SYSTEMS OPTIMIZATION THROUGH SIMULATION

by

ELI A. MATALON

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APPROVED:

J. W. SCHMIDT, chairman

H. D. SHERALI

R. A. WYSK

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Blacksburg, Virginia

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Chapter I

INTRODUCTION

1.1 BACKGROUND

1.1.1 The roll of simulation in systems modeling

Simulation modeling has proven to provide a considerable benefit in the development of optimal or near optimal policies for a system's operations. A simulation model can be viewed as a logical-mathematical-symbolic model which represents a complex real system, and can be manipulated in an experimental manner on a digital computer. By learning about the relationship between the input to and the output from the model, the analyst may acquire the necessary knowledge for evaluating alternate designs and different operating policies for the system.

Simulation offers the analyst the opportunity to evaluate systems: (a) with the ability to expedite the analysis, (b) with the ability to evaluate the system in those cases where a system's performance cannot be represented in a closed mathematical form, (c) without constructing and testing a costly real system to evaluate performances of a proposed one, and (d) without interupting the present operation or damaging the system, when the purpose of the analysis is to test performance and to evaluate effectiveness of an actual system in different operating environments.

A computer simulation model can be regarded as a "black box" that provides a set of output (response) variables $Y=[y_1,\ldots,y_m]$ for a given set of input (control) variables $X=[x_1,\ldots,x_n]$ (see Figure 1). Certain aspects of the system which are beyond the analyst's control, are represented by a set of variables $W=[w_1,\ldots,w_q]$. This set of uncontrollable variables may impose constraints on the system's behavior and limitations on the desired optimization process. The analyst's objective may be to optimize the system's performance by choosing that set of control variables which is likely to result in the most favorable system response under those limitations.

Generally, a given set X of n control variables, is assumed to generate a set Y of m responses, according to a relationship given by:

$$y_i=f_i(X,W) \qquad i=1,2,\ldots,m \qquad (1)$$

where W represents a vector of input variables which are beyond the analyst's control. For a given set W, one can replace equation 1 by:

$$y_i=g_i(X) \qquad i=1,2,\ldots,m \qquad (2)$$

where the response functions g_i (for $i=1,\ldots,m$) represent relationships between the controllable input to and the out-

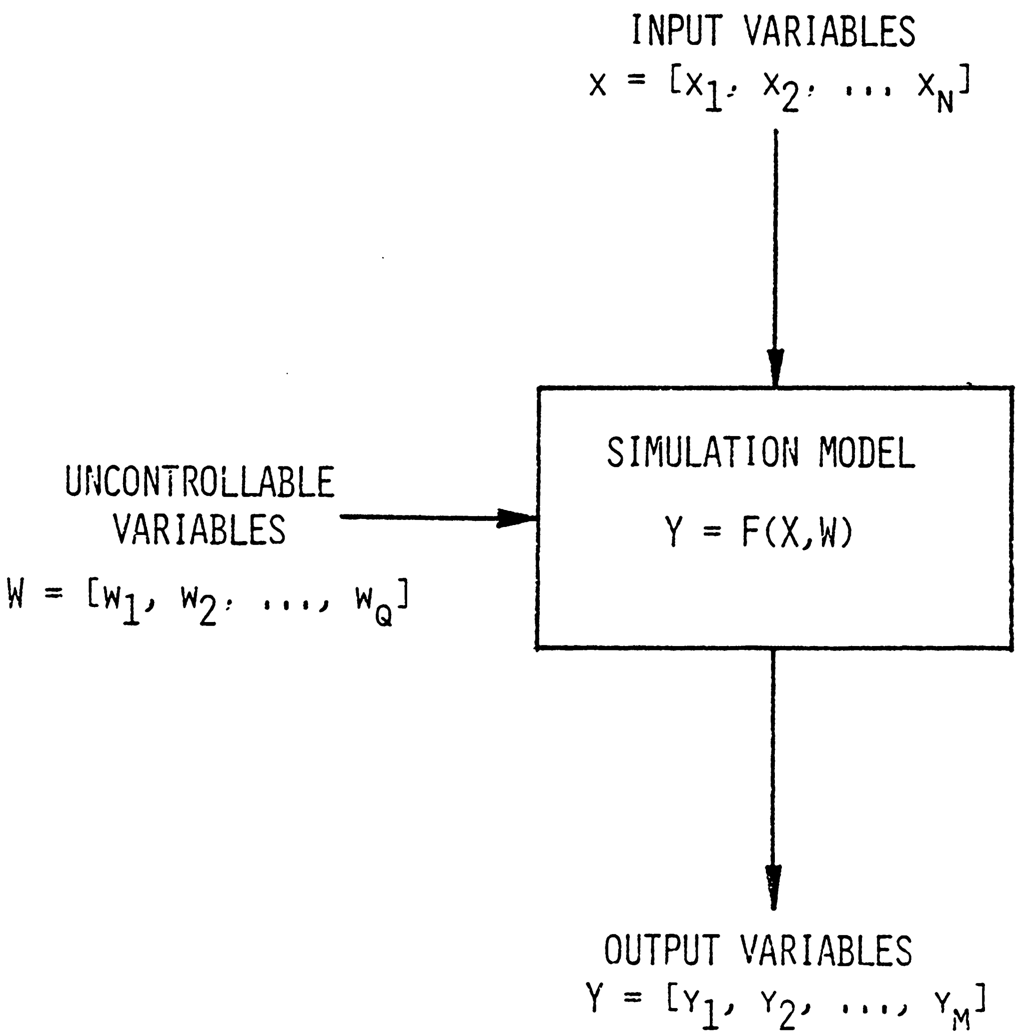


Figure 1: Representation of a simulation model as a "black box"

put from the complex system, which cannot be formulated in explicit mathematical form. These functions are investigated through simulation.

1.1.2 The approach to the optimization of a general complex system

The problem of optimizing the general multiple-variable, multiple-response system can be formulated in several ways. Two basic approaches are discussed by Biles in [14]. The first approach is a constrained optimization formulation, in which one of the simulation responses, y, is to be maximized or minimized subject to maintaining the remaining responses within predetermined bounds. A second approach is a single-objective formulation in which the m responses are combined in some manner to form a single objective function to be optimized. According to the latter approach, it is possible to formulate a single "achievement" or "performance" function of the form:

$$
\begin{aligned}
U(X) &= G[y_1, y_2, \ldots, y_m] \\
&= G[g_1(X), g_2(X), \ldots, g_m(X)] \qquad (3)
\end{aligned}
$$

for which a solution $X' = [x'_1, \ldots, x'_n]$ is considered optimal if the corresponding value $U(X)$ is the same as, or preferred to, any other value $U(X)$.

This research concentrates on the latter approach assuming that the system's overall performance can be represented in terms of cost or profit for a given period of operation. This approach is rather common in evaluating the efficiency of real-life systems, and is therefore interesting from a practical point of view. The analyst's major goal is to optimize the system's performance using simulation in order to obtain a set of control variables which results in maximum profit or minimum cost for that period. As shown later, seeking that optimum by applying a search procedure to a simulation model imposes difficulties in both performing and terminating the optimization process. The purpose of this research is to formulate an optimization procedure in order to overcome these difficulties.

1.2 ORGANIZATION

This thesis is divided into eight chapters and two appendices. The remaining sections of chapter I provide an overview of the problem of optimization through simulation. The problem is formulated in section 1.3. Section 1.4 describes the approach to the problem, specifies the objectives of research which led to this thesis, and suggests a "modular" optimization procedure in order to achieve these objectives.

Chapter II contains a literature review and provides a background on search and ranking methods which may be utilized in the development of an optimization procedure for a simular response function.

Chapter III contains a description of the structure of the "Sampling Routine" - a part of the modular optimization procedure developed in this research. A general discussion concerning the purpose of this routine, as well as a derivation of a statistical inferential procedure on which it is based, are given.

Chapter IV contains a description of the "Control Module" - the major part of the modular optimization procedure. An approximate distribution for the "Gross Savings' Rate" obtainable during an optimization process is derived. This derivation is followed by the design of a statistical test used to determine the termination point of the optimization process.

Chapter V contains a discussion of the problem of selecting the major parameters of the "Sampling Routine" followed by examples and recommendations for choosing values for the parameters discussed.

Chapter VI contains a discussion of the problem of selecting the major parameters of the "Control Module". The effect of various possible selections is demonstrated using examples.

Chapter VII contains a demonstration of the procedure developed in this research in optimizing a real simulation model. A short description of a (Q;r) reorder point inventory system which was selected for this demonstration is provided. The results of applying the optimization process to the simulation model of this system are presented and discussed.

Chapter VIII concludes the thesis. This chapter contains a summary of the suggested optimization procedure, its advantages and its drawbacks. Areas for further research are also discussed.

1.3 OPTIMIZATION THROUGH SIMULATION, PROBLEM FORMULATION

1.3.1 The sampling error associated with systems optimization through simulation

The response functions $g_i(X)$;$(i=1,...,m)$ in equation 3 are generally unknown and are usually assumed to be nonlinear. Any proposed technique for solving the problem must provide experimental estimates of these unknown functions. When the experimental observations are produced by a simulation model each realization of the system's response is affected by random error causing "noise" in estimating the value of the performance function (equation 3), such that:

$$\hat{U}_j(X)=U(X)+e_j(X) \qquad j=1,2,...,r \qquad (4)$$

The quantity U(X) is the true system's performance in terms of cost or profit for a given set X of input variables and r is the number of realizations generated by the model in order to estimate U(X). $e_j(X)$ reflects the random variation inherent in the stochastic system under study, and can be interpreted as sampling error. The uncontrollable variables W in equation 1 usually contribute to the random variation $e_j(X)$.

A "good" or "well-behaved" simulation model is assumed to generate realizations affected by noise with zero expected value for any set X of input variables. Such a model provides the analyst with an unbiased estimator $\hat{U}(X)$ of the true performance level U(X). This estimator is based on a given set of r independent observations, as follows:

$$\hat{U}(X)=\frac{1}{r}\sum_{j=1}^{r}\hat{U}_j(X)=\frac{1}{r}\sum_{j=1}^{r}[U(X)+e_j(X)] \tag{5}$$

where

$$\begin{aligned} E[\hat{U}(X)]&=E\{\frac{1}{r}\sum_{j=1}^{r}[U(X)+e_j(X)]\} \\ &=U(X)+\frac{1}{r}\sum_{j=1}^{r}E[e_j(X)] \\ &=U(X) \end{aligned} \tag{6}$$

The simulation experiment may also provide an unbiased estimator of the variance of system performance:

$$
\begin{aligned}
S_U^2 &= \frac{1}{r-1} \sum_{j=1}^{r} [\hat{U}_j(X) - \hat{U}(X)]^2 \\
&= \frac{1}{r-1} \{ \sum_{j=1}^{r} [e_j(X)]^2 - r[\frac{1}{r} \sum_{j=1}^{r} e_j(x)]^2 \} \qquad (7) \\
&= S^2_{e_j}(x)
\end{aligned}
$$

provided $e_j(X)$ are independently distributed.

The true performance U(X) can therefore be viewed as a point on an unknown (n+1) dimensional function at a given point X. The simulated realization $\hat{U}_j(X)$ is a random variable distributed about its mean U(X) at the point X (see [14]). The distribution of $\hat{U}_j(X)$ depends on that of the random error $e_j(X)$. Hence, a preliminary assumption concerning the probability density function of $e_j(x)$ should be made before attempting to analyze the system's response. For any given set X of input variables, the unbiased estimator of the system's performance, $\hat{U}(X)$, is taken to be the sample mean. This estimator has an approximate normal distribution, especially when the sample size is large. In this research, $e_j(X)$ is assumed to have a normal distribution with mean zero (reflecting an unbiased simulation model), and variance $\sigma^2(X)$ such that:

$$E[\hat{U}_j(X)]=U(X)$$
$$Var[\hat{U}_j(X)]=\sigma^2(X) \tag{8}$$

The realizations of the simulation model are assumed to be subject to independent, normally distributed random error with mean zero. Later in this thesis, the assumption of normality is relaxed and the effect of relaxation is demonstrated by examples. In general, $\sigma^2(X)$ is assumed to depend upon the point X and the existence of a functional relation of the form:

$$\sigma^2(X)=h[U(X)] \tag{9}$$

is assumed. In the simplest case, where $h[U(X)]=C$, $e_j(X)$ has an homogeneous distribution over the domain of X. Other typical variations of this function include the cases where $h[U(X)]$ is either a monotonically increasing or decreasing function of U(X).

1.3.2 The cost-elements of the optimization procedure

According to Schmidt and Taylor [51], two conflicting aspects of a system's optimization must be considered when the analyst wishes to choose the most favorable ("optimal") system's design. First, an attempt to optimize the system implies that the analyst expects a reduction in cost or an increase in profit to result from applying an optimization procedure. Second, the optimization process itself may not be cost-free.

Let $U(X_0)$ be the cost of an initial system's design or starting point for the search, and $U(X^*)$ the cost of the suggested "best" design, defined as a result of applying a given search technique. The improvement in the system's operating cost resulting from the search may be given by:

$$Q=U(X_0)-U(X^*) \tag{10}$$

Let C_s be the cost of executing the search and performing the required simulation experimentation in achieving the optimal design, X^*. The net return R resulting from applying the search is given by:

$$R=Q-C_s=U(X_0)-U(X^*)-C_s \tag{11}$$

Consider the case where the cost C of applying a search for an optimum is not negligible compared to the improvement in the objective function. The analyst's goal should be, then, to maximize equation 11. Hence, a search should be performed in such a manner so as to maximize the net gain or net return from its application.

According to [51], the search cost C_s is a function of both the number of points in the solution space evaluated in the course of the search and of the duration of the simulation run at each point. A point is defined by a specific set of control variables for which system's performance is evaluated through simulation. Since each individual realization taken from the simulation model at any point contains random error it is necessary to estimate system performance at that point by taking a sample of observations. The sample size required to satisfactorily characterize the performance at a given point may vary as the search progresses. This variation may affect the necessary number of replicates and, therefore, may affect the necessary duration of the simulation run at different points in the search.

Assume that k points are evaluated in the course of a given search. Let C_{f_i} be the fixed cost of the simulation at point i. C_{f_i} may be viewed as a fixed cost which does not depend on the duration of the simulation run but is associated with identifying a point for investigation by the search method. C_{f_i} may or may not be the same at each point. Let C_{ν_i} be the variable cost of the simulation associated with generating n_i replicates at point i. C_{ν_i} is assumed to equal:

$$C_{\nu_i} = (n_i)(C_r) \tag{12}$$

where C_r is the cost of one replicate of the simulation at any point. The cost C_s of executing the search is, therefore,

$$C_s = \sum_{i=1}^{k} [C_{f_i} + C_{\nu_i}] \tag{13}$$

The updated value of the cumulative search cost C_s is assumed to be known at any point in the course of the search.

1.3.3 Performing and terminating the optimization process

The attempt to optimize the system's performance through simulation introduces two major difficulties. The first is a typical problem associated with using a simulation model. This problem results from the sampling error associated with the experimental evaluation of the objective function at any point. The second is an outcome of the conflict between the analyst's specific goal, to choose the most favorable system's design (in terms of cost or profit for a given operating period), and the application of a relatively costly method to do so.

The first problem prevents a simple application of a known "optimum seeking" method to a simulation model. Most of the exploratory methods perform in an iterative manner; that is, by experimentally evaluating the objective function at points of interest in the first stage, then comparing or ranking the points in the second stage, and, finally, making a decision concerning the next iteration. When a search method is applied to a deterministic function the experimental observations provide the exact values of the function. The comparison stage is performed at each iteration by simply ranking the points according to the values of the objective function associated with each of them. However, when the system's performance is evaluated through simulation, the values of the objective function can be estimated only through sampling observations from the model at each point. These estimates are random variables whose variation depends on the "sampling error" of the model and comparing pairs of these estimates and/or ranking a group of them must be accomplished by appropriate statistical methods. Hence, the first step toward applying any optimization procedure should be to properly modify the search technique to deal with a stochastic function.

The second problem is associated with the attempt to apply a costly optimization procedure in order to achieve the "best" system design in terms of cost or profit. Since the funds that are spent in the course of the search affects the overall gain from applying the procedure, the analyst's objective is to maximize the net return R, as given by equation 11. In order to confirm this goal it is necessary to apply the optimization procedure only as long as it is desirable from an economical point of view. This can be accomplished by defining a criterion which terminates the search when the net return R is no longer likely to increase. The development of such a termination procedure is a major goal of this research.

1.3.4 Assumptions

1. A single measure of system performance is to be optimized, reflecting the system's cost or profit for a given period of operation.
2. The underlying functional relationship, U(X), between the measure of system's performance and the set of control varables X, is convex for the minimization of U(X), and concave for the maximization of U(X).
3. The cost of simulating the system's performance at any point X is not negligible and should be taken into account when applying any optimization procedure.

4. The analyst's goal is to maximize the net return resulting from the application of an optimization procedure based on the expected value of the system's performance.
5. For any given set X of control variables, the random error in estimating U(X) is an independent normally distributed random variable with mean zero and variance whose value may depend on X, (homogeneity of variance is not required).

The last assumption requires that at any given point X in the search, $U(X_i)$ will be estimated through a sample of independent observations from the simulation model. The independence between observations at a given point can be achieved by an appropriate design of the sampling from the simulation model. The normality of the random error in estimating $U(X_i)$ is implied by the "Law of Large Numbers" since $\hat{U}(X_i)$ is taken to be an arithmetic mean of n_i independent random variables having the same expectation and the same variance for each replicate at X (see [30], p. 117).

In general, the assumptions given above are not restrictive and apply to a reasonably broad class of optimization problems.

1.4 APPROACH TO THE PROBLEM

1.4.1 Suggested requirements for an acceptable optimization procedure

The following requirements characterize the approach to the problem:

1. It is necessary to design an external optimization procedure which may be applied to any multivariate simulation model with a single response (given in terms of cost or profit).

2. The suggested procedure should enable, to the extent possible, the use of known optimum seeking methods which have been proven to be efficient in exploring deterministic multivariable response surfaces.

3. The procedure should be applied to the simulation model in order to maximize the net return from the optimization process.

4. The optimization procedure may involve optimum seeking methods which enable interactive evaluations of their performances during the search for the optimum. This requirement suggests the implementation of sequential search procedures.

In summary, two major problems should be solved by any suggested procedure: (a) the design of a proper modification technique for sequential search methods in order to deal with stochastic functions, and (b) the design of a termination criterion for the optimization process. Both problems should preferably be solved independently of any specific sequential search method.

1.4.2 Modifying a search technique to deal with stochastic functions

Following the requirements outlined above, it is necessary to design a procedure which can be used with various sequential search methods. The purpose of this procedure is to assist the search in identifying the "better" point using $\hat{U}(X^*)$ and $\hat{U}(X_i)$, the estimated values of the objective at the "best point thus far", and at a new point, respectively. This comparison is needed after each iteration in order to determine whether improvement was achieved before making a decision concerning the next step in the search.

Two comparison methods are examined. The first is based on a statistical test which applies to the random variables $\hat{U}(X^*)$ and $\hat{U}(X_i)$, in order to determine which is more favorable. The second method defines a confidence interval for the difference between $\hat{U}(X^*)$ and $\hat{U}(X_i)$. A decision is reached by evaluating the prospective upper and lower limits for the improvement in the value of the objective function U, and by comparing these limits with the cost of the step that was needed to investigate the new point. The derivation of these methods is given in chapter III.

1.4.3 Designing a termination procedure for the optimization process

The problem of developing a termination criterion for the optimization process appears to be exceptionally complicated. Assume that a given system is to be optimized by selecting the best set of n control variables, $X=[x_1,...,x_n]$. Let $U[x_1,...,x_n]$ be the expected value of the measure of the system's performance as a function of the set of control variables. Assume, for example, that U(X) represents the system's cost for a given operating period. This cost should be minimized. Let X_ℓ be the ℓ^{th} point investigated by the search such that:

$$U(X_\ell) < U(X_i) \qquad i=1,2,...,\ell-1 \qquad (14)$$

Hence, X_ℓ is defined as the "best point thus far" in the course of the search. Suppose X' is the minimizing point for U(X) and, therefore, represents the desired, or best, set of control parameters for the system under study. The cost of evaluating an additional point, the $\ell+1^{st}$, is given by $C_{f_{\ell+1}}+C_{v_{\ell+1}}$. As suggested by Schmidt and Taylor in [51], when

$$U(X_\ell)-U(X') < C_{f_{\ell+1}}+C_{v_{\ell+1}} \tag{15}$$

the search should not be continued since the maximal improvement that may result by searching for better points is less than the cost of investigating the next point. In this case, X_ℓ maximizes the net return R that results from applying the search (equation 11), and should be chosen as "optimal".

Termination of the search may also be called for when equation 14 holds but equation 15 does not (see [51]). The nature of the sequential technique used may be such that at a certain point in the course of the search, further improvement of the objective function is either obtained in small increments, or is not likely to be obtained at all. The inability of the search to obtain further significant improvements may be anticipated when the search reaches the neighborhood of the optimum. In this case, continuation of the search may improve the value of the objective function by a relatively small amount, or may not improve it at all, while search cost continues to accumulate.

The search should be terminated, therefore, in either one of two cases: (a) when the search identifies a point for which the associated value of the objective function is close to the optimum, or (b) when the search reaches a point at which further improvement in the objective function is unlikely.

This research focuses on applying an optimization procedure to a simulation model in order to determine the set of control variables which maximize the net return given by equation 11. All the experimental observations of the objective function are based, in this case, on samples of realizations and contain random error. In fact, the improvement in the system's performance given by $\hat{U}(X_0)-\hat{U}(X^*)$ is a random variable. Hence, equation 11, which represents the net return from the search, is a stochastic objective function of the form:

$$\hat{R}=\hat{U}(X_0)-\hat{U}(X^*)-C_s \tag{16}$$

Before making an attempt to apply known methods for performing and terminating an optimization process for stochastic functions, one should observe the structure of equation 16. At any point in the course of the search, the net estimated return $\hat{R}$ is a function of the random variable $U(X_0)$.

This random variable represents the cost of a system's design as defined by the initial set of control variables X_0. R is also a function of U(X*), a random variable representing the cost of the "best design thus far" defined by X*, and a function of C_s, the updated amount of funds depleted by applying the search up to that point, (a cumulative cost term). The realizations of R, as generated in the course of the search, are affected, therefore, by that cumulative cost term and do not form a sequence of independent random variables.

It is reasonable to assume that the analyst would like to continue the search for the optimum based on economic considerations. Hence, continuation is desired as long as the true values of R (in equation 11) form a non-decreasing sequence with a slope greater than, or equal to, a desired rate of return from applying the search. If the R's in equation 16 form a sequence of independent realizations, one could design a simple termination criterion based on the continuous examination and testing of the slope of a straight line fitted to the sequence using a linear regression technique. The observation that R does not produce a sequence of independent realizations complicates the design of a termination criterion. This observation calls for a statistical procedure for testing the slope of a relatively "small" sequence of correlated data.

Two assumptions may be made at this point:

1. It is reasonable to assume that: (a) the incremental improvements in the values of the objective function tend to decrease as the search gets closer to the optimum, and, (b) from a certain point in the course of the search, further improvement in the objective function are either obtained in small increments compared to the beginning of the search or are not obtained at all. It is particularly true when attempting to either minimize a convex function or to maximize a concave function, (see Figure 2/a).
2. At any point in the course of the search, the value of the objective function is estimated through simulation, and, therefore, contains random error. Any sequential search technique used by the analyst should be able to compare the estimated values of the objective function at the "best point thus far", $\hat{U}(X^*)$, with the value of the function as evaluated at a new point, $\hat{U}(X_i)$. This comparison is needed in order to determine whether improvement has been achieved. Since both estimators $\hat{U}(X^*)$ and $\hat{U}(X_i)$ are random variables, this comparison may be performed by using inferential statistical techniques. The incremental improvements in the value of the objective

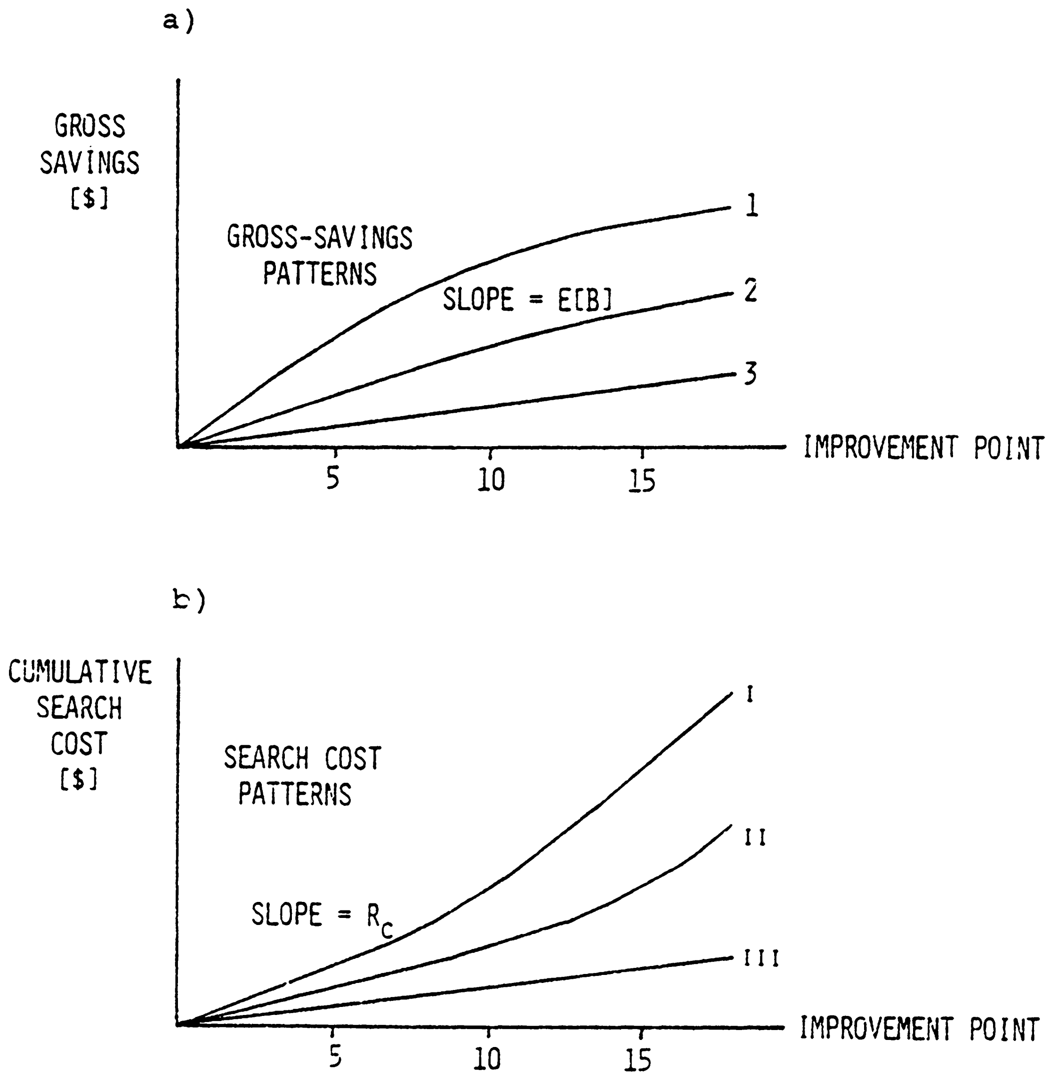


Figure 2: Possible accumulation patterns for the gross savings and search cost at improvement points along the search

function are assumed to decrease as the search approaches the optimum, thus reducing the difference between $\hat{U}(X^*)$ and $\hat{U}(X_i)$ (see assumption 1 above). Hence, a more powerful statistical technique with a higher resolution may be needed to investigate this difference, calling for larger sample sizes. Following this rationale, one may expect the search cost to form a pattern with a non-decreasing marginal value as shown in Figure 2/b.

The analyst reasonably wishes to maximize the net savings obtained by applying the search. In some cases, he/she may even wish to terminate the optimization procedure when the net return from the search is still positive but is less than some given value, (representing an alternative investment channel for the search funds). However, under no circumstances should a search be continued when it starts to accumulate cost at a higher rate than the rate at which the objective function is improving. Hence, following the assumptions outlined above, a termination criterion may be based on a continuous estimation and comparison of the slopes of the "savings graph" and the "cost graph" along the search. Once the cost slope is significantly larger than the savings' slope (implying that negative increments of net-savings are produced by the search) the optimization procedure is terminated. The final improvement point which was obtained up to the instant of termination is considered to be the "optimal", and the associated set of control parameters is taken to represent the "best" system's design.

Constructing a termination criterion based on comparing the rate of gross savings with the rate of search cost has two major advantages:

1. As opposed to the net savings sequence, the sequence of the gross savings associated with every improvement point is a sequence of independent random variables. Consequently, the slope of the gross savings' graph which represents the rate at which gross savings are accumulated, is a random variable whose probability density function may be approximated. It is possible, therefore, to use a relatively simple statistical procedure to determine the termination point.
2. A procedure which terminates the search whenever the savings are accumulated at a lower rate than that of the cost needed to obtain them provides the analyst with a sensitive termination criterion. This criterion may terminate a search which may otherwise pro-

ceed in an undesired manner. When, for example, a choice of the initial point, direction, and step size causes the search to improve the objective function in small increments, while cost is accumulating in large increments, this criterion terminates the search and calls for a revision of the initial conditions. This property represents a conservative approach in attempting to prevent an irreversible waste of funds in an unproductive search.

1.4.4 Research objectives

Following the assumptions and the approaches to the problem outlined above, the objectives of this research are:

1. To develop a procedure to compare, for each step of the search, the estimated value of the objective function at the "best point thus far" with that of a new point. This comparison is required in order to determine whether significant improvement was obtained by the search at each step.
2. To construct a decision rule to determine the sample size needed at each point in order to obtain the decision outlined in objective 1 above, and to design a default option for those points in which a decision cannot be reached.

To introduce an acceptable approximate distribution for the slope of the gross savings graph which can be updated in a recursive manner during the search as more improvement points are obtained.

4. To adopt a statistical approach, as well as to construct a test to compare the gross savings slope and the cost slope, in order to determine when to terminate the search.
5. To investigate the relationship between major parameters of the suggested termination criterion, and to provide recommendations for choosing their values.
6. To achieve objectives 1-5 independently, to the extent possible, of the type of the sequential search technique which is used in the course of the optimization process.

Objectives 1 and 2 may be achieved by constructing a "sampling routine" to assist the search procedure in dealing with the sampling error associated with the simulation model (see Figure 3). The input to the routine is a new set of control parameters ("new point") as defined by the search technique. The routine samples replicates from the model, follows a statistical inferential procedure, and compares the estimated values of the objective function at the new point with that of the "best point thus far". The routine

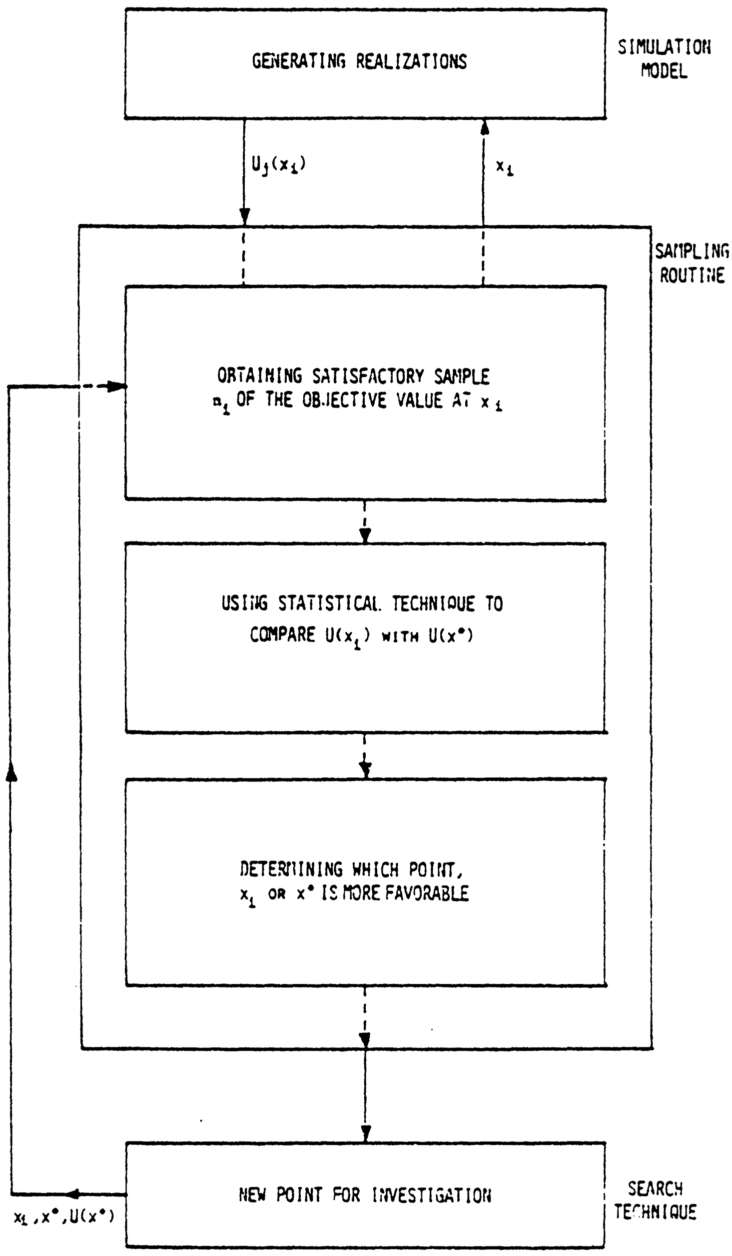


Figure 3: The relationship between the search technique, the simulation model, and the sampling routine

updates the sample sizes, if necessary, and then determines whether or not improvement has been achieved. This information is transferred back to the search technique which, in turn, determines the next point to be investigated. The structure of the sampling routine, as well as the development of the appropriate statistical technique on which it is based, is presented in Chapter III.

Objectives 3,4 may be achieved by constructing a major "control module" to keep track of, as well as to evaluate, the performance of the optimization procedure at any point in time along the search (see Figure 4). The control module allows the search to investigate an initial number of points and to achieve an initial improvement in the value of the objective function, and then determines whether additional funds should be spent in investigating new points, or whether the search should be terminated. This evaluation is performed before any point, suggested by the search procedure, is actually investigated. The structure of the control module and the derivation of the statistical procedure on which it is based, are given in Chapter V.

The performance of the suggested termination criterion is evaluated, later in this thesis, by testing whether it actually terminates a search whenever it is so desired from an economic point of view. The procedure is considered "suc-

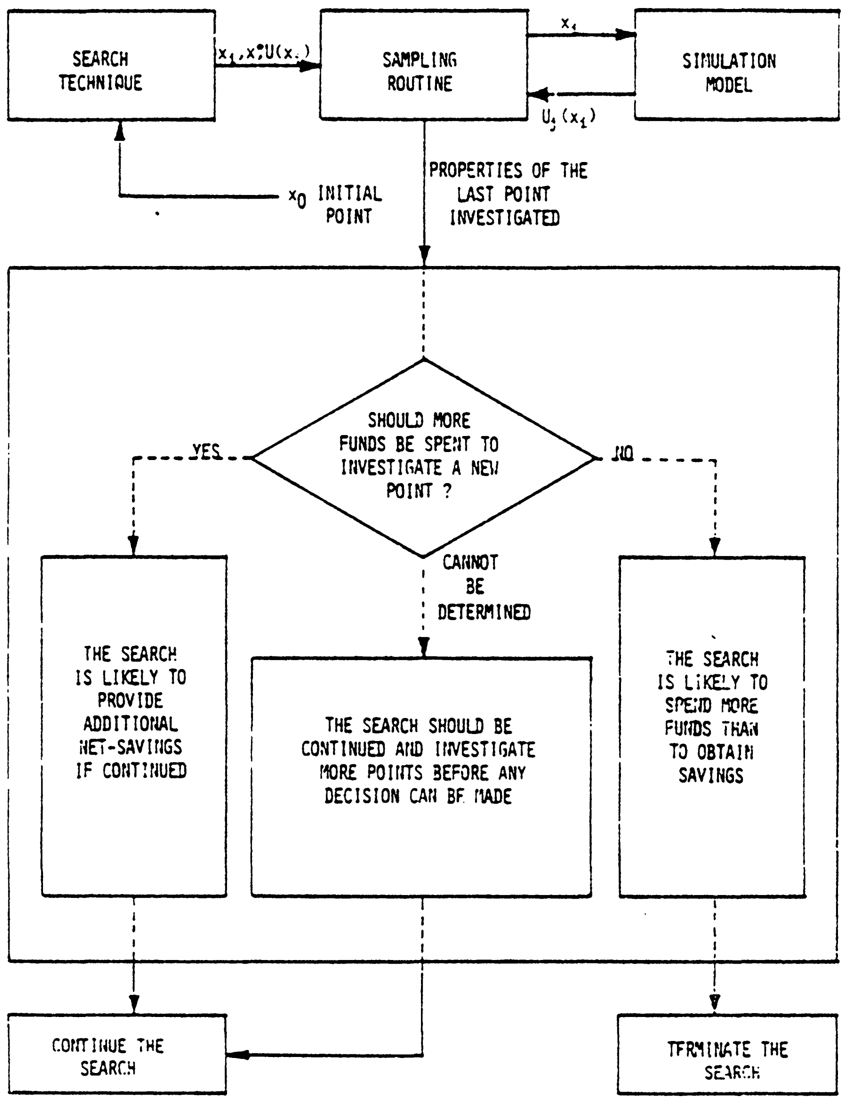


Figure 4: The relationship between the control module and the other modules of the optimization procedure

cessful" if it has the ability to determine when a search begins to accumulate cost at a higher rate than the rate in which further savings are likely to be achieved, and if this decision and the termination that follows, are performed before the search has unnecessarily spent too much funds. The ability to reach the true optimum of the objective function, however, is not considered as a measure of efficiency for the termination criterion. It may only be regarded as an indication of the compatibility of the search technique and the "structure" or "shape" of the investigated response surface. An efficient termination procedure is expected to allow the continuation of the search only as long as the net savings are produced with an acceptable rate. This procedure should conclude a search which does not provide this rate, regardless of whether it is still far from the optimum, realizing that the true optimum may not be reached at the termination point.

Chapter II

LITERATURE REVIEW

2.1 INTRODUCTION

Although simulation is widely used in connection with the problem of system's optimization, the literature on simulation does not reflect a full treatment of the special case on which this research focuses. In this case, the cost of performing a search for the optimal simular response is not negligible and may affect the overall effectiveness of the optimization process. Most of the optimization methods that may be found in today's literature represent, however, a good treatment of the "general" problem of optimizing a multivariable simular response. This problem may be defined as the problem of estimating the optimal value of a chosen measure of performance for the system under study, as well as locating a set of control variables which results in that optimal response. The majority of the existing methods are based on search techniques which were originally designed to explore deterministic functions and have been suitably modified to deal with stochastic functions. Such methods consist of successive evaluations of the system's response through simulation for various sets of input variables. They also prescribe a rule for choosing the sampling points and use an estimator for approximating the true, unknown, value of the system's response at each point.

When the system's performance is measured in either cost or profit for a given operating period, and when the cost of performing the optimization procedure cannot be neglected, the goal of any search method should be to maximize the net gain from its application. This net return is given by deducting the cumulative search cost from the gross improvement in the value of the objective function. The amount of resources that are available for the optimization procedure is, therefore, unknown a priori. This amount depends on the procedure's performances, and must be reevaluated during the search. Performing and terminating such an optimization procedure introduce problems that may not be easily solved by the existing search procedures.

Following Shubert [54], two basic types of experimental optimization methods exist:

1. Sequential search techniques,
2. Non-sequential, "random" or "grid" methods.

The first group consists of methods in which the sampling rule utilizes the values of the previous samples in determining the next sampling point, such as to obtain incremental improvements in the objective value and to proceed to the optimum. The main drawback of these methods is that they require that the underlying functional relationship between the control variables and the response be unimodal. This requirement is necessary since these methods are based on a principle of moving to the next point in the direction in which the function increases (for maximization) or decreases (for minimization). When the function is not unimodal, sequential methods may reach a local optimum only. The random or grid methods may not suffer this drawback. However, as long as only a limited amount of resources are available for the optimization procedure, these methods, as the sequential methods, may not obtain a global optimum with certainty.

This research concentrates on the optimization of a unimodal simular response where the availability of resources and, therefore, the total number of experimental evaluations, depend on the actual performance of the optimization procedure. In this case, sequential search methods, which perform in an iterative manner, are preferred to the non-sequential.

In an attempt to provide a background for the development of an optimization procedure for the specific case under study, a literature review was conducted. This review consists of:

1. A discussion of various sequential search techniques for investigating deterministic functions, which, after proper modifications may also be applicable in the case of optimizing stochastic simular responses (Section 2.2).
2. A short review of ranking procedures which may be used as a part of a proposed search method for stochastic functions (Section 2.3).
3. A discussion of exploratory techniques present in the literature for the purpose of investigating simular response surfaces (Section 2.4).

2.2 SEQUENTIAL SEARCH TECHNIQUES

2.2.1 General consideration

Search techniques play an important role in system optimization whenever the objective function cannot be explicitly represented in a mathematical form, as in the case of a simulation model. In this case, "exploratory methods" (or techniques for "optimization without derivatives") are often the only procedures that can be applied in an attempt to attain the best possible result. Consider those cases in which the function for which an optimum is desired, is the "performance" or "response" of some real-world system. Here, the response may be assumed to depend, perhaps exclusively, on the values of certain controllable parameters called "control variables". The precise relationship, however, between control variables and response is not completely known or cannot be quantified. To evaluate the response function at any specific point; that is, to measure the response for any set of values of the control variables, it is necessary to perform a costly and/or time consuming experiment. In most cases, the analyst has limited resources available to perform only a limited number of experiments. After observing all the results, he is expected to either recommend a single solution as the optimal or near optimal, or to state a set of ranges within which the optimal value of the experimental variables appears to lie.

2.2.2 Review of sequential search techniques

Search techniques applicable in the case of one control variable under the assumption of quasi-concave or quasi-convex functions (a full unimodal behavior) are discussed in [55] and [1]. Among these methods one can find the "Fibonacci method", originally suggested by Kiefer [39], and frequently approximated by a related technique, the "Golden section method" [1]. These interval elimination procedures are the most widely used among the single variable methods subject to inequality constraints. Several variations of these methods exist [11].

A procedure that may be used in order to find an unconstrained maximum of a single variable, non-linear function, is the "Coggin's Algorithm" presented in [41]. It is a combination of the single variable techniques proposed by Davies, Swann and Campey in [15] and Powell [48].

According to Simmons [55], the shift to higher dimensionality when dealing with experimental optimum seeking problems having two or more control variables, introduces certain difficulties that are not present in the single variable search problem. These difficulties are introduced since even for quasi concave (quasi convex) response functions, segments of the feasible region cannot be eliminated as easily as in the one-variable case. The simplest and most direct sequential search strategy, among the multivariable unconstrained methods, is the "Optimal Gradient Method", frequently known as the "Steepest Ascent" ("Steepest Descent" for minimization), suitably modified as required to deal with the unknown experimental response function, see [55].

One of the early discussions of some of the basic methods for seeking the optimum of a multivariable function, namely, the "Factorial Method", the "Random Method", the "Univariate Method" ("One at a time") and the "Steepest Ascent", is given by Brooks in [19]. In a later paper [18], these methods are compared in versions of two-factor situations, with or without experimental error. The sequential methods were generally found to be superior to the nonsequential methods. Quite a few other types of multivariable search strategies have been developed and studied since experimental seeking began to be of interest in the early 1950's. One rather straightforward sequential search strategy is the "Contour Tangent Elimination" method, described by Wilde in [58]. This method is applicable whenever one may assume that the value of the response function increases monotonically along a straight line from any feasible point X_i to the maximum X^*, (where a monotonically decreasing function is assumed for minimization problems). Quasi-concave response functions as well as certain other unimodal functions, possess this property. Other sequential methods tend to be somewhat more sophisticated than those discussed above; these methods were designed to obtain more rapid convergence to the optimum for certain special classes of response functions. Among the more interesting are the "Pattern Search" of Hooke and Jeeves [36], the "Method of Parallel Tangent" designed by Shah, Buehler and Kempthorne [53], the "Parallel Algorithm" of Chazan and Miranker [20], the "Rotating Directions Method" presented by Rosenbrock in [49], and the algorithm of Nelder and Mead [41], based on [46], as an extension of the simplex method of Splendley, Hext and Himsworth [57].

Sequential search techniques for the multivariable constrained case are discussed in [41]. One technique which is discussed, is the "Box Complex Algorithm" [16], proven to be effective in solving problems with nonlinear objective functions subject to non-linear inequality constraints, requires no derivatives and can be applied when the function may not be given in closed form. Another algorithm discussed is the "Constrained Rosenbrock" based on the "Automatic Method" proposed by Rosenbrock in [49] and [50]. This method has proven effective in solving problems where the variables are constrained. Another method, based on a procedure of Fiacco and McCormick [28], can be found in detail in [41]. This technique uses the problem constraints and the original objective function to form an unconstrained objective function which is then minimized by any appropriate unconstrained multivariable technique. The method can be used for functions which are not explicitly represented in a mathematical form, by choosing a "derivative free" unconstrained multivariable sequential search technique.

2.2.3 Summary and conclusions

Many sequential search techniques are known at the present. Some of them are specially designed to accomodate a special structure of the objective function. However, almost all of these techniques generally perform in a similar manner: all start from some feasible base point and move toward the optimum, based on sequential improvements in the value of the objective function. The step size may be fixed, or increased and decreased, subject to a given set of rules, and the success of using the method usually depends on the compatibility of the search technique with the function's structure, as well as on the selection of an initial base point.

Frequently, according to Simmons [55], the analyst wishes to stop experimenting whenever he feels he has both learned "enough" about the response function and located the optimum with sufficient accuracy. When applying a sequential search technique in an attempt to do so, the experimenter may face one or more of the following problems:

1. The total amount of experimental resources to be made available for the search may not be known until after experimentation has begun.
2. The resources required for an experiment may vary unpredictably, perhaps as a function of the control variables or the response.

It is, therefore, reasonable to assume that the analyst would prefer to be familiar with a procedure that will enable him to reevaluate the performance of his search procedure during the experimentation period. Such a procedure will be used to determine, at any given point, 1) whether to proceed with the search, and 2) the amount of additional resources to be made available for the search. The need for such a termination criterion and continuation policy is apparent when using costly search procedures to optimize a real-life large scale system using simulation.

2.3 RANKING PROCEDURES

2.3.1 General considerations

The problem of ranking several experimental evaluations and selecting the "best one" may be of considerable importance in connection with optimizing a simular response function. Although most of the sequential search techniques necessitate comparison of only two points at a time, other sequential search methods involve evaluations and comparisons of the simular response at several points at each iteration. (See, for example, the "Nelder and Mead" algorithm or "Box's Complex" algorithm, presented in [41].) Such a comparison is also vital to the non-sequential search methods. These methods, however, are designed to perform with a fixed number of experimental evaluations, and are of less interest in this research.

Ranking procedures are designed to select the "best" of several populations having the same distribution type but possibly different distribution parameters. The population with the largest (smallest) value of a given parameter is considered desirable and is, therefore, defined to be "the best". Such ranking procedures may be used in connection with the optimization of a simular response. It is commonly assumed that the simulated replicates are characterized by a single distribution type, with parameters that may depend on the set of input variables (the "sampling point"). In many practical cases, one may also assume that a simulation model produces normally distributed observations of the system's response, with mean representing the true value of the system's measure of performance at the sampling point, and variance that is usually unknown and may not be homogeneous over the solution space.

2.3.2 Review of ranking procedures

Bechhofer [2], considered the problem of ranking k normal populations with known variances. This single sample, multiple decision, procedure guarantees with probability P' that the selected population is the best whenever the standardized difference d between the largest (best) mean and the second largest mean is greater than, or equal to, a specific value d'.

The problem of ranking means of normal populations with a common but unknown variance is slightly more general than the one discussed in [2] and was considered by several authors. Bechhofer, Dunnett and Sobel [5], presented a two sample multiple desicion procedure for ranking means of such normal populations. In a later paper [3], Bechhofer presented a multiple desicion ranking procedure using an approach similar to [2]. This proposed sequential procedure was designed to terminate with probability unity and to guarantee that the probability of a correct selection is at least equal to some specified value P', whenever the largest population mean is greater than the second largest by some prespecified amount d'. A better computing formula, which considerably simplifies and reduces the computations required in applying the original procedure [3] was given four years later, by Bechhofer and Blumental in [10]. Gupta [31] considered two problems. The first deals with selecting a subset containing the population with the largest parameter, such that the probability that the selected subset contains that population is at least P'. The second problem deals with selecting a subset containing all populations better than a standard. Gupta's proposed ranking and selection procedure can be used to treat k normal populations with a common unknown variance.

Several papers concerning selection problems involving other parameters and/or types of populations were published. Bechhofer and Sobel [7], [8], presented two papers dealing with the problem of selecting the normal population with the smallest population variance, and a paper [9] concerning selection of multinomial populations. Selecting the binomial population with the smallest "probability of success" was considered by Huyett and Sobel in [35], while the problem of selecting the exponential population with the largest-scale parameter was presented by Sobel in [56].

The most general ranking and selection problem deals with selecting the best population from among k normal populations having unknown and unequal variances. It is also the most interesting problem in connection with the analysis of a simular response, whenever homogeneity of variance cannot be assumed. In an abstract published by Bechhofer, Dunnett and Sobel in 1953 [4], and in a paper by Bechhofer in 1954 [2], the authors noted that they were working on the case of k populations having unknown and unequal variances, via a generalization of the two-sample procedure that was used in [5], (for the case of a common and unknown variance). Refering to Bechhofer, Kiefer and Sobel [6], no such work appeared prior to 1968. Other authors, however, considered selecting problems involving k=2,3,4,5 populations, or presented modifications of procedures that were originally designed for the case of known and unequal, or unknown and equal, variances. These procedures merely incorporate some methods of variance estimation and may not satisfy, in most cases, the probability requirements concerning a correct decision [23].

Dudewicz and Dalal [23] were the first to present (1971), a ranking and selection procedure for k populations with unknown and unequal variances. This procedure solves the general ranking problem with unknown variances and may provide complete ranking of the k populations for k=1,...,25. It is the most useful ranking procedure which may be used with computer simulation experiments. The procedure consists of two stages: (a) taking an initial sample from each population in order to estimate the size of the final sample needed, and (b) computing the statistics needed for selecting the best population. The procedure can be performed in an iterative manner. The probability of a correct decision is defined as that of selecting the best population whenever the difference between the "best" and the "second best" is at least equal to a desired value. The method may also be modified for other goals, such as selection of the "t" best population and selection of a subset of the original k populations.

2.4 OPTIMIZATION PROCEDURES FOR SIMULAR RESPONSE FUNCTIONS

2.4.1 Introduction

Several techniques are presented in the literature for the purpose of locating "best" or "improved" solutions to the optimization through simulation problem. Farrell [26] presents a discussion and bibliography of these techniques and divides them into three major categories:

1. Naive techniques, which do not try to infer any mathematical properties of the objective function. (Such as running the simulation at several input values, generated at random.)
2. Methods appropriate for unimodal objective functions, mainly applications of several nonlinear programming techniques which may be classified as:
   a) Methods of approximating the shape of the objective function which utilize the approximated function to determine a steepest ascent (descent) direction using classical optimization procedures.
   b) Nonlinear programming techniques which do not require a derivative of the objective function.
3. Techniques useful for multimodal objective functions.

A discussion concerning these techniques is presented in the following sections.

2.4.2 Naive techniques

Naive techniques do not require knowledge of the objective function and may be classified, according to Farrell [26], as follows:

Heuristic Search Methods: For these methods some knowledge about the system is used to repeatedly guess at input values, run a simulation experiment and terminate the search when a "good" solution is assumed to be obtained. The success of such methods depends entirely on the analyst.

Complete Enumeration Techniques: These methods are applied in those cases where only a finite number of input values is feasible. The simulation may be run for all possible combinations and an optimal solution is assumed whenever no uncontrollable components exist in the model. Otherwise, an attempt to optimize an expected return from the model may be done by incorporating random variable generators to provide values for the set of uncontrollable factors. The simulation is then run several times for each feasible set of controllable variables. In this way, complete enumeration yields at least a "near optimal" solution. Schmidt, Taylor and Bennett [52] provide an example of the type of problems which may be practically solved by this method.

<u>Random Search Methods</u>: These methods investigate the simular response at a subset of the set of all possible input values, by choosing values at random for the controlable variables. Such techniques do not guarantee an optimum but increase their efficiency as the number of experiments increases.

