OPTIMIZATION OF MULTIPLE
CHILLER SYSTEMS

by

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(ABSTRACT)

The following investigation describes the method used in minimizing the power consumption of multiple chiller systems by determining not only the most efficient combination of chillers in the system but also the optimal operating levels of each chiller making up the combination. The general mathematical formulation for a system of N chillers with arbitrary performance characteristics is solved and applied to the special case of chillers having parabolic performance characteristics. The analysis is then applied to various two and six chiller systems using performance characteristics of centrifugal chillers in order to show the optimal operating conditions and to illustrate the potential energy savings of multiple chiller systems compared to a single chiller system. The effects of various combinations of chillers and the effects of entering condenser water temperature are systematically investigated. A particular six chiller combination, corresponding to portions of the existing chiller systems at Virginia Tech, is also investigated.
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>viii</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>ix</td>
</tr>
<tr>
<td>1. INTRODUCTION AND REVIEW OF LITERATURE</td>
<td>1</td>
</tr>
<tr>
<td>2. ANALYSIS</td>
<td>13</td>
</tr>
<tr>
<td>2.1 Problem Description</td>
<td>13</td>
</tr>
<tr>
<td>2.2 General Solution</td>
<td>14</td>
</tr>
<tr>
<td>2.3 Solution for Parabolic Power Curves</td>
<td>19</td>
</tr>
<tr>
<td>2.4 Solution for Two Chiller Case</td>
<td>22</td>
</tr>
<tr>
<td>3. APPLICATIONS AND RESULTS</td>
<td>30</td>
</tr>
<tr>
<td>3.1 Performance Characteristics</td>
<td>30</td>
</tr>
<tr>
<td>3.2 Two Chiller Systems</td>
<td>33</td>
</tr>
<tr>
<td>3.3 Six Chiller Systems</td>
<td>51</td>
</tr>
<tr>
<td>3.4 Virginia Tech System</td>
<td>60</td>
</tr>
<tr>
<td>4. SUMMARY AND CONCLUSIONS</td>
<td>71</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>74</td>
</tr>
<tr>
<td>APPENDIX A: Counting Schemes</td>
<td>76</td>
</tr>
<tr>
<td>VITA</td>
<td>81</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Cost increase of electrical power from August of 1970 to January of 1980</td>
<td>2</td>
</tr>
<tr>
<td>2.</td>
<td>Schematic illustration of the basic components of a simple liquid chiller</td>
<td>4</td>
</tr>
<tr>
<td>3.</td>
<td>Illustration of allowable operating levels of a two chiller system at various total loads</td>
<td>25</td>
</tr>
<tr>
<td>4a.</td>
<td>Performance characteristics of centrifugal chillers</td>
<td>31</td>
</tr>
<tr>
<td>4b.</td>
<td>Illustration showing the most efficient operating ranges of centrifugal chillers</td>
<td>32</td>
</tr>
<tr>
<td>4c.</td>
<td>Efficiency curves of centrifugal chillers for various capacities</td>
<td>32</td>
</tr>
<tr>
<td>5a.</td>
<td>Optimal operating levels of a 520 ton and 1200 ton chiller system at an ECWT of 65°F</td>
<td>35</td>
</tr>
<tr>
<td>5b.</td>
<td>Optimal operating levels of a 520 ton and 1200 ton chiller system at an ECWT of 75°F</td>
<td>36</td>
</tr>
<tr>
<td>5c.</td>
<td>Optimal operating levels of a 520 ton and 1200 ton chiller system at an ECWT of 85°F</td>
<td>37</td>
</tr>
<tr>
<td>6.</td>
<td>Percent savings of a 520 ton and 1200 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTs</td>
<td>40</td>
</tr>
<tr>
<td>7a.</td>
<td>Percent savings for various 2 chiller systems compared to a single chiller system in which all systems are operating at their optimal levels and have a total capacity of 1720 tons at an ECWT of 65°F</td>
<td>43</td>
</tr>
<tr>
<td>7b.</td>
<td>Percent savings for various 2 chiller systems compared to a single chiller system in which all systems are operating at their optimal levels and have a total capacity of 1720 tons at an ECWT of 75°F</td>
<td>44</td>
</tr>
<tr>
<td>7c.</td>
<td>Percent savings for various 2 chiller systems compared to a single chiller system in which all systems are operating at their optimal levels and have a total capacity of 1720 tons at an ECWT of 85°F</td>
<td>45</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>8a.</td>
<td>Percent savings of a 100 ton and 335 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTs</td>
<td>47</td>
</tr>
<tr>
<td>8b.</td>
<td>Percent savings of a 125 ton and 310 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTs</td>
<td>48</td>
</tr>
<tr>
<td>8c.</td>
<td>Percent savings of a 175 ton and 260 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTs</td>
<td>49</td>
</tr>
<tr>
<td>8d.</td>
<td>Percent savings of 217 ton and 218 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTs</td>
<td>50</td>
</tr>
<tr>
<td>9a.</td>
<td>Optimal operating levels for a six chiller system for a total load of 500 tons at various ECWTs</td>
<td>52</td>
</tr>
<tr>
<td>9b.</td>
<td>Optimal operating levels for a six chiller system for a total load of 1000 tons at various ECWTs</td>
<td>53</td>
</tr>
<tr>
<td>9c.</td>
<td>Optimal operating levels for a six chiller system for a total load of 1500 tons at various ECWTs</td>
<td>54</td>
</tr>
<tr>
<td>10a.</td>
<td>Percent savings for various six chiller systems compared to a single chiller system in which all systems are optimized and have a total capacity of 1970 tons at an ECWT of 65°F</td>
<td>56</td>
</tr>
<tr>
<td>10b.</td>
<td>Percent savings for various six chiller systems compared to a single chiller system in which all systems are optimized and have a total capacity of 1970 tons at an ECWT of 75°F</td>
<td>57</td>
</tr>
<tr>
<td>10c.</td>
<td>Percent savings for various six chiller systems compared to a single chiller system in which all systems are optimized and have a total capacity of 1970 tons at an ECWT of 85°F</td>
<td>58</td>
</tr>
<tr>
<td>11a.</td>
<td>Performance characteristics of absorption units</td>
<td>61</td>
</tr>
<tr>
<td>11b.</td>
<td>Illustration showing the most efficient operating ranges of absorption units</td>
<td>61</td>
</tr>
</tbody>
</table>
12a. Percent savings of single system compared to three separate systems at Virginia Tech using fractions of total load obtained from the actual chiller capacities ........................................... 64

12b. Percent savings of a single system compared to three separate systems at Virginia Tech using fractions of total load obtained from architect's estimate .................. 65

12c. Percent savings of a single system compared to three separate systems at Virginia Tech using fractions of total load obtained by averaging other two methods ........ 66

13. Percent savings of a single system compared to three separate systems at Virginia Tech for an ECWT of 85°F ................................................................. 68
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Number of operating combinations to be tested for a multiple chiller system considering the constraints given by Eqs. 4; obtained from Eq. A-6c in Appendix A</td>
<td>18</td>
</tr>
<tr>
<td>2.</td>
<td>Region of various total cooling loads with their allowable combinations as shown in Fig. 3</td>
<td>27</td>
</tr>
<tr>
<td>3.</td>
<td>Various two chiller combinations used in Figs. 7a-c, 8a-d and various six chiller combinations used in Figs. 10a-c</td>
<td>42</td>
</tr>
<tr>
<td>4.</td>
<td>Fractions used in dividing up the total load among the three chiller systems</td>
<td>63</td>
</tr>
</tbody>
</table>
NOMENCLATURE

\[ a_{ij} \quad \text{coefficients defined by Eqs. 22a-b} \]
\[ C_{ij} \quad \text{coefficients used in general parabolic power equation for the } \]
\[ \text{i}^{\text{th}} \text{ chiller} \]
\[ L_i \quad \text{load supplied by the } \text{i}^{\text{th}} \text{ chiller} \]
\[ L_{i,\text{min}} \quad \text{minimum operating level of the } \text{i}^{\text{th}} \text{ chiller} \]
\[ L_{i,\text{max}} \quad \text{maximum operating level of the } \text{i}^{\text{th}} \text{ chiller} \]
\[ \alpha \quad \text{possible chiller combinations} \]
\[ N \quad \text{number of chillers in a system} \]
\[ P \quad \text{total power consumed by a system} \]
\[ P_i \quad \text{power consumed by the } \text{i}^{\text{th}} \text{ chiller} \]
\[ P_{i,\text{max}} \quad \text{maximum power consumption of the } \text{i}^{\text{th}} \text{ chiller} \]
\[ P_{\text{sing}} \quad \text{power consumed by a single chiller system} \]
\[ P^* \quad \text{auxiliary power function defined by Eq. 5} \]
\[ \lambda \quad \text{Lagrange multiplier} \]
1. INTRODUCTION AND REVIEW OF LITERATURE

A primary electrical consumer in institutional, commercial, and many industrial buildings is the compressor of the air conditioning system [1,2]. Before the 1970's, the United States experienced an abundance of cheap energy which led to capital investments which did not favor energy conservation. Usually designs oriented towards energy conservation could not be justified economically [3].

