Effect of flexural and shear stresses simultaneously for optimized design of butterfly-shaped dampers: Computational study

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Abstract. Structural fuses are made up from oriented steel plates to be used to resist seismic force with shear loading resistance capabilities. The damage and excessive inelastic deformations are concentrated in structural fuses to avoid any issues for the rest of the surrounding elements. Recently developed fuse plates are designed with engineered cutouts leaving flexural or shear links with controlled yielding features. A promising type of link is proposed to align better bending strength along the length of the link with the demand moment diagram is a butterfly-shaped link. Previously, the design methodologies are purely based on the flexural stresses, or shear stresses only, which overestimate the dampers capability for resisting against the applied loadings. This study is specifically focused on the optimized design methodologies for commonly used butterfly-shaped dampers. Numerous studies have shown that the stresses are not uniformly distributed along the length of the dampers; hence, the design methodology and the effective implementation of the steel need revisions and improvements. In this study, the effect of shear and flexural stresses on the behavior of butterfly-shaped links are computationally investigated. The mathematical models based on von-Mises yielding criteria are initially developed and the optimized design methodology is proposed based on the yielding criterion. The optimized design is refined and investigated with the aid of computational investigations in the next step. The proposed design methodology meets the needs of optimized design concepts for butterfly-shaped dampers considering the uniform stress distribution and efficient use of steel.

Keywords: structural fuses; shear and flexural stresses; butterfly-shaped damper; von-Mises criteria; uniform stress state function; optimization

1. Introduction

Many structures around the world are designed and constructed to prevent collapses and permanent damages during large earthquakes. For this purpose, structural fuses have been implemented to concentrate inelasticity and damages in a specific part of the structure (Di Lauro et al. 2019, Farahi Shahr and Mousavi 2018, Liu et al. 2015, Mansouri et al. 2016, Martínez-Rueda 2002, Nuzzo et al. 2018, Zahrai et al. 2015) and protect the surrounding parts from damages and then be replaceable after any events (Eldin et al. 2018, Kim and Shin 2017, Mirzai et al. 2018, Shad et al. 2018, Zhan et al. 2017). To have appropriate adequacy in energy dissipation and ductility, structural fuses consisting plate with engineered cut-outs are implemented to be yielded as they subjected to shear loading (Daie et al. 2011, Latour 2017, Latour and Rizzano 2016, Nastri et al. 2017). Among many structural fuses, butterfly-shaped (BF) fuses shown in Figs. 1 and 2 are implemented due to having advantages. This fuse system is recently being proposed as a substitute for conventional EBF systems (Ashtari and Erfani 2016), or slit dampers. It is indicated that the implementation of butterfly-shaped fuses in structural applications leads to the reduction of shakes and disturbances caused by earthquakes (Luth et al. 2008). If these fuses are appropriately designed (Farzampour and Eatherton 2018b, Teruna et al. 2015, Vargas and Bruneau 2006), then yielding limit states will govern the brittle ones, which enhance the fuse resistance and energy dissipation capability (Ke and Yam 2016, Sun et al. 2017). These fuses are generally being used in high-rises buildings (as is shown in Fig. 2, in the USC School of Cinema, Los Angles) to make a spacious room for residence and keep the inside and outside the building unchanged.

The butterfly-shaped links are initially designed to align moment capacity with the shape of the moment demand diagram for efficient use of the steel. These hysteric dampers traditionally are added to the structural applications in the out-of-plane format as added damping and stiffness or stiffness device (TADAS) to be bent over the weak axis (Tsai et al. 1993, Whittaker et al. 1991). However, these links can work and bend over the major axis (in-plane bending) to create larger stiffness.

The planar use of the butterfly-shaped links for space-constrained applications is studied previously, (Farzampour...
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and Eatherton 2018a, b). The limitation of the desired load carrying capacity related to the brittle buckling limit states is addressed in some previous studies. The substantial energy dissipating, ductility and large uniform yielding distribution are observed with the in-plane use of butterfly-shaped fuses (Farzampour and Eatherton 2018a, b), which leads to the implementation of these fuses in high-rise buildings controlling drift responses and reducing the demands on the framing members (Hitaka and Matsui 2006, Kim et al. 2018, Luth et al. 2008). On the other hand, there are some implementation limitations reported previously. The buckling limit state at early drifts stage cracks propagation at the sharper geometrical changes, and the manufacturing cost is mentioned as the general limitations of use related to butterfly-shaped dampers.

