Towards Detecting Atmospheric Coherent Structures using Small Fixed-Wing Unmanned Aircraft

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Dissertation submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Aerospace Engineering

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2019 May 13
Blacksburg, Virginia

Keywords: Airborne Wind Measurement, Flight Dynamic Modeling, Unmanned Aircraft Systems, Lagrangian Coherent Structures
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Towards Detecting Atmospheric Coherent Structures using Small Fixed-Wing Unmanned Aircraft

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(ABSTRACT)

The theory of Lagrangian Coherent Structures (LCS) enables prediction of material transport by turbulent winds, such as those observed in the Earth’s Atmospheric Boundary Layer. In this dissertation, both theory and experimental methods are developed for utilizing small fixed-wing unmanned aircraft systems (UAS) in detecting these atmospheric coherent structures. The dissertation begins by presenting relevant literature on both LCS and airborne wind estimation. Because model-based wind estimation inherently depends on high quality models, a Flight Dynamic Model (FDM) suitable for a small fixed-wing aircraft in turbulent wind is derived in detail. In this presentation, some new theoretical concepts are introduced concerning the proper treatment of spatial wind gradients, and a critical review of existing theories is presented. To enable model-based wind estimation experiments, an experimental approach is detailed for identifying a FDM for a small UAS by combining existing computational aerodynamic and data-driven approaches. Additionally, a methodology for determining wind estimation error directly resulting from dynamic modeling choices is presented and demonstrated. Next, some model-based wind estimation results are presented utilizing the experimentally identified FDM, accompanied by a discussion of model fidelity concerns and other experimental issues. Finally, an algorithm for detecting LCS from a single circling fixed-wing UAS is developed and demonstrated in an Observing System Simulation Experiment. The dissertation concludes by summarizing these contributions and recommending future paths for continuing research.
Towards Detecting Atmospheric Coherent Structures using Small Fixed-Wing Unmanned Aircraft

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(GENERAL AUDIENCE ABSTRACT)

In a natural or man-made disaster, first responders depend on accurate predictions of where the wind might carry hazardous material. A mathematical theory of Lagrangian Coherent Structures (LCS) has shown promise in ocean environments to improve these predictions, and the theory is also applicable to atmospheric flows near the Earth’s surface. This dissertation presents both theoretical and experimental research efforts towards employing small fixed-wing unmanned aircraft systems (UAS) to detect coherent structures in the Atmospheric Boundary Layer (ABL). These UAS fit several “gaps” in available sensing technology: a small aircraft responds significantly to wind gusts, can be steered to regions of interest, and can be flown in dangerous environments without risking the pilot’s safety. A key focus of this dissertation is to improve the quality of airborne wind measurements provided by inexpensive UAS, specifically by leveraging mathematical models of the aircraft. The dissertation opens by presenting the motivation for this research and existing literature on the topics. Next, a detailed derivation of a suitable Flight Dynamic Model (FDM) for a fixed-wing aircraft in a turbulent wind field is presented. Special attention is paid to the theories for including aerodynamic effects of flying in non-uniform winds. In preparation for wind measurement experiments, a practical method for obtaining better quality FDMs is presented which combines theoretically based and data-driven approaches. A study into the wind-measurement error incurred solely by mathematical modeling is presented, focusing on simplified forms of the FDM which are common in aerospace engineering. Wind estimates which utilize our best available model are presented, accompanied by discussions of the model accuracy and additional wind measurement concerns. A method is developed to detect coherent structures from a circling UAS which is providing wind information, presumably via accurate model-based estimation. The dissertation concludes by discussing these conclusions and directions for future research which have been identified during these pursuits.
Acknowledgments

The road to completing my doctoral degree has been an unusually long one, filled with difficulties as well as opportunities. I have been fortunate to work with a diverse array of wonderful folks, to all of whom I am extremely grateful.

To the members of my first research group: D. Hong, Viktor, Eric, Derek, Mike H, Bryce, Jason, Joe, Semi, Niko, Paul, Jack, Steve, Chris, Hak, Coleman, JK, Mike R, and Taylor. From you I learned many invaluable lessons about effective teamwork, and it was a pleasure working together. I appreciate the research advise and discussions with J. Burns, M. Chung, A. Leonessa, and E. Cliff. I am also very grateful for the support of M. McKay and Dean DePauw: without your assistance I would not have completed my degree at Virginia Tech.

Outside of work, I will always treasure the delightful coffee hour discussions with Ennis, Darren, Monika, and Jeremy.

Thank you Dr. Kurdila for helping me transition research roles, for enabling me to teach my first course, and for connecting me to the Shandong University research team. These were some of my most enlightening experiences during my journey at Virginia Tech. Thank you Dr. Mueller and the folks at SDU for a productive and unforgettable summer experience.

To Adam, Garret, Labiba, Chuong, Christine, Paola, and Gail: It was a pleasure to work with you all. To Shirin and Matt, my collaborators and office mates: Thank you for all the maths and all the laughs.

In the Nonlinear Systems lab, thank you to Casey, Charles, Andrew K, Andrew R, JP, Dany, Mekonen, D. Schmale, Regina, Craig P, and Peter for great collaboration. I’m giving a special thanks to office mates Micah, Deva, Javier, Billy, and Jean-Michel for so much fun as well as so many thoughtful discussions. Thank you to George Z.P. for a delightful software collaboration. I also want to thank S. Ross for supporting my research across a wide variety of roles, consistently providing thoughtful advice and useful suggestions, no matter the topic.

Finally, the most important academic supporter during my journey and growth at Virginia Tech has been my mentor and advisor, Craig Woolsey. I will forever appreciate your support, your exemplary mentorship, and for your tireless dedication to my success despite all obstacles.

Thank you all very much.
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Chapter 1

Introduction

Wind in the Earth’s atmospheric boundary layer (ABL) affects a diverse variety of scientific and technological pursuits. R. B. Stull emphasizes both the difficulty and importance of understanding boundary layer turbulent flow \[144, \S1.0\]

It is here [in the ABL] where our crops are grown, our dwellings are constructed, and much of our commerce takes place. [...] Such near-earth characteristics, however, are not typical of what we observe in the rest of the atmosphere. One reason for this difference is the dominating influence of the earth on the lowest layers of air.

While a variety of wind measurement technologies exist, the combined difficulty of modeling and the outsized impact of the atmospheric boundary layer on human endeavors make studying it an important area for scientific development.

Airborne wind measurement from aircraft is a well established concept \[11, 85, 107\], although one that is costly to use. In contrast, for less than $1000 a small unmanned aircraft system (UAS) can be launched in a remote location and overseen by a single operator. Additionally, UAS enable wind measurement missions which were originally limited by human-pilot capability, especially dull or dangerous ones. In a dull scenario, an unmanned aircraft (or team of aircraft) might fly in repetitive circuits for long periods without fatigue. In dangerous situations a UAS can, for example, obtain meteorological data about severe tornadic storm cells by flying into these developing cells \[8, 34\]. Although larger aircraft can host larger, more accurate sensors, the size of a small UAS enhances its sensitivity to flow fields, thus a higher bandwidth of atmospheric disturbances register directly in measurements of the aircraft’s motion. Small UAS are able to operate at lower altitudes than general aviation (GA) aircraft, providing measurements of the turbulent ABL. With more resources, a fleet of UAS could persistently cover an operations area or densely sample a region of interest. Based on these advantages, small UAS present a clear opportunity to complement existing weather measurement technology in a heterogeneous sensor network.
1.1 Dissertation Contributions

This dissertation presents theory and methodology contributions towards measuring winds in the atmospheric boundary layer (ABL) using an inexpensive fixed-wing unmanned aircraft system (UAS). The following specific contributions are presented:

- Formulation of a flight dynamic model (FDM) for a fixed-wing aircraft in a non-uniform wind field.
- Derivation and critical review of theories for including wind-gradient effects in the aerodynamic equations.
- Summary of an experimental method [139] augmenting data-driven system identification with computational aerodynamics to produce a FDM for a Bix3 UAS.
- Summary of a method and Monte Carlo study [95] quantifying wind estimation error incurred by common FDM assumptions.
- Model-based wind estimates from an experimentally determined Bix3 fixed-wing FDM.
- Demonstration of model fidelity concerns for the experimentally determined Bix3 FDM.
- Experimental evidence that a multi-rotor UAS can disturb SoDAR wind sensing.
- Summary and supporting details for [111] which presents a method and observing system simulation experiment (OSSE) for detecting LCS from wind measurements obtained from an airborne fixed-wing UAS.

Throughout this dissertation, no significant distinction is drawn between the terms “unmanned aircraft”, “unmanned air vehicle” (UAV), and “unmanned aircraft system” (UAS). We use the latter acronym simply to remind readers of the ground-based equipment, communication systems, and personnel required for unmanned aircraft operations. While the descriptor “small” for UAS is defined by the FAA [153], in this dissertation it also connotes the inexpensive nature of small platforms and corresponding limitations on sensor quality and capabilities.

1.2 Atmospheric Coherent Structures

Understanding the complex interactions in atmospheric flow, from planetary-scale weather phenomena to the turbulent dissipating Kolmogrov microscale, has remained a key scientific pursuit for over 100 years [93]. Though Earth’s atmosphere has been long and well studied, predicting its evolution remains a cardinal challenge because the dynamics are chaotic [43].
It is understood that small scale turbulence has an outsized role in affecting larger flow patterns [166], but there is still no feasible computational method that can capture atmospheric motion at all spatial scales over time intervals of interest to forecasters [128].

Three types of approaches can be identified for computational modeling of atmospheric wind fields. The first approach, direct numerical simulation (DNS) [86, 167], is exact to numerical accuracy but computationally expensive and sensitive to uncertainty. Another approach is to ensemble-average atmospheric turbulence, obtaining useful stochastic models [63] but (by definition) uncertain estimates. The third approach focuses on the recurrent forms which dominate energy flow — the coherent structures — to gain predictive understanding of the turbulent flow [66]. This reduced-order modeling approach is intended to enable real-time forecasting (and hindcasting) for weather and material-transport applications by only considering the most significant turbulent flow characteristics.

A new mathematical methodology for understanding chaotic flow, called Lagrangian Coherent Structures (LCS) [4, 114] has been developed in the past 20 years. LCS differ from traditional coherent structures — such as regions where vorticity dominates strain [12, 97] — because they identify objective material surfaces (i.e. independent of the reference frame [59]) and are defined from the Lagrangian\(^1\) perspective [117]. The mathematical definition of LCS invokes concepts from dynamical systems theory [60, 135], but Ravela et al. [118] connect with familiar examples:

Many well-known mesoscale and sub-mesoscale structures are coherent including but not limited to vortices, plumes, clouds, fronts, jets, and storms. They include some benign but poorly understood phenomena such as sea breezes, and mature dangerous ones such as hurricanes.

Figures 1.1 and 1.2 show examples of atmospheric LCS. Throughout this dissertation, no significant distinction is drawn between an “Atmospheric Coherent Structure” (ACS) and a “Lagrangian Coherent Structure” (LCS) observed in the atmosphere, despite the two terms each having a distinct history and research community.

1.2.1 Literature on LCS

The term “Lagrangian Coherent Structures” was coined by Haller and Yuan [61]. The key property separating these LCS from previous ideas about coherent structures is their objectivity, that is, that the structures exist no matter whether the observer is moving or stationary with respect to the flow. Additional researchers continued development of the

\[^1\text{The “Lagrangian” name signifies their calculation from advected particle trajectories rather than “Eulerian” velocity field information, but objectivity [59] is the key property separating these from traditional coherent structures. Recent work has proposed a unified approach: Objective Eulerian coherent structures [133].}]](\text{)}
mathematics supporting this idea, and 15 years later, Haller authored a review of these developments [60]. One important work by Shadden et al. [135] demonstrates the most widely accepted definition of LCS as ridges of the Finite Time Lyapunov Exponent field. This theory was later generalized beyond 2 dimensions [84] by retaining the underlying concepts rather than the specific mathematics. Later, Shadden authored a book chapter [134] providing a more gentle introduction to the mathematics and LCS literature. Other noteworthy review/survey articles include an article in The Economist magazine [4] and an article in Physics Today [114].

Paralleling the development of LCS theory, research works began exploring the application of LCS to a variety of problems. In oceanic flows, LCS have been utilized to provide near-optimal trajectories for an underwater glider [68, 168] to transit between locations. Another work finds LCS in Monterey Bay [83] from surface-velocity radar data. In atmospheric flows, LCS are detected at the Hong Kong airport [75, 146] as well as correlated to onboard turbulence [147] experienced by the aircraft. LCS are observed to play a role in the atmospheric
transport of plant pathogens [145]. A review by Schmale and Ross [130] provides additional literature on plant pathogen transport as well. Research on airborne plant pathogens appears to be the first time unmanned aircraft systems (UAS) are utilized to study LCS phenomena [91, 131]. The use of UAS is seen again in work by Ravela et al. [118] explicitly seeking coherent structures by the vertical motion of a gliding aircraft.

### 1.2.2 Motivating Project: NSF-Hazards

One major source of funding for this work is NSF Award #AGS1520825\(^2\) Hazards SEES: Advanced Lagrangian Methods for Prediction, Mitigation and Response to Environmental Flow Hazards. In summary, this multi-university effort proposes to apply the theory of LCS to predicting hazardous material transport in both oceanic and atmospheric domains. For example, consider a nuclear-power disaster where a repelling-manifold LCS might naturally form an advection boundary, blocking airborne radioactive contamination from reaching one nearby city but not others. In such an emergency, redirecting disaster-response resources appropriately based on quick forecasts from LCS-based models would save lives. This multi-organization, multi-disciplinary research project [120] has nicknamed itself “ALPHA” standing for Advanced Lagrangian Prediction for Hazards Assessment, an acronym which may be mentioned in related research publications.

In collaboratively developing the ALPHA team’s research direction, focus shifted from using passive unmanned aircraft (e.g. balloons) towards using fixed-wing and rotary-wing UAS (i.e. multi-copters and helicopters). These UAS could simply serve as steerable sensor platforms, or perhaps even collaborate to actively locate and track an LCS of interest [67, 94, 99]. One key difficulty in experimentally finding ACS is the requirement for many high resolution wind velocity measurements, which is expensive, or the release of an airborne tracer around the area of interest, which is impractical for large scale and low threat situations.

This produced an early idea: to discover a naturally existing tracer for air masses distinguished by ACS. The tracer might be a scalar quantity such as altitude-adjusted air pressure or turbulent kinetic energy (TKE) dissipation rate. More likely, the tracer is actually some unique “signature” combining several atmospheric properties, such as the ratio of relative humidity to density. One key inspiration for this idea is work by Ravela et al. [118] where the vertical kinetic energy during glide is applied as a structure indicator. Using ground-based atmospheric sensors, Schols [132] uses sharp temperature gradients as indicators of atmospheric coherent structure. During the research efforts supporting this dissertation, the idea of seeking a naturally occurring tracer was abandoned due to the difficulty of experimentally observing ACS on any suitable scale for UAS exploration. Instead, a different idea for observing LCS in the atmosphere became a central theme: to measure the wind accurately while airborne and develop methods to detect LCS in the sequence of measurements.

Chapter 1. Introduction

obtained by an on-site UAS.

1.3 Airborne Wind Measurement

Because atmospheric LCS are mathematically defined by the trajectories of particles which are carried by the wind velocity field, measurements of the wind field itself constitute key information about LCS. Accurate wind measurement from aircraft during flight is a well-researched problem in aerospace technology [11, 85].

Remark 1.1. For the purposes of this dissertation, developing methods for accurate airborne wind estimation/measurement and detecting atmospheric coherent structures are analogous efforts. Thus much of this dissertation focuses on the “traditional aerospace problem” of improving airborne wind measurement accuracy.

Many works exist in flight dynamics research literature about airborne wind measurement from unmanned aircraft. Perhaps the most influential work in developing this dissertation’s research direction is by Langelaan et al. [77]. In that work, the dependence of model-based wind estimation accuracy on sensor error is explored both in theory and simulation. The authors conclude that bothairspeed and attitude accuracy are important factors for model-based wind estimation. A follow on paper analyzes experimental data [78], and the estimation data only somewhat match the balloon-based “truth” data. Another important work [126] which informed our research effort is a comparison of 4 estimation approaches which increase in complexity. The simulation engine underlying this study is provided in [124, 125]. The problem of estimating an aircraft’s angle of attack (AoA, \( \alpha \)) and sideslip angle (SSA, \( \beta \)) along with airspeed \( (V_r) \) is exactly analogous to wind estimation if the inertial velocity is known accurately\(^3\). One research effort [161] approaches this problem using a Moving Horizon Estimator to not only estimate the AoA and SSA, but also provide information about the aerodynamic parameters in the flight dynamic model. At the time of this dissertation’s writing, the authors of [161] have a journal draft in review; the reader is encouraged to search for works by Andreas Wenz and Tor Arne Johansen as well as investigate their precursors [71, 162]. Another promising approach to AoA/SSA estimation is provided by Lie and Gebre-Egziabher [89] with a sensitivity study [90] and additional details [88]. While the majority of these methods approach the estimation problem in the time domain, the frequency domain approach provided by Grauer and Boucher [47] is also recommended. Considering the problem of turbulent wind wakes during airborne refueling, a series of works by Lee [80, 81, 82] use an advanced FDM and some interesting filters. It appears the advanced flight dynamic model was developed for earlier works [30, 73] by Dogan’s research group and is described in detail by Lewis [87]. Many other works exist on

\(^3\)Accelerometers, GPS, and other typically available technologies can be combined to provide relatively accurate inertial velocity information.
wind estimation (and aerodynamic angle estimation) with simulation demonstrations such as [18, 106, 136, 150, 151].

A key advantage of unmanned aircraft, especially small and inexpensive ones, is the ability for experimental testing of wind measurement approaches. Some of the most influential experimental wind measurement works for this dissertation are [20, 121, 122, 154, 165]. In 2015, Elston et al. published a survey of UAS for weather research [32] which provides additional information as well as some sources not included in this dissertation, building on experience in UAS-based storm measurement [8, 33, 34]. Another review of UAS-based weather measurement can be found in Wildmann’s dissertation [163]. Because some of these methods assume that low-cost air data measurements are available, investigations into the accuracy of low-cost air data measurement sensors such as [171] become relevant. Thus having presented much literature which guided our research development, we would also like to mention other less influential sources that we considered [21, 22, 24, 41, 79, 112, 116, 123, 164].

All wind estimation for small UAS is fundamentally derived from parameter and disturbance estimation of full size aircraft. The earliest papers we found formulating the wind measurement equations date back to the 1960’s [11, 141] and perhaps the most cited in 1989 [85] formulates the “Lenschow equations.” Many wind measurement approaches, such as [54] were developed largely to provide useful models of turbulent wind fields above the Earth’s surface. Likewise identifying the aerodynamic parameters for an aircraft, known often as Aircraft System Identification (SysID) [70, 102] has a rich history. In this dissertation, we were heavily influenced by the time-domain formulations of Grauer, Morelli, and Klein in works such as [48, 50, 101, 104]. Hoffer et al. provide a survey of literature applying SysID to inexpensive aircraft [65] as well as some classification schemes. An influential work in developing our SysID methods of Chapter 3 is by Dorobantu et al. [31]. Previous work in our research group by Hale [55, 56, 57, 58] was also influential on our thinking. Finally, the reader might appreciate knowing of a recently available flight dynamic model [52] for the popular Skywalker X8 aircraft.
Chapter 2

Flight Dynamic Modeling

This chapter presents a detailed derivation of the mathematical equations which comprise an aircraft’s flight dynamic model in arbitrary wind. The equations are expressed in several useful forms, and theories for including the impact of the wind derivatives are discussed in detail. Important key references which contributed to these equations are Etkin’s *Dynamics of Atmospheric Flight* [35], Boiffier’s *The Dynamics of Flight: The Equations* [17], the Generic Nonlinear Aerodynamic model from Grauer and Morelli [49]. We were also influenced by Beard and McClain’s *Small Unmanned Aircraft* [15] and the collection of equations by Zogopoulos-Papaliakos [169].

2.1 Assumptions

The relevant assumptions used in deriving this flight dynamic model are:

1. Earth is a flat, inertial reference.
   - Neglect Earth’s curvature, rotation, movement through space.
   - For speeds lower than 800 m/s, this is a good choice [35, §5.7].

2. The aircraft is a rigid body with constant mass.
   - Neglect aircraft shape-change due to structural flexing and/or control surface deformations.
   - Neglect the dynamics of rotating subsystems such as engines/propellers.

3. Thrust is an instantaneously controllable force acting body-forward from the aircraft’s center of mass (CoM)
   - Neglect any dependence on altitude, speed, throttle-lag, etc.

4. The aircraft is symmetric about its $x_Bz_B$ plane.

5. Environmental parameters do not vary with altitude.
2.2 Rigid Body Dynamics

- Neglect the dependence of air density $\rho$ and gravitational acceleration $g$ on altitude.

6. The wind is described by a $C^1$-smooth kinematic vector field.
   - More formally: $W : \mathbb{R}^3 \times \mathbb{R}^+ \to \mathbb{R}^3$ is a $C^1$ vector-valued function.

7. Unsteady aerodynamic effects are neglected.
   - Aerodynamic forces and moments depend on current values of the state and control, but not their time histories.

8. The aircraft does not affect the wind field.
   - This “Apparent Wind Field” idea and its justification are detailed in Section 2.3.1.
   - The wind vector to which the aircraft responds is the value of $W$ at the current time $t$ and aircraft position $X$. This quantity is specified by $W(X, t)$.
   - The wind derivatives (spatial $\frac{\partial W}{\partial X}$ and temporal $\frac{\partial W}{\partial t}$) to which the aircraft responds are (likewise) evaluated at the current time $t$ and aircraft position $X$.

2.2 Rigid Body Dynamics

The flight dynamic model represents a composition of rigid body dynamic equations and aerodynamic equations. This presentation begins by presenting some relevant definitions and then deriving the rigid body dynamic equations.

2.2.1 Standard Definitions and Kinematics

Figures 2.1 and 2.2 depict standard notation for an aircraft in a uniform wind field. While this dissertation is not restricted to uniform wind fields, the definitions from the figures are still applicable given Assumption 8 above.

The Earth-referenced position of the aircraft’s center of mass in North-East-Down coordinates is

$$X = \begin{pmatrix} X_N \\ X_E \\ X_D \end{pmatrix}$$  \hspace{1cm} (2.1)

The Earth-referenced orientation, encoded via standard roll-pitch-yaw Euler angles is expressed in vector form as

$$\Theta = \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix}$$  \hspace{1cm} (2.2)
where we use the coordinate notation $\Theta \in \mathbb{R}^3$ instead of the intrinsic representation $R(\Theta) \in SO(3)$. Note that no distinction is drawn between a physical vector in $n$ dimensions (an object which has magnitude and direction) and an $n$-element mathematical vector (a group of $n$ elements). The reader determines which is correct from context.

The Earth-referenced translational velocity $V$ and angular velocity $\omega$ are expressed in body coordinates (forward-right-down) as

$$V = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \omega = \begin{pmatrix} p \\ q \\ r \end{pmatrix} \tag{2.3}$$

Throughout this document, we use the notation $R_{XY}(\Theta)$ to refer to the ($\Theta$-dependent) rotation matrix which expresses vectors from the $Y$-frame in the $X$-frame via multiplication

$$x^X_A = [R_{XY}(\Theta)](Y_A) \tag{2.4}$$

where a pre-superscript is used to indicate the frame in which the vector is expressed. In
2.2. Rigid Body Dynamics

In this dissertation, the frame of expression for a vector is omitted unless explicit specification is necessary.

Standard translational kinematics for an aircraft are

\[ \dot{X} = R_{EB}(\phi, \theta, \psi) \begin{pmatrix} u \\ v \\ w \end{pmatrix} = R_{EB}(\Theta) V \]  

(2.5)

where

\[ R_{EB} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \]

\[ = \begin{bmatrix} \cos \psi \cos \theta & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi \\ \sin \psi \cos \theta & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{bmatrix} \]  

(2.6)

and standard angular kinematics are

\[ \dot{\Theta} = \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = L^{-1}(\phi, \theta) \begin{pmatrix} p \\ q \\ r \end{pmatrix} = L^{-1}(\phi, \theta) \omega \]  

(2.7)

where

\[ L^{-1}(\phi, \theta) = \begin{bmatrix} 1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) \sec(\theta) & \cos(\phi) \sec(\theta) \end{bmatrix} \]  

(2.8)

Define the wind vector-field in the North-East-Down frame as

\[ W(X, t) = \begin{pmatrix} W_N(X, t) \\ W_E(X, t) \\ W_D(X, t) \end{pmatrix} \]  

(2.9)

and define standard air-relative velocity and corresponding flow-angles

\[ V_r = \begin{pmatrix} u_r \\ v_r \\ w_r \end{pmatrix} = R_{BE}(\Theta)(\dot{X} - W) = V - R_{BE}(\Theta) W \]  

(2.10)

\[ \alpha = \tan^{-1} \left( \frac{w_r}{u_r} \right), \quad \beta = \sin^{-1} \left( \frac{v_r}{V_r} \right) \]  

(2.11)

Some other useful identities also follow from these definitions

\[ \sin(\alpha) = \frac{w_r}{\sqrt{(u_r)^2 + (w_r)^2}}, \quad \cos(\alpha) = \frac{u_r}{\sqrt{(u_r)^2 + (w_r)^2}} \]  

(2.12)
\[
\cos(\beta) = \frac{\sqrt{u_r^2 + w_r^2}}{V_r}, \quad \tan(\beta) = \frac{v_r^2}{u_r^2 + w_r^2}
\] (2.13)

In this dissertation, two different notations are used to denote the standard Euclidean 2-norm of vectors: Either the standard norm operator \(\| \cdot \|\) or the vector’s symbol in non-bold type. Some common examples are

\[
\sqrt{u^2 + v^2 + w^2} = \| V \| = V
\] (2.14)

\[
\sqrt{u_r^2 + v_r^2 + w_r^2} = \| V_r \| = V_r
\] (2.15)

\[
\sqrt{W_N^2 + W_E^2 + W_D^2} = \| W \| = W
\] (2.16)

Finally, define the aircraft inputs (following standard notation) as thrust percentage, aileron deflection, elevator deflection, and rudder deflection

\[
u = \begin{pmatrix} T\% \\ \delta_a \\ \delta_e \\ \delta_r \end{pmatrix}
\] (2.17)

### 2.2.2 Rigid Body Dynamic Equations

Having defined the appropriate quantities, applying Newton’s second law of motion and expressing the resulting translational equations in the body frame yields

\[
R_{BM}(\alpha) \begin{pmatrix} C_D(\ldots) \\ C_Y(\ldots) \\ C_L(\ldots) \end{pmatrix} \frac{1}{2} \rho V_r^2 S + \begin{pmatrix} T\%(\frac{T_{max}}{100\%}) \\ 0 \\ 0 \end{pmatrix} + R_{BE}(\Theta) \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} = m \left( \dot{V} + (\omega \times V) \right)
\] (2.18)

where the rotation matrix \(R_{BM}(\alpha)\) between the body and modified stability frames is explicitly detailed in Section 2.4.1. The quantity \(T_{max}\) is the aircraft’s maximum available thrust. The drag, side force, and lift aerodynamic forces are resolved in the modified-stability frame. The corresponding aerodynamic force coefficients \(C_D, C_Y, C_L\) are defined in Section 2.4 and ellipses are used to remind the reader of their dependence on other variables such as \(\alpha, \beta, V_r, u\), and elements of \(\frac{dW}{dt}\).

