A QUARTERLY FEEDER CATTLE PRICE FORECASTING MODEL
WITH APPLICATION TOWARDS THE DEVELOPMENT
OF A FUTURES MARKET STRATEGY

by

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Dissertation submitted to the Graduate Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Agricultural Economics

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I wish to express appreciation to those who have been most helpful to me during this study. My major professor, David E. Kenyon, was an excellent source of imaginative approaches, sound critical commentary, and was exceedingly generous with his time. Professor Robert V. Foutz was an indispensable aid to the understanding of the Box-Jenkins methods of time series analysis and the computer packages which accompanied them. He, too, was very generous with his time. It was a pleasure working with both of these individuals.

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CHAPTER I

INTRODUCTION

The sale of meat animals provides almost 30% of the income earned by U.S. farmers. However, while the number of hogs, sheep and lambs on farms has decreased over the past 30 years, the number of cattle and calves has increased dramatically (Table 1). The increased importance of the beef producing sector has been in response to a significant upward trend in the demand for beef product (Table 2). The pronounced rise in production and consumption of beef during the last three decades has had a profound effect on the agricultural community. In short, beef production is now the major agricultural enterprise in the United States based on cash receipts from marketings of livestock and crops.

Because of its importance, the beef sector presented itself as the most promising area in which significant marketing research contributions could be made. These contributions took the form of facilitating production decisions under uncertainty, using futures markets for

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1 Agricultural Outlook, USDA, ERS, AO-10, May, 1976.
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<tr>
<td></td>
<td>(1000 head)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cattle and calves</td>
<td>81,204</td>
<td>94,241</td>
<td>104,488</td>
<td>121,534</td>
</tr>
<tr>
<td>Sheep and lambs</td>
<td>55,150</td>
<td>31,900</td>
<td>29,176</td>
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<tr>
<td>Hogs</td>
<td>73,881</td>
<td>51,755</td>
<td>58,695</td>
<td>61,106</td>
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<th>1943</th>
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<th>1963</th>
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<tr>
<td>Beef and veal</td>
<td>61.5</td>
<td>87.1</td>
<td>99.4</td>
<td>111.4</td>
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<tr>
<td>Lamb and mutton</td>
<td>6.4</td>
<td>4.7</td>
<td>4.9</td>
<td>2.7</td>
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<tr>
<td>Pork</td>
<td>78.9</td>
<td>63.5</td>
<td>65.4</td>
<td>61.6</td>
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^Agricultural Statistics.
hedging purposes in the live cattle and feeder cattle markets, analyzing the optimal regional distribution patterns for producing and slaughtering cattle, as well as structuring models of the beef or livestock sector. Yet despite all the research that has been


conducted there exists a need for more information at the farm level. More specifically, the cow-calf operator wants the kind of data that will facilitate his production decisions.

According to Trierweiler and Erickson, cow-calf operators are slow to respond to market indicators such as changes in the price of feeder cattle. They suggest that this is the result of two inherent characteristics of the cow-calf industry. These are the length of time necessary for the expansion of the industry's capacity and the length of time from the beginning of gestation to the marketing of the beef calves as feeders. During the planning period, producers must speculate on the price of feeders two years in advance. One piece of data which would facilitate the cow-calf operator's production decisions would be a forecast of feeder cattle prices. Such a forecast would assist cow-calf operators and backgrounders in timing calf purchases and feeder sales.

**Problem Statement**

Cattle and calf sales constitute the largest single source of income to farmers in Virginia, accounting for approximately 18% of

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Trierweiler and Erickson, p. 25.

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the value of all farm commodities produced.\textsuperscript{9} There are over 5,000 farm operators engaged in the enterprise of backgrounding. Backgrounders buy calves, usually in the fall, and feed them for up to one year. They then sell the heavier feeder cattle to feedlots in the East and Midwest.

There are also over 30,000 cow-calf operators in Virginia who handle the preparation of calves to feeder cattle weight. Oftentimes the cow-calf operations in the state will elect to feed their calves to feeder cattle weight if the price of calves offered by backgrounders is considered too low.

Whether the enterprise of producing feeder cattle is undertaken by backgrounders or cow-calf operators, at a certain point in time their basic problem is the same -- price uncertainty. The cow-calf operator has a longer planning and production period since it is he who breeds the cows and has capital already invested during the gestation period. However, at the time those calves are weaned, approximately six to nine months after birth, he would like to know what price he can reasonably ask for his weaned calves from the backgrounders. If that price is not met he would like to know what price he can expect if he were to hold the calves to feeder weight and instead sell to a feedlot. At this same point in time, the backgrounder would like to know what price he may offer for the weaned calves in order to make a reasonable profit for bringing the calves to feeder weight. Both

\textsuperscript{9}Virginia Crops and Livestock, Volume 46, No. 9, September, 1974.
groups, in other words, want a reasonable estimate of what price they would be able to attain for feeder calves approximately two to four quarters later.

Traditionally, cattle producers have had little indication of what price their investment of capital and time will return. From the time the production decision is made (the decision of purchasing calves by backgrounders, or the decision to retain rather than sell the calves by cow-calf operators) until the time to sell the feeder cattle to feedlots, the price of their investment can change dramatically. Recent history provides an excellent example of an adverse price movement.

Using Winchester, Virginia prices, on October 1, 1973, a backgrounder who bought a choice or fancy calf in the 300-500 pound weight class would have paid an average of $42.50 per cwt. If he had sold that calf one year later as a 500-750 pound feeder, he would have received a market price of $28.50 per cwt. on the average. Using an annual weight gain of 300 pounds, the backgrounder would have made an outlay of $191.25 (450 x $42.50) and would have received $213.75 (750 x $28.50) for selling the calf at feeder weight. This represents a substantial loss, since there is no doubt that the operator would not even have been able to cover feed costs. An indication of the price variability in feeder cattle is presented in the graph below.

One might ask whether the futures market for feeder cattle could be used as an accurate predictor of feeder cattle prices twelve months

\(^{10}\text{Virginia Crop Reporting Service.}\)
Figure 1-A. Average Price per cwt. Received in 10 National Markets for Feeder Steers, All Weights Combined, 1960-1975.
hence. In fact, this particular possibility has not been examined although a very closely related study indicates that the futures market is not a very accurate predictor of distant prices when a nonstor-able commodity is involved. Leuthhold found that beef cattle futures prices at time $t_1$ offered a poor indicator of distant cash prices relative to cash prices at time $t_1$.

"Further evaluation of live beef cattle price relationships revealed that for distant futures, the cash price is a more accurate indicator of future cash price conditions than is the futures price ... The producer who looks at the futures price routinely to establish a feeding margin so that he can decide whether or not to purchase and feed cattle may receive false signals and be misled into a costly decision, either a money loss or foregone profits."\(^{11}\)

Evidently there is a need for a forecast which would predict better than either the cash or futures price. The example presented above demonstrated that relying on the cash quotations could produce poor estimates of future prices due to the volatile nature of prices in the feeder calf business. The agricultural community in Virginia could benefit from a price forecasting model for feeders which is more accurate than using either the futures market quotations or current cash prices as a predictor of future feeder prices.

**Objectives and General Approach**

The specific objectives of this study are:

(1) To develop a price forecasting equation(s) for feeder cattle.

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(2) To examine the possibility of reducing the mean square error of income and raising the level of revenue to the feeder producer relative to that which would be achieved under the common practice of consistently selling feeder cattle in the cash market. A selected strategy incorporating forecasted prices and the use of the futures market would be developed to this end.

A structural model of the feeder cattle market was derived from a block recursive theoretical framework. The theoretical derivation served to indicate the more important variables which then would form the basis for statistical analysis. Rather than concentrating on a statistical derivation of the structure of the feeder cattle industry, most of the effort was devoted to refining the predictive equation through the use of statistical techniques developed expressly for forecasting purposes.

The forecast assumed that the backgrounder or cow-calf operator would be marketing a beef animal weighing approximately 750 pounds after having purchased (or having withheld from sale) a calf weighing approximately 450 pounds.

Feeder producers should benefit from knowledge of a strategy which could reduce the variability of their year-to-year income on sales of feeders, especially if the reduction in variability accompanied a rise in mean income. To this end, the futures market was employed in conjunction with the forecast of feeder prices in order to form a strategy to stabilize producers' income and perhaps raise the mean income from feeder sales. Even if mean income were not higher
under the strategy, the producer should be aware of the tradeoff involved between a higher income and a stable income, assuming the latter could be obtained.

The general approach employed in addressing these objectives is as follows: Chapter II is a presentation of the review of literature which includes a discussion of some background on the beef industry, hedging in the futures market and evaluation of forecasting methods. Chapter III is a presentation of the theoretical development of the structural model. Chapter IV presents the results of the empirical estimation of the structural equations for the feeder cattle market and a computation of the analytically reduced form for the price of feeders. It was the intent initially to use the reduced form as an input in the transfer function forecasting method developed later. Ultimately the reduced form series was abandoned as a candidate for the input series in favor of the first stage of a two stage least squares estimation of the feeder cattle market.

The development of the forecasting theory is given in Chapter IV, wherein the Box-Jenkins forecasting methods are outlined. An explanation of how the input series is employed in the more sophisticated transfer function forecasting model is presented. In Chapter V, the empirical results of the forecasting models are displayed.

The development of three strategies which have as their goal the reduction of income variance to feeder cattle producers, is presented in Chapter VI. One of the strategies incorporates the transfer function forecasts presented in Chapter V. Chapter VII offers the summary and conclusions of the study.
CHAPTER II

REVIEW OF LITERATURE

Most of the studies pertaining to the beef cattle sector reported in journals and technical bulletins have been directed toward the enhancement of cattle producers' economic position. In a review of such literature, it might facilitate discussion to impose a categorization on the material. Accordingly, the studies have been separated into three overlapping groups: interregional competition in the beef industry, the facilitating of production decisions, and hedging in the futures markets. Each will be discussed in turn.

Interregional Competition in Beef Industry

Regional Distribution Patterns

Beef cattle are produced in virtually every state in the union. Because of geographic differences there has developed a natural gravitation towards specialization. For example, "... the Southeastern states increased farm production of beef cattle 81% from the 1947-49 average to the 1960-62 average ... The Corn Belt still ranks first in the production of fed cattle". ¹ More recent data indicates that for 1975, almost two-thirds of projected U.S. fed cattle would be finished

in the Texas-Oklahoma Panhandle, Kansas and Nebraska, with Iowa and Illinois accounting for another one-sixth of the total. These projections are borne out by figures from the 1975 supplement of Livestock and Meat Statistics.

In the last few decades there have been extensive changes in the distribution patterns of these functions partly as a result of the increasing degree of specialization and partly due to institutional constraints such as the existing transportation network. Goodwin examined the optimum distribution patterns of feeder cattle from production areas to consumption areas in 1962. He used reactive programming analysis, using as inputs quarterly demand functions for feeder cattle, fixed supplies at 1962 production levels plus imports, and existing truck transportation rates. In general, Goodwin's study suggested that more feeder cattle be shipped to the North Central Region at the expense of the Western region under the optimum distribution. He estimated that returns for the industry could have been increased by 6.4% over the actual pattern of distribution for 1962.

Aylor and Juillerat, using 1962 data, sought to obtain the least cost movements of cattle to slaughter, calves to slaughter, and beef and veal from slaughter to consumption areas. The analytical tool employed was a linear programming transportation model. The solution

2Dietrich, Raymond A., Interregional Competition in the Cattle Feeding Economy, Texas A & M University, B-1115, September, 1971.

suggested that the level of slaughtering cattle be increased by 2/3 in the Southeast Region. The reasons for urging this dramatic shift were primarily due to the lower estimated slaughter costs in the Southeast and the fact that the Southeast was a low beef production region relative to their consumption. Another interesting conclusion for purposes of the present analysis was that the Southeast produced 27% more live calves than it had capacity to handle. Consumption of veal in the region was low, leaving open the speculation that the Southeast region and Virginia in particular already had the facility for producing feeder calves in substantial numbers.

The two above studies indicate that the Southeast Region's major role in cattle production is to raise calves to feeder weight, and then to ship them to the feedlots in the East and Midwest. The potential for developing the Southeast into a feedlot region as well was researched by Liu and West. They developed a multidimensional transshipment model to obtain the location of beef feeding and slaughtering which would minimize aggregate industry costs, given the areas of feeder cattle production, beef consumption, area differences in slaughter and feed costs and transportation charges. They concluded that Virginia and three other Southern states were in a good position in 1965 to expand their feedlot operations. However, they also noted that feeder cattle production would continue to be the most important function for the cattle-beef enterprises in the South.

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A more recent work indicates that there has in fact been even a greater degree of specialization in cattle marketing. Van Arsdall and Skold\(^5\) present a primarily descriptive report on the changing cattle industry from 1960 to 1970. They also present extensions of trends forward to 1980. One important finding which is germane to the present study was that the placement of cattle on feed is now more evenly distributed throughout the year. In 1960 the percentage of cattle placed on feed by quarters was 21-16-21-42 with the bulk of feeders being placed during the fourth quarter. In 1970 the percentage distribution by quarters was 21-22-25-32. The change indicates that the demand for feeders is becoming more evenly distributed throughout the year. This is most likely due to the increased specialization in the industry and the resultant expanded feedlot capacity.

**Structural Supply and Demand Models**

Identifying the sources of variation in the price and quantity of a commodity is often an important goal in itself. Such structural models perform this function and serve to give advance notice of the effects on price and quantity of taxes, foreign grain deals and other public policy decisions. In addition, the structural models provide the individual farmer with a basis from which more efficient allocation decisions can be made.

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Hildreth and Jarrett\textsuperscript{6} worked with livestock aggregates in attempting to estimate a number of relations at the farm level. The livestock aggregate quantity included hogs, cattle, calves, milk, sheep, chickens, eggs, and turkeys. A simultaneous system was estimated using both least squares and limited information maximum likelihood. The system included a production relation, demand functions for feed grain, protein feed, and livestock products. Supply functions included those for the corresponding demands. There were a number of arguments mentioned in the Hildreth and Jarrett study that could be used to forecast feeder prices. The authors included time as a variable to account for important unobserved factors which had increased fairly smoothly over time. As had been mentioned previously, the cattle industry has expanded dramatically between 1950 and 1970. Whether this expansion has been fairly smooth in order to make use of an explicit time variable will be investigated. They also noted that current price may also serve as an indicator of expectation of future price movements, in accordance with a cobweb theorem. "It has often been asserted that, in the absence of offsetting factors, an increase in price (particularly of cattle) leads to favorable anticipations, to an attempt to build up inventories and lower current sales and to a strengthening of the tendency for current price to rise".\textsuperscript{7} This serves to indicate that a forecast of feeder prices may have to include lag operators.


\textsuperscript{7}Hildreth and Jarrett, p. 106.
Havlicek and Myers developed a simultaneous system to describe the livestock-meat sector of the United States. The monthly model determined the interrelationships between pork, broilers, and beef at the retail level as well as specified the relationship between retail and farm levels in the hog and cattle sectors. Myers and Havlicek included an expectational element in the supply response functions of hogs and cattle. This was especially important when predictions were made. Despite the expectational formulations the authors found that "the cattle-beef component of the model exhibited the greatest deviations between predicted and observed values". The model, however, was tested for the months in 1971-72 when cattlemen were holding back their cattle from market despite record prices, anticipating a lift of the price freeze. Their research leaves unanswered to what extent supply responds to current rather than expected prices in the beef-cattle sector.

Myers and Havlicek reported that marketing margins for beef and pork have been studies by a number of researchers but no general conclusions have been reached as to how they respond to changes in other variables. By and large those studies which have specified marketing margins include most of the same types of variables as would have been explicitly formulated for each market in the system under consideration.

