Chapter 4  Effect of Material Orthotropy on Cylinder Response

Thus far, the influences of elliptical geometry have been studied using a linear analysis and a quasi-isotropic cylinder, and the influences of geometric nonlinearities have been studied using a quasi-isotropic cylinder. In this chapter the focus is shifted from the influence of elliptical geometry and geometric nonlinearities to the influence of material orthotropy. The quasi-isotropic laminate considered in the previous chapter will be compared with axially-stiff and circumferentially-stiff laminates using a geometrically nonlinear analysis for an elliptical cylinder. Each laminate has a different response to internal pressure due to the percentages of fibers running in the axial and circumferential directions. The axially-stiff laminate has almost 50% of the fibers aligned with the axial direction, the circumferentially-stiff laminate has almost 50% of the fibers aligned with the circumferential direction, and the quasi-isotropic has an equal number of fibers aligned with the axial, circumferential, and ±45° direction. For this study of the influence of orthotropy, a three-dimensional format figure for each of the three laminates will again be shown, along with two-dimensional format figures along a line at a particular $s/C$ or $x/L$ location for the purpose of a closer examination of an issue that may be difficult to discern from the three-dimensional format figures. Only those responses which show any significant differences due to orthotropy are discussed.
4.1 Displacements

Recall from the boundary conditions of eq. 1.3 that the axial displacement is zero at \( x/L = -0.5 \), and at \( x/L = 0.5 \) the axial displacement is determined by eq. 1.4. It appears that for all the laminates the axial displacement at \( x/L = 0.5 \) is approximately twice the value at \( x/L = 0 \). However, the magnitude of the axial displacement response differs for the three laminates. As seen in fig. 4-1, the circumferentially-stiff laminate requires a higher axial end displacement, or \( \Delta \), to satisfy the axial equilibrium given in eq. 1.4 than either the quasi-isotropic or axially-stiff laminates. In fact, the axially-stiff elliptical cylinder under internal pressure evaluated using nonlinear analysis requires a negative axial displacement to satisfy the axial equilibrium equation. Though the overall characters of the axial displacement responses are the same, the displacement difference at \( x/L = 0.5 \) is evident.
The circumferential displacements for the various laminates, as seen in fig. 4-2, have a similar overall behavior. The circumferential displacement is zero at $x/L=0.5$ due to the boundary conditions given in eq. 1.3, and increases in magnitude to a local extreme at $s/C = 0.1563$ at the midspan. However, the magnitude of this local extreme varies between the quasi-isotropic, axially-stiff, and circumferentially-stiff laminates. In fig. 4-2a, the three laminates are closely examined along the line at $x/L=0.0$. The circumferential displacement for the axially-stiff laminate is the smallest in magnitude, while it is largest in magnitude for the quasi-isotropic laminate.

**Figure 4-1. Influence of orthotropy on the axial displacement.**
Although the overall behavior of the normal displacement is unaffected by orthotropy, as seen in fig. 4-3, the magnitude of the normal displacement at the midspan is controlled by the orthotropy. At the crown of the cylinder the normal displacement for the axially-stiff laminate is greater than for the circumferentially-stiff laminate, but less than for the quasi-isotropic laminate. However, at the side of the cylinder the situation is somewhat reversed and the magnitude of the normal displacement for the circumferentially-stiff laminate is greater than for the axially-stiff laminate, but again less than the magnitude of the normal displacement for the quasi-isotropic laminate.

Figure 4-2. Influence of orthotropy on the circumferential displacement.

Although the overall behavior of the normal displacement is unaffected by orthotropy, as seen in fig. 4-3, the magnitude of the normal displacement at the midspan is controlled by the orthotropy. At the crown of the cylinder the normal displacement for the axially-stiff laminate is greater than for the circumferentially-stiff laminate, but less than for the quasi-isotropic laminate. However, at the side of the cylinder the situation is somewhat reversed and the magnitude of the normal displacement for the circumferentially-stiff laminate is greater than for the axially-stiff laminate, but again less than the magnitude of the normal displacement for the quasi-isotropic laminate.
laminate. In general, the circumferentially-stiff laminate best controls expansion at the crown of the cylinder and the axially-stiff laminate best controls contraction at the side of the cylinder.

