A Numerical Comparison of Atmospheric Density Models for Near-Earth Satellite Motion

by

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Chapter 1
Introduction

The theory of satellite motion prediction\(^1\) is directed toward:

(i) the computation of satellite position and velocity as a function of time, permitting realistic and adequate mission planning.

(ii) the prediction of re-entry position and re-entry time so as to avoid aircraft hazards, bodily injuries, etc.

In order to precisely predict the position of a satellite at any future time, it is necessary to accurately model the environmental perturbations which affect the orbit of the satellite. For satellites in relatively low orbits (perigee altitude below 500 km), the disturbance due to aerodynamic drag can be many times the magnitude of other orbital perturbations\(^2\). For this reason, proper modeling of the aerodynamic drag force which acts on a satellite becomes fundamentally important.

The basic expression for the satellite drag force involves a number of "dispersion parameters". Dispersion parameters are defined as "any parameter, variable or constant, which affects the dynamic behavior of a satellite, its trajectory, and lifetime". The uncertainty regarding some of these parameters makes the atmospheric drag a difficult perturbation to model.
accurately. For example, the atmospheric density and its variation in space and time must be modeled correctly if realistic results are to be obtained. A great deal of work has been done in an attempt to develop an accurate atmospheric density model. Theoretical and empirical techniques have been used, and combined into semi-empirical techniques, in order to obtain numerical values of the density as a function of the spatial variable(s) (and time if required)\(^3,4,5,6\).

In addition to the atmospheric density, the expression for the drag force includes the following dispersion parameters:

(i) the drag coefficient
(ii) the magnitude of the velocity of the satellite relative to the atmosphere
(iii) the satellite reference area upon which the drag coefficient is based.

The initial task of this investigation has been to perform a survey of the available literature dealing with the aerodynamic drag force which acts upon satellites in near-earth orbits. The purpose of this survey has been to review comments concerning all of the dispersion parameters involved in the expression for the drag force, and make recommendations for each. The atmospheric density is discussed thoroughly, and the recommendation concerning this parameter is supported by detailed numerical studies. Specifically, an attempt is made to quantify the differences, in both density and its effect on
satellite motion, between the recommended density model and a "popularly used" density model.
Chapter 2

Results of Literature Survey

The basic expression for the satellite drag force $D$ is:

$$D = - C_D \left( \frac{1}{2} \rho V_R^2 \right) A_{Ref} \cdot e$$

(2.1)

where

$C_D = \text{drag coefficient}$

$\rho = \text{local atmospheric density}$

$V_R = \text{magnitude of the velocity of the satellite relative to the atmosphere}$

$A_{Ref} = \text{satellite reference area upon which } C_D \text{ is based}$

$e = \text{unit vector in the direction of relative velocity } V_R \cdot$

From the form of Eq. (2.1), it is evident that accurate values of the four dispersion parameters are needed. The parameters $C_D$, $V_R$, and $A_{Ref}$ will now be discussed briefly, while $\rho$ will be considered in detail. This discussion parallels the work done by Lafontaine in Reference 2.

2.1 Determination of the Drag Coefficient $C_D$

In general, $C_D$ is affected by the following:

(i) the physical and dynamical properties of the surrounding medium.

(ii) The nature of the interaction between the surface of the body and the medium.
(iii) The attitude motion and surface properties of the satellite.

Each of the above items complicates the determination of $C_D$. For example, the mean free molecular path is a physical property of the fluid which affects the value of the drag coefficient. The mean free path is a strong function of altitude; its value ranges from $.142 \text{ m}$ at an altitude of 100 km to $240 \text{ m}$ at an altitude of 200 km. The degree of energy transfer between a gas molecule and a surface is usually expressed in terms of the accommodation coefficient. Theoretical investigations of this coefficient are inconclusive because of the assumptions made regarding the motion of the gas atoms and the configuration of the surface atoms. Furthermore, due to the dependence of $C_D$ upon the attitude of a satellite, the drag can vary by a factor of 2 or 3 as a function of attitude for some satellite configurations.

Therefore, because of the lack of decisive information there is not sufficient reason to abandon the value of 2.2 which has been widely used in recent years for the drag coefficient. It should be recognized, however, that this value is subject to some uncertainty and may be too low.

2.2 Determination of the Relative Velocity $V_R$

The upper atmosphere rotates at a slightly different angular rate than that of the earth. Thus, in order to
determine the velocity of the satellite relative to the atmosphere, the atmospheric angular velocity, \( \omega_A \), must be established. This is usually expressed in the form of the rotational ratio, \( \Lambda = \frac{\omega_A}{\omega_E} \) where \( \omega_E \) is the angular velocity of the earth. In general, the measurements of \( \Lambda \) are subject to some degree of uncertainty; however, it is generally recognized that \( \Lambda \) increases with altitude in the 200 km - 400 km region.

Note that with the angular velocity, \( \omega_A \), established the velocity of the satellite relative to the atmosphere is

\[
V_R = V - \omega_A \times r
\]

(2.2)

where \( r \) and \( V \) are the position and the velocity of the satellite relative to a geocentric but non-rotating coordinate system.

2.3 Determination of the Reference Area \( A_{Ref} \)

If a constant drag coefficient value, such as that suggested in Section 2.1, is utilized, then \( A_{Ref} \) (as used in Eq. 2.1) must be the instantaneous cross-sectional area presented to the flow.

Lafontaine showed how a detailed formulation of the variation of \( A_{Ref} \) with the attitude motion of the satellite can be avoided by using a value of \( A_{Ref} \) averaged over one orbit. For simple shapes and either inertial or spin stabilization, this is not difficult. For an arbitrary tumble, the use of the
The total surface area divided by four might be considered. The error in this case would tend to zero as the shape becomes more and more spherical.

2.4 Atmospheric Density Models

The main techniques used to obtain atmospheric density models are classified as empirical, theoretical, or semi-empirical. In the first, one assumes a mathematical function with undetermined coefficients and then utilizes numerical values of density derived from satellite observations in order to determine the coefficients. In the second, the density is obtained from a solution to the equations of state. With the semi-empirical approach, one obtains a mathematical function with undetermined coefficients from the equations of state, and then uses observational data to determine the coefficients.

A density model obtained using any of the three techniques mentioned above is categorized as either static or dynamic; each type will now be discussed.

2.4.1 Static Atmospheric Density Models

A model in which the only independent variable is the altitude, h, is referred to as a static density model. Such a model represents the mean density of the atmosphere averaged over all the time dependent density fluctuations.

With the assumption of hydrostatic equilibrium, constant temperature T, constant molecular weight M, constant
gravitational acceleration $g_E$, and homogeneous mixing of the atmospheric constituents, the density variation is found to be

$$\rho = \rho_0 \exp \left( \frac{h_0 - h}{H} \right)$$  \hspace{1cm} (2.3)

where

$$H = \frac{RT}{Mg_E}$$

and

$$R = \text{ideal gas constant.}$$

Equation (2.3) represents the exponential density model which is widely used because of its mathematical simplicity. However, the assumption of homogeneous mixing of the various constituents is not valid for altitudes above 85 km (53 miles). Therefore, this form of the exponential model is limited to low altitudes.

In order to both extend the applicable range and improve accuracy, several improvements can be applied to Eq. (2.3):

1. The use of a layering technique, with $H$ determined for each layer, improves accuracy but becomes numerically inefficient when many layers are traversed within an orbit. Also the mathematical simplicity of a pure exponential model is lost.

2. The use of a variational rule for the scale height, $H$, as a function of altitude gives a wider range of accuracy since the temperature and molecular weight
vary. But again, the introduction of a varying scale height complicates the mathematical computations.

(3) The use of an earth flattening factor, to account for the oblateness of the earth, transforms Eq. (2.3) into an oblate exponential density model which is referred to the origin of the inertial frame (rather than the earth's surface). This makes the density model useful in orbital theory.

The density can be represented by functions other than exponential. An example of another model is the power-law density model of the form:

\[ \rho = \rho_0 \left( \frac{h}{h_0} \right)^\eta. \]  

(2.4)

Eq. (2.4) is purely an empirical expression where \( \eta \) and \( \rho_0 \) are undetermined coefficients, and \( h_0 \) is a reference altitude.

Any of the static models suffer from the fact that well-known dynamic variations are not directly accounted for. For example, a static model will predict a density which is constant over one circular orbit, when in fact the density may vary by a factor of five or more (depending upon the altitude) due to the diurnal bulge. For this reason, many investigators have worked to develop a model which directly accounts for the dynamic variations in the atmospheric density.
2.4.2 Dynamic Atmospheric Density Models

The independent variables in a dynamic density model are the spatial variables and time. Several types of time dependent density fluctuations are recognized in these models. They can be classified as follows:

1. The daily, or diurnal, variation.
2. Variation with the 11-year solar cycle.
3. Variation with the daily changes in solar disk activity.
4. Variation with geomagnetic activity.
5. Seasonal-latitude variations.
6. The semiannual variation.
7. Rapid density fluctuations (gravity waves, etc).

These density variations are mainly induced by the heating of the atmosphere due to the sun's electromagnetic radiation. The first variation, the diurnal bulge, accounts for the expansion of the atmosphere due to this solar heating. The atmosphere on the sunlit side of the earth may bulge as high as 100 km at an altitude of 500 km representing a ratio \( \rho_{\text{max}}/\rho_{\text{min}} \) of five or more. The bulge, however, is not at the solar sub-point. It lags behind at an angle of 30°-40°, so that the maximum density at a given altitude occurs at approximately 2:20 PM. Typical of the expressions for the diurnal bulge is that from Jacchia's 1964 atmospheric density model:

\[
\rho = \rho_s(h)[1 + .19(e^{0.0055h} - 1.9)\cos^6(\phi/2)]
\] (2.5)
where

\[ \rho_s(h) = \text{static density profile}, \]

\[ \phi = \text{bulge angle (angular distance between bulge maximum and evaluation point)}, \]

and

\[ h = \text{altitude in kilometers}. \]

The second and third variations listed above account for the amount of the sun's energy incident on the atmosphere. The 10.7 cm solar flux index \( (F) \) is used as the measure of this solar activity along with its value averaged over several solar rotations \( (F_{11}) \). The instantaneous value \( F \) ranges between \( 70 \times 10^{-22} \) and \( 220 \times 10^{-22} \) watts/m\(^2\) Hz, and has a 27 day period (which coincides with the rotation period of the sun). The averaged value \( \bar{F} \) has an eleven year period (which coincides with the sunspot cycle period). The solar activity level can cause density variations of up to a factor of 20 at an altitude of 600 km; at 200-300 km, the factor can range between two and five.

The geomagnetic variation is due to the interaction of the solar wind with the earth's magnetic field. This produces a latitude-dependent increase in temperature which then affects the density.

The seasonal-latitudinal variation shows that density profiles are not symmetrical with respect to the equatorial
plane due to seasonal changes in the declination of the sun. The effect due to this variation is small relative to those already mentioned.

The semi-annual variation is important in that it extends to altitudes as low as 150 km. As an example of its magnitude, King-Hele and Walker\textsuperscript{12} report the following ratios at an altitude of 185 km:

\[
\frac{\rho_{\text{max}}}{\rho_{\text{min}}} = 1.43 \quad \text{(March/July)}
\]
\[
= 1.32 \quad \text{(October/July)}
\]
\[
= 1.22 \quad \text{(March/January)}
\]
\[
= 1.22 \quad \text{(October/January)}
\]

The seventh classification of variations, the rapid density fluctuation, is unpredictable and will not be considered in this review.

Each of the above variations results in space-dependent and/or time-dependent asymmetries in atmospheric density. These asymmetries can affect estimates of satellite lifetimes tremendously. Estimates may be affected by up to a factor of three due to the diurnal bulge alone. Therefore, in order to precisely predict the motion of a satellite in the atmosphere, it is necessary to take into account these dynamic variations of density.
2.5 Summary of Recommendations

(1) Use $2.2$ as the value of the drag coefficient $C_D$. Numerical refinement (based upon satellite observations) should be an attractive possibility, especially with an accurate density model.

(2) Use the values for atmospheric rotation given by King-Hele et al.\textsuperscript{9} in the computation of the satellite relative velocity $V_{R}$. Above the altitudes for which $\omega_A$ is known, use $\omega_A = \omega_E$.

(3) If possible, use an instantaneously valid reference area $A_{\text{ref}}$. The use of an average value should be used cautiously, especially under such conditions as highly eccentric orbits.

(4) Use a dynamic, rather than a static, model for the density. The Jacchia 1977 atmospheric model\textsuperscript{5} is singled out by the majority of investigators as the most accurate.

The remainder of this thesis is organized into three parts:

1. A brief review of the Jacchia 1977 density model, including an abbreviated example of a density calculation, followed by a short discussion concerning analytic representations of that model.

2. A detailed numerical study quantifying the differences, in both density and its effect on
satellite motion, between a particular analytic representation of the Jacchia 1977 density model and a popularly used static density model.

3. General conclusions based upon the results of the numerical studies.
Chapter 3

Characteristics of the Jacchia-Bass Atmospheric Density Model

3.1 Jacchia 1977 Atmospheric Density Model

The Jacchia 1977 atmospheric density model was developed from the analysis of approximately 30,000 densities derived from observations of ten different satellites. The model consists of two parts: 1) the basic static models, which give temperature and density profiles for the relevant atmospheric constituents for any specified exospheric temperature, and 2) a set of formulae to compute the exospheric temperature and the expected deviations from the static models as a result of all the recognized types of thermospheric variations.

The static model assumes that the atmosphere is composed only of nitrogen, oxygen, argon, helium, and hydrogen, in a condition of mixing up to 100 km and in diffusion above this height. The number densities up to 100 km are obtained by integration of the barometric equation. The densities at 100 km are taken as boundary values in the integration of the diffusion equation, which is used to compute number densities for heights above 100 km. The variations of the number densities of the various atmospheric constituents and of the total density with temperature and height are illustrated in Figures 3.1 and 3.2.

