Problems in *Electromagnetics, Vol. 1*
Version 1.2

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Change History

- Version 1.1: First publicly-available version.
- Version 1.2: Added Problems 3.8-4; 3.10-2; 3.12-4; 3.14-3; 3.15-2,3; 3.16-3; 3.19-4; 3.23-3; 5.3-2; 5.4-2; 5.5-5; 5.12-3; 5.15-3; 5.23-3; 6.5-2; 7.8-1; 8.4-2; 8.7-2; 9.4-2. Clarified 5.15-3, 7.3-1.

Contents

Chapter 2

Electric and Magnetic Fields

[m0002]

[1]

2.2-1 1.5 V is applied across the terminals of a thin parallel plate capacitor. The separation between the plates of the capacitor is 30 μm. Estimate the electric field intensity deep inside the capacitor.

[m0011]

[1]

2.4-1 12 V is applied across the terminals of a thin parallel plate capacitor. The plates are separated by a dielectric layer 90 μm thick, having relative permittivity 6. Estimate the electric flux density deep inside the capacitor.

Chapter 3

Transmission Lines

[m0027]

[1]

3.6-1 RG-59 coaxial transmission line can be modeled as having the following equivalent circuit parameters: $R' \cong 0.164\ \Omega$/m, $G' \cong 200\ \mu$S/m, $C' \cong 67.7$ pF/m, and $L' \cong 370$ nH/m. Let us consider the attenuation in voltage over one meter of RG-59. Assume the source frequency is 100 MHz.
(a) If the cable is perfectly impedance-matched at both ends, and the voltage magnitude is 1 V at the source end, then what is the voltage magnitude at the other end?
(b) Calculate the phase introduced by the cable. In other words, if the voltage phase is 0° at the source end, then what is the voltage phase at the other end?
(c) Even though you may not yet have formally encountered radio waves, you already know how to compute the answers to parts (a) and (b) for a radio wave propagating in free space. Compare your answers to parts (a) and (b) for RG-59 to those for a radio wave at the same frequency that propagates the same distance in free space.

[2]

3.6-2 It is claimed that

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

is a solution to the TEM transmission line wave equation

$$\frac{\partial^2}{\partial z^2}\widetilde{V}(z) - \gamma^2 \widetilde{V}(z) = 0$$

where V_0^+, V_0^-, and γ are complex-valued constants. Prove it.

[m0052]

[1]

3.7-1 True or false: The real part of the characteristic impedance of a transmission line must be positive. Justify your answer using a mathematical argument and, separately, a simple physical argument.

[m0080]

[1]

3.8-1 The current on a transmission line is

$$i(z,t) = (2\text{ A})\sin((3\text{ rad/s})t + (4\text{ rad/m})z + 5\text{ rad})$$

What is this current in phasor representation?

[2]

3.8-2 A voltage wave exists on a transmission line. This wave is expressed as the phasor

$$\widetilde{V}(x) = V_0 e^{+j\beta x}$$

The phase of V_0 is $\pi/3$ radians. What is this voltage wave as a function of both position and time, and in what direction is the wave traveling?

[3]

3.8-3 A voltage wave $V_0 e^{-j\beta\phi}$ travels along a transmission line. The voltage is maximum at

$\phi = \lambda/4$ and $t = 0$. The wavelength λ is 10 cm. Write an expression for this voltage wave as a function of both position and time. Be sure to indicate numerical values in your solution whereever possible.

[4]

3.8-4 An ideal sinusoidal voltage source is attached to a transmission line of infinite length. The point of attachment is at $z = 0$ and the transmission line lies along positive z axis. The magnitude of the voltage source is 2 mV and the potential at the input of the transmission line is -2 mV at time $t = 0$. The magnitude of the voltage wave is 1 mV at $z = 10$ m. The frequency of the voltage source is 15 MHz. The wavelength in the line is 0.4 times the wavelength in free space.
(a) What is the (physical, real-valued) voltage at all points along the line?
(b) What is the answer to (a) in phasor form?

[m0083]

[1]

3.9-1 A transmission line is known to have a characteristic impedance of 72 Ω and inductance per unit length equal to 0.5 μH/m. In a particular application, the frequency is 80 MHz, and the line may be considered low-loss at this frequency. Determine phase velocity and phase propagation constant in the line.

[m0143]

[1]

3.10-1 An air-filled coaxial line exhibits a characteristic impedance of 90 Ω. It is desired to modify the cable to reduce the characteristic impedance to 62 Ω. Describe two different ways to accomplish this. One way should involve *geometry*, and the other way should involve *material*. You may assume the inner and outer conductors exhibit negligible resistance. Please be specific; give numbers.

[2]

3.10-2 A certain coaxial cable has a characteristic impedance equal to 75 Ω. The relative permittivity of the spacer is equal to 2.25. A design change is being considered to increase the characteristic impedance. However it is not possible to change the geometry of the cable; only the spacer material may be changed. What is the maximum value that characteristic impedance can achieve, assuming "low-loss" conditions?

[m0084]

[1]

3.12-1 A CW transmitter is connected to an antenna by a lossless coaxial cable having characteristic impedance 75 Ω. ("CW" means "continuous wave", which is simply another term for "sinusoidal".) The coaxial cable is perfectly-matched to the transmitter. The input impedance of the antenna is 500 Ω, so there is a reflection from the antenna. If the peak voltage from the output of the transmitter is 30 V, what is the peak voltage of the reflected wave at the output of the transmitter?

[2]

3.12-2 A voltage wave having magnitude 7 mV and phase 180° is traveling along a lossless transmission line having characteristic impedance 60 Ω. The line is terminated in a load having input impedance 20 Ω. What is the magnitude and phase of the reflected voltage wave?

[3]

3.12-3 A voltage wave having magnitude 3 V and phase 170° exists on a 140 Ω transmission line. The wave reaches a terminating impedance of 33 Ω and is reflected. What is the magnitude and phase of the reflected voltage wave?

[4]

3.12-4 A particular system measures the impedance of a device by measuring the voltage reflection coefficient at the device when it is accessed through a coaxial cable. Derive the measurement equation; i.e., the formula that gives the device impedance (on the left side of the equation) in terms of the known and measured quantities (on the right side of the equation). Check your equation using three special cases for which the result is known independently of the derived formula.

[m0086]

[1]

3.13-1 Consider a short-circuited transmission line. The characteristic impedance of this line is 30 Ω. Moving away from the short-circuit along the line, a voltage maximum of 3 mV is identified at a distance of 8 cm from the short circuit. (a) What is the current at this point? (b) What is the distance between the short circuit and the next voltage maximum?