Applying Newton’s second law of motion in the angular quantities and expressing in the body frame yields

\[
\begin{bmatrix} b & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & b \end{bmatrix} \begin{pmatrix} C_t(\ldots) \\ C_m(\ldots) \\ C_n(\ldots) \end{pmatrix} \frac{1}{2} \rho V_r^2 S = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} \dot{\omega} + (\omega \times \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix} \omega)
\] (2.19)

where similarly the aerodynamic moment coefficients \(C_t, C_m, C_n\) are defined in Section 2.4.
Wind-Relative Rigid Body Dynamic Equations

An alternate form of the translational velocity dynamics is obtained by transforming from Earth-relative \( V \) to wind-relative \( V_r \). Utilizing the substantial derivative of \( W \) (defined in Section A.3.3)

\[
\frac{DW}{Dt} = \frac{\partial W}{\partial t} + \frac{\partial W}{\partial X} \dot{X}
\]  

(2.20)

note that

\[
\dot{V}_r = \frac{B_d}{dt} V_r
\]  

(2.21)

\[
= \frac{B_d}{dt} \left( V - R_{BE}(\Theta)W \right)
\]  

(2.22)

\[
= \dot{V} + (\omega \times R_{BE}(\Theta)W) - R_{BE}(\Theta) \frac{DW}{Dt}
\]  

(2.23)

by applying the wind-triangle equation (2.10). In this equation, we use a left superscript (following [1129, §1.3.3] or [74, §2.23]) to remind the reader that the time differentiation is in a moving frame. Note that existing literature is inconsistent whether the over-dot operator represents time differentiation in the inertial frame (see [1129, eq. 1.22] and [114, eq. 2.6.5]) or instead time differentiation in the same frame as a vector’s expression (see [35, eq. 4.6.1] and [1102, eq. 3.10]). The next steps are to solve equation (2.18) for \( V \), substitute that into equation (2.23), and apply a reorganization of the wind-triangle equation (2.10) to eliminate dependence on \( V \). With these substitutions, the translational dynamics are expressed in wind-relative form as

\[
\dot{V}_r = R_{BM}(\alpha) \begin{pmatrix}
C_D(\ldots) \\
C_Y(\ldots) \\
C_L(\ldots)
\end{pmatrix} \frac{\rho V_r^2 S}{2m}
\]

\[
+ \frac{1}{m} \left( T_\% \left( \frac{T_{\text{max}}}{100\%} \right) \right) + R_{BE}(\Theta) \begin{pmatrix}
0 \\
0 \\
g
\end{pmatrix} - (\omega \times V_r)
\]

\[
- R_{BE}(\Theta) \left( \frac{\partial W}{\partial t} + \frac{\partial W}{\partial X} (R_{EB}(\Theta)V_r + W) \right)
\]  

(2.24)

This agrees with a common form used in many publications

\[
F_{\text{aero}} + F_{\text{thrust}} + F_g = B_{\text{XYZ}} F + B_g = m \left( \dot{V}_r + (\omega \times V_r) + (R_{BE}) \frac{DW}{Dt} \right)
\]  

(2.25)

Another common form used in the literature (e.g. [16]) is obtained by applying the proper realization of the transport theorem equation (A.23) to express \( F = m \ddot{X} \) in Etkin’s wind frame [35, §4.2.5] as

\[
\sum \frac{wF}{m} = \begin{pmatrix}
V_r \\
0 \\
0
\end{pmatrix} + \left( \begin{pmatrix}
p \\
q - \dot{\alpha} \\
r
\end{pmatrix} + \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix} \right) \times \begin{pmatrix}
V_r \\
0 \\
0
\end{pmatrix} + (R_{WE}) \frac{DW}{Dt}
\]  

(2.26)
Preferred Form of Wind-Relative Rigid Body Dynamic Equations  The form of the translational dynamics utilized in this dissertation is obtained by transforming from the rectangular representation of air-relative velocity \( \mathbf{V}_r \) to the spherical-coordinate representation which is denoted

\[
\mathbf{V}_R = \begin{pmatrix} V_r \\ \beta \\ \alpha \end{pmatrix}
\]  

(2.27)

Notice this is *not* a physical vector, but instead a mathematical-vector of one scalar magnitude and two scalar angles. Equations (2.15) and (2.11) define the transformation, and its inverse is explicitly provided for later use

\[
\mathbf{V}_r(V_r, \beta, \alpha) = \begin{pmatrix} u_r \\ v_r \\ w_r \end{pmatrix} = \begin{pmatrix} \cos \alpha \cos \beta \\ \sin \beta \\ \sin \alpha \cos \beta \end{pmatrix}
\]  

(2.28)

Obtaining the desired form of the translational kinematics begins with time derivatives of each component [102, §3.5].

\[
\frac{d}{dt}(\mathbf{V}_r) = \frac{\dot{\mathbf{V}}_r^T \mathbf{V}_r}{V_r} = \dot{\mathbf{V}}_r^T \begin{pmatrix} \cos \alpha \cos \beta \\ \sin \beta \\ \sin \alpha \cos \beta \end{pmatrix}
\]  

(2.29)

\[
\dot{\beta} = \frac{\dot{v}_r V_r - v_r \left( \frac{d}{dt} (V_r) \right)}{V_r \sqrt{V_r^2 - (v_r)^2}} = \frac{\dot{v}_r V_r - v_r \left( \frac{d}{dt} (V_r) \right)}{V_r^2 \cos \beta} = \frac{\dot{\mathbf{V}}_r^T}{V_r} \begin{pmatrix} -\cos \alpha \sin \beta \\ \cos \beta \\ -\sin \alpha \sin \beta \end{pmatrix}
\]  

(2.30)

\[
\dot{\alpha} = \frac{\dot{w}_r u_r - w_r \dot{u}_r}{(u_r)^2 + (w_r)^2} = \frac{\dot{\mathbf{V}}_r^T}{V_r \cos \beta} \begin{pmatrix} -\sin(\alpha) \\ 0 \\ \cos(\alpha) \end{pmatrix},
\]  

(2.31)

which are valid for \( V_r \neq 0, \beta \neq \pm \pi/2 \). Adopting notation from publications by Dogan’s group (e.g. [30, 87]) define

\[
\mathbf{E}_R = \mathbf{R}_{BW} \begin{bmatrix} 1 & 0 & 0 \\ 0 & V_r & 0 \\ 0 & 0 & V_r \cos \beta \end{bmatrix}
\]  

(2.32)

\[
= \begin{bmatrix} \cos \alpha \cos \beta & -V_r \cos \alpha \sin \beta & -V_r \sin \alpha \cos \beta \\ \sin \beta & V_r \cos \beta & 0 \\ \sin \alpha \cos \beta & -V_r \sin \alpha \sin \beta & V_r \cos \alpha \cos \beta \end{bmatrix}
\]  

(2.33)
2.3 Generalizing Aerodynamic Equations in Wind

In a flight dynamic model, the aerodynamic equations assign a set of forces and moments based on how the air flows around the aircraft. In this dissertation, the aerodynamic equations are assumed to be explicit functions of aircraft state, input, and environment variables. The aerodynamic roll, pitch, and yaw moments \( L; M; N \) are consistently expressed in the body frame across the literature. For the aerodynamic forces, however, several expressions are commonly used. One standard expression, found in \([35, \text{§5.6}]\) and \([102, \text{§3.2}]\) is body-frame forward-force \( X \), right-force \( Y \), and down-force \( Z \). Another standard expression is in the aircraft’s wind frame \([35, \text{§4.2.5}]\) where the force acting opposite the air-relative velocity vector is drag \( D_W \) and the lift force \( L_W \) acts upward in the aircraft’s plane of symmetry. Two common representations of the wind-frame side force are used, denoted \( C \) when acting to the “aircraft’s left” direction \([35, \text{Fig. 4.5}]\) or \( Y_W \) \([102, \text{§3.5}]\) when acting to the “aircraft’s right” wind axis\(^2\). Noting that some notational confusion exists between the various systems, other literature may define less common symbols (e.g. \( A \) in \([17, \text{§4.3.3}]\)). This dissertation adopts a representation in the “modified stability frame” in agreement with \([49]\) which is presented in detail in Section 2.4.1.

\(^1\)Some authors group the thrust inside the aerodynamic forces as well.

\(^2\)We use the \( W \) subscript to disambiguate from the body-frame side-force \( Y \)
2.3.1 Aerodynamic Equivalence

Because many sources of aerodynamic models assume zero-wind or uniform wind conditions, they need to be generalized for application in non-uniform wind situations. Thus here we critically review some existing theories as well as present some new theoretical concepts which formalize the adaptations.

It is self-evident that straight-and-level flight at airspeed \( V_r \) directly into a uniform, steady headwind field \( W \) which has magnitude \( V_r \) (i.e. in the opposite direction of forward flight) causes the aircraft to have zero inertial velocity (i.e. the aircraft “hovers”). The aerodynamic forces in this situation must be exactly equivalent to all other situations where the air-relative velocity is \( V_r \), such as forward flight at speed \( V_r \) in zero wind. This concept of aerodynamic equivalence permits transferring knowledge of aerodynamic forces/moments from one flight condition to all other aerodynamically equivalent ones.

Most aerodynamic models depend also on angular velocity terms, for example the common coefficient\(^3\) \( C_{m\varphi} \) is termed “pitch damping.” However, few discussions exist about the proper substitution which should replace \( \omega = (p, q, r)^T \) inside the aerodynamic equations for arbitrary \( C^1 \)-class wind fields. Here, a theory is presented for an expression of “air-relative angular velocity” \( \omega_r \) to substitute for \( \omega \) when extending a uniform wind FDM into non-uniform wind conditions. In development of this argument, we also demonstrate that the shear and dilation components of the wind’s gradient are implicitly assumed to be zero for all aerodynamic models learned in zero-wind conditions. The reader should note that the \( \omega_r \) we develop is a separate concept from the relative angular velocity between the wind and body frames, which Etkin [35, eq. 5.2,12] denotes \( \omega_{rel} \).

The idea is mathematically formulated by first defining an Apparent Wind Field (AWF) and then demonstrating some important aerodynamically equivalent situations.

At any time \( t \), consider a spatially infinite arbitrary wind field expressed in body coordinates as viewed from the pilot’s perspective. This vector field is named the Apparent Wind Field (AWF). It is the aircraft-relative wind velocity field defined over every point \( z \) in physical 3d space. We denote it as a relative wind field

\[
W_r : \mathbb{R}^3 \to \mathbb{R}^3
\] (2.39)

where it maps positions \( z \) expressed in the body-frame to velocity vectors \( W_r(z) \) which are also expressed in body frame coordinates. Note that the vector field may change with time \( t \), but for simplicity this is not reflected in the notation. To simplify some following expressions, we denote the space of \( C^1 \)-smooth maps from some space \( A \) to another space \( B \) as

\[
C^1(A; B) = \{ f(\cdot) | f : A \to B \text{ is class } C^1 \}\] (2.40)

\(^3\)Following conventional notation, \( C_{m\varphi} \) denotes the linear dependence of pitch moment coefficient \( C_m \) on the non-dimensionalized pitch rate \( \dot{\varphi} \).
Now, since Assumption 6 provides that the wind field is $C^1$-smooth, we can additionally state that
\[ W_r \in C^1(\mathbb{R}^3; \mathbb{R}^3) \] (2.41)

It is true that in a physical sense, inserting any object (an aircraft) into a flow field represented by $W_r$ would disrupt the flow significantly. As examples, the AWF would become undefined on the interior of the aircraft, it would have a non-penetrating boundary condition on the aircraft’s hull, and the flow will be distorted beyond the aircraft’s boundary. Furthermore, viscous flow effects create chaotic turbulence, the aircraft engine’s inflow may violate the boundary condition, and other disruptions may be observed. To justify neglecting these flow disruptions in compliance with Assumption 8, we introduce the concept of the “actual wind” operator $W_a$.

Define an operator $W_a$ which maps the AWF (which is undisturbed by the aircraft) to the expected true aircraft-relative wind field. With a slight abuse of notation, we use $W_a$ to refer both to the operator as well as the resulting actual vector field, as functionally the two are equivalent. To avoid some complexities inherent in determining $W_a$ the following assumptions are utilized:

1. An aircraft is a non-penetrable rigid body with a smooth boundary.
   - Define the body’s boundary via a scalar function $F \in C^1(\mathbb{R}^3; \mathbb{R}^1)$.
   - The closure of the aircraft body becomes $z_{body} = \{ z \in \mathbb{R}^3 : F(z) \leq 0 \}$.
   - Thus the non-penetration boundary condition becomes
     \[ R_{BE}W_a(z) \cdot \frac{\partial F}{\partial z}(z) = 0 \quad \forall \quad \{ z : F(z) = 0 \} \]

2. The airflow disturbance from the aircraft disappears far from the aircraft
   - Written mathematically: $\lim_{\|z\| \to \infty} W_a(z) = W_r(z)$

The reader should also recall Assumption 7 in which unsteady aerodynamic effects are neglected. These assumptions combine with the physical laws for a body in low-speed compressible flow to produce an aircraft-relative wind field corresponding to each possible $W_r$ into which the aircraft is inserted. This relation can be expressed as
\[ W_a : C^1(\mathbb{R}^3; \mathbb{R}^3) \to C^1(\mathbb{R}^3 \setminus z_{body}; \mathbb{R}^3) \] (2.42)

Then from integrating pressure and friction associated with $W_a$ across the aircraft’s boundary, the aerodynamic force and moment relations can be expressed
\[ F_a : C^1(\mathbb{R}^3 \setminus z_{body}; \mathbb{R}^3) \to \mathbb{R}^3 \] (2.43)
\[ M_a : C^1(\mathbb{R}^3 \setminus z_{body}; \mathbb{R}^3) \to \mathbb{R}^3 \] (2.44)
By composing equations (2.42) and (2.43) the desired result is achieved: an AWF is said to “produce aerodynamic forces” and the relationship is expressed by

\[ F : C^1(\mathbb{R}^3; \mathbb{R}^3) \to \mathbb{R}^3 \times \mathbb{R}^3 \]  

where

\[ F \triangleq F_a \circ W_a \]  

An identical result follows for an AWF producing aerodynamic moments

\[ M : C^1(\mathbb{R}^3; \mathbb{R}^3) \to \mathbb{R}^3 \times \mathbb{R}^3 \]  

\[ M \triangleq M_a \circ W_a \]  

Notice that the physical disruption of the AWF by the aircraft body, which is known to exist in practice, is safely ignored because it is immaterial to the aerodynamic expressions as seen by equations (2.45) and (2.47).

For a FDM determined in no wind conditions, such as Grauer [49], the aerodynamically relevant flight state variables are \((V, \omega)\). Next, we demonstrate that this set of states is aerodynamically equivalent to the set of AWF from superposing uniform wind fields and circulatory-flow wind fields.

**Important Example: Zero-Wind AWF**  
Consider an aircraft flying in zero wind conditions at some linear and angular velocity

\[ V_0 = (V_0, \beta_0, \alpha_0)^T, \quad \omega_0 = (p_0, q_0, r_0)^T \]

To indicate that the AWF corresponds to zero-wind and is parameterized by the velocity terms, we denote it \(W_{r,0}(z|V_0, \omega_0)\). Because \(W = 0\), the AWF is equal-and-opposite of the velocity of each point in the body frame under general rigid-body motion. Denoting the air-relative velocity \(V_r\) of each point \(z\) parameterized by the translational and rotational velocity of the body frame as \(V_r(z|V_0, \omega_0)\) we can write

\[ W_{r,0}(z|V_0, \omega_0) = -V_r(z|V_0, \omega_0) \]

\[ = -\left( V_0 + \omega_0 \times z \right) \]

\[ = -\left( R_{BW}(\alpha_0, \beta_0) \begin{pmatrix} V_0 \\ 0 \\ 0 \end{pmatrix} + \omega_0 \times z \right) \]  

To find an equivalent flow field for all points on an inertially fixed aircraft, rearrange to the form

\[ W_{r,0}(z|V_0, \omega_0) = -\omega_0 \times \left( z - \bar{z} \right) \]  

(2.50)
2.3. Generalizing Aerodynamic Equations in Wind

where

\[
\ddot{z} = \frac{V_0}{\|\omega_0\|} e_{\omega_0} \times \begin{pmatrix}
\cos \alpha_0 \cos \beta_0 \\
\sin \beta_0 \\
\sin \alpha_0 \cos \beta_0
\end{pmatrix}
\] (2.51)

and \(e_{\omega_0}\) denotes the unit vector in the direction of \(\omega_0\). Thus every combination of \((V, \omega)\) flown in zero wind is aerodynamically equivalent to an AWF generated by the sum of a pure-rotational flow field with a uniform flow field. This result is the conceptual link by which aerodynamic models derived in zero-wind conditions are generalized to non-zero wind fields.

2.3.2 Frost’s Theory of Wind Angular Rotation

The concepts of aerodynamic equivalence and Apparent Wind Field (AWF) suggest a theory for computing the apparent angular velocity of a wind field \(\omega_W\). This theory was first proposed by Frost and Bowles [38] and is presented with additional detail here. The reader should note this is not the same concept as the angular velocity of the wind frame as presented in [35, eq. 5.2,13].

An observer riding on the body frame through an arbitrary wind field experiences the total AWF \(W_r\) as the vector-field sum of the apparent wind due to the body-frame motion \(W_{r,0}\) (from equation (2.49)) and the inertially defined wind \(W\).

\[
W_r = W_{r,0} + R_{BE} W. \tag{2.52}
\]

Since both are at least \(C^1\)-smooth, the AWF near the body-frame origin \(z = 0\) is approximated by\(^4\) the first order terms of a Taylor series expansion

\[
W_r(z) \simeq W_r(0) + \left[ \frac{\partial W_r}{\partial z} \right]_0 z. \tag{2.53}
\]

Expanding the gradient \(\frac{\partial W_r}{\partial z}\) into its 3 constituent parts [74, §2.25] shows

\[
W_r(z) \simeq W_r(0) + \left[ \begin{array}{c}
\text{dilation part} \\
\text{shear part} \\
\text{rotation part}
\end{array} \right] + 
\left[ \begin{array}{c}
\frac{1}{2} \left( \frac{\partial W_r}{\partial z} + \frac{\partial W_r^T}{\partial z} \right) - \text{diag} \left( \frac{\partial W_r}{\partial z} \right) \\
\frac{1}{2} \left( \frac{\partial W_r}{\partial z} - \frac{\partial W_r^T}{\partial z} \right)
\end{array} \right] z. \tag{2.54}
\]

\(^4\)A private communication (Reference 38 of [36]) is claimed as evidence to justify this approximation.
To simplify notation, we employ the common “hat notation” which is detailed in Appendix A.1. To produce the important conclusion, examine terms which match the form “\( \omega_r \times z \)” (equivalently “\( \hat{\omega}, z \)” ) to demonstrate

\[
\left[ \frac{\partial W_r}{\partial z} \right]_{\text{rot}} = -\hat{\omega}_r = R_{BE} \left[ \frac{1}{2} \left( \frac{\partial W}{\partial X} - \frac{\partial W^T}{\partial X} \right) \right] R_{BE}^T - \hat{\omega}
\]

(2.55)

where the (\( ^\vee \) ) operation (notation from [105]) is simply the inverse of the standard skew-symmetric “hat” operator. By defining “the angular velocity of the wind field”

\[
\omega_W = \left( R_{BE} \left[ \frac{1}{2} \left( \frac{\partial W}{\partial X} - \frac{\partial W^T}{\partial X} \right) \right] R_{BE}^T \right) ^\vee
\]

(2.56)

we recognize the form in parity with the translational wind triangle

\[
\omega_r = \omega - \omega_W
\]

(2.57)

The similarities in form between equations (2.10) and (2.57) is instructive:

**Remark 2.1.** Both translational air-relative velocity \( V_r \) and rotational air-relative velocity \( \omega_r \) have equal-and-opposite contributions from the aircraft’s inertial motion \( (V, \omega) \) and from the wind field at the body-frame’s origin. This is the mathematical demonstration of aerodynamic equivalence.

**Limitation of the Frost theory** An important fact is observed from these equations: An aircraft flying in zero-wind conditions can encounter only a limited subset of aerodynamically important situations. The possible subset is aerodynamically equivalent to an inertially fixed aircraft experiencing only uniform wind and the rotational part of the wind gradient. Stated differently, flight experiments conducted in zero wind cannot produce information about the aerodynamic contributions of the dilation and/or shear contributions to the wind-gradient which are also present in a first-order representation of a wind field via equation (2.54).

As summarized in the next subsection, ideal flow theory for simple airfoils predicts non-negligible aerodynamic effects from a shear wind gradient. If the same is true for a full aircraft, the Frost theory is an incomplete aerodynamic representation in this regard.

### 2.3.3 Ideal Flow Airfoils in Shear Flow

The following question becomes relevant: “Does any theory predict that an aircraft’s aerodynamics should have non-negligible contributions from the shear and/or dilation components of the AWF?” The answer is “Yes.” In this section, we present historical work on 2d uniform-shear flow over airfoils which formulate theory and experiments suggesting that the shear-gradient aerodynamic effect is relevant.
The seminal work in the area by Tsiens [152] uses assumptions similar to ideal flow to obtain the theory for a symmetric Joukowsky airfoil in uniform-shear flow. A situational diagram (except for a non-symmetric airfoil) is presented in Figure 2.3. Equations (60) of [152] for the lift and moment coefficients are reproduced below.

\[
C_L = 2\pi \left[ l_0 \sin \alpha + K(l_1 + l_2 \cos 2\alpha) + K^2(l_3 \sin \alpha + l_4 \sin 3\alpha) \right]
\]

\[
C_m = \frac{\pi}{2} \left[ m_0 \sin 2\alpha + K(m_1 \cos \alpha + m_2 \cos 3\alpha) + K^2(m_3 \sin \alpha + m_4 \sin 4\alpha) \right]
\]

In these equations, \(K\) is the magnitude of the shear and all coefficients \((l_i, m_i)\) are specified in Equations (62)–(63) of [152] as functions of the airfoil thickness. In the very same year, Tsiens colleague Kuo [76] formulates the same results in a simpler form as an extension of Blasius’ theorem.

Two other papers present alternate mathematical variations of Tsiens’s theory. The first [25] extends Tsiens’ work using complex-variable techniques, thus becoming applicable to a more general class of airfoils. The second [69] formulates the theory suitable for a moving airfoil, generalizing past the stationary case. Note that while it is denoted “Part 1 of 2”, after exhaustive search it seems that Part 2 was never published.

The next significant innovation comes from [140] where the theory is extended to cambered\(^5\) Joukowsky airfoils in uniform shear. The application of interest is STOL aircraft, where the wing airfoil is subjected to high shear in part due to being in a propeller slipstream. It appears the Cornell Aeronautical Laboratory (CAL) led research in this area for a few years, under contracts from the U.S. Army Transportation and Engineering Research Command. A resulting CAL/TREC report [155] describes wind tunnel experiments for airfoils in uniform, uniform-shear, and non-uniform-shear flows, showing good matches with the theory especially for small \(\alpha\).

In 1966, Chen [23] formulates a simplified second-order theory which seems to be sufficiently accurate for reasonable \(\alpha\) and shear-strength values. One important contribution is being able to handle arbitrary airfoils (not only Joukowsky). Another is that this simplified theory is better at providing intuition for practitioners.

\(^5\)Recall a symmetric airfoil has zero camber.
Chapter 2. Flight Dynamic Modeling

Following the history of the significant CAL/TREC research, several additional reports in the same project pursue theory suitable for Short/Vertical Takeoff and Landing (STOL/VTOL) aircraft. Two important effects are demonstrated:

- An airfoil in nonuniform shear (see Fig 6 of [156]) experiences different aerodynamic forcing based on its vertical position in the slipstream.

- A propeller slipstream decays radially as one moves spanwise away from the aircraft center along the wing. This requires a 3-dimensional aerodynamic theory.

Reports [156, 157] explore this first effect, the non-uniform-shear effects on an airfoil, with both theory and experimental results. A following report [19] includes wall effects. Later work by Gupta [53] (outside CAL/TREC) reproduces and validates these results.

2.3.4 Etkin’s Theory of Wind Angular Rotation

In 1981, Etkin [36] presented an alternate theory for the angular velocity of the wind field $\omega_W$. Etkin’s argument is re-created here with additional details:

- Expressed in the body frame, the spatial wind-gradient has 9 components.

- Since most aircraft are approximately planar (in the $x_By_B$ plane), neglect the wind-gradient components w.r.t. the $z_B$ direction

$$\frac{\partial W_x}{\partial z} = \frac{\partial W_y}{\partial z} = \frac{\partial W_z}{\partial z} = 0$$

- Notice that dilation terms are expected to have little longitudinal aerodynamic effect and no lateral-directional effect, thus neglect them.

$$\frac{\partial W_x}{\partial x} = \frac{\partial W_y}{\partial y} = 0$$

- Next, express the apparent downwash velocity $W_{r,z}(x,y)$ on any point in the $x_By_B$ plane (i.e. the wing or horizontal stabilizer) from a spatial wind gradient as well as from wind angular velocity components $p_W, q_W$

$$W_{r,z}(x,y) = \frac{\partial W_{r,z}}{\partial x} x + \frac{\partial W_{r,z}}{\partial y} y$$

$$W_{r,z}(x,y) = -q_W x + p_W y$$

---

As this is an invited lecture, the theory may likely exist in earlier publications. Also, Boiffier [17, §3.2.2] presents the same theory, indirectly referencing an [Etkin 1990] publication which was unable to be located.

Correspondingly, restrict this theory to such nearly planar aircraft.
By equating the downwash amounts it is determined that

\[ p_W = \frac{\partial W_{r,z}}{\partial y}, \quad q_W = -\frac{\partial W_{r,z}}{\partial x} \]  

(2.58)

- The same principle demonstrates equivalence for yawing to two separate gradient components

\[ W_{r,x}(x,y) = \left( \frac{\partial W_{r,x}}{\partial y} \right) y = -r_W y \]

\[ W_{r,y}(x,y) = \left( \frac{\partial W_{r,y}}{\partial x} \right) x = r_W x \]

so declare two new aircraft coefficients \( N_W, N_F \) (suggestion the “wing” and “tailfin” contributions, respectively) and declare the yawing angular velocity of the wind to be the parametrically weighted sum

\[ r_W = -N_W \frac{\partial W_{r,x}}{\partial y} + N_F \frac{\partial W_{r,y}}{\partial x} \]  

(2.59)

this completes Etkin’s argument.