The various dynamic density variations are represented by empirical equations that have been devised from observational
Figure 3.1. Number Densities of Individual Atmospheric Constituents as a Function of Height for Three Representative Exospheric Temperatures [taken from Reference 5].
Figure 3.2. Total Density as a Function of Exospheric Temperature for Various Heights [taken from Reference 5].
data. These equations are summarized in Appendix A. A good way to illustrate the use of these equations is to consider the following abbreviated example taken directly from Reference 5.

Consider the problem of determining the density at the following point and time:

Longitude: 45° West of Greenwich (315° E)
Latitude: \( \phi = 40° \) N
Height: \( z = 320 \) km
Time: \( 140^h_0^m, \) May 4, 1974

For that instant, Eqs. (A1-A8) along with the static model tables give the pseudo-temperatures \( (\Theta_i) \) and corresponding number densities \( (n_i) \) listed in Table 3.1 for each of the atmospheric constituents. The deviations from these number densities, due to the thermospheric variations, are computed from Eqs. (A9-A28). The density variations are summarized in Table 3.1. In this table:

\( \Delta_G \) = the change in number density due to the geomagnetic variation,

\( \Delta_{SL} \) = the change in number density due to the seasonal-latitudinal variations,

\( \Delta_{sa} \) = the change in number density due to the semiannual variation.

The total density is then obtained from:

\[
\rho = \frac{\sum M_i n_i}{A}
\]
Table 3.1. Pseudo-Temperature, Number Densities, and Changes in the Number Densities for Each of the Atmospheric Constituents.

<table>
<thead>
<tr>
<th>Constituent</th>
<th>$\theta_i$</th>
<th>$\log(n_i)_0$</th>
<th>$\Delta \log n_i$</th>
<th>$\Delta L \log n_i$</th>
<th>$\Delta S \log n_i$</th>
<th>$\Delta a \log n_i$</th>
<th>Final $\log n_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N$_2$</td>
<td>952.6</td>
<td>13.670</td>
<td>.330</td>
<td>0</td>
<td>.037</td>
<td></td>
<td>14.037</td>
</tr>
<tr>
<td>O$_2$</td>
<td>950.8</td>
<td>12.224</td>
<td>.445</td>
<td>0</td>
<td>.037</td>
<td></td>
<td>12.716</td>
</tr>
<tr>
<td>O</td>
<td>963.9</td>
<td>14.587</td>
<td>-.180</td>
<td>-.070</td>
<td>.037</td>
<td></td>
<td>14.374</td>
</tr>
<tr>
<td>Ar</td>
<td>948.2</td>
<td>9.765</td>
<td>.703</td>
<td>0</td>
<td>.037</td>
<td></td>
<td>10.505</td>
</tr>
<tr>
<td>He</td>
<td>996.8</td>
<td>12.719</td>
<td>-.410</td>
<td>-.346</td>
<td>.037</td>
<td></td>
<td>12.000</td>
</tr>
<tr>
<td>H</td>
<td>939.3</td>
<td>11.265</td>
<td>-.098</td>
<td>-</td>
<td>.037</td>
<td></td>
<td>11.204</td>
</tr>
</tbody>
</table>
where

\[
A = \text{Avogadro's number}
\]

\[
M_i = \text{molecular weight of the } i\text{th constituent}
\]

\[
n_i = \text{the number density of the } i\text{th constituent}
\]

The final value of \( \rho \) in this example is

\[
\rho = 1.164 \times 10^{-11} \text{ kg/m}^3.
\]

### 3.2 Jacchia-Bass Analytic Representation of the Jacchia 1977 Density Model

A fully analytic solution to the diffusion equation is not possible for the temperature profiles specified in the Jacchia 1977 density model. Hence numerical results must be tabulated, requiring extensive computer storage. For this reason, other temperature profiles which allow for analytic integration of the diffusion equation have been explored\(^\text{13,14}\). Bass has shown that analytically solvable temperature profiles can be adjusted to realistically represent temperature vs height variations above 125 km, allowing one to reduce storage requirements and avoid the inconvenience of interpolation.

In addition to investigating analytic temperature profiles, Bass\(^\text{15}\) has shown that it is necessary to tabulate the density for only one of the constituents at the reference altitude as a function of the exospheric temperature. From this the densities of the other constituents are derived, thus reducing storage requirements further.
Chapter 4

Numerical Comparisons Between Static and Dynamic Models

The numerical studies contained within this chapter support recommendation (4) of Section 2.5 that a dynamic density model can be utilized to obtain a more accurate prediction of near-earth satellite motion. Section 4.1 contains a brief description of two density models selected for comparison. In Section 4.2, the density predictions of the two models are contrasted. Finally, in Section 4.3, the most critical issue is addressed: How do the two density models compare on the basis of predicted satellite motion?

4.1 Density Model Descriptions

The density models which will be compared in this study are:

1. The model currently utilized by the Naval Surface Weapons Center (NSWC) in orbit integration routines (referred to as the Celest model), and

2. The Jacchia-Bass model described in Section 3.2.

The Celest atmospheric density model is a static model defined by

\[ \rho = \exp[Ah - B - (Ch^2 + Dh - E)^{1/2}] \]  

(4.1)

where \( h \) is the satellite's geocentric altitude above its sub-point (\( h > 125 \) km) and \( A, B, C, D, \) and \( E \) are constants. For the purposes of this comparison these constants have the following values:
The reader should note that the densities obtained from Eq. 4.1 are non-dimensional. A density constant with appropriate units will be discussed in Section 4.3.

The Jacchia-Bass density model algorithm was obtained through NSWC. Modifications to this were made in order to reduce storage requirements and computation time. For this reason, density values are only available for altitudes above 125 km, and the hydrogen constituent only contributes to the total density above altitudes of 500 km. A FORTRAN program encompassing the Jacchia-Bass formulation is presented in Appendix B. The following is a list of input values used for the density model. Unless otherwise stated, these input values remain constant throughout the comparisons.

- Modified Julian Date = 42128.0
- Right Ascension of Sun = 0.9065°
- Declination of Sun = 0.3932°
- Obliquity of Ecliptic = 23.4425°
- Earth-Sun Distance = 1 A.U.
- $F_{10.7} = 114.0 \times 10^{-22} \text{ W m}^{-2} \text{ Hz}^{-1}$
- $\tilde{F}_{10.7} = 84.14 \times 10^{-22} \text{ W m}^{-2} \text{ Hz}^{-1}$
- Geomagnetic Index = 1.0
The first five input values correspond to the Julian Date and the sun's position for 12:00 noon on March 22, 1974 as given in The American Ephemeris and Nautical Almanac\textsuperscript{17}. The final three input values were chosen arbitrarily and are not the actual values for this date.

For reasons of continuity, the reference frames which are pertinent to the comparisons to be made are now discussed:

(i) The geocentric-equatorial coordinate system has its origin at the earth's center. The fundamental plane is the equatorial plane and the x-axis points in the vernal equinox direction. The z-axis points in the direction of the north pole and the y-axis is directed so as to form a right-handed coordinate system.

(ii) The right ascension-declination system also has its origin at the center of the earth. The right ascension, $\alpha$, is measured eastward along the equatorial plane from the vernal equinox direction. The declination, $\delta$, is measured northward from the equator to the line-of-sight.

(iii) The tangential-radial coordinate system has its origin on the satellite which is moving in a Jacchia-Bass density. The unit vector, $e_t$, points along the velocity vector while the unit vector, $e_r$, points away from the geocenter.
These coordinate systems are illustrated in Figure 4.1. Reference to this figure may be helpful in visualizing the results of the comparisons presented in Sections 4.2 and 4.3.

4.2 Density Model Comparisons

Density profiles at constant altitudes along the equator have been generated using the Jacchia-Bass model, by setting the satellite declination to 0°, and varying the satellite right ascension from 0 to 360°. Figure 4.2 illustrates that the overall shape of the total density profile comes from the diurnal component, and that the diurnal bulge occurs behind the sub-solar point. Figure 4.3 indicates that the shape of the density profiles remains the same for various altitudes, but the magnitude of the densities decreases as the altitude increases.

The magnitude of the atmospheric density exhibits variations other than diurnal. The variations in density due to variations in solar activity are also significant. Figure 4.4 shows the effect of an increase in the solar activity index from the values, $F_{10.7} = 114.0$ and $F_{10.7} = 84.14$, to the values $F_{10.7} = F_{10.7} = 213.4$ (all in units of Wm$^{-2}$Hz$^{-1}$). Dowd and Tapley illustrated this same effect and also the effect of density profile variations due to changes in the geomagnetic activity index.

In contrast to the density profiles generated by the Jacchia-Bass density model, the density profiles predicted by the Celest model are constant. This illustrates the inability
Figure 4.1. Geocentric-Equatorial and Right Ascension-Declination Coordinate Systems
Figure 4.2. Total Density and Diurnal Component of Density as a Function of Satellite Right Ascension.

altitude = 200 km
Figure 4.3. Total Density as a Function of Satellite Right Ascension for Various Altitudes.
Figure 4.4. Total Density as a Function of Satellite Right Ascension for Increased Solar Activity.
of a static density model to accurately represent the recognized
dynamic variations in the atmospheric density.

4.3 Satellite Motion Comparisons

A comparison of the atmospheric density models alone is not
enough to appreciate the differences between the models. In
this section, the interaction of two systems, the atmosphere and
the orbiting satellite, will be considered.

A simulation program was created which generates satellite
positions as a function of time by integrating (via a fifth and
sixth order Runge Kutta routine) the differential equations
which govern the motion of the satellite. These equations of
motion in rectangular coordinates \((x,y,z)\) are:

\[
\ddot{x} + \mu \frac{x}{r^3} = P_x \tag{4.2a}
\]

\[
\ddot{y} + \mu \frac{y}{r^3} = P_y \tag{4.2b}
\]

\[
\ddot{z} + \mu \frac{z}{r^3} = P_z \tag{4.2c}
\]

where \(r^2 = x^2 + y^2 + z^2\)

\(\mu = \) gravitational constant, and

\((P_x, P_y, P_z) = \) total perturbing accelerations.

In this study the only perturbations considered are those
associated with the atmospheric drag, given by Eq. 2.1, and
those associated with corrections to the basic spherical
potential field, given by

\[
\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \text{ and } \frac{\partial V}{\partial z}, \text{ where } V \text{ is}
\]

assumed to be 19
\[ V = \left( J_2 \mu R^2 \right) \frac{1}{2r^3} (3 \frac{r^2}{r^2} - 1) . \]  

(4.3)

The earth model constants used in all programs are:

- Gravitational Constant \( \mu = 398600.8 \text{ km}^3/\text{s}^2 \)
- Earth Radius \( R = 6378.135 \text{ km} \)
- Second Zonal Harmonic Coefficient \( J_2 = 0.0010826157 \)

The assumed values of the dispersion parameters associated with atmospheric drag (other than density) are as follows:

- Drag Coefficient \( C_D = 2.2 \)
- Reference Area \( A_{\text{ref}} = 1.26 \text{ m}^2 \)
- Satellite Mass \( m = 625 \text{ Kg} \)

Relative Velocity \( \frac{V_r}{V} = 1 \)

where \( V \) is the velocity of the satellite relative to the geocentric non-rotating coordinate system.

In the simulations, two sets of satellite positions as a function of time are generated. One of these sets corresponds to the integration of Eqs. (4.2a–c) using the Celest density model. The other set corresponds to the integration of Eqs. (4.2a–c) using the Jacchia-Bass density model. Assuming the positions generated by the Jacchia-Bass integration to be correct and those generated by the Celest integration to be in error, the components of the error vector are calculated in the tangential-radial coordinate system. The error vector is given by

\[ \mathbf{r}_e = \mathbf{r}_C - \mathbf{r}_{JB} \]

(4.4)
where

\[ \mathbf{r}_C = \text{position vector of satellite using the Celest density model and} \]

\[ \mathbf{r}_{JB} = \text{position vector of satellite using the Jacchia-Bass density model.} \]

The components of \( \mathbf{r}_e \) are referred to as along track error (ET) which points in the \( \mathbf{e}_t \) unit vector direction, and the altitude error (ER) which points in the \( \mathbf{e}_r \) unit vector direction. With this coordinate system, a positive along track error corresponds to the Celest satellite being ahead of the Jacchia-Bass satellite, and a positive altitude error corresponds to the Celest satellite being above the Jacchia-Bass satellite. Plots of these error vector components as a function of time are presented in Section 4.3 for various satellite orbits.

For all orbits generated, the initial position of the satellite is on the x-axis of the geocentric coordinate system. The initial velocities are calculated to give the desired orbital shape and inclination. These calculations are done assuming a \( J_2 \) gravity field and no drag. The final time for integrations corresponds to the time required for four vacuum orbit periods. In order to reduce integration time, types of orbits are limited to equatorial and polar. Also, for the case of elliptical orbits, perigee is located on the positive x-axis. The various satellite orbits investigated are summarized in Table 4.1.
### Table 4.1. Summary of Orbit Test Cases

<table>
<thead>
<tr>
<th>Case #</th>
<th>Perigee (km)</th>
<th>Eccentricity</th>
<th>Inclination (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150.0</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>200.0</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>500.0</td>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>150.0</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>150.0</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>200.0</td>
<td>0.0</td>
<td>90</td>
</tr>
</tbody>
</table>
4.3.1 **Averaged Density Constant**

The ratios of the Jacchia-Bass density (averaged over 18 values of right ascension along the equator) to the Celest non-dimensional density are listed in Table 4.2 for various altitudes. From this table, it is evident that the Celest non-dimensional density values are much smaller than the density values predicted by the Jacchia-Bass model. For this reason the Celest model is modified as shown:

\[
p = p_0 \exp[Ah - B - (Ch^2 + Dh - E)^k],
\]

where \( p_0 \) is a density constant which must be determined for each orbital comparison. It should be noted that significant figure requirements in all constants, such as initial conditions, gravitational constants, and density values (such as those appearing in Table 4.2) depend upon the final time and the accuracy desired in the final integrated values of position and velocity.

