

High resolution scattering measurements for stationary particles

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**Particle characterization is important to the aerospace field because particle ingestion in propulsion engines can lead to catastrophic failures. It has been shown laser based methodologies can determine size and concentration of spherical particles by using light extinction. However, when one moves to increasingly complex shapes one must take into consideration not only light extinction but multi angle light scattering. Cylindrical particles scatter light in a way that can be quantified by electromagnetic wave theory. This scattering distribution is directly related to the cylinders diameter and material properties, as well as the wavelength of the incident light. This project designed and implemented a rig that measures the scattering distribution of single static cylindrical particles. It was shown that the scattering distribution for cylinders can be measured and compared to computational expected values, especially in the forward scattering region. Future work in measuring the scattering distribution of increasingly complex geometries and in flow conditions is proposed.**

## Table of Contents

Introduction.....	6
Review of Literature .....	7
Experiment:.....	10
Experimental Set-up.....	10
Procedure .....	11
Data Collection and Analysis.....	11
Results.....	13
Experimental limitations.....	15
Discussion.....	16
Conclusion .....	17
References.....	19
Appendix I : Data Reduction Code.....	20

## Table of Figures

Figure 1: Scattering distributions varying size parameter .....	9
Figure 2: Scattering distributions varying index of refraction.....	9
Figure 3: Scattering Distribution measurement rig.....	10
Figure 4: Geometry of set up (not to scale) .....	11
Figure 5: Angle references top down rig view (not to scale).....	12
Figure 6: Raw data scattering profile snapshot 77um sample .....	13
Figure 7: 77um experimental and expected values.....	13
Figure 8: 67um experimental and expected values.....	14
Figure 9: 77um percent error .....	14
Figure 10: 67um percent error .....	15
Figure 11: Microscope photos of 67um sample along axis .....	15

## Introduction

With the advancement of globalization and travel technologies, there has been an increase in travel in areas that contain high levels of sand particulates in the air. Areas high in sand particulates pose great threat to commercial and military aircraft. They are proven to cause damage due to erosion, deposition, and in most severe cases total catastrophic loss of aircraft when ingested in the engine. Even so, people need to travel through these harsh environments to accomplish humanitarian aid efforts, global trade, and for personal travel. Due to this, there needs to be a way in which aircraft pilots can tell if an environment is safe to fly in at any given time regarding particulate matter being ingested into aircraft engines. There is a need for monitoring the sand ingestion in the engine, a major component that contains millions of subcomponents that if damaged by sand could lead to catastrophic failure.

To date there is no way in which pilots on an aircraft can deduce how much particulate matter is entering their engine in real time. Investigating, creating, and implementing a system that does this can make travel safer for people around the world. It can also defray the costs of maintaining and repairing one of the most expensive components on the airplane (for a Boeing 787, the cost of the engine is about 22 percent of the total cost of up to 300million dollars). Having a more effective way to prevent or predict damage due to ingested particles has effects for airline companies. It can increase their cost savings and potentially pass these lower costs to the consumer. This has an effect of opening travel to those who previously could not afford it.

A laser based methodology was explored in this Rolls Royce sponsored project. The long-term goal of this project is to devise a line-of-sight optical diagnostic sensor to characterize sand particles being ingested on an engine. When light is incident on a particle, the interaction between the particle and the light wave causes a measurable reaction proportionate to the size, material properties, and concentration of particles the light is incident on. The particles absorb and scatter the incident light. Proof of concept for the characterization of spherical particles in static conditions was developed and proven by Barboza et al [1]. This study further develops this research to incorporate measurements of cylindrical particles, to move to increasingly complex shapes. This research project was to design a scattering measurement device to further the fundamental research in creating a particulate diagnostic sensor.

## Review of Literature

There is a direct relationship between the interaction of light and physical matter. This interaction can physically be seen and measured in the extinction and scattering of light. The extinction of light is defined as the total degradation of incident light onto a material. The scattering of light is the combined optical effects of internal and external reflection, refraction, and diffraction. Differences in material and geometric properties of the physical matter will cause light to interact in distinct ways. Index of refraction and size parameter are two dimensionless parameters which allow these distinct interactions to be explored in a deterministic manner.

Size parameter describes the relationship between geometric properties and the wavelength of incident light by equation 1,

$$\alpha = \frac{\pi a}{\lambda} \quad (1)$$

where  $a$  is the particles radius,  $\lambda$  is the wavelength of light, and  $\alpha$  is the size parameter. This property delineates the different regimes for light scattering, regimes which demarcate what physics describes the nature of scattering.

Rayleigh scattering is for very small size parameters, whereby the very small particles act as radiating dipoles of the incident electromagnetic (light) interference. They then radiate this incident energy as vibrations. If these vibrations happen to be at special frequencies between 390-700nm, one can see them with their eyes in the forms of light! One can see this effect in the blue sky, or in the many colors in a sunset.

