

## Large U(1) charges in F-theory

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**ABSTRACT:** We show that massless fields with large abelian charges (up to at least  $q = 21$ ) can be constructed in 6D F-theory models with a U(1) gauge group. To show this, we explicitly construct F-theory Weierstrass models with nonabelian gauge groups that can be broken to U(1) theories with a variety of large charges. Determining the maximum abelian charge allowed in such a theory is key to eliminating what seems currently to be an infinite swampland of apparently consistent U(1) supergravity theories with large charges.

**KEYWORDS:** F-Theory, Supergravity Models, Superstring Vacua

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**1 Introduction****1.1 Charged matter in string theory and F-theory**

While string theory can produce a vast range of consistent supergravity theories in four and higher space-time dimensions, there are nonetheless constraints on what kinds of low-energy theories can arise from string theory. These constraints are particularly strong in higher dimensions, and have recently been explored in 10D [1], 8D [2, 3], and 6D [4–12]. More generally, the set of low-energy theories that look consistent but cannot be realized in string theory have been referred to as the “swampland” [13, 14].

A particularly interesting question that is relevant in every dimension is: what kinds of light or massless matter fields can arise in compactifications of string theory? A priori, one might think that a matter field could transform under any representation of a gauge group

$G$  in a consistent low-energy theory of gravity. This is not the case, however, at least in higher dimensions with supersymmetry. The highest dimension in which matter fields can arise in any representation other than the adjoint in a supersymmetric theory is 6D. In this paper, we consider possible charges for massless fields charged under a gauge group in a 6D supergravity theory that has only a single  $U(1)$  factor, like the familiar four-dimensional theory of electromagnetism.

In six dimensions, for supersymmetric theories of gravity with nonabelian gauge groups, there are strong constraints on the possible matter representations that can arise. For theories with fewer than 9 tensor multiplets, anomaly cancellation conditions alone restrict the set of possible nonabelian gauge groups and charged matter fields to a finite set [5, 15]. In six dimensions, F-theory [16–18] (see [2, 19, 20] for recent reviews and further background on F-theory) gives the most general class of known supersymmetric string vacuum constructions, and recent work has focused on what kinds of matter representations can be realized for massless fields in 6D F-theory models. The simplest F-theory models give only a simple generic set of massless matter fields; for an  $SU(N)$  gauge group, these fields are in just the singlet, fundamental, adjoint, and two-index antisymmetric representations. A few more exotic representations can be constructed in F-theory (see [8, 21] for recent work and further references), but the constraints both from anomaly cancellation and from F-theory on the allowed representations are generally quite strong. For example, it seems that in F-theory no massless matter field can transform in any representation of  $SU(2)$  of dimension higher than 5.

For abelian charges, however, the story is quite different and less well understood. As far as low-energy consistency conditions go, there seems to be an infinite family of 6D supergravity models with a  $U(1)$  gauge group, even in theories without tensor multiplets, in which the abelian charges  $q$  can be arbitrarily large [10]. On the other hand, from the finiteness of the set of elliptic Calabi-Yau threefolds [5, 22], it is clear that there is a finite upper bound on the largest abelian charge  $q_{\max}$  that can be realized in any F-theory construction. This raises the natural question of what is the largest abelian charge that can arise in a 6D F-theory vacuum with a single-factor  $U(1)$  gauge group. Little is known about the answer to this question. The most well understood F-theory  $U(1)$  models [23] have abelian charges of only  $q = 1, 2$ . Explicit F-theory models with abelian charge  $q = 3$  were first found in [24] and constructed more generally in [25], along with some explicit models with abelian charge  $q = 4$ . The F-theory models with larger abelian charges, however, contain increasingly complicated singularity structures, and are hard to analyze analytically.

In this paper, we use an indirect method to show that F-theory must allow the construction of 6D supergravity theories with a  $U(1)$  gauge group and massless fields with large abelian charges. Our strategy is to explicitly construct F-theory models with nonabelian gauge groups that, according to field theory arguments, can be Higgsed to  $U(1)$  groups admitting large charges. With this technique, we show that F-theory admits abelian charges as large as  $q = 21$ .

In section 1.2, we describe our strategy in further detail. In section 2.1, we discuss some general aspects of Higgsing processes and present a specific Higgsing chain that is used throughout the rest of the analysis. Section 2.2 reviews the 6D anomaly cancellation

conditions for  $SU(N)$  and  $U(1)$  gauge groups, which are then used in section 2.3 to constrain the scope of F-theory models considered. We then turn to explicit constructions of the F-theory models with nonabelian gauge groups. Section 3 focuses on F-theory models on a  $\mathbb{P}^2$  base, which allow us to demonstrate that abelian charges  $q = 1$  through 7 are realized in F-theory. In section 4, we discuss F-theory models on Hirzebruch surfaces  $\mathbb{F}_n$  bases. These models allow us to realize the largest abelian charges found in this paper. We conclude in section 5 by presenting some open questions and directions for future work.

## 1.2 General strategy

Ideally, one would establish that a certain charge can be realized in F-theory by finding an explicit  $U(1)$  model admitting the desired charge. However, constructing F-theory models with large charges is a challenging enterprise. Weierstrass models admitting charges<sup>1</sup> larger than  $q = \pm 2$  involve algebraically complex non-UFD structures. As one attempts to obtain larger and larger charges, the Weierstrass models become more and more unwieldy. The currently known F-theory models with just a  $U(1)$  gauge group only admit charges  $\pm 1$  through  $\pm 4$ , and there are few, if any, tractable techniques available for systematically constructing models with arbitrarily large charges.

Given these difficulties, we use an indirect approach to determine that any other specific charges must be realized in F-theory. Our strategy is to explicitly realize F-theory models with nonabelian gauge groups that can be Higgsed down to a  $U(1)$  gauge symmetry admitting large charges. In particular, we focus on 6D F-theory models having an  $SU(N)$  gauge group and at least two adjoint hypermultiplets. As described in section 2.1, such a low-energy  $SU(N)$  supergravity model can be Higgsed down to a  $U(1)$  model. If the  $SU(N)$  model can be realized in F-theory, it must be therefore possible to deform it to the corresponding  $U(1)$  F-theory model. In other words, constructing the  $SU(N)$  model in F-theory demonstrates that the corresponding  $U(1)$  model must exist in F-theory, even if we cannot determine the exact deformations necessary to Higgs the  $SU(N)$  symmetry. And if field-theoretic considerations show that the  $U(1)$  theory has hypermultiplets with large charges, those large charges must be realizable in F-theory.

We therefore focus on constructing explicit  $SU(N)$  Weierstrass models. The Higgsing process, which can be understood purely from field-theoretic considerations, then implies that certain  $U(1)$  charges can be realized. Thus, we can establish that particular charges occur in F-theory without explicitly constructing  $U(1)$  F-theory models. Of course, this strategy has some limitations. While we can show that certain charges occur in F-theory, we cannot prove that certain charges are ruled out in F-theory. As a result, we will not be able to establish an upper bound on the charges in F-theory. Nevertheless, this technique demonstrates that the highest possible charge must be at least  $\pm 21$ , significantly larger than the charges that have currently been realized in explicit F-theory models. Even if they cannot rule out certain charges, these  $SU(N)$  models provide new information about the possible charge spectra in F-theory.

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<sup>1</sup>Note that in 6D a matter hypermultiplet containing a field of charge  $q > 0$  also contains a field of charge  $-q$ .

Note that in this paper when we speak of “large”  $U(1)$  charges, we mean relative to the natural unit of charge in the theory. In most cases we deal with the natural unit of charge is the greatest common divisor of the nonzero massless charges, and is generally 1 in the units we use. We discuss this issue a little further in section 2.2.

## 2 The Higgsing process, anomalies, and F-theory models in 6D

In this section we go over some basic aspects of the Higgsing process, constraints from anomaly cancellation, and F-theory models for 6D supergravity theories. In general, 6D supergravity theories have some number  $T$  of tensor multiplets, a gauge group  $G$ , and hypermultiplet matter fields transforming in some representation  $\mathcal{R}$  of  $G$ . Here, we focus primarily on theories with zero or one tensor multiplets ( $T = 0, 1$ ), and gauge groups of the form  $SU(N)$ ,  $U(1)$ , or products of such factors. In particular, we are interested in starting with a theory having a gauge group  $SU(N)$  and at least two matter fields in the adjoint representation, which can be broken by Higgsing processes down to a theory with a  $U(1)$  gauge group and various charged matter representations.

### 2.1 The Higgsing process

We begin with a few generalities on Higgsing processes in 6D theories with  $\mathcal{N} = (1, 0)$  supersymmetry. While there are some differences, such processes can be understood in analogy with Higgsing processes in  $\mathcal{N} = 1$  4D gauge theories. In the latter context, a process in which a field or fields  $\phi_i$  acquire nonzero expectation values and break a gauge group  $G$  can be described either in terms of supersymmetric D-term constraints or geometric invariant theory. In the former context, the field expectation values must satisfy the conditions  $\sum_i \phi_i^\dagger T^A \phi_i = 0$ , where each generator  $T^A$  acts on the fields  $\phi_i$  according to the appropriate representation. In the context of geometric invariant theory, the vacua are parameterized by gauge-invariant polynomials in the fields  $\phi_i$  [26]. From each point of view one can see that a gauge group can be broken by Higgsing on a single field in the adjoint representation; for example, from the D-term point of view this follows from the fact that  $T^A$  acts through the adjoint action, so that the D-term conditions automatically vanish. On the other hand, one needs two fields in the fundamental representation to break  $SU(N)$  through Higgsing, since a single nonzero VEV cannot solve the D-term constraints for all generators, and cannot be used to form a gauge-invariant polynomial.

Our ultimate goal is to break gauge groups, such as  $SU(N)$ , down to  $U(1)$  in a way that generates large charges. To accomplish this, we use a specific  $SU(N) \rightarrow U(1)$  Higgsing process described in [10]. The preserved  $U(1)$  corresponds to the  $SU(N)$  generator

$$\text{diag}(1, 1, \dots, 1, -N + 1), \tag{2.1}$$

which is written in the fundamental representation. The details of this Higgsing process are summarized below for convenience. In most instances, we consider the resulting charge spectrum when an  $SU(N)$  gauge group undergoes this exact Higgsing process. Even when we consider alternative Higgsing processes, the steps outlined below form part of the Higgsing sequence.

The starting point for this Higgsing process is a 6D supergravity theory with an  $SU(N)$  gauge symmetry and at least two hypermultiplets in the adjoint representation. Giving a generic VEV to one of the adjoint hypermultiplets breaks  $SU(N)$  to its Cartan subgroup,  $U(1)^{N-1}$ . We can describe how a hypermultiplet is charged under this  $U(1)^{N-1}$  symmetry using a charge vector  $\vec{q} = (q_1, \dots, q_{N-1})$ , where  $q_i$  denotes the charge under the  $i$ th  $U(1)$ . After giving a VEV to the first adjoint multiplet, an  $SU(N)$  representation  $\mathbf{R}$  branches to a collection of  $U(1)^{N-1}$  charge vectors corresponding to the weight vectors of  $\mathbf{R}$ .

We then want to give VEVs to hypermultiplets charged under the  $U(1)^{n-1}$  symmetry to break it to a single  $U(1)$ . This is not possible if one uses the remnant hypermultiplets from only the first adjoint hypermultiplet. Most of the degrees of freedom in this first adjoint hypermultiplet are eaten, and those that remain after the Higgsing are neutral under the  $U(1)^{N-1}$  symmetry. But all the degrees of freedom from the second adjoint hypermultiplet still remain, and many of them are charged under the  $U(1)^{N-1}$  symmetry. After giving the VEV to the first adjoint hypermultiplet, the second adjoint hypermultiplet branches to  $N^2 - N$  charged hypermultiplets whose charge vectors  $\vec{q} = (q_1, \dots, q_{N-1})$  under  $U(1)^{N-1}$  are the  $SU(N)$  root vectors.<sup>2</sup> We can work in the Dynkin basis, where the root vectors for the simple roots are the rows of the Cartan matrix:

$$\begin{aligned}
 \alpha_1 &= (2, -1, 0, 0, \dots) \\
 \alpha_2 &= (-1, 2, -1, 0, \dots) \\
 \alpha_3 &= (0, -1, 2, -1, \dots) \\
 &\vdots \\
 \alpha_{N-1} &= (0, \dots, 0, -1, 2)
 \end{aligned}
 \tag{2.2}$$

We can now give VEVs to the charged hypermultiplets whose charge vectors are the simple roots  $\alpha_2$  through  $\alpha_{N-1}$  (along with their negative counterparts to satisfy the D-term constraints). This breaks the  $U(1)^{N-1}$  symmetry down to a single  $U(1)$  corresponding to the direction in root space orthogonal to  $\alpha_2$  through  $\alpha_{N-1}$ . In the Dynkin basis, this direction is given by the vector  $(N-1, N-2, \dots, 2, 1)$ .

This explicit description of the Higgsing process allows us to calculate the resulting  $U(1)$  charges. For instance, the fundamental representation of  $SU(N)$  consists of weights of the form

$$[1, 0, 0, \dots], [-1, 1, 0, 0 \dots], [0, -1, 1, 0 \dots], \dots [0, \dots, 0, -1, 1].
 \tag{2.3}$$

When one takes the inner product of these weights with  $(N-1, N-2, \dots, 2, 1)$ , the highest weight  $[1, 0, 0 \dots]$  leads to charge  $N-1$ , while the other weights lead to charge  $-1$ . These charges agree with the diagonal entries in (2.1), at least up to sign and normalization, indicating that we have preserved the desired generator. Hypermultiplets in a representation  $\mathbf{R}$  include fields in both  $\mathbf{R}$  and  $\overline{\mathbf{R}}$ , and the hypermultiplets charged under the final

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<sup>2</sup>In addition to these  $N^2 - N$  charged hypermultiplets, there are  $N - 1$  neutral hypermultiplets coming from the second adjoint hypermultiplet.

U(1) include fields with both positive and negative charges. Therefore, a fundamental hypermultiplet branches to U(1) hypermultiplets in the following way:

$$\square \rightarrow (\mathbf{q} = \mathbf{N} - \mathbf{1}) + (N - 1) \times (\mathbf{q} = \mathbf{1}). \tag{2.4}$$

Note that this result can also be derived easily in the fundamental basis; if the unbroken adjoint field takes the form  $\text{diag}(1, 1, \dots, -N + 1)$ , then clearly a field in the fundamental representation breaks up into  $N - 1$  fields of charge  $(\mathbf{q} = \mathbf{1})$  and one field of charge  $(\mathbf{q} = \mathbf{N} - \mathbf{1})$ . All the calculations here can be carried out in a straightforward fashion in either basis; the Dynkin basis may be more useful for generalization to other groups.