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9 Myers and Havlicek, p. 44.
Oftentimes marketing margins were employed because this specification leads to shortcuts in estimation. Since the present study uses economic theory rather than statistical estimation to derive the final form to be estimated, it was considered that the marketing margin specification would be unnecessary.

Hayenga and Hacklander\textsuperscript{10} developed a monthly supply and demand relationship for cattle and hogs using two-stage least squares. They hypothesized that farmers react more to the direction of the monthly price change rather than the current level of prices. The writers considered the overall fit of the demand, supply and storage equations to be quite good. The only major flaw in the article was that the cross-flexibilities of demand for beef and pork had opposite signs. In the live cattle demand equation the quantity of hogs coefficient was negative, indicating substitutability. In the live hog demand equation, however, the coefficient of quantity of cattle was positive, indicating complementarity. The authors concede that there was perhaps a spurious statistical result. Bullock, in a criticism of the article, opined that their use of ratios as independent variables in regression analysis can sometimes lead to biased results. Ratios are inappropriate when the relationships between other variables and the numerator of the ratio is the relationship of interest. The problem

does not arise when the hypothesis to be tested has initially been formulated in terms of ratios.¹¹

The above studies dealt with problems on a macro level in that the intent was to allocate resources in a large context and isolate variables important to the industry as a whole. The emphasis in the following section seems to be directed more toward the individual operator.

Facilitating Production Decisions

Decision Theory Models

Bullock and Logan¹¹a applied decision theory to cattle feedlot marketing. They developed decision rules for whether to sell or feed cattle another month. The method of analysis employed was Bayesian decision theory. An historical distribution of monthly price changes constituted the a priori probabilities. Price change predictions conditioned the a posteriori probabilities. Cost estimates were made for feeding cattle an additional month. Decision rules could then be formulated using the estimated direction of price change in combination with the cost estimates to determine whether feeding for an additional month would be profitable. For example, if a predicted price change


for cattle over the next month was $1.00 per cwt. and the cost for another month was $0.80, the decision would obviously be to feed the cattle for an additional month. The price prediction data would allow the a priori distribution of actual price changes to condition a more accurate a posteriori distribution of predicted price changes which had a closer correspondence with actual price changes. Unfortunately it has been reported that the technique met with unsatisfactory results when the model was tested with data subsequent to the data base used to formulate the model. However, the testing was done in the period of abnormal conditions when there was an exceptionally high degree of market interference by the government in terms of selected price freezes.

Y. C. Chiang, R. B. Jensen, D. E. Kenyon, and R. G. Kline used Bayesian decision theory to develop "sell or keep" decisions regarding the cattle on a representative large beef farm in the Shenandoah area of Virginia. Linear programming was used to derive the data needed to develop the payoff matrix. The states of nature included pasture conditions and price levels. Historical information on prices was conditioned by forecasts of high, medium or low prices. One problem encountered in this analysis was the arbitrary setting of high, medium and low prices at a certain absolute level and applying that same scale to differing weight groups of cattle. The authors footnote that

"It is recognized, however, that the prices for heavier weights of cattle tend to increase or decrease less than the prices for lighter weights of cattle; thus, this method only serves as an approximation of investigating the impact of differing levels of price projection on the production and marketing decision." The study indicated that under certain conditions it would probably be more desirable to finish heavy slaughters rather than sell cattle at feeder weight depending upon the relative prices of feeders and slaughters and the level of corn production.

Price Prediction Models

Foote, Williams and Craven formulated an impressive study of a set of quarterly three-equation models which were fitted statistically via three stage least squares to represent the economic forces which determine prices for cash pork bellies at Chicago, movement into consumption for the 48 states, and end-of-period U.S. stocks. The models were formulated to allow price predictions to be made for cash and futures quotations. The models performed satisfactorily for most periods from the second quarter of 1971 forward. However, at times the models yielded prices which varied greatly from the actual values. The authors concluded that a month was too short a time span to allow the economic forces in the pork belly economy to be adequately

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13 Chiang, et al., p. 19.

described in an econometric model of the type which was developed. A Nerlove model in which individuals adjust expectations according to the proportion of the error made in the previous period was used in the estimation of supply responses.

Davis\textsuperscript{15} constructed an econometric model in which the primary goals were to predict feeder cattle prices after discovering the underlying relationships within and among the feeder cattle, slaughter cattle, and retail sectors of the beef industry. The model as specified had at least three signs on coefficients which did not correspond to what economic theory would suggest \textit{a priori}. Some of these "incorrect" signs perhaps could have been rationalized by a discussion of the expectation argument in supply response as presented a number of times in the preceding studies. In the actual forecasts, a substitution of lagged values for current values was performed in order that data could be made available for predictions. This seemed quite irregular since the coefficients for the current variables were retained and the lagged values were used after having minimized the squared errors with respect to the current values. Finally, the model correctly predicted only two out of five directions of change for price and three out of five directions of change for quantity. Apparently the Davis study is the only attempt made to predict feeder cattle prices. In light of Davis' results, the present study, which makes explicit use of

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\textsuperscript{15}Davis, Joe T., An Econometric Analysis of the Quarterly Demand and Supply Relationships for Feeder Cattle in the United States, unpublished Ph.D. dissertation, University of Tennessee, August, 1974.
\end{flushright}
forecasting methods, could conceivably add a second approximation to the knowledge of feeder cattle price prediction.

Walter Myers\(^{16}\) developed a sinusoidal function for predicting hog prices. He argued for de-emphasis of exposing and explaining the statistics of a given technique in favor of stressing the production of useful information. To this end, he stated that several statistical properties of his model either were violated or fell outside the bounds of conventional acceptability. More specifically, the model indicated extreme positive serial correlation due to the omission of a few important variables. These were eliminated when it was found that the error involved in forecasting future values exceeded their additional contribution to reducing the standard error of the predictive equation. In the evaluation of his model, Myers sought to determine whether hog futures should be bought or sold. In a 24 month period, he made only two identification errors, although he cautioned that he used exact rather than estimated values of the independent variables.

**Forecasting Evaluation**

Burch and Stekler\(^{17}\) have written a note which evaluates the forecasting accuracy of the index of consumer sentiment constructed by the Michigan Survey Research Center. To evaluate the accuracy of the index, mean square error and mean absolute errors were calculated. For


purposes of the present study, it was decided that the mean square error would be used to evaluate the forecasting models since that statistic includes the two components, bias and variance.

The authors used the Cochrane-Orcult iteration technique to eliminate autocorrelated disturbances which arose in the initial estimations. Using Box-Jenkins methods makes unnecessary the separate treatment of autocorrelated residuals since the methods explicitly consider the autocorrelation function in the formulation of parameter estimation.

Stekler in a separate article evaluates the forecasts of a number of econometric models using Theil's U-coefficient. He notes that it is inappropriate merely to count the number of true turning points forecast for two reasons. First, a model that is expected to predict quantitatively should be judged on that basis, and attention should not merely be confined to the few turns that occurred. Second, the right or wrong searing system for accuracy is inappropriate because the extent of the error matters. For these reasons, he argues for use of the U-coefficient. Calculation of the U-coefficient for a forecast model can be compared with the U-coefficient generated by a naive random walk model as a benchmark. A random walk model yields a U-coefficient of 1. The predictive performance of any forecasting model which also predicts change will always be equal to or better than that of the random walk model. The Box-Jenkins models developed later will

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be compared to a random walk model in terms of both mean square errors and U-coefficients.

Theil goes into greater detail by decomposing the U-coefficient into errors due to central tendency, unequal variation and incomplete covariation. Errors of central tendency occur when the average predicted change differs from the average realized change. Errors of unequal variation occur when the standard deviation of predicted and realized changes are unequal. Errors of incomplete covariation exist when the correlation coefficient between predicted and realized changes is less than one. The purpose of decomposition is to make linear corrections in the forecasts.

When the error due to incomplete covariation constitutes most of the total error, the opportunity for making corrections in the forecast are dim. When the forecast error stems mostly from errors in central tendency or unequal variation, corrections can be made when it is believed that the forecaster will continue to make the same kind of systematic error in the future. However, there is considerable controversy over whether trend adjustments should be made. Only in the case where the trend is believed to be permanent should these types of adjustments be made. For this reason, no trend terms were employed in the Box-Jenkins methods and hence there is little point in decomposing the U-coefficient. In fact, the existence of a systematic error over a small range of forecast values would be spurious.

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since the Box-Jenkins methods should account for all persistent sys-
tematic error.

Futures Trading as a Hedging Device

Theory of Futures Trading as Related to Hedging

A hedge is defined as the taking of equal and opposite (quantity) positions in the cash and futures markets. Since cash and futures prices in some markets tend to approach a fairly stable basis as the futures contract matures, the gains or losses in the cash market approximately offset the losses or gains, respectively, in the futures market position. By hedging, a producer can "lock-in" on a certain price at the beginning of a production period as long as the cash and futures prices converge to a stable basis as the contract approaches its expiration. Cash and futures prices should ultimately be equated during the last month of a contract. Arbitrage between the two markets insures this, for at that time the futures contracts can be fulfilled in the cash market.

For some commodities, the cash and futures markets bear a consistent functional relationship during the life of a contract. These are the markets for storable commodities. The theory behind this functional relationship has been well developed. The nonstorable

20A basis is defined as the difference between cash and futures prices at any point in time.

commodities have had only a relatively recent existence on the futures markets and there has been generated a substantial amount of research in the attempt to clarify the hedging potential of nonstorable commodities.  

Futrell presented a paper in 1966 which noted some major differences between the typical storable commodity futures contract and the newborn live beef futures contract. At that time the live cattle contract had been in existence a little less than two years. Some of the limitations on the degree to which livestock futures markets can perform hedging and pricing functions were offered. One problem he noted was the wide range of quality and weight encompassed in the choice grade. The producer may not be able to accurately estimate the correlation between his cattle and that of the terms in the futures contract. This would create a problem in estimating the price that is "locked-in". He also conjectured that it would be difficult to closely estimate supply patterns in the future since the producers have great flexibility as to what weight they may opt to bring the stock to slaughter. This would make future supply and demand a difficult assessment. Futrell also pointed out that there may be a lack of hedging incentive if the cash market consistently exceeds the futures

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22 The storable commodities include the grain crops, soybeans, and their by-products. The nonstorable commodities are those for which production is continuous throughout the year.

price during the term of the contract. In fact, this has occurred quite frequently in cattle futures and the short hedgers have generally suffered.

A paper presented by Allen Paul purported to argue that the cash and futures markets for livestock should be functionally related in much the same way that the storable commodities are related via the cost of storage. He contended that a live beef futures contract was a substitute for "contract farming" in which a feedlot operator agrees to fatten a feeder on his lot for a certain price. The difference between the futures price of a choice 1100 pound steer and the spot cash price would be the cost of the feeder plus feed cost to bring the animal to 1100 pounds by the time the contract matures. The logic has some appeal but unfortunately, history has shown that oftentimes the cash price is greater than the futures price.

Elder sought to develop a theoretical hedging decision model for cattle feeders. He derived an optimal hedge by minimizing the variance of income with respect to hedging proportion, given expected net returns from cattle feeding. He concluded that the level of hedging

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24 Hedging short refers to selling futures and buying in the cash market at time $t_0$ then buying futures and selling cash at $t_1$.


as a proportion of expected output is negatively related to the correlation between finished weight of cattle and the price received for finished cattle, and also negatively related to the correlation between the variable costs of production and the price received for finished cattle.

Tomek and Gray, 27 have argued that hedging has historically been thought of primarily as an aid in the carrying of inventories and only recently have futures markets been thought of in terms of a forward pricing function. The futures markets for non-inventory commodities which are continuously produced involve production hedges rather than inventory hedges. Whether an optimal hedge can be placed is questioned since if the futures price is more variable than the cash price, a routine hedge does not stabilize revenue. The gains in income stability for a production hedge, they hold, while minimal for continuously stored commodities, may be substantial for other commodities. For live beef cattle and other commodities, this of course is an empirical question. Leuthhold addressed the forward pricing function in an article discussed in the next section.

Application of Hedging to the Livestock Sector

Heifner\textsuperscript{28} has argued that price risks can be shifted through hedging almost as effectively in livestock as in grain. He found that overall price risk was minimized by hedging less than 100% of the cash position. In cattle feeding minimizing risk hedging levels and hedging level effectiveness tended to decline as distance from the delivery point increased. Quality differences affecting hedging potential was relatively less important in livestock than in grain storage. Apparently at least one of Futrell's caveats was unwarranted.

Heifner employed portfolio theory to obtain a preferred combination of expected profits and risks. The model he developed was quite sensitive to futures profit expectations and cash profit expectations in determining the optimal hedging levels. But in determining the expectations for cash profits he admitted that there may be substantial problems in defining a cash profit level. This problem notwithstanding, Heifner derived the optimal hedging level range for slaughter cattle to be between .56 and .88 unit of short futures per unit of slaughter cattle, disregarding trading costs and futures price bias.

Ehrich\textsuperscript{26} tested the theory that the cash-futures price spread should be related to production costs. He concluded that the


cash-futures spread was appropriately specified as a function of the cost of feeding a calf from feeder weight to finished weight. Under the theory of perfect competition, price spreads will be forced into line with the cost structure of low cost firms. The cash price of feeder cattle are tied by economic forces to prices of futures contracts. The spread would be the price of feedlot services and as such the spread coordinates decision making in the industry. "However, unlike the case of storable commodities, where cash-futures spreads signal adjustments in quantity stored, it is not expected that quantities placed on feed will adjust to cash-futures price spreads for beef cattle. Rather, the price of feeder cattle will adjust to expected fed cattle prices, the latter being determined by inventories of available feeder cattle and expected demand".  

30 As will be seen in the theory section discussed shortly, fed cattle prices are used as a signal to the producers of feeder cattle in the prediction model presented herein.  

Leuthhold 31 examined the efficiency with which the beef cattle futures market fulfilled the forward pricing role by computing the mean square error on the difference between the price at delivery and the corresponding price j weeks prior to delivery. He found that live beef cattle cash prices are a more accurate indicator of cash

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30 Ehrich, p. 38.

price conditions after approximately four month than is the futures price. He also concluded that futures price becomes less and less efficient both absolutely and relative to the cash price estimates as distance from maturity increases.

If a reasonable price forecasting model can be developed, it will contribute additional information to the function of forward pricing. The economic theory which serves as a basis for the development of the price forecasting model is presented in the next section.
CHAPTER III

STRUCTURAL DERIVATION OF THE FEEDER CATTLE MARKET

Only the most important variables in the feeder cattle market should be selected for use in a forecasting model. The analytically reduced form for the price of feeders will be considered for possible use as an input series in a quarterly time series forecasting model. For these reasons, it is necessary to develop a structural model of the beef-cattle sector.

