DESIGN OF A FRAMELESS TORQUE MOTOR

by

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I. INTRODUCTION

Specifications

A torque motor is a motor primarily used for slow speed or positioning applications. The type being investigated is a permanent magnet direct current motor. This is a frameless unit which is directly connected to its load. The response is very fast since there is no gear backlash and the torque to inertia ratio is much higher than for geared systems.

These motors are normally specified as follows:

1. Maximum volts and maximum amperes. Usually the motor is supplied by a constant current source.
2. Stator outside diameter.
3. Rotor inside diameter.
4. Overall axial length of assembled unit.
5. Peak torque.

The design specifications for the torque motor considered in this investigation are: 2.22 amperes at 50 volts, stator outside diameter 4.089 inches, rotor inside diameter 2.00 inches, axial length 1.093 inches, peak torque 1.0 pound-foot.

Design Considerations

The electrical specifications are to be considered as minimum requirements and should be exceeded as much as possible. However, the primary objective is to keep the output torque per watt as high as possible. Therefore the engineer must design a motor which will always meet these minimum requirements, considering variations in raw materials, manufacturing tolerances, etc.

The possibility of deriving a set of simultaneous equations which would describe the design parameters of the most efficient motor in terms of torque per watt input was considered. The nonlinearity of ferrous materials makes the design choice very difficult. Also physical limitations may make the motor calculated by this method impossible to manufacture. Therefore the iteration method, where each design possibility is explored, seems most desirable.

II. DESIGN PROCEDURE

Physical Dimensions Calculations

The following information will provide a detailed explanation of a torque motor design procedure, including derivations of the design equations. Figure 1 is a drawing of a typical torque motor with the various dimensions named.

A minimum rotor outside diameter, minimum rotor stack thickness, and minimum teeth width are chosen from past experience, so their size will be considerably less than that of an optimum design. All the other internal dimensions, such as insulation thickness, are very nearly fixed because of the physical limitations of materials.

First a complete design calculation is made for a motor which has the minimum dimensions for the rotor outside diameter, rotor stack thickness, and teeth width. The teeth width is increased in small increments to a value larger than optimum. The complete calculations are made and performance data is printed for each value of teeth width. The rotor outside diameter and the rotor axial thickness are held constant for these calculations.

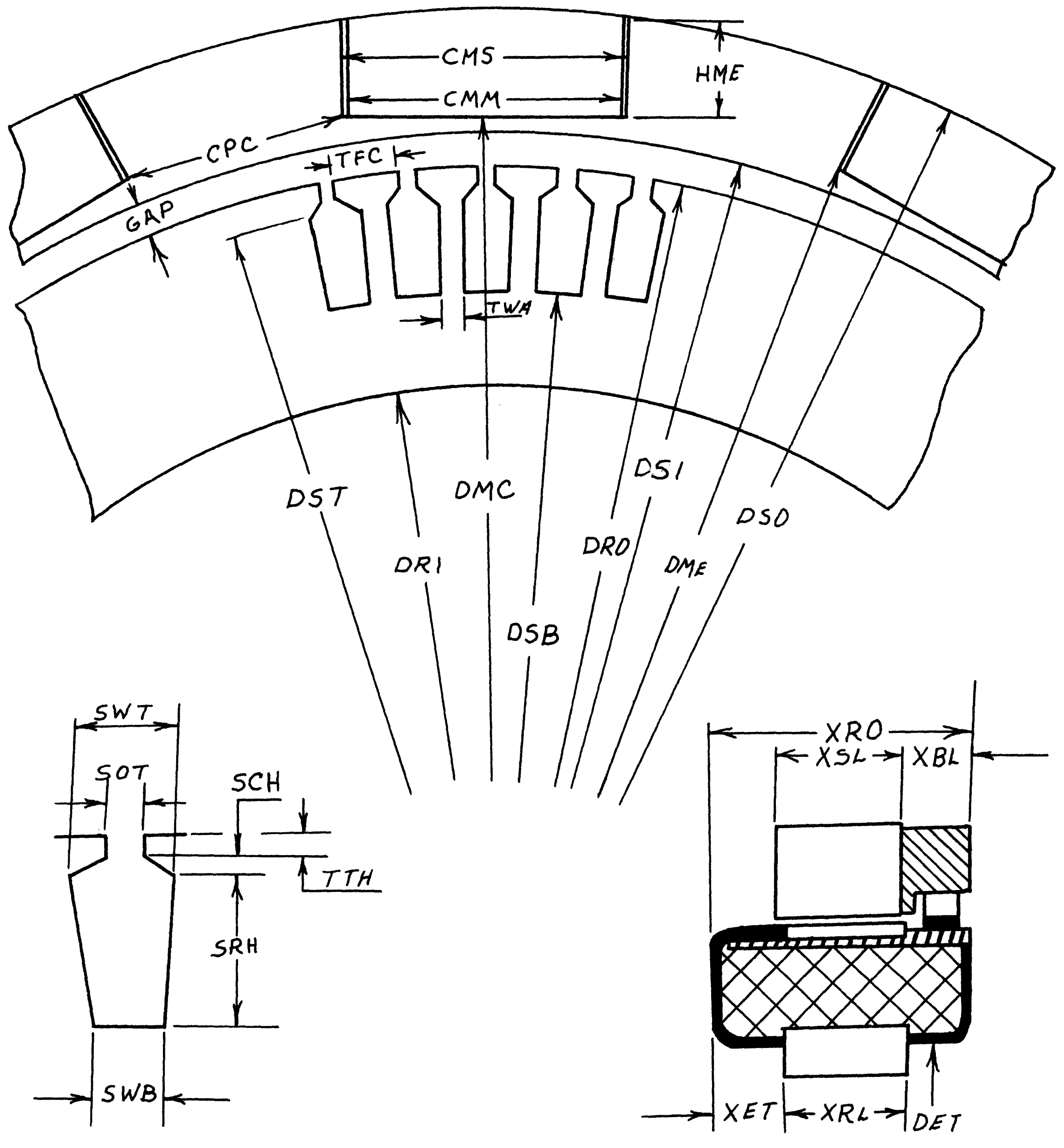
CMS
CMM
HME
CPC
TFC
GAP
TWA
DST
DMC
DSI
DRI
DRO
DME
DSO
DSB
SWT
SOT
SCH
TTH
SRH
SWB
XRO
XSL
XBL
XET
XRL
DET

Figure 1.

Next the rotor axial length is increased in small increments to a value larger than optimum. For each value of the rotor axial length, the tooth width is varied over its complete range as above. The rotor outside diameter is held constant. The complete performance data is printed for each design.

Finally the rotor outside diameter is increased in small increments to a value larger than optimum. At each increment of the rotor outside diameter, the rotor axial length is varied over its complete range, and the tooth width is varied over its complete range for each increment of the rotor axial length.

The computer output data is a tabulation of the performance characteristics for each calculated design. The optimum design is chosen from this data by the engineer. The optimum design motor is the one which will produce the highest torque per watt input.

All the internal parts and dimensions of a typical torque motor are defined below.

The stator axial thickness (XSL) is determined by the motor overall axial length (XRO) and the brush rigging (XBL). The brush rigging is normally made as thin as possible so the magnet volume in the stator can be increased to a maximum.

The number of poles and slots are determined primarily by the torque ripple requirements and the overall radial thickness of the motor. The torque ripple amplitude decreases as the number of slots per pole increases. The normal torque motor has between four and six slots per pole. The stator radial thickness and outside diameter (DSO) physically control the number of poles. The magnet slot length (CMS) is primarily determined by the size of the mounting holes in the stator. The magnet slot length is made as large as possible, and the magnet length (CMM) allows minimum clearance. The bridge between the magnet and the stator inside diameter (DSI) short-circuits the magnet flux, so the bridge thickness (BT) is made as small as physically possible. The flux through this bridge is limited only by the gap in the magnet slot and the saturation characteristics of the stator iron. The most efficient design allows minimum flux to be wasted through this bridge.

All the calculations for this procedure will be made on a per pole basis.

Magnet

The magnet area is two times the cross-sectional area of the magnet because two magnets work in parallel to provide the pole flux.

$$\text{RMA} = 2(\text{XSL})\ \text{HME}$$

The minimum magnet height is used in determining the magnet area to make calculated performance data tend

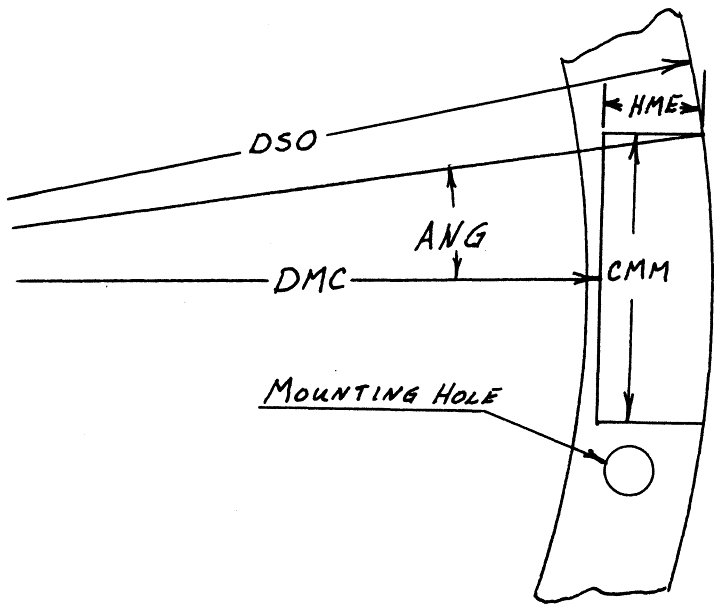

Figure 2.

to be conservative. Figure 2. illustrates the trigonometry involved in calculating the minimum magnet thickness.

$$HME = \frac{DSO\ COS\ (ANG)\ -\ DMC}{2}$$

The magnet length (CMM) is as long as possible allowing room for mounting holes for the stator. The effective magnet length per pole is actually one half the physical magnet length. Two magnets work in parallel to furnish the flux for one pole. Therefore only one half the magnet length provides magnetomotive force for one pole as shown in Figure 3.