<u>Variations on Complete Enumeration</u>: These techniques are applicable to problems with only a small set of feasible values of the controllable variables, but which contain uncontrollable variables as well. According to [26], these techniques aim to aid in determining the number of replicates needed to estimate the expected value of the response at any given set of input values, and utilize a complete enumeration of the controllable variables. A method described by Kleijnen, Naylor and Seaks [40] samples the simular output at each input point, estimates the expected values of the means, ranks them and provides a correct selection with some prespecified probability. Another method, suggested by Schmidt, et al. [52], runs several replications of the simular response for each vector of input variables. This method utilizes confidence intervals in an iterative manner in order to eliminate points from further consideration, until only one set of input values remains, or until the cost of

further investigation is greater than the prospective gain. Such a method is usefully applied to a certain, though rather limited, class of problems.

Variation on Random Search: A technique presented by Luus and Jaakola [43] which utilizes a sequence of random searches, each of which run over a range of input variables centered at the most favorable point of the previous search in order to proceed toward the optimum. The technique is apparantly simple and has demonstrated successful application on several problems, (six such problems are described in [43]).

2.4.3 Techniques for unimodal objective functions

The first group of techniques in this category consists of methods which approximate the shape of the unknown response surface, and apply steepest ascent (descent) algorithms to the approximated function in order to determine the next point for investigation. Most of these methods perform in an iterative manner, by first computing the value of the objective function at points near an intermediate solution point, and then using those points to fit a hyperplane which approximates the objective function at the neighborhood of the intermediate solution. A steepest ascent (descent) vector is defined by taking partial derivatives of the hyperplane function with respect to the input variables, new points are investigated along this direction and a new intermediate solution point is obtained. This process is repeated until no further significant improvement can be achieved. According to Farrell, several methods for determining the points at which to investigate the objective value in the neighborhood of the intermediate solution exist. Experimental designs for that purpose can be found in [17] and [57] (simplex), [21], [38], [22] (fractional factorial), and [37] (full factorial). Three examples of search methods for unimodal response surfaces are described below.

Miharm [44] introduces an interesting approach to the search for a description of the response surface of a simulation model. The surface is defined as the locus of the expectations of the simular responses as a function of a p-dimentional control variable $X=[x_1,...,x_p]$. The exploration procedure uses simplex experimental designs of the p control variables to estimate planer and quadratic approximations of the simular response surface, and performs in two stages. The first stage approximates the simular response function by p-dimensional hyperplanes, proceeds toward a possible location of the optimum in an iterative manner, and terminates when an approximation plane is found to be devoid of any significant tilt. The second stage performs an intensive search during which the nature of the simular response function is revealed by the estimation of approximating surfaces of higher degree, and the locus of the optimal response is more accurately situated. At the end of the second stage the response function is estimated by the last available quadratic approximation, and the designed contours of the response surface are plotted. The paper presents simplex experimental designs for the case of p=2. Designs of higher dimensionality may be found in Box and Behnken [17]. The assumptions under which the procedure is valid include the existance of: 1) a "good" simulation model that has been adequatly validated, 2) an ability to define a set of independent, quantitative and unbounded control factors, and 3) an independent, normally distributed, sampling error with mean zero and a fixed variance, regardless of the set of control factors. The accumulation of search costs, including the cost of each encounter with the simulation model, is not taken into consideration in evaluating the overall performance of the optimization process. The procedure appears to require substantial modifications in order to handle these costs. Its usefullness is, therefore, limited to those cases where costs are negligible.

Schmidt and Taylor [51], present an optimization technique which may be used in optimizing a measure of systems performance given in terms of cost or profit. The procedure performs in an iterative manner and uses successive quadratic approximations in order to search for the optimum. At each iteration, such an approximation is develpoed through least squares and the optimum of the approximating function is located. The search technique determines the number of replications at that point, and the simulation is executed. Both the sample mean and the sample standard deviation of the measure of system performance are calculated. The point yielding the least favorable value of the sample mean among the original n points (to which the quadratic function is fit), is replaced by the new optimal point. A new quadratic surface is fit to the current set of points, and the entire process is repeated. The search is terminated when two criteria are met. At each iteration a statistical test is conducted to determine whether the approximating equation fails to fit the experimental data. If a significant "lack of fit" is indicated, the search continues. Otherwise, a hyperplane is fit to the data through the method of least squares. If this plane is found to be "horizontal", the optimum is assumed to lie within the region represented by the current experimental data and near the optimum for the fitted quadratic function and the search is terminated. The number of replications to be taken at a given point is defined in conformity with pre-specified statistical criteria with respect to the test for lack of fit. The difference between the expected value of the measure of effectiveness estimated from the approximating quadratic equation at the new point, and the expected value estimated through simulation at that point is tested. The test takes into account the total cost of evaluating system performance at the new point and defines that cost to be the critical difference for which the approxomating equation should be considered as "inadequate". Thus, the degree of lack of fit is defined by economic considerations. The method is applicable whenever the functional relationship between the measure of system effectiveness and the control variables is convex. Homogeneity of the variance of the measure of system performance over the solution space is also necessary.

The procedures discussed above may represent a class of "Response Surface Analysis" techniques. These techniques approach the problem of optimizing a function, which cannot be explicitly represent in a mathematical form, by searching for a discription of its response surface. In general, an approximating functon is obtained through an exploratory search which evaluates the objective function at various points. The approximating function is then optimized using classiccal optimization procedures. When a large set of control variables is assumed to affect the objective function, a large sample of points is necessary in order to fit an approximate function. In addition, the application of classical optimization procedures may require a substantial computational effort in order to obtain the optimal point in each iteration. When search costs are not negligible, a need for large samples and a relatively long computation time may be regarded as a drawback which limits the usefulness of these methods.

Several other comuterized multiple variable optimization procedures, developed for use in connection with simulation models, are presented by Biles in [14]. Each of these methods involve an adaptation of the original procedure on which it was based in order to accomodate multiple simulation responses. The original search techniques where "Box's Constrained Simplex" [16] and the "Method of Steepest Ascent" modified to create four multiple variable, multiple response, optimization procedures. The author demonstrates and compares these techniques using different simulation models. Although the suggested techniques are developed for use in connection with simulation models, the cost associated with the implementation of each of these methods in particular, or with any optimization procedure which involves simulation runs in general, is not taken into consideration. The efficiency of each procedure is measured by the ability to locate the optimal value within certain number of iterations or within some limits on the computation time. This approach is reasonable when search costs are either negligible, or cannot be related to the desired measure of system performance.

The second group of methods applicable for unimodal simular response surfaces consists of techniques without derivatives as follows:

Coordinate search ("One at a time"): This is a simple method according to which one variable is chosen at a time. A search along that coordinate is performed until no further improvement is achieved, before a second variable is considered. Lefkowitz and Schriber [42] present a problem of optimizing a simular response in which such a method is applicable.

Several other techniques are chosen in [26] to be classified in this group. However, no experience of their performances in connection with optimization through simulation is reported. Three of these methods are given below:

Pattern search: This is a sequential method based on Hooke and Jeeves [36] which uses a coordinate search to determine a "pattern" direction along which a new point is tested in each iteration.

Rotating coordinates: This method is based upon Rosenbrock's procedure [49] and changes coordinate systems so that the pattern direction becomes a coordinate direction and all other coordinate directions are orthogonal to the pattern direction. The method has capability of detecting directional changes of a ridge and may be used in cases where other methods may fail.

Mugele's "Ridge Follow": This method is based on [45] and moves in a coordinate direction as long as a move improves the objective function and utilizes a pattern only when coordinate directions fail. (The technique mainly introduces an alternative for the "one at a time" technique when it fails.)

The last technique which yields a local optimum and may be applicable for optimizing unimodal functions is: The "Razor Search" described for simulation by Nelson and Krisebergh [47] as a variation of the pattern search. This technique uses pattern search until it fails, and then makes a random jump in order to obtain a point from which pattern search can be reinitialized. When that search fails, the technique uses the two failure points to generate a new pattern, thus decreasing the probability of terminating at a suboptimum.

2.4.4 Techniques for multimodal surfaces

Two techniques for investigating response surfaces having several local optima in attempt to locate their global optimum, are presented in [26]. Elridge (see also [25]) utilizes a random factorial design and regression to devide the solution space into several unimodal surfaces which can then be optimized by any applicable technique. Hartman (see also [34]) introduces a method of spreading several starting points for any search technique, thus increasing the chance of finding the global optimum. Several other techniques exist. The reader may refer to Farrell, McCall and Russell [27] for more details on techniques applicable in optimization through simulation problems.

Chapter III

THE STRUCTURE OF THE SAMPLING ROUTINE

3.1 INTRODUCTION

The solution methodology, presented in Chapter I, ssuggests the development of a modular optimization procedure consisting of four major modules as follows:

1. A simulation model.
2. A sequential search technique.
3. A "sampling routine".
4. A "control module".

This chapter contains a discussion of two possible structures for the "sampling routine". The purpose of this routine is to assist the search technique in identifying an improvement in the value of the objective function at each iteration. The input to this routine is a new set of control parameters (defined by the search technique). The "sampling routine" controls the process of sampling replicates from the model and follows a statistical procedure in order to compare the estimated values of the objective function at the new point with that of the "best point thus far". The interfaces between the sampling routine, the simulation model, and the serach technique is presented in Figure 3 in

Chapter I. These interfaces are generalized in Figure 5 below.

The sampling routine considers the new point X_i as better than the "best point thus-far" X^*, whenever $E[U(X_i)]$ is preferred to $E[U(X^*)]$. Assume, without loss of generality, that $U(X)$ is to be minimized. Hence, a direction of the search will be defined as "promising" if the true difference $E[D_i]$ defined as $E[U(X^*)]-E[U(X_i)]$ is greater than some specified value d'. Since both $E[U(X^*)]$ and $E[U(X_i)]$ are unknown, only their estimators, $\hat{U}(X^*)$ and $\hat{U}(X_i)$ respectively, may be provided by the simulation model and only an unbiased estimator for $E[D_i]$, given by $D_i=\hat{U}(X^*)-\hat{U}(X_i)$, may be obtained.

Two basic approaches are used in evaluating and testing the random variable D_i. The first uses a statistical test which compares D_i to d'. The second estimates a confidence interval for $E[D_i]$ and compares its limits to d'. Both methods assume that D_i is the difference between two independent normal random variables. The value of d' has a profound effect on the classification of an investigated direction as "promising" or "not promising", and should be cautiously chosen. One rather simple and acceptable approach is to choose d' to reflect the minimum acceptable rate of improvement required to justify the investment of

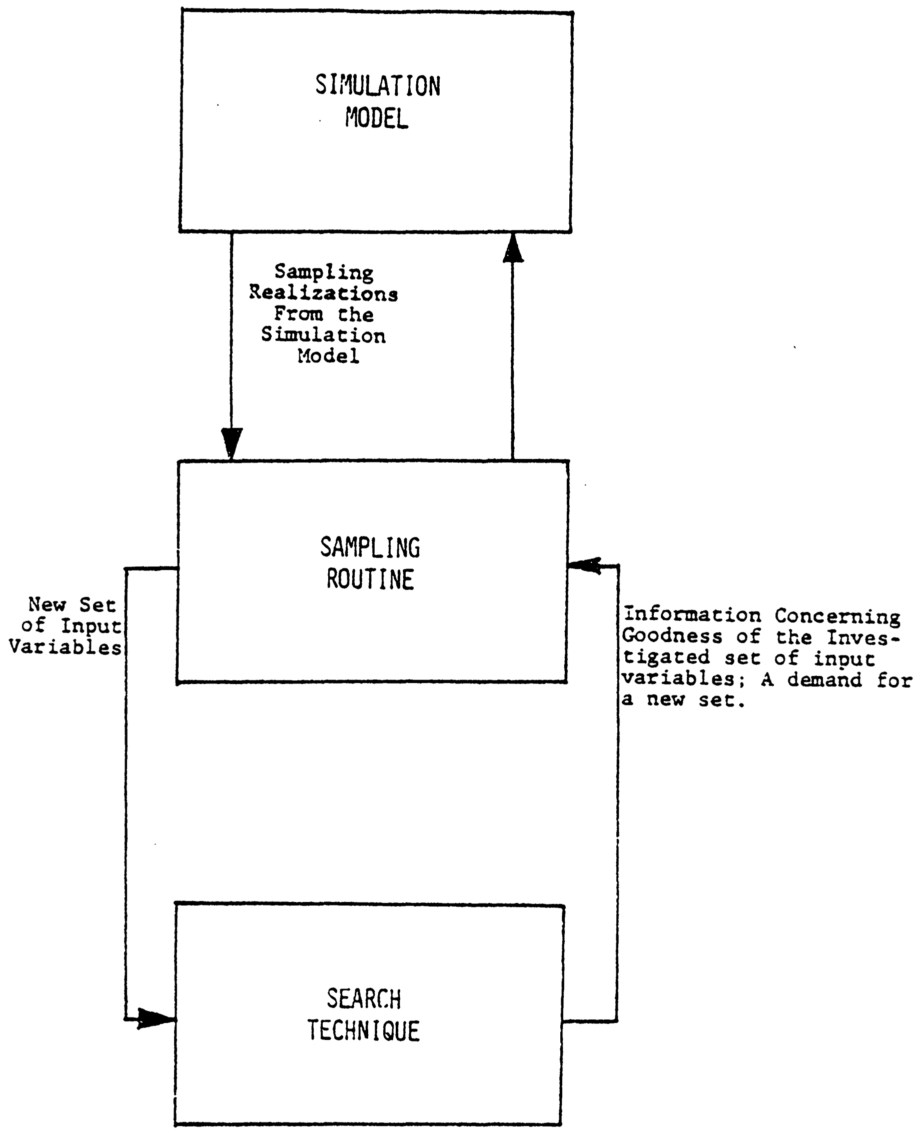


Figure 5: The sampling routine as an external modification for the serach technique

funds involved in investigating the new point. Let $(C_r)(n_i)$ be the variable cost associated with n_i replicates of the system's measure of performance at point i, at a cost C_r per replication. Assume that the fixed cost associated with defining a new point is negligible compared to the cost of replication. Let r denote the desired rate of return from the optimization process. Hence, the minimum acceptable improvement in the simular response may be defined as:

$$d'=(1+r)(C_r)(n_i) \tag{17}$$

When the improvement achieved by a step in the course of the search is less than that defined in equation 17, the current direction of the search may be considered as "unpromising", and point i is defined as "not an improvement". The sampling routine transfers that information to the search technique which consequently determines a new point X_{i+1} for investigation. Otherwise, the search technique treats the new point as "an improvement" and continues to perform accordingly.

3.2 EVALUATING THE "GOODNESS" OF A DIRECTION USING A STATISTICAL TEST

At each point in the search, check whether improvement is achieved by testing:

H_0 : $E[\hat{U}(X^*)] \leq E[\hat{U}(X_i)]$ no-improvement

H_1 : $E[\hat{U}(X^*)] > E[\hat{U}(X_i)]$ improvement

If H_0 is rejected, system's cost at X_i is assumed to be lower than at X* (the "best point thus far"), and the new point is defined as an "improvement". When H_0 is not rejected, the new point does not improve the system's cost and X* remains the "best thus far". After performing the test, this information is transfered to the search-method module, which then determines the next point for investigation.

The test statistic is the estimated amount of improvement, given by the difference between the estimators of system's cost at X* and X_i as follows:

$$D_i=\hat{U}(X^*)-\hat{U}(X_i) \tag{18}$$

<u>Type I error</u> is given by:

$$\alpha=Pr\{\text{reject } H_0 \quad | \quad E[D_i]=0\}$$

<u>The power of the test</u> is defined by:

$$1-\beta=Pr\{\text{reject } H_0 \quad | \quad E[D_i]=d'>0\}$$

where d' represents the desired difference to be detected by the test.

The i^{th} point is defined to be the "best point thus far" only if it provides an improvement of at least d' in system's cost, compared to the current "best point". d' is chosen to reflect the minimum acceptable rate of improvement, required to justify investing funds in exploring a new point, as given by equation 17. Hence,

$$1-\beta = \Pr\{\text{reject } H_0 \mid E[D_i]=(1+r)(C_r)(n_i)\} \tag{19}$$

The unbiased estimator of $U(X_i)$, the true system's cost, is given by $\hat{U}(X_i)$ in equation 5. For large n_i:

$$\hat{U}(X_i) \overset{ap}{\sim} N\{\ E[\hat{U}(X_i)]\ ;\ Var[\hat{U}(X_i)]\ \} \tag{20}$$

where

$$Var[\hat{U}(X_i)] = \sigma_i^2/n_i \tag{21}$$

$$\sigma_i^2 = \sigma^2(X_i) = Var[\hat{U}_j(X_i)] \tag{22}$$

and $\hat{U}_j(X_i)$ represents the j^{th} realization of system's cost at X_i, $j=1,\ldots n_i$. Hence, for large n_i and $n*$, the distribution of the test statistic is given by:

$$D_i = \hat{U}(X*) - \hat{U}(X_i) \overset{ap}{\sim} N\{\ E[D_i]\ ; Var[D_i]\ \} \tag{23}$$

where

$$E[D_i] = E[\hat{U}(X*)] - E[\hat{U}(X_i)] \tag{24}$$

and,

$$Var[D_i] = Var[\hat{U}(X*)] + Var[\hat{U}(X_i)] = \sigma_i^2/n_i + \sigma_*^2/n_* \tag{25}$$

D_c, the "<u>critical</u> <u>value</u>" for the test, may be computed using

$$\alpha = Pr\{ D_i > D_c \mid E[D_i]=0 \}$$
$$= Pr\{[(D_i - 0)/\sigma_{D_i}] > [(D_c - 0)/\sigma_{D_i}] \mid E[D_i]=0\}$$
$$= Pr[Z > D_c/\sigma_{D_i}]$$

yielding,

$$D_c = (\sigma_{D_i})(Z_{1-\alpha}) \tag{26}$$

H_0 should be rejected when $D_i > D_c$. Using equation 19, the power of the test can be expressed as:

$$1-\beta = Pr\{D_i > D_c \,/\, E[D_i]=d'=(n_i)(C_r)(1+r)\}$$
$$= Pr\{[D_i - (n_i)(C_r)(1+r)]/\sigma_{D_i} > [D_c - (n_i)(C_r)(1+r)]/\sigma_{D_i}\}$$
$$= Pr\{Z > [D_c - (n_i)(C_r)(1+r)]/\sigma_{D_i}\}$$

Hence,

$$Z_\beta = [D_i - (n_i)(C_r)(1+r)]/\sigma_{D_i}$$

or,

$$Z_\beta = [(\sigma_{D_i})(Z_{1-\alpha}) - (n_i)(C_r)(1+r)]/\sigma_{D_i} \tag{27}$$

where,

$$\sigma_{D_i} = [\sigma_i^2/n_i + \sigma_*^2/n*]^{1/2} \tag{28}$$

Since both σ_i^2 and σ_*^2 are unknown, their unbiased estimators S_i^2 and S_*^2 respectively, are used and σ_{D_i} is approximated by:

$$S_{D_i} = \hat{\sigma}_{D_i} = [S_i^2/n_i + S_*^2/n_*]^{1/2} \tag{29}$$

Thus,

$$Z_\beta = [(S_{D_i})(Z_{1-\alpha}) - (n_i)(C_r)(1+r)]/S_{D_i} \tag{30}$$
$$= Z_{1-\alpha} - [(n_i)(C_r)(1+r)]/S_{D_i}$$

For given values of $\alpha, \beta, C_r, r, n*$, and S_*^2, equation 30 represents a necessary condition which should be satisfied before the test can be performed.[1] n_i should therefore be fixed as the least positive integer which satisfies:

$$Z' = Z_{1-\alpha} - [(n_i)(C_r)(1+r)]/S_{D_i} < Z_\beta \tag{31}$$

Based on the above derivation, one may propose a procedure consisting of two stages as follows:

<u>Stage 1</u>: in which equation 31 is used to determine the sample size needed in order to satisfy the power requirements, using the following steps:

1. Compute $Z_{1-\alpha}$; Z_β ; $(1+r)(C_r)$.
2. Sample $n_i = n_o$ observations at point i.
3. Estimate σ_i^2 using unbiased estimator S_i^2 based on sample of n_i.
4. Compute S_{D_i} using equation 29.
5. Compute $Z' = Z_{1-\alpha} - [(n_i)(C_r)(1+r)]/S_{D_i}$.

----

1 Since both S_i and S_* are unknown, the power of the test should actually be computed using a "t" distribution for the test's statistic. The normal distribution is used here as an approximation only. A more accurate formulation which uses a "t" variate with f degrees of freedom, as well as a computational formula for f are given in the next section.

6. Compare the required value Z_β with Z':

   a) If $Z'>Z_\beta$, the power requirement is not satisfied. Increase the sample size $n_i=n_i+1$, go to step no. 3.

   b) If $Z'\leq Z_\beta$, the power requirement is satisfied, go to stage 2.

<u>Stage</u> <u>2</u>: once the procedure has reached step 6-b, the sample size is considered to be large enough and the test may be conducted:

1. Compute:
$$D_i=\hat{U}(X*)-\hat{U}(X_i)$$
$$S_{D_i}=(S_i^2/n_i + S_*^2/n_*)^{1/2}$$
$$D_c=(S_{D_i})(Z_{1-\alpha})$$

2. Compare D_i with D_c:

   a) If $D_i\leq D_c$ reject H_0, X_i is an "improvement" point.

   b) If $D_i>D_c$ do not reject H_0, X_i is not an improvement.

3. Transfer the information to the search routine.

Equation 17 represents an economic consideration which leads to the definition of the critical difference d'. This value appears also in equation 31 and affects the sample size needed for given α and β. Unfortunately, defining d' using equation 17 may result in a very large sample size whenever the cost per simulated replication (C_r) is small compared to the value of S_{D_i}. In this case the procedure may not exit stage 1, unless one of the following events occurs:

1. The needed sample n_i is achieved.
2. A lower power (larger β) is defined to be acceptable.
3. Anothei critical difference, larger than d', is defined.

The first case requires that the test will be performed only when the needed large sample n_i is actually obtaind. Since the value of the statistic D_i (equation 18), is not taken into account in deciding to increase n_i in stage 1, this increase may involve an unnecessary investment of funds. Case 2 requires reducing the needed power in order to satisfy equation 31 with a smaller n_i, whenever d' happens to be too small. This demand imposes inconsistency in terms of test power along the search, can not be easily justified, and is hard to implement. Case 3 requires an artificial increase in the critical difference d' in order to use a smaller sample size. Since the true value and the variability of the statistic D are unknown, there is no basis for choosing other values for d' as long as one wants to adopt the same economic considerations expressed by equation 19. A modification of the above test procedure, which was designed in order to overcome some of these difficulties, is presented in the next section.

3.3 EVALUATING THE "GOODNESS" OF A DIRECTION VIA A CONFIDENCE INTERVAL APPROACH

At each point i, compute a confidence interval and obtain a lower limit L_D and an upper limit U_D for D_i, the estimated amount of improvement associated with point i. Define the desired return d' as before:

$$d' = (1+r)(C_r)(n_i) \tag{32}$$

Define a point to be an "improvement point" whenever

$$L_D > d' \tag{33}$$

and if

$$U_D < d' \tag{34}$$

do not consider point i as an "improvement point". If neither equation 33 nor 34 is satisfied, increase the sample size at point i (n_i) and/or the sample size at the "best point thus far", (n^*), in order to provide a narrower interval ($L_D;U_D$). Continue this process until either equation 33 or equation 34 is satisfied, or an upper limit of n' observations at a point is reached. If the latter event occurs, a conservative approach according to which the point is defined as "not-an-improvement" is adopted, provided that n' is chosen to be "large enough" (20-40 etc.).

D_i has an approximate normal distribution as given by equation 23. When $Var[D_i]$ is unknown, D_i may be treated as "t" distributed

$$D_i \sim t(f) \tag{35}$$

As given by Gibra [30] p. 259, the degrees of freedom, f, are equal to:

$$f = (S_i^2/n_i + S_*^2/n^*)^2/SS - 2 \tag{36}$$

where SS is given by

$$SS = \{ [(S_i^2/n_i)^2/(n_i+1)] + [(S_*^2/n^*)^2/(n^*+1)] \}$$

The standard deviation of D_i is given by equation 28, and can be approximated by equation 29. Hence,

$$L_D = \hat{U}(X^*) - U(X_i) - (S_{D_i}) t_{1-\alpha/2}(f) \tag{37}$$

$$U_D = \hat{U}(X^*) - U(X_i) + (S_{D_i}) t_{1-\alpha/2}(f) \tag{38}$$

By increasing the sample size one can decrease the value of S_{D_i} (equation 29) and produce a narrower confidence interval. However, since it is reasonable to assume that the quantities S_i^2 and S_*^2 are not likely to differ radically, it appears that whenever such an increase is needed one should first increase either n_i or n^* until they become equal, then continue to increase them simultaneously until n' is reached. In this manner search funds may more efficiently be used. Alternatively (but perhaps less efficiently), one could choose a higher value for n_0, the initial sample size at any point, and increase only n_i whenever neither equation 33 nor 34 is satisfied with n_0.

The procedure can be summarized as follows:

1. Set nn=0,
2. Sample $n_i = n_0$ observations at point i.
3. Compute $d' = (1+r)(C_r)(n_i)$.
4. Estimate σ_i^2 using unbiased estimator S_i^2 based on sample of n_i.
5. Compute S_{D_i} using equation 29.
6. Compute f using equation 36.
7. Compute $t_{1-\alpha/2}(f)$.
8. Compute L_D and U_D using equation 37 and equation 38.
9. Check:
   a) if equation 33 is satisfied, point i is an improvement point, go to step 18.
   b) if equation 34 is satisfied, point i is not an improvement point, go to step 18.
   c) otherwise, go to step 10.
10. Compute n_i and n^*, and check:
    a) if $n_i < n^*$, set $n_i = n_i + 1$, simulate an additional observation at point X_i, go to step 3.

b) if $n^* < n_i$, set $n^* = n^* + 1$, $nn = nn + 1$, go to step 11.

c) otherwise, $n^* = n_i$, go to step 15.

11. Simulate an additional observation at point X^*.
12. Compute $d' = (1+r)(C_r)(n_i + nn)$.
13. Estimate σ_*^2 using S_*^2 based on sample of n^*.
14. Go to step 4.
15. Compare $n_i = n^*$ with n':

    a) if $n_i \geq n'$, define point i to be "not an improvement", go to step 18.

    b) if $n_i < n'$, set $n^* = n^* + 1$, $n_i = n_i + 1$, $nn = nn + 2$, go to step 16.
16. Simulate an additional observation at each of the points X^* and X_i.
17. Go to step 12.
18. Transfer the information to the search routine.

The problem of selecting the proper values for the major parameters introduced by the above "sampling routine" (α, n_0, n'), is discussed in Chapter V.

Chapter IV

THE STRUCTURE OF THE CONTROL MODULE

4.1 INTRODUCTION

The control module is the major part of the modular optimization procedure presented in Chapter I. The relationship between this module and the other modules of the optimization procedure is presented in Figure 4 in Chapter I. The purpose of the control module is to determine whether or not to proceed with the search by keeping track of its performance as follows. After evaluating the objective function at a point, and before calling the search technique to suggest the next point for investigation, the expected value of the gross savings rate, E(B), is compared statistically with the updated rate of search cost, R_c. Both $B=\hat{E}(B)$ and R_c are evaluated by applying "moving regression" techniques to the cumulative gross savings data and actual search cost data, respectively. By definition, the cost of performing an "unsuccessful" search between any two successive improvement points (i.e. investigating a set of p_i points which have not improved the objective value, between the i^{th} and the $i+1^{st}$ improvement points) is associated with point i. In this way, the incremental increase in the cumulative cost at the i^{th} improvement point equals the cost of evaluating the i^{th} point, plus the total amount of funds spent to proceed with the search until the next improvement is obtained (see Figure 6).

R_c represents the average actual cost of the attempt to "exit" an improvement point in order to explore and discover a "better" point. The gross savings data are based on the estimated values of the improvement in the objective function as obtained by the simulation. The value of B is taken, therefore, as an estimator of the true savings' rate. A procedure for computing B, as well as an approximate distribution for the true gross-savings rate, are given in Section 4.2. The statistical test for comparing the gross-savings rate with the cost rate is presented in Section 4.3. A summary of the proposed termination procedure on which the operation of the control module is based is given in section 4.4.

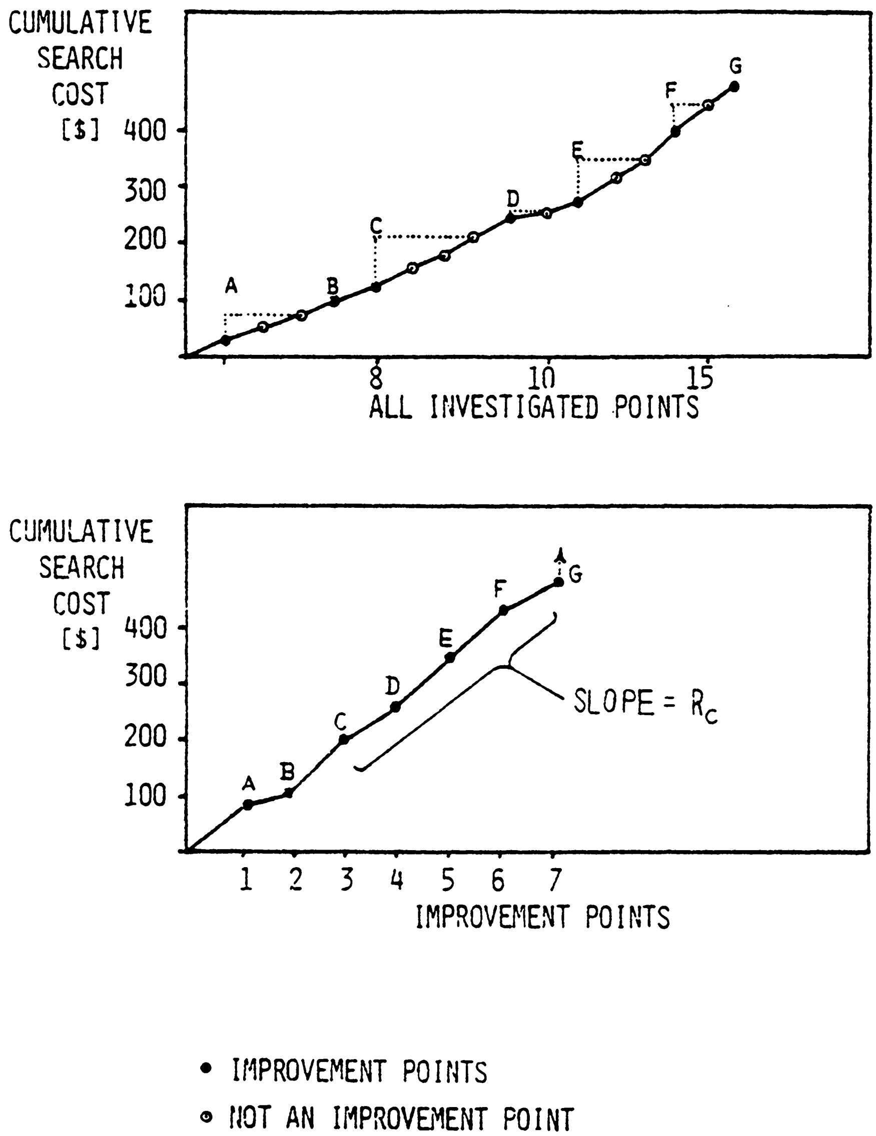


Figure 6: Presenting the cumulative search cost at improvement points

4.2 APPROXIMATING A DISTRIBUTION FOR THE GROSS-SAVING RATE

4.2.1 Applying a moving regression technique in order to estimate the gross savings rate

The estimated cumulative gross savings at the k^{th} improvement point in the course of the search is given by:

$$\hat{Q}_k = \hat{U}(X_o) - \hat{U}(X_k) \tag{39}$$

where $\hat{U}(X_0)$ and $\hat{U}(X_k)$ represent the estimated cost at the initial point X_0 and at X_k respectively (see equations 5 and 6). The straight line

$$\hat{Q}_k = \psi + \delta k$$

is estimated based on the most recent m improvement points (Q_k) for k=1,2,...,m using a "moving regression" technique. k=1 denotes the first point to which the line is fit. This point, however, may not necessarily be the first point in the search. The estimators A= $\hat{\psi}$ and B= $\hat{\delta}$ are chosen to minimize

$$SS_e = \sum_{k=1}^{m} \{\hat{Q}_k - [A + (B)(k)]\}^2 \tag{40}$$

The partial derivatives of SS with respect to A and B are:

$$\frac{\partial SS_e}{\partial A} = -2 \sum_{k=1}^{m} [\hat{Q}_k - A - (B)(k)]$$

$$\frac{\delta SS_e}{\delta B} = -2 \sum_{k=1}^{m} k[\hat{Q}_k - A - (B)(k)] \tag{41}$$

Setting the partial derivatives to equal zero and solving for A and B yields,

$$B = \frac{12}{m(m^2+1)} \sum_{k=1}^{m} \hat{Q}_k (k - \frac{m+1}{2}) \tag{42}$$

and

$$A = \frac{2(2m+1)}{m(m-1)} \sum_{k=1}^{m} \hat{Q}_k - \frac{6}{m(m-1)} \sum_{k=1}^{m} (\hat{Q}_k)(k) \tag{43}$$

Equations 42,43 provide the estimators A and B for ψ and δ, respectively, based on the m most recent improvement points.

4.2.2 Obtaining the distribution parameters for the gross-savings rate

Equation 42 provides an estimator $B=\hat{E}(B)$ for the true gross-savings rate. The true rate E(B) is given by:

$$E(B) = \frac{12}{m(m^2-1)} \sum_{k=1}^{m} E(\hat{Q}_k)(k - \frac{m+1}{2}) \tag{44}$$

where

$$E(\hat{Q}_k) = U(X_o) - U(X_k) \tag{45}$$

The variance of B is given by:

$$Var(B) = E(B) - [E(B)]^2 \tag{46}$$

substituting for B and E(B) from equations 42 and 44 respectively,

$$Var(B) = \frac{144}{m^2(m^2-1)^2} E\{\sum_{k=1}^{m} (k-\frac{m+1}{2})[\hat{Q}_k - E(\hat{Q}_k)]\}^2 \tag{47}$$

and by using equations 39 and 45,

$$\text{Var}(B) = \frac{144}{m^2(m^2-1)^2} \; E\{\sum_{k=1}^{m} (k-\frac{m+1}{2})[\hat{U}(X_o)-\hat{U}(X_k) - U(X_o) + U(X_k)]\}^2 \tag{48}$$

Now, since

$$U(X_k) = U(X_k) + \frac{1}{n_k} \sum_{j=1}^{n_k} e_{kj} \qquad k = 1,2,\ldots,m \tag{49}$$

where e_{kj} = the "noise" at the j^{th} replicate at point k, equation 48 may be rewritten as follows:

$$\text{Var}(B) = \frac{144}{m^2(m^2-1)^2} \; E\{\sum_{k=1}^{m} [(k-\frac{m+1}{2})(\frac{1}{n_o} \sum_{j=1}^{n_0} e_{oj} - \frac{1}{n_x} \sum_{j=1}^{n_k} e_{kj})]\}^2 \tag{50}$$

Let

$$\bar{e}_k = \frac{1}{n_k} \sum_{j=1}^{n_k} e_{kj} \qquad k = 1,2,\ldots,m \tag{51}$$

be the average estimated "noise" at point k. Hence,

$$\text{Var}(B) = \frac{144}{m^2(m^2-1)^2} \; E\{\sum_{k=1}^{m} [(k-\frac{m+1}{2})(\bar{e}_o-\bar{e}_k)]\}^2 \tag{52}$$

Now, since

$$\sum_{k=1}^{m} (k-\frac{m+1}{2})\bar{e}_o = \bar{e}_o \left[\frac{m(m+1)}{2} - \frac{m(m+1)}{2}\right] = 0$$

then

$$\mathrm{Var}(B) = \frac{144}{m^2(m^2-1)^2} E\{-\sum_{k=1}^{m} (k-\frac{m+1}{2})\bar{e}_k]\}^2$$

$$= \frac{144}{m^2(m^2-1)^2} E\{\sum_{k=1}^{m} [(k-\frac{m+1}{2})\bar{e}_k]^2 \tag{53}$$

$$+ 2 \sum_{k=1}^{m-1} \sum_{i=k+1}^{m} [(k-\frac{m+1}{2})(i-\frac{m+1}{2})\bar{e}_k \bar{e}_i]\}$$

Since $\bar{e}_k$ and $\bar{e}_i$ are independent random variables for $k \neq i$,

$$E[(\bar{e}_k)(\bar{e}_i)] = E[\bar{e}_k] E[\bar{e}_i] = 0$$

and equation 53 reduces to:

$$\mathrm{Var}(B) = \frac{144}{m^2(m^2-1)^2} E\{\sum_{k=1}^{m} [(k-\frac{m+1}{2})^2(\bar{e}_k)^2]\} \tag{54}$$

Using the assumption of normality,

$$e_{x_j} \sim N[0;\sigma^2(X_k)]$$

Now, denoting $\sigma^2(X_k)$ by σ_k^2 yields,

$$\bar{e}_k = [\frac{1}{n_k} \sum_{j=1}^{n_k} e_{kj}] \sim N[0; \frac{\sigma_k^2}{n_k}]$$

and

$$E[(\bar{e}_k)^2] = Var[\bar{e}_k] = \frac{\sigma_k^2}{n_k} \tag{55}$$

Substituting $E[(\bar{e}_k)^2]$ from equation 55 in equation 54 provides:

$$Var[B] = \frac{144}{m^2(m^2-1)^2} \sum_{k=1}^{m} [(k-\frac{m+1}{2})^2(\frac{\sigma_k^2}{n_k})] \tag{56}$$

Hence,

$$\sigma_B = \frac{12}{m(m^2-1)} \sqrt{\sum_{k=1}^{m} (k-\frac{m+1}{2})^2 \ \frac{\sigma_k^2}{m_k}} \tag{57}$$

which may be estimated by

$$S_B = \frac{12}{m(m^2-1)}\sqrt{\sum_{k=1}^{m}\left(k-\frac{m+1}{2}\right)^2 \frac{S_k^2}{n_k}} \tag{58}$$

B, the gross-savings rate is assumed to be normally distributed

$$B \sim N[E(B);\ Var(B)] \tag{59}$$

where E(B) and Var(B) are given by equations 44 and 56 respectively. E(B) is estimated by $B=\hat{E}(B)$ using equation 42. Var(B) is estimated by S_B^2 in equation 58. These estimates are evaluated whenever the statistical test for the gross-savings is performed.

4.3 DESIGNING A TEST TO DETERMINE THE TERMINATION POINT

A conservative approach would require a statistical test to be conducted at each point of the search as follows:

H_0 : $E(B) \leq b_0$

H_1 : $E(B) > b_0$

Under H_0, the rate of improvement of U(X) is less then b_0. b_0 is defined by the user to be the minimal gross-savings rate for which a search should apply, and the optimization process should be terminated. Rejction of H_o ["accepting" H_1], indicates that the rate of improvement is greater than the needed value, in which case the search should be continued.

Using the assumption that B is normally distributed with E(B) and var(B) given by equations 44 and 56, respectively,

$$Z = \frac{B-E[B]}{\sigma_B} \sim N\ (0;1) \tag{60}$$

<u>Under H_o</u> $E(B)=b_o$ and

$$Z = \frac{B-b_o}{\sigma_B} \sim N\ (0;1)$$

<u>Under H_1</u> $E(B)=b_o+d>b_o$ for $d>0$; and

$$Z = \frac{B-b_o}{\sigma_B} \sim N\ (d;1)$$

Using statistical inference theory [30], a random variable defined as

$$H = \frac{(n-1)S^2}{\sigma^2}$$

will have a chi-square distribution with n-1 degrees of freedom whenever S^2 is an unbiased estimator of the unknown variance σ^2, based on n independent observations from a normal distribution. In the case of the gross-savings rate random variable, σ_B is given by equation 57 and is estimated by S_B using equation 58. Let

$$Y = \frac{f_2 S_B^2}{\sigma_B}$$

and assume that Y has an approximated chi-square distribution with f_2 degrees of freedom. Define the test statistic:

$$T_{exp} = \frac{Z}{\sqrt{Y/f_2}} = \frac{\left(\frac{B-b_o}{\sigma_B}\right)}{S_B/\sigma_B} \quad \frac{B - b_o}{S_B} \tag{61}$$

<u>Under</u> $\underline{H}_o$ $\quad Z \sim N(0;1)$. and,

$$T_{exp} \sim t(f_2)$$

(a central t distribution with f_2 d.f.)

<u>Under</u> $\underline{H}_1$ $\quad Z \sim N(d;1)$, and,

$$T_{exp} \sim t(f_2;d)$$

(a non-central t distribution with f_2 d.f. and a non-centrality parameter d)

The non-central t distribution can be evaluated using numerical integration at each point. Such an evaluation may involve a long "trial and error" process in an attempt to compute the inverse of the t in order to obtain a critical value T_c which satisfies:

$$\Pr[\ T_{exp} > T_c \ \mid \ H_1 \ ] = \beta_T$$

a necessary condition derived from the power requirement.

The derivation that follows provides an approximation which may be used to satisfy the power requirements without computing a non-central t variate.

<u>The type I error</u> associated with the test is given by:

α_T =Pr[reject H_o / H_o is true]

=Pr[continue the search / the search should be terminated]

$=\Pr[T_{exp} > T_c \ / \ T_{exp} \sim t(f_2)]$

hence,

$$T_c = t_{1-\alpha}(f_2)$$

<u>The power of the test</u> is given by

$$1 - \beta_T = \Pr[\text{reject } H_o \mid H_o \text{ is false}]$$

$= \Pr[$continue the search | the search should be continued$]$

$$= \Pr[T_{exp} > Tc \mid T_{exp} \sim t(f_2;d)]$$

$$= \Pr\left[\frac{Z}{\sqrt{Y/f_2}} > t_{1-\alpha}(f_2) \;/\; \frac{Z}{\sqrt{Y/f_2}} \sim t(f_2;d) \right]$$

Now, $Z = \dfrac{B-b_o}{\sigma_B}$ and $Y = \dfrac{f_2 S_B^2}{\sigma_B^2}$

Therefore

$$\frac{Z}{\sqrt{Y/f_2}} = \frac{[(B-b_o)/\sigma_B]}{\sqrt{S_B^2/\sigma_B^2}} = \frac{B - b_o}{S_B}$$

Hence

$$1-\beta_T = \Pr\{ [(B-b_o)/S_B] > t_{1-\alpha_T}(f_2) \mid E(B)=b_o+d \}$$

$$= \Pr[B-(S_B)t_{1-\alpha_T}(f_2) > b_o \mid E(B)=b_o+d]$$

Now, let $W=B-(S_B)t_{1-\alpha_T}(f_2)$, and assume that

$$W \sim N[E(W) ; Var(W)]$$

$E(W)$ and $Var(W)$ should be derived in order to be able to compute $1-\beta_T$.

a) <u>Derivation of E(W)</u>:

Based on the assumption that $Y = \frac{f_2 S_B^2}{\sigma_B} \sim \chi^2(f_2)$

where

$$f_Y(y) = \frac{1}{\Gamma(f_2/2)2^{f_2/2}} \; y^{f_2/2-1} \quad \exp(-y/2)$$

$$0 < y < \infty$$

one may define

$$A = \sqrt{Y} = \sqrt{\frac{f_2 \; S_B^2}{\sigma_B^2}}$$

Hence,

$$S_B = \frac{A\sigma_B}{\sqrt{f_2}}$$

The p.d.f. of A is given by the transform:

$$g_A(a) = |J| f_Y[\phi(a)]$$

where

$$Y = \phi(A) = A^2 \quad => \quad dy = 2AdA \quad => \quad |J| = |2A| = 2A$$

Hence

$$g_A(a) = \frac{1}{\Gamma(f_2/2)2^{(f_2/2)/2}} \; (a)^{f_2-1} \; \exp(-a^2/2)$$

$$0 < a < \infty$$

Now, since

$$S_B = \frac{A\sigma_B}{\sqrt{f_2}} \;;\quad A = \frac{S_B \; f_2}{\sigma_B} \Rightarrow dA = \left(\frac{f_2}{\sigma_B}\right) dS_B \Rightarrow \; |J| = \frac{f_2}{\sigma_B}$$

and $h_{S_B}(S_B)$, the p.d.f. of S_B, is given by:

$$h_{S_B}(S_B) = |J|\, g_A[\psi(S_B)], \qquad \text{where } \psi(S_B) = A$$

then,

$$h_{S_B}(S_B) = \left(\frac{\sqrt{f_2}}{\sigma_B}\right)\left(\frac{1}{\Gamma(f_2/2)2^{(f_2-2)2}}\right)\left(\frac{S_B\sqrt{f_2}}{\sigma_B}\right)^{f_2-1} \exp\left[-\left(\frac{S_B\sqrt{f_2}}{\sigma_B}\right)^2/2\right]$$

$$= \frac{2(f_2/2)^{f_2/2}}{(\sigma_B)^{f_2}\Gamma(f_2/2)} \, S_B^{f_2-1} \quad \exp\left(\frac{-f_2 \; S_B^2}{2\sigma_B^2}\right) \qquad 0 < S_B < \infty$$

Using this p.d.f., the expected value of S_B is

$$E(S_B) = \frac{2(f_2/2)^{f_2/2}}{(\sigma_B)^{f_2}\Gamma(f_2/2)} \int_0^\infty (S_B)^{f_2} \exp\left(\frac{-f_2 \; S_B^2}{2\sigma_B^2}\right) dS_B$$

Now, by using the transformation

$$\ell = \frac{\sqrt{f_2 S_B}}{\sigma_B} \quad \Rightarrow \quad S_B = \frac{\ell\sigma_B}{\sqrt{f_2}} \quad \Rightarrow dS_B = \frac{\sigma_B}{\sqrt{f_2}} \, d\ell$$

and by substituting: $\ell = \sqrt{p}$, $d_\ell = (1/2)p^{-1/2}\,dp$

$$E(S_B) = \frac{2\sigma_B}{\sqrt{f_2}\,(2)^{f_2/2}\,\Gamma(f_2/2)}\left(\frac{1}{2}\right)\int_o^\infty p^{(f_2-1)/2}\exp(-p/2)\,dp$$

$$= \sqrt{\frac{2}{f_2}}\left(\frac{\Gamma(\frac{f_2+1}{2})}{\Gamma(f_2/2)}\right)\sigma_B \qquad (62)$$

This expression can be approximated by:

$$E(S_B) \cong \sqrt{1-1/2f_2}\;\sigma_B \qquad (63)$$

hence, under H_1, $E(B) = b_o + d$ and,

$$E(W) = E[B-(S_B)t_{-\alpha_T}(f_2)]$$

$$= b_o+d-[E(S_B)]t_{1-\alpha_T}(f_2)$$

$$= b_o+d-\sigma_B\sqrt{1-1/2f_2}\;\,t_{1-\alpha_T}(f_2) \qquad (64)$$

b) <u>Derivation of Var(W)</u>:

Since

$$Var(W)=Var[\ B-(S_B)t_{1-\alpha_T}(f_2)\ ]$$

and since B is a normally distributed random variable, one may assume independence between B and S_B. Hence,

$$Var(W)=Var(B)+[(t_{1-\alpha_T}(f_2)]^2 Var(S_B)$$

$$=\sigma_B^2+[t_{1-\alpha_T}(f_2)]^2 E[S_B-E(S_B)]^2$$

where

$$E[S_B-E(S_B)]^2=E[S_B^2-2S_B^2E(S_B)+E^2(S_B)]$$

$$=E(S_B^2)-E^2(S_B)=\sigma_B^2-[1-1/(2f_2)]\sigma_B^2$$

$$=\sigma_B^2/(2f_2)$$

(since S_B^2 is unbiased estimator for σ_B^2 and $E(S_B)$ is given by $\sqrt{1-1/(2f_2)}\sigma_B$.

Hence,

$$\mathrm{Var}(W) = \sigma_B^2 + [t_{1-\alpha_T}(f_2)]^2 \quad \frac{\sigma_B^2}{2f_2} \tag{65}$$

$$= \sigma_B^2 \left[1 + \frac{t^2_{1-\alpha_T}(f_2)}{2f_2} \right]$$

which finally provides:

$$W \sim N\{\ b_o + d - \sigma_B \sqrt{1-1/(2f_2)}\ t_{1-\alpha_T}(f_2)\ ; \sigma_B^2\ [1 + \frac{t^2_{1-\alpha_T}(f_2)}{2f_2}]\ \} \tag{66}$$

The power of the test may now be reformulated as:

$$1 - \beta_T = \Pr[\ B-(S_B)t_{1-\alpha_T}(f_2) > b_o \mid E(B)=b_o+d\ ]$$

$$= \Pr[\ W > b_o \mid H_1\ ]$$

$$= \Pr[\frac{W-E(W)}{\sqrt{\mathrm{Var}(W)}} > \frac{b_o - E(W)}{\sqrt{\mathrm{Var}(W)}} \mid H_1]$$

$$= \Pr\left[\ Z > \frac{-d + \alpha_B \sqrt{1 - \frac{1}{2f_2}}\ t_{1-\alpha_T}\ (f_2)}{\alpha_B \sqrt{1 + \frac{t^2_{1-\alpha_T}\ (f_2)}{2f_2}}}\ \right]$$

$$= \Pr[\ Z > Z_{\beta_T}\ ]$$

where $Z \sim N[0;1]$.

By estimating σ_B by S_B, $Z_{1-\beta_T}$ may be approximated by:

$$Z_{1-\beta_T} \cong \frac{d - S_B \sqrt{1 - \frac{1}{2f_2}}\, t_{1-\alpha_T}(f_2)}{S_B \sqrt{1 + \frac{t^2_{1-\alpha_T}(f_2)}{2f_2}}} \tag{68}$$

Recall that S_B is a function of m (equation 58). f_2 is also a function of m (as shown in the next section).
Hence, the desired power is achieved for the least integer m which satisfies:

$$Z^* = \frac{d - S_B \sqrt{1 - \frac{1}{2f_2}}\, t_{1-\alpha_T}(f_2)}{S_B \sqrt{1 + \frac{t^2_{1-\alpha_T}(f_2)}{2f_2}}} \geq Z_{1-\beta_T} \tag{69}$$

c) <u>Derivation of f_2</u>:

Recall that

$$Y = \frac{f_2 S_B^2}{\sigma_B^2} \sim \chi^2(f_2) \tag{70}$$

Hence

$$E(Y) = E\left(\frac{f_2 S_B^2}{\sigma_B^2}\right) = f_2 \; ; \quad Var\ (Y) = Var\ \left(\frac{f_2 S_B^2}{\sigma_B^2}\right) = 2f_2 \qquad (71)$$

But

$$Var\ \left(\frac{f_2 S_B^{\ 2}}{\sigma_B^2}\right) = \frac{(f_2)^2}{(\sigma_B)^4} \quad Var\ (S_B^2) \qquad (72)$$

where

$$Var\ (S_B^2) = E[S_B^2 - E(S_B^2)]^2$$

For S_B as defined by equation 58,

$$Var\ (S_B^2) = \left(\frac{144}{m^2(m^2-1)^2}\right)^2 E\left\{\sum_{k=1}^{m} \left[\left(k - \frac{m+1}{2}\right)^2 \frac{S_k^2 - \sigma_k^2}{n_k}\right]\right\}^2 \qquad (73)$$

where

$$E\{\sum_{k=1}^{m}[(k-\frac{m+1}{2})^2 \quad \frac{S_k^2 - \sigma_k^2}{n_k}]\}^2 =$$

$$= E\{\sum_{k=1}^{m}[(k - \frac{m+1}{2})^4 \ (\frac{S_k^2 - \sigma_k^2}{n_k})]$$

$$+ 2 \sum_{k=1}^{m-1} \sum_{\ell=k+1}^{m} [(k-\frac{m+1}{2})^2(\ell-\frac{m+1}{2})^2(\frac{S_k^2-\sigma_k^2}{n_k})(\frac{S_\ell^2-\sigma_\ell^2}{n_\ell})]\}$$

Since S_k^2 and S_ℓ^2 are independent random variables for $\ell \neq k$, then

$$E[(\frac{S_k^2 - \sigma_k^2}{n_k}) \ (\frac{S_\ell^2 - \sigma_\ell^2}{n_\ell})] = 0$$

and

$$\mathrm{Var}(S_B^2) = (\frac{144}{m^2(m^2-1)^2})^2 \sum_{k=1}^{m} [\frac{(k - \frac{m+1}{2})^4}{n_k^2} \ \mathrm{Var}\ (S_k^2)] \qquad (74)$$

Now, since S_k^2 is an unbiased estimator for σ_k^2 and the simulation model is assumed to produce independent normally distributed replicates, then

$$\frac{(n_k - 1)S_k^2}{\sigma_x^2} \sim \chi^2 \; (n_k - 1) \quad \text{for } k = 1,2,\ldots,j$$

which implies:

$$\text{Var} \; (\frac{(n_k-1)S_k^2}{\sigma_k^2}) = 2(n_k - 1)$$

Hence

$$\frac{(n_k - 1)^2}{\sigma_k^2} \text{Var} \; (S_k^2) = 2(n_k - 1)$$

or

$$\text{Var}(S_k^2) = \frac{2 \; \sigma_k^4}{(n_k-1)} \tag{75}$$

Substituting this expression in equation 74 yields:

$$\text{Var} \; (S_B^2) = (\frac{144}{m^2(m^2-1)^2})^2 \sum_{k=1}^{m} [\frac{(k-\frac{m+1}{2})^2 \; 2 \; \sigma_k^4}{(n_k)^2(n_k - 1)}]$$

Now, substituting $Var(S_B^2)$ in equation 72 yields:

$$Var\ (Y) = \frac{(f_2)^2}{(\sigma_B)^4} \quad Var\ (S_B^2)$$

$$= [\frac{f_2^2}{(\sigma_B)^4}][\frac{144}{m^2(m^2-1)^2}]^2 \sum_{k=1}^{m} [(k-\frac{m+1}{2})^4 \quad \frac{2(\sigma_k)^4}{(\ _k)^2(\ _k-1)}] \quad (76)$$

$$= 2f_2$$

Finally, substituting for σ_B using equation 57 yields:

$$f_2 = \frac{\left[\sum_{k=1}^{m} (k - \frac{m+1}{2})^2 \quad \frac{\sigma_k^2}{n_k}\right]^2}{\sum_{k=1}^{m}\left[(k - \frac{m+1}{2}) \quad \frac{(\sigma_k)^4}{(n_k)^2(n_k-1)}\right]} \quad (77)$$

Using the last expression, f_2 is approximated by $\hat{f}_2$ as follows:

$$\hat{f}_2 = \frac{\left[\sum_{k=1}^{m} (k - \frac{m+1}{2})^2 \ \frac{S_k^2}{n_k}\right]^2}{\sum_{k=1}^{m}\left[(k - \frac{m+1}{2})^4 \quad \frac{(S_k)^4}{(n_k)^2(n_k-1)}\right]} \quad (78)$$

where S_k is the estimated standard deviation of the system's cost at point k, as obtained by sampling n_k replicates using simulation.