[m0081]

[1]

3.14-1 A datasheet for a particular amplifier indicates that the VSWR at the input of the amplifier is ≤ 1.2. The amplifier is designed to interface to a source impedance of 50 Ω. Let us assume that the imaginary part of the amplifier's input impedance is negligible. What is the expected range of input impedances of this amplifer?

[2]

3.14-2 A transmission line exhibiting characteristic impedance 72 Ω is terminated into a 60 Ω load. What are the voltage reflection coefficient and the standing wave ratio on the line?

[3]

3.14-3 A transmission line exhibiting characteristic impedance 50 Ω is terminated into a load having impedance $20-j35$ Ω. What are the voltage reflection coefficient and the standing wave ratio on the line?

[m0087]

[1]

3.15-1 A lossless transmission line having characteristic impedance 50 Ω is terminated into a load having input impedance $72+j42$ Ω. The line is determined to be 1.5λ long. What is the input impedance at the open (unterminated) end of the transmission line?

[2]

3.15-2 How important is it to account for the characteristics of a transmission line, really? Consider the following experiment. You wish to measure the resistance of an ideal resistor having resistance R_{DUT}. To do this, you connect the resistor across one end of a coaxial cable. The other end of the coaxial cable is connected to a device which perfectly measures impedance. The coaxial cable has characteristic impedance 50 Ω, and wavelength within the cable is 60% of the free space wavelength. Plot the real and imaginary components of the measured impedance using the relationship

$$Z = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

For this experiment, assume the actual value of R_{DUT} is 10 Ω and the length of the cable is 10 cm.
(a) Plot the result as a function of frequency from 1 MHz to 10 GHz. *Important*: The horizontal axis **must** be in *log* scale (in MATLAB/Octave, you can do this by using `semilogx()` in lieu of `plot()`). Make sure you sample appropriately!
(b) Roughly what is the maximum frequency at which the measured resistance is not significantly in error?
(c) Roughly what is the maximum frequency at which the measured reactance is not significantly in error?

[3]
3.15-3 A lossless coaxial cable having characteristic impedance 50 Ω is terminated by a load with impedance $25 + j25$ Ω.
(a) What is the voltage reflection coefficient at the load?
(b) Plot the input impedance for transmission line lengths from 0 to 0.45λ. Please plot as a single curve in the complex plane, with the horizontal axis as the real component and the vertical axis as the imaginary component. Indicate at each end of the curve the associated lengths.
(c) For what lengths is the input impedance completely real-valued, and what is the impedance at these lengths?

[m0088]

[1]
3.16-1 A 13 cm section of coaxial cable having characteristic impedance 75 Ω is open-circuited at one end. At 900 MHz, what is the input impedance looking into the end opposite the open circuit termination? Assume velocity factor $0.55c$.

[2]
3.16-2 A PCB trace having characteristic impedance 75 Ω is short-circuited at one end. At 1.5 GHz, an input reactance of 300 Ω looking into the end opposite the short-circuit termination is desired. What is the minimum length that accomplishes this? Assume phase velocity $0.6c$, and give your answers in meters or associated units.

[3]
3.16-3 Design a transmission line stub that can be used to replace an 83 pF capacitor in a circuit that is to operate at 5.8 GHz. Assume characteristic impedance 50 Ω with velocity factor 0.7. Indicate the shortest possible length (in meters or associated units) and load impedance.

[m0145]

[1]

3.17-1 You can make a bandpass filter using a transmission line stub attached in parallel to an existing transmission line. Design such a filter for 200 MHz center frequency using RG-58 coaxial cable ($Z_0 = 50\ \Omega$, $v_p = 0.67c$). Assume the existing transmission line is also RG-58.
(a) Specify minimum stub length and whether the stub is open- or short-circuited.
(b) Sketch the connection details in a manner that would be understandable to a layperson.

[2]

3.17-2 An open-circuited transmission line stub is to be used to replace an inductor at 3 GHz. The phase velocity of the transmission line is $0.7c$. What is the smallest contiguous range of transmission line length that might be used? Express your answers in units of distance; e.g. meters.

[3]

3.17-3 A notch filter has zero response at one frequency (i.e., it rejects that frequency), and greater-than-zero response at adjacent frequencies. You can make a notch filter using an open- or short-circuited transmission line stub attached in parallel to an existing transmission line. Design such a filter for 1.3 GHz center frequency using transmission lines with with characteristic impedance 30 Ω and phase velocity $v_p = 0.6c$. Specify the minimum stub length and whether the stub is open- or short-circuited.

[m0091]

[1]

3.19-1 Design a microstrip transmission line that connects the output of an RF source to the input of an RF load. The transmission line can be a straight line; however, the devices are separated by 5 cm, so the transmission line must be exactly this long. The connection between the devices must be reflectionless at 1.5 GHz; please accomplish this using a quarter-wave match *adjacent to the source*. The source output impedance is 50 Ω. The load input impedance is 300 Ω. Both devices are on the same FR4 substrate with $\epsilon_r = 4.5$ and thickness 1.6 mm. Provide your answer in the form of a sketch of the transmission line, and show your work.

[2]

3.19-2 A transmission line is terminated into a 200 Ω load. The line is a quarter-wavelength long and exhibits a characteristic impedance of 100 Ω. What is the input impedance of this line?

[3]

3.19-3 Figure 3.1 shows a bandpass filter implemented using microstrip on a printed circuit board. The filter consists of 6 stubs connected to the "main line" that runs between the connectors on either end of the board. The characteristic impedance of the main line is equal to the input and output impedance of the filter at its center frequency. The length of each stub is 3.38 mm, and each stub ends in a via (a plated through-hole) that connects the end of the stub to the ground plane on the other side of the board. The phase velocity in each stub is $0.6c$. Estimate the center frequency of this bandpass filter.

