SELECTING EXPANSION FACTOR AND NUMBER OF SAMPLING POSITIONS FOR POINT AND PLOT SAMPLING

by

Elizabeth A. Jones

Thesis submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Forestry

APPROVED:

R. G. Oderwald, Chairman

H. E. Burkhart

T. E. Burk

August, 1982
Blacksburg, Virginia

ACKNOWLEDGEMENTS

I wish to thank my committee of Drs. Harold E. Burkhart and Thomas E. Burk for their support and guidance. I also wish to thank Dr. Rickard H. Hokans for his interest and encouragement. I wish to express special thanks to my chairman Dr. Richard G. Oderwald whose patience and ideas helped to produce this thesis.

Special thanks are extended to Kenneth Farrar for his help and advise in the course of this study. I also thank my fellow graduate students whose humor and friendship have made these past two years so enjoyable. Finally, I thank my parents and David Reed for their love and support without which this would not have been possible.

CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

Over the years the demand for forest resources has greatly increased. This growing demand produces a need for accurate and efficient inventory procedures; management decisions are made on the basis of information acquired from these inventories. Point and plot sampling are used extensively in the United States for gathering inventory information. Many authors have compared the relative merits of point to plot sampling; however few of these comparisons have considered the effects that existing stand conditions play in the efficiency of a sampling procedure. By using all known information about a given stand, it is hoped that the efficiency of different sampling methods may be more easily evaluated.

Tree spatial distribution, diameter distribution, density, age, and site quality all affect the estimation efficiency of stand characteristics. Given varying types of stand conditions, measures of the efficiency and the associated costs need to be derived in order to evaluate the two sampling procedures. To achieve such measures approximations of the point and plot sampling variance need to be developed. These variances should reflect the role that stand characteristics and expansion factor play in both sampling methods. From these derivations, insight can be gained into the effects of expansion factor and number of sampling positions on the sample variance as well as the cost of sampling for each sampling method under varying stand conditions. The objectives of this study, then, were:

1) To develop approximations to the point and plot sampling variance which show explicitly the role of stand characteristics, expansion factor, and number of sampling positions.
2) To use the approximations to develop guidelines for selection of the expansion factor and the number of sampling positions for point and plot sampling.

LITERATURE REVIEW

Many studies have been concerned with evaluating sampling efficiency. In the past this problem has been approached in two different ways. Some studies have considered the optimization of plot sizes, shapes and arrangement as a way of evaluating the efficiency of a sampling scheme. Other studies have compared the relative efficiency of point and plot sampling in their estimation of particular stand characteristics.

Several studies have been conducted to optimize the use of plots. The efficiency of varying plot sizes,shapes, and arrangements has been evaluated. Johnson and Hixon (1952) found rectangular plots of size 0.20 acres to 0.33 acres to be more efficient in estimating total net sawtimber volume for old-growth Douglas-fir than other plot sizes or shapes. Mesavage and Grosenbaugh (1956) optimized plot size and arrangement for a given tract when sampling for volume. As the size of the plot decreased and the number of plots increased, the sampling error decreased enough to offset the increased cost of measuring more plots. Freese (1961) used the data from the two previously mentioned studies and developed a formula relating plot size to variability; this relationship varies depending on the nature of the population. O'Reagan and Palley (1965), O'Reagan and Arvanitis (1966), and Prodan(1968) also used a formula to determine the optimal number of plots as a function of size, but found it to be inefficient for forest data. Zeide (1980) defined the optimal plot size as a function of the total time necessary for location and measurement of the plot; this was applied to simple random sampling and systematic sampling on a square lattice.

Some studies have compared the efficiency of different plot sizes and basal area factors (BAF). Husch (1955) examined the efficiency of point sampling estimates of basal area and volume with the use of BAF 2.5, 10, and 40; he also considered the time factor required to sample a point. His results indicated that a small BAF tended to underestimate basal area and volume, and based on accuracy and required time, BAF 40 consistently gave the best results. Point sampling appeared to work best with a high expansion factor used at the greatest number of sampling points. Avery and Newton (1965) compared BAF 10 point samples to varying plot sizes in bottomland hardwood and loblolly and shortleaf pine stands in Georgia. Expansion factors of size 10 in either point or plot samples were recommended in the bottomland hardwood stands where the average dbh was 12 inches, tenth and twentieth acre plot samples were recommended in the pine stands where the average dbh was 9 inches, and if cost could be ignored, fifth acre plots were recommended in the high value sawtimber pine stands. Kulow (1966) compared several plot sizes to several basal area factors and found that the sampling precision and accuracy was proportional to the size of the expansion factor. Zeide and Troxwell (1979) attempted to minimize the bias in point samples in Appalachian hardwoods through optimizing BAF and number of trees per point.

Comparisons between point and plot sampling have been made since point sampling was first introduced into the United States by Grosenbaugh (1952). These comparisons have ranged from repeated sampling surveys to simulations of sampling surveys. Generally these evaluations have dealt with determining which sampling method was more efficient for estimating a particular stand characteristic.

Grosenbaugh and Stover (1957) compared point to plot sampling and found point sampling to have the highest efficiency relative to other sampling methods. Kirby (1965) justified the use of point sampling economically; it was faster and concentrated on the larger crop trees. In each case, the comparisons of point to plot sampling were made on the same land areas.

Several studies were based on simulation trials. Palley and O'Regan (1961) and O'Regan and Palley (1965) used simulation trials in mapped stands. They found that the relative variability of total basal area and total volume were lower for point sampling than for plot sampling. The reverse was true when considering the variability of number of trees. The variance of plot sampling estimators was inversely proportional to the number of trees included in the sample and the variance of point sampling estimators was proportional to the basal area factor. O'Regan and Arvanitis (1966) and Arvanitis and O'Regan (1967) considered the cost of estimation as well as the variance of the estimates. These costs were expressed in terms of walking time between sampling positions, establishment of point or plots, and tree measurement on that point or plot. They found point sampling to be more cost effective than plot sampling in the estimation of basal area, but less cost effective in the estimation of tree frequency. Sukwong et al. (1971) used simulated forest stands to determine the effect of spatial distribution, stand density, diameter distribution, and size and type of sampling plot on the efficiency of the point and plot sampling methods for basal area. The coefficient of variation was used to evaluate the comparisons; point sampling was found to be more efficient in estimation of basal area.

Some comparisons between the two sampling methods have considered tree spatial patterns. Analytically deriving the variance of point and plot sample basal area estimates, Holgate (1967) found point sampling to be more precise than plot sampling for the same degree of sampling effort. These comparisons were developed for trees with random spatial patterns only. Sukwong et al. (1971) and Matern (1972) developed studies that considered both clumped random and completely random spatial patterns and found that as the degree of clumping increased, the coefficient of variation increased. Oderwald (1979, 1981) showed point sampling to be more precise for basal area estimates in random and clumped stands; plot sampling was more precise in square lattice stands.

MATERIALS AND METHODS

The initial step in this study was to develop approximations to the variance of estimates of total basal area per acre and total volume per acre for both point and plot sampling. The variance approximations were evaluated numerically, and cost functions and confidence intervals were then applied to develop guidelines for point and plot sampling in varying stand conditions.

Variance Approximation

A single estimator was applied to both point and plot sampling; this may be an estimate of total basal area per acre or total volume per acre. All variance approximations were for the random spatial pattern. The notation used throughout this study was:

$\hat{n}$ = estimate of the number of trees included at each sampling position

n = expected number of trees included at each sampling position

n_i = number of trees included at sampling position i

EF = size of expansion factor (basal area factor in point sampling, reciprocal of plot size in plot sampling)

m = number of sampling positions

λ = true number of trees per acre

$\hat{r}$ = estimate of the average volume basal area ratio on a tree in the stand

B = true total basal area per acre

$\hat{B}$ = estimate of the true total basal area per acre

V = true total volume per acre

$\hat{V}$ = estimate of the true total volume per acre

Looking first at the variances from a plot sample, the variance for total basal area per acre derived by Oderwald (1975) was:

$$V(\hat{B}) = \lambda\ (EF)\ E(\bar{b}^2)$$

where $\bar{b}$ is the estimate of the average basal area of a tree in the stand and b is the true average basal area of a tree in the stand. This may be rewritten as

$$V(\hat{B}) = \lambda(EF)\ (S^2_{\bar{b}} + b^2)$$

Similarly, the variance for the estimate of total volume per acre in a plot sample may be written as

$$V(\hat{V}) = \lambda(EF)\ (S^2_{\bar{v}} + v^2)$$

where $\bar{v}$ is the estimate of the average volume of a tree in the stand and v is the true average volume of a tree in the stand. These results were restricted to a random spatial pattern.

In obtaining point sampling variances, the variance of n was needed. Assuming $\hat{n}$ was from a Poisson distribution, then for any spatial pattern

$$n = \frac{B}{EF}$$

for a point sample where EF was the basal area factor, and

$$n = \frac{\lambda}{EF}$$

for a plot sample, where EF was the reciprocal of the plot size. From this the variance of $\hat{n}$ in a random spatial pattern may be written as

$$V(\hat{n}) = \frac{B}{EF}$$

for point sampling and

$$V(\hat{n}) = \frac{\lambda}{EF}$$

for plot sampling (Oderwald,1975).

