THE SOUND OF FRACTIONS

teaching inherently abstract representations from an aural and embodied approach

Christopher Special Frisina

Thesis submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Master of Science

In

Computer Science

Deborah Tatar - Advisor

Kurt Luther – Committee Member

Steve Harrison – Committee Member

Tuesday, May 3, 2016

Blacksburg, Virginia

specialorange.org/thesis

Keywords:

Music Performance; Fraction Learning; Mathematics; Drumming; Coordination; Entrainment; Multi Representations

THE SOUND OF FRACTIONS

teaching inherently abstract representations from an aural and embodied approach

Christopher Special Frisina

ABSTRACT

Learning fractions is the focus for much of elementary school mathematics instruction because it is important and can be difficult. Fractions constitute a system of thinking about numbers and representations that differs in important ways from counting numbers. To understand fractions requires, for example, perceiving that a symbol such as "6" is not automatically associated with a larger quantity than "5" if they are denominators. In the system that constitutes fractions, 1/5 is bigger than 1/6. When students fail to master the system of fractions by a certain age, the inherent difficulty of the concepts can become confounded with discouragement, boredom, and humiliation. Music, especially percussion, not only provides an engaging context for many students but musical patterning can also provide deep analogic experiences to fractions at embodied and representational levels. Reasonable questions about musical patterns can both motivate and guide students towards understanding the properties of systems of fractions and their representations. We utilize this possibility in a new tool and associated curriculum called Sound of Fractions (SoF). SoF incorporates three main ideas to leverage musical interest and skill to provide an alternative approach to teaching fractions:

1. Experiencing the whole and the part at the same time is crucial to learning fractions;
2. Drumming is a compelling, embodied, culturally-relevant activity that allows students to experience the wholes, the parts, and the relationships between them at the same time;
3. A new computer-based representational infrastructure utilizing aural, visual, physical, and temporal components that scaffolds classroom-based activities that bridge the relationship between percussion-related and mathematics activities in such a way as to gradually bring the student towards more standard mathematical representations and usages.

We conducted preliminary testing of this approach in two series of after school programs with 5th-8th grade children who were significantly behind in learning fractions. Preliminary indications are that the approach is promising and ready to be tried in more formal contexts. This work illustrates that academic instruction rich in representational infrastructure and domains continues to be an important component of how technology can have positive impact.

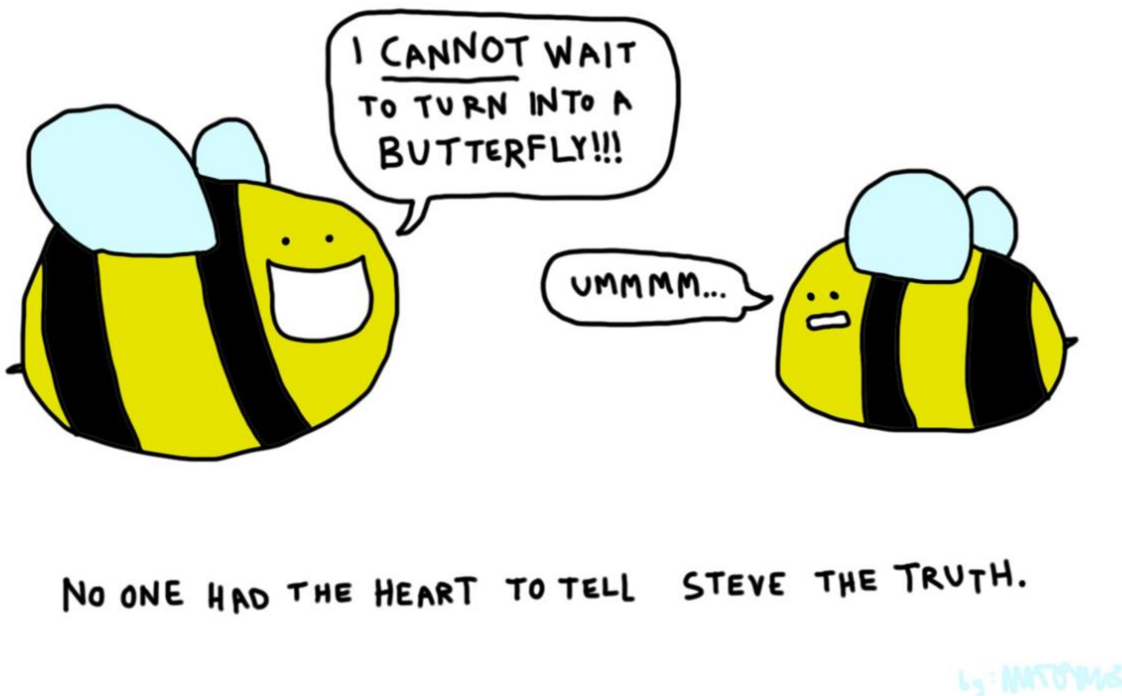


Figure 0-: http://plus5mace.tumblr.com/image/9924002940 below: http://arseniic.deviantart.com/art/What-do-you-call-an-alligator-in-a-vest-288568405

Special thanks to : Dr. Jonathan Crane, Phil Stewart Jr., Paweł Woźniak, Anna Kötteritzsch, Ella Grace Frisina, Justin Greenberg, Justin Hall, Nancy Edwards, Rebecca Welch, Phil Stewart Sr., the colors **orange** and **gween**, Hiko Lillpopp, Steve Cheyney, my parents, Gerald Stewart, Vicki Ramsey, Jackson Frisina, Gary Zeitz, Jessica Frisina, Dwight Hartsell, Jessica Self, Nathan Self, Dr. Heather Gallardo, Jim Currie, Jim Steele, Eric Welch, Dr. Karen DePauw, Drusiano Scerbo, Jenny Zartman, David Frisina, Larry Ramsey, Patrick County Schools, Winnie Welch, Emily Stewart, Randy Zartman, Dwayne Boone, Edie Frisina, Dr. Troy Abel, Siroberto Scerbo, Dr. Manuel Perez-Quiñones, Mark Large, Dan Rogers, Stephen Ramsey, Jack Fairley, Peter Jothen, the designers and tools of Sublime Text, Spotify, Linux, OSX, TotalSpaces, OmniGraffle, BibDesk, Excel, & Alfred, Julia Scerbo, Patricia Stewart, Heather Ramsey, Samantha Iglesias, Staci Kuntzman, the Third Lab, Jeff Hall, Bobby Beaton, Mike Cantrell, Debbie Carano, Christopher Riat, Dan Grano, Joseph Taylor, Charlie Mays, Jennifer Frisina, Martin Stein, Emma Welch, Diane Large, Kate Felix, Nathan Yau, God, Anne Steele, Mihika Daga, Eduardo Dewalt, Angie McFarland, Jessica McFrisina, the Lutheran Church, Dr. Volker Wulf, Dr. Steve Edwards, Dr. Warren DiBiase, Erik Jothen, Debbie Slatkin, Steve Harrison, the communities of Stack Overflow, EggheadIO, & RailsCasts, Aakash Gautam, Jay Layne, George Albansoder, Lena Lillpopp, Valerie Justice, Dr. Deborah Tatar, Robert Hernandez, Vincent Frisina Sr., Music from Eliot Morris, Wake Owl, Sleeper Agent, The Band Perry, Fences, The Greeting Committee, Sia, Mike McFarland, Tiffany Jothen, Petey Mainardi, the Reynolds Homestead, Wendy Hall, Dr. Steve McKrickard, Megan Wright, Hailey McLeod, Prerna Daga, ICAT, Shun Coaster, Diana Greenberg, Ann Zeitz, Karen Jothen, Kelly Finley, Melanie Darden, Mayank Daga, Dr. David Knight, Katie Zartman, Virginia Tech, Charles Frisina, Vincent Frisina Jr., Michael Stewart

Siroberto Scerbo
David Frisina
Jessica Self Nathan Self
my parents
Rebecca Welch

Special thanks to:
Paweł Woźniak

orange and gween

Eric Welch
Emily Stewart

Heather Ramsey
Martin Stein
Dr. Warren DiBiase
Dr. Deborah Tatar
Petey Mainardi

Michael Stewart

LIST OF TABLES

LIST OF FIGURES

1 INTRODUCTION

Fractions are arguably the most important topic in upper-elementary mathematics. Furthermore, they are surprisingly difficult to learn. For this reason, there is a large body of literature in the learning sciences, mathematics education, and other related fields exploring early conceptualization and approaches to teaching [45]. In the United States (US), the struggle to learn fractions is ``pervasive and an obstacle to further progress in mathematics and other domains'' and can have detrimental effects on adulthood, including failure to comprehend medication regimens [45]. Difficulties many children experience while learning fractions are compounded when the children live in situations of high poverty, attend low-performing schools, have personal experiences of failure, and succumb to boredom with what appears to be repeated presentations of the same material [15].

We observed students struggling with fractions in the classroom and noticed some children's reaction to their teacher's instruction, where students were individually tapping on their desk using fingers and pencils. We used this observation as a possible design pathway to offer a different instructional path, one that begins with music as an area of interest and for students as well as an area related to mathematics.

We identified important parts of successful learning by running small sequential studies and design sessions, and created a comprehensive curriculum to approach fractions through the lens of music. We used the foremost math learning strategies, leveraged the students' interest in music, designed novel representations with inspiration from traditional math learning resources, and organized them into a new socio-technical web application that is strengthened by activities both with and without the use of a computer. Our robust approach also supports children's interests towards learning fractions and mathematics more so than the current learning environment that they are underperforming in.

We ran two sessions of this study with 8 students over 7 weeks and 12 students over 6 weeks. We began by allowing the students to explore music and making representations of rhythmic performances, highlighting the important components of the rhythm when considering them in a different representational form. Our curriculum then allowed students to perform rhythms individually and collectively in an environment that undergirds embodied interactions before introducing them to SoF, which then adds additional rich visual representations and external audio representations. Each stage is carefully planned so that students progress from creation to identification, followed by manipulation of rhythms. This allows students to be able to contextualize, identify, quantify, iterate, separate, and order the unit fractions, parts, whole(s), and the relationship of the parts and wholes as they interact with the system of fractions in different domains.

2 LITERATURE REVIEW

Math education is the subject of a tremendous amount of research covering curriculum topics, technique, and order of topic presentations [15,28,44]. Music educators follow a separate set of influences in educating students. Computer Science, a major influence in the history of this research, also has many thoughts about education, instruction, and topic discussion. In this section we discuss the wealth of research that we cover in our research efforts.

2.1 Common Themes in Mathematical Instruction

Education in the US varies from state to state, from county to county, and in some cases, from city to city. The diversity of education services and resources has a strong impact on districts that already struggle to offer both the best and equal services for all students across schools within the district. One common method adopted by districts for consistent curriculum is the Common Core (CC) [28]. The CC concretely suggests what should be known by all K-12 students upon completing a particular grade, barring any particular hindering learning disability. States are typically left with how to qualify students' knowledge of the CC milestones. Districts and schools, however, are left to decide what curriculum and tools to use to meet these standards, and also how to handle students who do not meet the CC guidelines. Grade retention, a common choice by administrators for students falling behind, is on the rise since the 1970s [19], and sometimes applied to a specific course despite progressing the student on to the next grade level. The segmented and abstracted hierarchical structure of the educational system leaves too much distance between those that teach and those that assess, and adds layers of complexity when teachers are left to decide how to approach learning in a meaningful way to their classrooms as a whole and students as an individual, and has been criticized by scholars from other countries that have a high literacy rate in mathematics [54].

In regards to math, the CC begins to introduce fractions in grade 3 (~8 years old), by familiarizing students to the idea of identifying numbers by their amount/size (via location on the number line), whole numbers in fraction form, and equivalence (same location on a number line)[28]. A brief simplification of the CC's approach to fractions shows the following progression from representations (fraction, decimal, percentage) to more integrated math schemes in Table 2-1:

Common Core Math Levels		
Grade	~Age (years)	Topics covered
K	5	Counting, Cardinality, Shapes, Measurement
1	6	Operations (add, subtract), Base 10,

2	7	Standard units, place value, foundation for multiplication/division
3	8	Operations (multiple, divide), fractions as numbers, unit fractions
4	9	Fractions, decimals, and percentage representations, equivalence and ordering
5	10	Multiplication and division of fractions
6	11	Variables, ratios, and proportions
7	12	Applying equations to geometry, rates, and more integrated math schemes
8	13	Use equations and expressions to form higher order functions and statistics

Table 2-1 : Common Core topics covered in math curriculum for grades K-8

The complexity of mathematical language that students are expected to master grows with each grade. Thus, there is a clear need to understand a previous grade level's concepts prior to progressing to the next level. This structure is flawed in its motivation to undergird progression, highlighted by the lack of guidance for students who fall behind. The current typical response for schools, administration, and teachers is to have the students repeat the class or curriculum, and in some cases develop severely delayed individual development plans of the same curriculum often with little variance in instructional design or presentation. This unaddressed area is the focus of our approach.

Students who fail to pass the tests given to them are usually not aware of the concepts that they are failing to understand. They are aware of their failing only from the numerical or alphabetical grade returned to them. Since students fail to understand the concepts that underpin the system of fractions, they fail to see their mistakes, evidenced by their grade. Rather, students may attempt to make sense of the various mnemonics, rules, shortcuts, and sing-a-long memorization strategies that are presented to them over time (Muhammed cites a rule `Copy-Change-Flip’ as a way to address fraction reduction when the mnemonic is used for dividing fractions). The problem of this mechanism is more `helping tools' are continually added without understanding the current structure and meaning of how the tools help. In Figure 2-1 (Common mistakes students make in math), students know they have to add the numerator when adding, but don't understand you can only do this when they are of equal sizes, so they also add the denominator. When multiplying in long division style, students know that alignment is important, but forget what to align (e.g. align to the left, align to the right, align the decimals or align the last digit). The difficulties may be traced to simple arithmetic as well, when students continue to persist when `borrowing'.

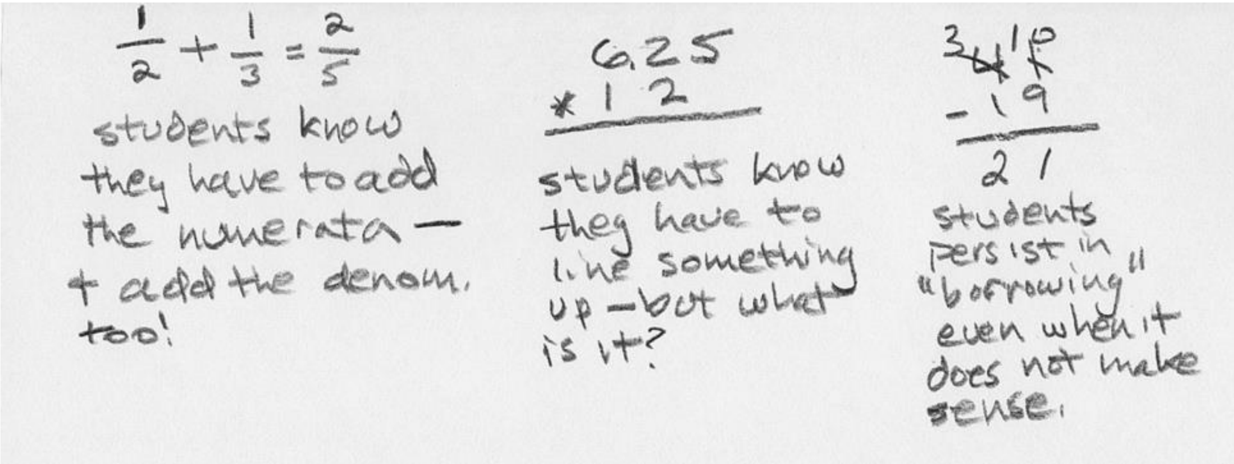


Figure 2-1 Common mistakes students make in math

This problematic area of how to help remedial math students is a focus of our approach. Starting from an approachable and attractive area of music allows remedial students to begin to use their musical skills to understand the system of fractions they are already exposed to. It is also a different approach, so the novelty factor plays an important roll for engagement. Providing our tool and accompanying curriculum allows for clear and productive interactions that foreground previous common mistakes and opportunities to understand the system as a whole to prevent future mistakes.

2.2 Common Themes in Musical Instruction

The curriculum for musical education in elementary school grades follows a very similar time schedule as mathematical education. However, unlike math education, music education begins with basic vocabulary and guided imitation. Students then progress to use more descriptive vocabulary to address the increased difficulty, wider range of musical performance, collaborate with other students who play the same instrument, and coordinate the complexities of playing with other musicians in an organized manner. Novice-level vocabulary words describe (although not completely exact, robust, or distinguishable) the characteristics of the sounds, such as tone, pitch, frequency, or their associated name (e.g. A♭, A, and A#, ...). Other words are reserved for describing the mechanics, such as *measure*, *beat*, *rest*, and *tempo*. Some describe the length and duration, such as whole note, whole rest, half [note/rest], quarter [note/rest], eight [note/rest], 16th, 32nd, and 64th, but require knowledge of the tempo (120 beats per minute, 180 bpm, 240 bpm, …), and the signature, such as 4/4, common time, or more complex signatures to know the exact length in time. There may even be discussion about the performance, including, staccato (playing the the sound sharply with distinguishable separation from the next note), crescendo (playing successive notes louder than the previous), allegro (playing at a faster tempo temporarily), and other distinguishable actions. As students develop their musical talent, the teacher (who is sometimes a professional conductor or musician who teaches) begins to use a language that goes beyond the elementary vocabulary, mimicking, or rote repetitive call and answer. This language uses more abstract thoughts and allegories, such as one conductor used to describe a staccato rhythm, ``these little accents... ...it's as if a little cat came in to touch you'' [3].

The differences between *how to teach math* and *how to teach music* reveal varied emphasis on progression from simple to complex mental events and novice-level practice of the subject matter. *How to teach math* is dependent on a multi-tiered progression of complexity, a student starts at the arguably simpler concepts of amount and size and must construct the cognitive steps necessary to climb to the next level of complexity. Teaching strategies used for more simple concepts often have problematic interactions when coupled with each other. In comparison, *how to teach music* depends on a similar progression from simple to more complex activities, but this process is mitigated with novice-level strategies like imitation, mimicking, and rote repetitive call and answer that collectively and consistently reinforce each other and do not bring the added confusion and problems. A student in math can grasp multiplication without understanding other mathematical properties (such as it being the inverse of division, or the same as repeated addition). This is different than a student playing a small piece twice as fast without being able to preserve other characteristics of the musical piece (the relation of the spacing of the beats to the musical piece [whole] being the same, but the sound being different). Said differently, to progress in in either domain requires well rounded and intricate knowledge of relationships and representations. Musical education does a better job of constantly reinforcing the relationships consistently and collectively because the music is performed and heard. Math education struggles where students can perform well in a given area, but ultimately will stymie their progress if they can not relate it to a great number [and representational] sense, especially if they memorize rules and not understand concepts. A more comprehensive approach is needed to support the areas [seemingly separate - from a novice's perspective] of math education (such as number sense, functions [add, subtract, multiply, divide], fractions, geometry, algebra, and beyond) to collectively reinforce the relationship(s) between them. Previous work in the areas of math and music education (both individually and collectively) attempt to address how to teach each subject simultaneously (in parallel and mixed settings). The interesting approaches are how elements of musical education (such as novice-level practice strategies, notation/representations, and form/structure) might bridge the gap between enabling students to climb and enabling teachers to pull students up in math [2,9,22,63,64].

2.3 Math, Music, and in Between

Previous work encompasses evaluation and revision of academic instruction, musical performances, and influential design strata. The problem of how to help children struggling with fractions is addressed through developing the theoretical elements in teaching approaches, revising the instructional framework, building better representational references, and utilizing elements of musical theory that provide useful insight into mathematical theory. Our aim is to present a culturally relevant, academically grounded, and design oriented perspective to include student agency, both afforded and enacted, an overlooked component of the learning process. We found that there is room for improvement in what representational affordances we provide students, and that music can function both as a motivating feature and framework for mathematical theory.

2.3.1 Revisiting Educational Approach to Mathematics

The following theories are the correct prominent attempts to the cognitive elements of mathematics learning in such a way as to indicate how math should be taught without incorporating elements of musical theory.

2.3.1.1 Rational Number Project (RNP)

The numerous and varied descriptions of the system of fractions emphasize the difficulties some children experience in the learning process. Fractions encompass the five basics of the Rational Number Project (RNP) (1976 - current):

1. relationships between the part(s) and the whole,
2. how to measure them,
3. ratios,
4. operations, and
5. quotients [5] based on the rational number components from Kieren [32].

Arguably, a student cannot even attempt the last three basics (ratios, operations, and quotients) if he or she fails to understand the initial relationship between parts and wholes or the language or mechanisms to quantify them.

2.3.1.2 The Splitting Loope

Research on the way that students come to understand the system of fractions has progressed since the introduction of RNP. Major work was done by Lamon to show the strongest component, or `subconstruct', is measurement, the second of the five basics of the RNP [34][37]. Two separate longitudinal projects investigated the cognitive reasoning involved in the overall complete process as a part of ``The Fractions Project'' by Steffe and Olive (1990 - 2002). These two studies deconstructed the many cognitive steps employed by children [23][44]. Building on these works, Norton and Wilkins have collected and organized these cognitive steps into a more cohesive and comprehensible structure of fraction schemes, referred to as ``The Splitting Loope'' [66]. Norton and Wilkins identify the following main schemes [43]:

a) Part/Whole
b) Partitive Unit Fraction Scheme(PUFS)
c) EquiPartition Scheme (EPS)
d) Splitting
e) Advanced fraction schemes

The progression of these schemes can be viewed in Figure 2-2 (The Splitting Loope schemes according to Norton and Wilkins (2013)). Students move from the beginning stages on the left to more complex stages on the right. Norton and Wilkins point out that to have a strong number sense and fraction acumen relies on the ability to split the wholes independent of partitioning and iterating, better supporting number sense.

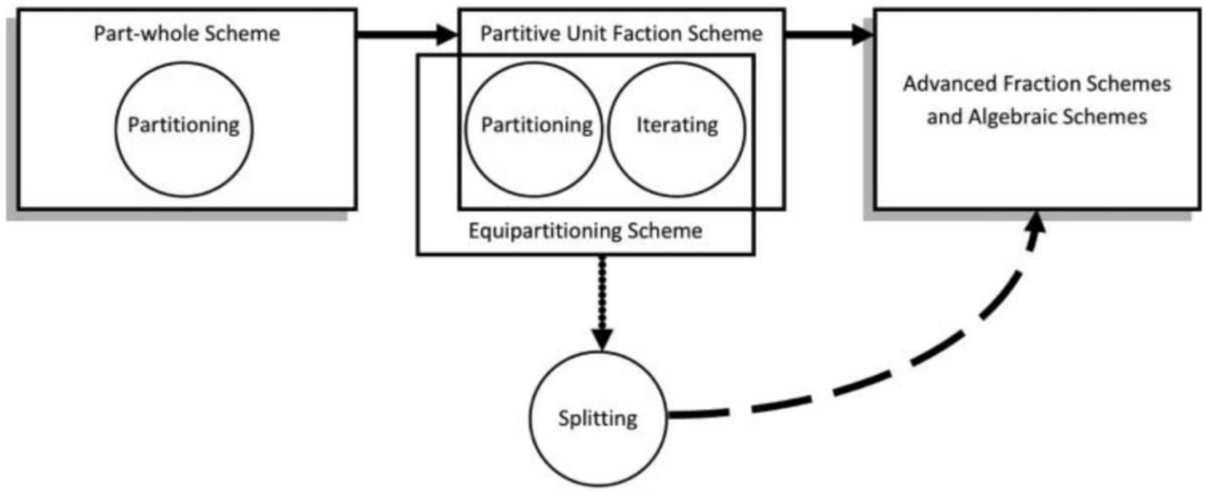


Figure 2-2 The Splitting Loope schemes according to Norton and Wilkins (2013)

Part/Whole includes partitioning (separating the whole *W* into *n* parts), but does not include *iteration*. Iteration plays an integral role in PUFS and EPS schemes. EPS is a scheme that students use to partitioning a whole into `equal' pieces, and they iterate over ONE of the pieces to verify that they are equal, comparing the whole against an iterated piece, not comparing the pieces themselves. PUFS is a way that students partition a whole into pieces, and identify one of the pieces as a part of the whole, understanding the quantity outside of the fractional language. To understand the difference of the closely related PUFS and EPS, Steffe explains, ``The partitive unit fractional scheme differs from the equipartitioning scheme in the explicit numerical one-to-many comparison (one-to-ten) and in the explicit use of fractional language "one-tenth" to refer to that relation'' [59]. Splitting occurs by students who can split iteratively, not recursively [8]. That is, students split by starting with a best guessed sized piece of a whole and do so iteratively of the remaining `whole' until the `whole' is exhausted, NOT splitting the whole into pieces, and then splitting those pieces into smaller pieces, the way a rolling pizza cutter slices pizza.

The five schemes are important but nuanced and can be confusing to non-mathematics education experts. It should be noted that they all have in common both the concepts of part and the whole. The method(s) that the students choose to enact to perform or validate the relationships speaks to how well they understand the system of fractions, but even a novice who fails to move beyond the part/whole scheme is still aware that there are two distinct components to be understood, the part and the whole. We leverage this novice-level knowledge of the student to include in our curriculum and accompanying technical implementation, SoF, to allow students to experience the part [beats] and the whole [rhythm] at the same time.

2.3.1.3 Equal Sharing

On a separate research track, Susan B. Empson approached studying fraction learning with first graders with exercises based on equal sharing [16]. [Equal] sharing is a skill that young children have experience and informal knowledge with [13] [49]. Empson was also able to identify that the students could solve problems using strategies that involved order, equivalence, addition, subtraction, and multiplication (division, although it can be considered as the inverse of multiplication, was not covered). The curriculum followed the progression of partitioning from easier situations (2 cookies among 4 people, 4 pizzas among 8 people), to larger numbers and non-halving (8/16 and 16/20), and then to more complex partitions (thirds, ninths, fifths, sevenths) [16]. This greatly influences our curriculum for particular stages and what musical activities to challenge the students with and in what order (see the section 5: Curriculum for a comprehensive curriculum overview).

2.3.2 Musical Theory as an Analogy to Mathematical Theory

Musical theory and performance has a wealth of features that can be approached and incorporated into mathematical learning. We however think it would be disadvantageous to the math domain, music domain, and students if we try and align the two domains in parallel, or to just include both in the other. To provide a solid curriculum, interface, and therefore learning environment with the highest positive impact, we studied many components of the musical education domain as it relates to music theory, including performance (creation, listening, entrainment, and emotions), structure (notation, vocabulary, descriptions, and composition) and educational approaches (how they are scaffolded, what they include, and what they avoid).

We identified the major contributions of music theory as it pertains to math learning:

- the natural embodied interactions when performing music,
- the act of teaching how to perform the music prior to presenting vocabulary to students and the vocabulary itself used to describe representations of music,
- the transfer of cognitive, temporal, and physical components into a unified musical domain

Previous work on the utility of musical theory or components of musical theory cover both the benefits that a synthesized teaching approach could create for struggling students and the consequences such an approach would have on music education - namely the possibility that flaws in math education may bleed into student's perception of music and act the part of a parasite and not a symbiotic partner.

2.3.2.1 Useful Components of Musical Theory

Music was well integrated in to the earliest of our civilizations, hence there are many varying ideas for the order in which students should come to learn how to perform music themselves [1] likely to predate some of our citations. A major concern for teachers today is not what to teach (tempo, tone, representations or names, hearing or distinction, performance or practice, rote repetition or mimicking, etc.), but the order in which to teach them. Joseph H. Naef, the student and later colleague of Johann Heinrich Pestalozzi (1746-1827) who professed experience before introducing names and symbols, advocated for `Sound before Sign', and outlined seven recommendations to music teachers in ``Principles of the Pestalozzian System of Music'' at a meeting of the American Institute of Instruction in 1830 [39]. The first of these recommendations notable for our research is that students should learn more active activities such as [hearing and performing] sounds before more passive activities such writing symbols [notes] or names. The other is to teach incrementally, one concept at a time and practice separately, before more difficult tasks of attending to multiple concepts simultaneously [39].

Two of the most popular approaches to musical education in the US adhere to these recommendations for active before passive activity and incremental learning, the Suzuki and Kodály methods. The Suzuki method, a pedagogy developed by Shin'ichi Suzuki, focuses on education students at an early age with rote performance prior to introducing or learning note [symbol] reading. This approach is grounded on the way children learn their native language, surrounded by actors performing the language, and the child speaks [producing meaningful sound or music] before learning to read [35]. The Kodály method, originating from Zoltán Kodály, incorporates hand gestures, *solfège* (voice/tonal) syllables (e.g. ``*Do-Re-Mi-Fa-So-La-Ti-Do*"), and cultural-relevant approaches, often determined by the teacher him/herself [58].

Previous work uses both methods to determine that music experience prior to math improves math learning and spatial-temporal skills, specifically tasks that require mental rotation or multiple solution steps for two-/three-dimensional mental representations, ones that lack physical models [41][65][64]. Lois Hetland's research shows that music instruction which introduces notation yields higher spatial-temporal skills, especially in younger (< 9 year olds) children [25]. However, given the research in this field with the largest impact size does not decouple music education prior to or concurrently with math instruction to understand their separate contributions [22], Vaughn suggest ``further research is required to test the possibility that concurrent but separate math instruction using spatial-temporal methods improves math performance.'' [64].

In addition to Vaughn's hint towards future research, music education professor Bennett Reimer offers a very import warning about linking musical education or experiences to increased mathematical performance and other domains. Reimer vehemently believes that the integrity of the music education should not be exploited purely for gains in other domains, as it undermines the value to improve people's abilities to gain ``meaningful, gratifying musical experiences.''[53]. An abstract representation of Reimer's teleological objection is that the ends (gains in other non-music domains) shouldn't justify the means (violating the integrity of musical education), because the unintended result is the depreciation of musical experiences and degradation of music education. However, we argue that using music as one motivating feature in a broader more culturally-relevant teaching approach for other educational domains does not devalue musical education or an individual's enjoyment of music. Such an approach would instead provide individuals with a more holistic perspective on the aesthetics of human experience. In this sense, incorporating elements of musical theory in mathematics is inclusive not exploitative.

2.3.2.2 Entrainment

One culturally evolved social activity is entrainment - the ability to perceive and synchronize internal thoughts or external movement to an external third party sound. Mostly considered unique to humans [33], entrainment allows individuals to perceive sound, including `noise' with no inherent rhythm, as having a rhythm. This ability to perceive rhythmic patterns as external musical performances begins in the first year of infancy and develops to discern between *grouping* (perceiving boundaries between other groups, or between subgroups), *rhythm* (discerning the temporal intervals or separation of beats), and *meter* (an abstract hierarchy of beats, denoted by the strength of beats) [62]. Kirschner and Tomasello tested infant response time to synchronous tasks in a social setting; their results showed that children at least 2.5 years old were able to adjust their drumming tempo outside the range of their spontaneous motor tempo. Furthermore, all children performed better at the synchronous tasks when performing in a social environment as opposed to a physical drum machine or a sound playing from a speaker [33].

The role of third party external noise in signal processing arguably demonstrates the humans are be able to process non-aural signals and perceive them as having meaning, substance, and musical properties. Examples of this are the usefulness of visual metronomes as an alternative channel to communicate a prescribed rhythm [29], and haptic limb bands used for performance stimulation, coordination, and feedback of polyphonic rhythms to train novice drummers using multiple limbs without sounds that conflict to the drummers' own learning performance [26].

2.3.2.3 Embodied Interactions

Musical performance is inherently embodied, as is entrainment. Entrainment, synchrony, and rhythm play an important role in embodied interactions, especially in communication that is mediated [20]. Paul Dourish reminds us that `embodied phenomena are ones we encounter directly rather than abstractly' [14] and occur in real time and real space. Polanyi also claims that tacit experiences have two properties, proximal and distal, together which enable a deeper sense of knowledge that are harder to communicate explicitly [48]. By focusing our curriculum design on tasks that share qualities of proximal, collaborative, embodied, and explorative, we hope to create a learning environment that welcomes physically expressive ideas and inquires for traditional representations.

2.3.2.4 Expectation

Bobby McFerrin, a vocal virtuoso, gave a presentation to the World Science Festival in 2009 under the topic of ``Notes & Neurons: In Search of a Common Chorus'' and presented the idea of expectations in a musical context. He began by standing in one location on the front of the stage, facing the audience, and singing a medium pitched note in sync with jumping in spot, and signaled with his left hand pointed to the audience to mimic in real time. After he felt that the audience was performing in sufficient synchrony (after about 5 hops and accompanying notes), he signaled with his left hand to the left [to the audience's right], then he moved to that spot (to his left) and began singing a note of one step higher pitch. The majority of the audience automatically followed with the higher note, while the remaining caught up on the next beat. After jumping in the second higher position for a few times, McFerrin pointed back to the first spot and jumps back, and adjusted his tone to the lower original tone, and the audience followed in suit with no delay or mix up. He then proceeded to jump back and forth, with no notification given before jumping between the notes, and the audience followed along perfectly. His tempo increased quite rapidly, and the audience kept up. Then, still with no notice, he jumped from the higher note [right-most position from the audience's perspective] to an even higher note [farther position to the audience's right], but made no sound into the microphone himself. The audience not only didn't miss a beat in timing, they also flawlessly increased their pitch by one note's frequency of the same increase between the previous two notes.

The audience's performance follows musical expectations according to McFerrin, and occurs universally because of the [pervasively pleasing] pentatonic scale (a scale with 5 notes, instead of the other traditional heptatonic scale, comprised of 7 notes) [38]. We seek to align mathematical scaffolding progressions in the curriculum with natural musical expectations to strengthen the understanding of the part/whole relationship. The idea of universal expectations of what is stronger when a rhythm is present, as followers are able to mimic the leader more with gestures as simple as hand movement ($p<0.001$), and also head movement [56].

2.3.3 Elements of Educational Approach

The following elements of teaching music and math are focused on in previous work as key points to creating a better teaching approach.

2.3.3.1 Scaffolding

Scaffolding is an important feature in educational curriculum, that helps balance educational goals with student performance and expectations. This concept is especially needed in scenarios that involve multiple students. Classroom activities often involve planned group tasks. Tasks scaffolding should include:

1. the initial setup/condition of the activity [environment],
2. information about the activity, and
3. what tools [stimuli] are available to the students during the task.

To better understand the cognitive processes, it is also suggested that the instructor be prepared for possible responses. Teachers should also be aware and have a mental model for the different cognitive states of individual students between responses from the students collectively [42]. Michael Cole clarifies that the task is not necessarily limited to the exact initial communicated assignment, but rather its interpretation is co-developed among students during the execution of task in practice, and their task emerges as an ``interpersonal, public resource for coordinated action'' [7]. Understanding the stages of scaffolding of the curriculum, cognitive stages and process of the children, and that the meaning of a task from the students' perspective changes during the task is integral in developing a robust curriculum and technologically mediated tool to present to students to help improve their comprehension in any domain they are struggling with. The task of developing a robust system is even more difficult when mixing domains [53], but also strengthens the need for paying attention to the nuances of the cognitive stages and potential miscommunication that come along with making analogies.

2.3.3.2 Engagement

Engagement is another important factor in educating students, especially children. Our original motivation for this research, from the initial prototype, is that the students were highly engaged in discussion, and motivated to follow teacher prescribed classroom rules to help facilitate learning that were previously ignored.

2.3.3.3 Proximity

Musical performances most often occur with all musicians co-located and in relative immediate proximity. With more recent advances of electronic technology and the music industry itself, the musical community now features musicians that are separated from one another, separated from the audience, and separated from the instrument. Close proximity allows for robust nonverbal communication on channels to be fully utilized between musicians.

Mustafa Radha categorizes the channels into three main groups: (1) Visual, (2) Verbal, and (3) Auditory [51] and Robert Beaton supports this in his research of how leaders perform during music making performances [4]. Visual channels include eye contact, breathing, head nods, facial expressions, arm movement, and body movement [56]. Verbal channels include words, breathing, sounds produced by the mouth, or tapping. Auditory channels include loudness, pitch, rhythm, tapping (of a foot or hand), snapping, non traditional musical performances (e.g. an extra tap of the cymbal, a non tonal breath pushed through a wind instrument, a hammer-on of a stringed instrument, etc.). Some actions can be communicated over more than one channel such as tapping, as it has a visual performance, an verbal direction, and auditory rhythm.

Beaton explains that future work in remote proximity may have important insight into the role of leadership and collaborative coordination performances [4]. There are even more considerations for communication channels between musicians as a group and the audience as a group that are not discussed in this paper.

2.3.3.4 Synchronous Coordination

Drumming performances can grow to become quite complex, starting with easier patterns such as **simple meter** (each beat has two equal parts), and growing more difficult with **compound meter** (each beat is divided equally into three parts), **mixed meter** (a.k.a. additive rhythms: adding different rhythms played in a cycle [group]), to quite challenging **polyrhythms** (a.k.a. cross rhythms: using two or more rhythms simultaneously). There is also **metric modulation** (where the drummer switches or segues between rhythms at intervals matching the grouping), **polymeter** (involving two separate beats played simultaneously, the beats and tempo being equal, but the measure size differing [the meter {strength of the first beat in a measure} being different]), or the meta-metric **hypermeter** (the measures themselves act as individual beats)[40]. All of these rhythm patterns require copious amounts of attention to muscle movement and memory, external stimuli, and other performers altogether; they are all forms of coordination.

2.3.3.5 Communication

While our newest implementation shares many characteristics with the growing trend of collaborative tabletop music interfaces, it may or may not fully fit the classification [6]. Having a single electronic interface for several people to interact with forces students to discuss what actions are taken, before or after they are enacted. This keeps the conversation going, as students rearrange the representations to elicit particular phenomena for discussion. It does however pose problems for classroom management and personal agency. With individual computers, students are able to make choices on their own. With an accompanying curriculum, their actions and thoughts can be guided to focus on particular fractional properties that wills scaffold towards a stronger understanding of the system of fractions. Individual devices also allow participants to focus on the communication required to complete the activity and understand the takeaway message. This differs from coordination in that they may be able to perform the musical component of the exercises, but will be challenged to communicate the reasoning behind their performance choices, hopefully in more mathematical descriptions as activities progress.

2.3.3.6 Micro-coordination

Understanding the taxonomy of micro-coordination interactions allows us to understand the different coordination interactions are being done, design for them, and have the design be meaningful in the communication between musicians [51]. Some tasks that musicians perform while making music are *synchronization* of tempo, rhythm, harmony, or a more complex complementary synchronization, where the musicians collectively produce a continuous sound, but alternate (with or without a pattern) making sound individually. This form of fast-paced non-verbal communication and synchronization can come from repetitive practicing, sometimes found in drumming circles, and can include a mixture of novice and expert performers [4]. In more formal settings of music making, notations of shared resources help clarify goals, mood, and musical direction for reference when playing the musical piece through at a later time, helping memorialize all of the feelings in an easily consumable form.

2.3.4 Building Better Representational References

Literacy, dscribed as the ability to Read and Rite (and colloquially perform Rithmetic to achieve the three R’s) is pinned/under by the ability too comprehend and compose ideas in waze that are meaningful, so that it may be communicated (back two one’s self or too others). While the previous sentence is both lacking in vocabulary and grammatical correctness, if read aloud, it should be interpreted correctly given the power of homonyms, slurring of words, and other literary devices. This is the strongest tenet and motivation for of our work. While one representation can be confusing, another can exemplify the focal meaning and ideas easily. A call to use the three C’s (comprehension, composition [21], and communication) is more appropriate than just rote arithmetic and traditional instruction. Using the CCC's are how teachers [and other knowledge holders] can ascertain what students [and peers] actually mean. This shift of focus from Rs to Cs (like a pirate's favorite letter, thought to be the Rrrrr, when their true love is the Ceeeea) is an important step in educating the students who are already behind, as it returns their own agency for them to create and compose, contextualize, compare, and communicate their comprehension from a domain they are already excited and familiar with.

Empson's *a posteriori* approach to mathematics mitigates the cognitive steps of solving math problems with meaningful personal experiences. This meeting between mathematical concepts and real-world implications introduces the approach to helping struggling students with fractions - representational references.

With students being forced to repeat the same curriculum after unsuccessfully comprehending the previous year or semester, the curriculum loses the power of novelty. As students continually are exposed to the same representations and information about the concepts and characteristics, the features become more difficult to discern and appreciate. This concern motivates us to include new and novel representations in SoF. Well-designed two-dimensional objects can be very expressive of multidimensional musical concepts for music making interfaces [47]. Jorda et al. make progress with representations by mapping geometrical shapes to particular musical sounds, filters, or features such as a circle as a metronome or a decagon (geometrical shape with 10 equal length sided and 10 equal angles) for a decimator (a noise reducer) [29].

2.3.4.1 Culture & Ethnomathematics

One strategy of successful math teachers is to adapt their approaches to students at an individual level, interacting one-on-one with a student to address specific needs. Teacher perception of one-on-one educational approaches directly affects the instructional interactions and mechanisms they use in inclusive learning environments [30]. A key concept to consider and give weight to is *ethnomathematics*, the relationships between culture and mathematics [12]. The compound word ``ethnomathematics'' can be deconstructed to the terms *ethno* describing ``all of the the ingredients that make up the cultural identity of a group [or individual]: language, codes, values, jargon, beliefs, food and dress, habits, and physical traits'', and *mathematics* expressing a ``broad view of mathematics which includes ciphering, arithmetic, classifying, ordering, inferring, and modeling'' representations [11]. Teachers who embrace ethnomathematic approaches with students address student needs through leveraging students' past personal experiences, rich cultural past, and introducing cultures that are foreign to the individual student or the class altogether. Ethnomathematic approaches help create periods of empathic immersion for students to help understand math from the context of their own culture, reaffirm their understanding of their culture, and experience opportunities to explore other unfamiliar cultures. Therefore, teachers and students can use a wider range of cultural artifacts (or references) in their exploration of new academic material, as well as traditional curriculum material to compare or contrast against.

2.3.4.2 Cross-domain teaching

Fractions are already taught alongside other educational areas [9][69]. Some common domains that have been used to teach fractions are food, music, dance, visual arts, and photography. Each of these approaches uses a particular domain to appeal to the students, and is described further in 2.4 Other Cross-Domain Approaches.

2.3.5 The Role of Technology in Mathematics Learning

Technology to help teach mathematics in the classroom is approaching its 50th year [31], and has proven to be of great help. Technology also has had negative effects[18]. Researchers, academics, and professionals have been trying to identify and add beneficial technology while avoiding the negative attributes ever since.

Technology in the mathematics learning domain:

- Reinforces the pedagogical shift towards active learning [31]
- Aids in graphical representation of complex or abstract modeling [18]
- Creates external symbolic and cognitive representations [57]
- Allows rapid deployment and trials
- Provides reusable, distributable, configurable, and potentially personalizable resources
- Has the potential to reduce costs associated with education

Our research follows in this tradition by approaching from a socio-technical approach, guided by our own curriculum, and engaging in a domain appealing to students.

2.4 Other Cross-Domain Approaches

Given that barrier to understanding fractions exists for many students, it is not surprising that many educators, academics, parents, and administrators look to alternative ways of teaching the subject and associated concepts in and outside of the classroom. While these examples do highlight other domains, they lack a robust approach with complementary curricula.

2.4.1 Music

The music domain coincides with strong fraction understanding when using terms such as whole note, half note, quarter note, … However, musical notation can be confusing, leaving ambiguous what a quarter note is a quarter of.

2.4.2 Food

Some manipulatives are used to elicit fractions in terms of food, ordering, making, or sharing, such as pizza, pies, cookies, and meal sharing.

2.4.3 Dance

Dance is used to teach mathematics, with attention to agency given to the performer in actions or steps that can be described with fractional qualities. [10]

2.4.4 Embodied

Action, touch, and other embodied interactions support cognitive learning in multiple domains, especially math [55]. One example is where students use hand movements to mark and generalize relationships between parts and the whole with visual feedback [27].

2.4.5 Hair Braiding

Ron Eglash has studied ethnomathematics, and how the culture and livelihood of certain groups provides ways to count using diagrams and figures. These figures and ways of representing mathematics have even been used when braiding hair, by braiding the patterns into the hair weave of other community members.

3 OUR PRIOR WORK

This thesis document is a culmination of many prior works, ideas, experiments, and the byproducts of larger related grants and funded by many smaller grants and institutions. To understand how we came to the most recent study (as described in 4 The MAIN Study), it is best to look at the work we did that led us to our main study.

3.1 Proto Computational Thinking

We, the authors, worked with teachers in a study to help incorporate computational thinking in the classrooms of a low socioeconomic status (SES) middle school in the US. Circumstances were such that a school program provided every child with access to a laptop for the school day and to take home. The students we worked with, aged 11 to 13 years old, were in 6th grade of a middle school in the northern region of Virginia, United States (US), who are guided by the Virginia SOL, not the CC. The majority of the students in the class and in the school as a whole were of a low SES, and participated in free or reduced lunch programs. This school was of particular interest because of the administrative program providing one laptop per child; each laptop was fairly new, with a large color screen, several gigabytes of RAM, powerful processor, and modern web browsers (Google Chrome and Mozilla Firefox). This enabled computer exposure on a daily basis at school as well as at home, with a deposit of under $50 USD, mainly required for asset tracking purposes.

We were working with a NSF grant to study how to include components of computational thinking into the classroom from each of major education domains (math, science, language arts, history, etc..) to help seed the future potential for understanding computational thinking. Computational thinking, a term coined by Jeannette Wing, describes the properties of thinking, problem solving, and structures that are fundamental to computer science, and can be extended and applied in all domains [67]. We believe that teaching computational thinking begins without the use of computers, especially in the context of computer science, and should be approached concurrently alongside other major education domains.

The math teacher we collaborated with noted to us that fractions were a major hurdle for his students. Many of his students had failed the same class for one or more years prior to the current year and showed continuous lack of engagement for traditional curriculum. Over the course of the observational phase, we observed disengaged students not participating in classroom discussion of the math curriculum but rather tapping on their desks with their fingers and pencils, effectively drumming to pass the time. We then identified that numerous students in the class were also enrolled in music classes, predominately band, and even participated in extra musical performances such as marching and jazz band. This discovery was our motivation to explore the area of music and fractions in the same setting; our conception of this area influenced the design phase of our initial prototype. When we introduced our prototype, a rudimentary beat-box machine automatically calculated fractions of the ratio of beats to a measure. Student and teacher interest soared, despite the lack of strong accompanying curriculum. To address the issues surrounding the lack of mathematically grounded curriculum and maintaining student engagement, the existing body of literature on these topics must be reviewed.

3.1.1 Proof of Concept

To test the viability of teaching fractions to students with a new approach of using their musical prowess and interest, we developed an interactive web application for the teacher to use as an accessory in the classroom. Students were mostly African-American and of a low SES as measured by the provision of free or reduced school lunches. The students also were several years behind in the state's learning standards that have similar progressions at the country wide level of the CC.

The initial prototype we created provided functionality for students to :

- See the relationship between the beats [parts] and the whole [measures].
- Interact with the software with the mouse, NOT with tapping or recording.
- See multiple representations of this representation: several mathematical (proper fractions, improper fractions, mixed numbers, decimals, and percentages), one aural (the sound), and one graphical (bars).
- Create their own music by adjusting the number of beats in the measure (this is equal to the PUFS fractional scheme).
- And operate in a novel and different classroom environment than they are used to.

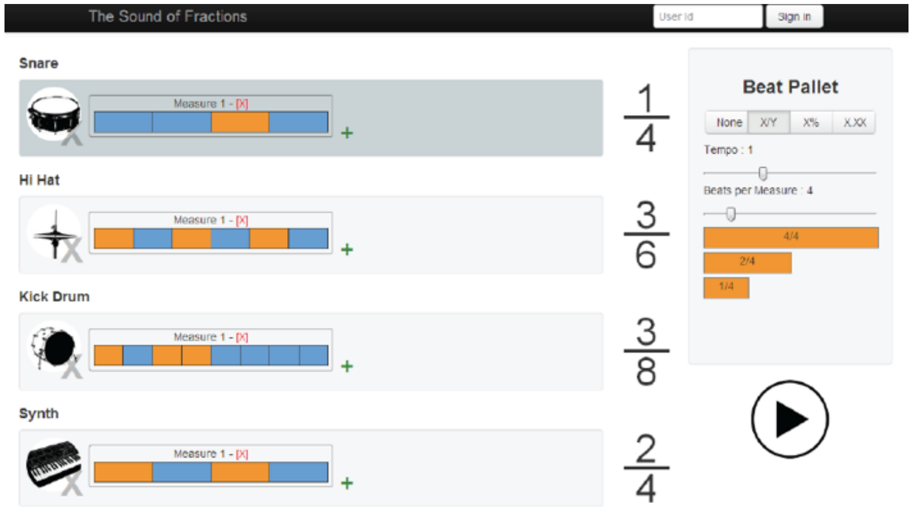


Figure 3-1 The initial prototype of the Sound of Fractions

The response from the students was overwhelmingly positive. Student engagement was non-stop. Questions were raised by students and answers were given in reply to teacher requests. This positive experience fueled our initiative to pursue meaningful curriculum, as we quickly realized we lacked a curriculum and formal structure to approach mathematics with this new tool.

3.2 Redesign and Conjectures

After implementing an initial proof of concept, we noticed several issues when testing with students, mostly surrounding the lack of any curriculum, let alone a curriculum that corresponds to the interface or goals of teaching fractions. With new insights for consideration, we sought out to redesign a new approach from the ground up.

3.2.1 Agency

In the US, typical responses to [continued] student failure by administrators are repeating the same curriculum, double time blocks of the same curriculum, or ongoing remedial classes with the same approach time after time. These approaches are common when following the Common Core. When teachers are forced to meet criteria that doesn't account for or adapt to the needs of a large group of students, then it actually marginalizes students and sets teachers up for failure. Pasi Sahlberg, a scholar and expert on Finnish education, attacks the Common Core as an experiment on children and has said many times that `accountability is what is left when responsibility is taken away from teachers' [54]. These guidelines have turned into requirements that in turn restrict learning, not expand. This has a devastating effect on the students we are educating, as they have been stripped of their agency in the environment that has been created for them. We seek to open up the learning environment and give the students affordances to exercise their agency.

3.2.2 Design

We focused the redesign on two main non-mutually-exclusive perspectives, the individual student, and group tasks. Taking these into consideration, the design manifests itself in the curriculum, the web program (its user interface (UI), and user experience (UX)), and guiding the classroom interactions.

3.2.2.1 For The Individual

Allowing the students to create their own elements of music on their own returns some agency to them. Not all activities can be an open play area for creativity, as some tasks will need to be designed to bring forward a particular mathematical or musical phenomena, but bearing the importance of play and ownership in mind during our design process allowed many of the tasks to start with rhythms and representations created by the individual students themselves.

3.2.2.2 With The Group

A number of aspects of group behavior were considered in the redesign. With groups that have shared goals or tasks, leaders will emerge, and others will follow [52]. It is important for the students' interpersonal, social, and personal skills to be in small leadership roles. It is equally important to have experienced in a non-leadership role, to hear others' thoughts, experience empathy, and have time to reflect. Experiences negotiating different roles will also help student negotiation skills.

Moving between tasks that are designed to be done individually, as a group, and as a class allow the the students to become socially aware of their peers, environment, and context; students also can reflect on their position in situ respective of their social arena. Working in groups also allows the chance to imitate one another to enhance their skills and understanding of music and math. The students will be faced with questions and challenges from their peers that they must defend or change their own concepts and approaches to account for alternative viewpoints. Teacher oversight will help guide the students towards the important phenomena that is at the heart of the task, as well as avoid the negative situations such as constant public failure, ridicule, distraction, frustration, and disengagement.

These highly complex multi-channeled communication networks will be accessible to students given their social and academic experiences and face-to-face classroom situation. This level of robust communication can not occur in distal and remote situations. As the distance increases, more technologically mediated solutions must come into play, and although some implementations maybe able to increase clarity and affordances for particular communication features, they come at a cost of impeding other channels. This exchange and loss of communication and instruction maybe not acceptable for students that are already in a hostile learning environment, and trying to focus on mathematical phenomena.

3.2.3 Brainstorming Sessions

We brainstormed for several months around the key components of fraction learning, embodied interactions, and entrainment with experts in design, curriculum, education, micro-worlds, and technology. The main ideas that came out of our brainstorming sessions concerned layout and representations which led to a scaffolded curriculum, balancing off-computer interactions and slowly increasing computer usage. This allowed increased representational complexity as the identification and description skills of the students improved.

3.2.3.1 Representations

The idea of providing multiple representations stems both from an abundance of traditional mathematical representations in current standard curricula and from the previous version of the software that had buttons for allowing each instrument's corresponding fraction to be represented in different forms ([improper]/fractions {5/3}, MIXED num/bers {1 2/3}, percentage% {166%} and de.cimal {1.66}). Upon further review, having the system auto-calculate the fractions of beats that are on out of the total number of beats available has many problems. When the computer completes these 'simple' tasks for the students, the students have less agency, are not challenged to identify what truly is important about the representation at hand, are distracted from thinking beyond the provided calculation, are reinforced that fractions 'are' easy and not 'can be' easy.

We brainstormed several designs, that were based on steam rollers (rolling out copies of the same representation), rosary bead, necklaces, paint[brushes], and other creation mechanisms. Once we had several different representations that we deepened discussion through card sorting to make sense of the mathematical properties we wanted to highlight. We began to focus on the interaction design, and it became obvious that we would also have to be aware of and potentially account for representational qualities such as size, shape, colors, and perspectives that would influence students' manipulations and understanding of mathematical perspectives.

We designed five graphical representations and three aural representations to interact with. The graphical representations include the Audio, Bead, Line, Pie, and Bar. The aural representations include the Snare, High-Hat, and Kick Drum. With these multiple representations in both graphical and aural domains, the students' would be provided with multiple ways to see, hear, feel, manipulate, experience, interact with, and study the fractional properties.

The Audio representation is a default view provided with all representations of particular percussive patterns. It is the first way that we provide a mechanism of graphically representing something that is normally tapped, heard, internally discerned, or experienced. It consists of a circular 'light' that flashes with each sounding beat.

The Bead, Line, Pie, and Bar are the remaining graphical representations, and we refer to them as the Core Four. We planned on working with these representations the most, with each representation having strengths that are unique, and some sharing certain properties that will be highlighted during certain exercises.

The Bead representation is our strongest novel contribution, with several properties thought to enhance the learning experience. It has a circular 'chord' in the background to 'hold' all the beads in an ordered group, with equal spacing of each bead. Each bead represents a beat (sounding or not), and collectively they represent a rhythm. Its circular fashion helps reiterate the idea of repetition. Its beads with appropriate separation highlight the distinct separation between the parts. Its resemblance to a necklace provides the possibility of transformation into a linear form. It highlights circular, repetitive, cyclical, segmented, divisional, scalar, ordered, grouped, prototypical, colorful, interactive, shrink-able, stretch-able, transition-able, and manipulable characteristics, with static, active, attractive, simple, and approachable visuals for students.

The Line representation, another contribution by our team, is subtly different than typical number line representations presented to students. It is comprised of an infinite number line in the background, to help reinforce the placement on a number line and the relationship to numbers. There is a thicker line that is on top of the number line that reflects the rhythm duration, with vertical hash marks for each beat (sounding or not). The Line is purposefully not labeled between two numbers (whole or otherwise), as fractions can exist between any two referents. Additionally, we provide a labeling mechanism for students.

The Pie representation, a very common form of presenting fraction problems, is presented in the similar form that students have already been exposed to previously, except that it is animatable and easily editable. The pie consist of a thicker bordered circle with equal pie slices, each colored in the same order as the other Core Four.

The Bar representation is also similar to standard representation that students have been exposed to. It has a length-wise rectangle with a thick border, and equally sized rectangles inside. For this reason, both the Pie and the Bar representations are presented last as options to represent their rhythms. They still have importance, as these representations will allow students to visualize rhythm creations in a more standard representational form of fractions, as well as a visual reference when working with multiple representations simultaneously or in stepped progression.

All graphical representations graphically and aurally distinguish between beats that are turned off (non-sounding) and beats that are on (sounding). The graphical representations also auto-space, and properly reflect equal unit fractions. Undoubtedly students will want to move beats closer together (with less space between them) to make them appear faster, but we discourage this so that the students can understand the other related parts (the division of the whole and the other beats [rests] that need to be considered) when rearranging unit fraction based rhythms, the primary structure of our application for study. All graphical components/representations (accompanied by a corresponding aural representation [with each set of graphical representations grouped by instrument type/sound]) represents each of the underlying beat patterns of the matching rhythm being interacted with.

The Core Four provide mechanisms to stretch (to lengthen the duration – sounding as if to slow down) or shrink (to shorten the duration – sounding as if it speeds up) the representation. Instances can transition between each other (discussed in 3.2.4.2 Transitions), be added (multiple times even to show consistency and abstraction) or removed, and rearranged. Each of the Core Four individual representations will also have the functionality to transition between them to show what is conserved, what is lost, and hopefully highlighting what is important when analyzing representations in a fraction or mathematical context. Additional information can be found in 4.7 User Interface (UI))

3.2.3.2 Layout

Dealing with multiple representations brought about a problem of screen real estate, and what to include and what not. While zensign (the conscious effort and decision of what not to include [24]) is just as important of what to include to help the students learn, design choices about the layout go beyond graphical elements. With our target student audience, their current limitations, and the entire learning environment that the students are being subjected to, we need to consider how these all considerations weigh on each other and in relation to and our goals of incorporating music, motion, and agency.

To help choose what should be included in the electronic interface and what should be presented over other channels, we devised some basic math questions, potential labeling mechanisms, physical material design based prototypes, and representations (as described above in 3.2.3.1 Representations) to separate the large amount of information that is presented to be scaffolded, limited, direct, cohesive, and meaningful. Using a material design approach, we cut out scaled medium-fidelity paper representations to be rearranged representing the digital screen dimensions. This allowed rapid prototyping of additional representations, and to study the intended interactions in conjunction with the curriculum, which would include off-computer activities.

When including computers and other digital tools into the environment where students would be learning, we know that it is important to include the complications that go along with such electronic devices. For our study, these complications of course include power requirements, internet usage, logging of data, Ethernet cable layout, setup and distribution, a novel learning tool and the potential for a distraction, and other numerous complications associated with electronic devices. It is also important to consider the implications of incorporating devices that are meant to help communicated between people. Emotions can be negatively impacted using computers, while discussing highly complex or novel perspectives also are a factor [36,61].

3.2.4 User Interface (UI) Components

After several brainstorming sessions (described in further detail in 3.2.3 Brainstorming Sessions guided by the principles discussed in 3.2.2 Design), several major UI components emerged: instruments, labels, and representations. Other less essential UI components helped manage the electronic interfaces, such as logging in, a reset button, the name heading, and light demarcations to help the user separate components.

3.2.4.1 Instruments

Students could select from among three instruments: Snare, High Hat, Kick Drum. All interactions start with a Snare, but can be removed from (and added back to) the instrument. We provide three instruments to allow the students to have options, take advantage of the screen real estate, and to use the multiple instruments for deeper investigation into fraction learning. Some activities allow for comparison between two representations within the same instrument [rhythm]. Others allow students to compare representations with different underlying rhythms. A few activities utilize the third instrument to make hypotheses and adjustments to their hypotheses without affecting the other two instruments, or as a reference representation to reach as a goal.

3.2.4.2 Transitions

To help students strengthen their knowledge about fractions and that fractions are abstract representations of real world things, we highlight what is conserved between representations that have the same underlying structure (i.e. 3/5 or 6/7) of relationships between the parts and the whole. We attempt to accomplish this through synchronous animations of the various components that portray our representations (as described in 4.7.4 Transitions).

3.2.4.3 Labels

With a rhythm made in SoF, the students could be interpreting it one way when the task is to discuss it in another way. To help gain insight of how the students are interpreting the graphical representation of their rhythm, we provide a mechanism to label graphical elements with by dragging and dropping single numbers (0-9) and a slash (/) so that they can label the representation in any numerical way they want, and with multiple representations, they can label the same representation multiple times, hopefully different. Labeling allows the students to begin to interact with their mental models of the part and the whole and try and concretize it in a way that is both meaningful and mathematically accurate. The ability to change their labels is also important, as they begin to interact with or gravitate towards particular ways of labeling rhythms, it may not apply to rhythms with different patterns, sizes, or representations.

To provide the best balance of options and interaction, we provide the vinculum (the bar between the numerator and the denominator in a fraction) and whole numbers 0-9, in that order. This allows students to label things as whole, order, odd, even, and more just using the whole numbers. It also allows them to construct their own fractions to count beats relative to the rhythm, beats with regards to their state (on/off), improper fractions, or other non-traditional labels and descriptions. Providing every possible prefilled labels, would cost too much screen real estate, as we would need to account for whole numbers (0-16), as well as every unit fraction (1/2 through 1-16) and fraction (1/1, 2/2, …) and (2/3, 2/4, 3/4, 2/5, 3/5, 4/5,

… [n/m while n<m and n approaches m]) for every possible rhythm, up to 16 beats, and also possibly decimals and percentages. This approach also does not allow the students to fail, for example if we provide only relevant fractions for a rhythm with three beats, the student would not have an option to label with a 4, when there may be an argument for that, especially if the student has a misconception about the system of fractions.

3.2.4.4 Technology

The students have access to a technologically current laptop, and we also supply additional computers for backups. Planning for wide-spread deployment, we developed a web application to avoid varying technology and IT related barriers at different schools and instruction locations and avoid purchasing a standard computer that may age faster than the technical development of the web application. We only have to develop for a particular web browser version and can avoid different operating systems as time progresses. Given multiple study locations and the potential for students to engage with SoF outside of school, especially in rural areas, we account for students having intermittent Internet access at the house by allowing the web page to be run in the `off-line' mode as well as capturing their data while they are disconnected and it uploads once reconnected.

3.3 Iterative Redesigns

We used our major redesign in several smaller usability studies that were often conducted during community exposure events and seeking potential study participants. Ages ranged from 5 to 14 years old, and targeted activities typically lasted five to 30 minutes. Many of these events involved walk-up use of the system with little or no instruction.

Despite a relatively wide range of age groups, some common themes were observed:

- Discussing fractions in terms of rhythm is initially difficult, but becomes easier
- Describing rhythms using math terms is easier than describing math terms in terms of rhythms
- The children had difficulty with accurate mouse movement
- The children became confused and sometimes disengaged when presented with the technological environment with no goal or priming activity
- The children need stateful feedback (such as instruction to continue a process) of the technology when performing multi stage interactions (such as 1 – adding a beat to 2- a particular representation)
- Having colors is more attractive than no colors
- Animation coupled with sound is attractive

3.4 Iterating on the Interactive Design

Using the information gathered from the small studies, we:

- adjusted the interface with larger buttons and accompanied roll-over text descriptions.

- added messages that help students when adding or transitioning a representation.
- used a consistent color scheme.
- adjusted the curriculum to better scaffold the progression of topics, with each activity helping prime the next, such as adding an instrument, and then adding a representation to that instrument.

These enhancements were beneficial; however other hurdles arose and needed to be addressed. Our prior work was based in an urban setting with support from teachers, school level and district level administrators, and funded through national grants. This strong support network came to an abrupt end when a key leader in the team passed away suddenly. This stopped progress in that particular setting in which so much of our work had been grounded. We no longer had access to students in an urban setting, or with access to a 'one laptop per child' program. This required a great shift in the recruitment and target audience.