Mie Scattering is the complete solution of Maxwell's electromagnetic equations in describing the light scattering and extinction of homogeneous spheres with a size parameter close to one.

Finally, Geometric scattering is the regime for size parameters much larger than one. In this regime the laws of geometric optics can describe the interaction of light. The scattering is more determined by the geometry of the particle. These well known regimes help determine the mechanics of the electromagnetic behavior around geometrical objects.

The index of refraction is a material property defined by equation 2,

$$m = \frac{c}{v} \quad (2)$$

where  $c$  is the speed of light in a vacuum,  $v$  is the speed of light in the medium, and  $m$  is the index of refraction. It describes how light propagates through different mediums. The index of refraction and size parameter of the materials measured were used as independent variables in this experiment.

By measuring the extinction of laser light intensity in a monodisperse polystyrene sample, one can determine the diameter and size distribution of the suspended particles. (Barboza et al, 2015). This fundamental research laid the groundwork for this project in developing the particulate diagnostic sensor. Barboza used Mie extinction theory to size the spherical particles by measuring light extinction. This validates that a laser based technique can measure particulate size and concentration in monodisperse static aqueous solutions. However, to investigate increasingly complex shapes one must not only measure extinction but also the scattering effects around the particles.

Though Mie scattering can describe the nature of light around spherical particles, more information is needed when it comes to non-spherical particles. Hull et al [3] determined the approximate effective size of small soot like particles using a light scattering device they developed called the Deiseal particle scatterometer. This device contained an array of fixed light detectors around a baffle containing a flowing diesel particle stream. It obtained multi-angular measurements around the flowing particles. They utilize Mie theory to model experimental scattering data at various aspect ratios, and could determine that for spheres the predicted radii remained constant, whereas for ellipsoids the predicted radii remained constant up to aspect ratios of 3. As aspect ratio increases, the particle becomes more ellipsoid. This study shows how shape can affect the expected measurement of scattering as well.

Asano and Sato [9] observed this same phenomenon in their computational research to determine the effects of ellipsoid shape on concentration determination in a monodisperse mixture. They determined that looking at extinction alone can create up to 250 percent uncertainty in concentration measurements around ellipsoid shaped particles. To obtain more accurate measurements in particles whose shape is more complex than a sphere, one must use not only the extinction information but also the scattering information.

Cai et al [2] determined significant shape characteristics and size distribution of spheroidal particles by measuring the scattering of particles at multiple wavelengths and multiple fixed angles. This study provided insight into analyzing information from scattering for spherical particles, as well as novel way as to the design of instruments for particle characterization utilizing the scattering methodology. This study also demonstrated how orientation of particles non-spherical in shape produce scattering profiles quantifiable through numerical simulations, and that there is a direct relationship between the two parameters.

These researchers have all demonstrated that there are unique characteristics between the interaction of light and small micro sized particles. They have demonstrated that one can utilize scattering in fixed locations to obtain information of spherical as well as ellipsoid particles in flow conditions. However none, to this researchers knowledge, have measured the scattering profile around single cylindrical cylinders. It is believed that the uniqueness of a particles scattering profile contains information specific to the size and shape of the particle. While light extinction alone can sufficiently resolve characteristics of spherical particles, one need not utilize the scattering nature of the particle to determine information concerning the particle. Which makes measuring spherical particles very convenient, however spherical is not the norm for sand particles that are ingested inside of an engine. Those particles have vastly varied geometrical shapes. To resolve particles with complex shapes, one must utilize not only light extinction, but the scattering as well to obtain the complete light particle interaction.

Fortunately, Schäfer and Lee [3] determined the near and far field scattering of electromagnetic waves by infinite cylinders, and created a computational tool, MatScat, which calculates the effects (much like Mie scattering does for spherical particles) across various size parameters and index of refractions utilizing Maxwell's electromagnetic equations. These equations were developed in 1862 when he first proposed light interacts with the environment as waves. The expected values of scattering profiles were

determined using this tool and these profiles were used as a validation of scattering measurements taken.

Preliminary attempts were made to observe the scattering profile of cylindrical particles utilizing MatScat. The expected values of scattering profile of infinite cylinders varying size parameter and index of refraction were calculated. One can see the effect of size parameter holding index of refraction constant in figure 1.

