Strategic Planning Models and Approaches to Improve Distribution Planning in the Industrial Gas Industry

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(ABSTRACT)

The industrial gas industry represents a multi-billion dollar global market and provides essential product to manufacturing and service organizations that drive the global economy. In this dissertation, we focus on improving distribution efficiency in the industrial gas industry by addressing the strategic level problem of bulk tank allocation (BTA) while considering the effects of important operational issues. The BTA problem determines the preferred size of bulk tanks to assign to customer sites to minimize recurring gas distribution costs and initial tank installation costs. The BTA problem has a unique structure which includes a resource allocation problem and an underlying vehicle routing problem with split deliveries.

In this dissertation, we provide an exact solution approach that solves the BTA problem to optimality and recommends tank allocations, provides a set of delivery routes, and determines delivery amounts to customers on each delivery route within reasonable computational time. The exact solution approach is based on a branch-and-price algorithm that solves problem instances with up to 40 customers in reasonable computational time.

Due to the complexity of the problem and the size of industry representative problems, the solution approaches published in the literature rely on heuristics that require a set of potential routes as input. In this research, we investigate and compare three alternative route generation algorithms using data sets from an industry partner. When comparing the routes generation algorithms, a sweep-based heuristic was the preferred heuristic for the data sets evaluated.

The existing BTA solution approaches in the literature also assume a single bulk tank can be allocated at each customer site. While this assumption is valid for some customers due to space limitations, other customer sites may have the capability to accommodate multiple tanks. We propose two alternative mathematical models to explore the possibility and potential benefits of allocating multiple tanks at designated customer site that have the capacity to accommodate more than one tank. In a case study with 20 customers, allowing multiple tank allocation yield 13% reduction in total costs.

In practice, industrial gas customer demands frequently vary by time period. Thus, it is important to allocate tanks to effectively accommodate time varying demand. Therefore, we develop a bulk tank allocation model for time varying demand (BTATVD) which captures changing demands by period for each customer. Adding this time dimension increases complexity. Therefore, we present three decomposition-based solution approaches. In the first two approaches, the problem is decomposed and a restricted master problem is solved. For the third approach, a two phase periodically restricting heuristic approach is developed. We evaluate the solution approaches using data sets provided by an industrial partner and solve the problem instances with up to 200 customers. The results yield approximately 10% in total savings and 20% in distribution cost savings over a 7 year time horizon.

The results of this research provide effective approaches to address a variety of distribution issues faced by the industrial gas industry. The case study results demonstrate the potential improvements for distribution efficiency.

To my mom, I owe you everything.
This work is dedicated to you.

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1 Introduction

The global market for the industrial gas sector was approximately \$77 billion in 2014 and is projected to increase to \$116.6 billion in 2020 (Industry Experts, 2014). Industrial gases such as hydrogen, oxygen, argon, helium, and nitrogen are manufactured for use in industries including petrochemicals, chemicals, power, mining, metal production, environmental protection, healthcare, food and beverages, agriculture, electronics fabrication, and aerospace. The use of industrial gases is expected to increase as they are used to support more environmentally conscious processes in petrochemicals, agrochemicals, and renewable energy production due to increased environmental legislation (BCC Research LLC, 2009).

Industrial gases are generally produced and then delivered to customers by pipeline, transported in gas cylinders, or delivered as bulk liquid by truck. Bulk liquid distribution revenues account for approximately 34% of revenues in this industry (Baker and Garvey, 2004) and is the focus of this research.

For gases that are transported by cryogenic trailers to bulk tanks at customer sites, industrial gas producers often implement a vendor managed inventory (VMI) system with their customers. The producers are responsible for inventory management as well as gas delivery to customer sites. Customers assume ownership of the gas inventory upon delivery to bulk tanks located at their sites. The industrial gas producer typically owns and monitors the tanks and replenishes the tanks using cryogenic trailers. These trailers and storage tanks are generally high value assets and their effective utilization is important for the company. Also, industrial gas producers must meet strict customer service levels while coping with demand fluctuations. Thus, efficient distribution is a main driver to achieve lower costs and remain competitive. Supply chain managers face unique logistical challenges mainly due to disparate customer profiles, high asset values, non-aligned strategic and operational goals, wide geographic spread of customers, demand seasonality, and consumption patterns (Chowhan, 2013).

When distributing gases, trailers typically depart their depots, obtain liquefied products at gas sources, and deliver the gas product to bulk tanks at one or more customer sites. The trailers may then either conclude the route and return to the depot or may obtain additional product during delivery by refilling at a source. When trailers visit multiple sources during routes, this is referred to as a continuous route.

The customer tank size is a determining factor for the quantity and volume of deliveries that a customer requires. When determining the preferred tank for a customer site, industrial gas producers consider estimated consumption patterns, safety stock requirements of each customer, and proximity to other customers. Swapping tanks between customers involves removing a tank from one customer site, moving it to a tank warehouse, refurbishing then transporting it to another customer location, and installing it at the new location. This is an expensive process which is justified only if the benefits of a more efficient distribution plan exceed the costs of tank exchanges.

With a significant investment in bulk tanks, industrial gas companies face an important strategic level decision of bulk tank allocation which directly affects the operational efficiency of their distribution networks. Installing proper size tanks at customer locations provides improved utilization of their assets (bulk tanks and vehicle fleet) and better service quality to their customers.

In this dissertation, we focus on improving distribution efficiency in the industrial gas industry by addressing the strategic level problem of bulk tank allocation (BTA) while considering the effects of important operational issues. Our objectives include the following:

- provide an exact solution approach to solve the problem to optimality and recommend tank allocations, provide a set of delivery routes, and determine delivery amounts to customers on each delivery route within reasonable computational time,

- provide multiple ways to capture the operational level decision of delivery route generation using an exact column generation-based approach and multiple heuristic approaches,

- explore the potential benefits of assigning multiple resources at a site by developing two alternative mathematical models, and

- address time varying customer demand by developing solution approaches for the BTA problem that incorporate multiple time periods.

The primary contributions of this research include novel models and solution approaches to provide efficient resource allocation decisions at the strategic level while considering important operational level characteristics for the industrial gas industry. This research is motivated by Air Liquide, a leading international industrial gas company, that produces and distributes industrial gases such as hydrogen, oxygen, argon, helium, carbon dioxide, and nitrogen for several business segments in eighty countries.

We review the related literature and describe the previous research addressing BTA in section 2. In section 3, we present a mathematical model for the BTA problem and describe a branch and price solution approach for solving the model to optimality. Then in section 4 we review a decomposition based heuristic approach to solve the model for larger industry instances and compare the performance of alternative route generation modules. We extend the BTA model to allow multiple tank allocations at customer sites in section 5. Section 6 presents the BTA problem for customers with time varying demand and proposes a two-phase periodically restricting heuristic method.

2 Literature Review

The bulk tank allocation (BTA) problem allocates or reallocates bulk tanks to customer sites in order to minimize the sum of the net present value of investment costs in the logistics network and distribution costs over a time horizon. The BTA problem involves resource allocation with an underlying vehicle routing problem with split deliveries (VRPSD). VRPSD is a generalized vehicle routing problem (VRP) in which a customer's demand can be satisfied from multiple vehicle visits rather than just one. The VRPSD can also be viewed as a special case of inventory routing problem (IRP) with known demand where no back orders are allowed. In the following sections, we present a brief survey of the literature on VRPSD and IRP and then review the previous research on bulk tank allocation problem in the industrial gas industry.

2.1 Vehicle Routing Problem with Split Deliveries

In a classic VRP, a set of customers with known demand are served by a fleet of vehicles. The objective is to minimize the total travel cost while requiring that each customer's demand is satisfied by exactly one vehicle visit. In a VRPSD, the restriction of one visit for each customer is relaxed, and a customer's demand can be fulfilled by several deliveries on multiple vehicles routes. This problem was first proposed in the literature by Dror and Trudeau (1989), where they show the cost saving potentials through split deliveries in a VRP.

Archetti et al. (2008) and Archetti and Speranza (2012) provide surveys of the state of the art on this problem. They provide a description of the mathematical formulation along with analysis of the properties and complexity of the problem. They review the exact solution approaches for solving a VRPSD for small instances of the problem. These approaches include a dynamic programming model (Lee et al., 2006), a two-stage solution approach including iteratively solving assignment problems and traveling salesman problems (Jin et al., 2007), and an approach based on a set covering formulation of the problem and a column generation approach (Feillet et al., 2006). Archetti et al. (2011) propose a branch-and-price-and-cut scheme for solving VRPSD. They use column generation approach to generate routes and determine delivery amounts in the subproblem using a labeling algorithm and then master problem selects the optimal routes.

Archetti et al. (2008) also review some effective heuristics that have been designed and tested on larger data sets. They compare the performance of major heuristics available in the literature on several instances ranging from 50 to 199 customers. More recently, several metaheuristic approaches are proposed by Aleman and Hill (2010), Aleman et al. (2010), and Derigs et al. (2010) to solve the VRPSD. Most of the VRPSD literature reports solving instances with less than 200 customers. The additions of tank allocation decisions along with time varying customer demand introduce additional complexity to the problem that efficient heuristic approaches are necessary for solving industry size problem.

2.2 Inventory Routing Problem

In general, the IRP involves the integration and coordination of two components of the logistics systmes: inventory management and vehicle routing. It is concerned with the distribution of products from facilities to a set of customers over a given planning horizon. Customers consume the product and can maintain an inventory of the product up to a certain level. A fleet of vehicles of known capacities are available for the distribution of the product. The objective is to minimize the distribution and inventory costs during the planning period.

Inventory management and routing are important decisions within a logistic system that are widely discussed in the literature. There are different versions of the complex inventory routing problem in the literature. Golden et al. (1984) is one of the first papers that addresses this problem and uses the term inventory routing. They develop a simulation model to investigate the interaction between inventory management and routing and also perform a cost/benefit analysis. In a survery paper, Moin and Salhi (2006) present a logistical overview of the IRP and classify the papers into single-period, multi-period, infinite time horizon, and problems with stochastic demand. Single-period models are popular in the literature, as they are simpler to solve and often provide a basis for solving multi-period models. The multi-period models are often used for long term planning as they better capture the trade-offs between short-term and long-term decisions. While multi-period models provide a more realistic representation of the problem, they are more complex and therefore most of the papers consider deterministic demand for the customers and solve the problem using heuristic methods. Some research has been published on stochastic IRP (see Bard et al. (1998); Kleywegt et al. (2002, 2004)). Campbell et al. (1998) discuss the complexity and practical issues of the IRP with stochastic demand and present different solution approaches.

In a more recent paper, Bertazzi et al. (2008) review the inventory routing literature and discuss the characteristics and complexity of the problem and describe the challenges encountered when trying to simultaneously minimize the inventory holding and routing costs. Andersson et al. (2010) also provide a comprehensive survey of over 90 scientific papers for IRP where one party in the supply chain is responsible for transportation and inventory management. The papers are categorized by time horizon into the following three categories: instant time horizon, finite time horizon, and infinite time horizon. The papers are compared with respect to problem description, assumptions, and solution approaches proposed in major papers within each category. In most of the cases, the complexity of the problem makes it difficult to solve to optimality and heuristic or decomposition approaches are often employed. For most industry-sized problems, a computational time limit is enforced for exact approaches and then the best integer solution found is reported or a heuristic has been implemented. They emphasize the importance of using advanced decision support tool for inventory management and routing decisions for businesses with complex systems, but found no commercial decision support tools available for the combined inventory management and routing problem. Thus, optimization-based decision support systems are needed for the IRP. Aksoy and Derbez (2003) survey the available software systems for supply chain management. They identify 160 software companies providing supply chain management software. Among these they find several software systems that have separate modules for inventory management and routing, but no software systems where it is combined.

The IRP has been studied in supply chains for different industries. Zhao et al. (2007) study the IRP for a distribution system that provides constant demand replenished from a single warehouse for a set of retailers. They minimize the long-term average costs using the best inventory and routing strategies, while making sure all the demand is satisfied. They test their methodology on problem instances with 50-75 retailers without considering any limit on the inventory capacity at the warehouse or the retailers sites. Bard and Nananukul (2009) present heuristic solution approaches for the IRP in a manufacturing supply chain.

They maximize the net benefits while ensuring that all the demand is met. One of their assumptions is that inventory can accumulate at the customer sites, whereas for industrial gas the amount of inventory held at customer location is limited by the size of the tank that is located at each customer site. Rusdiansyah and Tsao (2005) model the IRP of a vending machine supply chain over multiple time periods considering time windows and they present solution procedures along with computational experiments. Gaur and Fisher (2004) develop a multi-period IRP for a super market chain in Netherlands and analyze the tactical and strategic advantages of their approach for the firm. They use a matching algorithm over a finite (6 days) planning horizon. Bertazzi et al. (2002) study a multi-period, multi-product with deterministic but time-varying demand for each discrete time period, IRP with order-up-to level inventory policy. They explore various objective functions for different decision policies and propose a two-step heuristic algorithm to solve the problem.

Bell et al. (1983) discuss a decision support tool for inventory management and vehicle routing problem for an industrial gas company. They generate vehicle routes with up to four customers using a heuristic which incorporates Lagrangian relaxation of a mixed integer mathematical program and then use a shortest path formulation to solve the problem. Trudeau and Dror (1992) consider the IRP with specific application in oil and gas distribution, in which a single product is delivered to multiple customers with stochastic demand from a single depot. They minimize the average transportation cost over the time horizon by developing a heuristic solution approach based on solving MIP submodels. The idea is extended in Dror and Trudeau (1996) for both stochastic and deterministic demand with an objective to minimize the present value of the cash flow associated with distribution of a product over a long period of time. Campbell and Savelsbergh (2004), in collaboration with an international industrial gas company, develop a two phase decomposition approach for the IRP. A high-level base plan is developed using integer programming to construct delivery schedules and routes in the first phase. Then in the second phase, routing and scheduling heuristics generate a more detailed operational plan. They consider the long-term implications and try to adjust the short term decisions based on the long term perspective. Savelsbergh and Song (2007) study a deterministic IRP with continuous moves and develop a randomized greedy heuristic, in which at each step they determine the next visit to each customer instead of obtaining a delivery schedule. Further, Savelsbergh and Song (2008) incorporate some practical consideration from the industrial gas industry and develop an integer programming-based optimization approach for this problem over multiple periods. Song and Savelsbergh (2007) also develop practical performance measures to assess the long-term effectiveness of inventory routing strategies and establish methodologies to evaluate those measures. Although there has been research on the IRP in a variety of industries, none has yet been reported as a decisions support system that is implemented by industry (Andersson et al., 2010).

2.3 Bulk Tank Allocation Problem in Industrial Gas Industry

Integrating distribution and inventory routing problem with tank allocation decisions results in a large scale optimization problem that is very difficult to solve. If solved, the solution will provide economic benefits, increased flexibility, improved robustness, and better coordination in the supply chain. While there is abundant research on the VRPSD and IRP, minimal research was found that integrates this problem with tank allocation for the industrial gas industry. Ellis et al. (2014), You et al. (2011a), and You et al. (2011b) are among the few researchers who address this problem.

The BTA problem as defined by Ellis et al. (2014) allocates or reallocates bulk tanks to customer sites in order to minimize the present value of distribution costs and tank investment costs. The objective of this problem is to minimize the present value of gas distribution, tank installation, tank reallocation, tank refurbishment, and tank purchasing costs. The locations of customers, depots, and tank warehouses are known, as well as each customer's demand and safety stock requirements. Some of the given tank and trailer associated parameters include tank capacities, quantity available of tanks of different types in the warehouse, trailer capacities, and transportation, installation, and refurbishment costs of each tank. Distribution costs include a variable cost based on distances and a fixed access cost to each site. The MIP model described by Ellis et al. (2014) assigns appropriate size tanks to customers and selects a set of suggested delivery routes and amounts while ensuring that all the customer demands are satisfied and none of the practical and financial constraints are violated. The implementation of their BTA model requires the results of a vehicle routing problem, which is known to be NP-hard (Campbell and Savelsbergh, 2004). Given the complexity, Ellis et al. (2014) propose to solve a restricted problem that contains a promising subset of possible routes. The subset of routes is generated through a cluster and route generation subproblem which relies on a sweep-based heuristic. The routes are then input to a tank allocation and route selection subproblem, where a restricted mixed-integer programming model is solved permuting only the previously generated set of routes. The model assigns an appropriate size tank to each customer, suggests a set of delivery routes and amount, and estimates the total distribution cost of the resulting tank allocation.

The estimates of distribution costs using the sweep-based heuristic compares closely to the actual distribution costs in practice. This approach was evaluated using several data sets ranging from a small data set involving a single depot with 5-10 customers to a larger national data set with 18 depots and 1287 customers provided by Air Liquide. The result of the solution approach is an allocation of tanks to customer sites and an estimate of the investment and distribution costs.

You et al. (2011a) address inventory-distribution planning problem in industrial gas distribution, including bulk tank allocation. The BTA problem is modeled as a large scale MIP model to minimize investment costs, change-out costs, and distribution costs. They propose heuristic solution approaches, with a primary focus on a continuous approximation approach. With the continuous approximation approached, detailed vehicle routing parameter and variables are approximated by functions to represent distribution of customer demands and locations, thus simplifying some aspects of the problem. When included in the model, however, the functions result in a mixed integer nonlinear problem (MINLP). The problem is linearized and used to select tank sizes. A comparison of the continuous approximation distribution costs with detailed routing distribution costs is not provided. In addition, the comparisons of the resulting tank selections or distribution costs to actual practice are not provided.

You et al. (2011b) continue exploration of inventory-distribution problems in industrial gas distribution. In this paper, the bulk tank allocation problem considers uncertain customer demand and the loss or addition of new customers with an objective to minimize investment costs, tank change-out costs, and distribution costs. For this problem, customer demand fluctuations are assumed to follow a normal distribution. Customer demand can vary by year, but seasonal demand within a year is not considered. They present a stochastic MINLP model, which relies on a continuous approximation approach. The paper primarily focuses on the computational aspects of the stochastic MINLP model, with problem instances varying from 4 to 200 customers. A comparison of the continuous approximation distribution costs with the detailed routing distribution costs is not provided. In addition, the comparisons of the resulting tank selections or distribution costs to actual practice are not provided. Thus, the suitability of the continuous approximation is not clear. In addition, the effectiveness of the approach in practice is not clear.