4.4 SUMMARY OF THE PROPOSED TERMINATION PROCEDURE

After investigating a new point in the course of the search, and before proceeding to a suggested next point, perform the following procedure:

Stage 1: (in this stage the value of m is determined in order to satisfy the power requirements)

1. Choose $m=m_o$.
2. Conduct the search until m improvement points are discovered.
3. Calculate S_B using equation 58 based on most recent m points.
4. Calculate $\hat{f}_2$ using equation 78 based on most recent m points.
5. Choose values for d, b_o, α_T, and β_T.
6. Compute Z* using equation 69.
7. Compute $Z_{1-\beta_T}$
8. Check:

   a) if $Z_{1-\beta_T} < Z^*$, conduct the test, go to step 11.

   b) otherwise, go to step 9.

9. The power requirement is not satisfied by m points, set m=m+1, go to step 10.

10. Recall NN=the total number of improvement points discovered thus far, and check:

    a) if NN < m go to step 2.

    b) otherwise, go to step 3.

<u>Stage 2</u>: (in this stage the test is performed)

11. Conduct the test:

    a) compute B using equation 42.

    b) check:

       i) if $T_{exp} = \frac{B-b_0}{S_B} > t_{1-\alpha_T}(f_2)$,

       continuation of the search is desired, go to step 12.

       ii) otherwise, continuation of the search is undesired, go to step 13.

12. Allow investigation of an additional point which will be provided by the search technique, and go to "stand-by" position.

13. Terminate the search.

After investigating the new point, the control module exits the "stand-by" state and applies the procedure again, starting at step 1.

The problem of selecting the major parameters of the above termination procedure is discussed in Chapter VI.

Chapter V

SELECTING THE MAJOR PARAMETERS USED BY THE SAMPLING ROUTINE

5.1 MOTIVATION

In this chapter, a discussion of the problem of selecting the major parameters of the sampling routine described in Chapter III is presented. Section 5.2 provides information for choosing the confidence level 1-α introduced by the "Confidence Interval Approach" of Section 3.3. The effect of different choices of α is demonstrated through various examples. Section 5.3 below, discussed the selection of n_0, the initial sample size of any pcint and of n', the maximal sample size along the search.

5.2 SELECTING THE CONFIDENCE LEVEL (1-α)

5.2.1 General considerations

The confidence level, given by (1-α) in ection 3.3, may be defined as the probability that the true difference $E[D_i]$, given by equation 24, will lie betwen the lower limit, L_D, and the upper limit, U_D, given by equations 37 and 38, respectively. $E[D_i]$ represents the expected amount of improvement in the value of the objective function achieved by performing a single step in a given direction. The sampling routine compares the limits of $E[D_i]$ with the desired return d', given by equation 32, and determines whether or not to continue searching in the same direction. Assuming that a direction should be defined as a "good direction" if there is a probability of at least γ^* that a single step along this direction improves the value of the objective function by at least d'. Hence, a point should be considered as an improvement point if:

$$\begin{aligned} \Pr[\text{ point i is an imprv. pt. }] &= \Pr\{\, E[D_i] > d' \,\} \\ &= \Pr\left[\frac{D_i - [D_i]}{S_{D_i}} < \frac{D_i - d'}{S_{D_i}}\right] \\ &= \Pr[\, t < T_{exp} \,] \geq \gamma^* \end{aligned}$$

where

$$T_{exp} = \frac{D_i - d'}{S_{D_i}} > t_{\gamma}{}^*$$

or, equivalently, consider point i as an improvement if

$$d' < D_i + (S_{D_i}) t_{1-\gamma^*}$$

The right hand side of the last inequality may be regarded as the lower limit of a $1-\alpha$ confidence interval for $E[D_i]$ for which $\alpha = 2(1-\gamma^*)$. Such a confidence interval, based on an initial sample of n_0 observations taken from the simulation model, may be utilized in forming a sampling rule which checks whether:

1. $d' < L_D$, in which case:
   Pr[True difference > d'] > $1-\alpha/2=\gamma^*$
   and point i may be considered as an improvement point.
2. $d' > U_D$, in which case:
   Pr[True difference < d'] > $1-\alpha/2=\gamma^*$
   and point i cannot be considered as an improvement point.
3. $L_D < d' < U_D$, in which case no decision is made and the sample size is increased.

If neither condition 1 nor condition 2 above is satisfied, the sample size is increased, a new, narrower, interval $(L_D;U_D)$ is computed, and the check is repeated. The maximal sample size, n', is prespecified by the user. If no decision can be made with a maximum of n' observations, point i does not provide the necessary probability γ^* and is not considered as "an improvement".

The approach outlined above, led to the formualtion of the sampling procedure suggested in Section 3.3. This approach explains the relationship between the minimal probability, γ^*, required in order to define a point as an improvement, and the confidence level $1-\alpha$ used by the sampling routine. The desired minimal probability γ^* may be defined by the user. This definition will lead to choosing an appropriate value for which will be used in estimating lower and upper limits for $E[D_i]$. If one wishes to use, for example, γ*=.60, .70, .80 or .90, the corresponding values for should be =.80, .60, .40, or .20, respectively. It is important to note, however, that a conservative approach in choosing values for γ* (and consequently for α) is unnecessary. The purpose of the sampling routine is merely to control the sampling process at each point along the search. Its purpose is not to evaluate the efficiency of the search, or to determine when to terminate the optimization process. The economic considerations on which the sampling routine is based was formed only in order to enable the routine to increase the sample size at a given point. This increase is acceptable as long as there is a probability of at least γ* that a net improvement in the value of the objective function will result by doing so. The overall performance of the search, however, is continuously checked by the control module. Hence, γ* for the sampling routine may be defined less conservatively. A conservative approach leads to choosing a larger value for γ* and therefore, a smaller value for α and resutls in a wide interval $(L_D;U_D)$. This approach may require a larger sample size at each point in order to finally satisfy either one of the conditions given in (1) or (2) above. A need for a larger sample size will result in a higher cost rate, which may cause the control module to terminate the search earlier. It may also cause an increase in the number of times no decision will be made even with a maximum of n' observations (condition 3 above). Under this condition, the associated point is not considered an improvement by default. In this case, the search may start to converge around a point which may be "far" from the optimum. On the other hand, an approach according to which γ^* is chosen too small may result in a narrow interval $(L_D;U_D)$, having a low confidence level. In many cases, this may allow the sampling routine to satisfy either one of conditions (1) or (2), based on the original initial sample size n_0, and may reduce its effect in adjusting the sample sizes throughout the search. When, for example, γ^* is chosen as .50, the associated value of α is 1.00 and $L_D=U_D=D_i$. A decision is simply made here, by comparing the sample mean D_i with the value of d' . Such a comparison may cause the search to continue "wandering" around the optimal point by producing "pseudo improvments" due to the random variations inherent in the simulation model, when such "improvements" are actually insignificant. The effect of different choices of γ^*, (and consequently of α), is demonstrated through different examples in Section 5.2.2 below. The parameters used throughout these examples are as follows:

1. $\underline{X}^0 = (x_1^0; x_2^0)$ = the initial point.
2. $S = (S_1; S_2)$ = the step size used by the search in the directions of $(x_1; x_2)$ respectively.
3. T.O.E. = the type of the sampling error added to the value of the function.
4. ρ = the coefficient of variation, σ/μ, of the stochastic function.
5. α = the confidence level used by the sampling routine
6. C_r = the cost of obtaining one observation from the simulation model, (here represented by a stochastic function).
7. n_i = initial sample size at x_0.
8. n_0 = initial sample size at any additional point.
9. n' = maximal sample size at any point.
10. α_T = type I error for the termination test.
11. β_T = type II error for the termination test.

The notations above are used throughout the remaining examples in this thesis (including those of Sections 6.3.2 and Section 6.3.3).

5.2.2 Examples

The examples below demonstrate the effect of α (and γ^*) in optimizing the response surface of stochastic objective functions as follows:

1. <u>Minimization case</u>:

$$f_1(x_1;x_2) = (|x_1|^{1.5} + |x_2|^{1.5} + 500)\cdot 20 + e_1(x_1;x_2)$$

with optimal value of $f_1^*(0;0)=\$10000$.

2. <u>Maximization case</u>:

$$f_2(x_1;x_2) = \$95000 - f_1(x_1;x_2) + e_2(x_1,x_2)$$

with optimal value of $f_2^*(0;0)=\$85000$.

$e_i(x_1,x_2)$, the sampling error associated with function i, is normally distributed with mean $\mu_i = 0$ and standard deviation given by:

$$\sigma_i = \sigma_i(x_1,x_2)$$

$$= (\rho)E[f_i(x_1,x_2)]$$

The different parameters of the optimization procedure where chosen to be:

$\underline{X}^0 = (150;150)$

$\underline{S} = (15;15)$

$\rho = .15$

$C_r = 10.0$

$n_1 = 25$

$n_C = 15$

$n'=30$

$\alpha_T=.01$

$\beta_T=.95$

A "One-at-a-time" search procedure was applied to the first function above as a minimization algorithm, and to the second function, as a maximization algorithm. In each case, the optimization process was repeated five times, each time with a different value of α, while the values of the other parameters were kept fixed. The results are summarized in Table 1. For each example, these results include: a) the value of γ^* and the associated value of α, b) the total number of points investigated by the search and the number of improvement points among them, c) the average cost per point (which may indicate the average sample size used), d) the final improvement point which was discovered by the search, and e) the appropriate values of the maximal and the final net-savings obtained by applying the optimization process.

Example 5.1 reflects an optimization process for which $\gamma^*=.505$ or, equivalently, for which $\alpha=.99$. This higher value of α resulted in a search through 37 points (24 of which are improvement points) which terminated at point

TABLE 1

The effect of α on the efficiency of the optimization process - examples

a) A minimization case:

Ex. No.	γ^*	α	Total Pts.	Imprv. Pts.	$/pt.	Final Point	Net Savings Max	Net Savings Final
5.1	.505	.99	37	24	152.7	-7.2,.3	67891	67114
5.2	.600	.80	37	22	153.8	.3,4.1	68981	67886
5.3	.700	.60	38	22	165.0	-7.2,.3	67971	66921
5.4	.800	.40	38	4	162.7	135.3,120.3	18868	12686
5.5	.900	.20	35	4	203.7	131.5,120.3	13202	9372

b) A maximization case:

Ex. No.	γ^*	α	Total Pts.	Imprv. Pts.	$/pt.	Final Point	Net Savings Max	Net Savings Final
5.6	.505	.99	48	22	152.1	7.8,19.1	71581	69610
5.7	.600	.80	46	21	152.2	15.3,28.5	70387	67315
5.8	.700	.60	49	18	154.3	30.3,15.3	71579	67299
5.9	.800	.40	28	9	174.6	30.3,150.3	35009	31759
5.10	.900	.20	37	3	153.2	135.3,135.3	14578	9918

(-7.17;.33) and yielded \$67,114 net savings. Choosing other values for α, is reflected in examples 5.2-5.5. By observing the results of these examples it is apparently clear that the effectiveness of the sarch decreases when α is chosen to be small. Values of α =.80 (example 5.2) and α =.60 (example 5.3), yielded searches which terminated relatively close to the true optimum $(x_1*;x_2*)=(0;0)$ and resulted in high net savings. Choosing α =.40 (example 5.4), or α =.20 (example 5.5), resulted in poor performance of the search. In these cases, the associated level of confidence in defining a point as an improvement was so high, that only 4 points, in the beginning of the search could satisfy it. This caused the search to start converging around points which were relatively far from the optimum. Having a requirement, stated by the user, to obtain at least 4 improvement points, before applying the termination test for the first time, this convergence could not be stopped by the termination test before accumulating enough cost to justify termination.

The second group of examples (5.6-5.10) represent the application of a maximiation algorithm to the second objective function. The same effect of α is reflected through these examples. Small values for α resulted in a conservative decision rule which required points to provide a substantial improvement in the value of the objective function in order to be considered as "improvement points". This high requirement could not be satisfied by the search beyond a certain point, and caused the search to start converging relatively far from the optimum.

Examples 5.1-5.10 may lead to formulating the following conclusions:

1. Small values for α (such as $0<\alpha<.50$), may result in an increasing inability of the sampling routine to identify improvements along the search. This inability may cause the search to converge around a point which is relatively far from the optimum. In some extreme cases, the convergence may occur early in the course of the search, before obtaining the minimal number of improvement points necessary for applying the termination test for the first time.
2. An extremely large value for α, ($\alpha=.99$) results in a low level of confidence associated with the sampling routine. Decisions concerning "goodness" of points along the search may simply be made, then, by observing the value of the statistic D_i obtained by the initial sample at these points. Almost no adjustments of the sample size are made with this large α and the choice of the initial sample size, n_0, may determine the success of performing the search.

3. It is necessary to incorporate a "default stopping rule" which will be in effect early in the course of the search. The purpose of such a stopping rule is to terminate the search in those cases in which convergence occurs before obtaining the minimal number of improvement points necessary for applying the termination test. This stopping rule may be formed by specifying the maximal number of consecutive unsuccessful iterations allowed.

The above conclusions may suggest choosing values for α from the range $(.50<\alpha<.90)$. A value of $\alpha=.80$ was selected for the remaining examples in this thesis.

5.3 SELECTING THE SAMPLE SIZES n_0 AND n'

The confidence interval approach of section 3.3 uses two additional parameters: n_0, the initial sample size at any given point, and n', the maximal sample size allowed along the search. In determining values for these parameters one should be aware of the trade off between the "quality" of estimating system's performance through sampling, and the sampling cost. Large samples provide more accurate estimators, require more encounters with the simulation model and result in a higher search cost. Small samples provide less accurate estimators but result in a low cost per point.

Fixing n_0, the initial sample size, to be "large" enough may provide an ability to obtain "good" initial estimators at each point. It may, however, limit the total number of points that can be investigated, due to a high sampling costs per point. Choosing n_0 small at each point may cause the estimators to be less accurate but may also provide an ability to investigate more points, due to a smaller sampling cost per point. The sampling routine, however, is designed to adjust the necessary sample sizes in accordance with the level of difficulty in reaching a decision concerning "goodness" of points along the search. The principle on which this routine is based is to utilize relatively small samples as long as the comparison between the "best point thus far" and any new point can be made with the desired, pre-specified, level of confidence. Larger samples are necessary, therefore, only if no decision can be made with n_0. Hence, n_0 should be defined, such as to provide an "acceptable" initial estimation of systems performance at any given point. If this estimation needs to be improved, more observations are taken up to a maximum of n' observations per point. n' represents, therefore, the size of a "large" sample, assumed to provide a "good" final estimator of system's performance. This final estimator is the "best" that may be used in evaluating the quality of a given point. If the point cannot be defined as an improvement after taking n' observations, no further attempt to increase the sample size at this point is made, and the next point is introduced for investigation.

As a criterion for determining values for n_0 and n' one may consider the "width" of a confidence interval $(L_T;U_T)$ containing the true value of the system's measure of performance, obtainable by these sample sizes. (U_T-L_T), the width of the interval, may be expressed as a fraction q of the true mean. The value of q depends on the desired confidence level, $1-\alpha'$, and the true coefficient of variation, $\rho=\sigma/\mu$, of the simualtion model. For given values of α' and ρ, increasing the sample size will result in a narrower confidence interval for the true mean around the sample mean, and the sample mean may then be regarded as a better estimator of the true value. Two cases may be considered. The first, assumes a simulation model with symetric, normally distributed, random error. The second, assumes that the error has an exponential shape distribution with mean zero. For each of these cases, the sample size needed in order to provide a confidence interval with a requird width q, for various values of α' and ρ, was computed and is given below.

5.3.1 Normally distributed sampling error

Let $\bar{X}_i$ and S_i^2 be the estimated values of μ_i, the true mean, and σ_i^2 the true variance, of systems performance at point i, respectively, based on a sample of n_i observations. $\bar{X}_i$ and S_i^2 may be used to design a $1-\alpha'$ confidence interval $(L_T;U_T)$ for the true mean, such as:

$$U_T=\bar{X}_i+t_{1-\alpha'/2}(n_i-1)S_i/\sqrt{n_i}$$
$$L_T=\bar{X}_i-t_{1-\alpha'/2}(n_i-1)S_i/\sqrt{n_i}$$

The width (U_T-L_T) of the interval $(U_T;L_T)$ may be defined as a fraction q of the estimated mean, such that:

$$U_T \leq \bar{X}_i(1+q/2)$$

and

$$L_T \geq \bar{X}_i(1-q/2)$$

for

$$0 < q < 1$$

Hence, for a given value of q, n_i, the necessary sample size, is given by:

$$n_i \geq \frac{1}{(q/2)^2}(S_i/\bar{X}_i)^2 t^2_{1-\alpha'/2}\ (n_i-1)$$

which may be approximated as:

$$n_i \geq \frac{1}{(q/2)^2}(S_i/\bar{X}_i)^2 z^2_{1-\alpha'/2}$$

The lower bound of n_i was computed for various values of α', q, and $s_i/\overline{x}$ and is given in table 2.

Table 2 may assist in choosing the required values of n_0 and n'. If, for example, the simulation model is assumed to have a coefficient of variation of .20, a sample of 44 observations can provide an interval $(L_T;U_T)$ for which $L_T = (.95)\overline{x}$ and $U_T = (1.05)\overline{x}$. According to Table 1, this interval contains the true mean with a probability of $1-\alpha'=.90$. If only a probability of $1-\alpha'=.80$ is required, a sample of 26 observations is necessary. However, the examples in section 5.2.2 concerning the $(1-\alpha)$ confidence interval for $E[D_i]$, have demonstrated that a high confidence in that case was not necessary. A value of .60 for γ^* provided $\alpha=.80$ and was shown to be satisfactory. Since the value of α, used by the sampling routine, controls the sample size at any given point, it has the same effect as α'. Hence, a recommended high value for (i.e. $\alpha=.80$) may suggest that n_0 need not be defined conservatively and can be taken to be as small as 5-15 observations at a point. (Values of n_0 may vary in accordance with the value of the coefficient of variation of the simulation model). n', the maximal sample size at any point, may then be 20-50, (again, depending on the coefficient of variation). Starting with a small sample n_0 at any point, will enabale the sampling routine to use large sam-

TABLE 2

The required sample size for a normally distributed sampling error

a) confidence level, $(1-\alpha') = .90$:

q	S/$\bar{x}$ = .05	.10	.15	.20	.25	.30
.02	n = 68	271	609	1083	1693	2435
.05	n = 11	44	98	174	271	390
.10	n = 3	11	25	44	68	97
.20	n = 1	3	7	11	17	24

b) confidence level, $(1-\alpha') = .80$:

q	S/$\bar{x}$ = .05	.10	.15	.20	.25	.30
.02	n = 41	164	369	656	1024	1475
.05	n = 7	27	59	105	164	236
.10	n = 2	7	15	26	41	59
.20	n = 1	2	4	7	11	15

c) confidence level, $(1-\alpha') = .60$:

q	S/$\bar{x}$ = .05	.10	.15	.20	.25	.30
.02	n = 18	71	159	283	441	631
.05	n = 3	12	26	46	71	102
.10	n = 1	3	7	12	18	25
.20	n = 1	1	2	3	5	6

ples only in those cases in which they are necessary, and will enable the search to accumulate costs at a lower rate.

5.3.2 Exponentially distributed sampling error

The above recommendation for choosing n_0 and n' also holds in those cases in which the random error, inherent in the simulation model, cannot be assumed to have a normal distribution. Assume, for example, that the random error w_i at point i has a p.d.f. given by:

$$h_{w_i}(t) = \lambda_i e^{-\lambda_i (t + \frac{1}{\lambda_i})}$$

for which $1/\lambda_i$ is equal to a fixed fraction $0 < v < 1$ of the true mean at point i. v can be viewed, therefore, as the coefficient of variation of the simulation model. Now, $E[w_i]=0$, $Var[w_i]=1/(\lambda_i)$, and the sample mean $\bar{X}_i$ can be expressed as:

$$\bar{X}_i = \mu_i + \frac{1}{n_i} \sum_{j=1}^{n_i} [u_j - \frac{1}{\lambda_i}] = \mu_i + \bar{u}_i - \frac{1}{\lambda_i}$$

where

$$u_i \sim \exp(\lambda_i), \quad w_{ij} = u_j - \frac{1}{\lambda_i}$$

and

$$\bar{u}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} u_j$$

It can be shown that

$$2\,\lambda_i \sum_{j=1}^{n_i} u_j \sim \chi^2(2n_i)$$

Hence,

$$\Pr[\chi^2_{\alpha'/2}\ (2n_i) < 2\lambda_i n_i \bar{u}_i < \chi^2_{1-\alpha'/2}\ (2n_i)\ ] = 1-\alpha'$$

Dividing by $2\lambda_i n_i$ and substracting $1/\lambda_i$ from all the elements of the inequality yields:

$$\Pr[\frac{1}{\lambda_i}\ (\frac{\chi^2_{\alpha'/2}}{2n_i} - 1) < w_j < \frac{1}{\lambda_i}\ (\frac{\chi^2_{1-\alpha'/2}}{2n_i} - 1)] = 1 - \alpha'$$

Now, replacing $1/\lambda_i$ by $(v)(\mu_i)$ yields:

$$\Pr[\ \mu_i(\frac{\chi^2_{\alpha'/2}}{2n_i} -1) < w_i < v\mu_i(\frac{\chi^2_{1-\alpha'/2}}{2n_i} - 1)] = 1 - \alpha'$$

or

$$\Pr[\mu_i \cdot h_1 < \bar{w}_i < \mu_i h_2] = 1 - \alpha'$$

where:

$$h_1 = (v)\left(\frac{\chi^2_{\alpha'/2}}{2n_i} - 1\right)$$

$$h_2 = (v)\left(\frac{\chi^2_{1-\alpha'/2}}{2n_i} - 1\right)$$

Based on the sample mean $\overline{X}_i = \mu_i + \overline{w}_i$, a $(1-\alpha')$ confidence interval for the true mean μ_i is given by:

$$\Pr[\overline{X}_i(\frac{1}{1+h_2}) < \mu_i < \overline{X}_i\ (\frac{1}{1+h_1})] = 1 - \alpha'$$

or

$$\Pr[\overline{X}_i(1 - P_L) < \mu_i < \overline{X}_i\ (1 + P_u)] = 1 - \alpha'$$

where:

$$L = \left(\frac{1}{1+h_2}\right)\overline{X}_i = (1-P_L)\overline{X}_i$$

$$U = \left(\frac{1}{1+h_1}\right)X_i = (1+P_U)\overline{X}_i$$

P_L and P_U represent the deviation of U and L, respectively, from the sample mean as a fraction of the sample mean. The value of $P_L + P_U$ can be interpreted as the value of q in the case of the normally distributed random error. The coefficient of variation ρ may be estimated by $S_i/\overline{X}_i$. For the exponential case, the relationship between these parameters and the necessary sample size n is given in Table 3.

TABLE 3

The required sample size for exponentially distributed sampling error

confidence level, $(1-\alpha')=.90$:

n	v=.10 P_L	v=.10 P_U	v=.20 P_L	v=.20 P_U	v=.30 P_L	v=.30 P_U
5	.08	06	.14	.11	.20	.15
10	.05	.04	.10	.08	.14	.12
15	.04	.04	.08	.07	.12	.10
20	.04	.03	.07	.06	.10	.09
30	.03	.03	.06	.05	.09	.08
40	.03	.02	.05	.05	.07	.07

confidence level, $(1-\alpha')=.80$:

n	v=.10 P_L	v=.10 P_U	v=.20 P_L	v=.20 P_U	v=.30 P_L	v=.30 P_U
5	.06	.05	.10	.09	.15	.13
10	.04	.04	.08	.07	.11	.10
15	.03	.03	.06	.06	.09	.09
20	.03	.03	.06	.05	.08	.08
30	.02	.02	.04	.04	.07	.06
40	.02	.02	.04	.04	.06	.05

confidence level, $(1-\alpha')=.60$:

n	v=.10 P_L	v=.10 P_U	v=.20 P_L	v=.20 P_U	v=.30 P_L	v=.30 P_U
5	.04	.04	.07	.07	.10	.10
10	.03	.03	.05	.05	.07	.08
15	.02	.02	.04	.04	.06	.06
20	.02	.02	.04	.04	.05	.06
30	.02	.02	.03	.03	.04	.05
40	.01	.01	.03	.03	.04	.04

The required sample size n in table 3, for given confidence level $1-\alpha'$, coefficient of variation ρ and width P_L+P_U, is similar to the required sample obtained in table 2, for the same values of $1-\alpha'$, $S/\bar{X}$ and q, respectively. Hence, violation of the assumption of normality of the sampling error has almost no effect on the required sample size.

The recommendations for choosing n_0 and n', which were previously made may serve therefore as a general guideline for determining the sample size.

Chapter VI

SELECTING THE MAJOR PARAMETERS USED BY THE CONTROL MODULE

6.1 MOTIVATION

This chapter discusses the problem of selecting the major parameters of the termination test, the principle algorithm which forms the "Control Module". This test is presented in Chapter IV, and is summarized in Section 4.4. Section 6.2 below discusses the selection of b_0, the minimal acceptable saving's rate, the slection of d, the critical difference used by the test, and the selection of type I & II errors. The choice of type II error is shown to have an important effect in determining the termination point. This effect is deomonstrated through various examples in Section 6.3. Section 6.4 discusses the determination of the sample size needed for estimating the costs rate along the search.

6.2 SELECTING THE PARAMETERS b_0, d, AND THE TYPE I AND II ERRORS

The termination test is intentionally designed to be conservative by having null and alternative hypotheses as follows:

$$H_0 : E[B] \leq b_0$$

$$H_1 : E[B] > b_0$$

Under H_0, the expected value of the gross-savings rate, E[B], is less then b_0, (representing the minimum acceptable savings rate), and the search should be terminated. The alternative case, represents a high, acceptable gross-savings rate for which the search should be continued. Two types of errors are associated with the test:

<u>Type I error</u>:

α_T =Pr[reject H_0 | H_0 is true]

=Pr[continue the search | the search should be terminated]

<u>Type II error</u>:

β_T =Pr[do not reject H_0 | H_0 is false]

=Pr[terminate the search | the search should be continued]

or, alternatively,

$1-\beta_T$=Pr[continue the search | the search should be continued]

b_0 represents a minimum acceptable gross-savings rate. The search should undoubtedly be terminated when the expected value of the gross-savings rate, E[B], is less than b_0. However, when E[B] is greater than b_0+d for $d>0$, the search should be continued, since a net gain of at least d is expected from discovering an additional improvement point. Hence,

$$1-\beta_T=\Pr[\text{continue the search} \mid E[B]=b_0+d]$$

$$1-\alpha_T=\Pr[\text{terminate the searrch} \mid E[B]=b_0]$$

Assume that the user wishes to perform the search as long as $E[B]>b_0+d$ and wants to have a high probability of terminating the search when $E[B]<b_0$. An operation characteristics curve, designed to represent these considerations, is presented in Figure 7. A conservative approach in designing the test implies a need for a small α_T. In this case, whenever the search provides savings in a rate smaller than the minimum acceptable rate, b_0, it should be terminated with high probability $1-\alpha_T$. However, if the search provides savings with a high rate such as b_0+d, it should be continued. The probability of continuing a "good" search is given by the power of the test, $1-\beta_T$.

An efficient termination procedure should, therefore, incorporate a small type I error (small α_T) in order to avoid additional efforts and more expenses when the net return from applying the search drops below the acceptable value. On the other hand, a large power (small β_T) is also needed to avoid a premature termination of a promising search (when additional net savings can still be achieved).

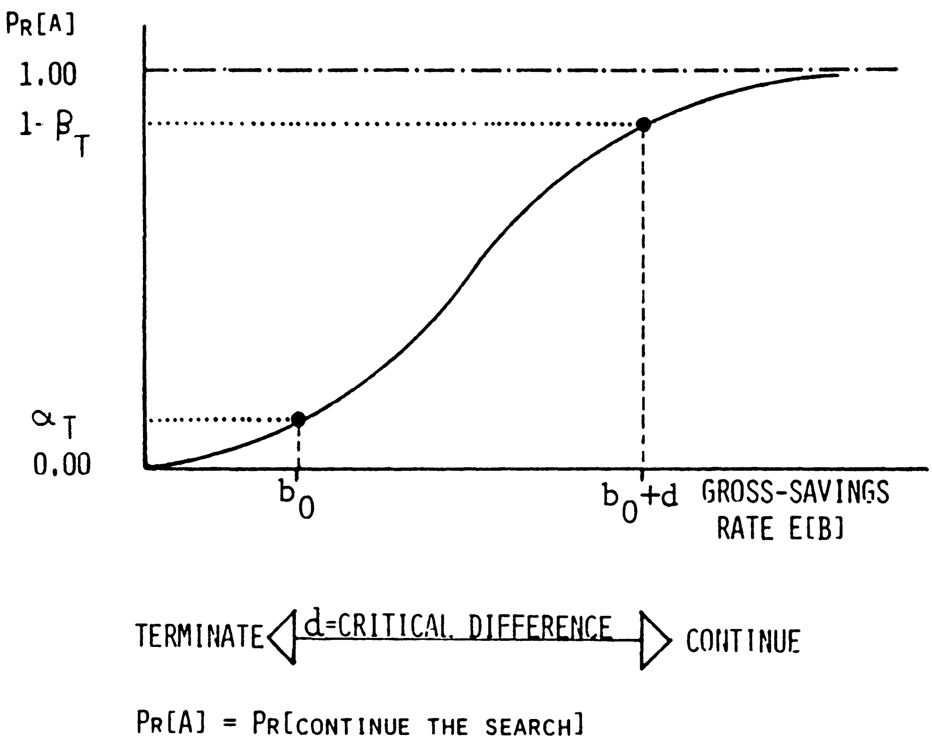


Figure 7: Designed operation characteristics curve for the termination test

The values of b_0 and d may be determined based on economic considerations. b_0 should represent the minimal rate of savings below which a user will not be able to justify a continuation of the search. b_0+d should represent a high, desired rate of savings for which continuation of the search is undoubtedly desired. Relating these values to the values of the search costs, one may suggest that:

1. When the savings rate becomes smaller than the cost rate, the user would certainly like to terminate the search since no further net savings are likely to be gained. Hence, b_0 may be set equal to R_c, the actual cost rate ($ per improvement point) at a decision point.
2. The search should be continued as long as it provides rate of return higher than r, where r represents an alternative investment channel for search funds. b_0+d may therefore be set equal to $[R_c][1+r]=[b_0][1+r]$, (see Figure 8).

The above recommendation for choosing α_T, β_T, d, and b_0, appears to be quite straightforward. However, an attempt to implement it may introduce difficulties. In most cases, a typical search technique will initially provide large improvements in the objective value. These successive improvements tend to decrease significantly as the search proceeds and approaches the neighborhood of the optimum. On

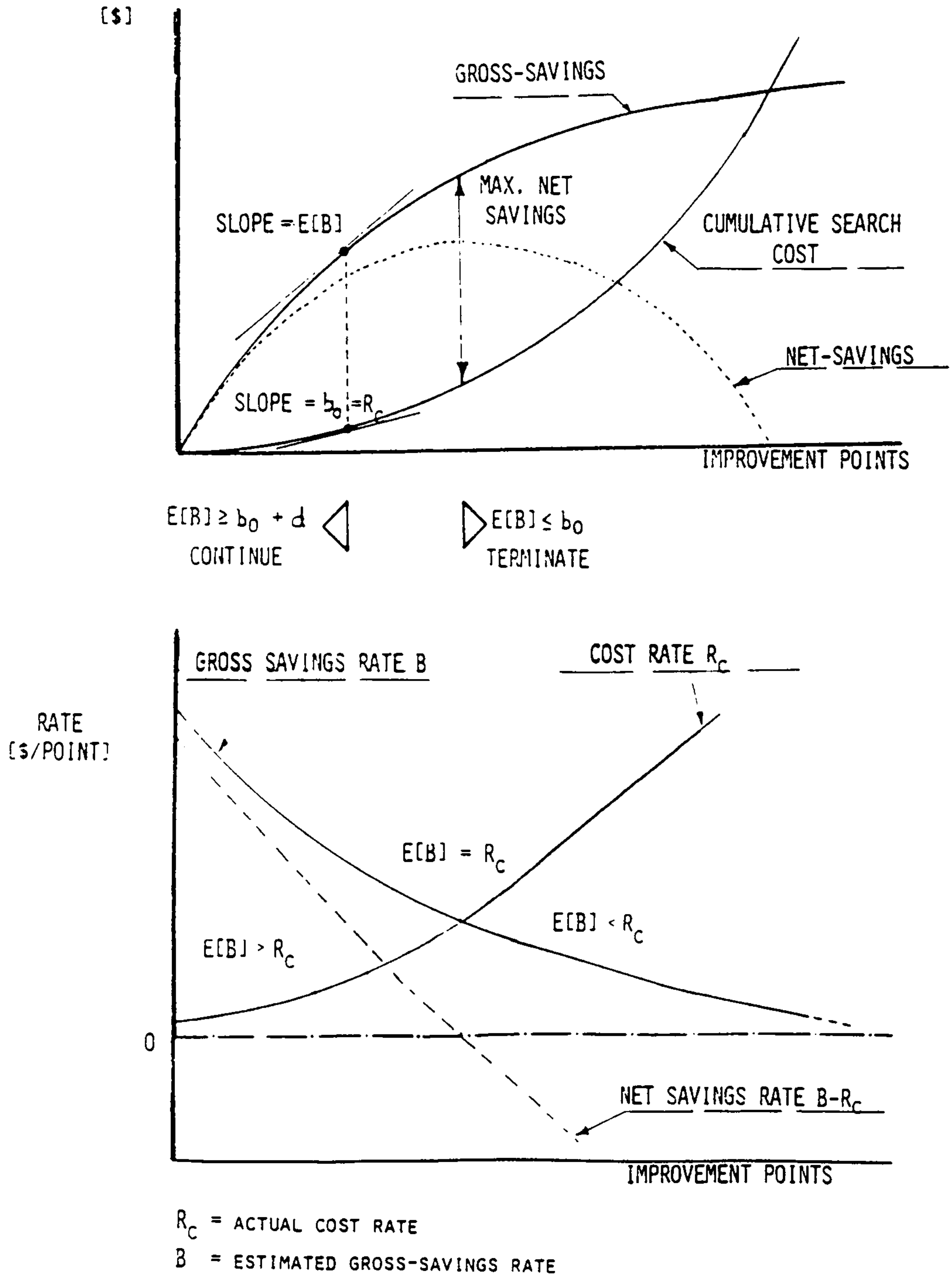


Figure 8: Selecting a termination point based on comparison of gross-savings rate and cost rate

the other hand, the search cost is likely to increase the closer the search gets to the optimum, due to increasing sample sizes (number of simulated replicates at a point). These "large" samples may be needed in order to provide satisfactory resolution between two successive points. By designing a termination test in which $b_0=R$, and therefore $d=[b_0][r]=[R_c][r]$, one may face a situation in the early stages of the search in which the savings rate is much larger than the cost rate, yielding a large value for B and relatively small values for b_0 and for the critical difference d. If, at that point, one uses the desired small values for α_T and β_T to design a test to detect this difference d, a large sample size may be needed to satisfy the power requirements. This sample size, in terms of number of improvement points, is not likely to be available early in the search. Therefore the test will not be conducted before enough improvement points are discovered and only then will provide a decision whether to terminate the search. If the search has proceeded in an undesired direction, this decision may come too late and funds may be unnecessarily wasted. Obviously, there is a need for a more sensative termination criterion.

A possible way to avoid this difficulty is to adjust the values of d and β_T along the search. Adopting a conservative approach, the user will pay more attention to the possibility of continuing an undesired search which would certainly lead to a waste of funds, than to that of making a premature decision to terminate a "good" search through which greater net-savings might be provided. Since the costly optimization procedure is applied only once, one would prefer to design a test which indicates, with a very high probability, a need for termination whenever the search starts providing savings with a rate smaller than the cost rate $b_0=R_c$. In this case, if the test allows continuation, the user can be almost certain that the search is proceeding in a promising manner. Hence, the test should be designed to detect the case of $E[B]<b_0$ (when the search should be terminated), by having a small type I error (small α_T). A more liberal approach may be adopted in choosing β_T, especially in the begining of the search, and the test may initially be performed with a relatively low power.

In addition to the above, one can use the same rationale in choosing d. As mentioned before, in the early stages of the search, one would expect the gross-savings rate B to be larger than the cost rate $b_0=R_c$. Hence, the user may choose a critical difference d much larger than the desired $(R_c)(r)$ value, by defining d to be equal to $(B)(r)$, a fraction of the gross-savings rate. As long as $B>>R_c$, the critical difference d need not be smaller than (B)(r). In addition, using a low power (large β_T), will significantly reduce the sample size needed to detect the difference d. It will then be possible, to perform the test even in the early stages of the search. As long as no termination occurs, the analyst can "almost guarantee" that the gross-savings rate is higher than the cost rate by at least (B)(r). Termination may occur prematurely, due to the low power of the test and the high value of d. In such cases the most recent estimators of the cost and gross-savings rates should be examined, new smaller values for d and β_T may be introduced, and the search may proceed from this point. The process of updating the value of d along the search may be "self-conducted", by choosing:

$$d=\mathrm{Max}\{\ (R_c)(r)\ ;\ (B)(r)\ \} \tag{79}$$

thus, keeping the critical value d to be a fraction of the gross-savings rate B, as long as it is higher than the cost rate R_c. By doing so, the test is allowed to use a small sample and may check the profitability of the search more frequently. Obviously, the value of d is reduced along the search, and is finally equal to the desired fraction of the accumulating cost rate R . At this point, termination using the last definition of d, may be due to the low power of the test. The decision whether or not to risk more funds at that point is left to the user, who is able to force the search to continue by reducing the value of β_T (increasing the power). When termination finally occurs with a reasonably high power, or when the total net savings has reached a satisfactory level, no more adjustments of β are necessary.

6.3 THE EFFECT OF TYPE II ERROR IN DETERMINING THE TERMINATION POINT

6.3.1 General considerations

The value of β_T, type II error of the termination test, has an important effect in determining the termination point. Recall that $1 - \beta_T$, the power of the test, is defined as the probability of continuing a "good" search. The continuation of such a search is desired, since it is likely to result in increasing the net-savings obtained from the optimization process. High power in this case, may cause the search to continue investigating points even when the probability of obtaining additional net savings is low. Low power however, requires less cost to be accumulated in order to justify the termination of the search. With b_0, the minimal acceptable savings rate equals the cost rate and d, the critical difference equals to the desired return, (as defined in Section 6.2), the highest power that may be used is

$1-\beta_T=.50$. This value represents a probability of .50 to continue a good search which provides the same rate of return as of an alternative investment channel, (certainly, a case of indifference). The values of β_T should therefore be chosen, from the range $.50 < \beta_T < 1.00$. The effect of choosing different values for β_T from this range is demonstrated in Sections 6.3.2 and 6.3.3. Throughout these sections the simulation model is represented by a stochastic function whose general form is given by

$$f_j(X) = E[f(X)] + e_j(X)$$

for:

$$j=1,2,\ldots,r$$

$$X=[x_1,x_2,\ldots,x_n]$$

where $e_j(X)$ is the "sampling error" added to the true value of the function. The performance of the optimization procedure is tested for a normally distributed sampling error with mean $\mu=0$ and standard deviation $\sigma=\sigma(X)$. The examples of Section 6.3.2 are also repeated with sampling error having an "exponential shape" distribution function with expected value of zero. The distribution of the sampling error is shown to have almost no effect on the efficiency of the optimization process. The implementation of the optimization process in investigating a real simulation model, and the effect of β_T in this case, may be found in Chapter VII. The examples of Section 6.3.2 below utilize a "Coordinate Search" ("One-at-a-time") to investigate an objective function with circular contours. Examples which demonstrate an application of the "Pattern Search" in investigating objective functions with eliptic, non-circular contours are given in Section 6.3.3.

6.3.2 Examples using a "One-at-a-time" search

This set contains 12 examples which utilize the search technique in order to optimize the following objective functions:

1. Minimization case:

$$f_1(x_1;x_2) = (|x_1|^{1.5} + |x_2|^{1.5} + 50)(100) \quad [\$]$$

with optimal value of $f_1^*(0;0) = \$5000$

2. Maximization case:

$$f_2(x_1;x_2) = \$100000 - f_1(x_1;x_2) \quad [\$]$$

with optimal value of $f_2^*(0;0) = \$95000$

The parameters used throughout these examples were chosen as follows (refer to Section 5.2.1 for the list of notations):

$\underline{X}^0 = (60;60)$

$\underline{S} = (8;8)$

$\rho = .15$

α = .80

C_r = 15

n_1 = 25

n_0 = 15

n' = 30

α_T = .01

β_T = various values from [.50,.95]

The optimization process was applied to the first function, as a minimization algorithm, and to the second function, as a maximization algorithm. Each of these optimization types yielded a group of 6 examples. The first three examples in each group reflect a normally distributed random error as associated with the stochastic function. Repeating these examples with sampling error having an exponential shape distribution is reflected by the other three examples in each group. In order to demonstrate the effect of β_T, all the other parameters used through these examples remained unchanged, thus the effect reflected is only due to the selection of β_T.

Table 4 presents a summary of these 12 examples, (see Appendix 2 for the computer output of these examples). The first three examples (6.1 - 6.3), demonstrate a minimization algorithm applied to the appropriate function with normal

TABLE 4

Examples using a "One-at-a-time" search technique

Ex. No.	Opt. Type	Error Type	β_T	Total Pts.	Imprv. Pts.	Final Point	Net Savings Max.	Net Savings Final
6.1	Min.	Norm.	.50	47	17	.33,.33	86584	82145
6.2	Min.	Norm.	.80	37	17	.33,.33	86584	84395
6.3	Min.	Norm.	.95	37	17	.33,.33	86584	84395
6.4	Min.	Exp.	.50	43	17	.33,.33	86063	82438
6.5	Min.	Exp.	.80	36	17	.33,.33	86063	84028
6.6	Min.	Exp.	.95	36	17	.33,.33	86063	84028
6.7	Max.	Norm.	.50	66	20	-.42,-3.67	90617	84992
6.8	Max.	Norm.	.80	61	20	-.42,-3.67	90617	86117
6.9	Max.	Norm.	.95	38	19	-.67,-3.67	90486	89208
6.10	Max.	Exp.	.50	58	21	4.33,-1.67	89062	84030
6.11	Max.	Exp.	.80	53	21	4.33,-1.67	89062	85155
6.12	Max.	Exp.	.95	34	20	4.33,.33	89062	88148

sampling error. Choosing a value of .50 for β_T (example 6.1) resulted in a search which investigated a total of 47 points, (17 of which are improvement points), terminated at (.33;.33) and provided $82,145 net savings. Using a β_T of .80 (example 7.2) caused the search to terminate after investigating 37 points and resulted in final net savings of $84,395. Increasing β_T to be equal .95 (example 6.3), did not have an effect on terminating the search. Replacing the normal distribution of the sampling error by an exponential distribution and repeating examples 6.1 - 6.3, yielded examples 6.4 - 6.6. These examples do not vary much from those with the normal error. The next three examples (6.7 - 6.9), demonstrate a maximization algorithm, and a model with a normally distributed random error. Hence, a β_T of .50 (example 6.7) yielded a search through 66 points (20 of which were improviment points), terminated at (-.42;-3.67) and provided final net savings of $84,992. Using a higher value for β_T resulted in a more sensitive termination test which concluded the search after investigating 61 points for β_T=.80 (example 6.8), and only 38 points for β_T=.95 (example 6.9). As in the minimization case, replacing the normally distributed sampling error by an exponentially distributed error (examples 6.10 - 6.12), had only a small effect on the efficiency of the terminiation procedure (see Figures 9, 10, 11, 12, 13, at the end of this section).

Examples 6.1 - 6.12 may lead to formulating the following conclusions:

1. Although choosing a power of .50 for the termination test can be intuitively justified on an economic basis (see Section 6.3.1) this power may still be too high in some cases. It appears that the initial power for the test should be chosen as low as .05 (β_T=.95). This low power may provide a sensitive termination test which will conclude the search closer to the point where the net savings reach their peak.
2. The search procedure and the termination test appear to be insensitive to the distribution type of the random error inherent in the simulation model. This insensitivity is a result of using the sample mean as the principle statistic by both the sampling routine and the termination test.
3. The results presented in Table 4 reflect also a relationship between the ability of the search to get close enough to the true optimal point (before being terminated) and the type of the optimization process (minimization or maximization). This relationship may be explained by the fact that the simulation model was assumed to have a coefficient of variation

(standard deviation/mean) which does not vary much along the search. The variance of the sample mean tends therefore, to decrease during a minimization process, and to increase, during a maximization process. The variation of the sample mean affects the variation of the statistic D_i, used by the sampling routine in evaluating the "goodness" of new points along the search (see equations 18, 23, 24, 25). Small $Var[D_i]$ enables the search to detect small differences in the value of the objective function. The search may be able, therefore, to identify further improvements even when it gets relatively close to the minimum point. In a maximization process, however, $Var[D_i]$ tends to increase as the search approaches the maximum. Large $Var[D_i]$ may cause the search to face increasing difficulties in detecting small differences between adjacent points. These difficulties are reflected in examples 6.7 - 6.12. However, the termination of the search less close to the optimum is justified in these cases. The increasing disability of the search to obtain additional significant improvements beyond a certain point, causes the net savings' sequence to start decreasing. This negative trend was observed by the control module which concluded the search, as required.