In summary, the resulting Etkin’s theory for angular wind velocity is

\[ \omega_W = \left( \begin{array}{c} \rho_W \\ q_W \\ r_W \end{array} \right) = \left( \begin{array}{c} \frac{\partial W_{r,z}}{\partial y} \\ -\frac{\partial W_{r,z}}{\partial x} \\ -N_W \frac{\partial W_{r,z}}{\partial y} + N_F \frac{\partial W_{r,z}}{\partial x} \end{array} \right) \]  

(2.60)

Two simpler versions following Etkin’s logic are published in U.S. military documents MIL-HDBK-1797 [3] and MIL-F-8785C [2]; these are well summarized in [39]. Note that these assume only one of the two yaw effects presented by Etkin and differ from each other only by sign conventions. Additionally, Boiffier provides a generalized version which accommodates both of these as special cases [17]. In this dissertation, we refer to this family of theories as “Etkin” models.

### 2.3.5 Critique of Wind Gradient Aerodynamic Theories

To recap, Section 2.3.1 presents the principle of “aerodynamic equivalence” to demonstrate that aerodynamic equations must have dependence on spatial wind gradients when being applied in non-uniform wind fields. From there, the available theories diverge.
Section 2.3.2 presents the theory of Frost [38], supported by the concept of the Apparent Wind Field (AWF). This theory provides a logically consistent wind-field angular velocity $\omega_W$ in equation (2.56). The major shortcoming of this theory is that it cannot account for the shear and dilation wind-gradient components. Thus it produces unintuitive (i.e. clearly incorrect) results for some cases of interest, as demonstrated in [37].

In Section 2.3.3 a review of the historical theory and experiments of a 2d airfoil in a shearing flow are presented. While shear effects on a 2-dimensional airfoil are not the same as shear effects on a 3-dimensional aircraft, their existence strongly suggests that non-negligible shear-flow effects correspondingly exist for aircraft.

Section 2.3.4 presents an alternate theory, proposed by Etkin. The result is captured by equation (2.60). The drawback of Etkin’s theory is its basis in intuition. For example, Etkin claims that the “wind roll” contribution from $\frac{\partial W_{x,z}}{\partial y}$ is negligible because most aircraft are planar, but most aircraft contain a vertical stabilizer which violates this assumption. Despite its shortcomings, for some easily visualized test cases Etkin’s theory produces answers which are clearly superior to the Frost theory. In fact, Etkin published a Technical Comment [37] in which he demonstrates that Frost’s theory predicts an unintuitive result for the case where $\frac{\partial W_{x,z}}{\partial y} \neq 0$ but all other components are zero. In this case, Frost’s theory predicts exactly half the result of Etkin’s. As the cross-sectional area of an aircraft’s vertical stabilizer shrinks in comparison to the wing + horizontal stabilizer planform area, Etkin’s theory increases in accuracy.

In a comprehensive Technical Report [13, §4.2.1.2] it is claimed that a wind gradient (specifically a shear) causes additional physical effects which must be modeled, and the suggestion is to include new aerodynamic derivatives. Likely this is a reasonable approximation for linear analysis and small deviations from the trim state. Accordingly, linear equations are presented which include these additional terms.

Having analyzed each of these theories, we understand that each is a simplistic representation of a much more complicated truth. Further investigation would be required to seek a suitable alternative. While we apply Frost’s theory in the works accompanying this dissertation, we have come ultimately to suggest that others use Etkin’s theory instead. We hope that future research may identify a superior alternative.

### 2.4 Aerodynamic Equations

Having completed our discussion of adapting aerodynamic models for non-uniform wind fields, we present two aerodynamic models which comprise FDMs used in later chapters of this dissertation. For our aerodynamic models, we utilize a less common frame of expression, which we name the “modified stability frame.”
2.4.1 Modified Stability Frame Aerodynamics

Motivated by equations (3.40a, 3.40b) of [102], a “modified stability frame” is defined for the directions of the aerodynamic forces. This frame is an intermediate between the standard body-frame and the standard wind-frame. The “modified” term is used because Etkin’s stability frame [35, §4.2.7] is body-fixed, while this frame rotates with respect to the body frame with changes in angle of attack $\alpha$. Its first axis $\mathbf{x}_M$ points negatively along the projection of $\mathbf{V}_r$ onto the aircraft’s $x_Bz_B$ plane (the “forward-and-down” plane, pointing in the direction of drag) and its second direction is identical to the aircraft’s $y_B$ (“right-wing”) direction. Notice that in the case of symmetric flight ($\beta = 0$) the modified stability frame axes are parallel with the wind frame, but some have reversed directions. The direction cosine matrix (DCM) that transforms vectors from the modified stability frame to the body frame is

$$R_{BM}(\alpha) = 
\begin{bmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1 \\
\end{bmatrix}
\begin{bmatrix}
\cos(\alpha) & 0 & -\sin(\alpha) \\
0 & 1 & 0 \\
\sin(\alpha) & 0 & \cos(\alpha) \\
\end{bmatrix}
= 
\begin{bmatrix}
-\cos(\alpha) & 0 & \sin(\alpha) \\
0 & 1 & 0 \\
-\sin(\alpha) & 0 & -\cos(\alpha) \\
\end{bmatrix}
(2.61)$$

(Note that equation (1) of [49] has a sign error.) For completeness, equations (3.40c, 3.40d) of [102] identify the DCM between the modified stability and the wind frames as

$$R_{MW}(\beta) = 
\begin{bmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1 \\
\end{bmatrix}
\begin{bmatrix}
\cos(\beta) & -\sin(\beta) & 0 \\
\sin(\beta) & \cos(\beta) & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
= 
\begin{bmatrix}
-\cos(\beta) & \sin(\beta) & 0 \\
\sin(\beta) & \cos(\beta) & 0 \\
0 & 0 & -1 \\
\end{bmatrix}
(2.62)$$

The product of these DCM’s recovers the standard result [35, eq. 4.5.5]

$$R_{BW}(\alpha, \beta) = R_{BM}(\alpha)R_{MW}(\beta) = 
\begin{bmatrix}
\cos \alpha \cos \beta & -\cos \alpha \sin \beta & -\sin \alpha \\
\sin \beta & \cos \beta & 0 \\
\sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha \\
\end{bmatrix}
(2.63)$$

This dissertation follows the notation of [102] by defining the drag force $D$ acting along the first modified stability axis $\mathbf{x}_M$, the side force $Y$ along $\mathbf{y}_M$, and the lift force $L$ along $\mathbf{z}_M$. Finally, it is standard practice to work with non-dimensionalized force and moment coefficients which are related to the body-frame aerodynamic forces and moments via

$$F_{\text{aero}} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R_{BM}(\alpha) \begin{bmatrix} D \\ Y \\ L \end{bmatrix} = R_{BM}(\alpha) \begin{bmatrix} C_D(...) \\ C_Y(...) \\ C_L(...) \end{bmatrix} \frac{1}{2} \rho V_r^2 S
(2.64)$$

$$M_{\text{aero}} = \begin{bmatrix} b & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & b \end{bmatrix} \begin{bmatrix} C_t(...) \\ C_m(...) \\ C_n(...) \end{bmatrix} \frac{1}{2} \rho V_r^2 S
(2.65)$$

The flight model’s aerodynamic forces are represented by coefficients $(C_D, C_Y, C_L)$ and its moments by $(C_t, C_m, C_n)$ as found in equations (2.64) and (2.65) respectively. Some explicit expressions for these coefficients are provided next in Sections 2.4.2 and 2.4.3, where we generalize them for use in arbitrary wind fields according to the discussions of Section 2.3.
2.4.2 GGA Aerodynamic Coefficients

In Chapters 6 and 4 we utilize the Generic Global Aerodynamic (GGA) model from Grauer and Morelli [49]. Per the discussions of Section 2.3, the GGA model is modified for use in non-uniform wind by substituting in air-relative velocity and angular velocity

\[ V \leftarrow V_r, \quad \text{and} \quad \omega \leftarrow \omega_r \]  \hspace{1cm} (2.66)

With these modifications, the aerodynamic coefficients from equation 17 of Grauer and Morelli [49] are

\[
\begin{align*}
C_D &= \theta_1 + \theta_2 \alpha + \theta_3 \alpha \dot{q}_r + \theta_4 \alpha \delta_c + \theta_5 \alpha^2 + \theta_6 \alpha^2 \dot{q}_r + \theta_7 \alpha^2 \delta_e + \theta_8 \alpha^3 + \theta_9 \alpha^3 \dot{q}_r + \theta_{10} \alpha^4 \\
C_Y &= \theta_{11} \beta + \theta_{12} \ddot{p}_r + \theta_{13} \ddot{r}_r + \theta_{14} \delta_a + \theta_{14} \delta_r \\
C_L &= \theta_{16} + \theta_{17} \alpha + \theta_{18} \dot{q}_r + \theta_{19} \delta_e + \theta_{20} \alpha \ddot{q}_r + \theta_{21} \alpha^2 + \theta_{22} \alpha^3 + \theta_{23} \alpha^4 \\
C_l &= \theta_{24} \beta + \theta_{25} \ddot{p}_r + \theta_{26} \ddot{r}_r + \theta_{27} \delta_a + \theta_{28} \delta_r \\
C_m &= \theta_{29} + \theta_{30} \alpha + \theta_{31} \dot{q}_r + \theta_{32} \delta_c + \theta_{33} \alpha \ddot{q}_r + \theta_{34} \alpha^2 \delta_e + \theta_{35} \alpha^2 \ddot{q}_r + \theta_{36} \alpha^3 \delta_e + \theta_{37} \alpha^3 \dot{q}_r + \theta_{38} \alpha^4 \\
C_n &= \theta_{39} \beta + \theta_{40} \ddot{p}_r + \theta_{41} \ddot{r}_r + \theta_{42} \delta_a + \theta_{43} \delta_r + \theta_{44} \beta^2 + \theta_{45} \beta^3
\end{align*}
\]  \hspace{1cm} (2.67)

where

\[
\begin{pmatrix}
\ddot{p}_r \\
\dot{q}_r \\
\ddot{r}_r
\end{pmatrix} = \tilde{\omega}_r = \frac{1}{2V_r} \begin{bmatrix}
b & 0 & 0 \\
0 & \ddot{c} & 0 \\
0 & 0 & b
\end{bmatrix} \omega_r = \frac{1}{2V_r} \begin{bmatrix}
b & 0 & 0 \\
0 & \ddot{c} & 0 \\
0 & 0 & b
\end{bmatrix} (\omega - \omega_W) \]  \hspace{1cm} (2.68)

In these equations \( \theta_1, \theta_2, \ldots, \theta_{45} \) are the aircraft-specific parameters as given in Tables 3 and 4 of [49]. Aerodynamic angles \( (\alpha, \beta) \) are defined in equation (2.11), and \( (\delta_a, \delta_c, \delta_r) \) are aileron, elevator, and rudder deflections from equation (2.17). Recall that \( \omega_W \) might come either from the Frost model or from the Etkin model, and is not the same concept as the angular velocity of the wind frame from [35, eq. 5.2,13].

As a brief summary of [49], the GGA model structure is a sort of “best overall fit” for data which span even into stall-flight regimes for 8 different aircraft. As stated, it is likely not the best model for any single aircraft of the 8 considered, but it is an “optimal compromise” to obtain a single model structure which represents all 8 aircraft.

A few relevant details of the model derivation are also reproduced here. In deriving this model, the lateral and longitudinal dynamics were decoupled, each being calculated with the other set to null. The model was developed on subsonic \( (M=0.1) \) speeds with any additional control surface deflections (flaps, canards) set to null. Some data near/during stall were included in the analysis, as were asymmetries during high sideslip. Thrust effects on the aerodynamic coefficients were neglected. Table 2 of [49] provides training-data bounds on the independent variables, which suggest the models should be trustworthy in similar ranges.
2.4. Aerodynamic Equations

2.4.3 Bix3 Aerodynamic Coefficients

Chapter 3 of this dissertation presents a summary and some details of a paper by Simmons et al. [139] wherein an aerodynamic model for the Bix3 small fixed-wing UAS is determined. For the reader’s convenience, the original equations of the Bix3 model structure are

\[ C_X = C_{X_0} + C_{X_u} \dot{u} + C_{X_w} \dot{w} + C_{X_{w2}} \dot{w}^2 \]
\[ C_Y = C_{Y_o} + C_{Y_v} \dot{v} + C_{Y_p} \dot{p} + C_{Y_r} \dot{r} + C_{Y_{a_o}} \delta_a + C_{Y_{r_r}} \delta_r \]
\[ C_Z = C_{Z_o} + C_{Z_u} \dot{u} + C_{Z_q} \dot{q} + C_{Z_{w2}} \dot{w}^2 + C_{Z_{e_e}} \delta_e \]
\[ C_l = C_{l_o} + C_{l_v} \dot{v} + C_{l_p} \dot{p} + C_{l_r} \dot{r} + C_{l_{a_o}} \delta_a + C_{l_{r_r}} \delta_r \]
\[ C_m = C_{m_o} + C_{m_u} \dot{w} + C_{m_q} \dot{q} + C_{m_{e_e}} \delta_e \]
\[ C_n = C_{n_o} + C_{n_u} \dot{v} + C_{n_p} \dot{p} + C_{n_r} \dot{r} + C_{n_{a_o}} \delta_a + C_{n_{r_r}} \delta_r \]

(2.69)

For comparison with the GGA model, those equations are transformed to equivalent variables and presented here. Since the Bix3 model assumes zero-wind conditions, the first transformation necessary is to substitute in air-relative velocities per equation (2.66). Next, note that a constant nominal velocity \( V_o \) is defined in the non-dimensionalization of translational and rotational velocities, in contrast to the actual velocity \( V_r \). By applying equations (2.28) and (2.64) the Bix3 model can be formulated in the same form as the GGA model in equation (2.67) to become

\[ C_D = - \left( C_{Z_o} + C_{Z_q} \dot{q} \frac{V_r}{V_o} + C_{Z_{e_e}} \delta_e \right) \]
\[ + C_{Z_w} \cos \beta \sin \alpha \frac{V_r}{V_o} + C_{Z_{w2}} \left( \sin \alpha \cos \beta \frac{V_r}{V_o} \right)^2 \]
\[ - \left( C_{X_o} + C_{X_u} \cos \alpha \cos \beta \frac{V_r}{V_o} + C_{X_{w2}} \sin \alpha \cos \beta \frac{V_r}{V_o} \right) \cos \alpha \]
\[ + \left( C_{X_o} + C_{X_u} \cos \alpha \cos \beta \frac{V_r}{V_o} + C_{X_{w2}} \frac{V_r}{V_o} \right) \frac{V_r}{V_o} \]
\[ + C_{Y_o} \sin \beta \frac{V_r}{V_o} + C_{Y_{a_o}} \delta_a + C_{Y_{r_r}} \delta_r \]
\[ C_Y = C_{Y_o} + C_{Y_v} \dot{v} \frac{V_r}{V_o} + C_{Y_r} \dot{r} \frac{V_r}{V_o} + C_{Y_{a_o}} \delta_a + C_{Y_{r_r}} \delta_r \]
\[ C_L = - \left( C_{Z_o} + C_{Z_q} \dot{q} \frac{V_r}{V_o} + C_{Z_{e_e}} \delta_e \right) \]
\[ + C_{Z_w} \sin \alpha \cos \beta \frac{V_r}{V_o} + C_{Z_{w2}} \left( \sin \alpha \cos \beta \frac{V_r}{V_o} \right)^2 \]
\[ + \left( C_{X_o} + C_{X_u} \cos \alpha \cos \beta \frac{V_r}{V_o} + C_{X_{w2}} \frac{V_r}{V_o} \right) \frac{V_r}{V_o} \]
\[ + C_{Y_o} \sin \beta \frac{V_r}{V_o} + C_{Y_{a_o}} \delta_a + C_{Y_{r_r}} \delta_r \]
\[ C_l = C_{l_o} + C_{l_v} \dot{v} \frac{V_r}{V_o} + C_{l_p} \dot{p} \frac{V_r}{V_o} + C_{l_r} \dot{r} \frac{V_r}{V_o} + C_{l_{a_o}} \delta_a + C_{l_{r_r}} \delta_r \]
\[ C_m = C_{m_o} + C_{m_{e_e}} \delta_e + C_{m_q} \dot{q} \frac{V_r}{V_o} + C_{m_w} \sin \alpha \cos \beta \frac{V_r}{V_o} \]
\[ C_n = C_{n_o} + C_{n_{a_o}} \delta_a + C_{n_{r_r}} \delta_r + C_{n_p} \frac{b_{r_r}}{2V_o} + C_{n_r} \frac{b_{r_r}}{2V_o} + C_{n_e} \sin \beta \frac{V_r}{V_o} + C_{n_{w2}} \left( \sin \beta \frac{V_r}{V_o} \right)^2 \]
2.5 State-space Forms

The standard form for expressing nonlinear flight dynamic models is the state-space form. This form is defined by the fundamental first-order ordinary differential equation

\[ \dot{x} = f(x, u, d) \]  

(2.71)

where the variables describing the aircraft’s state are in \( x \), the (known) inputs \( u \) are defined by equation (2.17), and the disturbances \( d \) represent the unknown “inputs” to the dynamic system.

2.5.1 Preferred FDM State-Space Form

In this dissertation the state vector is chosen as

\[
\begin{bmatrix}
X_N \\
X_E \\
X_D \\
\phi \\
\theta \\
\psi \\
V_r \\
\beta \\
\alpha \\
p \\
q \\
r
\end{bmatrix}
= \begin{bmatrix}
X \\
\Theta \\
V_R \\
\omega
\end{bmatrix}
\]  

(2.72)

and the disturbance vector is composed of the wind and its derivatives

\[
d = \begin{bmatrix} W^T, & \text{colvec} \left( \frac{\partial W}{\partial X} \right)^T, & \frac{\partial W^T}{\partial t} \end{bmatrix}^T
\]  

(2.73)

where the colvec(·) operator reshapes a matrix into a column vector. To express the flight dynamic model in standard state-space form, here we explicitly identify the components of \( f \) inside equation (2.71) which are differential equations corresponding to \( \dot{X}, \dot{\Theta}, \dot{V}_R \), and \( \dot{\omega} \) which depend only on \( x, u, \) and \( d \).

The translational velocity equations are given by combining equations (2.5) and (2.10) to obtain

\[
\dot{X} = R_{EB}(\Theta) \begin{bmatrix}
V_r \cos \alpha \cos \beta \\
V_r \sin \beta \\
V_r \sin \alpha \cos \beta
\end{bmatrix} + W
\]  

(2.74)
The rotational velocity equations expressing Euler-angle derivatives \( \dot{\Theta} \) are provided directly by equation (2.7) and rewritten here

\[
\dot{\Theta} = \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = L^{-1}(\phi, \theta) \begin{pmatrix} p \\ q \\ r \end{pmatrix} = L^{-1}(\phi, \theta)\omega
\]

The final development of Section 2.2.2 is equation (2.38), which provides the quantities in \( \dot{V}_R \). It is re-written in expanded form as

\[
\dot{V}_R = \begin{bmatrix} V_r & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sec \beta \end{bmatrix} \begin{bmatrix} R_{WM}(\beta) \\ R_{Y}(\cdots) \\ R_{L}(\cdots) \end{bmatrix} \begin{bmatrix} C_D(\cdots) \\ C_Y(\cdots) \\ C_L(\cdots) \end{bmatrix} \frac{p V_r S}{2m} + \frac{R_{WB}(\alpha, \beta)}{m V_r} \begin{pmatrix} T(\frac{T_{max}}{100\%}) \\ 0 \end{pmatrix} \\
+ \frac{R_{WB}(\alpha, \beta) R_{BE}(\Theta)}{V_r} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - R_{WB}(\alpha, \beta) \begin{pmatrix} \omega \times \begin{pmatrix} \cos \alpha \cos \beta \\ \sin \beta \\ \sin \alpha \cos \beta \end{pmatrix} \end{pmatrix} \\
- \frac{R_{WB}(\alpha, \beta) R_{BE}(\Theta)}{V_r} \begin{pmatrix} \frac{\partial W}{\partial t} + \frac{\partial W}{\partial X} \end{pmatrix} V_r R_{EB}(\Theta) \begin{pmatrix} \cos \alpha \cos \beta \\ \sin \beta \\ \sin \alpha \cos \beta \end{pmatrix} + W \right] (2.75)
\]

The final state derivative equations for \( \dot{\omega} \) are obtained from re-organizing equation (2.19) to become

\[
\dot{\omega} = [I^{-1}] \left[ -(\omega \times I \omega) + \begin{bmatrix} b & 0 & 0 \\ 0 & \bar{c} & 0 \\ 0 & 0 & b \end{bmatrix} \begin{pmatrix} C_l(\cdots) \\ C_m(\cdots) \\ C_n(\cdots) \end{bmatrix} \frac{\rho S V_r^2}{2} \right] (2.76)
\]

The state-space equations are completed by choosing an aerodynamic model for \((C_D, C_Y, C_L, C_l, C_m, C_n)\) such as the GGA of equations (2.67) or the Bix3 of equations (2.70).

### 2.5.2 Alternate Forms

**Inertial Velocity** Because many works assume zero wind in their FDM formulation, one common state formulation contains inertial velocity \( V \) in the state vector instead of air-relative velocity \( V_R \). The advantage of this formulation is that the translational dynamics of equation (2.75) are replaced by the following reorganization of equation (2.18)

\[
\dot{V} = R_{BM}(\alpha) \begin{bmatrix} C_D(\cdots) \\ C_Y(\cdots) \\ C_L(\cdots) \end{bmatrix} \frac{p V_r^2 S}{2m} + \begin{pmatrix} T(\frac{T_{max}}{100\%}) \\ 0 \end{pmatrix} + R_{BE}(\Theta) \begin{pmatrix} 0 \\ 0 \end{pmatrix} - (\omega \times V) (2.77)
\]
The drawback of this formulation in non-zero wind is that the air-relative quantities (inside the aerodynamics and $\mathbf{R}_{BM}$) are unwieldy expressions of $\Theta, \mathbf{V},$ and $\mathbf{W}$

$$V_r = \left( (u + W_D s \theta - W_N c \psi c \theta - W_E c \theta s \psi)^2 
+ (v - W_E (c \phi c \psi + s \phi s \psi \theta) + W_N (c \phi s \psi - c \psi s \phi \theta) - W_D c \theta s \phi)^2 
+ (w + W_E (c \psi s \phi - c \phi s \psi \theta) - W_N (s \phi s \psi + c \phi c \psi \theta) - W_D c \phi c \theta)^2 \right)^{\frac{1}{2}} \tag{2.78}$$

$$\beta = \arcsin \left( \frac{v - W_E (c \phi c \psi + s \phi s \psi \theta) + W_N (c \phi s \psi - c \psi s \phi \theta) - W_D c \theta s \phi}{V_r} \right) \tag{2.79}$$

$$\alpha = \arctan \left( \frac{w + W_E (c \psi s \phi - c \phi s \psi \theta) - W_N (s \phi s \psi + c \phi c \psi \theta) - W_D c \phi c \theta}{u + W_D s \theta - W_N c \psi c \theta - W_E c \theta s \psi} \right) \tag{2.80}$$

where shorthand notation $s, c$ is utilized for the $\sin(\cdot)$ and $\cos(\cdot)$ functions.

**Cartesian Air-relative Velocity** Another alternate form used often in constant-wind situations is to use the Cartesian form of the air-relative velocity vector $\mathbf{V}_r$ in the state vector. The translational dynamics are expressed in equation (2.24) and the air-relative angles can be found from their definitions in equations (2.11). We note that this formulation is preferred for body-frame aerodynamic formulations which depend on air-relative velocities, as is the case in [139].
Chapter 3

Experimental Dynamic Model Identification

To complement other chapters' theory and simulation contributions, this chapter presents experimental methodology and a tutorial-style procedure to obtain a flight dynamic model (FDM) for a specific small low-cost fixed-wing unmanned aircraft. This work can be found in published journal-article form in [139] and many additional details are available in Simmons' M.S. Thesis [138]. The most significant research contribution of [139] is improving the fidelity of the identified FDM by both using a vortex lattice method to supplement data-driven identification methods as well as synthesizing data from distinct types of flight maneuvers. The UAS of interest is the HobbyKing™ Bix3 aircraft [64] into which we have installed a PixHawk [98] flight control system running the ArduPilot [5, 6] software. Figure 3.1 is a picture of the author holding the UAS. It is worth noting that no aerodynamic angle sensors were utilized, and the airspeed measurement is low quality. The application of these procedures for the Bix3 are explicitly detailed in order that they might readily be adapted to other aircraft at a similar scale.

Figure 3.1: McClelland holding the Bix3 aircraft, the primary UAS for this dissertation’s experimental efforts
The collaborative work presented in [139] contains primary contributions from the first two authors, B. Simmons and H. McClelland. Simmons was primarily responsible for preliminary modeling and experiment design, flight test data processing, and the resulting parameter estimation procedure. McClelland was responsible for obtaining the SIDPAC software, configuring and flying the UAS platform, and collaborating about issues encountered during system identification. As an overview of our methodology, the parameter estimation strategy is based on the time-domain formulation of the output-error system identification (SysID) method using the nonlinear flight dynamic equations for a rigid aircraft with a nonlinear aerodynamic model. We also utilize a computational aerodynamics approach, the Vortex Lattice Method (VLM) implemented via XFLR5 software, to both design the flight experiments as well as improve the resulting longitudinal model quality. The result of combining the VLM approach with SysID techniques is a more accurate FDM than either technique produces in isolation. For the lateral-directional model, we observe no improvement from using VLM, but creating a sequence to integrate Dutch-Roll and Bank-to-Bank Roll maneuvers does improve the final model quality.

3.1 Motivation

In this dissertation, the purpose of an aircraft FDM is to improve model-based wind estimation. Briefly, we also note that aircraft system identification is useful for many other flight-related capabilities such as model-based risk assessment, control system design, flying qualities analysis, flight simulation, and validation of experimental results. System identification methods have been well developed for characterizing the flight dynamics of full-sized fixed-wing aircraft [70, 102]. Section 1.3 of this dissertation discusses many published works on the topic for both full-sized and small aircraft.

Model identification for small aircraft amplifies the presence of several key challenges compared to full-scale aircraft. One challenge is that due to their inexpensive nature, small UAS often utilize sensors which provide fewer and noisier measurements. Another challenge is that small aircraft are more susceptible to atmospheric disturbances. While this is exactly what makes them well suited as wind sensors — the goal of this dissertation’s efforts — it likewise degrades the assumption of no-wind during model identification experiments.

These factors reduce the quality of experimental data, which in turn limits the quality of the model that is obtained through system identification [70]. Despite the increased challenges of model identification for small UAS, the agility and lower risk associated with their small scale affords a larger flight envelope, which suggests applications that might benefit from nonlinear modeling over this larger flight regime. The mass and wing loading of the Bix3 are notably smaller than for aircraft considered in earlier discussions of fixed-wing UAS system identification [9, 31, 51, 113, 137]. Model identification from small perturbations is untenable due to the low signal-to-noise ratio (SNR) of the onboard sensors around the trim condition. Thus our SysID approach bypasses any methodology restricted to linear
modeling and instead seeks a nonlinear aerodynamic model valid over a large portion of the flight envelope.

Because physical principles are sufficient to generate the structure of the rigid body dynamic equations (See Section 2.2.2), aircraft system identification focuses on determining explicit expressions for the aerodynamic forces and moments in terms of the state and input variables.