This situation has turned around ever since the Arab oil embargo of 1973 resulting in an increase in energy costs. Figure 1 shows the cost increase in electrical power from August of 1970 until 1980 [3]. Due to these every increasing electrical costs many corporations and businesses are looking for realistic ways to reduce their consumption of electricity [4]. Since many large businesses and institutions are becoming aware of the importance of energy savings, the formation of energy management programs is quite common. An example of this is at Virginia Polytechnic Institute and State University (Va. Tech) where an energy management project was initiated on September 13, 1982 [5]. Two major purposes of this project are to provide a basis for future planning decisions and to implement energy saving techniques for immediate savings. One of the first things that an energy management program does is to conduct detailed energy audits from which a list of energy conservation recommendations, including the cost of implementation, savings and payback can be made. These recommendations are then listed by priorities.

Another effect of the cost increases in electrical power is the trend towards the use of low-energy chillers. In 1975 specifications on
Figure 1. Cost increase of electrical power from August of 1970 to January 1980
centrifugal chillers ranged from .80 to .90 kilowatts per ton of refrigeration (kw/TR). In 1980, the specifications dropped to about .70 to .75 kw/TR, while at present it has fallen to an average of .60 to .70 kw/TR [6].

The purpose of this investigation is to reduce the electrical consumption (operating cost) of a multiple chiller system by optimizing the loading of the chillers. The main objective is to develop the theory and computational means to determine, at any desired total load, the optimal operating levels of each chiller in a multiple chiller system and to investigate the relative advantages or disadvantages of various arrangements of chillers. These optimal levels are those which result in the minimization of power consumption.

The general characteristics of liquid chillers can be summarized as follows. A liquid chilling system cools a secondary refrigerant liquid such as water or brine for the purpose of air conditioning or refrigeration. The basic components of a liquid chilling system consist of a compressor, a compressor drive, an evaporator, a condenser, a refrigerant flow control device (expansion valve) and a control center. A schematic showing the basic components appears in Fig. 2 [7].

The principles of operation begin with the secondary refrigerant liquid entering the evaporator where it is chilled by the evaporation of the colder primary liquid refrigerant. The refrigerant gas is then compressed to a higher temperature and pressure in the compressor before entering the condenser. Heat is removed from the refrigerant by the use of condenser water which in turn exhausts the heat to the atmosphere by the use of a cooling tower. The result is that the refrigerant drops in
Figure 2. Schematic illustration of the basic components of a simple liquid chiller
temperature and then condenses at the higher pressure. The refrigerant is then throttled through an expansion valve which results in a drop of pressure and temperature. The refrigerant then enters the evaporator and the cycle continues [7].

For a chiller system the flowrate may be either constant or variable. For a system with a constant flowrate, 3-way valves are used at the chilled water coils. These valves direct the amount of water through the coils while the remainder is bypassed, thus maintaining a constant flowrate through the system while varying the load. The variable flow system uses 2-way valves which are controlled to vary the flowrate through the cooling coils in response to changes in load. Load variation in a constant flow system is temperature related; therefore, the temperature control can sense changes in load which leads to a stable system. However, in a variable flow system the temperature control cannot sense a variation of flow due to a changing load therefore resulting in control instability [8].

Many systems consist of multiple chillers in order to match load requirements more efficiently and to provide a backup in case one unit must be removed to be serviced. The two types of arrangement for multiple chiller systems consist of series and parallel combinations.

Parallel piped arrangements are generally used when the chilling coils are designed to yield a relatively low water temperature rise of about 8 to 12 degrees across the coils [8]. In order to accommodate a desired cooling requirement with this relatively low temperature rise, a large chilled water flowrate is necessary. Under these conditions a parallel piping arrangement is advantageous in order to keep the pres-
sure drop at reasonable levels. However, this arrangement has the disadvantage of mixing. When the system is operating at part load, where a single chiller handles the load, return water passes through the idle chiller without a temperature change and mixes with the chilled water from the active machine. The active machine must chill the water to a very low temperature in order to bring the temperature of the mixture to an acceptable level. As a precaution, a minimum temperature control must be used to prevent freeze-up of the chilled water. This mixing results in a waste since the active chiller must deliver lower temperature water than the system requires without the presence of mixing [8].

The other option is the series piping arrangement. This is used when the chilled water coils are designed for a relatively high total temperature rise, in the range of 14 to 20 degrees. Due to this high temperature rise, a lower flowrate is needed to obtain a desired cooling load. This arrangement eliminates wasteful mixing which results in compressor savings. Further compressor savings are experienced since it is more efficient to reduce the chilled water temperature in several stages. A disadvantage to the series arrangements is that the pressure drop through the chillers is additive, resulting in more pumping energy, therefore increasing the electrical consumption (8).

There are many ways in which the performance of the chillers themselves may be increased. Throughout the 1970's, major improvements were attained such as increased heat transfer surfaces, advances in heat exchanger technology, open motors, changes in cooling tower water temperatures and the development of heat recovery chillers. Due to the
already available technology these advances generally had low development costs. However, these improvements are reaching their economic limits very quickly and further advances must now be developed by the use of new technology which is more costly. Compressor improvement and electronic controls are two examples of such improvements [6].

Lee [1] suggest four changes which can reduce energy consumption in mechanical chillers. The first change is eliminating low load operation by running chillers at higher loads for shorter periods of time. Two other changes are lowering condensing pressures and raising evaporator pressures. The fourth change is cooling directly with tower water during design operation using the theory of Dewpoint and Hybrid Cooling.

Gerhold (9) claims that there are three main energy saving areas in water-cooled air conditioning systems. These being regulating operating temperatures, treating water and cleaning condenser tubes, and operating cooling towers properly. The chilled water temperature controller should be set to maintain the highest possible temperature and still supply the necessary load since efficiency increases about 1 1/2 percent for each degree increase in chilled-water temperature (9,10). The condenser-water temperature should also be maintained as low as possible since efficiency improves about 1 1/2 percent for each decreased degree in condenser water temperature. Cooling tower controls should be set to maintain a 70 or 75 F minimum temperature. Water treatment should be used to prevent serious condenser scaling and corrosion of heat exchanger tubes. If this is not done, efficiency will be reduced due to a decreased transfer of heat, and operating costs will be increased.
If there is more than one chiller in the system, proper loading of these chillers should be done by the use of their part load characteristics. Under conventional control this is done manually. However, by the use of an energy management control system this process is improved. This control system can determine the building load from the temperature difference between the leaving and returning chilled water along with the water flowrate. With these data it is possible to automatically select the proper loading of each chiller [11].

Due to the arrival of the microprocessor it is now cost effective to have automated air conditioning systems. One component of a control system is known as adaptive control which uses a feedback signal to change a setting on a system as operating conditions change. An example of a system having adaptive control would be one that measures the indoor and outdoor temperature and automatically changes the operation of the HVAC system to match the present building load more efficiently. Adaptive control has three major areas which are optimum start time, duty cycling and night setback [12,13].

Other components of a load management system are separated into the following groups; centralized control, demand control, load scheduling, utility monitoring and reporting, and maintenance management programs.

A complete understanding of the minimization of electrical costs extends beyond the engineering aspects of a particular system. For instance, a proper understanding of the local power company's rate structure is necessary. For most industrial users the electric bill consists of a charge for the amount of electricity consumed and also a demand charge for the peak monthly consumption during a specified time
interval, typically about 15 minutes. The reason for a demand charge is to cover the cost of the generating capacity that a utility must have on line to meet peak load conditions. This capacity is only needed for peak load conditions which in some cases only occur a small portion of the year.

Many utilities have a minimum demand charge based on a percentage of the highest monthly demand over an extended period of about 11 months. This rate structure (ratchet clause) increases the costs of electricity if demand fluctuates significantly. When a ratchet clause is in effect, the reduction of one 15 minute peak can save demand dollars for a year or more afterwards. Typical electric rates range up to $0.04 per kilowatt hour for consumption and range from $1.30 to $8.50 per kilowatt for demand. In February of 1982, VPI's electric rates were $0.01883 KWH for energy and $8.434 per KW for demand.

A necessity to minimize electrical costs is monitoring the demand level, which can be accomplished in three different ways. First, there is a system of current transformers along with a watt transducer. The watt transducer monitors current, voltage and phase angle to yield a time power reading. The second method uses an ampere-hour transducer power monitor which monitors only the three phase current. This method only gives an approximation of power since it only measures current flow alone and not voltage or phase angle. The final method is known as the utility company pulse generator where the utility provides the pulses to the user. This is the most common method used as well as the most accurate method although it does not detect errors in the utility's monitoring [12].
Once the demand is monitored, the user must know which method the utility uses to calculate the demand level in order to achieve the greatest electrical savings. The three methods used are the rate mode, the fixed interval and the sliding window interval. The rate mode of calculation considers only instantaneous power and neglects the time interval. The fixed interval method determines the average demand for a particular period [12] while the sliding window method treats each instant as the end of a demand period and at each instant calculates an average demand for the last 15 or 30 minute period [14].