Similar calculations show that other  $SU(N)$  representations branch as<sup>3</sup>

$$\mathbf{Adj} \rightarrow 2(N - 1) \times (\mathbf{q} = \mathbf{N}) + (N - 1)^2 \times (\mathbf{q} = \mathbf{0}) \tag{2.5}$$

$$\begin{array}{|c} \square \\ \hline \square \end{array} \rightarrow (N - 1) \times (\mathbf{q} = \mathbf{N} - \mathbf{2}) + \frac{(N - 1)(N - 2)}{2} \times (\mathbf{q} = \mathbf{2}) \tag{2.6}$$

$$\begin{array}{|c} \square \\ \hline \square \\ \hline \square \end{array} \rightarrow \frac{(N - 1)(N - 2)}{2} \times (\mathbf{q} = \mathbf{N} - \mathbf{3}) + \frac{(N - 1)(N - 2)(N - 3)}{6} (\mathbf{q} = \mathbf{3}) \tag{2.7}$$

Table 1 summarizes the charges coming from different  $SU(N)$  representations under this Higgsing process.<sup>4</sup> Already, one can make interesting observations about the charge spectra. Many of the  $SU(N)$  gauge groups lead to massless charged spectra that skip over certain charges. For instance, consider Higgsing an  $SU(8)$  model with hypermultiplets in the representations listed in table 1. The resulting U(1) charge spectrum includes all of the charges from  $\pm 1$  to  $\pm 8$  *except* for charge  $\pm 4$ . This fact might naively seem to contradict the completeness hypothesis [28, 29], which states that all possible charges must be realized in the Hilbert space of a theory. However, as discussed in [10], the charge spectra we consider here involve only massless states, whereas the completeness hypothesis considers both massless and massive states. Massless U(1) charge spectra that seem to skip over charges therefore do not directly contradict the completeness hypothesis. Nevertheless, one might be tempted to conjecture that F-theory U(1) models obey some massless equivalent of the completeness conjecture in which all charges between  $\pm 1$  and some large value occur. Examples such as the  $SU(8)$  model above, which we explicitly construct in F-theory in section 4, contradict these sort of conjectures.

It is important to note that this Higgsing process can be seen directly and explicitly in  $SU(3)$  and  $SU(4)$  F-theory models [25]. In appendix B, we give a specific example in which an a U(1) F-theory model with charge  $\pm 4$  matter is unHiggsed to an  $SU(4)$  model admitting

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<sup>3</sup>Even though these formulas allow one to compute all of the resulting spectra in this paper by hand, many of the calculations of specific spectra quoted later were also performed using LieART [27] as an additional check.

<sup>4</sup>While we restrict our attention to the representations listed in table 1, one could consider other representations, namely the symmetric representation. Under the Higgsing process that we have described here, the symmetric representation would give charges as large as  $\pm(2N - 2)$ . However, if we require that there are at least two adjoints, the largest  $SU(N)$  model that we have been able to obtain using techniques similar to those in [8] is  $SU(5)$ , at least for a  $\mathbb{P}^2$  base. Therefore, including symmetric matter does not provide an obvious way of obtaining significantly larger charges, although it would be interesting to systematically explore the charges possible when one includes the symmetric representation; we leave this for future work.

| <b>Gauge Group</b> | <i>Fundamental</i> | <i>Adjoint</i> | <i>Two-Index Antisymmetric</i> | <i>Three-Index Antisymmetric</i> |
|--------------------|--------------------|----------------|--------------------------------|----------------------------------|
| SU(2)              | $\pm 1$            | $0, \pm 2$     | —                              | —                                |
| SU(3)              | $\pm 1, \pm 2$     | $0, \pm 3$     | —                              | —                                |
| SU(4)              | $\pm 1, \pm 3$     | $0, \pm 4$     | $\pm 2$                        | —                                |
| SU(5)              | $\pm 1, \pm 4$     | $0, \pm 5$     | $\pm 2, \pm 3$                 | —                                |
| SU(6)              | $\pm 1, \pm 5$     | $0, \pm 6$     | $\pm 2, \pm 4$                 | $\pm 3$                          |
| SU(7)              | $\pm 1, \pm 6$     | $0, \pm 7$     | $\pm 2, \pm 5$                 | $\pm 3, \pm 4$                   |
| SU(8)              | $\pm 1, \pm 7$     | $0, \pm 8$     | $\pm 2, \pm 6$                 | $\pm 3, \pm 5$                   |

**Table 1.** U(1) charges realized by Higgsing SU( $N$ ) according to the Higgsing process on two adjoints described in the text. Each entry denotes the charges coming from a hypermultiplet in a particular representation of SU( $N$ ). Note that hypermultiplets include fields in a representation  $\mathbf{R}$  and its conjugate  $\bar{\mathbf{R}}$ , allowing one to obtain both positive and negative charges from a single hypermultiplet. Dashes indicate representations that either do not occur for a particular gauge group or are equivalent to some other representation. Not all of the representations listed in this table appear in the F-theory models considered later.

two adjoint hypermultiplets. The explicit realization of this Higgsing/unHiggsing process provides additional confirmation of the general arguments presented above. Of course, our ultimate goal is determine whether larger charges can be realized in F-theory. Table 1 already suggests that SU( $N$ ) models should lead to charges beyond those currently realized in F-theory U(1) models. But before we can establish that certain U(1) charges occur in F-theory, we must show that the corresponding SU( $N$ ) models can be realized in F-theory. We turn to this issue next.

## 2.2 Anomaly cancellation conditions for SU( $N$ ) and U(1) models

Clearly, knowing the types of SU( $N$ ) models that can be realized tells us information about the possible U(1) charges. We therefore must determine which 6D SU( $N$ ) supergravity models can be realized in F-theory. In particular, larger SU( $N$ ) models allow us to obtain larger U(1) charges, so we are most interested in determining the largest suitable SU( $N$ ) models that occur in F-theory with at least two hypermultiplets of adjoint matter. A worthwhile first step is to determine the 6D SU( $N$ ) supergravity models that satisfy the anomaly cancellation conditions. All F-theory constructions should satisfy these conditions, allowing us to narrow the scope of F-theory models to investigate. Of course, a model that satisfies the anomaly cancellation conditions may not have an F-theory realization. Nevertheless, the anomaly analysis provides interesting insights into the SU( $N$ ) F-theory models and their implications for the U(1) charge spectra.

6D (1,0) supergravity theories have chiral spectra and may therefore suffer from anomalies. These anomalies can be canceled via the Green-Schwarz mechanism [30, 31], which uses tree-level diagrams involving tensors to cancel contributions from chiral fermions. However, the massless spectrum must satisfy certain conditions for the anomalies to can-



cel. Suppose that our theory has one graviton multiplet,  $T$  tensor multiplets,  $V$  vector multiplets, and  $H$  hypermultiplets. Gravitational anomalies are canceled only if

$$H - V + 29T = 273. \tag{2.8}$$

Gauge anomalies need to be canceled as well. Let us first focus on cases where the gauge group is  $SU(N)$  to simplify the anomaly cancellation conditions. Suppose that there are  $x_R$  full hypermultiplets in the  $R$  representation of  $SU(N)$ . The gauge anomaly conditions depend on two vectors,  $a$  and  $b$ , living in a lattice  $\Gamma$  of signature  $(1, T)$  with an inner product denoted by  $\cdot$ . Gravitational anomaly cancellation imposes the condition that  $a \cdot a = 9 - T$ . Gauge and mixed gauge-gravitational anomalies cancel if the following equations are satisfied:

$$-a \cdot b = \frac{1}{6} \left( \sum_R x_R A_R - A_{\text{adj}} \right), \tag{2.9}$$

$$0 = \sum_R x_R B_R - B_{\text{adj}}, \tag{2.10}$$

$$b \cdot b = \frac{1}{3} \left( \sum_R x_R C_R - C_{\text{adj}} \right). \tag{2.11}$$

Here we have used the group theory coefficients  $A_R$ ,  $B_R$ , and  $C_R$  defined by the relations

$$\text{Tr}_R F^2 = A_R \text{tr} F^2 \qquad \text{Tr}_R F^4 = B_R \text{tr} F^4 + C_R (\text{tr} F^2)^2, \tag{2.12}$$

where  $\text{tr}$  represents a trace in the fundamental representation and  $\text{Tr}_R$  represents a trace in the  $R$  representation.

If the gauge group is the product of  $SU(N)$  factors, there is an additional anomaly constraint. Suppose we consider two of the  $SU(N)$  factors,  $SU(N)_i$  and  $SU(N)_j$ , with corresponding vectors  $b_i$  and  $b_j$ . Let  $x_{(R_i, R_j)}$  denote the number of hypermultiplets in the representation  $(R_i, R_j)$  of  $SU(N)_i \times SU(N)_j$ . Then, the additional anomaly constraint takes the form

$$b_i \cdot b_j = \sum_{(R_i, R_j)} x_{(R_i, R_j)} A_{R_i} A_{R_j}. \tag{2.13}$$

For theories with a  $U(1)$  gauge group, the anomaly conditions take a similar but simpler form [32, 33],

$$a \cdot \tilde{b} = -\frac{1}{6} \sum_i q_i^2, \tag{2.14}$$

$$\tilde{b} \cdot \tilde{b} = \frac{1}{3} \sum_i q_i^4. \tag{2.15}$$

Here  $q_i$  is the  $U(1)$  charge of the  $i$ th charged multiplet and  $\tilde{b}$  is again a vector in the lattice  $\Gamma$ . When there are multiple  $U(1)$  factors, or abelian and nonabelian factors there are further conditions analogous to (2.13), but we will not need those here.

Note that for every spectrum that satisfies the abelian anomaly equations, there is an infinite family of solutions, which can be achieved by multiplying all charges by  $n$  and multiplying the anomaly coefficient  $\tilde{b}$  by  $n^2$ . While these may seem to be equivalent theories, under a simple rescaling of the charge, the different value of  $\tilde{b}$  in the anomaly lattice distinguishes the theories. This is related to the fact that in some F-theory models determining the charge unit can be subtle. For example, as discussed in [10], there are two distinct F-theory models with no tensor multiplets that have 108 charges  $q = \pm 1$  and  $q = \pm 2$  respectively. The values of  $\tilde{b}$  differ between these theories by a factor of 4. This can be seen in F-theory from the fact that an unHiggsing of the U(1) model gives a nonabelian theory with gauge group SO(3) instead of SU(2).

### 2.3 6D F-theory models

While the anomaly cancellation equations are a low-energy condition, the  $a$  and  $b$  parameters have a geometric interpretation in F-theory.  $a$  can be viewed as the canonical class  $K_B$  of the base of the F-theory model's elliptic fibration.  $b$ , meanwhile, can be viewed as the homology class of the divisor on which the SU( $N$ ) gauge group is tuned. The inner product  $\cdot$  then represents the intersection product between homology classes. In fact, one can solve the gauge and mixed anomaly conditions solely in terms of properties of the gauge divisor, such as its self-intersection  $n = b \cdot b$  and its arithmetic genus

$$g = 1 + \frac{1}{2}b \cdot (a + b) \tag{2.16}$$

In other words, the charged spectrum can be determined without specifying the base. Here, we restrict our attention to the fundamental, the adjoint, the two-index antisymmetric, and three-index antisymmetric representations. The resulting charged matter spectra for SU(2) through SU(9) are given in table 2 [34, 35].

The gravitational anomaly condition, however, requires more global information. The number of tensor multiplets and neutral hypermultiplets depends on the specific base chosen. Moreover, some bases have non-Higgsable clusters [36] which contribute to the number of vector multiplets. Thus, even if there is no matter charged under both the SU( $N$ ) group and the non-Higgsable gauge group, the non-Higgsable cluster can affect the gravitational anomaly condition. Therefore, to definitively determine which SU( $N$ ) models that come from F-theory are consistent with anomalies, we need to consider specific bases.

#### 2.3.1 F-theory models with $T = 0$ (compactifications on $\mathbb{P}^2$ )

We start by considering SU( $N$ ) F-theory models on  $\mathbb{P}^2$ . Models on  $\mathbb{P}^2$  have zero tensor multiplets, and thus the vectors  $a$  and  $b$  live in a one-dimensional lattice. Alternatively, one can say that the basis of homology classes of  $\mathbb{P}^2$  consists of a single element  $H$  with self-intersection number 1. There are no genus-two algebraic curves on  $\mathbb{P}^2$ , but quartic curves on  $\mathbb{P}^2$  have genus  $g = 3$ . Thus, if we are willing to have extra adjoint hypermultiplets, we can construct appropriate SU( $N$ ) models on  $\mathbb{P}^2$ .

For quartic curves,  $g = 3$  and  $n = 16$ . Suppose we assume that there are no three-index antisymmetric hypermultiplets. The three-index antisymmetric representation is

| <b>Gauge Group</b> | <i>Fundamental</i>   | <i>Adjoint</i> | <i>Two-Index Antisymmetric</i> | <i>Three-Index Antisymmetric</i> |
|--------------------|----------------------|----------------|--------------------------------|----------------------------------|
| SU(2)              | $16 - 16g + 6n$      | $g$            | —                              | —                                |
| SU(3)              | $18 - 18g + 6n$      | $g$            | —                              | —                                |
| SU(4)              | $16 - 16g + 4n$      | $g$            | $2 - 2g + n$                   | —                                |
| SU(5)              | $16 - 16g + 3n$      | $g$            | $2 - 2g + n$                   | —                                |
| SU(6)              | $16 - 16g + 2n + r$  | $g$            | $2 - 2g + n - r$               | $\frac{1}{2}r$                   |
| SU(7)              | $16 - 16g + n + 5r$  | $g$            | $2 - 2g + n - 3r$              | $r$                              |
| SU(8)              | $16 - 16g + 9r$      | $g$            | $2 - 2g + n - 4r$              | $r$                              |
| SU(9)              | $16 - 16g - n + 14r$ | $g$            | $2 - 2g + n - 5r$              | $r$                              |

**Table 2.** Charged matter multiplicities for  $SU(N)$  gauge groups. The multiplicities are given in terms of the arithmetic genus  $g$  and the self-intersection  $n = b \cdot b$  of the  $SU(N)$  divisor. Entries with a dash indicate representations that are not relevant for the gauge group in question. Note that  $r$  is a free integer.

somewhat exotic from an F-theory perspective, as the corresponding models are more challenging to construct and involve additional fine-tuning of the Weierstrass coefficients. For instance,  $SU(N)$  models constructed using Tate’s algorithm [34, 37] typically admit only the fundamental, two-index antisymmetric, and adjoint representations. Restricting our attention to these representations is therefore a natural first step in the analysis. Under this assumption, table 2 suggests that for  $N$  larger than 6, the  $SU(N)$  model has a negative number of fundamental hypermultiplets. And for higher degree curves, the number of fundamental hypermultiplets becomes negative for even smaller values of  $N$ . This result would naively suggest that  $SU(6)$  is the largest consistent  $SU(N)$  group on  $\mathbb{P}^2$  admitting at least two adjoint hypermultiplets.

