reference point approaches \$8, the effect of uncertainty will gradually decrease, given that all profits are now perceived as losses. We note that even a small difference in perception ($\$1.5$) caused the total grid load to shift from 145 kWh to 130 kWh. This highlights the importance of behavioral analysis and prosumer subjectivity when assessing the performance of dynamic pricing strategies.

Fig. 6.3 shows the effect of the loss multiplier λ on the total energy purchased, for a fixed power company strategy. The loss multiplier maps the loss aversion of prosumers when assessing their utility outcomes. The effect of framing is more prominent as the loss multiplier increases. In fact, the prosumers will exhibit more risk averse behavior for a reference point in the range of $[-\$0.5, \$2.5]$. As seen in Fig. 6.3, as λ increase from 2

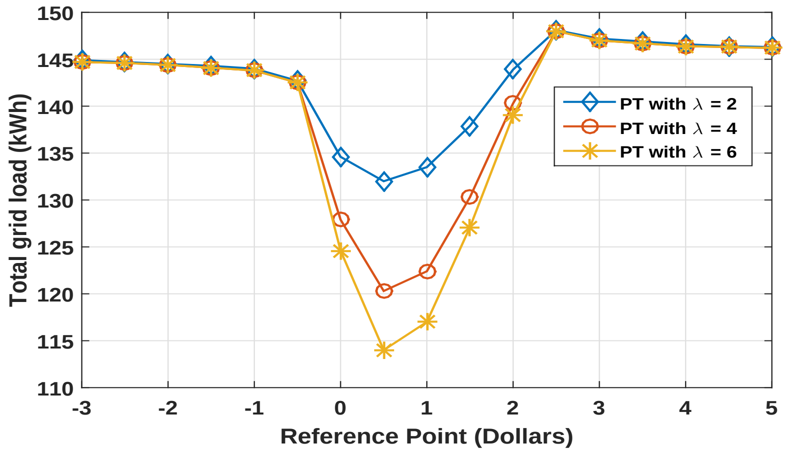

Figure 5.2: Effect of varying the loss multiplier λ.

to 6, the total load would decrease by up to 14%. In fact, to avoid the large losses, the prosumers will decrease the energy they purchase at the current risk free energy price.

Fig. 6.4 compares the company's profits for the scenario in which the power company accounts for prosumer irrationality to the scenario in which the power company assumes that prosumers are rational. In both scenarios, the prosumers are irrational. For a reference point below $\$1$, the company's profits are barely affected. However, as the reference point crosses $\$1$, the company's profits start to show a clear decrease between the two scenarios. In fact, as the power company is not accounting for the prosumer's actual subjective behavior, its pricing strategy is no longer optimal. As was seen in Fig. 6.2, this is the reference point range where the total consumption mostly differs between CGT and PT. The decrease in profits reaches a peak value of 15 % at a reference point of \$2. Clearly, the power company will experience a decrease in profits, if it neglects the subjective perception of prosumers.

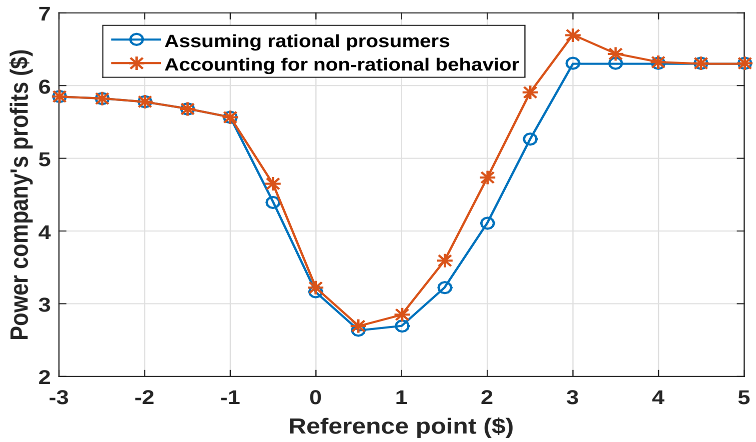

Figure 5.3: Power company's profits

Fig. 5.4 shows the total grid load energy consumption as function of the number of prosumers. The figure highlights the difference in consumption between rational prosumers and subjective prosumers with $R_n = \$1$, which increases significantly with the number of prosumers in the grid. This difference reaches 100 kWh for 50 prosumers. This highlights the impact of irrational behavior, which is prominent for larger grids.

Fig. 5.5 shows the energy consumption of different groups of prosumers, with different reference points, inside a single grid. For a very small ρ_{base}, the different groups have equal consumption. As ρ_{base} is increased to -5 cents, the prosumers with $R_n = \$1$ start to decrease their consumption at equilibrium, while the other groups' consumption remains unchanged. This is similar to what was discussed in Fig 6.2, where prosumers with reference points close to $\$1$, exhibit risk averse behavior and thus lower energy consumption. On the other hand, rational prosumers will start decreasing their consumption at $\rho_{\text{base}} = 2$ cents, while risk seeking prosumers ($R_n = \$3$) will start decreasing their consumption at $\rho_{\text{base}} = 5$ cents.

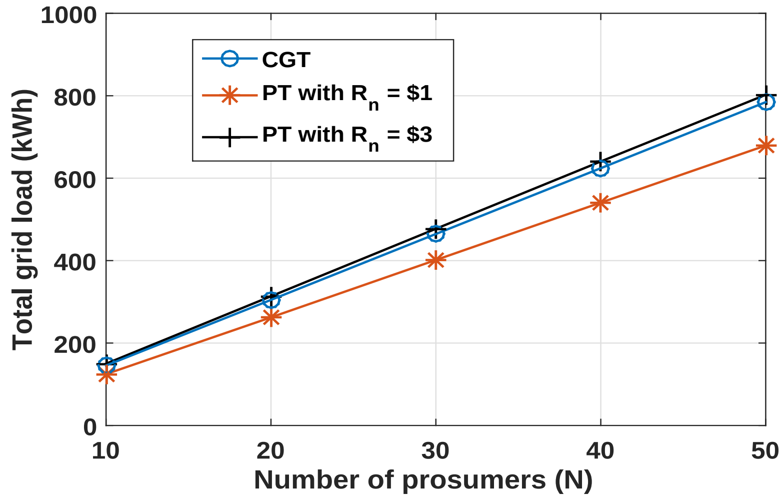

Figure 5.4: Total grid load for different number of prosumers

Fig. 6.1 shows the number of iterations needed for the best response algorithm to converge to a followers' NE for different number of prosumers, under PT. Clearly, the best response algorithm converges, for all these cases. In addition, the number of iterations needed for convergence is reasonable, even as the number of prosumers significantly increases from 10 to 70.

5.6 Conclusion

In this chapter, we have proposed a novel framework for analyzing energy trading of prosumers with the power grid, while accounting for the uncertainty of the future price of energy. We have formulated the problem as a Stackerberg game between the power company (leader), seeking to maximize its profits by setting its optimal pricing strategy, and multiple prosumer (followers), attempting to choose the optimal amount of energy to trade.

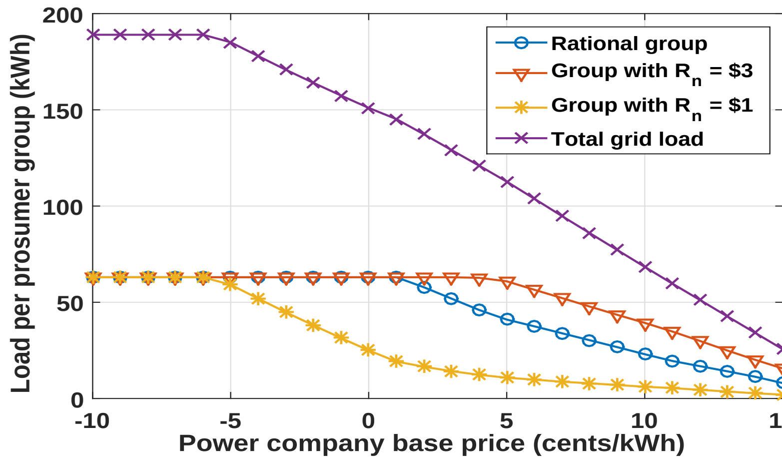

Figure 5.5: Load of different groups in a single grid.

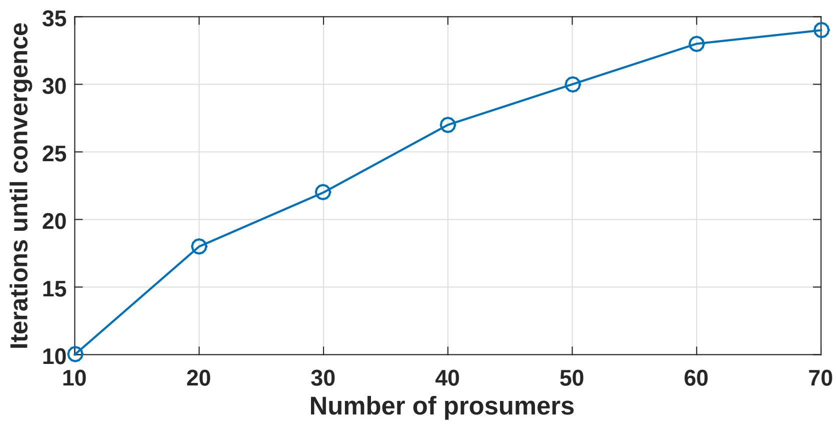

Figure 5.6: Number of iterations for convergence

The prosumers game was shown to have a unique pure strategy Nash equilibrium under classical game theoretic analysis. Subsequently, we have used the novel concept of utility framing from prospect theory to model the behavior of prosumers when faced with the uncertainty of future energy prices. Simulation results have highlighted the impact of behavioral considerations on the overall energy trading process.

Chapter 6

Enhanced Smart Grid Resilience Using Microgrid Energy Storage

The proliferation of distributed generation and storage units is leading to the development of local, small-scale distribution grids, known as microgrids. In this chapter, the problem of optimizing the energy trading decisions of MG operators is studied using game theory. In the formulated game, each MGO chooses the amount of energy that must be sold immediately or stored for future emergencies, given the prospective market prices which are influenced by other MGOs' decisions. The problem is modeled using a Bayesian game to account for the incomplete information that MGOs have about each others' levels of surplus. The proposed game explicitly accounts for each MGO's subjective decision when faced with the uncertainty of its opponents' energy surplus. In particular, the so-called framing effect, from the framework of prospect theory, is used to account for each MGO's valuation of its gains and losses with respect to an individual utility reference point. The reference point is typically different for each individual and originates from its past experiences and future aspirations. A closed-form expression for the Bayesian Nash equilibrium is derived for the standard game formulation. Under PT, a best response algorithm is proposed to find the equilibrium. Simulation results show that, depending on their individual reference points, MGOs can tend to store more or less energy under PT compared to classical game theory. In addition, the impact of the reference point is found to be more prominent as the emergency price set by the power company increases.

6.1 Introduction

The recent increase in the installation of microgrid systems has accelerated the evolution of the power grid into a distributed generation infrastructure, compared to the traditional centralized generation structure [103]. The autonomous nature of microgrids, who are capable of operating while connected to the macrogrid as well as in island mode, would ultimately result in a more resilient overall power system. In fact, there has been a number of recent reports encouraging and analyzing the use of microgrid storage devices to enhance grid resilience, by supplying the power grid's ciritical loads in case of power outage, as discussed in Chapter 2. However, in order to fully leverage the storage capability of microgrids, sufficient incentives must be offered to microgrid operators, to encourage them into storing part of their energy excess, to be used in case of blackout. Furthermore, given that microgrid operators are human decision makers, their irrational behavior must be accounted for, in order to insure that any proposed energy storing incentive reaches its goal, and insures continuous power supply to the critical loads.