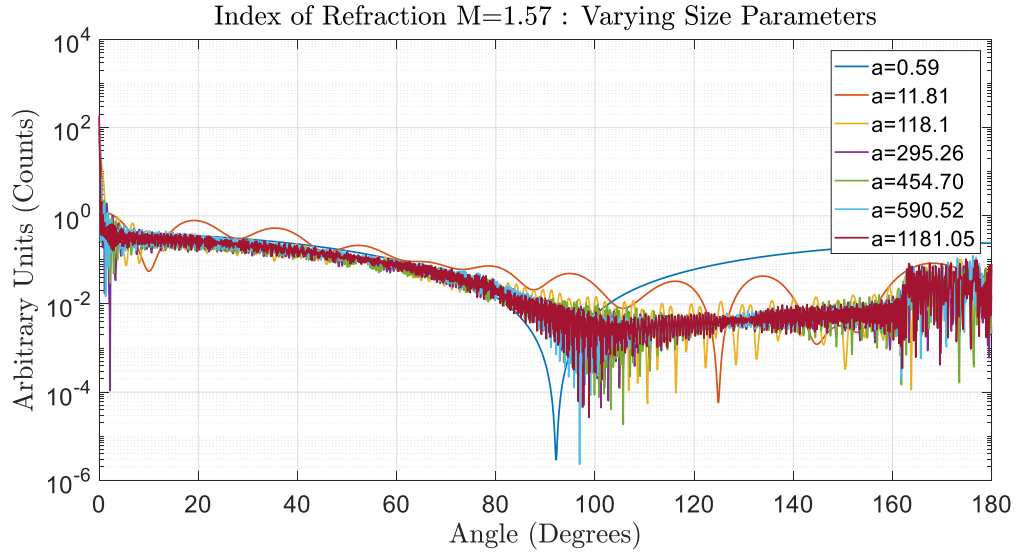


Figure 1: Scattering distributions varying size parameter

One can see the effect of index of refraction holding size parameter constant in figure 2.

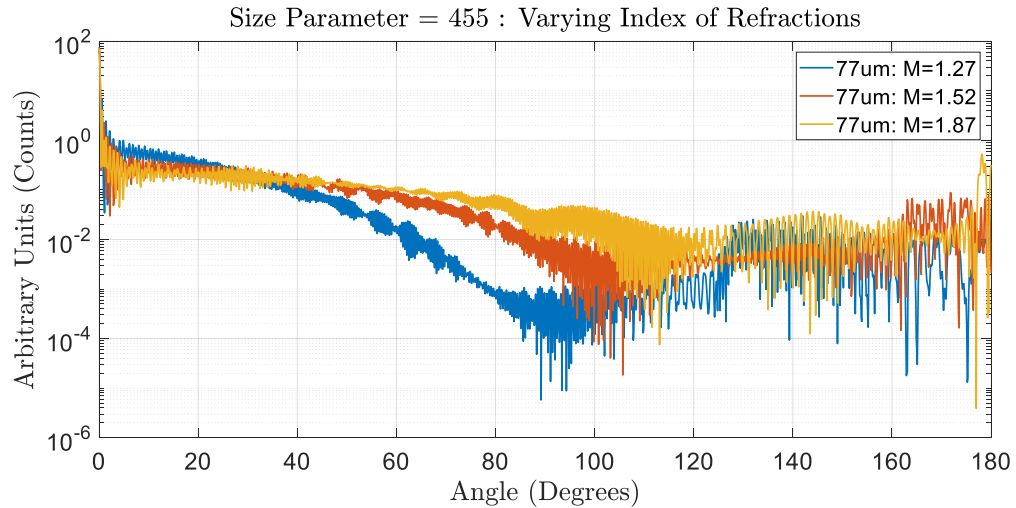


Figure 2: Scattering distributions varying index of refraction

It was observed in both plots that changing the set parameters drastically changed the pattern of scattering. It is the purpose of this study to detect and measure the differences in scattering profiles for particles at different size parameters and different indices of refraction. These measurements will further aid in the design of the diagnostic sensor, by learning how one can measure scattering of particles that are not spherical in shape for the first time.



## Experiment:

### Experimental Set-up

Measurements of scattering intensity around a cylindrical particle subjected to a 532nm parallel polarized laser were taken using a Pco.edge sCMOS camera sensor. The data collected from the sensor are images of the light intensity taken at discrete 2 degree intervals in the range of 10-140 degrees around the mounted particle. The cylindrical particles measured were 67um and 77um fishing line. Fishing line was used because it is a cylindrical shape, mechanically sound to handle the testing stand, and was readily available for use.

The sCMOS camera was attached and mounted on a motorized rotation stage (MRS). The MRS was made of concentric circles, the inner circle was stationary, and the outer circle was subject to rotation. This stage was connected to a computer interface that rotated the camera around the particle according to how many degrees the rotation stage was told to rotate. The particle was mounted over and attached to the center static circle of the rotation stage, ensuring the centeredness of the particle. The other end of the particle was fixed to piece which ensured it was vertically mounted perpendicular to the face of the rotation stage. One can see a picture of a mounted particle, the camera, and the rotation stage in figure 3.