### 3.2 Description of Methods

The order of steps used in the identification procedure is:

1. Physical aircraft measurements
2. Flight test experiment design
3. Flight testing (data collection)
4. Data post-processing
5. Model structure identification
6. Iterative parameter estimation
7. Model validation

Brief descriptions are provided here and further details are found in [138, 139]. First, the geometric quantities (e.g. wingspan, mean chord, center of gravity position, mass) were measured by hand. The moments of inertia were obtained using the compound pendulum method for all axes [9, 100]. Table 3.1 presents values for these parameters. Next, the XFLR5 program [29] was used to develop a preliminary linear aerodynamic model. By varying

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>$m$</td>
<td>1.202</td>
<td>kg</td>
</tr>
<tr>
<td>Mean aerodynamic chord</td>
<td>$\bar{c}$</td>
<td>0.188</td>
<td>m</td>
</tr>
<tr>
<td>Projected wing span</td>
<td>$b$</td>
<td>1.54</td>
<td>m</td>
</tr>
<tr>
<td>Projected wing area</td>
<td>$S$</td>
<td>0.285</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Roll moment of inertia</td>
<td>$I_x$</td>
<td>0.095</td>
<td>kg·m$^2$</td>
</tr>
<tr>
<td>Pitch moment of inertia</td>
<td>$I_y$</td>
<td>0.045</td>
<td>kg·m$^2$</td>
</tr>
<tr>
<td>Yaw moment of inertia</td>
<td>$I_z$</td>
<td>0.12</td>
<td>kg·m$^2$</td>
</tr>
<tr>
<td>Product of inertia</td>
<td>$I_{xx}$</td>
<td>$\sim0$</td>
<td>kg·m$^2$</td>
</tr>
</tbody>
</table>
selected parameter values (such as the center of gravity location or trim airspeed) within reasonable bounds, a preliminary set of aerodynamic parameter estimates were generated. These preliminary models guided the design of flight test experiments designed such that the aircraft’s significant modes would be well-excited by the pilot’s input. Because the estimated phugoid and spiral modes are too slow for observation, each flight test maneuver’s duration was specified to excite the remaining short period, Dutch roll, and roll modes. Table 3.2 presents the desired control inputs which were human-piloted using manual flight control to capture the aircraft’s “bare airframe” dynamic response.

Table 3.2: Control inputs used for the Bix3 system identification flight tests

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Control Input</th>
<th>Mode Frequency (Hz)</th>
<th>Time Step (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Period</td>
<td>Elevator Doublet</td>
<td>2.1 ± 0.5</td>
<td>0.24 ± 0.06</td>
</tr>
<tr>
<td>Dutch Roll</td>
<td>Rudder Doublet</td>
<td>0.75 ± 0.1</td>
<td>0.67 ± 0.1</td>
</tr>
<tr>
<td>Bank-to-Bank Roll</td>
<td>Aileron 1-2-1</td>
<td>–</td>
<td>Roll to 30°-60°</td>
</tr>
</tbody>
</table>

The flight testing was performed at the Kentland Experimental Aerial Systems (KEAS) Laboratory [158] in Blacksburg, VA in calm wind conditions. A metronome was utilized to communicate the maneuver timing to the pilot, and control inputs were performed at half and full control deflections of varying polarity. While 53 Short Period maneuvers, 75 Dutch Roll maneuvers and 87 Bank-to-Bank Roll maneuvers were executed in total, approximately half were discarded as being poor quality. As discussed in the conclusions in Section 3.4, some autopilot assistance would have significantly raised the percentage of retained maneuvers. Finally, the flight data was post-processed to reduce noise, synchronize time-stamps, and ensure kinematic consistency. The reader should note that this was a very important step [138, §3.2] despite the details being omitted in this dissertation.

### 3.3 System Identification Results

This section details the system identification methods to produce the Bix3 FDM (originally presented Section 2.4.3). Several choices which impact the identification are worth remark at this point. First, the explanatory variables in the aerodynamic model were non-dimensionalized by the nominal velocity $V_o = 12 \text{ m/s}$ rather than the instantaneous airspeed $V_r$. Also, the Cartesian velocity components $V = (u, v, w)^T$ were selected in lieu of the spherical representation $V_R = (V_r, \beta, \alpha)^T$ due to the absence of an aerodynamic angle sensor and the low quality of the airspeed measurement. The lateral-directional and longitudinal dynamics were explicitly decoupled by assumption. Also, the nominal thrust was absorbed into the aerodynamic model due to difficulty in identifying a reliable thrust model for the Bix3. Justifications for these choices are discussed in [139].

The aerodynamic model structure was determined by assuming a form polynomial in the
explanatory variables and then employing a stepwise regression method. We employed the equation-error [102, §6.4] SysID method, quantifying a model’s fit by the coefficient of determination, $R^2$. We also used the partial $F$ statistic and partial correlation to examine if each candidate term contributed significant value to justify its inclusion in the model structure. As seen in equation (2.69), the lateral-directional model structure included all 18 linear terms and 1 nonlinear term. The longitudinal model has 2 nonlinear terms, and 11 (of the 15 possible) linear terms.

### 3.3.1 Longitudinal Parameter Identification

Having determined the model structure, we advanced to using the output-error method [102, §6.2] to estimate the longitudinal parameter values from the Short Period flight maneuver. The equation-error results (from model structure selection) were used to obtain an initial guess for the output-error method, which takes much longer to compute. An important note: the axial acceleration $a_x$ was not included as an output in the method because it was dominated by noise. Because each maneuver produces a slightly different value for each parameter, the set of values (one for each maneuver) is treated as a distribution on the actual aircraft’s value. Note that this provides two separate ideas of “uncertainty:"

- For each parameter value, as computed from a single maneuver, its accompanying Cramér-Rao lower bound [103] gives an idea of that value’s quality across the single maneuver.

- For each parameter in the aerodynamic model structure, the distribution of values (one for each distinct maneuver) gives an idea of that parameter’s uncertainty for the aircraft.

After rejecting parameter values which either have abnormally large Cramér-Rao bounds, or are outliers compared to the values computed from other maneuvers, the distribution of possible values for each parameter are obtained. The decision to accept or reject each value was based on quantified justifications, see [138, §3.3.5] for details. The final value for each longitudinal parameter (as reported in Table 3.3) is the median across this distribution, and the uncertainty is calculated from the median absolute deviation (MAD) as computed across the set of Cramér-Rao bounds. The uncertainty for each value is quantified in Table 3.3 using a 95% confidence interval.

An innovative aspect of our approach is now described. If the output-error method is applied to compute values for all parameters in the model structure, a resulting set of values and uncertainties results. We term this the “Flight Test Data only” (FTD only) set of values and uncertainties accordingly. However, we discover that by fixing some of the parameter values to the estimates originally produced from XFLR5 via the Vortex Lattice Method (VLM), and using the output error method only for those which remain free, we obtain better models
Table 3.3: Longitudinal parameter estimates obtained by fixing the parameters $C_{xw}$, $C_{zw}$, $C_{zq}$ to their VLM values and then estimating the remaining terms from FTD

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Value</th>
<th>95% CI</th>
<th>Coef.</th>
<th>Value</th>
<th>95% CI</th>
<th>Coef.</th>
<th>Value</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{xo}$</td>
<td>+0.197</td>
<td>±0.195</td>
<td>$C_{zo}$</td>
<td>−0.179</td>
<td>±0.152</td>
<td>$C_{mo}$</td>
<td>+0.0134</td>
<td>±0.0149</td>
</tr>
<tr>
<td>$C_{xu}$</td>
<td>−0.156</td>
<td>±0.157</td>
<td>$C_{zw}$</td>
<td>−5.32</td>
<td>(VLM)</td>
<td>$C_{mw}$</td>
<td>−0.240</td>
<td>±0.195</td>
</tr>
<tr>
<td>$C_{xw}$</td>
<td>+0.297</td>
<td>(VLM)</td>
<td>$C_{zq}$</td>
<td>−8.20</td>
<td>(VLM)</td>
<td>$C_{mq}$</td>
<td>−4.49</td>
<td>±2.13</td>
</tr>
<tr>
<td>$C_{xw2}$</td>
<td>+0.960</td>
<td>±1.90</td>
<td>$C_{zq^2}$</td>
<td>−0.308</td>
<td>±0.351</td>
<td>$C_{m_\delta e}$</td>
<td>−0.364</td>
<td>±0.119</td>
</tr>
<tr>
<td>$C_{zq^2}$</td>
<td>+7.02</td>
<td>±4.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

overall. Again a stepwise regression approach, analogous to that used for determining the model structure, is utilized in determining which parameters to fix at their VLM values and which to identify from the output-error SysID method. To compare models — which are each the result of applying output-error after different parameters have been fixed to their VLM values — we use the average normalized root mean square error (ANRMSE) as well as the average normalized mean absolute error (ANMAE) as specified in [138, §6.3.3]. Note that these comparisons were done on a small set of validation maneuvers (7 total) which were excluded from the training set used in the output-error process.

The improvement gained by fixing some parameters (specifically $C_{xw}$, $C_{zw}$, and $C_{zq}$) to their VLM values and estimating the others from flight test data (FTD) is shown by Figure 3.2. As seen, the parameter estimates from FTD alone have larger uncertainty than those using VLM to initially fix some values. The uncertainty in estimates of parameters in $C_z$ and $C_m$ generally decreases while terms associated with $C_x$ are less affected by using VLM values. Recall that $C_x$ is the noisiest aerodynamic coefficient — this clearly correlates with large uncertainty in its parameter estimates. It is apparent that models created by combining FTD and VLM results are better able to predict the aerodynamic force and moment coefficients than the model created from FTD alone. The force coefficients $C_x$ and $C_z$ show the most improvement when VLM parameter estimates are incorporated. The pitching moment coefficient $C_m$ shows only marginal improvement.

In Section 5.3 of this dissertation we present figures demonstrating the fidelity of this best FDM’s predictions on the validation maneuvers where we compare with similar figures for flight data taken under various other circumstances.

### 3.3.2 Lateral-Directional Parameter Identification

The majority of the approach used in longitudinal parameter identification is also used to identify the lateral-directional half of the FDM. However, two distinct types of maneuvers (Dutch Roll and Bank-to-Bank Roll) provide information useful for identifying parameter values. While the Bank-to-Bank Roll maneuver is useful to identify the aileron parameters $C_{y\alpha a}$, $C_{\alpha\delta a}$, $C_{n\delta a}$ and the Dutch Roll maneuver is useful to identify the rudder parameters $C_{y\delta r}$,
3.3. System Identification Results

Figure 3.2: Longitudinal parameter estimates and uncertainty calculated using only FTD (blue) and from fixing $C_{xw}$, $C_{zw}$, $C_{zq}$ to VLM estimates before using FTD (red). (Reprinted from [139])

Both maneuvers produced estimates of the remaining lateral-directional parameters. As demonstrated in Figure 3.3, some values of these estimates (specifically $C_{np}$ and $C_{y}$) conflict. The phenomenon of conflicting parameter estimates is not uncommon [42]. It is one reason why such maneuvers are often performed in rapid succession, or even combined, so that all parameters may be estimated from this single maneuver [42, 70, 102]. Because combining the maneuvers is infeasible for the manually piloted Bix3, instead we developed a procedure to integrate information from the two different maneuvers in a single parameter estimation process. Overall, four output-error estimations are performed, two for each maneuver type, but at each step only some of the dependent variables’ values are retained, while the remaining parameters are re-identified in the next step. The procedure, developed through exhaustive analysis and outlined in Table 3.4, produces the best modeling results for the lateral-directional dynamics.

The lateral-directional parameter estimates obtained following our stepwise procedure are shown in Table 3.5. The median parameter estimate is shown with error bounds computed in the same manner as in the longitudinal case. The nonzero lateral force bias $C_{y0}$ accounts
for aerodynamic asymmetries and/or measurement system offsets.

Again, we utilized a stepwise regression process to investigate applying VLM values for some parameters to improve the model. However in contrast to our longitudinal findings, for the lateral-directional half no notable improvement was observed. Thus we recommend following the stepwise procedure to combine the separate maneuvers, and learning all parameter values directly from FTD.

Finally, for both longitudinal and lateral-directional dynamics, their performance was quantitatively validated via the goodness of fit (GOF) and Theil’s inequality coefficient (TIC) [149] metrics. Across the set of validation maneuvers (the subset of total maneuvers which were excluded from the training set) Tables 3.6 and 3.7 present the results. The high values of GOF and low values of TIC for all output quantities indicate good modeling performance.
3.4 Conclusions

As discussed, several factors contribute unique challenges to identifying flight dynamic models (FDMs) for small fixed-wing UAS. These include (i) low SNR due to inexpensive sensors, (ii) requiring large perturbations and nonlinear model structure(s), and (iii) increased difficulty in obtaining ideal maneuvers and undisturbed flight conditions. To improve system identification performance, we utilize a computational aerodynamic approach — the vortex lattice method (VLM) — for both flight testing experimental design and to supplement the output-error-based parameter identification process for the longitudinal dynamics. For the lateral-directional dynamics, we developed a procedure for combining distinct maneuver types (Bank-to-Bank Roll and Dutch Roll) in a sequential fashion to improve model fidelity.

One encountered difficulty is that the aerodynamic bias terms ($C_{x_0}$, $C_{z_0}$, $C_{m_o}$, $C_{y_o}$, $C_{l_o}$, and $C_{n_o}$) appear to vary from flight to flight. This variability could be attributed to flaws in the

<table>
<thead>
<tr>
<th>Step</th>
<th>Maneuver</th>
<th>Estimated Model Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bank-to-Bank Roll</td>
<td>$C_{y_o}$, $C_{y_p}$, $C_{y_r}$, $C_{l_o}$, $C_{n_o}$, $C_{n_{z_2}}$</td>
</tr>
<tr>
<td>2</td>
<td>Dutch Roll</td>
<td>$C_{l_p}$, $C_{n_p}$, $C_{n_r}$</td>
</tr>
<tr>
<td>3</td>
<td>Bank-to-Bank Roll</td>
<td>$C_{l_r}$, $C_{y_ba}$, $C_{l_ba}$, $C_{n_{ba}}$, $C_{y_o}$, $C_{l_o}$, $C_{n_o}$</td>
</tr>
<tr>
<td>4</td>
<td>Dutch Roll</td>
<td>$C_{y_{ba}}$, $C_{l_{ba}}$, $C_{n_{ba}}$, $C_{y_r}$, $C_{l_r}$, $C_{n_o}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Value</th>
<th>95% CI</th>
<th>Coef.</th>
<th>Value</th>
<th>95% CI</th>
<th>Coef.</th>
<th>Value</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{y_o}$</td>
<td>+0.0286</td>
<td>±0.0382</td>
<td>$C_{l_o}$</td>
<td>−3 × 10⁻³</td>
<td>±0.00832</td>
<td>$C_{n_o}$</td>
<td>−9 × 10⁻³</td>
<td>±0.00480</td>
</tr>
<tr>
<td>$C_{y_p}$</td>
<td>−0.251</td>
<td>±0.119</td>
<td>$C_{l_p}$</td>
<td>−0.0756</td>
<td>±0.0459</td>
<td>$C_{n_p}$</td>
<td>+0.0408</td>
<td>±0.0158</td>
</tr>
<tr>
<td>$C_{y_r}$</td>
<td>+0.170</td>
<td>±0.199</td>
<td>$C_{l_r}$</td>
<td>−0.319</td>
<td>±0.0933</td>
<td>$C_{n_r}$</td>
<td>−0.242</td>
<td>±0.150</td>
</tr>
<tr>
<td>$C_{y_{ba}}$</td>
<td>+0.350</td>
<td>±0.377</td>
<td>$C_{l_{ba}}$</td>
<td>+0.183</td>
<td>±0.126</td>
<td>$C_{n_{ba}}$</td>
<td>−0.166</td>
<td>±0.0998</td>
</tr>
<tr>
<td>$C_{y_{r}}$</td>
<td>+0.0157</td>
<td>±0.0482</td>
<td>$C_{l_{r}}$</td>
<td>−0.0117</td>
<td>±0.0220</td>
<td>$C_{n_{r}}$</td>
<td>−0.0618</td>
<td>±0.0177</td>
</tr>
</tbody>
</table>

| $C_{n_2}$ | +0.0126 | ±0.0269 |

<table>
<thead>
<tr>
<th>Coef.</th>
<th>Value</th>
<th>95% CI</th>
<th>Coef.</th>
<th>Value</th>
<th>95% CI</th>
<th>Coef.</th>
<th>Value</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{x_o}$</td>
<td>+0.963</td>
<td>±0.991</td>
<td>$C_{z_o}$</td>
<td>±0.864</td>
<td>±0.984</td>
<td>$C_{m_o}$</td>
<td>±0.940</td>
<td></td>
</tr>
<tr>
<td>$C_{y_o}$</td>
<td>+0.097</td>
<td>±0.086</td>
<td>$C_{l_o}$</td>
<td>±0.195</td>
<td>±0.063</td>
<td>$C_{n_o}$</td>
<td>±0.112</td>
<td></td>
</tr>
</tbody>
</table>
measurement system, where calibration of the measured parameters drifts between flights, or it could be caused by slight changes in mass distribution, e.g., when the battery is replaced between flights. In addition, slight changes in control surface trim settings between flights will affect the bias terms. The difficulty with estimating bias terms might be resolved by re-estimating these parameters in each flight, though this may be impractical. The phenomenon is not uncommon for full-scale aircraft [102], but its effect may be more pronounced at smaller scales.

Some areas for future development have become apparent during this research effort. First, significant improvements might be found by a more accurate characterization of thrust. Another obvious improvement might be found from obtaining a larger set of training maneuvers. A reasonable future development is to implement autopilot-assist techniques for performing higher-quality flight maneuvers. Under manual control by a ground-based pilot, obtaining straight-and-level pre-maneuver flight conditions was a challenge which can be eliminated via autopilot assistance. Likewise, it was common that the human pilot accidentally nudges the elevator when moving the ailerons, because the two inputs are physically on the same control stick. An assistance mode could easily reject the unintentional control surface input. Finally, the inclusion of an aerodynamic angle sensor, or the improvement of sensor quality during identification tests would likely improve the final model’s fidelity.
Chapter 4

Model-Incurred Wind Estimation Error

This chapter describes a methodology for demonstrating the airborne wind estimation error which is incurred by common simplifications of the flight dynamic model. The methodology is demonstrated by performing a Monte-Carlo study of reconstruction error as wind turbulence intensity is varied from “light” to “severe.” A number of theoretical predictions are both noted during the method development and observed in the resulting data.

A portion of this work containing only open-loop reconstructions was presented at the AIAA SciTech conference in January 2019 [96].

4.1 Motivation

To infer the wind from airborne data, modern estimation algorithms combine sensor measurements with dynamic models for an accurate reconstruction [18, 77, 126, 150]. Inexpensive small UAS rely on low cost (and therefore less accurate) sensors, which correspondingly degrade the accuracy of any reconstructed quantities. Because sensor error is likely the most significant source of reconstruction error, previous studies have explored its impact on estimate accuracy.

In contrast, this work considers the largely unstudied reconstruction error due to dynamic modeling assumptions. Since total reconstruction error is a combination of sensor error and modeling error, this makes our study complementary to all sensor-focused work. To summarize the methodology, a nonlinear flight dynamic model is simulated in a non-uniform wind field, and the unsensed states and disturbances are reconstructed from common simplified forms of the dynamic model. Because these reconstruction methods use exact-but-incomplete sensor information, the discrepancy between the reconstructed and original values is solely caused by the modeling simplifications. To demonstrate the methodology, this work presents details of the simulation environment which is used to compare model-based reconstruction errors as wind turbulence intensity is varied.
4.1.1 Literature Review

As a study of state and disturbance reconstruction for an aircraft during flight, this work has its roots in the established topics of dynamic systems and estimation \[26, 40, 142\], flight dynamic modeling \[35, 143\], system identification \[70, 102\] and the use of UAS for weather research \[32\]. Since a review of the dynamic systems and estimation literature is beyond this section’s scope, relevant wind reconstruction studies are presented to provide context for this work.

Employing an exact flight dynamic model, Langelaan et al. \[77\] analyze the wind reconstruction error caused by sensor inaccuracy. Conversely, this study assumes exact sensor data and quantifies the error caused by flight dynamic model inaccuracy, using a similar simulation environment. Borup et al. \[18\] derive a nonlinear observer for reconstructing the motion variables considered here, but in a constant (or slowly varying) wind field. Our work applies in turbulent wind fields which cannot be modeled as slowly varying. Additionally, Borup et al. specifically acknowledge that modeling errors give reconstruction errors \[18, §1.1\]; it is our goal to understand and quantify this unavoidable effect. A recent work by Rhudy et al. \[126\] compares, in simulation and experiment, several wind reconstruction approaches based on “available sensors.” Adopting that language, our work conversely seeks the impact of “available flight dynamic models” on the reconstruction accuracy. Tian and Chao \[150\] experimentally demonstrate 3d flow-angle and wind reconstruction utilizing a flight dynamic model inside the Kalman filter in lieu of a flow-angle sensor. While they demonstrate practical implementation, our methodology provides a theoretical foundation for examining error sources in this type of application. Practitioners typically select the Kalman filter covariance information to provide good performance. The methodology we demonstrate shows how modeling simplifications may be framed as emergent process noise, thus providing quantified justification for selecting covariance values in a filter implementation.

A large number of other experimental and simulation efforts in wind measurement have been published which may have similarly benefited from a quantified understanding of model-based reconstruction error \[21, 24, 82, 89, 116, 154, 162, 165\]. Finally a key thrust of this research effort is to help practitioners (like ourselves) determine whether to spend available resources on improving sensor capabilities for inexpensive UAS \[127, 160, 171\] or on obtaining more accurate flight dynamic models \[49, 50, 65\].

4.2 Methodology

A nonlinear flight dynamic model is simulated in a turbulent wind field. This original flight dynamic model is treated as the “truth” model, and the realization of the turbulent wind field is the “true” disturbance. To demonstrate reconstruction error stemming solely from dynamic modeling, each reconstruction algorithm utilizes error-free state information but a
simplified dynamic model. This reflects an assumption of perfect sensors, and isolates the source of any reconstruction error as the modeling simplification.

4.2.1 Longitudinal Flight Dynamic Model

Only the longitudinal portion of Chapter 2’s flight dynamic model (FDM) is utilized in this work. To simplify notation, the aircraft is assumed to fly North at all times, thus the traditional “aircraft-forward” direction is synonymous with “North” in this chapter. In order to remove the lateral-directional dynamics, plug in zeros for lateral-directional (aileron and rudder) inputs

\[
\delta_a = \delta_r = 0
\]

and disturbance elements containing East (lateral-directional) dependence

\[
W_E = \frac{\partial W_E}{\partial N} = \frac{\partial W_E}{\partial E} = \frac{\partial W_N}{\partial E} = \frac{\partial W_D}{\partial E} = \frac{\partial W_E}{\partial t} = 0
\]

Additionally, choose initial conditions corresponding to wings-level Northbound flight

\[
X_E(0) = \phi(0) = \psi(0) = v(0) = p(0) = r(0) = \beta(0) = 0
\]

Making these substitutions to equations (2.75) and (2.76) shows that all lateral-directional accelerations are zero, and thus the lateral-directional variables remain at their initial (zeroed) values.

For the remainder of this chapter, a number of vectors are re-defined to only contain longitudinal components. These include the state vector \(x\), the position vector \(X\), the velocity vector \(V_R\), the input vector \(u\), the wind vector \(W\), and the disturbance vector \(d\) per the following equations

\[
x = \begin{bmatrix} X_N \\ X_D \\ \theta \\ V_r \\ \alpha \\ q \end{bmatrix}
= \begin{bmatrix} X' \\ \theta \\ V_R \\ q \end{bmatrix}
\]

\[
u = \begin{bmatrix} T_e \\ \delta_e \end{bmatrix}
\]

\[
W = \begin{bmatrix} W_N \\ W_D \end{bmatrix}
\]

\[
d = \begin{bmatrix} W^T, \frac{\partial W^T}{\partial X_N}, \frac{\partial W^T}{\partial X_D}, \frac{\partial W^T}{\partial t} \end{bmatrix}^T
\]
The longitudinal translational kinematics are
\[
\dot{X} = V_r + W = V_r \left( \cos(\alpha - \theta) \right) + \left( \begin{array}{c} W_N \\ W_D \end{array} \right) \tag{4.8}
\]
and the rotational kinematics are simply
\[
\dot{\theta} = q \tag{4.9}
\]
The convected derivative of the wind field \( W(X, t) \) along the flight path of the aircraft’s center of mass \( X(t) \) is
\[
\frac{DW}{Dt} = \left( \begin{array}{c} \frac{DW_N}{Dt} \\ \frac{DW_D}{Dt} \end{array} \right) = \left( \begin{array}{c} \frac{\partial W_N}{\partial t} + \frac{\partial W_N}{\partial X} \dot{X} \\ \frac{\partial W_D}{\partial t} + \frac{\partial W_D}{\partial X} \dot{X} \end{array} \right) \tag{4.10}
\]
The longitudinal rigid-body dynamic equations for translation become
\[
\left( \begin{array}{c} \dot{V}_r \\ (\dot{\alpha} - q)V_r \end{array} \right) = -\rho Sv_r^2 \left( \begin{array}{c} C_D(\ldots) \\ C_L(\ldots) \end{array} \right) + g \left( \begin{array}{c} \sin(\alpha - \theta) \\ \cos(\alpha - \theta) \end{array} \right) + \frac{T\%}{100\%} \left( \begin{array}{c} \cos \alpha \\ \sin \alpha \end{array} \right) - \left[ \begin{array}{cc} \cos(\alpha - \theta) & \sin(\alpha - \theta) \\ \sin(\alpha - \theta) & -\cos(\alpha - \theta) \end{array} \right] \frac{DW}{Dt} \tag{4.11}
\]
and the longitudinal rotational dynamic equation is
\[
\dot{q} = \frac{\rho \bar{c} Sv_r^2}{2I_{yy}} C_m(\ldots) \tag{4.12}
\]
Recall that the aerodynamic coefficients \( C_D(\ldots) \), \( C_L(\ldots) \), and \( C_m(\ldots) \) are written with ellipses to indicate that they are functions of other variables per equation (2.67). Inside these aerodynamic equations, the Frost model of wind angular velocity (detailed in Section 2.3.2) is used. The quantity \( \tilde{q}_r \) is obtained from combining the longitudinal part of equations (2.56), (2.57), and (2.68) to become
\[
\tilde{q}_r = \frac{\bar{c}}{2V_r} \left( q - \frac{1}{2} \left( \frac{\partial W_D}{\partial X_N} - \frac{\partial W_N}{\partial X_D} \right) \right) \tag{4.13}
\]
for substitution into the aerodynamic expressions.
Equations (4.4) through (4.13) constitute the reasonably sophisticated longitudinal flight dynamic model which is named the “true” flight dynamic model (TRU) for this chapter
\[
\dot{x} = f_{\text{TRU}}(x, u, d) \tag{4.14}
\]
4.2. Methodology

4.2.2 Simplifications of the Flight Dynamic Model

Next, three simplifications to the longitudinal flight dynamic model are presented: linearizing about a trim point (LIN), neglecting the angular velocity of the wind (NAW), and the combination of neglecting the angular wind and then linearizing (NAWLIN).