The recent design methodologies for butterfly-shaped dampers are purely based on the flexural stresses only or shear stresses, which overestimate the dampers capability for resisting against applied loadings. Numerous studies have shown that the stresses are not uniformly distributed along the length of the dampers (Lee et al. 2015); hence, the design methodology and the effective implementation of the steel need improvements. In this study, the new design methodology will be developed and computationally investigated.

Also, both shear and flexure limit states are investigated under simultaneous shear and flexural stresses. The von-Mises criterion is used to develop the upper limit for the total stress imposed on the link. The flexural stresses are formulated based on the developed concepts in this study. Along the same lines, the shear stresses are developed considering the critical section along the length of dampers. With the aid of flexural and shear stresses prediction equations, the von-Mises criterion is implemented to generate the stress function, and the resulting function is optimized for finding the appropriate geometrical properties to have the uniform stresses along the length of the links. This study aims to propose an optimized design methodology to improve the behavior of the dampers, especially the commonly used butterfly-shaped dampers while having the economical implementation of the steel. By implementation of the proposed methodology, the
fundamental knowledge about the yielding behavior of the damping system is developed. Also, the plastic mechanisms governing the strength and spread of the plasticity along the length of the dampers are investigated leading to seismic performance improvement of the shear fuses and damage reduction due to earthquakes.

2. Methodology

In what follows briefly the general analytical investigations of shear and flexure is explained: both shear and flexure limit states are considered to be applied simultaneously. The von-Mises criterion is used to develop the upper limit for the total stress imposed on the link. The moment along the length \( M(z) \) formulated from the middle point as it is shown in Fig. 1, and the end moment \( M_0 \) is indicated in Eq. (1). The varying width \( w(z) \) and sectional inertia \( I(z) \) of the butterfly-shaped link are indicated in Eqs. (2) and (3), respectively.

\[
M(z) = \frac{2Ma_z}{L} \quad \text{and} \quad M_0 = PL/2 \tag{1}
\]

\[
w(z) = \frac{2(b - a)z}{L} + a \tag{2}
\]

\[
I(z) = \frac{1}{12}w(z)^3t = \frac{1}{12} \left[ \frac{2(b - a)}{L} z + a \right]^3 t \tag{3}
\]

In which the geometrical parameters, \( a, b, L, t \) are defined in Fig. 1. Therefore, the flexural stress at a section is as shown in Eq. (4).

\[
\sigma = \frac{M(z)w(z)}{I(z)} = \frac{Pz(w(z)/2 - y)}{\frac{1}{12}\left[ 2(b - a)/L z + a \right]^3 t} \tag{4}
\]

where \( P \) is the shear force applied to the end length of the butterfly-shaped link. Along the same lines shear stresses \( (\delta) \) for each section of the butterfly-shaped links as it is shown in Fig. 3, is derived based on Eq. (5).

\[
\eta = \frac{VQ}{P} = \frac{Pyt(w(z)/2 - y)/2}{I(z)t} = \frac{Pyt(\frac{1}{L}z + a - y)/2}{\frac{1}{12}\left[ 2(b - a)/L z + a \right]^3 t^2} \tag{5}
\]

Also, yielding criterion von-Mises stress, which is shown in Eq. (6)

\[
\sigma_y^2 = \frac{1}{2} \left[ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2) \right] \tag{6}
\]

Therefore, re-establishing the equation would yield to Eq. (7)

\[
\sigma_y^2 = \frac{1}{2} \left[ \frac{Pz}{\frac{1}{12}\left[ 2(b - a)/L z + a \right]^3 t} \left( \frac{2(b - a)}{L} z + a \right)^2 - y \right] \tag{7}
\]

\[
\quad + 6 \left[ \frac{Pz}{\frac{1}{12}\left[ 2(b - a)/L z + a \right]^3 t} \left( \frac{2(b - a)}{L} z + a \right)^2 - y \right]^2 \tag{7}
\]

