Essays on Applied Game Theory and Public Economics

Tsung-Han Yang

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Thorwald N. Tideman, Co-chair
Eric Bahel, Co-chair
Sheryl Ball
Alexander Charles Smith

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ABSTRACT

The first chapter presents a theoretical model of electoral competition where two parties can increase campaign contributions by choosing policies benefiting a significant interest group. However, such decision will shrink their hardcore vote base where voters are well informed about the policy. The parties can then allocate the funds between campaigning and personal wealth. Different from the core voters, independent voters can be attracted by advertisements funded by campaign spending. Using a multi-stage extensive form game, I investigate how electoral competition interacts with diversions and policy distortions. My result shows that a higher level of electoral competition helps mitigate policy distortions, but prompts the parties to divert more funds.

Perfectly informed signal senders need to communicate their true type (productivity or ability) which is often private information to potential receivers. While tests are commonly used as measures of applicants' productivity, the accuracy of them has been questioned. Beginning with the framework of a two-type labor market signaling game, the second chapter investigates how tests of limited reliability affect the nature of equilibria in signaling games with asymmetric information. Our results show that, if a test is inaccurate and costly, only pooling PBE exists given certain conditions. Different forms of test inaccuracy may allow a separating PBE to exist. We also study the case of three types and find different PBEs.

The central issue of siting noxious facilities is that the host community absorbs potential costs, while all others can share the benefits without paying as much. The third chapter presents a modified Clarke mechanism to facilitate the siting decision, taking into account all residents' strategies. Suppose that a social planner is able to reasonably estimate the possible costs, depending on the host location, to each resident created by the facility. Our proposed Clarke mechanism is characterized by strategy-proofness and yields an efficient siting outcome. The issue of budget imbalance is mitigated when the total cost, including the compensation scheme, is fully funded with tax revenues. We then use a simple example to show that a weighted version of the Clarke mechanism may yield a different outcome.

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GENERAL AUDIENCE ABSTRACT

People worry that lobbying may affect legislative decision-making in ways that disadvantage ordinary citizens. The first chapter presents a theoretical model of electoral competition where two parties can increase campaign contributions by choosing policies benefiting a significant interest group. However, such decision will shrink their hardcore vote base where voters are well informed about the policy. The parties can then allocate the funds between campaigning and personal wealth. Different from the core voters, independent voters can be attracted by advertisements funded by campaign spending. The result shows that a higher level of electoral competition helps mitigate policy distortions, but prompts the parties to divert more funds.

Signaling games have been widely used for decades. Perfectly informed signal senders need to communicate their true type (productivity or ability) which is often private information to potential receivers. While tests are commonly used as measures of applicants' productivity, the accuracy of them has been questioned. Beginning with the framework of a two-type labor market signaling game, the second chapter investigates how tests of limited reliability affect the nature of equilibria in signaling games with asymmetric information. Our results show that, if a test is inaccurate and costly, both high- and low-productivity workers voluntarily take the test given certain conditions. Different forms of test inaccuracy may allow the existence of a specific equilibrium where only high-productivity workers are willing to take the test. We also study the case of three types and find different types of equilibria.

The central issue of siting noxious facilities is that the host community absorbs potential costs, while all others can share the benefits without paying as much. The third chapter presents a modified Clarke mechanism to facilitate the siting decision, taking into account all residents' strategies. Suppose that a social planner is able to reasonably estimate the possible costs, depending on the host location, to each resident created by the facility. Our proposed mechanism motivates all citizens to honestly report their preferences and yields an efficient siting outcome. The issue of budget imbalance is mitigated when the total cost, including the compensation scheme, is fully funded with tax revenues. We then use a simple example to show that a wealth-weighted version of our proposed Clarke mechanism may yield a different outcome.

Dedication

To my parents for their endless love,

To my wife, Yi-An, for her sacrifice and support, and

To my lovely kids, Hannah and Anderson, for their smiles.

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Chapter 1

Interest Group, Electoral Competition and Diversion

1.1 Introduction

Political parties seek to maximize their power to secure their political benefits. They typically compete for seats in a legislature, which come from the ballots they obtain in an election. Interest groups can also gain benefits by politics. Influences of interest groups on electoral competition and policy outcomes have been widely discussed. Grossman and Helpman (1996) develop a theoretical model to explore how contributions from interest groups affect the equilibrium policy positions. While the authors offer a comprehensive and interesting analysis, they do not consider the possibility that political parties may divert some of the campaign contributions to increase the wealth level. This possibility does not exemplify the usual meaning of "commission", but it captures some version of it. In order to investigate the impact of diversions on the political equilibrium, I employ a modified framework of Grossman and Helpman (1996), that allows political parties to divert campaign contributions to personal uses. My paper will focus on the nature of the interaction between political lobbying and political parties' decisions on the level of diversion of funds and on announced policies. I am also interested in the impact of important electoral parameters on the nature of equilibria in a three-stage noncooperative political game.

Consider a policy that most voters (core voters or informed voters) do not like, for example something related to causing pollution. Without any influence of interest groups, the parties would choose the policy preferred by their followers. However, the existence of interest groups may change the story. Assume that there is an interest group, say a big tech industry that prefers this policy (the more the better). This interest group can move the policy closer to their preferred one by offering contribution schedules to the parties.

It would be better if the political parties would choose the policies that best served people's needs. However, it is commonly accepted that special interest groups may distort the parties' policy decisions by offering contributions. There have been many papers studying rent-seeking behavior by interest groups. Instead of discussing the rent-seeking model, I focus on how they affect electoral outcomes and policy decisions by making contributions. In the absence of lobbying, the two parties will choose policies in accordance with their core (or informed) voters' preferences. If they want to take an offer from an interest group, they have to sacrifice some ballots, due to choosing what the interest group wants, which is different from their core voters' wishes.

Interest groups have two motives for making donations to parties. First, they intend to influence the parties' policy announcements. Second, they intend to influence the electoral outcomes. Even though this paper focuses on the first incentive, my model allows them to contribute to the parties for both reasons. My first interest is in how the political parties' behavior and the interest group's contribution schedules will be affected by some important electoral parameters, for example a higher level of electoral competition.

Austen-Smith (1987) studies policy determination in a theoretical model of electoral competition by two political parties. The author assumes that that the parties can use funds contributed by interest groups to alleviate voters' uncertainty about candidates' policy positions. The key distinction between my paper and Austen-Smith's is the motive that interest groups are assumed to have for donating to the parties. While Austen-Smith assumes that interest groups view policy announcements as given and make contributions to influence the probability of winning the election, my model does not impose the restriction. I allow interest groups to affect the policy platforms directly. The author shows that interest groups can benefit from affecting candidates' electoral strategies.

Portugal et al. (2007) also give interest groups an opportunity to influence the parties' policy platforms. They develop an electoral competition game, where parties, lobbyists, and voters determine their optimal strategies in different stages. They assume that electoral campaigns may be financed by public funds and lobbyists' contributions. Lobbying groups are distinguished in terms of income. The authors show that parties will distinguish themselves by choosing different policies. In equilibrium, ideology rigidity determine how much private funds a party will receive from lobbyists. Poor and rich groups tend to make more contributions than the middle class.

It is important to discuss the relevant characteristics of the voters and their behavior first. Even though information can be circulated and exchanged easily nowadays, it is difficult to assure that people can be perfectly informed about candidates' characteristics and policies. Moreover, some people may not be sensitive to or interested in the information they are exposed to. Consider a specific policy, for example a law about pollution reduction. Even if they are informed about the policy, no matter from the mass media or from the campaign advertisement, these voters will not cast their ballots based on the candidates' proposal about this policy. Hence, it is not unusual that some voters are concerned about the candidates' policy platforms, but the others are not.

In this paper, the category of the voters plays an important role. Different from the literature, which generally assumes that voters are distributed in a continuum, I assume that there are two different types of voters: informed and uninformed ones. . The first group of voters are core voters of political parties. Voters in this category are well informed about the positions of the parties. They only vote for their favorite party. If they are not satisfied with the party's policy platforms, they will abstain, but they never vote for the other parties. This assumption that some voters only stick to one party may seem unreasonable. However, before the further discussion, a quote of the lyrics from Iolanthe, a comic opera with music by A. Sullivan and libretto by W. S. Gilbert, may support the idea:

> "I often think it's comical - Fal, lal, la!
> How Nature always does contrive - Fal, lal, la!
> That every boy and every gal
> That's born into the world alive
> Is either a little Liberal
> Or else a little Conservative!" (Sullivan, 1882)

Intuitively, if a party has a relatively large hardcore voter-base, it has a great chance to obtain more votes. It is not unusual to see that in some jurisdictions, residents predominantly prefer a party to another. For example, there are so-called red states and blue states in the U.S. Voters living in the red states are likely to vote out candidates belonging to the Republican Party, while those in blue ones prefer Democrats. According to a calculation by Campbell

(2008) from American National Election Study (ANES) data, in the 2004 U.S. Presidential election, around 40% of voters participating in the election identify themselves with either the Republican or the Democratic parties. Needless to say, the group of hardcore supporters play a significant role in elections.

Besides the voters belonging to the groups of the two opposite sides, there should be some uninformed or independent people. These voters do not have enough information about the parties or do not have specific tastes for them. Sometimes, these undecided and weak supporters represent a valuable and winnable prize in elections. They may potentially prefer one party to the others, but their primary concern is whether a party can bring them better life. In reality, voters cannot observe their payoff at the moment of making their decisions on whom to vote. All they can observe is the signals sent by parties or broadcasted by the mass media. Based on how much information they receive, they may choose to stay at their original position or switch to the other one. I will use the terms "independent voter" or "persuadable voter" to refer to voters in this group.

There have been several papers focusing on electoral competition with informed and uninformed voters. Baron (1994) uses a model of electoral competition to show how candidates choose their policies targeting informed and uninformed voters. The author claims that informed voters cast ballots based on specific policies benefiting them, while those uninformed ones are only concerned about campaign expenditures. He concludes that, in the case of particularistic policies, candidates separate their policies in equilibrium if the fraction of uninformed voters is sufficiently great. Furthermore, the degree of separation increases as that fraction becomes larger.

Grossman and Helpman (1996) develop a two-stage game to analyze how interest groups interact with two political parties in an election. The interest groups first propose their contribution schedules, which maximize their expected net payoffs. Then the parties choose their policy positions to maximize their seats in the legislature. They assume that, while informed voters cast their ballots for the part announcing the policy they relatively prefer, those uninformed ones may ignore the policy and only respond to the parties' messages or advertisement funded by their campaigning expenditures. The result shows that the party with greater expected seats will amass more campaign funding and adopt the policy more favorable to the interest group.

If some voters are informed about a policy, the candidates will try to choose what these voters like, to maximize the chance of winning. There are several papers addressing the issues about how incumbents respond to voters' needs, to enhance their chance to win. Cole et al. (2012) use weather disasters in India to test hypotheses about electoral outcomes and government responsiveness to weather crises. Their empirical findings show that voters may rationally punish incumbents for economically signicant events beyond the incumbents' control. These results also indicate how possible failures in electoral performance can lead to suboptimal levels of policies.

Instead of using voters' predispositions toward candidates as the standard of categorization, Majumdar et al. (2004) divide voters into two groups– rural and urban residents. Assuming that people in these two sectors have different access to information about sectoral resource allocation, they propose a theoretical model to investigate how incumbents optimize their chance to win by allocating resources to the two sectors. Their theoretical results imply that the incumbent will inefficiently over-allocate resources to urban areas if the number informed urban citizens sufficiently exceeds that of informed rural ones. By examining the effect of voter informativeness on the role of the electoral system, they conclude that greater information availability does not necessarily improve the potential function of the electoral process in weeding out incompetent incumbents.

I will address the issue that the parties can divert some funds to their private uses, which is not fully investigated in the theoretical literature of electoral studies. The amount of the diversions can be viewed as the price of "knocking the door." That is, *if you need me to offer some help, you should pay me for it*. Except for the political parties and the interest group, no one knows the truth about parties responding to interest groups, or no one has evidence of it. Based on this interpretation, the amount of funds diverted can be viewed as a measure of commissions. I assume that the remainder of the funds goes to their campaigns. This part of the payment from the interest group is a legal campaign contribution. The parties will use it on campaigning to attract more independent (or uninformed) voters.

The effect of campaign expenditures and advertisements is important in this paper. However, I only use the rough idea that a party can attract more persuadable voters if it spends more on campaigning. I do not exactly include "advertisement" into the model. Prat (2002) studies the role of campaign advertising in electoral competitions. He assumes that candidates can spend the contributions they obtain on informative advertisement. His results show that different forms of equilibrium exist depending on corresponding conditions. Strömberg (2001) claims that the mass media provides most of the information people need to determine for whom they vote. His theoretical result indicates that the a political party aiming to increase the chance to win will assign higher redistributive spending on those programs covered intensely by the mass media.

Dix and Santore (2003) do not consider campaign advertising in their model. Nevertheless, they assume that campaign expenditures can influence voters' perceptions about a candidate's ability. Different from my model, the authors do not assume swing voters to be uninformed and they do not consider hardcore supporters. A more important distinction is that their model ignores the impact of contributions on policy decisions. Using an all-pay-auction model, they find that campaign expenditure may be greater when the amount of swing voters decreases.

In line with the the discussion above, I allocate voters into three groups. Each party has its own potential core voters who care about the parties' policy positions. These voters are well informed about the policies. I assume that the other voters are all independent voters not tied to either party. Suppose that voters in the same hardcore group share a common preferred policy position. As long as the party chooses its core voters' preferred policy position, it can obtain all of their votes. If the party decides to deviate from its core voters' ideal position, it will lose some of the hardcore vote base. Here, I assume that those core voters who decide not to vote for their preferred party will abstain.[1] As the degree of the deviation in the announced policy position increases, more core voters choose to abstain.

Different from the hardcore supporters, the independent voters may vote for either party. Moreover, they are assumed to not have enough knowledge or not care about the policies chosen by the parties. Hence, while hardcore supporters care about the positions chosen by their preferred parties, the independent ones are concerned only about the messages or advertisements delivered by the parties. It is assumed that they can be persuaded to vote for a party through its advertisements funded by campaigning expenditures.

In addition, I assume that there is one interest group seeking to influence the policy outcome by donating to the parties. For simplicity, let policy positions be measured along the unit interval, i.e. $P \in [0, 1]$. To make my analysis less complicated without losing the main focus, I assume that the interest group does not directly participate in the election. That is, they are not counted as voters in the election. This group has its own ideal policy position, and will propose a contribution "schedule" to the two parties in order to push the expected policy outcome closer to its ideal one. The interest group may have two motivations to make

[1] It is somewhat reasonable to say that core voters will not vote for the party on the opposite side, even if they are disappointed with their preferred one.

contributions in my model. First, if a party's policy position is close enough to its ideal one, the interest group may donate more to this party to enhance its prospect to win more ballots. Second, the campaigning contributions can push the two parties to choose policies better serving the group's own interest.

The two parties have two concerns in this model: the ballots they can get and the expected cash benefit. During the election, the parties announce their policy positions, which will affect their ballots and the political contributions they may receive. On the one hand, to keep their core voters, the parties have to avoid policies too far from to the supporters' ideal position. On the other hand, to attract more contributions, the parties are likely to choose a position closer to the interest group's ideal one. However, by doing this, they may lose a significant level of the ballots from their hardcore supporters. Their loss in ballots depends on how far the announced policy is away from their supporters' ideal one.

I further assume that the parties can divert some contributions they receive to private uses. For example, the funds could be used for their own interests or some other political purposes, instead of being used for campaigning. How the politicians use the funds they divert is not important in my model, so I just assume that those diversions become wealth of the parties. Suppose that more funds used for campaigning can raise the number of votes by attracting more independent voters. In such a case, the parties confront another trade-off situation here. If they divert more, they can undoubtedly increase their wealth level. Meanwhile, they will lose some independent voters by reducing their campaigning expenditures.

The parties have to choose between keeping core voters and losing some ballots but getting some funds from the interest group. Moreover, if they take the money offered by the interest group, they will then have to choose the level of diversion to optimize their expected benefits. The purpose of this paper is to investigate how the players' strategies interact with each other, and how the important parameters affect the equilibrium. I will focus on the link between the incidence of electoral commissions and the interest group's activities. Also, I will discuss how electoral competition plays an important role in determining the players' strategies.

This paper is organized as follows. In the next section, I present a multi-stage, noncooperative political game to analyze the issue. My theoretical results come in Section 3. The last section offers concluding remarks.

1.2 Theoretical Analysis

1.2.1 The Basic Setup

I employ parts of Grossman and Helpman (1996)'s theoretical framework and propose a modified version that allows the parties to divert the campaign contributions. Consider an economy where there are only two self-interested political parties, denoted A and B, and there is a continuum of voters of mass $N > 0$. The two parties are assumed to run against one another in the election. I assume that seats in the parliament are allocated by proportional representation. If party A obtains 60% of ballots in the election, it will control exactly 60% of the legislature.[2]

Political parties can gain some political benefits according to the votes they obtain in the election (or the seats they control in the parliament). A party that obtains more than half of the ballots is more powerful and has a "bonus" based on how much it's ballots exceeds 1/2. By contrast, a party that has less than half of the seats has a smaller power in the parliament. Suppose that the total political benefit from the ballots is $R > 0$. Each party's expected electorate benefit is assumed to be $[\theta_j + d(\theta_j - 1/2)]R$, where $\theta_j \geq 0$ is the fraction of votes gained by party j and a small $d > 0$ measures the magnitude of the extra benefit if a party wins more than half of the ballots.

Consider a multi-stage extensive form game. In the first period, the parties choose the proportion of contributed funds diverted for private uses. The amount of diversions can be viewed as the political commissions. In the second period, the interest group proposes contribution schedules to the parties. Then the parties announce their policy positions and then obtain the funds. In the last period, the voters decide whom to vote for.[3]

I will explain the model more in-depth in the following discussion. My detailed description begins with the voters.

[2] Here, 60% of the seats may not be an integer. However, we can view it as how powerful party A is in the parliament. Hence, I will not make further discussion about the number of seats.

[3] To focus on the strategies by the two parties and the interest group, I do not model voters' strategic reacts in my equilibrium condition. The voters just follow a decision rule at the individual level which is not dependent on the others' actions. In other words, voters' responses to the parties' strategies are modeled as the the change in the share siding with the one or the other. They will simply vote according to the distribution.

1.2.2 The Voters

A fraction a of voters are potential hardcore supporters of party A. I will call these voters group a in the following parts. Analogously, a fraction b of voters potentially support party B, and they will be called group b. They are concerned only about the parties' policy position. Also, once they belong to the hardcore vote base, they will not vote for the other party. For instance, people in group a will not vote for party B in any case. If party A does not choose a satisfying policy position, some of them will leave group a and abstain in the election. The number of voters who end up abstaining is dependent on how far the announced policy is from their ideal one. For simplicity, let policy positions be measured along the unit interval, i.e. $P \in [0, 1]$. Assume that the voters in group a all prefer a policy of 0, and those in group b also prefer a policy of 0.[4] To make the problem more interesting, I assume that a and b are both smaller than 1/2.[5]

The share of voters' votes that party A can eventually obtain from group a is assumed to be $a(1 - P_A)^2$, where P_A is party A's announced policy position. Similarly, party B is assumed to get a share of $b(1 - P_B)^2$ from group b. It can be seen that, if party A chooses $P_A = 0$, then it can obtain all votes from group a. Analogously, party B can obtain all votes from its hardcore supporters, if $P_B = 0$. No party can attract any votes form their opponent's hardcore supporters, no matter which policy position it announces.

The rest of the voters, numbering $I = 1 - a - b$, are uninformed independent voters who do not care about the policy position but respond to the parties' efforts or advertisements. I assume that the party spending more on its campaign can attract a greater number of independent voters. Let λ be the expected fraction of independent voters who will vote for party A, which is assumed to be independent of a and b. To simplify the analysis, assume that no independent voters will quit. Then, we know that the fraction $1 - \lambda$ of them will vote for party B.

Consider a representative independent voter with label i, who is impacted by campaigning messages according to $g(E_j)$, where E_j is party j's campaigning spending. Here, I use a modified version of the functional form demonstrated by Grossman and Helpman (1996).

[4] One can view the policy as some pollution related one which most voters do not like.

[5] If a is too large, say greater than 1/2, then candidate A can easily win the election by choosing the policy that group a prefers. This assumption sounds more reasonable, if we consider party identification as the key factor of determining core voters. According to the survey conducted by the American National Election Studies, the ratio of people who claim themselves as "strong" Democrats or Republicans never exceeds 20% since 1970. Even if we include "weak" ones, the largest ratio since 2000 is just around 1/3.

Assume that voter i, decides to vote for party A if and only if $g(E_A) - g(E_B) \geq v_i$, where v_i measures voter i's assessment of the superiority (or inferiority if negative) of party B prior to campaign spending. The two parties do not observe the exact value of v_i for any particular voter i. However, they believe that for all i, v_i is an i.i.d. random variable drawn from a known distribution, $F(v)$. I assume that the distribution is independent of the two parties policy positions. Furthermore, suppose that v_i's are uniformly distributed on the range

$$\left(\frac{-1}{2h} - \frac{\epsilon}{h}, \frac{1}{2h} - \frac{\epsilon}{h}\right),$$

where ϵ, a parameter small enough, centered around zero, can be viewed as the ex ante voter bias in favor of party A, and $h > 0$ is a parameter measuring effectiveness of campaign spending. I will explain these two parameters later.

To simplify the case, I assume that $g(E_j) = E_j$. The probability that voter i will vote for party A is $F(E_A - E_B)$. Since all v_i's are drawn from the same distribution, we know that the expected fraction of the independent voters that party A can obtain is also $\lambda = F(E_A - E_B)$.[6]

1.2.3 The Political Parties

Each party is endowed with $W_j > 0$, which will be used for campaigning. In addition, they can receive political contributions C_A and C_B from the interest group. The amounts of C_A and C_B depend on the policy they propose and the interest group's ideal policy position. For any level of political contribution, I assume that the politicians can divert some of the funds to their pockets, and the remainder goes to their campaign.

We can see a trade-off confronted by the parties. For $j = A, B$, assume that $\delta_j C_j$ is the proportion of the contributions which is used for campaigning. Regardless of the seats in the parliament, the party can always have the diverted funds, $(1 - \delta_j)C_j$. In other words, the more they choose to put in their pockets; the higher the wealth level they will have. However, increasing the funds for personal uses also decreases their expected ballots, and thereby their control over the legislature, because less of the funds will be used for campaigning. In such a case, each party has to choose an optimal level of δ_j. The campaign spending of party j is then $E_j = W_j + \delta_j C_j$.

[6] According to the law of large numbers, with the continuum of independent voters, the expected fraction is that party A can get is $(1/n_I)\int_0^{n_I} F(E_A - E_B)di = F(E_A - E_B)$, where $n_I = NI$ is the total amount of independent voters.

For $j = A, B$, party j aims to maximize its payoff function

$$\Pi_j = \left[\theta_j(\cdot) + d\left(\theta_j(\cdot) - \frac{1}{2}\right)\right] R + (1 - \delta_j)C_j, \tag{1.1}$$

where $\theta_j(\cdot)$ is the expected share of ballots that party j can obtain in the election.

1.2.4 The Special Interest Group

The interest group can be viewed as an outside company or organization. The members do not vote in the election, but they can affect the outcome by their contributions. Assume that all members share a common interest about the policy. They have the same ideal policy position, defined as $P_L = 1$. However, they can hardly achieve this goal, since all informed voters do like the policy to be *zero*. Still, the interest group can push the policy closer to *one* as possible as they can by offering contribution schedules to the two parties.

Suppose that the final policy position that is expected to be implemented is $\bar{P} = \theta_A P_A + \theta_B P_B$.[7] Basically, as the expected policy outcome is farther away from $P_L = 1$, the interest group becomes worse off. I assume that all members in the interest group share identical preference concerning the policy issue, and the representative utility function of the interest group is

$$u_L = -\eta|\bar{P} - P_L| - C_A - C_B, \tag{1.2}$$

where $\eta > 0$ can be viewed as the pecuniary measurement of the marginal loss from the deviations from the ideal policy.

The contribution schedules, $C_A(P_A)$ and $C_B(P_B)$, are continuous, differentiable, and non-negative. The interest group is allowed to withhold the contribution, but they cannot tax the parties.

[7] I also assume that the fact that those voters who quit strongly prefer $P = 0$ will also affect the expected outcome, i.e. $\bar{P} = \theta_A P_A + \theta_B P_B + (1 - \theta_A - \theta_B)0$.

