Geosynthetic Reinforced Soil: Numerical and Mathematical Analysis of Laboratory Triaxial Compression Tests

Karla Johanna Santacruz Reyes

Dissertation submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
In
Civil Engineering

George M. Filz
Katerina Ziotopoulou
Thomas L. Brandon
Joseph E. Dove
Adrian Rodriguez-Marek

December 13, 2016
Blacksburg, VA

Keywords: geosynthetic reinforced soil, three-dimensional numerical analyses, constitutive model for dense coarse-grained soil, constitutive model for geosynthetic and interfaces with soil, numerical and mathematical analyses of stress-strain and strength response of GRS under triaxial tests conditions
Geosynthetic Reinforced Soil: Numerical and Mathematical Analysis of Laboratory Triaxial Compression Tests

Karla Johanna Santacruz Reyes

ACADEMIC ABSTRACT

Geosynthetic reinforced soil (GRS) is a soil improvement technology in which closely spaced horizontal layers of geosynthetic are embedded in a soil mass to provide lateral support and increase strength. GRS is popular due to a relatively new application for bridge support, as well as long-standing application in mechanically stabilized earth walls. Several different GRS design methods have been used, and some are application-specific and not based on fundamental principles of mechanics. Because consensus regarding fundamental behavior of GRS is lacking, numerical and mathematical analyses were performed for laboratory tests obtained from the published literature of GRS under triaxial compression in consolidated-drained conditions.

A three-dimensional numerical model of GRS was developed using FLAC3D. An existing constitutive model for the soil component was modified to incorporate confining pressure dependency of friction angle and dilation parameters, while retaining the constitutive model's ability to represent nonlinear stress-strain response and plastic yield. Procedures to obtain the parameter values from drained triaxial compression tests on soil specimens were developed. A method to estimate the parameter values from particle size distribution and relative compaction was also developed. The geosynthetic reinforcement was represented by two-dimensional orthotropic elements with soil-geosynthetic interfaces on each side.

Comparisons between the numerical analyses and laboratory tests exhibited good agreement for strains from zero to 3% for tests with 1 to 3 layers of reinforcement. As failure is approached at larger strains, agreement was good for specimens that had 1 or 2 layers of reinforcement and soil friction angle less than 40 degrees. For other conditions, the numerical model experienced convergence problems that could not be overcome by mesh refinement or reducing the applied loading rate; however, it appears that, if convergence problems can be solved, the numerical model may provide a mechanics-based representation of GRS behavior, at least for triaxial test conditions.

Three mathematical theories of GRS failure available in published literature were applied to the laboratory triaxial tests. Comparisons between the theories and the tests results demonstrated that all three theories have important limitations.

These numerical and mathematical evaluations of laboratory GRS tests provided a basis for recommending further research.
Geosynthetic Reinforced Soil: Numerical and Mathematical Analysis of Laboratory Triaxial Compression Tests

Karla Johanna Santacruz Reyes

PUBLIC ABSTRACT

Sometimes soils in nature do not possess the strength characteristics necessary to be used in a specific engineering application, and soil improvement technologies are necessary. Geosynthetic reinforced soil (GRS) is a soil improvement technology in which closely spaced horizontal layers of geosynthetic material are placed in a soil mass to provide lateral support and increase the strength of the reinforced mass. The geosynthetic materials used in GRS are flexible sheets of polymeric materials produced in the form of woven fabrics or openwork grids. This technology is widely used to improve the strength of granular soil to form walls and bridge abutments.

Current design methods for GRS applications are case specific, some of these methods do not rely on fundamental principles of physics, and consensus regarding the fundamental behavior of GRS is lacking. To improve understanding of GRS response independent of application, the three dimensional response of GRS specimens to axisymmetric loading were investigated using numerical and mathematical analysis.

A numerical model using the finite difference method in which the domain is discretized in small zones was developed, and this model can capture the response of GRS laboratory specimens under axisymmetric loading with reasonably good accuracy at working strains (up to 3% strain). This numerical model includes a robust constitutive model for the soil that is capable of representing the most important stiffness and strength characteristics of the soil.

For large strains approaching failure loading, the numerical model encountered convergence difficulties when the soil strength was high or when more than two layers of reinforcement were used. As an alternative to discretized numerical analysis, three mathematical theories available in the published literature were applied to the collected GRS laboratory test data. These evaluations demonstrated that all three theories have important limitations in their ability to represent failure of GRS laboratory test specimens.

This study is important because it proposed a numerical model in 3D to represent the GRS behavior under working strains, and it identified several limitations of mathematical theories that attempt to represent the ultimate strength of GRS. Based on these findings, recommendations for further research were developed.
Dedication

For my family
Acknowledgements

I would like to express my gratitude towards my advisors: Dr. George Filz and Dr. Katerina Ziotopoulou for their patience and guidance; both have greatly contributed to the completion of this work.

I would like to thank my committee members, Dr. Thomas Brandon, Dr. Joseph Dove, and Dr. Adrian Rodriguez-Marek for their ideas and suggestions.

I wish to extend many thanks and much appreciation for the financial and institutional support provided by SENESCYT in Ecuador, the Universidad de Cuenca in Ecuador, the Fulbright Commission in Ecuador, and Virginia Tech.

Finally, my sincere gratitude to my family: my kids, my husband, my mom, and my sister. You all inspire me! Your love and support always guide me everywhere.
Table of Contents

ACADEMIC ABSTRACT .............................................................................................................. ii
PUBLIC ABSTRACT .................................................................................................................. iii
Dedication ................................................................................................................................ iv
Acknowledgements .................................................................................................................. v

1. INTRODUCTION .................................................................................................................. 1
   1.1. Problem Statement ......................................................................................................... 1
   1.2. Research Goal, Purpose, Objectives, and Limitations .................................................. 2
   1.3. Report Organization ..................................................................................................... 3

2. LITERATURE REVIEW ........................................................................................................ 5
   2.1. Basic Behavior of Reinforced Soil ............................................................................... 6
   2.2. Theories of Reinforced Soil Behavior .......................................................................... 13
   2.3. Reinforced Soil Behavior through Large Scale Experimentation ................................ 15
   2.4. Numerical Investigation of Geosynthetic Reinforced Soils .......................................... 18
   2.5. Geotextile Properties and Interface Soil-Reinforcement through Experimentation .... 23
   2.6. Unanswered Questions about Geosynthetic Reinforced Soil ...................................... 25

3. SOIL MODEL ....................................................................................................................... 26
   3.1. Chsoil Model in FLAC3D (Itasca 2012) ..................................................................... 26
       3.1.1. Chsoil Model in FLAC3D Applied to 3D Conditions (Itasca 2012) ....................... 27
       3.1.2. Chsoil Model in FLAC3D for anAxisymmetric Loading Condition ..................... 32
   3.2. Development of Equations for a Triaxial Test ............................................................. 33
   3.3. Discussion of Model Performance ............................................................................. 44
       3.3.1. Chsoil Model Behavior ......................................................................................... 44
       3.3.2. Comparison of Chsoil Model and Hyperbolic Model .......................................... 46
   3.4. Modifications to Chsoil Model ..................................................................................... 50
   3.5. Procedure to Obtain Input Parameters ....................................................................... 53

4. GEOSYNTHETIC AND INTERFACE MODELS ................................................................. 75
   4.1. Structural Elements in FLAC3D ................................................................................. 75
   4.2. Liner Elements of FLAC3D ....................................................................................... 76
   4.3. Element Tests on Interfaces and Structural Elements .................................................. 77

5. NUMERICAL MODELING PROCEDURES ........................................................................ 83
   5.1. FLAC3D Computer Program ....................................................................................... 83
APPENDIX B: FLAC3D CODE FOR LINER ELEMENT TESTS ........................................ 199
B.1. Direct Shear Test Code ................................................................................ 199
B.2. Pullout Test Code ......................................................................................... 200
B.3. Structural Element Test Code ....................................................................... 201
APPENDIX C: GRS FLAC3D CODE ....................................................................... 203
C.1. Stage 1: Code to Collect Data ....................................................................... 203
C.2. Stage 2: Code to Generate the GRS Model .................................................. 205
C.3. Stage 3: Code to Generate and Store Results ............................................... 208
CHAPTER 1:
INTRODUCTION

1.1. Problem Statement

Geosynthetic reinforced soil (GRS) is a soil improvement technology in which closely spaced horizontal layers of geosynthetic material are placed in a soil mass to provide lateral support and increase the strength of the reinforced mass. This type of soil improvement technology has been used since ancient times, but lately GRS has gained popularity in a new application for bridge abutments in which the bridge load bears directly on the GRS rather than on piles that extend through the reinforced soil mass (Adams et al. 2011a). GRS also continues to be widely used for mechanically stabilized earth walls (MSEWs) in other applications.

In terms of material components and composition, GRS bridge abutments can be closely compared to MSEWs reinforced with geosynthetics. However, some of the design methods employed for these two types of applications and systems are considerably different, and they can thus result in very different outcomes in terms of the amount of reinforcement required for the same wall height and surcharge load (Phillips et al. 2015).

Researchers who have studied the use of GRS for bridge abutments have introduced the concept of a special “composite behavior” of the GRS mass, when the vertical distance between reinforcement layers is less than 0.3 m (Wu et al. 2014, Adams et al. 2011b), and they have proposed an application-specific design methodology based on that postulate. In this context, the composite behavior is a special but predictable behavior of the GRS mass resulting from the close spacing of the reinforcement layers, the aggregate size, and the friction angle of the soil (Adams et al. 2011b). A design methodology consistent with these assumptions has been presented by Adams et al. (2011a), who proposed a design equation (1-1) that relates the required reinforcement tensile strength ($T_{req}$) to the geosynthetic spacing ($S_v$), the maximum grain size of the reinforced soil ($d_{max}$), and the total horizontal stress ($\sigma_h$) in the GRS mass at the level of interest. Equation 1-1 introduces a normalized spacing-over-maximum-particle-size term, for which a fundamental basis has not been provided, to the author's knowledge. Neither has a fundamental basis been provided for the limitation of 0.3 m (12 in.) on geosynthetic spacing for the special composite behavior to occur.
\[ T_{req} = \left[ \frac{\sigma_h}{S_v} \right] S_v \]

where:

\( T_{req} \) is the required reinforcement tensile strength
\( \sigma_h \) is the total lateral stress at a given depth and location
\( S_v \) is the reinforced spacing
\( d_{max} \) is the maximum grain size of the soil

A comparison study performed using four existing design methodologies, two for MSE walls and two for GRS abutments, showed that those design methodologies rely on different assumptions and produce widely different outcomes (Phillips et al. 2015). At present, there is no consensus regarding the fundamental mechanisms producing the observed behavior of reinforced soil, and thus it should be further investigated, with the goal of improving confidence in the mechanics that underlie design methods.

Studies on soil reinforced with geosynthetics have been published since the 1970s, yielding a vast amount of literature resources. Recently, significant research has been performed on GRS behavior, with large amounts of laboratory testing on GRS masses, and some theories of GRS behavior have been based on these tests. However, there is no widely accepted reinforced soil strength theory for GRS. Instead there are application-specific design methodologies, like the one described above for GRS bridge abutments.

1.2. Research Goal, Purpose, Objectives, and Limitations

The overall goal of this research is to improve fundamental understanding of GRS response to load, including: load distribution among GRS components, deformations in response to load, and margin of safety against collapse.

The purpose of pursuing this goal is to permit development and/or improvement of mechanistically sound theories of GRS response, which in turn would reduce analysis uncertainty and permit more reliable and/or economic design of GRS systems.

The specific research objectives and tasks to make progress towards the overall goal include:

- Review relevant GRS literature
- Numerical modeling:
  - Identify appropriate software for 3D analysis of GRS systems
  - Identify/develop a 3D constitutive model and properties for compacted coarse-grained soil in GRS applications
- Identify/develop a suitable numerical model and properties for geosynthetic reinforcement, including soil-geosynthetic interfaces, in GRS applications
- Develop a 3D numerical model that can be applied to laboratory tests of GRS
- Compare response of the 3D numerical model to the results of laboratory tests and determine limitations of the 3D numerical model

- Analytic modeling:
  - Identify existing analytic failure theories for GRS
  - Compare the failure theories to the results of laboratory tests, and assess the reliability and limitations of the analytic failure theories

- Draw conclusions and develop recommendations for further study

There are significant limitations on the scope and outcomes of this research, including:

- No new physical tests have been performed as part of this research
- During this research, it was discovered that, even though the numerical model could represent the pre-failure load-deformation response of GRS masses under simple triaxial loading conditions, realistic representation of failure conditions was only achieved for GRS triaxial test specimens with one or two layers of reinforcement and soil friction angles less than 40 degrees. Because the numerical model did not accurately represent failure conditions for GRS triaxial test specimens with three or more layers of reinforcement or with soil friction angle greater than 40 degrees, mathematical failure theories were investigated for their ability to represent failure conditions. The three mathematical failure theories that were investigated each have limitations in their ability to represent failure conditions for GRS triaxial test specimens. Based on these outcomes, some recommendations for additional research were developed.

1.3. Report Organization

Chapter 2 presents the literature review of geosynthetic reinforced soils. As the overall subject is extensive, the literature review has focus on five specific topics of most interest for the present investigation: 1) Basic behavior of reinforced soils, 2) Theories of reinforced soil behavior, 3) Large scale experimentation on reinforced soils, 4) Numerical investigation of reinforced soils, and 5) Geotextile and soil-geotextile interface properties. Chapter 2 provides a brief summary of the principal sources for each topic, and the following chapters provide additional detailed information on singular topics as needed.

Chapters 3, 4, 5 and 6 present the stages on the development of a numerical model for GRS soil.

Chapter 3 contains details of the development, validation, and application procedure for the soil constitutive model (one of the GRS components). The chapter includes examples and tables with information about how the model can be applied to track real test data.
Chapter 4 presents details of the geosynthetic reinforcement and the soil-geosynthetic interface numerical modeling. The chapter presents a discussion of the models and features available in the numerical analysis program used in this research to represent the geosynthetic behavior, the applicability of those features, and examples showing how the final geosynthetic model performs.

Chapter 5 shows the assemblage of a numerical model of GRS including the soil, geosynthetic, and soil-geosynthetic interfaces presented in the two previous chapters.

Chapter 6 shows the validation procedure for the developed GRS numerical model. It begins with the collection of information regarding real data on triaxial tests of GRS. A second stage in the chapter discusses how the properties for each material in the GRS mass are obtained. After this, the developed GRS numerical model is applied to real tests, and the results are compared with test data to demonstrate its usability and limitations. General comments and recommendations about the GRS model capabilities are discussed.

Chapter 7 focuses on the analysis of existing GRS failure theories. The available theories are explained, applied to real cases, and comparisons between the theories and experimental results are discussed.

Chapter 8 provides conclusions and recommendations based on the findings from this research, as well as recommendations for future research.
CHAPTER 2:  
LITERATURE REVIEW

This section presents information from published literature about geosynthetic-reinforced soil, and it sheds some light on the issues that are of most importance to the present study.

Having been used even in ancient societies, reinforced soil is one of the oldest methods to provide lateral restraint and increase the strength of the reinforced soil mass. In more recent times, many authors agree that the reinforced soil technology was formally introduced by Henri Vidal in his paper “The Principle of Reinforced Earth” (Vidal 1969). This document proposed the reinforcement of earth with linear components such as rigid strips, and it discussed topics like friction between the soil-reinforcement interfaces and theories of influence of reinforcement on stress distribution, among other general topics, and thus introduced a formal background and a first formulation for the engineering behavior of reinforced soil.

A few years after Vidal's (1969) publication, Zen Yang's PhD dissertation (Yang 1972) presented a significant work about soil with layers of reinforcement. This document described layers of high tensile strength materials to reinforce the soil. This work included an important experimental investigation using triaxial tests of soil reinforced with fiberglass and stainless steel plates, as well as circular plate load tests to represent model footings on reinforced soil. Yang's (1972) work also incorporated numerical investigations. Based on the results, Yang (1972) proposed a semi-empirical method to analyze stress distributions in the reinforced soil. Yang's (1972) method stipulates an increase of confining stress due to the reinforcement effect, and calculates it as a function of the frictional characteristics of the soil-reinforcement interface surface and of the region of influence, which depends on the amount of reinforcement and geometry of the specimen.

Following these initial investigations, a vast amount of research regarding reinforced soil has been performed, including laboratory tests, field tests, numerical analyses, and case studies. Analytical studies constitute another important branch of the investigation of reinforced soil, from simple equilibrium based theories to entire design methodologies for specific applications. The work most relevant to the present research can be grouped into five general categories:

- Basic behavior of reinforced soil mainly analyzed by laboratory experimentation;
- Important theories of reinforced soil behavior;
- Reinforced soil behavior through large scale experimentation;
- Numerical simulations; and
- Geotextile properties and soil-reinforcement interface studies.

This chapter presents a summary of the state of the art of reinforced soil, classified into these five categories. Some specific aspects of the literature that are especially pertinent to Chapters 3, 4, 5, 6, and 7 are described in more detail and analyzed in those chapters: description of the numerical
program features in Chapters 3, 4 and 5, laboratory triaxial tests on reinforced soils and material properties in Chapter 6, and strength theories for reinforced soil behavior in Chapter 7.

2.1. Basic Behavior of Reinforced Soil

The basic behavior of reinforced soil has been investigated by many authors, mostly using laboratory experimentation on unreinforced and reinforced soils and contrasting their outcomes. Laboratory tests like triaxial, plane strain, and direct shear are the most common tests used. The main topics of investigation in such analyses are the effect of the reinforcement on the soil behavior and the influence of specific parameters such as reinforcement spacing, reinforcement properties, imposed stresses, soil properties, and interface characteristics. Table 1 illustrates the major sources summarized in chronological order. Some of the principal sources are described in more detail after the table.
Table 1: Summary of references that experimentally study the basic behavior of reinforced soils

<table>
<thead>
<tr>
<th>Source</th>
<th>Test type</th>
<th>Soil</th>
<th>Geosynthetic</th>
<th>Main Subject of Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yang (1972)</td>
<td>Triaxial and model footing</td>
<td>Silica sands</td>
<td>Fiberglass nets and rigid stainless steel plates</td>
<td>Influence of: confining pressure, grain size, soil density, reinforcement variables. It becomes an introductory study of soil with reinforced layers.</td>
</tr>
<tr>
<td>Broms (1977)</td>
<td>Triaxial</td>
<td>Dry sand</td>
<td>Fabric</td>
<td>Influence of soil relative density, reinforcement spacing and confining pressure. Focus on changes in lateral expansion</td>
</tr>
<tr>
<td>McGown and Andrawes (1977)</td>
<td>Plane strain</td>
<td>Sand</td>
<td>Non-woven fabrics</td>
<td>Influence of the orientation of the reinforcement, and porosity of the soil.</td>
</tr>
<tr>
<td>Long et al. (1983)</td>
<td>Triaxial</td>
<td>Uniform sand</td>
<td>Bronze circular discs</td>
<td>Monitors reinforcement strain, radial and tangential stresses as a function of radius. Lateral pressure coefficient vs. radial distance</td>
</tr>
<tr>
<td>Gray and Al-Refai (1986)</td>
<td>Triaxial</td>
<td>Dry dune sand</td>
<td>Woven and non-woven geotextiles, and fibers</td>
<td>Influence of: amount of reinforcement, confining pressure, type of reinforcement, and soil density</td>
</tr>
<tr>
<td>Chandrasekaran et al. (1989)</td>
<td>Triaxial and direct shear</td>
<td>River sand</td>
<td>Woven and non-woven geotextiles</td>
<td>Influence of confining pressure on: interface friction and the tensile strength of the reinforcement</td>
</tr>
<tr>
<td>Futaki et al. (1990)</td>
<td>Large triaxial tests</td>
<td>River sand</td>
<td>Different grids</td>
<td>Influence of aperture of grids, grids’ cross section shape, strength and stiffness of grid. Influence of confining pressure</td>
</tr>
<tr>
<td>Al-Omari et al. (1995)</td>
<td>Triaxial</td>
<td>Dry sand</td>
<td>Polymer geogrid</td>
<td>Influence of geosynthetic stiffness, number of layers and soil density</td>
</tr>
<tr>
<td>Boyle (1995)</td>
<td>Plane strain</td>
<td>Sand</td>
<td>Woven and non-woven geotextiles and steel sheets</td>
<td>Dilation behavior, creep, and influence of reinforcement modulus</td>
</tr>
<tr>
<td>Haeri et al. (2000)</td>
<td>Triaxial</td>
<td>SP</td>
<td>Three different non-woven geotextiles</td>
<td>Influence of geotextile amount, geotextile type, geotextile arrangement, confining pressure, sample size on stress-strain and dilation characteristics</td>
</tr>
<tr>
<td>Ketchart and Wu (2002)</td>
<td>Plane strain</td>
<td>Road base soil</td>
<td>Woven and non-woven geotextiles</td>
<td>Focus on lateral and vertical strains to investigate preloading influence and creep behavior.</td>
</tr>
<tr>
<td>Latha and Murthy (2006)</td>
<td>Undrained triaxial tests</td>
<td>SP river sand</td>
<td>Woven geotextile, geogrid and polyester film</td>
<td>Influence of geotextile type, geotextile configuration, surface roughness</td>
</tr>
<tr>
<td>Ziegler et al. (2008)</td>
<td>Large triaxial tests</td>
<td>Base coarse material</td>
<td>Welded polypropylene geogrid</td>
<td>Radial strains at different levels, stress - strain curves</td>
</tr>
</tbody>
</table>
Early studies related restriction on lateral expansion to the presence of reinforcement in the soil. Broms (1977) performed triaxial tests on dry sand reinforced with fabric. The specimens had dimensions of 137 mm in height and 69 mm in diameter and were performed using different relative densities of the sand, different spacing between the reinforcement layers, and different confining pressures. The objective was to analyze the changes in lateral earth pressure, lateral expansion of the reinforced samples and the influence of the analyzed variables on the reinforced soil behavior. Broms (1977) found a strength increase of 25% for one reinforcement layer in the middle of the sample and 75% strength increase for two layers of reinforcement. Broms (1977) also placed reinforcement layers at the top and the bottom of the sample. However, they had no influence on the samples behavior. The influence of the confining pressure on the results of these tests was significant, while the relative density of the soil barely affected the outcomes of the testing program. Broms (1977) finally correlated the effects of the reinforcement to a reduction of the lateral movement of the soil. Also, the author interpreted the existence of an increase in internal confining pressure due to the reinforcement as the cause of increased strength of the reinforced soil. Broms (1977) derived a coefficient of lateral earth pressure and related it to Rankine's active coefficient. The resulting lateral earth pressure coefficient was 40% greater than Rankine's value. With this, and as the paper states “other test results”, the author recommended working with a lateral coefficient of 0.5 and with the same friction angle of the unreinforced soil, to determine the influence of the reinforcement on the soil behavior. It is worth noting that this lateral earth pressure is internal to the geosynthetic reinforced soil mass, and it does not represent the lateral earth pressure that a GRS mass would apply to a wall or to GRS facing.

McGown and Andrawes (1977) concentrated their efforts on the influence of different types of reinforcement on the reinforced soil behavior, and on the influence of the orientation of the geosynthetics. The authors reference the orientation of the geosynthetic relative to the major principal strain plane. The methodology of the study involved plane strain tests on unit cells of sand, with dimensions of 152 mm by 102 mm by 102 mm. Three different types of non-woven fabrics were used as reinforcements. The authors also studied the behavior of a model footing located over the sand and subjected to load under plane strain conditions. The model footing, with dimensions 80 by 76 mm, was placed over a box of 610 by 260 by 76 mm that was filled with sand. For both configurations (with and without the footing), different reinforcements were tested: unreinforced sand and sand with one layer of the non-woven reinforcement at different orientations.

For the tests without footings, different porosities of the soil were considered and the behavior of the peak friction angle and the axial strain at the peak were recorded. The results showed an increase of the friction angle of the reinforced soil with the non-woven geotextile with respect to the unreinforced soil, with this result being repeatable at different soil porosities. However, when the density of the sand was high, the weaker non-woven inclusion generated a decrease on the peak friction angle response. Regarding the axial strain at the peak strength, all the inclusions (non-woven geotextiles) made the reinforced soil increase its peak axial strain by about 1%. Also, the post-peak loss of strength was reduced with the presence of the inclusions in the soil. When the inclusions were placed at different angles with respect to the horizontal, the response of the system
varied considerably to the point that even some orientations cause an opposite effect than “reinforcing” the soil. That means steeper orientations, when the interface friction angle is less than the soil friction angle, can get sliding along the interface. This was the reason behind the authors using the term “inclusions” rather than reinforcements. If the inclusions are located perpendicular to the major principal strain direction, the optimum effect would be obtained.

Regarding the tests by McGown and Andrawes (1977) with the model footing, those involved: one layer of inclusion at different locations among the different specimens being tested, two layers at different locations, and one layer at the interface of two soils with different densities. The results of the tests showed an increase of strength of the soil with the inclusions compared to the soil without any inclusions (baseline test); however, the influence of the inclusions in loose soil was much more pronounced than for the dense soil. It was also shown that the location of the inclusions affected the magnitude of the footing settlement. Most of the tests showed the greatest improvement when the reinforcement was located at a depth of 0.5 times the width of the footing.

McGown et al. (1978) continued the study on reinforced soil behavior with an objective to understand the effect of the inclusion surface characteristics, shape, and stress-strain behavior, on the soil-inclusion system. McGown et al. (1978) used different types of inclusions on the same plane strain unit cell test. These included: aluminum foil, aluminum mesh, and non-woven fabric. Based on these tests, the authors classified inclusions into two categories: extensible and inextensible. This classification was made as a function of their tensile strain at rupture, relative to the soil's tensile strain at failure (without inclusions and at the same stress levels). However tensile strains at failure in the soil were not determined or measured. The authors considered the inclusion had a lower or higher tensile strain than the soil when the inclusion ruptured or not (respectively) during the failure of the test. When the inclusion ruptured at tensile strains less than the maximum tensile strains of the soil, it was called an inextensible inclusion, and the opposite would be the extensible inclusion. McGown et al. (1978) referred to soil with inextensible inclusions as “reinforced soils”. Reinforced soils systems showed an increase in strength compared with the unreinforced soil. However, the loss of strength after failure was significant. System failure would occur when the reinforcement reached its strength. Therefore, the strength of the system would be a function of the maximum strength of the reinforcement. McGown et al. (1978) referred to soil with extensible inclusions as “plysoil,” which means the inclusions strengthen the soil at strain levels, such that the strain at failure increases, and the loss of stress after failure is decreased. Finally, no matter which inclusion type is used, the authors pointed out the reduction of lateral deformation of the soil-inclusion system with respect to the soil alone.

Analyzing similar topics but using triaxial tests, Gray and Al-Refeai (1986) studied the influence of fiber and fabric inclusions on sandy soil. Effects of the orientation of the fibers were observed, and the influences of confining pressures, amount of reinforcement, and different soil densities on the reinforced soil system behavior were investigated. Principal findings that Gray and Al-Refeai (1986) pointed out were: (a) the presence of the reinforcement produced a higher peak strength, a higher strain at failure, and a decrease in the post failure stress for dense sand; (b) the importance of the location of the geosynthetic reinforcement in the soil mass, with the reinforcement located
in the zones of high stress demand producing a larger effect than for reinforcement located in other zones; and (c) the authors observed the presence of a critical confining stress that marked a change in the slope of the failure envelope for reinforced soil giving it a bilinear result as shows Figure 1.

![Graph showing the Major Principal Stress at Failure vs. Confining Pressure](image)

Figure 1: Unreinforced and reinforced soil envelopes on Muskegon dune sand. Reinforced sample has 3 layers of geosynthetic Typar 3401 (after Gray and Al-Refai 1986)

Chandrasekaran et al. (1989) focused on the soil-geosynthetic interface behavior and on the development of tensile stress in the reinforcement. These authors performed triaxial tests and direct shear tests to develop an expression that relates the force in the geosynthetic to the strength increase. They also described two types of failures of reinforced material, one is the failure due to excessive compression of the soil, and the other one is due to fabric rupture while the soil strength has not been reached yet. These two types of failure directly influence the final strength the system can reach, as well as the principal parameters influencing the system stress-strain behavior.

Gray and Al-Refai (1986) and Chandrasekaran et al. (1989), among others, provide the basis for the work presented in Chapter 6 where they are used as reference triaxial tests for the numerical analysis. As such, other pertinent details are presented in Chapter 6.
Many authors used laboratory tests, mostly triaxial tests, to study the behavior of soils reinforced with geogrids. Futaki et al. (1990) used large triaxial tests on sand reinforced with grids to investigate the optimum size of the grid's openings, as well as the influence of confining pressure, grid strain, and grid strength. Al-Omari et al. (1995) also used triaxial testing to study the influences of grid stiffness, the number of grid layers, and the influence of the density of the soil on the reinforced soil response. Other authors like Ziegler et al. (2008) developed analytical equations based on laboratory experimentation on large triaxial tests and pullout tests on soil reinforced with grids. An objective of this study was to calculate the deformation of the geogrid as the reinforced mass was loaded in numerical calculations.

Athanasopoulos (1993) presented a study of geosynthetic reinforced soil, with emphasis on the soil-geosynthetic interfaces, which he indicated have an important influence on the reinforced soil strength. Athanasopoulos (1993) studied the influence of soil particle size (using the average size particle) in relationship to the geosynthetic texture (using a woven geotextile) and the influence of confining pressure. Athanasopoulos (1993) stated that the relationship between the geosynthetic's fiber spacing and the soil's particle size plays an important role in the reinforced soil behavior. As such, the author defined the “aperture ratio” which is the ratio of the woven geotextile fiber spacing over the average sand particle size as a parameter of analysis. Athanasopoulos (1993) also proposed an alternative laboratory test to study the soil-reinforcement interface behavior. This test consisted of direct shear tests on sand reinforced with a vertical (perpendicular to the shear plane) geosynthetic layer. This shear test box had dimensions of 63 by 63 by 37 mm and the geotextile was 63 by 37 mm in size. The author wanted, with the special test configuration, to generate a mixture between direct shear test and pullout test, where the geotextile would undergo sliding and extensive deformation at the same time. The methodology followed in these tests used different particle size ranges for each shear test, and the sand was tested unreinforced at different normal stresses and reinforced with a woven geotextile under the same normal stresses.

The principal findings reported by Athanasopoulos (1993) are grouped into 4 categories:

- **Failure envelope:** The author found a bilinear strength envelope. The change in slope developed occurred at a normal stress that depends on the aperture ratio. The bilinear envelope exhibited two types of failure modes. The first failure mode (for lower normal stresses) involved sliding between the reinforcement and the soil particles, and the test results showed a friction angle of the reinforced soil greater than the friction angle of the unreinforced soil. The second failure mode corresponded to deformation of the reinforcement together with the soil, for which the friction angle of the reinforced soil resulted close to the friction angle of the unreinforced soil with a cohesion intercept. Athanasopoulos (1993) stated that the interlocking between soil particles and the geosynthetic can get improved with higher normal stresses and generate the second failure mode.

- **Shear-stress-shear-displacement-behavior:** Similar to other authors, Athanasopoulos (1993) determined that a reinforced soil exhibits the following with respect to an unreinforced soil:
major strength (increase in the applied major principal stress at failure), increase in the failure displacement, and a decrease in post-peak strength loss (i.e., an increase in the post-peak strength). The author also reported changes in stiffness when the soil is reinforced. At small shear displacements, the stiffness of the reinforced soil decreased with respect to the unreinforced soil stiffness. The opposite behavior was observed at larger displacements. These previous effects were less pronounced when the aperture ratio was low.

- **Volumetric-deformation-versus-shear-displacement:** All the reinforced and unreinforced tests demonstrated an initial compression and a later dilation for the volumetric deformation behavior. However, the point of transition between contractive and dilative behavior was influenced by the aperture ratio, wherein the transition occurred earlier if the aperture ratio was lower. The aperture ratio also influenced the degree of dilation: greater dilation occurred when the aperture ratio was larger.

- **Apparent friction angle:** the aperture ratio was found to affect the interface friction angle between the soil and the geotextile, and therefore the aperture ratio affected the final friction angle of the reinforced soil. The author proposed an optimum aperture ratio of 1.6 which generated the greatest geotextile-soil friction. The author also mentioned the influence of the normal pressure on the friction angle of the system.

Three more studies were identified that used laboratory triaxial tests and address different topics with regards to the behavior of GRS, such as: cyclic behavior of reinforced soils (Ashmawy and Bourdeau 1998), influence of the surface roughness of the reinforcement (Latha and Murthy 2006), and influence of sample size on the reinforced soil behavior (Haeri et al. 2000). The latter study incorporated an extensive laboratory investigation with about 160 triaxial tests designed to determine the influence of different parameters on the reinforced soil behavior: geotextile amount, geotextile type, geotextile arrangement, confining pressure, and sample size. A detailed summary of these three sources can be found in Chapter 6 since these triaxial tests are used in the numerical model comparison process.

An important topic that authors have studied with regards to reinforced soils is the influence of preloading on the reinforced-soil system's behavior. Ketchart and Wu (2001) focused their investigations on the study of the reinforced soil deformation behavior where preloading played an important role. These authors performed triaxial tests, geosynthetic extension tests, and interface tests with a direct shear apparatus. The same authors also developed a “soil geosynthetic performance (SGP) test” which is a plane strain test with a reinforced soil mass under monotonic and cyclic loading. The focus of the testing program was to study the reinforced soil behavior in terms of strength, stiffness, and deformation. The studies also included a finite element analysis to investigate the geosynthetic reinforced soil behavior in the SGP tests.

The key finding of this study with regards to the reinforcement influence on the soil behavior was that reinforced soil showed higher strength and higher stiffness than the soil alone. However, some deformation was required to mobilize this influence of the reinforcement and obtain the stiffness
increment effect. Finite element analysis also showed that reinforcement layers affected the distribution of horizontal and shear stresses. The effect was more pronounced closer to each reinforcement layer. The distribution of vertical stresses did not show important changes with the inclusion of reinforcement. Preloading increased the GRS stiffness by factors of more than 2. However, it did not influence the maximum load the GRS could withstand under unloading-reloading tests.

2.2. Theories of Reinforced Soil Behavior

Through experimentation and/or analytical derivation based on general principles, several theories have been developed to describe the behavior of reinforced soil. The following table presents a brief summary of the nature of each study and the principal findings. A more extensive summary on four of these theories is included in Chapter 7, where those theories are analyzed and three of them are applied to real triaxial results. Yang (1972), Schlosser and Long (1974), Tatsuoka (2004) and Wu et al. (2014) are chosen for discussion in Chapter 7. This choice is based on the significance of the contribution made and the completeness of the work. The contribution of each work is evaluated based on whether a theory has implemented new criteria or new variables into the reinforced-soil strength determination. Completeness means theories that present a complete procedure to calculate the strength of the GRS system are included.
### Table 2: Summary of principal reinforced soil behavior theories

<table>
<thead>
<tr>
<th>Source</th>
<th>Studies</th>
<th>Key Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yang (1972)</td>
<td>Studied soil reinforced with layers of high tensile strength material. Performed triaxial tests to investigate the behavior of reinforced soil and finite element analyses of triaxial tests with different length/diameter ratios. Also tested a model foundation with reinforced soil.</td>
<td>Deformation modulus and strength of the reinforced system increased with respect to the unreinforced material. Reinforcement properties played an important role in system's behavior. Source proposed a semi-empirical theory about how reinforcement influences the composite's behavior. It is based on determination of an increment in confining pressure due to the reinforcement.</td>
</tr>
<tr>
<td>Schlosser and Long (1974)</td>
<td>Schlosser and Long (1974) presented a state of the art summary of reinforced soil work done in France. It mentioned: experimentation in the laboratory, in the field, case studies, and numerical investigations. However, its primary focus was on analytical theories.</td>
<td>Important theories on reinforced earth behavior: • Mechanisms of reinforced earth response (active and passive) • Friction at the soil-geosynthetic interface • Calculation of traction forces in reinforcement • Reinforced soil as a foundation material: characterized the reinforced soil with soil friction angle and enhanced cohesion due to reinforcement.</td>
</tr>
<tr>
<td>Hausmann (1976)</td>
<td>Analysis of previous theories about behavior of reinforced soil. Triaxial tests on reinforced soils with slippage on the interface as failure mechanism. Pieces of geosynthetic material of different size were used as reinforcement.</td>
<td>Proposed two theories to explain the reinforced soil behavior at failure. One theory resulted when the failure occurs due to rupture of the reinforcement, in this case the reinforcement generated a restriction on lateral displacements in the soil prior to failure. The second theory would be applicable when the failure in the reinforced mass is due to slippage on the interface. In this case the reinforcement generated shear stresses in the horizontal and vertical directions.</td>
</tr>
<tr>
<td>Elton and Patawaran (2004)</td>
<td>Large unconfined cylindrical tests were performed on geosynthetic reinforced soil to investigate strength behavior. Studied tensile strength of reinforcement, and compared test results with theories.</td>
<td>Developed a new equation to calculate tensile stress in reinforcement as a function of: the reinforcement spacing; lateral stress within the geosynthetic reinforced soil, which depends on the location of reinforcement layer and soil properties; and a strain distribution factor.</td>
</tr>
<tr>
<td>Tatsuoka (2004)</td>
<td>Analytical procedure, including many simplifications, to analyze the reinforced soil system behavior.</td>
<td>Proposed a “closed-form approximate solution” to find the maximum load a reinforced soil can withstand under two types of failure mechanisms: soil failure and reinforcement rupture.</td>
</tr>
<tr>
<td>Wu et al. (2014)</td>
<td>Studied volume change behavior through field-scale experiments and finite element analysis. It used a parametric study to investigate the influence of some parameters on volumetric change behavior.</td>
<td>Proposed a theory to explain the effect of reinforcement in the soil mass through suppression of dilation. Reinforcement spacing, soil, and reinforcement stiffness influenced the mechanism of suppression of dilation in the mass.</td>
</tr>
</tbody>
</table>
2.3. Reinforced Soil Behavior through Large Scale Experimentation

Large scale experimentation has been performed to overcome the limitations of small scale testing and get closer to the actual behavior of GRS. Some authors recognize that small testing (regular triaxial tests, direct shear tests) cannot capture the behavior of reinforced soil behavior because those tests do not use a representative sample of reinforced soil. Therefore, they propose larger tests such as: mini piers, and large plane strain tests. The main topics the authors address with large-scale tests are: improving understanding of creep behavior, influence of preloading, volume change behavior, and “composite behavior”. Some of the principal sources on this topic are summarized in what follows.

“Performance of a Prestrained Geosynthetic Reinforced Soil Bridge Pier” (Adams 1997)

A bridge pier was constructed by the US Federal Highway Administration Turner Fairbank Highway Research Center. The bridge pier was a 5.4 m high structure, with dimensions in plan view of 3.6 by 4.8 m at the bottom and at the top about 3 by 4.3 m. The structure used segmental blocks at the face, and road base soil reinforced with woven fabric in the body, as shown in Figure 2. Vertical strains, vertical and lateral stresses, and strains in some reinforcement layers were monitored. The objective was to demonstrate the structure’s performance, and study its behavior under vertical loading. The loading was applied through hydraulic jacks and loading pads as Figure 3 shows.

The bridge pier was pre-strained using a loading system composed by hydraulic jacks and the loading pads located at the bottom and at the top of the pier. The resulting effects on the strain behavior of the structure are discussed. The pre-straining procedure was implemented to help mobilize the reinforcement action and to decrease the vertical settlements on the reinforced soil. The instrumentation on the bridge pier monitored stresses to demonstrate the high stress capabilities of the pier and to document creep strains in the geosynthetic.

The study outcomes included that:

- The test showed the high stress capabilities of the structure with an easy construction process
- The study recorded less than 0.17% creep strains in the body of the pier.
- Performance of the facing blocks was satisfactory due to good frictional connections with the geosynthetic layers

To complement this study, a new publication titled “Mini Pier Experiments: Geosynthetic Reinforcement Spacing and Strength as Related to Performance” (Adams et al. 2007) was prepared that focused on the relative influences of the reinforcement spacing and the reinforcement strength. The authors determined that the spacing of the reinforcement layers is more important than the strength of the reinforcement. The authors suggested that the reinforced soil exhibited a special behavior when it has close spacing reinforcement (less than 16 inches). Finally, the publication stated that GRS with a close spacing of the reinforcement should be designed using an appropriate
design methodology that incorporates the special behavior produced by close reinforcement spacing.

“Vegas GRS Mini Pier Experiment and the Postulate of Zero Volume Change” (Adams et al. 2002)

A large scale GRS mass (“Vegas Mini Pier”) was constructed and instrumented for the National Concrete Masonry Association Exposition. It consisted of an 8 ft high structure that was square in plan view with each side of the square being 3.5 ft long. The soil was compacted coarse grained fill, classified as poorly graded-silty gravel and reinforced with layers of woven geotextile. The spacing of the reinforcement layers was 6 inches in the main portion of the pier body, and the top layers were at 3-inchs spacing. The faces of the pier consisted of modular blocks, and the geosynthetic layers extended between the layers of blocks to provide frictional connections between the geotextile layers and the facing blocks.

The pier was monitored during vertical loading. The vertical loading consisted on hydraulic jacks that compressed the reinforced soil through a base reaction pad at the base of the structure and a concrete pad of 3 by 3 ft on the top. Vertical and lateral deformations were determined during loading. From this experiment, the postulate of “zero volume change” was formulated, which means there is no net change in the volume of GRS mass because the volume change due to vertical deformation was fully compensated by the lateral deformations.

This experiment has a high aspect ratio and the concrete pad extends close to the concrete block facing. It seems likely that loads from the concrete pad were transferred to the concrete facing blocks by shear stresses acting on the vertical contact between the fill soil and the facing blocks in a classical soil arching action. The impacts that this phenomenon could have on the test results is not discussed by Adams et al. (2002).