It is hypothesized that the beef-cattle sector can be subdivided into four sectors: the retail market for beef, the slaughter market for carcasses, the fed, or finished, cattle market, and the feeder cattle market. Each of these markets will be analytically examined in turn. Endogenous variables in the model are denoted by an asterisk. Only the variables in the feeder cattle market are precisely defined and that data appears in Appendix Table A. The demand for feeder cattle ultimately originates at the retail level in the demand for beef. It is assumed that the quantity of beef demanded is primarily a function of the price of beef, and the prices of its two traditionally named substitutes, chicken and pork. In addition, income is also cited as an important argument. Thus, the quantity demanded of beef may be represented as:

\[ Q_{Bd}^d = f(PB^*, PCH^*, PP^*, Y) \]
where

\[ \text{QB}^{d*} = \text{quantity of beef demanded, per capita} \]
\[ \text{PB}^* = \text{retail price of beef per lb.} \]
\[ \text{PCH}^* = \text{retail price of chicken per lb.} \]
\[ \text{PP}^* = \text{retail price of pork per lb.}, \text{ and} \]
\[ Y = \text{disposable personal income per capita} \]

Since beef, pork and chicken all compete for consumers' meat dollar, it can reasonably be argued that at the retail level, the prices of chicken and pork are determined simultaneously with the quantity and price of beef. This is explicitly taken into account by formulating the following demand equations for pork and chicken:

\[
\text{(2)} \quad \text{PCH}^* = f(\text{QCH}, \text{QP}^*, \text{QB}^*, Y)
\]
\[
\text{(3)} \quad \text{QP}^{d*} = f(\text{PP}^*, \text{PCH}^*, \text{PB}^*, Y)
\]

where variables not defined above include

\[ \text{QCH} = \text{quantity of chicken demanded, per capita} \]
\[ \text{QP}^* = \text{quantity of pork demanded, per capita} \]

The choice of price or quantity as the dependent variable in a demand equation is arbitrary in a mathematical sense especially in a simultaneous system where both price and quantity are deemed endogenous. In many studies, price has been expressed as a function of quantities; in others, quantity has been expressed as a function of prices. In an analytical sense, it has been argued, the selection of normalized
variables is not arbitrary. In the present formulation, QB was ex-
pressed as a function of prices at each level because it was felt
that in the elimination of explicit identities ($QB^d = QB^s$), price
should be the variable that is maintained. It seemed reasonable to
suggest that price rather than quantity is the more likely signal
which is monitored at each level from wholesale to the feeder cattle
market.

Supply functions at the retail level were specified for beef and
pork. No supply function was specified for chicken because evidence
suggests that at least monthly variations in cold storage holdings of
broilers are quite small in relation to production.¹ For convenience,
it did not seem unreasonable to assume this fact would hold for a
quarterly model as well.

According to basic economic theory, the quantity supplied of a
commodity is a function of its own final product price and the cost
of other inputs in the production process. Hence, the quantity sup-
plied of beef was considered to be a function of the price of beef,
cold storage in the current quarter and the price of carcasses in the
current quarter. The retail quantity supplied of beef is represented
as:

(4) \[ QB^s* = f(PB^*, CSB, PC^*) \]

where

¹L. H. Myers, and Joseph Havlicek, Jr.
CSB = cold storage of beef in the current quarter

PC* = price of carcasses in the current quarter

The supply function for pork is treated similarly except that an index of the cyclical nature of hog quantities and prices is explicitly included along with the retail price of hogs, cold storage of pork lagged one period and the price of slaughter hogs lagged one period. The retail quantity supplied of pork is thus

\[(5) \quad Q_{PS}^* = f(P_{P}^*, \text{CSP}_{t-1}, \text{HCY}, \text{PSH}_{t-1})\]

where

\[\text{CSP}_{t-1} = \text{cold storage of pork in the previous quarter}\]
\[\text{HCY} = \text{an index of relative position on the hog production cycle}^2\]
\[\text{PSH}_{t-1} = \text{price of slaughter hogs in the previous quarter}\]

Included in the above five equations are six endogenous variables, \(PB^*, QB^*, PP^*, QP^*, PCH^*,\) and \(PC^*\). Thus the retail block of five equations cannot be solved simultaneously. The sixth endogenous variable which renders the first five equations insoluble is the price of carcass beef \((PC^*)\). In order to explain the determination of \(PC^*\), the slaughter and fed cattle markets must be considered.

In the slaughter market the quantity demanded of carcasses is explicitly treated. Carcass beef may be considered a factor in the production of retail beef. The quantity demanded of carcasses is

\[^2\text{See article by Myers and Havlicek previously footnoted.}\]
therefore considered to be a function of its own price; the price of the final product, beef; another major input expense in the production of beef, butchers' wages; and cold storage holdings of beef. The quantity supplied of carcasses in the slaughter market is taken to be a function of its own price; and two major input costs in the slaughter market, packers' wages and the price of fed cattle. The slaughter market is thus specified:

\[
QC^d* = f(PC^*, PB^*, BW, CSB)
\]

(6)

\[
QC^s* = f(PC^*, PW, PFC^*)
\]

(7)

where

- \(QC^*\) = quantity demanded (d) or supplied (s) of carcasses
- \(BW\) = butchers' hourly wage rate
- \(PW\) = packers' hourly wage rate
- \(PFC^*\) = price of fed cattle

Again the problem arises that there are \(N\) equations and \(N+1\) endogenous variables both when the slaughter market is considered in itself [equations (6) and (7)] and when it is considered along with the retail block of equations [equations (1) through (5)]. The problem is remedied, however, by the introduction of the next step in the derived demand sequence.

The variable not explained by any of the previous equations, yet introduced endogenously into the model, is the price of fed cattle. This variable is determined in the fed cattle market which describes the demand and supply for that commodity. The quantity demanded of
fed cattle is assumed to be a function of the price of fed cattle. Since the demand for fed cattle is also a derived demand and can be considered a factor of production of carcass beef, the remaining arguments in the demand for fed cattle are the factors of production in the slaughter cattle market. They include packing plant wages and the price of carcasses. The quantity supplied of fed cattle is a function of the price of fed cattle and variables indicative of other input costs in converting a 750 pound beef animal to approximately 1100 pounds. These would include the price of corn, the price of hay, and the number of head on feed. The price of feeders lagged two quarters would also be an important variable in the supply function. This price is lagged two quarters since feeders are generally placed in feedlots for approximately six months or two quarters. The fed cattle market can then be expressed as:

(8) \[ QF_{Cd}^* = f(PFC^*, PW, PC^*) \]

(9) \[ QF_{Cs}^* = f(PFC^*, PCN, HOF, PF_{t-2}) \]

where variables not defined previously are:

\( QFC^* \) = quantity of fed cattle, demanded (d), or supplied (s),

\( PCN \) = average quarterly price of No. 2 yellow corn received by farmers,

\( HOF \) = head on feed,

\( PF_{t-2} \) = average quarterly price of 750 lb. feeders lagged two quarters.
By virtue of the production lag in the placing of feeders into feedlots for six months, the price of feeders lagged two periods can be considered predetermined. This fact considerably simplifies the model. The first nine equations involve nine endogenous variables and can be solved independently of the final factor of production, the feeder cattle market.

The noteworthy feature at this juncture is that the derivation of the feeder cattle market can now be treated as a block-recursive system. A block-recursive system is a model composed of distinct groups of equations. What distinguishes these groups is the fact that the errors in one group are assumed independent of other groups. The error in one equation is dependent on the errors of other equations within the same group and hence the endogenous variables within the group must be solved simultaneously. The labor-saving upshot of a block-recursive system is that the endogenous variables in one group may be treated as predetermined in succeeding groups.

The block-recursive construct was employed in this research for the theoretical development only. The first block was not empirically estimated but only served in the logical development as to which arguments should be considered in the feeder cattle market.

The block-recursive framework was used for three reasons:

1. The other feasible alternative for a derived demand factor of production was a simultaneous model of approximately ten equations. The data requirements for such a model are too demanding for predictive work. The block-recursive system simplifies the empirical estimation.
2. The block-recursive framework lends itself easily to derived demand. The demand for feeder cattle is a derived demand.

3. It is intuitively appealing that the factor market groups monitor prices in finished product markets and then react. The block recursive model seems justified in the present case because of the two quarter production lag between buying the input factor, feeder cattle and the production of the final output, fed cattle.

Since the feeder cattle market is the primary focus of analysis in this research, the selection of arguments in the demand and supply functions involved greater attention. In the formulation of this block of major interest, it was assumed that the most important signal to the production of feeders coming from the fed cattle market was the price of fed cattle. This variable enters the second block as predetermined, having been solved in the first block. The second block, composed of the demand and supply functions for feeder cattle, forms the basis of variables which will be analyzed in the predictive model.

The quantity of feeders supplied is hypothesized to be a function of its own price; the cost of other inputs: the price of hay and and the price of calves lagged four quarters; and the inventories of heifers, steers, and bulls at the beginning of each quarter. The quantity of feeders demanded is assumed to be a function of its own price; the major input cost at the fed cattle level, i.e., the price of corn; the price of fed cattle; expectations of the price of fed cattle; and a fourth quarter seasonal dummy. In addition, the variable time was included in both the demand and supply functions to
account for slowly changing trends over the time period studied. The demand and supply functions for feeder cattle are thus:

\[ Q_{Fd}^d = f(P_f^*, P_CN, P_FC, P_FC^E, Q_4, TM) \]

\[ Q_{Fs}^s = f(P_f^*, P_H, PCV_{t-4}, HSB, HSB_2, HSB_3, HSB_4, TM) \]

where variables not defined previously are:

- \( P_f^* \) = average quarterly price of choice feeder steers, all weights combined, eight markets,
- \( Q_F \) = quantity of steers, placed on feed by quarters, 22 states,
- \( PCV_{t-4} \) = average quarterly price of calves 300-500 lbs., good and choice grades, lagged four periods, Omaha market,
- \( P_FC^E \) = expected price of fed cattle,
- \( TM \) = time,
- \( HSB \) = number of heifers, steers, and bulls on farms January 1, U.S. and
- \( HSB_i (i = 2, 3, 4) \) = slope shifter for HSB in respective quarter,
- \( P_H \) = price of hay except alfalfa, per ton, received by farmers, U.S.
- \( Q_4 \) = fourth quarter dummy variable.

The eleven equations presented above constitute the theoretical underpinnings of an analysis of the feeder cattle market. The whole theoretical block-recursive construct can be represented in a simple diagram.
Block No. 1

Retail Beef Mkt.
Carcass Beef Mkt.
Fed Cattle Mkt.

The first block represents 9 equations and 9 unknowns. This generally implies the system is soluble. From this solution, the price of fed cattle can enter Block No. 2 as a predetermined variable. The implicit assumption is that the errors in equations of Block 1 are independent of the errors of equations in Block 2.

Block No. 2

Feeder Cattle Mkt.

Derived Demand Nature of the Feeder Cattle Market

The preceding sections have indicated the derived demand nature of feeder cattle through a description of the block-recursive framework. To further clarify theoretical foundations, a graphical analysis of the demand for feeder cattle is also presented. It should be borne in mind that the diagrammatic presentation assumes a perfectly competitive structure in that each firm in the industries presented is considered a price taker, and market equilibrium takes place where the value of marginal product equals supply.

Consider first the industry demand for retail beef. Explicit in diagram (a), Figure 3-A, is the quantity of beef as a function of price. Implicit are the shift parameters which include the price of substitute meats: chicken and pork, and personal income. Diagram (b) shows an industry supply curve as the summation of marginal cost (MC) schedules for retail beef which lie above the average variable cost curve (AVC). Explicit in the diagram again are quantity on the horizontal axis and dollar amount on the vertical. Implicit in the supply schedule are the shift parameters, comprised of other major
Figure 3-A. Derived Demand Nature of the Feeder Cattle Market.
input costs in retail beef production: butchers' wages and the price of carcass beef. If diagram (a) is superimposed on (b), the intersection of D and S gives the equilibrium quantity and price of beef at the retail level.

The intersection of supply and demand in the retail beef market yields an equilibrium quantity as well as an equilibrium price. The technical relationship between the equilibrium quantity of retail beef ($Q^*$) and an input, the quantity of carcasses is shown in diagram (c). Implicit in this diagram are the fixed levels of other inputs in the production of retail beef such as the number of butcher-hours employed and the amount of capital equipment ($K_1$) which exists in the industry. The slope of the total product curve at each point multiplied by the price of retail beef indicates the value of marginal product (VMP) curve for carcasses. The case for more than one variable input in deriving a VMP curve is analogous except that the schedule is comprised of a locus of points from shifting VMP curves. The production function can be expressed as $QB = f(QC, BMH, K_1)$.

The VMP curve constitutes the derived demand for carcasses which is depicted in diagram (d). The implicit arguments in the derived demand function for carcass beef include: the price of the final product for which carcass beef is a factor of production, i.e.,

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3 It may be argued that the VMP relation multiplied by output price breaks down in the aggregate in which case the derived demand for a product should be expressed as a function of prices. The standard derived demand discussion using quantities as arguments is presented here for simplicity of exposition. The estimated feeder cattle derived demand employs prices as arguments.
the price of retail beef; the other major input price in the production of retail beef, butchers' wage rate. The demand for carcass beef can be shown then as $Q_C^d = f(PC, PB, BW)$.

The industry supply schedule shown in (e) is composed of a horizontal summation of identical $MC$ schedules when $MC > AVC$. Implicit shift parameters include other major input costs in the production of carcass beef: packers' wages and the price of fed cattle. The superimposition of schedule (d) on (e) renders the equilibrium price and quantity of carcass beef.

The technical relation between the amount of carcasses produced and the number of fed cattle required as input, given all other inputs is given by the production function in diagram (f). The important implicit inputs include packing plant man-hours ($PPMH$), and capital in the beef slaughtering industry, ($K_2$). Thus the production relation may be represented as $QC = f(QFC, PPMH, K_2)$.

The fed cattle market describes the demand and supply for that commodity. Since the demand for fed cattle is also a derived demand, and can be considered a factor of production of carcass beef, the logic of diagrams (g), (h), and (i) are exactly analogous to (a) through (c) and to (d) through (f).

The industry demand for fed cattle, shown in diagram (g) includes the implicit "fixed variables" packers' wages and price of carcass beef.

The industry quantity supplied shown in diagram (h) includes the shift "fixed variables" price of corn, price of hay, head on feed, and the price of feeder cattle lagged two periods (since the major factor
in the production of fed cattle is feeder cattle which is held in feedlots approximately six months).

The technical relation shown in diagram (i) relates the output, fed cattle, with the input requirement, feeder cattle, given the level of all other inputs in the production of fed cattle. These other inputs include the quantity of corn (C) and soybean meal (S) available, the number of steers on feed (SOF), and a measure of feed-lot capacity (K_3). The production function can thus be written:

\[ Q_{FC} = F(Q_F, C, S, SOF, K_3) \]

Finally, the demand for feeder cattle, derived thrice removed from the retail beef demand may be shown in diagram (j). The diagram depicted can be obtained by taking the first derivative of the total product curve in (i) and multiplying the resultant marginal physical product of feeder cattle by the "final" product price of fed cattle. This diagrammatic presentation, of course, again assumes all arguments other than feeder cattle in the production relation remain unchanged. In the actual aggregate derived demand relation which is estimated, prices rather than the quantities of a physical product relation are employed.

Having discussed verbally and diagrammatically the derivation of feeder cattle demand, a presentation of how some of these variables interact with the supply of feeder cattle and ultimately affect the price of feeder cattle is now in order.
A Priori Signs on Coefficients in the Feeder Cattle Market

The supply and demand function which comprise the feeder cattle market are treated as a block independent of the rest of the system. Changes in any of the endogenous variables in the retail, carcass, or fed cattle market affect the feeder cattle market only through the effect which these variables have on the price of fed cattle. This can be recognized by observing that the price of fed cattle is simultaneously determined with other endogenous variables in equations (1) through (9). The relationship which the feeder cattle market has with the first block enters recursively into the demand for feeders, equation (10), through the price of fed cattle. A theoretical analysis of the coefficients in the first block of equations will generally be made through an analysis of how the price of fed cattle is affected.

The analysis may be presented diagrammatically. First, consider a few arguments at the retail level. Economic theory would dictate that the demand for retail beef would shift outward and to the right given an increase in income or the prices of retail beef substitutes, chicken and pork. In the series of diagrams presented earlier, this is indicated by a broken line labeled $D_1D_1'$ in diagram (a). When diagram (b) is superimposed on (a), a new, higher equilibrium price ($P'$) and quantity ($Q'$) are achieved. In terms of the technical relationship in diagram (c), there is an outward shift in this curve since it is assumed that a change in the retail demand brings to bear forces which would tend to alter the proportion of factors to the variable input, carcasses. The implicit assumption is
that profit maximization criterion assures movement along an expansion path which employs more than one input. As a result of a change in demand, there will be a movement of the curve in diagram (c) to the new higher level of carcasses QC'.