1.2.5 The Expected Share of Ballots

The expected share of ballots that party A can get is

$$\theta_A := a(1-P_A)^2 + \lambda I = a(1-P_A)^2 + \lambda(1-a-b),$$

and similarly, we know

$$\theta_B := b(1-P_B)^2 + (1-\lambda)(1-a-b),$$

We know that $\lambda = F(E_A - E_B)$. Since $F(\cdot)$ is the cumulative density function of the uniform distribution, $U(-1/2h - \epsilon/h, 1/2h - \epsilon/h)$, we can derive λ:

$$\lambda = F(E_A - E_B) = \frac{1}{2} + \epsilon + h(E_A - E_B), \text{ for all } E_A - E_B \in \left(\frac{-1}{2h} - \frac{\epsilon}{h}, \frac{1}{2h} - \frac{\epsilon}{h}\right).$$

If $\epsilon > 0$, we might expect party A to be the incumbent party. If instead, party B is running the office, ϵ is expected to be negative. There are other reasons that ϵ might be different from zero. For example, party A is more competent because of the average quality of its candidates is higher or its ideology agenda is more appealing to the public.

We can then obtain

$$\theta_A = a(1-P_A)^2 + \left[\frac{1}{2} + \epsilon + h(E_A - E_B)\right](1-a-b), \tag{1.3}$$

and

$$\theta_B = b(1-P_B)^2 + \left[\frac{1}{2} - \epsilon - h(E_A - E_B)\right](1-a-b). \tag{1.4}$$

1.3 Political Equilibrium

Again, the timing of the sequential game can be summarized as follows:

1. The two parties, $j = A, B$, choose the degree of diversion, $\delta_j \in [0, 1]$.
2. The interest group proposes the contribution schedules, $\{C_A(P_A), C_B(P_B)\}$, simultaneously to the two parties.

3. The parties announce their policy platforms, P_j, and then the contributions are paid.

4. Based on the distribution, the voters decide whom to vote for.

I will use backward induction to solve the political equilibrium.

1.3.1 Participation Constraints

We know that both parties have incentives to alter their policies only when the interest group offer the contributions sufficient enough to cover their expected loss from losing the ballots. Let us first consider the case without any contribution schedule, i.e. $C_j = 0$. It can be easily seen that party A's optimal choice is $P_A = 0$ and party B's is $P_B = 0$ as well. In other words, they will choose their core voters' preferred policy positions to keep all the ballots. I will use the subscript "0" to represent the case in the absence of lobbying, and "1" to stand for the case of lobbying. Then, we can derive party A's expected payoff:

$$\Pi_{A,0} = \left[\theta_{A,0}(\cdot) + d\left(\theta_{A,0}(\cdot) - \frac{1}{2}\right)\right] R.$$

Suppose that the interest group offers party A a contribution in exchange of announcing P_A, which is associated with its own utility. If party A takes the offer, the expected payoff will be

$$\Pi_{A,1} = \left[\theta_{A,1}(\cdot) + d\left(\theta_{A,1}(\cdot) - \frac{1}{2}\right)\right] R + (1 - \delta_A)C_A.$$

Each party has the option of denying the interest group's offer. To prevent from this situation, we have to consider the participation constraint. In other words, party A takes the offer only when $\Pi_{A,1} - \Pi_{A,0} \geq 0$. A similar condition holds for party B. Thus, we can find that the contribution to party A should satisfy

$$C_A \geq \frac{R(1+d)[a - a(1-P_A)^2]}{Rh\delta_A(1+d)(1-a-b) + 1 - \delta_A}.\text{[8]} \tag{1.5}$$

Similarly, we can derive the participation constraint for party B, which is

$$C_B \geq \frac{R(1+d)[b - b(1-P_B)^2]}{Rh\delta_B(1+d)(1-a-b)+1-\delta_B}. \tag{1.6}$$

1.3.2 Equilibrium Policy Positions

According to Eq. (1.2), we know that the utility level of the interest group is decreasing as the contributions paid to each party increase. Assume that the parties will choose to cooperate with the interest group when equality of the two constraints, (1.5) and (1.6), holds. Hence, the interest group will offers the possible minimum contributions, i.e. (1.5) and (1.6) with equality.

Given the interest group's utility function and the participation constraints, the optimization problem in the second stage can be written as

$$\begin{aligned}
\max_{\{P_A, P_B\}} \quad & u_L = -\eta|\bar{P} - P_L| - C_A - C_B \\
\text{s.t.} \quad & C_A = \frac{R(1+d)[a - a(1-P_A)^2]}{Rh\delta_A(1+d)(1-a-b)+1-\delta_A}; \\
& C_B = \frac{R(1+d)[b - b(1-P_B)^2]}{Rh\delta_B(1+d)(1-a-b)+1-\delta_B}; \\
& \bar{P} = \theta_A P_A + \theta_B P_B; \\
& P_A \in [0,1], \quad P_B \in [0,1].
\end{aligned} \tag{1.7}$$

The first order condition is

$$\begin{aligned}
\frac{\partial u_L}{\partial P_A} &= \eta\left(\frac{\partial \theta_A}{\partial P_A} P_A + \theta_A\right) - \frac{\partial C_A}{\partial P_A} = 0; \\
\frac{\partial u_L}{\partial P_B} &= \eta\left(\frac{\partial \theta_B}{\partial P_B} P_B + \theta_B\right) - \frac{\partial C_B}{\partial P_B} = 0.
\end{aligned} \tag{1.8}$$

We can solve P_A^* and P_B^*. Rather than the explicit solution of P_j^*, I am more interested in the properties in equilibrium. In order to obtain the equilibrium properties at the first stage of the game, we need to derive $\partial P_j^*/\partial \delta_i$ for $i = A, B$ and $j = A, B$. Using the first order

[8] See Appendix A.1 for the detailed derivation.

condition, as well as the Cramer's rule, we can obtain

Lemma 1. *For $j = A, B$, the following condition is true if $\partial^2 u_L / \partial P_j^2 < 0$ at $P_j = P_j^*$.*

$$sign\left\{\frac{\partial P_j^*}{\partial \mu}\right\} = sign\left\{\frac{\partial^2 u_L}{\partial P_j \partial \mu}\right\} \quad for\ \mu \in \{a, b, \delta_A, \delta_B, \epsilon, h, R, W_A, W_B\}.$$

Proof. See Appendix A.3. □

Lemma 1 is important in the theoretical analysis for the first stage of the game. Before the further discussion, I derive some important comparative statics properties, based on Lemma 1. The results are summarized in Proposition 1.

Proposition 1. *At the second stage, i.e. the stage at which the interest group determines the optimal policy requirement given the participation constraint, the interest group will offer party j a contribution schedule that requires greater P_j^*, and the parties will take the offer, if*

(i) party j chooses lower level of diversions, i.e. $\partial P_j^* / \partial \delta_j > 0$;

(ii) party $-j$ chooses higher level of diversions, i.e. $\partial P_j^* / \partial \delta_{-j} < 0$;

(iii) ϵ increases and $j = A$, i.e. $\partial P_A^* / \partial \epsilon > 0$.

Proof. See Appendix A.4. □

From Proposition 1, we can find that the interest group has greater influence on the announced policy of a party if this party decides to divert less or its opponent decides to divert more. Intuitively, a party with a great level of diversion can be viewed as the one that requires more political commissions. It is more likely for this party to choose wealth over the votes. Hence, the contributions will be less effective in delivering the policy preferred by the interest group. In contrast, if a party uses more contributions on campaigning, the interest group is willing to donate more and has greater influence on the announced policy.

The explanation for the second property in Proposition 1 is similar. If a party diverts more funds into private uses, the interest group knows that the contribution will be less effective. It will then have an incentive to offer the other party more contributions and require a policy position closer to its ideal one. Therefore, a party with greater diversions (that desires more commissions) is likely to indirectly push its opponent's announced policy toward the interest group's preferred position.

With respect to the third result, note that positive ϵ implies that party A is more popular among independent voters than party B is. As ϵ increases, party A becomes even more popular and electoral competition becomes less intense. The interest group then finds it more profitable to contribute to the popular party. Therefore, the interest group will induce party A to announce a policy closer to its preferred position with a greater amount of contributions.

1.3.3 Equilibrium Level of Diversions

Now, I will discuss the first stage of the game. At this stage, both parties simultaneously choose the proportion of campaign contributions to maximize their expected political benefits. Given the solution in the previous section, we can then derive the properties in equilibrium.

$$\begin{aligned} \max_{\{\delta_j\}} \quad & \Pi_j = \left[\theta_j(\cdot) + d\left(\theta_j(\cdot) - \frac{1}{2}\right)\right] R + (1-\delta_j)C_j \\ \text{s.t.} \quad & P_j = P_j^*(\delta_j, \delta_{-j}), \quad P_{-j} = P_{-j}^*(\delta_j, \delta_{-j}); \\ & \delta_j \in [0,1], \quad \delta_{-j} \in [0,1]. \end{aligned} \tag{1.9}$$

The corresponding first order condition for the two parties is

$$\frac{d\Pi_A}{d\delta_A} = 0 \text{ and } \frac{d\Pi_B}{d\delta_B} = 0. \tag{1.10}$$

Solving the two equations above gives rise to the equilibrium δ_j^*. Since the solution is messy and my focus is on the interesting properties in equilibrium, I will temporarily ignore the explicit solution of δ_j^* here. Consider the case where the interior solution exists. The first order condition implies Lemma 2.

Lemma 2. *For $j = A, B$, the following condition is true if $\partial^2\Pi_j/\partial\delta_j^2 < 0$ at $\delta_j = \delta_j^*$.*

$$\mathit{sign}\left\{\frac{\partial\delta_j^*}{\partial\rho}\right\} = \mathit{sign}\left\{\frac{\partial^2\Pi_j}{\partial\delta_j^*\partial\rho}\right\} \quad \mathit{for}\ \rho \in \{a, b, \delta_{-j}, \epsilon, h, R, W_A, W_B\}.$$

Proof. See Appendix A.5. □

Then, we can obtain Proposition 2 by deriving the comparative statics.

Proposition 2. *At the first stage of the game, the equilibrium level of the diversion of party j increases, if*

(i) given all other things being equal, party $-j$ reduces its diversion, i.e. $\partial\delta_j^*/\partial\delta_{-j} < 0$;

(ii) electoral competition becomes more intense, i.e. $\partial\delta_j^*/\partial\epsilon > 0$, assuming that $\epsilon > 0$, if $j = A$;

Proof. See Appendix A.6. □

In this model, the only reason for either party to lower the diversion is to attract more uninformed independent voters by increasing the campaign expenditures. The first property shows that the two parties' equilibrium activities are strategic substitute. In other words, when a party raises the proportion of its funds to campaigning, the opponent will choose a different strategy. One plausible explanation is that, if a party, say party A, finds that its rival competes more aggressively by increasing the campaign spending (or reducing the diversion), party A knows that its own campaign expenditure has a less influence on attracting the ballots from independent voters. Hence, parry A has a profitable move to divert more funds to personal uses.

Before the discussion about the second result, note that party A has an ex ante electoral advantage if ϵ is positive. In this case, if ϵ goes down, the electoral competition becomes more intense. Similarly, the marginal benefit of more campaign spendings becomes smaller, which implies that the campaign spending becomes less profitable. Therefore, party A will have less incentive to deviate from its core voters' preferred policy and will be likely to choose a greater level of the diversion.

1.4 Conclusion

It is generally believed that interest groups can gain benefits by politics. They offer money to candidates or parties to move policy platforms closer to their preferred ones. Through the contributions, interest groups may influence the electoral outcome and the political parties' announced policies. I use a modified version of Grossman and Helpman (1996)'model to show the interaction between the interest group's activity of lobbying and the political parties' decision about the policy platforms. Different from their model, the theoretical model proposed in this paper also allows the two parties to divert the campaign donations into personal uses, which means that they do not have to spend all the funds on campaigning. The more they allocate the funds to campaigning, the more votes from the persuadable uninformed voters they can acquire.

This paper does not put much emphasis on the explicit form of equilibrium strategies due to messy algebra. Fortunately, the properties in equilibrium still shows some interesting results. First, my model shows that the interest group is less influential on a party's announced policies if the amount of this party's diversions is great. The lobbyists find contributions to be less effective in moving the announced policy to their preferred one. As a result, this party will receive less contribution.

Moreover, a higher level of electoral competition reduces the parties' incentives to deviate from their followers' preferred policy positions. In contrast, if electoral competition becomes less intense, the interest group will contributes more to the popular party and move the parties' announced policy platforms closer to its desired position. Also, more intense competition limits the ability of a party to choose policies in favor of the interest group, and thus gives the party an incentive to divert more funds to private uses. The results suggest that policy distortions that come from lobbying activities may be mitigated if we enhance the level of electoral competition. However, the distortions cannot be eliminated.

There are some important issues that I have not been able to address or to solve. First, I assume that voters cast their ballots according to which group they belong to. Voters in different groups are not relevant with each other and make decisions based on different standards. However, I believe that we can build a more general model with less restrictions on voters. Second, it is fairly interesting to consider multiple policy dimensions or multiple interest groups in the analysis.

Another important topic that deserves more attentions but has not been considered in this paper is uncertainty. It is not realistic that every player in a political game, especially a game of elections, has full information about under-table activities. We can use a signaling game where the lobbyists cannot observe the degree of diversion with certainty. For instance, interest groups can only receive some signals or messages from the parties. They do not know exactly how much the parties will divert.

Finally, it is interesting to consider a repeated game instead of a one-shot game. If the parties take reelections into account, the result may be altered. Furthermore, it is also useful to investigate my topic with a model that incorporates the possibility that candidates or parties renege their campaign promises. I believe that such a model captures more comprehensive coverage of the study.

Chapter 2

Signaling Games with Costly and Imperfect Tests

2.1 Introduction

Signaling games have been widely used for decades, with some remarkable applications to labor markets, school admissions, and product grading certificates. Perfectly informed signal senders need to communicate their true type which is often private information to potential receivers. A commonly used signal is to submit test results that may fully or partially reveal their type. In the literature, tests are always assumed to be accurate, while in reality no test can be guaranteed to be perfect. Therefore, we want to investigate how tests of limited reliability affect the nature of equilibriums in signaling games with asymmetric information. In this paper, we use a framework of labor market signaling game to do the analysis.

The potential employers have no idea about workers' true productivity. One of the convincing ways to sort workers based on their abilities is through specific tests. There are many examples of tests that can reveal people' abilities. Many job seekers use Microsoft Certified Professional (MCP) certification as a signal to show that they are capable of doing high-skilled jobs. In addition, many IT firms use MCP certification as a tool to sort more able workers from the less able ones. In the context of university entrance admissions, most potential foreign students are required to take the TOEFL test or IELTS test to prove their english fluency.[1] Even if the tests are not mandatory, job seekers or students may still take them and use the results as signals to show that they are more capable than other candidates.

When tests are perfectly accurate, a commonly accepted prediction is separating equilibrium or at least some degree of separating equilibrium. To our best knowledge, not much work has been done on the scenario where tests are costly and imperfect. It becomes even less if we focus on the case that workers play an active role and are able to decide whether to take the test. Under the framework of product certification, De and Nabar (1991) claim that a pooling equilibrium where sellers of good or bad products are willing to certify their products exists when the cost is low enough. While De and Nabar's result is similar to one of our findings, this paper presents a more comprehensive investigation. First, like most previous relevant studies, they seem to have overlooked the effect of some important parameters like prior distribution of all the types. These parameters may also play important roles in determining existence and forms of possible equilibria. Therefore, in addition to test inaccuracy, we will also put some focus on the impacts of prior distribution and productivity.

One of De and Nabar's findings draws our attention. They conclude that there exists no separating equilibrium in pure strategies if tests are imperfect and costly. However, our result shows that a separating equilibrium exists if the test is costly and "partially" imperfect. Literally, we find that if the test is so easy that only low-productivity workers may be misreported as high-productivity ones, then we can obtain a separating equilibrium where only high-productivity ones take the test is the cost lies in the required range. This type of inaccuracy is called "upward biased" in our discussion later. Although the test is inaccurate, it can still correctly report a high-productivity worker's type. We also consider the other different forms of test inaccuracy and obtain several interesting results.

To investigate the issue more in-depth, we also consider the case where there are three possible types. Our finding reinforces the claim that the impacts of the parameters play important roles. In the two-type model, the workers (signal senders) play a win-or-lose game. Although their goal is to maximize the expected payoffs, their main concern is actually the winning chance if there is no test cost. We can see that, given any suitable belief, the required conditions for existence of possible equilibria are solely contingent on the prior and the inaccuracy rate. Nevertheless, as the number of types increases, each type has to consider the productivity of other types as well as her own. The probability of being recognized as

[1]The TOEFL test is a test of English as a foreign language. The IELTS test is the abbreviation of "The International English Language Testing System," which is commonly used in UK universities, but some US universities also admit the scores.

a high- or low-productivity worker is not the only thing that matters. Each equilibrium is accompanied by specific requirements involving priors, inaccuracy rates, and productivities.

Before going to the further discussion, let us get a quick review of the relevant literature. Sometimes, firms offer a wage schedule for given job classifications and use some exams to test potential employees to guarantee an acceptable level of worker performance. Even though we are interested in signaling models rather than screening ones, it is still helpful to capture some of the ideas from screen models. Guasch and Weiss (1980) consider an accurate pass-fail test offered by firms as workers' self-selecton device. They assume that test fees are paid only by workers, because of their beliefs that paying test costs discourages workers whose probability of passing the tests is low. Their paper shows that while there is no pooling equilibrium, depending on the costs and the distribution of workers, a semi-separating equilibrium may exist, where some higher-productivity workers take the tests and are sorted according to their productivities. Guasch and Weiss (1981) demonstrate a partial equilibrium model to investigate firms' behavior, including wage structures and organization of the firms. They show that if risk-neutral or risk averse workers are perfectly informed about their chance of passing the tests, there always exists a separating equilibrium where a wage-fee combination only attracts high-skill workers. Burdett and Mortensen (1981) studies the case that, in addition to signals attached to workers, the firms can choose some workers to test. While the signal can be used to predict workers' ability, the costly test can reveal workers' true ability. He shows that informed high-ability workers may be encouraged to take the test, and a competitive equilibrium exists.

While tests are commonly used as measure of applicants' ability, accuracy of them has been questioned. Is a potential worker with MCP certification more skillful than others without that certification? Can we easily conclude that a student with 110 TOEFL scores speaks English more fluently than some other one who only gets 90? It is nearly impossible to construct a test that can perfectly reveal a person's true ability. Were there such a test, we could still find many factors that can affect the test results, for example the test taker's physical or mental status. Also, some people might have special tricks to get high scores or some might just be luckier in the exam. Due to such background, we are interested in the impact of inaccurate tests on how workers and firms interact with each other in equilibrium.

Based on Mirrlees (1974)'s analytic framework, Nalebuff and Scharfstein (1987) study the role of tests in models of asymmetric information. It is assumed in their paper that only a fraction of workers who may have high or low productivity are selected to take the test, and that workers' test performance are publicly observable. Firms offer a wage contract which specifies a probability of being tested, test fees, and wages conditional on test performance, and then a potential worker decides which contract to accept. Given inaccuracy of the tests, they find that there exists no pooling equilibrium. If the tests become accurate as test expenditures approach to infinity, then there exists a separating equilibrium where low- and high-productivity workers accept different contracts.

So far, we have seen several important papers about the role of tests in labor markets with asymmetric information. Under specific circumstances, a separating or semi-separating equilibrium exists. While they offer good insights to deal with adverse selection, those papers all assume that uninformed agents like firms move first. Nonetheless, in many cases, we can see that, instead of passively being required to take the tests, informed agents sometimes want to signal themselves by submitting specific test results. As can be seen in the previous discussion, some people acquire MPC certification because it gives a signal to their potential employees that they may be better than those without MPC.

Weiss (1983) extends Spence's analytical framework by making assumptions that workers are not perfectly informed about their productivity and are tested upon completion of schooling. He maintains that firms also care about workers' success or failure in school, and tests can be used as a measure to sort workers. One critical difference between his paper and the previous work on sorting models is that he allows workers to be active players in the game and choose their strategies prior to firms' wage offers. Weiss finds that Nash equilibria always exist although some are unreasonable in terms of firms' low wage offers. By proposing a new concept of equilibria and assuming that schooling is productive, he concludes that if a pooling equilibrium exists, it may be characterized by too little schooling. Similar to Weiss's paper, Daley and Green (2014) also consider a job market signaling model with both an imperfect test and a costly signal like educational level. They generalize the structure for external information, allowing the informative ability of the test to vary with the worker's costly signal. Their results show that the stable equilibrium involves some degree of pooling on the educational level, which is different from commonly believed prediction of separating.

While Weiss provides a new angle about job market signaling models, he still focuses on the signaling effect of education. His assumption about mandatory tests cannot explain workers' incentives to take the test. In Daley and Green's paper, the test also plays an indirect role and is not the the signal that the sender uses to communicate her true type. Different from their papers, our model focuses on senders' choice of taking the test. Then, we want to see if inaccuracy of a test would diminish high-productivity workers' incentive to take the test or encourage low-productivity ones to take it in equilibrium. We are also interested in whether or not an imperfect test can carry enough information by sorting workers in terms of their strategies. Therefore, it is better to assume voluntary testing, which can be widely seen in markets of used cars or agricultural products. Some food companies may seek public third-party certification or labeling to signal food safety to uninformed buyers. In these cases, they try to obtain the certification first and then customers decide whether or not to buy from them and their willingness to pay.

Even though we put our focus on labor markets, there are still other examples that can help illustrate the case we are interested in. If you were a sport card fan, how much would pay for a Michael Jordan rookie-year card which usually costs several thousand dollars? Normally, when we look at the card trade market, we can find that some people may pay more to get one that has been graded high or pay less to buy a raw (ungraded) card. The higher the grade, the more expensive the card. The fact that consumers may be willing to pay a greater amount of money gives sellers incentives to submit their cards to a third party to get certificates or gradings. The seller may be lucky to get a higher grade than it is supposed to be. However, it is also possible that the card gets an unexpectedly lower grade.[2] If it turns out that the grade is not satisfactory, the value of that card will be even less than a similar-level raw card. Obviously, inaccuracy of gradings may give a seller with poor cards some hopes to get high grades and prevent the one with outstanding cards from submitting.

In the context of product certification or grading, the following studies may give us a good insight about this issue. De and Nabar (1991) investigate the impact of test errors on sellers' incentives to voluntarily certify their product. They find that low-quality sellers have no incentive to undertake the certification under perfect testing, but existence of test errors gives them some hope to obtain a wrongly graded high-quality certification. In other words, inaccuracy of tests offers low-quality sellers incentives to undertake the certification. Assuming that buyers believe that uncertified products are definitely of lower quality, they concludes that there is a differentiated pooling equilibrium where all suppliers, with high- or low-quality products, opt for certification. This result is strongly grounded on the assumption of buyers' off-equilibrium posterior belief. Similar to De and Nabar, Mason and Sterbenz (1994) show that inaccurate tests tend to create pooling outcomes where both high-and

[2] Of course, a card with better condition is more likely to get higher gradings, but there is no guarantee. It also depends on the credit of the grading company. Some companies like Beckett Grading Service (BGS) and Professional Sport Authenticator (PSA) are trustworthy and seem to give fair gradings.

low-quality producers take the test.

Although the structure and environment of job markets are different from markets of products, some interesting question can still be discussed. First, does inaccuracy of tests reduce more able job candidates' incentives to take the test or increase less able one' incentive to do so? Some studies mentioned above show that pooling equilibrium does not exist when tests are accurate. We want to investigate if inaccuracy of tests leads to pooling outcomes in job markets, and whether or not a separating equilibrium remains. Furthermore, tests are always costly. It is also our interest to study the role of test costs in job market signaling games.

In this paper, we demonstrate a theoretical model of a sequential game with incomplete information in the context of labor markets. We assume that workers have different types, either with low or high productivity, that would only be known by themselves. The firms cannot observe the potential employee's types directly. However, they can infer some information through a test that may reveal the worker's ability. The worker decides whether or not to take the test, and the competitive firms then make wage offers. To be more general, we do not restrict the test to be 100% accurate. Our interest is in how inaccuracy of a test would affect the equilibrium condition. Focusing on the concept of perfect Bayesian equilibrium (PBE), we want to see if there exist PBEs in pooling and separating strategies.

Furthermore, we extend the model to several particular cases. The first one is the case where the test exhibits only one-way inaccuracy. That is, this test may mistakenly report a high type as a low type or the other way. We can think of this test as one that is either too difficult or too easy. For the difficult test, while it is nearly impossible for a low-productivity worker to get a high grade, even a high-productivity worker may fail to get a satisfactory result. By contrast, an easy test may give low-productivity workers high grades but is not likely to fail a high-productivity taker. We show that the prior distribution of types and the rates of inaccuracy play an important role in determining existence of a pooling equilibrium where both types voluntarily take the test. Moreover, even if the test is not perfectly accurate, a separation PBE where only high type workers choose to take the test still exists if the derived conditions are satisfied.