Thus, in this chapter, a novel resilience enhancement scheme is proposed to incentivize microgrid operators into storing part of their energy excess, to supply the power grid's critical loads, in case of emergency. This scheme is modeled by a Bayesian game model, to accurately study the energy trading decisions of microgrid operators, given their incomplete information regarding each others' energy surplus.

6.2 System Model and Bayesian Game Formulation

Consider a large-scale smart grid managed by a power company that integrates a set $\mathcal{N}$ of N microgrids, each of which is managed by an MG operator. Microgrids are small-scale distribution grids which typically include renewable generation units, storage devices, and energy consumers. Each MG operator manages all energy trades conducted by its own MG. Each MG $n \in \mathcal{N}$, managed by its MGO n, includes a storage unit with capacity $Q_{n,\text{max}}$ which can be used to store the excess of energy produced. Given the intermittent nature of renewable energy sources, each MG's energy surplus $Q_n \in [0, Q_{n,\text{max}}]$ is unknown beforehand and will vary over time. A positive Q_n indicates that an MG has extra energy while $Q_n = 0$ indicates that no surplus is available. Given an amount of energy surplus, Q_n, an MGO n has the option of selling this stored energy to the grid at the corresponding retail price, ρ, or saving it for later use in case of emergency, for improved resilience. In this regard, each MGO will choose a portion $\alpha_n \in [0, 1]$ of its MG's Q_n to store and will consequently sell the rest. In case of emergency or blackout, the power company will purchase the stored energy to cover a certain required critical load L_c, until normal power supply is restored.

In order to increase the resilience of the power grid against emergency events, the power company will encourage the MGOs to store part of their MGs' excess by offering a price ρ_c per unit of stored energy purchased in case of emergency. Typically, ρ_c must be significantly larger than ρ to incentivize the MGOs to store the excess. If the total stored energy exceeds the needed L_c, the power company will no longer purchase the entire energy stored by each MG.

Let $\boldsymbol{\alpha}$ and $\boldsymbol{Q}$ be the vectors that represent, respectively, the storage strategy and the available energy surpluses of all the MGOs in the set $\mathcal{N}$. In this respect, when $\boldsymbol{\alpha}^\intercal \boldsymbol{Q} > L_c$, the power company will purchase, from each MG n, an amount of energy D_n given by:

$$D_n = \left(\alpha_n Q_n - \frac{\boldsymbol{\alpha}^\intercal \boldsymbol{Q} - L_c}{N} \right)^+, \tag{6.1}$$

where $(q)^+ = \max(0, q)$. $\boldsymbol{\alpha}^\intercal \boldsymbol{Q} - L_c$ is the amount by which the total stored energy exceeds the required L_c. Let θ be the expected probability of an emergency event occurring. Then, each MGO n will choose its optimal storage strategy α_n to optimize the following utility function:

$$U_n(\boldsymbol{\alpha}, \boldsymbol{Q}) = \begin{cases} \rho\left(Q_n - \alpha_n Q_n\right) + \theta \rho_c \alpha_n Q_n, & \text{if } \boldsymbol{\alpha}^\intercal \boldsymbol{Q} \leq L_c, \\ \rho\left(Q_n - \alpha_n Q_n\right) + \theta \rho_c D_n, & \text{otherwise.} \end{cases} \tag{6.2}$$

Note that, when $\theta \rho_c < \rho$, the MGOs will have no incentive to store their MGs' excess and, hence, they will sell all the available surplus at the current market price. Thus, hereinafter, we restrict our analysis to the case $\theta\rho_c > \rho$. As seen from (6.2), the driving factor in determining an MGO's optimal strategy is the total energy stored by its opponents. In fact, as $\boldsymbol{\alpha}^\intercal \boldsymbol{Q} - L_c$ increases, so will the amount of stored energy which will not be bought in case of emergency. Indeed, the MGO could have instead sold that energy at the current market price and made a profit. Given this trade-off between selling at the current market price and storing the excess for a potentially higher profit in case of emergency, each MGO aims at maximizing its utility function by choosing the optimal storage strategy α_n, while also accounting for the actions of its opposing MGs.

Each MGO is typically fully aware of the presence of all N MGs in the power grid and knows the size of their storage devices. In addition, each MGO knows the exact amount of energy excess available to its own MG. However, an MGO cannot determine the energy excess of other MGs. In fact, obtaining such information is not possible especially given the intermittent renewable energy sources and the time-varying nature of energy consumption. Each MGO thus assumes the excess of energy Q_m of other MGs to be a random variable that follows a certain probability distribution function $f_n(Q_m)$ over $[0, Q_{m,\max}]$ where $m \in \mathcal{N} \setminus \{n\}$. We refer to Q_n as the *type* of MGO n and, to $f_n(Q_m)$, as MGO n's *belief* of another MGO m's type. In fact, when MGO n chooses a certain storage strategy α_n, it is uncertain of the profit it will gain. This uncertainty stems from its incomplete information regarding the type of its opponents, originating from the intermittent renewable energy and the time-varying nature of energy consumption, as well as from randomness of an emergency event.

Given the competition over the financial incentives offered by the power company for emergency energy, the MGOs' actions and utility are highly interdependent thus motivating a game-theoretic approach [82]. In addition, given the incomplete information of the opponents' excess of energy that directly affects the MGOs' utility, each MGO will maximize its expected utility given its own beliefs $f_n(Q_m)$. MGO n's expected utility, $E_n(\boldsymbol{\alpha}, Q_n)$, will therefore be given by

$$E_n(\boldsymbol{\alpha}, Q_n) = \mathbb{E}_{\boldsymbol{Q}_{-n}}\left[U_n(\boldsymbol{\alpha}, \boldsymbol{Q})\right], \tag{6.3}$$

where $\boldsymbol{Q}_{-n}$ is the vector that represents the energy excess of all MGs in the set $\mathcal{N} \setminus \{n\}$. The strategic interactions between the various MGOs under incomplete information can be modeled using Bayesian game models [82].

6.2.1 Bayesian Game Formulation

We formulate a static noncooperative Bayesian game [82] between the different MGOs in the set $\mathcal{N}$. In this game, each MGO seeks to maximize its expected utility given its beliefs of its opponents' energy excess by choosing its optimal storage strategy. Since the decisions on the portion of energy to store are coupled, as captured by (6.2), we adopt a game-theoretic approach. Formally, we define a strategic game
$\Xi = \{\mathcal{N}, \{\mathcal{A}_n\}_{n\in\mathcal{N}}, \{\mathcal{T}_n\}_{n\in\mathcal{N}}, \{\mathcal{F}_n\}_{n\in\mathcal{N}}, \{U_n\}_{n\in\mathcal{N}}\}$ where $\mathcal{N}$ is the set of all MGOs, $\mathcal{A}_n$ is the action space which represents the possible storage strategies of each player n, $\mathcal{T}_n$ is the set of types of MGOs that represent the possible energy surplus for each their MGs, $\mathcal{F}_n$ is the set of beliefs of player n represented by the probability distributions of each of its

opponents' types, and U_n is the utility function of player n defined in (6.2). In order to find the solution of the proposed game, we first define the two key concepts of *best response strategy* and *Bayesian Nash equilibrium*.

Definition 7. *The set of* best response strategies *of an MGO $n \in \mathcal{N}$ to the strategy profile $\boldsymbol{\alpha}_{-n}$, $r(\boldsymbol{\alpha}_{-n})$, is defined as*

$$r_n(\boldsymbol{\alpha}_{-n})=\{\alpha_n^* \in \mathcal{A}_n | \mathbb{E}_{\boldsymbol{Q}_{-n}}\left[U_n(\alpha_n^*, \boldsymbol{\alpha}_{-n}, \boldsymbol{Q})\right] \geq \mathbb{E}_{\boldsymbol{Q}_{-n}}\left[U_n(\alpha_n, \boldsymbol{\alpha}_{-n}, \boldsymbol{Q})\right], \forall \alpha_n \in \mathcal{A}_n\},$$

where $\boldsymbol{\alpha}_{-n}$ is the vector that represents the storage strategy of all MGOs in the set $\mathcal{N} \setminus \{n\}$.

In other words, when the strategies of the opponents are fixed to $\boldsymbol{\alpha}_{-n}$, any best response strategy would maximize player n's expected utility, given its beliefs $\mathcal{F}_n$ of its opponents' types. In our analysis, we assume that an MGO's belief $f_n(Q_m)$ over its opponent's energy surplus follows a uniform distribution over the domain $[0, Q_{m,\max}]$. We next define the concept of a pure strategy Bayesian Nash equilibrium.

Definition 8. *A strategy profile $\boldsymbol{\alpha}^*$ is said to be a* pure strategy Bayesian Nash equilibrium *if every MGO's strategy is a best response to the other MGOs' strategies, i.e.*

$$\alpha_n^* \in r_n(\boldsymbol{\alpha}_{-n}^*)\, \forall n \in \mathcal{N}. \tag{6.4}$$

In the proposed game, at the BNE, no MGO n, can increase its expected utility by unilaterally deviating from its storage strategy α_n^*.

In what follows, we will derive closed-form expressions of the BNEs for the case in which two MGs are located in the proximity of the critical load. In fact, power supply to the critical load from distant MGs might not be feasible due to transmission barriers and significant power losses. As such, given these limitations and the scale of a given microgrid, the analysis for two MGs will be quite representative.

6.3 Two-player Game Solution under Classical Game-Theoretic Analysis

For the case in which two MGs ($N = 2$) are capable of supplying the critical load, the expected utility of MGO 1 given its belief of MGO 2's type can be written as

$$E_1(\boldsymbol{\alpha}, Q_1) = \int_0^{Q_{2,\max}} U_1(\boldsymbol{\alpha}, \boldsymbol{Q}) f_1(Q_2) dQ_2, \tag{6.5}$$

where $\boldsymbol{\alpha} = [\alpha_1 \ \alpha_2]$ and $\boldsymbol{Q} = [Q_1 \ Q_2]$. For the two-MG case, we have

$$U_1(\boldsymbol{\alpha}, \boldsymbol{Q}) = \begin{cases} \rho Q_1 (1 - \alpha_1) + \theta \rho_c \alpha_1 Q_1 & \text{if } \alpha_2 \leq \frac{L_c - \alpha_1 Q_1}{Q_2}, \\ \rho Q_1 (1 - \alpha_1) + \theta \rho_c D_1 & \text{otherwise.} \end{cases} \tag{6.6}$$

Next, we assume that neither of the MGs owns a large enough storage device to fully supply the critical load on its own. Under this assumption, D_1 will be given by

$$D_1 = \alpha_1 Q_1 - \frac{1}{2} (\alpha_1 Q_1 + \alpha_2 Q_2 - L_c). \tag{6.7}$$

In order to find the solution of the proposed game, we first derive the best response strategy of each player which we then use to compute the different BNEs.

6.3.1 Derivation of the Best Response

The best response strategy of each MGO is characterized next. In fact, we present the following propositions that analyze MGO 1's best response for different values of α_2.