If inelasticity is needed to be concentrated far from the edges, and to reduce the possibility of crack propagation and brittle modes, \( z \) should be equalized to \( L/4 \) which is the recommended farthest point away from the sharp edges (Ma et al. 2011); therefore, by substituting the \( L/4 \) for \( z \), Eq. (8) is obtained.

\[
\sigma_y = \frac{1}{2} \left[ \left( \frac{4L}{b + a} \right) \left( \frac{b + a}{4} - y \right)^2 + 6 \left( \frac{4L}{b + a} \right)^2 \right] \tag{8}
\]

The right-hand side of the Eq. (8) indicates stress state function indicated by \( F(y) \), which combines the effect of shear with flexure stresses at a specific section located at \( L/4 \) from the midpoint of a butterfly-shaped link as it is determined in Eq. (9).

\[
F(y) = \frac{1}{2} \left[ \left( \frac{PL}{4L} \right) \left( \frac{b + a}{4} - y \right)^2 + 6 \left( \frac{PL}{4L} \right)^2 \right] \tag{9}
\]

To have an efficient and economic fuse system, it is required to have the ductile yielding limit states occurred for all the points along the length of the link. The stress state function is a continuous function over the length of the link section which is schematically shown in Fig. 4. To have equal state of stresses over the specified section, the difference between the minimum and the maximum critical point should be approaching to zero (as it is determined in Fig. 4), which, indicates that the stresses reach the limit state simultaneously along the length of the section.

To find the minimum and maximum values of the stress state function indicated in Eq. (9), the derivation of the stress function should be equalized to zero, which
eventually would lead to three real roots as shown in Eq. (10). The three roots are as follows

\[
I. \quad \frac{a}{4} + \frac{b}{4}
\]

\[
II. \quad \frac{a}{4} + \frac{b}{4} - \frac{\sqrt{3}}{12} \left( \sqrt{3a^2 + 6ab + 3b^2} - L^2 \right)
\]

\[
III. \quad \frac{a}{4} + \frac{b}{4} + \frac{\sqrt{3}}{12} \left( \sqrt{3a^2 + 6ab + 3b^2} - L^2 \right)
\]

Therefore, based on the concept mentioned in Fig. 4, the difference between the minimum and maximum values of the stress state function should approach zero to have the same stress state distribution for all the points along the length of the link, which is mathematically described as shown in Eq. (11).

\[
f(I) - f(II) \approx 0
\]

\[
III) - f(II) \approx 0
\]

\[
f(III) - f(I) \approx 0
\]

By investigating the set of equations proposed in Eq. (11), and simplifying them, Eq. (12) is derived.

\[
\pm \frac{6P^2(3a^2 + 6ab + 3b^2 - L^2)^2}{2t^2(a + b)^6} = 0
\]

Eq. (12) is further simplified in Eq. (13). Therefore, the appropriate geometrical condition for having the inelasticity concentrated at the quarter points is derived according to Eq. (13) and further simplified as shown in Eq. (14).

\[
3(a + b)^2 = L^2
\]

\[
a + b = \frac{L}{\sqrt{3}}
\]

To understand the effect of the proposed geometry criteria in Eq. (14) on the behavior of the links, two different geometrical variations of \( L = (a + b)/\sqrt{4} \) and \( L = (a + b)/\sqrt{2} \) are considered for the comparison with the proposed criterion \( L = (a + b)/\sqrt{3} \). The stress state function determining the stress components variation for all the cases along the length of the link is as shown in Fig. 5. It is concluded that the proposed geometry which considers the effect of shear and flexural stresses together for having the inelasticity located in quarter points in design, the less variations through the length of the section which is shown Fig. 5.

The finite element analysis is used to further investigate and compare the behavior of the models in the next parts.