The estimate for total basal area per acre in a point sample may be written as

$$\hat{B} = EF * \hat{n}$$

and the variance of the estimate would be

$$V(\hat{B}) = V(EF * \hat{n})$$

$$= EF^2 \; V(\hat{n})$$

but, as seen previously,

$$V(\hat{n}) = \frac{B}{EF}$$

and thus,

$$V(\hat{B}) = EF * B$$

Similarly, the estimate of total volume per acre in a point sample may be written as

$$\hat{V} = \hat{B} * \hat{r}$$

where $\hat{r}$ is as previously defined. Thus

$$V(\hat{V}) = V(\hat{B} * \hat{r})$$

$$= E(\hat{B}\hat{r} - Br)^2$$

This may be rewritten using a method presented by Sukhatame and Sukhatame (1970):

$$V(\hat{V}) = E(\hat{B}\hat{r} - Br)^2$$

$$= E\{(B + \varepsilon_B)(r + \varepsilon_r) - Br\}^2$$

$$= E(Br + r\,\varepsilon_B + \varepsilon_B\,\varepsilon_r - Br)^2$$

$$= E(r\varepsilon_B + B\varepsilon_r + \varepsilon_B\varepsilon_r)^2$$

where ε_B = error associated with average basal area per acre and ε_r = error associated with the ratio r ; $E(\varepsilon_B) = E(\varepsilon_r) = E(\varepsilon_B\varepsilon_r) = Cov(\varepsilon_B\varepsilon_r) = 0$. Taking the expectation of the estimator,

$$V(\hat{V}) = E(r^2\varepsilon^2_B + B^2\varepsilon^2_r + \varepsilon^2_B\varepsilon^2_r + 2rB\varepsilon_B\varepsilon_r$$
$$+ 2r\varepsilon^2_B\varepsilon_r + 2B\varepsilon_B\varepsilon^2_r)$$

and looking only at errors to the first approximation,

$$V(\hat{V}) = r^2 E(\varepsilon^2_B) + B^2 E(\varepsilon^2_r)$$
$$= r^2 V(\hat{B}) + B^2V(\hat{r})$$

where $E(\varepsilon^2_B)$ = the variance of $\hat{B}$ and $E(\varepsilon^2_r)$ = the variance of $\hat{r}$.

Validation of the Variance

The validity of the approximation was determined by comparing the approximations with the results of simulated samples. The simulations were performed using a modified version of a computer program INDEX FORTRAN developed by Watson (1981). Because of the wide variation in simulation results, twenty samples were simulated for each set of stand conditions and the results were then averaged; graphs indicated that the variances stabilized after twenty samples. The average was then compared to the approximated variance.

Stands of differing conditions were generated as input for the simulation program. These conditions varied across a range of site indices, densities, and ages. The site indices used were 40, 50, and 60 feet (base age 25) and the densities considered were 200, 300, and 400 trees per acre. The ages of the stands were 15 to 25 years.

These generated stands were square with 524 feet on a side for a total of 9 acres. Each tree was randomly assigned an X and Y coordinate from a discrete uniform (0,524) distribution. In this manner a random spatial pattern was achieved.

The trees within these generated stands were given the characteristics of planted loblolly pine. Given specific stand conditions, the heights of dominant trees were calculated from a formula presented by Smalley and Bower (1971)

$$\log H_D = \log SI - 2.460976\ (\ 1/\sqrt{A} - 1/\sqrt{25}\ )$$

where H_D = height of dominants, SI = site index, and A = age. The diameters for each tree were then calculated from a 3 parameter Weibull function. The parameters for the Weibull function were calculated from equations presented by Smalley and Bailey (1974).

$$a = -1.5254 + 0.06394H_D$$

$$a + b = -6.6951 - 0.0008T_s + 7.5104H_D$$

$$c = 3.3542 + 0.0002A$$

where T_s = trees surviving and all other variables are as previously defined. Total height for each tree, given its diameter, was calculated from an equation presented by Feduccia et al. (1979) and was as follows

$$\log (H_D/H_i) = -0.032876 + (1/D_i - 1/DMAX) \{1.9930 + 2.5047AT_s (10^{-5}) + 0.0043249T_s/A - 1.0360\log (T_s/A) - 0.026038\log (H_D/A)\}$$

where H_i = height of i^{th} tree, D_i = dbh of i^{th} tree, DMAX = maximum diameter in stand, and all other variables are as previously defined. Volume for each tree was computed to a 4-inch top. The volume equation used was developed by Burkhart et al. (1972):

$$CV_{ob} = -3.7097 + 0.00233 (D^2H)$$

In this simulation every tree has a known diameter, total height, basal area, cubic-foot volume, and position. Table 1 contains the average tree values and the average stand characteristics per acre for each simulated stand. Point and plot samples were then taken from each 9 acre tract based on a ten percent cruise with the percent cruise of the point samples based on average dbh.

Table 1. Tree and stand values for different stand conditions resulting from simulations.

Stand Type[a]	Basal Area (sq.ft./ac.)	Basal Area (sq.ft./tree)	Volume (cu.ft./tree)	Ratio (vol/ba/tree)	Variance of Basal Area (sq.ft./tree)	Variance of Volume (cu.ft./tree)	Variance of Ratio (vol/ba/tree)
(200,15,40)	84.69	0.4424	5.41	11.99	0.0418	7.3828	0.3158
(200,25,40)	105.22	0.5365	9.02	16.40	0.0541	18.9438	1.0535
(200,15,50)	97.63	0.5004	7.71	15.55	0.0495	13.8793	0.5438
(200,25,50)	119.78	0.6029	12.65	20.45	0.0715	38.8094	1.7057
(200,15,60)	109.78	0.5570	10.30	18.13	0.0604	24.3067	0.8327
(200,25,60)	134.67	0.6756	17.17	24.88	0.0746	59.0253	2.0552
(300,15,40)	79.66	0.2952	3.67	12.27	0.0136	2.4618	0.1966
(300,25,40)	105.80	0.3647	6.09	16.35	0.0216	7.5615	0.8958
(300,15,50)	96.72	0.3410	5.27	15.22	0.0198	5.5842	0.3868
(300,25,50)	125.94	0.4253	8.90	20.48	0.0281	15.3757	1.3514
(300,15,60)	112.20	0.3851	7.11	18.15	0.0257	10.3015	0.6179
(300,25,60)	143.15	0.4789	12.12	24.81	0.0343	27.2700	1.8000

Table 1 (continued)

Stand Type[a]	Basal Area (sq.ft./ac.)	Basal Area (sq.ft./tree)	Volume (cu.ft./tree)	Ratio (vol/ba/tree)	Variance of Basal Area (sq.ft./tree)	Variance of Volume (cu.ft./tree)	Variance of Ratio (vol/ba/tree)
(400,15,40)	77.50	0.2345	2.97	12.56	0.0100	1.3600	0.1000
(400,25,40)	110.30	0.2935	5.00	16.76	0.0128	4.5570	0.5757
(400,15,50)	99.43	0.2745	4.29	15.46	0.0108	3.0334	0.2156
(400,25,50)	134.68	0.3459	7.37	20.96	0.0167	9.2468	0.9029
(400,15,60)	118.46	0.3116	5.86	18.60	0.0149	6.0546	0.3332
(400,25,60)	156.72	0.3960	10.13	25.17	0.0210	16.6228	1.4525

[a] Stand Type - (trees per acre, age, site index)

Point centers and plot centers were randomly chosen X and Y coordinates from a discrete uniform (0,524) distribution and the size of the samples was dependent on the expansion factor desired. The expansion factors used were basal area factors of 5, 10, 20, and 40 square feet per acre and plot sizes of 0.025, 0.05, 0.10, and 0.20 acres. In each simulation five plots or points were taken for an expansion factor 5, nine plots or points were taken for an expansion factor 10, 20 plots or points were taken for an expansion factor 20, and 40 plots or points were taken for an expansion factor 40. Equal sample sizes were used in order to standardize the variances for comparisons. Simulation results included estimates of the total basal area per acre and the total volume per acre and their associated variances.

Cost Analysis

Once validation had been completed two approaches were taken to determine the appropriate expansion factor and number of sampling positions for a particular situation. The first approach was to minimize the variance for a given fixed cost and to minimize the cost for a given fixed variance; this was applied to both point and plot sampling estimates of basal area and volume. The second approach was to control confidence interval width.

The cost function used in this study was exponential in form; the more trees there were per sampling point, the more each tree would cost to measure. The cost function chosen was used by Oderwald (1975).

$$C = c_o + mc_m + mnc_n$$

where c_o was the fixed cost, c_m was the cost of establishing a sampling point, m was the number of sampling points or plots, n was the average number of trees per sampling position, and c_n was the cost of measuring a tree on that sampling position. This function was rewritten to show the relationship of sample size to measurement cost per tree.

$$C = c_o + mc_m + mn\alpha e^{\beta n}$$

where

$$c_n = \alpha e^{\beta n}$$

The coefficents α and β were estimated from time to sample data in Kulow (1966) which resulted in $\alpha = 0.271$ and $\beta = 0.022$ where time is in minutes. In the analysis that follows C and c_0 were assumed to be constant.

Minimization of the variance under the constraint of the cost function and minimization of the cost function under the constraint of the variance were with respect to m, the number of sampling positions, and EF, the expansion factor. This was accomplished through the use of the Lagrange multiplier procedure.

Another expression used to determine the number of sampling positions was

$$m = \frac{t^2 S^2}{E^2}$$

where S^2 was the derived variance approximation, t was the t value taken from student's t-tables, and E was the bounds within which estimation was desired. These bounds were developed for the estimation of volume and basal area given particular stand conditions.

Comparisons between point and plot sampling were made using these results. For given stand conditions and stand type, the number of sampling positions and the expansion factor were evaluated for each sampling procedure and the point where point sampling trades optimality with plot sampling was determined.