Our academic institution, Virginia Tech, is located in Blacksburg, Virginia, US, considered a relatively rural area despite the large university's presence. Our recruitment efforts to contact relatively close participants was met with many additional hurdles. The immediate community has many protections for children given the numerous research projects stemming from the university and its affiliates. Paperwork and other administrative hurdles alone made it difficult, not including the common request to have initial promising data, something that we could not solidly provide. Considering after-school programs, private and charter schools, and other extra curricular communities also proved challenging to our research efforts. Many of the population around the university is of a median or higher SES, and thusly, the students typically are excelling in math and other subjects in and out of school. This eliminates the focus group, as students who already understand the system of fractions do not fit our target audience, and have less of a reason to entertain alternative approaches for learning 'old' material.

With these limitations, we sought to expand our community to include rural, low-SES communities, and not restrict our population to urban settings. Through a university outreach program, we were able to establish promising connections with a community 2 hours away. This community is quite rural with a generally low-SES, low education populace. This opportunity was pursued to approach our research interests, even thought the goals and approaches had to be adapted to meet the needs and challenges of the newer community.

4 THE MAIN STUDY

After several smaller individual sessions, we were poised to use the data collected to fully develop a longer study lasting several sessions. To make a meaningful educational tool and learning environment, we needed to work on developing a solid curriculum, address the technology and the hurdles identified in the earlier studies, and find students who both need the help and were able to work with us.

4.1 Motivation and Goals

Our areas of interest for this study were:

- Providing a study that provides positive impact on students in learning fractions.
- Have students experience the part(s) [beat(s)] and the whole [rhythm] at the same time.
- Study the compelling, embodied, culturally-relevant activity of drumming and making rhythms.
- Study how performing rhythms allows students to experience the wholes, the parts, and the relationships between them at the same time.
- Provide and study the inclusion of a new computer-based representational infrastructure.
- Introduce a new representational infrastructure comprised of aural, visual, physical, and temporal components.
- Provide a curriculum that scaffolds classroom-based activities that bridge the relationship between percussion-related and mathematics activities.
- Have students be able to create, identify, and manipulate [gradually] more standard mathematical representations and usages.
- Provide multiple open and accessible data sources for analysis by students, parents, administration, and academics alike.
- Leave students with skills that extend beyond our study to enable critical and computational thinking with representations and other abstract ideas in multiple domains.

4.2 Research Questions

We developed three main research questions to continually observe during our study.

4.2.1 Research Question 1: How can a micro-world that utilizes multiple representations provide learning resources to address student misconceptions?

4.2.2 Research Question 2: How can a musical approach to fraction learning help students progress in their composition, comprehension, and communication?

4.2.3 Research Question 3: How do multiple interactive culturally-relevant activities affect engagement in a learning environment?

4.3 Setting and Setup

Through a community connection with a local nearby county, we began working with a school (Group 1) and a church (Group 2) who both host afterschool programs. Both face the same challenges with rural, low-SES struggling students and communities. More detailed information about the two groups is provided in 6.1 Demographic Information.

4.3.1 Technology

To be able to run our study with the accompanying curriculum, the technology needed to be designed to fulfill many of the goals we were focusing on. First and foremost, we seek to provide an educational tool that has positive effect for students and their community. To accomplish this, we have to consider, the curriculum as an accompanying tool, the embodied nature of rhythm making, classroom management and layout, and the technological restraints of school systems. The user interface also must be considered as how it plays into the user experience as a whole with all of our goals. See 4.6 Sound of Fraction Software for a complete description of the development of the technology.

4.4 Procedure

Each week we arrived at the locations at least an hour ahead of time to:

- Setup the learning environment with personal Ethernet hardwired laptops, with a mouse and headphones
- Place notebooks, writing utensils, and other relevant materials for the day's activities
- Distribute curriculum packets to helping facilitators, and discuss and address potential issues
- Setup audio/video recording instruments, and verify technological requirements
- Collect items and organize them for the next session

4.5 Curriculum

We developed the curriculum so that students could involve themselves in an environment that fosters creation, identification, and manipulation. This approach allows for inspiration and exploration by the students themselves. To market our study to schools and administrators, we needed to account for various sized groups, different locations, and varying class and study durations. To accommodate this wider potential audience, we expanded our target audience to include all students who struggle with fractions, regardless of musical skill. Music interest was still strong within this population, and only including those who are musically talented was to restrictive. We approached the activities from a more musically-inclusive approach, and kept drumming activities approachable and interesting for all skill levels. See 5 Curriculum for a detailed description of the development of the curriculum and I.C Blank Lesson Plan and I.D Curriculum for the actual curriculum.

4.6 Sound of Fraction Software

To address the programming concerns and requirements for this study, we addressed the architecture and UI to match our intended curriculum and UX goals.

4.6.1 Architecture

To accommodate the robust requirements of multiple manipulable graphical representations and synchronized animations in a highly configurable microworld, the architecture quickly grew complex, especially as we accommodated recording of students' tapping their own rhythms. We needed to develop reusable components that took advantage of DRY (Do not Repeat Yourself) coding styles and functions that fit in a proper structure. The architecture soon moved to a structure that is analogous to a live symphony, and so many of our components are named after the matching roles or actions of an orchestra with some references to the recording industry as well. Some of the names are displayed in the UI, while many exist only in the code base, not user facing, but aids development with stronger naming conventions.

The **Stage** is where the magic happens. It has a conductor, and instruments (used and unused), and a button to reset everything back to the initial state, often used when starting a new activity, and sometimes used when a bug prevents further interaction.

The **Conductor** is represented by the Play/Stop button, as it communicates to all instruments when to play or stop. It is up to each instrument to manage its own animations and sounds.

The **Instrument** (also known as a H[orizontal]Track or Track) consists of several UI elements and an underlying matching sound. There is an icon to help students reinforce what they are interacting with, a text label, as some students may not recognize the instrument by its look and shape, and a container for all of the Measure Representations to be aligned. We call this container the Measure Representation Collection, and this is where the default Audio representation will be, along with any other representations the user adds.

The **Measure Representations** are individual representations located within a particular instrument, in the form of one of the five representation types: Audio, Bead, Line, Pie, or Bar. The Measure Representations for each instrument all share the same underlying rhythm, and have graphical components that represent the whole (via the larger circle [for the audio, bead, and pie], the line length, or the bar outline) and the parts (each individual beat via a flash, small bead, line tick, pie slice, or bar piece).

The **Whole** is the graphical representation of the entire rhythm's duration. The term measure is avoided in the UI, curriculum, and discussion, as it pertains strongly to musical notation and terminology, and can confuse students who need to focus on the whole, and not what constitutes a musical measure.

Each **Beat** is individual to its Measure Representation, but it is also the SAME beat [sound, order, and idea) across all Measure Representations of the same Instrument. This allows each corresponding beat to animate during playback or toggle (turn on or off) during interaction at the same time. In other words, the third graphical beat in one Measure Representation of a Bead Representation (the third bead) shares the same underlying sound as the third graphical beat in another Measure Representation of a Pie Representation (the third pie slice), and will both turn on or off if either is clicked, and will both move at the same time when the rhythm's order comes to the third beat unit.

The **Instrument Generator** container allows the users to add a new instrument when needed or desired, and it is automatically returned here when the user removes an instrument from the stage, providing a consistent approach to dealing with a finite number of mutable instances.

The **Stamps** container is holds all of the labels that we allow the students to drag and drop (stamp) onto the representations when trying to

The **Representations** container holds the icons of the different representations that are available. These are available to use when a user wants to Add a representation to a current instrument, or when the user wants to Transition a current Core Four representation into another.

Other general background requirements include a link to the homepage via the title, and a login button to provide more specific user tracking.

4.6.2 Application Specifics

This research has been developed for many years, and as a result, there have been many implementations for different areas of the research, mainly documentation, SoF application, and the data viewer. To support this robust approach, we used many technologies, frameworks, and libraries.

4.6.2.1 Sound of Fractions Main Application

Backend is run by:

- Ubuntu: 14.2 Virtual Machine (VM) server
- Node.js : A JavaScript server to provide easier communication between the clients, server, and database
- Upstart : Serves continuous updates and handles server crashes
- Git : Deployment of production code
- Sequelize : A node package to communicate with the database.
- JS-Cookie, Node-UUID, Crypto-JS : A node package to handle logging of users until they log in, as well as if thy lose internet connection temporarily.
- HTTPS Certificate : an SHA-256 certificate under Virginia Tech for a secure connection
- MySQL : a database for storing the users, their rhythms, and log each interaction, served on a different port of the same server
- Winston : A node package for extra verbose logging for debugging

Front end consists of:

- Framework

    - Backbone.js : manage the data structure, and document object model (DOM) layout and views
    - Underscore.js : a requirement for backbone, as well as knowing the order of the beats that students manipulate on their rhythms
- UI
    - D3.js : To draw, animate, and transition the different graphical components required for the students to interact with
    - Bootstrap : Framework for creating organized and responsive pages with structured HTML, CSS, and JS
    - JQuery (JQueryUI) : Manipulation of the different graphical components required to label the representations and manipulate the rhythms
    - Font Awesome : a text font that provides graphical icons as a selectable font
- Audio
    - HTML5 Audio – Mozilla's code to utilize the laptop audio (HTTPS required)
    - Soundcloud : a database of open source sounds to make the audio rhythms
- Browser
    - Chrome : Google's we browser that takes advantage of Mozilla's HTML5 Audio
    - Chrome DevTools : used to develop and validate SoF web application

4.6.2.2 SoF Data Viewer

- AngularJS – A framework for developing sites focused on making front end developer's work easier.

- CrossFilter – For filtering multidimensional data

- Parse – An online backend database that uses RESTful approaches

- Lightbox – for providing a photo viewer and browser

- HTML5 Video – For displaying the videos of transitions

4.6.2.3 Documentation and General Development Tools

- GitHub : a repository to mange code for redundancy and multiple developers

- Sublime Text 3 : a text editor for writing code
- OmniFocus : for making visualizations of the curriculum, data, and teaching material for developers
- Word (2016) – a word processor for writing this document
- Homebrew – A mac based package manager
- LaTeX (MacTeX) – An open source writing tool and compiler for making complex written documents
- Fake Data Generator – A Sublime Text package to generate structured seed data
- MySQL Workbench – to manage, view, and edit the database

- Gource – An open source visualization tool for look at code contributions by people over time (see I.F **Main Code Branch Development Timeline Video** for the video itself)
- Adobe Creative Suite (CS6 – Photoshop, Illustrator, InDesign) – A suite of applications for making posters, pictures, drafting, and public facing documents
- Mendeley – an online tool for managing citations
- BibDesk – a open source Mac based citation manager
- ZamZar – Word to HTML converter. Free, and AWESOME. Word 2016 fails with large documents, ZamZar delivers

4.6.3 Data Collection

The four data types (video, diaries, artwork, and logs) were collected during the study via a method suitable for each type. Classroom activities were recorded on a digital camcorder and organized into meaningful contributions from the students with loose transcriptions. The making of the physical representations (Artwork) was supplied with general crafting materials and pictures taken with accompanying notes if provided by the student. Interaction logs were stored in a backend MySQL database in sentential form with every action accompanied by the state of their rhythm and accompanied graphical representations in JSON (JavaScript Object Notation) format.

4.7 User Interface (UI)

To account for all of the different main components (as discussed in **Error! Reference source not found. Error! Reference source not found.**), the final UI came together with the play/stop button, the Stamps section to label, the graphical representations to add or transition to, the Stage area where instruments were placed, and the collection of measure representations for each instrument. Instruments not needed were held in an area below instruments that were being used. A reset button and login area were also provided. You can see many different situations that are occurring during a typical activity in Figure 4-1 .

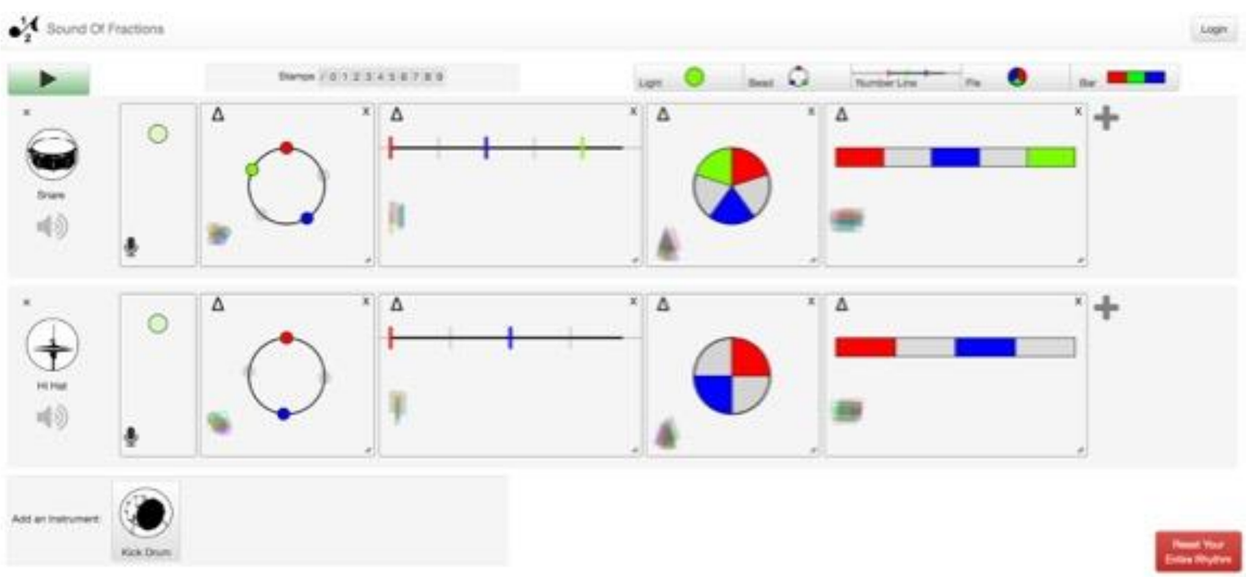


Figure 4-1 The final UI design of SoF with a Snare instrument with 5 alternating beats, a High Hat with 4 alternating beats, an unused Kick Drum, and both used instruments having all 5 representations present, currently not playing/animating.

With several smaller usability and mathematical oriented studies conducted, we chose to focus on the color, stage, representations, transitions between representations, and animation/sound of the interface to properly create an environment that

4.7.1 Color

The color choice was chosen as a result of several discussions and smaller UX studies. We are not confident that our choice is the best, nor are we confident that there is a stronger candidate, or that including color itself has the strongest gains.

4.7.1.1 Our Color Choice

Our major study included a color palette choice for beats that are ordered, and cycle through sets (arrays) of three based on red, green, and blue (RGB) variations, so that there is always distinction between the individual, with the last (blue) variation is perceptually different from the next following first (red) variation in the array, with problematic colors removed. The Hexadecimal values of our array are: 1: "#FF0000" (red), 2: "#00ff00" (bright green), 3: "#0000ff" (blue), 4: "#ff0080" (hot pink), 5: "#80ff00" (light green), 6: "#00ffff" (light blue), 7: "#660000" (maroon), 8: "#FF7722" (light orange), 9: "#0080ff" (med blue), 10: "#808000" (olive), 11: "#00ff80" (Turquoise), 12: "#ff00ff" (magenta), 13: "#EE4000" (dark orange), 14: "#FFD700" (yellow), 15: "#006600" (michaels green), 16: "#8000ff" (purple). These can be seen in Figure 4-2. Unfortunately, these colors were not chosen with accessibility as a concern, and later were identified as a problem if we were to address scalability.

Figure 4-2 The color palette of beats in order 1 through 16

The main motivation for this choice is to emphasize the order, scalar, proportionate properties, consistency, conservation, and supporting polymorphism. This ensures that during the instruction, the facilitator can both inquire and reiterate these fractional properties across multiple representations and activities. This can be accomplished through questions like

- Which comes first?
- Describe the red beat.
- Which green beat comes first?
- What color (or position) is the eighth (or light orange) beat before and after you transition the representation?
- Why are the last beats in all of your individual representations the same color?
- Why is the last beat of your representation(s) a different color from your neighbor's last beat color?

This helps us emphasize:

- The first beat is red no matter which representation.

- The red beat is the first beat no matter which representation (1/n always comes first)
- The second beat (bright green) follows the first beat (red) (2/n always comes after 1/n)
- The first beat (red) precedes the second beat (bright green) (1/n always comes before 2/n)
- The order remains the same no matter how many beats (parts) (1/n is followed by 2/n, followed by …, until n/n no what the size of n)
- The order remains the same no matter what size of the rhythm (whole) (1/n is followed by 2/n, followed by …, until n/n no what the size of n/n)

4.7.1.2 Other Color Options

We also considered to describe the colors representative of the color wheel, so that a beat in a particular angular or radial degree would always represent the same color (e.x. the color at the 3 o'clock position would always be green). This would then translate from linear progression along linear representations relative to circular representations' circumference in radians. We also considered greyscale, but the look seemed to bland and disengaging. We did include the greyscale beats to signify beats that are off. We unfortunately did not account for protanopia, deutaronopia, or tritanopia color blindness for representing the colors of the beats.

4.7.1.3 Color in Current Typical Mathematics Representations

Looking at printed textbooks available to students, standard graphical mathematical representations that are presented to elicit or inquire about fractional qualities are typically in greyscale. Some are provided in color, but all are most commonly binary with foregrounding and backgrounding color, no matter how many divisions or parts are being presented. This supports the incorrect notion that there are only two ways to consider a group. Our labeling system supports several ways to identify the components of a fractions and whole.

4.7.2 Stage

We limit the number of instruments to three, as it is an optimal number that provides the following:

- Maximum usage of the vertical real estate available on a typical laptop resolution and aspect ratio
- Distinct sounds that are perceivable by students regardless of their musical skills
- An excellent number to pose mathematical problems in an abstract form, such as a + b = c, a(b) = c, a>b>c, and other equations (in different forms such as word, sound, and static and animated visuals) that are important for a strong understanding of the system of fractions

4.7.3 Representations

We have developed five different graphical representations [Audio, Bead, Line, Pie, Bar] to elicit particular mathematical and computational properties [68] for a musical rhythm. All representations have unique properties, but the Audio is different from the other four [Bead, Line, Pie, Bar], as it focuses on representing the temporal and cognitive idea of the rhythm, not focusing on graphical elements. We call the other four the `Core Four'.

All representations can be presented on screen for a particular instrument [Snare, High-Hat, or Kick Drum], and the representations are linked, meaning that they each represent the **same** rhythmic pattern recorded for that instrument. If the underlying rhythm is changed, all representations will update. This can happen when a new rhythm is recorded, a beat is added to or removed from the rhythm, a beat is toggled between playing (sounding) or when resting beats (not sounding) occur.

All representations can be removed from the instrument by clicking the minus (-) sign in the upper right, allowing students to create arrangements of representations with particular representations juxtaposed next to other. The Audio representation has a recording button (finger tapping logo) in the lower left to allow the student to record over the current rhythm for a particular instrument. The Core Four have a pile of remaining beats in the lower left, allowing the students to add beats to their rhythm; the Audio rhythm is spatial-temporal based, and therefore can't have beats to grab from, making it the candidate for the recording button, as a button that represents the translation from the temporal and cognitive rhythm in their head to the computer.

The Core Four also have the ability to stretch or shrink, changing the playback length, which gives the effect of speeding up or slowing down the tempo of the rhythm pattern. The handle to adjust the size (limited to horizontal manipulation for the one dimension [linear] representations [Line and Bar] and limited to equal linked horizontal and vertical manipulation for the two dimension [circular] representations [Bead and Pie]) is in the lower right. The stretch or shrink manipulation waits until the student is done dragging to adjust all other representations to the same final size. This is the only manipulation that does not propagate to the other Core Four representations in real time, because if they did, and had at least one Core Four representation to the left, that representation would grow simultaneously, causing the manipulated representation to shift.

4.7.3.1 Audio Representation

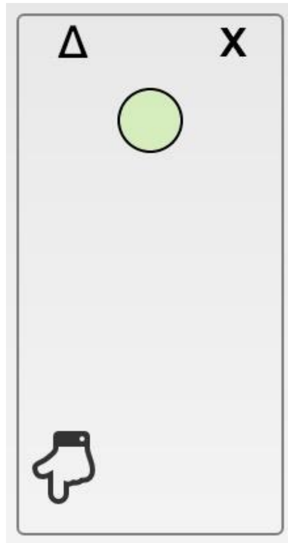


Figure 4-3 The Audio representation present in SoF

The spatial-temporal component of a rhythm should not be ignored, as a beat that resides solely in a cognitive state can not and will not be communicated for others. Children who are exposed to music and music education show improvement in spatial temporal reasoning skills in fractions [22]. We provided this visual metronome that is linked to the performance of any beat as a way to aid the student in seeing and feeling the music as a real time performance by the computer. It also can serve as a feedback notification when tapping the beats on the table near the computer, as microphone settings or the distance to the computer may need adjustment to ensure proper recording.

The Audio representation, always at least one present in every instrument, also serves as the place where you can record your own representation by clicking on the hand facing down, as if it were tapping against a desk. This allows students to record their rhythm just by tapping it, and stopping. The system recognizes the first beat, and as the user is tapping, audio processing begins processing for the average beats per minute (BPM) and tempo. When the system recognizes that there have been no more taps for three cycles using the average BPM and tempo of the rhythm up to that point, it automatically stops recording, and draws the instruments' representations based on the rhythms structure. This allows students to experience their music without being encumbered from technological requirements. They just enjoy making music , whether just creating music or having a particular rhythmic goal.

4.7.3.2 Bead Representation

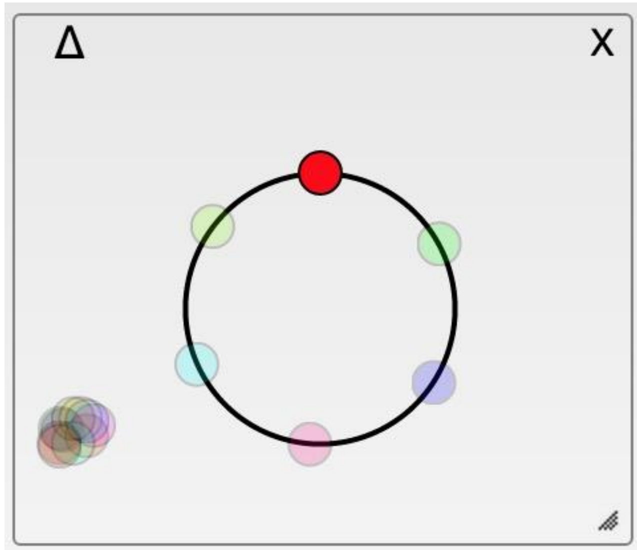


Figure 4-4 The Bead representation present in SoF

This is a particularly novel contribution. Modeled from necklaces, bead making, and the rosary beads during our ideation design sessions, it has many features that are attractive and informative. Beads [and by virtue beats] can be added to or removed from the circle [whole], completely changing the properties of whole circle. Simply increasing or decreasing the count by one drastically makes the mathematical properties quite different. Even if a student taps an easy pleasing pattern in halves, fourths, unlikely sixths, or pushing the system and going to the full limitation of 16 beats, a single manipulation will enter them into much harder musical patterns and mathematical concepts of a whole, thirds, fifths, sevenths, or fifteenths.

4.7.3.3 Line Representation

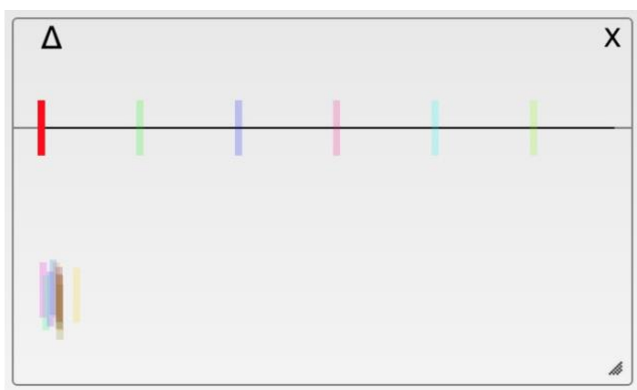


Figure 4-5 The Line representation present in SoF

The Line representation is the first in the selection of representations that the students will notice as familiar, but not totally the same. We do not call it a number line as there are no numbers, as the student can make the determination themselves where to place the reference points. This will be illustrated by taking a straw and threading it over a taught string tied between to points, with the straw having delimiting marks but no labels. The string will have marks evenly spaced at the length of the straw, allowing the students to move the straw along the string and choosing the reference points.

4.7.3.4 Pie Representation

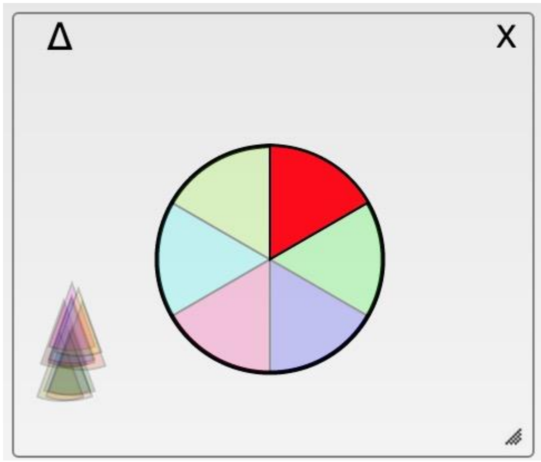


Figure 4-6 The Pie representation present in SoF

The Pie representation, the fourth button available to the students to choose, is the first fully recognizable representation, as the pie shape is the most prevalent representation used to teach fractions, as it can be referenced as a food (e.g. a pizza, a circular fruit, a cookie, etc...), a geometrical circle, or any other circular object that could be feasibly split (e.g. a clock, a bag of marbles, coins/tokens, etc...).

4.7.3.5 Bar Representation

Figure 4-7 The Bar representation present in SoF

The Bar representation is another fully recognizable form factor that students in the US will have already been exposed to extensively in math curriculum and class. This representational for easily maps to word problems that involve candy bars, sticks or rods, and certain distance problems.

4.7.4 Transitions

The Core Four also can be transitioned from one representation to another in a graphical transition that shows the important stages highlighting what is conserved and what is lost, and strengthening the understanding of what each representation has to offer. The idea of conservation originates from Piaget's conservation experiment, in particular Grouping VII, which deals with equilibrium [equivalence] (what is the same), combinativity (the concept of things as combined group rather than individual pieces), associativity (order doesn't matter in a group of equal parts), identity (a unique part), and reversibility (a part has an inverse) [46]. We provide mechanisms to highlight conservation during the graphical animations of the representations, which have clear delineated stages to show *transformation*, a graphical transition where no forefront properties are added or removed, only the shapes are deformed [a circle is cut and unrolled into a line]), *replacement*, where one form of a beat (a bead circle) is replaced by another form (a number line tick), *removal*, when an important feature is removed (a graphical element representing a division is removed), *addition*, when a new important feature is included in the representation (when a graphical element representing duration is added).

Upon reviewing the transitions, we have come to realize that each of the Core Four offer different yet important characteristics. As previously stated, the Line and Bar share linear characteristics, while the Bead and Pie share circular characteristics. Two other characteristics the Core Four afford are duration and division. The Pie and Bar representations highlight the duration of each beat, where the eye is drawn to the size of each beat compared to the whole, and the divisional lines are minimized graphically. The Bead and Line representations highlight the divisions in the whole rhythm pattern, as the spacing between each beat is emphasized more strongly than the beat, drawing the student to see the separation as more important.

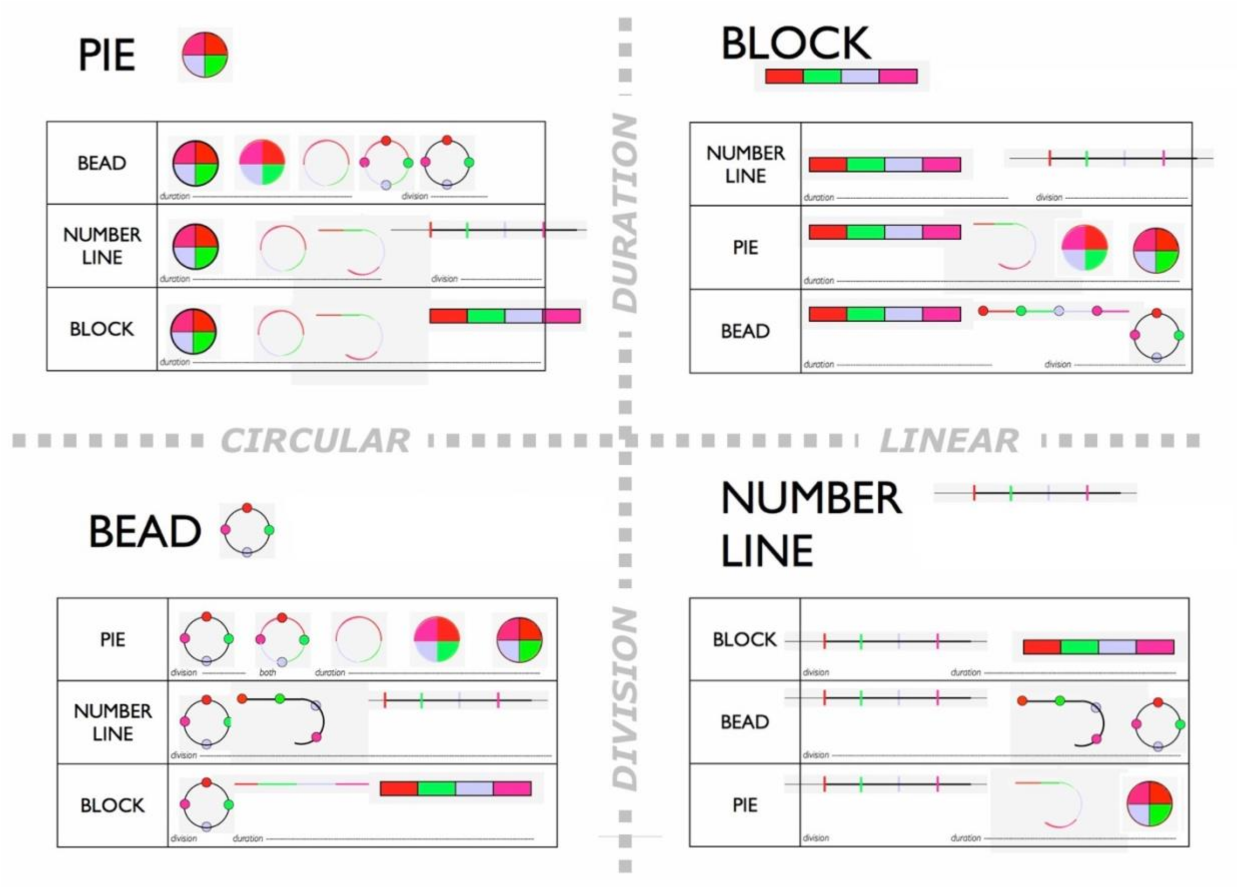


Figure 4-8 A comparison of the representations and their transitions with relative properties

To our knowledge, these animations and arrangements are also a novel contribution, particularly within the D3 (Data Driven Documents JavaScript) community. To accomplish the animation of a circle unrolling into a line, we make a path (consisting of a series of coordinates) of a line, and add a path of a circle that moves horizontally along path of the line over several stages (each stage representing a frame in the animation). We then combine the first part of the line with the remaining front part of the circle to achieve the needed coordinates for the path to appear unrolling. If you were to consider the circle (known to be a geometrical shape with an infinite number of sides) to have less sides, such as eight, the stages will look like Figure 4-9:

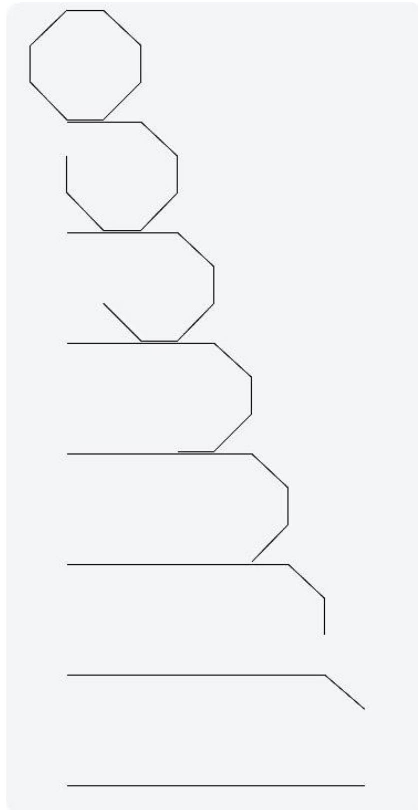


Figure 4-9 Unrolling a circle into a line with fewer frames and less reference points.

To accommodate both the different (generally not strong) computational capacity of various computers used during the study and the different number of beats that are on a representation (trying to get even degrees of separation around the circle with one to sixteen beats), we dynamically generate the path to provide the best possible combination of the highest number of frames (between 39 and 48 [calculated by the number of beats that divides equally into a number closest to 40 without exceeding 50]) and the number of coordinates along the path (approximately 120) to provide the highest fidelity circle with the most fluid simultaneous transition animation of the path AND the beats along the path.

4.7.5 Beats and Rests

During exploration of parts and wholes, it is inherent that the problems begin around the parts that make up the whole, and interpretation varies. Students struggle to see beyond the homogeneous whole. In traditional curricula, the majority of mathematical problems distinguish the parts in a binary manner (i.e. `haves' and `have nots'). Thus, our implementation has to discern any differences we choose to support. We decided to also maintain binary parts, and thus we support the `haves' as *beats*, a graphical representation whose shape is based on the representational type of the rhythm (e.g. a Bead beat is a small bead as in a necklace, a Pie beat is a small pie slice) with full opacity with an accompanying sonification of a particular instrument for a particular amount of time, and the `have nots' as *rests*, the same graphical representation size and shape of a counterpart beat with partial opacity and an accompanying perceived amount of time where no sound is produced. For this paper it should be noted that in our discussion, unless otherwise noted, we refer to BOTH beats and rests as a beat or beats.

4.7.6 Playback

Playback is quite important for the students. A play and stop button allow for repeated sonification and graphical animation from the beginning, as well as visual and aural cues about the mathematical and musical properties of the rhythm they have physically create using their hands and recorded or using the interface to arrange beats into a rhythm representative of their cognitive rhythm. During playback of the rhythm, the beat representations animate in ordered fashion, the first beat in each representation moves respective of its whole measure and returns to the original position during the time allotted for the individual beat or rest, followed by the second, and so on. This ordered animation supports the idea that the graphical beat occupies a specific amount of temporal space noted by the time it plays, a physical space noted by the physical graphical space it occupies, as well as having an auditory landscape of a particular instrument represented by its sound.

5 CURRICULUM

Our first implementation to aid fraction learning for middle schoolers with drumming from a computational thinking perspective shined light on a very important area for improvement, the accompanying curriculum. With a better understanding of the previous research conducted (see 2 Literature Review) and several targeted design sessions (see 3 Our Prior Work), we aimed at addressing the educational needs of the students while considering the culturally and socially relevant perspectives they own.

5.1 Education Requirements

Quality education has many qualities. To begin to approach teachers, administrators, and school systems, we needed to consider the external requirements imposed by these roles and their individual and collective goals. Many of the people in these roles share the same overarching goals, but how they interact, participate, and execute to achieve these goals separates them. In order to communicate to each person what work we have done and how it relates to their goals, we studied the different standards proposed by the states that we were pursuing to run the studies in, as well as a common national standard. We then cross referenced each curriculum requirement to identify which areas of learning our curriculum targeted. We pursued schools, after school programs, and community programs in the state of Virginia (southern United States [US]) which use the Virginia Standards of Learning (SOLs).

5.1.1 Virginia (VA) Standards of Learning (SOLs)

The VA SOLs are the state-wide educational requirements students are expected to learn by leaving each grade level (Kindergarten-12) until graduation, specific to each major subject matter (English, Fine Arts, History, Math, Science). We focused on the math SOL requirements, and became familiar with the music related fine arts sections. Each grade level has major themes that are intertwined through the students' academic career, and some themes have several component measures.

The state of Virginia tests students at the end of the year with statewide standardized tests that are designed to test students' comprehension of all themes and many if not all of the components. The structure to identify themes and components within the Math SOLs are Grade Level – Theme – Component – Description. So for example, ``4.2.b Identify equivalent fractions'' means in the Fourth grade (aged ~9 years old), the second theme has several components, the second one of which is to "identify equivalent fractions". We used each of these identifiers to mark each of our own curriculum activities with the corresponding VA SOLs, and can be seen on the top of each activity in 5.6 Lesson Plans.

The following themes and components from the VA SOLs were identified and incorporated with our curriculum and lesson plans to administer the study and teach with SoF:

Grade 2

- 2.3 Identify Fractions as real numbers
    1. identify the parts of a set and/or region that represent fractions for halves, thirds, fourths, sixths, eighths, and tenths
    2. write the fractions
    3. compare the unit fractions for halves, thirds, fourths, sixths, eighths, and tenths
- 2.20 Identify and create patterns
    - The student will identify, create, and extend a wide variety of patterns

Grade 3

- 3.7 Add/Subtract Like denominators
    - The student will add and subtract proper fractions having like denominators of 12 or less
- 3.19 Extend Pattern Representations
    - The student will recognize and describe a variety of patterns formed using numbers, tables, and pictures, and extend the patterns, using the same or different forms

Grade 4

- 4.2.a Compare and order fractions
    - compare and order fractions and mixed numbers
- 4.2.b Identify Equivalent Fractions
    - represent equivalent fractions
    - identify the division statement that represents a fraction

- 4.5 Factors and Simplification
    - determine common multiples and factors, including least common multiple and greatest common factor
    - add and subtract fractions having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors
    - solve single-step and multi-step practical problems involving addition and subtraction with fractions and with decimals
- 4.12 Introduction to Polygons
    - define polygon
    - identify polygons with 10 or fewer sides

Grade 5

- 5.2 Equivalence and order of fractions
    - recognize and name fractions in their equivalent decimal form and vice versa
    - compare and order fractions and decimals in a given set from least to greatest and greatest to least
- 5.6 Fractions and mixed Numbers
    - Solve single-step and multi-step practical problems involving addition and subtraction with fractions and mixed numbers and express answers in simplest form

Grade 6

- 6.2 Order, Equivalence, and representation form of fractions
    - investigate and describe fractions, decimals and percents as ratios
    - identify a given fraction, decimal or percent from a representation
    - Demonstrate equivalent relationships among fractions, decimals, and percents
    - Compare and order fractions, decimals, and percents
- 6.4 Representations of multiplication and division
    - The student will demonstrate multiple representations of multiplication and division of fractions
- 6.6 Mixed Number operations
    - Multiply and divide fractions and mixed numbers

Grade 7

- 7.12 Making Representations
    - The student will represent relationships with tables, graphs, rules, and words

Grade 8

- 8.14 Identify relationships between representations

- The student will make connections between any two representations (tables, graphs, words, and rules) of a given relationship

5.1.2 Common Core

The Common Core (discussed in further detail in 2.1 Common Themes in Mathematical Instruction) is a set of guidelines that is guided by national research and sponsored by the US federal government, and adopted individually by most states. Each state also manages how students are assessed against these guidelines, mostly by statewide standardized tests.

The following guidelines from the Common Core were identified and incorporated with our curriculum and lesson plans to administer the study and teach with SoF:

Grade 1

- 1.MD.C.4
  - Represent and Interpret Data
  - Categorize data and compare categories in terms of more/less and total/whole
- 1.MD.A.1
  - Measure Lengths indirectly and by iterating length units
  - Order 3 objects by length, using a third object as reference

Grade 3

- 3.NF.A.1
  - Develop Understanding of fractions as Numbers
  - Understand 1/b as 1 part of one whole partitioned into `b' equal parts
  - a/b means `a' parts of size `a/b'
- 3.NF.A.3
  - Explain equivalence of fractions and compare fractions by reasoning about their size
  - Explain equivalence and comparison of fractions by size
    1. a) Two fractions are equivalent if they have the same size
    2. b) recognize and generate simple equivalent fractions
    3. c)recognize whole numbers as fractions
    4. d) Compare 2 fractions with the same numerator or denominator with <, >, =
    5. Recognize comparisons only valid when wholes are the same

Grade 4

- 4.NF.A.1
  - Extend Understanding of Fraction Equivalence and Ordering
  - Understand a/b is equivalent to same fraction multiplied by a constant number
- 4.NF.A.2

    - Compare two fractions with >, <, = with different numerator and/or denominator
    - Recognize comparisons only valid when wholes are the same
- 4.NF.B.3
    - Build Fractions from Unit Fraction
    - a/b understand as `a’ sum of `1/b’ fractions
    - understand addition and subtraction of fractions as joining and separating parts of the same whole
    - take apart a fraction that is not a unit fraction into a sum of fractions with the same denominator in more than one way
    - add and subtract mixed numbers by converting into their equivalent fraction
    - add and subtract fractions having the same denominator by utilizing visual fractions and equations
- 4.NF.B.4 - Multiplying a fraction by a whole number
    - understand a/b as `a’ multiple of 1/b
    - whole number multiplied by 1/b is a/b
    - solve problems with multiplying a fraction by a whole numbers by using visual models and equations
- 4.MD.B.4 - Represent and interpret Data
    - Create a line plot to display unit fractions, and use this for addition and subtraction

Grade 5

- 5.NF.A.1 - Use Equivalent fractions as a strategy to add and subtract fractions
    - Add and subtract fractions with unlike denominators
- 5.NF.A.2 - Solve word problems with addition and subtraction
    - Include fractions with unlike denominators by using visual models and equations
- 5.NF.B.3
    - Apply and extend previous understandings of multiplication and division
    - Interpret a fraction as a division of the numerator by denominator
- 5.NF.B.4 - Multiplying a fraction by a whole number or fraction
    - interpret a/b x `q’ as ‘a’ parts of a part of `q’ into `b’ equal parts
- 5.NF.B.F.B
    - Multiplying a number by a fraction greater than 1 creates a product greater than the given number
- 5.NF.B.6
    - Solve problems with multiplication of fractions and mixed numbers by using fraction models and equations
- 5.NF.B.7 - Divide unit fractions by whole numbers and whole numbers by unit fractions
    - division of unit fraction by a non-zero whole number
    - division of whole number by unit fraction
    - solve problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions

- 5.MD.B.2 - Represent and interpret Data
    - Create line plot of unit fractions and use this information and fraction operations to solve problems using the data presented

5.2 Goals and Tasks

As we designed the curriculum, we wanted to take advantage of all of the previous research (discussed further in 2 Literature Review). Some critical goals were:

- The experience of embodied interactions via tapping and drumming with other students and SoF
- Open ended questions and environments where the students can explore and expand their thoughts
- Group discussion, with voting, individual hypotheses, experimenting, and discussion about hypotheses
- A scaffolded yet separate approach to allow students to make gains from previous experience to make new knowledge
- Avoiding music terminology (quarter note, whole note, etc..) and other music properties (pitch, timbre, A through G, tempo or rate), as they conflict with other goals
- Guide the students towards what is important all why asking students to identify what is important themselves

While considering these goals, we started to identify certain tasks and activities that would address particular properties of the system of fractions while also fulfilling themes, components, and guidelines provided by the SOLs and CC. Some of the major tasks we wanted the students to participate in were:

- Making their own physical representation of their own rhythm, and try and learn what is important to communicate to others.
- Tapping and recording their own rhythms, and seeing if they could recognize their own rhythms when played back.
- Getting them to identify which beat comes first when presented with different information.
- Writing down their own predictions, then interacting with SoF and discussing the outcomes.
- Moving between (transition, visualize, conceptualize, and describe) representations to see what is conserved, what changes, and what is important.
- Interacting with the part (beats) and the whole (rhythm) at the same time.

These tasks helped us finalize each activity that is present in 5.6 Lesson Plans, but needed to be arranged in a meaningful order (as described in 5.5 Organization and Order).

5.3 Stages

We have designed our curriculum into three stages: (1) Creation, (2) Identification, and (3) Manipulation. Each stage is carefully structured to properly scaffold the curriculum based on mathematical progressions, leverage the musical skills of the students both individually and collaboratively, present the students with meaningful musical questions that highlight the mathematical properties needed to understand the system of fractions, and to provide a fun and engaging ecosystem that encourages and rewards the students throughout the assignments.

Our lab's successful history with educational curriculum, computational thinking approaches, and computer-mediated learning tools [4,36,61] provides us a solid starting point to start our curriculum from a complete hands-on approach, one where computers are not used. This allows for an environment that is familiar, and grounded in action while avoiding distractions, technological problems, and other unwanted situations [36,61]. Since the study and accompanying curriculum is a novel and foreign situation to students, we especially try to focus on the concepts we want to stress throughout the entire curriculum, and want to capture and maintain the novelty to leverage throughout the study sessions.

Each stage will also be somewhat recursively nested, that is the Creation stage will of course have activities that highlight creativity, but they will also have smaller activities or questions that ask students to identify and manipulate their creation. Likewise, the Identification stage will predominately ask students to use association skills (ex mapping a pie representation to a fraction, and also to a bead representation), but will do so on creations of theirs before and after basic manipulations. The Manipulation stage will focus on different process the students can do to their creations, once they have identified what is important. For detailed information about the lesson plans, please review I.D Curriculum.

5.3.1 Stage 1 : Creation

To help students learn fractions without them realizing it initially, we structured the tasks in stages that help the students enter into a creative mode that acknowledges their musical skills and doesn't focus on their mathematical skills.

Math is about patterns. We engage the students in a number of activities that focus on noticing and describing patterns. Most of our activities are about the underpinnings of fractions, but some of them are about other kinds of patterns. The overarching goals of Stage 1 are to have students (1) gain familiarity with description terminology for rhythms(s), (2) maintain engagement, and (3) have an embodied experience of parts and wholes at the same time.

5.3.1.1 Curriculum & Tasks

Day 1 – What are Fractions

We started simply by asking students to write their own definition(s) of what fractions are in their notebooks.

Task 1 – What Do Fractions Mean to You?

We inquired about what fractions meant to them, and how the relate to them, conceptually and on a daily basis.

Task 2 – Listening to a Rhythm

To introduce the idea of music as the central starting point of the curriculum, we began by playing popular rhythms recognizable by students of their age group, with any lyrics removed, such as small snippets of Taylor Swift's Shake It Off [60] and Queen's We Will Rock You[50]. We ask the students to describe the rhythm aloud to the teacher, allowing all students to hear the main words, concepts, and descriptions of their peers.

Task 3 – Make Your Own Rhythm and Representation

The teacher will then distribute sets of supplies including pens, pencils, paper, play putty, pipe cleaners, tongue depressors, tape, and other fast prototypical materials to each student. Each student will then individually tap a rhythm that they themselves have created, and make a physical representation using the supplies that resembles the rhythm they created. The students will then exchange their physical representations, and their partner will try to tap the rhythm from purely from the visual information presented by the physical representation. The creating student will try and listen to how their rhythm is played by other students in their group. The challenge for students is that their rhythms will likely not be played by their group members the way that the original creating student intended. They can repeat this task as many times as the class time allows. The group discussions will be focused on identifying what is important and how to strengthen their representations to communicate their rhythms nonverbally to elicit as close to a musical performance of the rhythm as possible.

After discussion, we will then ask students to come up with a new rhythm and a modified representation they will then again exchange. A whole-class discussion will investigate what students discovered about the challenges of getting another person to play what they wanted. We want the students to experience the varied representations that people have of rhythms, and that key common vocabulary exist to describe particular features of musical rhythm, which hopefully they retain and can provide a stronger foundation to make math analogies later. We will ask the students to write down the key vocabulary words they learn for future reference in addition to strengthening their remembering of the description [14] in a diary that will be collected by the teacher in between sessions. If students try and share `traditional musical' representations that come from standard musical education, we will encourage them to make additional representations that differ substantially.

Day 2 – Tapping Along

Task 1 – Describe What is Important

We asked students to describe what is important about fractions, and to help the get in the mindset of learning about them, as they have transitioned from other school activities and environments.

Task 2 – Mimicking Beats

Another task will be to ask one student in a group to create a rhythm, and then have all the members of the group to mimic the beat being tapped. We then asked questions surrounding what it means to be the same, and identify things that are different.

Task 3 – What Size!?

For this task we pose questions and scenarios that force students to consider the parts more discretely.

Task 4 – Make Your Own Representations

We then ask students to make their own representations of their music again, and see what has changed, and why.

Task 5 – Play with SoF

We then Invite the students to just play with the SoF program, and we discuss their findings briefly.

The tasks that are posed to the students in the creation stage focus on the students beginning to use their own representations, vocabulary, definitions, and descriptions of processes of fraction concepts in the context of rhythm, record their findings, and discuss and defend their concepts with peers. This prepares them for the next stage with a stronger collaboratively produced and academically guided concept of fractions.

5.3.2 Stage 2 : Identification

The students are now challenged with tasks that may be easy to perform musically, depending on their current skills, but will challenge them to describe the key features and process in mathematical terms they identified in the previous stage as well as mathematical concepts that they may not have been able to comprehend previously. Also, tasks that utilize SoF challenge students to use digital representations of their music that more closely resemble classical mathematical representations.

Day 3 – Making music and Representations with SoF

Task 1 – What are Part(s) and Whole(s)?

Our questions and activities challenge students to identify what a part and what a whole is both conceptually and applied to the new graphical domain of SoF.

Task 2 – Make a Bead Representation

This is the first activity to prompt students to create a digital representation using SoF. It is a notable moment, as it is similar enough to begin discussion, but also novel to allow for discovery.

Task 3 – Record Your Rhythm, Do You Hear What You Tapped?

To ensure that SoF is working properly, and that students can recognize what they are making, we test their tapping and recognition skills.

Task 4 – Complementary Rhythms

We then asked students to make a rhythm that complements their tapped rhythm in another instrument, priming them to understand a whole in a new context of music.

Task 5 – What is a Whole?

We asked the students to identify a whole in SoF and defend their answer.

Day 4 – Comparing Rhythms

Task 1 – Compare a Small and a Large Item

With several created rhythms and discussion of wholes and parts, we challenged students to compare them for the first time, priming them for more mathematical analysis later.

Task 2 – Compare all the Representation Types

With growing usage in SoF, we challenged the analytical skills of students to identify what is the same and what is different across all of the representations types of the same rhythm and instrument.

Task 3 – Complementary Rhythms (distributed) and Playing Twice as Long

Students will begin to create rhythms that are twice as long, and compare them.

Task 4 – Which Comes First

To help students understand the inverse properties of fractions, we provided different rhythms and asked them to ask which beat will come first between them, and then play the rhythms to verify their hypothesis.

5.3.2.1 Curriculum & Tasks

To help segue to the new stage, we will start with a pigtailed task that allows students to play around with the interface to get acclimated with the interface and simply play. Many students will be able to pick up on the majority of the functionality during playing around, but each directed task will provide detailed instructions for task completion throughout the stage. This will involve individual tasks such as adding an instrument, discussion on how to add a representation to one instrument, and breaking down the instructions into manageable chunks. Students will record be able to record their rhythm and perform their rhythm by tapping on the desk. Students will then be asked to describe and manipulate their rhythms, and to create complementary rhythms with additional instruments. Many activities involve switching roles, such as from producing music to describing music, to help focus on both parts of the whole, what is theirs, and what is not. Just as in the first stage, some discussion will take place in person, and some hypotheses and descriptions will be written in their notebooks.

Other scaffolded activities include carefully structured problems that challenge students to identify the particular attributes of the beats (individually and collectively, both on and off), the different patterns for each instrument, and the rhythm as a whole. Guided discussion alongside the scaffolded activities will allow them to see alternate descriptions of the fractional components of the rhythms, as well as the preparing them to change the rhythms with greater accuracy in future dynamic activities. Strengthening students' common vocabulary allows for them to investigate the changing situations that come along with manipulating their rhythms in the next stage.

5.3.3 Stage 3 : Manipulation

In our proposed new curriculum, students tapped individually and collectively in a group by themselves with increasing amounts of instruction, and then progressed to activities that involved guided identification of specific mathematical properties. Now the students will move into a technologically mediated environment that challenges them to use their robust musical skills to grow and defend more mathematical language by manipulating multiple visual and aural representations that represent their musical performances, including recording of their tapping.

Upon completing the final stage and thus all activities and the entire study, students will gain a stronger ability to create, identify and manipulate their own representations, those created by others, and more traditional representations that come up in various contexts and scenarios. The third stage helps with this by focusing on moving between representations, describing the important attributes that surround the parts and whole in situ (not sure if this is clear, I meant the contexts that the fractions exist within), recalling situations with similar problem spaces, and considering multiple perspectives of the parts and whole.

5.3.3.1 Curriculum & Tasks

Day 5 – Splitting a Rhythm

Task 1 – Comparing Two Objects (small and large)

Students will begin to manipulate their rhythms by modifying the duration by stretching or shrinking the graphical representations to match an aural representation challenge, such as playing it twice as fast (or half as slow) or half as fast (or twice as slow).

Task 2 – Comparing Individual recorded rhythms with a (SoF supplied) rhythm using beads and lines

Limiting students to just the bead and the line, we inquired what

Task 3 – Sharing a Rhythm

Another task will be to ask one student in a group to create a rhythm, and then have all the members of the group to split the rhythm so that each person is responsible for an equal part. We will then ask the students what 'equal' is, as there are multiple ways to answer this within the context of a musical rhythm. Further questioning will happen to ask the students how they divided the rhythm equally, and have them write down their process in their diary.

Day 6 – Transitioning and Labeling

Task 1 – Complementary Rhythms (non-distributed tapping) and in-depth complementary Fractions

This task will be to ask each student to create a rhythm for one instrument, and then create a rhythm with another instrument that complements their first rhythm. We then asked students what 'equal' is, and what how to know what is complementary. We also asked what happens when you put both together.

Task 2 – Transition a Representation

For this activity, students can transition a Core Four representation into another, and then take notes about what they see, focusing on what stays the same, and what changes.

Task 3 – Label a Rhythm

The final activity allows students to label their rhythms with the supplied stamps (0-9 and /) to describe their rhythm. We challenged them to label their rhythm a different way, and justify why they labeled it a particular way. *To acknowledge what is conserved and what is not,* students will also be challenged to label their graphical SoF representations using the label facility provided with SoF. This allows students to label what the see as important, which may be whole numbers, counting strategies, and stronger number sense especially in the context of a fraction system.

5.4 Do, Ask, Takeaway

In order to plan transportation, purchase and organize materials, delegate responsibilities, identify important goals, provide important questions to ask, and provide for on the fly changes, we designed a lesson plan structure that follows the Do, Ask, Takeaway strategy. Each lesson plan begins by asking the students to 'Do' some task, sometimes accompanied by instruction. After performing the tasks, the students are then posed questions that are designed to target particular attributes, ideas, features, or mental processes that align with the activity and goal. Both the 'Do' actions and the 'Ask' goals are oriented to get the students to 'Takeaway' a particular idea after completing the activity.

A blank activity sheet in Figure 5-1 (originally done by hand as shown in Figure 5-2) that the activities are all structured around has a title section for the name of the activity, along with subheadings for the associated SOL themes and components, the location, day that the activity occurs on, the number of the activity it occurs in relation to the other activities for that day, and the projected duration of the activity. The left side has an area to include all materials needed for the particular activity, the setup instructions for that activity, any facilitator notes (and an area for the facilitator to take notes during the activity). The main section is divided up into three sections of Do, Ask, and Takeaway. Some activities require two or three instances of the Do activity, so there is an appropriate number of subsections if the activities or questions differ slightly based on which iteration is occurring.

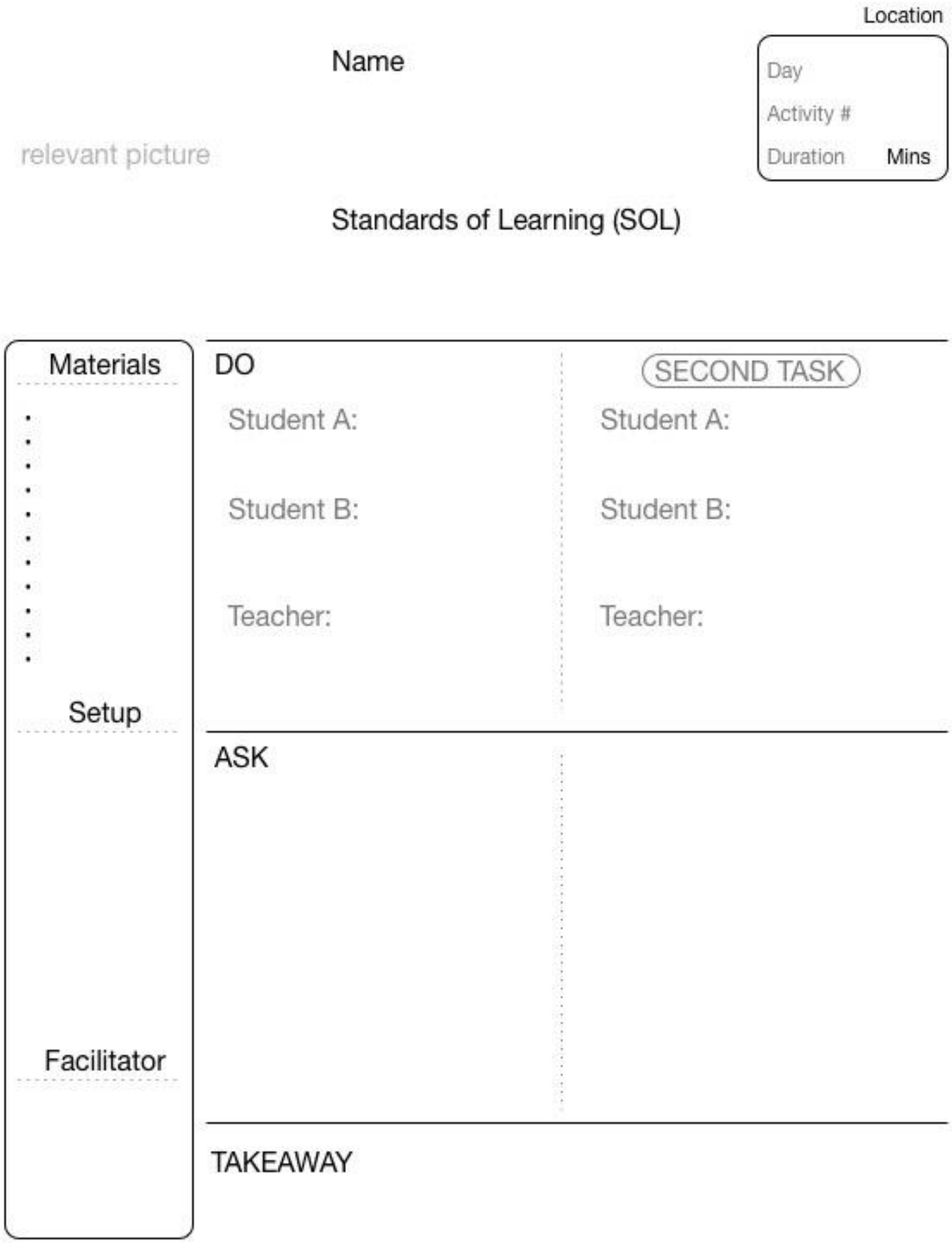


Figure 5-1 A Blank Activity sheet that follows the Do, Ask, Takeaway model. It has areas for materials, setup, notes for the facilitator, location, day, activity number, projected duration, a, the name of the activity, and the associated SOL themes and components

5.5 Organization and Order

Using our goals and ideas of tasks (described by 5.2 Goals and Tasks) as a guide to elaborate on our activities and three stage structure (described in 5.3 Stages), we needed to work through designing each activity and the order to properly scaffold the students' learning progressions.

5.5.1 Cut ups, Card Sorting, and David Bowie

Matching appropriate lessons to our available study participants (number of students, groups, locations, and resources available) is not an easy task. It can not simply be made a rigid structure, nor a top-down approach to solving the structured order. It also can not be a loose approach or a bottom-up approach to design the scenarios. The rapid changing situations of schools, administrators, and after school programs requires flexibility, but this characteristic can not affect the students in a negative manner.

To provide the most flexibility for real time adjustments while adhering to the three stage structure of our curriculum, we made individual papers of each activity and [re]arranged them to categorize and order them. This also allowed us to adjust them by splitting some activities into multiple activities,

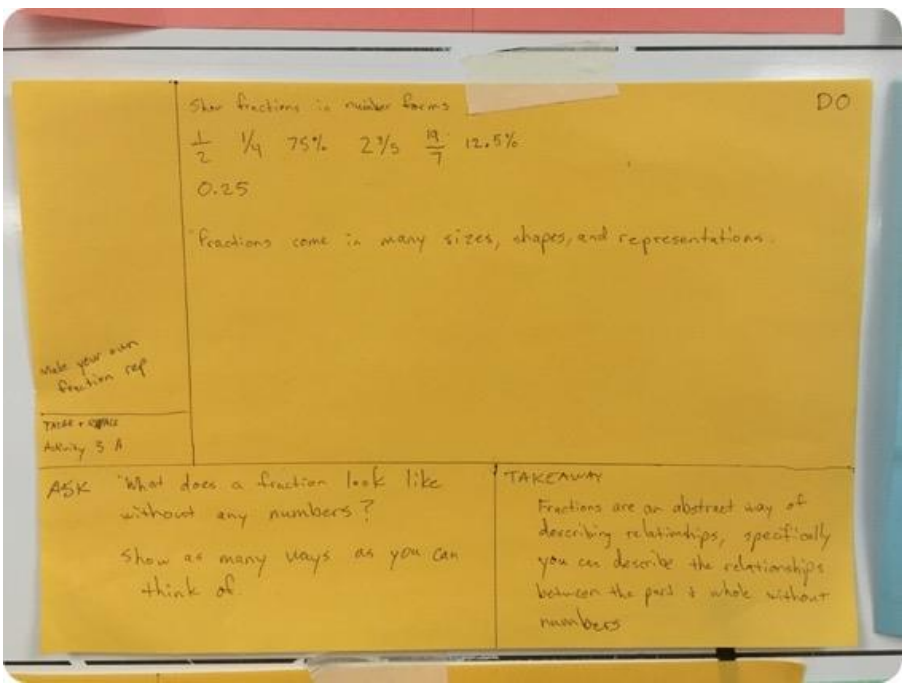


Figure 5-2 An example of an activity prior to being complete, with sections for Do, Ask, Takeaway, Supplies, and general notes

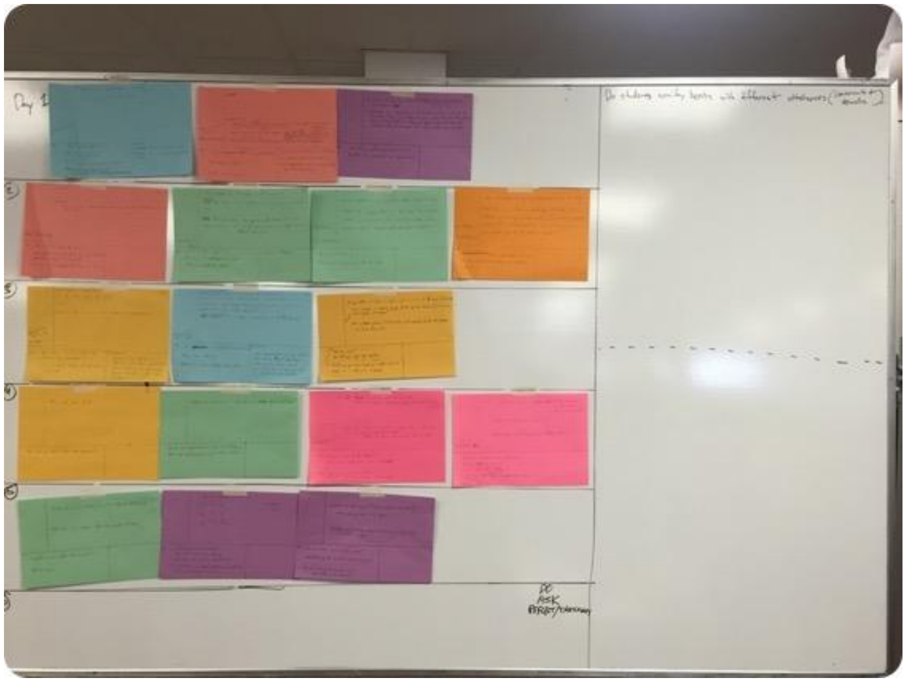


Figure 5-3 Most activities on a temporarily taped on a dry erase board to allow for moving around, categorization, order, and editing during the final stages of planning for the overall curriculum and lesson plans.