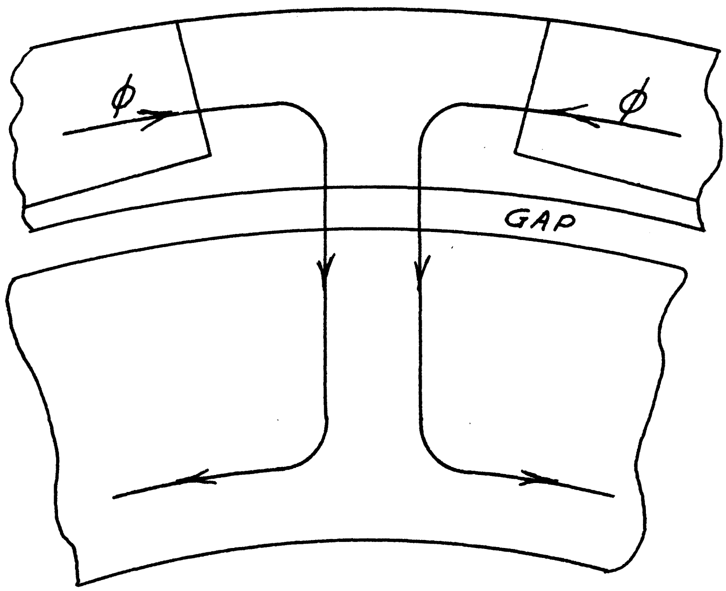

Figure 3.

Air Gap

The pole pitch (PP) is the circumference of the circle formed by the diameter in the center of the air gap divided by the number of poles (PNO).

$$PP = \frac{(DRO + GAP)\pi}{PNO}$$

The air gap area is the product of the pole pitch (PP), pole embrace (PE), and the effective axial length of the air gap (XRL + GAP).

$$RAG = PP\ (PE)\ (XRL + GAP)$$

The pole embrace is a factor which accounts for the fact that the physical air gap area is not 100% effective.

This is caused by the flux density over the pole not being uniform and the necessary neutral point between a north and a south pole. Actual measurements of the flux pattern show the pole effective area on standard motor configurations to be approximately 75% of the actual pole area. A typical flux pattern is shown on Figure 4.

The effective length of the air gap is longer than the physical air gap. The effective length is greater because the rotor has slots instead of a smooth iron cylindrical surface. The increase in effective length is dependent upon the physical gap length, slot opening, and tooth pitch. One method of correcting for this apparent increase in air gap length is the introduction of a constant known as Carter's Coefficient. The equation for determining Carter's Coefficient is as follows:

$$CCO = \frac{5\ (GAP + STO)\ TP}{5\ (GAP + STO)\ TP - SOT\ (SOT)}$$

The tooth pitch (TP) is the outside circumference of the rotor divided by the number of teeth.

$$TP = \frac{DRO\ \pi}{SNO}$$

The slot opening (SOT) is determined by the winding wire size and the thickness of the slot insulation which must pass through it. This is kept as small as possible to keep the effective air gap short.

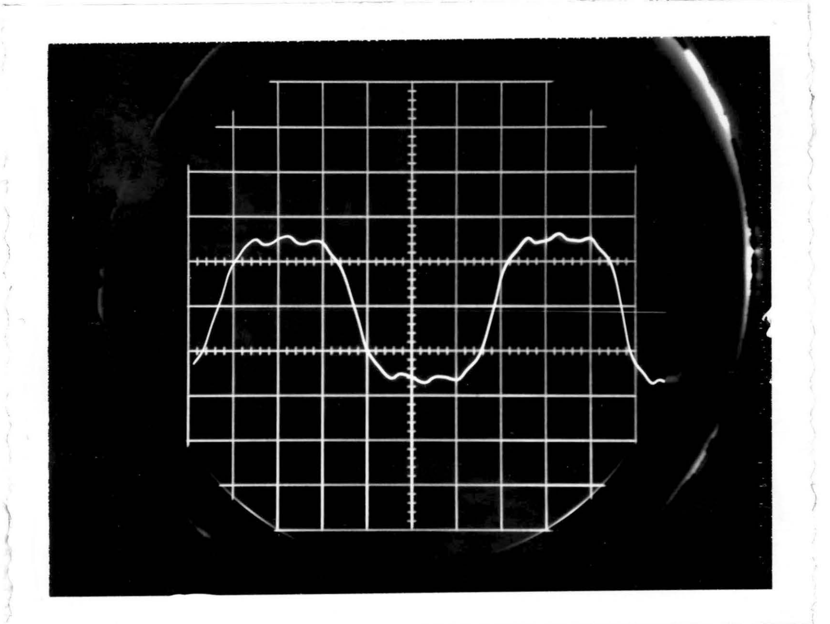

Figure 4.

The calculated effective air gap (GAPE) is the product of the physical air gap and Carter's Coefficient.

$$GAPE = GAP\ (CCO)$$

The minimum physical air gap is determined by the eccentricities in mounting the motor components. When maximum performance is required, the air gap length must be a minimum. However when lower performance is acceptable or smaller torque ripple is required, the air gap may be made larger.

Teeth

The tooth area per pole is the product of the tooth width, rotor stack length, and the number of teeth divided by the number of poles. The effective tooth area per pole (RTH) is the actual area multiplied by the pole embrace and a stacking factor. The pole embrace reduces the effective area because all of the air gap area is not effective. The stacking factor reduces the effective area because part of the rotor iron volume is occupied by a bonding agent and oxidized iron between the laminations.

$$RTH = \frac{TWA\ (SNO)\ PE\ (SF)\ XRL}{PNO}$$

The most efficient use of the rotor teeth as magnetic circuits is obtained by making the sides of a tooth parallel. Therefore the average tooth width (TWA) is the actual tooth width. This investigation will not consider slots which do not form parallel tooth sides.

The area of the tooth tip (TTH) and (SCH) affects the total magnetic circuit very little; hence, the tooth tip areas will be neglected and the teeth will be considered straight.

The magnetic length of the teeth is determined by the rotor outside diameter (DRO), tooth tip dimensions (TTH) and (SCH), number of teeth (SNO), slot width at the bottom (SWB), and average tooth width (TWA).

The diameter at the top of the slot (DST) is the rotor outside diameter (DRO) minus two times the tooth tip dimensions (TTH) and (SCH).

$$DST = DRO - 2\ (TTH + SCH)$$

The diameter at the bottom of the slot (DSB) makes a circle whose circumference is the slot width at bottom (SWB) plus the average tooth width (TWA) times the number of teeth (SNO).

$$DSB = \frac{SNO\ (SWB + TWA)}{\pi}$$

The slot width at bottom (SWB) is generally determined by the physical limitations of the slot punch for small motors. However this will increase for larger diameter motors.

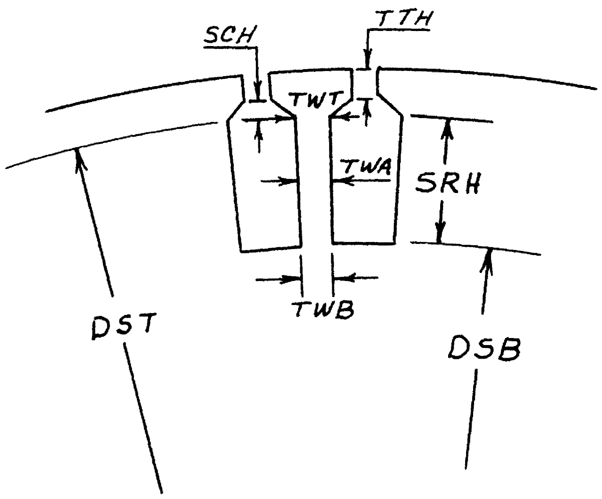

Figure 5.

The tooth length (SWT) is one half the difference between the diameter at the top of the slots (SWT) and the diameter at the bottom of the slots (SWB).

$$SRH = \frac{DST - DSB}{2}$$

Rotor Core

The rotor core area (RCR) is actually double the cross sectional area of the iron below the slots because the flux per pole splits and flows two directions in the core. The effective rotor core area (RCR) is the difference between the diameter at the bottom of the slots (DSB) and the rotor inside diameter (DRI) multiplied by the rotor core axial length (XRL) and the stacking factor (SF).

$$RCR = (DSB - DRI)\ XRL\ (SF)$$

Since two sections of the core are in parallel for the flux per pole, the magnetic core length (CRL) is one half the physical distance between poles as shown in Figure 3. The effective core length is the circumference of the average core diameter (DSB + DRI)π/2 divided by two times the number of poles.

$$CRL = \frac{(DSB + DRI)\pi}{4\ PNO}$$

Yoke

The yoke area is assumed to be the smallest area in the stator through which the flux must pass. The distance between adjacent magnet slots (CPC) must be determined in order to calculate the yoke area (RYK) as shown in Figure 6.

BANG is the angle whose tangent is the magnet slot length (CMS) divided by the diameter at the center of the magnet (DMC).

$$BANG = \tan^{-1} CMS/DMC$$

The diameter at the end of the magnet (DME) is equivalent to the square root of the sum of the squares of the diameter at the center of the magnet (DMC) and the magnet slot length (CMS).