Ellis et al. (2014) report the results of solving the BTA problem to optimality for small test case with up to 7 customers but do not present a solution approach. We formulate the BTA problem as a mixed integer programming model and present a branch-and-price exact solution approach to solve the problem to optimality for instances with up to 40 customers. In the branch-and-price solution approach, the problem is reformulated and decomposed to a tank allocation and route selection master problem and a route generation and cost estimation subproblem. Even with the column generation approach, the problem is not solvable for large instances. Therefore, we also propose alternative heuristic approaches for route generation and compare their performance.

The current BTA solution approaches in the literature assume a single tank can be allocated at each customer site. At some customers only a single tank can be assigned due to space limitations, however other customer sites may have the feasibility for allocation of multiple tanks. Due to tank inventory and budget limitations on new tank purchases, it may be beneficial to allocate multiple smaller tanks at a customer site instead of one large tank. This could be particularly beneficial when considering the seasonality in customer demand as one or more tanks installed at a customer can be utilized in different time periods. We propose two alternative mathematical models to explore the possibility and potential benefits of allocating multiple tanks at designated customers with the capacity to accommodate more than one tank.

Furthermore, the approaches presented by Ellis et al. (2014) and You et al. (2011a) assume constant demand for customers. In reality, customer demand frequently varies by time period. Thus, it is important to allocate tanks to effectively accommodate time varying demand. Therefore, we extend the previous model of Ellis et al. (2014) to consider changing demands by period for each customer, resulting in the bulk tank allocation model for time varying demand (BTATVD). Introducing this new dimension to the problem increases complexity, and therefore, we investigate a periodically restricting heuristic (PRH) solution approach. The PRH approach is compared to the results of the BTATVD model to evaluate the effectiveness.

3 Branch-and-price Approach for Bulk Tank Allocation Problem

The bulk tank allocation problem is a complex planning problem with the goal of allocating bulk tanks to customer locations while determining a set of vehicle routes to deliver products to customers and making sure their demand and safety stock requirements are met. In this chapter, the BTA problem is formulated as a mixed integer programming model and then solved to optimality using a decomposition approach, based on column generation. The algorithm presented here is the first known exact solution approach for this problem that solves instances for up to 40 customers. The approach provides a valuable framework for analyzing strategic level decisions while incorporating operational characteristics of the distribution system.

The remainder of this chapter is organized as follows. In section 3.1, the objective of the problem is discussed and the problem is described as an extension to the vehicle routing problem with split deliveries (VRPSD) that incorporates both theoretical and practical constraints. Then problem is modeled as a mixed integer programming (MIP) model, followed by the branch-and-price framework in section 3.2. Details of the route generation subproblem are presented in section 3.3. We conclude with computational results in section 3.4.

3.1 The BTA Problem

We consider an industrial gas distribution network for a single gas product distributed by trailers from multiple depots. The industrial gas company provides bulk tanks to customers and delivers the products to these tanks. The customers assume ownership of the delivered product while the tank itself is owned by the industrial gas company. Tank allocation is an expensive process that involves the costs of purchasing new tanks or refurbishing existing tanks, transporting the tanks, and installing them at customer location. The capacity of the tanks, which are assumed to be allocated at the beginning of the time horizon, directly affect distribution planning. Each customer must be assigned a tank that is large enough to hold each delivery plus the safety stock requirement. Smaller tanks generally require more frequent but smaller deliveries.

Trailers start at depots with a full trailer and then deliver the gas to customers assigned to the depots. In this study, we consider a homogeneous trailer fleet with a given capacity in each depot region. The location of customers, depots, and bulk tank warehouse are known. The tanks initially assigned to customers and the initial number of tanks available in the warehouse are given, as well as a budget for purchasing new tanks and refurbishing existing tanks located at customer locations.

The objective of the BTA model captures the complex trade-off between strategic level investment decisions and operational level routing decision. The model (re-)allocates bulk tanks to customer sites in order to minimize the total of the initial tank investment costs and the present value of the recurring distribution costs. The initial tank investment costs include tank change out or installation, purchasing and refurbishment of the tanks, and transportation of the tanks to customer sites.

The primary assumptions and constraints that we consider for the BTA problem include the following:

- Inventory and assignment constraints:

  We assume that for the delivery process, the trailers leave the depots, visit customers, deliver liquefied product to bulk tanks allocated at customer sites, and then return to the depot. One or more customers can be visited on a delivery route. In addition to the routing constraints, constraints are needed to ensure that a tank is assigned to each customer and that the tank has the capacity to hold the inventory larger than the safety stock requirement for each customer in addition to the replenishment deliveries. We have also included valid inequalities to tighten the formulation and improve solvability by ensuring each customer receives the minimum number of visits required based on its bulk tank capacity and trailer capacity. Constraints are also included to ensure that each delivery is smaller than the trailer capacity and the largest tank capacity minus safety stock requirement for each customer.

- Tank allocation:

The model accounts for the number of allocated tanks of each type. The number of tanks of each type assigned to all customers must not exceed the total number at current customer sites, available in the warehouse, plus new purchased tanks of that type.

- Budgetary constraints:

  We assume budgets for purchasing new tanks and refurbishing the existing tanks. The cost of the tanks purchased and refurbished cannot exceed the allocated budgets.

- Practical constraints:

  The route durations are limited based on the hours of service allowed except for drivers. When route duration is computed, time needed for visiting and filling customer tanks is considered and added to the travel time between stops at customer sites. We assume a fixed estimate of replenishment time per stop. We also limit the number of customers visited on a route.

3.1.1 Mathematical Model

The mathematical model for the BTA problem is formulated using the following notation:
Sets and indices:

P	set of depots indexed by p.
T	set of tank types, indexed by t.
R	set of possible routes, indexed by r.
R_p	subset of routes pertaining to depot p.
I	set of customers, indexed by i, j.
I_p	subset of customers assigned to depot p.
0_p	index for depot $p, \forall p = 1, \ldots, \|P\|$.

Depot and Trailer Parameters:

m	distribution cost per unit distance traveled for trailers.
g_p	volume capacity of each trailer in the fleet of depot p.

ν_p average travel speed of trailers in depot p area.

k_p number of trailers available at the depot p.

ζ_p max number of customers that can be visited on a route in depot p region.

Tank Parameters:

n_t the number of tanks of type t available at the warehouse.

v_t volume of tank type t.

$v_{t\max}$ maximum tank volume where $v_{t\max} = \max_{t\in T} v_t$.

c_t cost to purchase tank type t.

a_{it} indicates if tank type t is initially at customer site i.

b_{wt} cost to change from tank type w to tank type t at a customer site.

ϕ_t cost to refurbish tank type t.

λ_{it} cost per unit distance to transport tank type t to customer site i.

Customer Parameters:

δ_i mass demand for customer i.

σ_i mass amount of safety stock for customer i.

d_{ij} distance from site i to site j, where $i, j \in I \cup p, i \neq j$.

ρ_i working density for customer i (determined by working pressure and product type).

f_i fixed cost to visit customer i.

h_i fixed time to visit customer i.

w_i distance to customer i's assigned warehouse.

Economic Parameters:

PVA monthly discount rate.

β_{purch} total budget allocated to purchase new tanks during the time horizon.

β_{refurb} total budget allocated for refurbishment during the time horizon.

Time Parameters:

η	length of the planning horizon in time periods.
$\tau_{\max}$	maximum allowable time for a route.
k	number of trailers.

Decision Variables:

X_{it}	binary variable that equals 1 if customer i is allocated a tank of type t, and 0 otherwise.
N_{it}	binary variable that indicates if a new tank of type t is purchased for customer i.
M_{it}	binary variable that indicates if customer i receives a tank of type t from the warehouse.
Ψ_r	binary variable that equals 1 if route r is selected, and 0 otherwise.
D_{ir}	amount of gas delivered to customer i on route r (continuous).
Z_{ijr}	binary variable that equals 1 if customer j immediately follows customer i on route r, and 0 otherwise.
Y_{ir}	binary variable that equals 1 is customer i is on route r, and 0 otherwise.
${F_{ij}}^{pqr}$	indicates flow on arc (i,j) for the q^{th} commodity flowing from depot p to node q within route r.

Using the introduced notation, the BTA problem is formulated as follows:

BTA Problem:

$$\text{Minimize} \quad PVA \sum_{p \in P} \sum_{i \in I_p} \sum_{r \in R_p} f_i Y_{ir} + PVA \sum_{p \in P} \sum_{(i,j) \in \{I_p \cup p\}, i \neq j} \sum_{r \in R_p} m d_{ij} Z_{ijr}$$
$$+ \sum_{p \in P} \sum_{i \in I_p} \sum_{\{w,t\} \subseteq T} a_{iw} b_{wt} X_{it} + \sum_{p \in P} \sum_{i \in I_p} \sum_{t \in T} 2\lambda_{it} \omega_i (M_{it} + N_{it}) + \sum_{p \in P} \sum_{i \in I_p} \sum_{t \in T} \phi_t M_{it}$$
$$+ \sum_{p \in P} \sum_{i \in I_p} \sum_{t \in T} c_t N_{it} \tag{1}$$

subject to:

$$\sum_{i\in I} M_{it} \leq n_t + \sum_{i\in I} a_{it}(1 - X_{it}) \qquad \forall t \in T \qquad (2)$$

$$X_{it}(1 - a_{it}) = M_{it} + N_{it} \qquad \forall i \in I, t \in T \qquad (3)$$

$$\sum_{i\in I}\sum_{t\in T} \phi_t M_{it} \leq \beta_{refurb} \qquad (4)$$

$$\sum_{i\in I}\sum_{t\in T} c_t N_{it} \leq \beta_{purch} \qquad (5)$$

$$\sum_{t\in T} X_{it} = 1 \qquad \forall i \in I \qquad (6)$$

$$D_{ir} + \sigma_i \leq \sum_{t\in T} \rho_i v_t X_{it} \qquad \forall i \in I, r \in R \qquad (7)$$

$$\sum_{i\in I} D_{ir} \leq g_p \qquad \forall r \in R \qquad (8)$$

$$\sum_{r\in R} D_{ir} \geq \delta_i \qquad \forall i \in I \qquad (9)$$

$$D_{ir} \leq \min\{g_p, \rho_i v_{\max} - \sigma_i\} y_{ir} \qquad \forall i \in I, r \in R \qquad (10)$$

$$\sum_{r\in R_p} Y_{ir} \geq \sum_{t\in T} \max\left\{\left\lceil\frac{\delta_i}{g}\right\rceil, \left\lceil\frac{\delta_i}{\rho_i v_t - \sigma_i}\right\rceil\right\} X_{it} \qquad \forall i \in I, p \in P \qquad (11)$$

$$\sum_{i\in I_p} h_i Y_{ir} + \sum_{(i,j)\in\{I_p\cup p\}, i\neq j} \frac{d_{ij} Z_{ijr}}{v_p} \leq \tau_{\max} \qquad \forall r \in R, p \in P \qquad (12)$$

$$\sum_{i\in I_p} Y_{ir} \leq \zeta_p \qquad \forall r \in R, p \in P \qquad (13)$$

$$\sum_{r\in R_p}\Big(\sum_{i\in I_p} h_i Y_{ir} + \sum_{(i,j)\in\{I_p\cup p\}, i\neq j} \frac{d_{ij} Z_{ijr}}{v_p}\Big) \leq \eta k_p \qquad \forall p \in P \qquad (14)$$

$$\sum_{i\in I_p\cup p} Z_{ijr} = Y_{jr} \qquad \forall j \in I_p, r \in R_p, p \in P \qquad (15)$$

$$\sum_{j\in I_p\cup p} Z_{ijr} = Y_{ir} \qquad \forall i \in I_p, r \in R_p, p \in P \qquad (16)$$

$$\sum_{i\in I_p}((g+1)(Y_{ir} - Y_{i,r+1}) + (D_{ir} - D_{i,r+1})) \geq 0 \qquad \forall r = 1, ..., |R_p| - 1, p \in P \qquad (17)$$

$$F_{ij}^{pqr} \leq Z_{ijr} \qquad \forall q \in I_p, \{i \in \{I_p \cup\ p,\ i \neq q\}\}, \{j \in I_p,\ j \neq i\}, r \in R_p, p \in P \qquad (18)$$

$$F_{ij}^{pqr} = Z_{ijr} \qquad \forall q \in I_p, i \in I_p \cup 0_p i \neq q, r \in R_p, p \in P \qquad (19)$$

$$\sum_{i \in I_p} F_{0_p i}^{pqr} = Y_{qr} \qquad \forall q \in I_p, r \in R_p, p \in P \tag{20}$$

$$\sum_{i \in I_p \cup 0_p, i \neq q} F_{iq}^{pqr} = Y_{qr} \qquad \forall q \in I_p, r \in R_p, p \in P \tag{21}$$

$$\sum_{j \in I_p i \neq j} F_{ij}^{pqr} = \sum_{j \in I_p \cup 0_p, i \neq j, j \neq q} F_{ji}^{pqr} \qquad \forall q \in I_p, i \in I_p i \neq q, r \in R_p, p \in P \tag{22}$$

X, Y, Z binary

N, M integer

$$0 \leq F \leq 1, D \geq 0 \tag{23}$$

As described, the model includes tank allocation variables and constraints, as well as customer clustering and routing type constraints to provide estimates of distribution costs when a trailer visits more than one customer during a delivery route. Objective function (1) minimizes the net present value of the distribution costs over the planning horizon as well as tank change-out, transportation, refurbishment, and purchase costs. Constraint (2) ensures that the number of tanks of each type t from the warehouse does not exceed the initial inventory plus the tanks of that type removed from customer sites. Constraint (3) ensures that if a customer receives a tank different than its original, the tank is either purchased new or refurbished from the warehouse. Constraints (4) and (5) enforce the budget limits on new tank purchases and tank refurbishment costs. Each customer must be assigned a tank as required by constraint (6). Constraint (7) ensures that the tank assigned to each customer is large enough to hold the delivery amounts plus the safety stock. The total amount delivered on each route should be less than the trailer capacity as enforced by constraint (8). Constraint (9) requires that all customer demands are satisfied. Constraint (10) ensures that each delivery is smaller than the trailer capacity and the capacity of the largest tank minus the safety stock requirement for that customer. It also makes sure that if a route is not selected, there are no deliveries on it. Constraint (11) is a tightening constraint that ensures that each customer receives the minimum number of required delivery visits. The duration of the delivery routes should be less than the maximum allowable time stated in constraint (12). Constraint (13) enforces a practical limit on the number of customers visited on a route in each depot area. Also, as enforced by constraint (14), the total time required to traverse all the selected delivery routes in each depot area should be less than or equal to the study time horizon multiplied by the number trailers available at that depot. The purpose of constraints (15) and (16) are to assign values to route variables. The model is linear with respect to the decision variables and includes symmetry defeating constraints (17) to improve solvability. These symmetry defeating constraints are described by Sherali and Smith (2001). Constraints (18) - (22) are subtour elimination constraints that are polynomial in number (Sherali et al., 2006).

3.2 Branch-and-price Framework

The BTA formulation has an underlying VRPSD problem which is known to be an NP hard problem (Dror et al., 1994). As reported by Ellis et al. (2014), the model is challenging to solve to optimality for even 10 customers within reasonable time. We explore a branch-and-price solution approach to solve larger problems to optimality.

For the branch-and-price approach, we reformulate the BTA problem to form a master problem with all the possible routes as input parameters in order to remove the complicating variables related to routes and their associated constraints. The master problem determines an optimal allocation of tanks to customer sites, selects optimal delivery routes, and determines delivery amounts to customers on different routes. The number of possible delivery routes in the BTA model increases exponentially as the number of customers increases. Thus, the master problem quickly becomes intractable. Therefore, using a branch-and-price approach, we decompose the problem into a reduced master problem (RMP) and a route generation subproblem. The reduced master problem is a mixed-integer program with both binary and continuous decision variables that is solved within a branch-and-bound framework.

Initially the linear programming (LP) relaxation of the master problem is solved within a column generation framework. First, the LP relaxation of the RMP is solved using a subset of feasible delivery routes. Then, in an iterative procedure, new routes are generated by solving a route generation (pricing) subproblem based on the dual variable values to the RMP. New routes (columns) are added and the procedure is repeated. If no integer solution is derived at the current node, then we branch on a non-integer variable and solve the problem again with the addition of branching constraints until reaching an integer solution. The branch-and-price solution algorithm is illustrated in Figure 1.

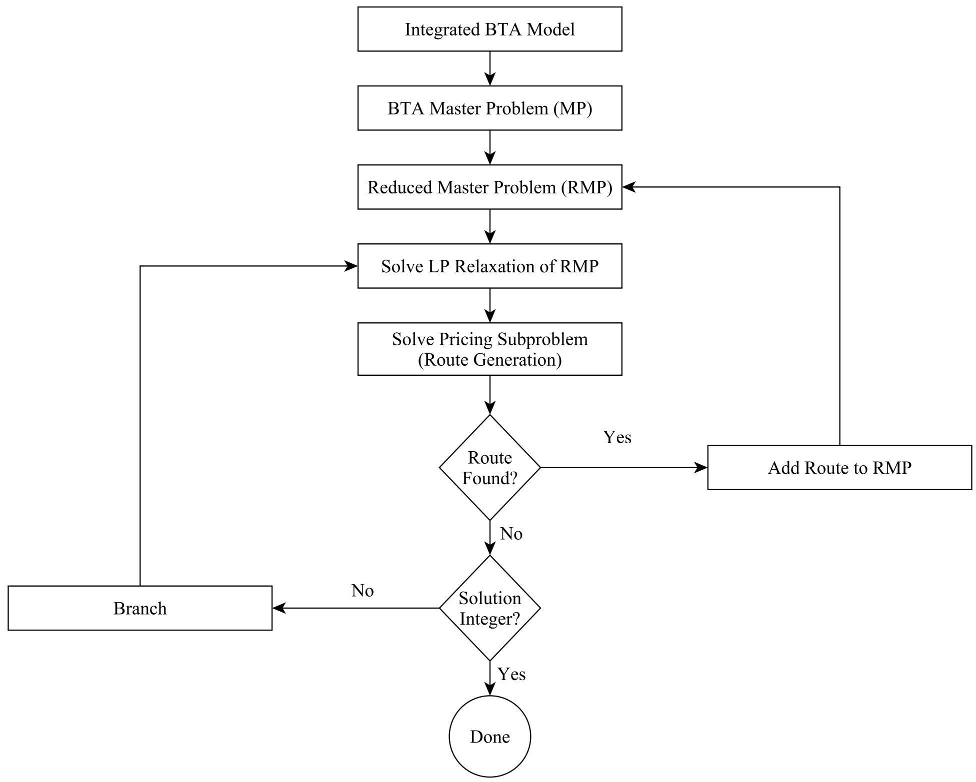


Figure 1: Branch-and-price solution approach for the BTA problem

The following sections describe the branch-and-price framework for solving the BTA problem to optimality. In section 3.2.1, we present a reformulation of the BTA problem to form the master problem assuming that all of the possible delivery routes are input to the model. The problem is decomposed into a reduced master problem and a pricing subproblem. Then in section 3.2.2, a method for obtaining an initial feasible solution to the RMP is described. In section 3.2.3 the LP relaxation of the RMP is considered and solved within a column generation framework. Section 3.2.4 described the branch-and-bound framework that is used to obtain optimal integer solutions. The details of the pricing subproblem is described in section 3.3.