For cases 1, 2, and 3 the values of \( p_0 \) are ratios given in Table 4.2 corresponding to the altitudes of the respective orbits. Plots of error components as a function of time for the cases are presented in Figures 4.5 - 4.7. The following points concerning these plots should be emphasized:

(i) the magnitudes of the error components decrease for satellites in higher orbits,
Table 4.2. Jacchia-Bass Density Values (Averaged Over 18 Values of Right Ascension Along the Equator) and Ratios of the Jacchia-bass Density to the Celest Non-Dimensional Density for Various Altitudes.

<table>
<thead>
<tr>
<th>ALTITUDE (KM)</th>
<th>AVERAGE JB DENSITY (KG/KM³)</th>
<th>P(JB)/P(CELEST)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150.0</td>
<td>0.21083669D+01</td>
<td>0.1118lBl1i1D Ol1</td>
</tr>
<tr>
<td>160.0</td>
<td>0.12542469D+01</td>
<td>0.14284050D 04</td>
</tr>
<tr>
<td>170.0</td>
<td>0.79797529D+00</td>
<td>0.14193915D 04</td>
</tr>
<tr>
<td>180.0</td>
<td>0.53099589D+00</td>
<td>0.14295148D 04</td>
</tr>
<tr>
<td>190.0</td>
<td>0.36499643D+00</td>
<td>0.111480082D 04</td>
</tr>
<tr>
<td>200.0</td>
<td>0.25728082D+00</td>
<td>0.111698273D 04</td>
</tr>
<tr>
<td>210.0</td>
<td>0.18510698D+00</td>
<td>0.111193915D 04</td>
</tr>
<tr>
<td>220.0</td>
<td>0.13549684D+00</td>
<td>0.11193915D 04</td>
</tr>
<tr>
<td>230.0</td>
<td>0.10066464D+00</td>
<td>0.11193915D 04</td>
</tr>
<tr>
<td>240.0</td>
<td>0.75759555D-01</td>
<td>0.15546242D 04</td>
</tr>
<tr>
<td>250.0</td>
<td>0.57667472D-01</td>
<td>0.15711840D 04</td>
</tr>
<tr>
<td>260.0</td>
<td>0.44338817D-01</td>
<td>0.15857825D 04</td>
</tr>
<tr>
<td>270.0</td>
<td>0.34395768D-01</td>
<td>0.15972237D 04</td>
</tr>
<tr>
<td>280.0</td>
<td>0.26894674D-01</td>
<td>0.16055486D 04</td>
</tr>
<tr>
<td>290.0</td>
<td>0.21178442D-01</td>
<td>0.16105360D 04</td>
</tr>
<tr>
<td>300.0</td>
<td>0.16782620D-01</td>
<td>0.16120065D 04</td>
</tr>
<tr>
<td>310.0</td>
<td>0.13734090D-01</td>
<td>0.16098273D 04</td>
</tr>
<tr>
<td>320.0</td>
<td>0.10712339D-01</td>
<td>0.16039155D 04</td>
</tr>
<tr>
<td>330.0</td>
<td>0.86191809D-02</td>
<td>0.15942431D 04</td>
</tr>
<tr>
<td>340.0</td>
<td>0.69634434D-02</td>
<td>0.15808344D 04</td>
</tr>
<tr>
<td>350.0</td>
<td>0.56466099D-02</td>
<td>0.15637676D 04</td>
</tr>
<tr>
<td>360.0</td>
<td>0.45941879D-02</td>
<td>0.15431709D 04</td>
</tr>
<tr>
<td>370.0</td>
<td>0.37493717D-02</td>
<td>0.15192189D 04</td>
</tr>
<tr>
<td>380.0</td>
<td>0.30685025D-02</td>
<td>0.14921268D 04</td>
</tr>
<tr>
<td>390.0</td>
<td>0.25177820D-02</td>
<td>0.14621445D 04</td>
</tr>
<tr>
<td>400.0</td>
<td>0.20708747D-02</td>
<td>0.14295513D 04</td>
</tr>
<tr>
<td>450.0</td>
<td>0.12043699D-03</td>
<td>0.12383085D 04</td>
</tr>
<tr>
<td>500.0</td>
<td>0.37276345D-03</td>
<td>0.10283239D 04</td>
</tr>
<tr>
<td>550.0</td>
<td>0.16040837D-03</td>
<td>0.83642586D 03</td>
</tr>
<tr>
<td>600.0</td>
<td>0.63868625D-04</td>
<td>0.67908077D 03</td>
</tr>
<tr>
<td>650.0</td>
<td>0.31431667D-04</td>
<td>0.56628357D 03</td>
</tr>
<tr>
<td>700.0</td>
<td>0.17038618D-04</td>
<td>0.49729850D 03</td>
</tr>
<tr>
<td>750.0</td>
<td>0.10255328D-04</td>
<td>0.46619883D 03</td>
</tr>
<tr>
<td>800.0</td>
<td>0.67986458D-05</td>
<td>0.46493275D 03</td>
</tr>
<tr>
<td>850.0</td>
<td>0.48654856D-05</td>
<td>0.48527414D 03</td>
</tr>
<tr>
<td>900.0</td>
<td>0.36745848D-05</td>
<td>0.51985995D 03</td>
</tr>
<tr>
<td>950.0</td>
<td>0.28748449D-05</td>
<td>0.56258246D 03</td>
</tr>
<tr>
<td>1000.0</td>
<td>0.23006337D-05</td>
<td>0.60868767D 03</td>
</tr>
</tbody>
</table>
Figure 4.5. Along Track Error (ET) and Altitude Error (ER) vs Time for Case 1.

\[
\rho_0 = 1480.3471 \text{ kg/km}^3
\]
Figure 4.6. Along Track Error (ER) and Altitude Error (ER) vs Time for Case 2.
Figure 4.7. Along Track Error (ET) and Altitude Error (ER) vs Time for Case 3.

\[ \rho_0 = 1028.8392 \text{ kg/km}^3 \]
(ii) the shape of the error components as a function of time repeats itself over each orbital period, and

(iii) the maximum values of the error components grow as time goes on.

The orbits of the satellites in Cases 4 and 5 are elliptical, hence the satellite altitudes range from the perigee height to the apogee height. In Case 4 the apogee height is 260 km. Therefore, the value of $p_0$ is obtained by averaging the ratios given in Table 4.2 corresponding to the altitudes 150 km (perigee) and 260 km (apogee). The error component plots generated for this case, as illustrated in Figure 4.8, demonstrate similar behavior as those for the previous circular orbit cases.

For Case 5 the apogee height is approximately 400 km. Table 4.2 shows that the ratios of Jacchia-Bass density to Celest density do not increase with altitude. For this reason, averaging the ratios corresponding to apogee and perigee altitudes does not yield a value for $p_0$ which is representative of densities encountered. Instead, an average of the maximum and minimum ratios between altitudes 150 and 400 km is used as the value of $p_0$.

The error component plots generated for this case, Figure 4.9, show less accumulated error than Case 4. This is due to the fact that the satellites are at higher altitudes. These plots also show that the Celest satellite "falls behind" the
$\rho_0 = 1533.0648 \text{ kg/km}^3$

Figure 4.8. Along Track Error (ET) and Altitude Error (ER) vs Time for Case 4.
\[ \rho_0 = 1546.1768 \text{ kg/km}^3 \]

Figure 4.9. Along Track Error (ET) and Altitude Error (ER) vs Time for Case 5.
Jacchia-Bass satellite, and then "catches up" after approximately 2 1/2 orbits. This type of behavior is not obviously anticipated.

The orbit of the satellite in Case 6 is polar. Although the orbit is also circular, the altitude of the satellite is not constant due to the oblateness of the earth. The value of $\rho_0$ for this case is obtained as follows:

1. the Jacchia-Bass density is averaged over an orbit of constant radius (The altitude of the orbit varies from 200 km at the equator to approximately 222 km at the poles).

2. The Celest density is calculated at an average altitude of approximately 211 km.

3. The ratio of the Jacchia-Bass density to the Celest density is then computed.

The error component plots are presented in Figure 4.10.

4.3.2 Corrected Density Constant

In the previous section the Celest model was modified by Eq. 4.5. The density constant, $\rho_0$, was determined by an averaging process so that the effect of the different density models could be observed. This method of determining $\rho_0$ may not yield values which are best for predicting positions at a given time, or for all time. In this section a least square differential correction (LSDC) process is utilized to determine the value of
Figure 4.10. Along Track Error (ET) and Altitude Error (ER) vs Time for Case 6.

\[ \rho_0 = 1538.1251 \text{ kg/km}^3 \]
which best fits the Jacchia-Bass satellite positions generated. Weighted and unweighted versions of the LSDC process are used to determine the value of $\rho_o$ which will minimize the error of the Celest integration. This gives the Celest model its best chance for accurate prediction of satellite motion.

Cases 1 and 4 are repeated here using values of $\rho_o$ obtained by fitting the Celest model to 31 satellite positions predicted by the Jacchia-Bass model. These 31 observed positions were taken at equally spaced time intervals spread over the last three orbits of the Jacchia-Bass integration. Plots of the error components as a function of time are presented in Figures 4.11 - 4.14.

Figures 4.11 and 4.12, corresponding to the satellite orbit of Case 1, indicate that the value chosen for $\rho_o$ can greatly effect the positions predicted by the Celest model. Compared to Figure 4.5, these figures show much less accumulated error after the same amount of time. However, the difference between the dynamic density model and the static density model is still evident in the behavior of the error components. Figures 4.13 and 4.14 correspond to the satellite orbit of Case 4. These figures should be compared to Figure 4.8.

The plots in Figures 4.12 and 4.14 were generated using the value of $\rho_o$ obtained from a weighted LSDC process. In this process the observed position at the final time is weighted 10,000 times more than the positions at all other times. These plots indicate that the Celest model can be used to predict
Figure 4.11. Along Track Error (ET) and Altitude Error (ER) vs Time for Case 1 Using Corrected, Unweighted Density Constant.

\[ \rho_0 = 1484.0912 \text{ kg/km}^3 \]
Figure 4.12. Along Track Error (ET) and Altitude Error (ER) vs Time for Case 1 Using Corrected, Weighted Density Constant.

\[ \rho_0 = 1494.5692 \text{ kg/km}^3 \]
\[ \rho_0 = 1491.5484 \text{ kg/km}^3 \]

Figure 4.13. Along Track Error (ET) and Altitude Error (ER) vs Time for Case 4 Using Corrected, Unweighted Density Constant.
\( \rho_0 = 1491.4256 \text{ kg/km}^3 \)

Figure 4.14. Along Track Error (ET) and Altitude Error (ER) vs Time for Case 4 Using Corrected, Weighted Density Constant.
satellite positions at a specified time with a comparatively fair degree of accuracy. However, for times other than the time specified, the Celest positions predicted are still in error.

A least square differential correction process to determine the values of the Celest model constants A, B, C, D, and E that best fit the Jacchia-Bass positions was attempted. Problems arose that rendered this effort undesirable. The form of the Celest model, as given by Eq. 4.5, is such that the matrix of partial derivatives needed in the fitting process is singular. Correcting this problem would entail changing the form of the Celest model. Since this was not a suitable alternative, the effort was abandoned.
Chapter 5
Summary and Conclusions

A numerical comparison of atmospheric density models has been presented which quantifies the differences between a popularly used static model and a recommended dynamic model. Density profiles were plotted to illustrate the changes that can occur in atmospheric density due to the different dynamic variations. The effect of the different density models on the orbital motion of near-earth satellites was evaluated by comparing the satellite positions predicted by each model for various satellite orbits.