RESULTS AND DISCUSSION

Variance Approximation

Values for the variables comprising the developed variance approximations were generated from simulations of various stand types. Given these stand and tree values, certain trends were found in the values of the variances. In a point sample, variance values increased as total basal area per acre increased. The same trend was true as the expansion factor increased or as the number of trees per acre increased. The variances for both total volume and total basal area per acre in plot samples decreased as the number of trees per acre increased; the greater available growing space in stands with fewer trees per acre provided for larger values of basal area and volume per tree which in turn increased the respective variances. As in point samples, the variances values increased as total basal area per acre increased and as the expansion factor increased.

Validation

Comparisons were made between the variance formula approximation and the variances from the simulations for point and plot sampling for expansion factors of 5, 10, 20, and 40. These comparisons were made for point and plot sample variances of total volume and total basal area per acre respectively. The differences between the formula approximation and the simulation results were standardized by dividing through by the simulation results and presented as percentage differences; comparisons were then made between the percentage differences and the absolute value percentage differences (Table 2).

In a point sample with variances of either total volume per acre or total basal area per acre few trends in the percent differences between the variance approximations and the variances of the simulations existed. Graphs have shown that for expansion factors 5 and 40, as the number of trees per acre increased the mean percent differences increased. For an expansion factor 10, as the number of trees per acre initially increased, the mean percent differences increased accordingly; however, at increasingly higher numbers of trees per acre, the mean percent differences then began to decrease. No trend in the percent differences was apparent as the number of trees per acre increased when an expansion

Table 2. Summary of the differences expressed as percentages between the formula approximated variances and the simulated variances across all populations and replications.

Variable	Minimum Percent Difference	Maximum Percent Difference	Mean Percent Difference	Standard Deviation of Mean Percent Difference	Mean Absolute Percent Difference
EF 5[a]					
PT VOL[b]	-30.1	+26.1	- 7.64	19.55	18.19
PT BA[b]	-32.6	+23.6	- 8.74	19.40	18.36
PL VOL[b]	-53.6	-26.2	-37.30	9.18	37.30
PL BA[b]	-54.4	-25.8	-37.70	8.70	37.70
EF 10					
PT VOL	-25.2	+34.6	- 4.77	16.28	14.63
PT BA	-25.6	+37.5	- 3.27	17.37	15.46
PL VOL	-27.8	+19.5	- 6.37	14.46	12.94
PL BA	-27.3	+19.9	- 6.44	13.12	13.91
EF 20					
PT VOL	-13.3	+ 5.4	- 3.75	17.56	7.00

Table 2 (continued)

Variable	Minimum Percent Difference	Maximum Percent Difference	Mean Percent Difference	Standard Deviation of Mean Percent Difference	Mean Absolute Percent Difference
EF 20 continued					
PT BA	-10.2	+ 6.9	- 1.30	17.34	6.30
PL VOL	- 9.7	+34.2	+ 4.40	14.64	11.00
PL BA	- 9.0	+39.2	+ 4.90	16.00	11.30
EF 40					
PT VOL	-16.6	+16.9	- 1.85	10.41	8.51
PT BA	-13.6	+20.3	- 0.71	10.49	7.29
PL VOL	-11.7	+23.2	+ 3.69	17.17	9.84
PL BA	-11.3	+28.5	+ 4.35	13.26	10.20
ALL EF					
PT VOL	-30.1	+34.6	- 4.57	16.30	12.69
PT BA	-32.6	+37.5	- 3.18	16.73	9.21
PL VOL	-53.6	+34.2	- 8.29	14.24	16.62
PL BA	-54.4	+39.2	- 8.18	13.06	17.24

Table 2 (continued)

a/ EF - Expansion Factor

b/ PT VOL - Point Sample Variance of Total Volume Per Acre

PT BA - Point Sample Variance of Total Basal Area Per Acre

PL VOL - Plot Sample Variance of Total Volume Per Acre

PL BA - Plot Sample Variance of Total Basal Area Per Acre factor 20 was applied. No trends existed for any expansion factor in any other varied stand or tree condition.

Variances of total basal area per acre in plot samples produced several trends in the percent differences. An expansion factor 5 gave low percent differences or underestimates of the variance for all values of total volume or total basal area per acre, average volume or average basal area per tree, as well as for all average volume-basal area ratios per tree. Expansion factor 20 had high mean percent differences or overestimates of the variance at higher values of total volume or total basal area per acre and at higher values of the average volume to basal area ratio per tree. No trends were apparent for expansion factors 10 and 40.

In plot samples with a variance of total volume per acre, expansion factor 5 again produced low percent differences or underestimates of the variance for all stand conditions. With an expansion factor 10 low mean percent differences, or underestimates of the variance, were found at higher values of total volume per acre, average volume or average basal area per tree, and average ratio of volume to basal area per tree; however, there is no apparent reason for these particular results. Higher values of total volume and total basal area per acre again resulted in higher percent differences or overestimates of the variance with an expansion factor 20 and, again, no trends in the percent differences were visible for any stand condition with an expansion factor 40.

As the value of the expansion factor increased from 5 to 10 to 20 to 40, the range between the maximum and the minimum percent differences decreased and the mean percent difference in variances came closer to zero. This was true for both point and plot samples with variances of either total volume per acre or total basal area per acre. All variance approximations seemed to improve as the size of the tree tally on either a point or a plot decreased.

Tests were made to determine if the mean percent differences could be considered not different from zero or unbiased. Variance approximations for either total volume per acre or total basal area per acre in both point and plot samples with expansion factors of 20 and 40 were considered unbiased ($\alpha > 0.10$); point samples with variances of either stand characteristic and expansion factors of 5 and 10 were considered unbiased as well ($\alpha > 0.10$). For the variance approximations of total volume and total basal area per acre, plots with an expansion factor 10 had p-values between 0.05 and 0.025. The resulting p-values for an expansion factor 5 with plot sample variance approximation of either stand characteristic were very low (α < 0.005) indicating that the mean percent differences can not be assumed unbiased. It appears that some factor or variable is unaccounted for in these plot sample variance formula approximations which has resulted in these biased results.

The point sample variance of total basal area per acre generally had smaller mean percent differences than the point sample variance of total volume per acre; there was little difference between the two plot sample variance approximations. Point sample variances for either stand characteristics had smaller mean percent differences than the respective plot sample variances and an overall comparison of all variance approximations found the approximation for total basal area per acre in a point sample to more closely estimate the true variance. Though the point sampling variance approximation for total basal area per acre was the best, all were considered good in estimation of the variance and all were generally considered unbiased.

Cost Analysis

Two approaches were used to evaluate point and plot sampling procedures and determine where they traded optimality. The initial approach in this cost analysis was to minimize the variance for a given fixed cost and to minimize the cost for a given fixed variance through the use of Langrange multipliers. Another approach taken involved the use of confidence intervals.

Minimization of Variance and Cost

The functions u_1 and u_2 minimizing the variance under constraint of a fixed cost and minimizing the cost under constraint of a specified variance respectively and were

$$u_1 = \frac{\text{Var}}{m} + w_1 \left(c_o + mc_m + mn\alpha e^{\beta n} - C\right)$$

$$u_2 = c_o + mc_m + mn\alpha e^{\beta n} + w_2 \left(\frac{\text{Var}}{m} - S^2\right)$$

where w_1 and w_2 were the respective Langrange multipliers, Var was the respective variance approximation, and all other terms were as previously defined. the expected value of n at a randomly chosen point center is n=B/EF, and in a plot sample the expected value of n at a randomly chosen plot center is n= λ/EF. These two functions were substituted for all terms of n depending on which sampling technique was used; the respective variance formula approximations for total volume per acre and total basal area per acre in either sampling procedure were substituted for the Var terms of u_1 and u_2.

In a point sample with the variance of total basal area per acre, the functions u_1 and u_2 become

$$u_1 = \frac{(EF)(B)}{m} + w_1 \left(c_o + mc_m + \frac{mB}{EF}\alpha e^{\beta B/EF} - C\right)$$

$$u_2 = c_o + mc_m + \frac{mB}{EF}\alpha e^{\beta B/EF} + w_2\left(\frac{(EF)(B)}{m} - S^2\right)$$

Differentiating u_1 with respect to m, EF, and w_1, setting each equal to zero, and solving for optimal values of m and EF resulted in the functions

$$m = \frac{C - c_o}{(c_m + B\alpha e^{\beta B/EF}/EF)}$$

and

$$\frac{e^{\beta B/EF}}{EF^2} = \frac{c_m}{B^2\alpha\beta}$$

In the same manner, differentiation of u_2 gave

$$m = \frac{(EF)(B)}{S^2}$$

and

$$\frac{e^{\beta B/EF}}{EF^2} = \frac{c_m}{B^2 \alpha\beta}$$

The optimal function of EF was the same regardless of whether a specified variance was minimized or a fixed cost was minimized.

A point sample with a variance of total volume per acre resulted in

$$u_1 = \frac{(r^2)(EF)(B) + (B^2)V(\hat{r})}{m} + w_1(c_o + mc_m + \frac{mB}{EF}\alpha e^{\beta B/EF} - C)$$

$$u_2 = c_o + mc_m + \frac{mB}{EF}\alpha e^{\beta B/EF} + w_2(\frac{(r^2)(EF)(B) + (B^2)V(\hat{r})}{m} - S^2)$$

Differentiation of u_1 with respect to m, EF, and w_1, setting of each to zero, and solving for m and EF yielded

$$m = \frac{C - c_o}{(c_m + B\alpha e^{\beta B/EF}/EF)}$$

and

$$\frac{\alpha e^{\beta B/EF}}{EF^2}\left(\beta + \frac{V(\hat{r})}{r^2} + \frac{\beta B V(\hat{r})}{r^2 EF}\right) = \frac{c_m}{B^2}$$

With the same approach, differentiation of u_2 gave

$$m = \frac{(r^2)(EF)(B) + (B^2)V(\hat{r})}{S^2}$$

and

$$\frac{\alpha e^{\beta B/EF}}{EF^2} \left(\beta + \frac{V(\hat{r})}{r^2} + \frac{\beta B V(\hat{r})}{r^2 EF} \right) = \frac{c_m}{B^2}$$

Both optimal functions of m and expansion factor when minimizing a specified variance for a given fixed cost were the same for both the variance of total basal area per acre and total volume per acre.