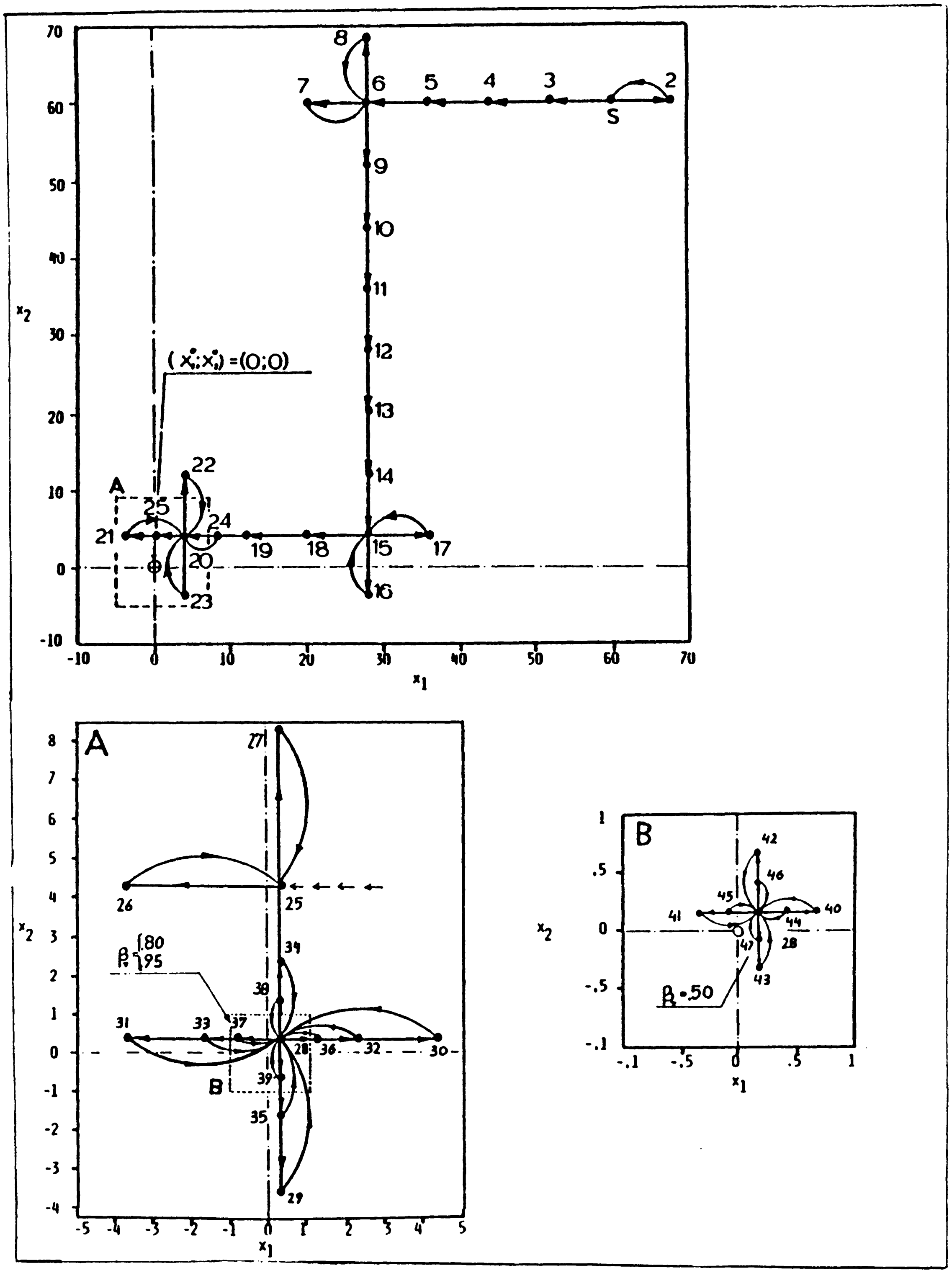


Figure 9: The search pattern of examples 6.1-6.3 (One-at-a-time, minimization, normal error)

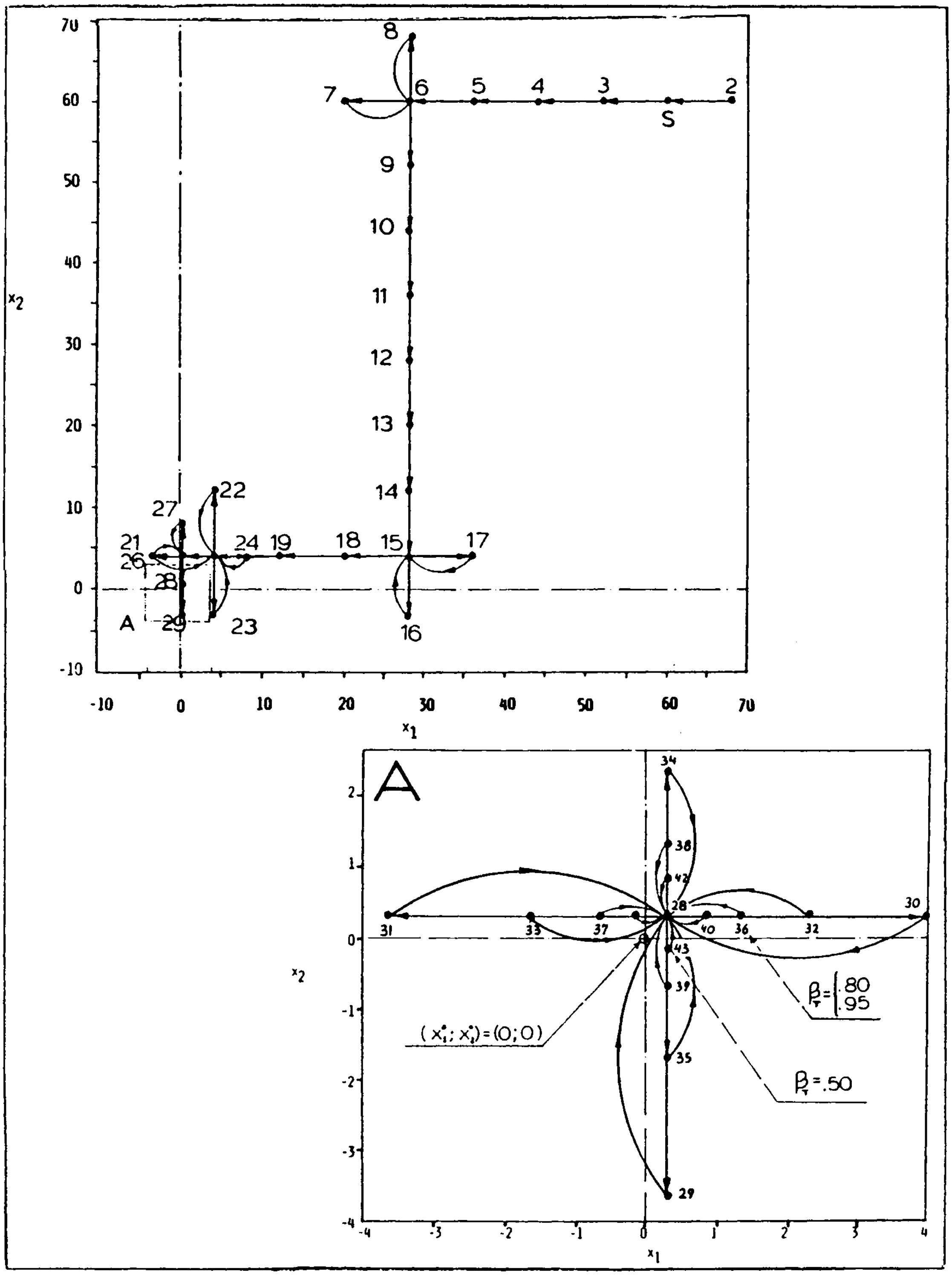


Figure 10: The search pattern of examples 6.4-6.6 (One-at-a-time, minimization, exponential error)

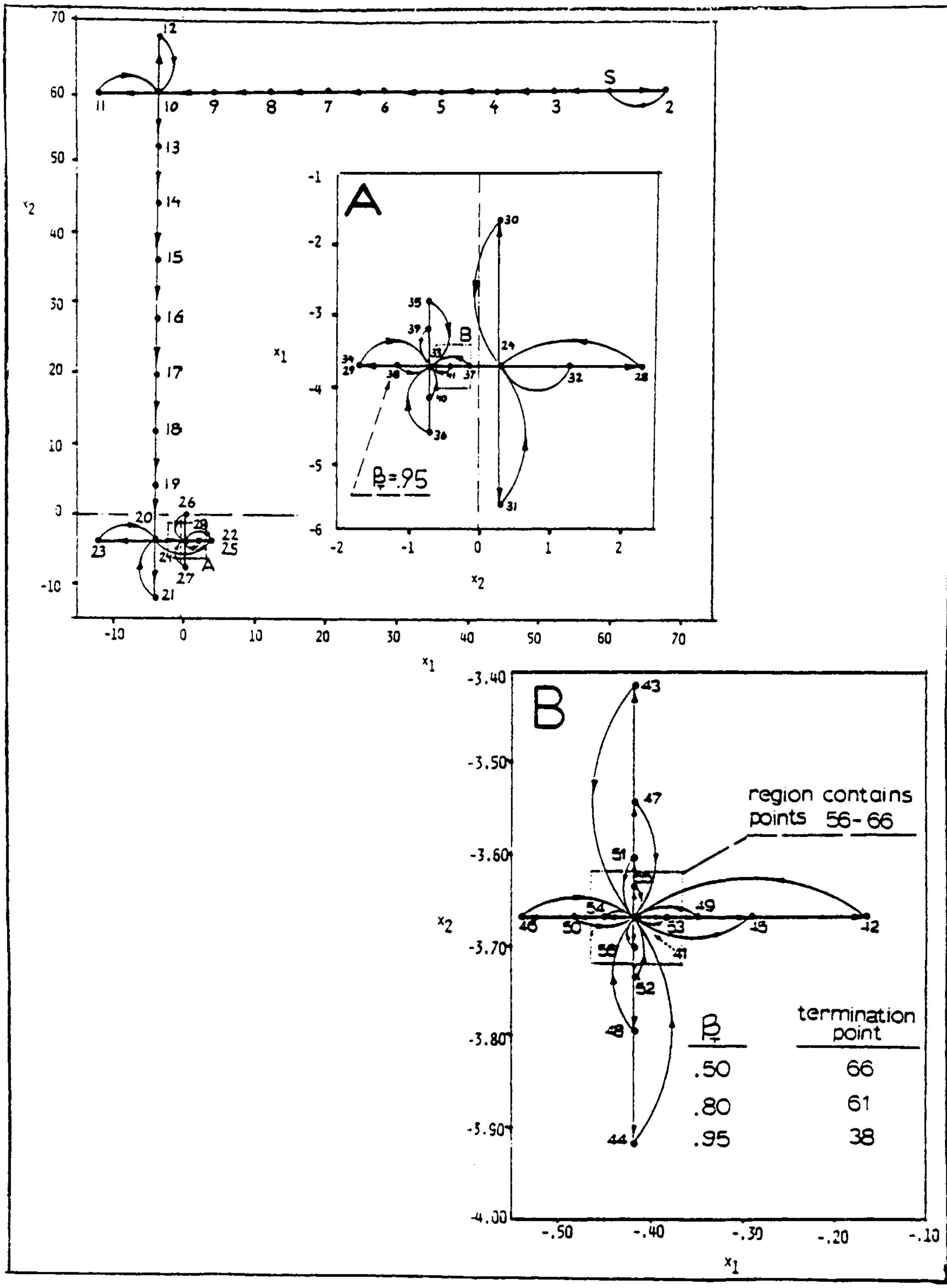


Figure 11: The search pattern of examples 6.7-6.9 (One-at-a-time, maximization, normal error)

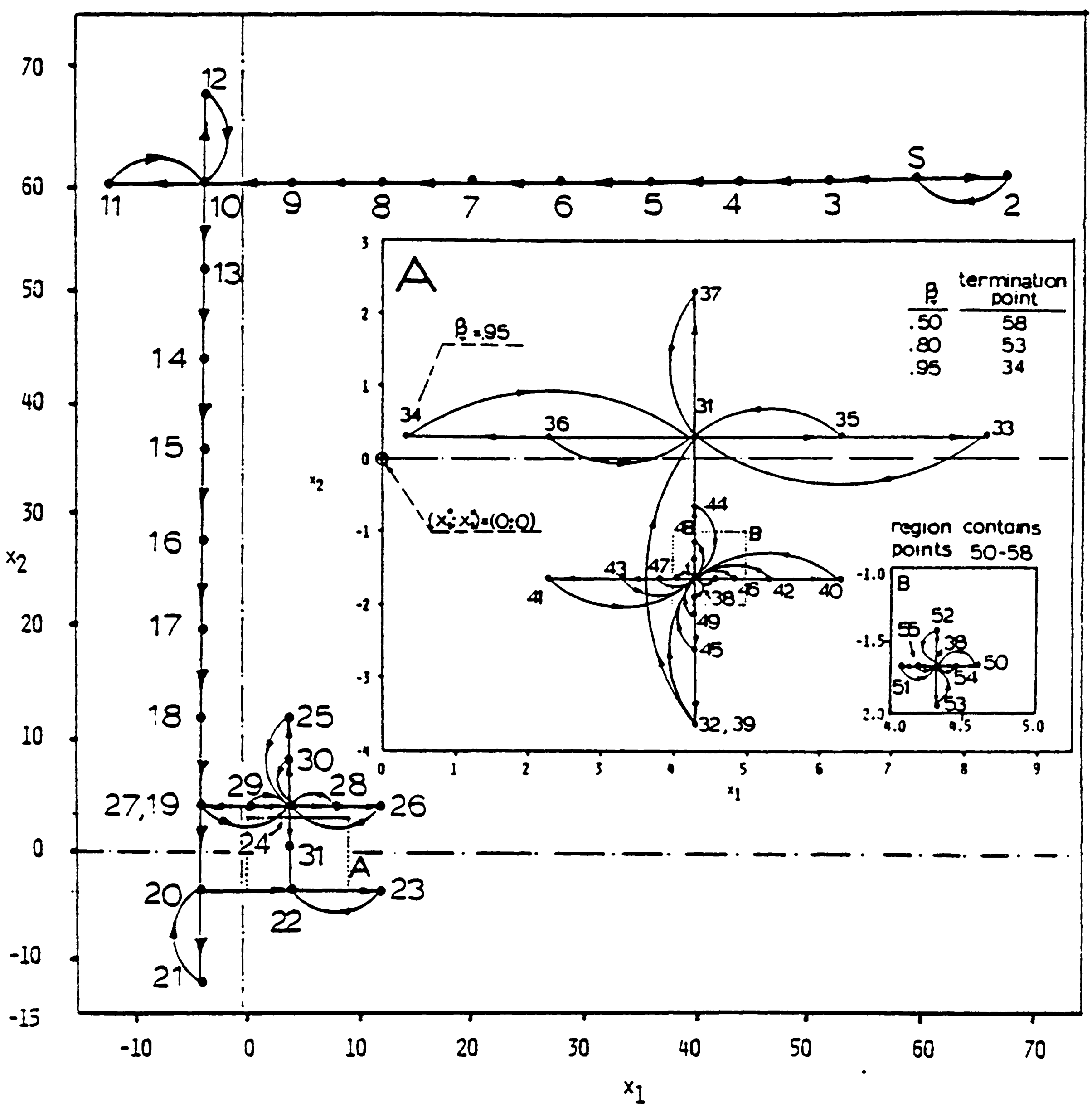


Figure 12: The search pattern of examples 6.10-6.12 (One-at-a-time, maximization, exponential error)

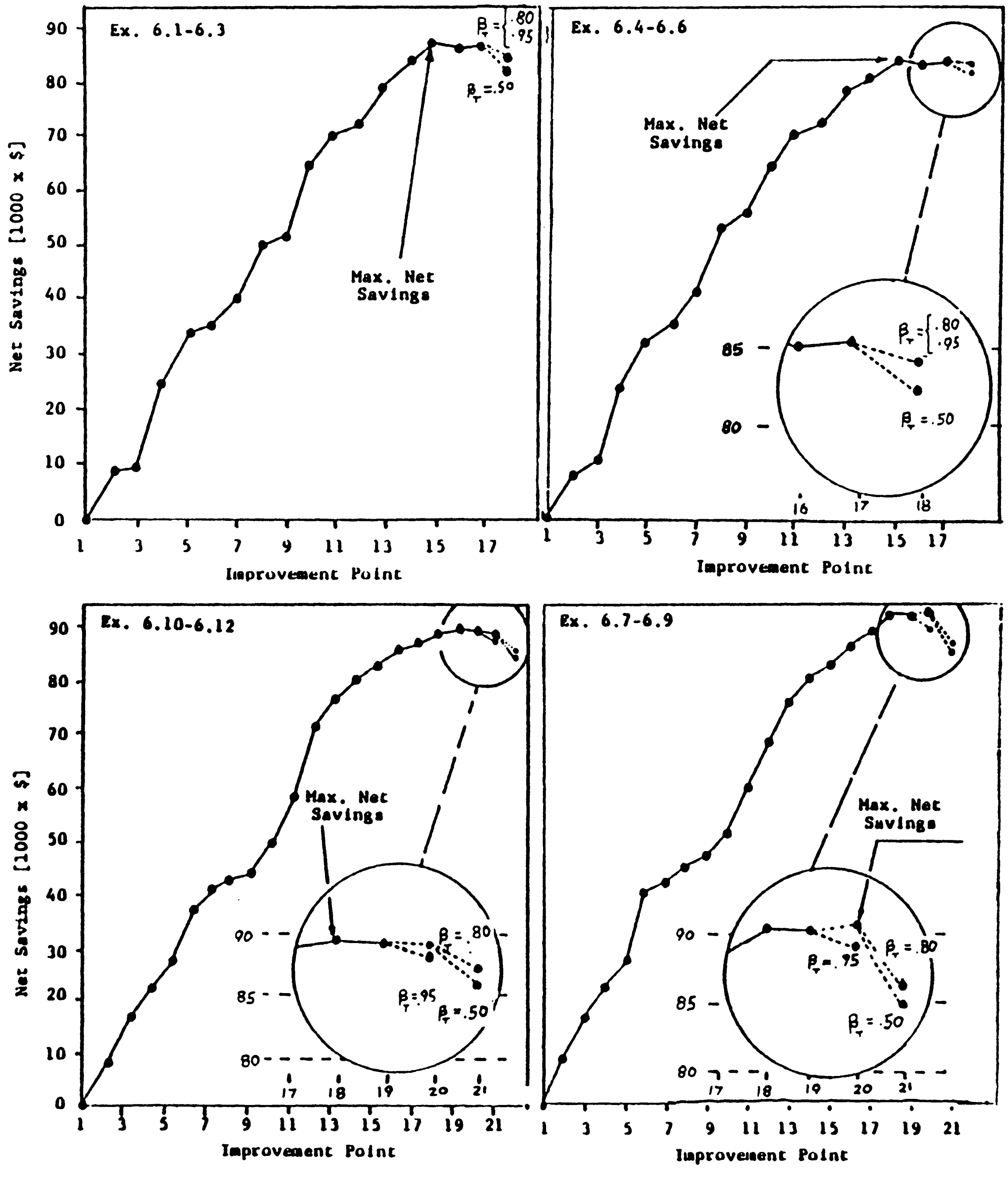


Figure 13: The net savings obtained at improvement points along the search in examples 6.1-6.12

6.3.3 Examples using a "Pattern" search

The following sets of examples uses a "Pattern Search" to investigate stochastic objective functions which have non-circular contours. These examples demonstrate the same effect of β_T on the duration of the search as the effect reflected by the examples of Section 6.3.2. They may also suggest that this effect is independent of the type of the sequential search technique used throughout the optimization process.

Consider the following objective functions:

1. Minimization case:

$$f_1(x_1;x_2) = [\frac{(x_1)^2}{1.8} + \frac{(x_2)^2}{1.8} - \frac{(x_1)(x_2)}{1.125} + 50] \cdot (10)$$

with optimal value of $f_1^*(0;0) = \$500$

2. Maximization case:

$$f_2(x_1,x_2) = \$40000 - f_1(x_1,x_2) \quad [\$]$$

with optimal value of $f_2^*(0;0) = \$39500$

These functions have eliptic contours whose major axes form an angle of 45 degrees with the $x_1;x_2$ axes.

The parameters used by the optimization procedure were chosen as follows:

$\underline{X}^0 = (100;80)$

$\underline{s} = (8;8)$

ρ = .15

α = .80

C_r = 5.0

n_1 = 25

n_0 = 15

n' = 30

α_T = .01

The optimization process was applied as a minimization algorithm to the first function above, and as a maximization algorithm, to the second. Each type of these optimization processes is reflected by a group of 4 examples. The first example in each group used β_T =.50. This parameter was increased up to .95 in the fourth example. Table 5 summarizes the major results of these examples (see Appendix 2 for the computer output of these examples).

The first example in Table 5 (example 6.13) used β_T =.50. This value resulted in a search through 58 points (27 of which are improvement points) which terminated at (-3.67;-3.67), and provided \$15513 net savings. Using values of .70 and .80 (examples 6.14 and 6.15) provided the same results. A value of .95, however, caused the search to terminate after investigating a total of 37 points (18 of which were improvement points) as shown in example 6.16. This large value of β_T caused the search to terminate at (40.33;32.33) after providing \$13754 net savings. Here, as

TABLE 5

Examples using a "Pattern" search technique

Ex. No.	Opt. Type	Error Type	β_T	Total Pts.	Imprv. Pts.	Final Pt.	Net Savings Max.	Net Savings Final
6.13	Min.	Norm.	.50	58	27	-3.67;-3.67	15638	15513
6.14	Min.	Norm.	.70	58	27	-3.67;-3.67	15638	15513
6.15	Min.	Norm.	.80	58	27	-3.67;-3.67	15638	15513
6.16	Min.	Norm.	.95	37	18	40.33;32.33	13754	13754
6.17	Max.	Norm.	.50	65	15	28.33;14.33	18037	16116
6.18	Max.	Norm.	.70	62	15	28.33;14.33	18037	16341
6.19	Max.	Norm.	.80	59	15	28.33;14.33	18037	16566
6.20	Max.	Norm.	.95	48	15	28.33;14.33	18037	17391

in the previous examples (Section 6.3.2), β_T is shown to have an effect on determining the termination point. Obviously, by observing the search pattern and the sequence of the net savings provided by β_T=.95, in cases similar to example 6.16, the analyst may decide whether or not to continue the search. Such a decision may be favorable in the case of example 6.16, and may improve the net savings. Hence, recommendation no. 1 of Section 6.3.2 can be adopted as a general rule for choosing β_T. This recommendation provides a sensitive search and may be reevaluated once termination occurs (see Figure 14).

Examples 6.17 - 6.20 demonstrate the application of a maximization algorithm. Again choosing different values for affected the termination point. A value of β_T=.50 resulted in a search through 65 points, which provided \$16116 net savings, where β_T=.95 terminated the same search after 48 points and yielded \$17391 net savings. Examples 6.17 - 6.20 demonstrate, therefore, the effect of β_T in determining the number of points that should be investiaged beyond the last improvement point which can be significantly obtained. In these examples, the search reached the point (28.3;14.3) and could not obtain further improvements due to the large variation of the sample mean in the region of the optimum. This point was the 43rd along the search. Here

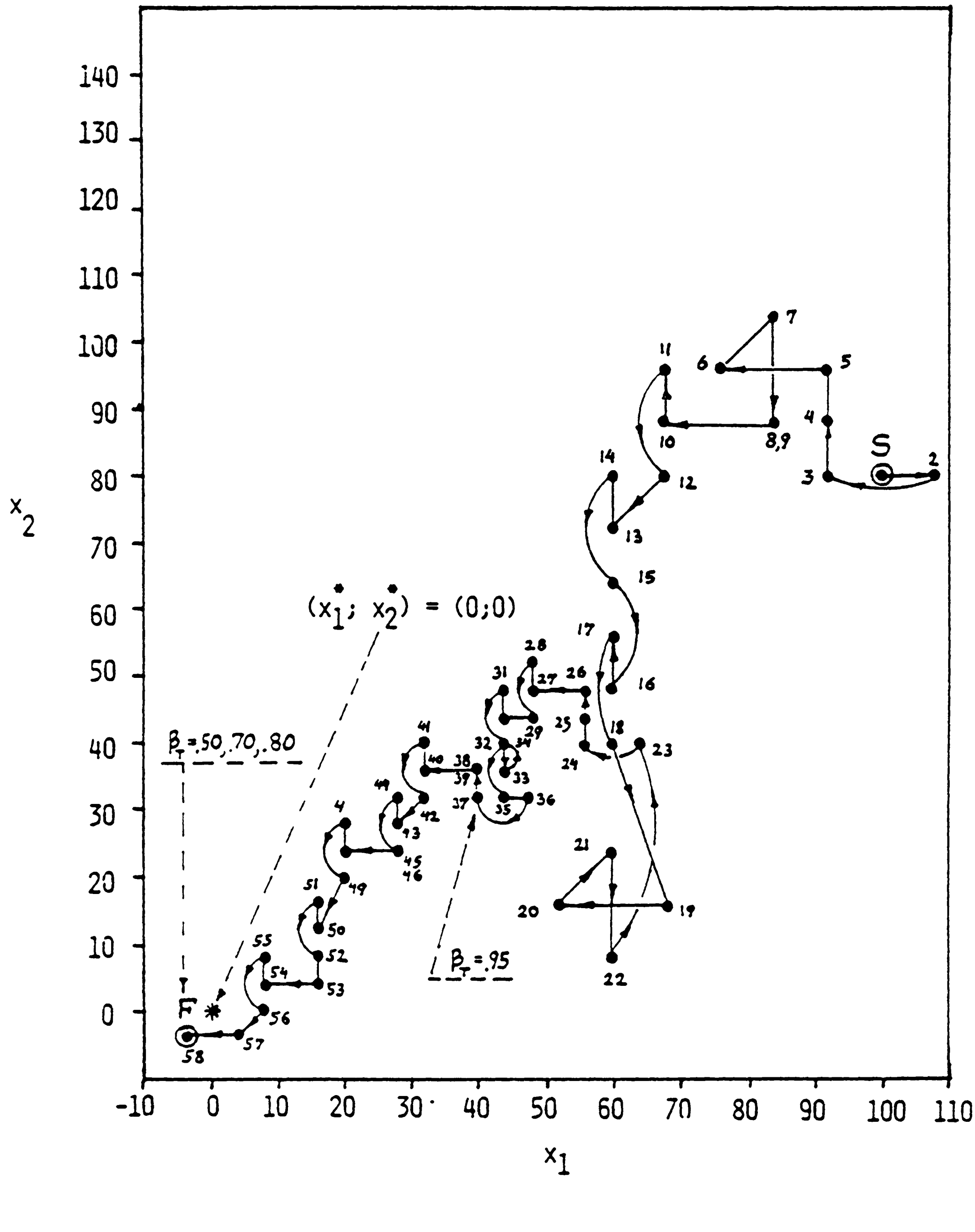


Figure 14: The search pattern of examples 6.13-6.16 (Pattern Search, minimization, normal error)

β_T=.50 required additional 65-43=22 points to be investigated before terminating the search, while β_T=.95 required only 48-43=5 points. These different levels of sensitivity are clearly reflected by the final net savings which were obtained in each of this cases (see Figures 15, and 16).

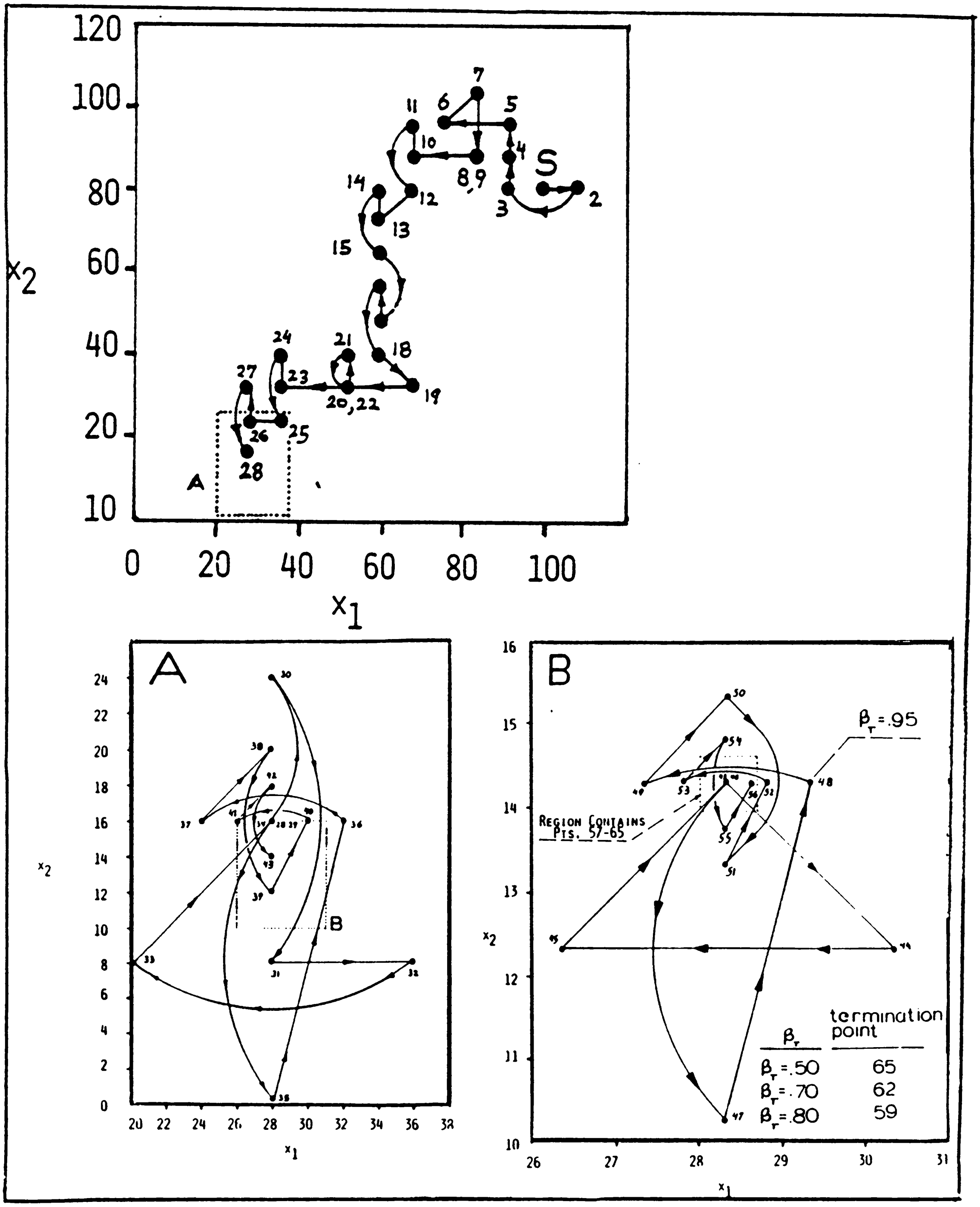


Figure 15: The search pattern of examples 6.17-6.20
(Pattern Search, minimization, normall error)

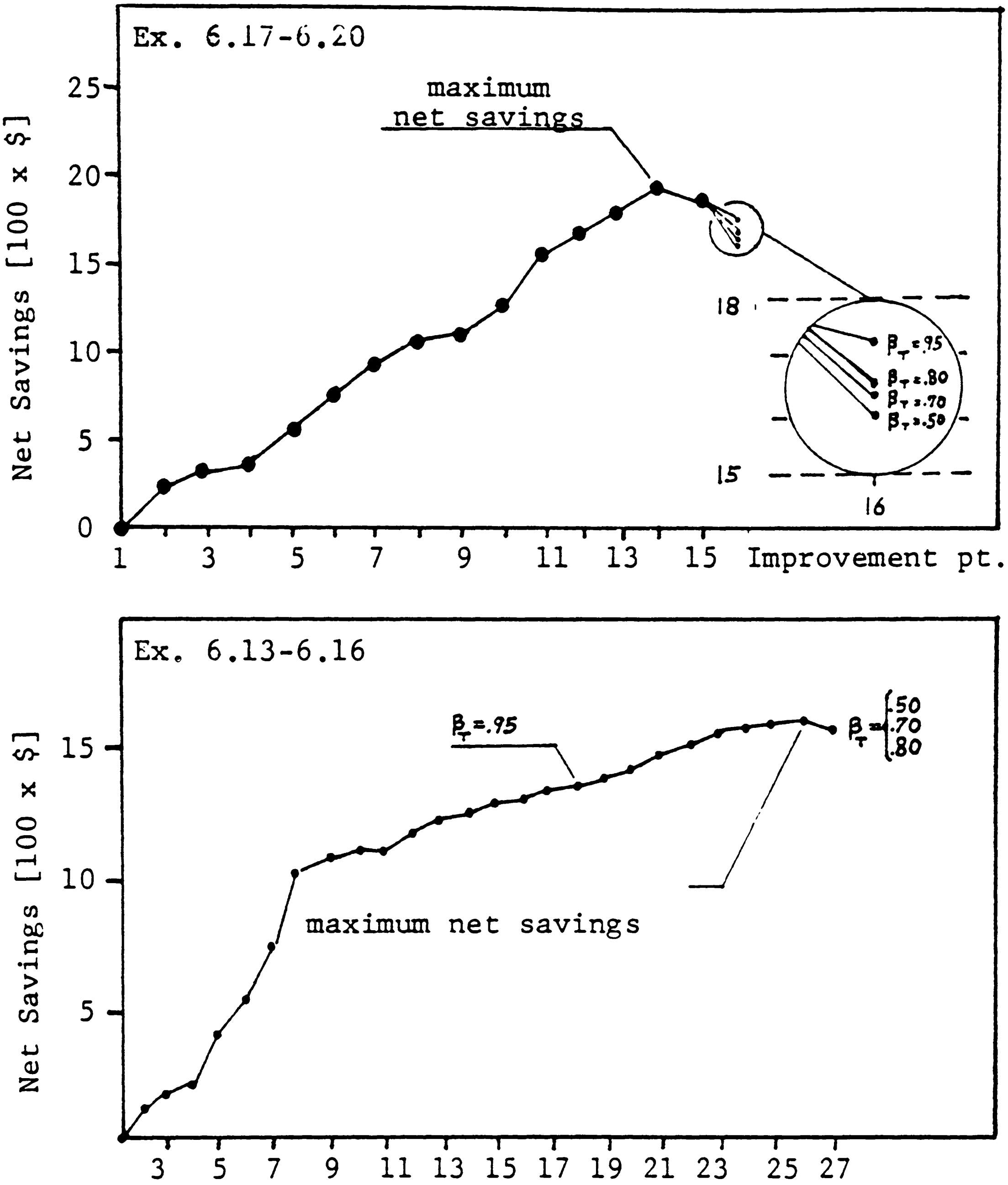


Figure 16: The net savings obtained at improvement points along the search in examples 6.13-6.20

Chapter VII

OPTIMIZING A (Q,R) REORDER POINT INVENTORY SYSTEM

7.1 MOTIVATION

In this chapter, the implementation of the optimization procedure is demonstrated in investigating the simulation model of a real system. A (Q;r) reorder point inventory system was selected for this purpose. This system was chosen for the following reasons:

1. A mathematical model of the system can be developed and may be used to validate the results obtained through the optimization process. (Such a model may be built if the demand can be assumed to have either a Poisson or a Normal distribution.)
2. The response surface of this system has non-circular contours and is relatively flat in the neighborhood of the optimum.
3. The system has a two dimensional set of input variables, and a graphic presentation of its response surface and the search path is possible.

Section 7.2 below, provides a mathematical model which represents a (Q;r) inventory system with either a Poisson or a Normal demand and is used in order to obtain a graphical presentation of its response surface. Section 7.3 provides the results obtained by applying the suggested optimization procedure to the simulation model of the (Q;r) system. The procedure was applied twice, using two different search techniques. The different parameters of both the sampling routine and the control module were selected to conform with the recommendations made in Chapters V and VI. The effect of changing the value of the parameter β_T was also checked and found to be similar to the one described in Chapter VI.

3.2 THE MATHEMATICAL MODEL OF A (Q,R) REORDER POINT INVENTORY SYSTEM

The (Q;r) reorder point inventory system operates as follows; units are demanded one at a time in an apparently random manner. If inventory on hand is positive, a unit is withdrawn and the demand is met. When there is no stock on hand at the time of the demand, a unit is specially ordered, involving an extra "Backorder" cost of C_B for each unit. An order for Q units is placed whenever the total inventory position (units on hand plus on outstanding orders) falls to exactly r units, causing an "Ordering Cost" of C_0 for each order placed. Lead time for orders placed is considered to be constant and equal to t months. "Inventory Carrying

Cost" is computed based on average inventory level, and equals to C_I per unit per month.

The cost components of the inventory system may be approximated by:

E_0 (ordering cost) = $C_0(1/T)$

E_B (backorders cost) = $C_B[b(r)](1/T)$

E_I (inventory carrying cost) = $C_I[\overline{I}(r)]$

where T, $\overline{b}(r)$ and $\overline{I}(r)$ are the cycle length, the average number of backorders at the end of each cycle, and the average inventory on-hand, respectively. The number of cycles per time unit is given by:

$$N = 1/T = D/[Q+\overline{b}(r)]$$

where Q is the order quantity and D is the demand per time unit. Now let $X=X|_t$ be the demand during lead time and f(x) be the p.d.f. of X, then:

$$\overline{b}(r) = \int_r^\infty (x-r)f(x)dx$$

which for Poisson demand is given by:

$$\sum_{x=r}^{\infty} (x-r)\left(\frac{e^{-\lambda}\lambda^x}{x!}\right)=\lambda[Pr(X=r-1)]+(\lambda-r)[1-Pr(X\leq r-1)]$$

where $\lambda = \mu_{X|_t} = E[X|_t]$. Using a normal approximation one may obtain:

$$b(r) = \sigma_{X|_t} \quad L'(\frac{r-\mu_{X|_T}}{\sigma_{X|_T}})$$

where

$$L'(u) = \int_u^\infty (Z-u)\phi(Z)dZ = u \cdot \Phi(u) + \phi(u) - u$$

Z is a standard normal r.v., Φ (z)=the c.d.f. of Z ; ϕ (z)=the p.d.f. of Z. $\bar{I}(r)$, the average level of inventory, is given by:

$$\bar{I}(r) = [\ Q/2+r-\mu_{X|_T}+\bar{b}(r)\ ][\ \frac{Q}{Q+\bar{b}(r)}\ ]$$

and the total cost is:

$$TC = K(Q;r) + C_O(1/t) + C_B[\bar{b}(r)](1/T) +$$
$$+C_I[Q/2+r-\mu_{X|_t}+\bar{b}(r)][\ \frac{Q}{Q+\bar{b}(r)}\ ]$$

A similar model can be found in [33]. An optimal solution to the above total cost equition may be found iteratively, by assuming $1/T = [Q+\bar{b}(r)]/D \cong Q/D$, taking first partial derivatives with respect to Q and r, setting them to equal zero, and obtaining:

$$Q^* = \sqrt{\frac{2D[C_0 + C_L\ b(r^*)}{C_I}}$$

$$F(r^*) = \frac{C_I Q^*}{C_I Q^* + C_L D}$$

where

$$F(r^*) = 1 - \Pr[X|_t \leq r^*]$$

The above three equations may be solved iteratively, by first setting $\bar{b}(r^*)=0$ and obtaining an initial value for Q^*, then using the second equation to obtain an initial value for r^*, updating the value of $\bar{b}(r^*)$, obtaining a new value for Q^* etc. For Poisson demand with an average of 200 units per month, and cost elements of C_I = \$5; C_0 = \$40; C_L = \$20 the optimal solution is at $(Q^*;r^*) = (56;26)$ for which TC = \$3260.6 per month. The contours of TC, the total cost function, given the above cost elements are presented in Figure 17.

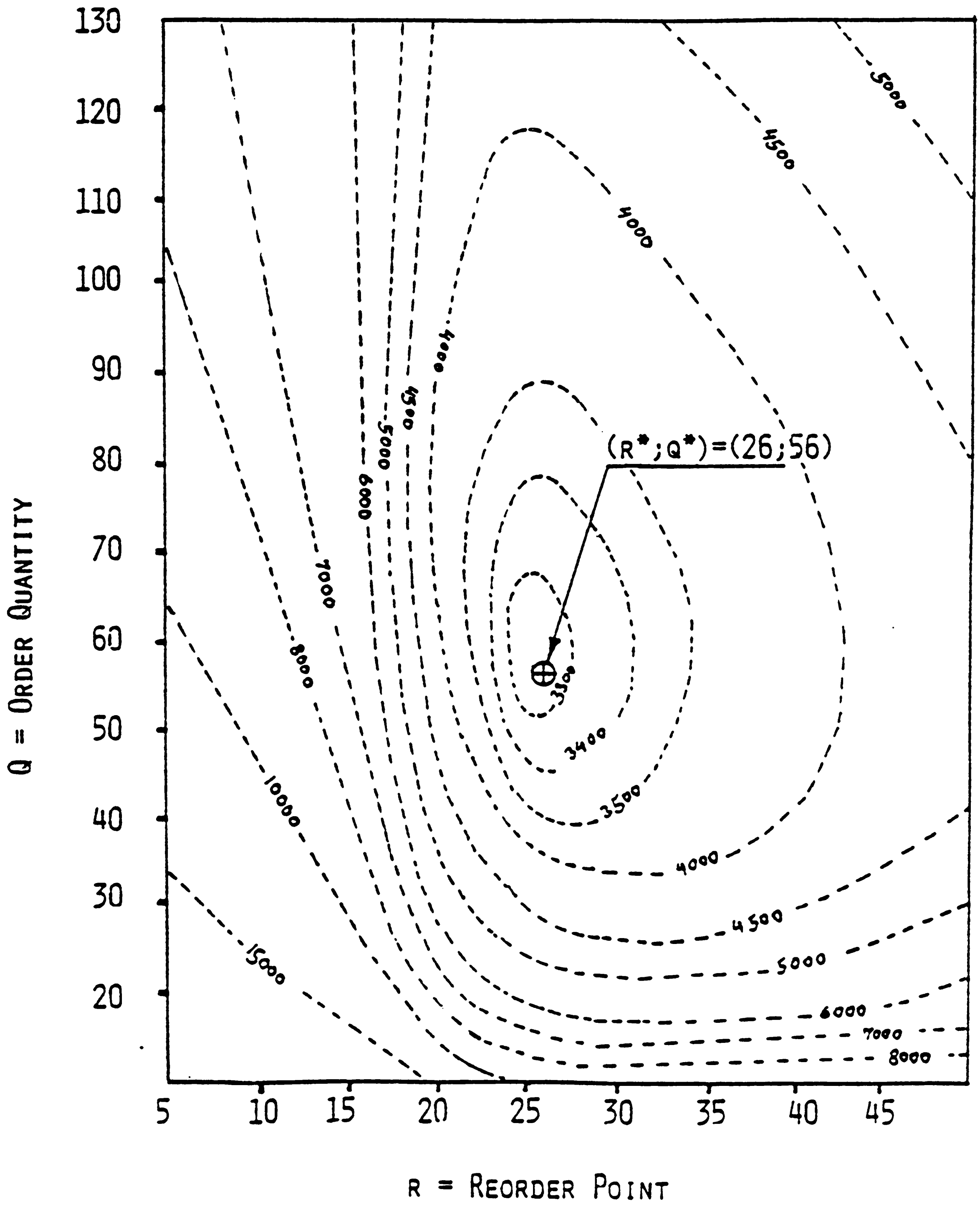


Figure 17: The total cost function for the (Q,r) reorder point inventory system, [$/month]

7.3 OPTIMIZING THE INVENTORY SYSTEM USING A SIMULATION MODEL

This section presents an application of the optimization procedure to a properly validated simulation model of the (Q,r) inventory system. This model provides independent realizations of the total monthly cost of the system, for any given set (Q,r) of input variables. Each of these realizations is obtained by simulating the operation of the system for a sequence of several months, using a new seed, and observing the total monthly cost which occurs at the end of the simulated period. A planning horizon of 5 years is assumed. The time value of money during that period is taken to be 15% per year, and the rate of inflation, 10% per year. The purpose of the optimization process is to minimize the present value of the expected total costs over the planning period. The following parameters were used throughout the optimization process (using the notations defined in Chapter V):

$\underline{X}^0=(r_0;Q_0)=(5;10)$

$\underline{S}=(5;10)$

$\alpha=.80$

$\alpha_T=.01$

$C_r=15.0$

$n_1=10$

$n'=30$

$n_0=15$

Each search technique was applied to the simulation model, first, with $\beta_T=.50$, then, with $\beta_T=.80$, and finally, with $\beta_T=.95$. The search pattern and the net savings' graph for each of the examples are illustrated in Figures 18, 19, 20. The major results are summarized in Table 6. The net savings given in this table, reflect the present value of the amount of improvement in the system's monthly cost, for a five year period of operation. This present value was computed by multiplying the monthly cost by the following factor:

$$(P/A,i',j',n) = \begin{cases} \dfrac{1-(1+j')^{n}(1+i')^{-n}}{i'-j'} & i' \neq j' \\ \dfrac{n}{1+i} & i' = j' \end{cases}$$

where:

A=the present montly cost.

n=the length of the planning period in months.

j=the expected rate of inflation per year during the planning period.

i=the time value of money per year during the planning period.

j'=the equivalent monthly rate of inflation given by $i'=i^{1/12}-1$.

i'=the equivalent monthly rate of return given by $j'=j^{1/12}-1$.

For j=10%, i=15%, and n=(5)x(12)=60 [months], this factor equals to 53.275.

In the first example of Table 6, a value of .50 for β_T yielded a search through 37 points, 12 of which were improvement points. This search was terminated at (r;Q)=(25;55) and provided net savings (present value) of \$121503 for the planning period or, \$121503/53.275=\$2280.6 per month. Using a larger value of β_T (examples 7.2 and 7.3), is shown to improve the net savings obtainable from the search. A β_T of .80 (example 7.2), yielded a search through 29 points only (12 of these points were improvement points as in example 7.1). This search provided net savings of \$122703 for the planning period or \$2303.2 per month. β_T=.95 (example 7.3) improved the net savings even better. This value caused the search to terminate closer to the point in which the net savings reached their peak. Here, the search was conducted through 17 points only, and provided the highest net savings (present value of \$124178 for

TABLE 6

Optimizing the (Q,r) inventory system - examples

Ex. No.	Search type	β_T	Total Pts.	Imprv. Pts.	Final pt (r,Q)	Net Savings Max.	Net Savings Final
7.1	One-at-a-time	.50	37	12	(25,55)	124628	121503
7.2	One-at-a-time	.80	29	12	(25,55)	124628	122703
7.3	One-at-a-time	.95	17	11	(25,50)	124628	124178
7.4	Pattern	.50	35	9	(27,55)	126598	124129
7.5	Pattern	.80	29	9	(27,55)	126598	124978
7.6	Pattern	.95	25	9	(27,55)	126598	125629

five years, or, \$2330.8 per month). As in the case of investigating a stochastic function (see Chapter VII), β_T is shown to have an importnt effect on the determination of the termination point. Example 7.3 shows that although the final point (r,Q)=(25;50), obtained by β_T =.95, is less close to the true optimum (r*;Q*)=(26;56), as compared to the final point obtained in examples 7.1 and 7.2, its overall performance is better. The discovery of the point (r;Q)=(25;55) in examples 7.1 and 7.2 necessitated additional cost and reduced the total net savings obtained by the search. Here, even though the inventory system may operate with a "better design", (r;Q)=(25;55), for 5 years, the overall net savings obtained by this design do not exceed \$123000, due to the high cost which was involved in searching for this design. Operating the system for 5 years with the less optimal final design obtained by example 7.3 yields total net savings of more than \$124000, due to a cheaper optimization process.

In examples 7.4-7.6 the optimization process is repeated using a "Pattern" search. This search yielded a final design of (r;Q)=(27;55) for the inventory system. The "Pattern" search was apparently more efficient, and provided higher net savings for each value of β_T, as compared to the "One-at-a-time" search technique.

The above demonstration of applying the optimization procedure to a real simulation model may be used to strengthen the conclusions made in Chapter V. Again β_T is shown to have an effect on the length of the search, providing the higher net-savings for a value as high as $\beta_T = .95$. The sampling error is shown to have almost no effect on the efficiency of the sampling routine and of the termination procedure. Finally, examples 7.1-7.6 demonstrate the ability of the procedure to utilize more than a single type of sequential search technique in the optimization process.

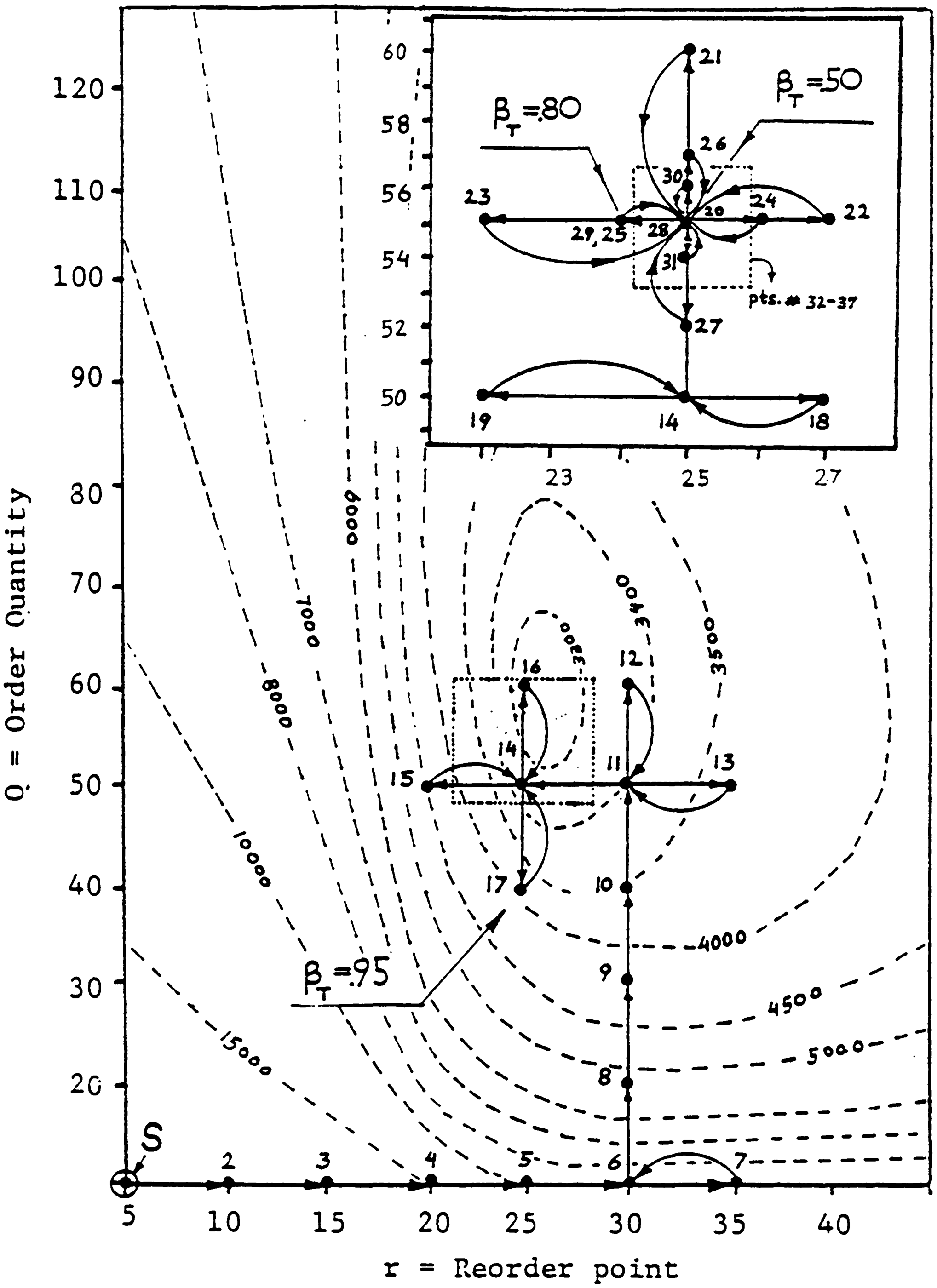


Figure 18: The search pattern of examples 7.1-7.3 (One-at-a-time, minimization, inventory model)

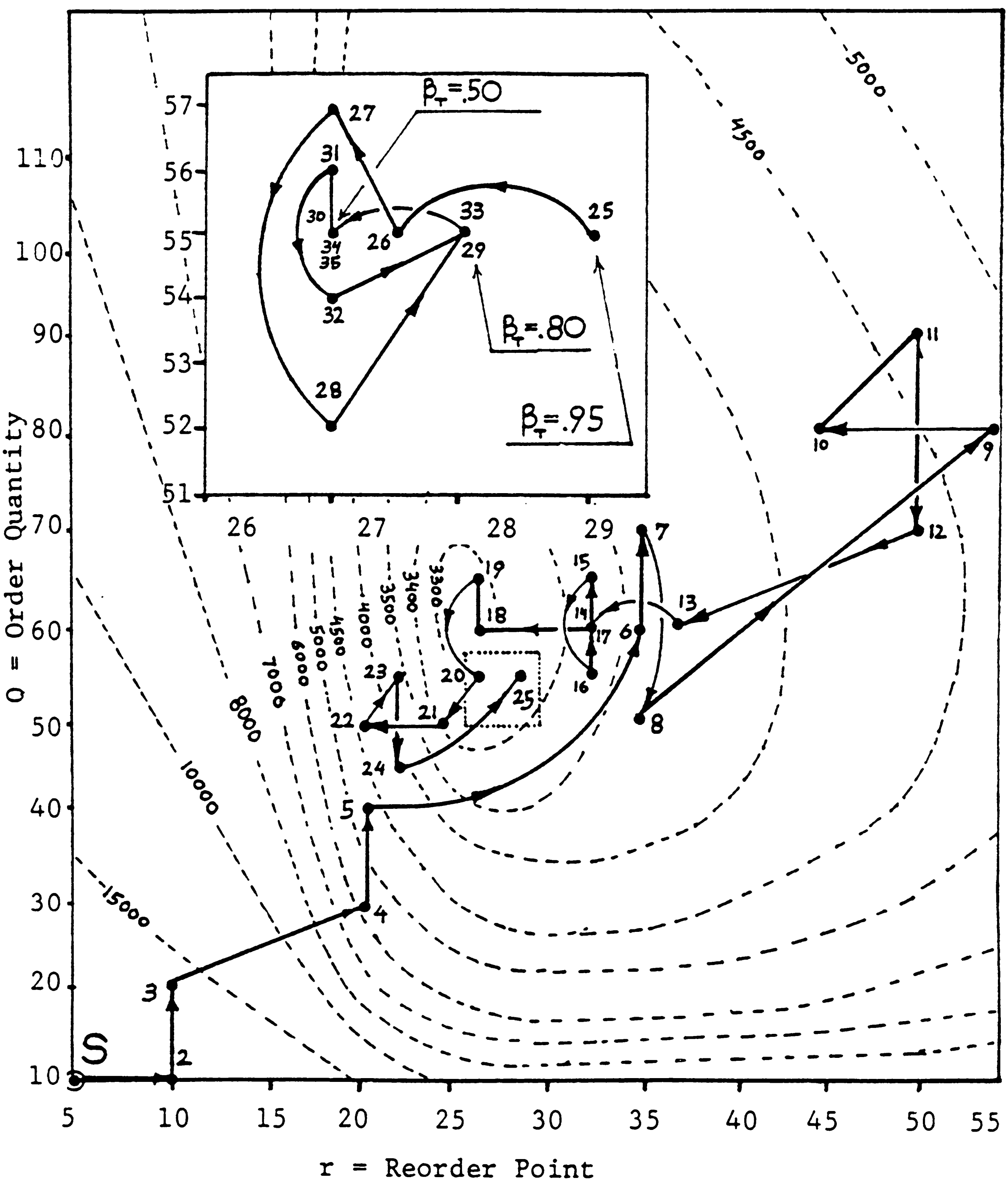
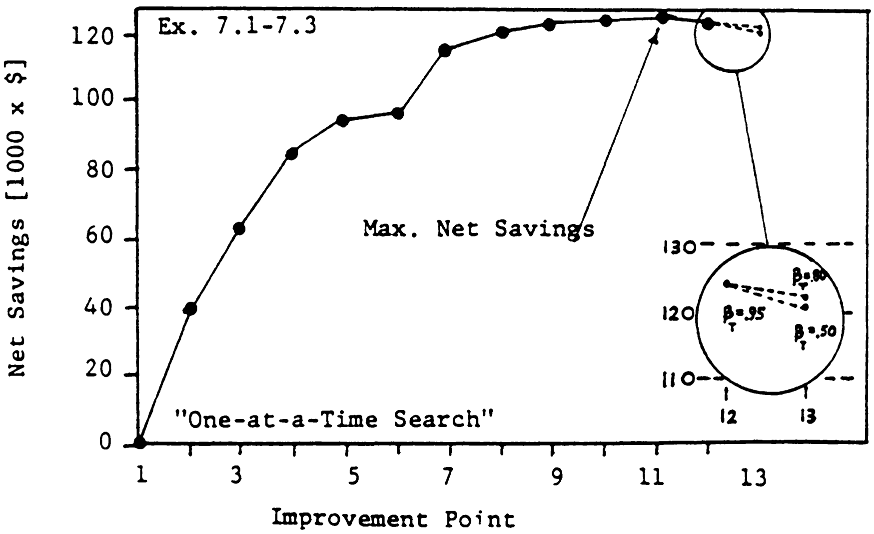


Figure 19: The search pattern of examples 7.4-7.6 (Pattern Search, minimization ,inventory model)



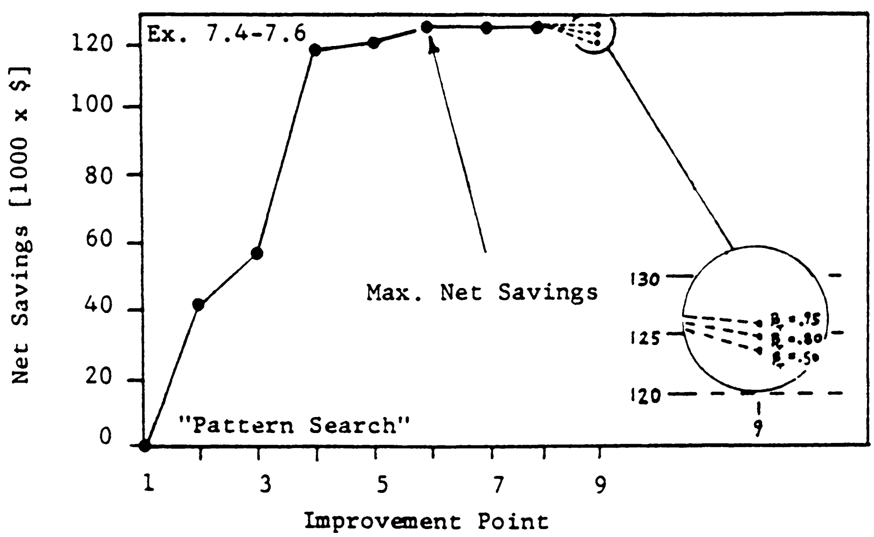


Figure 20: The net savings obtained at improvement points along the search in examples 7.1-7.6

Chapter VIII

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS FOR FURTHER RESEARCH

8.1 AN OVERVIEW OF THE OBJECTIVES OF THE RESEARCH

In this thesis, a procedure for performing and controling the process of system's optimization through simulation was presented. The objective of the research was to develop an external optimization procedure which can be applied to the simulation model of any multiple-variable, single-response, system whose overall performance is defined in terms of cost or profit. Such an optimization procedure should enable, to the extend possible, the use of known optimum seeking methods which have been proven to be efficient in investigating multivariable response surfaces.