Once the user knows which method of demand calculation the utility uses, he can now control demand to his best advantage. For example, if the fixed interval is in effect, two popular submethods, the prediction method or the ideal rate curve method may be used [14].

The control of the peak demand can result in substantial savings. A 1000 KW load operated unnecessarily during a time of peak demand will cost over $70,000 the following year [2]. Schiebout [15] suggests using absorption chillers which operate with steam instead of electricity in conjunction with electric centrifugal chillers in order to control peak demand. Dubin [16] suggests the idea of installing chilled water storage in new buildings to control peak demand since electric chillers could then be turned off in times of peak demand. Choi [17] suggests the use of an emergency diesel engine generator to control peak demand.

Due to these significant savings, and the age of the microprocessor, it is generally advantageous to purchase a demand control system. The main objective of a control system is the capability of predicting load trends so that shedding may take place throughout the demand period
instead of having heavy shedding near the end of the demand period. The system should also possess the ability to assign priorities to the different energy consuming systems and prevent excessive cycling of certain loads. The system should be capable of being easily reprogrammed to satisfy changing conditions [3].

It is clear from the considerations and literature cited above, that energy conservation is playing a major role in industry today due to the never ending increase of electrical costs that the world is facing. Also technological advances, such as the advent of the microprocessor are making the use of control systems more and more economically sound which at this time prove to be the best way to conserve energy especially for large HVAC systems.

This work shows the method of determining the optimal operating levels of multiple chiller systems which has the potential of developing predetermined load schedules which can be used in HVAC control.

Chapter 2 contains the derivation of the general solution for a system consisting of N chillers, utilizing the theory of Lagrange multipliers [18]. The method is shown which incorporates the various constraints on the individual chillers and the general solution is then applied to the case of a two chiller system.

Chapter 3 presents and discusses the results and applications of this work. Both two and six chiller systems are investigated and compared to single chiller systems having equal system capacities as the multiple chiller systems in order to obtain the energy savings. This study considers the operating savings only and does not consider the added initial investment that is inherent in a multiple chiller
system. Three chiller systems at Va. Tech are then investigated and the potential operating savings which are possible by combining these three individual systems into one system are presented. Chapter 4 concludes the investigation with a summary and general observations.
2. ANALYSIS

This section describes the procedure for determining the optimal operating conditions of a multiple chiller system. The general mathematical formulation for a system consisting of N chillers with arbitrary power vs. load characteristics is solved and applied to the special case of a system where the power consumption equations of the chillers are parabolic, since the performance characteristics of many chillers in use today match this condition [11]. The general results are further applied to the special case of a two chiller system.

2.1 Problem Description

For a multiple chiller system, it is desired to determine the most efficient way to operate the chillers in order to minimize the overall system power consumption for any cooling load requirement. Therefore, for a given load, the combination of chillers as well as their operating levels must be determined.

In order to accomplish this, the total power requirement for the system, which is merely the sum of the power consumption of each chiller, must be minimized. It is also required that certain constraints be satisfied simultaneously. The constraints require that the sum of the chiller operating levels equal the total load and that each chiller operates within a specified range, typically 20 percent to 100 percent of its maximum capacity. The minimum power consumption for every possible combination must be calculated and tested in order to determine the optimal combination. This includes the operation of a single chiller as a possible combination.
2.2 General Solution

The first step in the analysis is to obtain the power vs. load characteristics for each chiller in the system. These equations are typically determined by curve fitting performance data obtained from the manufacturer. Once this is done a power equation may be written for each chiller in the form,

\[ P_i = P_i(L_i) \quad i = 1, 2, \ldots, N \]  \hspace{1cm} (1)

The total power, \( P \), can then be expressed as

\[ P = \sum_{i=1}^{N} P_i(L_i) \]  \hspace{1cm} (2)

The total power (Eq. 2) must be minimized subject to the condition that the individual loads add up to the desired total load

\[ L = \sum_{i=1}^{N} L_i \]  \hspace{1cm} (3)

and that each individual chiller operates within a certain range

\[ L_{i,\text{min}} < L_i < L_{i,\text{max}} \quad \text{or} \quad L_i = 0, \quad i = 1, 2, \ldots, N \]  \hspace{1cm} (4)

while Eq. 2 must be minimized subject to constraints 3 and 4, the solution will be carried out by initially ignoring the operating constraints given by Eqs. 4. Since constraint 3 is expressible as an equality, it can be incorporated into the minimization process directly. However, constraints 4 are inequalities and cannot be directly incorporated, but must be tested against the various solution possibilities.
The general solution is obtained by using the theory of Lagrange multipliers [18]. The summation of the individual loads is subtracted from the total load and this quantity is multiplied by the Lagrange multiplier, \( \lambda \), which is then used to form the following function

\[
P^* = P + \lambda \left[ L - \sum_{i=1}^{N} L_i \right]
\] (5)

Since the total load equals the summation of the individual loads, any finite value of \( \lambda \) will result in \( P^* = P \); therefore, if \( P^* \) is minimized, \( P \) will also be minimized. \( P^* \) is considered to be a function of the individual loads \( (L_i) \) and \( \lambda \) only, since the total load \( (L) \) is a parameter. We minimize \( P^* \) by requiring that all partial derivatives be simultaneously set equal to zero,

\[
\frac{\partial P^*}{\partial L_i} = 0 \quad i = 1, 2, \ldots, N \quad (6)
\]
\[
\frac{\partial P^*}{\partial \lambda} = 0 \quad (7)
\]

By using Eq. 5 in conditions 6 and 7, the following is obtained

\[
\frac{dP_1(L_1)}{dL_1} = \lambda
\]
\[
\frac{dP_2(L_2)}{dL_2} = \lambda
\]
\[
\cdot
\]
\[
\cdot
\]
\[
\cdot
\]
\[
\frac{dP_N(L_N)}{dL_N} = \lambda
\] (8)
This shows that in order to obtain the greatest operational efficiency all chillers should operate at an equal incremental power consumption, \( \lambda \), known as the system lambda [19].

This set of equations must be solved simultaneously in order to determine \( L_1, L_2, \ldots L_N, \) and \( \lambda \). If the incremental power consumption equations are nonlinear, an iterative procedure must be used to obtain a solution. Since linear incremental power consumption equations often occur in practice, this case will be examined in detail in section 2.3.

The solution to Eqs. 8 and 9 may result in a multiple number of operating levels for each chiller, i.e., the solution may have multiple roots. If only one root is found the second derivative test can be used to determine if the root produces a relative minimum or relative maximum. If the root gives a relative minimum and constraints 4 are satisfied, the boundaries of the chiller's operating range need not be tested since a relative minimum is also an absolute minimum for a solution of one root. However, if the single root gives a relative maximum, all boundaries of the chillers' operating range must be tested to obtain the absolute minimum. It should be noted that the case of obtaining a single root which gives a relative maximum is very unusual since performance curves for chillers are usually concave upwards. However, the most general case is a solution from Eqs. 8 and 9 which yields multiple roots and each root (operating level) must be tested to see if the constraints given by Eq. 4 are satisfied. For all roots that satisfy these constraints, a corresponding power consumption is calculated. In
order to ensure that an absolute minimum is obtained all boundaries of the chillers' operating ranges formed from constraints 4 must be systematically tested. This results in the following number of tests that must be performed.

\[ n = \sum_{x=1}^{N} \frac{N!}{(N-x)!x!} 3^{(N-x)} \]  

(10)

The derivation of this equation is shown in Appendix A. Table 1 shows the vast numbers of tests that must be performed for systems having one through ten chillers.

Each test is performed by subtracting the minimum capacities of the chillers, which are set at the low point of their operating range, and the maximum capacities of the chillers, which are set at the high point of their operating range from the total load. The operating levels of the remaining chillers are then calculated using the new value of total load and these operating levels are tested against their constraints. If any of the operating levels of these chillers fall outside of their constraints the test is failed immediately. However, if the operating levels all fall within their constraints the power consumption is calculated. After all tests are performed, the values of the power consumptions are compared to each other and the combination which gives the smallest value is chosen as the optimal one.

This procedure is straightforward as long as a logical sequence of steps are taken to obtain all the possible combinations as well as the vast numbers of arrangements formed from testing the boundaries of the chillers' operating ranges.
Table 1. Number of operating combinations to be tested for a multiple chiller system considering the constraints given by Eqs. 4; obtained from Eq. A-6c in Appendix A, or Eq. 10.

