DELAY ANALYSIS OF SATELLITE PACKET BROADCASTING SYSTEMS:

A QUEUEING THEORETIC APPROACH

by

Sarvamangala Hendi

Thesis submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

Master of Science

in

Electrical Engineering

APPROVED:

Martin A. Wortman, Chairman

Charles E. Nunnally

Jeffrey D. Tew

August 25, 1988

Blacksburg, Virginia

DELAY ANALYSIS OF SATELLITE PACKET BROADCASTING SYSTEMS:

A QUEUEING THEORETIC APPROACH

by

Sarvamangala Hendi

Martin A. Wortman, Chairman

Electrical Engineering

(ABSTRACT)

This thesis develops a stochastic model for satellite packet switching networks, using results from queueing theory that have been previously explored in modeling communication networks. This thesis also analyzes message queueing delay when users of the network are generating data at moderate to high rates. Average packet delay and average number of packets in the system are formulated.

The model developed herein is applied to two cases. In the first case packet transmission and back off times are deterministic. In the second case packet transmission and back off times are exponentially distributed. The input parameters to this model are packet arrival rate, average packet transmission time, average back off time and probability of packet collision. The model yields average packet delay and average number of packets in the system. Methods to compute the probability of collision are presented.

Acknowledgements

I would like to thank my advisor, Dr. M. A. Wortman for giving me direction as I worked on this thesis. He helped me keep going when it seemed that every idea had turned into a dead end. I also thank my other committee members for their many helpful comments.

I want to thank my brother and his family, for encouraging me to pursue higher studies.

My husband has been a special source of encouragement, and I am thankful for his patience and support.

I am most of all grateful to my parents for the unconditional love that I have enjoyed for so many years.

I am thankful to all my friends for keeping me cheerful through out my stay here.

Table of Contents

List of Illustrations

List of Tables

Chapter 1

Introduction

1.1 Radio packet broadcasting systems

Data transmission using computers has grown enormously in the past two decades initiating a need for communication channels with higher bandwidth and lower error rate than exists in circuit switched systems. Random access systems were developed to meet this demand. Random access systems employ radio packet broadcasting. Packets can be broadcast either by a ground station or by a satellite depending upon the application. These packet broadcasting systems find extensive application in long haul communication networks.

Packet broadcasting is attractive as a method of communication from a central site to remote computer users in cases where:

1. Users have high peak to average data rate. (In most computer communication systems data is extremely bursty.)

2. Users are located in the areas where circuit switching system is poorly developed or nonexistent.

3. Users are mobile. (i. e., a fleet of ships that is inherently mobile.)

Circuit switching systems have some drawbacks if they are used in the above mentioned situations. The data from computer systems comes in a sequence of bursts with long periods of silence between the bursts. In such cases the cost of communication can easily exceed the cost of computation. The long distance between the remote users adds to this cost; having a dedicated channel becomes uneconomical in such cases. Errors caused by random noise are common in circuit switching lines.

If conventional multiplexing techniques are used for channel allocation, then the common channel from remote users to the central site is divided into a large number of low speed channels and one channel is assigned to each user whether he is active or not. Since at any given time only a fraction of the total number of users in the system are active and because of the bursty nature of the data from each user, such a scheme would lead to similar inefficiencies as those found in a circuit switched communication system. The channel allocation may, in some instances, be improved by employing polling techniques. However, it takes 270 mseconds for the satellite to discover that a polled user has nothing to say . With N users, the full polling cycle takes 0.270N seconds. If a ground station issues polls, this delay is doubled. As the number of users in the system increases, the delay increases [3].

Another alternative of channel allocation for remote users is frequency division multiplexing. Again, for large number of users and largely varying data rate, a large piece of valuable spectrum will be wasted if few users are active. Because of the bursty nature of the data most of the channel will be idle most of the time.

Radio packet broadcasting systems were developed to alleviate the problem of wasted bandwidth. Users in such systems access the channel whenever they have to transmit data packets irrespective of the activity of other users in the system. This method of accessing the channel provides better channel utilization when compared to other techniques [1].

1.2 Principle of operation

Whether it is ground based packet broadcasting or satellite packet broadcasting, the basic idea of such system is the same. In a packet broadcasting system a group of independent users (e.g., different plants of a company) share a common communication channel. Users who wish to communicate with one or more of the other users in the system transmit data packets through a common channel to the satellite. The satellite retransmits the packets. In case of ground based packet broadcasting, no satellite is employed. The users in such systems receive only those packets that are addressed to them. If two or more users transmit simultaneously, then the packets will collide. These colliding packets are destroyed. If the sender of the packet does not receive acknowledgement after one round trip delay, it is confirmed that the packet is destroyed. Now the sender waits for certain amount of time, called back off time and transmits again. The back off time is randomly selected for different users; otherwise, the same packets will collide again and again. The process of retransmission is continued until the packet is transmitted successfully.

The performance of a radio packet broadcasting systems depends upon the following factors:

- Number of users in the system that are contending for the channel. (Probability of packet collision depends upon the active users in the system).
- Channel error.

1.3 Literature review

Much research has been carried out in the past decade, on the performance of radio packet broadcasting systems. In the following section, we review the work contributed by different researchers on the performance analysis of radio packet broadcasting systems and the factors influencing the performance.

Abramson proposed the pure ALOHA system [1], with the combined features of packet switching and broadcast channels for data communication networks. He introduced the the pure ALOHA system as an alternative to point to point communication. In his paper on packet broadcasting channels [1], he analyzes the performance of pure ALOHA system under the following assumptions:

1. Users generate packets at a light rate [i. e., the rate at which packets are generated is such that the average time between packets from a single user is much

greater than the time needed to transmit a single packet]. Therefore at any time, no packet is waiting to access channel.

2. The start times between packets in the channel (both newly generated and re-transmitting) comprise a Poisson point process.

Abramson's work focuses on estimating the maximum number of of users that can share a common channel, and the maximum channel throughput. He defines channel throughput as the ratio of carried packet traffic in the channel to offered packet traffic to the channel. He shows that carried traffic is a function of probability of collision. From the second assumption, he develops an approximate analytical expression for probability of packet collision. He also discusses four methods of dealing with packets lost due to collision. Three of these methods are 1) positive acknowledgements, 2) transponder packet broadcasting, and 3) carrier sense packet broadcasting. These methods make use of a feedback channel. The fourth method is based on packet recovery codes. He presents theoretical results of channel throughput vs channel traffic for the pure ALOHA system.

Tannenbaum [15] and Klienrock [9] proceed along the same lines as Abramson, to determine the maximum channel utilization and maximum number of users for a satellite broadcasting system.

Another factor that influences the performance of radio packet broadcasting is the delay experienced by a packet in reaching its destination successfully. Delay is introduced because of repeated transmissions of a packet in case of collision. Ferguson [5] deals with the analysis of mean delay for both fixed and variable length packets in the pure ALOHA system. This analysis is for a lightly loaded system. An expression for average delay is developed in terms of packet transmission time and number of re-transmissions. Simulation results for delay vs re-transmission time are presented. His analysis of delay indicates that the fixed length packets experience less delay than variable length packets.

Haye's [6] work on the pure ALOHA system deals with the analysis of throughput and average packet delay. His model for the pure ALOHA system is derived from queueing theory concepts. The system is modeled as a M/G/1/1 queue. He makes the same assumptions as in [1], in computing the probability of packet collision and the system throughput. His delay analysis is for a system with fixed length packets and exponentially distributed back off times. He discusses the dependence of delay on back off time. From this analysis he derives an optimum value for the mean of exponentially distributed back off times.

Note that the following two assumptions are common to all the research work discussed so far:

1. Users generate packets at a light rate.

2. The start times of packets in the channel (both newly generated and re - transmitting) comprise a Poisson point process.

When the users generate packets at a light rate, the generated packets need not queue up. Every packet is transmitted as soon as it is generated. During heavy packet generation there would be need for a queue. A Queue is also necessary if there is high probability of packet collision. In such cases the users will transmit only backlogged packets and newly generated packets have to wait for access to the channel. In such cases it is appropriate to model the system as M/G/1 queue.

The second assumption is only an approximation to the true distribution of number of packets in the channel. The start times between the retransmitting packets does not strictly form a Poisson process [14], because they are not independent, exponentially distributed random variables.

1.4 Description of model

In this thesis we develop a model for satellite packet broadcasting system under the assumption that users have moderate to high data rates. This system operates on the same principle as that of the pure ALOHA system and employs Abramson's positive acknowledgement method. We are interested in analyzing average packet delay and number of packets for this system. The average packet delay here, is not only due to repeated transmission of packet but also due to time spent waiting for access to the channel. While estimating the probability of packet collision, we use the approximate analytical expression like other researchers and also estimate probability of collision by simulating the system.

Several analytical tools have been used in the past to model packet communication networks. The major tools are queueing theory and theory of stochastic processes. As far as queueing theory is concerned, M/G/1, M/M/1 and G/G/1 queues can be employed depending upon the system assumptions. Stochastic processes include renewal theory, Markov chain theory, semi - Markov, regenerative processes and Markov decision theory [16]. These modeling techniques, their capabilities and limitations are reviewed in [16]. Different network optimization techniques, basic performance measurement tools, and their applications to different network environment are also presented in this reference. A Markov chain approach has been used to model a queueing system with delayed feedback and constant service time in [2]. This procedure is lengthy, but is suitable for deterministic packet lengths.

In this work we consider the analysis of average packet delay for fixed and variable length packets and back off times. In modeling the channel and buffer as an M/G/1 queue, we follow Haye's [6] work with some modification. Haye's model is developed for a system with light packet generating rate; hence the buffer capacity is one packet size. Since we are developing a model for a system with moderate to heavy packet generating rates, we consider a buffer of infinite capacity and model the system as M/G/1 queue. With this model we analyze the the average packet delay for deterministic and exponentially distributed packet lengths and back off times. Since both deterministic and exponential packet service times can be analyzed under generally distributed service time model, we employ M/G/1 queuing model for our system.

The parameters for the model developed of this thesis are arrival rate, packet transmission time, back off time (random amount of time a packet waits in case of collision) and the probability of packet collision. The model yields average packet delay, average number of packets in the system and packet traffic intensity in the channel.

The results of this work are two sets of curves for the packet delay as a function of the probability of collision, for different arrival rates. One set of curves is for deterministic packet service times and the other set of curves is for exponentially distributed service times. This work focuses on providing useful information on average packet delay and average number of packets in the system.

1.5 Outline of the thesis

The structure of this thesis can be outlined as follows: In Chapter 2, a queueing model is developed and an analytical expression for average packet delay is derived. Chapter 3 presents different methods to compute the probability of packet collision. The results for average packet delay, probability of packet collision are analyzed in Chapter 4. Conclusions and recommendations for further study are presented in the same Chapter.