For the particular solar and geomagnetic conditions, and satellite orbits presented here, it has been shown that the use of a static density model can result in predicted satellite positions which can be quite different from those predicted using a dynamic density model. From the results of all the simulated satellite orbits, it was found that the effect of the different density models is most severe for satellites in lower orbits. Improvement of the static model (via a parameter fit) resulted in smaller accumulated error; however, significant differences in predicted positions remained. Further improvement of the static model (via a weighted parameter fit) illustrated the ability to force the position predicted by this model close to the observed position at a specified time, but the error in position at other times increased.
In conclusion, based on the comparisons presented in this thesis, it is evident that a static atmospheric density model is unable to accurately predict low-altitude satellite positions as a function of time. In order to obtain more accurate predictions of satellite positions, an atmospheric model which accounts for the recognized dynamic variations in density should be utilized. In this thesis a modification of the Jacchia 1977 density model was used. It should be noted that although this model is regarded as the most accurate, it may not be the most desirable in some applications due to the relatively high execution time requirements.
REFERENCES


Appendix A

Appendix A summarizes equations used in the Jacchia 1977 atmospheric density model. For further details, see Reference 5.
Solar Activity

\[ T_{1/2} = 5.48 F^{0.08} + 101.8 F^{0.4} \]  
(A1)

F to be taken at time \( t - \Delta t \), where

\[ \Delta t = 1^d.26 + 0^d.37 \sin(H - 92^\circ) \]  
(A2)

\[ \overline{F} = \frac{\sum wF}{\sum w} \]  
(A3)

\[ w = \exp \left[-\left(\frac{t-t_0}{\tau}\right)^2\right], \quad (\tau = 71 \text{ days}) \]  
(A4)

Diurnal Variation

\[ \frac{\delta}{T_{1/2}} = 1 + 0.15 \frac{\delta}{\varepsilon} \sin \phi + 0.24 \cos \phi [f_1(H) - \frac{1}{2}] \]  
(A5)

\[ f_1(H) = \cos^n \frac{1}{2} (H + \beta_1) + 0.08 \cos [3(H + \beta_1) - 75^\circ] \]  
(A6)

\[ n = 2 + \cos^2 \left(\frac{\phi}{90^\circ}\right) \]  
(A7)

\[ \beta_1 = 35^\circ + 27 \left(\frac{\theta}{\tau}\right) \]  
(A8)

(for actual temperature, \( \beta_T = -60^\circ \))

Geomagnetic Activity

\[ \Delta G \log n_i = \Delta_T \log n_i + \Delta_H \log n_i + \Delta_e \log n_i \]  
(A9)

\[ \Delta G \to = A \sin^4 \phi_i \]  
(A10)

\[ A = 57^\circ.5 K_p \left[1 + 0.027 \exp(0.4 K'_p)\right] \]  
(A11)

\[ K_p' = K_p \text{ at time } t - \tau, \text{ where} \]  
(A12)

\[ \tau = 0^d.1 + 0^d.2 \cos^2 \phi_i \]

\[ \Delta_G T(z) = \Delta_G T_\infty \tanh[0.006(z - 90)] \]  
(A13)

\[ \Delta_H \log n_i = a_i \Delta z_H \]  
(A14)

\[ \Delta z_H = 5.0 \times 10^3 \sinh^{-1} (0.010 \Delta_G T_\infty), \text{ (meters)} \]  
(A15)

\[ a(Ar) = +3.07 \times 10^{-5}, \quad a(O_2) = +1.03 \times 10^{-5}(?) \quad a(N_2) = 0, \]  
(A16)

\[ a(0) = -4.85 \times 10^{-5}, \quad a(He) = -6.30 \times 10^{-5} \text{ (mks)} \]  
(A17)

\[ \Delta_e \log n_i = 5.2 \times 10^{-4} A \cos^4 \phi_i \]  
(A17)
Seasonal-Latitude Variations

a) Thermospheric:

\[ \Delta_{SL} \log n_i = c_i \frac{\theta}{\varepsilon} \sin \phi \quad (A18) \]

Values of \( c_i \): \( c(N_2) = 0 \), \( c(O) = -0.16 \), \( c(He) = -0.79 \), \( c(Ar) = 000 \quad (A19) \)

b) "Mesospheric":

\[ \Delta_{SL} \log \rho = |\frac{\phi}{\phi}| S \rho \sin^2 \phi \quad (A20) \]

\[ S = 0.014(z - 91) \exp \left[ -0.0013(z - 91)^2 \right], \quad \text{(z in km)} \quad (A21) \]

\[ P = \sin(2\pi \frac{t - t_0}{365} + 1.72), \quad (t \text{ in days}, \; t_0 = \text{Jan.1}) \quad (A22) \]

\[ \Delta_{SL} T = -2.9P(z - 102.5)\exp(-7.8 \times 10^{-5} |z - 102.5|^{2.7}) \quad (A23) \]

Semiannual Variation

J71 model:

\[ \Delta_{sa} \log \rho = f(z) \; g(t) \quad (A24) \]

\[ f(z) = [0.04\left(\frac{z}{100}\right)^2 + 0.05] \exp(-0.25 \frac{z}{100}), \quad (z \text{ in km}) \quad (A25) \]

\[ g(t) = 0.0284 + 0.382 \left[ 1 + 0.467 \sin(2\pi T + 4.14) \right] \sin(4\pi T + 4.26) \quad (A26) \]

\[ \tau = \phi + 0.0954 \left[ \frac{1}{2} + \frac{1}{2} \sin(2\pi \phi + 6.04) \right]^{1.65} \frac{1}{2} \quad (A27) \]

\[ \phi = (t - \text{Jan. 1})/365 \quad (A28) \]
Appendix B

Appendix B contains the FORTRAN programming used to generate Jacchia-Bass satellite positions, Celest satellite positions, and error vector components.
THIS PROGRAM GENERATES SATELLITE POSITIONS AS A FUNCTION OF TIME
BY INTEGRATING A SET OF FIRST ORDER DIFFERENTIAL EQUATIONS OF
MOTION WHICH GOVERN THE SATELLITE.

THERE ARE TWO INTEGRATIONS HEREIN. ONE INTEGRATION UTILIZES
THE JACCHIA-BASS DENSITY MODEL FORMULATION TO OBTAIN THE
DENSITY VALUES NECESSARY FOR THE DRAG PERTURBATION TERM. THE
OTHER INTEGRATION USES THE CELEST STATIC DENSITY MODEL TO
OBTAIN THE DENSITY VALUES.

ASSUMING THE \((J\beta)\) POSITIONS TO BE CORRECT AND THE CELEST
POSITIONS TO BE IN ERROR, THE ERROR VECTOR COMPONENTS ARE
CALCULATED.

IMPLICIT REAL*8(A-H,O-Z)
REAL*8 MU,J2
INTEGER N,IND,IND1,NW,IER,IER1,K,LAST,STSZ
DIMENSION Y(4),C(24),W(4,9),S(4),SC(24),SW(4,9)
EXTERNAL FCN1
EXTERNAL FCN2
DIMENSION SAT(2),JOUT(8)
DIMENSION DENS(5),DDNS(5)
COMMON /CONGEO/ CGE07(2),CGE010(2),CGE03(2),CGE05
COMMON /JSDMPR/ ALZSTP(8),MTITLE(8),AMWMD(7),ALDZ(7)
COMMON /MODRLN/ GNMOD,RMODE,RSTMD
COMMON /AVOGMD/ AVOGMD
COMMON /JUMP/ NTSTEP,NSTEP,MTITLE(8),NREG,NSUM
COMMON /DIURN/ CDGAMA(7),CDH(2),CD3,CD1
COMMON /PGE/ CDGAMA,CD3,CD1
COMMON /PS/ SMFGM,DTGKP2
COMMON /PSEM/ CSEM(15)
COMMON /PSL/ CSLAT(7),SLFUDG(7)
COMMON /PTHLF/ CTHALF(10)
COMMON /SU/ SM,CDR,PAS,MU,J2,RAD,CA,CB,CE
COMMON /MATCN/ RTOD,DTOR,PI,TWOP1
COMMON /MATC/ RPI,FOURPI,HALFP1,P1OR3,P1OR4
COMMON /VARNAM/ NSPNAM,NTYPNM,NVARNM,NTOTNM
COMMON /JB/ TX,GX,SIG,SIGPR,CH1,AL1,AL2,C2,EMAL1,EMAL2,CNST,TLJWB,
TLOC,TLOCC,CHI,EMAL11,EMAL22,TLJWB
COMMON /INIT/ AMJD,SUN(4),GE0(3)
MODJC FORTRAN A1 06/29/82 12:27 CLONAD F 80 1232 RECS VA TECH

X = 0.00
T = 0.00
NW = 4
N = 4
TOL = 1.0D-4
IND = 1
IND1 = 1

C... PHYSICAL PARAMETERS AND CONSTANTS.
C...
SM = 625.00
CDR = 2.200
PAS = 1.260-6
RAD = 6378.13500
MU = 398600.800
J2 = 0.001082615700
CA = 0.013620
CB = -0.08335
CC = 0.0001018
CD = 1.083
CE = 89.39

C... THE VARIABLES IN THE FOLLOWING READ STATEMENT ARE:
C...
1. INITIAL VALUES TO THE FIRST ORDER DIFFERENTIAL EQUATIONS
C... WHICH MODEL THE FORCES ON A SATELLITE IN ORBIT.
C...
2. THE TIME IN MODIFIED JULIAN DAYS.
C...
3. SUN CONSTANTS.
C...
4. GEOMAGNETIC CONSTANTS.
C...
5. THE FINAL TIME FOR A CIRCULAR ORBIT INTEGRATION.
C...
6. THE STEPSIZE FOR THE INTEGRATION OF THE DIFFERENTIAL EQUATIONS.
C...

READ(5,100) Y,AMJD,SUN,GEO,TFIN,STSZ
100 FORMAT(1X,2E15.7,/,1X,2E18.10,/,1X,E13.5,/,1X,4E15.7,/,1X,3E12.4,/
       1, 1X,E18.11,/, 1X,13)
WRITE(6,101) Y
101 FORMAT(/1X, 'INITIAL VALUES:',/18X, 'X',16X,'Y',16X,'XDOT ',12X,'YDO
       1T',//15X,4E17.9)
SUN(1) = SUN(1)*DTOR
SUN(2) = SUN(2)*DTOR
SUN(3) = SUN(3)*DTOR
DO 11 I=1,4
   S(I) = Y(I)
11 CONTINUE
LAST = IFIX(SNGL(TFIN/STSZ))*STSZ+STSZ
ITER = 0
DO 10 K=STSZ,LAST,STSZ
   ITER = ITER+1
   IF(DFLOAT(K).LT.TFIN) GO TO 111
   XEND = TF
   TEND = TF
   GO TO 112
111 CONTINUE
   XEND = DFLOAT(K)
   TEND = DFLOAT(K)
112 CONTINUE
DVERK IS A RUNGE-KUTTA FIFTH AND SIXTH ORDER DIFF. EQ. SOLVER.

CALL DVERK(N, FCN1, X, Y, XEND, TOL, IND, C, NW, W, IER)
CALL DVERK(N, FCN2, T, S, TEND, TOL, IND1, SC, NW, SW, IER1)

IF(K.EQ.STSZ) WRITE(6,102) IND, IER, IND1, IER1
102 FORMAT(/1X, 'IND= ',15,4X, 'IER= ',15,4X, 'IND= ',15,4X, 'IER1= ',15)

CALCULATE THE ERROR VECTOR COMPONENTS.

DELX = S(1)-Y(1)
DELY = S(2)-Y(2)
VJ = DSQRT(Y(3)*Y(3)+Y(4)*Y(4))
RJ = DSQRT(Y(1)*Y(1)+Y(2)*Y(2))
ET = 1000.*((S(1)-Y(1))*Y(3)+(S(2)-Y(2))*Y(4))/VJ
ER = 1000.*((S(1)-Y(1))*Y(1)+(S(2)-Y(2))*Y(2))/RJ

IF(K.EQ.STSZ) WRITE(6,200) X,Y(1),Y(2),S(1),S(2),ET,ER,DELX,DELY
200 FORMAT(//2X, TIME' ,4X, 'X (KM)',9X, 'Y (KM)',9X 'XCEL (KM)',6X 'YCEL
1 (KM)',/4X, 'ET (M)',9X, 'ER (M)',9X, 'DELX (KM)',8X, 'DELY (KM) '/,1X,
2F7.1,4E15.8,/,2X,E15.8,/,2X,E15.8,2X,2E15.8)
IF(ITER.NE.5 .AND. X.NE.TFIN) GO TO 10
IF(K.GT.STSZ) WRITE(6,201) X,Y(1),Y(2),S(1),S(2),ET,ER,DELX,DELY
201 FORMAT(1X,F7.1,4E15.8,/,1X,2E15.8,2X,2E15.8)
ITER = 0
10 CONTINUE
STOP

SUBROUTINE FCN1(N,X,Y,YPRIME)

THE JACCHIA-BASS DENSITY MODEL FOR THE DRAG TERM.

SM = SATELLITE MASS
CD = DRAG COEFFICIENT
PAS = PROJECTED AREA OF SATELLITE (KM2)
ROW = DENSITY (KG/KM3)
J2 >
OBCON > CONSTANTS USED IN OBTAINING GRAVITATIONAL
OBLAT > FORCE DUE TO AN OBLATE EARTH.
MU = GRAVITATIONAL CONSTANT

IMPLICIT REAL*8(A-H,O-Z)

REAL*8 MU, J2
DIMENSION SAT(2),Y(N),YPRIME(N)
COMMON /INITAL/ AMJD, SUN(4), GEO(3)
COMMON/PARAM/ SM, CDR, PAS, MU, J2, RAD, CA, CB, CC, CD, CE
R1 = DSQRT(Y(1)*Y(1)+Y(2)*Y(2))
R3 = R1**3
R5 = R1**5
V = DSQRT(Y(3)*Y(3)+Y(4)*Y(4))
OBLAT = (1.5*J2*MU*RAD*RAD)/R5
OBCON = 5.*Y(2)*Y(2)/(R1*R1)
OBCON = 0.0

CONVERSION OF THE INPUT VALUES FROM A GEOCENTRIC-EQUATORIAL
COORDINATE SYSTEM TO A RIGHT ASCENSION-DECLINATION COORDINATE SYSTEM.

```
SAT(1) = DATAN2(Y(2),Y(1))
CALL CONVRT(Y,SAT,Z)
Z = R1-6378.135
GLONGL = SAT(1)-X*(6.2831853/86400. )-5.0789081
GMAGLT = DARSIN(.9792*(Y(2)/R1)+.2028*(Y(1)/R1)*DCOS(GLONGL))
CALL ATMDEN(AMJD,SUN,SAT,GMAGLT,Z,GE0,CMDPNS,DRHODH)
ROW = CMDPNS*1.D9
DTERM = (CDR*ROW*PAS/(2.*SM))*V
YPRIIME(1) = Y(3)
YPRIIME(2) = Y(4)
YPRIIME(3) = (-MU/R3)*Y(1)-OBLAT*Y(1)*(1.-0BCON)-DTERM*Y(3)
YPRIIME(4) = (-MU/R3)*Y(2)-OBLAT*Y(2)*(1.-0BCON)-DTERM*Y(4)
RETURN
END
SUBROUTINE FCN2(N,T,S,SPRIME)
```

SUBROUTINE FCN2 EVALUATES THE DIFF. EQS. FOR THE SYSTEM, USING THE CELEST DENSITY MODEL FOR THE DRAG TERM.

