THE INFLUENCE OF LOAD DISTRIBUTION
ON THE RELIABIITY ANALYSIS
OF
LUMBER PROPERTIES DATA

by

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CHAPTER I

INTRODUCTION

Lumber, like all other structural materials, displays an inherent variability. Variability is not readily apparent from the design viewpoint since allowable stresses and nominal loads are established by codes and design specifications. For example, the allowable bending stress of a No. 2 KD Southern Pine 2" x 4" is 1550 psi (47). In actuality, because of lumber's natural variability, the bending strength of a randomly chosen No. 2 KD Southern Pine 2" x 4" is probably much higher than the tabulated value. However, it may also be lower.

Allowable design values of structural lumber have been derived from tests of small clear wood specimens. These tests were performed in order to describe the distribution of strength of green, small clear wood specimens (7). Historically, the normal distribution was assumed to adequately describe the strength distribution because random samples from a large and unbiased population tend toward normality (32). Also, the normal distribution and its statistical moments are well defined, thereby allowing simple mathematical manipulation and application.

Early designers did not have the analytical tools to design structures using strength distributions. Therefore, the fifth percentile of the small clear wood strength distribution was chosen as a reasonable estimate of the clear wood strength. In other words, if the wood strengths are normally distributed, then only five percent of all wood strengths would be weaker than the fifth percentile strength value. This decision was based on the consequences of failure of lumber structures fabricated by typical light frame construction practices (32).

Small clear wood specimens are, in general, stronger than dimension lumber of the same species. Once the design values for small clear wood strengths had been calculated, conversion factors were derived and applied to develop the allowable design strengths of dimension lumber. These conversion factors fell into three main categories as listed by Hoyle (32). One is a factor allowing for an increase in the property value due to the effect of seasoning; in other words, a moisture adjustment factor. Another factor accounts for the effect of strength reducing defects permitted in the grade of lumber involved. This category is used to partition structural lumber into varying grades (5). The last is a general adjustment factor which is the composite result of a systematic consideration of other influences known to affect lumber strength. For example, the adjustment factor for bending strength consists of a load duration factor and a manufacture and use factor (32). The load duration factor accounts for the reduced load carrying capacity of lumber under a sustained load. The manufacture and use factor consolidates such diverse considerations as shrinkage variability, broken edges and other damage, the effect of fasteners, small end splits occurring after construction and the probability of error in grading. The manufacture and use factor may be interpreted as a safety factor.

Similar logic was employed to develop nominal loads for use in structural design. The nominal value of a load is the value specified by the code authorities (59). Loads acting on a structure during its design lifetime are highly variable. The basic concept is that a structure or structural component should be designed to withstand the largest load or load combination to which the structure will be subjected during its life. Therefore, nominal loads were developed to reflect a slight probability of occurrence (4). In other words, by equating the nominal load to an upper percentile value, the chance of a larger load occurring on the structure is maintained at an acceptable level. For instance, in moderate snow regions of the United States, the nominal value of a 20 psf annual extreme ground snow load has a 2 percent probability of being exceeded.

By utilizing the allowable design values for structural lumber and a prescribed load or load combination, engineers have an established procedure to design light frame lumber structures. These structures have an excellent record of service; however, recent research indicates this is not due uniquely to the allowable strength values (25, 43, 52). Under close study are the adjustment factors converting small clear wood strengths to dimension lumber strengths. Also, it is now recognized that small clear specimen data is often inadequate for predicting strength properties of structural lumber and that present grading practices may not produce lumber in which the allowable stress is a consistent percentile of the probability distribution for strength (18). Therefore, lumber properties research is now focused on tests of full size structural lumber.

In order to provide leadership and guidance in the research of structural lumber strength properties, the United States Forest Products

Laboratory, in cooperation with other agencies concerned with lumber research, initiated the In-grade testing program in 1977 (22). The main thrust of the program is to develop the mechanical properties of lumber produced by the rules of the American Lumber Standard (2). As a result, a considerable amount of structural lumber strength data has been collected. This data not only encompasses the usual strength properties of structural lumber according to size or grade but also includes test results concerning the moisture adjustment factor, the rate of loading, and comparisons of visually graded lumber to machine stress rated lumber.

The results from in-grade tests have already focused concern on present allowable design values for structural lumber and the associated adjustment factors. Madsen (43) has noted that the actual allowable strengths for dimension lumber may be lower in some cases than the tabulated design values. Also, Madsen (44) and Wilson (68) have independently concluded that the bending strength of green and dry structural lumber is the same at the fifth percentile. Both of these studies focused on the effect of moisture content on the bending strength of dry lumber at 19 percent maximum moisture content and green lumber. At the fifth percentile, which is the historical approach to allowable design stresses, these results suggest an adjustment factor of unity. The present adjustment from green lumber to dry lumber of 19 percent maximum moisture content is an allowable increase in bending strength of 25 percent (32).

With such strong experimental evidence, designers can no longer confidently use the 25 percent allowable increase in strength for seasoning; however, to accept an adjustment factor of unity will not efficiently utilize the lumber resource since the dry lumber strength does increase over the green lumber strength at the middle and higher percentile levels. Hence, it seems appropriate to utilize the entire strength distribution in lumber properties research since the distribution reflects an entire range of strengths available in lumber of a particular size and grade from the mill.

Two popular concepts today which acknowledge the influence of the probability distributions are the limit states design concept and the differential reliability concept. Limit states design is a probabilistic approach particularly well suited for design formats. Once resistance distributions for structural materials and load distributions reflecting loads acting on the structure are defined, a safety index, β, can be calculated which reflects a certain limit state. A limit state can either be a serviceability limit such as deflection or an ultimate limit such as maximum allowable bending strength. If the calculated safety index is less than the allowable safety index, the design is inadequate and a different size, grade, or type material must be selected (57,73).

The differential reliability concept has been applied to material properties research only recently. If a resistance distribution and a load distribution are known, along with their relative positions on a coordinate axis, then a probability of failure associated with the two distributions can be determined. This probability of failure may have no significance by itself; however, if another resistance distribution is known which is related to the first, then comparative probabilities of failure can focus on differences between the two resistance distributions. Suddarth, Woeste and Galligan (61) note that when studies of two or more cases are made, the contrast between the probabilities of failure for these cases allows strong analytical focus on the case differences. This strong analytic advantage occurs because one case can also be carried through for the others in a completely formal way. In other words, the effects of errors or biases induced by incomplete data describing load or resistance distributions should be minimized by the differential reliability analysis.

The form and location of the load distribution can have a great impact on the reliability analysis of lumber properties. Accurate load information is necessary to ensure a proper reliability analysis of lumber properties data. It has been illustrated that additional properties research may not improve product performance unless realistic load information is also available (26).

As a consequence, the goal of this study is to evaluate the use of a relative reliability procedure in wood engineering research. The form and location of a load distribution impacts upon the results of any reliability analysis; therefore, this study will concentrate on this problem. Load distributions for load combinations will be developed to reflect the state of the art on loads found in light-frame structural applications. Combining the above data sets with appropriate resistance distributions, the effect of load distributions on lumber property adjustment factors will be explored. Also, the overall effects of the reliability analysis approach on lumber properties data will be assessed. A set of loads for analyzing lumber data properties will be recommended.

CHAPTER II

REVIEW OF LITERATURE

The review of literature is presented in three sections. The first section describes the differential reliability concept and previous applications of the analytical method. A short discussion of the concept of probability of failure and its calculation is included. The second section describes several important probabilistic concepts. Important statistical techniques and descriptions of necessary probability distributions are outlined. Finally, the last main division is a summary of commonly acting loads on light frame structures. Various models for predicting loads are discussed. Also, load survey data which has an impact on this study will be presented.

2.1 Differential Reliability

Zahn (72) has commented on the goal and advantages of probabilistic design methods. He notes

> "The goal of probabilistic design methods is to provide a unified procedure applicable to all materials, all loads, and all types of uncertainty in design. These methods are still evolving but already they offer several outstanding advantages over traditional deterministic methods: they are more rational, logically consistent, nearly universally applicable, realistic, relatively free of semantic confusion, and explicit in their treatments of uncertainities. The adoption of improved design procedures would facilitate rational working of the market place".

Using probabilistic methods in lumber properties research is logical since all results can be compared to a benchmark safety level. Differential reliability analysis is a rational method to compare lumber properties.

Suddarth, et al. (61) recognized the potential of differential reliability analysis in wood engineering research. They list three immediate applications of the treatment to wood engineering research and design. The first application tests the sensitivity of failure probability when a structural design is modified. In this manner, researchers are able to ascertain the key elements in a design as reflected by the probability of failure. Major and minor components can be defined in terms of their impact. A more efficient use of research funds can be realized. Another use is the possible quantification of material quality control. The control of variability of lumber grades is a significant concern of lumber manufacturers. Differential reliability analysis can measure the effects of variability in a product. The last application relates to code calibration. As new codes are developed, it is imperative that the design safety be at least that provided by the older, established codes. The probability of failure of a structure designed by a new code should be approximately equal to the structural probability of failure calculated using presently accepted design specifications.

In lumber properties research, the calculated probability of failure of a single member is not the probability of failure calculated for end-use situations. As an example, the probability of failure of a population of No. 2 KD Southern Pine 2" x 8" lumber is not equal to the probability of failure of a floor constructed with joists taken from the population. The probability of failure of the floor will be less than the probability of failure of the lumber because composite action, load-sharing, and the effects of non-structural items are not included in the analysis.

Situations similar to the above floor example do not preclude a differential reliability analysis of lumber properties data. Even though the calculated probability of failure does not exactly characterize the system or component, the relationship between failure probabilities does maintain a significant meaning once a benchmark probability has been established. Suddarth, et al. (61) have recommended that the notion of equal reliability be accepted. For instance, if a material with high variability has a certain probability of failure, then how much more load can the same material exhibiting a lower variability carry? One way to answer this question is to say the low variability material can be awarded a higher design stress than the high variability material because there is less fluctuation in strength related variables. This approach was used by Suddarth, et al. (61) to study the effect of variability of modulus of elasticity on roof truss reliability.

Marin and Woeste (45) used the differential reliability approach to study product reliability as affected by reverse proof loading. For reverse proof loading to be effective in the quality control of lumber manufacturing, all weak pieces of lumber below a specified level must be removed. Marin and Woeste showed that a 27 percent mill drift is acceptable without reducing bending strength quality when proof loading is used. This result was based on a comparison of a control lumber sample and a lumber sample with the weaker pieces removed by a reverse proof loading scheme.

Green (26) has proposed evaluating the moisture adjustment factor utilizing a differential reliability analysis. It has been suggested that a suitable adjustment factor can be specified through this probability approach (65). By requiring equal probabilities of failure for green and dry lumber, the effect of moisture content on lumber strength can be assessed.

Probability of failure is a measure of underlying risk associated with a structure, member, or material due to the uncertainities of engineering design (45). A failure event occurs whenever the strength of a structure or member (R) is less than the load (S). This can be described by the following equation.

$$P_f = \Pr\ (\text{failure}) = \Pr\ (R<S) \tag{2.1}$$

If the load and the strength are described by continuous probability distribution functions and are mutually independent, then the probability of failure can be shown to be

$$P_f = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{s} f_R(r)\,dr \right] f_S(s)\,ds \tag{2.2}$$

where

$f_R(r)$ is the probability density function of the resistance variable, strength

$f_S(s)$ is the probability density function of the load variable.

Theoretically, the limits on the probability of failure are as shown above. However, the limits of integration can change depending on the distributions of load and strength. For instance, if the strength distribution is non-negative, then the lower limit of integration would be zero. The expression in brackets, [ ], of equation 2.2 is the cumulative distribution function of strength, $F_R(r)$. Also, the strength or resistance, and the load must be expressed in the same units such as psi or psf. Only in a few cases can equation 2.2 be solved analytically. Usually, a numerical technique based on the concept of integration is used.

The probability of failure can be described as the sum of many small incremental probabilities of failure. An incremental probability of failure can be written as

$$d_{P_f} = \int_{x}^{x + dx} F_R(x)\ f_S(x)\ dx \qquad (2.3)$$

where

$F_R(x)$ is the cumulative distribution function of the resistance variable

$f_S(x)$ is the probability density function of the load variable.

Figure 2.1 taken from Suddarth, et al. (61) demonstrates this concept graphically. The load and resistance distributions are depicted on two separate graphs for clarity. The distributions could have been drawn on one graph. Also, the two horizontal axes are labeled X to denote that the resistance and the load should have the same units.

Numerically, this incremental probability of failure can be approximated by two equations. The first equation is the upper limit and the second equation is the lower limit of the incremental probability of failure. They are

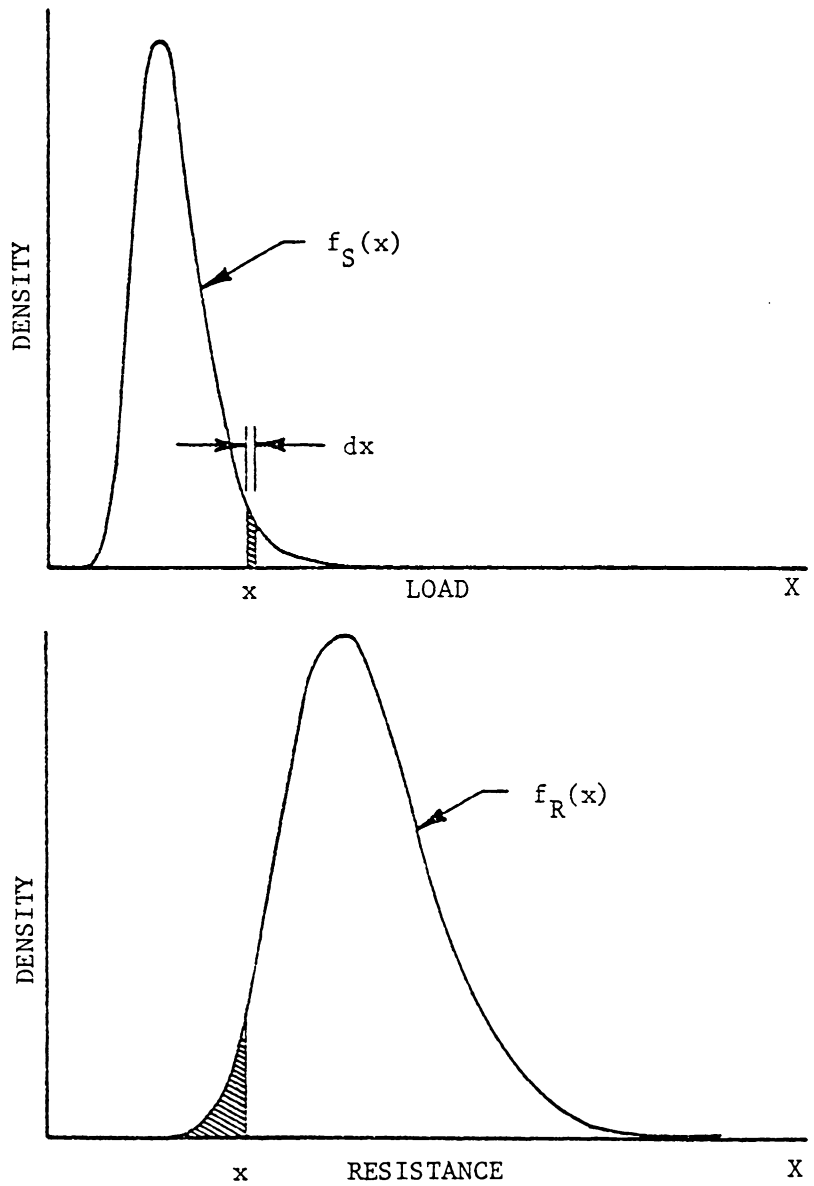

Figure 2.1 A load distribution is shown above and an associated resistance below. The product of the shaded areas is an incremental probability of failure. The total of such products for all possible values of x is the probability of failure for the prescribed load and resistance distributions. Figure reproduced from Suddarth, et al. (61).

$$d_{P_{f_{upper}}} = F_R(x + dx) \left[F_S(x + dx) - F_X(s) \right] \quad (2.4)$$

$$d_{P_{f_{lower}}} = F_R(x) \left[F_S(x + dx) - F_S(x) \right] \quad (2.5)$$

where

F_R = the cumulative probability distribution of the resistance variable evaluated at x or x + dx

F_S = the cumulative probability distribution of the load variable evaluated at x or x + dx

By choosing an appropriate increment, dx, and solving equations 2.4 and 2.5 iteratively over a selected range of the resistance function, the probability of failure can be approximated. In other words, starting at a suitable point on the horizontal axis of the resistance distribution, such as the location parameter of a 3-parameter Weibull distribution, and stepping from increment to successive increment, infinitesimal probabilities of failure can be calculated. The sum of these infinitesimal probabilities of failure is the probability of failure of the resistance variable under the influence of the load variable. If the increment, dx, is small enough the lower and upper limits will approach one another. The range of calculation must be selected by trial and error. The range is the distance on the horizontal axis for which the calculation of incremental probabilities of failure is made. When the lower and upper limits of the probability of failure are no longer significantly changed by an increase in the range, the range is sufficient.

In the case of infinite load and resistance probability distributions, the upper and lower limits on the probability of failure are bounds of the probability of failure for the range of calculation. They are not the bounds of the theoretical probability of failure obtained by closed integration to plus infinity. The calculation of upper and lower limits serves to check the adequacy of the increment as a step size.

Research (63) has shown that the form and location of the load distribution impacts upon any probability of failure analysis. Of special importance are the tail portions of these distributions. Necessarily, the load distribution should reflect the actual maximum loads that the lumber would experience during its structural lifetime. Also, the location of the load distribution should reflect the interaction of design load to design stress.

Suddarth, et al. (61) investigated the effects of concomitance of bending and tension stress in lumber. If a combined load produced a value greater than 1.0 on the right side of a modified interaction equation, failure occurred. The study focused on the influence of variability in modulus of elasticity (E) on bending and tension stresses and the resulting concomitance. High variability material with a coefficient of variation of E, Ω_E, of 0.21 and material of low variability with Ω_E of 0.10 were chosen and compared. Since bending and tension strengths are both positively correlated to E, the variability of E is related to the probability of failure of a truss.

Corotis (12) had concluded that the present design load levels are close to the 99.9 percent cumulative levels for observed floor loads.

Therefore, the area under the load curve located to the right of 1.0 on the horizontal axis was taken to be 0.001. Also, to study the influence of shifting the load, the area under the load curve located to the right of 1.0 on the horizontal axis was changed to 0.01 and 0.1, respectively. The load distribution was chosen to be adequately represented by the lognormal distribution. The coefficient of variation of the load was taken to be 0.30.

Suddarth, et al. (61) also studied the effect of truncating the load distribution in the right tail. This simulated a loading situation where "disaster" level loads were not involved. While the probabilities of failure were much different between this case and the previous analysis, the allowable increase in the loads remained essentially the same in both cases. For the previous analysis, the allowable design stresses could be increased 8.35 percent while maintaining the same safety. With the truncated load distribution, an 8.50 percent increase was noted. The engineering design result is therefore the same.

Marin and Woeste (45) based the choice of load distribution on research conducted by Sexsmith and Fox (57). The location of the load distribution was chosen on a different basis than the location chosen by Suddarth, et al. (61). Sexsmith and Fox (57) had reported on data from six Canadian cities. The expected thirty year lifetime maximum ground snow load was calculated for each location and this quantity was divided by the thirty year recurrent interval ground snow load. The mean of this ratio, γ, was 1.12. Assuming a snow load of 41.7 psf and a rain load of 5 psf, as given by Sexsmith and Fox (57), Marin and Woeste noted this to be analogous to subtracting 0.12 from the ratio

$\gamma=1.12$. Now γ equals 1.00 which infers the mean of the maximum lifetime snow load equals the thirty year recurrence interval value. Marin and Woeste then set the allowable fifth percentile equal to the thirty year recurrence interval value, thus establishing the relationship between the load and the resistance distributions. This is consistent with present design practices where the design load is set equal to design resistance. The load distribution was assumed to be extreme value type I. The coefficient of variation was 0.18 as reported by Sexsmith and Fox (57).

Green (26) set the 95th percentile of the load distribution equal to the fifth percentile of the resistance distribution. Comparing the bending strength of green lumber to the bending strength of dry lumber at 19 percent maximum moisture content, he found no difference in the probabilities of failure; however, when the load distribution was shifted to the left, differing probabilities of failure resulted. Green concluded that further research on the form and location of the load distribution was needed. The load distribution was again taken as extreme value type I reflecting maximum lifetime load values.

2.2 Probability Concepts

The form of the probability distribution of a random variable may be deduced theoretically on the basis of physical considerations or inferred empirically on the basis of observational data (8). Therefore, some distributions are more naturally suited to model real world situations. In structural design, the maximum load which acts on the structure during its lifetime is of special interest. A distribution which models maximum values or extreme values would therefore be a possibility.

2.2.1 Probability Distributions

The extreme value type I distribution can model either maximum or minimum extreme values. It has been referred to as Gumbel's extreme value distribution, the extreme value distribution, the Fisher-Tippet-Type I distribution, and the double exponential distribution (28). Gumbel (27) has done extensive research on the statistics of extremes and has categorized many of the extreme value distributions according to their properties. The type I asymptotic distribution for maximum values is the limiting model as n approaches infinity for the distribution of the maximum of n independent values from an initial distribution whose right tail is unbounded and which is of exponential type; that is, the initial cumulative distribution approaches unity with increasing values of the random variable as quickly as the exponential distribution approaches unity (8). Underlying distributions which fulfill this criteria are the normal, lognormal, gamma, and exponential distributions.

The definition of the extreme value type I distribution for maximum values can be explained heuristically. First, choose an acceptable underlying distribution which describes a load acting on a structure at any time such as the lognormal distribution. Hence, for one structure, a random lognormal variable will describe the load acting on that structure at some particular point in time. Now draw a random sample from that lognormal distribution, choose the largest value, and save it.

This value would represent the largest load acting on the structure during its lifetime. Assume another identical structure with the same lognormally distributed load acting upon it. Again draw a random sample and choose the largest value. If this process is repeated many times, then the distribution of these largest values will be modeled by the extreme value type I distribution.

The probability density function for the extreme value type I distribution is

$$f_X(x;\alpha,\beta) = \frac{\exp\left\{-\left(\frac{x-\beta}{\alpha}\right) - \exp\left[-\left(\frac{x-\beta}{\alpha}\right)\right]\right\}}{\alpha} \tag{2.6}$$

$$-\infty < x < \infty; \; -\infty < \beta < \infty; \quad \alpha > 0$$

where

α = the scale parameter

β = the location parameter

The cumulative distribution function can therefore be derived as

$$F_X(x;\alpha,\beta) = \exp\left(-\exp\left[-\left(\frac{x-\beta}{\alpha}\right)\right]\right) \tag{2.7}$$

The mean and variance of the extreme value type I distribution are

$$E(x) = \beta + 0.577\alpha \tag{2.8}$$

$$Var(x) = 1.645\ \alpha^2 \tag{2.9}$$

If the mean and variance of the extreme value type I distribution are calculated by the method of moments, the scale and location parameters can be estimated by

$$\alpha = s/1.283 \tag{2.10}$$

$$\beta = \bar{x} - 0.45\ s \tag{2.11}$$

where

$\bar{x}$ = the calculated mean of the data

s = the calculated standard deviation of the data

Lowery and Nash (42) have compared several methods of estimating the type I parameters and have concluded that the above method is as satisfactory as other methods.

The extreme value type I distribution has the particular property that the n-th power of the distribution is algebraically related to the first power of the distribution. Expressed in terms of the scale and location parameters, the transformations are

$$\alpha_n = \alpha \tag{2.12}$$

$$\beta_n = \beta + \alpha\ \ln(n) \tag{2.13}$$

where

α_n = the scale parameter of the extreme value type I distribution raised to the n-th power

β_n = the location parameter of the extreme value type I distribution raised to the n-th power

α = the scale parameter of the original extreme value type I distribution

β = the location parameter of the original extreme value type I distribution.

Distributions other than extreme value distributions can be used to model maximum lifetime loads. As a result of the physical nature of loads, the lognormal distribution has appeal because it is non-negative. Simulation studies are enhanced since negative variables cannot be generated. The lognormal probability density function is

$$f_X(x;\lambda,\zeta) = \frac{1}{\sqrt{2\pi}\, x\zeta} \quad \exp\left[-\tfrac{1}{2}\left(\frac{\ln(x) - \lambda}{\zeta}\right)^2\right] \tag{2.14}$$

$x > 0$

0 elsewhere

where

$\lambda = E(\ln(x))$

$\zeta^2 = VAR\ (\ln(x))$

The lognormal cumulative distribution function cannot be evaluated in closed form. A useful property of the lognormal distribution is if X is distributed lognormally, then $Y = \ln X$ is distributed normally. Therefore, considering the standard normal variate

$$Z = \frac{\ln(x) - \lambda}{\zeta} \tag{2.15}$$

the cumulative probability at any value of x may be found using a standard normal table.

Often, material properties cannot be physically lower than some minimum value. Lumber strength properties exhibit this trait. The three parameter Weibull distribution (66), developed by W. Weibull in 1939, has been extensively used in this respect. Pierce (51) and

Woeste, et al. (69) have recommended its use in modeling lumber strength and material properties. The 3-parameter Weibull probability density function is

$$f_X(x;\eta,\sigma,\mu) = \frac{\eta}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{\eta-1} \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\eta}\right] \qquad (2.16)$$

$x \geq \mu, \quad -\infty < \mu < \infty, \quad \sigma > 0, \quad \eta > 0$

0 elsewhere

where

η = the shape parameter

σ = the scale parameter

μ = the location parameter

The resulting Weibull cumulative distribution function is

$$F_X(x;\eta,\sigma,\mu) = 1 - \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\eta}\right] \qquad (2.17)$$

$x \geq \mu$

The parameters of the 3-parameter Weibull distribution can be estimated by several different methods (28); however, the computational methods are tedious and rather difficult. A system-independent computer program based on the method of maximum likelihood was developed by Simon and Woeste (58) to estimate parameters and it has been used on many data sets with success.

Often the variable under study is a function of several other random variables. If these other random variables have known probability density functions, then the probability density function of the study variable may be specified. Occasionally, the function is algebraically simple and the new probability density function can be mathematically derived or numerically calculated by a convolution integral. For the most part, the function is too complicated as the ingredient probability density functions are too complex to ensure a mathematical solution. Hence, a simulation is necessary.

2.2.2 Deriving the Probability Distribution of a Function of Random Variables

Commonly, two loads act on a structure simultaneously creating a need to calculate the probability density function of the sum of two random variables. Each of these random variables has its own specified probability density function. Specifically, if

$$Z = X + Y \tag{2.18}$$

where

X = a random variable described by a continuous probability density function

Y = a random variable described by a continuous probability density function

and X and Y are statistically independent, then from Ang (8) the cumulative distribution function of Z is

$$F_Z(z) = \int_{-\infty}^{+\infty} \int_{-\infty}^{z} F_X(z-y) f_Y(y)\, dz\, dy \tag{2.19}$$

It can be shown, therefore, that the resulting probability density function, the convolution integral, is

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(z-y)\ f_Y(y)\ dy \tag{2.20}$$

Equations 2.19 and 2.20 cannot always be expressed as explicit functions; however, numerical integration can be used to compute the value of the cumulative distribution function at any z. Details of this procedure are shown and a sample program in listed in Appendix A.

Frequently a distribution may be a function of several random variables and the convolution cannot be solved explicitly or is the solution amendable to numerical integration. An approximate method of solution is Monte Carlo simulation. The histogram of these results will approximate the desired probability distribution (10).

Monte Carlo simulation has been outlined by Hahn and Shapiro (29). The procedure is

(1) Select a random value from each of the statistical distributions for each component variable.

(2) Using the relationship between component variables, calculate the value of the system performance for a system composed of components with values obtained from step one.

(3) Repeat steps one and two the desired number of times.

(4) Summarize and plot resulting values of system performance.

This provides an approximation of the distribution of system performance.

The accuracy of the simulation depends upon the validity of the ingredient probability distributions and the correct assessment of their interrelationship. Also the number of simulation trials will influence the error involved in describing the system performance. An application of the Monte Carlo method to evaluate the performance of a complex system is demonstrated by Bender (9).

A crucial item in Monte Carlo simulation is the generation of random variables. A random variable is defined as a number selected at random from a population of numbers in such a fashion that every number in the population has an equal chance of being selected (28). Since the cumulative distribution function of any continuous variate is uniformly distributed over the interval zero to one (9, 28), random observations may be generated from any probability density function.

The procedure, as illustrated in Figure 2.2, for generating a random variable from an arbitrary probability density function is

(1) Select a random number, R_u from a uniform distribution in the interval (0,1).

(2) Set $F_Y(y) = R_u$, where $F_Y(y)$ is the cumulative distribution function of y.

(3) Solve for y.

Hahn and Shapiro (29) and Haan (28) have tabulated the relationship for a random observation for several common distributions.

In some cases, the cumulative distribution function cannot be solved in closed form. The normal and lognormal distributions are examples of this. Fortunately, numerical generation routines of standard random normal deviates, R_n, are available on most computer systems. The standard normal distribution has a mean equal to zero and a variance equal to one. In order to generate random x-values from a normal distribution with mean, μ and variance, σ^2, the standard

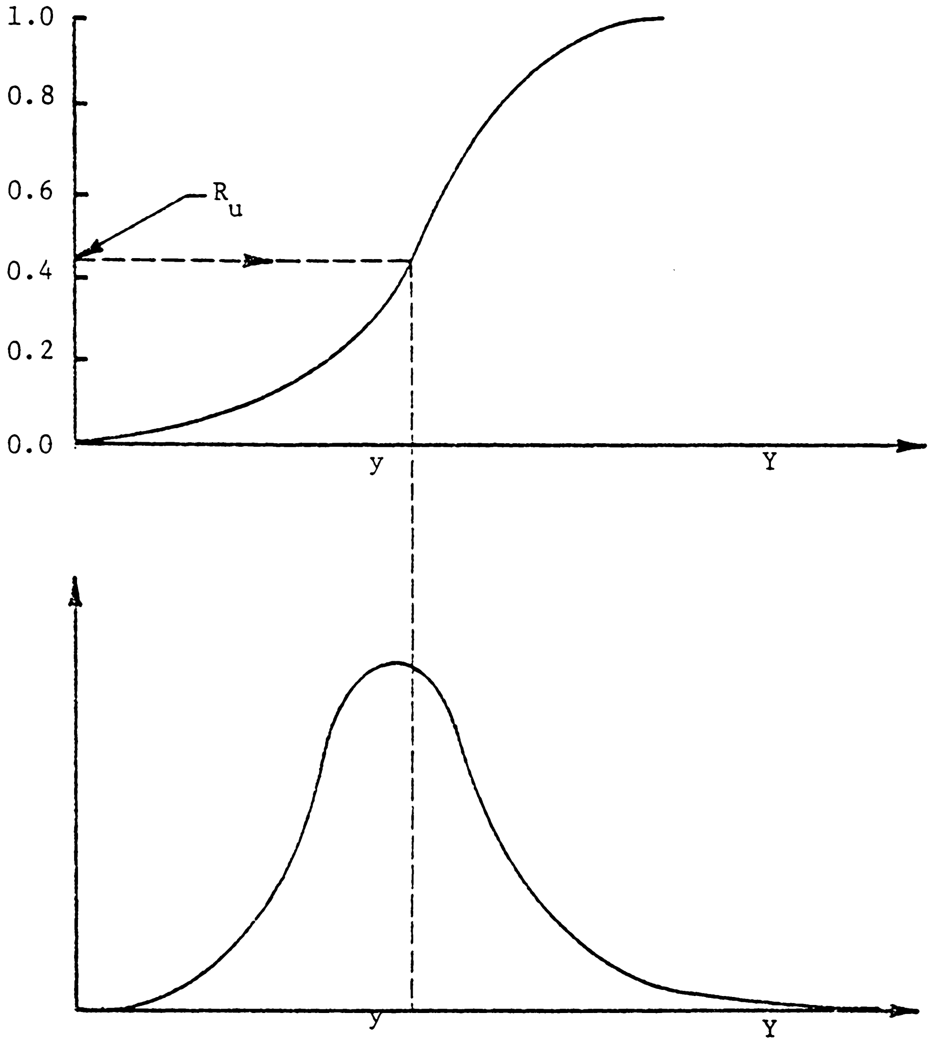

Figure 2.2 The interrelationship between a uniform deviate, a cumulative density function and the associated probability density function is shown. Based on the analytical form of the cumulative distribution function, a value y may be calculated from the inverse of $F_Y(y)$ when R_u is substituted for $F_Y(y)$. Since cumulative distribution functions are derived from an associated probability density function, the random value y from the cumulative distribution function is also the same random value from the probability density function.

normal transformation is utilized.

$$x = \mu + Z\sigma \tag{2.22}$$

Since Z is a standardized variable, it can be replaced by R_n and random x-values result.

Once histograms of density functions have been constructed, hypothesized distributions are usually selected to describe or fit the data. Often, the form of the histogram and physical nature of the data will suggest one or several possible distributions. Necessarily, some criteria must be employed to select the distribution which yields the best fit. Several visual and statistical tests are available. Two methods will be discussed; a visual test and the Kolmogorov-Smirnoff goodness of fit test (8).

2.2.3 Goodness of Fit Tests for Distributions

The first step in choosing a probability distribution is the visual inspection of the histogram. Once data has been accumulated, a histogram can be constructed. From its form or the physical nature of the data, a likely distribution is selected and its parameters are calculated. By overlaying a graph of the calculated probability density function on the histogram, a visual comparison can be made. Any obvious lack of fit can be detected in this manner.

The above visual procedure is not a statistical test in the strict sense. It can be useful, though, to narrow the choice from several possible distributions down to one or two distributions. One drawback is the influence of histogram interval width on the visual impact of the overlayed graph. Too few classes will eliminate detail and obscure the basic pattern of the data, while too many classes result in erratic patterns of alternating high and low frequencies (28). Sturges (60) has suggested the number of classes be determined from

$$c = 1 + 3.3 \log_{10} n \tag{2.23}$$

where

c = the number of classes

n = the number of data values

An available statistical test is the Kolmogorov-Smirnoff (K-S) test. The K-S test compares a theoretical cumulative distribution function to the actual cumulative distribution function calculated from the data. If the difference is too great, the theoretical distribution is rejected. Note that the test signifies whether a particular distribution can be rejected; it does not provide information with regard to the true distribution.

Ang and Tang (8) describes the test as follows. First, the data is ordered in ascending order. Then a sample cumulative distribution function $P_n(x)$ is calculated from the relationship

$$P_n(x) = k/n \tag{2.24}$$

where

k = the number of observations less than or equal to x

n = the sample size

If $F_X(x)$ is the completely specified theoretical cumulative distribution function, the difference between $F_X(x)$ and $P_n(x)$ can be calculated over the range of x. The maximum absolute difference, D is determined as

$$D = \max \mid F(x) - P_n(x) \mid \tag{2.25}$$

If D is greater than the critical tabulated value of the K-S statistic at a chosen significance level, then the hypothesized distribution is rejected.

Haan (28) notes the insensitivity of the K-S test in the tails of the distributions. Also, the K-S test is not very powerful in the sense that the probability of accepting the hypothesis, when in fact it is false, is very high.

It is important to note that $F_X(x)$ must be a completely specified theoretical cumulative probability distribution. In other words, no parameters for the distribution should be estimated from observed data. Crutcher (14) notes that the K-S test has been misused by being applied to parametric distributions in this manner. When standard tables of critical values for the K-S test are used for parametric cases, the test is conservative with respect to the Type I error. In other words, if the critical value is exceeded by the test statistic, D, obtained from the observed values, the hypothesized distribution is rejected with considerable confidence. Crutcher (14) presents a table of critical values for the exponential, gamma, normal and extreme value distributions when the respective parameters are estimated from the observed data. The table is for samples sizes of 25, 30, and large values of n. In light of these facts, the Kolmogorov-Smirnoff test still remains useful.

2.3 Loads

Loads exhibit temporal and spatial variability. Assessing the distributional nature of loads is therefore a complex and difficult problem. In order to study loads, costly and time-consuming surveys and field studies are necessary. Surveys often require ten or twenty years of continuous monitoring to adequately describe various loads. Nevertheless, surveys have been performed; however, earlier surveys did not always treat loads within a framework of statistics (55). Hence, it is difficult to compare results of all available load surveys. In regards to quantifying the stochastic nature of loads, load surveys may have limitations; however, they are helpful in validating probabilistic load models. From various load models or simulations, distributions describing commonly acting loads on structures may be specified.

Of specific interest are the distributions of the maximum lifetime load for dead, snow and floor live loads. These loads are typically considered in light frame structural design. The design life of a typical light frame structure as specified by ANSI A58.1-81 (4) is normally fifty years. All further references to lifetime in this study will therefore refer to fifty years unless otherwise noted.

The maximum lifetime load distribution may be considered as a distribution of maximums from a population of stochastic realizations. Suppose all identical light frame wood structures were equipped to measure roof snow loads. Also, considering the design life of the structure, these measurements are to be taken over a fifty year period. For each structure, a maximum load can be identified during the fifty year period. Then, combining all maximum loads, the resulting data would describe the distribution of maximum lifetime roof snow load. Naturally, this is not physically possible, nor economically feasible. However, load surveys and models combined with simulation techniques and theoretical considerations can adequately define distributional forms of desired maximum lifetime loads.

2.3.1 Dead Load

Dead load is the weight of the structure and all permanently affixed equipment, machines and fixtures either installed initially, or anticipated for future installation (34). It has been noted (63) that the subject of dead loads is relatively simple probabilistically and has not been treated to any extent in the literature except for Johnson (37). Sentler (54) notes that a slight reduction in dead load occurs in a new structure due to moisture evaporation; however, this reduction is so modest that it can be ignored in most cases. Dead load is therefore considered constant during the life of the structure.

Most researchers feel the probability distribution of dead load is approximately normal (17). Many researchers have assumed the ratio of mean load to nominal load, $\bar{D}/D_n$, is unity and that the coefficient of variation, Ω_D, varies between 0.06 to 0.15 with a typical value of 0.10 (1, 16, 21, 41). Ellingwood, et al. (17) proposes $\bar{D}/D_n = 1.05$ and $\Omega_D = 0.10$ because many design professionals feel that designers tend to underestimate the total dead load.

All the above estimates of mean dead load to nominal dead load are applicable to heavy structural applications. Hence, the nominal dead load is the calculated weight of the structural members and permanent fixtures. For most light frame structural applications, the nominal dead load is usually assumed to be 10 psf (32). Since the dead load is considered constant over the life of the structure, the mean dead load would be the calculated dead load for light frame structures. Therefore, the ratio of mean dead load to nominal dead load should be calculated for each individual case such as floor dead load or roof dead load.

The coefficient of variation of dead load reflects two uncertainties. First, variability in the weight of the structural members and non-structural components is included. And secondly, the designers inability to accurately calculate the dead load effect adds to the total variability.

2.3.2 Snow Load

Snow loads are derived using climatological data and field studies which relate the roof snow load to the ground snow load and the roof exposure, geometry, and thermal characteristics (17). An estimate of the roof snow load can be expressed as

$$S = C_S q \tag{2.26}$$

where

S = maximum lifetime roof snow load

C_S = snow load coefficient relating roof snow load to ground snow load; C_S is determined by roof exposure, geometry and thermal factors

q = maximum fifty year ground snow load

All available data relate to the annual extreme ground snow load; however, the maximum fifty year ground snow load can be derived from

$$F_{50}(q) = [F(q)]^{50} \tag{2.27}$$

where

$F_{50}(q)$ = the cumulative distribution function of the fifty year maximum ground snow load

$F(q)$ = the cumulative distribution function of the annual extreme ground snow load

Elligwood, et al. (17) developed a distribution of maximum roof snow load using equations 2.26 and 2.27. Utilizing a recent analysis of annual extreme ground snow loads by the U.S. Army Cold Regions Research and Engineering Laboratory (64), a more detailed analysis was conducted for a number of sites across the U.S. in which there was measurable snow accumulation in each of the years of record. The CRREL analysis indicates that the cumulative distribution function for annual extreme ground snow load is lognormal with parameters that vary from site to site. The selected sites and the parameters for the lognormally distributed annual extreme ground snow load are shown in Table 2.1 (17).

The C_S conversion coefficient adds further uncertainties to the conversion of ground snow load to roof snow load. It has been noted that the C_S factors in the current and proposed A58 standards (3, 4) have been selected based on the results of field surveys, adjusted to a considerable degree by professional judgement, and are conservative (17). The best estimate of the probabilistic aspects of C_S is that it is symmetrical with a mean, $\bar{C}_S$ = 0.50 and a coefficient of variation,

Ω_{C_S} = 0.23. The distribution of C_S is assumed to be normal. These estimates have been basically confirmed; however, the estimated coefficient of variation may be low (19).

Ellingwood, et al. (17) computed the maximum roof snow load by numerical quadrature using equations 2.26 and 2.27. Numerical quadrature is a numerical integration technique which can solve a convolution integral with the aid of a high speed digital computer. The integral describing maximum roof snow load at the different sites was solved for various values. The normal, extreme value type I and extreme value type II distributions were fit to the calculated values. None of these distributions fit over the entire range; however, the extreme value type II distribution did provide an excellent fit in the upper percentiles.

The type II distribution was chosen as the distribution of maximum roof snow load, Ellingwood, et al. (17). The characteristic extreme and shape (u, α) are listed in Table 2.1 by site. A single set of parameters was calculated from the results presented in Table 2.1. They are

$$u = 0.72 \tag{2.28}$$

$$\alpha = 5.82 \tag{2.29}$$

These parameters correspond to a mean snow load to nominal snow load ratio, $\bar{S}/S_n$ and coefficient of variation, Ω_S of

$$\bar{S}/S_n = 0.82 \tag{2.30}$$

$$\Omega_S = 0.26 \tag{2.31}$$

Table 2.1 The tabulated values are the sites and the parameters for the analysis of snow load conducted by Ellingwood, et al. (17). The annual extreme ground snow load for each site is lognormally distributed with the tabulated parameters shown based on the available number of years of record. The tabulated value, q_n, is the nominal ground snow load specified by the standard A58.1-72. The fifty year maximum roof load was fit by an extreme value type II distribution for each site and the tabulated parameters u and α were calculated from the fit.

Site	Annual Extreme Ground Snow Load			A58.1-72	50-yr. Maximum Roof Load	
	Years of Record	λ	ζ	q_n	u	α
Green Bay, WI	26	2.01	0.70	28	0.87	5.07
Rochester, NY	26	2.49	0.56	34	0.83	6.16
Boston, MA	25	2.28	0.51	30	0.70	6.63
Detroit, MI	20	1.63	0.58	18	0.69	5.97
Omaha, NB	26	1.50	0.69	25	0.62	5.20
Cleveland, OH	26	1.50	0.58	19	0.60	6.30
Columbia, MO	25	1.21	0.84	20	0.69	4.05
Great Falls, MN	26	1.77	0.49	15	0.80	7.16

Ellingwood had some confirmation of these results from a Monte Carlo simulation of Canadian snow load data by Isyumov (36).

The Joint Committee on Structural Safety (39) recommended the use of the type I distribution to model the maximum ground snow load in 50 years. The European ground to roof snow load conversion factor, C_S, is 0.80. The committee made no recommendation of a mean value for maximum 50 year ground snow load since the variation across the continent was so great. However, if data was available, statistical values could be calculated from recommended extreme value equations. The uniformly distributed maximum roof snow load could then be calculated from deterministic equations.

2.3.3 Live Load

The maximum lifetime floor live load is composed of two components, the sustained load and the extraordinary load. The sustained load is the weight of all furnishings and movable fixtures and includes the weight of all normal occupants. It is characterized by its long duration and essential uniformity over time. The extraordinary, or transient load is of short duration, usually only several hours long. The transient load represents occasions when many people are gathered or furniture is placed together for some reason such as remodeling. This load is the most unpredictable since all transient loads may not be measurable or even identified (54).

While the transient load may be difficult to measure, the sustained load is not. Numerous live load surveys have concentrated on the sustained load. Heaney (31) presented a synopsis of 59 live load surveys conducted up to 1971; however, the use of these surveys in a probabilistic analysis is severely restricted because statistical principles were not always applied in the data analysis (35). Hasofer (30) states

> "The published results of past surveys are unfortunately not given in a suitable form (for analysis). The main methodological mistake was the statistical misconception that one can obtain a reasonable picture of possible maxima of loads by looking at the observed maxima of the survey. The statistical theory of extreme values has shown that this is an unreliable and inefficient method of analysis."

Nonetheless, live load surveys do provide useful information. While a maximum lifetime load cannot be derived, statistical parameters of the sustained load can be developed from properly conducted live load surveys.

Karmen (40) appears to have performed the first residential load survey in which a serious effort was made to analyze the data from a statistical standpoint. From a sampling of 183 dwellings in Budapest, Hungary, he found the mean live load to be 11.35 psf and a standard deviation equal to 4.02 psf. Sentler (53) performed a similar study in Sweden. Dwellings were divided into two categories according to age; prior to 1940 and post 1940. Sentler found for the prior to 1940 division a mean of 6.0 psf and a standard deviation of 2.5 psf. For the post 1940 division, the mean was 4.7 psf and the standard deviation equaled 1.9 psf. The combined statistics of the entire sample of 341 rooms are therefore a mean of 5.35 psf and a standard deviation of 2.32 psf. Both Sentler and Karmen employed the statistics of extremes in their analysis (35).

Sentler (53) made several important observations concerning his survey. First, he noted that his results were lower than Karmen's results; a fact he attributed to differences in life styles between countries. Second, the live load for sitting rooms and bedrooms is independent of size. Third, there is no justification for reducing live load for larger rooms. And lastly, design loads should include the effect of short term heavy loads.

Johnson (37) conducted a load survey in the 1940's to obtain a sample which could be analyzed statistically. The survey consisted of 139 apartments in Stockholm and the results were a mean sustained load of 5.18 psf and a standard deviation of 2.07 psf. This sustained load did not include the load of the tenants.

Paloheimo (49) conducted a load survey in Helsinki, Finland in 1973. The mean sustained load was 4.93 psf with a standard deviation equal to 2.05 psf. The sustained load, in this case, did include the weight of normal occupants.

Sentler (53) fitted the gamma distribution and the lognormal distribution to his survey data. The gamma distribution appeared to fit the data better; however, the final conclusion was that both distributions provided a good fit to residential load survey data. Johnson (37) only fit his survey data with the lognormal distribution. Corotis and Doshi (13) conducted an extensive analysis of five live load surveys. After obtaining the histograms and basic statistics, Corotis and Doshi fit the normal, lognormal and gamma distributions to the data by the method of moments and compared them to one another.

They concluded that the gamma distribution provided the best overall fit to the distribution of sustained load.

The best method of surveying the transient load is on a continuous time basis (11). However, the expense and logistics necessary to conduct a study of this sort are prohibitive. Hence, none have been conducted in this manner. The next best alternative is a personal survey of occupants. This method introduces uncertainties, but it is the only one which has produced results.

Only two surveys attempted to quantify the extraordinary load in residential buildings (55). Johnson and Paloheimo both questioned families concerning the maximum number of persons who had visited the apartment at the same time. Johnson assumed each adult weighed 70 kg (154 lbs.) and each child weighed 35 kg (47 lbs.). Sentler (55) does not say whether or not Paloheimo made any assumptions of this nature. Johnson found a mean transient load of 6.1 psf with a standard deviation of 3.1 psf. Paloheimo calculated a mean of 5.7 psf for transient load and a standard deviation of 1.4 psf. No attempts were made to fit a distribution to either of these sets of data; however, several researchers (11, 55) feel the gamma distribution is a good choice.

Several live load models have been developed to quantify the stochastic nature of loads (11, 54, 67). Considering the total floor live load as a probabilistic combination of a sustained load and an extraordinary load, these models attempt to stochastically develop the maximum lifetime floor live load. It is generally believed (11) that the maximum lifetime floor live load can occur by one of three possible cases. Chalk and Corotis (11) list the three cases. The first case

(Case I) is the sum of the maximum sustained load, L_s, and the largest extraordinary load, L_{e_1}, occurring during the duration of the maximum sustained load denoted by subscript 1. The second case (Case II) is the largest extraordinary load, L_e, during the life of the structure plus the sustained load, L_i, acting at the time of this extraordinary load. Both of these cases have the same likelihood of occurrence. The third case (Case III) has a reduced likelihood of occurrence. Case III is the sum of the maximum lifetime extraordinary load, L_e, and the maximum lifetime sustained load, L_s. Case III is the largest possible floor load which can occur during the lifetime of the structure. However, the possibility of two extreme events occurring simultaneously is slight; therefore, Case III has a reduced probability of occurrence.

Even though the sustained load is of long duration, it is usually not constant over the entire life of the structure. Due to occupancy changes or major remodeling efforts, the sustained load will change. During some periods of time, the sustained load may not even be present, an event which could be described by a vacant house.