Chapter 2

Stochastic Modeling

In this Chapter an analytical expression for average packet delay in a satellite broadcasting system is derived. The problem of delay analysis for satellite packet broadcasting system is stated and then the system is modeled as a M/G/1 queue. In satellite communications different "types" of transmission times are important. The behavior of all these times can be studied under a general case. The distribution of time spent in transmitting a packet is characterized as general. Mean service time and mean square service times are derived for this general packet transmission time distribution. These derivations are used to examine systems having deterministic and exponentially distributed packet transmission times. The expression for average packet delay is obtained from the Pollaczek - Khinchin formula [4]. In the latter section of this Chapter, the satellite broadcasting system is referred to simply as the system, user buffers in the system are referred to simply as stations, and the packet generating rate for a station as packet arrival rate.

2.1 Problem statement

A typical system is as shown in Figure 1. We are given a system where N different stations are contending for the same channel. The packet generating rate for every station is high. Channel A is for transmission of packets from stations to the satellite. Channel B is for transmission of acknowledgements from satellite to stations. The reason for having two separate channels is to avoid the interference of packets propagating in both directions [7]. In order to avoid collision of the same packet again and again, each station is assigned a different back off time. This time may be fixed or may be varying for an individual station. If the back off time is fixed for a station, then a packet held for re-transmission will wait for this fixed amount of time prior to next attempt of transmission. If the back off time is varying for a station, then a retransmitting packet will wait for different time before every attempt. For the system discussed below we consider both types and analyze the average packet delay. Every station is assumed to have an infinite capacity buffer. The stations access the channel whenever messages are available for transmission.

The process of packet transmission from uncoordinated users in a typical system is shown in Figure 2. The communication between different users in the system is only through satellite. Whenever a station generates a packet, it transmits regardless of the activity of other stations, provided the previous packets from that station have successfully reached the destination; otherwise, new packets are queued. The satellite acts as repeater, amplifying and rebroadcasting whatever it receives. After transmitting a packet, the station waits for an acknowledgement. An important characteristic of satellite packet broadcasting is that, sender can listen to his own packet after one round trip delay. Since the sender can tell from this whether or not a colli-

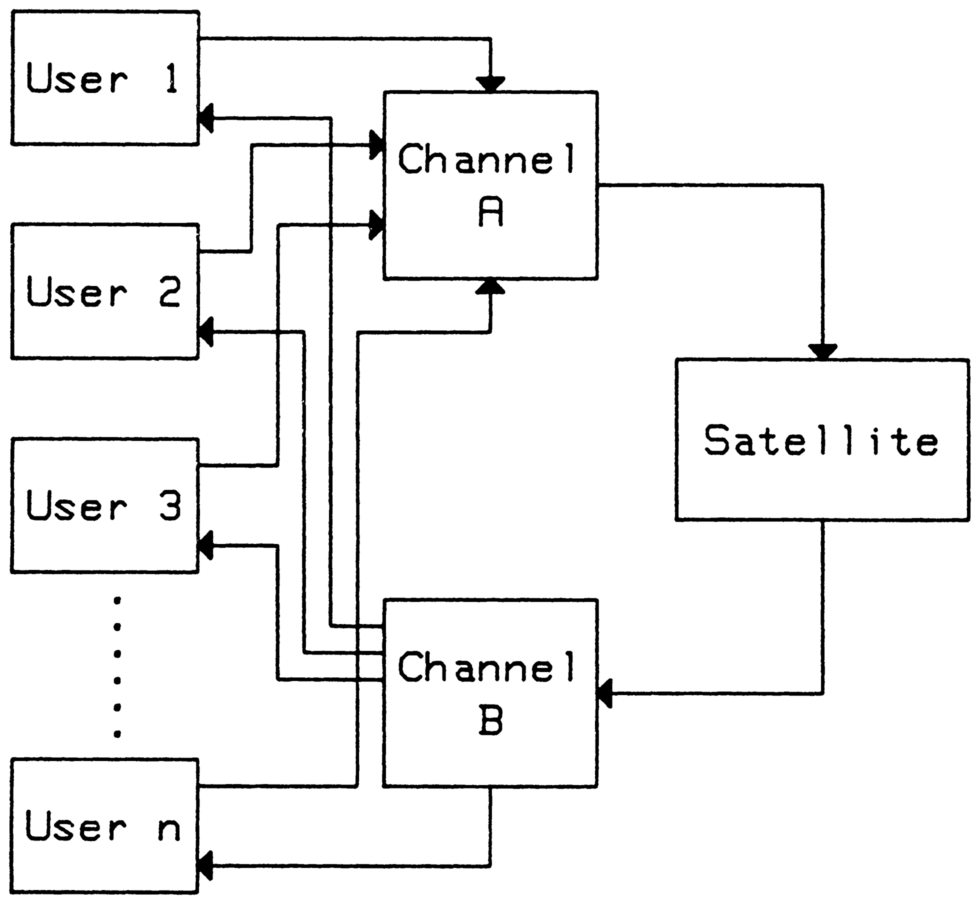

Figure 1. **Signal flow for a large number of users linked with a satellite.**

sion has occurred, there is no need for destination to source acknowledgements; the acknowledgement is the rebroadcasted packet itself. If two or more stations transmit simultaneously, the satellite will rebroadcast the sum of all incoming signals, resulting in chaos. The addressed stations will not be able to receive these garbled packets. If a packet is garbled, the sender learns of the problem simultaneously with receiver and takes the appropriate action. All damaged packets are held for retransmission. After waiting for corresponding back off times, packets are retransmitted from their respective stations. This process is repeated until a successful transmission occurs.

While the process of packet re-transmission continues, packets generated during this time must wait in a buffer. Because of repeated transmissions and waiting in the queue, a delay is introduced between a packet generation and its successful transmission.

The problem addressed here is the analysis of this delay for a system with moderate to heavy packet generating stations. In the following section we model this system as a M/G/1 queue and derive an expression for average packet delay.

2.2 System modeling

Packet collision is a major issue in systems with uncoordinated stations. Because of packet collision, the average transmission time of a packet increases. In modeling the system as M/G/1 queue, we must characterize packet collision and packet transmission time.

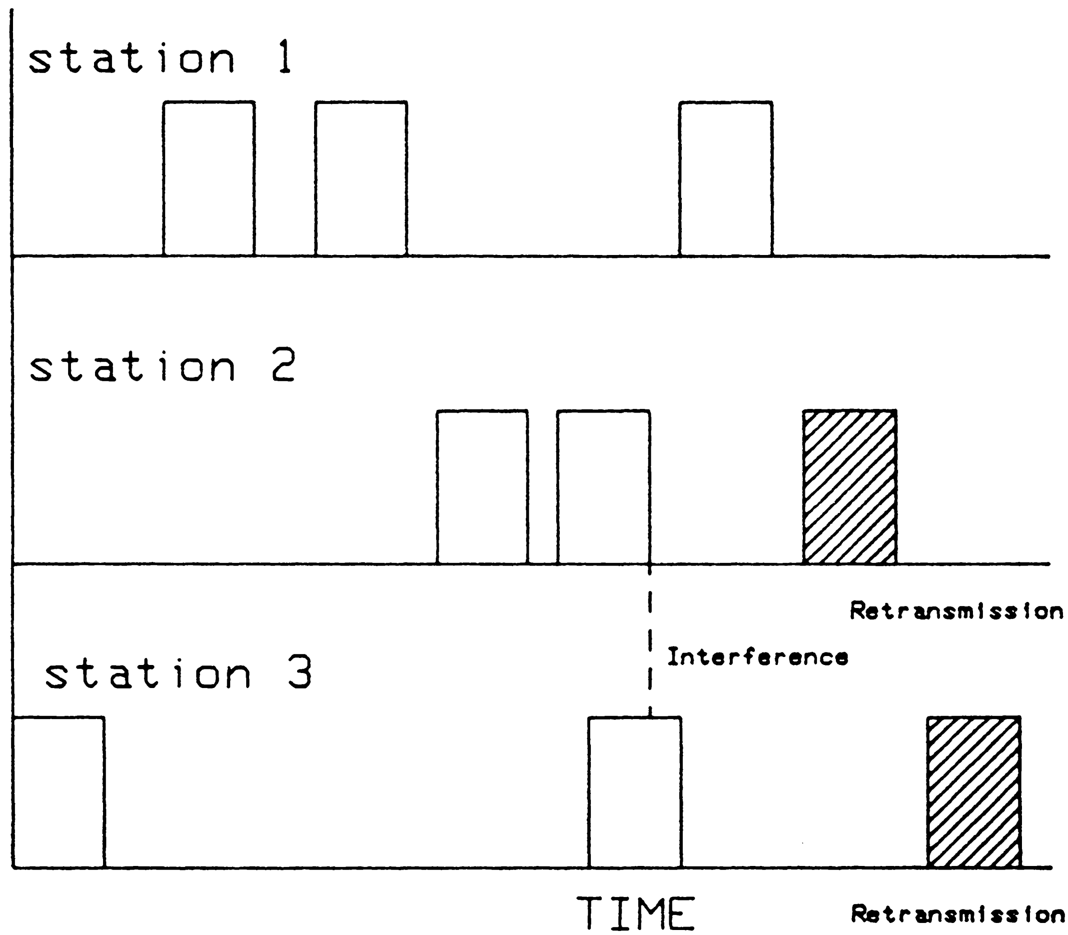

Figure 2. Packet transmission from uncordinated users in the system.

2.21 Packet collision characterization

Errors in packet transmission result from two primary causes:

- Random noise in the channel
- Packet collision due to interference

The first type of error is not a serious problem on UHF channel [15]. The second type of error is of importance, when a large number of stations are trying to use the channel at the same time. It is assumed here that there is no error in acknowledgement packets.

The process of packet collision is shown in Figure 3. When two or more stations transmit in an interval between t_0 to $t_0 + 2t$ (where t is packet transmission time) all the packets will be destroyed, and all must be retransmitted at a latter time. A packet will not suffer a collision, if no other packets are sent within one packet transmission time of its start as shown in Figure 3. The packet in transmission (the shaded one in Figure 3) will be damaged under the following conditions:

- If any other station has generated a packet between t_0 and $t_0 + t$, the end of that packet will collide with the beginning of the shaded one.
- If any other packet begins transmission between $t_0 + t$ and $t_0 + 2t$, then it will overlap the shaded packet.

No other packet should begin transmission during the entire vulnerable period (as marked in Figure 3) for the the shaded packet to reach satellite without damage.

The process of packet collision can be characterized by defining the probability of successful transmission for an arbitrary packet as

$p = P$[No other packet is transmitted in the vulnerable period];

the probability of collision is defined as

$q = 1 - p$

q depends upon the number of stations having packets to be transmitted in the vulnerable period of the packet in transmission. We have N stations with particular packet generation rate, and each contending for the same server (channel) for service. The availability of the server depends upon the probability of successful transmission of the packet already in transmission.

Now we make the following assumptions in modeling the system.