C... SM = SATELLITE MASS
C... CDR = DRAG COEFFICIENT
C... PAS = PROJECTED AREA OF SATELLITE
C... DENS = DENSITY
C... MU = GRAVITATIONAL CONSTANT
C... J2 >
C... OBCON > CONSTANTS USED IN OBTAINING GRAVITATIONAL FORCE DUE TO AN OBLATE EARTH.
C... RAD >
C...

```
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 MU,J2
INTEGER N
DIMENSION S(N),SPRIME(N)
COMMON/PARAM/ SM, CDR, PAS, MU, J2, RAD, CA, CB, CC, CD, CE
R1 = DSQRT(S(1)*S(l)+S(2)*S(2))
R3 = R1**3
R5 = R1**5
V = DSQRT(S(3)*S(3)+S(4)*S(4))
OBLAT = (1.5*J2*MU*RAD*RAD)/R5
OBCON = 5.*S(2)**2/(R1*R1)
OBCON = 0.0
H = R1-6378.135
CNEG = CC*H*H+CD*H-CE
IF(CNEG.LT.0.) WRITE(6,123) CNEG
CSQRT = DSQRT(CC*H*H+CD*H-CE)
ALNDN = (CA*H-CB-CSQRT)
DENS = DEXP(ALNDN)**1491.5484
DTERM = (CDR*DENS*PAS/(2.*SM))*V
YPRIIME(1) = S(3)
YPRIIME(2) = S(4)
YPRIIME(3) = (-MU/R3)*S(1)-OBLAT*S(1)*(1.-OBCON)-DTERM*S(3)
```
**SUBROUTINE CONVRT**

CONVERTS SATELLITE POSITIONS FROM A GEOCENTRIC COORDINATE SYSTEM TO A RIGHT ASCENSION-DECLINATION COORDINATE SYSTEM FOR INCLINED ORBITS.

**IMPLICIT REAL*8(A-H,O-Z)**

**DIMENSION** Y(4), SAT(2)

**R1 = DSQRT(Y(1)**2**+Y(2)**2])**

**SAT(1) = DATAN2(Y(2),Y(1))**

**AE = 6378.135**

**ECC = 0.08181**

**SIN2 = (Y(2)/R1)**2**2**

**R = AE/(1.+ECC***(1.-ECC**SIN2))**

**Z = R1-R**

**RETURN**

**END**

***CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/***

DATA CGE01 /0.100/
DATA CGE02 /0.200/
DATA CGE03 /57.500/, 46.000/
DATA CGE05 /0.02700/
DATA CGE06 /0.400/
DATA CGE07 /4.000/, 3.000/
DATA CGE09 /5.20-4/
DATA CGE10 /4.000/, 3.000/
DATA CGE12 /5.03/
DATA CGE13 /5.03/
DATA CGE14 /800.00/
DATA CGE15 /1.700/
DATA CGE16 /0.00500/
DATA CGE17 /100.00/
DATA CGE18 /0.005400/
DATA CGE19 /0.2100/
DATA CGE20 /0.000200/
DATA CGE021 /130.00/
DATA CGE022 /-.717D0/
DATA CGE023 /0.003D0/
DATA ALF02 /1.03D-5/
DATA ALFO /-4.85D-5/
DATA ALFN2 /0.0D/
DATA ALFN /-3.70D-5/
DATA ALFHE /-6.30D-5/
DATA ALFAR /3.07D-5/
DATA ALF /0.0D/
DATA BETAO2 /1.1600/
DATA BETAO /0.5200/
DATA BETAN2 /1.0000/
DATA BETAN /0.4600/
DATA BETAH /0.1000/
DATA BETAA /1.4900/
DATA JAC77

COMMON /JSDMPR/ ALZSTP(8), MTITLE(8), AMWMOD(7), ALDZ(7),
$ NZSTEP(7), MODRLN(7), GNMOD, RNMOD, RSTAMD,
$ AVOGMD, DTMOD, TMINMD, TMAXMD
$ NTSTEP, NSPROM, LABEL, NREG, NSUM
DATA TMINMD, TMAXMD, NTSTEP, DTMOD, NREG, 475.00, 1975.00, 25.00, 60.00, 4/
DATA GNMOD, RNMOD, RSTAMD, AVOGMD, NSPROM, AMWMOD, JAC77

1 .98066D0-02, 6356.766D0, .0831432D0, .602257D+27, 7,
2 31.9988D0, 15.9994D0, 28.0134D0, 14.0067D0, 4.0026D0, 39.948D0,
3 1.60797D0/
DATA ALZSTP/
1 4.6051701859881,5.298317366548,6.6846117276679,8.0063675676503,
DATA NZSTEP/30, 20, 10, 4, 1, 1, 1/
DATA ALDZ/
1 .23104906018665D-01, .69314718055993D-01, .13217558399824,
2 .478779996228147, .0000000000000, .0000000000000, .0000000000000/
COMMON /PD1URN/ CDGAMA(7), CD4(2), CD3, CD1,
$ CD6, CDBETO, CDBET1, CDBETH, CD2
REAL*8 CDIURN(16)
EQUIVALENCE (CDIURN(1), CD1)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
DATA CD1 /0.15D0/
DATA CD2 /0.24D0/
DATA CD3 /-1.047198D0/
C = -60. DEG (nominal diurnal phase shift)
DATA CD4 /0.08D0, 0.04D0/
DATA CD6 /-1.308997D0/
C = -75. DEG
DATA CDBETO /-0.610865D0/
C = -35. DEG
DATA CDBET1 /-0.471239D0/
C = -27. DEG
DATA CDBETH /-1.047198D0/
C = -60. DEG
DATA CDGAMA(1) /1.00/
DATA CDGAMA(2) /1.00/
DATA CDGAMA(3) /1.00/
DATA CDGAMA(4) /1.00/
DATA CDGAMA(5) /1.0D0/
DATA CDGAMA(6) /1.0D0/
DATA CDGAMA(7) /1.0D0/
COMMON /PGEM/ SMFGM, DTGKP2
REAL*8 CGEOM(2)
EQUIVALENCE (CGEOM(1), SMFGM)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
DATA SMFGM /2.0D0/
C KP SMOOTHING FACTOR
DATA DTGKP2 /56.0D0/
C ADOPTED ROUND VALUE FOR KP = 2
COMMON /PSEM/ CSEM(15)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
DATA CSEM(1) /0.02835D0/
DATA CSEM(2) /0.3817D0/
DATA CSEM(3) /0.4671D0/
DATA CSEM(4) /4.137D0/
DATA CSEM(5) /4.259D0/
DATA CSEM(6) /0.09544D0/
DATA CSEM(7) /6.035D0/
DATA CSEM(8) /1.650D0/
DATA CSEM(9) /36204.0D0/
DATA CSEM(10) /4.0D-6/
DATA CSEM(11) /0.050D0/
DATA CSEM(12) /-2.5D-3/
COMMON /PSLT/ CSLAT(7), SLFUDG(7)
DATA CSLAT /$ 0.0D0, $ -0.160D0, $ 0.0D0, $ -0.220D0, $ -0.790D0, 
$ 0.0D0, $ 0.0D0/
DATA SLFUDG /7*0.0D0/
COMMON /PTHLF/ CTHALF(10)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
DATA CTHALF(1) /0.0D0/
DATA CTHALF(2) /5.480D0/
DATA CTHALF(3) /0.80D0/
DATA CTHALF(4) /101.8D0/
DATA CTHALF(5) /0.4D0/
COMMON /SUNCOE/ SUNCOF(20)
REAL*8 TROPYR
EQUIVALENCE (SUNCOF(20), TROPYR)
DATA SUNCOF(1) /15019.5D0/
C EPOCH (1900 JAN 0.5)
DATA SUNCOF(2) /281.22083D0/
DATA SUNCOF(3) /4.70684D-5/
DATA SUNCOF(4) /3.39D-13/
DATA SUNCOF(5) /7.0D-20/
C MEAN LONGITUDE OF PERIGEE, DEGREES,
C MEAN EQUINOX OF DATE
C (FOR THIRD DEGREE POLYNOMIAL)
DATA SUNCOF(6) /358.47584500/
DATA SUNCOF(7) /0.985600267000/
DATA SUNCOF(8) /-1.12D-13/
DATA SUNCOF(9) /-7.0D-20/
C         MEAN ANOMALY, DEGREES
C (FOR THIRD DEGREE POLYNOMIAL)
DATA SUNCOF(10) /0.0167510400/
DATA SUNCOF(11) /-1.1444D-9/
DATA SUNCOF(12)/-9.4D-15/
DATA SUNCOF(13) /-1.03D-20/
C         ECCENTRICITY
C (FOR SECOND DEGREE POLYNOMIAL)
DATA SUNCOF(17)/33281.923000/
DATA SUNCOF(18)/-6.11907D-7/
DATA SUNCOF(19)/-3.5626D-7/
DATA SUNCOF(20)/-1.23D-15/
DATA SUNCOF(21)/1.03D-20/
C         MEAN OBLIQUITY, DEGREES
C (FOR THIRD DEGREE POLYNOMIAL)
DATA SUNCOF(17)/23.45229400/
DATA SUNCOF(18)/9.92413600/
DATA SUNCOF(19)/-23.45229400/
DATA SUNCOF(20)/-9.92413600/
COMMON /MATCN/, RTOD, DTOR, PI, TWOP1, RPI, RTWOP1, RHLFPI, BASENT, ALGE10
DATA RTOD /57.29577951308200/
DATA DTOR /0.01745329251994300/
DATA PI /3.141592653589800/
DATA TWOP1 /6.283185307179600/
DATA TREPI /9.424777960769300/
DATA FOURP1 /12.56637061435900/
DATA HALFP1 /1.570796326794900/
DATA PI0VR3 /1.047197551196600/
DATA PI0VR4 /.7853981633974500/
DATA RPI /.3183098861837900/
DATA RTWOP1 /.159154943091900/
DATA RHLFPI /.636617723675800/
DATA BASENT /2.71828182845900/
DATA ALGE10 /2.302585092994000/
COMMON /VARNAM/, NSPNAM, NTYPNM, NVARNM, NTOTNM, NINVVR, NINVTP
$ *** IF CHANGES ARE MADE TO THE CONTENTS OF /VARNAM/ THEN
C *** THE /DATUM/ COMMON IN THE CRUNCH OVERLAY SHOULD BE ADJUSTED
C *** TO REFLECT THE CHANGES
C *** NSPNAM = NUMBER OF SPECIES (+ DENSITY) IN MODEL
DATA NSPNAM /8/
C *** NTYPNM = NUMBER OF DATA TYPES FOR THE SPECIES
DATA NTYPNM /6/
C *** NVARNM = TOTAL NUMBER OF MODEL VARIABLES THAT MAY BE REQUESTED
DATA NVARNM /19/
C *** NTOTNM = NSPNAM+NTYPNM+NVARNM
DATA NTOTNM /33/
**Subroutine ATMDEN(A1) SUN, SAT, GMAGL, Z, GEO, CMPDNS, DRHODH)**

**IMPLICIT REAL*8(A-H,O-Z)**

**INTEGER HF AM PL**

**REAL*8 SUN(4),SAT(2),GE0(3),CMPLGN(7),DLOGGM(7),TEMP(2)**

**DIMENSION SATD(2),JOUT(8),DHN(2)**

**COMPUTES MODEL DENSITIES AND TEMPERATURES**

**ARGUMENTS:**

**-INPUT-**

A1 TIME IN MODIFIED JULIAN DAYS

SUN(1) RIGHT ASCENSION OF SUN

(2) DECLINATION OF SUN

(3) OBLIQUITY OF SUN

(4) DISTANCE TO SUN (A.U.)

SAT(1) RIGHT ASCENSION OF SATELLITE

(2) DECLINATION OF SATELLITE

GMAGL GEOMAGNETIC LATITUDE

**NOTES - ALL ANGLES IN RADIANS.**

IF GEO(1) AND GEO(2) (SEE BELOW) ARE AT 1 A.U. FROM THE SUN, THEN SUN(4) MUST BE ACTUAL SUN-EARTH DISTANCE.