[3]

3.19-4 Figure 3.2 shows a bandpass filter constructed from microstrip transmission lines having characteristic impedance 50 Ω and velocity factor $0.67c$. Specifically, the filter consists of a quarter-wavelength stub at the input, a quarter-wavelength line, and a quarter-wavelength

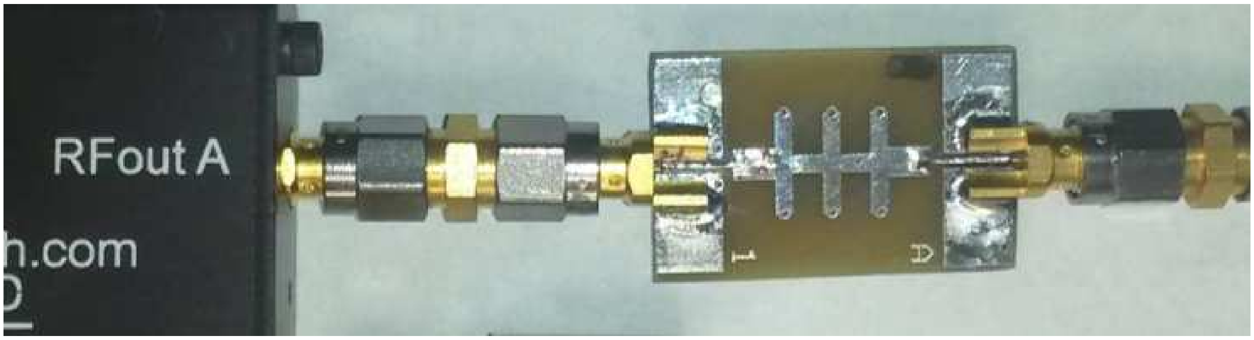


Figure 3.1: A 6-stub bandpass filter.

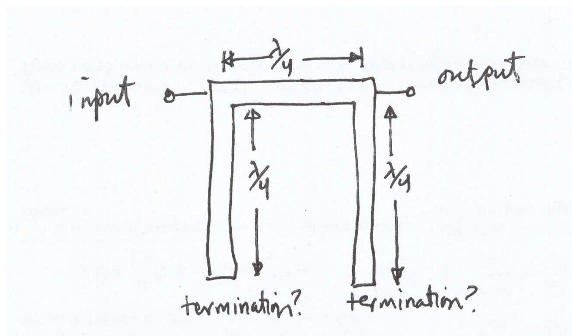


Figure 3.2: Scheme for a bandpass filter consisting of 2 quarter-wave stubs.

stub at the output. The center frequency of the filter is specified to be 2.4 GHz and the input and output impedances are specified to be 50 Ω.
(a) Design the filter. Your answer should consist of a diagram indicating the physical length (i.e., in meter-like units) of each of the three sections, as well as the termination for each of the stubs.
(b) Explain how to calculate the frequency response of your design. In this case, "frequency response" is defined as power delivered to a 50 Ω load at the output, relative to power incident on the input. Be specific; describe an algorithm that can be used to determine the response at each frequency of interest, and which can then be iterated over frequency to obtain frequency response.
(c) Plot the frequency response your design. Your answer should consist of a plot in dB for frequencies from 1.0 GHz to 3.8 GHz. Include your source code.

[m0090]

[1]

3.20-1 A load is attached to a coaxial cable. The power arriving from the coaxial cable is 5 W. The power delivered to the load is 4.6 W. What is the standing wave ratio on the coaxial cable?

[1]

3.20-2 The voltage reflection coefficient at the interface between a transmission line and a load is found to be $0.3 + j0.4$. Time-average power of 3 W is incident on the load from the transmission line. What power is delivered to the load?

[m0094]

[1]

3.23-1 Design a single-stub match that connects a dipole antenna to RG-58 coaxial cable (nominal $Z_0 = 50\ \Omega$, $v_p = 0.67c$) at 220 MHz. The antenna impedance at 220 MHz is $73 + j42\ \Omega$. The stub must also be RG-58, and the length of this stub should be minimized. Be sure to indicate (1) the distance from antenna terminals to stub, (2) the stub length, and (3) whether the stub is open- or short-circuited.

[2]

3.23-2 In the process of designing a single-stub matching structure, one first selects a length of line to attach to the load, resulting in a load-terminated transmission line. In a particular design, the input admittance of this load-terminated transmission line is $0.0128 - j0.0040\ \Omega^{-1}$. To complete the design, a transmission line stub is added in parallel. In this particular design, the result is intended to be a real-valued impedance. (a) What is this impedance? (b) What is the input impedance of the stub?

[3]

3.23-3 Design the smallest possible single-stub match that transforms a $35 - j10\ \Omega$ load impedance to $50\ \Omega$. Use transmission lines with characteristic impedance $100\ \Omega$ throughout. You may assume negligible loss.

(a) Your answer should indicate the length of the primary line in wavelengths, length of the stub in wavelengths, and termination (short or open) of the stub.

(b) If the design is implemented at 1.5 GHz in microstrip having velocity factor 65%, then what are the lengths in SI base units?

(c) If the design is implemented at 1.5 GHz, what is the power delivered to the load as a fraction of the power incident from a transmission line with characteristic impedance $50\ \Omega$? Plot your answer for frequencies from 1.0 to 2.0 GHz. Be careful: The physical dimensions of the structure do not change as you sweep over frequency!

Chapter 5

Electrostatics

[m0102]

[1]

5.1-1 A point charge at the origin has a charge -24 nC. The medium surrounding the charge has relative permittivity equal to 2. What is the electric field intensity at ($x = 1$ m, $y = 2$ m, $z = 3$ m)? Please give your answer in Cartesian coordinates.

[m0103]

[1]

5.2-1 Point charges equal to 3 nC are located at $(x, y, z) = (0, 0, -0.5 \text{ m})$ and $(0, 0, +0.5 \text{ m})$ in free space. (a) Find the electric field intensity at $(+1.5 \text{ m}, 0, 0)$. (b) What single point charge, located at $(0, 0, 0)$, would result in the same electric field intensity?

[m0100]

[1]

5.3-1 The charge density everywhere is Kr^{-2} where $K = 2$ C/m. What is the total charge inside a spherical shell centered at the origin with interior radius 1 m and exterior radius 2 m?

[2]

5.3-2 The density of charge in a spherical shell with an inner radius of 1 m and an outer radius of 2 m is $(1.3 \text{ C}\cdot\text{m}^{-1})/(r^2 \sin\theta)$ in the region $\pi/4 < \theta < 3\pi/4$, and zero elsewhere. What is the total quantity of charge?

[m0104]

[1]

5.4-1 Three infinite flat sheets of charge exist in a medium with permittivity equal to twice that of free space. The first sheet lies in the $x = 0$ plane and has constant charge density $+4$ nC/m^2. The second sheet lies in the $y = 0$ plane and has constant charge density $+16$ nC/m^2. The third sheet lies in the $z = 0$ plane and has constant charge density $+64$ nC/m^2. What is the electric field intensity in the region $\{x > 0, y > 0, z > 0\}$? Please do not leave your answer in terms of physical constants, and instead reduce to values as much as possible.