Finally, we consider the case of three types rather than just high and low type. Unfortunately, we have not been able to obtain a satisfactory result from a completely general three-type model with positive test costs. However, from the study of some particular cases, it can still give us some interesting implications. Under some circumstances, we find existence of a special PBE where only mid-productivity workers choose to take the test. Intuitively, if the firms believe that a random pick from those who do not take the test yields the same expected payoff as any of the mid-productivity worker, they will offer the same wage. Therefore, no worker has a profitable deviation.

The remainder of this paper is organized as follows. In the next section, we introduce test errors to a job-market signaling model and present the setup in the economic environment. In Section 3, the PBE solution concept will be described. The equilibrium analysis comes in Section 4. We then extend the model by allowing different forms of test errors and including three types. The results are shown in Section 5. The last section offers concluding remarks.

2.2 Model Setup with Two Types

- Consider a framework with two firms and one worker. The worker may have a high or low productivity, denoted by θ_H and θ_L respectively. By definition, θ_H is greater than θ_L. Nature moves first by determining type θ_i for the worker according to some probability distribution p defined over $\Theta := \{\theta_H, \theta_L\}$ with a probability $p = \text{Prob}(\theta_H) \in (0, 1)$.

- After the potential worker observes her own type, she decides whether or not to take a test which will reveal her true type with probability $(1 - \alpha_t) \in (1/2, 1]$, where $t \in \{H, L\}$. [3] For example, if $\alpha_H = 0.1$, we can say that the test may reveal a wrong report with probability 0.1 if a high-type worker takes the test. Similarly, $\alpha_L = 0$ means that the test will perfectly reveal the true productivity of a low-type worker if she decides to take the test. To simplify the case, I assume that $\alpha_L = \alpha_H = \alpha$.[4] We will then consider the case where only $\alpha_H = 0$ or $\alpha_L = 0$. Since our focus is on imperfect tests, the case where $\alpha_H = \alpha_L = 0$ is excluded.

  To summarize, there are three possible signals that any type of the worker may send: $m \in M := \{N, T_H, T_L\}$, where N means "not to take the test," and T_H and T_L stand for the test results if the test is taken.[5]

[3] α can be viewed as the rate of inaccuracy. $\alpha = 0$ implies that the test is exactly accurate. We assume that $\alpha_t < 1/2$, since an exam which is significantly inaccurate seems unlikely to be used.

[4] Here I assume that the likelihood of a test wrongly reporting a high type as a low type is the same as that in the other way. This assumption does not affect our results of the two-type model significantly.

[5] Note that the worker may not be certain about the test result, if α is not zero. As α goes up, it is more

- Assume that the marginal cost of taking the test is $c \geq 0$, which is borne by the worker.
- Firms do not observe the worker's true type, but they can update their beliefs about the worker's type based on the signal received. Let $\mu_m : M \to [0,1]$ be the firms' posterior belief about the worker being of high type upon seeing signal m. For example, μ_L represents the probability that the firms believe the worker has high productivity given that she takes the test and the result shows that she has low productivity.
- Knowing the signal, but not the true type, each firm j offers a wage contract $w_j(m)$. Assume that both firms bid for the worker's services as in a first-price auction, and that the only cost for hiring the worker is the wage.

The game tree of this signaling game is shown in Figure 2.1.

likely that the high-type (or low-type) worker who takes the test will get the result of T_L (or T_H).

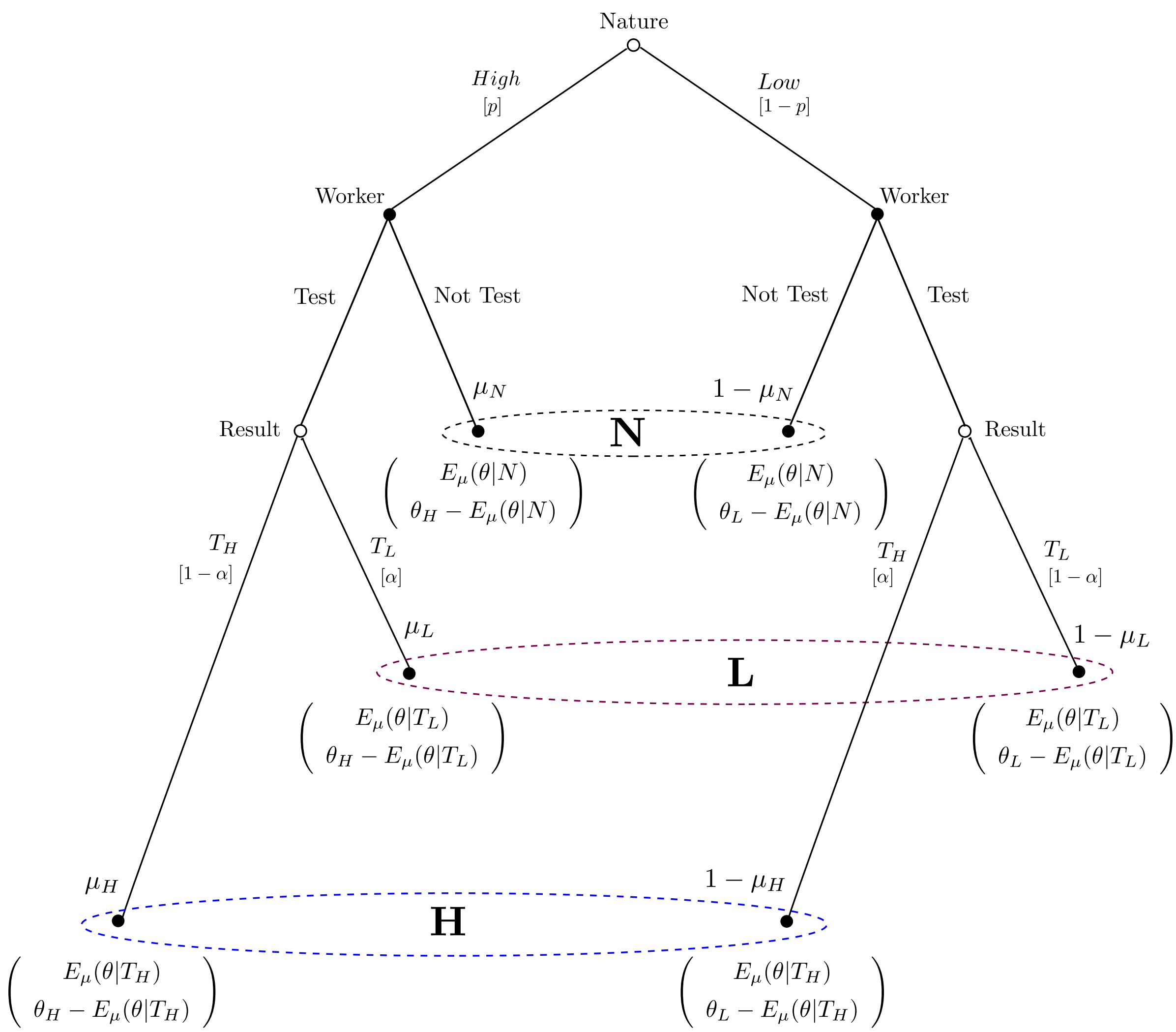

Figure 2.1: Job Market Signaling Game with Test Errors

2.2.1 Strategy Profiles

- The worker's strategies:
  Each type of the worker has two actions: taking the test (T), or not to (N). Let $s_t \in [0,1]$ be the probability that the worker takes the test if she is of type $t \in \{H, L\}$. Hence, the strategy set is denoted by $\Sigma \equiv [0,1] \times [0,1]$. Then, the worker's strategy can be written as $s_w := (s_H, s_L)$. Focusing on pure strategies, we can further assume $s_t \in \{0,1\}$. For instance, $s_w = (1,0)$ represents the strategy that the worker takes the test if she has high productivity and she does not take the test otherwise, while $(1,1)$

means that the worker takes the test regardless of her true type. The pure-strategy set is $S \equiv \{(1,0),(0,1),(1,1),(0,0)\} \subset \Sigma$.

- The firms' strategies:
  Given the worker's strategy, the two firms choose the wage schedules conditional upon the signal m. Denote $w_j^\eta(m)$ firm j's wage offer, where the superscript "η" represents the prescribed strategy of the worker with which the firms offer the wage plan.[6] For example, $w_j^{1,0}(T_H)$ means that when the worker's strategy profile is "(taking the test if high type, not taking the test if low type)," firm j pays $w_j^{1,0}(T_H)$ upon observing that the worker takes the test and the result shows high type. .

2.2.2 Expected Payoffs

Given their beliefs and the worker's prescribed strategy profile, the firms j's payoffs are

$$\pi_j^\eta(m) = \begin{cases} \theta_H - w_j^\eta(m) & \text{if the worker sending signal } m \text{ is of high type;} \\ \theta_L - w_j^\eta(m) & \text{if the worker sending signal } m \text{ is of low type.} \end{cases}$$

The worker decides whether or not to take the test before the result is shown, which means that she cannot be certain about the result unless $\alpha = 0$. Hence, her expected payoff is conditional on the probability of occurrence of test errors, i.e. α. Also, taking the test costs the worker, with high or low productivity, $c \geq 0$. Denote type i's expected payoff after taking the test $E_i^\eta(T)$, and that after deciding not to test $E_i^\eta(N)$. Suppose that the worker accepts the offer from firm $j \in \{1,2\}$. Given the strategy profile η, if the high-type worker takes the test, her expected payoff is

$$\begin{aligned} E_H^\eta(T) &= (1-\alpha)w_j^\eta(T_H) + \alpha w_j^\eta(T_L) - c \\ &= \theta_L + [(1-\alpha)\mu_H^\eta + \alpha\mu_L^\eta](\theta_H - \theta_L) - c. \end{aligned} \tag{2.1}$$

Similarly, we can derive the low-type worker's expected payoff when taking the test

$$\begin{aligned} E_L^\eta(T) &= (1-\alpha)w_j^\eta(T_L) + \alpha w_j^\eta(T_H) - c. \\ &= \theta_L + [(1-\alpha)\mu_L^\eta + \alpha\mu_H^\eta](\theta_H - \theta_L) - c. \end{aligned} \tag{2.2}$$

[6]The use of η is to avoid too many parentheses when we have to mention the prescribed strategy s_w in mathematical ways. Basically, $w_j^\eta(m) \equiv w_j(m|s_w)$. However, to simplify the notation, we will take out the parentheses.

If the worker decides not to take the test, the firms have no information about her true productivity. Regardless of the type, her payoff is

$$E_H^\eta(N) = E_L^\eta(N) = \mu_N^\eta \theta_H + (1 - \mu_N^\eta)\theta_L. \tag{2.3}$$

2.2.3 Perfect Bayesian Equilibrium

In this section, we want to find possible equilibriums in this signaling game. For a sequential game with incomplete information, perfect Bayesian equilibrium (PBE) seems to be an ideal equilibrium concept. A strategy profile and a system of beliefs over the nodes at all information sets constitute a PBE if:

1. At all information sets, each player who moves chooses a strategy that optimizes her payoff, given her beliefs and all the opponents' subsequent strategies.
2. The beliefs at all information sets are determined by Bayes' rule and the equilibrium strategies if they are on the equilibrium path. At the information sets off the equilibrium path, the beliefs are determined by Bayes' rule and the players' equilibrium strategies whenever possible.

Sequential Rationality

The first condition is the so-called sequential rationality. At each information set, the action chosen by a player has to maximize her expected payoff given her beliefs and the others' strategies. For example, given Eq. (2.1)-(2.3), a high ability worker will test if $E_H^\eta(T) \geq E_H^\eta(N)$.

Consistency of Beliefs (Bayesian Updating)

Let us consider the firms' optimal behavior. Since the labor market is assumed competitive, in equilibrium the wage is equal to the worker's marginal productivity. That is, were there perfect information about the worker's true type, we would have $w_j^\eta(\theta_H) = \theta_H$ and $w_j^\eta(\theta_L) = \theta_L$ for $j = 1, 2$.

Since the two firms have the same information, they should have identical beliefs in equilibrium (given the worker's strategy). The Bertrand-like competition will then result in an expected profit of zero for both, i.e., in equilibrium each firm should offer a wage equal to the expected productivity of the worker (conditional on their common belief). We will therefore drop the subscript of $w_j^\eta(m)$. We can then rewrite the firms' strategy in equilibrium as the following:

$$w^\eta(m) = \begin{cases} E_\mu(\theta|N, s_w) = \mu_N\theta_H + [1-\mu_N]\theta_L & \text{if the worker does not take the test} \\ E_\mu(\theta|T_L, s_w) = \mu_L\theta_H + [1-\mu_L]\theta_L & \text{if the test result is low type} \\ E_\mu(\theta|T_H, s_w) = \mu_H\theta_H + [1-\mu_H]\theta_L & \text{if the test result is high type.} \end{cases}$$

In this model, there are three information sets in which the firms have to assign their beliefs over all the nodes. The key point is how the firms form their beliefs. Two different cases should be considered: an information set can be either on or off the equilibrium path, based on the worker's strategy profiles. While the firms can assign an arbitrary belief on the information set off the equilibrium path, those beliefs formed along the equilibrium should be consistent with Bayes' rule.

For simplicity of the notation, define $P_N := p(1-s_H) + (1-p)(1-s_L)$ as the probability that the worker does not take the test. Also, define $P_H := ps_H(1-\alpha) + (1-p)s_L\alpha$ and $P_L := ps_H\alpha + (1-p)s_L(1-\alpha)$. Literally, P_H and P_L represent the probability that the worker takes the test and obtains the high result and low result respectively.[7] Then, according to Bayes' rule, the firms' belief, μ_m, should be

$$\begin{aligned} \mu_N^\eta &= \text{Prob}\,(\text{high type}\,|\text{not test}\,) &&= \frac{p(1-s_H)}{P_N} &&= \frac{p(1-s_H)}{p(1-s_H)+(1-p)(1-s_L)} \\ \mu_H^\eta &= \text{Prob}\,(\text{high type}\,|\text{test and get } T_H\,) &&= \frac{ps_H(1-\alpha)}{P_H} &&= \frac{ps_H(1-\alpha)}{ps_H(1-\alpha)+(1-p)s_L\alpha} \\ \mu_L^\eta &= \text{Prob}\,(\text{high type}\,|\text{test and get } T_L\,) &&= \frac{ps_H\alpha}{P_L} &&= \frac{ps_H\alpha}{ps_H\alpha+(1-p)s_L(1-\alpha)} \end{aligned} \tag{2.4}$$

[7] Note that one of the following three events should occur: the worker does not take the test, the worker takes the test and gets T_H, or the worker takes the test and gets T_L. Therefore, $P_H + P_L + P_N = 1$ has to hold.

2.3 Theoretical Results

2.3.1 Existence of PBE When The Test Cost Is Zero

First, let us consider the case where $c = 0$. We will focus on the following two types of equilibria: separating equilibria and pooling equilibria. For each strategy "s_w," we will check if it can be supported as part of a PBE. It can also be shown that existence of these different types of PBEs is contingent on the value of all parameters including α, p, θ_H, and θ_L. In order to make the explanation succinct, we will use "T-pooling PBE" to represent the pooling PBE where both types take the test, i.e. $s_w = (1, 1)$. Analogously, "N-pooling PBE" stands for the pooling PBE where neither type takes the test, i.e. $s_w = (0, 0)$.

It is not difficult to show that, without the test cost, a separating PBE exists only when the test can perfectly reveal the true type.[8] Then, we find the T-pooling PBE when the test is inaccurate. This is the same as De and Nabar's result if we consider zero cost as "low enough." However, our finding includes a more detailed explanation about the required condition for firms' posterior beliefs which does not seem to appear in their paper. Furthermore, we also show that the N-pooling PBE exists even without any cost. Note that in the literature, possible existence of the N-pooling PBE usually follows sufficiently high test costs.

Proposition 3. *Suppose that the test cost $c = 0$ and the likelihood of test error $0 < \alpha < 1/2$.*

1. *There exists a class of T-pooling PBE, $\{s_w = (1, 1), \mu_N^{1,1} \leq M(p, \alpha)\}$, where*

$$M(p, \alpha) = \frac{p\alpha(1-\alpha)}{[p(1-\alpha) + (1-p)\alpha][p\alpha + (1-p)(1-\alpha)]} < p.$$

2. *There exists a class of N-pooling PBE, $\{s_w = (0, 0), (\mu_H^{0,0}, \mu_L^{0,0}) \in \Psi\}$, where*

$$\Psi := \{(\mu_H, \mu_L) | \mu_L \leq \mu_H, (1-\alpha)\mu_H + \alpha\mu_L \leq p\}.$$

[8] It is not difficult to find the high-type worker's dominant strategy is to take the test while the low-type worker's dominant strategy is not to take the test. Therefore, we can find that the only strategy that can be supported as part of a PBE is $s_w = (1, 0)$. Intuitively, since the test is accurate and there is no cost for taking the test, a worker with a high productivity is always inclined to take the test in order to reveal her true type. However, a worker with a low productivity cannot gain any benefit by taking the test since she cannot hide her true productivity.

Proof. See Appendix B.1. □

Existence of Separating PBE When $0 < \alpha < 1/2$

Then, let us consider the case where the test is inaccurate, i.e. $0 < \alpha < 1/2$. There are two possible candidates for the worker's strategies in a separating PBE, $s_w = (1, 0)$ or $(0, 1)$. Since only one type takes the test, the firms can build their beliefs merely based on the worker's prescribed strategy. We can first rule out the possibility of $(0, 1)$ being supported as part of a PBE. All information sets are reached in this case, so we can apply Bayes' rule to update the firms' posterior beliefs. Given the prescribed strategy, it can be found that $\mu_N^{0,1} = 1$ and $\mu_L^{0,1} = \mu_H^{0,1} = 0$ by Eq. (2.4). The set of beliefs implies that the firms believe that test-takers are all low types, regardless of their test results. Based on the beliefs, we can then derive the firms' wage offers:

$$w^{0,1}(m) = \begin{cases} E_\mu(\theta|N) = \theta_H & \text{if the worker does not take the test ;} \\ E_\mu(\theta|T_L) = \theta_L & \text{if the test result is low type ;} \\ E_\mu(\theta|T_H) = \theta_L & \text{if the test result is high type.}^{9} \end{cases}$$

Behaving as the prescribed strategy, i.e. taking the test, the low-type worker can only get θ_L. She has a profitable deviation toward "not taking the test," by which she could obtain θ_H. Therefore, $s_w = (0, 1)$ cannot be supported as a PBE when the test cannot perfectly reveal the worker's productivity.

Similarly, with $0 < \alpha < 1/2$, we can find that $s_w = (1, 0)$ also cannot be supported as a PBE. According to Eq. (2.4), we know $\mu_N^{1,0} = 0$ and $\mu_L^{1,0} = \mu_H^{1,0} = 1$. Literally, the worker's prescribed strategy makes the firms believe that the worker who does not take the test has a low productivity while the worker who takes the test has a high productivity regardless of the results. Thus, the low-type worker's payoff is θ_L based on the prescribed strategy. If she deviates toward "taking the test," she can anticipate to get a greater payoff, θ_H. In conclusion, the separating strategy $s_w = (1, 0)$ cannot be supported as a PBE when there exists uncertainty about test results.[10]

[9] From now on, we will skip most of the detailed wage schedules and expected payoffs. Actually, these values depend on the firms' beliefs. With the system of beliefs, we can easily get the information about expected payoffs by applying Eq. (2.1), (2.2), and (2.3).

[10] Note that if the test were perfectly accurate, i.e. $\alpha = 0$, then the firms would realize the low-type worker's real productivity, θ_L, when she takes the test. In that case, when the firms observe the result being

The key point here is $\alpha > 0$. Inaccuracy of the test prevents the firms from utilizing the information carried by different test results if only one type chooses to take the test. Were the test perfectly accurate, i.e. $\alpha = 0$, the firms would offer wages in accordance with the test results. Under that situation, there exists a separating PBE in pure strategies: only high type workers take the test. To any any worker who test, the firms offer exactly their productive ability. And any worker who refuses to test is offered θ_L by both firms.

Existence of Pooling PBE When $0 < \alpha < 1/2$

Suppose that $0 < \alpha < 1/2$. There are two possible candidates of the worker's strategies which may be supported as a pooling PBE, $(s_H, s_L) = (1,1)$ or $(0,0)$. Before deriving the equilibrium condition, we need to obtain the firms' beliefs first. Consider $(s_H, s_L) = (1,1)$, with which the worker decides to take the test regardless of her true type. Information set $\boldsymbol{N}$ is not reached, so the belief here can be arbitrarily assigned. If we focus on the framework of sequential equilibrium, the initial prior p should be the best candidate for $\mu_N^{1,1} = p$.[11] However, it can be proven that with $\mu_N^{1,1} = p$, the T-pooling PBE breaks down since the low type worker has a profitable deviation to not taking the test.

Were the test perfectly accurate, the T-pooling PBE would exist with firms' posterior belief being $\mu_N^{1,1} = 0$. In other words, if firms believe that only low-productivity workers would choose not take the test, then they offer the possibly lowest wage to those who do not take the test. As can be seen in Proposition 1, this result holds even if the test is not accurate. However, the posterior belief, $\mu_N^{1,1}$, at information set $\boldsymbol{N}$ is not strictly required to be zero in the case of inaccurate tests. One plausible reason is that the firms believe that inaccuracy may reduce some high-productivity workers' willingness to take the test due to the decrease in their expected payoffs.

Since $\mu_N^{1,1} = 0$ is an acceptable belief in the T-pooling PBE, we can look at this case from the other angle. If the test is perfectly accurate, we know that in the T-pooling PBE with $\mu_N^{1,1} = 0$, the low-productivity worker's payoff is θ_L regardless of her choice. Consider a small increase in α from $\alpha = 0$. It seems reasonable to say that the firms may not adjust their

θ_L, they believe that the worker is low type, and then offer $w^{1,0}(T_L) = \theta_L$.

[11] We use the concept from Kreps and Wilson (1982). An assessment (μ, s) is consistent if there exists a sequence of totally mixed strategies, s^k, and corresponding beliefs, μ^k, derived from Bayes' rule such that $\lim_{k\to\infty}(\mu^k, s^k) = (\mu, s)$. See Appendix B.2 for further discussion.

beliefs, i.e. keeping $\mu_N^{1,1} = 0$. In this case, her payoff when choosing not to take the test is still θ_L. However, this low-productivity worker can obtain $\theta_L + M(p,\alpha)(\theta_H - \theta_L)$. We can view $M(p,\alpha)$ as a multiplier on the premium from inaccuracy of the test. Intuitively, if the firms believe that only low-productivity workers choose not to take the test and offer the lowest wage, then the low-productivity workers will stick to taking the test since inaccuracy raise their expected payoff.

Now we consider how changes in α and p affect the nature of the T-pooling PBE. Figure 2.2 presents a simple example showing the relation between $M(p,\alpha)$ and the values of α given different values of p. It can be found that when α increases, $M(p,\alpha)$ also increases.[12] This result implies that the firms may guess that more high-productivity workers choose not to take the test if the rate of inaccuracy goes up. Moreover, the increase in α will raise low-productivity workers' expected payoffs of taking the test since they have a higher chance to get a high grade. Even if the firms increase the wage offer at information set $\boldsymbol{N}$, they still choose to take the test. To sum up, the set of beliefs $\mu_N^{1,1}$ becomes larger if the test is getting less accurate. Then, let us turn the focus to p. We can find that $\partial M(p,\alpha)/\partial p > 0$ holds whenever $0 < p < 1$. All other things been equal, if there is a solid reason allowing the firms to believe that the portion of high-productivity workers becomes greater, the firms will then increase their expectation for those who take the test. Therefore, workers with high or low productivity are more willing to take the test. In this case, even if the firms raise $\mu_N^{1,1}$ and thereby the expected payoffs at information set $\boldsymbol{N}$, workers have no incentive to deviate unless $\mu_N^{1,1}$ is sufficiently large.

[12] $\partial M(p,\alpha)/\partial\alpha > 0$ holds whenever $0 < \alpha < 1/2$.