Looking at plot samples with variances of total basal area per acre resulted in the functions

$$u_1 = \frac{\lambda EF(S^2_{\bar{b}} + b^2)}{m} + w_1(c_o + mc_m + \frac{m\lambda}{EF} \alpha e^{\beta\lambda/EF} - C)$$

and

$$u_2 = c_o + mc_m + \frac{m}{EF} \alpha e^{\beta\lambda/EF} + w_2(\frac{EF(S^2_{\bar{b}} + b^2)}{m} - S^2)$$

Solving for the optimal values of m and EF, differentiation of u_1 with respect to m, EF, and w_1 gave the equations

$$m = \frac{C - c_o}{(c_m + \lambda\alpha e^{\beta\lambda/EF}/EF)}$$

and

$$\frac{e^{\beta\lambda/EF}}{EF^2} = \frac{c_m}{\lambda^2\alpha\beta}$$

Following the same approach with u_2 yielded the equations

$$m = \frac{\lambda EF(S^2_{\bar{b}} + b^2)}{B^2}$$

and

$$\frac{e^{\beta\lambda/EF}}{EF^2} = \frac{c_m}{\lambda^2\alpha\beta}$$

Variances of total volume per acre in plot sampling resulted in the equations

$$u_1 = \frac{\lambda EF(S^2_{\bar{v}} + v^2)}{m} + w_1(c_o + mc_m + \frac{m\lambda}{EF}\alpha e^{\beta\lambda/EF} - C)$$

$$u_2 = c_o + mc_m + \frac{m\lambda}{EF}\alpha e^{\beta\lambda/EF} + w_2(\frac{\lambda EF(S^2_{\bar{v}} + v^2)}{m} - S^2)$$

Differentiating u_1 with respect to m, EF, and w_1 and solving for the optimal values of m and EF resulted in the equations

$$m = \frac{C - c_o}{(c_m + \lambda\alpha e^{\beta\lambda/EF}/EF)}$$

and

$$\frac{e^{\beta\lambda/EF}}{EF^2} = \frac{c_m}{\lambda^2\alpha\beta}$$

Solving for the optimal values of m and EF when differentiating u_2 gave

$$m = \frac{\lambda EF(S^2_{\bar{v}} + v^2)}{S^2}$$

and

$$\frac{e^{\beta\lambda/EF}}{EF^2} = \frac{c_m}{\lambda^2\alpha\beta}$$

The optimal function of EF was the same regardless of whether the variance was minimized or the cost was minimized for plot samples of either stand characteristic. The optimal function of EF in the point sample with a variance of total basal area per acre and the plot samples with variances of either total basal area or total volume per acre were similar; B and λ were interchangeable depending only on the sampling procedure used. Also, the optimal function of m when minimizing a specified variance for a given cost was similar in both point and plot sampling procedures for variances of either total basal area per acre or total volume per acre; again, B and λ were interchangeable in the function depending on the sampling procedure employed.

Optimal values of EF must be found through iteration. These values for the expansion factors can then be placed in the respective equations for m to find the optimal values of m.

Some limitations were encountered in using this particular cost function. Estimates of coefficients of α and β had to be within a certain range in order to produce a reasonable cost curve; there was a great deal of difficulty in obtaining these values. Several different data sets were examined, but all resulted in a negative value for β ; this resulted in a negative exponential curve where as the number of trees per acre increased, the cost in minutes of sampling increased at a decreasing rate. Also, the negative β values did not allow the iterative process to converge to zero when solving for expansion factor values. The only data set producing a positive value for β came from Kulow (1966) and thus, these values for α and β were chosen for this study. Other cost functions were tried with the various data sets, but if expansion factors could be calculated at all, they were not realistic values.

Optimal Expansion Factors

Figure 1 shows the optimal expansion factor values for a point sample with a variance of total basal area per acre at varying values of total basal area per acre and for different time costs when establishing a sampling point. These values were applicable to both the minimization of a given cost or the minimization of a specified variance. The values of the expansion factors decreased both as the time to establish a sampling point increased and as the total basal area per acre decreased.

Regardless of whether the variance of total volume per acre was minimized for a specified cost or a cost was minimized for a given variance the optimal expansion factor values were the same. Again, as the time to establish a sampling point increased and as the total basal area per acre decreased, the optimal expansion factor values decreased (Figure 2). These expansion factor values with a variance of total volume per acre were slightly larger for a given total basal area per acre than those found with the variance of total basal area per acre. Fewer trees needed to be sampled per point when either a given cost or variance was minimized for total volume per acre in comparison to total basal area per acre.

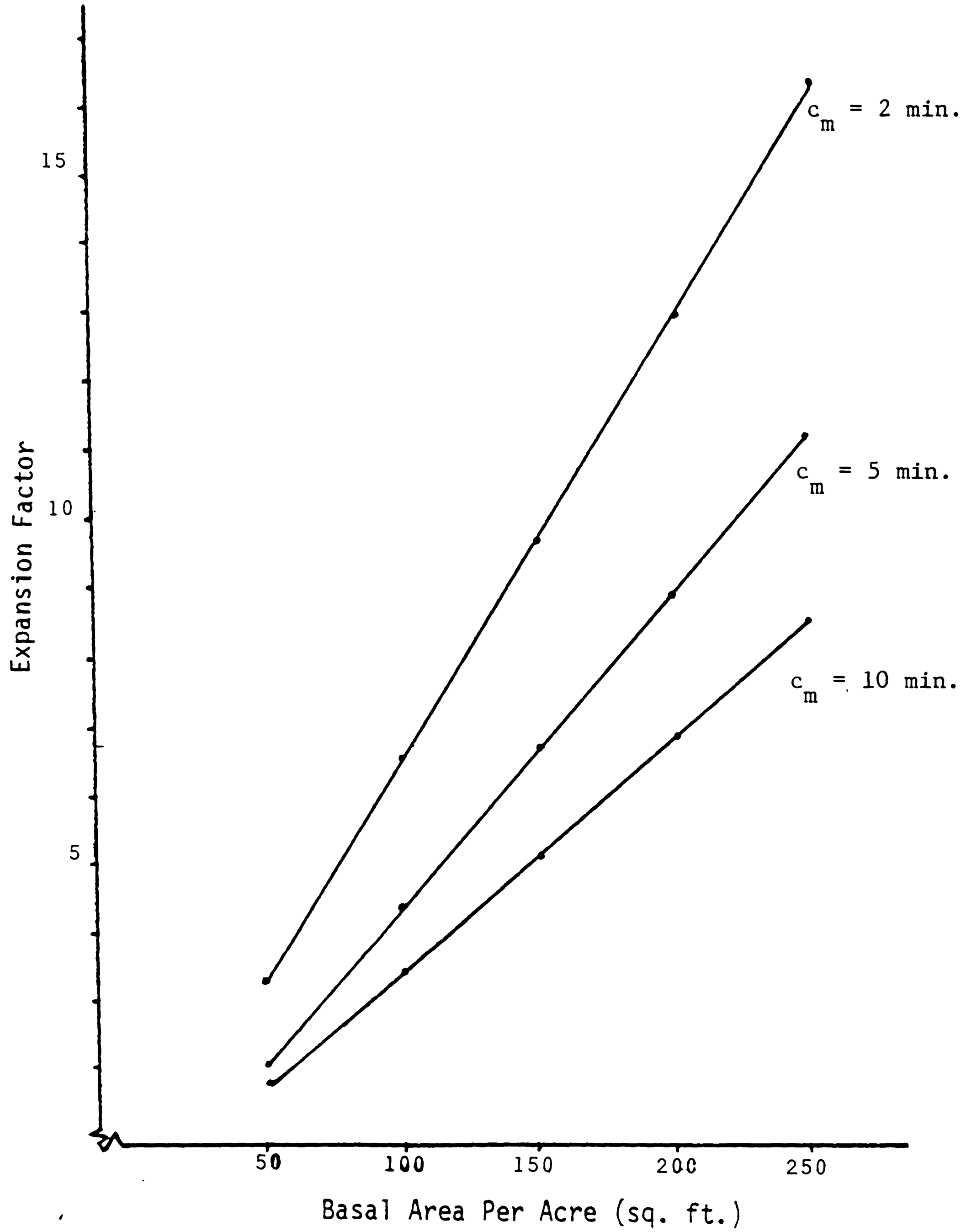

Figure 1. Optimal expansion factor values for a point sample with a variance of total basal area per acre at different time costs of establishing a sampling point.