3.2.1 Reformulation of the model

For the branch-and-price approach, we consider a reformulation of the model with a restricted set of routes to form a reduced master problem. We introduce additional notation to describe route related parameters and decision variables. Then, given a set of potential feasible routes the reduced master problem is defined as follows:

Route Parameters and Variables:

φ_r	distribution cost of route r.
t_r	time for route r to be executed by a trailer.
y_{ir}	binary indicator that equals to 1 if customer i is visited on route r, and 0 otherwise.
Ψ_r	binary variable that equals 1 if route r is selected, and 0 otherwise.

BTA Reduced Master Problem:

$$\text{Minimize} \quad PVA \sum_{p \in P} \sum_{r \in R_p} \varphi_r \Psi_r + \sum_{p \in P} \sum_{i \in I_p} \sum_{\{w,t\} \subseteq T} a_{iw} b_{wt} X_{it}$$

$$+ \sum_{p \in P} \sum_{i \in I_p} \sum_{t \in T} 2\lambda_{it}\omega_i (M_{it} + N_{it}) + \sum_{p \in P} \sum_{i \in I_p} \sum_{t \in T} \phi_t M_{it} + \sum_{p \in P} \sum_{i \in I_p} \sum_{t \in T} c_t N_{it} \tag{24}$$

subject to:

$$\sum_{i \in I} M_{it} \leq n_t + \sum_{i \in I} a_{it}(1 - X_{it}) \qquad \forall t \in T \tag{25}$$

$$X_{it}(1 - a_{it}) = M_{it} + N_{it} \qquad \forall i \in I, t \in T \tag{26}$$

$$\sum_{i \in I} \sum_{t \in T} \phi_t M_{it} \leq \beta_{refurb} \tag{27}$$

$$\sum_{i \in I} \sum_{t \in T} c_t N_{it} \leq \beta_{purch} \tag{28}$$

$$\sum_{t \in T} X_{it} = 1 \qquad \forall i \in I \tag{29}$$

$$D_{ir} + \sigma_i \leq \sum_{t \in T} \rho_i v_t X_{it} \qquad \forall i \in I, r \in R \tag{30}$$

$$\sum_{i \in I_p} Y_{ir} D_{ir} \leq g \Psi_r \qquad \forall r \in R_p, p \in P \tag{31}$$

$$\sum_{r \in R_p} Y_{ir} D_{ir} \geq \delta_i \qquad \forall i \in I, p \in P \tag{32}$$

$$D_{ir} \leq \min\{g_p, \rho_i v_{t_{\max}} - \sigma_i\} Y_{ir} \qquad \forall i \in I, r \in R \tag{33}$$

$$\sum_{r \in R_p} Y_{ir} \Psi_r \geq \sum_{t \in T} \max\left\{\left\lceil \frac{\delta_i}{g} \right\rceil, \left\lceil \frac{\delta_i}{\rho_i v_t - \sigma_i} \right\rceil\right\} X_{it} \qquad \forall i \in I, p \in P \tag{34}$$

$$\sum_{r \in R^p} t_r \Psi_r \leq \eta k_p \qquad \forall p \in P \tag{35}$$

X, Ψ binary,

$$D \geq 0 \tag{36}$$

Note that we enforce that

$$\Psi_r = 0 \qquad \text{if } t_r > \tau_{\max}, \qquad \forall r \in R \tag{37}$$

and,

$$X_{it} = 0 \qquad \text{if } \sigma_i > \rho_i v_t, \qquad \forall t \in T, i \in I \tag{38}$$

3.2.2 Initial feasible solution

To initialize the branch-and-price algorithm for the BTA problem, we need an initial set of feasible delivery routes to solve the RMP. Each route corresponds to a variable or column in the RMP. The initial columns are generated by considering direct delivery routes from depots to customer sites and back to the depot (single customer on each route). To ensure feasibility, sufficient number of columns (routes) need to be input to the model. The number of direct (single customer) routes needed for each customer is obtained by using the following expression:

$$max\{\lceil \frac{\delta_i}{g} \rceil, \lceil \frac{\delta_i}{a_{it} \rho_i \nu_t - \sigma_i} \rceil\} \tag{39}$$

In this expression, the maximum of customer demand divided by trailer capacity and customer demand divided by the effective capacity (tank capacity minus the safety stock) of the current assigned tank is rounded up to the next integer value.

3.2.3 Solving the LP relaxation of the master problem

After finding an initial feasible solution, the LP relaxation of the master problem is solved in an iterative procedure using column generation. In each iteration, a restricted master problem is solved containing a subset of delivery routes. The problem is solved in CPLEX and the values of the dual variables for constraints (34) and (35) are collected.

3.2.4 Branch-and-bound framework

The traditional column generation algorithm solves a linear RMP to optimality. Here, we solve the relaxed RMP using column generation, but our route selection variables are required to be integer. Thus, we need to implement a branch-and-bound algorithm to find an optimal integer solution. The branch-and-bound algorithm starts with the initial solution obtained from the algorithm described in Figure 1. If non-integer route selection variables remain, one is picked and two new RMPs are generated with proper bounding constraints. Generated RMPs are added to a list, waiting to be processed. At each iteration of the branch-and-bound algorithm, a problem from the list is solved to optimality using the procedure described in Figure 1. Then, if the solution still includes any non-integer route selection variables, the process is repeated. Otherwise, the integer solution is compared to the best solution so far. If better, the best solution found is updated and a pruning process is implemented to compare the objective function values of the problems waiting to be processed to the new best solution found. If the objective function values of the problem remaining to be solved are worse, then they are removed from the list. The process continues until there are no problems remaining to be processed.

3.3 Pricing Subproblem

For any solution to a linear programming problem, each constraint has an associated dual variable which is known as the reduced cost. In the BTA problem, the reduced cost can be interpreted as the marginal improvement in the objective function if a route is included in the solution. Thus, we define the subproblem with the objective of finding the route with the minimum reduced cost to include in the set of potential routes that are considered by the reduced master problem. We denote the dual variable of the reduced master problem's constraint (n) as π_n. We define the subproblem using the following modified decision variables for each route:

Route Variables:

Z_{ij}	binary variable that equals 1 if customer j immediately follows customer i on route, and 0 otherwise.
Y_i	binary variable that equals 1 if customer i is visited, and 0 otherwise.
${F_{ij}}^{pq}$	indicates flow on arc (i,j) for the q^{th} commodity flowing from depot p to node q.

Pricing Subproblem:

$$\text{Minimize} \textstyle\sum_{i\in I}\sum_{j\in I}(PVA(d_{ij}m + f_iY_i) - \pi_{31i}Y_i - \pi_{32}(d_{ij}/\nu_p)Z_{ij}) \tag{40}$$

subject to:

$$\sum_{i\in I_p\cup p} Z_{ij} = Y_j \qquad \forall j \in I_p, p \in P \tag{41}$$

$$\sum_{j\in I_p\cup p} Z_{ij} = Y_i \qquad \forall j \in I_p, p \in P \tag{42}$$

$$\sum_{i\in I_p} h_iY_i + \sum_{(i,j)\in\{I_p\cup p\}, i\neq j} \frac{d_{ij}Z_{ij}}{v_p} \leq \tau_{\max} \qquad \forall p \in P \tag{43}$$

$$\sum_{i\in I_p} Y_i \leq \zeta_p \qquad \forall r \in R, p \in P \tag{44}$$

$$F_{ij}^{pq} \leq Z_{ij} \qquad \forall q \in I_p, i \in I_p \cup p\ i \neq q, j \in I_pj \neq i, p \in P, \tag{45}$$

$$F_{ij}^{pq} = Z_{ij} \qquad \forall q \in I_p, i \in I_p \cup p\ i \neq q, p \in P, \tag{46}$$

$$\sum_{i\in I_p} F_{pi}^{pq} = Y_q \qquad \forall q \in I_p, p \in P, \tag{47}$$

$$\sum_{i\in I_p\cup p, i\neq q} F_{iq}^{pq} = Y_q \qquad \forall q \in I_p, p \in P, \tag{48}$$

$$\sum_{j \in I_p i \neq j} F_{ij}^{pq} = \sum_{j \in I_p \cup p, i \neq j, j \neq q} F_{ji}^{pq} \qquad \forall q \in I_p, i \in I_p i \neq q, p \in P, \tag{49}$$

Y, Z binary

$$0 \leq F \leq 1 \tag{50}$$

In the subproblem, all the delivery routes start and end at the depot, and each customer can be visited only once on a route. The number of customers that can be visited on a route are limited by capacity of the trailer, route duration, and practical number of customer deliveries on a route. Therefore, the subproblem can be viewed as an elementary shortest path problem with resource constraints (ESPPRC), which is NP-hard in the strong sense (Dror, 1994). The standard approach for solving ESPPRCs in practice is based on dynamic programming with pseudopolynomial complexity (Feillet et al., 2004). By solving the subproblem, the route with the minimum reduced cost is identified and added to the master problem. In order to add the new route to the master problem, we need the associated cost for the objective function, route duration, and the list of customers visited on that route.

3.3.1 Dynamic programming model for the shortest path problem

The pricing subproblem, using the reduced costs, is solved using dynamic programming. The objective of the model is to find the route with minimum reduced cost considering the resource constraint (trailer capacity). Also, considering industrial gas distribution in practice, we limit the number of customers visited on a route.

DP State Variables:

rq	set of remaining nodes to be explored.
rg	remaining trailer capacity.
pr	list of nodes in a partial route.

The value function, V, with the four mentioned state variables is defined as follows:

$$\begin{aligned} V(i, rq, rg, pr) = \min\nolimits^{+}_{j \in rq: \delta_j \leq rg} &((PVA(d_{ij}m + f_i) - \pi_{7i} - \pi_8(d_{ij}/\nu_p)) \\ &+ V(j, rq - \{j\}, rg - \delta_j, pr + \{j\}), \end{aligned} \tag{51}$$

The DP process is terminated either when there are no remaining nodes to be added ($rq = \emptyset$) or the current node is the depot and there are at least two nodes on the partial route ($i \in P$ and $|pr| \geq 2$) as all the single customer routes are generated separately. For the last case, since we end the route at the depot, no other node should be visited after the depot. To find the optimal solution, we calculate the value function with the following variables:

- $i \in P$, route starts from the depot.
- $rq = I \cup P$, set of remaining nodes contains all customers and depots.
- $rg = g$, remaining capacity is equal to the trailer capacity.
- $pr = \emptyset$, only depot is added to the partial route.

In the DP process, decision epochs are node visits. At each decision epoch, only nodes with demand less than remaining trailer capacity are considered and the node with the lowest positive reduced cost is added to the pr. The added node is then removed from the rq and the remaining capacity (rg) is updated. If the demand for the new node violates the remaining capacity, it will not be considered to be added the route. The value function with initial state variables as defined in DP (51) returns the reduced cost for the optimal route and the partial routes variable (pr) will contain the optimal route.

3.3.2 Label correcting algorithm for the shortest path problem

The developed DP (51) considers all the possible routes and becomes intractable quickly as the number of the nodes (i.e., customers) increases. Thus, we use a label correcting algorithm, which finds an elementary shortest path in networks with resource constraints more efficiently than a general DP. We use a modified version of the label correcting algorithm for the elementary shortest path problem with resource constraints (ESPPRC) developed by Feillet et al. (2004). This algorithm is an extension of the Ford-Bellman algorithm (Desrochers, 1988) considering resource constraints. In the algorithm, nodes are iteratively evaluated until no label is changed. When a node is evaluated, all its labels are considered for extension towards all of the node's reachable successors. Only valid extensions are added (i.e., extensions that do not violate any resource constraints), and affected nodes get marked for reevaluation. In contrast to the DP algorithm discussed in the previous section that only generates a single route at each subproblem iteration, the label correcting algorithm finds multiple routes.

Next, we introduce the modified labels used in our modified algorithm, as adopted from Feillet et al. (2004).

Definition 1 *With each path U_{pi} from the depot p to customer i, associate a state $R_i = (rg_i, rd_i, s_i, V_i^1, \ldots, V_i^n)$ corresponding to the remaining trailer capacity (rg_i), remaining duration (rd_i), the number of visited customers, and the visitation vector V_i^j, which is 1 if the path visits customer j and 0 otherwise. Then, a label is defined for each path U_{pi} as (R_i, C_i), where C_i is the cost of the route.*

The modified dominance relation for comparison of two labels is defined as follows:

Definition 2 *Let U'_{pi} and U^*_{pi} be two distinct paths from depot p to customer i with associated labels (R'_i, C'_i) and (R^*_i, C^*_i). U'_{pi} dominates U^*_{pi} if and only if $C'_i \leq C^*_i$, $s'_i \leq s^*_i$, $rg'_i \leq rg^*_i$, $rd'_i \leq rd^*_i$, $V_i^{\prime j} \leq V_i^{*j}$ for $j = 1, \ldots, n$, and $(R'_i, C'_i) \neq (R^*_i, C^*_i)$.*

At each iteration of the algorithm, labels of the node under evaluation are considered for extension towards reachable successors (for definitions of reachable and unreachable nodes refer to Feillet et al. (2004)). Reachability considers both resource usage violations and direct path on the network. Following extension of labels, only non-dominated labels are kept and dominated ones are discarded. The notation used in the algorithm is defined as follows:

Λ_i	List of labels associated with customer i.
$Succ(i)$	Set of successors of depot or customer i. (For each depot i, $Succ(i)$ contains the customers assigned to that depot. Similarly, for each customer i, $Succ(i)$ only contains the customers assigned to the same depot).
E	List of depots and customers waiting to be evaluated.

$Extend(\lambda_i, j)$	Function that returns the label resulting from the extension of label $\lambda_i \in \Lambda_i$ towards customer j when the extension is possible, nothing otherwise. The function first updates the remaining trailer capacity, route duration, and the number of visited customers. If all of the constraints are satisfied, it explores the set of outgoing arcs to update the vector of unreachable nodes and the number of unreachable nodes.
F_{ij}	Set of labels extended from customer i to customer j.
$EFF(\Lambda)$	Procedure that removes dominated labels in the list of labels Λ.

Given the modified definition of labels, dominance relation, reachability, and required notations, we now present the modified version of the Feillet et al. (2004) algorithm for solving the underlying ESPPRC for the BTA problem.

Algorithm 1: ESPPRC label correcting algorithm from Feillet et al. (2004)

begin
  $\Lambda_p \leftarrow \{(0,\ldots,0)\}$;
  for $i \in I$ **do**
    $\Lambda_p \leftarrow \emptyset$
  $E = P$;
  while $E \neq \emptyset$ **do**
    choose $i \in E$;
    for $j \in Succ(i) \cup P$ **do**
      $F_{ij} \leftarrow \emptyset$;
      for $\lambda_i = (rg_i, rd_i, s_i, V_i^1, \ldots, V_i^n, C_i) \in \Lambda_i$ **do**
        if $V_i^j = 0$ **then**
          $F_{ij} \leftarrow F_{ij} \cup Extend(\lambda_i, j)$
      $\Lambda_j \leftarrow EFF(F_{ij} \cup \Lambda_j)$;
      if Λ_j *has changed* **then**
        $E \leftarrow E \cup \{v_j\}$;
    $E \leftarrow E \setminus \{v_i\}$;

Result: Elementary non-dominated paths considering resource constraints

Algorithm 1 initially starts with all the depots in list E for evaluation. At each iteration, one item (depot or customer) is removed from list E and all its labels are extended towards its successors, considering resource constraints. After removing dominated labels from the successors, if any of the successor labels are changed, they are added to E. We keep track of dominated labels that are removed to avoid reevaluating them again. This procedure continues until list E is empty.

3.4 Computational Results

The purpose of the computational study is to evaluate the performance and effectiveness of the proposed optimal solution approach. Therefore, a series of data sets of varying size are generated in collaboration with Air Liquide. The results obtained from the exact solution approach are compared against estimations obtained from the BTA heuristic approach proposed by Ellis et al. (2014) to further evaluate both solution approaches in terms of solution quality and solvability.

In each test case, the number in the Case ID represents the number of customers in that case. We consider 2 depots which are assumed to be co-located with gas sources (production facility). The trailers depart from the depots, visit customers to deliver products, and then return to the depots. Monthly demand for customers ranges from 2,504 (kg) to 1,181,658 (kg). We consider 18 different tank types ranging from 3 m^3 to 195 m^3. Refurbishment costs are assumed to range from 7,422 to 34,551 units. We use industry representative cost data, omit specific information about customers, and omit monetary units to protect sensitive information from our industrial partner. The length of the study time horizon is three years. Additional parameters are summarized in Table 1. The data sets are analyzed on a workstation with two Intel Xeon 3.10GHz quad-core processors, 32GB RAM, and CPLEX v12.5.