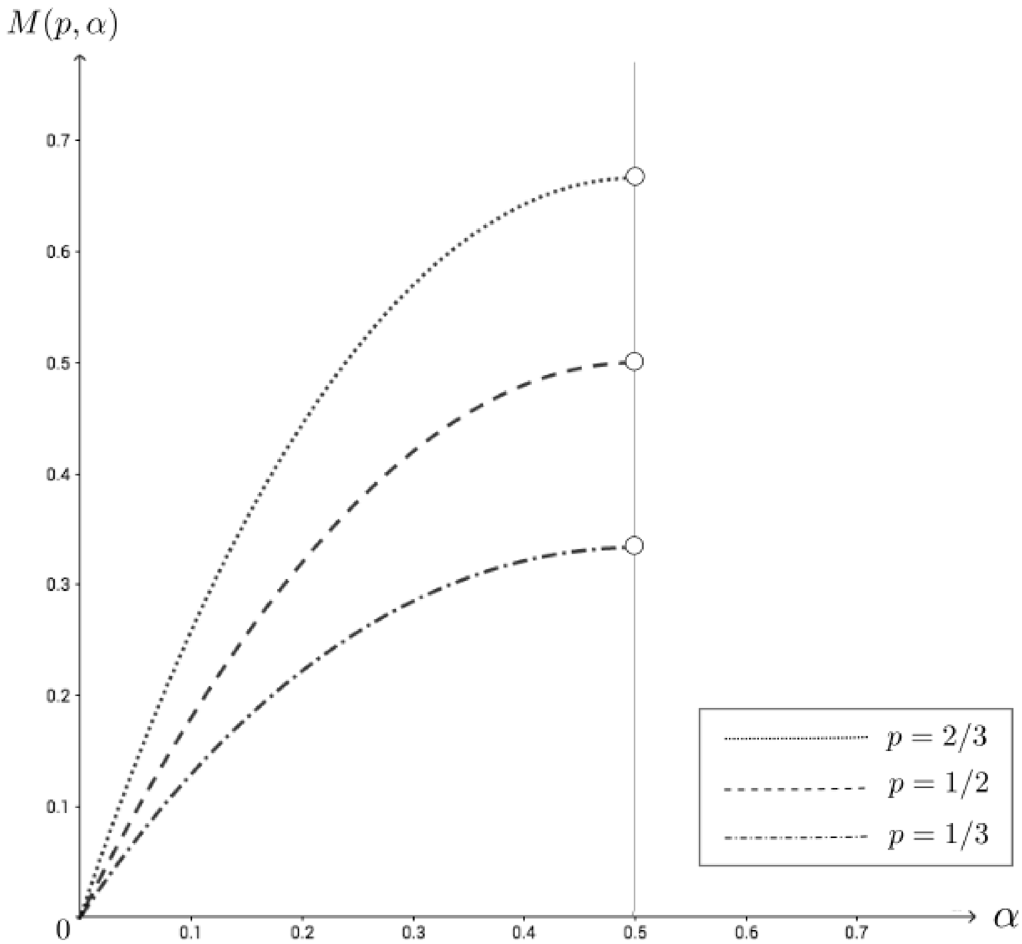

Figure 2.2: The values of $M(p, \alpha)$ based on different values of p

With respect to the N-pooling PBE, we can see that one possible combination of the beliefs that can support the equilibrium is $(\mu_H^{0,0}, \mu_L^{0,0}) = (p, p)$. Intuitively, if the firms do not believe that the test can sort the potential employees due to inaccuracy, they update their posterior beliefs with the prior distribution. In that case, we can obtain a weak PBE such that all the workers are indifferent between taking or not taking the test. The other candidate for the beliefs is $(\mu_H^{0,0}, \mu_L^{0,0}) = (1, 0)$. That is, the firms undoubtedly believe that those who get high grades in the test are of high type and those who get low grades are of low type. Nonetheless, this kind of N-pooling PBE can survive only if $p \geq 1 - \alpha$. This is a strong requirement since $\alpha < 1/2$, which then implies that p cannot be smaller than $1/2$.

2.3.2 Existence of PBE When The Test Cost is Strictly Positive

Since only the worker bears the cost, the wage schedule that the firms offer, as well as their posterior beliefs, are the same as those under $c = 0$. However, existence of the cost will affect the worker's expected payoff when she chooses to take the test. Again, Eq. (2.1) - (2.3) give us a general form of the worker's expected payoffs based on different signal m.

In this section, we find that the result is similar to the case with $c = 0$. However, the equilibrium condition is not only determined by the value of α but also contingent upon the value of c. There are two trivial results that we would like to mention. First, we can find a threshold $\bar{C} > 0$ such that the N-pooling PBE exists if $c > \bar{C}$. The other one is that a separating PBE with $s_w = (1, 0)$ exists as long as $c = \theta_H - \theta_L$.[13] However, we can find that this separating PBE is Pareto dominated by the N-pooling PBE. These two results are straightforward, so we will skip the detailed discussion here. Also, it is not difficult to see that $s_w = (0, 1)$ cannot be support as a PBE.

Proposition 4. *Suppose that the test cost $c > 0$ and $0 < \alpha < 1/2$.*

1. *There exists a class of T-pooling PBE, $\{s_w = (1,1), \mu_N^{1,1} \in \Phi\}$, where*

$$\Phi := \{(\mu_N \mid c \leq [M(p, \alpha) - \mu_N](\theta_H - \theta_L)\},$$

which implies that $c \leq M(p, \alpha)(\theta_H - \theta_L)$ is a necessary condition for existence of the T-pooling PBE.

2. *There exists a class of N-pooling PBE, $\{s_w = (0,0), (\mu_H^{0,0}, \mu_L^{0,0}) \in \Psi\}$, where*

$$\Psi' := \{(\mu_H, \mu_L) \mid \mu_L \leq \mu_H, c \geq [(1 - \alpha)\mu_H + \alpha\mu_L - p](\theta_H - \theta_L)\},$$

which implies that $c \geq (1 - p)(\theta_H - \theta_L)$ is a sufficient condition for existence of the N-pooling PBE.

Proof. See Appendix B.3. □

[13] Intuitively, the high-type worker utilizes the test to communicate her true productivity to the firms. Based on her productivity, the greatest payoff she can receive is θ_H and the lowest is θ_L. If the test cost is too high, any benefit from taking the test will be totally offset. The worker will incur some loss even if she can get high result. Hence, there is no separating PBE when $c > \theta_H - \theta_L$ no matter how accurate the test is.

Benchmark Case With $\alpha = 0$

Before the further discussion, let us consider the case about accurate tests and see how appearance of positive test costs affect the equilibrium condition. It can be seen that, with positive test costs, choosing not to take the test is a dominant strategy for low-type workers regardless of the other players' action. Hence, the only two strategies that may be supported as part of a PBE are $s_w = (1, 0)$ and $(0, 0)$.

Were the test cost zero, $(0, 0)$ would be ruled out as a strategy in a pooling PBE since the high type worker would deviate. However, a positive test cost may prevent the high type worker from taking the test if the cost is large enough. Therefore, different from the previous case, we now have a pooling PBE with $s_w = (0, 0)$ under $\alpha = 0$ if $c \geq (1 - p)(\theta_H - \theta_L)$.

$c > 0$ limits the chance of $s_w = (1, 0)$ being supported as part of a PBE. Since the test is perfectly accurate, the high-type worker can obtain θ_H if taking the test and only θ_L if not taking the test. We need to guarantee that the cost is not too large. It is easy to see that the required condition is $c \leq (\theta_H - \theta_L)$. We can find that when $(1 - p)(\theta_H - \theta_L) \leq c \leq \theta_H - \theta_L$ there exist both pooling and separating PBEs. Nonetheless, it can shown that $s_w = (1, 0)$ is Pareto dominated by $s_w = (0, 0)$.

We can view $(1 - p)(\theta_H - \theta_L)$ as a threshold $\bar{C}$. An interesting point here is that as the prior p decreases, the value of this threshold increases. In other words, given any $c > 0$, if the portion of high-type workers becomes smaller, the chance of obtaining the pooling equilibrium will decrease. Intuitively, the importance of the test is strengthened if the firms believe that the chance of hiring a worker with a high productivity from those who do not take the test is getting smaller. The high-type worker knows that she can promote herself a lot by taking the test and obtaining a satisfactory result. Hence, she will become more tolerant toward the cost.

Hence, we can conclude that, given $\alpha = 0$, there is a Pareto dominant PBE in pooling strategies, $\left\{s_w = (0, 0), \mu_N^{0,0} = p\right\}$, if and only if $c \geq (1 - p)(\theta_H - \theta_L)$. There exists a PBE in separating strategies, $\left\{s_w = (1, 0), \mu_N^{1,0} = 0\right\}$, if and only if $c < (1 - p)(\theta_H - \theta_L)$.

Existence of Pooling PBE When $1 < \alpha < 1/2$

First, we consider $s_w = (1, 1)$. In the case without the cost, we know that $s_w = (1, 1)$ can be supported as a PBE only if $\mu_N \leq M(p, \alpha)$. Existence of a positive cost further strengthens the importance of this requirement. Again, one plausible belief at information set $\boldsymbol{N}$ is $\mu_N^{1,1} = 0$. In this case, the test cost has to be smaller than $M(p, \alpha)(\theta_H - \theta_L)$. As $\mu_N^{1,1}$ becomes greater, we have a more strict requirement for c. That is, if the firms start to believe that part of those who choose not to take the test may come from the high-productivity workers, we need a even lower cost to guarantee that the low-productivity workers have no incentive to deviate. We can also see that the smallest possible value of $\mu_N^{1,1}$ is zero. It implies that the T-pooling PBE does not exist if c is greater than $M(p, \alpha)(\theta_H - \theta_L)$. In fact, $M(p, \alpha)(\theta_H - \theta_L)$ can be viewed as the maximum acceptable test cost for low-productivity workers.

Note that $\partial M(p, \alpha)/\partial p > 0$ and $\partial M(p, \alpha)/\partial \alpha > 0$. As the test becomes less accurate, the low-productivity worker is more tolerant toward the test cost. It is straightforward since they have a greater chance to get a high result when taking the test. With respect to p, we can see that the test cost can be higher without breaking down the equilibrium if the portion of high-productivity workers becomes greater. Intuitively, since it is more likely that a test taker is of high productivity, the firms will offer a higher wage associated with the test results. Hence, even if there is no change in test accuracy, the low-productivity workers still can obtain a greater expected payoff from taking the test give any chance that they may get high results.

If the worker's prescribed strategy is not to take the test for both types, one of the acceptable firms' beliefs is $\mu_N^{0,0} = \mu_L^{0,0} = \mu_H^{0,0} = p$. Putting the cost aside temporarily, both types can obtain $p\theta_H + (1 - p)\theta_L$ in any case. It can be seen that a positive cost implies that there is no profitable deviation for both types. Taking the test brings costs without gaining any benefit. Therefore, $s_w = (0, 0)$ can be supported as part of a PBE whenever c is positive.

The other candidate of the belief in the N-pooling PBE is $\{\mu_N^{0,0} = p, \mu_L^{0,0} = 0, \mu_H^{0,0} = 1\}$. Again, this set of belief implies that the firms seemingly neglect inaccuracy of the test. Without the test cost, this N-pooling PBE requires $p \geq 1 - \alpha$. Nevertheless, here we only need the cost to be large enough. Moreover, even if p is smaller than $1 - \alpha$, this PBE still survive as long as c is large enough. At last, note that the possible maximum value of $\mu_L^{0,0}$ or $\mu_H^{0,0}$ is one. Then, we can find the threshold $\bar{C} := (1 - p)(\theta_H - \theta_L)$ such that the N-pooling PBE exists as long as the set of beliefs satisfies consistency.

2.4 Discussion of Some Particular Cases

2.4.1 The case of one-way error in the model of two types

In this section, we first extend the model to examine the equilibrium condition under different forms of test inaccuracy. Instead of solely using α, we assume that the test will reveal a worker's true productivity with probability $(1 - \alpha_t) \in (1/2, 1]$ if she is of type $t \in \{H, L\}$. For instance, $\alpha_H = 1/3$ and $\alpha_L = 0$ imply that this test will reveal the worker's true type if she is of low type. However, it will mistakenly report "low type" with probability 1/3 if the worker is of high type. The case where $\alpha_H = \alpha_L = 0$ is excluded since we focus on imperfect tests. Hence, we only focus on tests with inaccuracy rates $\{\alpha_H = 0, 0 < \alpha_L < 1/2\}$ or $\{0 < \alpha_H < 1/2, \alpha_L = 0\}$. In fact, the former one can be viewed as a relatively easy test and the latter one can be viewed as a really difficult test.

Proposition 5. *Suppose that the test cost* $c \geq 0$ *and the likelihood of test error* $(1 - \alpha_t) \in (1/2, 1]$*, where* $t \in \{H, L\}$*. There exists a separating PBE,* $\left\{s_w = (1, 0), \mu_H^{1,0} = 1\right\}$*, if the three conditions hold.*

1. $\alpha_H = 0$;
2. $0 \leq \alpha_L < 1/2$;
3. $\alpha_L(\theta_H - \theta_L) \leq c \leq \theta_H - \theta_L$.

Proof. See Appendix B.4. □

Considering the case with no test cost, i.e. $c = 0$, we have the following results.

1. There is no separating PBE in pure strategies whether only $\alpha_H = 0$ or only $\alpha_L = 0$.
2. There exists a T-pooling PBE, $\left\{s_w = (1, 1), \mu_N^{0,0} = 0\right\}$, whether only $\alpha_H = 0$ or only $\alpha_L = 0$.
3. If $\alpha_H = 0$ and $0 < \alpha_L < 1/2$, the necessary condition for existence of the N-pooling PBE is $p \leq \alpha_L$.
4. If $\alpha_L = 0$ and $0 < \alpha_H < 1/2$, the necessary condition for existence of the N-pooling PBE is $p \geq 1 - \alpha_H$.

Similar to the previous cases where the test is inaccurate, we cannot find any separating equilibrium even though the test is somewhat "semi-accurate." With respect to the T-pooling PBE, the result is analogous to the previous cases; and the latter two results give two interesting findings about the N-pooling PBE.

Even though theoretically the N-pooling PBE may exist in either case, $\alpha_L = 0$ or $\alpha_H = 0$, both necessary conditions are very strict. We assume that $\alpha_t < 1/2$ but it is reasonable to say that α_t should be much smaller. Under this circumstance, the N-pooling PBE exists only if the portion of high-productivity workers is really large or really small. It is getting harder for these conditions to hold if the test is getting more accurate. Furthermore, the N-pooling PBE exists only if the firms believe that those who take the test are very likely to be of low-productivity

A positive test cost changes the results significantly. For example, in the case where $\alpha_H = 0$ and $0 < \alpha_L < 1/2$, the necessary condition for the N-pooling PBE is no longer required. In addition to the value of α_H and α_L, how large the cost is also affects the equilibrium condition. We can find a class of pooling PBEs similar to our previous discussion. We skip the detail here since we only see slight algebra differences in the requirement for existence of PBEs. In order to see how the test cost plays a role in this signaling game, we only focus on the following posterior beliefs. If information set $\boldsymbol{N}$ is not reached, then the firms' belief is $\mu_N^{1,1} = 0$. If information set $\boldsymbol{H}$ is not reached, then the firms' belief is $\mu_H^{0,0} = 1$. If information set $\boldsymbol{L}$ is not reached, then the firms' belief is $\mu_L^{0,0} = 1$. Note that this set of the beliefs is not the only one in the PBE. We use this specific set only for illustrating the impact of costs.

1. Suppose that $\alpha_L = 0$ and $0 < \alpha_H < 1/2$.
   - There exists a T-pooling PBE, $\left\{s_w = (1,1), \mu_N^{0,0} = 0\right\}$, for all $c \leq p\alpha_H(\theta_H - \theta_L)/(p\alpha_H + 1 - p)$.
   - There exists a N-pooling PBE, $\left\{s_w = (0,0), \mu_L^{0,0} = 1\right\}$, for all $c \geq (1-p)(\theta_H - \theta_L)$.[14]
2. Suppose that $\alpha_H = 0$ and $0 < \alpha_L < 1/2$.
   - There exists a separating PBE, $\left\{s_w = (1,0), \mu_H^{1,0} = 1\right\}$, if $\alpha_L(\theta_H - \theta_L) \leq c \leq \theta_H - \theta_L$.

[14] There exists a separating PBE, $\left\{s_w = (1,0), \mu_H^{1,0} = 1\right\}$, if $c = \theta_H - \theta_L$. However, this PBE is Pareto dominated by the N-pooling PBE.

- There exists a T-pooling PBE, $\{s_w = (1,1), \mu_N^{0,0} = 0\}$, for all $c \leq p\alpha_L(\theta_H - \theta_L)/[p + (1-p)\alpha_L]$.
- There exists a N-pooling PBE, $\{s_w = (0,0), \mu_H^{0,0} = 1\}$, for all $c \geq (1-p)(\theta_H - \theta_L)$.

We can find an interesting result that a separating PBE with $s_w = (1,0)$ exists only if $\alpha_H = 0$ and $0 < \alpha_L < 1/2$. The other requirement is $\alpha_L(\theta_H - \theta_L) \leq c \leq \theta_H - \theta_L$. One implication here is to make the test relatively easy so that high-productivity workers are willing to take the test because they can obtain high grades easily. Even though low-productivity workers also have a chance to benefit from wrong reports, we can use the test cost as a tool to prevent them from taking the test. Note that the cost cannot be set too high or even high-productivity workers would be locked out of the door.

We can also see that $\bar{C} := (1-p)(\theta_H - \theta_L)$ is indeed a threshold with which the N-pooling PBE exists regardless of the form of inaccuracy. That is to say, as long as the test is inaccurate and a positive test cost is allowed, the N-pooling PBE exists whenever the test cost is not smaller than $(1-p)(\theta_H - \theta_L)$. It can also be found that as p goes up $\bar{C}$ goes down. We know that the test cost is basically used to prevent high-productivity workers from taking the test. Intuitively, if the portion of the high-productivity workers increases, i.e. p increases, then the firms will offer a greater wage to those who choose not to take the test. Thus, we need a smaller cost to discourage the high-productivity workers from deviating to taking the test.

Note that for the two pooling PBEs, we can find different beliefs that still survive in the equilibrium but the requirement may be different due to different kind of inaccuracy. For instance, in the case where only $\alpha_H = 0$, if the firms use the prior to update their beliefs on information set $\boldsymbol{H}$, i.e. $\mu_H^{0,0} = \mu_N^{0,0} = p$, they will offer the same wage schedule as that on information set $\boldsymbol{N}$. Therefore, there is a pooling PBE where no one takes the test as long as $c > 0$. In the case where only $\alpha_L = 0$, if the firms use the prior to update their beliefs on information set $\boldsymbol{L}$, i.e. $\mu_L^{0,0} = \mu_N^{0,0} = p$, then we need $c \geq (1-\alpha_H)(1-p)(\theta_H - \theta_L)$ to guarantee existence of this N-pooling PBE.

Note that when only $\alpha_L > 0$ and the firms use $\mu_H^{0,0} = p$, it is implied that they believe that the test is useless in recognizing types upon a high test result. Similarly, the firms do not think the test as a useful tool in reporting true types upon a low result if they know only $\alpha_H > 0$ and decide to use $\mu_L^{0,0} = p$. One implication is that when the test is relatively easy, i.e. only $\alpha_L > 0$, high type workers cannot benefit from the test given firms' belief $\mu_H^{0,0} = p$

even though they can surely obtain a high result. Therefore, even a small amount of positive test cost can stop them from deviation. By contrast, when the test is relatively difficult, i.e. only $\alpha_H > 0$, high type workers may fail in the test but they have a great chance to get high grades which can differentiate them from others. Hence, we need a greater cost (at least $(1-\alpha_H)(1-p)(\theta_H-\theta_L)$) to prevent the high-productivity workers from deviation.

2.4.2 The case of three types with a costless test

It is assumed that a worker may have three possible types, high-productivity (H), mid-productivity (M), and low-productivity (L). Suppose that there is no test cost. Let the pure-strategy set be $S \equiv \{s_w = (s_H, s_M, s_L) | s_H, s_M, s_L \in \{0,1\}\}$.[15] Also, denote $\mu^{\eta}_{t|m}$ the firms' belief at information set $\boldsymbol{m}$ given a prescribed strategy η. For example, $\mu^{1,1,1}_{M|L}$ represents the probability that the firms believe that this worker is mid-type if they observe a low test result given that the worker claims to take the test for all three types.

We begin with the benchmark case with no test error. It is not difficult to see that the only strategy that can be supported as a PBE is $s_w = (1,1,0)$. If we allow the test to be one-way inaccurate so that it only misreports a worker's productivity to a lower level or a higher level, the result will be different. Suppose that the test can only be biased with one degree, which means that a high-productivity worker may obtain a test result indicating her true type or mid-productivity but never the low productivity. Analogously, if the test is upward biased, then the test may report a low-productivity worker as a mid-productivity one but never a high type. The game trees of these two forms of bias are shown in Figure 2.3 and Figure 2.4.

[15] Most of the notations will follow the previous setting in an analogous method.

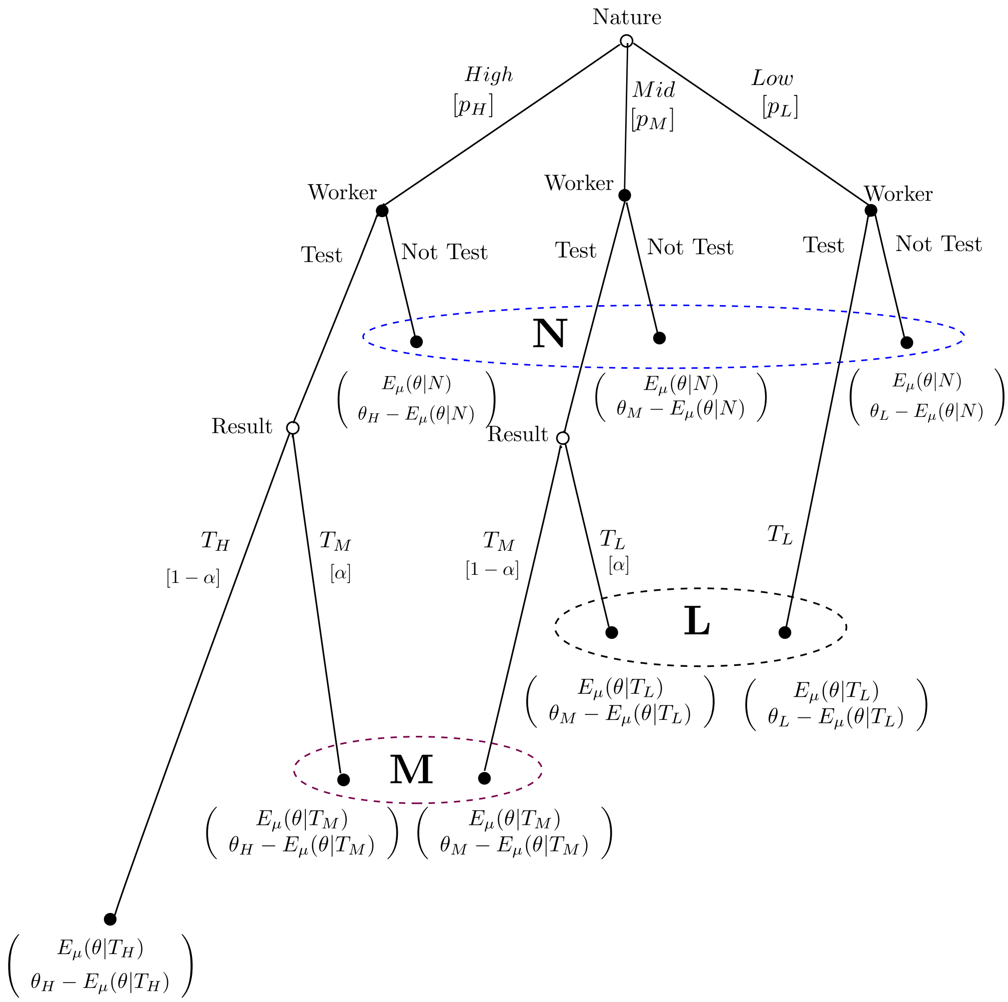

Figure 2.3: Three Types with Downward Test Errors

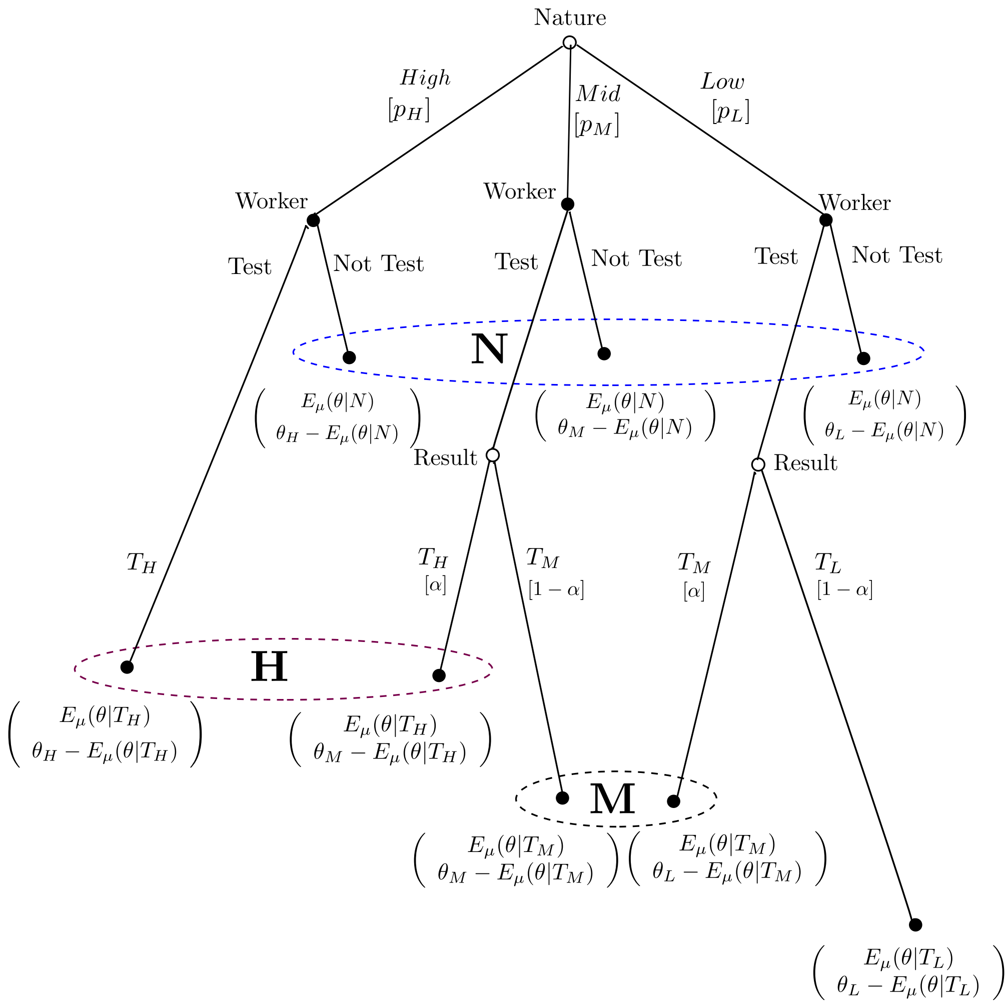

Figure 2.4: Three Types with Upward Test Errors

Proposition 6. *In the case of three types, suppose that the test cost $c = 0$. Assume that the test will reveal a worker's true type with probability $(1-\alpha) \in (1/2, 1)$ and report a one-level wrong result with probability α.*

1. *Suppose that the test is downward biased up to one level.*

   (a) *There is a semi-separating PBE, $\left\{s_w = (1,0,0), \mu^{1,0,0}_{M|L} = 0\right\}$, in pure strategies if*

$$(1-\alpha)(\theta_H - \theta_L) \le \frac{p_M}{p_M + p_L}(\theta_M - \theta_L). \tag{2.5}$$

   Two necessary conditions for existence of this PBE are $p_M > p_L$ and $\theta_H - \theta_M < \theta_M - \theta_L$.