5.6 Lesson Plans

After all of the conceptualization, planning, and organization, the final curriculum was devised. We designed activities to be flexible for the needs of the study participants, and meaningful progress. Please see I.D Curriculum for the entire curriculum.

6 RESULTS

During smaller trials, research of prior works, involvement in other research projects within the Third Lab and other projects, certain data was planned to be collected and analyzed. This section discusses the data that we collected.

6.1 Demographic Information

We conducted the current main study in the Fall of 2015 in southern Virginia. The county is predominately of a low-SES in education where the large majority of jobs are outside of the county itself. We worked with a local community center to communicate with the school system and active community members to identify two locations for the study. We worked with a local elementary school and a local church afterschool program.

6.1.1 Census Data

To fully understand the struggles that this county faces, it should be noted that 75% of the population that is 25 years old or more have a high school diploma, while only 10% have a bachelors or higher.   The county has a population of about 20,000 and 92% are Caucasian and 6% Black or African American. The average time to work is 26 minutes. The average number of people in a household is 2.32 and the average total household income is $35K USD. [70].

6.1.2 Education System Data

The county has four elementary schools (K-7), one primary school (K-3), one upper elementary school (4-7), and one high school (8-12). All schools have been fully accredited by the state board of education, meeting or exceeding state expectations for public schools. The county school system operates on a budget of $29-30M [71]. The average cost per pupil is reported as $10K.

	School System
Number of students	2600
% on Free Lunch	47%
% on Reduced Lunch	10%

Table 6-1 Education system data of the number of students, and the percentage on free and reduced lunch

6.1.3 Student Demographic Data

We conducted one study at an elementary school, referred to as Group 1, and one at a local church's afterschool program with children from a different elementary, referred to as Group 2.

Group 1 students were identified by the principal and teachers as needing extra help in mathematics, and were already enrolled in a special extra remedial math class to help them individually with their math development. The research instruction occurred for about 45 minutes before school officially ended, followed by a break to receive a small snack (juice and crackers) and then return for an afterschool session. Some students were unable to participate on certain days due to school wide functions, transportation issues, other afterschool program conflicts, or sickness.

Group 2, at the local church, is an ongoing effort from members of the community to offer additional educational opportunities and provide a meal to families who would like to take advantage of this resource. For some students, this service will include an important meal. The students ride a bus to the church provided by the local school system, eat a meal, and then participate in activities. The students are then picked up by their parents/guardians.

	Group 1 Remedial Class/Afterschool	Group 2 Afterschool
Number of students Starting	8	12
Number of students Ending	7 (1 white male grade 6 left after Day 4)	11 (1 white female grade 6 left after day 3)
# | % of White	8 | 100%	8 | 66%
# | % of Black	0 | 0%	3 | 25%
# | % Hispanic	0 | 0%	1 | 8%
Average total time of instruction per session	93 minutes (with a break of about 10 minutes)	66 minutes
Number of Sessions	7	6
Age Range	11 - 13	11 - 13
Grade Range	6 - 7	6 - 7
Boys # | %	4 | 50%	7 | 58%
Girls # | %	4 | 50%	5 | 42%

Table 6-2 Student Demographic Information of the two groups

6.1.4 Attendance and Participation

The consent and assent conditions of our Institutional Review Board (IRB) allow students and parents to abstain from a single activity, the entire class, or the program as a whole, at their own discretion. As a result of this, one student in Group 1 was requested by the parents to no longer participate in the study midway through. One student in Group 2 also was no longer allowed to participate due to behavioral issues outside of our sessions. Transportation issues also restricted participation to the first remedial class session consistently for one student in Group 1.

6.2 Data Sources

The main types of data that were collected were

- Video (notes and observations of in class interactions),
- Computer logs of interactions with SoF (every important interaction coupled with the state [before and after] of their rhythm and representations),

- Journal notes (hand written and drawn answers to questions asked during class), and
- Artwork and other manipulations (their physical constructions and completed handouts)

These four data sources were then each analyzed, compared, and contrasted with each other to initially identify common themes.

6.3 Themes Present in Each Data Source

Reviewing the different data sources provided a start to the analysis of our research interests. We must evaluate our data and study from several perspectives to understand our learning environment that we created, along with the various situational micro-worlds students participated in throughout the curriculum. Some perspectives relate to the people involved: students, teachers and administrators, and researchers. Other perspectives include the addition of our aural and physical contexts. And of course, we must inspect the mathematical concerns as a primary motivation for our work.

We grouped these themes based on their overarching perspectives, which highlight what we are trying to incorporate. We considered educational, design, sensory, musical, and mathematical perspectives. We also considered the similarities between particular themes and grouped them by similar concepts within their own perspective. Educational perspectives include teaching [teachers and administrators] as well as [student] learning. Design has many facets, but we focus on the visual elements for our analysis. This helps us see and verify what areas are likely to provide useful insight into what to investigate in further detail.

We found evidence for visual, physical, aural, and gustatory senses. Like our literature search, there remained little to no evidence of olfactory data in our observations. Musical perspectives include the patterns of the music as well as the musical musical components and attributes that actually make up the music students create and work with. The mathematical perspective includes the system of fractions and the current perspectives of cognitive schemes children employ to work with fractions.

We judged how strong of a presence a particular concept exists within each data source, and can be seen in Table 6-3 and Table 6-4. These tables show that the strongest concepts come out of aural senses, physical senses, and visual senses, in mathematical and learning perspectives. The richest data type was the video, and the motivation for Figure 6-1 and Figure 7-1.

Perspective	Concept	Theme present in data source	Data Sources			
			Video	Notebook	Logs	Artwork
Education	Teaching	Spend more time on a subject [5]	**STRONG**	-	-	Medium
		Clarification (Teacher to students) [6]	**STRONG**	-	-	**STRONG**
		Clarification (student to student) [8]	**STRONG**	-	Medium	**STRONG**
	Learning	Confusion [10]	**STRONG**	**STRONG**	weak	**STRONG**
		Incorrect Information [8]	**STRONG**	**STRONG**	weak	weak
		Correction [12]	**STRONG**	**STRONG**	**STRONG**	**STRONG**
		Notable Learning Moment [10]	**STRONG**	Medium	Medium	**STRONG**
Design	Visual	Orientation (linear/circular) [12]	**STRONG**	**STRONG**	**STRONG**	**STRONG**
		Colors [9]	Medium	weak	**STRONG**	**STRONG**
Senses	Aural	Descriptions [8]	**STRONG**	weak	weak	**STRONG**
		Sound [11]	**STRONG**	Medium	**STRONG**	**STRONG**
	Visual	Label [11]	**STRONG**	**STRONG**	**STRONG**	Medium
		Animations [8]	**STRONG**	weak	**STRONG**	weak
	Physical	Tangible [7]	**STRONG**	weak	**-**	**STRONG**
		Manipulations [11]	**STRONG**	Medium	**STRONG**	**STRONG**
		Embodied [8]	**STRONG**	weak	Medium	Medium
	Gustatory [5]		Medium	Medium	**-**	weak

Table 6-3 Section A of themes, concepts, and perspectives present in each data source

Perspective	Concept	Theme present in data source [Weighting score of all data sources]	Data Sources			
			Video	Notebook	Logs	Artwork
Musical	Pattern	Type/Categorization [10]	**STRONG**	**STRONG**	**STRONG**	weak
		Size/Quantity [12]	**STRONG**	**STRONG**	**STRONG**	**STRONG**
	Component	Pitch/Timbre [3]	weak	-	**-**	Medium
		Speed/tempo [10]	**STRONG**	Medium	**STRONG**	Medium
		Volume/Intensity [8]	Medium	Medium	weak	**STRONG**
Mathematical	Fractional	Order [9]	**STRONG**	Medium	**STRONG**	weak
		Scalar [11]	**STRONG**	Medium	**STRONG**	**STRONG**
		Part [12]	**STRONG**	**STRONG**	**STRONG**	**STRONG**
		Whole [9]	Medium	**STRONG**	**STRONG**	weak
	Literature	PUFS (Partitive Unit Fraction Scheme) [4]	Medium	Medium	-	-
		EPS (Equipartitioning Scheme) [4]	Medium	Medium	-	-
		Splitting Loop [5]	**STRONG**	Medium	-	-
			79	54	52	61

Table 6-4 Section B of Themes present in each data source. A **STRONG** *rating equals 3 points, a* Medium *rating equals 2 points, a* weak *rating equals 1, and 0 points are assigned to data that does not show evidence of being present with -. The totals are tallied vertically for each Data source, and horizontally for each theme.*

6.3.1 Evaluation of themes and data sources

To provide a basic quantifiable measurement of the strength of perspectives, concepts, themes, and data sources, a weighted evaluation was assigned. Their weighting was rated based on existence, number of occurrences, quality of data, and basic analysis of student contributions such as descriptions or thought processes. 3 points were awarded for **STRONG**, 2 points for Medium, 1 point for weak, and 0 points for no evidence of presence. The points were then totaled, shown in Table 6-5. It became clear that all themes that were of interest to us exist primarily in at the video and artwork.

6.3.1.1 Themes, Concepts, and Perspectives

The weakest themes surround pitch/timbre of music, and the evaluation of the presence of mathematical learning schemes. They may be stronger in the Artwork form than the other data sources, as students had full creative license for their artwork, but our interface was restrictive for pitch and timbre. This may also highlight the actual lack of presence in the data sources in addition to our lack of expertise in these particular fields to recognize meaningful presence.

The strongest themes were Correction (learning moments when students corrected their statements, hypotheses, findings, approaches, etc…), Orientation (discussion and interaction surrounding the linear or circular placement of objects), Size/Quantity (discussion and interaction including the size or quantity of elements or groups), and Parts (discussion and interaction involving the parts of a rhythm or fraction).

Other noticeably present themes include Sound (performing or interacting with sounds), Labels (attempts to use alternative representations to describe an ideas or objects), Manipulations (acts of direct manipulations of something already completed or presented to students) and Scalar (discussion and interaction surrounding the scalar properties of rhythms and fractions). This highlights that our approach has both a varied and important presence in each of the perspectives that we want to discuss during our curriculum.

To consider the concepts and perspectives as a whole, we aggregated the themes by their concepts, and tabulated the results, shown in Table 6-5. The weakest concept remains gustatory (and may be argued as an outlier against our overall study and goals). Other lower scores (Teaching, Visual [Design], Aural, Visual [Senses], Patterns, and Components) do not show signs of being weaker than the others, rather a consistent baseline, scoring very close to each other and accounting for sixty percent of the concepts (6/10) and fifty percent percent of the themes (14/28) we identified.

We also aggregated the perspectives with regards to their concepts in the same method. This highlights that our focus is not as strong in the design perspective. It also shows that the sensory perspective is quite strong in relation to our other perspectives.

Perspective	Concept	Data Sources			
		Video	Notebook	Logs	Artwork
Education [59]	Teaching[19]	9	-	2	8
	Learning[40]	12	11	7	10
Design [21]	Visual[21]	5	4	6	6
Senses [69]	Aural[19]	6	3	4	6
	Visual[19]	6	4	6	3
	Physical[26]	9	4	5	8
	Gustatory [5]	2	2	-	1
Musical [43]	Pattern [22]	6	6	6	4
	Component [21]	6	4	4	7
Mathematical [44]	Fractional[31]	11	10	12	8
	Literature[13]	7	6	-	-
		79	54	52	61

Table 6-5 Weighted ratings of a higher level Perspectives and their underlying Concepts per each data source.

6.3.1.2 Data Sources

Adding the scores for each data source also makes a contribution. The Video data source has the largest concentration of themes present, followed by Artwork, and the Notebook and Logs are relatively equivalent. This motivates us to focus more on the artwork and the Video data more so than the notebooks and logs. This is beneficial as there are more (and richer) data points to consider in the front runners. This doesn't not deter us from the other two, as they have very rich contributions relevant to their medium and context.

6.4 Curriculum

The curriculum, exercises, and study as a whole as described in the Curriculum section was analyzed for consistency, effectiveness, areas of opportunity, student engagement, current knowledge, and learning. In this section we deconstruct the days and activities as it pertains to the all students, the groups, and a select few students from each group who highlight the benefits of using SoF as a robust teaching curriculum.

6.4.1 Group information

We designed each curriculum activity with a projected amount of time each activity should last, accounting for, transitional periods, setup, questions risen by students, and other classroom management concerns. Given the differences in the structure between both groups, the order of the activities was adjusted slightly to accommodate the time students were available, student attendance and participation, school maintenance closings, national holidays, the direction of the students' interest and questions, technical problems, and other unforeseen or uncontrollable events. The overview of both groups can be seen in Figure 6-1.

6.4.1.1 Group 1

Students in Group 1 had seven sessions with an average session length of 101 minutes (1 hour and 41 minutes) with an average of 93 minutes (1 hour and 33 minutes) of actual instruction time. Due to the nature of the class structure, Group 1's sessions were setup to span across the school day's official end time. The first part, approximately 40 minutes, was part of a remediation class, and the second was part of an after school class, lasting approximately 60 minutes. This structure is the main reasoning behind the intermediate break(s) between class instruction during the study. Most breaks consisted of the students going to another area for a short time (10 minutes or less) to grab a snack and bring back with them.

Given the extra time that Group 1 has in comparison to Group 2 (approximately 30 minutes) we were more relaxed on pushing the curriculum with regards to timing, and allowed for more instruction that was directed by students' inquiries.

For a complete overview and detail documentation of the events that occurred in the classroom, please see I.E Group 1 Video Data and Notes.

6.4.1.2 Group 2

Students in Group 2 had six sessions with an average session length of 66 minutes (1 hour and 6 minutes). The structure of Group 2 being in an afterschool program at a local church, the students would receive a school provided bus ride to the church, and then would eat a large snack (sometimes serving as the student's only dinner meal). The students that participated in our study only made up about thirty percent (30%) of all students, and so some weeks we provided music-based activities that were appropriate for all ages at the afterschool program, as most students and volunteer understood us to be associated with music learning activities. This is the reasoning for some of the shorter sessions that have substantially less time (days 4 and 6).

For a complete overview and detail documentation of the events that occurred in the classroom, please see 11.1 Group 2 Video Data and Notes.

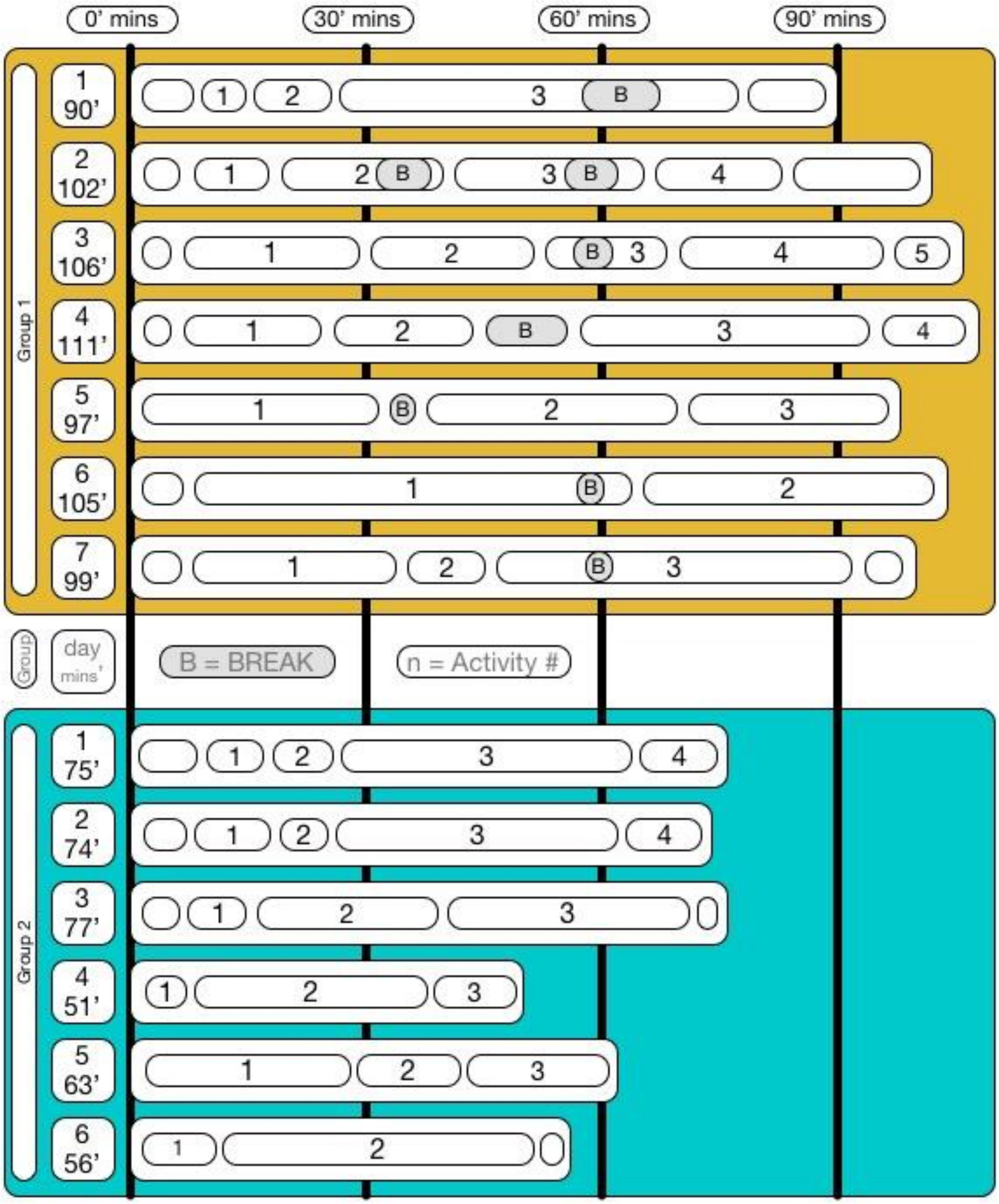


Figure 6-1 Overview of both groups study sessions, including the overall elapsed time of the day and individual times for activities

6.5 Learning Moments : Evidence from Pre-Post Inquiries and Teachable Moments

The main purpose of our research is to make meaningful tools to help children learn fractions. The answer is not as simple as yes or no, but we believe that overall we definitely made strong positive measures and improvements in the students' knowledge and understanding of fractions. We can say this from look at pre-post inquiries, the explanations students provided as the study progressed, the struggle and perseverance displayed along with the 'ah hah!' moments, and the analogies created and communicated provided by students during instruction.

6.5.1 Pre-post Test – What are fractions

When we began the study, we began with music playing in the background playing, and talked a bit about our research. Interestingly enough, several students began tapping (feet and writing utensils) while signing the assent papers. This let us know that they were engaged. To help us understand where the students are, both individually and collectively as a group, we asked the students to define what a fractions is, what it means to them, why it is important, and what their favorite fractions was if they had one.

Student descriptions of fractions were varied but predominately short, misspelled definitions. At the end of the study, their definitions were longer, more direct, and focused on more important information regarding fractions. One student described fractions as being `hard' to begin with, but later described many components, including that they have `parts and wholes' and they can be used in life for 'cooking breakfast', while another student pointed out it is helpful for `changing oil [in a car]'.

7 DATA ORGANIZATION AND VISUALISATIONS

Given the plethora of data types and ensuing data, a need arose to present the data in a structured, meaningful, and useful manner to allow for analysis, recall, and discussion. We separated the data into:

- Classroom activities – video analysis which were digitized into graphical visualizations
- Diaries - their notes, hypothesis, and written answers to curriculum activities and questions
- Logs – sentential structured data of students interacting with SoF
- Artwork - Physical representations the students created of their own rhythms

7.1 Video Data

Each group was recorded during the study with two video cameras, a digital camera, and instructional notes. To be able to quickly reference the information that occurred at a high level with relevant information (such as student participation, voting, absenteeism, answers, general observations, etc …), we notated each interesting idea on paper by hand, and evaluated this method after two sessions. This enabled us to concentrate and refactor our observations according to particular cues. We actively avoided full transcriptions, as the software and techniques associated with them focus on more quantitative analysis, where we are interested in more qualitative approaches. We then organized this data by each group, each day, and each activity to show each student's contribution (as noted by us according to our cues noted in the initial reviews) during each activity (and sub-activity). Several thoughts, discussions, and guesses were left out of these data records, mainly those that were not seen as a noticeable or verifiable clue as to what the student was thinking (such as off topic discussions, shouting random guesses, or sayings of 'I don't know' without a hint of what the student does know). You can find a legend of the layout and the many components of the video data in Figure 7-1.

For a complete overview and detail documentation of the events that occurred in the classroom, please see the corresponding sections for each group: I.E Group 1 Video Data and Notes and 11.1 **Group 2 Video Data and Notes**.

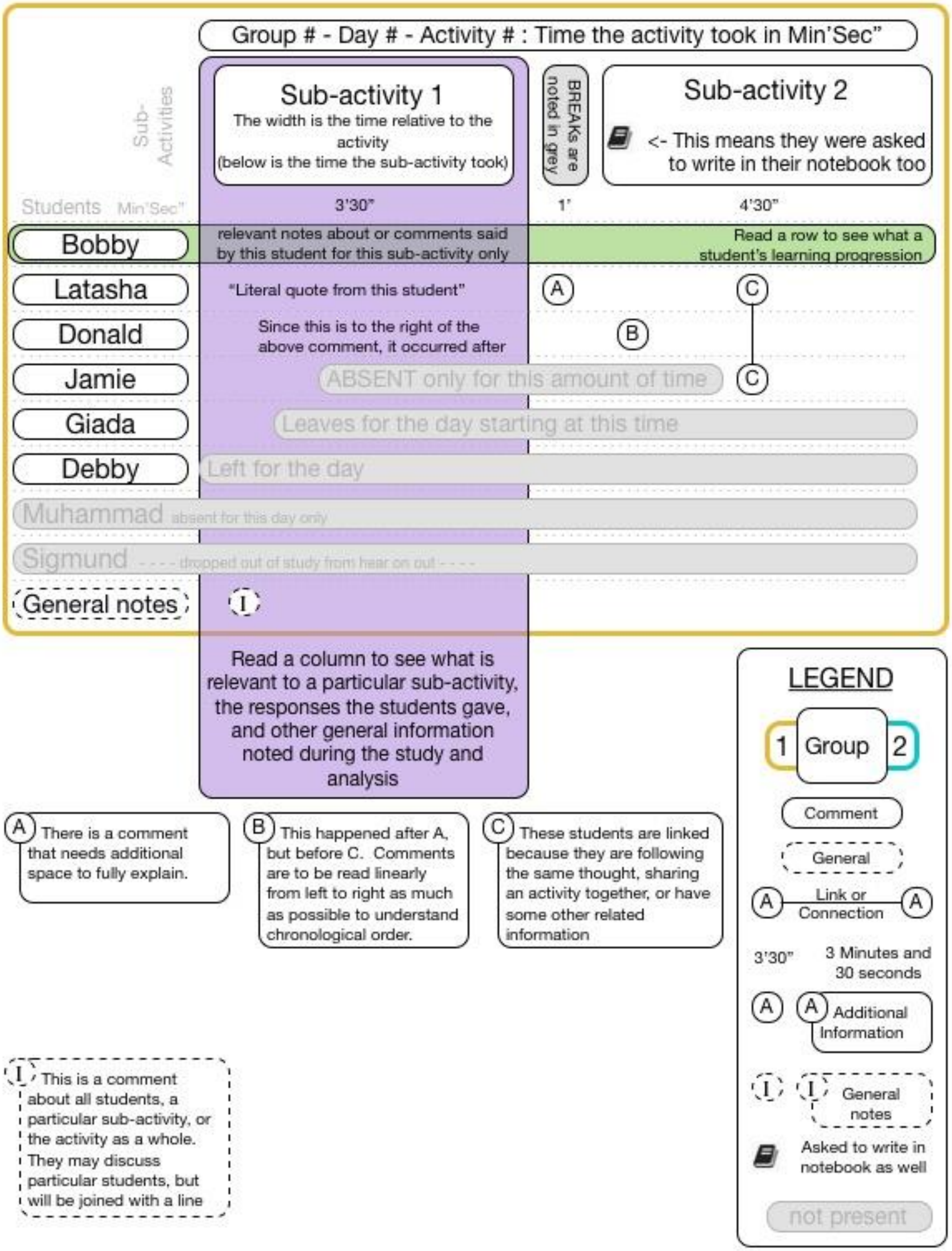


Figure 7-1 A legend of the structure of data of the video analysis of each activity of each day of a each group, including students, sub activities, contributions, and general notes, with notations of accompanying diary entries.

7.2 Notebooks and Artwork Data

The notebooks were distributed daily and students were often asked to write down their answers and reasoning on a page. Pictures were taken of each student's contributions per activity, and compared with each student's prior contributions as well as other student's contributions for that particular activity. The physical representations that students made can be seen at specialorange.org/sof-data , and cross-filter sorted by the attributes you are interested in. This allowed for interesting progression with definitions of fractions, insight into student hypotheses and observations, confidence, and their ability to translate fractions between the different domains that the fraction questions were posed.

7.3 Log Data

Log Data was collected during usage of the online interface of SoF. Students were told that the interactions were being logged, but that they didn't need to worry about it. The logging system kept track of interactions we were interested in, such as

- adding, removing, or toggling a beat,
- adding, deleting, or transitioning a representation,
- adding or deleting an instrument,
- starting, stopping, and duration of playing a rhythm,
- stretching or shrinking a rhythm,
- adding, adjusting, or removing labels,
- recording a rhythm,
- or general administration information such as logging in

The log message was stored with information about

- The student performing the interaction,
- The interaction itself in sentential form with pertinent information,
- A JSON string of the student's rhythm,
- And a Date and Time stamp of when the interaction occurred,

and sent to the database for storage, with backup plans for internet connectivity issues. We collected 17851 interactions during the study.

A sample log sentence looks like:

Stopped playing music. Duration of playback in seconds: 3.648 and the song lasts 3.204 seconds for a total playback number of 1.138576779026217 times.

And a sample JSON String of their rhythm looks like:


```
{
  "usedInstruments": [
    {
      "label": "Snare",
      "active": true,
      "signature": 4,
      "tempo": 120,
      "measures": [
        {
          "beatsArray": ["ON","OFF","ON","OFF"],
          "numberOfBeats": 4,
          "measureRepresentations": [
            { "currentRepresentationType": "audio", "transitions": 0 },
            { "currentRepresentationType": "bead", "transitions": 0 }
          ]
        }
      ]
    },
    {
      "label": "Hi Hat",
      "active": true,
      "signature": 8,
      "tempo": 120,
      "measures": [
        {
          "beatsArray": ["ON","OFF","OFF","ON","OFF","OFF","ON","OFF”],
          "numberOfBeats": 8,
          "measureRepresentations": [
            { "currentRepresentationType": "audio", "transitions": 0 },
            { "currentRepresentationType": "pie", "transitions": 0 },
            { "currentRepresentationType": "line", "transitions": 0 }
          ]
        }
      ]
    }
  ],
  "unusedInstrumentsCount": 1
}
```


From the collected data, we were able to analyze the following:

- The distribution of rhythms made by students with a particular signature
- The distribution of recorded rhythms by duration
- The comparison of rhythmic complexity (how many equal parts [beats on and off]) against the number of beats tapped (how many times the student actually tapped)
- The number of instruments used in each rhythm
- The representations being visualized in each rhythm
- The number of times a rhythm was transitioned
- Speeding Up versus Slowing Down Rhythms
- Rhythm duration scaled in terms of original duration in terms of x

7.3.1 The distribution of rhythms made by students with a particular signature

Every interaction was analyzed for the instruments used, and each of their signatures (the number of beats [on and off] that are in each measure of each instrument) was tallied. Each instrument's default signature was 4 beats. The signatures of 5 and 16 were the most prevalent. The signatures of 16 was the upper limit of SoF due to graphical real estate limitations, and are likely also attractive for students given that it was the upper limit, used themost colors used, and had the most musical complexity. The rhythms with a signature of 5 are most likely due to the curriculum design and student choices focusing on this rhythm to solve the questions posed by the curriculum. These results can be seen in Figure 7-2.

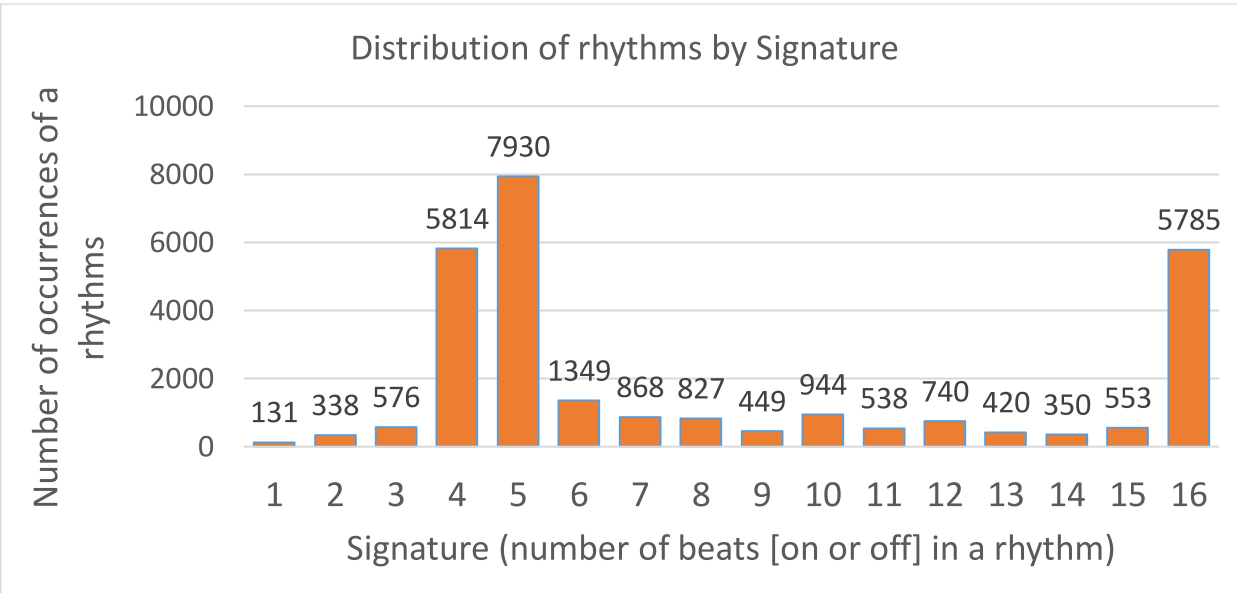


Figure 7-2 Distribution of Rhythms by Signature

7.3.2 The distribution of recorded rhythms by duration

To look at the recordings of each student, we studied the duration of each student's recorded rhythms. In general, students tapped shorter (less than 4 seconds of percussion) rhythms. Altogether, there were 125 recorded rhythms during our study. These results can be found in Figure 7-3.

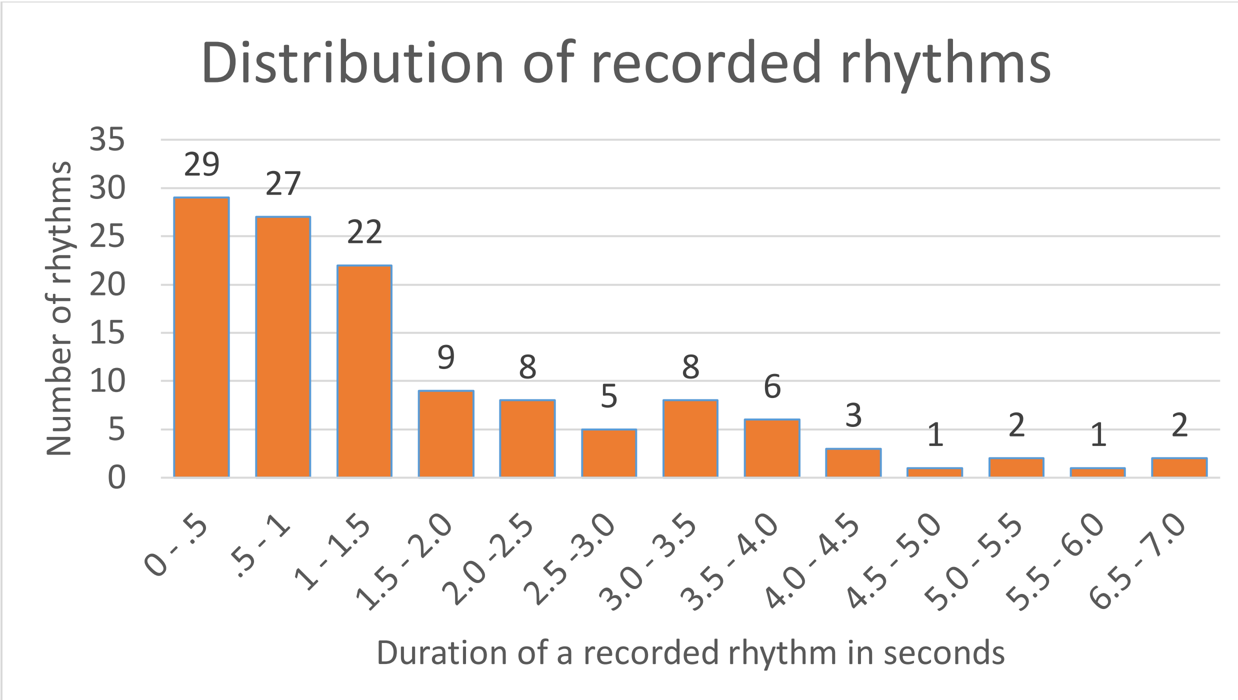


Figure 7-3 Distribution of recorded rhythms by duration

7.3.3 The comparison of rhythmic complexity against the number of beats tapped

Thirty-six rhythms were complex enough that they needed to be trimmed to the first sixteen beats to be compatible with SoF. However, students did not always tap beats evenly. Some students tapped rhythms that had few beats, but with varying musical patterns (tempo, spacing/beat sizes), reflected in a rhythm with more parts than tapped. Although complex to perform, SoF would only play back the first sixteen equal units, so students often adjusted their rhythms to something less complex. This data can be seen in Figure 7-4. For example, the normalized rhythms are rhythms that were tapped by a student with

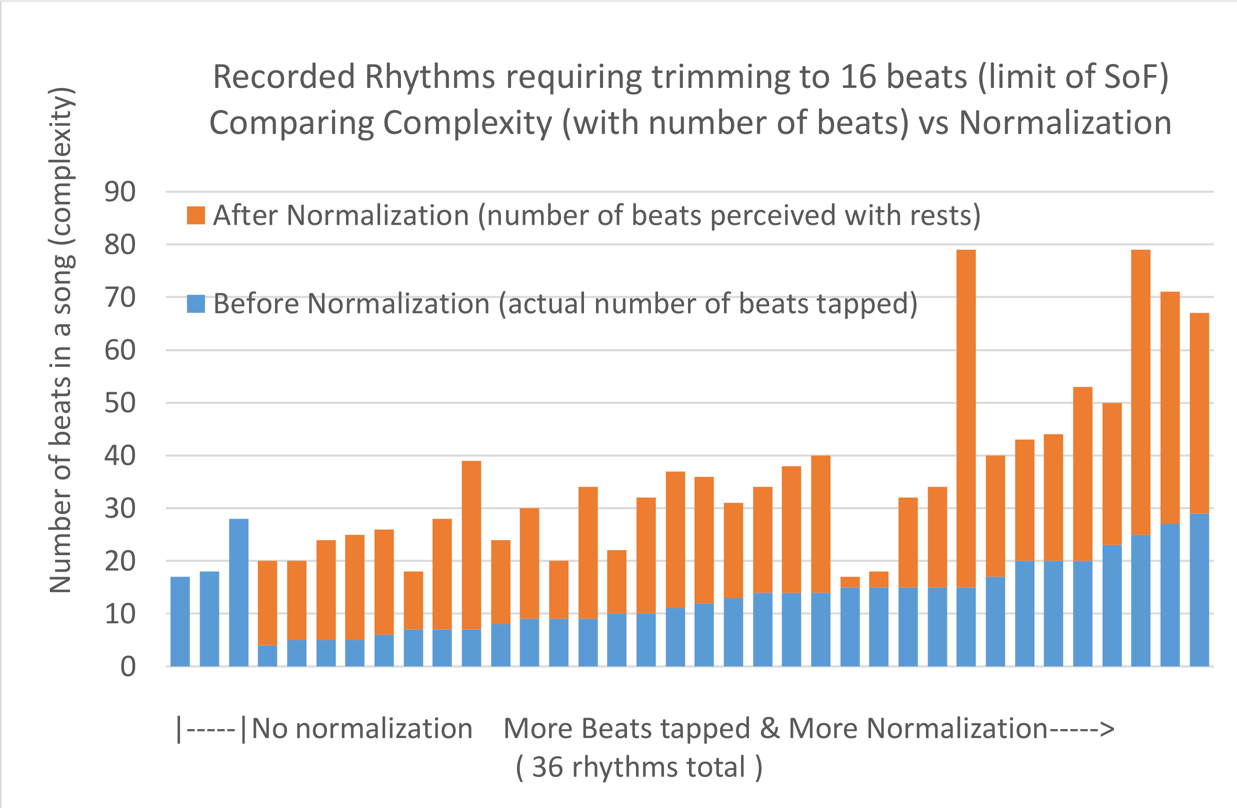


Figure 7-4 The comparison of rhythmic complexity against the number of beats tapped

7.3.4 The number of instruments used in each rhythm

Students were able to navigate the Stage quite well, adding and removing instruments as they interacted with SoF. Although their stage was seeded with 1 instrument, and some activities included interacting with multiple, it is informative to know that students were aware of the lower (0 instruments) and upper (3 instruments) boundaries of SoF, but predominately interacted consistently with 1 instrument. This can be seen in Figure 7-5. Some curriculum questions require working with two and three instruments to answer, but none involve zero instruments in SoF.

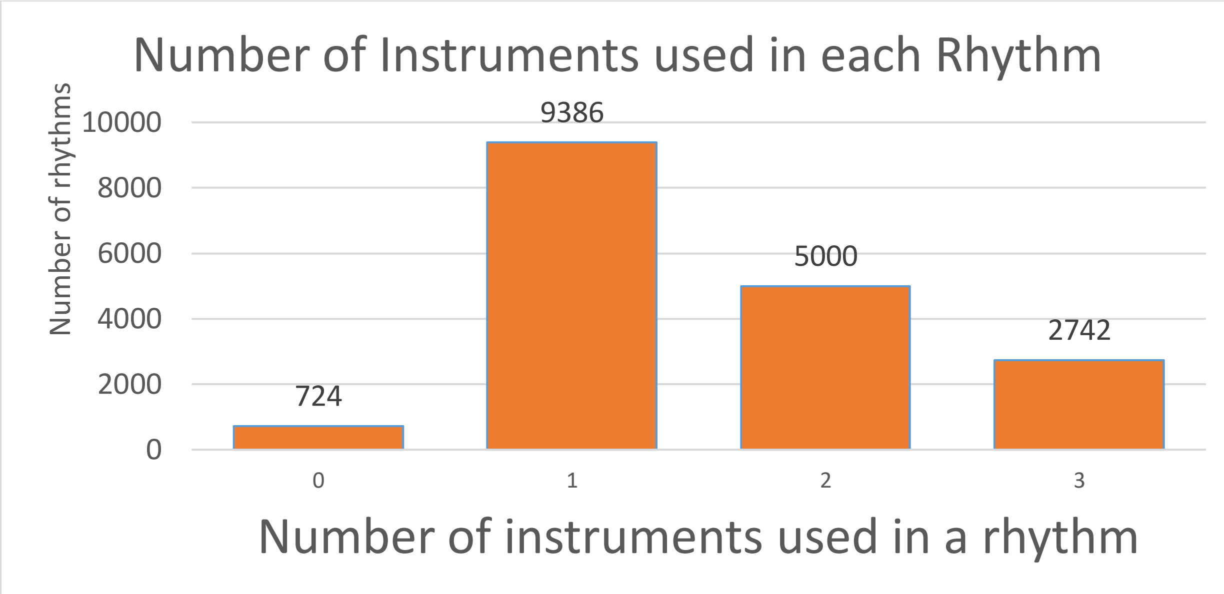


Figure 7-5 The number of instruments used in each rhythm

7.3.5 The representations being visualized in each rhythm

When students start SoF or add an instrument, an Audio representation is automatically supplied. Many of our curriculum activities focus on the bead representation. SoF helps take a cognitive and embodied rhythm and visualizes it in another form. It is a great parallel that the most common representation used by students is the bead, but also more traditional mathematical representations are visualized. This can be seen in Figure 7-6.

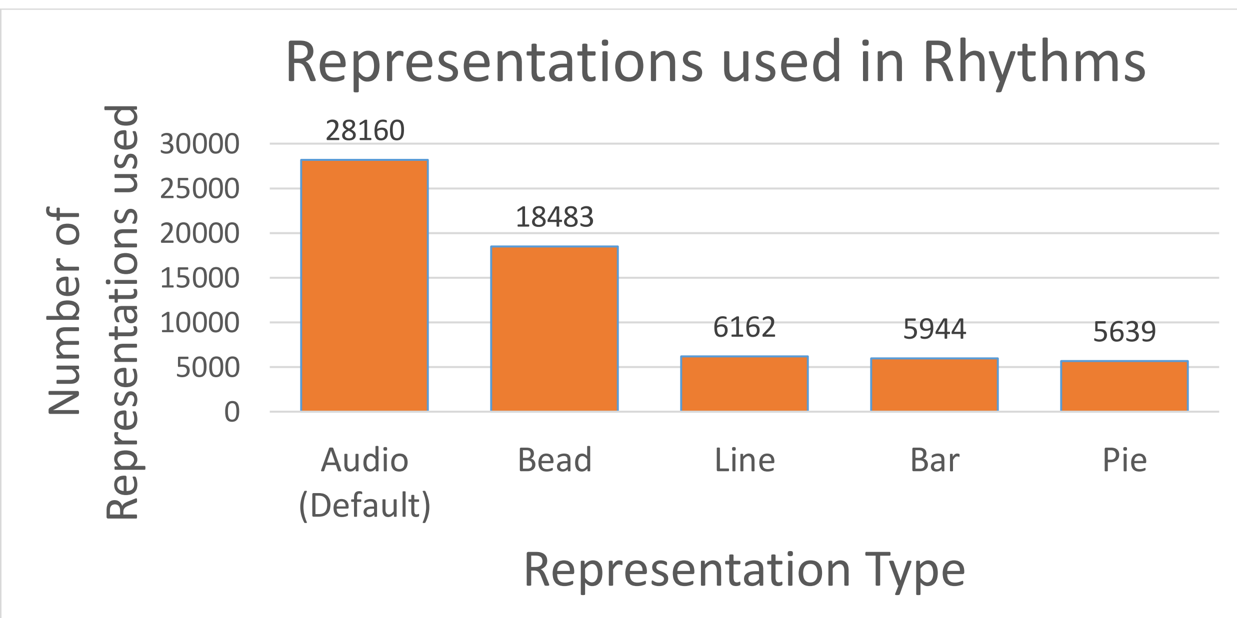


Figure 7-6 The representations being visualized in each rhythm

7.3.6 The number of times a rhythm was transitioned

Understanding what a rhythm is can be experienced through an embodied, cognitive, entrained, visual, and mathematical approach. Each of these approaches have multiple perspectives. SoF supports multiple mathematical and visual perspectives through our representations, and transitions between them. Our curriculum only asked to transition once, so it is important to see that students transitioned many of the representations multiple times. This can be seen in Figure 7-7.

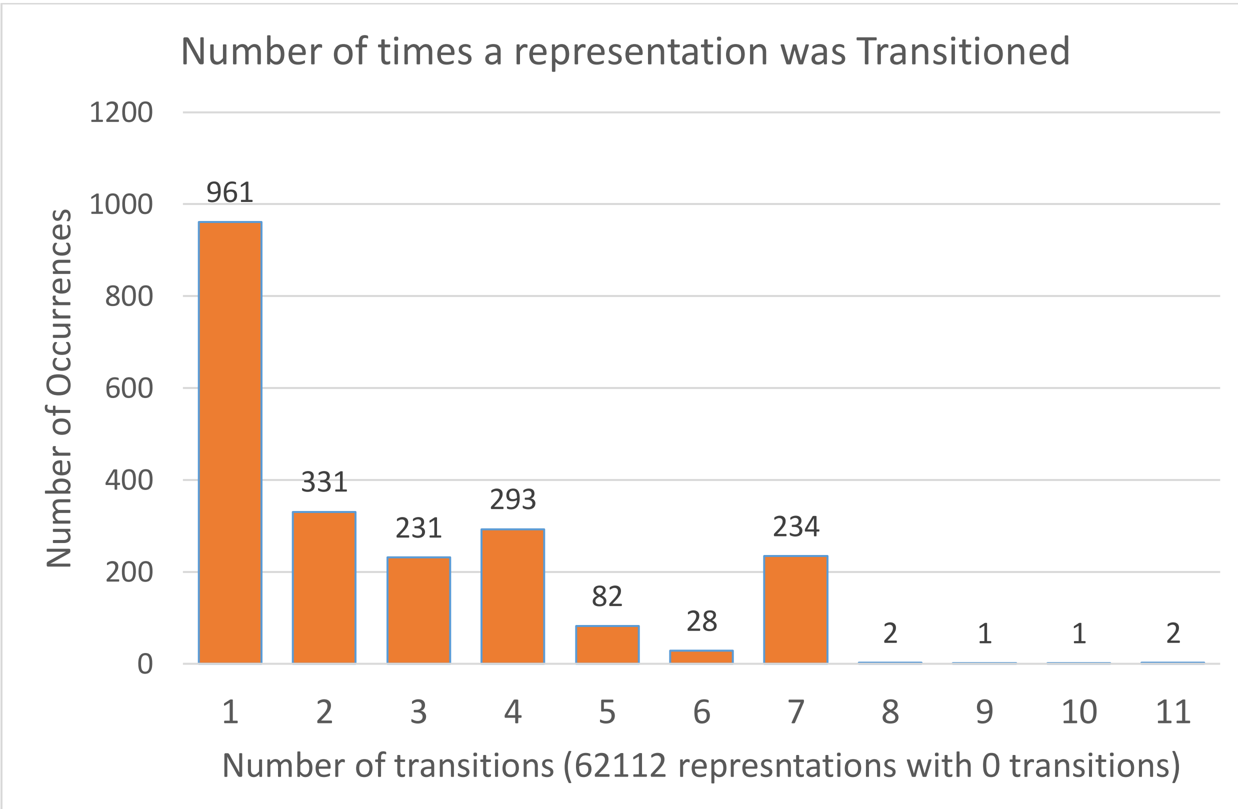


Figure 7-7 The number of times a rhythm was transitioned

7.3.7 Speeding Up versus Slowing Down

SoF supports many mechanisms to adjust the musical characteristics of their rhythm. An important mathematical and musical adjustment is the ability to stretch or shrink the representation to increase or decrease the playback duration, giving the aural and visual appearance of speeding up or slowing down the rhythm as a whole. Students stretched the rhythms more than shrinking them, as seen in Figure 7-8. This means they made the rhythm appear visually larger. This however makes the rhythm play slower (more distance to travel at a constant rate. Students ultimately operated with songs that were less than 1x of the original duration of their rhythm (recorded or graphically made). This can be seen in Figure 7-9. This shows that the visual interaction supports two different concepts: usability teaching, and musical interest/complexity. The interface correctly teaches the students through visual and aural feedback that the larger the representation means the slower of the rhythm and the smaller the representation means the faster the rhythm. Each students' musical interest and complexity is show by the fact that students chose to operate with rhythms that are more complex in [perceived] tempo.

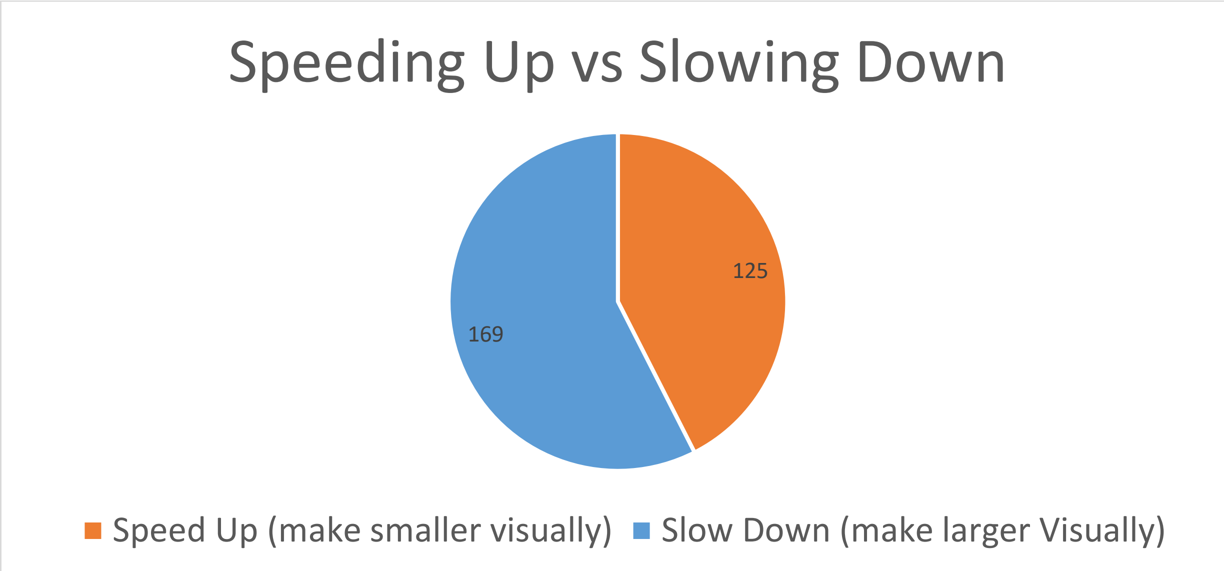


Figure 7-8 Occurrence Counts of Rhythms that were Sped Up and Slowed Down

Noticing whether they used SoF graphical interaction to stretch (slow down) or shrink (speed up) the [perceived] tempo does not tell us their final state, as a student could have sped up a rhythm once, and then slowed it down twice, or any other combination. We looked at what the final rhythm was scaled to to see what students' final rhythms were, mostly faster.

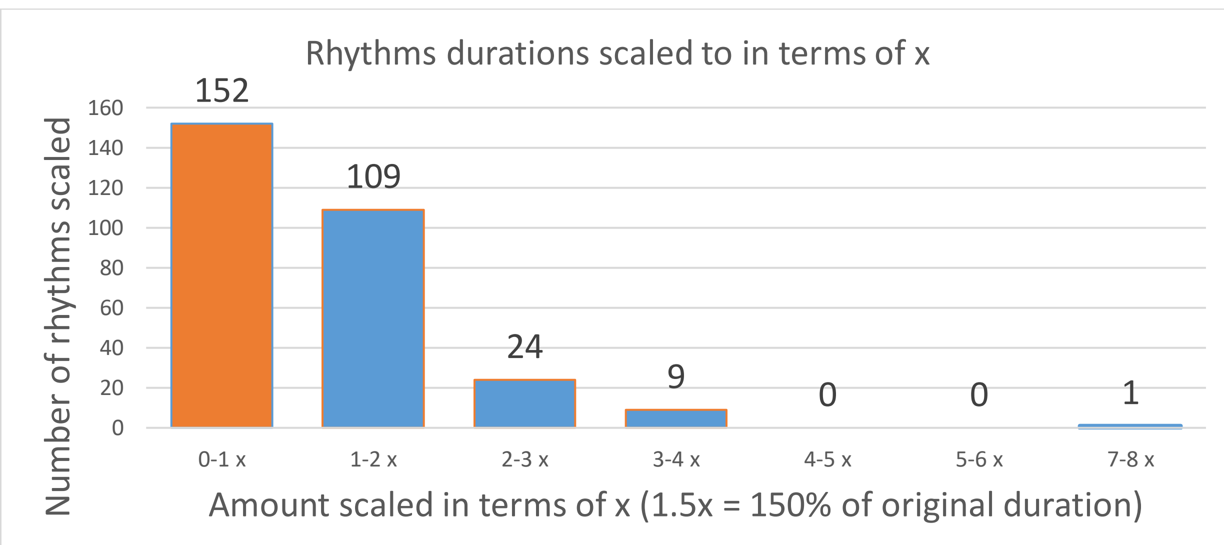


Figure 7-9 Rhythm duration scaled in terms of original duration in terms of x

8 DISCUSSION

With the curriculum made (see 5 Curriculum) and the data collected and organized (see 6 Results), we began to analyze everything for important themes, data that proves our hypotheses, and other notable phenomena.

XXXXX

We support full transitions between each of the Core Four. We do not support transitioning between the Audio and the Core Four. The Audio representation provides a strong mechanism to communicate to the user that the microphone picked up a beat when recording, and also when any beat is making a sound, however, beyond that, we allow each individual to internalize the music as they experience it. Each transition between the Core Four has distinct stages to help the user see what is going is being transitioned, what is being removed, and what is being added.

XXXXX

8.1 Research Questions

During the development of SoF as a whole, we began to realize that there are certain interactions that have great potential for learning moments, direct questions that will challenge the students to think abstractly and beyond their traditional mechanisms for solving problems, and areas of exciting potential when moving from musical and embodied interactions towards the goal of more standard mathematical representations and discussions. We began to inquire about several questions based on our conjectures:

- How can a micro-world that utilizes multiple representations provide learning resources to address student misconceptions ?
- How can a musical approach to fraction learning help students progress in their composition, comprehension, and communication?
- How do multiple interactive culturally-relevant activities affect engagement in a learning environment?

8.2 Research Question 1: How can a micro-world that utilizes multiple representations provide learning resources to address student misconceptions?

We want to investigate how SoF provides meaningful advantages in contrast to the current ecology that is not fully addressing certain students in their academic endeavors. This requires us to look at the current environment students are exposed to, what students choose to operate with, and what our approach supports.

8.2.1 Contexts in Math [Education] Domain

Our study started with extensive reviews of math literature on current and comprehensive math education resources. Out of these readings we cam to understand the current academic thinking of how students perceive and approach fractions, and where some main barriers exist. Reviewing the resources available to teachers via textbooks and other media, it is clear that the majority of education in the math domain with regards to fractions is not robust.

8.2.1.1 Where Does Sound of Fractions Fit In

Our research indicates that the main components of fractions are the the unit fraction (1/n), the parts (all the 1/n s), the whole, the part and the whole together, and the relationship(s) between the part(s) and whole(s). To understand fractions at a competent level, one must be able to contextualize the situation, identify the important components and ignore the non-important ones, and describe the identified components in a meaningful way. The person also must be able to manipulate these components by iterating the parts, separating the whole into parts, and ordering them appropriately for the context of the situation they identified.

To teach these skills, fractions are taught in the classroom with word problems, numbers and equations, graphical elements, or stories about real life examples that students likely have some form of varying participatory experience. Teachers and administration can measure the student's fraction competency by asking the student to solve an equation or problem, properly label a graphical element, or perform mental math to understand how well the student knows fractions. We would like to note that it is quite difficult for any one person to properly design and assess a particular facet of any fraction learning scheme that a student may be using. It is difficult because the instructor must have a well rounded understanding of fractions themselves, as well as how to provide a teachable moment to a student when there is a struggle to understand or perform, as well as a way to test the newly communicated concept in a different way that strongly highlights if a student understands the concept, and not a rigid process (i.e. not just changing the variables or numbers in an equation, but changing the equation itself). Our outline of what fractions consist of and the approach in the math domain are shown in Figure 8-1.

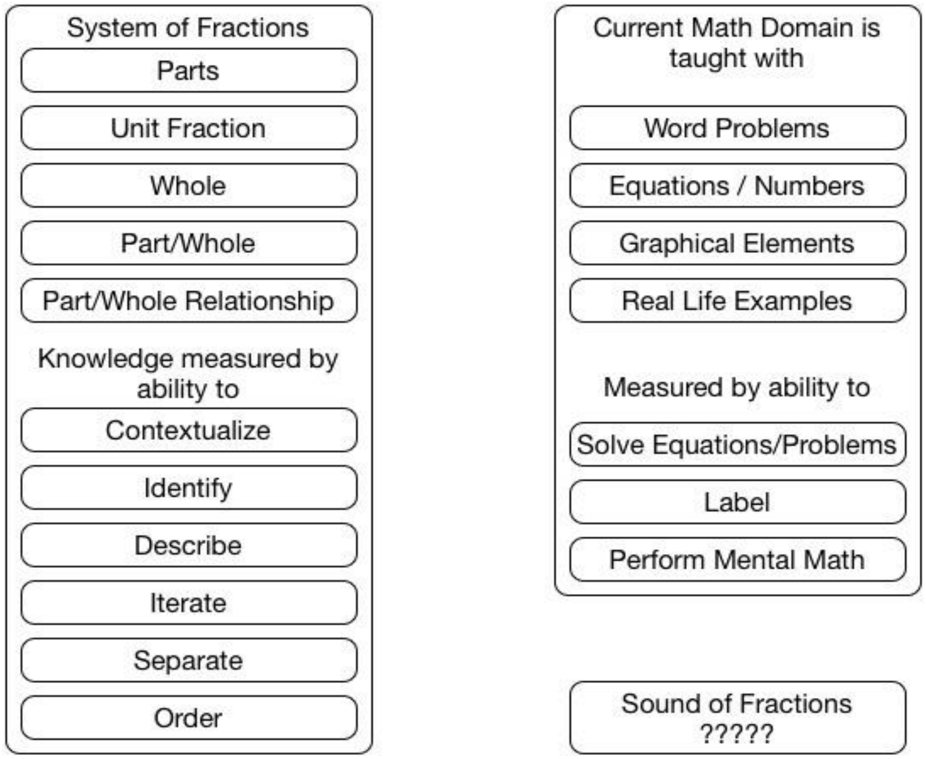


Figure 8-1 The system of fractions and its major components on the left. A representation of how the math domain is currently represented, implemented, and measured on the right, with the Sound of Fractions unsure of how to properly fit in the math domain.

Using this understanding of how math is currently being approached in an educational setting is what fueled our initial design. The initial prototype failed academically however because we failed to have academically structured lessons. It did however flourish with opportunities beyond engagement, highlighting the students' creativity, exploration, and continuous interest. This leaves an interesting question. How does Sound of Fractions fit into the math domain? By trying to include SoF into the math domain with appropriately guided curriculum, it became apparent that it fits quite well. During the study it became clear that students also excel in understanding fractions in other contexts too.

8.2.1.2 Other Learning Contexts That Students Can Operate and Learn in

Current approaches in the math domain are meant to teach, with the knowledge ultimately ending up in the COGNITIVE domain of the body, also known as knowledge. There are many ways that we can gauge knowledge that exists in the cognitive realm through different communication mediums. People often use hand gestures and utterances in communicating the ideas and problems we are working through in the mathematical domain to help aid our efforts with higher fidelity channels (such as verbal and written). We identify five contexts that are relevant to math education, some that have particular attractive features for fraction learning: Visual, Aural, Physical, Gustatory, and Olfactory, all based on the five senses.

We also identify the temporal qualities of each of these contexts as they relate to the cognitive domain.

Most of the current math domain is represented in VISUAL contexts. They are exhibited in the form of word problems, equations, numbers, shapes, graphs, icons. These representations can be performed by the student by viewing or creating animations, labeling them, or solving equations or problems.

SoF brings in an entirely new context, AURAL. We do this by providing curriculum that starts with listening to music first. This experience provides a base for an environment that highlights the entraining nature of music, so the students can experience the whole rhythm and the individual beats at the same time. The study participants, like even younger children, are able to perceive the repeating rhythm rather than just individual beats or chaotic noise.

Our curriculum goes one step beyond listening to music. We provide PHYSICAL representations and performances by allowing students to participate in music making with embodied interactions, both individually and as a group. We allow them to tap multiple rhythms, and adjust them in the interface by dragging new beats on to a rhythm, dragging beats off of a rhythm, stretching the rhythm as a whole to lengthen the duration, and shrink the representations as a whole to decrease the playback duration of the rhythm. These approaches allow them to approach analysis from the whole towards the parts (from each of the Core Four representations), the parts (by interacting with the beats individually), or both the whole and the part at the same time (while tapping their rhythm, managing the).

During our study, several students repeatedly communicated GUSTATORY references and experiences. These descriptions went beyond typical word problems and highlighted that the students had experience with making food, challenges with different situations when dealing with food, and had a strong interest in food like they do in music. Taste, a somewhat subjective sense, made it difficult to pose questions during the study that students could answer accurately in relation to fractions, but recipes, and

Olfactory: Given the we have identified four of the other contexts as they relate to the five senses, we briefly identified other representations that exist in the context relating to smell. Smell (more so than taste) is very subjective, and lends itself as to a reason why we were not able to identify many representations or performances in this context, or find research that highlights bridging mathematics education with smell.

We arrange these contexts in Figure 8-2 to highlight the additional contexts that exist infraction learning. This arrangement IS NOT meant to codify the all components of what it means to understand the system of fractions, highlight one fraction scheme over another, or a canonical list of the methods and skills required to perform fractions. It is also not to be construed as a comprehensive or codified hierarchal relationship of the contexts that exist cognitively or in the mathematical education area. Contexts are organized within the cognitive domain (because we know and experience each of these contexts) from what is strongly represented in the math domain, followed by our contribution of aural and physical, and the other two sense which are yet to be discussed in detail in the academic community as having a link to math or fraction learning. We also do not intend to say that these contexts, representations, or performances are completely isolated. We argue that this arrangement highlights the key differences in the ways that students can come to understand fractions, and need to be separated and given an identity to fully take advantage of the performances, representations, and contexts that students find themselves participating in.

Just as a words `bouncy ball' on a page is physical ink on a physical page, it can also be seen visually, elicit the sound and feel of a bouncing ball, and be recalled cognitively with common stages (dropping, bouncing, catching) during the bouncing of the ball. This example highlights a person who has had more or stronger experiences with a bouncy ball. We hope to also provide the same descriptions and experiences of fractions with students by allowing them to experience and interact with fractions in more contexts than traditional approaches.

This figure IS a visual representation to highlight some common and compelling representations in different contexts that students are exposed to and familiar with. We also want to communicate that this figure helps academics understand the many different performances that are available to manipulate those representations. The temporal context is purposefully on the edge of the cognitive area, as there is an argument that temporal exists both internally and externally of the brain, hence why it is tied to several of the other sensory contexts in varying strengths.