During the period of time over the life of the structure when the maximum sustained load is acting, several transient load events may occur. These events could be parties when people are crowded in several rooms of the house or a remodeling of several rooms when materials and furniture would be stored in other rooms. The largest of these transient loads is L_{e_1}, the maximum extraordinary load acting during the maximum sustained load. This largest extraordinary load, L_{e_1} may not be the maximum extraordinary load occurring during the lifetime of the structure, L_e. The maximum lifetime extraordinary load could occur at any time even when the sustained load is small. For instance, this event could occur in a new unfurnished house where the occupants are having a house warming party with a large number of guests. These events are graphically depicted in Figure 2.3.

The maximum lifetime total load distribution can be calculated utilizing the above three cases relative to their respective probabilities of occurrence. Noting that L_s, L_e and L_{e_1} are well modeled by the type I extreme value distribution, Chalk and Corotis (11) have derived the distribution of total maximum floor load based on two assumptions. First, the sustained load was considered to be deterministic. In other words, the mean of a sustained load survey would be considered the sustained load on a structure. The second assumption was that the type I extreme value distribution would adequately model the combined distributions of case I and case III.

The moments of the three cases were therefore found to be

$$m_I = E[L_s + L_{e_1}] = m_{L_s} + m_{L_{e_1}} \tag{2.32}$$

$$\sigma_I^2 = VAR[L_s + L_{e_1}] = \sigma_{L_s}^2 + \sigma_{L_{e_1}}^2 \tag{2.33}$$

$$m_{II} = m_{L_e} \tag{2.34}$$

$$\sigma_{II}^2 = \sigma_{L_e}^2 \tag{2.35}$$

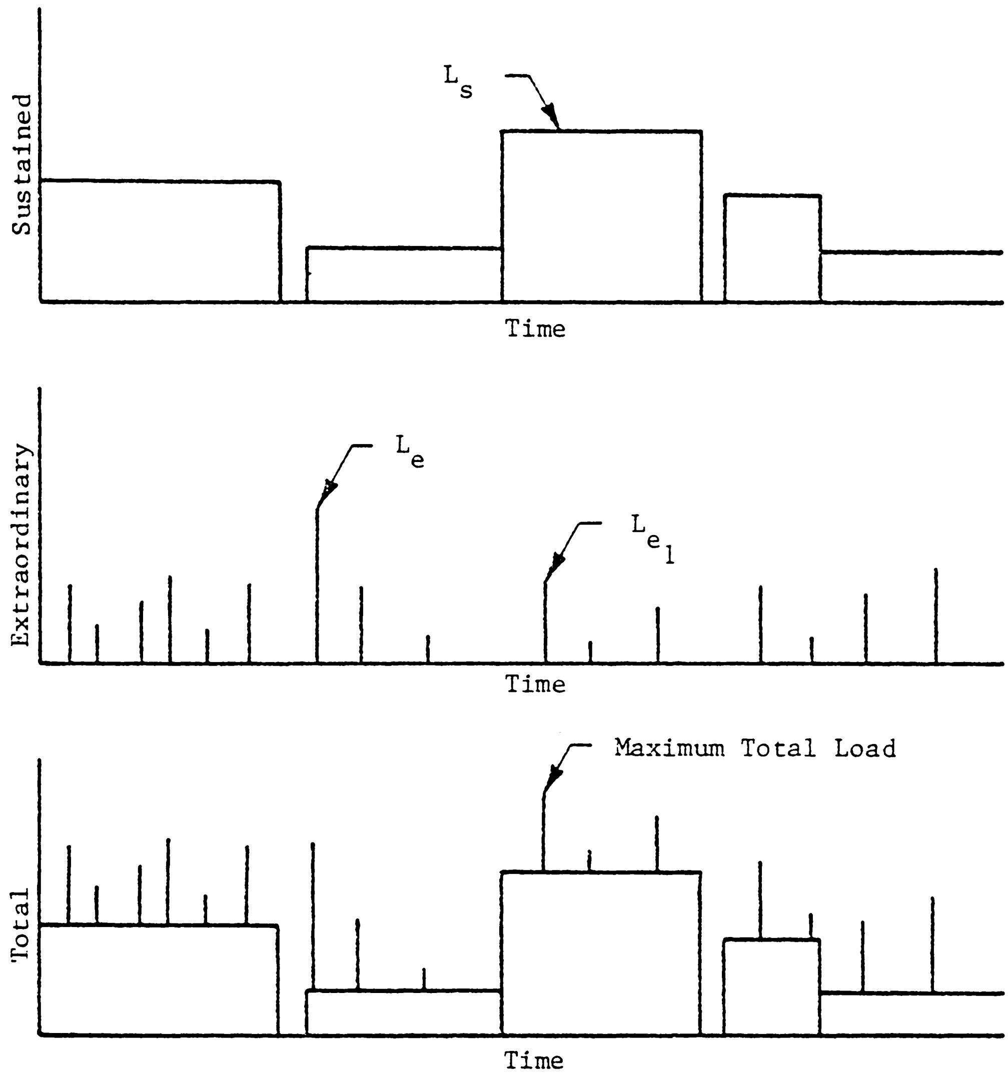

Figure 2.3 A possible stochastic load realization for a light frame structure is shown. The top figure represents the sustained load history over the life of the structure. The middle figure represents the extraordinary load history over the life of the structure. The history of the combined load is shown in the bottom figure.

$$m_{III} = E[L_s + L_e] = m_{L_s} + m_{L_e} \tag{2.36}$$

$$\sigma^2_{III} = VAR[L_s \div L_e] = \sigma^2_{L_s} + \sigma^2_{L_e} \tag{2.37}$$

where

m = the mean and the subscript denotes the load case or the type of load

σ^2 = the variance and the subscript denotes the load case or the type of load

Equations 2.32, 2.33, 2.36 and 2.37 assume statistical independence between L_e, L_s and L_{e_1} (11).

Since all three cases are modeled by the type I extreme value distribution, the extreme value distribution parameters, α and β can be calculated from the mean and variance using equations 2.10 and 2.11. The distribution of maximum lifetime floor live load can now be calculated as

$$F_{L_t}(\ell) = \exp\left[-\exp(-w_1)\right] \exp\left[-\exp(-w_2)\right] \frac{T - E(\tau)}{T} + \exp\left[-\exp(-w_3)\right] \frac{E(\tau)}{T} \tag{2.38}$$

where

$F_{L_t}(\ell)$ = the cumulative distribution function of the maximum lifetime floor live load evaluated at ℓ.

$w_1 = (\ell-\beta_1)/\alpha_1$, the reduced variate for Case I

$w_2 = (\ell - (\beta_2 + m_{L_i}))/\alpha_2$, the reduced variate for Case II;

m_{L_i} = the mean of the sustained load survey

$w_3 = (\ell - \beta_3)/\alpha_3$, the reduced variate for Case III

T = the design life of the structure

$E(\tau)$ = the expected duration of the sustained load

In order to describe the sustained load, Chalk and Corotis (11) pooled the data from several live load surveys. These surveys included work by Dunham (15), Johnson (37), Karmen (40), Sentler (53), and Paloheimo (49). All of these surveys except Dunham have been previously described. For the extraordinary load, since only two surveys were available, Chalk and Corotis (11) utilized a model developed by Pier and Cornell (50) to describe the extraordinary load parameters. This model gave good results when compared to the mean of available data (11).

Chalk and Corotis (11) used their model to calculate maximum lifetime loads for several different types of building occupancies including residential. For residential structures, two categories were selected based on occupancy; owner occupied and renter occupied. The difference between the two categories is the average length of duration of occupancy. The renter occupied residences averaged two years between occupant changes. For owner occupied dwellings the average occupancy duration was found to be about ten years. Based on these facts, the calculated mean of the maximum lifetime floor live load, m_{L_t}, was 35 psf for renter occupied residences. The mean for owner occuped residences was 39 psf. Both renter occupied and owner occupied residences had a calculated standard deviation, σ_{L_t}, equal to 7.7 psf.

Chalk and Corotis did not speculate on the distribution of maximum lifetime floor live load from this load model. Other researchers have ventured recommendations concerning the probability distribution. Ellingwood, et al. (17) advocates the use of the extreme value type I distribution. Sentler (54) felt the distribution of maximum lifetime floor load did not need to be chosen specifically as long as it was unimodal. The Joint Committee on Structural Safety of Europe (38) recommended the extreme value type I distribution as adequately describing the maximum lifetime floor live load.

Sentler (54) also developed a stochastic model for floor live loads for application to offices, hotels and apartments. He developed probability distributions based on live load surveys for sustained and extraordinary loads. The data in a survey by Sentler (53) was used for sustained loads and the transient load survey by Paloheimo (49) was considered adequate for extraordinary loads. The development of the maximum sustained and maximum extraordinary floor live loads were based on equations which considered the temporal variation in live load. This rendered a closed form development of the maximum lifetime floor live load distribution too complicated for practical use. Therefore, Sentler (54) simplified the model by assuming no time dependence and calculated the maximum lifetime floor live load using both methods. He found no significant differences between the two methods and therefore recommended the simplified model.

The Joint Committee on Structural Safety (38) based their recommendations of floor live load parameters on Sentler's model (54) and survey (53). For dwellings, a mean value of 20.89 psf and a standard deviation of 8.35 psf resulted for maximum lifetime floor live load.

Canadian researchers (1, 59) have advocated that the ratio of the mean maximum lifetime floor live load to nominal floor live load be taken as 0.70, independent of tributary area. This value was developed based on the fact that an office building in Canada is designed for a 50 psf live load. Allen (1) states that the expected maximum live load in thirty years would be 35 psf. Hence, the value of 0.7 was assumed. Based on the indication of several live load surveys, a coefficient of variation of 0.30 was assumed for maximum floor load.

Ellingwood, et al. (17) felt that the maximum floor live load varied significantly with respect to the tributary area; therefore, he made the recommendation that the mean of the maximum floor load be calculated from one of two equations based on the A58 standards (3, 4). The coefficient of variation of the maximum floor live load was taken to be 0.25. This result was comparable to the Canadian results based on the transformation from a 30 year return period to a 50 year return period (17).

CHAPTER III

THE DEVELOPMENT OF LOAD DISTRIBUTIONS AND DEMONSTRATION OF THEIR APPLICATION IN DIFFERENTIAL RELIABILITY ANALYSIS

As a result of the In-grade testing program, data describing material and strength properties of structural lumber of various grades and sizes has been collected. Utilizing this data, researchers have developed evidence which demonstrates that present factors employed in assigning and adjusting lumber strengths may be unsatisfactory. It also appears that present allowable design values are not a consistent percentile of the distributions of full size structural lumber strength. An alternative method for developing realistic lumber strength adjustment factors and lumber size and grade relationships is the differential reliability approach to lumber data analysis.

The differential reliability technique is an integrated approach relying on the resistance distributions, the associated load distributions, and a correct assessment of the position of the load distribution relative to the resistance distribution. As previously stated, various distributions describing material and strength properties of full size structural lumber have been identified through the In-grade testing program. Also, distributions of commonly acting loads on light frame structures have been developed; however, no attempt has been made to measure the impact or appropriateness of these load distributions in a differential reliability analysis.

As outlined in the review of literature, several different methods of positioning the load distribution relative to the strength distri bution have been utilized in differential reliability analyses. One promising method places the mean of the load distribution relative to the adjusted fifth percentile of the strength distribution. This choice of relative placement reflects the presently accepted design practice of setting the design load equal to the allowable design strength.

Once the load distribution has been positioned relative to the strength distribution, two or more contrasting sets of lumber data may be compared using a probability of failure analysis. The contrasting lumber data sets should exhibit similar characteristics except the property under study. For instance, to study the effect of moisture content on lumber strength, the same species, size, and grade of lumber should be utilized; only the average moisture content between lumber sets should differ. The analytical probability of failure technique outlined in the review of literature can be employed to calculate the failure probabilities for each lumber data set. Each set will have, in general, a different probability of failure which reflects the effect of the particular study variable. For the example above, the different probabilities of failure would reflect the effect of moisture content on lumber strength.

To compare the probabilities of failure, the first step is to establish a particular reference base. If one of the data sets is chosen as the reference data set, then its probability of failure is the benchmark safety level. All other calculated probabilities of failure are then compared to the benchmark probability of failure. A rational method of comparing the data sets is to alter the strength values of each resistance data set by some factor, k, until a similar probability of failure as the benchmark safety level is attained. This k factor is then the factor which relates each data set to the reference data set.

The above comparison method is employed in all previous applications of differential reliability analysis described in the review of literature. The choice of load distribution, however, has been contested. The load distribution should reflect actual loads applied to the structure during its lifetime. Therefore, distributions describing the maximum lifetime roof snow load and maximum lifetime floor live load are important since these loads significantly affect the design of light frame structures. The distribution of dead load should be included in some fashion because it is the permanent load on the structure and is an integral part of all design calculations. Therefore, an explanation of the development of the distributions of dead load, maximum lifetime roof snow load, and maximum lifetime floor live load follows.

3.1 Development of the Dead Load Distribution

Dead load is the weight of the structure and all permanently affixed equipment, machines and fixtures either installed initially, or anticipated for future installation (34). For structural members used in light frame applications such as floor joists, ceiling joists and low slope rafters not supporting a finished ceiling, the nominal dead load, D_n, is 10 psf (32). For farm building roof trusses, the nominal dead load is 4 psf for top chords and 1 psf for bottom chords (32). Since the dead load is considered constant during the life of the structure, the mean dead load is assumed to be the calculated dead load for each particular structural application assumed in the differential reliability analysis. As an example, for a typical low slope roof using rafter design, 2" x 8" No. 2 Douglas Fir rafters, 16 inches on center with ½ inch plywood sheathing and asbestos shingles, a dead load of 5.7 psf can be calculated for the roof. Based on the review of literature, the coefficient of variation of dead load, Ω_D, is taken to be 0.10 for this study. Therefore, the dead load parameters for this study case are

$$\bar{D}/D_n = 0.57 \tag{3.1}$$

$$\Omega_D = 0.10 \tag{3.2}$$

Since most researchers feel the dead load is only approximately normal, the distribution of dead load will be assumed to be lognormal. For small coefficients of variation, the differences between the normal distribution and the lognormal distribution are negligible. This is graphically depicted in Figure 3.1. Also, the lognormal distribution has the added advantage of being non-negative. Based on these two observations, the lognormal distribution appears to be the best choice to model the dead load.

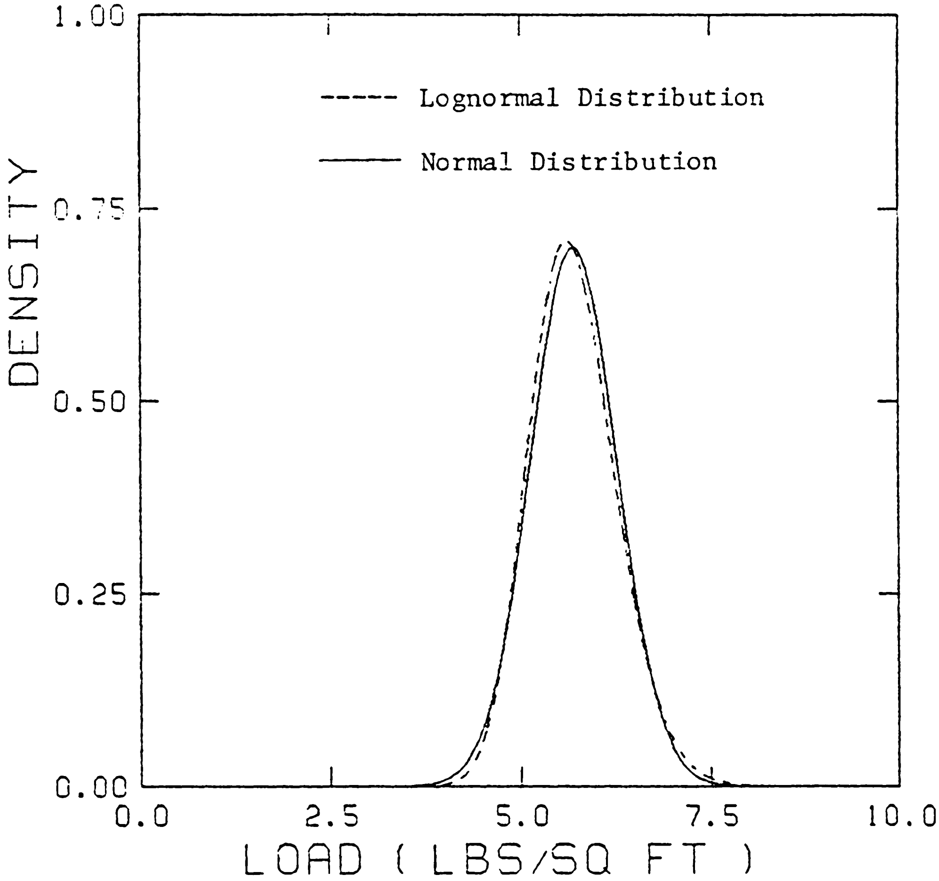

Figure 3.1 The comparison between a normal distribution and a lognormal distribution is shown. Both distributions have a mean value of 5.7 psf and a coefficient of variation of 0.10.

3.2 Development of the Maximum Lifetime Roof Snow Load Distribution

As outlined in the review of literature, Ellingwood, et al. (17) has derived the distribution of roof snow load based on an extensive analysis of annual extreme ground snow load by the U.S. Army Cold Regions Research and Engineering Laboratory (64) and the present ANSI standard, A58.1-72 (3). The CRREL analysis is the best available at this time based on United States snow load records. The conclusion of Ellingwood, et al. (27) was that the extreme value type II distribution provided the best fit as the distribution of maximum lifetime roof snow load; however, the distribution did not fit the data over the entire range. The choice of best fit was based on the fit in the upper percentiles, 85th and above. The results of this analysis were used to develop load factors for application in a reliability based design format (17).

In a reliability based design format, the fit of the tail of a distribution to the data is very important; however, for a differential reliability analysis, a distribution should provide a good fit over the entire range of data. Based on these observations, an analysis was conducted similar to Ellingwood, et al. (17) in order to determine a distribution which provided a good overall fit to the roof snow load data. New information from ANSI standard A58.1-81 (4) was incorporated into this analysis. The two analyses differed in that Ellingwood, et al. (17) solved the convolution integral describing the cumulative distribution function of roof snow load with the technique of numerical quadrature. A Monte Carlo simulation was employed in the present analysis to describe the probability density function of the roof snow load. A detailed explanation of the maximum lifetime roof snow load analysis follows.

First, it was necessary to identify the parameters of the distributions of ground snow load and the conversion factor of maximum lifetime ground snow load to maximum lifetime roof snow load, C_S. These parameters are used in equation 2.26 and 2.27 which describe the roof snow load. Since Ellingwood, et al. (17) utilized ground snow load data from the latest available analysis by CRREL (64), the same data was chosen for the Monte Carlo analysis. As mentioned in the review of literature, the annual extreme ground snow load is lognormally distributed with parameters that vary from site to site. The data from CRREL is the basis for the proposed ANSI load standard A58.1-81 (4); therefore, the nominal ground snow load specified by that standard for each site was utilized. The sites and parameters of the annual extreme ground snow load are listed in Table 3.1. The nominal ground snow loads taken from the proposed ANSI standard A58.1-81 for each site are also listed in Table 3.1.

The conversion factor, C_S as described in the review of literature is normally distributed with a mean, $\bar{C}_S$ equal to 0.50 and a coefficient of variation, Ω_{C_S} equal to 0.23. In order to be consistent, the nominal ground-to-roof conversion factor of $C_{S_n} = 0.7$ as taken from ANSI standard A58.1-81 (4) was used.

Combining equations 2.26 and 2.27, the distribution of maximum lifetime roof snow load for each site listed in Table 3.1 can be found using the equation

$$S/S_n = C_S/C_{S_n} \quad q_{50}/q_n \qquad (3.3)$$

Table 3.1 The tabulated values are the sites and parameters for the analysis of snow load. The annual extreme ground snow load for each site is lognormally distributed with parameters λ and ζ as shown. λ is the mean of the logarithms of the snow load in psf and ζ is the corresponding standard deviation. The tabulated value, q_n, is the nominal ground snow load specified by the proposed standard A58.1-81. The maximum lifetime roof snow load parameters are calculated from random roof snow load deviates generated from a Monte Carlo simulation.

Site	Annual Extreme Ground Snow Load			A58.1-81	Maximum Lifetime Roof Snow Load	
	Years of Record	λ	ζ	q_n	$\bar{S}/S_n$	Ω_S
Green Bay, WI	26	2.01	0.70	40	0.68	0.47
Rochester, NY	26	2.49	0.56	40	0.79	0.37
Boston, MA	21	2.28	0.51	35	0.65	0.35
Detroit, MI	20	1.63	0.58	20	0.70	0.38
Omaha, NB	26	1.60	0.59	25	0.70	0.44
Cleveland, OH	26	1.50	0.58	25	0.49	0.38
Columbia, MO	25	1.21	0.84	20	0.86	0.56
Great Fall, MN	26	1.77	0.49	20	0.65	0.34
Average					0.69	0.44

where

S/S_n = the normalized maximum lifetime roof snow load

C_S = the normally distributed ground-to-roof snow load conversion factor

C_{S_n} = the nominal ground-to-roof snow load conversion factor

q_{50} = the 50 year maximum ground snow load which has the distribution of maximum lifetime ground snow load derived by equation 2.27

q_n = the nominal ground snow load specified by ANSI standard A58.1-81

A histogram of the maximum lifetime roof snow load for each site was constructed utilizing equation 3.3 in the following manner. First, 1000 random C_S deviates and 1000 maximum lifetime ground snow load deviates were generated for each site utilizing the normal parameters for C_S listed above and the lognormal parameters of the annual extreme ground snow load for each site listed in Table 3.1. These deviates were then substituted in equation 3.3 to calculate random observations of normalized maximum lifetime roof snow load, S/S_n for each site. The constructed histograms of S/S_n for each site are shown in Figures 3.2 through 3.9. The means and coefficients of variation for each site are listed in the last two columns of Table 3.1. Also, the average mean and resulting coefficient of variation are given.

A visual inspection of Figures 3.2 through 3.9 suggested that the

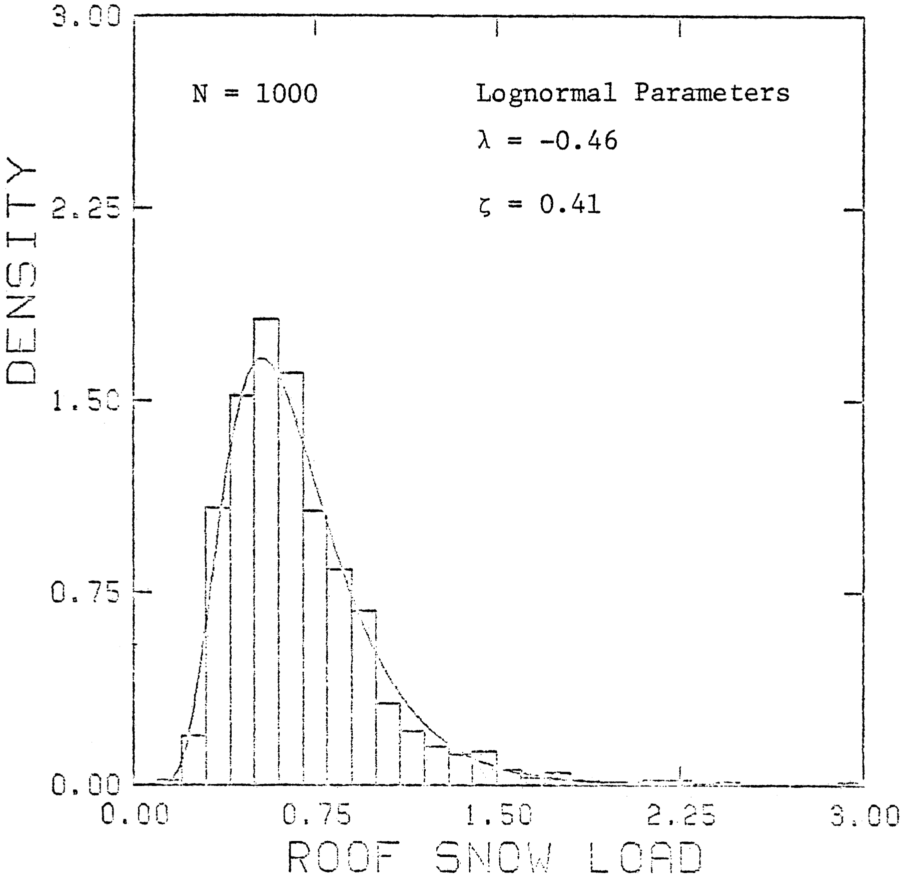

Figure 3.2 A histogram of the normalized maximum lifetime roof snow load for Green Bay, WI calculated from the results of a Monte Carlo simulation is given with an overlay of the estimated lognormal density function.

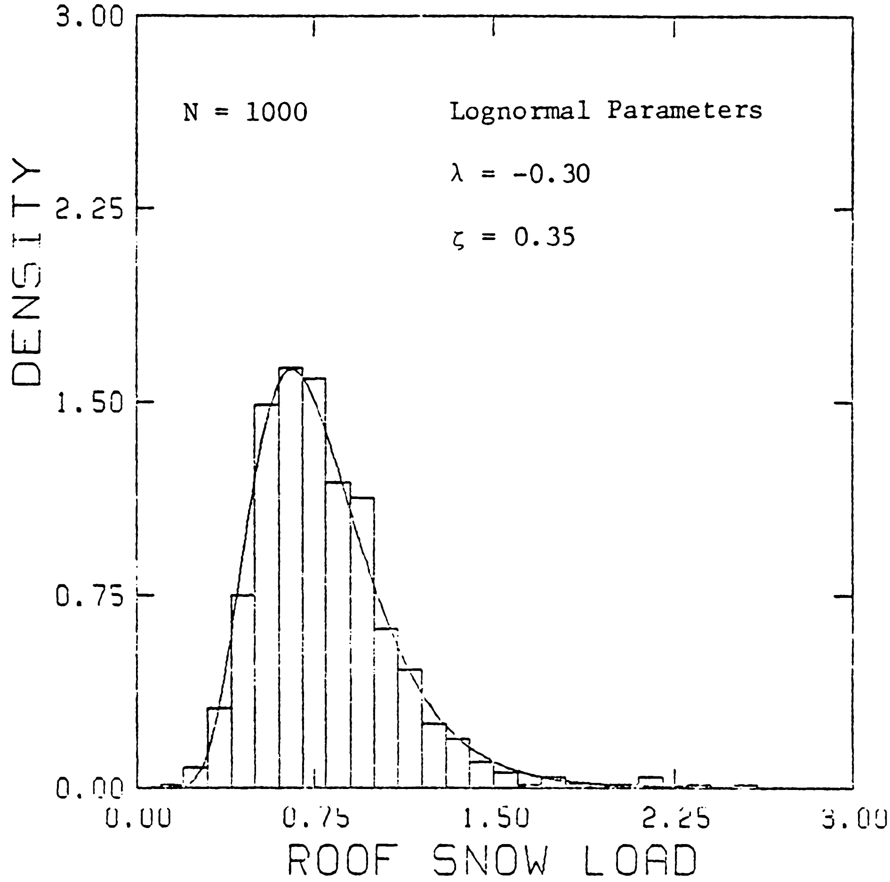

Figure 3.3 A histogram of the normalized maximum lifetime roof snow load for Rochester, NY calculated from the results of a Monte Carlo simulation is given with an overlay of the estimated lognormal density function.

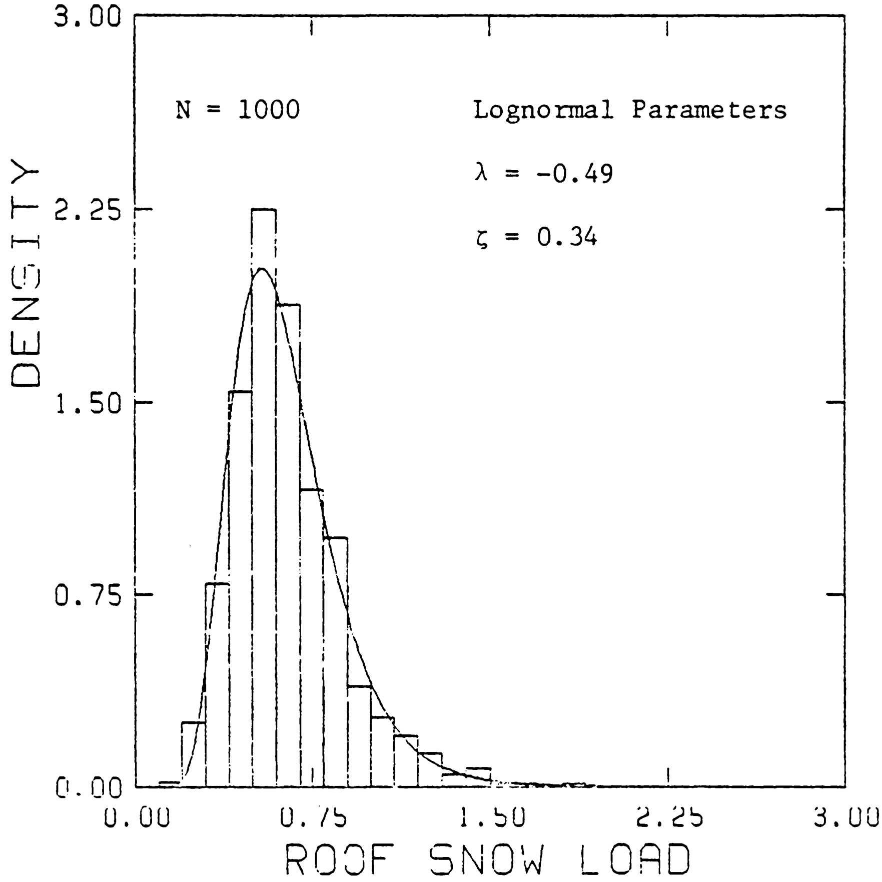

Figure 3.4 A histogram of the normalized maximum lifetime roof snow load for Boston, MA calculated from the results of a Monte Carlo simulation is given with an overlay of the estimated lognormal density function.

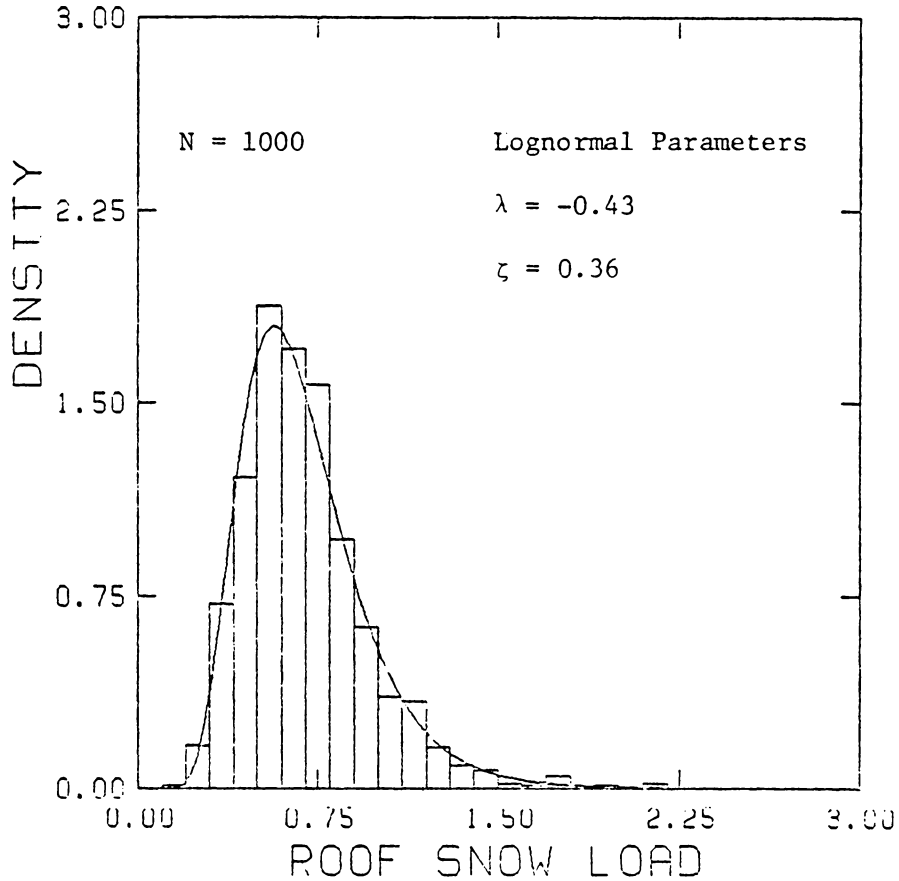

Figure 3.5 A histogram of the normalized maximum lifetime roof snow load for Detroit, MI calculated from the results of a Monte Carlo simulation is given with an overlay of the estimated lognormal density function.

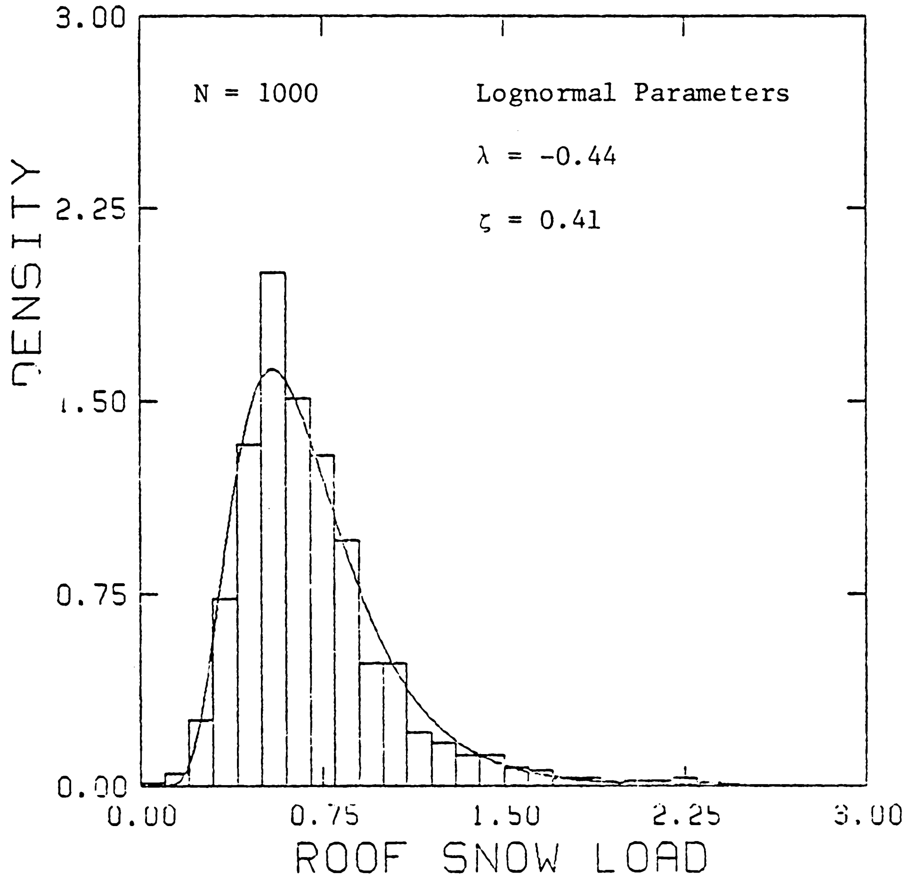

Figure 3.6 A histogram of the normalized maximum lifetime roof snow load for Omaha, NB calculated from the results of a Monte Carlo simulation is given with an overlay of the estimated lognormal density function.

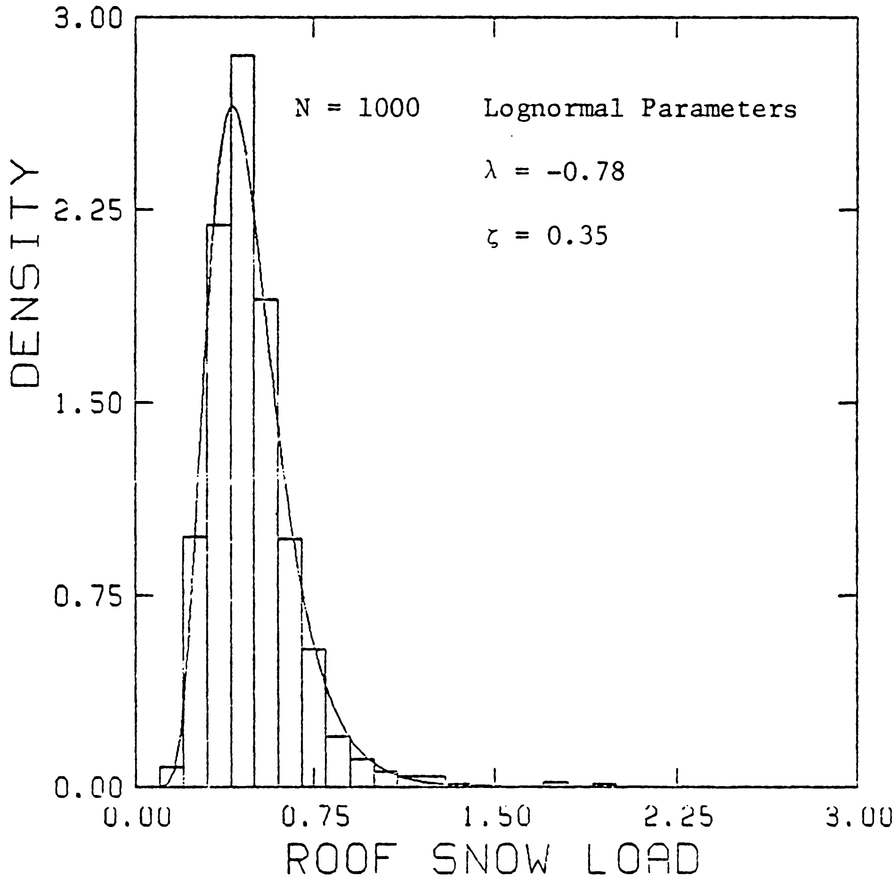

Figure 3.7 A histogram of the normalized maximum lifetime roof snow load for Cleveland, OH calculated from the results of a Monte Carlo simulation is given with an overlay of the estimated lognormal density function.

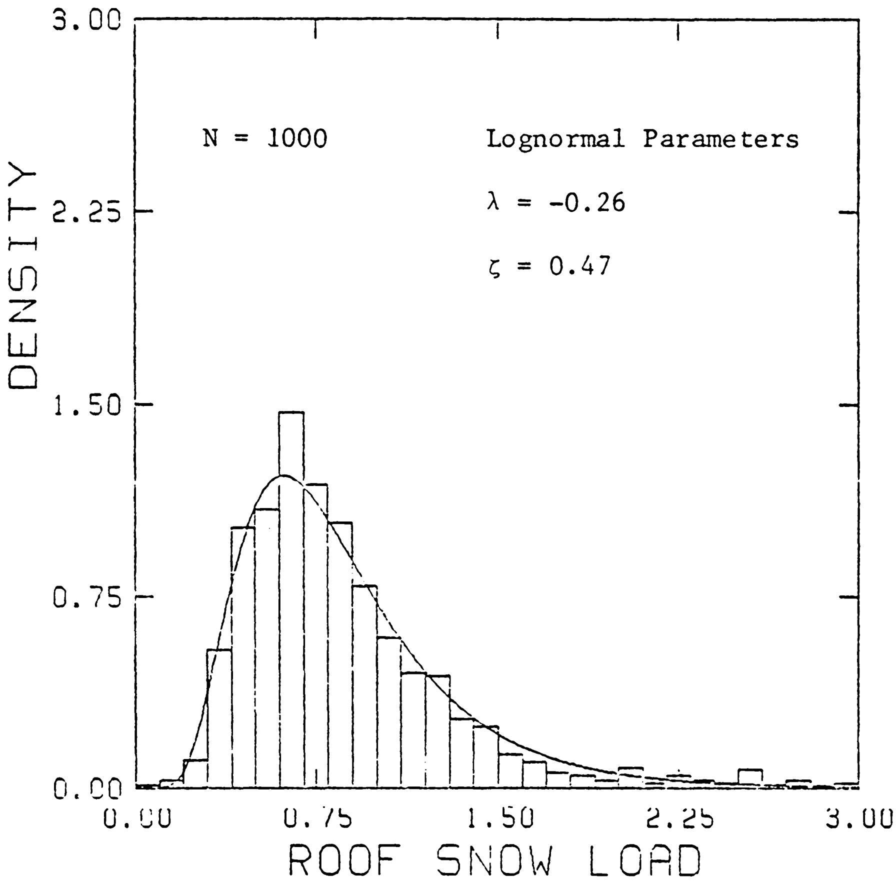

Figure 3.8 A histogram of the normalized maximum lifetime roof snow load for Columbia, MO calculated from the results of a Monte Carlo simulation is given with an overlay of the estimated lognormal density function.

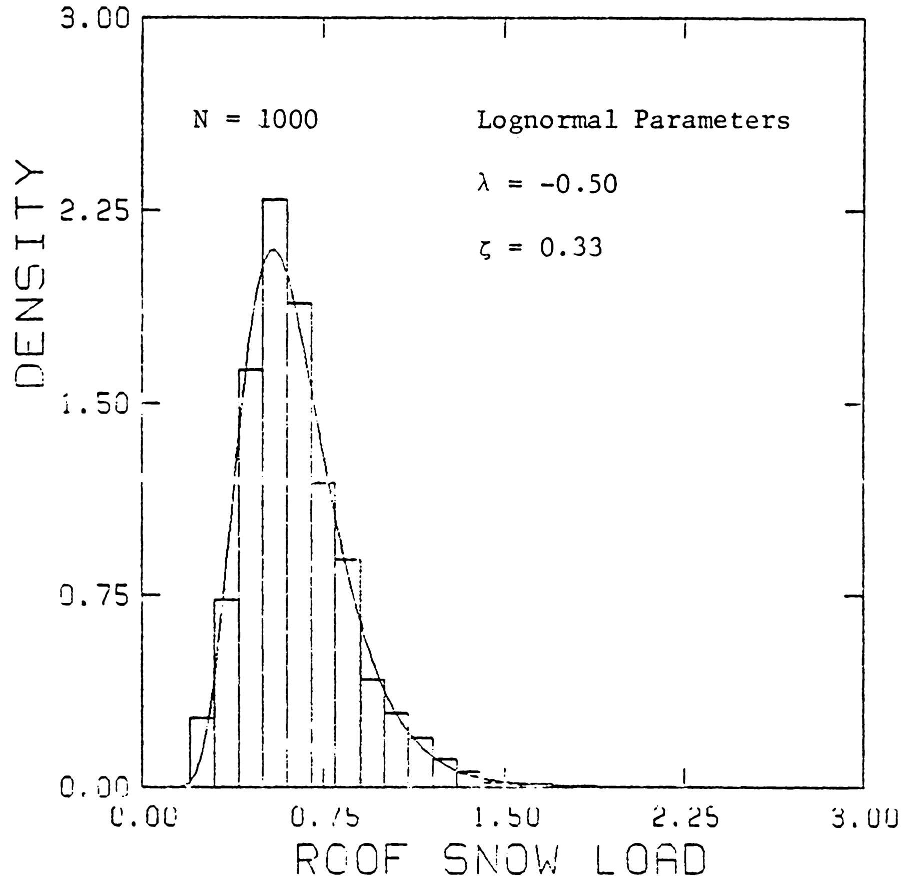

Figure 3.9 A histogram of the normalized maximum lifetime roof snow load for Great Falls, MN calculated from the results of a Monte Carlo simulation is given with an overlay of the estimated lognormal density function.

lognormal distribution might provide a good fit. Accordingly, the lognormal distribution was overlayed on the normalized maximum lifetime roof snow load histograms shown in Figure 3.2 through 3.9. The calculated parameters of each lognormal distribution are listed by site in Table 3.2.

The final step was to measure the goodness of fit of the resulting lognormal functions. Visual inspection of the distributions indicated a good fit. In addition to the visual test, a Kolmogorov-Smirnoff goodness of fit test was conducted for each site. The lognormal function was rejected only once at the 20 percent level of significance. The K-S statistics are given in Table 3.2 for each site.

Since a modified table of K-S critical values was available (14), it was also used to determine the adequacy of the lognormal distribution. Based on the explanation given in the review of literature, the modified K-S critical values are smaller than the critical values of the non-parametric K-S test listed in column 5 of Table 3.2. The significance levels of the modified K-S goodness of fit tests for each site are listed in Table 3.2. A significance level or p-value indicates the weight of evidence for rejecting a null hypothesis. If the p-value is small, the probability is great for rejecting the null hypothesis. For this case, the null hypothesis is that the distribution of normalized maximum lifetime roof snow load is lognormal.

According to Ott (48), statistical significance of a particular level does not dictate practical significance. Furthermore, he states that after determining the statistical level of significance of a test,

Table 3.2 The tabulated values are the estimated lognormal parameters, non-parametric K-S values, and p-values based on modified K-S critical values for the normalized maximum lifetime roof snow load data. The hypothesized distribution is rejected if the computed value of the K-S statistic exceeds the critical value or if the p-value is less than 0.01. A sample size, N = 1000 was utilized for each site.

Site	λ	ζ	K-S (computed)	K-S[1] (critical)	p[2]
Green Bay, WI	-0.46	0.41	0.333	0.034	$p<0.01$
Rochester, NY	-0.30	0.35	0.028	0.034	$0.05<p<0.10$
Boston, MA	-0.49	0.34	0.033	0.034	$p<0.01$
Detroit, MI	-0.43	0.36	0.026	0.034	$0.05<p<0.10$
Omaha, NB	-0.44	0.41	0.030	0.034	$0.01<p<0.05$
Cleveland, OH	-0.78	0.35	0.035	0.034	$p<0.01$
Columbia, MO	-0.26	0.47	0.028	0.034	$0.05<p<0.10$
Great Falls, MN	-0.50	0.33	0.025	0.034	$0.10<p<0.15$

[1] Critical non-parametric K-S value at 20 percent level of significance.

[2] p-value for the modified K-S critical values as given by Crutcher (14).

the experimenter should always consider the practical significance of the finding. In three cases, the level of significance of the goodness of fit test was less than 0.01. Based on this, the lognormal distribution was formally rejected 3 out of a possible 8 times. However, none of the two parameter distributions tested by Ellingwood, et al. (17) fitted over the entire range. Therefore, the lognormal distribution was chosen as the best distribution of normalized maximum lifetime roof snow load. The parameters as listed in Table 3.1 are

$$\bar{S}/S_n = 0.69 \tag{3.4}$$

$$\Omega_S = 0.44 \tag{3.5}$$

3.3 Development of the Maximum Lifetime Floor Live Load Distribution

The temporal and spatial aspects of the maximum lifetime floor live load render characterization of that load extremely difficult. Live load surveys are available which describe the sustained load with a good degree of accuracy. The transient load, however, has only been quantified twice in load surveys. Both of these surveys were based on the occupants' memory of the largest unusual personnel load event in the dwelling and therefore reflect a large degree of uncertainty. Unfortunately, the sustained and transient load surveys cannot be directly combined and extrapolated to determine the maximum lifetime floor live load. Nevertheless, several models as described in the review of literature are available which model the maximum lifetime floor live load based on a stochastic analysis.

The models usually assume that the maximum lifetime floor live load distribution is extreme value type I or no assumption regarding the distribution is given. Based on recommendations from current literature (1,13,16,17,21), the maximum lifetime floor live load is assumed to follow an extreme value type I distribution. The parameters of the maximum floor live load distribution are not so apparent.

Based on the discussion of live loads in the review of literature, no single set of parameters appear to be the "best" set of parameters. Chalk and Corotis (4) and Sentler (54) developed live load model parameters based on the stochastic nature of loads. Chalk and Corotis (11) calculated a mean of 36 psf for renter occupied dwellings and a mean of 39 psf for owner occupied dwellings. Both renter and owner occupied residences had a calculated standard deviation equal to 7.7 psf. Sentler's results (54), adopted by the Joint Committee on Structural Safety (38), were a mean of 20.89 psf and a standard deviation of 8.35 psf. Also, Canadian researchers (1,59) have utilized a ratio of mean maximum lifetime floor live load to nominal floor live load equal to 0.70 with an assumed coefficient of variation equal to 0.30.

As described in the review of literature, Suddarth, et al. (61) had studied the effects of concomitance of bending and tension stresses in lumber. The load distribution was lognormal. Also, the effect of truncating the load distribution in the right tail was studied. Employing a differential reliability analysis, Suddarth, et al. (61) found that the engineering design result was the same.