But if we relax the assumption that there are no three-index antisymmetric multiplets, one can obtain higher values of  $N$ . For instance, there is an anomaly-free  $SU(7)$  model on  $\mathbb{P}^2$  with three **48** hypermultiplets, four **35** multiplets, four **7** hypermultiplets, and nine singlets. However, obtaining  $SU(8)$  groups and beyond on  $\mathbb{P}^2$  with a sufficient number of adjoints appears difficult and likely impossible: even when we consider all four of the representations mentioned, one cannot obtain a suitable  $SU(8)$  model or beyond without having a negative number of fundamental or two-index antisymmetric multiplets.<sup>5</sup> To obtain higher  $SU(N)$ , we must consider bases other than  $\mathbb{P}^2$ .

### 2.3.2 F-theory models with $T = 1$ (compactifications on $\mathbb{F}_n$ )

Models on  $\mathbb{F}_n$  have one tensor multiplet, and  $a$  and  $b$  live on a two-dimensional lattice. The basis for the homology classes consists of two elements,  $S$  and  $F$ , with

$$S \cdot S = -n \qquad S \cdot F = 1 \qquad F \cdot F = 0. \qquad (2.17)$$

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<sup>5</sup>Even including the two-index symmetric representation does not make such a model possible.

Additionally, we define a homology class  $\tilde{S} \equiv S + nF$ , with

$$\tilde{S} \cdot \tilde{S} = n \qquad \tilde{S} \cdot S = 0 \qquad \tilde{S} \cdot F = 1. \qquad (2.18)$$

The canonical class is  $K_B = -2S - (n + 2)F$ .

Unlike  $\mathbb{P}^2$ , at least some of the  $\mathbb{F}_n$  have algebraic curves of genus two. Our analysis will rely in particular on smooth curves of class  $2\tilde{S}$  on  $\mathbb{F}_3$ , which have self-intersection  $n = 12$ . As can be verified with (2.16), such curves have genus  $g = 2$ , and  $SU(N)$  groups tuned on these curves admit two adjoint hypermultiplets. Note that the curve  $S$  with self-intersection  $S \cdot S = -3$  on  $\mathbb{F}_3$  gives a non-Higgsable cluster: an  $SU_3$  gauge algebra with no matter. Since  $S \cdot \tilde{S} = 0$ , there is no jointly charged matter, and this non-Higgsable cluster plays no role in the model except that it contributes an additional 8 vector multiplets to  $V$ , which increases the number of matter hypermultiplets available and is relevant in some extreme cases as we encounter below.

If we assume there are no hypermultiplets of three-index antisymmetric matter, table 2 suggests that the largest  $SU(N)$  we can tune on  $2\tilde{S}$  on  $\mathbb{F}_3$  is  $SU(6)$ . Beyond this, the number of fundamental hypermultiplets would become negative. However, if we include three-index antisymmetric matter, the anomaly cancellation conditions allow for groups as large as  $SU(9)$  on  $2\tilde{S}$ . The  $SU(9)$  model, with a charged spectrum of

$$2 \times \mathbf{80} + 2 \times \mathbf{84}, \qquad (2.19)$$

will not be discussed much here for a few different reasons. First, the  $\mathbf{84}$  representation is difficult (perhaps impossible) to obtain in F-theory; if it can be realized, this representation would likely involve complicated mechanisms that may not be visible in the Weierstrass model [8, 21, 38]. Moreover, this  $SU(9)$  model would not give interesting charges under the Higgsing process of section 2.1. Even though the resulting spectrum naively includes charge  $\pm 9$  matter, the resulting charges are all multiples of 3, and the true maximum charge of the resulting spectrum, in the natural charge units, is  $\pm 3$ . But the  $SU(8)$  model on  $2\tilde{S}$  can be cleanly realized in F-theory, as discussed further in section 4.

### 3 Explicit F-theory models on $\mathbb{P}^2$ (charges $q = 1$ through 7)

So far, we have shown that certain  $SU(N)$  spectra satisfy the anomaly cancellation conditions and can be Higgsed down to  $U(1)$ . But a model that satisfies the anomaly cancellation conditions may not necessarily be realized in F-theory. Therefore, we now turn to explicit F-theory constructions of  $SU(N)$  models. This section focuses on F-theory models on a  $\mathbb{P}^2$  base, which have no tensor multiplets. In section 4, we discuss F-theory models with  $\mathbb{F}_n$  bases, which admit one tensor multiplet.

#### 3.1 $SU(5)$ and $SU(6)$ (charges 1 through 6)

As noted earlier, the fundamental, two-index antisymmetric, and adjoint representations of  $SU(N)$  are relatively easy to realize in F-theory. The anomaly analysis in section 2.2 suggests that, at least for our purposes, the largest consistent  $SU(N)$  groups admitting

only these representations are SU(5) and SU(6) in theories with no tensor multiplets. According to table 1, Higgsing these SU( $N$ ) models leads to charges  $\pm 1$  through  $\pm 6$ . Thus, by explicitly constructing the appropriate SU(5) and SU(6) F-theory models, we can demonstrate that charges  $\pm 1$  through  $\pm 6$  can be realized in F-theory.

We construct these F-theory models over a  $\mathbb{P}^2$  base. In order to obtain the two adjoint hypermultiplets necessary for the Higgsing process, we must tune the SU( $N$ ) symmetries on a curve  $\sigma = 0$  of genus  $g \geq 2$ . As mentioned previously, there are no algebraic curves of genus 2 on  $\mathbb{P}^2$ , but quartic curves on  $\mathbb{P}^2$  have genus 3. We therefore let  $\sigma$  be a smooth quartic curve i.e. a curve with homology class  $4H$ . The resulting SU( $N$ ) models have three adjoint hypermultiplets, one more than necessary to Higgs the gauge group down to U(1).

Fortunately, there is already a known recipe to construct SU( $N$ ) models with only the three representations mentioned above. The simplest construction of a model with gauge group SU( $N$ ) proceeds by tuning the coefficients  $a_i$  in the ‘‘Tate form’’  $y^2 + a_1yx + a_3y = x^3 + a_2x^2 + a_4x + a_6$  in a way that automatically guarantees the appropriate Kodaira singularity type for SU( $N$ ) [18, 34, 37]; the models constructed in this way have precisely the three representations we want. A more general approach to tuning Weierstrass models with SU( $N$ ) gauge groups directly was developed in [39]; because we will be interested in models with other representations we follow that approach here. The expressions are different for even and odd  $N$ , so let us focus on the SU(5) model first. According to the formulas in [39], the SU(5) Weierstrass model is<sup>6</sup>

$$y^2 = x^3 + \left( -\frac{1}{3}\Phi^2 + \frac{1}{2}\phi_0\psi_2\sigma^2 + f_3\sigma^3 \right) x + \left( \frac{2}{27}\Phi^3 - \frac{1}{6}\Phi\phi_0\psi_2\sigma^2 - \frac{1}{3}\Phi f_3\sigma^3 + \frac{1}{4}\psi_2^2\sigma^4 \right), \tag{3.1}$$

where  $\Phi$  is given by

$$\Phi = \frac{1}{4}\phi_0^2 + \phi_1\sigma. \tag{3.2}$$

The homology classes for the various parameters are

$$\begin{aligned} [\sigma] &= 4H & [\Phi] &= -2K_B = 6H \\ [\phi_0] &= -K_B = 3H & [\psi_2] &= -3K_B - 2[\sigma] = H \end{aligned} \tag{3.3}$$

$$[f_3] = -4K_B - 3[\sigma] = 0H \quad [\phi_1] = -2K_B - [\sigma] = 2H. \tag{3.4}$$

The discriminant meanwhile is given by

$$\Delta \equiv 4f^3 + 27g^2 = \frac{1}{16}\sigma^5 \left[ \phi_0^4\psi_2(\phi_1\psi_2 - \phi_0f_3) + \mathcal{O}(\sigma) \right] \tag{3.5}$$

$\Delta$  is proportional to  $\sigma^5$ , while  $f$  and  $g$  are not proportional to  $\sigma$ . Moreover, the split condition [34, 37, 40] is satisfied, as  $\Phi|_{\sigma=0}$  is a perfect square. The Kodaira classification [18, 34, 41–43] therefore indicates that we have tuned an  $I_5$  singularity on  $\sigma = 0$ , signaling the

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<sup>6</sup>The original expressions include a  $g_5\sigma^5$  term in the  $g$  for the Weierstrass model. However,  $g_5$  would be ineffective for  $[\sigma] = 4H$ . To address this issue, we simply set  $g_5$  to zero, which does not cause any problems in the Weierstrass model.

expected presence of an  $SU(5)$  gauge group. Additionally, the only other component of the discriminant is an  $I_1$  locus, suggesting that there are no other nonabelian gauge factors.

To verify the  $SU(5)$  model's matter spectrum, we first note that, because  $\sigma$  is a smooth curve of genus 3, there are three adjoint (**24**) hypermultiplets. The remaining charged hypermultiplets are localized at codimension-two loci in the  $\mathbb{P}^2$  base with enhanced fiber singularities. Enhancements occur at  $\phi_0 = \sigma = 0$  and  $\psi_2(\phi_1\psi_2 - \phi_0f_3) = \sigma = 0$ . At  $\phi_0 = \sigma = 0$ , the singularity type enhances from  $I_5$  to  $I_1^*$ , indicating that the  $[\phi_0] \cdot [\sigma] = 12$  points where  $\phi_0 = 0$  and  $\sigma = 0$  intersect support two-index antisymmetric (**10**) multiplets. Finally, at the  $\psi_2(\phi_1\psi_2 - \phi_0f_3) = \sigma = 0$  loci, the singularity type enhances to  $I_6$ . Therefore, the 16  $\psi_2(\phi_1\psi_2 - \phi_0f_3) = \sigma = 0$  points support fundamental (**5**) multiplets. In summary, the charged spectrum for the  $SU(5)$  model is

$$3 \times \mathbf{24} + 12 \times \mathbf{10} + 16 \times \mathbf{5}, \tag{3.6}$$

in line with the expectations from the anomaly cancellation conditions.

The Higgsing procedure outlined in section 2.1 leads to a charged  $U(1)$  spectrum of

$$16 \times (\mathbf{q} = \mathbf{5}) + 16 \times (\mathbf{q} = \mathbf{4}) + 48 \times (\mathbf{q} = \mathbf{3}) + 72 \times (\mathbf{q} = \mathbf{2}) + 64 \times (\mathbf{q} = \mathbf{1}). \tag{3.7}$$

This spectrum satisfies the  $U(1)$  anomaly cancellation conditions with  $\tilde{b} = 5 \times 4 \times [\sigma]$ , as expected from the analysis in [10], where it was shown that this kind of Higgsing of an  $SU(N)$  model gives a  $U(1)$  model with anomaly coefficient  $\tilde{b} = N(N - 1)[\sigma]$ . We have therefore explicitly constructed an  $SU(5)$  model in F-theory that can be Higgsed down to a  $U(1)$  model with charges  $\pm 1$  through  $\pm 5$ . This demonstrates that charges  $\pm 1$  through  $\pm 5$  can be realized in F-theory.

Now let us turn to the  $SU(6)$  theory, which allows us to demonstrate that charge  $\pm 6$  matter can be realized in F-theory. There are in fact two ways to obtain an  $SU(6)$  model. The first approach is to set  $\psi_2$  to 0 in (3.1), giving

$$y^2 = x^3 + \left(-\frac{1}{3}\Phi^2 + f_3\sigma^3\right)x + \left(\frac{2}{27}\Phi^3 - \frac{1}{3}\Phi f_3\sigma^3\right). \tag{3.8}$$

This Weierstrass model again corresponds to the Tate form, and also matches that derived from the expressions in [39].<sup>7</sup> The discriminant is now

$$\Delta = -\frac{1}{16}f_3^2\sigma^6 \left[(\phi_0^2 + 4\phi_1\sigma)^2 - 64f_3\sigma^3\right]. \tag{3.9}$$

The  $\sigma^6$  factor indicates the expected presence of an  $SU(6)$  gauge symmetry on  $\sigma = 0$ . Meanwhile,  $[f_3]$  is  $0H$ , so the  $f_3^2$  factor does not represent an additional nonabelian gauge group. Thus, the gauge group is simply  $SU(6)$ .

The matter content analysis resembles that for  $SU(5)$ . Since  $\sigma = 0$  is still a genus 3 curve, there are three adjoint (**35**) hypermultiplets. And the  $\phi_0 = \sigma = 0$  loci still contribute twelve two-index antisymmetric (**15**) multiplets. However, there are no codimension-two

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<sup>7</sup>Again, a  $g_6\sigma^6$  term has been dropped because  $g_6$  would be ineffective.

loci where the singularity type enhances from  $I_6$  to  $I_7$ , indicating that there are no fundamental  $(\mathbf{6})$  hypermultiplets. The charged matter spectrum is therefore

$$3 \times \mathbf{35} + 12 \times \mathbf{15}, \tag{3.10}$$

in agreement with the anomaly cancellation conditions. The corresponding  $U(1)$  spectrum is

$$20 \times (\mathbf{q} = \mathbf{6}) + 60 \times (\mathbf{q} = \mathbf{4}) + 120 \times (\mathbf{q} = \mathbf{2}). \tag{3.11}$$

This spectrum hints that  $\pm 6$  matter can be realized in F-theory, although one might argue that this model truly contains  $\pm 1$ ,  $\pm 2$ , and  $\pm 3$  matter because of the common factor between the charges.<sup>8</sup> As expected, the spectrum satisfies the  $U(1)$  anomaly conditions with  $\tilde{b} = 6 \times 5 \times [\sigma]$ .