<table>
<thead>
<tr>
<th>Number of Chillers</th>
<th>Number of Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>37</td>
</tr>
<tr>
<td>4</td>
<td>175</td>
</tr>
<tr>
<td>5</td>
<td>781</td>
</tr>
<tr>
<td>6</td>
<td>3,367</td>
</tr>
<tr>
<td>7</td>
<td>14,197</td>
</tr>
<tr>
<td>8</td>
<td>58,975</td>
</tr>
<tr>
<td>9</td>
<td>242,461</td>
</tr>
<tr>
<td>10</td>
<td>989,527</td>
</tr>
</tbody>
</table>
2.3 Solution for Parabolic Power Curves

Quite often the power consumption equations for chillers are parabolic in nature [14] and are expressible as,

\[ P_i(L_i) = C_{10} + C_{11} L_i + \frac{1}{2} C_{12} L_i^2, \quad i = 1, 2, \ldots, N \]  \hspace{1cm} (11)

For this case of parabolic power curves, the incremental power consumption equations (Eqs. 8) become linear.

\[ \frac{dP_i(L_i)}{dL_i} = C_{i1} + C_{i2} L_i, \quad i = 1, 2, \ldots, N \]  \hspace{1cm} (12)

Substituting for \( \frac{dP_i}{dL_i} \) into Eq. 8, the system of N+1 equations with N+1 unknowns can be written as

\[
\begin{align*}
C_{12} L_1 - \lambda &= -C_{11} \\
C_{22} L_2 - \lambda &= -C_{21} \\
&\quad \cdots \\
\vdots &\quad \vdots \\
C_{N2} L_N - \lambda &= -C_{N1} \\
\sum_{i=1}^{N} L_i &= L
\end{align*}
\]  \hspace{1cm} (13)

This set of equations can be written more compactly in the following matrix form,
These equations may now be solved by performing the following matrix manipulations. Multiply the first row by $1/C_{12}$, the second row by $1/C_{22}$, ..., and row $N-1$ by $1/C_{N2}$. Next, multiply the first row by $-1$ and add to the last row, then multiply the second row by $-1$ and add to the last row, ..., and finally multiply row $n-1$ by $-1$ and add to the last row. This sequence of manipulations produces the following matrix equation:

$$
\begin{bmatrix}
C_{12} & 0 & 0 & \ldots & 0 & -1 \\
0 & C_{22} & 0 & \ldots & 0 & -1 \\
0 & 0 & C_{32} & \ldots & 0 & -1 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & C_{N2} & -1 \\
1 & 1 & 1 & \ldots & 1 & 0
\end{bmatrix}
\times
\begin{bmatrix}
L_1 \\
L_2 \\
L_3 \\
\vdots \\
L_n \\
\lambda
\end{bmatrix}
=
\begin{bmatrix}
-C_{11} \\
-C_{21} \\
-C_{31} \\
\vdots \\
-C_{n1} \\
L
\end{bmatrix}
$$

(15)

$$
\begin{bmatrix}
1 & 0 & 0 & \ldots & 0 & -1/ C_{12} \\
0 & 1 & 0 & \ldots & 0 & -1/ C_{22} \\
0 & 0 & 1 & \ldots & 0 & -1/ C_{32} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & 1 & -1/ C_{n2} \\
0 & 0 & 0 & \ldots & 0 & A
\end{bmatrix}
\times
\begin{bmatrix}
L_1 \\
L_2 \\
L_3 \\
\vdots \\
L_n \\
\lambda
\end{bmatrix}
=
\begin{bmatrix}
C_{11}/ C_{12} \\
C_{21}/ C_{22} \\
C_{31}/ C_{32} \\
\vdots \\
C_{n1}/ C_{n2} \\
B
\end{bmatrix}
$$

(16)
where

\[ A = \sum_{i=1}^{N} \frac{1}{C_{i1}} \]

\[ B = L + \sum_{i=1}^{N} \frac{C_{i1}}{C_{i2}} \]

From this matrix, the desired solution is conveniently determined since the system lambda can be immediately expressed as

\[ \lambda = \frac{L + \sum_{i=1}^{N} \frac{C_{i1}}{C_{i2}}}{\sum_{i=1}^{N} \frac{1}{C_{i2}}} \]  \quad (17)

and the operating level of each chiller can then be expressed

\[ L_i = \frac{\lambda - C_{i1}}{C_{i2}}, \quad i = 1, 2, \ldots, N \]  \quad (18)

This solution yielding the operating levels must be considered merely as a possibility, since the constraints given by Eq. 4 have not yet been taken into account. The methodology needed to integrate these inequality constraints with the possible solutions given by Eqs. 17 and 18 is described now.

Equations 17 and 18 are linear and therefore the solution for each combination yields only one operating level per chiller, i.e., the solution gives a single root. Since the power curves are concave upwards this single root will always give a relative minimum. For the
case where all the operating levels satisfy constraints 4, the end points need not be checked because this relative minimum is also the absolute minimum. However, if the operating levels are calculated for a particular combination and one or more of the chiller's constraints are not satisfied then tests on the boundaries must be performed to obtain the minimum power consumption which satisfies the constraints. This is done by following the same methodology as for the general solution except that if the calculated operating levels for a particular combination are all within their operating limits then the absolute minimum power consumption is obtained and no further testing need be done for that combination.

2.4 Solution for Two Chiller Case

For the case of two chillers having parabolic performance curves, the equations for the operating levels of each chiller will be derived in terms of the various system parameters.

The total power equation reduces to

\[ P = P_1(L_1) + P_2(L_2) \quad (19) \]

and the constraints become

\[ L = L_1 + L_2 \quad (20) \]

\[ L_{1,\min} < L_1 < L_{1,\max} \quad \text{or} \quad L_1 = 0 \quad (21a) \]

\[ L_{2,\min} < L_2 < L_{2,\max} \quad \text{or} \quad L_2 = 0 \quad (21b) \]
Since performance curves are often given in terms of percent power versus percent load, the following form is used,

\[ P_1 = \frac{p_{1, \text{max}}}{100} \left[ a_{10} + a_{11} \cdot \frac{100}{L_{1, \text{max}}} \cdot L_1 + a_{12} \cdot \left( \frac{100}{L_{1, \text{max}}} \cdot L_1 \right)^2 \right] \]  

(22a)

\[ P_2 = \frac{p_{2, \text{max}}}{100} \left[ a_{20} + a_{21} \cdot \frac{100}{L_{2, \text{max}}} \cdot L_2 + a_{22} \cdot \left( \frac{100}{L_{2, \text{max}}} \cdot L_2 \right)^2 \right] \]  

(22b)

where the general coefficients of Eq. 11 have taken the form

\[ C_{i0} = \frac{p_{i, \text{max}}}{100} \cdot a_{i0} \]

\[ C_{i1} = \frac{p_{i, \text{max}}}{L_{i, \text{max}}} \cdot a_{i1} \]

\[ C_{i2} = \frac{p_{i, \text{max}}}{(L_{i, \text{max}})^2} \cdot 200 \cdot a_{i2}, \quad i = 1, 2 \]

The coefficients \( a_{i0}, a_{i1}, \) and \( a_{i3} \) must be specified or determined for each particular model chiller and are affected by such things as entering condenser water temperature [ECWT].

The optimal operating levels for a two chiller system are readily available from general solutions 17 and 18 and are expressed as
This shows that for any two chiller case, the possible operating levels as given by Eqs. 23a and 23b are functions of the maximum capacity of each chiller along with the maximum power consumption, the total load, and the constants, $a_{ij}$, obtained from the performance characteristics which in turn are functions of the entering condenser water temperature (ECWT).

Since constraints 21a and 21b have not yet been incorporated into the solution, the operating levels must be checked to make sure they are in their allowable operating range. A testing procedure incorporating these constraints into Eqs. 23a and 23b will be illustrated for the special case of a two chiller system.

In order to clarify this testing procedure for a two chiller system, Fig. 3 will be used which shows the range of allowable operating levels. Each line, representing a specific cooling load, has a slope of 45 degrees since the operating levels of the two chillers must sum to this total load. The solid portions of these lines show the allowable
Figure 3. Illustration of allowable operating levels of a two chiller system at various total loads.
operating levels, since this is where the chiller constraints are satisfied, while the dotted portions show the unacceptable operating levels. There are seven distinct regions each corresponding to the different allowable chiller combinations. These regions, along with their allowable combinations, are shown in Table 2.

In region I of Fig. 3, all combinations fail since the total load is below the minimum operating levels of both chillers. The only possible combination in region II is operating chiller one alone; therefore, the testing procedure eliminates all other possible combinations since their constraints can not be satisfied. The power consumption is calculated for chiller one and since it is the only allowable combination for this total load range, it is chosen as the optimal combination. In region III, two possible combinations exists; the operation of chiller one alone or that of chiller two. The power consumption for both combinations is calculated and compared in order to determine the combination giving the least power consumption.