It is believed, therefore, that the choice of parameters of the floor live load distribution may not be critical if contrasting data sets are analyzed utilizing the differential reliability technique. The calculated probabilities of failure may vary significantly depending on the parameters chosen; however, the calculated k factors may be the same from an engineering design standpoint. In order to study the sensitivity of the differential reliability analysis to the parameters of the load distribution, and in order to determine a recommended set or sets of floor live load parameters, all three sets of above mentioned maximum lifetime floor live load parameters will be utilized in the differential reliability analyses to follow. Also, a fourth set of floor live load parameters will be developed from the Chalk and Corotis model (11). Existing live load data describing the sustained and transient loads will be utilized in the parameter analysis. A thorough discussion of the development of the fourth set of parameters follows preceded by a short discussion of each of the three presently available sets of parameters.

3.3.1 Live Load Parameters Calculated by Chalk and Corotis

As previously mentioned, Chalk and Corotis (11) calculated a mean maximum lifetime floor live load for owner and renter occupied residences. In their discussion, Chalk and Corotis (11) note that the difference between the two occupancy types is the average length of duration of occupancy. Therefore, owner occupied and renter occupied live loads were developed separately. However, it was noted that such a division might be impractical due to the changing nature of housing use. Hence, for this analysis, the parameters for owner occupied residences is combined with the parameters for renter occupied residences. The combined mean and standard deviations are

$$\bar{L} = 37.5 \text{ psf} \tag{3.6}$$

$$s_L = 7.7 \text{ psf} \tag{3.7}$$

The proposed ANSI load standard A58.1-81 cites the nominal floor live load as 40 psf. Therefore, the combined floor live load parameters normalized with respect to the nominal floor live load are

$$\bar{L}/L_n = 0.94 \tag{3.8}$$

$$\Omega_L = 0.21 \tag{3.9}$$

It is important to note several analytical aspects of the model employed by Chalk and Corotis. First, the sustained load parameters were a combined average of the parameters from five live load surveys. Only one survey, Dunham (15) was conducted for residences in the United States; the rest (37, 40, 49, 53) were residential surveys conducted in Europe. Secondly, Chalk and Corotis (11) did not feel survey information concerning the transient load was sufficient to warrant its use. Therefore, a model developed by Pier and Cornell (50) was utilized to determine the transient load parameters. Based on the combined sustained load parameters and the modeled transient load parameters, the maximum lifetime floor live load parameters were developed for owner and renter occupied dwellings.

3.3.2 Live Load Parameters Calculated by Sentler

The Joint Committee on Structural Safety (38) recommended the use of Sentler's floor live load parameters for dwellings. As previously stated in the review of literature, the parameters calculated by Sentler (54) are

$$\bar{L} = 20.89 \text{ psf} \tag{3.10}$$

$$s_L = 8.35 \text{ psf} \tag{3.11}$$

Normalizing the mean by the nominal floor live load, L_n equal to 40 psf and calculating the coefficient of variation results in the parameters

$$\bar{L}/L_n = 0.52 \tag{3.12}$$

$$\Omega_L = 0.40 \tag{3.13}$$

The sustained load survey utilized by Sentler was the background survey for Sentler (53). The survey by Sentler is described in the review of literature. Sentler felt that the transient load survey by Paloheimo (49) was adequate to describe the transient load parameters. The survey by Paloheimo is also described in the review of literature. Sentler (54) noted that while accurate estimates of the actual maximum lifetime floor live load parameters might not be determined from the model, the model was still useful as an aid in understanding the total load process and as a guide to code authorities.

3.3.3 Live Load Parameters Assumed by Canadian Researchers

As mentioned in the review of literature, Canadian researchers (1,59) have advocated that the ratio of mean maximum lifetime floor live load to nominal floor live load be taken as 0.70. A coefficient of variation of 0.30 was assumed. In Canada, the lifetime of a structure is assumed to be 30 years.

These values are somewhat suspect in that the mean maximum lifetime load value was not derived from an analysis which accounted for the effect of the extraordinary load. Also, the coefficient of variation was assumed based only on sustained load survey results. However, it is believed that the utilization of these parameters may provide useful knowledge concerning the sensitivity of the differential reliability analysis to the load distribution.

Since the lifetime for the above parameters is 30 years, a transformation is necessary to remain consistent with the accepted definition of structural life in the United States. Since the load distribution is extreme value type I, equations 2.10 and 2.11 can be utilized to calculate the parameters in terms of the type I parameters, α and β. The calculated parameters are

$$\alpha_{30} = 0.16 \tag{3.14}$$

$$\beta_{30} = 0.61 \tag{3.15}$$

Then, equations 2.12 and 2.13 can be utilized to transform the 30 year lifetime type I parameters. The resulting 50 year lifetime extreme value type I parameters are

$$\alpha_{50} = 0.16 \tag{3.16}$$

$$\beta_{50} = 0.69 \tag{3.17}$$

and the resulting mean and coefficient of variation are

$$\bar{L}/L_n = 0.78 \tag{3.18}$$

$$\Omega_L = 0.27 \tag{3.19}$$

3.3.4 Live Load Parameters Developed Utilizing the Chalk and Corotis Model

As previously discussed, Chalk and Corotis calculated live load parameters based on a pooled sustained load and a modeled transient load. It was felt than an alternative set of parameters could be calculated by combining transient load survey information and the results from four individual sustained load surveys in the Chalk and Corotis model. In this manner, four sets of floor live load parameters could be calculated; then an average of these parameters could be obtained and utilized in the differential reliability analysis. The four sustained load surveys selected have been described in the review of literature. All four surveys were used by Chalk and Corotis (11) in the pooled estimate of the sustained floor live load. A description of the input data essential for this analysis follows.

First, live load surveys describing the parameters of the sustained load, L_i, are required. The four surveys that are utilized in the analysis are Johnson (37), Karmen (40), Paloheimo (49) and Sentler (53). The parameters are listed in columns two and three of Table 3.3. Secondly, estimates of the transient load parameters are required. Two surveys are available which describe the transient floor live load on a structure, Johnson (37) and Paloheimo (49). As described in the review of literature the transient load model developed by Pier and Cornell (50) gives good results when compared to the mean of the above transient load surveys; therefore, the transient load results calculated by Chalk and Corotis (11) using the Pier and Cornell model will also be employed. The parameters of the transient load for the three cases are

Table 3.3 The statistical parameters from available live load surveys are tabulated. The moments of L_{e_1} are the combined moments for renter occupied and owner occupied dwellings from data noted by Chalk and Corotis (11).

Survey	Sustained Load		Transient Load		$L_{e_1}^*$	
	m_{L_i}	σ_{L_i}	m_e (psf)	σ_e (psf)	$m_{L_{e_1}}$ (psf)	$\sigma_{L_{e_1}}$ (psf)
Chalk and Corotis (11)			6.1	6.6	16.4	5.7
Johnson (37)	5.18	2.07	6.1	3.1		
Karmen (40)	11.35	4.02				
Paloheimo (49)	4.93	2.05	5.7	1.4		
Sentler (53)	5.35	2.32				

$^*L_{e_1}$ = the maximum extraordinary load occurring during the maximum sustained load listed in columns four and five of Table 3.3.

The sustained load and the transient load are only two components required in the analysis. The moments of maximum lifetime sustained load, L_s and the maximum lifetime extraordinary load, L_e need to be identified. Also, the maximum extraordinary load occurring during the maximum sustained load denoted as L_{e_1} is required. Chalk and Corotis list the only available parameters concerning L_{e_1}. For owner occupied residences, the moments are

$$m_{L_{e_1}} = 20.8 \text{ psf} \tag{3.20}$$

$$\sigma_{L_{e_1}} = 6.3 \text{ psf} \tag{3.21}$$

For renter occupied residences, the moments are

$$m_{L_{e_1}} = 11.9 \text{ psf} \tag{3.22}$$

$$\sigma_{L_{e_1}} = 5.0 \text{ psf} \tag{3.23}$$

Following Ott (48), the moments are averaged for use in the analysis assuming equal sample sizes. The resulting moments of the maximum extraordinary load are listed in column six and seven of Table 3.3.

As mentioned in the previous paragraph, the moments of maximum lifetime sustained load and maximum lifetime extraordinary load need to be evaluated. Wen (67) derived an analytical solution for the mean and standard deviation of the maximum of a family of independent gamma variables. If L_{max} represents the maximum lifetime sustained or maximum lifetime extraordinary load, then

$$E[L_{max}] = m[1 + \delta(C_1 + 0.5772C_2)] \tag{3.24}$$

$$\sigma_{L_{max}} = \frac{m\delta\pi C_2}{6} \tag{3.25}$$

where

$$C_1 = \frac{6}{\pi} \ln(N)$$

$$C_2 = \frac{1 + C_1\delta}{2\delta + C_1}$$

m = the mean value of the sustained or transient load

δ = the coefficient of variation of the sustained or transient load

N = the mean number of independent repetitions

As mentioned in the review of literature, the gamma distribution is favored as the distribution of the sustained load and transient load. Chalk and Corotis (11) compared the above method with more exact solutions. They found that for Wen's method the means are within 10 percent and are always conservative and the standard deviations are close. Therefore, Chalk and Corotis (11) adopted Wen's method in their analysis.

Utilizing equations 3.24 and 3.25, the moments of maximum sustained load and maximum extraordinary load can be calculated. As mentioned in the review of literature, Chalk and Corotis note that the average duration of occupancy for owner occupied dwellings is 10 years. Based on a 50 year design life, the number of independent repetitions for sustained load is N equal to 5. For the four surveys listed in Table 3.3, the moments of the maximum lifetime sustained load were calculated and are tabulated in Table 3.4.

Chalk and Corotis (11) note that the frequency of extraordinary events in residential units can, at best, only be estimated. They suggest that one or two extraordinary occurrences per year might be typical. Therefore, for the extraordinary load, N was calculated to be equal to 50 based on the assumption that one extraordinary load event occurred per year. The moments of the maximum lifetime extraordinary load are calculated using equations 3.24 and 3.25 for the three transient loads listed in Table 3.3. The results are tabulated in Table 3.4. Since all sustained load surveys utilized did not have an accompanying transient load survey, the moments of the maximum lifetime extraordinary load were pooled for use in the analysis. The results are tabulated at the bottom of columns four and five in Table 3.4.

As described in the review of literature, the maximum lifetime floor live load can occur by one of three different modes. Case I is the sum of the maximum lifetime sustained load, L_s, and the largest extraordinary load, L_{e_1}, occurring during the duration of the maximum lifetime sustained load. Case II is the largest extraordinary load, L_e, during the life of the structure plus the sustained load, L_i, acting at the time of this extraordinary load. Case III is the sum of the maximum lifetime extraordinary load, L_e, and the maximum lifetime sustained load, L_s. Employing equations 2.32 and 2.33 the moments for Case I are calculated utilizing the moments listed in the second and third columns of Table 3.4 and the

Table 3.4 The tabulated values are the maximum lifetime sustained load and maximum lifetime extraordinary load parameters calculated by the method derived by Wen (67) for the survey loads tabulated in Table 3.3. The pooled parameters listed at the bottom of the table are taken as the parameters of the maximum lifetime extraordinary load utilized in the floor live load analysis.

Survey	Maximum Lifetime Sustained Load N = 5		Maximum Lifetime Extraordinary Load N = 50	
	m_{L_s} (psf)	σ_{L_s} (psf)	m_{L_e} (psf)	σ_{L_e} (psf)
Chalk and Corotis (11)			29.4	7.0
Johnson (37)	8.7	1.9	16.7	2.5
Karmen (40)	16.5	4.4		
Paloheimo (49)	8.4	1.9	10.4	0.9
Sentler (53)	9.2	2.2		
AVERAGE			18.8	4.3

moments listed in the sixth and seventh columns of Table 3.3. The case I moments are tabulated by survey in the third and fourth columns of Table 3.5. For the case II moments, equations 2.34 and 2.35 are employed. The moments for case II are tabulated by survey in columns five and six of Table 3.5. Finally, the moments for case III are calculated utilizing equations 2.36 and 2.37. The moments of the maximum sustained load needed to solve equations 2.36 and 2.37 are tabulated in the second and third columns of Table 3.4. The moments of the maximum extraordinary load utilized in equations 2.36 and 2.37 are the average moments listed at the bottom of Table 3.4. The resulting moments for case III are tabulated in columns seven and eight of Table 3.5. In order to calculate the maximum lifetime load utilizing equation 2.38, the mean of the sustained load is also needed. These parameters, tabulated in Table 3.3 are reproduced in the second column of Table 3.5. Also, the average number of years between occupany changes $E(\tau)$, is assumed to be 10 years as recommended by Chalk and Corotis (11).

Now the parameters of the maximum lifetime floor live load can be calculated utilizing equation 2.38 and the moments listed in Table 3.5. Equation 2.38 calculates the cumulative distribution function of the maximum lifetime floor live load. Since the distribution of maximum lifetime floor live load is assumed to be the extreme value type I distribution, the mean of the distribution corresponds approximately to the 57-th cumulative percentile level. The corresponding standard deviation can be found by computing the load at a cumulative level equal to one standard deviation above the mean, a cumulative level of approximately 0.856.

Table 3.5 The tabulated values are the mean of the sustained load survey and the moments of the three cases for each survey. These values are utilized in equation 2.38 to calculate the parameters of the floor live load. The moments of the three cases are calculated utilizing equations 2.32 through 2.37.

Survey	Sustained Load	Case I		Case II		Case III	
	m_L (psf)	m_I (psf)	σ_I (psf)	m_{II} (psf)	σ_{II} (psf)	m_{III} (psf)	σ_{III} (psf)
Johnson (37)	5.18	25.1	6.0	18.8	4.3	27.5	4.7
Karmen (40)	11.35	32.9	7.2	18.8	4.3	35.3	6.2
Paloheimo (49)	4.93	24.8	6.0	18.8	4.3	27.2	4.7
Sentler (53)	5.35	25.6	6.1	18.8	4.3	28.0	4.8

The mean and standard deviation for each survey listed in Table 3.5 can be calculated in the following manner. First, the type I parameters are calculated for the moments of the three cases listed in Table 3.5 utilizing equations 2.10 and 2.11. Substituting the type I parameters into equation 2.38 along with the appropriate mean sustained load, m_{L_i} and $E(\tau)$ of 10, the mean of the maximum lifetime floor live load can be iteratively calculated by substituting $F_{L_t}(\ell) = 0.57$. A value equal to one standard deviation above the mean can be calculated by setting $F_{L_t}(\ell) = 0.856$. The standard deviation is then this value minus the mean value. A computer program to solve equation 2.38 iteratively was written in Fortran IV language and is listed in Appendix B. The program was tested using appropriate input values taken from Chalk and Corotis (11). The results of the test listed in Table 3.6 are the same as the results obtained by Chalk and Corotis. Therefore, the verified computer program was utilized to determine the mean and standard deviation for each of the four survey loads. The results and the combined parameters are listed in Table 3.7. Normalizing the combined mean by the nominal floor live load, L_n equal to 40 psf results in the fourth set of live load parameters

$$\bar{L}/L_n = 0.73 \tag{3.26}$$

$$\Omega_L = 0.19 \tag{3.27}$$

Four sets of parameters are now available to describe the maximum lifetime floor live load distribution. The distribution of floor live

Table 3.6 The tabulated values are a comparison between the results tabulated by Chalk and Corotis (11) and the results calculated by the computer program listed in Appendix B.

Model Results	Owner Occupied		Renter Occupied	
	m_{L_t} (psf)	σ_{L_t} (psf)	m_{L_t} (psf)	σ_{L_t} (psf)
Listed by Chalk and Corotis	39.0	7.0	36.0	7.0
Obtained from computer program	38.6	6.9	36.0	6.9

Table 3.7 The tabulated values are the results of the floor live load analysis utilizing the Chalk and Corotis model (11). The cumulative level values are calculated employing the computer program listed in Appendix B. The average parameters are the final results of the floor live load analysis.

Survey	Cumulative Level 0.57 Mean (psf)	Cumulative Level 0.856 (psf)	Standard Deviation (psf)	Coefficient of Variation
Johnson (37)	27.4	32.5	5.1	0.1861
Karmen (40)	27.1	32.2	5.1	0.1861
Paloheimo (49)	24.8	41.0	6.2	0.1782
Sentler (53)	27.7	33.0	5.3	0.1913
AVERAGE	29.3		5.4	0.19

load is assumed extreme value type I in all four cases. The load parameters are derived from the following. Load A is the combination of renter occupied and owner occupied floor live load parameters from Chalk and Corotis. Load B is the set of parameters calculated by Sentler and recommended by the Joint Committee on Structural Safety. Load C is the set of parameters assumed by the Canadian researchers adjusted to a 50 year lifetime. Load D is the result of an analysis utilizing the Chalk and Corotis model. The preceding floor live load parameters and the roof snow load parameters which are utilized in the differential reliability analysis are tabulated in Table 3.8. The dead load is also tabulated in Table 3.8.

3.4 Application of the Differential Reliability Technique Utilizing a Dead Plus Snow Load Combination

In order to demonstrate the application of a dead load plus snow load combination in a differential reliability analysis, two contrasting lumber data sets are utilized. The first data set, taken from Green (26), consists of two samples of 2" x 8" No. 2 Douglas fir lumber which was tested in bending for modulus of rupture. One sample was green, in other words, the moisture content was above 30 percent for all lumber in the sample. The dry sample was at 19 percent maximum moisture content. Both the green and dry lumber distributions are described by the 3-parameter Weibull distribution. The Weibull parameters and the calculated fifth percentile are shown in Table 3.9. Figure 3.10 accentuates the differences in the green and dry lumber samples. The effect of moisture content on bending strength can be demonstrated utilizing a differential

Table 3.8 The tabulated values are the load distributions and parameters which are utilized in the differential reliability analyses.

Load	Distribution	$\bar{X}/X_n$	Ω_X
Dead	Lognormal	varies	0.10
Snow	Lognormal	0.69	0.44
Floor Live			
Load A	Type I	0.94	0.21
Load B	Type I	0.52	0.40
Load C	Type I	0.78	0.27
Load D	Type I	0.73	0.19

Table 3.9 The tabulated values are the Weibull parameters of the modulus of rupture and the calculated fifth percentile of 2" x 8" green and dry No. 2 Douglas fir.

Sample	Weibull Parameters			Fifth Percentile
	μ (ksi)	σ (ksi)	η	(ksi)
Green	0.903	4.309	2.586	2.269
Dry	1.304	4.597	1.845	2.223

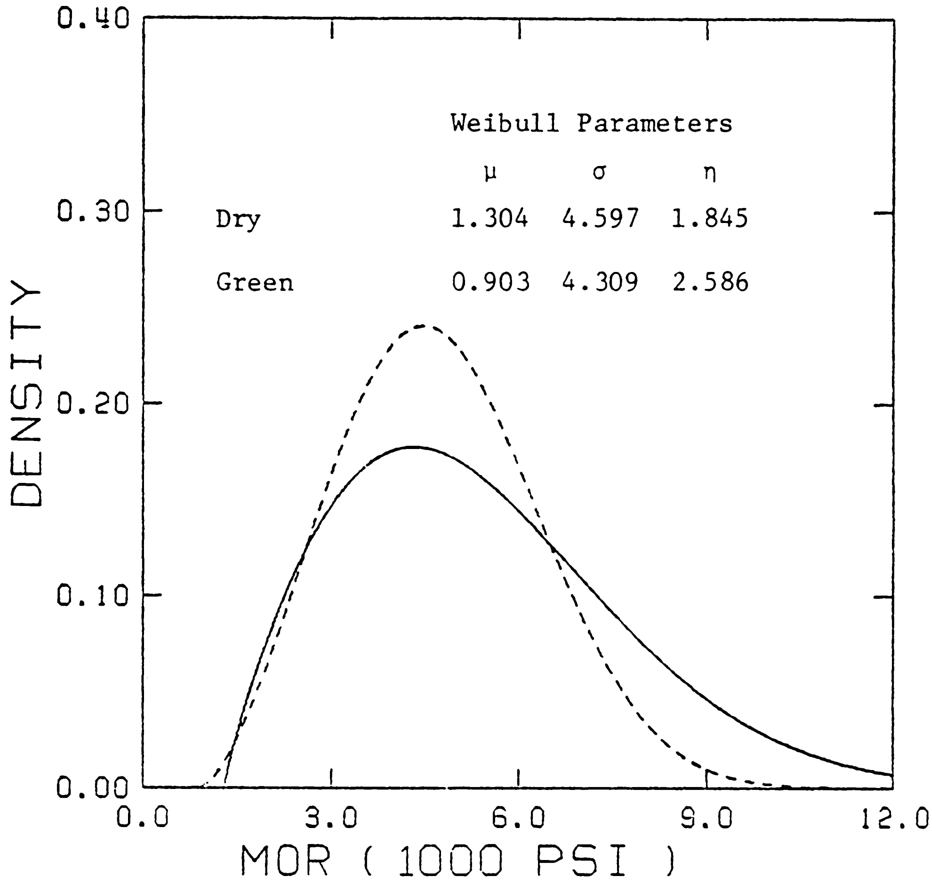

Figure 3.10 The 3-parameter Weibull functions describe green and dry 2" x 8" No. 2 Douglas fir lumber utilized in the analysis of a moisture adjustment factor. The dry lumber, represented by the solid line, is the material used to establish the benchmark safety level. The Weibull parameters, μ and σ are given in ksi.

reliability analysis involving the green and dry lumber data.

The second set of contrasting lumber data is taken from Hoyle, et al. (33). The set consists of two grades of lumber, Select Structural and No. 1. Both grades of lumber are 2" x 8" Hem-Fir which has been tested in bending. All lumber sets analyzed by Hoyle, et al. (33) had been adjusted to 12 percent average moisture content (23). The data sets were fitted to a 3-parameter Weibull distribution (33). The Weibull parameters and the calculated fifth percentile are tabulated in Table 3.10. Figure 3.11 accentuates the differences between the grades of Select Structural and No. 1 Hem-Fir lumber.

Equal safety is implied between design values of lumber grades, sizes and species as tabulated in the Supplement to the National Design Specification (47). A differential reliablity analysis can demonstrate whether lumber of differing sizes or grades exhibit the same safety in similar design applications. Hence, an application of the differential reliability technique to lumber data of different grades can highlight the relative safety of one set of lumber versus another set of lumber.

When conducting a differential reliability analysis, the load and resistance distributions should reflect a certain design situation. Since both samples consist of 2" x 8" lumber suitable for utilization as rafters, the design assumed is that of a low slope roof utilizing rafter construction. A typical load combination for this design is dead plus snow load.

The conventional design load for a dead plus snow load combination is nominal dead load plus nominal roof snow load if the resistance

Table 3.10 The tabulated values are the Weibull parameters of the modulus of rupture and the calculated fifth percentile of 2" x 8" Select Structural and No. 1 Hem-Fir at 12 percent average moisture content.

Sample	Weibull Parameters			Fifth Percentile
	μ (ksi)	σ (ksi)	η	(ksi)
Select Structural	1.526	6.450	2.628	3.609
No. 1	1.352	4.215	1.713	2.096

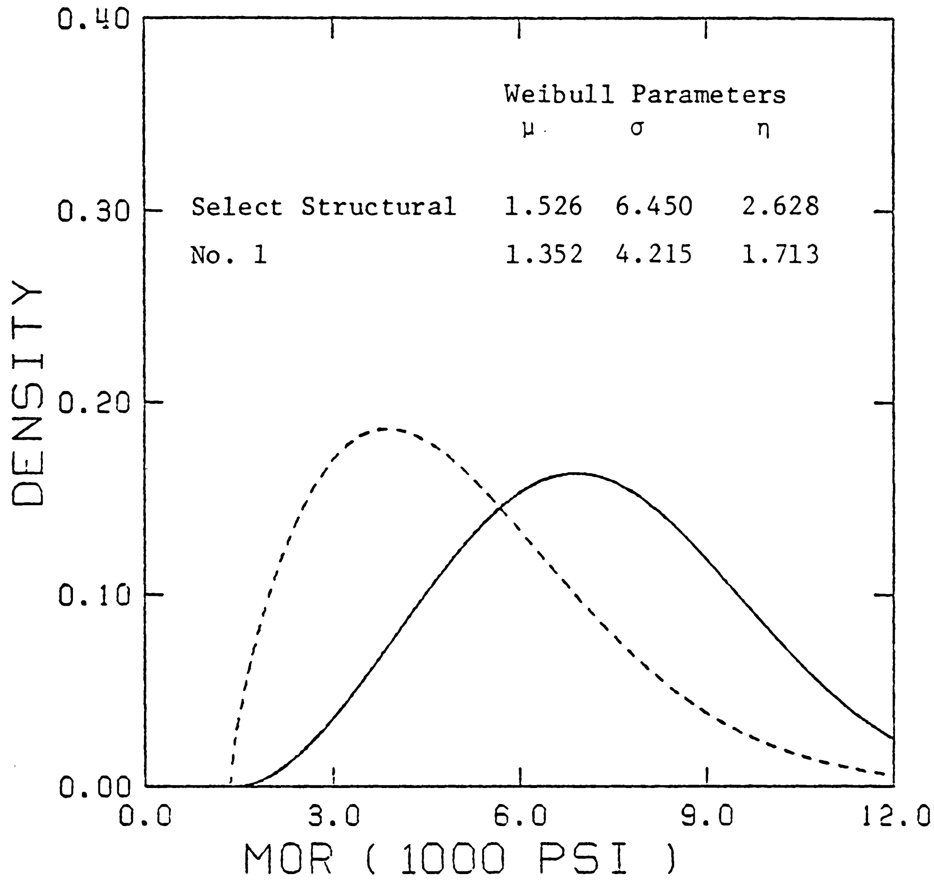

Figure 3.11 The 3-parameter Weibull functions describe Select Structural and No. 1 2" x 8" Hem-Fir lumber utilized in the determination of relative safety of the two grades. The Select Structural grade, represented by the solid line, serves as the reference material. The Weibull parameters μ and σ are given in ksi.

variable is appropriately adjusted by the correct load duration factor. The nominal design dead load for a low slope roof is 10 psf (32). In the moderate snow region of the United States, a typical nominal design roof snow load can be calculated by the method outlined in the proposed ANSI standard A58.1-81 (4) to be 20 psf. This reflects a probability of exceeding the design snow load once every fifty years. Therefore the total nominal design load is the sum of the nominal loads or 30 psf.

3.4.1 Determination of a Moisture Adjustment Factor

For a typical low slope roof using rafter design, 2" x 8" Douglas fir rafters sixteen inches on center with ½ inch plywood sheathing and asbestos shingles, a dead load of 5.7 psf was calculated. This was assumed to be the mean dead load. The normalized statistics of the dead load based on Table 3.8 and a nominal dead load of 10 psf are

$$\bar{D}/D_n = 0.57 \tag{3.28}$$

$$\Omega_D = 0.10 \tag{3.29}$$

The maximum lifetime roof snow load statistics taken from Table 3.8 are

$$\bar{S}/S_n = 0.69 \tag{3.30}$$

$$\Omega_S = 0.44 \tag{3.31}$$

Utilizing the method outlined in the review of literature, the probability of failure analysis is simple when the load and resistance distributions are known. In this example, the distribution of the sum of two lognormal variates cannot be derived to a form useful in the reliability analysis. However, noting that the coefficient of variation of the maximum lifetime roof snow load is large compared to the coefficient of variation of dead load, the parameters of the maximum lifetime total load may be approximated by adding the means and variances of the two lognormal distributions. The resulting distribution of maximum total load is assumed to be lognormal. Therefore, the parameters are calculated by

$$\mu_T = \mu_D + \mu_S \tag{3.32}$$

$$\sigma_T^2 = \sigma_D^2 + \sigma_S^2 \tag{3.33}$$

where

μ_T = the average maximum lifetime load

μ_D = the average dead load

μ_S = the average maximum lifetime roof snow load

σ_T^2 = the variance of the maximum lifetime total load

σ_D^2 = the variance of the dead load

σ_S^2 = the variance of the maximum lifetime roof snow load

Using equations 3.32 and 3.33, the second moment parameters of the total load are expressed as

$$\mu_T = (\bar{D}/D_n)\ (D_n) + (\bar{S}/S_n)\ (S_n) \tag{3.34}$$

$$= 0.57(10) + 0.69(20) = 19.5 \text{ psf}$$

$$\sigma_T^2 = [\Omega_D(\bar{D}/D_n)\ (D_n)]^2 + [\Omega_S(\bar{S}/S_n)\ (S_n)]^2 \tag{3.35}$$

$$= [0.10(0.57)\ (10)]^2 + [0.44(0.69)\ (20)]^2$$

$$= 37.19\ (\text{psf})^2$$

The addition of the above variances implies independence between roof snow load and dead load which is believed to be a good assumption.

In order to test the above assumptions, a simulation study was conducted. First, random lognormal deviates representing dead load and random lognormal deviates representing snow load were generated using their respective parameters as employed in equations 3.34 and 3.35. These deviates were summed and a relative frequency histogram constructed. Next, a lognormal function described by the combined parameters calculated above, was overlayed on the histogram as shown in Figure 3.12. The visual test indicates no obvious lack of fit.

As a final test, a Kolmogorov-Smirnoff test was performed at a 20 percent level of significance. The K-S test statistic, non-parametric K-S critical value and p-value for the modified K-S test described in the review of literature are tabulated in Table 3.11. The distribution parameters are also listed in the table. Based on the test results, the lognormal distribution was accepted as a suitable distribution of dead plus snow load. Since the lognormal distribution adequately models

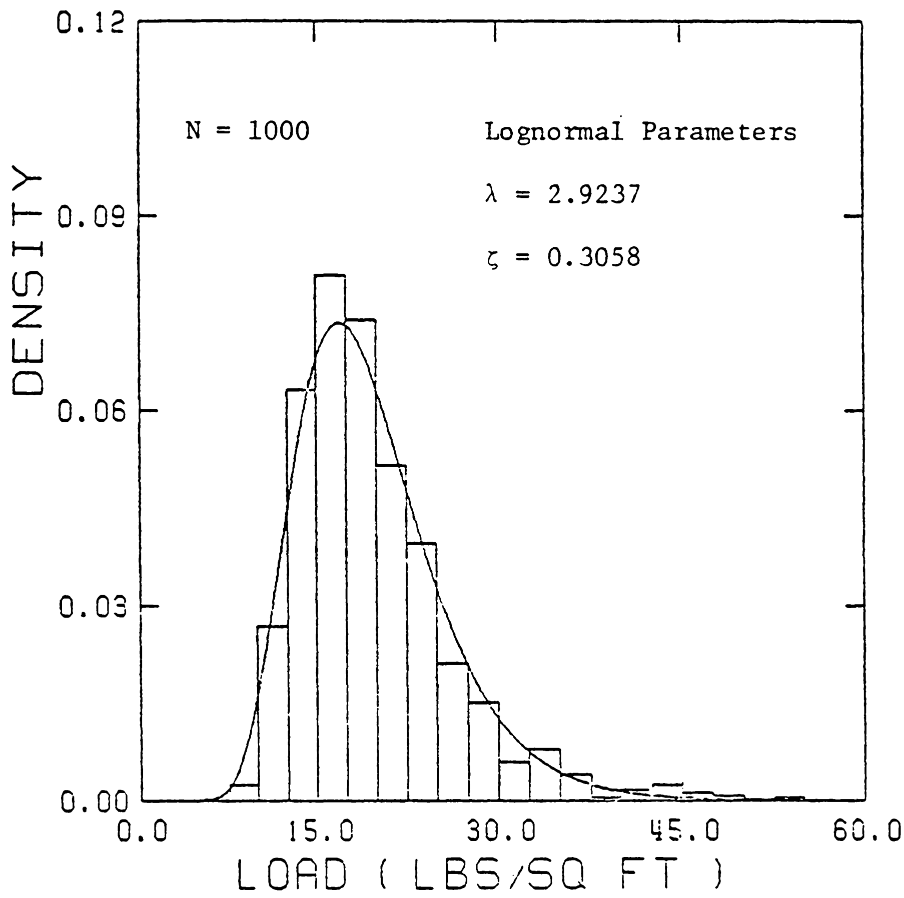

Figure 3.12 The histogram of the sum of two random lognormal variables representing the dead load and the maximum lifetime roof snow load, respectively of a low slope roof using rafter design is shown. The parameters of the overlayed lognormal distribution were calculated from the sum of the second moment parameters of the two lognormal distributions representing dead and snow load. The distribution fit the histogram very well.

Table 3.11 The tabulated values are the estimated lognormal parameters of the distribution of the dead plus snow load combination and the K-S test values. The second moment parameters of the dead plus snow load combination were calculated using equations 3.34 and 3.35. If the computed K-S statistic is greater than the critical K-S value then the distribution is rejected at the particular level of significance. The p-value indicates the significance when using the modified K-S test.

Lognormal Parameters λ	ζ	K-S (computed)	K-S[1] (critical)	Significance Level
2.9237	0.3058	0.0309	0.0338	$0.01<p<0.05$

[1] K-S critical value at a 20 percent level of significance.

the load combination, the probability calculations can now be conducted once the load and resistance distributions are expressed in similar units.

For this reliability analysis, the units are expressed as pounds per square inch. This is favored because resistance or strength data is usually expressed in these units. Also, it is relatively simple to convert the units of the load distribution from psf to psi.

To provide compatible units for the load distribution, two basic steps are employed. First, the design load should be related to the design resistance. As mentioned previously, in conventional design situations, the design load is assumed equal to the design resistance. In reliability analysis, the normalized mean value, $\bar{X}/X_n$, is utilized to relate the load distribution to the resistance distribution. Since setting design load equal to design resistance is engineering practice, the nominal design load is equal to the allowable design resistance when each is expressed in identical units. The load distribution will then be shifted by the parameter $\bar{X}/X_n$ because the mean value represents a more realistic load applied to a structure rather than the nominal load. The second step is to represent in proportion to the total load each type of load involved in the load combination. The total load is the total nominal design load since adjustments for distribution have already been made by the previous factor.

For the dead plus snow load combination, the equations for the parameters of total load are

$$\mu_T = \frac{D_n}{T_n} (\bar{D}/D_n) F_b + \frac{S_n}{T_n} (\bar{S}/S_n) F_b \qquad (3.36)$$

$$\Omega_T = \frac{\left[(\mu_D \Omega_D)^2 + (\mu_S \Omega_S)^2\right]^{\frac{1}{2}}}{\mu_T} \qquad (3.37)$$

where

μ_T = the mean total lifetime load expressed in psi

Ω_T = the coefficient of variation of the total lifetime load

D_n = the nominal dead load expressed in psf

S_n = the nominal snow load expressed in psf

$T_n = D_n + S_n$ = the total nominal load expressed in psf

$\bar{D}/D_n$ = the normalized mean of the dead load distribution

$\bar{S}/S_n$ = the normalized mean of the maximum lifetime roof snow load distribution

Ω_D = the coefficient of variation of the dead load

Ω_S = the coefficient of variation of the snow load

F_b = the adjusted allowable design value

The adjusted allowable design strength, F_b, is calculated from the contrasting data sets. The calculated fifth percentile from the reference data set is divided by the general adjustment factor of 2.1 and multipled by 1.15 because a 15 percent increase in allowable stress is used for duration of load when a snow load is applied. The adjusted fifth percentile historically has been the allowable design value. For this reason, the fifth percentile of the strength distribution is utilized to position the load distribution.

Only one value of F_b needs to be calculated in a moisture analysis. Green and dry lumber denote different physical states of the same lumber population. Since F_b is the factor in equations 3.36 and 3.37 which positions the load distribution in relation to the resistance distribution, F_b determines which load distribution is to be utilized. Either the green or dry lumber must be chosen for a base reliability level which then defines the data set from which F_b is calculated. An adjusted F_b was calculated from the dry lumber set. The load parameters are then calculated and the base probability of failure determined. Then, the green lumber strength parameters were altered by a factor until the same probability of failure as the base probability of failure was obtained.

It is suggested that the dry lumber always be chosen as the base. In this fashion no adjustment would be made to the dry allowable values.

Utilizing equations 3.36 and 3.37, the parameters of the total maximum lifetime load are calculated as

$$\mu_T = \frac{10}{30}\,(0.57)(2.223)\left(\frac{1.15}{2.1}\right) + \frac{20}{30}\,(0.69)(2.223)\,\frac{1.15}{2.1} \qquad (3.38)$$

$$= 0.23 + 0.56 = 0.79 \text{ psi}$$

$$\Omega_T = \frac{\left\{[(0.23)(.1)]^2 + [(0.56)(.44)]^2\right\}^{\frac{1}{2}}}{0.79} = 0.32 \qquad (3.39)$$

The second moment parameters of the dead load, maximum lifetime roof snow load, and maximum lifetime total load are tabulated in Table 3.12. By using a computer program developed for the probability of failure analysis, the probabilities of failure for the green and dry lumber are calculated. These are shown in the first two lines of Table 3.13 as lower and upper limits. As described in the review of literature, lower and upper limits are calculated to show that the step size of the numerical integration is adequate.

Using an iterative approach, the green lumber Weibull resistance parameters, μ and σ are altered by a factor k until the probabilities of failure for green and dry lumber are the same. Altering the location and scale parameters of the 3-parameter Weibull strength distribution by k is analogous to altering all of the green lumber strength values by k. A proof of this fact is offered in Appendix C. It is this k factor which is the moisture conversion factor for green and dry lumber. For this example, k is 1.100 which is to say dry lumber is 1.100 times stronger than green. Line 3 of Table 3.13 shows that the correct value of k was obtained since when the green lumber strength is increased by a factor of k it yields almost an identical probability of failure as the dry lumber. The two computer programs utilized for this analysis are listed in Appendix D. The probability of failure program is denoted as PFS1. The iterative program which calculates k is labelled KLN.

The lognormal function calculated from the combined second moment parameters utilizing equations 3.34 and 3.35 fit the relative frequency histogram of the simulated data generated from the lognormal distributions of the dead and snow load quite well. However, it was thought that the probabilities of failure and the k factor could be different

Table 3.12 The tabulated values are the second moment parameters of the dead load, maximum lifetime roof snow load, and the total maximum load. The total maximum load is a dead plus snow load combination commonly utilized in roof design. The distributions of the loads are also listed.

Load	Distribution	Second Moment Parameters	
		μ (ksi)	Ω
Dead	Lognormal	0.23	0.10
Snow	Lognormal	0.56	0.44
Dead + Snow	Lognormal	0.79	0.32

Table 3.13 The tabulated values are the probabilities of failure calculated for a 2" x 8" Douglas fir low sloping rafter under a dead plus snow load combination. The distribution of total load used in the probability of failure calculation was shown by the K-S test to be adequately described by the lognormal distribution.

Moisture Condition	Lower Limit	Upper Limit
Dry	1.57×10^{-4}	1.58×10^{-4}
Green	3.28×10^{-4}	3.29×10^{-4}
Green, adjusted by a k factor of 1.100	1.56×10^{-4}	1.57×10^{-4}

if a convolution integral describing the dead plus snow load was employed to characterize the total load. This thought is further supported by the known weakness of the K-S goodness of fit test for discriminating between data and a hypothesized distribution.

A computer routine was developed to numerically solve the convolution integral of the cumulative distribution function of the maximum lifetime total load. The total load is the sum of two independent lognormal variables describing the dead and maximum lifetime roof snow load. The convolution integral is described by equation 2.19 in the review of literature and derived in Appendix A. The listed computer program in Appendix A solves the convolution integral when the two variables are normally distributed. The developed routine is used in a computer program which calculates the probability of failure when the load distribution is the sum of two independent lognormal variables. The program, PFS2, is listed in Appendix F.

The green and dry probabilities of failure can now be calculated using the exact integral approach. Using the dry lumber to establish a benchmark reliability, the green lumber strength was altered artificially by a k factor until it had a probability of failure equal to the dry. This k equaled 1.080. Table 3.14 shows that when the strength of the green lumber is artificially increased 8 percent, the same probability of failure results as that of the dry lumber. It is interesting to note that the calculated probabilities of failure for the two methods as listed in Table 3.13 and 3.14 are quite similar. The exact integral approach provides a more conservative probability of failure than the

Table 3.14 The tabulated values are the probabilities of failure of the rafter structure of Table 3.13 using an exact integral approach. In this case, the load distribution is the sum of two independent lognormal variables; one representing dead load and the other representing snow load. These distributions were added or convoluted by equation 2.19 before the standard load and resistance reliability analysis was conducted. The k factor equaled 1.080.

Moisture Condition	Lower Limit	Upper Limit
Dry	2.44×10^{-4}	2.50×10^{-4}
Green, adjusted	2.39×10^{-4}	2.40×10^{-4}

simpler approach as shown by the larger failure probability.

Since the k value result obtained by the more simple method summarized in Table 3.13 is close enough to be considered equal to the above k result of 1.080, the simpler approach should be used. This conclusion is based on the fact that the engineering results obtained by utilizing the two factors would be the same. However, if a researcher feels that the dead and snow load will not combine in a simple mathematical way, the more theoretical but complicated approach might be used. For example, if the coefficients of variation of dead and snow load are approximately equal, then one may suspect that the two log-normal distributions may not combine simply.

3.4.2 Determination of Relative Safety Between Lumber Grades

In order to demonstrate the differential reliability technique to determine the relative safety between lumber grades, the contrasting data set of Hem-Fir lumber described at the beginning of section 3.4 was employed. The Weibull parameters are shown in Table 3.10. Figure 3.11 graphically depicts the distribution of bending strength for the two grades, Select Structural and No. 1. As previously mentioned, the design situation is taken as a low slope roof using rafter design. If the design specifies 2" x 8" Hem-Fir rafters, sixteen inches on center with ½ inch plywood sheathing and asbestos shingles, the calculated dead load is 5.2 psf. The nominal dead load remains 10 psf. Hence, the normalized dead load statistics are

$$\bar{D}/D_n = 0.52 \tag{3.40}$$

$$\Omega_D = 0.10 \tag{3.41}$$

These parameters are shown in Table 3.15 with the snow load parameters and the specified nominal design loads.

Since the load parameters and nominal loads are the same as those utilized in the previous analysis, the simple approach employing the second moment parameters will be utilized in the probability of failure analysis. Based on equations 3.36 and 3.37, load parameters are calculated for the Select Structural and No. 1 grades of Hem-Fir lumber. An adjusted F_b is calculated from the fifth percentile value of each lumber grade. The adjustment factor is (1.15/2.1) which is the load duration factor divided by the general adjustment factor. The calculated second moment load parameters of the dead load, snow load and dead plus snow load combination are shown in Table 3.16. The adjusted F_b used in the calculation of the load parameters of each grade is also shown.

It is important to note the difference between this analysis and the analysis described in the preceding section. In the differential reliability analysis for a moisture adjustment factor only one set of load parameters was needed; a result of green and dry lumber being two different physical states of the same lumber population. In the present analysis, the Select Structural and No. 1 Hem-Fir grades described by the 3-parameter Weibull functions depicted in Figure 3.11 are from two separate populations of lumber. Therefore, each lumber grade should be analyzed with respect to a load whose parameters are determined utilizing the calculated allowable strength from the respective lumber grade. This

Table 3.15 The tabulated values are the normalized parameters of the dead load and maximum lifetime roof snow load. The nominal design values are also shown.

Load	Nominal Design Value	Second Moment Parameters	
		$\bar{X}/X_n$	Ω_X
Dead	10	0.52	0.10
Snow	20	0.69	0.44

Table 3.16 The tabulated values are the calculated second moment load parameters for use in the probability of failure analysis of Select Structural and No. 1 grades of 2" x 8" Hem-Fir lumber. The combined load parameters are calculated using equations 3.36 and 3.37 with the adjusted fifth percentile strength value listed and the second moment parameters shown in Table 3.15.

Load	Second Moment Parameters		Adjusted Design Strength
	μ (ksi)	Ω	F_b (ksi)
Select Structural			1.976
Dead	0.34	0.10	
Snow	0.91	0.44	
Dead + Snow	1.25	0.32	
No. 1			1.148
Dead	0.20	0.10	
Snow	0.53	0.44	
Dead + Snow	0.73	0.32	

approach eliminates the effect of the different allowable stresses for the grades and focuses on the influence of distribution shape. As an example, the second moment load parameters for Select Structural Hem-Fir listed in Table 3.16 are calculated using the adjusted fifth percentile value of the Select Structural strength distribution shown in the fourth column of Table 3.16.

The probabilities of failure for the Select Structural and No. 1 Hem-Fir are now calculated utilizing the total load parameters listed in Table 3.16 and the resistance parameters given in Table 3.10. The computer program, PFS1 is again utilized to calculate the probabilities of failure. The upper and lower limits of the probabilities of failure for the two lumber grades are shown in the first two lines of Table 3.17. For comparison purposes, the Select Structural grade is chosen for the base reliability level. Using the iterative computer program, KLN, the location and scale parameters of the distribution of No. 1 Hem-Fir are modified by a factor k until equal probabilities of failure for the No. 1 and Select Structural lumber result. The k factor is 0.900. The third line of Table 3.17 shows that the correct factor is 0.900 because when the No. 1 strength values are artificially decreased by 10 percent, the same probability of failure as the Select Structural probability of failure results.

Two important facts need to be clarified concerning this result. First, the parameters of the load distribution utilized in the iterative program to calculate k are those calculated for the No. 1 Hem-Fir lumber. This is logical since the strength values of the No. 1 grade are being

Table 3.17 The tabulated values are the probabilities of failure calculated for a 2" x 8" Hem-Fir low sloping rafter under a dead plus snow load combination. The distribution of total load used in the probability of failure calculation was shown by the K-S test to be adequately described by the lognormal distribution.

Lumber Grade	Probability of Failure Lower Limit	Probability of Failure Upper Limit
Select Structural	2.69×10^{-4}	2.74×10^{-4}
No. 1	1.02×10^{-4}	1.05×10^{-4}
No. 1, adjusted by a k factor of 0.900	2.70×10^{-4}	2.78×10^{-4}

adjusted to determine the factor k. The load distribution associated with the grade must be used since different grades have different allowable stresses hence different allowable loads. Secondly, the k factor of 0.900 does not infer that the No. 1 grade is 10 percent stronger than the Select Structural grade. The correct interpretation is that the No. 1 grade has to be adjusted 10 percent in strength to have the same reliability as the Select Structural grade when both are subjected to their respective design loads. In other words, the two grades do not exhibit the same safety.

As in the previous analysis, the assumptions concerning the addition of the second moment parameters of the dead and the maximum lifetime roof snow load are tested by calculating the probabilities of failure and the resulting k factor from the convolution integral computer program, PFS2. The Select Structural grade was again chosen as the base reliability level. The probability of failure of the No. 1 grade of lumber was altered by a k factor until equal to the Select Structural grade probability of failure. This k equaled 0.915. Table 3.18 shows that when the No. 1 grade strength values are artificially decreased by 8.5 percent, the same probability of failure results.

Since the k factor of 0.915 is close enough to the tabulated k value calculated by the simpler method shown in Table 3.17, the simpler approach should be used. Again, if a researcher feels that the situation does not warrant combining the second moment parameters, the more exact convolution technique can be used. Based on this result and the result from the moisture analysis, all further differential reliability analyses

Table 3.18 The tabulated values are the probabilities of failure of the rafter structure of Table 3.17 using an exact integral approach. In this case, the load distribution is the sum of two independent lognormal variables; one representing dead load and the other representing snow load. These distributions were added or convoluted by equation 2.19 before the standard load and resistance reliability analysis was conducted. The k factor equals 0.915.

Lumber Grade	Probability of Failure	
	Lower Limit	Upper Limit
Select Structural	3.53×10^{-4}	3.58×10^{-4}
No. 1, altered by k	3.54×10^{-4}	3.62×10^{-4}

utilizing a dead plus snow load combination are conducted employing the simpler approach.

3.5 Application of the Differential Reliability Technique Utilizing a Dead Plus Floor Live Load Combination

To demonstrate the differential reliability technique utilizing a load distribution comprised of a dead load and the maximum lifetime floor live load, the same contrasting lumber sets are employed as in the previous main section. Table 3.9 shows the strength parameters of the 2" x 8" Douglas fir lumber. The strength parameters of the two grades of Hem-Fir lumber are shown in Table 3.10. Figures 3.10 and 3.11 graphically demonstrate the differences between the contrasting lumber sets.

A typical design situation for a dead plus floor live load combination is the design of floor joists in a light frame structure. For shorter spans, 2" x 8" lumber is often utilized. A typical design is usually 2" x 8" lumber, sixteen inches on center. Based on present construction practices, ½" plywood underlayment is overlayed on the floor joists. If carpet is to be installed, ½" pressed particleboard is overlayed next, followed by the carpet pad and carpet. Hence, the previous contrasting lumber sets are utilized in the following differential reliability analyses since both are 2" x 8" lumber.

The conventional design load for a dead plus floor live load combination is nominal dead load plus nominal floor live load. No adjustments are necessary for duration of load since none are specified (46). The nominal design dead load for the floor of a light frame structure is 10 psf (32). The nominal design live load is 40 psf (32) for the floor of a light frame structure. Therefore the total nominal design load is the sum of the nominal loads or 50 psf.