The  $SU(5)$  and  $SU(6)$  examples considered so far demonstrate that matter with charges  $\pm 1$  through  $\pm 5$ , and possibly  $\pm 6$ , can be realized in F-theory. At this point, there seems to be an obstruction to tuning larger  $SU(N)$  gauge groups. All the ways of enhancing the  $SU(6)$  singularity of (3.8) force  $(f, g, \Delta)$  to simultaneously vanish on  $\sigma = 0$ , indicating that  $\sigma = 0$  would no longer support an  $SU(N)$  symmetry. This observation is in line with the expectations from anomaly cancellation: for  $\mathbb{P}^2$  models with only fundamental, adjoint, and two-index antisymmetric matter, the largest possible consistent gauge group is  $SU(6)$ .  $SU(6)$  is also the largest  $SU(N)$  that can be tuned on a quartic in  $\mathbb{P}^2$  using the Tate tuning approach. Naively, this might suggest that our approach can at best demonstrate that charges  $\pm 1$  through  $\pm 6$  occur in F-theory. But these results depend on the artificial assumption that we consider only the fundamental, adjoint, and two-index antisymmetric representations. While the constructions become more complicated, there are still consistent F-theory models with the three-index antisymmetric representation, allowing us to show that charges larger than  $\pm 6$  can be realized in F-theory.

### 3.2 $SU(6)$ and $SU(7)$ with three-index antisymmetric matter (charges 6 and 7)

Let us now consider models admitting the three-index antisymmetric representation. The anomaly conditions suggest that, when one includes three-index antisymmetric matter, an  $SU(N)$  gauge symmetry on a quartic on  $\mathbb{P}^2$  can be as large as  $SU(7)$ . According to table 1, the resulting  $U(1)$  symmetry would support charge  $\pm 7$  matter. Thus, if we can explicitly construct this  $SU(7)$  model in F-theory, we know that charge  $\pm 7$  matter can be realized in F-theory.

To actually find this  $SU(7)$  F-theory model, we must consider the second method for obtaining an  $SU(6)$  Weierstrass model from (3.1). Instead of setting  $\psi_2$  to zero, we let

$$\phi_1 = f_3\beta \qquad \phi_0 = \beta\psi_2, \tag{3.12}$$

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<sup>8</sup>In later examples, charge  $\pm 6$  matter appears in spectra without a common factor, more rigorously establishing that charge  $\pm 6$  matter can be realized in F-theory.

where  $[\beta] = 2H$ . The Weierstrass model is now

$$y^2 = x^3 + \left(-\frac{1}{3}\Phi^2 + \frac{1}{2}\beta\psi_2^2\sigma^2 + f_3\sigma^3\right)x + \left(\frac{2}{27}\Phi^3 - \frac{1}{6}\Phi\beta\psi_2^2\sigma^2 - \frac{1}{3}\Phi f_3\sigma^3 + \frac{1}{4}\psi_2^2\sigma^4\right) \quad (3.13)$$

with

$$\Phi = \beta \left(\frac{1}{4}\beta\psi_2^2 + f_3\sigma\right), \quad (3.14)$$

and the discriminant is

$$\Delta = -\frac{1}{16}\sigma^6 [\beta^3\psi_2^4(\psi_2^2 + f_3^2\beta) + \mathcal{O}(\sigma)]. \quad (3.15)$$

The  $\sigma^6$  factor indicates that we have tuned an  $SU(6)$  symmetry on  $\sigma = 0$ , and since  $\sigma = 0$  has genus  $g = 3$ , there are three adjoint (**35**) hypermultiplets in the spectrum. At  $\psi_2 = \sigma = 0$ , the singularity type enhances from  $I_6$  to  $I_2^*$ , so these four points contribute four hypermultiplets of two-index antisymmetric (**15**) matter. The eight  $\psi_2^2 + f_3\beta = \sigma = 0$  points, where the singularity types enhances from  $I_6$  to  $I_7$ , contribute eight hypermultiplets of fundamental (**6**) matter. But there is a third codimension-two locus,  $\beta = \sigma = 0$ , where the singularity type enhances from  $I_6$  to  $IV^*$ , a behavior not seen in the previous models. These eight points contribute eight half-hypermultiplets of three-index antisymmetric (**20**) matter. In summary, the total charged spectrum is

$$3 \times \mathbf{35} + 8 \times \frac{1}{2}\mathbf{20} + 4 \times \mathbf{15} + 8 \times \mathbf{6}. \quad (3.16)$$

The resulting  $U(1)$  spectrum would be

$$20 \times (\mathbf{q} = \mathbf{6}) + 8 \times (\mathbf{q} = \mathbf{5}) + 20 \times (\mathbf{q} = \mathbf{4}) + 80 \times (\mathbf{q} = \mathbf{3}) + 40 \times (\mathbf{q} = \mathbf{2}) + 40 \times (\mathbf{q} = \mathbf{1}). \quad (3.17)$$

As expected, this  $U(1)$  spectrum satisfies the anomaly conditions with  $\tilde{b} = 6 \times 5 \times [\sigma]$ . With this explicit  $SU(6)$  model, we have unambiguously shown that charge  $\pm 1$  through  $\pm 6$  can be realized in F-theory. Importantly, the greatest common factor of the charges is 1, indicating that the  $U(1)$  model would genuinely have charge  $\pm 6$  matter.

We can then derive an  $SU(7)$  Weierstrass model by letting

$$\psi_2 = f_3\delta \qquad \beta = -\delta^2, \quad (3.18)$$

where  $[\delta] = H$ . The Weierstrass model is now

$$y^2 = x^3 + \left(-\frac{1}{3}\Phi^2 - \frac{1}{2}f_3^2\delta^4\sigma^2 + f_3\sigma^3\right)x + \left(\frac{2}{27}\Phi^3 + \frac{1}{6}\Phi f_3^2\delta^4\sigma^2 - \frac{1}{3}\Phi f_3\sigma^3 + \frac{1}{4}f_3^2\delta^2\sigma^4\right) \quad (3.19)$$

with

$$\Phi = f_3\delta^2 \left(\frac{1}{4}f_3\delta^4 - \sigma\right), \quad (3.20)$$



and the discriminant is

$$\Delta = \frac{1}{16} f_3^3 \sigma^7 [2f_3^2 \delta^8 - 13f_3 \delta^4 \sigma + 64\sigma^2]. \quad (3.21)$$

The  $\sigma^7$  factor indicates that the gauge group is  $SU(7)$ , while the  $f_3^3$  does not signal the appearance of an extra gauge factor since  $[f_3] = 0H$ . Again, there are three adjoint **(48)** hypermultiplets because  $\sigma = 0$  is a genus-3 curve. The only codimension-two singularities occur at  $\delta = \sigma = 0$ , where the  $I_7$  singularity type enhances to  $III^*$ . Each  $\delta = \sigma = 0$  point therefore contributes a three-index antisymmetric **(35)** hypermultiplet and a fundamental **(7)** hypermultiplet. The charged matter spectrum is therefore

$$3 \times \mathbf{48} + 4 \times \mathbf{35} + 4 \times \mathbf{7}, \quad (3.22)$$

in line with the expectations from the anomaly conditions. The corresponding  $U(1)$  charge spectrum would be

$$24 \times (\mathbf{q} = \mathbf{7}) + 4 \times (\mathbf{q} = \mathbf{6}) + 60 \times (\mathbf{q} = \mathbf{4}) + 80 \times (\mathbf{q} = \mathbf{3}) + 24 \times (\mathbf{q} = \mathbf{1}), \quad (3.23)$$

which, as expected, satisfies the  $U(1)$  anomaly conditions with  $\tilde{b} = 7 \times 6 \times [\sigma]$ . The explicit  $SU(7)$  F-theory construction therefore shows that charge  $\pm 7$  matter can be realized in F-theory.

For the four types of representations considered so far,  $SU(7)$  is the largest  $SU(N)$  gauge group that can be tuned on a quartic on  $\mathbb{P}^2$ . The anomaly conditions suggest that models with  $N > 7$  would have a negative number of two-index antisymmetric multiplets and would therefore be inconsistent. However, by again expanding the scope of constructions considered, we can obtain charges larger than  $\pm 7$ . In particular, we then focus on models with  $\mathbb{F}_n$  bases, for which anomaly cancellation suggests one can obtain a satisfactory  $SU(8)$  group with two adjoint matter hypermultiplets.

## 4 Explicit F-theory models on $\mathbb{F}_n$ (charges up to $q = 21$ )

### 4.1 $SU(8)$ with three-index antisymmetric matter (charge 8)

So far, we have considered curves on  $\mathbb{P}^2$  of genus three, which give us one more adjoint hypermultiplet than needed for the Higgsing process. In principle, we require only a genus-two curve to perform the Higgsing. While there are no algebraic curves of genus two on  $\mathbb{P}^2$ , there are algebraic genus-two curves on some of the  $\mathbb{F}_n$ , as mentioned in section 2.3.2. For example, a curve  $\sigma = 0$  of homology class  $2\tilde{S} = 2S + 6F$  on  $\mathbb{F}_3$  has genus

$$g = 1 + \frac{1}{2}[\sigma] \cdot (K_B + [\sigma]) = 1 + \tilde{S} \cdot F = 2. \quad (4.1)$$

The smaller genus allows us to obtain larger  $SU(N)$  groups on  $2\tilde{S}$ , which in turn suggest higher charges should exist in F-theory.

In particular, we can tune  $SU(8)$  on  $2\tilde{S}$ , implying that charge  $\pm 8$  matter can occur in F-theory. To construct the explicit model, we introduce a coordinate  $u$  of homology class  $\tilde{S}$

and a coordinate  $v$  of homology class  $S$ . We start with an  $SU(6)$  Weierstrass model, which can be constructed by using the formulas in [39] and accounting for the fact that certain parameters are reducible, which leads in particular to the explicit appearance of powers of  $v$  in  $f$  and  $g$ :

$$\begin{aligned}
 f &= -\frac{1}{48}v^2 \left[ \alpha^4 \beta^4 v^2 + 8\alpha^2 \beta^3 \nu v^2 \sigma + 8\beta (2\beta \nu^2 v^2 + \alpha^2 \phi_2) \sigma^2 + 16(9\beta \lambda + \nu \phi_2) \sigma^3 \right], \quad (4.2) \\
 g &= \frac{1}{864}v^2 \left[ \alpha^6 \beta^6 v^4 + 12\alpha^4 \beta^5 \nu v^4 \sigma + (12\alpha^4 \beta^3 \phi_2 v^2 + 48\alpha^2 \beta^4 \nu^2 v^4) \sigma^2 \right. \\
 &\quad + (72\alpha^2 \beta^2 (3\beta \lambda + \nu \phi_2) v^2 + 64\beta^3 \nu^3 v^4) \sigma^3 \\
 &\quad \left. + (24\alpha^2 \phi_2^2 + 96\beta \nu (9\beta \lambda + \nu \phi_2) v^2) \sigma^4 + 864\lambda \sigma^5 \phi_2 \right]. \quad (4.3)
 \end{aligned}$$

The discriminant is proportional to  $\sigma^6 v^4$ , indicating that we have an  $SU(6)$  symmetry on  $\sigma = 0$  and an  $SU(3)$  symmetry tuned on  $v = 0$ . The  $SU(3)$  symmetry is the well-known non-Higgsable cluster on  $\mathbb{F}_3$ , and since  $[v] = S$ , there is no matter charged under both the  $SU(6)$  and  $SU(3)$  gauge groups, as discussed above. We take the various parameters (which are locally functions of the coordinates  $u, v$ ) to have homology classes

$$[\beta] = 2F \quad [\alpha] = S + 3F \quad [\nu] = 2F \quad [\lambda] = 0F \quad [\phi_2] = 0F. \quad (4.4)$$

Note that  $\lambda$  and  $\phi_2$  are essentially constants.

We can now enhance the  $SU(6)$  symmetry to  $SU(7)$  by letting

$$\beta = \delta^2 \quad \alpha = \delta \xi v \quad \phi_2 = 3\kappa_0^2 \quad \lambda = \rho_0 \kappa_0^3 \quad \nu = \kappa_0 (3\delta^2 \rho_0 + \xi), \quad (4.5)$$

where

$$[\delta] = F \quad [\xi] = 2F \quad [\kappa_0] = 0F \quad [\rho_0] = 0F. \quad (4.6)$$

The discriminant is now

$$\Delta = v^4 \sigma^7 \left[ \frac{1}{8} v^6 \delta^8 \kappa_0^7 \xi^4 (6\rho_0 \delta^2 - \xi) + \mathcal{O}(\sigma) \right]. \quad (4.7)$$

To obtain an  $SU(8)$  model, we therefore must let

$$\xi = 6\rho_0 \delta^2. \quad (4.8)$$

This redefinition gives us an  $SU(8)$  tuned on  $\sigma = 0$ . In fact, we can set  $\rho_0$  and  $\kappa_0$  to 1 without loss of generality, giving us a Weierstrass model of the form

$$\begin{aligned}
 y^2 &= x^3 - 3v^2 \delta^2 (9\delta^{18} v^6 + 18\delta^{12} v^4 \sigma + 15\delta^6 v^2 \sigma^2 + 4\sigma^3) x \\
 &\quad + 3v^2 (18\delta^{30} v^{10} + 54\delta^{24} v^8 \sigma + 72\delta^{18} v^6 \sigma^2 + 48\delta^{12} v^4 \sigma^3 + 15\delta^6 v^2 \sigma^4 + \sigma^5). \quad (4.9)
 \end{aligned}$$

The discriminant is now

$$\Delta = 27v^4 \sigma^8 (9\delta^{12} v^4 + 14\delta^6 v^2 \sigma + 9\sigma^2). \quad (4.10)$$

The  $\sigma^8$  factor indicates that we have successfully tuned an SU(8) gauge group, while the  $v^4$  factor represents the expected non-Higgsable SU(3) symmetry.