In region IV, all three combinations are possible solutions since they all can satisfy their constraints. For the two combinations of operating each chiller alone, the operating constraints are immediately satisfied; however, for the combination of running both chillers, the constraints may or may not be satisfied immediately depending on where the point of their operating levels falls on the total load line. If the point falls within the rectangle of Fig. 3, such as between points b and c on the total load line between regions IV and V, the constraints are satisfied and the power consumption is calculated immediately. However, if the point falls outside of the rectangle such as on point a,
Table 2. Regions of various total cooling loads with their allowable combinations as shown in Fig. 3.

<table>
<thead>
<tr>
<th>Region Number</th>
<th>Allowable Chiller Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>none</td>
</tr>
<tr>
<td>II</td>
<td>1</td>
</tr>
<tr>
<td>III</td>
<td>1, 2</td>
</tr>
<tr>
<td>IV</td>
<td>1, 2, 1 and 2</td>
</tr>
<tr>
<td>V</td>
<td>2, 1 and 2</td>
</tr>
<tr>
<td>VI</td>
<td>1 and 2</td>
</tr>
<tr>
<td>VII</td>
<td>none</td>
</tr>
</tbody>
</table>
then the constraints are violated, i.e., chiller two is operating below its minimum. This results in shifting the operating levels along the load line from point a to point b, i.e., the operating level of chiller two is set equal to its minimum and the operating level of chiller one is set equal to $L - L_{2,\text{min}}$. Since the constraints are now satisfied the power consumption is calculated. If the initial operating level fell on point d, it would be forced to point c in a similar fashion. The three values of power consumption, one for each combination, are compared to determine the optimal combination.

The same process is carried out for region V where the allowable combinations are operating chiller two alone or both together. For region VI the only allowable combination is operating both chillers together and in region VII the loads are greater than the sum of the two chillers' maximum capacities so all combinations fail when the constraints are applied.

For the simple case of two chillers, the possible combinations obtained by incorporating operating constraints 4 are numerous, as just described with the aid of Fig. 3. If the graph of Fig. 3 was extended for three chillers, the rectangle would become a three dimensional box and the 45 degree load lines would become 45 degree planes. A graph for a four chiller system can not be visualized since it is four dimensional. The number of possible combinations become increasingly greater as the number of chillers is increased as shown in Table 1 and the need for a systematic testing procedure is quite evident.

For the comparison of running two chillers versus a single chiller of equal capacity, the percent savings may be determined from the
following expression

\[
\% \text{ Savings} = \left[ \frac{P_{\text{sing}} - (P_1 + P_2)}{P_{\text{sing}}} \right] \cdot 100\% \quad (24)
\]

If all chillers have the same performance characteristics on a percent power consumption versus percent load basis, the constants \( a_{ij} \) in Eqs. 22a and 22b become identical if they are operating at the same ECWT and it can be shown that the percent savings approach a constant as the total load approaches its maximum value. This is done by substituting the expressions for \( P_{\text{sing}}, P_1, \) and \( P_2 \) found through Eqs. 22a-b. As the load approaches its maximum value, all chillers approach their maximum capacities. Noting these conditions Eq. 24 simplifies to

\[
\% \text{ Savings} = [1 - \frac{P_{1,\text{max}} + P_{2,\text{max}}}{P_{\text{sing},\text{max}}}] \cdot 100\% = \text{Constant} \quad (25)
\]

The results of this analysis will now be applied to a variety of cases in order to demonstrate the optimization of several combinations of multiple chiller systems. In particular the results will be demonstrated on two and six chiller systems and each will be compared to a single chiller with a capacity equal to the sum of the system's capacity in order to determine the sizes of chillers which, while working together, consume the smallest amount of energy for various total loads and entering condenser water temperatures (ECWTs). The results are further applied to certain chilling systems at Virginia Polytechnic Institute and State University.
3. APPLICATIONS AND RESULTS

The results obtained in the previous analysis will now be applied to a variety of multiple chiller systems. First, a two chiller system is investigated in order to show the optimum operating levels for any given total cooling requirement and to demonstrate the effects of entering condenser water temperature (ECWT). Then a number of two chiller systems are compared with a single chiller system having the same total capacity in order to assess the relative savings in power consumption.

Next, the investigation looks at a variety of six chiller systems in order to compare with a portion of the existing system presently in use on the campus of Virginia Polytechnic Institute and State University (Va. Tech). In this way the relative advantages of a more energy efficient arrangement of the existing chillers can be assessed.

3.1 Performance Characteristics

The performance characteristics used are those of centrifugal Trane chillers [20] which are given in terms of percent power versus percent load as shown in Fig. 4a. Figure 4b shows the percent of full load cooling capacity divided by the percent full load power consumption versus percent load. This figure clearly illustrates the operating ranges of highest efficiency for the different ECWT's. For an ECWT of 65°F, the most efficient operating range occurs between 35 and 65 percent of total load, at 75°F this range is between 40 and 70 percent of total load and for an ECWT of 85°F this range is from 50 to 80 percent of total load. These chillers operate most efficiently at part load
Figure 4a. Performance characteristic of centrifugal chillers
Figure 4b. Illustration showing the most efficient operating ranges of centrifugal chillers

Figure 4c. Efficiency curves of centrifugal chiller for various capacities
conditions and as ECWT increases the most efficient operating range shifts to higher loads. Since these ranges of greatest efficiency occur at part load conditions, it immediately seems advantageous to divide a total cooling load requirement among several chillers as opposed to operating one large chiller.

Two typical examples from the Trane catalog [20] are used to obtain the maximum power consumed by the chillers through the use of linear interpolation. The examples include a 315 ton chiller consuming 196 kw and a 1,450 ton chiller consuming 991 kw of power. It should be noted that the tons per kw for the smaller machine is 1.61 while it is only 1.46 for the larger one. Figure 4c shows the efficiency curves for three different chiller capacities at an ECWT of 85°F. Due to this fact, the smaller chillers will be more efficient than the larger ones throughout this investigation.

3.2 Two Chiller Systems

Figures 5a-c show the operating levels versus total load, given on a percentage basis, for a system consisting of a 520 and a 1200 ton chiller at three different ECWT's, respectively. A minimum operating level of 20 percent full load capacity is placed on the chillers since the performance curves, shown in Fig. 4a, begin near this point. All graphs of Figs. 5a-c therefore start at a total load equal to 20 percent of the smaller chillers maximum capacity at which point there is no alternative but to operate the 520 ton chiller. At 44 percent of the system's total capacity there are two choices; operate either the 520 ton chiller or the 1200 ton chiller. At all ECW Ts the graphs show that
Figure 5a. Optimal operating levels of a 520 ton and 1200 ton chiller system at an ECWT of 65°F.
Figure 5b. Optimal operating levels of a 520 ton and 1200 ton chiller system at an ECWT of 75°F.
Figure 5c. Optimal operating levels of a 520 ton and 1200 ton chiller system at an ECWT of 85°F.
it is more economical to continue to run the 520 ton chiller at 46 percent of its maximum capacity instead of operating the 1200 ton chiller at 20 percent of its maximum capacity.

At 20 percent of the total system load there exists three choices; operate either the 520 ton chiller, the 1200 ton chiller or both chillers together. The graphs show that operating the 520 ton is the most economical choice for all ECWTs. This follows from Fig. 4b since the 520 ton chiller is operating at 66.2 percent of its maximum capacity which is a more efficient point than either the 28.7 percent level at which the 1200 ton chiller would be operating or the 20 percent level at which both chillers together would be operating. At small loads the figures show that it is best to run the smaller chiller up to its maximum capacity except for an ECWT of 65°F in order to take advantage of a more efficient operating range as shown in Fig. 4b. After the smaller chiller can no longer handle the total load, the larger chiller assumes the load up to the point where it becomes more economical to run both chillers at part load conditions. It is less efficient to run the larger chiller up to its maximum capacity before using both chillers, as can be explained by Fig. 4b. It is also apparent from Figs. 5a-c that the point at which both chillers come into operation occurs at higher total loads for larger ECWTs.

Once both chillers begin operating simultaneously their operating levels increase linearly as the total cooling requirement increases until the smaller chiller reaches its maximum capacity. This behavior is due to the parabolic nature of the power equations given by Eqs. 22a and 22b which results in a linear increase in operating levels as can be
seen by Eqs. 23a and 23b of the analysis section. At the point where the smaller chiller has reached its maximum capacity it continues to operate at full load and the larger chiller supplies the additional required cooling load until both chillers are operating at their maximum capacities.

Since Figs. 5a-c are given on a percentage basis, the results and trends are more universal than merely giving absolute operating levels versus absolute total loads. However, this method of presentation tends to obscure the fact that the larger chiller is supplying a greater portion of the cooling requirement when both machines are in operation even though the dashed line (representing the 1200 ton chiller) is below the solid line (representing the 520 ton chiller).