In diagram (d), the derived demand for carcass beef is obtained by multiplying the final product price, \( PB \) times the marginal physical product of carcass beef, \( MPP_C \). Since \( PB \) has increased and the \( MPP_C \) has changed only slightly, (implicitly assuming that employment of carcasses as a factor is not close to stage III of production), the derived demand for carcasses shifts outward to \( D_2 \)'s. The final product price argument acts as a shift parameter in all derived demand equations. In this case, for example, \( PB \) is included in the derived demand for carcass beef equation (6).

Tracing the effect of a shift in retail demand with analogous logic through the remaining market levels leads to outward shifts in the demand for fed cattle and finally to feeders. Thus arguing from the source at the retail level, changes in personal income, the price of chicken or the price of pork is seen to have a positive sign with respect to the price of feeders.

Changes in arguments affecting the supply schedule are more complicated than changes affecting demand functions. This is because changes in supply factors shift the marginal physical product schedules, as well as the price of the final product, due to factor substitutions. For example, a decrease in butchers' wages cause two opposing forces to come into play. First, the industry supply schedule
for retail beef shifts to the right. Given the unchanged retail demand for beef, the retail price of beef will fall. However, as more butchers' man-hours are employed due to lower wages, the marginal physical product of carcasses (MPPₜₜ) would increase. If the increase in MPPₜₜ outweighs the decrease in PB, the derived demand for carcasses would increase, and conversely. The derived demand for fed cattle and for feeders will shift in the same direction as that for carcasses via the same reasoning as presented above in the income, chicken and pork prices argument. Thus it is not possible to state absolutely what a priori sign should be posited for changes in the prices of final product supply functions with respect to changes in the demand for feeders.

Other arguments in the retail, carcass, and fed cattle markets can be similarly traced as to their effect on the demand for feeders via the price of fed cattle.

Now consider equation (10). Each argument in the demand function for feeders will be examined as to its a priori sign.

\[ Q_{Fd}^* = f(PF^*, PCN, PFC, PFC^E, Q4, TM) \]

The first argument, PF*, needs little explanation. The demand for any normal good is a negative function of its own price.

The price of corn, PCN, is a factor in the production of fed cattle. An increase in the price of corn is thus an increase in the factor cost of fed cattle. Ceteris paribus, the increase in factor cost would decrease the supply of fed cattle and concomitantly decrease
the demand for all factors in the supply of fed cattle, including feeder cattle. Hence, the a priori sign on the price of corn with respect to the demand for feeders is expected to be negative.

Implicit in the above arguments, it has been established that the a priori sign of the price of fed cattle with respect to the quantity of feeders is positive. This conclusion obtains from the fact that the derived demand for a factor is equal to final product price multiplied by the factor's marginal physical product.

The expected price of fed cattle consists of a variable which will hopefully capture anticipated final product price. Perhaps a relevant argument in this regard would be the price of fed cattle two quarters hence since this is the point at which 750 lb. feeders would be sold as fed cattle. The variable used to capture this expectation is the price of fed cattle lagged two periods. This reasoning implies that feedlots look at the price of the quarter in which they will sell their fed cattle as of one year ago to form an opinion of what price they may expect. The a priori sign with respect to quantity of feeders demanded would again be positive.

Preliminary investigation indicated the feeder placements increased in the fourth quarter but price did not consistently fall in that quarter. The latter would be expected if demand did not exhibit a seasonal change and the supply function did. This suggested the possibility of including a dummy variable in the fourth quarter to account for a seasonal change in demand. In fact, the underlying reason for an increased demand in the fourth quarter may be the lower expense of feed. The dummy, Q4, was expected to have a positive sign.
The final argument in the demand for feeders is time (TM). The time variable involves the sequential numbering of successive quarters to account for slowly changing trends in the feeder cattle industry. Since the number of feeder cattle marketed has increased monotonically for the last 100 years, the a priori sign is expected to be positive.

Equation (11) represents the supply function for feeders. Each of the variables in the equation will be examined as to its a priori sign with respect to the quantity supplied of feeders.

\[(11) \quad QF_s = f(PF, PCV_{t-4}, HSB, HSB2, HSB3, HSB4, TM)\]

The first argument in the supply function, PF, is the price of feeders. Basic economic theory dictates that the price of a good in a supply function should be positive if the schedule is to be upward sloping.

The price of hay, as a factor of production was expected to enter the supply function negatively. On a national basis, however, the price of hay can be dramatically different and the variable was ultimately dropped due to lack of significance.

The third variable, the price of calves lagged four quarters (PCV_{t-4}), represents one of the basic costs in the production of feeders, i.e., the cost of calves lagged four quarters. Items involved in the cost of production are expected to have a negative sign with respect to the quantity of feeders supplied.

The next four variables (HSB, HSB_i, i = 2, 3, 4) represent the inventory situation. The supply of feeders in any given year is for the most part biologically determined. HSB refers to the number of
heifers, steers, and bulls under 500 lbs. on farms January 1. In order for this variable to provide information for each quarter, a dummy slope shifter was introduced to account for the differing effects the January 1 stock would have on feeder marketings throughout the year. The quarterly coefficients on this variable structure can be determined by the data.

A diagram of how this dummy structure works is presented below:

<table>
<thead>
<tr>
<th>Year</th>
<th>HSB</th>
<th>HSB2</th>
<th>HSB3</th>
<th>HSB4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960 I</td>
<td>X</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1960 II</td>
<td>X</td>
<td>X</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1960 III</td>
<td>X</td>
<td>0</td>
<td>X</td>
<td>0</td>
</tr>
<tr>
<td>1960 IV</td>
<td>X</td>
<td>0</td>
<td>0</td>
<td>X</td>
</tr>
</tbody>
</table>

X represents the reported January 1, 1960, inventory of heifers, steers, and bulls under 500 lbs. The same variable appears as a slope shifter in the second, third, and fourth quarters. The value for X changes each year. No a priori sign can be given for this set of variables other than HSB4. Since historically, fall has been the most active trading season for feeders, HSB4 should have a positive sign with respect to the quantity supplied of feeders.

Time (TM) is also included in the supply function. Since the number of feeders traded has increased almost monotonically over time, the a priori sign on the coefficient should be positive.

A priori signs for the demand and supply functions for feeders have been presented above. The empirical estimation of these equations will be presented in the next Chapter. In addition, since the primary
goal of this research is the forecasting of feeder cattle prices, analytically reduced forms will also be presented in order that PF may be presented as a function of predetermined variables.
CHAPTER IV

EMPIRICAL ESTIMATION OF STRUCTURAL EQUATIONS AND THE
CONVERSION TO ANALYTICALLY REDUCED FORMS

Every empirical piece of work has its share of data problems. The research herein was no exception. Some of the more significant problems encountered are noted below.

Data

The data base employed in this quarterly model were the years 1960 through 1975. 1960 was chosen as the starting point since a few of the more important series involved in the study originated in that year.

Considerable time and effort was devoted to discovering a structural model which remained fairly stable from 1971 through 1975. The quixotic objective of those ill-spent months was to confront the predictive model with an acid test by extrapolating beyond the range of the data for five consecutive years starting in 1971. Anyone familiar with the cattle industry will appreciate the fact that 1971 separates the "easy years" from the "model scrapping years". Starting in 1972, the industry was subjected to selective price controls, acute shortages of feed grains, significant foreign grain deals, and unprecedented inflation.

Although a structural model was obtained for which most of the variables remained significant and of the correct theoretical sign for
the five models estimated from 1971 through 1975, the analytically reduced forms derived from the structure failed grotesquely to provide a satisfactory fit of the actual data in the turbulent years beyond 1971. Ultimately it was decided to extend the data base through 1975 in order to concentrate on a predictive model. Recall that initially the intent was to use the analytically reduced form for the price of feeders as an input series in the application of Box-Jenkins methods.

The nature of the predictive model also argued for an extension of the data base through 1975. Since a quarterly model was estimated, only 64 observations were available. In the structural regression model, this was not found to be a significant problem. The limited number of observations did, however, present some difficulty in attempting to apply Box-Jenkins methods in identifying and fitting a time series model to the data. Fifty observations is considered the bare minimum required to apply such techniques.

Considerable difficulty was encountered in the construction of the individual data series of the model in two respects:

1) There was a lack of exact correspondence between the two most important endogenous variables, the price and quantity of feeders. Whereas the price series employed was obtained from a weighted average cost per 100 pounds for eight national markets combined and referred only to steers, the quantity variable consisted of cattle and calves, regardless of sex, placed on feed in 22 states. Although these two variables do not correspond exactly, it was felt that these series were the best choice of available data.
2) Perhaps the most important single independent variable in the determination of the price of feeders was the inventory of heifers, steers and bulls. Unfortunately, until very recently, this variable was only available once a year, on January 1. In order to handle this problem, the dummy variable construct mentioned at the end of Chapter II was devised.

Empirical Estimates

Two Stage Least Squares (TSLS) method of analysis was used to obtain the empirical estimation of equations (10) and (11). The coefficients of a simultaneously estimated system are known to be biased. The adequacy of the model is difficult to judge since there has really been no definitive criteria set on the magnitude of the standard errors in relation to their associated coefficients. Christ has suggested that t-values as low as 1.0 on an independent variable could be considered to have a statistically significant effect on the dependent variable.\(^1\) However, a more conservative view would be to ascribe the more stringent criteria of single equation estimation to the coefficients and demand t-values of approximately 2.0, that value representing the 95% level of significance. The point is, that in viewing the significance of the following coefficients, the acceptable conservative standard seems to be t-values of approximately 2.0.

The Structural Demand Estimation for Feeder Cattle

Recall the demand equation for feeders posited in the last Chapter was:

\[ Q_{d}^* = f(P^*, PCN, PFC, PFC^E, Q4, TM) \]

The result of the empirically estimated demand equation through 1975 was:

\[ Q_{Fd} = -1701.08 - .5985 PF + .8962 PFC + .9914 PFC^E - 1.09 1.77 2.23^{t-2} \]

\[ -138.07 PCN -1126.60 SLPCRN + 2562.86Q4 - .14 - 2.10 9.63 \]

\[ + 69.295 TM \]

\[ 5.23 \]

\[ R^2 = .83 \]

The a priori sign on the price of feeders in the demand equation was, as expected, negative. The t-statistic of 1.09 meets the more liberal significance criterion of > 1.01.

The final product price for which feeder cattle serve as an input, the price of fed cattle, had a positive coefficient as expected. It can be argued that a one-tailed test of significance could be applied, in which case, the coefficient for the price of fed cattle was significant at the 95% level.

The price of fed cattle lagged two periods served as a proxy for the expected price of fed cattle which feedlot operators could obtain.
at the time they sold an 1100 to 1300 pound steer. The estimated coefficient on this variable was positive and consequently corresponded to a priori expectations. The t-value was 2.23.

The price of corn had a negative coefficient and conformed to a priori expectations. This variable did not exhibit a significant t-value. An examination of the data indicates that this should not be surprising. Not until 1972 did the price of corn play an important role in the cattle marketing decisions. The price of corn was historically very stable but came to the fore starting in 1972. For this reason, a slope shifter on the price of corn, SLPCRN was included in the model for 1972 and succeeding years. As was expected, SLPCRN had a negative sign and entered the equation with a significant t-value of -2.10.

The fourth quarter dummy variable in the demand equation, Q4 entered the equation positively as expected, with a highly significant t-value at 9.63.

The time coefficient had a positive sign with a t-value of 5.23. Since demand by feedlots for feeder cattle has increased secularly, the coefficient conformed to a priori expectation.

The Structural Supply Estimation for Feeder Cattle

Recall the supply equation hypothesized in the last chapter was:

\[
QFS^* = f(PF^*, PH, PCV_{t-4}, HSB, HSB2, HSB3, HSB4, TM)
\]

The result of the empirically estimated supply equation was:
The a priori sign on the price of feeders was positive as expected in the supply equation. The t-value obtained was significant at 2.75. Although cow-calf operators and backgrounders cannot produce many more 750 pound steers than they have during any quarter, they can elect to respond positively to an increase in the price of feeders with their available stock on hand. The sign indicates that they do so.

The price of hay, as mentioned previously was not significant and the variable was dropped from the equation. The price of calves lagged four quarters represents an input cost to the producer of 750 pound feeder cattle, whether the cost is explicit, as in the case of a backgrounder, or implicit as in the case of a cow-calf operator. The price of calves lagged four quarters has a negative sign which conforms to a priori expectations.

The inventory situation, as has been alluded to previously, was perhaps one of the most important variables in the determination of price. "Heifers, steers, and bulls under 500 pounds on farms January 1"
has been the only figure released on this important series until recently. (Now the inventory situation is reported semi-annually.)

The coefficients on the inventory variables fluctuated considerably for estimations based on five annual data extensions from 1960-1971 to 1960-1975. The coefficient for HSB in the 1975 estimation was unusually large and negative. Although no a priori sign was hypothesized, it was assumed that the slope shifter HSB4, when added to HSB, would result in a positive coefficient. 1975 is the only year in which this did not occur. However, it was encouraging to note that HSB4 had the correct positive sign in relation to the first quarter placements, HSB.

Time as a variable in the supply equation was expected to be positive as the supply of feeder cattle has increased secularly. The coefficient corresponded to expectation with a t-value of 5.58.

Analytically Reduced Forms

The estimated equations for the demand and supply of feeders are for the most part, sterile in the form cited above. In order that the equations may be used for the input series of the forecasting model, analytically reduced forms were calculated. The purpose of this transformation is to make the endogenous variables in the model (PF and QF) solely a function of the predetermined variables. The analytically reduced forms may be obtained as follows:

Rewriting equations (12) and (13) with the endogenous variables on the left-hand side of the equations,
\[ QF^d + 0.5985 \, PF = 1701.08 = 0.8962 \, PFC + 0.9914 \, PFC_{t-2} \]
\[-138.07 \, PCN - 1126.60 \, SLPGRN + 2562.86Q4 + 69.295 \, TM \]

\[ QF^s - 0.5085 \, PF = 0.5696.32 - 0.6270 \, PCV_{t-4} - 0.146 \, HSB \]
\[-0.0140 \, HSB2 + 0.0084 \, HSB3 + 0.0800 \, HSB4 + 85.38 \, TM \]

Letting \( QF^s = QF^d = QF \), and rewriting equations (14) and (15) in matrix form yields the desired analytically reduced forms after inverting the 2 x 2 matrix. The operation is indicated in Table 4-A.

In this form, impact multipliers, the simultaneous system's conceptual substitute for elasticities may be computed. Since the primary motivation of this research is forecasting, a calculation of the impact multipliers along with a discussion of the biasness which is to be expected as a result of simultaneity has been relegated to an appendix.