3.5.1 Determination of a Moisture Adjustment Factor

For the design situation described at the beginning of section 3.5, a dead load of 7.2 psf was calculated. This was assumed to be the mean dead load. The normalized statistics of the dead load based on Table 3.8 are

$$\bar{D}/D_n = 0.72 \tag{3.42}$$

$$\Omega_D = 0.10 \tag{3.43}$$

The maximum lifetime floor live load parameters are given in Table 3.8. Each set of live load parameters is combined with the dead load parameters listed above. The differential reliability analysis is then conducted utilizing each of the four sets of combined parameters.

As in the previous examples, the distribution of the sum of the two loads must be known in order to perform the probability of failure analysis. The distribution of the sum of a lognormal variable and an extreme value type I variable cannot be derived in closed form. However, since the coefficient of variation of dead load is relatively small compared to the coefficient of variation of the maximum lifetime floor live load in all four cases, the parameters of the maximum lifetime total load may be approximated by adding the means and variances of the lognormal and extreme value type I distributions. The resulting distribution of maximum total load is assumed to be extreme value type I. This assumption is further supported by the following discussion.

The dead load is only one fifth of the total load combination. Based on the mathematical relationship between the mean, variance and coefficient of variation, the ratio of the variance of dead to the variance of live load is extremely small. Therefore, the variance of the total load is approximately equal to the variance of the floor live load. Hence, the addition of the second moment parameters of dead and live load can be approximated by adding a point estimate of the dead load to the mean floor live load. It is therefore believed that the probabilities of failure which are calculated by the simple method will be similar to the probabilities of failure calculated by the exact integral method and hence the k factors will also be similar.

Based on equations 3.32 and 3.33, the moments of the total maximum load are calculated by

$$\mu_T = (\bar{D}/D_n)(D_n) + (\bar{L}/L_n)(L_n) \tag{3.44}$$

$$\sigma_T^2 = [\Omega_D(\bar{D}/D_n)D_n]^2 + [\Omega_L(\bar{L}/L_n)L_n]^2 \tag{3.45}$$

where

μ_T = the mean of the total maximum load

σ_T^2 = the variance of the total maximum load

$\bar{D}/D_n$ = the normalized mean of the dead load

$\bar{L}/L_n$ = the normalized mean of the maximum lifetime floor live load

Ω_D = the coefficient of variation on the dead load

Ω_L = the coefficient of variation of the maximum lifetime floor live load

D_n = the nominal dead load

L_n = the nominal floor live load

The above addition of variances implies independence between the dead load and the floor live load which is believed to be a good assumption. The combined second moment parameters and the resulting type I parameters of the total load are shown in Table 3.19. The total load is identified in Table 3.19 in conjunction with the live load utilized in the calculation of the total load. In other words, Load A is the total floor load comprised of the dead load and live load A. The above results are utilized to test the assumptions regarding the addition of the dead load and floor live load parameters.

A simulation study similar to the one executed in section 3.4.1 was conducted. First, random lognormal deviates representing dead load and random extreme value type I deviates representing floor live load were generated using their respective parameters. This was done for each set of floor live load parameters shown in Table 3.8. The dead load and each set of floor live load deviates were summed and four relative frequency histograms constructed. Next, four extreme value type I functions described by the total load parameters shown in Table 3.19 were overlayed on their respective histograms as shown in Figures 3.13 through 3.16. A visual test conducted on each of the four figures indicates no obvious lack of fit.

As a final test, a Kolmogorov-Smirnoff test was performed at a 20 percent level of significance for each total load. The higher the sig-

Table 3.19 The combined second moment parameters and resulting type I parameters are tabulated for the four different total loads. The load combination is dead plus floor live load combined according to equations 3.44 and 3.45. The live load statistics are given in Table 3.8.

Load Case	Total Load: Second Moment Parameters μ_T (psf)	σ_T (psf)	Type I Parameters α (psf)	β (psf)
A	44.7	7.7	6.0	41.2
B	28.1	8.4	6.5	25.1
C	38.6	8.4	6.6	34.8
D	36.5	5.5	4.3	34.0

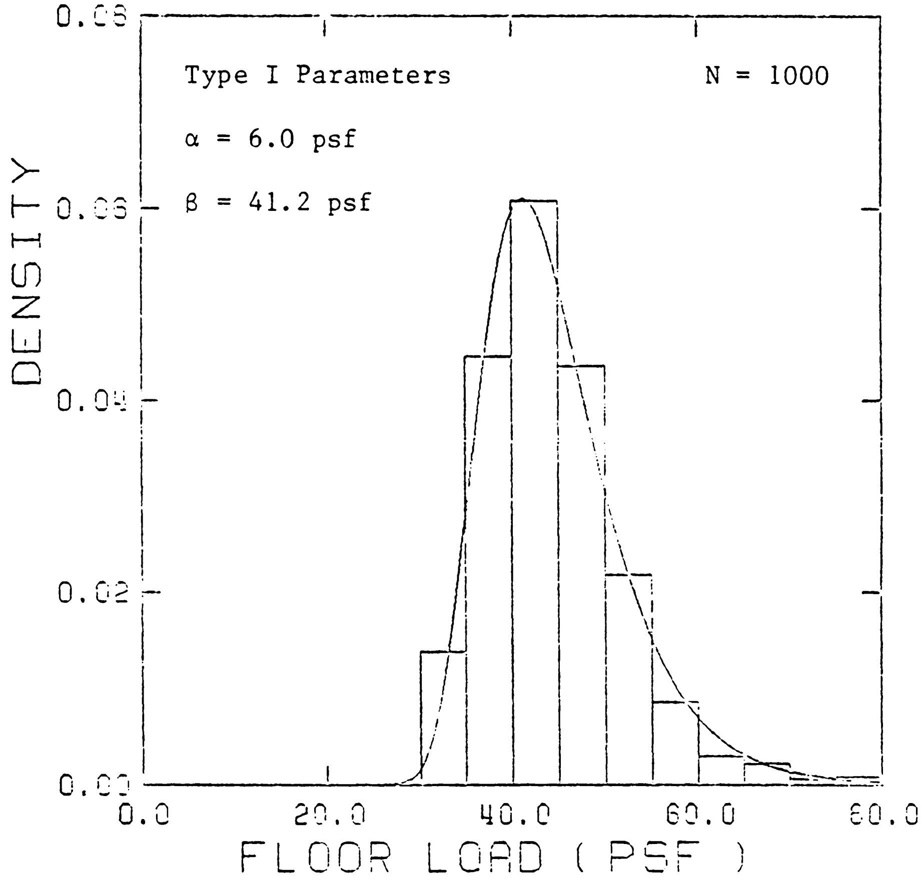

Figure 3.13 A histogram of the simulated total load is shown with an overlay of an extreme value type I function with parameters calculated as the sum of the parameters of the dead load and the floor live load. The second moment parameters of the floor live load are those of Live Load A which are the average of the load parameters of owner occupied and renter occupied dwellings as listed by Chalk and Corotis and tabulated in Table 3.8.

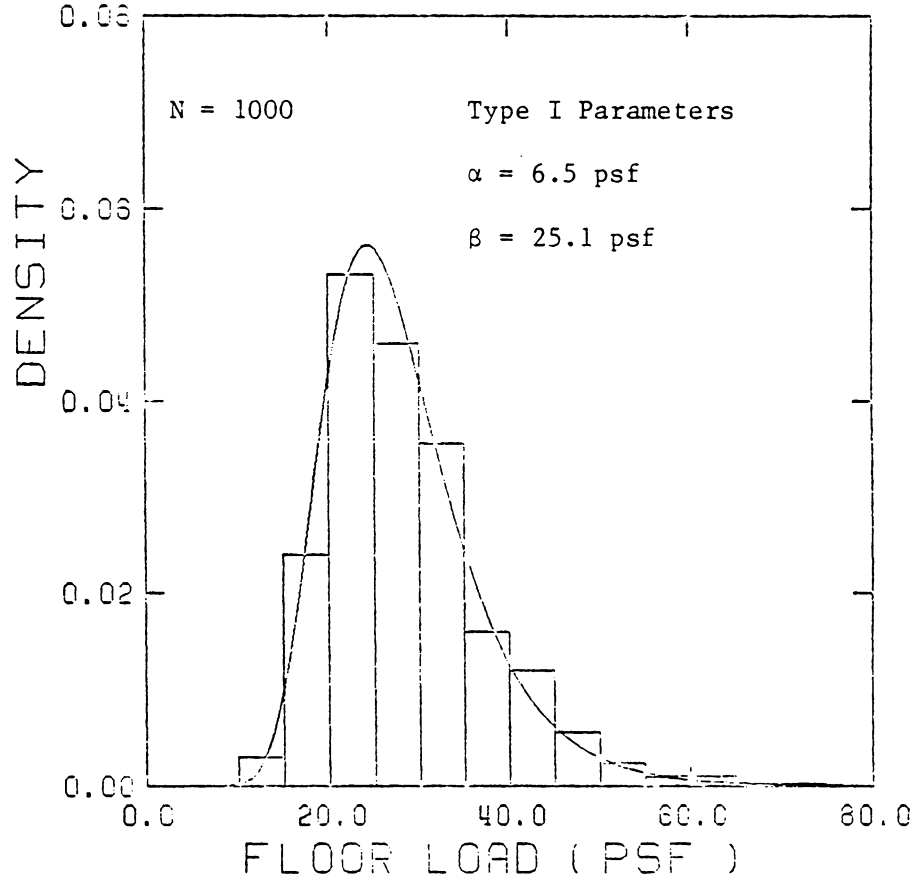

Figure 3.14 A histogram of the simulated total load is shown with an overlay of an extreme value type I function with parameters calculated as the sum of the parameters of the dead load and the floor live load. The second moment parameters of the floor live load are those of Live Load B which are the load parameters developed by Sentler and tabulated in Table 3.8.

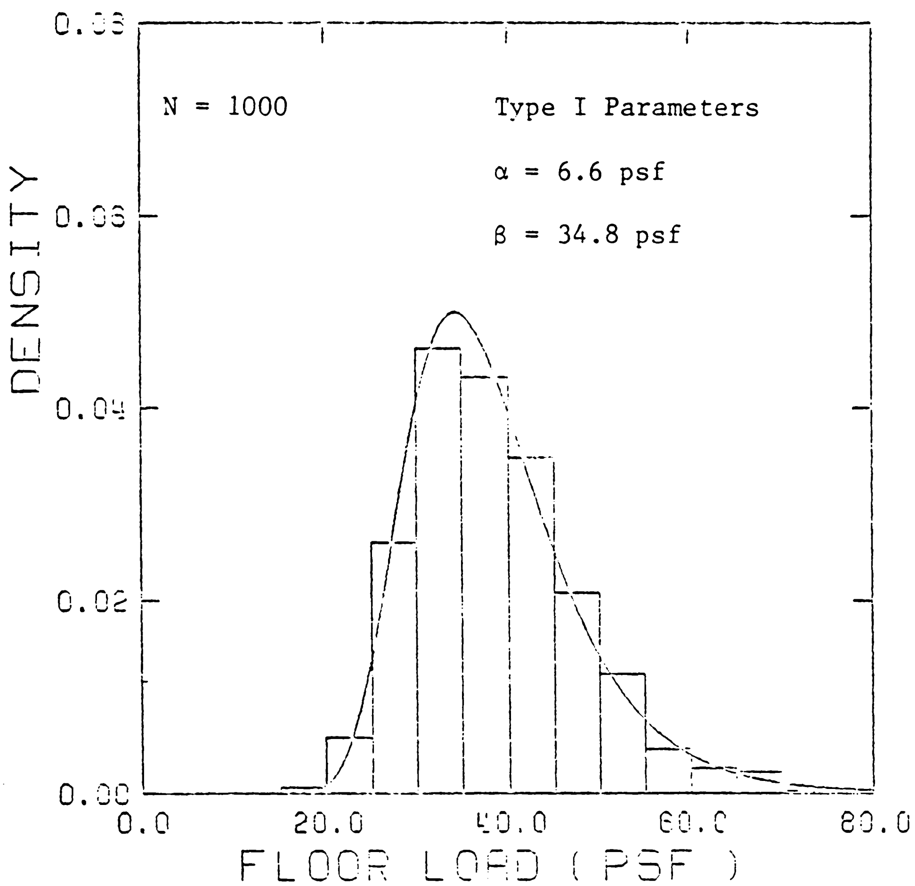

Figure 3.15 A histogram of the simulated total load is shown with an overlay of an extreme value type I function with parameters calculated as the sum of the parameters of the dead load and the floor live load. The second moment parameters of the floor live load are those of Live Load C which are the Canadian floor live parameters adjusted to a 50 year lifetime as shown in Table 3.8.

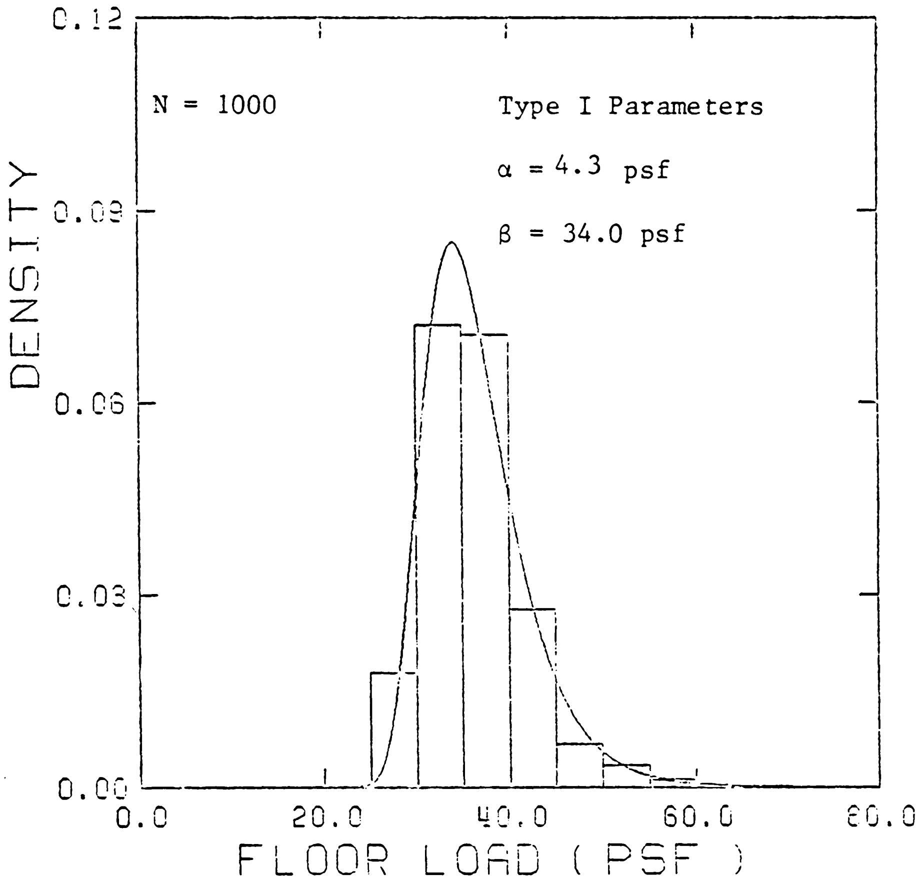

Figure 3.16 A histogram of the simulated total load is shown with an overlay of an extreme value type I function with parameters calculated as the sum of the parameters of the dead load and the floor live load. The second moment parameters of the floor live load are those of Live Load D developed from the Chalk and Corotis model as listed in Table 3.8.

nificance level, the easier it is to reject the assumed distribution. The K-S test was also conducted using the modified critical values as listed by Crutcher (14). The K-S test statistics, non-parametric K-S critical value and the p-values for the modified K-S test are shown for each of the four total loads in Table 3.20. The extreme value type I distribution was rejected once at a 20 percent level of significance which coincided with a p-value of less than 0.01 for the modified K-S test. Based on these results, the extreme value type I distribution was accepted as a suitable distribution of dead plus floor live load. Since the type I function adequately models the combined load, the probability of failure calculations can now be conducted once the parameters of the combined load are expressed in units of psi.

To convert the total load to units of psi, the procedure outlined in section 3.4.1 is again employed. First, the design load is set equal to the design resistance. In conventional design, this is achieved by calculating the load effect based on the nominal load. The calculated load effect is then compared to allowable design strength values in order to determine a suitably sized structural member. For the probability of failure analysis, the design resistance is the adjusted fifth percentile, F_b of the strength distribution. Since the design resistance is set equal to the design load, the adjusted fifth percentile is set equal to the nominal design load in the probability of failure analysis. When the normalized mean values of the dead load and floor live load are multipled by the adjusted design resistance, the loads are expressed in units of psi and they reflect the relative position of the dead load and the floor live load to the strength distribution.

Table 3.20 The tabulated values are the results of the Kolmogorov-Smirnoff goodness of fit test performed at a 20 percent level of significance as well as the significance levels based on the modified K-S test. The p-value indicates the significance when using the modified K-S test.

Load Case	K-S (computed)	K-S (critical)	Significance Level
A	0.0212	0.0338	$0.20<p$
B	0.0274	0.0338	$0.05<p<0.10$
C	0.0347	0.0338	$p<0.01$
D	0.0320	0.0338	$0.01<p<0.05$

Secondly, each load in the load combination should be represented in proportion to the total nominal load. For this load combination, the dead load is one fifth of the total nominal load and the maximum lifetime floor live load is four fifths of the total nominal load.

For the dead plus floor live load combination, the equations for the total load parameters are equations 3.46 and 3.47 and they are

$$\mu_T = \frac{D_n}{T_n} (\bar{D}/D_n) F_b + \frac{L_n}{T_n} (\bar{L}/L_n) F_b \tag{3.46}$$

$$\Omega_T = \frac{\left[(\mu_D \Omega_D)^2 + (\mu_L \Omega_L)^2\right]^{\frac{1}{2}}}{\mu_T} \tag{3.47}$$

where

μ_T = the mean of the maximum total load

Ω_T = the coefficient of variation of the maximum total load

D_n = the nominal dead load

L_n = the nominal floor live load

$T_n = D_n + L_n$ = the total nominal load

$\bar{D}/D_n$ = the normalized mean of the dead load

$\bar{L}/L_n$ = the normalized mean of the maximum lifetime floor live load

Ω_D = the coefficient of variation of the dead load

Ω_L = the coefficient of variation of the maximum lifetime floor live load

F_b = the adjusted allowable design value

The adjusted allowable design strength, F_b is the fifth percentile of the dry lumber strength divided by the general adjustment factor of 2.1. As previously discussed, only one set of parameters describing the load is calculated for use in the reliability analysis of lumber strength as affected by moisture content. The load parameters of the total load are calculated using equations 3.46 and 3.47. The adjusted allowable strength and the calculated parameters of the dead load, floor live load, and total load are given in Table 3.21.

By using a computer program developed for the probability of failure analysis when the load distribution is extreme value type I and the resistance distribution is the 3-parameter Weibull distribution, the probabilities of failure for the green and dry lumber are calculated. These are shown in the first two lines for each total load in Table 3.22 as lower and upper limits. The computer program, PFL1 is listed in Appendix E.

Using an iterative approach, the green lumber Weibull strength parameters, μ and σ are altered by a factor k until the probabilities of failure for the adjusted green and the dry lumber are the same. For each of the total loads, the k factor is shown in the third line of each section of Table 3.22 with the probabilities of failure. Since the adjusted green probabilities of failure are similar to the dry probabilities of failure, the correct k factors have been achieved. The computer program, KEV1 developed to conduct this analysis is listed in Appendix E.

In order to test the assumption that the probabilities of failure and the k factor would be similar regardless of which approach, simple or

Table 3.21 The tabulated values are the various load parameters and adjusted allowable strengths utilized in the probability of failure analysis to determine a moisture adjustment factor. The adjusted allowable strength value is the fifth percentile of the strength distribution of the dry lumber whose parameters are shown in Table 3.9.

Load Case	Allowable Strength F_b (ksi)	Dead Load μ_D (ksi)	Ω_D	Live Load μ_L (ksi)	Ω_L	Total Load μ_T (ksi)	Ω_T
A	1.059	0.72	0.10	0.79	0.21	0.95	0.17
B	1.059	0.72	0.10	0.44	0.40	0.60	0.30
C	1.059	0.72	0.10	0.66	0.26	0.82	0.22
D	1.059	0.72	0.10	0.66	0.19	0.77	0.15

Table 3.22 The tabulated values are the failure probabilities calculated for a 2" x 8" Douglas fir floor joist under a dead plus floor live load combination. The distribution of total load was shown by the K-S test to be adequately modeled by the extreme value type I distribution.

Moisture Condition	Probability of Failure Lower Limit	Probability of Failure Upper Limit
Load A		
Dry	7.78×10^{-5}	8.08×10^{-5}
Green	2.97×10^{-4}	3.09×10^{-4}
Green, adjusted by k = 1.145	7.82×10^{-5}	8.12×10^{-5}
Load B		
Dry	8.90×10^{-6}	9.23×10^{-6}
Green	2.98×10^{-5}	3.09×10^{-5}
Green, adjusted by k = 1.135	8.95×10^{-6}	9.28×10^{-6}
Load C		
Dry	4.52×10^{-5}	4.68×10^{-5}
Green	1.47×10^{-4}	1.52×10^{-4}
Green, adjusted by k = 1.135	4.49×10^{-5}	4.65×10^{-5}
Load D		
Dry	1.91×10^{-6}	2.01×10^{-6}
Green	2.14×10^{-5}	2.26×10^{-5}
Green, adjusted by k = 1.195	1.95×10^{-6}	2.06×10^{-6}

exact integral was utilized, the computer subroutine mentioned in section 3.4.1 was altered to calculate the cumulative distribution function of the total load when the individual variables are distributed lognormal and extreme value type I, respectively. This subroutine is utilized in the computer program, PFL2 listed in Appendix F. PFL2 is used to calculate the probabilities of failure for the green and dry lumber which are shown in Table 3.23. The probabilities of failure of the green lumber adjusted by k are also listed. In the reliability analyses previously discussed and in the present analysis, the k factor is incremented in the computer programs by $\Delta = 0.005$. The chosen k factor is therefore an estimate of the true k factor with an error of 0.005. Utilizing a smaller increment than Δ to calculate closer probability of failure estimates to the benchmark reliability level is not justified considering the end use of the k factor. In other words, a more accurate k factor is not needed considering the engineering results which would be obtained by utilizing a k factor incremented by less than 0.005. A comparison of the results listed in Tables 3.22 and 3.23 shows that the k values are the same within an error of Δ = 0.005. Also, the probabilities of failure calculated by the two methods are very close. Therefore the simpler approach should be utilized. It is noted that the researcher may wish to use the more exact integral approach if he feels that the dead and floor live load will not combine in a simple mathematical way.

Table 3.23 The tabulated values are the probabilities of failure of the rafter structure of Table 3.22 using an exact integral approach. In this case, the load distribution is the sum of two independent distributions; a lognormal distribution representing dead load and an extreme value distribution representing maximum lifetime floor live load. The parameters of these distributions are shown in Table 3.21 and were added or convoluted by equation 2.19 before the standard load and resistance reliability analysis was conducted.

Moisture Condition	Probabilities of Failure Lower Limit	Upper Limit
Load A		
Dry	7.57×10^{-5}	7.98×10^{-5}
Green	2.96×10^{-4}	3.07×10^{-4}
Green, adjusted by k = 1.145	7.76×10^{-5}	8.06×10^{-5}
Load B		
Dry	8.76×10^{-6}	9.08×10^{-6}
Green	2.95×10^{-5}	3.06×10^{-5}
Green, adjusted by k = 1.135	8.85×10^{-6}	9.17×10^{-6}
Load C		
Dry	4.47×10^{-5}	4.63×10^{-5}
Green	1.46×10^{-4}	1.52×10^{-4}
Green, adjusted by k = 1.135	4.45×10^{-5}	4.61×10^{-5}
Load D		
Dry	1.83×10^{-6}	1.94×10^{-6}
Green	2.10×10^{-5}	2.22×10^{-5}
Green, adjusted by k = 1.195	1.90×10^{-6}	2.00×10^{-6}

3.5.2 Determination of Relative Safety Between Lumber Grades

The Select Structural and No. 1 2" x 8" Hem-Fir lumber described in section 3.4 is studied with a dead plus floor live load combination. The design situation is described in section 3.5. If the 2" x 8" Hem-Fir lumber is placed sixteen inches on center with identical construction as section 3.5 indicates, a dead load of 6.7 psf is calculated. The normalized dead load statistics are

$$\bar{D}/D_n = 0.67 \tag{3.48}$$

$$\Omega_D = 0.10 \tag{3.49}$$

The parameters of the floor live are shown in Table 3.8. The nominal loads are those specified for dead load and live load on floors, 10 psf and 40 psf, respectively (32). The Weibull strength parameters for the Select Structural and No. 1 grades of Hem-Fir are shown in Table 3.10.

Since the load parameters and nominal loads are the same as previously utilized in section 3.5.1, the simpler approach employing the sum of the second moment parameters of dead and floor live load will be used. Using equations 3.46 and 3.47, total load parameters for each grade utilized in the reliability analysis are calculated. As mentioned in section 3.4.2, each grade is analyzed with respect to its own load distribution. The calculated dead load parameters, floor live load parameters, and total load parameters are shown in Table 3.24. The allowable design strength, F_b is the fifth percentile strength value listed in Table 3.10 divided by the general adjustment factor of 2.1.

Table 3.24 The tabulated values are the calculated second moment parameters for use in the probability of failure analysis of Select Structural and No. 1 grades of 2" x 8" Hem-Fir lumber. The combined load parameters are calculated using equations 3.46 and 3.47 and the allowable strength listed.

Lumber	Allowable Strength	Second Moment Parameters					
	F_b	Dead Load		Floor Live Load		Total Load	
		μ_D	Ω_D	μ_L	Ω_L	μ_T	Ω_T
	(ksi)	(ksi)		(ksi)		(ksi)	
Select Structural							
Load A	1.719	0.23	0.10	1.29	0.21	1.52	0.18
Load B	1.719	0.23	0.10	0.71	0.40	0.94	0.31
Load C	1.719	0.23	0.10	1.07	0.27	1.30	0.22
Load D	1.719	0.23	0.10	1.00	0.19	1.23	0.16
No. 1							
Load A	0.998	0.13	0.10	0.75	0.21	0.88	0.18
Load B	0.998	0.13	0.10	0.42	0.40	0.55	0.30
Load C	0.998	0.13	0.10	0.62	0.27	0.75	0.22
Load D	0.998	0.13	0.10	0.58	0.19	0.71	0.16

The reliability analysis of Select Structural versus No. 1 2" x 8" Hem-Fir lumber is now conducted using the total load parameters shown in Table 3.24 and the resistance parameters listed in Table 3.10. The probabilities of failure are calculated using the computer program PFL1. The calculated upper and lower limits of the probabilities of failure are shown in Table 3.25 for the Select Structural and No. 1 lumber according to the floor live load utilized in the calculation of total load. The Select Structural lumber is again chosen to represent the base reliability level. Using the iterative computer program KEV1, the location and scale parameters of the No. 1 Hem-Fir Weibull distribution are altered by a factor k until the same probability of failure as the Select Structural probability of failure results. The altered No. 1 probabilities of failure are also listed in Table 3.25.

As previously discussed in section 3.4.2, the load distribution utilized in the calculation of the k factor is the load distribution associated with the No. 1 grade of lumber. Also, the k factor does not represent the relative strength between the lumber grades. It does represent the relative safety of each grade under the influence of its design load.

As in the previous analyses, the assumptions regarding the addition of the dead load parameters to the floor live load parameters is tested utilizing the exact integral approach. With the computer program PFL2, the individual load parameters of the dead load and floor live load listed in Table 3.24 are utilized to calculate the probabilities of failure of the Select Structural and No. 1 Hem-Fir lumber. These probabilities of failure and the probabilities of failure of the adjusted No.

Table 3.25 The tabulated values are the probabilities of failure calculated for a 2" x 8" Hem-Fir floor joist under a dead plus floor live load combination. Four different sets of floor live load parameters are utilized to calculate the total load. The distribution of total load used in the probability of failure calculation was shown by the K-S test to be adequately described by the extreme value type I distribution.

Lumber Grade	Probability of Failure Lower Limit	Probability of Failure Upper Limit
Load A		
Select Structural	2.54×10^{-4}	2.60×10^{-4}
No. 1	4.39×10^{-5}	4.58×10^{-5}
No. 1, adjusted by k = 0.865	2.45×10^{-4}	2.55×10^{-4}
Load B		
Select Structural	2.43×10^{-5}	2.49×10^{-5}
No. 1	4.23×10^{-6}	4.40×10^{-6}
No. 1, adjusted by k = 0.875	2.45×10^{-5}	2.54×10^{-5}
Load C		
Select Structural	1.09×10^{-4}	1.12×10^{-4}
No. 1	2.01×10^{-5}	2.08×10^{-5}
No. 1, adjusted by k = 0.865	2.06×10^{-4}	1.10×10^{-4}
Load D		
Select Structural	1.65×10^{-5}	1.71×10^{-5}
No. 1	8.05×10^{-7}	8.52×10^{-7}
No. 1, adjusted by k = 0.825	1.62×10^{-5}	1.71×10^{-5}

1 Hem-Fir are shown in Table 3.26. A comparison of the k factors listed in Table 3.25 and 3.26 reveals that since the factors are similar, the simpler approach to the probability of failure analysis should be utilized. Accordingly, based on this result and the similar results of the preceding section, all future differential reliability analyses utilizing a dead plus floor live load combination are conducted employing the simpler approach.

3.6 Summary of the Differential Reliability Technique

The previous analyses have confirmed that for the load combinations of dead plus maximum lifetime roof snow load and dead plus maximum lifetime floor live load, the simpler method entailing the addition of the second moment parameters of the loads results in similar k factors as those obtained from an exact integral approach. The following steps provide a summary of a differential reliability analysis on contrasting lumber sets utilizing the simpler method.

1. An appropriate design situation for the contrasting lumber sets give meaning to the comparison.
2. The fifth percentiles of the contrasting lumber data are calculated and then adjusted by the general adjustment factor of 2.1 and any applicable load factors to obtain the allowable design strength, F_b.
3. The actual dead load for the design situation is calculated and assumed to be the mean dead load. The normalized dead load is therefore the mean dead load divided by the nominal dead load.

Table 3.26 The tabulated values are the probabilities of failure of the floor joist structure of Table 3.25 using an exact integral approach. In this case, the load distribution is the sum of a lognormal distribution representing dead load and an extreme value type I distribution representing maximum lifetime floor live load. The distributions were added or convoluted by equation 2.19 before the standard load and resistance reliability analysis was conducted.

Lumber Grade	Probability of Failure Lower Limit	Probability of Failure Upper Limit
Load A		
Select Structural	2.49×10^{-4}	2.55×10^{-4}
No. 1	4.25×10^{-5}	4.43×10^{-5}
No. 1, adjusted by k = 0.865	2.39×10^{-4}	2.49×10^{-4}
Load B		
Select Structural	2.14×10^{-5}	2.18×10^{-5}
No. 1	4.90×10^{-6}	5.08×10^{-6}
No. 1, adjusted by k = 0.880	2.09×10^{-5}	2.17×10^{-5}
Load C		
Select Structural	1.14×10^{-4}	1.17×10^{-4}
No. 1	2.19×10^{-5}	2.28×10^{-5}
No. 1, adjusted by k = 0.865	1.13×10^{-4}	1.18×10^{-4}
Load D		
Select Structural	1.42×10^{-5}	1.47×10^{-5}
No. 1	6.33×10^{-7}	6.70×10^{-7}
No. 1, adjusted by k = 0.820	1.47×10^{-5}	1.55×10^{-5}

4. The total load parameters are calculated utilizing the dead load parameters, live load parameters, nominal loads, and the allowable design strength, F_b in equations 3.36 and 3.37 or equations 3.46 and 3.47 depending on the design situation. The assumption is that the dead load and the live load are independent.
5. One of the contrasting lumber sets is chosen for the base or benchmark reliability level.
6. Using the appropriate computer programs, the probabilities of failure and the k factor of the contrasting lumber sets are calculated and the appropriate k factor is chosen as the one which produces a probability of failure approximately equal to the base probability of failure.

As discussed in the reveiw of literature, the calculated probabilities of failure are representative of the structural element when subjected to design load acting as a single member. The single members as part of a system will not simultaneously experience design stress since the stress analysis does not account for interior partitions, sheathing and load sharing, all contributing factors in reducing the actual stress of structural members. Consequently, the failure probabilities must only be considered in a comparative sense.

CHAPTER IV

EXAMPLE DIFFERENTIAL RELIABILITY ANALYSES OF LUMBER DATA

The technique of differential reliability analysis has been explained in detail in the previous chapter. Utilizing the developed load distributions, contrasting lumber data sets can be analyzed by the simpler method of differential reliability analysis. In other words, by combining the second moment parameters of the individual load distributions, the probability of failure analysis can be conducted without using the more costly and time consuming exact integral approach.

In this chapter, the differential reliability method is applied to various contrasting sets of lumber. In all but one case, a dead plus roof snow load combination or dead plus floor live load combination is used based on the design situation.

The computer programs utilized in the probability of failure solution are the same as previously discussed except the equations involving the resistance are either for the lognormal distribution or the three parameter Weibull distribution. For a lognormally distributed strength distribution, the equivalent of altering all strength values by a factor of k is the addition of the natural logarithm of k to the mean of the resistance distribution, λ. This fact is derived in Appendix C. The computer programs are listed in Appendix D and E for a dead plus roof snow load combination and a dead plus floor live load combination, respectively.

4.1 Relative Safety Between Lumber Grades - Bending Strength Comparison

In the preceding chapter the differential reliability analysis of Select Structural and No. 1 2" x 8" Hem-Fir lumber was explained in detail. The technique is now used to analyze two more grades of 2" x 8" Hem-Fir lumber. The Select Structural grade used in the previous chapter is again chosen as the reference material. The lumber to be analyzed are the No. 2 and No. 3 grades of 2" x 8" Hem-Fir (33), both at 12 percent average moisture content. The Weibull parameters and fifth percentiles of each grade are shown in Table 4.1. Figures 4.1 and 4.2 graphically depict the differences between the Select Structural and No. 2 grades and the Select Structural and No. 3 grades of Hem-Fir lumber, respectively.

Since the 2" x 8" lumber can be used in either floor or roof construction, both the dead plus snow load combination and the dead plus floor live load combination are utilized in the differential reliability analyses. The design situation and the dead load parameters have been described in sections 3.4.2 and 3.5.2 for each of the combinations. The dead load and roof snow load parameters are shown in Table 3.15 while equations 3.48 and 3.49 and Table 3.8 show the dead load and floor live load parameters. Utilizing these parameters in equations 3.36 and 3.37 for the snow load combination and equations 3.46 and 3.47 for the floor

Table 4.1 The tabulated values are the Weibull parameters and fifth percentile of various grades of 2" x 8" Hem-Fir lumber at 12 percent average moisture content (33). The Select Structural grade serves as the reference material in the differential reliability analysis.

	Weibull Parameters			Fifth Percentile
	μ (ksi)	σ (ksi)	η (ksi)	(ksi)
Select Structural	1.526	6.450	2.628	3.609
No. 2	0.051	6.335	2.778	2.226
No. 3	0.663	3.854	1.505	1.199

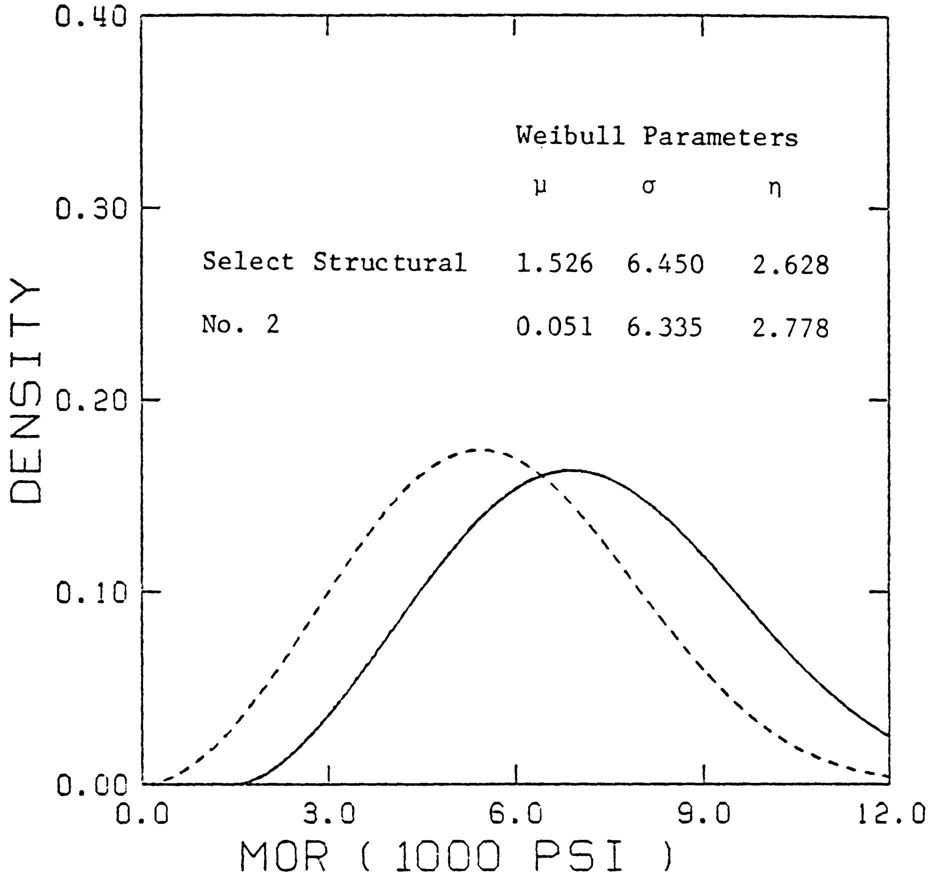

Figure 4.1 The Weibull functions describing Select Structural and No. 2 grades of 2" x 8" Hem-Fir lumber are shown (33). The Select Structural grade, shown as a solid line, serves as the reference material. Parameters μ and σ are given in ksi.

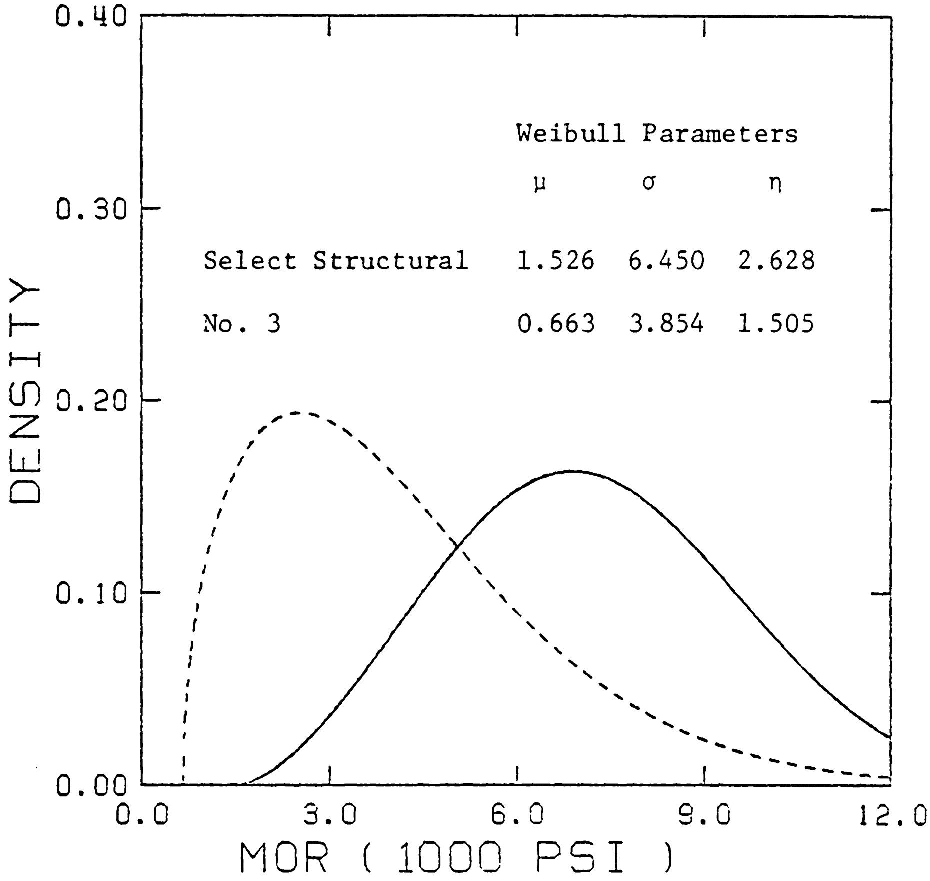

Figure 4.2 The Weibull functions describing the Select Structural and No. 3 grades of 2" x 8" Hem-Fir lumber are shown (33). The Select Structural grade, shown as a solid line, is used to calculate the benchmark safety level. Parameters μ and σ are given in ksi.

live load combination, the parameters of the total load are calculated. These parameters and the allowable design values utilized in the calculation of the total loads are shown in Table 4.2.

The probabilities of failure are calculated for each Hem-Fir grade utilizing each of the five load combinations. The Weibull parameters of the No. 2 and No. 3 grades are adjusted by k until the same probabilities of failure result as the Select Structural probabilities of failure. The k factor is a measure of safety of the No. 2 and No. 3 lumber grades relative to the Select Structural grade of Hem-Fir. The probabilities of failure and the resulting k factors are shown in Table 4.3 and Table 4.4.

The k factors reflect the effect of the shape of the resistance distribution on lumber strength. The effects of sampling are minimized by placing the load distribution relative to the fifth percentile in all cases. The k factors represent the relative safety of each lumber grade compared to the Select Structural grade; they do not represent the relative strength of each lumber grade.

The k factor is the value by which the strength distribution of various lumber grades is altered to obtain comparative failure probabilities. The k factor is not a ratio to equate any particular percentile of a strength distribution to the same percentile of the reference material distribution. The results of Table 4.3 suggest that the grading rules for No. 2 lumber be altered to delete the lower quality material in order to have the same safety as the Select Structural grade.

Table 4.2 The tabulated values are the total load parameters used in the comparison of Select Structural and No. 2 grade of Hem-Fir and Select Structural and No. 3 grade of Hem-Fir and the individual load parameters utilized to calculate the total load. F_b is the adjusted fifth percentile of the lumber data shown in Table 4.1.

Load Combination	F_b	Second Moment Parameters					
		Dead Load		Live Load		Total Load	
		μ_D	Ω_D	μ_L	Ω_L	μ_T	Ω_T
	(ksi)	(ksi)		(ksi)		(ksi)	
Select Structural							
Snow	1.976	0.34	0.10	0.91	0.44	1.25	0.32
Load A	1.719	0.23	0.10	1.29	0.21	1.52	0.18
Load B	1.719	0.23	0.10	0.71	0.40	0.94	0.31
Load C	1.719	0.23	0.10	1.07	0.27	1.30	0.22
Load D	1.719	0.23	0.10	1.00	0.19	1.23	0.16
No. 2							
Snow	1.219	0.21	0.10	0.56	0.44	0.77	0.32
Load A	1.060	0.14	0.10	0.80	0.21	0.94	0.18
Load B	1.060	0.14	0.10	0.44	0.40	0.58	0.31
Load C	1.060	0.14	0.10	0.66	0.27	0.80	0.22
Load D	1.060	0.14	0.10	0.62	0.19	0.76	0.16
No. 3							
Snow	0.656	0.11	0.10	0.30	0.44	0.41	0.33
Load A	0.571	0.08	0.10	0.43	0.21	0.51	0.18
Load B	0.571	0.08	0.10	0.24	0.40	0.32	0.31
Load C	0.571	0.08	0.10	0.36	0.27	0.44	0.22
Load D	0.571	0.08	0.10	0.33	0.19	0.41	0.16

Table 4.3 The tabulated values are the probabilities of failure for Select Structural and No. 2 grades of 2" x 8" Hem-Fir lumber when utilized as a rafter or a floor joist as described in Chapter 3. The Weibull parameters μ and σ are adjusted by k for the No. 2 lumber grade until the probabilities of failure are the same as the Select Structural probabilities of failure. The distribution of the dead plus snow load combination is lognormal. The extreme value type I distribution is the distribution of the dead plus floor live load combination.

Design Situation	Probabilities of Failure Lower Limit	Upper Limit
Roof		
Select Structural	2.71×10^{-4}	2.72×10^{-4}
No. 2	3.09×10^{-3}	3.11×10^{-3}
No. 2, adjusted by k = 2.235	2.71×10^{-4}	2.72×10^{-4}
Floor		
Load A		
Select Structural	2.57×10^{-4}	2.58×10^{-4}
No. 2	4.65×10^{-3}	4.67×10^{-3}
No. 2, adjusted by k = 2.600	2.56×10^{-4}	2.57×10^{-4}
Load B		
Select Structural	2.45×10^{-5}	2.46×10^{-5}
No. 2	1.32×10^{-3}	1.33×10^{-3}
No. 2, adjusted by k = 3.415	2.45×10^{-5}	2.48×10^{-5}
Load C		
Select Structural	1.10×10^{-4}	1.11×10^{-4}
No. 2	3.03×10^{-3}	3.04×10^{-3}
No. 2, adjusted by k = 2.910	1.10×10^{-5}	1.11×10^{-4}
Load D		
Select Structural	1.67×10^{-5}	1.68×10^{-5}
No. 2	2.44×10^{-3}	2.45×10^{-3}
No. 2, adjusted by k = 4.565	1.68×10^{-5}	1.68×10^{-5}

Table 4.4 The tabulated values are the probabilities of failure for Select Structural and No. 3 grades of 2" x 8" Hem-Fir lumber when utilized as a rafter or a floor joist as described in Chapter 3. The Weibull parameters, μ and σ of the No. 3 lumber grade are adjusted by k until the probabilities of failure are the same as the Select Structural probabilities of failure. The distribution of the dead plus snow load combination is lognormal. The extreme value type I distribution is the distribution of the dead plus floor live load combination.

Design Situation	Probabilities of Failure Lower Limit	Upper Limit
Roof		
Select Structural	2.71×10^{-4}	2.72×10^{-4}
No. 3	2.66×10^{-4}	2.69×10^{-4}
No. 3, adjusted by k = 0.995	2.77×10^{-4}	2.80×10^{-4}
Floor		
Load A		
Select Structural	2.57×10^{-4}	2.58×10^{-4}
No. 3	2.14×10^{-4}	2.17×10^{-4}
No. 3, adjusted by k = 0.985	2.51×10^{-4}	2.54×10^{-4}
Load B		
Select Structural	2.45×10^{-5}	2.46×10^{-5}
No. 3	2.00×10^{-5}	2.03×10^{-5}
No. 3, adjusted by k = 0.980	2.46×10^{-5}	2.50×10^{-5}
Load C		
Select Structural	1.10×10^{-4}	1.11×10^{-4}
No. 3	1.03×10^{-4}	1.04×10^{-4}
No. 3, adjusted by k = 0.995	1.08×10^{-4}	1.10×10^{-4}
Load D		
Select Structural	1.67×10^{-5}	1.68×10^{-5}
No. 3	7.84×10^{-7}	8.00×10^{-7}
No. 3, adjusted by k = 0.950	1.62×10^{-5}	1.65×10^{-5}

4.2 Relative Safety Between Lumber Sizes - Bending Strength Comparison

As previously mentioned, design values of lumber grades, sizes and species are tabulated (47) based on an implied equal safety that is inherent in each grade, size and species of lumber utilized in a design. Chapter 3 and Section 4.1 demonstrate differential reliability analyses of 2" x 8" Hem-Fir lumber by grade. This section demonstrates the differential reliability analysis of No. 1 Hem-Fir by size. For this analysis the bending strength of 2" x 6", 2" x 8", and 2" x 10" No. 1 Hem-Fir is utilized. These lumber sets reflect a 12 percent average moisture content (23). The parameters are taken from Hoyle, et al. (33) and are shown in Table 4.5. The differences in the bending strength of the No. 1 grade of Hem-Fir based on size are depicted in Figures 4.3 and 4.4. The 2" x 8" No. 1 Hem-Fir is arbitrarily chosen as the reference material. In other words, in the reliability analysis, the probabilities of failure of the 2" x 6" and 2" x 10" lumber will be compared to the probabilities of failure of the 2" x 8" lumber.

The design situations are the same as previously utilized in the analysis of relative safety between lumber grades. For the combination of dead plus snow load, a roof constructed with low slope rafters is assumed as the design situation. For the dead plus floor live load combination, a typical floor joist construction is assumed as the design situation. Both of these design cases are reasonable since all three lumber sizes are typically used in these situations based on the required span. Depending upon joist spacing, span and grade, deflection may, in fact, be the controlling design criteria; however, stress can also

Table 4.5 The tabulated values are the Weibull parameters and the fifth percentile values of No. 1 Hem-Fir lumber by size (33).