- Generation of packets for a station forms the Poisson process with rate λ.

- There is a separate buffer of infinite capacity for every station to accommodate the queue for that station.

- Once a packet is in transmission, no other packet enters the channel from that station, until the previous one is transmitted successfully. The rest of the packets generated during this period wait in queue.

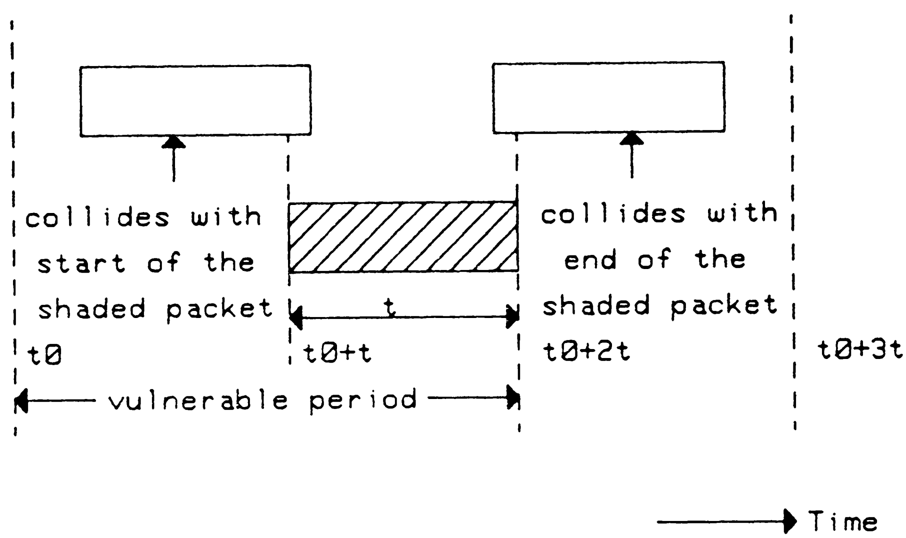

Figure 3. Packet interference in vulnerable period.

- All the stations in the system have same packet generation rates.

- Individual stations have different back off times. They may be either fixed or may be varying.

- Probability of collision is stationary for every station.

In making the first and the last assumption we follow Abramson. The justification for the first assumption is, in the close resemblance of Poisson distribution to the process of packet generation in computer networks [1]. The probability of collision becomes constant for every station in steady state. With these assumptions, every station in the system has same set of characteristics. The packet delay analysis for any one station applies to all other stations in the system, provided we have information of the probability of packet collision. The probability of collision is function of the number of active stations in the system. In the following section we characterize the time spent transmitting a packet.

2.22 Transmission time characterization

The packet processing time is divided in to two stages; the transmitting stage and the waiting stage. The processing time for transmitting stage is t_1 and that for waiting stage is t_2. t_1 includes packet transmission time and acknowledgement time. t_2 is back off time for any station in the system.

If there are no packets in the queue, a packet arriving to the station enters the first stage immediately. If there is no collision then that packet leaves the system with a probability 1. If there is collision, then the packet enters second stage with probability q, where it waits for time t_2, then re-enters the first stage for another transmission attempt. This process continues until the packet is successfully transmitted. This is shown in Figure 4.

As can be observed from the transmitting process, the transmission time starts when a packet begins transmission and ends when the packet is acknowledged. It may include number of re-transmissions. The probability distribution of the total time spent in transmitting a packet is complicated and does not seem to follow any standard distribution. Thus, it is appropriate to analyze this situation for a general distribution.

To proceed with the analysis of packet delay, we use the Pollaczek-Khinchin formula given below.

$$\bar{d} = \bar{m} + \lambda \frac{\bar{m}^2}{[2(1-\rho)]} \qquad [2.1]$$

Where

$\bar{d}$ = average packet delay

$\bar{m}$ = mean service time

$\bar{m}^2$ = mean square service time

ρ = packet traffic intensity in channel

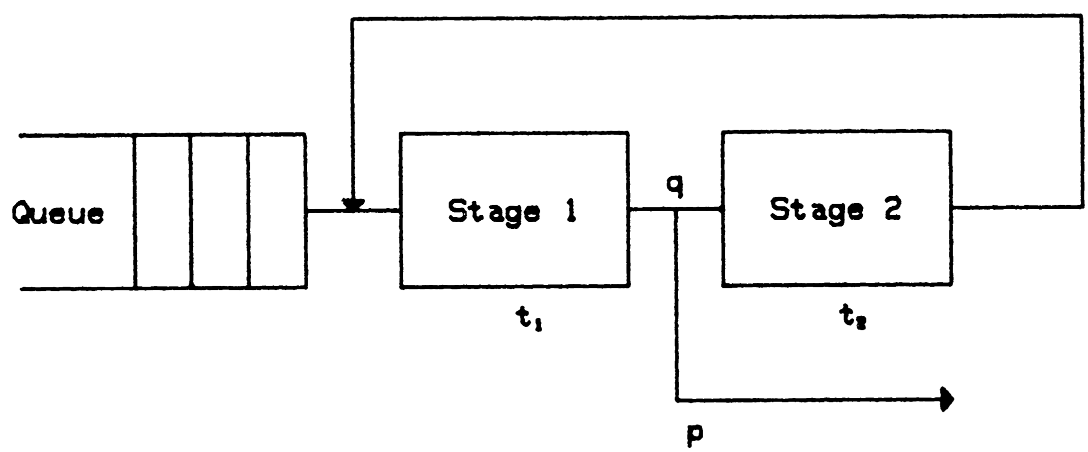

Figure 4. Queuing System With Two stages.

It is only necessary to know the mean and mean square values of service time distribution to obtain average packet delay. In the following section the general service time distribution is derived in terms of packet transmission time distribution, back off time distribution and probability of collision. From this the mean service time and mean square service times are obtained in terms of t_1, t_2 and q. The information about t_1 , t_2 and λ comes from system specifications and methods to compute q are provided in Chapter 3.

2.3 Delay formulation

In formulating the average packet delay, we first analyze the case, where the first stage and second stage service times are generally distributed. In order to find the mean service time and mean square service times, a Laplace transform method is adopted.

The behavior of the system is depicted in the state diagram of Figure 5. Any packet under transmission, will be in any one of the three states shown in Figure 5 (state implies the status of the packet). The packet is transmitted in the origin state. If the packet is in the intermediate state it will be waiting for another attempt to transmit. A packet successfully transmitted, goes to the absorbing state. Any packet that is transmitted will either go to the absorbing state with a probability p or re-enter the origin state with a probability q, following one back off time for re-transmission.

In the following section regular letters represent distribution functions and italic letters represent Laplace transforms. Let M(t) be the first stage service time distribution function and H(t) be the back off time distribution function. The first stage service time and back off time are mutually independent. Let G(t) be the distribution function of the time a packet spends between successive re - transmission attempts (in case of collision). Now the mean service time and mean square service time are derived in the following manner. Assume M(t), H(t) and G(t) have densities m(t), h(t) and g(t) respectively. Let the Laplace transforms of m(t), h(t) and g(t) be represented by $M(s)$, $H(s)$ and $G(s)$ respectively. These Laplace transforms can be obtained as follows:

$$M(s) = \int_0^{\infty} m(t) e^{-st} dt \qquad [2.2]$$

$$H(s) = \int_0^{\infty} h(t) e^{-st} dt \qquad [2.3]$$

Packet transmission times and back off times are independent. The distribution of the time spent between successive transmission attempts is the convolution of the packet transmission time distribution and back off time distribution.

i. e.,

G(t) = M(t) * H(t)

and hence

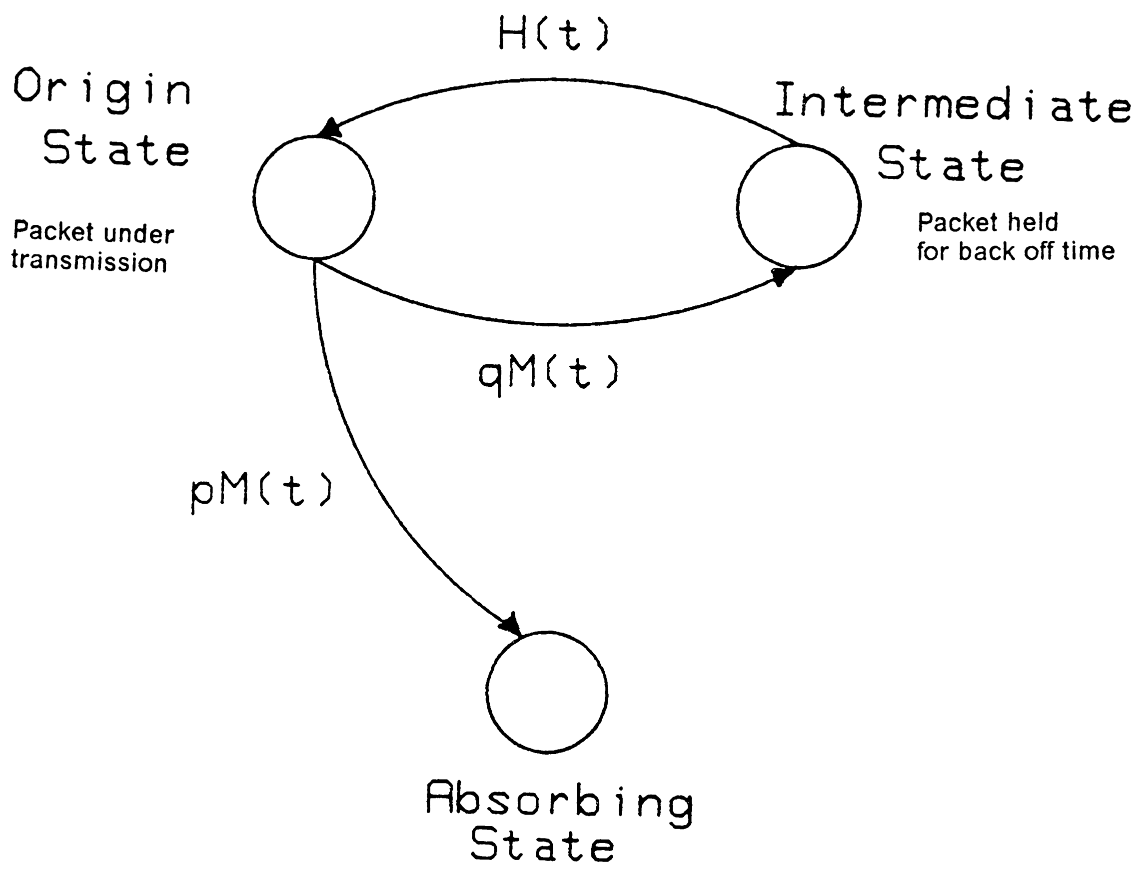

Figure 5. Modeling of general service time system.