The more important function of analytically reduced forms is the possibility of using the forms for predicting the price of feeder cattle. Since PF is a function of predetermined variables, there is no conceptual difficulty in using values of the predetermined variables in order to forecast. Using the analytically reduced form for PF as an input to the forecast function (discussed subsequently) proved satisfactory until the data base was extended beyond 1972. In the turbulent years that followed, the analytically reduced forms produced egregious results which ultimately led to their rejection in favor of employing the first stage OLS equations used in the TSLS estimation of the structural model. The first stage OLS equation presents no
TABLE 4-A

ANALYTICALLY REDUCED FORMS FOR THE PRICE AND QUANTITY OF FEEDERS

\[
\begin{bmatrix}
\begin{bmatrix}
\hat{Q}_t \\
\hat{P}_t
\end{bmatrix}
\end{bmatrix} =
\begin{bmatrix}
1 - .5085 \\
1 - .5985
\end{bmatrix}^{-1}
\begin{bmatrix}
5696.32 & -.0140 & .0084 & .0802 & 85.38 & 0 & 0 & 0 & 0 & 0 \\
-1701.08 & 0 & 0 & 0 & 0 & 0 & 0 & 69.295 & .8962 & .9914 & -138.07 & -1126.60 & 2562.86
\end{bmatrix}
\begin{bmatrix}
1 \\
FCV_{t-4} \\
HSB \\
HSB2 \\
HSB3 \\
HSB4 \\
TM \\
PFC \\
PFC_{t-2} \\
PCN \\
SLPCR1 \\
Q4
\end{bmatrix}
\]
conceptual problem insofar as PF is expressed as a function of pre-determined variables. There is, however, an almost insurmountable practical difficulty - data on most of the independent variables does not become available until the same time the dependent variable data becomes available, thus making current period forecasts unnecessary and future period forecasts impossible.

This overwhelming practical difficulty suggests the use of methods developed expressly for predictive purposes. Box and Jenkins have recently developed a body of purely mechanical techniques which may serve our purposes. A heuristic explanation of the models used to forecast feeder cattle prices will be presented in the next Chapter.
APPENDIX CHAPTER IV

ELASTICITIES IN A SIMULTANEOUS MODEL

In a single equation model, with the dependent variable as the only endogenous variable, measures such as elasticities have meaning, practical value, and are calculated in a straightforward manner. The elasticity indicates the responsiveness of the dependent variable to a change in one of the exogenous variables, *ceteris paribus*. The computation has obvious policy implications in that a desired response in the dependent variable can be evoked by an appropriate change in one of the exogenous variables.

The computation of elasticities in a simultaneous system are of dubious meaning, questionable value, and are not calculated in a straightforward manner. This conclusion obtains from the fact that 1) simultaneous system coefficients are biased and 2) the *ceteris paribus* assumption required for elasticity computational validity is explicitly violated by virtue of simultaneity.

An example will serve to illustrate the point. The elasticity of a variable $Q$ with respect to variable $X$ is given by

\[
E_i = \frac{dQ}{dX_i} \cdot \frac{X_i}{Q}
\]

where $\bar{X}_i$ and $\bar{Q}$ represent respective means.

---

1 The content of this appendix borrows heavily from Chapter 19 of *Lecture Notes in Econometric Methods and Analysis* by Joseph Havlicek, Jr.
The assumptions which allow this interpretation in a straightforward way as an elasticity are that the quantity is endogenous and functionally related to the X variables and that the responsiveness of quantity to a change in any \( i^{th} \) exogenous variable may be interpreted individually with all other exogenous variables non-variant.

Now consider a simultaneous system with two endogenous variables \((Y_1, Y_2)\) and two equations. Let \( Y_1 \) represent quantity and \( Y_2 \) represent price. \( Z_i, i=0, 1, 2, 3 \) are exogenous variables and \( u_1 \) and \( u_2 \) are random disturbances.

\[
\begin{align*}
(2) \quad B_{11}Y_1 + B_{12}Y_2 + \delta_{11}Z_1 + \delta_{12}Z_2 + \delta_{10}Z_0 &= U_1 \\
(3) \quad B_{21}Y_1 + B_{22}Y_2 + \delta_{23}Z_3 + \delta_{24}Z_4 + \delta_{20}Z_0 &= U_2
\end{align*}
\]

Normalizing equation (2) with respect to \( Y_1 \) and equation (3) with respect to \( Y_2 \) and estimating by a simultaneous method, the two equations may be rewritten as:

\[
\begin{align*}
(4) \quad Y_1 &= \hat{B}_{12}Y_2 + \hat{\delta}_{11}Z_1 + \hat{\delta}_{12}Z_2 + \hat{\delta}_{10} - \hat{U}_1 \\
(5) \quad Y_2 &= \hat{B}_{21}Y_1 + \hat{\delta}_{23}Z_3 + \hat{\delta}_{24}Z_4 + \hat{\delta}_{20} - \hat{U}_2
\end{align*}
\]

In this form, the ambiguities of elasticity computations are more readily apparent. Equation (1) representing the calculation for an elasticity involves partial differentiation. In equations (4) and (5) partial differentiation is improper since \( Y_1 \) and \( Y_2 \) affect each other and change simultaneously. Consider in equation (4), an exogenous one unit change in say \( Z_1 \). \( Y_1 \) changes by \( \hat{\delta}_{11} \). In equation (5), this brings about a change in \( Y_2 \) of \( \hat{\delta}_{11}B_{21} \). This in turn affects
Y_1 via equation (4) through Y_2. The ceteris paribus condition of elasticities has not been fulfilled.

In addition, the estimated coefficient are biased. Monte Carlo studies have indicated that this is an inescapable result in using simultaneous methods. Therefore, even if elasticities were computed with abandon on simultaneous equations, they would be wrong to an unknown extent.

Having decried the use of elasticities in simultaneous models, a conceptual substitute will be proffered. In Chapter III, analytically reduced forms were derived in which the endogenous variables were presented as a function of only predetermined variables. Net impacts of changes in exogenous variables on the endogenous variables can be evaluated. The technique of impact multipliers is common parlance in the area of monetary theory and these derived net impacts are commonly used in making policy decisions in many fields.

Any impact multiplier or elasticity desired can be obtained by taking the partial derivatives of the dependent variables with respect to the independents or applying equation (1), respectively, to the equations set out below. The following equations are the analytically reduced forms of Table 3-A, Chapter III, after having determined the coefficients:

\[
(6) \quad QF = 2298.41 - .3390 PCV_{t-4} - 0.79 HSB - .0076 HSB^2 + .0046 HSB^3 \\
+ .0433 HSB^4 + 78.0 TM + .4117 \times PFC + .455 PFC_{t-2} - 63.43 PCN \\
- 517.56 SLPCRN + 1177.38 Q4
\]
(7) \[ PF = -6683 + 0.5665 \, PCV_{t-4} + 0.1319 \, HSB + 0.0127 \, HSB2 - 0.0076 \, HSB3 \]
\[ - 0.0723 \, HSB4 - 14.53 \, TM + 0.809 \, PFC + 0.895 \, PFC_{t-2} - 124.73 \, PCN \]
\[ - 1017.77 \, SLPCRN + 2315.29 \, Q4 \]
CHAPTER V

THE FORECASTING MODEL

Forecasting has been of great importance in a variety of fields including physics and economics. While the area of physics is generally considered to be a deterministic (exact) science, to a varying degree almost all phenomenon in the universe are subject to random variation. As such, a mathematical model which purports to explain a particular phenomenon is almost always stochastic to some extent. In the case of economics, the stochastic nature of a mathematical model is more apparent since a social science encompasses a myriad of nonquantifiable "human" elements. Accordingly, it may be possible to develop stochastic rather than deterministic models which can be used to calculate the probability of a future value lying between two specified limits.

Box and Jenkins have developed a number of stochastic models for describing and forecasting time series. In this Chapter, a heuristic presentation of some of their models will be given followed by the empirical results of applying those models to feeder cattle data.

1The explanation of forecasting models in this Chapter draws heavily from Time Series Analysis Forecasting and Control, George E. P. Box and Gwilym M. Jenkins, Holden-Day, 1970.
The Logical Framework of Time Series Models

The stochastic models employed by Box and Jenkins are based on the hypothesis that a time series in which successive values are highly dependent can be regarded as having been generated from a series of independent "shocks", \( a_t \). These shocks are random selections from a fixed distribution which is assumed to be normal, having mean zero and variance \( \sigma^2 \). Such a sequence of random variables, \( a_t, a_{t-1}, a_{t-2}, \ldots \) is called a white noise process.

The white noise is supposedly transformed to the observed time series \( Z_t \) by a linear filter, as shown in Figure 5-A. The linear filtering operation is a weighted (by \( \psi(B) \)) sum of previous values of \( Z_t \). Stochastic model construction involves finding the weights in the linear filter which renders a white noise series from a particular time series of interest discussed later. The model shown in the figure can be expressed as

\[
Z_t = \psi_0 a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots
\]

In regression analysis, a dependent variable \( Z \) is regarded as a function of several independent variables \( X_1, X_2, \ldots \). In the basic time series models, however, \( Z_t \) is considered to be a function of past values of itself, \( Z_{t-1}, Z_{t-2}, \ldots \). Hence, these models are called autoregressive.

The series \( Z_t \) which is observed is viewed as one realization of a stochastic process. A stochastic process is a statistical phenomenon that evolves in time according to probabilistic laws. In constructing
Figure 5-A. White Noise as a Transformation of an Observed Time Series Through a Linear Filter.
such a model, the underlying probabilistic generating mechanism is sought.

Maximum likelihood estimation is the technique employed to obtain model specification and estimates of the parameters of a time series model. Once the model has been specified, optimal forecasts can be made. The optimality refers to minimizing the mean square error of forecasts. The observations \( Z_t \) composing the realization can be described by an N-dimensional random variable \((Z_1, Z_2, \ldots, Z_n)\) with joint probability distribution \(p(Z_1, Z_2, \ldots, Z_n)\). This follows from the fact that an observation \( Z_t \) at a given time, say \( t = 25 \), is a realization of a random variable \( Z_t \) with probability density function \( p(Z_t) \).

Similarly, the observations at any two times, say \( t_1 = 25 \) and \( t_2 = 27 \), may be regarded as realizations of two random variables \( Z_{t_1} \) and \( Z_{t_2} \) with joint probability density \( p(Z_{t_1}, Z_{t_2}) \).

The joint density of the N-dimensional random variable will be considered a likelihood function from which the population parameters will be estimated. Since the \( Z_t \) series is a linear combination of the white noise process, which is assumed to be normal, the \( Z_t \) series is completely described by a knowledge of the mean and autocovariances of \( Z_t \). The autocovariances and mean of the process must be estimated from the data.

The principal tools used in constructing a model deal with the analysis of the autocovariance structure of an observed series. These are the Autocovariance Function (ACF) and the Partial Autocovariance Function (PACF). The Autocovariance Generating Function yields the
autocovariances of a linear process at succeeding lags. Initially it may not be known which order of autoregressive process to fit to an observed time series. The problem is analogous to deciding on the number of independent variables to be included in a multiple regression. The PACF is a device which by its very nature can describe an autoregressive process of \( p \)th order in terms of \( p \) non-zero functions of the autocorrelations. What constitutes an autoregressive process of order \( p \) may be clarified in the next section wherein a few models are presented.

An intuitive feel for what these models attempt to describe may be gained by considering a bicycle being propelled over a level surface. Let \( Z_t \) represent the speed of the bicycle after \( t \) revolutions of the bike's pedals. The \( a_t \)'s are the force applied on the \( t \)th revolution. In this example, the \( \psi \) weights would most likely be exponentially decreasing from 1.0. This would indicate that the most recent stroke of the pedal would have the greatest influence on the present speed of the bicycle. The model would be represented as

\[
Z_t = u + a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \ldots
\]

The Basic Models and Some Additional Concepts

Before describing the models, a few notational definitions are in order:

- \( B \) is a backshift operator.
- \( BZ_t = Z_{t-1} \)
- \( B^2 Z_t = B(BZ_t) = BZ_{t-1} = Z_{t-2} \)
In general,
\[ B^k \tilde{z}_t = \tilde{z}_{t-k}. \]
\( \nabla \) is a backward difference operator which can be written in terms of \( B \).
\[ \nabla z_t = z_t - z_{t-1} = (1 - B)z_t. \]

The Autoregressive Model

In the autoregressive stochastic model, the current value of a process may be expressed in terms of a finite number of previous values of that process and a shock \( a_t \). Let \( \tilde{z}_t = z_t - \mu \) where \( \mu = \) population mean. The autoregressive process of order \( p \), \( \text{AR}(p) \) can then be written.

(2) \[ \tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \ldots + \phi_p \tilde{z}_{t-p} + a_t \]

If the autoregressive operator of order \( p \) is written

\[ \phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p \]

(3) \[ \phi(B) \tilde{z}_t = a_t \]

We can also write (2) in terms of previous shocks. For example \( \tilde{z}_{t-1} \) can be eliminated from the right-hand side of equation (2) by substituting \( \tilde{z}_{t-1} = \phi_1 \tilde{z}_{t-2} + \phi_2 \tilde{z}_{t-3} + \ldots + \phi_p \tilde{z}_{t-p-1} + a_{t-1} \).

Other substitutions can likewise be made for \( \tilde{z}_{t-2}, \tilde{z}_{t-3}, \ldots \) to yield an infinite series in the \( a \)'s.

So equation (3) is equivalent to

(4) \[ \tilde{z}_t = \phi^{-1}(B) a_t = \psi(B) a_t \]
The Moving Average Model

Another model very similar to the above is the moving average model of order q, MA(q). While the AR(p) model expressed the current value \( Z_t \) as a finite number p of previous \( Z_t \) values, the moving average model of order q expressed \( Z_t \) as linearly dependent on a finite number of previous a's.

The moving average process of order q can thus be expressed as:

\[
(5) \quad \tilde{Z}_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q}
\]

If the moving average operator of order q is defined

\[
\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q
\]

then the moving average model can be written as

\[
\tilde{Z}_t = \theta(B)a_t.
\]

Mixed Autoregressive-Moving Average Models

To minimize the number of parameters required to describe a particular observed series and to provide greater flexibility in identifying a stochastic model, it is sometimes desirable to include both autoregressive and moving average terms in the model. The mixed autoregressive-moving average (ARMA) model of order p and q, respectively, can be written

\[
(6) \quad \tilde{Z}_t = \phi_1 \tilde{Z}_{t-1} + \ldots + \phi_p \tilde{Z}_{t-p} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}
\]

This model can be more economically expressed as

\[
\phi(B) \tilde{Z}_t = \theta(B)a_t.
\]
Discussion of all three models has made reference to the parsimonious expression of a series in terms of its required parameters. The value of the AR model is that it can be expressed as a finite number \( p \) of past values rather than an infinite MA series. Similarly, the value of the MA model is that it can be considered as a finite number \( q \) of random shocks rather than an infinite AR series. The ARMA model considers the case between the two extremes. The parsimonious representation is very important in practical estimation. If it were not for the opportunity to express an observed series as a finite linear combination rather than an infinite linear series, it would be impossible to estimate the infinite number of parameters. In fact, the parsimonious representation offers a lot of intuitive appeal. For example, the daily stock price of General Motors most likely depends more on what the price was yesterday rather than on what the prices were in the infinite past.

**Stationarity**

The stochastic processes described above may be either stationary or nonstationary. A stationarity stochastic process is in a particular state of statistical equilibrium such that its properties are unaffected by a change of time origin. More specifically, the joint probability distribution associated with \( m \) observations \( Z_{t1}, Z_{t2}, \ldots, Z_{tm} \) made at
any set of times $t_1, t_2, \ldots, t_m$ is the same as that associated with $m$ observations $Z_{t_1+k}, Z_{t_2+k}, \ldots, Z_{t_m+k}$ made at times $t_1+k, t_2+k, \text{etc.}$

Stationarity imposes conditions on the autocovariance matrix.

Any linear function of random variables $Z_t$ has a variance which can be expressed as a positive definite quadratic form. The positive definite-ness implies that the determinant and all the principal minors of the autocovariance matrix are positive.

All of the above stationarity conditions can be embodied in the single condition that the series $\psi(B)$ in equation (1) must converge for $|B| < 1$, when viewing the equation as a polynomial in $B$. A more pedestrian explanation of stationarity would be that the process has a finite variance, and the $\psi$ weights die out "sufficiently rapidly".