There are no hypermultiplets charged under the SU(3) symmetry, but there are hypermultiplets charged under the SU(8). Since  $\sigma = 0$  is a genus-two curve, there are two hypermultiplets in the adjoint (**63**) representation of SU(8). Additionally, the singularity type enhances from  $I_8$  to  $II^*$  at  $\sigma = \delta = 0$ . Each of the two  $\sigma = \delta = 0$  points therefore supports hypermultiplets in the **56** + **28** + **8**. In summary, the spectrum of hypermultiplets charged under the SU(8) is

$$2 \times \mathbf{63} + 2 \times \mathbf{56} + 2 \times \mathbf{28} + 2 \times \mathbf{8}, \tag{4.11}$$

which agrees with the SU(8) anomaly cancellation conditions. If this SU(8) is Higgsed according to the Higgsing procedure of section 2.1, the resulting charge spectrum becomes

$$14 \times (\mathbf{q} = \mathbf{8}) + 2 \times (\mathbf{q} = \mathbf{7}) + 14 \times (\mathbf{q} = \mathbf{6}) + 42 \times (\mathbf{q} = \mathbf{5}) \\ + 70 \times (\mathbf{q} = \mathbf{3}) + 42 \times (\mathbf{q} = \mathbf{2}) + 14 \times (\mathbf{q} = \mathbf{1}), \tag{4.12}$$

satisfying the U(1) anomaly conditions for  $\tilde{b} = 8 \times 7 \times 2\tilde{S}$ . Therefore, our explicit SU(8) construction demonstrates that charge  $\pm 8$  matter can be realized in F-theory.

#### 4.2 Alternative Higgsings of SU(8) (charges 9, 10, 11, 12, 14)

In fact, one can obtain charges larger than  $\pm 8$  through alternative Higgsings of this SU(8) model. We will not perform an exhaustive investigation of all possible Higgsing chains, instead focusing on a few specific Higgsing processes that roughly follow the pattern

$$\text{SU}(8) \xrightarrow{\text{Higgs on } \mathbf{56}} \text{SU}(5) \times \text{SU}(3) \xrightarrow{\text{Higgs on } (\mathbf{24}, \mathbf{1}), (\mathbf{1}, \mathbf{8})} \text{U}(1) \times \text{U}(1) \xrightarrow{\text{Higgs on } (q_1, q_2)} \text{U}(1) \tag{4.13}$$

Admittedly, this procedure is somewhat ad-hoc, suggesting that other Higgsing processes might produce even larger charges.

We start by giving VEVs to two full hypermultiplets of three-index antisymmetric (**56**) matter in the SU(8) model considered above.<sup>9</sup> This VEV breaks the SU(8) symmetry to SU(5)  $\times$  SU(3), and the SU(8) representations branch as<sup>10</sup>

$$\mathbf{56} \rightarrow (\overline{\mathbf{10}}, \mathbf{1}) + (\mathbf{10}, \mathbf{3}) + (\mathbf{5}, \overline{\mathbf{3}}) + (\mathbf{1}, \mathbf{1}) \tag{4.14}$$

$$\mathbf{63} \rightarrow (\mathbf{24}, \mathbf{1}) + (\mathbf{5}, \overline{\mathbf{3}}) + (\overline{\mathbf{5}}, \mathbf{3}) + (\mathbf{1}, \mathbf{8}) + (\mathbf{1}, \mathbf{1}) \tag{4.15}$$

$$\mathbf{28} \rightarrow (\mathbf{10}, \mathbf{1}) + (\mathbf{5}, \mathbf{3}) + (\mathbf{1}, \overline{\mathbf{3}}) \tag{4.16}$$

$$\mathbf{8} \rightarrow (\mathbf{5}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}). \tag{4.17}$$

<sup>9</sup>D-term constraints for this breaking suggest we must give VEVs to two hypermultiplets instead of just one, just as for the Higgsing on two fundamental matter representations as described in section 2.1.

<sup>10</sup>Note that the branching patterns distinguish between representations and their conjugates. Even though full hypermultiplets still contain fields in  $\mathbf{R}$  and  $\overline{\mathbf{R}}$ , it is important to keep track of representations and their conjugates for jointly charged matter. For instance,  $(\mathbf{5}, \overline{\mathbf{3}})$  is not the conjugate representation of  $(\mathbf{5}, \mathbf{3})$ , and the two represent different types of hypermultiplets.

The two  $(\mathbf{5}, \bar{\mathbf{3}})$  hypermultiplets coming from the two  $\mathbf{56}$  multiplets are eaten during the Higgsing process. Thus, after noting that hypermultiplets in conjugate representations are essentially the same, the  $SU(5) \times SU(3)$  spectrum is

$$2 \times (\mathbf{10}, \mathbf{3}) + 4 \times (\mathbf{5}, \bar{\mathbf{3}}) + 2 \times (\mathbf{5}, \mathbf{3}) + 2 \times (\mathbf{24}, \mathbf{1}) + 2 \times (\mathbf{1}, \mathbf{8}) + 4 \times (\mathbf{10}, \mathbf{1}) + 2 \times (\mathbf{5}, \mathbf{1}) + 4 \times (\mathbf{1}, \mathbf{3}). \quad (4.18)$$

As expected, this spectrum is consistent with the anomaly conditions for  $SU(5)$  and  $SU(3)$  models tuned on two distinct divisors in the homology class  $2\tilde{S}$ . In fact, we construct an explicit F-theory realization of this  $SU(5) \times SU(3)$  model in appendix A.

Since there are two  $SU(5)$  adjoint hypermultiplets and two  $SU(3)$  adjoint hypermultiplets, we can now Higgs the  $SU(5)$  and  $SU(3)$  groups individually using the Higgsing process in section 2.1. The end result is a  $U(1) \times U(1)$  gauge group. Hypermultiplets charged under this  $U(1) \times U(1)$  symmetry are labeled as  $(q_1, q_2)$ , where  $q_1$  and  $q_2$  denote the charges under the two  $U(1)$  symmetries. However, note that a  $(q_1, q_2)$  hypermultiplet includes fields with charges  $(q_1, q_2)$  and  $(-q_1, -q_2)$ . The branching patterns for the  $SU(5) \times SU(3)$  representations are similar to the individual  $SU(5)$  and  $SU(3)$  branching described in section 2.1. But it is important to note that charges coming from a conjugate representation  $\bar{\mathbf{R}}$  are the negative of those coming from  $\mathbf{R}$ . To illustrate the effects of this fact, consider the branching patterns for the  $(\mathbf{5}, \mathbf{3})$  and  $(\mathbf{5}, \bar{\mathbf{3}})$  representations. First considering the  $SU(5)$  and  $SU(3)$  representations individually, the rules in section 2.1 suggest that the  $\mathbf{5}$ ,  $\mathbf{3}$ , and  $\bar{\mathbf{3}}$  representations branch as

$$\mathbf{5} \rightarrow 4 \times (\mathbf{q} = \mathbf{1}) + (\mathbf{q} = -\mathbf{4}) \quad (4.19)$$

$$\mathbf{3} \rightarrow 2 \times (\mathbf{q} = \mathbf{1}) + (\mathbf{q} = -\mathbf{2}) \quad (4.20)$$

$$\bar{\mathbf{3}} \rightarrow 2 \times (\mathbf{q} = -\mathbf{1}) + (\mathbf{q} = \mathbf{2}). \quad (4.21)$$

Note that we have kept track of the signs of the charges, and the signs for the charges coming from  $\mathbf{3}$ , and  $\bar{\mathbf{3}}$  are negatives of each other. From these individual branching patterns, the  $(\mathbf{5}, \mathbf{3})$  representation should branch as

$$(\mathbf{5}, \mathbf{3}) \rightarrow (-4, -2) + 2 \times (-4, 1) + 4 \times (1, -2) + 8 \times (1, 1). \quad (4.22)$$

In contrast, the  $(\mathbf{5}, \bar{\mathbf{3}})$  representation should branch as

$$(\mathbf{5}, \bar{\mathbf{3}}) \rightarrow (-4, 2) + 2 \times (-4, -1) + 4 \times (1, 2) + 8 \times (1, -1). \quad (4.23)$$

These branching patterns are distinct from one another. For instance  $(-4, -2)$  and  $(-4, 2)$  represent different types of multiplets, as the relative sign between the  $q_1$  and  $q_2$  charges differs. It is therefore important to distinguish between representations and their conjugates when considering the branching patterns.

In the end, the branching patterns for the  $SU(5) \times SU(3)$  hypermultiplets are

$$(\mathbf{10}, \mathbf{3}) \rightarrow 4 \times (-3, -2) + 8 \times (-3, 1) + 6 \times (2, -2) + 12 \times (2, 1) \quad (4.24)$$

$$(\mathbf{5}, \mathbf{3}) \rightarrow (-4, -2) + 2 \times (-4, 1) + 4 \times (1, -2) + 8 \times (1, 1) \quad (4.25)$$

$$(\mathbf{5}, \bar{\mathbf{3}}) \rightarrow (-4, 2) + 2 \times (-4, -1) + 4 \times (1, 2) + 8 \times (1, -1) \quad (4.26)$$

$$(\mathbf{24}, \mathbf{1}) \rightarrow 4 \times (5, 0) + 4 \times (-5, 0) + 16 \times (0, 0) \quad (4.27)$$

$$(\mathbf{1}, \mathbf{8}) \rightarrow 2 \times (0, 3) + 2 \times (0, -3) + 4 \times (0, 0) \quad (4.28)$$

$$(\mathbf{10}, \mathbf{1}) \rightarrow 4 \times (-3, 0) + 6 \times (2, 0) \quad (4.29)$$

$$(\mathbf{5}, \mathbf{1}) \rightarrow (-4, 0) + 4 \times (1, 0) \quad (4.30)$$

$$(\mathbf{1}, \mathbf{3}) \rightarrow (0, -2) + 2 \times (0, 1). \quad (4.31)$$

To find the charged  $U(1) \times U(1)$  spectrum, we must account for the fact that some of the  $(\pm 5, 0)$  and  $(0, \pm 3)$  multiplets are eaten as part of the Higgsing process. Additionally, we are free to identify  $(q_1, q_2)$  and  $(-q_1, -q_2)$  hypermultiplets. Taking these facts into account, the charged  $U(1) \times U(1)$  spectrum is

$$\begin{aligned} & 8 \times (3, 2) + 16 \times (3, -1) + 12 \times (2, -2) + 24 \times (2, 1) \\ & \quad + 2 \times (4, 2) + 4 \times (4, -1) + 8 \times (1, -2) + 16 \times (1, 1) \\ & \quad + 4 \times (4, -2) + 8 \times (4, 1) + 16 \times (1, 2) + 32 \times (1, -1) \\ & \quad + 8 \times (5, 0) + 4 \times (0, 3) + 16 \times (3, 0) + 24 \times (2, 0) \\ & \quad + 2 \times (4, 0) + 8 \times (1, 0) + 4 \times (0, 2) + 8 \times (0, 1). \end{aligned} \quad (4.32)$$

This spectrum satisfies the  $U(1) \times U(1)$  anomaly cancellation conditions described in, for instance, [32, 33].

Finally, we can Higgs  $U(1) \times U(1)$  down to a single  $U(1)$  by giving a VEV to a charged hypermultiplet. Suppose we give a VEV to a hypermultiplet with charge  $(q'_1, q'_2)$ . A hypermultiplet with  $U(1) \times U(1)$  charge  $(q_1, q_2)$  would then have a  $U(1)$  charge given by

$$q = q'_2 q_1 - q'_1 q_2. \quad (4.33)$$

Of course, the overall sign of  $q$  is not too important, since  $U(1)$  charged hypermultiplets with charge  $q$  have fields with charges  $+q$  and  $-q$ . Note that, at least for the charged  $U(1)$  spectrum, we need not worry about the eaten degrees of freedom, as they would have charge  $q = 0$ . For particular  $(q'_1, q'_2)$ , the resulting  $U(1)$  charges can be higher than  $\pm 8$ , as we illustrate with three examples:

**Higgsing on charge  $(1, 2)$  matter.** If we give a VEV to  $(q'_1, q'_2) = (1, 2)$  matter, the resulting  $U(1)$  charges are  $2q_1 - q_2$ . Therefore, the  $(4, -2)$  matter in the  $U(1) \times U(1)$  spectrum would become charge  $\pm 10$  matter, while the  $(4, -1)$  matter would become charge  $\pm 9$  matter. Indeed, the resulting charged  $U(1)$  spectrum is

$$\begin{aligned} & 12 \times (\mathbf{q} = \mathbf{10}) + 4 \times (\mathbf{q} = \mathbf{9}) + 2 \times (\mathbf{q} = \mathbf{8}) + 24 \times (\mathbf{q} = \mathbf{7}) + 30 \times (\mathbf{q} = \mathbf{6}) \\ & \quad + 40 \times (\mathbf{q} = \mathbf{4}) + 60 \times (\mathbf{q} = \mathbf{3}) + 12 \times (\mathbf{q} = \mathbf{2}) + 24 \times (\mathbf{q} = \mathbf{1}). \end{aligned} \quad (4.34)$$

This spectrum satisfies the  $U(1)$  anomaly conditions with  $\tilde{b} = 86 \times 2\tilde{S}$ .

**Higgsing on charge (4, -1) matter.** Alternatively, if we Higgs on  $(q'_1, q'_2) = (4, -1)$  matter, the resulting U(1) charges are  $q_1 + 4q_2$ . Then, the (0, 3) matter becomes charge  $\pm 12$  matter, while the (3, 2) matter becomes charge  $\pm 11$  matter. The charged U(1) spectrum is

$$\begin{aligned}
 &6 \times (\mathbf{q} = \mathbf{12}) + 8 \times (\mathbf{q} = \mathbf{11}) + 16 \times (\mathbf{q} = \mathbf{9}) + 12 \times (\mathbf{q} = \mathbf{8}) + 8 \times (\mathbf{q} = \mathbf{7}) \\
 &\quad + 36 \times (\mathbf{q} = \mathbf{6}) + 24 \times (\mathbf{q} = \mathbf{5}) + 14 \times (\mathbf{q} = \mathbf{4}) \\
 &\quad + 48 \times (\mathbf{q} = \mathbf{3}) + 24 \times (\mathbf{q} = \mathbf{2}) + 24 \times (\mathbf{q} = \mathbf{1}),
 \end{aligned} \tag{4.35}$$

which satisfies the U(1) anomaly equations with  $\tilde{b} = 116 \times 2\tilde{S}$ .