4.2 Strains and Curvatures

As seen in fig. 4-4, the degree to which the circumferential strain varies with the \( s \) coordinate at the midspan is completely affected by the laminate considered. Recall from fig. 2-3a-b, the circumferential strain for a circular cylinder has no variation with the \( s \) coordinate, while the circumferential strain for the elliptical cylinder varies considerably with the \( s \) coordinate. As seen in

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**Figure 4-3. Influence of orthotropy on the normal displacement.**
fig. 4-4, at midspan the circumferentially-stiff laminate mitigates, to a high degree, the effect of ellipticity, as the strain does not vary much with $s$ there. The circumferential strain for the axially-stiff and quasi-isotropic laminates varies more with the $s$ coordinate at the midspan. Therefore, it appears that the percentage of fibers in the circumferential direction controls the degree of variation of the circumferential strain with the $s$ coordinate at the midspan.

Figure 4-4. Influence of orthotropy on the circumferential strain.

As seen in fig. 4-5, the degree to which the axial strain varies with spatial location is also affected by the laminate considered. Comparatively, the axial strain for the axially-stiff laminate
varies least with both the $x$ and $s$ coordinates and it varies the most for the circumferentially-stiff laminate. Recall from fig. 2-3c-d, the axial strain for a circular cylinder has no variation with the $s$ coordinate, while for the elliptical cylinder it varies considerably with both the $x$ and $s$ coordinate. Although the axially-stiff laminate doesn’t completely mitigate the effect of ellipticity on the axial strain at midspan, the increased percentage of fibers in the axial direction controls the degree of spatial variation for the axial strain there. To be noted, the degree of orthotropy has little if any influence on the shear strain distribution with $x$ and $s$.

![Figure 4-5. Influence of orthotropy on the axial strain.](image)
As seen in fig. 4-6, in the midspan region the axial curvature is similar for all orthotropies, namely zero. In the boundary region, however, the variation with $s$ of the axial curvature depends on the orthotropy. Recall that the behavior of the axial curvature at the boundary is due to the clamped boundary conditions imposed on the cylinder ends, and that the elliptical shape forces a reversal in curvature at the boundary as $s/C$ changes from 0 to 0.25. The orthotropy affects the degree of the reversal in curvature at the boundary. Comparatively, at the boundary, the axial curvature for the axially-stiff laminate varies least with the $s$ coordinate, and it varies the most for the circumferentially-stiff laminate. Therefore, the percentage of fibers in the axial direction controls the degree of reversal of the curvature at the boundary. Interestingly, the axially-stiff elliptical cylinder evaluated using a linear analysis instead of a nonlinear analysis does not show this reversal of curvature.
As seen in fig. 4-7, the overall behavior of the twist curvature is similar for all orthotropies. However, the magnitude of the local extreme in the twist curvature, and its location with $x$ and $s$, changes with the degree of orthotropy. The twist curvature for the circumferentially-stiff laminate has a minimum value of -0.3796 located at $x/L=0.4758$ and $s/C=0.1458$, the twist curvature for the quasi-isotropic laminate has a minimum value of -0.3920 located at $x/L=0.4678$ and $s/C=0.1458$, and the twist curvature for the axially-stiff laminate has a minimum value of -0.4285 located at $x/L=0.4678$ and $s/C=0.1563$. Therefore, as the percentage of fibers in the axial direction

Figure 4-6. Influence of orthotropy on the axial curvature.
increases, the magnitude of the local minimum value of the twist curvature increases and shifts toward the side of the cylinder, and as the percentage of fibers in the circumferential direction increases, the magnitude of the local minimum value of the twist curvature decreases and shifts toward the clamped boundary. The circumferential curvature does not depend on the degree of orthotropy.