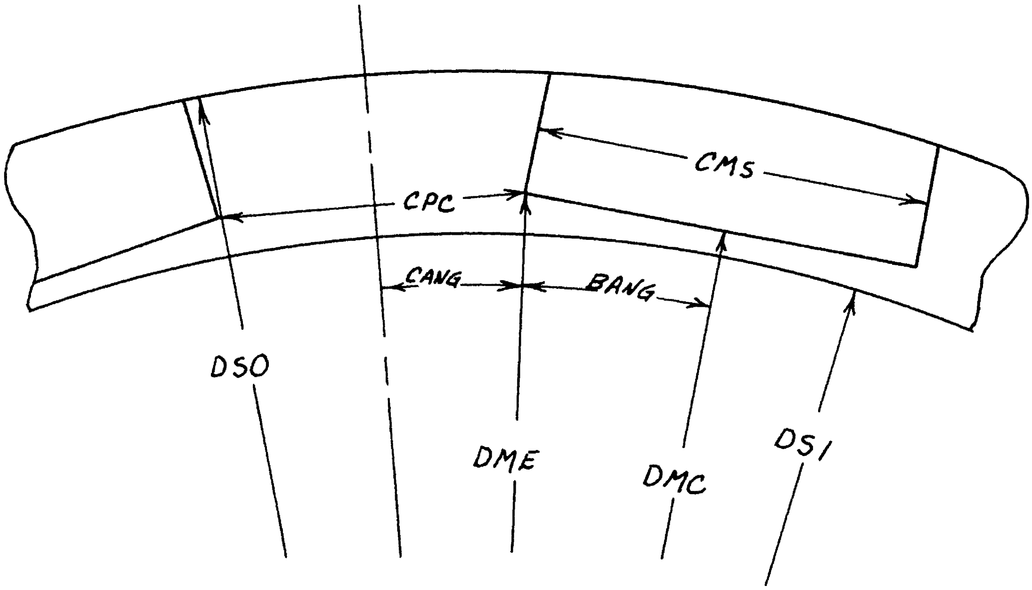

Figure 6.

$$DME = \sqrt{CMS^2 + DMC^2}$$

The angle CANG is one half the degrees per pole minus angle BANG.

$$CANG = 180/PNO - BANG$$

Therefore the distance between adjacent magnet slots (CPC) is the diameter at the end of the magnet times the sine of angle CANG.

$$CPC = DME \sin (CANG)$$

The minimum yoke area (RYK) is the product of the stator axial length (XSL) and the distance between magnet slots (CPC).

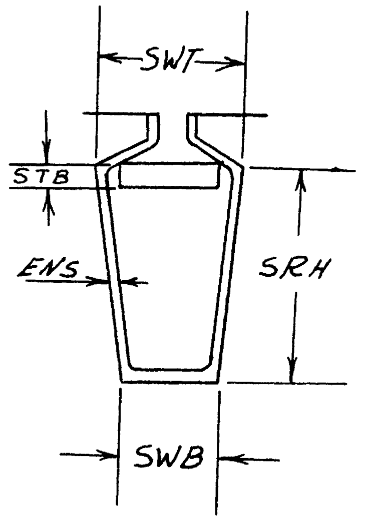

Figure 7.

RYK = CPC (XSL)

A stacking factor is not used for this area because the stator is normally made of solid iron.

Since the actual yoke area is not constant and the flux is constricted at one point, the effective length (YKL) is assumed to be one third of the stator radial thickness.

YKL = (DSO - DSI)/6

Winding

The lower slot width (SWB) is given and the slot length (SRH) has been previously determined. The upper slot width (SWT) must be calculated in order to determine the effective slot area (RSE).

The upper slot width (SWT) is the slot pitch at the diameter at the top of the slots (DST) minus the tooth width (TWA).

$$SWT = (DST)\ \pi/SNO - TWA$$

The slot is assumed to be a parallelogram whose area is the average width times the height allowing for slot insulation and commutator bar shank thickness (STB) which passes through the slot as shown in Figure 7. This effective area is divided by 2 because each slot is shared by two coil sides.

$$RSE = \frac{[SRH - STB - 2(ENS)]\left[\frac{SWB + SWT}{2} - 2(ENS)\right]}{2}$$

The effective end turn area (EWA) is the product of the end turn extension and radial thickness of end turns minus the thickness of the epoxy over the end turns (TEE) and the thickness of the insulation on the rotor stack (ENE). The winding area is shown in Figure 8.

$$EWA = \left[\frac{XRO - XRL}{2} - (ENE + TEE)\right]\left[\frac{DST - (DET + 2(TEE))}{2}\right]$$

Using the effective area of the slot and end turns the maximum number of turns for each area is determined. Each turn in the coil is assumed to occupy a square whose sides equal the diameter over insulation (DWO) of the wire. The fill factors of the slot (CFS) and end

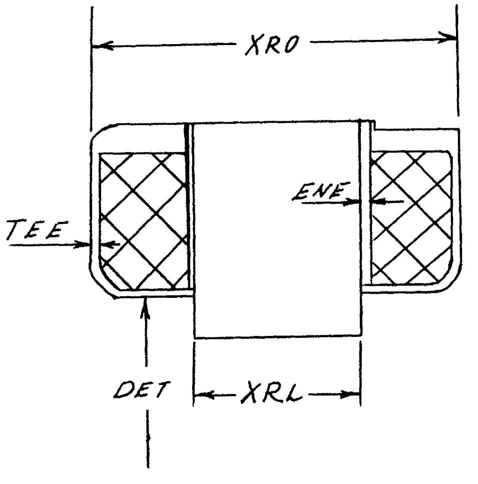

Figure 8.

turns (CFE) are determined from experience and depend upon the tightness of the winding desired in the motor. The fill factors account for the fact that the available area can never be completely filled by the round wire. Usually these factors are different with the end turn fill factor being smaller.

The allowable turns for the slot (KTPC1) equals the effective area (RSE) times the fill factor (CFE) divided by the area taken by one turn (DWO^2).

$$KTPC1 = \frac{RSE\ (CFS)}{(DWO)^2}$$

The allowable turns for the end turns (KTPC2) equals the effective area (EWA) times the fill factor (CFE) divided by the maximum number of crosses in the end turns

(CP) and the area taken by one turn (DWO^2). The maximum number of crosses in the end turns (CP) is usually the coil pitch plus one.

$$KTPC2 = \frac{EWA\ (CFE)}{CP\ (DWO)^2}$$

The two allowable turns per coil (KTPC1 and KTPC2) are compared and the smaller one becomes the actual turns per coil.

Flux Calculations

The flux calculations determine the amount of flux which will result from the previously defined permanent magnet and magnetic circuit. This flux is found by assuming a small initial value of flux. The MMF drop in the complete circuit is calculated and compared with the MMF rise in the magnet. The flux is then increased in predetermined steps until the drop in the magnetic circuit equals the rise in the magnet.

Since the saturation curves of the iron and the magnets are not linear, their straight line approximations have been assumed for the computer calculations. Their actual curves along with the straight line approximations are shown on Figures 9 and 10.

Air Gap

The air gap flux (FAG) is defined as the lines of flux which link the active conductors in the armature.

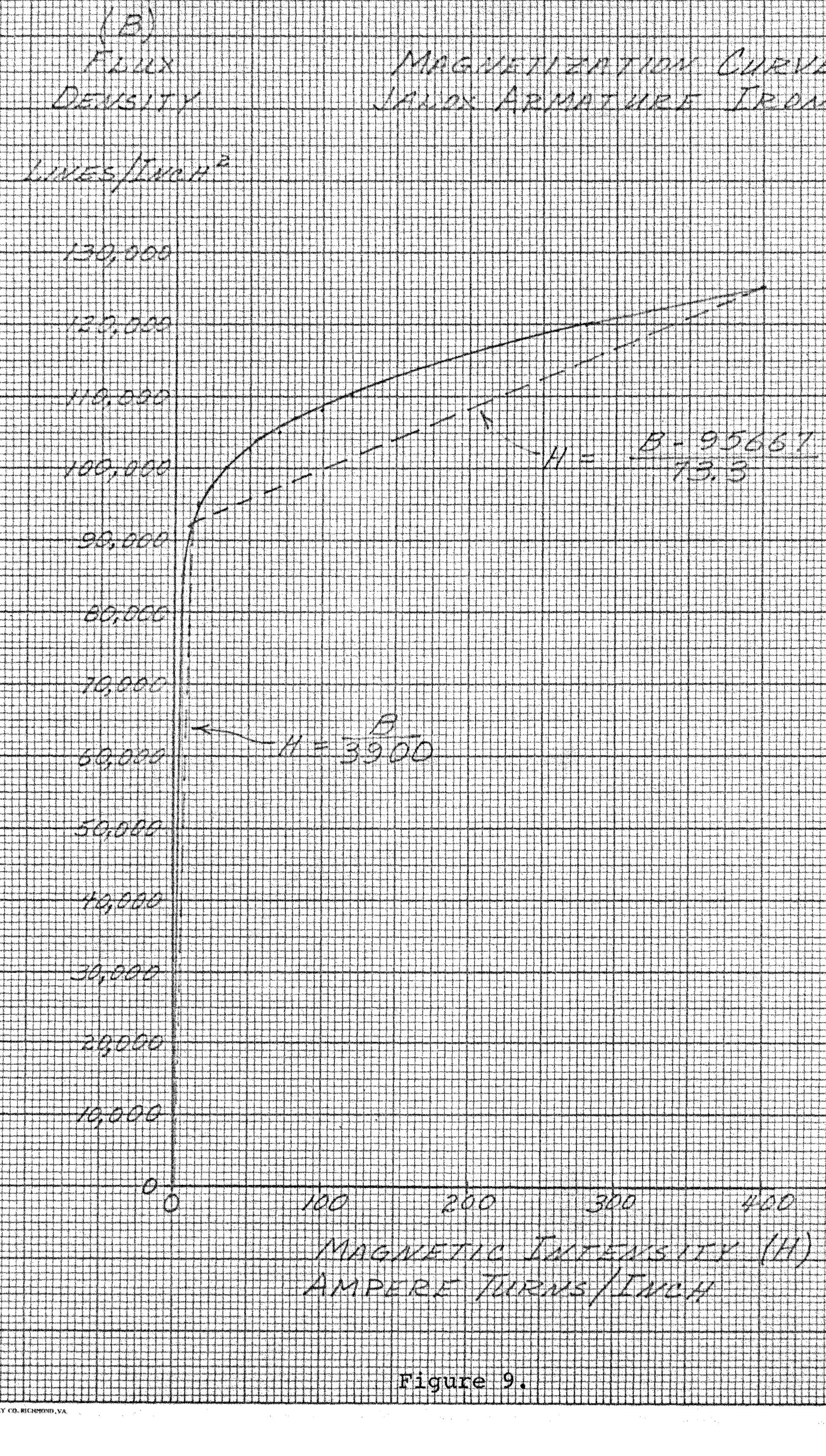
(B)
FLUX
DENSITY
LINES/INCH²
MAGNETIZATION CURVE
JALOX ARMATURE IRON
130,000
120,000
110,000
100,000
90,000
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
0
100
200
300
400
H = (B - 95667) / 73.3
H = B / 3900
MAGNETIC INTENSITY (H)
AMPERE TURNS/INCH

Figure 9.