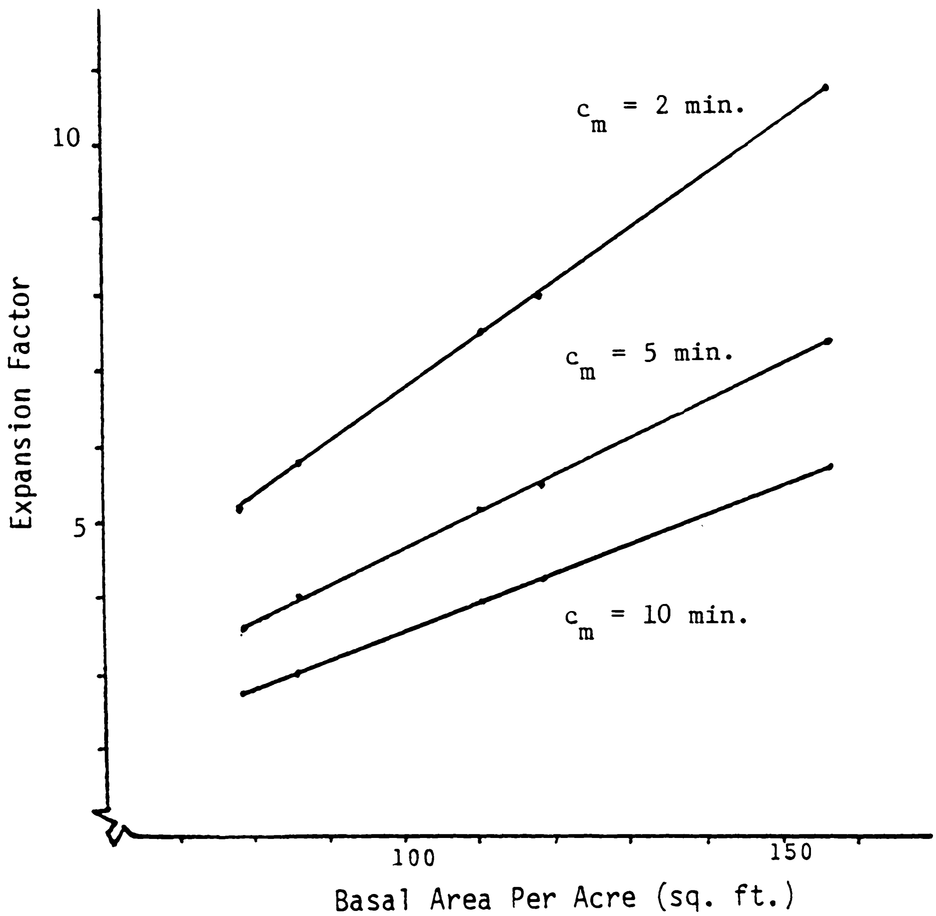

Figure 2. Optimal expansion factor values for a point sample with a variance of total volume per acre at different time costs of establishing a sampling point.

Values for the expansion factors in plot samples of both total basal area per acre and total volume per acre were the same regardless of whether the variance was minimized or the cost was minimized. As before, these expansion factor values decreased as the time to establish a sampling plot increased and as the total number of trees per acre decreased (Figure 3). As noted earlier, B and λ were interchangeable in the minimized function of EF and depended only on whether a plot or a point sample was employed when the respective variances were minimized; the expansion factor values are comparable for the point and plot samples only when λ and B are equal. However, values of λ will generally be larger than the values of B and thus, expansion factors should generally be larger for plot samples and fewer plots than points need to be taken.

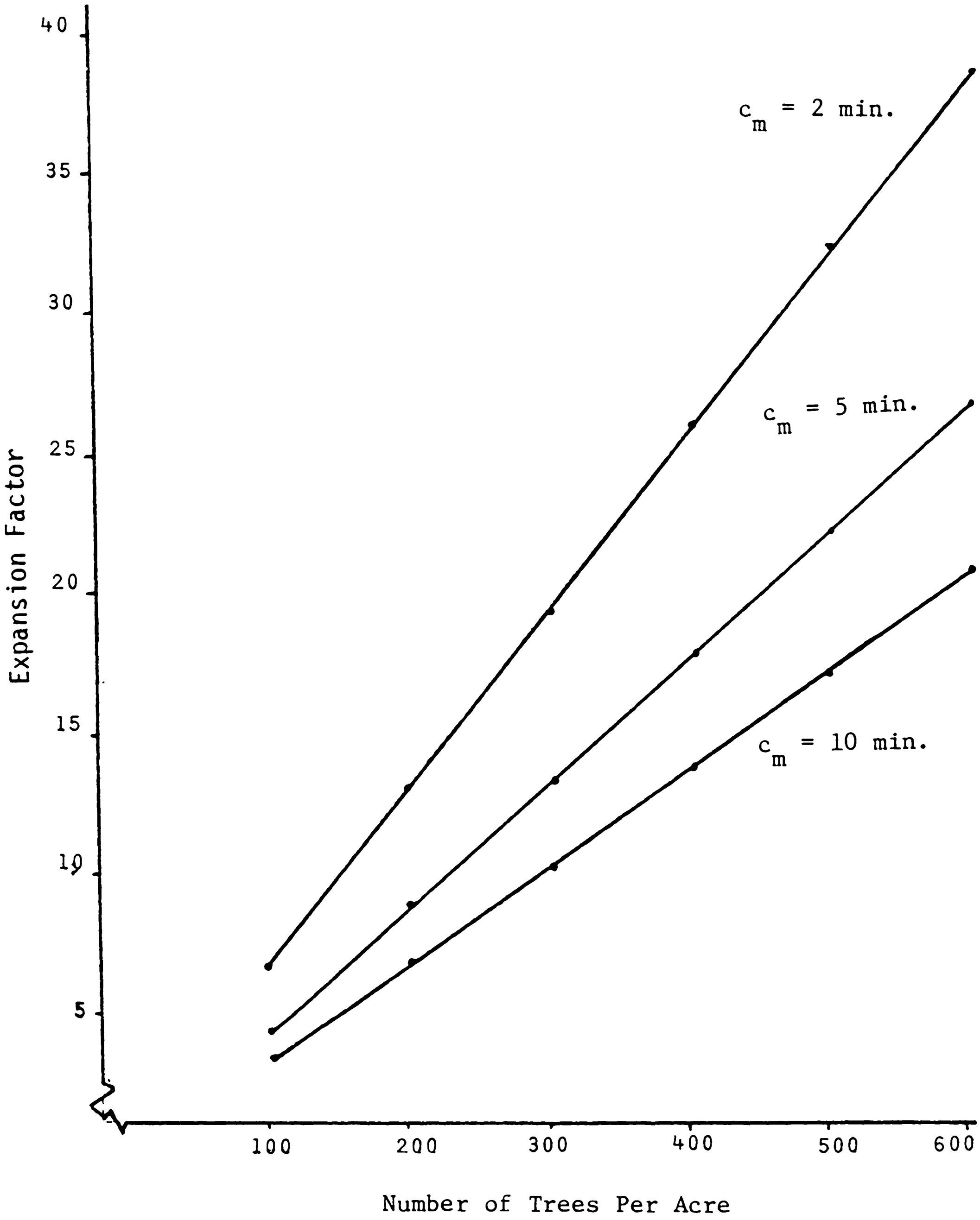

Figure 3. Optimal expansion factor values for plot samples with variances of either total basal area or total volume per acre at different time costs of establishing a sampling point.

Optimal Number of Sampling Positions

When minimizing a specified variance under the constraint of a fixed cost, the optimal number of sampling positions was calculated as a multiplier of the constant term $C-c_0$ (Table 3). This was done for both the point variances of total basal area and total volume per acre as well as the plot variances of total basal area and total volume per acre. These multipliers decreased as the cost in minutes to establish a sampling point increased. With point sample variances the multipliers generally remained constant over all values of total basal area per acre and with plot sample variances, the multipliers remained constant for any given number of trees per acre; in each case, they were presented as averages for a given expansion factor. There was little difference in the values of the multipliers when either a point sample or a plot sample of total basal area per acre or total volume per acre was minimized despite changes in the stand conditions and in the expansion factor values for each situation; the same number of sampling positions are needed regardless of whether a point or plot sample is taken.

When minimizing a fixed cost under the constraint of a given variance the relationships between the number of sampling positions in a point sample to a plot sample were

Table 3. Optimal number of points or plots calculated as an average multiplier of the constant term $C-c_0$ when the variance approximation was minimized under the constraint of a fixed cost.

Variable	Cost in minutes of establishing a sampling point		
	2	5	10
Point Sample Variance of Total Basal Area Per Acre	0.1265	0.0658	0.0391
Point Sample Variance of Total Volume Per Acre	0.1366	0.0722	0.0431
Plot Sample Variances of Total Basal Area and Total Volume Per Acre	0.1269	0.0664	0.0394

examined. If m_v = the optimal number of sampling positions in a point sample with a variance of total basal area per acre and m_f = the optimal number sampling positions in a plot sample with a variance of total basal area per acre then

$$\frac{m_v}{m_f} = \frac{EF_v B}{\lambda EF_f(S^2_{\bar{b}} + b^2)}$$

where EF_v is the expansion factor used in a point sample and EF_f is the expansion factor used in a plot sample. Simplifying

$$\frac{m_v}{m_f} = \frac{EF_v b}{EF_f(S^2_{\bar{b}} + b^2)}$$

$$= \frac{EF_v}{bEF_f(CV^2_{\bar{b}} + 1)}$$

where B is λb and $CV^2_{\bar{b}}$ (coefficient of variation) is $s^2_{\bar{b}} / b$ and assuming $EF_v = EF_f$

$$\frac{m_v}{m_f} = \frac{1}{b(CV^2_{\bar{b}} + 1)}$$

The function is dependent on the average basal area per tree or the average dbh found in the stand. Until the value of b is greater than 1, or the average dbh is larger than 14 inches, fewer plots than points are required for the same expansion factor.