“Geosynthetic Reinforced Soil Performance Testing – Axial Load Deformation Relationships” (Nicks et al. 2013)

Continuing with the Vegas Mini Pier experimentation, a modified version of tests referred to as “performance tests” was developed by the FHWA (Federal Highway Administration). This new version of the test used concrete masonry unit blocks as facing elements while the mini pier test used segmental retaining wall blocks. These new performance test's dimensions are 6.4 ft tall, with 3.2 ft on side of a square cross section. The materials were similar to the mini pier tests, except for the soil, and open-graded and well-graded materials were used. The spacing among reinforcement layers varied from 4 to 16 inches. Nineteen performance tests were run in total and the principal objectives for this study, among others, were to: generate a data base to calibrate design procedures, study the relationship between strength and spacing on the reinforcement, and analyze the influence of the new blocks as the facing elements.

The study presented a database on the stress-strain behavior of the reinforcement soil subjected to vertical loading. The study also identified the parameters that affect the stress-strain response: preloading, soil angularity, compaction level, bearing type, and facing confinement. Another
outcome from the study resulted the differences between using open graded and well graded granular soils. Both can be used into GRS; however, each soil type will exhibit its own particularities. Many tests showed that the well-graded soil behaves stiffer and stronger than the open-graded soil under GRS with vertical loading. Similar to the previous study, this investigation emphasized the importance of reinforcement spacing on capacity of the GRS system. Finally, this investigation proposed guidelines to be applied to existing design methodologies.

Like the pier described by Adams et al. (2002), the performance tests described by Nicks et al. (2013) also have high aspect ratios and may be subject to arching and load transfer to the block facing elements, which would reduce the vertical stress at the bottom of the GRS mass compared to the applied stress at the top. If this effect is significant, the "stress-strain" relationships from these tests would not represent the stress-strain response the GRS itself.

“Composite Behavior of Geosynthetic Reinforced Soil Mass” (Wu et al. 2013)

Large scale plane strain tests called “generic soil geosynthetic composite (GSGC)” tests on granular reinforced soil were developed for this study. The test dimensions were: 2 m in height, and 1.4 m in depth and 1.2 m long. The reinforcement layers had a spacing of 0.2 m or 0.4 m. A sophisticated apparatus was used to maintain the plane strain condition. Regarding the materials used in the tests, the soil was well graded gravel (USCS classification) and the geosynthetic was a woven polypropylene geotextile. The GRS facing consisted of hollow concrete blocks.

The principal research objectives were to: investigate existing theories of GRS behavior, analyze compaction-induced stress effects, and determine the effects of some parameters on GRS behavior.

The test program consisted of five GSGC tests using different geosynthetic configurations (one, two, or zero geotextile sheets at each reinforcement level), different confining pressures, and two different reinforcement spacings. All the tests included monitoring of vertical and lateral movements, internal soil movements, and reinforcement strains. This laboratory experimentation was complimented with finite element numerical simulations.

Some of the main contributions of this study are the test data itself and the development and assessment of analytical models to account for compaction-induced stress and compute lateral displacements. The study produced a new equation to determine the increment in confining stress due to reinforcement. This expression includes the particle size parameter, and it provides required reinforcement tensile strength.

2.4. Numerical Investigation of Geosynthetic Reinforced Soils

Several published references describing numerical analyses of GRS were reviewed. The objective of most of these studies was to represent the response of a physical test or tests through numerical simulation to validate the numerical methods employed. Some of the documented cases use older version of the two-dimensional explicit finite difference numerical program FLAC (Itasca 2011) for their studies.
Four of the most important references describing numerical analyses of GRS are summarized here. Additional information about the three dimensional explicit finite difference numerical program FLAC3D (Itasca 2012), which was used in this research is provided in Sections 3.1, 4.1, 4.2, and 5.1.

“Behaviour of Geosynthetic Reinforced Soil Retaining Walls Using the Finite Element Method” (Karpurapu and Bathurst 1995)

Finite element numerical modeling in two dimensions was performed to improve understanding of the behavior of GRS retaining walls. The model was calibrated against two full size walls constructed at the Royal Military College of Canada (RMCC). The code used for the numerical modeling was developed by the authors.

The GRS walls used for calibration purposes were 4 m high, 2.4 m wide, and 6 m long, the materials used in construction of those walls included dense granular sand and layers of biaxial geogrid reinforcement. The facing elements for two of the walls were reinforced concrete panel units. One wall had the panel units bolted together while the other had layers of soft foam rubber between the panels in order to permit vertical compressibility. The wall construction methods also differed. For the wall with foam rubber between panels, the panels were supported externally as the fill was placed and compacted, and the external support was then removed after the fill for that layer of panels was completed. For the wall with bolted panels, all of the panels over the entire wall height were supported during the placing of all the fill soil. Then, the support was removed after all of the fill had been placed. The walls were loaded by inflating air bags located between the top soil of the walls and the facility until failure was reached. Failure was determined by excessive lateral displacements on the wall.

Regarding the constitutive models for each wall material component, the soil was represented using a modified hyperbolic model including dilative behavior. The model was able to capture stiffness, strength, and plastic behavior characteristics. The soil model required a relatively small number of soil properties, and those can be determined by simple laboratory testing. The reinforcement used a model that replicates the geosynthetic stiffness as a function of strain and time. The interfaces (soil-panel and soil-geosynthetic interfaces) use six-noded joint elements which stiffness matrix used different normal and tangential stiffness. Separation between materials at the interfaces was possible when the normal stress dropped to zero, and during compression, perfect bonding was assumed. The tangential behavior was modeled using a “stick-slip” type of formulation, in which perfect bonding is assumed when the tangential stresses at the interface are lower than the interface strength, and when shear stresses reach the interface shear strength, relative displacements of the materials are allowed. The interface strength followed a frictional law.

The numerical model used a finite element code developed at the RMCC called GEOFEM. The model included different types of mesh elements, including quadrilateral, triangular, interface, uniaxial, and nodal link elements to represent different material components of the walls. The
model code updated the stiffness matrix at every interaction, and large strain mode was allowed. The construction procedure for each wall was captured by the numerical model by turning on the gravity forces on the respective mesh elements and at the respective times as construction progresses. The external support each wall received during construction was simulated in the model using springs.

Two dilation angles were used in the numerical modeling, one run used zero dilation and other used 15 degrees. The first model over-predicted lateral displacements and reinforcement strains. The dilation of 15 degrees produced good agreement between the calculations and the test data, including displacements and stresses. The numerical model was able to reproduce the effects of the different construction method as well as the failure type.

“A Calibrated FLAC model for Geosynthetic Reinforced Soil Modular Block Walls at End of Construction” (Hatarni et al. 2003) and “Development and Verification of a Numerical Model for the Analysis of Geosynthetic-Reinforced Soil Segmental Walls under Working Stress Conditions” (Hatami and Bathurst 2005)

Numerical modeling using the two-dimensional FLAC version 3.4 program (Itasca 2011) was carried out to simulate the construction process of three large scale GRS walls constructed at the Royal Military College of Canada.

The physical models for calibration analysis were three walls reinforced with polypropylene geogrid in three different configurations: the first wall had six layers of reinforcement at spacing of 0.6 m, the second wall had the same layers but the reinforcement had only 50% of the strength and stiffness of the first wall, and the third wall had reinforcement with the same strength properties as the first wall but only four reinforcement layers at a spacing of 0.9 m. The reinforcement layers had rigid connections with the facing elements. The facing material of the walls was solid masonry concrete blocks with “continuous concrete shear keys”. The fill was clean uniform sand that classified as SP. The walls were 3.6 m high, and the configuration of the walls permitted a plane strain behavior at the monitored location. The instrumentation on the walls measured the horizontal displacement of the facing, forces at the toe of the walls, reinforcement strains, and backfill settlements.

The numerical models were calibrated against the end-of-construction stages of the walls. The soil was modeled as a cohesionless material with a Mohr-Coulomb failure criterion. Dilation behavior was included in the model. Regarding the soil elastic response, it used two models: (1) elastic-perfectly-plastic, and (2) stress-dependent hyperbolic model. Reinforcements were modeled with elastic-plastic cable elements with nonlinear stiffness, where the load-strain behavior was represented with a parabolic type of response. The cable elements of the reinforcement layers were rigidly attached to the grid points on the soil mesh. In this form, interface interaction between reinforcement and soil was not permitted. The facing elements were represented with a linear elastic model and with null elements having zero thickness in the separation between block rows. The interfaces between the soil and blocks and between the blocks were modeled with a spring-
slider system where the normal and shear stiffness were chosen to reduce overlap while still limiting computation time.

The numerical model results showed an acceptable match with physical results. The best results were obtained when the numerical model used the hyperbolic nonlinear model for the soil, with back calculation of the soil stress-strain properties. The calculated lateral displacements were in good agreement with the physical results at working stage loading (end of wall construction) for the three analyzed walls. The reinforcement strains resulting from the numerical model mostly matched the distribution and magnitude of the physical results at the end of construction, with a few relatively small discrepancies. The measured forces at the wall toes were compared with the results of the numerical calculations, and some differences occurred at early stages of construction. However, at the end of wall construction, the match was very good. One of the important conclusions of these studies was the usefulness of determining elastic modulus values for plane strain conditions instead of using results from triaxial tests.

“Effects of Geosynthetic Reinforcement Spacing on the Performance of Mechanically Stabilized Earth Walls” (Vulova and Leshchinksy 2003)

This reference constitutes a large FHWA report that details a numerical model developed using the two-dimensional computer program FLAC version 3.4 (Itasca 2011) to simulate the behavior of mechanically stabilized earth walls (MSEW). The main objective was to study the influence of various parameters on wall behavior, with emphasis on reinforcement spacing. The study also identified different failure mechanisms and related them mainly to reinforcement spacing. A parametric study changing reinforcement spacing and soil properties, among other parameters, is presented and some conclusions from them are derived.

The numerical model represented a wall with modular block facing and geosynthetic reinforcement, and it included construction stages of the wall until the wall fails due to gravitational loading. The construction stages incorporated in the model include: modeling of the actual wall foundation, placement of the first row of facing blocks, placement of the first layer of backfill soil, placement of the first reinforcement layer, and then continuing with subsequent layers of facing block, soil, and reinforcement until failure under gravity forces was reached. Soil below and behind the reinforced wall was incorporated to simulate real conditions. Failure was detected in the numerical model through excessive deformations identified by the computer program's “bad geometry” error message.

This study used specific models for each material. The soil had a cohesionless plastic constitutive model with a Mohr-Coulomb failure criterion and with a hyperbolic stress-strain relationship. Interfaces for block-to-block, block-to-soil and soil-to-reinforcement are modeled in such a way that sliding and separation was permitted using interaction elements available in the computer program. Reinforcement layers were modeled using structural elements. Each reinforcement layer was divided into two sections, one for the geosynthetic between blocks and the other for the geosynthetic embedded in the soil. The geosynthetic reinforcement between the blocks were modeled using beam elements that can sustain tension and bending moments.
reinforcement embedded in the soil was modeled using cable elements that can only sustain tension. The facing blocks used a linear elastic constitutive model.

The parametric study included variables such as reinforcement spacing, soil type, reinforcement type, and connection type. From this parametric study, some conclusions were obtained. Four wall failure mechanisms were identified, and these could be related to physical failure mechanisms considered in limit equilibrium slope stability analysis. The four failure modes are presented in Figure 4 and are classified basically as a function of the sliding surface position. The reinforcement spacing was the parameter with the largest influence on the wall behavior, including direct influence on the failure mechanism. Effects of other parameters such as reinforcement stiffness, connection strength, foundation stiffness, and presence of a secondary reinforcement layer were also assessed. Decreasing reinforcement stiffness generated a decrease in the critical wall height, and affected the mode of failure. Stronger connection strength increased the wall stability and decreased the wall displacements. Similar effect was showed if the foundation stiffness and strength was increased. The overall stability of the wall is increased if secondary reinforcement layers are incorporated into the model.

![Figure 4: Failure modes](image)

Figure 4: Failure modes: (a) external slip surface; (b) deep-seated slip surface; (c) compound slip surface; and (d) internal slip surface (after Vulova and Leshchinksy 2003)
2.5. Geotextile Properties and Interface Soil-Reinforcement through Experimentation

Literature sources that present information about soil-geotextile interfaces and geotextile properties are included in this section. Most of the sources present interface friction coefficient values, and few refer to the stiffness of the interface. Regarding geotextile properties, this review focus on Poisson's ratio because this property is not often included in the property data sheets provided by geosynthetic manufacturers. A brief summary of the principal sources is presented in Table 3, and then in Section 6.2, more details and specific values are presented as needed for the analyses described later in Section 6.3.
Table 3: Literature sources on geotextile and interface properties

<table>
<thead>
<tr>
<th>Source</th>
<th>Studies</th>
<th>Key Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long et al. (1983)</td>
<td>Triaxial tests on soil reinforced with bronze discs are investigated; the objective is to determine what happens at the interface zone. Shear strains on reinforcement faces and friction with soil are evaluated.</td>
<td>Evolution from $K_o$ (at rest lateral coefficient) at the center of the sample to $K_a$ (active coefficient) at the edges of the reinforcement. Influence of the spacing of reinforcements (less space makes the reinforcement more effective)</td>
</tr>
<tr>
<td>Lee and Manjunath (2000)</td>
<td>Large direct shear tests on soil-reinforcement interface. Granular soil with three different types of geotextiles was tested. The objective is to investigate the interface friction coefficient.</td>
<td>Addresses peak and residual shear stresses, geotextile surface damage after testing. Presents results of friction angles in the interface. Contrasts initial stiffness of interface with unreinforced soil. Results from a modified apparatus for direct shear tests are included.</td>
</tr>
<tr>
<td>Basudhar (2010)</td>
<td>Direct shear tests on soil-geotextile interfaces to investigate shear-force-displacement behavior. Two geotextiles with different textures are tested.</td>
<td>Coarse textured geotextile (no openings) presents higher shear strength. Non-linear constitutive model to predict interface behavior is proposed. Methodology differs for pre-peak and post-peak stages.</td>
</tr>
<tr>
<td>Koerner (2012)</td>
<td>This book includes general discussions about geosynthetic composition, properties, uses, and design of different structures using geosynthetics.</td>
<td>Extended explanations on geotextile classification. General overview of geotextile properties, and more in depth analysis of soil-geosynthetic interface properties. Especially useful for determining interface friction angle (in Chapters 2 and 5 of the book).</td>
</tr>
<tr>
<td>Shukla et al. (2009)</td>
<td>Theoretical analysis to obtain an expression of the Poisson's ratio at zero strain.</td>
<td>A chart of Poisson's ratio as a function of axial strain to obtain Poisson's ratio at zero strain, is developed.</td>
</tr>
<tr>
<td>Kutay et al. (2006)</td>
<td>Twelve tensile tests on different geosynthetics were performed to understand strain behavior of geosynthetics. The influence of production defects, punctured zones and seams in the strain behavior were analyzed. Differences in results due to type of clamping techniques (hydraulic grips and roller grips) were analyzed too.</td>
<td>Optical flow techniques were successfully used to monitor results. Type of clamping analysis returned a better performance for roller grips. Strain distribution on geosynthetics resulted highly influenced by seam type, and presence of punctured zones. Higher strains were measure around seam zones. Presented results on lateral strains and axial strain, from which Poisson's ratios can be calculated, with results between 0.2 and 2.</td>
</tr>
<tr>
<td>Veldhuijzen van Zanten (1986)</td>
<td>Book on geotextiles and geomembranes. It analyzes topics like geosynthetics history, product manufacture, properties, and applicability.</td>
<td>Important source on geotextiles properties, including typical ranges of variation for Young's modulus for different geotextiles. Also includes different biaxial tests results (in graph form: stress vs. strain) for some geotextiles.</td>
</tr>
<tr>
<td>Ingold (1994)</td>
<td>Book on geotextiles and geomembranes. Includes discussion about manufacturing process, properties, design, standards, and specifications.</td>
<td>Excellent source on data for older commercial geotextiles (product directory). Presents some useful properties as well as material descriptions. Presents information on typical ranges of variation of elastic modulus as a function of the composition.</td>
</tr>
</tbody>
</table>
2.6. Unanswered Questions about Geosynthetic Reinforced Soil

The publications summarized in this chapter demonstrate how extensive the research performed on reinforced soil systems has been over the years. However, several questions remain unanswered:

- Why there is no widely accepted theory on reinforced soil behavior?
- Why there is a special composite behavior when small spacing reinforcement layers are used?
- Why and how is volumetric strain important for reinforced soil systems?
- Does the zero volume change postulate represent real GRS response? Some authors indicate that there is a decrease in volumetric strain due to reinforcement, while others show an increase on volumetric strain after the soil has been reinforced.

All these questions indicate there is more to learn about the basic behavior of geosynthetic reinforced soil systems.
CHAPTER 3:
SOIL MODEL

This chapter details the process to obtain an adequate soil constitutive model, which is an important component of a complete GRS mass numerical model. The commercial three dimensional explicit finite difference program FLAC3D is used to simulate GRS behavior in this research. This program has been chosen due it is designed for soil-structure interaction analyses, it contains some useful constitutive models and elements for GRS, and it is a 3D program, so it can be used for full GRS representation.

The present chapter is divided into five sections. The Section 3.1 contains an overview on the chosen FLAC3D constitutive model to represent the soil component (Chsoil model). The operation of the model is explained for general application conditions in 3D into Section 3.1.1, and then for application to axisymmetric loading conditions into Section 3.1.2. The second, Section 3.2 presents manipulations of the model equations to express the Chsoil model in terms of triaxial test outcomes.

The third section of the chapter, Section 3.3, analyses how the Chsoil model works by discussing some of its main features (Section 3.3.1) and by contrasting this model with the well-known Duncan and Chang hyperbolic model (Section 3.3.2).

Section 3.4, describes improvements made to the Chsoil model so it can better represent the stress-strain and volume change response of compacted coarse grained soils like those typically used in GRS.

Section 3.5 describes two methodologies to obtain parameter values for the improved Chsoil model from triaxial compression test data or from general soil characteristics.

3.1. Chsoil Model in FLAC3D (Itasca 2012)

FLAC3D Version 5.0 (Itasca 2012) has 15 implemented and readily available constitutive models to characterize material behavior. These constitutive models can be grouped into three categories: the null model, the elastic models (3 in total), and the elasto-plastic models (11 in total). The differences among the group of elasto-plastic models lie in the yield function, hardening/softening law, and the plastic flow rule used (Itasca 2012). The aforementioned features are key in the simulation capabilities of a constitutive model/law and the behaviors that it is able to reasonably capture. Therefore, different elasto-plastic models in FLAC3D are recommended for different applications. For the granular soil that is the subject of this investigation, the "Chsoil" model is the most appropriate one because: it has the capacity to better fit the response of granular soils in a relatively simple way due to its built-in strain-hardening law, its includes useful options for incorporating dilation behavior, and the necessary set of input parameters can be obtained with a reasonable amount of engineering effort through laboratory testing and use of correlations.
The Chsoil Model is also referred to as the Simplified Cysoil Model (Itasca 2012), and it is a simplified version of the FLAC3D cap-yield (Cy) soil model. The Chsoil Model includes a nonlinear stress-strain law, a Mohr-Coulomb type of yielding criterion, a non-associated potential function to prescribe plastic flow direction, and it can capture dilative behavior with two built-in dilation laws. In the following, all of the model's features are explained in detail using the equations in the FLAC3D Constitutive model manual (Itasca 2012) for consistency. All the equations use effective stresses represented by $\sigma$.

### 3.1.1. Chsoil Model in FLAC3D Applied to 3D Conditions (Itasca 2012)

This section explains the Chsoil model features when the model is applied to 3D conditions.

#### Elastic behavior:

The Chsoil model uses Hooke's law to establish the relationship between elastic stress increments and the corresponding strain increments as follows:

\[
\begin{align*}
\Delta \sigma_1 &= \alpha_1 \Delta e^e_1 + \alpha_2 (\Delta e^e_2 + \Delta e^e_3) \\
\Delta \sigma_2 &= \alpha_1 \Delta e^e_2 + \alpha_2 (\Delta e^e_1 + \Delta e^e_3) \\
\Delta \sigma_3 &= \alpha_1 \Delta e^e_3 + \alpha_2 (\Delta e^e_1 + \Delta e^e_2)
\end{align*}
\]

where:

- $\Delta \sigma_i$ is the principal effective stress increment in direction $i$
- $\Delta e^e_i$ is the principal elastic strain increment in direction $i$
- $\alpha_1 = K^e + 4 \frac{G^e}{3}$
- $\alpha_2 = K^e - 2 \frac{G^e}{3}$
- $K^e$ is the tangent elastic bulk modulus
- $G^e$ is the tangent elastic shear modulus

These two elastic moduli exhibit the following equations:

\[
K^e = K_{ref} \frac{p_{ref}^m}{p_{ref}'}
\]

where:

- $K_{ref}$ is the bulk modulus number
- $p_{ref}$ is the reference pressure (atmospheric pressure)
- $m$ is a constant modulus exponent, $m \leq 1$
- $p_{ref}'$ is the initial mean effective stress
\( G^e = G_{ref} p_{ref} \left( \frac{p_m'}{p_{ref}} \right)^n \)

where:

- \( G_{ref} \) is the shear modulus number
- \( n \) is a constant modulus exponent, \( n \leq 1 \)

Note that the product \( G_{ref} p_{ref} \) is the value of the tangent elastic shear modulus at the reference pressure, and the product \( K_{ref} p_{ref} \) is the value of the tangent elastic bulk modulus at the reference pressure.

- **Nonlinear stress-strain behavior:**

Nonlinear stress-strain behavior uses a hyperbolic formulation for the plastic shear strain component only (Puebla et al. 1997). This plastic shear strain is controlled by the sine of the mobilized friction angle: \( \sin \phi_m \). The following differential equation shows the relationship between these two parameters:

\[
d(\sin \phi_m) = \frac{G^P}{p_m'} d(\gamma^P)\]

where:

- \( \phi_m \) is the mobilized friction angle
- \( \gamma^P \) is the plastic shear strain, which from now on will be called: hardening parameter for shear yielding
- \( G^P \) is the plastic shear modulus. This is equal to:

\[
G^P = G^e \left( 1 - \frac{\sin \phi_m}{\sin \phi_f} R_f \right)^2 \quad \text{with } \phi_m \leq \phi_f
\]

- \( \phi_f \) is the ultimate friction angle
- \( R_f \) is the failure ratio

Failure ratio \( (R_f) \) is defined as sine of the failure friction angle over sine of ultimate friction angle. Where this last parameter represent the ultimate strength from the best fit hyperbola (Puebla et al. 1997):

\[
R_f = \frac{\sin \phi_f}{\sin \phi_{ult}}
\]
The resultant equation Chsoil model uses to generate a nonlinear stress-strain behavior is a built-in friction strain-hardening law that uses a mobilized friction angle as a function of the hardening parameter for shear yielding, as presented in Equation 3-9. This equation comes after substitution of Equation 3-5 into Equation 3-7, then into Equation 3-6, and after integration with zero mobilized friction angle at zero hardening parameter for shear yielding.

\[
sin\phi_m = \frac{sin\phi_f}{R_f} \left(1 - \frac{1}{1 + \gamma^p \left(\frac{G_e}{p'_m} \left(\frac{R_f}{sin\phi_f}\right)\right)}\right)
\]

where:

\[\phi_m\] is the mobilized friction angle
\[\phi_f\] is the ultimate friction angle
\[R_f\] is the failure ratio
\[\gamma^p\] is the hardening parameter for shear yielding

- **Yield criterion:**

The Chsoil model uses the classical Mohr-Coulomb criterion to define the yield surface, expressed as:

\[
\sigma_1 = \sigma_3 \frac{1 + sin\phi_m}{1 - sin\phi_m} - 2c \sqrt{1 + sin\phi_m} \left(\frac{1}{1 - sin\phi_m}\right)
\]

where:

\[\sigma_1\] is the major principal effective stress
\[\sigma_3\] is the minor principal effective stress
\[c\] is the cohesion
\[\phi_m\] is the mobilized friction angle

The model controls if yield criterion has been reached through the mobilized friction angle attained into each step. Unloading is elastic into the model and so it is reloading until the maximum yield envelope previously attained is reached.
• **Failure criterion:**

Failure occurs when the mobilized friction angle reaches a maximum value, called final friction angle ($\phi_f$). The final friction angle is an input for the model while mobilized friction angle changes as a function of the hardening parameter for shear yielding following Equation 3-9.

• **Plastic flow:**

Elasto-plastic constitutive models require the definition of an elastic function, a yield criterion and a plastic flow rule to represent the material behavior. The elastic function determines the stress-strain relationship under the yield criterion where the strains are fully recovered when the stresses are removed. The yield criterion determines the limit under which only elastic behavior applies and over which both elastic response and plastic flow occur. Plastic behavior induces an irrecoverable strain response. This irrecoverable strain response needs to be specified or determined, and a flow rule is used to do that. The flow rule uses a “potential function” usually represented by the letter “g”, to specify the direction of the plastic strain increments, which are normal to the plastic potential function. When the potential function is parallel to the yield criterion function, it is called “associated”, otherwise it is called a “non-associated potential function”. (Wood 1990)

Plastic flow for the Chsoil model is described by the following non-associated potential function:

$$g = \sigma_1 - \sigma_3 N_{\psi} \quad \text{3-11}$$

where:

$$N_{\psi} = \frac{1 + \sin \psi_m}{1 - \sin \psi_m}$$

$\psi_m$ is the mobilized dilation angle

As a consequence of Equation 3-11 and with the principal plastic shear-strain increment in direction 2 equal to zero ($\Delta e_2^p = 0$), Equation 3-12 can be obtained. This expression relates other two principal plastic shear-strain increments in directions 1 and 3 for a 3D application.

$$\Delta e_3^p = -\Delta e_1^p \left( \frac{1 + \sin \psi_m}{1 - \sin \psi_m} \right) \quad \text{3-12}$$

where:

$\Delta e_i^p$ are the principal, plastic shear-strain increments in the directions $i=1,3$
• **Hardening parameter for shear yielding:**

The increment of the hardening parameter for shear yielding is an important parameter within the framework of the Chsoil constitutive model. It directly influences the non-linear stress strain behavior through Equation 3-9, and it can also be used to generate input for the mobilized dilation angle via a user-defined table correlating the two parameters. The increment of the hardening parameter for shear yielding is equal to the following equation, which is the second invariant of the plastic shear-strain increment tensor:

\[
\Delta \gamma_p = \sqrt{\frac{(\Delta e_p^1 - \Delta e_m)^2 + (\Delta e_p^3 - \Delta e_m)^2 + (\Delta e_m)^2}{2}}
\]

where:

- \(\Delta e_p^i\) are the principal, plastic strain increments in the directions \(i=1,2,3\)
- \(\Delta e_m\) is the mean plastic strain increment, which equals \((\Delta e_p^1 + \Delta e_p^2 + \Delta e_p^3)/3\) this reduces to \((\Delta e_p^1 + \Delta e_p^3)/3\) because \(\Delta e_p^2 = 0\)

• **Dilative behavior:**

Three methods are available to model dilative behavior by means of the mobilized dilation angle. The first option requires the specification of a table with different values of dilation angles at discrete hardening parameters for shear yielding. The second option is to use a built-in equation based on Rowe's stress-dilatancy theory (Rowe 1962). The third option is a rather simple method, wherein:

\[
\psi_m = 0 \text{ when } \phi_m < \phi_{cv} \quad \text{and} \quad \psi_m = \psi_f \text{ when } \phi_m > \phi_{cv}
\]

The ultimate dilation angle (\(\psi_f\)) and \(\phi_{cv}\) are required as inputs. It is worth noting that \(\phi_{cv}\) in FLAC3D is called the constant volume (cv) angle, but it refers to the mobilized friction angle that serves as the limit between application of a dilation of zero and a dilation different from zero (Itasca 2012); \(\phi_{cv}\) as used in FLAC3D is not a constant volume friction angle. The present research uses the third option to model the dilative behavior of soils. This option is easy to implement and gives good results. When \(\psi_m = 0\), the soil is non-dilative, and when \(\psi_m = \psi_f = \phi_m\) the plastic flow is associated. Typically, \(\psi_f < \phi_f\), and \(\psi_f\) increases as the relative density of a compacted granular soil increases.
3.1.2. Chsoil Model in FLAC3D for an Axisymmetric Loading Condition

The present investigation deals with reinforced soil subjected to conventional triaxial loading of laboratory specimens, which is axisymmetric loading. Therefore, the main interest in this section is to present details of the Chsoil model under axisymmetric loading conditions, which produces simplifications of the equations. In addition, plastic strain increments can occur in all three principal stress directions because the intermediate and minor principal stresses are the same. It is useful to present the differences between the formulation for the general case and the axisymmetric case:

- **Elastic behavior:**

  The elasticity formulation of the Chsoil constitutive model is simplified for axisymmetric loading according to Equations 3-14 to 3-18. The elastic strain increment in the intermediate direction 2 would be equal to the increment in direction 3, and the same holds true for the stress increments. Equations 3-14 and 3-15 correspond to Equations 3-1 to 3-3:

  \[
  \Delta \sigma_1 = \alpha_1 \Delta e_1^e + 2\alpha_2 (\Delta e_3^e) \quad 3-14
  \]

  \[
  \Delta \sigma_2 = \Delta \sigma_3 = \alpha_1 \Delta e_3^e + \alpha_2 (\Delta e_1^e + \Delta e_3^e) \quad 3-15
  \]

  When the effective stress confining pressure is held constant, the increments in \( \sigma_2 \) and in \( \sigma_3 \) are zero, therefore Equation 3-15 yields:

  \[
  \Delta e_3^e = \frac{-\alpha_2 (\Delta e_1^e)}{\alpha_1 + \alpha_2} \quad 3-16
  \]

  And then the increment in the major principal stress can be expressed just as a function of its respective strain increment, as shown in Equation 3-17. The inverse is presented in Equation 3-18:

  \[
  \Delta \sigma_1 = \alpha_1 \Delta e_1^e + 2\alpha_2 \left( \frac{-\alpha_2 (\Delta e_1^e)}{\alpha_1 + \alpha_2} \right) \quad 3-17
  \]

  \[
  \Delta e_1^e = \frac{\Delta \sigma_1}{\alpha_1 - \frac{2\alpha_2^2}{\alpha_1 + \alpha_2}} = \frac{\Delta \sigma_1}{\frac{9G^eK^e}{G^e + 3K^e}} \quad 3-18
  \]
**Plastic strains:**

To produce equal plastic strains in all radial directions for axisymmetric loading, the Chsoil model uses two potential functions, which are defined in Equations 3-19 and 3-20. The resulting relationship between the plastic strain increments is shown in Equation 3-21.

\[
\begin{align*}
    g^1 &= \sigma_1 - \sigma_2 (N_{\phi}) \\
    g^2 &= \sigma_1 - \sigma_3 (N_{\phi}) \\
    \Delta e_3^p &= \Delta e_2^p = -\frac{\Delta e_1^p}{2} \left(1 + \sin \psi_m\right) \left(1 - \sin \psi_m\right)
\end{align*}
\]

The mean plastic strain increment will become:

\[
\Delta e_m = \frac{\Delta e_1^p + 2\Delta e_3^p}{3}
\]

The increment in the hardening parameter for shear yielding for axisymmetric conditions is obtained by substituting Equations 3-21 and 3-22 into Equation 3-13:

\[
\Delta y^p = \sqrt{\frac{3(\Delta e_1^p)^2 (N_{\psi}^2 + N_{\psi} + 1)}{3}}
\]

### 3.2. Development of Equations for a Triaxial Test

Section 3.1 illustrates the formulation of the Chsoil constitutive model as a function of the hardening parameter for shear yielding, elastic strains, plastic strains, dilation angles, friction angles, and stresses. For the Chsoil model to be utilized in a numerical simulation of a triaxial test, plastic strains and elastic strains are difficult to obtain directly from a laboratory triaxial test to obtain model parameter values, and thus the validation and calibration of the model can become difficult. This section presents a recasting of the Chsoil formulation, in order for it to better fit results that are typically obtained from triaxial tests and thus correlate deviatoric stress to axial strains, as well as volumetric strains versus axial strains. This permits model parameter values to be obtained from triaxial compression test data.

The first objective is to obtain an equation that relates the axial strain to the major principal stress (the mobilized friction angle is used instead of the major principal stress for simplicity of the equations, but the mobilized friction angle can be converted back to the major principal stress with
knowledge of the minor principal stress using Equation 3-10). In order to obtain the axial stress-strain relationship, the axial strain is decomposed into elastic and plastic strains (Equation 3-24). Then, the variation of each term with respect to the variation of the mobilized friction angle $\phi_m$ (Equation 3-25) is determined. Following the sign convention of FLAC3D and its constitutive models, it is noted that compression is negative:

$$e_1 = e^e_1 + e^p_1 \quad (3-24)$$

$$\frac{\Delta e_1}{\Delta \sin \phi_m} = \frac{\Delta e^e_1}{\Delta \sin \phi_m} + \frac{\Delta e^p_1}{\Delta \sin \phi_m} \quad (3-25)$$

Now it is necessary to find expressions for each term on the right-hand side of Equation 3-25 as a function of triaxial test variables.

First the variation of the plastic strain with respect to the variation on the mobilized friction angle needs to be determined. From the increment of the hardening parameter for shear yielding equation applied to an axisymmetric loading case (Equation 3-23):

$$\frac{\Delta \gamma^p}{\Delta \sin \phi_m} = \sqrt{\frac{3\Delta e^p_1}{\Delta \sin \phi_m}^2 \left(N_\psi^2 + N_\psi + 1\right)} \quad (3-26)$$

Taking the derivative of the hardening parameter for shear yielding with respect to $\sin \phi_m$ (from Equation 3-9 after solving for $\gamma^p$):

$$\frac{d\gamma^p}{d\sin \phi_m} = \frac{-\sigma_3 (\sin \phi_f)^2}{G_e (\sin \phi_f - R_f \sin \phi_m)^2} \quad (3-27)$$

By equating Equation 3-26 to Equation 3-27 and rearranging:

$$\frac{\Delta e^p_1}{\Delta \sin \phi_m} = \sqrt{\frac{3\sigma_3 \sin \phi_f^2}{G_e (\sin \phi_f - R_f \sin \phi_m)^2 \sqrt{3(N_\psi^2 + N_\psi + 1)}}} \quad (3-28)$$

With regards to Equation 3-25, the second term on the right-hand side has been expressed as a function of triaxial parameters in Equation 3-28. Now it is necessary to determine the variation of the elastic strain with respect to the variation on the mobilized friction angle, so that the first term on the right-hand side of Equation 3-25 can be expressed in a similar manner:
From the Mohr-Coulomb yield equation (Equation 3-10), taking the derivative with respect to $sin\phi_m$:

$$\frac{d\sigma_1}{d sin\phi_m} = \frac{2\sigma_3}{(sin\phi_m - 1)^2} + \frac{2c}{(sin\phi_m - 1)(1 - (sin\phi_m)^2)^{1/2}}$$ 3-29

From Hooke's law, using Equation 3-18:

$$\frac{\Delta e^e_1}{\Delta \sigma_1} = \frac{G^e + 3K^e}{9G^e K^e}$$ 3-30

Therefore, from Equations 3-29 and 3-30:

$$\frac{\Delta e^e_1}{\Delta sin\phi_m} = \frac{G^e + 3K^e}{9G^e K^e} \left( \frac{2\sigma_3}{(sin\phi_m - 1)^2} + \frac{2c}{(sin\phi_m - 1)(1 - (sin\phi_m)^2)^{1/2}} \right)$$ 3-31

Substituting 3-28 and 3-31 in 3-25 yields 3-32:

$$\frac{\Delta e_1}{\Delta sin\phi_m} = \left( \frac{2\sigma_3}{(sin\phi_m - 1)^2} + \frac{2c}{3\sigma_3 sin\phi_f^2} \right) \frac{G^e + 3K^e}{9G^e K^e} \frac{3\sigma_3 sin\phi_f^2}{G_e(sin\phi_f - R_f sin\phi_m)^2} \sqrt{3(N^2 + N^2 + 1)}$$ 3-32

Now, the axial strain as a function of the major principal stress (or as a function of $sin\phi_m$) can be obtained. The Chsoil model response can be divided into three stages, and the axial strain formulation for each stage is presented in the following three equations. The notation $e_{1,i}$ is used to distinguish the axial strain ($e_1$) for each of the three different stages, using $i = 0,1,2$ for the three stages. It should be remembered that the sign convention: compression as negative.

- **Axial strain when $\sigma_3 > \sigma_1 > \sigma_3 - 2c$:**

  When the major principal strain falls into this range, $sin\phi_m = 0$, there is no plasticity and the stress-strain behavior follows an elastic relationship. From Equation 3-30:

  $$e_{1,0} = \frac{G_e + 3K^e}{9G^e K^e}(\sigma_1 - \sigma_3)$$ 3-33
- **Axial strain when \( \sigma_1 \leq \sigma_3 - 2c \) and \( \phi_m < \phi_{cv} \)**

This range of stresses, include plastic deformation. Therefore, integration of Equation 3-32, which includes yield criterion into its development, gives Equation 3-34. The limits of integration are \( e_1 \) from \( e_{1sz} \) to an arbitrary \( e_{1,1} \) and \( sin \phi_m \) from zero to any \( sin \phi_m \) (inside the established range \( \phi_m < \phi_{cv} \)); also, and as the specified range suggests, dilation is zero.

\[
e_{1,1} = \frac{R_f B(sin \phi_f - R_f sin \phi_m) - I(sin \phi_m - 1)}{DR_f[R_f sin \phi_m ^2 - (R_f + sin \phi_f)sin \phi_m + sin \phi_f]} - \frac{A(1 + sin \phi_m)}{\sqrt{1 - sin \phi_m ^2}}
\]

\[
e_{1sz} = \left( \frac{I + R_f B sin \phi_f}{DR_f sin \phi_f} - A \right) + e_{1sz}
\]

- **Axial strain when \( \sigma_1 \leq \sigma_3 - 2c \) and \( \phi_m \geq \phi_{cv} \)**

This range exhibits plastic deformation and also dilation. Integration of Equation 3-32, using \( e_1 = e_{1cv} \) at \( sin \phi_m = sin \phi_{cv} \), gives Equation 3-35

\[
e_{1,2} = \frac{R_f B(sin \phi_f - R_f sin \phi_m) - F(sin \phi_m - 1)}{DR_f[R_f sin \phi_m ^2 - (R_f + sin \phi_f)sin \phi_m + sin \phi_f]} - \frac{A(1 + sin \phi_m)}{\sqrt{1 - sin \phi_m ^2}}
\]

\[
e_{1cv} = \left[ \frac{R_f B(sin \phi_f - R_f sin \phi_{cv}) - F(sin \phi_{cv} - 1)}{DR_f[R_f sin \phi_{cv} ^2 - (R_f + sin \phi_f)sin \phi_{cv} + sin \phi_f]} - \frac{A(1 + sin \phi_{cv})}{\sqrt{1 - sin \phi_{cv} ^2}} \right] + e_{1cv}
\]

where:

\[
A = \frac{2G^e c + 6K^e c}{9G^e K^e}
\]

\[
B = \sigma_3(2G^e + 6K^e)
\]

\[
D = 9G^e K^e
\]

\[
I = 9K^e \sigma_3(sin \phi_f)^2
\]

\[
F = \frac{9\sqrt{3}K^e \sigma_3(sin \phi_f)^2}{N_\psi} with N_\psi = \frac{1 + sin \psi_f}{1 - sin \psi_f}
\]

\[
e_{1sz} \text{ is } e_{1,0} \text{ evaluated at } \sigma_1 = \sigma_3 - 2c
\]

\[
e_{1cv} \text{ is } e_{1,1} \text{ evaluated at } \phi_m = \phi_{cv}
\]

\[
\psi_f \text{ is the ultimate dilation angle}
\]
Equations 3-33, 3-34 and 3-35 present the relation between axial strain and mobilized friction angle (which is a direct function of the major and minor principal stresses). When a soil has zero cohesion the model would not present \( e_{1,0} \). When a soil has zero dilation, the model would not present \( e_{1,2} \), and when a soil can be represented with \( \phi_{cv} = 0 \) (\( \phi_{cv} \) cannot be negative) the model would not present \( e_{1,1} \). Otherwise, the model would show all ranges of axial strain and sequentially. With this, the first objective set forward when looking for expressions to fit triaxial test outcomes is satisfied.

The second objective that needs to be fulfilled is the development of a relation between volumetric and axial strains. Relationships between volumetric strain and mobilized friction angle are established in the following pages, and then these expressions can combine with Equations 3-33, 3-34 and 3-35 (which relate mobilized friction angle with axial strains) to obtain the correspondence between volumetric and axial strains.

Based on the decomposition of strain into their elastic and plastic components, the volumetric strain increment is expressed as follows for axisymmetric conditions:

\[
\Delta e_v = \Delta e^e_1 + 2\Delta e^e_3 + \Delta e^p_1 + 2\Delta e^p_3
\]

As done previously, the solution will be expressed in terms for \( \Delta \sin \phi_m \).

\[ \frac{\Delta e^e_1}{\Delta \sin \phi_m} \] is already determined in Equation 3-31.