	Weibull Parameters			Fifth Percentile
	μ (ksi)	σ (ksi)	η	(ksi)
2" x 6"	1.926	3.815	1.511	2.460
2" x 8"	1.352	4.215	1.713	2.096
2" x 10"	1.514	2.284	1.365	1.773

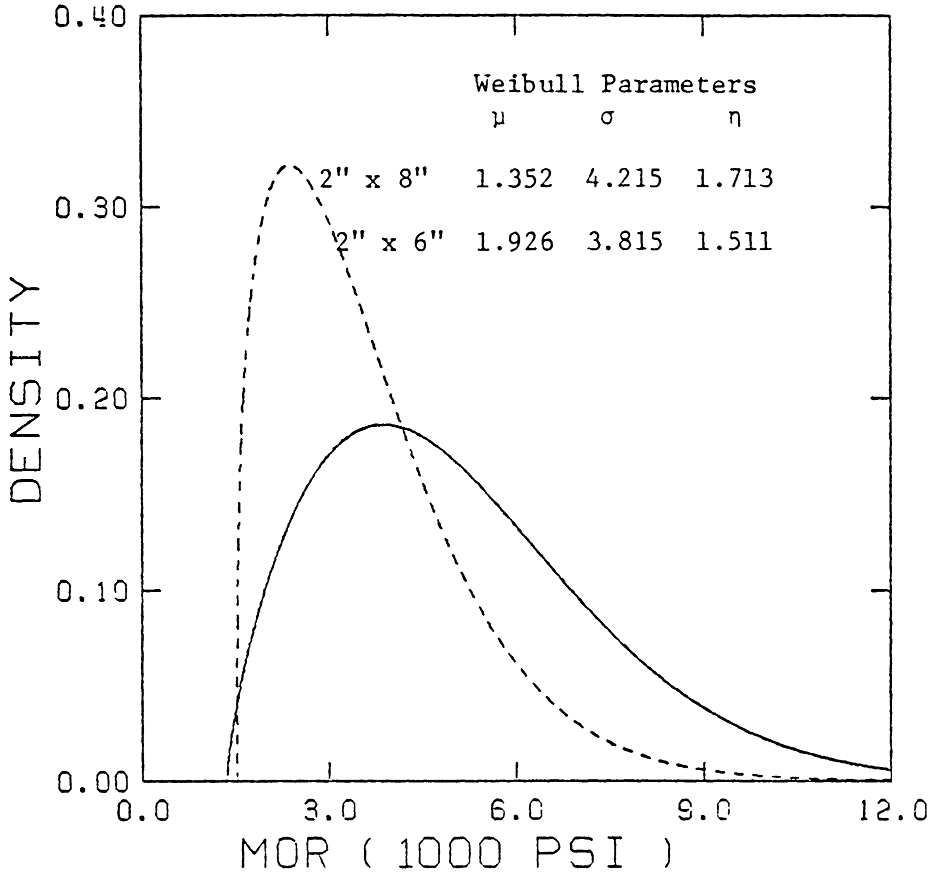

Figure 4.3 The Weibull functions describing 2" x 8" and 2" x 6" No. 1 Hem-Fir lumber are shown (33). The 2" x 8" lumber, shown as the solid line, serves as the reference material. The parameters μ and σ are given in ksi.

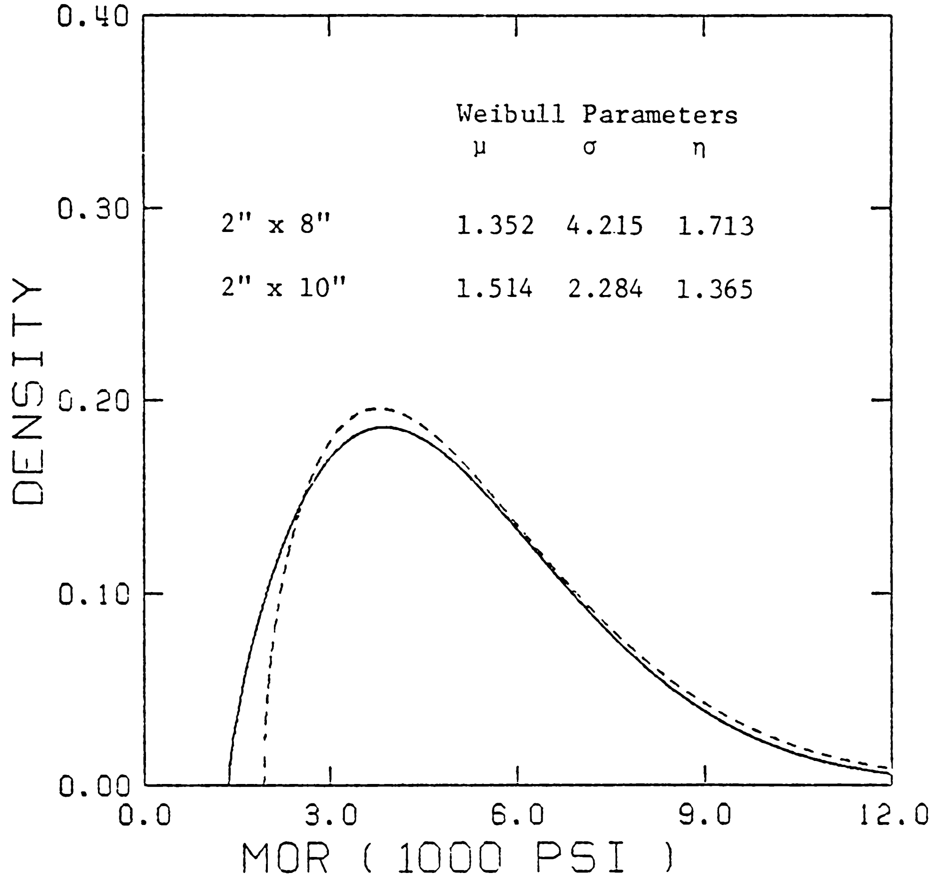

Figure 4.4 The Weibull functions describing 2" x 8" and 2" x 10" No. 1 Hem-Fir lumber are shown (33). The 2" x 8" lumber, shown by the solid line, is used to calculate the benchmark safety level. The parameters μ and σ are given in ksi.

control the design.

The calculated dead load for each design situation depends upon the lumber size utilized in the design. The calculated dead load parameters are shown in Table 4.6 based on each design case and lumber size. The snow load and floor live load parameters are given in Table 3.8. The nominal loads are as specified previously; 10 psf for dead load, 20 psf for roof snow load, and 40 psf for floor live load.

Utilizing the calculated fifth percentile values shown in Table 4.5, allowable design values are calculated for each design situation based on lumber size. Equations 3.36, 3.37, 3.46 and 3.47 are utilized to calculate the total load parameters employing the parameters listed in Tables 3.8 and 4.6. The parameters of the total load as well as the parameters of dead load, maximum lifetime floor live load, and maximum lifetime roof snow load are shown in Table 4.7.

The probabilities of failure of the three sizes of No. 1 Hem-Fir are calculated using the strength parameters shown in Table 4.5 and the associated total load parameters shown in Table 4.7. The Weibull strength parameters, μ and σ of the 2" x 6" and 2" x 10" lumber are altered by k until the probabilities of failure are the same as the failure probabilities of the 2" x 8" lumber. The results are shown in Tables 4.8 and 4.9 for the two comparisons for each load combination utilized in the analysis. The k values are a measure of the safety of each lumber size relative to the reference material. Each strength distribution was analyzed with its own calculated load distribution based on the fifth percentile of the strength distribution. As a result, the k factors

Table 4.6 The tabulated values are the calculated normalized means of the dead load, $\bar{D}/D_n$ of various sizes of No. 1 Hem-Fir used in two design situations. One design is a roof utilizing low slope rafters; the other design is a typical floor joist assembly. Detailed descriptions of the design situations are given in Chapter 3. The coefficient of variation is 0.10 in all cases.

	Mean Normalized Dead Load		
	2" x 6"	2" x 8"	2" x 10"
Roof	0.52	0.52	0.56
Floor	0.67	0.67	0.71

Table 4.7 The tabulated values are the total load parameters used in the reliability analysis of various sizes of No. 1 Hem-Fir lumber and the individual load parameters utilized to calculate the total load. F_b is the adjusted fifth percentile of the lumber parameters in Table 4.5. The 2" x 8" lumber is arbitrarily chosen as the reference material.

		Second Moment Parameters					
	F_b	Dead Load		Live Load		Total Load	
		μ_D	Ω_D	μ_L	Ω_L	μ_T	Ω_T
	(ksi)	(ksi)		(ksi)		(ksi)	
2" x 8"							
Snow	1.148	0.20	0.10	0.53	0.44	0.73	0.32
Load A	0.998	0.13	0.10	0.75	0.21	0.88	0.18
Load B	0.998	0.13	0.10	0.42	0.40	0.55	0.30
Load C	0.998	0.13	0.10	0.62	0.27	0.75	0.22
Load D	0.998	0.13	0.10	0.58	0.19	0.71	0.16
2" x 6"							
Snow	1.347	0.22	0.10	0.62	0.44	0.84	0.33
Load A	1.171	0.15	0.10	0.88	0.21	1.03	0.18
Load B	1.171	0.15	0.10	0.49	0.40	0.64	0.31
Load C	1.171	0.15	0.10	0.73	0.27	0.88	0.22
Load D	1.171	0.15	0.10	0.68	0.19	0.83	0.16
2" x 10"							
Snow	0.971	0.18	0.10	0.45	0.44	0.63	0.31
Load A	0.841	0.12	0.10	0.63	0.21	0.75	0.18
Load B	0.841	0.12	0.10	0.35	0.40	0.47	0.30
Load C	0.841	0.12	0.10	0.53	0.27	0.65	0.22
Load D	0.841	0.12	0.10	0.49	0.19	0.61	0.15

Table 4.8 The tabulated values are the probabilities of failure for 2" x 8" and 2" x 6" No. 1 Hem-Fir lumber (33) when utilized as a rafter or a floor joist as described in Chapter 3. The Weibull parameters, μ and σ of the bending strength of the 2" x 6" lumber are adjusted by k until the probabilities of failure are the same as the failure probabilities of the reference lumber, the 2" x 8" Hem-Fir. The distribution of load acting on the roof is lognormal and the distribution of load acting on the floor is extreme value type I.

Design Situation	Probabilities of Failure Lower Limit	Upper Limit
Roof		
2" x 8"	1.03×10^{-4}	1.03×10^{-4}
2" x 6"	4.85×10^{-5}	4.87×10^{-5}
2" x 6", adjusted by k = 0.930	1.04×10^{-4}	1.04×10^{-4}
Floor		
Load A		
2" x 8"	4.47×10^{-5}	4.50×10^{-5}
2" x 6"	1.06×10^{-5}	1.07×10^{-5}
2" x 6", adjusted by k = 0.905	4.38×10^{-5}	4.41×10^{-5}
Load B		
2" x 8"	4.29×10^{-6}	4.33×10^{-6}
2" x 6"	1.42×10^{-6}	1.43×10^{-6}
2" x 6", adjusted by k = 0.920	4.35×10^{-6}	4.38×10^{-6}
Load C		
2" x 8"	2.03×10^{-5}	2.05×10^{-5}
2" x 6"	5.46×10^{-6}	5.50×10^{-6}
2" x 6", adjusted by k = 0.910	1.98×10^{-5}	1.99×10^{-5}
Load D		
2" x 8"	8.23×10^{-7}	8.33×10^{-7}
2" x 6"	8.03×10^{-8}	8.10×10^{-8}
2" x 6", adjusted by k = 0.885	8.19×10^{-7}	8.27×10^{-7}

Table 4.9 The tabulated values are the probabilities of failure for 2" x 8" and 2" x 10" No. 1 Hem-Fir lumber (33) when utilized as a rafter or a floor joist as described in Chapter 3. The Weibull parameters, μ and σ of the bending strength of the 2" x 10" lumber are adjusted by k until the probabilities of failure are the same as the probabilities of failure of the reference lumber, the 2" x 8" Hem-Fir. The load distribution acting on the roof is lognormal and the load distribution acting on the floor is extreme value type I.

Design Situation	Probabilities of Failure Lower Limit	Upper Limit
Roof		
2" x 8"	1.03×10^{-4}	1.03×10^{-4}
2" x 10"	2.98×10^{-5}	3.00×10^{-5}
2" x 10", adjusted by k = 0.900	1.01×10^{-4}	1.02×10^{-4}
Floor		
Load A		
2" x 8"	4.47×10^{-5}	4.50×10^{-5}
2" x 10"	6.96×10^{-6}	7.03×10^{-6}
2" x 10", adjusted by k = 0.880	4.63×10^{-5}	4.68×10^{-5}
Load B		
2" x 8"	4.29×10^{-6}	4.33×10^{-6}
2" x 10"	7.86×10^{-7}	7.93×10^{-7}
2" x 10", adjusted by k = 0.890	4.17×10^{-6}	4.21×10^{-6}
Load C		
2" x 8"	2.03×10^{-5}	2.05×10^{-5}
2" x 10"	4.60×10^{-6}	4.64×10^{-6}
2" x 10", adjusted by k = 0.900	2.06×10^{-5}	2.08×10^{-5}
Load D		
2" x 8"	8.23×10^{-7}	8.33×10^{-7}
2" x 10"	1.84×10^{-8}	1.86×10^{-8}
2" x 10", adjusted by k = 0.830	8.71×10^{-7}	8.83×10^{-7}

demonstrate the effect of the shape of the strength distribution on the safety of each lumber size relative to the reference material.

4.3 The Effect of Rate of Loading on Tensile Strength

A farm truss with no ceiling has a very low bottom chord dead load. The lower chord is therefore subjected to mostly tensile stress by the conventional pin joint analysis. This contention is supported by the following examples taken from Woeste, et al. (70).

The nominal loads for a truss utilized in a farm application in a 20 psf ground snow load region are 16 psf for snow, 4 psf for top chord dead load and 1 psf for bottom chord dead load. The ANSI standard A58.1-81(4) specifies an importance factor, I of 0.80 for category IV structures. Category IV structures include all structures that represent a low hazard to human life or to property in the event of failure. Agricultural buildings comprise a major portion of this category. Therefore the usual 20 psf nominal snow load is reduced to 16 psf.

Utilizing these loads, a double W truss constructed of No. 1 KD Southern Yellow Pine with a 2" x 10" top chord and a 2" x 12" bottom chord was analyzed. The truss had a 54'4" span and a 3-in-12 slope. A combined stress index, CSI, of 0.892 for the top panel point, 0.862 for the top mid-panel point and 1.000 for the bottom panel was calculated. Hence, the lower chord is the critical member for this design. The stress components of the interaction equation for the lower chord were calculated separately. The bending stress component of the interaction equation was 0.02 while the tensile stress component was 0.98.

A similar case study utilized a farm truss constructed of 1950-1.7E MSR lumber with a 2" x 10" top chord and a 2" x 8" bottom chord. The span was 59'2" and the slope was 3-in-12. Resulting from the analysis, the top panel point CSI was 0.886, the top mid-panel point CSI was 0.907, and the bottom panel CSI was 0.999. As in the previous example, the bottom chord is the critical structural member of the truss based on the combined stress index. In the bottom chord, the tensile stress component calculated from the interaction equation was 0.971 and the bending stress component was 0.028.

It is reasonable to assume that the bottom chord of a farm truss is subject to tensile stress only, thus lumber tested in tension can be analyzed with a reliability approach. In this case, the design situation is assumed to be a double W farm truss. A good design assumption is a truss located four feet on center with a clear span of 48 feet. The roof consists of 2" x 4" purlins placed two feet on center and 26 gauge galvanized steel roofing.

The nominal loads assumed in the design of a farm truss as previously mentioned are 16 psf for snow, 4 psf for top chord dead load and 1 psf for bottom chord dead load. Applying these loads to the farm truss, the calculated reaction in the bottom chord is 5053 lbs. If the nominal snow load is applied singly, the calculated tensile force in the bottom chord is 3850 lbs. Likewise, if the nominal top chord dead load and nominal bottom chord dead load are applied separately, the calculated tensile forces in the bottom chord are 962 lbs. and 240 lbs., respectively.

These values are substituted for the nominal loads in the calculations of the second moment parameters.

Based on the design situation, an actual top chord dead load of 1.94 psf and a bottom chord dead load of 0.51 psf were calculated. Dividing by the nominal dead loads, 4 psf and 1 psf, respectively, the normalized dead load parameters for the top and bottom chords are

$$\bar{D}/D_{n_T} = 0.49 \tag{4.1}$$

$$\bar{D}/D_{n_B} = 0.51 \tag{4.2}$$

$$\Omega_D = 0.10 \tag{4.3}$$

The contrasting lumber sets to be analyzed are 2" x 6" tension laminating material which was tested in tension parallel to grain by procedures outlined in ASTM D198 (6) except that three rates of loading were used (24). Tension laminating material would never be used as a truss chord but it would respond to the influence of rate of loading in somewhat the same way as the Select Structural grade which is a common lower chord farm truss lumber. The rates of loading were the standard rate, 10 times the standard rate, and 25 times the standard rate. ASTM D198 (6) specifies the standard rate of loading for tension tests to be greater than 5 minutes but no more than 20 minutes. The Weibull parameters of the lumber tested at the three rates of loading and the fifth percentile values are given in Table 4.10. Figures 4.5 and 4.6 show the Weibull density functions of the tensile strength of the lumber loaded at 10 times the standard rate and at 25 times the standard rate each compared to the Weibull density function of lumber loaded at the standard rate.

Faster rates of loading are desired; therefore, tensile strengths at various rates need to be standardized with respect to the present allowable rate of loading. In order to compare the tensile strengths by reliability methods, the load distribution and calculated load parameters associated with the tensile strengths of lumber loaded at the standard rate are utilized. The failure probability of the tensile strength loaded at the standard rate is the benchmark reliability level. The probability distributions of the tensile strengths at other rates of loading are then utilized with the load distribution and parameters associated with the standard loading rate to obtain comparative probabilities of failure. If the usual technique of altering the resistance distribution parameters by k is utilized until equal failure probabilities are obtained, the resulting k factors are the required adjustment factors to equate faster loading rates to the standard loading rate.

To calculate the total load parameters, it was assumed that top and bottom chord dead loads can be combined with the roof snow load in a manner similar to equations 3.34 and 3.35. Since the dead load coefficients of variation are small compared to the snow load coefficient of variation, it is believed that this assumption is valid. Using equations 3.34 and 3.35 for the parameters of total load associated with

Table 4.10 The tabulated values are the Weibull parameters and calculated fifth percentile of Douglas fir 2" x 6" tension laminating material. The lumber was tested in tension using three rates of loading (24).

Rate of Loading	Weibull Parameters μ (ksi)	σ (ksi)	η	Fifth Percentile (ksi)
Standard	3.780	3.682	1.803	4.489
10 x Standard	1.283	7.396	2.644	3.688
25 x Standard	2.334	5.821	2.567	4.164

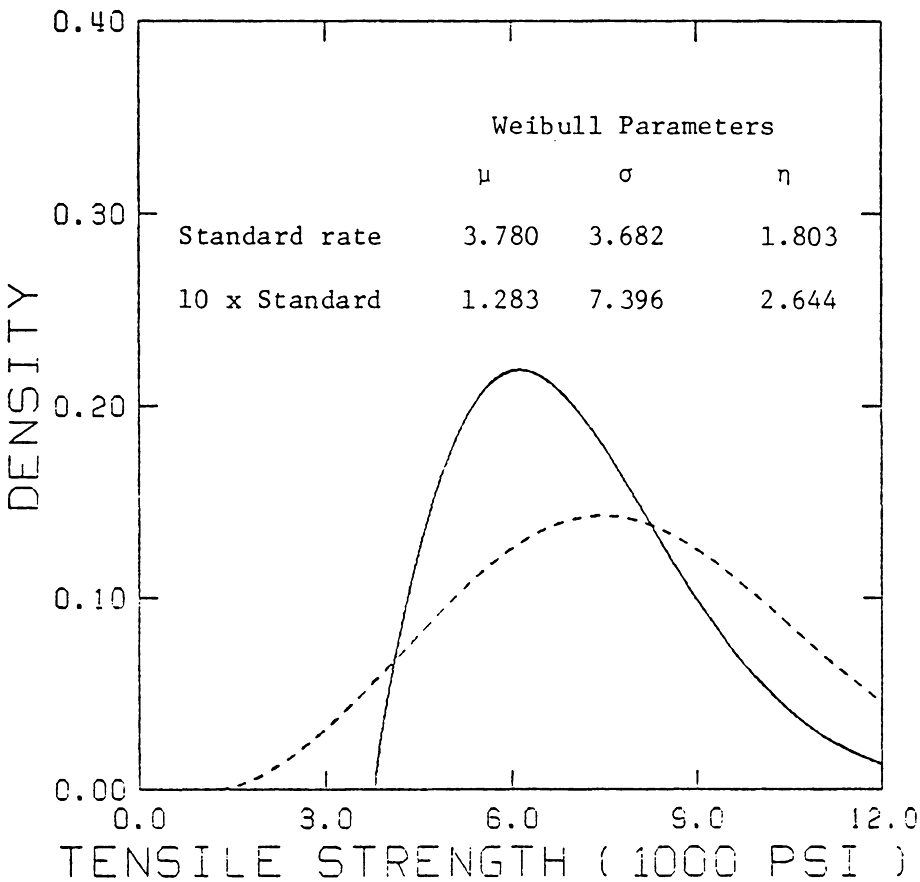

Figure 4.5 The 3-parameter Weibull functions describe 2" x 6" Douglas-fir tension laminating material tested at a standard rate of loading and at 10 times the standard rate. The lumber tested at the standard rate, represented by the solid line, serves as the reference material. The parameters μ and σ are given in ksi.

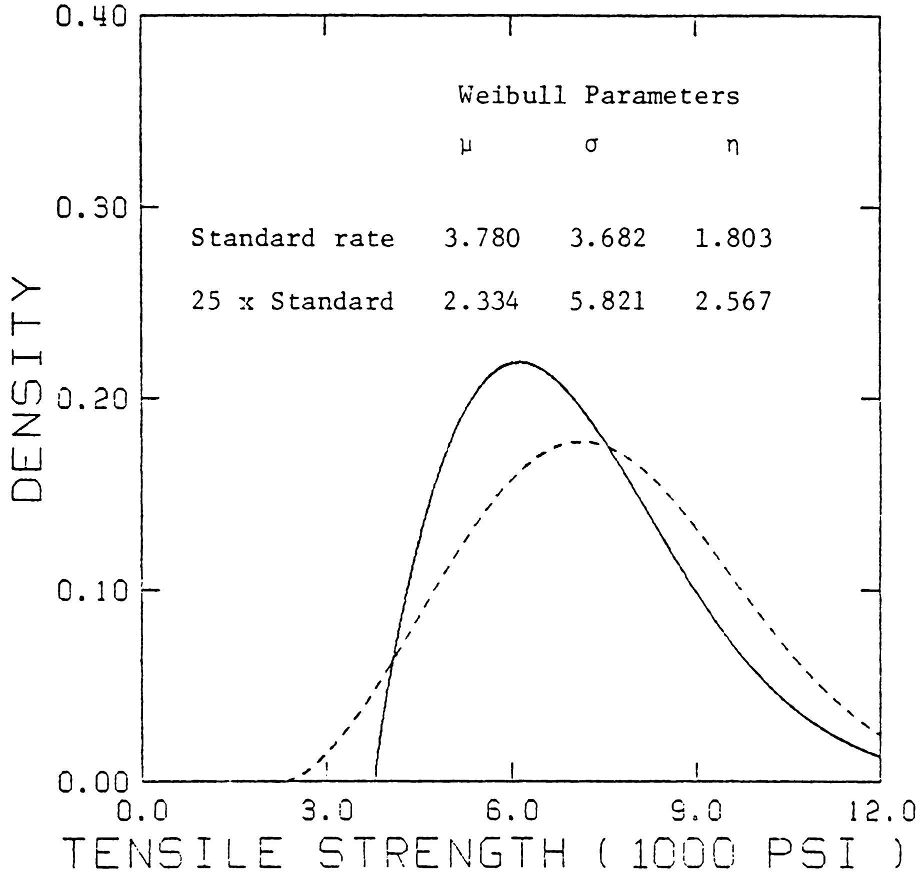

Figure 4.6 The 3-parameter Weibull functions describe 2" x 6" Douglas-fir tension laminating material tested at a standard rate of loading and at 25 times the standard rate. The lumber tested at the standard rate, represented by the solid line, is the material used to establish the benchmark safety level. The parameters μ and σ are given in ksi.

the standard loading rate the result is

$$\mu_T = \frac{241}{5053}(0.51)\ (2.458) + \frac{962}{5053}(0.49)\ (2.458) + \frac{3850}{5053}(0.69)(2.458)$$

$$= 0.06 + 0.23 + 1.29 = 1.58 \text{ ksi} \qquad (4.4)$$

$$\Omega_T = \frac{\left\{[0.06(.10)]^2 + [0.23(.10)]^2 + [1.29(.44)]^2\right\}^{\frac{1}{2}}}{1.58} = 0.36 \qquad (4.5)$$

The distribution of the total load is assumed to be lognormal. F_b is the fifth percentile of the lumber loaded at the standard rate adjusted by various factors. These factors include the general adjustment factor and the allowable 15 percent increase for snow.

The probabiliites of failure for the 2" x 6" lumber tested in tension at various loading rates are calculated and shown in Table 4.11. The Weibull parameters, μ and σ of the faster loading rates are altered by k until the probabilities of failure are the same as the probabilities of failure of the lumber loaded at the standard rate. The probabilities of failure for the loading rates altered by k and the k factors are also shown in Table 4.11.

The conventional analytical technique to compare two or more sets of lumber is to calculate the fifth percentile of each set and compare the values. For the tensile strength lumber, the fifth percentiles of each data set are shown in Table 4.10. A conventional adjustment factor is the ratio, r of the fifth percentile of the faster loading rate strength distribution to the fifth percentile of the standard test strength distribution. The fifth percentile values and the ratio values, r are

Table 4.11 The tabulated values are the probabilities of failure calculated for 2" x 6" tension laminating material (24) which simulates Select Structural with respect to rate of loading used as the bottom chord of a farm truss under a dead plus snow load combination. The load distribution is assumed to be lognormal.

Rate of Loading	Probabilities of Failure Lower Limit	Upper Limit
Standard	1.33×10^{-4}	1.33×10^{-4}
10 x Standard	2.28×10^{-3}	2.28×10^{-3}
10 x Standard, adjusted by k = 1.585	1.32×10^{-4}	1.32×10^{-4}
25 x Standard	4.67×10^{-4}	4.68×10^{-4}
25 x Standard, adjusted by k = 1.165	1.34×10^{-4}	1.34×10^{-4}

shown in Table 4.12.

Wood (71) derived an equation describing the increase in lumber strength as a function of the duration of stress. The equation is

$$y = \frac{108.4}{x^{0.04635}} + 18.3 \qquad (4.6)$$

where

y = lumber stress expressed as a percentage of the standard test strength

x = the duration of the stress in seconds.

The assumed standard duration of stress is 7.5 minutes in this equation (71). The above equation was derived based on Douglas fir lumber strength data; however, Wood (71) states that the expression may be used for other lumber species used in construction.

Since the standard rate of loading for tensile strength tests is between 5 and 20 minutes, an average time to failure of 10 minutes is assumed for this analysis. Ten times the standard rate is therefore 1 minute and 25 times the standard rate is 24 seconds. Substituting these values into equation 4.6 yields the lumber stress expressed as a percentage of the standard test strength. These values are shown in column three of Table 4.12. The conversion factors from the differential reliability method are given in the last column of Table 4.12 as a direct comparison to the results obtained from a conventional analysis and Wood's equation.

Table 4.12 The tabulated values are the calculated results from various methods used to compare faster rates of loading to the standard rate of loading. The value of y is calculated using equation 4.6 and the tabulated x value. The ratio, r is the conversion factor resulting from a conventional fifth percentile analysis while the reciprocal of the k factor is the conversion factor based on a reliability analysis.

Load Rate	Duration of Stress x (sec)	Percentage of Standard Test[1] Strength y	Fifth Percentile (ksi)	r	$\frac{1}{k}$
Standard	600	98.9	4.489		
10 x Standard	60	108.0	3.688	0.822	0.631
25 x Standard	24	111.9	4.164	0.928	0.858

1
The duration of stress for the standard test used in the development of equation 4.6 is 7.5 minutes (71).

4.4 The Effect of Failure Mode on Allowable Stresses

Lumber has different allowable stresses in bending, tension, and compression. Using lumber tested in bending, tension and compression, allowable stresses can be calculated based on the concept of equal reliability. If lumber tested in bending is chosen as the reference material, the allowable stresses for the lumber tested in tension and compression can be calculated based on k factors and the allowable stress of the lumber tested in bending.

For this analysis, the contrasting lumber consists of 1650-1.5E MSR 2" x 4" Hem-Fir lumber (33) tested in bending, tension and compression, respectively. The grade indicates the lumber is machine stress rated with an allowable bending design stress of 1650 psi and a modulus of elasticity, E, of 1.5 million psi. The lognormal parameters of the strength distributions of the Hem-Fir lumber are shown in Table 4.13. The lumber tested in bending is chosen as the reference material. The distributions of lumber tested in tension and compression are graphically contrasted to the reference material in Figures 4.7 and 4.8, respectively.

Since a similar design situation is not apparent for all three cases based on a failure mode, the reliability analysis is conducted based on load parameters calculated for a roof snow load or a floor live load without consideration to the dead load. The assumption of a design situation gives meaning to reliability comparisons; however, it is believed that the use of load distributions reflecting live loads is sufficient for this comparison. The parameters are calculated by

Table 4.13 The tabulated values are the lognormal parameters and fifth percentiles of 2" x 4" 1650-1.5E Douglas fir lumber tested in bending, tension, and compression (33). The parameters λ and ζ are the mean and corresponding standard deviations of the logarithms of the lumber strengths given in ksi.

	Lognormal Parameters λ	ζ	Fifth Percentile (ksi)
Bending	1.758	0.351	3.250
Tension	1.525	0.387	2.430
Compression	1.516	0.178	3.400

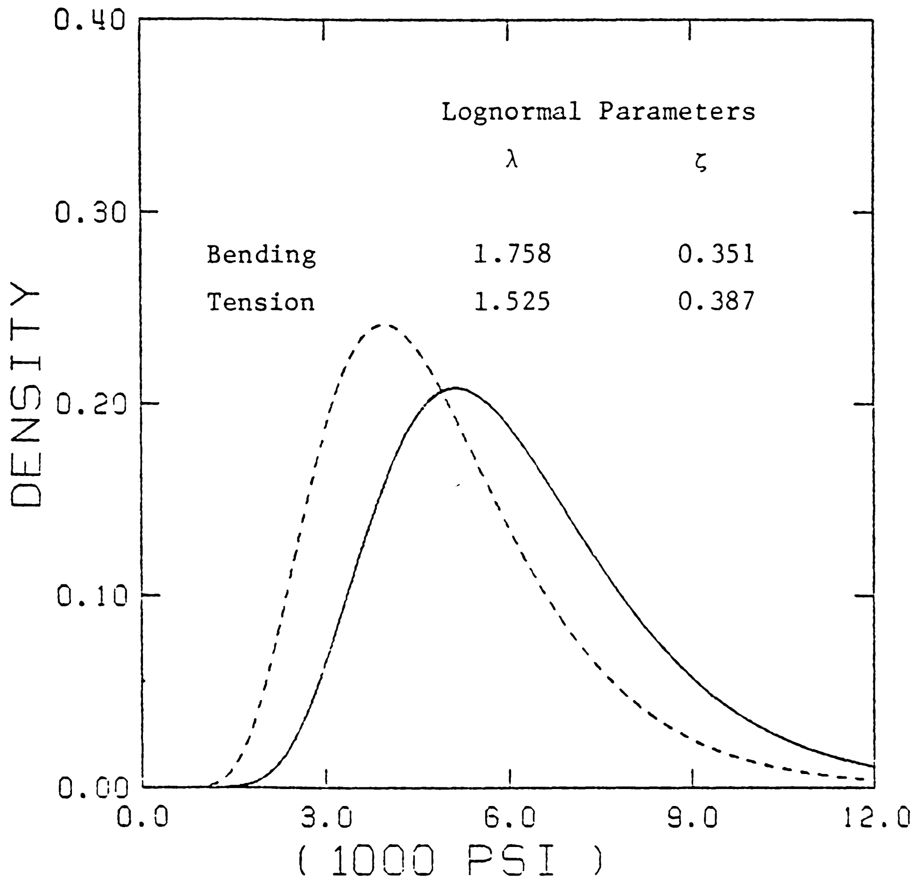

Figure 4.7 The lognormal functions describe 2" x 4" 1650-1.5E Hem-Fir lumber tested in bending and in tension (33). The lumber tested in bending, shown as the solid line, is the material used to establish the benchmark safety level. The lognormal parameters are the means and corresponding standard deviations of the logarithms of the data given in ksi.

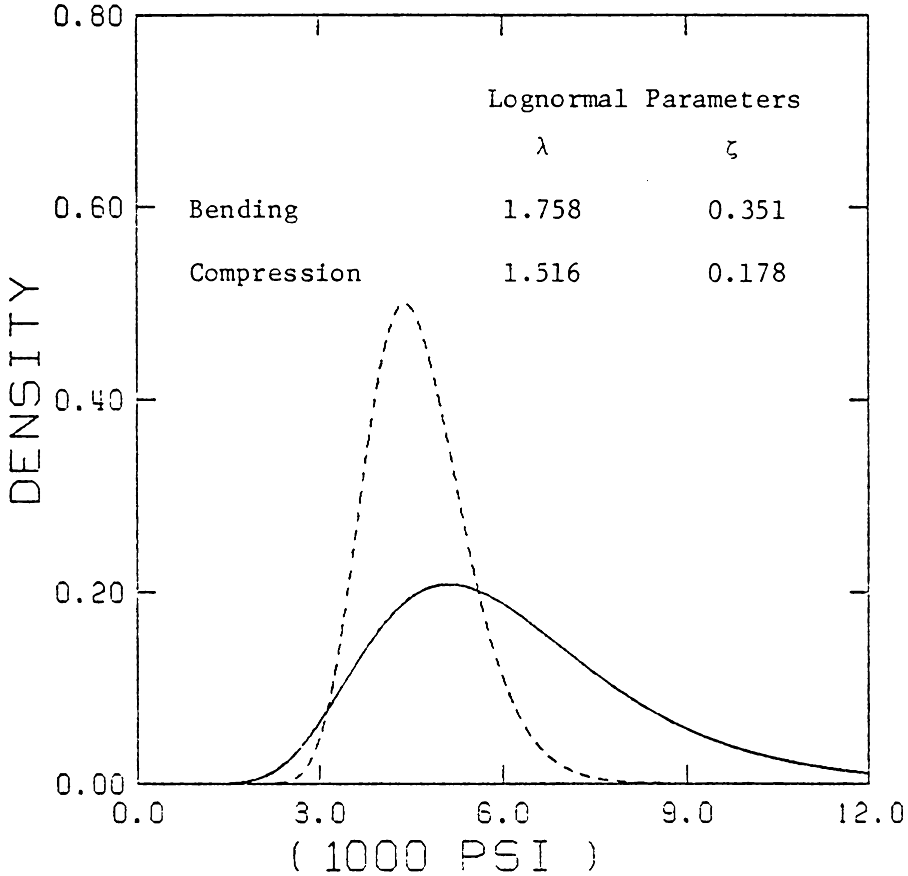

Figure 4.8 The lognormal functions describe 2" x 4" 1650-1.5E Hem-Fir lumber tested in bending and in compression (33). The lumber tested in bending, shown as a solid line, is chosen as the reference material. The lognormal parameters are the means and corresponding standard deviations of the logarithms of the data given in ksi.

$$\mu_T = \bar{X}/X_n \ (F_b) \tag{4.7}$$

$$\Omega_T = \Omega_X \tag{4.8}$$

where

μ_T = the mean of the total load expressed in psi

Ω_T = the coefficient of variation of the total load

$\bar{X}/X_n$ = the mean of the roof snow load or floor live load as given in Table 3.8

Ω_X = the coefficient of variation of the roof snow load or floor live load as given in Table 3.8

F_b = the allowable design stress adjusted for duration of load expressed in psi

The lumber tested in tension and compression is compared to the lumber tested in bending. Therefore, the calculated load parameters utilized in the reliability analysis are the load parameters associated with the Hem-Fir tested in bending. The calculated load parameters based on equations 4.7 and 4.8 are shown in Table 4.14.

The probability of failure analysis is conducted on the three lumber sets. The failure probabilities for the lumber tested in bending and tension are given in Table 4.15. Table 4.16 shows the failure probabilities of the lumber tested in bending and compression. The k factors for the tension and compression cases are also given in Tables 4.15 and 4.16 with the calculated probabilities of failure obtained from the k factor

Table 4.14 The tabulated values are the calculated load parameters used in the probability of failure analysis of 2" x 4" 1650-1.5E Douglas fir which has been tested in bending, tension and compression. The load distributions are also given.

	Distribution	Second Moment Parameters	
		μ_T (ksi)	Ω_T
Snow	Lognormal	1.23	0.44
Floor			
Load A	Type I	1.45	0.21
Load B	Type I	0.81	0.40
Load C	Type I	1.22	0.27
Load D	Type I	1.23	0.19

Table 4.15 The tabulated values are the probabilities of failure calculated for 2" x 4" 1650-1.5E lumber tested in bending and tension (33). No design situation is specified. The loads utilized in the analysis are the snow and live loads described in Chapter 3.

	Probability of Failure Lower Limit	Upper Limit
Snow		
Bending	1.38×10^{-3}	1.39×10^{-3}
Tension	6.93×10^{-3}	6.94×10^{-3}
Tension, adjusted by k = 1.355	1.38×10^{-3}	1.39×10^{-3}
Floor		
Load A		
Bending	3.56×10^{-4}	3.58×10^{-4}
Tension	3.99×10^{-3}	4.00×10^{-3}
Tension, adjusted by k = 1.395	3.60×10^{-4}	3.62×10^{-4}
Load B		
Bending	3.58×10^{-5}	3.60×10^{-5}
Tension	4.19×10^{-4}	4.21×10^{-4}
Tension, adjusted by k = 1.400	3.54×10^{-5}	3.56×10^{-5}
Load C		
Bending	1.88×10^{-4}	1.89×10^{-4}
Tension	2.03×10^{-3}	2.04×10^{-3}
Tension, adjusted by k = 1.395	1.88×10^{-4}	1.88×10^{-4}
Load D		
Bending	2.61×10^{-5}	2.62×10^{-5}
Tension	5.61×10^{-4}	5.64×10^{-4}
Tension, adjusted by k = 1.425	2.64×10^{-5}	2.66×10^{-5}

Table 4.16 The tabulated values are the probabilities of failure calculated for 2" x 4" 1650-1.5E Hem-Fir lumber tested in bending and compression (33). No design situation is specified. The loads utilized in the analysis are the snow and live loads described in Chapter 3.

	Probabilities of Failure	
	Lower Limit	Upper Limit
Snow		
Bending	1.38×10^{-3}	1.39×10^{-3}
Compression	1.11×10^{-3}	1.11×10^{-3}
Compression, adjusted by k = 0.970	1.38×10^{-3}	1.38×10^{-3}
Floor		
Load A		
Bending	3.56×10^{-4}	3.58×10^{-4}
Compression	4.41×10^{-4}	4.43×10^{-4}
Compression, adjusted by k = 0.840	3.64×10^{-4}	3.66×10^{-4}
Load B		
Bending	3.58×10^{-5}	3.60×10^{-5}
Compression	5.31×10^{-6}	5.33×10^{-6}
Compression, adjusted by k = 0.850	3.52×10^{-5}	3.53×10^{-5}
Load C		
Bending	1.88×10^{-4}	1.89×10^{-4}
Compression	3.05×10^{-5}	3.06×10^{-5}
Compression, adjusted by k = 0.855	1.85×10^{-4}	1.86×10^{-4}
Load D		
Bending	2.61×10^{-5}	2.62×10^{-5}
Compression	5.14×10^{-7}	5.17×10^{-7}
Compression, adjusted by k = 0.905	2.50×10^{-5}	2.51×10^{-5}

adjustment.

A possible method to calculate allowable stresses for 2" x 4" lumber loaded in tension and compression is described as follows. Using the k factors listed in Tables 4.15 and 4.16, the allowable design values for tension and compression, respectively, could be calculated by

$$F_b = F_t \, k_{tb} \tag{4.9}$$

$$F_b = F_c \, k_{cb} \tag{4.10}$$

where

F_b = the calculated allowable bending stress from the data (33) in psi

F_t = the calculated allowable tensile stress in psi

F_c = the calculated allowable compressive stress in psi

k_{tb} = the k factor for conversion from tensile allowable strength to bending allowable strength as determined by the reliability analysis.

k_{cb} = the k factor for conversion from compressive allowable strength to bending allowable strength as determined by the reliability analysis.

The k values are given in Table 4.17.

The conventional analytical technique is to calculate the fifth percentiles and compare the values in a ratio of tensile or compressive strength to bending strength. These ratios could be denoted as r_{tb} and r_{cb} respectively. These ratios are calculated based on the resistance parameters of Table 4.13 and are given in the first footnote of Table 4.17. Also, footnote 2 of Table 4.17 gives the results of the conventional analysis based on the specified NDS (47) values. The allowable bending strength of 1650-1.5E MSR lumber is 1650 psi (47).

4.5 Relative Safety Between Lumber Grades - Tensile Strength Comparison

It has been demonstrated in Section 4.3 that the bottom chord of a farm truss with low dead load is subjected to mostly tensile stress. Therefore, the farm truss described in Section 4.3 under a snow load is utilized as the design situation. Section 4.3 lists the nominal loads for this situation as 16 psf for snow load, 4 psf for top chord dead load, and 1 psf for bottom chord dead load. The tensile forces present in the bottom chord of the truss due to the nominal loads are 5053 lbs. for total load, 3850 lbs. for snow load, 962 lbs. for top chord dead load, and 241 lbs. for bottom chord dead load. These values are substituted for the nominal loads in the equations used to calculate the total load parameters.

The lumber to be analyzed is machine stress rated 2" x 6" Hem-Fir (33) of 3 grades, 1650-1.5E, 2100-1.8E and 2400-2.0E. The strength parameters and the fifth percentile of each lumber grade are given in Table 4.18. Figures 4.9 and 4.10 highlight the differences in the MSR lumber grades compared to the 1650-1.5E grade of lumber. As previously mentioned, the design situation is the same as in Section 4.3; a double W truss, four feet on center, 48 foot clear span with 2" x 4" purlins

Table 4.17 The tabulated values are the differential reliability k factors describing the conversion from tensile or compressive strength to bending strength. The conversion factors, r_{tb} and r_{cb} resulting from a conventional fifth percentile analysis of the lumber parameters[1] given in Table 4.13 and the allowable values from NDS (47)[2] are also shown for comparison.

	Snow	Live			
		A	B	C	D
k_{tb}	1.355	1.395	1.400	1.395	1.425
k_{cb}	0.970	0.840	0.850	0.855	0.905

[1] The fifth percentiles of the lumber parameters in Table 4.13 are 3250 psi for bending, 2430 psi for tension and 3400 psi for compression. These values yield an r_{tb} of 1.34 and an r_{cb} of 0.96.

[2] The allowable tensile strength as given by NDS (47) is 1020 psi which yields a value of r_{tb} equal to 1.62. The allowable compressive strength as given by NDS (47) is 1320 psi which yields a value of r_{cb} equal to 1.25.

Table 4.18 The tabulated values are the lognormal parameters describing the distribution of tensile strength of 3 grades of 2" x 6" Hem-Fir (33). The parameters λ and ζ are the mean and standard deviation of the logarithms of tensile strengths expressed in ksi.

Grade	Lognormal Parameters λ	ζ	Fifth Percentile (ksi)
1650-1.5E	1.161	0.351	1.793
2100-1.5E	1.645	0.274	3.301
2400-2.0E	1.926	0.285	4.294

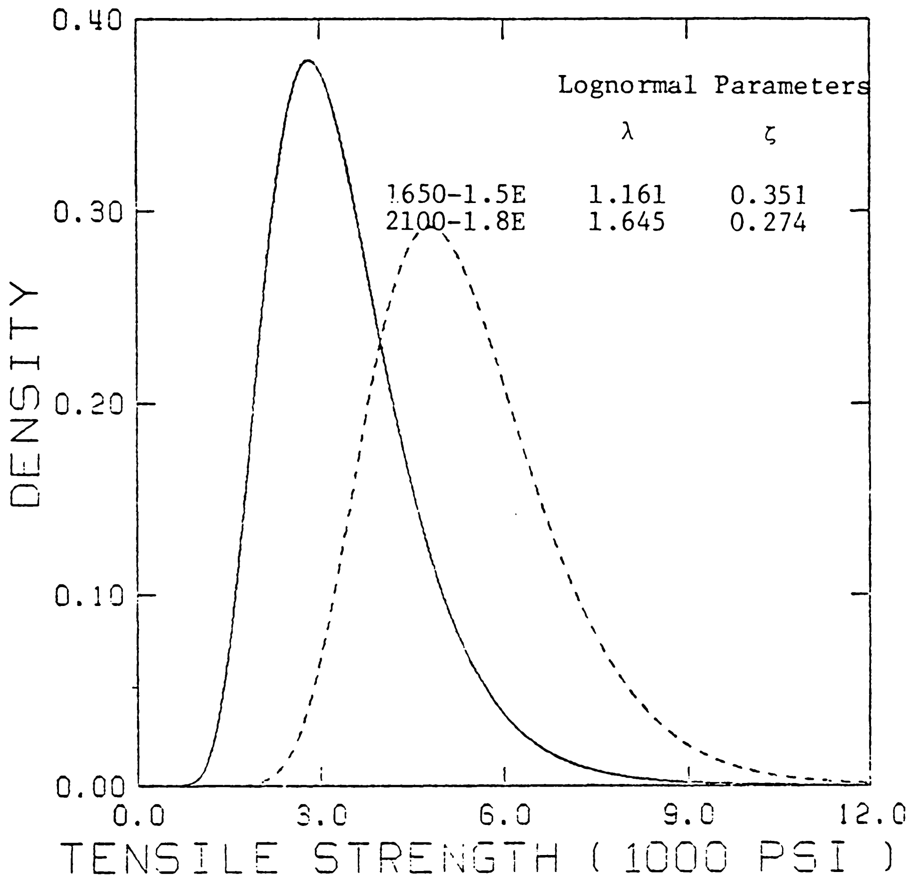

Figure 4.9 The lognormal functions describe two grades of 2" x 6" Hem-Fir lumber (33) tested in tension. The MSR grade, 1650-1.5E, represented as a solid line, serves as the reference material. The parameters λ and ζ are the mean and standard deviation of the logarithms of the tensile strengths given in ksi.

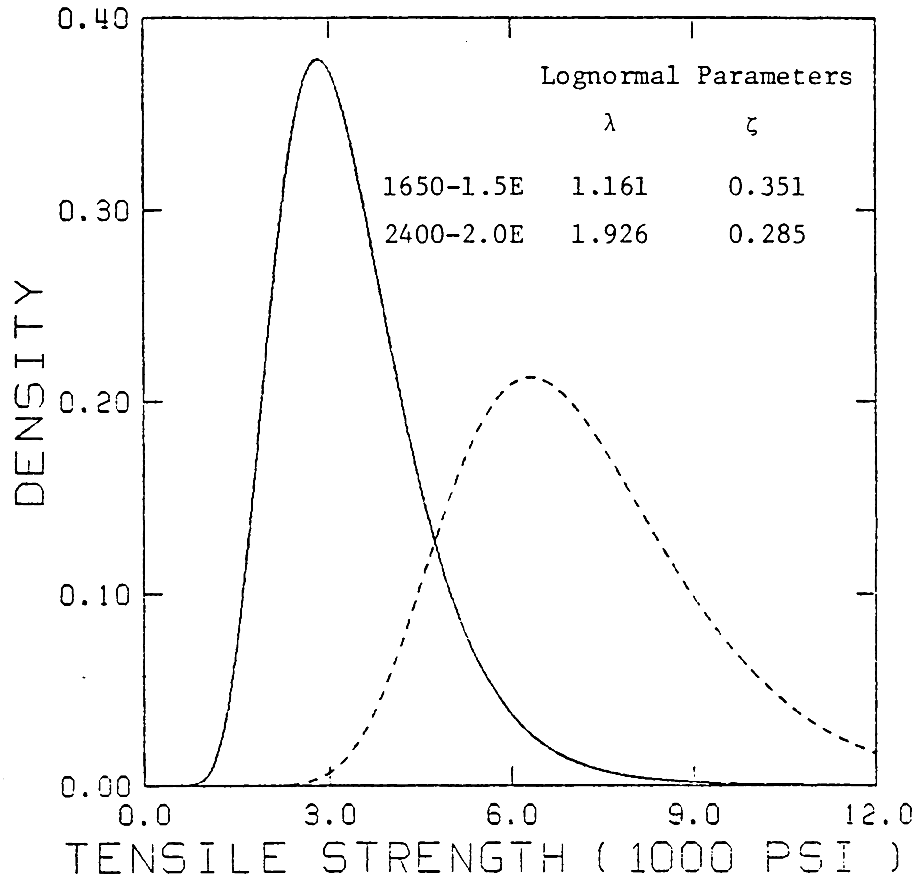

Figure 4.10 The lognormal functions describe two grades of 2" x 6" Hem-Fir lumber (33) tested in tension. The MSR grade, 1650-1.5E, represented as a solid line, is the material used to establish the benchmark safety level. The parameters λ and ζ are the mean and standard deviation of the logarithms of the tensile strengths given in ksi.

covered by 26 gauge galvanized steel. The calculated dead loads are 1.86 psf for top chord and 0.42 psf for bottom chord. The nominal dead load parameters for the two chords are

$$\bar{D}/D_{n_T} = 0.46 \qquad (4.11)$$

$$\bar{D}/D_{n_B} = 0.42 \qquad (4.12)$$

$$\Omega_D = 0.10 \qquad (4.13)$$

The 1650-1.5E lumber grade is chosen as the reference material. The load parameters are calculated following equations 4.4 and 4.5 for each of the lumber grades. The same logic concerning load distributions applies for this case as to the previously discussed cases of relative safety between grades and sizes based on bending strength. Each grade has its own associated load distribution calculated based on the adjusted fifth percentile value. The fifth percentile is adjusted by the general adjustment factor and the duration of load factor as described in Section 4.3. The load parameters and F_b are given in Table 4.19. The load distribution is assumed to be lognormal as in previous analyses.