We also acknowledge that the arrangement of entities in Figure 8-2 does have problems. From the cognitive sciences field, our organization may conflict with current understanding of the organization of the brain, and is not as simple as the five senses. Also, we do not include additional physiological senses such as equilibrioception (sense of balance), proprioception (sense of one's own position), exteroception (sense of the world external to the body), interoception (sense of the internal body), or kinesthesia (sense of one's own movement) that may be of interest in further evaluation, future work, or other literature.

Academics may argue that this representation is problematic and does not properly represent their viewpoint. This argument highlights the justification to teach the same subject in different contexts. As the experiences of students are expanded by the interactions with varied representations, which are inherently abstract and possibly foreign, when considered from another context or perspective, students become more skilled at identifying the important components of the different representations to achieve a properly contextualized abstract idea.

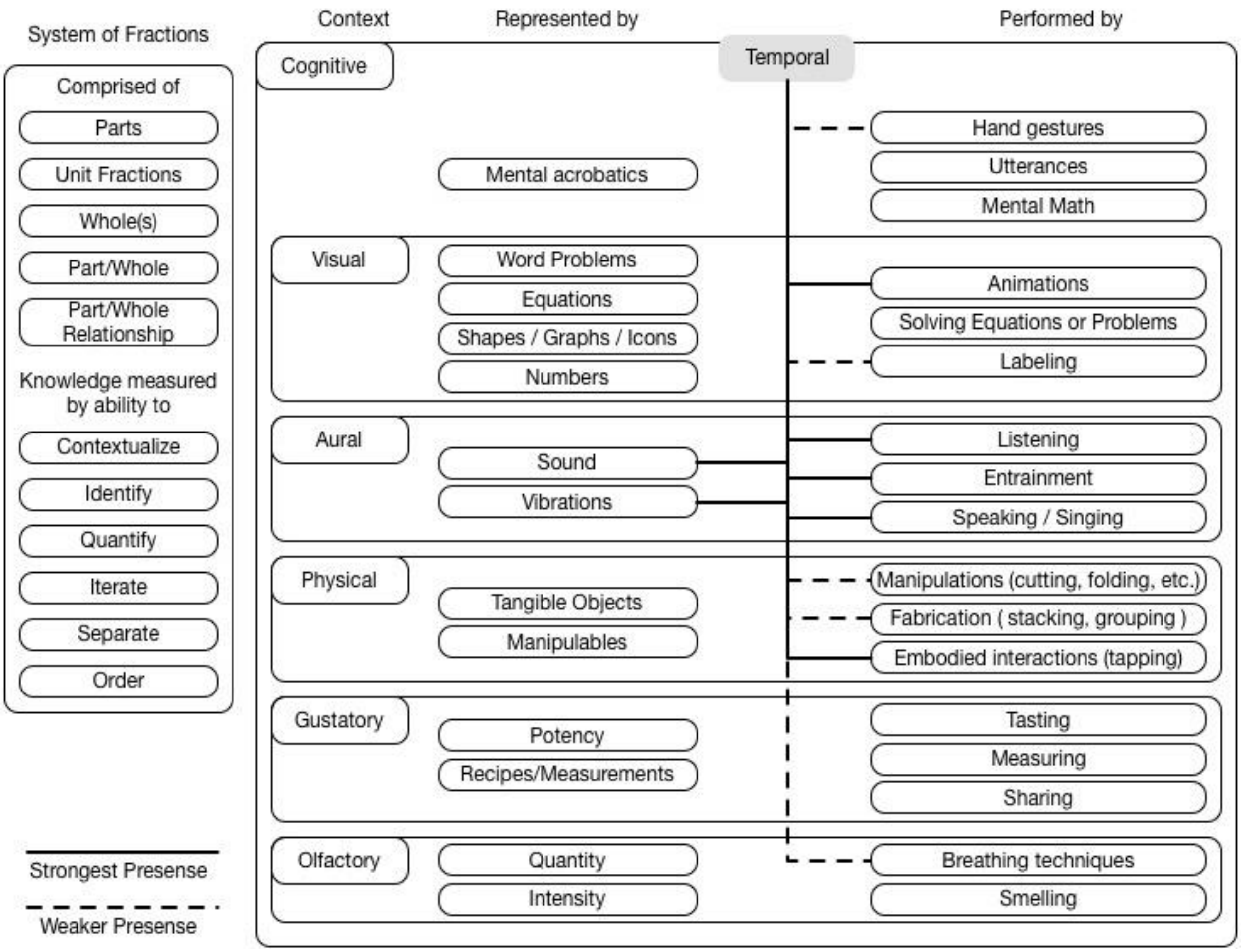


Figure 8-2 By introducing a wholly different approach to fraction learning (particularly using aural and embodied components) we noticed that other contexts exist to teach fractions with. These were confirmed by students during the study, as well as other contexts were given as examples by students (Gustatory) during discussion and interactions. This led us to consider how math is represented in different contexts, their affordances and weaknesses, and what contexts provide strong and meaningful learning opportunities with fewer misconceptions and difficulty when learning fractions.

8.2.1.3 What SoF Supports

With a better understanding of what contexts exist in the math domain, currently, perceived, and potentially in the future, we want to identify what components, measurements, contexts, representations, and performances are supported by SoF.

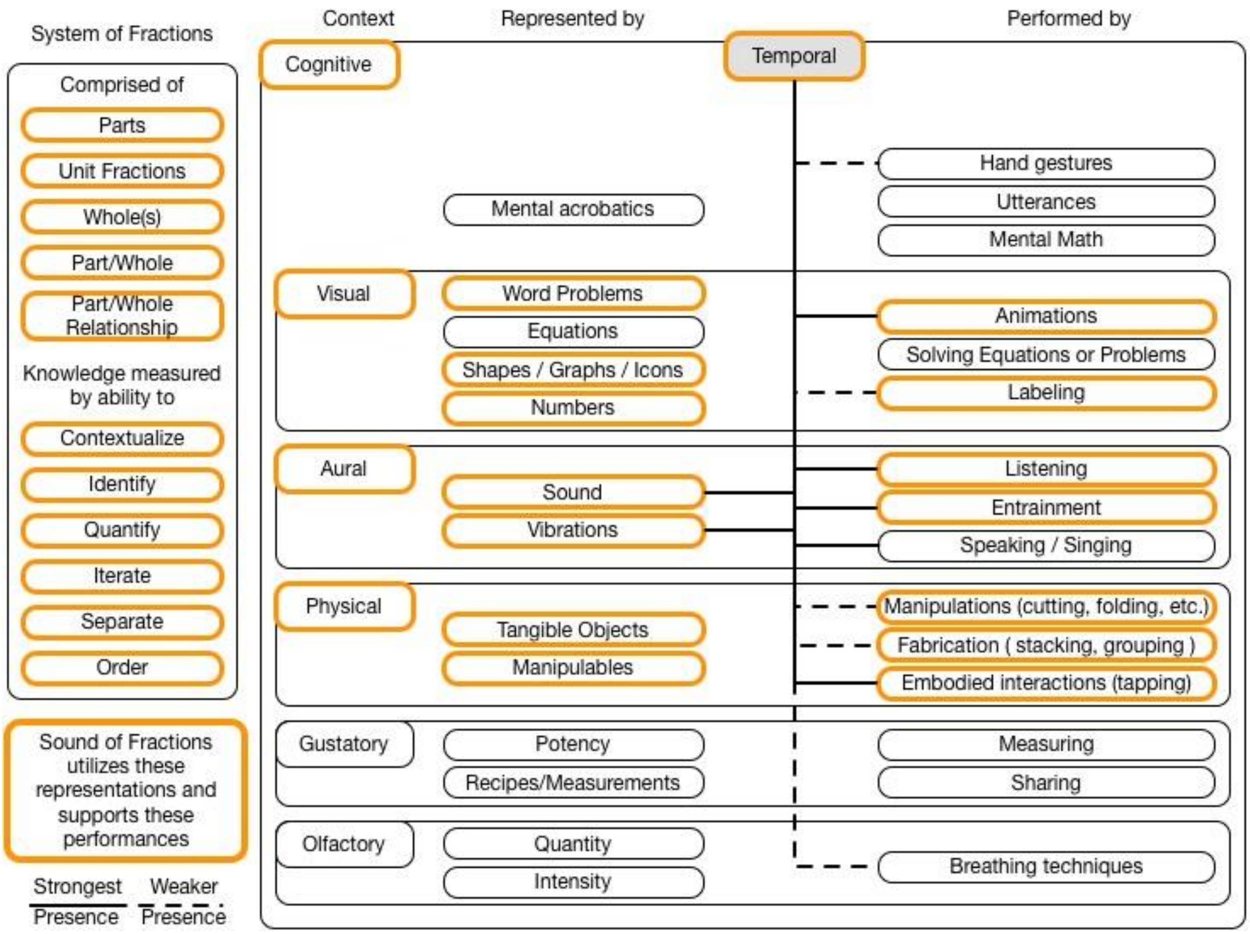


Figure 8-3 The contexts, representations, and performances that SoF engages and challenges students in, supports via the curriculum and socio-technical interface, and operates within a dynamic environment to support different students' approaches.

8.2.2 Fractions, Representations, and Abstraction

Fractions are inherently abstract. They describe the relationships between the part(s) and a greater unit whole. Fractions exist in many forms (ratios and descriptions, pictures or graphical elements, percentage, decimal, simple [numerator/denominator], mixed number, complex) and the seminal way is hieroglyphic [17]. The need for many representations to describe the `same' relationship is not by chance. Each representation highlights special cases that exist when describing parts and wholes. For example, mixed numbers and improper fractions deal with multiple unit wholes, ratios and descriptions focus on the relationship(s) rather than the quantity of the part(s), percentages imply 100 equal units, and decimals adhere to powers of ten. The hieroglyphic representations even take the form of adding unit fractions of canonical [mostly unit] rational fractions to achieve non unit fractions.

Given the ability and need for us to describe the same relationship in different forms requires a person to be able to understand each of the contexts in which one representational form is affords benefits better than another. This requires both experience in different contexts as well as an ability to think abstractly. Simply having experience in multiple domains or contexts itself is not enough, as evidenced by the students who experience fractions in their everyday lives and self reportedly understand their importance, but fail to identify the important components in the various contexts or apply their knowledge correctly. Also, the skill of thinking abstractly does not come without experiencing multiple situations and being able to correctly identify and link the similarities and differences that exist in both contexts as well as differentiating across contexts. This skill is also a major hurdle for students, as they may not see the benefits of different fractional representations as they may not be properly situated to see the strengths and weaknesses of particular representations.

Our approach of using a wholly separate context of aural and embodied interactions with stronger temporal requirements forces provides more facets of the situation to identify and develop abstract reasoning about fractions and the varying forms and requirements in separate situations. Allowing the students to create their own representations also allows the students to communicate with their peers and discover what features are important to include when trying to convey their rhythm in an environment with shared goals. Moving beyond their own artwork, students are introduced to our novel bead representation and can begin to apply their knowledge from their own artwork. Taking another step, we provide animations to help solidify the correct associations of what is conserved across representations, what features are important, what is lost when moving between representations, and what representations elicit stronger characteristics as they relate to fractions and the various forms fractions take. This path of discovery, introduction, and guidance help students not only experience the embodied and entrained characteristics of rhythms, but also helps them develop skills to correctly perceive relationships between objects and scenarios in abstract ways.

8.3 Research Question 2: How can a musical approach to fraction learning help students progress in their composition, comprehension, and communication?

With a formidable approach to a new learning environment, we need to gauge what works, what doesn't, and how can this approach provide skills that can be used beyond fractions and in other domains.

8.3.1 Hurdles Present in Students' Misconceptions

Our curriculum is designed to start at a common shared interest for all students, and progress in scaffolded discussion that utilizes the advantageous properties, representations and embodied musical performances. This has proved to be great for ongoing engagement, interest, and consideration. We struggle like many educators with helping those that are struggling with concepts that are well below our curriculum's expectations. Since we had not formally tested our curriculum in a longer and more formal setting, we were surprised when we came across certain clear hurdles.

8.3.1.1 n/n is confusing when n>20

We discuss what is the same, what is different, and what is important; A theme that we use repeatedly during the curriculum to ultimately hone the skills of students to identify what important components are needed to properly contextualize their situation. Early on in Group 1 when discussing what the artwork that students made to represent their rhythm, many students were able to cite differences between the representation artwork. The could pick out count number of beats, rests, colors or shapes used, size of the beats, and some legend information such as direction or order of play.

But it was increasingly harder for students to identify what is the same between students' artwork. Students tended to describe the beats (parts), and not the rhythm (the whole/as a whole). To help aid the students see some similarities, in particular that each person made a [whole] rhythm, the conversation was guided towards mathematical properties of the rhythms as expressed in the artwork.

Most students are able to properly identify the placement of a numerator and a denominator, but their explanations of what they are vary widely, with few extending beyond a shallow explanation of the numerator being `how many you have' and the denominator being `how many there are [total]'. This exhibited itself when we were discussing rhythms with fewer beats, and how would we represent a rhythm of just 1 beat. Ultimately the students settled on describing it in a fraction form of `1/1'. However, their understanding of their chosen representation as it relates to both math and other rhythms is where we were able to highlight an interesting phenomenon.

Since students chose `1/1', it was implied (by both students and instructor) that students meant there was 1 beat (numerator) and there was only 1 beat in the rhythm (denominator). This assumption was fueled by reducing the previous different rhythms further and further down to one. All students in this group except one did not explicitly represent rests, so the majority of the rhythms (shared on the whiteboard in front of the classroom) 'fit' into this description of having n/n beats. Our hope was to use this to pose a situation that highlighted that each rhythm is a whole, and that no matter how many beats there are, as long as they are all sounding and equal, then the rhythm is whole. This could be expressed and verified through the reflexive and symmetrical mathematical properties of their rhythms, that 1 is 1, 1/1 is the same as 1, 1/1 is the same as 2/2, is the same as 10/10, is then same as *n*/*n*. However, when increasing n, students were not as quick to respond that 10/10 is 1, and became less confident when presented with 21/21 and higher.

Knowing that the students are familiar with the rule of reduction (supported by other activities) shows their exposure to current math education. Their hesitance as *n* grows shows that they lack a true understanding of what *n*/*n* really means, because they likely were only exposed to *n*/*n* when *n* was 10 or less and memorized that rule, not understanding that it can be extended to or abstractly thought of as a whole.

8.3.2 Reducing Complexity

Students acknowledge many times, in many of our data forms, that they know understanding fractions is important for them and their future. They also acknowledge in all forms of our data that the subject is hard, something that they do not fully comprehend, and not interesting.

To tackle the lack of interest in math, as stated by the students and observed in previous studies, we started this study from an area that is of high interest to them, music. This has many positives as we have discussed (see 2.3 Math, Music, and in Between, 3 Our Prior Work, and 8.2 Other Domains and Approaches. It also has many challenges. As we all learn, there are several pedagogical tools and approaches that students and teachers use to communicate ideas. Some tools include (certainly not limited to) examples (in the form of oral, written, illustrative, and video), experiments (manipulables), literature (online, handheld, and handouts), and experiences. Some approaches that teachers use are providing comparisons, allegories, similes, and reducing complexity (as seen from the instructor's view of what the student knows, does not know, and is familiar and experienced in), or presenting the problem from a completely different perspective.

When these tools and approaches initially fail, teachers continue to try and approach the cognitive area that the troubled student(s) is in. This can only go so far, as classroom management limits a significant amount of direct and tailored education. This limits the teacher's attention and potential for making meaningful progress. The students, in particular those that have low SES resources, have less skill and resources to apply their own agency to manage their own meta-cognition to move towards the teacher's approach and `meet-in-the-middle'. This creates a huge impasse. Students are frustrated and unable to explain why, and teachers continue to teach for the larger class and recommend additional opportunities that are exterior of the classroom (tutoring, after-school, or additional homework or activities) that students and their parent(s) may not be able to accommodate.

SoF challenges students to experience music first, and inquire about the properties that can be expressed in the math domain. We start off our curriculum by allowing the students to create their own rhythm (see Figure 11-3 and Figure 11-4), and challenge them to create a representation of that rhythm so that another can reproduce their rhythm from looking at their representation, with little to no guidance at all. This is not an easy task. Most students immediately are interrupting the instructor that they do not understand, despite being prequalified that (a) it is challenging, (b) We will repeat the task many times and successfully slower, and (c) will help the students separate the task into smaller blocks.

Despite initial misunderstandings of the task to create their own representation of their rhythm, the students begin to make their own rhythms that were rather complex (in number of beats, speed, bi-modal, wide range of beat sizes [small, medium, and long size/duration], rhythm-less [not a perceivable synchronized rhythm or poly-rhythm], poly-rhythmic, or not cyclical). When the students then try to analyze their rhythm to begin to make their own representation, they hit a road block that is expressed in their face, their body, and their voice. When they revisit the task at hand though, prior to the teacher suggesting alternatives, the students self-regulate their initial preference for complexity to manage the creative micro-world that they are exploring. They do this by making rhythms that are simpler (with less beats, slower, and with a narrow range of beat lengths/sizes) so that they can understand how to translate the features that they want into a foreign context.

This phenomena of self-regulated learning in a foreign context is of particular interest to us. First, this is an example of them being able to self regulate. Students are able to do this in a domain that they are not normally able to navigate by themselves. Second, we know that their interest helps fuel their progress, as they would have been disengaged, their typical response to challenging math education. Third, students can navigate a task in a domain they struggle with by approaching it from a domain they can perform in. Fourth, students are able to make representations that are meaningful, and they continuously ideate over their representations of their musical domain to decide what is important to include and exclude in the problem domain of math. This translation allows for a solid building block to begin other exercises in the curriculum that require students to contextualize, identify, quantify, iterate, separate, and order the components of percussive rhythms.

8.3.2.1 Go Speed Racer Go!

Students initially want to create rhythms that are fun, resulting in one that is more complex in nature, with more beats, rests, divisions, tempo, and polyrhythms. This is evidenced in 7.3.1 The distribution of rhythms made by students with a particular signature, 7.3.2 The distribution of recorded rhythms by duration, 7.3.3 The comparison of rhythmic complexity against the number of beats tapped, 7.3.4 The number of instruments used in each rhythm, and 7.3.7 Speeding Up versus Slowing Down. But what is also relevant is that students can't always operate in the complex musical world when asked mathematical questions, especially as the questions become complex, nuanced, and particular. Students struggle to properly identify all of the musical parts, let alone properly translate what parts are mathematical and which ones of those are important.

However, students are able to reduce the complexity of their rhythms to a level that still elicits the mathematical question at hand, and approach the mathematical question from a musical perspective. This helps solidify that our approach is appropriate, culturally relevant, mathematically meaningful, and relevant for learning.   It is also important to not that the students themselves regulate the rhythms they tap before the teacher instructs them to do so, showing they have agency within their own learning environment, as well as the appropriate skills to navigate towards meaning with less pedantic or demeaning instructions.

8.3.3 What is the numerator?

During our redesign of the interface and curriculum, we came to the decision to limit the interface to initially only support one manipulable whole unit per instrument (this may be thought of as a `measure' in a musical domain), and not support additional measurements to help teach more advanced concepts that surround mixed numbers. This decision was supported by technical limitations, limitations in scheduling school and student participants for the study, and being beyond the scope of a strong initial study. With a design that does not fluidly undergird multiple manipulables in the same manner as the rest of the mathematical concepts we support, we were surprised to find that students still operated with less confidence when considering the size of the unit.

When students created rhythms by tapping on the desk to artwork that they crafted themselves, or when SoF analyses music they tapped into the microphone, the concept of a rest [non sounding beat] is not the forefront in the new representation (artwork or SoF visual). For the artwork representations that the students made, only two included explicit rests (one in each group). All others represented the size (often related to intensity) of the beat by the space between the items representing the beats. In the representations that SoF creates, rests are in greyscale with lower opacity. While both choices seem to align in their respective visual representations, this posed a problem for students, one that challenges how to represent multiple beats of varying size in the same rhythm.

Some students tried to rearrange SoF graphical beats so that they were closer to each other, leaving a larger non uniform space between beats, as to give the beat the appearance (graphical and aural) of being larger and louder. In representations in SoF that highlight division (the bead and the line), the graphical beat does not change size, only location, reinforcing the idea that the beat remains the same size in relation to all of its beats, the same way that the representations that highlight duration (Pie and Bar) have graphical beats that change size uniformly and immediately when a beat is added to or removed from the rhythm. This calls attention to the lack of understanding from students to recognize the true size of a unit. The students struggle to understand initially that a bead and a line (mark) remain constant graphical sizes (diameter in relation to circumference and height in relation to width respectively) with relative equal unit distance between regardless of the size (or duration) of a whole (rhythm).

8.3.3.1 What happens to the size and fraction of the numerator as the whole changes?

To help work through describing an individual beat's size, we illustrated their verbal descriptions on the white board. When asked which representation to work with, students chose to focus on the bead and the pie representations. Prior to using SoF, students were challenged in an activity to describe what a representation (both parts and whole) of a rhythm (beats and rhythm respectively) looks like after it is doubled in size. Group 1's descriptions were sufficient enough to draw a pie representation that, according to their descriptions, included 11 pie slices arranged around the circle with the same distance as it was previously. This led to a pie representation that only accounted for one half of the whole circle, as seen in Figure 8-4. When the students then were questioned about the remaining half, they were able to work through discussions that they had to separate the actual size of the pie slice from the relationship to its whole. Understanding that there are numerous ways to describe a beat was a pivotal point in their understanding of fractions, and supported by the affordance of SoF with regards to supplying multiple representation types that are manipulable as a whole. This learning moment was supported when it came time to draw the bead representation twice the size of a previous representation, where students both correctly identified the order and placement of the beats, but also correctly and confidently described the size and fraction of each beat with relation to each other and the whole rhythm.

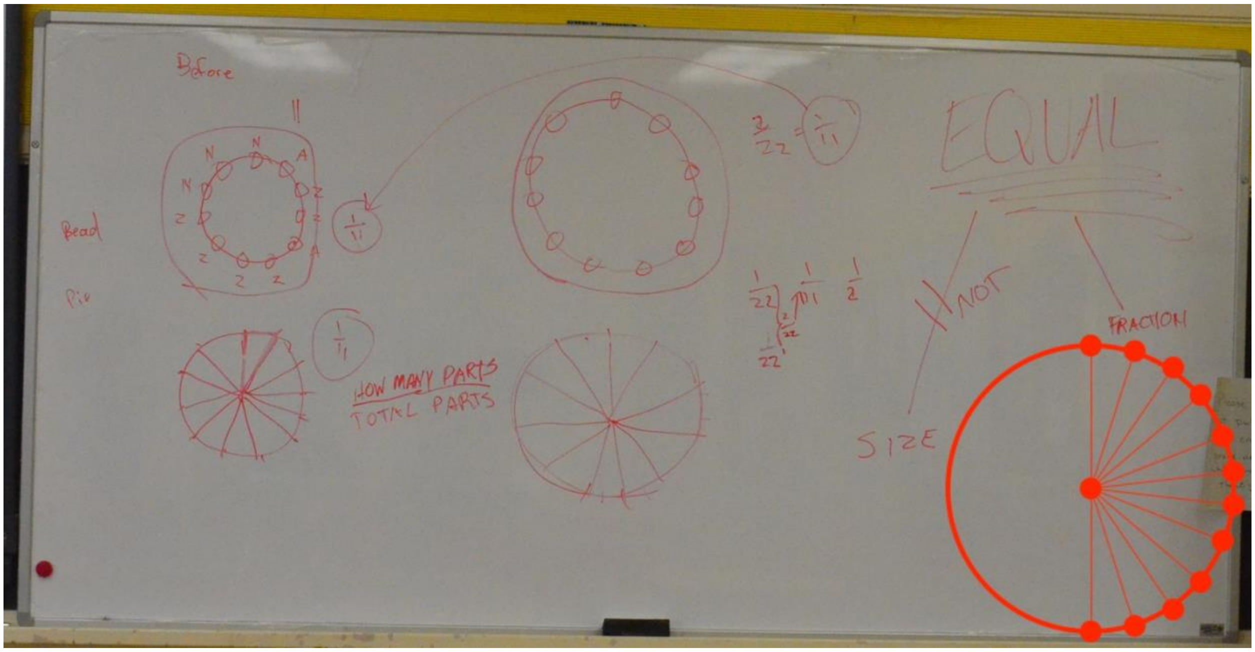


Figure 8-4 The whiteboard showing the bead and the pie representations (chosen by the students as preferable to work with) before and after doubling the size of them. The pie representation previously (shown in the bottom right) was divided into 11 equal parts, but only occupying 50% of the doubled circle. The number of iterated parts (made by the students) of the now doubled Pie representation equaled the previous smaller Pie, but the design was starkly different and required reworking to ensure that the unit fraction remained the same after the whole (rhythm) was doubled (in duration).

When performing the same activity with SoF and manipulating the representations, students were accurately able to stretch the representations to become twice as large and describe the position and size of the beats in both circular and linear representations.

8.3.3.2 Which comes first?

Students descriptions of order of the beats (first, second, third, etc…) were correct, but to ensure that they were not just guessing (given the third remains the third when manipulating the size of the whole), we posed questions about which beat will come first when comparing two different rhythms. In Group 1 we asked if the second beat of three comes before the second beat of four, and for Group 2 we asked if the second beat of two comes before the second beat of three (as seen in Figure 8-5). Both groups had students that incorrectly thought the beat further (larger) along the rhythm came before the former beat (smaller).

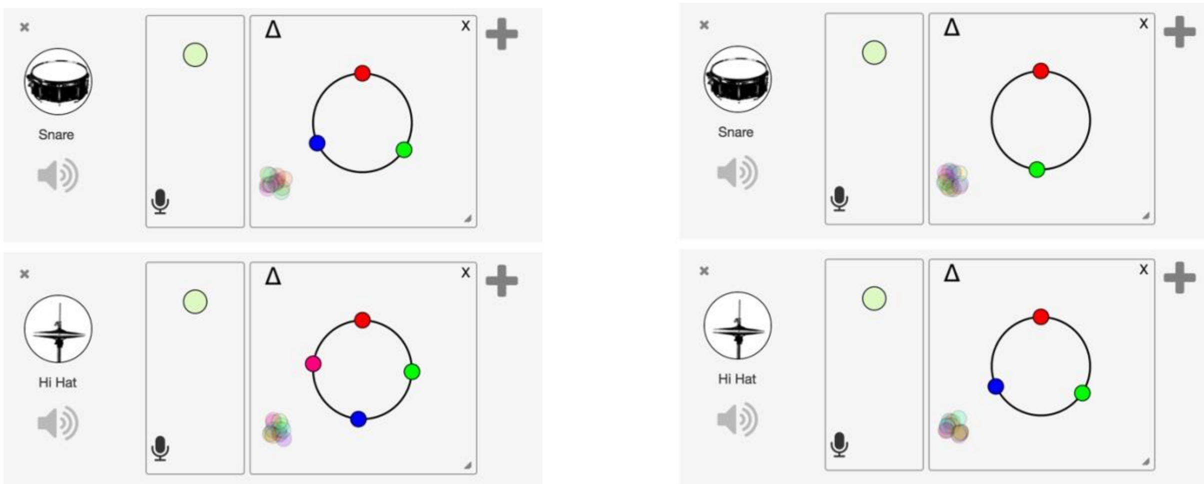


Figure 8-5 Which second beat comes first? The left is what was asked of group 1, the right is what was asked of Group 2.

When questioned, the students who hypothesized incorrectly gave answers that were related to its position on the circle. While their statements were correct ("one is closer to the bottom than the other" and "the one with three is closer to the right than the one with the two") they are not pointing toward the ordered and scalar attributes that we want them to see. Their answers hint that they have a competing understanding of which beat is first in the circular bead representations. Rhythms, especially those represented as a Bead, which has a strong suggestion of a never-ending repeating cycle, are harder for students to identify the beginning of a rhythm during inspection at any given point during playback other than the initial playback, which must include aural feedback, visual feedback, or both. This phenomenon is confirmed form the beginning of the study when listening to rhythms of Taylor Swift (Shake it Off) and Queen (We Will Rock You), which both were looped indefinitely, and students chose different starting points, forgetting what the first sound was when the music started playing.

This does not mean that the students are unable to identify it the correct answer, only that they are not yet confident in their answers in the situation when they have [circular] representations that conflict with their understanding of fractions and their experience of the rhythm. To help them confirm or change their answer, we allowed discussion and to add an accompanying Line representation. This greatly helped the students to see the more important attributes when comparing the unit fractions, size and order (by placement). This is because the linear representations have more clear start (left most mark) and end (the rightmost part of the rightmost beat) positions that the students can use to consider the quantity (by size) and order, by both position. The line also helps students map events temporally. After adding the line representation, students were able to more accurately provide reasons as to why the smaller units with larger denominators come before the larger units with smaller denominators. They provided descriptions similar to "it depends on where the beat is on the line [or circle]". So they are being forced by the representations to think about the correct characteristics.

Upon playing the rhythm in SoF, it was clear to students which beat[s] came first. This is an excellent case of how SoF provides a way for students hypothesize about math related questions that they are not as confident in, and to identify the core concepts when both the embodied, aural, temporal, and visual representations are share a uniform, proximal, synchronized, and meaningful performance.

8.3.4 What is the denominator?

Another major component of fractions is the denominator and what it represents. A denominator is more complex than just saying how many parts there are. It helps explain what parts to consider, what parts to ignore, that the parts are all equal, how big the the whole is (by virtue of how big the part(s) are), and how big each part is (by virtue of how big the whole is). Our study helped understand a consistent phenomenon that exists when using embodied rhythmic performances to approach math, that students tend to look identify some parts (beats that make sound) more often than all parts (beats and rests that do not make sound).

Throughout the activities in both groups, students were asked to describe the rhythms (whether they were created by SoF or the student's themselves). The portion of their descriptions that dealt with identifying how many parts there were were predominantly one sided towards describing how many parts were beats (made sound and had full opacity color). We had to consistently ask again (sometimes more than twice) for students to consider the entire rhythm and all of its parts, including those parts that are 'turned off'. Students are categorizing the parts based on their state.

Most questions asked students to describe their rhythms and representations including all parts (beats and rests). Even basic descriptions allow the students to delve further into more complex descriptions and relationships that help describe situations in meaningful ways and communicate them more clearly. When students only focus on one portion of the rhythm, they undermine their progress because they are failing to see the more important complete larger whole rhythm and whole portion of a fraction. Problems posed to students in textbooks are often also focused on a limited scope, asking how many apples in the basket of fruit. When one understands fractions properly, one can identify what is important and what is not. When a student is confused about the system of fractions, they can be confused as to what to consider and what is important, making it more difficult for them to answer questions that seemingly direct focus on one portion, without clearly identifying all the components. SoF helps counteract this by enforcing unit fraction rhythms, providing a labeling mechanism for the entire rhythm, constant inclusion of both beats and rests (two inherent properties in non-metronomic rhythms), allowing the state of the beat to be binary, and providing aural feedback for verification.

8.3.5 What is a fraction?

Students entered into our study with their own preconceived notion of what a fraction is and their own experiences with the system of fractions. They also already were already familiar with music and have skills related to making music and rhythms. As noted in 6.5.1 Pre-post Test – What are fractions, after students participated in our study, they were able to provide more descriptions than before, descriptions that are more robust, and highlight new abstract thinking in contexts they previously had not explored. It is also important to look at how the students were able to map the musical and math patterns to each other.

8.3.5.1 pattern.map(function(Music||Math){ return Music ? Math : Music })

When students are asked to make a rhythm, whether it is reiterated to be repetitive or not, they always made one rhythm, and did not repeat it. We propose this is for many reasons. First, students are concentrating on performing the physical actions of tapping their internal creation, and managing that performance feedback loop. This feedback loop involves tapping and listening to what it sounds like and making adjustments as needed. The tapping persists within the rhythm. This highlights their creative skills and abilities, and their interest in music with regards to perceiving, creating, performing, and entraining patterns. Second, students are trying to concentrate on the [curricular] task at hand, and therefore pause after each performance to evaluate if their rhythm and performance achieve the goal. This shows that students are able to analyze, mimic, repeat, and critique patterns. Thirdly, students adjust their rhythms over time. They do this to experiment and adjust towards a goal. These actions highlight students' abilities to manipulate, enhance, perfect, and familiarize musical patterns.

Understanding the complexity of musical performances is important, especially as it pertains to knowledge transfer across domains. In the mathematical domain, students also share many of the skills related to patterns. Infants understand logarithmic change. Very young children learn to count. Kindergarten aged students being to categorize and measure. Younger primary education students begin to understand common units and manipulations. Towards the end of primary education, students will work with higher order functions, ratios, and applied approaches. As students progress in their academic careers, their skills for contextualizing, identifying, creating, and manipulating patterns grow.

Both domains have skills that are challenging. This is confirmed by students disengaging in either or both domains, in addition to displaying lower grades or performances. A compelling strength of including both math and music is that students can draw upon experiences, rules, representations, and vocabulary in one domain, and concretely map them to the same in the other domain. We focus primarily on meaningful progression in the math domain, to helps students understand the system of fractions, but we do so with experiences in both domains. Helping students move between both domains helps students perform wholly in a single domain better. This is analogous to language acquisition. Learning and performing in a second language helps you in your native tongue.

With students actively participating in a structured curriculum supported by aural, visual, and physical context that have strong embodied and temporal components, students can develop stronger understandings in each domain, as well as more abstract reasoning to apply their knowledge beyond the classroom setting. This is evidenced by students' performances of music making in clearer subsequent performances coupled with their ability to recognize and describe mathematical patterns.

8.3.5.2 Descriptions and Definitions of Fractions as they Relate to SoF

Students were asked throughout the study to explain what fractions mean to them, what they are, identify the parts and whole(s), make representations of fractions, and apply their understanding in different contexts. Student definitions are often initial terse, off-topic, not comprehensive, and fail to identify important component or relationships about fractions or the system that they describe. This is discussed in further detail in 6.5.1 Pre-post Test – What are fractions. It is of no surprise that when tasked to make physical representations of their aural rhythms so that others can replicate them, that students are able to provide rich representations based on music [63]. It is also no surprise that students can make complex rhythms as described in 8.3.2.1 Go Speed Racer Go!. What is interesting is the explanations of fractions towards the end of the study, after incorporating SoF and the accompanying curriculum.

But why are the mathematical definitions initially shorter and less complex initially, but the musical representations (aural, embodied, and tangible) are not, and then the mathematical definitions longer, stronger, and more meaningful to the students and also mathematically? The short answer is that our approach is promising. Our reasoning is because:

- Students can use their own creation and skills in one domain and with guided instruction can identify the important components for a given context
- Students who are exposed to and operate within richer environments experience and provide richer descriptions
- Representations and embodied interactions with targeted mathematical components provide meaningful avenues to understand relationship and representational similarity

8.4 Research Question 3: How do multiple interactive culturally-relevant activities affect engagement in a learning environment?

With several examples of how our approach provides meaningful academic progress and skills that enhance their problem solving skills, we need to evaluate the engagement of students and the impact it has.

8.4.1 Hey Mikey, I think s/he likes it!

During our study we asked questions to determine what the students found fun, interesting, motivating, and educational. Throughout the study, students repeatedly replied to inquiries about what they liked, and they replied that they liked listening to and making music, making tangible representations, hearing and interacting with their rhythms, seeing different representations and their similarities (see I.E Group 1 Video Data and Notes 11.1 Group 2 Video Data and Notes for occurrences). Students appreciated the examples tied to music that they themselves created, individually or as a class group or class, evidenced by statements such as ‘Now I get it’, and acknowledgement of using smaller steps. They commented positively on popular musical references. Students also rushed into the classroom with excitement, even after returning from break. They also enjoyed making references learned in class to other experiences outside of school.

8.4.2 Multiple Representations

Math has many representations, even before reaching fractions, and especially reaching far beyond. They each have their purpose too. As with all representations, there is a trade off, highlighting one feature and backgrounds another. Utilizing a single feature risks losing another altogether. But creating, identifying, and manipulating each representation challenges people to be engaged with the underlying principle(s) that the representations seek to represent. The beauty of our approach is that the students operate within the educational environment first without knowing the underlying ideas, concepts, and principles that we are trying to educate them on. They simply have so many to work with that they are excited by the idea to choose, an area of agency that is often overlooked in some settings. By providing students with multiple representations and allowing them the opportunity to choose is a strong reason why we are able to capture their attention in our curriculum.

8.4.3 Novel Representations

When we present the Bead representation, its novelty is challenging. Many of our exercises focus on interacting with the Bead, but students also chose it when given the option to work with any representation type. The Bead provides circular, repetitive, cyclical, segmented, divisional, scalar, ordered, grouped, prototypical, colorful, interactive, shrink-able, stretch-able, transition-able, manipulable, static, active, attractive, simple, and approachable properties. Some of these attributes are conflicting, such as stretch-able and shrink-able, active and static, and grouped and divisional. These juxtapositional properties, along with the others, are carefully integrated into this single representation to allow contextual exploration of what a fraction can describe. By providing multiple various activities for students to discover and interact with these properties, the students are not only interested, but remain intrigued.

8.4.4 Rich [mathematically oriented and musically related] Representations

Just like the Bead representation, the other three of the Core Four provide mathematically rich properties. SoF also helps provide musical representations that are accessible to the students, who are interested in music. Students are able to hear and recognize their rhythms that they record in SoF. This provides a sense of accomplishment and successful communication to another, albeit themselves via the computer (especially since the challenge of making tangible representations of their rhythms is quite complex, unguided, and generally unreproducible when sharing with a classmate).

Students tendency to gravitate towards faster or more complex rhythms in non directed situations also shows that we capture their attention. The complexity of their rhythms in comparison to the activity at hand keeps their curiosity, and the scaffolding and structure of the curriculum helps ensure that the mathematical idea is properly communicated. SoF and the curriculum also provide mechanisms to consistently build upon and accurately hone their descriptions of their rhythms and other representations in mathematical and fractional terms.

8.4.5 Transitions

'Woah!.......' This expression was observed many times and in many forms when students discovered that the Core Fore could be graphically transitioned between each other. This awesome property also provides many computational thinking properties, such as abstraction, functions, and modeling. Its power of making clearly defined one to one associations also lends its hand to describing non math domain situations into standard mathematical representations, especially in conjunction with our labeling mechanism. Given the Core Four can be transitioned between each other, SoF provides twelve separate transitions alone that students can operate and study. It also reinforces the idea that representations are not immutable.

8.4.6 Engagement as a whole

Engagement in the classroom is powerful. If you can capture a student's interest, curiosity, and perseverance consistently, then the learning environment and the topics being discussed are engaging, meaningful, and rewarding (not just for the student). Students also commented that they struggled with certain activities, but were able persevere with answers and reasoning on their own given the proper amount of time to think and explore.

In short, students experienced excitement and engaging participation with SoF in a way that made it possible for them to interact with math in an academically meaningful way they had previously lost.

9 FUTURE WORK

We have shown that there is a great need to approach fraction learning from a musical, embodied, and culturally relevant domain that students maintain interest, can perform accurately in, and make meaningful academic progress. This study, while quite extensive, does not provide for all domains or contexts thoroughly, and our research has shown that there is strong potential for more progress.

9.1.1 Enhancements for additional studies

To enhance our analysis, we would like to study log data in more robust way, and need a way to conceptualize and record data in a synchronized way across video and queryable data structures.

9.1.2 Other Domains and Approaches

- Calculus before arithmetic. Include music?
- Mixed numbers, multiple measures, non unit fraction rhythms.
- Better scheduling, handouts, and a button to separate tasks for the database

- Random patterns of new representations, to prevent pushing a standard way of representing

10 CONCLUSION

We focused on an important and challenging area of mathematics education where many students fail to make meaningful progress, fractions. By focusing on the social and economic backgrounds of the students who needed the most help, we developed a socio-technical program with an accompanying curriculum to address the situation from the students' perspective, and progressed using contexts familiar, approachable, performable, and interesting to students. This robust approach showed promising results of learning while providing an experience that highlights the whole and the parts at the same time in a culturally relevant set of activities. These embodied activities were performed alongside our aural and visual application that helped scaffold students' knowledge to gradually bring students into more standard mathematical representations and situations.

We have provided the history, a literature review, our motivations and prior work, and the data for review. We ran the study and analyzed the data to discover new phenomena that highlights future potential and possible areas of improvement. SoF provides a compelling, socio-technical microword, supporting the whole and the part at the same time, with embodied, culturally-relevant, strong representational infrastructure with aural, visual, physical, and temporal component classroom-based activities, oriented towards more standard mathematical representations and usages. We look forward to continuing this project as another tool for educators and motivated individuals to learn something new and share with the community at large.

11 APPENDIX

A. Abbreviations

SoF – Sound of Fractions - our socio-technical application to help teach fractions

CC – Common Core – The United States based Kindergarden-12 curriculum

SOL – The Virginia state based Standards Of Learning Kindergarden-12 curriculum

SES – SocioEconomic Status

UI – User Interface

UX – User Experience

B. Accessibility of this Thesis Document

This thesis is available at http://www.specialorange.org/thesis in many formats with accompanying slides, presentation, and links to data sources.

All figures should have alternate text for screen readers.

The colors of the document text are chosen to support color-blindness, and other colors in figures. The figures of the interface are not designed to support color blindness. Please accept our apology.

C. Blank Lesson Plan

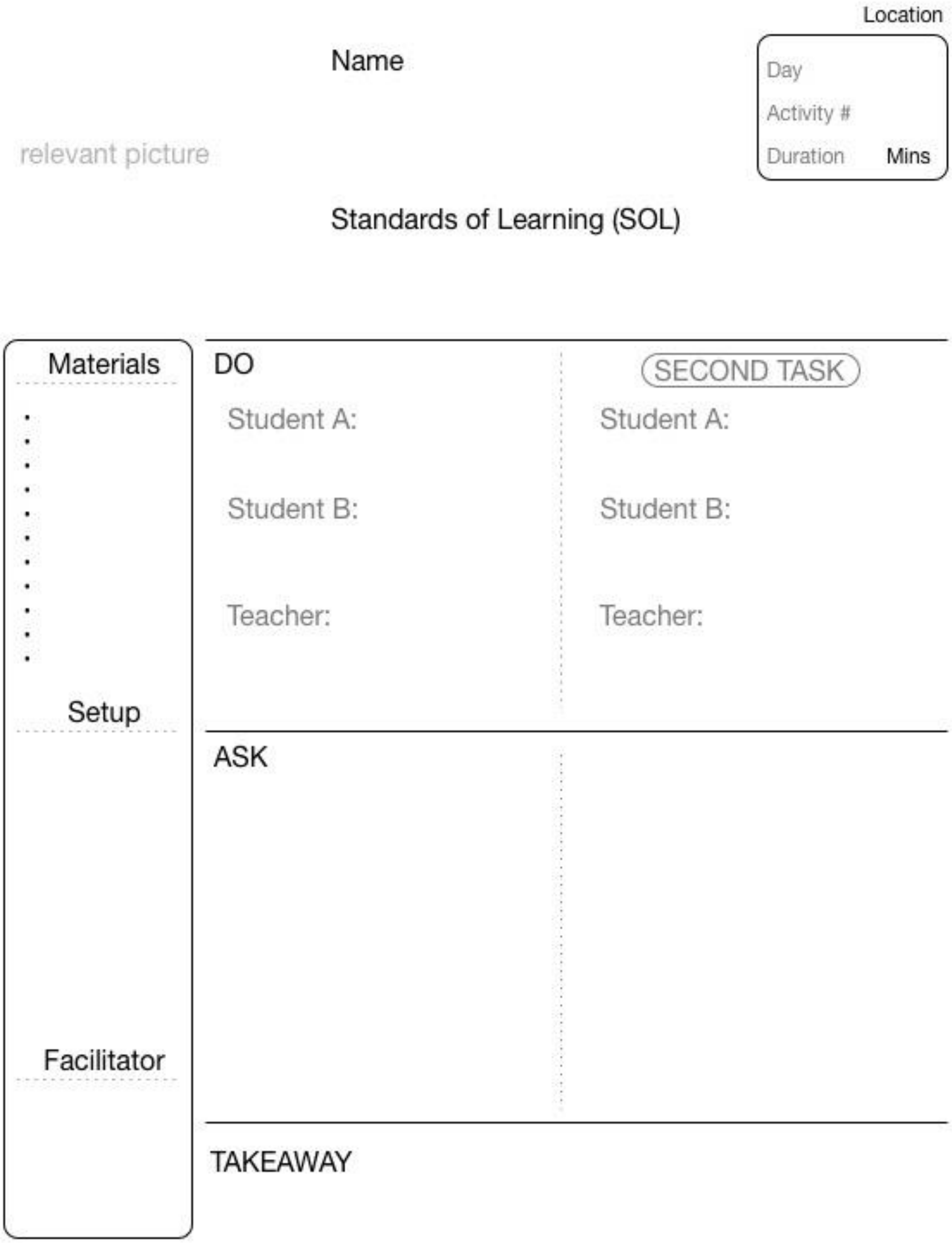


Figure 11-1 : The Lesson Plan planning page. This allows researchers to identify the meaningful components of a well designed lesson and communicate them easily for a teacher to review and reference during a class session.

D. Curriculum

What do Fractions mean to you?

Day	1
Activity #	1
Duration	5 Mins

Standards of Learning (SOL)
2.3 Identify Fractions as real numbers

Materials

- Speakers
- Notebooks
- Name placards

Setup

Distribute notebooks

Write names on placards and place so everyone can see

Facilitator

DO

Teacher:
Have some music playing when they enter.

Introduce ourselves, music, and math.

ASK

What is a fraction to you? [Write it in your notebook]

What is your favorite number? [write it down]

Why do we need fractions?

Do you have a favorite fraction? [write it down]

TAKEAWAY

What is a fraction to you?

Figure 11-2 What do Fractions mean to you? - Lesson Plan Day 1, Activity 1

Listening to a Rhythm

Day	1
Activity #	2
Duration	10 Mins

Standards of Learning (SOL)
2.20 Identify and Create Patterns
3.19 Extend Pattern Representations

Materials

- Speakers
- Samples
- Notebooks

Setup

Place speakers so that the whole class can hear the music.

Distribute notebooks

Facilitator

Pay attention to the vocabulary used

DO

Teacher:

"Listen to this:"
Play repeating song - is Taylor Swift Shake it Off
Let it play repeatedly for at least 30 seconds.
Start swaying/moving to the music to get the kids interested.
Halfway through invite them to clap along if they want to
Play again if needed.

Other songs
Happy Pharell Williams clap riff
We Will Rock You by Queen

ASK

What do you hear?
When does it start? How do you know?
When does it end [How long is it]? [make sure to focus on non time-based answers]
What is it like or similar to?
How do you know it is different from another rhythm [use previous vocabulary]?
How would you describe this rhythm [use previous vocabulary]? (W & P)
Write your description in your notebook.
[Teacher reiterates the important vocabulary words]

What is important?

TAKEAWAY

What is a whole. What is a part. How to describe them.

Figure 11-3 Listening to a Rhythm - Lesson Plan Day 1, Activity 2

Make your own beat and representation

Day 1
Activity # 3
Duration 30 Mins

Standards of Learning (SOL)

2.3 Identify Fractions as Real Numbers
2.20 Identify and Create Patterns
3.19 Extend Pattern Representations

Materials

- Tape
- Popsicle sticks
- Stickers
- Paper
- Pencils
- Colored Pencil
- Markers
- Straws
- Play-doh
- Cotton Balls
- Rubber Bands
- Plastic bags
- Containers

Setup

Distribute the materials to each group desk

Facilitator

Walk around the classroom. Avoid musical terminology/ representations

Look if they are tapping with one or two hands [Howison]

DO

Teacher:

Teacher taps own rhythm.
Now tap your own rhythm. We want everyone to create your own rhythm.
Tap it several times so that you are familiar with it.
Ask if everyone can do it or needs more time
Use the materials in front of you to create a physical representation of your rhythm so that another student can reproduce tapping your rhythm by looking at your representation.

SECOND TASK

Teacher:

Switch representations with your partner.
Try to play your partners rhythm while they listen.

NOTE Do not tap your own rhythm to help your partner reproduce your rhythm. Use words!

ASK

What is important about your physical representation?
Write your description in your notebook.

What was helpful of your representation to your partner?

Was your partner's representation helpful to you?

What was missing?

Ask some students to share with the class

TAKEAWAY

Representations are models of real world situations and things. It is hard to express everything. Some representations show the situation better than others.

Figure 11-4 Make your own rhythm and representation - Lesson Plan Day 1, Activity 3

Make a New Representation

Day	1
Activity #	4
Duration	20 Mins

Standards of Learning (SOL)

- 2.3 Identify Fractions as Real Numbers
- 2.20 Identify and Create Patterns
- 3.19 Extend Pattern Representations

Materials

- Tape
- Popsicle sticks
- Stickers
- Paper
- Pencils
- Colored Pencil
- Markers
- Straws
- Play-doh
- Cotton Balls
- Rubber Bands
- Plastic bags
- Containers
- Notepads

Setup

Materials should already be on the desk.

Facilitator

DO

Teacher:

Make a completely new rhythm.
Tap it several times so that you are familiar with it.
Make a new physical representation for your new rhythm, still so your partner can reproduce tapping your rhythm by looking at your representation.

SECOND TASK

Teacher:

Switch representations with your partner.
Try and play your partners rhythm while they listen.

NOTE Do not tap your own rhythm to help your partner reproduce your rhythm. Use words!

ASK

What did you add to your representation from last time?
What did you remove from your representation from last time?
What was helpful?
Write in your notepads

Was your partner's representation helpful to you? How?

What was missing from your partner's representation?

Ask some students to share with the class

TAKEAWAY

Representations/models can be modified. You can change the representation to better describe the actual item.

Figure 11-5 Make a new representation - Lesson Plan Day 1, Activity 4

Day 1 Homework

Day	1
Activity #	HW
Duration	Mins

Standards of Learning (SOL)

relevant picture

Materials

- HW card

Setup

Distribute the HW Cards

Facilitator

DO

Teacher:

Patterns are everywhere! They are in music, art, architecture, biology - both plants and animals, physics and chemistry. Most importantly they also exist in math. Two important patterns are wholes and parts.

During the next week, look for many different patterns and choose one you really enjoy. Then draw the whole and the parts on the back.

ASK

TAKEAWAY

Figure 11-6 Day 1 Homework Patterns - Lesson Plan Day 1, Homework

Describe what's important?

Day	2
Activity #	1
Duration	15 Mins

Standards of Learning (SOL)

asdf

relevant picture

Materials

- Notebooks

Setup

Tell to turn to page 2.1 in their book.

Facilitator

...

DO

Teacher:

Please hand me your HW cards from last week if you have them. (as they are walking in as well)
We talked about lots of different things last week. We made some different representations of your music, and we looked at each of them.

ASK

What is important about fractions?
Write in your notebook.

Describe what a whole is to you.

TAKEAWAY

Recap from last week and to get their mind prepared

Figure 11-7 Describe What's important - Lesson Plan Day 2, Activity 1

Mimicking Rhythms

Day	2
Activity #	2
Duration	15 Mins

Standards of Learning (SOL)

2.20 Identify and Create Patterns
4.2.b Identify Equivalent Fractions

relevant picture

Materials

- Notebooks

Setup

Tell to turn to page 2.1 in their book.

Facilitator

...

DO

Teacher:

We want you to work in pairs. Write your process in your notebook

Student A:

Tap a rhythm and keep tapping it over and over so that it is consistent.

Student B:

Mimic your partner so that you are both tapping the same rhythm at the same time.

SECOND TASK

Teacher:

Switch roles.

Student B:

Tap a rhythm and keep tapping it over and over so that it is consistent.

Student A:

Mimic your partner so that you are both tapping the same rhythm at the same time.

ASK

Are each person's actions equal?

How do you ensure that they are equal?

What needs to be changed?

Did you do the exact same thing as last time?

What was different?

What was the same?

TAKEAWAY

Recap from last week and to get their mind prepared

Figure 11-8 Mimicking Rhythms - Lesson Plan Day 2, Activity 2

What Size!?

Day	2
Activity #	3
Duration	15 mins

relevant picture

Standards of Learning (SOL)

2.20 Identify and create patterns
3.19 Extend Pattern Representations
4.2a Compare and order Fractions
4.2b Identify Equivalent Fractions
6.2 Order,Equivalence, and representation form of fractions

Materials

- Previous representations
- Pens
- Pencils
- Paper

Setup

Have students' representations labeled with their names to easily grab if they want to use it.

Facilitator

Ask about the parts AND the whole

DO

Teacher:

Take a rhythm you are familiar with, or make another one and get familiar with it. You can use a rhythm you tapped and make a representation or a representation you have made prior.
Identify the beats in your rhythm and label them on a separate piece of paper.
Share your rhythm representation with a partner nearby. Identify their beats of their rhythm in your notebook.
Identify other important features!

note : Try and get them to pair with a student that has different beat numbers and sizes.

Show the diagram of representations and labels (medical)

ASK

How do you know the size of a beat?
How do you label a bigger beat?
Did your names/labels match?
What labels do you need to include so that your partner can recreate the rhythm by tapping?

note : They may not label the whole, and ask them to

TAKEAWAY

Recognizing different units

Figure 11-9 What Size!? - Lesson Plan Day 2, Activity 3

Names	Numbers	Abbreviations	Icons	Physical	Aural
Doctor		Dr.			
Nurse		CNA, NP			
Ambulance	911				Siren
Neurosurgeon	180 cc				
heart	$\frac{105}{75}$				
heart rate	110 bpm				Bum Bum Bum....

Figure 11-10 The Medical Representations Handout - Lesson Plan Day 2, Activity 3

Make your own fraction representation

Day	2
Activity #	4
Duration	15 Mins

Standards of Learning (SOL)

2.20 Identify and create patterns
3.19 Extend Pattern Representations
6.2 Order,Equivalence, and representation form of fractions

relevant picture

Materials

- Pens
- Pencils
- Paper

Setup

...

Facilitator

...

DO

Teacher:

Look at these real numbers:

Show the diagram of number representations (numbers)

Circle the ones have you seen?
Which ones are new to you?
All of these are real numbers, and exist in the world. (show some pictures of them)
"Fractions come in many sizes, shapes, and representations.

Make a number representation of your favorite rhythm.

ASK

Q: What does a fraction look like without numbers?
Q: What is important to include when you make a fraction without numbers?

TAKEAWAY

Fractions are a concrete abstract way of describing relationships, specifically you can describe the relationships between the part and the while without numbers.

Figure 11-11 Make your own fraction representation - Lesson Plan Day 2, Activity 4

$\frac{1}{2}$ ¼ 75% 2¾ $\frac{19}{2}$ 12.5% 0.22 1.73 35×10^{0} $1 * 10^{3}$ 1.3333… $.\overline{6}$

.6 3.35 6.022 e-23 ½ 50% 0½ 0.5% 0.5 0.50 .5 .50 $5 * 10^{-1}$

Figure 11-12 The numbers representation handout - Lesson Plan Day 2, Activity 4

Play with the Sound Of Fractions

Day	2
Activity #	5
Duration	10 Mins

Standards of Learning (SOL)

relevant picture

Materials

- Laptops
- Internet Connection
- Pen
- Pencils
- Paper

Setup

Make sure the computers are connected. They have a group name signed in, and we have the UUID noted, along with who is in the group.

Facilitator

...

DO

Teacher:

We have made this program to help you make your own rhythms and representations. We call it, 'The Sound of Fractions' and hope you enjoy it as much as we do.

We want you to play around with The Sound Of Fractions.

ASK

What did you discover about the Sound of Fractions?

TAKEAWAY

An introduction to the program and getting them to think about new ways of representing parts and wholes.

Figure 11-13 Play with SoF - Lesson Plan Day 2, Activity 5

Day 2 Homework

Day	2
Activity #	HW
Duration	Mins

Standards of Learning (SOL)

relevant picture

Materials

- HW card

Setup

Distribute the HW Cards

Facilitator

DO

Teacher:

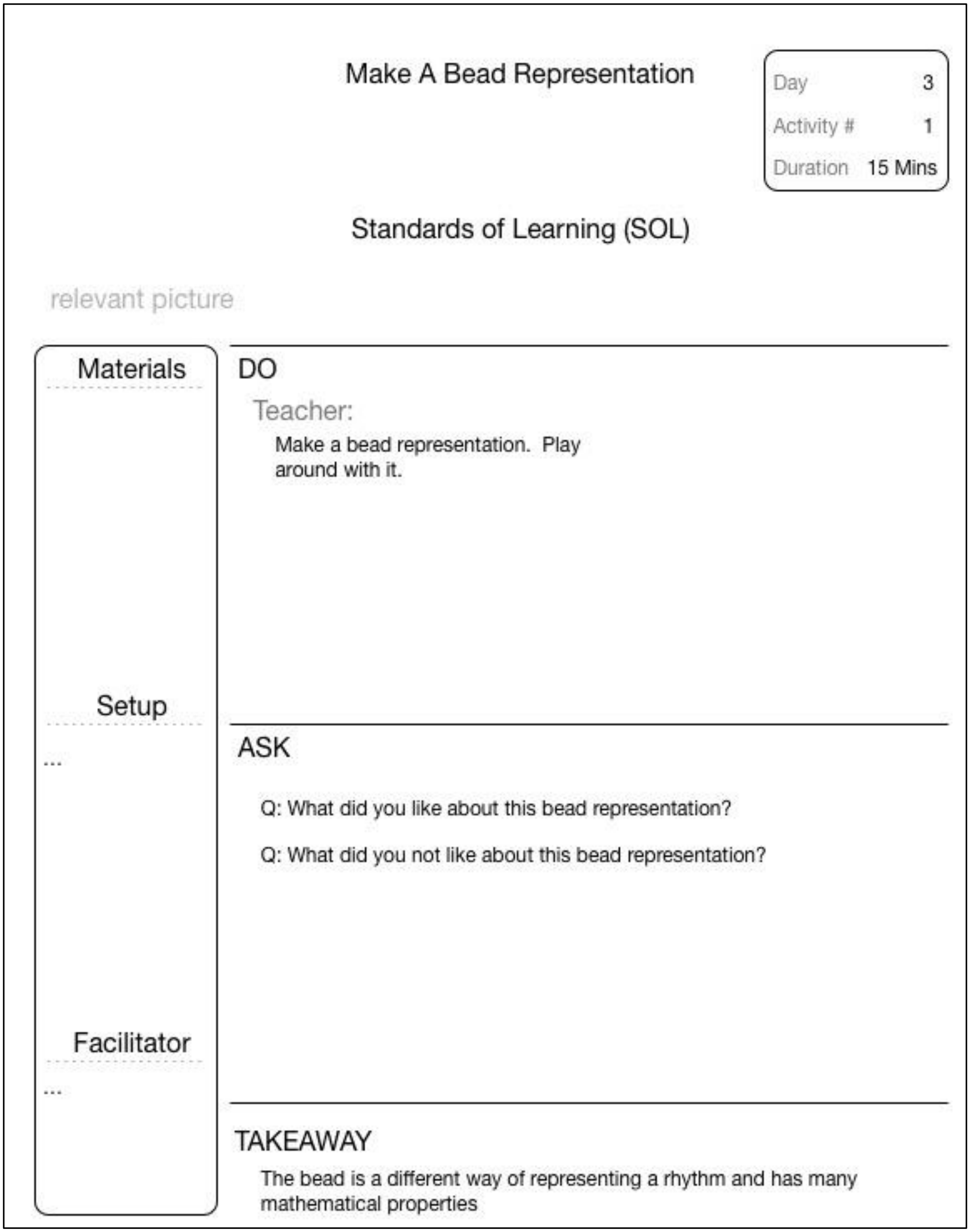
Two important patterns in fractions are wholes and parts. You can

During the next week, look for two different patterns and come up with as many names as you can for the wholes and parts. Then write the names of the whole and the parts on the back.

ASK

TAKEAWAY

Figure 11-14 Day 2 Homework



Make A Bead Representation

Day	3
Activity #	1
Duration	15 Mins

Standards of Learning (SOL)

relevant picture

Materials

Setup

...

Facilitator

...

DO

Teacher:

Make a bead representation. Play around with it.

ASK

Q: What did you like about this bead representation?

Q: What did you not like about this bead representation?

TAKEAWAY

The bead is a different way of representing a rhythm and has many mathematical properties

Figure 11-15 Make a Bead representation - Lesson Plan Day 3, Activity 1

Tap in your rhythm

Day	3
Activity #	2
Duration	15 Mins

Standards of Learning (SOL)

relevant picture

Materials

- Laptops
- Camera
- Video Camera
- Paper
- Pencils

Setup

Do something

Facilitator

Look for something

DO

Teacher:

Make sure you are logged in to SoF
Create a drum rhythm by tapping it into the computer.

Play the song back

SECOND TASK

Teacher:

Adjust the song to make it the way you want or change the rhythm to another pattern.

ASK

Q: Was the song played back what you expected?
Q: What did you not expect?

Q: Was the song played back what you expected?
Q: What did you not expect?

TAKEAWAY

Identifyng a whole and parts at the same time

Figure 11-16 Tap in your rhythm - Lesson Plan Day 3, Activity 2

What is a Whole?

Day	3
Activity #	3
Duration	15 Mins

Standards of Learning (SOL)

relevant picture

Materials

- Rhythm Cards
  - X X
  - X . X .
  - X X . X X .
- Notepads

Setup

Distribute the notepads.

Facilitator

...

DO

Teacher:

I am going to tap. Please listen:
X X (tap this just once: two beats with relatively no rest between them.)

Show the rhythm as "X" on a card

SECOND TASK

Teacher:

I am going to tap. Please listen:
X . X (tap this just once: two beats with a middle rest, the same duration of one of the beats)
Show the rhythm as "X . X" on a card

THIRD TASK

Teacher:

I am going to tap. Please listen:
X . X . (tap this just once: a beat with a rest the same duration, twice)
Show the rhythm as "X . X ." on a card

X = Beat
. = rest

ASK

Q: How many total parts are there?

If they say anything but '2':
Q: How many beats are there?
Q: How many rests are there?

Q: How many total parts are there?
Q: When does it end? How do you know?

Q: How many beats are there?
Q: How many rests are there?

Q: Was there a whole? Describe it. Write your description in your notepads.
Q: Was it played once, or was it the first rhythm played twice?

TAKEAWAY

You can't determine the whole without its parts

Figure 11-17 What is a whole - Lesson Plan Day 3, Activity 3

Day 3 Homework

Day	3
Activity #	HW
Duration	Mins

Standards of Learning (SOL)

relevant picture

Materials

- HW card

Setup

Distribute the HW Cards

Facilitator

DO

Teacher:

Patterns are everywhere! You can even adjust a pattern if you know how. Two important components of a pattern are **whole** and **parts**.

ASK

Look around you this week and find a pattern that you want to change. Draw the pattern before and after, and highlight what you changed.

TAKEAWAY

You can change a pattern, but you must know what you are changing and how to change it appropriately.

Figure 11-18 Day 3 Homework

All the representations!

Day	4
Activity #	1
Duration	15 Mins

relevant picture

Standards of Learning (SOL)

2.20 Identify and create patterns
3.19 Extend pattern representations
6.2 Order, equivalence, and representation form of fractions

Materials

- Laptops
- Physical representations from Day 1
- Photos for each student of their representation

Setup

Distribute the photos of each students' physical representation from day 1.

Facilitator

...