(b) *There is a N-pooling PBE,* $\left\{s_w = (0,0,0), \mu^{0,0,0}_{M|M} = \mu^{0,0,0}_{M|L} = 1\right\}$, *in pure strategies if*

$$p_H\theta_H + p_M\theta_M + p_L\theta_L \geq \theta_M + (1-\alpha)(\theta_H - \theta_M). \tag{2.6}$$

One necessary conditions for existence of this PBE is $p_H > 1-\alpha$.

2. *Suppose that the test is upward biased up to one level.*

(a) *A semi-separating PBE,* $\left\{s_w = (0,1,0), \mu^{0,1,0}_{M|H} = \mu^{0,1,0}_{M|M} = \mu^{0,1,0}_{M|L} = 1, \mu^{0,1,0}_{H|N} = 1 - \mu^{0,1,0}_{L|N} = p_H/(p_H+p_L)\right\}$, *in pure strategies exists if*

$$\theta_M = \frac{p_H\theta_H}{p_H+p_L} + \frac{p_L\theta_L}{p_H+p_L}. \tag{2.7}$$

(b) *There is a N-pooling PBE,* $\left\{s_w = (0,0,0), \mu^{0,0,0}_{H|H} = \mu^{0,0,0}_{M|M} = 1\right\}$, *in pure strategies if* $p_L \leq 1-\alpha < p_H$ *and*

$$p_H\theta_H + p_M\theta_M + p_L\theta_L \geq \theta_M + (1-\alpha)(\theta_H - \theta_M).$$

Proof. See Appendix B.5. □

As can be seen in the previous discussion, we can find a trivial result that the T-pooling PBE exists if the firms set their beliefs at information set $\boldsymbol{N}$ as $\mu^{1,1,1}_{L|N} = 1$. That is to say, as long as the firms believe that those who choose not take the test are definitely low-productivity workers, the T-pooling PBE always exists. Any one who decides not to take the test can only obtain the lowest wage offer θ_L. Nevertheless, since the test is inaccurate, even the low-productivity workers have a greater expected payoffs when taking the test. It can be shown that this T-pooling PBE also exists when the test is allowed to be two-way inaccurate. Moreover, we can reasonably expect that this T-pooling PBE exists even when the test is costly. The only requirement is that the test cost is sufficiently low.

One interesting finding here is that we have a semi-separating PBE where only high-productivity workers take the test if the firms believe that those who obtain low test results are definitely low-productivity workers. Note that we need two strict necessary conditions. The first one is that the portion of the low-productivity workers has to be sufficiently small. In addition, the gap between the productivity of high- and mid-type workers has to be much smaller than the gap between the productivity of mid- and low-type workers. It is easier for Eq. (2.5) to hold if the test is more difficult, i.e. a greater α. The key point here is the mid-productivity worker's decision. Intuitively, even though the mid-productivity workers can benefit from test inaccuracy, they have a risk to lose a huge amount if they obtain low test results.[16]

If we further allow the test to be two-way biased, the result is similar to that of the case with an upward-biased test. In this case, it is still assumed that the test only carries one-level inaccuracy. That is, type H may get a test result wrongly showing that she is of mid-type but never the low type. Similarly, type L may be lucky and obtain a result showing that she is type M but never type H. Of course, type M may obtain a high or low result. We assume that the test is inaccurate, and the probability of indicating a correct result is $(1-\alpha) \in (1/2, 1)$. Type M may get a high or low result with the same probability $\alpha/2$. This game is illustrated in Figure 2.5.

[16] They obtain θ_H if the test results truly report their type. However, they only obtain θ_L given the firms' belief if this downward-biased test reports a wrong type. Since the difference between $\theta_H - \theta_M$ is much smaller than $\theta_M - \theta_L$, the mid-productivity workers have no incentive to deviate due to a possible huge expected loss.

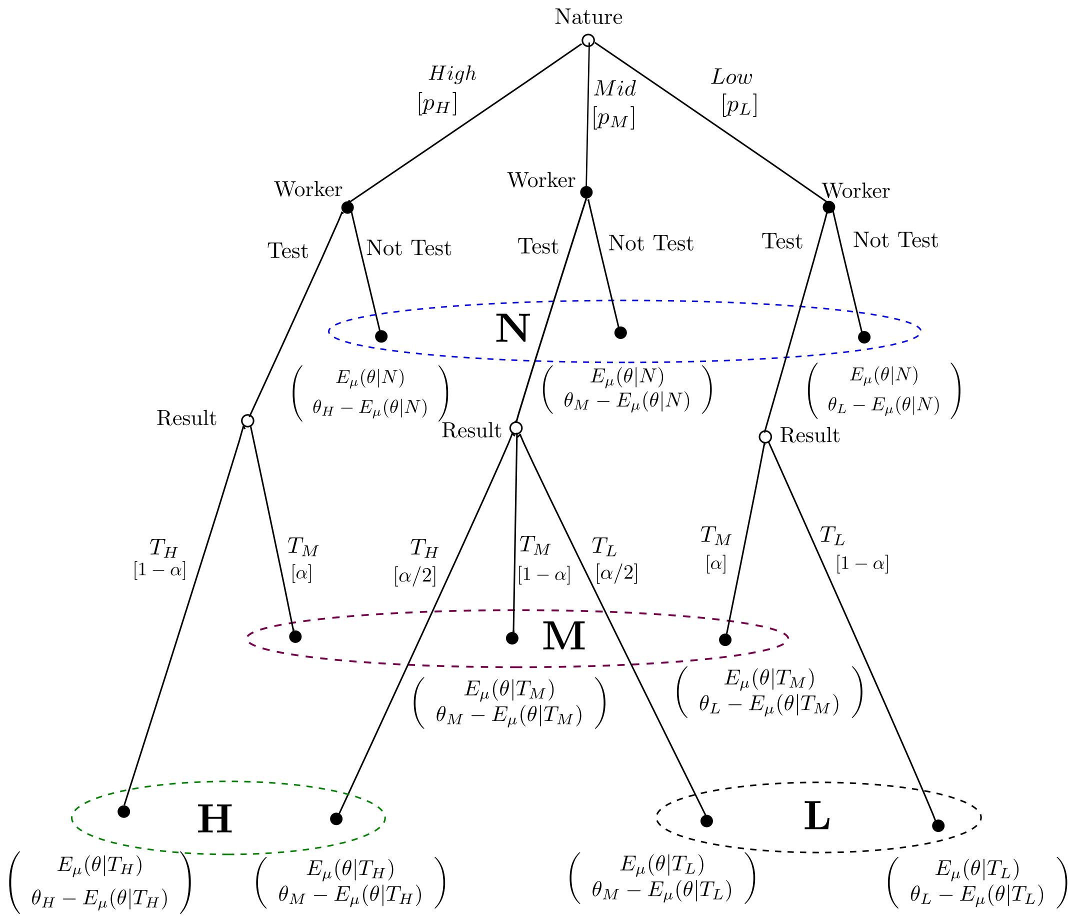

Figure 2.5: Three Types with Two-way Test Errors

Proposition 7. *In the case of three types, suppose that the test cost $c = 0$. Assume that the test will reveal a worker's true type with probability $(1-\alpha) \in (1/2, 1)$. It may indicate type H as type M or indicate type L as type M with probability α, and report type M as type H or type L with equal probability $\alpha/2$.*

1. *There is a semi-separating PBE, $\left\{s_w = (1,0,0), \mu^{1,0,0}_{M|L} = 0\right\}$, in pure strategies if*

$$\left(1 - \frac{\alpha}{2}\right)(\theta_H - \theta_L) \leq \frac{p_M}{p_M + p_L}(\theta_M - \theta_L). \tag{2.8}$$

   Two necessary conditions for existence of this PBE are $p_M > p_L$ and $\theta_H - \theta_M < (\theta_M - \theta_L)/3$.

2. *A semi-separating PBE,*

$\left\{s_w = (0,1,0), \mu^{0,1,0}_{M|H} = \mu^{0,1,0}_{M|M} = \mu^{0,1,0}_{M|L} = 1, \mu^{0,1,0}_{H|N} = 1 - \mu^{0,1,0}_{L|N} = p_H/(p_H + p_L)\right\}$, *in pure strategies exists if*

$$\theta_M = \frac{p_H \theta_H}{p_H + p_L} + \frac{p_L \theta_L}{p_H + p_L}.$$

3. *There is a N-pooling PBE,* $\left\{s_w = (0,0,0), \mu^{0,0,0}_{H|H} = 1, \mu^{0,0,0}_{M|M} = 1, \mu^{0,0,0}_{L|L} = 0\right\}$, *in pure strategies if*

$$p_H\theta_H + p_M\theta_M + p_L\theta_L \geq \theta_M + (1-\alpha)(\theta_H - \theta_M).$$

One necessary conditions for existence of this PBE is $p_H > 1 - \alpha$.

Proof. See Appendix B.6. □

Let us take a further look at the three cases of test inaccuracy: upward, downward, and two-way biased up to one level. If the test is upward-biased or two-way biased, we can find a semi-separating PBE where only type M takes the test, as long as θ_M is equal to the weighted average of θ_H and θ_L based on the priors. Note that if the test is only downward biased, we cannot obtain this result. The key point here is that the only type H can obtain a high test result if the test is downward biased. Therefore, the firms will realize that a test taker is of high type if the result shows exactly it. Then, if only type M chooses to take the test, type H can get a higher expected payoff if deviating to taking the test. One implication is that the endowment of productivity for the different types plays an important role in the case of three types. Eq. (2.7) can be rewritten as $p_H(\theta_H - \theta_M) = p_L(\theta_M - \theta_L)$. It can seen that if $\theta_H - \theta_M$ is significantly greater than $\theta_M - \theta_L$, then it requires a small enough p_H so that the PBE exists. In other words, if productivity of type H (or L) is very high (or very low), then the proportion of high-type (or low-type) workers cannot be too large.

We have shown that there exists a semi-separating PBE where only high-productivity workers choose to test if the test is downward biased as can be seen in Proposition 4. This result stays true even if the test is two-way biased. However, it does not hold when the test is upward biased. With upward biased tests, those mid-productivity workers have no chance to be misreported as low-productivity ones. Therefore, taking the test is not as risky. For the high-productivity workers, they can obtain high test results, and thereby a payoff equal to θ_H, so they will surely take the test. Now consider the low-productivity workers. We can find that if the test is accurate enough ($\alpha < p_M/(p_M + p_L)$), then the firms will believe that none of the low-productivity workers chooses to take the test. In this case, the mid-productivity workers can certainly obtain a payoff greater than θ_M by taking the test. On the contrary, if the test is sufficiently inaccurate ($\alpha \geq p_M/(p_M + p_L)$), then these mid-productivity workers will have a higher chance to be misreported as high-productivity ones. Hence, a profitable deviation exists.

We can also find that if the test is inaccurate, the strategy that only type L chooses not to take the test is not in a PBE. This result is different from the common observation upon a perfectly accurate test. Intuitively, if all other types choose to take the test, then type L can hide behind the other types and benefit from the imperfect information. Such benefit may come from her luck of getting a higher result. It can also result from the higher type's miss in the test. Since the firms cannot perfectly distinguish a worker's type due to inaccuracy, they can only offer a weighted wage schedule based on their beliefs. Instead of being the only one choosing not take the test and getting θ_H, type L will deviate and obtain a greater payoff.

2.4.3 Further discussion on the effects of parameters in the three-type case

First, we want to show that the parameters play an important role in determining existence of PBEs. Here we do not consider the trivial T-pooling PBE. According to Proposition 4 and Proposition 5, we know that no PBE can survive if $p_L > 1/2$ and $\theta_H - \theta_M < \theta_M - \theta_L$, regardless of the form of test inaccuracy. Literally, if most workers in the society have low productivity and their productivity is much lower than the higher type workers, these low-type workers' decision of taking the test may carry no useful information as long as the test is inaccurate. Figure 2.6 (a) shows possible relation among the parameters in this case. The other special case which leads to a similar conclusion is when $1/4 < p_L \simeq p_H < 1/2$ and $p_L > p_M$. In this case, if the gap between θ_M and θ_L is quite different from that between θ_M and θ_H, we can obtain a similar result that no PBE in pure strategies exists as long as $\alpha \neq 0$. Figure 2.6 (b) illustrates a possible case discussed here.

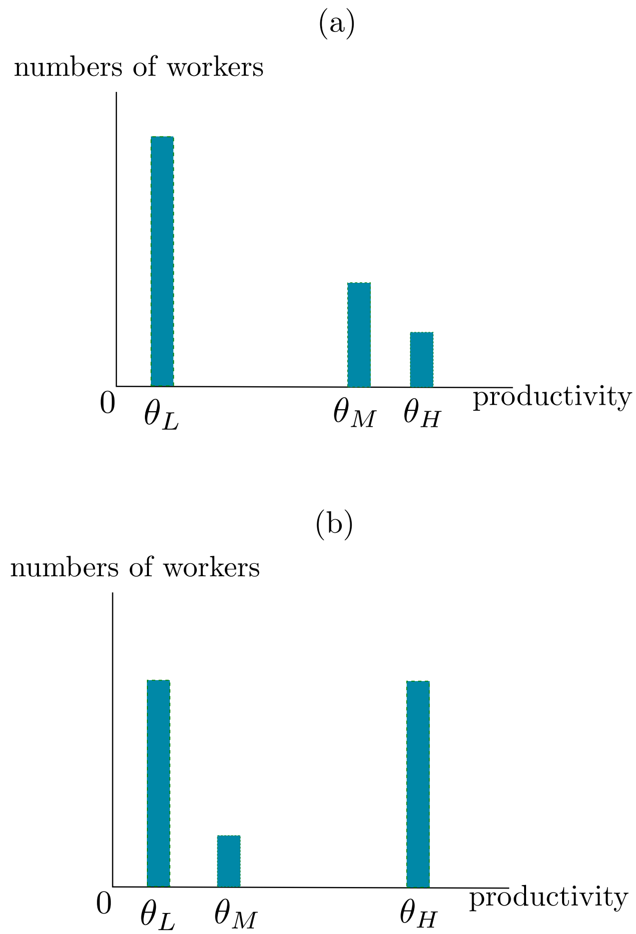

Figure 2.6: Special cases

If we take a closer look at the effects of the parameters on the equilibrium condition, we can see that the test error rate α plays a really important role in both two-type and three type models. One might be interested in a question: "how about the other parameters like prior distribution or endowment productivity?" Before discussing more in depth, let us first do a further comparison between the two-type and three-type models. We only focus on the cases with no test costs.

It is clear that whether α is zero or not may change the equilibrium condition no matter in the model of two or three types. The requirement of α is basically the same in both models. If the test cost is strictly positive, then all the parameters may affect the equilibrium status through the required condition of c. With no test cost, we can find that no further condition other than the value of α (zero or positive) is required for existence of the possible PBEs in most of the two-type model. The only two exception occurs when the test is only one-way inaccurate where the value of the prior distribution p also matters. Nonetheless, in the three-type model, it requires more conditions to determine existence of PBEs even in the absence of test costs. For instance, if the test is downward biased, we need Eq. (2.6) as well as $1 - p_L \geq \alpha$ to guarantee that the N-pooling PBE exists when the test is upward biased up to one level.

One possible explanation for this result is that as the number of types increases it is more difficult for any player in this game to only focus on any one type. In the model with only two types, we can easily find that the expected wage that the firms offer based on their beliefs definitely lies between θ_H and θ_L. In this model, all types of workers can be viewed as players who play a simple win-or-lose game with different probabilities based on prior distribution or firms' beliefs. However, appearance of the mid-type in the model with three types eliminates such simplicity.

2.5 Conclusion

In this paper, we consider signaling models with costly and imperfect tests. While this model can be applied to analyzing many issues, a competitive labor market is chosen to be our background context. Some previous studies in similar frameworks also consider imperfect tests, but they view tests as an indirect tool. Our motivation is different. In many cases, workers (or signal senders) utilize test results to show their true type to employers (or signal receivers). Presence of inaccuracy of the test may even give some types of the workers hopes to get better results, and thereby higher expected payoffs. We are interested in how tests of limited reliability affect interaction of senders and receivers in signaling games with asymmetric information. In order to have more comprehensive discussion about this issue, we also put some focus on the impacts of the important parameters on equilibrium status.

We begin with the benchmark two-type model with no test costs. It can be easily shown that the separating PBE in pure strategies where only high-productivity workers take the test exists if the test is perfectly accurate. If each type chooses different strategies, the firms can update their beliefs based on the worker's prescribed strategy. Since each information set is only achieved by one type given the prescribed strategy, inaccuracy of the test gives the low-type worker a chance to hide in the group of high-type workers. Therefore, in the presence of test inaccuracy, we can find that there exists no separating PBE if the test is costless.

With respect to the pooling PBEs, we find that the T-pooling PBE always exists if the firms believe that only those with low productivity will choose not to take the test. This result remains true whether the test is accurate or not. De and Nabar (1991) conclude that even if the test is costly, this T-pooling PBE exists as long as the test cost is sufficiently low. We obtain a similar result, and further show that existence of the T-pooling PBE depends only on how confidently that the firms believe those who do not take the test are indeed low-productivity workers. The best candidate of the firms' believe is that they believe ONLY low-productivity workers refuse to take the test. This result can be easily extended to a model with more types.

By contrast, we can find that, without test costs, the N-pooling PBE in pure strategies exists only if the test is inaccurate, i.e. $\alpha > 0$. This result can be explained by the firms' lack of useful information. There are two pieces of information that help the firms determine the wage offer. The first is the worker's prescribed strategy and the other one is the test result if it is taken. Since both types choose the same strategy and no enough information can be drawn from the test results, it is plausible that the firms can offer the same wage for every signal. That is, the firms have the same updated beliefs according to the prior distribution in all unreached information set. Therefore, the worker feels indifferent between taking and not taking the test if there is no test cost. Note that we can find different beliefs that can support the N-pooling strategy as a PBE. However, as the firms' beliefs change, we may need a more strict constraint on α and p.

While a positive test cost may affect the equilibrium condition when the test is accurate, allowing $c > 0$ does not change the results significantly when the test is inaccurate. Moreover, with $c = 0$, the value of α plays an extremely important role. Existence of different PBEs depends on whether or not $\alpha = 0$. Presence of positive test costs somehow lessens the importance of α. Although some combinations of α and p imply non-existence of certain form of PBEs, the test cost c can be an adjusted term to pull the equilibrium back. In addition, we know that when the test cost is high enough, neither type of workers will take the test. We find that the threshold is $\bar{C} = (1-p)(\theta_H - \theta_L)$; that is the sufficient condition for existence of the N-pooling PBE is $c \geq (1-p)(\theta_H - \theta_L)$. It can also be seen that as the portion of the high-productivity workers decreases, we need a greater cost to guarantee existence of the N-pooling PBE. A similar conclusion can be found as the gap between θ_H and θ_L increases.

We further extend the model to incorporate all different forms of test errors. This is interesting because sometimes we can find that it is relatively hard for a person with lower ability to get high scores in the test. However, it seems easier that one with higher ability gets unsatisfactory scores in the test due to some reasons like physical or mental status. By contrast, if a test is easy enough, then we may expect the result in the other direction. Workers with high ability will not fail in the test and those with low ability have a better chance to get a satisfactory test result. We find a situation under which the separating PBE where only high-productivity workers take the test exists. This finding deserves some attention because it is the only case that a separating PBE with $s_w = (1, 0)$ exists when the test is inaccurate. One implication of our finding is to design the test easy enough and set the test cost within the reasonable range. Then workers' decision about whether to take the test will be informative.

At last, considering only costless tests, we increase possible types of a worker to three so that the study can be more comprehensive. Different forms of inaccuracy are also considered here. It can be seen that the required condition for existence of any form of the PBE becomes more complicated in the three-type model. We first find that there exists a semi-separating PBE where only high-type workers take the test if the test is either two-way or downward biased. The other finding shows that if the test is upward or two-way biased, there is a semi-separating PBE including a strategy that only type M takes the test. We can then find some interesting implication from this condition. For instance, when θ_H is significantly high compared to other types' productivity, we need p_H to be small enough to guarantee existence of the PBE. Both results require different constraints on the parameters.

While this paper offers a more comprehensive view, there are still some possible interesting extensions. For instance, it is more reasonable to include more types in the model. In the real world, ability of a worker or a student is not just high or low, but ranges widely. We hope to extend the model to a more general case that allows $N > 3$ types of a signal sender. Moreover, although we have not been able to solve the whole model of three or more cases with positive test costs, we still believe that the value of c plays an important role in this signaling game. The other issue that draws our attention is how the results may change if we allow the message sender to retake the test. One good example is when college applicants decide to retake SAT. It can also be seen that many international students take TOEFL for several times.

Chapter 3

Collective Decision-making in Siting Noxious Facilities

3.1 Introduction

Siting facilities that are nationally relevant but locally obnoxious presents a major political challenge for policy makers over the recent decades. The issue, often termed the NIMBY (not in my backyard) issue, arises because people mostly agree that their society needs such facilities but they refuse those useful facilities to be sited in their neighborhoods. Thanks to a substantial amount of previous research, many schemes have been proposed to facilitate the siting of noxious facilities. However, most of the studies treat communities as individual players and presume public participation and acceptability of proposed compensation by all residents in each community. We believe that the issue can be better solved if we consider the players to be the residents instead of communities.

One of the pioneer research papers on this topic, Kunreuther et al. (1987), proposes a low-bid auction mechanism. While the auction mechanism provides a new policy tool to deal with the location process, what arouses our interest is that the authors discuss the difficulties of using the Clarke mechanism for the noxious facility problem. Some of their concerns about using the simple Clarke mechanism seem plausible. However, we believe that the Clarke mechanism is still serviceable after some minor modifications. This paper proposes a modified Clarke mechanism and focuses on how residents in each community participate and react in the siting procedure.