**Linearizing the Dynamic Equations (LIN)** The first simplification is linearizing the TRU dynamic equation \((\ref{eq:trudyn})\) about an equilibrium-flight trim point \((x^t, u^t, d^t)\). This is a common simplification for flight dynamic models; Etkin (§5.10 \([35]\)) states that linearization “has been used with enormous success ever since the beginnings of this subject.” Briefly, the linearization is obtained from the Taylor series expansion\(^1\)

\[
\dot{x} = f_{\text{TRU}}(x^t, u^t, d^t) + \left[ \frac{\partial f_{\text{TRU}}}{\partial x} \right]_A (x - x^t) + \left[ \frac{\partial f_{\text{TRU}}}{\partial u} \right]_B (u - u^t) + \left[ \frac{\partial f_{\text{TRU}}}{\partial d} \right]_G (d - d^t) + \text{(higher order terms)} \tag{4.15}
\]

First, since the trim solution is an equilibrium flight condition, one realizes that

\[
f_{\text{TRU}}(x^t, u^t, d^t) = 0 \tag{4.16}
\]

Then, neglecting the higher order terms yields the standard explicit form of the linearized dynamic model

\[
\dot{x} = A(x - x^t) + B(u - u^t) + G(d - d^t) \tag{4.17}
\]

where \(A\) is the system matrix, \(B\) is the input matrix, and \(G\) is the disturbance matrix.

**Neglecting Angular Wind (NAW)** Another common simplification of the TRU flight dynamic model is neglecting the apparent angular velocity of the wind as experienced by the aircraft. Inside the aerodynamic equations if one “neglects the angular wind” by assuming that \(q_w = 0\) the result is

\[
q_{r,\text{NAW}} = q \tag{4.18}
\]

This approximation is common in flight dynamic modeling literature, often the result of assuming the wind field is irrotational or contains only low frequency components. Specific examples can be found in MIL-HDBK-1797 \([3, \S 4.9.3]\), in Borup et al. \([18, \text{eq. (12)}]\), and in Rhudy et al. \([124, \text{eq. (13), (16)}]\).

\(^1\)The kinematic equations are omitted from the linearization process, but for the sake of brevity, we have not introduced additional notation to emphasize this fact.
Replacing \( q_r \) with \( q \) inside the \( C_D, C_L, \) and \( C_m \) expressions removes their dependence on the spatial wind gradient \( \frac{\partial W}{\partial X} \). The dynamic equations under this simplification are written as

\[
\dot{x} = f_{\text{NAW}}(x, u, d) \tag{4.19}
\]

Notice that the NAW dynamics are still dependent on the wind gradient through the \( \frac{DW}{Dt} \) term in equation (4.11), so wind-gradient reconstructions are still possible for the NAW model.

**Remark 4.1.** When reconstructing wind-gradient components from the NAW model, more accurate reconstructions are expected for \( \frac{\partial W_N}{\partial X_N} \) than for \( \frac{\partial W_D}{\partial X_N} \). The influence of the former term on the aerodynamics is unchanged by the simplification, but the latter term is entirely neglected. This effect is observed in the numerical results of Section 4.3.

**Both Neglecting Angular Wind and then Linearizing (NAWLIN)** A third simplified flight dynamic model for analysis is the composition of the previous two assumptions: A linearized version of the NAW flight dynamic model. Briefly, it is written

\[
\dot{x} = \left[ \frac{\partial f_{\text{NAW}}}{\partial x} \right]^\dagger (x - x^\dagger) + \left[ \frac{\partial f_{\text{NAW}}}{\partial u} \right]^\dagger (u - u^\dagger) + \left[ \frac{\partial f_{\text{NAW}}}{\partial d} \right]^\dagger (d - d^\dagger) \right) f_{\text{NAWLote}}(x, u, d) \tag{4.20}
\]

**Remark 4.2.** Since the NAWLIN model is a combination of the NAW and LIN assumptions, the reconstruction error is expected to compound the error of the two respective simplifications. Indeed, this is observed in the numerical results of Section 4.3.

**4.2.3 Reconstructing the Wind**

By combining a simplified version of the flight dynamic model with available sensor information along a simulated flight in wind, a reconstruction of the wind may be calculated. To isolate wind reconstruction error caused solely by the mathematical simplifications of the dynamic model, all “sensed” states are known exactly to the reconstruction methods. While perfect sensing might seem to be a major simplification, the wind reconstruction is not trivial because not all of the states are sensed, and therefore these unsensed states must be reconstructed as well. To proceed, the general wind reconstruction problem is mathematically posed, and then necessary details for reconstructing from each of the modeling simplifications are presented.

**General Wind Reconstruction Problem Statement**

To begin, partition the elements of the state vector \( x \) into a sensed group \( x_s \) and unsensed group \( x_u \). Since the “truth” state trajectory \( x(t) \) is generated by forward simulation of the
4.2. Methodology

TRU flight dynamic model (4.14) from initial time \(t_0\) to final time \(t_f\), the dynamic model \(f_{\text{TRU}}(\cdot)\) can be interpreted as a constraint on the states \((x_s, x_u)\), their derivatives \((\dot{x}_s, \dot{x}_u)\), the input \(u\) and the “true” disturbance \(d\). Reordering vectors \(x, \dot{x}, f\) and writing \(f_{\text{TRU}}(\cdot)\) as the equivalent function of 4 arguments yields

\[
0 = \left(\begin{array}{c} \dot{x}_s \\ \dot{x}_u \end{array}\right) - f_{\text{TRU}}\left(x_s(t), x_u(t), u(t), d(t)\right) \quad \forall t \in [t_0, t_f] \tag{4.21}
\]

This constraint form is used in the general statement of the wind reconstruction problem.

**Wind Reconstruction Problem**

- **Given** a trajectory of (perfectly sensed) state information \(x_s(t)\) and inputs \(u(t)\) and

- **Given** a (simplified) flight dynamic model \(f_{\text{SIMP}}(\cdot)\),

- **Solve** for the reconstructions of the unsensed states \(\hat{x}_u(t)\), derivatives \(\dot{\hat{x}}_u(t)\), and disturbances \(\hat{d}(t)\) such that

\[
0 = \left(\begin{array}{c} \dot{x}_s \\ \dot{\hat{x}}_u \end{array}\right) - f_{\text{SIMP}}\left(x_s(t), \hat{x}_u(t), u(t), \hat{d}(t)\right) \quad \forall t \in [t_0, t_f] \tag{4.22}
\]

Individual algorithms for reconstructing the wind depend on the mathematical form of the flight dynamic model and which states are sensed/unsensed. These implementation details are presented next for each of the simplified models, LIN, NAW, and NAWLIN.

**Remark 4.3.** This work’s aim is not to propose a strategy for implementing model-based wind reconstruction in practice, but rather to assess the impact of flight dynamic model simplifications on reconstruction accuracy. In application, where sensors would never be perfectly accurate, one would instead use an appropriate filter/smoothing algorithm [26, 40, 142] to calculate the reconstructions.

**Model-Specific Reconstruction Details**

Since inexpensive UAS often lack accurate directional air data systems, this work takes the angle of attack as the (scalar) unsensed state

\[
x_u = \alpha \tag{4.23}
\]

The remaining state components are sensed without error

\[
x_s = \left(X_N \ X_D \ \theta \ V_r \ q\right)^T \tag{4.24}
\]
Because these states are sensed perfectly, their time derivatives $\dot{x}$ are also known. To simplify notation, the sensor/input information is expressed as a single vector of known quantities

$$\mathbf{z}(t) = \left( \dot{x}_s^T(t) \; \mathbf{x}_s^T(t) \; \mathbf{u}_s^T(t) \right)^T \quad \forall \ t \in [t_0, t_f]$$

(4.25)

Additionally, two important assumptions are made about the disturbance vector $\mathbf{d}$:

1. The wind field is “frozen” (Taylor’s hypothesis [144, §1.4])

$$\frac{\partial \mathbf{W}}{\partial t} = \left( \frac{\partial W_N}{\partial t} \; \frac{\partial W_D}{\partial t} \right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(4.26)

2. The wind field has no variation with altitude

$$\frac{\partial \mathbf{W}}{\partial X_D} = \left( \frac{\partial W_N}{\partial X_D} \; \frac{\partial W_D}{\partial X_D} \right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

(4.27)

Thus the remaining disturbance quantities to be reconstructed are

$$\hat{\mathbf{d}} = \begin{pmatrix} \hat{W}_N \\ \hat{W}_D \\ \frac{\partial \hat{W}_N}{\partial X_N} \\ \frac{\partial \hat{W}_D}{\partial X_N} \end{pmatrix}^T$$

(4.28)

Figure 4.1 summarizes the reconstruction process. The first step is to reconstruct the angle of attack estimate $\hat{\alpha}$. The implementation details depend on which simplified model form (LIN, NAW, or NAWLIN) is being used. These details are explained in each model-specific paragraph below. After reconstructing $\hat{\alpha}$, there are two distinct approaches for reconstructing $\hat{\mathbf{d}}$, named the “kinematic” and “dynamic” reconstructions. In the context of this work, these are competing alternatives for calculating $\hat{\mathbf{d}}$ estimates, but for model-based estimation in practice an appropriate filtering algorithm would combine them to improve reconstruction accuracy.

For the kinematic (KIN) approach, the estimate $\hat{\alpha}$ is combined with known information $(\mathbf{X}, \theta, V_r)$ in the translational kinematics (4.8) to reconstruct the wind $\hat{\mathbf{W}}_{\text{KIN}}$. Then, due to the assumptions of frozen wind (4.26) and no altitudinal variation of wind (4.27), the kinematic wind estimate can be differentiated and combined with $\mathbf{X}$ in equation (4.10) to compute the remaining wind gradient $\frac{\partial \mathbf{W}}{\partial X_N}$.\text{KIN}.

**Remark 4.4.** Since this approach utilizes perfect sensing for airspeed $V_r$, inertial velocity $\dot{\mathbf{X}}$, and pitch angle $\theta$, the kinematic reconstruction of the wind gradient $\frac{\partial \mathbf{W}}{\partial X_N}$ is expected to be highly (unrealistically) accurate. In practical situations, differentiation of the noisy estimate $\hat{\mathbf{W}}_{\text{KIN}}$ would significantly magnify error. The box is dashed in the reconstruction flowchart to remind the reader that this reconstruction’s accuracy would degrade significantly compared to the ideal circumstances of this work. For completeness, comparative plots of the kinematic and dynamic wind-gradient reconstruction results are provided by Figures 4.25 and 4.26 in Section 4.3.5.
4.2. Methodology

"Kinematic" reconstructions

\[ \dot{W}_\text{KIN}(t) \]

from translational kinematics

\[ \frac{\partial \dot{W}}{\partial R_N \text{ KIN}}(t) \]

from \( \frac{D}{Dt}\dot{\mathbf{W}}/R_N \)

"Dynamic" reconstructions

\[ \frac{\partial \dot{W}}{\partial R_N \text{ DYN}}(t) \]

from \( \dot{V}_r, \dot{q} \) dynamic equations

\[ \dot{W}_\text{DYN}(t) \]

from \( \int^t \frac{\partial \dot{W}}{\partial R_N} \left( \tau \right) R_N(R_N(\tau)) \, d\tau \)

Figure 4.1: A flowchart depicting the "kinematic" (top) and "dynamic" (bottom) approaches for reconstruction. (Reprinted from [95])

For the dynamic (DYN) approach, the estimate \( \dot{\alpha} \) is combined with other perfectly sensed information \( z \) in the airspeed and pitch rate components of equation (4.22) to reconstruct the wind gradient \( \frac{\partial \dot{W}}{\partial X_N \text{ DYN}} \). In this approach, note that the (simplified) dynamic equations are used two separate times, which is expected to create additional model-based reconstruction error. To finish, a corresponding wind estimate \( \dot{W}_\text{DYN} \) can be computed from integrating equation (4.10).

Remark 4.5. Notice that the dynamic wind estimate \( \dot{W}_\text{DYN} \) is calculated by integrating the (erroneous) dynamic gradient reconstruction. The gradient’s error gets accumulated during integration, and thus the dynamic wind estimate drifts away from the truth as time progresses. In practice, this would not be used as a stand-alone estimation technique, but rather as a complement to the kinematic reconstruction over short time intervals. To remind the reader of this, the reconstruction box is dashed. Figures 4.23 and 4.24 in Section 4.3.5 include comparative plots, where the dynamic wind reconstruction error is observed to be at least an order of magnitude larger than the kinematic.

LIN Specific Reconstruction Details For the linearized version of the flight dynamic model (LIN), an additional condition is necessary for reconstruction:

1. The trim point values of the wind’s spatial gradients are all zero.

\[
\frac{\partial \dot{W}^\dagger}{\partial X} = \begin{bmatrix}
\frac{\partial W_N}{\partial X_N} & \frac{\partial W_N}{\partial X_D} \\
\frac{\partial W_D}{\partial X_N} & \frac{\partial W_D}{\partial X_D}
\end{bmatrix} = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\] (4.29)

Under this condition, the three non-trivial dynamic equations from equation (4.17) form a system of differential-algebraic equations (DAEs). Recall that derivatives \( \dot{V}_r(t) \) and \( \dot{q}(t) \) are
known to the reconstruction calculations from perfect sensing of \( V_r(t) \) and \( q(t) \). The system of equations which is differential in \( \dot{\alpha}(t) \), algebraic in \( \frac{\partial \dot{W}}{\partial X} \), and parameterized by \( z \) is

\[
\begin{pmatrix}
0 \\
\dot{\alpha} \\
0
\end{pmatrix} = \begin{pmatrix}
f_{V_r,\text{LIN}}(\dot{\alpha}, \frac{\partial \dot{W}}{\partial X}, \frac{\partial \dot{W}}{\partial X}, z) - \dot{V}_r \\
f_{\alpha,\text{LIN}}(\dot{\alpha}, \frac{\partial \dot{W}}{\partial X}, z) \\
f_{q,\text{LIN}}(\dot{\alpha}, \frac{\partial \dot{W}}{\partial X}, z) - \dot{q}
\end{pmatrix}
\] (4.30)

The key to solving this DAE system is that the second and third equations can be combined into a single linear ordinary differential equation (ODE) of the form

\[
\dot{\alpha} = C_1(z) \dot{\alpha} + C_0(z)
\] (4.31)

This ODE is solved by providing an initial condition \( \dot{\alpha}(t_0) \) and numerically integrating in time\(^2\). The solution \( \dot{\alpha}_{\text{LIN}} \) can then be used to produce both kinematic (KIN) and dynamic (DYN) reconstructions of \( \dot{W}_{\text{LIN}} \) and \( \frac{\partial W}{\partial X}_{\text{LIN}} \).

**NAW Specific Reconstruction Details** For the No Angular Wind (NAW) model, a simpler reconstruction computation exists. Since the NAW dynamic equation for \( \dot{q} = f_{q,\text{NAW}}(\cdot) \) inside equation (4.19) does not depend on any elements of \( d \) nor on \( \dot{\alpha} \), solving it for \( \dot{\alpha}_{\text{NAW}}(t) \) is reduced to finding zeros of a nonlinear function

\[
\dot{\alpha}_{\text{NAW}}(t) = \alpha(t) \text{ such that } f_{q,\text{NAW}}(\alpha(t), z(t)) - \dot{q}(t) = 0 \quad \forall \ t \in [t_0, t_f]
\] (4.32)

which is purely algebraic, rather than differential-algebraic. Further, if the aerodynamic moment for \( C_m(\cdot) \) is polynomial in \( \alpha \) as in equation (2.67), solving for \( \dot{\alpha}_{\text{NAW}}(t) \) reduces to finding the zeros of a polynomial, for which efficient numerical methods exist. For the DYN reconstruction, differentiating the result (numerically) in time provides the corresponding \( \dot{\alpha}_{\text{NAW}}(t) \). The remaining NAW dynamic equations for \( \dot{V}_r \) and \( \dot{\alpha} \) inside equation (4.19) are affine in \( \frac{\partial W}{\partial X} \), thus they can be rearranged analytically to obtain explicit reconstruction equations

\[
\frac{\partial \dot{W}}{\partial X}_{\text{NAW}} = \left( \frac{\partial \dot{W}}{\partial X}, \frac{\partial \dot{W}}{\partial X} \right)_{\text{NAW}} = \frac{\partial W}{\partial X}_{\text{NAW}}(\dot{\alpha}, \dot{\alpha}, z)
\] (4.33)

The KIN reconstruction proceeds as usual after obtaining \( \dot{\alpha}_{\text{NAW}}(t) \).

**NAWLIN Specific Reconstruction Details** For the flight dynamic model where the NAW model is subsequently linearized, the \( \dot{q} = f_{q,\text{NAWLIN}}(\alpha, z) \) dynamic equation becomes affine in \( \alpha \) and can therefore be rearranged analytically to the form

\[
\dot{\alpha}_{\text{NAWLIN}} = a_{\text{NAWLIN}}(z)
\] (4.34)

\(^2\)If the equation is numerically unstable because \( C_1 \) is positive, instead provide a final condition \( \dot{\alpha}(t_f) \) and integrate backwards in time.
4.3 Simulation Details & Results

From there, completing the DYN reconstruction proceeds identically to the NAW model: differentiate $\dot{\delta_{\text{NAWLIN}}}$ to get $\dot{\delta_{\text{NAWLIN}}}$ and then solve analytical expressions for $\frac{\partial W}{\partial X_{\text{NAWLIN}}}$ from inverting the NAWLIN dynamics for $\dot{V}_r$ and $\dot{\alpha}$. The KIN reconstruction requires no special modification.

4.3 Simulation Details & Results

A MATLAB simulation is used to demonstrate the wind reconstruction error incurred from the flight dynamic model simplifications (NAW, LIN, and NAWLIN) detailed above. MATLAB’s Symbolics Toolbox [148] is used to generate the equations of motion and wind reconstruction, and MATLAB’s numerical methods are used for simulation and wind reconstruction. This section describes the specific details of the simulation environment, presents an example of a typical simulation in detail, and then discusses Monte Carlo results quantifying how wind reconstruction errors from modeling assumptions vary with wind turbulence magnitudes.

4.3.1 Simulation Details

In the simulation environment, the T-2 Generic Transport Model aircraft parameters for Grauer and Morelli’s [49] GGA model are used. To give a sense of the aircraft’s scale, some key physical quantities are

$$m = 22.5 \text{ kg} \quad b = 2.09 \text{ m} \quad \bar{c} = 0.28 \text{ m}$$

The remaining parameters are available in [49]. The model is flown near its minimum-thrust trim point

$$V_r^\dagger = 42.268 \text{ m/s} \quad \alpha^\dagger = 3.667 \text{ deg} \quad \delta_e^\dagger = 2.177 \text{ deg} \quad T^\dagger = 17.465 \text{ N}$$

The simulated aircraft is initialized at

$$X(t_0) = (0, 0)^T \text{ m}, \quad V_r(t_0) = V_r^\dagger \quad \alpha(t_0) = \theta(t_0) = \alpha^\dagger \quad q(t_0) = \dot{\theta}(t_0) = 0 \text{ deg/s}$$

Two separate flight types are simulated: open-loop flight and feedback-controlled flight using a PID-based controller [109] tuned for disturbance rejection. In the open-loop simulations, the controls $\mathbf{u} = (T, \delta_e)^T$ remain at their trim values for the entire simulation, and the aircraft is perturbed during flight by a realization of the random wind field with prescribed spectral properties. Since the T-2 aircraft is dynamically stable and the perturbations do not drive the dynamic states outside the basin of attraction, the aircraft state stays in a region in which the GGA model remains valid. Reconstructions from open-loop flight data are labeled with the descriptor “OL.” For feedback-controlled flight, a “successive loop closure”
PID-based control scheme [15] is also implemented. The details of the control scheme are presented Section 4.3.6. Reconstructions from this closed-loop flying data are labeled with the descriptor “CL.”

To generate a wind field realization, the high-altitude Dryden wind model described by §4.9.2 of Appendix A in MIL-HDBK-1797 [3] is used. The horizontal and vertical turbulence spectra equations in spatial frequency \( \Omega \) are reproduced here

\[
\Phi_N(\Omega) = \sigma^2 \frac{2L_N}{\pi} \frac{1}{1 + (L_N \Omega)^2} \quad \Phi_D(\Omega) = \sigma^2 \frac{2L_D}{\pi} \frac{1 + 12(L_D \Omega)^2}{[1 + 4(L_D \Omega)^2]^2}
\]

with \( L_N = 533.4 \) m and \( L_D = 266.7 \) m as specified in §4.9.2.2.a. The wind field varies spatially but is temporally frozen. The turbulence intensity value \( \sigma \) is chosen between 0 and 8 m/s. Figure 262 of [3] provides guidance on how to classify this range of \( \sigma \) values into “light,” “moderate,” and “severe” turbulence.

To generate a continuous turbulence realization, the process suggested by §4.4 of Lloyd [92] for generating random waves as a sum of sinusoids is used: First, partition the frequency spectrum into a bounded interval of the lowest frequencies containing 99% of the spectral energy and the remaining open-ended 1% high-frequency tail. To obtain the actual realization frequencies, discard the tail and randomly select \( N = 1000 \) frequency values from a uniform distribution on the 99% frequency range. For each of these \( N = 1000 \) frequencies, randomly select a phase shift from a uniform distribution over \([0, 2\pi)\) and calculate that sinusoid’s magnitude from the spectral density function. Summing these \( N = 1000 \) sinusoids together results in the continuous frozen Dryden turbulence realization. A constant \( W_N = -5 \) m/s headwind is added to the sinusoids, completing the continuous wind field realization.

Though Lloyd’s method is perfectly legitimate, future work might consider a modern alternative: the Lánczos method as presented by Grauer [46].

To give the reader a clearer sense of open-loop (OL) and closed-loop (CL) flights in the Dryden wind field, the results and reconstructions from one typical simulation are presented in detail.

### 4.3.2 An Example Simulation

Figures 4.2–4.5 present true values of flight quantities for one example simulation. To begin, Figure 4.2 shows the forward position \( X_N \) and altitude \( -X_D \) histories for a typical 100 s simulation in moderate turbulence (\( \sigma = 3 \) m/s). In both the open-loop and closed-loop flights, the aircraft makes forward progress over the ground at \( \dot{X}_N \simeq V_r + W_N \simeq 37 \) m/s as expected. For the vertical direction, the wind drives the aircraft \(~170\) m above its initial altitude during open-loop flight. The wind disturbance excites the aircraft’s phugoid mode

\[^3\text{Note that in the Monte Carlo study of Section 4.3.4, each simulation is 200 s.}\]
which has a period of 20.1 s. In the closed-loop flight, the controller setpoint is the initial altitude \(X_D = 0\), and the aircraft remains within 30 m of this value. Figure 4.3 shows the wind \(W\) along the flight path, which is a realization of the Dryden wind model with a \(W_N = -5\) m/s average horizontal component. The slight difference in the OL and CL plots is due to the small horizontal position difference during the OL and CL flights. Notice that the updraft \((-W_D)\) in this example is more positive than negative; this corresponds to the OL aircraft being blown above its initial altitude as observed in Figure 4.2. The corresponding wind gradient plots are included for the interested reader in Figure 4.14 of Section 4.3.5. Figure 4.4 shows the control inputs during this simulated flight. The OL inputs are constant, as expected. In CL flight, the airspeed PID controller regulates the thrust (expressed as a percentage of the maximum thrust) during the simulation, generally keeping it below the open-loop value. The elevator deflection angle is also PID-regulated.
Chapter 4. Model-Incurred Wind Estimation Error

Figure 4.4: Thrust percent and elevator deflection $\delta_e$ control inputs during the example simulation. (Reprinted from [95])

within two degrees around its trim value, as expected. Figure 4.5 shows two of the dynamic

Figure 4.5: Air-relative velocity $V_r$ and angle of attack $\alpha$ during the example simulation. (Reprinted from [95])

state histories $(V_r(t), \alpha(t))$. The aircraft’s phugoid mode response is observed in the open-loop airspeed plot, as expected. As predicted by the aircraft’s open-loop dynamic stability, the OL states do not diverge from the initial condition, which is also the trim condition, despite being continuously affected by the wind disturbance. For the CL flight, the controller regulates these states in order to track its altitude, airspeed, and pitch-angle setpoints. As expected, the CL airspeed remains closer to its setpoint than the OL. In contrast, the CL drives angle of attack further away from its trim value than the OL in order to regulate aircraft altitude.

To complete the full set of plots, the final two states $(\theta(t), q(t))$ are provided in Section 4.3.5 as Figure 4.15.
4.3.3 Reconstruction Errors for the Example Simulation

The reconstruction error for some generic quantity \( y \) is denoted as \( \Delta y \). It is computed as the difference between the true value \( y \) and the reconstructed estimate \( \hat{y} \) from some simplified model

\[
\Delta y(t) = y(t) - \hat{y}(t) \tag{4.35}
\]

To quantify reconstruction error \( \Delta y(t) \) with a single measure over the simulation duration, the root mean square (rms) value is used

\[
\Delta y_{\text{rms}} = \sqrt{\frac{1}{t_f - t_0} \int_{t_0}^{t_f} [\Delta y(\tau)]^2 d\tau} \tag{4.36}
\]

Angle of Attack Reconstruction Errors

For the OL flight, the true angle of attack (from Figure 4.5) and all three reconstructions (LIN, NAW, NAWLIN) are plotted in the top pane of Figure 4.6. The bottom pane shows the corresponding reconstruction errors. The NAW and NAWLIN reconstructions are nearly identical, and both of them show more error than the LIN reconstruction. For the closed-loop flight the LIN error is again lower than NAW and NAWLIN, the plots are shows in Figure 4.16 in Section 4.3.5. The root mean square values for all six reconstruction errors are presented in Table 4.1.

Figure 4.6: Angle of attack and reconstructions (top) and reconstruction errors (bottom) for the OL flight. (Reprinted from [95])
Table 4.1: Angle of attack reconstruction error rms values ($\Delta \alpha_{\text{rms}}$) for the example simulation

<table>
<thead>
<tr>
<th></th>
<th>LIN</th>
<th>NAW</th>
<th>NAWLIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>OL</td>
<td>$2.33 \times 10^{-2}$ deg</td>
<td>$5.52 \times 10^{-2}$ deg</td>
<td>$5.82 \times 10^{-2}$ deg</td>
</tr>
<tr>
<td>CL</td>
<td>$1.11 \times 10^{-2}$ deg</td>
<td>$5.55 \times 10^{-2}$ deg</td>
<td>$5.55 \times 10^{-2}$ deg</td>
</tr>
</tbody>
</table>

(Fig. 4.6, bottom)

(Fig. 4.16, bottom)

Wind Reconstruction Errors

Each wind (and wind-gradient) component is reconstructed twelve different ways. Table 4.2 presents the labeling terminology and also depicts how the twelve reconstructions are organized according to the relevant categories. When plotting these quantities for the example simulation, only the reconstruction errors are shown since each reconstruction would be visually indistinguishable from the true values of Figures 4.3 and 4.14.