3. The computational investigation

3.1 Finite element modeling of butterfly-shaped fuses

In this section of the computational study, the nonlinear static load-deformation behavior and cumulative plastic strain of the fuses are investigated with FE package software, ABAQUS (Simulia 2014). A general butterfly-shaped model is shown in Fig. 6 in which the butterfly-shaped link meshes with 5mm four nodded shell elements (Farzampour and Eatherton 2017). It is worthy of notice that the effect of initial imperfections is considered by applying out of plane deformation similar to first buckling mode. The initial imperfection is conducted by Eigenvalue analysis and then scaled to have maximum out-of-plane displacement value of \( L/250 \) based on the previous studies (Farzampour and Eatherton 2018a). As it is shown in Fig. 6, the boundary condition at the bottom is fully fixed, and the boundary condition at the top is fixed for out-of-plane movement, and vertical displacements.
The mentioned boundary conditions are chosen based on the previous laboratory tests conducted by (Ma et al. 2010). The displacement controlled loading is then applied to the top edge of the butterfly-shaped link. It is noted that the mesh sensitivity analysis is furthermore conducted on the models to find the appropriate mesh size.

### 3.2 Finite element methodology validation

The computational study is conducted by ABAQUS and is verified with for two laboratory tests. Different element types are used for verification purposes. Specimen B10-36W is one of the butterfly-shaped structural fuses tested by (Ma et al. 2010). The specimen is shown in Fig. 7(a), had six links with length, \( L = 229 \) mm, width at link ends, \( b = 64 \) mm, width at link middle, \( a = 25 \) mm, and thickness, \( t = 6 \) mm. The measured yield stress of 273 MPa and ultimate stress of 380 MPa as given by Ma et al. with kinematic hardening for post yielding and slope of 0.2 mm/mm plastic strain, and shell elements (S4R) are considered for establishing the computation models. The mesh sensitivity analysis is done to find the appropriate mesh size. The mesh is approximately 30mm for loading beam and 10 mm for BF fuse on a side which is shown in Fig. 7(a). The cyclic displacement history as shown in Fig. 7(b) was applied to the top loading beam matching experimentally applied displacements. Fig. 7 plots the story shear-story drift responses of the specimens done by links inside of the beam web experimentally, and compare it with the finite element results. It is indicated that the finite element analysis in this study, can predict the butterfly-shaped links behavior obtained from the laboratory tests.

Fig. 8 plots the computed story shear-story drift responses of the specimens done by links inside of the beam web done by (Shin et al. 2017). ABAQUS element C3D20R is implemented to avoid any hour-glass effect. A bilinear stress-strain relationship was assumed for the steel, with the yield strength of 379 MPa, elastic modulus of 200 GPa, and strain-hardening modulus of 1.38 GPa. Based on the geometry of the loading frame, the story shear is taken as 1.43 times the beam shear acquired from the FEA. The story drift is taken as the beam chord rotation divided by 1.34, in which the chord rotation is the transverse displacement divided by the clear span length of the beam. In addition, to understand the effect of initial imperfection, the first buckling mode is considered and the imperfection of \( L/250 \) is applied.

### Table 1 The initial design values for the uniform design concept

<table>
<thead>
<tr>
<th>Criterion for comparison ( a + b = \frac{L}{\sqrt{2}} )</th>
<th>( a ) (m)</th>
<th>( b ) (m)</th>
<th>( L ) (m)</th>
<th>( t ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a + b = \frac{L}{\sqrt{2}} )</td>
<td>0.14</td>
<td>0.43</td>
<td>1.00</td>
<td>0.02</td>
</tr>
<tr>
<td>( a + b = \frac{L}{\sqrt{3}} )</td>
<td>0.18</td>
<td>0.53</td>
<td>1.00</td>
<td>0.02</td>
</tr>
<tr>
<td>( a + b = \frac{L}{\sqrt{4}} )</td>
<td>0.13</td>
<td>0.38</td>
<td>1.00</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Fig. 7 Verification of the finite element modeling methodology against laboratory test (Ma et al. 2010)

The results show that this amount of imperfection slightly changes the curves and the overall values for strength of the links would not be different. It is confirmed that the buckling has been occurred by having as the significant out-of-plane displacement represented in Fig. 8. The strength values mentioned by authors are 76 (kN) and 57.4 (kN) for before and after buckling occurred. While, the FE model strength values are 73.2 (kN) and 52.7 (kN), respectively.