Most of the standard demand-revealing mechanism require voters to report their willingness to pay (WTP) or willingness to accept (WTA) in terms of money. Good (1977) mentions that using money in the demand-revealing method gives richer voters more voting power. Focusing on the two-option case, he proposes a continuous strictly increasing unbounded function as a transformation of WTP or WTA. Since he aims to decrease rich people's voting power, he uses the inverse of wealth as the weights and obtain all necessary weighted measurements. His result shows that the transformation does not affect the required properties, especially incentive compatibility. However, the author does not explicitly show whether or not the social choices remains the same after using the weighted version of the Clarke mechanism. We present a simple example of siting a noxious facility and show that the socially chosen site may be different under the weighted version of the Clarke mechanism.

A commonly seen NIMBY argument is: "I know that we need (something), but do we really need to build it in our neighborhood?" It is important to know why the NIMBY phenomenon exists. The noxious facilities can be viewed as local public goods in the sense that all the residents can share the benefits of them. On the other hand, they also have a private "bad" aspect that only those who live close enough to the facilities may be exposed to potential harmful effects like health risks. According to Sandman (1987), other concerns may include the overburdening of community services, the decrease in property values, and the decline in quality of life.

Usually, most people are able (though not perfectly) to perceive an unwanted cost imposed on nearby residents of the community or the jurisdiction selected to host those facilities. As a result, the NIMBY syndrome reflects the propensity of local citizens to insist on siting unwelcome but necessary facilities somewhere else other than in their own community. The key idea of the NIMBY problem is thus the chance of not providing (or building) something that is good for the whole society, given that only residents in the host community absorb the costs. Hamilton (1993) finds evidence to illustrate how the NIMBY phenomenon may delay the process of siting:

> "A 1987 nationwide survey found that of 81 recent siting applications for commercial facilities that treat, store, or dispose of hazardous waste (TSDs), 31 had been denied or withdrawn, 36 were still under review, and 14 had received operating permits; 6 of the approved permits had not been implemented, however, because of judicial review or market circumstances. (101)"

If those facilities are desirable but every community prefers not to accept them due to local opposition, then how should a site be chosen as the host? This question has drawn growing attention since early 1980s. Compensation seems to be one of the most commonly proposed solutions. However, it is fairly difficult to determine who should pay how much compensation to whom. Note that the answer to this question is dependent on people's true preferences. Therefore, the primary focus for economists interested in solving this siting problem is to design a mechanism that creates an incentive for people to truthfully reveal their preferences. This property is so-called incentive compatibility (IC) or strategy-proofness, and will be included in the main discussion.

Being pioneers in the siting literature, Kunreuther and Kleindorfer (1986) and Kunreuther et al. (1987) apply an auction with a compensation scheme to the siting problem. They propose using a sealed-bid auction, where the community submitting the lowest bid is the"winner" and receives its own bid as compensation for being the host. The non-host communities have to pay an amount equal to their bid, divided by the number of non-host communities. The theoretical result indicates that a one-shot, sealed-(low) bid auction is an individually rational (IR) and a coalition-free mechanism. Also, assuming all the candidates (communities) are risk-averse, a max-min bidding strategy is prudent. In such case, the outcome is efficient if each community is indifferent to the location of the facility, as long as it is not sited in their backyard. An experiment is then conducted to examine how close individual participants are to the max-min solution shown in the theoretical result. The experimental results show that a single-shot trial could be quite misleading, but by allowing more trials, the outcomes from the experiments are consistent with their max-min theoretical prediction. While this paper offers a new direction of siting schemes, their mechanism does not induce communities to truthfully report their preferences.

Kunreuther et al. (1987) assert that the Clarke mechanism is not applicable to the case of siting noxious facilities. However, our analysis shows that the Clarke mechanism can be used to facilitate the siting. Note that, it is generally expected that a facility is to be built only when the expected benefit is greater than the expected cost. In that case, each citizen is required to pay a tax based on the estimated benefits.[1] The tax revenue can then be used to fully fund the compensation scheme.[2] In addition, if the social planner can capture all the observable negative impacts and there are no spillover effects to other communities, then

[1]In order to simplify the model, I assume that each citizen obtains identical benefits from the facility.

[2]This paper only focuses on the siting of the facility according to residents' reaction. Assume that there is no significant spatial variation and that the construction cost is fully covered by the project.

people will only need to be compensated if the facility is in their backyard.

Sullivan (1992) puts more focus on the compensation scheme than the siting decision. Instead of proposing a mechanism that can facilitate the siting, he suggests using a lottery to pick a site for the noxious facilities. After the site is determined, the regional government will transfer income from tenants to the host-city land owners, to at least partially offset the loss in property values. Minehart and Neeman (2002) especially emphasize the importance of balancing the budget during the siting procedure. They claim that, in practice, self-financing is a more difficult goal than truthful revelation of preferences. Based on this belief, their approach is to consider a mechanism that is always budget-balanced, but might not be incentive compatible.

It is our interest to investigate some mechanisms that always result in balanced budgets. However, truthful revelation still plays a more important role in our study. This is not a right-or-wrong question. In this paper, the mechanism proposed still retain the property of incentive compatibility. Nevertheless, due to the importance of budget balancing, we also take budget surplus minimization into account. In my analysis, reduction in budget surplus caused by the mechanism depends highly on how accurately the social planner can evaluate the impacts (or costs) of all siting options.

One important presumption in this paper is that the social planner must be able to estimate the impact well. It might not have been applicable a decade or two decades ago. However, nowadays technological progress and much more historical survey or experimental data have significantly improved the accuracy of estimating the impacts. For instance, the US Environmental Protection Agency (EPA) is one of the sources that conducts surveys to help assess the negative impact of hazardous sites. Hamilton and Viscusi (1999) use 1990 census data to determine the the number of residents living within a certain distance of the hazardous wast sites. They also conduct risk assessments to determine potential risk, in terms of cancer, of living at the neighborhood of the sites. Sun and Zhu (2014) focus on the case with respect to public perception of nuclear power in China. Based on the contingent valuation survey, they obtain an evaluation of people's willingness to pay (WTP), indicating that people in China are willing to pay extra US$ 80.1 (up to 116.6) annually to avoid the construction of a nuclear power plant in their neighborhood. This work on evaluating the impact of noxious facilities gives us tools with which to explore better mechanisms.

In this paper, we use the framework of the Clarke mechanism. It is assumed that a noxious facility is going to be built. The only issue is to determine where to site it. Instead of using communities as the agents in the mechanism, we allow all the residents to cast their "bids." The social planner first estimates the possible impacts and then announces the amount each resident can receive if the facility is sited in their city. Based on the estimated values for each possible option, all the citizens are required to pay a tax in advance to fund the compensation scheme and the construction cost.

The estimation will not be perfectly accurate. Therefore, the social planner asks all the residents to report extra compensation they need (which can be negative), to let the facility be in their neighborhood. After collecting all the numbers, the city where residents submit the lowest aggregate extra compensations is selected as the host. The mechanism looks similar to the traditional auction-compensation scheme proposed by Kunreuther et al. (1987) . However, there are several points of difference. First, in the auction mechanism, individual residents' risk perceptions and preferences are not observed. Second, in our model, every resident whose report switches the outcome has to pay a fee (Clarke tax) in addition to their share of costs. Our results show that the modified Clarke mechanism is characterized by efficiency and strategy-proofness. Honest reporting is a weak dominant strategy for all the residents. The budget surplus issue can be mitigated if the effectiveness of estimation is better.

The remainder of this paper is organized as follows. In the next section, we introduce the setup in the economic environment. Some preliminary results come in Section 3. We then discuss a special case of incorporating wealth effects and present some comparison and contrast between the weighted and the unweighted mechanisms in Section 4. The last section offers concluding remarks.

3.2 Theoretical Analysis

Consider a society of K cities. Denote the set of cities $\Phi = \{1, 2, \ldots, K\}$. Assume that the number of citizens in city k is n_k, where $k \in \Phi$. Let N be the total number of citizens in this society, i.e. $N = \sum_{k=1}^{K} n_k$. The social planner decides to build a hazardous waste facility which can benefit all the citizens in this society but may create some extra cost , e.g. decreasing environmental quality or increasing potential physical risk, only to people in the host city. Assume that people have to pay much more for similar services if the facility does not exist. It is reasonable to expect that all citizens, regardless of the living location, want the facility to be built, but do not want it to be in their own city.

3.2.1 Negative impacts of the facility on the host city

For simplicity, we normalize each city to a unit interval. The negative impact on a citizen if the facility is built in her city is negatively correlated with the distance between the location of this citizen and the location of the facility. Assume that there is no effect on any citizen who does not live in the host city; that is, there is no spillover effect. Let the location of each citizen be $k_i \in (0,1)$, where $k \in \Phi$ stands for the index of each city. For example, $2_i = 0.5$ means that person i lives in the middle point of city 2.

The social planner can estimate a major part of the negative impacts based on historic data, geographic characteristics, and other relevant information. However, for each citizen, there is still some unobservable effect that is only known to the citizen herself. Suppose that the true negative impact if city k is chosen is $-\Lambda_i^k = -\eta[f(k_i) + \epsilon_i^k] \leq 0$,[3] where $\eta > 0$ can be viewed as the pecuniary measurement of marginal loss and $f(k_i)$ is the part that can be estimated. Since there is no spillover effect, we know $\Lambda_i^k = 0$ if the facility is sited in any other city. Λ_i^k can also be viewed as each citizen i's value of willingness to accept (WTA) if the facility is built in her city. Without loss of generality, let $\eta = 1$, and thus $-\Lambda_i^k = -[f(k_i) + \epsilon_i^k]$. Assume that $f(k_i)$ follows a multiple of the probability density function of the beta distribution $B(\alpha, \beta) = x^{\alpha-1}(1-x)^{\beta-1}/B(\alpha,\beta)$ with $\alpha > 1$ and $\beta > 1$, where $B(\alpha,\beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha+\beta)$.[4] Since the mode of $f(\cdot)$ is $(\alpha-1)/(\alpha+\beta-2)$,[5] the location of the facility determines the ratio of $\alpha - 1$ to $\beta - 1$. Moreover, the values of α and β also determine the value of $f(H_i)$, and thereby the total estimated impacts on city H, i.e. $\sum_{i=1}^{n_H} f(H_i)$, if city H is chosen. Finally, assume that ϵ_i^H follows the an i.i.d. uniform distribution, $U(a,b)$. The values of a and b (which are assumed to be reasonably small) determines the bias of the social planner's estimation.[6]

[3] We assume that any citizen in the host city does not have a positive utility about the siting. The best situation for a citizen in the host city is that she does not care about the siting at all. Note that the main concern here is the siting issue only.

[4] $\Gamma(\cdot)$ is the Gamma function, defined as $\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx$.

[5] For example, if the mode is at 1/3 (meaning that the facility is sited at the 1/3 point of the unit interval), then we know that the ratio of $\alpha - 1$ to $\beta - 1$ is 1 to 2.

[6] $a \geq 0$ implies that the social planner always underestimates the impact and $b \leq 0$ tells an opposite story. The expected estimation error is $(a+b)/2$.

3.2.2 Total costs of building the facility

To fully fund establishment of this facility, all the citizens, regardless of where they live, are required to pay a tax based on the total cost of building this facility. Suppose that the facility is sited in city k. Let the tax each citizen has to pay be t_k. The total cost includes the compensation scheme (based on the estimated negative impacts on the host city) and the cost of building this facility. Denote C_k the total cost and P_k the construction cost or the price of this facility. For simplification, assume that there is no spatial difference in terms of the construction cost, i.e. $P_k = P$ for all $k \in \Phi$. Thus the total estimated cost of the facility if being sited in city k is $C_k = \sum_{i=1}^{n_k} f(k_i) + P$.

For simplification, assume that all the citizens receive the same level of benefit once the facility is built. Let $u > 0$ be the utility for each citizen from utilizing this facility and $\Pi = Nu$ be the total social benefit. The facility is worth of being built if the benefits it will bring outweigh the estimate costs. Therefore, it is reasonable to assume that $\Pi = Nu > \max\{\sum_{i=1}^{n_1} f(1_i), \sum_{j=1}^{n_2} f(2_j), \cdots, \sum_{t=1}^{n_N} f(K_j)\} + P$. In other words, the social benefit brought by the facility is greater than the total social cost, regardless of the location where it is sited. It is further assumed that the tax revenue, denoted R_k, is only used to cover C_k if the facility is sited in city k. Then, we can obtain $t_k = C_k/N$, or more clearly

$$t_k = \begin{cases} \dfrac{\sum_{i=1}^{n_1} f(1_i) + P}{N}, & \text{if the facility is sited at city 1 (or } k = 1) \\ \dfrac{\sum_{i=1}^{n_2} f(2_i) + P}{N}, & \text{if the facility is sited at city 2 (or } k = 2) \\ & \vdots \\ \dfrac{\sum_{i=1}^{n_K} f(K_i) + P}{N}, & \text{if the facility is sited at city K (or } k = K) \end{cases} \tag{3.1}$$

Without loss of generality, suppose that $t_1 \leq t_2 \leq \ldots t_K$.

3.2.3 The modified pivotal mechanism

The social planner knows each k_i and the corresponding $f(k_i)$.[7] However, only each citizen i knows her own ϵ_i^k. The planner then announces a compensation scheme that person i, who lives in city k, can obtain $\delta_i^k = f(k_i)$ if city k is chosen to be the host city. Let H be the index of the host city. There exists no spillover effect, so there is no negative impact on any citizen j who does not live in the host city, and thereby $\delta_j^l = 0$ for all $l \neq H$.

Based on the compensation scheme, the social planner also announces the tax each citizen has to pay. The tax depends on where the facility is located, which can be seen in Eq. (3.1). The combination of taxes and compensations are summarized in Table 3.1.

Table 3.1: Summary of compensations and taxes

	Sited in City 1	Sited in City 2	$\cdots$	Sited in city K
citizen i in city 1	(δ_i^1, t_1)	$(0, t_2)$	$\cdots$	$(0, t_K)$
citizen j in city 2	$(0, t_1)$	(δ_j^2, t_2)	$\cdots$	$(0, t_K)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
citizen k in city K	$(0, t_1)$	$(0, t_2)$	$\cdots$	(δ_k^K, t_K)

$*$ Each entry stands for $(compensation, tax)$.

Putting aside the benefit from the facility, each citizen has a "given" net compensation or tax, determined by the social planner's announcement. We use Table 3.2 to illustrate the relationship between the siting decision and the compensations (or taxes) of citizens in different cities.

Note that a positive entry can be viewed as a net compensation while a negative entry can be viewed as a tax. Since there is no spillover effect, citizens do not obtain any compensation but have to pay a tax if their city is not the host city. Normally, a citizen has a net compensation

[7] Suppose that the social planner hires a professional team to estimate the real impacts, and obtains a very close result.

Table 3.2: Citizens' net compensations or taxes determined by the social planner

	Sited in City 1	Sited in City 2	$\cdots$	Sited in city K
citizen i in city 1	$\delta_i^1 - t_1$	$-t_2$	$\cdots$	$-t_K$
citizen j in city 2	$-t_1$	$\delta_j^2 - t_2$	$\cdots$	$-t_K$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
citizen k in city K	$-t_1$	$-t_2$	$\cdots$	$\delta_k^K - t_K$

if the facility is sited in her city. However, it is also possible that even a citizen in the host city has a net tax, as long as her estimated compensation is small enough.

Since δ_i^k is not the exact value of the true WTA, each citizen is asked to report an adjusted term. The social planner asks a question like: "how much more do you need to obtain besides the announced compensation in order to allow the facility to be sited in your city?" Meanwhile, after knowing the tax scheme, all citizens are also required to report adjusted terms with respect to the taxes. For instance, in the case of three cities, a citizen i who lives in city 1 is charged $(t_1, t_2, t_3) = (6, 7, 9)$ if the facility is sited at city 1, city 2 and city 3 respectively. These figures imply that, to pay the extra taxes of not having the facility in city 1, she will need extra 1 dollar as a compensation to let the social planner site the facility in city 2 or extra 3 dollars to allow it to be sited in city 3.

Actually, we can combine the two questions. Table 3.2 can be viewed as a summary of citizens' WTP or WTA for letting the facility be sited in different cities. Nonetheless, these WTP's or WTA's are assigned by the social planner and do not represent the citizens' true preference. Therefore, each citizen will be asked to report an adjusted term about the net value of the combination of compensations and taxes. Consider an arbitrary citizen i who lives in city k. Denote $X_{k_i}^H$ this citizen i's reported adjusted value if the facility is sited in city H and $X_{k_i} = (X_{k_i}^1, X_{k_i}^2, \ldots, X_{k_i}^K)'$ the vector of the reported values for different cities. Note that each citizen has to pay at least t_1 no matter where the facility is sited. Also, even though there is no unobservable value about taxes, the difference of taxes among cities still may affect citizens' incentives about the siting decision. Define $\mu_k = t_k - t_1$ as every citizen's willingness to accept to site the facility in city k instead of city 1.[8] For the case of siting in city 1, $\mu_1 = 0$.

Citizen i will take her true ϵ_i^k into consideration if her city k is chosen to be the host, i.e. if $k = H$. All citizens have to pay a tax determined by the location, to fund establishment of this facility. In the case where city k is not the host city, the tax is her only concern. Therefore, her reported value of $X_{k_i}^H$ is based on her true uncompensated impact, ϵ_i^k, and the tax rate, t_H.[9] The social planner views μ_l as citizen i's WTA if $l \neq k$ is the host city and $X_{k_i}^k = \bar{\epsilon}_i^k + \mu_k$ if the facility is sited in city k. Here $\bar{\epsilon}_i^k$ is the reported adjusted term of citizen i in city k. Then, we have $X_{k_i} = (\mu_1, \mu_2, \dots, X_{k_i}^k, \dots, \mu_K)'$

Note that for all $h \in \Phi$ and $i = 1, 2, \dots, n_k$, the reported $X_{k_i}^h$ are only used to incentivize one to honestly report the willingness to accept (or to pay if $X_{k_i}^h < 0$). Reporting a positive $X_{k_i}^h$ does not mean that citizen i will receive an additional compensation if city h is chosen. Similarly, reporting a negative $X_{k_i}^h$ does not require her to pay an extra tax. Only those whose reported values switch the outcome that would have been if their reports do not count need to pay a tax.[10]

Let $X \equiv \cup_{k=1}^K X^k$, where $X^k = \{X_{k_1}, \dots, X_{k_{n_k}}\}$, be all the reported values from citizens in city k. Also, let $L(X) \in \Phi$ be the city that the social planner chooses after considering all citizens' announcements X. To apply the transfer rule, denote $L(X \setminus k_i) \in \Phi$ as the location that would be chosen when considering all citizens' reports except the one of citizen i in city k. The modified Clarke mechanism operates as follows:

- The social planner announces the tax and the compensation scheme, $\big((t_1, t_2, \dots, t_K), \delta_i^k\big)$, where $k \in \{1, 2, \dots, K\}$ and $i = 1, 2, \dots, n_k$. If city H is chosen, then all citizens pay a tax t_H. Each citizen i in city H can obtain δ_i^H as compensation.
- According to the scheme and her true value, ϵ_i^k, each citizen i in city k reports adjustment terms, $X_{k_i}^h \in \mathbb{R}$, for all $h \in \Phi$.

[8] If there were no negative impact, all the citizen would prefer to site the facility in city 1 since they only have to pay the lowest tax rate.

[9] $X_{k_i}^H > 0$ means that citizen i would like to obtain extra compensation if the facility is sited in city h while $X_{k_i}^H < 0$ implies that this citizen is willing to pay extra money so that the facility can be sited in city h. For example, if this citizen i knows that the social planner overestimates the damage and offers a greater compensation (δ_i^k) than her true damage, then she may have an incentive to pay extra money to keep it in city k.

[10] $X_{k_i}^k$ can be either positive or negative. $X_{k_i}^k > 0$ describe a subsidy as person i claims that she needs a higher level of compensation, while $X_{k_i}^k < 0$ means a tax as person i believes that she obtains more than what she deserves and wants to return some amount to the government.

- The social planner chooses $L(X)$ that minimizes the total reported damages on the society (or the social cost)

$$SC(L) = \begin{cases} RD_1 = & \sum_{k=1}^{K}\sum_{i=1}^{n_k} X_{k_i}^1, \quad \text{if } L(X) = 1 \\ RD_2 = & \sum_{k=1}^{K}\sum_{i=1}^{n_k} X_{k_i}^2, \quad \text{if } L(X) = 2 \\ \quad\vdots & \\ RD_K = & \sum_{k=1}^{K}\sum_{i=1}^{n_k} X_{k_i}^K, \quad \text{if } L(X) = K \end{cases}. \tag{3.2}$$

  For instance, city 1 is chosen if $RD_1 = \min_{k=1}^{K}\{RD_k\}$.

- Consider a specific adjust function, $A(X_{M_j}) = \min_{k=1}^{K}\{RD_k - X_{M_j}^k\}$. In other words, $A(X_{M_j})$ is the lowest value of RD_k taking out the reported values of citizen j in city M. We can further express $A(X_{M_j})$ as

$$A(X_{M_j}) = \begin{cases} RD_H - X_{M_j}^H, & \text{if } L(X \setminus M_j) = H \\ RD_G - X_{M_j}^G, & \text{if } L(X \setminus M_j) = G \neq H \end{cases}. \tag{3.3}$$

  When the facility is sited at any city $H \in \{1, 2, \ldots, K\}$, i.e. $L(X) = H$, this citizen j pays a surtax following the transfer rule $T : \mathbb{R}^K \to \mathbb{R}$ as

$$T(X_{M_j}) = \left[\min_{k=1}^{K}\{RD_k\} - X_{M_j}^H\right] - A(X_{M_j}) = RD_H - X_{M_j}^H - A(X_{M_j}). \tag{3.4}$$

  It can be seen that $T(X_{M_j}) = 0$ if $L(X \setminus M_j) = H$ and $T(X_{M_j}) > 0$ if $L(X \setminus M_j) \neq H$.

3.3 Theoretical results

Before going further into the results, let us discuss all relevant objective functions. An arbitrary citizen j that lives in city M aims to maximize her true utility function

$$U_j^M(X_{M_j}|\epsilon_j^M) = \begin{cases} u - [f(M_j) + \epsilon_j^M] + T(X_{M_j}) + \delta_j^M - t_M, & \text{if } L(X) = M \\ u - t_H + T(X_{M_j}), & \text{if } L(X) = H \neq M. \end{cases} \tag{3.5}$$

Since $\delta_j^M = f(M_j)$ for all citizens in the host city, the individual utility function can be written as

$$U_j^M(X_{M_j}|\epsilon_j^M) = \begin{cases} u - \epsilon_j^M - t_M + T(X_{M_j}), & \text{if } L(X) = M \\ u - t_H + T(X_{M_j}), & \text{if } L(X) = H \neq M. \end{cases} \tag{3.6}$$

The social planner does not observe ϵ_i^H, so she faces a set of the reported values of all citizens' utilities as can be seen in Table 3.3.

Table 3.3: Summary of reported utility by each citizen

[11]

	Sited in City 1	Sited in City 2	$\cdots$	Sited in city K
citizen i in city 1	$u - X_{1_i}^1$	$u - X_{1_i}^2$	$\cdots$	$u - X_{1_i}^K$
citizen j in city 2	$u - X_{2_j}^1$	$u - X_{2_j}^2$	$\cdots$	$u - X_{2_j}^K$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
citizen s in city K	$u - X_{K_s}^1$	$u - X_{K_s}^2$	$\cdots$	$u - X_{K_s}^K$

Now, consider the social welfare function. Suppose that the social planner aims to maximize

[11]Note that t is included in the project, but $T(X_i^H)$ is just a transfer schedule used to incentivize citizens to report their true preference.

a simple utilitarian welfare function $SW(\cdot) : \Phi \to \mathbb{R}$,

$$SW(L) = \begin{cases} \sum_{k=1}^{K} \sum_{i=1}^{n_k} \left(u - X_{k_i}^{1}\right), & \text{if } L(X) = 1 \\ \sum_{k=1}^{K} \sum_{i=1}^{n_k} \left(u - X_{k_i}^{2}\right), & \text{if } L(X) = 2 \\ \quad\vdots \\ \sum_{k=1}^{K} \sum_{i=1}^{n_k} \left(u - X_{k_i}^{K}\right), & \text{if } L(X) = K \end{cases} . \tag{3.7}$$

The first result shows that this modified pivotal mechanism in solving the siting of noxious facilities satisfies three important properties.