Proposition 1. *The best response of MGO* 1*, for* $\alpha_2 \in \left[0, \frac{L_c - Q_1}{Q_{2,max}}\right]$*, is given by* $r_1(\alpha_2) = 1$*. MGO* 1 *thus maximizes its expected utility by storing its MG's entire energy excess.*

Proof. For $\alpha_2 \leq \frac{L_c - Q_1}{Q_{2,\max}}$, the total stored energy is below the critical load for all types of MGO 2 and all strategies of MGO 1 since $\alpha_2 Q_{2,\max} + Q_1 \leq L_c$. Thus, MGO 1's best response is to store its entire energy excess which is fully sold in case of emergency. In fact, here, $E_1(\boldsymbol{\alpha}, Q_1) = U_1(\boldsymbol{\alpha}, \boldsymbol{Q}) = \rho\left(Q_1 - \alpha_1 Q_1\right) + \theta \rho_c \alpha_1 Q_1$ since $U_1(\boldsymbol{\alpha}, \boldsymbol{Q})$ is independent of Q_2 for this case, as seen in (6.6). $E_1(\boldsymbol{\alpha}, Q_1)$ is clearly an increasing function, given that $\rho_c\theta > \rho$, which is maximized at its upper boundary ($\alpha_1 = 1$). Thus $r_1(\alpha_2) = 1$ for $\alpha_2 \in \left[0, \frac{L_c - Q_1}{Q_{2,\max}}\right]$. $\square$

Proposition 2. *The best response of MGO* 1*, for* $\alpha_2 \in \left[\frac{L_c - Q_1}{Q_{2,max}}, 1\right]$*, is given by:*

$$r_1(\alpha_2) = \begin{cases} \frac{L_c \rho_c \theta + (\rho_c \theta - 2\rho)\alpha_2 Q_{2,max}}{Q_1 \rho_c \theta}, & \textit{if } \left[\frac{2\rho}{\rho_c \theta} - 1\right] \alpha_2 > \frac{L_c - Q_1}{Q_{2,max}}, \\ 1, & \textit{if } \left[\frac{2\rho}{\rho_c \theta} - 1\right] \alpha_2 \leq \frac{L_c - Q_1}{Q_{2,max}}. \end{cases} \tag{6.8}$$

Proof. The proof of this proposition is in Appendix B.1. $\square$

Given the previous propositions, an MGO's best response strategy is thus summarized in the following theorem.

Theorem 5. *The best response strategy of MGO* 1*,* $r_1(\alpha_2)$*, is given by*

$$r_1(\alpha_2) = \begin{cases} 1, & \textit{if}\ \alpha_2 \leq \frac{L_c - Q_1}{Q_{2,max}}, \\ \alpha_{1,r}, & \textit{if}\ \alpha_2 > \frac{L_c - Q_1}{Q_{2,max}}\ \textit{and}\ \left[\frac{2\rho}{\rho_c\theta} - 1\right]\alpha_2 > \frac{L_c - Q_1}{Q_{2,max}}, \\ 1, & \textit{if}\ \alpha_2 > \frac{L_c - Q_1}{Q_{2,max}}\ \textit{and}\ \left[\frac{2\rho}{\rho_c\theta} - 1\right]\alpha_2 \leq \frac{L_c - Q_1}{Q_{2,max}}. \end{cases} \tag{6.9}$$

MGO 2*'s best response strategy* $r_2(\alpha_1)$ *is derived similarly and is the same as (6.9) but with indices* 1 *and* 2 *interchanged.*

Proof. The proof follows from Propositions 1 and 2. □

6.3.2 Derivation and Interpretation of the Equilibria

Given the MGOs' best response function in (6.9), we will compute all possible BNEs for this game. We will then derive and interpret the conditions needed for each BNE to exist.

Theorem 6. *The proposed MGO game admits four possible Bayesian Nash equilibria for different conditions that relate the MG parameters,* Q_n *and* $Q_{n,max}$*, with power grid parameters* ρ, ρ_c, θ*, and* L_c*. The strategy profiles* (α_1^*, α_2^*)*, that constitute the four BNEs, are the following:*

1) First BNE: (1,1).

2) Second BNE: $\left(1, \dfrac{L_c\rho_c\theta + (\rho_c\theta - 2p)Q_{1,max}}{Q_2\rho_c\theta}\right)$.

3) Third BNE: $\left(\dfrac{L_c\rho_c\theta + (\rho_c\theta - 2p)Q_{2,max}}{Q_1 p_c\theta}, 1\right)$.

4) Fourth BNE: $\left(\alpha^*_{1,4}, \alpha^*_{2,4}\right)$ *is the strategy profile that constitute the fourth BNE, where*

$$\alpha^*_{1,4} = \frac{-L\rho_c\theta(Q_2\rho_c\theta - 2Q_{2,max}\rho + Q_{2,max}\rho_c\theta)}{Q_{1,max}Q_{2,max}\left(4\rho^2 + \rho_c^2\theta^2 - 4\rho\rho_c\theta\right) - Q_1Q_2\rho_c^2\theta^2},$$
$$\alpha^*_{2,4} = \frac{-L\rho_c\theta(Q_1\rho_c\theta - 2Q_{max,1}\rho + Q_{1,max}\rho_c\theta)}{Q_{1,max}Q_{2,max}\left(4\rho^2 + \rho_c^2\theta^2 - 4\rho\rho_c\theta\right) - Q_1Q_2\rho_c^2\theta^2}.$$

Proof. The strategy profiles of the BNEs are derived by solving the set of best-response equations, $\alpha^*_1 = r_1(\alpha^*_2)$ and $\alpha^*_2 = r_2(\alpha^*_1)$, for the different possible combinations of the best response strategies. □

The conditions under which each BNE is defined are further summarized and interpreted next.

First BNE

the strategy profile (1,1) constitutes a BNE of the proposed game if any of the following four conditions is satisfied:

a) $L_c \geq Q_{2,\max} + Q_1$ and $L_c \geq Q_{1,\max} + Q_2$. Here, each MGO is aware that the total stored energy is below the critical load, regardless of the type and strategy of its opponent.

b) $L_c \geq Q_{2,\max} + Q_1$ and $\frac{2\rho}{\rho_c\theta} - 1 \leq \frac{L_c - Q_2}{Q_{1,\max}} < 1$. Here, MGO 1 knows that the total stored energy is always below the critical load regardless of the type and strategy of its opponent. On the other hand, MGO 2 is aware that part of its MG's stored energy might not be sold in case of emergency. However, ρ_c is large enough compared to ρ to satisfy the condition under which MGO 2 stores its MG's entire excess.

c) $\frac{2\rho}{\rho_c\theta} - 1 \leq \frac{L_c - Q_1}{Q_{2,\max}} < 1$ and $L_c \geq Q_{1,\max} + Q_2$. The analysis of this condition is the same as condition b) with the order of the players reversed.

d) $\frac{2\rho}{\rho_c\theta} - 1 \leq \frac{L_c - Q_1}{Q_{2,\max}} < 1$ and $\frac{2\rho}{\rho_c\theta} - 1 \leq \frac{L_c - Q_2}{Q_{1,\max}} < 1$. In this case, both MGOs are aware that part of their stored energy might not be sold. However, ρ_c is large enough compared to ρ to satisfy the conditions for which both MGOs store their MGs' entire excess.

Second BNE

the strategy profile $\left(1, \frac{L_c\rho_c\theta + (\rho_c\theta - 2p)Q_{1,\max}}{Q_2\rho_c\theta}\right)$ constitutes a BNE of the proposed game if any of the following two conditions are satisfied:

a) $L_c \geq \frac{L_c\rho_c\theta + (\rho_c\theta - 2p)Q_{1,\max}}{Q_2\rho_c\theta} Q_{2,\max} + Q_1$ and

$\frac{2\rho}{\rho_c\theta} - 1 > \frac{L_c - Q_2}{Q_{1,\max}}$. In this case, MGO 1 knows that given MGO 2's storage strategy, the total stored energy is always below the critical load. Meanwhile, MGO 2 is aware that, given MGO 1's strategy, the total stored energy might exceed the critical load and part of its stored energy might not be sold in case of emergency. MGO 2 will not store the entire excess given that ρ_c is not large enough compared to ρ.

b) $\left[\frac{2\rho}{\rho_c\theta} - 1\right] \frac{L_c\rho_c\theta + (\rho_c\theta - 2\rho)Q_{1,\max}}{Q_2\rho_c\theta} \leq \frac{L_c - Q_1}{Q_{2,\max}}$,

$\frac{L_c - Q_1}{Q_{2,\max}} < \frac{L_c\rho_c\theta + (\rho_c\theta - 2\rho)Q_{1,\max}}{Q_2\rho_c\theta}$ and $\frac{2\rho}{\rho_c\theta} - 1 > \frac{L_c - Q_2}{Q_{1,\max}}$. Here, both MGOs know that given their opponent's strategy, part of their MG's stored energy might not be sold. The emergency price ρ_c is large enough compared to ρ to satisfy the condition for which MGO 1 stores the entire excess, however, it is not large enough for MG 2 to fully store its MG's entire excess.

Third BNE

The interpretation of the third BNE is similar to that of the second but with index 1 swapped with 2.

Fourth BNE

The strategy profile $\left(\alpha^*_{1,4}, \alpha^*_{2,4}\right)$, defined in Theorem 2, constitutes a BNE which is obtained by solving the set of equations $\alpha^*_1 = \alpha_{1,r}$ and $\alpha^*_2 = \alpha_{2,r}$, in the case where the following condition is satisfied:

a) $\alpha^*_{2,4}\left[\frac{2\rho}{\rho_c\theta} - 1\right] > \frac{L_c - Q_1}{Q_{2,\max}}$ and $\alpha^*_{1,4}\left[\frac{2\rho}{\rho_c\theta} - 1\right] > \frac{L_c - Q_2}{Q_{1,\max}}$.

Under this condition, both MGOs know that given their opponent's strategy, part of their MG's stored energy might not be sold. The emergency price ρ_c is not large enough to satisfy the conditions under which either MGO stores the entire excess.

Our previous analysis assumes that all MG operators are fully rational and their behavior can thus be modeled using classical game-theoretic analysis. However, this assumption might not hold true in a real smart grid, given that the operators of the MGs might have different subjective valuations of the payoffs gained from selling their energy surplus. Next, we will use the framework of prospect theory [77] to model the behavior of MGOs when faced with such uncertainty and subjectivity of profits, stemming from the presence of renewable energy and the uncertainty it imposes on the volume of energy surplus that other MGOs generate.

6.4 Prospect Theoretic Analysis

In a classical noncooperative game, a player evaluates an objective expected utility. However, in practice, individuals tend to subjectively perceive their utility when faced with uncertainty [77]. In our model, an MGO's uncertainty originates from the presence of renewable energy and the uncertainty it imposes on the volume of energy surplus that the opposing MGOs generate. In fact, an MGO is uncertain of the portion of its MG's stored energy that will be sold in case of emergency, which is directly related to the energy surplus available to its opponents. Since MGOs are humans, they will perceive the possible profits of energy trading, in terms of gains and losses.