[2]

5.4-2 An infinite line of charge having uniform density $+8$ mC/m exists along the z-axis. In addition, a infinite sheet of charge having uniform charge density $+12$ mC/m^2 lies in the $z = 0$ plane. If the permittivity of the medium is twice that of free space, then what is the electric field intensity in the region $z > 0$? Express your answer in *cylindrical* coordinates.

[m0014]

[1]

5.5-1 Show that Coulomb's Law is a solution to the integral equation that expresses Gauss' Law. Here's Coulomb's Law:

$$\mathbf{F}_{12} = \hat{\mathbf{R}}_{12} \frac{Q_1 Q_2}{4\pi\epsilon R_{12}^2}$$

where $\mathbf{F}_{12}$ is the force exerted on a point charge Q_2 by a charge Q_2. Here's Gauss' Law:

$$\oint_{\mathcal{S}} \mathbf{D} \cdot d\mathbf{s} = Q_{encl}$$

where Q_{encl} is the total charge enclosed by the surface $\mathcal{S}$. By solving this problem, you will be demonstrating that Coulomb's Law is redundant information given Gauss' Law. Here are some tips: (1) Remember that electric field intensity is – by definition – force divided by charge. (2) You'll find this easiest in spherical coordinates where charge is placed at the origin.

[4]

5.5-2 An infinitely long cylindrical shell of charge is centered on the z-axis and extends between $\rho = 1$ m and $\rho = 3$ m. The volume charge density within this shell is uniform and equal to ρ_v. Use the integral form of Gauss' Law to find the electric field intensity everywhere (Note: There are three regions to consider here). Leave the permitivitty of the medium as an independent variable.

[2]

5.5-3 A spherical shell of charge is centered at the origin and extends from $r = 2$ m to $r = 4$ m. The volume charge density ρ_v within this shell is uniform. Use the integral form of Gauss' Law to find the electric field intensity everywhere (Note: There are three regions to consider here). Leave ρ_v and the permittivity of the medium as independent variables.

[3]

5.5-4 An electric field $\mathbf{E}(x, y, z) = \hat{\mathbf{x}}Axz^2 - \hat{\mathbf{y}}Byz + \hat{\mathbf{z}}Cx$ exists in free space.
(a) If distances are in units of m, and if the electric field intensity is in units of V/m, what are the units of A, B, and C?
(b) Using the integral form of Gauss' Law, calculate the total charge inside the flat-walled region defined by -1 m $\leq x \leq +1$ m, $0 \leq y \leq +1$ m, and -1 m $\leq z \leq 0$.

[5]

5.5-5 A distribution of electric charge in the shape of a cube 1 cm $\times$ 1 cm $\times$ 1 cm gives rise to the electric field intensity $\hat{\mathbf{r}}(3 \text{ V} \cdot \text{m})/r^2$ in free space. The charge cube is centered at the origin. What is the total charge enclosed in the cube?

[m0149]

[1]

5.6-1 An infinite line of charge having uniform charge density -2.1 mC/m exists along the z-axis; i.e., along $x = y = 0$. The surrounding media is a plastic, which is well-characterized as a homogeneous medium having relative permittivity 2.5 times that of free space. What is the electric flux density everywhere? Please reduce your answer to a numerical value as much as possible.

[m0045]

[1]

5.7-1 The electric field intensity is $\hat{\mathbf{x}}(6 \text{ V/m}^2)x + \hat{\mathbf{y}}(2 \text{ V/m}^3)yz + \hat{\mathbf{z}}(1 \text{ V/m}^3)xy$ inside an ideal dielectric material having relative permittivity of 4.5. What is the electric charge density in the material?

[2]

5.7-2 For each of the electric fields given below, determine the electric charge density in the same region. Assume free space.
(a) $\mathbf{E}(\mathbf{r}) = \hat{\mathbf{x}}\,(2 \text{ V/m})\sin x \cos y - \hat{\mathbf{y}}\,(2 \text{ V/m})\cos x \sin y$.
(b) $\mathbf{E}(\mathbf{r}) = \hat{\mathbf{x}}\,(3 \text{ V/m})\cos xy + \hat{\mathbf{y}}\,(3 \text{ V/m})\sin xy$.

[3]

5.7-3 An electric field $\mathbf{E}(x,y,z) = \hat{\mathbf{x}}Axz^2 - \hat{\mathbf{y}}Byz + \hat{\mathbf{z}}Cx$ exists in free space.
(a) If distances are in units of m, and if the electric field intensity is in units of V/m, what are the units of A, B, and C?
(b) Calculate the total charge inside the flat-walled region defined by $-1 \text{ m} \leq x \leq +1$ m, $0 \leq y \leq +1$ m, and $-1 \text{ m} \leq z \leq 0$. Do this using the differential form of Gauss' Law followed by an integration over the region of interest.

[m0061]

[1]

5.8-1 A -4 mC point charge is moved through a region of uniform electric field intensity equal to $3\hat{\mathbf{z}}$ V/m. The position of the charge at time t is $\hat{\mathbf{x}}2\cos \pi t + \hat{\mathbf{y}}2\sin \pi t + \hat{\mathbf{z}}4t$ meters, where t is in seconds. How much power does this require? *Hint:* Power is energy per unit time.

[m0064]

[2]

5.12-1 An infinite line of charge having line charge density ρ_l exists along the z-axis. Find the electric potential difference V_{21} between points at distances ρ_1 and ρ_2 along a radial extending from the line of charge. Leave ρ_l and the permittivity of the medium (ϵ) as independent variables. Note: You do not need to derive an expression for the electric field due to the line of charge; feel free to use an expression from a textbook for this.

[1]

5.12-2 Infinite lines of charge having the same uniform line charge density ρ_l exist along the x and y axes. Find the electric potential difference V_{21} at point 2, which is at $x = 1$ m and $y = 1$ m; relative to point 1, which is at $x = 2$ m and $y = 4$ m. Leave ρ_l and the permittivity of the medium (ϵ) as independent variables. Note: You do not need to derive an expression for the electric field due to the line of charge; feel free to use an expression from a textbook for this.

[3]

5.12-3 A $+3$ μC point charge is placed at the origin. The medium has a permittivity twice that of free space. A $+2$ μC point charge is moved in a straight line from $(x,y,z) = (3,-4,0)$ m to $(x,y,z) = (0,0,1)$ m. What is the electrical potential difference at the end point relative to the start point?