Table 1: Case study description

Data Set Description	Monthly Discount Rate	Tank Purchase Budget	Tank Refurbishment Budget	Max Number of Tank Swaps
Different subsets of customers	1%	400,000	100,000	Unrestricted
	Trailer Capacity (kg)	Tank Change-out Cost	Tank Transportation Cost (per km)	Gas Transportation Cost (per km)
	18,000	4,600	1.5	1.5

To validate the branch-and-price solution approach, we compared the obtained results for small cases against the results obtained from solving the BTA model in CPLEX. A test case with 4 customers was solved in 647 seconds in CPLEX while our proposed solution approach solved the same case in 1 seconds. In Table 2, we present the results obtained for 4 test cases using the branch-and-price algorithm as well as the results obtained from

Table 2: Comparison of the results

Case ID	Method	Total Cost	Dist Cost	Tank Exchange Costs	Tank Swaps	Solution Time (Sec)
AB10	B&P	488,237	10,951	158,533	6	<1
AB10	Ellis	494,137	11,146	158,533	6	<1
AB20	B&P	988,757	21,900	329,399	13	6
AB20	Ellis	998,985	22,239	329,399	13	1
AB30	B&P	3,553,730	103,760	429,774	17	83
AB30	Ellis	3,595,595	104,536	448,277	18	1
AB40	B&P	8,270,043	255,274	584,381	23	1480
AB40	Ellis	8,336,165	257,268	590,446	23	1

applying the decomposition approach of Ellis et al. (2014) to the same cases. Note that CPLEX was not able to solve any of these cases. As expected, the exact solution approach generated lower objective function values but required more computational time. In all of the cases the objective function value is within 2% of the results from the heuristic approach proposed by Ellis et al. (2014). This difference is mainly due to lower distribution cost. Our proposed label correcting algorithm was not able to solve the subproblem of a test case with 50 customers within one hour time limit.

4 Alternative Route Generation Modules

In the exact branch and price solution approach of section 3, routes are generated via column generation in an iterative procedure which becomes intractable as the number of customers increase in realistic industry problems. Decomposition-based heuristic solution approaches have been proposed for solving the BTA problem. Specifically, the approach proposed by Ellis et al. (2014) establishes a restricted master problem for the tank allocation problem that relies on a restricted set of routes, generated by a clustering and route construction subproblem as illustrated in Figure 4. The approach requires that a sufficient number of quality routes are generated to support effective tank allocation decisions while still maintaining computational efficiency. The objective of the clustering and route construction subproblem is to generate feasible routes while avoiding impractical or ineffective ones. The approach proposed by Ellis et al. (2014), is based on a sweep algorithm.

The route construction subproblem considers trailer capacities, customer demands, locations, depot, and sourcing assignments when generating routes (along with their cost and time estimates). This subproblem clusters the customers using their location and demand requirements for each depot area and generates delivery routes and provides and estimated cost and duration for each route. These routes are input to the route selection and tank allocation model which allocates tanks to customer sites, selects delivery routes, and determine delivery amounts on each route to each customer. The objective function of the model provides a total estimate for the tank investment costs and the net present value of the recurring distribution cost over the planning time horizon.

The routes provide an estimate of the cost to deliver product to each customer and play a pivotal role in the bulk tank allocation model. Accurately estimating the cost to deliver product to each customer is important to determine the effect of a tank change on the overall distribution network. We evaluate the following three routing algorithms, which have been successfully applied to solve vehicle routing problems (VRP):

- Sweep,
- Nearest neighbor, and

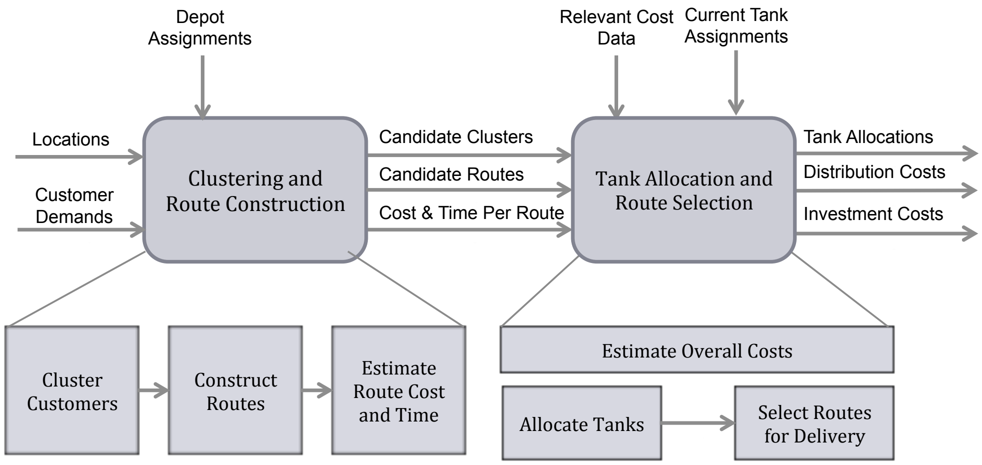


Figure 2: Tank Allocation Solution Approach

- Clark-Wright savings.

In this chapter, our goal is to assess and compare the performance of these three algorithms to determine the preferred algorithm (or combination of algorithms) for the BTA solution approach. The tank allocation and route selection (the restricted master problem for the BTA) is presented in section 4.1. Section 4.2 contains the clustering procedure that is used in all the route generation algorithms. Then, in section 4.3, the algorithmic steps of the three route generation heuristic are presented and their performances are compared in several test cases.

4.1 BTA Restriced Master Problem

The notation used for the tank allocation and route selection model is presented in this section. Using this notation, the BTA restricted master problem is formulated.

Sets and indices:

P	set of depots indexed by p.
Γ	set of sources indexed by γ.
T	set of tank types, indexed by t.
R	set of possible routes, indexed by r.
S	set of trips, indexed by s.
S_r	subset of trips composing route r.

R_p	subset of routes pertaining to depot p.
I	set of customers, indexed by i, j.
I_p	subset of customers assigned to depot p.
T_i	subset of tanks considered for customer i.
Γ_p	subset of sources assigned to serve customer in I_p.

Depot and Trailer Parameters:

m	distribution cost per unit distance traveled for trailers.
g_p	volume capacity of each trailer in the fleet of depot p.
ν_p	average travel speed of trailers in depot p area.
k_p	number of trailers available at the depot p.

Tank Parameters:

n_t	the number of tanks of type t available at the warehouse.
v_t	volume of tank type t.
$v_{t\max}$	maximum tank volume where $v_{t\max} = \max_{t \in T} v_t$.
c_t	cost to purchase tank type t.
a_{it}	indicates if tank type t is currently at customer site i.
b_{wt}	cost to change from tank type w to tank type t at a customer site.
ϕ_t	cost to refurbish tank type t.
λ_{it}	cost per distance to transport tank type t to customer site i.

Customer Parameters:

δ_i	mass demand for customer i.
σ_i	mass amount of safety stock for customer i.
d_{ij}	distance from site i to site j, where $i, j \in I \cup \{0\}, i \neq j$.
ρ_i	working density for customer i (determined by working pressure and product type).
f_i	fixed cost to visit customer i.
h_i	fixed time to visit customer i.

w_i distance to customer i's assigned warehouse.

Route Parameters (route variables when generating routes):

φ_r distribution cost of route r.

t_r time for route r to be executed by a trailer.

y_{is} binary indicator that equals 1 if customer i is visited on trip s, and 0 otherwise.

z_{ijs} binary indicator that equals 1 if customer i is immediately followed by customer j on trip s, and 0 otherwise.

Economic Parameters:

PVA economic conversion factor to convert regularly occurring costs (such as monthly costs) to present value where n represents the time horizon in months and $rate$ represents the monthly discount rate:

$$PVA = \frac{(1 + rate)^n - 1}{rate(1 + rate)^n} \tag{52}$$

β_{purch} total budget allocated to purchase new tanks during the time horizon.

β_{refurb} total budget allocated for refurbishment during the time horizon.

Time Parameters:

η length of the planning horizon.

$\tau_{\max}$ maximum allowable time for a route.

Decision Variables:

X_{it} binary variable that equals 1 if customer i is allocated a tank of type t, and 0 otherwise.

N_{it} binary variable that indicates if a new tank of type t is purchased for customer i.

M_{it} binary variable that indicates if customer i receives a tank of type t from the warehouse.

Ψ_r binary variable that equals 1 if route r is selected, and 0 otherwise.

D_{is} amount of gas delivered to customer i on trip s (continuous).

Using the notation and assuming a set of potential routes is given, the BTA restricted master problem (RMP) is formulated as follows:

BTA-RMP:

$$\text{Minimize } PVA\sum_{r\in R}\varphi_r\Psi_r + \sum_{i\in I}\sum_{\{w,t\}\subseteq T} a_{iw}b_{wt}X_{it} + \sum_{i\in I}\sum_{t\in T} 2\lambda_{it}\omega_i(1-a_{it})X_{it}$$
$$+\left(\sum_{i\in I}\sum_{t\in T}\gamma_t(1-a_{it})X_{it} - \sum_{t\in T}\gamma_t N_t\right) + \sum_{t\in T} c_t N_t \tag{53}$$

subject to:

$$\sum_{i\in I} X_{it} \leq \sum_{i\in I} a_{it} + n_t + N_t, \qquad \forall t\in T \tag{54}$$

$$\sum_{t\in T} c_t N_t \leq \beta_{purch} \tag{55}$$

$$\sum_{t\in T} c_t M_t \leq \beta_{refurb} \tag{56}$$

$$\sum_{t\in T} X_{it} = 1, \qquad \forall i\in I \tag{57}$$

$$X_{it} = 0 \;\text{ if }\; \sigma_i > s_t, \qquad \forall t\in T, i\in I \tag{58}$$

$$D_{is} + \sigma_i \leq \sum_{t\in T} s_t X_{it}, \qquad \forall i\in I_p, s\in S, p\in P \tag{59}$$

$$\sum_{i\in I_p} y_{is}D_{is} \leq g\Psi_r, \qquad \forall s\in S_r, r\in R_p, p\in P \tag{60}$$

$$\sum_{s\in S_r}\sum_{r\in R_p} y_{is}D_{is} = \delta_i, \qquad \forall i\in I_p, p\in P \tag{61}$$

$$D_{is} \leq \min\{g, s_{\max} - \sigma_i\}y_{is}, \qquad \forall i\in I_p, s\in S, p\in P \tag{62}$$

$$\sum_{r\in R^p} y_{is}\Psi_r \geq \sum_{t\in T} max\left\{\left\lceil\frac{\delta_i}{g}\right\rceil, \left\lceil\frac{\delta_i}{s_t-\sigma_i}\right\rceil\right\} X_{it}, \qquad \forall i\in I_p, p\in P \tag{63}$$

$$\Psi_r = 0 \;\text{ if }\; t_r > \tau_{\max}, \qquad \forall r\in R \tag{64}$$

$$\sum_{r \in R^p} t_r \Psi_r \leq \eta k_p, \qquad \forall p \in P \tag{65}$$

$$X, \Psi \text{ binary},$$

$$D \geq 0,$$

$$N \geq 0, \text{ integer} \tag{66}$$

The objective function (53) minimizes the present value of the recurring distribution costs and the one-time tank investment costs. Constraint set (54) ensures that the number of tanks of type t assigned to customers does not exceed the total number of that type available at current customer sites, in the warehouse, plus newly purchased tanks. Constraints (55) and (56) enforce budget limits on the cost of the new tanks purchased and tanks refurbishment. Constraint set (57) makes sure that each customer i is assigned a tank. As delineated by constraint set (58), the tank size at customer i must be at least as large as the safety stock, σ_i, which is specified for each customer. Constraint set (59) estates that the amount delivered to customer i on trip s plus the safety stock, σ_i, must not exceed the allocated tank size. Constraint set (60) ensures that the amount delivered to all the customers on trip s is less than or equal to the trailer capacity, g, when the route containing the trip is selected, or zero otherwise. The delivery amounts to customer i across all routes r must satisfy the demand for customer i, as stated by constraint set (61). Constraint set (62) ensures that the amount of gas delivered to customer i on trip s is less than or equal to the minimum of the trailer capacity or the largest tank size less the safety stock for customer i, or zero if customer i is not visited on route r. The demand for customer i divided by the trailer capacity, or divided by the tank capacity less the safety stock (all rounded up), provide the minimum number of visits required to the customer site, as stated in constraint set (63).Even though 63 is not a required constraint, it serves as a valid inequality that tightens the solution space. Constraint set (64) ensures that the time required for any selected route r does not exceed the maximum allowable route time. Constraint set (65) ensures that the time required for all the selected routes is less than the time horizon η multiplied by the number of available vehicles, k_p, for each depot p. Constraint set (66) represents the logical restrictions on the decision variables.

4.2 Clustering Customers

Before routes are constructed, customers are grouped based primarily on business logic. We assume that customers are served from pre-assigned depots and each customer is also assigned to one or more sources for replenishment. Because there is no direct connection between depots and sources, we create clusters of customers that are served from the same depot and sources. A customer is assigned to a single depot but may be assigned to more than one source. Consider a route that begins by visiting a source that provides low-purity nitrogen. Customers that require high-purity product should not be considered in the set of potential customers for this route. The clustering process ensures that customers are properly grouped with a depot and source before they are considered in the route construction phase. The clustering process is considered as step 0 and is described as follows:

Step 0: Defining clusters of depots, customers, and sources.

For each depot $p \in P$,

- Select the customers, I_p, assigned to depot p.
- Select the set of all sources, Γ_p, that serve customers in I_p.

For each source $\gamma \in \Gamma_p$,

- Select the customers served by source $\gamma \in \Gamma_p$ from depot p to form a cluster.

The construction methods then generate individual routes using the depot, source, and set of customers from a single cluster. On each route, a trailer leaves the cluster depot, obtains product from a source within the cluster, and then delivers product to cluster customers. The trailers may then either conclude the route and return to the depot or may obtain additional product during delivery by refilling at a source. When trailers visit multiple sources during routes, this is referred to as a continuous route. The inclusion of continuous movement adds additional decisions to the route construction process: when to refill, where to refill, and which customers to visit following a refill.

We divide the routes used in the gas distribution system into two categories: single customer routes and multiple customer routes. On a single customer route, gases are delivered on direct routes starting at a depot, visiting a source, replenishing a customer, and then returning to the depot. The costs and durations of these routes are easily estimated using the locations of depots, sources, customers, and the relevant delivery costs. For multiple customer routes where more than one customer is visited on a route, the cost and durations are more difficult to obtain, because both the customer clusters and the sequences that the customers are visited affect the costs and durations of the routes. Customer demand may be satisfied by a combination of single customer or multiple customer routes over the time horizon. Therefore, we need to generate a sufficient number of each type of route to input to the model. We initially normalize all demands δ_i using full load drops (fld_i) and partial load drops (pld_i) where

$$fld_i = \lfloor \frac{\delta_i}{g} \rfloor \tag{67}$$

$$pld_i = \frac{\delta_i}{g} - fld_i \tag{68}$$

The remainder of the demand (pld) is used by the route generation algorithms to generate routes on which multiple customer are visited, as described in section 4.3.

4.3 Route Generation

Estimating the cost to deliver product to each customer plays a pivotal role in the bulk tank allocation model. The more accurate the estimate of the cost to serve a customer, the easier it is to determine the impact of a tank change on the overall distribution network. To more thoroughly represent the industrial gas distribution practices, we improve the routing subproblem of Ellis et al. (2014) by differentiating between depots and gas sources and considering the practice of continuous delivery.

- Depots and sources: The routing subproblem in Ellis et al. (2014) generates candidate routes assuming depots and sources are co-located such that a full trailer departs from a depot, delivers product to up to five customers, then returns to the depot. In reality, a depot is the location in which trailers park and receive maintenance. In order to deliver product, trailers must first obtain liquefied product from a source, which may or may not be co-located with the depot. Customers may have additional product requirements such as purity of product provided by certain sources. The allocation of customers to sources is provided as input to ensure that these requirements are met.

- Continuous delivery: The consideration of sources allows for the possibility for a trailer to refill product during a delivery route. Under certain circumstances, a trailer may refill at a source and serve additional customers before returning to the depot. Modeling this continuous delivery practice provides a more reflective estimate of the distribution costs in practice. Two common delivery practices considered in the network are shown in Figure 3. A trailer following the route (a) fills at source S1, visits two customers, refills at source S1, then delivers to three more customers before returning to the depot. A trailer following route (b) fills at source S1, visits two customers, refills at source S2, then delivers to three customers before returning to the depot. Note that in route (b), only customers assigned to receive product from this different source may be considered after refill.

  When generating routes for the model to evaluate, we incorporate this continuous delivery practice. A trailer obtains product from a source, then it begins a polar sweep to add additional customers to the trip. Eventually, the trailer may consider refilling on product if the product level is below a threshold. Then the algorithm searches the set of sources allocated to serve customers assigned to the current depot for the one nearest the trailer's current location. If this source is different from the last one, the trailer refills on product and proceeds to serve customers assigned to this new source.

Using this modified route generation algorithm, we solve the restricted bulk tank allocation problem to determine the allocation of bulk tanks to customer sites for each period. The resulting tank assignments for all the individual periods are used to construct a set of candidate tanks, T_i, for each customer i.

In this section, we briefly review the steps involved in the sweep, nearest neighbor, and Clark-Wright savings heuristic algorithms to generate input routes for BTA. We apply the methods to several data sets and compare the results.

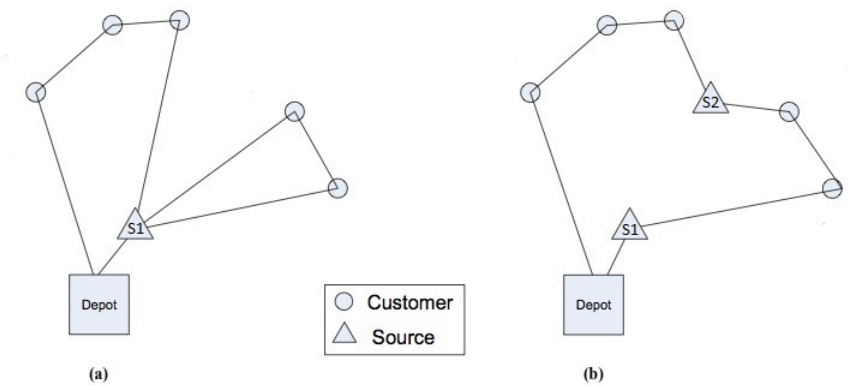


Figure 3: Two Examples of Routes with Sources

As a large number of potential routes render the model too difficult to solve, we use the following limits to reduce the number and increase the feasibility and practicality of the generated routes:

- Limited Trip Size: Because a trailer has finite capacity, only a limited number of customers are served between refills in practice. The parameter, *maximum trip size*, is considered by the algorithms to limit the number of customers on each trip.