$$G(s) = M(s)H(s) \qquad [2.4]$$

Now we can derive total time required to transmit a packet in terms of $G(s)$ and $M(s)$ as given below:

Packet transmission time and the time spent between successive transmission attempts are independent. Thus the Laplace transform of the sum is the product of individual transforms. Assume that $F(s)$ represent the Laplace transform of the density of total time spent in transmission. Averaging over the number of transmissions we have,

$$F(s) = \sum_{n=0}^{\infty} M(s)G(s)^n pq^n \qquad [2.8]$$

Using the geometric series approximation, we obtain

$$F(s) = \frac{pM(s)}{(1 - G(s)q)} \qquad [2.9]$$

In order to find the mean of total transmission time, we use the definition

$$\lim_{s \to 0} F'(s) = -E[X] \qquad [2.10]$$

where X is total transmission time. But

$$\bar{m} = E[\text{total transmission time}] \qquad [2.11]$$

Second derivative of $F(s)$ gives the mean square time as given below:

$$\lim_{s \to 0} F''(s) = E[X^2] = \bar{m}^2 \qquad [2.12]$$

From [2.9]

$$F'(s) = \frac{pM'(s)}{[1 - G(s)q]} + \frac{pqM(s)G'(s)}{[1 - G(s)q]^2}. \qquad [2.13]$$

From [2.13]

$$F''(s) = \frac{pM''(s)}{[1 - G(s)q]} + \frac{2pqG'(s)M'(s)}{[1 - G(s)q]^2} + \frac{pqM(s)G''(s)}{[1 - G(s)q]^2} + \frac{2pq^2M(s)[G'(s)]^2}{[1 - G(s)q]^3} \qquad [2.14]$$

In the following section we apply the above derived relations to compute the mean and mean square service times for packets of fixed length and variable length respectively.

2.31 Deterministic service times

In this case we consider fixed length packets. Every station has fixed back off time. The first stage service time is packet transmission time plus the acknowledgement time. Let this be denoted by t_1. The second stage service time is the back off time. Let this be denoted by t_2.

The distribution function of deterministic service time is a unit step function given by

$$u(t) \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases} \qquad [2.14]$$

where a is the value the function takes with probability 1. The density function of a unit step function is delta function given by

$$\delta(t-a) \qquad [2.15]$$

The Laplace transform of this density function be given by $U(s)$

$$U(s)=\int_0^\infty \delta(t-a)e^{-st}dt=\int_0^\infty \delta(t-a)e^{-sa}dt=e^{-sa} \qquad [2.16]$$

The service times of stage 1 and stage 2 are deterministic values t_1 and t_2 respectively. From the above discussion the Laplace transform of the density functions of first stage and second stage service time distributions are

$$M(s)=e^{-st_1} \qquad [2.17-a]$$

$$H(s)=e^{-st_2} \qquad [2.17-b]$$

respectively. Then, we obtain

$$G(s)=e^{-s(t_1+t_2)} \qquad [2.17-c]$$

From [2.17-a] and [2.17-c] the first derivative and the second derivative of $M(s)$ and $G(s)$ are obtained as given below:

$$M'(s)=-t_1e^{-st_1} \qquad [2.18-a]$$

$$G'(s) = -(t_1 + t_2)e^{-s(t_1 + t_2)} \quad [2.18-b]$$

and

$$M''(s) = t_1^2 e^{-st_1} \quad [2.19-a]$$

$$G''(s) = (t_1 + t_2)^2 e^{-s(t_1 + t_2)} \quad [2.19-b]$$

Using the set of equations [2.17], [2.18] and [2.19] in equations [2.13] and [2.14] we get the mean service time and mean square service time as given below:

$$\bar{m} = \frac{(t_1 + qt_2)}{p} \quad [2.20a]$$

$$\bar{m}^2 = \frac{(1+q)(t_1^2 + qt_2^2) + 4qt_1t_2}{p^2} \quad [2.20b]$$

The packet traffic intensity is

$$\rho = \lambda \bar{m} \quad [2.21]$$

by definition. Substituting $\bar{m}$, $\bar{m}^2$ and ρ in Pollaczek-Khinchin formula yields expression for average packet delay as given below:

$$\bar{d} = \frac{(t_1 + qt_2)}{p} + \frac{\lambda(1+q)(t_1^2 + qt_2^2) + 4qt_1t_2}{p^2[2(1-\rho)]} \quad [2.22]$$

The mean number of packets in the system is found using the definition

$$\bar{n} = \bar{d}\lambda \quad [2.23]$$

The equations [2.22] and [2.23] are plotted in Figure 6 and Figure 8 in Chapter 4.

2.32 Exponentially distributed service times

In this case packets of variable lengths are considered. The time to transmit a packet and receive an acknowledgement is assumed to be exponentially distributed. Every station has back off times which are exponentially distributed with different mean values. Assume that the service times of stage 1 and 2 are exponentially distributed with mean values μ_1 and μ_2 respectively. The distribution functions for first stage service time and second stage service time are

$$M(t) = 1 - e^{-\mu_1 t}, \qquad H(t) = 1 - e^{-\mu_2 t} \qquad [2.24]$$

respectively. Now G(t) = M(t) * H(t).

The Lalace transforms of m(t), h(t) and g(t) are given by

$$M(s) = \frac{\mu_1}{\mu_1 + s}, \quad H(s) = \frac{\mu_2}{\mu_2 + s}, \quad G(s) = M(s)H(s) = \frac{\mu_1 \mu_2}{(\mu_1 + s)(\mu_2 + s)} \qquad [2.25]$$

respectively. From [2.25] first derivative and second derivative of M(s) and G(s) can be obtained as shown below.

$$M'(s) = \frac{-\mu_1}{(\mu_1 + s)^2}, \qquad M''(s) = \frac{2\mu_1}{(\mu_1 + s)^3} \qquad [2.26]$$

$$G'(s) = \frac{-\mu_1 \mu_2 [(\mu_1 + s) + (\mu_2 + s)]}{(\mu_1 + s)^2 (\mu_2 + s)^2} \qquad [2.27]$$

$$G''(s) = \frac{-2\mu_1 \mu_2}{(\mu_1 + s)^2 (\mu_2 + s)^2} + \frac{2(\mu_1 + \mu_2 + 2s)}{(\mu_1 + s)^3 (\mu_2 + s)^2} + \frac{2(\mu_1 + \mu_2 + 2s)}{(\mu_2 + s)^3 (\mu_1 + s)^2} \qquad [2.28]$$

Using [2.25] and [2.26] in [2.11] the mean service time is obtained as given below:

$$\bar{m} = \frac{\frac{1}{\mu_1} + q\frac{1}{\mu_2}}{p} \qquad [2.29]$$

Using [2.25], [2.26], [2.27] and [2.28] in eq [2.12] the mean square sarvice time is obtained in terms of μ_1 and μ_2 as follows:

$$\bar{m}^2 = \frac{2[\frac{1}{\mu_1^2} + \frac{q}{\mu_2^2}] + \frac{4q}{\mu_1\mu_2}}{p^2} \qquad [2.30]$$

Then the packet traffic intensity is obtained using the definition

$$\rho = \bar{m}\lambda \qquad [2.31]$$

Substituting [2.29], [2.30] and [2.31] in [2.1] (the expression for $\bar{d}$ from Pollaczek-Khinchin formula), we get the average packet delay as

$$\bar{d} = \frac{\frac{1}{\mu_1} + q\frac{1}{\mu_2}}{p} + \frac{2\lambda[\frac{1}{\mu_1^2} + \frac{q}{\mu_2^2}] + \frac{4q}{\mu_1\mu_2}}{p^2[2(1-\rho)]} \qquad [2.32]$$

The mean number of packets in the system is obtained using Little's formula as given below:

$$\bar{n} = \bar{d}\lambda \qquad [2.33]$$

Figure 7 shows the plot of delay vs probability of collision for different arrival rates and Figure 9 shows mean number of packets vs probability of collision for different arrival rates in Chapter 4.

Chapter 3

Simulation Of Probability Of Collision

In this chapter a simulation procedure is presented to compute the probability of collision. Another method based on Poisson assumption of the total traffic in the channel is also provided to compute the probability of collision. The logic behind the simulation procedure is explained below.

The number of times every packet is transmitted until it is successfully received is counted. The probability of successful transmission for any packet is estimated as

$$\lim_{n \to \infty} P[\text{Successful transmission}] = \frac{1}{n}$$

where $n =$ number of transmissions.

The steps followed in running the simulation for deterministic and exponentially distributed times are presented separately. A common flow chart for these methods is given in appendix A.

3.1 Deterministic model

The packets arriving to every station have the same fixed lengths and individual stations have different fixed back off times. The following are the definitions that have been used in the simulation procedure.

- Vulnerable period = twice the transmission time.
- Minptr = The station that has the first packet to transmit.
- p = P [Successful transmission]

Step 1: Packets are generated.

Store the arrival times for every station in an array of dimension (number of stations, number of arrivals). Store the back off times in an array of dimension (number of stations).

Step 2: The first arrived packet starts transmission.

Choose the minimum arrival time among all the stations in the system. Call the corresponding station as minptr. If the minimum time is 99999.0 go to step 7. Otherwise find the difference between minimum arrival time and every other arrival time in the top row of the arrival times array.

Step 3: Check if there is collision.

If this difference is less than the vulnerable period, then all those packets are colliding with the one in transmission. Store a count of one in the count array for a colliding packet from every station. Otherwise go to step 5.

Step 4: Collided packets wait for next transmission.

Transmission time, acknowledgement time and back off times are added to all the collided packet arrival times. Now they are ready for next transmission. Add the same amount of time to the packets behind them if they are waiting for transmission. Go to step 2.

Step 5: If there is no collision

If none of the differences is less than the vulnerable period, then the packet in transmission goes out successfully. Check the counts corresponding to that packet. If the count is 1, then p is 1, otherwise $p = 1/\text{count}$ Accumulate this p and count the number of packets. Set the transmission count to 0 for this station.

Step 6: Next packet is ready for transmission.

Check if the next packets were already waiting when the first was in transmission. If so set them to (minimum arrival time + vulnerable period). Move the arrival times in the minptr column one row up. Set the last arrival time in this column to 99999.0. Now the next packet is ready for transmission. Go to step 2.

Step 7: Find the average probability of collision.

3.2 Exponential model

Here the stations have variable length packets which are exponentially distributed. But every station transmits packets of same mean length. Individual stations have different mean valued back off times that are also exponentially distributed. The steps explained in deterministic case can be used with some modification. The difference here is every time a packet experiences a collision, it will wait for different back off times from the exponential distribution. Whenever there is a successful transmission, the next packet will have different length which is picked from the exponential distribution.

Step 1: Packets are generated.

Store the arrival times for every station in array of dimension (number of stations, number of arrivals). Store the back off times for every station in array of dimension (number of stations, 100000). Store the packet lengths in array of the same dimensions as the arrival times array.