IF THEY ARE AT THE EARTH'S SURFACE, THEN SUN(4) MUST BE 1.0

Z HEIGHT OF SATELLITE (KM)

GEO(1) DAILY VALUE OF 10.7 CM FLUX

(2) SMOOTHED VALUE OF 10.7 CM FLUX

(3) KP - THE GEOMAGNETIC INDEX

JOUT COMPUTE ITH DENSITY IF AND ONLY IF JOUT(1) = 1

1. MASS

2. O2

3. O

4. N2

5. N

6. HE

7. AR

8. H

**-OUTPUT-**

CMPDNS COMPUTED TOTAL DENSITY (KG/M3)

DRHODH PARTIAL DERIVATIVE OF ATMOSPHERIC DENSITY WITH RESPECT TO SATELLITE ALTITUDE (KG/M4)

**VERSION 1.2 LEVEL 770509**

**COMMON /CONGEO/ CGEO02, CGEO01, CGEO09, CGEO02, CGEO12, CGEO13, CGEO14, CGEO15, CGEO16, CGEO17, CGEO18, CGEO19, CGEO20, CGEO21, CGEO22, ALFHE, ALFAR, ALFH, BETAO, BETAO
$ BETAN2, BETAN, BETAHE, BETAAR, BETAH
REAL*8 ALFGE0(7), BETGE0(7)
EQUIVALENCE (ALFGE0(1),ALF02), (BETGE0(1),BETA02)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
C
COMMON /JSDMPR/ ALZSTP(8), MTITLE(8), AMWMD(7), ALDZ(7),
$ NZSTEP(7), MODRLN(7), GNMOD, RNMOD, RSTAMD,
$ AVOGMD, DTMOD, TMINMD, TMAXMD
$ NTSTEP, NSPMOD, LABEL, NREG, NSUM
C *** PARAMETERS USED IN GENERATING AND INTERPOLATING THE TABULAR
C *** MODEL
C VARIABLE USE
--------------------
LABEL LABEL I.D. FOR THIS VERSION OF THE MODEL
MTITLE TITLE OF MODEL
TMXMD LOWEST EXOSPHERIC TEMPERATURE IN MODEL
TMAXD HIGHEST EXOSPHERIC TEMPERATURE IN MODEL
NTSTEP NUMBER OF TEMPERATURE STEPS BETWEEN MIN AND MAX
DTMOD ((TMAXD-TMINMD)/NTSTEP
NREG NUMBER OF REGIONS OF HEIGHT
NSUM (NOT USED)
ALZSTP NATURAL LOG OF THE BOUNDARIES OF THE REGIONS
NZSTEP NUMBER OF INTEGRATION STEPS FOR EACH REGION
ALDZ CHANGE IN LN(Z) PER INTEGRATION STEP FOR EACH
REGION
GNMOD GRAVITATIONAL ACCELERATION AT EARTH'S SURFACE
RNMOD RADIUS OF EARTH
RSTAMD UNIVERSAL GAS CONSTANT
AVOGMD AVOGADRO'S NUMBER
NSPMOD NUMBER OF SPECIES IN MODEL (7)
AMWMD MOLECULAR WEIGHS OF SPECIES
MODRLN LENGTH OF EACH RECORD ON THE TABULAR MODEL FILE
COMMON /PDIURN/ CDGAMA(7), CD4(2), CD3, CD2, CD1,
$ CD6, CD3ETO, CD3ET1, CD3ETH, CD2
REAL*8 CDIURN(16)
EQUIVALENCE (CDIURN(1),CD1)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
COMMON /PGEM/ SMFGM,
$ DGKP2
REAL*8 CGEOM(2)
EQUIVALENCE (CGEOM(1),SMFGM)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
COMMON /PSEM/ CSEMl(15)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
COMMON /PSLT/ CSLAT(7), SLFUDG(7)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
COMMON /PTHLF/ CTHALF(10)
C *** CHANGES IN THIS COMMON BLOCK SHOULD BE REFLECTED IN /PRNAME/
COMMON /SUNCOE/ SUNCOF(20)
REAL*8 TROPYR
EQUIVALENCE (SUNCOF(20),TROPYR)
COMMON /MATCH/ RTOD, DTOR, PI, TWOPI,
$ TREEPI, FOURPI, HALFP, PIOVR3, PIOVR4,
$ RPI, RTWOP1, RHLFI, BASENT, ALGEO10
COMMON /VARNAM/ NSPNAM, NTYPNM, NVARNM, NTOTNM,
S
REAL*8 LOGNR(7),DLOGSL(7),DLOGSA ,DLGNO(7)
REAL*8 LOGN(7),DLG(7),DGMDH(7)
REAL*8 INVLAT, KP
REAL*8 N
REAL*8 NUMDEN, NUMTOT
DATA JOUT/1,7*0/,HFAMPL/1/

ASINH(X) = DLOG(X+DSQRT(X**2+1.))

*** PUT INPUT VARIABLES INTO LOCAL VARIABLES:

ALFSUN = SUN(1)
DELSUN = SUN(2)
OBLIQ = SUN(3)
RSUN = SUN(4)
ALFSAT = SAT(1)
DELSAT = SAT(2)
CSDLST = DCOS(DELSAT)
INVLAT = GMAGLT
ZSAT = DABS(Z)

IF(ZSAT.LE.125.0)CALL FATAL(24HZSAT BELOW TABULAR MODEL)
IF(ZSAT.GE.20000.0) CALL FATAL(24HZSAT ABOVE TABULAR MODEL)

FDAILY = GEO(1)
FBAR = GEO(2)
KP = GEO(3)

****************************************************************** ***

*** DIURNAL VARIATION ***

ARGUMENTS

-INPUT-

FDAILY = DAILY VALUE OF 10.7 CM FLUX
FBAR = SMOOTHED VALUE OF 10.7 CM FLUX
RSUN = RADIUS VECTOR TO SUN (A.U.)
ALFSUN = RIGHT ASCENSION OF SUN
DELSUN = DECLINATION OF SUN
OBLIQ = OBLIQUITY OF SUN
ALFSAT = RIGHT ASCENSION OF SATELLITE
DELSAT = DECLINATION OF SATELLITE
CSDLST = COSINE OF SATELLITE DECLINATION
ZSAT = HEIGHT OF SATELLITE

-OUTPUT-

THALF: MEAN GLOBAL EXOSPHERIC TEMPERATURE
AMBAR: MEAN VALUE OF MEAN MOLECULAR WEIGHT
H: LONGITUDINAL ANGLE BETWEEN SATELLITE AND SUN
N: EXPONENT FOR DIURNAL LONGITUDINAL TERM
SLFAC: SEASONAL-LATITUDINAL FACTOR
LOGNR: NUMBER DENSITIES FOR DIURNAL VARIATION

****************************************************************** ***

FIRST COMPUTE MEAN GLOBAL EXOSPHERIC TEMPERATURE: T(#)

FBARO = FBAR/RSUN**2
FDALYO = FDAILY/RSUN**2
TH = CTHALF(1) + CTHALF(2)*FBARO**CTHALF(3)
THALF = TH + CTHALF(4)*FDALYO**CTHALF(5)
THAFBR = TH + CTHALF(4)*FBARO**CTHALF(5)

NOW GET MEAN VALUES FOR TOTAL DENSITY AND MEAN MOLECULAR WEIGHT
C *** USING SMOOTHED VALUE OF T(#) AND ASSUMING KP=2
CALL INTPMD(THAFBR+DTGP2,ZSAT,LOGN,NSPNAM,DLG)
NUMTOT = 0.
WTTOT = 0.
DO 220 IS=1,NSPMOD
NUMDEN = 10.**LOGN(IS)
NUMTOT = NUMTOT + NUMDEN
220 WTTOT = WTTOT + NUMDEN*AMWMOD(IS)
AMBAR = WTTOT/NUMTOT
SLFAC = DELSUN*DSIN(DELSAT)/OBLIQ
H = ALFSAT-ALFSUN
N = 2. + DCOS(RHLFP1*DELSAT**2)**2
C *** DIURNAL VARIATION FOR O2, O, N2, N, HE, AR
DO 240 IS=1,6
IF(JOUT(1).NE.1.AND.JOUT(IS+1).NE.1) GO TO 240
TAU = H + CDBETO+CDBET1*(1.-AMBAR/AMWMOD(IS))*CDGAMA(IS)
TINF = THALF*DIURF(TAU,SLFAC,CSDLST,N)
CALL INTPRT(TINF,ZSAT,LOGNDR(IS),IS,DLGNO(IS))
240 CONTINUE
C *** DIURNAL VARIATION FOR HYDROGEN:
IF(JOUT(8).NE.1.AND.JOUT(1).NE.1) GO TO 260
TAU = H + CDBETH
TINF = THALF*DIURF(TAU,SLFAC,CSDLST,N) + DTGP2
CALL INTPRT(TINF,ZSAT,LOGNDR(7),7,DLGNO(7))
260 CONTINUE
C END OF DIURNAL CALCULATIONS

C***********************************************************************
C *** SEASONAL-LATITUDINAL VARIATION ***
C ARGUMENTS FOR THIS SECTION:
C -INPUT- AMJD TIME IN MODIFIED JULIAN DAYS
C ZSAT HEIGHT OF SATELLITE (KM)
C -OUTPUT- DLOGSL CHANGE IN DENSITIES DUE TO S.L. VARIATION
C***********************************************************************
TIMLES = AMJD-CSEMl(9)
YRFRAC = DMOD(TIMLES,TROPYR)/TROPYR

C***********************************************************************
C *** SEMIANNUAL VARIATION ***
C ARGUMENTS FOR THIS SECTION:
C -INPUT- AMJD TIME IN MODIFIED JULIAN DAYS
C ZSAT HEIGHT OF SATELLITE (KM)
C -OUTPUT- DLOGSA CHANGE IN LOGARITHM OF NUMBER DENSITY (TO BE
C APPLIED TO ALL SPECIES)
C***********************************************************************
STERM = (0.5*(1.+DSIN(TWOP!YRFRAC+CSEMI(7))))**CSEMI(8) - 0.5
TAU = YRFRAC + CSEMI(6)*STERM
A1 = (1.+CSEMI(3)*DSIN(TWOP!TAU+CSEMI(4))) *$
S = DSIN(FOURPI*TAU+CSEMI(5))
TDEP = CSEMI(1) + CSEMI(2)*A1
ZDEP = (CSEMI(10)*ZSAT**2+CSEMI(11)) * DEXP(CSEMI(12)*ZSAT)
DLOGSA = TDEP-ZDEP
DSADH = ((2.*CSEMI(10)*ZSAT+CSEMI(12)*(CSEMI(10)*ZSAT*ZSAT
1+CSEMI(11)))*DEXP(CSEMI(12)*ZSAT))*TDEP

END OF SEMIANNUAL CALCULATIONS

C***********************************************************************
C*** GEOMAGNETIC EFFECT ***
C***********************************************************************
C ARGUMENTS FOR THIS SECTION:
C -INPUT-
C CSOLST COSINE OF SATELLITE DECLINATION
C H LONGITUDINAL ANGLE BETWEEN SATELLITE AND SUN
C GLAT GEOMAGNETIC LATITUDE OF SATELLITE
C ZSAT HEIGHT OF SATELLITE
C KP GEOMAGNETIC INDEX
C SLFAC SEASONAL-LATITUDINAL FACTOR
C THALF T(#): MEAN GLOBAL EXOSPHERIC TEMPERATURE
C N EXPONENT FOR DIURNAL LONGITUDINAL TERM
C -OUTPUT-
C DLOGGM CHANGES IN NUMBER DENSITIES OF EACH SPECIES DUE TO GEOMAGNETIC EFFECT
C
C***********************************************************************
C*** GET EXOSPHERIC TEMPERATURE FOR DIURNAL PHASE ANGLE = -60 DEG
TOGEOM = THALF * DIURF(H+CD3, SLFAC, CSOLST, N)
AGEOM = CGE03(HFAMPL)*KP*(1.+CGE05*DEXP(CGE06*KP))
DGMAG = AGEOM*DABS(DSIN( INVLAT))**CGE07(HFAMPL)
C FOR ALL HEIGHTS GET CHANGE IN DENSITIES DUE TO TEMPERATURE
C CHANGE BY USING THE PARAMETERIZED RESULTS OF THE INTEGRATIONS
FGM = (CGE014/TOGEOM)**(CGE015*DTANH(CGE016*(ZSAT-CGE017))) *$
S (CGE018*ZSAT+CGE019/(1.+CGE020*(ZSAT-CGE021)**2)+CGE022) *$
$ ASINH(CGE023*DGMAG)
C FACTOR FOR DENSITY WAVE:
DELNS = CGE02*AGEOM*DCOS(INVLAT)**CGE010(HFAMPL)
C FACTOR FOR CHANGE IN HEIGHT OF HOMOPAUSE:
DELZH = CGE012*ASINH(CGE013*DGMAG)
C NOW COMBINE ALL THREE:
DO 580 IS=1,NSPMOD
DGMH(IS)=BETGEO(IS)*((CGE014/TOGEOM)**(CGE015*DTANH(CGE016*(ZSAT=1CGE017))))*(CGE015*CGE016*DSH(CGE014/TOGEOM)*((1./2DCDSH(CGE016*(ZSAT-CGE017))))**2.)*(CGE018*ZSAT+3CGE019/(1.+CGE020*(ZSAT-CGE021)**2)+CGE022)+4(CGE018-2.*CGE019*CGE020*(ZSAT-CGE021)/(1.+CGE020*(ZSAT=5CGE021)**2.))*ASINH(CGE023*DGMAG)
580 DLOGCM(IS) = BETGEO(IS)*FGM + DELDNS + ALFGEO(IS)*DELZH
C END OF GEOMAGNETIC CALCULATIONS
C
C *** NOW PUT IT ALL TOGETHER ***
C
ARGUMENTS FOR THIS SECTION:

* -INPUT-

- LOGNDR: NUMBER DENSITIES FROM DIURNAL VARIATION
- DLOGSL: CHANGE IN NUMBER DENSITIES OF EACH SPECIES DUE TO SEASONAL-LATITUDINAL VARIATION
- DLOGSA: CHANGE IN DENSITIES OF ALL SPECIES DUE TO SEMIANNUAL VARIATION
- DLOGGM: CHANGES IN NUMBER DENSITIES OF EACH SPECIES DUE TO GEOMAGNETIC EFFECT
- H: LONGITUDINAL ANGLE BETWEEN SATELLITE AND SUN
- CSLFAC: SEASONAL-LATITUDINAL FACTOR
- CSDLST: COSINE OF SATELLITE DECLINATION
- N: EXPONENT FOR DIURNAL LONGITUDINAL TERM
- THALF: T(#): MEAN GLOBAL EXOSPHERIC TEMPERATURE

* -OUTPUT-

- CMPLGN: LOGARITHMS OF NUMBER DENSITIES OF SPECIES
- CMPDNS: LOGARITHM OF TOTAL DENSITY

**********************************************************************

IF(ZSAT .GT. 530.) GO TO 609

DHN(1) = 0.0
DHN(2) = 0.0

609 CONTINUE
DNIDH = 0.0
DWTDH = 0.0
NUMTOT = 0.
WTTOT = 0.
WTTWOGM = 0.
DO 620 IS=1,7
CLNWM = LOGNDR(IS) + DLOGSL(IS) + DLOGSA
CMPLGN(IS) = CLNWM + DLOGGM(IS)
NUMDEN = 10.**CMPLGN(IS)
DN = 2.30258509*NUMDEN*(DLGNO(IS)+DSADH+DGMDH(IS))
DNIDH = DN+DNIDH
DWTDH = DWTDH+AMWMOD(IS)*DN
NUMTOT = NUMTOT + NUMDEN
WTTOT = WTTOT + NUMDEN*AMWMOD(IS)

620 CONTINUE
CMPDNS = WTTOT/AVOGMD
DRHODH = DWTDH/AVOGMD
RETURN
END

FUNCTION DIURF(TAU, SLFAC, CSDLST, N)
IMPLICIT REAL*8(A-H,O-Z)
INTEGER HFAMPL
REAL*8 N

**********************************************************************

COMPUTES THE RATIO TINF/T(#): EXOSPHERIC TEMPERATURE OVER MEAN GLOBAL EXOSPHERIC TEMPERATURE

ARGUMENTS:

* -INPUT-

TAU: LONGITUDINAL ANGLE BETWEEN SATELLITE AND SUN
MINUS AN ANGLE THAT VARIES FOR EACH SPECIES
SLFAC = H + BETA(IS)  

SLFAC = SEASONAL-LATITUDINAL FACTOR  

CSDLST = COSINE OF SATELLITE DECLINATION  

N EXPONENT IN LONGITUDINAL TERM OF DIURNAL VAR.  