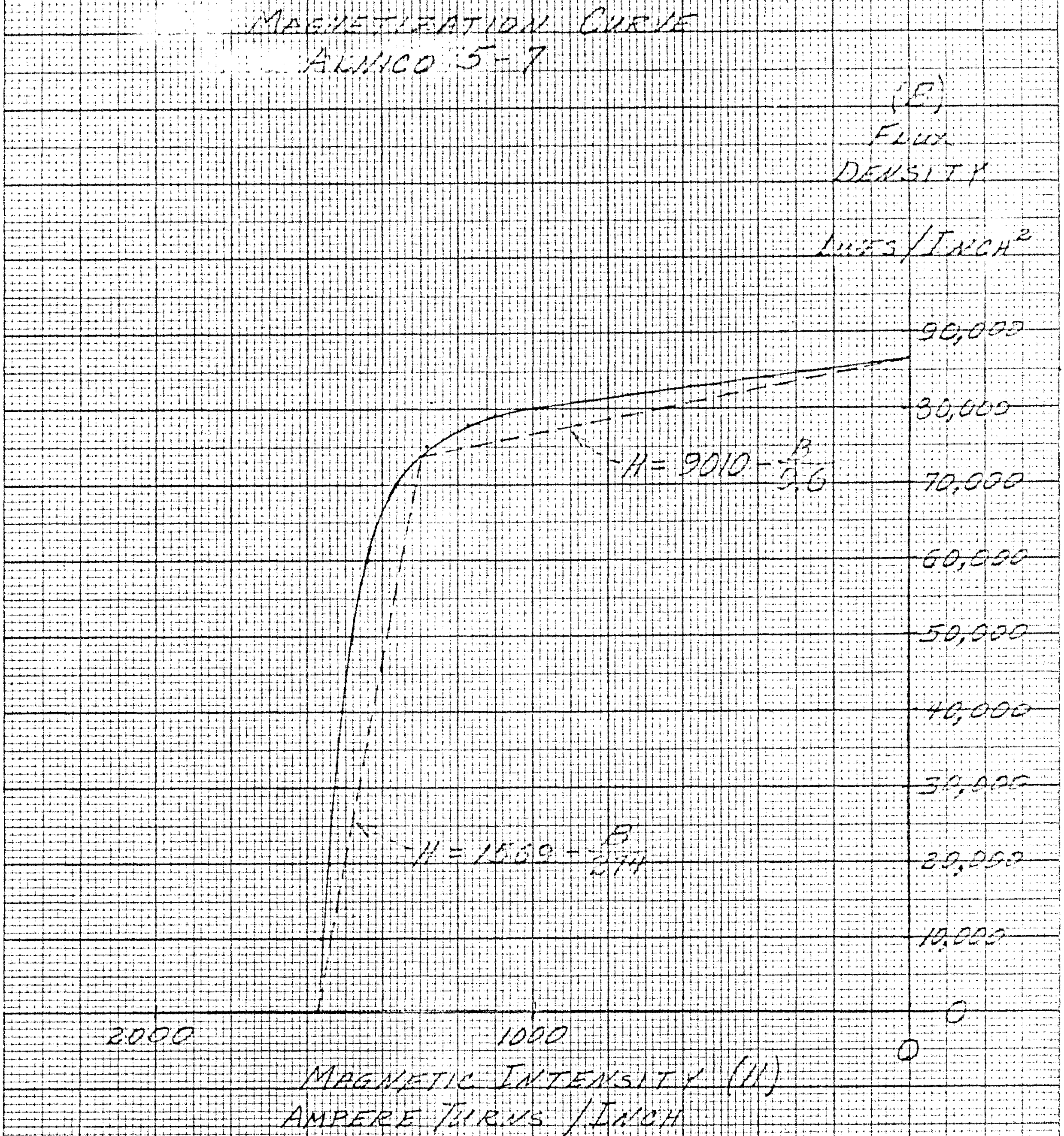
MAGNETIZATION CURVE
ALNICO 5-7
(B)
FLUX
DENSITY
LINES / INCH^2
90,000
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
0
H = 9010 - B/2.6
H = 1569 - B/24
2000
1000
0
MAGNETIC INTENSITY (H)
AMPERE TURNS / INCH

Figure 10.

This flux is assumed to be completely contained in the iron portions of the armature. This value of flux is used in all the performance calculations.

The air gap flux density is the air gap flux (FAG) divided by the effective air gap area (RAG).

$$BAG = FAG/RAG$$

Flux density in any material is equal to the permeability times the magnetic intensity ($B = \mu H$). The permeability of air is 3.19 in the English system of units. The MMF drop equals the magnetic intensity multiplied by the length. Therefore the MMF drop in the air gap (AAG) equals the flux density multiplied by the effective length divided by 3.19.

$$AAG = \frac{BAG\ (GAPE)}{3.19}$$

Magnet

The flux passing through the magnet is the air gap flux plus the flux passing through the bridge beneath the magnet and the leakage flux which is not contained in the motor iron. It is assumed that the thinnest part of the bridge under the magnet will always have a flux density of 115,000 lines per square inch. The bridge flux (FBR) is the effective bridge area times the bridge flux density (115,000 lines per square inch).

$$FBR = 115{,}000\ (BT)\ XSL\ (2)$$

The leakage flux is assumed to be proportional to the air gap flux (FAG) plus the bridge flux (FBR). The coefficient of leakage (COL) accounts for all the flux the magnet produces that is not contained in the magnetic circuit of these calculations. The flux passing through the magnet (FMA) equals the sum of the air gap flux (FAG) and the bridge flux (FBR) multiplied by the coefficient of leakage (COL).

$$FMA = (FAG + FBR)\ COL$$

The coefficient of leakage is determined from test results of similar motors. This value depends upon the physical dimensions of the motor.

The flux density of the magnet (BMA) equals the magnet flux (FMA) divided by the effective magnet area (RMA).

$$BMA = FMA/RMA$$

Since the purpose of this investigation is to design a motor with maximum performance, Alnico 5-7 magnet material was used because it has the highest energy product of any magnet material commercially available today.

Two straight lines are used to approximate the B-H curve of Alnico 5-7 for the computer calculations. One line defines the magnet magnetic intensity (HMA) for a flux density (BMA) of less than 74,000 lines per square inch.

$$HMA = 1569 - BMA/274$$

For a magnet flux density (BMA) of greater than 74,000 lines per square inch, the equation defining the straight line approximation is:

$$HMA = 9010 - BMA/9.6$$

The total magnetomotive force (AMA) available in the magnet is the magnetic intensity (HMA) multiplied by the effective length of the magnet.

$$AMA = HMA\ (CMM)/2$$

<u>Teeth</u>

Jalox armature iron is used for all the iron portions of the magnetic circuit because it has a very high permeability and a relatively low hysteresis loss. This means the rotor can carry high flux densities with relatively small hysteresis loss. The teeth flux density equals the flux in the teeth (FAG) divided by the effective area of the teeth (RTH).

$$BTH = FAG/RTH$$

Two straight lines are used to approximate the B-H curve of Jalox armature iron for the computer calculations.

One line defines the magnetic intensity (HTH) for flux densities (BTH) of less than 97,500 lines per square inch.

$$HTH = BTH/3,900$$

For a teeth flux density (BTH) of greater than 97,500 lines per square inch, the equation defining the straight line approximation is:

$$HTH = \frac{BTH - 95,667}{73.3}$$

The MMF drop in the teeth (ATH) equals the magnetic intensity of the teeth (HTH) multiplied by the tooth length (SRH).

$$ATH = HTH\ (SRH)$$

Core

The relationships for the core MMF drop are the same as for the teeth, since the magnetic material is the same.

$$BCR = FAG/RCR$$

For densities less than 97,500 lines per square inch:

$$HCR = BCR/3,900$$

For densities more than 97,500 lines per square inch:

$$HCR = \frac{BCR - 95{,}667}{73.3}$$

The core MMF drop (ACR) is:

$$ACR = HCR\ (CRL)$$

Yoke

All of the magnet flux is assumed to pass through the yoke. This figure is pessimistic in determining the MMF drop in the yoke, but the performance calculations will tend to be conservative.

The relationships for the yoke MMF drop are the same as for the teeth, since the magnetic material is the same.

$$BYK = FAG + FBR$$

For densities less than 97,500 lines per square inch:

$$HYK = BYK/3{,}900$$

For densities more than 97,500 lines per square inch:

$$HYK = \frac{BYK - 95{,}667}{73.3}$$

The yoke MMF drop is:

$$AYK = HYK\ (YKL)$$

Magnet Gap

The magnet gap (GML) is the clearance between the magnet and the magnet slot. The magnet is assumed to be centered in the slot, so the effective gap length per pole is one half the clearance between the magnet and its slot.

The magnet gap is assumed to have the same flux and area as the magnet. Therefore its flux density will be equal that of the magnet (BMA) which has been previously calculated.