- Refill Threshold: As the trailer continues delivering product to customers, the amount of product in its tank decreases. When the amount of product in a trailer tank drops bellow a pre-defined *refill threshold*, we consider refilling at product sources.

- Limited Number of Refills: As the number of trailer refills at sources increases, routes are more likely to exceed allowed work hours of drivers. The parameter, *maximum source visits*, limits the number of additional source visits to refill to provide additional control over the construction process.

- Maximum Route Time: Government or company-specific regulations often limit the maximum time a driver may work. Routes with times that exceed this limit, *maximum route time*, are removed from the set of potential routes.

After the route construction algorithms are executed, the multiple customer routes constructed by the algorithms may include a large set of customers and multiple source visits. These routes are referred to as master routes. A post processing step of route generation, the sub-routes created while constructing a master route will also be added as candidates routes. For example, suppose the following route was output from one of the construction algorithm: $Depot \rightarrow s_1 \rightarrow i_1 \rightarrow i_2 \rightarrow s_1 \rightarrow i_3 \rightarrow Depot$ where $s_1$is a source and i_1, i_2, and i_3 are customer sites. Then the following ordered subsets would also be considered candidate routes:

$Depot \rightarrow s_1 \rightarrow i_1 \rightarrow Depot$

$Depot \rightarrow s_1 \rightarrow i_1 \rightarrow i_2 \rightarrow Depot$

$Depot \rightarrow s_1 \rightarrow i_1 \rightarrow i_2 \rightarrow s_1 \rightarrow i_3 \rightarrow Depot$

As each route can only be selected once by the tank allocation and route selection master problem, copies of each route are created to allow quality routes to be utilized multiple times.

4.3.1 Sweep Heuristic

The first algorithm that we explore for multiple customer route construction is a modification of the sweep algorithm introduced by Ellis et al. (2014). The original algorithm was an adaptation of the sweep and petal methods used to solve VRPs (Gillett and Miller, 1974; Ryan et al., 1993). A modified version of the algorithm has been adapted to allow continuous movement and addresses the decision of when to visit a source to refill, which source to visit, and which customers to serve following a refill. Before constructing routes for a depot-source cluster, customers are sorted by polar coordinates. All customers in this cluster (formed in step 0) have been assigned to the source, and the algorithm begins by representing each customer i in polar coordinates (ρ_i, θ_i) with the source as the pole or origin and a polar axis $(1, 0)$. The customers are then sorted in non-decreasing order of θ_i. In the following algorithm, the route construction heuristic starts from a source, $\gamma \in \Gamma_p$. Let N be the number of customers assigned to this source and R be the set of routes generated. Each customer has the opportunity to lead the sweep procedure, indicated by i_{start}.

Initialization Set $i_{start} = 1$ and $R = \emptyset$.

Step 1 (Route Start) Create a new route r, which begins at the cluster depot, visits the cluster source, and start to consider customers in the clockwise direction from the i_{start} position on the ordered list.

Step 2 (Sweep) If the number of customers served on r has reached some limit, go to Step 3 to consider a refill. Otherwise, consider adding customer i_{next} to the current route. If the trailer capacity can serve i_{next} then add i_{next} to r, set $i_{next} = i_{next} + 1$, and repeat Step 2. If the remaining trailer capacity cannot serve i_{next} and the capacity is below a pre-specified threshold, go to Step 3 to refill. If the remaining capacity cannot serve customer i_{next} but is above the refill threshold, set $i_{next} = i_{next} + 1$ and repeat Step 2.

Step 3 (Consider Refilling) If some maximum number of refills has been reached, go to Step 4 and end the route. Otherwise, search the set of sources serving customers from this depot, Γ_p, for the source nearest to the last customer served. If the nearest source is the original cluster source, visit to refill the trailer with product and go to Step 2 to continue serving customers. If this source is different from the last one, the trailer refills on product and proceeds to serve customers assigned to this new source. This process is denoted as *hopping* and is described later in this section.

Step 4 (End Route) Add the route to the set of completed routes $R = R \cup r$. If $i_{start} < N$ then set $i_{start} = i_{start} + 1$ and go to Step 1.

The algorithm is applied to each source and the associated customers. In the procedure shown above, customers are ordered and swept through in the clockwise direction. The heuristic is also run in the counterclockwise direction to generate additional candidate routes.

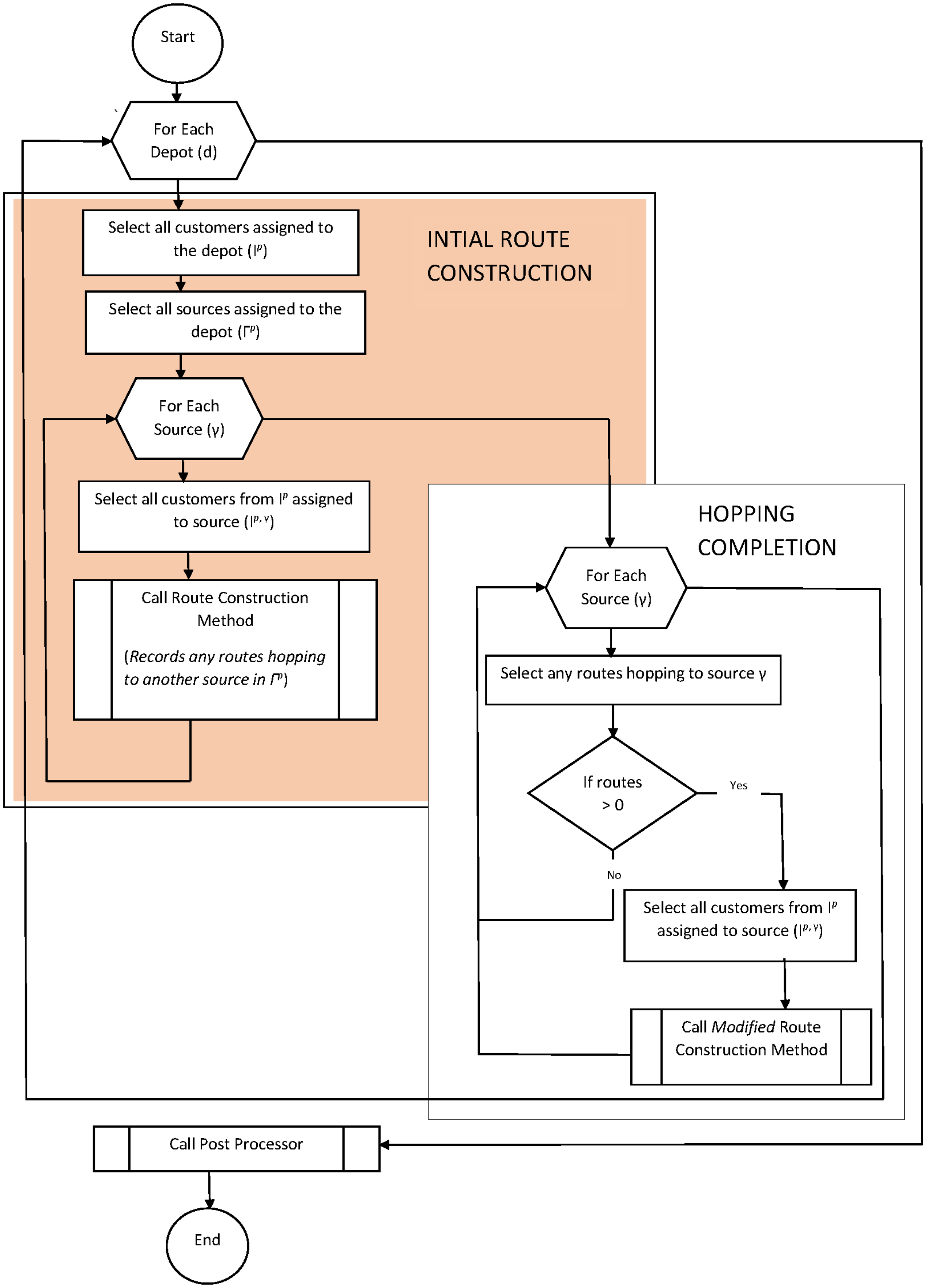
During the process of route construction, it is possible for a trailer to obtain product from multiple sources on a single route. After a trailer obtains product from a source, it begins a polar sweep to add additional customers to the trip. Eventually, the trailer may consider refilling on product. The algorithm then searches the set of sources allocated to serve customers assigned to the current depot for the one nearest the trailers current location. If this source is different from the last one, the trailer refills on product and proceeds to serve customers assigned to this new source. This process is denoted as *hopping* because the trailer is moving to serve a new set of customers. When a route hops from one source to another, a new set of set of customers must be considered. The figure below details the implementation of the clustering and route generation procedures to incorporate hopping. The *Initial Route Construction* phase proceeds exactly as described. The key detail is that when the algorithm constructs a route hopping to another source, the route and hop source are recorded. After initial route have been constructed for each source, the *Hopping Completion* phase iterates over each source once more and checks for any route hopping into this source.

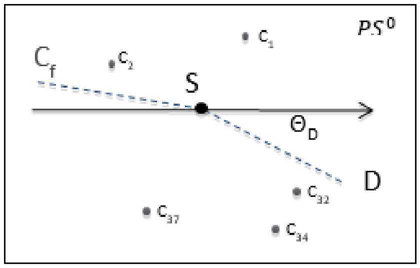
After refilling on product at a new source, the next customer to visit is determined. The assumption made is that after hopping, the trailer is most likely low on time. Therefore, any sweep should be in the direction of the depot. The *modified route construction method* is identical to the original route construction method with the following exceptions:

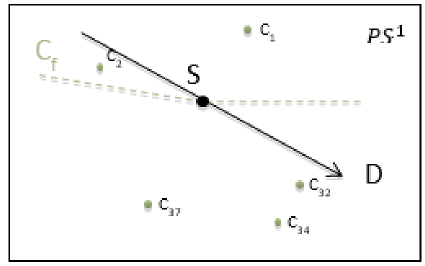
- Hopping or refilling is not allowed
- The customers considered during this sweep are in the direction of the depot

In order to determine which customers should be considered in the polar sweep after hopping, the following algorithm is used to place an upper and lower bound on the sweep angle:

1. Select all customers assigned to receive product from this particular source.
2. Order customers in polar system PS^0
   - Each customer i is represented in polar coordinates (ρ_i, θ_i) with the source as the pole and a polar axis $(1, 0)$.
   - The customers are sorted in non-decreasing order of θ.
3. Rotate PS^0 such that the line (SD) becomes the polar axis for polar system PS^1. This new system eases the clustering of customers and the determination of the sweep direction.
   - For each customer i, calculate $(\theta_i)^1 = (\theta_i)^0 - (\theta_D)^0$


Start
For Each Depot (d)
Select all customers assigned to the depot (I^p)
INTIAL ROUTE CONSTRUCTION
Select all sources assigned to the depot (Γ^p)
For Each Source (γ)
Select all customers from I^p assigned to source ($I^{p,\gamma}$)
Call Route Construction Method
(*Records any routes hopping to another source in Γ^p*)
HOPPING COMPLETION
For Each Source (γ)
Select any routes hopping to source γ
If routes > 0
Yes
No
Select all customers from I^p assigned to source ($I^{p,\gamma}$)
Call *Modified* Route Construction Method
Call Post Processor
End





- If $(\theta_i)^1 < -\pi$ then $(\theta_i)^1 + = 2\pi$
- If $(\theta_i)^1 > \pi$ then $(\theta_i)^1 - = 2\pi$

4. For each partially constructed route hopping to this source, select the appropriate set of customers to consider using the rotated PS^1 and construct the next trip.

   - The last customer visited on route k before hopping is customer C_f.
   - Calculate $\theta_f^1 = \theta_f^0 - \theta_D^0$
   - Determine the bounds of the customers we will consider
     - If $\theta_f^1 > 0$
       * $\theta_{LB}^1 = min[0, (\theta_f^1 - \pi/2)]$
       * $\theta_{UB}^1 = \theta_f^1$
     - If $\theta_f^1 < 0$
       * $\theta_{LB}^1 = \theta_f^1$
       * $\theta_{UB}^1 = max[0, (\theta_f^1 + \pi/2)]$

4.3.2 Nearest Neighbor

The second algorithm that we explore for route generation is the nearest neighbor algorithm. With the nearest neighbor algorithm, the idea is to combine customers that are close in proximity based on pld_i to utilize trailers more efficiently. This algorithm begins with a customer and searches for the nearest customer in the same cluster until certain criteria are met. In our case the termination criterion are the *maximum route duration* and the *trailer capacity*. The route construction heuristic starts from a source, $\gamma \in \Gamma_p$. Let N be the number of customers assigned to this source and R be the set of routes generated. Each customer has the opportunity to lead the nearest neighbor procedure, indicated by i_{start}. Let $NN_{i_{start}}$ indicate the nearest neighbor list starting from customer i_{start}, and i_{next} indicate the customer at the top of the $NN_{i_{start}}$. The algorithm is described as follows:

Initialization Set $i_{start} = 1$ and $R = \emptyset$.

Step 1 (Nearest neighbor list) To form $NN_{i_{start}}$, we start by adding the nearest customer to i_{start} in $I_p - i_{start}$ to the list. Then, we continue by adding the nearest customer in $I_p - (NN_{i_{start}} \cup i_{start})$ to the last added customer to the list. We continue until $I_p - (NN_{i_{start}} \cup i_{start}) = \emptyset$.

Step 2 (Route start) Create a new route r, which begins at the cluster depot, visits the cluster source, and visits customer i_{start}.

Step 3 (Visit nearest neighbor) If the number of customers served on r has reached the *maximum trip size* limit, go to Step 4 to consider a refill. Otherwise, consider adding the customer i_{next} to r.

If the trailer capacity can serve i_{next} (if remaining trailer capacity is greater than $pld_{i_{next}}$), then add i_{next} to r and remove it from $NN_{i_{start}}$, and repeat Step 3. If the remaining trailer capacity cannot serve i_{next} and is below a *refill threshold*, go to Step 4 to refill. If the remaining capacity cannot serve customer i_{next} but is above the refill threshold, remove i_{next} from $NN_{i_{start}}$.

Step 4 (Consider refilling) If the *maximum source visits* has been reached, go to Step 5 and end the route. Otherwise, search the set of sources serving customers from this depot, Γ_p, for the source nearest the last customer served and i_{next}. Visit the nearest source and go to Step 3.

Step 5 (End route) Add the route to the set of completed routes $R = R \cup r$. If $i_{start} < N$ then set $i_{start} = i_{start} + 1$ and go to Step 1.

The hopping procedure is similar the one describe for sweep algorithm and only considers the customer in the direction of the depot.

4.3.3 Clark-Wright Savings

The third heuristic algorithm that we explore is a savings algorithm which is widely used for solving VRPs. In this algorithm, we start with the set of single customer routes and only consider the pld_is. The savings associated with combining each pair of customers and visiting them on a single combined route is calculated. Then, the routes are sequentially combined based on the savings until one of the stopping criteria defined for routes are met.

Initialization Build a single-customer route for each customer.

Step 1 (Savings list) Calculate saving S_{ij} for every pair of customer i and customer j as defined by the following:

$$S_{ij} = d_{i,depot} + d_{depot,j} - d_{ij} \tag{69}$$

Sort the list in non ijs sort the list in non-increasing order.

Step 2 (Combine routes) Consider every S_{ij} from the top of the savings list. Determine whether there exist two routes that can be combined. The criteria include:

- Customer i is the last customer visited on route r.
- Total partial load drops for the two routes is less than or equal to 1.
- Total time of the resulting combined route is less than the maximum route time.

If all criteria are satisfied, two routes are combined by removing the connections $(depot, i)$ and $(j, depot)$ and connecting (i, j). The total partial load drops and the total route time are updated. Step 2 is repeated until all (i, j) pairs are evaluated and no route can be combined.

Step 3 (Consider refilling) If the *maximum number of refills* has been reached for all the routes, then end the routes. Else, for all the routes, search the set of sources serving customers from this depot, Γ_p for the source nearest the last customer served on that route. Visit the nearest source and go to Step 2.

The hopping procedure is similar the one describe for sweep and nearest neighbor algorithm. We only consider the customer in the direction of the depot to be added and select the customer that results in largest savings when included in the route.

4.3.4 Performance Comparison

In this section, we compare the performance of these route generation algorithms when used separately and together. We present the results obtained using test cases with 12 to 100 customers. The results are summarized in Table 3. The number in the name of each test case represent the number of customers in that data set. All of the parameters are the same for each case except the input routes. For each case, we present the results obtained from only using routes generated by sweep based method (S), nearest neighbor (NN), or Clark-Wright savings algorithm (CW). Then we input combination of routes generated by two or all three of the algorithms. We also compare the results of including a single copy of routes generated by two or three algorithms against multiple copies of routes generated by individual algorithm. The multiplier represents the number of copies of each route that are input to the model. In all of the test cases, we include same sufficient number of single customer routes to ensure feasibility.

Based on the results obtained from using individual algorithms, the sweep heuristic consistently generates the most efficient routes and results in the lowest objective function values. The next best performing algorithm is the nearest neighbor, and the Clark-Wright savings algorithm has the poorest performance. Due to the structure of the Clark-Wright savings algorithm, it generates fewer routes. Thus the other two algorithms, each customer is set as the initial customer visited on a route and this will result in producing alternative routes. With Clark-Wright savings algorithm, routes are generated based on the sorted list of savings and this result in fewer multiple customer routes. Thus, the model is required to select more single customer routes when the Clark-Wright savings algorithm is used to generate multiple customer routes. Therefore, we do not recommend using this algorithm individually.