Figure 4-7. Influence of orthotropy on the twist curvature.
4.3 Force and Moment Resultants

As seen in fig. 4-8, the boundary region for the circumferential force resultant differs among the circumferentially-stiff, axially-stiff, and quasi-isotropic laminates. At the clamped boundary, the circumferential force resultant for the quasi-isotropic laminate varies more with the $s$ coordinate than for the axially-stiff laminate, but it varies less than the circumferential force resultant for the circumferentially-stiff laminate. Therefore, as the percentage of fibers in the axial direction increases, the variation with the $s$ coordinate of the circumferential force resultant at the boundary decreases.
The circumferential moment resultant, as seen in fig. 4-9, also differs among the circumferentially-stiff, axially-stiff, and quasi-isotropic laminates. The variations with $s$ along $x/L=0$ and 0.5 are examined for a closer look at these differences. The circumferential moment resultant is not large in magnitude at the midspan, but there are differences among the orthotropies. The circumferential moment resultant for the circumferentially-stiff laminate is significantly greater in magnitude at the side of the cylinder, changes sharply between $s/C=0.20$ and 0.15, then flattens out at the crown to a magnitude greater than those for the axially-stiff and quasi-isotropic lami-
nates. Along the clamped boundary the circumferential moment resultant for the circumferentially-stiff laminate is greater in magnitude at the side and the crown than for the other two laminates. The axial and twist moment resultants have results similar to the circumferential moment resultant in that the magnitude at the side and crown, and at the midspan and clamped boundary depend to some degree on orthotropy.
Figure 4-9. Influence of orthotropy on the circumferential moment resultant.
As seen in fig. 4-10, in the midspan region the circumferential transverse shear force resultant is similar for all three orthotropies, namely, almost zero. In the region of the clamped boundary, however, the variation of the force resultant depends on the orthotropy. Recall that the axial and circumferential transverse shear force resultants enforce the $w^0=0$ condition at the boundary of the cylinder, and with the elliptical geometry they are forced to a change sign at the boundary because the cylinder moves outward at the crown and keel, and inwards at the sides. The orthotropy affects the degree of the sign reversal of the circumferential transverse shear force resultant at the boundary. Comparatively, the force resultant for the axially-stiff laminate varies least with the $s$ coordinate at the boundary and that for the circumferentially-stiff varies most. In general, the percentage of fibers in the axial direction controls the degree of the sign reversal of the circumferential transverse shear force resultant at the boundary.
As seen in fig. 4-11, in the midspan region the axial transverse shear force resultant is similar for all orthotropies, being close to zero there in all cases, but in the clamped boundary region the variation of the force resultant depends on the orthotropy. Again the boundary conditions on \( w^0 \) combined with the elliptical geometry force a change of sign in the force resultant, the location of this sign change depending on the orthotropy. Furthermore in the side region of the cylinder, the force resultant for the circumferentially-stiff laminate is greatest in magnitude and that for the

\[ \text{Figure 4-10. Influence of orthotropy on the circumferential transverse shear force resultant, } Q_s. \]

As seen in fig. 4-11, in the midspan region the axial transverse shear force resultant is similar for all orthotropies, being close to zero there in all cases, but in the clamped boundary region the variation of the force resultant depends on the orthotropy. Again the boundary conditions on \( w^0 \) combined with the elliptical geometry force a change of sign in the force resultant, the location of this sign change depending on the orthotropy. Furthermore in the side region of the cylinder, the force resultant for the circumferentially-stiff laminate is greatest in magnitude and that for the
axially-stiff laminate is smallest in magnitude. In the crown region of the cylinder, this trend reverses. The force resultant for the circumferentially-stiff laminate is smallest in magnitude and that for the axially-stiff laminate is greatest.