Step 2: First arrived packet starts transmission.

Choose the minimum time from the arrival times in the first row of the arrival times array. Recognize this station as minptr. If the minimum time is equal to 99999.0. Go to step 7. Otherwise find the difference between this time and the other arrival times in the first row

Step 3: If there is collision.

If these differences are less than 2*transmission time of the minptr station, then store a one to account for collision of each of these packets and also for the packet in transmission. Other wise go to step 5.

Step 4: Collided packets wait for next transmission.

Corresponding transmission times, acknowledgement times and back off times are added to the collided packet arrival times. If there are any packets waiting for transmission, their arrival times are set to the above times. The top packets are ready for next transmission. Go to step 2.

Step 5: If there is no collision.

If none of the differences are less than (2*minptr transmission time) then the packet in transmission goes out successfully. Check the transmission count. If it is 1, then p = 1 otherwise p = 1/transmission count. Set the corresponding transmission count to 0.

Step 6: Next packet is ready for transmission.

Move the arrival times in the minptr station one row up. If there are any packets waiting when the previous one is in transmission, then there arrival times are set to minimum arrival time +(2*transmission time). Set the last elcment in the column to 99999.0. Move the transmission times in the minptr column one row up. Set the last time to 99999.0 Move the back off times in the minptr column one row up. set the last element to 99999.0. Now the top packet is ready for transmission. Go to step 2.

Step 7: Compute the average probability of collision.

3.3 Probability of collision by analytical approximation

In this section an expression for q is developed under the assumption that packets are retransmitted such that the total traffic in the channel is the sum of two Poisson processes and hence, is Poisson. In deriving this expression we follow reference [5].

Let λ be the rate at which packets are generated in packets / sec. Let the total arrival rate of flow of new and retransmitted packets be denoted as λ' packets / sec. A packet will be involved in collision, only if another station transmits in the vulnerable period of the packet under transmission. From the Poisson assumption we have

$$P[\text{No packet is transmitted in vulnerable period}] = e^{-2\lambda' m} \qquad [3.1]$$

There fore

$$P[\text{collision}] = 1 - e^{-2\lambda' m} \qquad [3.2]$$

The rate at which packets are retransmitted is $\lambda'(1 - e^{-2\lambda' m})$. Adding the newly generated and retransmitted packet traffic, we have

$$\lambda' = \lambda + \lambda'(1 - e^{-2\lambda' m}) \qquad [3.3]$$

The above equation can be solved for λ' and from that the probability of collision can be found using [3.2]. The results obtained from both simulation and assumption are presented in Chapter 4.

Chapter 4

Analysis Of Results

In Chapter 2 we derived expressions for

- Average packet delay as a function of packet generation rate, packet transmission time distribution, back off time distribution and probability of collision.

- Average number of packets in the system as a function average packet delay and arrival rate.

for deterministic and exponentially distributed packet service times respectively. In Chapter 3 we proposed two methods to estimate the probability of packet collision. In this chapter we present the analytical results for average packet delay and average number in the system. Simulation and analytical results for the probability of packet collision are also presented.

4.1 Delay analysis

In order to study the variation of average packet delay and the average number of packets in the system as functions of the probability of collision, we use two sets of relations derived in Chapter 2. These relations are given below:

$$\bar{d} = \frac{(t_1 + qt_2)}{p} + \frac{\lambda(1 + q)(t_1^2 + qt_2^2) + 4qt_1t_2}{p^2[2(1 - \rho)]} \qquad [4.1]$$

which corresponds to deterministic packet length and back off times, and

$$\bar{d} = \frac{\frac{1}{\mu_1} + q\frac{1}{\mu_2}}{p} + \frac{2\lambda[\frac{1}{\mu_1^2} + \frac{q}{\mu_2^2}] + \frac{4q}{\mu_1\mu_2}}{p^2[2(1 - \rho)]} \qquad [4.2]$$

which corresponds to exponentially distributed packet length and back off time. The variation of average number of packets in the system is analyzed using Little's Formula.

$$\bar{n} = \bar{d}\lambda \qquad [4.3]$$

In the following section we present a discussion of the analytical results for average delay and average number in the system.

The results given in Table 1, Table 2, Table 3, Table 4 and Table 5 are displayed in Figure 6 and Figure 8. Since the packet transmission time and backoff time are fixed for a particular system we have analyzed the variation of delay as a function arrival rate and probability of collision. Figure 6 and Figure 8 represent packet delay vs probability of collision as a function of arrival rate for deterministic and exponential cases respectively. These results are obtained for a stable system. The measure of stability is $\rho < 1$ [6]. By allowing a maximum probability of collision of 0.2, we choose upper limit for λ for a stable system. A fixed packet transmission time of 0.032 sec is considered for deterministic case and a mean transmission time of 0.032 sec is considered for exponential case. The backoff time of 0.2 sec is fixed for deterministic case and it varies with a mean of 0.2 in the exponential case. For all these observations q is varied from 0 to 0.19 and λ is varied from 2 to 10 packets/sec.

From Figure 6 and Figure 8, it can be observed that for λ varying between 2 to 6 packets/ sec, the increase in the delay with respect to q is small, but for λ between 8 to 10 packets/sec the delay is very sensitive to q in the range 0.1 to 0.18. This behavior can be reasoned as follows. In the delay expression

$$\bar{d} = \bar{m} + \lambda \frac{\bar{m}^2}{[2(1-\rho)]}$$

as ρ approaches 1, $\bar{d}$ increases dramatically. Now, by definition

$$\rho = \lambda \bar{m} \qquad \text{where} \qquad \bar{m} = \frac{t_1 + q t_2}{p}$$

With the high probability of collision, the same packet is transmitted again and again, hence average packet transmission time becomes more. With a high packet generating rate, the second term (queueing delay) in the Pollaczek - Khinchin formula increases without bound. Hence the overall delay is very large in the range for λ of 8 - 10 packets / sec and q of 0.1 to 0.18.

For each of the arrival rates, the average delay in exponential case is slightly more than in the deterministic case. Since the average packet transmission time is same for both cases, this difference is due to high second moment of exponential distribution. Similarly, the average number of packets in the system in exponential case is greater than in the deterministic case.

The nature of the graphs indicate that, the delay increases very rapidly with the increase in the arrival rate than the probability of collision. This indicates the delay increases more because of arrival rate, than the number of stations in the system; i. e. queueing delay predominates average delay.

4.2 Probability of collision analysis

The simulation procedure developed in Chapter 3 is used to estimate the probability of collision. These results are obtained for a specific system, using the program in Appendix B. A system with the following specifications is considered for simulation. The channel data rate is 10 kb/sec, fixed length packets are 320 bits long, backoff times for the stations in the system vary from 0.1 to 0.9 seconds. The simulation was run for a maximum of 25 stations sharing a single channel. Each station generates packets at a rate of 4 packets/sec. The primary purpose of running this simulation was to have an estimation of the value of q and to identify the factors affecting q. The simulation and analytical results are given in Table 6. The analytical results are obtained for [3.2] derived in Chapter 3. The computer program given in Appendix C is used for analytical results of probability of collision.

The analytical results show that the probability of collision increases with the increase in the number of stations. Simulation results show the same trend. But there is considerable difference obtained in the results from two methods. The simulation was run as an experimental technique to study the variation of probability of collision. To validate the output from simulation run, a confidence interval is constructed for each set of stations. A fixed sample size procedure is used in constructing the confidence interval. Since the batch means method (one of the fixed size sample procedures) appear to have the greatest promise in terms of proven performance and applicability to realistic scenarios [11], we have chosen this method to construct the confidence interval. In order to avoid the possibility of batch means being non - normal, the simulation run is divided into more than 40 batches [11]. The batch size is chosen large enough so that batch means are approximately uncorelated [11]. With these properties the batch means can be treated as near normally distributed random variables.

The simulation results can be claimed with approximately 74% confidence. The estimated range of confidence intervals for simulation results are given in Table 7. The following behavior was observed from these results. The interval becomes more narrow as the number of stations become more, implying that, the procedure might perform well if enough data were available. However in this case the data set available was not large enough to make any comparison with analytical results.

To summarize, queueing delay is a significant part of the total delay under moderate and heavy loads. Identifying packet arrival rate in the range 2 - 6 packets/sec as moderate for the case we have considered, the system behaves such that the delay will not increase dramatically, with slight increase in q. Few more stations can be added in this range without affecting the delay. The average delay depends more on the arrival rate than on the number of stations in the system. Deterministic times exhibit less delay compared to exponential times. The average number of of packets in the system for exponentially distributed packet lengths and back off time is more than for deterministic packet length and back off time. The probability of collision is a function of number of stations in the system.

4.3 Conclusions

In the following section we summarize the main findings contained in this thesis. We also discuss the possible extensions.

In this thesis we developed an extension of the original model for satellite broadcasting systems. This model is based on queueing theory. Although many researchers have worked on average delay analysis, their analysis did not include queueing delay. We developed a model for a system with moderate to heavy packet generating rate. With this model we analyzed the average delay (considering queueing delay) and the average number of packets in the system. We computed the probability of collision by simulation and by analytical expression.

We consider, the estimation of average packet delay as a function of average packet transmission time and queueing delay for a stable system to be the most important contribution in this thesis. This estimation offers intuition regarding the buffer design and the maximum allowable arrival rate for the system to remain stable. The simulation procedure we have adopted here, has few limitations. This procedure can't be run for more than 45 stations and 1000 packets/station, because of memory limitations. The computation time is considerably high. For each set of stations we have considered, the CPU time was at least 600 seconds.

The work contained in this thesis brings to light some recommendation for further study. One area that warrants further study is that of devising a more accurate and simple method to compute the probability of collision. This research can further be extended to explore the other systems such as slotted ALOHA, CSMA and CSMA/CD.