From Equation 3-16, and knowing that:

\[
\alpha_1 = K^e + 4 \frac{G^e}{3}
\]

\[
\alpha_2 = K^e - 2 \frac{G^e}{3}
\]

Then:

\[
\Delta e^e_3 = \frac{(-3K^e + 2G^e)(\Delta e^e_1)}{6K^e + 2G^e}
\]

Using the established relationships between Young's Modulus \( E^e \) and Poisson's ratio \( \nu^e \) as functions of the bulk and shear moduli (\( E^e = \frac{9K^eG^e}{3K^e + G^e} \), \( \nu^e = \frac{3K^e - 2G^e}{6K^e + 2G^e} \)) we can obtain 3-40 from 3-31 and we can obtain 3-41 from 3-39 and 3-40.
\[
\frac{\Delta e_1^e}{\Delta \sin \phi_m} = \frac{1}{E^e} \left( \frac{2\sigma_3}{(\sin \phi_m - 1)^2} + \frac{2c}{\sin \phi_m - 1}(1 - (\sin \phi_m)^2)^{1/2} \right)  
\]

\[
\frac{\Delta e_3^e}{\Delta \sin \phi_m} = \frac{-\nu}{E^e} \left( \frac{2\sigma_3}{(\sin \phi_m - 1)^2} + \frac{2c}{\sin \phi_m - 1}(1 - (\sin \phi_m)^2)^{1/2} \right)  
\]

\[
\frac{\Delta e_1^P}{\Delta \sin \phi_m} \text{ is given in Equation 3-28, and } \frac{\Delta e_3^P}{\Delta \sin \phi_m} \text{ can be determined from 3-28 and 3-21, producing the expression:}
\]

\[
\frac{\Delta e_3^P}{\Delta \sin \phi_m} = \frac{-N_p}{2} \left( \frac{3\sigma_3 \sin \phi_f^2}{G_e (\sin \phi_f - R_f \sin \phi_m)^2 \sqrt{3(N_p^2 + N_p + 1)}} \right) 
\]

Substituting these four expressions (3-40, 3-41, 3-42 and 3-28) in Equation 3-36 divided by \( \Delta \sin \phi_m \) yields:

\[
\frac{\Delta e_v}{\Delta \sin \phi_m} = \frac{(1 - 2\nu)}{E^e} \left( \frac{2\sigma_3}{(\sin \phi_m - 1)^2} + \frac{2c}{\sin \phi_m - 1}(1 - (\sin \phi_m)^2)^{1/2} \right)  
\]

\[
+ \left( \frac{3\sigma_3 \sin \phi_f^2(1 - N_p)}{G_e (\sin \phi_f - R_f \sin \phi_m)^2 \sqrt{3(N_p^2 + N_p + 1)}} \right) 
\]

The same as for the stress-strain behavior, the model can be divided into three intervals to determine the volumetric strain equations, an additional sub index is used to specify which interval the volumetric strain equation belongs to.

- **Volumetric strain when \( \sigma_3 > \sigma_1 > \sigma_3 - 2c \):**

This range exhibits no plasticity, \( \sin \phi_m = 0 \), there is no plastic strain and elastic theory leads volumetric strain behavior. From 3-30, 3-36 and 3-39 Equation 3-44 is obtained. From this last equation, the integral with \( e_v = 0 \) at \( \sigma_1 = \sigma_3 \) gives the volumetric strain relationship in Equation 3-45.

\[
\Delta e_v = \frac{(\Delta \sigma_1)}{3K^e} 
\]
\[ e_{v,0} = \frac{(\sigma_1 - \sigma_3)}{3K^e} \]  

- **Volumetric strain when \( \sigma_1 \leq \sigma_3 - 2c \) and \( \phi_m < \phi_{cv} \)**

Integration of Equation 3-43 with \( e_v = e_{vsz} \) at \( \sin \phi_m = 0 \), gives Equation 3-46. This range does not present dilation, however it does present plastic strains.

\[ e_{v,1} = \frac{-2Jc(\sin \phi_m + 1)}{\sqrt{1 - \sin^2 \phi_m}} + \frac{2\sigma_3}{1 - \sin \phi_m} - \frac{(2\sigma_3 - 2c)}{(1 - \sin \phi_m)} + e_{vsz} \]  

- **Axial strain when \( \sigma_1 \leq \sigma_3 - 2c \) and \( \phi_m \geq \phi_{cv} \)**

Integration of Equation 3-43 with \( e_v = e_{vcv} \) at \( \sin \phi_m = \sin \phi_{cv} \), gives Equation 3-46. This range does present dilation and plastic strains.

\[ e_{v,2} = \frac{-2Jc(\sin \phi_m + 1)}{\sqrt{1 - \sin^2 \phi_m}} + \frac{2\sigma_3}{1 - \sin \phi_m} - \frac{H}{R_f^2\sin \phi_m - \sin \phi_f R_f} - \]

\[ \left[ \frac{-2Jc(\sin \phi_{cv} + 1)}{\sqrt{1 - \sin^2 \phi_{cv}}} + \frac{2\sigma_3}{1 - \sin \phi_{cv}} - \frac{H}{R_f^2\sin \phi_{cv} - \sin \phi_f R_f} \right] + e_{vcv} \]

where:

\[ H = \frac{3\sigma_3(\sin \phi_{f})^2(1 - N_\psi)}{G^e \sqrt{3(N_\psi^2 + N_\psi + 1)}} \]

\[ J = \frac{1 - 2\nu}{E^e} \]

\( \nu^e \) is Poisson's ratio  
\( E^e \) is the tangent initial Young's modulus  
\( e_{vsz} \) is the \( e_{v,0} \) when \( \sigma_1 = \sigma_3 - 2c \)  
\( e_{vcv} \) is the \( e_{v,1} \) when \( \phi_m = \phi_{cv} \)

When the cohesion of the soil is equal to zero, the equations derived above become a little easier to manage. Although it would be possible to extract the versions of the above equations for the
cohesionless case, some terms in the equations would become not determined (by zero denominator fractions). Therefore, it is useful to re-derive them directly as a function of the major principal stress, as follows.

Again, first, the equations that relate axial strain to the major principal stress are obtained. The procedure is very similar to the general case presented above. The axial strain is decomposed into its elastic and plastic components, and the variations on each strain with respect to the variation on the major principal stress (Equation 3-48) are determined:

$$\frac{\Delta e_1}{\Delta \sigma_1} = \frac{\Delta e^e_1}{\Delta \sigma_1} + \frac{\Delta e^p_1}{\Delta \sigma_1}$$  \hspace{1cm} 3-48

To determine the variation of the plastic strain with respect to the variation of the major principal stress:

From the increment of the hardening parameter for shear yielding (Equation 3-23):

$$\frac{\Delta \gamma^p}{\Delta \sigma_1} = \frac{\sqrt{3\Delta e^p_1}^2(N_\psi^2 + N_\psi + 1)}{3\Delta \sigma_1}$$  \hspace{1cm} 3-49

From the yield criterion with zero cohesion:

$$\sin \phi_m = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$  \hspace{1cm} 3-50

The derivative of 3-50 with respect to $\sigma_1$ gives

$$\frac{d \sin \phi_m}{d \sigma_1} = \frac{\sigma_3^2}{(\sigma_1 + \sigma_3)^2}$$  \hspace{1cm} 3-51

Using the chain rule of derivatives, with Equations 3-27 and 3-51 and substituting $\sin \phi_m$ with Equation 3-50:

$$\frac{d \gamma^p}{d \sigma_1} = \frac{-2\sigma_3^2 (\sin \phi_f)^2}{G^e(R_f \sigma_3 - R_f \sigma_1 + \sigma_1 \sin \phi_f + \sigma_3 \sin \phi_f)^2}$$  \hspace{1cm} 3-52
Equating Equations 3-49 and 3-52 and solving for the increment of plastic strain in the major principal stress direction as a function of the increment of major principal stress:

\[
\frac{\Delta e_p}{\Delta \sigma_1} = \frac{6\sigma_3^2 \sin \phi_f^2}{G^e (R_f \sigma_3 - R_f \sigma_1 + \sigma_1 \sin \phi_f + \sigma_3 \sin \phi_f)^2 \sqrt{3 (N_{\phi}^2 + N_{\phi} + 1)}}
\]  

3-53

Substituting Equations 3-53 and 3-30 into 3-48 yields:

\[
\frac{\Delta e_1}{\Delta \sigma_1} = \frac{G^e + 3K^e}{9G^eK^e} + \frac{2\sqrt{3} \sigma_3^2 \sin \phi_f^2}{G^e (R_f \sigma_3 - R_f \sigma_1 + \sigma_1 \sin \phi_f + \sigma_3 \sin \phi_f)^2 \sqrt{(N_{\phi}^2 + N_{\phi} + 1)}}
\]  

3-54

Dividing the model behavior into two intervals, with and without dilation; the following expressions are obtained. The same as it was made previously, an extra sub-index specifies the interval for the axial strain, \(e_{1,0}\) and \(e_{1,1}\) for interval with dilation and without dilation respectively.

- **Axial strain when \(\sigma_3 > \sigma_1 > \sigma_{cv}\):**

When \(\sigma_1\) falls into this range, dilation is zero due \(\sigma_1 > \sigma_{cv}\), where \(\sigma_{cv}\) is the major principal stress at \(\phi_m = \phi_{cv}\); therefore the friction angle does not reach \(\phi_{cv}\) yet. (Remember the sign convention, compression is negative)

Integrating Equation 3-54 using \(e_1 = 0\) at \(\sigma_1 = \sigma_3\) with zero dilation angle:

\[
e_{1,0} = \frac{(G^e + 3K^e)}{9G^eK^e} (\sigma_1 - \sigma_3)
\]

3-55

\[
- \left[ \frac{\sigma_3 \sin \phi_f}{G^e (R_f - \sin \phi_f)} \left( 1 - \frac{2\sigma_3 \sin \phi_f}{(R_f (\sigma_3 - \sigma_1) + (\sigma_1 + \sigma_3) \sin \phi_f)} \right) \right]
\]
• **Axial strain when \( \sigma_1 \leq \sigma_{cv} \)**

This interval would exhibit dilation different than zero if \( \psi_f \neq 0 \). Integrating Equation 3-54 using \( e_1 = e_{1cv} \) at \( \sigma_1 = \sigma_{cv} \):

\[
e_{1,1} = \frac{(G^e + 3K^e)}{9G^e K^e} (\sigma_1 - \sigma_{cv}) + \frac{2\sqrt{3}\sigma_3^2 \sin \phi_f^2}{G^e \sqrt{(N_\psi^2 + N_\phi + 1)}} \]

\[
\left( \begin{array}{c} (\sigma_1 - \sigma_{cv}) \\ R_f(\sigma_3 - \sigma_{cv}) + (\sigma_{cv} + \sigma_3)\sin \phi_f \\ R_f(\sigma_3 - \sigma_1) + (\sigma_1 + \sigma_3)\sin \phi_f \\ + e_{1cv} \end{array} \right)
\]

where \( e_{1cv} \) is \( e_{1,0} \) evaluated at \( \sigma_1 = \sigma_{cv} \)

In order to determine the relationships between volumetric strain and major principal stress:

The variation of volumetric strain is given by the Equation 3-36. Each term of the equation needs to be determined as a function of \( \Delta \sigma_1 \).

Working with the elastic terms of Equation 3-36, increment strain in direction 1 is already obtained into Equation 3-18, increment strain in direction 3 can be obtained as follows:

From Equations 3-2 and 3-3, with \( \Delta \sigma_2 = \Delta \sigma_3 = 0 \), it can be determined that \( \Delta e^{e_2} = \Delta e^{e_3} \). Using this condition into Equation 3-1, Equation 3-57 can be obtained

\[
\Delta e^{e_3} = \frac{\Delta \sigma_1 - \alpha_1 \Delta e^{e_1}}{2\alpha_2}
\]

Regarding the plastic strain terms, Equation 3-53 gives the plastic strain in direction 1 and Equation 3-21 the plastic strain in direction 3 as a function of the plastic strain in direction 1.
The four terms are used into Equation 3-36, after simplification, Equation 3-58 is obtained:

\[
\Delta e_v = 4\sqrt{3}\sigma_3^2 \sin\phi_f^2 \Delta \sigma_1 \left[ \frac{1 - N_\psi + \nu^e - \nu^e N_\psi}{\sqrt{(N_\psi^2 + N_\psi + 1) E^e [R_f(\sigma_3 - \sigma_1) + (\sigma_1 + \sigma_3)\sin\phi_f]^2}} \right] - \Delta \sigma_1 \frac{2\nu^e - 1}{E^e} \tag{3-58}
\]

where:

- \(E^e\) is the elastic tangent initial Young's modulus
- \(\nu^e\) is the elastic Poisson's ratio
- \(\sigma_{cv}\) is the major principal stress at \(\phi_m = \phi_{cv}\)

Two intervals are used for the volumetric strain description, the same as it was made with the stress-strain equations. The first interval has zero dilation and in the second the dilation is equal to the final dilation angle:

- **Volumetric strain when \(\sigma_3 > \sigma_1 > \sigma_{cv}\)**

This interval presents zero dilation, and \(\phi_m < \phi_{cv}\) while \(\sigma_1 > \sigma_{cv}\). Then, \(N_\psi = 1\)

Integrating Equation 3-58 using \(e_1 = 0\) at \(\sigma_1 = \sigma_3\) with zero dilation angle:

\[
e_{v,0} = \frac{(\sigma_1 - \sigma_3)(1 - 2\nu^e)}{E^e} \tag{3-59}
\]

- **Volumetric strain when \(\sigma_1 \leq \sigma_{cv}\)**

This interval would exhibit dilation different than zero if \(\psi_f \neq 0\). Integrating Equation 3-58 using \(e_1 = e_{1,cv}\) at \(\sigma_1 = \sigma_{cv}\):

\[
e_{v,1} = e_{v,0} + \frac{(\sigma_1 - \sigma_{cv})(1 - 2\nu^e)}{E^e} + \frac{4\sqrt{3}\sigma_3^2 \sin\phi_f^2 (1 - N_\psi + \nu^e - \nu^e N_\psi)}{\sqrt{(N_\psi^2 + N_\psi + 1) E^e [R_f - \sin\phi_f]^2}} \tag{3-60}
\]

\[
\left[ \frac{1}{R_f(\sigma_3 - \sigma_1) + (\sigma_1 + \sigma_3)\sin\phi_f} - \frac{1}{R_f(\sigma_3 - \sigma_{cv}) + (\sigma_{cv} + \sigma_3)\sin\phi_f} \right]
\]
3.3. Discussion of Model Performance

The equations defining the Chsoil model that simulates soil behavior have been presented, and they have been recast to better represent axial-stress-versus-axial-strain and volumetric-strain-versus-axial-strain behavior under triaxial compression test conditions with constant confining pressure. This section first describes important characteristics of the Chsoil model, and then comparisons are made between the Chsoil model and the well-established Duncan and Chang hyperbolic model from Duncan and Chang (1970). The description of the model presented in Duncan et al. (1980) is used.

3.3.1. Chsoil Model Behavior

Using the new arrangement of equations derived in the previous section, response of the Chsoil model to triaxial loading can be readily discussed, including the following major points:

- Initial slope of a stress-versus-strain curve:

  When the soil in analysis has cohesion greater than zero, the initial slope is affected just by the elastic parameters, which are the elastic shear modulus and the elastic bulk modulus. The initial slope is given by the following expression, which is the Young's modulus in terms of the shear and bulk modulus. Therefore, the initial slope in a stress-strain curve, $E_o$, of the Chsoil model with cohesion greater than zero is the elastic Young's modulus, $E^e$.

  $$E_o = E^e = \frac{9G^eK^e}{G^e + 3K^e} \quad 3-61$$

  When the soil in analysis has cohesion equal to zero, the initial slope of a stress-strain curve also includes just elastic parameters. However, the initial slope is not the elastic Young's modulus as in the previous case, it is a different expression presented in Equation 3-62. This expression can be obtained from Equation 3-54 using $\sigma_1 = \sigma_3$:

  $$E_o = \frac{18G^eK^e}{2G^e + 15K^e} \quad 3-62$$

  It can be seen by comparing Equations 3-61 and 3-62 that, for the same values of $G^e$ and $K^e$, the initial Young's modulus, $E_o$, is larger for a soil with cohesion than without. Equations 3-61 and 3-62 also show that variations in the shear modulus have a larger influence on the initial slope of the stress-strain curve than equal percentage variation in the bulk modulus.
• Unloading reloading behavior:

A stress-strain curve slope during unloading or reloading is a function of just the elastic parameters, i.e., the elastic shear and bulk moduli, as indicated in Equation 3-63:

\[ E_{ur} = \frac{9G^eK^e}{G^e + 3K^e} \]  

This is the same expression as provided previously in Equation 3-61 for the initial stress-strain slope for a soil with non-zero cohesion. Therefore, the unloading reloading behavior has a uniform slope equal to the initial Young's modulus for a cohesionless soil represented by the Chsoil model. As it was explained in the yield criterion description of the model, unloading-reloading are identified by the mobilized friction angle value.

• Importance of the initial effective mean pressure \( p'_m \):

The initial effective mean pressure is an important parameter for the Chsoil model. This parameter influences the elastic shear and bulk moduli (Equations 3-4 and 3-5), and it influences the strain-hardening law (Equation 3-9). Therefore, \( p'_m \) has a significant impact on the model behavior.

The value of \( p'_m \) is input as a parameter for the Chsoil model, and it is not automatically calculated based on initial stresses. Also, this parameter represents the initial mean pressure, not a mean pressure that changes as the loading conditions change. Because \( p'_m \) is kept constant through the analyses, the elastic shear modulus and the bulk modulus are constants according to Equations 3-4 and 3-5, and they do not change, regardless of changes in mean stress or any other stress change.

In a triaxial test for example, when the initial confining pressure is 5 kN/m\(^2\), the value of \( p'_m \) would be set equal to 5 kN/m\(^2\); however, if in the middle of the test, the confining pressure is increased to 10 kN/m\(^2\), \( p'_m \) is still 5 kN/m\(^2\), and the elastic parameters and the strain-hardening law still use that initial mean pressure of 5 kN/m\(^2\). However, as a consequence of the strain hardening law used by the Chsoil model, the model input parameters can be selected in a way that the soil response can be reasonably represented despite the fact that the elastic shear and bulk modulus do not change during loading. This is illustrated in more detail later in this chapter.
• Sensitivity of stress-versus-strain and volumetric-strain-versus-axial-strain curves to the input parameters:

Stress-strain curves for triaxial compression tests obtained from the Chsoil model are most sensitive to changes in elastic shear modulus and failure ratio, while the other parameters generate relatively little influence in the pre-failure stress-strain curves. Failure values of cohesion and friction angle are key to determine the maximum values in such curves.

Regarding volumetric-strain-versus-axial-strain curves, the parameters of greatest influence are: dilation angle, cv angle, and elastic bulk modulus.

The Chsoil constitutive model has some limitations. One is the fact that the model, as implemented in FLAC3D, requires $p'_m$ as an input parameter, as if $p'_m$ were a fundamental soil characteristic. Other limitations of the model include independence of friction angle and dilatancy parameters from confining pressure. The model would represent real soil response more closely if it calculated the friction angle and dilatancy parameters as a function of the confining pressure.

3.3.2. Comparison of Chsoil Model and Hyperbolic Model

The Chsoil model has characteristics similar to Duncan and Chang (1970) hyperbolic model, e.g., Itasca (2012) calls the Chsoil model an alternative to Duncan and Chang's hyperbolic model. It is worthwhile to contrast the actual response of these two models so their relationship can be understood and the advantages and limitations of the Chsoil model can be better determined.

Duncan and Chang's hyperbolic model is chosen for this comparison study because this model is widely used and well known, and as indicated by Itasca (2012), the Chsoil model is similar to Duncan and Chang's model.

In the following, a summary of the key aspects on the hyperbolic model is presented, and after that, a table of comparisons between the two methods is stated.

Duncan and Chang (Duncan and Chang 1970) developed a hyperbolic model to capture the stress-strain nonlinearity, stress-dependency, and inelasticity of soil behavior. In follows some of the principal aspects of the model are summarized from Duncan et al. (1980). The equations presented use the original nomenclature from the source, however when the symbols are the same than for Chsoil model, to avoid confusion, it is used a sub-index “DC” for the symbols belonging to Duncan and Chang's model. Also, all the stresses presented into the coming equations and explanations are effective stresses.

The model represents the stress-strain behavior of a soil by a hyperbolic curve with the equation:
\[(\sigma_1 - \sigma_3) = \frac{\varepsilon}{E_i} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_{ult}}\]  

where:

- \(E_i\) is the initial tangent modulus
- \(\varepsilon\) is the strain
- \((\sigma_1 - \sigma_3)_{ult}\) is the asymptotic value of stress difference

The initial tangent modulus is a function of the confining pressure with the following expression:

\[E_i = K P_a \left(\frac{\sigma_3}{P_a}\right)^{n_{DC}}\]  

where:

- \(K\) is the modulus coefficient
- \(P_a\) is atmospheric pressure
- \(\sigma_3\) is the confining pressure
- \(n_{DC}\) is the modulus exponent

To permit the hyperbolic Equation 3-64 to reasonably match real stress-strain curves, the asymptotic value of stress difference is larger than the maximum stress difference that can be sustained at soil failure. The relationship between the asymptotic stress difference and the stress difference at failure is specified by the failure ratio \(R_{fDC}\) in the following equation:

\[(\sigma_1 - \sigma_3)_f = R_{fDC}(\sigma_1 - \sigma_3)_{ult}\]  

The inelastic behavior is represented by using a different tangent modulus for unloading-reloading than for the loading portions of the stress-strain curve.

The unload-reload modulus is given by:

\[E_{ur} = K_{ur} P_a \left(\frac{\sigma_3}{P_a}\right)^{n_{DC}}\]  

Finally, the Duncan and Chang hyperbolic model uses a bulk modulus as a function of the confining pressure as shown in Equation 3-68. The bulk modulus value must satisfy some limits.
to avoid values of Poisson's ratio be less than zero or equal to or greater than 0.5. In particular, the bulk modulus cannot be less than $E_i/3$ nor greater than $17E_i$.

$$B = K_b P_a \left( \frac{\sigma_3}{P_a} \right)^{m_{DC}}$$

where:

$K_b$ is the bulk modulus number
$m_{DC}$ is the bulk modulus exponent

Based upon the descriptions of the Chsoil model and the Duncan and Chang hyperbolic model provided above, the essential features of the two models can be compared, as shown in the following table.
Table 4: Correspondence between Chsoil model and Duncan and Chang model equations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Duncan and Chang Model</th>
<th>Chsoil Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unload, reload slope</td>
<td>$E_{ur} = K_{ur} Pa \left( \frac{\sigma_2}{\sigma_a} \right)^{n_{DC}}$</td>
<td>$E_{ur} = \frac{9G^eK^e}{G^e + 3K^e}$</td>
</tr>
<tr>
<td>Initial tangent Young's modulus</td>
<td>$E_i = KPa \left( \frac{\sigma_2}{\sigma_a} \right)^{n_{DC}}$</td>
<td>$E_0 = \frac{9G^eK^e}{G^e + 3K^e}$ for $c &gt; 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$E_0 = \frac{18G^eK^e}{2G^e + 15K^e}$ for $c = 0$</td>
</tr>
<tr>
<td>Failure ratio</td>
<td>$R_{f_{DC}} = \frac{(\sigma_1 - \sigma_3)f}{(\sigma_1 - \sigma_3)_{ult}}$</td>
<td>$R_f = \frac{\sin \phi_f}{\sin \phi_{ult}}$</td>
</tr>
<tr>
<td>Bulk modulus (See Note 1)</td>
<td>$B = K_b Pa \left( \frac{\sigma_3}{\sigma_a} \right)^{m_{DC}}$</td>
<td>$K^e = K_{ref} P_{ref} \left( \frac{p_m}{P_{ref}} \right)^m$</td>
</tr>
<tr>
<td>Elastic shear modulus</td>
<td>—</td>
<td>$G^e = G_{ref} P_{ref} \left( \frac{p_m}{P_{ref}} \right)^n$</td>
</tr>
<tr>
<td>Stress-strain tangent slope</td>
<td>$E_t = \left[ 1 - \frac{R_{f_{DC}}(1 - \sin \phi_f)(\sigma_1 - \sigma_3)}{2c \cos \phi + 2 \sigma_3 \sin \phi} \right]^2 E_i$</td>
<td>See Note 2</td>
</tr>
<tr>
<td>Volumetric strain-axial strain at failure slope</td>
<td>$\frac{\Delta e_v}{\Delta e_1} \approx 0$</td>
<td>$\frac{\Delta e_v}{\Delta e_1} = 1 - \frac{1 + \sin \psi}{1 - \sin \psi}$</td>
</tr>
<tr>
<td>Failure criterion</td>
<td>$\sigma_1 = \sigma_3 \frac{1 + \sin \phi_f}{1 - \sin \phi_f} - 2c \sqrt{\frac{1 + \sin \phi_f}{1 - \sin \phi_f}}$</td>
<td>$\sigma_1 = \sigma_3 \frac{1 + \sin \phi_f}{1 - \sin \phi_f} - 2c \sqrt{\frac{1 + \sin \phi_f}{1 - \sin \phi_f}}$</td>
</tr>
<tr>
<td>Yield criterion</td>
<td>—</td>
<td>$\sigma_1 = \sigma_3 \frac{1 + \sin \phi_m}{1 - \sin \phi_m} - 2c \sqrt{\frac{1 + \sin \phi_m}{1 - \sin \phi_m}}$</td>
</tr>
</tbody>
</table>

Symbols:
- $K_{ur}$ is the unloading-reloading modulus number
- $Pa$ is the atmospheric pressure
- $n_{DC}$ is modulus exponent
- $K$ is the modulus number
- $(\sigma_1 - \sigma_3)_f$ is the stress difference at failure
- $(\sigma_1 - \sigma_3)_{ult}$ is the asymptotic value of stress difference
- $K_b$ is the bulk modulus number
- $m_{DC}$ is the bulk modulus exponent
- $\phi_f$ is the failure friction angle
- $c$ is the cohesion
- $G^e$ is the elastic shear modulus
- $K^e$ is the elastic bulk modulus
- $\phi_{ult}$ ultimate friction angle, which sine represent the ultimate strength from the best fit hyperbola
- $\phi_f$ is the failure friction angle
- $K_{ref}$ is the bulk modulus number
- $P_{ref}$ is the pressure of reference, atmospheric pressure
- $m$ bulk modulus exponent
- $G_{ref}$ is the shear modulus number
- $n$ is the shear modulus exponent
- $p'_m$ is the mean initial effective stress

49
Note 1: $K^e$ is the elastic bulk modulus, while $B$ is the bulk modulus not limited to the elastic component.

Note 2: The inverse of the stress-strain slope for the Chsoil model can be expressed as:

$$\frac{\Delta e_1}{\Delta \sigma_1} = \frac{1}{E_{ur}} - \frac{2\sqrt{3}p_m\sigma_3(sinf)}{G^e \sqrt{N_1^2+N_2^2+1(R_f\sigma_3-R_f\sigma_1+sinf\sigma_1+\sigma_3)^2}} \quad \text{with:} \quad N_1 = \frac{1+sin\psi_m}{1-sin\psi_m}$$

The first row in the table shows that the unload-reload slope for Duncan and Chang's model increases with confining pressure, while for Chsoil model it is constant. The same behavior is observed for the initial tangent Young's modulus, where for Duncan and Chang's model it changes as a function of the confining pressure while for Chsoil model it has a constant value.

The failure ratio (third row of the table) results as a function of stresses in Duncan and Chang's model while in Chsoil model it is a function of stress ratios. Both models use the failure parameter over the ultimate parameter, where for both models the ultimate parameter is used from the best fit hyperbola.

Regarding the bulk and shear moduli in Table 4, in Chsoil model moduli result constants as it was discussed in the previous section. Once a mean initial effective stress is given as an input to the model, the elastic shear modulus and the elastic bulk modulus result constants. Duncan and Chang's model instead exhibits a general bulk modulus calculation which varies as a function of the confining stress, while Duncan and Chang's model does not explicitly state an elastic shear modulus equation.

The stress-strain tangent slope for Duncan and Chang's model is a function of the failure ratio, friction angle, stresses, and initial tangent Young's modulus. For Chsoil model, similar parameters are used, however the elastic shear modulus and the dilation angle are incorporated.

A significant difference between the Duncan and Chang's model and the Chsoil model is that the latter incorporates dilation. That can be demonstrated into the volumetric-strain-versus-axial-strain at failure, whereby for Duncan and Chang's model this ratio is approximately zero, while for Chsoil model it varies as a function of the mobilized dilation angle.

Both models exhibit the same Mohr-Coulomb failure equation, and yield criterion is stated only for Chsoil model and has the same Mohr-Coulomb criterion.

### 3.4. Modifications to Chsoil Model

In this section, modifications of Itasca's (2012) Chsoil constitutive model for the soil are presented. The objective is to improve the stress dependent response of stiffness, friction angle, and dilatancy.
Compacted granular soil material is an important component of a GRS mass, and they have to be well represented by the soil constitutive model selected for numerical analysis. Aspects of model parameter dependencies that are expected to influence the overall response of the soil include: nonlinear stress-strain response, zone-by-zone assignment of pressure-dependent initial stiffness, confining-pressure dependent friction angle, and confining-pressure dependent dilation behavior.

The Chsoil model in FLAC3D is used to represent the granular soil behavior and is capable of capturing several of the response characteristics important to this application. The existing Chsoil model in FLAC3D can already capture the nonlinear stress-strain behavior, but it is lacking some of the other characteristics important to the GRS problem at hand, particularly that the initial stiffness, friction angle, and dilation behavior are pressure-dependent. As a result, some modifications were implemented and are explained herein:

The model requires the initial mean effective pressure $p'_m$ as a fixed input value for all soil of a particular type. Instead, it would be preferred for $p'_m$ to be calculated internally as a function of the stresses applied to the soil. Therefore, that change has been incorporated in the model, and now $p'_m$ is generated instead of being an input. The code of the model used in this research calculates the $p'_m$ value as the mean pressure of the initial stress values in the test.

Other changes made to the original Chsoil model concern the friction angle, dilation angle, and cv angle dependency on the confining pressure.

The friction angle is defined through 3-69 From Duncan and Wright (2005):

$$\phi_f = \phi_o - \Delta \phi \log \left( \frac{\sigma_3}{p_a} \right)$$  \hspace{1cm} 3-69

where:

- $\phi_f$ is the secant effective friction angle (ultimate friction angle)
- $\phi_o$ is the secant effective friction angle at the reference pressure of $\sigma_3 = 1$ atm.
- $\Delta \phi$ is the variation of friction angle for a ten-fold increase in $\sigma_3$
- $\sigma_3$ is the effective confining pressure. $\sigma_3$ can be replaced by $p'_m$ (initial mean effective pressure) in a triaxial test.
- $p_a$ is the atmospheric pressure in the same units as $p'_m$

After $p'_m (\sigma_3)$ is specified for each zone in the model, the friction angle is calculated using 3-69, with $\phi_o$ and $\Delta \phi$ given as material property inputs. These two input parameters are a function of the material type and its relative density (RD). As a result, the ultimate friction angle in the modified Chsoil model is no longer a fixed value. Instead it is calculated as a function of the input values $\phi_o$ and $\Delta \phi$ and the confining pressure for the specific test.
The dilation and cv angles follow equations of functional forms similar to that of Equation 3-69. Thus, similar to the inputs needed for the secant effective friction angle, \((\psi_o, \Delta \psi)\) and \((\phi_{cvo}, \Delta \phi_{cv})\) are the input pairs for the dilation and cv angles, respectively. Again, these parameters are expected to depend upon the material type and its relative density RD.

\[
\phi_{cv} = \phi_{cvo} - \Delta \phi_{cv} \log \left( \frac{p'_m}{p_a} \right)
\]

\[
\psi = \psi_o - \Delta \psi \log \left( \frac{p'_m}{p_a} \right)
\]

3-70

3-71

Through the modifications described above, the targeted constitutive responses and behavior characteristics are obtained. As a result, the modified Chsoil constitutive model can approximate real laboratory data. The following figures illustrate an example of the relative improvement in the model's responses by implementing the modifications. Figure 5 presents the best fit from FLAC3D using the unmodified Chsoil model applied to an example set of consolidated drained triaxial test results from Duncan et al. (2007). The material analyzed was limestone aggregate from Acco Stone Quarry in Blacksburg, VA, with a relative density of about 90%. Figure 6 presents the best fit from FLAC3D using the modified Chsoil model for the same set of triaxial test results.

![Figure 5: Original Chsoil model applied to triaxial results. Dashed lines: laboratory data, continuous lines: results from FLAC3D. The four different colors correspond to four different confining pressures. Comparison of simulation and experimental results in terms of: (a) Stress – strain response, (b) Volumetric strain versus axial strain response curves.](image)
Figure 6: Modified Chsoil model applied to triaxial results. Dashed lines: laboratory data, continuous lines: results from FLAC3D. The four different colors correspond to four different initial mean effective stresses. Comparison of simulation and experimental results in terms of: (a) Stress – strain response, and (b) Volumetric strain versus axial strain response.

From Figure 5(a) and Figure 6(a), it can be observed that both the unmodified and modified Chsoil models provide reasonable approximations to the stress-strain curves, although the modified Chsoil model shows better agreement with the test data. The improvement Figure 6(a) shows is due to the internal calculation of $p'_m$ which makes each curve depends on its own elastic shear and bulk moduli. The implementation of Equation 3-69 has also influenced the stress-strain results mainly at the failure stage.

With regards to the volumetric strain – axial strain curves, Figure 5(b) shows that the unmodified Chsoil model produces curves that are almost independent of the confining pressure, whereas Figure 6(b) shows that the modified Chsoil model is able to capture the effect of confining pressure on the dilation angle as well as on the onset of dilative behavior. This improvement is primarily due to the implementation of Equations 3-70 and 3-71. Thus, the modified Chsoil model is able to reasonably capture and reproduce the important constitutive responses of a dense granular soil subjected to triaxial loading.

3.5. Procedure to Obtain Input Parameters

With the modified Chsoil model able to fulfill the requirements of soil behavior response for application to GRS, it is important to establish ways in which the model input parameters can be obtained. Following, two step-by-step procedures are described for use when: (1) data from a full set of triaxial test is available and (2) only material descriptions and relative density/compaction are available.
Obtaining material property values when a full set of triaxial results is available (stress-strain curves and volumetric-strain-versus-axial-strain curves at different confining pressures)

Having a set of constitutive equations directly applicable to triaxial test results (developed in Section 3.2), the goal here is to obtain the input parameters for the modified Chsoil model that will provide a reasonable match with test results. The fitting parameters are the dilation angle parameters, the cv angle parameters, the friction angle parameters, the elastic properties, and the failure ratio. The available data that provide the calibration target are at least two sets of curves (stress-strain and volumetric-strain-versus-axial-strain curves) from different confining pressures for each test. The procedure can be described in the following five steps:

**Step 1:** Determine the parameters $\phi_0$ and $\Delta \phi$. In order to do so, it is necessary to calculate the secant ultimate friction angle for each confining pressure and make a graph friction angle vs. $\sigma_3/p_a$ (in log scale). The best fit line is obtained and its slope is the $\Delta \phi$ parameter. The angle that corresponds to $\sigma_3/p_a$ of 1 is $\phi_0$.

The secant friction angle can be conveniently obtained using the maximum deviator stress, $\sigma_{d,f}$, and the confining stress, assuming cohesion is zero, and using the following equation.

$$\phi = \sin^{-1} \left[ \frac{\sigma_{d,f}}{\sigma_{d,f} + 2\sigma_3} \right]$$ 3-72

**Step 2:** Select one set of laboratory curves at one confining pressure and from these, determine the cv angle, dilation angle, and bulk modulus. The cv angle is the value of the secant friction angle at the lowest value of volumetric strain (most contraction, smallest volume of the specimen). Similar equation than the previous one can be used to determine the cv angle, using the deviator stress at the lowest volumetric strain ($\sigma_{d,cv}$).

$$\phi_{cv} = \sin^{-1} \left[ \frac{\sigma_{d,cv}}{\sigma_{d,cv} + 2\sigma_3} \right]$$ 3-73

The elastic bulk modulus is the initial ratio of the deviator stress variation over three times its corresponding volumetric strain variation (Equation 3-74), and the process is illustrated in the example below.

$$K^e = \frac{\Delta \sigma_d}{3 \Delta \varepsilon_v}$$ 3-74

For the dilation angle, the following expression is used:
\[
\sin \psi = -\frac{d\varepsilon_1 + 2d\varepsilon_3}{d\varepsilon_1 - 2d\varepsilon_3}
\]

where \(d\varepsilon_1, d\varepsilon_3\) are the strain increments in the corresponding principal directions when the maximum deviator stress is reached. The increments should be large enough to produce a stable value of dilation angle.

In addition to the calculated parameters, it is necessary to record the axial strain at the cv point \((e_{1cv})\), the strain at the maximum deviator stress point \((e_{1f})\), and the stress at failure point \((\sigma_{1f})\).

**Step 3:** Determine the shear modulus and failure ratio values that produce the best fit with the data. This is done through applying the Equations 3-79 and 3-77. These two expressions are obtained by matching the set of equations applicable to the triaxial case (presented in Section 3.2) with the data curves (from the triaxial test) at two points: the failure point and the cv angle point, as follows:

Equation 3-55 is applied to the cv point on triaxial data: \(e_{1cv}\) and \(\sigma_{cv}\), these values are determined in the previous step. The following equation is obtained:

\[
e_{1cv} = \frac{(G^e + 3K^e)}{9G^e K^e} (\sigma_{cv} - \sigma_3) - \left[ \frac{(\sigma_3 - \sigma_{cv})\sigma_3 \sin \phi_f}{G^e R_f (\sigma_3 - \sigma_{cv}) + (\sigma_{cv} + \sigma_3) \sin \phi_f G^e} \right] 3-76
\]

Solving for \(G^e\):

\[
G^e = \frac{-3K^e (R_f (\sigma_3 - \sigma_{cv}) + (\sigma_{cv} + 4\sigma_3) \sin \phi_f)}{(R_f - \frac{\sin \phi_f}{\sin \phi_{cv}})(9K^e e_{1cv} + \sigma_3 - \sigma_{cv})} 3-77
\]

Equation 3-56 is applied to the failure data from the triaxial test, after solving for \(G^e\):

\[
G^e = \frac{3K^e (\sigma_{1f} - \sigma_{cv})}{\sqrt{N_\psi^2 + N_\psi + 1(\sigma_3 - \sigma_{cv}) (R_f - \frac{\sin \phi_f}{\sin \phi_{cv}})(R_f - 1)(\sigma_3 + \sigma_{1f})}} + 1 3-78
\]

\[
G^e = \frac{-6\sqrt{3}(\sigma_3^2 \sin \phi_f)}{\sigma_{cv} - \sigma_{1f} + 9K^e (-e_{1cv} + e_{1f})}
\]
Equating Equations 3-77 and 3-78, and solving for $R_f$, Equation 3-79 is obtained:

$$ R_f = \frac{S + \sqrt{S^2 - 4MN}}{2M} \quad \text{(3-79)} $$

where:

$$ S = \frac{\sin \phi_f}{\sin \phi_{cv}} + \frac{[\sigma_{cv} - \sigma_3 + \sin \phi_f (4\sigma_3 + \sigma_{cv})][\sigma_{1f} - \sigma_{cv} + 9K^e(e_{1cv} - e_{1f})]}{(\sigma_{1f} - \sigma_{cv})(\sigma_3 - \sigma_{cv} + 9K^e e_{1cv})} + 1 $$

$$ M = 1 - \frac{(\sigma_3 - \sigma_{cv})[\sigma_{1f} - \sigma_{cv} + 9K^e(e_{1cv} - e_{1f})]}{(\sigma_{1f} - \sigma_{cv})(\sigma_3 - \sigma_{cv} + 9K^e e_{1cv})} $$

$$ N = \frac{\sin \phi_f}{\sin \phi_{cv}} + \frac{\sin \phi_f (4\sigma_3 + \sigma_{cv})[\sigma_{1f} - \sigma_{cv} + 9K^e(e_{1cv} - e_{1f})]}{(\sigma_{1f} - \sigma_{cv})(\sigma_3 - \sigma_{cv} + 9K^e e_{1cv})} - \frac{6\sqrt{3}\sigma_3^2 \sin \phi_f}{(\sigma_3 - \sigma_{cv})(\sigma_3 + \sigma_{1f}) \sqrt{N_\phi^2 + N_\psi + 1}} $$

**Step 4:** Repeat Steps 1, 2 and 3 for each pair of curves at the different confining pressures.

**Step 5:** Formulate the equation that fits the test results best (cv angle, dilation angle, shear modulus and bulk modulus) as a function of the confining pressure. Failure ratio is the average of the values from all the confining pressures analyzed.

**Example:**

The outlined procedure is applied to a practical example. The information used in the present example comes from triaxial test data from Chandrasekaran (1992), and it consists of three consolidated-drained triaxial compression tests with confining pressures equal to 25, 50 and 100 kPa. The material used was uniform fine sand with a relative density of about 75%. Each test produced data for vertical strain, axial stress, and volumetric strain. The following table shows the actual information where all $\sigma$ represent effective stresses.
Table 5: Data of three triaxial laboratory tests for different confining pressures. Data compiled from: Chandrasekaran (1992)

<table>
<thead>
<tr>
<th>$\sigma_3$ (kPa)</th>
<th>$\sigma_1$ (kPa)</th>
<th>$\phi$ (degrees)</th>
<th>$\sigma_3/p_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>122</td>
<td>41.3</td>
<td>0.25</td>
</tr>
<tr>
<td>50</td>
<td>214</td>
<td>38.4</td>
<td>0.49</td>
</tr>
<tr>
<td>100</td>
<td>416</td>
<td>37.8</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Step 1: Determine the parameters $\phi_o$ and $\Delta \phi$.

First, the maximum values of $\sigma_1$ for each test are determined. Those values are in bold on Table 5 and are summarized in Table 6. From $\sigma_1$ and $\sigma_3$, the ultimate secant friction angle is calculated using Equation 3-72. The results are presented in Table 6.

Table 6: Results of calculation of ultimate friction angle and confining pressure ratios for the triaxial tests data in analysis.

<table>
<thead>
<tr>
<th>$\sigma_3$ (kPa)</th>
<th>$\sigma_1$ (kPa)</th>
<th>$\phi$ (degrees)</th>
<th>$\sigma_3/p_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>122</td>
<td>41.3</td>
<td>0.25</td>
</tr>
<tr>
<td>50</td>
<td>214</td>
<td>38.4</td>
<td>0.49</td>
</tr>
<tr>
<td>100</td>
<td>416</td>
<td>37.8</td>
<td>0.99</td>
</tr>
</tbody>
</table>

The obtained results are plotted in Figure 7. The best fit trend line is drawn, and the friction angle parameters can be determined from the equation for this trend line. The slope of the trend line
represents \( \Delta \phi \) and the value at \( \sigma_3 / p_a = 1 \) is \( \phi_0 \). Therefore, \( \phi_0 \) is 37.4 degrees and \( \Delta \phi \) is 5.8 degrees for this data.