Proposition 8. *The modified pivotal mechanism produces a collective decision $L(X)$. The decision is efficient, feasible and strategy-proof.*

Proof. See Appendix C.1. □

3.3.1 A Simple Example

Consider a society of three cities. For simplification, assume that there are 1,000 citizens living in each city. Suppose that the construction cost is 20,000. The social planner is going to build a hazardous waste facility. In order to site the facility in the best location, the social planner estimates the negative impacts on all 3,000 citizens and offer compensations based on the estimation. Since the estimation is not perfectly accurate, each citizen has their self-known damage cost. Assume that in city 1, the social planner underestimates nine hundred people's damage. The unobservable damage cost is 4 for each of the nine hundred people. Meanwhile, the remaining one hundred people's damage costs are overestimated. Assume that they all obtain 1 dollar more than the true damage cost. Then, we know that the total unobservable damage for city 1 is $900 * 4 + 100 * (-1) = 3,500$. Similarly, assume that in city 2 five hundred people's damage is underestimated and that they need 3 extra dollars to fully compensate the negative impacts. The other five hundred one's is overestimated and they get 2 more than they actually need from the compensation scheme. At last, people in city 3 all need extra 1 dollar to compensate their loss due to the facility. Table 3.4 summarizes the benefit and cost of this facility.

Table 3.4: Example of the modified Clarke mechanism

	City 1	City 2	City 3
construction cost	$20,000$	$20,000$	$20,000$
total estimated damage	$26,500$	$28,000$	$40,000$
tax	15.5	16	20
total unobservable damage	$3,500$	500	$1,000$
total social cost	$50,000$	$48,500$	$61,000$

If the social planner makes the decision by evaluating only the construction cost and total estimated damage in each city, she would site the facility in city A because of the lowest amount 46,500. However, if she had complete information about all the citizens' preferences, she would choose city 2 since the social cost of siting the facility in city 2 is the lowest, 48,500. Now consider the modified Clarke mechanism. All the citizens are asked to report an adjusted term. Then we have

$$\begin{aligned}
RD_1 &= 3500 \\
RD_2 &= 500 + (16 - 15.5)(3000) = 2000 \\
RD_3 &= 1000 + (20 - 15.5)(3000) = 14500.
\end{aligned}$$

It can be found that RD_2 is the smallest. According to the decision rule, city 2 is the where the facility should be sited.

So far, we have shown that this modified Clarke mechanism satisfies several important properties and how it can be operated. While it is a serviceable tool to deal with the siting issue, some of the concerns still draw our attention.

3.3.2 Unbalanced Budget

According to Maskin and Laffont (1979) and Kim (2003), it seems impossible to find an efficient mechanism, which is incentive compatible in dominant strategies and budget balanced.

Even though the mechanism in this paper can encourage people to honestly reveal their preference in the siting issue, unfortunately, it still falls into the category of budget surplus. The reason is that we cannot redistribute the actual surplus from the transfer $\sum_{k=1}^{K}[\sum_{i=1}^{n_k} T(X_{k_i})]$ among the citizens, because it will distort their incentives in the mechanism. If the surplus is distributed, the increase in the payoff will give citizens incentives to alter their reported values. Therefore, the property of incentive compatibility cannot be guaranteed.

One good thing about this mechanism is that the surplus will be reduced as the estimation of the impacts becomes more accurate. In most conventional methods, people are asked to report the total willingness to accept (or the total impact). In that case, the surplus from the transfer rule could be enormous. Recall that ϵ_i^H follows the a uniform distribution, $U[a, b]$, where $(a + b)/2$ can be viewed as the expected possible estimation error. As the social planner can more efficiently exclude the effect of unobservable impacts, i.e. $f(X_i^H)$ getting closer to the total impact Λ_i^H, we can expect that the values of $(a + b)/2$ and $b - a$ are also getting closer to zero. This is important because only pivotal agents need to pay a surtax, and that surtax would never be greater than any agent's reported value.[12]

To sum up, even though we cannot solve the issue of unbalanced budget, at least the method proposed in this paper can reduce the amount of budget surplus.

3.4 Special Case Study: Wealth-related Weighting

Standard demand-revealing mechanisms, including the modified version of Clarke mechanism, give richer people more power in the process of voting. Take a standard pivotal mechanism. When being asked for WTP for a public facility, rich people can be expected to announce great numbers due to their wealth. Sometimes rich people may even have veto power. Good (1977) discusses this feature that some people may regard as unjust and suggests a modified version of the pivotal mechanism. In order to solve the issue of "un-justice," he proposes the inverse of wealth as the weight to evaluate the votes or WTP. He shows that this modified method is incentive compatible. What seems interesting is whether or not the socially chosen site decided by our proposed Clarke mechanism will be different if we take wealth weights in to account.

We are not going to solve the whole general model here. Instead, we employ a modified

[12] If for citizen i, who is pivotal, $T(X_i^H)$ is greater than X_i^H, then this citizen cannot be pivotal.

version of Good (1977). Denote by Y_i^k the wealth level of citizen i in city k. It is assumed that the social cost in Eq. (3.2) is transformed into weighted social cost:

$$WSC(L) = \begin{cases} WRD_1 = & \displaystyle\sum_{k=1}^{K}\sum_{i=1}^{n_k} \frac{X_{k_i}^1}{Y_i^k}, \quad \text{if } L(X) = 1 \\ WRD_2 = & \displaystyle\sum_{k=1}^{K}\sum_{i=1}^{n_k} \frac{X_{k_i}^2}{Y_i^k}, \quad \text{if } L(X) = 2 \\ \quad\vdots & \\ WRD_K = & \displaystyle\sum_{k=1}^{K}\sum_{i=1}^{n_k} \frac{X_{k_i}^K}{Y_i^k}, \quad \text{if } L(X) = K \end{cases}. \tag{3.8}$$

The decision rule is similar. The site is decided by $L(X) = \text{argmin}_{L\in\Phi}\{WSC(L)\}$. In other words, city 2 is chosen if $WRD_2 = \min_{k=1}^{K}\{WRD_k\}$.

Now we use the example shown in Table 3.4. The only difference is that this time we will consider the wealth weights. Also, we concentrate on a simple case, where in each city people's wealth level only falls into only two categories. In city 1, five hundred of the citizens whose damage is underestimated have 500 dollars in wealth and the other five hundred all have 400 dollars. In city 2, the five hundred citizens whose damage is underestimated have 200 dollars in wealth while the other five hundred have 400 dollars. In city 3, eight hundred people have 800 dollars in wealth and the other two hundred people have 400 dollars. According to citizens' reported values, we have

$$\begin{aligned} WRD_1 &= \frac{4(500)}{500} + \frac{4(400)}{400} - \frac{1(100)}{400} = 7.75 \\ WRD_2 &= \frac{3(500)}{200} - \frac{2(500)}{400} + 0.5\left(\frac{500}{500} + \frac{500}{400} + \frac{500}{400} + \frac{500}{200} + \frac{800}{800} + \frac{200}{400}\right) = 8.75 \\ WRD_3 &= \frac{1(800)}{800} + \frac{1(200)}{400} + 4.5\left(\frac{500}{500} + \frac{500}{400} + \frac{500}{400} + \frac{500}{200} + \frac{800}{800} + \frac{200}{400}\right) = 35.25 \end{aligned}$$

.

We can find that the socially chosen site is city 1, which is different from city 2 decided by the original mechanism with no wealth weights. Note that the Clarke tax a citizen needs to pay if her report switches the result also has to be transformed. The adjustment function in

Eq. (3.3) becomes a weighted adjustment function

$$WA(X_{M_j}) = \begin{cases} WRD_H - \dfrac{X^H_{M_j}}{Y^M_j}, & \text{if } L(X \setminus M_j) = H \\ WRD_G - \dfrac{X^G_{M_j}}{Y^M_j}, & \text{if } L(X \setminus M_j) = G \neq H \end{cases}. \quad (3.9)$$

Then the corresponding transfer rule is

$$WT(X_{M_j}) = \left[\min_{k=1}^{K}\{WRD_k\} - \frac{X^H_{M_j}}{Y^M_j} \right] - WA(M_j) = WRD_H - \frac{X^H_{M_j}}{Y^M_j} - WA(M_j). \quad (3.10)$$

In this case, suppose that citizen i in city 2 with 400 dollars in wealth has a weighted transfer $WT(X_{2_i}) = -1.5$. Since her wealth level is 400, it can be seen that her Clarke tax should be 600 in terms of money.

The socially chosen site may be different if the social planner has a different goal. In this example, if reduction in richer people's voting power is the main concern, then we can use the weighted version of the Clarke mechanism and site the facility in city 1. If the planner believes that using dollars as the measurement is fair or desirable for some reason, then she can use the original mechanism and choose city 2 as the host city.

3.5 Conclusion

The key issue of the siting of noxious facilities is that all the citizens in a society can share the benefits but most potential costs are absorbed by the host community. As can be seen in the literature, many researchers propose different efficient and demand-revealing methods, including low-bid auctions, lottery and compensation, and others, to facilitate siting decisions. Nonetheless, most of the previous research presumes that citizens in each community participate and reach a consensus, and then treat communities as individual players in the mechanisms. We believe that a mechanism that can incorporate all citizens' strategies is likely to offer a more reliable tool to deal with the siting issue. Also, we concentrate on the Clarke mechanism because some previous researchers have mentioned the difficulties of using it for siting NIMBYs. Their points are not fully wrong, but we have shown that the Clarke mechanism is still serviceable, after some modification. In this paper we present a modified Clarke mechanism for the siting of noxious facilities, taking into account all citizens' strategies.

Establishment of a noxious facility creates two components of cost, construction costs and invisible negative impacts for the residents in the host city. Since it is assumed that all the people share the same level of benefits, our mechanism requires that all of them should pay a tax to fund this facility. The tax is determined by the total costs. We assume that the negative impacts on the citizens of each city can be reasonably estimated. The social planner then announces the compensation and tax scheme based on the estimation. The estimation is not perfectly accurate, so all the citizens are asked to report their adjusted terms, according to the estimation error, i.e. the difference between their true value and the estimated value.

We use a Clarke mechanism with the result that only a citizen whose reported value alters the socially chosen site has to pay a money transfer, or so-called Clarke tax. Our result shows that this modified Clarke mechanism is characterized by three important properties: strategy-proofness, feasibility, and efficiency. Unfortunately, like most discussion in the literature, our mechanism cannot avoid the issue of budget imbalance. However, we find that this problem can be mitigated as accuracy of the estimation increases.

Then, we consider a special case where the social planner take wealth into consideration. Our motivation comes from Good (1977), who discusses out the problem that using money as the measurement in a demand-revealing method may give too much power to rich people compared to poor people. We use the inverse of each citizen's wealth level as a weight and transform our mechanism into a weighted version. A simple three-city example is presented to illustrate different conditions.

First, we find that if a social planner makes the siting decision with only the observable cost then the outcome may not be optimal since it has been proved that the outcome from the Clarke mechanism maximizes the social welfare. If the social planner takes account of the effect of wealth on the marginal utility of money, citizens still report their true values, but the outcome may be different from the one generated by the standard pivotal mechanism with money values. One implication is that the social planner can affect the outcome according to her desirable goal. If she believes that rich people's voting power is too huge, then she can use estimates of the marginal utility of income as the weights. If this social planner is concerned more about environmental justice, then the weights can be determined by environmental variables.

There is no doubt that that the real siting of NIMBY facilities is a complicated issue. Nonetheless, we believe that the mechanism proposed in this paper could offer an acceptable and serviceable solution. There are still many relevant issues that we have not been able to address or solve. The first one is how to better evaluate the impacts. This issue is not the siting mechanism itself, but it plays an important role in the siting procedure. Next, we ignore spatial differences in our model. People may benefit from the facility, but the level of benefit may vary among regions due to geographical factors. This claim is also true for construction costs. At last, it is interesting to analyze the case of siting more than one noxious facilities.

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Appendix A

Mathematical proofs for chapter 1

A.1 Participation constraint

Since $\Pi_{A,1}$ is at least as large as $\Pi_{A,0}$, we have

$$\Pi_{A,1} - \Pi_{A,0} = R(1+d)(\theta_{A,1} - \theta_{A,0}) + (1-\delta_A)C_A \geq 0,$$

which implies

$$(1-\delta_A)C_A \geq R(1+d)(\theta_{A,0} - \theta_{A,1}).$$

According to Eq. (1.3), we know that

$$\theta_{A,0}(\cdot) = a + [1/2 + \epsilon + h(W_A - E_B](1-a-b),$$

and

$$\theta_{A,1}(\cdot) = a(1-P_A)^2 + [1/2 + \epsilon + h(E_A - E_B)](1-a-b).$$

Applying $\theta_{A,0}$ and $\theta_{A,1}$, we can get

$$\theta_{A,0} - \theta_{A,1} = a - a(1-P_A)^2 + h(W_A - E_A)(1-a-b).$$

Then, Eq. (1.5) can be obtained after necessary algebra manipulations.

A.2 Some Important First- and Second-Order Derivatives

These derivatives are useful for proving other results.

$$\frac{\partial C_A}{\partial \delta_A} = \frac{-C_A\left[Rh(1+d)(1-a-b)-1\right]}{Rh\delta_A(1+d)(1-a-b)+1-\delta_A} < 0$$

$$\frac{\partial \theta_A}{\partial \delta_A} = h(1-a-b)\left(C_A + \delta_A\frac{\partial C_A}{\partial \delta_A}\right) = \frac{hC_A(1-a-b)}{Rh\delta_A(1+d)(1-a-b)+1-\delta_A} > 0$$

$$\frac{\partial \theta_A}{\partial P_A} = \frac{-2a(1-\delta_A)(1-P_A)}{Rh\delta_A(1+d)(1-a-b)+1-\delta_A} < 0$$

$$\frac{\partial C_A}{\partial P_A} = \frac{2Ra(1+d)(1-P_A)}{Rh\delta_A(1+d)(1-a-b)+1-\delta_A} > 0$$

$$\frac{\partial^2 u_L}{\partial P_A^2} = \frac{2a}{Rh\delta_A(1+d)(1-a-b)+1-\delta_A}\left[\eta(1-\delta_A)(3P_A-2)+R(1+d)\right]$$

$$\frac{\partial\left(\partial \theta_A/\partial P_A\right)}{\partial \delta_A} = \frac{2Rha(1+d)(1-a-b)(1-P_A)}{\left[Rh\delta_A(1+d)(1-a-b)+1-\delta_A\right]^2} > 0$$

$$\frac{\partial\left(\partial C_A/\partial P_A\right)}{\partial \delta_A} = \frac{-2Ra(1+d)(1-P_A)\left[Rh(1+d)(1-a-b)-1\right]}{\left[Rh\delta_A(1+d)(1-a-b)+1-\delta_A\right]^2} < 0$$

A.3 Proof of Lemma 1

I derive $\partial P_j^*/\partial \mu$ here. The same method can be used to derive the other results.

$$\frac{\partial P_A}{\partial \mu} = \frac{-\left(\frac{\partial^2 u_L}{\partial P_A \partial \mu}\right)\left(\frac{\partial^2 u_L}{\partial P_B^2}\right) + \left(\frac{\partial^2 u_L}{\partial P_A \partial P_B}\right)\left(\frac{\partial^2 u_L}{\partial P_B \partial \mu}\right)}{\left(\frac{\partial^2 u_L}{\partial P_A^2}\right)\left(\frac{\partial^2 u_L}{\partial P_B^2}\right) - \left(\frac{\partial^2 u_L}{\partial P_A \partial P_B}\right)^2} = -\left(\frac{\frac{\partial^2 u_L}{\partial P_A \partial \mu}}{\frac{\partial^2 u_L}{\partial P_A^2}}\right);$$

$$\frac{\partial P_B}{\partial \mu} = \frac{-\left(\frac{\partial^2 u_L}{\partial P_B \partial \mu}\right)\left(\frac{\partial^2 u_L}{\partial P_A^2}\right) + \left(\frac{\partial^2 u_L}{\partial P_A \partial P_B}\right)\left(\frac{\partial^2 u_L}{\partial P_A \partial \mu}\right)}{\left(\frac{\partial^2 u_L}{\partial P_A^2}\right)\left(\frac{\partial^2 u_L}{\partial P_B^2}\right) - \left(\frac{\partial^2 u_L}{\partial P_A \partial P_B}\right)^2} = -\left(\frac{\frac{\partial^2 u_L}{\partial P_B \partial \mu}}{\frac{\partial^2 u_L}{\partial P_B^2}}\right).$$

Since I only focus on the interior solution that satisfies the first order condition, it follows that the sign of $\partial P_j^*/\partial \mu$ is the same as that of $\partial^2 u_L/(\partial P_j^* \partial \mu)$.

A.4 Proof of Proposition 1

I only show the proof of the case about party A, i.e. $j = A$. Similar condition holds for party B. Recall the first order condition:

$$\frac{\partial u_L}{\partial P_A} = \eta \left(\frac{\partial \theta_A}{\partial P_A} P_A + \theta_A\right) - \frac{\partial C_A}{\partial P_A} = 0.$$

A.4.1 Proposition 1(i)

I use Lemma 1 and Appendix A.2 to prove the result.

$$\frac{\partial^2 u_L}{\partial P_A \partial \delta_A} = \eta \left[\frac{\partial \left(\partial \theta_A / \partial P_A\right)}{\partial \delta_A} P_A + \frac{\partial \theta_A}{\partial \delta_A}\right] - \frac{\partial \left(\partial C_A / \partial P_A\right)}{\partial \delta_A} > 0$$

A.4.2 Proposition 1(ii)

The proof is similar to the one for Proposition 2(i).

$$\frac{\partial^2 u_L}{\partial P_A \partial \delta_B} = \eta \left(\frac{\partial \theta_A}{\partial \delta_B}\right) = -\eta h(1-a-b)\left(C_B + \delta_B \frac{\partial C_B}{\partial \delta_B}\right)$$

Applying the results in Appendix B, we know that $C_B + \delta_B(\partial C_B / \partial \delta_B) > 0$. The proof follows.

A.5 Proof of Lemma 2

Totally differentiating the first order condition, Eq. (1.10), we can obtain

$$\frac{\partial^2 \Pi_A}{\partial (\delta_A^*)^2} d\delta_A^* + \frac{\partial^2 \Pi_A}{\partial \delta_A^* \partial \rho} d\rho = 0,$$

which implies

$$\frac{d\delta_A^*}{d\rho} = -\left(\frac{\frac{\partial^2 \Pi_A}{\partial \delta_A^* \partial \rho}}{\frac{\partial^2 \Pi_A}{\partial (\delta_A^*)^2}} \right).$$

Since I only focus on the interior solution that satisfies the first order condition, it follows that the sign of $\partial \delta_A^* / \partial \rho$ is the same as that of $\partial^2 \Pi_A / (\partial \delta_A^* \partial \rho)$.

A.6 Proof of Proposition 2

I only show the proof of the case about party A, i.e. $j = A$. Similar condition holds for party B. To obtain the properties, we need to derive $d\Pi_A / d\delta_A$.

$$\frac{d\Pi_A}{d\delta_A} = \left[R(1+d)\frac{\partial \theta_A}{\partial P_A} + (1-\delta_A)\frac{\partial C_A}{\partial P_A} \right] \frac{dP_A}{d\delta_A} + R(1+d)\frac{\partial \theta_A}{\partial \delta_A} + (1-\delta_A)\frac{\partial C_A}{\partial \delta_A} - C_A + R(1+d)\frac{\partial \theta_A}{\partial P_B}\frac{dP_B}{d\delta_A}$$

Applying the results in Appendix A.2, we can obtain a clearer version of $d\Pi_A / d\delta$.

$$\frac{d\Pi_A}{d\delta_A} = \Psi_1 (1-P_A)\frac{dP_A}{d\delta_A} + \Psi_2 (1-P_B)\frac{dP_B}{d\delta_A} + \Psi_3 C_A,$$

where

$$\Psi_1 = \frac{-2R\delta_A a(1+d)}{Rh\delta_A (1+d)(1-a-b) + 1 - \delta_A} < 0$$

$$\Psi_2 = \frac{-2R^2 ah\delta_B (1+d)^2 (1-a-b)}{Rh\delta_B (1+d)(1-a-b) + 1 - \delta_B} < 0$$

$$\Psi_3 = \frac{Rh(1+d)(1-a-b)}{Rh\delta_A B(1+d)(1-a-b) + 1 - \delta_A} > 0.$$

A.6.1 Proposition 2(i)

I will use Lemma 2 to prove the result.

$$\begin{aligned}\frac{\partial^2 \Pi_A}{\partial \delta_A \partial \delta_B} &= \left(\frac{\partial^2 \Pi_A}{\partial \delta_A \partial P_A}\right)\frac{\partial P_A}{\partial \delta_B} \\ &= \left[-\Psi_1 \frac{dP_A}{d\delta_A} + \Psi_3 \frac{\partial C_A}{\partial P_A}\right]\frac{\partial P_A}{\partial \delta_B} < 0\end{aligned}$$

From Proposition 1, we know that $\partial P_A/\partial \delta_A > 0$ and $\partial P_A/\partial \delta_B < 0$. Also, the results in Appendix B show that $\partial C_A/\partial P_A > 0$. The proof of $d\delta_A/d\delta_B < 0$ follows.

A.6.2 Proposition 2(ii)

The proof is similar to the one for Proposition 2(i).

$$\begin{aligned}\frac{\partial^2 \Pi_A}{\partial \delta_A \partial \epsilon} &= \left(\frac{\partial^2 \Pi_A}{\partial \delta_A \partial P_A}\right)\frac{\partial P_A}{\partial \epsilon} \\ &= \left[-\Psi_1 \frac{dP_A}{d\delta_A} + \Psi_3 \frac{\partial C_A}{\partial P_A}\right]\frac{\partial P_A}{\partial \epsilon} > 0\end{aligned}$$

From Proposition 1, we know that $\partial P_A/\partial \epsilon > 0$. The proof of $d\delta_A/d\epsilon > 0$ follows.

Appendix B

Mathematical proofs for chapter 2

B.1 Proof of Proposition 3

1. Suppose $c = 0$ and $0 < \alpha < 1/2$. We first consider the firms' wage offer when the prescribed strategy is $s_w = (1, 1)$.

$$w^{1,1}(m) = \begin{cases} E_\mu(\theta|N) = \mu_N^{1,1}\theta_H + (1-\mu_N^{1,1})\theta_L = \theta_L + \mu_N^{1,1}(\theta_H - \theta_L) & \text{if not to take the test} \\ E_\mu(\theta|T_L) = \dfrac{p\alpha}{p\alpha + (1-p)(1-\alpha)}\theta_H + \dfrac{(1-p)(1-\alpha)}{p\alpha + (1-p)(1-\alpha)}\theta_L & \text{if the result is low} \\ E_\mu(\theta|T_H) = \dfrac{p(1-\alpha)}{p(1-\alpha) + (1-p)\alpha}\theta_H + \dfrac{(1-p)\alpha}{p(1-\alpha) + (1-p)\alpha}\theta_L & \text{if the result is high.} \end{cases}$$

Since the high-productivity worker's expected payoff when taking the test is greater than the low-productivity worker's, the required condition for the T-pooling PBE is $E_L^{1,1}(T) \geq E_L^{1,1}(N)$. Applying Eq. (2.2), if the low-type worker takes the test, her expected payoff is

$$\begin{aligned} E_L^{1,1}(T) &= \theta_L + \left[(1-\alpha)\mu_L^{1,1} + \alpha\mu_H^{1,1}\right](\theta_H - \theta_L) \\ &= \theta_L + \left[\frac{p\alpha(1-\alpha)}{P_L} + \frac{p\alpha(1-\alpha)}{P_H}\right](\theta_H - \theta_L) \\ &= \theta_L + \left[\frac{\alpha p(1-\alpha)}{P_H P_L}\right](\theta_H - \theta_L). \end{aligned}$$

Obviously, $E_L^{1,1}(T) \geq E_L^{1,1}(N)$ holds whenever $\mu_N^{1,1} \leq M(p,\alpha)$, where

$$M(p,\alpha) := \frac{\alpha p(1-\alpha)}{P_H P_L} = \frac{p\alpha(1-\alpha)}{[p(1-\alpha)+(1-p)\alpha][p\alpha+(1-p)(1-\alpha)]}.$$

2. In the case with $s_w = (0,0)$, the firms offer $w^{0,0}(N) = E_H^{0,0}(N) = \theta_L + p(\theta_H - \theta_L)$ to those who choose not to take the test. Again, since high-productivity workers have greater expected payoffs when taking the test, the required condition for existence of the N-pooling is that high-productivity workers have no profitable deviation, i.e. $E_H^{0,0}(N) \geq E_H^{0,0}(T)$. We can find that the high-productivity worker's expected payoff of taking the test is

$$\begin{aligned} E_H^{0,0}(T) &= (1-\alpha)[\theta_L + \mu_H^{0,0}(\theta_H - \theta_L)] + \alpha[\theta_L + \mu_L^{0,0}(\theta_H - \theta_L)] \\ &= \theta_L + [(1-\alpha)\mu_H^{0,0} + \alpha\mu_L^{0,0}](\theta_H - \theta_L). \end{aligned}$$

The N-pooling PBE exists as long as the firms' beliefs in information set $\boldsymbol{H}$ and $\boldsymbol{L}$ satisfy $p \geq (1-\alpha)\mu_H^{0,0} + \alpha\mu_L^{0,0}$.