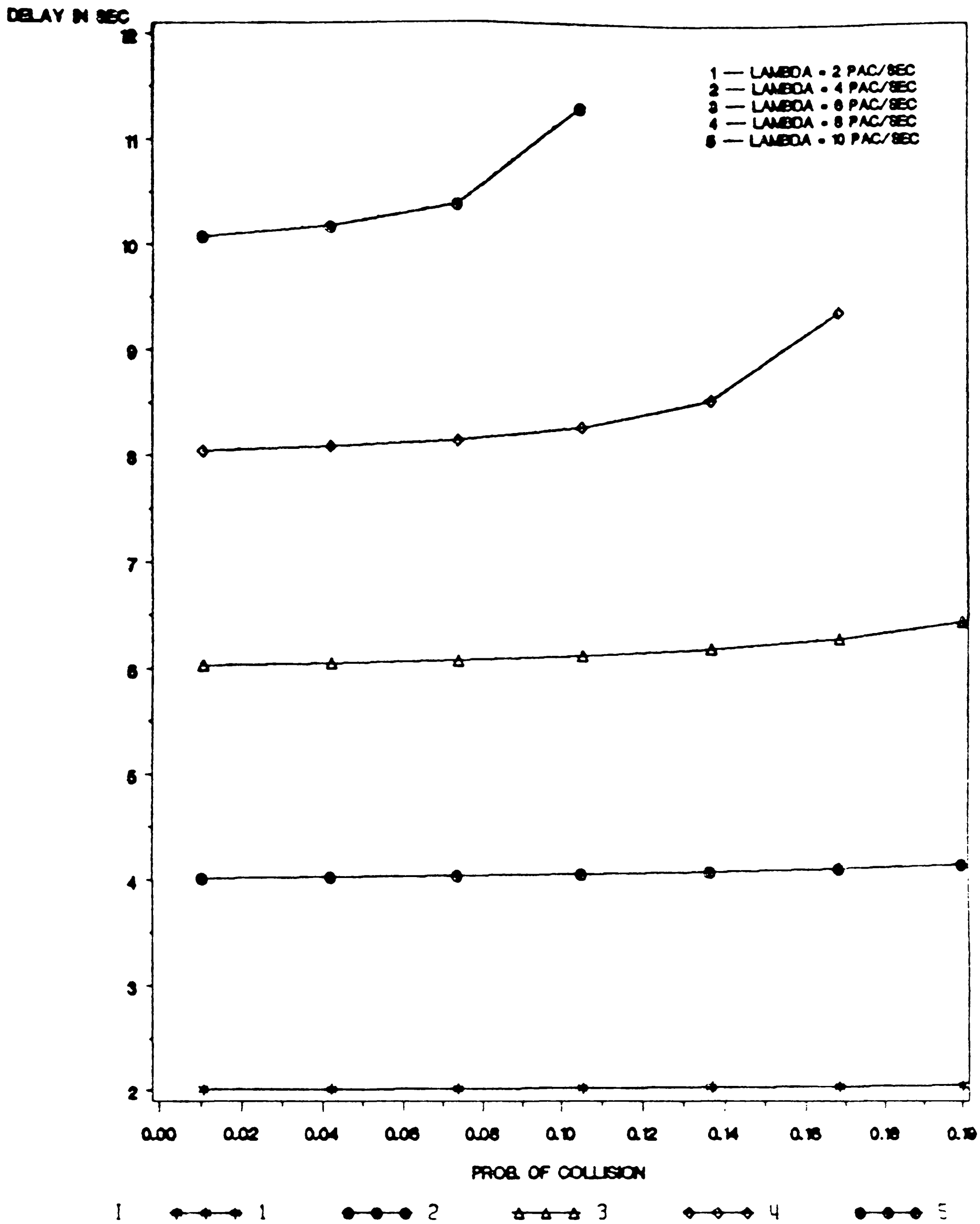

Figure 6. Average delay vs Probability of collision for deterministic transmission time and back off time

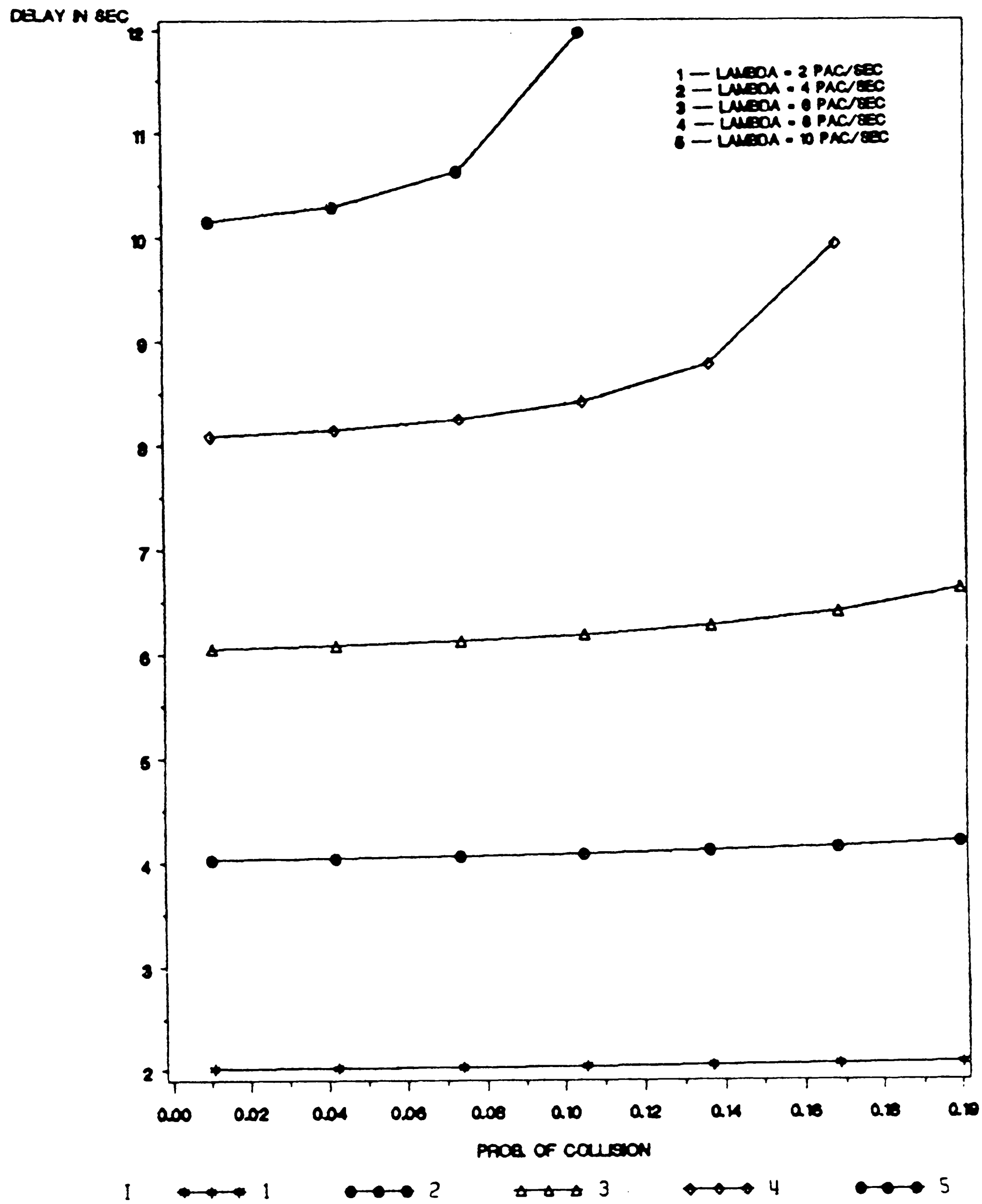

Figure 7. Average delay vs Probability of collision for exponentially distributed transmission time and back off time

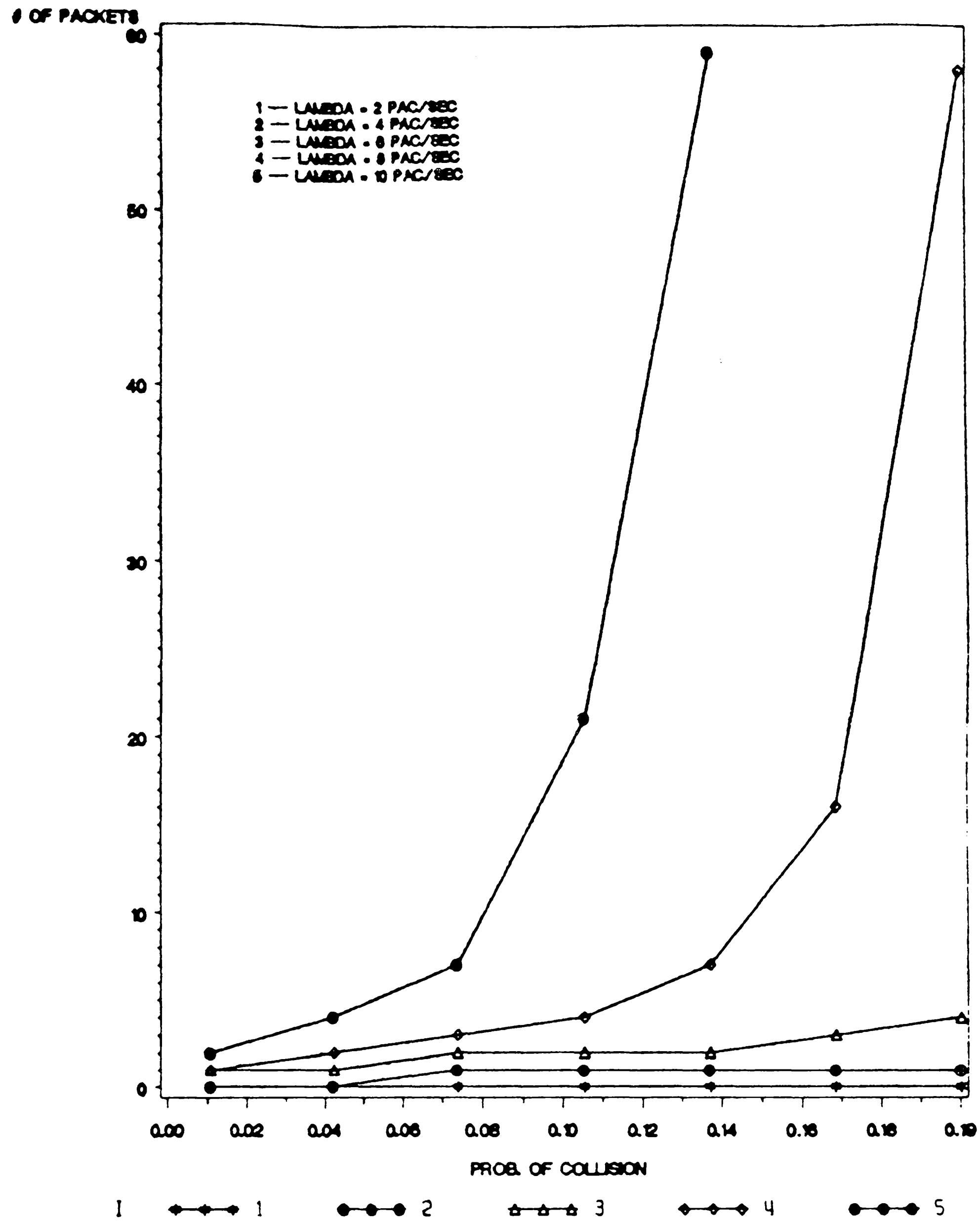

Figure 8. Average # of packets vs Probability of collision for deterministic transmission time and back off time