![Graph showing friction angle vs \( \sigma_3 / p_a \)](image)

Figure 7: Results of ultimate secant friction angle and confining pressures over atmospheric pressure ratios for the triaxial tests in analysis.

**Step 2:** Using the data of \( \sigma_3 = -25 \) kPa, which is presented in the following graphs (Figure 8, Figure 9), determine the cv angle, dilation angle, and bulk modulus.

![Stress-strain graph](image)

Figure 8: Stress-strain graph of triaxial test with confining pressure of 25 kPa
Dilation angle: In order to get the dilation angle, Equation 3-75 is used with the data from the two points in orange in Figure 9. Remember compression is negative

\[ d\varepsilon_1 = -\frac{9.79 - 4.52}{100} = -0.0527 \]

\[ d\varepsilon_v = \frac{1.799 - 0.726}{100} = 0.01073 \]

\[ d\varepsilon_3 = \frac{d\varepsilon_v - d\varepsilon_1}{2} = 0.0317 \]

\[
\sin\psi = -\frac{d\varepsilon_1 + 2d\varepsilon_3}{d\varepsilon_1 - 2d\varepsilon_3}
\]

\[
\sin\psi = -\frac{-0.0527 + 2(0.0317)}{-0.0527 - 2(0.0317)}
\]

\[ \sin\psi = 0.092 \]

\[ \psi = 5.3^\circ \]

cv angle: With the data from the yellow point in Figure 9.

\[ \sigma_1 = \sigma_{cv} = -73.39 \text{ kPa} \]
\[ \phi_{cv} = \sin^{-1} \left[ \frac{\sigma_{d,cv}}{\sigma_{d,cv} + 2\sigma_3} \right] \]

\[ \phi_{cv} = \sin^{-1} \left[ \frac{73.39 - 25}{73.39 - 25 + 2(25)} \right] \]

\[ \phi_{cv} = 29.46^\circ \]

Bulk modulus: Using the information from the first two rows of data:

\[ K_e = \frac{\Delta \sigma_d}{3\Delta \varepsilon_v} \]

\[ K_e = \frac{-59.78 - (-25)}{3(-0.0122/100)} \]

\[ K_e = 94794.8 \text{kPa} \]

Additional data recording for use in Step 3:

\[ e_{1,cv} = -0.0045 \]
\[ e_{1,f} = -0.113 \]

\[ \phi = 40.9^\circ \]

\[ \sigma_{1,f} = \sigma_3 \tan^2 \left( 45 + \frac{\phi}{2} \right) \]

\[ \sigma_{1,f} = -120 \text{kPa} \]

**Step 3:** With all the results obtained and applying Equations 3-79 and 3-77

\[ R_f = \frac{S + \sqrt{S^2 - 4MN}}{2M} \]

\[ S = \frac{\sin \phi_f}{\sin \phi_{cv}} + \left[ \frac{\sigma_{cv} - \sigma_3 + \sin \phi_f (4\sigma_3 + \sigma_{cv})}{(\sigma_{1,f} - \sigma_{cv})(\sigma_3 - \sigma_{cv} + 9K_e(e_{1,cv} - e_{1,f}))} \right] + 1 \]

\[ S = \frac{\sin 40.9}{\sin 29.5} + \]

\[ + \left[ -73.4 + 25 + \sin 40.9(4(-25) - 73.4)] -120 + 73.4 + 9(94795)(-0.0045 + 0.113) \right] \]

\[ \frac{(-120 + 73.4)(-25 + 73.4 + 9(94795)(-0.0045))}{1} \]
\[ S = -81 \]

\[ M = 1 - \frac{(\sigma_3 - \sigma_{cv})[\sigma_{1f} - \sigma_{cv} + 9K^e(e_{1,cv} - e_{1,f})]}{\sigma_{1f} - \sigma_{cv}} \frac{(\sigma_3 - \sigma_{cv} + 9K^e e_{1,cv})}{(\sigma_3 - \sigma_{cv})} \]

\[ M = 1 - \frac{(-25 + 73.4)(-120 + 73.4 + 9(94795)(-0.0045 + 0.113))}{(-120 + 73.4)(-25 + 73.4 + 9(94795)(-0.0045))} \]

\[ M = -23.8 \]

\[ N = \frac{\sin \phi_f}{\sin \phi_{cv}} + \frac{\sin \phi_f(4\sigma_3 + \sigma_{cv})[\sigma_{1f} - \sigma_{cv} + 9K^e(e_{1,cv} - e_{1,f})]}{\sigma_{1f} - \sigma_{cv}} \frac{(\sigma_3 - \sigma_{cv} + 9K^e e_{1,cv})}{(\sigma_3 - \sigma_{cv})} \]

\[ N = 1 + \frac{\sin \phi_f}{1 - \sin \phi_f} = 1 + \frac{0.092}{1 - 0.092} = 1.2 \]

\[ N = \frac{\sin 40.9}{\sin 29.5} + \frac{\sin 40.9(-100 - 73.4)(-120 + 73.4 + 9(94795)(-0.0045 + 0.113))}{(-120 + 73.4)(-25 + 73.4 + 9(94795)(-0.0045))} \]

\[ - \frac{6\sqrt{3}(\sigma_3 - \sigma_{cv})^2 \sin 40.9}{(-25 - 73.4)(-25 - 120)\sqrt{1.2^2 + 1.2 + 1}} \]

\[ N = -57 \]

\[ R_f = \frac{-81 + \sqrt{(-81)^2 - 4(-23.8)(-57)}}{2(-23.8)} \]

\[ R_f = 0.99 \]

\[ G^e = \frac{-3K^e(R_f(\sigma_3 - \sigma_{cv}) + (\sigma_{cv} + 4\sigma_3)\sin \phi_f)}{(R_f - \frac{\sin \phi_f}{\sin \phi_{cv}})(9K^e e_{1,cv} + \sigma_3 - \sigma_{cv})} \]
\[ G^e = \frac{-3(94795)(0.99(-25 + 73.4) + (-73.4 - 100)\sin 40.9)}{(0.99 - \sin 40.9/\sin 29.5)(9(94795)(-0.0045) - 25 + 73.4)} \]

\[ G^e = 14390 \ kPa \]

**Step 4:** Repeat the process for the other two triaxial tests (confining pressures of 50 and 100 kPa)

The results obtained are summarized in the following Table 7.

**Table 7: Results from the three triaxial tests**

<table>
<thead>
<tr>
<th>( \sigma_3 ) (kPa)</th>
<th>( G^e ) (kPa)</th>
<th>( K^e ) (kPa)</th>
<th>( R_f )</th>
<th>( \psi )</th>
<th>( \phi_{cv} )</th>
<th>( \sigma_3/p_a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>14390</td>
<td>94795</td>
<td>0.991</td>
<td>5.3</td>
<td>29.5</td>
<td>0.247</td>
</tr>
<tr>
<td>50</td>
<td>12600.7</td>
<td>34234.6</td>
<td>0.958</td>
<td>5.3</td>
<td>34.7</td>
<td>0.494</td>
</tr>
<tr>
<td>100</td>
<td>21777</td>
<td>42238</td>
<td>0.958</td>
<td>4.7</td>
<td>27.8</td>
<td>0.987</td>
</tr>
</tbody>
</table>

**Step 5:** Now calculate the shear modulus and bulk modulus parameters, the cv angle, and dilation angle parameters as a function of the confining pressure.

In order to determine the parameters for shear and bulk modulus, the values in Table 7 are plotted: each modulus value is plotted versus the normalized confining pressure. Because the modulus values are expressed as power functions in Equations 3-4 and 3-5, the best fit power functions for the data are determined; then, the next step is to relate the modulus equation to the best fit power function. In that way the modulus parameters can be determined.

![Figure 10: Shear modulus data for different confining pressures for the example.](image)
Figure 11: Bulk modulus data for different confining pressures for the example.

Shear modulus parameters:

The shear modulus Equation 3-5 is compared to the best fit power function:

\[ G^e = G_{ref} \frac{p}{p_{ref}} \left( \frac{p'}{p_{ref}} \right)^n \]

is compared to \( y = 19518x^{0.299} \)

Then, \( G_{ref} = \frac{19518}{101.28} = 192.7 \) and \( n = 0.299 \)

Bulk modulus parameters:

The bulk modulus Equation 3-4 is compared to the best fit power function:

\[ K^e = K_{ref} \frac{p}{p_{ref}} \left( \frac{p'}{p_{ref}} \right)^m \]

vs. \( y = 34163x^{-0.583} \)

Then, \( K_{ref} = \frac{34163}{101.28} = 337.3 \) and \( m = -0.583 \)

However, the modulus exponent is not less than zero for typical compacted coarse-grained soils, and the negative value calculated here may be due to variation from sample to sample or some type of measurement inaccuracy. For this case, an average constant bulk modulus is believed to produce a better approximation. So,

\( K_{ref} = 563.7 \) and \( m = 0 \)
Calculate the dilation and cv angle parameters: A procedure similar to the previous cases is followed. However, in this case the graphs are semi-log, and the trend line follows a logarithmic type of equation. In these cases, the necessary parameter values can be read directly from the coefficients of the best fit lines.

![Figure 12: Dilation angle data for different confining pressures for the example.](image)

![Figure 13: cv angle data for different confining pressures for the example.](image)

Dilation angle parameters: \( \psi_o = 4.8^\circ \) and \( \Delta \psi = 0.95^\circ \)

cv angle parameters result: \( \phi_{cvo} = 29.8^\circ \) and \( \Delta \phi_{cv} = 2.7^\circ \)

Regarding the failure ratio, this value is obtained through an average of the failure ratios obtained for all the confining pressures. Then \( R_f = 0.969 \)

With this, the entire set of input parameter values for the Chsoil model for this example result:
Table 8: Resultant input for Chsoil model for the case in study

<table>
<thead>
<tr>
<th>Input Modified Chsoil Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o$ (deg)</td>
<td>37.4</td>
</tr>
<tr>
<td>$\Delta \phi$ (deg)</td>
<td>5.83</td>
</tr>
<tr>
<td>$\phi_{cvo}$ (deg)</td>
<td>29.8</td>
</tr>
<tr>
<td>$\Delta \phi_{cv}$ (deg)</td>
<td>2.7</td>
</tr>
<tr>
<td>$\psi_o$ (deg)</td>
<td>4.8</td>
</tr>
<tr>
<td>$\Delta \psi$ (deg)</td>
<td>0.95</td>
</tr>
<tr>
<td>$G_{ref}$</td>
<td>193</td>
</tr>
<tr>
<td>$n$</td>
<td>0.299</td>
</tr>
<tr>
<td>$K_{ref}$</td>
<td>564</td>
</tr>
<tr>
<td>$m$</td>
<td>0</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.969</td>
</tr>
</tbody>
</table>

The results from FLAC3D using the obtained input is presented in the following graphs together with the data from the laboratory tests that have been used in the present example.

![Graph](image)

**Figure 14:** Vertical stress vs. strain curves for different confining pressures. Continuous lines from numerical model (modified Chsoil model), dashed lines from triaxial tests data.
It can be seen that the deduced parameter values for the modified Chsoil model provide a very close agreement with the original data, with the only significant discrepancy occurring for the volume change comparisons at a confining pressure of 50 kPa. For all other comparisons, the agreement is very good. Therefore, it can be concluded that the procedure worked for this case. Many other laboratory tests have been used to confirm the procedure works. Thirteen sets of triaxial tests performed on granular soil were obtained from published literature and were analyzed: 8 from Duncan et al. (2007) and 5 from other sources: Haeri et al. (2000), Chandrasekaran (1992), Wu et al. (2013), Ketchart and Wu (2001). All these laboratory data were analyzed with exactly the same procedure explained above, and the results are presented in Appendix A. In general, the agreement obtained between data and results from the modified Chsoil model is very good.

Obtaining material property values when limited soil information is available

A full set of triaxial results is often not available for specific projects in engineering practice, in which case the above procedure would not be usable.

The Duncan and Chang hyperbolic model (Duncan and Chang 1970) is a well-established and widely applied constitutive model of granular soil behavior; however, the Duncan and Chang model does not include dilative behavior, which is incorporated in the modified Chsoil model. By combining previously compiled data for the hyperbolic model with additional dilatancy parameters developed as part of this research, modified Chsoil model parameters can be estimated based on soil characteristics.
Duncan et al. (1980) present a compilation of parameter values for the hyperbolic model for different types of soil at different relative densities or relative compactions. Duncan and Chang model's equations and model parameters and their counterparts for the Chsoil model are compared in Table 4 in Section 3.3.2 for the case of no cohesion intercept. Using these equations and the Duncan et al. (1980) parameter summary, the corresponding inputs for the modified Chsoil model can be obtained. In cases where the cohesion is different from zero, parameters for the Duncan and Chang model can be used to calculate stress-strain curves, and the relevant portion of the previous step-by-step procedure can be applied, using the calculated-stress curves as if they were laboratory data.

In the following, the procedure to establish the correspondence between the two constitutive models is detailed. It is explained how each input parameter for Chsoil model is obtained. First, and similar to the previous procedure, modified Chsoil model parameters are obtained for one confining pressure, this procedure consists of: 1) Preliminary work only with the compiled parameters from Duncan et al. (1980), 2) Correspondence of Duncan and Chang's parameters with modified Chsoil model parameters. 3) Determination of additional modified Chsoil model parameters (regarding dilation and cv angle that are not comparable with any Duncan and Chang's model parameters).

The procedure should be repeated for different confining pressures, and then it can be calculated the general input for the modified Chsoil model following step 5 from the previously explained procedure.

1) Preliminary work:
First, relevant portion of the values presented by Duncan et al. (1980), for different types of soil at different relative densities or compactions is determined. The focus here is on granular soils with cohesion equal to zero and with relative compaction (with respect to standard Proctor compactive effort) greater than 95 percent. These types of soils would be suitable to apply in a geosynthetic reinforced system, while the soils with a lower relative compaction would not apply. Accordingly, the applicable portion of the compilation summary of Duncan and Chang model parameters from Duncan et al. (1980) is presented in Table 9.

Table 9: Part of the compilation summary of soil data for Duncan and Chang’s hyperbolic model. From Duncan et al. (1980). Some parameters include a sub-index “DC” to differentiate them from the Chsoil model symbols

<table>
<thead>
<tr>
<th>Unified</th>
<th>RC Stand.</th>
<th>$\phi_o$</th>
<th>$\Delta\phi$</th>
<th>k</th>
<th>$n_{DC}$</th>
<th>$R_{fDC}$</th>
<th>$\kappa_b$</th>
<th>$m_{DC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classification</td>
<td>AASHTO</td>
<td>deg</td>
<td>deg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GW, GP, SW, SP</td>
<td>105</td>
<td>42</td>
<td>9</td>
<td>600</td>
<td>0.4</td>
<td>0.7</td>
<td>175</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>39</td>
<td>7</td>
<td>450</td>
<td>0.4</td>
<td>0.7</td>
<td>125</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>36</td>
<td>5</td>
<td>300</td>
<td>0.4</td>
<td>0.7</td>
<td>75</td>
<td>0.2</td>
</tr>
<tr>
<td>SM</td>
<td>100</td>
<td>36</td>
<td>8</td>
<td>600</td>
<td>0.25</td>
<td>0.7</td>
<td>450</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>95</td>
<td>34</td>
<td>6</td>
<td>450</td>
<td>0.25</td>
<td>0.7</td>
<td>350</td>
<td>0</td>
</tr>
</tbody>
</table>
The values on Table 9 from Duncan et al. (1980) are applied to confining pressures equal to 5, 10, 15 and 30 psi; with these confining pressures, initial Young's modulus and bulk modulus for Duncan and Chang's model are calculated using Equations 3-65 and 3-68, respectively. The results are presented in Table 10; some of the calculated bulk modulus values were smaller than the established limits for Duncan and Chang's model; therefore, the minimum accepted value was used (\(E_i/3\)).

Table 10: Calculation of initial Young's modulus, bulk modulus, relative density (RD) for Duncan and Chang model parameters, and calculation of elastic bulk modulus, elastic shear modulus, and failure ratio for Chsoil model

<table>
<thead>
<tr>
<th>U. C.</th>
<th>RC Stand. AASHTO</th>
<th>(\sigma_3) psi</th>
<th>(k)</th>
<th>(n_{DC})</th>
<th>(E_i) psi</th>
<th>(R_{fDC})</th>
<th>(K_B) psi</th>
<th>(m_{DC})</th>
<th>(B)</th>
<th>RD</th>
<th>(K_e) psi</th>
<th>(G_e) psi</th>
<th>(R_f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GW, GP, SW, SP</td>
<td>105</td>
<td>5</td>
<td>600</td>
<td>0.4</td>
<td>5727</td>
<td>0.7</td>
<td>175</td>
<td>0.2</td>
<td>2072</td>
<td>100</td>
<td>3819</td>
<td>5727</td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5</td>
<td>600</td>
<td>0.4</td>
<td>5727</td>
<td>0.7</td>
<td>175</td>
<td>0.2</td>
<td>2072</td>
<td>100</td>
<td>3819</td>
<td>5727</td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>5</td>
<td>600</td>
<td>0.4</td>
<td>5727</td>
<td>0.7</td>
<td>175</td>
<td>0.2</td>
<td>2072</td>
<td>100</td>
<td>3819</td>
<td>5727</td>
<td>0.964</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>5</td>
<td>600</td>
<td>0.4</td>
<td>5727</td>
<td>0.7</td>
<td>175</td>
<td>0.2</td>
<td>2072</td>
<td>100</td>
<td>3819</td>
<td>5727</td>
<td>0.964</td>
</tr>
<tr>
<td>GW, GP, SW, SP</td>
<td>100</td>
<td>5</td>
<td>450</td>
<td>0.4</td>
<td>4295</td>
<td>0.7</td>
<td>125</td>
<td>0.2</td>
<td>1480</td>
<td>75</td>
<td>2863</td>
<td>4295</td>
<td>0.957</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5</td>
<td>450</td>
<td>0.4</td>
<td>4295</td>
<td>0.7</td>
<td>125</td>
<td>0.2</td>
<td>1480</td>
<td>75</td>
<td>2863</td>
<td>4295</td>
<td>0.957</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>5</td>
<td>450</td>
<td>0.4</td>
<td>4295</td>
<td>0.7</td>
<td>125</td>
<td>0.2</td>
<td>1480</td>
<td>75</td>
<td>2863</td>
<td>4295</td>
<td>0.957</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>5</td>
<td>450</td>
<td>0.4</td>
<td>4295</td>
<td>0.7</td>
<td>125</td>
<td>0.2</td>
<td>1480</td>
<td>75</td>
<td>2863</td>
<td>4295</td>
<td>0.957</td>
</tr>
<tr>
<td>GW, GP, SW, SP</td>
<td>95</td>
<td>5</td>
<td>450</td>
<td>0.4</td>
<td>2863.7</td>
<td>0.7</td>
<td>75</td>
<td>0.2</td>
<td>954.56</td>
<td>50</td>
<td>1909</td>
<td>2863.7</td>
<td>0.940</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5</td>
<td>450</td>
<td>0.4</td>
<td>2863.7</td>
<td>0.7</td>
<td>75</td>
<td>0.2</td>
<td>954.56</td>
<td>50</td>
<td>1909</td>
<td>2863.7</td>
<td>0.940</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>5</td>
<td>450</td>
<td>0.4</td>
<td>2863.7</td>
<td>0.7</td>
<td>75</td>
<td>0.2</td>
<td>954.56</td>
<td>50</td>
<td>1909</td>
<td>2863.7</td>
<td>0.940</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>5</td>
<td>450</td>
<td>0.4</td>
<td>2863.7</td>
<td>0.7</td>
<td>75</td>
<td>0.2</td>
<td>954.56</td>
<td>50</td>
<td>1909</td>
<td>2863.7</td>
<td>0.940</td>
</tr>
<tr>
<td>SM</td>
<td>100</td>
<td>5</td>
<td>600</td>
<td>0.25</td>
<td>6732.2</td>
<td>0.7</td>
<td>450</td>
<td>0</td>
<td>6610.5</td>
<td>75</td>
<td>6610.5</td>
<td>6326</td>
<td>0.949</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5</td>
<td>600</td>
<td>0.25</td>
<td>6732.2</td>
<td>0.7</td>
<td>450</td>
<td>0</td>
<td>6610.5</td>
<td>75</td>
<td>6610.5</td>
<td>6326</td>
<td>0.949</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>5</td>
<td>600</td>
<td>0.25</td>
<td>6732.2</td>
<td>0.7</td>
<td>450</td>
<td>0</td>
<td>6610.5</td>
<td>75</td>
<td>6610.5</td>
<td>6326</td>
<td>0.949</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>5</td>
<td>600</td>
<td>0.25</td>
<td>6732.2</td>
<td>0.7</td>
<td>450</td>
<td>0</td>
<td>6610.5</td>
<td>75</td>
<td>6610.5</td>
<td>6326</td>
<td>0.949</td>
</tr>
<tr>
<td>SM</td>
<td>95</td>
<td>5</td>
<td>450</td>
<td>0.25</td>
<td>5049.2</td>
<td>0.7</td>
<td>350</td>
<td>0</td>
<td>5141.5</td>
<td>50</td>
<td>5141.5</td>
<td>4723</td>
<td>0.933</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>5</td>
<td>450</td>
<td>0.25</td>
<td>5049.2</td>
<td>0.7</td>
<td>350</td>
<td>0</td>
<td>5141.5</td>
<td>50</td>
<td>5141.5</td>
<td>4723</td>
<td>0.933</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>5</td>
<td>450</td>
<td>0.25</td>
<td>5049.2</td>
<td>0.7</td>
<td>350</td>
<td>0</td>
<td>5141.5</td>
<td>50</td>
<td>5141.5</td>
<td>4723</td>
<td>0.933</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>5</td>
<td>450</td>
<td>0.25</td>
<td>5049.2</td>
<td>0.7</td>
<td>350</td>
<td>0</td>
<td>5141.5</td>
<td>50</td>
<td>5141.5</td>
<td>4723</td>
<td>0.933</td>
</tr>
</tbody>
</table>

In a similar way, friction angles for each confining pressure are calculated using Equation 3-69. Also relative density (RD) can be estimated from the relative compaction using recommendations presented in Duncan and Bursey (2007), the resultant RD values are also presented in Table 10.
2) Correspondence between Duncan and Chang's model and modified Chsoil model:

Here some of the modified Chsoil model parameters are determined as follows, and working with just one confining pressure:

- Modified Chsoil model elastic bulk modulus $K^e$ is made equal to the Duncan and Chang's bulk modulus $B$, calculated in the previous stage.

- Confining pressure is the same for both models. Also, for application to triaxial compression tests with constant confining pressure, the constant value of $p'_m$ in the Chsoil model is appropriate to equal to the confining pressure.

- In order to obtain the elastic shear modulus value for the Chsoil model, the initial slopes in the stress-strain curves for the two models are equated. Table 4 shows the initial tangent Young's modulus equations for the two methods. Then Equation 3-62 is equated to the calculated initial Young's modulus for Duncan and Chang's model, and the elastic shear modulus value, $G_e$, required to make the initial slopes be the same is found equal to:

$$G^e = \frac{15E_iK^e}{18K^e - 2E_i}$$

where:

- $E_i$ is the initial Young's modulus for Duncan and Chang's model calculated into Table 10 with Equation 3-65
- $K^e$ is the elastic bulk modulus in the modified Chsoil model

Now with $G^e$, it is possible to control $K^e$ stay into the acceptable limits. The value of $K^e$ for the modified Chsoil model is constrained to be between $2/3G^e$ and 49.66 $G^e$ for positive Poisson ratios less than 0.49 (Itasca 2012). If $K^e$ is outside the limits, the maximum or minimum value must be used. With the new $K^e$, $G^e$ should be calculated again and this procedure should be repeated, in case it is necessary, until reaching a bulk modulus between the acceptable limits.

When a correction of the bulk modulus is made, two consequences should be clarified:

1. When a bulk modulus is used equal to $2/3 \ G^e$. Applying Equation 3-80 gives $G^e = E_i$. This outcome is due to the initial stress-strain slope the Chsoil model uses for a cohesionless soil. This would not happen for a cohesive soil.

2. Modified Chsoil model bulk modulus would result different than Duncan and Chang's bulk modulus. Again, this is a consequence of the different initial slope equations for both methods.
These two consequences can be clearly noticed in Table 10.

- The failure ratio for the Chsoil model \((R_f)\) is determined by making the failure stress-strain point be the same for the two methods. First, the failure strain for the Duncan and Chang model is obtained by solving Equations 3-64 and 3-66 to produce:

\[
\varepsilon = \frac{(\sigma_1 - \sigma_3)(\sigma_3 - \sigma_{1f})}{E_i(\sigma_3 - \sigma_{1f} + R_{fDC}\sigma_1 - R_{df}\sigma_3)}
\]

where:

\(\sigma_{1f}\) is the major principal stress at failure point

\(R_{fDC}\) is the failure ratio for the Duncan and Chang model

Then, Equation 3-81 for the Duncan and Chang model is set equal to Equation 3-56 for the Chsoil model. Solving for the Chsoil failure ratio:

\[
R_f = \frac{2X\left(\frac{\sigma_3^2 - \sigma_{cv}\sigma_{1f}}{\sigma_3 - \sigma_{cv}}\right) + 1 + \left(\frac{2X\frac{\sigma_{cv} - \sigma_{1f}}{\sigma_3 - \sigma_{cv}} + 1}{\sigma_{cv} - \sigma_{1f}}\right)^2 + \frac{8\sqrt{3}X}{\sigma_{cv} - \sigma_3}\sqrt{\frac{\sigma_{cv} - \sigma_{1f}}{\psi^2 + N\psi + 1}}}{\sqrt{\frac{\sigma_3^2 + \sigma_{1f}^2}{\sigma_3^2 + \sigma_{1f}^2}}}
\]

where:

\(\sigma_{cv}\) is the major principal stress at \(\phi_{cv}\)

\[X = \frac{(G^e + 3K^e)}{9K^e} + \frac{G^e}{E_i(R_{fDC} - 1)}\]

Modified Chsoil model parameters are calculated in Table 10.

3) Calculus of additional parameters for modified Chsoil model:

Finally, dilation and cv angle must be determined. As part of this research, a method to estimate the dilation parameters for the Chsoil model from soil type and relative density or compaction was developed because the Duncan and Chang model does not incorporate dilation. From the thirteen sets of triaxial tests analyzed in the previous procedure and compiled in Appendix A, a set of data relating cv angle parameters (\(\phi_{cv0}\) and \(\Delta\phi_{cv}\)) with relative density of the soil, and dilation parameters (\(\phi_0\) and \(\Delta\psi\)) with relative density of the soil, were produced. The results are shown in Figure 16 for the cv angle parameters and in Figure 17 for the dilation parameters.
From Figure 16 it can be seen that $\phi_{cvo}$ demonstrates an increasing trend as relative density increases. Fitting these data points with a linear trend line, the approximation seems reasonable and the scatter around the trend line is acceptable. Regarding the variation of $\Delta\phi_{cv}$ with relative density, no defined trend can be distinguished, so introducing an average (blue line in Figure 16b) or a lightly sloping linear approximation (black line) would be a good enough approximation to the data points. The red data point in Figure 16 is considered as an outlier, consistently demonstrating a behavior outside of the otherwise observed limits (see Figure 16b and Figure 17).

![Graph](image)

(a)

![Graph](image)

(b)

Figure 16: cv angle parameters versus relative density of the material. (a) cv angle at confining pressure = 1 atm. (b) Variation of cv angle for a ten-fold increase in $p'_{m}$

The data points corresponding to dilation parameters demonstrate an even greater scatter so any attempt to provide a reasonable fit to them includes greater uncertainty. If two points (those shown in circles in Figure 17(a) are discarded, the parameter $\psi_{o}$ seems to increase with relative density until large relative density values are reached (~80%), and then, it seems approximately constant. $\Delta\psi$ exhibits the most variability, with its data seeming almost randomly distributed between 4 and 15 degrees. Although it is important to include dilation to realistically track volume change for dense coarse-grained soils, variation of the parameter values within the ranges of the data shown
in Figure 17 do not produce large changes in the model response over practical ranges of confining pressures. Consequently, values within the range of data shown in Figure 17 are believed to produce reasonable results.

![Graph](image)

**Figure 17:** Dilation parameters vs. relative density of the material. (a) Dilation angle at pressure = 1 atm. (b) Variation of dilation angle for a ten-fold increase in $p'_m$.

Figure 18 presents the variations of friction angle parameters with respect to the relative density for the 13 sets of triaxial test results analyzed as part of this research. Figure 18(a) shows a trend of increasing friction angle, $\phi_0$, with relative density, as expected. Figure 18(b) exhibits substantial scatter in $\Delta \phi$ for the entire group of 13 sets of triaxial test results, although trends seem more evident for each soil type considered separately. Values of $\phi_0$ and $\Delta \phi$ have been compiled by others (Duncan et al. 1980), and the values presented in Figure 18 are not meant to replace previous well-established recommendations for these parameters.
Therefore, and to conclude the determination of modified Chsoil model parameter for one confining pressure, Figure 16 and Figure 17 can be used to estimate the dilation and cv angle parameters, as a function of the relative density of the soil, for use in the modified Chsoil model when test results are not available. When little scatter is present, it would seem reasonable to use the trend lines presented in the figures. When there is substantial scatter, a typical value between the limits of the data shown on the figures can be used, or two values can be tried to investigate the influence of the parameter on the model response. In cases where the data in Figure 16 and Figure 17 exhibit substantial scatter, variations in parameter values within the predominant range of the data should be used to check if that produce large changes in the modified Chsoil model response.

In this way, all of the input for modified Chsoil model, and for only one confining pressure can be obtained as a function of Duncan and Chang model's parameters and with the help of Figure 16 and Figure 17 for dilatancy.
The next stage is to repeat the procedure for all the confining pressures in analysis, and belonging to the same soil type and relative compaction. With those results, step 5 in the previously explained procedure can be followed to determine all the modified Chsoil model input parameters.

The obtained results are summarized in the following Table 11, where part of the Duncan compilation summary is complemented with the modified Chsoil model parameters.

Table 11: Final results for modified Chsoil model inputs

<table>
<thead>
<tr>
<th>Unified Classif.</th>
<th>Data from Duncan and Chang Model</th>
<th>Calculated Modified Chsoil Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi_0$ deg $\Delta \phi$ deg $k$ $n_{DC}$ $R_{fDC}$ $K_b$ $m_{DC}$ $\psi_{0}$ deg $\Delta \psi$ deg $\psi_{cv}$ deg $\Delta \psi_{cv}$ deg $R_f$ $G_{ref}$ $n$ $K_{ref}$ $m$</td>
<td></td>
</tr>
<tr>
<td>GW, GP, SW, SP</td>
<td>42 9 600 0.4 0.7 175 0.2 13 10 35 4 0.954 600 0.4 400 0.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39 7 450 0.4 0.7 125 0.2 12 10 32 -2 0.948 450 0.4 300 0.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>36 5 300 0.4 0.7 75 0.2 3 10 30 -5 0.933 307 0.4 182 0.25</td>
<td></td>
</tr>
<tr>
<td>SM</td>
<td>36 8 600 0.25 0.7 450 0 12 10 32 -2 0.938 586 0.3 460 0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>34 6 450 0.25 0.7 350 0 3 10 30 -5 0.925 438 0.3 353 0.01</td>
<td></td>
</tr>
</tbody>
</table>

It must be mentioned that the cv angles at pressure of 1 atm have been decreased more than 10 degrees from the values recommended in the Figure 16(a). This was done due to the conservationism in the Duncan et al. (1980) recommended ultimate friction angles. If the friction angle parameters conservatively recommended by Duncan et al. (1980) are plotted on Figure 18(a), it can be seen the points fell below the trend shown in Figure 18(a). Accordingly, the cv angles must have a similar difference with respect to the trend in Figure 16(a). The values of $\Delta \phi_{cv}$, $\psi_0$, and $\Delta \psi$ are obtained directly from Figure 16(b), Figure 17(a), and Figure 17(b), respectively.
CHAPTER 4:
GEOSYNTHETIC AND INTERFACE MODELS

This chapter presents selection and simple confirmation analyses of a suitable numerical model to represent the geosynthetic behavior and its interface interaction with the soil in GRS. FLAC3D incorporates several options that could be used to represent the behavior of a geosynthetic material inside a soil mass; however, some of these are not ideal for representing geosynthetic reinforcement in GRS. Section 4.1 presents a discussion about the best available option in FLAC3D to represent a geosynthetic in GRS. Section 4.2 describes how this option works. Section 4.3 presents a summary of three simple numerical validation tests performed to ensure that the chosen option works as desired.

4.1. Structural Elements in FLAC3D

FLAC3D incorporates several structural elements that can be used in analysis of geotechnical systems. Examples of structural elements include cables, beams, piles, shells, geogrids and liners. For some of these structural elements, simulation of the behavior of the element's interaction with the soil within it is embedded is included without the need to assign separate interface elements.

Itasca (2012) recommends using “geogrid structural elements” to represent geotextile reinforcement in a soil mass. Using geogrid elements, geotextiles are represented by three-noded (triangular) flat plane stress elements that resist membrane stresses in their planar direction; in the normal direction these elements exactly follow the movements on the adjacent soil grid. Each of these triangular elements has 2 translational degrees of freedom per node.

The geogrid structural elements incorporate linear elastic behavior, where failure does not occur, and isotropic, orthotropic, or anisotropic characteristics can be assigned. The interface behavior between the geosynthetic and the surrounding soil mass is modeled by a spring-slider on each node in the direction tangent to the geotextile surface; each spring-slider uses the normal force from both sides of the geotextile surface over the area of influence to determine the shear strength of the interface that connects the geosynthetic to the surrounding soil. The interface behavior is cohesive and frictional (Itasca 2012).

A problem with this type of structural element is that it is linked to the soil zones by coupling springs, such that the soil zones on either side of the geosynthetic remain directly connected regardless of the presence of the geosynthetic. This type of coupling between the geosynthetic and the soil allows for the geosynthetic to be pulled out of the soil, and it does permit the geosynthetic to resist elongation of the soil in the plane of the geosynthetic. However, this type of coupling does not allow for the soil on just one side of the geosynthetic to slide against the geosynthetic because the soil on opposite sides of the geosynthetic remains as connected as if the geosynthetic were not present. This limitation restricts failure mechanisms that include sliding of soil against one side of the geosynthetic but not the other.
A better choice for representing geosynthetic reinforcement in GRS is the "embedded liner element" available in FLAC3D. The embedded liner element separates the soil zones on one side of the liner element from the soil zones on the other side, and it has soil-liner interface connections on both sides. These liner elements can be used to more realistically represent the geosynthetic-soil interaction, as described in the following section.

4.2. Liner Elements of FLAC3D

Liner elements are more capable structural elements than geogrids. Structural liner elements can act like membranes, and they can act like plates and shells that support bending moments. With regards to the interface interaction, liner elements can be implemented with interfaces on one side or on both sides of the liner. When interfaces are used on both sides, the liner element is termed an “embedded liner element” in the language of FLAC3D. To avoid bending stiffness, which is appropriate for geotextile reinforcing in GRS, the "membrane" version of the liner element is used.

The structural response of the liner material itself, with membrane behavior, acts exactly the same as the geogrid explained above. The three nodded flat elements represent elastic behavior in the shear direction (membrane loading). Isotropic, orthotropic, or anisotropic types of behavior can be chosen. Most of the geosynthetic materials used in GRS have different properties in machine and cross-machine directions; therefore, orthotropic representation of geosynthetic material is appropriate.

A liner element under membrane loading (plane stress condition) and with an orthotropic behavior follows Equation 4-1.

Therefore, the required data inputs for the structural behavior of the liner elements are: two Young's moduli, one Poisson's ratio (the other can be obtained from \(c_{12}\)), and the shear modulus. This yields four property values in total.

With regards to the interface behavior, the embedded liner elements have interfaces on both sides, with interface response in the normal and shear directions for each side. The interface behavior in the normal direction is controlled by a spring with a stiffness per unit area (the resultant dimensions of the spring stiffness are force per unit length cubed) and a maximum tensile strength (with dimensions of force per unit length squared). In the shear direction the behavior is similar to that of the geogrid, with springs to represent the shear stiffness (force per unit length cubed) and sliders to represent shear strength (force per unit length squared). The shear strength can be specified to include cohesive and frictional components. A non-zero residual cohesive strength can be specified when the liner fails in tension, although this would not normally be done for GRS. In summary, the required inputs for each side for the interface behavior are:

- For the normal interface: normal stiffness and tensile strength
- For the shear interface: stiffness, cohesive strength, residual cohesive strength, and friction angle
\[
\begin{align*}
\{\sigma_x\} &= \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{22} & 0 \\ 0 & 0 & c_{33} \end{bmatrix} \{\varepsilon_x\} \\
\{\sigma_y\} &= \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{12} & c_{22} & 0 \\ 0 & 0 & c_{33} \end{bmatrix} \{\varepsilon_y\}
\end{align*}
\]

where:

\[
\begin{align*}
c_{11} &= \frac{E_x}{1 - \nu_x\nu_y} \\
c_{22} &= \frac{E_y}{1 - \nu_x\nu_y} \\
c_{33} &= G \\
c_{12} &= \frac{E_x\nu_y}{1 - \nu_x\nu_y} = \frac{E_y\nu_x}{1 - \nu_x\nu_y}
\end{align*}
\]

- \(E_i\) is the Young's modulus in the \(i\) direction
- \(\nu_i\) is the Poisson's ratio in the \(i\) direction
- \(G\) is the shear modulus

### 4.3. Element Tests on Interfaces and Structural Elements

Three simple numerical tests were developed in order to validate the correct behavior of the embedded membrane liner elements to represent geosynthetic reinforcement in GRS:

- **Direct shear test:**

  Two direct shear tests were analyzed, each test having its own interface properties for each side of the liner elements. One test has the weaker interface properties on its upper face and the other on the lower face. During the test, it is checked that the weaker interface fails (slides).

  Each test consists of two soil blocks with a layer of liner structural elements between them. The soil strength and stiffness are high enough that there is no failure in the soil and there are not significant differential movements inside the soil mass neither (soil behaves essentially as a rigid block). The liner elements are embedded within the two soil blocks, and one of the liner faces is weaker than the other. The test imposes a horizontal movement of the upper soil block while the lower soil block is restrained. Then the weaker interface must fail.

  The normal interface properties are the same for both liner faces: a high normal stiffness (1E10 kN/m^3) and zero tension are specified. The tangential interface stiffness is also high (1E10 kN/m^3), the interface friction angle is zero, and the residual cohesion is zero on both faces. Different cohesive strengths are used on each side, with 50 kPa used for the weaker interface and 100 kPa for the other.
The test is run by moving the upper block to the right, and the liner layer is left without any restriction so it can move freely. The shear stress responses on each face of the liner are recorded as the sliding takes place.

One test has the weaker cohesive strength in the upper liner interface face, and this is where the sliding occurs. Figure 19 shows the deformed shape of the two blocks with their relative displacement and the liner layer remaining stuck to the lower block due to the stronger interface. Figure 20 and Figure 21 show the outcomes in shear-stress-versus-displacement behavior on both interfaces. Figure 20 shows the lower interface results. Because it is the stronger interface, minimum displacement results as a consequence of the tangential stiffness. When the upper interface fails, the lower interface does not generate any additional shear stress or displacement, and it stays attached to the soil block. Figure 21 shows the shear-stress-versus-displacement behavior for the upper face (the weaker one), and it can be seen the initial response follows the interface stiffness until the shear strength is reached. From there, sliding occurs, and the relative displacement increases while the shear stress remains constant. There is equilibrium in shear stresses in the upper and lower faces, and the main displacement occurs in the upper interface face when it reaches the cohesive strength limit. The lower face does not reach its cohesive strength limit.

![Figure 19: Direct shear test result with the weaker interface in the upper face of the liner. Geosynthetic in blue.](image)

![Figure 20: Stronger interface shear stress - displacement](image)
The second test, with the lower interface as the weaker one, exhibits the expected behavior. Figure 22 shows the deformed shape, in which the liner layer stays attached to the upper block and the lower interface fails. The shear-stress-versus-displacement behavior switched from the previous analysis, and Figure 21 would represent the lower face behavior and Figure 20 would represent the upper interface for the conditions of the test shown in Figure 22.

The “embedded liner element” exhibits the necessary behavior for geosynthetic reinforcement of GRS, where sliding takes place on the weaker side and the geosynthetic remains attached to the soil block on the stronger side. This behavior is impossible to reproduce in a geogrid type of element. The FLAC3D code used to run this test is presented in Appendix B.

- Pullout test:

On this test, the geosynthetic layer, which is represented by embedded liner structural elements, is pulled out of the soil mass. Each interface has its own strength properties and therefore each should respond with its respective strength during sliding.
Similar to the direct shear test, two soil blocks are created with an embedded geosynthetic layer in their interface. The soil is made very strong so it does not exhibit any deformation during the test. The geosynthetic itself is also made very stiff so it does not stretch during the pullout test. The interfaces have the same properties as in the direct shear test, except that the cohesive strength is 500 kPa for the lower side and 300 kPa for the upper side. The geosynthetic is pulled out in the direction of the positive x axis, and the shear strength generated on each face of the liner layer is checked.

Figure 23 shows the result of the pullout test, and it can be seen that the geosynthetic is displaced uniformly in the positive x direction, while the soil masses remain stationary. Figure 24 presents the shear stress – displacement response of the interfaces above and below the liner layer. The input shear stiffness controls the slope of the shear-stress-versus-displacement plots until the cohesive strengths for each side are reached, with the upper interface able to mobilize more shear stress than the lower interface. After reaching the shear strength on both sides, the geosynthetic moves freely without any additional shear stress development on either side. The total pullout force was also calculated, and it equals the sum of the forces acting on each interface, which equal the interface area times the interface shear strength. This test demonstrates that the liner elements can act with different properties on each interface, and not with one set of properties as the geogrid elements do. The FLAC3D code for this test is also included in Appendix B.
• Geosynthetic structural test.

This test has the objective of verifying the orthotropic behavior on an embedded structural element.