This motivates the application of PT to account for the MGO's subjectivity while choosing the optimal energy portion to store. PT is a widely used tool for understanding human behavior when faced with uncertainty of alternatives. In our analysis, we will inspect the effect of the key notion of utility framing from prospect theory. Utility framing states that a utility is considered a gain if it is larger than the reference point, while it is perceived as a loss if it is smaller than that reference point. We define R_n as the reference point of a given MGO n. The choice of R_n can be different between MGOs as it reflects personal expectations of profit from selling the energy surplus. In this regard, a certain profit, r, originating from a particular energy trade, will be perceived differently by an MGO used to reaping larger profits as opposed to an MGO that usually generates lower profits. In fact, an MGO n with historically high profits would have a high reference point, $R_n > r$, and will hence consider r to be a loss, whereas, an MGO m with relatively low historical profits would have a lower reference point, $R_m < r$ and would hence consider r to be a gain. Consequently, to model this subjective perception of losses and gains we need to redefine the utility function of the MGOs using PT framing [89]:

$$V\left(U_n\left(\boldsymbol{\alpha},\boldsymbol{Q}\right)\right)=\begin{cases}\left(U_n(\boldsymbol{\alpha},\boldsymbol{Q})-R_n\right)^{\beta^+} & \text{if } U_n(\boldsymbol{\alpha},\boldsymbol{Q})>R_n,\\ -\lambda_n\left(R_n-U_n(\boldsymbol{\alpha},\boldsymbol{Q})\right)^{\beta^-} & \text{if } U_n(\boldsymbol{\alpha},\boldsymbol{Q})<R_n,\end{cases} \tag{6.10}$$

where $0<\beta^-\leq 1, 0<\beta^+\leq 1$ and $\lambda\geq 1$.

$V(\cdot)$ is the framing value function that is concave in gains and convex in losses with a larger slope for losses than for gains [89]. In fact, PT studies show that the aggravation that an individual feels for losing a sum of money is greater than the satisfaction associated with gaining the same amount [77], which explains the introduction of the loss multiplier λ_n. In addition, the framing principle states that an individual's sensitivity to marginal change in its utility diminishes as we move further away from the reference point, which explains the introduction of the gain and loss exponents β^+ and β^-.

It is important to note that, as an MGO chooses to store a larger portion α of its MG's energy, its potential payoffs will now span a larger range of values. In other words, as an MGO stores more energy, it will now have the possibility to make higher expected profits by selling more in case of emergency. On the other hand, by storing more energy, the

MGO risks making less profit whenever its opponent has also stored a significant part of its own energy. These probable payoffs are related to the type of the opponent. In fact, the MGO would get a maximum profit for the case in which the opponent's type is small, i.e. the opponent did not have a significant energy surplus. For the case in which the opponent's type is large, a significant part of an MG's stored energy will not be sold in case of emergency, resulting in lower possible payoffs for its MGO, compared to smaller values of α. This concept is key in our PT analysis, given that payoffs are evaluated through comparison to the reference point. Similarly to our analysis for the CGT case, we will first derive the best response strategy of the MGOs.

Proposition 3. *The best response of MGO* 1 *under PT, for* $\alpha_2 \in \left[0, \frac{L_c - Q_1}{Q_{2,max}}\right]$*, is to store its entire energy excess, similarly to the classical game theory analysis.*

Proof. As seen from Proposition 1, for $\alpha_2 \in \left[0, \frac{L_c - Q_1}{Q_{2,\max}}\right]$, $U_1(\boldsymbol{\alpha}, \boldsymbol{Q})$ is an increasing function over its domain. Given that the framing function $V(\cdot)$ is an increasing function as well, MGO 1's expected utility, $E_{1,\mathrm{PT}}(\boldsymbol{\alpha}, Q_1) = V\left(U_1\left(\boldsymbol{\alpha}, \boldsymbol{Q}\right)\right)$, is thus maximized at its upper boundary of $\alpha_1 = 1$. $\square$

We next derive the expected utility of MGO 1 under PT for $\alpha_2 \in \left[\frac{L_c - Q_1}{Q_{2,\max}}, 1\right]$. MG 1's expected utility for $\alpha_2 \in \left[\frac{L_c - Q_1}{Q_{2,\max}}, 1\right]$ takes different values for $\alpha_1 \in \left[0, \frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}\right]$ and $\alpha_1 \in \left[\frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}, 1\right]$:

Proposition 4. *For* $\alpha_1 \in \left[0, \frac{L_c - \alpha_2 Q_{2,max}}{Q_1}\right]$ *and* $\alpha_2 \in \left[\frac{L_c - Q_1}{Q_{2,max}}, 1\right]$*, MGO 1's expected utility under PT,* $E_{PT,1,2a}$*, is given by*

$$E_{PT,1,2a}(\boldsymbol{\alpha}, Q_1) = \begin{cases} -\lambda_1 \left(R_1 - U_{1,2a}\right)^{\beta_1^-} & \textit{if } \alpha_1 \leq B, \\ \left(U_{1,2a} - R_1\right)^{\beta_1^+} & \textit{if } \alpha_1 > B, \end{cases} \tag{6.11}$$

where $U_{1,2a} = \rho\left(Q_1 - \alpha_1 Q_1\right) - \theta\rho_c\alpha_1 Q_1$, *and* $B = \frac{R_1 - \rho Q_1}{Q_1(\rho_c\theta - \rho)}$.

Proof. In Proposition 4, Equation (6.11) follows from the fact that for $\alpha_1 \leq B$, the original utility, $U_{1,2a}$, is below the reference point R_1 and is thus perceived as a loss. On the other hand, it is considered as a gain for $\alpha_1 > B$. $\square$

Proposition 5. *For* $\alpha_1 \in \left[\frac{L_c - \alpha_2 Q_{2,max}}{Q_1}, 1\right]$ *and* $\alpha_2 \in \left[\frac{L_c - Q_1}{Q_{2,max}}, 1\right]$*, player 1's expected utility under PT is given by*

$$E_{PT,1,2b}(\boldsymbol{\alpha}, Q_1) = I_1 + I_2, \tag{6.12}$$

where

$$I_1 = \begin{cases} -\dfrac{\lambda_1(L_c - \alpha_1 Q_1)}{\alpha_2 Q_{max,2}} \left[R_1 - U_{I,1}\right]^{\beta_1^-} & \textit{if } \alpha_1 \leq B, \\ \dfrac{L_c - \alpha_1 Q_1}{\alpha_2 Q_{max,2}} \left[U_{I,1} - R_1\right]^{\beta_1^+} & \textit{if } \alpha_1 > B, \end{cases} \tag{6.13}$$

$$U_{I,1} = \rho\left(Q_1 - \alpha_1 Q_1\right) + \theta\rho_c\alpha_1 Q_1, \tag{6.14}$$

$$I_2 = \begin{cases} M_l \left[(R_1 - U_{max,2})^{\beta_1^- + 1} - (R_1 - U_{A,2})^{\beta_1^- + 1}\right] & \textit{if } C_1, \\ M_g \left[(U_{r,2} - R_1)^{\beta_1^+ + 1} - (U_{A,2} - R_1)^{\beta_1^+ + 1}\right] + & \\ M_l \left[(R_1 - U_{max,2})^{\beta_1^- + 1} - (R_1 - U_{r,2})^{\beta_1^- + 1}\right] & \textit{if } C_2, \\ & \\ M_g \left[(U_{max,2} - R_1)^{\beta_1^+ + 1} - (U_{A,2} - R_1)^{\beta_1^+ + 1}\right] & \textit{if } C_3, \end{cases} \tag{6.15}$$

$M_g = \dfrac{-2}{\left(\beta_1^+ + 1\right)\rho_c\theta\alpha_2}$, $M_l = \dfrac{-2\lambda_1}{\left(\beta_1^- + 1\right)\rho_c\theta\alpha_2}$, $U_{max,2} = \rho\left(Q_1 - \alpha_1 Q_1\right) + \frac{1}{2}\theta\rho_c\left(\alpha_1 Q_1 + L_c - Q_{2,max}\right)$, $U_{A,2} = \rho\left(Q_1 - \alpha_1 Q_1\right) + \frac{1}{2}\theta\rho_c\left(\alpha_1 Q_1 + L_c - A\right)$, $A = \frac{L_c - \alpha_1 Q_1}{\alpha_2}$, *and* $U_{r,2} = \rho\left(Q_1 - \alpha_1 Q_1\right) + \frac{1}{2}\theta\rho_c\left(\alpha_1 Q_1 + L_c - Q_{2,r}\right)$. $Q_{2,r}$ *is given in (B.6).*

Condition C_1, C_2, *and* C_3 *are given by*

$$C_1 : \alpha_1 \le B, \tag{6.16}$$

$$C_2 : \alpha_1 > B \quad \textit{and} \quad Q_1\left(\theta\rho_c - 2\rho\right)\alpha_1 \le \theta\rho_c\alpha_2 Q_{max,2} - L_c\rho_c\theta - 2\rho Q_1 + 2R_1, \tag{6.17}$$

$$C_3 : \alpha_1 > B \quad \textit{and} \quad Q_1\left(\theta\rho_c - 2\rho\right)\alpha_1 > \theta\rho_c\alpha_2 Q_{max,2} - L_c\rho_c\theta - 2\rho Q_1 + 2R_1. \tag{6.18}$$

MGO 2's expected utility function is derived in a similar manner as MGO 1's with indices 1 and 2 reversed.

Proof. The proof is given in Appendix B.2. □

Given the complex structure of each MGO's expected utility function with framing, computing the closed-form expression of the best response strategy is difficult for the PT case. In particular, the analysis of $E_{\mathrm{PT},1,2b}$ is quite challenging due to the various forms that the function can take under different conditions as seen in (6.13) and (6.15). Therefore, in order to find the BNE under PT, a best response algorithm is proposed.

This iterative algorithm dictates that, in response to its opponent's current strategy, each MGO sequentially chooses its optimal storage strategy by numerically characterizing, from its action space, the action that maximizes its expected utility. In fact, given the closed-form expressions provided in Propositions 3, 4, and 5, an MGO can easily compute its expected utility for each of its strategies. In this respect, upon convergence, this algorithm is guaranteed to reach an equilibrium [82]. In fact, at the point of convergence, each MGO is playing the strategy that maximizes its expected PT utility facing its opponent's strategy. Hence, the MGOs will reach a BNE from which none has any incentive to deviate since such deviation would not improve their expected payoff. Indeed, as observed in our simulations in Section V, the algorithm always converged to an equilibrium.

6.5 Simulation Results and Analysis

For our simulations, we consider a smart grid with $N = 2$ MGs capable of supplying power to one of the power grid's critical loads which requires a total of $L_c = 200$ kWh to remain operational until regular power supply is restored. We also assume the regular price per unit of energy to be $\rho = \$0.1$ per kWh. In addition, we take $\theta = 0.01$, and $\rho_c = \$11.6$ per kWh unless stated otherwise. The exponents β^+ and β^- are taken to be both equal to 0.88 and the loss multiplier $\lambda = 2.25$ unless stated otherwise [89]. We simulate the system for two scenarios: CGT, and PT under utility framing.

Fig. 6.1 compares the effects of different MGO reference points on the total energy stored for both CGT and PT analysis. In the classical game theory case ($\beta^+ = \beta^- = \lambda = 1$), an MGO's reference point is irrelevant given that losses and gains are computed in an identical objective manner. For the PT case, for a reference point below $\$8$, the BNE action profile is not significantly affected compared to the classical game theory case, since most

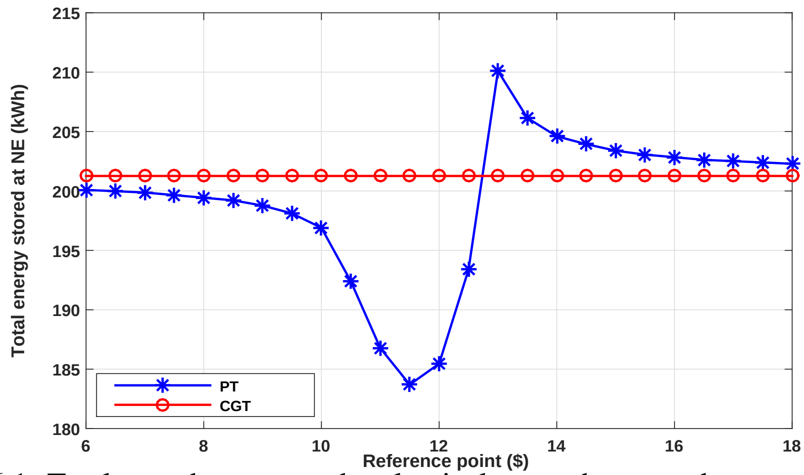

Figure 6.1: Total stored energy under classical game theory and prospect theory.