The relationship for the MMF drop in the magnet gap (AMG) is the same as that of the air gap between the rotor and the stator.

$$AMG = \frac{BMA\ (GML)}{3.19\ (2)}$$

Armature

A portion of the ampere turns resulting from the current in the armature coils must be considered as demagnetizing. The percentage of these ampere turns which are considered to be opposite those of the magnet are estimated from test results of previously built motors. The total ampere turns in the armature is the current through each coil multiplied by the number of turns per coil (TPC) and the number of coils (SNO). The current per coil is one half the armature current because the winding is a simplex wave winding which has two parallel paths. To calculate the armature demagnetizing ampere turns (ADM) the total ampere turns must be divided by the number of poles (PNO) and multiplied by the percentage of demagnetizing ampere turns (ANI).

$$ADM = \frac{AMP\ (TPC)\ SNO\ (ANI)}{2\ (PNO)}$$

Total

The total MMF requirement of the magnetic circuit (ATT) is the sum of all the drops in the teeth (ATH), core (ACR), yoke (AYK), magnet gap (AMG), air gap (AAG), and armature (ADM).

$$AAT = ATH + ACR + AYK + AMG + AAG + ADM$$

The computer program compares the total MMF drop in the magnetic circuit (ATT) with the MMF rise in the magnet (AMA). If the MMF drop in the magnetic circuit is the smallest, the air gap flux (FAG) is increased by a designated amount and the MMF in each portion is calculated again. This is repeated until the MMF drop in the magnetic circuit (AAT) is very near the MMF rise in the magnet (AMA).

III. PERFORMANCE CALCULATIONS

The basic force equation for an electric conductor may be written F = BLI, where the force, magnetic field and conductor are orthogonal. F is the force (dynes) on a conductor of length L (centimeters) having a current I (absolute amperes) flowing through a magnetic field B (lines per square centimeter). The torque produced by a conductor in a slot of a motor is the product of the force on the conductor and the radius of the air gap between the stator and the rotor.

The normal torque motor has a simplex wave winding with two parallel paths. The current per conductor is one half the armature current. Since the basic force equation includes absolute amperes (I) the armature current (AMP) must also be divided by ten.

$$I = \frac{AMP}{20}$$

The flux density (B) for a normal torque motor (BAG) is the air gap flux (FAG) divided by the effective air gap area (RAG). The effective area (RAG) is the rotor circumference per pole multiplied by the rotor core length (XRL) and the pole embrace (PE).

$$B = \frac{FAG\ (PNO)}{PE\ (DRO)\ XRL\ \pi}$$

The total effective length of the conductors (L) equals the effective conductors per slot (PE (TPC) 2) multiplied by the conductor length (rotor stack length - XRL) and the number of slots (SNO).

$$L = PE\ (TPC)\ 2\ (XRL)\ SNO$$

The effective radius at which the force is applied on the conductors is the outside radius of the rotor (DRO/2). Therefore the torque produced in a motor is a product of the current per conductor (I), the flux density at the conductors (B), the total length of conductors (L), and the outside radius of the rotor (DRO/2).

$$T = BLI\ (DRO/2)$$

By substituting the motor features and canceling terms, the following torque equation is derived.

$$TOR = \frac{FAG\ (PNO)\ AMP\ (TPC)\ SNO}{20\ \pi}\ \text{dyne-cm}$$

Dividing the above equation by 70,620 changes the torque to ounce - inches.

$$TOR = 22.5\ (PNO)\ FAG\ (SNO)\ TPC\ (AMP)\ 10^{-8}\ \text{oz-in}$$

The torque sensitivity per ampere (TPA) is derived from the torque (TOR) equation by dividing by the armature current (AMP).

$$TPA = 22.5\ (PNO)\ FAG\ (SNO)\ TPC\ 10^{-8}\ \text{oz-in/Amp.}$$

To calculate the terminal resistance of the armature (RTR), the mean length of one coil (CLT) must be determined. The mean coil length is a sum of the end turn lengths and the length of wire through the slots. The length for one end turn is the average end turn circumference ((DST + DSB) $\pi/2$) multiplied by the coil pitch (CP - 1) and divided by the number of slots (SNO).

$$\text{End Turn} = \frac{(DST + DSB)\ \pi\ (CP - 1)}{2\ (SNO)}$$

The average axial length of an inductor is the rotor stack length (XRL) plus the end turn extension (XET).

$$\text{Inductor} = XRL + XET$$

There are two inductors and end turns per coil. Therefore the mean length of turn (CLT) is as follows:

$$CLT = \frac{(DST + DSB)\ (CP - 1)}{SNO} + 2\ (XRL + XET)$$

The resistance of one coil is the product of the mean length of turn (CLT), the number of turns per coil (TPC), and the resistance per foot of the wire (RES).

$$RC = \frac{CLT\ (TPC)\ RES}{12}$$

Since there are two parallel paths, half of the coils are in each path. The resistance of the armature is therefore one fourth of the resistance if all the coils were connected in series. The terminal resistance is usually 90% of the armature resistance because some of the coils are shorted by the brushes.

$$\text{RTR} = .9\ (\text{CLT})\ \text{TPC}\ (\text{SNO})\ \text{RES}\ /48$$

The performance index (PKO) is a calculated quantity which indicates the efficiency of a particular design. The most efficient design is the one which has the highest torque output per watt input.

$$\frac{\text{Torque}}{\text{Watts}} = \frac{22.5\ (\text{PNO})\ \text{FAG}\ (\text{SNO})\ \text{TPC}\ (\text{AMP})\ 10^{-8}}{(\text{AMP})^2\ \text{RTR}}$$

The performance index (PKO) must be a quantity which is constant over the full performance range of the motor. From the above expression, the torque is directly proportional to the current and the watts are directly proportional to the current squared. However, when the torque per watt ratio is a maximum the torque per square root of watts ratio is also a maximum. Therefore the torque per square root of watts will be used as a performance index (PKO). The above equation becomes:

$$\text{PKO} = \frac{\text{TOR}}{\sqrt{\text{WATTS}}} = \frac{22.5\ (\text{PNO})\ \text{FAG}\ (\text{SNO})\ \text{TPC}\ (\text{AMP})\ 10^{-8}}{\text{AMP}\ \sqrt{\text{RTR}}}$$

$$TPA = 22.5\ (PNO)\ FAG\ (SNO)\ TPC\ 10^{-8}$$

$$PKO = \frac{TPA}{\sqrt{RTR}}$$

The computer prints the performance index along with all the necessary design information. The motor designer reviews all the values of the performance index and chooses the design with the largest index as the optimum motor.

IV. DATA COMPARISON

A sample motor was used to test the computer design procedure of this investigation. The tabulations below include actual test data from the sample motor and the computer calculated data for a motor with nearly the same dimensions as the sample. The third column is the computer calculated data for the optimized design.

	Actual Motor Test Data	Calculated	Optimum
Rotor Outside Diameter	3.239 inches	3.240	3.190
Diameter at Slot Bottoms	2.432 inches	2.314	2.197
Tooth Width	.102 inches	.102	.094
Rotor Stack Length	.500 inches	.502	.542
Mean Length of Turn	.308 inches	3.06	3.05
Turns Per Coil	64	62	69
Air Gap Flux	13,000 lines	12,500	12,900
Terminal Resistance	18.0 Ohms	16.5	18.3
Torque Per Ampere	86.5 oz-in/amp	80.2	92.1
Performance Index	20.4	19.7	21.5
Torque	192 oz-in	178	205

The primary reason for the slight difference between the calculated values and the test values for the sample motor is that the slot punch was not designed for a motor with this rotor outside diameter. The slot design resulted in teeth without parallel sides. The diameter at the bottom of the slots is much less for the calculated motor than for the actual sample motor. The resulting smaller rotor core area in the calculated motor restricts the flux per pole and reduces the air gap flux, torque per ampere, performance index, and output torque.

The turns per coil of the calculated motor were less than the turns of the sample motor because the slot winding fill factor of the sample motor is .685. A fill factor of .685 is considered to be too high for good manufacturing practice. Therefore a fill factor of .58 was used for the calculated data. The smaller fill factor resulted in fewer allowable turns in the slot. The fewer turns per coil result in lower resistance, torque per ampere, performance index, and output torque.

V. CONCLUSIONS

The design procedure formulated in this investigation provides calculated data which adequately describes the performance of the test sample. The differences between calculated data and test data are very small and are explained in the section titled Data Comparison. The manufacturing test limits for any given torque motor design are $\pm 12.5\%$ tolerance on the terminal resistance and $\pm 10\%$ tolerance on the torque per ampere. The calculated performance characteristics are easily within these tolerances.

If it were possible to derive a set of design equations which perfectly describe all the parts of a torque motor, it would be impossible to build a group of the motors with performance characteristics exactly as previously calculated. Variations in iron permeability and copper resistivity plus manufacturing tolerances will make the performance characteristics vary from one motor to the next.

Since the calculated data so closely agrees with actual test data the optimum design can be chosen using the performance index (PKO) as a check point. Even though the performance may vary within tolerances, the design with the largest performance index (PKO) will have the best performance which can be obtained with the given materials.

VI. COMPUTER PROGRAM FLOW CHART

1. Read in Input Data
2. Calculate:

    Rotor Outside Diameter (DRO) = Initial Value (DROM) + K - increments

    Diameter at Center of Magnet (DMC)

    Magnet Thickness at Ends (DME)

    Stator Inside Diameter (DSI)

    Magnet Area (RMA)

    Pole Pitch (PP)
3. Calculate:

    Rotor Stack Length (XSL) = Initial Value (XRLM) + J increments

    Air Gap Area (RAG)

    Teeth Pitch (TP)

    Teeth Face Length (TFC)

    Carter's Coefficient (CCO)

    Equivalent Air Gap Length (GAPE)
4. Calculate:

    Teeth Width (TWA) = Initial Value (TWAM) + N - increments

    Teeth Area (RTH)

    Diameter at Top of Slots (DST)

    Diameter at Bottom of Slots (DBS)

    Teeth Length (SRH)

Repeat 9 Times

Repeat 9 Times

Repeat 4 Times

Core Length (CRL)

Core Area (RCR)

Yoke Area (RYK)

Yoke Length (YKL)

Slot Winding Area (RSE)

End Turn Winding Area (EWA)

Turns Per Coil (TPC)

Armature Demagnetizing Ampere Turns (ADM)

5. Calculate: (Repeat until ATT AMA)

Air Gap Flux (FAG) = Initial Value
(FLUI) + L - Increments

Bridge Flux (FBR)

Magnet Flux (FMA)

Magnet Flux Density (BMA) and Magnetic
Intensity (HMA)

Magnet MMF Rise (AMA)

Teeth Flux Density (BTH) and Magnetic
Intensity (HTH)

Teeth MMF Drop (ATH)

Core Flux Density (BCR) and Magnetic
Intensity (HCR)

Core MMF Drop (ACR)

Yoke Flux Density (BYK) and Magnetic
Intensity (HYK)

Yoke MMF Drop (AYK)

Magnet Gap MMF Drop (AMG)

Air Gap Flux Density (BAG)

Air Gap MMF Drop (AAG)

Total All MMF Drops (ATT)

Compare MMF Drop (ATT) with MMF Rise (AMA)

If ATT $\leq$ AMA

6. Calculate:

Torque Current Sensitivity (TPA)

Mean Length of Turn (CLT)

Terminal Resistance (RTR)

Performance Index (PKO)

Output Torque (TOR)

7. Print:

Stator Inside Diameter (DSI)

Rotor Outside Diameter (DRO)

Diameter at Top of Slots (DST)

Diameter at Bottom of Slots (DBS)

Teeth Width (TWA)

Rotor Stack Length (XRL)

Mean Length of Turn (CLT)

Turns Per Coil (TPC)

Air Gap Flux (FAG)

Air Gap Flux Density (BAG)

Magnet Flux Density (BMA)

Terminal Resistance (RTR)

Torque Current Sensitivity (TPA)

Performance Index (PKO)

Output Torque (TOR)

Teeth Flux Density (BTH)

Stop.

