Appendix B   Axial Displacement Required for each Pressure

Due to the assumed rigid end plates, clamped-clamped boundary conditions are applied to each end of the cylinder, with the exception of allowing the end at $x = +L/2$ end to expand uniformly in the axial direction with displacement Δ. The end at $x = -L/2$ cannot move axially in order to restrict axial rigid body translation. Formally, the boundary conditions at the ends of the cylinder ($x = \pm L/2$) are as follows:

$$\begin{gathered} \text{i)}\ u^o = 0 \ @\ x = -\frac{L}{2},\ u^o = \Delta \ @\ x = +\frac{L}{2} \\ \text{ii)}\ v^o = 0 \\ \text{iii)}\ w^o = 0 \\ \text{iv)}\ \frac{\partial w^o}{\partial x} = 0. \end{gathered} \tag{B.1}$$

The end displacement Δ is determined by enforcing axial equilibrium of the end enclosure at $x = +L/2$, namely,

$$\int_0^C N_x ds = p_o \pi ab, \tag{B.2}$$

where N_x is the axial force resultant within the cylinder, C is the circumference of the cylinder reference surface, and the cross-sectional area of the ellipse is πab. Physically, eq. B.2 states that the net axial force due to the internal pressure times the cross-sectional area of the end enclosure must be balanced by the net axial force due to the axial force resultant.

Tables B-1 and B-2 show the end displacements Δ used for various internal pressure values for the quasi-isotropic, axially-stiff, and circumferentially-stiff laminates evaluated using linear and nonlinear analyses, respectively. Since table B-1 shows the end displacements for each laminate evaluated using linear analysis, only one internal pressure value is needed. The end displacement can be determined for any other internal pressure by linearly extrapolating. The circumferentially-stiff laminate requires the largest end displacement Δ to satisfy axial equilibrium, whereas the axially-stiff laminate requires the smallest end displacement Δ. This can be explained by considering the percentage of fibers along the axial direction. The circumferentially-stiff laminate has fewer fibers in the axial direction than the quasi-isotropic and axially-stiff laminates. Therefore, the circumferentially-stiff laminate provides less resistance to expansion in the axial direction.

Table B-1. End displacement required to satisfy axial equilibrium corresponding to an internal pressure evaluated using linear analysis

	Quasi-isotropic	Axially-stiff	Circumferentially-stiff
Pressure (psi)	100	100	100
Δ (in.)	0.002391	0.000326	0.005627

For the nonlinear analyses, the end displacements cannot be obtained through linear extrapolation, as seen in table B-2. In the nonlinear case, linearly extrapolating overestimates the end displacement Δ necessary to satisfy axial equilibrium. Therefore, for each pressure used the end displacement Δ must be determined. Again, the circumferentially-stiff laminate requires the largest end displacement Δ to satisfy axial equilibrium.

Table B-2. End displacement required to satisfy axial equilibrium corresponding to an internal pressure evaluated using nonlinear analysis

	Quasi-isotropic	Axially-stiff	Circumferentially-stiff
Pressure (psi)	100	100	100
Δ (in.)	0.002039	-0.000061	0.005349
Pressure (psi)	130	130	240
Δ (in.)	0.002536	0.000208	0.012180
Pressure (psi)	250	250	
Δ (in.)	0.004137	0.001240	

In general, the end displacement Δ required for the nonlinear case is smaller than for the linear case. The most significant difference is seen with the axially-stiff laminate for 100 psi. The axially-stiff laminate evaluated using linear analysis extends 0.000326 in. but contracts -0.000061 in. when evaluated using nonlinear analysis. Recalling the definition of axial reference surface strain,

$$\varepsilon_x^o = \frac{\partial u^o}{\partial x} + \underline{\frac{1}{2}\left(\frac{\partial w^o}{\partial x}\right)^2}, \tag{B.3}$$

the difference in the axial displacements between the linear and nonlinear case exist in the underlined term. With the addition of the underlined term for the nonlinear case, the end displacement is more sensitive to outward or inward wall deflection caused by the internal pressure.