DO

Teacher:

Make an instrument with one of each representation.

SECOND TASK

Teacher:

Teacher will also display some of the physical representations from day 1.

Write the main ideas/words on a white board.

ASK

What is important about each representation?
How are the representations in the SoF similar?
How are the representations in the SoF different?

How are the digital representations similar to the physical representations you made earlier?

TAKEAWAY

What is conserved between representations and what is important.

Figure 11-19 All the representations! - Lesson Plan Day 4, Activity 1

The Other Part

Day	4
Activity #	2
Duration	15 Mins

Standards of Learning (SOL)

3.19 Extend pattern representations
4.2.b Identify equivalent fractions
6.2 Order, equivalence, and representation form of fractions

relevant picture

Materials

- Laptops
-

Setup

Do something

Facilitator

Look for something

DO

Teacher:

Student A:
Create a rhythm that has some beats turned off.

Student B:
Create a complementary instrument that plays beats where your partners rhythm does not play a beat.

SECOND TASK

Teacher:
Switch roles.

Student B:
Create a rhythm that is 2 times longer than your previous rhythm

Student A:
Create a complementary instrument that plays beats where your partners rhythm does not play a beat.

ASK

Who is playing more of the rhythm?

How much is each person playing of the rhythm? [try and get in terms of both whole and parts]

Who is playing more of the rhythm?

How much is each person playing of the rhythm? [try and get in terms of both whole and parts]

Why are they the same amount as before?

TAKEAWAY

Percentages and unit fractions don't matter when dealing with wholes of different sizes.

Figure 11-20 The Other Part - Lesson Plan Day 4, Activity 2

Which Comes First?

Day 4
Activity # 3
Duration 15 Mins

Standards of Learning (SOL)

4.2.a Compare and order fractions
5.2 Equivalence and order of fractions
6.2 Order, equivalence, and representation form of fractions

relevant picture

Materials

- Laptops
- Notebooks

Setup

Tell to turn to page 4.3 in their book.

Facilitator

...

DO

Student A:
Create a rhythm using four(4) beats on the SoF

Student B:
Create a rhythm using three(3) beats on the SoF

Teacher:
Predict which beat will play first. Write it down.

Then hit play at the same time.

ASK

Was your prediction correct?
Can you explain why or why not?

Is the first beat that plays bigger or smaller than the other first beat?
Can you explain?

TAKEAWAY

The more parts, the smaller each part gets in a whole.

Figure 11-21 Which comes first? - Lesson Plan Day 4, Activity 3

HW 4

Day	4
Activity #	HW
Duration	10 Mins

Standards of Learning (SOL)

3.19 Extend pattern representations
4.2.b Identify equivalent fractions
6.2 Order, equivalence, and representation form of fractions

relevant picture

Materials

- HW 4 card

Setup

Distribute the HW cards

Facilitator

...

DO

Teacher:

There are many things in the world that are different sizes. Sometimes we need to describe only one part of the whole.

Look for two objects, one that is very small, and one that is very large. Draw them.

Now draw a part of each object so that they are the same fraction as each other.

ASK

...

TAKEAWAY

Percentages and unit fractions don't matter when dealing with wholes of different sizes.

Figure 11-22 Day 4 Homework

Recap

Day	5
Activity #	1
Duration	10 Mins

Standards of Learning (SOL)

Materials

- Notebooks

Setup

Facilitator

DO

Teacher:

There are many things in the world that are different sizes. Sometimes we need to describe only one part of the whole.

Look for two objects, one that is very small, and one that is very large. Draw them.

Now draw a part of each object so that they are the same fraction as each other.

ASK

What do you notice about the fractions?

How do you know they are the same fraction?

TAKEAWAY

Exposure and experience that you can manipulate the whole, the parts, or both.

Figure 11-23 Recap - Lesson Plan Day 5, Activity 1

Beads vs Lines

Day	5
Activity #	2
Duration	10 Mins

Standards of Learning (SOL)

Materials

- Notebooks

Setup

Facilitator

DO

Make one instrument and tap your own rhythm. Add a Bead representation.

Make a second instrument with a Bead representation and make it ony have 4 beats

SECOND TASK

Clear the SoF

Make one instrument and tap your own rhythm. Add a Line representation.

Make a second instrument with a Bead representation and make it ony have 4 beats

ASK

Describe your rhythms and representations in your notebook.

What is ‘exactly’ the same?

What is similar?

What is different?

Which has more?

Describe your rhythms and representations in your notebook.

Compare them.

TAKEAWAY

Figure 11-24 Beads vs Lines - Lesson Plan Day 5, Activity 2

Ordering your Rhythm

Day	5
Activity #	3
Duration	20 Mins

Standards of Learning (SOL)

Materials

- Notebooks

Setup

Facilitator

DO

Teacher:

Tap a rhythm with N beats where N equals the number of students. Get the students to clap/tap along with you.

Students:

Tap the rhythm

SECOND TASK

Teacher:

Help the students

Students:

Tap the rhythm back in a different order.

ASK

Describe the rhythm. {Try to get students to focus on the order}

Who plays first? second? next? last?

What is different?
What is the same?

Why can you rearrange the beats and still hear the same pattern?

TAKEAWAY

The order of unit fractions does not matter.

Figure 11-25 Ordering your rhythm - Lesson Plan Day 5, Activity 3

Identify and Label What is Important

Standards of Learning (SOL)

Day	6
Activity #	1
Duration	30 Mins

Materials

- Notebooks

Setup

Facilitator

DO

Teacher:

Ask for a favorite number between 1-16 from student A (preferably 5 or higher).
Ask for a favorite fraction from Student B.

Write in your notebook how many beats would be on with {fraction} of beats {number} in a rhythm. How many would be off?

SECOND TASK

Teacher:

Create as many additional representations you want of that same rhythm.

Label each representation differently.

ASK

Describe the beats and rhythm in your notebook using fractions.

How can you label them differently using numbers and fractions?

How can you label the parts/ beats? the whole/rhythm? the order/sequence? the duration? the elapsed amount? the size of the beats? the quantity of the beats? the on beats? the off beats?

TAKEAWAY

Many ways to describe parts and wholes using numbers and fractions.

Figure 11-26 Identify and Label what is important - Lesson Plan Day 6, Activity 1

Transitioning to other representations

Standards of Learning (SOL)

Day	6
Activity #	2
Duration	30 Mins

Materials

- Notebooks

Setup

Facilitator

DO

Teacher:

Make a rhythm with a bead representation.
Describe the representation using math terms in your notebook.

Add another bead representation, and transition it to a number line.

SECOND TASK

Teacher:

Write in your notebook what you think will be different/same about a pie representation.

Add another line representation and transition it to the pie.

THIRD TASK

Teacher:

Transition between all the representation types.

ASK

Describe the bead in your notebook. Can you point out the important parts using fractions?

What is similar/different between the bead and the line?

SECOND TASK

What is same?different? (write in your notebook)

THIRD TASK

What are the important features that all the representations share?

TAKEAWAY

Noticing the important parts.

Figure 11-27 Transitioning to other representations - Lesson Plan Day 6, Activity 2

E. Group 1 Video Data and Notes

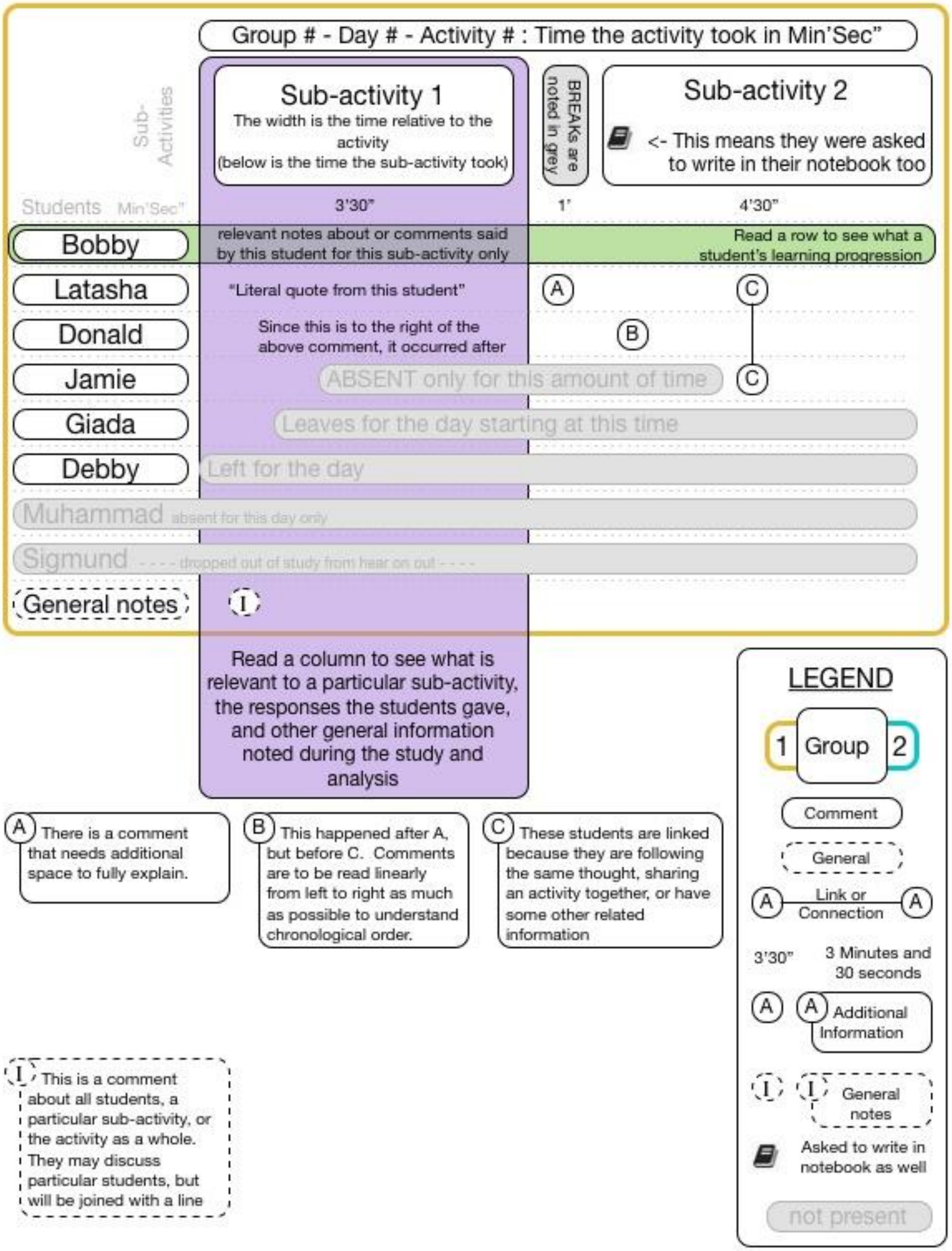


Figure 11-28 A Legend and description for reviewing video data of activities and sub-activities

1. Group 1 Day 1

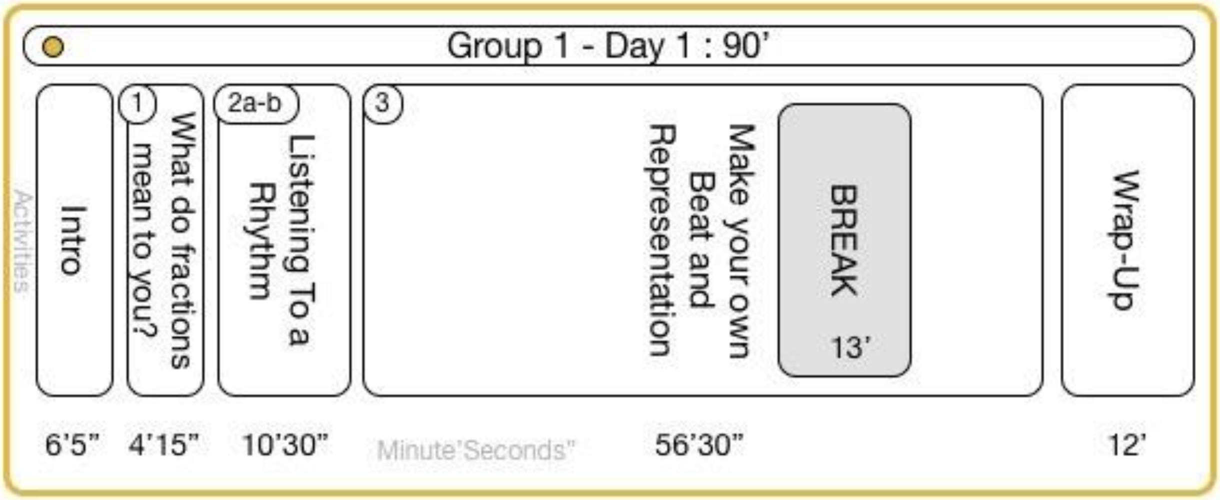


Figure 11-29 An Overview of Group 1 Day 1 Classroom video data

a) Group 1 Day 1 Activity 1

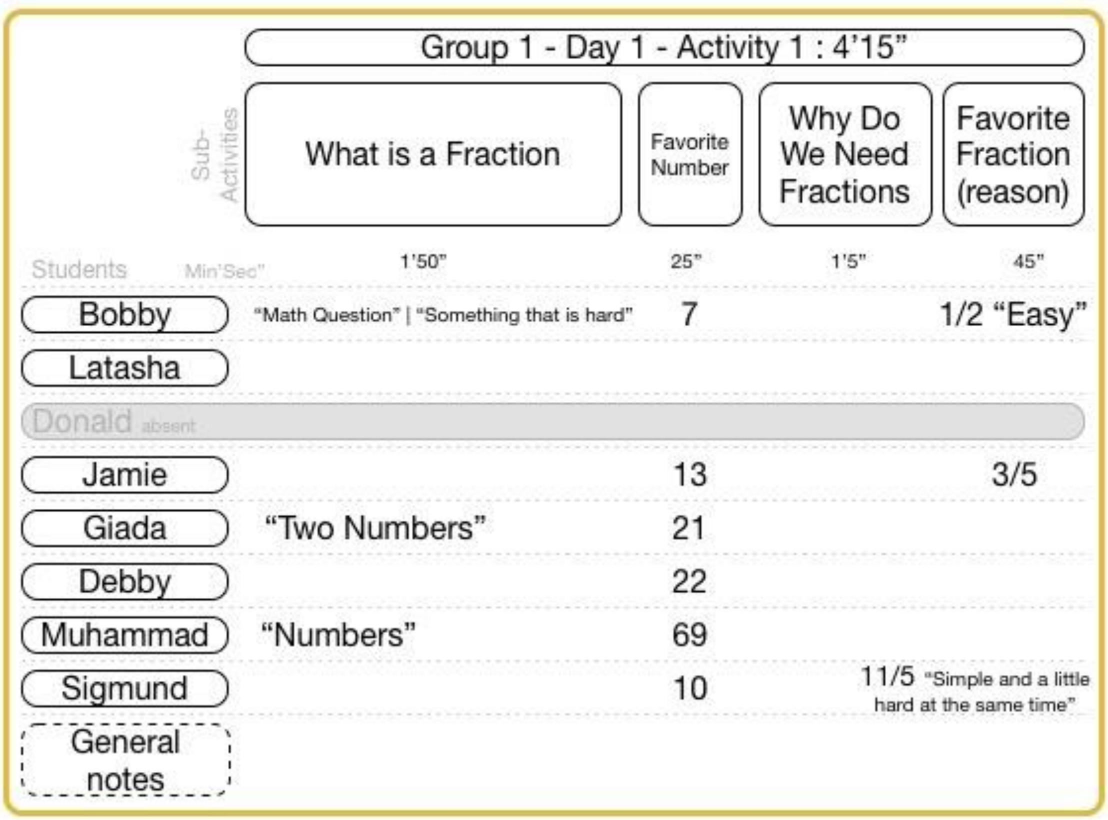


Figure 11-30 Group 1 Day 1 Activity 1 Classroom video data

b) Group 1 Day 1 Activity 2 (a and b)

Group 1 - Day 1 - Activity 2a : 5'						
Sub-Activities	What do you hear?	When Does it Start?	When Does it Stop?	How Long does it last?	What do you hear in between?	Do they participate in clapping along?
Students / Min'Sec"	1'20"	1'10"	20"	30"	30"	1'20"
Bobby						✓
Latasha						✓
Donald absent						
Jamie	Shake It Off by Taylor Swift				Drums	✓
Giada	Taylor Swift					✓
Debby						✓
Muhammad						
Sigmund		On the Third Beat	I Don't Know	2 minutes	Nothing	✓
General notes		The wording of this question is bad, students have too much difficulty				