VII. SAMPLE COMPUTER CALCULATION

The following sample calculation will step by step go through the first trial calculation for a motor with a rotor diameter of approximately 3.2 inches.

Input data is read in as follows:

Initial Rotor Outside Diameter DROM = 3.188 inches

Air Gap Length GAP = .010 inch

Bridge Thickness BT = .0215 inch

Initial Rotor Stack Length XRLM = .450 inch

Pole Embrace PE = .75

Rotor Slot Opening SOT = .058 inch

Initial Teeth Width TWAM = .092 inch

Teeth Tip Length TTH = .021 inch

Slot Width at Bottom SWB = .062 inch

Magnet Slot Length CMS = .793 inch

Commutator Bar Shank Thickness STB = .040 inch

Slot Insulation Thickness ENS = .005 inch

Epoxy Thickness Over End Turns TEE = .010 inch

Wire Diameter Over Insulation DWO = .0116 inch

Rotor Overall Length XRO = 1.093 inches

Insulation Thickness on End of Rotor Core ENE = .010 inch

Diameter at Bottom of End Turns DET = 2.125

Slot Fill Factor CFS = .58

Initial Flux Value FLUI = 12,000 lines

Coefficient of Leakage COL = 1.9

Magnet Length CMM = .790 inch

Magnet Gap Length GML = .003 inch

Resistance Per Foot of Wire RES = .101 ohms

Armature Current AMP = 2.22 amperes

Lamination Stacking Factor SF = .95

End Turn Fill Factor CFE = .55

Portion of Armature MMF Demagnetizing ANI = .35

Teeth Tip Dimension SCH = .010 inch

Stator Outside Diameter DSo = 4.030 inches

Rotor Inside Diameter DRI = 2.000 inches

Stator Axial Length XSL = .599 inch

End Turn Coil Crosses CP = 5

Number of Poles PNO = 10

Number of Slots SNO = 41

Using the data read in the computer performs the following calculations.

The rotor outside diameter (DRO) for the first calculation is the initial value (DROM) plus one increment of .002 inch.

$$DRO = 3.188 + .002 = 3.190 \text{ inches}$$

Diameter at Center of Magnet (DMC) -

$$DMC = DRO + 2\ (GAP + BT)$$

$$DMC = 3.190 + 2\ (.010 + .015) = 3.240 \text{ inches}$$

Minimum height of magnet (HME) -

$$X = \sqrt{DSO^2 - CMM^2}$$

$$X = \sqrt{4.030^2 - .790^2} = 3.96 \text{ inches}$$

$$HME = \frac{X - DMC}{2} = \frac{3.96 - 3.240}{2} = .360 \text{ inches}$$

Stator inside diameter (DSI) -

$$DSI = DRO + 2\ (GAP)$$

$$DSI = 3.190 + 2\ (.010) = 3.210 \text{ inches}$$

Magnet Area (RMA) -

$$RMA = 2(XSL)\ HME$$

$$RMA = 2(.599)\ .360 = .431 \text{ square inch}$$

Pole Pitch (PP) -

$$PP = \frac{\pi(DRO + GAP)}{PNO}$$

$$PP = \frac{\pi(3.190 + .010)}{10} = 1.003 \text{ inches}$$

The rotor stack length (XRL) for the first calculation is the initial value (XRLM) plus one increment of .002 inch.

$$XRL = .450 + .002 = .452 \text{ inch}$$

Air Gap Area (RAG)

$$RAG = PP\ (PE)\ (XRL + GAP)$$

$$RAG = 1.003\ (.75)\ (.452 + .010)\ .349 \text{ square inch}$$

Teeth Pitch (TP) -

$$TP = \frac{\pi DRO}{SNO}$$

$$TP = \frac{\pi\ 3.190}{41} = .245 \text{ inch}$$

Teeth Face Length (TFC) -

$$TFC = TP - SOT$$

$$TFC = .245 - .058 = .187 \text{ inch}$$

Carter's Coefficient (CCO) -

$$CCO = \frac{5(GAP + SOT)}{5(GAP + SOT) - SOT^2}$$

$$CCO = \frac{5(.010 + .058)}{5(.010 + .058) - (.058)^2} = 1.005$$

Equivalent Air Gap (GAPE) -

$$GAPE = GAP\ (CCO)$$

$$GAPE = .010\ (1.005) = .01005 \text{ inch}$$

The teeth width (TWA) for the first calculation is the initial value (TWAM) plus one increment of .002 inch.

$$TWA = .092 + .002 = .094 \text{ inch}$$

Teeth Area (RTH) -

$$RTH = \frac{TWA\ (SNO)\ PE\ (SF)\ XRL}{PNO}$$

$$RTH = \frac{.094\ (41)\ .75\ (.95)\ .452}{10} = .1235 \text{ square inches}$$

Diameter at the Top of Slots (DST) -

$$DST = DRO - 2\ (TTH + SCH)$$

$$DST = 3.190 - 2\ (.021 + .010) = 3.128 \text{ inches}$$

Diameter at Bottom of Slots (DSB) -

$$DSB = \frac{SNO\ (SWB + TWA)}{\pi}$$

$$DSB = \frac{41\ (.062 + .094)}{\pi} = 2.197 \text{ inches}$$

Slot Radial Height (SRH) -

$$SRH = \frac{DST - DSB}{2}$$

$$SRH = \frac{3.128 - 2.197}{2} = .465 \text{ inch}$$

Core Magnetic Length (CRL)

$$CRL = \frac{\pi\,(DSB + DRI)}{4\ PNO}$$

$$CRL = \frac{\pi\ (2.197 + 2.000)}{4\ (10)} = .330 \text{ inch}$$

Core Area (RCR) -

$$RCR = (DSB - DRI)\ XRL\ (SF)$$

$$RCE = (2.197 - 2.000)\ .452\ (.95) = .0845 \text{ square inch}$$

Distance Between Magnet Slots (CPC)

$$X2 = \frac{CMS}{DMC}$$

$$X2 = \frac{.793}{3.240} = .245$$

$$BANG = \tan^{-1}\ (X2)$$

$$BANG = \tan^{-1}\ (.245) = .241 \text{ radians}$$

$$CANG = \frac{\pi}{PNO} - BANG$$

$$CANG = \frac{\pi}{10} - .241 = .0731 \text{ radians}$$

$$DME = \sqrt{CMS^2 + DMC^2}$$

$$DME = \sqrt{.793^2 + 3.240^2} = 3.34 \text{ inches}$$

$$CPC = DME \text{ Sin } (CANG)$$

$$CPC = 3.34 \text{ Sin } (.0731) = .245 \text{ inch}$$

Yoke Area (RYK) -

$$RYK = CPC\ (XSL)$$

$$RYK = 2.45\ (.599) = .147 \text{ square inches}$$

Yoke Length (YKL) -

$$YKL = \frac{DSO - DSI}{6}$$

$$YKL = \frac{4.030 - 3.210}{6} = .137 \text{ inches}$$

Slot Width at Top (SWT) -

$$SWT = \frac{DST}{SNO} - TWA$$

$$SWT = \frac{3.128}{46} - .094 = .121 \text{ inch}$$

Slot Winding Area (RSE) -

$$RSE = [\frac{SRH - STB - 2\ ENS}{2}]\ [\frac{SWB + SWT}{2} - 2\ ENS]$$

$$RSE = [\frac{.465 - .040 - 2\ (.005)}{2}]\ [\frac{.056 + .121}{2} - 2\ (.005)]$$

$$= .0162 \text{ square inches}$$

End Turn Winding Area (EWA) -

$$EWA = [\frac{XRO - XRL}{2} - ENE - TEE]\ [\frac{DST - DET - 2\ TEE}{2}]$$

$$EWA = [\frac{1.093 - .452}{2} - .010 - .010]\ [\frac{3.128 - 2.125 - 2\ (.010)}{2}]$$

$$= .148 \text{ square inch}$$

Axial Length of End Turns (XET) -

$$XET = \frac{XRO - XRL}{2} - ENE - TEE$$

$$XET = \frac{1.093 - .452}{2} - .010 - .010 = .3005 \text{ inch}$$

Possible Turns in Slots (KTPC1) -

$$KTPC1 = \frac{RSE\ (CFS)}{DWO^2}$$

$$KTPC1 = \frac{.0162\ (.58)}{(.0116)^2} = 69 \text{ turns}$$

Possible Turns in End Turns (KTPC2) -

$$KTPC2 = \frac{.148\ (.55)}{5\ (.0116)^2} = 120 \text{ turns}$$

The maximum allowable turns in the end turns and slots are compared. The smallest number of turns (TPC) is chosen for the winding because a larger number of turns cannot be accommodated in both the slots and the end turns.