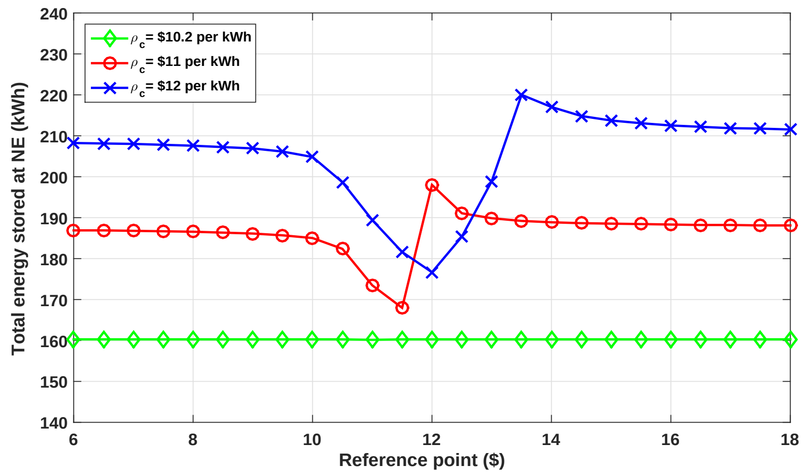

Figure 6.2: Effect of emergency price on PT sensitivity to the reference point.

potential payoffs of the BNE actions are still viewed as gains above the reference point. As the reference point increases from $\$8$ to $\$11.5$, the total stored energy will decrease from around 200 to 184 kWh, since some of the potential payoffs of the current BNE will start to be perceived as losses, as they cross the reference point. Given that losses have a larger weight under PT compared to classical game theory, the expected utility of the current strategy profile will significantly decrease, thus causing the BNE to drift towards lower storage strategies. The MGOs will exhibit risk averse behavior as they sell more of their energy at the current risk-free retail market price ρ. In fact, as previously mentioned, by decreasing α, the minimum potential payoffs are larger, compared to the larger values of α, and are still above the reference point.

The described behavior is reversed in the $[11.5, 13]$ range where the MGOs start exhibiting more risk seeking behavior, i.e., storing more energy, to reach a total stored energy of 210 kWh. In fact, the low risk strategies' potential payoffs are now fully perceived as losses causing a significant devaluation of their expected utility values. The BNE will thus go towards higher values of α with larger maximum payoffs, compared to lower values of α, which are partially still considered as gains. Finally, when the reference point is above $\$13.5$, most potential payoffs of most strategies are now perceived as losses and the effect of PT will diminish gradually, and the total energy stored will reach 202 kWh, identically to classical game theory. It is important to note that the critical load energy requirements are 200 kWh, which is met with the stored energy of the MGs under classical game theory but not necessarily under PT analysis. This highlights the need for an accurate behavioral analysis of the studied system.

Fig. 6.2 shows the effect of changing the emergency price ρ_c on the role of the reference point in an MGO's decision, for $\lambda = 4$. For a price of $\rho_c = \$10.2$ per kWh, the total energy stored does not vary with the reference point. In fact, the expected future profits gained from storing energy are close to the profits incurred by selling at the current market price. On the other hand, when the price is increased to $\rho_c = \$11$ per kWh, the total stored energy will vary with the reference point by up to 10% from its original value. In fact, storing energy will now yield significantly higher expected future profits, compared to selling at the current market price. Thus, an MGO's risk-seeking or risk-averse behavior

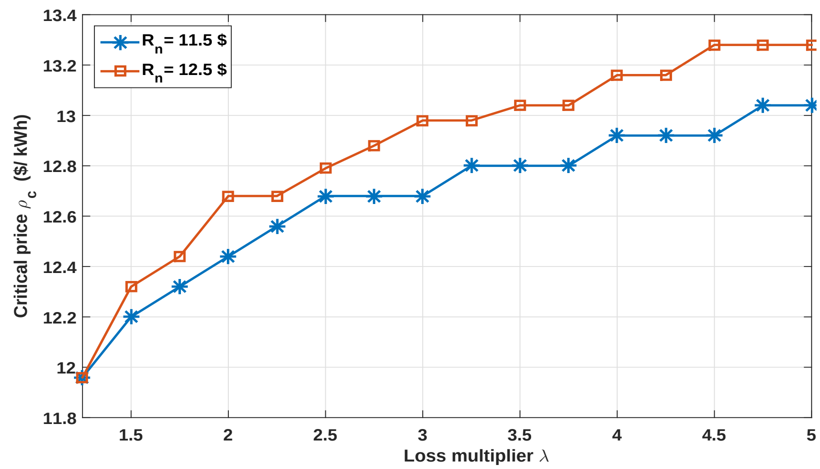

Figure 6.3: Emergency price needed to cover L_c as a function of λ.

is justified given the increasing uncertainty in profits. Similarly, when $\rho_c = \$12$ per kWh, the total stored energy would vary further with the changing reference point, by up to 17% from its original value.

Fig. 6.3 shows the effect of the loss multiplier λ on the emergency price ρ_c needed to cover the critical load for the reference points of $\$11.5$ and $\$12.5$. The effect of framing is more prominent as the loss multiplier increases. In fact, the MGOs will exhibit more risk averse behavior for the specified reference points as λ increases, thus prompting the power company to increase the critical price in order to cover the critical load. In fact, as λ increases, so will the valuation of the MGOs' losses. To avoid the large losses, the MGOs will decrease the energy stored by their MGs and will tend to sell more energy at the current risk free market price. This highlights the importance of behavioral analysis in choosing the proper pricing mechanism in smart grid resilience planning.

Fig. 6.4 illustrates the storage strategies at equilibrium for the case in which one of the MGOs is fully rational, while the second is subjective. The rational MGO will naturally

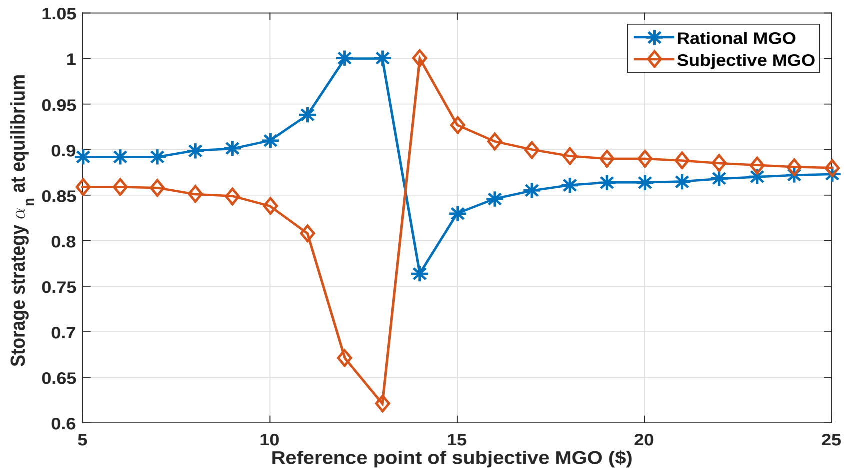

Figure 6.4: Storage strategies at equilibrium for a case with one subjective (PT) MGO and one rational (CGT) MGO.

have no reference point. Here, both MGs have the same size of storage $Q_{\max} = 150\,\text{kWh}$ and energy excess available $Q = 120\,\text{kWh}$. As seen in Fig. 6.4, as the reference point of the subjective MGO increases from $\$5$ to $\$13$, it will exhibit risk averse behavior and decrease the portion of energy it stores, to reach a value of 0.625. This is similar to the analysis of Fig. 6.1. To respond, the rational MGO will hence increase the portion of energy stored to reach its maximum of 1, given the lower stored energy of its opponent. As the reference point increases from $\$13$ to $\$14.5$, the subjective MGO will exhibit more risk seeking behavior and increase the portion of energy stored to reach its maximum of 1. The rational MGO, will thus decrease its MG's stored energy, given the storage strategy of its opponent. Finally, as the reference point increases from $\$14.5$ to $\$25$, the effect of utility framing will gradually decrease, and the storage strategy of both MGOs will reach a value of 0.88. Given the negligible effect of PT at the high reference point of $\$25$, both MGOs, rational and subjective, will have equal strategies at equilibrium and thus similar behavioral patterns.

6.6 Conclusion

In this chapter, we have proposed a novel framework for analyzing the storage strategy of micorgrid operators in an attempt to enhance smart grid resilience. We have formulated the problem as a Bayesian game between multiple MGOs, who must choose the portion of their microgrids' excess to store, in order to maximize their expected profits. The MGOs play a noncooperative game, which is shown to have four Bayesian Nash equilibria for the two MG case, under different conditions. Subsequently, we have used the novel concept of utility framing from prospect theory to model the behavior of MGOs when faced with the uncertainty of their opponents' energy surplus. Simulation results have highlighted the impact of behavioral considerations on the overall process of enhancing the resilience of a smart grid by exploiting distributed, microgrid energy storage.

Chapter 7

Summary and Future Extensions

In this chapter, we summarize our work and contribution. In addition, we discuss possible extensions and relevant future work.

7.1 Prosumer-Centric Energy Trading: The Effect of Wind Generation Uncertainty

7.1.1 Summary

The increased penetration of renewable energy generators and storage devices at the prosumer's side of the power grid, is bringing forth challenges in the study and analysis of DSM schemes. In fact, DSM models must henceforth account for the capability of prosumers to generate, store, and consumer energy, when assessing the effectiveness of a given incentive scheme. As mentioned in Chapter 2, while the current DSM literature extensively accounts for the integration of utility scale renewable generation, the topic of small-scale renewable generator at the prosumer's side is understudied. In fact, widely used day-ahead DSM scheduling schemes must now be altered to mitigate possible deviations of prosumers, from their day-ahead energy bids, which are based on probabilistic renewable generation forecasts. In Chapter 4, we have proposed a PT-based game-theoretic energy trading model, which introduces grid consumption-based dynamic pricing, and imposes financial penalties on prosumers for any deviation. Our simulation results have shown the difference in prosumer decisions in day-ahead based energy trading, under CGT analysis, where prosumers are assumed to be rational, and PT analysis. In addition, irrational prosumers tend to exaggerate the possible deviation penalties which lead to an increase in the declared energy consumption, compare to a grid of rational prosumers. This contrast in energy trading must be taken into account, in order to design efficient and effective DSM schemes.

7.1.2 Future Extensions

The main future extensions related to for our DSM energy trading model in Chapter 4, are summarized as follows:

- The energy trading model in Chapter 4 can be extended to account for a continuous prosumer action space, where prosumer n can now pick any energy bid in the range $[B_{n,\text{min}}, B_{n,\text{max}}]$, which is more realistic. This would make an analytical characterization of the NE possible, and would thus provide a clearer picture on the main drivers of the prosumers' energy trading decisions at equilibrium. However, to accurately account for both the PT weighting and framing effects in this case, the framework of *cumulative prospect theory* (CPT) must be applied. CPT is an extension to the original PT framework, where cumulative probabilities are weighted, rather than the actual probabilities.