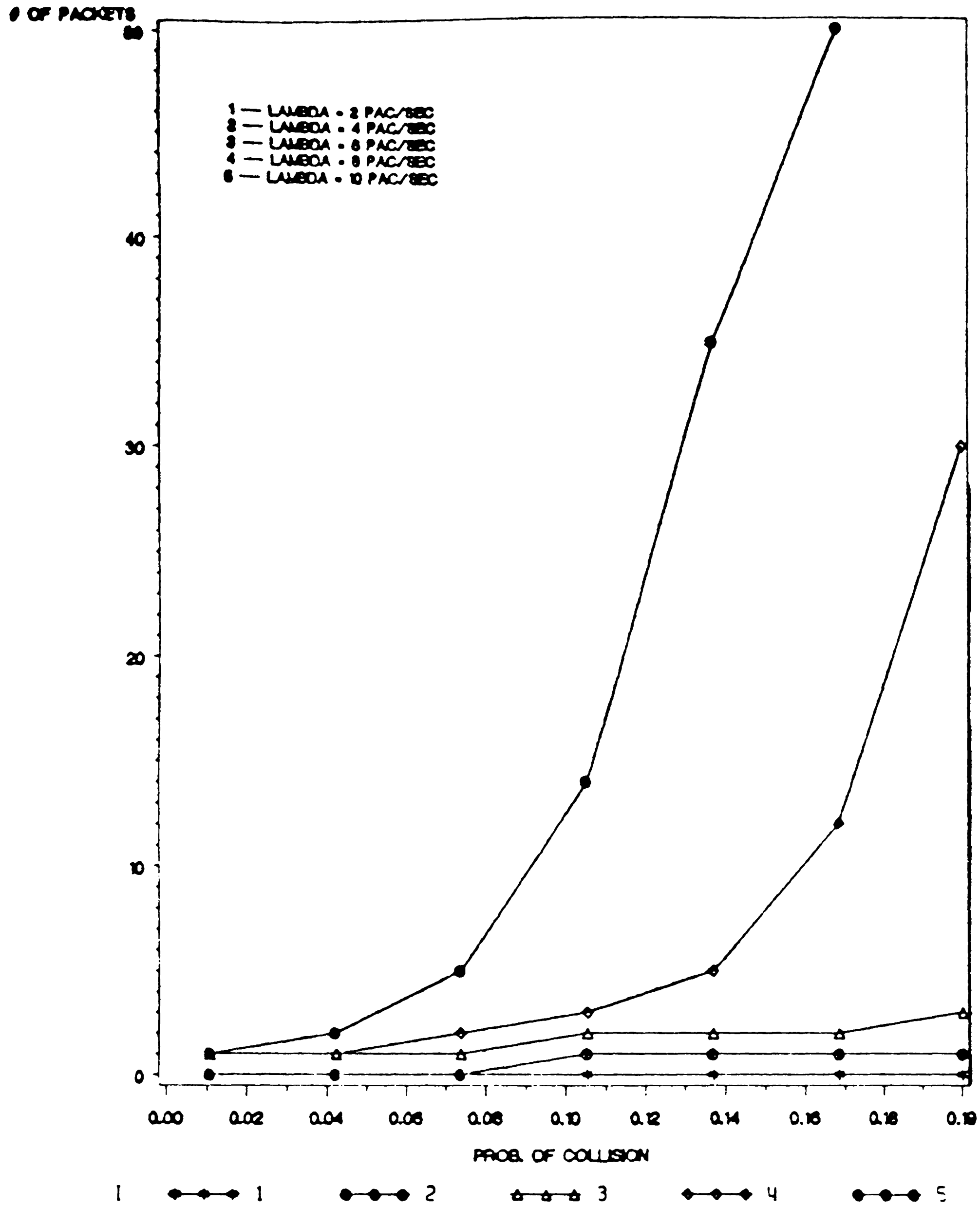

Figure 9. Average # of packets vs Probability of collision for exponentially distributed transmission time and back off time

Table 1. Results of Average Delay and Average # of Packets

$\lambda =$ 2 packets / sec

Prob. of packet collision	Ave. Delay in sec Det. time	Ave. Delay in sec Expo. time	Ave. # of Packets Det. time	Ave. # of Packets Expo. time
0.01	2.006	2.011	0	0
0.04	2.010	2.017	0	0
0.07	2.015	2.024	0	0
0.10	2.021	2.032	0	0
0.13	2.029	2.042	0	0
0.16	2.037	2.054	0	0
0.19	2.048	2.067	0	0

Table 2. Results of Average Delay and Average # of Packets

$\lambda =$ 4 packets / sec

Prob. of packet collision	Ave. Delay in sec Det. time	Ave. Delay in sec Expo. time	Ave. # of Packets Det. time	Ave. # of Packets Expo. time
0.01	4.014	4.026	0	0
0.04	4.025	4.042	0	0
0.07	4.038	4.060	0	1
0.10	4.055	4.084	1	1
0.13	4.077	4.114	1	1
0.16	4.106	4.152	1	1
0.19	4.145	4.203	1	1

Table 3. Results of Average Delay and Average # of Packets

$\lambda =$ 6 packets / sec

Prob. of packet collision	Ave. Delay in sec Det. time	Ave. Delay in sec Expo. time	Ave. # of Packets Det. time	Ave. # of Packets Expo. time
0.01	6.026	6.048	1	1
0.04	6.047	6.080	1	1
0.07	6.077	6.121	1	1
0.10	6.118	6.179	1	2
0.13	6.179	6.264	2	2
0.16	6.276	6.395	2	3
0.19	6.442	6.620	3	4

Table 4. Results of Average Delay and Average # of Packets

$\lambda =$ 8 packets / sec

Prob. of packet collision	Ave. Delay in sec Det. time	Ave. Delay in sec Expo. time	Ave. # of Packets Det. time	Ave. # of Packets Expo. time
0.01	8.044	8.083	1	1
0.04	8.086	8.146	1	2
0.07	8.154	8.244	2	3
0.10	8.273	8.415	3	4
0.13	8.526	8.774	5	7
0.16	9.347	9.930	12	16

Table 5. Results of Average Delay and Average # of Packets

$\lambda =$ 10 packets / sec

Prob. of packet collision	Ave. Delay in sec Det. time	Ave. Delay in sec Expo. time	Ave. # of Packets Det. time	Ave. # of Packets Expo. time
0.01	10.08	10.15	1	2
0.04	10.17	10.29	2	4
0.07	10.39	10.62	5	7
0.10	11.30	11.97	14	21

Table 6. Probability of collision

$\lambda =$ 4 packets / sec

# of stations	From simulation	From assumption
5	0.03344	0.3421
15	0.04308	0.6922
25	0.05852	0.8756

Table 7. Estimated Confidence Intervals

Degree of Confidence = 0.74

# of stations	Mean of p	Variance of p	Confidence Interval
5	0.03344	0.1421	0.049
15	0.04308	0.1302	0.047
25	0.05852	0.0856	0.030

References

1. N. Abramson, "The throughput of packet broadcasting channels.", IEEE Transactions on Communications, Vol. Com-25, No.1, January 1977, pp. 117 - 127.

2. F. Borgonovo, L. Fratta and F. Maffoli, "Queueing systems with delayed feed back and computer communication networks.", The Europian computing conference on communication networks - London, Communication networks, pp. 379 - 393. September 1975,

3. A. Carleial, M. Hellman, "Bistable behavior of ALOHA type systems.", IEEE Transactions on Communications, Vol.Com - 20, No 7, April 1975. pp. 401 - 409.

4. D. Gross, C. M. Harris, "Single Server Queues", Fundamentals of Queueing theory, John Wiley & Sons., pp 223 - 289, 1974.

5. M. Ferguson, "An approximate analysis of delay for fixed and variable length packets in an unslotted ALOHA channel.", IEEE Transactions on Communications, Vol. Com - 25, No 7, July 1977, pp. 644 - 653.

6. J. Hayes, "Random access systems." Modeling and analysis of computer communication networks, 1984, Plenum press, New York, pp. 207 - 220.

7. R. Kahn, "The organization of computer resources in to a packet radio network.", IEEE Transactions on Communications, January 1977, pp. 167 - 178.

8. L. Klienrock, "Satellite packet switching.", Queueing systems Vol II: Computer applications. A Wiley - Interscience publication. 1976, pp. 360 - 365.

9. L. Klienrock, S. Lam, "Packet switching in a multi access broadcast channel: Performance evaluation.", IEEE Transactions on Communications, Vol. Com - 23, No 4 April 1975, pp. 401 - 422.

10. L.Klienrock and F. A. Tobagi, "Packet switching in radio channels: Part I-Carrier sense multiple-access modes and their throughput-delay Charecteristics", IEEE Transactions on Communications., Vol. COM-23, pp. 1400-1416, Dec. 1975.

11. A. Law and W. D. Kelton, "Output Data Analysis for a Single System", Simulation, Modeling and Analysis, McGraw - Hill, Inc., pp. 279 - 314, 1982.

12. J. Medith, C. Lea, " Stability and optimization of the CSMA and CSMA/CD channels.", IEEE Transactions on Communications, Vol. Com - 31, No 6. June 1983.

13. L. G. Roberts, "ALOHA packet system with and without slots and capture", Computer Communication Review, Vol. 5, pp. 28-42, Apr. 1975.

14. L. Takacs, "A single server queue with feedback.", The Bell Systems Technical Journal, VOL 42, NO 2, March 1963, pp. 505 - 519.

15. A. Tannenbaum, "Satellite and Radio broadcasting.", Computer networks, Prentice Hall Inc., pp. 252 - 258.

16. F. Tobagi, M. Gerla, R. Peebles, E. Manning, "Modelling and measurement techniques in packet communication networks.", Proceedings of the IEEE, Vol. 66, No. 11, November 1978, 1423 - 1447.