Armature Demagnetizing Ampere Turns (ADM) -

$$ADM = \frac{AMP\ (TPC)\ SNO\ (ANI)}{2\ PNO}$$

$$ADM = \frac{2.22\ (69)\ 46\ (.35)}{2\ (10)} = 123.$$

The air gap flux (FAG) for the first trial is assumed to be a low value (FLUI) plus one increment of 100 lines.

$$FAG = FLUI + L \text{ increments}$$

$$FAG = 12{,}000 + 100 = 12{,}100 \text{ lines}$$

Bridge Flux (FBR) -

$$FBR = 115{,}000 \ (BT) \ XSL \ (2)$$

$$FBR = 115{,}000 \ (.015) \ .599 \ (2) = 2070 \text{ lines}$$

Magnet Flux (FMA) -

$$FMA = (FAG + FBR) \ COL$$

$$FMA = (12{,}100 + 2070) \ 1.9 = 26{,}000 \text{ lines}$$

Magnet Flux Density (BMA) -

$$BMA = \frac{FMA}{RMA}$$

$$BMA = \frac{26{,}800}{.431} = 62{,}500 \text{ lines per square inch}$$

If the magnet flux density (BMA) is less than 74,000 lines per square inch the magnetic intensity (HMA) is defined by the following equation:

$$HMA = 1569 - \frac{BMA}{294}$$

$$HMA = 1569 - \frac{62{,}500}{294} = 1357 \text{ ampere turns per inch.}$$

If the magnet flux density (BMA) is more than 74,000 lines per square inch the magnetic intensity (HMA) is defined by the following equation:

$$HMA = 9010 - \frac{BMA}{9.6}$$

Ampere Turn Rise in Magnet (AMA) -

$$AMA = \frac{HMA\ (CMM)}{2}$$

$$AMA = \frac{1357\ (.790)}{2} = 536 \text{ ampere turns.}$$

Teeth Flux Density (BTH) -

$$BTH = \frac{FAG}{RTH}$$

$$BTH = \frac{12{,}100}{.1235} = 98{,}000 \text{ lines per square inch}$$

If the teeth flux density (BTH) is less than 97,500 lines per square inch, the magnetic intensity (HTH) is defined by the following equation:

$$HTH = \frac{BTH}{3900}$$

If the teeth flux density (BTH) is more than 97,500 lines per square inch, the magnetic intensity (HTH) is defined by the following equation:

$$HTH = \frac{BTH - 95{,}667}{73.3}$$

$$HTH = \frac{98{,}000 - 95{,}667}{73.3} = 31.8 \text{ ampere turns per inch.}$$

Ampere Turn Drop in Teeth (ATH) -

$$ATH = HTH\ (SRH)$$

$$ATH = 31.8\ (.465) = 14.8 \text{ ampere turns}$$

Core Flux Density (BCR) -

$$BCR = \frac{FAG}{RCR}$$

$$BCR = \frac{12,100}{.0845} = 143,500 \text{ lines per square inch}$$

If the core flux density (BCR) is less than 97,500 lines per square inch, the magnetic intensity (HCR) is defined by the following equation:

$$HCR = \frac{BCR}{3900}$$

If the core flux density (BCR) is more than 97,500 lines per square inch, the magnetic intensity (HCR) is defined by the following equation:

$$HCR = \frac{BCR - 95,667}{73.3}$$

$$HCR = \frac{143,500 - 95,667}{73.3} = 652 \text{ ampere turns per inch}$$

Ampere Turn Drop in Core (ACR) -

$$ACR = HCR\ (CRL)$$

$$ACR = 652\ (.330) = 215 \text{ ampere turns.}$$

Yoke Flux Density (BYK) -

$$BYK = \frac{FAG + FBR}{RYK}$$

$$BYK = \frac{14,170}{.147} = 96,000 \text{ lines per square inch}$$

If the yoke flux density (BYK) is less than 97,500 lines per square inch, the magnetic intensity (HYK) is defined by the following equation:

$$HYK = \frac{BYK}{3900}$$

$$HYK = \frac{96,000}{3900} = 24.6 \text{ ampere turns per inch}$$

If the yoke flux density (BYK) is more than 97,500 lines per square inch, the magnetic intensity (HYK) is defined by the following equation:

$$HYK = \frac{BYK - 95,667}{73.3}$$

Ampere Turn Drop in Yoke (AYK) -

$$AYK = HYK\ (YKL)$$

$$AYK = 24.6\ (.137) = 3.37 \text{ ampere turns}$$

Ampere Turn Drop in Magnet Gap (AMG) -

$$AMG = \frac{.313\ FMA\ (GML)}{2\ RMA}$$

$$AMG = \frac{.313\ (26,800)\ .003}{2\ (.431)} = 29.3 \text{ ampere turns}$$

Air Gap Flux Density (BAG) -

$$BAG = \frac{FAG}{RAG}$$

$$BAG = \frac{12,100}{.349} = 34,700 \text{ lines per square inch}$$

Ampere Turn Drop in Air Gap (AAG) -

AAG = .313 (BAG) GAPE

AAG = .313 (34,700) .01005 = 109.5 ampere turns

Total Ampere Turn Drop (ATT) -

ATT = ATH + ACR + AYK + AMG + AAG + ADM

ATT = 14.8 + 215 + 3.37 + 29.3 + 109.5 + 123. = 495.

The total ampere turn drop (ATT) is compared with the magnet ampere turn rise (AMA). The initial value of flux (FLUI) was chosen to make the ampere turn drop in the magnetic circuit (ATT) less than the rise in the magnet (AMA).

AMA = 536 ampere turns

If the total ampere turn drop (ATT) is less than the magnet rise, the air gap flux (FAG) is increased by 100 lines. This is repeated until the total ampere turn drop (ATT) is greater than the magnet ampere turn rise (AMA).

This final value of flux will be used in the performance calculations below.

Sensitivity Torque Per Ampere (TPA) -

TPA = .000000225 (PNO) FAG (SNO) TPC

TPA = .000000225 (10) 12,200 (46) 69 = 87.1 oz-in/amp

Mean Length of Turn (CLT) -

$$CLT = \frac{\pi\ (DST + DSB)\ (CP - 1)}{SNO} + 2\ (XRL + XET)$$

$$CLT = \frac{\pi\ (3.128 + 2.197)\ (5 - 1)}{46} + 2\ (.452 + .300)$$

$$= 2.959 \text{ inches}$$

Terminal Resistance (RTR) -

$$RTR = \frac{.9\ (CLT)\ TPC\ (SNO)\ RES}{48}$$

$$RTR = \frac{.9\ (2.959)\ 69\ (46)\ .101}{48} = 17.8 \text{ ohms}$$

Performance Index (PKO) -

$$PKO = \frac{TPA}{\sqrt{RTR}}$$

$$PKO = \frac{87.1}{\sqrt{17.8}} = 20.7$$

Peak Rated Torque (TOR) -

$$TOR = AMP\ (TPA)$$

$$TOR = 2.22\ (87.1) = 193. \text{ oz-in}$$

The output data is written out as follows:

Stator Inside Diameter (DSI)

Rotor Outside Diameter (DRO)

Diameter at Top of Slots (DST)

Diameter at Bottom of Slots (DSW)

Teeth Width (TWA)

Axial Rotor Length (XRL)

Mean Length of Turn (CLT)

Turns Per Coil (TPC)

Air Gap Flux (FAG)

Air Gap Flux Density (BAG)

Magnet Flux Density (BMA)

Terminal Resistance (RTR)

Torque Pere Ampere (TPA)

Performance Index (PKO)

Peak Rated Torque (TOR)

Teeth Flux Density (BTH)

This completes one complete design calculation. The teeth width (TWA) is increased by an increment four more times. Each time all the calculations are repeated. Then the rotor axial length (XRL) is increased by an increment ten times. At each increment of the rotor axial length (XRL), the tooth width is varied over its complete range. The rotor outside diameter (DRO) is increased by an increment ten times. At each increment of the rotor outside diameter (DRO), the rotor axial length is varied over its complete range. Also the tooth width (TWA) varies over its complete range at each increment of the rotor axial length.

The output data is printed at each increment of each variable. For this set of calculations, there were five hundred possible designs. The design with the largest Performance Index (PKO) will be the optimum design because it will produce more torque per watt than any of the others.