An attempt to optimize the system's performance using a simulation model introduces two major problems. The first problem results from the sampling error associated with any experimental evaluation of the objective function through simulation. This problem prevents a simple application of known search methods which could otherwise be utilized in investigating the response surface of the function. A solution to this problem may be provided by constructing a sampling procedure which modifies sequential search techniques to deal with stochastic functions. This routine should be utilized with various sequential search methods. The development of such a routine was the first objective of this research. The second problem is associated with the attempt to apply a costly optimization procedure in order to achieve the "best" system design (in terms of cost or profit over a planning period). The funds that are expended in the course of a search for that "best" design affects the overall net gain from applying the search, and requires redefinition of the goal of the optimization process to be the maximization of the "overall net savings". This objective does not necessarily imply the discovery of the global optimum of the system's performance function. It may be achieved by concluding the search whenever the "net-savings" reach their peak, regardless of whether the search is still far from the global optimum. A solution to this problem may be provided by designing a control procedure to continuously evaluate the performance of the optimization process in order to determine the desired termination point. The design of such a module is the second objective of the thesis.

8.2 THE STRUCTURE OF THE OPTIMIZATION PROCEDURE

In attempting to achieve the objectives of the thesis, a modular structure for the optimization procedure is suggested. This structure consists of four basic modules, as follows:

1. The simulation model: provides realizations $\hat{U}_j(X_i)$ of the system's performance for a given set X_i of control parameters, $(j=1,2,...n_i)$.
2. The search technique: provides the "sampling rule" for the experimental search (determines the next point to be investigated) in order to obtain incremental improvements in the objective value.
3. The sampling routine: controls the sampling proces (determines the sample size n_i), for any new set X_i of input parameters; provides estimators for system's performance $U(X_i)$; tests and determines whether improvement has been achieved, and finally transfers the information to the search technique.
4. The control module: keeps track of the search's performance by evaluating, comparing, and testing the rate of improvement with the rate of accumulating cost in order to determine when to terminate the search.

The simulation model (1) and the search technique (2) are assumed to be given and are treated as "black boxes". The sampling routine (3) and the control module (4) are designed to provide external modifications for (1) and (2) in order to deal with the optimization of a simular response. The interface between these modules is demonstrated in Figures 3 and 4 in Chapter I.

The "sampling routine" was designed to assist the search technique in identifying the "better" point using estimated values of the objective function at the "best point thus far", and at a new point. Such a comparison is performed after each iteration of the search, in order to determine whether improvement was achieved. This information is transfered to the search method which, in turn, makes a decision concerning the next point for investigation. Two comparison methods were examined in this research. The first, is based on a statistical test which applies to the above estimated values in order to determine which one is more favorable. The second method defines a confidence interval for the difference between these two estimated values. A decision is reached by evaluating the prospective upper and lower limits for the improvement in the value of the objective function, and by comparing these limits with the cost of the step that was needed to investigate the new point. Both of these comparison methods provide dynamic adjustments of the necessary sample sizes along the search, in accordance with the level of difficulty in which a decision concerning the "goodness" of a new point should be made. The confidence interval approach, however, requires smaller sample sizes along the search and was found superior to the statistical test approach.

The "control module" was designed to terminate the search in either one of two cases: a) when the search identifies a point for which the associated value of the objective function is close to the optimum, or, b) when the search reaches a point at which further significant improvements in the value of the objective function is unlikely. In each of these cases, the control module terminates the search whenever the estimated costs, associated with further improvements of the objective function, are found to be higher than the prospective amount of improvement. This decision is made by continuously approximating the distribution of the rate of "gross-savings" obtained during the serach, and comparing this rate with the rate of costs using a statistical test. Continuation of the search is allowed as long as the "gross-savings" rate is found to be significantly larger than the cost rate. Otherwise, the search is terminated, and the final improvement point which was obtained, is considered to be the "optimal". The associated set of control parameters is taken to represent the system's design which provides the "best" overall performance over the planning period. The "power" of the control module to terminate the search on time is designed to be adjustable by the user. This adjustment is achieved by choosing proper values for the parameters of the termination test.

8.3 THE MAJOR ADVANTAGES OF THE OPTIMIZATION PROCEDURE

The suggested optimization procedure provides several advantages to the user. The major advantage is an outcome of the modular structure of this procedure. Both the simulation model and the sequential search technique are treated as "black-boxes". The only assumption which was made in develping the procedure, concerned the ability of the simulation module to provide stochastically independent observations of the system's measure of performance, for each set of input variables. The search technique was assumed to provide a new set of input variables for investigation, whenever a decision concerning the "goodness" of the current set, is made. This decision is supplied to the search technique by the sampling routine. Hence, no special modification of either the simulation model or the sampling routine are necessary. The procedure can be applied to many multiple-variable, single-response simulation models and may utilize various sequential search techniques. Moreover, designing both the sampling routine and the control module to utilize a sample mean as their principle statistic, reduces the sensitivity of these modules to the distribution type of the random error inherent in the simulation model. This insensitivity provides a flexible optimization procedure which does not require the simulation model to have a normally distributed random error. Finally, the optimization procedure may be applied both as a maximization algorithm (in those cases in which the system's measure of performance is given in terms of cost over an operating period), or, as a maximization algorithm (whenever the measure of performance is given in terms of profit). In summary, the optimization procedure appears to be insensitive in regard to the:

1. Violation of the assumption of normality of the random error inherent in the simulation model.
2. Mild departures from the assumption of convexity (concavity) of the system's measure of performance function (as suggested by examples 7.1-7.6 in Chapter VII).

When the search cost are negligible and may not be taken into considerations, the optimization proces and its termination test may be easily modified to incorporate a different termination criterion. This criterion may be defined in terms of maximal number of iterations allowed, or, in terms of a fixed desired rate of improvement (or amount of improvement) below which the search should be concluded, or, in terms of locating the optimal set of control variables within pre-specified ranges.

8.4 SUGGESTED AREAS FOR FURTHER RESEARCH

This thesis focuses on a subset of the existing problems of system's optimization through simulation. The procedure developed here, may be utilized in optimizing the simulation model of a real system whenever:

1. An unbounded set of control variables exists.
2. A single measure of performance for the system under study may be defined and represented in terms of cost or profit.
3. The functional relation between the set of control variables and the single measure of performance is unimodal, convex or concave for minimization or maximization problems, respectively.
4. A "well-behaved" simulation model which provides independent realizations of the system's measure of efficiency, for any given set of input variables, exists.

Other problems of systems' optimization through simulation, which do not satisfy the above conditions, remain unsolved. These problems include cases in which a large set of input variables affects the system's response. In these cases, the efficiency of the existing search methods which were utilized in this thesis is questionable. An application of simple sequential search methods in problems which involve many input variables, is less efficient, may result in high search cost, and may not provide an "optimal" or "near optimal" solution before its termination. In these cases, it is necessary to apply some methods of variables elimination, and to apply the search method to that subset of input variables which appears to have the greatest effect on the system's performance. Problems of this nature may, therefore, require a design of a special search method, as well as a different termination criterion for the optimization process.

Other cases which were not treated in this thesis include: a) the investigation of the general multi-response system, b) the investigation of multi-modal simular response functions, c) the optimization of a simular response over a bounded set of input variables, and d) the utilization of simulation models which do not necessarily provide uncorrelated data. The first case may introduce a problem of constrained optimization, in which one of the system's responses is optimized subject to maintaining the remaining responses within pre-determined bounds. The second case may require to incorporate a search method which guarantees convergance at a "global" optimum, or a "near global" optimum, for a simular response function. The third case also introduces a problem of constrained optimization in which parts of the solution space are infeasible. The last case may require to provide the optimization procedure with the ability to use correlated realizations in estimating the system's response at each point along the search. These realizations may be utilized in some manner in estimating the correlation inherent in the model, and consequently, in obtaining an estimator of the system's response at any point.

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Appendix A

COMPUTER PROGRAMS

```
C
C        ==================================================
C        THIS PROGRAM APPLIES A PATTERN SEARCH PROCEDURE
C        TO THE SIMULATION MODEL OF A (Q;R) INVENTORY SYSTEM
C        ==================================================
C
C
C
C
C        THE FOLLOWING DIMENSION COMMANDS BELONG TO THE SIMULATION MODEL
C        ***************************************************************
      DIMENSION TOT(50)
      DIMENSION ACORD(5),ACSPC(5),ACTOT(5)
      DIMENSION TIM(102)
      DIMENSION CINV(50),CORD(50),CSPC(50),ISAL(50),ILOS(50),IDEMN(50),I
     OORDD(50),IORCC(50),IORDP(50)
      DIMENSION CTO(50),DIFFS(3),DD(3)
      DIMENSION AINV(3,3),S(3),WK(6)
      DIMENSION ACV(5)
C
C        THE FOLOWING DIMENSION COMMANDS BELONG TO THE SEARCH PROCEDURE
C        **************************************************************
      DIMENSION X(150,5),SD(2),XBEST(2),FX(150),VAR(150)
      DIMENSION N(150),R(150),LAM(150),XX(5),STD(150)
      DIMENSION CMPRV(100),FMPRV(100),VMPRV(100),NMPRV(100),SMPRV(100)
      DIMENSION XMPRV(100,5),STDPRV(100),SNET(100)
      DIMENSION XB1(2),XB2(2),XT(2)
      COMMON NR,N1,ALFA,RATE,CR,FINTL
C
C
C
C        INITIAL DATA:
C        *************
C          DEFINE: TOTAL NO. OF ITERATIONS, INITIAL PT., STEP SIZE,
C          ITERATION NO. INDICATOR;
C          -------------------------------------------------------
      M=10
      X(1,1)=5.33
      X(1,2)=10.33
      SD(1)=5.
      SD(2)=10.
      JJ=1
C          DEFINE: BASE POINT, TEMPORARY HEAD POINT.
C          ---------------------------------------
      DO 8 I=1,2
      XB1(I)=X(1,I)
      XT(I)=X(1,I)
    8 CONTINUE
      IBASE=0
C
C          DEFINE: NO. OF VARIABLES, MAX. TOTAL NO. OF PTS SEARCHED
C          -------------------------------------------------------
      NVARBL=2
      MAXNPT=150
      MAXIMP=100
C
C          DEFINE BEST PT. THUS FAR:
C          ------------------------
      XBEST(1)=X(1,1)
      XBEST(2)=X(1,2)
C
```

```
C           DEFINE: MIN. NO. OF REPLICATES FOR INITIAL PT., FOR ALL THE
C           FOLLOWING PTS., TYPE 1 ERROR, DESIRED RATE OF RETURN,
C           COST PER REPLICATION.
C           ------------------------------------------------------------
      N1=15
      NR=10
      ALFA=.80
      RATE=.15
      CR=15.0
C
C           SET: INITIAL CUM. SEARCH COST=ZERO, FOR ALL POINTS OF IMPROV
C           -------------------------------------------------------------
      DO 9 I=1,MAXIMP
      CMPRV(I)=0.
    9 CONTINUE
C
C           DEFINE ALFA & BETA FOR THE TERMINATION TEST
C           ------------------------------------------
      ALFAT=.01
      BETAT=.95
C
C           DEFINE MIN. NO. OF IMPRV. PTS. TO BE SEARCHED BEFORE APPLYIN
C           THE TERMINATION PROCEDURE.
C           -------------------------------------------------------------
      MIN=4
C
C           DEFINE THE NO. OF IMPRV. PTS. FOR ESTIMATING THE COST RATE
C           -----------------------------------------------------------
      MC=3
C
C
C         THE CONTROL MODULE:
C         *******************
C
C           SIMULATE AND OBTAIN ESTIMATED MEASURE OF EFFICIENCY FOR
C           THE INITIAL PT.
C           ------------------------------------------------------
      JPOINT=1
      CALL SIMULA (NVARBL,MAXNPT,JPOINT,XBEST,0.,0.,0,X,FX(1),VAR(1),N(1
     )),R(1),LAM(1),XX)
      DO 10 I=1,NVARBL
      XBEST(I)=X(1,I)
   10 CONTINUE
C
C            DEFINE THE CHARACTERISTICS OF THE "BEST PT. THUS FAR"
C            ----------------------------------------------------
      FXBEST=FX(1)
      VXBEST=VAR(1)
      NBEST=N(1)
C
C            START THE SEARCH:
C            INITIALIZATION OF VARIABLES USED BY THE SEARCH TECHNIQUE:
C            ---------------------------------------------------------
   20 ISUCSS=0
      L=1
   25 IBACK=1
C
C            TEST WHETHER ADDITIONAL PT. SHOULD BE INVESTIGATED
      GOTO 31
C
C            THE TEST:
C            APPLICATION OF THE TERMINATION PROCEDURE TO
C            DETERMINE WHETHER ADDITIONAL POINT SHOULD BE EVALUATED
C            ------------------------------------------------------
C               UPDATE THE LIST OF IMPROVEMENT POINTS AND SEARCH COSTS
   31 IF(JPOINT.NE.1) GOTO 501
C               JPOINT=1, THIS IS THE INITIAL POINT. (1/ST IMPRV POINT.)
```

```
      ICONT=0
      KKK=1
      III=1
      IGOOD=1
C              UPDATE DATA OF THE FIRST PT. OF IMPRUVEMENT
      CMPRV(1)=CR*FLOAT(N(1))
      FMPRV(1)=FX(1)
      VMPRV(1)=VAR(1)
      NMPRV(1)=N(1)
      SMPRV(1)=0.
      SNET(1)=R(1)
      DO 502 J=1,NVARBL
      XMPRV(1,J)=X(1,J)
  502 CONTINUE
      GOTO 550
C              THIS IS NOT THE 1ST POINT, CHECK: GOOD OR BAD?
  501 IF(LAM(JPOINT).NE.1) GOTO 503
C              THIS IS AN IMPROVEMENT POINT,UPDATE DATA
      KKK=KKK+1
      III=KKK
C              CHECK THE TYPE OF THE LAST POINT:
      IF(IGOOD.EQ.1) GOTO 701
C              THIS IS A "GOOD" PT. JUST AFTER A "BAD" PT.
      CMPRV(KKK)=CMPRV(KKK)+CR*FLOAT(N(JPOINT))
      GOTO 702
C              THIS IS A "GOOD" PT. JUST AFTER A "GOOD" PT.
  701 CMPRV(KKK)=CMPRV(KKK-1)+CR*FLOAT(N(JPOINT))
C              UPDATE IGOOD TO INDICATE AN IMPROVEMENT
  702 IGOOD=1
      FMPRV(KKK)=FX(JPOINT)
      VMPRV(KKK)=VAR(JPOINT)
      NMPRV(KKK)=N(JPOINT)
      SMPRV(KKK)=FMPRV(1)-FMPRV(KKK)
      SNET(KKK)=R(JPOINT)
      DO 504 J=1,NVARBL
      XMPRV(KKK,J)=X(JPOINT,J)
  504 CONTINUE
      GOTO 550
C
C              THIS IS A BAD POINT, CHECK THE TYPE OF THE PREVIOUS POINT
  503 IF(IGOOD.NE.1) GOTO 505
C
C              IGOOD=1, THIS IS THE 1/ST BAD POINT AFTER AN IMPROVEMENT
      IGOOD=0
      III=KKK+1
      CMPRV(III)=CMPRV(KKK)+CR*FLOAT(N(JPOINT))
      GOTO 550
C
C              THIS IS AN ADDITIONAL BAD POINT,(THE PREVIOUS PT.=BAD)
  505 CMPRV(III)=CMPRV(III)+CR*FLOAT(N(JPOINT))
      GOTO 550
C
C          STAGE 1:
C          --------
C          BASED ON KKK IMPRV. PTS. AND III PTS. OF THE COST
C          GRAPH, CHECK WHETHER A TERMINATION TEST CAN BE PERFORMED.
  550 IF(ICONT.NE.1) MPT=MIN
  511 IF(KKK.GE.MPT) GOTO 506
C
C              KKK < MPT, THE TEST CANNOT BE PERFORMED YET
      ICONT=0
C
C              CONTINUE THE SEARCH IN ORDER TO DISCOVER MORE IMPROVEMENT
C              POINTS. (TRANSFER CONTROL TO 30)
      GOTO 30
C
```

```
C          STAGE 2:
C          --------
C          THERE ARE ENOUGH IMPROVEMENT POINTS TO
C          PERFORM THE TEST:
C          -------------------------------------
C
C               CALCULATION OF SB BASED ON MOST RECENT MPT IMPRV. POINTS
C               FROM AMONG THE KKK AVAILABLE THUS FAR
  506 IK=KKK-MPT+1
      SUM=0.
      DO 507 I=IK,KKK
      JX=I-IK+1
      SUM=SUM+((FLOAT(JX)-(MPT+1.)/2.)**2)*VMPRV(I)/FLOAT(NMPRV(I))
  507 CONTINUE
      SB=(12./FLOAT(MPT*(MPT**2-1)))*SQRT(SUM)
C
C               COMPUTATION OF THE D.F. F2 BASED ON THE MOST RECENT MPT
C               POINTS OF IMPROVEMENT
      IK=KKK-MPT+1
      SUM1=SUM
      SUM2=0.
      DO 508 I=IK,KKK
      JS=I-IK+1
      US=(VMPRV(I)**2)/((FLOAT(NMPRV(I))**2)*FLOAT(NMPRV(I)-1))
      SUM2=SUM2+((FLOAT(JS)-(MPT+1.)/2.)**4)*US
  508 CONTINUE
      F2=(SUM1**2)/SUM2
C
C               CALCULATION OF THE COST SLOPE BO AND THE CRITICAL DIFF.
C               TO BE DETECTED BY THE TEST, BASED ON THE MOST RECENT MC
C               POINTS OF THE COST GRAPH .
C               CHOOSING THE MOST RECENT MC POINTS FROM THE III POINTS
C               OF THE COST GRAGH
      IX=III-MC+1
      SUMB=0.
      DO 509 I=IX,III
      IG=I-IX+1
      SUMB=SUMB+CMPRV(I)*(FLOAT(IG)-(MC+1.)/2.)
  509 CONTINUE
      BO=(12./FLOAT(MC*(MC**2-1)))*SUMB
C
C               COMPUTATION OF B, THE GROSS SAVINGS SLOPE BASED ON
C               THE MPT MOST RECENT PTS. AMONG THE KKK IMPROV. PTS.
      IS=KKK-MPT+1
      SUMBB=0.
      DO 512 I=IS,KKK
      IR=I-IS+1
      SUMBB=SUMBB+SMPRV(I)*(FLOAT(IR)-(MPT+1.)/2.)
  512 CONTINUE
      BB=(12./FLOAT(MPT*(MPT**2-1)))*SUMBB
C        <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
C     PRINT,JPOINT,'COST=  ',BO,' SAVE=  ',BB
C
C               THE DIFF. TO BE DETECTED IS THE MAX.OF EITHER COST OR
C               SAVINGS SLOPE MULTIPLIED BY THE RATE OF RETURN
      DIFF=AMAX1(BO,BB)*RATE
C
C               CALCULATION OF Z
C               FIRST COMPUTE T2=T(1-ALFAT) WITH F2 D.F.
      AA=2.*ALFAT
      CALL MDSTI (AA,F2,T2,IER)
      A1=SB*SQRT(1.-(1./(2.*F2)))*T2
      B1=SB*SQRT(1.+(T2**2)/(2.*F2))
      Z=(DIFF-A1)/B1
C
```

```
C              COMPUTATION OF Z(1-BETAT)
      BX=1.-BETAT
      CALL MDNRIS (BX,Z1,IER)
      ZB=Z1
C
C              CHECK WHETHER MPT POINTS CAN PROVIDE THE NECESSARY POWER
      IF(ZB.LE.Z) GOTO 510
C
C              Z(1-BETA) IS >= Z, MPT PTS. ARE NOT ENOUGH, THE TEST
C              CANNOT BE PERFORMED YET, INCREASE MPT.
      MPT=MPT+1
      GOTO 511
  510 ICONT=0
C
C              MPT IS ENOUGH, CONDUCT THE TEST FOR THE FIRST TIME.
C              COMPUTE THE TEST STATISTIC:
      TEXP=(BB-BO)/SB
C
C              PERFORM THE TEST
      IF(TEXP.GT.T2) GOTO 30
C
C              THE SEARCH SHOULD CNTINUE IF TEXP > T(1-ALFAT) .
C              WHEN TEXP <= T(1-ALFAT), THE SEARCH SHOULD BE TERMINATED.
C
C              TERMINATE THE SEARCH
      WRITE(6,1000)
 1000 FORMAT(1H1,10X,'SEARCH TERMINATED',/,11X,17('='))
      GOTO 999
C
C              CONTINUE THE SEARCH (END OF THE TEST PROCEDURE)
C              ---------------------------------------------
C
C         THE SEARCH TECHNIQUE:
C         *********************
C
C         DETERMINE WHICH PART OF THE SEARCH PROCEDURE SHOULD BE
C         PERRFORMED NEXT
C         -------------------------------------------------------
C
   30 GOTO(32,34,59),IBACK
C
C         DETERMINE THE NO. FOR THE NEXT PT. TO BE SEARCHED
C         ----------------------------------------------
   32 D=1.
   33 JPOINT=JPOINT+1
C
C         DETERMINE THE NEXT PT. TO BE SEARCHED
C         ---------------------------------
C         OPTIONAL OUTPUT:
C     PRINT,'                        '
C     PRINT,'  JPOINT    I      X     XBEST     L     D         SD'
C
C         DETERMINATION OF NEXT PT.
      DO 40 I=1,NVARBL
      X(JPOINT,I)=XT(I)
      IF(I.EQ.L) X(JPOINT,I)=XT(I)+SD(L)*D
C          OPTIONAL OUTPUT:
      WRITE(6,111) JPOINT,I,X(JPOINT,I),XBEST(I),L,D,SD(L)
  111 FORMAT(3X,I4,3X,I4,3X,2(F6.3,3X),I2,3X,F6.2,2X,F6.2)
   40 CONTINUE
C
C     THE CONTROL MODULE (CONT.)
C     **************************
C
C         INVESTIGATE SYSTEM PERFORMANCE AT THE NEW PT. AND DETERMINE
C         WHETHER IMPROVEMENT EXISTS, BY CALLING THE SAMPLING ROUTINE.
C         -----------------------------------------------------
          CALL SIMULA (NVARBL,MAXNPT,JPOINT,XBEST,FXBEST,VXBEST,NBEST,X,
     FFX(JPOINT),VAR(JPOINT),N(JPOINT),R(JPOINT),LAM(JPOINT),XX)
```

```
      IF(LAM(JPOINT).EQ.1) GOTO 60
      IBACK=2
C         TRANSFER CONTROL TO THE TEST PROCEDURE
      GOTO 31
C
C         THE NEW PT. IS AN IMPROVEMENT, APDATE VARIABLES.
   60 ISUCSS=1
      IBASE=1
      DO 41 I=1,NVARBL
      XT(I)=X(JPOINT,I)
   41 CONTINUE
C         CHECK IN DIRECTION OF A DIFFERENT VARIABLE
      GOTO 44
C
C         INVESTIGATE ALONG PREVIOUS COORDINATE IN OPPOSITE DIRECTION
C         -----------------------------------------------------------
   34 D=-1.
C         DETERMINE THE NO. FOR THE NEXT PT. TO BE SEARCHED
      JPOINT=JPOINT+1
C         DETERMINE THE NEXT POINT TO BE SEARCHED
C         --------------------------------------
C         OPTIONAL OUTPUT
C     PRINT,'                   '
C     PRINT,'   JPOINT    I     X     XBEST    L    D    SD'
C         DETERMINATION OF NEXT PT.
      DO 43 I=1,NVARBL
      X(JPOINT,I)=XT(I)
      IF(I.EQ.L) X(JPOINT,I)=XT(I)+SD(L)*D
C         OPTIONAL OUTPUT
      WRITE(6,111) JPOINT,I,X(JPOINT,I),XBEST(I),L,SD(L)
   43 CONTINUE
C
C     THE CONTROL MODULE (CONT.)
C     **************************
C
C        INVESTIGATE SYSTEM PERFORMANCE AT THE NEW PT. AND DETERMINE
C        WHETHER IMPROVEMENT EXISTS, BY CALLING THE SAMPLING ROUTINE.
C        ------------------------------------------------------------
         CALL SIMULA (NVARBL,MAXNPT,JPOINT,XBEST,FXBEST,VXBEST,NBEST,X,
     FFX(JPOINT),VAR(JPOINT),N(JPOINT),R(JPOINT),LAM(JPOINT),XX)
      IF(LAM(JPOINT).NE.1) GOTO 45
C
C         THE NEW PT. IS AN IMPROVEMENT, APDATE VARIABLES.
      ISUCSS=1
      IBASE=1
      DO 42 I=1,NVARBL
      XT(I)=X(JPOINT,I)
   42 CONTINUE
C
C        RECORD THE BEST PT. THUS FAR
   44 DO 50 I=1,NVARBL
      XBEST(I)=X(JPOINT,I)
   50 CONTINUE
      FXBEST=FX(JPOINT)
      VXBEST=VAR(JPOINT)
      NBEST=N(JPOINT)
C
C         DETERMINE THE NEXT COORDINATE ALONG WHICH TO SEARCH
C         ---------------------------------------------------
   45 L=L+1
      IF(L.LE.NVARBL) GOTO 25
C
C         L > NVARBL, NO FURTHER SEARCH AROUND XT IS NECESSARY
C         ----------------------------------------------------
C
C         CHECK WHETHER IMPROVEMENT WAS ACHIEVED
      IF(ISUCSS.NE.1) GOTO 46
```

```
C
C          IMPROVEMENT- DEFINE A NEW BASE POINT
   52 DO 47 I=1,NVARBL
      XB2(I)=XT(I)
      XT(I)=2*XB2(I)-XB1(I)
      XB1(I)=XB2(I)
      XBEST(I)=XB1(I)
   47 CONTINUE
      IBASE=0
C          CONTINUE A NEW ITERATION
      GOTO 20
C
C          NO IMPROVIMENT WAS OBTAINED BY SEARCHING AROUND XBEST.
C          CHECK WHEIHER THE CURRENT HEAD PT. IS ALSO THE CURRENT BASE.
   46 IF(IBASE.EQ.0) GOTO 49
C
C          THE CURRENT HEAD IS NOT A BASE PT. INVESTIGATE THE FUNCTION AT
C          THE CURRENT HEAD
      IBACK=3
C          TRANSFER CONTRROL TO THE TEST TO DETERMINE WHETHER ADDITIONAL
C          PT. SHOULD BE EVALUATED
      GOTO 31
C
C          ADDITIONAL POINT IS NEEDED
C          DETERMINE THE NO. OF THE NEXT PT. TO BE SEARCHED
C          ------------------------------------------------
   59 JPOINT=JPOINT+1
C
C          DETERMINE THE NEXT PT. TO BE SEARCHED
C          -------------------------------------
C          OPTIONAL OUTPUT
C     PRINT,'          '
C     PRINT,'  JPOINT     I      X      XBEST    L    D     SD'
C          DETERMINATION OF THE NEXT PT.
      DO 48 I=1,NVARBL
      X(JPOINT,I)=XT(I)
   48 CONTINUE
C
C
C     THE CONTROL MODULE (CONT.)
C     **************************
C
C         INVESTIGATE SYSTEM PERFORMANCE AT THE NEW PT. AND DETERMINE
C         WHETHER IMPROVEMENT EXISTS, BY CALLING THE SAMPLING ROUTINE.
C         -----------------------------------------------------------
           CALL SIMULA (NVARBL,MAXNPT,JPOINT,XBEST,FXBEST,VXBEST,NBEST,X,
     FFX(JPOINT),VAR(JPOINT),N(JPOINT),R(JPOINT),LAM(JPOINT),XX)
      IF(LAM(JPOINT).NE.1) GOTO 49
C
C          THE NEW PT. IS AN IMPROVEMENT, DEFINE A NEW BASE PT.
      GOTO 52
C
C          THE NEW PT. IS NOT AN IMPROVEMENT, END OF ITERATION
   49 JJ=JJ+1
C
C          CHECK THE NO. OF ITERATIONS PERFORMED
      IF(JJ.GT.M) GOTO 999
C          IN THIS CASE TERMINATE THE SEARCH
C
C          OTHERWISE, MORE ITERATION CAN BE PERFORMED, REDUCE THE
C          STEP SIZE
      DO 53 I=1,NVARBL
      XT(I)=XB1(I)
      SD(I)=SD(I)/2.
   53 CONTINUE
```

```
C
C          START A NEW ITERATION
C          ---------------------
      GOTO 20
C
C          END OF THE SEARCH TECHNIQUE.
C
C
C        TERMINATION POINT
C        *****************
C          TERMINATION OF THE SEARCH DUE TO:
C           1) TERMINATION BY THE STATISTICAL TEST.
C           2) NO. OF ITERATIONS REACHED THE UPPER LIMIT
C
C          PRINT RESULTS
C          -------------
  999 WRITE(6,400)
  400 FORMAT(1H1,35X,'LIST OF POINTS SEARCHED',/,36X,23('='))
      WRITE(6,200)
  200 FORMAT(1X,/,1X,'POINT NO.',5X,'X(N)',12X,'F(X(N))',5X,'STD(F)',5X,
     A'SAMPLE',2X,'NET SAVINGS',2X,'GOOD/BAD 1/-1',/,90('='))
      DO 110 I=1,JPOINT
      STD(I)=SQRT(VAR(I))
      WRITE(6,300) I,(X(I,J),J=1,NVARBL),FX(I),STD(I),N(I),R(I),LAM(I)
300   FORMAT(4X,I3,2X,2(F7.2,1X),2X,F10.2,3X,F10.2,4X,I4,4X,F10.2,4X,I3)
  110 CONTINUE
C
C         OPTIONAL OUTPUT
C     WRITE(6,305) MPT,ICONT,KKK,III,SB,F2,DIFF,BO,BB,ZB,Z
C 305 FORMAT(1H1,//,1X,'MPT,ICONT,KKK,III,SB,F2,DIFF,BO,BB,ZB,Z',/,3X,4I
C    D7,7(F8.3,1X))
      WRITE(6,305) N1,ALFAT,NR,BETAT,RATE,MIN,ALFA,CR,MPT,F2,SB,BO,DIFF,
     ZBB,ZB,Z,MC
  305 FORMAT(1H1,/,3X,65('='),/,30X,'DATA SUMMARY',/,3X,65('='),/,3X,'MI
     NN. INITIAL SAMPLE=',I3,14X,'ALPHAT (TEST)=',F5.3,/,3X,'MIN. ADDITI
     OONAL SAMPLE=',I3,11X,'BETAT (TEST)=',F5.3,/,3X,'DESIRED RATE OF RE
     TTURN=',F5.3,9X,'MIN NO. OF IMPRV. PTS.=',I3,/,3X,'ALPHA (CONFID. I
     NNTERVAL)=',F5.3,7X,'COST PER REPLICATE=',F5.1,/,3X,('=========='),
     N('================'),39('='),/,25X,'TEST PROCEDURE SUMMARY',/,3X,6
     55('='),/,3X,'NO.OF PTS. IN REGRESSION=',I3,8X,'DEGREES OF FREEDOM=
     D',F6.2,/,3X,'EST. SAVINGS RATE STD.DEV.=',F6.1,3X,'BO, ACTUAL COST
     SS RATE=',F6.1,/,3X,'DIFF. BEING DETECTED=',F6.1,9X,'BB, EST. SAVIN
     GGS RATE=',F6.1,/,3X,'DESIRED Z(BETAT)=',F6.3,13X,'ACTUAL Z*=',F6.3
     ),/,3X,'NO. PTS. FOR COST RATE=',I2)  .
      WRITE(6,308)
  308 FORMAT(3X,65('='),/,30X,'CASE SIMULATED',/,3X,65('='))
      WRITE(6,3088)
 3088 FORMAT(3X,'F(X1;X2)=COST FUNCTION OF A SIMULATED (Q;R) INVENTORY S
     YYSTEM',/,3X,'CI=5.0,  CO=40.0,  CL=20.0,    MONTHLY DEMAND=P(200)'
     S,/,3X,'SEARCH TECHNIQUE= PATTERN SEARCH                          '
     C,/,3X,'DIFF=(MAX(BB;BO))*RATE',/,3X,'MODEL STD. DEV.- UNKNOWN',/,3
     CX,'TYPE OF ERROR- UNKNOWN')
C     +++++++++++++ CASE SIMULATED ++++++++++++
C     PRINT,'  DIFF=(MAX(BB;BO))*RATE'
C     PRINT,'  F(X1;X2)=(|X1|**1.5+|X2|**1.5+5)*100'
C     PRINT,'  MODEL STD. DEV.=F(X1;X2)*.15'
C     PRINT,'  TYPE OF ERROR=NORMAL'
      WRITE(6,306)
  306 FORMAT(1H1,19X,'TERMINATION PROCEDURE RESULTS:',/,20X,30('='),/)
      WRITE(6,401)
  401 FORMAT(3X,'POINT',4X,'F(X)',9X,'STD(F(X))',4X,'CUM. COST',3X,'GROS
     NS SAVINGS',2X,'NET SAVINGS',/,2X,75('='))
      DO 307 I=1,KKK
      STDPRV(I)=SQRT(VMPRV(I))
      WRITE(6,402) I,FMPRV(I),STDPRV(I),CMPRV(I),SMPRV(I),SNET(I)
  402 FORMAT(4X,I3,2X,3(F10.2,3X),2X,F10.2,5X,F10.2)
  307 CONTINUE
      WRITE(6,403)
  403 FORMAT(1H1,'END OF PROGRAM')
```

```
      DO 9999 I=1,MAXIMP
C     PRINT,I,'COST    ',CMPRV(I)
 9999 CONTINUE
C
      STOP
      END
C



C
C
C
C
C
C
C
C
C
C
      SUBROUTINE SIMULA (NVARBL,MAXNPT,JPT,XBEST,FXBEST,VXBEST,NBEST,X,F
     F,V,N,R,LAM,XX)
      DIMENSION XBEST(NVARBL),X(MAXNPT,NVARBL),XX(NVARBL)
      COMMON NR,N1,ALFA,RATE,CR,FINTL
C
C               THE SAMPLING ROUTINE
C               ********************
C
C         CHECK WHETHER SIMULA IS CALLED TO EVALUATE INITIAL POINT
      IF(JPT.NE.1) GOTO 30
C
C         THIS CALL EVALUATES THE INITIAL POINT
      N=N1
      SUM=0.
      SSQR=0.
C
C         START SIMULATE REPLICATES
      DO 10 I=1,NVARBL
      XX(I)=X(1,I)
   10 CONTINUE
      DO 20 I=1,N1
      CALL SMODEL (XX,NVARBL,FXX)
      SUM=SUM+FXX
      SSQR=SSQR+FXX**2
   20 CONTINUE
      F=SUM/FLOAT(N1)
      FINTL=F
      V=(SSQR-FLOAT(N1)*(F**2))/FLOAT(N1-1)
      NTOTAL=N1
      R=-CR*FLOAT(NTOTAL)
      LAM=1
      RETURN
C
C         THIS CALL EVALUATES A NEW PT. (NOT INITIAL PT.)
   30 N=NR
      SUM=0.
      SSQR=0.
C
C         START SIMULATE REPLICATES
      DO 40 I=1,NVARBL
      XX(I)=X(JPT,I)
   40 CONTINUE
      DO 50 I=1,NR
      CALL SMODEL (XX,NVARBL,FXX)
      SUM=SUM+FXX
      SSQR=SSQR+FXX**2
   50 CONTINUE
```

```
      F=SUM/FLOAT(NR)
      V=(SSQR-FLOAT(NR)*(F**2))/FLOAT(NR-1)
      NTOTAL=NTOTAL+NR
C
C        USING CONFIDENCE INTERVAL AS A STOPPING RULE.
C        ------------------------------------------
C
      SN=FLOAT(N)
      SNBST=FLOAT(NBEST)
C
C              EVALUATE D.F. FOR THE "T" R.V.
   55 A=(V/SN+VXBEST/SNBST)**2
      B=((V/SN)**2)/(SN+1.)
      C=((VXBEST/SNBST)**2)/(SNBST+1.)
      DF=A/(B+C)-2.
C
C              COMPUTE T(1-ALFA/2) WITH DF DEGREES OF FREEDOM
      CALL MDSTI (ALFA,DF,T,IER)
C
C              COMPUTE CONFIDENCE INTERVAL LIMITS FOR THE ACTUAL RETURN
      ALLMT=FXBEST-F-SQRT(V/SN+VXBEST/SNBST)*T
      ULMT=FXBEST-F+SQRT(V/SN+VXBEST/SNBST)*T
C
C              COMPUTE THE DESIRED RETURN FROM THE SEARCH AT THIS PT.
      DRTN=(1+RATE)*CR*SN
C
C              COMPARE LIMITS WITH ACTUAL RETURN
      IF(ALLMT.LE.DRTN) GOTO 60
C
C              THE SEARCH MOVES IN A PROMISING DIRECTION.
      LAM=1
      R=FINTL-F-CR*ILOAT(NTOTAL)
      GOTO 81
   60 IF(ULMT.GE.DRIN) GOTO 70
C              THE SEARCH MOVES IN AN UNDESIRED DIRECTION
      LAM=-1
      R=FINTL-FXBESI-CR*FLOAT(NTOTAL)
      GOTO 81
C
C              THIS IS A PT. OF UNCERTAINTY, SINCE ULMT < DRTN < ALLMT
C              INCREASE THE SAMPLE SIZE
   70 N=N+1
      NTOTAL=NTOTAL+1
      CALL SMODEL (XX,NVARBL,FXX)
      SUM=SUM+FXX
      SSQR=SSQR+FXX**2
      SN=FLOAT(N)
      F=SUM/SN
      V=(SSQR-SN*(F**2))/(SN-1.)
      IF(N.LT.30) GOTO 55
C              A MAXIMUM OF 30 REPLICATES HAS BEEN REACHED
C              APPROXIMATE A DESICION
      NMAX=(ULMT/(CR*(1.+RATE))+1.)
      DO 75 I=N,NMAX,5
      RI=FLOAT(I)
      SL=FXBEST-F-SQRT(V/RI+VXBEST/SNBST)*T
      SU=FXBEST-F+SQRT(V/RI+VXBEST/SNBST)*T
      DR=RI*CR*(1.+RATE)
      IF(SL.GT.DR)GOTO 76
      IF(DR.GT.SU)GOTO 77
   75 CONTINUE
C              DRTN < ALLMT, AND THE POINT MAY BE "GOOD"
   76 LAM=1
      R=FINTL-F-CR*FLOAT(NTOTAL)
      GOTO 78
```

```
C               DRTN > ULMT, AND THE POINT MAY BE "BAD"
   77 LAM=-1
      R=FINTL-FXBEST-CR*FLOAT(NTOTAL)
C         OPTIONAL OUTPUT
   78 CONTINUE
C  78 WRITE(6,801) RI,SL,SU,DR
C 801 FORMAT(1X,'APRX DECISION',1X,'NN=',F5.0,1X,'SL=',F8.2,1X,'SU=',F8.
C    D2,1X,'DR=',F8.2,1X,18('<'))
C               END OF THE APPROXIMATION SECTION
C
C
C         OPTIONAL OUTPUT
   81 CONTINUE
C  81 WRITE(6,800) JPT,DF,T,ALLMT,ULMT,DRTN
C 800 FORMAT(2X,'POINT',3X,'D.F.',3X,'T(1-A/2)',4X,'ALLMT',4X,'ULMT',4X,
C    F'DRTN',/,4X,I3,1X,F6.2,3X,F8.2,3(1X,F8.2))
      RETURN
      END
C
C
C
C
C
C
C
C
C
      SUBROUTINE SMODEL (XX,NVARBL,FXX)
      DIMENSION XX(NVARBL)
C
C                THE SIMULATION MODEL
C                ********************
C
      DIMENSION TOT(50)
      DIMENSION ACORD(5),ACSPC(5),ACTOT(5)
      DIMENSION TIM(102)
      DIMENSION CINV(50),CORD(50),CSPC(50),ISAL(50),ILOS(50),IDEMN(50),I
     OORDD(50),IORCC(50),IORDP(50)
      DIMENSION CTO(50),DIFFS(3),DD(3)
      DIMENSION AINV(3,3),S(3),WK(6)
      DIMENSION ACV(5)
C ************************************
      IR=XX(1)
      IQ=XX(2)
C ************************************
      IHAND=IR+3
      DELLT=1000.
      H=3.
      DEM=200.
      NOBS=20
      MNX=50
      KMONTH=10
      IU=12
      MONTH=KMONTH
      LENGTH=61*12
      LEADTM=6*12
      IOUT=0
      IORD=0
      CI=5.
      CO=40.
      CL=20.
      TIME=0.
      DO 33 I=1,102
      TIM(I)=999999999.
   33 CONTINUE
```

```
      TLAST=0.
      CALL DEMAN(TNEXT)
      TIM(1)=TNEXT
      AVDEM=0.
      AVLOST=0.
      AVSALE=0.
      AVORDP=0.
C  START SIMULATING AN OBSERVATION.
C     WRITE(6,112)
C 112 FORMAT(//,3X,'MONTH      T. COST    INV. COST ORD. COST LOST COST N
C    HOF SALES ORDS.OUT ORDS. RECV ORD. PLACED N.LOST  T.DEM',/,112('=')
C    J,/)
      KG=1
      KMONTH=MONTH
      DO 100 IT=KG,KMONTH
      XSI=0.
      IORRC=0
      IDEM=0
      ITORD=0
      ISALE=0
      ILOST=0
      CSTORD=0.
      CSTSPC=0.
C  SEARCH FOR THE TIME AND TYPE OF THE NEXT EVENT
   20 TIME=TIM(1)
      MIN=1
      TIMEK=TIM(1)-(IT-1)*LENGTH
C  IF DEMAND AND ORDER ARRIVAL OCCURS AT THE SAME TIME, ARRIV. IS TREATED FIRST
      IF(TIME.LT.TIM(2))GOTO10
      TIMEK=TIM(2)-(IT-1)*LENGTH
      TIME=TIM(2)
      MIN=2
   10 IF(TIMEK.LT.LENGTH)GOTO11
      TIME=IT*LENGTH
      MIN=3
      TIMEK=0.
   11 GOTO(30,40,80),MIN
C  THE NEXT EVENT IS "DEMAND";
   30 IDEM=IDEM+1
      IF(IHAND.LE.0)GOTO60
C  THE ON-HAND INVENTORY IS G.T. ZERO, A "SALE" IS GOING TO BE DONE;
C  RECORDING THE NO9 OF SALES:
      ISALE=ISALE+1
C  COMPUTE THE CUM. (INVENTORY ON HAND)*(TIME INVENTORY G.T. 0)
      SI=(TIME-TLAST)*IHAND
      XSI=XSI+SI
C  UP DATING THE INVENTORY ON HAND
      IHAND=IHAND-1
C  RECORDING THE "CURRENT" LAST MOMENT ON-HAND INV. WAS POSITIVE(G.T. 0)
      TLAST=TIME
C  CHECK THE INVENTORY LEVEL ( ON HAND + OUTSTANDING ORDERS ):
      ITOTAL=IHAND+IOUT
      IF(ITOTAL.NE.IR)GOTO 50
C  WHEN TOTAL INV. EQ. IR , AN ORDER IS PLACED ( IQ ITEMS ARE ORDERED)
C  COMPUTE TOTAL INVENTORY ON OUTSTANDING ORDERS
      IOUT=IOUT+IQ
C COMPUTE TOTAL ORDERS CURRENTLY OUTSTANDING:
      IORD=IORD+1
C  COMPUTE TOTAL NO. OF ORDERS PLACED ON THIS MONTH:
      ITORD=ITORD+1
C  COMPUTE CUM. ORDIRING COST
      CSTORD=CSTORD+CO
C RECORD THE TIME FOR THE NEW ORDER TO ARRIVE;
      TIM(IORD+1)=TIME+LEADTM
      GOTO 50
```

```
C  THE ON-HAND INVENTORY IS ZERO, A "LOST SALE" IS DONE:
C  LOST SALE= DEMAND DURING STOCKOUT CONDITION (ON HAND INVENTORY=0),IS MET
C  BY SPECIAL ORDER, THE ITEM IS NOT INCLUDED IN ANY OUTSTANDING ORDER. THERE IS
C  NO NEED TO UPDATE THE INVENTORY ON OUTSTANDING ORDERS,(AS OPPOSED TO "BACK
C  ORDERS").
   60 ILOST=ILOST+1
C  SPECIAL ORDER IS MADE AND CUM LOST SALES COST IS COMPUTED:
      CSTSPC=CSTSPC+CL
C  SET THE TIME FOR THE NEXT DEMAND
   50 CALL DEMAN(TNEXT)
      TIM(1)=TNEXT+TIME
      GOTO 20
C THE NEXT EVENT IS "ORDER ARRIVAL";
C  THE NO OF ORDERS ARRIVED  SO FAR ON THIS MONTH IS COMPUTED:
   40 IORRC=IORRC+1
      IF(IHAND.LE.0)GOTO 70
C  THE INVENTORY ON HAND IS POSITIVE WHEN THE ORDER ARRIVED-
C  COMPUTE THE CUM. (INVENTORY ON HAND)*(TIME INVENTORY G.T. 0)
      SI=(TIME-TLAST)*IHAND
      XSI=XSI+SI
C  UPDATING THE INVENTORY ON HAND
   70 IHAND=IHAND+IQ
C  RECORDING THE "CURRENT" LAST MOMENT ON-HAND INV. WAS POSITIVE(G.T. 0)
      TLAST=TIME
C  UPDATING THE TOTAL INVENTORY ON OUTSTANDING ORDERS
      IOUT=IOUT-IQ
C  UPDATING ARRIVAL TIMES FOR ALL OUTSTANDING ORDERS LEFT.
      IF(IORD.LT.1)GOTO 20
      KS=IORD+1
      DO 81 J=2,KS
      TIM(J)=TIM(J+1)
   81 CONTINUE
C  UPDATING TOTAL OF OUTSTANDING ORDERS.
      IORD=IORD-1
      GOTO 20
C  THE NEXT EVENT IS END OF MONTH.
   80 IF(IHAND.LE.0)GOTO 90
C  INVENTORY ON HAND IS POSITIVE
C  COMPUTE THE CUM. (INVENTORY ON HAND)*(TIME INVENTORY G.T. 0)
      SI=(TIME-TLAST)*IHAND
      XSI=XSI+SI
      TLAST=TIME
C COMPUTE AND PRINT MONTHLY RESULTS:
   90 AVGINV=XSI/LENGTH
      CSTINV=AVGINV*CI
      TOTCST=CSTINV+CSTORD+CSTSPC
C     WRITE(6,111)IT,TOTCST,CSTINV,CSTORD,CSTSPC,ISALE,IORD,IORRC,ITORD,
C    IILOST,IDEM
C 111 FORMAT(3X,I3,3X,4F10.2,4(6X,I4),8X,I4,4X,I4)
      TOT(IT)=TOTCST
  100 CONTINUE
C         THE ESTIMATOR IS BASED ON THE AVERAGE OF THE LAST 4 MONTHS
      AVTOT=0.
      NZ=KMONTH-3
      DO 2222 J=NZ,KMONTH
      AVTOT=AVTOT+TOT(J)/4.
 2222 CONTINUE
C         THE PRESENT VALUE OF COSTS OVER A FIVE YRS. PLANNING PERIOD
      FXX=AVTOT*53.275
C         THE P.V. OF TOTAL COST IS RETURNED
      RETURN
      END
```

```
      SUBROUTINE DEMAN(TNEXT)
      DOUBLE PRECISION DSEED
      DATA DSEED/456531.D0/
C     ***********************************
C     GENERATE A UNIFORM R.V. IN (0;1)
      CALL GGUBS(DSEED,1,R)
C     ***********************************
      TNEXT=(-1.)*(ALOG(R))/(.2732240)
      RETURN
      END
```

```
C          ====================================================
C          THIS PROGRAM APPLIES A ONE-AT-A-TIME SEARCH PROCEDURE
C          TO THE SIMULATION MODEL OF A (Q;R) INVENTORY SYSTEM
C          ====================================================
C
C          THE FOLLOWING DIMENSION COMMANDS BELONG TO THE SIMULATION MODEL
C          ***************************************************************
       DIMENSION TOT(50)
       DIMENSION ACORD(5),ACSPC(5),ACTOT(5)
       DIMENSION TIM(102)
       DIMENSION CINV(50),CORD(50),CSPC(50),ISAL(50),ILOS(50),IDEMN(50),I
      OORDD(50),IORCC(50),IORDP(50)
       DIMENSION CTO(50),DIFFS(3),DD(3)
       DIMENSION AINV(3,3),S(3),WK(6)
       DIMENSION ACV(5)
C
C          THE FOLOWING DIMENSION COMMANDS BELONG TO THE SEARCH PROCEDURE
C          **************************************************************
       DIMENSION X(150,2),SD(2),XBEST(2),IX(100),VAR(100)
       DIMENSION N(100),R(100),LAM(100),XX(2),STD(100)
       DIMENSION CMPRV(100),FMPRV(100),VMPRV(100),NMPRV(100),SMPRV(100)
       DIMENSION XMPRV(100,5),STDPRV(100),SNET(100)
       COMMON NR,N1,ALFA,RATE,CR,FINTL
C
C
C
C          INITIAL DATA:
C          *************
C            DEFINE: TOTAL NO. OF ITERATIONS, INITIAL PT., STEP SIZE,
C            ITERATION NO. INDICATOR;
C            -------------------------------------------------------
       M=10
       X(1,1)=5.33
       X(1,2)=10.33
       SD(1)=5.
       SD(2)=10.
       JJ=1
C
C            DEFINE: NO. OF VARIABLES, MAX. TOTAL NO. OF PTS SEARCHED
C            -------------------------------------------------------
       NVARBL=2
       MAXNPT=150
       MAXIMP=100
C
C            DEFINE BEST PT. THUS FAR:
C            ------------------------
       XBEST(1)=X(1,1)
       XBEST(2)=X(1,2)
C
C            DEFINE: MIN. NO. OF REPLICATES FOR INITIAL PT., FOR ALL THE
C            FOLLOWING PTS., TYPE 1 ERROR, DESIRED RATE OF RETURN,
C            COST PER REPLICATION.
C            -----------------------------------------------------------
       N1=15
       NR=10
       ALFA=.80
       RATE=.15
       CR=15.
C
```

```
C          SET: INITIAL CUM. SEARCH COST=ZERO, FOR ALL POINTS OF IMPROV
C          -------------------------------------------------------------
      DO 9 I=1,MAXIMP
      CMPRV(I)=0.
    9 CONTINUE
C
C          DEFINE ALFA & BETA FOR THE TERMINATION TEST
C          -------------------------------------------
      ALFAT=.01
      BETAT=.95
C
C          DEFINE MIN. NO. OF IMPRV. PTS. TO BE SEARCHED BEFORE APPLYIN
C          THE TERMINATION PROCEDURE.
C          -------------------------------------------------------------
      MIN=3
C
C          DEFINE THE NO. OF IMPRV. PTS. FOR ESTIMATING THE COST RATE
C          -----------------------------------------------------------
      MC=3
C
C
C        THE CONTROL MODULE:
C        *******************
C
C          SIMULATE AND OBTAIN ESTIMATED MEASURE OF EFFICIENCY FOR
C          THE INITIAL PT.
C          -------------------------------------------------------
      JPOINT=1
      CALL SIMULA (NVARBL,MAXNPT,JPOINT,XBEST,0.,0.,0,X,FX(1),VAR(1),N(1
     )),R(1),LAM(1),XX)
      DO 10 I=1,NVARBL
      XBEST(I)=X(1,I)
   10 CONTINUE
C
C           DEFINE THE CHARACTERISTICS OF THE "BEST PT. THUS FAR"
C           -------------------------------------------------------

      FXBEST=FX(1)
      VXBEST=VAR(1)
      NBEST=N(1)
C
C           START THE SEARCH:
C           INITIALIZATION OF VARIABLES USED BY THE SEARCH TECHNIQUE:
C           ---------------------------------------------------------
   20 L=1
      D=1.
      IDELTA=0
      NN=0
C
C           APPLICATION OF THE TERMINATION PROCEDURE TO
C           DETERMINE WHETHER ADDITIONAL POINT SHOULD BE EVALUATED
C           ------------------------------------------------------
C              UPDATE THE LIST OF IMPROVEMENT PCINTS AND SEARCH COSTS
   31 IF(JPOINT.NE.1) GOTO 501
C              JPOINT=1, THIS IS THE INITIAL POINT. (1/ST IMPRV POINT.)
      ICONT=0
      KKK=1
      III=1
      IGOOD=1
C              UPDATE DATA OF THE FIRST PT. OF IMPRUVEMENT
      CMPRV(1)=CR*FLOAT(N(1))
      FMPRV(1)=FX(1)
      VMPRV(1)=VAR(1)
      NMPRV(1)=N(1)
      SMPRV(1)=0.
      SNET(1)=R(1)
      DO 502 J=1,NVARBL
      XMPRV(1,J)=X(1,J)
  502 CONTINUE
      GOTO 550
```

```
C             THIS IS NOT THE 1ST POINT, CHECK: GOOD OR BAD?
  501 IF(LAM(JPOINT).NE.1) GOTO 503
C             THIS IS AN IMPROVEMENT POINT,UPDATE DATA
      KKK=KKK+1
      III=KKK
C             CHECK THE TYPE OF THE LAST POINT:
      IF(IGOOD.EQ.1) GOTO 701
C             THIS IS A "GOOD" PT. JUST AFTER A "BAD" PT.
      CMPRV(KKK)=CMPRV(KKK)+CR*FLOAT(N(JPOINT))
      GOTO 702
C             THIS IS A "GOOD" PT. JUST AFTER A "GOOD" PT.
  701 CMPRV(KKK)=CMPRV(KKK-1)+CR*FLOAT(N(JPOINT))
C             UPDATE IGOOD TO INDICATE AN IMPROVEMENT
  702 IGOOD=1
      FMPRV(KKK)=FX(JPOINT)
      VMPRV(KKK)=VAR(JPOINT)
      NMPRV(KKK)=N(JPOINT)
      SMPRV(KKK)=FMPRV(1)-FMPRV(KKK)
      SNET(KKK)=R(JPOINT)
      DO 504 J=1,NVARBL
      XMPRV(KKK,J)=X(JPOINT,J)
  504 CONTINUE
      GOTO 550
C
C             THIS IS A BAD POINT, CHECK THE TYPE OF THE PREVIOUS POINT
  503 IF(IGOOD.NE.1) GOTO 505
C
C             IGOOD=1, THIS IS THE 1/ST BAD POINT AFTER AN IMPROVEMENT
      IGOOD=0
      III=KKK+1
      CMPRV(III)=CMPRV(KKK)+CR*FLOAT(N(JPOINT))
      GOTO 550
C
C             THIS IS AN ADDITIONAL BAD POINT,(THE PREVIOUS PT.=BAD)
  505 CMPRV(III)=CMPRV(III)+CR*FLOAT(N(JPOINT))
      GOTO 550
C
C          STAGE 1:
C          --------
C          BASED ON KKK IMPRV. PTS. AND III PTS. OF THE COST
C          GRAPH, CHECK WHETHER A TERMINATION TEST CAN BE PERFORMED.
  550 IF(ICONT.NE.1) MPT=MIN
  511 IF(KKK.GE.MPT) GOTO 506
C
C             KKK < MPT, THE TEST CANNOT BE PERFORMED YET
      ICONT=0
C
C             CONTINUE THE SEARCH IN ORDER TO DISCOVER MORE IMPROVEMENT
C             POINTS. (TRANSFER CONTROL TO 30)
      GOTO 30
C
C         STAGE 2:
C         --------
C         THERE ARE ENOUGH IMPROVEMENT POINTS TO
C         PERFORM THE TEST:
C         ------------------------------------
C
C             CALCULATION OF SB BASED ON MOST RECENT MPT IMPRV. POINTS
C             FROM AMONG THE KKK AVAILABLE THUS FAR
  506 IK=KKK-MPT+1
      SUM=0.
      DO 507 I=IK,KKK
      JX=I-IK+1
      SUM=SUM+((FLOAT(JX)-(MPT+1.)/2.)**2)*VMPRV(I)/FLOAT(NMPRV(I))
  507 CONTINUE
```

```
      SB=(12./FLOAT(MPT*(MPT**2-1)))*SQRT(SUM)
C
C               COMPUTATION OF THE D.F. F2 BASED ON THE MOST RECENT MPT
C               POINTS OF IMPROVEMENT
      IK=KKK-MPT+1
      SUM1=SUM
      SUM2=0.
      DO 508 I=IK,KKK
      JS=I-IK+1
      US=(VMPRV(I)**2)/((FLOAT(NMPRV(I))**2)*FLOAT(NMPRV(I)-1))
      SUM2=SUM2+((FLOAT(JS)-(MPT+1.)/2.)**4)*US
  508 CONTINUE
      F2=(SUM1**2)/SUM2
C
C               CALCULATION OF THE COST SLOPE BO AND THE CRITICAL DIFF.
C               TO BE DETECTED BY THE TEST, BASED ON THE MOST RECENT MC
C               POINTS OF THE COST GRAPH .
C               CHOOSING THE MOST RECENT MC POINTS FROM THE III POINTS
C               OF THE COST GRAGH
      IX=III-MC+1
      SUMB=0.
      DO 509 I=IX,III
      IG=I-IX+1
      SUMB=SUMB+CMPRV(I)*(FLOAT(IG)-(MC+1.)/2.)
  509 CONTINUE
      BO=(12./FLOAT(MC*(MC**2-1)))*SUMB
C
C               COMPUTATION OF B, THE GROSS SAVINGS SLOPE BASED ON
C               THE MPT MOST RECENT PTS. AMONG THE KKK IMPROV. PTS.
      IS=KKK-MPT+1
      SUMBB=0.
      DO 512 I=IS,KKK
      IR=I-IS+1
      SUMBB=SUMBB+SMPRV(I)*(FLOAT(IR)-(MPT+1.)/2.)
  512 CONTINUE
      BB=(12./FLOAT(MPT*(MPT**2-1)))*SUMBB
C
C     PRINT,JPOINT,'COST=  ',BO,' SAVE=  ',BB
C
C               THE DIFF. TO BE DETECTED IS THE MAX.OF EITHER COST OR
C               SAVINGS SLOPE MULTIPLIED BY THE RATE OF RETURN
      DIFF=AMAX1(BO,BB)*RATE
C
C               CALCULATION OF Z
C               FIRST COMPUTE T2=T(1-ALFA1) WITH F2 D.F.
      AA=2.*ALFAT
      CALL MDSTI (AA,F2,T2,IER)
      A1=SB*SQRT(1.-(1./(2.*F2)))*T2
      B1=SB*SQRT(1.+(T2**2)/(2.*F2))
      Z=(DIFF-A1)/B1
C
C               COMPUTATION OF Z(1-BETAT)
      BX=1.-BETAT
      CALL MDNRIS (BX,Z1,IER)
      ZB=Z1
C
C               CHECK WHETHER MPT POINTS CAN PROVIDE THE NECESSARY POWER
      IF(ZB.LE.Z) GOTO 510
C
C               Z(1-BETA) IS >= Z, MPT PTS. ARE NOT ENOUGH, THE TEST
C               CANNOT BE PERFORMED YET, INCREASE MPT.
      MPT=MPT+1
      GOTO 511
  510 ICONT=0
C
C               MPT IS ENOUGH, CONDUCT THE TEST FOR THE FIRST TIME.
C               COMPUTE THE TEST STATISTIC:
```

```
      TEXP=(BB-BO)/SB
C
C               PERFORM THE TEST
      IF(TEXP.GT.T2) GOTO 30
C
C               THE SEARCH SHOULD CNTINUE IF TEXP > T(1-ALFAT) .
C               WHEN TEXP <= T(1-ALFAT), THE SEARCH SHOULD BE TERMINATED.
C
C               TERMINATE THE SEARCH
      WRITE(6,1000)
 1000 FORMAT(1H1,10X,'SEARCH TERMINATED',/,11X,17('='))
      GOTO 999
C
C               CONTINUE THE SEARCH (END OF THE TEST PROCEDURE)
C               -----------------------------------------------
C
C         THE SEARCH TECHNIQUE:
C         *********************
C
C         DETERMINE THE NO. FOR THE NEXT PT. TO BE SEARCHED
C         -------------------------------------------------
   30 JPOINT=JPOINT+1
C
C         DETERMINE THE NEXT PT. TO BE SEARCHED
C         -------------------------------------
C         OPTIONAL OUTPUT:
C     PRINT'                                        '
C     PRINT'  JPOINT     I       X      XBEST      L      D          SD'
C
C         DETERMINATION OF NEXT PT.
      DO 40 I=1,NVARBL
      X(JPOINT,I)=XBEST(I)
      IF(I.EQ.L) X(JPOINT,I)=XBEST(I)+D*SD(L)
C          OPTIONAL OUTPUT:
C     WRITE(6,111) JPOINT,I,X(JPOINT,I),XBEST(I),L,D,SD(L)
C 111 FORMAT(3X,I4,3X,I4,3X,2(F6.3,3X),I2,3X,F6.2,2X,F6.2)
   40 CONTINUE
C
C     THE CONTROL MODULE (CONT.)
C     **************************
C
C         INVESTIGATE SYSTEM PERFORMANCE AT THE NEW PT. AND DETERMINE
C         WHETHER IMPROVEMENT EXISTS, BY CALLING THE SAMPLING ROUTINE.
C         ------------------------------------------------------------
          CALL SIMULA (NVARBL,MAXNPT,JPOINT,XBEST,FXBEST,VXBEST,NBEST,X,
     FFX(JPOINT),VAR(JPOINT),N(JPOINT),R(JPOINT),LAM(JPOINT),XX)
      IF(LAM(JPOINT).NE.1) GOTO 60
C
C          THE NEW PT. IS AN IMPROVEMENT, APDATE VARIABLES.
      DO 50 I=1,NVARBL
      XBEST(I)=X(JPOINT,I)
   50 CONTINUE
      FXBEST=FX(JPOINT)
      VXBEST=VAR(JPOINT)
      NBEST=N(JPOINT)
C
C          THE SEARCH TECHNIQUE
C          ********************
      NN=0
      IDELTA=1
C
C          THE CONTROL MODULE
C          ******************
C           TRANSFER CONTROL TO APPLY THE TEST AGAIN
      GOTO 31
```

```
C
C         THE NEW PT. IS NOT AN IMPROVEMENT
C         TRANSFER TO THE SEARCH TECHNIQUE.
C
C        THE SEARCH TECHNIQUE
C        ********************
   60 NN=NN+IDELTA+1
      IF(NN.LT.(2*NVARBL)) GOTO 80
C              CHECK WHETHER A PREDETERMINED MAX. NO. OF ITERATIONS IS
C              REACHED. OTHERWISE, START WITH A NEW ITERATION:
      IF(JJ.GE.M) GOTO 999
C              START A NEW ITERATION
      JJ=JJ+1
      DO 70 I=1,NVARBL
      SD(I)=SD(I)/2.
   70 CONTINUE
C              CONTINUE SEARCHING WITH A REDUCED STEP SIZE
      GOTO 20
C              DETERMINE THE DIRECTION OF THE NEXT STEP
   80 IF(D.NE.1) GOTO 90
      IF(IDELTA.NE.0) GOTO 100
      D=-1.
      GOTO 31
   90 D=1.
C
C              SEARCH IN DIRECTION OF THE NEXT VARIABLE
  100 L=L+1
      IDELTA=0
      IF(L.LE.NVARBL) GOTO 31
      L=1
      GOTO 31
C         NEW POINT FOR INVESTIGATION IS TRANSFERED TO THE CONTROL
C         MODULE (LABEL 31).
C
C        END OF THE SEARCH TECHNIQUE.
C
C
C       TERMINATION POINT
C       *****************
C         TERMINATION OF THE SEARCH DUE TO:
C          1) TERMINATION BY THE STATISTICAL TEST.
C          2) NO. OF ITERATIONS REACHED THE UPPER LIMIT
C
C         PRINT RESULTS
C         -------------
  999 WRITE(6,400)
  400 FORMAT(1H1,35X,'LIST OF POINTS SEARCHED',/,36X,23('='))
      WRITE(6,200)
  200 FORMAT(1X,/,1X,'POINT NO.',5X,'X(N)',12X,'F(X(N))',5X,'STD(F)',5X,
     A'SAMPLE',2X,'NET SAVINGS',2X,'GOOD/BAD 1/-1',/,90('='))
      DO 110 I=1,JPOINT
      STD(I)=SQRT(VAR(I))
      WRITE(6,300) I,(X(I,J),J=1,NVARBL),FX(I),STD(I),N(I),R(I),LAM(I)
300   FORMAT(4X,I3,2X,2(F7.4,1X),2X,F10.2,3X,F10.2,4X,I4,4X,F10.2,4X,I3)
  110 CONTINUE
C
C        OPTIONAL OUTPUT
C     WRITE(6,305) MPT,ICONT,KKK,III,SB,F2,DIFF,BO,BB,ZB,Z
C 305 FORMAT(1H1,//,1X,'MPT,ICONT,KKK,III,SB,F2,DIFF,BO,BB,ZB,Z',/,3X,4I
C    D7,7(F8.3,1X))
      WRITE(6,305) N1,ALFAT,NR,BETAT,RATE,MIN,ALFA,CR,MPT,F2,SB,BO,DIFF,
     ZBB,ZB,Z,MC
  305 FORMAT(1H1,/,3X,65('='),/,30X,'DATA SUMMARY',/,3X,65('='),/,3X,'MI
     NN. INITIAL SAMPLE=',I3,14X,'ALPHAT (TEST)=',F5.3,/,3X,'MIN. ADDITI
     OONAL SAMPLE=',I3,11X,'BETAT (TEST)=',F5.3,/,3X,'DESIRED RATE OF RE
     TTURN=',F5.3,9X,'MIN NO. OF IMPRV. PTS.=',I3,/,3X,'ALPHA (CONFID. I
```

```
     NNTERVAL)=',F5.3,7X,'COST PER REPLICATE=',F5.1,/,3X,('=========='),
     N('================'),39('='),/,25X,'TEST PROCEDURE SUMMARY',/,3X,6
     55('='),/,3X,'NO.OF PTS. IN REGRESSION=',I3,8X,'DEGREES OF FREEDOM=
     D',F6.2,/,3X,'EST. SAVINGS RATE STD.DEV.=',F6.1,3X,'BO, ACTUAL COST
     SS RATE=',F6.1,/,3X,'DIFF. BEING DETECTED=',F6.1,9X,'BB, EST. SAVIN
     GGS RATE=',F6.1,/,3X,'DESIRED Z(BETAT)=',F6.3,13X,'ACTUAL Z*=',F6.3
     ),/,3X,'NO. PTS. FOR COST RATE=',I2)
      WRITE(6,308)
  308 FORMAT(3X,65('='),/,30X,'CASE SIMULATED',/,3X,65('='))
      WRITE(6,3088)
 3088 FORMAT(3X,'F(X1;X2)=COST FUNCTION OF A SIMULATED (Q;R) INVENTORY S
     YYSTEM',/,3X,'CI=5.0,  CO=40.0,  CL=20.0,    MONTHLY DEMAND=P(200)'
     C,/,3X,'DIFF=(MAX(BB;BO))*RATE',/,3X,'MODEL STD. DEV.- UNKNOWN',/,3
     CX,'TYPE OF ERROR- UNKNOWN')
C     +++++++++++++ CASE SIMULATED ++++++++++++
C     PRINT,'  DIFF=(MAX(BB;BO))*RATE'
C     PRINT,'  F(X1;X2)=COST FUNCTION OF A SIMULATED INENTORY SYSTEM '
C     PRINT,'  MODEL STD. DEV. UNKNOWN '
C     PRINT,'  TYPE OF ERROR UNKNOWN'
      WRITE(6,306)
  306 FORMAT(1H1,19X,'TERMINATION PROCEDURE RESULTS:',/,20X,30('='),/)
      WRITE(6,401)
  401 FORMAT(3X,'POINT',4X,'F(X)',9X,'STD(F(X))',4X,'CUM. COST',3X,'GROS
     NS SAVINGS',2X,'NET SAVINGS',/,2X,75('='))
      DO 307 I=1,KKK
      STDPRV(I)=SQRT(VMPRV(I))
      WRITE(6,402) I,FMPRV(I),STDPRV(I),CMPRV(I),SMPRV(I),SNET(I)
  402 FORMAT(4X,I3,2X,3(F10.2,3X),2X,F10.2,5X,F10.2)
  307 CONTINUE
      WRITE(6,403)
  403 FORMAT(1H1,'END OF PROGRAM')
C
C     <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
      DO 9999 I=1,MAXIMP
C     PRINT,I,'COST    ',CMPRV(I)
 9999 CONTINUE
C
      STOP
      END
```