In conjunction with the results of Figs. 5a-c the percent savings versus percent total load of this two chiller system compared to a single chiller having the same total capacity are shown in Fig. 6 for the various ECWTs. The graphs begin at 20 percent of the system's total capacity as opposed to Figs. 2a-c which start at 20 percent of the small chiller's capacity. At low loads greater percent savings are experienced for higher ECWTs. The reason for this can be seen from Fig. 1b since the greatest efficiency range occurs at higher percent loads as ECWT increases and the single chiller is operating on the low end of its range so that it experiences its worst efficiency at high ECWTs while the 520 ton chiller experiences good efficiency. The percent savings are greater for the higher ECWTs up until about 50 percent of system capacity where all three curves converge to a savings of 2.1 percent. The reason for this convergence is shown in section 2.5 of the analysis by Eq. 25 which becomes
Figure 6. Percent savings of a 520 ton and 1200 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTs.
\[
[1 - \frac{339.6 + 815.9}{1180.1}] \times 100\% = 2.1\%
\]

It is also evident from Fig. 6 that the lower the ECWT the quicker the curve converges to its final value. This is due to the fact that the single chiller reaches its most efficient range quicker for a lower ECWT. Due to this converging behavior, after a load of 50 percent of system capacity, ECWT has little or no effect on percent savings although constant percent savings result in greater absolute savings as total load increases in magnitude.

It is clear from Fig. 6 that the greatest advantages occur at small to moderate part load conditions where the flexibility of a multiple chiller system can result in relatively high percent savings. Even at high to full load conditions the multiple chiller system still operates more efficiently than a single chiller.

Figures 7a-c show the percent savings versus percent total load for various two chiller combinations all having system capacities of 1720 tons compared with a single chiller of equal capacity for different ECWTs. Each graph begins at 20 percent of the system's total capacity. Table 3 lists the chiller capacities for each of the combinations. In the low load range the combinations having chiller capacities of nearly equal magnitudes experience greater percent savings compared to the combinations whose capacities differ greatly in magnitude. At these low loads, combination 3 (the 520 with the 1200 ton chiller) shows the greatest percent savings for all ECWTs since the 520 ton chiller is operating in a good efficiency range. Once the load increases enough so
Table 3. Various two chiller combinations used in Figs. 7a-c, 8a-d and various six chiller combinations used in Figs. 10a-c

<table>
<thead>
<tr>
<th>Combination</th>
<th>Chiller capacities (tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200, 1520</td>
</tr>
<tr>
<td>2</td>
<td>300, 1420</td>
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<tr>
<td>3</td>
<td>520, 1200</td>
</tr>
<tr>
<td>4</td>
<td>860, 860</td>
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<tr>
<td>5</td>
<td>100, 335</td>
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<tr>
<td>6</td>
<td>125, 310</td>
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<tr>
<td>7</td>
<td>175, 260</td>
</tr>
<tr>
<td>8</td>
<td>217, 218</td>
</tr>
<tr>
<td>9</td>
<td>328, 328, 328</td>
</tr>
<tr>
<td></td>
<td>328, 329, 329</td>
</tr>
<tr>
<td>10</td>
<td>200, 250, 300</td>
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<td></td>
<td>350, 400, 470</td>
</tr>
<tr>
<td>11</td>
<td>100, 100, 300</td>
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<tr>
<td></td>
<td>300, 585, 585</td>
</tr>
<tr>
<td>12</td>
<td>75, 75, 100</td>
</tr>
<tr>
<td></td>
<td>260, 260, 1200</td>
</tr>
</tbody>
</table>
Figure 7.1. Percent savings for various 2 chiller systems compared to a single chiller system in which all systems are operating at their optimal levels and have a total capacity of 1720 tons at an ECWT of 65°F.
Figure 7b. Percent savings for various 2 chiller systems compared to a single chiller system in which all systems are operating at their optimal levels and have a total capacity of 1720 tons at an ECWT of 75°F.
Figure 7c. Percent savings for various 2 chiller systems compared to a single chiller system in which all systems are operating at their optimal levels and have a total capacity of 1720 tons at an ECWT of 85°F.
that the 1200 ton chiller of combination 3 assumes the load, combination 4 has the greatest savings since the 860 ton chiller handles the load more efficiently. The point where combination 4 experiences the greatest savings occurs at higher loads for higher ECWTs. The 200 and 1520 ton chiller combination and the 300 and 1420 ton chiller combination produce the least percent savings at these low loads since the smaller chillers cannot handle the loads but instead the larger chillers are forced to operate in a very low efficiency range.

At an ECWT of 65°F and a load of 35 percent of the system's total capacity all combinations converge within a band of 0.6 percent savings, whereas at 75°F, the combinations do not all converge to this bandwidth until a load of 44 percent and at 85°F, a load of 49 percent is required before this convergence is met. This again shows that the curves converge quicker for lower ECWTs. It is clear from Figs. 7a–c that after 50 percent of the system's total capacity all combinations produce approximately the same savings for all ECWTs. This illustrates that for high total loads the ECWT and combination type has little or no effect on the percent savings. However, there is still an advantage in having a multiple chiller system since the absolute savings are significant due to the large loads involved even though the percent savings are small.

Figures 8a–d show the percent savings versus percent load for various two chiller combinations having total system capacities of 435 tons compared with a single chiller of the same capacity at various ECWTs. Table 3 lists the combination numbers along with the corresponding capacities of the chiller systems. Again the curves begin at 20 percent of the system's maximum capacity (435 tons), and they clearly
Figure 8a. Percent savings of a 100 ton and 335 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTs.
Figure 8b. Percent savings of a 125 ton and 310 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTS.
Figure 8c. Percent savings of a 175 ton and 260 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTs.
Figure 3d. Percent savings of 217 ton and 218 ton chiller system compared to a single chiller system where all systems are operating at their optimal levels at various ECWTS.
indicate that at low loads greater savings are experienced at higher ECWTs. Here the four graphs show that at low loads the greatest percent savings are produced by the combinations in which the chiller capacities differ the greatest in magnitude (a small chiller with a larger chiller). The chiller combination in Fig. 8a produces the greatest percent savings at loads less than a 100 tons, since the 100 ton chiller operates the most economically; however, at a load greater than 100 tons the percent savings in Fig. 8a drop down to the lowest savings of all combinations, since the 100 ton chiller can no longer handle the load and the 335 ton chiller is forced to come into operation in a poor efficiency range. The combination which produces the greatest savings for a particular low load is the one having the smallest chiller which can handle the load.

As the maximum capacities of the two chillers in a system approach equal values the family of curves converge smoother and quicker to their final value. Figure 8d shows that the three curves for the 217 and 218 ton chiller system converge to the final value at 46 percent of the system's total load. This is due to the fact that both chillers have come into operation and so they are now operating at the same maximum capacity percentage level as the single chiller. The 175 and 260 ton chiller combination converges within 0.1 of its final value at 44 percent of the system's total load. As the two chillers' capacities differ greater in magnitude, the curves require higher total loads before they converge to their final value. The curves of the 100 and 335 ton chiller system do not converge to their final value until reaching a load of 98 percent of the system's maximum capacity.
It is quite evident that the chillers operate most efficiently at part load conditions and their most efficient load range depends on the ECWT. For the two chiller system it is clear that at low loads the percent savings are greater for larger ECWTs. At high loads the ECWT has more of an effect on percent savings for the chiller combinations with different size capacities than the ones that were nearly equal. Also, as the two chiller systems approach their maximum capacities, the percent savings approach a constant which depends on the chillers' maximum power consumption alone and for lower ECWTs the curves converge quicker to their final value; however, at very high loads the combination type and ECWT has little or no effect on the percent savings.

3.3 Six Chiller Systems

The study of two chiller systems is now further extended to six chiller systems where various combinations are investigated and compared with a single chiller system having the same total capacity. First, a typical six chiller combination was chosen whose capacities in tons are 200, 250, 300, 350, 400, and 470 (1970 tons total) in order to determine the optimal operating levels at different ECWTs. The chillers' percent of full load operating levels are bar graphed for loads of 500, 1000 and 1500 tons and for ECWTs of 65, 75, and 85°F in Figs. 9a-c.

For a relatively small total load of 500 tons, Fig. 9a shows that the optimal combination consists of the three smallest chillers only. The operating level of the 250 ton chiller, whose capacity is in between the other two operating chillers, remains at 66.6 percent of its maximum capacity for all three ECWTs. The largest chiller (300 ton) of the
Figure 9a. Optimal operating levels for a six chiller system for a total load of 500 tons at various ECWTs.
Figure 9b. Optimal operating levels for a six chiller system for a total load of 1000 tons at various ECWTs.
Figure 9c. Optimal operating levels for a six chiller system for a total load of 1500 tons at various ECWTs.
three operating at 500 tons load, operates at 61.9 percent of its maximum capacity at an ECWT of 65°F and increases to 63.7 percent at 85°F. The smallest chiller operates at 74.0 percent at 65°F and decreases to 71.2 percent at 85°F. Since the efficiency ranges increase with ECWT so does the operating level of the largest chiller, since it consumes the greatest portion of energy. The smaller machine operates at a higher percent full load compared to the larger machine since the smaller machine has a greater work output to energy consumption ratio and should take up as much of the total load as it can while still operating in a reasonably good efficiency range.