[m0063]

[1]

5.14-1 The scalar electric potential $V(\mathbf{r}) = V_0 r^2 \cos\theta$ where $V_0 = 5$ V/m^2. Find the (a) potential, (b) electric field intensity, and (c) electric charge density at $\mathbf{r}_0 \triangleq \hat{\mathbf{x}}x_0 + \hat{\mathbf{y}}y_0 + \hat{\mathbf{z}}z_0$, where $x_0 = 2$ cm, $y_0 = 3$ cm, and $z_0 = 4$ cm. Assume free space, and reduce all values in final answers to numbers as much as possible.

[2]
5.14-2 The electric scalar potential is $\left(4\ \mathrm{V}\cdot\mathrm{m}^{1/2}\right)/\sqrt{r}$ (spherical coordinates). What is the electric field intensity?

[m0067]

[1]
5.15-1 An axial semiconductor device is arranged along the x-axis, with a p-n junction at $x = 0$. (NOTE: This problem can be done using simple electromagnetic principles. You do not need any additional, special knowledge about semiconductor physics or devices.) The volume density of free charge, $\rho_v(x)$, is equal to zero for $x < -b$, equal to a constant value of $-a$ for $-b \leq x < 0$, equal to a constant value $+a$ for $0 \leq x \leq +b$, and equal to zero for $x > +b$, where a and b are a positive real-valued constants. Let the electric potential at $x = +b$ relative to the potential at $x = -b$ be V_d.
(a) Use Poisson's Equation to develop an expression for the electric potential as a function of x relative to the potential at $x = -b$.
(b) Obtain an expression for a in terms of V_d.
(c) The relative permittivity of silicon is roughly 12. Assuming $b = 100\ \mu$m and $V_d = 0.4$ V, what is magnitude of the volume charge density in the vicinity of the p-n junction?

[2]
5.15-2 Two perfectly-conducting concentric spherical shells are centered on the origin. The radii of the two shells are 1 m and 3 m. A battery is applied to the two shells such that the inner shell is at a potential of 100 V and the outer shell is at a potential of 20 V. The region between the two shells is free space. Use Laplace's Equation to find the potential in the region between the two shells.

[3]
5.15-3 Consider a perfectly-conducting sphere of radius 2 m, centered at the origin, and surrounded by free space. If the potential on the surface of the sphere is 20 V, what is the electrical potential field $V(\mathbf{r})$ for $r > 2$ m? Solve this problem using Laplace's Equation.

[m0068]

[1]
5.16-1 A coaxial cable consists of two perfectly-conducting concentric cylindrical conducting surfaces that are centered on the z-axis. ρ is the distance from the common axis. The radii of the two conducting surfaces are 1 mm and 2 mm. A voltage is applied to the two surfaces such that the inner conductor is at a potential of 50 mV and the outer conductor is at a potential of 20 mV. The region between the two conductors is filled with a low-loss teflon material (assume negligible conductivity, relative permittivity 2.1). Use Laplace's Equation to find the potential in the region between the two conductors.

[m0021]

[1]
5.18-1 Continuing Problem 5.16-1: Find the charge density on the inner conductor.

[2]
5.18-2 A sphere of radius 2 m, and centered at the origin, contains a constant charge density 3 pC/m^3 throughout the sphere. The charge density is zero outside the sphere. The medium

has relative permittivity equal to 4.5.
(a) Using Poisson's equation, find the electrical potential at any distance greater than 2 m from the origin.
(b) Calculate the electrical potential at 3 m from the origin.

[3]

5.18-3 Continuing Problem 5.15-2: Find the charge density on the inner shell.

[m0112]

[1]

5.22-1 A parallel-plate capacitor has a capacitance of 20 pF. If 3 V is applied to this capacitor, what is the net charge in the capacitor? What is the charge on positively-charged plate?

[m0070]

[1]

5.23-1 One (among many) of the things that makes RF electronics design challenging is that SMT (surface mount technology; AKA "chip") resistors do not act like ideal resistances at RF frequencies. Here's an example: Consider the 0603 SMT thin-film resistor shown in Figure 5.1. The advertised nominal (DC) resistance is 200 Ω. The ceramic holding the resistor together has a relative permittivity of about 37. One of the things that causes a problem is the capacitance of the resulting structure; let's explore this. (a) Considering just resistance and capacitance, draw an equivalent circuit for this component. Based on just this much, do you expect the effective resistance (i.e., the real part of the impedance) to increase, decrease, or remain constant with increasing frequency? Why? (b) Estimate the value of the capacitor in this equivalent circuit. (c) Estimate the effective resistance of this chip resistor at 10 GHz.

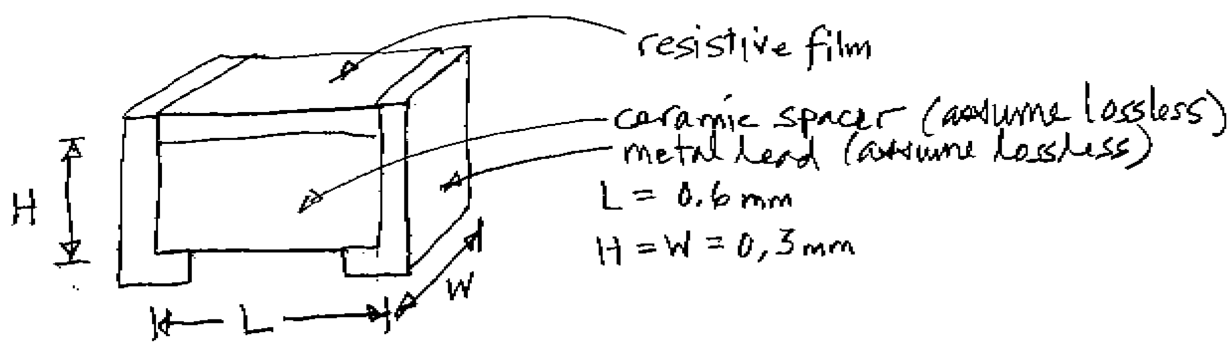


Figure 5.1: An SMT resistor.

[2]

5.23-2 A certain design requires the unintended capacitance contributed by a two-layer printed circuit board (PCB) to be less than 3 pF. The PCB has thickness 2 mm and relative permittivity 3. How much area may be in common between the top layer and the bottom layer?