- A further extension of this model, would be to account for various types of consumers in the smart grid. This would include, conventional generator owners, and passive consumers that cannot generate or store energy. Thus, in order to accurately model and study the energy trading process in any realistic grid, a more generic DSM model is needed, which can account for the various types of consumers.

- The single stage static game model developed in Chapter 4 can be extended into a multistage dynamic game. In other words, the single stage described in this model can be one of 24 stages, while each stage represents an hour of the day. This would provide more precise modeling of the day-ahead energy trading process, when consumption is broken down on an hourly scale. Such a scenario can be modeled with a stochastic game model [104]. In such a scenario, the wind energy output is still uncertain for all stages, and the prosumer does not know the amount of energy left in its storage at each stage. While this analysis might prove to be challenging, with the introduction of a stochastic game framework, it would render the DSM scheme analysis more realistic and relevant to a real-life application.

7.2 Price Uncertainty in Prosumer-Centric Energy Trading: A Stackelberg Game Approach

7.2.1 Summary

Given that DSM schemes are typically introduced by the power company, its role as decision maker must be extensively accounted for when modeling the prosumer-centric energy trading process. To this end, in Chapter 5, we have used the framework of Stackelberg games to capture the hierarchical interaction between the power company and prosumers. In this analysis, another important component of DSM, the variable and uncertain future energy price, has been accounted for. In fact, given the storage capability of prosumers, one must accurately account for their valuation of the unsold energy in their storage, which might significantly impact their decision to buy, sell, or store energy. In addition, as previously mentioned, prosumers are human players, typically irrational. The impact of the uncertain future energy price was thus accounted for in our analysis, using the framing effect from prospect theory. Our simulation results have highlighted the various effects that irrational prosumer behavior would have on the effectiveness of DSM schemes. In fact, simulation results show that the total grid load, under PT, decreases for certain ranges of prosumers' reference points and increases for others, when compared to CGT. This decrease will greatly affect the power company's profits, when it fails to account for the prosumers' irrational behavior under PT. Ultimately, this might influence the power company's ability to properly manage energy delivery.

7.2.2 Future Extension

The main future extensions related to this topic are summarized as follows:

- Our hierarchical energy trading model can be extended to account for the uncertainty associated with the prosumers' energy generation. This added uncertainty will surely affect the decision of prosumers and might result in a different aggregate grid consumption under PT. This would make our analysis under PT further challenging since one must now deal with two random variables that affect the expected utility of the prosumers. On the other hand, this analysis would be more realistic, given the stochastic renewable energy forecasts based on which a real-life prosumer would make its trading decision.

- Another future extension would be to account for the charging/discharging efficiency of the storage device, as well as its energy leakage over time, which might discourage the prosumers from keeping their energy stored for a longer period of time, and thus directly impact their decision to buy/sell energy at the current price.

- Finally, the prosumer's energy storage capability can be modeled as an EV's battery, given the extensive projected growth in the EV market. For a storage device to be accurately modeled as an EV, a number of factors will need to be considered, which include the time availability of the EV at the prosumer's premises, and the minimum charge level that must be kept in storage, for the EV's next trip.

7.3 Prospect Theory for Enhanced Smart Grid Resilience Using Distributed Energy Storage

7.3.1 Summary

The ever increasing number of installed microgrids, will bring forth major opportunities, for enhancing grid resilience and mitigating power outages. In Chapter 7, in order to fully leverage the storage capability of microgrids, we have proposed an energy management model where microgrid operators are incentivized to store part of their energy excess, to be used in case of emergency. Once a possible blackout scenario is expected to occur, the power company would offer microgrid operators high energy prices, to supply the critical loads through their stored energy. Given the limited nature of the needed critical power supply, we have used the framework of Bayesian games to study the competitive nature of the energy trading decision process, as well as to account for each MGO's incomplete information, regarding its opponents' energy excess. Furthermore, given that microgrid operators are human decision makers, we have applied the framework of utility framing to account for their irrational behavior, in order to insure that any proposed energy storing incentive reaches its goal. In fact, our simulations results have shown that, for a certain critical energy price, rational MGOs stored enough of their energy to supply the critical load, where irrational MGOs did not. The power company must therefore quantify the subjective behavior of the MGOs before choosing the optimal emergency energy price, in order to meet the critical load at minimal cost.

7.3.2 Future Extension

The main future extensions related to this topic are summarized as follows:

1- Our microgrid resilience enhancement model can be extended to the more generic case, applicable to smart grids that can include any number of microgrids and critical loads. This would provide a holistic model that can be used to analyze any possible scenario related to the application of microgrid storage for supplying the grid's critical loads. This would however require revisiting the proposed incentive model to make it more suitable for a large scale implementation.

2- Another possible extension is to consider the power company as a player and analyze this model as a hierarchical Stackelberg game. In this scenario, the power company, as a leader, would move first by announcing its incentive strategy, to which the microgrid operators would respond by choosing the portion of their energy excess to store. The power company's utility function would thus account for both the price paid to the operators, as well as the power company's ability to provide electricity for the critical loads.

3- In the future extension of this model, another factor that should be accounted for is the probability of losing transmission between each microgrid and the critical load. In fact, in the current model, we only account for the probable event of losing power supply from the macrogrid's generation units. Accounting for the probability of losing transmission capability will add another layer of uncertainty into our analysis. However, in a real-life application, the loss of transmission will most likely affect the MGO’s decision and must thus be accounted for.

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Appendices

Appendix A

Appendices for Chapter 5

A.1 Proof of Theorem 1

First, we analyze the strictly concave expected utility of prosumer n in (5.7). By taking the second derivative of (5.7) with respect to x_n, we get: $\frac{\partial U_n^{\mathrm{EUT}}}{\partial^2 x_n} = -2\alpha$, which is a strictly negative term, as $\alpha > 0$. The optimal solution is either an interior point obtained by solving the necessary and sufficient optimality condition given by $\frac{\partial U_n^{\mathrm{EUT}}}{\partial x_n} = 0$, or is at one of the boundaries, in case the interior solution is not feasible. Solving the optimality solution gives a unique solution $x_n^r = -\frac{\theta}{2\alpha} - \frac{\bar{x}_{-n}}{2}$. x_n^r maximizes each prosumer's expected utility function given that it lies in the feasible range of $\mathcal{X}_n$.

A.2 Proof of Theorem 2

First, we show that the followers' game is a *concave game* with closed and convex action sets in which the utility of player n is a concave function of its own action x_n, for any fixed actions of others $\boldsymbol{x}_{-n}$. From (5.7), one can see that the utility function of each prosumer n is quadratic, and thus concave, in terms of its own action variable x_n. Moreover, the action set of each prosumer $\mathcal{X}_n$ is clearly a closed convex set. Using [101, Theorem 1], we can show that the prosumers' game admits at least one pure strategy NE. For NE uniqueness, we use [101, Theorem 2] to show that the prosumers game is diagonally strictly concave. This means that one can find a fixed nonnegative vector $\boldsymbol{r} \geq 0$ such that for every two action profiles $\boldsymbol{x}^o, \tilde{\boldsymbol{x}} \in \mathcal{X}_1 \times \cdots \times \mathcal{X}_n$, $(\tilde{\boldsymbol{x}} - \boldsymbol{x}^o)' g(\boldsymbol{x}^o, \boldsymbol{r}) + (\boldsymbol{x}^o - \tilde{\boldsymbol{x}})' g(\tilde{\boldsymbol{x}}, \boldsymbol{r}) > 0$, where $g(\boldsymbol{x}, \boldsymbol{r}) = (r_1 \nabla_{x_1} U_1^{\mathrm{EUT}}(\boldsymbol{x}), \ldots, r_n \nabla_{x_n} U_n^{\mathrm{EUT}}(\boldsymbol{x}))'$. We let $r_j = 1$ for each $j \in \mathcal{N}$. Using (5.7), we have

$$g_j(\boldsymbol{x}, \boldsymbol{r}) = -2\alpha x_j + \theta + \alpha \bar{x}_{-j}, \;\; j \in \mathcal{N}.$$

We let I be the identity matrix, and J be a square matrix with all entries equal to 1. Then we can write $g(\boldsymbol{x}, \boldsymbol{r}) = \boldsymbol{K}\boldsymbol{x} + \boldsymbol{c}$, where $\boldsymbol{K} := -\alpha(\boldsymbol{I} + \boldsymbol{J})$. $\boldsymbol{K}$ is a negative definite matrix due to the positive definiteness of $\boldsymbol{I} + \boldsymbol{J}$ and the fact that $-\alpha < 0$. By checking the diagonally strict concavity condition we get

$$
\begin{aligned}
&(\tilde{\boldsymbol{x}} - \boldsymbol{x}^o)' g(\tilde{\boldsymbol{x}}, \boldsymbol{r}) + (\boldsymbol{x}^o - \tilde{\boldsymbol{x}})' g(\boldsymbol{x}^o, \boldsymbol{r}) \\
&\qquad = (\tilde{\boldsymbol{x}} - \boldsymbol{x}^o)'[\boldsymbol{K}\boldsymbol{x}^o + \boldsymbol{c}] + (\boldsymbol{x}^o - \tilde{\boldsymbol{x}})'[\boldsymbol{K}\tilde{\boldsymbol{x}} + \boldsymbol{c}] \\
&\qquad = -(\tilde{\boldsymbol{x}} - \boldsymbol{x}^o)'\boldsymbol{K}(\tilde{\boldsymbol{x}} - \boldsymbol{x}^o) > 0,
\end{aligned} \tag{A.1}
$$

where the last inequality is due to the negative-definiteness of the matrix $\boldsymbol{K}$. Using [101, Theorem 2] the NE will be unique.

A.3 Auxililiary Lemma for the Proof of Theorem 3

Lemma 1. There exists a constant $K > 0$ for which the Nikaido-Isoda function $\Psi(\boldsymbol{x}, \boldsymbol{y})$ associated with the prosumers' game satisfies $\Psi(\boldsymbol{x}, \boldsymbol{y}) \leq K\|\boldsymbol{x} - \boldsymbol{y}\|$. Moreover, $\Psi(\boldsymbol{x}, \boldsymbol{y})$ is convex in $\boldsymbol{x}$ and strongly concave in $\boldsymbol{y}$ such that

$$
\begin{aligned}
\Psi(\boldsymbol{x}, \lambda\tilde{\boldsymbol{y}} + (1-\lambda)\hat{\boldsymbol{y}}) = \lambda\Psi(\boldsymbol{x}, \tilde{\boldsymbol{y}}) + (1-\lambda)\Psi(\boldsymbol{x}, \hat{\boldsymbol{y}}) \\
+ \alpha\lambda(1-\lambda)\|\hat{\boldsymbol{y}} - \tilde{\boldsymbol{y}}\|^2, \ \forall\lambda \in [0, 1].
\end{aligned} \tag{A.2}
$$

Proof. For any two action profiles of the prosumers $\boldsymbol{x} = (x_1, \ldots, x_n) \in \Omega$ and $\boldsymbol{y} = (y_1, \ldots, y_n) \in \Omega$, the Nikaido-Isoda function adopted for the utility in (5.7) will be:

$$
\begin{aligned}
\Psi(\boldsymbol{x},\boldsymbol{y}) :&= \sum_{n\in\mathcal{N}} [U_n^{\text{EUT}}(y_n, \boldsymbol{x}_{-n}, \rho_{base}) - U_n^{\text{EUT}}(x_n, \boldsymbol{x}_{-n}, \rho_{\text{base}})] \\
&= \sum_{n\in\mathcal{N}} [\alpha(x_n^2 - y_n^2) + (\theta + \alpha\bar{x}_{-n})(x_n - y_n)]. \qquad \text{(A.3)}
\end{aligned}
$$