VERSION 1.2  

LEVEL 761002  

CSOLST USED INSTEAD OF DELSAT  

******************************************************************

COMMON /PDIURN/ CDGAMA(7), CD4(2), CD3, CD1, CD6, CDBETO, CDBET1, CDBETH, CD2  

REAL*8 CDIURN(16)  

EQUIVALENCE (CDIURN(1),CD1)  

DATA HFAMPL /1/  

FOFH = (DABS(DCOS(0.5*TAU)))**N+CD4(HFAMPL)*DCOS(3.*TAU+CD6)  

DIURF = 1. + CD1*SLFAC + CD2*CSDLST*(FOFH-0.5)  

RETURN  

END  

SUBROUTINE INTPMD(TINF,ZSAT,RESULT, INDEX,DRSLT)  

IMPLICIT REAL*8(A-H,O-Z)  

REAL*8 RESULT(7),DRSLT(7)  

******************************************************************  

INTERPOLATES THE TABULAR STATIC DIFFUSION MODEL BELOW 125 KM  

USES THE JACCHIA-BASS FORMULATION ABOVE 125 KM  

ARGUMENTS  

-INPUT-  

TINF EXOSPHERIC TEMPERATURE FOR INTERPOLATION  

ZSAT ALTITUDE FOR INTERPOLATION  

INDEX SPECIFIES WHAT TO INTERPOLATE  

0 LOCAL TEMPERATURE  

1 O2 NUMBER DENSITY  

2 O NUMBER DENSITY  

3 N2 NUMBER DENSITY  

4 N NUMBER DENSITY  

5 HE NUMBER DENSITY  

6 A NUMBER DENSITY  

7 H NUMBER DENSITY  

8 (NSPNAM) ALL NUMBER DENSITIES  

-OUTPUT-  

RESULT INTERPOLATED RESULTS,  

EITHER NUMBER DENSITIES OR LOCAL TEMPERATURE  

******************************************************************  

COMMON /JSOMPR/ ALZSTP(8), MTITLE(8), AMMMD(7), ALDZ(7),  

NZSTEP(7), MODRLN(7), GNMOD, RNMOD, RSTAMD,  

AVGMD, DTMD, TMNMD, TMAXMD,
COMMON/MATCN/RTOD,DTOR,Pl,REST(11)
COMMON/VARNAM/NSPNAM,NTYPNM,NVARNM,NTOTNM

DIMENSION AN100(26),ANLB(26),FLB(26),DLOGLB(6),ALFMOD(7)

EQUIVALENCE(AN100(1),ANLB(1))

COMMON/JB/TX,GX,SIG,SIGPR,CHl,AL1,AL2,C2,EMAL1,EMAL2,CNST,TLJWB,
TLOC,TLOCC,CHIL,EMAL11,EMAL22,TLJWBB

DATA DLOGLB/-0.644979D0,-1.077811D0,0.0D0,-4.017491D0,-5.173171D0,
-1.922228D0/
DATA IRGLST/-1/
DATA TLAST/-10000.DO/
DATA ZLAST/-10000.DO/
DATA ALFMOD/4*0.D0,-0.38DO,O.OD0,-0.25DO/

C***** F AT 125 KM
DATA FLB/1.53531918D-01, 0.52271760D-01, 0.51226885D-01,
0.49586107D-01, 0.48925949D-01, 0.48344062D-01,
0.47360123D-01, 0.46938362D-01, 0.4653792D-01,
0.4587506D-01, 0.4557492D-01, 0.45293822D-01,
0.44786348D-01, 0.44555520D-01, 0.44337987D-01,
0.44132159D-01, 0.43407966D-01, 0.43247561D-01,
0.4247561D-01, 0.4094117D-01/

C***** N2 DENSITY AT 100 KM
DATA AN100/18.97441D0, 18.97387D0, 18.97339D0, 18.97296D0, 18.97258D0,
18.97224D0, 18.97192D0, 18.97163D0, 18.97136D0, 18.97111D0,
18.97087D0, 18.97066D0, 18.97045D0, 18.97026D0, 18.97007D0,
18.9699D0, 18.96973D0, 18.96957D0, 18.96942D0, 18.96927D0,
18.96913D0, 18.9690D0, 18.96887D0, 18.96875D0, 18.96863D0,
18.96851D0/

DATA AJWBFC/1.134435198/ IINT(X) = IFIX(SNGL(X-0.5D0))
IBOUND(LOW,X,INHIGH) = MAXO(LOW,MINO(IHIGH,IINT(X)))
AJWB(X,T) = AJWBFC*(DLOG10(T*EXP(SIG*X))-OLOG10( TX))/(SIG*TINF)
FALPHA(Q)=Q*(1.0-Q+.33333333*Q*Q)
GALPHA(Q)=Q*(1.0-.5*Q+0.11111111*Q*Q)
DFALOQ(Q) = 1.0-2.*Q+Q*Q
OGALQ2(Q) = 1.0-Q+.333333333333333*Q*Q
DQOH(X) = X*EXP((-X)*CHI )*CHI
DLOGO = 0.00
DRSLT(7) = 0.0
ICOUNT=1

IF(X=0.0045* (TINF-188. ) ) DLSOLS
ASINH=DLOG(X+DSQRT(1. + X**2)) DLSOLS
TX=188. + 110.5*ASINH DLSOLS
GX=1.9*(TX-188. )/35. DLSOL
TLB=TX-.9743252532*(TX-188. )
15 CALL TJB(TINF,ZSAT,DCHI,TLJWBB,DTLOC)
18 TLOG=DLOG10(TLOC/TLB)
TLOGG = DLOG10(TLOC/TLCC)
IF(ICOUNT.NE.0) GO TO 30 DLSOLS
RESULT(1)=TLOC DLSOLS
RETURN DLSOLS
```fortran
C 30 CONTINUE

IF(ZSAT.EQ.ZLAST) GO TO 20
ALZSAT=DLOG(ZSAT)
IF(ALZSAT.LT.ALZSTP(1)) CALL FATAL(24HZSAT BELOW TABULAR MODEL)
DO 25 IREG=1,NREG
IF(ALZSAT.LT.ALZSTP(IREG+1)) GO TO 20
25 CONTINUE
ANZSTP=(ALZSAT-ALZSTP(IREG))/ALDZ(IREG)
NZSTP1=0
IF(IREG.LE.1) GO TO 35
NND=IREG-1
DO 36
NZT=1,NND
NZSTP1=NZSTP1+NZSTEP(NZT)
36 CONTINUE
IZMOD=IBOUND(1,ANZSTP,NZSTEP(IREG)-2)
DELZ=ANZSTP-IZMOD
IZMOD=IZMOD+NZSTP1
ANZSTP=(TINF-TMINMD)/DTMOD
ITMOD=IBOUND(1,ANZSTP,NTSTEP-2)
DELT=ANZSTP-ITMOD
40 IF(TINF.EQ.TLAST) GO TO 45
IF(TINF.LT.TMINMD) CALL FATAL(24HTINF BELOW TABULAR MODEL)
IF(TINF.TMAXMD) CALL FATAL(24HTINF ABOVE TABULAR MODEL)
ANZSTP=(TINF-TMINMD)/DTMOD
ITMOD=IBOUND(1,ANZSTP,NTSTEP-2)
DELT=ANZSTP-ITMOD
45 IF(ICOUNT.NSPMOD) 50,70,70
50 CALL BESSEL(ANLB(ITMOD),1,1,-1.DO,DELT,BESLB,1)
C ABOVE LOWER BOUNDARY OF JACCHIA-BASS REGION
54 BSANAL=AJWB(CH1,TLJWB)+(CNST*(0.33333333-FALPHA(EMAL1))/AL1
 2 -C2*(CH1/AL2*FALPHA(EMAL2)+(GALPHA(EMAL2)-0.61111111)
 2 /(AL2*AL2)))*AJWBF/DLOG(10.DO)
 1/*(AL2*AL2)) Ajwbf/DLOG(10.DO)
DAJWB=(AJWBF/(SIG*TINF))*(.43429448/(TLJWB*DEXP(SIG*CH1))**DTL
 1/DTLJWB**SIG*DEXP(SIG*CH1)**DCHI)
DBSAN=DAJWB+((CNST/AL1)*DFALDQ(EMAL1)*DQDH(AL1)-C2*((DCH1/AL2)*
 1FALPHA(EMAL2)+(CH1/AL2)*DFALDQ(EMAL2)*DQDH(AL2)+(DGALDQ(EMAL2)*
 2DQDH(AL2)/(AL2*AL2)))*AJWBF/DLOG(10.DO)
CALL BESSEL(ANLB(ITMOD),1,1,-1.DO,DELT,BESLB,1)
BESONE=BSANAL+BESLB
55 DNDIZ=-AMWMOD(ICOUNT)*BESONE-(1.+ALFMOD(ICOUNT))**1
 1 TLOG
XLNZST=DNDIZ+BESLB+DLOGNO*(ICOUNT)
DLOGNO=-AMWMOD(ICOUNT)*DBSAN+(1.+ALFMOD(ICOUNT))*(.43429448*DTL0
C/TLOG)
IF(ICOUNT.EQ.1) XLNZST=XLNZST-0.07*(1+DTANH(0.18*(ZSAT-111.)))
JBFX1 IFCOUNT.NE.2) GO TO 60
IF(ZSAT.LE.200.) XLNZST=XLNZST-0.24*DEXP(-.009*(ZSAT-97.7)**2)
JBFX1 IF(ZSAT.LE.200) DLOGNO=DLOGNO+.00432*(ZSAT-97.7)*DEXP(-.009*(ZSAT-97.7)**2.)
60 CONTINUE
IF(INDEX.NE.NSPNAM) GO TO 80
RESULT(ICOUNT)=XLNZST
DRSLT(ICOUNT)=DLOGNO
ICOUNT=ICOUNT+1
JBFX1 IF(ICOUNT.LT.NSPMOD) GO TO 55
C
C ATOMIC H
```
COMMON /JSMPR/ ALZSTP(8) , MTITLE(8) , AMWMOD(7) , ALDZ(7),
  $ NZSTEP(7) , MODRLN(7) , GNMOD , RNMOD , RSTAMD ,
  S AVOGMD , DTMOD , TMNMD , TMAXMD ,
  S NTSTEP , NSPMOD , LABEL , NREG , NSUM,
 COMMON /VARNAM/ NSPNAM , NYPNM , NVARNM , NTOTNM , JAC77
 S INNVVR , INVTP
 DIMENSION AN100(26),ANLB(26),FLB(26),DLOGLB(6),ALFMOD(7)
 EQUIVALENCE (AN100(1),ANLB(1))
 COMMON/MATCN/RTOD,DTOR,PI,REST(11) CLEAN2
COMMON/JB/TX,GX,SIG,SIGPR,CHl,AL1,AL2,C2,EMAL1,EMAL2,CNST,TLJWB,
 1 TLOC,TLOC,CH1,EMAL11,EMAL22,TLJWBB
 DATA DLOGLB/-0.644979D0,-1.077811DD0,0.0D0,-4.017491D0,-5.173171D0,
 1 -1.922288D0/
 DATA TLAST /-10000.DO/     JAC77
 DATA ZLAST /-10000.DO/
 DATA ALFMOD/4*0.D0,-0.38DO,O.D0,-0.25DO/
 C***** F AT 125 KM
 DATA FLB/
 1 .5351918DO-01, .52271768DO-01, .51226885DO-01, .50344144DO-01,
 1 .49586107DO-01, .48925949DO-01, .48344062DO-01, .47825825DO-01,
 1 .47360123DO-01, .46938362DO-01, .46553792DO-01, .46201023DO-01,
 1 .45875706DO-01, .45574287DO-01, .45293822DO-01, .45031864DO-01,
 1 .44786348DO-01, .44555520DO-01, .44337887DO-01, .44132159DO-01,
 1 .43937127DO-01, .4375920D0-01, .43575920DO-01, .43407966DO-01,
 1 .43247561DO-01, .43094117DO-01/
 C***** N2 DENSITY AT 100 KM
 DATA AN100/
 1 18.971410DO, 18.97387DD0, 18.97339DD0, 18.9729DD0, 18.97258DD0,
 1 18.9722D0, 18.97192D0, 18.97163D0, 18.97136D0, 18.97111D0,
 1 18.97087D0, 18.97066D0, 18.97045D0, 18.97025D0, 18.97007D0,
 1 18.9699D0D0, 18.96973DD0, 18.96957DD0, 18.96942DD0, 18.96927D0,
 1 18.96913D0, 18.96900DD0, 18.96887DD0, 18.96875D0, 18.96863D0,
 1 18.96851DD0/
 DATA AJWBF(1.134435198)/
 1 INT(X) = IFIX(SNGL(X-0.5D0))
 IBOUND(LOW,X,HIGH) = MAXO(LOW,MINO(HIGH,INT(X))
 AJWB(X,T) = AJWBF((DLOG10(T*DEXP(SIG*X))-DLOG10(TX))/(SIG*TINF)
 FALPHA(Q)=Q*(1.0-Q+.3333333333*Q*Q)
 GALPHA(Q)=Q*(1.0-.5*Q+0.11111111Q*Q)
 DFALDQ(Q) = 1.0-2.*Q+Q*Q
 DGALDQ(Q) = 1.0-Q+.333333333333333*Q*Q
 DQDH(X) = -X*EXP((-X)*CHI)*DCHI
 DLOGNO = 0.00
 DRSLT(1) = 0.0
 ICONT=1
 IF(INDEX .NE. NSPNAM) ICOUNT=INDEX
 X=0.0045 *(TINF-188.)
 ASINH=DLOG(X + SQRT(1. + X**2))
 TX=188. + 110.5*ASINH
 GX=1.9*(TX-188.)/35.
 TLB=TX-.9743252532*(TX-188.)
 15 CALL TJB(TINF,ZSAT,CH1,DTLJWB,TLLOC)
 18 TLOG=DLOG10(TLOC/TLB)
 TLOGG = DLOG10(TLOC/TLOC)
 IF(ICOUNT.NE.0) GO TO 30
 RESULT(1)=TLOC
 RETURN
 30 CONTINUE
 C
 IF(ZSAT .EQ. ZLAST) GO TO 20
 ALZSAT=DLOG(ZSAT)
 IF(ALZSAT.LT.ALZSTP(1)) CALL FATAL (24HZSAT BELOW TABULAR MODEL)
 DO 25 IREG=1,NREG
 25 CONTINUE
IF(ALZSAT.LT.ALZSTP(IREG+1)) GO TO 20
25 CONTINUE
CALL FATAL (24HZSAT ABOVE TABULAR MODEL)
20 ANZSTP=(ALZSAT-ALZSTP(IREG))/ALDZ(IREG)
NZSTP1=0
IF(IREG.LE.1) GO TO 35
NND=IREG-1
DO 36 NZT=1,NND
36 NZSTP1=NZSTP1+NZSTEP(NZT)
35 IZMOD=BOUND(1,ANZSTP,NZSTEP(IREG)-2)
DELZ=ANZSTP-IZMOD
IZMOD=IZMOD+NZSTP1
40 IF(TINF.EQ.TLAST) GO TO 45
IF(TINF.LT.TMINMD) CALL FATAL(24HTINF BELOW TABULAR MODEL)
IF(TINF.TMAXMD) CALL FATAL (24HTINF ABOVE TABULAR MODEL)
ANZSTP=(TINF-TMINMD)/DTMOD
ITMOD=BOUND(1,ANZSTP,NTSTEP-2)
DELT=ANZSTP-ITMOD
45 IF(ICOUNT.NSPMOD) 50,70,70
50 CALL BESSEL(ANLB(ITMOD),1,1,-1.D0,DELT,BESLB,1)
C ABOVE LOWER BOUNDARY OF JACCHIA-BASS REGION
54 BSANAL=AJWB(CHI,TLJWB)+(CNST*(0.33333333-FALPHA(EMAL1))/AL1
 2  -C2*(CHI/AL2*FALPHA(EMAL2)+GALPHA(EMAL2)-0.61111111)
 2  /(AL2*AL2))*AJWBC/DLOG(10.D0)
DAJWB = (AJWBC/(SIG*TINF))*(.4342948/(TLJWB*DEXP(SIG*CHI)))*(DTL
1JWB*DEXP(SIG*CHI)+TLJWB*SIG*DEXP(SIG*CHI)*DCHI)
DBSAN = DAJWB+(-CNST/AL1)*DFALDQ(EMAL1)*DQDH(AL1)-C2*((DCHI/AL2)*
1FALPHA(EMAL2)+(CHI/AL2)*DFALDQ(EMAL2)*DQDH(AL2)+DGALDQ(EMAL2)*
2DQDH(AL2)/((AL2*AL2)))AJWBC/DLOG(10.D0)
CALL BESSEL(FLB(ITMOD),1,1,-1.D0,DELT,BESLB,1)
DNDIZ=-AMWMOD(ICOUNT)*BESONE-(1.+ALFMOD(ICOUNT))*
1 TLOG
XLNZST=DNDIZ+BESLB+DLOGLB(ICOUNT)
DLOGNO = -AMWMOD(ICOUNT)*DBSAN-(1.+ALFMOD(ICOUNT))*(.43429484*DTLO
1C/TLOC)
1C IF(ICOUNT.EQ.1) XLNZST=XLNZST-0.07*(1+DTANH(0.18*(ZSAT-111.0))))
1C IF(ICOUNT.NE. 2) GO TO 60
JBFX1
1C IF(ZSAT.LE.200.) XLNZST=XLNZST-0.24*DEXP(-.009*(ZSAT-97.7)**2)
1C IF(ZSAT.LE.200.) DLOGNO=DLOGNO+.00432*(ZSAT-97.7)**2*DEXP(-.009*(ZSAT-
197.7)**2.)
60 CONTINUE
JBFX1
1C IF(INDEX.NE.NSPNAM) GO TO 80
RESULT(ICOUNT)=XLNZST
DRLT(ICOUNT) = DLOGNO
ICOUNT=ICOUNT+1
1C IF(ICOUNT.LT.NSPMOD) GO TO 55
C C ATOMIC H
C
XLNZST = 0.0
DLOGNO = 0.0
70 IF(ZSAT.LE.500.) GO TO 80
BSANLL = AJWB(CHI,TLJWBB)+(CNST*(0.33333333-FALPHA(EMAL11))/AL1-
1C2*(CHI/AL2*FALPHA(EMAL22)+(GALPHA(EMAL22)-0.61111111)/(AL2*AL2)))
2)*AJWBFC/DLOG(10.DO)
DNDIZ = -AMWMOD(ICOUNT)*(BSANAL-BSANLL)-(1.+ALFMOD(ICOUNT))*TLOGG
XLNZST = DNDIZ*5.94*28.9*TINF**(-.25)
DLOGNO = -AMWMOD(ICOUNT)*(DBSAN)-(1.+ALFMOD(ICOUNT))*
1.34329448*DTLOC/TLOC