Figure 4.7 shows the kinematic open-loop reconstruction errors for the horizontal (top pane) and vertical (bottom pane) wind components. In both horizontal wind $W_D$ and vertical wind $W_N$, the LIN model shows lower error than either the NAW or NAWLIN. Also, the NAWLIN error is observed to act like a combination of the NAW and LIN error, as predicted.
in Remark 4.2. In Figure 4.8, plots of the same quantities are shown for the closed-loop results. Similar trends are observed to the open-loop results: the LIN error is the lowest and

![Figure 4.8: Example kinematic, closed-loop, horizontal (top) and vertical (bottom) wind reconstruction errors. (Reprinted from [95])](image)

the NAWLIN error compounds the LIN and NAW errors.

As previously stated in Remark 4.5, the dynamic wind-component reconstructions are prone to random drift, thus they are poor estimates. Plots of these quantities are included for the interested reader Section 4.3.5 via Figure 4.17 (open-loop) and Figure 4.18 (closed-loop). The rms values for all twelve wind reconstructions are shown in Table 4.3 (horizontal) and Table 4.4 (vertical).

Table 4.3: Horizontal wind reconstruction error rms values ($\Delta W_{N,rms}$) for the example simulation

<table>
<thead>
<tr>
<th></th>
<th>LIN</th>
<th>NAW</th>
<th>NAWLIN</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OL</td>
<td>CL</td>
<td>DYN</td>
</tr>
<tr>
<td>KIN</td>
<td>14.18 $\times 10^{-4}$ m/s</td>
<td>36.10 $\times 10^{-4}$ m/s</td>
<td>35.88 $\times 10^{-4}$ m/s</td>
</tr>
<tr>
<td>CL</td>
<td>3.48 $\times 10^{-4}$ m/s</td>
<td>21.38 $\times 10^{-4}$ m/s</td>
<td>21.10 $\times 10^{-4}$ m/s</td>
</tr>
<tr>
<td>DYN</td>
<td>48.02 $\times 10^{-2}$ m/s</td>
<td>1.59 $\times 10^{-2}$ m/s</td>
<td>48.01 $\times 10^{-2}$ m/s</td>
</tr>
<tr>
<td>CL</td>
<td>45.14 $\times 10^{-2}$ m/s</td>
<td>1.57 $\times 10^{-2}$ m/s</td>
<td>45.12 $\times 10^{-2}$ m/s</td>
</tr>
</tbody>
</table>

**Wind Gradient Reconstruction Errors**

Following the same organizational layout as the wind reconstruction errors, the wind gradient reconstruction error figures are described and included in Section 4.3.5 of this paper. The rms values for all wind gradient reconstruction errors are shown in Tables 4.5 and 4.6. The reader is reminded that the kinematic reconstructions of the wind gradient were expected to be unrealistically accurate as suggested previously in Remark 4.4; this is observed in the KIN error values.
Table 4.4: Vertical wind reconstruction error rms values ($\Delta W_{D,\text{rms}}$) for the example simulation

<table>
<thead>
<tr>
<th></th>
<th>LIN</th>
<th>NAW</th>
<th>NAWLIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>KIN</td>
<td>$16.36 \times 10^{-3}$ m/s</td>
<td>$40.79 \times 10^{-3}$ m/s</td>
<td>$41.64 \times 10^{-3}$ m/s</td>
</tr>
<tr>
<td>CL</td>
<td>$8.23 \times 10^{-3}$ m/s</td>
<td>$41.04 \times 10^{-3}$ m/s</td>
<td>$41.01 \times 10^{-3}$ m/s</td>
</tr>
<tr>
<td>DYN</td>
<td>$4.51 \times 10^{-1}$ m/s</td>
<td>$3.39 \times 10^{-1}$ m/s</td>
<td>$4.70 \times 10^{-1}$ m/s</td>
</tr>
<tr>
<td>CL</td>
<td>$4.64 \times 10^{-1}$ m/s</td>
<td>$3.00 \times 10^{-1}$ m/s</td>
<td>$4.86 \times 10^{-1}$ m/s</td>
</tr>
</tbody>
</table>

Table 4.5: Horizontal wind gradient reconstruction error rms values ($\Delta \frac{\partial W_N}{\partial X_N,\text{rms}}$) for the example simulation

<table>
<thead>
<tr>
<th></th>
<th>LIN</th>
<th>NAW</th>
<th>NAWLIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>KIN</td>
<td>$19.12 \times 10^{-5}$ s$^{-1}$</td>
<td>$39.48 \times 10^{-5}$ s$^{-1}$</td>
<td>$40.83 \times 10^{-5}$ s$^{-1}$</td>
</tr>
<tr>
<td>CL</td>
<td>$5.49 \times 10^{-5}$ s$^{-1}$</td>
<td>$24.49 \times 10^{-5}$ s$^{-1}$</td>
<td>$24.14 \times 10^{-5}$ s$^{-1}$</td>
</tr>
<tr>
<td>DYN</td>
<td>$49.54 \times 10^{-4}$ s$^{-1}$</td>
<td>$4.40 \times 10^{-4}$ s$^{-1}$</td>
<td>$49.51 \times 10^{-4}$ s$^{-1}$</td>
</tr>
<tr>
<td>CL</td>
<td>$39.96 \times 10^{-4}$ s$^{-1}$</td>
<td>$2.54 \times 10^{-4}$ s$^{-1}$</td>
<td>$39.82 \times 10^{-4}$ s$^{-1}$</td>
</tr>
</tbody>
</table>

Reconstruction Error Conclusions

This simulated example flight demonstrates a number of general trends which are observed when performing model-based reconstruction from flight simulation data for a variety of Dryden wind-field realizations. For each observed trend, an explanation based in theory is also provided:

1. The reconstructions from the kinematic equations (KIN) are more accurate than those from dynamics (DYN) across all models.
   - The DYN models utilize the (simplified) dynamic equations twice, incurring model-based error first to estimate $\hat{\alpha}$ and again to estimate $\frac{\partial W}{\partial X_N}$. In contrast, the KIN models use the (exact) kinematic equations and (perfect) time differentiation for

Table 4.6: Vertical wind gradient reconstruction error rms values ($\Delta \frac{\partial W_D}{\partial X_N,\text{rms}}$) for the example simulation

<table>
<thead>
<tr>
<th></th>
<th>LIN</th>
<th>NAW</th>
<th>NAWLIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>KIN</td>
<td>$21.75 \times 10^{-4}$ s$^{-1}$</td>
<td>$43.90 \times 10^{-4}$ s$^{-1}$</td>
<td>$46.69 \times 10^{-4}$ s$^{-1}$</td>
</tr>
<tr>
<td>CL</td>
<td>$9.58 \times 10^{-4}$ s$^{-1}$</td>
<td>$44.00 \times 10^{-4}$ s$^{-1}$</td>
<td>$44.09 \times 10^{-4}$ s$^{-1}$</td>
</tr>
<tr>
<td>DYN</td>
<td>$7.31 \times 10^{-3}$ s$^{-1}$</td>
<td>$4.92 \times 10^{-3}$ s$^{-1}$</td>
<td>$9.24 \times 10^{-3}$ s$^{-1}$</td>
</tr>
<tr>
<td>CL</td>
<td>$6.27 \times 10^{-3}$ s$^{-1}$</td>
<td>$4.92 \times 10^{-3}$ s$^{-1}$</td>
<td>$8.61 \times 10^{-3}$ s$^{-1}$</td>
</tr>
</tbody>
</table>
4.3. Simulation Details & Results

the second calculation, thus only “carrying forward” the initial reconstruction error into the wind and wind-derivative calculations.

2. Reconstructions during closed-loop (CL) flight are more accurate than those during open-loop (OL) flight.
   • The closed-loop flight experiences smaller deviations from the linearization trim point, thus remaining nearer to state trajectories where the simplified models are designed to be accurate.

3. The NAWLIN model error is commensurate with the larger error exhibited by either the LIN or NAW model.
   • Since NAWLIN is a combination of both NAW and LIN simplifications, its error is expected to track whichever error source is more significant for any reconstructed quantity.

These observations suggest that more underlying trends may be found from further study of reconstruction performance. To investigate, a batch of randomized simulations were performed for a set of uniformly spaced turbulence intensity ($\sigma$) values. The ensemble-averaged rms errors are used to demonstrate each model’s average reconstruction accuracy. These results are presented in the next section.

4.3.4 Study of Reconstruction Error vs Turbulence Strength

The error for any reconstructed quantity is expected to grow with the magnitude of the unknown disturbances. In this work the disturbance magnitudes are prescribed for each simulation via the Dryden turbulence intensity $\sigma$. Guidance for selecting an appropriate value of $\sigma$ is provided in Figure 262 of Appendix A, MIL-HDBK-1797 [3], where turbulence with $\sigma < 2.5$ m/s is called “light,” turbulence with $\sigma > 5$ m/s is called “severe,” and values in between are labeled “moderate.” To study reconstruction errors across turbulence magnitudes, $N = 100$ simulations with independently generated Dryden wind fields were performed for each of 25 $\sigma$ values equally spaced between 0 and 8 m/s. While “severe” turbulence amounts are uncommon in reality, this study’s range extends into these values because unmanned aircraft might be tasked for flight in such wind environments. In order to contextualize the wind and wind-gradient reconstruction error amounts, a normalization is adopted

$$\tilde{\Delta}y_{\text{rms}} = \frac{\Delta y_{\text{rms}}}{(y - \bar{y})_{\text{rms}}} \times 100\%$$ (4.37)

where $\bar{y}$ denotes the time-averaged value of $y$ across the simulation’s 200 s duration, and the normalized result is expressed as a percentage.
Figures 4.9–4.13 show the ensemble-averaged results as \( \sigma \) is varied. The open-loop reconstruction errors in extreme turbulence \((\sigma > 6 \text{ m/s})\) are omitted, because flying open-loop in these extreme wind environments would be unreasonable in practice, and the results did not contribute useful information in the analysis\(^4\). For angle of attack reconstruction in

![Graph showing angle of attack reconstruction errors across turbulence intensity.](image)

Figure 4.9: Angle of attack reconstruction errors across turbulence intensity. (Reprinted from [95])

Figure 4.9 expressed in degrees, the error associated with every model grows with increasing turbulence severity as expected. For open-loop flight, the LIN model shows less than 10% rms error during light turbulence, but the error grows as turbulence increases, surpassing the NAW for severe turbulence simulations. For closed-loop flight, the linearized model (LIN) contributes less than 0.001° of rms error (below 6%) even in extreme turbulence. When expressed in degrees, the NAW error grows linearly with turbulence severity. However when normalized, the NAW model causes \(~20\%\) error for open-loop flights and \(~15\%\) for closed-loop flights, regardless of the turbulence magnitude.

Figure 4.10 shows the kinematic (KIN) rms errors for horizontal wind reconstruction across the ensemble of trials. The rms error values are largely below 0.02 m/s (or \(~0.5\%\)) for all quantities. While comparisons between various models can be drawn from this data, this amount of reconstruction error is negligible for most applications. For kinematic vertical wind reconstruction in Figure 4.11, the rms errors are mostly below 0.1 m/s (3%). The LIN simplification in closed-loop simulation creates error which grows with turbulence magnitude, but the error is less than 1% even in extreme conditions. The NAW model creates \(~2\%\) error across all turbulence values for both open-loop and closed-loop flight. Because the dynamic (DYN) reconstructions were observed to drift significantly from the true values, comparative plots which include the DYN reconstructions are included in Section 4.3.5 as Figures 4.23 and 4.24 for the interested reader.

\(^4\)Some of these severe-turbulence OL errors are included in the results of [96].
4.3. Simulation Details & Results

Figure 4.10: Horizontal wind kinematic (KIN) reconstruction errors (left) and normalized values (right). (Reprinted from [95])

For dynamic wind gradient reconstructions, Figure 4.12 shows the horizontal rms error amounts. Linearization incurs between 10% and 40% error for dynamic wind gradient reconstruction, as observed in the LIN and NAWLIN reconstructions. The NAW model produces an order of magnitude lower error than the LIN model, across all turbulence values. In the vertical direction, Figure 4.13 shows rms errors in the wind gradient estimate. Linearization incurs the same 10% and 40% error as the horizontal wind gradient. The NAW model incurs 12% error regardless of the turbulence amount. This is significantly more than in the horizontal case, as predicted in Remark 4.1. The NAWLIN error consistently behaves like a combination of the NAW and LIN errors, in agreement with Remark 4.2.

4.3.5 Additional Example-Simulation Figures

For the example simulation of Section 4.3.2, time plots of the wind gradients are provided in Figure 4.14, and time plots of the omitted state components $\theta(t)$, $q(t)$ are shown in Figure 4.15. The aircraft’s phugoid mode response is observed in the pitch angle $\theta$ as a large-amplitude oscillation with a period of 20 s. The closed-loop angle of attack reconstruction errors are presented in Figure 4.16. Figures 4.17 and 4.18 show the DYN wind reconstruction error. It is large because the DYN wind estimation drifts from the true value as explained in Remark 4.5.

All four wind gradient reconstruction error figures accompanying the example of Section 4.3.2 are included and briefly described here. The kinematic open-loop wind gradient reconstruction errors are shown in Figure 4.19. The LIN model appears to have slightly smaller error amounts, but all 3 are comparable. For the kinematic closed-loop results, Figure 4.20 shows corresponding wind gradient reconstruction error plots. Again the LIN model has a smaller
average error than the NAW or NAWLIN. For reconstructions based on the aircraft’s dynamics, the open-loop errors are shown in Figure 4.21 and for closed-loop in Figure 4.22. In both cases, the NAW model has significantly smaller reconstruction error for the horizontal wind gradient, and this was expected from Remark 4.1. For the vertical gradient, the NAW model has only slightly smaller error than the LIN (and NAWLIN) models.

Figures 4.23 and 4.24 present all twelve of the horizontal and vertical wind reconstructions for varying $\sigma$ values. Note that a logarithmic scale is used for the vertical axes. The dynamic (DYN) reconstruction errors are at least an order of magnitude larger than the corresponding kinematic (KIN) amounts.

Figures 4.25 and 4.26 present all twelve of the wind gradient reconstructions for varying $\sigma$ values using a logarithmic vertical axis. Recall that Remark 4.4 provides the expectation for lower kinematic (KIN) gradient error than dynamic (DYN) error. For LIN models the KIN error is much lower than the DYN error, but for the NAW models this effect is small.

### 4.3.6 Controller Implementation Details

For closed-loop flight in a turbulent wind field, a “successive loop closure” PID-based control scheme [15] is implemented. Figure 4.27 is a diagram of the controller layout, and Table 4.7 presents the numerical values for all controller gains used in this work’s results. The gains were hand-selected for reasonable disturbance rejection performance in randomized simulations at moderate turbulence. While the controller gains could instead be selected in some more principled manner (e.g. robust or optimal control) the reconstruction accuracy is not expected to be sensitive to incremental control-performance differences.
Each PID controller follows the same (standard) form presented in Appendix A.2. The controller’s input signal is formed by subtracting the quantity of interest \( y \) from its setpoint value \( y^\dagger \)

\[
e(t) = y(t) - y^\dagger(t)
\]  

(4.38)

and the PID equations (A.5),(A.7) produce the desired controller output \( u(t) \). In each controller, the initial conditions for the integrator \( I_0 \) and filtered derivatives \( x_d(t_0) \) are set to 0.

For controlling airspeed \( V_r \) by varying thrust \( T \), the thrust percentage change \( \delta T_p \) is the output of the “Airspeed PID” controller following the above equations. The thrust \( T \) is calculated from thrust percent change \( \delta T_p \) and the maximum thrust \( T_m = 115.65 \) N via

\[
T = T^\dagger + \left( \frac{\delta T_p}{100\%} \right) T_m
\]  

(4.39)

The maximum thrust value was obtained from Cunningham et al. [27]. Note that the PID controller may set the total thrust to a negative value. In practice this might represent air brakes or other unmodeled systems, but nonetheless it is possible for an aircraft and likely does not affect the reconstruction results.

The elevator command \( \delta_e \) is formed via the output of two successive PID controllers. An altitude error is formed as the difference between the current altitude \(-X_D\) and the altitude setpoint \(-X_D^{\dagger}\). This altitude error is the input to the first “altitude PID” controller whose output is pitch deviation \( \delta \theta \). Then, this pitch deviation is combined with the actual pitch \( \theta \) and trim value \( \theta^\dagger \) to form a pitch error. The pitch error is input to the second “pitch PID” controller, and that controller produces the elevator command \( \delta_e \).
4.4 Conclusions

Error in wind estimations from flight data of a fixed-wing aircraft comes from two primary sources: sensor error and modeling error. This work isolates and studies error incurred by using common simplified forms of the flight dynamic model. Section 4.2 presents a methodology for quantifying this error. Reconstructions are performed during open-loop and closed-loop simulated flights, following the “kinematic” and “dynamic” approaches defined in Section 4.2.3. A single example simulation’s results are presented in Section 4.3.2 for the reader to understand the reconstruction process. Section 4.3.4 shows how ensemble-averaged reconstruction errors vary with turbulence severity across a batch of Monte Carlo simulations.

Our methodology is used to present a rich set of conclusions which practitioners might find useful. For example, linearization incurs less than 1% wind reconstruction error during closed-loop flight, even in extreme turbulence. Another conclusion is that for reconstructing the wind gradient, linearization injects between 10% and 25% during closed-loop flight. For the aircraft we selected from literature, neglecting the angular wind’s effect injects more error than linearization to wind velocity reconstruction, but less error than linearization for
wind gradient reconstruction. Since model-based reconstruction error as we present herein is only one component of total reconstruction error, practitioners may use this work to make an informed decision about what dynamic modeling choices are acceptable for their application.

For future work a number of modeling assumptions of this work might be relaxed to increase realism; the value of relaxing modeling assumptions might be realized by weighing the potential insight which might be gained against the increased complexity cost. For example, a standard frozen Dryden wind field (Section 4.3.1) is utilized, but more realistic gust and turbulence models could be employed if desired. While this work specialized to the case of longitudinal flight, the authors expect that the trends discussed would persist when incorporating lateral-directional motions. However this remains a direction open for future investigation.
Another direction for research is the impact of the aircraft’s flight trajectory on reconstruction accuracy. Langelaan et al. [77] show that sensor-based error grows with airspeed, suggesting that slow flight is better for sensor-based wind reconstruction. Our methodology could be extended to determine the effect of trim point variation (including airspeed) on model-based reconstruction error. Another open question is if significant aircraft maneuvering, for example repeated climbing and descending, might improve reconstruction performance. Weather scientists would often like to measure the altitudinal variation of the wind field, so this study’s methodology could be applied to learn if the best measurement strategy is to continuously ascend and descend while measuring, or perhaps to maintain periods of steady flight at each sampled altitude.

A final direction for future study is to better understand how model-based error findings, such as those presented in this work, compound with other unavoidable error sources dur-
4.4. Conclusions

Figure 4.18: Example dynamic, closed-loop, horizontal (top) and vertical (bottom) wind reconstruction errors. (Reprinted from [95])

Figure 4.19: Example kinematic, open-loop, horizontal (top) and vertical (bottom) wind gradient errors. (Reprinted from [95])

ing airborne estimation. For example, error synergies between sensor and modeling error likely exist inside the estimation methods, so these deserve study in addition to separately quantifying the sources.

Significant resource investment is required to improve sensors and modeling capabilities for any given aircraft and measurement mission. Studies contributing to a more complete understanding of estimation error inform practitioners (like ourselves) how to improve airborne wind measurement in a cost-effective manner.
Figure 4.20: Example kinematic, closed-loop, horizontal (top) and vertical (bottom) wind gradient errors. (Reprinted from [95])

Figure 4.21: Example dynamic, open-loop, horizontal (top) and vertical (bottom) wind gradient errors. (Reprinted from [95])
Figure 4.22: Example dynamic, closed-loop, horizontal (top) and vertical (bottom) wind gradient errors. (Reprinted from [95])

Figure 4.23: All twelve horizontal wind reconstruction errors (left) and normalized values (right). (Reprinted from [95])
Figure 4.24: All twelve vertical wind reconstruction errors (left) and normalized values (right). (Reprinted from [95])

Figure 4.25: All twelve horizontal wind gradient reconstruction errors (left) and normalized values (right). (Reprinted from [95])
4.4. Conclusions

Figure 4.26: All twelve vertical wind gradient reconstruction errors (left) and normalized values (right). (Reprinted from [95])

Figure 4.27: Diagram of successive PID loop-closure controller. (Reprinted from [95])
Chapter 5

Model-Based Wind Estimation

The central goal of this dissertation, as expressed in Remark 1.1, is to improve the accuracy of airborne wind estimation by utilizing more accurate flight dynamic models (FDMs). In this chapter, we present results of performing wind estimation based on the Bix3 FDM as described in Chapters 2 and 3. These results have not been published elsewhere.

Briefly, the model-based wind estimates are calculated from a parameter-adaptive Extended Kalman-Bucy Filter (EKF) with the wind velocity as a zero-dynamic appended state. These estimates are compared against other available wind information, but due to the uncertainty from each sensor, we don’t make any conclusions of accuracy improvement. Additionally, we demonstrate some model fidelity concerns by comparing the predicted behavior from the FDM with the actual sensed behavior on several flights.

ArduPilot flight data logs are encoded in a custom binary format. In order to utilize that data in a MATLAB environment, we co-developed a log translator named ArduPilog [170] and released it open-source for public use.

5.1 Model-Based Wind Estimation Process

Our wind-estimation filter processes Bix3 flight data recorded by the ArduPilot flight control software. The Extended Kalman-Bucy Filter (EKF) estimation state \( x_e \) is the combination of the aircraft’s relevant state vector \( x \) from equation (2.72) where wind vector components are appended as adaptive parameters

\[
    x_e = \begin{pmatrix} x \\ W \end{pmatrix} = \begin{pmatrix} X^T & \Theta^T & V_R & \omega^T & W^T \end{pmatrix}^T
\]

(5.1)

The dynamic equations for the original 12 components of \( x \) are the Bix3 FDM from Chapter 2; specific expressions are found in Sections 2.4.3 and 2.5.1. The appended wind states are given zero-dynamic equations

\[
    \dot{W} = 0
\]

(5.2)

which is standard in parameter-adaptive Kalman Filtering. An alternative would be to give a very slight dynamic bias towards zero such as \( \dot{W} = -W/100 \) to reduce estimation drift, but this is not utilized in the results below. Additionally, we assume all wind derivative
information is identically zero in this estimation method

\[
\begin{align*}
\frac{\partial W}{\partial X} &= 0 \\
\frac{\partial W}{\partial t} &= 0
\end{align*}
\] (5.3) (5.4)

Thus with the wind in the EKF state and the wind derivatives zeroed, the disturbance vector \(d\) is eliminated from the estimation equations.

Recalling that at timestep \(t\), the state vector \(x(t)\) in an EKF is a random vector, we denote the associated covariance matrix \(P(t)\).

Because the ArduPilot system does not have any aerodynamic angle \((\alpha, \beta)\) sensing, the sensor function \(h(x)\) simply selects the other 10 aircraft state elements. The sensor-measurement function can be written

\[
h(x_e(t)) = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} x_e(t)
\] (5.5)

which simplifies identifying the jacobian \(H = \frac{\partial h}{\partial x_e}\) of the sensor function. The final set of equations needed for the EKF implementation is the jacobian of the dynamics with respect to the EKF state \(F = \frac{\partial f_e}{\partial x_e}\), but many elements in this matrix cannot be written out due to their length. The MATLAB Symbolic Toolbox [148] is used to perform the differentiation and obtain a symbolic expression for \(F\). This is then written in an optimized form to a function-file for numerical computation.

As a pre-processing step, the flight data log is re-sampled at the fastest observed log rate (50+ Hz) via linear interpolation. Note that care must be taken when sampling the Euler angles to correctly handle interpolation when the value “wraps” from one extreme to the other.

Recall from Chapter 3 that the ArduPilot airspeed sensor has several problems. It is very noisy, and also it measures \(u_r\) rather than absolute airspeed. While Section A.5 presents the transformation between these two quantities, knowledge of aerodynamic angles is required, but this is not measured by the Bix3 ArduPilot system. We explored three approaches to handle the poor airspeed sensor:
1. Neglect the airspeed entirely. Thus the sensor function $h(\cdot)$ only returns 9 components, and its jacobian $H$ is modified accordingly.

2. Use the airspeed despite it being poor quality, and give it a high uncertainty in $R$.

3. Utilize ArduPilot’s internal wind estimate (discussed in Section A.4) and combine it with the inertial velocity information to obtain a pseudo-measurement of airspeed. Note that equations (2.10) and (2.15) enable this transformation. Note that in this case a high uncertainty in $R$ is also justified.

As the uncertainty associated with the airspeed measurement increases, it is essentially ignored by the EKF. Thus the results presented herein take the first approach, simply neglecting it altogether.

We denote a sensor measurement at time $t$ by $z(t)$. The measurement vector is formed by concatenating the inertial position and orientation estimates from the flight data log with the angular velocity information from the gyros

$$z(t) = (X^T(t), \Theta^T(t), \omega^T(t))^T$$ \hspace{1cm} (5.6)

Now we specify the discrete-continuous EKF algorithm following [142, §4.6]. The initial state estimate $x_e(t_0)$ is set to the first sensor values $z(t_0)$ and the unknown quantities are initialized to 0. The covariance matrix is initialized based on operator intuition to

$$P(t_0) = \text{diag} (1, 1, 1, 0.1, 0.1, 0.1, 0.1, 1, 1, 1, 1, 1)$$ \hspace{1cm} (5.7)

where we adopt the “diag” notation from MATLAB to indicate a square matrix with the given values on the main diagonal, and zeros for every other element. For the timestep $\Delta t$ between $t_0$ and $t_1$, the predicted state $\hat{x}_e$ is calculated by numerically integrating the dynamic model

$$\hat{x}_e(t_1) = \int_{t_0}^{t_1} f_e(x(\tau), u(\tau))d\tau + x_e(t_0)$$ \hspace{1cm} (5.8)

using the “ode45” MATLAB capability. Likewise, the covariance prediction dynamics

$$\dot{P} = FP^T + PP^T + Q$$ \hspace{1cm} (5.9)

are also numerically integrated\(^1\) to produce $\dot{P}(t_1)$. Note that the process noise matrix is chosen by hand as

$$Q = \text{diag} (1, 1, 1, 0.1, 0.1, 0.1, 0.1, 1, 1, 1, 1, 1, 0.1, 0.1, 0.1)$$ \hspace{1cm} (5.10)

and the sensor noise matrix is also specified by hand to be

$$R = \text{diag} (1, 1, 1, 0.1, 0.1, 0.1, 1, 1, 1)$$ \hspace{1cm} (5.11)

\(^1\)To utilize the ode solvers, MATLAB requires reshaping the $P$ matrix and dynamics to be column vectors.
The Kalman gain matrix $K$ is computed from the covariance prediction via

$$
K = PH^T (HPH^T + R)^{-1} = (I - KH) P \tag{5.12}
$$

The state correction equation to bring in the impact of the sensor data $z$ is applied

$$
x_e(t_1) = \hat{x}_e(t_1) + K (z(t_1) - h\hat{x}_e(t_1)) \tag{5.13}
$$

and the covariance correction completes the EKF computation

$$
P(t_1) = \left( \hat{P}(t_1)^{-1} + H^T R^{-1} H \right)^{-1} \tag{5.14}
$$

For clarity, these equations are written to advance from $t_0$ to $t_1$, but they are trivially generalized to any timestep $t_{k-1}$ to $t_k$.