Appendix B

COMPUTER OUTPUTS

1) Example 6.1:

```
=================================================================
                 EXAMPLE 6.1 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.500
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
=================================================================
                     TEST PROCEDURE SUMMARY.
=================================================================
NO.OF PTS. IN REGRESSION=  5          DEGREES OF FREEDOM= 20.71
EST. SAVINGS RATE STD.DEV.= 143.1     BO, ACTUAL COSTS RATE=2475.0
DIFF. BEING DETECTED= 371.2           BB, EST. SAVINGS RATE=2336.5
DESIRED Z(BETAT)=-0.000               ACTUAL Z*= 0.096
NO. PTS. FOR COST RATE= 3
=================================================================
                          CASE SIMULATED
=================================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.3300	60.3300	97897.75	16319.62	25	-375.00	1
2	68.3300	60.3300	110185.10	15279.79	15	-600.00	-1
3	52.3300	60.3300	89147.13	13649.18	15	7925.63	1
4	44.3300	60.3300	87254.69	12974.54	15	9593.06	1
5	36.3300	60.3300	69789.75	12407.80	15	26833.00	1
6	28.3300	60.3300	60911.85	5705.82	15	35485.90	1
7	20.3300	60.3300	65963.06	9273.64	15	35260.90	-1
8	28.3300	68.3300	78598.00	11581.39	15	35035.90	-1
9	28.3300	52.3300	58838.86	10184.58	15	36883.89	1
10	28.3300	44.3300	51932.48	6828.77	15	43565.27	1
11	28.3300	36.3300	41411.55	6317.50	15	53861.20	1
12	28.3300	28.3300	36631.29	4096.10	15	58416.46	1
13	28.3300	20.3300	28438.77	4672.00	15	66383.94	1
14	28.3300	12.3300	24054.82	4026.26	15	70542.88	1
15	28.3300	4.3300	21326.22	2762.56	15	73046.50	1
16	28.3300	-3.6700	21281.39	2429.21	20	72746.50	-1
17	36.3300	4.3300	28060.65	4972.65	15	72521.50	-1
18	20.3300	4.3300	14387.78	2499.92	15	79234.94	1
19	12.3300	4.3300	10522.59	1746.36	15	82875.13	1
20	4.3300	4.3300	6587.76	1212.68	15	86584.94	1
21	-3.6700	4.3300	6894.10	1107.73	15	86359.94	-1
22	4.3300	12.3300	10159.67	1595.16	15	86134.94	-1
23	4.3300	-3.6700	6701.43	914.84	15	85909.94	-1
24	8.3300	4.3300	8520.88	1279.61	15	85684.94	-1
25	0.3300	4.3300	6029.59	863.85	15	86018.13	1
26	-3.6700	4.3300	6702.43	1017.96	15	85793.13	-1
27	0.3300	8.3300	7311.08	1131.14	15	85568.13	-1
28	0.3300	0.3300	4951.80	696.90	15	86420.94	1
29	0.3300	-3.6700	5964.82	857.99	15	86195.94	-1
30	4.3300	0.3300	6164.17	726.08	15	85970.94	-1
31	-3.6700	0.3300	5343.10	955.46	15	85745.94	-1
32	2.3300	0.3300	5504.74	742.38	15	85520.94	-1
33	-1.6700	0.3300	5431.14	756.71	15	85295.94	-1
34	0.3300	2.3300	5462.21	666.40	15	85070.94	-1
35	0.3300	-1.6700	5420.73	946.75	15	84845.94	-1

Example 6.1 (continued):

36	1.3300	0.3300	5008.18	729.89	15	84620.94	-1
37	-0.6700	0.3300	5164.07	665.28	15	84395.94	-1
38	0.3300	1.3300	5479.85	854.40	15	84170.94	-1
39	0.3300	-0.6700	5121.91	1016.00	15	83945.94	-1
40	0.8300	0.3300	5131.45	554.06	15	83720.94	-1
41	-0.1700	0.3300	5066.55	1012.83	15	83495.94	-1
42	0.3300	0.8300	5359.43	678.26	15	83270.94	-1
43	0.3300	-0.1700	5071.00	812.78	15	83045.94	-1
44	0.5800	0.3300	5492.69	821.15	15	82820.94	-1
45	0.0800	0.3300	5048.42	530.83	15	82595.94	-1
46	0.3300	0.5800	5261.26	743.23	15	82370.94	-1
47	0.3300	0.0800	4968.23	661.44	15	82145.94	-1

TERMINATION PROCEDURE RESULTS:
==============================

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	97897.75	16319.62	375.00	0.00	-375.00
2	89147.13	13649.18	825.00	8750.63	7925.63
3	87254.69	12974.54	1050.00	10643.06	9593.06
4	69789.75	12407.80	1275.00	28108.00	26833.00
5	60911.85	5705.82	1500.00	36985.90	35485.90
6	58838.86	10184.58	2175.00	39058.89	36883.89
7	51932.48	6828.77	2400.00	45965.27	43565.27
8	41411.55	6317.50	2625.00	56486.20	53861.20
9	36631.29	4096.10	2850.00	61266.46	58416.46
10	28438.77	4672.00	3075.00	69458.94	66383.94
11	24054.82	4026.26	3300.00	73842.88	70542.88
12	21326.22	2762.56	3525.00	76571.50	73046.50
13	14387.78	2499.92	4275.00	83509.94	79234.94
14	10522.59	1746.36	4500.00	87375.13	82875.13
15	6587.76	1212.68	4725.00	91309.94	86584.94
16	6029.59	863.85	5850.00	91868.13	86018.13
17	4951.80	696.90	6525.00	92945.94	86420.94

2) Example 6.2:

```
==================================================================
                  EXAMPLE 6.2 - DATA SUMMARY
==================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.800
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
==================================================================
                     TEST PROCEDURE SUMMARY
==================================================================
NO.OF PTS. IN REGRESSION=  4          DEGREES OF FREEDOM= 20.92
EST. SAVINGS RATE STD.DEV.= 150.6     BO, ACTUAL COSTS RATE=1350.0
DIFF. BEING DETECTED= 259.1           BB, EST. SAVINGS RATE=1727.1
DESIRED Z(BETAT)=-0.842               ACTUAL Z*=-0.716
NO. PTS. FOR COST RATE= 3
==================================================================
                         CASE SIMULATED
==================================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.3300	60.3300	97897.75	16319.62	25	-375.00	1
2	68.3300	60.3300	110185.10	15279.79	15	-600.00	-1
3	52.3300	60.3300	89147.13	13649.18	15	7925.63	1
4	44.3300	60.3300	87254.69	12974.54	15	9593.06	1
5	36.3300	60.3300	69789.75	12407.80	15	26833.00	1
6	28.3300	60.3300	60911.85	5705.82	15	35485.90	1
7	20.3300	60.3300	65963.06	9273.64	15	35260.90	-1
8	28.3300	68.3300	78598.00	11581.39	15	35035.90	-1
9	28.3300	52.3300	58838.86	10184.58	15	36883.89	1
10	28.3300	44.3300	51932.48	6828.77	15	43565.27	1
11	28.3300	36.3300	41411.55	6317.50	15	53861.20	1
12	28.3300	28.3300	36631.29	4096.10	15	58416.46	1
13	28.3300	20.3300	28438.77	4672.00	15	66383.94	1
14	28.3300	12.3300	24054.82	4026.26	15	70542.88	1
15	28.3300	4.3300	21326.22	2762.56	15	73046.50	1
16	28.3300	-3.6700	21281.39	2429.21	20	72746.50	-1
17	36.3300	4.3300	28060.65	4972.65	15	72521.50	-1
18	20.3300	4.3300	14387.78	2499.92	15	79234.94	1
19	12.3300	4.3300	10522.59	1746.36	15	82875.13	1
20	4.3300	4.3300	6587.76	1212.68	15	86584.94	1
21	-3.6700	4.3300	6894.10	1107.73	15	86359.94	-1
22	4.3300	12.3300	10159.67	1595.16	15	86134.94	-1
23	4.3300	-3.6700	6701.43	914.84	15	85909.94	-1
24	8.3300	4.3300	8520.88	1279.61	15	85684.94	-1
25	0.3300	4.3300	6029.59	863.85	15	86018.13	1
26	-3.6700	4.3300	6702.43	1017.96	15	85793.13	-1
27	0.3300	8.3300	7311.08	1131.14	15	85568.13	-1
28	0.3300	0.3300	4951.80	696.90	15	86420.94	1
29	0.3300	-3.6700	5964.82	857.99	15	86195.94	-1
30	4.3300	0.3300	6164.17	726.08	15	85970.94	-1
31	-3.6700	0.3300	5343.10	955.46	15	85745.94	-1
32	2.3300	0.3300	5504.74	742.38	15	85520.94	-1
33	-1.6700	0.3300	5431.14	756.71	15	85295.94	-1
34	0.3300	2.3300	5462.21	666.40	15	85070.94	-1
35	0.3300	-1.6700	5420.73	946.75	15	84845.94	-1
36	1.3300	0.3300	5008.18	729.89	15	84620.94	-1
37	-0.6700	0.3300	5164.07	665.38	15	84395.94	-1

Example 6.2 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	97897.75	16319.62	375.00	0.00	-375.00
2	89147.13	13649.18	825.00	8750.63	7925.63
3	87254.69	12974.54	1050.00	10643.06	9593.06
4	69789.75	12407.80	1275.00	28108.00	26833.00
5	60911.85	5705.82	1500.00	36985.90	35485.90
6	58838.86	10184.58	2175.00	39058.89	36883.89
7	51932.48	6828.77	2400.00	45965.27	43565.27
8	41411.55	6317.50	2625.00	56486.20	53861.20
9	36631.29	4096.10	2850.00	61266.46	58416.46
10	28438.77	4672.00	3075.00	69458.94	66383.94
11	24054.82	4026.26	3300.00	73842.88	70542.88
12	21326.22	2762.56	3525.00	76571.50	73046.50
13	14387.78	2499.92	4275.00	83509.94	79234.94
14	10522.59	1746.36	4500.00	87375.13	82875.13
15	6587.76	1212.68	4725.00	91309.94	86584.94
16	6029.59	863.85	5850.00	91868.13	86018.13
17	4951.80	696.90	6525.00	92945.94	86420.94

3) Example 6.3:

```
=================================================================
                    EXAMPLE 6.3 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.950
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
=================================================================
                        TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION=  4          DEGREES OF FREEDOM= 20.92
EST. SAVINGS RATE STD.DEV.= 150.6     BO, ACTUAL COSTS RATE=1350.0
DIFF. BEING DETECTED= 259.1           BB, EST. SAVINGS RATE=1727.1
DESIRED Z(BETAT)=-1.645               ACTUAL Z*=-0.716
NO. PTS. FOR COST RATE= 3
=================================================================
                           CASE SIMULATED
=================================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.3300	60.3300	97897.75	16319.62	25	-375.00	1
2	68.3300	60.3300	110185.10	15279.79	15	-600.00	-1
3	52.3300	60.3300	89147.13	13649.18	15	7925.63	1
4	44.3300	60.3300	87254.69	12974.54	15	9593.06	1
5	36.3300	60.3300	69789.75	12407.80	15	26833.00	1
6	28.3300	60.3300	60911.85	5705.82	15	35485.90	1
7	20.3300	60.3300	65963.06	9273.64	15	35260.90	-1
8	28.3300	68.3300	78598.00	11581.39	15	35035.90	-1
9	28.3300	52.3300	58838.86	10184.58	15	36883.89	1
10	28.3300	44.3300	51932.48	6828.77	15	43565.27	1
11	28.3300	36.3300	41411.55	6317.50	15	53861.20	1
12	28.3300	28.3300	36631.29	4096.10	15	58416.46	1
13	28.3300	20.3300	28438.77	4672.00	15	66383.94	1
14	28.3300	12.3300	24054.82	4026.26	15	70542.88	1
15	28.3300	4.3300	21326.22	2762.56	15	73046.50	1
16	28.3300	-3.6700	21281.39	2429.21	20	72746.50	-1
17	36.3300	4.3300	28060.65	4972.65	15	72521.50	-1
18	20.3300	4.3300	14387.78	2499.92	15	79234.94	1
19	12.3300	4.3300	10522.59	1746.36	15	82875.13	1
20	4.3300	4.3300	6587.76	1212.68	15	86584.94	1
21	-3.6700	4.3300	6894.10	1107.73	15	86359.94	-1
22	4.3300	12.3300	10159.67	1595.16	15	86134.94	-1
23	4.3300	-3.6700	6701.43	914.84	15	85909.94	-1
24	8.3300	4.3300	8520.88	1279.61	15	85684.94	-1
25	0.3300	4.3300	6029.59	863.85	15	86018.13	1
26	-3.6700	4.3300	6702.43	1017.96	15	85793.13	-1
27	0.3300	8.3300	7311.08	1131.14	15	85568.13	-1
28	0.3300	0.3300	4951.80	696.90	15	86420.94	1
29	0.3300	-3.6700	5964.82	857.99	15	86195.94	-1
30	4.3300	0.3300	6164.17	726.08	15	85970.94	-1
31	-3.6700	0.3300	5343.10	955.46	15	85745.94	-1
32	2.3300	0.3300	5504.74	742.38	15	85520.94	-1
33	-1.6700	0.3300	5431.14	756.71	15	85295.94	-1
34	0.3300	2.3300	5462.21	666.40	15	85070.94	-1
35	0.3300	-1.6700	5420.73	946.75	15	84845.94	-1
36	1.3300	0.3300	5008.18	729.89	15	84620.94	-1
37	-0.6700	0.3300	5164.07	665.38	15	84395.94	-1

Example 6.3 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	97897.75	16319.62	375.00	0.00	-375.00
2	89147.13	13649.18	825.00	8750.63	7925.63
3	87254.69	12974.54	1050.00	10643.06	9593.06
4	69789.75	12407.80	1275.00	28108.00	26833.00
5	60911.85	5705.82	1500.00	36985.90	35485.90
6	58838.86	10184.58	2175.00	39058.89	36883.89
7	51932.48	6828.77	2400.00	45965.27	43565.27
8	41411.55	6317.50	2625.00	56486.20	53861.20
9	36631.29	4096.10	2850.00	61266.46	58416.46
10	28438.77	4672.00	3075.00	69458.94	66383.94
11	24054.82	4026.26	3300.00	73842.88	70542.88
12	21326.22	2762.56	3525.00	76571.50	73046.50
13	14387.78	2499.92	4275.00	83509.94	79234.94
14	10522.59	1746.36	4500.00	87375.13	82875.13
15	6587.76	1212.68	4725.00	91309.94	86584.94
16	6029.59	863.85	5850.00	91868.13	86018.13
17	4951.80	696.90	6525.00	92945.94	86420.94

4) Example 6.4:

```
=================================================================
                   EXAMPLE 6.4 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.500
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
=================================================================
                       TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION=  5         DEGREES OF FREEDOM= 22.27
EST. SAVINGS RATE STD.DEV.= 131.1    BO, ACTUAL COSTS RATE=2032.5
DIFF. BEING DETECTED= 350.9          BB, EST. SAVINGS RATE=2339.3
DESIRED Z(BETAT)=-0.000              ACTUAL Z*= 0.186
NO. PTS FOR COST RATE= 3
=================================================================
                           CASE SIMULATED
=================================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=EXPONENTIAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.3300	60.3300	97362.00	15167.68	25	-375.00	1
2	68.3300	60.3300	110326.80	17018.98	15	-600.00	-1
3	52.3300	60.3300	89283.69	13442.70	15	7253.31	1
4	44.3300	60.3300	85392.13	10361.79	15	10919.88	1
5	36.3300	60.3300	69291.63	12556.87	15	26795.38	1
6	28.3300	60.3300	62853.66	7331.88	15	33008.34	1
7	20.3300	60.3300	64517.19	9100.03	15	32783.34	-1
8	28.3300	68.3300	78591.81	8703.46	15	32558.34	-1
9	28.3300	52.3300	58133.30	7906.01	15	37053.70	1
10	28.3300	44.3300	51883.94	5619.67	15	43078.06	1
11	28.3300	36.3300	41472.06	7061.84	15	53264.94	1
12	28.3300	28.3300	36953.36	3179.64	15	57558.64	1
13	28.3300	20.3300	28373.91	5242.81	15	65913.06	1
14	28.3300	12.3300	23887.69	4366.83	15	70174.25	1
15	28.3300	4.3300	21545.69	2526.70	15	72291.25	1
16	28.3300	-3.6700	21472.25	2092.34	18	72021.25	-1
17	36.3300	4.3300	27456.49	5266.45	15	71796.25	-1
18	20.3300	4.3300	14685.46	2241.93	15	78431.50	1
19	12.3300	4.3300	10104.78	1580.66	15	82787.19	1
20	4.3300	4.3300	6603.21	1423.17	15	86063.75	1
21	-3.6700	4.3300	7016.53	477.23	15	85838.75	-1
22	4.3300	12.3300	9684.28	1804.89	15	85613.75	-1
23	4.3300	-3.6700	6818.14	705.25	15	85388.75	-1
24	8.3300	4.3300	8519.41	993.61	15	85163.75	-1
25	0.3300	4.3300	6005.66	698.37	15	85536.31	1
26	-3.6700	4.3300	6713.56	975.59	15	85311.31	-1
27	0.3300	8.3300	7297.66	1275.83	15	85086.31	-1
28	0.3300	0.3300	5038.57	820.89	15	85828.38	1
29	0.3300	-3.6700	5979.41	508.67	15	85603.38	-1
30	4.3300	0.3300	6094.88	662.05	15	85378.38	-1
31	-3.6700	0.3300	5298.11	1133.30	15	85153.38	-1
32	2.3300	0.3300	5576.45	450.52	15	84928.38	-1
33	-1.6700	0.3300	5411.38	669.03	15	84703.38	-1
34	0.3300	2.3300	5508.03	563.68	15	84478.38	-1
35	0.3300	-1.6700	5474.48	478.91	15	84253.38	-1

Example 6.4 (continued):

36	1.3300	0.3300	4993.35	807.61	15	84028.38	-1
37	-0.6700	0.3300	5129.51	759.73	15	83803.38	-1
38	0.3300	1.3300	5349.76	777.83	15	83578.38	-1
39	0.3300	-0.6700	4959.07	1124.90	15	83353.38	-1
40	0.8300	0.3300	5290.88	620.78	15	83128.38	-1
41	-0.1700	0.3300	4866.01	1102.81	16	82888.38	-1
42	0.3300	0.8300	5356.69	557.81	15	82663.38	-1
43	0.3300	-0.1700	5094.26	642.94	15	82438.38	-1

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	97362.00	15167.68	375.00	0.00	-375.00
2	89283.69	13442.70	825.00	8078.31	7253.31
3	85392.13	10361.79	1050.00	11969.88	10919.88
4	69291.63	12556.87	1275.00	28070.38	26795.38
5	62853.66	7331.88	1500.00	34508.34	33008.34
6	58133.30	7906.01	2175.00	39228.70	37053.70
7	51883.94	5619.67	2400.00	45478.06	43078.06
8	41472.06	7061.84	2625.00	55889.94	53264.94
9	36953.36	3179.64	2850.00	60408.64	57558.64
10	28373.91	5242.81	3075.00	68988.06	65913.06
11	23887.69	4366.83	3300.00	73474.25	70174.25
12	21545.69	2526.70	3525.00	75816.25	72291.25
13	14685.46	2241.93	4245.00	82676.50	78431.50
14	10104.78	1580.66	4470.00	87257.19	82787.19
15	6603.21	1423.17	4695.00	90758.75	86063.75
16	6005.66	698.37	5820.00	91356.31	85536.31
17	5038.57	820.89	6495.00	92323.38	85828.38

5) Example 6.5:

EXAMPLE 6.5 - DATA SUMMARY

MIN. INITIAL SAMPLE= 25
MIN. ADDITIONAL SAMPLE= 15
DESIRED RATE OF RETURN=0.150
ALPHA (CONFID. INTERVAL)=0.800
ALPHAT (TEST)=0.010
BETAT (TEST)=0.800
MIN NO. OF IMPRV. PTS.= 4
COST PER REPLICATE= 15.0

TEST PROCEDURE SUMMARY

NO.OF PTS. IN REGRESSION= 4
EST. SAVINGS RATE STD.DEV.= 143.9
DIFF. BEING DETECTED= 236.9
DESIRED Z(BETAT)=-0.842
NO. PTS FOR COST RATE= 3
DEGREES OF FREEDOM= 24.71
BO, ACTUAL COSTS RATE=1237.5
BB, EST. SAVINGS RATE=1579.6
ACTUAL Z*=-0.769

CASE SIMULATED

DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=EXPONENTIAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.3300	60.3300	97362.00	15167.68	25	-375.00	1
2	68.3300	60.3300	110326.80	17018.98	15	-600.00	-1
3	52.3300	60.3300	89283.69	13442.70	15	7253.31	1
4	44.3300	60.3300	85392.13	10361.79	15	10919.88	1
5	36.3300	60.3300	69291.63	12556.87	15	26795.38	1
6	28.3300	60.3300	62853.66	7331.88	15	33008.34	1
7	20.3300	60.3300	64517.19	9100.03	15	32783.34	-1
8	28.3300	68.3300	78591.81	8703.46	15	32558.34	-1
9	28.3300	52.3300	58133.30	7906.01	15	37053.70	1
10	28.3300	44.3300	51883.94	5619.67	15	43078.06	1
11	28.3300	36.3300	41472.06	7061.84	15	53264.94	1
12	28.3300	28.3300	36953.36	3179.64	15	57558.64	1
13	28.3300	20.3300	28373.91	5242.81	15	65913.06	1
14	28.3300	12.3300	23887.69	4366.83	15	70174.25	1
15	28.3300	4.3300	21545.69	2526.70	15	72291.25	1
16	28.3300	-3.6700	21472.25	2092.34	18	72021.25	-1
17	36.3300	4.3300	27456.49	5266.45	15	71796.25	-1
18	20.3300	4.3300	14685.46	2241.93	15	78431.50	1
19	12.3300	4.3300	10104.78	1580.66	15	82787.19	1
20	4.3300	4.3300	6603.21	1423.17	15	86063.75	1
21	-3.6700	4.3300	7016.53	477.23	15	85838.75	-1
22	4.3300	12.3300	9684.28	1804.89	15	85613.75	-1
23	4.3300	-3.6700	6818.14	705.25	15	85388.75	-1
24	8.3300	4.3300	8519.41	993.61	15	85163.75	-1
25	0.3300	4.3300	6005.66	698.37	15	85536.31	1
26	-3.6700	4.3300	6713.56	975.59	15	85311.31	-1
27	0.3300	8.3300	7297.66	1275.83	15	85086.31	-1
28	0.3300	0.3300	5038.57	820.89	15	85828.38	1
29	0.3300	-3.6700	5979.41	508.67	15	85603.38	-1
30	4.3300	0.3300	6094.88	662.05	15	85378.38	-1
31	-3.6700	0.3300	5298.11	1133.30	15	85153.38	-1
32	2.3300	0.3300	5576.45	450.52	15	84928.38	-1
33	-1.6700	0.3300	5411.38	669.03	15	84703.38	-1
34	0.3300	2.3300	5508.03	563.68	15	84478.38	-1
35	0.3300	-1.6700	5474.48	478.91	15	84253.38	-1
36	1.3300	0.3300	4993.35	807.61	15	84028.38	-1

Example 6.5 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	97362.00	15167.68	375.00	0.00	-375.00
2	89283.69	13442.70	825.00	8078.31	7253.31
3	85392.13	10361.79	1050.00	11969.88	10919.88
4	69291.63	12556.87	1275.00	28070.38	26795.38
5	62853.66	7331.88	1500.00	34508.34	33008.34
6	58133.30	7906.01	2175.00	39228.70	37053.70
7	51883.94	5619.67	2400.00	45478.06	43078.06
8	41472.06	7061.84	2625.00	55889.94	53264.94
9	36953.36	3179.64	2850.00	60408.64	57558.64
10	28373.91	5242.81	3075.00	68988.06	65913.06
11	23887.69	4366.83	3300.00	73474.25	70174.25
12	21545.69	2526.70	3525.00	75816.25	72291.25
13	14685.46	2241.93	4245.00	82676.50	78431.50
14	10104.78	1580.66	4470.00	87257.19	82787.19
15	6603.21	1423.17	4695.00	90758.75	86063.75
16	6005.66	698.37	5820.00	91356.31	85536.31
17	5038.57	820.89	6495.00	92323.38	85828.38

6) Example 6.6:

```
==============================================================
                  EXAMPLE 6.6 - DATA SUMMARY
==============================================================
MIN. INITIAL SAMPLE= 25               ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15            BETAT (TEST)=0.950
DESIRED RATE OF RETURN=0.150          MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800        COST PER REPLICATE= 15.0
==============================================================
                  TEST PROCEDURE SUMMARY
==============================================================
NO.OF PTS. IN REGRESSION=  4         DEGREES OF FREEDOM= 24.71
EST. SAVINGS RATE STD.DEV.= 143.9    BO, ACTUAL COSTS RATE=1237.5
DIFF. BEING DETECTED= 236.9          BB, EST. SAVINGS RATE=1579.6
DESIRED Z(BETAT)=-1.645              ACTUAL Z*=-0.769
NO. PTS FOR COST RATE= 3
==============================================================
                      CASE SIMULATED
==============================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=EXPONENTIAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.3300	60.3300	97362.00	15167.68	25	-375.00	1
2	68.3300	60.3300	110326.80	17018.98	15	-600.00	-1
3	52.3300	60.3300	89283.69	13442.70	15	7253.31	1
4	44.3300	60.3300	85392.13	10361.79	15	10919.88	1
5	36.3300	60.3300	69291.63	12556.87	15	26795.38	1
6	28.3300	60.3300	62853.66	7331.88	15	33008.34	1
7	20.3300	60.3300	64517.19	9100.03	15	32783.34	-1
8	28.3300	68.3300	78591.81	8703.46	15	32558.34	-1
9	28.3300	52.3300	58133.30	7906.01	15	37053.70	1
10	28.3300	44.3300	51883.94	5619.67	15	43078.06	1
11	28.3300	36.3300	41472.06	7061.84	15	53264.94	1
12	28.3300	28.3300	36953.36	3179.64	15	57558.64	1
13	28.3300	20.3300	28373.91	5242.81	15	65913.06	1
14	28.3300	12.3300	23887.69	4366.83	15	70174.25	1
15	28.3300	4.3300	21545.69	2526.70	15	72291.25	1
16	28.3300	-3.6700	21472.25	2092.34	18	72021.25	-1
17	36.3300	4.3300	27456.49	5266.45	15	71796.25	-1
18	20.3300	4.3300	14685.46	2241.93	15	78431.50	1
19	12.3300	4.3300	10104.78	1580.66	15	82787.19	1
20	4.3300	4.3300	6603.21	1423.17	15	86063.75	1
21	-3.6700	4.3300	7016.53	477.23	15	85838.75	-1
22	4.3300	12.3300	9684.28	1804.89	15	85613.75	-1
23	4.3300	-3.6700	6818.14	705.25	15	85388.75	-1
24	8.3300	4.3300	8519.41	993.61	15	85163.75	-1
25	0.3300	4.3300	6005.66	698.37	15	85536.31	1
26	-3.6700	4.3300	6713.56	975.59	15	85311.31	-1
27	0.3300	8.3300	7297.66	1275.83	15	85086.31	-1
28	0.3300	0.3300	5038.57	820.89	15	85828.38	1
29	0.3300	-3.6700	5979.41	508.67	15	85603.38	-1
30	4.3300	0.3300	6094.88	662.05	15	85378.38	-1
31	-3.6700	0.3300	5298.11	1133.30	15	85153.38	-1
32	2.3300	0.3300	5576.45	450.52	15	84928.38	-1
33	-1.6700	0.3300	5411.38	669.03	15	84703.38	-1
34	0.3300	2.3300	5508.03	563.68	15	84478.38	-1
35	0.3300	-1.6700	5474.48	478.91	15	84253.38	-1
36	1.3300	0.3300	4993.35	807.61	15	84028.38	-1

Example 6.6 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	97362.00	15167.68	375.00	0.00	-375.00
2	89283.69	13442.70	825.00	8078.31	7253.31
3	85392.13	10361.79	1050.00	11969.88	10919.88
4	69291.63	12556.87	1275.00	28070.38	26795.38
5	62853.66	7331.88	1500.00	34508.34	33008.34
6	58133.30	7906.01	2175.00	39228.70	37053.70
7	51883.94	5619.67	2400.00	45478.06	43078.06
8	41472.06	7061.84	2625.00	55889.94	53264.94
9	36953.36	3179.64	2850.00	60408.64	57558.64
10	28373.91	5242.81	3075.00	68988.06	65913.06
11	23887.69	4366.83	3300.00	73474.25	70174.25
12	21545.69	2526.70	3525.00	75816.25	72291.25
13	14685.46	2241.93	4245.00	82676.50	78431.50
14	10104.78	1580.66	4470.00	87257.19	82787.19
15	6603.21	1423.17	4695.00	90758.75	86063.75
16	6005.66	698.37	5820.00	91356.31	85536.31
17	5038.57	820.89	6495.00	92323.38	85828.38

7) Example 6.7:

```
=================================================================
                EXAMPLE 6.7 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 25              ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15           BETAT (TEST)=0.500
DESIRED RATE OF RETURN=0.150         MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800       COST PER REPLICATE= 15.0
=================================================================
                    TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION= 11        DEGREES OF FREEDOM=105.12
EST. SAVINGS RATE STD.DEV.=  277.9  BO, ACTUAL COSTS RATE= 3757.5
DIFF. BEING DETECTED=  658.6        BB, EST. SAVINGS RATE= 4390.6
DESIRED Z(BETAT)=-0.000             ACTUAL Z*= 0.013
NO. PTS FOR COST RATE= 3
=================================================================
                         CASE SIMULATED
=================================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=100000-(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MAXIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.3300	60.3300	1269.91	211.68	25	-375.00	1
2	68.3300	60.3300	-8200.71	1176.61	15	-600.00	-1
3	52.3300	60.3300	10219.97	1564.80	15	8125.06	1
4	44.3300	60.3300	19970.76	2969.53	15	17650.86	1
5	36.3300	60.3300	24830.96	4414.61	15	22286.05	1
6	28.3300	60.3300	30084.71	2818.18	15	27314.80	1
7	20.3300	60.3300	42126.52	5922.36	15	39131.62	1
8	12.3300	60.3300	44975.86	6627.28	15	41755.95	1
9	4.3300	60.3300	47976.95	8304.48	15	44532.05	1
10	-3.6700	60.3300	49673.82	6531.83	15	46003.91	1
11	-11.6700	60.3300	43559.30	6645.11	15	45778.91	-1
12	-3.6700	68.3300	39398.85	4405.48	15	45553.91	-1
13	-3.6700	52.3300	54884.88	9016.62	15	50539.97	1
14	-3.6700	44.3300	63843.16	10685.73	15	59273.25	1
15	-3.6700	36.3300	73594.31	9533.33	15	68799.38	1
16	-3.6700	28.3300	80708.19	10584.60	15	75688.25	1
17	-3.6700	20.3300	86028.88	12822.29	15	80783.94	1
18	-3.6700	12.3300	88414.50	15998.04	18	82899.56	1
19	-3.6700	4.3300	91920.44	14004.49	15	86180.50	1
20	-3.6700	-3.6700	95036.81	18285.11	18	89026.88	1
21	-3.6700	-11.6700	92247.69	16319.34	16	88786.88	-1
22	4.3300	-3.6700	92213.25	11174.71	15	88561.88	-1
23	-11.6700	-3.6700	92637.06	12502.13	15	88336.88	-1
24	0.3300	-3.6700	97456.50	16710.04	18	90486.56	1
25	4.3300	-3.6700	95990.19	11452.96	15	90261.56	-1
26	0.3300	0.3300	93810.56	13349.05	15	90036.56	-1
27	0.3300	-7.6700	89989.38	13298.63	15	89811.56	-1
28	2.3300	-3.6700	95709.56	15400.93	15	89586.56	-1
29	-1.6700	-3.6700	96393.25	12081.88	18	89316.56	-1
30	0.3300	-1.6700	95675.31	14580.06	17	89061.56	-1
31	0.3300	-5.6700	91129.50	14307.40	15	88836.56	-1
32	1.3300	-3.6700	95985.88	14211.16	22	88506.56	-1
33	-0.6700	-3.6700	99553.63	11110.64	15	90378.69	1
34	-1.6700	-3.6700	96318.50	16279.95	15	90153.69	-1
35	-0.6700	-2.6700	91480.44	13233.00	15	89928.69	-1

Example 6.7 (continued):

36	-0.6700	-4.6700	95354.44	12184.32	15	89703.69	-1
37	-0.1700	-3.6700	97865.00	17338.09	18	89433.69	-1
38	-1.1700	-3.6700	96405.00	16450.86	15	89208.69	-1
39	-0.6700	-3.1700	96083.75	15783.12	15	88983.69	-1
40	-0.6700	-4.1700	93892.31	16074.94	15	88758.69	-1
41	-0.4200	-3.6700	101682.00	12676.68	18	90617.06	1
42	-0.1700	-3.6700	93937.38	12149.31	15	90392.06	-1
43	-0.4200	-3.4200	99705.75	16484.40	15	90167.06	-1
44	-0.4200	-3.9200	97361.25	9961.94	15	89942.06	-1
45	-0.2950	-3.6700	94865.00	14842.01	15	89717.06	-1
46	-0.5450	-3.6700	92421.06	15874.21	15	89492.06	-1
47	-0.4200	-3.5450	92002.13	17871.86	15	89267.06	-1
48	-0.4200	-3.7950	91017.38	15170.48	15	89042.06	-1
49	-0.3575	-3.6700	94798.81	16294.61	15	88817.06	-1
50	-0.4825	-3.6700	87386.56	10946.38	15	88592.06	-1
51	-0.4200	-3.6075	90729.50	21610.94	15	88367.06	-1
52	-0.4200	-3.7325	92539.88	18126.87	15	88142.06	-1
53	-0.3887	-3.6700	97857.13	14734.86	15	87917.06	-1
54	-0.4512	-3.6700	84735.69	8120.25	15	87692.06	-1
55	-0.4200	-3.6387	92396.19	11985.61	15	87467.06	-1
56	-0.4200	-3.7012	97325.56	14347.09	15	87242.06	-1
57	-0.4044	-3.6700	91308.25	10403.04	15	87017.06	-1
58	-0.4356	-3.6700	88312.19	11573.71	15	86792.06	-1
59	-0.4200	-3.6544	96957.75	18188.99	15	86567.06	-1
60	-0.4200	-3.6856	95686.38	11300.72	15	86342.06	-1
61	-0.4122	-3.6700	90332.50	14816.59	15	86117.06	-1
62	-0.4278	-3.6700	93315.31	16413.54	15	85892.06	-1
63	-0.4200	-3.6622	95539.25	12333.82	15	85667.06	-1
64	-0.4200	-3.6778	93412.88	11196.05	15	85442.06	-1
65	-0.4161	-3.6700	98027.44	12486.96	15	85217.06	-1
66	-0.4239	-3.6700	98822.88	19125.61	15	84992.06	-1

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	1269.91	211.68	375.00	0.00	-375.00
2	10219.97	1564.80	825.00	8950.06	8125.06
3	19970.76	2969.53	1050.00	18700.86	17650.86
4	24830.96	4414.61	1275.00	23561.05	22286.05
5	30084.71	2818.18	1500.00	28814.80	27314.80
6	42126.52	5922.36	1725.00	40856.62	39131.62
7	44975.86	6627.28	1950.00	43705.95	41755.95
8	47976.95	8304.48	2175.00	46707.05	44532.05
9	49673.82	6531.83	2400.00	48403.91	46003.91
10	54884.88	9016.62	3075.00	53614.97	50539.97
11	63843.16	10685.73	3300.00	62573.25	59273.25
12	73594.31	9533.33	3525.00	72324.38	68799.38
13	80708.19	10584.60	3750.00	79438.25	75688.25
14	86028.88	12822.29	3975.00	84758.94	80783.94
15	88414.50	15998.04	4245.00	87144.56	82899.56
16	91920.44	14004.49	4470.00	90650.50	86180.50
17	95036.81	18285.11	4740.00	93766.88	89026.88
18	97456.50	16710.04	5700.00	96186.56	90486.56
19	99553.63	11110.64	7905.00	98283.69	90378.69
20	101682.00	12676.68	9795.00	100412.00	90617.06