**Higgsing on charge (3, 2) matter.** Finally, if we Higgs on  $(q'_1, q'_2) = (3, 2)$  matter, the resulting U(1) charges are  $2q_1 - 3q_2$ . The (4, -2) matter would become charge  $\pm 14$  matter. The charged U(1) spectrum is

$$\begin{aligned}
 &4 \times (\mathbf{q} = \mathbf{14}) + 4 \times (\mathbf{q} = \mathbf{11}) + 20 \times (\mathbf{q} = \mathbf{10}) + 20 \times (\mathbf{q} = \mathbf{9}) + 10 \times (\mathbf{q} = \mathbf{8}) \\
 &\quad + 20 \times (\mathbf{q} = \mathbf{6}) + 40 \times (\mathbf{q} = \mathbf{5}) + 40 \times (\mathbf{q} = \mathbf{4}) \\
 &\quad + 8 \times (\mathbf{q} = \mathbf{3}) + 10 \times (\mathbf{q} = \mathbf{2}) + 40 \times (\mathbf{q} = \mathbf{1}),
 \end{aligned} \tag{4.36}$$

which satisfies the U(1) anomaly cancellation conditions with  $\tilde{b} = 134 \times 2\tilde{S}$ .

To summarize, various Higgsing of the explicit SU(8) F-theory model above lead to charges  $\pm 9, \pm 10, \pm 11, \pm 12$ , and  $\pm 14$  (in addition to charges  $\pm 1$  through  $\pm 8$ ). Therefore, these charges should be realizable in F-theory. Unluckily, the Higgsing processes considered here do not produce charge  $\pm 13$  matter. But, given the ad-hoc nature of this Higgsing process, it is likely that charge  $\pm 13$  can be realized through some other means.

### 4.3 SU(5) × SU(4) (charges 15, 16, 20, 21)

Even higher charges can be obtained by enhancing the SU(5) × SU(3) gauge group to SU(5) × SU(4). To obtain an explicit F-theory model realizing this SU(5) × SU(4) group, we take the SU(5) × SU(3) Weierstrass model described in appendix A and set

$$\epsilon = 3\delta^2. \tag{4.37}$$

The Weierstrass model is now

$$\begin{aligned}
 y^2 = &x^3 - 3v^2\delta^2 (144v^6\delta^{18} - 360v^4\delta^{12}\sigma + 105v^2\delta^6\sigma^2 + 10\sigma^3) x \\
 &+ 3v^2 (1152v^{10}\delta^{30} - 4320v^8\delta^{24}\sigma + 3960v^6\delta^{18}\sigma^2 - 330v^4\delta^{12}\sigma^3 + 150v^2\delta^6\sigma^4 + \sigma^5),
 \end{aligned} \tag{4.38}$$

where, as above,  $[v] = S, [\delta] = F$ , and  $[\sigma] = 2\tilde{S}$ . The discriminant meanwhile is

$$\Delta = 27v^4 (9\sigma - 4v^2\delta^6) (\sigma - 36v^2\delta^6)^4 \sigma^5, \tag{4.39}$$

signaling an SU(5) × SU(4) gauge group with each factor tuned on a divisor in the class  $2\tilde{S}$ .

The only codimension-two locus with enhanced singularities is  $\sigma = \delta = 0$ , where the singularity type enhances to  $II^*$ . By the Katz-Vafa analysis [44], in which one breaks the

**248** representation of  $E_8$  to  $SU(5) \times SU(4)$  representations, each of the two  $\sigma = \delta = 0$  points contributes

$$(\mathbf{10}, \mathbf{1}) + (\mathbf{10}, \mathbf{4}) + (\mathbf{5}, \mathbf{6}) + (\mathbf{1}, \mathbf{4}) + (\mathbf{5}, \bar{\mathbf{4}}) \quad (4.40)$$

hypermultiplets. Since both the  $SU(5)$  and  $SU(4)$  are tuned on genus-two curves, there are also two  $(\mathbf{24}, \mathbf{1})$  hypermultiplets and two  $(\mathbf{1}, \mathbf{15})$  hypermultiplets. Thus, the charged  $SU(5) \times SU(4)$  spectrum is

$$2 \times [(\mathbf{10}, \mathbf{1}) + (\mathbf{10}, \mathbf{4}) + (\mathbf{5}, \mathbf{6}) + (\mathbf{1}, \mathbf{4}) + (\mathbf{5}, \bar{\mathbf{4}}) + (\mathbf{24}, \mathbf{1}) + (\mathbf{1}, \mathbf{15})]. \quad (4.41)$$

It is interesting to note that there is no anomaly-consistent model with this gauge group and  $b$  coefficients without the exotic  $(\mathbf{10}, \mathbf{4})$  matter. Solving the anomaly equations for generic matter fields and including only bifundamental  $(\mathbf{5}, \mathbf{4})$  fields would give rise to negative multiplicities for some matter content. Thus, this seems to be the only Weierstrass model that realizes these gauge groups on curves in the class  $2\tilde{S}$ .

We can now give VEVs to the adjoint hypermultiplets as in section 2.1, breaking  $SU(5) \times SU(4)$  to  $U(1) \times U(1)$ . The  $SU(5) \times SU(4)$  representations branch as follows:

$$(\mathbf{10}, \mathbf{4}) \rightarrow 4 \times (-3, -3) + 12 \times (-3, 1) + 6 \times (2, -3) + 18 \times (2, 1), \quad (4.42)$$

$$(\mathbf{5}, \mathbf{6}) \rightarrow 3 \times (-4, -2) + 3 \times (-4, 2) + 12 \times (1, -2) + 12 \times (1, 2), \quad (4.43)$$

$$(\mathbf{5}, \bar{\mathbf{4}}) \rightarrow 3 \times (-4, -1) + 1 \times (-4, 3) + 12 \times (1, -1) + 4 \times (1, 3), \quad (4.44)$$

$$(\mathbf{10}, \mathbf{1}) \rightarrow 4 \times (-3, 0) + 6 \times (2, 0), \quad (4.45)$$

$$(\mathbf{1}, \mathbf{4}) \rightarrow 1 \times (0, -3) + 3 \times (0, 1), \quad (4.46)$$

$$(\mathbf{24}, \mathbf{1}) \rightarrow 4 \times (5, 0) + 16 \times (0, 0) + 4 \times (-5, 0), \quad (4.47)$$

$$(\mathbf{1}, \mathbf{15}) \rightarrow 3 \times (0, 4) + 9 \times (0, 0) + 3 \times (0, -4). \quad (4.48)$$

Accounting for the hypermultiplets eaten during the Higgsing process, the charged  $U(1) \times U(1)$  spectrum is<sup>11</sup>

$$\begin{aligned} & 8 \times (3, 3) + 24 \times (3, -1) + 12 \times (-2, 3) + 36 \times (2, 1) \\ & + 6 \times (4, 2) + 6 \times (4, -2) + 24 \times (-1, 2) + 24 \times (1, 2) \\ & + 6 \times (4, 1) + 2 \times (4, -3) + 24 \times (-1, 1) + 8 \times (1, 3) \\ & + 8 \times (3, 0) + 12 \times (2, 0) + 2 \times (0, 3) + 6 \times (0, 1) + 8 \times (5, 0) + 6 \times (0, 4). \end{aligned} \quad (4.49)$$

We now give a VEV to the charge  $(4, -3)$  matter, which breaks  $U(1) \times U(1)$  down to a single  $U(1)$ . The charges are given by  $q = 3q_1 + 4q_2$ , leading to a charged spectrum of

$$\begin{aligned} & 8 \times (\mathbf{q} = \mathbf{21}) + 6 \times (\mathbf{q} = \mathbf{20}) + 12 \times (\mathbf{q} = \mathbf{16}) + 16 \times (\mathbf{q} = \mathbf{15}) + 2 \times (\mathbf{q} = \mathbf{12}) \\ & + 24 \times (\mathbf{q} = \mathbf{11}) + 36 \times (\mathbf{q} = \mathbf{10}) + 8 \times (\mathbf{q} = \mathbf{9}) + 24 \times (\mathbf{q} = \mathbf{6}) \\ & + 48 \times (\mathbf{q} = \mathbf{5}) + 12 \times (\mathbf{q} = \mathbf{4}) + 24 \times (\mathbf{q} = \mathbf{1}). \end{aligned} \quad (4.50)$$

<sup>11</sup>Note that we have changed some of the signs by identifying hypermultiplets of charge  $(-q_1, -q_2)$  with those of charge  $(q_1, q_2)$ .

The spectrum satisfies the U(1) anomaly equations with  $\tilde{b} = 372 \times 2\tilde{S}$ . The spectrum includes charges as large as  $\pm 15$ ,  $\pm 16$ ,  $\pm 20$ , and  $\pm 21$ . Moreover, the charges above do not share any common factors, showing that the model genuinely realizes these large charges. Therefore, the largest charge that can be realized in F-theory must be at least charge  $q = \pm 21$ .

## 5 Open questions and further directions

By combining explicit Weierstrass constructions of 6D supergravity theories having non-abelian gauge groups with the basic physics of Higgsing processes, we have shown that F-theory can give rise to charges as large as  $q = \pm 21$  in 6D supergravity models with an abelian U(1) gauge group. We list here some questions and open problems for further research in this direction.

- While we have shown that charges up to  $q = 21$  are possible, we have not proven that this is the upper bound. It would be interesting to explore whether more exotic constructions can give even higher U(1) charges in 6D F-theory models, and/or to prove an upper bound on the charges allowed through F-theory constructions.
- We have noted that the constructions up to charge  $q = 6$  follow from simpler F-theory models with less exotic singularity types. It would be nice to understand if there is a qualitative difference between the geometric structure needed for charge  $q \leq 6$  and that needed for charge  $q > 6$ .
- Since the abelian anomaly conditions allow for an infinite set of solutions with U(1) gauge group and increasingly large charges, even for models with no tensor multiplets ( $T = 0$ ) [10], while only a finite number of F-theory models are possible, one may look for new quantum consistency conditions on the low-energy theory that may place an upper bound on the U(1) charge allowed in a consistent theory.
- For nonabelian gauge groups such as  $SU(N)$ , the Kodaira constraint from F-theory [5] imposes a strict upper bound on the anomaly coefficient  $b$ , namely  $-12a \geq Nb$ , meaning that  $-12a - Nb$  lies in the positivity cone of the theory. It is tempting to speculate that there is a natural geometric constraint on the anomaly coefficient  $\tilde{b}$  for an abelian U(1) factor; the large size of these coefficients for some of the constructions here, however (e.g.  $744\tilde{S}$  for the model with abelian charges  $q = \pm 21$ ), makes it clear that if there is such a bound it is quite large. It would be nice to either prove the existence of such a bound from geometry or give a convincing argument for the absence of such a bound.
- The constructions here are indirect and rely on the physical mechanism of Higgsing. To the best of our knowledge the largest  $q$  that has been explicitly constructed in a Weierstrass model for a 6D theory with only a U(1) gauge group is  $q = 4$  [25]. It would be good to have explicit constructions of the Weierstrass models for higher abelian charges and to investigate the singularity structure of the corresponding geometries.



- In particular, we have focused here on breaking  $SU(N)$  gauge groups to achieve high abelian charges. It would be interesting to explore whether breaking other groups such as exceptional groups  $E_7, E_6, F_4$  could also give large abelian charges. Because the constructions used here rely on exotic matter representations of  $SU(N)$ , and such exotic matter representations cannot be realized in a straightforward fashion for the exceptional groups [8], it may be harder to get large charges from other nonabelian groups; nonetheless, this avenue should be explored more thoroughly. (See also the following related point.)
- In the constructions in this paper we have relied on the presence of nonabelian theories with exotic matter that has an explicit construction through a Weierstrass model without non-resolvable  $(4, 6)$  codimension 2 points that may be associated with superconformal field theories. It is possible that there may be consistent abelian models with even higher charges that are related in a similar way to “unHiggsed” nonabelian models with exotic matter that gives rise to  $(4, 6)$  singularities in the geometry. For example, similar to the  $SU(5) \times SU(4)$  model in the last section, one may consider trying to construct a model with  $SU(6) \times SU(3)$  gauge group and matter in the  $(\mathbf{15}, \mathbf{3})$  representation. According to the logic of [8], such a model would have a singularity corresponding to an extended  $\hat{E}_8$  Dynkin diagram, which necessitates a  $(4, 6)$  point. Even if this model is not consistent as a nonabelian model, the Higgsed model with an abelian factor may still be a valid F-theory construction. There has also been some recent suggestion that some exotic matter of this type may give a consistent F-theory model as the singular features of the  $(4, 6)$  point can be compensated by T-brane degrees of freedom [21]. These questions would be interesting to understand further.
- The general structure of abelian charges when the gauge group has both an abelian and nonabelian factor like  $SU(N) \times U(1)$  has been investigated in [9, 45, 46]. It could be interesting to study the range of charges that may be realized in explicit constructions with such gauge groups.
- While the constructions here were carried out in 6 dimensions, where we have the strongest analytic control over F-theory and the low-energy constraints are strongest, in principle the Weierstrass constructions that lead to large  $q$  charges should be equally valid in four dimensions, although the story is complicated by the presence of fluxes and the superpotential. It would be interesting to attempt explicit constructions of four-dimensional F-theory models that give vacua with an abelian  $U(1)$  gauge theory and similar large charges.
- The abelian charges constructed here are much larger than those realized in most other approaches to string compactification. It would be interesting to systematically analyze other constructions such as heterotic, type II and M theory on  $G_2$  to see if a clear upper bound on abelian charges can be demonstrated in those frameworks.