Utilizing the parameters in Tables 4.18 and 4.19, the probability of failure analysis is conducted for each grade. The strength parameter, λ of the 2100-1.8E and 2400-2.0E lumber grades is incremented by the logarithm of k until failure probabilities result which are equal to the probabilities of failure of the reference lumber grade. These probabilities

Table 4.19 The tabulated values are the total load parameters and calculated allowable design value for each of the 3 grades of 2" x 6" Hem-Fir lumber (33) given in Table 4.18. The distribution of total load is assumed to be lognormal.

Grades	F_b (ksi)	Total Load Parameters μ_T (ksi)	Ω_T
1650-1.5E	0.982	0.62	0.37
2100-1.8E	1.808	1.15	0.37
2400-2.0E	2.351	1.49	0.37

of failure and the k factors are shown in Table 4.20. The k factors demonstrate that the shape of the distribution of tensile strength impacts upon the relative safety of each lumber grade. Since the load distribution is placed relative to the fifth percentile of the strength distribution in all three cases, the effect of sampling error is minimized.

4.6 Relative Safety Between Lumber Sizes - Tensile Strength Comparison

An analysis of the relative safety between lumber sizes as related to the tensile strength is conducted in the same manner as the analysis was conducted in Section 4.2. The design situation and nominal dead loads are the same as discussed in Sections 4.3 and 4.5. The contrasting lumber consists of 2" x 4", 2" x 6" and 2" x 8" 1650-1.5E Hem-Fir (33). The lognormal parameters describing the distribution of strength of each size are shown in Table 4.21. The dead load parameters calculated for the design situation according to lumber size are shown in Table 4.22. The snow load parameters are given in Chapter 3, Table 3.8. Figures 4.11 and 4.12 highlight the differences between the different sizes compared to the chosen reference material, the 2" x 4" lumber.

The total load parameters are calculated following equation 4.4 and 4.5. The parameters are listed for each lumber size in Table 4.23 with the calculated F_b. The distribution of total load is assumed to be lognormal. The failure probabilities and the k factors are calculated for all lumber sizes and the results are shown in Table 4.24. Since the load distribution is placed relative to the allowable fifth percentile of each

Table 4.20 The tabulated values are the probabilities of failure of different grades of 2" x 6" Hem-Fir (33) lumber used as a bottom chord in a farm truss as described in Section 4.3. The total load distribution is assumed to be lognormal.

Grade	Probabilities of Failure Lower Limit	Upper Limit
1650-1.5E	3.27×10^{-4}	3.29×10^{-4}
2100-1.8E	2.22×10^{-4}	2.23×10^{-4}
2100-1.8E, adjusted by k = 0.955	3.25×10^{-4}	3.26×10^{-4}
2400-2.0E	2.33×10^{-4}	2.34×10^{-4}
2400-2.0E, adjusted by k = 0.960	3.25×10^{-4}	3.26×10^{-4}

Table 4.21 The tabulated values are the lognormal parameters describing the distribution of tensile strength and the calculated fifth percentiles of 3 sizes of 1650-1.5E Hem-Fir (33). The parameters λ and ζ are the mean and standard deviation of the logarithms of tensile strength expressed in ksi.

Size	Lognormal Parameters λ	ζ	Fifth Percentile (ksi)
2" x 4"	1.525	0.387	2.431
2" x 6"	1.151	0.351	1.793
2" x 8"	1.174	0.292	2.001

Table 4.22 The tabulated values are the calculated mean dead loads of the top and bottom chords of a double W truss used in a farm application. The lumber is 1650-1.5E Hem-Fir (33) and the design is described in Section 4.3 in detail. The mean dead load is assumed to be the actual dead load normalized by the nominal dead load. The coefficient of variation of the dead loads is 0.10.

	Dead Load	
	Top Chord	Bottom Chord
2" x 4"	0.27	0.42
2" x 6"	0.42	0.46
2" x 8"	0.56	0.50

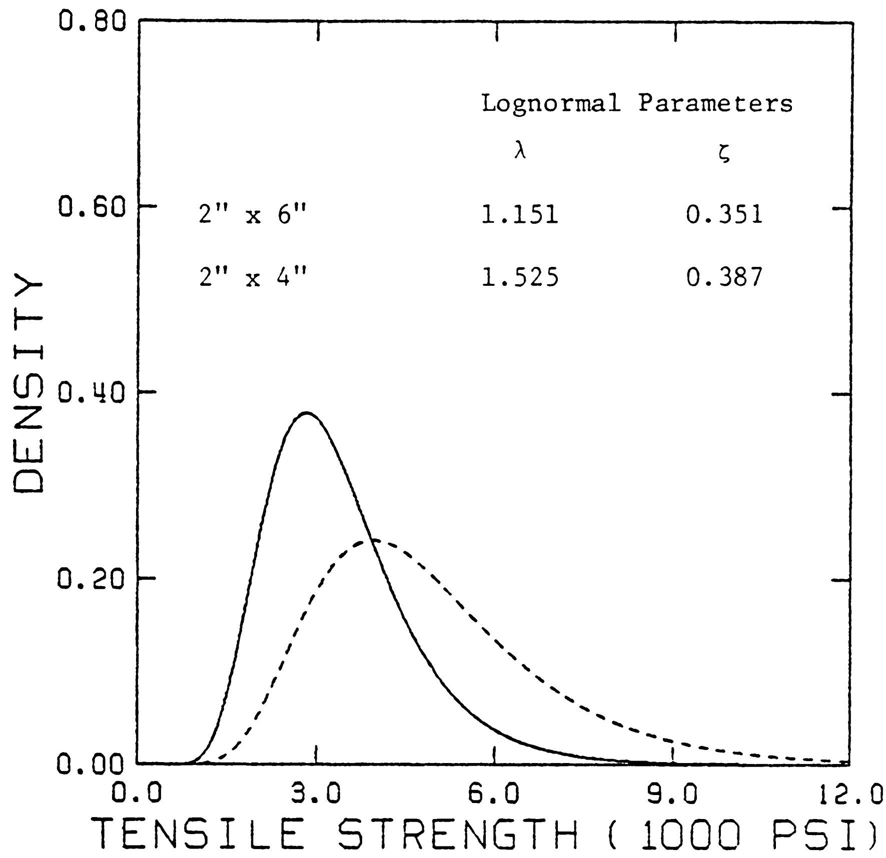

Figure 4.11 The lognormal functions describe two sizes of 1650-1.5E MSR Hem-Fir lumber (33) tested in tension. The 2" x 6" Hem-Fir, represented as a solid line, is the material used to establish the benchmark safety level. The parameters λ and ζ are the mean and standard deviation of the logarithms of the tensile strengths given in ksi.

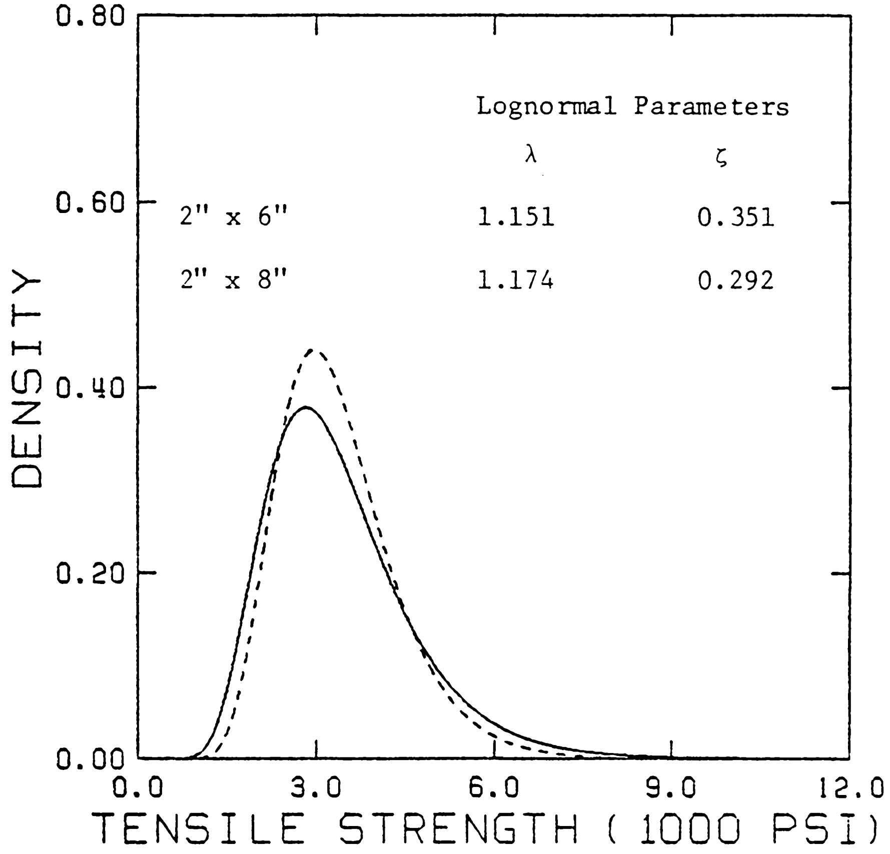

Figure 4.12 The lognormal functions describe two sizes of 1650-1.5E MSR Hem-Fir lumber (33) tested in tension. The 2" x 6" Hem-Fir, represented as a solid line, is the material used to establish the benchmark safety level. The parameters λ and ζ are the mean and standard deviation of the logarithms of the tensile strengths given in ksi.

Table 4.23 The tabulated values are the total load parameters and F_b for each of the 3 sizes of 1650-1.5E Hem-Fir lumber (33) given in Table 4.21. The distribution of total load is assumed to be lognormal.

Size	Total Load Parameters μ_T (ksi)	Ω_T	F_b (ksi)
2" x 4"	0.82	0.37	1.331
2" x 6"	0.62	0.37	0.982
2" x 8"	0.71	0.36	1.096

Table 4.24 The tabulated values are the probabilities of failure of different sizes of 1650-1.5E Hem-Fir lumber (33) used as the bottom chord in a farm truss as described in Section 4.3. The total load distribution is assumed to be lognormal.

Size	Probability of Failure Lower Limit	Upper Limit
2" x 6"	3.27×10^{-4}	3.29×10^{-4}
2" x 4"	3.79×10^{-4}	3.81×10^{-4}
2" x 4", adjusted by k = 1.020	3.31×10^{-4}	3.32×10^{-4}
2" x 8"	2.50×10^{-4}	2.52×10^{-4}
2" x 8", adjusted by k = 0.970	3.21×10^{-4}	3.22×10^{-4}

strength distribution, the resulting k factors demonstrate that the shape of the strength distribution could influence the relative safety of lumber loaded in tension. However, the 2" x 4" MSR lumber exhibits only 2.0 percent greater safety in use while the 2" x 8" MSR lumber is only 3.0 percent less safe in use than the 2" x 6" lumber. Based on the consequences of using these results in an engineering analysis, the safety of MSR lumber sizes tested in tension is similar.

4.7 The Relative Safety Between Lumber Grades and Sizes - Modulus of Elasticity Comparison

In a long column analysis the modulus of elasticity is the governing design factor. This is due to the fact that a long column will buckle at loads which are too low to cause compressive crushing (32). Conventional analytic design of a long column is based on Euler buckling theory.

In a truss, unbraced webs generally act as long columns. Therefore, a reliability analysis of the relative safety of lumber based on modulus of elasticity (E) data can be conducted assuming an unbraced truss web as the design situation. However, unlike distributions of strength, the distribution of E does not define a failure state. It is necessary to convert the modulus of elasticity into an allowable buckling stress based on Euler buckling theory in order to conduct the reliability analysis.

For the analysis of E data, the design situation is the farm truss previously described in Section 4.3. The truss is a double W truss, four feet on center with a 48' span. The web shown in Figure 4.13 is utilized to calculate the forces caused by the nominal loads. The basic assumptions are that the web is unbraced and the truss plates provide connections which can be modeled as pinned ends.

The nominal loads on a farm truss as previously mentioned are 16-4-1 psf. Utilizing these nominal loads, the compressive force in the web is calculated to be 933 lbs. for the total nominal load, 731 lbs. for the snow load, 183 lbs. for the top chord dead load and 19 lbs. for the bottom chord dead load. These forces are substituted for the nominal loads in the calculation of the load parameters utilized in the differential reliability analysis.

Two analyses are conducted. The first is a reliability analysis to determine the relative safety of long columns based on E data of differing lumber grades. The second analysis focuses on the relative safety based on E data of differing lumber sizes. The contrasting lumber sets include visually graded and machine stress rated lumber. The distribution parameters of E are given in Tables 4.25 through 4.28 grouped according to the comparisons made in the differential reliability analysis. The reference material is given in the first line of each table. Figures 4.14 through 4.22 graphically depict the differences in the distribution of E as related to the chosen reference material. Some of the lumber sizes and grades analyzed in this section would not be used as a truss web; however, the impact of an impractical design on the differential reliability analysis is believed to be negligible considering the comparative nature of the technique. All the lumber grades and

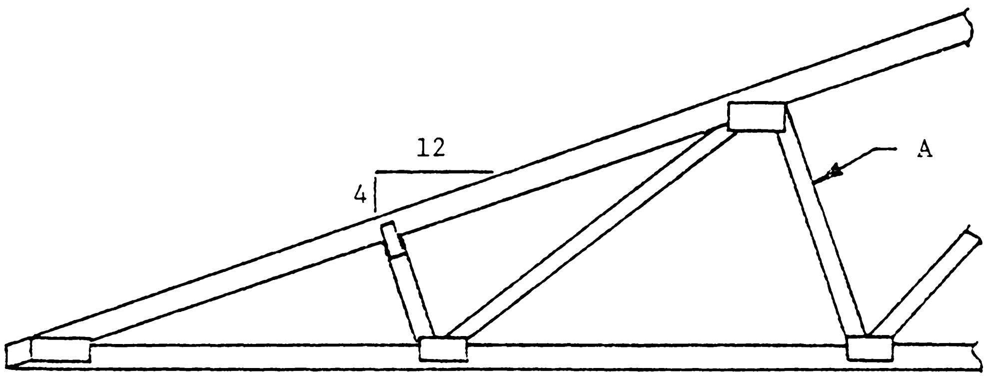

Figure 4.13 A section of a double W farm truss is shown. The forces in web A are calculated for use in the reliability analysis of E data assuming four different load situations. These four load situations are snow load only, top chord dead load only, bottom chord dead load only, and all three loads acting simultaneously.

Table 4.25 The tabulated values are the modulus of elasticity Weibull parameters of visually graded lumber and the allowable compressive stress, F_c, specified by NDS (47). The lumber is 2" x 8" Hem-Fir (33). The calculated mean, $\bar{E}$ is also given.

Grade	F_c (ksi)	Weibull Parameters μ (10^6psi)	σ (10^6 psi)	η	$\bar{E}$ (10^6 psi)
Select Structural	1.150	0.956	0.958	3.231	1.816
No. 1	1.050	0.967	0.791	2.408	1.668
No. 2	0.875	0.279	1.326	4.828	1.494
No. 3	0.550	0.741	0.835	2.536	1.482

Table 4.26 The tabulated values are the lognormal parameters of the modulus of elasticity of machine stress rated lumber and the allowable compressive stress, F_c specified by NDS (47). The lumber is 2" x 6" Hem-Fir (33). λ is the mean and ζ is the corresponding standard deviation of the logarithms of the E data given in million psi. The calculated mean, $\bar{E}$ is also given.

Grade	F_c (ksi)	Lognormal Parameters λ	ζ	$\bar{E}$ (10^6 psi)
1650-1.5E	1.320	0.325	0.100	1.391
2100-1.8E	1.700	0.615	0.029	1.850
2400-2.0E	1.925	0.744	0.052	2.107

Table 4.27 The tabulated values are the Weibull parameters describing the distribution of the modulus of elasticity of visually graded lumber and the allowable compressive stress, F_c specified by NDS (47). The lumber is No. 1 Hem-Fir (33). The calculated mean, $\bar{E}$ is also given.

Size	F_c (ksi)	Weibull Parameters μ (10^6 psi)	σ (10^6 psi)	η	$\bar{E}$ (10^6 psi)
2" x 4"	1.050	0.803	0.972	3.844	1.684
2" x 6"	1.050	0.603	0.998	4.069	1.508
2" x 8"	1.050	0.967	0.791	2.408	1.668

Table 4.28 The tabulated values are the lognormal parameters describing the distribution of the modulus of elasticity of MSR lumber and the allowable compressive stress, F_c specified by NDS (47). The lumber is 1650-1.5E MSR Hem-Fir (33). λ is the mean and ζ is the corresponding standard deviation of the logarithms of the E data given in million psi. The calculated mean, $\bar{E}$ is also given.

Size	F_c (ksi)	Lognormal Parameters λ	ζ	$\bar{E}$ (10^6 psi)
2" x 4"	1.320	0.493	0.135	1.653
2" x 6"	1.320	0.325	0.100	1.391
2" x 8"	1.320	0.392	0.098	1.486

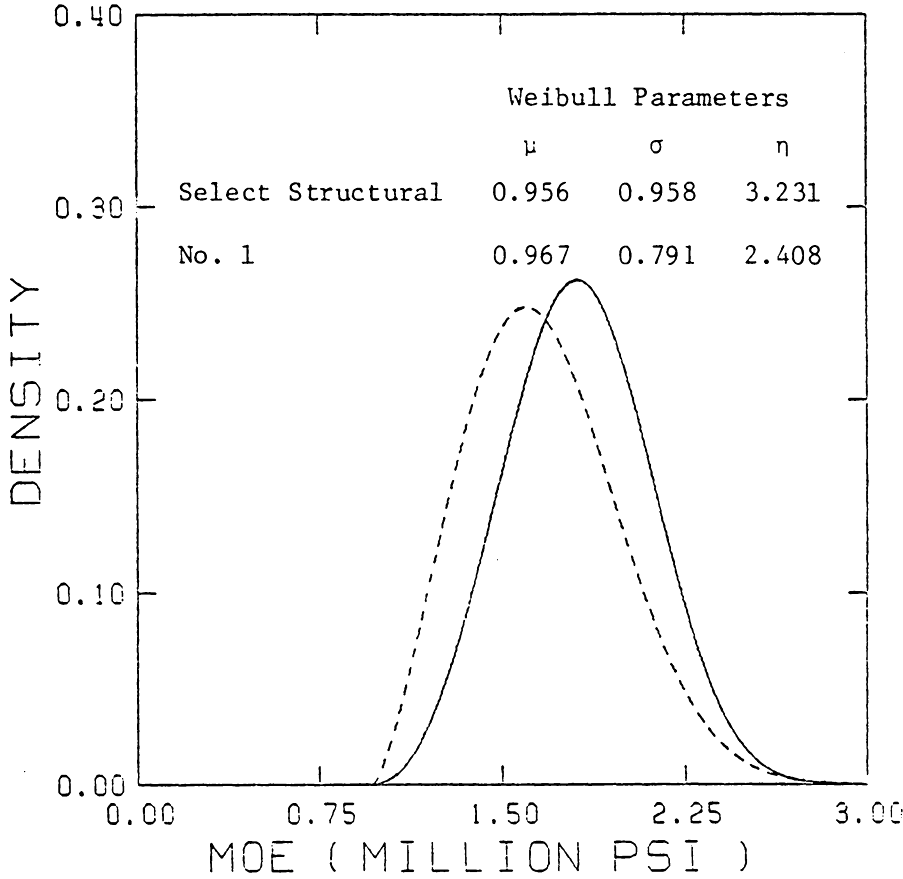

Figure 4.14 The 3-parameter Weibull functions describe the modulus of elasticity of two grades of 2" x 8" Hem-Fir lumber. The Select Structural grade, represented as the solid line, is the reference material. The parameters μ and σ are given in million psi.

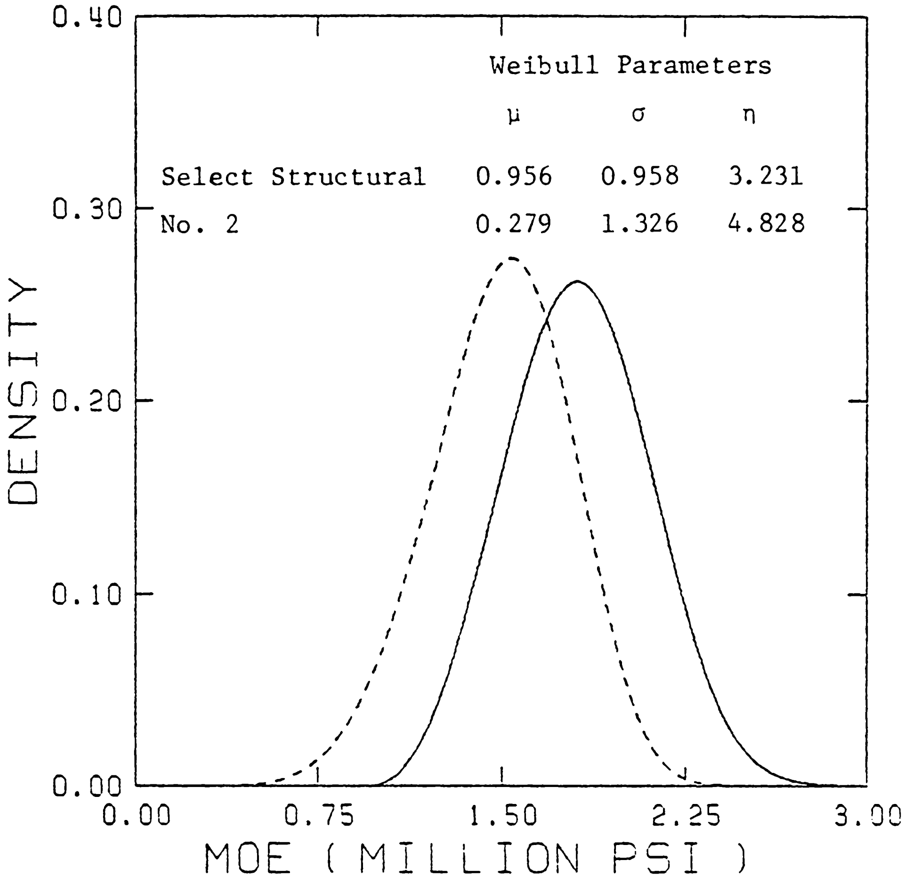

Figure 4.15 The 3-parameter Weibull functions describe the modulus of elasticity of two grades of 2" x 8" Hem-Fir lumber. The Select Strctural grade, represented as the solid line, is the reference material. The parameters μ and σ are given in million psi.

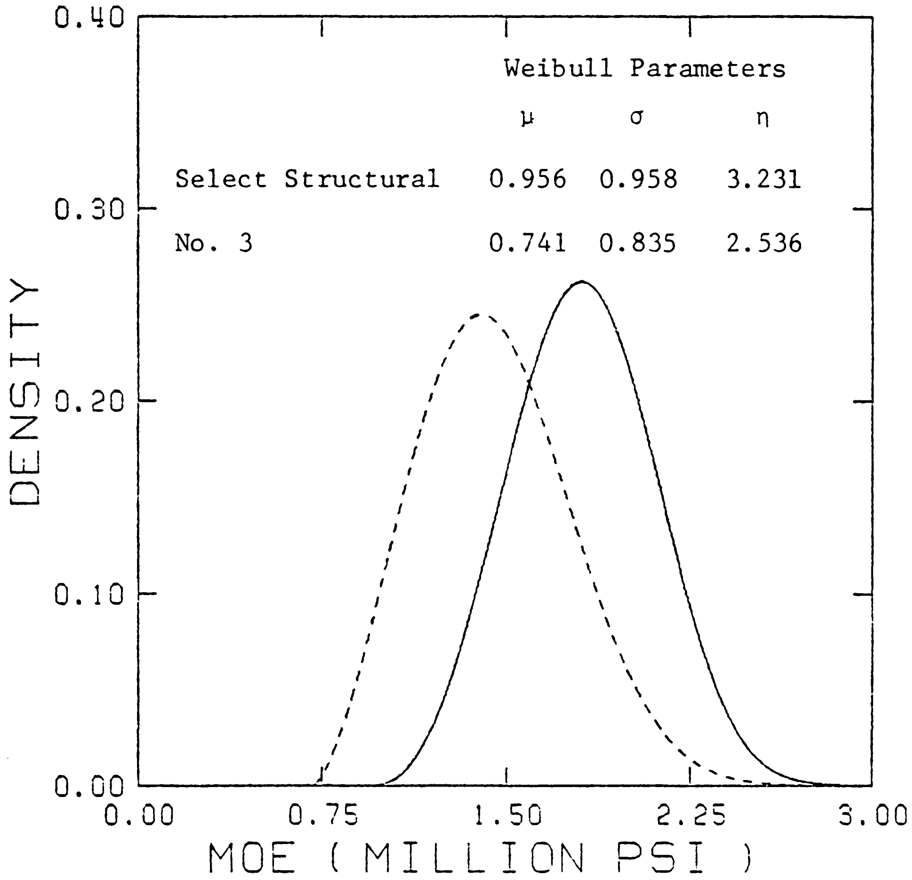

Figure 4.16 The 3-parameter Weibull functions describe the modulus of elasticity of two grades of 2" x 8" Hem-Fir lumber. The Select Structural grade, represented as the solid line, is the reference material. The parameters μ and σ are given in million psi.

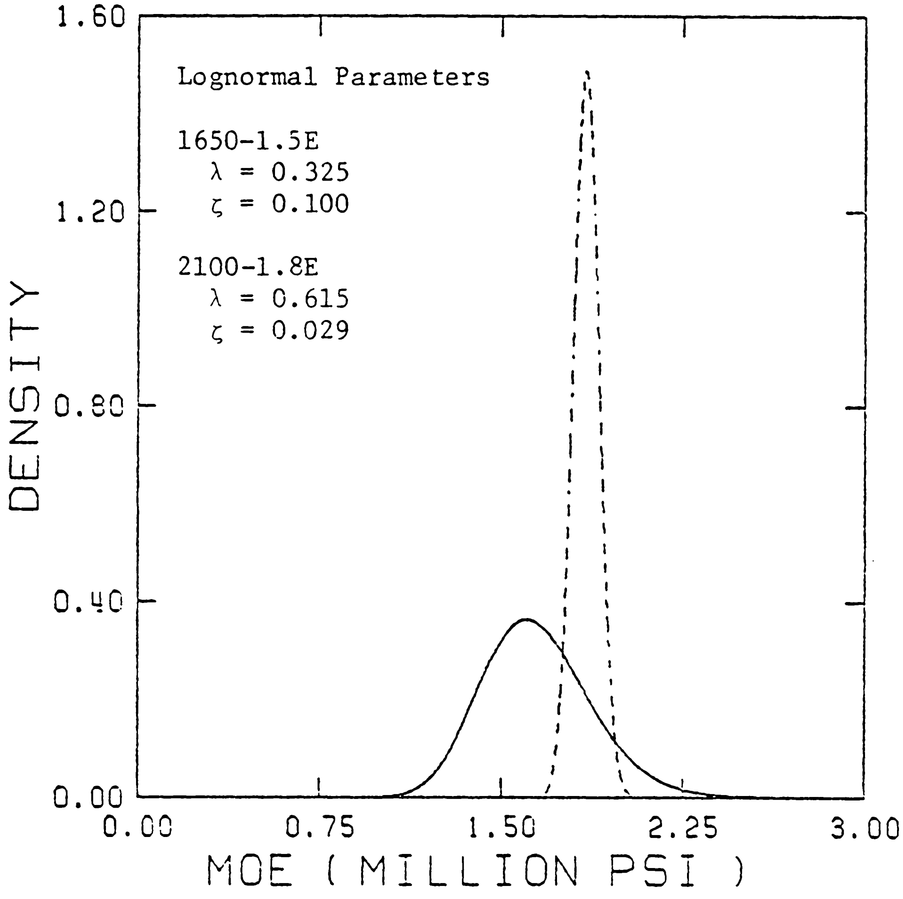

Figure 4.17 The lognormal functions describe the modulus of elasticity of 2" x 6" Hem-Fir lumber of two different grades. The 1650-1.5E grade, represented as the solid line, is the reference material. The parameters λ and ζ are the mean and standard deviation of the logarithms of the E data given in million psi.

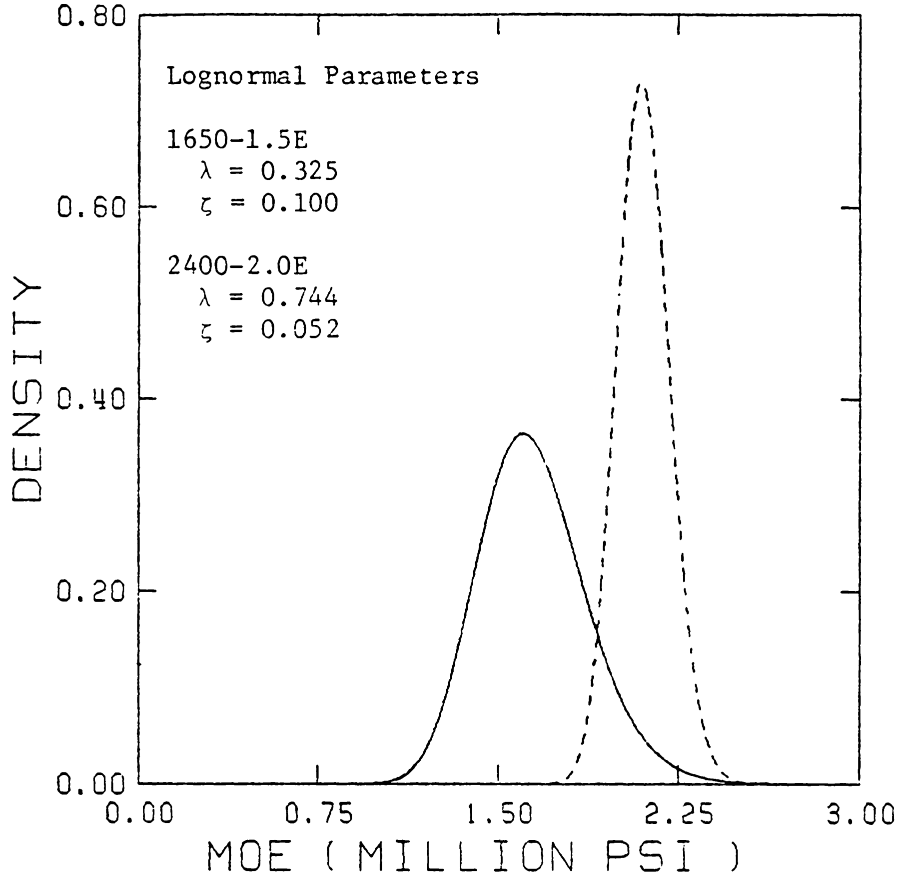

Figure 4.18 The lognormal functions describe the modulus of elasticity of 2" x 6" Hem-Fir lumber of two different grades. The 1650-1.5E grade, represented as the solid line, is the reference material. The parameters λ and ζ are the mean and standard deviation of the logarithms of the E data given in million psi.

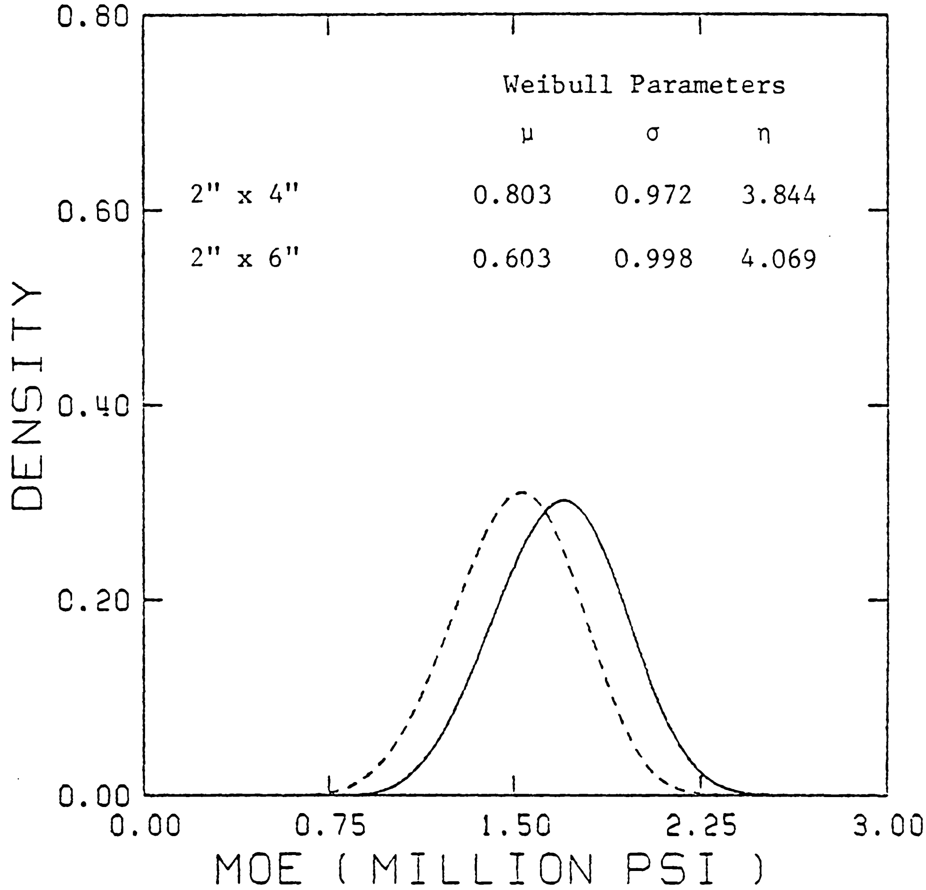

Figure 4.19 The 3-parameter Weibull functions describe the modulus of elasticity of two sizes of No. 1 Hem-Fir lumber. The 2" x 4" Hem-Fir, represented as the solid line, is used to calculate the benchmark safety level. The parameters μ and σ are given in million psi.

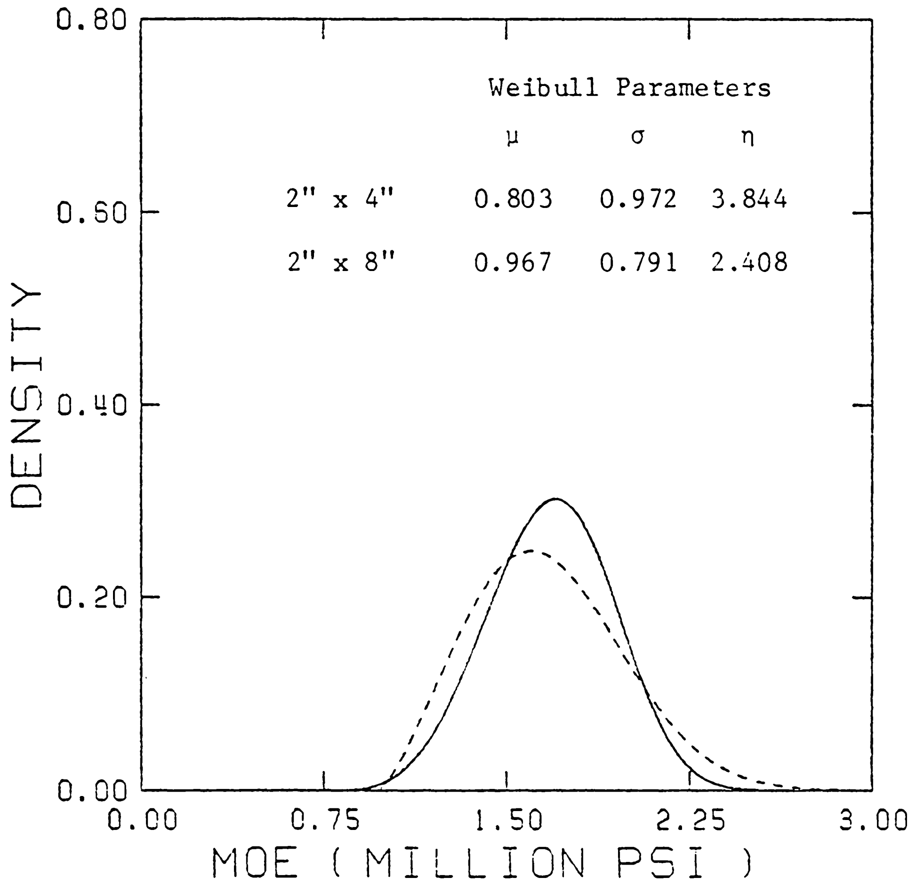

Figure 4.20 The 3-parameter Weibull functions describe the modulus of elasticity of two sizes of No. 1 Hem-Fir lumber. The 2" x 4" Hem-Fir, represented as the solid line, is used to calculate the benchmark safety level. The parameters μ and σ are given in million psi.

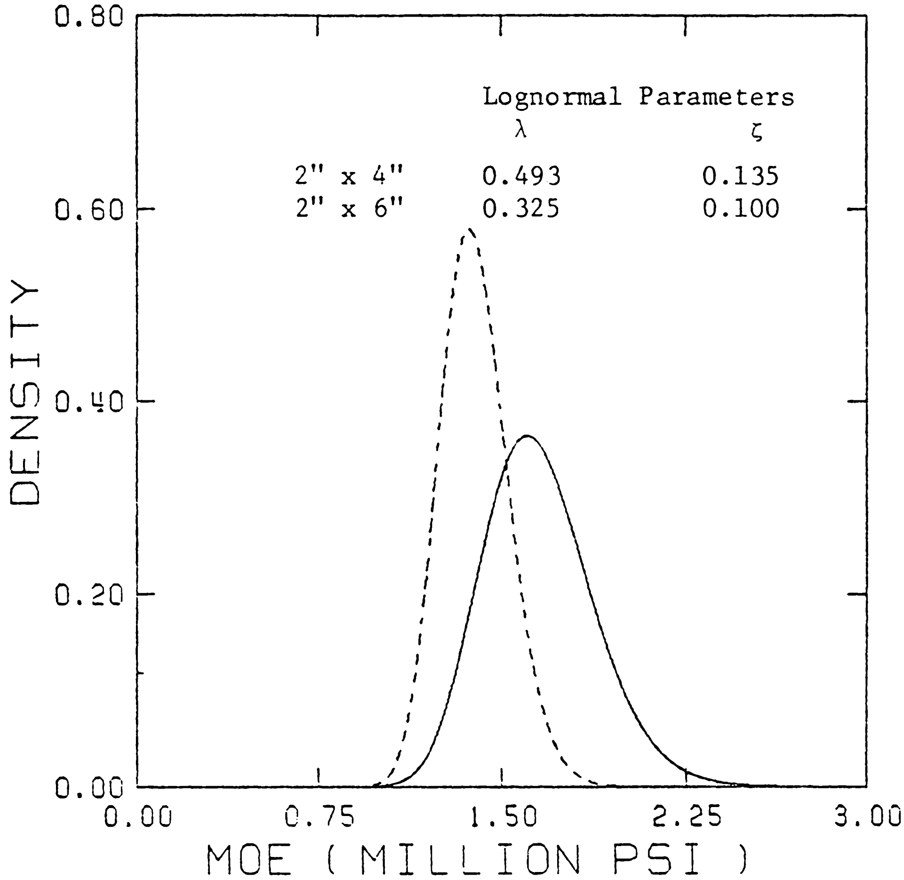

Figure 4.21 The lognormal functions describe the modulus of elasticity of two sizes of 1650-1.5E MSR Hem-Fir lumber. The 2" x 4" Hem-Fir, represented as the solid line, is used to calculate the benchmark safety level. The parameters λ and ζ are the mean and standard deviation of the logarithms of the E data given in million psi.

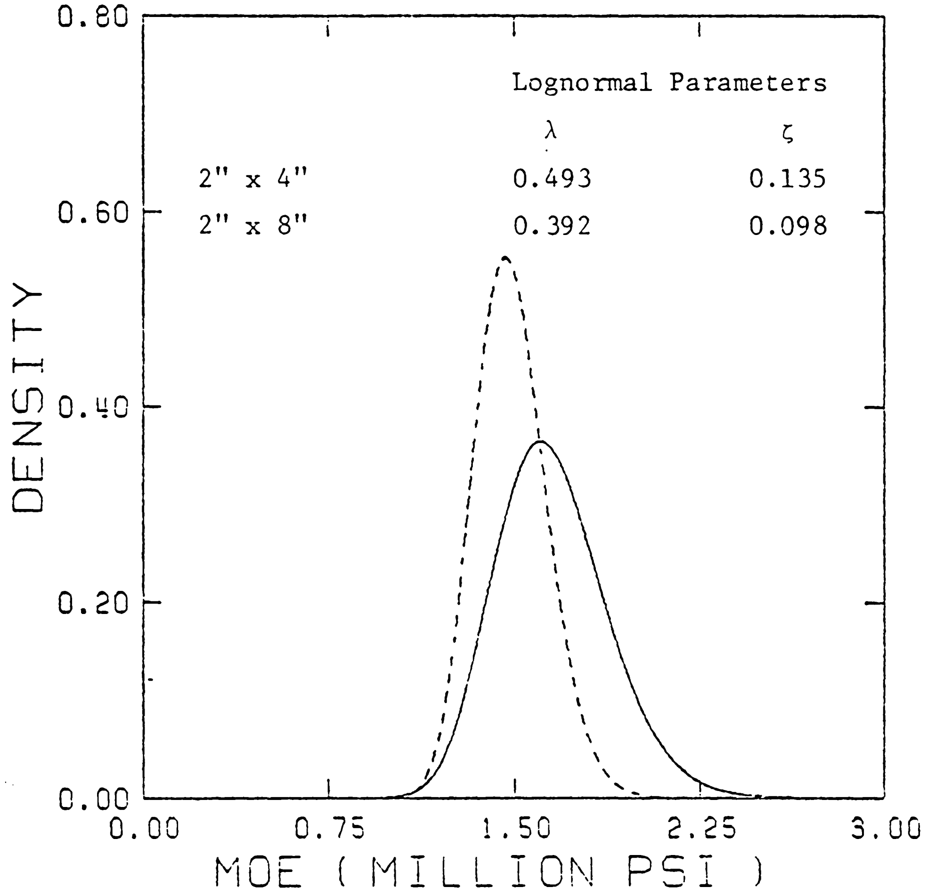

Figure 4.22 The lognormal functions describe the modulus of elasticity of two sizes of 1650-1.5E MSR Hem-Fir lumber. the 2" x 4" Hem-Fir, represented as the solid line, is used to calculate the benchmark safety level. The parameters λ and ζ are the mean and standard deviation of the logarithms of the E data given in million psi.

sizes are therefore assumed to be loaded to full design stress.

It would be interesting to compare the visually graded lumber to the machine stress rated lumber; however, the resistance distribution describing the visually graded E is different than the distribution of the MSR E. Further study needs to be conducted in order to determine the effect of differing resistance distributions on a reliability analysis. If the distributions of E were the same for visually graded lumber as for the machine stress rated lumber, the differential analysis could be conducted to compare the lumber grading methods.

4.7.1 Reliability Analysis of E Data by Lumber Grade

As previously mentioned, the modulus of elasticity is the governing factor in an analysis of a long column. Since modulus of elasticity is not an ultimate property of lumber, the modulus of elasticity parameters are transformed into parameters describing the distribution of buckling stress utilizing Euler buckling theory. Also, an allowable buckling stress, F_c' is calculated in order to determine the parameters of the load distribution. F_c' is substituted for F_b in the load parameter equations. The equations used to calculate the parameters employed in the reliability analyses are described in the following section.

To be classified as a long column, the slenderness ratio of the column, K must be longer than a minimum prescribed value. For visually graded lumber, the minimum slenderness ratio is

$$K = 0.671 \frac{E}{F_c} \tag{4.14}$$

where

K = the slenderness ratio, length divided by least unbraced dimension for a rectangular member

E = the modulus of elasticity expressed in psi

F_c = the allowable compressive strength expressed in psi

If the lumber is machine stress rated, the National Design Specification (46) states that the above equation may be altered to account for the reduced variability in the lumber. The equation is

$$K = 0.792 \frac{E}{F_c} \tag{4.15}$$

where the parameters are the same as in equation 4.14.

Modulus of elasticity is an average property of lumber; therefore, in the reliability analysis the E utilized in equations 4.14 and 4.15 is the calculated mean, $\bar{E}$ given in Tables 4.25 and 4.26. F_c is the allowable compressive stress tabulated in the supplement to the NDS (47) and given in Tables 4.25 and 4.26 for each lumber grade. F_c is adjusted by 1.15 to account for snow load before substituting in equations 4.14 and 4.15. The calculated slenderness ratios are given in the second column of Table 4.29 according to grade.

The allowable buckling stress of the visually graded lumber is calculated (32) as

$$F_c' = \frac{0.3E}{K^2} \tag{4.16}$$

where

F_c' = the allowable buckling stress expressed in psi

E = the modulus of elasticity expressed in psi

K = the calculated slenderness ratio

The allowable buckling stress for machine stress rated lumber (46) is

$$F_c' = \frac{0.418E}{K^2} \tag{4.17}$$

where the parameters are the same as in equation 4.16.

The allowable buckling stresses for the visual and MSR lumber grades are calculated by equations 4.16 and 4.17 utilizing the slenderness ratios given in Table 4.29 and $\bar{E}$ given in Tables 4.25 and 4.26. The resulting F_c' values are given in the third column of Table 4.29.

The allowable buckling stress, F_c' is utilized to calculate the total load parameters associated with each lumber grade. The assumption that the upper and lower chord dead loads will combine simply with the snow load is again employed as in the previous sections. Utilizing the dead load parameters listed in the third row of Table 4.22 for the visual grades, the dead load parameters listed in equations 4.11 through 4.13 for the MSR grades and the snow load parameters given in Table 3.8, the total load parameters are calculated utilizing equations 4.4 and 4.5 for all grades of lumber. The forces in the web resulting from the

Table 4.29 The tabulated values are the slenderness ratios and allowable buckling stresses for different grades of Hem-Fir lumber. The visually graded lumber is 2" x 8" Hem-Fir. The MSR lumber is 2" x 6" Hem-Fir. The total load parameters are calculated using the allowable buckling stress in equations 4.4 and 4.5 for each grade given. The distribution of total load is assumed to be lognormal.

Grade	K	F'_c (ksi)	Load Parameters μ_T (ksi)	Ω_T
Visually Graded				
Select Structural	24.9	0.881	0.58	0.36
No. 1	24.9	0.805	0.52	0.36
No. 2	25.9	0.670	0.44	0.36
No. 3	32.5	0.421	0.28	0.36
Machine Stress Rated				
1650-1.5E	20.3	1.011	0.65	0.37
2100-1.8E	20.6	1.303	0.83	0.37
2400-2.0E	20.7	1.475	0.94	0.37

applied nominal loads as described in Section 4.7 are utilized to proportion each load in equation 4.4 The total load parameters are tabulated in Table 4.29 for each lumber grade. The total load distribution is assumed to be lognormal.