**Invertibility**

A very similar condition to that of stationarity, though independent of the stationarity condition, is applicable both to stationary and non-stationary models. This is the invertibility condition and applies to the $\pi(B)$ series where $\pi(B)$ is defined by $\pi(B) = \psi^{-1}(B)$.

A series of $\tilde{Z}_t$ satisfies the invertibility condition if $\pi(B)$ converges for all $|B| < 1$. This condition prevents the $\pi$ weights from increasing as they become more remote.

To illustrate the basic idea of invertibility, consider the model

$$\tilde{Z}_t = (1 - \theta B)a_t$$

(Note this is a moving average model as in equation (5) where $\theta_0 = 1$ and $\theta_1 = \theta$). Expressing the $a$'s in terms of the $Z$'s, equation (7)
(8) \[ a_t = (1 - \theta B)^{-1} \tilde{Z}_t = (1 + \theta B + \theta^2 B^2 + \ldots + \theta^k B^k)(1 - \theta B)^{-1} \tilde{Z}_t \]

or

(9) \[ \tilde{Z}_t = -\theta \tilde{Z}_{t-1} - \theta^2 \tilde{Z}_{t-2} - \ldots - \theta^k \tilde{Z}_{t-k} + a_t - \theta^{k+1} a_{t-k-1} \]

and if \(|\theta| < 1\), on letting \(k\) tend to infinity, we obtain the infinite series

(10) \[ \tilde{Z}_t = -\theta \tilde{Z}_{t-1} - \theta^2 \tilde{Z}_{t-2} - \ldots + a_t \]

and the \(\pi\) weights of the model in (9) are \(\pi_j = -\theta^j\). Whatever the value of \(\theta\), (7) defines a perfectly stationary process. However, if \(|\theta| > 1\), the current deviation \(\tilde{Z}_t\) in (9) depends on \(\tilde{Z}_{t-1}, \tilde{Z}_{t-2}, \ldots, \tilde{Z}_{t-k}\) with weights which increase as \(k\) increases. This situation is avoided by requiring that \(\pi(B) = (1-B)^{-1}\) converges for all \(|B| < 1\). The series is then said to be invertible.

Expansions on the Basic Models

Nonstationary Models

An important group of series encountered empirically have no fixed mean as do the stationary models described above, and yet they do exhibit some sense of homogeneity over time. If this homogeneity involves one part of the series behaving much like any other part except for its local level or perhaps local level and trend, then the series may be accommodated by a differencing operation. Taking first or second differences will transform these series to stationary processes. Many economic series exhibit this kind of homogeneous nonstationary behavior. When differencing is employed, the models are
called "integrated". An example of two autoregressive integrated moving average (ARIMA) models are given below in Figure 5-B.

Seasonal Models

In representing a seasonal model it is desirable for the forecast function to trace out a periodic pattern. The fundamental fact of a seasonal series is that it repeats itself for each season k intervals apart. Box and Jenkins offer a general multiplicative seasonal model in which the interseasonal autocorrelations identify one of the models already described in exactly the same manner as the intraseasonal autocorrelations identify a possible model.

The seasonal model thus emphasizes two important time intervals. Specifically, the forecast function should show (a) relationships between observations for successive intervals within a season and (b) relationships between observations for successive intervals between seasons. Let it suffice to say that if (p, d, q) represent respectively the number of autoregressive parameters, differencing, and moving average parameters for a within season stationary model and (P, D, Q) represent analogous parameters for a between season stationary model, then the general multiplicative seasonal model (p, d, q) x (P, D, Q) requires examination of the autocorrelation function and partial autocorrelation function in order to derive estimates for the differencing and parameter requirements of p, P, q, and Q.

A hypothetical stationary ACF with a seasonal component of 12 months may look like Figure 5-C below.
a) A Series Showing Nonstationarity in Level Such as can be Represented by $\phi(B)\nabla Z_t = \theta(B)a_t$

b) A Series Showing Nonstationarity in Level and Slope Such as can be Represented by $\phi(B)\nabla^2 Z_t = \theta(B)a_t$

Figure 5-B. Two Types of Nonstationary Time Series
Figure 5-C. Hypothetical Stationary Autocorrelation Function With a Seasonal Component at Twelve Months.
Transfer Function Models

The transfer function model is dynamic in nature and is applicable in situations where it is impossible to realistically assume that a series is a function of only itself. The transfer function model expresses an output series $Y_t$ as the function of an input series $X_t$ plus a noise series $N_t$. The model is most effective when the input series is a natural "leading indicator" of the output series. In the case of price forecasting for feeder cattle, the futures market price of feeders would be a natural input series save for the fact that the data base would be seriously eroded since feeder cattle futures have only been traded since 1971.

Recall that in the univariate cases discussed previously, the observed series was assumed to be generated by a linear stochastic process with an unknown autocorrelation function (which was estimated from the data). In the case of transfer function models, the ordered pairs of observations $(X_1, Y_1), \ldots, (X_m, Y_m)$ are treated as the realization of a bivariate stochastic process. As a result, not only the ACF's of $X_t$ and $Y_t$ must be examined, but so must the cross-covariances between $X$ and $Y$.

In most respects, the transfer function is very analogous to the univariate models in the identification and fitting stages in model building. Both the $X$ and $Y$ series must be properly differenced in order to obtain a stationary bivariate series. Stationarity is assumed when the estimated autocorrelations for $X$ and $Y$ and the cross-correlations between $X$ and $Y$ die out "sufficiently rapidly".
The objective of the transfer model is the same as that of the univariate case to transform X and Y to white noise, or randomness, via a judicious selection of parameter values. Clues to the judicious selection are provided by estimations of the ACF's, PACF's of X and Y, cross-correlations between X and Y, and the ACF and PACF for N. Equation (11) expresses the general form of the transfer function model:

\[
(1 - \delta_1 B - \ldots - \delta_R B^R)Y_t = (W_0 - W_1 B - \ldots - W_s B^s)B^b X_t
\]

and

\[
Y_t = \frac{(W_0 - W_1 B - \ldots - W_s B^s)}{(1 - \delta_1 B - \ldots - \delta_R B^R)} B^b X_t + \phi^{-1}(B)\theta(B)a_t
\]

when a noise series is added to the model.

\(b\) represents the number of lags between a change in the input and the beginning of an output response, and \(\phi^{-1}(B)\theta(B)a_t = N_t\).

Model fitting involves estimation of the \(W\)'s, \(\delta\)'s, \(\phi\)'s, \(\theta\)'s, and \(b\).

For all of the above models, there are formulas available for computing the standard errors of the coefficients. It is also possible to compute confidence intervals for the forecasts which are made from the derived models.

Empirical Estimations

The Seasonal Model

A cursory glance at the feeder price time series shown on the graph in Chapter I indicates that the series is nonstationary from
1960 through 1975. This is evidenced by the fact that there is an obvious upward trend in the data, the local mean level changes over time. It was found that application of the differencing operator displayed in Figure 5-B (a) was able to bring about the desired stationarity. In more common parlance, first differences were taken.

Working with the series of feeder prices only, a simple model such as the autoregressive and moving average types discussed earlier in the chapter, were attempted. The identification program specified an intraseasonal random walk model. This means the feeder price series (differenced once to obtain stationarity) was a white noise process and no specification of autoregressive parameters (p) or moving average parameters (q) were required to obtain a random residual series. The forecast function of a random walk model implies that the optimum forecast for period $t + 1$ is the value of the series in period $t$ plus a random shock, $a_{t+1}$. Since the expected value of the random shock is zero, the optimum forecast for period $t + 1$ is the price in period $t$. The model can be represented as (12) $\tilde{z}_{t+1} = \tilde{z}_t + a_{t+1}$.

The above model has limited practical value. Fortunately the autocorrelation function (ACF) for the feeder price series exhibited a high correlation, or spike, between the current price and that at $t = 6$, indicating the possibility of a significant seasonal component of six quarters. In terms of the previous seasonal model discussion, the existence of a $P$ or $Q$ parameter was suggested. When this possibility was explored using the data base 1960-1971, a multiplicative seasonal model was fitted to the data and the forecast function
provided more information than the simple random walk model. Now in addition to the current price and a random component, the price one period hence was found to be correlated with the price 6 periods ago. The autocorrelation function of the feeder price series is shown in Figure 5-D.

When the data base was extended through 1975, the spike at $t = 6$ in the ACF was lost and a white noise series was again obtained. However, examination of the white noise residuals indicated marked heteroskedasticity as feeder prices climbed to higher levels in the 1970's. A log transformation on the data succeeded in reestablishing a seasonal component at 6 quarters and ridding the white noise series of the heteroskedastic characteristic.

The seasonal moving average model with a season of 6 quarters and transformed by logs can be represented as:

$$
\nabla \log (Z_t) = (1 - 0 B^6) \log a_t
$$

The actual value assumed by $\theta$, determined by maximum likelihood estimation, was .4757, using the data base through 1975. The 95% confidence limits for this parameter did not include zero, implying that a moving average seasonal component of 6 quarters was significantly different from zero at the 95% level.

Tracing through an example based on equation (13) may help to clarify its interpretation. Regrettably, some new notation is required:

$\tilde{Z}_t$ (1) represents the forecast from base $t$, 1 period ahead.

Where brackets indicate conditional expectation:
Figure 5-D. Autocorrelation Function for Feeder Cattle Price Series, Differenced Once

Note: The $2\hat{\sigma}$ bounds represent the 95% level of significance on the autocorrelations. Any autocorrelation exceeding these bounds is different from zero at the 95% level. One such autocorrelation occurs at $t = 6$. 
\[ [a_t] = z_t - \hat{z}_{t-1} \quad (1) \]
\[ [a_{t+j}] = \begin{cases} 
  a_{t+j} & j \leq 0 \\
  0 & j > 0 
\end{cases} \]
\[ [z_{t+j}] = \begin{cases} 
  z_{t+j} & j \leq 0 \\
  \hat{z}_{t+j} & j > 0 
\end{cases} \]

Rewriting (13) in difference equation form,

\[ \log z_t - \log z_{t-1} = \log a_t - \theta \log a_{t-6} \]
\[ \log z_t = \log z_{t-1} + \log a_t - \theta \log a_{t-6} \]

Similarly,

\[ \log z_{t+\lambda} = \log z_{t+\lambda-1} + \log a_{t+\lambda} - .4757 \log a_{t+\lambda-6} \]

Now to obtain the mean square error forecast at lead time \( \lambda = 1 \) quarter and origin say 1976 I, the above expression can be written

\[ (14) \quad \log \hat{z}_{t}(1) = \log z_t + \log a_{t+1} - .4757 \log a_{t+1-6} \]

More explicitly,

\[ \log \hat{z}_{1976I} (1) = \log z_{1976I} + \log a_{1976II} - .4757 \log a_{1974IV} \]
\[ = \log 3919 + 0 - .4757 (\log 2825 - \log 3173) \]
\[ = 8.273592 - .4757 (7.946264 - 8.062438) \]
\[ = 8.3288559 \]
The antilog of 8.3288559 = 4139 which means that the forecast for the second quarter of 1976 at origin 19761 is $41.39. Equation (14) above can be used to make one step ahead forecasts as new data becomes available. Through appropriate substitutions of the conditional expectations $\hat{Z}_t$ for $Z_t$ in equation (14), updated forecasts of all future quarters can be made. Via this method, forecasted prices for the third and fourth quarters of 1976 at time origin 1976(1) are $42.22 and $38.88, respectively.

A comparison of actual and predicted values for this model are presented in Table 5-A. An examination of the mean square errors (MSE) indicate that the forecast error gets larger as the number of quarters ahead, for which the forecasts are made, increases. This makes intuitive sense. The farther away from the data base is the forecast, the more tenuous is the prediction since there is more opportunity for a variety of influences to come into play.

The one quarter ahead forecast has a MSE of 18.64. This means that the root mean square (RMS) error during the turbulent years from 1972 through 1975 is approximately $\sqrt{18.64} \approx \$4.32$.

The two quarter ahead forecast has a MSE of 38.52, indicating the RMS error for the given years is $\sqrt{38.52} \approx \$6.21$.

The RMS error for a three quarter ahead forecast is $\approx \$8.82$. The RMS error for a four quarter ahead forecast is $\approx \$10.30$.

Figure 5-E is a graphical comparison of the two quarter ahead and four quarter ahead forecasts for the output series model versus the actual series.
Figure 5-E. Actual Feeder Cattle Prices per Cwt. Versus Two Quarter Ahead Forecast Output Series Model and Four Quarter Ahead Forecast Output Series Model, 1972-1975
**TABLE 5-A**

**OUTPUT SERIES FORECASTS**

**SEASONAL MODEL**

Price of Feeder Cattle, All Weights, in Cents per Cwt.

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The Transfer Function Model

In a transfer function model, it is assumed that it is impossible to realistically consider a series to be a function of only itself. The transfer function expresses the output series, $Y_t$, as a function of an input series, $X_t$, plus a noise series, $N_t$.

Transfer functions work best when the input series is a "leading indicator" of the output series. Since futures prices for feeder cattle are not available for a suitably large data base, another input series was used.

Initially, it was felt that the analytically reduced form for the price of feeders derived from the structural model would serve well as an input series since the reduced form is really a linear combination of variables considered to be important in the determination of the price of feeders. In fact, the analytically reduced form input series did render satisfactory results until 1973. Beginning in 1973 though, the reduced forms produced a gruesome substitute for the actual series, having a significant downward bias accompanied by a large variance. The reduced forms were thus abandoned as a candidate for the input series of the transfer function in favor of the first stage OLS equations from the TSLS estimation of the feeder cattle structural model. Figure 5-F is a comparison of the actual price of feeder series with the analytically reduced form series and the OLS series. The graph indicates why the OLS series was favored over the analytically reduced form series.
Figure 5-F. Comparison of Actual Price of Feeders, Analytically Reduced Form for Price of Feeders, and the First Stage OLS Equation for Price of Feeders, 1972-1975.
The first stage of a TSLS estimation expresses one endogenous variable in terms of all the predetermined variables in the system. Recalling equations (12) and (13) from Chapter III, the endogenous price of feeders is expressed as a function of the predetermined variables:

PCN, SLPCRN, PFC, PFC_{t-2}, Q4, TM, PCV_{t-4}, HSB, HSB2, HSB3, and HSB4.

The input series used in the transfer function was generated by the OLS coefficients estimated from the following equation, based on data from 1960 through 1975, but omitting the four quarters in 1973 and the first quarter of 1974 as outliers.

\[ PF = 1589.98 + .2543 \text{PCV}_{t-4} - .0938 \text{HSB} - .0007 \text{HSB2} \]

\[ + 3.81 \text{HSB3} + .0019 \text{HSB4} + 18.62 \text{TM} + .8913 \text{PFC} \]

\[ - .74 \text{PCN} - 891.06 \text{SLPCRN} - 227.63 \text{Q4} \]

\[ R^2 = .91 \]

\[ DW = .86 \]

The transfer function ultimately identified used a log transformation on the data due to the same reasons which appear in the seasonal
model discussion. Expressed in the general form of equation (11), the empirically estimated transfer function is:

\[
(16) \quad \nabla \log (Y_t) = [(.8601 + .2728B) \log X_t] + [(.3319B^6) \log a_t]
\]

The parameters .8601 and .3319 were significant at the 95% level; the other parameter was significant at the 90% level. This was considered very satisfactory considering the small number of observations and the fact that the latter parameter assumed a value which was more difficult to discriminate from zero. Although the former model lent itself to a fairly intuitive description in terms of applying the parameter estimates to the feeder cattle series, the transfer function model offers no such intuitive appeal. A one quarter ahead forecast equation is available but a somewhat complicated discussion of transformations and conditional expectations would be required. As an alternative, the interested reader should consult Box and Jenkins text, pp. 403-405 for a more thorough treatment.