When comparing the results of including a single copy of the multiple customer routes against including 3 copies, as expected, the value of the objective function increases when

Table 3: Comparison of route generation algorithms for small test cases

Case	Algorithm	Obj Fn	Dist Cost	Swaps	Sol time	Multiplier
12C	S	564215.88	11892.78	8	0.02	3
12C	NN	566744.49	11976.77	9	0.02	3
12C	CW	572298.06	12161.22	8	0.01	3
12C	S+NN	565330.19	11929.79	8	0.04	3+3
12C	S+NN+CW	565104.02	11922.28	8	0.03	3+3+3
24C	S	1625310.01	40589.58	16	0.21	3
24C	NN	1628536.08	40805.71	16	0.11	3
24C	CW	1645931.44	40647.98	17	0	3
24C	S+NN	1626030.77	41469.85	15	0.66	3+3
24C	S+NN+CW	1629939.66	41746.01	15	0.79	3+3+3
I50	S	1048951.90	28387.08	4	0.03	3
I50	NN	1066340.39	28913.30	4	0.01	3
I50	CW	1133214.18	30336.35	5	0.01	3
I50	S+NN	1059107.77	29205.24	3	0.03	3+3
I50	S+NN+CW	1051401.84	28980.05	3	0.02	3+3+3
I100	S	1560478.13	43137.68	4	0.09	3
I100	NN	1528956.10	41670.17	5	0.09	3
I100	CW	1863848.83	49723.12	8	0.01	3
I100	S+NN	1525617.76	42123.90	4	0.27	3+3
I100	S+NN+CW	1523097.67	42057.21	4	0.22	3+3+3

* The highlighted objective function values represent the three best values for each data set

including a single copy due to selection of less efficient routes. When comparing the results of including single copies of the collection of routes generated using all algorithms against multiple copies of routes generated only using each individual algorithm, an improvement in solution quality is often observed without much sacrifice in the computational time.

Overall, the best solution in terms of objective function is observed when using multiple copies of the collection of routes from all algorithms. Including a single copy of the combined set of routes generated using all algorithms often outperforms using one of the algorithms to generate input routes. To conclude, it is recommended to use the routes generated using all three algorithms collectively and inputing multiple copies of them if the computational resources are available. When comparing the use of routes generated using individual algorithms, the sweep algorithm is the recommended heuristic followed by nearest neighbor, and Clark-Wright savings.

5 Allocating Multiple Tanks at Customer Sites

In this chapter, the BTA model described in section 4.1 is extended to explore the potential benefits of allocating multiple tanks at designated customer sites that have the capacity to accommodate more than one tank. Due to tank inventory and budget limitations on new tank purchases, it may be beneficial to allocate multiple smaller tanks at a customer site, rather than a single larger tank. We present two generalizations of the BTA model to support the allocation of multiple tanks to customer sites and compare the performance of the models. The first model focuses on the number of tanks of each type assigned to each customer, namely a tank-typed-based approach. The tank-type-base model assumes each customer has a single tank allocated initially. The second model explicitly considers the potential for multiple pads to accommodate tanks at each customer, namely a pad-based approach. With the pad-based approach, customers may have multiple tanks allocated initially. We demonstrate the potential benefits and compare the effectiveness of the developed approaches using a cases study with 20 customers.

5.1 Tank-type-based Approach (MBTA-I)

In the tank typed-based approach, we assume that all of the customers initially have at most one assigned tank. We explore the benefits of exchanging the current tank and adding additional tanks to customer locations. We introduce an additional parameter for the maximum number of tanks that a customer can accommodate based on available space at customer locations. We also modify the decision variable types to allow the assignment of multiple tanks as follows:

Additional Parameters:

MT_i maximum number of tanks that customer i can accommodate.

Modified Decision Variables:

X_{it} integer variable indicating the number of tanks of type t allocated to customer i.

Z_{it} integer variable indicating the number of tanks of type t allocated to customer i in addition to the initially assigned tank t.

W_{it} binary variable indicating change in initial tank of type t for customer i.

N_{it} integer variable indicating the number of new tanks of type t purchased for customer i.

M_{it} integer variable indicating the number of tanks of type t moved from the warehouse to customer i.

Using this notation, the multiple bulk tank allocation - tank type based model (MBTA-I) is formulated as follows:

MBTA-I:

$$\text{Minimize } PVA \sum_{p\in P}\sum_{r\in R_p} \varphi_r \Psi_r + \sum_{p\in P}\sum_{i\in I_p}\sum_{\{w,t\}\subseteq T, t\neq w} a_{iw} b_{wt} X_{it}$$
$$+ \sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T} b_{tt} Z_{it} + \sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T} 2\lambda_{it}\omega_i (M_{it} + N_{it})$$
$$+ \sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T} \phi_t M_{it} + \sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T} c_t N_{it} \tag{70}$$

subject to:

$$Z_{it} \geq a_{it}(X_{it} - 1), \qquad \forall i \in I, t \in T, \tag{71}$$
$$W_{it} \geq a_{it}(1 - X_{it}), \qquad \forall i \in I, t \in T, \tag{72}$$
$$\sum_{i\in I} M_{it} \leq n_t + \sum_{i\in I} W_{it}, \qquad \forall t \in T, \tag{73}$$
$$X_{it} - a_{it} \leq M_{it} + N_{it}, \qquad \forall i \in I, t \in T, \tag{74}$$
$$\sum_{i\in I}\sum_{t\in T} \phi_t M_{it} \leq \beta_{refurb}, \tag{75}$$
$$\sum_{i\in I}\sum_{t\in T} c_t N_{it} \leq \beta_{purch}, \tag{76}$$
$$\sum_{t\in T} X_{it} \leq MT_i, \qquad \forall i \in I, \tag{77}$$
$$D_{is} + \sigma_i \leq \sum_{t\in T} \rho_i \upsilon_t X_{it}, \qquad \forall i \in I, s \in S \tag{78}$$

$$\sum_{i \in I_p} y_{is} D_{is} \leq g_p \Psi_r, \qquad \forall r \in R_p, s \in S_r, p \in P \tag{79}$$

$$\sum_{s \in S_r} \sum_{r \in R_p} y_{is} D_{is} = \delta_i, \qquad \forall i \in I, p \in P \tag{80}$$

$$D_{is} \leq \min\{g_p, MT_i \rho_i v_{t_{\max}} - \sigma_i\} y_{is}, \qquad \forall i \in I, s \in S \tag{81}$$

$$\sum_{s \in S_r} \sum_{r \in R_p} y_{is} \Psi_r \geq \left\lceil \frac{\delta_i}{g_p} \right\rceil, \qquad \forall i \in I, p \in P \tag{82}$$

$$\sum_{r \in R^p} t_r \Psi_r \leq \eta k_p, \qquad \forall p \in P, \tag{83}$$

$$X, Z \text{ Integer},$$

$$\Psi \text{ binary},$$

$$D \geq 0, \tag{84}$$

The first term in the objective function (70) minimizes the reoccurring distribution costs that are converted to present value using the PVA factor. The second term calculates the installation costs of tanks that are different than the initial tank allocated at customer sites. The third term calculates the cost of allocating additional tanks of the same type as the initial tank at a customer location. The remaining terms are associated with transportation costs of purchased and refurbished tanks moved from the warehouse to customers, total refurbishment costs, and total new tank purchase costs respectively. Constraint (71) defines the Z_{it} variable in relation to the X_{it} variable, and constraint (72) does the same for variable W_{it}. Constraint (73) keeps track of the number of tanks of each type that are moved to and from the warehouse. Constraint (74) makes sure that new or additional tanks of type t assigned to customer i are either newly purchased or moved from the warehouse inventory. Constraints (75) and (76) enforce budget limits on purchase and refurbishment costs. Constraint (77) ensures that the number of tanks allocated at each customer are not more than the available space at that customer. Constraint (78) requires that the total tank capacity installed at customer i is larger than each deliver to that customer plus its safety stock. The total delivery amount on each trip must be less than the trailer capacity. If the trip is not selected, no deliveries should be made on that trip, as stated in constraint (79). Constraint (80) ensures that customer demands are met. The amount delivered to customer i on each trip should be less than or equal to the minimum of the trailer capacity or the maximum total tank mass capacity available to be located at customer i (estimated by the maximum tank capacity available times the maximum number of tanks that customer i can accommodate) as stated by constraint (81). Constraint (82) serves to tighten the lower bound and ensures that each customer receives a minimum number of required trailer visits based on its demand and trailer capacity. Constraint (83) ensures that the time required for all the selected routes is less than the time horizon multiplied by the number of available vehicles for each depot. Constraint set (84) represents the logical restrictions on the decision variables. In this model the integer decision variables X_{it} allow for tanks of type t to be selected several times for customer i.

5.2 Pad-based Approach (MBTA-II)

In the pad-based approach, we differentiate between the tanks that are assigned at each customer site by explicitly considering multiple pads available for tanks at designated customers. Each pad can accommodate a single tank and the maximum number of pads for each customer is pre-determined. With this modeling approach we can relax the assumption of customers having a single tank initially. We can also capture the cost of multiple tank swaps at each customer by keeping track of the tanks assigned to each pad at a customer location. Additional sets and parameters, modifications to decision variables, and the mathematical model are presented as follows:

Additional Sets, Indices, and Parameters:

Θ_i	set of pads available for tanks at customer i indexed by θ.
$a_{i\theta t}$	indicates if tank type t is currently assigned to pad θ at customer site i

Modified Decision Variables:

$X_{i\theta t}$	binary variable indicating if tanks of type t is allocated to pad θ at customer i.

$N_{i\theta t}$ binary variable indicating if new tank of type t is purchased to pad θ at customer i.

$M_{i\theta t}$ binary variable if tank of type t is moved from the warehouse to pad θ customer i.

MBTA-II:

$$\text{Minimize} \quad PVA \sum_{p\in P}\sum_{r\in R_p} \varphi_r \Psi_r + \sum_{p\in P}\sum_{i\in I_p}\sum_{\theta\in\Theta_i}\sum_{\{w,t\}\subseteq T} a_{i\theta w} b_{wt} X_{i\theta t}$$
$$+\sum_{p\in P}\sum_{i\in I_p}\sum_{\theta\in\Theta_i}\sum_{t\in T} 2\lambda_{it}\omega_i (M_{i\theta t} + N_{i\theta t}) + \sum_{p\in P}\sum_{i\in I_p}\sum_{\theta\in\Theta_i}\sum_{t\in T} \phi_t M_{i\theta t}$$
$$+\sum_{p\in P}\sum_{i\in I_p}\sum_{\theta\in\Theta_i}\sum_{t\in T} c_t N_{i\theta t} \tag{85}$$

subject to:

$$\sum_{i\in I}\sum_{\theta\in\Theta_i} M_{i\theta t} \leq n_t + \sum_{i\in I}\sum_{\theta\in\Theta_i} a_{i\theta t}(1 - X_{i\theta t}), \qquad \forall t \in T \tag{86}$$

$$X_{i\theta t}(1 - a_{i\theta t}) = M_{i\theta t} + N_{i\theta t}, \qquad \forall i \in I, t \in T, \theta \in \Theta \tag{87}$$

$$\sum_{i\in I}\sum_{\theta\in\Theta_i}\sum_{t\in T} \phi_t M_{i\theta t} \leq \beta_{refurb}, \tag{88}$$

$$\sum_{i\in I}\sum_{\theta\in\Theta_i}\sum_{t\in T} c_t N_{i\theta t} \leq \beta_{purch}, \tag{89}$$

$$\sum_{t\in T} X_{i\theta t} = 1, \qquad \forall i \in I, \theta \in \Theta_i \tag{90}$$

$$D_{is} + \sigma_i \leq \sum_{t\in T}\sum_{\theta\in\Theta_i} \rho_i v_t X_{i\theta t}, \qquad \forall i \in I, s \in S \tag{91}$$

$$\sum_{i\in I_p} y_{is} D_{is} \leq g_p \Psi_r, \qquad \forall r \in R_p, s \in S_r, p \in P \tag{92}$$

$$\sum_{s\in S_r}\sum_{r\in R_p} y_{is} D_{is} = \delta_i, \qquad \forall i \in I, p \in P \tag{93}$$

$$D_{is} \leq \min\{g_p, MT_i \rho_i v_{t_{\max}} - \sigma_i\} y_{is}, \qquad \forall i \in I, s \in S \tag{94}$$

$$\sum_{s\in S_r}\sum_{r\in R_p} y_{is} \Psi_r \geq \left\lceil \frac{\delta_i}{g_p} \right\rceil, \qquad \forall i \in I, p \in P \tag{95}$$

$$\sum_{r\in R^p} t_r \Psi_r \leq \eta k_p, \qquad \forall p \in P \tag{96}$$

$$X, \Psi \text{ binary},$$

$$D \geq 0, \tag{97}$$

In MBTA-II, objective function 85, minimizes the total cost of selected routes for distribution as well as total costs associated with exchanging, transporting, refurbishing, and purchasing tanks allocated in all the pads at all customer locations. Constraint (86) - (89) serve similar roles as constraints (73) - (76). Constraint (90) requires that each pad is assigned exactly one tank. Note that we have defined an additional "dummy" tank type with negligible capacity, purchase, and refurbishment costs in order to maintain validity of this constraint. We also assume there is abundant inventory of this tank type available in the warehouse. The remaining constraints (91) - (97) serve similar purposes as the corresponding constraints (78)-(84) for MBTA-I.

5.3 Case Study

To evaluate and compare the MBTA-I and MBTA-II models, we use a data set with 20 customers. All customers in the data set are currently allocated a single tank. We assume that 6 customers have space to accommodate up to 3 tanks, 8 customers can accommodate upto 2 tanks, and the remaining customers are limited to a single tank. The tank parameters are summarized in Table 5.

We have solved this case using BTA, MBTA-I, and MBTA-II, and the results are summarized in Table 6. All cases are solved on a workstation with two Intel Xeon 3.10GHz quad-core processors, 32GB RAM, and CPLEX v12.5. Model BTA assumes at most one tank type is assigned to each customer and suggests two tank swaps for this data set. The computational time required is less than 1 second. Model MBTA-I suggests three tank changes, two of which are second tanks allocated to customer sites. Model MBTA-I requires 10.41 minutes of computational time. Model MBTA-II yields results similar to MBTA-I and solves in 5.25 minutes. Both the MBTA-I and MBTA-II results prescribe multiple tanks at two customer sites, yield 13% savings over the BTA results in which each customer is limited to a single tank.

Table 5: Tank Parameters

Tank ID	Inventory	Water Volume (m3)	Purchase Cost	Refurbishment Cost
0	1000	0	0	0
1200	16	1.2	10000000	6800
3000	32	3	26535	6800
5000	15	5	10000000	6800
6000	0	6	30470	7500
8200	10	8.2	10000000	7500
10000	18	10	37642	7500
16000	6	16	10000000	9000
18000	0	18	10000000	9000
20000	4	20	55098	9000
23000	0	23	10000000	9000
27000	0	27	10000000	10000
33000	2	33	10000000	10000
35000	0	35	72203	10000
40000	0	40	10000000	12000
50000	2	50	90794	12000
60000	0	60	97834	12000
70000	0	70	10000000	12000
80000	0	80	10000000	12000
100000	0	100	10000000	12000
120000	0	120	10000000	12000
150000	0	150	10000000	12000

To evaluate the capability of MBTA-II to support the initial allocation of multiple tanks at customer sites, we modify the test case IB20 to create IB20A. With IB20A, two customers (customers 4 and 12) initially have two tanks. This model generates results similar to the results of MBTA-II applied to IB20 and requires 4.91 minutes of computational time.

Details of the final tank allocations for these cases are presented in Tables 7 - 10. Table 7 contains the resulting tanks from the BTA (single tank) approach. In this case, two tank exchanges are recommended for customer 4 and customer 7. The model suggests replacing customer 4's tank type 1200 with a larger tank (type 33000). The model also suggests replacing customer 7's 8200 tank with a large tank (type 50000). When applying MBTA-I and MBTA-II, multiple tank allocations are allowed. Customer 4 receives tank type 33000 in addition to its original tank type 1200 (rather than replacing it). Customer 7 receives

Table 6: Multiple tank allocation results for a 20 customer case

Case	**Model**	**# of Pads**	**Obj Fn Value**	**Dist Cost**	**Sol Time (min)**	**Tank Swaps**	**Opt Gap**	**Savings**
Ib20	BTA	1	311,260.12	12,320.06	0.01	2	0.0112	
Ib20	MBTA-I	3	270,731.91	9,571.52	10.41	3	0.02	13%
Ib20	MBTA-II	3	270,731.91	9,571.52	5.25	3	0.02	13%
Ib20A	MBTA-II	3	270,731.91	9,571.52	4.91	3	0.02	13%

two tanks, one tank type 50000 similar to BTA results and an additional tank type 20000. Although the total capacity of these two tanks is equal to the capacity of tank type 70000, the expensive purchase cost of tank type 70000, results in the prescription of two smaller tanks. In Ib20A customers 4 and 12 initially have two tanks, and the suggested changes are similar to the changes suggested by MBTA-II for Ib20.

In this section, we have addressed the bulk tank allocation problem and have extended the previous BTA model to account for possibility of allocating multiple tanks at some customer sites. We proposed two mathematical models for multiple tank allocation and have studied their performance on several test cases. Initial analysis shows potential savings of 13% in a test case with 20 customers.