In this test, an embedded liner is placed in the middle of a soil block with a unit area and a height of two units, as shown in Figure 25. The interface properties are set to have zero strength so that the geosynthetic can slide freely between the soil blocks, and the interfaces do not have any effect on the embedded liner response. The liner is stretched in x and y directions while it is fixed in the y direction along the x-axis and in the x direction along the y axis, as shown in Figure 25. The resultant forces generated in the affected liner nodes are captured. The resultant forces and total displacements in the x and y directions are verified to satisfy Equation 4-1 with the liner material properties used to provide the values in the stiffness matrix, as shown in Table 12. These results show an orthotropic liner responds as expected. The code used for this test is presented in Appendix B.

Figure 25: Liner structural element test.
Table 12: Geosynthetic structural element test: data and results

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x$</td>
<td>psf</td>
<td>180000</td>
</tr>
<tr>
<td>$E_y$</td>
<td>psf</td>
<td>130000</td>
</tr>
<tr>
<td>$u_x$</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>$G$</td>
<td>psf</td>
<td>60000</td>
</tr>
<tr>
<td>Thickness</td>
<td>ft</td>
<td>0.001</td>
</tr>
<tr>
<td>Area</td>
<td>ft$^2$</td>
<td>1</td>
</tr>
<tr>
<td>$D_x$</td>
<td>ft</td>
<td>0.09</td>
</tr>
<tr>
<td>$D_y$</td>
<td>ft</td>
<td>0.009</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
<td></td>
<td>0.009</td>
</tr>
<tr>
<td>Total force in x</td>
<td>lbs</td>
<td>17.88</td>
</tr>
<tr>
<td>Total force in y</td>
<td>lbs</td>
<td>5.08</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>psf</td>
<td>17880</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>psf</td>
<td>5080</td>
</tr>
</tbody>
</table>

These three tests show that the behavior of the embedded structural liner element is adequate to be used to represent geosynthetic reinforcement in GRS. The interfaces will act independently, failure can be reached on any interface, and the structural behavior of the liner element ensures an orthotropic representation. Therefore, liner structural elements are used to represent the geosynthetic and the geosynthetic-soil interfaces in the analyses of GRS in this research.
CHAPTER 5:
NUMERICAL MODELING PROCEDURES

This chapter explains the general numerical model to represent the behavior of a geosynthetic reinforced soil mass subjected to triaxial compression loading with constant confining pressure. The chapter begins with an introduction to the computer program used, and then it describes how the particular model of GRS is assembled and analyzed.

5.1. FLAC3D Computer Program

The present study uses the commercial three-dimensional explicit finite difference program FLAC3D (Itasca 2012) to develop the numerical model for the GRS mass. As it was explained into Chapter 3, this computer program is chosen due to its applicability to geotechnical problems, its capabilities for analysis of GRS masses, and its broad use (and thus relevance) in the geotechnical industry. The three-dimensional version is used due to its ability to include the orthotropic behavior of the geotextile, which would not be possible in two-dimensional axisymmetric analysis. This subsection presents a summary of the computer program's basic features. A complete description of the program is provided by Itasca (2012). Some of the computer program's tools that are particularly important for the GRS analyses done as part of this research were discussed at the beginning of Chapters 3 and 4.

FLAC3D (Fast Lagrangian Analysis of Continua in 3D) is a commercially available three-dimensional explicit, finite difference program for engineering mechanics computation that performs Lagrangian analysis. It can simulate the behavior of different materials such as soil and rock using polyhedral elements in a 3 dimensional grid with specific constitutive models assigned to them, while the material is subjected to forces or boundary conditions.

The computer program default mode of operation is a “command-driven mode” where the user writes the code that defines geometry, material's behavior, and boundary conditions or forces. The code also controls the analysis of the studied problem. FLAC3D has available an extensive list of commands for the user to write the code; there are available many material constitutive models and also structural elements. However, there is the possibility for the user to write their own code through the use of FISH, a build-in programming language within FLAC3D.

The general calculation procedure in FLAC3D follows an explicit time-marching scheme which, in summary, operates as a loop of the following steps during each timestep (Itasca 2011):

- The numerical finite difference grid is created or updated, material properties are assigned or updated, and boundary conditions are applied or updated.
- The equations of motion are invoked to derive new velocities and displacements from stresses and forces at every grid point.
- The velocities are utilized so as to derive new strain rates which in turn will yield new stresses through the application of the constitutive laws.
In every step within the above loop for one timestep all of the grid variables are updated for use in the next timestep using fixed values from the previous timestep. Thus, all the equations operate on fixed values, and the scheme is marching forward in time. The velocities are assumed to remain unchanged for the operation of the computation cycle (or timestep), such that the newly calculated stresses do not affect the velocities used in calculations for the current timestep. This may seem counterintuitive because it would be expected that if a stress changes in one zone, its neighboring zones' velocities would change too. The key parameter used to overcome this apparent omission is the choice of a timestep so small that there is actually no time to pass information from one element to another, of course during the period of calculation. Since the timestep actually defines how much time the calculation around the “loop” can last, if we assign a small timestep, neighboring elements do not have enough time to affect each other. After several timesteps information is propagated among zones, and then it is possible to reproduce how disturbances spread physically in the material body. In this way, iteration process is not used to reach compatibility and equilibrium with any type of constitutive law. The advantage of this forward marching scheme is that it does not require stiffness matrix storage, which requires less computational memory. (Itasca 2011)

5.2. Geometry of the Test, Boundary Conditions and Stages

This section describes the assembly of the general 3D model for the analysis of GRS tests in triaxial compression. Pertinent information on the grid generation and discretization, as well as the boundary conditions, is provided.

The general numerical model represents a triaxial specimen subjected to a monotonically increasing axial deviatoric stress until failure is reached. The main components of the model are the soil and the geotextile. The modified Chsoil constitutive model, presented in Chapter 3, is applied to the soil zones. The geotextile and its interface is modeled through the embedded liner structural elements, using a constitutive material behavior that is elastic and orthotropic for the geotextile itself, and using a linear elastic perfectly plastic shear interfaces between each side of the geotextile and the adjacent soil, as discussed in Chapter 4.

- Geometry and boundary conditions:

In order to reduce the calculation time, the GRS specimen is represented by a quarter of its entire volume, taking advantage of the double symmetry of its cylindrical configuration. A one-eighth portion based on symmetry about the horizontal plane at mid-height was not used so that both an even and odd number of geosynthetic layers could be included. The two vertical planes and the bottom horizontal plane of the quarter section have movement restrained in the direction normal to these planes, and therefore the bottom corner is fully constrained. Also the top and bottom faces have lateral movement restriction. Figure 26 shows a sample figure of the triaxial specimen, with three equally spaced geosynthetic layers. The boundary conditions are shown in the figure, including the velocity applied to the top of the element. The velocity at the top face represents the effect of the deviatoric stress in a triaxial test. The height to diameter ratio in the model follows
the geometry of each specific case in analysis, most of the time the samples in a triaxial test present height to diameter ratios of about 2 because of triaxial tests' requirements.

![FLAC3D 5.01](image)

Figure 26: Grid for a GRS specimen with three layers of geosynthetic, showing the soil, the geosynthetic layers, the boundary conditions, and the velocity applied at the top.

- **Discretization:**

  Numerical difficulties can arise in the finite different calculations if the mesh zones have exaggerated aspect ratios with height much larger than width. These difficulties can be avoided by using mesh zones that are close to cubic. A cylindrical type of mesh generates very long zones with height much bigger than the radial dimension near the center of the cylinder. Therefore, this type of discretization is avoided, and a combination of approximately parallelepiped zones and radial sector zones is used to generate the mesh. Figure 27 shows a comparison of one mesh using only radial sector zones and another mesh using a combination of approximately parallelepiped and radial sector zones. The combination mesh is more stable numerically. Some comparative analyses have been performed to confirm this outcome. Moreover, the computer program FLAC3D has commands to evaluate the suitability of the chosen mesh for a numerical simulation. Those commands can also be used to determine the mesh improvement.

  The number of zones and mesh refinement to be used for the material's discretization is another topic of interest when evaluating the sensitivity of the model. This topic will be analyzed in Chapter 6 where the material properties for each constituent element of the GRS mass are
determined. Chapter 6 includes convergence studies and determination of the adequate velocity (that represents the deviatoric stress at the top of the element) to be applied to generate a stable numerical analysis and avoid generation of strong unbalanced forces.

![Figure 27: Mesh in the section of the element. (a) Mesh using only radial sector zones (b) Mesh using a combination of approximately parallelepiped and radial sector zones.](image)

- Stages of the test:

With the geometry, boundary conditions and an adequate mesh ready, the numerical analysis of the domain is performed for triaxial test conditions. The confining cell pressure is applied in the model, using a command in FLAC3D that initializes stress condition in the zones. This creates a stable model prior to application of the deviator stress. The deviator stress is generated by applying a constant velocity on the upper face of the specimen. The velocity must be low enough so the software can solve the model without generating unwanted errors. The test can be run until failure is reached. There are two possible failure scenarios: (1) excessive axial deformation and (2) tensile strength limit of the geosynthetic. The maximum deformation is fixed to 14% for most of the tests. The strengths of the geosynthetic in the machine direction and the cross-machine directions are obtained from manufacturer information and provided as input data. Once the tensile strength of the geosynthetic is reached, the analysis is stopped and a message of geosynthetic failure is reported. Finally, the results are stored for post-processing and interpretation.

An example of the code is presented in Appendix C. Basically the code uses three stages. The first stage collects the data for the test, including all about property values, geometry, initial conditions, boundary conditions, and analysis control. All the data the model requires stays into this stage, and posterior stages ask for this information while the model is generated and the numerical problem is solved. The second stage contains all the commands to generate the full model. This implies: geometry, boundary conditions and determination of material's behavior. The third stage run the model, collects and stores the results and applies the termination process on the analysis. The results collected into this stage are used for interpretation and to draw conclusions. Only stage one needs to be updated for each numerical simulation the other two are automatically adapted to the
new information for each test in analysis. Following it is explained what is contained into each stage in more detail:

The first stage collects information regarding:

- Geometry of the sample, confining pressure, number of equally spaced geotextile layers, and the number of zones (discretization).
- Failure information: maximum strain and the strength of the geosynthetic in the two directions.
- Applied velocity, to develop the deviatoric stress.
- Soil input properties.
- Geosynthetic input properties.
- Interface input strength properties.

The second stage in the model generates the sample itself using information from the first stage. The second stage includes:

- Mesh generation. As shown in Figure 26 and in Figure 27(b), the mesh consists of a combination of approximately parallelepiped zones and radial sector zones to avoid zones with large aspect ratios. That configuration is automatically created through the written code.
- Generation of the geosynthetic layers, with automatic location of equally spaced reinforced layers.
- Application of boundary conditions. Fixing the soil mesh zones against displacement does not automatically fix the corresponding liner element nodes, which need to be fixed by means of a separate command.
- Assignment of model types and properties for each mesh zone and structural element.

The third stage specifies how the results must be calculated during the numerical analyses process, including:

- The volumetric strain is obtained by running a loop through all the zones in the model and adding their volumetric changes during each timestep of the model. The resulting total volume changes in the quarter cylinder are divided by the initial total volume of the quarter cylinder. In this way, the volumetric strain is obtained for each timestep of the test.
- In order to find the vertical stress, a certain level on the total height of the quarter cylinder is chosen. The vertical forces in each zone at that level are summed, and the result is divided by the area of the quarter cylinder to determine the vertical stress. For greatest accuracy, the area of each zone is calculated because the discretization does not produce a perfectly circular shape.
- The axial strain is calculated by dividing the vertical displacement of the uppermost nodes by the initial height of the quarter cylinder.
This third stage runs the numerical analysis applying the stopping conditions (maximum strain and tensile strengths of the geotextiles) for the two types of failure mechanisms discussed previously. Also during the run stage, the desired output parameters (volumetric strain, vertical stress, and axial strain) are calculated using the mechanisms explained before. Finally, this stage generates the desired result curves, which include the volumetric-strain-versus-axial-strain curve and the stress-strain curve.

All the stages are stored in different files, called by a run file in order to perform the analysis. Many analyses (composed of these three stages) can be performed with the same FLAC3D file so that different confining pressures, for example, can be parametrically investigated for a specific GRS configuration.
CHAPTER 6:
NUMERICAL ANALYSIS OF LABORATORY TESTS

Performance of the 3D numerical model described in Chapters 3, 4 and 5 is compared to data collected from laboratory tests of GRS described in the available literature. The first step in this process is to find good quality data sets for the comparisons, and the next step is to determine the properties for the materials and interfaces in each suitable laboratory test. The last step is to perform numerical analyses to compare them with the laboratory data. This last step is divided into two parts, small strain response and large strain (failure) response. Some conclusions are presented at the end of the chapter.

6.1. Published Laboratory Tests of GRS

The objective of this section is to summarize suitable laboratory tests that will allow for comparison with results from the numerical model explained in the previous chapters. After a thorough search, 25 papers/thesis describing GRS laboratory tests were found, including a wide range of test conditions and geosynthetic types. The most common tests are plane strain and triaxial tests. References like McGown et al. (1978), Boyle (1995), Wu et al. (2013), Ketchart and Wu (2002) show test details and results for plane strain test on GRS. Fourteen references present test details and results from triaxial tests, and some of those are analyzed later in this chapter. Elton and Patawaran (2004) present an unconfined test of GRS. Adams et al. (2002) and Adams et al. (2007) describe mini-pier tests, which are pilot-scale tests than incorporate concrete masonry units as facing around a column of GRS.

Among the geosynthetics used in these investigations, there is an even broader range of variability than the variability in test condition. The tests include different types of woven and nonwoven geotextiles, different geogrids, and even fiberglass nets, aluminum meshes, and coir geotextiles. Besides the type of test and the geosynthetic in use, another important consideration for choosing published test results for use in analysis is the amount of information available. Some references present detailed information on the laboratory results, while others do not. Some of the references do not even describe the materials used in the tests.

In the present investigation, the numerical model development focuses on triaxial compression loading with constant confining pressure. Therefore, only the published test results involving this type of testing are considered suitable. Moreover, most applications of GRS technology use geotextiles; then, only tests that incorporate geotextiles are used for the comparisons with numerical analyses described later in this chapter.

Based on these criteria, five sources of appropriate GRS tests were identified. In the following, a summary of each of the five references is presented. Each summary emphasizes the objectives of the tests, the laboratory test details, and the test results. After the summary descriptions of the five sources, a table presents the principal and most useful information from each test for this research,
including: soil type and relative density, basic information about the geotextiles used, and information regarding other pertinent variables such as confining pressure, sample size, and number of geotextile layers in the test.


As the title suggests, the main purpose of this work was to contrast the behavior of a soil reinforced with layers of fabrics to a soil reinforced with randomly distributed fibers. In addition to comparing the effect of these two types of reinforcements on the soil behavior, this work also investigated the influence of variables such as the relative density of the soil, confining pressure, properties of the reinforcement, and amount of reinforcement on the reinforced soil behavior.

The methodology involved triaxial compression tests on both types of reinforced soils, where the stress-strain response was contrasted. The research used: (1) one type of soil, dune sand from Muskegon, Michigan; (2) five types of geotextiles, three woven and two non-woven geotextiles; and three different types of fiber filaments, two natural reed fibers and one glass fiber.

The principal findings include that: both types of reinforcement methods increase the strength of the soil and the axial strain at failure; increasing the amount of reinforcement generates an increase in the soil strength until a limit where the response becomes asymptotic; and both reinforcement methods were interpreted as increasing the normal stresses in the soil above the externally applied confining pressure, thereby producing strength envelopes of the reinforced soil that lie above and parallel (in many cases) to the strength envelope for the unreinforced envelope.


The main purpose of this study was to investigate how various parameters, such as: geosynthetic type, confining pressure, and geosynthetic-layers separation influence the frictional response of the soil-reinforcement interface and the tension in the reinforcement for a reinforced soil system.

Triaxial tests, direct shear tests, and analytical developments were used to fulfill the research objectives. In the triaxial tests, two sample sizes were used: 100 and 200 mm diameter, both with a ratio height to diameter of two. The soil used was a medium dense river sand; as the reinforcement: one woven and two non-woven geotextiles were used. Strain distribution in the reinforcement layers, for the 200 mm diameter samples only, was monitored through the use of strain gauges. Strength, and dilatancy behavior were monitored during the tests for both sample sizes, while parameters such as confining pressure, spacing, fabric properties (strength and stiffness) were varied.

Direct shear tests were performed using the same soil and the woven geotextile. The interface friction behavior between the soil and the geotextile was studied, and interface friction parameters were obtained.
Among the principal results the reference presents:

- Increasing the amount of layers of fabric in the sample generates increases in the maximum deviator stress that can be applied, increases in the strain to failure, and increases in the tendency for the soil to expand.

- Two types of failure modes are recognized. The first one is a failure due to compression where the reinforced soil strength is governed by the interface friction angle. This interface friction generates a confining in the soil between the reinforcement layers, which directly influences the strength of the system. The second failure mechanism was due to reinforcement rupture. The strength is governed by the tensile strength of the reinforcement.

- The soil-reinforcement mobilized interface friction is affected by the spacing of the reinforcement layers and the stiffness of the reinforcement. While the spacing decreases, the mobilized interface friction increases. The same happens with increasing reinforcement stiffness.


The objective of this research was to compare behavior of monotonically loading reinforced specimens and cyclic loading specimens. It was proposed to investigate strength for the monotonic loading and deformation for cyclic loading specimens.

Experiments with triaxial monotonic and cyclic loading on sand were done to fulfill the objectives. The soil used in the experiments consisted of a well-graded, fine to coarse sand referred to as “Concrete Sand”. Tests on unreinforced and reinforced soil were performed. Two geotextiles were used as reinforcement, one woven in the cyclic test and one non-woven for the monotonic and cyclic tests. During monotonic tests, axial stress and volume change were controlled. During cyclic tests axial deformation was recorded.

The obtained results showed an increase in monotonic strength on the reinforced soil compared to the unreinforced soil, and an increase in ductility and increase in strain at failure was also observed. In cyclic loading, the reinforced soil accumulates less axial deformation than the unreinforced soil. There is a reduction in volume change for both monotonic and cyclic tests due to reinforcement.

Reference 4: "Effect of Geotextile Reinforcement on the Mechanical Behavior of Sand" by Haeri et al. (2000)

This research's objective was to investigate the mechanical behavior of a geotextile-reinforced soil: its stress-strain and dilative behavior. The research proposed to investigate the influence of parameters such as: number of geotextile layers, geotextile type, geotextile arrangement, confining pressure, and sample size.
In order to fulfill the objectives, this research developed 160 triaxial tests using dry beach sand with a relative density of 70%. The soil was reinforced using three different types of non-woven geotextiles. Two sample sizes were used: 38 and 100 mm in diameter with height to diameter ratios of two. The layers of geotextile varied from one to four layers. Also their location within the soil mass was varied. The confining pressures varied from 60 to 500 kPa.

The principal findings include: reinforcement of sand increased the peak strength, axial strain at failure, and ductility, while dilation was reduced. Mentioned reinforcement effects were more noticeable for small size tests (38 mm diameter) than for the larger samples. As the number of geotextile layers increased, the peak strength increased too. This effect decreased with an increase in confining pressure. Geotextile type influenced the stiffness of the reinforced sand. While the stiffness in the geotextile was increased, the stiffness in the reinforced soil increased too. The arrangement of geotextile layers influenced the results, with the strongest and stiffest results obtained when the geotextiles were located at the mid-height of the specimens where the strength demands were the greatest.


The purpose of this research was to determine the influence of mainly two parameters into the soil-reinforcement system. These two parameters in study were: number of reinforcement layers and tensile strength of the reinforcement. Also, the influence of geosynthetic surface roughness on the composite strength was studied.

Thirty-six triaxial tests were done with different geosynthetic types and at different configurations or arrangements into the samples. The geosynthetics used in this study are: a woven geotextile, a geogrid, and a polyester film. The soil was a poorly graded river sand at 70% relative density. Interface direct shear tests were run to determine geosynthetic-soil interface properties.

Among the results, it was found that, independently of the type of geosynthetic used, the reinforced sand exhibited an increase in peak deviator stress and failure strain with respect to the unreinforced sand. These reinforced-soil properties did not result a function of the tensile strength of the reinforcement. An unexpectedly high efficiency of the polyester reinforcement material led to investigation of the influence of the geosynthetic surface roughness on the reinforcement efficiency. The study found that irregularities (which generate roughness) on the polyester surface made it more efficient than the geotextile material, and the surface roughness of the geosynthetic is an important parameter to be considered.

The following table shows in a compact form the most relevant information regarding the tests from each of the five selected references. Some important information should be noticed from Table 13:

- All references describe triaxial tests on sand reinforced with geotextiles.
• All tests but one, the test by Ashmawy and Bourdeau (1998), are run under different confining pressures.
• All the tests except Ashmawy and Bourdeau (1998) test use different amounts of geotextiles.
• Sample diameters include 38 mm, 71 mm, 100 mm, and 200 mm.
• In most cases, the height-to-diameter ratio is about 2. Two cases present greater ratios: 2.2 and 2.4 for Gray and Al-Refeai (1986) and Ashmawy and Bourdeau (1998), respectively.
• Out of the listed geotextiles, 3 are woven and 8 are non-woven.

Regarding the tests summarized in Table 13, two points considering test selection for comparison of the numerical model: the size of the specimens and the number of reinforcement layers. For specimen size, it would seem to be difficult to fabricate representative small diameter specimen that include small hand-cut reinforcement discs. Also, when specimens have a large number of reinforcement layers and small spacing between layers, specimen preparation would seem difficult. Therefore, the laboratory chosen tests for comparison with the numerical model should prefer bigger tests and avoid tests with very small spacing between geosynthetic layers. Small tests, less than 70 mm diameter are not used for comparison purposes.
Table 13: Summary of the selected papers on GRS laboratory tests

<table>
<thead>
<tr>
<th>Reference</th>
<th>Variables in the test*</th>
<th>Soil</th>
<th>Geotextile**</th>
<th>Useful Results for Comparison Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Confining pressure: 49, 98, 196, 294 and 392 kPa&lt;br&gt;Layers of geotextile: 1 to 6&lt;br&gt;Sample size: D = 36 mm, H = 80 mm</td>
<td>Dry, uniform, dune sand at 86% and 21% relative densities</td>
<td>GEOLON 200: W PP tape&lt;br&gt;TYPAR 3601: NW PP multifilament&lt;br&gt;TYPAR 3401: NW PP multifilament</td>
<td>Stress-strain curves with fixed confining pressure and soil density. The variables in the curves: two geotextile arrangements and three types of geotextiles.</td>
</tr>
<tr>
<td>2</td>
<td>Confining pressure: 25, 50, 80 kPa&lt;br&gt;Layers of geotextile: 1 to 5&lt;br&gt;Sample size: D = 100 mm, H = 200 mm and D = 200 mm, H = 400 mm</td>
<td>Medium dense river sand</td>
<td>FOV 600-8650: W PET fabric&lt;br&gt;Propex 4545: NW (in large tests only)&lt;br&gt;Propex 4553: NW (in large tests only)</td>
<td>Two sets of stress-strain and volumetric strain curves with fixed confining pressure, sample size, and geotextile type. Variables: amount of geotextile. Another set with fixed sample size and geotextile type. Variables: confining pressures and amount of geotextile.</td>
</tr>
<tr>
<td>3</td>
<td>Confining pressure: 50 kPa&lt;br&gt;Layers of geotextile: 5&lt;br&gt;Sample size: D = 71 mm, H = 169.8 mm</td>
<td>&quot;Concrete sand&quot; at 35% relative density</td>
<td>TYPAR 3801: NW</td>
<td>Stress-strain and volumetric strain curves with fixed confining pressure and amount of geotextile.</td>
</tr>
<tr>
<td>4</td>
<td>Confining pressure: 60, 100, 300, and 500 kPa&lt;br&gt;Layers of geotextile: 1 to 4&lt;br&gt;Sample size: D = 38 mm, H = 76 mm and D = 100 mm, H = 200 mm</td>
<td>Uniform, clean, quartz, beach SP sand, at 70% relative density</td>
<td>Hoechst, 11/180: NW&lt;br&gt;Husker, B500: NW&lt;br&gt;TYPAR 3407: NW</td>
<td>Six sets of stress-strain and volumetric strain curves mostly changing number of geotextile layers at different confining pressures, sample size, and geotextile types.</td>
</tr>
<tr>
<td>5</td>
<td>Confining pressure: 100, 150, 200 kPa&lt;br&gt;Layers of geotextile: 2, 3, 4 and 8&lt;br&gt;Sample size: D = 38 mm, H = 76 mm</td>
<td>SP river sand at 70% relative density</td>
<td>W PP geotextile</td>
<td>Two sets of stress-strain and volumetric strain curves, changing confining pressure on the first set and amount of geotextile on the second.</td>
</tr>
</tbody>
</table>

* D = diameter, H = height

**W: woven, NW: nonwoven, PP: polypropylene, PET: polyester
Using the information summarized from the five references and the considerations discussed above, the following table shows the specific tests to be used in the numerical comparison procedure. A total of 30 tests are listed in the table and judge suitable for the comparisons.

Table 14: Chosen tests for the comparison analyses

<table>
<thead>
<tr>
<th>Reference</th>
<th>Soil, relative density</th>
<th>Geotextile</th>
<th>Sample size Diameter (mm), high (mm)</th>
<th>Confining pressure (kPa)</th>
<th># Layers</th>
<th>Total number of tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Medium dense sand</td>
<td>FOV 600-8650</td>
<td>100, 200</td>
<td>25, 50, 100</td>
<td>1, 2, 3</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>Medium dense sand</td>
<td>FOV 600-8650</td>
<td>200, 400</td>
<td>25, 50, 80</td>
<td>1, 3</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>Sand medium dense</td>
<td>Propex 4545</td>
<td>200, 400</td>
<td>80</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Sand, 35%</td>
<td>TYPAR 3801</td>
<td>71, 169.8</td>
<td>50</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Sand, 70%</td>
<td>Husker, B500</td>
<td>100, 200</td>
<td>300</td>
<td>1, 2, 3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Sand, 70%</td>
<td>Husker, B500</td>
<td>100, 200</td>
<td>60</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Sand, 70%</td>
<td>Hoechst, 11/180</td>
<td>100, 200</td>
<td>60</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Sand, 70%</td>
<td>TYPAR 3407</td>
<td>100, 200</td>
<td>60</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Sand, 70%</td>
<td>Hoechst, 11/180</td>
<td>100, 200</td>
<td>100</td>
<td>1, 2, 3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>Sand, 70%</td>
<td>Husker, B500</td>
<td>100, 200</td>
<td>100</td>
<td>1, 2, 3</td>
<td>3</td>
</tr>
</tbody>
</table>

6.2. Material Properties for Each Source

To analyze the tests listed in Table 14, material property values are necessary for each component. This section explains the procedures used to obtain all the input parameters necessary, and it presents the resulting values for each case.

This section begins with the determination of the soil input values. Next, the geosynthetic parameters are determined, including a discussion of the relevance of parameters obtained from a one-dimensional loading test. Finally, this section presents the determination of the soil-geosynthetic interface parameter values.

6.2.1. Soil Parameters

Some input parameters are already clearly stated in their respective reference; others can be calculated using the data presented in their reference to directly determine the property. Table 15 presents the situation for the three references in analysis, showing how the soil data is obtained.
Table 15: Soil input parameters, type of determination

<table>
<thead>
<tr>
<th>Soil Properties</th>
<th>Reference:</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c$, $\Delta \phi$</td>
<td>calculated</td>
<td>given</td>
<td>calculated</td>
<td></td>
</tr>
<tr>
<td>$\psi_c$, $\Delta \psi$</td>
<td>calculated</td>
<td>calculated</td>
<td>calculated</td>
<td></td>
</tr>
<tr>
<td>$\phi_{cvo}$, $\Delta \phi_{cvo}$</td>
<td>calculated</td>
<td>calculated</td>
<td>calculated</td>
<td></td>
</tr>
<tr>
<td>Cohesion</td>
<td>calculated</td>
<td>given</td>
<td>calculated</td>
<td></td>
</tr>
<tr>
<td>Elastic properties</td>
<td>calculated</td>
<td>calculated</td>
<td>calculated</td>
<td></td>
</tr>
<tr>
<td>$R_f$</td>
<td>calculated</td>
<td>calculated</td>
<td>calculated</td>
<td></td>
</tr>
</tbody>
</table>

The three references used for the comparison analyses present results of stress-strain and volumetric-strain-axial-strain curves. References 2 and 4 include tests with at least three different confining pressures, which means that all the required information is available to calculate the input parameters for the numerical soil model. Reference 3 shows test results for only one consolidation pressure. For References 2 and 4, the method explained in Section 3.5 was used to obtain the input parameter values for the Chsoil constitutive model. The results are included in Appendix A, and a summary of the resulting inputs is in Table 16. For Reference 3, as there are only curves for one confining pressure, only the first part of the method described in Section 3.5 is used, which means that the resulting input parameters does not have a regular formulation that applies to any confining pressure, and they are only useful for the confining pressure used in the analysis. On this case, the input parameters for the model are chosen in a way that the influence of the confining pressure is eliminated and only constant values are used. That is the reason why all the parameters that meant variation are zero for this reference. See Table 16.

Table 16: Soil input parameters summary

<table>
<thead>
<tr>
<th>Soil Properties</th>
<th>Reference:</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c$, $\Delta \phi$</td>
<td>37.36; 5.83</td>
<td>38.84; 0.617</td>
<td>43.95; 0.535</td>
<td></td>
</tr>
<tr>
<td>$\psi_c$, $\Delta \psi$</td>
<td>4.81; 0.95</td>
<td>2.88; 0</td>
<td>18.67; 8.47</td>
<td></td>
</tr>
<tr>
<td>$\phi_{cvo}$, $\Delta \phi_{cvo}$</td>
<td>29.83; 2.69</td>
<td>33.46; 0</td>
<td>29.2; -11.12</td>
<td></td>
</tr>
<tr>
<td>Cohesion</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Elastic properties</td>
<td>192.7; 0.299</td>
<td>188.7; 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G_{ref}$, $n$</td>
<td>563.7; 0</td>
<td>49165; 0</td>
<td>395.1; 0.75</td>
<td></td>
</tr>
<tr>
<td>$K_{ref}$, $m$</td>
<td>0.969</td>
<td>0.965</td>
<td>0.914</td>
<td></td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.969</td>
<td>0.965</td>
<td>0.914</td>
<td></td>
</tr>
</tbody>
</table>

6.2.2. Geosynthetic Parameters

As stated in Section 4.2, the geosynthetic model requires four input parameters: two Young's moduli (in machine and cross-machine directions), one Poisson's ratio (the second can be calculated), and the shear modulus. For the tests in analysis, information regarding six geosynthetics is required, as indicated in Table 14. Therefore, four properties for each of six
geosynthetics are necessary. Unfortunately, complete data was not found for any of the geotextiles in Table 14. The manufacturer's information, when available, typically provides thicknesses and Young's modulus or stiffness, which is the product of thickness and modulus. No product-specific information was found regarding Poisson's ratios or shear moduli. Therefore, information on similar types of geotextiles was investigated.

Most of the geosynthetic elastic properties are estimated based on other similar products. The idea is that a similar geosynthetic composed of the same material and fabricated in the same way (woven, non-woven, heat-bonded, needle punched, etc.) should have similar elastic properties. However, insufficient information was uncovered about one parameter, the shear modulus, to provide a sound basis for estimating its value. In this case, the shear modulus was estimated as a fraction of the Young's modulus, and parameter studies indicate that the shear modulus of the geosynthetic has little impact on GRS loaded in triaxial compression. Table 17 shows information regarding the characteristic of the geosynthetics and how the parameters for each one are obtained. Detailed explanations for each property value are provided in what follows, and a tabular summary of the property values is provided.

Table 17: Geosynthetics in analysis, and specification of the procedure to be used in order to obtain each geosynthetic property

<table>
<thead>
<tr>
<th>Geosynthetic</th>
<th>Reference 2</th>
<th>Reference 3</th>
<th>Reference 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOV 600-8650</td>
<td>Propex 4545</td>
<td>TYPAR 3801</td>
<td>Hoechst, 11/180</td>
</tr>
<tr>
<td>Amoco</td>
<td>TYPAR 3407</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|-----------------|-----------------|----------------------------------------|------------------------|-----------|-----------|------------------------|

<table>
<thead>
<tr>
<th>Properties</th>
<th>Thickness:</th>
<th>Ex, Ey:</th>
<th>vx, vy:</th>
<th>G:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>given</td>
<td>calculated</td>
<td>estimated</td>
<td>imposed</td>
</tr>
<tr>
<td></td>
<td>estimated</td>
<td>calculated</td>
<td>estimated</td>
<td>imposed</td>
</tr>
<tr>
<td></td>
<td>estimated</td>
<td>estimated</td>
<td>estimated</td>
<td>imposed</td>
</tr>
<tr>
<td></td>
<td>estimated</td>
<td>estimated</td>
<td>estimated</td>
<td>imposed</td>
</tr>
</tbody>
</table>

Thickness is typically available in the manufacturer's information and, in such cases, this information is used for the analyses. For two cases, Propex 4545 and TYPAR 3801, the information is not given and then the thickness is estimated from similar products with similar weight per unit area. It is also important to note that, when load versus strain information is provided, the geosynthetic thickness is unimportant because thickness has the same role in the reported stiffness that it has in the numerical analyses. In particular, stiffness is the product of modulus times thickness. This reasoning applies in the case of the TYPAR 3801. However, both thickness and Young's modulus are estimated for the Propex 4545.

Young's modulus values are calculated from secant stiffness data or tensile-load-versus-strain curves. When this information is not available, information about similar products was used, based
on material type, fabrication method, and weight per unit area. The Young's modulus values are obtained as follows:

- **FOV 600-8650:** Reference 2 presents a curve of tensile load vs. axial strain. From this curve the Young's modulus at 5\% strain is determined. The stiffness corresponding to 5\% strain is 344 kN/m, and using a thickness of 1 mm (data from same reference), the Young's modulus value is 344,000 kN/m².

- **TYPAR 3801:** Reference 3 gives a stiffness of 120 kN/m for 10\% strain. With this data, the Young's modulus at 10\% strain is obtained using a thickness of 0.4 mm (from other non-woven TYPAR materials), and the result is a modulus of 300,000 kN/m².

- The four other Young's moduli were determined using the following process. The only data for these geosynthetics gives the ultimate strength and its respective strain. That information is used to make an approximation of the Young's modulus for each case. The stiffness at failure is affected by a factor of 0.25 compared to the stiffness at working strain levels, and this factor is used to obtain the Young's modulus at working strains by dividing the factor into the Young's modulus at failure. The factor of 0.25 was obtained through analyses of other non-woven geotextile load-versus-strain data. The following table shows the calculation process.

Table 18: Young modulus calculation for the case where only ultimate tensile strength is available. Dual numbers for Husker and Hoechst indicate the geosynthetic properties in the machine and cross-machine directions. Other products have the same property on both directions and therefore those do not need dual numbers

<table>
<thead>
<tr>
<th>Geosynthetic</th>
<th>Propex 4545 Amoco</th>
<th>Husker, B500</th>
<th>Hoechst, 11/180</th>
<th>TYPAR 3407</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate tensile strength (kN/m)</td>
<td>16</td>
<td>12/20</td>
<td>10.4/10.4</td>
<td>8.4</td>
</tr>
<tr>
<td>Ultimate strain (%)</td>
<td>50</td>
<td>130/120</td>
<td>75/80</td>
<td>60</td>
</tr>
<tr>
<td>Ultimate stiffness (kN/m)</td>
<td>32</td>
<td>9.2/16.7</td>
<td>13.9/13</td>
<td>14</td>
</tr>
<tr>
<td>Estimated stiffness at low strains (kN/m)</td>
<td>128</td>
<td>36.8/66.8</td>
<td>55.6/52</td>
<td>56</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>1</td>
<td>5.5</td>
<td>2.1</td>
<td>0.49</td>
</tr>
<tr>
<td>Young's modulus (kN/m²)</td>
<td>128000</td>
<td>6690/12145</td>
<td>26476/24762</td>
<td>114286</td>
</tr>
</tbody>
</table>

Poisson's ratio ended up being a difficult parameter to find in the available literature. No information regarding the geosynthetics used in the laboratory tests of the five selected references have been found. However, a few additional sources have been identified, and the available
reported values for Poisson's ratio are summarized in Table 19. The table includes some information characterizing the material (geotextile) type and its corresponding Poisson's ratio.

Table 19: Reported Poisson's ratios for geotextiles in the available literature

<table>
<thead>
<tr>
<th>Source</th>
<th>Geotextile type</th>
<th>Mass per unit area (g/m²)</th>
<th>Ultimate strength (kN/m)</th>
<th>Poisson's ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kutay et al. (2006)</td>
<td>Non woven needle punched polypropylene</td>
<td>278</td>
<td>39</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>Woven monofilament polypropylene</td>
<td>190</td>
<td>39</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>Woven monofilament polypropylene</td>
<td>250</td>
<td>47</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>Woven fibrillated yarn polypropylene</td>
<td>284</td>
<td>48</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>Woven multifilament polyester</td>
<td>290</td>
<td>70</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>Woven fibrillated yarn polypropylene</td>
<td>490</td>
<td>70</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>Woven fibrillated yarn polypropylene</td>
<td>578</td>
<td>105</td>
<td>1.46</td>
</tr>
<tr>
<td></td>
<td>Woven multifilament polyester</td>
<td>1907</td>
<td>632</td>
<td>1</td>
</tr>
<tr>
<td>Perkins (2000)</td>
<td>Woven geotextile polypropylene</td>
<td></td>
<td>31/31</td>
<td>0.5/0.1245</td>
</tr>
<tr>
<td>Shukla et al. (2009)</td>
<td>Non woven needle punched geotextile</td>
<td>80</td>
<td>5.8</td>
<td>1.75</td>
</tr>
</tbody>
</table>

As can been noticed in Table 19, several of the reported values of Poisson's ratio are very high, and such high values can generate unrealistic outcomes and numerical difficulties when the material (geotextile) is loaded in two directions.

Additional information from biaxial tests was sought to help resolve this issue. Veldhuijzen van Zanten (1986) describes development of a biaxial testing machine that is able to test geotextiles by applying loads in both directions. Test results on some geotextiles are presented in his book (Chapter 6) together with graphs showing strains and stresses in both directions for three different biaxial tests (for each geotextile): one with strain in the x-direction equal to zero, the second with strain in the y-direction equal to zero, and the third finds the stress required to keep strains in both directions the same. Those results were used to back-calculate properties that would generate the measured results. The orthotropic elastic material equations for plane stress were applied (Equations 4-1). Because the stiffnesses in all directions are equal to the corresponding modulus values times the thicknesses, these equations can be applied by substituting stiffnesses and loads (force per unit length) for moduli and stresses (force per unit area). The procedure details are presented in the following example for one of the tests described by Veldhuijzen van Zanten (1986):

Data from the reference:

Geotextile: Polypropylene needlepunched.
Tensile stress (listed values are actually loads) at 10% strain (from biaxial test curves):
Test with $\varepsilon_x = 0$, $\sigma_y = 4.9 \, kN/m$
Test with $\varepsilon_y = 0$, $\sigma_x = 6.7 \, kN/m$
Test with $\varepsilon_y = \varepsilon_x$, $\sigma_y = 6.8 \, kN/m$ and $\sigma_x = 9.18 \, kN/m$

From Equations 4-1, parameters $c_{11}$, $c_{22}$ and $c_{12}$ can be obtained as a function of the type of biaxial test:

- Test with $\varepsilon_y = 0$, $c_{11} = \frac{\sigma_x}{\varepsilon_x}$
- Test with $\varepsilon_x = 0$, $c_{22} = \frac{\sigma_y}{\varepsilon_y}$
- Test with $\varepsilon_y = \varepsilon_x$, $c_{12} = \frac{\sigma_x}{\varepsilon_x} - c_{11}$, or $c_{12} = \frac{\sigma_y}{\varepsilon_y} - c_{22}$

Therefore, applying the equation to the example (listed values are based on stiffnesses, not moduli):

- $c_{11} = \frac{6.7}{0.1} = 67 \, kN/m$
- $c_{22} = \frac{4.9}{0.1} = 49 \, kN/m$
- $c_{12} = \frac{9.18}{0.1} - 67 = 24.8 \frac{kN}{m}$, or $c_{12} = \frac{6.8}{0.1} - 49 = 19 \, kN/m$, the average result: 21.9 kN/m

Again, from Equations 4-1:

- $c_{11} = \frac{E_x}{1-\nu_x\nu_y}$, $c_{22} = \frac{E_y}{1-\nu_x\nu_y}$, and $c_{12} = \frac{E_x\nu_y}{1-\nu_x\nu_y} = \frac{E_y\nu_x}{1-\nu_x\nu_y}$

From $c_{11}$ and the first expression of $c_{12}$, $\nu_y = \frac{c_{12}}{c_{11}}$.

From $c_{22}$ and the second expression of $c_{12}$, $\nu_x = \frac{c_{12}}{c_{22}}$.

And then, $E_x$ can be obtained from $c_{11}$ equation and $E_y$ from $c_{22}$.

Finally, Poisson's ratio in $x = 0.45$, Poisson's ratio in $y = 0.33$, $E_x = 57 \, kN/m$, and $E_y = 42 \, kN/m$. It is recognized that these "Young's modulus" values are actually stiffnesses, which can be converted to moduli if desired by dividing by the thickness.