Looking at the ratio of point to plot samples with variances of total volume per acre

$$\frac{m_v}{m_f} = \frac{r^2 EF_v B + B^2 V(\hat{r})}{\lambda EF_f (S^2_{\bar{V}} + v^2)}$$

$$= \frac{r^2 (EF_v B + B^2 CV^2_r)}{\lambda EF_f v^2 (CV^2_{\bar{V}} + 1)}$$

where $CV^2_{\bar{V}}$ is $s^2_{\bar{V}} / v$. As shown earlier V is rB, B is **λb,** and V is λ v, thus simplifying

$$\frac{m_v}{m_f} = \frac{r^2 EF_v b}{EF_f v^2 (CV^2_{\bar{V}} + 1)} + \frac{r^2 B^2 CV^2_r}{\lambda EF_f v^2 (CV^2_{\bar{V}} + 1)}$$

$$= \frac{r^2 EF_v b}{EF_f v^2 (CV^2_{\bar{V}} + 1)} + \frac{\lambda CV^2_r}{EF_f (CV^2_{\bar{V}} + 1)}$$

Assuming $EF_v = EF_f$

$$\frac{m_v}{m_f} = \frac{r^2 b}{v^2 (CV^2_{\bar{V}} + 1)} + \frac{\lambda^2 CV^2_r}{EF_f (CV^2_v + 1)}$$

$$= \frac{1}{CV^2_{\bar{v}} + 1} \left(\frac{r^2 b}{v^2} + \frac{\lambda^2 CV^2_r}{EF_f} \right)$$

Generally, the optimal number of sampling positions in a point sample will be greater than the optimal number of sampling positions in a plot sample for any average dbh when the expansion factors are equal. A possible exception could occur in the case of a low density and a small plot size; only in this instance could the optimal number of points be less than the optimal number of plots.

Controlled Confidence Interval Width

Determining the number of points or plots through control of confidence interval widths gave an indication of the point where each sampling procedure traded in optimality. Values for a bounds, E, and a set of variances of each characteristic from each respective sampling scheme, all from the same stand type and with the same expansion factor, were arbitrarily chosen to illustrate the technique; the student's t was set equal to 2. In a point sample with a variance of total basal area per acre, the optimal number of sampling points was

$$m = \frac{t^2(EF)(B)}{E^2}$$

where E = 5 and 10 square feet and for a variance of total volume per acre

$$m = \frac{t^2}{E^2}\left((r^2)(EF)(B) + (B^2)V(\hat{r})\right)$$
$$= \frac{t^2 r^2}{E^2}\left((EF)(B) + (B^2)(CV^2{}_r)\right)$$

where r = 25 and $CV^2{}_r$ = 0.15 and E = 250 cubic feet. For plot samples with a variance of total basal area per acre the function was

$$m = \frac{t^2 \lambda EF}{E^2}\left(S^2{}_{\bar{b}} + b^2\right)$$

$$= \frac{t^2 \lambda EF b^2}{E^2}\left(CV^2{}_{\bar{b}} + 1\right)$$

where b = 0.4 , $CV^2{}_{\bar{b}}$ = 0.40, and E = 5 and 10 square feet while for a variance of total volume per acre

$$m = \frac{t^2 \lambda EF}{E^2}\left(S^2{}_{\bar{v}} + v^2\right)$$

$$= \frac{t^2 \lambda EF v^2}{E^2}\left(CV^2{}_{\bar{v}} + 1\right)$$

where v = 11 and $CV^2{}_{\bar{v}}$ = 0.50 and E = 250 cubic feet.

Fewer plots than points were required for either respective stand characteristic (Figures 4, 5, 6, and 7).

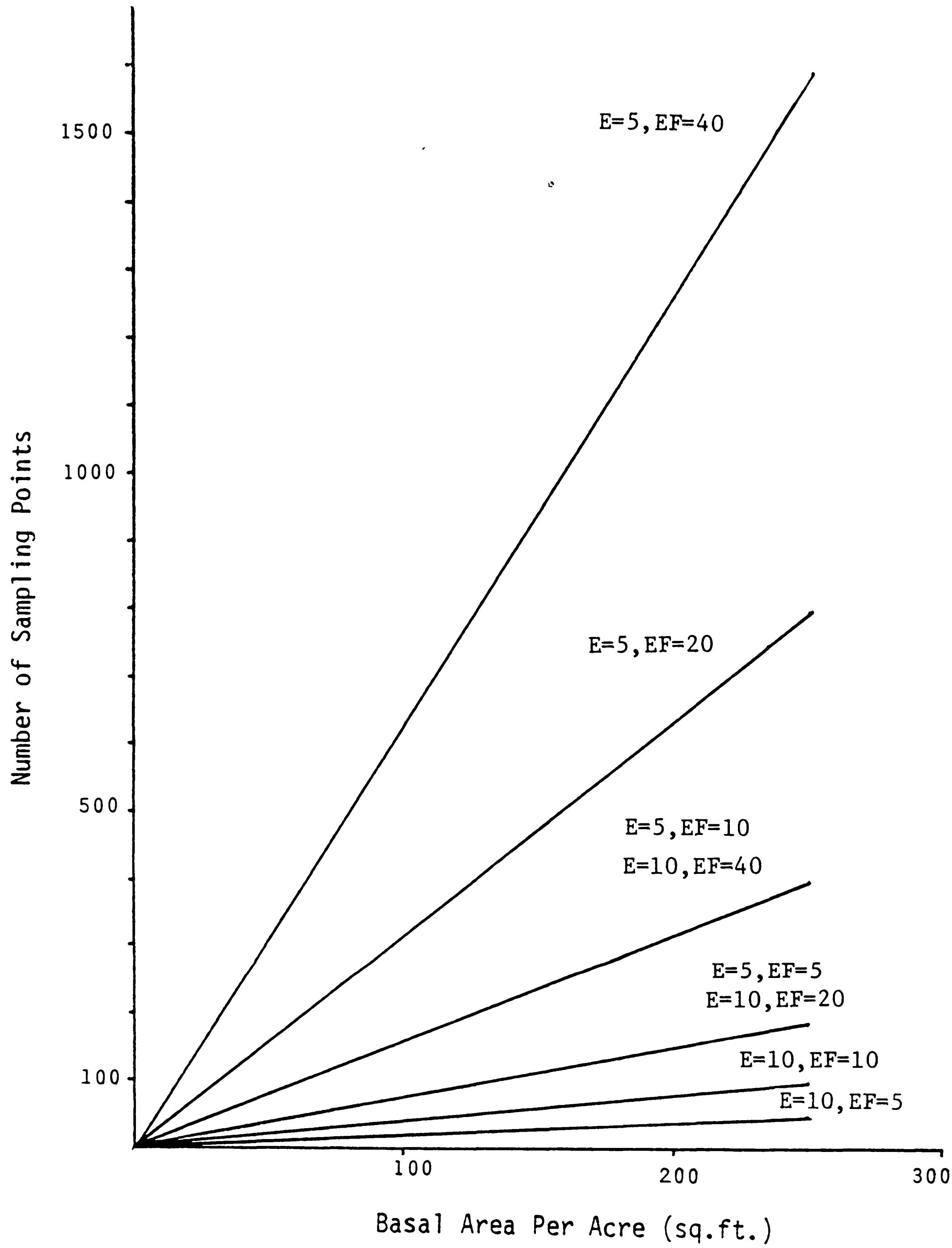

Figure 4. Optimal number of sampling points for a given bounds, E, and expansion factor, EF, with a variance of total basal area per acre.

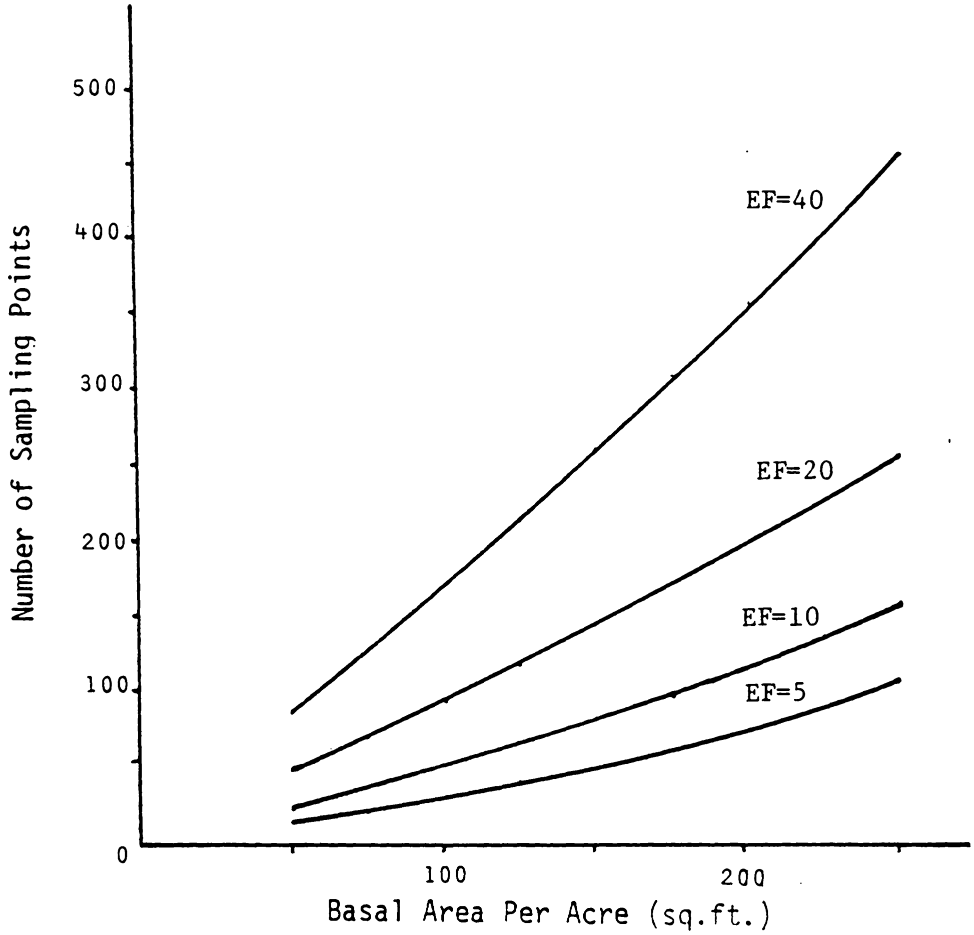

Figure 5. Optimal number of sampling points for a given bounds, E, and expansion factor, EF, with a variance of total volume per acre.