### 3.3 Discussion of the results

The schematic illustration of initial design examples and the capacity-demand moment diagrams are shown in Fig. 1. Table 1 shows the proposed first three configurations for further investigations on uniform stress distribution design methodology, which the computational results are extracted from FE analysis. Three sets of butterfly-shaped dampers are taken to investigate the applicability of the design methodology in previous sections. In all of the models, the loading is initially applied as the monotonically increasing pushover load. The computational study is done with the aid of the static general module. It is noted that the thickness is chosen in a way that the buckling would be prevented and yielding limit states have only occurred. The suggested details for the models are summarized in Table 1. The material model is based on the SS400 steel, which yields at 235 MPa, and reaches to ultimate strength value at 21% elongation and 400 MPa stress. All the computational models are developed based on the procedures mentioned in section 3.1.
For these models, \( a+b \) is equal to \( \frac{L}{\sqrt{4}} \), \( \frac{L}{\sqrt{3}} \), and \( \frac{L}{\sqrt{2}} \). The results indicate that having the uniformed stress distribution based on mathematical concepts proposed in this study will improve the general features of the sacrificial steel fuses. In this case, the accumulation of plastic strains are monitored and the maximum equivalent plastic strain is reported. This parameter, \( \dot{\varepsilon}^p \), is recorded within the 0.05 rad shear angle. The equation with which the plastic strain is estimated with Eq. (15).

\[
\dot{\varepsilon}^p_l = \sqrt{\frac{2}{3}} \dot{\varepsilon}_{ii}^p \dot{\varepsilon}_{ij}^p
\]

where the term \( \dot{\varepsilon}_{ij}^p \) is the rate of the \( ij \) component of the plastic strain matrix.

It is concluded that the uniform distribution of the plastic strain is obtained by the proposed methodology in which the effect of shear and flexural stresses are simultaneously considered, and uninformed. Table 2 indicates that the stress distribution based on the proposed concept works and if \( a + b \) is equalized to \( \frac{L}{\sqrt{3}} \), the total stress distribution would be uniform and almost the whole section reaches to the plastic level as compared to other geometries that only some portions of the dampers are plastified.

In addition, the pushover behavior of the models are compared, and it is concluded that ratio of the ultimate displacement value over the yielding displacement is higher for the uniform stress model. Fig. 9 shows the normalized pushover curves. It is concluded that the proposed model with uniform distribution is able to experience larger drifts after being yielded without losing strength resistance capacity, which is shown in Fig. 9.

### Table 2

<table>
<thead>
<tr>
<th>( (a+b)/L )</th>
<th>( 1/\sqrt{4} )</th>
<th>( 1/\sqrt{3} )</th>
<th>( 1/\sqrt{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized ( (u_{\text{max}}/u_y) )</td>
<td>1.00</td>
<td>1.20</td>
<td>1.12</td>
</tr>
</tbody>
</table>

4. Conclusions

Structural fuses are implemented to concentrate the damages and inelasticity in a desired part of the structure. This will protect the surrounding elements from the high force demands, leading to the economical implementation of the resources. This study investigates the design methodology for general structural fuses considering the simultaneous effect of shear and flexural stresses. For this purpose, commonly used in structural applications, butterfly-shaped dampers are considered to be designed for uniform stress distribution. The flexural and shear stresses combination is considered and subsequently von-Mises criterion for the upper limit associated with the total stress imposed on the link. The stress state function is subsequently derived and optimized for having uniformity of stress distribution over the length of the link. Ultimately, the new design properties for typical butterfly-shaped dampers are proposed and computationally investigated. It is noted that the procedures used in this study could be implemented for improving any general structural fuse shape under the effect of simultaneous shear and flexural stresses.
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References