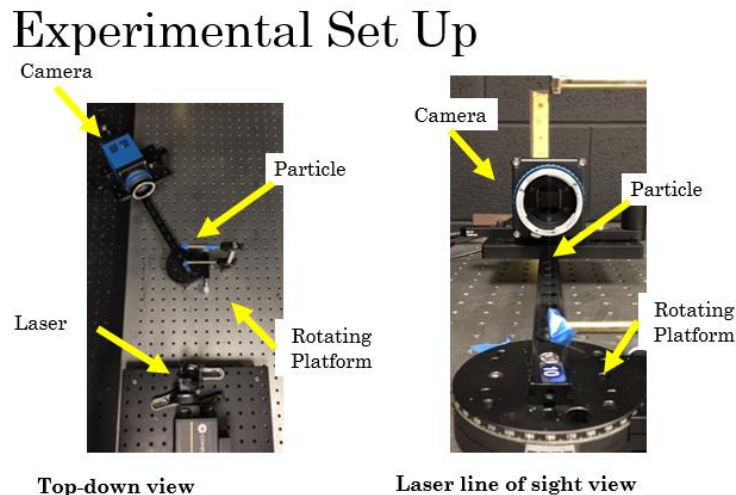


Figure 3: Scattering Distribution measurement rig

The 77 um sample had a size parameter of 455 and was made of fluorocarbon with an index of refraction of 1.42. The 67um sample was made of nylon and had an index of refraction of 1.57. The size parameter of this sample was 396. These samples were diverse in both size parameter and index of refraction.

To determine the angular width of the sensor, geometrical relations were used. The distance from the camera to the particle was 11.5 inches. The dimensions of the camera sensor are 16.6millimeters in width and 14 millimeters in height. With these geometrical conditions the angular distribution for the camera sensor at its location was calculated to be 3.177 degrees. A figure describing the geometry of this calculation is show in figure 4.

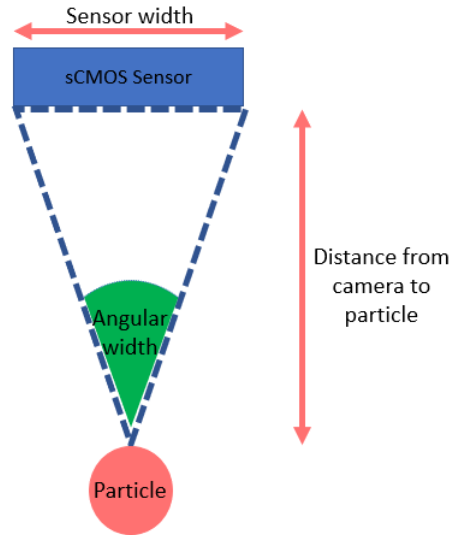


Figure 4: Geometry of set up (not to scale)

#### Procedure

To start, a sample was carefully mounted to the rotation stage. The laser, the camera, and the rotation stage were then all turned on and calibrated to the zero-starting condition at 180 degrees. At this angle the laser beam is centered on the camera sensor. It was also verified that the sample was within the line of sight of the laser beam at this zero starting condition. Once all the initial starting conditions were verified, all extraneous light sources were turned off and blocked out of the testing room, the laser beam was turned on to 5mW, and images of the scattering distribution were collected. For both the 77um and 67um samples, 70 images were taken spanning 10-140 degrees in 2-degree intervals.

#### Data Collection and Analysis

The images from the sCMOS camera have information as to the constructive and destructive interference of the light waves interacting with the particle. This interference of waves creates the pattern called the scattering profile, a sample of which is depicted in the image contained in figure 6.

A Matlab code was created to reduce the raw picture data into quantitative data describing the intensity of the scattering in the image. Each picture contains a scattering profile that has a vertical width. The code averages all vertical pixel intensities to obtain the information regarding the intensity of the light at each horizontal pixel location in the image. This averaged vertical intensity is matched to its angular location with respect its location to the profile determined by its horizontal location. A figure describing the nature of angular locations utilized for this experiment are highlighted in figure 5.