Figure 11-31 Group 1 Day 1 Activity 2a Classroom video data

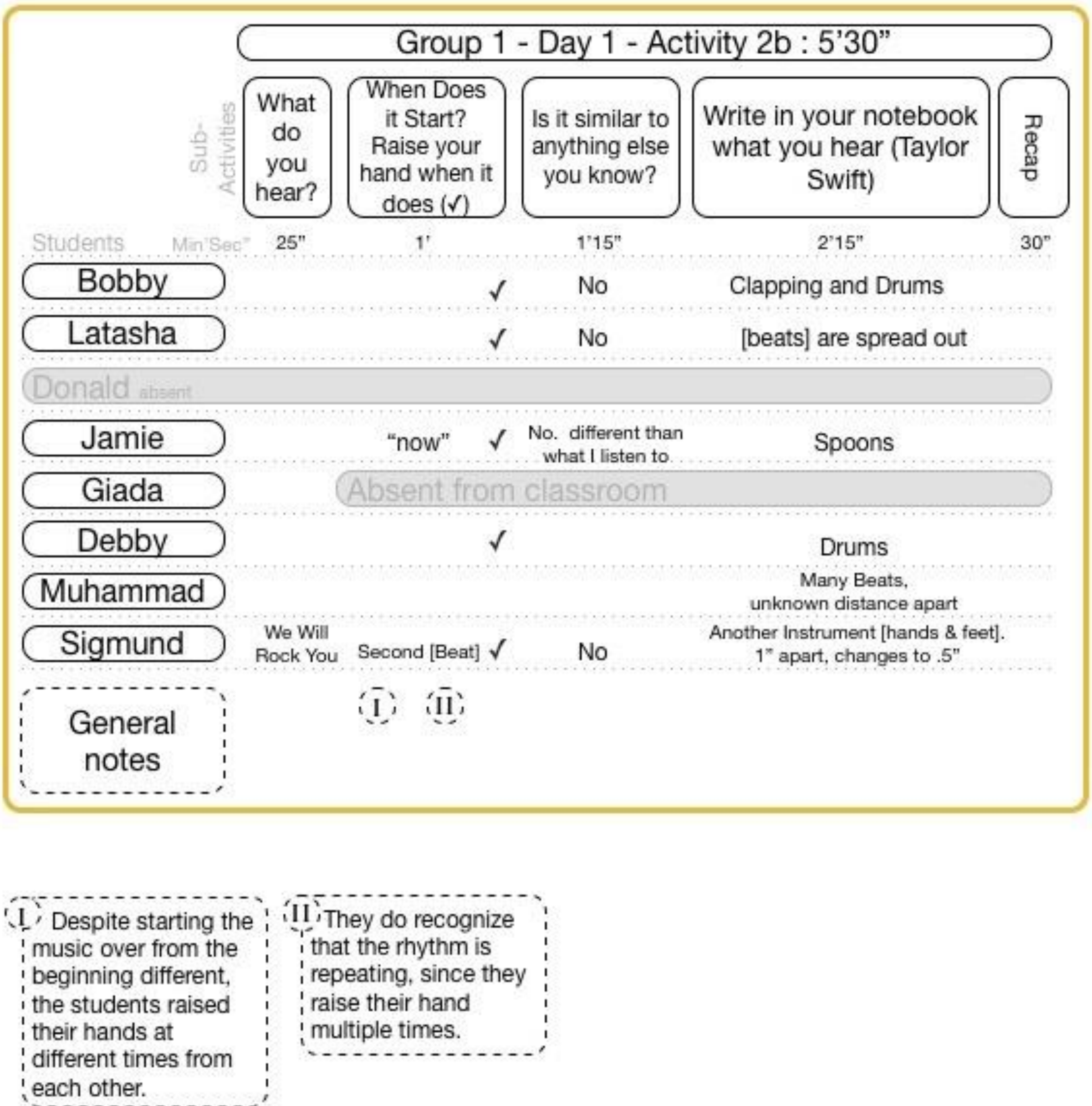


Figure 11-32 Group 1 Day 1 Activity 2b Classroom video data

c) Group 1 Day 1 Activity 3

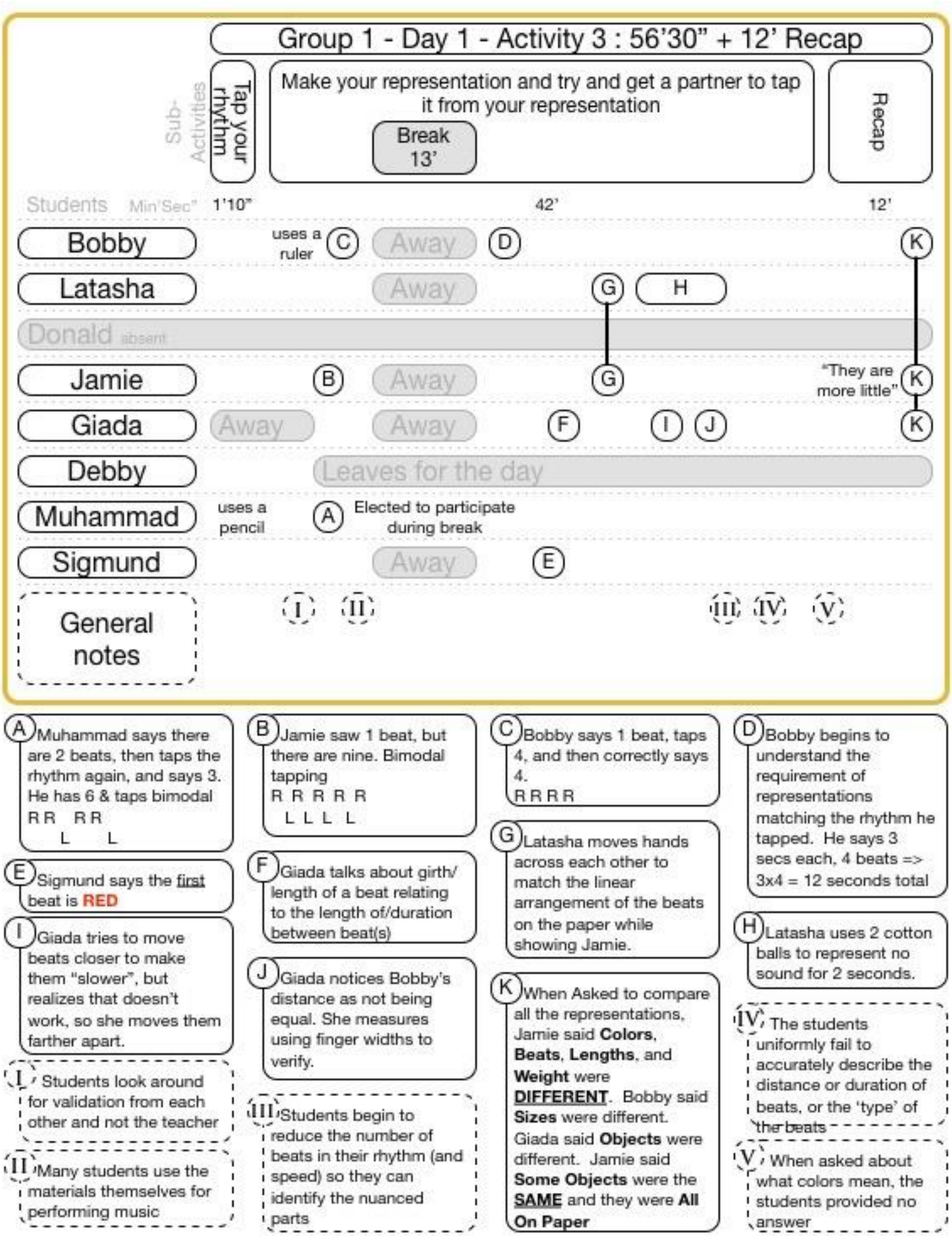


Figure 11-33 Group 1 Day 1 Activity 3 video data

2. Group 1 Day 2

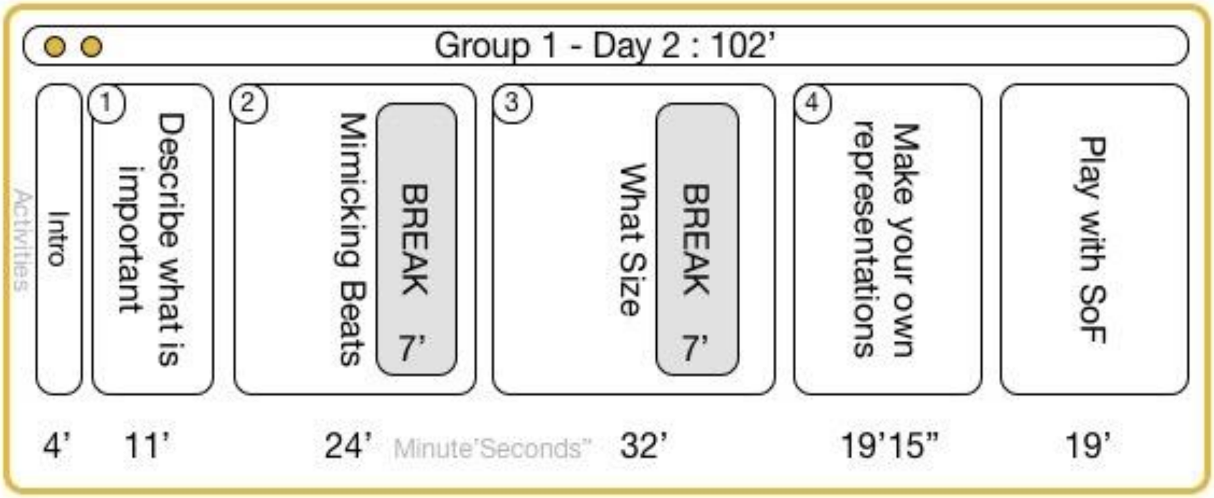


Figure 11-34 Group 1 Day 2 Overview Classroom video data

d) Group 1 Day 2 Activity 1

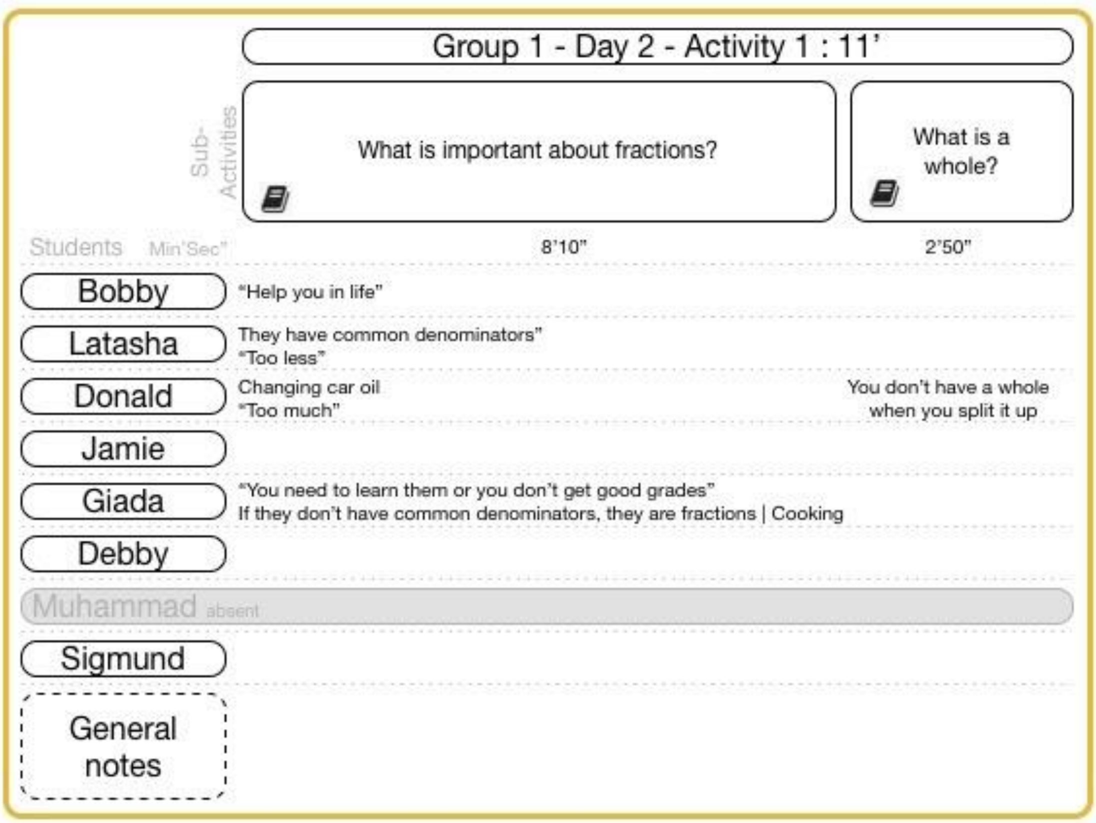


Figure 11-35 Group 1 Day 2 Activity 1 Classroom video data

e) Group 1 Day 2 Activity 2

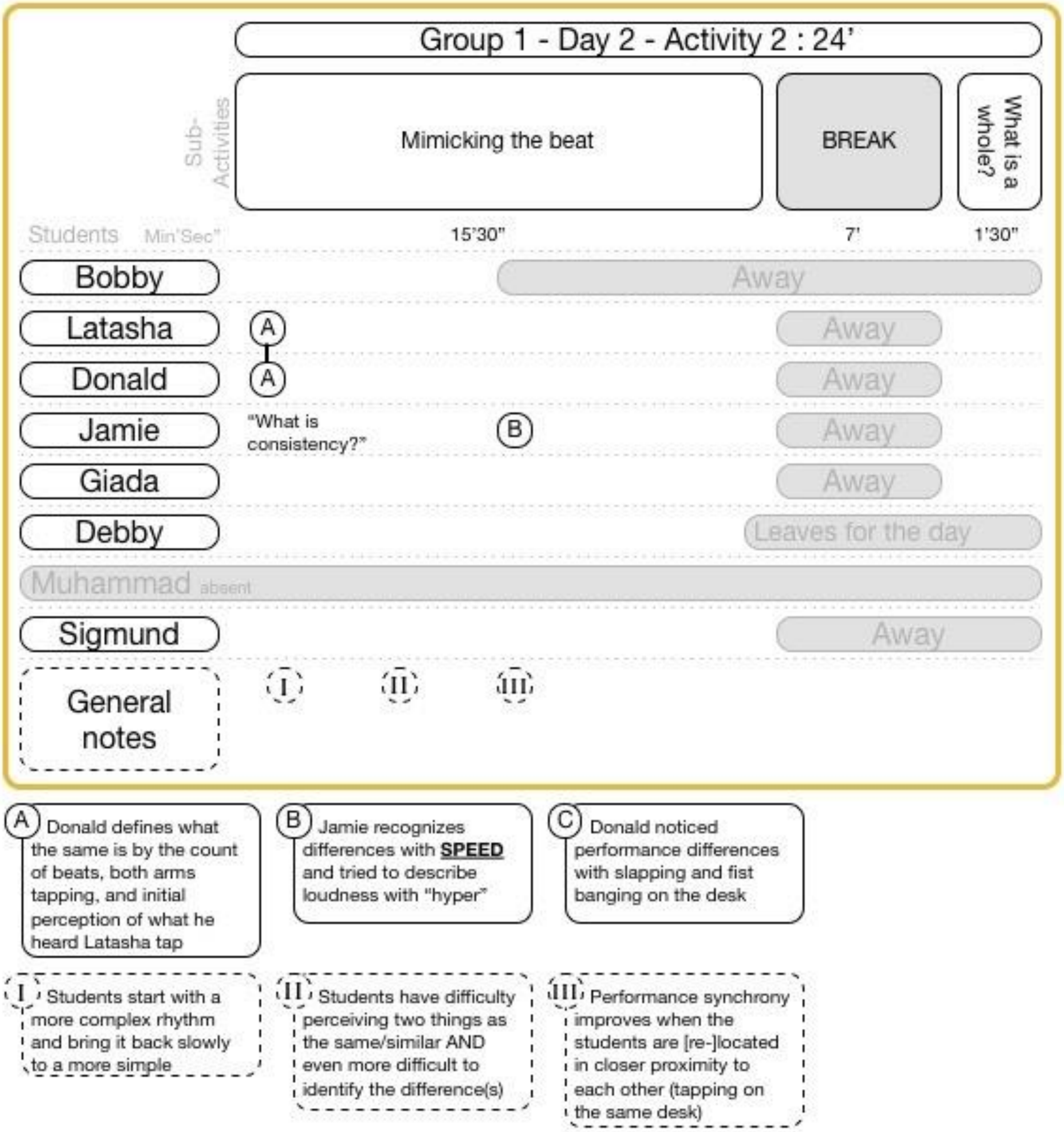


Figure 11-36 Group 1 Day 2 Activity 2 Classroom video data

f) *Group 1 Day 2 Activity 3*

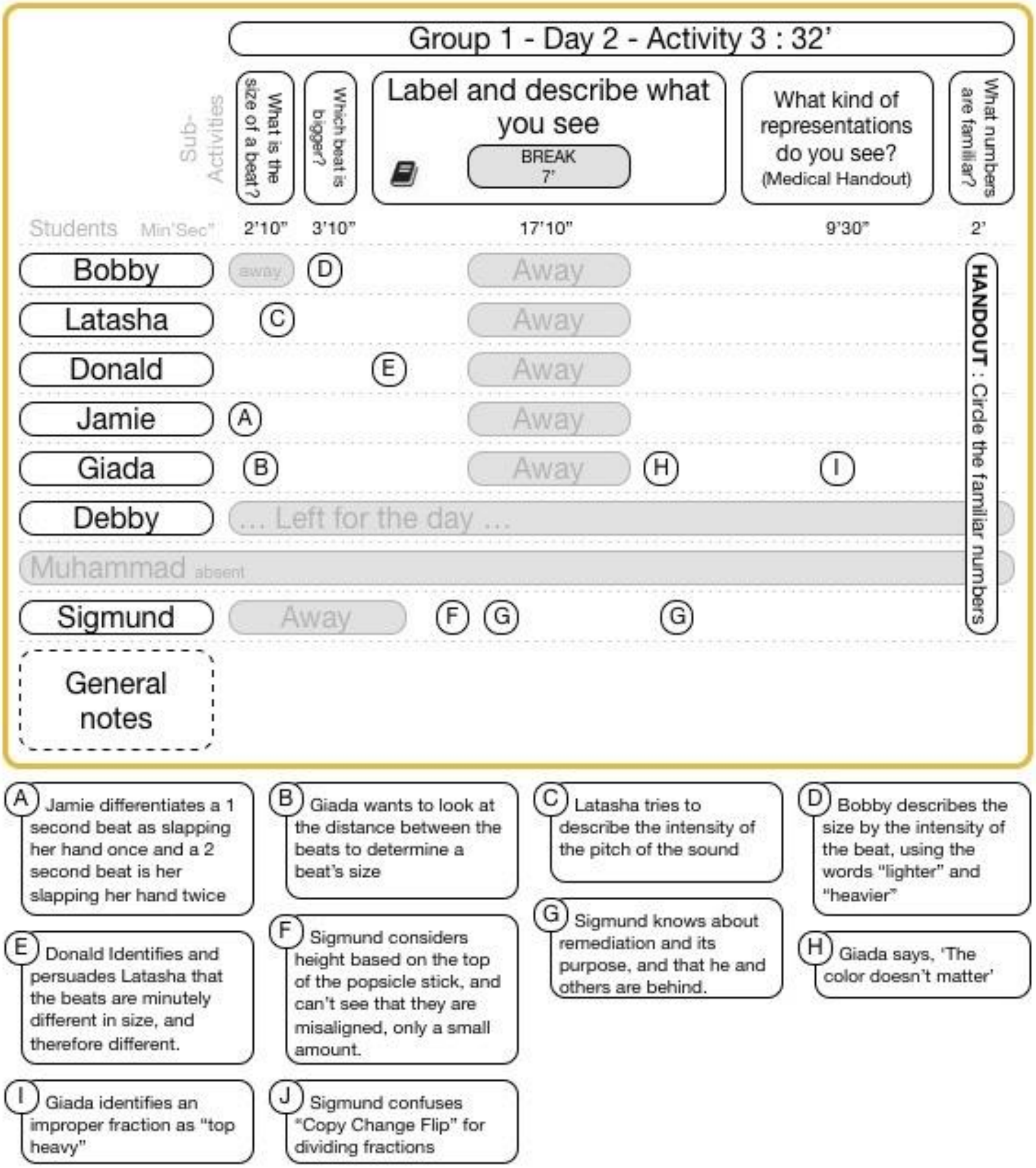


Figure 11-37 Group 1 Day 2 Activity 3 Classroom video data

g) Group 1 Day 2 Activity 4

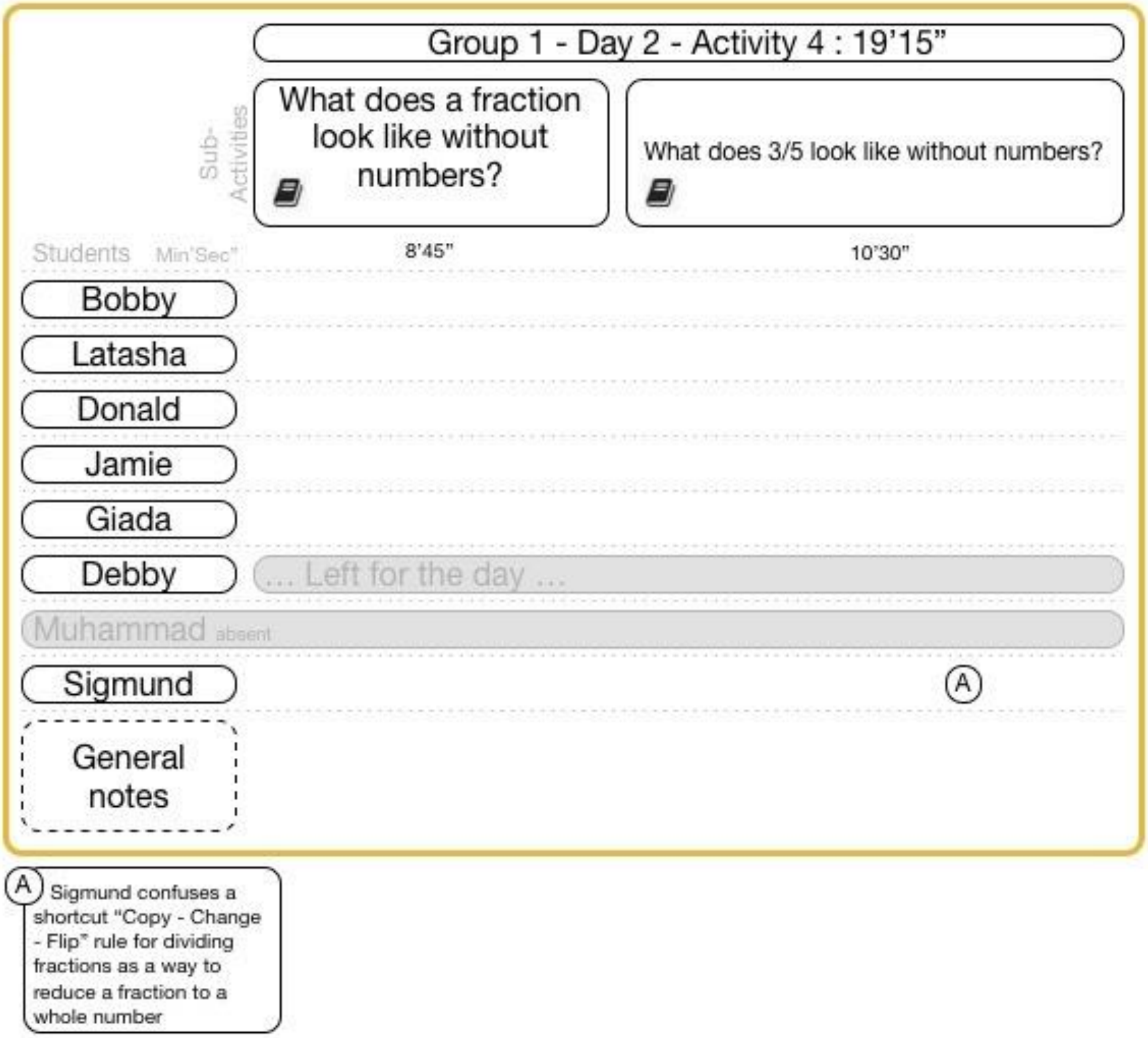


Figure 11-38 Group 1 Day 2 Activity 4 Classroom video data

3. Group 1 Day 3

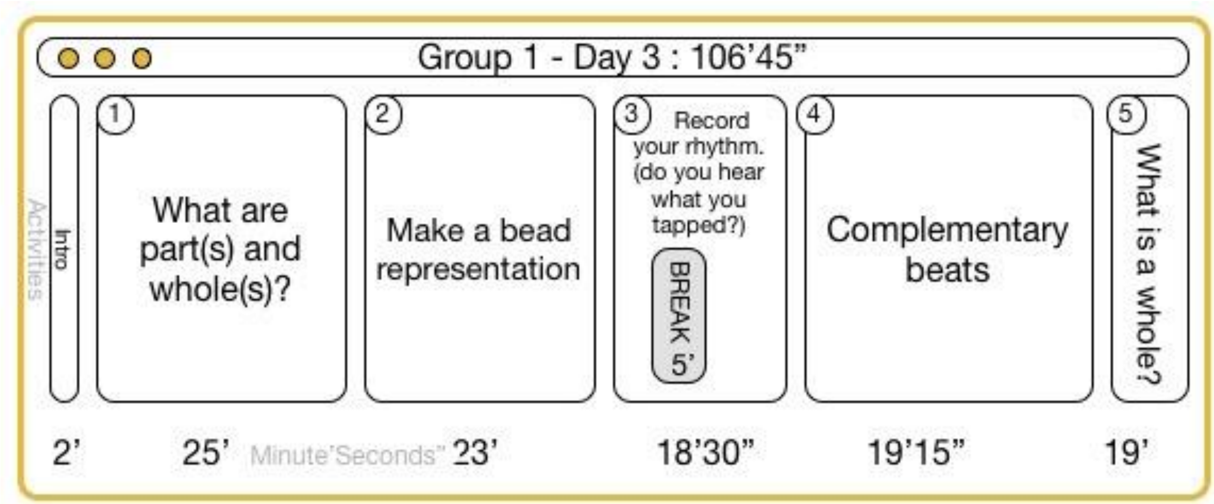


Figure 11-39 Group 1 Day 3 Overview Classroom video data

h) Group 1 Day 3 Activity 1

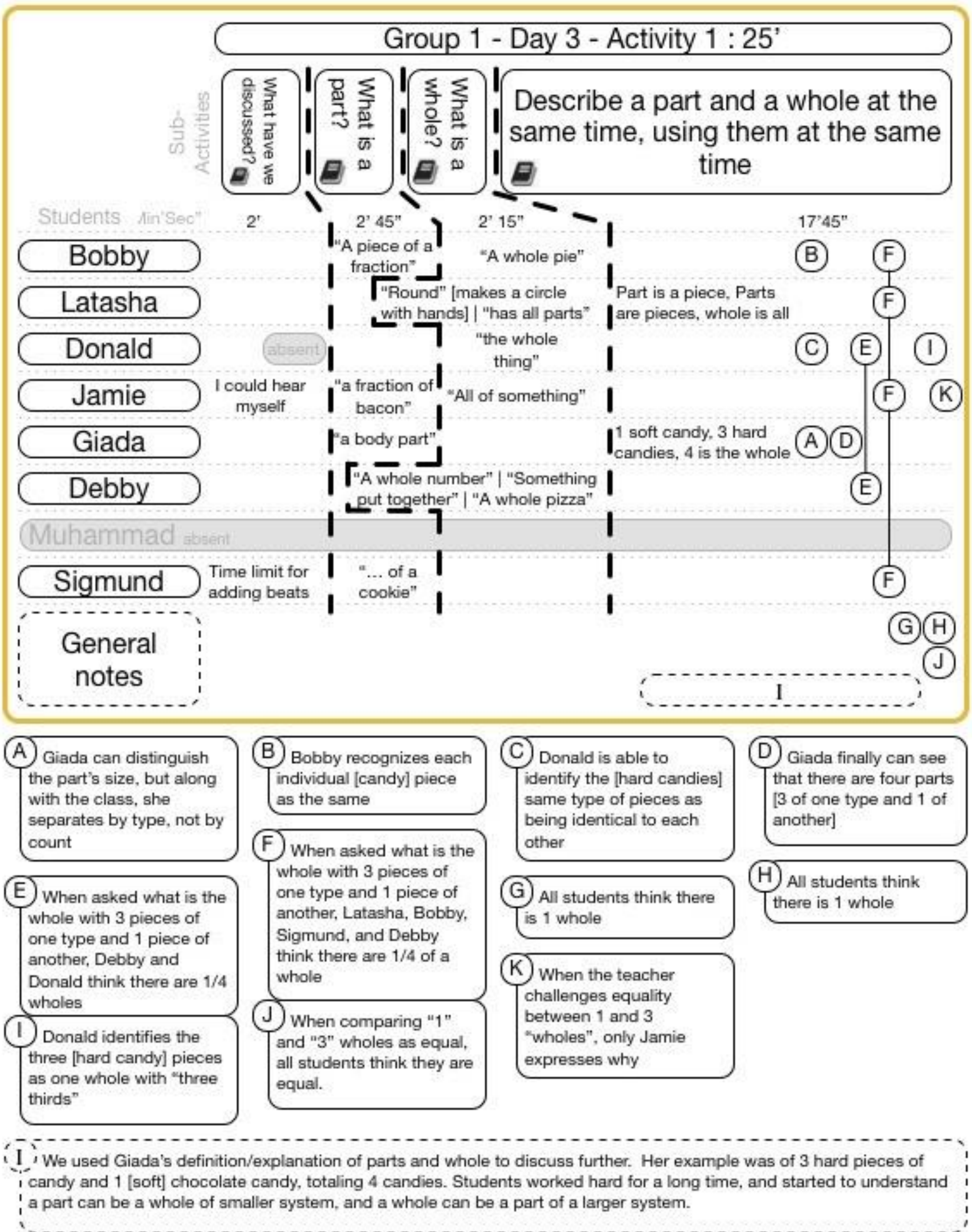


Figure 11-40 Group 1 Day 3 Activity 1 Classroom video data

i) Group 1 Day 3 Activity 2

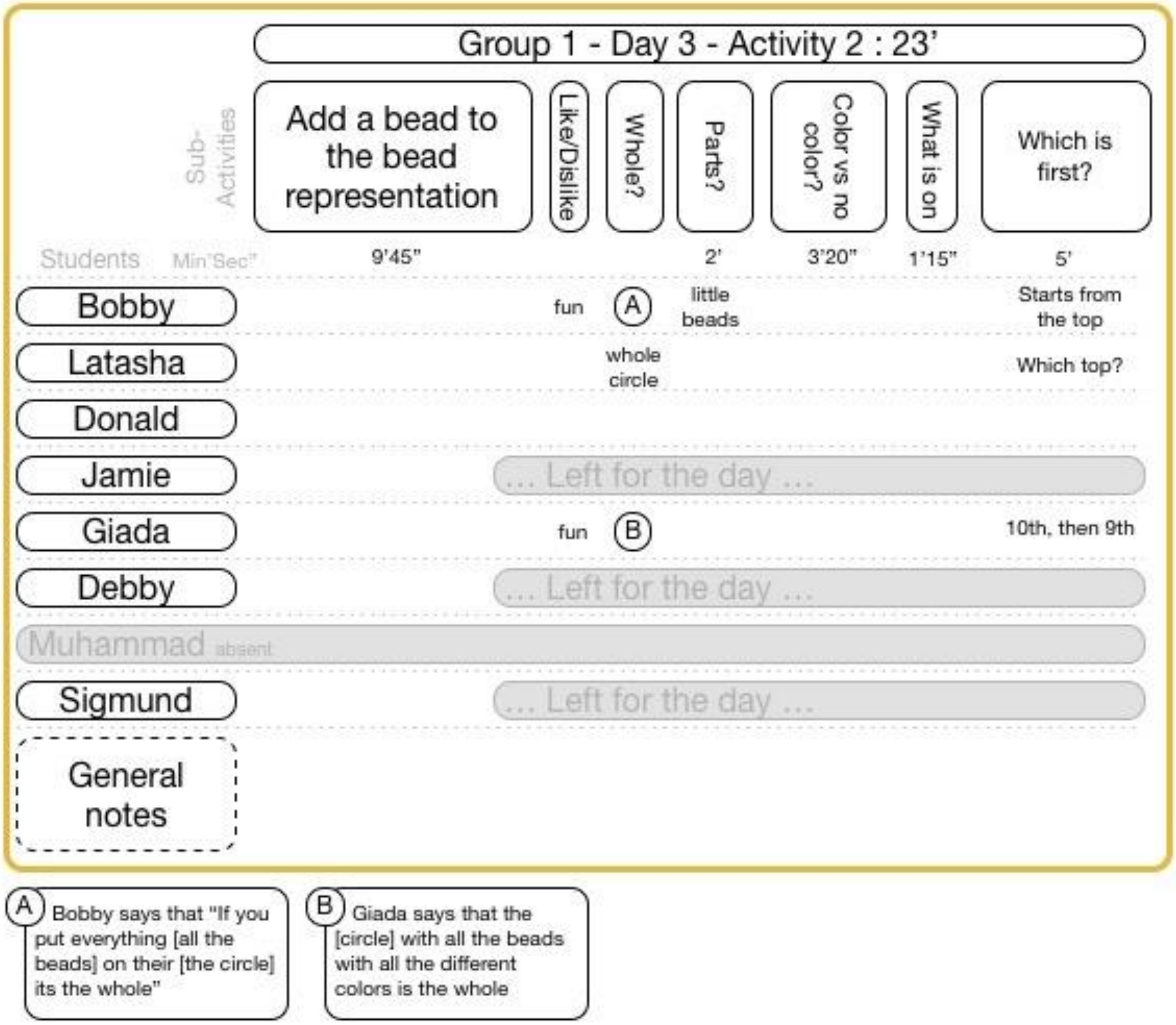


Figure 11-41 Group 1 Day 3 Activity 2 Classroom video data

j) Group 1 Day 3 Activity 3

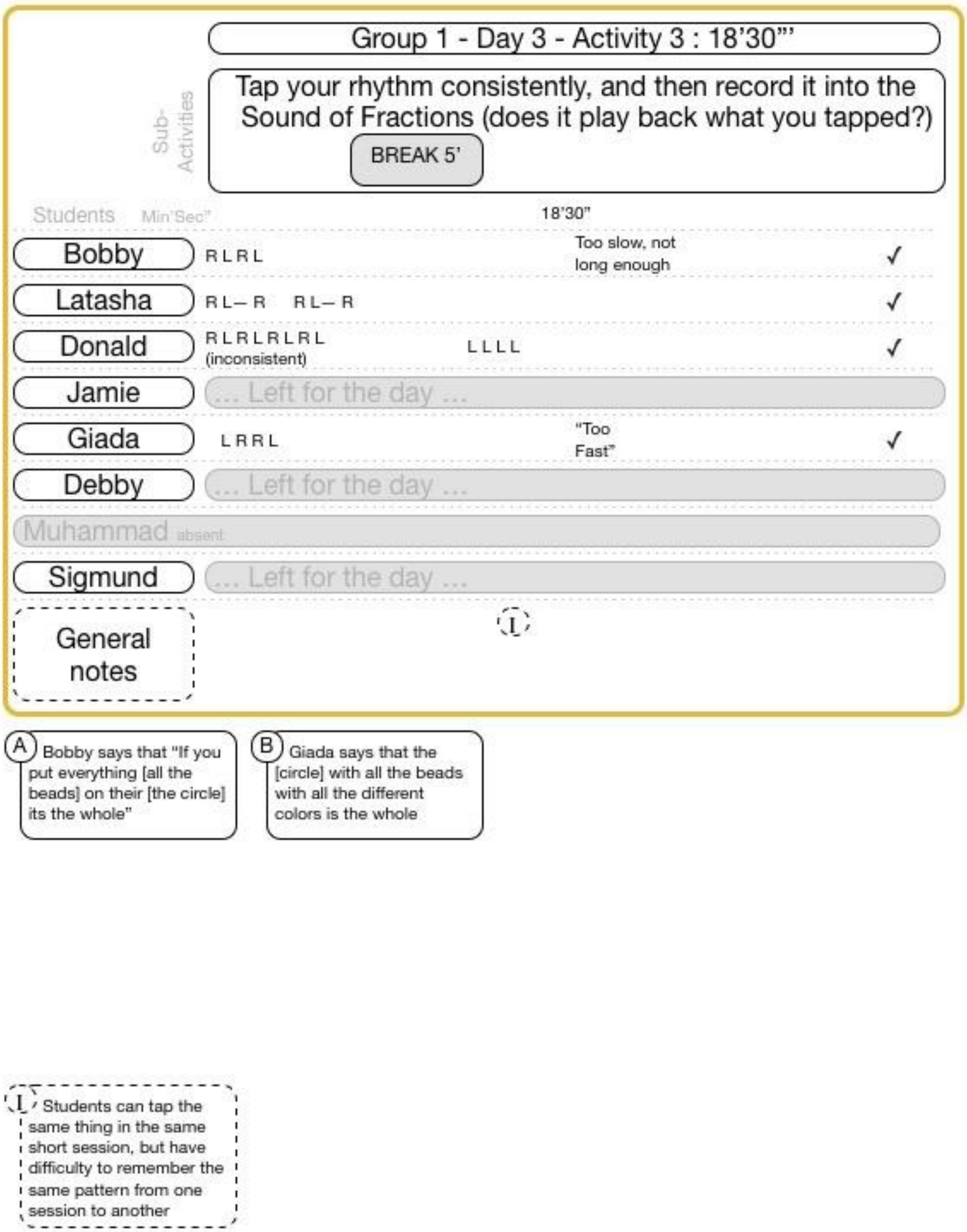


Figure 11-42 Group 1 Day 3 Activity 3 Classroom video data

k) Group 1 Day 3 Activity 4

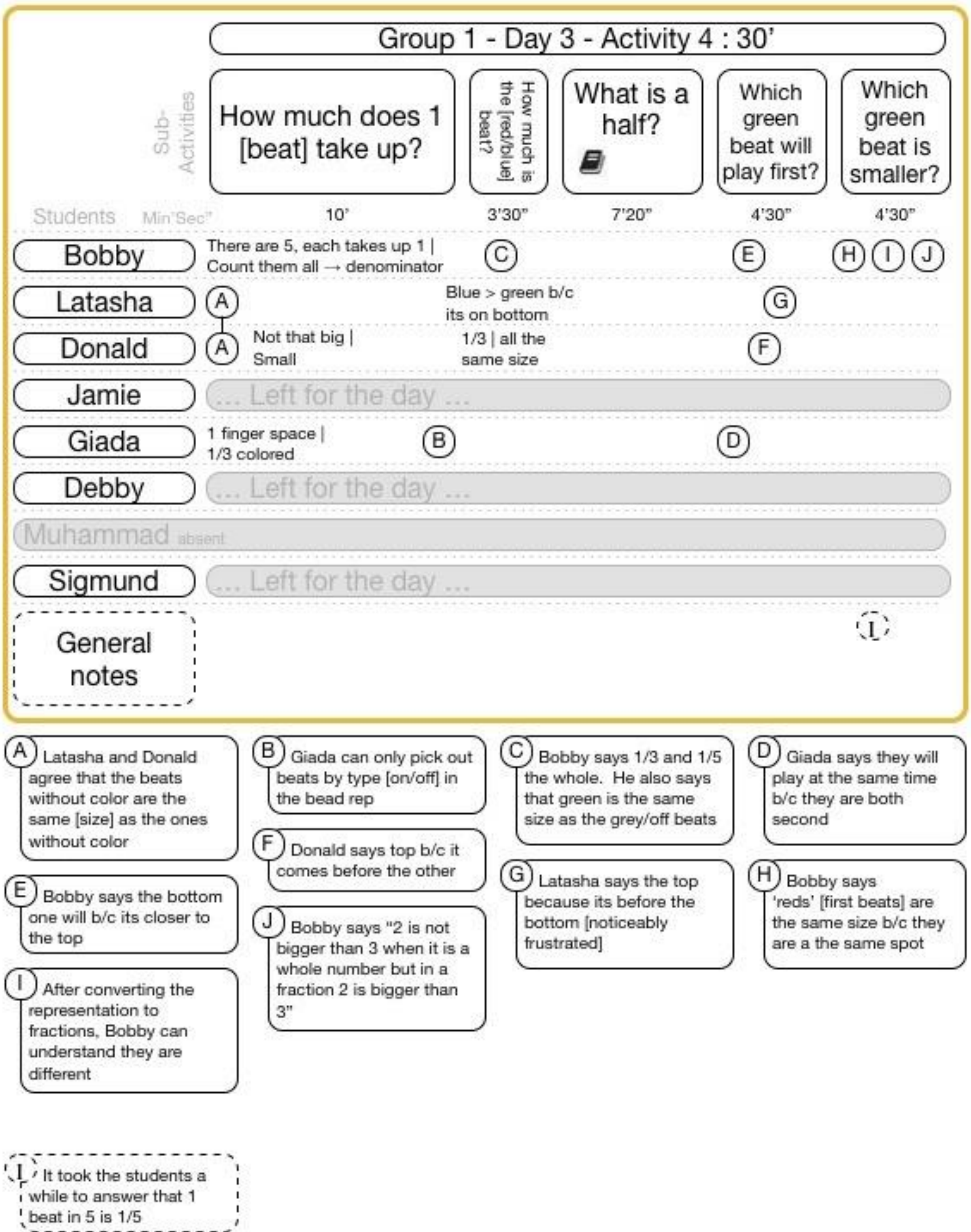


Figure 11-43 Group 1 Day 3 Activity 4 Classroom video data

l) Group 1 Day 3 Activity 5

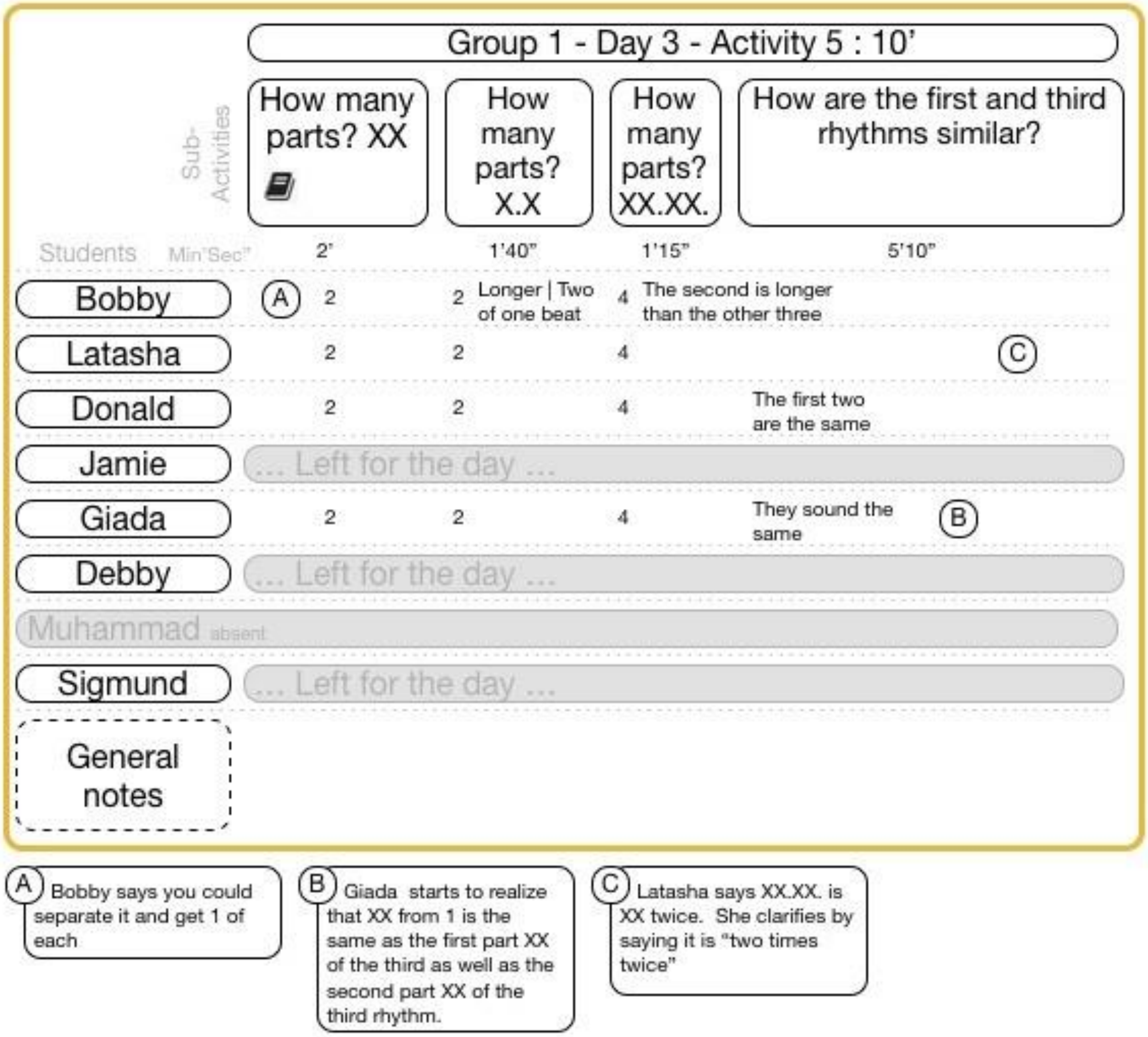


Figure 11-44 Group 1 Day 3 Activity 5 Classroom video data

4. Group 1 Day 4

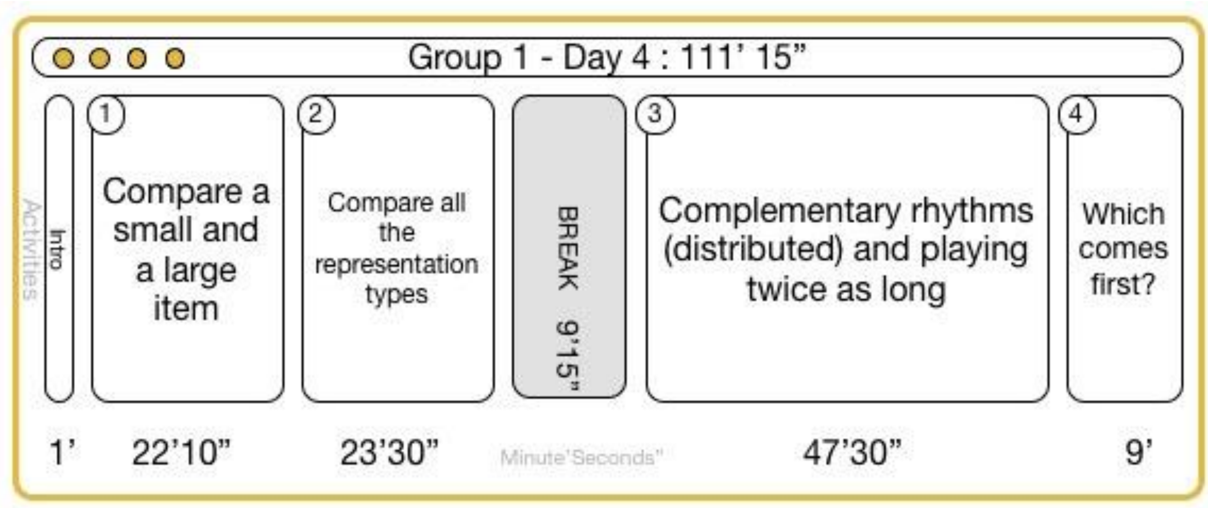


Figure 11-45 Group 1 Day 4 Overview Classroom video data

m) Group 1 Day 4 Activity 1

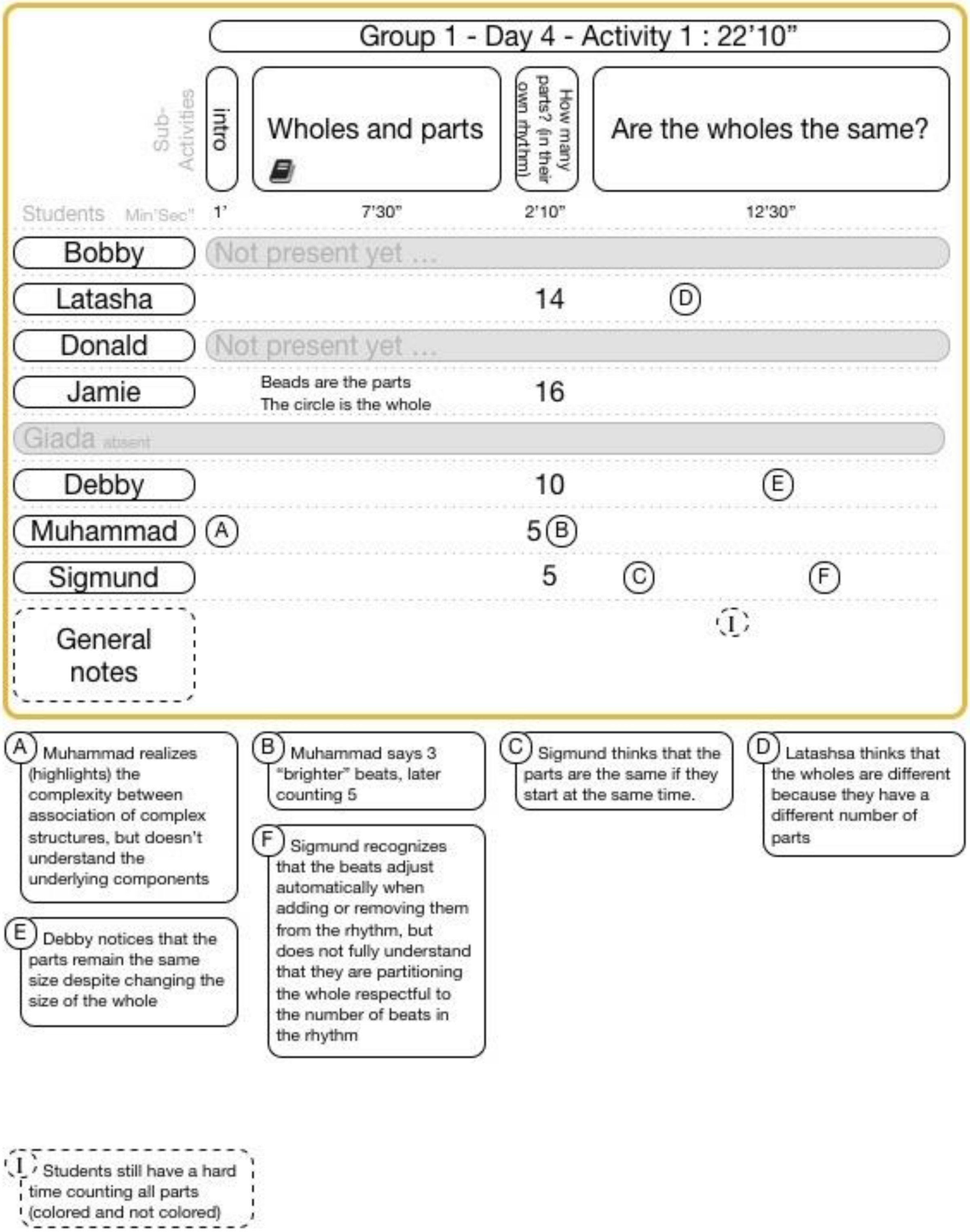


Figure 11-46 Group 1 Day 4 Activity 1 Classroom video data

n) Group 1 Day 4 Activity 2

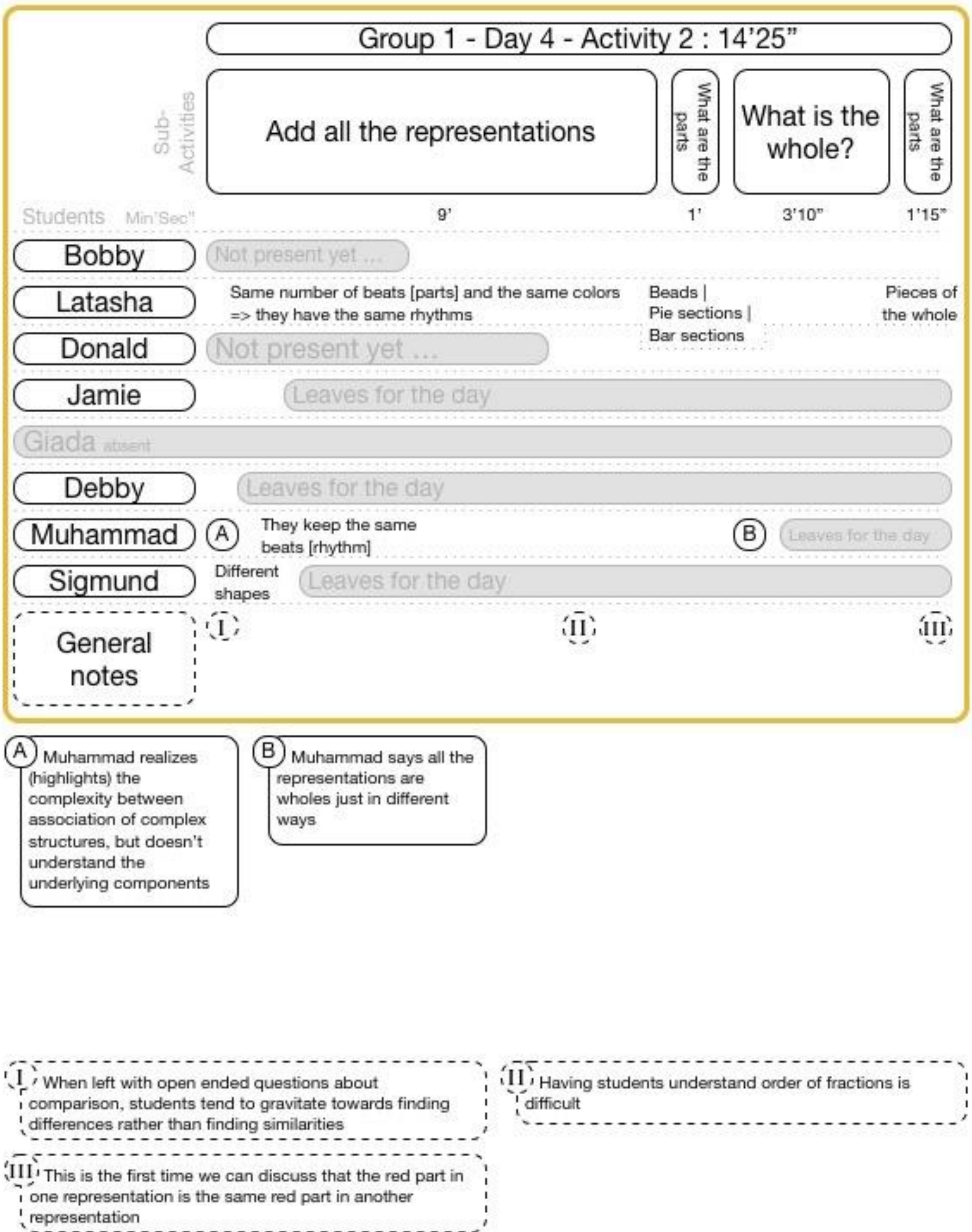
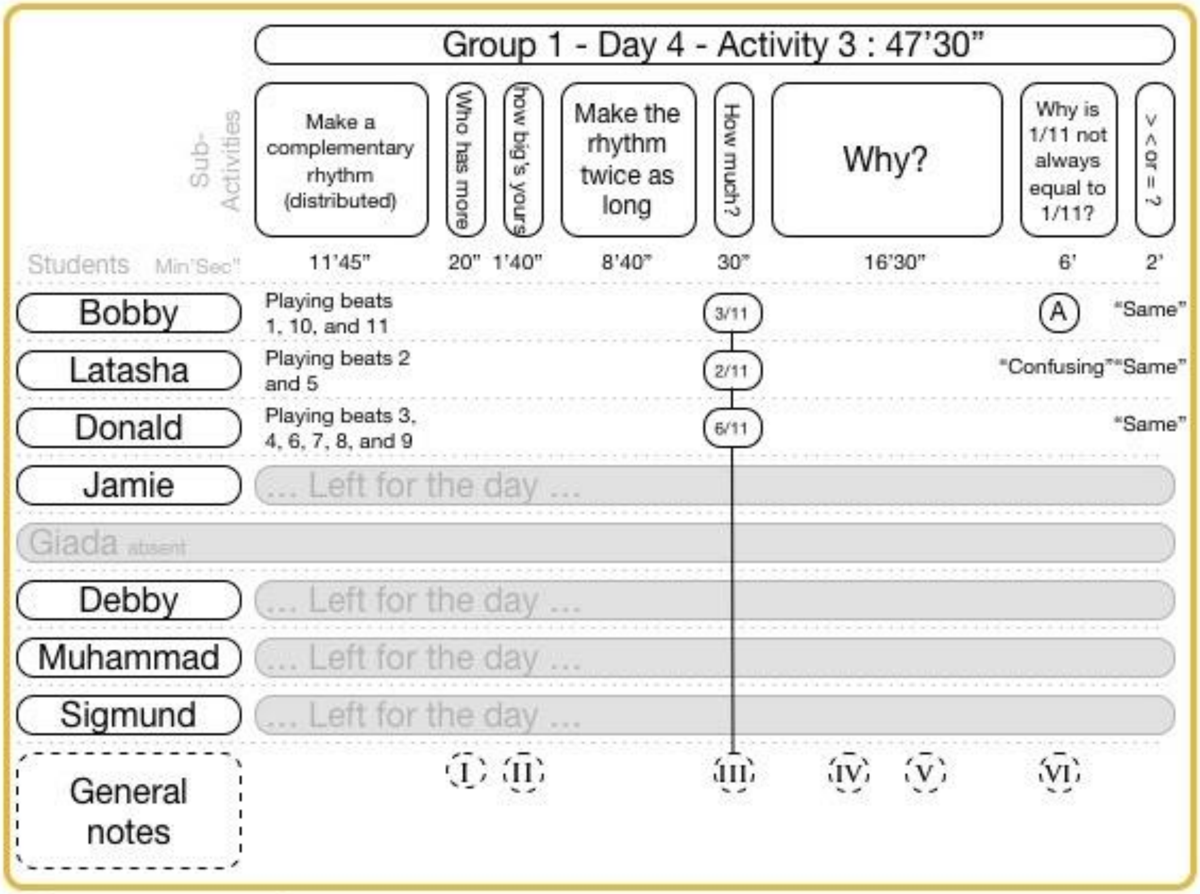


Figure 11-47 Group 1 Day 4 Activity 2 Classroom video data

o) *Group 1 Day 4 Activity 3*



(A) Bobby says that the whole equal in fraction but not equal in size

(I) The students are able to understand their own portion of the rhythm

(II) The saying 'Twice as Big' is problematic, just as is 'Twice as long'. Using a more concrete description such as 'Make the whole twice as big so that the parts are also twice as big' is also difficult because they don't understand the relationship between all of the components

(III) When asked "How much are you playing" all students answered "**SAME**" meaning they were comparing how much they were individually playing of the whole prior to making the rhythm twice as long (in duration), despite each person playing a different amount of beats in comparison to each other

(IV) When using the whiteboard to discuss doubling the rhythm graphically, the students are confident and adamant about placement, sizing and quantity of the parts and the whole

(V) They are less confident about comparing two representations so the have "EQUAL" fractions (i.e. 5/11 & 6/11 of a small object and 5/11 & 6/11 of a LARGE object)

(VI) They were shocked when we applied this idea of "a quarter" to money and change the whole from $1 to $100, $1M, $1T *p)* *Group 1 Day 4 Activity 4*

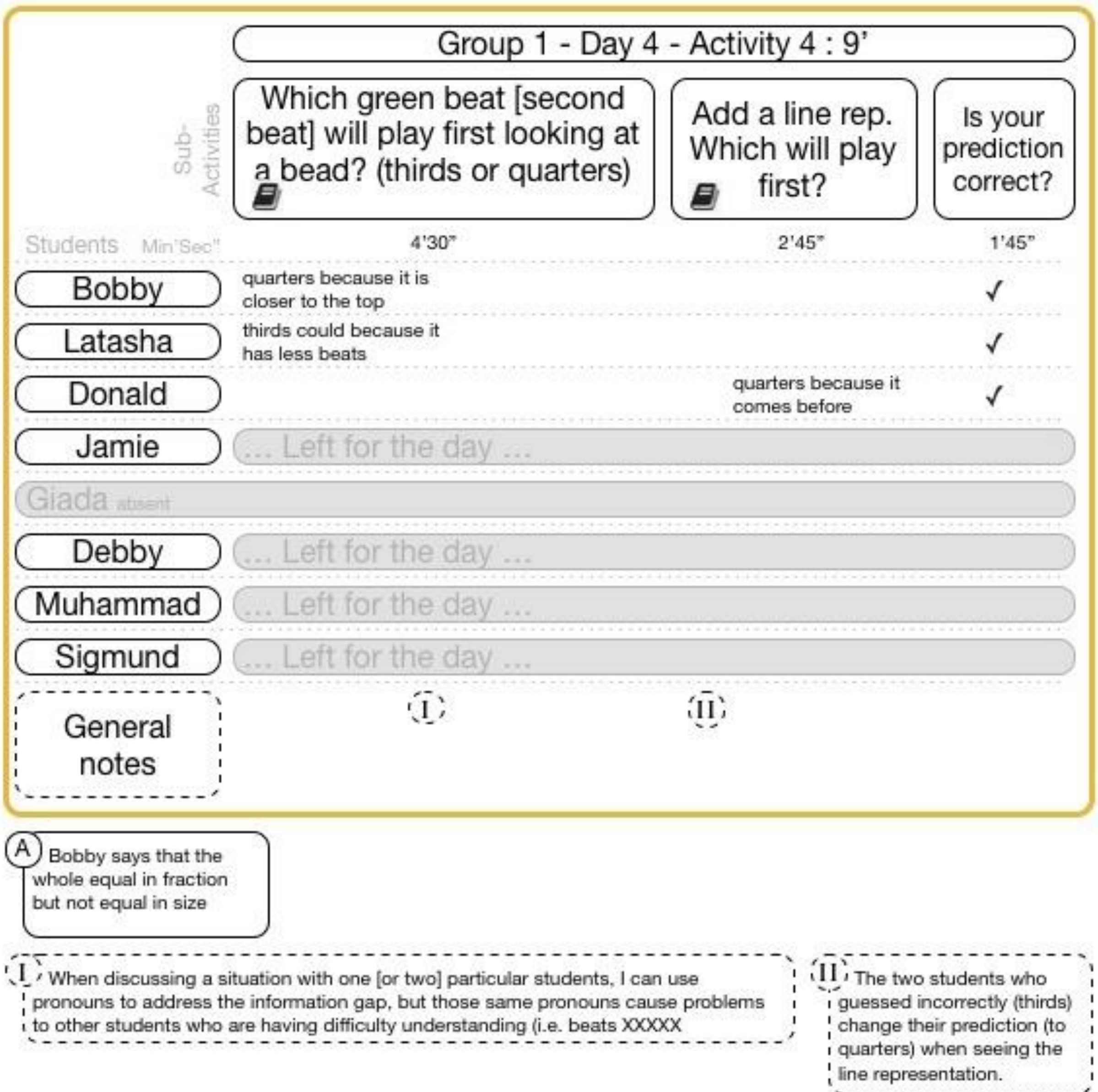


Figure 11-48 Group 1 Day 4 Activity 4

5. Group 1 Day 5

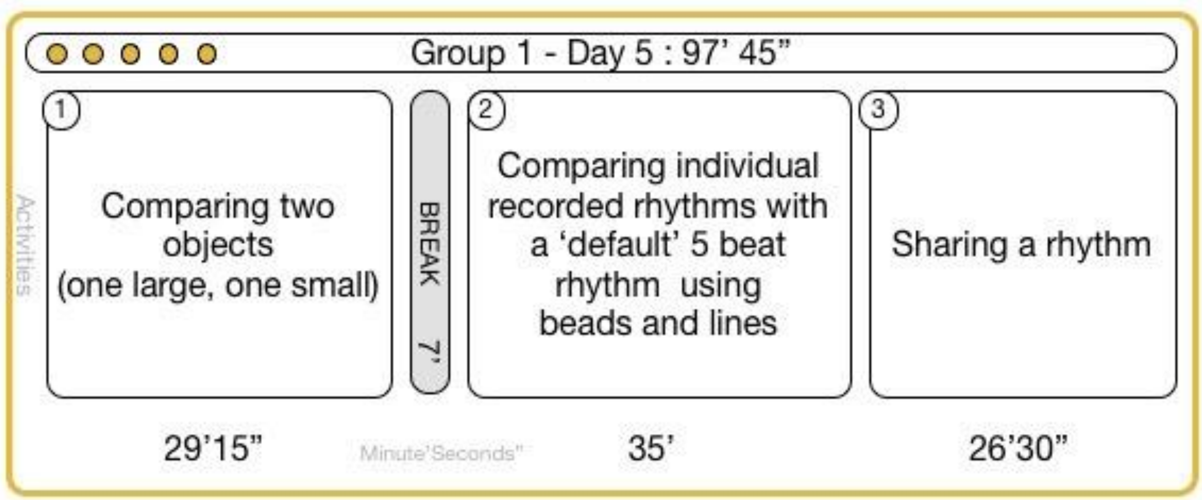


Figure 11-49 Group 1 Day 5 Overview Classroom video data

q) Group 1 Day 5 Activity 1

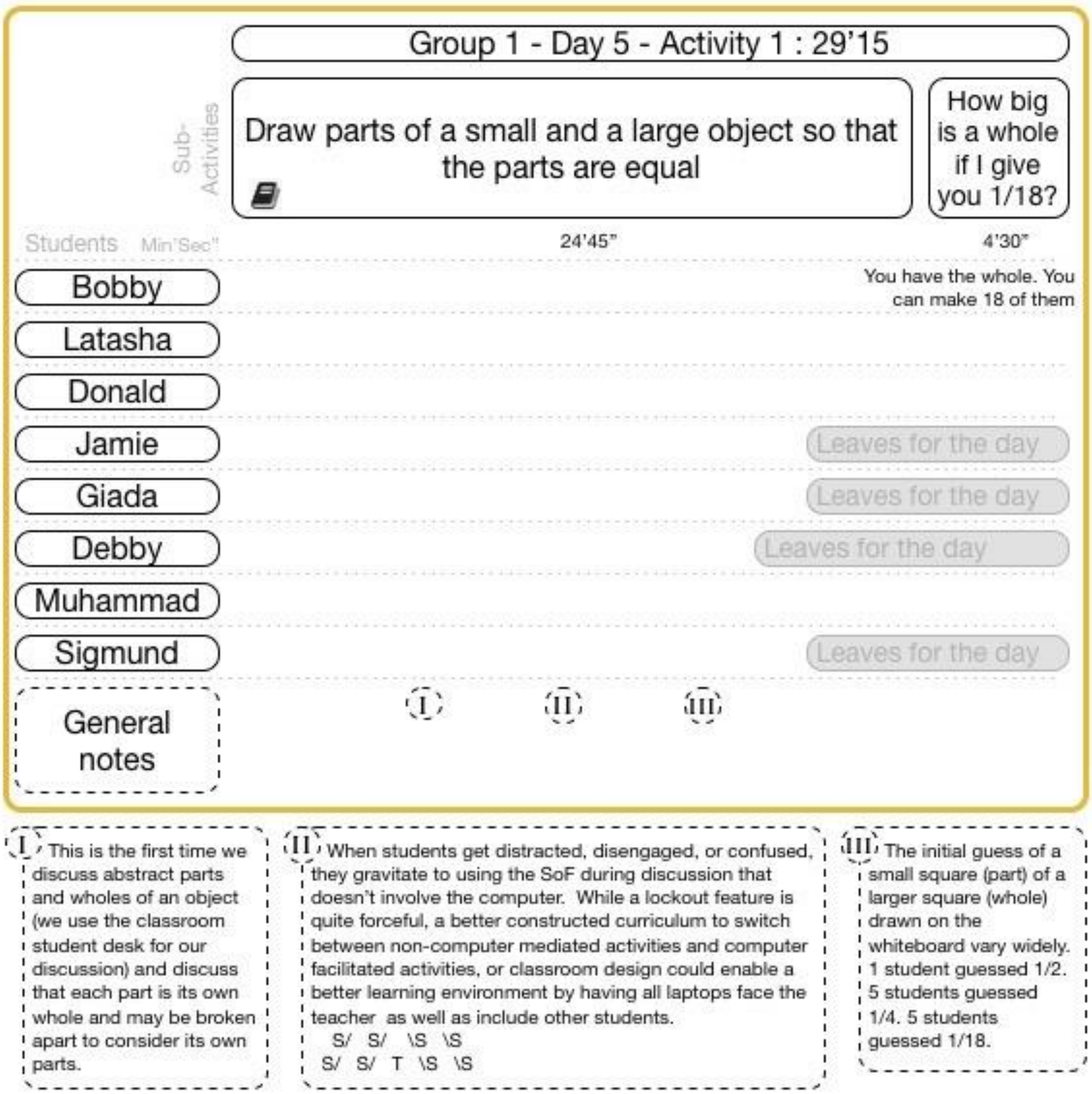


Figure 11-50 Group 1 Day 5 Activity 1 Classroom video data

r) Group 1 Day 5 Activity 2

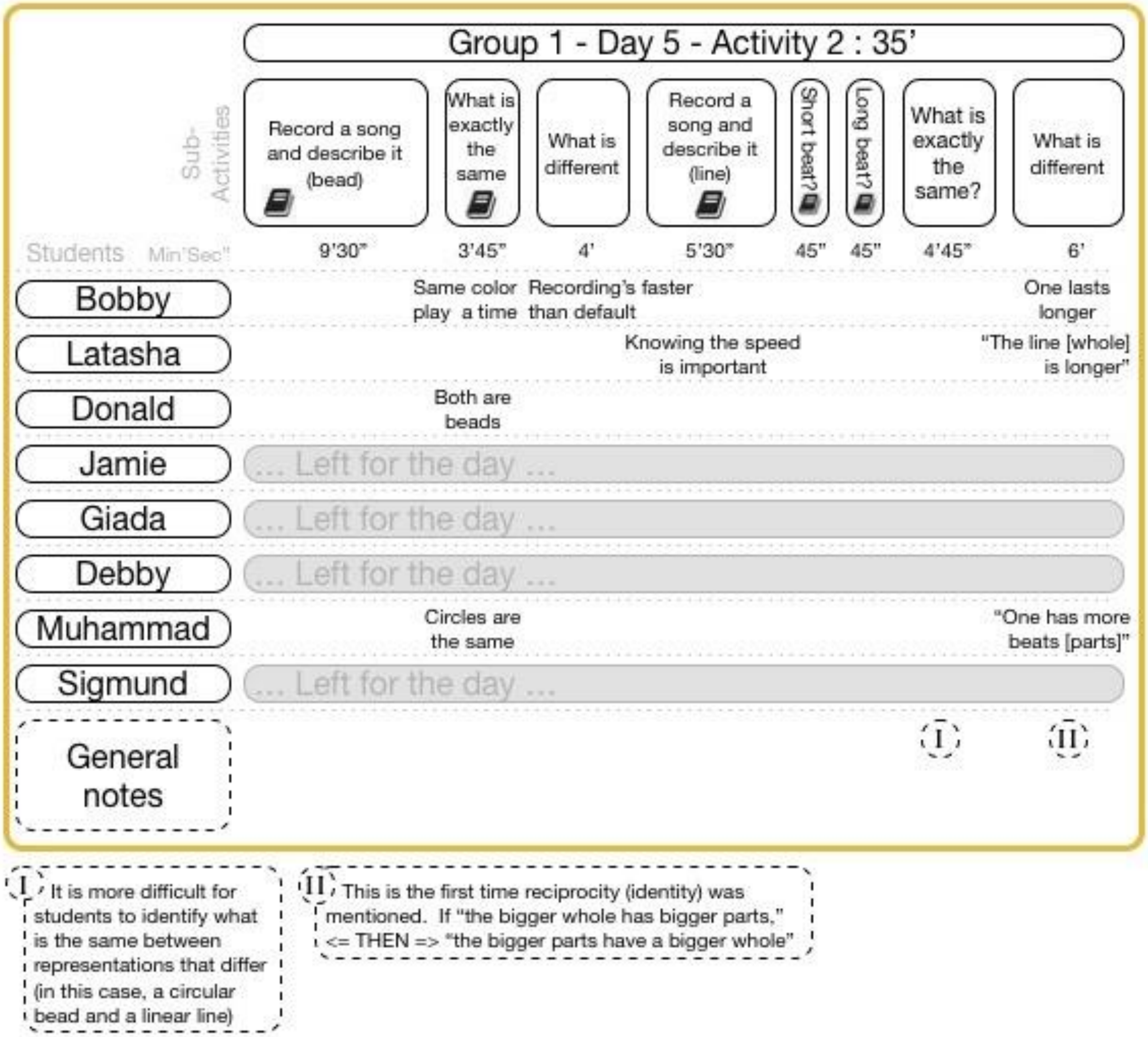


Figure 11-51 Group 1 Day 5 Activity 2 Classroom video data

s) Group 1 Day 5 Activity 3

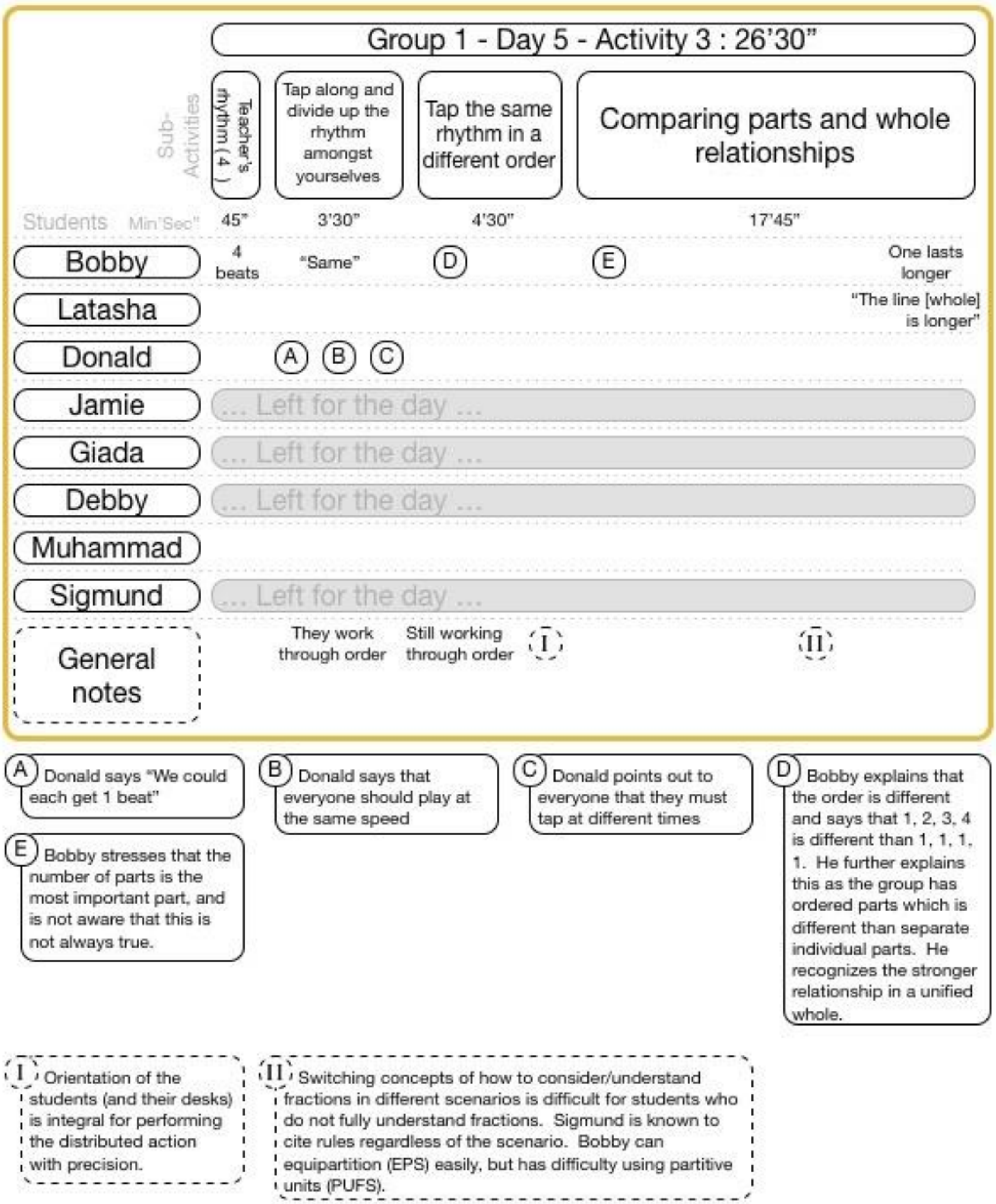


Figure 11-52 Group 1 Day 5 Activity 3 Classroom video data

6. Group 1 Day 6

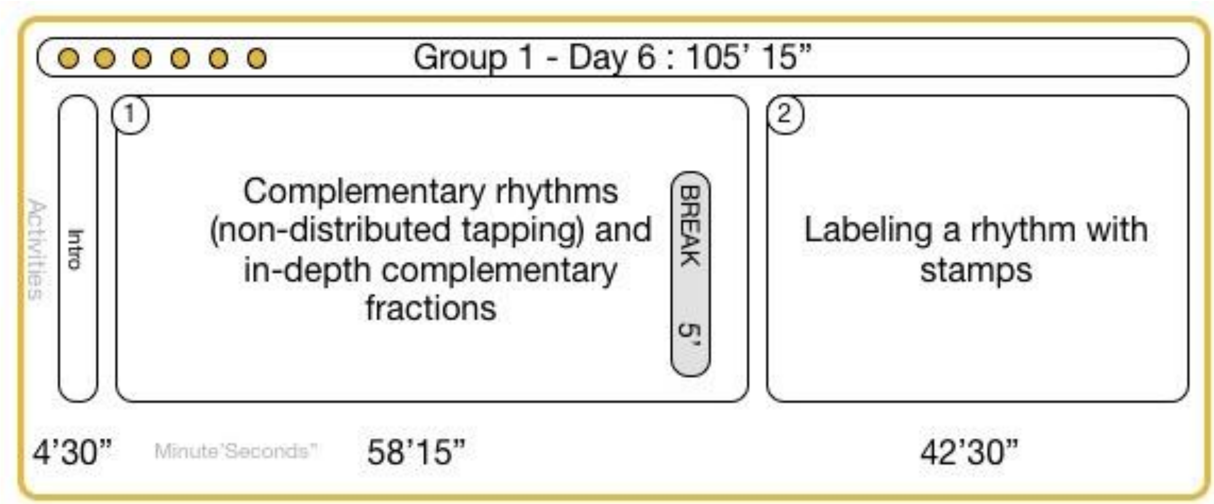


Figure 11-53 Group 1 Day 6 Overview Classroom video data

t) Group 1 Day 6 Activity 1

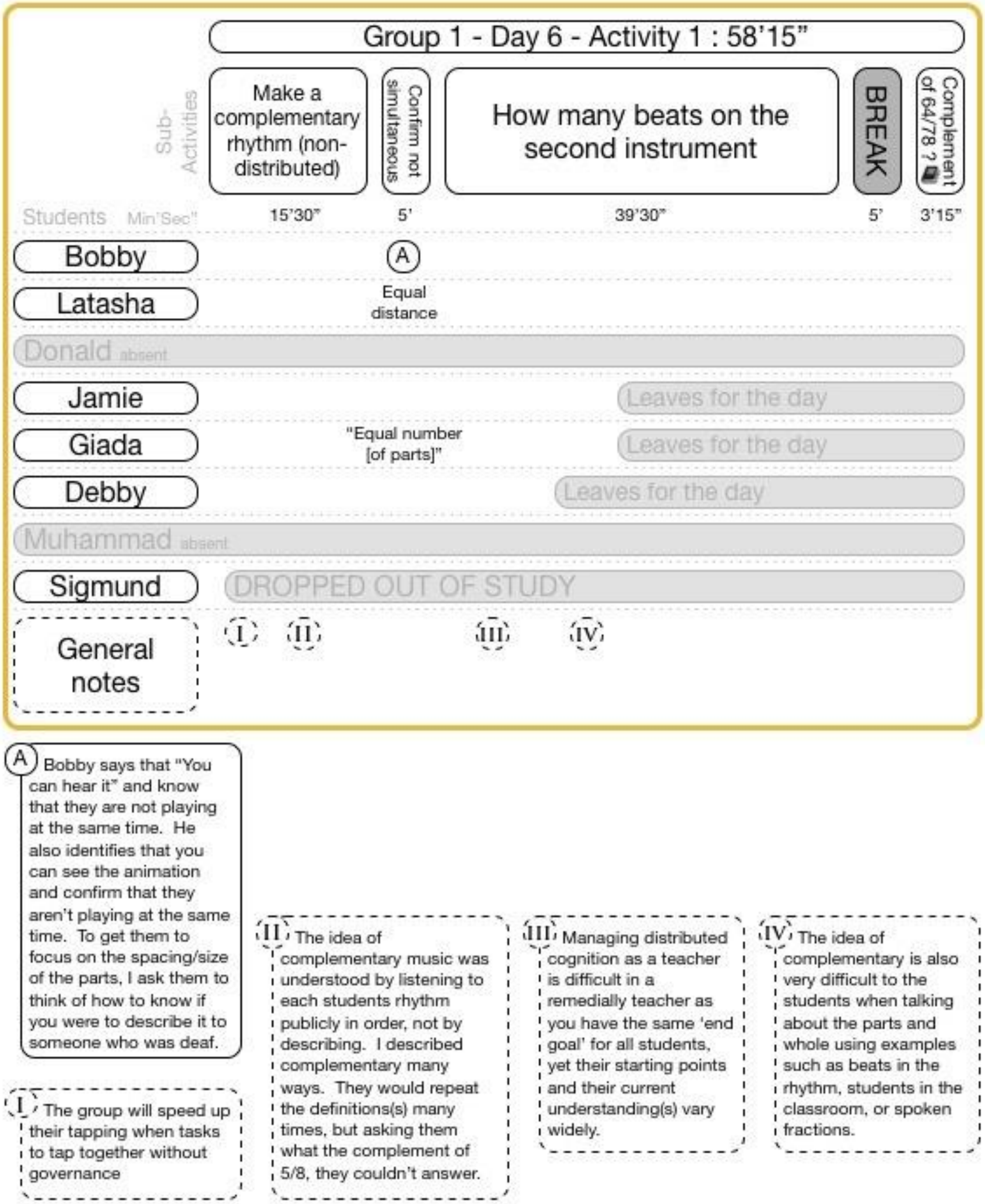


Figure 11-54 Group 1 Day 6 Activity 1 Classroom video data

u) Group 1 Day 6 Activity 2

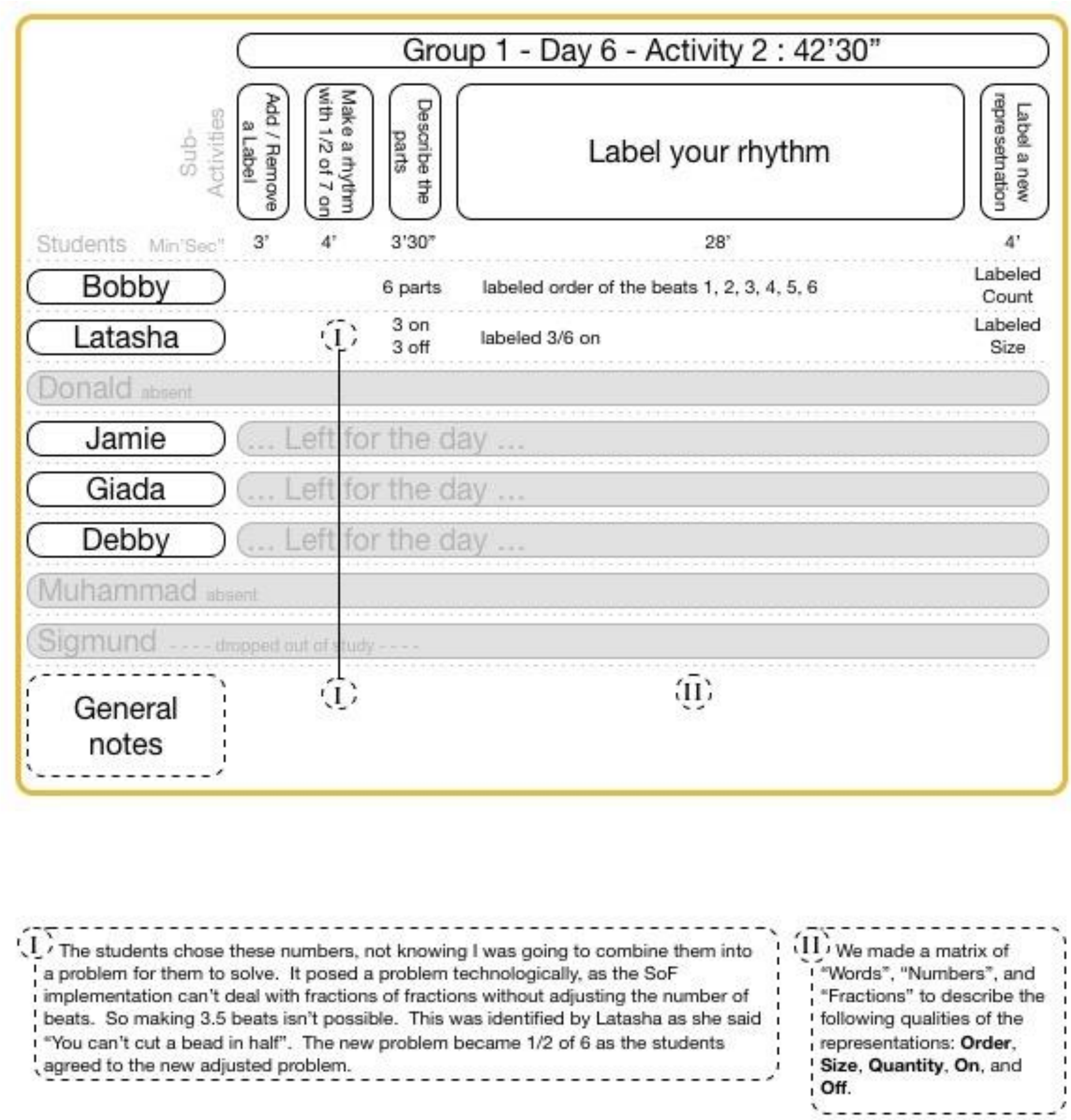


Figure 11-55 Group 1 Day 6 Activity 2 Classroom video data

7. Group 1 Day 7

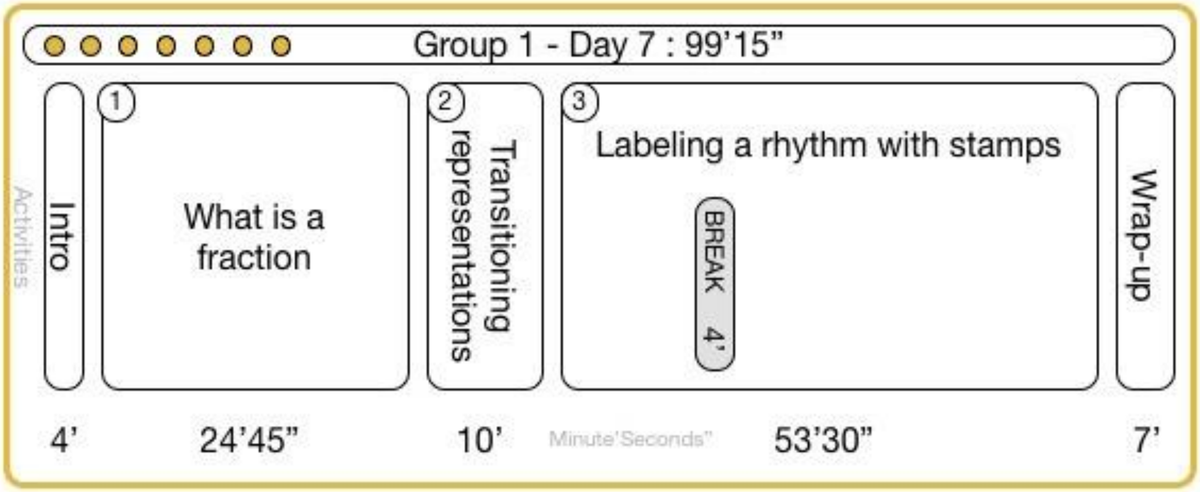


Figure 11-56 Group 1 Day 7 Overview Classroom video data

v) Group 1 Day 7 Activity 1

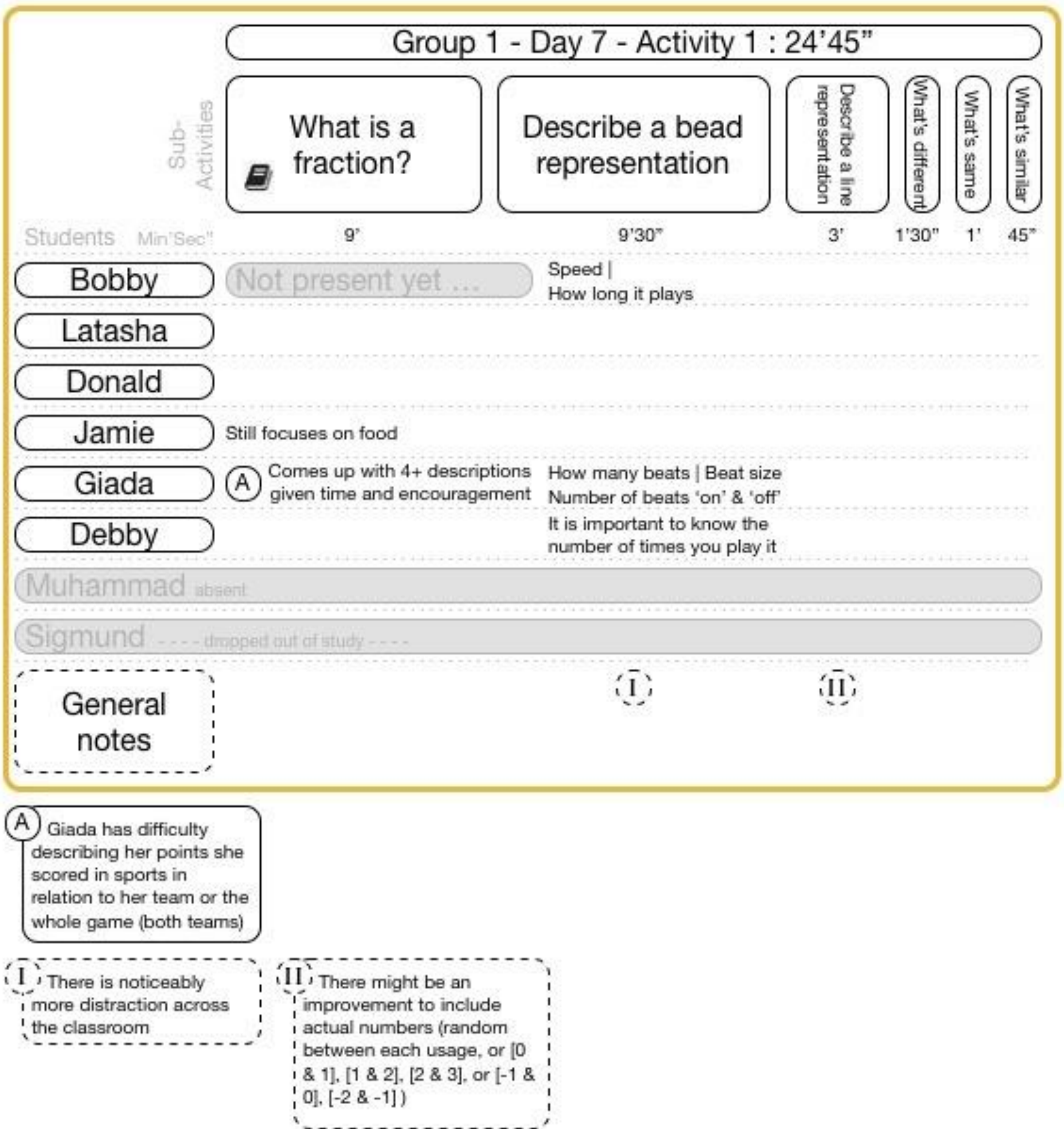


Figure 11-57 Group 1 Day 7 Activity 1 Classroom video data

w) Group 1 Day 7 Activity 2

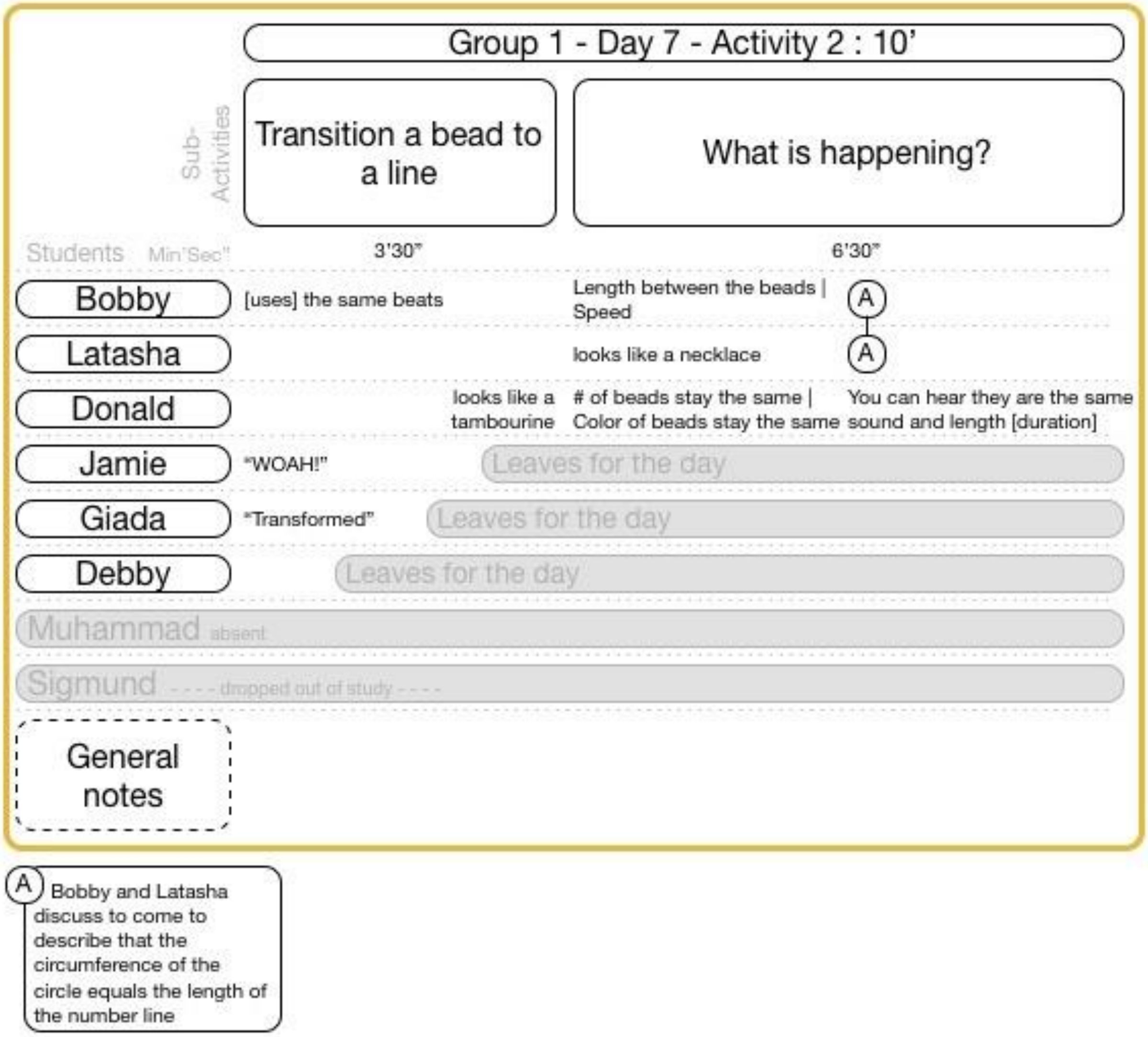


Figure 11-58 Group 1 Day 7 Activity 2 Classroom video data

x) Group 1 Day 7 Activity 3

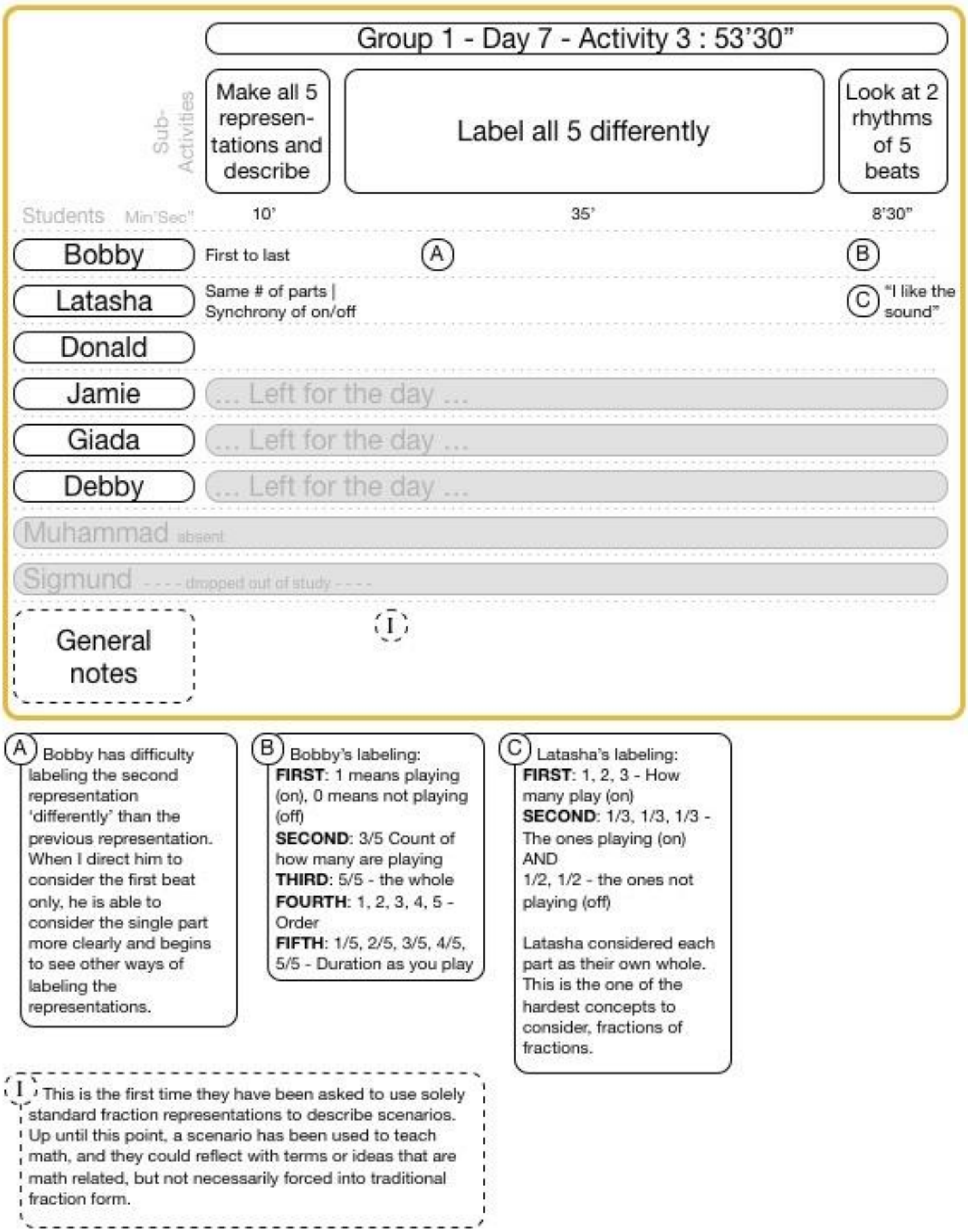


Figure 11-59 Group 1 Day 7 Activity 3 Classroom video data

11.1 Group 2 Video Data and Notes

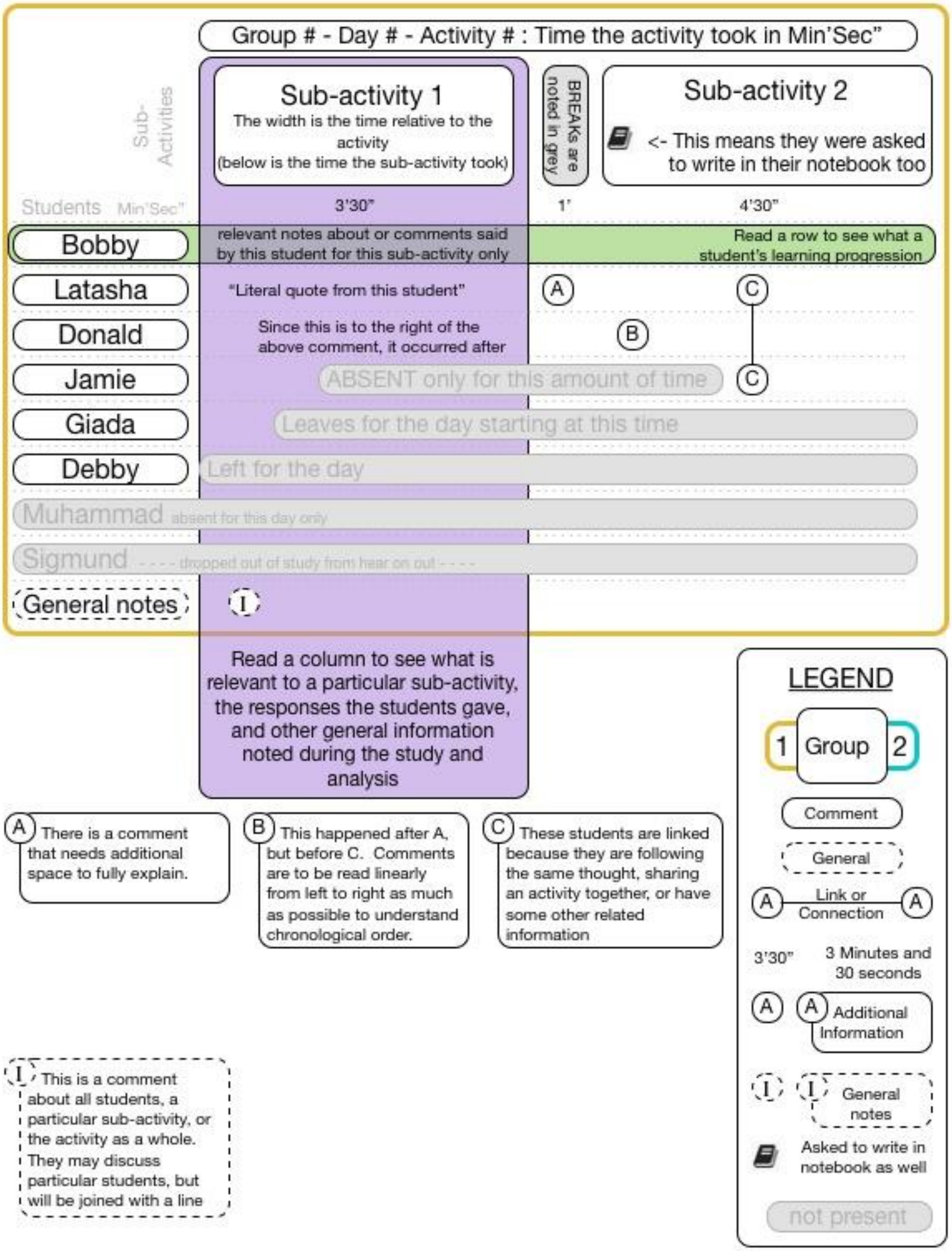


Figure 11-60 A Legend and description for reviewing video data of activities and sub-activities

8. Group 2 Day 1

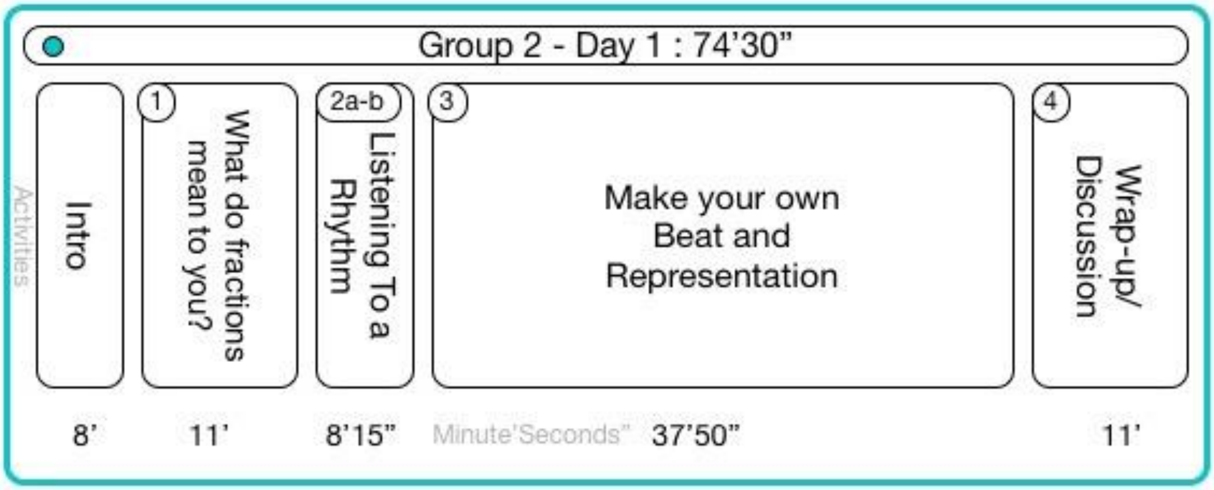


Figure 11-61 Group 2 Day 1 Overview Classroom video data

y) Group 2 Day 1 Activity 1

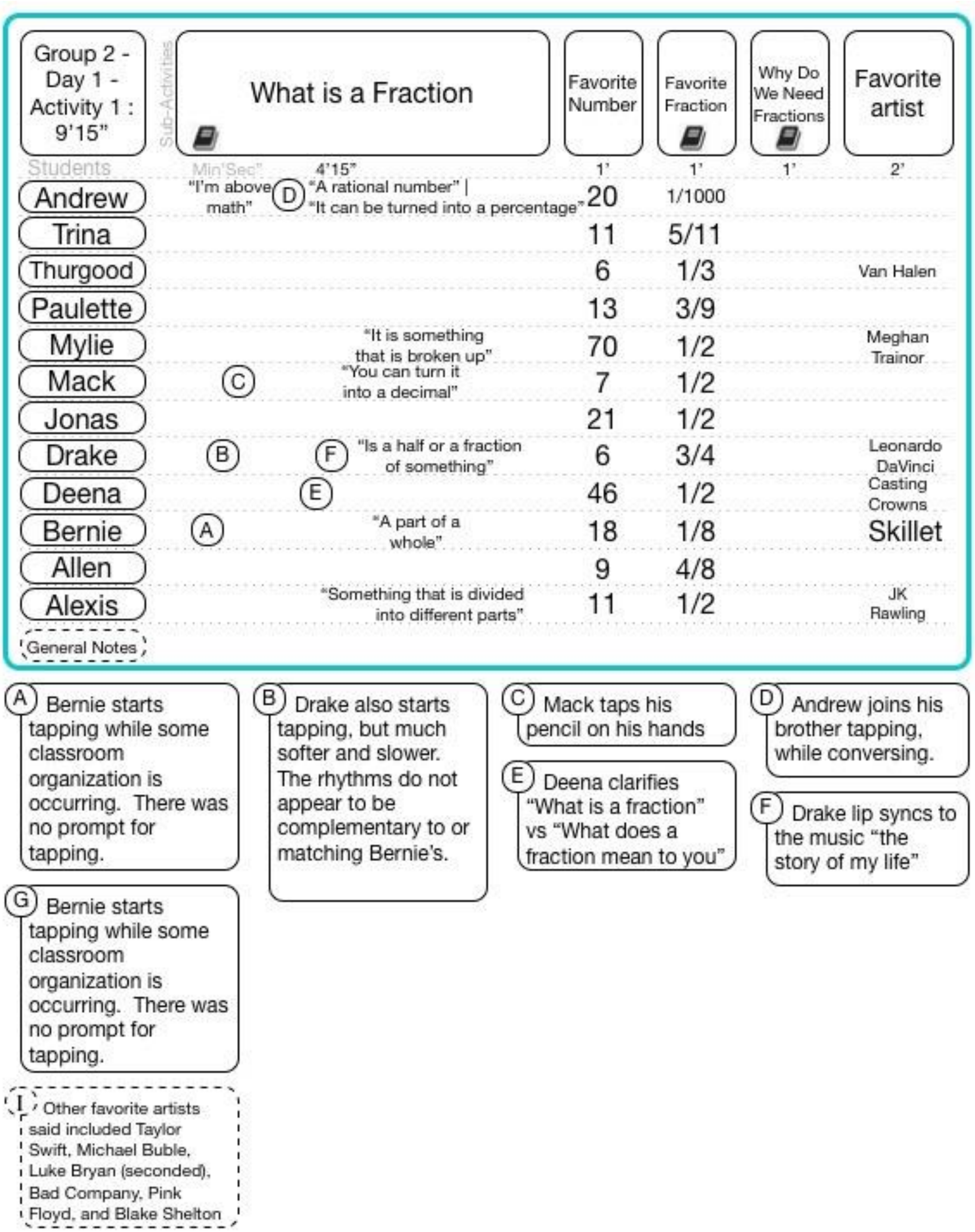


Figure 11-62 Group 2 Day 1 Activity 1 Classroom video data

z) Group 2 Day 1 Activity 2 (a and b)