This procedure produced the results presented in Table 20. The resulting Poisson's ratios vary from about zero to about 0.5, which are values that seem much more realistic and useful for the present investigation.
Table 20: Geotextiles properties calculated from biaxial tests data presented by Veldhuijzen van Zanten (1986). \( v = \) Poisson's ratio, \( E = \) Young's modulus

<table>
<thead>
<tr>
<th>Material type, all with mass per unit area = 200 g/m²</th>
<th>( \nu_x )</th>
<th>( \nu_y )</th>
<th>( E_x ) (kN/m)</th>
<th>( E_y ) (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NW needle-punched polypropylene</td>
<td>0.33</td>
<td>0.44</td>
<td>41.71</td>
<td>56.13</td>
</tr>
<tr>
<td>NW needle-punched and chemically bonded polyester</td>
<td>0.08</td>
<td>0.06</td>
<td>101.20</td>
<td>74.31</td>
</tr>
<tr>
<td>NW heat-bonded polypropylene and polyethylene</td>
<td>0.08</td>
<td>0.07</td>
<td>104.75</td>
<td>91.65</td>
</tr>
<tr>
<td>W tape polypropylene</td>
<td>0.39</td>
<td>0.53</td>
<td>136.74</td>
<td>185.00</td>
</tr>
<tr>
<td>W monofilament gauze polyethylene</td>
<td>0.25</td>
<td>0.26</td>
<td>109.16</td>
<td>113.43</td>
</tr>
<tr>
<td>W multifilament polyester</td>
<td>-0.02</td>
<td>-0.01</td>
<td>663.32</td>
<td>324.56</td>
</tr>
</tbody>
</table>

Regarding shear modulus, no information has been found, and very few references present a shear modulus, or the description of its determination. In the publications reviewed as part of this research, when a shear modulus was presented in association with description of calculations, no basis for the shear modulus was presented. For this research, an arbitrary value shear modulus equal to one-third of the Young's modulus was used. Parameter studies showed that the value of the shear modulus does not have a significant impact on the calculated response of geosynthetic reinforced soils subjected to triaxial loading cases. With this discussion, the final inputs for geosynthetic properties are summarized in the following table.

Table 21: Summary geosynthetic input properties

<table>
<thead>
<tr>
<th>Geosynthetic</th>
<th>Reference 2</th>
<th>Reference 3</th>
<th>Reference 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOV 600-8650</td>
<td>Propex 4545</td>
<td>TYPAR 3801</td>
<td>Husker, B500</td>
</tr>
<tr>
<td>Thickness (mm):</td>
<td>1</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>Ex, Ey (kN/m²):</td>
<td>344000</td>
<td>128000</td>
<td>300000</td>
</tr>
<tr>
<td>vx, vy:</td>
<td>0.4</td>
<td>0.4</td>
<td>0.27</td>
</tr>
<tr>
<td>G* (kN/m²):</td>
<td>114660</td>
<td>42660</td>
<td>100000</td>
</tr>
</tbody>
</table>

*Imposed as 1/3 of the Young's modulus, which is the value that would result from a homogenous, isotropic, linear-elastic material with Poisson's ratio equal to 0.5.

6.2.3. Interface Parameters

The soil-geosynthetic interface model requires 6 input parameters: normal stiffness, normal tensile strength, tangent stiffness, tangent cohesive strength, tangent residual cohesive strength, and tangent friction angle.
Regarding the normal properties in the interface, the normal tensile strength is kept at zero for all cases, which means the interface model does not permit any tensile stress to develop in the normal direction. In compression, the interface stiffness in the normal direction needs to be stiff enough to avoid undesirable deformation. On the other hand, the normal stiffness should not be so high that it slows the computation time. Itasca (2012) recommends that the stiffness should be limited to the value given by Equation 6-1, and this value was adopted as the normal stiffness on both sides of the geotextile for the analyses done in this research.

\[
k = \max \left[ \frac{K + \frac{4}{3}G}{\Delta z_{\text{min}}} \right]
\]

where:

- \( k \) is the normal stiffness of the interface
- \( K \) is the bulk modulus of the adjacent soil zone
- \( G \) is the shear modulus of the adjacent soil zone
- \( \Delta z_{\text{min}} \) is the smallest width of an adjacent zone in the normal direction

Regarding the tangential interface properties, the cohesive strength is taken equal to zero for all cases. Most of the analyzed literature shows an approximately zero cohesive strength when a geotextile is in contact with granular soil. The residual cohesive strength is also made equal to zero, for all cases.

Two interface parameters remain: the friction angle of the interface and the tangent interface stiffness. In order to make a good estimate of these parameter values, some information has been collected from available literature. A summary of the data obtained is presented in Table 22, including a summary of the values for efficiency or the interface friction angle as a function of the geotextile type.
### Table 22: Interface properties from available literature

<table>
<thead>
<tr>
<th>Soil data</th>
<th>Geosynthetic data</th>
<th>Interface Friction angle (degrees) and/or efficiency ((\tan\delta/\tan\phi))</th>
<th>Interface stiffness</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beach sand (SP)</td>
<td>Televev 70/70: woven, multifilament polyester. Tensile strength 70 kN/m</td>
<td>31.1, 31.2 (for soil with RD = 50%, RD = 80%) Efficiency: 0.91, 0.87</td>
<td></td>
<td>Lee and Manjunath (2000)</td>
</tr>
<tr>
<td></td>
<td>Amoco 2000: woven, monofilament polyester geotextile. Grab tensile strength 625 N</td>
<td>32.5, 33 Efficiency: 0.96, 0.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reemay 3401: non-woven, spun-bound, polypropylene, grab tensile strength 648/720 N (machine/cross-machine directions)</td>
<td>33.2, 32.2 Efficiency: 0.99, 0.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravely sand at 97% of optimum Proctor density. (\phi = 39) degrees</td>
<td>Woven</td>
<td>34-35. Efficiency: 0.83-0.86</td>
<td></td>
<td>Krieger and Thamm (1991)</td>
</tr>
<tr>
<td>Concrete sand. (\phi = 30) degrees</td>
<td>Nonwoven needle punched Nonwoven heat bonded Woven monofilament Woven slit film</td>
<td>Efficiencies: 1 0.84 0.84 0.77</td>
<td></td>
<td>Koerner (2012)</td>
</tr>
<tr>
<td>Rounded sand ((\phi = 28) degrees)</td>
<td>Nonwoven needle punched Woven slit film</td>
<td>0.92 0.84</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silty sand ((\phi = 26) degrees)</td>
<td>Nonwoven needle punched Woven slit film</td>
<td>0.96 0.87</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SP with RD = 70% (\phi = 46) degrees</td>
<td>Multifilament polyester woven with fine texture Multifilament polyester woven with coarse texture</td>
<td>36.7 (efficiency = 0.72) Adhesion 4 kPa 42.2 degrees (effic = 0.87) Adhesion 6 kPa</td>
<td></td>
<td>Basudhar (2010)</td>
</tr>
<tr>
<td>SP (angular grains) with RD = 70% (\phi = 46) degrees</td>
<td>Multifilament polyester woven 58 kN/m Multifilament polyester woven 320 kN/m</td>
<td>39.3 (efficiency: 0.79) 41.1 (0.84) Adhesion zero all cases 34.5 (0.95) 35.8 (0.99)</td>
<td></td>
<td>Anubhav and Basudhar (2013)</td>
</tr>
<tr>
<td>SP (rounded grains) RD = 70% (\phi = 36) degrees</td>
<td>Multifilament polyester woven 58 kN/m Multifilament polyester woven 320 kN/m</td>
<td>39.3 (efficiency: 0.79) 41.1 (0.84) Adhesion zero all cases 34.5 (0.95) 35.8 (0.99)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
From the table, the following summary applies:
Sand and woven monofilament: 0.84–0.96
Sand and woven slit film: 0.77
Sand and woven multifilament 0.72 – 0.99
Gravely sand and woven: 0.83–0.86
Sand and nonwoven needle punched: 0.92 – 1
Sand and nonwoven spun-bonded: 0.91 – 0.99
Sand and nonwoven heat bonded: 0.84

Estimates of the interface friction angles for the cases analyzed are made based on the collected information. Table 23 shows the values used. In the case of FOV 600-8650, the values listed in the table correspond to the actual measured results from Reference 2 (Chandrasekaran 1992). Also, the geosynthetics corresponding to Reference 4 (Haeri et al. 2000) have values coming from direct measurements presented in their reference.

Table 23: Interface friction angles for the cases analyzed in this research

<table>
<thead>
<tr>
<th>Geosynthetic</th>
<th>Reference 2</th>
<th>Reference 3</th>
<th>Reference 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOV 600-8650</td>
<td>Propex 4545</td>
<td>TYPAR 3801</td>
<td>Husker, B500</td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.88</td>
<td>0.95</td>
<td>0.84</td>
</tr>
<tr>
<td>Friction angle (degrees)</td>
<td>35</td>
<td>38</td>
<td>32</td>
</tr>
<tr>
<td>TYPAR 3407</td>
<td>0.89</td>
<td>0.84</td>
<td>Hoechst, 11/180</td>
</tr>
<tr>
<td></td>
<td>0.82</td>
<td></td>
<td>37</td>
</tr>
</tbody>
</table>

The last parameter is the shear stiffness of the interface. For the FOV 600-8650 geotextile in analysis, the interface shear stiffness can be obtained from a curve of shear stress vs. horizontal displacement available on its reference, and the initial stiffness is 480,000 kPa/m. For the other geotextiles, the selected GRS test references do not provide such information. Itasca (2012) recommends that the same interface stiffness be used in the normal and shear directions, as given by Equation 6-1. The obtained values result greater than 100,000,000 kPa/m. Numerical parameter studies (which will be explained in detail in the following sections) demonstrate that the interface stiffness do not produce important changes in GRS response when subject to triaxial loading, as long as the parameters are kept in a reasonable range; therefore, a good enough approximation is to use the recommendation by Itasca (2012), as represented in Equation 6-1, which was used in this research.

6.3. Comparison Analysis

Having the GRS model ready and the appropriate material properties for each GRS component, the comparison procedure can be run. The basic procedure is: run analyses in FLAC3D, compare with available test results, check if the numerical model needs changes, make appropriate and reasonable changes, and then start again. Estimated material and interface property values can also be legitimately adjusted within reasonable ranges during this process. The soil properties are not
changed when they are obtained directly from unreinforced triaxial tests on the same soil as used in the GRS tests.

The comparison procedure begins with analysis at low strains, which are called working strains herein. Most real GRS applications will work under low strains during their entire operational lives. Therefore, it is important to analyze this range of strains first. Axial strains less than 3% were selected as the range for this working strain level. In addition to the significance of this strain range to real applications, it is easier to control during numerical analyses within this strain range than at larger strains. The numerical model becomes less stable as it approaches failure.

After the low strain comparison, a comparison process on large strain is developed. Its principal objective is to analyze how the numerical model can capture failure.

The tests to be analyzed are described in Section 6.1, and the properties to be used are presented in Section 6.2. Besides the model and the material properties, there are two additional issues that have not yet been discussed. Those are the mesh refinement and the velocity of deformation applied at the top of the specimens to develop the deviator stress. Discussions of discretization and imposed velocity are presented, and then results of the validation process are shown and discussed.

A numerical model needs enough refinement to represent the system geometry and avoid abrupt changes in deformations and stresses from one zone to the next. Also, for these analyses, the numerical model uses a deformation imposed at the top of the specimen to develop the deviator stress in the sample. This velocity must be slow enough to avoid numerical instabilities. An initial set up of the model is run repeatedly to find the number of zones and the imposed velocity that reliably give a stable result, but do not make the model to run too slowly. These convergence studies are run prior to executing any comparison procedure.

The test from reference 2 with a diameter of 100 mm, 2 layers of geosynthetic and 100 kPa confining pressure is used to run this convergence study. The procedure consists on running the test with different velocities but keeping constant all the other parameters, then the best velocity is chosen. With that velocity, the refinement of the sample is modified. The smallest number of zones that reliably produces a stable result is chosen.

The following Figure 28 shows the differences in results when the velocity of deformation is changed, all the tests use exactly the same parameters. The mesh refinement uses 12 zones in the radial direction and 48 in the height. The velocities used are 1E-6, 1E-7, 1E-8 and 1E-10 m/timestep. As the velocity decreases, more timesteps would be needed to deform the body and the model would become more stable. The Figure 28 shows only three results, the two last results are almost the same; there is no appreciable difference between them. Therefore, velocity of 1E-7 m/timestep is used.
Figure 28: Results of changes in velocity. (a) Stress strain results with different velocities of deformation. (b) Volumetric strain results with different velocities of deformation.

Regarding the mesh refinement, a series of analyses were performed with increasing number of zones in the radial direction of 10, 12, 14, and 16. In each case, the ratio of the height to the radial length of each zone was unity. Because the overall sample height to diameter ratio was two, and because a quarter specimen was used in the numerical model, the number of zones in the vertical direction is four times the number in the radial direction. Thus, the 10 to 16 zones in the radial direction correspond to 40 to 64 zones in the vertical direction. In the tangential direction, the lengths of zones were kept approximately equal to the radial and vertical dimensions using the combined zone configuration shown in Figure 27(b). There were no noticeable differences among the stress-strain results of the analyses as a function of refinement. For the volumetric-strain-
versus-axial-strain curves, there is a slight difference between the results for the mesh with 10 zones compared with the other meshes; therefore, the mesh refinement with 12 zones in the radial direction and with its respective 48 zones in the vertical direction was selected. The tangential and radial refinement is shown in Figure 27(b) and Figure 26. The numerical model of the quarter specimen thus contains 5,184 three-dimensional mesh zones, which would correspond to 20,736 mesh zones for a full-diameter specimen. In addition, there are 432 two-dimensional structural embedded liner elements, with their corresponding interfaces, for each layer of geosynthetic in the numerical model of the quarter specimen.

6.3.1. Results at Working Strains

With the chosen mesh refinement and the velocity of deformation, the comparison process is applied to all the tests selected in Section 6.1. The results with the initial material properties are presented in the following graphs. Some cases volumetric strain results are not included due there is not such information for the laboratory test, therefore there is not possible a comparison process.

![Stress-strain curves](image_url)

Figure 29: Stress-strain curves for Reference 2, specimen with 200 mm diameter, 1 layer of geosynthetic, at different confining pressures. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 30: Stress-strain curves for Reference 2, specimen with 200 m diameter, 3 layers of geosynthetic, at different confining pressures. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 31: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 2, specimen of 100 mm diameter, 1 layer of geosynthetic, at different confining pressures. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 32: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 2, specimen with 100 mm diameter, 2 layers of geosynthetic. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 33: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 2, specimen with 100 mm diameter, 3 layers of geosynthetic. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 34: Stress-strain curves for Reference 2, geosynthetic Propex 4553, specimen with 200 mm diameter, 80 kPa confining pressure, 3 layers. Dashed line presents laboratory data and continuous line the results from the numerical model.
Figure 35: (a) Stress-strain curve and (b) volumetric-strain-versus-axial-strain curve for Reference 3, geosynthetic TYPAR 3801, specimen with 71 mm diameter, 169.8 mm height, 50 kPa confining pressure, 5 layers. Dashed line presents laboratory data and continuous line the results from the numerical model.
Figure 36: (a) Stress-strain curve and (b) volumetric-strain-versus-axial-strain curve for Reference 4, geosynthetic: Typar 3407, specimen with 100 mm diameter, 60 kPa confining pressure and 3 layers of geosynthetic. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 37: (a) Stress-strain curve and (b) volumetric-strain-versus-axial-strain curve for Reference 4, Geosynthetic Hoechst, 100 mm diameter, 60 kPa confining pressure, and 3 layers of geosynthetic. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 38: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 4, Geosynthetic Hoechst, 100 mm diameter, 100 kPa confining pressure, 1 and 2 layers. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 39: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 4, Geosynthetic Hoechst, 100 mm diameter, 100 kPa confining pressure, 3 and 4 layers. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 40: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 4, Geosynthetic Husker, 100 mm diameter, 300 kPa confining pressure, 1, 2 and 3 layers. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 41: (a) Stress-strain curve and (b) volumetric-strain-versus-axial-strain curve for Reference 4, Geosynthetic Husker, 100 mm diameter, 60 kPa confining pressure, 3 layers. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 42: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 4, Geosynthetic Husker, 100 mm diameter, 100 kPa confining pressure, 1, 2 and 3 layers. Dashed lines present laboratory data and continuous lines the results from the numerical model.

Figure 29 through Figure 42 show the results obtained from the numerical GRS model analyses compared with the corresponding results from the laboratory tests. As can be seen, the degree of agreement varies from test to test. The overall agreement is noticeable. Some figures show very close agreement such as Figure 29, Figure 30, and Figure 37. Other figures show moderately good
agreement, particularly for some individual tests within the set shown in figure. However, other figures show important differences between numerical calculations and laboratory test results, such as in Figure 35 and Figure 40 through Figure 42. These results are discussed further below.

It must be noted all the results are presented above are without any calibration, i.e., without any changes to the material or interface properties, although in several instances, property values were assumed based on information about similar materials and interfaces.

The following comments can be made at this stage of the comparisons:

- There is only one test with 4 layers of geosynthetic (Figure 39) and one test with 5 layers (Figure 35). Results for 4 layers are good however results for 5 layers do not show a good agreement with the laboratory data. It seems the behavior of the numerical model for tests with more than three layers needs to be further analyzed.

- Volumetric and axial strains results from laboratory tests include the influence of compression of the geotextile thickness, whereas the numerical analyses do not. It would be possible to include this effect in a revised numerical model; however, the present model does not incorporate geotextile compressibility in the normal direction. This effect may be significant for thick geotextiles, such as the Husker non-woven geosynthetic used in the tests for which results are presented in Figure 40 through Figure 42. The thickness of this non-woven geosynthetic is 5.5 mm, and compression of the geosynthetic very likely contributed to the axial deformations in the tests and it could account for the larger axial strains in the laboratory tests compared to the axial strains at the same deviator stresses in the numerical analyses, as shown in Figure 40 through Figure 42. As the number of geosynthetic layers increases, the deviation between the laboratory test results and the numerical model results increases.

- There is an overall agreement between numerical analysis results and laboratory test results. In particular, the numerical model works reasonably well at working strains levels, and it can be used to analyze triaxial tests of GRS composites subject to axisymmetric loading with constant confining pressure and up to 3 layers of reinforcement. Numerical parametric studies could be performed to help investigate the response of GRS as it is influenced by geosynthetic spacing, geosynthetic strength, etc.

- The initial slopes of the stress-strain curves from the numerical analyses of the GRS tests (presented in Figure 29 through Figure 39, except Figure 35) closely follow the slopes of the respective unreinforced soil represented by the Chsoil model (as shown in Appendix A). These initial slopes are almost the same, independent of the number of geosynthetic layers. Regarding the laboratory results, the same trends can be seen in several cases. Major exceptions are in Figure 40 through Figure 42, where compression of the geosynthetic thickness leads to softening of the stress-strain response as the number of geosynthetic layers is increased.
6.3.2. Results at Large Strains

The second step in the comparison procedure is to compare the failure stage of the GRS laboratory tests with numerical model results. The comparisons are presented in the same manner as in the previous section, except that now the outcomes are compared until failure is reached and not just over the 0% to 3% strain range as before.

The present section begins by comparing numerical results to laboratory test data from Reference 2. These comparisons are discussed, and later tests from other references are presented and discussed. The comparisons for Reference 2 are presented in the following figures:

Figure 43: Stress-strain curves for Reference 2, specimen with 200 mm diameter, 1 layer of geosynthetic, at different confining pressures. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 44: Stress-strain curves for Reference 2, specimen with 200 mm diameter, 3 layers of geosynthetic, at different confining pressures. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 45: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 2, specimen of 100 mm diameter, 1 layer of geosynthetic, at different confining pressures. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 46: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 2, specimen of 100 mm diameter, 2 layers of geosynthetic, at different confining pressures. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 47: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 2, specimen of 100 mm diameter, 3 layers of geosynthetic, at different confining pressures. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 48: Stress-strain curves for Reference 2, geosynthetic Propex 4553, specimen with 200 mm diameter, 80 kPa confining pressure, 3 layers. Dashed line presents laboratory data and continuous line the results from the numerical model.

Some results present good agreement between stress-strain curves for laboratory data and numerical analyses for large strains, such as shown in Figure 43, Figure 45, and Figure 46, which comprise nine tests in total.

On the other hand, numerical results for other tests presented in Figure 44, Figure 47, and Figure 48 (seven tests in total) show significant deviations from the laboratory data at large strains, with the strengths from the numerical analyses generally exceeding the strengths determined in the laboratory. The reasons for the differences were investigated, as described in the following.

The conditions for the tests and analyses in Figure 43 and Figure 44 are the same in every respect, except that one layer of geosynthetic is used for the tests and analyses in Figure 43 while 3 layers are used in Figure 44. The agreement between laboratory tests and numerical analyses is good for Figure 43 with one layer of geosynthetic reinforcement, but the numerical analyses overestimate the GRS capacity in Figure 44 with three layers of geosynthetic reinforcement. A similar situation exists for Figure 45, Figure 46, and Figure 47, where everything is the same except that the number of geosynthetic layers increases from 1 to 2 to 3, respectively. The agreement between laboratory tests and numerical analyses is good for Figure 45 and Figure 46 with one and two layers of geosynthetic reinforcement, respectively, but the numerical analyses overestimate the GRS capacity in Figure 47 with three layers of geosynthetic reinforcement.

To investigate the reason for the differences between analyses and laboratory tests for the cases with three layers of geosynthetic reinforcement, first the influence of the properties of the geosynthetic reinforcement and the soil-geosynthetic interfaces were investigated. However, no
realistic variations in these property values produced good agreement between numerical analyses and laboratory tests for 1, 2, and 3 layers of geosynthetic reinforcement.

The next numerical modeling issue addressed was the mesh refinement. The mesh refinement convergence study described above was done using 2 layers of geosynthetic reinforcement for the test at a confining pressure of 100 kPa shown in Figure 46 from Reference 2. This convergence study was repeated using 3 layers of geosynthetic reinforcement. As the mesh is made more refined, the imposed vertical velocity at the top of the specimen must be decreased to produce numerical stability. With a reduced velocity and a refined mesh, the computer run times to reach large strains increase dramatically. Nevertheless, enough refinement runs were done to demonstrate that refinement tended to improve agreement between numerical analyses and laboratory test results, although the degree of agreement never reached that shown for the stress-strain curves in Figure 43 and Figure 45.

Personnel from Itasca were contacted about this, and they attributed the difficulty to the potential for shear band formation in physical specimens, which requires high refinement in numerical analyses, and for which small difference in initial conditions can produce significant differences in outcomes.

A puzzling aspect of this investigation of the numerical analyses relates to the number of zones between geosynthetic layers. For mesh used to generate the results shown in Figure 29 through Figure 48, there are 16 vertical zones between adjacent layers of geosynthetic when the specimen contains two layers of geosynthetic. This same mesh results in 12 vertical zones between adjacent layers of geosynthetic when the specimen contains three layers of geosynthetic. One might suspect that, if the refinement increased to the extent that there were 16 vertical zones (with aspect ratios maintained to about 1:1:1) between adjacent layers of geosynthetic when the specimens contain three layers of geosynthetic, good agreement between numerical analyses and laboratory tests would result, as occurred numerical analyses of tests with two layers of geosynthetic reinforcement. However, that was not the case.

It seems that the numerical model developed in this research does not provide a good representation of large-strain response of a GRS laboratory test specimen subjected to triaxial compression loading for specimens containing more than 2 layers of geosynthetic.

An appropriate continuation of the comparisons is to analyze tests with a maximum of 2 layers of geosynthetic. That is the next step. Figure 49 through Figure 51 show the comparisons between numerical analyses and laboratory tests from Reference 4 for cases with 1 and 2 layers of geosynthetic, which represents a total of six tests altogether.
Figure 49: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 4, Geosynthetic Hoechst, 100 mm diameter, 100 kPa confining pressure, 1 and 2 layers. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 50: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 4, Geosynthetic Husker, 100 mm diameter, 300 kPa confining pressure, 1 and 2 layers. Dashed lines present laboratory data and continuous lines the results from the numerical model.
Figure 51: (a) Stress-strain curves and (b) volumetric-strain-versus-axial-strain curves for Reference 4, Geosynthetic Husker, 100 mm diameter, 100 kPa confining pressure, 1 and 2 layers. Dashed lines present laboratory data and continuous lines the results from the numerical model.

It can be seen in Figure 49 through Figure 51 that there is not a good agreement between numerical analyses and laboratory results. Therefore, adjustments to input data are considered. As discussed previously in connection with the analyses of the tests from Reference 2, the soil properties are not good candidates to change because a sound basis exists for material property values from
laboratory tests on the soils. On the other hand, most of the geosynthetic and interface properties were estimated rather than tested, and they can fairly be modified while still remaining reasonable.

Regarding geosynthetic properties, many analyses were performed using variations in Young's modulus, Poisson's ratios, and shear moduli. The following outcomes were obtained:

- Variation in shear modulus does not generate any important change in the outcomes, even if the shear modulus is changed by a factor of 100.
- Poisson's ratios within the range from about zero to one-half do not generate any important changes in outcomes.
- Young's modulus values of the geosynthetic do influence the outcomes by modifying portions of the stress-strain curves, with stiffer geosynthetics producing increased vertical stresses in the GRS mass after initial loading but before the failure point. However, the failure stress does not change, as shown in Figure 52. It must be noticed the “reference” curve in Figure 52 slightly differs from the original numerical result from Figure 50(a). This variation is due to an increase in velocity for this last comparison analysis in order to reach failure in an acceptable time frame. Young's modulus for the reference are 6690 and 12145 kN/m in machine and cross-machine directions (from Table 21).

![Stress-strain curves. Influence of change in geosynthetic stiffness (Young's modulus).](image)

This analysis demonstrates that non geosynthetic property would influence the results in a way as to get closer to the laboratory outcomes in Figure 48 through Figure 51. Interface properties should be analyzed now:
• Variation in normal interface properties does not generate changes in results.
• Interface tangential stiffness has been analyzed, and tests changing its value from about 500,000 kPa (about the value used for reference 2) to 100,000,000 kPa/m (average recommendation from Equation 6-1). The results were contrasted and there is no noticeable difference.
• Interface friction angle. Numerical parameter studies were performed in which the interface friction was varied, and a demonstrative outcome is shown in Figure 53, where it can be seen that reducing the interface friction angle produces reduction in the vertical stress that the GRS specimen can sustain in the numerical analyses. However, very low friction angle on the interface would need to be chosen so the numerical analysis outcomes fit the laboratory curves for Figure 48 through Figure 51, such that interface friction values in the range of 10 to 18 degrees in the numerical analyses produce good agreement with the laboratory curves. Also, different values of interface friction angle would have to be chosen for tests with different numbers of geosynthetic layers (with all the other properties been the same) in order for failure in the numerical analyses to match failure stresses in the laboratory test. This is not appropriate. Consequently, the possibility of changing interface properties within reasonable limits to produce agreement between the numerical analyses and the laboratory tests for failure conditions is abandoned.

Figure 53: Stress-strain curves. Influence of interface friction angle for reference 4, Geosynthetic Hoechst, 100 mm diameter, 100 kPa confining pressure, 1 layer of geosynthetic
Finally, the mesh refinement issue is investigated again. The following figure shows the impact of mesh refinement, and the corresponding necessary reduction in the imposed velocity, on the stress-strain curves from the numerical analyses.

![Stress-strain curves](image)

**Figure 54**: Stress-strain curves. Influence of number of zones for reference 4, Geosynthetic Hoechst, 100 mm diameter, 100 kPa confining pressure, 1 layer of geosynthetic. The number of zones presented in the graph's legend represent the zones in the radial direction of the quarter specimen. In the vertical direction, the specimen has four times as many zones as in the radial direction because the specimen is twice as tall as its diameter.

These analyses suggest that a combination of increasing the stiffness of the geosynthetic (which influence is presented in Figure 52) and large number of zones for the element (influence in Figure 54) could return the right results at large strains. However, for the analyses in Figure 54, the stress-strain curve with 36 zones took more than three weeks to generate, and it appears that an even slower velocity would be necessary to avoid the irregular stress-strain response. Therefore, to reach a point where mesh refinement is sufficient that convergence is no longer an issue at large strains, and smooth stress-strain response is produced, would require impractically slow analyses with the software and hardware used in this research.

There is still a question have not been answered yet. Why the numerical analysis did not have convergence problems in reference 2 for the same conditions than reference 4. This is: same number of layers and equal mesh refinement? Analyzing the variables used into both references, the only explanation is that the stronger soil used into reference 4 must requires more mesh refinement than the relatively weaker soil from reference 2.
6.4. Conclusions

The following conclusions are drawn from the numerical analyses and their comparison with the laboratory tests:

- Variations in properties of the geosynthetics and interfaces had relatively little impact on stress-strain response at small strains.

- Even at large strains, Poisson's ratio and shear modulus of the geosynthetic do not produce important differences in the GRS response as long as those properties remain in realistic ranges.

- Properties that have more influence the GRS response at large strains (apart from soil properties) are the geosynthetic stiffness and the strength of the soil-geosynthetic interface.
  - The geosynthetic stiffness influences the intermediate part of the stress-strain curve, with increasing geosynthetic stiffness producing stiffer GRS response in that range. The geosynthetic stiffness does not significantly influence the small strain response of the GRS, and it does not affect the failure stress on the GRS at all.
  - The interface strength has a substantial influence on the vertical stress the GRS can sustain at failure under triaxial test conditions. As the interface strength increases, the ability of the GRS to support vertical stress increases until a limit from where no change is observed.

- The necessary mesh refinement changes as a function of the number of layers and the properties of the elements involved in the process. Closer spacing of geosynthetics requires a more refined mesh, and it is believed that a stronger soil requires greater number of zones to work appropriately.

- The present numerical model is not able to capture failure for a wide range of materials and conditions without very long run times. Consequently, it is not practical to use this model to run a parametric study, which would have been informative if the numerical model could have been validated under a wide range of conditions. To represent failure conditions for GRS, another method should be used. The following chapter applies ultimate capacity theories to the laboratory tests identified for analysis in this chapter.

- The numerical model is useful to capture the behavior of reinforced soil under working strain conditions.
CHAPTER 7:
COMPARISON OF THE LABORATORY TEST RESULTS WITH ULTIMATE CAPACITY THEORIES

Available ultimate capacity theories are used to analyze their applicability to geosynthetic reinforced soil behavior at failure under triaxial compression test conditions with constant confining pressure. In the present chapter, four theories are explained and three of them are applied to available laboratory test results. The four theories are: increment of confining pressure (Yang 1972), equivalent cohesion (Schloesser and Long 1974), approximate isotropic perfectly plastic solution (Tatsuoka 2004), and suppression of dilation theory (Wu et al. 2014). This last one is just explained due this theory lacks of a direct application procedure. Results of the other three theories are compared with available laboratory results, and the potential applicability of the theories is discussed.

7.1. Summary of Available Theories

7.1.1. Increment of Confining Pressure (Yang 1972)

This theory, which is also known as equivalent confining pressure, was proposed by Yang (1972) in his PhD thesis. This theory is a semi empirical method to approximate the strength of a reinforced soil. Yang (1972) considers the strength of a reinforced soil is a function of an equivalent confining pressure generated by the interaction between the reinforcement and the soil. As the soil receives increasing normal stresses in a triaxial test for example, its soil particles tend to displace horizontally; however, the reinforcement restrains that movement, generating an increment of horizontal stress in the soil in addition to the externally applied confining pressure (hence the name "increment of confining pressure theory").

The increment in confining pressure is directly related with the shearing stresses on the contact interface between the soil and the reinforcement. The magnitudes of these shearing stresses are a function of the normal stress, frictional properties of the interface, and the area of the interface.

Based on laboratory and numerical experimentation, Yang (1972) determined the important parameters that influence the increment in confining pressure, $\Delta \sigma_3$, due to geosynthetic reinforcement according to the following equation. All the stresses are effective and all $\sigma$ refer to effective stresses.
\[ \Delta \sigma_3 \propto \sigma_1 \frac{f(Sr)}{f(Ss)} f \]

where:

- \( \sigma_1 \) is the maximum major principal stress at failure acting vertically on the top of a triaxial specimen of GRS, which Yang (1972) assumes to be equal to the normal stress acting on the geosynthetic, even though the existence of shear stresses at the soil-geosynthetic interface are recognized.
- \( Sr \) is the total surface area of reinforcement between two consecutive layers of reinforcement.
- \( Ss \) is the lateral boundary surface area between two consecutive layers of reinforcement.
- \( f(Sr) \) is a function that represents the influence of the area of the interface in contact with the soil.
- \( f(Ss) \) is a function that represents the influence of the lateral boundary surface area.
- \( f \) is the friction coefficient.

Yang (1972) proposed that empirical constants be incorporated to account for the three last terms defined above, such that Equation 7-1 becomes:

\[ \Delta \sigma_3 = C_0 \cdot \sigma_1 \left( \frac{Sr}{Ss} \right)^m \]

where:

- \( C_0, m \) are empirical constants.

Now, the maximum vertical stress that a reinforced soil specimen can sustain is given by:

\[ \sigma_1 = \left[ \sigma_3 + C_0 \cdot \sigma_1 \left( \frac{Sr}{Ss} \right)^m \right] \cdot \tan^2 \left[ 45 + \frac{\phi}{2} \right] \]

where:

- \( \sigma_3 \) is the confining pressure.
- \( \phi \) is the friction angle of the soil without reinforcement. Yang (1972) includes variation of the friction angle with respect to the confining pressure.

If Equation 7-3 is divided by \( \sigma_3 \), and it is used that \( \sigma_1 = N_\phi \sigma_{3e} \) due to zero cohesion, the following equation is obtained.
\[
\frac{\sigma_{3e}}{\sigma_3} = \frac{1}{1 - Co \cdot N\phi \cdot \left(\frac{Sr}{Ss}\right)^m}
\]

where:

- \(\sigma_{3e}\) is the equivalent confining pressure due the effects of the applied GRS specimen confining pressure and the effects of the geosynthetic reinforcement
- \(\sigma_3\) is the confining pressure applied to the GRS specimen in a triaxial compression test
- \(N\phi = \tan^2\left(45 + \frac{\phi}{2}\right)\)

With reinforcement in the horizontal direction of a cylindrical triaxial test:

\(Sr = 2\pi D^2\) and \(Ss = 4\pi DL\), with \(D = \) diameter and \(L = \) distance between two reinforcement layers.

Finally, Yang (1972) establishes that, the equivalent confining pressure can be calculated as follows:

\[
\frac{\sigma_{3e}}{\sigma_3} = \frac{1}{1 - Co \cdot N\phi \cdot \left(\frac{1}{2} \cdot \frac{D}{L}\right)^m}
\]

This last equation summarizes the increment confining pressure theory for a reinforced soil when the reinforcement has not reach failure during the test.

The determined equivalent confining pressure lets the calculation of the reinforced-soil strength as:

\[
\sigma_1 = \sigma_{3e} \left[\frac{1 + \sin(\phi)}{1 - \sin(\phi)}\right]
\]

where:

- \(\phi\) is the unreinforced soil friction angle
- \(\sigma_1\) is the failure major principal stress for the GRS tests

### 7.1.2. Equivalent Cohesion (Schlosser and Long 1974)

This characterizes the strength increase in a reinforced soil as an equivalent cohesion generated due to the presence of the reinforcement in the soil mass.

This equivalent cohesion is a function of the strength of the reinforcement, the spacing of the reinforcement, and the soil strength properties, principally the friction angle. This theory assumes that the friction angle of the reinforced soil is equal to the friction angle of the unreinforced soil.
The equivalent cohesion theory proposes a maximum major principal stress for the reinforced soil equal to the unreinforced maximum major principal stress (for cohesionless soil) plus an increment due to the reinforcement presence:

\[ \sigma_1 = \sigma_3 K_p + \Delta \sigma_1 \]  

where:

- \( \sigma_1 \) is the major principal stress at failure for a reinforced soil
- \( \sigma_3 \) is the confining pressure (same for reinforced and unreinforced soil)
- \( \Delta \sigma_1 \) is the major principal stress increment due to the reinforcement
- \( K_p = \tan^2 \left( 45 + \phi/2 \right) \)
- \( \phi \) is the friction angle from the unreinforced soil envelop

Contrasting Equation 7-7 with Rankines' lateral earth pressure equation, Equation 7-8 can be determined:

\[ \Delta \sigma_1 = 2c \sqrt{K_p} \]  

where:

- \( c \) is the cohesion intercept for the reinforced soil envelop

Therefore:

\[ c = \frac{\Delta \sigma_1}{2 \sqrt{K_p}} \]  

From a force equilibrium analysis of a reinforced soil component, another expression for major principal stress at failure was developed. It was considered the reinforcement reaches its tensile strength:

\[ \sigma_{1r} = \sigma_3 K_p + \frac{R_T K_p}{L} \]  

where:

- \( R_T \) is the tensile strength of the reinforcement
- \( L \) is the spacing between reinforcement layers
Now, the increment in major principal stress ($\Delta \sigma_1$) would result equal to the second term in the right hand side of the Equation 7-10. This result into Equation 7-9 gives the equivalent cohesion for Schlosser and Long (1974) theory:

$$c = \frac{R_T \cdot \sqrt{K_p}}{2L} \quad \text{(7-11)}$$

With the equivalent cohesion, the major principal stress at failure for the reinforced-soil can be calculated as follows:

$$\sigma_1 = \sigma_3 K_p + 2c \sqrt{K_p} \quad \text{(7-12)}$$

### 7.1.3. An Approximate Isotropic Perfectly Plastic Solution (Tatsuoka 2004)

Tatsuoka (2004) developed a closed-form solution to approximate the strength of a geosynthetic reinforced soil through a comprehensive analytical development that incorporates several simplifications. Some of the important simplifications are: the stress-strain response of the soil is considered isotropic and perfectly plastic, the weight of the soil is ignored, the strength of the soil is represented by a constant friction angle and the cohesion of the soil is zero, the friction angle along the entire soil-reinforcement interface is constant. The resulting equations are said to be valid if the relationship ratio of the width (diameter for triaxial tests) of the reinforced soil to the spacing between reinforcing layers is sufficiently large. The width-to-spacing ratio is considered large when the width of the specimen is greater than $\tan(45 - \phi/2)$ times the spacing between geosynthetic layers (most of the time this condition is easily satisfied).

Tatsuoka (2004) began this work by determining equations of stress equilibrium at failure for a general case of GRS with vertical loading, and using some of the simplifications explained previously. Tatsuoka (2004) used the stress characteristics method, and other author's work to mathematically reach some equilibrium conditions applicable to the GRS case with vertical loading and at failure. Those equilibrium conditions related strength properties of the soil, normal stresses, and principal stresses. Those conditions were used to analyze two failure mechanisms: failure in the backfill and failure in the reinforcement. It is necessary to mention; the author began with establishment of equations applicable to a 2D condition. Later on, the author simplified or adjusted those equations to be applicable to axisymmetric conditions. In follows, only main equations relevant to the present investigation are included, that means equations applicable to axisymmetric loading condition. Few 2D definitions or equations are included when those apply to the axisymmetric case too.

**Solution for soil failure with no rupture in the reinforcement:**

Working with a 2D condition, Tatsuoka (2004) analyzed the major principal stress distribution over two locations of a GRS: the soil-reinforcement interface and at the mid-height between
reinforcement layers, as Figure 55 shows. All the established parameters in the figure apply to axisymmetric condition too.

Figure 55: Stresses in a soil between two reinforcement layers after Tatsuoka (2004)

In the figure:
- \( \varepsilon = 45 - \phi/2 \). In follows a different symbol (\( \xi \)) is used to avoid confusion with strains. Therefore: \( \xi = 45 - \phi/2 \)
- \( \sigma_c \) is the confining pressure
- \( \sigma_1 \) is the major principal stress. It is vertical at the middle of the soil layer and it has an inclination \( \delta_b \) with the vertical at the soil-geosynthetic interface (called \( \sigma_{1b} \) in this case).
- \( \delta_b \) is the angle between the major principal stress direction and the vertical. Tatsuoka (2004) presented two conditions for its calculation:
  1) From the stress distribution at the soil-geosynthetic interface and considering the interface has a constant friction angle, Tatsuoka (2004) obtained the following relationship for \( \delta_b \) and the friction angle of the soil-reinforcement interface and the friction angle of the soil:

\[
\delta_b = \frac{1}{2} \left[ \mu + \sin^{-1} \left( \frac{\sin \mu}{\sin \phi} \right) \right]
\]

where:
- \( \mu \) is the soil-reinforcement interface friction angle in radians
- \( \phi \) is the friction angle of the soil

2) Tatsuoka (2004) stated that when the reinforcement is much stiffer than the soil, and there is no slippage in the interface, Equation 7-14 applies. It should be noticed that the
stiffness of the geosynthetic is not comparable with the stiffness of the soil. Geosynthetic stiffness has units of force/length while soil stiffness has units of force/squared length. No clarification is stated in this regard into Tatsuoka's (2004) work.

\[ \delta_b = \frac{\pi}{4} + \frac{\nu}{2} \]

where:

\[ \nu \] is the dilatancy angle at failure

In order to obtain the major principal stress at failure for a GRS with vertical loading, Tatsuoka (2004) determined the major principal stress distribution at BD (Figure 55) and defined a maximum normalized compressive load \( P_1 \):

\[ P_1 = \frac{(\sigma_1 - 1)}{BD \over d/2} \]

where:

\[ \sigma_1 \] is the unreinforced soil maximum major principal stress

\[ BD \] is the distance between points B and D in Figure 55

In order to determine the major stress distribution at BD, Tatsuoka (2004) analyzed the field stress in the triangle ABC on Figure 55 and the stress characteristics method together with the equilibrium conditions determined at the beginning of his work. After extensive mathematical derivations and using \( P_1 \) concept, Tatsuoka (2004) obtained equations that relate the maximum major principal stress (\( \sigma_1 \)) with confining pressure, \( \delta_b \), the friction angle of the soil, and the geometry of the reinforced specimen (width and layers spacing). The resultant equations regarding axisymmetric conditions are:
\[ R_1 = \frac{P_1 \left( 1 - \frac{h}{d} \tan \xi \right)^2 + 1}{(\tan \xi)^2} \]  

\[ P_1 = \frac{2(\exp(B) - B - 1)}{B^2} - 1 \]

\[ B = 2 \delta_b \left( \frac{\cos \delta_b + \sin \phi}{\cos \phi} \right) \tan \phi \left( \frac{d}{h} - \tan \xi \right) \]

where:

- \( R_1 \) is the maximum stress ratio. The ratio of the maximum value of the major principal stress that the GRS specimen can sustain to the confining pressure applied to the GRS specimen. It must be noted Tatsuoka (2004) assumes the maximum \( \sigma_1 \) the GRS can sustain is equal to the \( \sigma_1 \) developed at the middle of the soil layer, drawn as \( \sigma_1 \) in Figure 55.
- \( P_1 \) is called the maximum normalized compressive load that the reinforced soil will be able to sustain.
- \( \delta_b \) is the angle between the major principal stress and the vertical at the geosynthetic-soil interface. It is expressed in radians.
- \( \phi \) is the friction angle of the soil.
- \( d \) is the diameter of the reinforced soil.
- \( h \) is the spacing of the reinforcement layers (L for the other theories).
- \( \xi = 45 - \phi / 2 \)

Multiplying Equation 7-16 by the confining pressure produces the maximum major principal stress the GRS can sustain.