As seen in fig. 4-12, in the midspan region the circumferential transverse force resultant is similar for all orthotropies, but in the boundary region of the cylinder it varies. The orthotropy affects the location of the sign reversal and the peak-to-peak variations of the circumferential transverse force resultant at the boundary. In general, as the percentage of fibers along the axial

**Figure 4-11. Influence of orthotropy on the axial transverse shear force resultant, Q_x.**

As seen in fig. 4-12, in the midspan region the circumferential transverse force resultant is similar for all orthotropies, but in the boundary region of the cylinder it varies. The orthotropy affects the location of the sign reversal and the peak-to-peak variations of the circumferential transverse force resultant at the boundary. In general, as the percentage of fibers along the axial
direction increases, the variation of the circumferential transverse force resultant with the $s$ coordinate decreases at the boundary.

As seen in fig. 4-13, in the midspan region the axial transverse force resultant is similar for all orthotropies, being very close to zero, but in the boundary region it varies, depending on the orthotropy. Again, the boundary conditions on $w^o$ and the elliptical geometry force a change of sign in the axial transverse force resultant at the boundary. The orthotropy affects the location of the sign reversal and the peak-to-peak variation of the axial transverse force resultant at the

\[ \text{Figure 4-12. Influence of orthotropy on the circumferential transverse force resultant, } V_s. \]
boundary. In the side region of the cylinder, the force resultant for the circumferentially-stiff laminate is greatest in magnitude, and for the axially-stiff laminate it is smallest in magnitude. In the crown region of the cylinder, these characteristics reverse such that the force resultant for the circumferentially-stiff laminate is smallest in magnitude, and for the axially-stiff laminate it is greatest in magnitude.

Figure 4-13. Influence of orthotropy on the axial transverse force resultant, $V_x$. 
4.4 Summary of the Effects of Orthotropy

The effects of orthotropy seen in this section included several key issues. The axially-stiff, circumferentially-stiff, and quasi-isotropic laminates resulted in an overall difference in magnitude for the axial, circumferential, and normal displacements. In fact, the axially-stiff elliptical cylinder evaluated using nonlinear analysis contracts axially in response to internal pressure, whereas, for the other two cases there is axial extension. For some responses, orthotropy mitigates the effect of ellipticity. For example, the circumferential strain behaves like that of a circular cylinder in the midspan region of the circumferentially-stiff laminate. For the axially-stiff laminate, the axial strain displays less spatial variation with both $x$ and $s$ compared to the axial strains for the circumferentially-stiff and quasi-isotropic laminates. The variation with $s$ at the clamped boundary of the axial curvature, the circumferential force and moment resultants, and the transverse force resultants depends significantly on orthotropy. Compared to the circumferentially-stiff and quasi-isotropic laminates, for these responses the axially-stiff laminate does not generally exhibit as much variation with $s$.

This chapter has presented a through discussion of the character of the response to internal pressure of elliptical cylinders with three different levels of orthotropy. A complete catalogue of all the geometrically nonlinear responses of the three cylinders is presented in Appendix A, along with a comparison of the responses as predicted by the finite element code STAGS [6]. The latter comparison is for the purpose of verifying the present analysis. As noted in Appendix A, by the nature of finite element analysis, many of the important responses are not computed exactly at the ends of the cylinder, the location where many responses assume a maximum or minimum value. For this reason there appears to be a lack of agreement between STAGS predictions and the predictions of the present analysis near the ends of the cylinder. This issue becomes important when
failure is studied, as it is in the next chapter. Whereas the present analysis may predict failure to occur at the exact ends of the cylinder due to a certain pressure level, STAGS would predict failure to occur slightly inwards of the ends at a different pressure level. However, STAGS is not used here to study failure so the issue never arises.

Also by way of a catalogue, Appendix B provides a listing of the axial displacement $\Delta$ for each of the cases discussed here. Recall, $\Delta$ is determined by eq. 1.4.