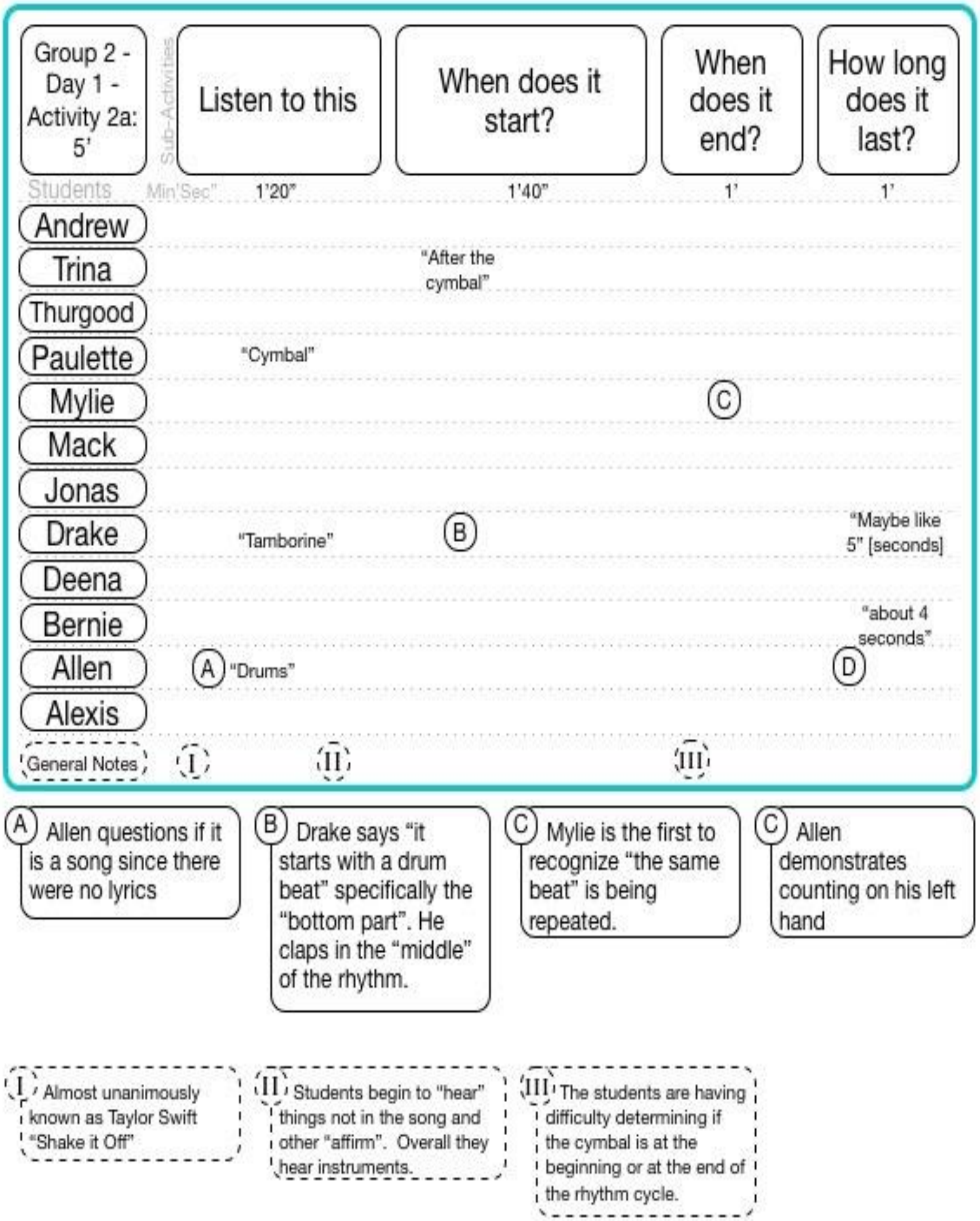


Figure 11-63 Group 2 Day 1 Activity 2a Classroom video data

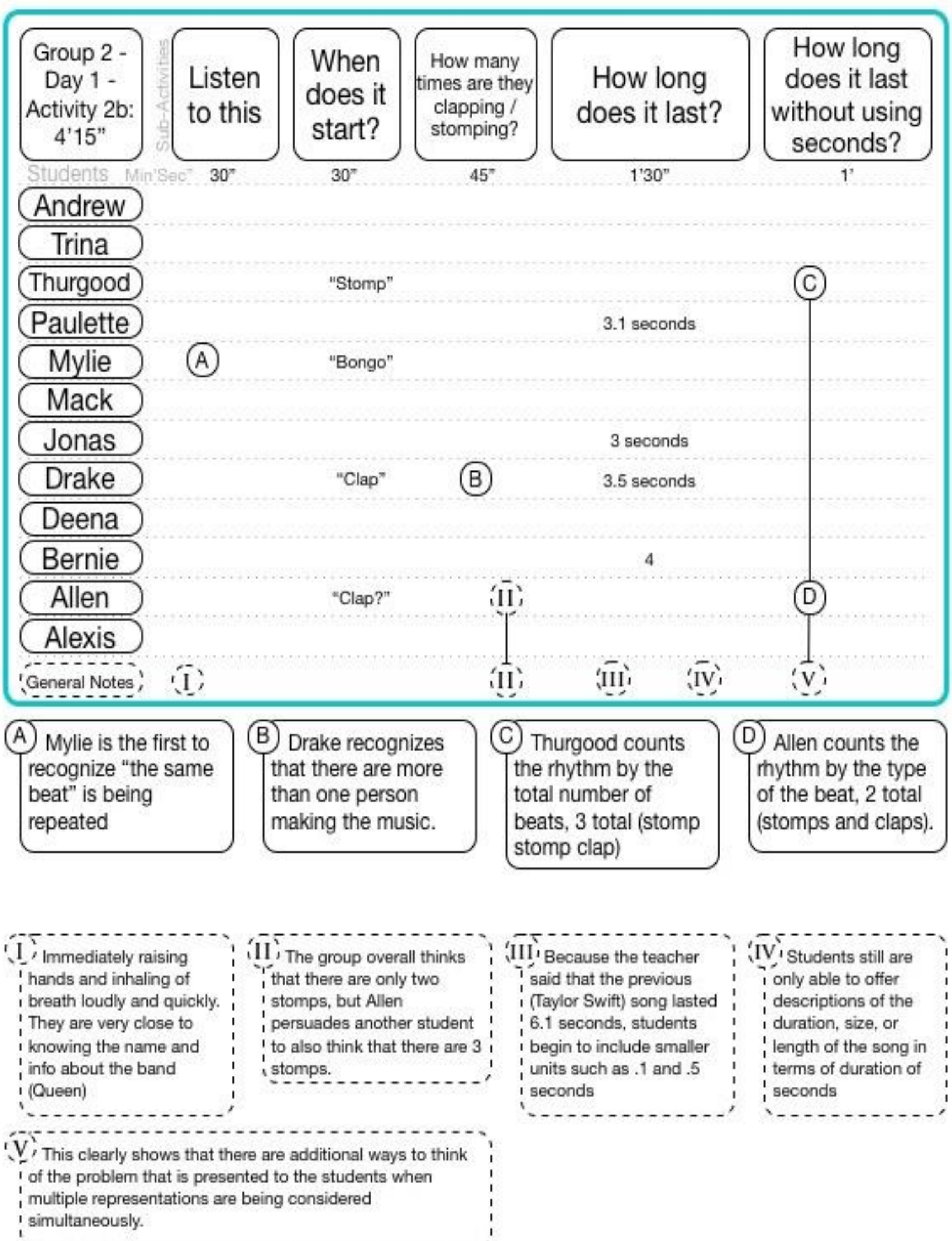


Figure 11-64 Group 2 Day 1 Activity 2b Classroom video data

aa) Group 2 Day 1 Activity 3

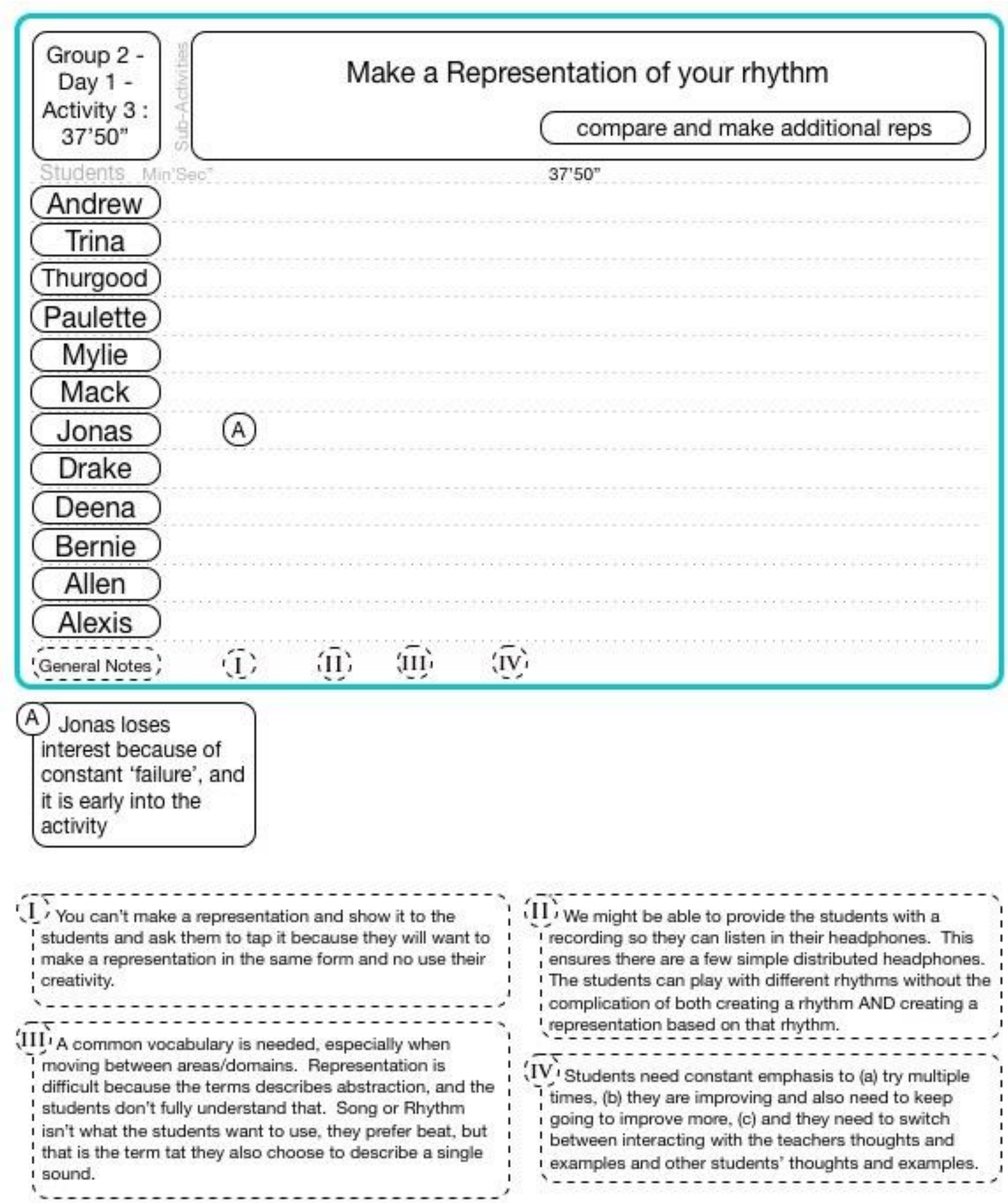


Figure 11-65 Group 2 Day 1 Activity 3 Classroom video data

bb) Group 2 Day 1 Activity 4

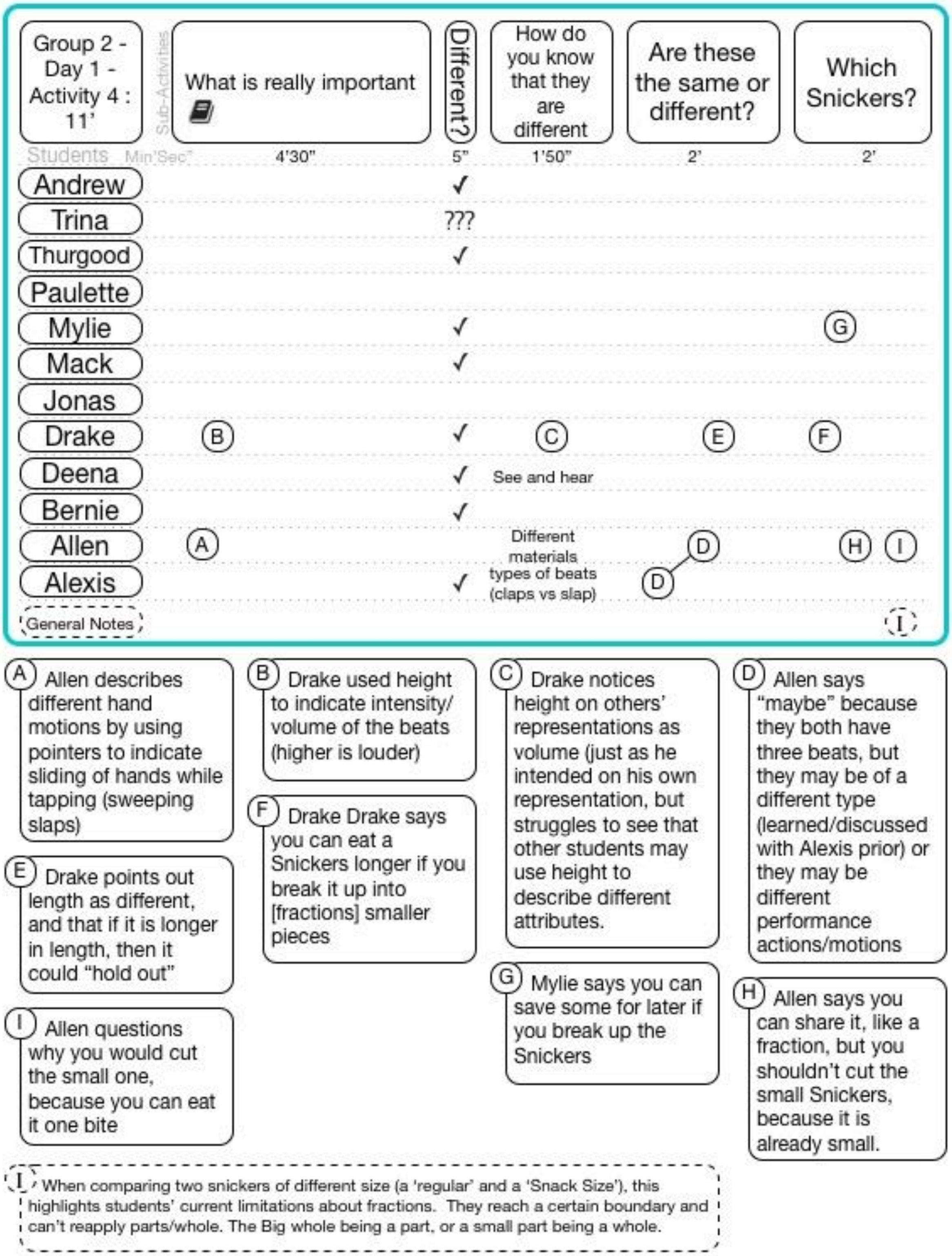


Figure 11-66 Group 2 Day 1 Activity 4 Classroom video data

9. Group 2 Day 2

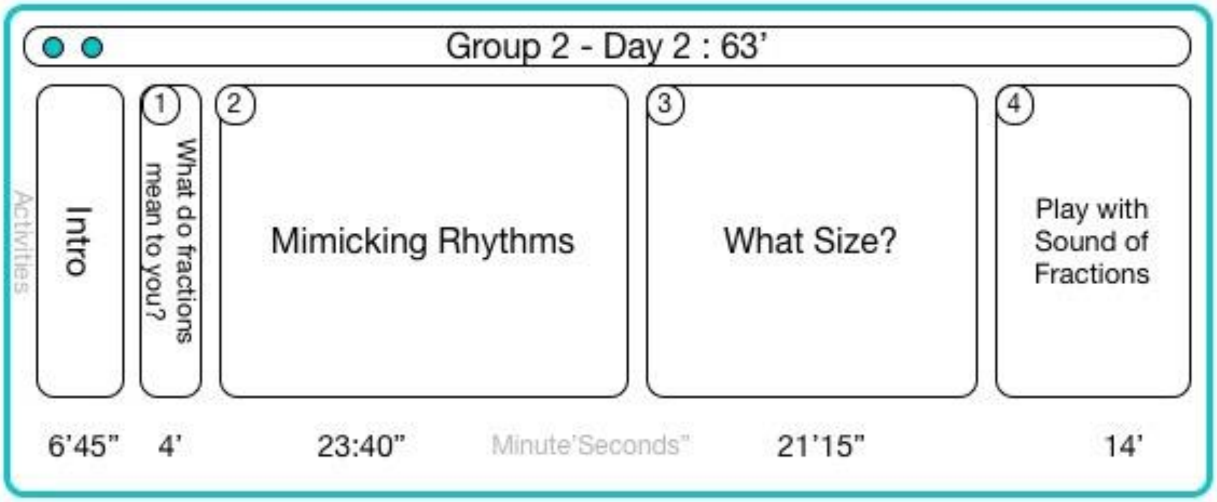


Figure 11-67 Group 2 Day 2 Overview Classroom video data

cc) Group 2 Day 2 Activity 1

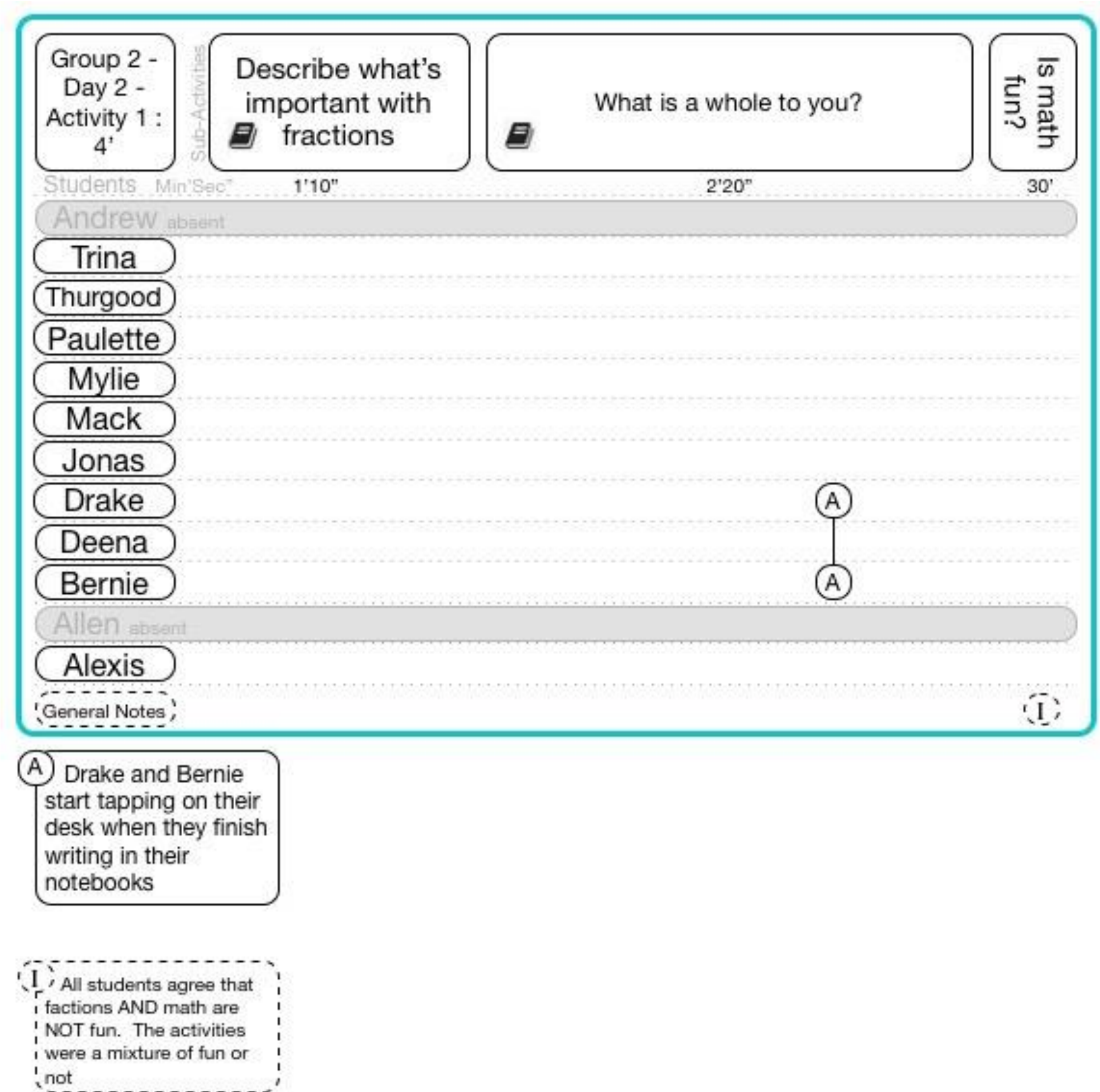


Figure 11-68 Group 2 Day 2 Activity 1 Classroom video data

dd) Group 2 Day 2 Activity 2

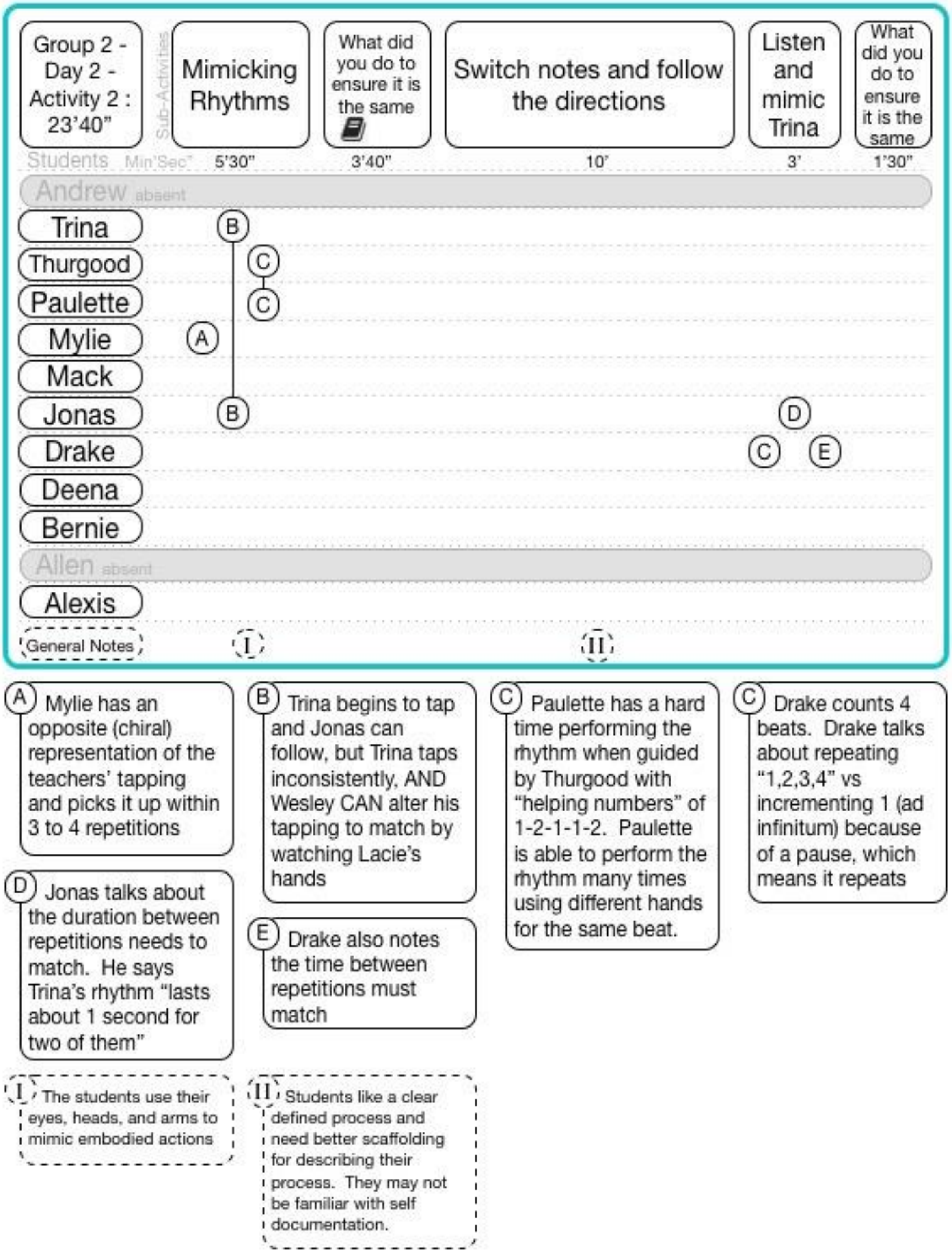


Figure 11-69 Group 2 Day 2 Activity 2 Classroom video data

ee) Group 2 Day 2 Activity 3

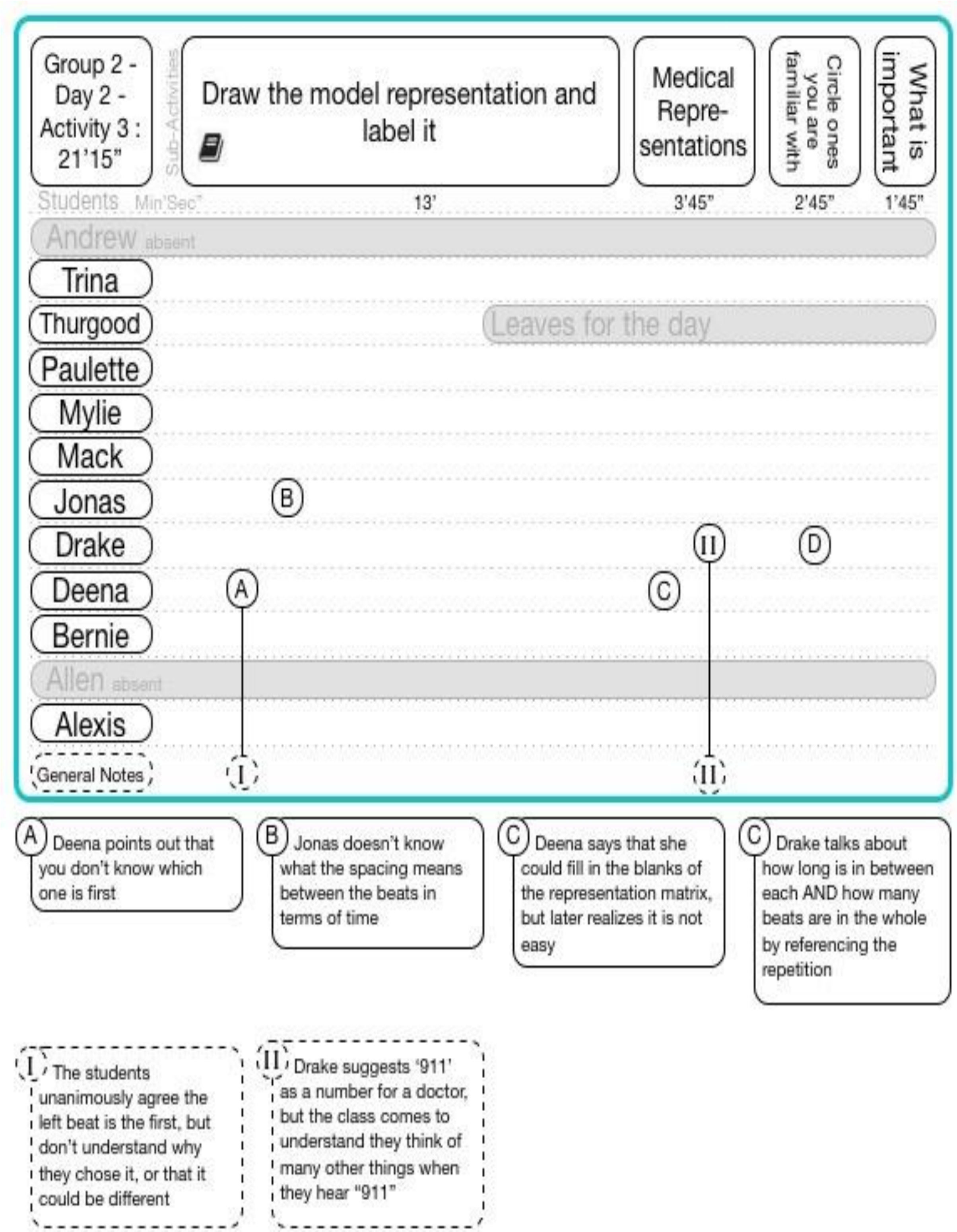


Figure 11-70 Group 2 Day 2 Activity 3 Classroom video data

ff) Group 2 Day 2 Activity 4

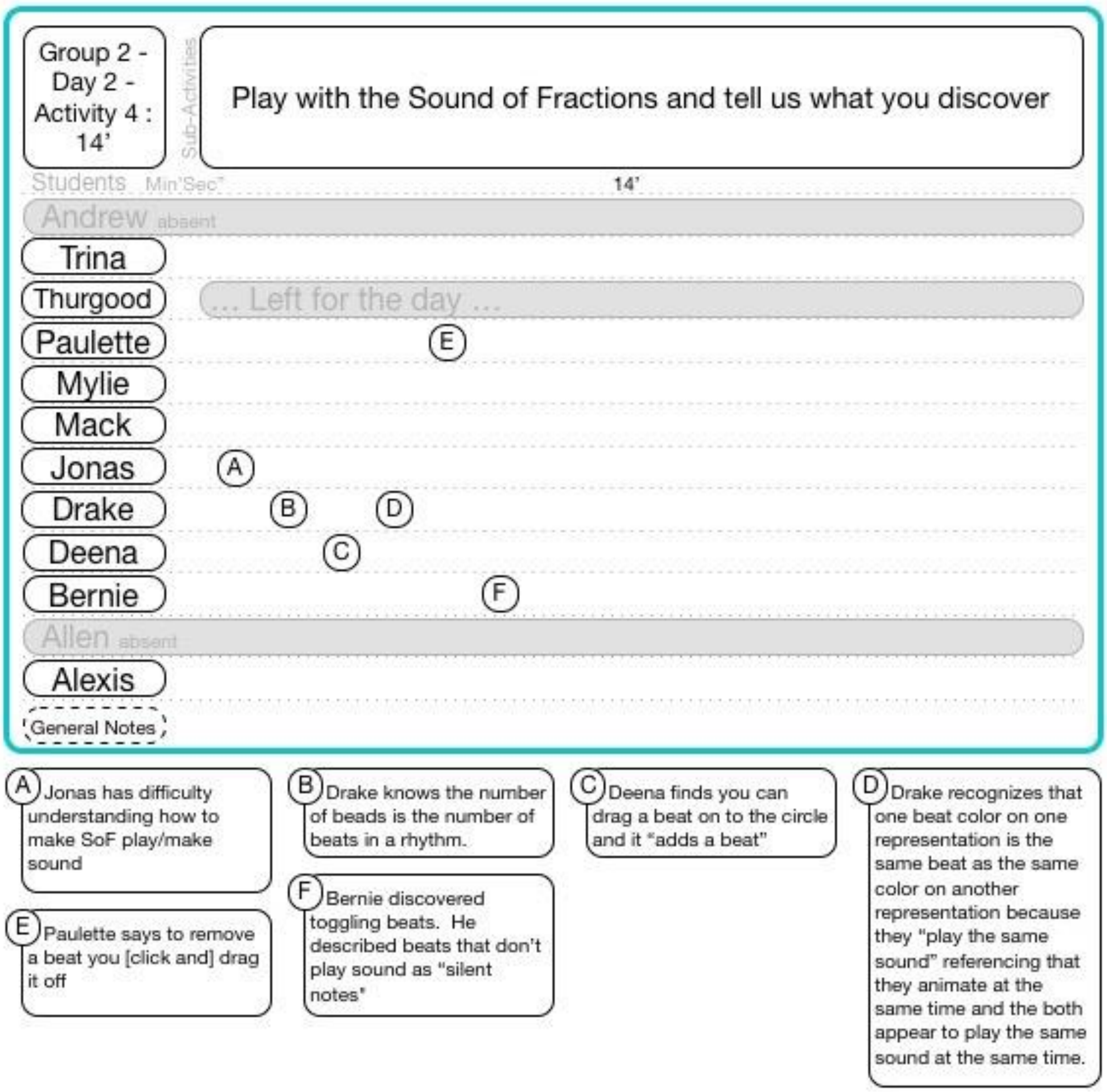


Figure 11-71 Group 2 Day 2 Activity 4 Classroom video data

10. Group 2 Day 3

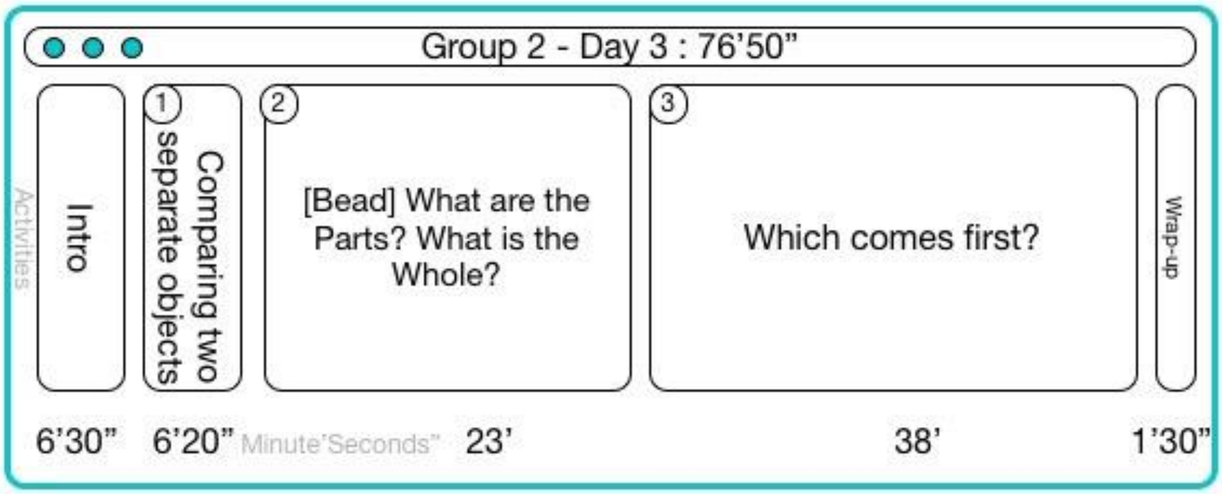


Figure 11-72 Group 2 Day 3 Overview Classroom video data

gg) Group 2 Day 3 Activity 1

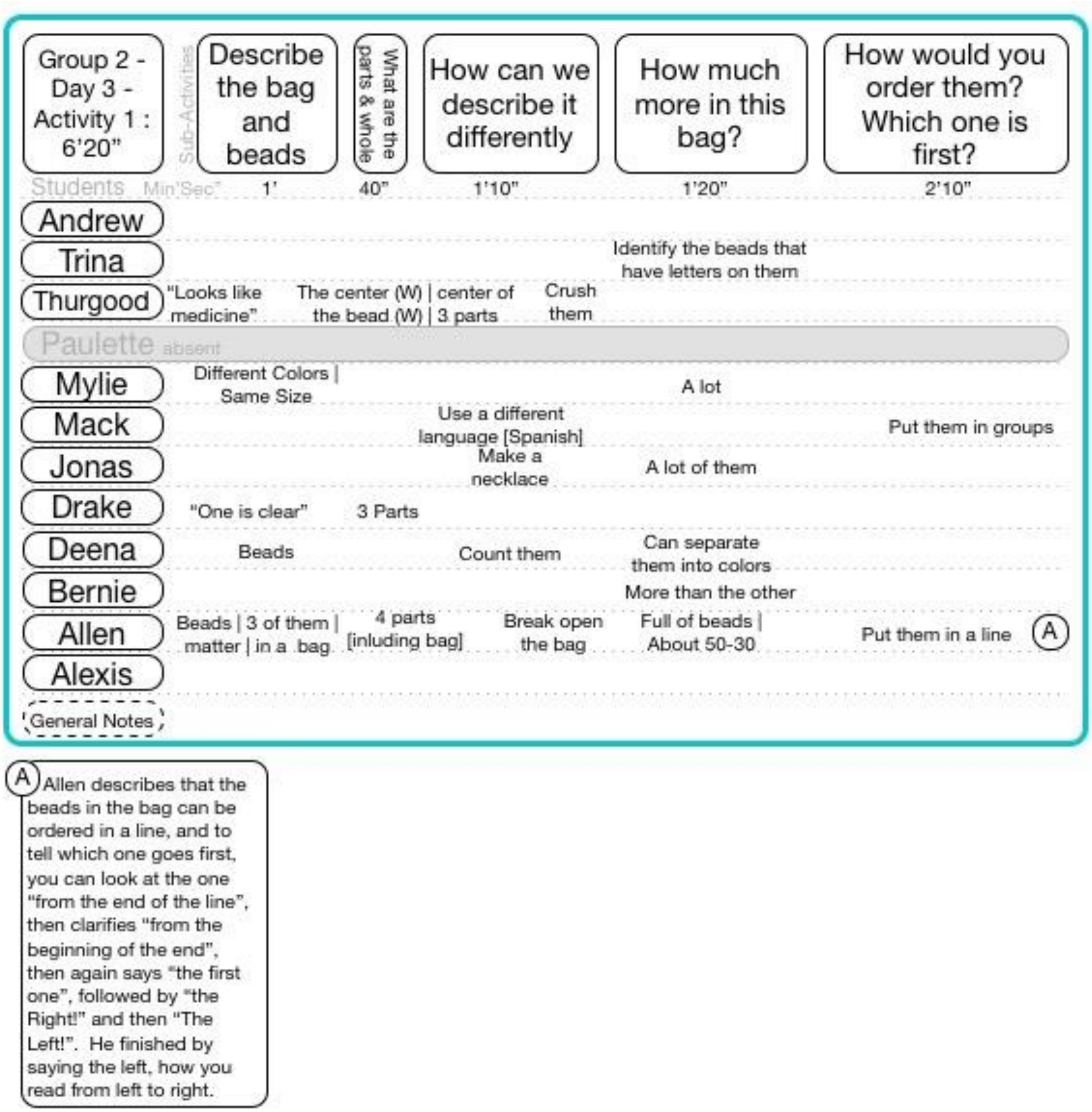


Figure 11-73 Group 2 Day 3 Activity 1 Classroom video data

hh) Group 2 Day 3 Activity 2

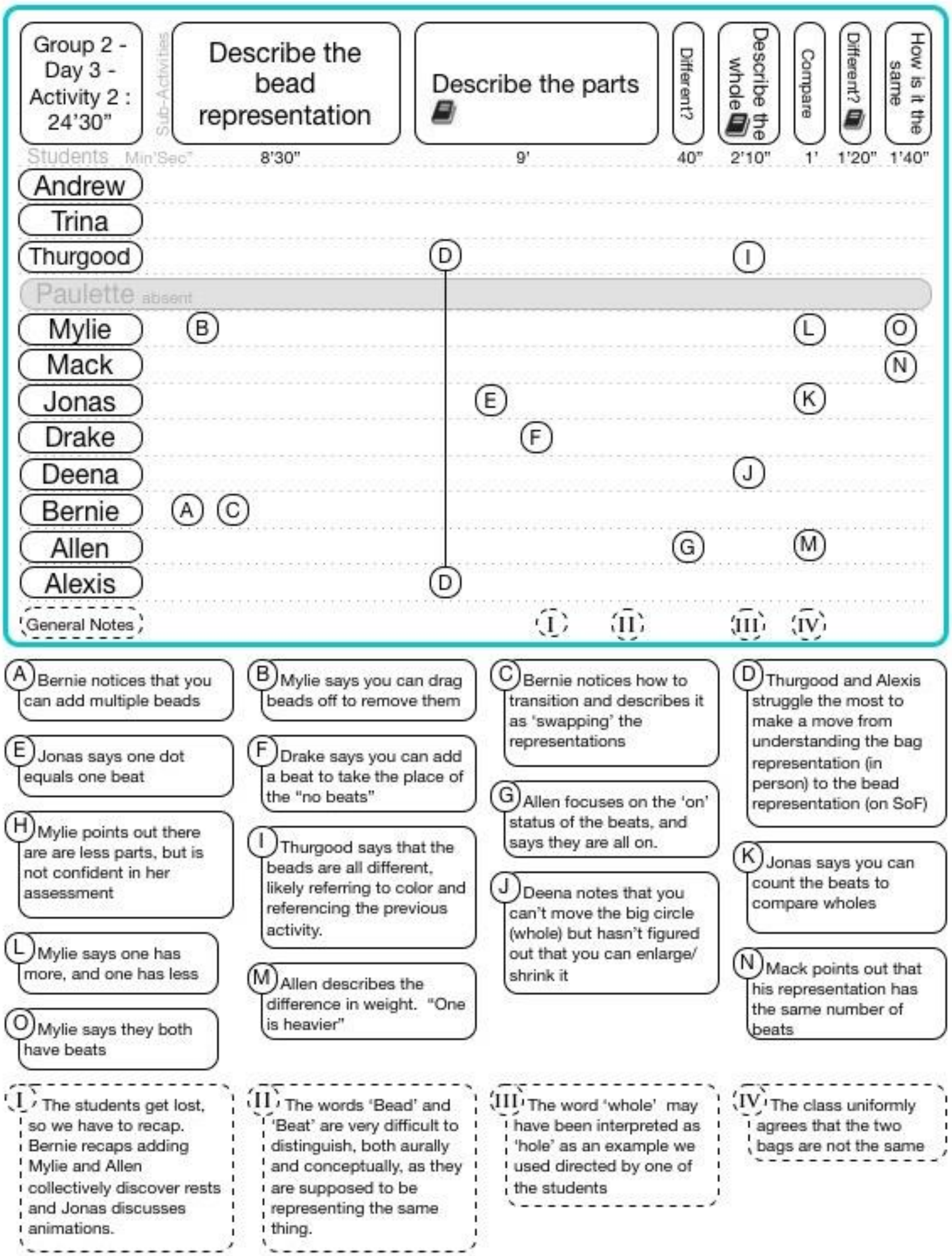


Figure 11-74 Group 2 Day 3 Activity 2 Classroom video data

ii) *Group 2 Day 3 Activity 3*

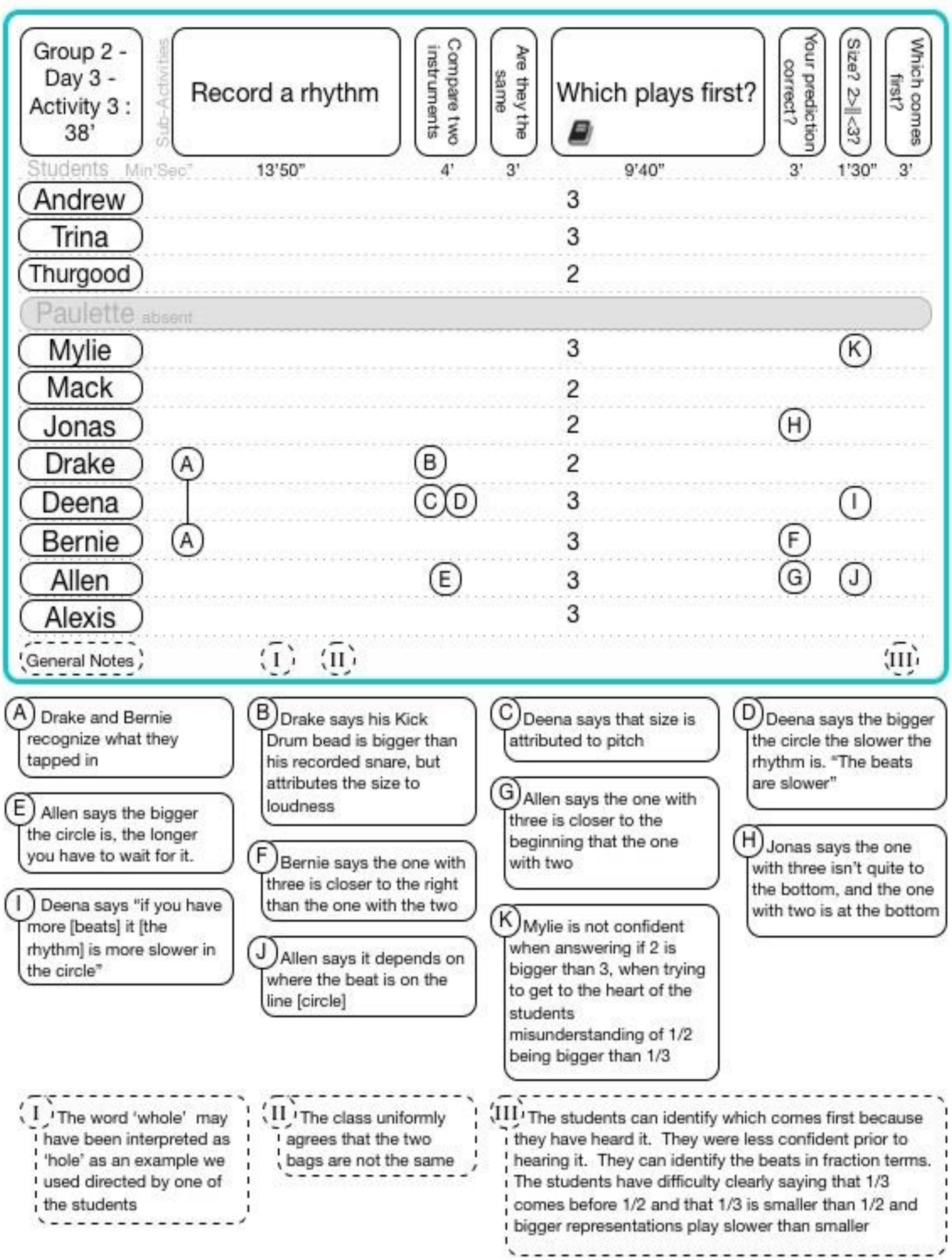


Figure 11-75 Group 2 Day 3 Activity 3 video data

11. Group 2 Day 4

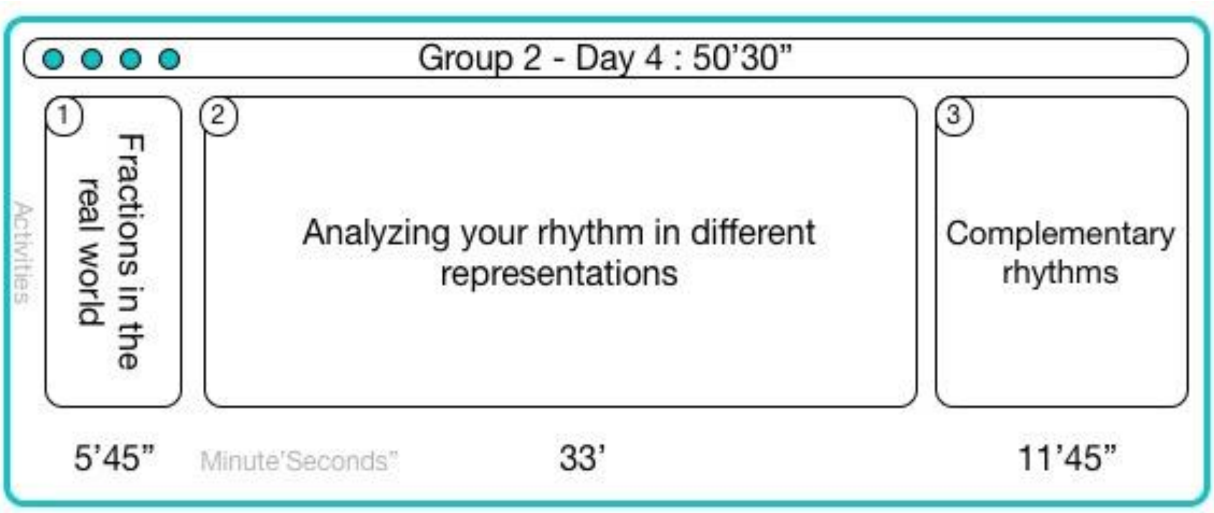


Figure 11-76 Group 2 Day 4 Overview Classroom video data

jj) Group 2 Day 4 Activity 1

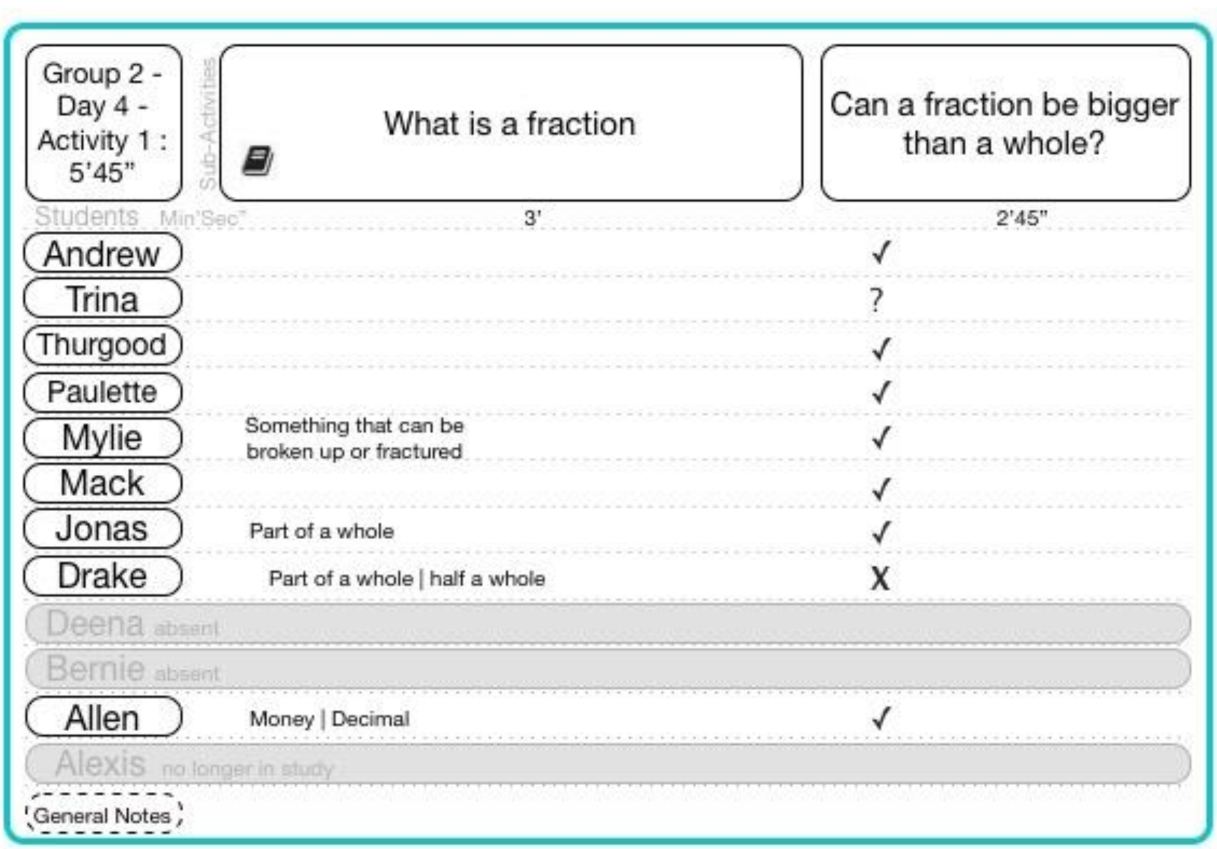


Figure 11-77 Group 2 Day 4 Activity 1 Classroom video data

kk) Group 2 Day 4 Activity 2

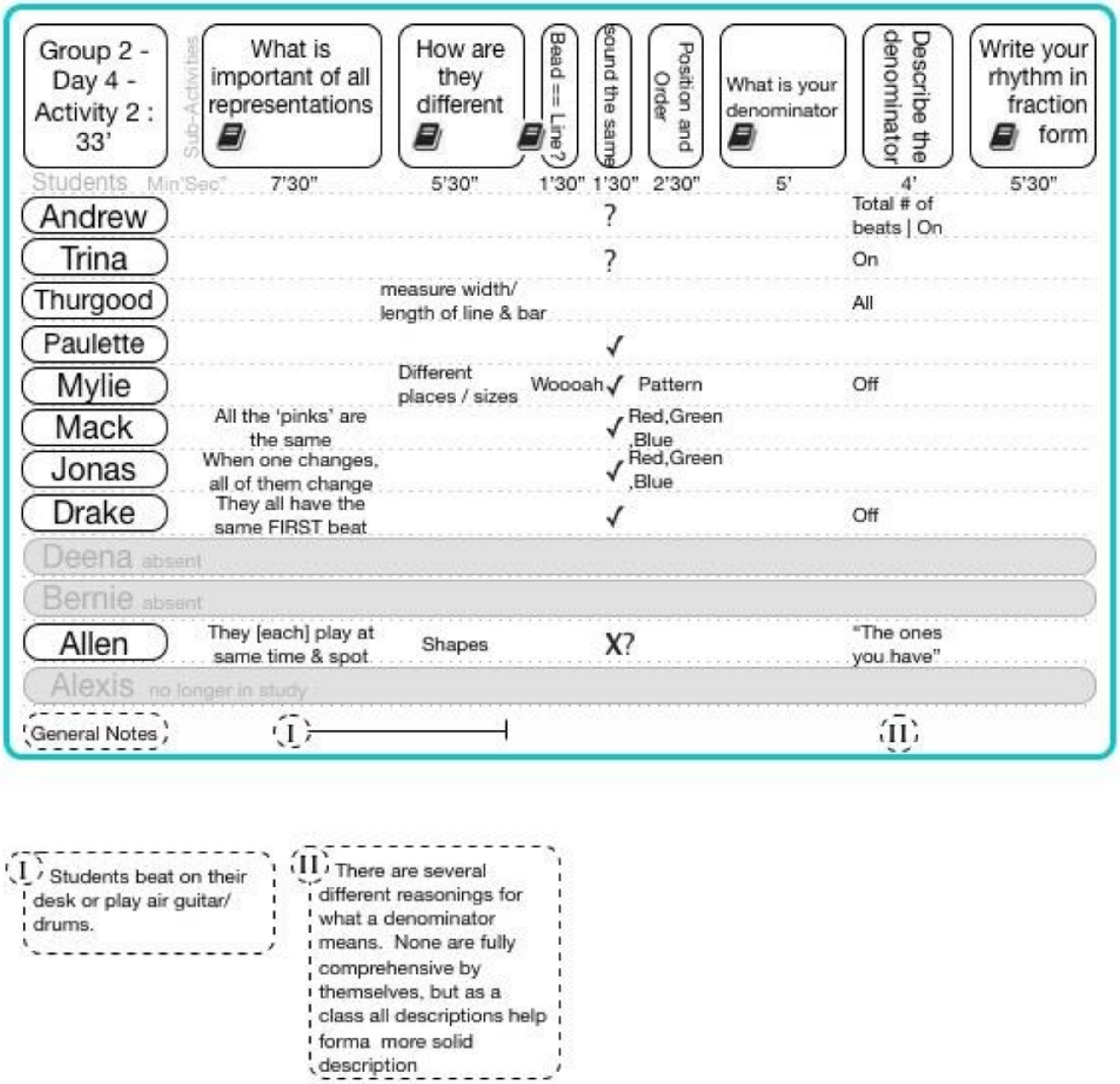


Figure 11-78 Group 2 Day 4 Activity 2 Classroom video data

II) Group 2 Day 4 Activity 3

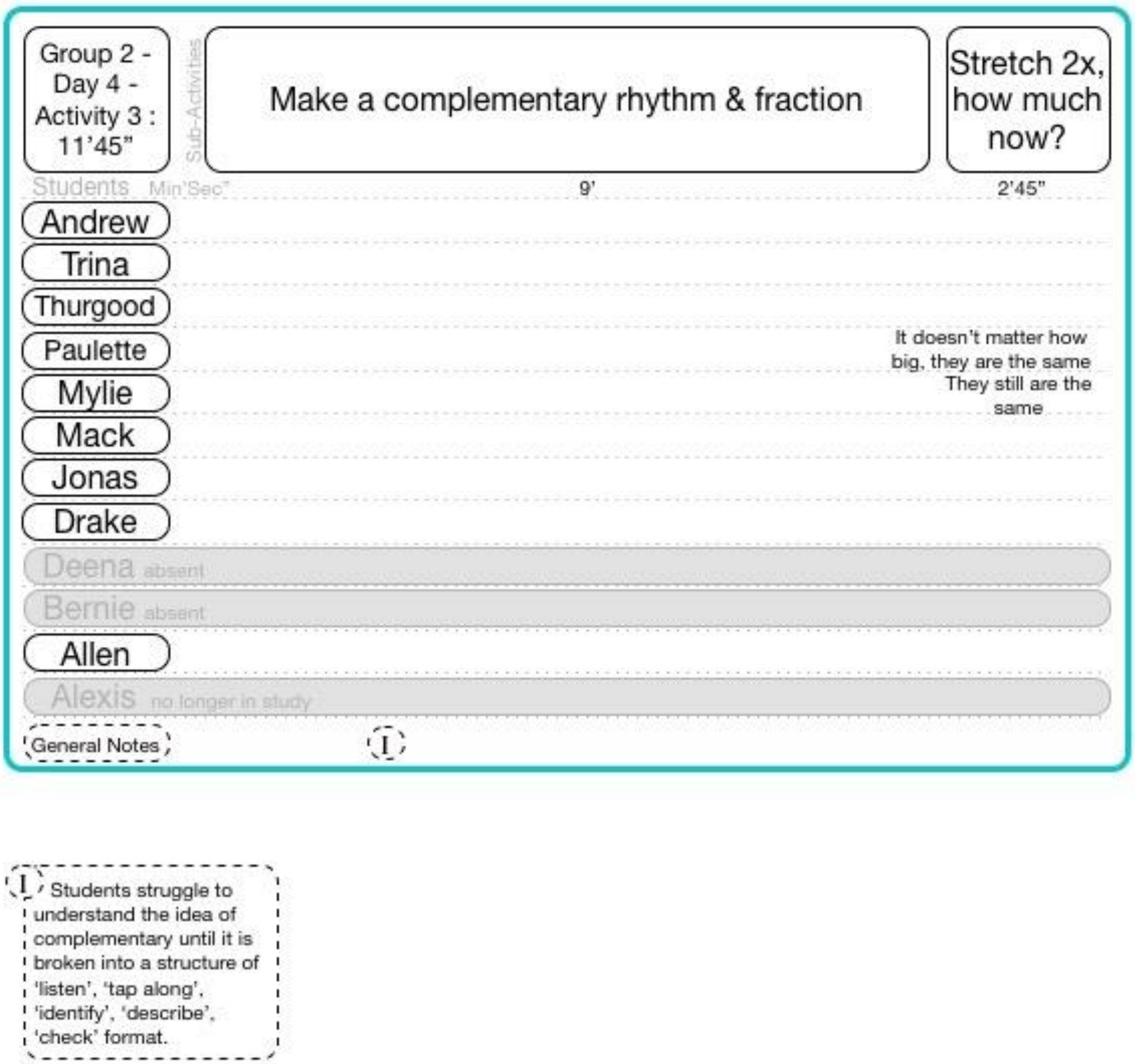


Figure 11-79 Group 2 Day 4 Activity 3 video data

12. Group 2 Day 5

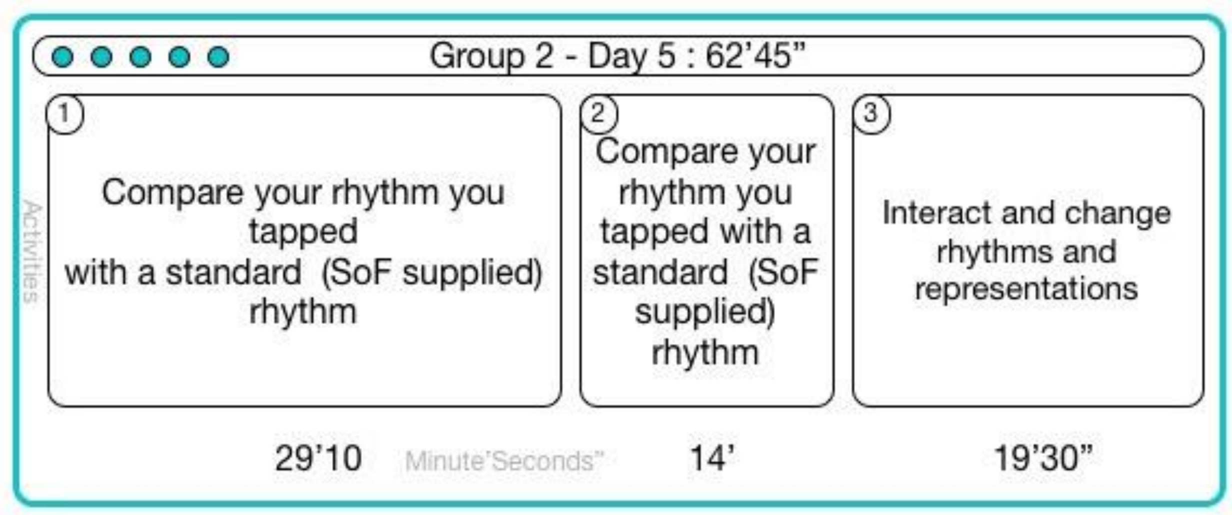


Figure 11-80 Group 2 Day 5 Overview video data

mm) Group 2 Day 5 Activity 1

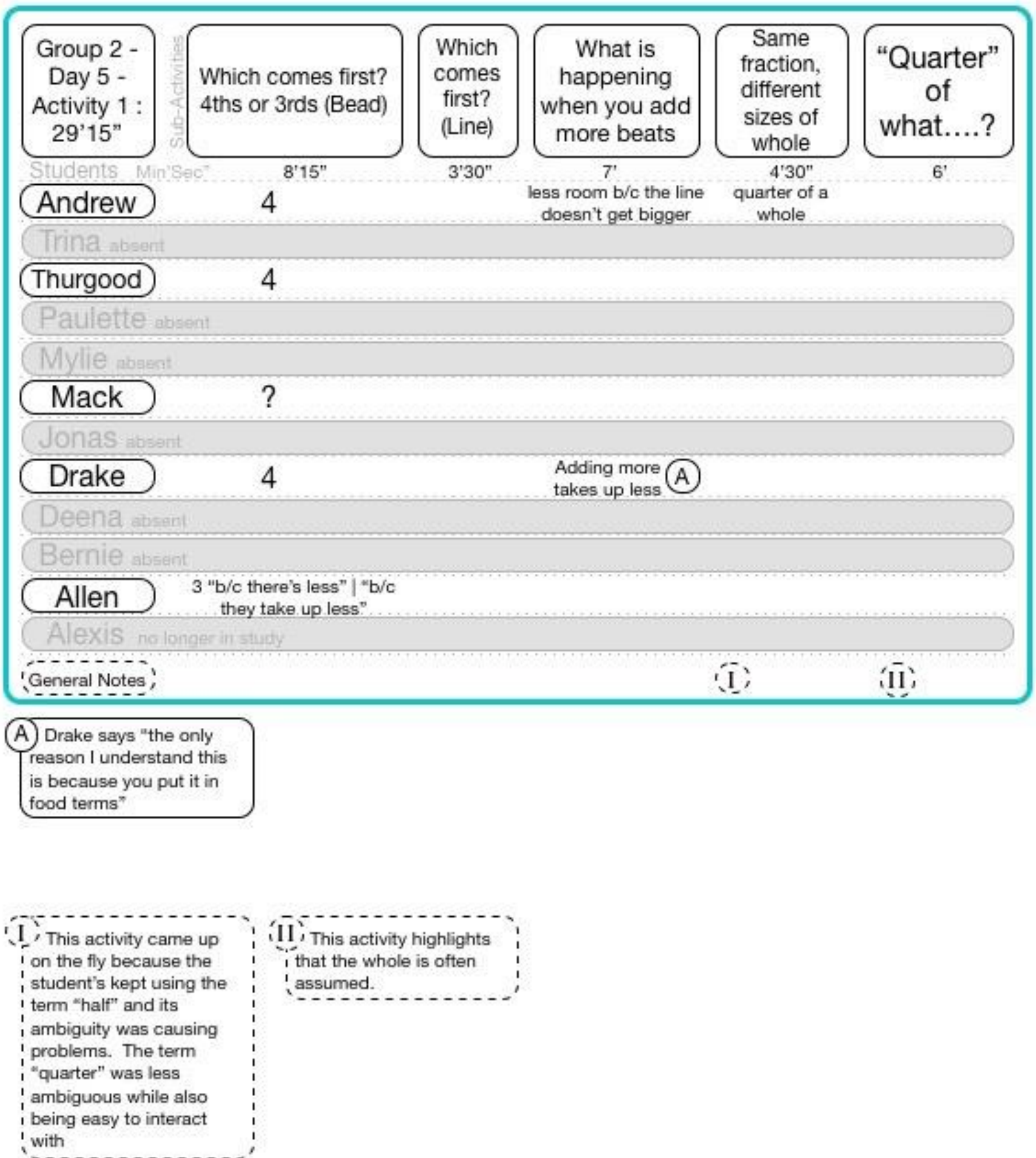


Figure 11-81 Group 2 Day 5 Activity 1 Classroom video data

nn) Group 2 Day 5 Activity 2

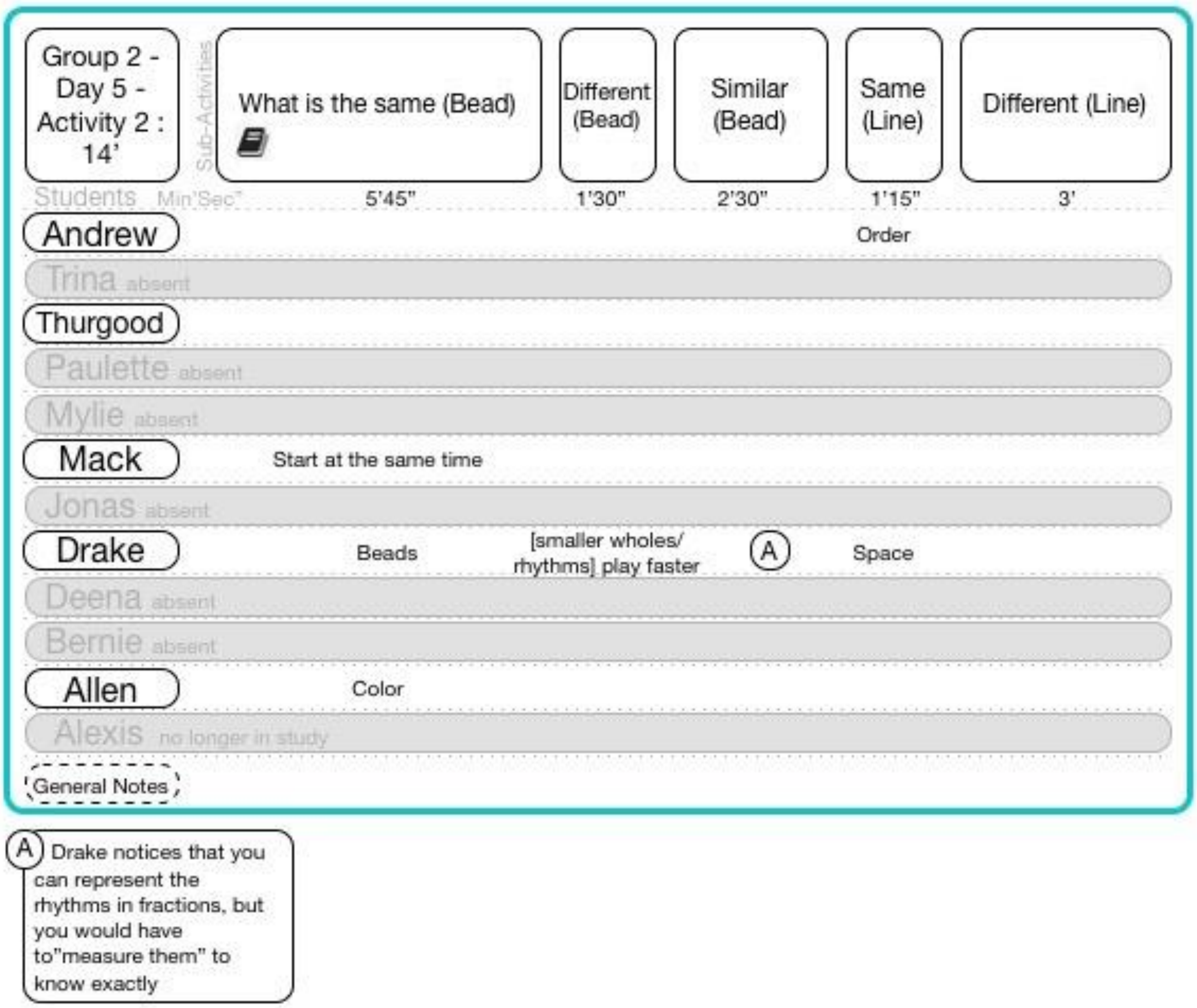


Figure 11-82 Group 2 Day 5 Activity 2 Classroom video data

oo) Group 2 Day 5 Activity 3

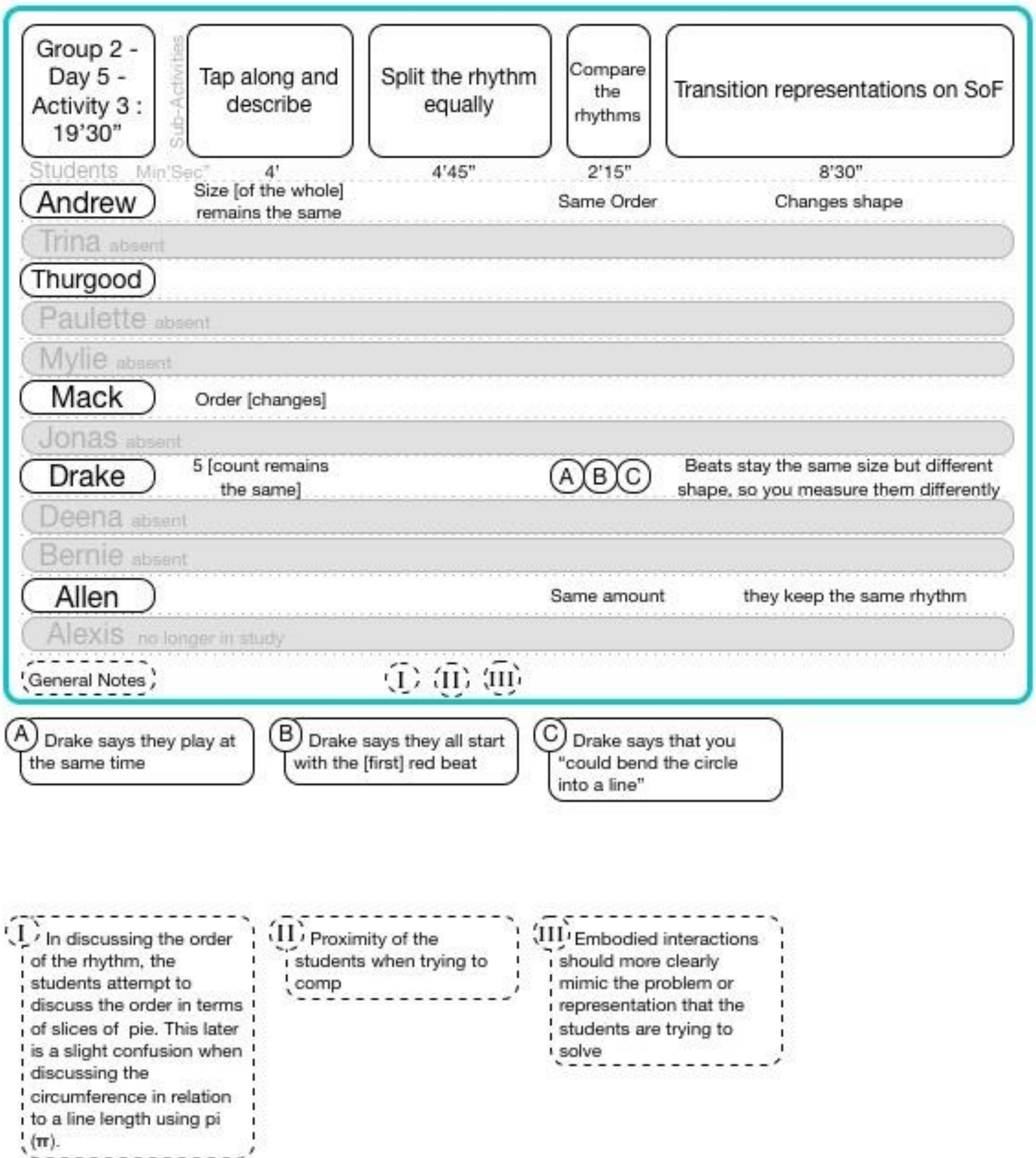


Figure 11-83 Group 2 Day 5 Activity 3 video data

13. Group 2 Day 6

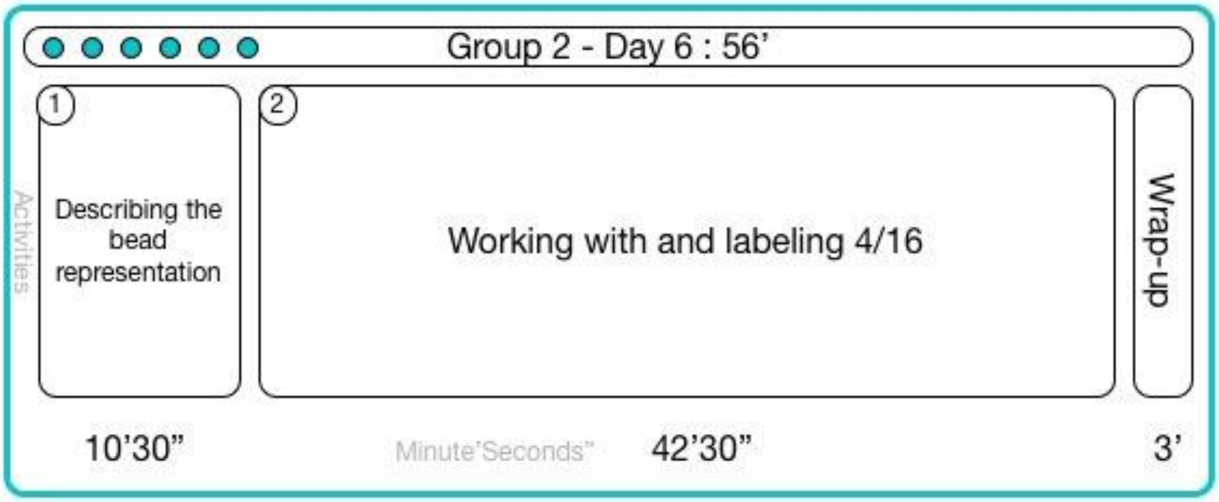


Figure 11-84 Group 2 Day 6 Overview Classroom video data

pp) Group 2 Day 6 Activity 1

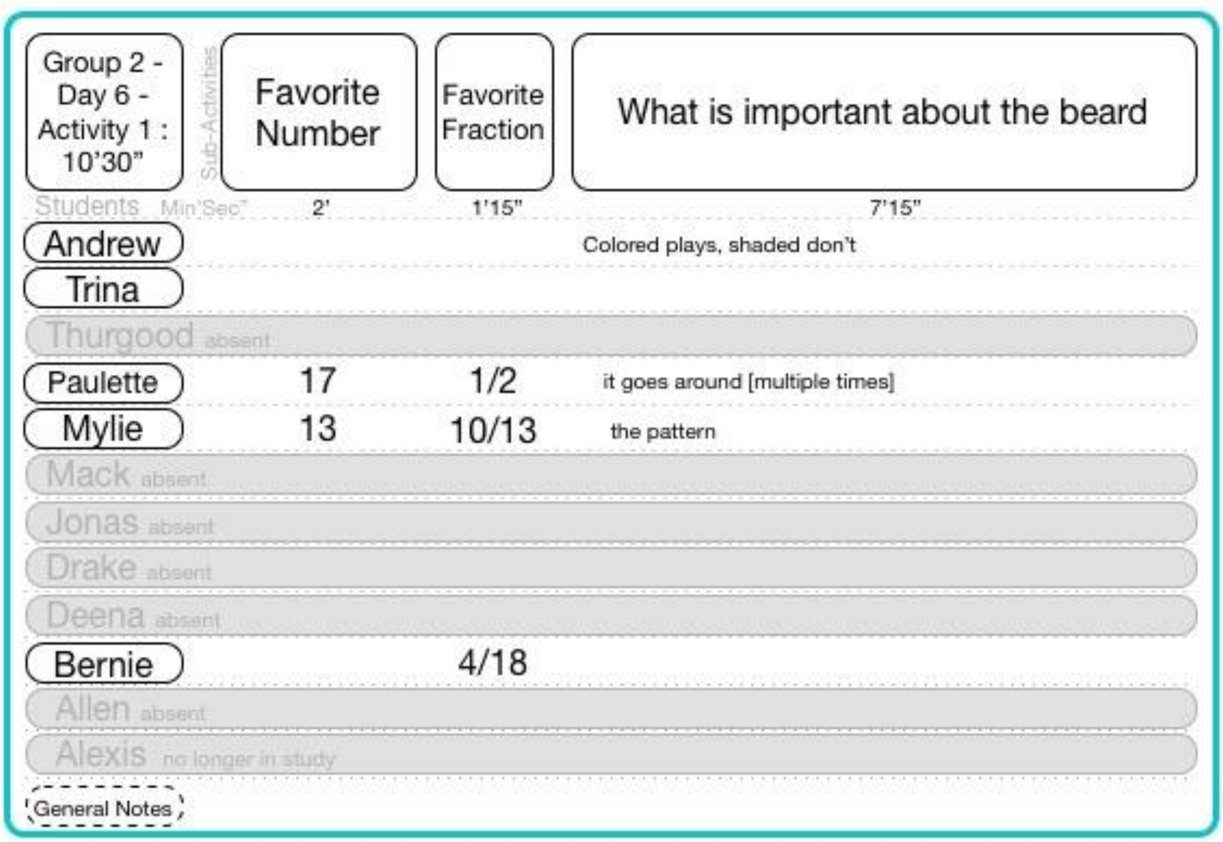


Figure 11-85 Group 2 Day 6 Activity 1 Classroom video data

qq) Group 2 Day 6 Activity 2

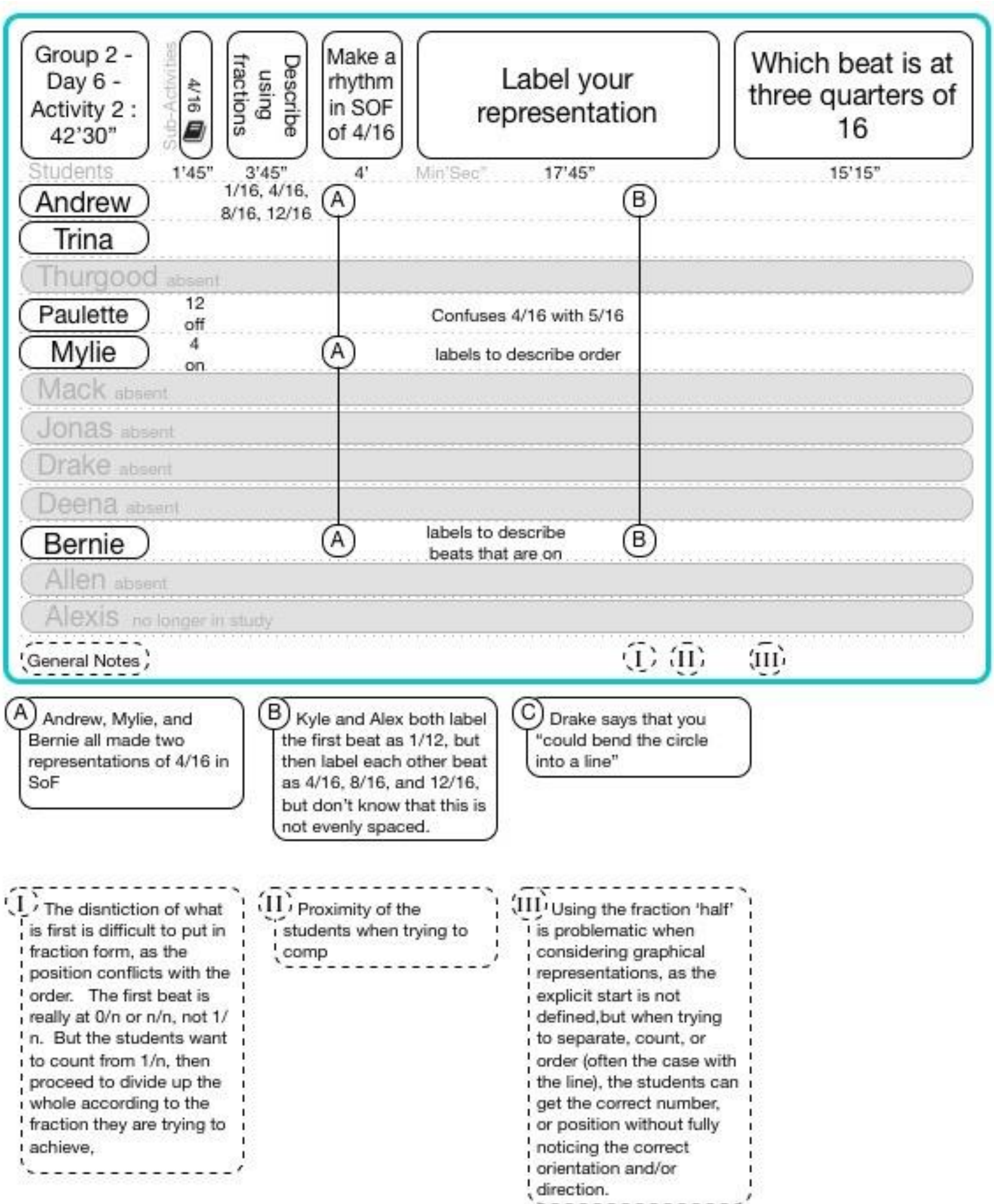


Figure 11-86 Group 2 Day 6 Activity 2 Classroom video data

F. Main Code Branch Development Timeline Video

To see the filesystem develop over time, with the many users contribute to its source, you can watch the following video. One tenth of one second equals one day.

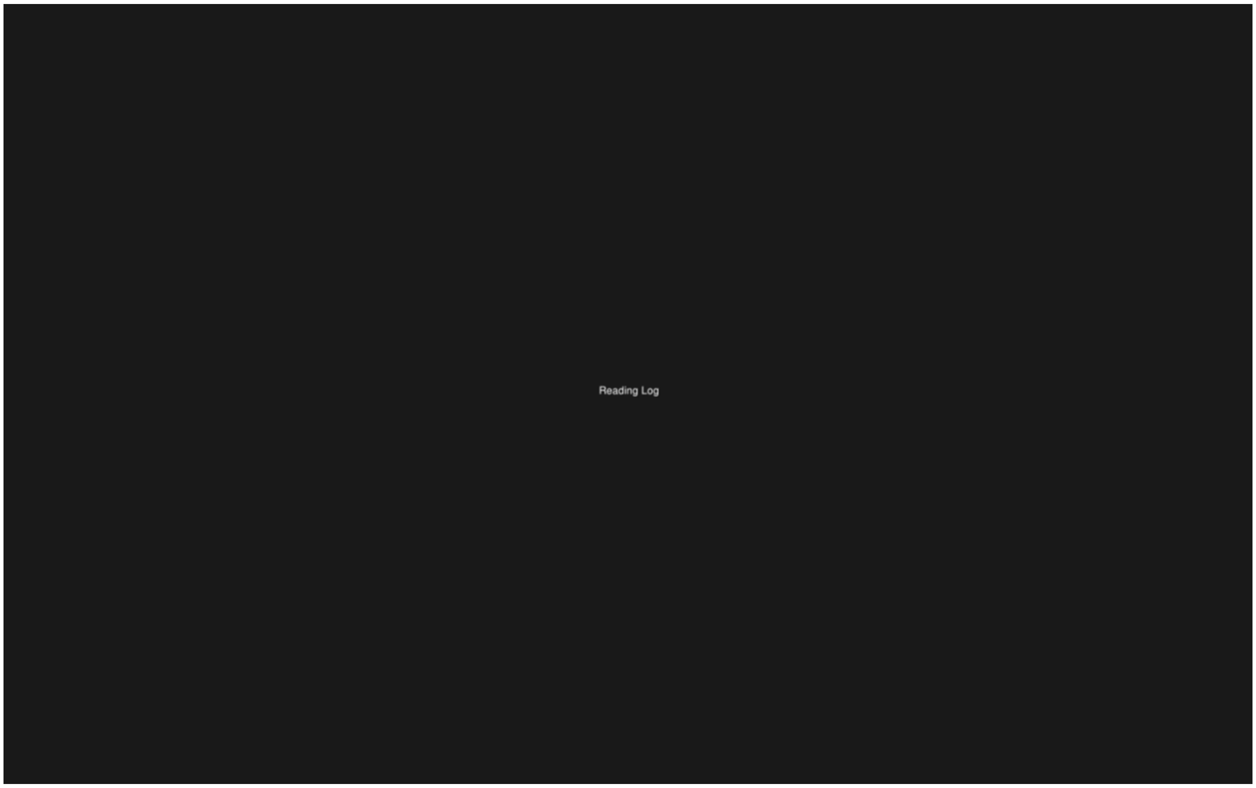


Figure 11-87 A video rendering of a force directed layout of the master branch development of code for the Sound of Fractions. Also viewable at https://youtu.be/fSM2UkWs2Vs

G. Institutional Review Board Certification

VirginiaTech

Office of Research Compliance
Institutional Review Board
North End Center, Suite 4120, Virginia Tech
300 Turner Street NW
Blacksburg, Virginia 24061
540/231-4606 Fax 540/231-0959
email irb@vt.edu
website http://www.irb.vt.edu

MEMORANDUM

DATE: August 22, 2014

TO: Deborah Gail Tatar, Christopher Special Frisina, Steve Harrison

FROM: Virginia Tech Institutional Review Board (FWA00000572, expires April 25, 2018)

PROTOCOL TITLE: Sound Of Fractions User Testing

IRB NUMBER: **14-831**

On August 21, 2014, the Virginia Tech IRB Chair, David M Moore, approved the interim application for the above-mentioned research protocol under 45 CFR 46.118.

This Interim approval only provides permission to being the initial planning required in developing study procedures and forms under this protocol and does not provide permission to begin human subject related activities. This Interim approval is being provided based on confirmation received from you that study procedures involving human subjects will not be initiated until regular (i.e., non-Interim) IRB approval is obtained.

Failure to obtain VT IRB approval prior to conducting human subject activities may result in serious sanctions such as the destruction of data, termination of research and loss of privilege to conduct research at Virginia Tech.

PROTOCOL INFORMATION:

Approved As:	**Interim**
Protocol Approval Date:	**August 21, 2014**
Protocol Expiration Date:	**N/A**
Continuing Review Due Date*:	**N/A**

*Date a Continuing Review application is due to the IRB office if human subject activities covered under this protocol, including data analysis, are to continue beyond the Protocol Expiration Date.

FEDERALLY FUNDED RESEARCH REQUIREMENTS:

Per federal regulations, 45 CFR 46.103(f), the IRB is required to compare all federally funded grant proposals/work statements to the IRB protocol(s) which cover the human research activities included in the proposal / work statement before funds are released. Note that this requirement does not apply to Exempt and Interim IRB protocols, or grants for which VT is not the primary awardee.

The table on the following page indicates whether grant proposals are related to this IRB protocol, and which of the listed proposals, if any, have been compared to this IRB protocol, if required.

Invent the Future

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
An equal opportunity, affirmative action institution

Figure 11-88 : IRB Certificate Page

H. Indirect References

The Following were referenced during research. Although we are not directly citing them, we would like to acknowledge that their research came up during our efforts and may have also helped steer us, albeit unconsciously.

Simmel, G. (1950). The stranger. *The Sociology of Georg Simmel*, 402–408.

Simon, H. A. (1969). *The Sciences of the Artificial* (Vol. 136). Cambridge, MA, USA: MIT Press.

Weeden, R. E. (1972). *A comparison of the academic achievement in reading and mathematics of Negro children whose parents are interested, not interested, or involved in a program of Suzuki violin.* ProQuest Information & Learning.

Fuller, R. B. (1975). *Synergetics*. Pacific Tape Library.

Shaw, M. (1980). The impact of abstraction concerns on modern programming languages. *Proceedings of the IEEE, 68*(9), 1119–1130.

Landers, R. (1984). The Talent Education School of Shinichi Suzuki: An Analysis: Application of Its Philosophy and Methods to All Areas of Instruction.

d'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the Learning of Mathematics*, 44–48.

Neufeld, K. A. (1986). Understanding of selected pre-number concepts: Relationships to a formal music program. *Alberta Journal of Educational Research*.

Alexander, E. R. (1987). Design in planning and decision making: theory and its implications for practice. In *Proceedings of the 1987 conference on planning and design in urban & regional planning (1987 international congress on planning and design theory, Boston, MA, 17--20 August 1987). ASME, New York* (pp. 25–30).

Schön, D. A. (1987). Educating the reflective practitioner: Toward a new design for teaching and learning in the professions. *San Francisco*.

D'Ambrosio, U. (1987). Reflections on ethnomathematics. *International Study Group on Ethnomathematics Newsletter*, *3*(1), 3–5.

Nielsen, J., & Molich, R. (1990). Heuristic evaluation of user interfaces. In *Proceedings of the SIGCHI conference on Human factors in computing systems* (pp. 249–256).

Shaw, M. (1990). Prospects for an engineering discipline of software. *Software, IEEE*, *7*(6), 15–24.

Fish, R. S., Kraut, R. E., Root, R. W., & Rice, R. E. (1992). Evaluating video as a technology for informal communication. In *Proceedings of the SIGCHI conference on Human factors in computing systems* (pp. 37–48).

Carroll, J. M., & Rosson, M. B. (1992). Getting around the task-artifact cycle: how to make claims and design by scenario. *ACM Transactions on Information Systems (TOIS)*, *10*(2), 181–212.

Barten, S. S. (1992). The language of musical instruction. *Journal of Aesthetic Education*, 53–61.

Buchanan, R. (1992). Wicked problems in design thinking. *Design Issues*, 5–21.

Feldman, E. B. (1994). *Practical art criticism*. Prentice Hall New York.

ABELES, H. F., HOFFER, C. F., & KLOTMAN, R. H. (1994). Social Psychological Foundations Of Music Education. *Schimer Book, New York, ABD*.

Williams, N. (1995). The Comic Book as Course Book: Why and How.

Kafai, Y. B., & Resnick, M. (1996). *Constructionism in practice: Designing, thinking, and learning in a digital world*. Routledge.

Bødker, S. (1996). Creating conditions for participation: Conflicts and resources in systems development. *Human-Computer Interaction*, *11*(3), 215–236.

Constant, D., Sproull, L., & Kiesler, S. (1996). The kindness of strangers: The usefulness of electronic weak ties for technical advice. *Organization Science*, *7*(2), 119–135.

Wellman, B. (1997). An electronic group is virtually a social network. *Culture of the Internet*, *4*, 179–205.

Jordan, A., Lindsay, L., & Stanovich, P. J. (1997). Classroom teachers' instructional interactions with students who are exceptional, at risk, and typically achieving. *Remedial and Special Education*, *18*(2), 82–93.

Sinor, J. (1997). The Ideas of Kod{á}ly in America This article on the application of Zolt{á}n Kod{á}ly's teaching method in the United States was first published in the Music Educators Journal in February 1986. *Music Educators Journal*, *83*(5), 37–41.

Story, M. F. (1998). Maximizing usability: the principles of universal design. *Assistive Technology*, *10*(1), 4–12.

Herbsleb, J. D., & Grinter, R. E. (1999). Architectures, coordination, and distance: Conway's law and beyond. *IEEE Software*, *16*(5), 63–70.

Reimer, B. (1999). Facing the Risks of the“ Mozart Effect.” Grand Masters Series. *Music Educators Journal*, *86*(1), 37–43.

MacKay, W. E. (1999). Is paper safer? The role of paper flight strips in air traffic control. *ACM Transactions on Computer-Human Interaction (TOCHI)*, *6*(4), 311–340.

Hollan, J., Hutchins, E., & Kirsh, D. (2000). Distributed cognition: toward a new foundation for human-computer interaction research. *ACM Transactions on Computer-Human Interaction (TOCHI)*, *7*(2), 174–196.

Czerwinski, M., Cutrell, E., & Horvitz, E. (2000). Instant messaging and interruption: Influence of task type on performance. In *OZCHI 2000 conference proceedings* (Vol. 356, p. 361).

Hetland, L. (2000). Learning to make music enhances spatial reasoning. *Journal of Aesthetic Education*, 179–238.

Vaughn, K. (2000). Music and mathematics: Modest support for the oft-claimed relationship. *Journal of Aesthetic Education*, 149–166.

Sherin, B. L. (2001). A comparison of programming languages and algebraic notation as expressive languages for physics. *International Journal of Computers for Mathematical Learning*, *6*(1), 1–61.

Hartson, H. R., Andre, T. S., & Williges, R. C. (2001). Criteria for evaluating usability evaluation methods. *International Journal of Human-Computer Interaction*, *13*(4), 373–410.

Somervell, J., Srinivasan, R., Vasnaik, O., & Woods, K. (2001). Measuring distraction and awareness caused by graphical and textual displays in the periphery. In *Proceedings of the 39th Annual ACM Southeast Conference*.

D’Ambrosio, U. (2001). What Is Ethnomathematics, and How Can It Help Children in Schools?. *Teaching Children Mathematics*, *7*(6), 308–310.

Baker, K., Greenberg, S., & Gutwin, C. (2002). Empirical development of a heuristic evaluation methodology for shared workspace groupware. In *Proceedings of the 2002 ACM conference on Computer supported cooperative work* (pp. 96–105).

Wilson, M. (2002). Six views of embodied cognition. *Psychonomic Bulletin & Review*, *9*(4), 625–636.

Russell, D. M., Drews, C., & Sue, A. (2002). Social aspects of using large public interactive displays for collaboration. In *UbiComp 2002: Ubiquitous Computing* (pp. 229–236). Springer.

Norman, D. A. (2002). *The psychpathology of everyday things*. Basic books.

Rosson, M. B., & Carroll, J. J. M. (2002). *Usability engineering [electronic resource]: scenario-based development of human-computer interaction*. Morgan Kaufmann.

Zhao, Q. A., & Stasko, J. T. (2002). What's happening?: promoting community awareness through opportunistic, peripheral interfaces. In *Proceedings of the working conference on advanced visual interfaces* (pp. 69–74).

Collective, T. D.-B. R. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, 5–8.

Denning, P. J. (2003). Great principles of computing. *Communications of the ACM*, *46*(11), 15–20.

Gulliksen, J., Göransson, B., Boivie, I., Blomkvist, S., Persson, J., & Cajander, Å. (2003). Key principles for user-centred systems design. *Behaviour and Information Technology*, *22*(6), 397–409.

Trafton, J. G., Altmann, E. M., Brock, D. P., & Mintz, F. E. (2003). Preparing to resume an interrupted task: Effects of prospective goal encoding and retrospective rehearsal. *International Journal of Human-Computer Studies*, *58*(5), 583–603.

Amrit, C., Hillegersberg, J., & Kumar, K. (2004). A social network perspective of Conway's law. In *Proceedings of the CSCW Workshop on Social Networks, Chicago, IL, USA*.

Collins, A., Joseph, D., & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *The Journal of the Learning Sciences*, *13*(1), 15–42.

Somervell, J. (2004). *Developing heuristic evaluation methods for large screen information exhibits based on critical parameters*. Virginia Polytechnic Institute and State University.

Erler, M. F. (2004). Exploring World Cultures with Music. *Social Studies and the Young Learner*, *17*(1), 30–32.

De Souza, C., Dourish, P., Redmiles, D., Quirk, S., & Trainer, E. (2004). From technical dependencies to social dependencies. In *Workshop on Social Networks for Design and Analysis: Using Network Information in CSCW*.

Bayazit, N. (2004). Investigating design: A review of forty years of design research. *Design Issues*, *20*(1), 16–29.

Harel, G., & Lim, K. (2004). Mathematics teachers' knowledge base: Preliminary results. In *Proceeding of the 28th annual meeting of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 25–32).

Rogers, Y. (2004). New theoretical approaches for HCI. *Annual Review of Information Science and Technology*, *38*(1), 87–143.

Viégas, F. B., & Donath, J. (2004). Social network visualization: Can we go beyond the graph. In *Workshop on Social Networks, CSCW* (Vol. 4, pp. 6–10).

Tatum, J. S. (2004). The challenge of responsible design. *Design Issues*, *20*(3), 66–80.

Abras, C., Maloney-Krichmar, D., & Preece, J. (2004). User-centered design. *Bainbridge, W. Encyclopedia of Human-Computer Interaction. Thousand Oaks: Sage Publications*, *37*(4), 445–456.

Dourish, P. (2004). *Where the action is: the foundations of embodied interaction*. MIT press.

Boehner, K., DePaula, R., Dourish, P., & Sengers, P. (2005). Affect: from information to interaction. In *Proceedings of the 4th decennial conference on Critical computing: between sense and sensibility* (pp. 59–68).

Van Der Aalst, W. M. P., Reijers, H. A., & Song, M. (2005). Discovering social networks from event logs. *Computer Supported Cooperative Work (CSCW)*, *14*(6), 549–593.

Terrenghi, L., Kranz, M., Holleis, P., & Schmidt, A. (2006). A cube to learn: a tangible user interface for the design of a learning appliance. *Personal and Ubiquitous Computing*, *10*(2-3), 153–158.

Wing, J. M. (2006). Computational Thinking. *COMMUNICATIONS OF THE ACM*, *Vol. 49*(3), 33–35.

Klemmer, S. R., Hartmann, B., & Takayama, L. (2006). How bodies matter: five themes for interaction design. In *Proceedings of the 6th conference on Designing Interactive systems* (pp. 140–149).

Few, S. (2006). *Information dashboard design*. O'Reilly.

Crick, C., Munz, M., & Scassellati, B. (2006). Synchronization in social tasks: Robotic drumming. In *Robot and Human Interactive Communication, 2006. ROMAN 2006. The 15th IEEE International Symposium on* (pp. 97–102).

Gonçalves, M. A., Moreira, B. L., Fox, E. A., & Watson, L. T. (2007). ``What is a good digital library?''--A quality model for digital libraries. *Information Processing & Management*, *43*(5), 1416–1437.

Michalowski, M. P., Sabanovic, S., & Kozima, H. (2007). A dancing robot for rhythmic social interaction. In *Human-Robot Interaction (HRI), 2007 2nd ACM/IEEE International Conference on* (pp. 89–96).

Leutenegger, S., & Edgington, J. (2007). A games first approach to teaching introductory programming. In *ACM SIGCSE Bulletin* (Vol. 39, pp. 115–118).

Martin, T., Rivale, S. D., & Diller, K. R. (2007). Comparison of student learning in challenge-based and traditional instruction in biomedical engineering. *Ann Biomed Eng*, *35*(8), 1312–1323. doi:10.1007/s10439-007-9297-7

Owen, C. (2007). Design thinking: Notes on its nature and use. *Design Research Quarterly*, *2*(1), 16–27.

Cross, N. (2007). From a design science to a design discipline: Understanding designerly ways of knowing and thinking. *Design Research Now*, 41–54.

Suchman, L. (2007). *Human-machine reconfigurations: Plans and situated actions*. Cambridge University Press.

KRAMER, J. (2007). IS ABSTRACTION THE KEY TO COMPUTING. *COMMUNICATIONS OF THE ACM*, *50*(5), 27–42.

Petroski, H. (2007). *Small things considered: Why there is no perfect design*. Random House LLC.

Gamoran, A. (2007). *Standards-based reform and the poverty gap: Lessons for No Child Left Behind*. Brookings Institution Press.

Tatar, D. (2007). The design tensions framework. *Human--Computer Interaction*, *22*(4), 413–451.

Bonsiepe, G. (2007). The uneasy relationship between design and design research. *Design Research Now*, 25–39.

Quarles, J., Lampotang, S., Fischler, I., Fishwick, P., & Lok, B. (2008). Collocated AAR: augmenting after action review with mixed reality. In *Proceedings of the 7th IEEE/ACM International Symposium on Mixed and Augmented Reality* (pp. 107–116).

Wing, J. M. (2008). Computational thinking and thinking about computing. *Philos Trans A Math Phys Eng Sci*, *366*(1881), 3717–3725. doi:10.1098/rsta.2008.0118

Sicart, M. (2008). Defining game mechanics. *Game Studies*, *8*(2).

Harrison, S., & Tatar, D. (2008). Places: people, events, loci--the relation of semantic frames in the construction of place. *Computer Supported Cooperative Work (CSCW)*, *17*(2-3), 97–133.

Blackwell, A. (2008). The Abstract is “an Enemy”: Alternative Perspectives to Computational Thinking. *PPIG Lancaster*.

Greenberg, S., & Buxton, B. (2008). Usability evaluation considered harmful (some of the time). In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (pp. 111–120).

Saldana, J. (2009). An introduction to codes and coding. *The Coding Manual for Qualitative Researchers*, 1–31.

Pauwels, K., Ambler, T., Clark, B. H., LaPointe, P., Reibstein, D., Skiera, B., … Wiesel, T. (2009). Dashboards as a service: why, what, how, and what research is needed? *Journal of Service Research*.

Zaharias, P., & Poylymenakou, A. (2009). Developing a usability evaluation method for e-learning applications: Beyond functional usability. *Intl. Journal of Human--Computer Interaction*, *25*(1), 75–98.

Kirschner, S., & Tomasello, M. (2009). Joint drumming: social context facilitates synchronization in preschool children. *Journal of Experimental Child Psychology*, *102*(3), 299–314.

Denning, P. J. (2009). The profession of IT Beyond computational thinking. *Communications of the ACM*, *52*(6), 28–30.

Lu, J. J., & Fletcher, G. H. L. (2009). Thinking about computational thinking. In *ACM SIGCSE Bulletin* (Vol. 41, pp. 260–264).

Tan, W., Liu, D., & Bishu, R. (2009). Web evaluation: Heuristic evaluation vs. user testing. *International Journal of Industrial Ergonomics*, *39*(4), 621–627.

Palen, L., Anderson, K. M., Mark, G., Martin, J., Sicker, D., Palmer, M., & Grunwald, D. (2010). A vision for technology-mediated support for public participation & assistance in mass emergencies & disasters. In *Proceedings of the 2010 ACM-BCS visions of computer science conference* (p. 8).

Dork, M., Gruen, D., Williamson, C., & Carpendale, S. (2010). A visual backchannel for large-scale events. *Visualization and Computer Graphics, IEEE Transactions on*, *16*(6), 1129–1138.

Beaton, B., Harrison, S., & Tatar, D. (2010). Digital drumming: a study of co-located, highly coordinated, dyadic collaboration. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (pp. 1417–1426).

Hornbæk, K. (2010). Dogmas in the assessment of usability evaluation methods. *Behaviour & Information Technology*, *29*(1), 97–111.

Schmager, F., Cameron, N., & Noble, J. (2010). GoHotDraw: evaluating the Go programming language with design patterns. In *Evaluation and Usability of Programming Languages and Tools* (p. 10).

Secundo, G., Margherita, A., Elia, G., & Passiante, G. (2010). Intangible assets in higher education and research: mission, performance or both? *Journal of Intellectual Capital, 11*(2), 140–157.

Hwang, W., & Salvendy, G. (2010). Number of people required for usability evaluation: the 10\pm2 rule. *Communications of the ACM, 53*(5), 130–133.

Eckerson, W. W. (2010). *Performance dashboards: measuring, monitoring, and managing your business*. John Wiley & Sons.

Council, N. R. (2010). REPORT OF A WORKSHOP ON THE SCOPE AND NATURE OF Computational Thinking, 114.