[3]

5.23-3 Typically, the dielectric spacer material inside a thin parallel plate capacitor is assumed to have uniform permittivity throughout the material. However this is not the only possibility. Another possibility is that permittivity is minimum at one plate, increases linearly with distance from this plate, and is maximum at the other plate. Expressed mathematically, permittivity is $\epsilon(z) = \epsilon_0 (az + b)$, where z is the distance from a specified plate and a and b are non-negative real-valued constants.
(a) What are the units of a and b? (SI base units please.)
(b) If the dielectric material exhibited uniform permittivity, then the capacitance would be $\epsilon A/d$ where A is plate area and d is distance between the plates. Derive the corresponding expression for the capacitance of the new device.
(c) Show that your new expression for capacitance is dimensionally correct.
(d) Show that your new expression for capacitance reduces to the expected result when the permittivity is uniform.
(e) In the new device, the relative permittivity increases from 2 at one plate to 10 at the other plate. If the plate area is 4 cm^2 and the plate separation is 0.5 mm, then what is the capacitance of this device?

[m0113]

[1]
5.24-1 The capacitance of a particular coaxial cable is 30 pF/m. This cable uses polyethylene as the spacer material, having a relative permittivity of 2.25. What is the capacitance of this cable if the spacer material is changed from polyethylene to air?

[2]
5.24-2 A particular coaxial cable is comprised of inner and outer conductors having radii 1 mm and 3 mm respectively, separated by air. The potential at the outer conductor is +1.5 kV relative to the inner conductor. What is line charge density on the positively-charged conductor? What is the surface charge density on this conductor?

[m0114]

[1]
5.25-1 A particular capacitor is specified to be 4.7 mF with a working voltage of 16 V. How much energy can be safely stored in the capacitor?

[2]
5.25-2 A particular 3.5 pF capacitor is well-modeled as an ideal parallel plate capacitor having plate separation 0.1 mm and spacer relative permittivity equal to 10. Estimate the energy density inside this capacitor when 3 V is applied across the terminals.

Chapter 6

Steady Current and Conductivity

[m0071]

[1]

6.4-1 The design of a DC signal interconnect such as a power connector is a three-way tradeoff between resistance (which should be low), mechanical rigidity, and cost. Gold has low resistance but is soft and expensive. Steel is relatively rigid and relatively cheap but also has relatively high resistance. One solution is to use steel that is clad in (i.e., coated with) gold. To demonstrate this, consider a steel wire having circular cross-section with radius 0.1 mm and and conductivity 1.00×10^6 S/m. (a) What is the resistance per unit length of this wire? (b) Assume gold has conductivity 4.10×10^7 S/m. What is the minimum thickness of the gold clad in order to achieve resistance per unit length less than 10 Ω/m? Assume the gold clad is added to the existing wire, so the radius of the steel core remains 0.1 mm.

[2]

6.4-2 RG-59 is coaxial cable having inner conductor diameter $2a = 0.0584$ cm, inner conductor conductivity 2.28×10^7 S/m, and outer conductor mean diameter $2b = 0.371$ cm. Calculate as accurately as you can the equivalent circuit parameter R', the resistance per unit length. Assume that the radial thickness of the outer conductor is 5% of the mean radius, and that the outer conductor has the same conductivity as the inner conductor.

[3]

6.4-3 It is found that the DC voltage drop across a wire is twice the allowed amount. If the diameter of the wire is D_0, what should the diameter become in order to meet the voltage drop requirement?

[4]

6.4-4 A resistor is comprised of a homogeneous right circular cylinder of material having diameter 1 mm. What should the diameter become if the DC resistance is to be reduced to one-half the original value?

[m0105]

[1]

6.5-1 RG-59 coaxial cable is exposed to sea water. The sea water completely saturates the dielectric spacer. Estimate the conductance per unit length (G') of the cable under this condition. *Hint:* The relevant properties of sea water are addressed in an appendix.

[2]

6.5-2 A microstrip transmission line consists of a trace having width W which is distance h above a ground plane, separated by a dielectric medium which has conductivity σ_s. For this line, the trace and the ground plane may be assumed to be perfectly-conducting and $W \gg h$.
(a) A DC current I is applied to the input of the transmission line, with positive I defined to flow *into* the trace, and *from* the ground plane. The output is open-circuited. Derive an expression for the current density vector between the trace and the ground plane.
(b) Derive an expression for the potential difference V between the trace and the ground plane.
(c) Derive an expression for the conductance per unit length G'.
(d) Under the condition described in part (a), what is the input impedance of this transmission line?

[m0106]

[1]

6.6-1 A resistor consists of a cylinder of material having length 1.2 cm and radius 1.6 mm. The current density in the resistor is uniform, but the electric field in the resistor is

$$\hat{\mathbf{z}}\left(3\ \mathrm{V}\cdot\mathrm{m}^{-1/2}\right)/\sqrt{\rho}$$

where ρ is the distance from the axis of the cylinder. The resistor dissipates 5 W. What is the conductivity of the material comprising the resistor? *Hint:* You can't expect the material to be homogeneous in this problem!

Chapter 7

Magnetostatics

[m0115]

[1]

7.1-1 State Maxwell's Equations for electrostatics and magnetostatics *specifically*, in differential form, using only the electric field intensity and magnetic field intensity. Identify units of any quantities which are not electric or magnetic fields.

[m0047]

[1]

7.3-1 A measurement indicates that the magnetic field in a particular region has the form $\hat{\mathbf{x}} B_0 x^2$. Use Gauss' law for magnetism to show that the measurement must be flawed.

[m0119]

[1]

7.5-1 A thin wire lies along the z-axis and carries a current I, defined to flow in the $+z$ direction when I is positive. Nearby is a rectangular loop, 20 cm in the ρ dimension and 30 cm in the z direction, with the closest side being 3 cm from the wire. (Note: The wire lies in the plane of the loop, not perpendicular to it!) The magnetic flux through the loop is $3\ \mu\text{T}\cdot\text{m}^2$. What is I? Assume non-magnetic media (i.e., $\mu = \mu_0$). Note: You do not need to derive an expression for the magnetic field due to the line of current; feel free to use an expression from a textbook for this.

[2]

7.5-2 A thin wire in free space lies along the z-axis and carries a current of $I = 3$ A flowing in the $+z$ direction. In the same plane lies a rectangular loop, 1 cm in the ρ dimension and 10 cm in the z direction, with the closest side being 1 cm from the wire. What is the magnetic flux through the loop?

[3]

7.5-3 A wire having circular cross-section of radius $a = 5$ mm lies along the z-axis and has an internal magnetic field given by:

$$\mathbf{B} = \hat{\phi} \mu_0 J_0 \rho \quad \text{for } \rho < a \tag{7.1}$$

where $J_0 = 127.3\ \text{A/m}^2$. Use Ampere's Law to find the total current carried by the wire.