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## A Explicit $SU(5) \times SU(3)$ Weierstrass model

The  $f$  and  $g$  for the  $SU(5) \times SU(3)$  Weierstrass model obtained by Higgsing the  $SU(8)$  model above are

$$f = -3v^2 \left[ 9\delta^{12}v^6 (\delta^2 - \epsilon)^4 + 18\delta^8\sigma v^4 (\delta^2 - 2\epsilon) (\delta^2 - \epsilon)^2 + 3\delta^4\sigma^2 v^2 (5\delta^4 - 8\delta^2\epsilon + 6\epsilon^2) + 2\sigma^3 (2\delta^2 + \epsilon) \right] \quad (\text{A.1})$$

and

$$g = 3v^2 \left[ 18\delta^{18}v^{10} (\delta^2 - \epsilon)^6 + 54\delta^{14}\sigma v^8 (\delta^2 - 2\epsilon) (\delta^2 - \epsilon)^4 + 18\delta^{10}\sigma^2 v^6 (\delta^2 - \epsilon)^2 (4\delta^4 - 10\delta^2\epsilon + 9\epsilon^2) + 6\delta^6\sigma^3 v^4 (8\delta^6 - 21\delta^4\epsilon + 15\delta^2\epsilon^2 - 5\epsilon^3) + 15\delta^2\sigma^4 v^2 (\delta^4 + \epsilon^2) + \sigma^5 \right]. \quad (\text{A.2})$$

As before, the homology classes of  $v$ ,  $\sigma$  and  $\delta$  are respectively  $S$ ,  $2\tilde{S}$  and  $F$ . The homology class of  $\epsilon$  is  $2F$ . One recovers the original  $SU(8)$  model when  $\epsilon$  is taken to 0. In principle, one can obtain these expressions by tuning  $f$  and  $g$ , as in [38]: one expands  $f$  and  $g$  as series in  $\sigma$  and tunes the various parameters to force the discriminant to vanish to higher and higher orders.

The discriminant is

$$\Delta = -27v^4\sigma^5 (12\delta^4v^2\epsilon - \sigma)^3 \left[ 9\sigma^2 + 9v^4 (\delta^3 - \delta\epsilon)^4 + 2\sigma v^2 (7\delta^2 - 16\epsilon) (\delta^2 - \epsilon)^2 \right]. \quad (\text{A.3})$$

The  $v^4$  factor in the discriminant reflects the expected  $SU(3)$  non-Higgsable gauge group. Indeed, there are no codimension-two singularities along  $v = 0$  where the singularity type enhances, indicating that no matter is charged under the non-Higgsable  $SU(3)$ . However, the discriminant also has a  $\sigma^5$  factor and a  $(\sigma - 12\delta^4v^2\epsilon)^3$  factor. One can verify that  $f$  and  $g$  are not proportional to  $\sigma$  or  $(\sigma - 12\delta^4v^2\epsilon)$  and that the  $SU(N)$  split conditions are satisfied. These two factors in the discriminant therefore signal the presence of an  $SU(5)$

group and an additional SU(3) group, which together form the SU(5) × SU(3) symmetry coming from the Higgsed SU(8).

Since the SU(5) and SU(3) groups both occur on curves of genus two, there are two hypermultiplets of (24, 1) matter and two hypermultiplets of (1, 8) matter. There are also four types of codimension-two loci that contribute charged matter. On  $\sigma = \epsilon - \delta^2 = 0$ , the  $I_5$  singularity type for the SU(5) symmetry enhances to  $I_1^*$ . This locus therefore contributes four hypermultiplets of (10, 1) matter. On  $\sigma - 12\delta^4 v^2 \epsilon = \epsilon - 3\delta^2 = 0$ , the  $I_3$  singularity type for the SU(3) symmetry enhances to  $I_4$ , indicating that this locus contributes four hypermultiplets of (1, 3) matter. Note that while the singularity type does enhance from  $I_3$  to  $II$  on  $\sigma - 12\delta^4 v^2 \epsilon = 5\epsilon + \delta^2 = 0$ , this locus does not contribute any charged matter.

The remaining codimension-two loci correspond to intersections between  $\sigma = 0$  and  $\sigma - 12\delta^4 v^2 \epsilon = 0$ . Before describing the matter supported at these loci, it is worth mentioning a subtle point regarding the SU(3). We can interpret the  $I_3$  singularity on  $\sigma - 12\delta^4 v^2 \epsilon = 0$  as either an SU(3) or an  $\overline{\text{SU}}(3)$  gauge group. In other words, we can freely conjugate the SU(3) symmetry, which will change the field-theoretic interpretation of the matter content. The (1, 3) hypermultiplets would be unaffected by this conjugation, as a single (1, 3) hypermultiplet contains fields in both the (1, 3) and (1,  $\overline{3}$ ) representations. But the jointly charged representations would be affected by this conjugation. For instance, a (10,  $\overline{3}$ ) hypermultiplet would become a (10, 3) after conjugation, and vice versa. Similarly, a (5,  $\overline{3}$ ) hypermultiplet would become a (5, 3) hypermultiplet.

With this in mind, we can analyze the loci where the two curves intersect. At  $\sigma = \epsilon = 0$ , the singularity type enhances to  $I_8$  or  $A_7$ . This locus therefore supports four hypermultiplets bifundamental matter. But without performing an explicit resolution, we cannot determine whether this locus supports four hypermultiplets of (5, 3) matter or (5,  $\overline{3}$ ) matter. Meanwhile, the singularity type enhances to  $III^*$  at  $\delta = \epsilon = 0$ . The Katz-Vafa method would suggest that, to determine the matter content, we should break the adjoint (133) representation of  $E_7$  into SU(5) × SU(3) representations:

$$133 \rightarrow (24, 1) + (1, 8) + (10, \overline{3}) + (\overline{10}, 3) + (5, \overline{3}) + (\overline{5}, 3) + (5, 1) + (\overline{5}, 1) + 2 \times (1, 1). \quad (\text{A.4})$$

Naively, the  $\delta = \epsilon = 0$  locus would therefore seem to support two hypermultiplets of (10,  $\overline{3}$ ) matter, two hypermultiplets of (5,  $\overline{3}$ ) matter, and two hypermultiplets of (5, 1) matter. However, the Higgsing patterns described in section 4.2 suggests we should obtain (10, 3) matter, *not* (10,  $\overline{3}$ ). To match the Katz-Vafa result with the field theory expectations, we should therefore conjugate the SU(3). The  $\delta = \epsilon = 0$  locus then supports (10, 3) matter, two hypermultiplets of (5, 3) matter, and two hypermultiplets of (5, 1) matter, exactly as expected from the SU(8) branching patterns. Moreover, the  $\sigma = \epsilon = 0$  should support four hypermultiplets of (5,  $\overline{3}$ ) matter. The complete charged spectrum for SU(5) × SU(3) is thus

$$2 \times (10, 3) + 4 \times (5, \overline{3}) + 2 \times (5, 3) + 2 \times (24, 1) + 2 \times (1, 8) + 4 \times (10, 1) + 2 \times (5, 1) + 4 \times (1, 3), \quad (\text{A.5})$$

which exactly matches expectations.

## B Explicit Higgsing of SU(4)

While it is difficult to construct U(1) models with large charges, there are previous constructions admitting smaller charges. As a result, the Higgsing process of section 2.1 can be seen explicitly in F-theory for small SU( $N$ ). In this appendix, we focus on the Higgsing of SU(4) down to U(1) with charge  $\pm 4$  matter. Explicit F-theory constructions with charge  $\pm 4$  matter were described in [25], and models admit an unHiggsing that is the exact analogue of the SU(4)  $\rightarrow$  U(1) Higgsing process.

We start with an explicit charge-4 U(1) model on an  $\mathbb{F}_3$  base. The Weierstrass tuning (along with the section components, which we do not list here) was originally given in [25]:

$$\begin{aligned}
 f = & -\frac{1}{3}(s_5^2 - 3s_1s_8)(a_1^2(d_1^2 - 3d_0d_2) - a_1b_1d_0d_1 + b_1^2d_0^2) \\
 & -\frac{1}{3}(s_2^2 - 3s_1s_3)(a_1^2d_2^2 + b_1^2(d_1^2 - 2d_0d_2)) \\
 & +\frac{1}{6}(2s_2s_5 - 3s_1s_6)(a_1^2d_1d_2 + a_1b_1(d_1^2 - 2d_0d_2) + b_1^2d_0d_1) \\
 & +\frac{1}{6}(a_1d_1 + b_1d_0)(2b_1d_2(s_2^2 - 3s_1s_3) - 3s_2s_6s_8 + s_5(2s_3s_8 + s_6^2)) \\
 & +a_1d_0\left(b_1d_2(3s_1s_6 - 2s_2s_5) + s_2s_8^2 - \frac{s_5s_6s_8}{2}\right) \\
 & +\frac{1}{6}(a_1d_2 + b_1d_1)(s_3(2s_2s_8 - 3s_5s_6) + s_2s_6^2) \\
 & +\frac{1}{2}b_1d_2s_3(2s_3s_5 - s_2s_6) - \frac{1}{48}(s_6^2 - 4s_3s_8)^2, \tag{B.1} \\
 g = & \frac{1}{864}(s_6^2 - 4s_3s_8)^3 - \frac{1}{2}(d_0d_2^3a_1^4 + b_1^3d_0(d_1^3 - 3d_0d_1d_2)a_1)s_1^2 \\
 & +\frac{1}{4}(d_2^2(d_1^2 - 2d_0d_2)a_1^4 + b_1^2(d_1^4 - 6d_0^2d_2^2 - 4d_0d_2(d_1^2 - 2d_0d_2))a_1^2 + b_1^4d_0^2(d_1^2 - 2d_0d_2))s_1^2 \\
 & +\frac{1}{27}((d_1^3 - 3d_0d_1d_2)b_1^3 + a_1^3d_2^3)s_2(9s_1s_3 - 2s_2^2) \\
 & +\frac{1}{18}(d_1d_2^2a_1^3 + b_1^2(d_1^3 - 3d_0d_1d_2)a_1 + b_1^3d_0(d_1^2 - 2d_0d_2))((2s_2^2 - 3s_1s_3)s_5 - 3s_1s_2s_6) \\
 & +\frac{1}{18}(d_0d_1d_2a_1^3 + b_1d_0(d_1^2 - 2d_0d_2)a_1^2 + b_1^2d_0^2d_1a_1)(2s_5^3 - 9s_1s_8s_5 + 9b_1d_2s_1^2) \\
 & +\frac{1}{72}(d_1d_2a_1^2 + b_1(d_1^2 - 2d_0d_2)a_1 + b_1^2d_0d_1) \\
 & \times \left[4b_1d_2s_2(2s_2^2 - 9s_1s_3) + s_6(s_6(2s_2s_5 + 3s_1s_6) - 12s_3s_5^2)\right. \\
 & \left.+4(s_2s_3s_5 - 3(s_2^2 - 5s_1s_3)s_6)s_8\right] \\
 & +\frac{1}{18}(d_2(d_1^2 - 2d_0d_2)a_1^3 + b_1(d_1^3 - 3d_0d_1d_2)a_1^2 + b_1^3d_0^2d_1)(s_2(2s_5^2 - 3s_1s_8) - 3s_1s_5s_6) \\
 & +\frac{2}{9}(d_0d_2^2a_1^3 + b_1^2d_0(d_1^2 - 2d_0d_2)a_1)(3s_1s_5s_6 + s_2(3s_1s_8 - 2s_5^2)) \\
 & +a_1^2d_0^2\left(-\frac{3}{2}b_1^2d_2^2s_1^2 + \frac{1}{4}s_8^2(s_5^2 - 4s_1s_8) + \frac{2}{9}b_1d_2s_5(9s_1s_8 - 2s_5^2)\right) \\
 & +\frac{1}{36}((d_1^2 - 2d_0d_2)b_1^2 + a_1^2d_2^2)(3(3s_5^2 - 8s_1s_8)s_3^2 + (4s_2^2s_8 - 3s_6(2s_2s_5 + s_1s_6))s_3 + 2s_2^2s_6^2)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{24} b_1 d_2 s_3 (6b_1 d_2 s_3 (s_2^2 - 4s_1 s_3) + (s_2 s_6 - 2s_3 s_5) (s_6^2 - 4s_3 s_8)) \\
 & + \frac{1}{36} (d_0 d_2 a_1^2 + b_1 d_0 d_1 a_1) \left[ (s_6^2 + 2s_3 s_8) s_5^2 + 18s_2 s_6 s_8 s_5 - 6(s_2^2 + 2s_1 s_3) s_8^2 \right. \\
 & \left. + 4b_1 d_2 ((2s_2^2 - 3s_1 s_3) s_5 - 3s_1 s_2 s_6) - 33s_1 s_6^2 s_8 \right] \\
 & - \frac{1}{54} ((d_1^3 - 3d_0 d_1 d_2) a_1^3 + b_1^3 d_0^3) (4s_5^3 + 9s_1 (3b_1 d_2 s_1 - 2s_5 s_8)) \\
 & + \frac{1}{72} a_1 d_0 \left[ 16b_1^2 s_2 (9s_1 s_3 - 2s_2^2) d_2^2 \right. \\
 & \left. + 6b_1 (s_6 (6s_3 s_5^2 + s_6 (9s_1 s_6 - 8s_2 s_5))) + 2(3(s_2^2 + 2s_1 s_3) s_6 - 8s_2 s_3 s_5) s_8 \right] d_2 \\
 & \left. + 3s_8 (s_5 s_6 - 2s_2 s_8) (s_6^2 - 4s_3 s_8) \right] \\
 & + \frac{1}{18} (d_0 d_1 a_1^2 + b_1 d_0^2 a_1) \left( 2b_1 d_2 (s_2 (2s_5^2 - 3s_1 s_8) - 3s_1 s_5 s_6) \right. \\
 & \quad \left. - 3s_8 (s_6 s_5^2 + (s_2 s_5 - 6s_1 s_6) s_8) \right) \\
 & - \frac{1}{72} (b_1 d_1 + a_1 d_2) \left( 12b_1 d_2 s_3 (s_2 s_3 s_5 + (s_2^2 - 6s_1 s_3) s_6) \right. \\
 & \quad \left. + (s_6^2 - 4s_3 s_8) (s_2 s_6^2 + s_3 (2s_2 s_8 - 3s_5 s_6)) \right) \\
 & + \frac{1}{72} (b_1 d_0 + a_1 d_1) \left[ 2b_1 d_2 (-6(s_5^2 + 2s_1 s_8) s_3^2 + (2s_8 s_2^2 + 18s_5 s_6 s_2 - 33s_1 s_6^2) s_3 + s_2^2 s_6^2) \right. \\
 & \left. - (s_6^2 - 4s_3 s_8) (s_5 (s_6^2 + 2s_3 s_8) - 3s_2 s_6 s_8) \right] \\
 & + \frac{1}{36} ((d_1^2 - 2d_0 d_2) a_1^2 + b_1^2 d_0^2) \left[ 2(s_6^2 + 2s_3 s_8) s_5^2 - 6s_2 s_6 s_8 s_5 \right. \\
 & \left. + 8b_1 d_2 (-2s_5 s_2^2 + 3s_1 s_6 s_2 + 3s_1 s_3 s_5) - 3s_8 (s_1 (s_6^2 + 8s_3 s_8) - 3s_2^2 s_8) \right]. \tag{B.2}
 \end{aligned}$$