Using (A.3), for any two action profiles $\boldsymbol{x},\boldsymbol{y} \in \Omega$, we have

$$
\begin{aligned}
\Psi(\boldsymbol{x},\boldsymbol{y}) &= \sum_{n\in\mathcal{N}} (x_n - y_n)[\alpha(x_n + y_n) + \theta + \alpha\bar{x}_{-n}] \\
&\le \sqrt{\sum_{n\in\mathcal{N}} (x_n - y_n)^2}\sqrt{\sum_{n\in\mathcal{N}} [\alpha(x_n + y_n) + \theta + \alpha\bar{x}_{-n}]^2} \\
&= \|\boldsymbol{x} - \boldsymbol{y}\| \sqrt{\sum_{n\in\mathcal{N}} [\alpha(x_n + y_n) + \theta + \alpha\bar{x}_{-n}]^2} \\
&\le K\|\boldsymbol{x} - \boldsymbol{y}\|,
\end{aligned}
$$

where the first inequality is due to the Cauchy-Schwarz inequality, and $K := \sqrt{n(\theta + \alpha(n+1)B_{\max})^2}$ is an upper bound constant for the second term of the last equality. To show the convexity of $\Psi(\boldsymbol{x},\boldsymbol{y})$ with respect to x, let $\boldsymbol{J}$ be the $n \times n$ matrix with all entries equal to 1. Using (A.3), a simple calculation shows that $\nabla^2_{\boldsymbol{x}\boldsymbol{x}}\Psi(\boldsymbol{x},\boldsymbol{y}) = 2\alpha\boldsymbol{J}$, where $\nabla^2_{\boldsymbol{x}\boldsymbol{x}}\Psi(\boldsymbol{x},\boldsymbol{y})$ denotes the Hessian matrix of $\Psi(\boldsymbol{x},\boldsymbol{y})$ with respect to variable vector $\boldsymbol{x}$. Since $\alpha > 0$ and $\boldsymbol{J}$ is a positive semi-definite matrix, this shows that $\nabla^2_{\boldsymbol{x}\boldsymbol{x}}\Psi(\boldsymbol{x},\boldsymbol{y}) > 0$, which implies $\Psi(\boldsymbol{x},\boldsymbol{y})$ is a convex function of $\boldsymbol{x}$. Finally using (5.7), one can easily check that the equality in (A.2) holds, which shows that $\Psi(\boldsymbol{x},\boldsymbol{y})$ is strongly concave with respect to its second argument $\boldsymbol{y}$. $\square$

A.4 Proof of Theorem 3

We show that $\lim_{t\to\infty} \boldsymbol{x}(t) = \boldsymbol{x}^*$, from Algorithm 1, where $\boldsymbol{x}^*$ is a pure-strategy NE of the prosumers. To show that, we measure the distance of an action profile $\boldsymbol{x}(t)$ and its best response $\Pi_\Omega\big[\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}(t)\big]$ using the Nikaido-Isoda function and show that this distance decreases as t becomes large. In particular, we show that at the limit, this distance equals zero which shows that the limit point is an NE of the game.

$$\begin{aligned}\Psi(\boldsymbol{x}(t+1), \boldsymbol{x}^r(t+1)) &= \Psi\Big((1-\frac{1}{\sqrt{t}})\boldsymbol{x}(t) + \frac{\boldsymbol{x}^r(t)}{\sqrt{t}}, \boldsymbol{x}^r(t+1)\Big)\\ &\leq (1-\frac{1}{\sqrt{t}})\Psi\Big(\boldsymbol{x}(t), \boldsymbol{x}^r(t)\Big) + \frac{1}{\sqrt{t}}\Psi\Big(\boldsymbol{x}^r(t), \boldsymbol{x}^r(t+1)\Big).\end{aligned} \tag{A.4}$$

Using the first part of Lemma 1, we have

$$\begin{aligned}\Psi\Big(\boldsymbol{x}^r(t), \boldsymbol{x}^r(t+1)\Big) &\leq K\|\boldsymbol{x}^r(t) - \boldsymbol{x}^r(t+1)\|\\ &= K\|\Pi_\Omega\big[\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}(t)\big] - \Pi_\Omega\big[\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}(t+1)\big]\|\\ &\leq K\|[\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}(t)] - [\boldsymbol{a} + \boldsymbol{A}\boldsymbol{x}(t+1)]\|\\ &\leq K\|\boldsymbol{A}\|\|\boldsymbol{x}(t) - \boldsymbol{x}(t+1)\|,\\ &= \frac{K(n-1)}{2\sqrt{t}}\|\boldsymbol{x}(t) - \boldsymbol{x}^r(t)\|.\end{aligned} \tag{A.5}$$

where the first inequality is due to the nonexpansive property of the projection operator, the second inequality uses the matrix norm inequality, and the last equality is obtained by replacing the expression for $\boldsymbol{x}(t+1)$ and noting that the induced 2-norm of matrix $\boldsymbol{A}$ equals $\frac{n-1}{2}$. Substituting (A.5) into (A.4) we have

$$\begin{aligned}\Psi\Big(\boldsymbol{x}(t+1), \boldsymbol{x}^r(t+1)\Big) \leq (1-\frac{1}{\sqrt{t}})\Psi\Big(\boldsymbol{x}(t), \boldsymbol{x}^r(t)\Big) \\ + \frac{K(n-1)}{2t}\|\boldsymbol{x}(t) - \boldsymbol{x}^r(t)\|.\end{aligned}$$

Since $\Psi(\boldsymbol{x}(t), \boldsymbol{x}(t)) = 0$, we can write

$$\begin{aligned}\Psi\Big(\boldsymbol{x}(t+1), \boldsymbol{x}^r(t+1)\Big) &\leq (1-\frac{1}{\sqrt{t}})\Psi(\boldsymbol{x}(t), \boldsymbol{x}^r(t)) \\ &+ \frac{1}{\sqrt{t}}\Psi(\boldsymbol{x}(t), \boldsymbol{x}(t)) + \frac{K(n-1)}{2t}\|\boldsymbol{x}(t) - \boldsymbol{x}^r(t)\| \\ &= \Psi\left(\boldsymbol{x}(t), (1-\frac{1}{\sqrt{t}})\boldsymbol{x}^r(t) + \frac{1}{\sqrt{t}}\boldsymbol{x}(t)\right) \\ &- \alpha(1-\frac{1}{\sqrt{t}})\frac{1}{\sqrt{t}}\|\boldsymbol{x}(t) - \boldsymbol{x}^r(t)\|^2 \\ &+ \frac{K(n-1)}{2t}\|\boldsymbol{x}(t) - \boldsymbol{x}^r(t)\| \\ &\leq \Psi(\boldsymbol{x}(t), \boldsymbol{x}^r(t)) - \frac{\Psi^2(\boldsymbol{x}(t), \boldsymbol{x}^r(t))}{\frac{2K^2}{\alpha}\sqrt{t}} + \frac{K(n-1)D}{2t}\end{aligned}$$

where the first equality is due to Lemma 1, and the last inequality is due to first part of Lemma 1 and the fact that $\boldsymbol{x}^r(t)$ maximizes $\Psi(\boldsymbol{x}(t), \cdot)$. Multiplying both sides of the above inequality by $\frac{\alpha}{2K^2\sqrt{t}}$ and defining $c := \frac{\alpha(n-1)D}{4K}$ and $a_t := \frac{\alpha}{2K^2\sqrt{t}}\Psi(\boldsymbol{x}(t), \boldsymbol{x}^r(t))$, we get

$$a_{t+1} \leq a_t - a_t^2 + \frac{c}{t\sqrt{t}}. \tag{A.6}$$

Our goal is to show that $a_t < \sqrt{2c} \times t^{-\frac{3}{4}}$ for all $t \geq \frac{100}{c^2}$, in which case by definition of a_t we obtain $\Psi(\boldsymbol{x}(t), \boldsymbol{x}^r(t)) = O(t^{-\frac{1}{4}})$. This not only shows that $\lim_{t\to\infty} \Psi(\boldsymbol{x}(t), \boldsymbol{x}^r(t)) = 0$, implying that $\{\boldsymbol{x}(t)\}$ converges to a pure strategy NE of the prosumers game (note that $\Psi(\boldsymbol{x}, \boldsymbol{x}^r) = 0$ if and only if $\boldsymbol{x}$ is a NE), but it also shows that after t steps, the action profile of the prosumers $\boldsymbol{x}(t)$ is an ϵ-NE of the game where $\epsilon = O(t^{-\frac{1}{4}})$ (this is due to $\Psi(\boldsymbol{x}(t), \boldsymbol{x}^r(t)) = O(t^{-\frac{1}{4}})$ implies $U_n(\boldsymbol{x}_n^r(t), \boldsymbol{x}_{-n}(t), \rho_{\text{base}}) - U_n(x_n(t), \boldsymbol{x}_{-n}(t), \rho_{\text{base}}) = O(t^{-\frac{1}{4}})$ for all $n \in \mathcal{N}$, meaning that given the action profile $\boldsymbol{x}(t)$, no prosumer can increase its utility by more that $O(t^{-\frac{1}{4}})$ by playing its best response).

We complete the proof using induction on t to show that $a_t < \sqrt{2c} \times t^{-\frac{3}{4}}$. Assume that this relation is true for t. Then

$$\begin{aligned}
a_{t+1} &\leq a_t - a_t^2 + \frac{c}{t\sqrt{t}} \\
&\leq \sqrt{2c}t^{-\frac{3}{4}} - 2ct^{-\frac{3}{2}} + \frac{c}{t\sqrt{t}} \\
&= \sqrt{2c}t^{-\frac{3}{4}} - ct^{-\frac{3}{2}}.
\end{aligned}$$

Let $f(z) : [1, \infty) \to \mathbb{R}$ be a function defined by $f(z) = \sqrt{2c}z^{-\frac{3}{4}} - \sqrt{2c}(z+1)^{-\frac{3}{4}} - cz^{-\frac{3}{2}}$. We only need to show that $f(z) < 0$, for $t \geq \frac{100}{c^2}$. By writing the Taylor expansion of the first two terms of $f(z)$ for $z \geq 1$, we have $f(z) \leq 7\sqrt{2c}z^{-\frac{7}{4}} - cz^{-\frac{3}{2}}$, which is less than 0 for $t \geq \frac{100}{c^2}$. This completes the induction and shows that $a_t = O(t^{-\frac{3}{4}})$.

A.5 Proof of Theorem 4

We first find the conditions under which the expected utility function is uniform over each prosumer's action space. We then find the additional condition to ensure that the function is strictly concave.

Case 1: To have $R_n < \rho_{\text{min}}c + d$, for all of prosumer n's actions, we first rewrite the inequality in terms of x_n:

$$-\alpha x_n^2 + (\rho_{\text{min}} - \rho_b - \alpha\bar{x}_{-n})\, x_n + (\rho_{\text{min}} K_n - R_n) < 0,$$

where $k_n = W_n + Q_n - L_n$. By analyzing the second order polynomial, and its roots, r_1 and r_2, we get the condition for case 1. Under such a condition, the expected utility function of prosumer n under PT, simplifies to $U_n^{\text{PT}}(x_n, \bar{x}_{-n}, \rho_{\text{base}}) = U_n^{\text{EUT}}(x_n, \bar{x}_{-n}, \rho_{\text{base}}) - R_n$. This is clearly a concave function, given that, $U_n^{\text{EUT}}(x_n, \bar{x}_{-n}, \rho_{\text{base}})$ has been shown to be concave, and R_n is a constant.