VIII. FORTRAN PROGRAM

```
$IBJOB          NODECK
$IBFTC D1
  505 FORMAT(162H    DSI      DRO      DST      DSB      TWA
     1XRL      CLT      TPC      FAG      BAG      BMA
     2RTR      TPA      PKO      TOR      BTH      )
  204 FORMAT(F10.2)
  506 FORMAT (1H1)
  507 FORMAT(7F8.4,F6.1,3F9.1,3F8.3,F9.3,F9.1)
   82 READ(5,204) DROM,GAP,BT,XRLM,PE,SOT,TWAM,TTH,SWB,CMS,
     1STB,ENS,TEE,DW,OXRO,ENE,DET,CFS,FLUI,COL,CMM,GML,RES,
     2AMP,SF,CFE,ANI,SCH,DSO,DR,IXSL,CP,SNO,PNO
      WRITE (6,506)
      WRITE (6,505)
      DO 80 K=1,50,5
      AAA=K
      DRO=DROM+0.002*AAA
    1 DMC=DRO+2.*(GAP+BT)
      Y=DSO*DSO-CMM*CMM
      X=SQRT(Y)
    2 HME=(X-DMC)/2.0
      DSI=DRO+2.*GAP
    3 RMA=2.*XSL*HME
    4 PP= (3.141*(DRO+GAP))/PNO
      DO 80 J=1,50,5
      BBB=J
```

```
  5 XRL=XRLM+0.002*BBB
  6 RAG=PP*PE*(XRL+GAP)
  7 TP=(DRO*3.141)/SNO
  8 TFC=TP-SOT
  9 CCO=(5.*(GAP+SOT)*TP)/(5.*(GAP+SOT)*TP-SOT*SOT)
 10 GAPE=GAP*CCO
101 DO 80 N=1,10,2
    CCC=N
102 TWA=TWAM+.002*CCC
 11 RTH=TWA*SNO*PE*SF*XRL/PNO
 12 DST=DRO-2.*(TTH+SCH)
 13 DSB=SNO*(SWB+TWA)/3.141
 14 SRH=(DST-DSB)/2.
 15 CRL=3.141*(DSB+DRI)/(4.*PNO)
 16 RCR=(DSB-DRI)*XRL*SF
    X2=CMS/DMC
 17 BANG=ATAN(X2)
 18 CANG=3.141/PNO-BANG
    X4=CMS*CMS+DMC*DMC
 19 DME=SQRT(X4)
 20 CPC=DME*SIN(CANG)
 21 RYK=CPC*XSL
 22 YKL=(DSO-DSI)/6.
 23 SWT=DST*3.141/SNO-TWA
 24 RSE=(SRH-STB-2.*ENS)*((SWB+SWT)/2.-2.*ENS)/2.
```

```
25 EWA=((XRO-XRL)/2.-ENE-TEE)*(DST-DET-2.*TEE)/2.
   XET=(XRO-XRL)/2.-(ENE+TEE )
26 KTPC1=RSE*CFS/(DWO*DWO)
27 KTPC2=EWA*CFE/(CP*DWO*DWO)
28 KBEX=KTPC1-KTPC2
29 IF(KBEX)30,30,31
30 TPC=KTPC1
   GO TO 32
31 TPC=KTPC2
32 ADM=AMP*TPC*SNO*ANI/(2.*PNO)
33 DO 67 L=1,500
   DDD=L
34 FAG=FLUI+100.*DDD
35 FBR=115000.*BT*XSL*2.
36 FMA=(FAG+FBR)*COL
37 BMA=FMA/RMA
38 BMX=BMA-74000.
39 IF(BMX)40,40,41
40 HMA=1569.-BMA/274.
   GO TO 42
41 HMA=9010.-BMA/9.6
42 AMA=HMA*CMM/2.
43 BTH=FAG/RTH
44 BXT=BTH-97500.
45 IF(BXT)46,46,47
```

```
46 HTH=BTH/3900.
   GO TO 48
47 HTH=(BTH-95667.)/73.3
48 ATH=HTH*SRH
49 BCR=FAG/RCR
50 BXC=BCR-97500.
51 IF(BXC)52,52,53
52 HCR=BCR/3900.
   GO TO 54
53 HCR=(BCR-95667.)/73.3
54 ACR=HCR*CRL
55 BYK=(FAG+FBR)/RYK
56 BXY=BYK-97500.
57 IF(BXY)58,58,59
58 HYK=BYK/3900.
   GO TO 60
59 HYK=(BYK-95667.)/73.3
60 AYK=HYK*YKL
61 AMG=.313*FMA*GML/(2.*RMA)
62 BAG=FAG/RAG
63 AAG=.313*BAG*GAPE
64 ATT=ATH+ACR+AYK+AMG+AAG+ADM
65 XMMF=AMA-ATT
66 IF(XMMF)68,68,67
67 CONTINUE
```

```
68 TPA=.000000225*PNO*FAG*SNO*TPC
69 CLT=3.141*(DST+DSB)*(CP-1.)/SNO+2.*(XRL+XET)
70 RTR=.9*CLT*TPC*SNO*RES/48.
71 PKO=TPA/SQRT(RTR)
72 TOR=AMP*TPA
80 WRITE (6,507) DSI,DRO,DST,DSB,TWA,XRL,CLT,TPC,FAG,BAG,
  1BMA,RTR,TPA,PK,OTOR,BTH
81 GO TO 82
   END
$ENTRY          D1
$IBSYS
```

Input Data for Trial Design

3.188	DROM
0.010	GAP
0.0215	BT
0.45	XRLM
0.75	PE
0.058	SOT
0.092	TWAM
0.021	TTH
0.056	SWB
0.793	CMS
0.040	STB
0.005	ENS
0.010	TEE
0.0116	DWO
1.093	XRO
0.010	ENE
2.125	DET
0.58	CFS
8000.0	FLUI
1.90	COL
0.790	CMM
0.003	GML
0.101	RES
2.22	AMP
0.95	SF
0.55	CFE
0.35	ANI
0.010	SCH
4.030	DSO
2.00	DRI
0.559	XSL
5.00	CP
46.00	SNO
10.00	PNO

Variable Name List

AAG	Ampere Turns of Air Gap
ACR	Ampere Turns of Core
ADM	Ampere Turns Demagnetizing
AMA	Ampere Turns of Magnet
AMG	Ampere Turns of Magnet Gap
AMP	Current at Peak Torque (TOR)
ANI	Percent of Total Armature MMF Demagnetizing
ATH	Ampere Turns in Teeth
ATT	Ampere Turns Total Dropped in all of Magnetic Circuit Except Magnet Itself
AYK	Ampere Turns of Yoke
BAG	Air Gap Flux Density in K lines/in^2
BBR	Bridge Flux Density in K lines/in^2
BCR	Core Flux Density in K lines/in^2
BMA	Magnet Operating Flux Density in K lines/in^2
BMG	Magnet Gap Density in K lines/in^2
BT	Bridge Thickness
BTH	Tooth Flux Density in K lines/in^2
BYK	Yoke Flux Density in K lines/in^2
CCO	Carter's Coefficient
CFS	Slot Fill Factor
CFE	End Turn Fill Factor
CLT	Coil-Mean Length of One Complete Turn in Inches

CMM	Magnet Material Length in Direction of Orientation
CMS	Magnet Slot Length in Direction of Orientation
CP	Maximum Coil Crosses in End Turns
COL	Coefficient of Leakage
CPC	Chord Length at Pole Constriction
CRL	Core Length (Magnetic Length)
DBS	Diameter of Band on Stator (Outside Diameter) if there is no Band OD of Stator is DSO
DET	Inside Diameter of End Turns
DMC	Diameter of Magnets-Inside-At Center
DME	Diameter of Magnet End-Inside-
DRI	Diameter of Rotor-Inside-
DRO	Diameter of Rotor-Outside-
DROM	Initial DRO
DSB	Diameter of Slot Bottom
DSI	Diameter of Stator Inside
DSO	Diameter of Stator-Outside- if no Stator Band. If a Band is used this is Diameter over outside of Magnets but Under Band
DST	Diameter of Slot Top
DUK	Inductance in Henries
DWO	Diameter of Wire over Insulation
ENE	Insulation Thickness on End of Rotor Core
ENS	Insulation Thickness in Slot
EWA	End Turn Winding Area

FAG	Flux in Air Gap in K lines
FBE	Flux in Bridge
FCR	Flux in Core in K lines
FFE	Fill Factor in End Turns
FFS	Fill Factor in Slots
FLUI	Initial Flux Per Pole
FMA	Flux in Magnets in K lines
FMG	Flux in Magnet Gap in K lines
FTH	Flux in Teeth in K lines
FYK	Flux in Yoke in K lines
GAP	Radial Air Gap on one Side
GML	Gap Length at Ends of Magnet (Total of Both Ends)
HAG	Magnetic Intensity of Air Gap (.313B) 10^{-3} (BAG)
HCR	Magnetic Intensity of Core in NI/in
HMA	Magnetic Intensity of Magnet NI/in
HMC	Height of Magnet in Center
HME	Height of Magnet at End
HMG	Magnetic Intensity of Magnet Gap (.313) 10^{-3} (BMG)
HSC	Height of Slot Closure
HTH	Magnetic Intensity of Teeth in NI/in
HTT	Heights of Tooth Tip
HYK	Magnetic Intensity of Yoke
PE	Pole Embrace

PKO	Performance Index $TPA/\sqrt{RTR}$
PNO	Number of Poles when Floating Point Desired
PP	Pole Pitch at Air Gap in Inches
RAG	Area of Air Gap
RBR	Area of Bridge
RCR	Area of Core
RES	Resistance Per Foot of Wire
RMA	Area of Magnet
RMG	Area of Magnet Gap
RSE	Effective Slot Area
RTH	Area of Teeth
RTR	Terminal Resistance Measured Thru Brushes
RYK	Area of Yoke
SCH	Tooth Tip Dimension
SF	Staking Factor
SNO	Number of Slots When Floating Point Desired
SOT	Slot Opening at Top
SRH	Slot Radial Height
STB	Commutator Bar Thickness
SWA	Slot Winding Area
SWB	Slot Width Bottom
SWT	Slot Width Top
TEE	Epoxy Thickness Over End Turns

TFC	Distance Along Tooth Face
THL	Tooth Length
TOR	Peak Rated Torque
TP	Tooth Pitch in Inches
TPA	Sensitivity Torque per Ampere
TPC	Turns per Coil on Armature
TTH	Tooth Tip Length
TWA	Tooth Width Average
TWAM	Initial TWA
TWB	Tooth Width Bottom
TWT	Tooth Width Top
XSL	Axial Stator Length
XBL	Axial Brush Ring Length
XRL	Axial Rotor Length
XRLM	Initial XRL
XRO	Axial Rotor Overall
XET	Axial End Turn Dimensions
YKL	Yoke Length

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DESIGN OF A FRAMELESS TORQUE MOTOR

by John Jay Rinehart

ABSTRACT

The space industry has the ever increasing problem of the optimal utilization of all the volume inside the various spacecraft. Nearly every present day missile has an inertial guidance system which uses permanent magnet motors to compensate for friction loss in the moving parts. Since the space available for these motors is limited, the motor designer must obtain a design which will supply maximum torque for the available power input.

The designer must determine the optimum rotor outside diameter, magnet size, rotor stack length, slot size, and all other internal dimensions of the motor. Economics dictate that the engineer may work on a design for a given time only. At the end of this time he is forced to use his best design, without knowing whether it is the best one possible.

This investigation consists of the derivation of design equations for a permanent magnet direct current motor. The design procedure is programmed into a digital computer. The performance characteristics of each possible design, within practical limits, is calculated using an iteration process. The calculated performance data is compared with the actual test values from a similar motor.