We take the parameters to have the homology classes given in table 3. Note that  $d_0$  and  $s_1$  have trivial homology classes and are thus constants. Additionally, the homology classes suggest that many of the parameters are reducible. While not necessary, one can explicitly address this by setting

$$d_1 = v \tilde{d}_1 \quad d_2 = v^2 \tilde{d}_2 \quad s_5 = v \tilde{s}_5 \quad s_6 = \tilde{s}_6 \quad s_8 = v^2 \tilde{s}_8, \tag{B.3}$$

where  $[v] = S$ . Performing these redefinitions makes the SU(3) non-Higgsable cluster (NHC) on  $\mathbb{F}_3$  explicitly visible in the Weierstrass model. There are no nonabelian gauge groups other than this NHC, and there is no matter charged under the nonHiggsable SU(3). Based on the matter analysis from section 4.3 of [25], the charged matter spectrum of the model is

$$6 \times (\mathbf{q} = \mathbf{4}) + 32 \times (\mathbf{q} = \mathbf{3}) + 60 \times (\mathbf{q} = \mathbf{2}) + 96 \times (\mathbf{q} = \mathbf{1}). \tag{B.4}$$

We can then unHiggs the U(1) symmetry to SU(4) by setting  $a_1 \rightarrow 0$ ,  $s_2 \rightarrow 0$ ,  $s_3 \rightarrow 0$ . One can verify that these tunings cause the generating section for the U(1) to coincide with the zero section, suggesting that the U(1) has been unHiggsed to some nonabelian gauge group. The discriminant now takes the form

$$\Delta = -\frac{1}{16} b_1^4 v^4 d_0^3 s_1^2 \left[ \tilde{s}_6^4 \left( \tilde{d}_2 \tilde{s}_6^2 - \tilde{d}_1 \tilde{s}_6 \tilde{s}_8 + d_0 \tilde{s}_8^2 \right) v^4 + \mathcal{O}(b_1) \right]. \tag{B.5}$$

| Parameter | Homology Class         |
|-----------|------------------------|
| $a_1$     | $3F$                   |
| $b_1$     | $2\tilde{S} = 2S + 6F$ |
| $d_0$     | $0S + 0F$              |
| $d_1$     | $2S + 3F$              |
| $d_2$     | $4S + 6F$              |
| $s_1$     | $0S + 0F$              |
| $s_2$     | $F$                    |
| $s_3$     | $2F$                   |
| $s_5$     | $2S + 4F$              |
| $s_6$     | $2S + 5F$              |
| $s_8$     | $4S + 8F$              |

**Table 3.** Homology classes for the parameters of the charge-4 Weierstrass model on  $\mathbb{F}_3$ .

Because  $s_1$  and  $d_0$  are constants, the  $d_0^3$  and  $s_1^2$  factors in the discriminant do not represent any new nonabelian gauge groups. Meanwhile, the  $v^4$  factors corresponds to the expect SU(3) NHC on  $\mathbb{F}_3$ . But the  $b_1^4$  factor represents a new SU(4) gauge group.<sup>12</sup> The U(1) has therefore been unHiggsed to an SU(4) tuned on  $b_1 = 0$ . Since  $[b_1] = 2\tilde{S}$ ,  $b_1 = 0$  is a genus-two curve, and the spectrum includes two hypermultiplets in the adjoint **(15)** representation. Additionally, the codimension-two locus  $\tilde{s}_6 = b_1 = 0$  contributes ten **10** hypermultiplets, while the locus  $(\tilde{d}_2\tilde{s}_6^2 - \tilde{d}_1\tilde{s}_6\tilde{s}_8 + d_0\tilde{s}_8^2) = b_1 = 0$  contributes thirty-two **4** hypermultiplets. This charged spectrum agrees exactly with anomaly cancellation.

We therefore see that the F-theory charge-4 model admits a  $U(1) \rightarrow SU(4)$  unHiggsing. Therefore, the corresponding  $SU(4) \rightarrow U(1)$  unHiggsing also occurs in F-theory, providing further evidence that the  $SU(N) \rightarrow U(1)$  Higgsing of section 2.1 should be valid more generally.

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## References

- [1] A. Adams, O. DeWolfe and W. Taylor, *String universality in ten dimensions*, *Phys. Rev. Lett.* **105** (2010) 071601 [[arXiv:1006.1352](https://arxiv.org/abs/1006.1352)] [[INSPIRE](https://inspirehep.net/literature/87511)].
- [2] W. Taylor, *TASI lectures on supergravity and string vacua in various dimensions*, [arXiv:1104.2051](https://arxiv.org/abs/1104.2051) [[INSPIRE](https://inspirehep.net/literature/95111)].

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<sup>12</sup> $f$  and  $g$  are not proportional to  $b_1$  after the tunings, and the split condition is satisfied.

- [3] I. García-Etxebarria, H. Hayashi, K. Ohmori, Y. Tachikawa and K. Yonekura, *8d gauge anomalies and the topological Green-Schwarz mechanism*, *JHEP* **11** (2017) 177 [[arXiv:1710.04218](#)] [[INSPIRE](#)].
- [4] V. Kumar and W. Taylor, *String universality in six dimensions*, *Adv. Theor. Math. Phys.* **15** (2011) 325 [[arXiv:0906.0987](#)] [[INSPIRE](#)].
- [5] V. Kumar, D.R. Morrison and W. Taylor, *Global aspects of the space of 6D  $N = 1$  supergravities*, *JHEP* **11** (2010) 118 [[arXiv:1008.1062](#)] [[INSPIRE](#)].
- [6] N. Seiberg and W. Taylor, *Charge lattices and consistency of 6D supergravity*, *JHEP* **06** (2011) 001 [[arXiv:1103.0019](#)] [[INSPIRE](#)].
- [7] T.W. Grimm and A. Kapfer, *Anomaly cancelation in field theory and F-theory on a circle*, *JHEP* **05** (2016) 102 [[arXiv:1502.05398](#)] [[INSPIRE](#)].
- [8] D. Klevers, D.R. Morrison, N. Raghuram and W. Taylor, *Exotic matter on singular divisors in F-theory*, *JHEP* **11** (2017) 124 [[arXiv:1706.08194](#)] [[INSPIRE](#)].
- [9] M. Cvetič and L. Lin, *The global gauge group structure of F-theory compactification with  $U(1)_s$* , *JHEP* **01** (2018) 157 [[arXiv:1706.08521](#)] [[INSPIRE](#)].
- [10] W. Taylor and A.P. Turner, *An infinite swampland of  $U(1)$  charge spectra in 6D supergravity theories*, *JHEP* **06** (2018) 010 [[arXiv:1803.04447](#)] [[INSPIRE](#)].
- [11] S. Monnier, G.W. Moore and D.S. Park, *Quantization of anomaly coefficients in 6D  $N = (1, 0)$  supergravity*, *JHEP* **02** (2018) 020 [[arXiv:1711.04777](#)] [[INSPIRE](#)].
- [12] S.-J. Lee, W. Lerche and T. Weigand, *Tensionless strings and the weak gravity conjecture*, [arXiv:1808.05958](#) [[INSPIRE](#)].
- [13] C. Vafa, *The string landscape and the swampland*, [hep-th/0509212](#) [[INSPIRE](#)].
- [14] H. Ooguri and C. Vafa, *On the geometry of the string landscape and the swampland*, *Nucl. Phys. B* **766** (2007) 21 [[hep-th/0605264](#)] [[INSPIRE](#)].
- [15] V. Kumar and W. Taylor, *A bound on 6D  $N = 1$  supergravities*, *JHEP* **12** (2009) 050 [[arXiv:0910.1586](#)] [[INSPIRE](#)].
- [16] C. Vafa, *Evidence for F-theory*, *Nucl. Phys. B* **469** (1996) 403 [[hep-th/9602022](#)] [[INSPIRE](#)].
- [17] D.R. Morrison and C. Vafa, *Compactifications of F-theory on Calabi-Yau threefolds. 1*, *Nucl. Phys. B* **473** (1996) 74 [[hep-th/9602114](#)] [[INSPIRE](#)].
- [18] D.R. Morrison and C. Vafa, *Compactifications of F-theory on Calabi-Yau threefolds. 2*, *Nucl. Phys. B* **476** (1996) 437 [[hep-th/9603161](#)] [[INSPIRE](#)].
- [19] T. Weigand, *TASI lectures on F-theory*, [arXiv:1806.01854](#) [[INSPIRE](#)].
- [20] M. Cvetič and L. Lin, *TASI lectures on Abelian and discrete symmetries in F-theory*, *PoS(TASI2017)020*, (2018) [[arXiv:1809.00012](#)] [[INSPIRE](#)].
- [21] M. Cvetič, J.J. Heckman and L. Lin, *Towards exotic matter and discrete non-Abelian symmetries in F-theory*, [arXiv:1806.10594](#) [[INSPIRE](#)].
- [22] M. Gross, *A finiteness theorem for elliptic Calabi-Yau threefolds*, [alg-geom/9305002](#) [[INSPIRE](#)].
- [23] D.R. Morrison and D.S. Park, *F-theory and the Mordell-Weil group of elliptically-fibered Calabi-Yau threefolds*, *JHEP* **10** (2012) 128 [[arXiv:1208.2695](#)] [[INSPIRE](#)].
- [24] D. Klevers, D.K. Mayorga Pena, P.-K. Oehlmann, H. Piragua and J. Reuter, *F-theory on all toric hypersurface fibrations and its Higgs branches*, *JHEP* **01** (2015) 142 [[arXiv:1408.4808](#)] [[INSPIRE](#)].

- [25] N. Raghuram, *Abelian F-theory models with charge-3 and charge-4 matter*, *JHEP* **05** (2018) 050 [[arXiv:1711.03210](#)] [[INSPIRE](#)].
- [26] M.A. Luty and W. Taylor, *Varieties of vacua in classical supersymmetric gauge theories*, *Phys. Rev. D* **53** (1996) 3399 [[hep-th/9506098](#)] [[INSPIRE](#)].
- [27] R. Feger and T.W. Kephart, *LieART — a Mathematica application for Lie algebras and representation theory*, *Comput. Phys. Commun.* **192** (2015) 166 [[arXiv:1206.6379](#)] [[INSPIRE](#)].
- [28] T. Banks and N. Seiberg, *Symmetries and strings in field theory and gravity*, *Phys. Rev. D* **83** (2011) 084019 [[arXiv:1011.5120](#)] [[INSPIRE](#)].
- [29] D. Harlow and H. Ooguri, *Symmetries in quantum field theory and quantum gravity*, to appear.
- [30] M.B. Green, J.H. Schwarz and P.C. West, *Anomaly free chiral theories in six-dimensions*, *Nucl. Phys. B* **254** (1985) 327 [[INSPIRE](#)].
- [31] A. Sagnotti, *A note on the Green-Schwarz mechanism in open string theories*, *Phys. Lett. B* **294** (1992) 196 [[hep-th/9210127](#)] [[INSPIRE](#)].
- [32] J. Erler, *Anomaly cancellation in six-dimensions*, *J. Math. Phys.* **35** (1994) 1819 [[hep-th/9304104](#)] [[INSPIRE](#)].
- [33] D.S. Park and W. Taylor, *Constraints on 6D supergravity theories with Abelian gauge symmetry*, *JHEP* **01** (2012) 141 [[arXiv:1110.5916](#)] [[INSPIRE](#)].
- [34] M. Bershadsky, K.A. Intriligator, S. Kachru, D.R. Morrison, V. Sadov and C. Vafa, *Geometric singularities and enhanced gauge symmetries*, *Nucl. Phys. B* **481** (1996) 215 [[hep-th/9605200](#)] [[INSPIRE](#)].
- [35] S.B. Johnson and W. Taylor, *Enhanced gauge symmetry in 6D F-theory models and tuned elliptic Calabi-Yau threefolds*, *Fortsch. Phys.* **64** (2016) 581 [[arXiv:1605.08052](#)] [[INSPIRE](#)].
- [36] D.R. Morrison and W. Taylor, *Classifying bases for 6D F-theory models*, *Central Eur. J. Phys.* **10** (2012) 1072 [[arXiv:1201.1943](#)] [[INSPIRE](#)].
- [37] S. Katz, D.R. Morrison, S. Schäfer-Nameki and J. Sully, *Tate’s algorithm and F-theory*, *JHEP* **08** (2011) 094 [[arXiv:1106.3854](#)] [[INSPIRE](#)].
- [38] L.B. Anderson, J. Gray, N. Raghuram and W. Taylor, *Matter in transition*, *JHEP* **04** (2016) 080 [[arXiv:1512.05791](#)] [[INSPIRE](#)].
- [39] D.R. Morrison and W. Taylor, *Matter and singularities*, *JHEP* **01** (2012) 022 [[arXiv:1106.3563](#)] [[INSPIRE](#)].
- [40] J. Tate, *Algorithm for determining the type of a singular fiber in an elliptic pencil*, in *Modular functions of one variable IV*, Springer, Berlin, Heidelberg, Germany, (1975), pg. 33.
- [41] K. Kodaira, *On compact analytic surfaces, II*, *Ann. Math.* **77** (1963) 563.
- [42] K. Kodaira, *On compact analytic surfaces, III*, *Ann. Math.* **78** (1963) 1.
- [43] A. Grassi and D.R. Morrison, *Anomalies and the Euler characteristic of elliptic Calabi-Yau threefolds*, *Commun. Num. Theor. Phys.* **6** (2012) 51 [[arXiv:1109.0042](#)] [[INSPIRE](#)].
- [44] S.H. Katz and C. Vafa, *Matter from geometry*, *Nucl. Phys. B* **497** (1997) 146 [[hep-th/9606086](#)] [[INSPIRE](#)].
- [45] C. Lawrie, S. Schäfer-Nameki and J.-M. Wong, *F-theory and all things rational: surveying U(1) symmetries with rational sections*, *JHEP* **09** (2015) 144 [[arXiv:1504.05593](#)] [[INSPIRE](#)].
- [46] T.W. Grimm, A. Kapfer and D. Klevers, *The arithmetic of elliptic fibrations in gauge theories on a circle*, *JHEP* **06** (2016) 112 [[arXiv:1510.04281](#)] [[INSPIRE](#)].