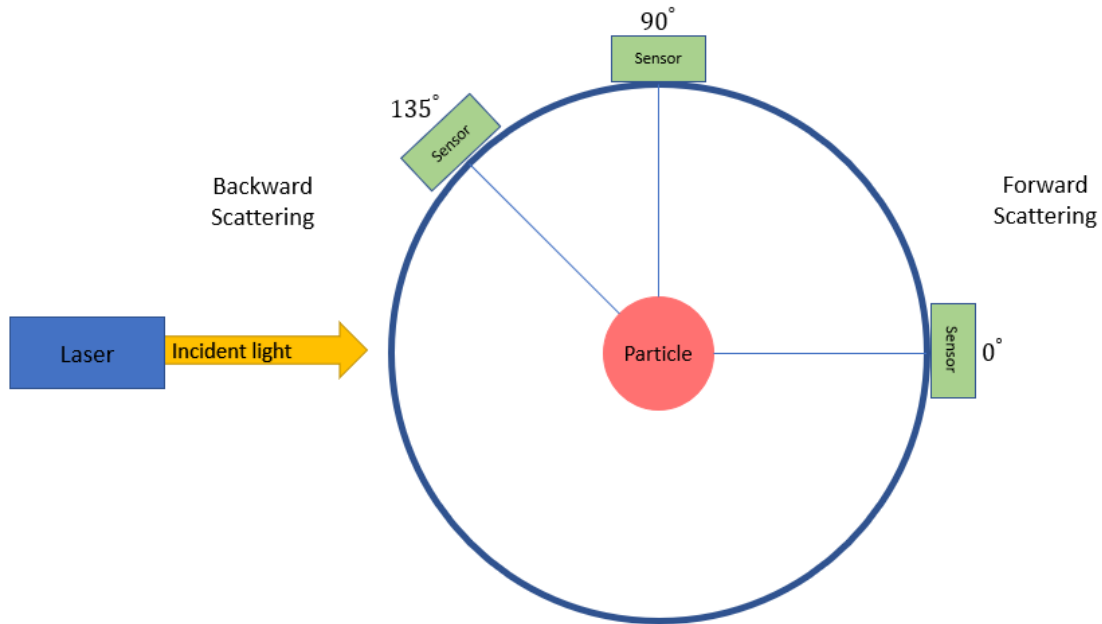
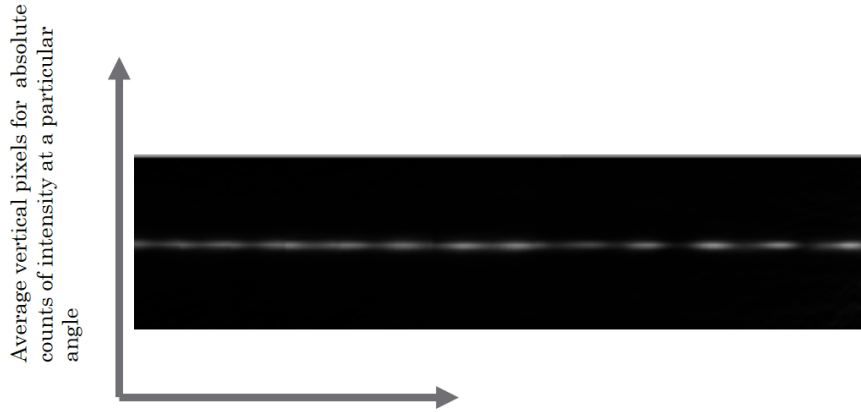


Figure 5: Angle references top down rig view (not to scale)

Once the average intensities are obtained, each photos intensity data is truncated at the point at which overlap occurs, and the entire scattering distribution is pieced together. This data is then normalized against itself in a trapezoidal transformation which takes the integral across all the data. This normalization is then plotted across the entire distribution to obtain the entire scattering profile. The units for the dependent variable of scattering intensity is thus in arbitrary units. Since it was normalized against itself, there are no dimensions. The same sort of normalization was applied to the expected value data. In this way a comparison between the relative intensity is made between the experimental and expected value distributions, and have a good basis for comparison regardless of laser power used.

Once the average intensity information is taken, it is then plotted in a log base along the angular traverse. A pictorial diagram of this data reduction process is seen in figure 6.



We normalize the intensity of each pixel against the average of the entire picture, and plot each normalized intensity across the horizontal.

Figure 6: Raw data scattering profile snapshot 77um sample

## Results

The scattering distribution for the 77um sample is displayed in figure 7, and the scattering distribution for the 67um sample is displayed in figure 8.