**Solution for a failure due to the rupture of the reinforcement:**

When the failure is controlled by the reinforcement strength, Tatsuoka (2004) determined the new maximum stress ratio through the maximum tensile stress in the geosynthetic developed for the previous failure mechanism. This tensile stress in the reinforcement was mainly obtained through integration of the developed shear stresses at the geosynthetic-soil interface. Its result is represented by Equation 7-19.
\[ T_{\text{max}} = \frac{2 \sin \phi \sin 2\delta_b}{d(1 + \sin \phi) \exp(\delta_b \tan \phi)} \frac{d^2 \sigma_c}{8} \left( R_1 - \frac{h(2d - h \tan \varepsilon)}{d^2 \tan \varepsilon} \right) \]

where:

\( T_{\text{connection}} \) is the tensile stress of the reinforcement at the connection to the facing elements (zero for the cases in this study).

The maximum tensile stress for the previous failure mechanism is transformed into tensile rupture of the geosynthetic, and then the maximum stress ratio for the rupture of the geosynthetic is obtained by solving Equation 7-19 for \( R_1 \), now transformed into \( R_2 \) (second failure mechanism):

\[ R_2 = \frac{8}{d \sigma_c} (1 + \sin \phi) \exp(\delta_b \tan \phi) \frac{2 \sin \phi \sin 2\delta_b}{h(2d - h \tan \varepsilon)} \frac{d^2 \sigma_c}{8} \left( T_{\text{rupture}} - T_{\text{connection}} \right) \]

This equation has been corrected from the original in Tatsuoka (2004). The original equation had a problem in the derivation process.

where:

\( R_2 \) is the maximum stress ratio for a reinforced soil when the reinforcement reaches failure
\( d \) is the diameter of the reinforcement soil sample
\( T_{\text{rupture}} \) is the tensile strength of the reinforcement
\( T_{\text{connection}} \) is the tensile stress of the reinforcement at the connection to the facing elements (zero for the cases in this study).

Equations 7-16 and 7-20 permit calculation of the maximum principal stress ratios for the two failure mechanisms considered by Tatsuoka (2004). The major principal stress at failure for each condition is determined by multiplying the maximum stress ratios by the confining pressure. The smallest resulting value of the major principal stress would become the controlling value.

### 7.1.4. Suppression of Dilation (Wu et al. 2014)

Wu et al. (2014) explained the importance of the tendency for change of volume of the soil during shearing on the behavior of GRS. The authors use laboratory experimentation and numerical analyses, in which a parametric study was performed to investigate the influence of spacing of the reinforcement layers, the stiffness of the reinforcement, and the soil stiffness. The authors stated that the largest influence is the spacing of the reinforcement layers. At smaller spacing, less dilation or even total suppression of dilation was found in the soil. An inflexion point in GRS
response as a function of reinforcement spacing was observed. The authors identify the inflexion point as the limit spacing below which the dilation is suppressed and the reinforced soil responds in a composite fashion.

Based on their analyses of the volume change in reinforced soil, Wu et al. (2014) developed a theory to represent reinforced soil behavior. According to this theory, the reinforcement suppresses dilation, which increases the strength of reinforced soil. Wu et al. (2014) described dilation suppression as a completely different mechanism for the contribution of reinforcement to GRS behavior, which has not been identified previously. Wu et al. (2014) explained that to apply this theory, the amount of dilation suppression can be determined from the dilation angle of the soil and the dilation angle of the reinforced soil. However, there is not suggested an explicit methodology to be used. Therefore, this theory is not further analyzed and also is not applied to the collected triaxial tests.

### 7.2. Applying Theories to Collected Laboratory Results

#### 7.2.1. Available Triaxial Test Data for Comparison Process

The first three of the four previously described theories are applied to the laboratory test results collected into Section 6.1. As explained into Section 6.1, there were chosen 30 triaxial tests as suitable for comparison analysis. Table 24 provides the principal information for each test: confining pressure ($\sigma_3$); major principal stress at failure or the maximum reported for the test ($\sigma_1$); geometry of the specimen including the diameter (D) and total height (H); and information regarding the geosynthetic in each test including the name of the geosynthetic, ultimate tensile strength ($T_{\text{ult}}$), number of layers of geosynthetic in each specimen, and the spacing between geosynthetic layers ($L$). Table 24 also provides the soil friction angle and the type of failure the specimen experienced. Two types of failures occurred: (1) "Soil" failure occurred when a peak stress or limiting strain was reached without geosynthetic failure and (2) "Fabric rupture" when the geosynthetic failed in tension. For some of the tests, a failure condition was not reached and those are not included in the analysis.
Table 24: Relevant information from laboratory tests collected in Section 6.1. All of this information is relevant to apply the first three reinforced soil theories summarized in Section 7.1.

<table>
<thead>
<tr>
<th>Reference and Soil Type</th>
<th>σ₃ kPa</th>
<th>σ₁ kPa</th>
<th>D cm</th>
<th>H cm</th>
<th>Geosynthetic</th>
<th>Tᵤₜₑ kN/m</th>
<th>Layers</th>
<th>L cm</th>
<th>Soil</th>
<th>φ</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chandrasekaran et al. (1989)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medium dense sand</td>
<td>25</td>
<td>155.7</td>
<td>10</td>
<td>20</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>1</td>
<td>10</td>
<td>40.9</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>295.2</td>
<td>10</td>
<td>20</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>1</td>
<td>10</td>
<td>39.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>540.2</td>
<td>10</td>
<td>20</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>1</td>
<td>10</td>
<td>37.4</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>239.9</td>
<td>10</td>
<td>20</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>2</td>
<td>6.7</td>
<td>40.9</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>410.6</td>
<td>10</td>
<td>20</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>2</td>
<td>6.7</td>
<td>39.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>761</td>
<td>10</td>
<td>20</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>2</td>
<td>6.7</td>
<td>37.4</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>375.7</td>
<td>10</td>
<td>20</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>3</td>
<td>5</td>
<td>40.9</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>705</td>
<td>10</td>
<td>20</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>3</td>
<td>5</td>
<td>39.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1200.6</td>
<td>10</td>
<td>20</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>3</td>
<td>5</td>
<td>37.4</td>
<td>Fabric rupture</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>152.2</td>
<td>20</td>
<td>40</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>1</td>
<td>20</td>
<td>40.9</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>277.06</td>
<td>20</td>
<td>40</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>1</td>
<td>20</td>
<td>39.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>446.4</td>
<td>20</td>
<td>40</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>1</td>
<td>20</td>
<td>38.0</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>332.9</td>
<td>20</td>
<td>40</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>3</td>
<td>10</td>
<td>40.9</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>575.2</td>
<td>20</td>
<td>40</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>3</td>
<td>10</td>
<td>39.1</td>
<td>Fabric rupture</td>
<td></td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>727</td>
<td>20</td>
<td>40</td>
<td>FOV 600-8641</td>
<td>48</td>
<td>3</td>
<td>10</td>
<td>38.0</td>
<td>Fabric rupture</td>
<td></td>
</tr>
<tr>
<td>Haeri et al. (2000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sand with 70% relative density</td>
<td>60</td>
<td>721.4</td>
<td>10</td>
<td>20</td>
<td>TYPAR 3407</td>
<td>8.4</td>
<td>3</td>
<td>5</td>
<td>44.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>555.2</td>
<td>10</td>
<td>20</td>
<td>HOECHST 11/180</td>
<td>10.4</td>
<td>3</td>
<td>5</td>
<td>44.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>722.5</td>
<td>10</td>
<td>20</td>
<td>HOECHST 11/180</td>
<td>10.4</td>
<td>1</td>
<td>10</td>
<td>44.0</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>839.7</td>
<td>10</td>
<td>20</td>
<td>HOECHST 11/180</td>
<td>10.4</td>
<td>2</td>
<td>6.7</td>
<td>44.0</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1024.7</td>
<td>10</td>
<td>20</td>
<td>HOECHST 11/180</td>
<td>10.4</td>
<td>3</td>
<td>5</td>
<td>44.0</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1305</td>
<td>10</td>
<td>20</td>
<td>HOECHST 11/180</td>
<td>10.4</td>
<td>4</td>
<td>4</td>
<td>44.0</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>470.9</td>
<td>3.8</td>
<td>7.6</td>
<td>TYPAR 3407</td>
<td>8.4</td>
<td>1</td>
<td>3.8</td>
<td>44.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>641.1</td>
<td>3.8</td>
<td>7.6</td>
<td>TYPAR 3407</td>
<td>8.4</td>
<td>2</td>
<td>2.5</td>
<td>44.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>965.7</td>
<td>3.8</td>
<td>7.6</td>
<td>TYPAR 3407</td>
<td>8.4</td>
<td>3</td>
<td>1.9</td>
<td>44.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1300.3</td>
<td>3.8</td>
<td>7.6</td>
<td>TYPAR 3407</td>
<td>8.4</td>
<td>4</td>
<td>1.5</td>
<td>44.1</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>2041.3</td>
<td>10</td>
<td>20</td>
<td>HUSKER B500</td>
<td>20</td>
<td>1</td>
<td>10</td>
<td>43.7</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>2147.2</td>
<td>10</td>
<td>20</td>
<td>HUSKER B500</td>
<td>20</td>
<td>2</td>
<td>6.67</td>
<td>43.7</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>781.9</td>
<td>10</td>
<td>20</td>
<td>HUSKER B500</td>
<td>20</td>
<td>1</td>
<td>10</td>
<td>44.0</td>
<td>Soil</td>
<td></td>
</tr>
<tr>
<td>Ashmawy and Boudeau (1998)</td>
<td>50</td>
<td>817.3</td>
<td>7.1</td>
<td>17</td>
<td>TYPAR 3801</td>
<td>20</td>
<td>5</td>
<td>2.83</td>
<td>38.84</td>
<td>Soil</td>
<td></td>
</tr>
</tbody>
</table>
7.2.2. Increment of Confining Pressure (Yang 1972)

Increment of confining pressure theory by Yang (1972) applies Equation 7-5 to calculate the equivalent confining pressure due to the reinforcement in the GRS mass. Prior to apply this equation, the empirical constants $C_0$ and $m$ must be determined. Yang (1972) recommends using the average of empirical constants obtained from the tests for one specific type of reinforcement and soil.

In order to calculate $C_0$ and $m$, the laboratory test information is grouped into tests with same soil and geosynthetic. Equivalent confining pressure from the laboratory data is calculated using Equation 7-21. Then a regression analysis in excel is used to determine the constants $C_0$ and $m$ that produce the best-fit for Equation 7-5.

$$\sigma_{3e} = \sigma_1 \left[ \frac{1 - \sin(\phi)}{1 + \sin(\phi)} \right]$$ \hspace{1cm} 7-21

where:

- $\sigma_{3e}$ is the equivalent confining pressure
- $\phi$ is the unreinforced soil friction angle
- $\sigma_1$ is the failure major principal stress for the GRS tests

When the reference presents only one test with a specific reinforcement type, the empirical constants cannot be calculated. Therefore, Ashmawy and Boudeau (1998) data is not considered. Table 25 presents the empirical constants obtained. Remember that each reference works with only one soil type.
Table 25: Evaluation of empirical constants for Yang's (1972) theory

<table>
<thead>
<tr>
<th>Reference and Geosynthetic</th>
<th>$\sigma_3$ kPa</th>
<th>$\sigma_1$ kPa</th>
<th>$\phi$ degrees</th>
<th>$N_\phi$</th>
<th>$\sigma_{3e,\text{Lab}}$ kPa</th>
<th>$D$ m</th>
<th>$L$ m</th>
<th>$Co$</th>
<th>$m$</th>
<th>$\sigma_{3e,\text{Theory}}$ kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chandrasekaran et al. (1989)</td>
<td>25</td>
<td>155.7</td>
<td>40.9</td>
<td>4.8</td>
<td>32.5</td>
<td>0.1</td>
<td>0.100</td>
<td></td>
<td></td>
<td>160.5</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>295.2</td>
<td>39.1</td>
<td>4.4</td>
<td>66.7</td>
<td>0.1</td>
<td>0.100</td>
<td></td>
<td></td>
<td>288.7</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>540.2</td>
<td>37.4</td>
<td>4.1</td>
<td>132.0</td>
<td>0.1</td>
<td>0.100</td>
<td></td>
<td></td>
<td>522.2</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>239.9</td>
<td>40.9</td>
<td>4.8</td>
<td>50.0</td>
<td>0.1</td>
<td>0.067</td>
<td></td>
<td></td>
<td>222.2</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>410.6</td>
<td>39.1</td>
<td>4.4</td>
<td>92.8</td>
<td>0.1</td>
<td>0.067</td>
<td></td>
<td></td>
<td>385.0</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>761</td>
<td>37.4</td>
<td>4.1</td>
<td>185.9</td>
<td>0.1</td>
<td>0.067</td>
<td></td>
<td></td>
<td>674.7</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>705</td>
<td>39.1</td>
<td>4.4</td>
<td>159.3</td>
<td>0.1</td>
<td>0.050</td>
<td></td>
<td></td>
<td>405.7</td>
</tr>
<tr>
<td>FOV 600-8641</td>
<td>100</td>
<td>1200.6</td>
<td>37.4</td>
<td>4.1</td>
<td>293.4</td>
<td>0.1</td>
<td>0.050</td>
<td></td>
<td></td>
<td>1027.2</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>152.2</td>
<td>40.9</td>
<td>4.8</td>
<td>31.8</td>
<td>0.2</td>
<td>0.200</td>
<td></td>
<td></td>
<td>160.5</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>277.06</td>
<td>39.1</td>
<td>4.4</td>
<td>62.6</td>
<td>0.2</td>
<td>0.200</td>
<td></td>
<td></td>
<td>288.7</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>446.4</td>
<td>38.0</td>
<td>4.2</td>
<td>106.4</td>
<td>0.2</td>
<td>0.200</td>
<td></td>
<td></td>
<td>431.3</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>332.9</td>
<td>40.9</td>
<td>4.8</td>
<td>69.5</td>
<td>0.2</td>
<td>0.100</td>
<td></td>
<td></td>
<td>405.7</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>575.2</td>
<td>39.1</td>
<td>4.4</td>
<td>130.0</td>
<td>0.2</td>
<td>0.100</td>
<td></td>
<td></td>
<td>632.9</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>727</td>
<td>38.0</td>
<td>4.2</td>
<td>173.3</td>
<td>0.2</td>
<td>0.100</td>
<td></td>
<td></td>
<td>875.9</td>
</tr>
<tr>
<td>Haeri et al. (2000)</td>
<td>60</td>
<td>555.2</td>
<td>44.1</td>
<td>5.6</td>
<td>99.6</td>
<td>0.1</td>
<td>0.050</td>
<td></td>
<td></td>
<td>603.6</td>
</tr>
<tr>
<td>H O E C H S T 11/180</td>
<td>100</td>
<td>722.5</td>
<td>44.0</td>
<td>5.5</td>
<td>130.5</td>
<td>0.1</td>
<td>0.100</td>
<td></td>
<td></td>
<td>719.8</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>839.7</td>
<td>44.0</td>
<td>5.5</td>
<td>151.7</td>
<td>0.1</td>
<td>0.067</td>
<td></td>
<td></td>
<td>836.3</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1024.7</td>
<td>44.0</td>
<td>5.5</td>
<td>185.1</td>
<td>0.1</td>
<td>0.050</td>
<td></td>
<td></td>
<td>994.0</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1305</td>
<td>44.0</td>
<td>5.5</td>
<td>235.7</td>
<td>0.1</td>
<td>0.040</td>
<td></td>
<td></td>
<td>1220.8</td>
</tr>
<tr>
<td>Haeri et al. (2000)</td>
<td>60</td>
<td>721.4</td>
<td>44.1</td>
<td>5.6</td>
<td>129.4</td>
<td>0.1</td>
<td>0.050</td>
<td></td>
<td></td>
<td>828.6</td>
</tr>
<tr>
<td>TYPAR 3407</td>
<td>60</td>
<td>470.9</td>
<td>44.1</td>
<td>5.6</td>
<td>84.5</td>
<td>0.038</td>
<td>0.038</td>
<td></td>
<td></td>
<td>475.7</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>641.1</td>
<td>44.1</td>
<td>5.6</td>
<td>115.0</td>
<td>0.038</td>
<td>0.025</td>
<td></td>
<td></td>
<td>604.3</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>965.7</td>
<td>44.1</td>
<td>5.6</td>
<td>173.3</td>
<td>0.038</td>
<td>0.019</td>
<td></td>
<td></td>
<td>828.6</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1300.3</td>
<td>44.1</td>
<td>5.6</td>
<td>233.3</td>
<td>0.038</td>
<td>0.015</td>
<td></td>
<td></td>
<td>1319.3</td>
</tr>
<tr>
<td>Haeri et al. (2000)</td>
<td>300</td>
<td>2041.3</td>
<td>43.7</td>
<td>5.5</td>
<td>374.0</td>
<td>0.1</td>
<td>0.100</td>
<td></td>
<td></td>
<td>2100.2</td>
</tr>
<tr>
<td>H U S K E R B 500</td>
<td>100</td>
<td>781.9</td>
<td>44.0</td>
<td>5.5</td>
<td>141.2</td>
<td>0.1</td>
<td>0.100</td>
<td></td>
<td></td>
<td>713.0</td>
</tr>
</tbody>
</table>

After obtaining the $Co$ and $m$ empirical coefficients, Equation 7-5 can be applied and the equivalent confining pressure is determined for each test, presented also in Table 25. With this confining pressure, the corresponding major principal stress is determined by Equation 7-6. The following Figure 56 shows the obtained calculated results compared to the laboratory test results.
It can be seen from Figure 56 that Yang's (1972) theory gives high accuracy when the coefficients are back-calculated for each geosynthetic type. Most of the calculated results fall very close to the laboratory data. The largest variation is about 20%. It appears that this theory provides very good predictions of the reinforced soil strength. However, it must be recognized that the coefficients are back-calculated from test results, and as yet, there does not seem to be a way to predict values of the empirical coefficients from characteristics of the soil and geosynthetics, nor the stress ranges or diameter to spacing ratios for which the particular set of coefficient values applies.

Regarding the empirical coefficients obtained, it can be seen from Table 25: The results of $Co$ and $m$ are in smaller ranges, except for the last group of tests. $m$ ranges from 0.94 to 1.47 and $Co$ from 0.08 to 0.147. Yang's (1972) own laboratory results produced values of the empirical coefficients that are within these ranges: $m = 1.18$ and $Co = 0.13$.

If the equivalent confining pressures for the tests listed in Table 25 are calculated using the coefficient values presented by Yang's (1972), i.e., $m = 1.18$ and $Co = 0.13$, the resulting comparison between measured and calculated major principal stresses is as shown in Figure 57.
Figure 57: Major Principal Stress at failure using Yang's (1972) theory, calculated with Yang's (1972) empirical coefficients versus laboratory results. (a) All test results (b) Portion of the figure, from 0 to 3500 kPa
Figure 57 indicates good accuracy of the theory for many of the points, particularly those where the calculated major principal stresses are less than about 3,500 kPa. The two eliminated values into Figure 57(b) have D/L equal to 2.5. This D/L value results the highest for all the analyzed tests and the two eliminated points are the only tests with that ratio. It suggests that high D/L ratios return unconservative high calculated strengths. At high calculated values of major principal stress, the calculated values can be unconservative by a very large margin.

7.2.3. **Equivalent Cohesion (Schlosser and Long 1974)**

This theory requires an equivalent cohesion determined according to Equation 7-11 to account for the strengthening effect of geosynthetic reinforcement on GRS. The theory also assumes that the friction angle for the reinforced soil is the same as for the unreinforced soil. To apply this theory to the collected laboratory data, the apparent cohesion is calculated and used with the soil friction angle and applied confining pressure to calculate the maximum major principal stress that the GRS specimen can sustain according to Equation 7-12. Table 26 shows the calculation results, and the Figure 58 compares the calculated stress with the laboratory test results.
Table 26: Application of Schlosser and Long's (1974) theory to the laboratory data

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\sigma_3$ kPa</th>
<th>$\sigma_1$ kPa</th>
<th>$T_{ult}$ kN/m</th>
<th>$L$ m</th>
<th>$\phi$ degrees</th>
<th>$K_p$</th>
<th>$c$ Theory kPa</th>
<th>$\sigma_{1r}$ Theory kPa</th>
<th>Type of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chandrasekaran et al. (1989)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>156</td>
<td>48</td>
<td>0.1</td>
<td>40.9</td>
<td>4.8</td>
<td>525.4</td>
<td>220.2</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>295</td>
<td>48</td>
<td>0.1</td>
<td>39.1</td>
<td>4.4</td>
<td>504.8</td>
<td>315.8</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>540</td>
<td>48</td>
<td>0.1</td>
<td>37.4</td>
<td>4.1</td>
<td>485.5</td>
<td>498.4</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>240</td>
<td>48</td>
<td>0.067</td>
<td>40.9</td>
<td>4.8</td>
<td>788.2</td>
<td>242.8</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>411</td>
<td>48</td>
<td>0.067</td>
<td>39.1</td>
<td>4.4</td>
<td>757.3</td>
<td>337.0</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>761</td>
<td>48</td>
<td>0.067</td>
<td>37.4</td>
<td>4.1</td>
<td>728.3</td>
<td>518.4</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>376</td>
<td>48</td>
<td>0.05</td>
<td>40.9</td>
<td>4.8</td>
<td>1050.9</td>
<td>261.8</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>705</td>
<td>48</td>
<td>0.05</td>
<td>39.1</td>
<td>4.4</td>
<td>1009.7</td>
<td>354.9</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1201</td>
<td>48</td>
<td>0.05</td>
<td>37.4</td>
<td>4.1</td>
<td>971.0</td>
<td>535.3</td>
<td>Fabric rupture</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>152</td>
<td>48</td>
<td>0.2</td>
<td>40.9</td>
<td>4.8</td>
<td>262.7</td>
<td>190.8</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>277</td>
<td>48</td>
<td>0.2</td>
<td>39.1</td>
<td>4.4</td>
<td>252.4</td>
<td>288.1</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>446</td>
<td>48</td>
<td>0.2</td>
<td>38.0</td>
<td>4.2</td>
<td>245.8</td>
<td>399.9</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>333</td>
<td>48</td>
<td>0.1</td>
<td>40.9</td>
<td>4.8</td>
<td>525.4</td>
<td>220.2</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>575</td>
<td>48</td>
<td>0.1</td>
<td>39.1</td>
<td>4.4</td>
<td>504.8</td>
<td>315.8</td>
<td>Fabric rupture</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>727</td>
<td>48</td>
<td>0.1</td>
<td>38.0</td>
<td>4.2</td>
<td>491.6</td>
<td>426.5</td>
<td>Fabric rupture</td>
</tr>
<tr>
<td>Haeri et al. (2000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>721</td>
<td>8.4</td>
<td>0.050</td>
<td>44.1</td>
<td>5.6</td>
<td>198.3</td>
<td>400.9</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>555</td>
<td>10.4</td>
<td>0.050</td>
<td>44.1</td>
<td>5.6</td>
<td>245.5</td>
<td>408.4</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>723</td>
<td>10.4</td>
<td>0.100</td>
<td>44.0</td>
<td>5.5</td>
<td>122.4</td>
<td>605.8</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>840</td>
<td>10.4</td>
<td>0.067</td>
<td>44.0</td>
<td>5.5</td>
<td>183.5</td>
<td>617.5</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1025</td>
<td>10.4</td>
<td>0.050</td>
<td>44.0</td>
<td>5.5</td>
<td>244.7</td>
<td>627.3</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1305</td>
<td>10.4</td>
<td>0.040</td>
<td>44.0</td>
<td>5.5</td>
<td>305.9</td>
<td>636.0</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>471</td>
<td>8.4</td>
<td>0.038</td>
<td>44.1</td>
<td>5.6</td>
<td>260.9</td>
<td>410.7</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>641</td>
<td>8.4</td>
<td>0.025</td>
<td>44.1</td>
<td>5.6</td>
<td>391.4</td>
<td>427.9</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>966</td>
<td>8.4</td>
<td>0.019</td>
<td>44.1</td>
<td>5.6</td>
<td>521.9</td>
<td>442.3</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1300</td>
<td>8.4</td>
<td>0.015</td>
<td>44.1</td>
<td>5.6</td>
<td>652.4</td>
<td>455.0</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>2041</td>
<td>20</td>
<td>0.100</td>
<td>43.7</td>
<td>5.5</td>
<td>233.6</td>
<td>1709.0</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>2147</td>
<td>20</td>
<td>0.067</td>
<td>43.7</td>
<td>5.5</td>
<td>350.5</td>
<td>1725.1</td>
<td>Soil</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>782</td>
<td>20</td>
<td>0.100</td>
<td>44.0</td>
<td>5.5</td>
<td>235.3</td>
<td>625.9</td>
<td>Soil</td>
</tr>
<tr>
<td>Ashmawy and Boudeau (1998)</td>
<td>50</td>
<td>817</td>
<td>20</td>
<td>0.028</td>
<td>38.8</td>
<td>4.4</td>
<td>738.2</td>
<td>331.7</td>
<td>Soil</td>
</tr>
</tbody>
</table>
Figure 58: Major Principal Stress at failure, from equivalent cohesion theory and laboratory information

Figure 58 shows the calculated results are much higher than the laboratory test results. The accuracy and the reliability of the theory is very low, and the calculated values are all unconservative, for these laboratory data. The theory over-predicts capacity even for the cases with geosynthetic rupture, for which good results might be expected.

7.2.4. An Approximate Isotropic Perfectly Plastic Solution (Tatsuoka 2004)

In order to apply this approximate solution to calculate the strength response of the collected laboratory tests, equations from 7-16 to 7-18 and 7-20 have been applied to the laboratory tests. The calculated major principal stresses at failure are plotted against the corresponding laboratory test results in Figure 59 for the cases in which reinforcement rupture did not occur. About 60% of the comparisons in Figure 59 exhibit good agreement between the laboratory test results and the calculated theoretical result. The best agreement occurs at low values of major principle stress, in particular when the calculated values are less than about 1,000 kPa. Otherwise, the differences are large and unconservative.
Table 27: Application of Tatsuoka's (2004) theory to the laboratory data

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\sigma_3$ kPa</th>
<th>$\sigma_1$ kPa</th>
<th>d (D) m</th>
<th>h (L) m</th>
<th>$\phi$ rad</th>
<th>$\mu$ degrees</th>
<th>$\delta_\theta$ rad</th>
<th>$\xi$ rad</th>
<th>B</th>
<th>$P_1$ kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chandrasekaran et al. (1989)</td>
<td>25</td>
<td>155.7</td>
<td>0.1</td>
<td>0.1</td>
<td>0.7</td>
<td>35</td>
<td>0.8</td>
<td>0.4</td>
<td>1.4</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>295.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.7</td>
<td>35</td>
<td>0.9</td>
<td>0.4</td>
<td>1.2</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>540.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.7</td>
<td>35</td>
<td>0.9</td>
<td>0.5</td>
<td>1.1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>239.9</td>
<td>0.1</td>
<td>0.07</td>
<td>0.7</td>
<td>35</td>
<td>0.8</td>
<td>0.4</td>
<td>2.7</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>410.6</td>
<td>0.1</td>
<td>0.07</td>
<td>0.7</td>
<td>35</td>
<td>0.9</td>
<td>0.4</td>
<td>2.4</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>761</td>
<td>0.1</td>
<td>0.07</td>
<td>0.7</td>
<td>35</td>
<td>0.9</td>
<td>0.5</td>
<td>2.2</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>375.7</td>
<td>0.1</td>
<td>0.05</td>
<td>0.7</td>
<td>35</td>
<td>0.8</td>
<td>0.4</td>
<td>3.6</td>
<td>3.8</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>705</td>
<td>0.1</td>
<td>0.05</td>
<td>0.7</td>
<td>35</td>
<td>0.9</td>
<td>0.4</td>
<td>3.9</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>152.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7</td>
<td>35</td>
<td>0.8</td>
<td>0.4</td>
<td>1.4</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>277.06</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7</td>
<td>35</td>
<td>0.9</td>
<td>0.4</td>
<td>1.2</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>446.4</td>
<td>0.2</td>
<td>0.2</td>
<td>0.7</td>
<td>35</td>
<td>0.9</td>
<td>0.5</td>
<td>1.1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>332.9</td>
<td>0.2</td>
<td>0.1</td>
<td>0.7</td>
<td>35</td>
<td>0.8</td>
<td>0.4</td>
<td>3.9</td>
<td>4.9</td>
</tr>
<tr>
<td>Haeri et al. (2000)</td>
<td>60</td>
<td>721.4</td>
<td>0.1</td>
<td>0.05</td>
<td>0.8</td>
<td>32</td>
<td>0.7</td>
<td>0.4</td>
<td>4.4</td>
<td>6.9</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>555.2</td>
<td>0.1</td>
<td>0.05</td>
<td>0.8</td>
<td>37</td>
<td>0.8</td>
<td>0.4</td>
<td>4.9</td>
<td>9.6</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>722.5</td>
<td>0.1</td>
<td>0.1</td>
<td>0.8</td>
<td>37</td>
<td>0.8</td>
<td>0.4</td>
<td>1.8</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>839.7</td>
<td>0.1</td>
<td>0.07</td>
<td>0.8</td>
<td>37</td>
<td>0.8</td>
<td>0.4</td>
<td>3.3</td>
<td>3.2</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1024.7</td>
<td>0.1</td>
<td>0.05</td>
<td>0.8</td>
<td>37</td>
<td>0.8</td>
<td>0.4</td>
<td>4.8</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>1305</td>
<td>0.1</td>
<td>0.04</td>
<td>0.8</td>
<td>37</td>
<td>0.8</td>
<td>0.4</td>
<td>4.8</td>
<td>9.3</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>470.9</td>
<td>0.038</td>
<td>0.038</td>
<td>0.8</td>
<td>36</td>
<td>0.8</td>
<td>0.4</td>
<td>1.8</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>641.1</td>
<td>0.038</td>
<td>0.03</td>
<td>0.8</td>
<td>36</td>
<td>0.8</td>
<td>0.4</td>
<td>3.3</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>965.7</td>
<td>0.038</td>
<td>0.019</td>
<td>0.8</td>
<td>36</td>
<td>0.8</td>
<td>0.4</td>
<td>4.8</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1300.3</td>
<td>0.038</td>
<td>0.0152</td>
<td>0.8</td>
<td>36</td>
<td>0.8</td>
<td>0.4</td>
<td>6.3</td>
<td>26.4</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>2041.3</td>
<td>0.1</td>
<td>0.1</td>
<td>0.8</td>
<td>39</td>
<td>0.9</td>
<td>0.4</td>
<td>1.8</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>2147.2</td>
<td>0.1</td>
<td>0.07</td>
<td>0.8</td>
<td>39</td>
<td>0.9</td>
<td>0.4</td>
<td>3.4</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>781.9</td>
<td>0.1</td>
<td>0.1</td>
<td>0.8</td>
<td>39</td>
<td>0.9</td>
<td>0.4</td>
<td>1.8</td>
<td>1.0</td>
</tr>
<tr>
<td>Ashmawy and Boudeau (1998)</td>
<td>50</td>
<td>817.3</td>
<td>0.071</td>
<td>0.03</td>
<td>0.7</td>
<td>32</td>
<td>0.8</td>
<td>0.4</td>
<td>4.4</td>
<td>6.8</td>
</tr>
</tbody>
</table>
Figure 59: Major Principal Stress at failure, from Tatsuoka's theory (2004) against laboratory data. (a) All data (b) Portion of the figure from 0 to 5,000 kPa
The solution for the failure mechanism when the reinforcement ruptures gives results that are completely off by factors of about 20 (calculated/laboratory), and those results have not been included in Figure 59.

7.3. Summary and Discussion

From application of the theories to the compiled laboratory test data, the following observations can be made:

- The best agreement between theory and test data is given by Yang's (1972) theory. The agreement between calculations and measurements is very good when the two empirical constants are obtained from a set of laboratory results using the same soil and geosynthetic. This indicates that the behavior of the reinforced soil in laboratory triaxial compression tests can be expressed in the fashion of Yang's (1972) equations, which incorporates dependency on the friction angle of the soil and the diameter/spacing ratio of the geosynthetic.

- Even when the empirical constants do not come from the same laboratory information, the agreement between Yang's (1972) theory and the laboratory data is very good for about 70% of the points, although the results exhibit more variation when the diameter/spacing ratio gets higher. For ratios of 2.5, the theoretical results are much higher than the laboratory data, which indicates an overrepresentation of the effects of geosynthetic reinforcement.

- For Tatsuoka's (2004) theory, about 60% of the calculated values are in good agreement with the test results. The agreement is generally good when the calculated major principal stress is less than about 1,000 kPa, and for larger values, the theory can be unconservative by a large margin.

- The equivalent cohesion theory predicts unconservative values that are significantly higher than the laboratory results for all cases. However, the excess capacity calculated using this theory does not reach the extremes from Yang's (1972) theory using Yang's empirical coefficients or the extremes from Tatsuoka's (2004) theory.

Regarding the parameters embedded in all three theories for which comparisons were made to laboratory test results in this research:

- All the theories agree that increasing the friction angle of the soil and the confining pressure should increase the capacity of the reinforced soil. Yang's (1972) theory and Tatsuoka's (2004) approximation use the diameter/spacing ratio as an important parameter of influence. For both theories, when the spacing gets very low, the calculated strength exceeds the measured strength of the reinforced soil. The equivalent cohesion theory only uses the spacing between reinforcement layers and does not
implement the width or the diameter of the reinforced specimen as an influencing parameter.

- The equivalent cohesion theory uses the reinforcement tensile strength as a key parameter that influences the reinforced soil behavior. It appears that this theory does not represent the actual behavior of the reinforced soil even if the failure mechanism is due to the reinforcement rupture.

- Yang's (1972) theory requires the use of empirical constants that depend on the geosynthetic and interface behavior, but the empirical constants are determined from GRS results rather than explicitly from the geosynthetic and interface behavior. Tatsuoka's (2004) theory directly incorporates the influence of the soil-reinforcement interaction through the use of the friction angle of the soil-geosynthetic interface.
CHAPTER 8:
CONCLUSIONS AND RECOMMENDATIONS

This chapter is divided into three sections: the first section presents a summary of the work accomplished, the second section presents conclusions derived from the research, and the third section presents recommendations for future research.

8.1. Summary of Accomplished Work

The work completed in the present investigation is summarized in the following items, which are organized in five categories: constitutive model for the soil, constitutive model for the geosynthetic and the soil-geosynthetic interfaces, numerical model of GRS, comparison of numerical analyses with laboratory test results from the published literature, and evaluation of GRS strength theories.

a) Constitutive Model for the Soil:

This category presents a summary of the work to identify, adapt, and implement a suitable constitutive model for the soil as a component of a GRS mass, including details for application to axisymmetric loading conditions, such as in laboratory triaxial compression tests. The soil constitutive model was developed by modifying the Chsoil model described by Itasca (2012). Work completed in this regard includes the following items:

- The Chsoil model described by Itasca (2012) was studied. Its characteristics for application in three dimensional loading conditions were explained, and equations specific to axisymmetric loading conditions were derived.

- The constitutive equations that regulate the Chsoil model behavior under axisymmetric conditions were reformulated to provide manageable expressions that use outcomes of drained triaxial compression tests under constant confining pressure conditions. Such outcomes include: axial stress versus axial strain and volumetric strain versus axial strain. The mathematical derivations of these new expressions were explained for soils with non-zero cohesion and for soils with zero cohesion.

- The behavior of the Chsoil model was applied to demonstrate its ability to generate the key parameters that result from a drained triaxial compression test, including: initial slope of the stress-strain curve, nonlinearity of the stress-strain curve, unloading-reloading slope, and sensitivity of these outcomes to variation of input parameters.

- A comparative mathematical analysis of the Chsoil model and Duncan and Chang's hyperbolic model (Duncan et al. 1970) was performed. This resulted in expressions that can be used to relate the parameter values from one model to the parameter values of the other model. Many of the parameters in the Chsoil model are presented by Itasca (2012) in identical or similar ways to parameters in Duncan and Chang's model, but the comparative mathematical analysis
performed as part of this research demonstrated that several of the parameters are different, and quantitative evaluation of realistic parameter values for each model showed how different they can be.

- During the study of the Chsoil model, it was found that the stress dependency response of the model could be improved. A modified model was developed that changes the stiffness, friction angle, and dilatancy behavior automatically as a function of the initial confining pressure. The modified model better represents real granular soil behavior under drained triaxial test conditions. Data from several sets of triaxial test were analyzed to correlate dilation property values for the modified Chsoil model with relative density of cohesionless soils like those used for GRS systems.

Two methods were developed to obtain the parameter values for the modified Chsoil model:

- A methodology was developed to obtain soil property values for the modified Chsoil model when a full set of drained triaxial test results is available, including axial-stress-strain curves and axial-stress-versus-volumetric-strain curves for multiple confining pressures. The procedure is explained step by step, and an example with real data is included.

- A different methodology was developed to obtain soil property values for the modified Chsoil model when limited strength test information is available. This procedure uses recommendations for the Duncan and Chang model parameters from Duncan et al. (1980) and converts them to modified Chsoil model parameters using the expressions described above that were developed as part of this research, as well as the correlations between dilation parameters and relative density that were also developed as part of this research.

b) Geosynthetic constitutive model

Itasca (2012) presents some alternatives to represent the geosynthetic and the geosynthetic-soil interface behavior characteristics. Those alternatives were analyzed, and the best representation for a geosynthetic and its interfaces with the soil was determined. The assessment included some basic tests to verify that the selected model has the necessary characteristics. Interestingly, the "geogrid" model is not suitable because it does not permit sliding of the soil adjacent to the geosynthetic along the soil-geosynthetic interface, although the "geogrid" elements can be pulled out of the soil while the soil zones on opposite sides of the geogrid remain fully connected. Instead, an "embedded liner" element does permit sliding of the soil adjacent to the geosynthetic along the soil-geosynthetic interface. Also, the "embedded liner" elements can be assigned orthotropic properties suitable for geosynthetic reinforcement.

c) GRS numerical model

After the determination of adequate constitutive models for the soil, the geosynthetic, and their interfaces, a numerical model was developed to represent the behavior of GRS under triaxial test
conditions. The model was implemented using FLAC3D (Itasca 2012), with specialized code developed as part of this research that includes:

- Automatic generation of the mesh geometry as a function of specimen diameter, height, number of geosynthetic layers, and number of zones in the radial direction. Recommendations regarding these inputs are provided.

- The use of adequate mesh elements to avoid computational problems.

- Two types of failure modes were considered: geosynthetic rupture and limiting axial strain.

- Results are expressed as stress-strain curves and volumetric-strain-versus-axial-strain curves.

d) Comparison of numerical results and laboratory triaxial test results.

Results obtained through the GRS numerical model were compared with results from real triaxial tests on GRS specimens obtained from published literature. These analyses required the following work:

- Information about GRS triaxial tests were collected from the available literature. Twenty-five sources with GRS triaxial information were identified. Of these, five sources included adequate information about the tests to permit numerical analysis. Thirty triaxial tests were selected from three of these five sources based on specimen size and reinforcement layer separation distance. Very small specimens and very close reinforcement spacings that were judged to make specimen fabrication difficult were not selected for numerical analysis.

- Material properties for each component (soil, geosynthetic, and interface) of the reinforced soil triaxial tests were determined. This included:
  
  - Calculation of the soil properties as a function of unreinforced soil stress-strain and volumetric-strain-versus-axial-strain curves presented in the literature.
  
  - Determination of geosynthetic properties from manufacturer's data and correlations with properties for similar products.
  
  - Geosynthetic-soil interface properties were obtained from the sources reporting the GRS test results when available, and through literature recommendations for interface properties for similar materials when the source of the GRS test data did not include interface properties.

- Numerical analyses of the GRS tests were performed, and the results were compared with the laboratory results.
- Two characteristics of the numerical model had significant impacts on stability of the analyses: (1) mesh refinement and (2) velocity applied to the top of the specimen to produce the deviatoric stress applied to the sample. Sufficient mesh refinement and suitably slow imposed velocity are necessary for stable results of the numerical analyses.

- The numerical model was applied to the thirty selected triaxial tests on GRS specimens over strains level up to 3% axial strain. The stress-strain and volumetric-strain-versus-axial-strain responses of the numerical analyses were compared with the laboratory test results for each case.

- The same analyses and comparisons were made for large strains to investigate whether the numerical model could reliably represent failure conditions.

e) GRS strength theories.

Three theories available in the published literature were applied to the selected laboratory triaxial tests on GRS specimens from published literature. The three theories are: increment of confining pressure theory proposed by Yang (1972), an apparent cohesion theory proposed by Scholosser and Long (1974), and an approximate isotropic perfectly plastic solution by Tatsuoka (2004). The three theories were explained, and details of the application of these theories to the selected laboratory tests were presented. The predictions from these theories were compared to the laboratory test results.