[4]

7.5-4 A line current flows along the y axis, in the $-\hat{\mathbf{y}}$ direction. In what direction does the magnetic field point at $(x, y, z) = (+1, +1, 0)$ m? Give your answer in Cartesian coordinates.

[5]

7.5-5 A straight current-bearing wire creates a magnetic field. At $x = +a$, the magnetic field is $+\hat{\mathbf{z}}$-directed. At $x = -a$, the magnetic field is $-\hat{\mathbf{z}}$-directed, and has the same magnitude. This is true for any value of a, where a is a positive constant. Where is the wire, and in what direction is the current flowing?

[m0120]

[1]

7.6-1 A cardboard tube is 10 cm long and 5 mm in diameter, with air inside ("air core").

100 windings of insulated wire are wound onto the tube, and connected to a 2 A current source. Also, 300 windings of insulated wire wound onto the *same* tube but in the *opposite* direction, and connected to a 4 A current source. The two sets of windings are overlapping, but they do not short out because they are insulated. What is the magnitude of the magnetic flux density in the coil?

[m0049]

[1]

7.7-1 I wish to increase the magnetic field in a coil by a factor of 2. The only parameter that can be changed is the conductivity of the wire. Precisely how should the conductivity be changed?

[m0121]

[1]

7.8-1 A microstrip transmission line is driven at $z = 0$ by an ideal DC voltage source and is terminated at $z = l$ by a resistor R. The microstrip consists of a perfectly-conducting trace at $y = h$ separated from a perfectly-conducting ground plane at $y = 0$ by a lossless non-magnetic dielectric material. The width of the trace is W, extending from $x = -W/2$ to $x = +W/2$. The voltage V of the voltage source is measured at the trace, with respect to the ground plane. We are interested in a particular class of such transmission lines for which $W \gg h$. Also, $l \gg W$.
(a) Use a simple "rule of thumb" to determine the reference direction of the magnetic field vector deep inside the transmission line.
(b) We are interested in estimating the magnetic field deep inside the transmission line using the integral form of Ampere's law. Determine the simplest choice of path $\mathcal{C}$ for this task. Show your answer as an unambiguous figure, with $\mathcal{C}$ precisely identified, and fully justify your answer.
(c) Following up part (b), obtain an expression for the magnetic field intensity deep inside the transmission line in terms of the variables identified in the problem statement.
(d) Determine the associated expression for the magnetic flux density deep inside the transmission line.
(e) Show that your answers to parts (c) and (d) are consistent with the differential form of Gauss' law for magnetism and the differential form of Ampere's law.
(f) Calculate the magnetic flux density vector deep inside the transmission line when $W =$ 6 mm, $h = 1.6$ mm, $V = +5$ mV, and $R = 50\ \Omega$.

[m0123]

[1]

7.12-1 Consider an inductor constructed from linear and time-invariant materials, and which has an inductance of 1 H. The magnetic field within the inductor is caused to double by changing the current. What does the inductance become?

[m0124]

[1]

7.13-1 Consider a coil consisting of two closely-spaced windings of wire bearing a current

I. If the inductance of the coil is L, then what is the magnetic flux through the coil? An answer in terms of L and I is expected.

[2]

7.13-2 What is the inductance of a solenoid which is 5 cm in length, 5 mm in diameter, and consists of 300 turns of wire, if the core of the solenoid is a ferromagnetic material having relative permeability equal to 200.

[m0127]

[1]

7.15-1 How much current is required to store 2 mJ in a 47 mH inductor?

[2]

7.15-2 A current of 2 mA flows through a 3 mH inductor. Then, the energy stored by the inductor is transferred, without loss, to a 4 nF capacitor. When this transfer is complete, what is the voltage across the capacitor?

Chapter 8

Time-Varying Fields

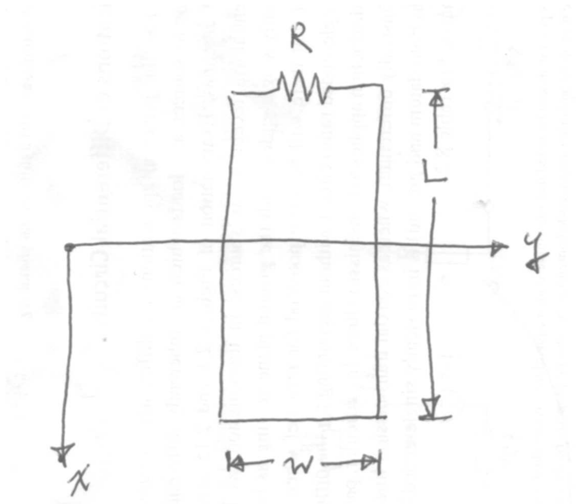
[m0055]

[1]

8.3-1 A coil consists of 50 turns around a ferromagnetic core having relative permeability of 2×10^5, is 10 cm long, and has a cross-sectional area of 200 cm^2. A time-invariant spatially-uniform magnetic field of 20.0 mA/m, oriented parallel to the axis of the coil, is impressed. Then, over a span of 200 ms, the magnetic field is reduced to one-fifth of its original value. After this 200 ms interval, the magnetic field is again held constant. Estimate as best you can the potential that is induced in the coil (a) before the reduction in the magnetic field begins, (b) while the magnetic field is being reduced, and (c) after the reduction of the magnetic field is completed.

[2]

8.3-2 In the scenario shown in the figure, the loop has length $L = 1$ m and width $w = 10$ cm. The loop is moving to the left at 250 m/s. An impressed magnetic field of $\hat{\mathbf{z}}\ B_0\ e^{ay}$ exists throughout the domain of the problem, with $B_0 = 0.8$ T and $a = -0.50$ m^{-1}. The resistance $R = 2.5\ \Omega$. What is the current induced in the loop at the moment that the left side of the loop reaches $y = 0.5$ m, and in what direction (clockwise or counter-clockwise) does this current flow?



[3]

8.3-3 A circular wire loop of time-varying radius $a = vt$ lies in the $x-y$ plane and is centered on the origin. The variable v is a positive constant; that is, the loop radius increases at a constant rate. The magnetic flux density is uniform and is given by

$$\mathbf{B} = \hat{\mathbf{z}} B_0 \left(1 + kt\right)$$

where B_0 and k are constants. A small gap is introduced into the loop at which a voltage V_g may be measured. The angular separation measured from the "$-$" terminal to the "+" terminal of the gap is positive with respect to ϕ. (Alternatively, you might say that to go from "$-$" to "+" through the gap is traveling in the counterclockwise direction as viewed from the positive z axis.)