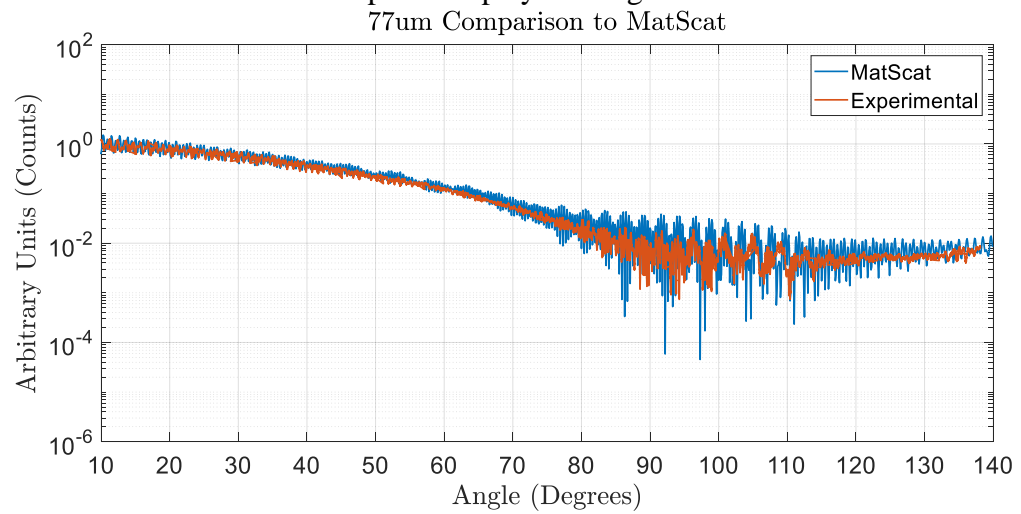


Figure 7: 77um experimental and expected values

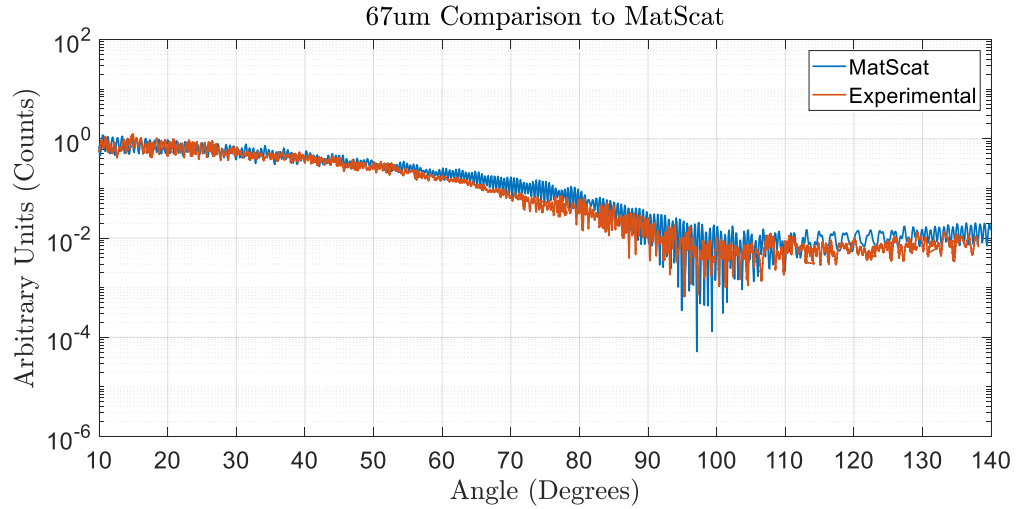


Figure 8: 67um experimental and expected values

Both graphs above contain the experimental data and the expected value data called MatScat.

Experimental error between the expected value MatScat and Experimental data was also calculated and is shown in figure 9 and figure 10 for the 77um and the 67um sample respectively.

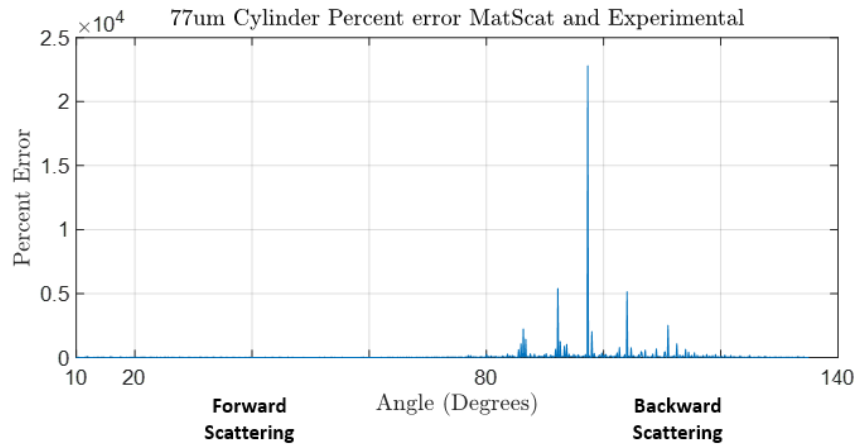


Figure 9: 77um percent error

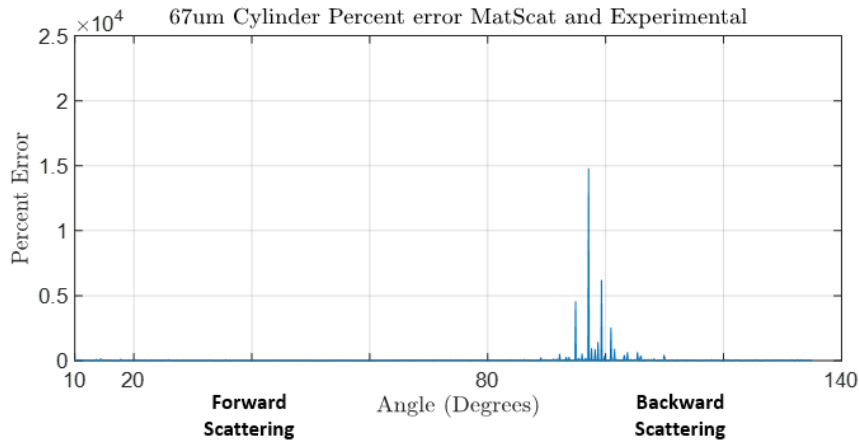


Figure 10: 67um percent error

One can see that the error is very high in the backward scattering regime. For this reason, the experimental error calculated was plotted on a logarithmic scale to show the relative error at different scattering locations. The reasons for this high numerical error are explained in the discussion with future implications for quantifying the fit of the scattering distributions.