A chi-square test at the 95% level confirmed that the estimated residuals were not significantly different from white noise. Evidently then the transfer function model succeeded in reducing the observed series to a white noise process through judicious selection of parameter values.

An additional check of whether the transfer model specification is correct is to examine the cross-correlations between the pre-whitened input series and the estimated residuals. These cross-correlations were not significantly different from zero.
Again the problem of comparing the actual series with the forecasted series arises as in the case of the seasonal model. In similar fashion, a few of the most recent years were selected and the forecasts of up to 4 quarters from the forecast origin are displayed in Table 5-B.

As in the case of the output series forecasts, the transfer function forecasts converge to the actual value as the length of the forecast period decreases. The one quarter ahead forecast for the volatile years 1972 through 1975 indicated a mean square error of 23.68 which implies a RMS of $\approx$4.87. The two quarter ahead forecast produced a mean square error of 46.72, or a RMS of $\approx$6.84. The three quarter ahead forecast produced a RMS of $\approx$9.03. The RMS for a four quarter ahead forecast was $\approx$10.70.

Although the MSE's of these two predictive models give some indication of how well the forecasts perform, another benchmark for comparison is offered in the form of a dissertation by Joe T. Davis and an article by Leuthhold, previously cited. Leuthhold found for live cattle that current cash price was a more accurate predictor of future cash prices than the futures market for periods longer than 4 months. For periods less than 4 months, he found that the futures market was a better predictor of future cash prices. Leuthhold used MSE as his basis for evaluation.

Apparently the only attempt, other than this study, at forecasting feeder cattle prices is the Davis study. However, direct comparison with Davis' forecasts would be difficult due to the technique he employed. Davis offered three predictive models. Recognizing that
### TABLE 5-B

**TRANSFER FUNCTION FORECASTS**

Price of Feeder Cattle, All Weights, in Cents per Cwt.

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</tr>
<tr>
<td>2 Qt. Ahead</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3452</td>
</tr>
<tr>
<td>3 Qt. Ahead</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3502</td>
</tr>
<tr>
<td>4 Qt. Ahead</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3949</td>
</tr>
</tbody>
</table>

*Note: The table is cut off at 3888, 2825, and 2949, respectively.*
the dependent variable and most of independent variables were contemporaneous as he specified his model, he reformulated the model to accommodate the lack of data on the independent variables for making forecasts. Keeping the same coefficients, all of the independent variables were lagged one quarter. The third predictive model lagged all of the independent variables two quarters. In essence, Davis was assuming a random walk for all of the independent variables. It would seem logical to extend that implicit assumption to the dependent variable as well, in which case, the Davis model would be equivalent to using the cash price in one quarter as the best forecast for the succeeding quarter. Had Davis assumed this, his model would approach Leuthhold's statement that the current cash is a better predictor than the futures market. Parenthetically, two other comments might be made on the Davis models. First, the models are only capable of forecasting two quarters in advance. Secondly, his results indicated a smaller MSE on the two quarter ahead forecast than on the one quarter ahead forecast. This seemed peculiar.

The Leuthhold study and the implicit nature of the Davis study argue for comparison of the MSE among the forecasting models presented herein and that of a random walk model. In the latter, of course, the actual price of feeders serves as the estimator for succeeding quarters.

Table 5-C presents a summary of the MSE's for the three models. It is seen that the output series model consistently has the lowest MSE, the transfer function ranks second in overall performance.
### TABLE 5-C

**MEAN SQUARE ERROR**

<table>
<thead>
<tr>
<th>Forecasts</th>
<th>Output Series Model</th>
<th>Transfer FCN Model</th>
<th>Random Walk Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahead 1 Qt.</td>
<td>18.64</td>
<td>23.68</td>
<td>21.44</td>
</tr>
<tr>
<td>Ahead 2 Qt.</td>
<td>38.52</td>
<td>46.72</td>
<td>49.35</td>
</tr>
<tr>
<td>Ahead 3 Qt.</td>
<td>77.79</td>
<td>81.50</td>
<td>101.41</td>
</tr>
<tr>
<td>Ahead 4 Qt.</td>
<td>106.14</td>
<td>114.42</td>
<td>154.63</td>
</tr>
</tbody>
</table>
U-coefficients based on data from 1972-1975 were also computed for the one, two, three, and four quarter ahead forecasts of both the output series model and the transfer function model. The U-coefficient is a measure of how well the forecast function picks up turning points. It accounts for the seriousness of a prediction error through the quadratic loss criterion in such a way that the zero corresponds with perfection and the unit with the loss associated with no-change extrapolation. The coefficient as defined below has no finite upper bound, which means that it is possible to do considerably worse than by extrapolating on no-change basis. A presentation of the U-coefficients are given in Table 5-D. It can be seen that only in one case, the two-quarter ahead forecast of the output series model, was the model a better predictor than a no-change extrapolation. The other coefficients were very close to unity, indicating a close correspondence to a random-walk model when judged on the basis of a quadratic loss criterion.

The input series in the transfer function model, recall, was an OLS series with a truncated data base (1973 and the first quarter of 1974 were omitted). Since the transfer function offered more flexibility and opportunity for improvement of forecasts under different input series, it was decided that the transfer function forecast would be used in formulating the hedging strategies discussed in the next Chapter.
<table>
<thead>
<tr>
<th>Output Series</th>
<th>Transfer Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter Ahead</td>
<td>Quarter Ahead</td>
</tr>
<tr>
<td>1 Quarter Ahead</td>
<td>1.29</td>
</tr>
<tr>
<td>2 Quarters Ahead</td>
<td>.96</td>
</tr>
<tr>
<td>3 Quarters Ahead</td>
<td>1.43</td>
</tr>
<tr>
<td>4 Quarters Ahead</td>
<td>1.22</td>
</tr>
</tbody>
</table>
INCORPORATING THE FORECAST FUNCTION INTO THE DEVELOPMENT
OF A FUTURES MARKET STRATEGY

Unstable prices not only make forecasting difficult, they have contributed to both substantial gains and losses to producers in the feeder cattle market during the last few years. The thousands of small cow-calf operators in Virginia may not be able to sustain heavy losses. It would seem desirable to develop, if possible, a strategy which has as its goal a reduction in income variance without a substantial reduction in average income in the trade-off.

In order to accommodate the majority of producers and to incorporate the futures market in the development of a management tool, it was assumed that the two months of greatest trading interest are October and April. In fact, they are the heaviest trading contract months in feeder cattle futures.

Most feeder producers do not make use of the futures market. In effect, they are speculating in the cash market, betting that the cash price at the time of delivery (April or October) will be high enough to cover costs and make an adequate rate of return.

The returns under three different conservative strategies will be compared to an exclusively cash market operation in the following situation. A producer is assumed to own 450 lb. calves in October.
He must decide at that time whether to hold the calves until April, at which time they will weigh about 600 lbs. He then must make a second decision on whether to hold the animals for another six months, until October, when he would sell them as 750 lb. feeders. The above two decisions will be determined by the following strategies and will be evaluated by calculating the difference between revenue generated under a particular strategy and a break-even point (henceforth BEP) which serves as a cost of operation estimate. Calculations of actual BEP's and estimates of BEP, designated BÊP, are presented in Table 6-A. The only difference between BEP and BÊP is that BÊP is an estimate of the break-even point based on the previous quarter's hay price. BEP is the actual break-even point which is based on the actual hay prices that exist during the quarters in which the cattle feeding operation takes place. BÊP is used in the strategy decision process while BEP serves as the basis against which profits and losses are calculated.

It should be noted that in the calculation of the profitability of the various strategies, the simplifying assumptions of zero basis on both 600 lb. and 750 lb. feeder cattle can be made in using the futures markets. In fact, there is a discount on animals of heavier weight.

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1BEP calculation is based upon a series of publications by James M. Moore of the Cooperative Extension Service, Virginia Polytechnic Institute and State University. His budgets have been modified somewhat for the sake of convenience.
TABLE 6-A
CALCULATION OF BREAK-EVEN PRICE

October - April For Raising a 450 lb. steer to 600 lbs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>October (Oklahoma City) Price 450 lb. calf</td>
<td>$___________</td>
</tr>
<tr>
<td>Feed</td>
<td></td>
</tr>
<tr>
<td>Hay</td>
<td></td>
</tr>
<tr>
<td>.375 tons @ ___ per ton</td>
<td>1.06</td>
</tr>
<tr>
<td>Salt</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Subtotal</td>
</tr>
<tr>
<td>Interest (12% per annum) = 1.06 x subtotal</td>
<td>14.46</td>
</tr>
<tr>
<td>Other variable costs</td>
<td></td>
</tr>
<tr>
<td>5.87¢/day x 180 days</td>
<td>10.57</td>
</tr>
<tr>
<td>Fixed cost and management</td>
<td>14.46</td>
</tr>
<tr>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>BEP = Total/6 cwt.</td>
<td></td>
</tr>
</tbody>
</table>

April - October For Raising a 600 lb. steer to 800* lbs.

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>April (Omaha) Price 600 lb. choice steer</td>
<td>$___________</td>
</tr>
<tr>
<td>Variable expense</td>
<td>8.60</td>
</tr>
<tr>
<td>Pasture rent</td>
<td>25.000</td>
</tr>
<tr>
<td></td>
<td>Subtotal</td>
</tr>
<tr>
<td>Interest (12% per annum) = 1.06 + subtotal</td>
<td>6.80</td>
</tr>
<tr>
<td>Marketing</td>
<td>3.50</td>
</tr>
<tr>
<td>Fixed cost and management</td>
<td>6.80</td>
</tr>
<tr>
<td></td>
<td>Total</td>
</tr>
<tr>
<td>BEP = Total/8 cwt.</td>
<td></td>
</tr>
</tbody>
</table>

*Additional assumptions: \( \text{BEP} \) is calculated by using average hay price from the previous quarter rather than the actual hay price for a 6 month period.

Also, 800 lb. choice steers are assumed to have approximately the same fixed and variable costs per cwt. as 750 lb. steers.
Strategy I

(1) The producer will hold the 450 lb. calves in a cash position until April when the April futures contract for feeders is > BEP. The April futures contract price used on the decision rule will be the average April contract price during the last week in October.

When trading in the April futures does not begin until November, the first April futures quotation that becomes available will be used. If the April futures contract price is < BEP, the producer will sell his 450 lb. calves at the current market price during October. In this latter event, the producer will not be engaged in the enterprise of raising feeders for at least 6 months and consequently he experiences zero profit (and loss) on his feeding operation for that time period. In all succeeding strategies as well, a decision to sell rather than feed will be indicated by a zero profit on the feeding operation for that time period.

(2) In April the producer may enter (or maintain) his feeder production. He will decide to hold his herd, now at 600 lbs. when the October futures contract price > BEP. It is assumed that the producer can purchase 600 lb. cattle in April if the situation looks favorable, even if he sold his 450 lb. calves due to decision rule (1). The October futures contract price used in the decision rule will be the average price for the contract during the last week of April. If

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2In some years, the April futures contract is not traded until November. In this case, the first April futures trade that becomes available is used in the decision rule.
the October futures price is $< \hat{B}E_P$, the producer should sell his 600 lb. animals at the current price for feeders during April.

**Strategy II**

(1) The producer will hedge using the April contract month if at any time during October (or the first trade in November if the April contract is not introduced until then) the April futures price $\geq \hat{B}E_P$. Otherwise the producer will sell his 450 lb. calves at the current price during October.

(2) In April, the producer decides to hedge his herd, now at 600 lbs., if at any time during April, the October futures price $\geq \hat{B}E_P$. Otherwise the producer will sell his 600 lb. calves at the current price during April.

**Strategy III**

(1) The producer will hedge using the April contract month if at any time during October (or the first trade in November if the April contract is not introduced until then) the April futures price is $\geq$ both the upper bound on the 75% confidence limit for the April (2 quarters ahead) price forecast and $\hat{B}E_P$.

He will maintain a cash position if the April futures price is $\geq \hat{B}E_P$ but $< \text{the 75% bound}$.

Otherwise the producer will sell his 450 lb. calves at the current price during October.

(2) In April, the producer decides to hedge his herd, now at 600 lbs., if at any time during April the October futures price $\geq$ both the upper bound on the 75% confidence limit for the October (2 quarter.
ahead) forecast and BEP. He will maintain a cash position if the October futures price, averaged during the last week of April > BEP.

Otherwise the producer will sell his 600 lb. feeders at the current price during April.

Table 6-B summarizes the data needed for calculation of profits and losses involved in each strategy. Table 6-C is a presentation of the results for each strategy in addition to the results generated under an exclusively cash operation. The cash operation essentially involves speculation in the cash market in the manner mentioned previously.

An example of the use of Table 6-B in the calculation of profits or losses shown in Table 6-C may better illuminate the essence of the different strategies.

Consider the October - April period for 1973-74 shown in the upper section of Table 6-B. $56.28 represents the estimated break-even point based on the budget given in Table 6-A with the hay price from July - September, the previous quarter, used in the computation of this estimate.

Strategy I dictates maintaining a cash position if the average April futures price during the last week of October is > BEP, otherwise sell. Since the April futures contract price during that month averaged $53.20 which is < $56.28, under strategy I, the producer sold his 450 lb. calves and hence made zero profit on a feeding operation from October 1973-April 1974. This is duly recorded in Table 5-C under strategy I, 1973-74, October - April.
### TABLE 6-B

DATA ON STRATEGY CALCULATIONS

#### OCTOBER - APRIL

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1972-73</td>
<td>$45.89</td>
<td>$45.52</td>
<td>$41.31</td>
<td>$44.19</td>
<td>$(no)</td>
<td>$(no)</td>
<td>$49.36</td>
<td>$52.72</td>
</tr>
<tr>
<td>1973-74</td>
<td>$56.74</td>
<td>$56.28</td>
<td>$57.77</td>
<td>$53.20</td>
<td>$(no)</td>
<td>$(no)</td>
<td>$62.04</td>
<td>$46.10</td>
</tr>
<tr>
<td>1974-75</td>
<td>$31.65</td>
<td>$31.63</td>
<td>$38.99</td>
<td>$32.50</td>
<td>$(no)</td>
<td>$32.50</td>
<td>$30.16</td>
<td>$31.69&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>1975-76</td>
<td>$35.65</td>
<td>$36.38</td>
<td>$38.78</td>
<td>$36.00</td>
<td>$(no)</td>
<td>$(no)</td>
<td>$36.04</td>
<td>$39.69&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

#### APRIL - OCTOBER

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>$37.64</td>
<td>$37.64</td>
<td>$39.48</td>
<td>$36.40</td>
<td>$(no)</td>
<td>$(no)</td>
<td>$40.63</td>
<td>$41.69</td>
</tr>
<tr>
<td>1973</td>
<td>$47.65</td>
<td>$47.65</td>
<td>$48.77</td>
<td>$51.95</td>
<td>$50.20</td>
<td>$50.20</td>
<td>$52.72</td>
<td>$51.33</td>
</tr>
<tr>
<td>1974</td>
<td>$42.39</td>
<td>$42.19</td>
<td>$46.96</td>
<td>$41.68</td>
<td>$47.00</td>
<td>$44.80</td>
<td>$46.10</td>
<td>$31.58</td>
</tr>
<tr>
<td>1975</td>
<td>$30.93</td>
<td>$30.93</td>
<td>$30.68</td>
<td>$32.50</td>
<td>$31.90</td>
<td>$31.90</td>
<td>$31.69</td>
<td>$38.09&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>1976</td>
<td>$37.29</td>
<td>$37.29</td>
<td>$42.76</td>
<td>$43.80</td>
<td>$44.88</td>
<td>$44.88</td>
<td>$39.69</td>
<td>?</td>
</tr>
</tbody>
</table>

<sup>a</sup>Price for 600-700 lb. steer, Kansas City.