8) Exammple 6.8:

EXAMPLE 6.8 - DATA SUMMARY

MIN. INITIAL SAMPLE= 25
ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15
BETAT (TEST)=0.800
DESIRED RATE OF RETURN=0.150
MIN NO. OF IMPRV. PTS.= 4
ALPHA (CONFID. INTERVAL)=0.800
COST PER REPLICATE= 15.0

TEST PROCEDURE SUMMARY

NO.OF PTS. IN REGRESSION= 10
DEGREES OF FREEDOM= 94.42
EST. SAVINGS RATE STD.DEV.= 326.8
BO, ACTUAL COSTS RATE= 3195.0
DIFF. BEING DETECTED= 578.7
BB, EST. SAVINGS RATE= 3857.8
DESIRED Z(BETAT)=-0.842
ACTUAL Z*=-0.581
NO. PTS FOR COST RATE= 3

CASE SIMULATED

DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=100000-(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MAXIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.3300	60.3300	1269.91	211.68	25	-375.00	1
2	68.3300	60.3300	-8200.71	1176.61	15	-600.00	-1
3	52.3300	60.3300	10219.97	1564.80	15	8125.06	1
4	44.3300	60.3300	19970.76	2969.53	15	17650.86	1
5	36.3300	60.3300	24830.96	4414.61	15	22286.05	1
6	28.3300	60.3300	30084.71	2818.18	15	27314.80	1
7	20.3300	60.3300	42126.52	5922.36	15	39131.62	1
8	12.3300	60.3300	44975.86	6627.28	15	41755.95	1
9	4.3300	60.3300	47976.95	8304.48	15	44532.05	1
10	-3.6700	60.3300	49673.82	6531.83	15	46003.91	1
11	-11.6700	60.3300	43559.30	6645.11	15	45778.91	-1
12	-3.6700	68.3300	39398.85	4405.48	15	45553.91	-1
13	-3.6700	52.3300	54884.88	9016.62	15	50539.97	1
14	-3.6700	44.3300	63843.16	10685.73	15	59273.25	1
15	-3.6700	36.3300	73594.31	9533.33	15	68799.38	1
16	-3.6700	28.3300	80708.19	10584.60	15	75688.25	1
17	-3.6700	20.3300	86028.88	12822.29	15	80783.94	1
18	-3.6700	12.3300	88414.50	15998.04	18	82899.56	1
19	-3.6700	4.3300	91920.44	14004.49	15	86180.50	1
20	-3.6700	-3.6700	95036.81	18285.11	18	89026.88	1
21	-3.6700	-11.6700	92247.69	16319.34	16	88786.88	-1
22	4.3300	-3.6700	92213.25	11174.71	15	88561.88	-1
23	-11.6700	-3.6700	92637.06	12502.13	15	88336.88	-1
24	0.3300	-3.6700	97456.50	16710.04	18	90486.56	1
25	4.3300	-3.6700	95990.19	11452.96	15	90261.56	-1
26	0.3300	0.3300	93810.56	13349.05	15	90036.56	-1
27	0.3300	-7.6700	89989.38	13298.63	15	89811.56	-1
28	2.3300	-3.6700	95709.56	15400.93	15	89586.56	-1
29	-1.6700	-3.6700	96393.25	12081.88	18	89316.56	-1
30	0.3300	-1.6700	95675.31	14580.06	17	89061.56	-1
31	0.3300	-5.6700	91129.50	14307.40	15	88836.56	-1
32	1.3300	-3.6700	95985.88	14211.16	22	88506.56	-1
33	-0.6700	-3.6700	99553.63	11110.64	15	90378.69	1
34	-1.6700	-3.6700	96318.50	16279.95	15	90153.69	-1
35	-0.6700	-2.6700	91480.44	13233.00	15	89928.69	-1

Example 6.8 (continued):

36	-0.6700	-4.6700	95354.44	12184.32	15	89703.69	-1
37	-0.1700	-3.6700	97865.00	17338.09	18	89433.69	-1
38	-1.1700	-3.6700	96405.00	16450.86	15	89208.69	-1
39	-0.6700	-3.1700	96083.75	15783.12	15	88983.69	-1
40	-0.6700	-4.1700	93892.31	16074.94	15	88758.69	-1
41	-0.4200	-3.6700	101682.00	12676.68	18	90617.06	1
42	-0.1700	-3.6700	93937.38	12149.31	15	90392.06	-1
43	-0.4200	-3.4200	99705.75	16484.40	15	90167.06	-1
44	-0.4200	-3.9200	97361.25	9961.94	15	89942.06	-1
45	-0.2950	-3.6700	94865.00	14842.01	15	89717.06	-1
46	-0.5450	-3.6700	92421.06	15874.21	15	89492.06	-1
47	-0.4200	-3.5450	92002.13	17871.86	15	89267.06	-1
48	-0.4200	-3.7950	91017.38	15170.48	15	89042.06	-1
49	-0.3575	-3.6700	94798.81	16294.61	15	88817.06	-1
50	-0.4825	-3.6700	87386.56	10946.38	15	88592.06	-1
51	-0.4200	-3.6075	90729.50	21610.94	15	88367.06	-1
52	-0.4200	-3.7325	92539.88	18126.87	15	88142.06	-1
53	-0.3887	-3.6700	97857.13	14734.86	15	87917.06	-1
54	-0.4512	-3.6700	84735.69	8120.25	15	87692.06	-1
55	-0.4200	-3.6387	92396.19	11985.61	15	87467.06	-1
56	-0.4200	-3.7012	97325.56	14347.09	15	87242.06	-1
57	-0.4044	-3.6700	91308.25	10403.04	15	87017.06	-1
58	-0.4356	-3.6700	88312.19	11573.71	15	86792.06	-1
59	-0.4200	-3.6544	96957.75	18188.99	15	86567.06	-1
60	-0.4200	-3.6856	95686.38	11300.72	15	86342.06	-1
61	-0.4122	-3.6700	90332.50	14816.59	15	86117.06	-1

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	1269.91	211.68	375.00	0.00	-375.00
2	10219.97	1564.80	825.00	8950.06	8125.06
3	19970.76	2969.53	1050.00	18700.86	17650.86
4	24830.96	4414.61	1275.00	23561.05	22286.05
5	30084.71	2818.18	1500.00	28814.80	27314.80
6	42126.52	5922.36	1725.00	40856.62	39131.62
7	44975.86	6627.28	1950.00	43705.95	41755.95
8	47976.95	8304.48	2175.00	46707.05	44532.05
9	49673.82	6531.83	2400.00	48403.91	46003.91
10	54884.88	9016.62	3075.00	53614.97	50539.97
11	63843.16	10685.73	3300.00	62573.25	59273.25
12	73594.31	9533.33	3525.00	72324.38	68799.38
13	80708.19	10584.60	3750.00	79438.25	75688.25
14	86028.88	12822.29	3975.00	84758.94	80783.94
15	88414.50	15998.04	4245.00	87144.56	82899.56
16	91920.44	14004.49	4470.00	90650.50	86180.50
17	95036.81	18285.11	4740.00	93766.88	89026.88
18	97456.50	16710.04	5700.00	96186.56	90486.56
19	99553.63	11110.64	7905.00	98283.69	90378.69
20	101682.00	12676.68	9795.00	100412.00	90617.06

9) Example 6.9:

```
=================================================================
                 EXAMPLE 6.9 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.950
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
=================================================================
                     TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION=  7          DEGREES OF FREEDOM= 70.58
EST. SAVINGS RATE STD.DEV.=  597.6    BO, ACTUAL COSTS RATE= 1687.5
DIFF. BEING DETECTED=  460.8          BB, EST. SAVINGS RATE= 3071.9
DESIRED Z(BETAT)=-1.645               ACTUAL Z*=-1.570
NO. PTS FOR COST RATE= 3
=================================================================
                          CASE SIMULATED
=================================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=100000-(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MAXIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.3300	60.3300	1269.91	211.68	25	-375.00	1
2	68.3300	60.3300	-8200.71	1176.61	15	-600.00	-1
3	52.3300	60.3300	10219.97	1564.80	15	8125.06	1
4	44.3300	60.3300	19970.76	2969.53	15	17650.86	1
5	36.3300	60.3300	24830.96	4414.61	15	22286.05	1
6	28.3300	60.3300	30084.71	2818.18	15	27314.80	1
7	20.3300	60.3300	42126.52	5922.36	15	39131.62	1
8	12.3300	60.3300	44975.86	6627.28	15	41755.95	1
9	4.3300	60.3300	47976.95	8304.48	15	44532.05	1
10	-3.6700	60.3300	49673.82	6531.83	15	46003.91	1
11	-11.6700	60.3300	43559.30	6645.11	15	45778.91	-1
12	-3.6700	68.3300	39398.85	4405.48	15	45553.91	-1
13	-3.6700	52.3300	54884.88	9016.62	15	50539.97	1
14	-3.6700	44.3300	63843.16	10685.73	15	59273.25	1
15	-3.6700	36.3300	73594.31	9533.33	15	68799.38	1
16	-3.6700	28.3300	80708.19	10584.60	15	75688.25	1
17	-3.6700	20.3300	86028.88	12822.29	15	80783.94	1
18	-3.6700	12.3300	88414.50	15998.04	18	82899.56	1
19	-3.6700	4.3300	91920.44	14004.49	15	86180.50	1
20	-3.6700	-3.6700	95036.81	18285.11	18	89026.88	1
21	-3.6700	-11.6700	92247.69	16319.34	16	88786.88	-1
22	4.3300	-3.6700	92213.25	11174.71	15	88561.88	-1
23	-11.6700	-3.6700	92637.06	12502.13	15	88336.88	-1
24	0.3300	-3.6700	97456.50	16710.04	18	90486.56	1
25	4.3300	-3.6700	95990.19	11452.96	15	90261.56	-1
26	0.3300	0.3300	93810.56	13349.05	15	90036.56	-1
27	0.3300	-7.6700	89989.38	13298.63	15	89811.56	-1
28	2.3300	-3.6700	95709.56	15400.93	15	89586.56	-1
29	-1.6700	-3.6700	96393.25	12081.88	18	89316.56	-1
30	0.3300	-1.6700	95675.31	14580.06	17	89061.56	-1
31	0.3300	-5.6700	91129.50	14307.40	15	88836.56	-1
32	1.3300	-3.6700	95985.88	14211.16	22	88506.56	-1
33	-0.6700	-3.6700	99553.63	11110.64	15	90378.69	1
34	-1.6700	-3.6700	96318.50	16279.95	15	90153.69	-1
35	-0.6700	-2.6700	91480.44	13233.00	15	89928.69	-1

Example 6.9 (continued):

36	-0.6700	-4.6700	95354.44	12184.32	15	89703.69	-1
37	-0.1700	-3.6700	97865.00	17338.09	18	89433.69	-1
38	-1.1700	-3.6700	96405.00	16450.86	15	89208.69	-1

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	1269.91	211.68	375.00	0.00	-375.00
2	10219.97	1564.80	825.00	8950.06	8125.06
3	19970.76	2969.53	1050.00	18700.86	17650.86
4	24830.96	4414.61	1275.00	23561.05	22286.05
5	30084.71	2818.18	1500.00	28814.80	27314.80
6	42126.52	5922.36	1725.00	40856.62	39131.62
7	44975.86	6627.28	1950.00	43705.95	41755.95
8	47976.95	8304.48	2175.00	46707.05	44532.05
9	49673.82	6531.83	2400.00	48403.91	46003.91
10	54884.88	9016.62	3075.00	53614.97	50539.97
11	63843.16	10685.73	3300.00	62573.25	59273.25
12	73594.31	9533.33	3525.00	72324.38	68799.38
13	80708.19	10584.60	3750.00	79438.25	75688.25
14	86028.88	12822.29	3975.00	84758.94	80783.94
15	88414.50	15998.04	4245.00	87144.56	82899.56
16	91920.44	14004.49	4470.00	90650.50	86180.50
17	95036.81	18285.11	4740.00	93766.88	89026.88
18	97456.50	16710.04	5700.00	96186.56	90486.56
19	99553.63	11110.64	7905.00	98283.69	90378.69

10) Example 6.10:

```
=================================================================
                  EXAMPLE 6.10 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 25              ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15           BETAT (TEST)=0.500
DESIRED RATE OF RETURN=0.150         MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800       COST PER REPLICATE= 15.0
=================================================================
                    TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION= 12        DEGREES OF FREEDOM=106.67
EST. SAVINGS RATE STD.DEV.=  211.1  BO, ACTUAL COSTS RATE= 3142.5
DIFF. BEING DETECTED=  544.4        BB, EST. SAVINGS RATE= 3629.1
DESIRED Z(BETAT)=-0.000             ACTUAL Z*= 0.219
NO. PTS FOR COST RATE= 3
=================================================================
                         CASE SIMULATED
=================================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=100000-(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=EXPONENTIAL
MAXIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.33	60.33	1262.96	196.72	25	-375.00	1
2	68.33	60.33	-8189.80	1310.49	15	-600.00	-1
3	52.33	60.33	10235.64	1541.02	15	8147.68	1
4	44.33	60.33	19544.44	2371.52	15	17231.48	1
5	36.33	60.33	24653.71	4467.64	15	22115.75	1
6	28.33	60.33	31043.78	3621.24	15	28280.82	1
7	20.33	60.33	41203.11	5811.78	15	38215.15	1
8	12.33	60.33	44972.38	4979.88	15	41759.42	1
9	4.33	60.33	47401.64	6446.62	15	43963.68	1
10	-3.67	60.33	49627.40	5375.32	15	45964.44	1
11	-11.67	60.33	43622.95	7428.11	15	45739.44	-1
12	-3.67	68.33	39745.24	3419.83	15	45514.44	-1
13	-3.67	52.33	54759.73	10118.19	15	50421.77	1
14	-3.67	44.33	63399.55	11589.88	15	58836.59	1
15	-3.67	36.33	74351.69	8719.04	15	69563.69	1
16	-3.67	28.33	81390.69	8651.13	15	76377.69	1
17	-3.67	20.33	85786.44	14556.67	15	80548.44	1
18	-3.67	12.33	87780.13	13375.19	16	82302.13	1
19	-3.67	4.33	91047.50	15800.02	15	85344.50	1
20	-3.67	-3.67	92957.31	17514.27	20	86954.31	1
21	-3.67	-11.67	90526.88	14681.10	16	86714.31	-1
22	4.33	-3.67	94502.81	11404.50	16	88019.81	1
23	12.33	-3.67	91035.06	10716.68	15	87794.81	-1
24	4.33	4.33	96040.00	12169.78	18	89062.00	1
25	4.33	12.33	93310.75	9710.90	15	88837.00	-1
26	12.33	4.33	85955.00	16424.09	15	88612.00	-1
27	-3.67	4.33	90883.13	15533.19	15	88387.00	-1
28	8.33	4.33	96281.88	8070.82	30	87937.00	-1
29	0.33	4.33	93833.38	13763.76	15	87712.00	-1
30	4.33	8.33	88447.56	16884.49	15	87487.00	-1
31	4.33	0.33	97631.06	7717.09	15	88853.06	1
32	4.33	-3.67	94123.13	12221.15	15	88628.06	-1
33	8.33	0.33	96907.63	9166.09	15	88403.06	-1
34	0.33	0.33	94511.56	14207.57	17	88148.06	-1
35	6.33	0.33	94901.56	12650.65	15	87923.06	-1

Example 6.10 (continued):

36	2.33	0.33	94751.88	16549.30	15	87698.06	-1
37	4.33	2.33	90848.56	20307.15	15	87473.06	-1
38	4.33	-1.67	99093.50	9961.00	24	88575.50	1
39	4.33	-3.67	88755.56	20797.08	15	88350.50	-1
40	6.33	-1.67	98201.31	9666.21	15	88125.50	-1
41	2.33	-1.67	96474.06	12247.17	15	87900.50	-1
42	5.33	-1.67	98285.00	10571.39	18	87630.50	-1
43	3.33	-1.67	97158.50	9452.21	15	87405.50	-1
44	4.33	-0.67	95585.38	16661.92	15	87180.50	-1
45	4.33	-2.67	94942.19	8140.41	15	86955.50	-1
46	4.83	-1.67	85369.31	27412.67	15	86730.50	-1
47	3.83	-1.67	94155.13	14389.94	15	86505.50	-1
48	4.33	-1.17	87598.38	21059.57	15	86280.50	-1
49	4.33	-2.17	90077.56	12985.35	15	86055.50	-1
50	4.58	-1.67	90189.13	22451.71	15	85830.50	-1
51	4.08	-1.67	87989.13	16320.45	15	85605.50	-1
52	4.33	-1.42	93180.69	11469.50	15	85380.50	-1
53	4.33	-1.92	89432.69	11648.10	15	85155.50	-1
54	4.46	-1.67	94913.56	8089.64	15	84930.50	-1
55	4.21	-1.67	95247.06	11104.95	15	84705.50	-1
56	4.33	-1.54	93313.50	10473.00	15	84480.50	-1
57	4.33	-1.79	90510.50	13076.24	15	84255.50	-1
58	4.39	-1.67	93407.06	17586.04	15	84030.50	-1

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	1262.96	196.72	375.00	0.00	-375.00
2	10235.64	1541.02	825.00	8972.68	8147.68
3	19544.44	2371.52	1050.00	18281.48	17231.48
4	24653.71	4467.64	1275.00	23390.75	22115.75
5	31043.78	3621.24	1500.00	29780.82	28280.82
6	41203.11	5811.78	1725.00	39940.15	38215.15
7	44972.38	4979.88	1950.00	43709.42	41759.42
8	47401.64	6446.62	2175.00	46138.68	43963.68
9	49627.40	5375.32	2400.00	48364.44	45964.44
10	54759.73	10118.19	3075.00	53496.77	50421.77
11	63399.55	11589.88	3300.00	62136.59	58836.59
12	74351.69	8719.04	3525.00	73088.69	69563.69
13	81390.69	8651.13	3750.00	80127.69	76377.69
14	85786.44	14556.67	3975.00	84523.44	80548.44
15	87780.13	13375.19	4215.00	86517.13	82302.13
16	91047.50	15800.02	4440.00	89784.50	85344.50
17	92957.31	17514.27	4740.00	91694.31	86954.31
18	94502.81	11404.50	5220.00	93239.81	88019.81
19	96040.00	12169.78	5715.00	94777.00	89062.00
20	97631.06	7717.09	7515.00	96368.06	88853.06
21	99093.50	9961.00	9255.00	97830.50	88575.50

11) Example 6.11:

```
=================================================================
                   EXAMPLE 6.11 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 25                 ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15              BETAT (TEST)=0.800
DESIRED RATE OF RETURN=0.150            MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800          COST PER REPLICATE= 15.0
=================================================================
                     TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION= 11           DEGREES OF FREEDOM= 89.28
EST. SAVINGS RATE STD.DEV.=  241.6     BO, ACTUAL COSTS RATE= 2580.0
DIFF. BEING DETECTED=  461.1           BB, EST. SAVINGS RATE= 3074.1
DESIRED Z(BETAT)=-0.842                ACTUAL Z*=-0.447
NO. PTS FOR COST RATE= 3
=================================================================
                        CASE SIMULATED
=================================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=100000-(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=EXPONENTIAL
MAXIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.33	60.33	1262.96	196.72	25	-375.00	1
2	68.33	60.33	-8189.80	1310.49	15	-600.00	-1
3	52.33	60.33	10235.64	1541.02	15	8147.68	1
4	44.33	60.33	19544.44	2371.52	15	17231.48	1
5	36.33	60.33	24653.71	4467.64	15	22115.75	1
6	28.33	60.33	31043.78	3621.24	15	28280.82	1
7	20.33	60.33	41203.11	5811.78	15	38215.15	1
8	12.33	60.33	44972.38	4979.88	15	41759.42	1
9	4.33	60.33	47401.64	6446.62	15	43963.68	1
10	-3.67	60.33	49627.40	5375.32	15	45964.44	1
11	-11.67	60.33	43622.95	7428.11	15	45739.44	-1
12	-3.67	68.33	39745.24	3419.83	15	45514.44	-1
13	-3.67	52.33	54759.73	10118.19	15	50421.77	1
14	-3.67	44.33	63399.55	11589.88	15	58836.59	1
15	-3.67	36.33	74351.69	8719.04	15	69563.69	1
16	-3.67	28.33	81390.69	8651.13	15	76377.69	1
17	-3.67	20.33	85786.44	14556.67	15	80548.44	1
18	-3.67	12.33	87780.13	13375.19	16	82302.13	1
19	-3.67	4.33	91047.50	15800.02	15	85344.50	1
20	-3.67	-3.67	92957.31	17514.27	20	86954.31	1
21	-3.67	-11.67	90526.88	14681.10	16	86714.31	-1
22	4.33	-3.67	94502.81	11404.50	16	88019.81	1
23	12.33	-3.67	91035.06	10716.68	15	87794.81	-1
24	4.33	4.33	96040.00	12169.78	18	89062.00	1
25	4.33	12.33	93310.75	9710.90	15	88837.00	-1
26	12.33	4.33	85955.00	16424.09	15	88612.00	-1
27	-3.67	4.33	90883.13	15533.19	15	88387.00	-1
28	8.33	4.33	96281.88	8070.82	30	87937.00	-1
29	0.33	4.33	93833.38	13763.76	15	87712.00	-1
30	4.33	8.33	88447.56	16884.49	15	87487.00	-1
31	4.33	0.33	97631.06	7717.09	15	88853.06	1
32	4.33	-3.67	94123.13	12221.15	15	88628.06	-1
33	8.33	0.33	96907.63	9166.09	15	88403.06	-1
34	0.33	0.33	94511.56	14207.57	17	88148.06	-1
35	6.33	0.33	94901.56	12650.65	15	87923.06	-1

Example 6.11 (continued):

36	2.33	0.33	94751.88	16549.30	15	87698.06	-1
37	4.33	2.33	90848.56	20307.15	15	87473.06	-1
38	4.33	-1.67	99093.50	9961.00	24	88575.50	1
39	4.33	-3.67	88755.56	20797.08	15	88350.50	-1
40	6.33	-1.67	98201.31	9666.21	15	88125.50	-1
41	2.33	-1.67	96474.06	12247.17	15	87900.50	-1
42	5.33	-1.67	98285.00	10571.39	18	87630.50	-1
43	3.33	-1.67	97158.50	9452.21	15	87405.50	-1
44	4.33	-0.67	95585.38	16661.92	15	87180.50	-1
45	4.33	-2.67	94942.19	8140.41	15	86955.50	-1
46	4.83	-1.67	85369.31	27412.67	15	86730.50	-1
47	3.83	-1.67	94155.13	14389.94	15	86505.50	-1
48	4.33	-1.17	87598.38	21059.57	15	86280.50	-1
49	4.33	-2.17	90077.56	12985.35	15	86055.50	-1
50	4.58	-1.67	90189.13	22451.71	15	85830.50	-1
51	4.08	-1.67	87989.13	16320.45	15	85605.50	-1
52	4.33	-1.42	93180.69	11469.50	15	85380.50	-1
53	4.33	-1.92	89432.69	11648.10	15	85155.50	-1

TERMINATION PROCEDURE RESULTS:
==

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	1262.96	196.72	375.00	0.00	-375.00
2	10235.64	1541.02	825.00	8972.68	8147.68
3	19544.44	2371.52	1050.00	18281.48	17231.48
4	24653.71	4467.64	1275.00	23390.75	22115.75
5	31043.78	3621.24	1500.00	29780.82	28280.82
6	41203.11	5811.78	1725.00	39940.15	38215.15
7	44972.38	4979.88	1950.00	43709.42	41759.42
8	47401.64	6446.62	2175.00	46138.68	43963.68
9	49627.40	5375.32	2400.00	48364.44	45964.44
10	54759.73	10118.19	3075.00	53496.77	50421.77
11	63399.55	11589.88	3300.00	62136.59	58836.59
12	74351.69	8719.04	3525.00	73088.69	69563.69
13	81390.69	8651.13	3750.00	80127.69	76377.69
14	85786.44	14556.67	3975.00	84523.44	80548.44
15	87780.13	13375.19	4215.00	86517.13	82302.13
16	91047.50	15800.02	4440.00	89784.50	85344.50
17	92957.31	17514.27	4740.00	91694.31	86954.31
18	94502.81	11404.50	5220.00	93239.81	88019.81
19	96040.00	12169.78	5715.00	94777.00	89062.00
20	97631.06	7717.09	7515.00	96368.06	88853.06
21	99093.50	9961.00	9255.00	97830.50	88575.50

12) Example 6.12:

```
=================================================================
                    EXAMPLE 6.12 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.950
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
=================================================================
                      TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION=  8          DEGREES OF FREEDOM= 74.55
EST. SAVINGS RATE STD.DEV.=  413.0    BO, ACTUAL COSTS RATE= 1252.5
DIFF. BEING DETECTED=  334.0          BB, EST. SAVINGS RATE= 2226.5
DESIRED Z(BETAT)=-1.645               ACTUAL Z*=-1.532
NO. PTS FOR COST RATE= 3
=================================================================
                          CASE SIMULATED
=================================================================
DIFF=(MAX(BB;BO))*RATE
F(X1;X2)=100000-(|X1|**1.5+|X2|**1.5+50)*100
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=EXPONENTIAL
MAXIMIZATION ALGORITHM
SEARCH TECHNIQUE= ONE-AT-A-TIME
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	60.33	60.33	1262.96	196.72	25	-375.00	1
2	68.33	60.33	-8189.80	1310.49	15	-600.00	-1
3	52.33	60.33	10235.64	1541.02	15	8147.68	1
4	44.33	60.33	19544.44	2371.52	15	17231.48	1
5	36.33	60.33	24653.71	4467.64	15	22115.75	1
6	28.33	60.33	31043.78	3621.24	15	28280.82	1
7	20.33	60.33	41203.11	5811.78	15	38215.15	1
8	12.33	60.33	44972.38	4979.88	15	41759.42	1
9	4.33	60.33	47401.64	6446.62	15	43963.68	1
10	-3.67	60.33	49627.40	5375.32	15	45964.44	1
11	-11.67	60.33	43622.95	7428.11	15	45739.44	-1
12	-3.67	68.33	39745.24	3419.83	15	45514.44	-1
13	-3.67	52.33	54759.73	10118.19	15	50421.77	1
14	-3.67	44.33	63399.55	11589.88	15	58836.59	1
15	-3.67	36.33	74351.69	8719.04	15	69563.69	1
16	-3.67	28.33	81390.69	8651.13	15	76377.69	1
17	-3.67	20.33	85786.44	14556.67	15	80548.44	1
18	-3.67	12.33	87780.13	13375.19	16	82302.13	1
19	-3.67	4.33	91047.50	15800.02	15	85344.50	1
20	-3.67	-3.67	92957.31	17514.27	20	86954.31	1
21	-3.67	-11.67	90526.88	14681.10	16	86714.31	-1
22	4.33	-3.67	94502.81	11404.50	16	88019.81	1
23	12.33	-3.67	91035.06	10716.68	15	87794.81	-1
24	4.33	4.33	96040.00	12169.78	18	89062.00	1
25	4.33	12.33	93310.75	9710.90	15	88837.00	-1
26	12.33	4.33	85955.00	16424.09	15	88612.00	-1
27	-3.67	4.33	90883.13	15533.19	15	88387.00	-1
28	8.33	4.33	96281.88	8070.82	30	87937.00	-1
29	0.33	4.33	93833.38	13763.76	15	87712.00	-1
30	4.33	8.33	88447.56	16884.49	15	87487.00	-1
31	4.33	0.33	97631.06	7717.09	15	88853.06	1
32	4.33	-3.67	94123.13	12221.15	15	88628.06	-1
33	8.33	0.33	96907.63	9166.09	15	88403.06	-1
34	0.33	0.33	94511.56	14207.57	17	88148.06	-1

Example 6.12 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	1262.96	196.72	375.00	0.00	-375.00
2	10235.64	1541.02	825.00	8972.68	8147.68
3	19544.44	2371.52	1050.00	18281.48	17231.48
4	24653.71	4467.64	1275.00	23390.75	22115.75
5	31043.78	3621.24	1500.00	29780.82	28280.82
6	41203.11	5811.78	1725.00	39940.15	38215.15
7	44972.38	4979.88	1950.00	43709.42	41759.42
8	47401.64	6446.62	2175.00	46138.68	43963.68
9	49627.40	5375.32	2400.00	48364.44	45964.44
10	54759.73	10118.19	3075.00	53496.77	50421.77
11	63399.55	11589.88	3300.00	62136.59	58836.59
12	74351.69	8719.04	3525.00	73088.69	69563.69
13	81390.69	8651.13	3750.00	80127.69	76377.69
14	85786.44	14556.67	3975.00	84523.44	80548.44
15	87780.13	13375.19	4215.00	86517.13	82302.13
16	91047.50	15800.02	4440.00	89784.50	85344.50
17	92957.31	17514.27	4740.00	91694.31	86954.31
18	94502.81	11404.50	5220.00	93239.81	88019.81
19	96040.00	12169.78	5715.00	94777.00	89062.00
20	97631.06	7717.09	7515.00	96368.06	88853.06

13) Example 6.13:

```
===================================================================
                  EXAMPLE 6.13 - DATA SUMMARY
===================================================================
MIN. INITIAL SAMPLE= 25              ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15           BETAT (TEST)=0.500
DESIRED RATE OF RETURN=0.150         MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800       COST PER REPLICATE=  5.0
===================================================================
                  TEST PROCEDURE SUMMARY
===================================================================
NO.OF PTS. IN REGRESSION=  4        DEGREES OF FREEDOM= 27.07
EST. SAVINGS RATE STD.DEV.=  11.7   BO, ACTUAL COSTS RATE= 300.0
DIFF. BEING DETECTED=  49.0         BB, EST. SAVINGS RATE= 326.5
DESIRED Z(BETAT)= 0.000             ACTUAL Z*= 1.632
NO. PTS. FOR COST RATE= 3
===================================================================
                      CASE SIMULATED
===================================================================
DIFF=(MAX(BB;BO))*RATE
F=(((XX(1)**2+XX(2)**2)/1.8-(XX(1)*XX(2))/1.125+50)*10
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= PATTERN SEARCH
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	100.33	80.33	20449.38	3199.79	25	-125.00	1
2	108.33	80.33	24382.20	4265.22	15	-200.00	-1
3	92.33	80.33	18683.32	2546.02	15	1491.06	1
4	92.33	88.33	18191.58	2494.59	15	1907.80	1
5	92.33	96.33	19364.84	2904.93	15	1832.80	-1
6	76.33	96.33	18460.12	2192.97	21	1727.80	-1
7	84.33	104.33	20404.40	2777.87	15	1652.80	-1
8	84.33	88.33	17421.04	2427.60	15	2348.34	1
9	84.33	88.33	18342.74	3058.92	15	2273.34	-1
10	68.33	88.33	15179.30	2116.81	15	4440.08	1
11	68.33	96.33	19305.18	2777.45	15	4365.08	-1
12	68.33	80.33	13749.09	1336.98	15	5720.29	1
13	60.33	72.33	11525.14	2043.37	15	7869.24	1
14	60.33	80.33	13959.94	2168.38	15	7794.24	-1
15	60.33	64.33	9125.41	1723.01	15	10118.97	1
16	60.33	48.33	7954.57	1075.98	15	11214.81	1
17	60.33	56.33	8115.86	1563.81	15	11139.81	-1
18	60.33	40.33	7493.83	1119.15	15	11525.55	1
19	68.33	16.33	18548.15	4029.68	15	11450.55	-1
20	52.33	16.33	9175.83	1633.51	15	11375.55	-1
21	60.33	24.33	11752.82	1197.73	15	11300.55	-1
22	60.33	8.33	16729.40	2118.53	15	11225.55	-1
23	64.33	40.33	10132.02	1899.46	15	11150.55	-1
24	56.33	40.33	7232.81	1145.31	15	11336.57	1
25	56.33	44.33	6617.86	984.81	15	11876.52	1
26	56.33	48.33	6846.68	1155.77	15	11801.52	-1
27	48.33	48.33	6041.39	736.42	15	12302.98	1
28	48.33	52.33	6330.96	805.49	15	12227.98	-1
29	48.33	44.33	5293.40	800.81	15	12900.98	1
30	44.33	44.33	4764.89	573.47	15	13354.48	1
31	44.33	48.33	5419.57	589.45	15	13279.48	-1
32	44.33	40.33	4419.89	499.03	15	13549.48	1
33	44.33	36.33	4148.67	759.99	15	13745.71	1
34	44.33	40.33	4753.44	760.28	15	13670.71	-1
35	44.33	32.33	4567.74	1049.15	15	13595.71	-1

Example 6.13 (continued):

36	48.33	32.33	5602.28	729.01	15	13520.71	-1
37	40.33	32.33	3839.97	524.21	15	13754.41	1
38	40.33	36.33	3682.14	474.73	15	13837.23	1
39	40.33	36.33	3754.84	712.15	15	13762.23	-1
40	32.33	36.33	3286.74	507.45	15	14082.64	1
41	32.33	40.33	3534.64	485.26	15	14007.64	-1
42	32.33	32.33	2710.95	407.92	15	14508.43	1
43	28.33	28.33	2158.24	442.72	15	14986.13	1
44	28.33	32.33	2420.25	286.02	15	14911.13	-1
45	28.33	24.33	2295.83	349.51	15	14836.13	-1
46	28.33	24.33	2183.14	375.16	15	14761.13	-1
47	20.33	24.33	1710.17	221.44	15	15134.20	1
48	20.33	28.33	2210.51	270.88	15	15059.20	-1
49	20.33	20.33	1472.25	125.56	15	15222.12	1
50	16.33	12.33	1045.08	126.18	15	15574.30	1
51	16.33	16.33	1081.16	170.58	15	15499.30	-1
52	16.33	8.33	1228.22	184.47	15	15424.30	-1
53	16.33	4.33	1437.73	189.28	15	15349.30	-1
54	8.33	4.33	680.56	89.78	15	15638.82	1
55	8.33	8.33	674.46	108.29	15	15563.82	-1
56	8.33	0.33	898.36	102.46	15	15488.82	-1
57	4.33	-3.67	808.19	132.10	15	15413.82	-1
58	-3.67	-3.67	505.42	67.48	15	15513.96	1

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	20449.38	3199.79	125.00	0.0	-125.00
2	18683.32	2546.02	275.00	1766.06	1491.06
3	18191.58	2494.59	350.00	2257.80	1907.80
4	17421.04	2427.60	680.00	3028.34	2348.34
5	15179.30	2116.81	830.00	5270.08	4440.08
6	13749.09	1336.98	980.00	6700.29	5720.29
7	11525.14	2043.37	1055.00	8924.24	7869.24
8	9125.41	1723.01	1205.00	11323.97	10118.97
9	7954.57	1075.98	1280.00	12494.81	11214.81
10	7493.83	1119.15	1430.00	12955.55	11525.55
11	7232.81	1145.31	1880.00	13216.57	11336.57
12	6617.86	984.81	1955.00	13831.52	11876.52
13	6041.39	736.42	2105.00	14407.98	12302.98
14	5293.40	800.81	2255.00	15155.98	12900.98
15	4764.89	573.47	2330.00	15684.48	13354.48
16	4419.89	499.03	2480.00	16029.48	13549.48
17	4148.67	759.99	2555.00	16300.71	13745.71
18	3839.97	524.21	2855.00	16609.41	13754.41
19	3682.14	474.73	2930.00	16767.23	13837.23
20	3286.74	507.45	3080.00	17162.64	14082.64
21	2710.95	407.92	3230.00	17738.43	14508.43
22	2158.24	442.72	3305.00	18291.13	14986.13
23	1710.17	221.44	3605.00	18739.20	15134.20
24	1472.25	125.56	3755.00	18977.12	15222.12
25	1045.08	126.18	3830.00	19404.30	15574.30
26	680.56	89.78	4130.00	19768.82	15638.82
27	505.42	67.48	4430.00	19943.96	15513.96

14) Example 6.14:

```
=====================================================================
                 EXAMPLE 6.14 - DATA SUMMARY
=====================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.700
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE=  5.0
=====================================================================
                   TEST PROCEDURE SUMMARY
=====================================================================
NO.OF PTS. IN REGRESSION=  4          DEGREES OF FREEDOM= 27.07
EST. SAVINGS RATE STD.DEV.=  11.7     BO, ACTUAL COSTS RATE= 300.0
DIFF. BEING DETECTED=  49.0           BB, EST. SAVINGS RATE= 326.5
DESIRED Z(BETAT)=-0.524               ACTUAL Z*= 1.632
NO. PTS. FOR COST RATE= 3
=====================================================================
                        CASE SIMULATED
=====================================================================
DIFF=(MAX(BB;BO))*RATE
F=(((XX(1)**2+XX(2)**2)/1.8-(XX(1)*XX(2))/1.125+50)*10
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= PATTERN SEARCH
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	100.33	80.33	20449.38	3199.79	25	-125.00	1
2	108.33	80.33	24382.20	4265.22	15	-200.00	-1
3	92.33	80.33	18683.32	2546.02	15	1491.06	1
4	92.33	88.33	18191.58	2494.59	15	1907.80	1
5	92.33	96.33	19364.84	2904.93	15	1832.80	-1
6	76.33	96.33	18460.12	2192.97	21	1727.80	-1
7	84.33	104.33	20404.40	2777.87	15	1652.80	-1
8	84.33	88.33	17421.04	2427.60	15	2348.34	1
9	84.33	88.33	18342.74	3058.92	15	2273.34	-1
10	68.33	88.33	15179.30	2116.81	15	4440.08	1
11	68.33	96.33	19305.18	2777.45	15	4365.08	-1
12	68.33	80.33	13749.09	1336.98	15	5720.29	1
13	60.33	72.33	11525.14	2043.37	15	7869.24	1
14	60.33	80.33	13959.94	2168.38	15	7794.24	-1
15	60.33	64.33	9125.41	1723.01	15	10118.97	1
16	60.33	48.33	7954.57	1075.98	15	11214.81	1
17	60.33	56.33	8115.86	1563.81	15	11139.81	-1
18	60.33	40.33	7493.83	1119.15	15	11525.55	1
19	68.33	16.33	18548.15	4029.68	15	11450.55	-1
20	52.33	16.33	9175.83	1633.51	15	11375.55	-1
21	60.33	24.33	11752.82	1197.73	15	11300.55	-1
22	60.33	8.33	16729.40	2118.53	15	11225.55	-1
23	64.33	40.33	10132.02	1899.46	15	11150.55	-1
24	56.33	40.33	7232.81	1145.31	15	11336.57	1
25	56.33	44.33	6617.86	984.81	15	11876.52	1
26	56.33	48.33	6846.68	1155.77	15	11801.52	-1
27	48.33	48.33	6041.39	736.42	15	12302.98	1
28	48.33	52.33	6330.96	805.49	15	12227.98	-1
29	48.33	44.33	5293.40	800.81	15	12900.98	1
30	44.33	44.33	4764.89	573.47	15	13354.48	1
31	44.33	48.33	5419.57	589.45	15	13279.48	-1
32	44.33	40.33	4419.89	499.03	15	13549.48	1
33	44.33	36.33	4148.67	759.99	15	13745.71	1
34	44.33	40.33	4753.44	760.28	15	13670.71	-1
35	44.33	32.33	4567.74	1049.15	15	13595.71	-1

Example 6.14 (continued):

36	48.33	32.33	5602.28	729.01	15	13520.71	-1
37	40.33	32.33	3839.97	524.21	15	13754.41	1
38	40.33	36.33	3682.14	474.73	15	13837.23	1
39	40.33	36.33	3754.84	712.15	15	13762.23	-1
40	32.33	36.33	3286.74	507.45	15	14082.64	1
41	32.33	40.33	3534.64	485.26	15	14007.64	-1
42	32.33	32.33	2710.95	407.92	15	14508.43	1
43	28.33	28.33	2158.24	442.72	15	14986.13	1
44	28.33	32.33	2420.25	286.02	15	14911.13	-1
45	28.33	24.33	2295.83	349.51	15	14836.13	-1
46	28.33	24.33	2183.14	375.16	15	14761.13	-1
47	20.33	24.33	1710.17	221.44	15	15134.20	1
48	20.33	28.33	2210.51	270.88	15	15059.20	-1
49	20.33	20.33	1472.25	125.56	15	15222.12	1
50	16.33	12.33	1045.08	126.18	15	15574.30	1
51	16.33	16.33	1081.16	170.58	15	15499.30	-1
52	16.33	8.33	1228.22	184.47	15	15424.30	-1
53	16.33	4.33	1437.73	189.28	15	15349.30	-1
54	8.33	4.33	680.56	89.78	15	15638.82	1
55	8.33	8.33	674.46	108.29	15	15563.82	-1
56	8.33	0.33	898.36	102.46	15	15488.82	-1
57	4.33	-3.67	808.19	132.10	15	15413.82	-1
58	-3.67	-3.67	505.42	67.48	15	15513.96	1

TERMINATION PROCEDURE RESULTS:
==============================

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	20449.38	3199.79	125.00	0.0	-125.00
2	18683.32	2546.02	275.00	1766.06	1491.06
3	18191.58	2494.59	350.00	2257.80	1907.80
4	17421.04	2427.60	680.00	3028.34	2348.34
5	15179.30	2116.81	830.00	5270.08	4440.08
6	13749.09	1336.98	980.00	6700.29	5720.29
7	11525.14	2043.37	1055.00	8924.24	7869.24
8	9125.41	1723.01	1205.00	11323.97	10118.97
9	7954.57	1075.98	1280.00	12494.81	11214.81
10	7493.83	1119.15	1430.00	12955.55	11525.55
11	7232.81	1145.31	1880.00	13216.57	11336.57
12	6617.86	984.81	1955.00	13831.52	11876.52
13	6041.39	736.42	2105.00	14407.98	12302.98
14	5293.40	800.81	2255.00	15155.98	12900.98
15	4764.89	573.47	2330.00	15684.48	13354.48
16	4419.89	499.03	2480.00	16029.48	13549.48
17	4148.67	759.99	2555.00	16300.71	13745.71
18	3839.97	524.21	2855.00	16609.41	13754.41
19	3682.14	474.73	2930.00	16767.23	13837.23
20	3286.74	507.45	3080.00	17162.64	14082.64
21	2710.95	407.92	3230.00	17738.43	14508.43
22	2158.24	442.72	3305.00	18291.13	14986.13
23	1710.17	221.44	3605.00	18739.20	15134.20
24	1472.25	125.56	3755.00	18977.12	15222.12
25	1045.08	126.18	3830.00	19404.30	15574.30
26	680.56	89.78	4130.00	19768.82	15638.82
27	505.42	67.48	4430.00	19943.96	15513.96

15) Example 6.15:

```
==================================================================
                  EXAMPLE 6.15 - DATA SUMMARY
==================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.800
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE=  5.0
==================================================================
                      TEST PROCEDURE SUMMARY
==================================================================
NO.OF PTS. IN REGRESSION=  4          DEGREES OF FREEDOM= 27.07
EST. SAVINGS RATE STD.DEV.=   11.7    BO, ACTUAL COSTS RATE= 300.0
DIFF. BEING DETECTED=  49.0           BB, EST. SAVINGS RATE= 326.5
DESIRED Z(BETAT)=-0.842               ACTUAL Z*= 1.632
NO. PTS. FOR COST RATE= 3
==================================================================
                           CASE SIMULATED
==================================================================
DIFF=(MAX(BB;BO))*RATE
F=(((XX(1)**2+XX(2)**2)/1.8-(XX(1)*XX(2))/1.125+50)*10
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= PATTERN SEARCH
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	100.33	80.33	20449.38	3199.79	25	-125.00	1
2	108.33	80.33	24382.20	4265.22	15	-200.00	-1
3	92.33	80.33	18683.32	2546.02	15	1491.06	1
4	92.33	88.33	18191.58	2494.59	15	1907.80	1
5	92.33	96.33	19364.84	2904.93	15	1832.80	-1
6	76.33	96.33	18460.12	2192.97	21	1727.80	-1
7	84.33	104.33	20404.40	2777.87	15	1652.80	-1
8	84.33	88.33	17421.04	2427.60	15	2348.34	1
9	84.33	88.33	18342.74	3058.92	15	2273.34	-1
10	68.33	88.33	15179.30	2116.81	15	4440.08	1
11	68.33	96.33	19305.18	2777.45	15	4365.08	-1
12	68.33	80.33	13749.09	1336.98	15	5720.29	1
13	60.33	72.33	11525.14	2043.37	15	7869.24	1
14	60.33	80.33	13959.94	2168.38	15	7794.24	-1
15	60.33	64.33	9125.41	1723.01	15	10118.97	1
16	60.33	48.33	7954.57	1075.98	15	11214.81	1
17	60.33	56.33	8115.86	1563.81	15	11139.81	-1
18	60.33	40.33	7493.83	1119.15	15	11525.55	1
19	68.33	16.33	18548.15	4029.68	15	11450.55	-1
20	52.33	16.33	9175.83	1633.51	15	11375.55	-1
21	60.33	24.33	11752.82	1197.73	15	11300.55	-1
22	60.33	8.33	16729.40	2118.53	15	11225.55	-1
23	64.33	40.33	10132.02	1899.46	15	11150.55	-1
24	56.33	40.33	7232.81	1145.31	15	11336.57	1
25	56.33	44.33	6617.86	984.81	15	11876.52	1
26	56.33	48.33	6846.68	1155.77	15	11801.52	-1
27	48.33	48.33	6041.39	736.42	15	12302.98	1
28	48.33	52.33	6330.96	805.49	15	12227.98	-1
29	48.33	44.33	5293.40	800.81	15	12900.98	1
30	44.33	44.33	4764.89	573.47	15	13354.48	1
31	44.33	48.33	5419.57	589.45	15	13279.48	-1
32	44.33	40.33	4419.89	499.03	15	13549.48	1
33	44.33	36.33	4148.67	759.99	15	13745.71	1
34	44.33	40.33	4753.44	760.28	15	13670.71	-1
35	44.33	32.33	4567.74	1049.15	15	13595.71	-1

Example 6.15 (continued):

36	48.33	32.33	5602.28	729.01	15	13520.71	-1
37	40.33	32.33	3839.97	524.21	15	13754.41	1
38	40.33	36.33	3682.14	474.73	15	13837.23	1
39	40.33	36.33	3754.84	712.15	15	13762.23	-1
40	32.33	36.33	3286.74	507.45	15	14082.64	1
41	32.33	40.33	3534.64	485.26	15	14007.64	-1
42	32.33	32.33	2710.95	407.92	15	14508.43	1
43	28.33	28.33	2158.24	442.72	15	14986.13	1
44	28.33	32.33	2420.25	286.02	15	14911.13	-1
45	28.33	24.33	2295.83	349.51	15	14836.13	-1
46	28.33	24.33	2183.14	375.16	15	14761.13	-1
47	20.33	24.33	1710.17	221.44	15	15134.20	1
48	20.33	28.33	2210.51	270.88	15	15059.20	-1
49	20.33	20.33	1472.25	125.56	15	15222.12	1
50	16.33	12.33	1045.08	126.18	15	15574.30	1
51	16.33	16.33	1081.16	170.58	15	15499.30	-1
52	16.33	8.33	1228.22	184.47	15	15424.30	-1
53	16.33	4.33	1437.73	189.28	15	15349.30	-1
54	8.33	4.33	680.56	89.78	15	15638.82	1
55	8.33	8.33	674.46	108.29	15	15563.82	-1
56	8.33	0.33	898.36	102.46	15	15488.82	-1
57	4.33	-3.67	808.19	132.10	15	15413.82	-1
58	-3.67	-3.67	505.42	67.48	15	15513.96	1

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	20449.38	3199.79	125.00	0.0	-125.00
2	18683.32	2546.02	275.00	1766.06	1491.06
3	18191.58	2494.59	350.00	2257.80	1907.80
4	17421.04	2427.60	680.00	3028.34	2348.34
5	15179.30	2116.81	830.00	5270.08	4440.08
6	13749.09	1336.98	980.00	6700.29	5720.29
7	11525.14	2043.37	1055.00	8924.24	7869.24
8	9125.41	1723.01	1205.00	11323.97	10118.97
9	7954.57	1075.98	1280.00	12494.81	11214.81
10	7493.83	1119.15	1430.00	12955.55	11525.55
11	7232.81	1145.31	1880.00	13216.57	11336.57
12	6617.86	984.81	1955.00	13831.52	11876.52
13	6041.39	736.42	2105.00	14407.98	12302.98
14	5293.40	800.81	2255.00	15155.98	12900.98
15	4764.89	573.47	2330.00	15684.48	13354.48
16	4419.89	499.03	2480.00	16029.48	13549.48
17	4148.67	759.99	2555.00	16300.71	13745.71
18	3839.97	524.21	2855.00	16609.41	13754.41
19	3682.14	474.73	2930.00	16767.23	13837.23
20	3286.74	507.45	3080.00	17162.64	14082.64
21	2710.95	407.92	3230.00	17738.43	14508.43
22	2158.24	442.72	3305.00	18291.13	14986.13
23	1710.17	221.44	3605.00	18739.20	15134.20
24	1472.25	125.56	3755.00	18977.12	15222.12
25	1045.08	126.18	3830.00	19404.30	15574.30
26	680.56	89.78	4130.00	19768.82	15638.82
27	505.42	67.48	4430.00	19943.96	15513.96

16) Example 6.16:

```
=====================================================================
                   EXAMPLE 6.16 - DATA SUMMARY
=====================================================================
MIN. INITIAL SAMPLE= 25                  ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15               BETAT (TEST)=0.950
DESIRED RATE OF RETURN=0.150             MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800           COST PER REPLICATE=  5.0
=====================================================================
                       TEST PROCEDURE SUMMARY
=====================================================================
NO.OF PTS. IN REGRESSION=  4            DEGREES OF FREEDOM= 35.92
EST. SAVINGS RATE STD.DEV.=  64.6       BO, ACTUAL COSTS RATE= 187.5
DIFF. BEING DETECTED=  45.7             BB, EST. SAVINGS RATE= 304.6
DESIRED Z(BETAT)=-1.645                 ACTUAL Z*=-1.644
NO. PTS. FOR COST RATE= 3
=====================================================================
                           CASE SIMULATED
=====================================================================
DIFF=(MAX(BB;BO))*RATE
F=(((XX(1)**2+XX(2)**2)/1.8-(XX(1)*XX(2))/1.125+50)*10
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MINIMIZATION ALGORITHM
SEARCH TECHNIQUE= PATTERN SEARCH
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	100.33	80.33	20449.38	3199.79	25	-125.00	1
2	108.33	80.33	24382.20	4265.22	15	-200.00	-1
3	92.33	80.33	18683.32	2546.02	15	1491.06	1
4	92.33	88.33	18191.58	2494.59	15	1907.80	1
5	92.33	96.33	19364.84	2904.93	15	1832.80	-1
6	76.33	96.33	18460.12	2192.97	21	1727.80	-1
7	84.33	104.33	20404.40	2777.87	15	1652.80	-1
8	84.33	88.33	17421.04	2427.60	15	2348.34	1
9	84.33	88.33	18342.74	3058.92	15	2273.34	-1
10	68.33	88.33	15179.30	2116.81	15	4440.08	1
11	68.33	96.33	19305.18	2777.45	15	4365.08	-1
12	68.33	80.33	13749.09	1336.98	15	5720.29	1
13	60.33	72.33	11525.14	2043.37	15	7869.24	1
14	60.33	80.33	13959.94	2168.38	15	7794.24	-1
15	60.33	64.33	9125.41	1723.01	15	10118.97	1
16	60.33	48.33	7954.57	1075.98	15	11214.81	1
17	60.33	56.33	8115.86	1563.81	15	11139.81	-1
18	60.33	40.33	7493.83	1119.15	15	11525.55	1
19	68.33	16.33	18548.15	4029.68	15	11450.55	-1
20	52.33	16.33	9175.83	1633.51	15	11375.55	-1
21	60.33	24.33	11752.82	1197.73	15	11300.55	-1
22	60.33	8.33	16729.40	2118.53	15	11225.55	-1
23	64.33	40.33	10132.02	1899.46	15	11150.55	-1
24	56.33	40.33	7232.81	1145.31	15	11336.57	1
25	56.33	44.33	6617.86	984.81	15	11876.52	1
26	56.33	48.33	6846.68	1155.77	15	11801.52	-1
27	48.33	48.33	6041.39	736.42	15	12302.98	1
28	48.33	52.33	6330.96	805.49	15	12227.98	-1
29	48.33	44.33	5293.40	800.81	15	12900.98	1

Example 6.16 (continued):

30	44.33	44.33	4764.89	573.47	15	13354.48	1
31	44.33	48.33	5419.57	589.45	15	13279.48	-1
32	44.33	40.33	4419.89	499.03	15	13549.48	1
33	44.33	36.33	4148.67	759.99	15	13745.71	1
34	44.33	40.33	4753.44	760.28	15	13670.71	-1
35	44.33	32.33	4567.74	1049.15	15	13595.71	-1
36	48.33	32.33	5602.28	729.01	15	13520.71	-1
37	40.33	32.33	3839.97	524.21	15	13754.41	1