#### Experimental limitations

To confirm the diameter of each sample, microscopic sizing was performed on the cylindrical samples. Each sample was examined using microscopy along its cylindrical length, and around its radial axis. It was observed that there were defects on both samples. One can see images of these defects for a 67um sample in figure 11.

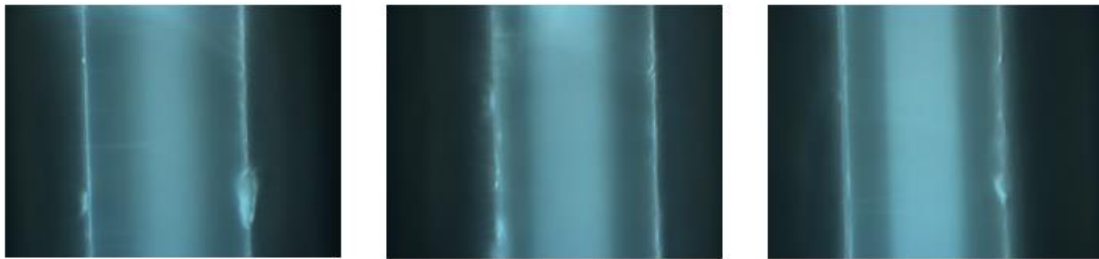


Figure 11: Microscope photos of 67um sample along axis

As our investigation claims that shape greatly determines scattering profile, these defects are a source of error. The defects will scatter light and influence the scattering profile and contribute to the resultant error.

This experiment is also making a small angle assumption in taking a 2-dimensional photo of a radial scattering profile. We are taking a picture of a curved distribution. Intensity at the far edges of the sensor are at a different distance to the particle than the midplane of the picture. With the small angle assumptions such effects are assumed to be negligible, but it is also a potential source of error and needs to be explored fully in next iterations of this experimental set up.

The arm holding the camera started to bend down backwards during testing as well. This bending could have potentially effected the field of view of the camera. Having the plane of the camera centered is critical for correct angle resolution. This was observed toward the end of testing.

## Discussion

It was found that the scattering profile collected experimentally patterned the scattering profile collected from MatScat, the expected value. The degree that the profiles matched was compared by calculating experimental error between the MatScat and experimental data point by point, as well as by visually observing scattering shape. The experimental errors are plotted in figures 9 and 10.

When one looks at the relative error across the angles of scattering, the forward scattering data is much closer to expected values as compared to the backward scattering data. This makes sense since the profile in the forward scattering regime is much more uniform. The intensity changes in the backward scattering regime are much more sensitive to the angular placement. One source of this error is hypothesized to stem from the way in which the data was reduced. The experimental data was truncated at portions of subsequent overlap explained earlier. This truncation was done as it was a means to adequately piece together the entire scattering profile in a uniform way. While data was collected the scattering profile would shift left and right by a few hundred pixels corresponding to about a third of a degree. This phenomena during data collection is explained by the knowledge of the wave-particle duality of light. When a light beam is scattered around an aperture such as a cylinder, light has characteristics of both waves and particles. That is, a scattering distribution is created that is a probabilistic measure of where the light “particles” will end up hitting that creates the scattering profile that comes from the constructive and destructive interference of the pattern. When the exposure rate is high, there is more averaging of the probabilistic placement of the light wave. At smaller exposure rates, there is less averaging. This explains the very high error percentages especially at the backward scattering area.

Looking at traditional methods of data reduction for these intensities is a good first step. However, this quantitative data does not adequately describe the degree of agreement between scattering structures that a visual method would, as this research demonstrates. The field of chemistry uses spectroscopy to determine chemical composition in a visual manner. Compounds are exposed to light radiation, and the resulting “pattern” it creates from the exposure is compared to a library of known compound patterns and matched to the correct compound. In a similar manner, a powerful future step for this research would be to consider similar types of data reduction and comparison methods. To determine an approximal scattering distribution shape for definitive shape, size parameters, and indices of refraction, and matching the shapes with a pre-existing library of shape profiles would give us a deterministic way of figuring out the characteristics of non-spherical particles. As of now, the quantitative measures used in this experiment are amazing first step into this new measurement field.

## Conclusion

During the course of this Masters of Engineering in Mechanical Engineering, a technique to measure the scattering of light around micro sized cylindrical particles was developed. The resulting scattering measurements were then compared to expected values of scattering generated from a solution of Maxwell's equations called MatScat.