8.2. Conclusions

The following key findings and recommendations are drawn from the research:

- The Chsoil model described by Itasca (2012) appears to be a capable constitutive model for granular materials. It has a built-in strain-hardening law to reproduce nonlinear stress-strain behavior, it incorporates different options to represent dilative behavior, and it requires a reasonable number of input parameters. However, the model presents some disadvantages. The model uses the initial mean effective stress as an input as if the initial stress were a soil property. The model does not recalculate this mean stress during an analysis, and it is instead kept constant. The mean initial stress is important in the strain-hardening law and in calculation of the elastic parameters. Another disadvantage the Chsoil model is that it does not incorporate variation of friction angle as a function of confining pressure. Finally, the simplest procedure native to the model for incorporating dilation consists of zero dilation below a certain mobilized friction angle that must be specified, and then a constant non-zero dilation angle when the mobilized friction angle exceeds the specified limit for zero dilation. This model does not capture well the volumetric-strain-versus-axial-strain behavior of dense granular soil. If a different dilation alternative native to the Chsoil model is selected, then the model becomes complicated to manage or too many difficult-to-determine input parameters are required.
• It is possible to establish mathematical comparisons between the Chsoil model and Duncan and Chang's hyperbolic model (Duncan et al. 1970). These two models share some characteristics and differ in others. Both models use a hyperbolic formulation. However, the Chsoil model incorporates a hyperbolic strain-hardening procedure only for plastic strains. Both models use a failure ratio. However, Duncan and Chang's model applies the failure ratio to the deviatoric stress, while the Chsoil model applies the failure ratio to the sine of the friction angle. The Chsoil model incorporates plasticity theory and yield behavior, while Duncan and Chang's model is based on nonlinear elasticity during increasing loading and includes inelastic response to unloading.

• Modifications to the Chsoil model were developed and implemented to produce a modified constitutive model able to more accurately represent the behavior of granular soil, including nonlinear stress-strain response, confining pressure-dependent initial stiffness, confining-pressure dependent friction angle, and confining pressure dependent dilative behavior. This modified model overcomes several disadvantages of the original Chsoil model.

• This investigation includes two methods to determine the input parameters for the modified soil model. The first methodology uses standard results of drained triaxial tests to directly obtain all the material property values for the modified Chsoil constitutive model. The second methodology uses correlations with soil classification and relative compaction to obtain all the material property values for the Chsoil model. The second methodology is based on material property values for Duncan and Chang's hyperbolic model recommended by Duncan et al. (1980) and correlations for dilatancy property values developed as part of this research. Both methods are easy to apply, with the first presented in a step-by-step procedure with examples, and the second consisting of simple look-up values from a table.

• Poisson's ratio values that were found in the published literature for geotextiles range from 0.39 to 2.1. Many of these values are from uniaxial loading tests on narrow strips of geotextile, which are not representative of response under biaxial loading conditions. To address this, mathematical calculations of Poisson's ratios were performed based on the results of biaxial tests on some types of geosynthetics presented in a published source. These calculations produced values of Poisson's ratio in the range from about 0 to 0.5, which are realistic values for biaxial loading.

• The GRS numerical model developed in this research is able to represent GRS behavior under triaxial test conditions for working strains up to about 3% axial strain for GRS specimens with up to 3 layers of geosynthetic. Correspondence between numerical analyses and test results has not been verified for more than three layers of geosynthetic in triaxial test specimens.

• When the numerical model is used to try to represent GRS failure, numerical convergence problems can arise when there are more than two layers of geosynthetic in the specimen or when the soil friction angle exceeds 40 degrees. To avoid such problems, a highly refined mesh and a low applied velocity would be necessary, such that very large periods of time would be
necessary for the software and hardware used in this research to complete an analysis. When the GRS numerical model was applied to GRS samples with less than three layers of reinforcement and with soil friction angles up to 40 degrees, the numerical analyses performed in this research did not experience convergence problems. In general, it is recommended that a convergence study be performed before accepting the results of numerical analyses of any specific GRS system.

- Variations in GRS component parameter values had the following influences on the numerical model response:

  - Regarding soil property values:
    - Increasing the elastic shear modulus produced stiffer stress-strain response of the GRS specimen.
    - Increasing the failure ratio produced a softer stress-strain response over most of the strain range to failure, although the initial stress-strain response was unaffected.
    - Increasing the friction angle increased the specimen strength.
    - Variations in the bulk modulus, cv angle, and dilation angle had relatively little to no impact on the stress-strain response of the GRS, although they did affect the volumetric strain response as expected.

  - Regarding the geosynthetic and soil-geosynthetic interface properties:
    - Increasing the geosynthetic stiffness generally increases the GRS stiffness.
    - Decreasing the soil-geosynthetic friction angle decreases the GRS strength.
    - The interface stiffnesses, geosynthetic Poisson's ratios, geosynthetic shear modulus did not play an important role in outcomes of the numerical analyses of GRS tests, as long as those properties are within reasonable values.

- Regarding strength theories for GRS:

  - Yang’s (1972) theory shows a high accuracy in comparison to the laboratory results. However, applying this theory depends on establishing two empirical parameters. Therefore, the disadvantage of this theory is the necessity of having laboratory test results to obtain the empirical parameters that would then be applied to other tests using the same soil and geosynthetic. These empirical parameters embody the influence of many other parameters (e.g., geosynthetic properties and interface behavior), such that a new set of empirical coefficients would have to be determined from laboratory tests for every
combination of soil and geosynthetic. The theory does appear to appropriately represent
the influences of specimen-diameter-to-geosynthetic-spacing ratio and applied confining
pressure, but since many other characteristics are embedded in the empirical coefficients,
this theory has limited potential to promote fundamental understanding of GRS behavior.

- Schlosser and Long's (1974) theory uses the ultimate tensile strength of the reinforcement
as an important parameter to calculate the GRS strength. This theory would be mostly
applicable to failure due to reinforcement rupture. However, in the comparisons performed
in this research, the theory and the laboratory results were not in good agreement,
regardless of the failure mode.

- Tatsuoka's (2004) theory provided strengths deviations of about ±20% from the laboratory
test results for about 60% of the comparisons made in this research. In general, the theory
was reasonably accurate for calculated values of major principal stress that were less than
about 1,000 kPa, but the results were unconservative for higher calculated values of major
principal stress. This theory considers two types of failure, soil strength failure and
geosynthetic rupture, which is a good characteristic. This theory seems promising, and it
may be worthy of further development.

8.3. Recommendations for Future Work

- Use of two-dimensional, axisymmetric numerical analyses of GRS triaxial compression tests
should be investigated. This research employed three-dimensional analyses to represent the
orthotropic character of the geosynthetic reinforcement. However, very long analysis times
are necessary for analyses conducted to failure for some cases of practical interest due to the
highly refined meshes and slow applied velocities that are necessary to avoid convergence
problems. If two-dimensional numerical analyses can be shown to be essentially equivalent to
three-dimensional analyses, highly refined meshes could be used with fewer mesh zones and
nodal points than necessary for three-dimensional analyses. A path forward would be to
perform three-dimensional analyses using isotropic properties of the geosynthetic and compare
them with three-dimensional analyses using orthotropic properties of the geosynthetic with an
isotropic Young's modulus equal to the average of the two orthotropic Young's moduli. If the
results are nearly the same for a realistic range of system characteristics, this would indicate
that two-dimensional axisymmetric numerical analyses would be suitable. Confirming two-
dimensional axisymmetric numerical analyses with convergence studies should also be
performed.

- If two-dimensional numerical analyses can be demonstrated to produce reliable results at
working strains and approaching failure in reasonable computation times, parametric studies
should be performed to investigate the influence of a wide range of geometries and property
values on GRS response. The results of such analyses would provide insight to the principal
factors controlling GRS response.
• Results of the recommended parameter study using two-dimensional numerical analyses could be compared with Tatsuoka's (2004) theory. This could shed light on the range of applicability of Tatsuoka's (2004) theory, and the theory could perhaps be modified to increase its range of applicability using the insights about GRS behavior gained from the parametric study.

• If the two-dimensional axisymmetric numerical model of laboratory GRS tests can be verified, then two-dimensional plane strain analyses of large scale tests such as those presented by Wu et al. (2013) could be performed. This would establish whether the soil, geosynthetic, and interface representation in small-scale models could also be used to represent response of large-scale plane-strain tests.
REFERENCES


APPENDIX A: APLICATION OF SOIL MODEL TO TRIAXIAL TESTS

This appendix presents the application of the constitutive soil model, explained in Chapter 3, to thirteen triaxial tests obtained from available literature. The appendix has three sections. The first section presents the procedure to follow in order to apply the numerical model to the laboratory data, the second section presents the results obtained from applying the procedure to the test data, and the third section summarizes the results and conclusions.

A.1. Modified Chsoil Model Applied to Triaxial Laboratory Tests

Chapter 3, in the present investigation, explains the development of the constitutive model for the soil. As part of the assessment of the constitutive model performance, the model was applied to triaxial test data, as follows:

1. Several sets of triaxial test data for granular soils with different confining pressures were compiled. The search for laboratory triaxial test data focused on granular soils similar to what would be used for GRS fill. In order to apply the constitutive model, the test data must include stress-strain curves and volumetric-strain-versus-axial strain curves at multiple confining pressures. Most of the selected tests were performed on soils at high relative density. Thirteen suitable sets of triaxial test data were obtained from 6 different sources.

2. Each set of triaxial test data was analyzed, and the information necessary to apply the step-by-step procedure described in Section 3.5 was compiled.

3. After the necessary information was compiled, the step-by-step procedure explained in Section 3.5 was applied to obtain the input parameters for the modified Chsoil model.

4. A simple numerical model of the triaxial test was then run using the input parameter values with the same confining pressures as used in the laboratory tests, and the results of the numerical model were compared with the results of the laboratory tests. The FLAC3D code for the modified Chsoil model is included into the GRS code in Appendix C.

A.2. Results

In the following pages, the results obtained from evaluation of parameter values and the comparisons between Chsoil model results and laboratory data are presented, including:

- Reference source of the triaxial test data
- Information regarding the soil used, such as the soil type and relative density
- Stress-strain and volumetric-strain-versus-axial-strain curves from the triaxial data
• Input parameter values for the modified Chsoil model obtained using the step-by-step procedure described in Section 3.5

• Stress-strain and volumetric-strain-versus-axial-strain curves from the numerical model. These curves are presented in the same figures used to present the laboratory curves for ease of comparison.
Test 1:

Material: Limestone, commercial gradation 21b, low density from Duncan et al. (2007)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_0$ (deg)</td>
<td>43.3</td>
</tr>
<tr>
<td>$\Delta \phi$ (deg)</td>
<td>10.1</td>
</tr>
<tr>
<td>$\phi_{cv0}$ (deg)</td>
<td>17</td>
</tr>
<tr>
<td>$\Delta \phi_{cv}$ (deg)</td>
<td>-28</td>
</tr>
<tr>
<td>$\psi_0$ (deg)</td>
<td>-2.7</td>
</tr>
<tr>
<td>$\Delta \psi$ (deg)</td>
<td>20.13</td>
</tr>
<tr>
<td>$G_{ref}$</td>
<td>446</td>
</tr>
<tr>
<td>$n$</td>
<td>0.36</td>
</tr>
<tr>
<td>$K_{ref}$</td>
<td>411</td>
</tr>
<tr>
<td>$m$</td>
<td>0</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.979</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-1: Stress vs. strain curves for different confining pressures Test 1. Continuous lines from numerical model, dashed lines from triaxial tests data.
Figure A-2: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 1. Continuous lines from numerical model, dashed lines from triaxial tests data.
**Test 2:**

Material: Granite 21b, low density from Duncan et al. (2007)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o$(deg)</td>
<td>45.8</td>
</tr>
<tr>
<td>$\Delta \phi$ (deg)</td>
<td>7.12</td>
</tr>
<tr>
<td>$\phi_{cv0}$(deg)</td>
<td>39.2</td>
</tr>
<tr>
<td>$\Delta \phi_{cv}$(deg)</td>
<td>1.69</td>
</tr>
<tr>
<td>$\psi_o$(deg)</td>
<td>14.1</td>
</tr>
<tr>
<td>$\Delta \psi$(deg)</td>
<td>3.74</td>
</tr>
<tr>
<td>$G_{ref}$</td>
<td>541</td>
</tr>
<tr>
<td>$n$</td>
<td>0.12</td>
</tr>
<tr>
<td>$K_{ref}$</td>
<td>394</td>
</tr>
<tr>
<td>$m$</td>
<td>0</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.977</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-3: Stress vs. strain curves for different confining pressures Test 2: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-4: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 2. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Test 3:

Material: Limestone 21b, high density from Duncan et al. (2007)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o$ (deg)</td>
<td>52.4</td>
</tr>
<tr>
<td>$\Delta \phi$ (deg)</td>
<td>12.8</td>
</tr>
<tr>
<td>$\phi_{cvo}$ (deg)</td>
<td>45.1</td>
</tr>
<tr>
<td>$\Delta \phi_{cvo}$ (deg)</td>
<td>-0.48</td>
</tr>
<tr>
<td>$\psi_o$ (deg)</td>
<td>12.5</td>
</tr>
<tr>
<td>$\Delta \psi$ (deg)</td>
<td>6.9</td>
</tr>
<tr>
<td>$G_{ref}$</td>
<td>949</td>
</tr>
<tr>
<td>$n$</td>
<td>0.53</td>
</tr>
<tr>
<td>$K_{ref}$</td>
<td>921</td>
</tr>
<tr>
<td>$m$</td>
<td>0</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.973</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-5: Stress vs. strain curves for different confining pressures Test 3: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-6: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 3. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Test 4:

Material: Granite 21b, high density from Duncan et al. (2007)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_0 ) (deg)</td>
<td>53.6</td>
</tr>
<tr>
<td>( \Delta \phi ) (deg)</td>
<td>12.9</td>
</tr>
<tr>
<td>( \phi_{cvo} ) (deg)</td>
<td>43.0</td>
</tr>
<tr>
<td>( \Delta \phi_{cvo} ) (deg)</td>
<td>-0.98</td>
</tr>
<tr>
<td>( \psi_0 ) (deg)</td>
<td>25.2</td>
</tr>
<tr>
<td>( \Delta \psi ) (deg)</td>
<td>4.78</td>
</tr>
<tr>
<td>( G_{ref} )</td>
<td>927</td>
</tr>
<tr>
<td>( n )</td>
<td>0.58</td>
</tr>
<tr>
<td>( K_{ref} )</td>
<td>934</td>
</tr>
<tr>
<td>( m )</td>
<td>0</td>
</tr>
<tr>
<td>( R_f )</td>
<td>0.977</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-7: Stress vs. strain curves for different confining pressures Test 4: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-8: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 4. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
**Test 5:**

Material: Limestone 57, low density from Duncan et al. (2007)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_o ) (deg)</td>
<td>47.0</td>
</tr>
<tr>
<td>( \Delta \phi ) (deg)</td>
<td>12.1</td>
</tr>
<tr>
<td>( \phi_{cv0} ) (deg)</td>
<td>36.6</td>
</tr>
<tr>
<td>( \Delta \phi_{cv} ) (deg)</td>
<td>0.18</td>
</tr>
<tr>
<td>( \psi_o ) (deg)</td>
<td>24.9</td>
</tr>
<tr>
<td>( \Delta \psi ) (deg)</td>
<td>6.08</td>
</tr>
<tr>
<td>( G_{ref} )</td>
<td>284</td>
</tr>
<tr>
<td>( n )</td>
<td>0.42</td>
</tr>
<tr>
<td>( K_{ref} )</td>
<td>310</td>
</tr>
<tr>
<td>( m )</td>
<td>0</td>
</tr>
<tr>
<td>( R_f )</td>
<td>0.975</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

![Stress vs. Strain Curves](image_url)

Figure A-9: Stress vs. strain curves for different confining pressures Test 5: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-10: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 5. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Test 6:

Material: Phyllite 57, low density from Duncan et al. (2007)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_o)(deg)</td>
<td>49.5</td>
</tr>
<tr>
<td>(\Delta \phi) (deg)</td>
<td>8.86</td>
</tr>
<tr>
<td>(\phi_{cv0})(deg)</td>
<td>49.7</td>
</tr>
<tr>
<td>(\Delta \phi_{cv}) (deg)</td>
<td>3.20</td>
</tr>
<tr>
<td>(\psi_o)(deg)</td>
<td>21.4</td>
</tr>
<tr>
<td>(\Delta \psi)(deg)</td>
<td>4.96</td>
</tr>
<tr>
<td>(G_{ref})</td>
<td>608.7</td>
</tr>
<tr>
<td>(n)</td>
<td>0.16</td>
</tr>
<tr>
<td>(K_{ref})</td>
<td>1581</td>
</tr>
<tr>
<td>(m)</td>
<td>0.97</td>
</tr>
<tr>
<td>(R_f)</td>
<td>0.992</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-11: Stress vs. strain curves for different confining pressures Test 6: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-12: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 6. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Test 7:

Material: Limestone 57, high density from Duncan et al. (2007)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_o )(deg)</td>
<td>48.20</td>
</tr>
<tr>
<td>( \Delta \phi )(deg)</td>
<td>10.0</td>
</tr>
<tr>
<td>( \phi_{cvo} )(deg)</td>
<td>44.7</td>
</tr>
<tr>
<td>( \Delta \phi_{cvo} )(deg)</td>
<td>1.24</td>
</tr>
<tr>
<td>( \psi_o )(deg)</td>
<td>25.5</td>
</tr>
<tr>
<td>( \Delta \psi )(deg)</td>
<td>6.04</td>
</tr>
<tr>
<td>( G_{ref} )</td>
<td>346</td>
</tr>
<tr>
<td>( n )</td>
<td>0.18</td>
</tr>
<tr>
<td>( K_{ref} )</td>
<td>360</td>
</tr>
<tr>
<td>( m )</td>
<td>0</td>
</tr>
<tr>
<td>( R_f )</td>
<td>0.978</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-13: Stress vs. strain curves for different confining pressures Test 7: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-14: Stress vs. strain curves for different confining pressures Test 7: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.

Figure A-15: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 7. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-16: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 7. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Test 8:

Material: Phyllite 57, high density from Duncan et al. (2007)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o$ (deg)</td>
<td>50.1</td>
</tr>
<tr>
<td>$\Delta \phi$ (deg)</td>
<td>10.2</td>
</tr>
<tr>
<td>$\phi_{cv0}$ (deg)</td>
<td>41.0</td>
</tr>
<tr>
<td>$\Delta \phi_{cv0}$ (deg)</td>
<td>-1.78</td>
</tr>
<tr>
<td>$\psi_o$ (deg)</td>
<td>18.41</td>
</tr>
<tr>
<td>$\Delta \psi$ (deg)</td>
<td>1.47</td>
</tr>
<tr>
<td>$G_{ref}$</td>
<td>486</td>
</tr>
<tr>
<td>$n$</td>
<td>0.62</td>
</tr>
<tr>
<td>$K_{ref}$</td>
<td>629</td>
</tr>
<tr>
<td>$m$</td>
<td>0.11</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-17: Stress vs. strain curves for different confining pressures Test 8: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-18: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 8. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
**Test 9:**

Material: Uniform clean quartz beach SP sand with 70% relative density from Haeri et al. (2000). Tests with diameter of 100 mm.

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_0 ) (deg)</td>
</tr>
<tr>
<td>( \Delta \phi ) (deg)</td>
</tr>
<tr>
<td>( \phi_{cv_0} ) (deg)</td>
</tr>
<tr>
<td>( \Delta \phi_{cv} ) (deg)</td>
</tr>
<tr>
<td>( \psi_0 ) (deg)</td>
</tr>
<tr>
<td>( \Delta \psi ) (deg)</td>
</tr>
<tr>
<td>( G_{ref} )</td>
</tr>
<tr>
<td>( n )</td>
</tr>
<tr>
<td>( K_{ref} )</td>
</tr>
<tr>
<td>( m )</td>
</tr>
<tr>
<td>( R_f )</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-19: Stress vs. strain curves for different confining pressures Test 9: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-20: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 9. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
**Test 10:**

Material: Uniform fine sand with relative density of 75% from Chandrasekaran (1992)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o$ (deg)</td>
</tr>
<tr>
<td>$\Delta\phi$ (deg)</td>
</tr>
<tr>
<td>$\phi_{cv0}$ (deg)</td>
</tr>
<tr>
<td>$\Delta\phi_{cv}$ (deg)</td>
</tr>
<tr>
<td>$\psi_o$ (deg)</td>
</tr>
<tr>
<td>$\Delta\psi$ (deg)</td>
</tr>
<tr>
<td>$G_{ref}$</td>
</tr>
<tr>
<td>$n$</td>
</tr>
<tr>
<td>$K_{ref}$</td>
</tr>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>$R_f$</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-21: Stress vs. strain curves for different confining pressures Test 10: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-22: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 10. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Test 11:

Material: Uniform clean quartz beach SP sand with 70% relative density from Haeri et al. (2000). Tests with diameter of 38 mm

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o$ (deg)</td>
<td>43.9</td>
</tr>
<tr>
<td>$\Delta \phi$ (deg)</td>
<td>0.617</td>
</tr>
<tr>
<td>$\phi_{cv0}$ (deg)</td>
<td>19.6</td>
</tr>
<tr>
<td>$\Delta \phi_{cv}$ (deg)</td>
<td>-8.3</td>
</tr>
<tr>
<td>$\psi_o$ (deg)</td>
<td>16.8</td>
</tr>
<tr>
<td>$\Delta \psi$ (deg)</td>
<td>6.9</td>
</tr>
<tr>
<td>$G_{ref}$</td>
<td>192</td>
</tr>
<tr>
<td>$n$</td>
<td>0.92</td>
</tr>
<tr>
<td>$K_{ref}$</td>
<td>194</td>
</tr>
<tr>
<td>$m$</td>
<td>0.7</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-23: Stress vs. strain curves for different confining pressures Test 11: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Figure A-24: Volumetric-strain-versus-axial-strain curves for different confining pressures Test 11. Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Test 12:

Material: Crushed diabase with maximum dry density of 24.1 kN/m$^3$ from Wu et al. (2013)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o$(deg)</td>
<td>57.7</td>
</tr>
<tr>
<td>$\Delta\phi$ (deg)</td>
<td>15.7</td>
</tr>
<tr>
<td>$\phi_{cv0}$(deg)</td>
<td>49.8</td>
</tr>
<tr>
<td>$\Delta\phi_{cv}$ (deg)</td>
<td>4.4</td>
</tr>
<tr>
<td>$\psi_o$(deg)</td>
<td>11.3</td>
</tr>
<tr>
<td>$\Delta\psi$(deg)</td>
<td>3.6</td>
</tr>
<tr>
<td>$G_{ref}$</td>
<td>747</td>
</tr>
<tr>
<td>$n$</td>
<td>0.68</td>
</tr>
<tr>
<td>$K_{ref}$</td>
<td>1004</td>
</tr>
<tr>
<td>$m$</td>
<td>0.13</td>
</tr>
<tr>
<td>$R_f$</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-25: Stress vs. strain curves for different confining pressures Test 12: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
Test 13:

Material: Road base silty sand at a dry unit weight of 17.8 kN/m$^3$ from Ketchart and Wu (2001)

Modified Chsoil model input data:

<table>
<thead>
<tr>
<th>INPUT MODIFIED CHSOIL MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_o$ (deg)</td>
</tr>
<tr>
<td>$\Delta\phi$ (deg)</td>
</tr>
<tr>
<td>$\phi_{cv0}$ (deg)</td>
</tr>
<tr>
<td>$\Delta\phi_{cv}$ (deg)</td>
</tr>
<tr>
<td>$\psi_0$ (deg)</td>
</tr>
<tr>
<td>$\Delta\psi$ (deg)</td>
</tr>
<tr>
<td>$G_{ref}$</td>
</tr>
<tr>
<td>$n$</td>
</tr>
<tr>
<td>$K_{ref}$</td>
</tr>
<tr>
<td>$m$</td>
</tr>
<tr>
<td>$R_f$</td>
</tr>
</tbody>
</table>

Laboratory and numerical model curves:

Figure A-26: Stress vs. strain curves for different confining pressures Test 13: Continuous lines from modified Chsoil model, dashed lines from triaxial tests data.
A.3. Summary and Conclusions

This appendix presents thirteen test sets where the modified Chsoil constitutive model has been applied to triaxial test data from available literature. Each set includes different confining pressures, such that a total of about 50 tests with stress-strain and volumetric-strain-versus-axial-strain curves are included. Thus, a significant number of tests have been compared with the constitutive model.

It can be seen by examining the stress-strain curves that the constitutive model matches the laboratory test results very closely. Regarding the volumetric-strain-versus-axial-strain curves, the agreement between the constitutive model and the laboratory data is not as good as for the stress-strain curves, although the agreement is still relatively good. About 50% of the curves show a very close approximation between the constitutive model and the laboratory tests, while others present slight deviations and a very few present large differences.

Overall, it can be concluded that the modified Chsoil model is appropriate to represent the behavior of typical granular soils under triaxial testing conditions.
APPENDIX B: FLAC3D CODE FOR LINER ELEMENT TESTS

Three simple tests were performed on the “embedded liner” elements from FLAC3D (Itasca 2012) to ensure these elements are able to successfully represent orthotropic geotextile behavior. These three simple tests are: direct shear, pullout, and structural element test. Tests details are described in Chapter 4. In the following, the FLAC3D codes used to generate the results explained in Chapter 4 are presented.

B.1. Direct Shear Test Code

new
; Generation of two soil blocks of dimensions: 1×1×0.2 m each one, with cubic elements:
gen zone brick size 10 10 4 p0 0 0 0 p1 1 0 0 p2 0 1 0 p3 0 0 0.4
  group up range z 0.2 0.4
  group down range z 0 0.2
; Soil model and properties (high values in properties to make soil very stiff)
model mech mohr
  prop bulk 3.33e5 she 2.5e5 coh 1e3 fric 45
; Boundary conditions of the soil
  fix y z
  fix x range group down
; Geosynthetic:
  ; Create a new face at the soil where the liner will be located
  group face Interface internal range group up group down
  gen separate face origin 0 0 0 group Newint range group Interface
  ; Generating the geosynthetic:
  sel liner id=1 ele cst internal embedded 0 0 0 range group Interface
  ; Applying geosynthetic's properties (elastic isotropic material)
  sel liner property thick=0.001 iso=(150000,0.3)
; Lower face properties:
  sel liner property side1 cs_nck 0 cs_nk 1e10 cs_scoh 100 cs_scohres 0 cs_sfric 0 cs_sk 1e10
; Upper face properties
  sel liner property side2 cs_nck 0 cs_nk 1e10 cs_scoh 50 cs_scohres 0 cs_sfric 0 cs_sk 1e10
  apply xvel 4e-12 range group up
plot sel lincontour cpsstress1
plot sel lincontour cpsstress2
  cycle 500000000
B.2. Pullout Test Code

new
; Lower face 500 cohesive strength and upper face 300 cohesive strength
; Soil:
gen zone brick size 10 10 4 p0 0 0 0 p1 1 0 0 p2 0 1 0 p3 0 0 0.4
group up range z 0.2 0.4
group down range z 0 0.2
; Soil model and properties (high values for a stiff soil)
model mech mohr
prop bulk 3.33e5 she 2.5e5 coh 1e3 fric 45
; Geosynthetic
; Create a new face at the soil where the liner will be located
group face Interface internal range group up group down
gen separate face origin 0 0 0 group Newint range group Interface
; Generating the geosynthetic
sel liner id=1 ele cst internal embedded 0 0 0 range group Interface
; Geosynthetic properties (elastic isotropic material)
sel liner property thick=0.001 iso=(150000,0.3)
; Lower geosynthetic face:
sel liner property side1 cs_ncut 0 cs_nk 1e10 cs_scoh 500 cs_scohres 0 cs_sfric 0 cs_sk 1e10
; Upper face:
sel liner property side2 cs_ncut 0 cs_nk 1e10 cs_scoh 300 cs_scohres 0 cs_sfric 0 cs_sk 1e10
; Boundary conditions:
fix x y z
; History shear stress vs shear strain
history add id=1 sel linersel coupling side1 stress shear nd3 cid=1
history add id=2 sel linersel coupling side1 disp shear nd3 cid=1
; History shear stress vs shear strain
history add id=3 sel linersel coupling side2 stress shear nd3 cid=1
history add id=4 sel linersel coupling side2 disp shear nd3 cid=1
history unbal
sel node fix x
sel node init xvel 5e-10
cycle 1200000
B.3. Structural Element Test Code

new
; Soil:
gen zone brick size 1 1 2 p0 0 0 0 p1 1 0 0 p2 0 1 0 p3 0 0 2
group up range z 1 2
group down range z 0 1
; Soil model and properties:
model mech mohr
prop bulk 3.33e5 she 2.5e5 coh 1e3 fric 45
; Boundary conditions of the soil
fix x y z
; Geosynthetic
group face Interface internal range group up group down
; Newint = new interface in upsoil
; Interface = old interface in down soil
gen separate face origin 0 0 0 group Newint range group Interface
sel liner id=1 ele cst internal embedded 0 0 0 range group Interface
sel liner property thick=0.001 orthotropic stiff 192700 42000 139200 60000 matx 1 0 0
sel liner property side1 cs_ncut 0 cs_nk 1e9 cs_scoh 0 cs_scohres 0 cs_sfric 0 cs_sk 1e9
sel liner property side2 cs_ncut 0 cs_nk 1e9 cs_scoh 0 cs_scohres 0 cs_sfric 0 cs_sk 1e9
; Geosynthetic boundary conditions:
sel node fix x y z
sel node init xvel 1e-9 range x 0.99 1.01
sel node init yvel 1e-10 range y 0.99 1.01
; Determination of results:
def po_stress
while_stepping
np = nd_head
_a11=nd_rfo(np,1,1)
_a12=nd_rfo(np,1,2)
_a13=nd_rfo(np,1,3)
np=nd_next(np)
_a21=nd_rfo(np,1,1)
_a22=nd_rfo(np,1,2)
_a23=nd_rfo(np,1,3)
np=nd_next(np)
_a31=nd_rfo(np,1,1)
_a32=nd_rfo(np,1,2)
_a33=nd_rfo(np,1,3)
np=nd_next(np)
_a41=nd_rfo(np,1,1)
_a42=nd_rfo(np,1,2)
_a43=nd_rfo(np,1,3)
sumx = _a31 + _a41
sumy = _a32 + _a22
sumz = _a33 + _a43 + _a13 + _a23
end
@po_stress
history unbalance
history @sumx
history @sumy
history @sumz
cycle 800000
APPENDIX C: GRS FLAC3D CODE

This appendix presents the FLAC3D (Itasca 2012) code developed for the GRS numerical model. Chapter 5 explained the general assembly and details of the GRS numerical model developed to reproduce GRS behavior under triaxial loading with constant confining pressure.

As explained in Chapter 5, the code consists of three main parts. The first part is the data collection, the second part is the model generation, and the last part includes running the test with the collection of analysis outcomes. The following code is divided into those three main parts, and brief explanation of some routines and commands is included.

C.1. Stage 1: Code to Collect Data

new ; Collecting data. For the present example the units are kN-m
def dimes
    ; Geometry, confining pressure and zones number
    d_r=0.05 ; ratio
    d_h=0.2 ; height
    n_l=2 ; number of geosynthetic layers
    g_f=0.0012 ; to improve the joint between the parallelepiped and radial zones
    c_p=-25 ; confining pressure in kPa
    r_z=6 ; number of zones in ratio of cylindrical shell (half of total)
    sp_z=16 ; number of zones between two reinforcement layers
    n_r=26 ; row number where the vertical stress is calculated
    n_ru=36 ; another row for stress calculation to check equilibrium

    ; Parameters to determine GRS failure:
    stlimit=-0.14 ; maximum axial strain 14%
    lim_sx = 47500 ; geosynthetic ultimate strength in machine direction
    lim_sy = 47500 ; geosynthetic ultimate strength in cross-machine direction

    ; Applied velocity to represent deviatoric stress
    vel_uz= -0.00000001

    ; Calculation of some parameters to use in a later stage:
    qc_z=2*r_z ; number of zones in angular direction
    h_z=(n_l+1)*sp_z ; number of zones in total height
    s_g=d_h/(n_l+1) ; spacing distance

    ; Soil properties:
    ; Dilation angle parameters:
    d_o=4.8
    Dd=0.95
; Friction angle parameters:
f_o=37.36
Df=5.83
; cv angle parameters:
cv_o=29.8
Dcv=2.7
;Shear modulus parameters:
p_shear_ref=192.72
p_n=0.299
;Bulk modulus parameters:
p_bulk_ref=563.67
p_m=0
; Reference pressure (atmospheric pressure)
p_a=101.28 ;kPa
;Cohesion and failure ratio
p_coh=0
p_rf=0.969
;Geosynthetic properties
p_c11=404040
p_c12=40404
p_c22=404040
p_c33=130000
g_th=0.001 ;thickness
;Interface properties:
g_s_coh=0 ;adhesion
g_s_fric=35 ;interface friction angle
g_s_st=480000;shear stiffness
;Normal stiffness calculation
p_ke=p_bulk_ref*p_a*(-c_p/p_a)^p_m
p_ge=p_shear_ref*p_a*(-c_p/p_a)^p_n
p_D_z=s_g/sp_z
g_st=10*(p_ke+4*p_ge/3)/p_D_z
end
@dimes
C.2. Stage 2: Code to Generate the GRS Model

def m_gen
    c_ir=d_r/2.0 ; ratio/2 to generate the combined mesh with parallelepiped and radial zones
    ; generating soil zones
    command
        gen zone cshell size @r_z @h_z @qc_z 2 ratio 1 1 1 p0 0,0,0 p1 0,@d_r,0 p2 0,0,@d_h p3 @d_r,0,0 p4 0,@d_r,@d_h p5 @d_r,0,@d_h p8 0,0,c_ir,0 p9 0,c_ir,0,0 p10 0,0,c_ir,0,@d_h
    ; all this command in one line
    end_command
    ; moving internal points of the shell to fit an internal polygon
    local p_gp=gp_head
    loop while p_gp #null
        local px=gp_xpos(p_gp)
        local py=gp_ypos(p_gp)
        local dist=sqrt(px*px+py*py) - 0.000001
        adit=c_ir/sqrt(2)
        if dist<c_ir then
            if px>adit then
                gp_ypos(p_gp)=int(py/adit*r_z)*adit/r_z
                gp_xpos(p_gp)=adit+g_f*(r_z-int(py/adit*r_z))
            else
                if py>adit then
                    gp_xpos(p_gp)=int(px/adit*r_z)*adit/r_z
                    gp_ypos(p_gp)=adit+g_f*(r_z-int(px/adit*r_z))
                end_if
            end_if
        end_if
    p_gp=gp_next(p_gp)
    end_loop
    adit2=adit+g_f*r_z
    nz_b=r_z
    ; Generation of internal polygon
    command
        gen zone brick size @nz_b @nz_b @h_z p0 0,0,0 p1 @adit2,0,0 p2 0,@adit2,0 p3 0,0,@d_h p4 @adit,0,0 p5 0,0,0 @adit,0 p6 @adit2,0,0 @d_h
    ; only one line
    attach face
        group up range z @s_g @d_h
    end_command
    ; Generating geosynthetic layers
    i=1
    loop while i<(n_l+1)
lim_up=s_g*i
iv=s_g*i-s_g/20000
fv=s_g*i+s_g/20000
command
group down range z 0 @lim_up
group face Interface internal range group up group down z @iv @fv
gen separate face origin 0 0 0 group Newinterface range group Interface z @iv @fv
sel liner id=1 ele cst internal embedded 0 0 0 range group Interface z @iv @fv
end_command
i=i+1
end_loop

; Boundary conditions
u_b=d_h+d_h/20000
d_b=d_h-d_h/20000
command
fix z range z -0.0000001 0.0000001
fix x range x -0.0000001 0.0000001
fix y range y -0.0000001 0.0000001
; Total restrain at bottom face and lateral movement restriction on top face:
fix x range z -0.0000001 0.0000001
fix y range z -0.0000001 0.0000001
fix x range z @d_b @u_b
fix y range z @d_b @u_b
; Fixing geosynthetic
sel node fix x range x -0.0000001 0.0000001
sel node fix y range y -0.0000001 0.0000001
; Assigning model
model mechanical chsoil
end_command
; Assigning properties: initial mean pressure, dilation angle, cv angle and friction angle
z_pm=-c_p
DDf=-Dd*log(z_pm/p_a)+d_o
CVA=-Dcv*log(z_pm/p_a)+cv_o
FA=-Df*log(z_pm/p_a)+f_o
command
prop shear_ref=@p_shear_ref rf=@p_rf bulk_ref=@p_bulk_ref ten 0 dilaw=2
prop m_k=@p_m n_g=@p_n p_ref=@p_a cohesion=@p_coh
prop p_ini=@z_pm dilf=@DDf fricv=@CVA fricf=@FA
sel liner prop thick @g_th
sel liner prop side1 cs_scoh @g_s_coh cs_sfric @g_s_fric cs_sk @g_s_st cs_scohes @g_s_coh ; continues from previous line
sel liner prop side1 cs_ncut 0 cs_nk @g_st
sel liner prop side2 cs_scoh @g_s_coh cs_sfric @g_s_fric cs_sk @g_s_st cs_scohres @g_s_coh; continues from previous line
sel liner prop side2 cs_ncut 0 cs_nk @g_st
sel liner prop orthotropic stiff @p_c11 @p_c12 @p_c22 @p_c33 matx 1 0 0
; Initializing confining stress
ini sxx @c_p syy @c_p szz @c_p
apply sxx @c_p syy @c_p
apply szz @c_p range z @d_b @u_b
apply sxx 0 syy 0 range x -0.000001 0.000001
apply sxx 0 syy 0 range y -0.000001 0.000001
apply sxx 0 syy 0 range z -0.000001 0.000001
apply sxx 0 syy 0 range z @d_b @u_b
end_command
j=1
loop while j<(n_l+1)
   iv=s_g*j-s_g/20000
   fv=s_g*j+s_g/20000
   command
      apply sxx 0 syy 0 range z @iv @fv
   end_command
   j=j+1
end_loop
command
solve
ini disp (0,0,0)
end_command
end
@m_gen
C.3. Stage 3: Code to Generate and Store Results

; Finding volumetric strain
def volresult
    sum=0.0 ;initializing variables in zero
    voltot=0
loop c_l (1,(r_z*qc_z+nz_b*nz_b)*h_z)
    pnt=z_find(c_l)
    sum=sum+z_vsi(pnt)*z_volume(pnt) ; determination of total volume change
    voltot=voltot+z_volume(pnt) ; determination of total volume
end_loop
volstr=sum/(voltot) ; volumetric strain
end

; Finding stresses
def cal_st
    strst=0 ;initialization of variables on zero
    artot=0
loop c_j (0,qc_z-1) ; loop for the parallelepided zones
loop c_k ((n_r-1)*r_z+1+c_j*r_z*h_z,n_r*r_z+c_j*r_z*h_z) ; stresses at row n_r
    zpnt=z_find(c_k) ; determination of zones of interest
    strsp=z_sz(zpnt)*z_volume(zpnt)/(d_h/h_z) ;stress × its respective area
    strst=strst+strsp ; total force
    arpar=z_volume(zpnt)/(d_h/h_z) ; calculation of area
    artot=artot+arpar ; total area
end_loop
end_loop
ini_b=r_z*qc_z*h_z+1+(n_r-1)*nz_b*nz_b
loop c_k (ini_b,ini_b+nz_b*nz_b-1) ; loop for the radial zones
    zpnt=z_find(c_k)
    strsp=z_sz(zpnt)*z_volume(zpnt)/(d_h/h_z)
    strst=strst+strsp
    arpar=z_volume(zpnt)/(d_h/h_z)
    artot=artot+arpar
end_loop
end
fst=strst/(artot) ; final vertical stress

; Stresses at different level, at row n_ru, with same procedure than before
local strstu=0
local artotu=0
loop c_j (0,qc_z-1)
loop c_k ((n_ru-1)*r_z+1+c_j*r_z*h_z,n_ru*r_z+c_j*r_z*h_z) ; stresses at row n_r
    zpnt=z_find(c_k)
    strspu=z_sz(zpnt)*z_volume(zpnt)/(d_h/h_z)
    strstu=strstu+strspu
    arparu=z_volume(zpnt)/(d_h/h_z)

208
artotu = artotu + arparu
end_loop
end_loop
ini_b = r_z*qc_z*h_z + 1 + (n_ru-1)*nz_b*nz_b
loop c_k (ini_b, ini_b + nz_b*nz_b-1)
zpnt = z_find(c_k)
strspu = z_szz(zpnt)*z_volume(zpnt)/(d_h/h_z)
strstu = strstu + strspu
arparu = z_volume(zpnt)/(d_h/h_z)
artotu = artotu + arparu
end_loop
fstu = strstu/(artotu)
; Finding axial strain
gp_pnt = gp_near(0,0,d_h)
axstr = gp_zdisp(gp_pnt)/d_h
end
@volresult
@cal_st
; To see histories every 3000 steps
hist id = 7 fish @cal_st n=3000
hist id = 8 fish @volresult n=3000
; Data to draw the stress-strain and volumetric-strain-versus-axial-strain curves
hist id=1 @volstr n=3000
hist id=2 @fst n=3000
hist id=3 @axstr n=3000
hist id=4 @fstu n=3000
hist id=6 unbal n=3000
; Applying deviatoric stress through velocity
apply zvel 0 range z @d_b @u_b
apply szz 0 range z @d_b @u_b
step 3000
; Stopping condition:
def StopLoad
gp_pntCheck = gp_near(0,0,d_h)
axstrCheck = gp_zdisp(gp_pntCheck)/d_h
if axstrCheck < stlimit then ; is negative, so smaller gives more strain
StopLoad = 1
else
loop i(0,n_l-1)
    command
        sel recover surf surfx 1 0 0
        sel recover stress
    end_command
f_id=2*r_z*2*r_z+1+i*((2*r_z)^2+(2*nz_b)*nz_b)
f_sp=find_s(f_id)
f_sx=sst_str(f_sp,0,1)
f_sy=sst_str(f_sp,0,2)
if f_sx > lim sx then
    StopLoad = 1
else
    if f_sy > lim_sy then
        StopLoad = 1
    else
        StopLoad = 0
    end_if
end_if
end_loop
endif
end
apply zvel @vel_uz range z @d_b @u_b
solve fishhalt @StopLoad

Each of these three stages of the code should be saved in .dat files, then theses three stages can be called in FLAC3D sequentially. In this way, results for a GRS triaxial test at a specific confining pressure can be obtained.