For a total load of 1000 tons, Fig. 9b shows that for an ECWT of 65°F, the optimal combination consists of all six chillers operating. Once again the smaller chillers are running at higher percent full loads than the larger ones. At ECWTs of 75 and 85°F, the optimal combination consists of 5 chillers, with the largest chiller not in operation. For these two temperatures it is evident that the operating levels in percent of full load of the two small chillers decrease as ECWT increases, the operating level of the middle chiller remains constant and the operating levels of the two larger chillers increase as ECWT increases with temperature as shown in Fig. 9b.

For a total load of 1500 tons, all six chillers are in operation for all three ECWTs as shown in Fig. 9c. Again, the operating levels in percent of full load for the two smaller chillers decrease, the middle two remain nearly constant, and the two largest increase as ECWT increases.
Figure 10a. Percent savings for various six chiller systems compared to a single chiller system in which all systems are optimized and have a total capacity of 1970 tons at an ECWT of 65°F.
Figure 10b. Percent savings for various six chiller systems compared to a single chiller system in which all systems are optimized and have a total capacity of 1970 tons at an ECWT of 75°F.
Figure 10c. Percent savings for various six chiller systems compared to a single chiller system in which all systems are optimized and have a total capacity of 1970 tons at an ECWT of 85°F.
Table 3 shows the combination number and the corresponding chiller capacities for the various six chiller systems. Figures 10a-c show the percent savings for these chiller combinations compared with a single chiller in which all systems have a capacity of 1970 tons. At an ECWT of 65°F combination 12, consisting of two 75, one 100, two 260 and one 1200 ton chillers, has the greatest percent savings for all total loads except for the very high loads where all combinations converge to a value of 9.1 percent. The final savings of 9.1 percent is significantly higher than the final savings that the two chiller systems experience. Combination 11, consisting of two 100, two 300 and two 585 ton chillers, produces the next greatest savings. The least percent savings occur for combination 9, which consists of four 328 ton chillers and two 329 ton chillers.

The fact that combination 12 has five small chillers having capacities of 260 tons and less shows that the more small chillers a system has the greater the percent savings are, since this type of system can generally match the total load more efficiently. Combination 9, consisting of chillers whose capacities are all greater than 260 tons, produces the smallest percent savings. This combination does converge the quickest to its final value at 46 percent of the system's maximum capacity. This quick convergence is due to the fact that the chillers of this combination practically have the same size capacities.

Similar trends occur for an ECWT of 75°F except for loads between 750 and 800 tons and between 950 and 1000 tons combination 11 instead of 12 shows the greatest savings as shown in Fig. 10b. At a temperature of 85°F, the trends are again similar except that combination 10 and 11
show greater savings than combination 12 at a load of 785 tons and combination 11 still experiences greater savings at a load of 1100 tons.

It is clear that the six chiller systems show potential savings compared with a single chiller system and in general it is advantageous to divide a total cooling requirement among several chillers, since a system of this type is more flexible in matching the load.

3.4 Virginia Tech System

Some of the existing chiller systems at Virginia Tech, which are studied in this part of the investigation, consist of six chillers operating in three separate systems. The first system consists of a single 1200 ton centrifugal chiller. The second system consists of two 260 ton absorption chillers and the third system consists of a 100 ton absorption and two 75 ton reciprocating chillers. The performance characteristics from Fig. 4a are used for the centrifugal as well as for the two reciprocating chillers while the performance characteristics for the three absorption units are taken from the Trane catalog [21] entitled "Absorption Cold Generator" and are shown in Fig. 11a. Figure 11b shows the ratio of percent load to percent power consumption and illustrates the operating ranges of highest efficiency for various ECW Ts. The performance characteristics of the absorption units are similar to the centrifugal units in that the most efficient operating ranges increase as ECWT increases; however, these operating ranges of greatest efficiency for the absorption units occur at higher loads for all ECW Ts. At 65°F, the range is from 50 to 80 percent of full load while at 75°F, it ranges from 50 to 90 percent and at 85°F, it ranges
Figure 11a. Performance characteristics of absorption units.

Figure 11b. Illustration showing the most efficient operating ranges of absorption units.
from 60 to 100 percent. Although absorption units use steam instead of electricity for the necessary energy supply in cooling the refrigerant, the analysis bases energy consumption in terms of kilowatts and it is assumed that the absorption units consume an equivalent amount of kilowatts at their maximum capacities as the other chillers.

The three separate systems operating independently are compared to one single combined system. In this way, the relative advantages of interconnecting the three separate systems can be investigated. The total load for the single system is divided up among the three separate systems by the use of (1) architect's estimated percent cooling load (2) the actual capacities of the chillers making up the systems and (3) averaging the values obtained from (1) and (2). Table 4 shows these three sets of fractions.

In this part of the analysis a 40 percent minimum is placed on all chillers since the manual controls on the chillers at Virginia Tech range from 40 to 100 percent of their total capacity.

Figures 12a-c show the percent savings versus percent total load for the single combined system compared to the three separate systems for ECWTs of 65, 75, and 85°F. Figure 12a shows the percent savings using the actual capacities in dividing up the total load among the three systems. Since this method allots 60.9 percent of the total load to system one, which solely consists of the 1200 ton chiller, a total load of 790 tons (40 percent of the system's total load) is necessary to satisfy the 40 percent minimum requirement. Using these fractions a total load of 1970 tons (100 percent of the system's maximum capacity) may be reached without any of the chillers going beyond their maximum
Table 4. Fractions used in dividing up the total load among the three chiller systems

<table>
<thead>
<tr>
<th>System Number</th>
<th>Chiller Capacities</th>
<th>Fractions Derived from</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Architect's Estimate</td>
</tr>
<tr>
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<td>1200</td>
<td>.751</td>
</tr>
<tr>
<td>2</td>
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<td>.155</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
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<td></td>
<td>75</td>
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Figure 12a. Percent savings of a single system compared to three separate systems at Virginia Tech using fractions of total load obtained from the actual chiller capacities.
Figure 12b. Percent savings of a single system compared to three separate systems at Virginia Tech using fraction of total load obtained from architect's estimate.
Figure 12c. Percent savings of a single system compared to three separate systems at Virginia Tech using fractions of total load obtained by averaging other two methods.
capacities. Initially as ECWT increases so does the percent savings. At 65°F and 41 percent of the system's maximum load, there is only a 0.3 percent savings where as at 85°F there is a 5.8 percent savings. At loads of 66 percent of the system's maximum capacity and greater, the ECWT does not have much of an effect on percent savings. As the total load approaches the system's maximum capacity, the percent savings approach zero, since all machines of both the separate systems as well as the combined system, are operating at 100 percent of their maximum capacity.

Figure 12b gives the percent savings using the architect's estimate in obtaining the portions of total load that three separate systems must supply. The curves start at 34 percent of the system's total load, since the two 260 ton chillers, which take 15.5 percent of the total load, just satisfy the 40 percent minimum constraint. The curves end at a load of 81 percent of the system's maximum capacity, since the 1200 ton chiller system, taking 75.1 percent of the total load, reaches its maximum capacity. Initially the greater the ECWT the greater the savings. For an ECWT of 85°F and a total load of 36 percent of the system's maximum capacity (700 tons) the percent savings is 17.4 while at 41 percent (800 tons) they are only 6.3. The reason for this sharp decrease in percent savings is that at 700 tons the five small chillers can handle the entire load while at a load of 800 tons the large 1200 ton chiller is forced to come into operation. This behavior also exists for ECWTs of 65 and 75°F.

At a load of 1,025 tons (52 percent of the system's maximum capacity), the 85°F curve goes below the other two curves and remains
Figure 13. Percent savings of a single system compared to three separate systems at Virginia Tech for an ECWT of 85°F.
below for the duration of the remaining loads. The 65°F curve shows the greatest savings at 1,025 tons but dips below the 75°F curve at 1,125 tons (57 percent of the system's maximum capacity). At loads of 1,250 tons (63 percent) and greater, the 65°F curve again shows the greatest savings.

Figure 12c shows the percent savings using the average of the architect's estimate and the actual capacities in obtaining the fractions of the total load for the three systems. As can be expected the 85°F curve again shows the greatest savings at low loads. At a load of 975 tons (49 percent of the system's maximum capacity), the 75°F curve shows the greatest savings and at a load of 1025 tons (52 percent) the 65°F curve produces the greatest savings. This behavior continues until a load of 1250 tons (63 percent) is reached where the 65°F curve gives the greatest savings all the way to 86 percent of the system's maximum load (1700 tons) where the 85°F curve crosses over.