Table 7: BTA tank allocations results for Ib20

Customer	**Max # of Tanks**	**Initial Tank**	**New Tank**
1	1	3000	3000
2	1	5000	5000
3	1	5000	5000
4	1	1200	33000
5	1	1200	1200
6	1	1200	1200
7	1	8200	50000
8	1	3000	3000
9	1	16000	16000
10	1	8200	8200
11	1	5000	5000
12	1	3000	3000
13	1	5000	5000
14	1	5000	5000
15	1	16000	16000
16	1	8200	8200
17	1	100000	100000
18	1	10000	10000
19	1	6000	6000
20	1	16000	16000

Table 8: MBTA-I tank allocations for Ib20

Customer ID	Max # of Tanks	Initial Tank	New Tank 1	New Tank 2	New Tank 3
1	3	3000	3000		
2	3	5000	5000		
3	3	5000	5000		
4	3	1200	1200	33000	
5	3	1200	1200		
6	3	1200	1200		
7	2	8200	20000	50000	
8	2	3000	3000		
9	2	16000	16000		
10	2	8200	8200		
11	2	5000	5000		
12	2	3000	3000		
13	2	5000	5000		
14	2	5000	5000		
15	2	16000	16000		
16	1	8200	8200		
17	1	100000	100000		
18	1	10000	10000		
19	1	6000	6000		
20	1	16000	16000		

Table 9: MBTA-II tank allocations for Ib20

Cust	**# of pads**	**Initial Tank Pad 1**	**New Tank Pad 1**	**Initial Tank Pad 2**	**New Tank Pad 2**	**Initial Tank Pad 3**	**New Tank Pad 3**
1	3	3000	3000	0	0	0	0
2	3	5000	5000	0	0	0	0
3	3	5000	5000	0	0	0	0
4	3	1200	1200	0	0	0	33000
5	3	1200	1200	0	0	0	0
6	3	1200	1200	0	0	0	0
7	2	8200	20000	0	50000		
8	2	3000	3000	0	0		
9	2	16000	16000	0	0		
10	2	8200	8200	0	0		
11	2	5000	5000	0	0		
12	2	3000	3000	0	0		
13	2	5000	5000	0	0		
14	2	5000	5000	0	0		
15	2	16000	16000	0	0		
16	1	8200	8200				
17	1	100000	100000				
18	1	10000	10000				
19	1	6000	6000				
20	1	16000	16000				

Table 10: MBTA-II tank allocations for Ib20A

Cust	**# of pads**	**Initial Tank Pad 1**	**New Tank Pad 1**	**Initial Tank Pad 2**	**New Tank Pad 2**	**Initial Tank Pad 3**	**New Tank Pad 3**
1	3	3000	3000	0	0	0	0
2	3	5000	5000	0	0	0	0
3	3	5000	5000	0	0	0	0
4	3	1200	1200	1200	1200	1200	33000
5	3	1200	1200	0	0	0	0
6	3	1200	1200	0	0	0	0
7	2	8200	20000	0	50000		
8	2	3000	3000	0	0		
9	2	16000	16000	0	0		
10	2	8200	8200	0	0		
11	2	5000	5000	0	0		
12	2	3000	3000	1200	1200		
13	2	5000	5000	0	0		
14	2	5000	5000	0	0		
15	2	16000	16000	0	0		
16	1	8200	8200				
17	1	100000	100000				
18	1	10000	10000				
19	1	6000	6000				
20	1	16000	16000				

6 Bulk Tank Allocation Problem for Customers with Time Varying Demand

In the practice of industrial gas distribution, bulk tanks that are allocated to customer sites typically remain in place for multiple years. Air Liquide reports that the expected average time for a particular tank to remain at a customer site is approximately seven years. Because customer demand often varies over time, it is important to allocate tanks that effectively accommodate time varying demand.

We consider the bulk tank allocation problem for industrial gas distribution systems where customer demand varies over time, extending previous research that assumed constant demand. We develop solution approaches to determine the preferred size of bulk tanks to assign to customer sites to minimize recurring gas distribution costs and initial tank installation costs while accommodating time varying customer demand. The problem is modeled as a mixed-integer program (MIP) and then solved using a two-phase decomposition heuristic. We evaluate the solution approaches using data sets provided by an industrial partner and show potential savings of the developed approaches over two different planning time horizons.

In this chapter, we formally present a MIP model for the bulk tank allocation problem for time varying demand (BTATVD) problem and explore modifications to improve the solvability of the model. Then, we introduce our two phase PRH solution approach and discuss the modifications that we made to the previously developed routing subproblem. The developed methodologies are demonstrated using modified data sets provided by Air Liquide, and we close with a scalability analysis.

6.1 Problem Description

We consider an industrial gas distribution network for a single gas product distributed by trailers from multiple gas sources. The tanks, which are assumed to be allocated at the beginning of the time horizon, directly affect distribution planning. Each customer must be assigned a tank that is large enough to hold each delivery plus the safety stock requirement of that customer. Smaller tanks will require more frequent but smaller deliveries. In different time periods, a customer may be replenished on different routes. For example, in a high demand period, if a customer has a tank large enough to receive a full trailer load, that customer may be visited on a direct route. Delivery to the same customer can be combined with others so that the customer is replenished on a multiple customer route in a time period with lower demand. Note that the high demand period for one customer may be the low demand period for another.

Trailers start at depots, obtain product from gas sources, and then deliver the gas to customers assigned to those depots and sources. Trailers can provide continuous delivery by visiting sources in between deliveries to refill their tanks. In this context, the entire tour from depot departure to arrival back at the depot is referred to as a *route*, and the set of customers served between source visits is a single *trip*. In this study we consider a homogeneous trailer fleet with a given capacity. The location of customers, depots, and bulk tank warehouse are known and customers are assigned to sources and depots. The initial number of tanks available in the warehouse is given as well as a budget for purchasing new tanks and refurbishing existing tanks located at customer locations. We assume that customer demand for each period is given and each customer is replenished from the depot to which it is assigned. Delivery routes along with their costs and duration are assumed to be parameters for a restricted model. However, parameters associated with routes are the decision variables when generating routes in a separate route generation module. The amount delivered on each route is a decision variable and is determined by the model.

The model minimizes the sum of the initial cost of tank installation, tank transportation from the warehouse, tank refurbishment, and new tank purchase cost and the present value of the periodic distribution costs. The challenge is to allocate an expensive resource over a long time horizon (e.g. 7 years) while accommodating customer demand that fluctuates over each time period (e.g. quarter). Although the focus of this study is for a single gas type, the model is adaptable for multiple gas types.

6.1.1 Notation

The notation used in the proposed model is summarized as follows:

Sets and indices:

P	set of depots indexed by p.
Q	set of periods indexed by q.
T	set of tank types, indexed by t.
R	set of possible routes, indexed by r.
S	set of trips, indexed by s.
S_r	subset of trips composing route r.
R_p	subset of routes pertaining to depot p.
I	set of customers, indexed by i, j.
I_p	subset of customers assigned to depot p.
T_i	subset of tanks considered for customer i.

Depot and Trailer Parameters:

m	distribution cost per unit distance traveled for trailers.
g_p	volume capacity of each trailer in the fleet of depot p.
ν_p	average travel speed of trailers in depot p area.
k_p	number of trailers available at the depot p.

Tank Parameters:

n_t	the number of tanks of type t available at the warehouse.
v_t	volume of tank type t.
$v_{t\max}$	maximum tank volume where $v_{t\max} = \max_{t \in T} v_t$.
c_t	cost to purchase tank type t.
a_{it}	indicates if tank type t is currently at customer site i.
b_{wt}	cost to change from tank type w to tank type t at a customer site.
ϕ_t	cost to refurbish tank type t.
λ_{it}	cost per distance to transport tank type t to customer site i.

Customer Parameters:

δ_{iq}	mass demand for customer i during period q.

σ_i mass amount of safety stock for customer i.

d_{ij} distance from site i to site j, where $i, j \in I \cup p, i \neq j$.

ρ_i working density for customer i (determined by working pressure and product type).

f_i fixed cost to visit customer i.

h_i fixed time to visit customer i.

w_i distance to customer i's assigned warehouse.

Route Parameters (route variables when generating routes):

φ_r distribution cost of route r.

t_r time for route r to be executed by a trailer.

y_{is} binary indicator that equals 1 if customer i is visited on trip s, and 0 otherwise.

z_{ijs} binary indicator that equals 1 if customer i is immediately followed by customer j on trip s, and 0 otherwise.

Economic Parameters:

ι periodical discount rate.

β_{purch} total budget allocated to purchase new tanks during the time horizon.

β_{refurb} total budget allocated for refurbishment during the time horizon.

Time Parameters:

η length of the planning horizon in time periods.

$\tau_{\max}$ maximum allowable time for a route.

κ length of each period.

ξ number of periods in one year.

Decision Variables:

X_{it} binary variable that equals 1 if customer i is allocated a tank of type t, and 0 otherwise.

N_{it} binary variable that indicates if a new tank of type t is purchased for customer i.

M_{it} binary variable that indicates if customer i receives a tank of type t from the warehouse.

Ψ_{rq} binary variable that equals 1 if route r is selected during period q, and 0 otherwise.

D_{isq} amount of gas delivered to customer i on trip s during period q (continuous).

6.1.2 Model formulation for BTATVD

The bulk tank allocation for time varying demand (BTATVD) model captures complex trade-offs between strategic level investment decisions and operational level routing decisions. The model simultaneously allocates tank types, selects distribution routes from a set of candidate routes, and determines delivery amounts. For a given set of potential routes, the problem can be formulated as follows:

BTATVD:

$$\text{Minimize} \sum_{p\in P}\sum_{r\in R_p}\sum_{q\in Q}\varphi_r\Psi_{rq}/(1+\iota)^q + \sum_{p\in P}\sum_{i\in I_p}\sum_{\{w,t\}\subseteq T} a_{iw}b_{wt}X_{it}$$

$$+\sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T}2\lambda_{it}\omega_i(M_{it}+N_{it}) + \sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T}\phi_t M_{it} + \sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T}c_t N_{it} \tag{98}$$

subject to:

$$\sum_{i\in I} M_{it} \leq n_t + \sum_{i\in I} a_{it}(1-X_{it}), \qquad \forall t\in T, \tag{99}$$

$$X_{it}(1-a_{it}) = M_{it}+N_{it}, \qquad \forall i\in I, t\in T, \tag{100}$$

$$\sum_{i\in I}\sum_{t\in T}\phi_t M_{it} \leq \beta_{refurb}, \tag{101}$$

$$\sum_{i\in I}\sum_{t\in T} c_t N_{it} \leq \beta_{purch}, \tag{102}$$

$$\sum_{t\in T} X_{it} = 1, \qquad \forall i\in I, \tag{103}$$

$$D_{isq} + \sigma_i \leq \sum_{t \in T} \rho_i v_t X_{it}, \qquad \forall i \in I, s \in S, q \in Q \qquad (104)$$

$$\sum_{i \in I_p} y_{is} D_{isq} \leq g_p \Psi_{rq}, \qquad \forall r \in R_p, s \in S_r, p \in P, q \in Q \qquad (105)$$

$$\sum_{s \in S_r} \sum_{r \in R_p} y_{is} D_{isq} = \delta_{iq}, \qquad \forall i \in I, p \in P, q \in Q \qquad (106)$$

$$D_{isq} \leq \min\{g_p, \rho_i v_{t_{\max}} - \sigma_i\} y_{is}, \qquad \forall i \in I, s \in S, q \in Q \qquad (107)$$

$$\sum_{s \in S_r} \sum_{r \in R_p} y_{is} \Psi_{rq} \geq \sum_{t \in T} \max \left\{ \left\lceil \frac{\delta_{iq}}{g_p} \right\rceil, \left\lceil \frac{\delta_{iq}}{\rho_i v_t - \sigma_i} \right\rceil \right\} X_{it}, \quad \forall i \in I, p \in P, q \in Q \qquad (108)$$

$$\sum_{r \in R^p} \sum_{q \in Q} t_r \Psi_{rq} \leq \eta k_p, \qquad \forall p \in P, \qquad (109)$$

$$X, \Psi \text{ binary},$$

$$D \geq 0 \qquad (110)$$

Note that we enforce that

$$\Psi_{rq} = 0 \qquad \text{if } t_r > \tau_{\max}, \qquad \forall r \in R, q \in Q. \qquad (111)$$

and,

$$X_{it} = 0 \qquad \text{if } \sigma_i > \rho_i v_t, \qquad \forall t \in T, i \in I, \qquad (112)$$

The objective function (98) minimizes the net present value of the recurring distribution costs and one-time tank installation, tank transportation from the warehouse, tank refurbishment, and tank purchase costs. Constraint set (99) ensures that the number of tanks of type t moved from the warehouse does not exceed the warehouse inventory and the tanks returned to the warehouse. For each customer i, with a new assigned tank type t, the tank can be either obtained from the warehouse or newly purchased, as stated by constraint set (100). Constraint sets (101) and (102) ensure that the budget limits for tanks refurbishment and procurement are not exceeded. Each customer i must be assigned a tank, as stated by constraint set (103). Constraint set (104) ensures that the amount of each delivery to customer i on trip s in period q plus the safety stock requirements for that customer does not exceed the allocated tank mass capacity. Constraint set (105) ensures that the amount delivered to all the customers on trip s in each period is less than or equal to the trailer capacity, g, when the route containing trip s is selected, or zero otherwise. The delivery amounts to customer i across all routes r in each period q must meet the requirements for customer i in that period, as enforced by constraint set (106). Constraint set (107) ensures that the amount of gas delivered to customer i on trip s in period q is less than or equal to the minimum of the trailer capacity or the largest tank size less the safety stock for customer i, or zero if customer i is not visited on trip s in period q. The minimum number of visits required to the customer site in period q, is determined by the requirements for customer i divided by the trailer capacity or the requirements for customer i divided by the tank capacity less the safety stock (all rounded up), as stated in constraint set (108). Constraint set (109) ensures that the time required for all the selected routes across all periods does not exceed the time horizon η multiplied by the number of available vehicles, k_p, for each depot p. Constraint set (110) represents the logical restrictions on the decision variables. Constraint set (111) ensures that routes with duration longer than the maximum allowable time for a route are not selected and constraint set (112) ensures that tanks with mass capacity less than the safety stock requirements of customer i are not considered for assignment to customer i.

Moreover, the cost for route $r \in R$ to be executed by a trailer includes the cost to access customer sites and the cost to transport the gas on the distribution route. This cost is defined as follows:

$$\varphi_{r_p} \equiv \sum_{i \in I} \sum_{s \in S_r} f_i y_{is} + \sum_{i,j \in I \cup p, i \neq j} \sum_{s \in S_r} m \cdot d_{ij} z_{ijs}. \tag{113}$$

The time for route $r \in R$, to be executed by a trailer is defined as follows:

$$t_{r_p} \equiv \sum_{i \in I} \sum_{s \in S_r} h_i y_{is} + \sum_{i,j \in I \cup p, i \neq j} \sum_{s \in S_r} \frac{d_{ij} z_{ijs}}{\nu_p}. \tag{114}$$

6.1.3 Model formulation for recurring demand (BTATVD-R)

For demand that varies within a year but is stationary over multiple years, the demand in each period of year one is equal to the demand in corresponding periods of subsequent years. In this case, we can solve the BTATVD model for a single year while considering the overall time horizon. We modify the objective function using an economic conversion factor (PV) to account for the time horizon length and convert recurring costs to present value using effective interest rate. Effective annual interest rate is calculated as $(1+\iota)^{\xi}-1$. When values recur annually, their present value is calculated as follows:

$$PV = 1 + \frac{(1+\iota)^{\eta-\xi} - 1}{((1+\iota)^{\xi} - 1)(1+\iota)^{\eta-\xi}} \tag{115}$$

Note that we assume that the first year cost occurs at the beginning of the year. The objective function is revised as follows:

$$\begin{aligned} \text{Minimize } PV \sum_{p\in P}\sum_{r\in R_p}\sum_{q\in Q} \varphi_r \Psi_{rq}/(1+\iota)^q + \sum_{p\in P}\sum_{i\in I_p}\sum_{\{w,t\}\subseteq T} a_{iw} b_{wt} X_{it} \\ + \sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T} 2\lambda_{it}\omega_i (M_{it}+N_{it}) + \sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T} \phi_t M_{it} + \sum_{p\in P}\sum_{i\in I_p}\sum_{t\in T} c_t N_{it} \end{aligned} \tag{116}$$

For the remainder of this document, we refer to BTATVD with recurring demand as BTATVD-R.

6.2 Solution Approach

The BTATVD problem can be viewed as a variant of a multidimensional knapsack problem with additional constraints. Considering the budget limits for tank purchase and tank (re-)allocations, the goal is to identify a subset of customers, for which different tanks will contribute the most to the distribution cost savings. Therefore, our problem also belongs to the category of NP-hard problems. Given the complexity of the BTATVD problem, we present a two-phase heuristic algorithm to solve BTATVD model. In the formulation, set R includes all the possible routes that start at a depot, visit a source, deliver to at least one customer, and return to the depot. The number of routes for each period increases exponentially as the number of customers increases which results in an intractable problem even for a small to moderate problem size.

The multiple time periods of model BTATVD also increases the number of time periods and adds complexity to the problem. Adapting the decomposition-based approach proposed by Ellis et al. (2014) for the BTA problem, our periodically restricting heuristic (PRH) for the BTATVD problem includes the following main phases:

- Solve bulk tank allocation period by period to determine candidate tanks
- Determine tank allocations for varying demand over multiple periods

For the expected demand in each period, the preferred tank size is allocated to each customer by solving the BTA model for expected demand in each period. Then across all periods, each customer has a set of candidate suitable tanks. These candidate tanks are considered when solving the tank allocation problem across the multiple time periods. Thus, the size of the solution space is decreased by restricting the number of tank types that the model considers for each customer.

6.2.1 Solve bulk tank allocation with constant demand period by period to determine candidate tanks

The objective of phase I in our solution approach is to develop a set of preferred candidate tanks for each customer. In this phase, the bulk tank allocation problem is solved for each period assuming that the monthly demand over the time horizon is equal to the monthly demand in that period. Then we implement the decomposition approach presented in Ellis et al. (2014) in which first a set of high quality routes are generated. In this approach, two types of routes are generated: single-customer routes that only contain a direct trip from a depot to a customer and back to the depot, and multiple-customer routes in which more than one customer is visited on a route. To generate the multiple-customer routes, the customers are clustered using a sweep heuristics. First, the customers are sorted by corresponding polar coordinates. Then the customers are clustered either clockwise or counterclockwise according to the polar coordinates until the total residual partial demands in a cluster reaches the trailer capacity. This process is repeated for all of the depots. Then a heuristic for the traveling salesman problem (TSP) is solved to determine the sequence in which the customers should be visited. The output is a set of routes with their corresponding cost and duration which are input to a restricted bulk tank allocation problem as presented in Ellis et al. (2014) to determine the allocation of bulk tanks to customer sites.

6.2.2 Determine tank allocations for varying demand over multiple periods

In phase II, we solve a restricted multi-period bulk tank allocation problem, where the model selects from one of the tanks in set T_i for each customer i. Potential routes are generated using the sweep algorithm assuming that each customer's demand is equal to their maximum demand across all time periods to ensure sufficient routes are available during each time period. Constraint sets (103) and (104) are revised as follows:

$$\sum_{t \in T_i} X_{it} = 1, \qquad \forall i \in I, \tag{117}$$

$$D_{irq} + \sigma_i \leq \sum_{t \in T_i} \rho_t v_t X_{it}, \qquad \forall i \in I, r \in R, q \in Q \tag{118}$$

Constraint set (117) assigns exactly one tank from each customer's set of candidate tanks to that customer, and constraint set (118) ensures that the amount of each delivery to customer i on route r in period q plus safety stock requirements for that customer does not exceed the mass capacity of the allocated tank from the set of candidate tanks. Since sets of T_i are obtained from solving the BTA model which includes constraint set (112), the set of candidate tanks T_i for each customer i only contains tank types that have larger mass capacity than safety stock requirements for customer i. Therefore, constraint set (112) can be eliminated from the restricted phase two model. Figure 4 summarizes the overall solution approach.