The use of fishing line was an ingenious idea for an infinite cylinder sample and was used to collect data. This material was inexpensive and was in abundant supply, therefore it was perfect for designing and troubleshooting a novel experimental setup to engage in the first generation of scattering measurements. In the future one should use a cylindrical sample with higher tolerances for defects. It was seen upon microscopy of the samples they contained defects like those shown in figure 11. Also, a single cylinder was measured in this set-up. This was done so that investigation into how to measure a single cylinder in relation to its scattering shape could be determined. Another cylinder adds another layer of scattering complexity since constructive and destructive interference between the cylinders occurs in this situation. Future work for this project entails increasing the number of cylindrical particles to be measured.

This set-up succeeded in taking measurements of scattering around cylindrical particles between 10-140 degrees. These measurements were then compared to a computational tool which calculated the scattering distribution of an infinite cylindrical particle at various diameters, indices of refraction, and incident wavelength of light. It was found that the entire profile shape for the measured scattering values followed the same trend in shape and distribution. A definitive way to calculate error between the measured and expected scattering distribution needs to be determined. The probabilistic nature of individual light photons appearing at any exact location is in direct contrast with the spatial resolution we are trying to measure. A way to compare the global scattering profiles with each other would be a much more robust measure. Notwithstanding the uncertainty in the error, the scattering profiles behaved as expected looking at the relative global errors. The values for the forward scattering regime have less error than in the backward scattering regime. This is because the effects of the particle nature of light makes less of an impact in the forward scattering regime measurements. In the backward scattering regime, the intensity of the light varies as much as three orders of magnitude between 80-100 degrees alone. The intensity of light within the forward scattering regime has a range of one order of magnitude between 10-80 degrees. The backward scattering is thus more sensitive to measurements for this reason. Future work would take care to incorporate this sensitivity to measurements along with the particle nature of light so that the relative error for the backward scattering area is lessened. Even though the measured error was high, the scattering profile of the measured values visually contained similar structures to their expected values. Future iterations of this experiment should take heed to developing a manner in reducing the pictorial data in a visual rather than a quantitative way.

The scattering profiles of cylindrical particles are unique to the geometric and material properties of the particle. This direct relationship can be quantified by the scattering intensity. This study devised a method to measure the scattering profile around cylindrical particles at different size parameters and indices of refractions. These measurements were then compared to computational expected values. Quantifying a way in which to determine the uniqueness of these profiles will be the guiding force to being



able to size particles of cylindrical shapes based off the scattering profile it creates alone. Future studies will go on to analyze more complex geometries to resolve completely non-spherical particles.

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## Appendix I : Data Reduction Code

```
%Get Photos
fileLocation='C:\Users\file \';
%Get photo information matrix
matrixfileLocation = 'C:\Users\matrix\ ';
matrix=xlsread(strcat(matrixfileLocation,'TestConditions.xlsx'));
%dir(fileLocation);
fileList=1:70;
filePost='.tif';
filePostlong=''.tif';
%Angular change in degrees:
dA=2;
%sensor angular range in degrees (adjust to get the curves to overlap):
aRange=3.175;
maxfwdstangle = (145-127)/80;
aRef=maxfwdstangle;
%Number of pixels to average vertically
pixAve=500;
b='rgbkc';
% prog = waitbar(0,'Calculating intensities...');

for i=1:length(fileList)
    fid=0;
    while(fid==0)
        fid=fopen(strcat(fileLocation,num2str(fileList(i)),filePost),'r');
    end
    fclose(fid);
    vars=double((imread(strcat(fileLocation,num2str(fileList(i)),filePost))));
    currentIm=vars(:, :, 1);

    [maxy,ind]=max(currentIm(:));

    [indy,jindy]=ind2sub(size(currentIm),ind);
    correction = matrix(i,4)/matrix(i,5);

    intens=correction*fliplr(mean(currentIm(max(1,indy-
pixAve/2):min(2160,indy+pixAve/2),474:2086),1));
    if i==1
        buildSig=intens;
        angle=linspace(0,dA,1613)+dA*(i-1)+maxfwdstangle+.79375;
    else
        buildSig=[buildSig intens];
        angle=[angle linspace(0,dA,1613)+dA*(i-1)+maxfwdstangle+.79375];
    end
end
```

```

    sig(i)=mean(currentIm(indy,:));
end

[angg,M11D67,M12D67]=test_calccylONLYYGB(120e-6,1.42,0.00,532e-9); %(particle
size, index of refraction, --,laser wavelength)
figure
semilogy(angle,abs(buildSig)/((trapz(angle*pi/180,abs(buildSig)))^2),'g-')
hold on
semilogy(angg(10.0:1550),(M11D67(10.0:1550)-
M12D67(10.0:1550))/(trapz(angg(10.0:1550)*pi/180,M11D67(10.0:1550)-
M12D67(10.0:1550))^2),'r-');

xlabel('arbitrary angle, degrees')
ylabel('scattering intensity, counts')
grid on

```