<sup>b</sup>Average price in March, since this was the latest quotation at this writing.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>II</td>
<td>III</td>
<td>Cash</td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>Cash</td>
</tr>
<tr>
<td>1971-72</td>
<td>Feed Futures not yet Available</td>
<td>$0</td>
<td>$0</td>
<td>$0</td>
<td>$0</td>
<td>$0+30.38</td>
<td></td>
</tr>
<tr>
<td>1972-73</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+40.98</td>
<td>+27.60</td>
<td>+19.20</td>
<td>+19.20+27.60</td>
</tr>
<tr>
<td>1973-74</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-63.84</td>
<td>0</td>
<td>+18.07</td>
<td>+34.58-81.08</td>
</tr>
<tr>
<td>1974-75</td>
<td>+.36</td>
<td>+5.22</td>
<td>+.36</td>
<td>+36</td>
<td>+53.70</td>
<td>+7.28</td>
<td>+7.28+53.70</td>
</tr>
<tr>
<td>1975-76</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+24.24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mean Income/Head</th>
<th>Standard error</th>
<th>Standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy I</td>
<td>10.21</td>
<td>18.75</td>
</tr>
<tr>
<td>Strategy II</td>
<td>6.22</td>
<td>7.56</td>
</tr>
<tr>
<td>Strategy III</td>
<td>7.68</td>
<td>11.97</td>
</tr>
<tr>
<td>Exclusively Cash</td>
<td>4.03</td>
<td>43.33</td>
</tr>
</tbody>
</table>

It should be noted that the criterion of maximizing profit on the feeding operations is not necessarily the best. A better criterion may be to maximize the returns to fixed resources.
Under strategy II, the producer hedges if the April futures price ever exceeds the $B_{EP}$ in October or November. In 1973, this never happened. As part of the strategy, the producer would elect again to sell the 450 lb. animals in October or November. Again this shows up in Table 6-C as a zero profit on cattle feeding operations in October to April.

Following strategy III, the producer would hedge if the April futures price ever exceeded both $B_{EP}$ and the 75% upper bound on the price forecast during the months of October or November. This did not happen in 1973 so this strategy also led to a zero profit in cattle feeding operations during this six month period.

The benchmark for comparison is the strategy presumably employed by a great number of cow-calf operators, i.e. engage in a cattle feeding operation at all times. Under this strategy, the producer always engages in the feeding enterprise, accepting the risk of a loss determined by prices which exist in the cash market at the time he sells. The revenue which is received when the producer sells in the cash market in April 1974, based on quotations at Omaha is $46.10 per cwt. This implies that a producer who engaged in a cattle feeding operation incurred a loss of $6 \cdot ($46.10 - $56.74) = -$63.84 on a 600 lb. animal. This loss is recorded in the Exclusively Cash column in Table 5-C which corresponds to the October - April period for 1973-74.

Another example more fully demonstrates the differences in profits which can occur. Again consider the year 1974 but this time the April to October feeding season. $B_{EP} = \hat{B}_{EP}$ during the April -
October period of the year based on the budgets presented in Table 6-A. The budget for this period assumes that no hay need be purchased.

Under strategy I, the decision would have been made not to feed cattle since the price of the October futures during the last week of April averaged $41.48 which was less than the BEP of $42.39. In the profit/loss Table 6-C, strategy I registers a zero for this period.

Strategy II on the other hand would have led to the locking in of a fairly good profit. Recall the strategy dictates the selling of a futures contract if the October futures in April ever exceeds BEP. In fact this occurred on April 1, at which time the October futures traded for $44.80 per cwt. Since actual BEP for the April - October period was calculated to be $42.39, the total profit received on the cattle feeding operation equaled (7.5 cwt) • ($44.80 - $42.39) = $18.07. This figure appears in the appropriate cell in Table 6-C.

Following strategy III in April 1973, the producer would hedge if the October futures price in April ever exceeded both BEP and the upper 75% bound on the October forecast. The upper bound of that forecast was $46.96. The October futures exceeded that bound on April 11 with a trade of $47.00. Thus under strategy III a per head profit of (7.5 cwt) • ($47.00 - $42.39) = $34.58 was locked in through the futures market. This contrasts markedly with the following exclusively cash operation.

If the producer decided to speculate in the cash market by feeding his 600 lb. steers to 750 lbs. during the April - October 1974 feeding period, he would have experienced the substantial loss of
(7.5 cwt.) \cdot ($31.58 - $42.39) = -$81.08 \text{ per head since the cash price per cwt. on choice feeders had fallen to $31.58 by October, 1974.}

The above examples illustrate the computations involved in arriving at Table 5-C. An overall evaluation of the different strategies may be gleaned from the summary statistics presented in the lower half of Table 5-C, though it should be noted that the computations were based on a very small number of observations.

The highest mean income of $10.21/head was generated through the use of strategy I. That ranking was obtained through an extremely profitable April - October 1975 season which resulted in a profit of $53.70. That fact is reflected in the standard error for strategy I. The standard error for strategy I including the feeding season in which the strategy dictated an abstention from a feeding cattle operation, hence a zero profit, was $18.75. If the standard error is calculated only for those seasons in which the producer actually engaged in the operation of feeding cattle, the standard error was a very large $27.63, though based on only 3 observations.

Strategy III offered a mean income of $7.68/head and ranked second over the 1972-75 period. Although the mean income was significantly lower than in strategy I, the standard errors were significantly lower as well with an $11.97 standard error for all eight seasons of the analysis and a $15.08 standard error for the four seasons in which the feeding operation was implemented under the strategy.

Strategy II ranked third with a mean income of $6.22 for the period of study but this strategy rendered the lowest standard errors
of $7.56 and $8.82 for all eight seasons and the four seasons in which the feeding operation was implemented, respectively.

It is noteworthy that most of the profit generated through strategies I, II, and III occur in the April - October feeding season when producers are not forced to purchase hay as must be done in the winter months.

The exclusively cash strategy resulted in a mean of $4.03 per head with a standard error of $43.33. Presumably this is the strategy that a substantial community of cow-calf operators employ. The results under this strategy under the volatile years 1972-1975 are perhaps the most telling justification for research which presents the producers with strategical alternatives which demonstrate the trade-offs between the mean and variance of income in the enterprise of cattle feeding.

An interesting footnote might be added to this study. It was implied through a comparison with Leuthhold's article, that the forecast models for feeder cattle prices may predict better than the futures market. A comparison of the MSE's among the forecasting models and the futures market was made for the eight semi-annual feeding periods during which the three strategies were evaluated, i.e., 1972-1975. The futures market forecast for October was taken to be the price of that contract averaged during the previous April. The futures market forecast for April was taken to be the price of that contract averaged during the previous October or the first available quotation in November when there were no trades on the April contract during October. The forecast models used two quarter ahead
forecasts to predict second and fourth quarter prices representing April and October, respectively.

Both the output series and transfer function forecasting models had lower mean square prediction errors than the futures market for this period. The futures market exhibited a MSE of 74.74, the output series model a MSE of 29.34, and the transfer function model a MSE of 5.80. It will be interesting to observe whether the forecasting models will continue to outperform the futures market as the data base expands.
CHAPTER VII

SUMMARY AND CONCLUSIONS

This study was an attempt to develop a price forecasting model for feeder cattle in the United States. A structural model was developed in order to provide the basis for isolating the most important variables affecting price in the feeder cattle sector. A linear combination of these variables would then serve as an input series in the more sophisticated methods developed expressly for forecasting. A brief summary of the findings and conclusions are presented in this Chapter.

Summary

Structural Model

The structural model of the feeder cattle market was hypothesized to be a demand derived from retail beef. A system of nine equations described the simultaneous interaction among three market levels: retail beef, wholesale carcass beef and fed cattle. By virtue of a six month production lag between feeder and fed weights, feeder cattle entered the supply of fed cattle as a predetermined variable. This allowed the feeder cattle market to be estimated separately from the first block of nine equations. The price of fed cattle could enter the demand for feeders recursively.
The simultaneous block of nine equations was theoretically discussed but not estimated empirically since it was intended only as the vehicle through which the derived demand nature of the feeder cattle market was presented. Primary emphasis was placed on developing a forecasting model rather than constructing a model of the entire beef-cattle sector.

The herculean objective initially was to obtain a stable structure for the feeder cattle market, reestimate the coefficients upon expanding the data base for the five successive years 1971-1975, and use the analytically reduced forms as input series in a forecasting model.

Through 1972, problems were minimal. A structure with all of the "correct" theoretical signs was achieved fairly readily and the analytically reduced forms provided very satisfactory results. When 1973 was added to the data base, the model began to strain in terms of weak t-statistics and unstable coefficients on the variables. Adding 1974 and 1975 to the data base produced such bizarre results that rebuilding the structure to accommodate all of the tumultuous 1970's was no mean task. (In another sense it was extremely mean). Eventually a structure which could contain the volatile years was achieved, although there were one or two variables with the wrong theoretical sign. However, when the analytically reduced forms were computed for 1975, the forecasted series which resulted was singularly disastrous. The variables employed in the structural estimation for the most part come into the equations estimated from 1971-1975 as significant and of the proper theoretical sign. This fact seems
remarkable given the unstable conditions and serves to justify the presence of the variables in a structure of the feeder cattle industry.

In order to obtain an input series for the forecast model, the original plan of depending on the structure only to indicate the most important variables affecting feeder prices was reinstated. The first stage of the two-state least squares estimation was employed as an estimator of the price of feeders. In fact, this single equation ordinary least squares technique provided a fairly good series of estimates for the price of feeders.

The so-called Box-Jenkins methods provided the basis for formulating three different forecasting models. The simplest model was the fitting of an autoregressive, moving average, or autoregressive-moving average model to the output series, feeder prices, itself. The model which was identified was a random-walk, which suggests that the best (minimum MSE) forecast of next quarter's price is the present quarter's price. This forecast of course proved to be of questionable value and the search continued for a better predictor.

A seasonal model was identified in which a moving average parameter was discovered at period 6. This meant that the present price is correlated with the price six quarters previous. The suggestion that there is a lag in production response of six quarters in the feeder cattle market seems very appealing. The forecasted series based on this relationship performed fairly well.

A transfer function model was constructed using the estimated price of feeders as an input series. A transfer function is warranted
when it is impossible to assume that a time series is a function only of itself. It was felt that a linear combination of variables which significantly affect the price of feeders would be a valuable adjunct to the output series itself in explaining the variation in the price of feeders. The forecast performed quite well given the limited number of observations which are required to estimate the parameters in the model.

Development of Futures Market Strategies

Three strategies were developed to reduce income variance of feeder cattle producers. They were evaluated by comparing the profits or losses generated over the six month intervals October - April and April - October. Cost estimates were obtained through budgets constructed by Cooperative Extension personnel. The revenue side of the ledger was of course determined by the particular strategy involved.

Perhaps the true benchmark for comparison of the strategies was the exclusively cash operation which many small cow-calf operators employ. These producers in effect speculate in the cash market by betting that the price of their finished product (a 600 lb. - 750 lb. feeder) will be high enough to reward their efforts.

In the years that were tested, 1972-1975, the strategy that paid the highest average return was the strategy which recommended a cash position when the two quarter ahead futures price exceeded the current BEP. Strategy III, which employed futures market entry when warranted by the forecast model predictions, ranked second in mean income per head but had a lower variance than Strategy I. Noteworthy
is the fact that the lowest average return was generated by the modus operandi engaged in by most small producers, the exclusively cash market operation.

Conclusions

Based on the structural model, some of the more important variables in the determination of the price of feeder cattle are the inventory of heifers, steers and bulls under 500 pounds, the price of corn, the price of fed cattle, the price of fed cattle lagged two periods.

In forecasting the price of feeder cattle, analytically reduced forms are of limited value and a better policy would seem to be using a simple OLS model to generate an estimator of the price of feeders.

An OLS estimator can then be used as an input series in the transfer function forecasting model. If a leading indicator series can be found which can be used as an input series the transfer function model will perform at its best. The futures market seems an ideal source for such a series but the data base is not large enough at this time to incorporate futures prices in a feeder cattle price forecast function.

A few simplistic strategies incorporating futures markets in conjunction with price forecasts hold promise for not only reducing the income variance which would be experienced under an exclusively cash market operation, but also in raising the average income received by producers of feeder cattle. Further research in the area of developing strategies seems encouraging.
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__________, "Do Live Cattle Futures Differ From Other Existing Futures Contracts?" in Futures Trading in Livestock, Mimis Press, 1969.


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## APPENDIX TABLE A

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### APPENDIX TABLE A
CONTINUED

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<tr>
<th>Year and Quarter</th>
<th>FDPL22&lt;sup&gt;a&lt;/sup&gt; (000) hd.</th>
<th>PALLFS&lt;sup&gt;b&lt;/sup&gt;</th>
<th>PFC113&lt;sup&gt;c&lt;/sup&gt;</th>
<th>PCV35&lt;sup&gt;d&lt;/sup&gt;</th>
<th>HSB&lt;sup&gt;e&lt;/sup&gt;</th>
<th>PCORN&lt;sup&gt;f&lt;/sup&gt;</th>
<th>SOF&lt;sup&gt;g&lt;/sup&gt;</th>
<th>PHAY&lt;sup&gt;h&lt;/sup&gt;</th>
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<td>1975 I</td>
<td>4376</td>
<td>25.72</td>
<td>25.72</td>
<td>26.75</td>
<td>36302</td>
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<td>35.18</td>
<td>35.38</td>
<td>35.18</td>
<td>2.44</td>
<td>1467</td>
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<sup>a</sup> Feeder placements in 22 states, Livestock and Meat Statistics, USDA, ERS/SRS, Agricultural Marketing Service.

<sup>b</sup> Feeder steer prices: weighted average cost per 100 pounds, 8 markets combined, Livestock and Meat Statistics, USDA, ERS/SRS, Agricultural Marketing Service.

<sup>c</sup> Price of choice fed cattle, 1100-1300 pounds, Omaha market, Livestock and Meat Statistics, USDA, ERS/SRS, Agricultural Marketing Service.

<sup>d</sup> Price of choice calves, 300-500 pounds, Omaha market, Livestock and Meat Statistics, USDA, ERS/SRS, Agricultural Marketing Service.

<sup>e</sup> Heifers, steers, and bulls under 500 pounds on farms January 1, in U.S., Livestock and Meat Statistics, USDA, ERS/SRS, Agricultural Marketing Service.

<sup>f</sup> Average price of corn per bushel received by farmers in U.S., Agricultural Prices Annual Summary, USDA, SRS, Crop Reporting Board.

<sup>g</sup> Steers and steer calves, number on feed 500-699 pounds, 23 states, Cattle on Feed, USDA, SRS, Crop Reporting Board.

<sup>h</sup> Average price of hay paid by farmers in U.S., Agricultural Prices Annual Summary, USDA, SRS, Crop Reporting Board.
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The specific objectives of this study were: 1) to develop a price forecasting equation for feeder cattle and 2) to examine the possibility of reducing the variance in income and raising the level of revenue relative to that which would be achieved under the common practice of consistently selling feeder cattle in the cash market.

To these ends, a structural model for the feeder cattle market in the United States was developed in order to isolate the important variables to be used in the construction of a quarterly price forecasting model. Time series methods described by Box and Jenkins were employed in the construction of the forecasting equations.

Selected strategies were developed which incorporated trading in the futures market for feeder cattle based on the price forecasts. The trade-off between mean income and the mean square error of income is indicated by a test, within the range of the data base, of the various strategies for the volatile years 1972-1975.