Figure 13 shows the percent savings for the three different sets of fractions for an ECWT of 85°F. The three curves can only be compared for loads between 790 tons and 1,595 tons (40 percent and 81 percent of the system's maximum capacity), since this is where all three curves are simultaneously present. At these loads, the curve developed using the fractions from the architect's estimate shows the greatest percent savings which range from 4.9 to 7.3. For these same loads the fractions obtained from using the actual capacities produce savings ranging from 2.1 to 5.8 percent. The savings produced using the average of the other two fractions ranged from 3.6 to 5.7 percent.

It is obvious that there are potential energy savings by combining
these three existing chiller systems at Virginia Tech into a single system and that these savings depend on how the total load is divided up among the three separate systems.

In order to determine if it is indeed economical to combine the three separate systems into a single system, a payback period must be determined. To accomplish this, the initial investment for the additional piping and installation needed to combine the systems must be determined. Also, a dollar estimate is needed of the yearly energy savings that would be experienced if the three systems were combined. This is done by first choosing the method of dividing the total load up among the three systems which most closely models the operating conditions and by predicting the amount of time that the chillers operate at each load. From this, the total annual amount of energy saved can be obtained and this value can be multiplied by the present electrical rate to obtain the dollar value. If the electrical energy consumed by the chillers contributes to the electrical demand charge, then extra savings would be experienced but it should be remembered that the absorption units run off of steam instead of electricity so these machines do not effect demand charge.
4. SUMMARY AND CONCLUSIONS

As was pointed out in the introduction, the primary electrical consumer in many large buildings is the compressor of the air conditioning system. Due to this fact and the ever increasing electrical costs, it is more and more advantageous to optimize the loading of multiple chiller systems. As shown in the analysis, this minimization process is carried out by obtaining the performance characteristics of each chiller and applying the principle of the Lagrange multiplier. It is found that all chillers should operate at an equal incremental power consumption known as the system lambda \[19\]. Once the optimal operating levels are determined for a particular combination they are subjected to the operating constraints of the chillers. This is done for each combination in order to determine the combination which produces the smallest amount of energy consumed.

The mathematical formulation is derived for the general case of \(N\) chillers having arbitrary performance characteristics and is then applied to chillers having parabolic performance characteristics, since this is usually what occurs in practice. The general results are then further applied to the special case of a two chiller system.

Based on the analysis, it is evident that the parameters needed to determine the optimal operating conditions of a multiple chiller system are the capacity, maximum power consumption, the performance characteristics and the entering condenser water temperature (ECWT) of each chiller. Once all these parameters are set, the optimal operating conditions depend only on the desired total cooling load.
Both two and six chiller systems were studied to demonstrate the effects that ECWT has on the optimal operating levels. These chiller systems were compared to a single chiller system having the same capacity to illustrate the potential savings of a multiple chiller system. All chillers were given performance characteristics of centrifugal chillers. It was shown that the chillers operate most efficiently at part load conditions and that the operating ranges of greatest efficiency increase as ECWT increases. For the two chiller system, the percent savings compared to a single chiller system increase as ECWT increases at low loads. The greatest advantages occur at small to moderate part load conditions where the flexibility of a multiple chiller system matches the load more efficiently. It was shown that the curves representing the percent savings converge to a final value which is solely dependent on the chillers' maximum power consumption. The lower the ECWT the quicker the curves converge to their final values. In a system where the chillers' maximum capacities approach equal values, their curves converge more rapidly. At high loads, ECWT and the combination type has little, if any, effect on percent savings.

For six chiller combinations where all chillers are in operation, the optimal operating levels of the small chillers increase as ECWT increases while the operating levels of the large chillers decrease. The chiller systems having nearly the same size capacities converge to their final values the quickest. In general, the more small chillers a system has the greater the percent savings are experienced, since this type of system can match the total load more efficiently. It is obvious that a multiple chiller system is more energy efficient than a single chiller if it is operated in the most efficient manner.
Three separate systems at Virginia Tech were compared to the same chillers in one combined system to determine the potential energy savings of interconnecting the separate systems. The total load of the single system was divided among the three separate systems by using the actual capacities of the chillers and by using the architect's cooling load estimate and by averaging these two methods. There are potential energy savings by combining the three existing chiller systems at Virginia Tech and the amount of savings depend on how the total load is divided among the three separate systems.

In general, a required cooling load can be matched more efficiently in terms of power consumption by dividing the load among several small chillers as opposed to running one large chiller. However, the overall decision in choosing a multiple chiller system to meet a cooling requirements becomes a compromise between the energy savings experienced and the initial investment. As electric rates rise, it becomes more economical to spend more on a multiple chiller system. It should be noted that the multiple chiller system must be optimized in order to achieve the maximum of the potential energy savings.
REFERENCES


Appendix A: Counting Schemes
The simultaneous solution of Eqs. 8 and 9 results in possible operating levels, \( L_1, L_2, \ldots, L_N \) for a multiple chiller system. However, these solutions must be considered only as mathematical possibilities since the operating constraints given by Eq. 4 have not been taken into account. For the case of arbitrary performance curves, the end points of these constraints (i.e., \( L_1 = 0, L_1 = L_1, \text{min}, \) or \( L_1 = L_1, \text{max} \)) must be systematically tested to ensure that an absolute minimum in power consumption is obtained. The derivation for the number of combinations to be tested, given by Eq. 10, will now be developed.

The total number of outcomes for \( \omega \) independent trials, each having \( m \) possible outcomes is

\[
n = m^\omega \tag{A-1}
\]

If the trials are dependent on each other then the total number of outcomes is expressed as

\[
n = m_1 \cdot m_2 \cdot m_3 \cdots m_\omega \tag{A-2a}
\]

For arranging \( m \) objects in sequence, \( m_1 = m, m_2 = m-1, m_3 = m-2, \ldots, m_\omega = 1 \) and Eq. A-2a becomes

\[
n = m! \tag{A-2b}
\]

This may be interpreted as the number of arrangements or permutations of \( m \) objects among themselves. The number of permutations of \( x \) objects from a total of \( m \) objects is
\[ m^P_x = m \cdot (m-1) \cdot (m-2) \ldots (m-x+1) \]

or

\[ m^P_x = \frac{m!}{(m-x)!} \quad 0 \leq x \leq m \]  \hspace{1cm} \text{(A-3)}

The number of combinations of \( x \) objects selected from \( m \) objects disregarding the order is simply the number of permutations divided by the number of arrangements of the \( x \) objects [22].

\[ \binom{m}{x} = \frac{m^P_x}{x!} \]

or

\[ \binom{m}{x} = \frac{m!}{(m-x)! \cdot x!} \quad 0 \leq x \leq m \]  \hspace{1cm} \text{(A-4)}

For the total number of combinations, Eq. A-4 is summed from \( x=1 \) to \( x=m \) written as

\[ n = \sum_{x=1}^{m} \frac{m!}{(m-x)! \cdot x!} \]  \hspace{1cm} \text{(A-5)}

The following principles above may be extended to determine the number of possible combinations that exist for \( N \) trials each having \( K+1 \) different outcomes illustrated in the matrix below
and satisfying the following two conditions. At least one outcome can be chosen from column 1 and only one outcome can be chosen from each row. This implies that once the outcomes from row one are chosen, the other trials are independent. The number of combinations from row 1 alone without regard to the order of arrangement is

\[
\sum_{x=1}^{N} \frac{N!}{(N-x)! \cdot x!}
\]

where \( x \) represents the number taken from column one. Once the outcome from the first column are specified there are \( N-x \) rows (trials) left, each having \( K \) possible outcomes. Since these trials are independent there are \( K^{(N-x)} \) additional outcomes. Therefore for \( x \) trials the number of possible outcomes is

\[
\frac{N!}{(N-x)! \cdot x!} \cdot K^{(N-x)}
\]

(A-6a)

and the total number of outcomes, \( n \), is
\[ n = \sum_{x=1}^{N} \frac{N}{(N-x)!} \frac{K^{N-x}}{x!} \quad (A-6b) \]

Applying this to the case of \( N \) chillers there are four possible outcomes; operating at the calculated level, not operating, operating at a minimum level or operating at a maximum level. Since there are four outcomes per chiller, \( K+1 = 4 \) or \( K = 3 \). The two conditions are: at least one operating level must be calculated and that each chiller can only operate at one of the four possible levels, i.e., only one possible outcome per chiller. Since there are four possible outcomes, \( K + 1 = 4 \) or \( K = 3 \) and Eq. A-6b becomes

\[ n = \sum_{x=1}^{N} \frac{N!}{(N-x)!} \frac{3^{N-x}}{x!} \quad (A-6c) \]

This proves the result used in Eq. 10.
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