(a) If $\mathbf{B}$ is in units of T (teslas) and t is in units of s (seconds), then what are the units of B_0, k, and v?
(b) Use Faraday's Law to develop an expression for $V_g(t)$.
(c) $V_g(t)$ may contain both motional and transformer emf. At what time(s) t, if any, are the contributions of the motional emf and transformer emf equal?

[m0056]

[1]

8.4-1 Consider a magnetic field probe consisting of a flat circular loop of wire with radius 10 cm. The probe's terminals correspond to a small gap in the loop. This probe is placed in a uniform magnetic field having magnitude $B(t) = B_0 \sin(2\pi f t + \alpha)$, where $f = 100$ kHz and α is an unknown constant. (By *uniform*, we mean that the magnetic field has the same magnitude and direction at all points in space.) The orientation of the loop with respect to the magnetic field vector is unknown. The voltage at the terminals is measured for all possible orientations of the probe, and it is found that the maximum voltage is 20 mV peak-to-peak. What is B_0?

[2]

8.4-2 The peak amplitude of the potential ("voltage") induced in a single-turn circular loop of diameter 20 cm is 15 V in response to a spatially-uniform sinusoidally-varying magnetic field. The loop is then replaced by a two-turn square loop which is 20 cm on a side and which lies in the same plane. Now what is the peak amplitude of the induced potential?

[m0031]

[1]

8.5-1 The transformer described in the book (consisting of coils on opposite sides of a toroidal core) is modified as follows: A slot is cut through the core on the secondary side, and the secondary coil is rewound through the slot. As a result, the secondary coil remains oriented in the same direction relative to the magnetic flux, but has exactly one-half the cross-sectional area as it did previously (i.e., is now one-half the cross-sectional area of the primary coil). The overall cross-sectional dimensions of the core remains nearly the same on both sides, so the flux density in the core is not significantly affected. The number of turns in the primary and secondary coils are 200 and 300, respectively. What is the potential V_2 on the secondary side in terms of the potential V_1 on the primary side?

[2]

8.5-2 Which of Maxwell's Equations most directly explains the operation of a transformer? Give your answer in differential time-domain form.

[m0030]

[1]

8.7-1 A generator consists of a loop rotating in a static uniform magnetic field of 2 T oriented in a direction perpendicular to the axis of rotation. The loop is circular with radius 2 m. The peak emf generated is 5 V. At time $t = 0$ s, the emf was zero, but increasing. Write an expression for the emf as a function of time.

[2]

8.7-2 A generator consists of a loop rotating in a static magnetic field of 2 T oriented in a direction perpendicular to the axis of rotation. The area of the loop is 4 m^2. The peak induced potential is 5 V. How fast is the loop rotating?

[m0053]

[1]

8.9-1 The electric field at a particular point in free space is $\hat{\mathbf{y}}\,(3\ \mathrm{V\ m^{-1}\ s^{-2}})\,t^2$ where t is time. What is the displacement current density at that point?

Chapter 9

Plane Wave Propagation in Lossless Media

[m0042]

[1]

9.1-1 Beginning with the most general possible time-domain form of Ampere's Law in differential form, derive the time-harmonic (phasor) version of this same equation.

[2]

9.1-2 Write the Maxwell's Equation that is associated with Faraday's Law in differential phasor form, using only field intensities (i.e., no flux densities).

[m0036]

[1]

9.2-1 Derive the time-harmonic (phasor) version of the wave equation for $\widetilde{\mathbf{H}}(\mathbf{r},\omega)$ in a source-free region. Begin by stating the relevant Maxwell's Equations in terms of $\widetilde{\mathbf{E}}$ and $\widetilde{\mathbf{H}}$ (only) in differential form. Then, show how you proceed from these equations to the desired wave equation for $\widetilde{\mathbf{H}}$.

[2]

9.2-2 The phase of an electromagnetic wave is found to advance by 90° for each meter of travel through a lossless, source-free material. Write a differential equation that may be solved to obtain $\widetilde{\mathbf{E}}$, the phasor form of the electric field intensity. Your equation should include no variables other than $\widetilde{\mathbf{E}}$ itself.

[m0038]

[1]

9.4-1 The textbook begins with $\nabla^2\widetilde{\mathbf{E}} + \beta^2\widetilde{\mathbf{E}} = 0$ (the homogeneous wave equation for $\widetilde{\mathbf{E}}$) and obtains one equation for each of the three cartesian components ($\widetilde{E}_x$, $\widetilde{E}_y$, and $\widetilde{E}_z$) of $\widetilde{\mathbf{E}}$.
(a) Now do this for each of the three components of $\widetilde{\mathbf{E}}$ in *cylindrical coordinates.*
(b) Assume that $\widetilde{\mathbf{E}}$ is uniform in ϕ and z and has no component in the ρ or ϕ direction. Determine the resulting wave equation for the remaining component of $\widetilde{\mathbf{E}}$.

[2]

9.4-2 A uniform plane wave propagates in the $+\hat{\mathbf{z}}$ direction through free space. The electric field has magnitude 2 μV/m and points in the $\hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ direction. What is the magnitude and direction of the magnetic field intensity?

[m0039]

[1]

9.5-1 A uniform plane wave propagating in free space has an electric field intensity equal to $\hat{\mathbf{e}}E_0 \cos\left(\omega t + [1\text{ rad/m}]\,x + [2\text{ rad/m}]\,y + [3\text{ rad/m}]\,z\right)$. The direction of "$\hat{\mathbf{e}}$" is not specified.
(a) What is the unit vector that points in the direction of propagation? Be as specific as possible.
(b) What is the wavelength in this material?
(c) What is the frequency of the wave?

[2]

9.5-2 A uniform plane wave propagates in the $-\hat{\mathbf{x}}$ direction. The magnetic field points in the $+\hat{\mathbf{y}}$ direction. In what direction does the electric field point?

[3]

9.5-3 The electric field of a uniform plane wave has magnitude 3 V/m. The wave travels through plastic having relative permittivity equal to 2. What is the magnitude of the magnetic field intensity in the plastic?

[m0031]

[1]

9.7-1 A laser is designed to deliver 3 W uniformly onto an aperture having area 1 mm^2. What is the peak magnitude of the electric field intensity in the laser beam in free space?