TERMINATION PROCEDURE RESULTS:
==============================

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	20449.38	3199.79	125.00	0.0	-125.00
2	18683.32	2546.02	275.00	1766.06	1491.06
3	18191.58	2494.59	350.00	2257.80	1907.80
4	17421.04	2427.60	680.00	3028.34	2348.34
5	15179.30	2116.81	830.00	5270.08	4440.08
6	13749.09	1336.98	980.00	6700.29	5720.29
7	11525.14	2043.37	1055.00	8924.24	7869.24
8	9125.41	1723.01	1205.00	11323.97	10118.97
9	7954.57	1075.98	1280.00	12494.81	11214.81
10	7493.83	1119.15	1430.00	12955.55	11525.55
11	7232.81	1145.31	1880.00	13216.57	11336.57
12	6617.86	984.81	1955.00	13831.52	11876.52
13	6041.39	736.42	2105.00	14407.98	12302.98
14	5293.40	800.81	2255.00	15155.98	12900.98
15	4764.89	573.47	2330.00	15684.48	13354.48
16	4419.89	499.03	2480.00	16029.48	13549.48
17	4148.67	759.99	2555.00	16300.71	13745.71
18	3839.97	524.21	2855.00	16609.41	13754.41

17) Example 6.17:

```
=================================================================
                EXAMPLE 6.17 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.500
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE=  5.0
=================================================================
                  TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION= 13          DEGREES OF FREEDOM= 78.43
EST. SAVINGS RATE STD.DEV.=   89.3    BO, ACTUAL COSTS RATE= 1397.5
DIFF. BEING DETECTED=  236.5          BB, EST. SAVINGS RATE= 1576.7
DESIRED Z(BETAT)=-0.000               ACTUAL Z*= 0.275
NO. PTS FOR COST RATE= 3
=================================================================
                      CASE SIMULATED
=================================================================
SEARCH TECHNIQUE=PATTERN SEARCH
DIFF=(MAX(BB;BO))*RATE
FUNCTION WITH ELIPTIC CONTOURS
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MAXIMIZATION ALGORITHM
F=40000-
   -((XX(1)**2+XX(2)**2)/1.8-(XX(1)*XX(2))/1.125+50)*10
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) \$	STD(F) \$	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	100.3300	80.3300	19206.60	3201.57	25	-125.00	1
2	108.3300	80.3300	16075.19	2229.25	15	-200.00	-1
3	92.3300	80.3300	22077.48	3380.16	15	2595.88	1
4	92.3300	88.3300	22825.93	3394.07	15	3269.32	1
5	92.3300	96.3300	18589.48	3305.06	15	3194.32	-1
6	76.3300	96.3300	19052.92	1784.88	15	3119.32	-1
7	84.3300	104.3300	19160.35	2693.71	15	3044.32	-1
8	84.3300	88.3300	23466.00	3457.73	15	3609.40	1
9	84.3300	88.3300	22929.53	4046.53	16	3529.40	-1
10	68.3300	88.3300	25484.49	2934.84	15	5472.89	1
11	68.3300	96.3300	20153.32	3018.63	15	5397.89	-1
12	68.3300	80.3300	27593.67	3085.02	15	7432.07	1
13	60.3300	72.3300	28338.00	5137.43	16	8096.40	1
14	60.3300	80.3300	26036.66	3738.58	15	8021.40	-1
15	60.3300	64.3300	31839.21	4316.96	15	11447.61	1
16	60.3300	48.3300	32318.50	3765.43	16	11846.90	1
17	60.3300	56.3300	31866.87	5358.47	15	11771.90	-1
18	60.3300	40.3300	31369.24	5742.47	15	11696.90	-1
19	68.3300	32.3300	26958.20	4107.09	15	11621.90	-1
20	52.3300	32.3300	33562.41	6791.18	15	12790.81	1
21	52.3300	40.3300	36368.82	4744.52	15	15522.22	1
22	52.3300	32.3300	31798.02	5285.46	15	15447.22	-1
23	36.3300	32.3300	37736.82	4916.08	15	16740.22	1
24	36.3300	40.3300	37222.00	5733.41	16	16660.22	-1
25	36.3300	24.3300	37101.20	5172.81	15	16585.22	-1
26	28.3300	24.3300	38836.60	6001.02	15	17610.00	1
27	28.3300	32.3300	36856.78	5697.69	15	17535.00	-1
28	28.3300	16.3300	37266.04	5119.91	15	17460.00	-1
29	28.3300	16.3300	39488.64	5329.46	15	18037.04	1
30	28.3300	24.3300	38481.32	4935.21	15	17962.04	-1

Example 6.17 (continued):

31	28.3300	8.3300	34719.30	6540.38	15	17887.04	-1
32	36.3300	8.3300	35286.79	4761.07	15	17812.04	-1
33	20.3300	8.3300	39016.65	5516.52	18	17722.04	-1
34	28.3300	16.3300	39025.38	4704.57	17	17637.04	-1
35	28.3300	0.3300	36535.05	5750.72	15	17562.04	-1
36	32.3300	16.3300	36319.46	5588.71	15	17487.04	-1
37	24.3300	16.3300	38461.73	6639.87	15	17412.04	-1
38	28.3300	20.3300	38259.08	6950.06	17	17327.04	-1
39	28.3300	12.3300	38575.78	5480.21	16	17247.04	-1
40	30.3300	16.3300	38150.21	6810.31	16	17167.04	-1
41	26.3300	16.3300	39101.02	5008.66	24	17047.04	-1
42	28.3300	18.3300	38912.44	6180.82	16	16967.04	-1
43	28.3300	14.3300	40363.23	5943.27	15	17766.63	1
44	30.3300	12.3300	36620.45	3880.70	15	17691.63	-1
45	26.3300	12.3300	39011.70	5653.15	15	17616.63	-1
46	28.3300	14.3300	37463.68	4784.37	15	17541.63	-1
47	28.3300	10.3300	35435.17	8311.15	15	17466.63	-1
48	29.3300	14.3300	37284.62	5449.21	15	17391.63	-1
49	27.3300	14.3300	35914.11	6226.61	15	17316.63	-1
50	28.3300	15.3300	36483.14	6016.36	15	17241.63	-1
51	28.3300	13.3300	37196.37	6997.38	15	17166.63	-1
52	28.8300	14.3300	34673.57	7556.82	15	17091.63	-1
53	27.8300	14.3300	38698.03	6752.16	15	17016.63	-1
54	28.3300	14.8300	35856.59	4740.45	15	16941.63	-1
55	28.3300	13.8300	36400.96	4271.66	15	16866.63	-1
56	28.5800	14.3300	37561.15	6667.94	15	16791.63	-1
57	28.0800	14.3300	36789.52	4138.17	15	16716.63	-1
58	28.3300	14.5800	34682.59	3687.13	15	16641.63	-1
59	28.3300	14.0800	38300.32	6419.08	15	16566.63	-1
60	28.4550	14.3300	38711.69	5560.19	15	16491.63	-1
61	28.2050	14.3300	35295.66	5161.63	15	16416.63	-1
62	28.3300	14.4550	36544.68	6448.23	15	16341.63	-1
63	28.3300	14.2050	38617.61	5011.65	15	16266.63	-1
64	28.3925	14.3300	37870.39	4954.46	15	16191.63	-1
65	28.2675	14.3300	38747.80	5172.67	15	16116.63	-1

TERMINATION PROCEDURE RESULTS:
======================================

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	19206.60	3201.57	125.00	0.00	-125.00
2	22077.48	3380.16	275.00	2870.88	2595.88
3	22825.93	3394.07	350.00	3619.32	3269.32
4	23466.00	3457.73	650.00	4259.40	3609.40
5	25484.49	2934.84	805.00	6277.89	5472.89
6	27593.67	3085.02	955.00	8387.07	7432.07
7	28338.00	5137.43	1035.00	9131.40	8096.40
8	31839.21	4316.96	1185.00	12632.61	11447.61
9	32318.50	3765.43	1265.00	13111.90	11846.90
10	33562.41	6791.18	1565.00	14355.81	12790.81
11	36368.82	4744.52	1640.00	17162.22	15522.22
12	37736.82	4916.08	1790.00	18530.22	16740.22
13	38836.60	6001.02	2020.00	19630.00	17610.00
14	39488.64	5329.46	2245.00	20282.04	18037.04
15	40363.23	5943.27	3390.00	21156.63	17766.63

18) Example 6.18:

```
================================================================
                 EXAMPLE 6.18 - DATA SUMMARY
================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.700
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE=  5.0
================================================================
                    TEST PROCEDURE SUMMARY
================================================================
NO.OF PTS. IN REGRESSION= 11          DEGREES OF FREEDOM= 69.23
EST. SAVINGS RATE STD.DEV.=  116.4    BO, ACTUAL COSTS RATE= 1285.0
DIFF. BEING DETECTED=  230.9          BB, EST. SAVINGS RATE= 1539.2
DESIRED Z(BETAT)=-0.524               ACTUAL Z*=-0.381
NO. PTS FOR COST RATE= 3
================================================================
                        CASE SIMULATED
================================================================
SEARCH TECHNIQUE=PATTERN SEARCH
DIFF=(MAX(BB;BO))*RATE
FUNCTION WITH ELIPTIC CONTOURS
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MAXIMIZATION ALGORITHM
F=40000-
   -((XX(1)**2+XX(2)**2)/1.8-(XX(1)*XX(2))/1.125+50)*10
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	100.3300	80.3300	19206.60	3201.57	25	-125.00	1
2	108.3300	80.3300	16075.19	2229.25	15	-200.00	-1
3	92.3300	80.3300	22077.48	3380.16	15	2595.88	1
4	92.3300	88.3300	22825.93	3394.07	15	3269.32	1
5	92.3300	96.3300	18589.48	3305.06	15	3194.32	-1
6	76.3300	96.3300	19052.92	1784.88	15	3119.32	-1
7	84.3300	104.3300	19160.35	2693.71	15	3044.32	-1
8	84.3300	88.3300	23466.00	3457.73	15	3609.40	1
9	84.3300	88.3300	22929.53	4046.53	16	3529.40	-1
10	68.3300	88.3300	25484.49	2934.84	15	5472.89	1
11	68.3300	96.3300	20153.32	3018.63	15	5397.89	-1
12	68.3300	80.3300	27593.67	3085.02	15	7432.07	1
13	60.3300	72.3300	28338.00	5137.43	16	8096.40	1
14	60.3300	80.3300	26036.66	3738.58	15	8021.40	-1
15	60.3300	64.3300	31839.21	4316.96	15	11447.61	1
16	60.3300	48.3300	32318.50	3765.43	16	11846.90	1
17	60.3300	56.3300	31866.87	5358.47	15	11771.90	-1
18	60.3300	40.3300	31369.24	5742.47	15	11696.90	-1
19	68.3300	32.3300	26958.20	4107.09	15	11621.90	-1
20	52.3300	32.3300	33562.41	6791.18	15	12790.81	1
21	52.3300	40.3300	36368.82	4744.52	15	15522.22	1
22	52.3300	32.3300	31798.02	5285.46	15	15447.22	-1
23	36.3300	32.3300	37736.82	4916.08	15	16740.22	1
24	36.3300	40.3300	37222.00	5733.41	16	16660.22	-1
25	36.3300	24.3300	37101.20	5172.81	15	16585.22	-1
26	28.3300	24.3300	38836.60	6001.02	15	17610.00	1
27	28.3300	32.3300	36856.78	5697.69	15	17535.00	-1
28	28.3300	16.3300	37266.04	5119.91	15	17460.00	-1
29	28.3300	16.3300	39488.64	5329.46	15	18037.04	1
30	28.3300	24.3300	38481.32	4935.21	15	17962.04	-1
31	28.3300	8.3300	34719.30	6540.38	15	17887.04	-1

Example 6.18 (continued):

32	36.3300	8.3300	35286.79	4761.07	15	17812.04	-1
33	20.3300	8.3300	39016.65	5516.52	18	17722.04	-1
34	28.3300	16.3300	39025.38	4704.57	17	17637.04	-1
35	28.3300	0.3300	36535.05	5750.72	15	17562.04	-1
36	32.3300	16.3300	36319.46	5588.71	15	17487.04	-1
37	24.3300	16.3300	38461.73	6639.87	15	17412.04	-1
38	28.3300	20.3300	38259.08	6950.06	17	17327.04	-1
39	28.3300	12.3300	38575.78	5480.21	16	17247.04	-1
40	30.3300	16.3300	38150.21	6810.31	16	17167.04	-1
41	26.3300	16.3300	39101.02	5008.66	24	17047.04	-1
42	28.3300	18.3300	38912.44	6180.82	16	16967.04	-1
43	28.3300	14.3300	40363.23	5943.27	15	17766.63	1
44	30.3300	12.3300	36620.45	3880.70	15	17691.63	-1
45	26.3300	12.3300	39011.70	5653.15	15	17616.63	-1
46	28.3300	14.3300	37463.68	4784.37	15	17541.63	-1
47	28.3300	10.3300	35435.17	8311.15	15	17466.63	-1
48	29.3300	14.3300	37284.62	5449.21	15	17391.63	-1
49	27.3300	14.3300	35914.11	6226.61	15	17316.63	-1
50	28.3300	15.3300	36483.14	6016.36	15	17241.63	-1
51	28.3300	13.3300	37196.37	6997.38	15	17166.63	-1
52	28.8300	14.3300	34673.57	7556.82	15	17091.63	-1
53	27.8300	14.3300	38698.03	6752.16	15	17016.63	-1
54	28.3300	14.8300	35856.59	4740.45	15	16941.63	-1
55	28.3300	13.8300	36400.96	4271.66	15	16866.63	-1
56	28.5800	14.3300	37561.15	6667.94	15	16791.63	-1
57	28.0800	14.3300	36789.52	4138.17	15	16716.63	-1
58	28.3300	14.5800	34682.59	3687.13	15	16641.63	-1
59	28.3300	14.0800	38300.32	6419.08	15	16566.63	-1
60	28.4550	14.3300	38711.69	5560.19	15	16491.63	-1
61	28.2050	14.3300	35295.66	5161.63	15	16416.63	-1
62	28.3300	14.4550	36544.68	6448.23	15	16341.63	-1

TERMINATION PROCEDURE RESULTS:
====================================

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	19206.60	3201.57	125.00	0.00	-125.00
2	22077.48	3380.16	275.00	2870.88	2595.88
3	22825.93	3394.07	350.00	3619.32	3269.32
4	23466.00	3457.73	650.00	4259.40	3609.40
5	25484.49	2934.84	805.00	6277.89	5472.89
6	27593.67	3085.02	955.00	8387.07	7432.07
7	28338.00	5137.43	1035.00	9131.40	8096.40
8	31839.21	4316.96	1185.00	12632.61	11447.61
9	32318.50	3765.43	1265.00	13111.90	11846.90
10	33562.41	6791.18	1565.00	14355.81	12790.81
11	36368.82	4744.52	1640.00	17162.22	15522.22
12	37736.82	4916.08	1790.00	18530.22	16740.22
13	38836.60	6001.02	2020.00	19630.00	17610.00
14	39488.64	5329.46	2245.00	20282.04	18037.04
15	40363.23	5943.27	3390.00	21156.63	17766.63

19) Example 6.19:

```
================================================================
                  EXAMPLE 6.19 - DATA SUMMARY
================================================================
MIN. INITIAL SAMPLE= 25                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15             BETAT (TEST)=0.800
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE=  5.0
================================================================
                     TEST PROCEDURE SUMMARY
================================================================
NO.OF PTS. IN REGRESSION= 10          DEGREES OF FREEDOM= 68.12
EST. SAVINGS RATE STD.DEV.=  140.0    BO, ACTUAL COSTS RATE= 1172.5
DIFF. BEING DETECTED=  224.6          BB, EST. SAVINGS RATE= 1497.1
DESIRED Z(BETAT)=-0.842               ACTUAL Z*=-0.754
NO. PTS FOR COST RATE= 3
================================================================
                        CASE SIMULATED
================================================================
SEARCH TECHNIQUE=PATTERN SEARCH
DIFF=(MAX(BB;BO))*RATE
FUNCTION WITH ELIPTIC CONTOURS
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MAXIMIZATION ALGORITHM
F=40000-
   -((XX(1)**2+XX(2)**2)/1.8-(XX(1)*XX(2))/1.125+50)*10
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	100.3300	80.3300	19206.60	3201.57	25	-125.00	1
2	108.3300	80.3300	16075.19	2229.25	15	-200.00	-1
3	92.3300	80.3300	22077.48	3380.16	15	2595.88	1
4	92.3300	88.3300	22825.93	3394.07	15	3269.32	1
5	92.3300	96.3300	18589.48	3305.06	15	3194.32	-1
6	76.3300	96.3300	19052.92	1784.88	15	3119.32	-1
7	84.3300	104.3300	19160.35	2693.71	15	3044.32	-1
8	84.3300	88.3300	23466.00	3457.73	15	3609.40	1
9	84.3300	88.3300	22929.53	4046.53	16	3529.40	-1
10	68.3300	88.3300	25484.49	2934.84	15	5472.89	1
11	68.3300	96.3300	20153.32	3018.63	15	5397.89	-1
12	68.3300	80.3300	27593.67	3085.02	15	7432.07	1
13	60.3300	72.3300	28338.00	5137.43	16	8096.40	1
14	60.3300	80.3300	26036.66	3738.58	15	8021.40	-1
15	60.3300	64.3300	31839.21	4316.96	15	11447.61	1
16	60.3300	48.3300	32318.50	3765.43	16	11846.90	1
17	60.3300	56.3300	31866.87	5358.47	15	11771.90	-1
18	60.3300	40.3300	31369.24	5742.47	15	11696.90	-1
19	68.3300	32.3300	26958.20	4107.09	15	11621.90	-1
20	52.3300	32.3300	33562.41	6791.18	15	12790.81	1
21	52.3300	40.3300	36368.82	4744.52	15	15522.22	1
22	52.3300	32.3300	31798.02	5285.46	15	15447.22	-1
23	36.3300	32.3300	37736.82	4916.08	15	16740.22	1
24	36.3300	40.3300	37222.00	5733.41	16	16660.22	-1
25	36.3300	24.3300	37101.20	5172.81	15	16585.22	-1
26	28.3300	24.3300	38836.60	6001.02	15	17610.00	1
27	28.3300	32.3300	36856.78	5697.69	15	17535.00	-1
28	28.3300	16.3300	37266.04	5119.91	15	17460.00	-1
29	28.3300	16.3300	39488.64	5329.46	15	18037.04	1
30	28.3300	24.3300	38481.32	4935.21	15	17962.04	-1

Example 6.19 (continued):

31	28.3300	8.3300	34719.30	6540.38	15	17887.04	-1
32	36.3300	8.3300	35286.79	4761.07	15	17812.04	-1
33	20.3300	8.3300	39016.65	5516.52	18	17722.04	-1
34	28.3300	16.3300	39025.38	4704.57	17	17637.04	-1
35	28.3300	0.3300	36535.05	5750.72	15	17562.04	-1
36	32.3300	16.3300	36319.46	5588.71	15	17487.04	-1
37	24.3300	16.3300	38461.73	6639.87	15	17412.04	-1
38	28.3300	20.3300	38259.08	6950.06	17	17327.04	-1
39	28.3300	12.3300	38575.78	5480.21	16	17247.04	-1
40	30.3300	16.3300	38150.21	6810.31	16	17167.04	-1
41	26.3300	16.3300	39101.02	5008.66	24	17047.04	-1
42	28.3300	18.3300	38912.44	6180.82	16	16967.04	-1
43	28.3300	14.3300	40363.23	5943.27	15	17766.63	1
44	30.3300	12.3300	36620.45	3880.70	15	17691.63	-1
45	26.3300	12.3300	39011.70	5653.15	15	17616.63	-1
46	28.3300	14.3300	37463.68	4784.37	15	17541.63	-1
47	28.3300	10.3300	35435.17	8311.15	15	17466.63	-1
48	29.3300	14.3300	37284.62	5449.21	15	17391.63	-1
49	27.3300	14.3300	35914.11	6226.61	15	17316.63	-1
50	28.3300	15.3300	36483.14	6016.36	15	17241.63	-1
51	28.3300	13.3300	37196.37	6997.38	15	17166.63	-1
52	28.8300	14.3300	34673.57	7556.82	15	17091.63	-1
53	27.8300	14.3300	38698.03	6752.16	15	17016.63	-1
54	28.3300	14.8300	35856.59	4740.45	15	16941.63	-1
55	28.3300	13.8300	36400.96	4271.66	15	16866.63	-1
56	28.5800	14.3300	37561.15	6667.94	15	16791.63	-1
57	28.0800	14.3300	36789.52	4138.17	15	16716.63	-1
58	28.3300	14.5800	34682.59	3687.13	15	16641.63	-1
59	28.3300	14.0800	38300.32	6419.08	15	16566.63	-1

TERMINATION PROCEDURE RESULTS:
=======================================

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	19206.60	3201.57	125.00	0.00	-125.00
2	22077.48	3380.16	275.00	2870.88	2595.88
3	22825.93	3394.07	350.00	3619.32	3269.32
4	23466.00	3457.73	650.00	4259.40	3609.40
5	25484.49	2934.84	805.00	6277.89	5472.89
6	27593.67	3085.02	955.00	8387.07	7432.07
7	28338.00	5137.43	1035.00	9131.40	8096.40
8	31839.21	4316.96	1185.00	12632.61	11447.61
9	32318.50	3765.43	1265.00	13111.90	11846.90
10	33562.41	6791.18	1565.00	14355.81	12790.81
11	36368.82	4744.52	1640.00	17162.22	15522.22
12	37736.82	4916.08	1790.00	18530.22	16740.22
13	38836.60	6001.02	2020.00	19630.00	17610.00
14	39488.64	5329.46	2245.00	20282.04	18037.04
15	40363.23	5943.27	3390.00	21156.63	17766.63

20) Example 6.20

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==================================================================
                EXAMPLE 6.20 - DATA SUMMARY
==================================================================
MIN. INITIAL SAMPLE= 25                  ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 15               BETAT (TEST)=0.950
DESIRED RATE OF RETURN=0.150             MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800           COST PER REPLICATE=  5.0
==================================================================
                    TEST PROCEDURE SUMMARY
==================================================================
NO.OF PTS. IN REGRESSION=  7           DEGREES OF FREEDOM= 54.11
EST. SAVINGS RATE STD.DEV.=  259.9     BO, ACTUAL COSTS RATE=  760.0
DIFF. BEING DETECTED=  206.0           BB, EST. SAVINGS RATE= 1373.4
DESIRED Z(BETAT)=-1.645                ACTUAL Z*=-1.553
NO. PTS FOR COST RATE= 3
==================================================================
                        CASE SIMULATED
==================================================================
SEARCH TECHNIQUE=PATTERN SEARCH
DIFF=(MAX(BB;BO))*RATE
FUNCTION WITH ELIPTIC CONTOURS
MODEL STD. DEV.=F(X1;X2)*.15
TYPE OF ERROR=NORMAL
MAXIMIZATION ALGORITHM
F=40000-
   -((XX(1)**2+XX(2)**2)/1.8-(XX(1)*XX(2))/1.125+50)*10
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	100.3300	80.3300	19206.60	3201.57	25	-125.00	1
2	108.3300	80.3300	16075.19	2229.25	15	-200.00	-1
3	92.3300	80.3300	22077.48	3380.16	15	2595.88	1
4	92.3300	88.3300	22825.93	3394.07	15	3269.32	1
5	92.3300	96.3300	18589.48	3305.06	15	3194.32	-1
6	76.3300	96.3300	19052.92	1784.88	15	3119.32	-1
7	84.3300	104.3300	19160.35	2693.71	15	3044.32	-1
8	84.3300	88.3300	23466.00	3457.73	15	3609.40	1
9	84.3300	88.3300	22929.53	4046.53	16	3529.40	-1
10	68.3300	88.3300	25484.49	2934.84	15	5472.89	1
11	68.3300	96.3300	20153.32	3018.63	15	5397.89	-1
12	68.3300	80.3300	27593.67	3085.02	15	7432.07	1
13	60.3300	72.3300	28338.00	5137.43	16	8096.40	1
14	60.3300	80.3300	26036.66	3738.58	15	8021.40	-1
15	60.3300	64.3300	31839.21	4316.96	15	11447.61	1
16	60.3300	48.3300	32318.50	3765.43	16	11846.90	1
17	60.3300	56.3300	31866.87	5358.47	15	11771.90	-1
18	60.3300	40.3300	31369.24	5742.47	15	11696.90	-1
19	68.3300	32.3300	26958.20	4107.09	15	11621.90	-1
20	52.3300	32.3300	33562.41	6791.18	15	12790.81	1
21	52.3300	40.3300	36368.82	4744.52	15	15522.22	1
22	52.3300	32.3300	31798.02	5285.46	15	15447.22	-1
23	36.3300	32.3300	37736.82	4916.08	15	16740.22	1
24	36.3300	40.3300	37222.00	5733.41	16	16660.22	-1
25	36.3300	24.3300	37101.20	5172.81	15	16585.22	-1
26	28.3300	24.3300	38836.60	6001.02	15	17610.00	1
27	28.3300	32.3300	36856.78	5697.69	15	17535.00	-1
28	28.3300	16.3300	37266.04	5119.91	15	17460.00	-1
29	28.3300	16.3300	39488.64	5329.46	15	18037.04	1
30	28.3300	24.3300	38481.32	4935.21	15	17962.04	-1

Example 6.20 (continued):

31	28.3300	8.3300	34719.30	6540.38	15	17887.04	-1
32	36.3300	8.3300	35286.79	4761.07	15	17812.04	-1
33	20.3300	8.3300	39016.65	5516.52	18	17722.04	-1
34	28.3300	16.3300	39025.38	4704.57	17	17637.04	-1
35	28.3300	0.3300	36535.05	5750.72	15	17562.04	-1
36	32.3300	16.3300	36319.46	5588.71	15	17487.04	-1
37	24.3300	16.3300	38461.73	6639.87	15	17412.04	-1
38	28.3300	20.3300	38259.08	6950.06	17	17327.04	-1
39	28.3300	12.3300	38575.78	5480.21	16	17247.04	-1
40	30.3300	16.3300	38150.21	6810.31	16	17167.04	-1
41	26.3300	16.3300	39101.02	5008.66	24	17047.04	-1
42	28.3300	18.3300	38912.44	6180.82	16	16967.04	-1
43	28.3300	14.3300	40363.23	5943.27	15	17766.63	1
44	30.3300	12.3300	36620.45	3880.70	15	17691.63	-1
45	26.3300	12.3300	39011.70	5653.15	15	17616.63	-1
46	28.3300	14.3300	37463.68	4784.37	15	17541.63	-1
47	28.3300	10.3300	35435.17	8311.15	15	17466.63	-1
48	29.3300	14.3300	37284.62	5449.21	15	17391.63	-1

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	19206.60	3201.57	125.00	0.00	-125.00
2	22077.48	3380.16	275.00	2870.88	2595.88
3	22825.93	3394.07	350.00	3619.32	3269.32
4	23466.00	3457.73	650.00	4259.40	3609.40
5	25484.49	2934.84	805.00	6277.89	5472.89
6	27593.67	3085.02	955.00	8387.07	7432.07
7	28338.00	5137.43	1035.00	9131.40	8096.40
8	31839.21	4316.96	1185.00	12632.61	11447.61
9	32318.50	3765.43	1265.00	13111.90	11846.90
10	33562.41	6791.18	1565.00	14355.81	12790.81
11	36368.82	4744.52	1640.00	17162.22	15522.22
12	37736.82	4916.08	1790.00	18530.22	16740.22
13	38836.60	6001.02	2020.00	19630.00	17610.00
14	39488.64	5329.46	2245.00	20282.04	18037.04
15	40363.23	5943.27	3390.00	21156.63	17766.63

21) Example 7.1:

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==================================================================
                  EXAMPLE 7.1 - DATA SUMMARY
==================================================================
MIN. INITIAL SAMPLE= 15                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 10             BETAT (TEST)=0.500
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  3
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
==================================================================
                    TEST PROCEDURE SUMMARY
==================================================================
NO.OF PTS. IN REGRESSION=  5          DEGREES OF FREEDOM= 18.32
EST. SAVINGS RATE STD.DEV.= 101.8     BO, ACTUAL COSTS RATE=1725.0
DIFF. BEING DETECTED= 258.7           BB, EST. SAVINGS RATE= 998.6
DESIRED Z(BETAT)= 0.0                 ACTUAL Z*= 0.025
NO. PTS. FOR COST RATE= 3
==================================================================
                       CASE SIMULATED
==================================================================
F(X1;X2)=COST FUNCTION OF A SIMULATED (Q;R) INVENTORY SYSTEM
CI=5.0,   CO=40.0,   CL=20.0,      MONTHLY DEMAND=P(200)
SEARCH TECHNIQUE= ONE-AT-A-TIME
DIFF=(MAX(BB;BO))*RATE
MODEL STD. DEV.- UNKNOWN
TYPE OF ERROR- UNKNOWN
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)	F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	5.3300 10.3300	144278.62	8221.95	15	-225.00	1
2	10.3300 10.3300	103577.75	6546.80	10	40325.87	1
3	15.3300 10.3300	80587.75	4826.66	10	63165.87	1
4	20.3300 10.3300	57893.17	2458.34	10	85710.44	1
5	25.3300 10.3300	48723.52	3770.87	10	94730.06	1
6	30.3300 10.3300	46819.29	1881.81	10	96484.31	1
7	35.3300 10.3300	47870.84	1577.66	10	96334.31	-1
8	30.3300 20.3300	27690.12	1230.69	10	115313.50	1
9	30.3300 30.3300	21438.93	931.12	10	121414.69	1
10	30.3300 40.3300	18885.22	414.97	10	123818.37	1
11	30.3300 50.3300	18181.87	372.00	10	124371.75	1
12	30.3300 60.3300	18227.17	503.03	10	124221.75	-1
13	35.3300 50.3300	19496.66	338.40	10	124071.75	-1
14	25.3300 50.3300	17475.59	596.74	10	124628.00	1
15	20.3300 50.3300	22400.09	2863.44	10	124478.00	-1
16	25.3300 60.3300	17856.11	827.84	10	124328.00	-1
17	25.3300 40.3300	19725.45	1387.31	10	124178.00	-1
18	27.8300 50.3300	17723.49	639.00	10	124028.00	-1
19	22.8300 50.3300	19779.41	1789.82	10	123878.00	-1
20	25.3300 55.3300	17150.46	1262.65	10	124053.12	1
21	25.3300 60.3300	17237.88	534.53	10	123903.12	-1
22	27.8300 55.3300	17610.61	475.59	10	123753.12	-1
23	22.8300 55.3300	18685.44	2039.71	10	123603.12	-1
24	26.5800 55.3300	17825.82	862.47	10	123453.12	-1
25	24.0800 55.3300	18042.84	1258.76	10	123303.12	-1
26	25.3300 57.8300	17410.78	1088.97	10	123153.12	-1
27	25.3300 52.8300	17448.57	986.83	10	123003.12	-1
28	25.9550 55.3300	18301.05	1308.00	10	122853.12	-1
29	24.7050 55.3300	18568.27	1507.35	10	122703.12	-1
30	25.3300 56.5800	17657.36	798.01	10	122553.12	-1
31	25.3300 54.0800	17405.82	891.16	10	122403.12	-1
32	25.6425 55.3300	18159.00	1299.61	10	122253.12	-1
33	25.0175 55.3300	17597.55	719.37	10	122103.12	-1
34	25.3300 55.9550	17838.62	1030.99	10	121953.12	-1

Example 7.1 (continued):

35	25.3300	54.7050	17324.57	551.04	10	121803.12	-1
36	25.4862	55.3300	17634.17	887.47	10	121653.12	-1
37	25.1737	55.3300	17868.75	1135.11	10	121503.12	-1

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	144278.62	8221.95	225.00	0.0	-225.00
2	103577.75	6546.80	375.00	40700.87	40325.87
3	80587.75	4826.66	525.00	63690.87	63165.87
4	57893.17	2458.34	675.00	86385.44	85710.44
5	48723.52	3770.87	825.00	95555.06	94730.06
6	46819.29	1881.81	975.00	97459.31	96484.31
7	27690.12	1230.69	1275.00	116588.50	115313.50
8	21438.93	931.12	1425.00	122839.69	121414.69
9	18885.22	414.97	1575.00	125393.37	123818.37
10	18181.87	372.00	1725.00	126096.75	124371.75
11	17475.59	596.74	2175.00	126803.00	124628.00
12	17150.46	1262.65	3075.00	127128.12	124053.12

22) Example 7.2:

```
==================================================================
                  EXAMPLE 7.2 - DATA SUMMARY
==================================================================
MIN. INITIAL SAMPLE= 15                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 10             BETAT (TEST)=0.800
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  3
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
==================================================================
                      TEST PROCEDURE SUMMARY
==================================================================
NO.OF PTS. IN REGRESSION=  5          DEGREES OF FREEDOM= 18.32
EST. SAVINGS RATE STD.DEV.= 101.8     BO, ACTUAL COSTS RATE=1125.0
DIFF. BEING DETECTED= 168.7           BB, EST. SAVINGS RATE= 998.6
DESIRED Z(BETAT)=-0.842               ACTUAL Z*=-0.789
NO. PTS. FOR COST RATE= 3
==================================================================
                          CASE SIMULATED
==================================================================
F(X1;X2)=COST FUNCTION OF A SIMULATED (Q;R) INVENTORY SYSTEM
CI=5.0,  CO=40.0,  CL=20.0,    MONTHLY DEMAND=P(200)
SERACH TECHNIQUE= ONE-AT-A-TIME
DIFF=(MAX(BB;BO))*RATE
MODEL STD. DEV.- UNKNOWN
TYPE OF ERROR- UNKNOWN
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)	F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	5.3300 10.3300	144278.62	8221.95	15	-225.00	1
2	10.3300 10.3300	103577.75	6546.80	10	40325.87	1
3	15.3300 10.3300	80587.75	4826.66	10	63165.87	1
4	20.3300 10.3300	57893.17	2458.34	10	85710.44	1
5	25.3300 10.3300	48723.52	3770.87	10	94730.06	1
6	30.3300 10.3300	46819.29	1881.81	10	96484.31	1
7	35.3300 10.3300	47870.84	1577.66	10	96334.31	-1
8	30.3300 20.3300	27690.12	1230.69	10	115313.50	1
9	30.3300 30.3300	21438.93	931.12	10	121414.69	1
10	30.3300 40.3300	18885.22	414.97	10	123818.37	1
11	30.3300 50.3300	18181.87	372.00	10	124371.75	1
12	30.3300 60.3300	18227.17	503.03	10	124221.75	-1
13	35.3300 50.3300	19496.66	338.40	10	124071.75	-1
14	25.3300 50.3300	17475.59	596.74	10	124628.00	1
15	20.3300 50.3300	22400.09	2863.44	10	124478.00	-1
16	25.3300 60.3300	17856.11	827.84	10	124328.00	-1
17	25.3300 40.3300	19725.45	1387.31	10	124178.00	-1
18	27.8300 50.3300	17723.49	639.00	10	124028.00	-1
19	22.8300 50.3300	19779.41	1789.82	10	123878.00	-1
20	25.3300 55.3300	17150.46	1262.65	10	124053.12	1
21	25.3300 60.3300	17237.88	534.53	10	123903.12	-1
22	27.8300 55.3300	17610.61	475.59	10	123753.12	-1
23	22.8300 55.3300	18685.44	2039.71	10	123603.12	-1
24	26.5800 55.3300	17825.82	862.47	10	123453.12	-1
25	24.0800 55.3300	18042.84	1258.76	10	123303.12	-1
26	25.3300 57.8300	17410.78	1088.97	10	123153.12	-1
27	25.3300 52.8300	17448.57	986.83	10	123003.12	-1
28	25.9550 55.3300	18301.05	1308.00	10	122853.12	-1
29	24.7050 55.3300	18568.27	1507.35	10	122703.12	-1

Example 7.2 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	144278.62	8221.95	225.00	0.0	-225.00
2	103577.75	6546.80	375.00	40700.87	40325.87
3	80587.75	4826.66	525.00	63690.87	63165.87
4	57893.17	2458.34	675.00	86385.44	85710.44
5	48723.52	3770.87	825.00	95555.06	94730.06
6	46819.29	1881.81	975.00	97459.31	96484.31
7	27690.12	1230.69	1275.00	116588.50	115313.50
8	21438.93	931.12	1425.00	122839.69	121414.69
9	18885.22	414.97	1575.00	125393.37	123818.37
10	18181.87	372.00	1725.00	126096.75	124371.75
11	17475.59	596.74	2175.00	126803.00	124628.00
12	17150.46	1262.65	3075.00	127128.12	124053.12

23) Example 7.3:

```
=================================================================
                  EXAMPLE 7.3 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 15                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 10             BETAT (TEST)=0.950
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  3
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
=================================================================
                      TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION=  3          DEGREES OF FREEDOM= 16.05
EST. SAVINGS RATE STD.DEV.= 114.9     BO, ACTUAL COSTS RATE= 450.0
DIFF. BEING DETECTED= 105.7           BB, EST. SAVINGS RATE= 704.8
DESIRED Z(BETAT)=-1.645               ACTUAL Z*=-1.476
NO. PTS. FOR COST RATE= 3
=================================================================
                          CASE SIMULATED
=================================================================
F(X1;X2)=COST FUNCTION OF A SIMULATED (Q;R) INVENTORY SYSTEM
CI=5.0,  CO=40.0,  CL=20.0,    MONTHLY DEMAND=P(200)
SEARCH TECHNIQUE= ONE-AT-A-TIME
DIFF=(MAX(BB;BO))*RATE
MODEL STD. DEV.- UNKNOWN
TYPE OF ERROR- UNKNOWN
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	5.3300	10.3300	144278.62	8221.95	15	-225.00	1
2	10.3300	10.3300	103577.75	6546.80	10	40325.87	1
3	15.3300	10.3300	80587.75	4826.66	10	63165.87	1
4	20.3300	10.3300	57893.17	2458.34	10	85710.44	1
5	25.3300	10.3300	48723.52	3770.87	10	94730.06	1
6	30.3300	10.3300	46819.29	1881.81	10	96484.31	1
7	35.3300	10.3300	47870.84	1577.66	10	96334.31	-1
8	30.3300	20.3300	27690.12	1230.69	10	115313.50	1
9	30.3300	30.3300	21438.93	931.12	10	121414.69	1
10	30.3300	40.3300	18885.22	414.97	10	123818.37	1
11	30.3300	50.3300	18181.87	372.00	10	124371.75	1
12	30.3300	60.3300	18227.17	503.03	10	124221.75	-1
13	35.3300	50.3300	19496.66	338.40	10	124071.75	-1
14	25.3300	50.3300	17475.59	596.74	10	124628.00	1
15	20.3300	50.3300	22400.09	2863.44	10	124478.00	-1
16	25.3300	60.3300	17856.11	827.84	10	124328.00	-1
17	25.3300	40.3300	19725.45	1387.31	10	124178.00	-1

Example 7.3 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	144278.62	8221.95	225.00	0.0	-225.00
2	103577.75	6546.80	375.00	40700.87	40325.87
3	80587.75	4826.66	525.00	63690.87	63165.87
4	57893.17	2458.34	675.00	86385.44	85710.44
5	48723.52	3770.87	825.00	95555.06	94730.06
6	46819.29	1881.81	975.00	97459.31	96484.31
7	27690.12	1230.69	1275.00	116588.50	115313.50
8	21438.93	931.12	1425.00	122839.69	121414.69
9	18885.22	414.97	1575.00	125393.37	123818.37
10	18181.87	372.00	1725.00	126096.75	124371.75
11	17475.59	596.74	2175.00	126803.00	124628.00

24) Example 7.4:

```
===============================================================
                   EXAMPLE 7.4 - DATA SUMMARY
===============================================================
MIN. INITIAL SAMPLE= 15                 ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 10              BETAT (TEST)=0.500
DESIRED RATE OF RETURN=0.150            MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800          COST PER REPLICATE= 15.0
===============================================================
                     TEST PROCEDURE SUMMARY
===============================================================
NO.OF PTS. IN REGRESSION=  4           DEGREES OF FREEDOM= 12.78
EST. SAVINGS RATE STD.DEV.=  69.2      BO, ACTUAL COSTS RATE=1275.0
DIFF. BEING DETECTED= 191.2            BB, EST. SAVINGS RATE= 656.4
DESIRED Z(BETAT)= 0.0                  ACTUAL Z*= 0.143
NO. PTS. FOR COST RATE= 3
===============================================================
                         CASE SIMULATED
===============================================================
F(X1;X2)=COST FUNCTION OF A SIMULATED (Q;R) INVENTORY SYSTEM
CI=5.0,  CO=40.0,  CL=20.0,    MONTHLY DEMAND=P(200)
SEARCH TECHNIQUE= PATTERN SEARCH
DIFF=(MAX(BB;BO))*RATE
MODEL STD. DEV.- UNKNOWN
TYPE OF ERROR- UNKNOWN
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	5.33	10.33	146760.44	6910.64	15	-225.00	1
2	10.33	10.33	101926.81	5858.86	10	44458.62	1
3	10.33	20.33	87293.06	6110.13	10	58942.37	1
4	20.33	30.33	26835.32	3976.42	10	119250.06	1
5	20.33	40.33	23014.36	3736.32	10	122921.06	1
6	35.33	60.33	19187.36	242.53	10	126598.06	1
7	35.33	70.33	19585.36	220.35	10	126448.06	-1
8	35.33	50.33	19358.59	264.74	10	126298.06	-1
9	55.33	80.33	25585.59	269.00	10	126148.06	-1
10	45.33	80.33	22916.67	275.69	10	125998.06	-1
11	50.33	90.33	25013.42	318.57	10	125848.06	-1
12	50.33	70.33	23546.57	284.62	10	125698.06	-1
13	37.83	60.33	19927.10	190.59	10	125548.06	-1
14	32.83	60.33	18517.62	365.48	10	126067.81	1
15	32.83	65.33	18650.98	210.31	10	125917.81	-1
16	32.83	55.33	18380.13	278.23	11	125752.81	-1
17	32.83	60.33	18477.96	480.21	10	125602.81	-1
18	27.83	60.33	17641.45	382.18	10	126328.94	1
19	27.83	65.33	18065.22	1267.88	10	126178.94	-1
20	27.83	55.33	17291.27	664.42	10	126379.12	1
21	25.33	50.33	17972.15	1211.61	10	126229.12	-1
22	20.33	50.33	21201.98	1634.61	10	126079.12	-1
23	22.83	55.33	18476.95	1263.02	10	125929.12	-1
24	22.83	45.33	21468.59	3590.41	10	125779.12	-1
25	29.08	55.33	17824.29	349.81	10	125629.12	-1
26	26.58	55.33	17457.67	786.70	10	125479.12	-1
27	27.83	57.83	18086.46	662.21	10	125329.12	-1
28	27.83	52.83	17660.21	482.95	10	125179.12	-1
29	28.45	55.33	17578.73	384.26	10	125029.12	-1
30	27.20	55.33	17484.79	660.32	10	124879.12	-1
31	27.83	56.58	17598.48	666.03	10	124729.12	-1
32	27.83	54.08	17900.36	818.40	10	124579.12	-1
33	28.14	55.33	17529.79	360.86	10	124429.12	-1
34	27.52	55.33	17530.82	1015.20	10	124279.12	-1
35	27.83	55.95	17616.17	892.53	10	124129.12	-1

Example 7.4 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	146760.44	6910.64	225.00	0.0	-225.00
2	101926.81	5858.86	375.00	44833.62	44458.62
3	87293.06	6110.13	525.00	59467.37	58942.37
4	26835.32	3976.42	675.00	119925.06	119250.06
5	23014.36	3736.32	825.00	123746.06	122921.06
6	19187.36	242.53	975.00	127573.06	126598.06
7	18517.62	365.48	2175.00	128242.81	126067.81
8	17641.45	382.18	2790.00	129118.94	126328.94
9	17291.27	664.42	3090.00	129469.12	126379.12

25) Example 7.5:

```
===================================================================
                    EXAMPLE 7.5 - DATA SUMMARY
===================================================================
MIN. INITIAL SAMPLE= 15                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 10             BETAT (TEST)=0.800
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
===================================================================
                        TEST PROCEDURE SUMMARY
===================================================================
NO.OF PTS. IN REGRESSION=  4          DEGREES OF FREEDOM= 12.78
EST. SAVINGS RATE STD.DEV.=  69.2     BO, ACTUAL COSTS RATE= 825.0
DIFF. BEING DETECTED= 123.7           BB, EST. SAVINGS RATE= 656.4
DESIRED Z(BETAT)=-0.842               ACTUAL Z*=-0.721
NO. PTS. FOR COST RATE= 3
===================================================================
                           CASE SIMULATED
===================================================================
F(X1;X2)=COST FUNCTION OF A SIMULATED (Q;R) INVENTORY SYSTEM
CI=5.0,  CO=40.0,  CL=20.0,     MONTHLY DEMAND=P(200)
SEARCH TECHNIQUE= PATTERN SEARCH
DIFF=(MAX(BB;BO))*RATE
MODEL STD. DEV.- UNKNOWN
TYPE OF ERROR- UNKNOWN
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	5.33	10.33	146760.44	6910.64	15	-225.00	1
2	10.33	10.33	101926.81	5858.86	10	44458.62	1
3	10.33	20.33	87293.06	6110.13	10	58942.37	1
4	20.33	30.33	26835.32	3976.42	10	119250.06	1
5	20.33	40.33	23014.36	3736.32	10	122921.06	1
6	35.33	60.33	19187.36	242.53	10	126598.06	1
7	35.33	70.33	19585.36	220.35	10	126448.06	-1
8	35.33	50.33	19358.59	264.74	10	126298.06	-1
9	55.33	80.33	25585.59	269.00	10	126148.06	-1
10	45.33	80.33	22916.67	275.69	10	125998.06	-1
11	50.33	90.33	25013.42	318.57	10	125848.06	-1
12	50.33	70.33	23546.57	284.62	10	125698.06	-1
13	37.83	60.33	19927.10	190.59	10	125548.06	-1
14	32.83	60.33	18517.62	365.48	10	126067.81	1
15	32.83	65.33	18650.98	210.31	10	125917.81	-1
16	32.83	55.33	18380.13	278.23	11	125752.81	-1
17	32.83	60.33	18477.96	480.21	10	125602.81	-1
18	27.83	60.33	17641.45	382.18	10	126328.94	1
19	27.83	65.33	18065.22	1267.88	10	126178.94	-1
20	27.83	55.33	17291.27	664.42	10	126379.12	1
21	25.33	50.33	17972.15	1211.61	10	126229.12	-1
22	20.33	50.33	21201.98	1634.61	10	126079.12	-1
23	22.83	55.33	18476.95	1263.02	10	125929.12	-1
24	22.83	45.33	21468.59	3590.41	10	125779.12	-1
25	29.08	55.33	17824.29	349.81	10	125629.12	-1
26	26.58	55.33	17457.67	786.70	10	125479.12	-1
27	27.83	57.83	18086.46	662.21	10	125329.12	-1
28	27.83	52.83	17660.21	482.95	10	125179.12	-1
29	28.45	55.33	17578.73	384.26	10	125029.12	-1

Example 7.5 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	146760.44	6910.64	225.00	0.0	-225.00
2	101926.81	5858.86	375.00	44833.62	44458.62
3	87293.06	6110.13	525.00	59467.37	58942.37
4	26835.32	3976.42	675.00	119925.06	119250.06
5	23014.36	3736.32	825.00	123746.06	122921.06
6	19187.36	242.53	975.00	127573.06	126598.06
7	18517.62	365.48	2175.00	128242.81	126067.81
8	17641.45	382.18	2790.00	129118.94	126328.94
9	17291.27	664.42	3090.00	129469.12	126379.12

26) Example 7.6:

```
=================================================================
                EXAMPLE 7.6 - DATA SUMMARY
=================================================================
MIN. INITIAL SAMPLE= 15                ALPHAT (TEST)=0.010
MIN. ADDITIONAL SAMPLE= 10             BETAT (TEST)=0.950
DESIRED RATE OF RETURN=0.150           MIN NO. OF IMPRV. PTS.=  4
ALPHA (CONFID. INTERVAL)=0.800         COST PER REPLICATE= 15.0
=================================================================
                  TEST PROCEDURE SUMMARY
=================================================================
NO.OF PTS. IN REGRESSION=  4          DEGREES OF FREEDOM= 12.78
EST. SAVINGS RATE STD.DEV.=  69.2     BO, ACTUAL COSTS RATE= 525.0
DIFF. BEING DETECTED=  98.5           BB, EST. SAVINGS RATE= 656.4
DESIRED Z(BETAT)=-1.645               ACTUAL Z*=-1.045
NO. PTS. FOR COST RATE= 3
=================================================================
                      CASE SIMULATED
=================================================================
F(X1;X2)=COST FUNCTION OF A SIMULATED (Q;R) INVENTORY SYSTEM
CI=5.0,  CO=40.0,  CL=20.0,     MONTHLY DEMAND=P(200)
SEARCH TECHNIQUE= PATTERN SEARCH
DIFF=(MAX(BB;BO))*RATE
MODEL STD. DEV.- UNKNOWN
TYPE OF ERROR- UNKNOWN
```

LIST OF POINTS SEARCHED

PT. NO.	X(N)		F(X(N)) $	STD(F) $	SAMPLE SIZE	NET SAVINGS	IMPRV. (1)
1	5.33	10.33	146760.44	6910.64	15	-225.00	1
2	10.33	10.33	101926.81	5858.86	10	44458.62	1
3	10.33	20.33	87293.06	6110.13	10	58942.37	1
4	20.33	30.33	26835.32	3976.42	10	119250.06	1
5	20.33	40.33	23014.36	3736.32	10	122921.06	1
6	35.33	60.33	19187.36	242.53	10	126598.06	1
7	35.33	70.33	19585.36	220.35	10	126448.06	-1
8	35.33	50.33	19358.59	264.74	10	126298.06	-1
9	55.33	80.33	25585.59	269.00	10	126148.06	-1
10	45.33	80.33	22916.67	275.69	10	125998.06	-1
11	50.33	90.33	25013.42	318.57	10	125848.06	-1
12	50.33	70.33	23546.57	284.62	10	125698.06	-1
13	37.83	60.33	19927.10	190.59	10	125548.06	-1
14	32.83	60.33	18517.62	365.48	10	126067.81	1
15	32.83	65.33	18650.98	210.31	10	125917.81	-1
16	32.83	55.33	18380.13	278.23	11	125752.81	-1
17	32.83	60.33	18477.96	480.21	10	125602.81	-1
18	27.83	60.33	17641.45	382.18	10	126328.94	1
19	27.83	65.33	18065.22	1267.88	10	126178.94	-1
20	27.83	55.33	17291.27	664.42	10	126379.12	1
21	25.33	50.33	17972.15	1211.61	10	126229.12	-1
22	20.33	50.33	21201.98	1634.61	10	126079.12	-1
23	22.83	55.33	18476.95	1263.02	10	125929.12	-1
24	22.83	45.33	21468.59	3590.41	10	125779.12	-1
25	29.08	55.33	17824.29	349.81	10	125629.12	-1

Example 7.6 (continued):

TERMINATION PROCEDURE RESULTS:

POINT	F(X)	STD(F(X))	CUM. COST	GROSS SAVINGS	NET SAVINGS
1	146760.44	6910.64	225.00	0.0	-225.00
2	101926.81	5858.86	375.00	44833.62	44458.62
3	87293.06	6110.13	525.00	59467.37	58942.37
4	26835.32	3976.42	675.00	119925.06	119250.06
5	23014.36	3736.32	825.00	123746.06	122921.06
6	19187.36	242.53	975.00	127573.06	126598.06
7	18517.62	365.48	2175.00	128242.81	126067.81
8	17641.45	382.18	2790.00	129118.94	126328.94
9	17291.27	664.42	3090.00	129469.12	126379.12

SYSTEMS' OPTIMIZATION THROUGH SIMULATION

by

Eli A. Matalon

(ABSTRACT)

The problem of systems' optimization through simulation was investigated. An external modular optimization procedure applicable to the simulation model of a multiple-variable, single-response, system whose overall performance is defined in terms of cost or profit in designed. The procedure consists of four modules: The simulation model, A sequential search technique, A "Sampling Routine", and A "Control Module". The first two modules were treated as "black-boxes" and were assumed to be given. The third module controls the sampling process. This module offers dynamic adjustments of the necessary sample size at any point in the course of the search for an optimal system's design. The fourth module controls the optimization process. This module utilizes a statistical inferential technique in determining the duration of the optimization process, such as to maximize the net return from its application.

The procedure offers the possibility of utilizing various sequential search techniques, requires no modifications of the simulation model, and can be applied to a reasonably broad class of optimization problems.

Descriptions of the procedure's modules, as well as illustrative examples of their implementation are included.

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