IF(INDEX.EQ.NSPNAM) RESULT(ICOUNT)=XLNZST
IF(INDEX.LT.NSPNAM) RESULT(1)=XLNZST
IF(INDEX.LT.NSPNAM) DRSLT(1)=DLOGNO
TLAST=TINF
ZLAST = ZSAT
IRGLST = IREG
RETURN
END JAC77

SUBROUTINE BESSEL(ARRAY,NITEMS,NXDIM,DELX,DELY,RESULT,NINTRP)
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 ARRAY(NITEMS,NXDIM,1), RESULT(NINTRP)

PERFORMS 16-POINT-BESSEL INTERPOLATION

ARGUMENTS%
-INPUT-
ARRAY FIRST WORD OF ARRAY FROM WHICH TO INTERPOLATE
NITEMS MAXIMUM NUMBER OF ITEMS THAT MAY BE
INTERPOLATED FROM ARRAY
NXDIM X DIMENSION OF ARRAY
DELX (X-X2)/(X3-X2)
DELY (Y-Y2)/(Y3-Y2)
NINTRP NUMBER OF ITEMS TO BE INTERPOLATED

-OUTPUT-
RESULT ARRAY FOR INTERPOLATED RESULTS

VERSIOIN 2.0 LEVEL 760811

REAL*8 XWTS(4), YWTS(4)
DATA DXLAST,DYLAST/2*-10.DO/

IF(DELY.EQ.DYLAST) GO TO 5
YWTS(1) = -(DELY+1.)*(DELY-2.)/6.
YWTS(2) = (DELY+1.)*(DELY-1.)*(DELY-2.)/2.
YWTS(3) = -(DELY+1.)*DELY*(DELY-2.)/2.
YWTS(4) = (DELY+1.)*DELY*(DELY-1.)/6.

DYLAST=DELY

5 IF(NXDIM.EQ.1.OR.DELX.EQ.DXLAST) GO TO 8
XWTS(1) = -(DELX+1.)*(DELX-1.)*(DELX-2.)/6.
XWTS(2) = (DELX+1.)*(DELX-1.)*(DELX-2.)/2.
XWTS(3) = -(DELX+1.)*DELX*(DELX-2.)/2.
XWTS(4) = (DELX+1.)*DELX*(DELX-1.)/6.

DXLAST=DELX

8 DO 20 IN=1,NINTRP
RESULT(IN) = 0.
MODJC FORTRAN A1 06/29/82 12:27 CLONAD F 80 1232 RECS VA TECH

DO 20 IY=1,4
XINTP = 0.
NX=MINO(NXDMOD,4)
IF(NX .EQ. 1) GO TO 15
DO 10 IX=1,NX
10 XINTP = XINTP + XWTS(IX)*ARRAY(IN,IX,IY)
GO TO 20
15 XINTP = ARRAY(IN,1,IY)
20 RESULT(IN) = RESULT(IN) + YWTS(IY)*XINTP
C *** RESULT(IN) = INTERPOLATION OF% ARRAY(IN,1-4,1-4)
C
RETURN
END

SUBROUTINE TJB(TINF,ZSAT,DCHl,DTLJWB,DTLOC)
IMPLICIT REAL*8(A-H,O-Z)
C COMPUTES JACCHIA-BASS LOCAL TEMPERATURE
AND ASSOCIATED CONSTANTS
C
COMMON /JSDMPR/ ALZSTP(8) , MTITLE(8) , AMMMD(7) , ALDZ(7),
$ NZSTEP(7) , MODRLN(7) , GNMOD , RNMOD , RSTAMD ,
$ AVOCMD , DTMOD , TMNMD , TMAXMD ,
$ NTSTEP , NSPMOD , LABEL , NREG , NSUM
COMMON/JB/TX,GX,SIG,SIGPR,CHl,AL1,AL2,C2,EMAL1,EMAL2,CNST,TLJWB,
1 TLOC,TLOCC,CHII,EMAL11,EMAL22,TLJWBB
EQUIVALENCE (TLB,TX)
DIMENSION A1(3),A2(3),AC2(3)
DATA A1/.0384D0,.0388D0,.0343D0/,A2/.0065D0,.0232D0,.0192D0/,
1 AC2/.0166D-5,-.4382D-5,.4769D-5/
TALPHA(Q)=Q*(1.0-2.0*Q+Q*Q)
DTALDQ(Q) = 1.-4.*Q+3.*Q*Q
DQDH(X) = -X*DEXP(-X*CHI)*DCHI
DQHDH(X) =-X*DEXP((-X)*CHI)*DCHI
CHII=(ZSAT-125.)*(RNMOD+125.)/(RNMOD+ZSAT)
DCHI = ((RNMOD+125.)/(RNMOD+ZSAT))**2.
SIG=GX/(TINF-TX)
IF(TINF-1100.0) 100,200,200
100 CONTINUE
AL1=2.0*SIG*(1.0+1.D-4*(TINF-800.0))
AL2=.0215-.005*(TINF-500.0)/200.0
C2=.0566-.08*(TINF-900.0)/200.0+.04*((TINF-900.0)/200.0)**2
C2=C2*1.D-5
GO TO 250
200 AN=(TINF-1100.0)/400.0
ITIN = IFIX(SNGL(AN))+1
N=M1NO(ITIN,2)
FRAC=AN-FLOAT(N-1)
NP1=N+1
AL1=AL1(N)+FRAC*(A1(NP1)-A1(N))
IF(N .EQ. 1) AL1=.0385-.012*FRAC+.0123*FRAC*FRAC
AL2=AL2(N)+FRAC*(A2(NP1)-A2(N))
C2= AC2(N)+FRAC*(AC2(NP1)-AC2(N))
IF(N . EQ. 1) C2=AC2(1)+(AC2(2)-AC2(1))*FRAC*FRAC
250 CONTINUE
SIGPR=GX/(TINF-TX)+.2/(RNMOD+125.)
CNST = -GX*SIGPR/(2.*AL1*AL1*TX*TX)
EMAL1 = DEXP(-AL1*CHI)
EMAL11 = DEXP(-AL1*CHI1)
EMAL2 = DEXP(-AL2*CHI)
EMAL22 = DEXP(-AL2*CHI1)
TLJWB = TINF-(TINF-TLB)*DEXP(-SIG*CHI)
TLBB = TINF-.0000002*TINF**2.4
TLJWBB = TINF-(TINF-TLB)*DEXP(-SIG*CHI1)
DTLJWB = (TINF-TLB)*SIG*DEXP(-SIG*CHI)*DCHI
TLOC = 1.0/(1.0/TLJWB+CNST*TALPHA(EMAL1)+C2*CHI*TALPHA(EMAL2))
TLOCC = 1.0/(1.0/TLJWBB+CNST*TALPHA(EMAL11)+C2*CHI1*TALPHA(EMAL22))
DTLOC = -TLOC*TLOC*(-(1./(TLJWB*TLJWB))*DTLJWB+CNST*DTALDQ(EMAL1)
1*DQDH(AL1)+C2*(DCHI*TALPHA(EMAL2)+CHI*DTALDQ(EMAL2)*DQDH(AL2)))
RETURN
END

SUBROUTINE FATAL(MSG)
IMPLICIT REAL*8(A-H,O-Z)
WRITE (6,6050) MSG
6050 FORMAT (/" FATAL ERROR DETECTED BY PROGRAM"/1X,13A10)
STOP
END
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A Numerical Comparison of Atmospheric Density Models for Near-Earth Satellite Motion

by

William F. Ormsby

(ABSTRACT)

The effect of the atmosphere on near-earth satellites is evaluated by consideration of the drag perturbation and the associated dispersion parameters. Recommendations are made for each of these dispersion parameters. The recommendation concerning the density is that a dynamic density model be utilized instead of a static model. Included are numerical comparisons which quantify the error in predicted satellite positions which can occur due to an inferior density model alone. These comparisons are made for a variety of satellite orbits.