The parameters of the E distribution are used to calculate the resistance parameters used in the reliability analysis. The resistance or ultimate buckling stress of the long column is defined for a rectangular column by the Euler equation (32) as

$$R = \frac{\pi^2 E}{12(\ell/d)^2} \tag{4.18}$$

where

R = the resistance of the column to buckling

E = the elastic modulus

ℓ/d = the slenderness ratio of the column

For this analysis, the slenderness ratio, ℓ/d is taken to be the minimum slenderness ratio of a long column, K. This substitution generalizes the analysis for any long column which satisfies the minimum requirement of length. Utilizing equation 4.18 and the E parameters given in Table 4.25, the Weibull parameters of ultimate buckling stress are calculated by

$$\mu_R = \frac{\pi^2}{12K^2}\,\mu_E \tag{4.19}$$

$$\sigma_R = \frac{\pi^2}{12K^2}\,\sigma_E \tag{4.20}$$

$$\eta_R = \eta_E \tag{4.21}$$

where

μ_E = the location parameter of the E data

σ_E = the scale parameter of the E data

η_E = the shape parameter of the E data

K = the minimum slenderness ratio calculated by equation 4.14

μ_R = the calculated location parameter of the ultimate buckling stress

σ_R = the calculated scale parameter of the ultimate buckling stress

η_R = the calculated shape parameter of the ultimate buckling stress

If the distribution of E is lognormal, then the resistance parameters are

$$\lambda_R = \lambda_E + \ln\left(\frac{\pi^2}{12K^2}\right) \tag{4.22}$$

$$\zeta_R = \zeta_E \tag{4.23}$$

where

λ_E = the mean of the logarithm of the E data

ζ_E = the standard deviation of the logarithm of the E data

K = the minimum slenderness ratio calculated by equation 4.15

λ_R = the calculated mean of the ultimate buckling stress

ζ_R = the calculated standard deviation of the ultimate buckling stress

Equations 4.19 through 4.23 follow from the proofs presented in Appendix C. The calculated parameters for the visually graded lumber using equations 4.19 through 4.21 and for MSR lumber using equations 4.22 and 4.23 are given in Tables 4.30 and 4.31, respectively.

The probability of failure analyses are conducted utilizing the load parameters given in Table 4.29 and the ultimate buckling stress parameters given in Tables 4.30 and 4.31. The reference material is then the Select Structural grade and the 1650-1.5E MSR grade for the visual and MSR lumber, respectively. The calculated failure probabilities are shown in Table 4.32 for the various grades. The contrasting lumber is artificially altered by k until similar probabilities of failure result as the benchmark safety levels. The k values and altered failure probabilities are also shown in Table 4.32.

The k factors of the visually graded lumber indicate that the relative safety of the No. 2 and No. 3 lumber grade is significantly affected by the shape of the strength distribution. The shape of the distribution of MSR lumber grades is not a significant factor in the relative safety of the lumber as shown by the k factors given in the

Table 4.30 The tabulated values are the resistance parameters calculated for visually graded 2" x 8" Hem-Fir lumber of various grades. The parameters are calculated using equations 4.19 through 4.21 and the Weibull parameters listed in Table 4.25.

Grade	Weibull Parameters μ (ksi)	σ (ksi)	η
Select Structural	1.272	1.275	3.231
No. 1	1.279	1.046	2.408
No. 2	0.343	1.631	4.828
No. 3	0.578	0.651	2.536

Table 4.31 The tabulated values are the resistance parameters calculated for machine stress rated 2" x 6" Hem-Fir lumber of various grades. The parameters are calculated using equations 4.22 and 4.23 and the lognormal parameters listed in Table 4.26. The parameters λ and ζ are the mean and standard deviation of the resistance parameters given in ksi.

Grade	Lognormal Parameters	
	λ	ζ
1650-1.5E	0.863	0.100
2100-1.8E	0.941	0.029
2400-2.0E	1.064	0.052

Table 4.32 The tabulated values are the probabilities of failure and the k factors for a differential reliability analysis based on E data. The Hem-Fir lumber is visually graded or MSR lumber used as a web in a farm truss as shown in Figure 4.13. The load distribution is assumed to be lognormal.

	Probability of Failure Lower Limit	Upper Limit
Select Structural	7.65×10^{-5}	7.60×10^{-5}
No. 1	7.46×10^{-5}	7.51×10^{-5}
No. 1, adjusted by k = 0.995	7.86×10^{-5}	7.92×10^{-5}
No. 2	4.01×10^{-4}	4.04×10^{-4}
No. 2, adjusted by k = 1.240	4.53×10^{-5}	7.59×10^{-5}
No. 3	1.86×10^{-4}	1.89×10^{-4}
No. 3, adjusted by k = 1.095	7.65×10^{-5}	7.74×10^{-4}
1650-1.5E	7.67×10^{-4}	7.71×10^{-4}
2100-1.8E	4.57×10^{-4}	4.58×10^{-4}
2100-1.8E, adjusted by k = 0.950	7.54×10^{-4}	7.57×10^{-4}
2400-2.0E	5.04×10^{-4}	5.06×10^{-4}
2400-2.0E, adjusted by k = 0.960	7.48×10^{-4}	7.50×10^{-4}

lower half of Table 4.32. This is perhaps indicative of a more uniform grading procedure in the machine stress rated lumber over the visually graded lumber.

4.7.2 Reliability Analysis of E Data by Lumber Size

The relative safety of various sizes of lumber used as a long column is conducted in the same manner as the previous analysis for visually graded and machine stress rated lumber. Utilizing the mean, $\bar{E}$ and allowable compressive stresses shown in Tables 4.27 and 4.28, slenderness ratios and allowable buckling stresses are calculated for the lumber data. Equations 4.14 and 4.16 are utilized for the visually graded lumber and equations 4.15 and 4.17 are utilized for the machine stress rated lumber. The results are shown in Table 4.33 for each lumber size.

The reliability analysis is conducted utilizing the dead load parameters shown in Table 4.22 for the various sizes of visual and MSR grades of lumber. The snow load parameters are given in Table 3.8. The design situation is the same as in the previous section. The total load parameters are calculated using equations 4.4 and 4.5 and are given in Table 4.33.

The buckling stress parameters are calculated by equations 4.19 through 4.21 for the visually graded lumber and equations 4.22 and 4.23 for the machine stress rated lumber for each lumber size. The resulting values are given in Tables 4.34 and 4.35.

Table 4.33 The tabulated values are the slenderness ratios, allowable buckling stresses, and calculated load parameters for various sizes of Hem-Fir lumber. The Hem-Fir is of two grades, No. 1 and 1650-1.5E MSR. The total load parameters are calculated using the allowable buckling stress in equations 4.4 and 4.5 for each size shown. The distribution of total load is assumed to be lognormal.

Size	K	F'_c (ksi)	Load Parameters μ_T (ksi)	Ω_T
No. 1				
2" x 4"	25.1	0.805	0.50	0.38
2" x 6"	23.7	0.805	0.52	0.37
2" x 8"	24.9	0.805	0.52	0.37
1650-1.5E				
2" x 4"	22.1	1.011	0.64	0.38
2" x 6"	20.3	1.011	0.65	0.37
2" x 8"	21.0	1.011	0.66	0.36

Table 4.34 The tabulated values are the Weibull resistance parameters calculated for No. 1 Hem-Fir lumber of various sizes. The parameters are calculated using equations 4.19 through 4.21 and the Weibull parameters given in Table 4.27.

Size	Weibull Parameters		
	μ (ksi)	σ (ksi)	η
2" x 4"	1.052	1.273	3.844
2" x 6"	0.882	1.460	4.069
2" x 8"	1.270	1.038	2.408

Table 4.35 The tabulated values are the lognormal resistance parameters calculated for various sizes of 1650f-1.5E MSR lumber. The parameters are calculated using equations 4.22 and 4.23 and the lognormal parameters listed in Table 4.28. The parameters λ and ζ are the mean and standard deviation of the logarithms of the resistance parameters given in ksi.

Size	Lognormal Parameters λ	ζ
2" x 4"	0.678	0.135
2" x 6"	0.684	0.100
2" x 8"	0.684	0.098

The differential reliability analysis is conducted to determine the relative safety of a web as affected by various lumber sizes. In other words, the relative safety between lumber sizes as determined by E is assessed. The benchmark safety level is calculated using the 2" x 4" Hem-Fir lumber for both grading systems, visual and MSR. The probabilities of failure and k factors of the various lumber sizes for the visually graded lumber are given in the top half of Table 4.36. The probability of failure and the k factors for the MSR lumber are given in the bottom half of Table 4.36. The k factors of Table 4.36 indicate that the effect of the shape of the E distribution is not critical between lumber sizes regardless of which grading technique is utilized. This indicates the uniformity of both grading techniques for MOE over a range of lumber sizes.

Table 4.36 The tabulated values are the probabilities of failure and the k factors for a differential reliability analysis based on E data. The Hem-Fir lumber is visually graded or machine stress rated lumber used as a web in a farm truss as shown in Figure 4.13. The load distribution is assumed to be lognormal.

	Probability of Failure	
	Lower Limit	Upper Limit
No. 1 Hem-Fir		
2" x 4"	7.40×10^{-5}	7.45×10^{-5}
2" x 6"	1.26×10^{-4}	1.27×10^{-4}
2" x 6", adjusted by k = 1.060	7.27×10^{-5}	7.32×10^{-5}
2" x 8"	1.07×10^{-4}	1.07×10^{-4}
2" x 8", adjusted by k = 1.035	7.53×10^{-5}	7.58×10^{-5}
1650-1.5E MSR		
2" x 4"	1.16×10^{-3}	1.16×10^{-3}
2" x 6"	7.60×10^{-4}	7.64×10^{-4}
2" x 6", adjusted by k = 0.955	1.16×10^{-3}	1.16×10^{-3}
2" x 8"	6.84×10^{-4}	6.88×10^{-4}
2" x 8", adjusted by k = 0.945	1.16×10^{-3}	1.17×10^{-3}

CHAPTER V

RESULTS

The results of the preceding differential reliability analyses are presented in three sections. The first section discusses the reliability analyses in which the calculated k factors are various adjustment factors for lumber. The second section reports on the various trends concerning lumber properties as evidenced by the k factors describing the relative safety between lumber grades or sizes. The last section is a general overview of all the reliability results in order to recommend various distributional choices for use in future reliability applications.

In a comparison of k factors, differences of five percent or less are essentially negligible. This is based on the engineering design results which would be obtained using the k factors. In any analysis, the results should always be interpreted based on the desired precision for the application or end use.

5.1 Lumber Property Adjustment Factors

The differential reliability technique was employed in three cases in order to determine various adjustment factors. In these cases, only one load distribution, based on the reference material, was utilized. The assumption is that the lumber distributions come from the same lumber population. The contrasting lumber data was compared on the basis of equal reliability to a benchmark safety level in order to determine the factor, k which relates a specific lumber property to the reference material. The k factors which were calculated in the previous chapter and are used as adjustment factors are shown in Table 5.1 as they are given in previous tables. These factors are a moisture adjustment factor, a rate of loading adjustment factor and an adjustment factor relating tensile or compressive stress to bending stress.

For the moisture comparison, the dry lumber was chosen as the reference material. The green lumber was artificially increased in strength by 10 to 20 percent depending on the load distribution to obtain the same probability of failure as the dry lumber. Based on the present moisture adjustment factor, green lumber is theoretically 25 percent weaker than dry lumber. Since the dry lumber is chosen as the reference material, no adjustment is made to the dry allowable values. Therefore, if the adjustment factors shown in the second column of Table 5.1 are utilized, the green allowable values would be increased to a level closer to the dry values. In other words, dry lumber is stronger than green lumber; however, it is believed that the strength difference is not as great as previously believed.

The results of the reliability analysis of lumber tensile strength tested at three rates of loading contradict those calculated by conventional practices. A differential reliability analysis provides a "yardstick" for comparing lumber strengths tested at different loading rates. The adjustment factor from a faster loading rate to the standard rate is therefore the reciprocal of the k factor if the standard loading rate is

Table 5.1 The tabulated values are the adjustment factors calculated in the previous chapters from a differential reliability analysis of lumber properties data. The type of comparison is listed in the first row of the table. The reference material is listed in the second row of the table and the contrasting lumber data set is identified in the third row of the table. The k factors are those calculated in the previous chapters.

Load	Moisture	Rate of Loading		Failure Mode	
	Dry	Standard		Bending	
	Green	10 x standard 1/k	25 x standard 1/k	Tension k_{tb}	Compression k_{cb}
Snow	1.100	0.631	0.858	1.355	0.970
Load A	1.145			1.395	0.840
Load B	1.135			1.400	0.850
Load C	1.135			1.395	0.855
Load D	1.195			1.425	0.905

the reference material. The results given in the third and fourth columns of Table 5.1 indicate an opposite trend to the results of the Wood (71) curve given in Table 4.12. Also, Figures 4.5 and 4.6 graphically demonstrate that the distribution of lumber strengths changes dramatically when faster rates of loading are utilized. A conventional analysis cannot account for this change in shape of the strength distribution. However, differential reliability can account for the shape of strength distributions. Hence, the differential reliability technique is a rational method to obtain loading rate adjustment factors.

Differential reliability was also used to obtain factors to compare allowable stresses as described in Chapter 4, section 4.4. The calculated k factors are the adjustments from tensile or compressive stress allowables to bending stress allowable. Table 4.17 gives the reliability results compared to the results from conventional analyses. Comparing the conventionally calculated factors, r to the differential reliability factors, k, it is suggested that the allowable tensile stress can be increased to a level closer to the allowable bending stress and the allowable compressive stress can be increased to a level greater than the allowable bending stress. The calculated k factors which are the adjustment factors, k_{tb} and k_{cb} are reproduced in Table 5.1. It is evidenced, based on only two data sets, that the conventional analyses do not account for the stronger lumber and therefore the allowable tensile and compressive stresses may be low compared to the allowable bending stress.

5.2 Lumber Property Trends Based on Relative Safety

The effect of the shape of the strength distribution can be studied by a differential reliability analysis. Table 5.2 and 5.3 lists all of the k factors from the relative safety study conducted in the previous chapters. Based on these two tables, trends which result from various lumber sizes, grades and grading techniques are discussed.

First, MSR lumber exhibits less variability than the visual lumber when coupled by a load distribution as shown by an examination of the k factors for different lumber grades in Tables 5.2 and 5.3. The range of k factors for the MSR tensile comparisons is 0.955 to 1.020 and the range for MSR elastic modulus comparisons is 0.945 to 0.960. Machine stress rating theoretically produces a more uniform product than visual grading (32). As a contrast the k factor for visual bending strength by grade comparisons varies from 0.900 to 2.235 under snow load. The k factors of the tensile strength and elastic modulus comparisons given in Table 5.2 and 5.3 demonstrate that the MSR data studied exhibit the same safety based on a typical engineering application. However, the left side of Table 5.2 shows that the relative safety of contrasting lumber grades can vary significantly from the reference material for the visually graded lumber.

In particular, the comparison of Select Structural and No. 2 grades suggests that different criteria for grading a No. 2 grade of lumber may be warranted. The large k factors are a result of a location parameter very close to zero. If this behavior was observed on many data sets a change in grading rules might qualify some of the weaker pieces of

Table 5.2 The tabulated values are the k factors resulting from the relative safety studies based on bending or tensile strength conducted in the two previous chapters. The second row of the table lists the chosen reference material. The grades or sizes listed in the third row are the contrasting lumber sets. The bending strength comparisons were conducted on visually graded lumber while the tensile strength comparisons were performed on machine stress rated lumber.

Load Case	Bending Strength Comparisons					Tensile Strength Comparisons			
	SS			2 x 8		1650-1.5E		2 x 6	
	No. 1	No. 2	No. 3	2 x 6	2 x 10	2100-1.8E	2400-2.0E	2 x 4	2 x 8
Snow	0.900	2.235	0.995	0.930	0.900	0.955	0.960	1.020	0.970
Load A	0.865	2.600	0.985	0.905	0.865				
Load B	0.875	3.415	0.980	0.920	0.875				
Load C	0.865	2.910	0.995	0.910	0.865				
Load D	0.825	4.565	0.950	0.885	0.825				

Table 5.3 The tabulated values are the k factors resulting from the relative safety studies based on the elastic modulus of visual and MSR lumber grades or sizes. Only the roof snow load was utilized in these differential reliability analyses. The second row of the table lists the chosen reference material. The grades or sizes listed in the third row are the contrasting lumber sets.

Visual					MSR			
SS			2 x 4		1650-1.8E		2 x 4	
No. 1	No. 2	No. 3	2 x 6	2 x 8	2100-1.8E	2400-2.0E	2 x 6	2 x 8
0.955	1.240	1.095	1.060	1.035	0.950	0.960	0.955	0.945

No. 2 lumber for the No. 3 grade thereby reducing the differences in the safety between the No. 2 and Select Structural grade. The k factor of the No. 3 grade would therefore be expected to increase reflecting an increase of lumber of near minimal strength being added to the No. 3 grade.

However, the distribution of strength for the No. 2 grade may be just an unusual data set. A more typical data set would probably result in k factors in a more reasonable range. This is supported by the fact that the large k factors are a result of the weaker lumber as represented by a minimal location parameter. This requires the No. 2 lumber strengths to be artifically increased a large amount to obtain a failure probability similar to the benchmark safety level.

Comparisons of relative safety based on size show similar trends as the comparisons of relative safety based on grade. The range of the k factors for MSR E data by size in Table 5.3 is 0.945 to 0.955 and the range for visually graded E data is 1.035 to 1.060. For the k factors based on strength comparisons, the range for visually graded lumber analyzed by size is 0.900 to 0.930. According to the Supplement to the NDS (47), 2" x 6", 2" x 8" and 2" x 10" lumber has the same allowable bending stress. Based on the two data sets of Table 5.2, both 2" x 6" and 2" x 10" lumber is safer than the 2" x 8". However, the values given in Table 5.2 for tensile strength comparisons by lumber size show that the 2" x 4", 2" x 6" and 2" x 8" MSR sizes have the same safety in use.

Equality in relative safety based on size is logical since the grade is the same for the lumber sizes. Theoretically, lumber of different sizes but of the same grade should not vary except for the variance in the grading technique. This is exhibited by the equal safety demonstrated in the MSR grades with the larger spread of k factors for the visually graded lumber.

5.3 Suggested Load Distributions for Differential Reliability Analyses

Table 5.4 gives all the k factors which are used to determine a suggested load distribution for reliability analyses. The table lists the k factors which were calculated for lumber used in two of the design situations of Chapters 3 and 4, a roof rafter and a floor joist. Comparisons of the factors are made by column to determine equality between k factors with suggested trends highlighted by comparisons between rows. Based on an overview of Table 5.4, it appears the results are relatively insensitive to the choice of load distribution. The snow load produces k factors closer to unity in all cases when compared with the floor live loads. Of the four floor live loads, the first three produce k factors which are in excellent agreement while Load D produces k factors which in most part are the most extreme of any load case. The exception to this is the last column where the k factor of Load D is between the snow load k factor and the other floor live load k factors.

Ideally, one would recommend the snow load distribution and the best available floor live load distribution as the load distributions to use in reliability analyses. A researcher could then choose a design

Table 5.4 The tabulated values are the k factors calculated from the previously described analyses in Chapters 3 and 4 which are used to gauge the sensitivity of the differential reliability analysis to changing load distributions. Only the results from analyses in which a roof snow load and a floor live load were utilized are shown.

	k Factors							
Load	Dry	Select Structural			2 x 8		Bending	
Case	Green	No. 1	No. 2	No. 3	2 x 6	2 x 10	Tension	Compression
Snow	1.100	0.900	2.235	0.995	0.930	0.900	1.355	0.970
Load A	1.145	0.865	2.600	0.985	0.905	0.865	1.395	0.840
Load B	1.135	0.875	3.415	0.980	0.920	0.875	1.400	0.850
Load C	1.135	0.865	2.910	0.995	0.910	0.865	1.395	0.855
Load D	1.195	0.825	4.565	0.950	0.885	0.825	1.425	0.905

situation which best describes the mode under which the data was tested and then utilize the associated load distribution. As an example, lumber tested in tension is assumed to be used as the lower chord of a farm truss. Therefore, the snow load combination is the load used in the reliability analysis. However, choosing a "best" floor live load distribution is difficult since there are no clear criteria to use as a foundation for such a choice.

Loads A through C produce k factors which are almost equal. The parameters of Load A were determined by Chalk and Corotis (11) using their model. Since this model is easy to use for determining load parameters and accounts for the stochastic nature of the floor live load, Load A is recommended for use in reliability analyses along with the roof snow load. The parameters of Load D were also calculated based on the Chalk and Corotis model. The statistical data base for Load A and Load D was essentially the same; however, the method of combining the load parameters was different. Hence, the final parameters of the maximum lifetime floor live load are different for Load A and Load D; nevertheless, there is no physical basis for distinguishing which of the two loads is the "best" load. Therefore, Load D is also recommended for use in reliability analyses. The parameters and distribution of the snow load and the floor live loads, Load A and Load D are given in Table 5.5.

If the reliability technique is used to determine an adjustment factor, then the chosen load distribution should be the one which results in a conservative k factor. As an example, column two of Table 5.4 gives

Table 5.5 The tabulated values are the recommended load distributions to be utilized in reliability analyses of lumber properties data. These distributions should be combined with the lognormally distributed dead load. The normalized mean of the dead load should be calculated for each design situation. The coefficient of variation of the dead load Ω_D is 0.10.

Load	Distribution	$\bar{X}/X_n$	Ω_X
Snow	Lognormal	0.69	0.44
Load A	Type I	0.94	0.21
Load D	Type I	0.73	0.19

the k factors which are used as moisture adjustment factors for lumber bending strength. The snow load produces a k equal to 1.100, Load A gives a k equal to 1.145, and a k of 1.195 results from the use of Load D in the reliability analysis. The conservative factor, k equals 1.195 is the k factor which gives the lowest allowable bending stress for the green lumber.

For an appraisal of relative safety as shown by the k factors in columns three through seven of Table 5.4, one would use all three load cases, snow, Load A and Load D.

A reliability analysis of tensile or compressive stress in relation to bending stress can define adjustment factors for tensile and compressive allowable values. These allowable values would reflect the same safety as the allowable bending stress. The assumption of equal safety between lumber grades, sizes, and species is presently implied for the tabulated design values in the Supplement to the NDS (47). Since it is generally believed that tensile and compressive stresses are less than bending stresses, the conservative approach would select k_{tb} equal to 1.425 as the tensile to bending stress adjustment factor. This k_{tb} represents the lowest allowable tensile value which can be calculated from the adjustment factors shown in column eight of Table 5.4.

One would expect the compression adjustment factors to be greater than 1.0 based on the Supplement to the NDS (47). Therefore, the data set of lumber tested in compression which produced these k factors is likely to be an anomaly. At any rate, the choice of a compression adjustment factor, k_{cb} would be made in the same fashion as the before mentioned tension adjustment factor, k_{tb} using the conservative approach.

CHAPTER VI

SUMMARY AND CONCLUSIONS

The distributions of maximum lifetime floor live load and maximum lifetime roof snow load were developed for use in differential reliability analyses of lumber properties data. The models, assumptions, and data utilized in the development of the distributions reflect the present state of the art on loads available. In order to give meaning to the reliability analyses, various design situations were assumed. These situations encompassed typical lumber design to reflect the physical state of the lumber when it was tested. Based on the design situation, the dead load parameters were calculated and then combined by a second moment approach with the appropriate live load parameters to render the total load parameters for the assumed design. The distribution of the total load was found to be adequately described by the same distribution as the live load. This simplification regarding the calculation of the total load had no effect on the results of the differential reliability analysis from an engineering viewpoint. Contrasting lumber sets were then compared based on the concept of equal reliability utilizing the calculated load distributions. A reference material was chosen from the lumber sets and this material was used to calculate the benchmark safety level. The other contrasting lumber data sets were artificially altered until a failure probability approximately equal to the benchmark safety level resulted. This technique is based on the assumption that all the lumber in the sample is fully stressed to the allowable design stress. This does not occur in actual design situations since the lumber strength is affected by cladding, non-structural components, and load sharing between members. However, any inaccuracies in the calculation of the failure probabilities are believed to be minimized by the comparative nature of the reliability technique.

The factor k resulting from this reliability analysis is a logical comparison method between lumber sets because it is a comparison of the calculated probabilities of failure of the lumber in service. The probability of failure analysis can be carried out in a strictly formal and consistent way in every case. The end result is a number which can be compared to others on a logically uniform basis. While the failure probabilities are not absolute in the sense that they are the true probabilities of failure based on the design situation, they indicate the magnitude of difference between the contrasting lumber sets and the reference sample. This difference is therefore a measure of the effect of the study variable of the contrasting lumber data sets. This type analysis is very powerful because the entire strength distribution is utilized. Also, unlike conventional methods, differential reliability analysis utilizes in a probabilistic framework the actual loads to which lumber may be subjected during its structural lifetime.

In conclusion, the differential reliability technique is an integrated technique which is particularly well suited for the analysis of lumber properties data. The technique formally accounts for the inherent variability of lumber data. Lumber strength data can be analyzed within a framework of statistical principles while reflecting realistic design situations. All properties that affect the strength of lumber can be studied by differential reliability. Even average properties such as modulus of elasticity can be studied once the property has been transfomred to an ultimate strength property in order to reflect a failure state.

Based on the analysis in the previous chapters, the results from a differential reliability analysis are not particularly sensitive to changes in load parameters or choice of distribution. In other words, the k factors show no great differences based on engineering design use. Also, there are no clear criteria for choosing a "best" load distribution for use in reliability analyses. Therefore, it is suggested that three load distributions be utilized for any reliability analyses of lumber properties data. The recommended load distributions are the lognormally distributed roof snow load and two floor live loads, Load A and Load D which are distributed extreme value type I. All are maximum lifetime loads and the design life is assumed to be fifty years. The parameters of these distributions are given in Chapter 5, Table 5.5.

It is believed that lumber properties data can be rationally analyzed utilizing these load distributions. In analyzing lumber properties data in order to determine an adjustment factor, all three loads should be utilized and the choice of load distribution determined by which one of the three produces conservative k factors. In the case of a relative safety study, all three load distributions should be utilized; however, no choice need be made from the results of the k factors.

Further Research Needs

1. Demonstrate the usefulness of the reliability approach in determining lumber adjustment factors by analyzing many sets of lumber properties data.
2. Determine better statistical information concerning the ground to roof snow load coefficient, C_S in order to better estimate roof snow load parameters.
3. Obtain additional information concerning the transient floor live load and its statistical parameters over the life of a structure. This would result in a better estimation of the maximum lifetime floor live load parameters.

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APPENDICES

APPENDIX A

SUM OF TWO RANDOM VARIABLES

The sum of two random variables can be calculated using a convolution integral if the distributions of the two variables are known. The convolution integral is derived below as described in Ang and Tang (8). However, the integral cannot always be solved in closed form and a numerical procedure must be utilized. Following the derivation of the convolution integral is a computer program developed to solve the sum of two random normal variables. Sample output from the program is checked against a standard normal table. The program logic forms the basis of the subroutines listed in Appendix F.

<u>PROOF</u>:

If X and Y are continuous random variables, then

$$F_Z(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{g^{-1}(z,y)} f_{X,Y}\ (x,y)\ dx\ dy \qquad (A.1)$$

Changing the variable of the integration from x to z,

$$F_Z(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{z} F_{X,Y}\ (g^{-1}(z,y),\ y)\ \frac{dg^{-1}(z,y)}{dz}\ dy \qquad (A.2)$$

Thus, the probability density function is

$$f_Z(z) = \int_{-\infty}^{\infty} F_{X,Y}\ (g^{-1}(z,y),y)\ \frac{dg^{-1}(z,y)}{dz}\ dy \qquad (A.3)$$

Specifically, if

$$Z = X + Y \tag{A.4}$$

then

$$g^{-1}(z,y) = z - y \tag{A.5}$$

$$\frac{dg^{-1}(z,y)}{dx} = 1 \tag{A.6}$$

Substituting, equation A.3 is

$$f_Z(z) = \int_{-\infty}^{\infty} f_{X,Y}\,(z - y,\ y)\ dy \tag{A.7}$$

If X and Y are independent, then

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z - y)\ f_Y(y)\ dy \tag{A.8}$$

which is the convolution integral of Z = X + Y.

```
C***********************************************************************
C
C                                PROB2
C
C
C                      CONVOLUTION INTEGRAL PROGRAM
C
C
C***********************************************************************
C
C
C   THIS PROGRAM WAS USED TO VERIFY THE NUMERICAL PROCEDURE AND PROGRAM
C   LOGIC USED TO CALCULATE THE CDF OF Z WHEN
C
C                             Z=X+Y
C
C   X AND Y ARE DISTRIBUTED NORMALLY AND THEREFORE SO IS Z. THE OUTPUT
C   CAN BE EASILY CHECKED AGAINST ANY STANDARD NORMAL TABLE ONCE Z HAS
C   BEEN NORMALIZED BY THE CORRECT PARAMETERS.
C
C
      REAL MU1,MU2
C
C   READ IN THE VALUES AS LISTED IN THE USERS GUIDE
C
C                             Z  VALUE AT WHICH CDF IS TO BE CALCULATED
C                             DX  INCREMENT VALUE
C                             XLOW,XHIGH  MODELS +/- INFINITY
C                             MU1,VAR1  MEAN AND VARIANCE OF X
C                             MU2,VAR2  MEAN AND VARIANCE OF Y
C
      READ(1,21) Z,DX,XLOW,XHIGH
      READ(1,21) MU1,VAR1,MU2,VAR2
      SUM1=0.0
      SUM2=0.0
   10 XLOW=XLOW+DX
C
C   FIND THE PDF OF Y
C
      FN1=(6.2831853*VAR2)**(-0.5)*EXP((-0.5)*((XLOW-MU2)**2)/VAR2)
      FN2=(6.2831853*VAR2)**(-0.5)*EXP((-0.5)*((XLOW+DX-MU2)**2)/VAR2)
      IF(FN1.LT.0.1E-25) GO TO 30
C
C   CALCULATE THE CDF OF X
C
      FN3=((Z-XLOW)-MU1)/SQRT(VAR1)
      IF(FN3.LT.-8.0) GO TO 30
      CALL MDNOR(FN3,CDF)
C
C   CALCULATE INCREMENTAL VALUES AND THEN SUM THEM TO FIND CDF AT Z
C
      FZ1=FN1*CDF*DX
      FZ2=FN2*CDF*DX
      SUM1=SUM1+FZ1
      SUM2=SUM2+FZ2
      IF(XLOW.GE.XHIGH) GO TO 20
   30 GO TO 10
   20 CONTINUE
C
C   WRITE THE TWO ESTIMATED VALUES OF THE CDF AT Z
C
      WRITE(2,31) SUM1,SUM2
   11 FORMAT(2F10.3)
   21 FORMAT(4F10.3)
   31 FORMAT(2(5X,E20.13))
   41 FORMAT(15X,2(E20.13,2X))
      STOP
      END
```

Suppose X and Y are two normal variates,

$$X \sim N\ (10,\ 2) \tag{A.9}$$

$$Y \sim N\ (30, 6) \tag{A.10}$$

If

$$Z = X + Y \tag{A.11}$$

then Z is distributed normally (8) with parameters

$$\mu_Z = 40 \tag{A.12}$$

$$\sigma_Z^2 = 40 \tag{A.13}$$

Suppose the cumulative value at z = 20 is needed, then

$$F_Z(20) = \phi^{-1}\left[\frac{20-40}{\sqrt{40}}\right] = \phi^{-1}\ (3.162) \tag{A.14}$$

Utilizing a standard normal table,

$$F_Z\ (20) = 7.83 \times 10^{-4} \tag{A.15}$$

If the previously listed computer program, PROB2 is used to solve for $F_Z(20)$, then

$$F_{Z_U}\ (20) = 7.869 \times 10^{-4} \tag{A.16}$$

$$F_{Z_L}\ (20) = 7.826 \times 10^{-4} \tag{A.17}$$

where

F_{Z_U} = the upper bound of the calculation

F_{Z_L} = the lower bound of the calculation

APPENDIX B

USER'S GUIDE AND PROGRAM LISTING FOR THE DETERMINATION OF FLOOR LIVE LOAD PARAMETERS BASED ON THE CHALK AND COROTIS MODEL.

As described in Chapter 2, Chalk and Corotis (11) developed an equation to describe the cumulative distribution function of the maximum lifetime floor live load. This equation is listed in Chapter 2 as equation 2.38. Since the distribution of the floor live load is assumed to be extreme value type I, a cumulative value of 0.57 corresponds to the mean of the floor load. Likewise, a cumulative value of 0.856 is analogous to one standard deviation above the mean. Utilizing these cumulative values, the mean and standard deviation of the floor live load can be calculated by an iterative computer program.

A user's guide and program listing of the program used to solve equation 2.38 follows. The input quantities are defined as they are read into the compputer. The program was tested utilizing available data from Chalk and Corotis (11). These test results are given in Chapter 3, Table 3.6.

Basic Parameters:

FORMAT (7F10.4)

ML	M1	SIG1	M2	SIG2	M3	SIG3

ML = m_{L_i} = the mean of the sustained load survey

M1 = m_{L_s} = the mean of the maximum lifetime sustained load

SIG1 = σ_{L_s} = the standard deviation of the maximum lifetime sustained load

M2 = $m_{L_{e_1}}$ = the mean of the largest extraordinary load occurring during the maximum lifetime sustained load

SIG2 = $\sigma_{L_{e_1}}$ = the standard deviation of the largest extraordinary load occurring during the maximum lifetime sustained load

M3 = m_{L_e} = the mean of the maximum lifetime extraordinary load

SIG3 = σ_{L_e} = the standard deviation of the maximum lifetime extraordinary load

Basic Parameters:

FORMAT (4F10.4)

TAU	EPS	STOP	START

TAU = τ, the expected duration of occupancy

EPS = the maximum allowable error for STOP

STOP = the cumulative value of the floor live load, $F_{L_t}(\ell)$ which is used to solve for ℓ

START = an arbitrary starting value of the floor live load, ℓ given in psf

```
C*****************************************************************
C
C                        FLOOR LIVE LOAD PROGRAM
C                        CHALK AND COROTIS MODEL
C
C*****************************************************************
C
C
C          THIS PROGRAM SOLVES FOR THE MEAN AND STANDARD DEVIATION OF
C    THE DISTRIBUTION OF THE FLOOR LIVE LOAD FOLLOWING THE EQUATION
C    DEVELOPED BY CHALK AND COROTIS. THE DISTRIBUTION OF THE FLOOR LIVE
C    LOAD IS ASSUMED TO BE EXTREME VALUE TYPE I. THE PROGRAM USES AN
C    ITERATIVE TECHNIQUE.
C
C
      DOUBLE PRECISION MI,MII,MIII,SIGI,SIGII,START
      DOUBLE PRECISION SIGIII,X,TAU,STOP,A1,A2,A3,B1,B2,B3,W1,W2,W3,FX
      DOUBLE PRECISION EPS
      DOUBLE PRECISION ML,M1,SIG1,M2,SIG2,M3,SIG3
C
C    READ IN THE INPUT VALUES AS LISTED IN THE USERS GUIDE.
C
      READ(1,11) ML,M1,SIG1,M2,SIG2,M3,SIG3
      WRITE(2,11) ML,M1,SIG1,M2,SIG2,M3,SIG3
      READ(1,11) TAU,EPS,STOP,START
      WRITE(2,11) TAU,EPS,STOP,START
C
C    SOLVE FOR THE MOMENTS OF THE THREE LOAD CASES, CASE I, CASE II, AND
C    CASE III.
C
      MI=M1+M2
      SIGI=DSQRT(SIG1**2+SIG2**2)
      MII=M3
      SIGII=SIG3
      MIII=M1+M3
      SIGIII=DSQRT(SIG1**2+SIG3**2)
C
C    DETERMINE THE EXTREME VALUE PARAMETERS OF THE THREE CASES.
C
      A1=SIGI/1.283
      A2=SIGII/1.283
      A3=SIGIII/1.283
      B1=MI-0.577*A1
      B2=MII-0.577*A2
      B3=MIII-0.577*A3
C
C    ITERATIVELY SOLVE FOR THE MEAN OR VARIANCE OF THE TOTAL LOAD.
C
      NFLAG=0
      DO 10 I=1,100
      X=FLOAT(I)/10.+START
      W1=(X-B1)/A1
      W2=(X-(B2+ML))/A2
      W3=(X-B3)/A3
      FX=(DEXP(-DEXP(-W1))*DEXP(-DEXP(-W2))*(50.-TAU)/50.)+(DEXP(-DEXP(-
     1W3))*TAU/50.)
      WRITE(2,11) X,W1,W2,W3,FX
      IF(NFLAG.EQ.1) GO TO 20
   10 IF(FX.GT.STOP+EPS) NFLAG=1
   20 CONTINUE
   11 FORMAT(7F10.4)
      STOP
```

APPENDIX C

PRODUCT OF A RANDOM VARIABLE AND A CONSTANT: THREE PARAMETER WEIBULL AND LOGNORMAL TRANSFORMATIONS

In a differential reliability analysis, the strength values of the contrasting data set are altered by k until a similar probability of failure as the benchmark safety level results. Often, the strength distribution is either a 3-parameter Weibull or lognormal distribution. The first proof below demonstrates that if the location and scale parameters of a 3-parameter Weibull distribution are multipled by k, the result is the same as altering all of the strength values by k. Similarly, the second proof demonstrates that adding the logarithm of k to the mean of a lognormal distribution is analogous to multiplying all of the strength values by k.

The proofs are based on a general derivation taken from Ang and Tang (8). In particular, if $f_X(x)$ is a probability density function (pdf), then the pdf of the function

$$y = kx \tag{C.1}$$

is

$$f_Y(y) = f_X(g^{-1}(y)) \frac{dg^{-1}(y)}{dy} \tag{C.2}$$

where

$g^{-1}(y)$ = the inverse function of y

$\frac{dg^{-1}(y)}{dy}$ = the first derivative of $g^{-1}(y)$

The inverse function of equation C.1 is

$$x = y/k \tag{C.3}$$

and the first derivative is

$$\frac{dg^{-1}(y)}{dy} = 1/k \tag{C.4}$$

PROOF:

If $f_X(x)$ is a 3-parameter Weibull distrribution with parameters μ, σ, and η, then

$$f_X(x) = \frac{\eta}{\sigma}\left(\frac{x-\mu}{\sigma}\right)^{\eta-1} \exp\left[-\left(\frac{x-\mu}{\sigma}\right)^{\eta}\right] \tag{C.5}$$

Substituting equations C.3, C.4 and C.5 into equation C.2 results in

$$f_Y(y) = \frac{\eta}{\sigma}\left(\frac{(y/k)-\mu}{\sigma}\right)^{\eta-1} \exp\left[-\left(\frac{(y/k)-\mu}{\sigma}\right)^{\eta}\right]\frac{1}{k} \tag{C.6}$$

Multiplying by k/k within the parenthesis results in

$$f_Y(y) = \frac{\eta}{k\sigma}\left(\frac{y-k\mu}{k\sigma}\right)^{\eta-1} \exp\left[-\left(\frac{y-k\mu}{k\sigma}\right)^{\eta}\right] \tag{C.7}$$

which is a three parameter Weibull distribution with parameters

$$\mu' = k\mu \tag{C.8}$$

$$\sigma' = k\sigma \tag{C.9}$$

$$\eta' = \eta \tag{C.10}$$

PROOF:

Now suppose $f_X(x)$ is a lognormal distribution with parameters λ and ζ, then

$$f_X(x) = \frac{1}{\sqrt{2\pi}\, x\zeta} \exp\left[-\tfrac{1}{2}\left(\frac{\ln(x) - \lambda}{\zeta}\right)^2\right] \qquad (C.11)$$

Substituting equations C.3, C.4 and C.11 into equation C.2 results in

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\, (y/k)\zeta} \exp\left[-\tfrac{1}{2}\left(\frac{\ln(y/k) - \lambda}{\zeta}\right)^2\right] \frac{1}{k} \qquad (C.12)$$

Applying the laws of logarithms and rearranging,

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\, y\zeta} \exp\left[-\tfrac{1}{2}\left(\frac{\ln(y) - (\lambda + \ln(k)}{\zeta}\right)^2\right] \qquad (C.13)$$

which is a lognormal distribution with parameters

$$\lambda' = \lambda + \ln(k) \qquad (C.14)$$

$$\zeta' = \zeta \qquad (C.15)$$

APPENDIX D

USER'S GUIDE AND PROGRAM LISTING FOR ANALYSIS OF LUMBER PROPERTIES DATA USING A DEAD PLUS ROOF SNOW LOAD COMBINATION

This appendix is divided in two sections. The first section is the user's guide and program listing for the programs used to calculate the probability of failure of lumber data under a dead plus roof snow load. The strength data follows either a 3-parameter Weibull or lognormal distribution. The second section is the user's guide and program listing for the calculation of the k factor. Again, the resistance distributions follow either the 3-parameter Weibull or lognormal distribution. In both sections the load distribution is lognormal.

The input quantities are defined in the order that they are read into the computer programs. The following is a guide for inputting the quantities for the probability of failure programs, PFS1 and PFS3.

Basic Parameters:

FORMAT (2F10.4)

STEP	RANGE

STEP = Stepsize of the increment, dx in the failure probability calculation

RANGE = Range of failure probability calculation given in ksi

Basic Parameters:

FORMAT (5F10.4)

MNLN	COVLN	RMU[1]	RSIG[1]	RETA[1]

MNLN = Mean of the load distribution given in ksi

COVLN = Coefficient of variation of the load distribution

RMU = Location parameter of the strength distribution given in ksi

RSIG = Scale parameter of the strength distribution given in ksi

RETA = Shape parameter of the strength distribution.

[1] If program PFS3 is used, then RMU is substituted by RALAM and RSIG is substituted by RVARLN. RETA is not used. RALAM is the mean of the logarithms of the lumber strength values and RVARLN is the corresponding standard deviation.

```
C**********************************************************************
C
C                                PFS1
C
C
C                   SECOND MOMENT COMBINATION OF LOAD
C                         PROBABILITY OF FAILURE
C                                PROGRAM
C
C
C**********************************************************************
C
C
C    THIS PROGRAM CALCULATES THE PROBABILITY OF FAILURE WHEN THE LOAD IS
C    DISTRIBUTED LOGNORMALLY AND THE RESISTANCE IS A THREE PARAMETER
C    WEIBULL DISTRIBUTION.
C
C
C
      DOUBLE PRECISION RANGE,STEP,ALAM,RMU,RSIG,RETA,SUMLL,SUMUL
      DOUBLE PRECISION FRMAX,FRMIN,FSMAX,FSMIN,SMAX,SMIN,RBAR,RNOR1
      DOUBLE PRECISION RNOR2,RINIT,VARLN
      INTEGER N
C
C    READ IN THE INPUT PARAMETERS AS LISTED IN THE USERS GUIDE
C
      READ(1,11)STEP,RANGE
      READ(1,21)MNLN,COVLN,RMU,RSIG,RETA
C
C    CALL SUBROUTINE TO FIND THE LOGNORMAL PARAMETERS OF THE LOAD
C
      CALL PARLN(MNLN,COVLN.ALAM,VARLN)
      WRITE(2,21)STEP,RANGE,MNLN,COVLN,RMU,RSIG,RETA,ALAM,VARLN
C
C     INITIALIZE THE PROBABILITIES OF FAILURE AND BEGIN CALCULATION
C
      SUMLL=0.0
      SUMUL=0.0
      N=RANGE/STEP
      RINIT=RMU
      DO 30 I=1,N
      SMIN=RINIT+FLOAT(I)*STEP
      SMAX=RINIT+FLOAT(I+1)*STEP
C
C    CALL SUBROUTINE TO CALCULATE THE LOAD DISTRIBUTION CDF
C
      CALL CDFLN(ALAM,VARLN,SMIN,SMAX,FSMIN,FSMAX)
C
C    CALL SUBROUTINE TO CALCULATE THE RESISTANCE DISTRIBUTION CDF
C
      CALL CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
C
C    SUM UP THE INCREMENTAL PROBABILITIES OF FAILURE
C
      SUMLL=SUMLL+FRMIN*(FSMAX-FSMIN)
   30 SUMUL=SUMUL+FRMAX*(FSMAX-FSMIN)
C
C    WRITE THE PROBABILITIES OF FAILURE
C
      WRITE(2,31)SUMLL,SUMUL
   11 FORMAT(2F10.4)
   21 FORMAT(6F10.4)
```

```
   31 FORMAT(//,20X,'THE PROBABILITIES OF FAILURE',//,17X,'LOWER BOUND',
     115X,'UPPER BOUND',//,10X,E20.10,5X,E20.10)
      STOP
      END
C*********************************************************************
C
C     SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
C
C*********************************************************************
      SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
      DOUBLE PRECISION MEAN,COV,MNLN,VARLN
      MNLN=0.5*DLOG((MEAN**2)/((COV**2)+1.))
      VARLN=DLOG((COV**2)+1.)
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFLN(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C*********************************************************************
      SUBROUTINE CDFLN(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION ALAM,VARLN,SMIN,SMAX,AMIN,AMAX,FRMIN,FRMAX
      AMIN=(DLOG(SMIN)-ALAM)/DSQRT(VARLN)
      AMAX=(DLOG(SMAX)-ALAM)/DSQRT(VARLN)
      CALL MDNORD(AMIN,FRMIN)
      CALL MDNORD(AMAX,FRMAX)
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
C
C*********************************************************************
      SUBROUTINE CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX
      FRMIN=1.0-DEXP(-((SMIN-RMU)/RSIG)**RETA)
      FRMAX=1.0-DEXP(-((SMAX-RMU)/RSIG)**RETA)
      RETURN
      END
```

```
C*********************************************************************
C
C                              PFS3
C
C
C                   SECOND MOMENT COMBINATION OF LOAD
C                        PROBABILITY OF FAILURE
C                              PROGRAM
C
C
C*********************************************************************
C
C
C   THIS PROGRAM CALCULATES THE PROBABILITY OF FAILURE WHEN THE LOAD IS
C   DISTRIBUTED LOGNORMALLY AND THE RESISTANCE IS A LOGNORMAL
C   DISTRIBUTION.
C
C
C
      DOUBLE PRECISION RANGE,STEP,ALAM,RALAM,RVARLN,RZETA,SUMLL,SUMUL
      DOUBLE PRECISION FRMAX,FRMIN,FSMAX,FSMIN,SMAX,SMIN,RBAR,RNOR1
      DOUBLE PRECISION RNOR2,RINIT,VARLN
      REAL MNLN
      INTEGER N
C
C   READ IN THE INPUT PARAMETERS AS LISTED IN THE USERS GUIDE
C
      READ(1,11)STEP,RANGE
      READ(1,21)MNLN,COVLN,RALAM,RVARLN
C
C   CALL SUBROUTINE TO FIND THE LOGNORMAL PARAMETERS OF THE LOAD
C
      CALL PARLN(MNLN,COVLN,ALAM,VARLN)
      WRITE(2,21)STEP,RANGE,MNLN,COVLN,ALAM,VARLN,RALAM,RVARLN
C
C     INITIALIZE THE PROBABILITIES OF FAILURE AND BEGIN CALCULATION
C
      SUMLL=0.0
      SUMUL=0.0
      N=RANGE/STEP
      XMEAN=DEXP(RALAM+(RVARLN**2)/2.)
      RINIT=XMEAN-25.*(XMEAN**2)*(DEXP(RVARLN**2)-1)
      DO 30 I=1,N
      SMIN=RINIT+FLOAT(I)*STEP
      SMAX=RINIT+FLOAT(I+1)*STEP
C
C   CALL SUBROUTINE TO CALCULATE THE LOAD DISTRIBUTION CDF
C
      CALL CDFLN(ALAM,VARLN,SMIN,SMAX,FSMIN,FSMAX)
C
C   CALL SUBROUTINE TO CALCULATE THE RESISTANCE DISTRIBUTION CDF
C
      CALL CDFLN1(RALAM,RVARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C   SUM UP THE INCREMENTAL PROBABILITIES OF FAILURE
C
      SUMLL=SUMLL+FRMIN*(FSMAX-FSMIN)
   30 SUMUL=SUMUL+FRMAX*(FSMAX-FSMIN)
C
C  WRITE THE PROBABILITIES OF FAILURE
C
      WRITE(2,31)SUMLL,SUMUL
   11 FORMAT(2F10.4)
```

```
 21 FORMAT(6F10.4)
 31 FORMAT(//,20X,'THE PROBABILITIES OF FAILURE',//,17X,'LOWER BOUND',
   115X,'UPPER BOUND',//,10X,E20.10,5X,E20.10)
    STOP
    END
C*********************************************************************
C
C     SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
C
C*********************************************************************
      SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
      DOUBLE PRECISION MEAN,COV,MNLN,VARLN
      MNLN=0.5*DLOG((MEAN**2)/((COV**2)+1.))
      VARLN=DLOG((COV**2)+1.)
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFLN(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C*********************************************************************
      SUBROUTINE CDFLN(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION ALAM,VARLN,SMIN,SMAX,AMIN,AMAX,FRMIN,FRMAX
      AMIN=(DLOG(SMIN)-ALAM)/DSQRT(VARLN)
      AMAX=(DLOG(SMAX)-ALAM)/DSQRT(VARLN)
      CALL MDNORD(AMIN,FRMIN)
      CALL MDNORD(AMAX,FRMAX)
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFLN1(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C*********************************************************************
      SUBROUTINE CDFLN1(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION ALAM,VARLN,SMIN,SMAX,AMIN,AMAX,FRMIN,FRMAX
      AMIN=(DLOG(SMIN)-ALAM)/VARLN
      AMAX=(DLOG(SMAX)-ALAM)/VARLN
      CALL MDNORD(AMIN,FRMIN)
      CALL MDNORD(AMAX,FRMAX)
      RETURN
      END
```

For the calculation of the k factors, the computer programs KLN and KLN3 are utilized. KLN is used in conjunction with PFS1 and KLN3 is used in conjunction with PFS3. As a guide, the input quantities are defined as they are read into the computer programs.