Repenning, A., Webb, D., & Ioannidou, A. (2010). Scalable game design and the development of a checklist for getting computational thinking into public schools. In *Proceedings of the 41st ACM technical symposium on Computer science education* (pp. 265–269).

Tate, B. A. (2010). *Seven languages in seven weeks: a pragmatic guide to learning programming languages*. Pragmatic Bookshelf.

Ruthmann, A., Heines, J. M., Greher, G. R., Laidler, P., & Saulters II, C. (2010). Teaching computational thinking through musical live coding in scratch. In *Proceedings of the 41st ACM technical symposium on Computer science education* (pp. 351–355).

Fisher, D., McDonald, D. W., Brooks, A. L., & Churchill, E. F. (2010). Terms of service, ethics, and bias: Tapping the social web for cscw research. *Computer Supported Cooperative Work (CSCW), Panel Discussion*.

Koh, K. H., Basawapatna, A., Bennett, V., & Repenning, A. (2010). Towards the automatic recognition of computational thinking for adaptive visual language learning. In *Visual Languages and Human-Centric Computing (VL/HCC), 2010 IEEE Symposium on* (pp. 59–66).

Hirano, S. H., Yeganyan, M. T., Marcu, G., Nguyen, D. H., Boyd, L. A., & Hayes, G. R. (2010). vSked: evaluation of a system to support classroom activities for children with autism. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (pp. 1633–1642).

Berland, M., & Lee, V. R. (2011). Collaborative strategic board games as a site for distributed computational thinking. *International Journal of Game-Based Learning*, *1*(2), 65.

Council, N. R. (2011). Computer Science: Reflections of the Field, Reflections from the Field. *National Research Council*.

Kafura, D., & Tatar, D. (2011). Initial experience with a computational thinking course for computer science students. In *Proceedings of the 42nd ACM technical symposium on Computer science education* (pp. 251–256).

Vavoula, G., & Sharples, M. (2011). Meeting the challenges in evaluating mobile learning: A 3-level evaluation framework. *Combining E-Learning and M-Learning: New Applications of Blended Educational Resources*, 178.

Council, N. R. (2011). NRC Report of a Workshop of Pedagogical Aspects of Computational Thinking. *National Academy of Sciences*.

Knapp, R. B., Kim, J., & André, E. (2011). Physiological signals and their use in augmenting emotion recognition for human--machine interaction. In *Emotion-Oriented Systems* (pp. 133–159). Springer.

Basawapatna, A., Koh, K. H., Repenning, A., Webb, D. C., & Marshall, K. S. (2011). Recognizing computational thinking patterns. In *Proceedings of the 42nd ACM technical symposium on Computer science education* (pp. 245–250).

Fisher, D. (2011). Social Media Visualization & Privacy. In *Proc. CSCW* (pp. 19–23).

Steen, M. (2011). Tensions in human-centred design. *CoDesign*, *7*(1), 45–60.

Choi, I., & Bargar, R. (2012). A playable evolutionary interface for performance and social engagement. In *Intelligent Technologies for Interactive Entertainment* (pp. 170–182). Springer.

Hassenzahl, M., Heidecker, S., Eckoldt, K., Diefenbach, S., & Hillmann, U. (2012). All you need is love: Current strategies of mediating intimate relationships through technology. *ACM Transactions on Computer-Human Interaction (TOCHI)*, *19*(4), 30.

van Barneveld, A., Arnold, K. E., & Campbell, J. P. (2012). Analytics in higher education: Establishing a common language. *EDUCAUSE Learning Initiative*.

Initiative, C. C. S. S., & others. (2012). *Common core state standards for English language arts & literacy in history/social studies, science, and technical subjects*. Common Core Standards Initiative.

Aho, A. V. (2012). Computation and Computational Thinking. *The Computer Journal*, *55*(7), 4.

Sebesta, R. W. (2012). Concepts of Programming Languages.

DiPiro, J. T. (2012). Keeping Your Eyes on the Dashboard. *American Journal of Pharmaceutical Education*, *76*(3).

Radha, M. (2012). Mediated interactions in musical expression.

Hossain, M. S., Butler, P., Boedihardjo, A. P., & Ramakrishnan, N. (2012). Storytelling in entity networks to support intelligence analysts. In *Proceedings of the 18th ACM SIGKDD international conference on Knowledge discovery and data mining* (pp. 1375–1383).

Kapp, K. M. (2012). *The gamification of learning and instruction: game-based methods and strategies for training and education*. Wiley. com.

Rooksby, J., & Sommerville, I. (2012). The management and use of social network sites in a government department. *Computer Supported Cooperative Work (CSCW)*, *21*(4-5), 397–415.

Murray, T., Wing, L., Woolf, B., Wise, A., Wu, S., Clark, L., … Xu, X. (2013). A Prototype Facilitators Dashboard: Assessing and visualizing dialogue quality in online deliberations for education and work. In *Submitted to 2013 International Conference on e-Learning, e-Business, Enterprise Information Systems, and e-Government*.

Scott, K. A., & White, M. A. (2013). COMPUGIRLS'Standpoint Culturally Responsive Computing and Its Effect on Girls of Color. *Urban Education*, *48*(5), 657–681.

Bennett, V. E., Koh, K., & Repenning, A. (2013). Computing creativity: divergence in computational thinking. In *Proceeding of the 44th ACM technical symposium on Computer science education* (pp. 359–364).

Mitchell, J. J., & Ryder, A. J. (2013). Developing and Using Dashboard Indicators in Student Affairs Assessment. *New Directions for Student Services*, *2013*(142), 71–81.

Kirsh, D. (2013). Embodied cognition and the magical future of interaction design. *ACM Transactions on Computer-Human Interaction (TOCHI)*, *20*(1), 3.

Division, N. C. P. S. A. S. (2013). NORTH CAROLINA END-OF-COURSE TESTS. Retrieved from http://www.ncpublicschools.org/accountability/testing/eoc/

Suri, J. F. (2013). *Rotman on Design: The Best on Design Thinking from Rotman Magazine*. University of Toronto Press.

Norman, D. A. (2013). *The Design of Everyday Things: Revised and Expanded Edition*. Basic Books. Retrieved from http://books.google.com/books?id=nVQPAAAAQBAJ

Johnson, L., Adams, S., Cummins, M., Estrada, V., Freeman, A., & Ludgate, H. (2013). The NMC horizon report: 2013 higher education edition.

Bardzell, J., & Bardzell, S. (2013). What is critical about critical design? In *Proceedings of the SIGCHI conference on human factors in computing systems* (pp. 3297–3306).

Wisnioski, M. (2014). ``Suppose the World Were Already Lost'': Worst Case Design and the Engineering Imagination at Harvey Mudd College. *Engineering Studies*, (ahead-of-print), 1–22.

Ferri, G., Bardzell, J., Bardzell, S., & Louraine, S. (2014). Analyzing critical designs: categories, distinctions, and canons of exemplars. In *Proceedings of the 2014 conference on Designing interactive systems* (pp. 355–364).

Hui, J. S., Gerber, E. M., & Dow, S. P. (2014). Crowd-based design activities: helping students connect with users online. In *Proceedings of the 2014 conference on Designing interactive systems* (pp. 875–884).

Fox, E. A., & da Silva Torres, R. (2014). Digital Library Technologies: Complex Objects, Annotation, Ontologies, Classification, Extraction, and Security. *Synthesis Lectures on Information Concepts, Retrieval, and Services*, (0), 1–205.

Marlow, J., & Dabbish, L. (2014). From rookie to all-star: professional development in a graphic design social networking site. In *Proceedings of the 17th ACM conference on Computer supported cooperative work & social computing* (pp. 922–933).

Swamy, N., Fournet, C., Rastogi, A., Bhargavan, K., Chen, J., Strub, P.-Y., & Bierman, G. (2014). Gradual typing embedded securely in JavaScript. In *Proceedings of the 41st annual ACM SIGPLAN-SIGACT symposium on Principles of programming languages* (pp. 425–438).

Knowles, B., Blair, L., Walker, S., Coulton, P., Thomas, L., & Mullagh, L. (2014). Patterns of persuasion for sustainability. In *Proceedings of the 2014 conference on Designing interactive systems* (pp. 1035–1044).

Tatar, D. (2014). Reflecting Our Better Nature. *Interactions*, *21*(3), 46–49. doi:10.1145/2594188

Kavanaugh, A., Tedesco, J. C., & Madondo, K. (2014). Social Media vs. Traditional Internet Use for Community Involvement: Toward Broadening Participation. In *Electronic Participation* (pp. 1–12). Springer.

Wiese, J., Min, J.-K., Hong, J. I., & Zimmerman, J. (2015). ``You Never Call, You Never Write'': Call and SMS Logs Do Not Always Indicate Tie Strength. In *Proceedings of the 2015 conference on Computer supported cooperative work-CSCW'15*.

Pierce, J., Sengers, P., Hirsch, T., Jenkins, T., Gaver, W., & DiSalvo, C. (2015). Expanding and Refining Design and Criticality in HCI. In *Proceedings of the 33rd Annual ACM Conference on Human Factors in Computing Systems* (pp. 2083–2092).

K. Luther J. Tolentino, W. W. A. P. B. B. M. A. B. H., & Dow, S. (2015). Structuring, Aggregating, and Evaluating Crowdsourced Design Critique. In , Vancouver (Ed.), .

Abel, T. (n.d.). Design Education: Incorporating Eye-Tracking Research into the Traditional Design Studio.

Rich, M. (n.d.). Finding Forrester.

Fox, S., Ulgado, R. R., & Rosner, D. K. (n.d.). Hacking Culture, Not Devices: Access and Recognition in Feminist Hackerspaces.

I. Additional thoughts from the author

After writing this thesis, I am confident of the following:

I still wouldn't spell rhythm correctly if it weren't for autocorrect, or a vice that it resembles an iconic old train facing the right, where the 'r' is a caboose, the 'y' is the center, the 't' is where the wood is held to be put into the last 'h' smokestack, and the 'm' is the guard that pushes objects out of the way of the train on the tracks. This may seem silly, but the word 'rhythm' appears 278 times in this document. That doesn't count my years of notes. So while it is funny that I don't remember how to spell it with precision immediately, I have a mnemonic to cover my basis.

These are the words I have learned while in grad school, and thought enough to record them for later reference - typhlostrophedontically, methystrophedontically, chiasmatically, henophthalmotyphlostrophedontically, boustrophedontically, orthogonally, penultimate, gruit, sable, proprioception, exteroceptive, interoceptive, lagniappe, hedonic, shirk, virgule, patronymic, neologism, semogenesis, lethologica, trichotillomania, miasma, bijection, heterozygosity, helen, gtts, pick, contumacious, cloying, lunule, petrichor, aglet, phloem, drupe, preantepenultimate, pulchritudinous, callipygian, antepenultimate, propreantepenultimate, somaesthetics, sentential, lexicographical, vartiwell, subsume, pentheraphobia, blaxploitation, purposive, praxiology, bisceps, bicepses, flummox, fledging, vacillate, dour, vamp, abecedarian, arity, blat, asymptotic, contrapositive, aleph, sybaritic, quixotic, bigram, hapax, hapax, velleity, veridical, obverse, hilt, pervade, absquatulate, caroom, ramify, propinquity, homophily, boon, micturate, explanandum, start, undergird, doatingly, ambisinister, lemma, ablation, nosocomial, polsemy, olfactory, and finally din.

12 BIBLIOGRAPHY

1. H F ABELES, C F HOFFER, and R H KLOTMAN. 1994. Social Psychological Foundations Of Music Education. *Schimer Book, New York, ABD*.

2. Edoardo Acotto and Moreno Andreatta. 2012. Between mind and mathematics. Different

kinds of computational representations of music. *Math{é}matiques et sciences humaines. Mathematics and social sciences*, 199: 7–25.

3. Sybil S Barten. 1992. The language of musical instruction. *Journal of Aesthetic Education*: 53–61.

4. Bobby Beaton, Steve Harrison, and Deborah Tatar. 2010. Digital drumming: a study of co-located, highly coordinated, dyadic collaboration. *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, 1417–1426.

5. Merlyn Behr, Guershon Harel, Thomas Post, R Lesh, and D A Grouws. 1992. Handbook of research on mathematics teaching and learning. *Rational number, ratio. and proportion. In D. Grouws (Ed.), Macmillan, New York*.

6. Tina Blaine and Tim Perkis. 2000. The Jam-O-Drum interactive music system: a study in interaction design. *Proceedings of the 3rd conference on Designing interactive systems: processes, practices, methods, and techniques*, 165–173.

7. Michael Cole. 1998. *Cultural psychology: A once and future discipline*. Harvard University Press.

8. Jere Confrey. 1994. Multiplication and Exponential Functions. *The development of multiplicative reasoning in the learning of mathematics*: 291.

9. Susan Joan Courey, Endre Balogh, Jody Rebecca Siker, and Jae Paik. 2012. Academic music: music instruction to engage third-grade students in learning basic fraction concepts. *Educational Studies in Mathematics* 81, 2: 251–278. http://doi.org/10.1007/s10649-012-9395-9

10. Fred Cummins. 2009. Rhythm as an affordance for the entrainment of movement. *Phonetica* 66, 1-2: 15–28.

11. Ubiratan D'Ambrosio. 1987. Reflections on ethnomathematics. *International Study Group on Ethnomathematics Newsletter* 3, 1: 3–5.

12. Ubiratan D'Ambrosio. 2001. What Is Ethnomathematics, and How Can It Help Children in Schools?. *Teaching children mathematics* 7, 6: 308–310.

13. Gary Ernest Davis and Anne Pitkethly. 1990. Cognitive aspects of sharing. *Journal for Research in Mathematics Education*: 145–153.

14. Paul Dourish. 2004. *Where the action is: the foundations of embodied interaction*. MIT press.

15. Susan B Empson and Linda Levi. 2011. *Extending Children's Mathematics: Fractions and Decimals:[innovations in Cognitively Guided Instruction]*. Heinemann Portsmouth, NH.

16. Susan B Empson. 1999. Equal sharing and shared meaning: The development of fraction

concepts in a first-grade classroom. *Cognition and instruction* 17, 3: 283–342.

17. Howard Eves. 1969. An introduction to the history of mathematics . New York: Holt, Rinehart and Winston.

18. James T Fey. 1989. Technology and mathematics education: A survey of recent developments and important problems. *Educational Studies in Mathematics* 20, 3: 237–272.

19. Adam Gamoran. 2007. *Standards-based reform and the poverty gap: Lessons for No Child Left Behind*. Brookings Institution Press.

20. Satinder P Gill. 2012. Rhythmic synchrony and mediated interaction: towards a framework of rhythm in embodied interaction. *AI & society* 27, 1: 111–127.

21. Chris Grainger. Coding is not the new literacy. Retrieved from http://www.chris-granger.com/2015/01/26/coding-is-not-the-new-literacy/

22. Amy B Graziano, Matthew Peterson, and Gordon L Shaw. 1999. Enhanced learning of proportional math through music training and spatial-temporal training. *Neurological research* 21, 2: 139–152.

23. Joyce Eastlund Gromko. 1994. Children's invented notations as measures of musical understanding. *Psychology of Music* 22, 2: 136–147.

24. Steve Harrison, Deborah Tatar, and Phoebe Sengers. 2007. The three paradigms of HCI. *Alt. Chi. Session at the SIGCHI Conference on Human Factors in Computing Systems San Jose, California, USA*, 1–18.

25. Lois Hetland. 2000. Learning to make music enhances spatial reasoning. *Journal of Aesthetic Education*: 179–238.

26. Simon Holland, Anders J Bouwer, Mathew Dalgelish, and Topi M Hurtig. 2010. Feeling the beat where it counts: fostering multi-limb rhythm skills with the haptic drum kit. *Proceedings of the fourth international conference on Tangible, embedded, and embodied interaction*, 21–28.

27. Mark Howison, Dragan Trninic, Daniel Reinholz, and Dor Abrahamson. 2011. The Mathematical Imagery Trainer: from embodied interaction to conceptual learning. *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*, 1989–1998.

28. Common Core State Standards Initiative and others. 2012. *Common core state standards for English language arts & literacy in history/social studies, science, and technical subjects*. Common Core Standards Initiative.

29. Sergi Jordà, Günter Geiger, Marcos Alonso, and Martin Kaltenbrunner. 2007. The reacTable: exploring the synergy between live music performance and tabletop tangible

interfaces. *Proceedings of the 1st international conference on Tangible and embedded interaction*, 139–146.

30. Anne Jordan, Linda Lindsay, and Paula J Stanovich. 1997. Classroom teachers' instructional interactions with students who are exceptional, at risk, and typically achieving. *Remedial and special education* 18, 2: 82–93.

31. James J Kaput and Patrick W Thompson. 1994. Technology in mathematics education research: The first 25 years in the JRME. *Journal for research in mathematics education* 25, 6: 676–684.

32. Thomas E Kieren. 1976. On the Mathematical, Cognitive and Instructional. *Number and Measurement. Papers from a Research Workshop.*, 101.

33. Sebastian Kirschner and Michael Tomasello. 2009. Joint drumming: social context facilitates synchronization in preschool children. *Journal of experimental child psychology* 102, 3: 299–314.

34. Susan J Lamon. 2007. Rational numbers and proportional reasoning: Toward a theoretical framework for research. *Second handbook of research on mathematics teaching and learning* 1: 629–667.

35. Ray Landers. 1984. The Talent Education School of Shinichi Suzuki: An Analysis: Application of Its Philosophy and Methods to All Areas of Instruction.

36. Joon Suk Lee and Deborah Tatar. 2014. Sounds of silence: exploring contributions to conversations, non-responses and the impact of mediating technologies in triple space. *Proceedings of the 17th ACM conference on Computer supported cooperative work & social computing*, 1561–1572.

37. Frank K Lester. 2007. *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics*. IAP.

38. Bobby McFerrin. Bobby McFerrin Demonstrates the Power of the Pentatonic Scale.

39. Gary E McPherson and Alf Gabrielsson. 2002. From sound to sign. *The science and psychology of music performance: Creative strategies for teaching and learning*: 99–116.

40. Jocelyn Neal. 2000. Songwriter's Signature, Artist's Imprint. *Country Music Annual 2000*: 112.

41. K Allen Neufeld. 1986. Understanding of selected pre-number concepts: Relationships to a formal music program. *Alberta Journal of Educational Research*.

42. Denis Newman, Peg Griffin, and Michael Cole. 1989. *The construction zone: Working for cognitive change in school*. Cambridge University Press.

43. A Norton and J L M Wilkins. 2013. Supporting Students' Constructions of the Splitting

Operation. *Cognition and Instruction* 31, 1: 2–28.

44. Anderson Norton and Jesse L M Wilkins. 2009. A quantitative analysis of children's splitting operations and fraction schemes. *The Journal of Mathematical Behavior* 28, 2: 150–161.

45. National Mathematics Advisory Panel. 2008. *Foundations for success: The final report of the National Mathematics Advisory Panel*. US Department of Education.

46. Seymour Papert. 1980. *Mindstorms: Children, computers, and powerful ideas*. Basic Books, Inc.

47. James Patten, Ben Recht, and Hiroshi Ishii. 2002. Audiopad: a tag-based interface for musical performance. *Proceedings of the 2002 conference on New interfaces for musical expression*, 1–6.

48. Michael Polanyi. 1967. The tacit dimension.

49. Yvonne Pothier and Daiyo Sawada. 1983. Partitioning: The emergence of rational number ideas in young children. *Journal for Research in Mathematics Education*: 307–317.

50. Queen. 1977. We Will Rock You.

51. Mustafa Radha. 2012. Mediated interactions in musical expression.

52. Sean A Rands, Guy Cowlishaw, Richard A Pettifor, J Marcus Rowcliffe, and Rufus A Johnstone. 2003. Spontaneous emergence of leaders and followers in foraging pairs. *Nature* 423, 6938: 432–434.

53. Bennett Reimer. 1999. Facing the Risks of the" Mozart Effect." Grand Masters Series. *Music Educators Journal* 86, 1: 37–43.

54. Pasi Salsberg. 2014. No Title. Retrieved from https://www.youtube.com/watch?v=g0aM3qxR3E8

55. Ayelet Segal. 2011. Do gestural interfaces promote thinking? Embodied interaction: Congruent gestures and direct touch promote performance in math.

56. Capita Selecta. Synchronized clapping: a two-way synchronized process.

57. David Williamson Shaffer and James J Kaput. 1998. Mathematics and virtual culture: An evolutionary perspective on technology and mathematics education. *Educational Studies in Mathematics* 37, 2: 97–119.

58. Jean Sinor. 1997. The Ideas of Kod{á}ly in America This article on the application of Zolt{á}n Kod{á}ly's teaching method in the United States was first published in the Music Educators Journal in February 1986. *Music Educators Journal* 83, 5: 37–41.

59. Leslie P Steffe. 2002. A new hypothesis concerning children's fractional knowledge. *The*

Journal of Mathematical Behavior 20, 3: 267–307.

60. Taylor Swift. 2014. Shake It Off.

61. Deborah Tatar, Joon-Suk Lee, and Nouf Alaloula. 2008. Playground games: a design strategy for supporting and understanding coordinated activity. *Proceedings of the 7th ACM conference on Designing interactive systems*, 68–77.

62. Sandra E Trehub and Erin E Hannon. 2006. Infant music perception: Domain-general or domain-specific mechanisms? *Cognition* 100, 1: 73–99.

63. Rena Upitis. 1993. Children's invented symbol systems: Exploring parallels between music and mathematics. *Psychomusicology: A Journal of Research in Music Cognition* 12, 1: 52.

64. Kathryn Vaughn. 2000. Music and mathematics: Modest support for the oft-claimed relationship. *Journal of Aesthetic Education*: 149–166.

65. Robert E Weeden. 1972. A comparison of the academic achievement in reading and mathematics of Negro children whose parents are interested, not interested, or involved in a program of Suzuki violin.

66. J L M Wilkins and A Norton. 2011. The Splitting Loope. *Journal for Research in Mathematics Education* 42, 4: 386–416.

67. Jeannette M Wing. 2006. Computational Thinking. *COMMUNICATIONS OF THE ACM* Vol. 49, 3: 33–35.

68. Jeannette M Wing. 2008. Computational thinking and thinking about computing. *Philos Trans A Math Phys Eng Sci* 366, 1881: 3717–3725. http://doi.org/10.1098/rsta.2008.0118

69. Ellen Winner. 2007. Development in the arts: Drawing and music. *Handbook of Child Psychology*.

70. No Title. Retrieved from http://quickfacts.census.gov/qfd/states/51

71. No Title. Retrieved from Intentionally