B.2 Consistency

To verify if the assessment $\{\mu_N = p, (H^T, L^T)\}$ is consistent, let us consider a sequesce of totally mixed strategies $s_H^k = 1-1/k$ and $s_L^k = 1-1/k$ for both types, and the corresponding belief μ_N^k. According to Bayes' rule, we can find that $\mu_N = p$ is the limit of μ_N^k derived by s_H^k and s_L^k as k goes to infinity. In fact, $\mu_N^k = p$ for all k.

We also know that, as $k \to \infty$, s_H^k and s_L^k both approach to one. By our construction, this is equivalent to the pure strategy that both types choose to take the test, i.e. (H^T, L^T).

B.3 Proof of Proposition 4

The proof of Proposition follows that of Proposition 1.

1. Suppose $c > 0$. Now, all test takers have to take a positive c into consideration. The

low-productivity worker's expected payoff of taking the test becomes

$$E_L^{1,1}(T) = \theta_L + [M(p,\alpha)](\theta_H - \theta_L) - c.$$

Now, the required condition $E_L^{1,1}(T) \geq E_L^{1,1}(N)$ implies $\theta_L + [M(p,\alpha)](\theta_H - \theta_L) - c \geq \theta_L + \mu_N^{1,1}(\theta_H - \theta_L)$. This condition can be written as

$$c \leq [M(p,\alpha) - \mu_N^{1,1}](\theta_H - \theta_L).$$

Since $0 \leq \mu_N^{1,1} \leq 1$, we can conclude that the T-pooling PBE does not exist if $c > M(p,\alpha)(\theta_H - \theta_L)$.

2. The high-productivity worker's expected payoff of taking the test is $E_H^{0,0}(T) = \theta_L + [(1-\alpha)\mu_H^{0,0} + \alpha\mu_L^{0,0}](\theta_H - \theta_L) - c$. If she chooses not to take the test, her expected payoff is $E_H^{0,0}(N) = \theta_L + p(\theta_H - \theta_L)$. The required condition comes directly from $E_H^{0,0}(N) \geq E_H^{0,0}(T)$. Since the largest value of $\mu_H^{0,0}$ and $\mu_L^{0,0}$ is one, we can conclude that the N-pooling PBE always exists if $c \geq (1-p)(\theta_H - \theta_L)$.

B.4 Proof of Proposition 5

Suppose $\alpha_H = 0$. If $\alpha_L = 0$, then the test is perfectly accurate. It can be seen that $s_w = (1,0)$ can be supported as a PBE if $0 \leq c \leq \theta_H - \theta_L$. Hence, the conditions are satisfied. Then, we need to prove that this separating PBE exists when $0 < \alpha < 1/2$.

Since only the low-productivity worker claims not to take the test, the firms' belief at information $\boldsymbol{N}$ is $\mu_N^{1,0} = 0$. That is, those who decide not to take the test can only obtain θ_L. Moreover, since $\alpha_H = 0$, we know that the only candidate who may obtain the low result when taking the test is the low-productivity worker. Therefore, the firms offer θ_L to those who take the test and get the low result. At last, it can be easily seen the firms believe that only the high-productivity worker can obtain the high result, so they offer θ_H to those who take the test and get the high result. To summarize, we have $w^{1,0}(N) = w^{1,0}(L) = \theta_L$ and $w^{1,0}(H) = \theta_H$.

To guarantee that $s_w = (1,0)$ can be supported as a PBE, we need $E_H^{1,0}(T) \geq E_H^{1,0}(N)$ and

$E_L^{1,0}(N) \geq E_L^{1,0}(T)$, i.e.

$$E_H^{1,0}(T) = \theta_H - c \geq \theta_L = E_H^{1,0}(N)$$
$$E_L^{1,0}(N) = \theta_L \geq \alpha_L \theta_H + (1-\alpha_L)\theta_L - c = E_L^{1,0}(T).$$

Combining the two inequalities, we can obtain $\alpha_L(\theta_H) - \theta_L \leq c \leq \theta_H - \theta_L$.

B.5 Proof of Proposition 6

Suppose that $c = 0$.

1. (a) If the prescribed strategy is $s_w = (1,0,0)$, the firms will offer the wage schedule as

$$w^{1,0,0}(m) = \begin{cases} E_\mu^{1,0,0}(\theta|N) = \theta_L + \dfrac{p_M}{p_M + p_L}(\theta_M - \theta_L) & \text{if the worker does not take the test;} \\ E_\mu^{1,0,0}(\theta|T_L) = \theta_L + \mu_{M|L}^{1,0,0}(\theta_M - \theta_L) & \text{if the test result is low type;} \\ E_\mu^{1,0,0}(\theta|T_M) = \theta_H & \text{if the test result is mid type;} \\ E_\mu^{1,0,0}(\theta|T_H) = \theta_H & \text{if the test result is high type.} \end{cases}$$

The high-productivity worker has no incentive to deviate since she can obtain θ_H by taking the test. Basically, mid-productivity workers have greater expected payoffs when taking the test than low-productivity ones do. Therefore, the only condition needed to guarantee that $s_w = (1,0,0)$ can be supported as a PBE is $E_M^{1,0,0}(N) \geq E_M^{1,0,0}(T)$. The mid-productivity worker's expected payoff when taking the test is

$$\begin{aligned} E_M^{1,0,0}(T) &= (1-\alpha)\theta_H + \alpha\left[\theta_L + \mu_{M|L}^{1,0,0}(\theta_M - \theta_L)\right] \\ &= \theta_L + (1-\alpha)(\theta_H - \theta_L) + \alpha\mu_{M|L}^{1,0,0}(\theta_M - \theta_L). \end{aligned}$$

To guarantee that type M has no profitable deviation, we need $E_M^{1,0,0}(T) - E_M^{1,0,0}(N) \leq 0$. Note that

$$E_M^{1,0,0}(T) - E_M^{1,0,0}(N) = (1-\alpha)(\theta_H - \theta_L) + \left(\alpha\mu_{M|L}^{1,0,0} - \frac{p_M}{p_M + p_L}\right)(\theta_M - \theta_L).$$

Suppose that the firms believe that those who take the test and get low grades are low-productivity workers, i.e. $\mu^{1,0,0}_{M|L} = 0$. We can find that $E^{1,0,0}_M(T) - E^{1,0,0}_M(N) \leq 0$ holds if

$$(1-\alpha)(\theta_H - \theta_L) \leq \frac{p_M}{p_M + p_L}(\theta_M - \theta_L).$$

Since $1-\alpha > 1/2$ and $\theta_H > \theta_M$, we know that $p_M/(p_M + p_L)$ must be greater than $1/2$. Therefore, one necessary condition is $p_M > p_L$. Also, if $\theta_H - \theta_M \geq \theta_M - \theta_L$, then Eq. (2.5) implies $p_M/(p_M + p_L) > 1$, which is impossible. Hence, we can find that $\theta_H - \theta_M < \theta_M - \theta_L$ is the other necessary condition.

(b) If the prescribed strategy is $s_w = (0, 0, 0)$, the firms will offer the wage schedule as

$$w^{0,0,0}(m) = \begin{cases} E^{0,0,0}_\mu(\theta|N) = p_H\theta_H + p_M\theta_M + p_L\theta_L & \text{if the worker does not take the test;} \\ E^{0,0,0}_\mu(\theta|T_L) = \theta_L + \mu^{0,0,0}_{M|L}(\theta_M - \theta_L) & \text{if the test result is low type;} \\ E^{0,0,0}_\mu(\theta|T_M) = \theta_M + \mu^{0,0,0}_{H|M}(\theta_H - \theta_M) & \text{if the test result is mid type;} \\ E^{0,0,0}_\mu(\theta|T_H) = \theta_H & \text{if the test result is high type.} \end{cases}$$

The expected payoffs of the three types when taking the test are

$$\begin{cases} E^{0,0,0}_L(T) & = \theta_L + \mu^{0,0,0}_{M|L}(\theta_M - \theta_L) \\ E^{0,0,0}_M(T) & = \theta_L + (1-\alpha)\mu^{0,0,0}_{H|M}(\theta_H - \theta_M) + \left(1 - \alpha + \alpha\mu^{0,0,0}_{M|L}\right)(\theta_M - \theta_L) \\ E^{0,0,0}_H(T) & = \theta_M + \left(1 - \alpha + \alpha\mu^{0,0,0}_{H|M}\right)(\theta_H - \theta_M). \end{cases}$$

We can find that $E^{0,0,0}_H(T) \geq E^{0,0,0}_M(T) \geq E^{0,0,0}_L(T)$. Thus, the N-pooling PBE exists as long as $E^{0,0,0}_H(T) \leq E^{0,0,0}_H(N)$, which means

$$E^{0,0,0}_H(T) - E^{0,0,0}_H(N) = \left(1 - \alpha + \alpha\mu^{0,0,0}_{H|M} - p_H\right)(\theta_H - \theta_M) + p_L(\theta_M - \theta_L) \leq 0.$$

It is not difficult to see that this inequality requires $\left(1 - \alpha + \alpha\mu^{0,0,0}_{H|M} - p_H\right)$ to be negative. Suppose that the firms believe that the workers with mid-grades are exactly mid-productivity workers, i.e. $\mu^{0,0,0}_{H|M} = 0$. After some algebra manipulation, the inequality above becomes

$$\theta_M + (1-\alpha)(\theta_H - \theta_M) \leq p_H\theta_H + p_M\theta_M + p_L\theta_L.$$

We need $1-\alpha-p_H < 0$, or $\alpha > 1-p_H$, to guarantee that this condition can be satisfied.

2. (a) If the prescribed strategy is $s_w = (0,1,0)$, the firms will offer the wage schedule as

$$w^{0,1,0}(m) = \begin{cases} E_\mu^{0,1,0}(\theta|N) = \theta_L + \dfrac{p_H}{p_H+p_L}(\theta_H-\theta_L) & \text{if the worker does not take the test;} \\ E_\mu^{0,1,0}(\theta|T_L) = \theta_L & \text{if the test result is low type;} \\ E_\mu^{0,1,0}(\theta|T_M) = \theta_M & \text{if the test result is mid type;} \\ E_\mu^{0,1,0}(\theta|T_H) = \theta_M & \text{if the test result is high type.} \end{cases}$$

In order to prevent the high-productivity worker from deviating to taking the test, we need $E_H^{0,1,0}(N) - E_H^{0,1,0}(T) \geq 0$, i.e.

$$\theta_L + \frac{p_H}{p_H+p_L}(\theta_H-\theta_L) - \theta_M \geq 0.$$

This condition also guarantees that the low-productivity worker has no profitable deviation. Similarly, the mid-productivity worker has no incentive to deviate if $E_M^{0,1,0}(T) - E_M^{0,1,0}(N) \geq 0$, i.e.

$$\theta_M - \left[\theta_L + \frac{p_H}{p_H+p_L}(\theta_H-\theta_L)\right] \geq 0.$$

Combining the two inequalities gives rise to Eq. (2.7).

(b) If the prescribed strategy is $s_w = (0,0,0)$, the firms will offer the wage schedule as

$$w^{0,0,0}(m) = \begin{cases} E_\mu^{0,0,0}(\theta|N) = p_H\theta_H + p_M\theta_M + p_L\theta_L & \text{if the worker does not take the test;} \\ E_\mu^{0,0,0}(\theta|T_L) = \theta_L & \text{if the test result is low type;} \\ E_\mu^{0,0,0}(\theta|T_M) = \theta_L + \mu_{M|M}^{0,0,0}(\theta_M-\theta_L) & \text{if the test result is mid type;} \\ E_\mu^{0,0,0}(\theta|T_H) = \theta_M + \mu_{H|H}^{0,0,0}(\theta_H-\theta_M) & \text{if the test result is high type.} \end{cases}$$

The expected payoffs of the three types when taking the test are

$$\begin{cases} E_L^{0,0,0}(T) & = \theta_L + \alpha\mu_{M|M}^{0,0,0}(\theta_M - \theta_L) \\ E_M^{0,0,0}(T) & = \theta_L + \alpha\mu_{H|H}^{0,0,0}(\theta_H - \theta_M) + \left[\alpha + (1-\alpha)\mu_{M|M}^{0,0,0}\right](\theta_M - \theta_L) \\ E_H^{0,0,0}(T) & = \theta_L + (1-\alpha)\mu_{H|H}^{0,0,0}(\theta_H - \theta_M) + \left(1 - \alpha + \alpha\mu_{M|M}^{0,0,0}\right)(\theta_M - \theta_L). \end{cases}$$

We can find that $E_H^{0,0,0}(T) \geq E_M^{0,0,0}(T) \geq E_L^{0,0,0}(T)$. Thus, the N-pooling PBE exists as long as $E_H^{0,0,0}(T) \leq E_H^{0,0,0}(N)$, which means

$$E_H^{0,0,0}(T) - E_H^{0,0,0}(N) = \left[(1-\alpha)\mu_{H|H}^{0,0,0} - p_H\right](\theta_H - \theta_M) + \left[p_L - \alpha + \alpha\mu_{M|M}^{0,0,0}\right](\theta_M - \theta_L) \leq 0.$$

Consider that $1 - p_L \geq \alpha$. We can find that $E_L^{0,0,0}(N) \geq E_L^{0,0,0}(T)$ is always true. In other words, for low-productivity workers, not to take the test is always better than to take the test. Therefore, the firms should believe that $\mu_{M|M}^{0,0,0} = 1$. That is, only mid-productivity workers may get mid results in the test. If they also assign $\mu_{H|H}^{0,0,0} = 1$, we have

$$E_H^{0,0,0}(T) - E_H^{0,0,0}(N) = (1 - \alpha - p_H)(\theta_H - \theta_M) + p_L(\theta_M - \theta_L) \leq 0.$$

After some algebra manipulation, the inequality above becomes

$$\theta_M + (1-\alpha)(\theta_H - \theta_M) \leq p_H\theta_H + p_M\theta_M + p_L\theta_L.$$

Note that we also need $p_L \leq 1 - \alpha < p_H$ to guarantee that this condition can be satisfied.

B.6 Proof of Proposition 7

The proofs of Proposition 5.2 and 5.3 are similar to the corresponding parts in Proposition 4, so we will skip them here. Assume $c = 0$ and there is a two-way bias in the test with a probability $0 < \alpha < 1/2$. If the prescribed strategy is $s_w = (1, 0, 0)$, the firms will offer the wage schedule as

$$w^{1,0,0}(m) = \begin{cases} E_\mu^{1,0,0}(\theta|N) = \theta_L + \dfrac{p_M}{p_M + p_L}(\theta_M - \theta_L) & \text{if the worker does not take the test;} \\ E_\mu^{1,0,0}(\theta|T_L) = \theta_L + \mu_{M|L}^{1,0,0}(\theta_M - \theta_L) & \text{if the test result is low type;} \\ E_\mu^{1,0,0}(\theta|T_M) = \theta_H & \text{if the test result is mid type;} \\ E_\mu^{1,0,1}(\theta|T_H) = \theta_H & \text{if the test result is high type.} \end{cases}$$

Since the high-productivity worker can obtain θ_H for sure by taking the test, she will not deviate. Now, consider the other two types' expected payoffs of taking the test.

$$\begin{cases} E_L^{1,0,0}(T) & = \theta_L + \alpha(\theta_H - \theta_L) + (1-\alpha)\mu_{M|L}^{1,0,0}(\theta_M - \theta_L) \\ E_M^{1,0,0}(T) & = \theta_L + \left(1 - \dfrac{\alpha}{2}\right)(\theta_H - \theta_L) + \left(\dfrac{\alpha}{2}\right)\mu_{M|L}^{1,0,0}(\theta_M - \theta_L). \end{cases}$$

To guarantee that type M does not deviate, we need $E_M^{1,0,0}(N) = \theta_M \geq E_M^{1,0,0}(T)$, which implies

$$E_M^{1,0,0}(T) - E_M^{1,0,0}(N) = \left(1 - \frac{\alpha}{2}\right)(\theta_H - \theta_M) + \left[1 - \frac{\alpha}{2} + \left(\frac{\alpha}{2}\right)\mu_{M|L}^{1,0,0} - \frac{p_M}{p_M + p_L}\right](\theta_M - \theta_L) \leq 0.$$

If the firms believe that only the low-productivity workers may get low results in the test, i.e. $\mu_{M|L}^{1,0,0} = 0$, then the inequality becomes

$$\left(1 - \frac{\alpha}{2}\right)(\theta_H - \theta_M) + \left[1 - \frac{\alpha}{2} - \frac{p_M}{p_M + p_L}\right](\theta_M - \theta_L) \leq 0.$$

The inequality here can be rearranged to Eq. (2.8). Note that there are two necessary conditions for Eq. (2.8) to hold. The first one is

$$1 - \frac{\alpha}{2} < \frac{p_M}{p_M + p_L}.$$

Without this condition, Eq. (2.8) can never hold true because both terms are positive. The second one wee need is $\theta_H - \theta_M < (\theta_M - \theta_L)/3$. The reason is that the maximum value of $p_M/(p_M + p_L)$ is one (or really close to one if both p_M and p_L are positive). If $\theta_H - \theta_M \geq (\theta_M - \theta_L)/3$, we can find

$$\left(1 - \frac{\alpha}{2}\right)(\theta_H - \theta_M) + \left[1 - \frac{\alpha}{2} - \frac{p_M}{p_M + p_L}\right](\theta_M - \theta_L) \geq \left(1 - \frac{\alpha}{2}\right)\frac{(\theta_M - \theta_L)}{3} + \left[1 - \frac{\alpha}{2} - \frac{p_M}{p_M + p_L}\right](\theta_M - \theta_L).$$

The right-hand side of this inequality is indeed

$$\left[\frac{4}{3}\left(1-\frac{\alpha}{2}\right)-\frac{p_M}{p_M+p_L}\right](\theta_M-\theta_L) \geq \left[\frac{4}{3}\left(1-\frac{\alpha}{2}\right)-1\right](\theta_M-\theta_L) = \left(\frac{1}{3}-\frac{2\alpha}{3}\right)(\theta_M-\theta_L) > 0.$$

Therefore, $\theta_H - \theta_M < (\theta_M - \theta_L)/3$ is a necessary condition for this semi-separating PBE.

Appendix C

Proof of Proposition 8

C.1 Proof of Proposition 8

I first show that the modified pivotal mechanism satisfies feasibility, which means that the transfer rule exhibits no deficit given any reported values. In order to prove this property, we need to show $\sum_{k=1}^{K}[\sum_{i=1}^{n_k} T(X_{k_i})] \leq 0$. Suppose that the host city chosen via this mechanism is city H. Consider an arbitrary citizen j in city M. If her reported X_{M_j} does not change the socially chosen site, than we know that $T(M_j) = 0$. If this citizen j's report alters the social decision, say $L(X \setminus M_j) = G \neq H$, according to Eq. (3.4), we know that the Clarke tax

$$\begin{aligned} T(X_{M_j}) &= RD_H - X_{M_j}^H - A(M_j) \\ &= RD_H - X_{M_j}^H - \left(RD_G - X_{M_j}^G\right) \\ &= (RD_H - RD_G) + \left(X_{M_j}^G - X_{M_j}^H\right) < 0. \end{aligned}$$

By definition, since H is chosen by the decision rule, we know that $RD_H \leq RD_G$. Moreover, since G would have been chosen had citizen j made no report, we can conclude $X_{M_j}^G$ must be sufficiently smaller than $X_{M_j}^H$. The two inequalities imply that $T(X_{M_j}) < 0$ whenever citizen j's report changes the social decision. Therefore, the modified pivotal mechanism is feasible.

The next part of the proof shows why this mechanism satisfies the efficiency criteria. After some algebra manipulation, $SW(L)$ can be rewritten as

$$SW(L) = \begin{cases} \sum_{k=1}^{K}\sum_{i=1}^{n_k}\left(u - X_{k_i}^1\right) = Nu - RD_1, & \text{if } L(X) = 1 \\ \sum_{k=1}^{K}\sum_{i=1}^{n_k}\left(u - X_{k_i}^2\right) = Nu - RD_2, & \text{if } L(X) = 2 \\ \vdots & \\ \sum_{k=1}^{K}\sum_{i=1}^{n_k}\left(u - X_{k_i}^K\right) = Nu - RD_K, & \text{if } L(X) = K \end{cases}.$$

According to the decision rule, we know that $L(X) \in \text{argmin}_{L\in\Phi}\{SC(L)\}$. Hence, it must also be true that $L(X) = \text{argmax}_{L\in\Phi}\{SW(L)\}$.

The last part is to show that this mechanism is strategy-proof; that is, reporting $\bar{\epsilon}_j^M = \epsilon_j^M$ is citizen i's dominant strategy. Consider an arbitrary citizen i in city A. Suppose that this citizen i honestly reports her true X_{A_i}. There are four possible cases:

1. A is the socially chosen site and citizen i is pivotal. Let $L(X \setminus A_i)$ be any city $B \neq A$.
2. A is the socially chosen site and citizen i is NOT pivotal.
3. The other city B is chosen and citizen i is pivotal. Let $L(X \setminus A_i)$ be any city $M \neq B$.
4. The other city B is chosen and citizen i is NOT pivotal.

We want to show that citizen i has no incentive to deviate in all four cases.

Even though in this mechanism we allow citizens to report their WTP or WTA for all cities, the only reported value that is actually determined by a citizen is her unobservable damage, i.e. $X_{A_i}^A$ or $\bar{\epsilon}_i^A$ here. In the first case, citizen i's utility is $u - \epsilon_i^A - t_A + T(X_{A_i})$. There is no incentive for citizen i to lower her report $X_{A_i}^A$ because doing so does not change the outcome. If she chooses to raise the report but the increase of $X_{A_i}^A$ does not flip the siting decision, then there is no change in her utility. Consider that citizen i increases her report so that the facility is sited in city B. Her utility is $u - t_B$ since she is not pivotal and the facility is not sited in her city. Then, we know

$$u - \epsilon_i^A - t_A + T(X_{A_i}) - (u - t_B) = -(\epsilon_i^A + t_A - t_B) + T(X_{A_i}) > 0.$$

Since A_i is pivotal, it can be shown that $\epsilon_i^A + t_A < t_B$ and also $\left|\epsilon_i^A + t_A - t_B\right| > |T(X_{A_i})|$.

Now consider the second case. We use the similar method to prove the result. Compare citizen i's utility of telling the truth and making a sufficiently greater report so that the facility can be sited in city $B \neq A$. Her utility of telling the truth is $u - t_A - \epsilon_i^A$ and it becomes $u - t_B + T(X_{A_i})$ if she raise the report and alter the decision. We can find that

$$\begin{aligned} u - t_A - \epsilon_i^A - [u - t_B + T(X_{A_i})] &= -t_A - \epsilon_i^A + t_B - T(X_{A_i}) \\ &= -t_A - \epsilon_i^A + t_B - (RD_A - RD_B - \epsilon_i^A - \mu_A - \mu_B) > 0, \end{aligned}$$

so telling the truth is the best strategy.

In the third case, we have to consider two possibilities, $L(X \setminus A_i) = A$ and $L(X \setminus A_i) \neq A$. If $L(X \setminus A_i) = M \neq A$, we know $RD_B < RD_A$ and $RD_M - \mu_M < RD_B - \mu_B$. The only movement that may change citizen i's utility is to reduce $\bar{\epsilon}_i^A$ so that city A becomes the host city. By reporting the true value, citizen i's utility is $u - t_B$. If she decreases $X_{A_i}^A$ so that the facility is sited in city A, then her utility becomes $u - t_A - \epsilon_i^A + T(X_{A_i})$. It can be shown that

$$u - t_B - [u - t_A - \epsilon_i^A + T(X_{A_i})] = -\mu_B + RD_A - RD_M + \mu_M > RD_B - \mu_B - (RD_M - \mu_M) > 0.$$

Hence, citizen i's best strategy in this case is to report the true value. We can use a similar analysis to prove the other case.

Now consider the last case. Again, the only movement that can change citizen i's utility is to decrease X_{A_i} so that city A becomes the host city. If she reports the true value, she can obtain $u - t_B$. If she reports a sufficiently low X_{A_i}, her utility becomes $u - t_A - \epsilon_i^A + T(X_{A_i})$. We know

$$u - t_B - [u - t_A - \epsilon_i^A + T(X_{A_i})] = -\mu_B + \mu_A + \epsilon_i^A - [RD_B - \mu_B - RD_A + \mu_A + \epsilon_i^A] = RD_A - RD_B > 0.$$

Therefore, citizen i has no incentive to deviate.