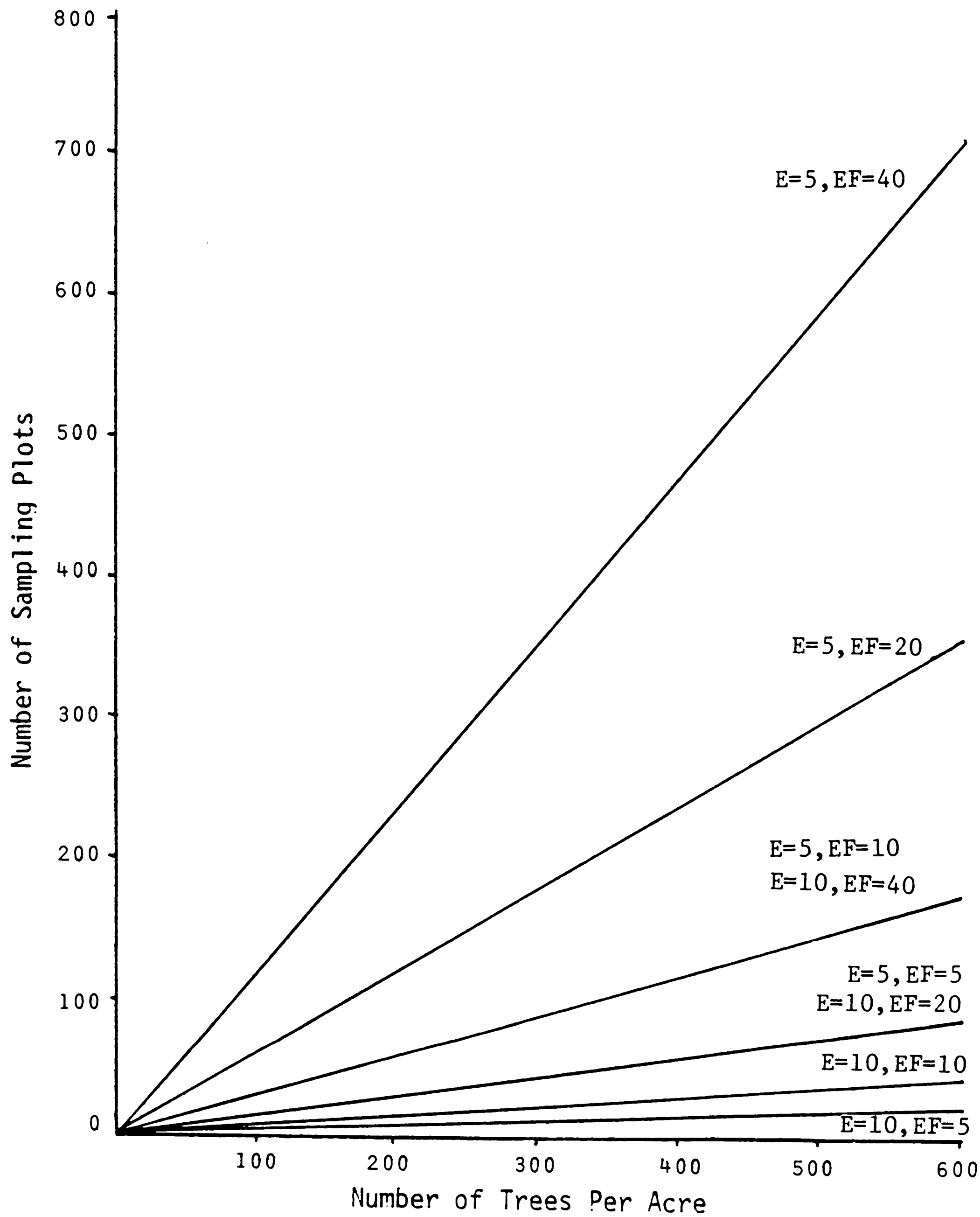

Figure 6. Optimal number of sampling plots for a given bounds, E, and expansion factor, EF, with a variance of total basal area per acre.

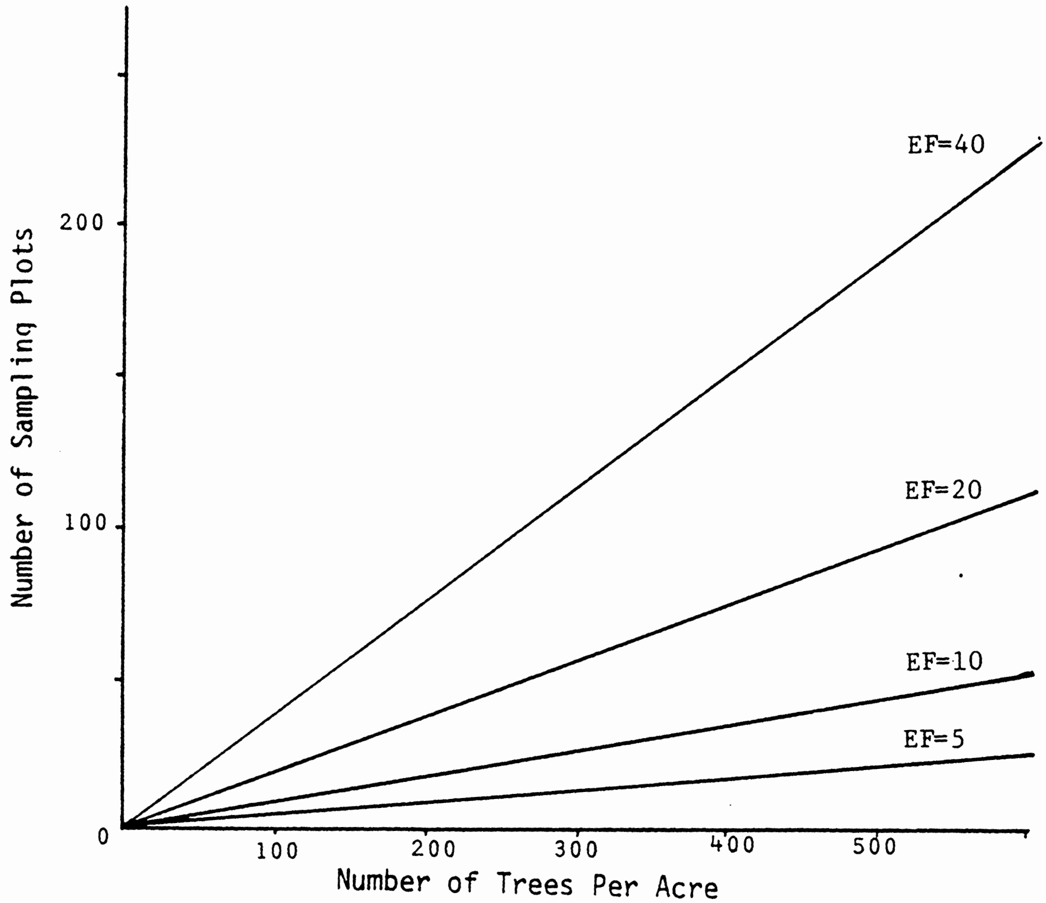

Figure 7. Optimal number of sampling plots for a given bounds, E, and expansion factor, EF, with a variance of total volume per acre.

This was the direct result of smaller plot than point variances for the same characteristic. This can be explained by looking at the ratio of expected values of n for each respective sampling technique and assuming equal expansion factors,

$$\frac{E(n)_{plot}}{E(n)_{point}} = \frac{\lambda/EF}{B/EF}$$

$$= \frac{\lambda}{B}$$

$$= \frac{1}{b}$$

where all variables are as previously defined. Avery and Newton (1965) found that tenth acre plot samples were generally equivalent in tree tally to BAF 10 point samples in stands having an average dbh of 13.5; as the average dbh decreased, smaller plots sizes were more equivalent to the same BAF 10 point samples. As seen by the ratio of expected values of n, until the value of b is greater than 1, or the quadratic mean dbh is larger than 14 inches, more trees are sampled in plot samples than in point samples; this increased sample size allows the variance of the estimate to decrease in a plot sample compared to a point sample for the same characteristic. In these particular stands, the average dbh was less than 13.5 inches; however, one would expect that in stands with average dbh's greater that 13.5 inches, this trend would reverse and fewer points than plots would be required for the same expansion factor.

SUMMARY AND CONCLUSIONS

Variance approximations were developed for total basal area per acre and total volume per acre in both point and plot sampling procedures. Variables comprising these approximations included stand and tree characteristics, expansion factors, and number of sampling positions for a given stand type, thus showing the effect each played on the variance approximation. These variance approximations were validated with a sampling simulation across a range of four different expansion factors.

In validating the variance approximation, several distinct trends developed. It appears that the formula approximation generally underestimated the variance in stands of lower numbers of trees per acre. In stands of higher numbers of trees per acre, the formula approximation underestimated the variance as frequently as it overestimated it. The formula approximation was determined to be best at expansion factor 40, the highest considered; with decreasing expansion factor values, the formula approximation was less accurate in estimating the variance.

The variance approximation performed best where fewer trees needed to be tallied at each sampling position. These trends were applicable to both the variances of total basal area per acre and total volume per acre in both point and plot sampling procedures.