Basic Parameters:

FORMAT (E10.3, 3F10.4)

ERROR	FK	STEP	RANGE

ERROR = One half the chosen maximum interval around the benchmark safety level.

FK = Starting value for k

STEP = Stepsize of the increment, dx in the failure probability calculation

RANGE = Range of failure probability calculation given in ksi

Basic Parameters:

FORMAT (6F10.4)

MNLN	COVLN	RMU[1]	RSIG[1]	RETA[1]

MNLN = Mean of the load distribution given in ksi

COVLN = Coefficient of variation of the load distribution

RMU = Location parameter of the strength distribution given in ksi

RSIG = Scale parameter of the strength distribution given in ksi

RETA = Shape parameter of the strength distribution

[1] If KLN3 is used, RMU is substituted by RALAM and RSIG is substituted by RVARLN. RETA is not used. RALAM is the mean of the logarithms of the lumber strength values and RVARLN is the corresponding standard deviation.

Basic Parameters:

FORMAT (2(2x, E16.9))

PROBL	PROBU

PROBL = Lower limit of the benchmark safety level.

PROBU = Upper limit of the benchmark safety level.

```
C*********************************************************************
C
C                                  KLN
C
C
C                         K CALCULATION PROGRAM
C
C
C*********************************************************************
C
C
C    THIS PROGRAM CALCULATES THE K VALUE WHEN THE LOAD DISTRIBUTION IS
C    DISTRIBUTED LOGNORMALLY. THE RESISTANCE DISTRIBUTION IS WEIBULL AND
C    THE FACTOR K IS INCREMENTED AND MULTIPLIED TO THE LOCATION AND SCALE
C    PARAMETERS OF THE STRENGTH DISTRIBUTION.
C
C
C
      REAL SSIG,FK
      DOUBLE PRECISION RANGE,STEP,SBAR,RMU,RSIG,RETA,SUMLL,SUMUL
      DOUBLE PRECISION FRMAX,FRMIN,FSMAX,FSMIN,SMAX,SMIN,RBAR,RNOR1
      DOUBLE PRECISION RNOR2,RINIT,PROBL,PROBU,RKSIG,RKETA,ERROR,RKMU
      DOUBLE PRECISION ALAM,VARLN
      INTEGER N
C
C    READ IN THE PARAMETERS AS LISTED IN THE USERS GUIDE
C
      READ(1,11)ERROR,FK,STEP,RANGE
      READ(1,21)MNLN,COVLN,RMU,RSIG,RETA
      READ(1,26)PROBL,PROBU
C
C    CALL SUBROUTINE TO FIND THE LOGNORMAL PARAMETERS OF THE LOAD
C
      CALL PARLN(MNLN,COVLN,ALAM,VARLN)
      WRITE(2,21)STEP,RANGE,MNLN,COVLN,RMU,RSIG,RETA,ALAM,VARLN
      WRITE(2,41)
C
C    INITIALIZE THE STARTING PARAMETERS
C
      N=RANGE/STEP
      NFLAG=0
   20 FK=FK+0.005
      SUMLL=0.0
      SUMUL=0.0
C
C    INCREMENT THE RESISTANCE PARAMETERS OF INTEREST
C
      RKSIG=RSIG*FK
      RKMU=RMU*FK
      RINIT=RKMU
C
C    FIND PROBABILITY OF FAILURE WITH CHANGED WEIBULL PARAMETERS.
C
      DO 30 I=1,N
      SMIN=RINIT+FLOAT(I)*STEP
      SMAX=RINIT+FLOAT(I+1)*STEP
      CALL CDFLN(ALAM,VARLN,SMIN,SMAX,FSMIN,FSMAX)
      CALL CDFWEI(RKMU,RKSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
      SUMLL=SUMLL+FRMIN*(FSMAX-FSMIN)
   30 SUMUL=SUMUL+FRMAX*(FSMAX-FSMIN)
C
C    WRITE THE K VALUE AND THE PROB. OF FAILURE.
C
```

```
      WRITE(2,31)FK,SUMLL,SUMUL
C
C   CHECK TO SEE IF PROBABILITY OF FAILURE IS CLOSE TO BENCHMARK LEVEL
C
      IF(NFLAG.EQ.1) GO TO 40
      IF(((PROBL-ERROR).LT.SUMLL.AND.SUMLL.LT.(PROBL+ERROR)).OR.((PROBU-
     %ERROR).LT.SUMUL.AND.SUMUL.LT.(PROBU+ERROR))) NFLAG=1
      GO TO 20
   40 CONTINUE
   11 FORMAT(E10.3,3F10.4)
   21 FORMAT(6F10.4)
   26 FORMAT(2(2X,E16.9))
   31 FORMAT(5X,F5.3,2X,E16.9,10X,E16.9)
   41 FORMAT(//,20X,'THE PROBABILITIES OF FAILURE',//,17X,'LOWER BOUND',
     115X,'UPPER BOUND',//)
      STOP
      END
C*********************************************************************
C
C     SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
C
C*********************************************************************
      SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
      DOUBLE PRECISION MEAN,COV,MNLN,VARLN
      MNLN=0.5*DLOG((MEAN**2)/((COV**2)+1.))
      VARLN=DLOG((COV**2)+1.)
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFLN(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C*********************************************************************
      SUBROUTINE CDFLN(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION ALAM,VARLN,SMIN,SMAX,AMIN,AMAX,FRMIN,FRMAX
      AMIN=(DLOG(SMIN)-ALAM)/DSQRT(VARLN)
      AMAX=(DLOG(SMAX)-ALAM)/DSQRT(VARLN)
      CALL MDNORD(AMIN,FRMIN)
      CALL MDNORD(AMAX,FRMAX)
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
C
C*********************************************************************
      SUBROUTINE CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX
      FRMIN=1.0-DEXP(-((SMIN-RMU)/RSIG)**RETA)
      FRMAX=1.0-DEXP(-((SMAX-RMU)/RSIG)**RETA)
      RETURN
      END
```

```
C*********************************************************************
C
C                                KLN3
C
C
C                        K CALCULATION PROGRAM
C
C
C*********************************************************************
C
C
C    THIS PROGRAM CALCULATES THE K VALUE WHEN THE LOAD DISTRIBUTION IS
C    DISTRIBUTED LOGNORMALLY. THE RESISTANCE DISTRIBUTION IS LOGNORMAL
C    AND THE LOG OF THE FACTOR K IS INCREMENTED AND ADDED TO THE MEAN
C    OF THE RESISTANCE DISTRIBUTION
C
C
      REAL SSIG
      DOUBLE PRECISION RANGE,STEP,SBAR,RALAM,RVARLN,SUMLL,SUMUL
      DOUBLE PRECISION FRMAX,FRMIN,FSMAX,FSMIN,SMAX,SMIN,RBAR,RNOR1
      DOUBLE PRECISION RNOR2,RINIT,PROBL,PROBU,ERROR
      DOUBLE PRECISION ALAM,VARLN,FK,SAVE
      INTEGER N
C
C    READ IN THE PARAMETERS AS LISTED IN THE USERS GUIDE
C
      READ(1,11)ERROR,FK,STEP,RANGE
      READ(1,21)MNLN,COVLN,RALAM,RVARLN
      READ(1,26)PROBL,PROBU
C
C    FIND SUBROUTINE TO FIND THE LOGNORMAL PARAMETERS OF THE LOAD
C
      CALL PARLN(MNLN,COVLN,ALAM,VARLN)
      WRITE(2,21)STEP,RANGE,MNLN,COVLN,ALAM,VARLN,RALAM,RVARLN
      WRITE(2,41)
C
C    INITIALIZE THE STARTING PARAMETERS
C
      N=RANGE/STEP
      NFLAG=0
      SAVE=RALAM
   20 FK=FK+0.005
      SUMLL=0.0
      SUMUL=0.0
C
C    INCREMENT THE RESISTANCE PARAMETERS OF INTEREST
C
      RALAM=SAVE+DLOG(FK)
      XMEAN=DEXP(RALAM+(RVARLN**2)/2.)
      RINIT=XMEAN-25.*(XMEAN**2)*(DEXP(RVARLN**2)-1)
C
C    FIND PROBABILITY OF FAILURE WITH CHANGED WEIBULL PARAMETERS.
C
      DO 30 I=1,N
      SMIN=RINIT+FLOAT(I)*STEP
      SMAX=RINIT+FLOAT(I+1)*STEP
      CALL CDFLN(ALAM,VARLN,SMIN,SMAX,FSMIN,FSMAX)
      CALL CDFLN1(RALAM,RVARLN,SMIN,SMAX,FRMIN,FRMAX)
      SUMLL=SUMLL+FRMIN*(FSMAX-FSMIN)
   30 SUMUL=SUMUL+FRMAX*(FSMAX-FSMIN)
C
C    WRITE THE K VALUE AND THE PROBABILITY OF FAILURE.
```

```
C
      WRITE(2,31)FK,SUMLL,SUMUL
C
C   CHECK TO SEE IF PROBABILITY OF FAILURE IS CLOSE TO BENCHMARK LEVEL
C
      IF(NFLAG.EQ.1) GO TO 40
      IF(((PROBL-ERROR).LT.SUMLL.AND.SUMLL.LT.(PROBL+ERROR)).OR.((PROBU-
     %ERROR).LT.SUMUL.AND.SUMUL.LT.(PROBU+ERROR))) NFLAG=1
      GO TO 20
   40 CONTINUE
   11 FORMAT(E10.3,3F10.4)
   21 FORMAT(6F10.4)
   26 FORMAT(2(2X,E16.9))
   31 FORMAT(5X,F5.3,2X,E16.9,10X,E16.9)
   41 FORMAT(//,20X,'THE PROBABILITIES OF FAILURE',//,17X,'LOWER BOUND',
     115X,'UPPER BOUND',//)
      STOP
      END
C***********************************************************************
C
C     SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
C
C***********************************************************************
      SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
      DOUBLE PRECISION MEAN,COV,MNLN,VARLN
      MNLN=0.5*DLOG((MEAN**2)/((COV**2)+1.))
      VARLN=DLOG((COV**2)+1.)
      RETURN
      END
C***********************************************************************
C
C     SUBROUTINE CDFLN(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C***********************************************************************
      SUBROUTINE CDFLN(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION ALAM,VARLN,SMIN,SMAX,AMIN,AMAX,FRMIN,FRMAX
      AMIN=(DLOG(SMIN)-ALAM)/DSQRT(VARLN)
      AMAX=(DLOG(SMAX)-ALAM)/DSQRT(VARLN)
      CALL MDNORD(AMIN,FRMIN)
      CALL MDNORD(AMAX,FRMAX)
      RETURN
      END
C***********************************************************************
C
C     SUBROUTINE CDFLN1(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C***********************************************************************
      SUBROUTINE CDFLN1(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION ALAM,VARLN,SMIN,SMAX,AMIN,AMAX,FRMIN,FRMAX
      AMIN=(DLOG(SMIN)-ALAM)/VARLN
      AMAX=(DLOG(SMAX)-ALAM)/VARLN
      CALL MDNORD(AMIN,FRMIN)
      CALL MDNORD(AMAX,FRMAX)
      RETURN
      END
```

APPENDIX E

USER'S GUIDE AND PROGRAM LISTING FOR ANALYSIS OF LUMBER PROPERTIES DATA USING A DEAD PLUS FLOOR LIVE LOAD COMBINATION

This appendix is divided into two sections. The first section is the user's guide and program listing for the programs used to calculate the probability of failure of lumber data under a dead plus floor live load. The strength data follows either a 3-parameter Weibull or lognormal distribution. The second section is the user's guide and program listing for the calculation of the k factor. Again, the resistance distributions follow either the 3-parameter Weibull or lognormal distribution. In both sections the load distribution is extreme value type I.

The input quantities are defined in the order that they are read into the computer program. The following is a guide for inputting the quantities for the probability of failure programs, PFL1 and PFL3.

Basic Parameters:

FORMAT (2F10.4)

STEP	RANGE

STEP = Stepsize of the increment, dx in the failure probability calculation

RANGE = Range of failure probability calculation given in ksi

Basic Parameters:

FORMAT (5F10.4)

MEAN	COV	RMU[1]	RSIG[1]	RETA[1]

MEAN = Mean of the load distribution given in ksi

COV = Coefficient of variation of the load distribution

RMU = Location parameter of the strength distribution given in ksi

RSIG = Scale parameter of the strength distribution given in ksi

RETA = Shape parameter of the strength distribution.

[1] If program PFL3 is used, then RMU is substituted by RALAM and RSIG is substituted by RVARLN. RETA is not used. RALAM is the mean of the logarithms of the lumber strength values and RVARLN is the corresponding standard deviation.

```
C***********************************************************************
C
C                               PFL1
C
C
C                    SECOND MOMENT COMBINATION OF LOAD
C                          PROBABILITY OF FAILURE
C                                 PROGRAM
C
C
C***********************************************************************
C
C
C    THIS PROGRAM CALCULATES THE PROBABILITY OF FAILURE WHEN THE LOAD IS
C    DISTRIBUTED EXTREME VALUE TYPE I AND THE RESISTANCE DISTRIBUTION IS
C    A THREE PARAMETER WEIBULL
C
C
C
      DOUBLE PRECISION RANGE,STEP,ALAM,RMU,RSIG,RETA,SUMLL,SUMUL
      DOUBLE PRECISION FRMAX,FRMIN,FSMAX,FSMIN,SMAX,SMIN,RBAR,RNOR1
      DOUBLE PRECISION RNOR2,RINIT,VARLN
      INTEGER N
C
C    READ IN THE INPUT PARAMETERS AS LISTED IN THE USERS GUIDE
C
      READ(1,11)STEP,RANGE
      READ(1,21)MEAN,COV,RMU,RSIG,RETA
C
C    CALL SUBROUTINE TO FIND THE TYPE I DISTRIBUTION PARAMETERS
C
      CALL PAREV1(MEAN,COV,A,B)
      WRITE(2,21)STEP,RANGE,MEAN,COV,RMU,RSIG,RETA,A,B
C
C    INITIALIZE THE PROBABILITIES OF FAILURE AND BEGIN CALCULATION
C
      SUMLL=0.0
      SUMUL=0.0
      N=RANGE/STEP
      RINIT=RMU
      DO 30 I=1,N
      SMIN=RINIT+FLOAT(I)*STEP
      SMAX=RINIT+FLOAT(I+1)*STEP
C
C    CALCULATE THE LOAD DISTRIBUTION CDF
C
      CALL CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
C
C    CALCULATE THE RESISTANCE DISTRIBUTION CDF
C
      CALL CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
C
C    SUM UP THE INCREMENTAL PROBABILITIES OF FAILURE
C
      SUMLL=SUMLL+FRMIN*(FSMAX-FSMIN)
   30 SUMUL=SUMUL+FRMAX*(FSMAX-FSMIN)
C
C    WRITE THE PROBABILITIES OF FAILURE
C
      WRITE(2,31)SUMLL,SUMUL
   11 FORMAT(2F10.4)
   21 FORMAT(6F10.4)
```

```
   31 FORMAT(//,20X,'THE PROBABILITIES OF FAILURE',//,17X,'LOWER BOUND',
     115X,'UPPER BOUND',//,10X,E20.10,5X,E20.10)
      STOP
      END
C*********************************************************************
C
C     SUBROUTINE PAREV1(MEAN,COV,A,B)
C
C*********************************************************************
      SUBROUTINE PAREV1(MEAN,COV,A,B)
      DOUBLE PRECISION MEAN,COV,A,B,STD
      STD=COV*MEAN
      A=STD/1.283
      B=MEAN-0.577*A
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
C
C*********************************************************************
      SUBROUTINE CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
      DOUBLE PRECISION A,B,SMIN,SMAX,FSMIN,FSMAX
      FSMIN=DEXP(-(DEXP(-(SMIN-B)/A)))
      FSMAX=DEXP(-(DEXP(-(SMAX-B)/A)))
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
C
C*********************************************************************
      SUBROUTINE CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX
      FRMIN=1.0-DEXP(-((SMIN-RMU)/RSIG)**RETA)
      FRMAX=1.0-DEXP(-((SMAX-RMU)/RSIG)**RETA)
      RETURN
      END
```

```
C***********************************************************************
C
C                             PFL3
C
C
C                  SECOND MOMENT COMBINATION OF LOAD
C                      PROBABILITY OF FAILURE
C                           PROGRAM
C
C
C***********************************************************************
C
C
C   THIS PROGRAM CALCULATES THE PROBABILITY OF FAILURE WHEN THE LOAD IS
C   DISTRIBUTED EXTREME VALUE TYPE I AND THE RESISTANCE DISTRIBUTION IS
C   LOGNORMAL
C
C
C
      DOUBLE PRECISION RANGE,STEP,MEAN,COV,RALAM,RVARLN,SUMLL,SUMUL
      DOUBLE PRECISION FRMAX,FRMIN,FSMAX,FSMIN,SMAX,SMIN
      DOUBLE PRECISION RINIT
      INTEGER N
C
C   READ IN THE INPUT PARAMETERS AS LISTED IN THE USERS GUIDE
C
      READ(1,11)STEP,RANGE
      READ(1,21)MEAN,COV,RALAM,RVARLN
C
C   CALL SUBROUTINE TO FIND THE TYPE I DISTRIBUTION PARAMETERS
C
      CALL PAREV1(MEAN,COV,A,B)
      WRITE(2,21)MEAN,COV,A,B,RALAM,RVARLN
C
C   INITIALIZE THE PROBABILITIES OF FAILURE AND BEGIN CALCULATION
C
      SUMLL=0.0
      SUMUL=0.0
      N=RANGE/STEP
      XMEAN=DEXP(RALAM+(RVARLN**2)/2.)
      RINIT=XMEAN-50.*(XMEAN**2)*(DEXP(RVARLN**2)-1)
      DO 30 I=1,N
      SMIN=RINIT+FLOAT(I)*STEP
      SMAX=RINIT+FLOAT(I+1)*STEP
C
C   CALCULATE THE LOAD DISTRIBUTION CDF
C
      CALL CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
C
C   CALCULATE THE RESISTANCE DISTRIBUTION CDF
C
      CALL CDFLN1(RALAM,RVARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C   SUM UP THE INCREMENTAL PROBABILITIES OF FAILURE
C
      SUMLL=SUMLL+FRMIN*(FSMAX-FSMIN)
   30 SUMUL=SUMUL+FRMAX*(FSMAX-FSMIN)
C
C   WRITE THE PROBABILITIES OF FAILURE
C
      WRITE(2,31)SUMLL,SUMUL
   11 FORMAT(3F10.4)
```

```
 21 FORMAT(6F10.4)
 31 FORMAT(//,20X,'THE PROBABILITIES OF FAILURE',//,17X,'LOWER BOUND',
   115X,'UPPER BOUND',//,10X,E20.10,5X,E20.10)
    STOP
    END
C**********************************************************************
C
C     SUBROUTINE PAREV1(MEAN,COV,A,B)
C
C**********************************************************************
      SUBROUTINE PAREV1(MEAN,COV,A,B)
      DOUBLE PRECISION MEAN,COV,A,B,STD
      STD=COV*MEAN
      A=STD/1.283
      B=MEAN-0.577*A
      RETURN
      END
C**********************************************************************
C
C     SUBROUTINE CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
C
C**********************************************************************
      SUBROUTINE CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
      DOUBLE PRECISION A,B,SMIN,SMAX,FSMIN,FSMAX
      FSMIN=DEXP(-(DEXP(-(SMIN-B)/A)))
      FSMAX=DEXP(-(DEXP(-(SMAX-B)/A)))
      RETURN
      END
C**********************************************************************
C
C     SUBROUTINE CDFLN1(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C**********************************************************************
      SUBROUTINE CDFLN1(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION ALAM,VARLN,SMIN,SMAX,AMIN,AMAX,FRMIN,FRMAX
      AMIN=(DLOG(SMIN)-ALAM)/VARLN
      AMAX=(DLOG(SMAX)-ALAM)/VARLN
      CALL MDNORD(AMIN,FRMIN)
      CALL MDNORD(AMAX,FRMAX)
      RETURN
      END
```

For the calculation of the k factors, the computer programs KEV1 and KEV13 are utilized. KEV1 is used in conjunction with PFL1 and KEV13 is used in conjunction with PFL3. As a guide, the input quantities are defined as they are read into the computer program.

Basic Parameters

FORMAT (E10.3, 3F10.4)

ERROR	FK	STEP	RANGE

ERROR = One half the chosen maximum interval around the benchmark safety level

FK = Starting value for k

STEP = Stepsize of increment, dx in the failure probability calculation

RANGE = Range of failure probability calculation given in ksi

Basic Parameters:

FORMAT (6F10.4)

MEAN	COV	RMU[1]	RSIG[1]	RETA[1]

MEAN = Mean of the load distribution given in ksi

COV = Coefficient of variation of the load distribution

RMU = Location parameter of the strength distribution given in ksi

RSIG = Scale parameter of the strength distribution given in ksi

RETA = Shape parameter of the strength distribution

[1] If KEV13 is used, RMU is substituted by RALAM and RSIG is substituted by RVARLN, RETA is not used. RALAM is the mean of the logarithms of the lumber strength values and RVARLN is the corresponding standard deviation.

Basic Parameters:

FORMAT (2(2x, E16.9))

PROBL	PROBU

PROBL = Lower limit of the benchmark safety level

PROBU = Upper limit of the benchmark safety level

```
C***********************************************************************
C
C                                KEV1
C
C
C                         K CALCULATION PROGRAM
C
C
C***********************************************************************
C
C
C   THIS PROGRAM CALCULATES THE K VALUE WHEN THE LOAD DISTRIBUTION IS
C   DISTRIBUTED EXTREME VALUE TYPE 1. THE RESISTANCE DISTRIBUTION IS
C   A WEIBULL AND THE FACTOR K IS INCREMENTED AND MULTIPLIED TO THE
C   LOCATION AND SCALE PARAMETERS OF THE WEIBULL DISTRIBUTION.
C
C
      REAL SSIG,FK
      DOUBLE PRECISION RANGE,STEP,SBAR,RMU,RSIG,RETA,SUMLL,SUMUL
      DOUBLE PRECISION FRMAX,FRMIN,FSMAX,FSMIN,SMAX,SMIN,RBAR,RNOR1
      DOUBLE PRECISION RNOR2,RINIT,PROBL,PROBU,RKSIG,RKETA,ERROR,RKMU
      DOUBLE PRECISION A,B
      INTEGER N
C
C   READ IN THE PARAMETERS AS LISTED IN THE USERS GUIDE
C
      READ(1,11)ERROR,FK,STEP,RANGE
      READ(1,21)MEAN,COV,RMU,RSIG,RETA
      READ(1,26)PROBL,PROBU
C
C   CALL SUBROUTINE TO FIND THE TYPE I PARAMETERS OF THE LOAD
C
      CALL PAREV1(MEAN,COV,A,B)
      WRITE(2,21)STEP,RANGE,MEAN,COV,RMU,RSIG,RETA,A,B
      WRITE(2,41)
C
C   INITIALIZE THE STARTING PARAMETERS
C
      N=RANGE/STEP
      NFLAG=0
   20 FK=FK+0.005
      SUMLL=0.0
      SUMUL=0.0
C
C   INCREMENT THE RESISTANCE PARAMETERS OF INTEREST
C
      RKSIG=RSIG*FK
      RKMU=RMU*FK
      RINIT=RKMU
C
C   FIND PROBABILITY OF FAILURE WITH CHANGED WEIBULL PARAMETERS.
C
      DO 30 I=1,N
      SMIN=RINIT+FLOAT(I)*STEP
      SMAX=RINIT+FLOAT(I+1)*STEP
      CALL CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
      CALL CDFWEI(RKMU,RKSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
      SUMLL=SUMLL+FRMIN*(FSMAX-FSMIN)
   30 SUMUL=SUMUL+FRMAX*(FSMAX-FSMIN)
C
C   WRITE THE K VALUE AND THE PROBABILITY OF FAILURE.
C
```

```
      WRITE(2,31)FK,SUMLL,SUMUL
C
C   CHECK TO SEE IF PROBABILITY OF FAILURE IS CLOSE TO BENCHMARK LEVEL
C
      IF(NFLAG.EQ.1) GO TO 40
      IF(((PROBL-ERROR).LT.SUMLL.AND.SUMLL.LT.(PROBL+ERROR)).OR.((PROBU-
     %ERROR).LT.SUMUL.AND.SUMUL.LT.(PROBU+ERROR))) NFLAG=1
      GO TO 20
   40 CONTINUE
   11 FORMAT(E10.3,3F10.4)
   21 FORMAT(6F10.4)
   26 FORMAT(2(2X,E16.9))
   31 FORMAT(5X,F5.3,2X,E16.9,10X,E16.9)
   41 FORMAT(//,20X,'THE PROBABILITIES OF FAILURE',//,17X,'LOWER BOUND',
     115X,'UPPER BOUND',//)
      STOP
      END
C*********************************************************************
C
C     SUBROUTINE PAREV1(MEAN,COV,A,B)
C
C*********************************************************************
      SUBROUTINE PAREV1(MEAN,COV,A,B)
      DOUBLE PRECISION MEAN,COV,A,B,STD
      STD=COV*MEAN
      A=STD/1.283
      B=MEAN-0.577*A
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
C
C*********************************************************************
      SUBROUTINE CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
      DOUBLE PRECISION A,B,SMIN,SMAX,FSMIN,FSMAX
      FSMIN=DEXP(-(DEXP(-(SMIN-B)/A)))
      FSMAX=DEXP(-(DEXP(-(SMAX-B)/A)))
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
C
C*********************************************************************
      SUBROUTINE CDFWEI(RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION RMU,RSIG,RETA,SMIN,SMAX,FRMIN,FRMAX
      FRMIN=1.0-DEXP(-((SMIN-RMU)/RSIG)**RETA)
      FRMAX=1.0-DEXP(-((SMAX-RMU)/RSIG)**RETA)
      RETURN
      END
```

```
C*********************************************************************
C
C                                KEV13
C
C
C                        K CALCULATION PROGRAM
C
C
C*********************************************************************
C
C
C   THIS PROGRAM CALCULATES THE K VALUE WHEN THE LOAD DISTRIBUTION IS
C   DISTRIBUTED TYPE 1. THE RESISTANCE DISTRIBUTION IS LOGNORMAL AND THE
C   LOG OF THE FACTOR K IS INCREMENTED AND ADDED TO THE MEAN OF THE
C   RESISTANCE DISTRIBUTION
C
C
      REAL SSIG
      DOUBLE PRECISION RANGE,STEP,SBAR,RALAM,RVARLN,SUMLL,SUMUL,FK
      DOUBLE PRECISION FRMAX,FRMIN,FSMAX,FSMIN,SMAX,SMIN,RBAR,RNOR1
      DOUBLE PRECISION RNOR2,RINIT,PROBL,PROBU,ERROR,SAVE
      DOUBLE PRECISION A,B
      INTEGER N
C
C   READ IN THE PARAMETERSAS LISTED IN THE USERS GUIDE.
C
      READ(1,11)ERROR,FK,STEP,RANGE
      READ(1,21)MEAN,COV,RALAM,RVARLN
      READ(1,26)PROBL,PROBU
C
C   CALL SUBROUTINE TO FIND THE TYPE I  PARAMETERS OF THE LOAD
C
      CALL PAREV1(MEAN,COV,A,B)
      WRITE(2,21)STEP,RANGE,MEAN,COV,A,B,RALAM,RVARLN
      WRITE(2,41)
C
C   INITIALIZE THE STARTING PARAMETERS
C
      N=RANGE/STEP
      NFLAG=0
      SAVE=RALAM
   20 FK=FK+0.005
      SUMLL=0.0
      SUMUL=0.0
C
C   INCREMENT THE RESISTANCE PARAMETERS OF INTEREST
C
      RALAM=SAVE+DLOG(FK)
      XMEAN=DEXP(RALAM+(RVARLN**2)/2.)
      RINIT=XMEAN-50.*(XMEAN**2)*(DEXP(RVARLN**2)-1)
C
C   FIND PROBABILITY OF FAILURE WITH CHANGED WEIBULL PARAMETERS.
C
      DO 30 I=1,N
      SMIN=RINIT+FLOAT(I)*STEP
      SMAX=RINIT+FLOAT(I+1)*STEP
      CALL CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
      CALL CDFLN1(RALAM,RVARLN,SMIN,SMAX,FRMIN,FRMAX)
      SUMLL=SUMLL+FRMIN*(FSMAX-FSMIN)
   30 SUMUL=SUMUL+FRMAX*(FSMAX-FSMIN)
C
C   WRITE THE K VALUE AND THE PROBABILITY OF FAILURE.
C
```

```
      WRITE(2,31)FK,SUMLL,SUMUL
C
C   CHECK TO SEE IF PROBABILITY OF FAILURE IS CLOSE TO BENCHMARK LEVEL
C
      IF(NFLAG.EQ.1) GO TO 40
      IF(((PROBL-ERROR).LT.SUMLL.AND.SUMLL.LT.(PROBL+ERROR)).OR.((PROBU-
     %ERROR).LT.SUMUL.AND.SUMUL.LT.(PROBU+ERROR))) NFLAG=1
      GO TO 20
   40 CONTINUE
   11 FORMAT(E10.3,3F10.4)
   21 FORMAT(6F10.4)
   26 FORMAT(2(2X,E16.9))
   31 FORMAT(5X,F5.3,2X,E16.9,10X,E16.9)
   41 FORMAT(//,20X,'THE PROBABILITIES OF FAILURE',//,17X,'LOWER BOUND',
     115X,'UPPER BOUND',//)
      STOP
      END
C*********************************************************************
C
C     SUBROUTINE PAREV1(MEAN,COV,A,B)
C
C*********************************************************************
      SUBROUTINE PAREV1(MEAN,COV,A,B)
      DOUBLE PRECISION MEAN,COV,A,B,STD
      STD=COV*MEAN
      A=STD/1.283
      B=MEAN-0.577*A
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
C
C*********************************************************************
      SUBROUTINE CDFT1(A,B,SMIN,SMAX,FSMIN,FSMAX)
      DOUBLE PRECISION A,B,SMIN,SMAX,FSMIN,FSMAX
      FSMIN=DEXP(-(DEXP(-(SMIN-B)/A)))
      FSMAX=DEXP(-(DEXP(-(SMAX-B)/A)))
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE CDFLN1(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
C
C*********************************************************************
      SUBROUTINE CDFLN1(ALAM,VARLN,SMIN,SMAX,FRMIN,FRMAX)
      DOUBLE PRECISION ALAM,VARLN,SMIN,SMAX,AMIN,AMAX,FRMIN,FRMAX
      AMIN=(DLOG(SMIN)-ALAM)/VARLN
      AMAX=(DLOG(SMAX)-ALAM)/VARLN
      CALL MDNORD(AMIN,FRMIN)
      CALL MDNORD(AMAX,FRMAX)
      RETURN
      END
```

APPENDIX F

USER'S GUIDE AND PROGRAM LISTING FOR ANALYSIS OF LUMBER DATA USING AN EXACT INTEGRAL APPROACH

The probability of failure of lumber strength data under a dead plus live load combination can be solved directly by incorporating a convolution integral in the failure probability calculations. The two load distributions, dead and live are convoluted or added at each successive incremental probability of failure calculation.

As a guide, the input quantities are listed as they are read into the computer. The programs are PFS2 and PFL2 which correspond to the dead plus snow load combination and the dead plus floor live load combination. The difference in the input quantities for the two programs is noted in the following user's guide.

Basic Parameters:

FORMAT (I5)

N

N = Number of incremental probabilities of failure to be calculated

Basic Parameters:

FORMAT (4F10.4)

MLN	COVLN	MLN2[1]	COVLN2[1]

MLN = Mean of the dead load given in ksi

COVLN = Coefficient of variation of the dead load

MLN2 = Mean of the roof snow load given in ksi

COVLN2 = Coefficient of variation of the roof snow load

[1]For program PFL2, the values MLN2 and COVLN2 are MEAN and COV, the mean and coefficient of variation of the floor live load, respectively.

Basic Parameters:

FORMAT (4F10.4)

DX	XMIN	XLOW	XHIGH

DX = Increment used to solve the convolution integral

XMIN = Minimum allowable value of the cumulative distribution function of the convoluted load. This value prevents machine underflow.

XLOW = Lower limit of the convolution integral

XHIGH = Upper limit of the convolution integral

Basic Parameters:

FORMAT (4F10.4)

DX1	RMU	RSIG	RETA

DX1 = Increment used in the probability of failure calculation

RMU = Location parameter of the strength distribution given in ksi

RSIG = Scale parameter of the strength distribution given in ksi

RETA = Shape parameter of the strength distribution

```
C***********************************************************************
C
C                                PFS2
C
C
C              CONVOLUTION INTEGRAL PROBABILITY OF FAILURE
C                             PROGRAM
C
C
C***********************************************************************
C
C
C    THIS PROGRAM CALCULATES THE PROBABILITY OF FAILURE WHEN THE LOAD
C    IS THE SUM OF TWO LOGNORMALLY DISTRIBUTED VARIABLES.
C    THE RESISTANCE IS A THREE PARAMETER WEIBULL DISTRIBUTION.
C
C
C
      INTEGER N
      DOUBLE PRECISION MLN,COVLN,MLN2,ALAM,VARLN,RMU,RSIG,RETA,Z
      DOUBLE PRECISION COVLN2,ALAM2,VARLN2
      DOUBLE PRECISION DX,XLOW,XHIGH,SUMUL,SUMLL,ZMIN,DIF
      DOUBLE PRECISION FSM(1000),FRMAX,FRMIN,FS,XMIN,SAVE,DX1,DUM
C
C    READ IN ALL THE PARAMETERS AS LISTED IN THE USERS GUIDE
C
      READ(5,1) N
      READ(5,11) MLN,COVLN,MLN2,COVLN2
      READ(5,11)DX,XMIN,XLOW,XHIGH
      READ(5,11) DX1,RMU,RSIG,RETA
      WRITE(6,16)
      WRITE(6,11) MLN,COVLN,MLN2,COVLN2
      WRITE(6,17)
      WRITE(6,11)DX,XMIN,XLOW,XHIGH
      WRITE(6,18)
      WRITE(6,11) DX1,RMU,RSIG,RETA
      SAVE=XLOW
C
C    FIND LOGNORMAL PARAMETERS LAMBDA AND PSI FOR BOTH DISTRIBUTIONS
C
      CALL PARLN(MLN,COVLN,ALAM,VARLN)
      CALL PARLN(MLN2,COVLN2,ALAM2,VARLN2)
      WRITE(6,19)
      WRITE(6,11)ALAM,VARLN,ALAM2,VARLN2
      SUMLL=0.D0
      SUMUL=0.D0
C
C    START THE OUTER INTEGATION WHICH STEPS OVER THE RANGE TO CALCULATE
C    THE PROBABILITY OF FAILURE.
C
      DO 10 I=1,N
      Z=FLOAT(I)*DX1+RMU
      XLOW=SAVE
C
C    CALL SUBROUTINE TO EVALUATE THE LOAD (CDF) AT EACH STEP OF THE
C    OUTER INCREMENT
C
      CALL FLNLN(ALAM,VARLN,ALAM2,VARLN2,Z,DX,XLOW,XHIGH,XMIN,FS)
C
C    SAVE THE CDF AT EACH STEP
C
      FSM(I)=FS
```

```
      IF(I.EQ.1) GO TO 10
      ZMIN=Z-DX
C
C   CALCULATE THE RESISTANCE DISTRIBUTION CDF ON BOTH SIDES OF THE
C   INCREMENTAL STEP
C
      FRMAX=1.0-DEXP(-((Z-RMU)/RSIG)**RETA)
      FRMIN=1.0-DEXP(-((ZMIN-RMU)/RSIG)**RETA)
C
C   CALCULATE THE INCREMENTAL AREA UNDER LOAD CURVE AND COMBINE WITH CDF
C   OF RESISTANCE DIST. THEN SUM TO FIND THE PROBABILITY OF FAILURE.
C
      DIF=FSM(I)-FSM(I-1)
      SUMLL=SUMLL+FRMIN*DIF
      SUMUL=SUMUL+FRMAX*DIF
   10 CONTINUE
C
C   WRITE THE PROBABILITIES OF FAILURE.
C
      WRITE(6,21) SUMUL,SUMLL
    1 FORMAT(I5)
   11 FORMAT(4F10.4)
   12 FORMAT(1X,2E20.13)
   15 FORMAT(1X,4(E16.9,2X))
   16 FORMAT(/,2X,'DEAD MEAN',2X,'DEAD COV',1X,'LIVE MEAN',1X,'LIVE COV
     &')
   17 FORMAT(/,3X,'CONV DX',5X,'XMIN',6X,'XLOW',6X,'XHIGH')
   18 FORMAT(/,3X,'PROBF DX',4X,'RMU',7X,'RSIG',6X,'RETA')
   19 FORMAT(/,8X,'DEAD LOAD',12X,'LIVE LOAD',/,4X,'LAMBDA',3X,'ZETA**2'
     &,5X,'LAMBDA',3X,'ZETA**2')
   21 FORMAT(//,12X,'THE PROBABILITIES OF FAILURE',//,10X,'UPPER BOUND',
     %10X,'LOWER BOUND',//,2(5X,E15.8))
      STOP
      END
C***********************************************************************
C
C     SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
C
C***********************************************************************
      SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
      DOUBLE PRECISION MEAN,COV,MNLN,VARLN
      MNLN=0.5*DLOG((MEAN**2)/((COV**2)+1.))
      VARLN=DLOG((COV**2)+1.)
      RETURN
      END
C***********************************************************************
C
C     SUBROUTINE FLNLN(ALAM,VARLN,ALAM2,VARLN2,Z,DX,XLOW,XHIGH,XMIN,FS)
C
C***********************************************************************
C
C   THIS SUBROUTINE CALCULATES THE CUMULATIVE DISTRIBUTION FUNCTION OF
C   THE VARIABLE Z, IF
C                             Z=X+Y
C
C                            WHERE
C                                  X IS LOGNORMAL
C                                  Y IS LOGNORMAL
C
      SUBROUTINE FLNLN(ALAM,VARLN,ALAM2,VARLN2,Z,DX,XLOW,XHIGH,XMIN,FS)
      DOUBLE PRECISION ALAM,VARLN,Z,DX,XLOW,XHIGH,FS,FN1,FN2,DUM,XMIN
      DOUBLE PRECISION ALAM2,VARLN2,DUMMY,FN3
      FS=0.D0
C
C   START THE OUTER NUMERICAL INTEGRATION; THEORETICALLY THIS IS FROM
C   MINUS INFINITY TO PLUS INFINITY. XLOW AND XHIGH SHOULD BE WISELY
```

```
C     CHOSEN SO THERE IS NO MACHINE UNDERFLOW BUT ACCURACY IS MAINTAINED.
C
   10 XLOW=XLOW+DX
      FN1=((6.2831853*VARLN)**(-0.5))*(1./XLOW)*DEXP((-0.5)*((DLOG(XLOW)
     &-ALAM)**2)/VARLN)
      FN2=((6.2831853*VARLN)**(-0.5))*(1./XLOW+DX)*DEXP((-0.5)*((DLOG(XL
     &OW+DX)-ALAM)**2)/VARLN)
C
C     CHECK LOCATION ON DENSITY CURVE AND THEN GUARD AGAINST UNDERFLOW
C
      IF(FN2.LT.FN1.AND.FN1.LT.0.1E-35) GO TO 40
C
C     CHECK TO GUARD AGAINST UNDERFLOW ON LOGNORMAL CDF CURVE
C
      IF(Z-XLOW.LT.1.0E-35) GO TO 30
      DUMMY=(DLOG(Z-XLOW)-ALAM2)/DSQRT(VARLN2)
      CALL MDNORD(DUMMY,FN3)
      IF(FN3.LT.0.0E-35) GO TO 30
C
C     ITERATE TO CALCULATE F(Z)
C
      FS=FS+FN1*FN3*DX
C
C     CHECK IF LOOP HAS GONE FAR ENOUGH
C
      IF(XLOW.GE.XHIGH) GO TO 40
   30 GO TO 10
   40 CONTINUE
      RETURN
      END
```

```
C***********************************************************************
C
C                                PFL2
C
C
C                 CONVOLUTION INTEGRAL PROBABILITY OF FAILURE
C                                  PROGRAM
C
C
C***********************************************************************
C
C
C   THIS PROGRAM CALCULATES THE PROBABILITY OF FAILURE WHEN THE LOAD
C   IS THE SUM OF A LOGNORMALLY DISTRIBUTED VARIABLE AND A TYPE I
C   EXTREME VALUE DISTRIBUTED VARIABLE. THE STRENGTH IS A THREE
C   PARAMETER WEIBULL DISTRIBUTION.
C
C
C
      INTEGER N
      DOUBLE PRECISION MLN,COVLN,MEAN,COV,VARLN,RMU,RSIG,RETA,Z
      DOUBLE PRECISION A,B
      DOUBLE PRECISION DX,XLOW,XHIGH,SUMUL,SUMLL,ZMIN,DIF
      DOUBLE PRECISION FSM(1000),FRMAX,FRMIN,FS,XMIN,SAVE,DX1,DUM
C
C   READ IN ALL THE PARAMETERS AS LISTED IN THE USERS GUIDE
C
      READ(5,1) N
      READ(5,11) MLN,COVLN,MEAN,COV
      READ(5,11)DX,XMIN,XLOW,XHIGH
      READ(5,11) DX1,RMU,RSIG,RETA
      WRITE(6,16)
      WRITE(6,11) MLN,COVLN,MEAN,COV
      WRITE(6,17)
      WRITE(6,11)DX,XMIN,XLOW,XHIGH
      WRITE(6,18)
      WRITE(6,11) DX1,RMU,RSIG,RETA
      SAVE=XLOW
C
C   FIND DISTRIBUTION PARAMETERS  FOR BOTH DISTRIBUTIONS
C
      CALL PARLN(MLN,COVLN,ALAM,VARLN)
      CALL PAREV1(MEAN,COV,A,B)
      WRITE(6,19)
      WRITE(6,11)ALAM,VARLN,A,B
      SUMLL=0.D0
      SUMUL=0.D0
C
C   START THE OUTER INTEGATION WHICH STEPS OVER THE RANGE TO CALCULATE
C   THE PROBABILITY OF FAILURE.
C
      DO 10 I=1,N
      Z=FLOAT(I)*DX1+RMU
      XLOW=SAVE
C
C   CALL SUBROUTINE TO EVALUATE THE LOAD (CDF) AT EACH STEP OF THE
C   OUTER INCREMENT
C
      CALL FLNEV1(ALAM,VARLN,A,B,Z,DX,XLOW,XHIGH,XMIN,FS)
C
C   SAVE THE CDF AT EACH STEP
C
      FSM(I)=FS
```

```
      IF(I.EQ.1) GO TO 10
      ZMIN=Z-DX
C
C   CALCULATE THE RESISTANCE DISTRIBUTION CDF ON BOTH SIDES OF THE
C   INCREMENTAL STEP
C
      FRMAX=1.0-DEXP(-((Z-RMU)/RSIG)**RETA)
      FRMIN=1.0-DEXP(-((ZMIN-RMU)/RSIG)**RETA)
C
C   CALCULATE THE INCREMENTAL AREA UNDER LOAD CURVE AND COMBINE WITH CDF
C   OF RESISTANCE DIST. THEN SUM TO FIND THE PROBABILITY OF FAILURE.
C
      DIF=FSM(I)-FSM(I-1)
      SUMLL=SUMLL+FRMIN*DIF
      SUMUL=SUMUL+FRMAX*DIF
   10 CONTINUE
C
C   WRITE THE PROBABILITIES OF FAILURE.
C
      WRITE(6,21) SUMUL,SUMLL
    1 FORMAT(I5)
   11 FORMAT(4F10.4)
   12 FORMAT(1X,2E20.13)
   15 FORMAT(1X,4(E16.9,2X))
   16 FORMAT(/,3X,'LOG MEAN',2X,'LOG COV ',1X,'EVT1 MEAN',2X,'EVT1 COV')
   17 FORMAT(/,3X,'CONV DX',5X,'XMIN',6X,'XLOW',6X,'XHIGH')
   18 FORMAT(/,3X,'PROBF DX',4X,'RMU',7X,'RSIG',6X,'RETA')
   19 FORMAT(/,4X,'LAMBDA',3X,'ZETA**2',5X,'ALPHA',5X,'BETA')
   21 FORMAT(//,12X,'THE PROBABILITIES OF FAILURE',//,10X,'UPPER BOUND',
     %10X,'LOWER BOUND',//,2(5X,E15.8))
      STOP
      END
C*********************************************************************
C
C     SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
C
C*********************************************************************
      SUBROUTINE PARLN(MEAN,COV,MNLN,VARLN)
      DOUBLE PRECISION MEAN,COV,MNLN,VARLN
      MNLN=0.5*DLOG((MEAN**2)/((COV**2)+1.))
      VARLN=DLOG((COV**2)+1.)
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE PAREV1(MEAN,COV,A,B)
C
C*********************************************************************
      SUBROUTINE PAREV1(MEAN,COV,A,B)
      DOUBLE PRECISION MEAN,COV,A,B,STD
      STD=COV*MEAN
      A=STD/1.283
      B=MEAN-0.577*A
      RETURN
      END
C*********************************************************************
C
C     SUBROUTINE FLNEV1(MNLN,VARLN,A,B,Z,DX,XLOW,XHIGH,XMIN,FS)
C
C*********************************************************************
C
C   THIS SUBROUTINE CALCULATES THE CUMULATIVE DISTRIBUTION FUNCTION OF
C   THE VARIABLE Z, IF
C                             Z=X+Y
C
      SUBROUTINE FLNEV1(MNLN,VARLN,A,B,Z,DX,XLOW,XHIGH,XMIN,FS)
```

```
      DOUBLE PRECISION B
      DOUBLE PRECISION MNLN,VARLN,A,Z,DX,XLOW,XHIGH,FS,FN1,FN2,DUM,XMIN
      FS=0.D0
C
C   START THE OUTER NUMERICAL INTEGRATION; THEORETICALLY THIS IS FROM
C   MINUS INFINITY TO PLUS INFINITY. XLOW AND XHIGH SHOULD BE WISELY
C   CHOSEN SO THERE IS NO MACHINE UNDERFLOW BUT ACCURACY IS MAINTAINED.
C
   10 XLOW=XLOW+DX
      FN1=((6.2831853*VARLN)**(-0.5))*(1./XLOW)*DEXP((-0.5)*((DLOG(XLOW)
     &-MNLN)**2)/VARLN)
      FN2=((6.2831853*VARLN)**(-0.5))*(1./XLOW+DX)*DEXP((-0.5)*((DLOG(XL
     &OW+DX)-MNLN)**2)/VARLN)
C
C   CHECK LOCATION ON DENSITY CURVE AND THEN GUARD AGAINST UNDERFLOW
C
      IF(FN2.LT.FN1.AND.FN1.LT.0.1E-35) GO TO 40
C
C   CHECK TO GUARD AGAINST UNDERFLOW ON TYPE I CDF CURVE
C
      IF((Z-XLOW).LT.XMIN) GO TO 40
      FN3=DEXP(-DEXP(-(((Z-XLOW)-B)/A)))
      IF(FN3.LT.0.0E-35) GO TO 30
C
C   ITERATE TO CALCULATE F(Z)
C
      FS=FS+FN1*FN3*DX
C
C   CHECK IF LOOP HAS GONE FAR ENOUGH
C
      IF(XLOW.GE.XHIGH) GO TO 40
   30 GO TO 10
   40 CONTINUE
      RETURN
      END
```

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THE INFLUENCE OF LOAD DISTRIBUTION

ON THE

RELIABILITY ANALYSIS

OF

LUMBER PROPERTIES DATA

by

Michael Bret Thurmond

(ABSTRACT)

Using state of the art information concerning the statistical nature of loads acting on light frame structures, distributions of maximum lifetime roof snow load and maximum lifetime floor live load were developed for use in differential reliability analyses of lumber properties data. The dead load was combined with the live load to obtain the total roof or floor load. Utilizing these total loads, contrasting sets of lumber data were analyzed based on the concept of equal reliability. The sensitivity of the reliability analysis to changing load distributions was studied. Subsequently, load distributions were recommended for use in differential reliability analyses of lumber properties data.