Appendix A

Flow Chart

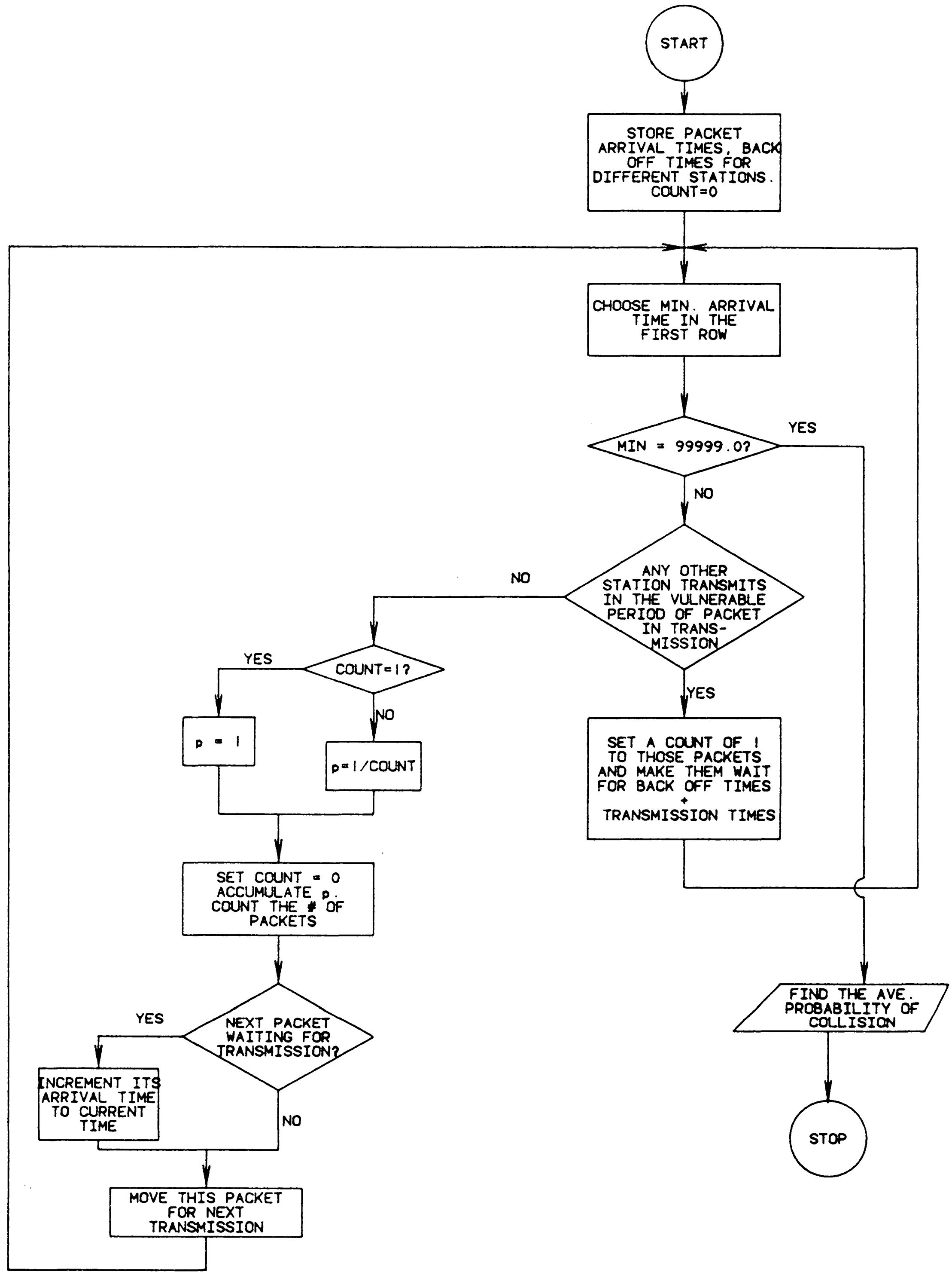

Figure 10. Flow Chart for the Simulation procedure

Appendix B
Program 1

Below we give the computer code to compute the probability of packet collision.

```
C
C THIS PROGRAM IS WRITTEN TO COMPUTE THE PROBABILITY OF SUCCESS FOR
C PURE ALOHA SYSTEM. NUMBER OF STATIONS IS 50. ALL THE STATIONS
C ARE HEAVILY
C
C
      INTEGER NUB,M,IER,COUNT,SCOUNT,NROW,NCOL,ISEED,NOUT
      REAL COMP(50),SUMCOL,ADDAMT,X,SS,SSTT,CSTT,PS, APP
      REAL B(50),READY(50),MIN,DIF,SCOLL,REAL BB(50),Q ,SPROB
      REAL STA(1000,50),CCTT,AQQ,AQ,TTT,TRA(50),AP
      REAL P,LAM,AVE,S,MST,D,TIME,TRACO
C
      DATA B,READY /50*0.0,50*0.0/
C
       ISEED=123457
       NCOL=50
       NROW=1000
       COUNT=0
       CCTT=0.0
       AP=0
       DIF=0.064
C
C IMSL ED 10 RNSET STAT VOL 3 P 953
C
       CALL RNSET (ISEED)
C
C IMSL ED 10 SUBROUTINE RNUN MATH LIB VOL 3 PAGE 1113
C
       CALL RNUN (NCOL,B)
C        WRITE (7, 1001) (B(I),I =1,10)
1001    FORMAT(10E10.3)
C
C RNNPP - IMSL ED 10 STAT VOL 3 P 1050
```

```
C
C     DO 3000 JJ=1,15
         JJ  =  4
        DO 5 I  =  1, NCOL
          CALL SCOPY (NROW,999999.0,0,STA(1,I),1)
   5    CONTINUE
         CALL SCOPY (NCOL,0.0,0,TRA,1)

C
C IMSL ED 10 RNSET STAT VOL 3 P 953
C
C        CALL RNSET (ISEED)
        X=1./JJ
        AP=0.0
        SUMCOL=0.0
        CCTT=0.0
        APP=0.0
        COUNT=0
      DO 20 I=1,NCOL
        CALL RNEXP (NROW,STA(1,I))
        CALL SSCAL(NROW,X,STA(1,I),1)
        TIME  =  0.0
        DO 10 J  =  1,NROW
          TIME  =  TIME  +  STA(J,I)
          STA(J,I)  =  TIME
  10    CONTINUE
  20  CONTINUE
2099    FORMAT(5E10.3)
C
C   GETTING THE TOP ROW READY FOR COMPARISION.
C
      DO 30 I=1,NCOL
         READY(I)=STA(1,I)
  30  CONTINUE
C        WRITE(6,2099)(READY(I),I=1,5)
C       WRITE(5,2099)((STA(I,J),J=1,5),I=1,5)
C
C TO FIND THE LEAST VALUE USE
C ESSL ROUTINE ISMIN (ESSL GUIDE AND REFERENCE P 153)
C
  40 CONTINUE
       MINPTR  =  ISMIN (NCOL,READY,1)
       MIN  =  READY(MINPTR)
       COUNT=0
C
C         WRITE (5,2099)(READY(I),I=1,5)
C       WRITE(5,2099)((STA(I,J),J=1,5),I=1,5)
1099       FORMAT(2E10.3)
C
       IF (MIN .EQ. 999999.0) GO TO 1000
```

```
      CALL SCOPY (NCOL,0.0,0,COMP,1)
C
      SUMCOL = 0.0
      DO 200 I = 1,NCOL
        IF ((READY(I) - MIN).LT.DIF) THEN
        COMP(I) = 1.0
        TRA(I) = TRA(I)+1.0
        END IF
        SUMCOL = SUMCOL + COMP(I)
 200   CONTINUE
C
      IF (SUMCOL.EQ.1.0) THEN
        ADDAMT = READY(MINPTR) + DIF
        DO 300 I=2,NROW
          IF (STA(I,MINPTR) .LT. ADDAMT) STA(I,MINPTR) = ADDAMT
 300     CONTINUE
C
        DO 400 I = 1,NROW - 1
          STA(I,MINPTR) = STA(I+1,MINPTR)
 400     CONTINUE
C
        STA(NROW,MINPTR) = 999999.0
        READY(MINPTR) = STA(1,MINPTR)
      ELSE
        DO 500 I = 1,NCOL
          IF (COMP(I).NE.0.0) READY(I) = READY(I) + B(I) + DIF
 500     CONTINUE
C
        DO 700 J = 1,NCOL
          IF (COMP(J).NE.0.0) THEN
            DO 600 I = 1,NROW
              IF (STA(I,J).LT.READY(J)) STA(I,J) = READY(J)
 600         CONTINUE
          END IF
 700     CONTINUE
      END IF
C
          IF (SUMCOL.EQ.1.0 .AND.  TRA(MINPTR) .EQ. 1.0) THEN
           SPROB = 1.0
           GO TO 3005
          END IF
        IF (SUMCOL. EQ. 1.0 .AND. TRA(MINPTR) . GT. 1.0) THEN
          SPROB = 1.0/TRA(MINPTR)
          GO TO 3005
        END IF
           TRA(MINPTR) = 0.0
     CCTT = CCTT + 1
3005  AP = AP + SPROB
5009  FORMAT(3E10.3)
```

```
       GO TO 40
C
C
 1000 APP = AP / CCTT
C      B(1)=3*DIF
C     P = 1 - AQQ
C     AVE = (DIF + (AQQ * B(1))) / P
     LAM =1./X
C     S = LAM * AVE
C     IF (S.GE.1.0) GOTO 5000
C     SS=LAM*DIF
C     MST = ((1.0 + AQQ) * (DIF**2 + B(1)**2 * AQQ)
C    $       + (4.0 * AQQ * DIF * B(1))) / P**2
C     NUM = NINT (S + (LAM**2 * MST)/ (2.0 * (1.0 - S)))
C     D =FLOAT(NUM) / LAM
     WRITE(6,2001)X,APP
2001  FORMAT(2E13.4)
2002  FORMAT (2E13.4)
3000  CONTINUE
5000  STOP
     END
```

Appendix C
Program 2

In the following section we give the code for computing the probability of collision by Poisson assumption.

```
C   THIS PROGRAM IS WRITTEN TO COMPUTE THE PROBABILITY OF COLLISION
C   UNDER THE ASSUMPTION THAT THE TOTAL TRAFFIC ON THE CHANNEL IS
C   POISSON.
C   THIS PROGRAM USES NETON-RAPHSON METHOD FOR FINDING THE ROOT
C   OF THE NONLINEAR EQUATION. FX IS THE FUNCTION AND F1X IS THE DERIVA
C   TIVE OF THE FUNCTION.
    INTEGER M
    REAL X,X1,LAM,EX,Q,P,AQQ,T1,DIF,AVE,MST,S,D,B(1)
    T1=0.064
    B(1)=0.2
    DO 330 JJ=5,50,10
    M=JJ
    LAM=4.0
    AQQ=0.0
 90 EPSI = 1E-3
    X = 15.0
110 DO 130 J = 1,1000
    X1 = X
    FX = LAM-X*(EXP(-0.064*X))
     F1X =-EXP(-0.064*X)+0.064*X*EXP(-0.064*X)
    IF (F1X.EQ.0.0) THEN
    F1X = 1E-4
    ENDIF
    X = X - (FX/F1X)
    ERROR = ABS(X - X1)
1    CHECK = ABS(X) - X
    IF(CHECK.NE.0.0) GO TO 110
    IF (ERROR.LE.EPSI) GO TO 120
130 CONTINUE
120 P=EXP(-X*0.064*M)
    DIF=T1
    AQQ = 1.0 - P
    AVE = (DIF + (AQQ * B(1))) / P
    S = LAM * AVE
```

```
      IF (S .GT. 1.0) GOTO 330
     MST = ((1.0 + AQQ) * (DIF**2 + B(1)**2 * AQQ)
    $       + (4.0 * AQQ * DIF * B(1))) / P**2
     NUM = NINT (S + (LAM**2 * MST)/ (2.0 * (1.0 - S)))
     D =FLOAT(NUM) / LAM
     WRITE(7,2001)LAM,X,AQQ
2001  FORMAT(3E13.4)
 220 CONTINUE
 330 CONTINUE
 150 STOP
     END
```

The vita has been removed from
the scanned document