Case 2: In order to have $R_n > \rho_{\text{max}}c + d$, for all of prosumer n's actions, we follow a similar approach in order to find the condition for case 2. Under this condition, the expected utility function under PT simplifies to $U_n^{\text{PT}}(x_n, \bar{x}_{-n}, \rho_{base}) = \lambda(U_n^{\text{EUT}}(x_n, \bar{x}_{-n}, \rho_{\text{base}}) - R_n)$, which is also strictly concave, given that λ is strictly positive.

Case 3: To have $\rho_{\text{min}}c + d < R_n < \rho_{\text{max}}c + d$, for all of prosumer n's actions, we follow a similar approach in order to find the condition for case 3. We next analyze the concavity of the expected utility function, given in the second line in (5.12). The second derivative is given by:

$$\frac{\partial U_{n,PT}}{\partial^2 x_n} = \frac{a_1}{m_1} x_n - \frac{a_1 m_1 - b m_1}{m_1^2} - \frac{(\lambda - 1) a_2^2}{(Q_n - L_n + W_n + x_n)^3},$$

where $a_2 = (R_n - (k_n \rho_{\text{base}}) + \alpha(L_n^2 + Q_n^2 + w_n^2) - 2L_n Q_n \alpha + L_n \alpha \bar{x}_{-n} - Q_n \alpha \bar{x}_{-n} - 2L_n \alpha w_n + 2Q_n \alpha w_n$. Note that $-\frac{(\lambda-1)a_2^2}{(Q_n - L_n + W_n + x_n)^3}$ is negative for all x_n. Next, we find the range of x_n for which $\frac{a_1}{m_1} x_n - \frac{a_1 m_1 - b_1 m_1}{m_1^2}$ is negative as well. Given that $\frac{a_1}{m_1}$ is negative, the utility function is thus guaranteed to be concave for $x_n > \frac{a_1 m_1 - b_1 m_1}{a_1 m_1}$.

Appendix B

Appendices for Chapter 6

B.1 Proof of Proposition 2

For the proof of Proposition 2, first, we analyze the expected utility of MGO 1, for $\alpha_1 \in \left[0, \frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}\right]$ and $\alpha_1 \in \left[\frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}, 1\right]$, with $\alpha_2 \in \left[\frac{L_c - Q_1}{Q_{2,\max}}, 1\right]$.

a) For $\alpha_1 \in \left[0, \frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}\right]$, the total energy stored is below the critical load L_c for all possible types of MGO 2. Here, MGO 1's expected utility is given by

$$E_{1,2a}(\boldsymbol{\alpha}, Q_1) = \rho\left(Q_1 - \alpha_1 Q_1\right) + \theta\rho_c\alpha_1 Q_1.$$

$E_{1,2a}$ is a strictly increasing function given that $\theta\rho_c > \rho$, hence, it is maximized at its upper boundary $\alpha^*_{1,2a} = \frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}$.

b) For $\alpha_1 \in \left[\frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}, 1\right]$, given MGO 2's strategy, the total energy stored is above the critical load for certain types of MGO 2. MGO 1's expected utility is given by

$$E_{1,2b}(\boldsymbol{\alpha}, Q_1) = \int_0^A U_1(\boldsymbol{\alpha}, \boldsymbol{Q}) f(Q_2) dQ_2 + \int_A^{Q_{2,\max}} U_1(\boldsymbol{\alpha}, \boldsymbol{Q}) f(Q_2) dQ_2, \tag{B.1}$$

with $A = \frac{L_c - \alpha_1 Q_1}{\alpha_2}$ which follows from (5). Under this assumption, $f_1(Q_2) = 1/Q_{2,\max}$ over its domain and $E_{1,2b}$ is now given by

$$\begin{aligned} E_{1,2b}(\boldsymbol{\alpha}, Q_1) = \frac{1}{Q_{2,\max}} \int_0^A \left[\rho\left(Q_1 - \alpha_1 Q_1\right) + \theta \rho_c \alpha_1 Q_1\right] dQ_2 + \\ \frac{1}{Q_{2,\max}} \int_A^{Q_{2,\max}} \left[\rho\left(Q_1 - \alpha_1 Q_1\right) + \frac{1}{2}\theta \rho_c \left(\alpha_1 Q_1 - \alpha_2 Q_2 + L_c\right)\right] dQ_2. \end{aligned} \tag{B.2}$$

By taking the second derivative of (B.2) with respect to the decision variable α_1, we get

$$\frac{\partial E_{1,2b}}{\partial^2 \alpha_1} = -\frac{Q_1^2 \rho_c \theta}{2\alpha_2 Q_{\max,2}}.$$

The function is strictly concave given that its second derivative is strictly negative. The optimal solution is, hence, obtained by the necessary and sufficient optimality condition given by $\partial E_{1,2b} \partial \alpha_1 = 0$. The optimality condition has a unique solution which is given by:

$$\alpha_{1,r} = \frac{L_c\rho_c\theta + (\rho_c\theta - 2\rho)\alpha_2 Q_{2,\max}}{Q_1\rho_c\theta}.$$

Given that $E_{1,2b}$ is a strictly concave function and that α_1 is restricted to $\left[\frac{L_c-\alpha_2 Q_{2,\max}}{Q_1}, 1\right]$, $\alpha^*_{1,2b}$ will be

$$\alpha^*_{1,2b} = \begin{cases} \frac{L_c-\alpha_2 Q_{2,\max}}{Q_1}, & \text{if } \alpha_{1,r} < \frac{L_c-\alpha_2 Q_{2,\max}}{Q_1}, \\ \alpha_{1,r}, & \text{if } \alpha_{1,r} \in \left[\frac{L_c-\alpha_2 Q_{2,\max}}{Q_1} \quad 1\right], \\ 1, & \text{if } \alpha_{1,r} > 1. \end{cases} \tag{B.3}$$

In fact, $\alpha_{1,r}$ is the optimal solution for $E_{1,2b}$ if it belongs to the feasible region of $E_{1,2b}$. On the other hand, if $\alpha_{1,r}$ is larger than the upper bound, then $E_{1,2b}$ is a strictly increasing function over the feasibility set and is maximized at its upper bound $\alpha^*_{1,2b} = 1$. Finally, if $\alpha_{1,r}$ is smaller than the domain's lower bound $\frac{L_c-\alpha_2 Q_{2,\max}}{Q_1}$, then $E_{1,2b}$ is a strictly decreasing function over the feasibility set and is maximized at its lower bound. However, the condition $\alpha_{1,r} < \frac{L_c-\alpha_2 Q_{2,\max}}{Q_1}$ cannot be satisfied for $\rho_c\theta > \rho$, and thus $\frac{L_c-\alpha_2 Q_{2,\max}}{Q_1}$ cannot be the maximizer of $E_{1,2b}$. We can thus rewrite (B.3) as

$$\alpha^*_{1,2b} = \begin{cases} \alpha_{1,r}, & \text{if } \left[\frac{2\rho}{\rho_c\theta} - 1\right]\alpha_2 > \frac{L_c-Q_1}{Q_{2,\max}}, \\ 1, & \text{if } \left[\frac{2\rho}{\rho_c\theta} - 1\right]\alpha_2 \le \frac{L_c-Q_1}{Q_{2,\max}}. \end{cases} \tag{B.4}$$

We first note that $E_{1,2a} = E_{1,2b}$ for $\alpha_1 = \frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}$ which is the maximizer of $E_{1,2a}$. However, as previously discussed, $E_{1,2b}$ cannot be maximized at $\frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}$. Thus, the maximizer of MGO 1's expected utility, for $\alpha_2 \in \left[\frac{L_c - Q_1}{Q_{2,\max}}, 1\right]$, belongs to the domain $\left[\frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}, 1\right]$. In other words, $r_1(\alpha_2) = \alpha^*_{1,2b}$ for $\alpha_2 \in \left[\frac{L_c - Q_1}{Q_{2,\max}}, 1\right]$.

B.2 Proof of Proposition 5

Player 1's expected utility under PT, for $\alpha_2 \in \left[\frac{L_c - Q_1}{Q_{2,\max}}, 1\right]$, and $\alpha_1 \in \left[\frac{L_c - \alpha_2 Q_{2,\max}}{Q_1}, 1\right]$, is given by

$$\begin{aligned} E_{\text{PT},1,2b}(\boldsymbol{\alpha}, Q_1) = &\int_0^A \frac{1}{Q_{2,\max}} V\left(\rho\left(Q_1 - \alpha_1 Q_1\right) + \theta \rho_c \alpha_1 Q_1\right) dQ_2 + \\ &\int_A^{Q_{2,\max}} \frac{1}{Q_{2,\max}} V\left(\rho Q_1 \left(1 - \alpha_1\right) + \frac{1}{2}\theta\rho_c\left(\alpha_1 Q_1 - \alpha_2 Q_2 + L_c\right)\right) dQ_2. \quad \text{(B.5)} \end{aligned}$$

We denote by I_1 the first integral in ($B.5$), and by I_2 the second. As previously mentioned, PT states that a utility is perceived in terms of gains and losses with respect to the reference point. Next, we analyze the possible values of both integrals I_1 (first integral) and I_2 (second integral) in (B.5) from that perspective. The original utility in I_1, $U_{I,1} = \rho\left(Q_1 - \alpha_1 Q_1\right) + \theta\rho_c\alpha_1 Q_1$, is only a function of α_1 and is independent of Q_2. Equation (6.13) follows from the fact that for $\alpha_1 \leq B$, $U_{I,1}$ is below the reference point R_1 and is thus perceived as a loss. On the other hand, it is considered as a gain for $\alpha_1 > B$.

We then assess the possible values of I_2. The original utility function in I_2, $U_{I,2} = \rho\left(Q_1 - \alpha_1 Q_1\right) + \frac{1}{2}\theta\rho_c\left(\alpha_1 Q_1 - \alpha_2 Q_2 + L_c\right)$ is considered a loss given that

$$\rho\left(Q_1-\alpha_1 Q_1\right)+\frac{1}{2}\theta\rho_c\left(\alpha_1 Q_1-\alpha_2 Q_2+L_c\right)<R_1,$$

which can be rewritten as $Q_{2,r}<Q_2$ with $Q_{2,r}$ given by

$$Q_{2,r}=\frac{2}{\rho_c\theta\alpha_2}\left[\rho\left(Q_1-\alpha_1 Q_1\right)+\frac{1}{2}\theta\rho_c\left(\alpha_1 Q_1+L_c\right)-R_1\right]. \tag{B.6}$$

Given that MGO 1's expected utility is taken over MGO 2's type (Q_2), we next analyze I_2 for different values of Q_2. (6.15) follows from the fact that I_2 is a loss integral for $Q_{2,r}<A$. Given that the lower bound of I_2 is larger than A, then the entire range of Q_2 values is as well. The condition $Q_{2,r}<A$ can be rewritten as C_1. On the other hand, I_2 is a gain integral for $Q_{2,r}>Q_{2,\text{max}}$ which can be rewritten as C_2. Finally, for $A<Q_{r,2}<Q_{2,\text{max}}$, I_2 is split into two parts: a gain integral on $[A,Q_{2,r}]$ and a loss integral on $[Q_{\text{ref2}},Q_{2,\text{max}}]$. $A<Q_{2,r}<Q_{2,\text{max}}$ can be rewritten as C_3. (6.13) and (6.15) are obtained by evaluating the integrals I_1 and I_2 for the described cases.