Noticing that the state $x_e$ contains the Euler-angle representation of aircraft orientation, the numerical implementation must handle the “angle wrap” phenomenon correctly. This is accomplished by wrapping both the angle differences inside equation (5.13) and the result to the desired range.

## 5.2 KEAS 2017-09 Wind Estimation Campaign

In September 2017, we conducted a 7-day wind measurement campaign at Kentland Farm, VA, USA. (Latitude: 37°11′49″N, Longitude: 80°34′41″W) This site is home to the Kentland Experimental Aerial Systems (KEAS) Lab [158]. During this campaign, we utilized 4 ground-based wind sensors, each of which are briefly described below. One might also notice that the North American Mesoscale (NAM) Weather Research and Forecasting (WRF) wind data utilized in Chapter 6 is centered on Kentland Farm. In fact, it is the exact same time and place: we used available wind measurement assets to get a sense of the NAM WRF data accuracy. To get an overview of the wind speed and direction during the week, Figure 5.1 is a comparison plot of all 4 wind sensors as well as the WRF model data. On the horizontal axis, the date marks are positioned at the beginning of each day (00:00) in the local timezone. Because the KFWS sensor is actually at an altitude of 3 m above ground level (AGL), while the others are measurements at 115 m AGL, it consistently reports lower average windspeed. The black vertical lines indicate the start and end times of the group of UAS flights featured in this Chapter. All wind measurement timestamps have been shifted to display at the center of the time interval over which they are calculated.

**HALO LiDAR** Dr. Stephen DeWekker’s laboratory [28] from the University of Virginia temporarily installed their HALO LiDAR [62]. It was configured to operate on 30° azimuthal steps with a 15° tilt from vertical, as well as a single vertical stare per circular sweep. Note
that this means 12 directional measurements per circular sweep, and the vertical stare was used to measure cloud height. Because full azimuthal circles are used, the center of the spatial measurement area is centered above the unit. The LiDAR returns along-beam averaged wind velocity (from the doppler shift of reflected light) with along-beam bins 30m in length, thus the altitudes of measurements are

\[ H_m = (\cos 15^\circ) \times [15, 45, 75, 105, \ldots] = [14.49, 43.47, 72.44, 101.42, \ldots] \text{ m AGL} \]

At each altitude, a 300 s (5 min) time window was selected to average the measurements. Any measurement with minimum intensity (SNR+1) less than 1.008 was rejected. For any 300 s group of measurements, if an azimuthal measurement gap of 240° exists or less than 4 total measurements are obtained, the group was rejected. For every remaining group, a
best-fit sine curve was computed for the along-beam measurements taking the form

$$V(\psi) = A \sin(\psi + \bar{\psi}) + w$$  \hspace{1cm} (5.15)

Measurement groups with sine-fit $R^2$ value less than 0.95 were rejected. Thus, for all 300 s groups at each altitude which passed the criteria, a single “wind measurement” was computed: the sin curve amplitude $A$ provides horizontal wind speed, the vertical offset $w$ provides vertical wind speed, and the phase shift $\bar{\psi}$ provides wind direction. For this chapter, we refer to this as a single “measurement” from the LiDAR.

**Remark 5.1.** Note that an important discrepancy exists regarding the definition of wind direction. The standard value for wind direction is to specify the compass heading *from which the wind blows* so we adopt this standard. However, the wind velocity is a physical vector, and the standard direction of a physical vector is *to which it points*. As these are exactly opposite definitions, care should be taken to understand which is being presented in scientific works.

A picture of the HALO LiDAR at KEAS is included in Figure 5.2. It was located at $37^\circ11'46.90''N, 80^\circ34'42.04''W$ during the campaign. Determined by cloud height and atmospheric conditions, the LiDAR’s maximum measurement altitude frequently exceeds our range of interest, which is 120 m AGL and below. For the curious reader, we obtained some brief (~30 min) periods of measurements as high as 5500 m AGL.

![Figure 5.2: A HALO LiDAR used in the KEAS2017-09 campaign, courtesy of the De Wekker Group [28] from the University of Virginia](image)

**ASC SoDAR** We also utilized Virginia Tech’s Atmospheric Systems Corporation (ASC) Model 400 SoDAR [10] in our measurement campaign. The ASC was set to 5 m altitude increments and 30 s averaging intervals to produce wind measurements. The ASC has one
vertical sonic beam and two directional, which are nominally deflected 15° from horizontal. The directional beams point at true headings ~330° (NNW) and ~60° (ENE). Note that in estimating the spatial area over which the ASC averages, the sonic beams are expected to have much more effective width than the LiDAR, although we have no experience nor information estimating the quantity. Also, the spatial area of measurement is not centered above the unit, but offset along the ~15° (NNE) direction between the directional beams. A picture of the ASC SoDAR is included in Figure 5.3. It was located at 37°11′46.88″N, 80°34′42.65″W during the campaign (approximately 15 m West-South-West of the LiDAR). We set the ASC configuration to its lowest-altitude/highest-resolution mode and limited the data to 200 m AGL or lower. Because 30 s is a faster sampling period than the other ground-based assets, in many plots we have averaged the data over 10 min intervals. The ASC provides a “reliability” scalar for every measurement, where 0 indicates very low reliability and 9 represents the most certain data point. Thus the 10 min ASC averages are in fact reliability-weighted averages, inherently neglecting measurements with a 0 reliability value. Additionally, the reliability-weighted averaging was conducted in cartesian coordinates to avoid problems inherent in averaging polar coordinate representation.

![Figure 5.3: A picture of the ASC SoDAR at KEAS](image)

**RemTech SoDAR** An alternate SoDAR was also utilized, a RemTech PA-XS [119]. The RemTech was configured to provide measurements every 10 m altitude and every 600 s (10 min). The RemTech has 4 directional beams, each of which are at a larger tilt from vertical than the HALO and ASC. The manufacturer recommends not having obstacles within a 70° tilt down from vertical along the beam directions. This is likely a conservative estimate, but it gives us indication that the tilt angle is likely much greater than 15° from vertical. Because it emits beams in 4 directions, the RemTech’s spatial measurement area is centered over the unit. A picture of the RemTech, with the ASC and LiDAR visible in the background, is provided in Figure 5.4. To prevent sonic interference with the ASC, the
RemTech was located approximately 35 m East-North-East of the LiDAR at 37°11′47.40″N, 80°34′42.80″W. While the RemTech’s maximum altitude was set to 200 m AGL, successful measurements above 150 m AGL were rarely recorded.

![Figure 5.4: A picture of the RemTech SoDAR at KEAS, with the ASC and HALO in the background](image)

**WeatherStem Anemometer**  A final ground-based sensor utilized was the onsite WeatherStem [159] measurement unit. This is a traditional cup anemometer and wind vane mounted 3 m AGL on a pole. The data are averaged to 60 s intervals, and the sensor is located 50 m East of the ASC SoDAR, at 37°11′46.91″N, 80°34′40.73″W. A picture of this sensor, termed the Kentland Farm Weather Stem (KFWS) is provided in Figure 5.5. The KFWS sensor measures significantly below the altitudes of the other assets. Additionally, it is in the wind shadow of a large barn when the wind blows from the prevailing WSW direction, so it was found to be less useful than the others.

**Unmanned Aircraft Systems**  During the course of the 7-day measurement campaign, flights were performed with a variety of UAS. The most significant aircraft (for this dissertation) was the Bix3, as it is the subject of the System Identification work summarized in Chapter 3. One other fixed-wing aircraft utilizing the ArduPilot control system was also flown, a foam flying-wing aircraft which we named Mako\(^2\). A picture of the Mako is included in Figure 5.6, and another picture of the Bix3 is included in Figure 5.7. The Mako UAS provides wind information from ArduPilot’s internal wind estimator, as described in Section A.4. In addition to these fixed-wing UAS, a 3DR Solo multi-rotor UAS was also flown, wind estimates from multi-rotor platforms are the research focus of Javier González-Rocha [44, 45]. A picture of the 3DR Solo is included in Figure 5.8. Wind estimates from

\(^2\)This is a popular aircraft, search for “Ready Made RC Mako” or “HobbyKing SkyRay” online.
the 3DR Solo are provided by Javier, in brief they are calculated from a model-based state observer when postprocessing the Solo data logs.

5.2.1 Experimental Data

The most significant UAS flight day was 2017-09-07 between 10:00 and 14:30 local time. During this period, 6 missions were flown with the Bix3 aircraft. Each mission consisted of circular flight around the point 37°11'48.06"N, 80°34'42.35"W maintaining an altitude of 110 m AGL. The radius of the flight path circle was varied between 30 m and 95 m, depending on the flight.

The first flight measured the wind between 10:32 and 10:43. The flight path radius was set
to 25 m, but an average radius of 30 m is observed across the flight duration. A comparison between the EKF model-based wind estimation, produced by the procedure of Section 5.1, and ArduPilot’s internal wind estimate are presented in Figure 5.9. For the EKF model-based wind estimates, dotted lines indicate the EKF’s ±2σ amounts. We observe that the slowly varying trends match well, and that the model-based estimate contains higher frequency variation than ArduPilot’s. The dominant frequency of the model-based estimate corresponds to the flight path circling frequency of the aircraft, which has a period of 15 s. While this could potentially be a measurement of the spatial variation in the wind field, it might also be due to modeling error. Zooming in and inspecting (not presented in this dissertation) demonstrates that the periodicity is often skewed, which corresponds to more time in upwind flight, compared to downwind flight, along the circle. Note that ArduPilot assumes 0 vertical wind, while our model-based EKF provides a vertical turbulence estimate. Because the EKF treats wind components in cartesian coordinates, an unscented transformation [72] was utilized to map the (cartesian) uncertainties to a (polar) windspeed and direction representation.

The second flight is between 11:03 and 11:20, and in this flight the circling radius was
increased to 40 m. Corresponding to the larger radius, the circling period increases to 22 s. A comparison of the wind estimates are presented in Figure 5.10. Similar observations to flight 1 are observed: The model-based estimate contains a dominant frequency corresponding to that of the circling flight path. Likewise the slowly varying trends agree with the Ardupilot estimate.

In the third flight, between 11:29 and 11:47, the flight radius is increased mid-flight from 40 m to 95 m at approximately 11:34. Comparisons between the wind estimates are provided in Figure 5.11. Two effects are observed during the radius change: First, a spike in the vertical wind corresponds to the transition period. We hypothesize that this is due to the non-circular flight while changing the flight behavior, not an actual wind artifact. Second, the period of wind speed oscillation changes from 22 s to correspond to the slower (larger) circular-path period, approximately 50 s.

The fourth Bix3 flight, between 12:46 and 13:00 local time, has the highest average wind velocity of the group. The radius for this flight is 95 m. Wind estimate comparisons are provided in Figure 5.12. In this comparison, we observe that the model-based estimate is biased higher than the ArduPilot estimate. Due to the increased total windspeed, both estimates agree on a more consistent wind direction during the 15 min duration than the other flights. Typical behavior is observed in the downdraft plot.

The fifth Bix3 flight is between 13:26 and 13:40 with a path radius of 95 m. Its wind estimate comparisons are provided in Figure 5.13. This flight begins with a fairly low sustained ArduPilot wind speed estimate in comparison with the others. Again, we observe the model-based estimate predicting slightly higher wind speed, as well as matching the general trend of the wind speed climbing near the end of the flight. We observe the wind direction estimate uncertainty decrease as the windspeed increases throughout the flight. This agrees with our expectations.

The sixth (and final) flight of the set is flown between 13:49 and 14:13 with a path radius of 95 m. Figure 5.14 provides wind estimate comparisons. Between the times 13:52:54 to 13:54:46, the pilot flew a manual deviation from circular flight before returning to standard circling. This can be observed in the dominant frequency of the wind estimate disappearing in this interval. Additionally, this flight is remarkable in that between 13:55 and 14:00, both estimates report a significant windspeed and direction shift. The model-based wind direction estimate records higher-than-usual wind direction variation during this shifting period, while the ArduPilot reports a more smooth transition. This is a good demonstration of the expected frequency-content difference between the two estimation approaches.

Each flight coincides with another UAS flight (either the Mako, the Solo, or both) at approximately the same altitude. Figure 5.15 presents the wind data from all the SoDAR and LiDAR sensors, as well as the estimates from the other UAS, containing all 6 Bix3 flights. The details of each flight are difficult to observe when all 6 are plotted together, so Figures 5.16 and 5.17 are zoomed-in versions of this plot, focusing on Flights 4 and 5. The disagreement between UAS wind measurements, and the disagreements between the
UAS and the ground-based assets, are of similar magnitude to the disparity between the ground assets themselves. Without any two sensors in repeated agreement, we are not able to conclude anything quantitatively about the wind measurement accuracy of our assets.

5.3 Model Fidelity Investigation

As previously observed, the model-based Bix3 wind estimates contain a dominant frequency which seems consistent with the flight path circling frequency. As the model’s ability to predict true flight behavior degrades, observations like this are expected. Thus to investigate the model fidelity of the Bix3 FDM, we compare forward simulations of the FDM to actual flight data logs.

To validate the simulation environment, we first re-create the model validation results used to identify the FDM in [139, Fig. 6b]. The same 3 maneuvers are presented in Figures 5.18–5.20, each showing 4 plots of longitudinal state quantities. The flight test data (FTD) is ArduPilot’s internal state estimate. ArduPilot estimates 1.7 m/s wind during this maneuver, but we assumed zero wind during model identification, which likely impacts our results. Accordingly, the FDM was simulated both neglecting the wind ($W=0$, pink line) and making use of ArduPilot’s wind estimate as the true value (blue line). Finally, when possible, the relevant raw sensor data is also included as red dots. The left-most two plots are pitch angle $\theta$ and pitch rate $q$, which show good agreement between the flight log and model predictions in all 3 maneuvers. These agree with the findings of [139] and suggest our simulation is performing correctly.

In our Figures 5.18–5.20 we have also included plots of altitude and airspeed $V_r$ on the right side. The altitude appears to be a good match between the FTD and FDM, regardless of whether wind is included or neglected. The airspeed plot (bottom right) reveals a number of interesting considerations. First, the 1.7 m/s wind is apparent as the initial difference between the forward simulation neglecting wind ($F_{W=0}$, pink) and the forward simulation which includes it ($F_W$, blue). In this plot we observe a truth about the ArduPilot internal estimator: its internal wind estimate is not kinematically consistent with airspeed data. The reader should avoid the temptation to believe the airspeed sensor, however, as it is known to be quite inaccurate [171].

To further examine the FDM’s fidelity for the Bix3 model, we consider a longer period of data (20 s) from the same flight day, but during a portion of the flight which does not contain maneuvers designed for model identification. Figure 5.21 longitudinal quantities resulting from both forward simulation of the FDM and from the data log. During this portion of the flight, the mean windspeed was only 0.7 m/s, and this is consistent with the forward simulations including and neglecting wind (blue and pink) agreeing closely with each other. During this segment of more gentle maneuvering, we see the sensor measurements (red) and flight test data (black) in relative agreement. However, the disagreement between the
forward simulation and the flight data serves as a typical demonstration of the model fidelity for the FDM identified in Chapter 3 when applied in a 3d setting.

The final model-validation plot is presented in Figure 5.22. This comparison is performed on flight data from the KEAS 2017-09 campaign, which was approximately 6 months prior to the Bix3 FDM identification experiments. During this 20 s period, the mean windspeed is 4.2 m/s, this is apparent in the difference between the wind-included and non-wind forward simulations of the FDM. Again, significant discrepancy is observed between the model’s predictions and the observed flight test data for pitch angle and pitch rate. The reader is cautioned not to over-emphasize the altitude and airspeed differences of the right-most plots. The altitude represents integration of aerodynamically relevant states, and is expected to drift away from the true value during forward simulation. The airspeed plot showcases the low quality airspeed sensor, and the reader is reminded that no thrust model was identified for the Bix3.

In summary, non-negligible discrepancy is observed between observed flight data for the Bix3 aircraft and the predictions generated by utilizing our flight dynamic model. This discrepancy is expected to contribute wind estimate error when following the procedure in Section 5.1.

### 5.4 SoDAR Disruption by a Multi-rotor UAS

When investigating the experimental data of the KEAS 2017-09 campaign, we found something noteworthy: The 3DR Solo multi-rotor aircraft is clearly observed to disrupt the wind measurements of the ASC SoDAR. This is observed in Figures 5.23 and 5.24 which are plots of the windspeed from wind measurement assets (including the ASC SoDAR) during two separate groups of Solo flight experiments. In each figure, we observe that all of the sodar windspeed outlier measurements (above 15 m/s) are obtained while the Solo UAS is flying, indicated by the shaded regions. While not included in this dissertation, significant distortion was also observed in the ASC’s wind direction information. This disruption effect was not noticed in the ASC data of the flight campaign featured in Section 5.1, so we cannot conclude if the ASC measurements in Figures 5.15–5.17 were affected or not.
Figure 5.9: Flight 1 of 6 from KEAS 2017-09-07 comparing Bix3 model-based wind estimates to ArduPilot’s internal estimates. The dotted lines indicate a ±2σ region (around the mean value)
Figure 5.10: Flight 2 of 6 from KEAS 2017-09-07 comparing Bix3 model-based wind estimates to ArduPilot’s internal estimates. The dotted lines indicate a $\pm 2\sigma$ region.
Figure 5.11: Flight 3 of 6 from KEAS 2017-09-07 comparing Bix3 model-based wind estimates to ArduPilot’s internal estimates. The dotted lines indicate a $\pm 2\sigma$ region.
Figure 5.12: Flight 4 of 6 from KEAS 2017-09-07 comparing Bix3 model-based wind estimates to ArduPilot’s internal estimates. The dotted lines indicate a ±2σ region.
Figure 5.13: Flight 5 of 6 from KEAS 2017-09-07 comparing Bix3 model-based wind estimates to ArduPilot’s internal estimates. The dotted lines indicate a $\pm 2\sigma$ region
Figure 5.14: Flight 6 of 6 from KEAS 2017-09-07 comparing Bix3 model-based wind estimates to ArduPilot’s internal estimates. The dotted lines indicate a ±2σ region.
Figure 5.15: A group of Bix3 wind estimates compared with other sensors and UAS
Figure 5.16: Flight 4 of 6 comparing the Bix3 wind estimates to other wind sensors
5.4. SoDAR Disruption by a Multi-rotor UAS

Figure 5.17: Flight 5 of 6 comparing the Bix3 wind estimates to other wind sensors
Figure 5.18: Comparison between FDM predictions and Flight Test Data (FTD) during an elevator doublet maneuver (1 of 3)
Figure 5.19: Comparison between FDM predictions and Flight Test Data (FTD) during an elevator doublet maneuver (2 of 3)
Figure 5.20: Comparison between FDM predictions and Flight Test Data (FTD) during an elevator doublet maneuver (3 of 3)
Figure 5.21: Comparison between FDM predictions and Flight Test Data (FTD) for a longer maneuver on the same day.
Figure 5.22: Comparison between FDM predictions and Flight Test Data (FTD) for data during the wind-measurement campaign
5.4. SoDAR Disruption by a Multi-rotor UAS

Figure 5.23: The ASC SoDAR produces outlier measurements while the multi-rotor is in flight, indicated by shaded gray periods (Demonstration 1 of 2)
Figure 5.24: The ASC SoDAR produces outlier measurements while the multi-rotor is in flight, indicated by shaded gray periods (Demonstration 2 of 2)
Chapter 6

Detecting LCS from Airborne Wind Measurements

This chapter summarizes research contributions originally published in [111] and presents some additional details which were omitted. As an overview, in this work an Observing System Simulation Experiment (OSSE) is conducted to explore a method for detecting Lagrangian Coherent Structures (LCS) from a fixed wing small UAS. The method uses airborne wind estimates at distinct spatial points along a circular flight path to approximate the spatial gradient at the center of the circle. This gradient estimate is utilized in two Eulerian diagnostics ($\hat{\rho}$ and $s_1$, to be explained shortly) to predict the existence of an LCS near the circle center. The predictive performance of these approaches is analyzed by varying several situational parameters (such as the flight path circle’s radius). This methodology requires only a single flying UAS, making field implementation easier than existing alternatives.

The work presented in [111] was collaborative, with primary contributions from the first two authors, P. Nolan and H. McClelland. Nolan’s contributions focused on computation of atmospheric flow and the Eulerian diagnostics; McClelland’s contributions focused on implementing a realistic flight-in-wind simulation and performing (numerical) experiments of the proposed sampling strategy using the UAS FDM of Chapter 2. Accordingly, the summary presented here focuses on the UAS-based wind estimation results with emphasis on the wind data analysis method. For example, the Eulerian diagnostics and the gradient estimation algorithm are rewritten with notation specific to a wind velocity gradient (in contrast to the paper’s more general forms) and discussions of intermediate, non-UAS results (e.g. for a particle traversing a perfectly circular path at a constant speed) are omitted. Additionally, this dissertation presents details of the closed-loop controller which are not included in [111].

6.1 Motivation

Lagrangian diagnostics have been used to discover the most significant regions of material collection or dispersion. Few works have attempted to find ways to directly detect atmospheric LCS from experimental measurements in the field. In [75, 146] the authors use wind velocity measurements from a Doppler light detection and ranging (LiDAR) to detect LCS passing near Hong Kong International Airport. The authors of [145] consider sudden
changes in pathogen concentrations in the atmosphere and link those changes to the passage of LCS using atmospheric velocity data from the North American Mesoscale (NAM) Weather Research and Forecasting (WRF) model. Recent advances in dynamical systems theory, such as new Eulerian diagnostics [108, 133], show promise for local detection of LCS using multiple small UAS [110].

In this effort, we propose a method to detect LCS using only a single fixed wing UAS, and demonstrate it in a simulated environment. Specifically, the flight dynamic model (FDM) of Chapter 2 is simulated in wind velocity data from a NAM-WRF model. A cascaded-PID controller [15] produces a realistic circular orbit about a portion of the NAM model centered on the coordinates of the Virginia Tech experimental site, Kentland Farm. We refer to the wind values along these circular orbits as “perfect wind measurements” and utilize them to calculate two Eulerian metrics, the local trajectory divergence rate and attraction rate as described in Section 6.2.1. These Eulerian metrics are used as indicators of nearby LCS, and the effectiveness of this approach is quantified across variations in relevant parameters.

6.2 Methods

For this dissertation Chapter, we restrict our consideration to a 2-dimensional wind vector field

\[ \mathbf{W} = \begin{pmatrix} W_N \\ W_E \end{pmatrix} \]

defined over inertial position

\[ \mathbf{X} = \begin{pmatrix} X_N \\ X_E \end{pmatrix} \]

where the subscripts indicate “North” and “East” directions.

6.2.1 Lagrangian-Eulerian Analysis

For a planar wind field \( \mathbf{W} \), the C-ridges of the FTLE field [115] define LCS. Briefly, the FTLE field is calculated by examining the eigenvalues of the Cauchy-Green strain tensor of the wind’s flow map. (The interested reader may find the calculation in [111].)

The first Eulerian metric, the minimum eigenvalue of the wind’s rate-of-strain tensor, is denoted \( s_1 \) and is calculated via

\[ s_1 = \frac{1}{2} \left( \frac{\partial W_E}{\partial X_E} + \frac{\partial W_N}{\partial X_N} \right) - \frac{1}{2} \sqrt{ \left( \frac{\partial W_E}{\partial X_E} - \frac{\partial W_N}{\partial X_N} \right)^2 + \left( \frac{\partial W_E}{\partial X_N} + \frac{\partial W_N}{\partial X_E} \right)^2 } \]

To provide intuition, Figure 6.1 demonstrates the evolution of a fluid parcel in the vicinity of an attracting LCS.
The trajectory divergence rate is denoted $\dot{\rho}$ and is calculated via

$$
\dot{\rho} = \frac{W_E^2 \left( \frac{\partial W_N}{\partial X_N} \right) + W_N^2 \left( \frac{\partial W_E}{\partial X_E} \right) - W_E W_N \left( \frac{\partial W_E}{\partial X_N} \frac{\partial W_N}{\partial X_E} \right)}{W_N^2 + W_E^2} \quad (6.4)
$$

To provide intuition, Figure 6.2 (also reproduced from [111]) demonstrates nearby fluid-particle trajectories for positive and negative $\dot{\rho}$ values.

![Figure 6.2: Schematic of the trajectory divergence rate: Where $\dot{\rho} < 0$, nearby trajectories are converging; where $\dot{\rho} > 0$ trajectories are diverging. (Reprinted from [111])](image)

The reader should be careful not to confuse the atmospheric density $\rho$ with the trajectory divergence rate $\dot{\rho}$. The symbol $\rho$ in the latter term does not denote density; moreover air density is assumed constant throughout this dissertation.

### 6.2.2 Gradient Approximation from UAS Flight

In order to calculate Eulerian diagnostics from wind information along a simulated UAS flight path, we utilize a simple algorithm to approximate the gradient of a scalar field based on measurements along a circular arc, Figure 6.3. The algorithm assumes the wind velocity is temporally fixed during the period of one full orbit, but still varies spatially. Note that
in the numerical implementation this assumption is not enforced, the wind field varies both temporally and spatially. A pseudo-code version of this algorithm is presented in Algorithm 1, modified slightly from [111] to be intuitively easier. The algorithm assumes a clockwise trajectory, but is easily modified for counter-clockwise flight.

![Figure 6.3: A schematic of the equally spaced locations of wind “measurements” from which the gradient is approximated; note also the rotating reference frame $x'y'$ and inertial frame $xy$. (Reprinted from [111])](image)

### 6.2.3 Simulation Details

The OSSE is conducted by simulating the FDM of Chapter 2 in a three-dimensional wind field generated from the NAM-WRF [1] model. This section focuses on UAS-specific information and the reader is referred to the original paper for specifics such as interpolation details.