Across the range of expansion factors evaluated, the variance approximations of both stand characteristics in all sampling procedures were generally considered unbiased; only on plot samples with expansion factors of 5 was this not true. Little or no difference was found in comparing the plot sample variances approximations of total volume per acre and total basal area per acre at any given expansion factor. The point sample variance approximations of both total basal area per acre and total volume per acre were usually closer in estimating their respective variances than the associated plot sampling variances. Overall, the formula approximation for total basal area per acre in a point sample more closely estimated its respective variance for any given expansion factor than any other variance formula approximation. All, however, were considered good in the estimation of their respective variances.

The variance formula approximations were evaluated through a cost analysis in order to select the optimal expansion factor and number of sampling positions required by either sampling procedure. From this one could determine where point and plot sampling procedures trade optimality.

Functions for determining optimal expansion factors in a point sample with a variance of total basal area per acre were the same regardless of whether the variance was minimized or a fixed cost was minimized. The same held true in point samples incorporating a variance of total volume per acre and in plot samples using variances of total basal area or total volume per acre respectively. The functions dealing with a point sample variance of total basal area per acre and plot sample variances of either total basal area or total volume per acre were quite similar; B, the true basal area per acre, and λ , the true number of trees per acre, were interchangeable in the functions where B was a variable in the point sample functions and λ was a variable the plot sample functions. Optimal functions for the number of sampling positions when minimizing a variance of either stand characteristic were the same for each respective sampling procedure, and again they were quite similar, differing only in the variables λ and B . Values calculated from these functions indicated that more trees would be tallied on a fewer number of sampling positions as more time was required to establish a sampling position and as the value of total basal area per acre decreased.

Values for the optimal expansion factors using a point sample and the variance of total basal area per acre were slightly less than those with the total volume per acre variance for a given total basal area per acre. The expansion factor values were the same when comparing plot sample variances. Values of the expansion factors will be higher for a plot sample variance than a point sample variance and fewer trees will need to be tallied on plots when the density of the stand is greater than the total basal area per acre. This was a direct result of having the variables B and λ interchangeable.

For the same expansion factor, little difference existed between the optimal number of sampling positions required in either sampling procedure for either stand characteristic when the variances were minimized. The same number of sampling positions are required regardless of whether a point or a plot sample is used. When cost was minimized under the constraint of a variance of total basal area per acre, and when the average dbh was less than 13.5 inches, fewer plots were required than points for equal expansion factors; under the constraint of a variance of total volume per acre fewer plots than points are required as well, except in the case of low densities with correspondingly small plot sizes.

In the second approach of the cost analysis arbitrarily chosen bounds were placed on the variance approximation to determine the optimal number of sampling positions. This technique was entirely dependent on the value of the variances; since the plot sample variances were smaller than the respective point sample variances, fewer plots than points were required for either stand characteristic. As the stand's average dbh increased to over 13.5 inches the reverse would be true; thus, for stands of smaller total basal area per acre plot sampling was recommended.

Where point and plot sampling trade optimality depends on the information required from the sample, how critical a wrong decision is, and the cost involved. All of the variance approximations did increasingly better as the size of the expansion factor increased. Point sample variance approximations did better than plot sample variance approximations for any given stand type at any expansion factor, but as the expansion factor increased the difference in the ability of the variance approximations for point and plot samples decreased. When cost is a consideration, fewer trees need to be tallied on equal or fewer plots than points. Thus, plot sampling would be more efficient where the cost factor was important and where possible biased variance approximations were not a critical factor; otherwise, the information required from the sample and the size of the expansion factor needed would determine which sampling scheme would be more desirable.

This study could be further extended in several directions. Only the random spatial pattern of trees was considered; it would be quite useful if other spatial patterns, such as clumped and lattice, could be analyzed in the same manner. Also, the simulations used in this study were based on loblolly pine diameter, height, and volume equations. A study of hardwoods applicable to different areas and analyzed for all spatial patterns would provide the sampler with beneficial information when working with those types of stands.

LITERATURE CITED

Arvanitis, L.G. and W.G. O'Regan. 1967. Computer simulation and economic efficiency in forest sampling. Hilgardia 38:133-164.

Avery, G. and R. Newton. 1965. Plot sizes for timber cruising in Georgia. J. Forest. 63:930-932.

Burkhart, H.E., R.C. Parker, M.R. Strub and R.G. Oderwald. 1972. Yields of old-field loblolly pine plantations. Div. of Forestry and Wildlife Resources, Va. Polytech. Inst. and State Univ., FWS-3-72, 51p.

Feduccia, D.P., T.R. Dell, W.F. Mann, T.E. Campbell and B.H. Polmer. 1979. Yields of unthinned loblolly pine plalntations on cutover sites in the West Gulf Region. U.S. Forest Service, Research Paper SO-148, 88p.

Freese, F. 1961. Relation of plot size to variability: an approximation. J. Forest. 59:679.

Grosenbaugh, L.R. 1952. Plotless timber estimates-new,fast,easy. J. Forest. 50:32-37.

Grosenbaugh, L.R. and W.S. Stover. 1957. Point-sampling compared with plot-sampling in southeast Texas. Forest Sci. 3:2-14.

Holgate, P. 1967. The angle-count method. Biometrika 54: 615-623.

Husch,B. 1955. Results of an investigation of the variable plot method of cruising. J. Forest. 53:570-574.

Husch,B. 1963. Forest Mensuration and Statistics. The Ronald Press Co., New York. 474pp.

Johnson, F.A. and H.J. Hixon. 1952. The most efficient size and shape of plot to use for cruising old-growth Douglas-fir timber. J. Forest. 50:17-20.

Kirby,C.L. 1965. Accuracy of point sampling in white spruce-aspen stands of Saskatchewan. J. Forest. 63:924-926.

Kulow, D.I. 1966. Comparison of forest sampling designs. J. Forest. 64:469-474.

Matern, B. 1972. The precision of basal area estimates. Forest Sci. 18:123-125.

Mesavage, C. and L.R. Grosenbaugh. 1956. Efficiency of several cruising designs on small tracts in North Arkansas. J. Forest. 54:569-576. Illus.

Oderwald, Richard G. (1975). The Effect of Spatial Distribution on the Efficiency of a Class of Probability Proportional to Size Sampling Estimators, Dissertation submitted in partial fulfillment of Ph.D. requirements at the University of Georgia, 63pp.

Oderwald, R.G. 1979. Comparison of efficiencies of point and plot sample basal area estimates in differing stand situations. In "Forest Resource Invertories-Workshop Proceedings," W.E. Frayer (ed.), Colorado State University, Fort Collins, Colorado, Vol. 1, 513pp. (250-258).

Oderwald, R.G. 1981. Comparison of point and plot sampling basal area estimators. Forest Sci. 27:42-48.

O'Regan, W.G. and L.G. Arvanitis. 1966. Cost-effectiveness in forest sampling. Forest Sci. 12:406-414.

O'Regan, W.G. and M.N. Palley. 1965. A computer technique for the study of forest sampling methods. Forest Sci. 11:99-114.

Palley, M.N. and W.G. O'Regan. 1961. A computer technique for the study of forest sampling methods. Forest Sci. 7:282-294.

Prodan, M. 1968. Forest Biometrics. Pergamon Press, New York. 447p.

Smalley, G.W. and R.L. Bailey. 1974. Yield tables and stand structure for loblolly pine plantations in Tennessee, Alabama, and Georgia highlands. U.S. Forest Service, Research Paper SO-96, 81p.

Smalley, G. W. and D.R. Bower. 1971. Site index curves for loblolly and shortleaf pine plantations on abandoned fields in Tennessee, Alabama, and Georgia highlands. U.S. Forest Service, Research Note SO-126, 6p.

Sukhatme, P.V. and B.V. Sukhatme. 1970. Sampling Theory of Surveys with Applications. Iowa State University Press, Ames, Iowa, U.S.A. 454pp.

Sukwong, S., W.E. Frayer, and E.W. Mogren. 1971. Generalized comparisons of the precision of fixed-radius and variable-radius plots for basal-area estimates. Forest Sci. 17:263-271.

Watson, M.S. 1981. An investigation of the use of spatial pattern information in forestry. Masters thesis. School of Forestry. Va. Polytech. Inst. and State Univ., Blacksburg, Va. 47p.

Zeide,B. and J.K. Troxwell. 1979. Selection of the proper metric basal area factor for Appalachian mixed hardwoods. In "Forest Resource Inventories-Workshop Proceedings," W.E. Frayer (ed.), Colorado State University, Fort Collins, Colorado, Vol. 1, 513pp. (261-269).

Zeide, B. 1980. Plot size optimization. Forest Sci. 26:251-257.

The vita has been removed from the scanned document

SELECTING EXPANSION FACTOR AND NUMBER OF SAMPLING POSITIONS FOR POINT AND PLOT SAMPLING

by

ELIZABETH A. JONES

(ABSTRACT)

Variance approximations were developed to estimate the variances of total basal area per acre and total volume per acre for both point and plot samples. These variance approximations reflect the role that stand characteristics and expansion factor play in either sampling procedure. Validation of the estimated variances was done through a sampling simulation.

Most of the variance approximations were considered unbiased and good in the estimation of their respective variances; only in plot samples with expansion factors of 5 was this not true. The variance approximation was considered best at expansion factor 40; with each decreasing expansion factor value, the approximations were less accurate in estimating the variance. Across the range of expansion factor values, point sampling approximations were generally more accurate than plot sampling approximations in estimating the variance; overall, the point sample variance approximation for total basal area per acre performed the best.

Variance approximations were used with a cost analysis in order to select the optimal expansion factor and number of sampling positions.

Plot samples require a higher expansion factor than point samples when the number of trees per acre is greater than the total basal area per acre for variances of either characteristic. For the same expansion factor, little difference existed between the otpimal number of sampling positions required in either sampling method when variances were minimized. When cost was minimized, generally fewer plots than points are needed in stands where the average dbh is less than 13.5 inches.