6.3 Case Study Results

To evaluate the performance and scalability of the proposed models and the PRH approach we conduct analyses based upon industrial data sets. We compare the performance of BTATVD model with the model with recurring demand assumption (BTATVD-R) and the

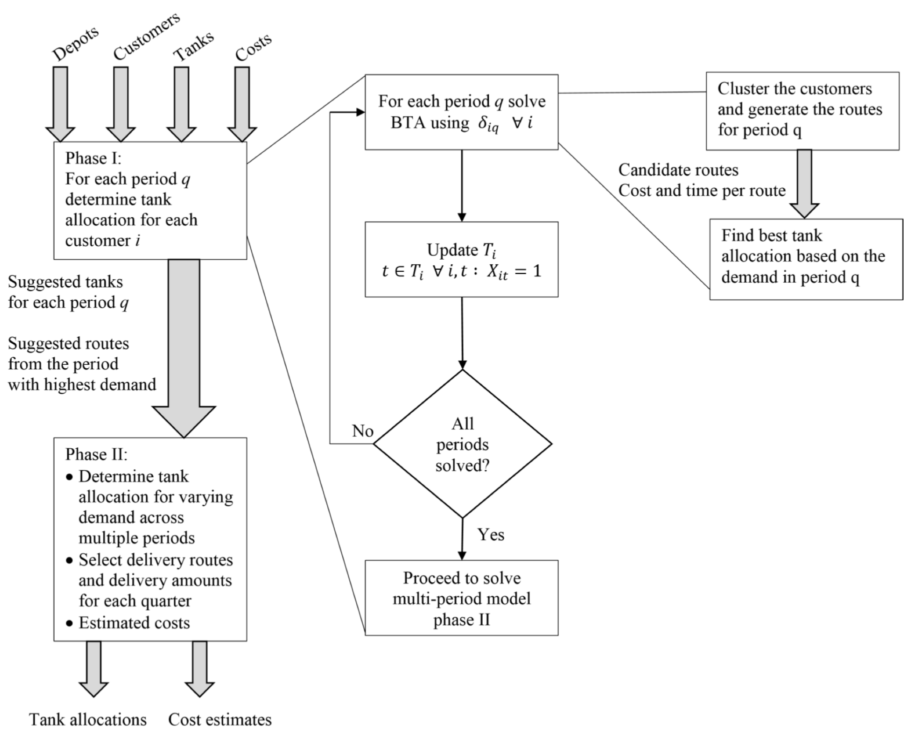


Figure 4: Bulk Tank Allocation for Time Varying Demand Approach

proposed PRH. In these case studies, we use industry representative cost data, omit specific information about customers, and omit monetary units to protect sensitive information from our industrial partner. These data sets range from 50 to 818 customers and the number included in the case name indicates the number of customers in that data set. In the test cases, the customers are assigned to 8 different depots and we consider 21 different tank types with capacities ranging from 1.2 m^3 to 150 m^3. Refurbishment costs are assumed to range from 6,800 to 12,000. In scenario I, we consider a 3 year time horizon with 3 month periods (i.e. 4 periods in a year). For scenario II, we extend the length of study time horizon to 7 years with 3 month periods. Table 11 summarizes the parameter values of our test cases. The data sets were analyzed on a workstation with two Intel Xeon 3.10GHz quad-core processors, 32GB RAM, and CPLEX v12.5.

In Table 99, we present the results obtained for each scenario. First we use the BTATVD model, with the original objective function (98) in which the demand is assumed to be changing for all the periods in the time horizon. Then we use the BTATVD-R model, with

Table 11: Case study description

Data Set Description	Periodic Discount Rate	Tank Purchase Budget	Tank Refurbishment Budget	Max Number of Tank Swaps
Different subsets of customers	3%	1,000,000	300,000	Unrestricted
	Trailer Capacity (kg)	Tank Change-out Cost	Tank Transportation Cost (per km)	Gas Transportation Cost (per km)
	22,000	8,000	1.8	0.8

the modified objective function (116) where we assume that the demand is recurring. In order for the values to be comparable, we assume that the demand for years 2 and 3 remain the same as the first year. Finally, we test the performance of the PRH by comparing the results obtained from this approach to the other two models.

For the BTATVD and BTATVD-R approach, we set the optimality gap tolerance to 2% and report the optimality gap at which the optimization model stopped. For the PRH approach, we use the lower bound obtained from CPLEX and determine the gap between the PRH solution and the lower bound as follows:

$$\text{PRH Gap} = (\text{PRH Estimated Final Cost - Lower Bound})/\text{Lower Bound} \tag{119}$$

To provide an estimate of the resulting savings yielded by our approaches for the company, we compare the total costs to the distribution costs with the initial tank allocation which we refer to as *fixed-tank approach.* To estimate the distribution costs of the initial state, we solve the BTATVD model but fix the initial tank allocation for each customer.

As shown in Table 12, the recurring demand assumption with the modified objective function results in substantially reduced computational time for model BTATVD-R compared to model BTATVD. The PRH approach provides comparable objective function values in substantially less computational time than either BTATVD or BTATVD-R. In all cases, the cost estimates are within 2% of the estimated objective function obtained from solving the other two model and within 3% of the best-known lower bound.

For scenario I, where the study time horizon is 3 years, all of approaches yield between 4%-6% reduction in the overall cost with annual distribution costs reduced by 9%-14% compared to the costs of initial tank allocations (obtained from the fixed-tank model). For scenario

Table 12: Performance comparison for Scenario I (3 years time horizon with 4 periods per year).

Case	model	Estimated Final Cost	Annual Dist Cost	Inst Cost	Refurb Cost	Transp Cost	Tank Swaps	Time (min)	Gap (%)	Total Savings (%)	Distribution Savings (%)
IA-50	Fixed	1,002,886	374,505	0	0	0	0	0.07	0.38	-	-
IA-50	BTATVD	952,732	325,654	32,000	41,000	4,614	4	12.52	1.00	5.00	13.04
IA-50	BTATVD-R	947,598	324,876	32,000	41,000	4,614	4	2.16	0.45	5.51	13.25
IA-50	PRH	954,311	334,752	24,000	29,500	4,380	3	0.08	1.16	4.84	10.61
IB-100	Fixed	1,537,990	574,328	0	0	0	0	2.96	0.41	-	-
IB-100	BTATVD	1,452,971	498,539	48,000	62,000	7,898	6	419.42	0.99	5.53	13.20
IB-100	BTATVD-R	1,453,074	498,591	48,000	62,000	7,898	6	50.92	0.84	5.52	13.19
IB-100	PRH	1,461,597	504,575	48,000	54,500	7,898	6	4.51	1.43	4.97	12.15
IC-150	Fixed	2,849,469	1,064,070	0	0	0	0	3.50	0.55	-	-
IC-150	BTATVD	2,727,771*	942,269	88,000	105,000	18,611	11	1,287.51	2.89	4.27	11.45
IC-150	BTATVD-R	2,682,977	921,845	88,000	106,500	19,872	11	750.98	1.00	5.84	13.37
IC-150	PRH	2,716,825	950,658	72,000	83,000	16,063	9	3.61	2.27	4.66	10.66
ID-200	Fixed	3,234,234	1,207,752	0	0	0	0	14.93	0.46	-	-
ID-200	BTATVD**	-	-	-	-	-	-	-	-	-	-
ID-200	BTATVD-R	3,065,313	1,072,233	80,000	97,500	16,484	10	1,372.04	1.00	5.22	11.22
ID-200	PRH	3,081,151	1,092,022	64,000	79,500	13,328	8	7.59	1.15	4.73	9.58

* The incumbent objective value upon premature termination due to memory limitations.

** Solution process was prematurely terminated due to memory limitations before finding a feasible solution.

Table 13: Performance comparison for Scenario II (7 years time horizon with 4 periods per year).

Case	model	Estimated Final Cost	Annual Dist Cost	Inst Cost	Refurb Cost	Transp Cost	Tank Swaps	Time (min)	Gap (%)	Total Savings (%)	Distribution Savings (%)
IA-50-7	Fixed	1,889,431	374,289	0	0	0	0	0.07	0.32	-	-
IA-50-7	BTATVD	1,683,962	298,743	72,000	90,000	13,947	9	15.17	0.86	10.87	20.18
IA-50-7	BTATVD-R	1,687,302	299,394	72,000	90,000	13,947	9	1.25	0.98	10.70	20.01
IA-50-7	PRH	1,706,968	311,253	56,000	69,500	10,245	7	0.07	2.16	9.66	16.84
IB-100-7	Fixed	2,907,591	575,983	0	0	0	0	1.14	0.71	-	-
IB-100-7	BTATVD	2,592,024	464,190	104,000	126,000	20,580	13	1,026.38	1.92	10.85	19.41
IB-100-7	BTATVD-R	2,598,255	472,887	88,000	107,000	16,098	11	7.73	1.91	10.64	17.90
IB-100-7	PRH	2,631,484	473,013	104,000	119,300	20,390	13	1.76	3.21	9.50	17.88
IC-150-7	Fixed	5,372,422	1,064,256	0	0	0	0	3.27	0.54	-	-
IC-150-7	BTATVD**	-	-	-	-	-	-	-	-	-	-
IC-150-7	BTATVD-R	4,859,519	890,488	152,000	181,500	30,787	19	203.69	1.96	9.55	16.33
IC-150-7	PRH	4,864,245	903,416	128,000	149,000	26,756	16	49.89	2.06	9.46	15.11
ID-200-7	Fixed	6,099,437	1,208,275	0	0	0	0	14.75	0.53	-	-
ID-200-7	BTATVD**	-	-	-	-	-	-	-	-	-	-
ID-200-7	BTATVD-R	5,721,307*	1,037,201	200,000	241,000	44,462	22	1,267.00	4.83	6.20	14.16
ID-200-7	PRH	5,556,286	1,029,227	152,000	175,500	33,195	19	8.36	1.81	8.90	14.82

* The incumbent objective value upon premature termination due to memory limitations.

** Solution process was prematurely terminated due to memory limitations before finding a feasible solution.

II, the study time horizon is increased to 7 years. The extended time horizon provides a longer payback period for the initial investments for the tank reallocation. Therefore, as expected, more tank exchanges are recommended and the total savings are more significant. The total cost savings are 8%-11%, with 14%-20% reduction in transportation costs. For these data sets, the PRH approach is capable of solving larger instances within reasonable computational time in comparison to the other two models and enables the industrial gas production company to consider larger clusters of their customers while capturing their demand fluctuations. Considering that an industrial gas production company typically has thousands of customers around the world, these approaches could improve their operational efficiency and result in significant savings in their distribution network.

6.4 Scalability Analysis

Using the PRH approach and the recurring demand assumption, we are able to solve instances for up to 200 customers in less than 10 minutes to 2% optimality gap. In reality, an international industrial gas company may have thousands of customers around the world and therefore needs to solve larger instances. To further improve the scalability of our solution approach, the formulation of the mathematical model is revised using a relaxation approach. In this approach, we relax the integrality of the variable for selecting routes, by changing the route selection variables, Ψ_{rq}, from binary to continuous, limited between 0 and 1. With this method, a route may be partially selected for fulfilling demand, leading to distribution costs that are potentially under-estimated. Thus solutions from this relaxed model will have objective function values (overall costs) less than or equal to the original models. Because the focus of the BTATVD model is on a strategic problem of allocating tanks, these estimates of distribution cost may be suitable. If desired, the distribution costs for the resulting tank allocations can be further evaluated. For this purpose, we use the resulting tank allocation decisions and input them to the BTATVD model as given (the fixed-tank approach).

We applied this relaxation approach to some of the test instances from Table 12 along with two larger case. As shown in Table 14, the relaxation approach solves the data sets in substantially less computational time. Using the tank allocations obtained from the relaxation method, we solve the fixed-tank model assuming that the allocated tanks are given and the route selection variables are binary. This fixed-tank approach provides estimates of distribution costs for the suggested tank allocation consistent with previous results. Table 14 summarizes fixed-tank approach results with new tank allocations for some of the test instances.

For example, case IB-100 was solved using BTATVD-R approach with resulting cost of 1,453,074 and computational time of 50.92 minutes (from Table 2). The relaxed model solves in approximately 0.09 minutes as shown in Table 4. The tank allocations from the relaxed model are then input as fixed solutions in the BTATVD-R model with binary route selection variables to obtain estimated final cost of 1,455,272, which is only 0.15% higher than the original BTATVD-R approach.

We use direct comparison of the results (objective functions and tank exchanges) for smaller data sets to evaluate the quality of the results. In all of these data sets, the relaxation approach resulted in objective function values within 3% of the values resulted by the PRH approach and within 1% of results of BTATVD-R approach. Thus, with the relaxed approach, we are able to solve larger data sets in reasonable time and the resulting tank allocation and distribution costs are similar to the approaches that use integrated variables. Given the complexity of the problem and size of industry problems, further research will be needed to explore alternative approaches to improve the scalability and quality of the obtained results.

Table 14: Results with relaxed variables

Case	Solution Approach	Relaxed Approach: Estimated Cost	Relaxed Approach: Tank Swaps	Relaxed Approach: Gap (%)	Relaxed Approach: Run Time (mins)	Relaxed - Fixed Tanks: Estimated Final Cost	Original: Estimated Final Cost	% Difference
IA-50	BTATVD-R	940,322	4	0.0059	0.02	952,560	947,598	0.52
IA-50	PRH	969,136	3	3.54	0.01	978,130	954,311	2.43
IB-100	BTATVD-R	1,426,087	6	0.0032	0.09	1,455,272	1,453,074	0.15
IB-100	PRH	1,475,237	4	3.64	0.02	1,503,454	1,461,597	2.78
IC-150	BTATVD-R	2,660,253	12	0.0129	0.24	2,706,151	2,682,977	0.85
IC-150	PRH	2,679,351	8	1.98	0.05	2,726,305	2,716,825	0.35
ID-200	BTATVD-R	3,015,844	12	0.40	1.22	3,075,787	3,065,313	0.34
ID-200	PRH	3,020,727	11	0.56	0.12	3,101,099	3,081,151	0.64
IE-400	BTATVD-R	4,940,615	16	0.45	4.53	5,002,467		
IE-400	PRH	5,024,092	16	2.10	0.96	5,078,950		
IF-818	BTATVD-R	10,556,324	31	0.07	28.36	10,699,679		
IF-818	PRH	10,652,538	29	0.97	4.6	10,832,886		

* This data set had premature termination due to memory limitation.

7 Conclusions and Future Work

For the industrial gas industry, the allocation of bulk tanks to customer sites is an important strategic level decision that also affects operational level distribution planning. In this work, we have focused on the bulk tank allocation (BTA) problem and developed models and exact and heuristic solution approaches for this problem under various assumptions.

The BTA problem is modeled as a mixed integer programming model under the assumption of constant customer demand. A branch-and-price algorithm is developed to solve the problem to optimality. In the branch-and-price solution approach, the problem is decomposed into a restricted master problem and a route generation subproblem. In the master problem, bulk tanks are assigned to customers and delivery routes are selected from a restricted set of potential routes. The master problem also determines the delivery amounts to customers on each route. The routes are generated in a pricing subproblem, and a column generation approach is used to identify the routes with negative reduced costs by solving an elementary shortest path problem with resource constraints (ESPPRC). We solve the ESPPRC using a label correcting algorithm. The solution approach is applied to data sets with up to 40 customers where some of the data sets are not solvable using the integrated model and CLPEX v12.5 on a computer with two Intel Xeon 3.10GHz quad-core processors, 32GB RAM.

Although the branch-and-price solution approach solves larger instances than the mixed-integer programming model, there is still a need for a more scalable approach for industry size problems. Because solving the pricing subproblem in the branch-and-price exact solution approach is the most resource intensive step, we have explored the use of alternative route generation methods based on sweep, nearest neighbor, and Clark-Wright savings heuristics to produce high quality routes with less computational time. The results of solving the BTA reduced master problem with these algorithms are compared.

We have extended the BTA model formulation and solution approach to assess the potential benefits of allocating multiple tanks at customer location. We have developed two mathematical models with different assumptions and demonstrated their application using a case study with modified data from Air Liquide.

Because industrial gas customers frequently have seasonal demands, we have developed a mathematical model for the BTA problem for customers with time varying demand. Given the complexity of the problem, we developed a two phase periodically restricting heuristic approach to address industry representative problem instances. In the first phase of this approach, the problem is solved for each period under the assumption that customer demand is constant and equals the demand in that period. The results of this phase includes a suggested tank size for each customer in each period and also a set of suggested routes which are generated using a sweep heuristic for the period with highest demand. In phase two, the bulk tank allocation model under varying demand is solved using CPLEX to determine a preferred tank allocation only from the restricted set of tanks from phase one. This phase also selects delivery routes, determines the amount of gas to be delivered to each customer on each route, and estimates the resulting distribution and investment costs. The solution approach is demonstrated using modified data sets provided by Air Liquide.

Future research will be devoted to exploring approaches to improve the scalability of the solution approaches and the quality of the obtained solutions. We plan on developing a nested Benders decomposition approach in an effort to solve the combined inventory routing and tank assignment problem in the industrial gas industry to optimality for larger instances. Furthermore, in this research, our main focus was on the effects of tank allocation on distribution cost savings, in future we also plan to assess additional benefits and effects of this strategic decision on other key performance measures.

Working with a leading international industrial gas company, we have identified additional problems where advanced operations research techniques can be used to study the dynamics and properties of their logistics system. An important strategic level challenge for industrial gas distributors is determining a suitable trailer fleet size for each area. In the future, we plan to conduct research to improve fleet management decisions by reducing total fleet investment costs and distribution costs while increasing flexibility. In an industrial gas distribution network, trailers obtain liquid product from a source. The efficiency of gas distribution is affected by sourcing decisions, which involve the assignment of customers to gas sources. We plan to develop mathematical models and algorithms to understand the effects of sourcing decisions on the overall industrial gas distribution system.

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