The wind field is centered at the Virginia Tech experimental site, Kentland Farm, for a 215 hr period beginning Sept 4th, 2017 at 00:00 UTC. The T-2 aircraft from Grauer [49] is simulated under closed-loop cascaded-PID control [15] to fly fixed-radius circles tracking the 850 mb isosurface with the wind field acting as a disturbance. While the altitude of the 850 mb isosurface varies in time, it is approximately at 1550 m above sea level. To give a sense of the aircraft’s scale, some of its physical properties are

$$m = 22.5 \text{ kg}, \quad b = 2.09 \text{ m}, \quad \bar{c} = 0.28 \text{ m}$$

A full circular orbit takes between 300 and 2250 seconds, depending on the orbit radius, at the T-2’s cruising airspeed of $\sim42 \text{ m/s}$. For comparison, the average horizontal windspeed at the 850 mb isosurface is $\sim10 \text{ m/s}$.
Algorithm 1: Approximate the value of the spatial gradient of a wind velocity component \( W \) at the center point of an arc from 4 samples along the arc’s path

**Input:** \( r, \theta, W(\theta) \)

**Output:** Approximation of \( \frac{\partial W}{\partial X} \) at center of circle

1. for \( i \leftarrow 1 \) to \( \text{length}(\theta) \) do
2.   if \( \theta(i) - \frac{3}{2} \pi \geq \theta(\text{end}) \) then
3.     Sample \( W(\cdot) \) along arc for \( W(\theta(i)), W(\theta(i) - \frac{1}{2} \pi), W(\theta(i) - \pi), W(\theta(i) - \frac{3}{2} \pi) \)
4.     \[
        \frac{\partial W}{\partial x} \approx \frac{W(\theta(i)) - W(\theta(i) - \pi)}{2r}
        \]
5.     \[
        \frac{\partial W}{\partial y} \approx \frac{W(\theta(i) - \frac{3}{2} \pi) - W(\theta(i) - \frac{1}{2} \pi)}{2r}
        \]
6.     \[
        \frac{\partial W}{\partial X}(i) \approx \left[ \begin{array}{cc} \cos(\theta(i)) & \sin(\theta(i)) \\ -\sin(\theta(i)) & \cos(\theta(i)) \end{array} \right] \left[ \begin{array}{c} \frac{\partial W}{\partial x} \\ \frac{\partial W}{\partial y} \end{array} \right]
        \]
7. return \( \frac{\partial W}{\partial X} \)

**Cascaded-PID Controller** The control system for the aircraft is a full cascaded-PID implementation, a modification of that presented by [15], comprised of 7 PID controllers. Each individual PID controller follows the form detailed in Section A.2. An organizational block diagram of the entire system is included in Figure 6.4. The simplest controller to explain is the sideslip controller, which actuates the rudder \( \delta_r \) to regulate the sideslip \( \beta \) to zero. Thus the controller is specified by its input \( e \) and output \( u \) signals

\[
    e(t) = 0 - \beta(t), \quad u(t) = \delta_r(t) \quad (6.5)
\]

The airspeed controller is also a single-level, regulating the thrust percentage \( T_{\%} \) to achieve the desired cruising airspeed \( V_{r,d} \). The controller is specified by

\[
    e(t) = V_{r,d} - V_r(t), \quad u(t) = T_{\%}(t) \quad (6.6)
\]

The elevator angle \( \delta_e \) is the result of a 2-level cascaded PID controller. At the higher level, the altitude PID controller is used to determine the desired pitch \( \theta_d \) via

\[
    e(t) = -X_{D,d} - (-X_D(t)), \quad u(t) = \theta_d(t) \quad (6.7)
\]

where recall that \( +X_D \) points downward, thus its negative is altitude. A slight modification is needed at this point, however, because the aircraft actually tracks a desired pressure value \( P_d \) rather than a true altitude. Because the aircraft flies very close to the target pressure, a linear pressure-to-altitude relationship is used with a constant factor of \(-1/9\) to convert between mb (milibars) and m (meters). The lower-level pitch controller regulates the elevator to attain the desired pitch via

\[
    e(t) = \theta_d(t) - \theta(t), \quad u(t) = \delta_e(t) \quad (6.8)
\]
Finally, the most complicated controller has 3 levels: turn, heading, and roll. The highest level “turn controller” is implemented to determine desired heading $\psi_d$. The concept is to set the desired aircraft heading as $\frac{\pi}{2}$ rad greater than the radial direction of the aircraft with respect to the circle’s center. The PID controller increases or decreases the desired heading by an amount $u_{\text{turn}}$ based on the aircraft’s distance inside (or outside) the intended flight circle radius $R$. The turn PID error signal is

$$e(t) = \frac{\|X(t) - X_c\| - R}{R}$$

where $X$ is the aircraft’s position and $X_c$ is position of the circle center. The controller calculation for $u_{\text{turn}}$ is given as equations (A.5) and (A.7) in Section A.2 using P, I, D, and M values from Table 6.1 below. The aircraft’s desired heading is thus calculated via

$$\psi_d = \text{atan2}(X - X_c) + 90^\circ + u_{\text{turn}}$$

The output of the turn controller is the reference input to the heading controller, which in turn outputs a desired roll angle

$$e(t) = \psi_d(t) - \psi(t), \quad u(t) = \phi_d(t)$$

that is directly fed into the calculations for the final roll controller

$$e(t) = \phi_d(t) - \phi(t), \quad u(t) = \delta_a(t)$$

Thus, through these 7 PID controllers, some of which are cascaded, the aircraft tracks the desired pressure and a circular path while maintaining its airspeed and zero sideslip.
Table 6.1 contains all the numerical values for these PID controllers. Simulink’s Control System Designer and Linear Analysis Tool were instrumental in obtaining good initial values for these parameters, from which they were further tuned by hand. Thus the cascaded-PID control system determines all four elements of the aircraft’s input vector \( \mathbf{u} \) which are specified in equation (2.17) of Section 2.2.1.

Table 6.1: PID controller numerical values, hand-tuned for reasonable performance

<table>
<thead>
<tr>
<th>Turn</th>
<th>Heading</th>
<th>Altitude</th>
<th>Airspeed</th>
<th>Roll</th>
<th>Pitch</th>
<th>Sideslip</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>20.0</td>
<td>0.2</td>
<td>0.001</td>
<td>4.0</td>
<td>-0.05</td>
<td>-0.02</td>
</tr>
<tr>
<td>I</td>
<td>0.015</td>
<td>0.0</td>
<td>0.032</td>
<td>0.1</td>
<td>0.8167</td>
<td>12.0</td>
</tr>
<tr>
<td>D</td>
<td>0.0001</td>
<td>0.5</td>
<td>0.9373</td>
<td>0.1</td>
<td>0.24</td>
<td>1.1</td>
</tr>
<tr>
<td>N</td>
<td>50.0</td>
<td>100</td>
<td>0.3259</td>
<td>1000</td>
<td>9.8852</td>
<td>100</td>
</tr>
</tbody>
</table>

6.3 Result Summary & Discussion

This section summarizes some results from the OSSE, specifically focusing on UAS-based information. The reader is referred to [111] for results which do not utilize the UAS and additional details.

Figure 6.5 demonstrates two findings. First, that the trajectory divergence rate \( \dot{\rho} \) (as calculated from Algorithm 1) is nearly identical comparing a perfect circle to the aircraft’s path as traversed under closed-loop control. Second, the \( \dot{\rho} \) estimate worsens as the flight path’s circle radius increases. Similar results were observed for the attraction rate \( s_1 \). The paper quantifies the similarity using Pearson correlation coefficients, ranging from 0.577 to 0.965 depending on the metric and the flight path radius.

To quantify the effectiveness of \( \dot{\rho} \) and \( s_1 \) for detecting LCS, we used the concept of a receiver operating characteristic (ROC) curve. The “truth” of detection is defined by a sufficiently strong LCS passing within a threshold radius of the center point, as in Figure 6.6. A true positive detection is defined by having a sufficiently strong LCS nearby and the Eulerian diagnostic successfully indicating the fact by crossing some threshold value. A false positive is when the Eulerian diagnostic crosses the threshold, but no strong LCS is nearby. Figure 6.7 shows an example of a true positive and a false positive from this study. The detection threshold for the Eulerian diagnostics determines the “sensitivity” of the sensor, thus serving as the horizontal parameter in the ROC curves. Figure 6.8 depicts an idealized ROC curve, where the dotted line represents random guessing. Thus a high-performing sensor has an ROC curve far from the dotted line. The area under the ROC curve (AUC) is used as a scalar measure of a parameter set’s performance, with higher AUC suggesting better performance.
Figure 6.5: Comparison of the trajectory divergence rate $\dot{\rho}$ at the desired center-point (red), along the path of the simulated UAS flight (black), and along a constant-altitude circle (blue). The simulated UAS flight produces nearly identical results to the ideal circle. As the flight path radius (lower right corner of each plot) increases, the $\dot{\rho}$ accuracy degrades. (Reprinted from [111])

Note that [111] examines the performance of exact Eulerian metrics $\dot{\rho}$ and $s_1$ as indicators of LCS in addition to their approximations from the UAS data. In this dissertation, we skip directly to the UAS-based results.

Simulated 3d UAS flights provided measurements in close agreement with those of perfect circular 2d paths. The main contributor to approximation error is radius of the circular arc. The trajectory divergence rate appears to be a more robust metric than the attraction rate, meaning that the trajectory divergence rate $\dot{\rho}$ can be better approximated at larger radii than the attraction rate $s_1$.

The ROC curves of Figures 6.9 and 6.10 show that both the trajectory divergence rate and attraction rate, as calculated from a simulated 2 km UAS flight, can be used to detect the passage of LCS. These results show that when approximated by a circling UAS, the
attraction rate $s_1$ slightly outperforms the divergence rate $\dot{\rho}$ as an LCS detection metric.

In conclusion, this work is a first step towards in-situ detection of LCS in the atmosphere. It provides a method (and some analysis) for a single fixed wing UAS to measure Eulerian diagnostics of an atmospheric flow. These Eulerian diagnostics serve as important indicators in turn towards inferring the Lagrangian dynamics of the flow field. Recommendations for future work involve a simulation-based evaluation the effects of sensor uncertainty on the accuracy of LCS detection, as well as application of approaches to experimental data.
Figure 6.7: Depiction of a true positive (left) and a false positive (right), from the WRF data. The true attraction rate field is displayed with darker greens indicating lower values of the attraction rate. Grid points where the value of the attraction rate is above the threshold value are masked (white). Strong attracting LCS are shown as blue lines. This example’s threshold radius is 5 km, indicated by a black circle. Both examples have an attraction rate which signals an LCS detection. In the true positive example there is an LCS passing within the threshold radius. Meanwhile, in the false positive there is no LCS within the threshold radius. (Reprinted from [111])
Figure 6.8: Schematic of an idealized ROC curve. In this (fictitious) example, thresholds of 60–80% provide a good quality sensor, balancing an ~0.9 true-positive rate with ~0.3 false-positive rate. (Reprinted from [111])
Figure 6.9: ROC curves for the trajectory divergence rate, $\rho$, as measured from a 2 km radius UAS simulation ability to detect 90th percentile LCS with integration times of 0.5 (green), 1 (red), and 2 (blue) hrs. Threshold radii are displayed in the upper left hand corner. The AUC for each integration time is given in the legend. (Reprinted from [111])
Figure 6.10: ROC curves for the attraction rate, $s_1$, as measured from a 2 km radius UAS simulation ability to detect 90th percentile LCS with integration times of 0.5 (green), 1 (red), and 2 (blue) hrs. Threshold radii are displayed in the upper left hand corner. The AUC for each integration time is given in the legend. (Reprinted from [111])
Chapter 7

Discussions of Future Work

While this dissertation demonstrates some research efforts towards detecting coherent structures in the ABL using fixed-wing unmanned aircraft, a plethora of challenges remain. To start, Lagrangian Coherent Structures (LCS) are fundamentally defined in a spatially dense manner. Because an aircraft measures wind information at a point, one may take advantage of the aircraft’s ability to move through the wind field. In this dissertation, we present an algorithm which utilizes an aircraft traversing a circular flight path to estimate the wind gradient at the center of the circle. As expected, the accuracy of the wind gradient estimate degrades as the radius of the flight path grows. One opportunity for future work is to explore alternate trajectories — perhaps “figure-eights” or similar — to determine if LCS detection might be improved by an alternative flight path choice. In a similar vein, upon detecting the passage of an LCS, future work might explore how an aircraft flies to track the LCS as it is advected and deformed by the wind field. A key assumption in our circling-UAS approach is that measurements taken at distinct time instants can be considered simultaneous. While this is clearly not true for a single agent, it would be true for a small team of aircraft working in collaboration. Future research might investigate how best to employ a team of UAS in LCS detection and tracking.

During this research progression, we considered another idea for LCS detection which did not appear in this dissertation. Based on the coherence of the air masses, we hypothesized that naturally existing tracers may exist which signal LCS boundaries. Such a tracer might be any detectable scalar quantity, for example the ratio of relative humidity to turbulent kinetic energy. A UAS might carry a suite of atmospheric sensors, not just for wind, and utilize them as well for detecting and tracking LCS. We did not pursue this research direction due to a lack of dense atmospheric sampling with which to confirm the presence of LCS on the spatial scales similar to that of the aircraft. In the future, researchers with improved capability to reliably detect LCS in the atmosphere might pursue this open question.

One of the fundamental assumptions in our LCS detection methodology is that the UAS provides exact wind information along its trajectory. This assumption is far from justified, as evidenced by the difficulty in obtaining accurate wind estimates during experiments. An immediate area for further research is to what extent our LCS detection approach degrades in the presence of realistic wind measurement noise and bias. A fundamentally related inquiry is to discover what constitutes typical airborne wind estimation error. These questions demonstrate the conceptual link, made explicit in Remark 1.1, between detecting atmo-
spheric coherent structures and improving the accuracy of wind measurement during flight. Many aspects of a small UAS which make it difficult to model also make it well-suited as a wind measurement sensor. Future research might explore what types of maneuvers provide the highest quality learning dataset from which to begin system identification. Our work suggests that wisely combining computational aerodynamic methods with data-driven approaches improves model fidelity, this is potentially a rich area for further inquiry.

In researching airborne wind estimation, we identified a previously unstudied topic: the impact of common dynamic modeling simplifications on the estimate accuracy. The methodology developed in this dissertation is a reasonable first step, but further exploration into maneuvering flight and more realistic wind fields is warranted. An interesting study would be to explore what aircraft designs (as reflected in their dynamic modeling) improve the accuracy of wind measurement. The most immediate opportunity for continuing this work is in exploring the synergy between modeling error and sensor error. It may be that the best FDMs are not the most “accurate,” but instead those which are robust to typical forms of sensor error. While this discussion has focused primarily on topics suited to theoretical and simulation study, ultimately truth is found in detailed experimental pursuits.

At the heart of every estimation task is combining sensor data with predictive models, and so it is for wind estimation as well. The majority of flight dynamic models in research literature assume uniform or slowly varying wind in order to greatly simplify the equations. This dissertation’s core theoretical contribution is the detailed formulation of a Flight Dynamic Model (FDM) for a fixed-wing aircraft which does not neglect spatial and temporal wind variation. While some of our assumptions have known generalizations, such as accounting for the Earth’s movement and curvature, others are the focus of ongoing research. The rigid-body assumption, which neglects structural flexing, may introduce significant error when operating in turbulent wind fields. Perhaps the most problematic modeling assumption we utilize is to neglect unsteady aerodynamic effects. Characterizing unsteady aerodynamic forcing in a suitable manner is the subject of ongoing work, and may significantly improve the FDM fidelity. In this dissertation, we provide some theoretical grounding and a critical review of some existing steady aerodynamic theories, but all admit clear opportunity for improvement.

In summary, wind measurement from an inexpensive fixed-wing UAS remains a rich area for future research and development. Our sincerest hope is that the contributions provided in this dissertation’s body of work contribute some foundational capabilities from which others might advance further.


[91] Binbin Lin, Amir E. BozorgMagham, Shane D. Ross, and David G. Schmale, III. Small fluctuations in the recovery of fusaria across consecutive sampling intervals with unmanned aircraft 100m above ground level. Aerobiologia, 29:45–54, May 2013. Published online 2012.


Appendices
Appendix A

Miscellaneous Information

This appendix contains some additional supporting concepts utilized within the dissertation.

A.1 The “Hat” Operator

The \((\cdot): \mathbb{R}^3 \to \mathbb{R}^{3x3}\) operator is defined for any vector \(\mathbf{v}\) as

\[
\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \quad \mathbf{\hat{v}} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}
\] (A.1)

The motivation for the “hat operator” is the connection between the cross product and matrix multiplication shown by

\[
\mathbf{v} \times \mathbf{u} = \mathbf{\hat{v}} \mathbf{u}
\] (A.2)

where \(\mathbf{u} \in \mathbb{R}^3\) is any arbitrary vector. Likewise, we borrow notation for the \((\cdot)^\vee\) operator from [105] to select the appropriate terms of the skew-symmetric matrix \(\mathbf{\hat{v}}\) to return the ordered components in the original vector

\[
\mathbf{v} = (\mathbf{\hat{v}})^\vee
\] (A.3)

To “rotate a hatted vector”, use the matrix transformation equation

\[
Y \mathbf{\hat{v}} = R_{YX}(X^\prime \mathbf{\hat{v}}) R_{XY}
\] (A.4)

A.2 PID Controller Equations

PID control [109] is likely the most common controller form utilized across engineering systems. This section presents the details of the relevant equations for PID controllers utilized within the dissertation’s research work.

Every PID controller accepts an error signal \(e(t)\) as its input. The proportional (P) and integral (I) contributions are obtained from this error and its integral, respectively. For the
A.2. PID Controller Equations

derivative (D) contribution, a filtered derivative of the error signal is used, its implementation is described in the next subsection. Thus the PID controller’s output signal \( u(t) \) is calculated as

\[
u(t) = P \left( e(t) + I \int_{t_0}^{t} e(\tau) d\tau + I_0 + D \dot{e}_f(t) \right)
\]

where \( I_0 \) is the initial condition on the integrator and \( \dot{e}_f(t) \) is the filtered derivative of the error signal.

The filtering-and-differentiation operation can be expressed in transfer function form as

\[
DF(s) = \frac{\dot{E}_f(s)}{E(s)} = \frac{Ms}{s + M}
\]

where \( M \) is the filter coefficient and we have slightly abused notation to express the Laplace transform of the filtered-derivative signal \( L\{\dot{e}_f(t)\} \) as \( \dot{E}_f(s) \). This filtering-and-differentiation is implemented in the time domain via a first-order dynamic system with input \( e \), output \( \dot{e}_f \), and an internal state \( x_d \)

\[
\begin{align*}
\dot{x}_d &= -M x_d - Me \\
\dot{e}_f &= M x_d + Me
\end{align*}
\]

where \( E(s) \) and \( E_f(s) \) are the Laplace transforms of (unfiltered) error \( e(t) \) and filtered error \( e_f(t) \) respectively. From the time-domain formulation, we see that an initial condition for the internal state \( x_d(0) \) must be specified, but in this dissertation it is always set to zero.

As a miscellaneous note, this state-space realization is not unique, for instance

\[
\begin{align*}
\dot{x}_d &= -M x_d + e \\
\dot{e}_f &= -M^2 x_d + Me
\end{align*}
\]

is also valid. This can be verified by computing the transfer function

\[
DF(s) = C (sI - A)^{-1} B + D
\]

where \((A, B, C, D)\) are defined in the standard LTI format.
A.2.1 PID Transfer Function

For the filtered PID controller available in Simulink\(^1\), the transfer function from the (PID input) error \(E(s)\) to (PID output) controller calculated result \(U(s)\) is

\[
\frac{U(s)}{E(s)} = C_{PIDF}(s) = P \left(1 + \frac{I}{s} + D \frac{N}{1 + Ns^2}\right) = P \frac{s(s + N) + I(s + N) + NDs^2}{s(s + N)} = P(ND + 1) \left(\frac{s^2 + \frac{N+I}{ND+1}s + \frac{NI}{ND+1}}{s(s + N)}\right)
\]

(A.12)  \hspace{1cm} (A.13)  \hspace{1cm} (A.14)

Two observations demonstrate useful (and expected) simplified forms. First, substituting \(I = 0\) (no I control) places a zero exactly at the origin to cancel the pole there. The remaining controller is recognized as filtered PD

\[
C_{PD}(s) = P(ND + 1) \left(\frac{s(s + \frac{N}{ND+1})}{s(s + N)}\right) = P(ND + 1) \left(\frac{(s + \frac{N}{ND+1})}{(s + N)}\right)\]

(A.15)  \hspace{1cm} (A.16)

Likewise, instead substituting \(D = 0\) (no D control) both eliminates the derivative term and causes the \((s + N)\) pole to cancel with a \((s + N)\) zero, becoming PI control

\[
C_{PI}(s) = P \left(\frac{(s + N)(s + I)}{s(s + N)}\right) = P \left(\frac{s + I}{s}\right)
\]

(A.17)  \hspace{1cm} (A.18)

A.3 Math Notation and Identities

This section used to record some math notation and identities used in this dissertation.

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\(^1\)We present Simulink’s “Ideal” form instead of the “Parallel” form.
### A.3.1 Jacobian Matrix Convention

As a Jacobian convention, for vector-function \( \mathbf{\beta}(\mathbf{\theta}) \) of vector \( \mathbf{\theta} \),

\[
\frac{\partial \mathbf{\beta}}{\partial \mathbf{\theta}} = \begin{bmatrix}
\frac{\partial \beta_1}{\partial \theta_1} & \frac{\partial \beta_1}{\partial \theta_2} & \cdots & \frac{\partial \beta_1}{\partial \theta_j} \\
\frac{\partial \beta_2}{\partial \theta_1} & \frac{\partial \beta_2}{\partial \theta_2} & \cdots & \frac{\partial \beta_2}{\partial \theta_j} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial \beta_i}{\partial \theta_1} & \frac{\partial \beta_i}{\partial \theta_2} & \cdots & \frac{\partial \beta_i}{\partial \theta_j}
\end{bmatrix}
\]  
(A.19)

Following this convention, for a scalar-function \( \alpha(\mathbf{\theta}) \) and vector-function \( \mathbf{\beta}(\mathbf{\theta}) \) it can be shown that

\[
\frac{\partial}{\partial \mathbf{\theta}} (\alpha \mathbf{\beta}) = \mathbf{\beta} \frac{\partial \alpha}{\partial \mathbf{\theta}} + \alpha \frac{\partial \mathbf{\beta}}{\partial \mathbf{\theta}}
\]  
(A.20)

### A.3.2 Time-Derivative of DCM

Another identity is the time derivative of a rotation matrix (a.k.a. Direction Cosine Matrix)

\[
\dot{\mathbf{R}}_{XY} = [\mathbf{R}_{XY}] \dot{\mathbf{\omega}}_{Y/X}.
\]  
(A.21)

For this equation, a vector \( \mathbf{v} \) in the \( X \)-frame (labeled \( X \mathbf{v} \)) is found from the same vector \( \mathbf{v} \) in the \( Y \)-frame (\( Y \mathbf{v} \)) via

\[
X \mathbf{v} = [\mathbf{R}_{XY}] Y \mathbf{v}
\]  
(A.22)

where \( \mathbf{\omega}_{Y/X} \) represents the angular velocity of the \( Y \)-frame with respect to the \( X \)-frame, expressed in the \( Y \)-frame. Using the preferred notation of this dissertation, this can be applied for \( E \) (inertial Earth) and \( B \) (body) frames as

\[
\frac{Ed}{dt}(\mathbf{R}_{EB} B \mathbf{X}) = \mathbf{R}_{EB}(B \mathbf{\omega}_{E/B} \times B \mathbf{X}) + \mathbf{R}_{EB} B \frac{Ed}{dt}(B \mathbf{X})
\]  
(A.23)

### A.3.3 Substantial Derivative

Consider a vector \( \mathbf{W}(\mathbf{X}, t) \) from a vector-field \( \mathbf{W} \) over space \( \mathbf{X} \) and at time \( t \). If the point-of-interest \( \mathbf{X} \) is moving in the field with velocity \( \dot{\mathbf{X}} \), the apparent time-rate-of-change of \( \mathbf{W} \) is called the substantial derivative

\[
\frac{D}{Dt} \mathbf{W} = \frac{\partial \mathbf{W}}{\partial t} + \left[ \frac{\partial \mathbf{W}}{\partial \mathbf{X}} \right] \dot{\mathbf{X}} = \frac{\partial \mathbf{W}}{\partial t} + (\dot{\mathbf{X}} \cdot \nabla) \mathbf{W}
\]  
(A.24)
A.4 ArduPilot’s Wind Estimator

At the core of ArduPilot’s state estimator is a discrete-time Extended Kalman Filter [7] which utilizes simple dynamic models. Several versions have been developed which include (or omit) various elements and thus often conflicting documentation exists. At the time we examined the derivations (2017 January), the state vector $x$ has the following elements:

- a quaternion (4-element) representation of orientation $q$
- a 3d inertial velocity $V$
- a 3d inertial position $X$
- 3 IMU angular biases $B_\omega = (B_p, B_q, B_r)^T$
- 3 IMU linear biases $B_X = (B_x, B_y, B_z)^T$
- a 2d wind velocity $W_2 = (W_N, W_E)^T$
- 3 Earth magnetic field components $m_E = (m_{E,N}, m_{E,E}, m_{E,D})^T$
- 3 aircraft magnetic field components $m_A = (m_{A,x}, m_{A,y}, m_{A,z})^T$

The sensor data $z$ utilized by the EKF are

- 3d inertial velocity $V$ (from GPS)
- 3d inertial position $X$ (from GPS)
- airspeed $V_r$ (from pitot-tube)
- 3d magnetic field vector $m_A = (m_{A,x}, m_{A,y}, m_{A,z})^T$ (from a magnetometer)

The measurement model relating the state components and sensor data is

$$h(x) = \begin{pmatrix}
V \\
X \\
\sqrt{(V_N - W_n)^2 + (V_E - W_E)^2 + V_D} \\
R(q)^T m_E + m_X
\end{pmatrix}$$ (A.25)

The “input” to this EKF consists of

- a timestep amount $\Delta t$
A.5. Pitot-tube vs Airspeed

- a 3-element change in orientation $\Delta \Theta$
- a 3-element change in body velocity $\Delta V$

Thus the discrete dynamic model at the core of ArduPilot is

$$\mathbf{x}_{t+\Delta t} = f(\mathbf{x}_t) = \begin{pmatrix}
    f_q(q, B_\omega, \Delta \Theta) \\
    V + g_E \Delta t + R(q)(\Delta V - B_X) \\
    X + V \Delta t
\end{pmatrix}$$

(A.26)

where $g_E$ is the Earth’s gravity vector, $R$ is a rotation matrix, and $f_q$ is the standard quaternion update equation for first order approximation of the rotational dynamics

$$f_q(\ldots) = q \cdot \begin{pmatrix}
    1 \\
    \frac{1}{2} \begin{pmatrix}
        \Delta \Theta^x - B_p \\
        \Delta \Theta^y - B_q \\
        \Delta \Theta^z - B_r
    \end{pmatrix}
\end{pmatrix}$$

(A.27)

where the ‘$\cdot$’ indicates quaternion multiplication.

This is the base-level EKF estimator which produces ArduPilot’s wind estimates. Two important qualities are its simplicity and that it is vehicle independent, making it useful for both multi-rotor and fixed-wing UAS.

A.5 Pitot-tube vs Airspeed

For an aircraft with nonzero airspeed $V_r$, angle of attack $\alpha$, and angle of sideslip $\beta$, it can be easy to confuse pitot-tube-measured speed $u_r$ with airspeed $V_r$, but they are not the same. To find the connection, rewrite equation (2.11) for $v_r = v_r(\beta; V_r)$ and $w_r = w_r(\alpha, u_r)$ and plug them into the definition $V_r = V_r(u_r, v_r, w_r)$ to find several potentially useful forms

$$V_r = \sqrt{u_r^2 + (V_r \sin \beta)^2 + (u_r \tan \alpha)^2}$$

(A.28)

$$V_r = u_r \sqrt{\frac{1 + \tan^2 \alpha}{1 - \sin^2 \beta}}$$

(A.29)

$$V_r = u_r \sqrt{\frac{1}{(\cos^2 \alpha)(\cos^2 \beta)}}$$

(A.30)