Solutions to Problems in *Electromagnetics, Vol. 2*
Version 1.0

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Change History

- Version 1.0: First publicly-available version.

Contents

Chapter 2

Magnetostatics Redux

[m0017] [1]

2.2-1

The force experienced by this segment of the wire is

$$\mathbf{F} = I\mathbf{l} \times \mathbf{B} \tag{2.1}$$

where, from the problem statement:

- $I = 2$ A,
- $\mathbf{l} = \hat{\mathbf{l}}L = -\hat{\mathbf{z}}L$ where $L = 25$ cm, and
- $\mathbf{B} = B_0\,(\hat{\mathbf{x}} - \hat{\mathbf{y}})$, where $B_0 = 3$ mT.

Thus:

$$\mathbf{F} = I\,(-\hat{\mathbf{z}})\,L \times B_0\,(\hat{\mathbf{x}} - \hat{\mathbf{y}}) \tag{2.2}$$

$$= ILB_0\,(-\hat{\mathbf{y}} - \hat{\mathbf{x}}) \tag{2.3}$$

$$= \underline{(1.5\text{ mN})\,(-\hat{\mathbf{x}} - \hat{\mathbf{y}})} \tag{2.4}$$

[m0017] [2]

2.2-2

The force on the loop is

$$\mathbf{F} = \int_{\mathcal{C}} I d\mathbf{l}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) \tag{2.5}$$

where $\mathcal{C}$ is the path followed by the current (in this case, in the $+\hat{\phi}$ direction along the loop) and $\mathbf{r}$ parameterizes the position of points along this path. Thus:

$$d\mathbf{l}(\mathbf{r}) = \hat{\mathbf{l}}(\mathbf{r})\ dl = +\hat{\phi}\,(a\ d\phi)$$

Next, note:

$$d\mathbf{l}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) = +\hat{\phi}\,(a\ d\phi) \times \frac{\hat{\rho}B_\rho \rho z + \hat{\mathbf{z}}\left(B_{zz}z^2 - B_{z\rho}\rho^2\right)}{\left(\rho^2 + z^2\right)^{5/2}} \tag{2.6}$$

$$= \frac{-\hat{\mathbf{z}}B_\rho \rho z + \hat{\rho}\left(B_{zz}z^2 - B_{z\rho}\rho^2\right)}{\left(\rho^2 + z^2\right)^{5/2}} a\ d\phi \tag{2.7}$$

Per the problem statement, we are only interested in the $\hat{\mathbf{z}}$ component:

$$\hat{\mathbf{z}} \cdot \left(d\mathbf{l}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})\right) = \frac{-B_\rho \rho z}{\left(\rho^2 + z^2\right)^{5/2}} a\ d\phi \tag{2.8}$$

Thus:

$$\hat{\mathbf{z}} \cdot \mathbf{F} = \int_{\mathcal{C}} I \frac{-B_\rho \rho z}{\left(\rho^2 + z^2\right)^{5/2}} a\ d\phi \tag{2.9}$$

The path $\mathcal{C}$ is a circle at $\rho = a$ increasing in the same direction as ϕ, with $z = z_0$. Thus:

$$\hat{\mathbf{z}} \cdot \mathbf{F} = \int_{\phi=0}^{2\pi} I \frac{-B_\rho a z_0}{\left(a^2 + z_0^2\right)^{5/2}} a\ d\phi \tag{2.10}$$

Factoring out quantities that are constant over the path:

$$\hat{\mathbf{z}} \cdot \mathbf{F} = I \frac{-B_\rho a z_0}{\left(a^2 + z_0^2\right)^{5/2}} a \int_{\phi=0}^{2\pi} d\phi \tag{2.11}$$

$$= I \frac{-B_\rho a z_0}{\left(a^2 + z_0^2\right)^{5/2}} a \cdot 2\pi \tag{2.12}$$

Cleaning up:

$$\boxed{\hat{\mathbf{z}} \cdot \mathbf{F} = -\frac{2\pi a^2 z_0 I B_\rho}{\left(a^2 + z_0^2\right)^{5/2}}} \tag{2.13}$$

Now part (b): SI base units of magnetic flux density are T, so the units of the magnetic field expression (repeated below):

$$\mathbf{B} = \frac{\hat{\rho}B_\rho \rho z + \hat{\mathbf{z}}\left(B_{zz}z^2 - B_{z\rho}\rho^2\right)}{\left(\rho^2 + z^2\right)^{5/2}} \tag{2.14}$$

are as follows:

$$\frac{B_\rho[\mathrm{m}][\mathrm{m}] + (B_{zz}[\mathrm{m}][\mathrm{m}] + B_{z\rho}[\mathrm{m}^2])}{[\mathrm{m}^5]} = [\mathrm{T}] \tag{2.15}$$

Therefore the units of B_ρ, B_{zz}, and $B_{z\rho}$ are $\underline{\mathrm{T{\cdot}m^3}}$ (b).
Units check for the final answer (Equation 2.13) is as follows:

$$\frac{[\mathrm{m}^2][\mathrm{m}][\mathrm{A}][\mathrm{T}\cdot\mathrm{m}^3]}{[\mathrm{m}^5]} = [\mathrm{A}][\mathrm{T}][\mathrm{m}] \tag{2.16}$$

Note that 1 A = 1 C/s and 1 T = 1 N·s / C· m, so:

$$[\mathrm{A}][\mathrm{T}][\mathrm{m}] = \left[\frac{\mathrm{C}}{\mathrm{s}}\right]\left[\frac{\mathrm{N}\cdot\mathrm{s}}{\mathrm{C}\cdot\mathrm{m}}\right][\mathrm{m}] = [\mathrm{N}] \tag{2.17}$$

as expected.

[m0017] [3]

2.2-3

(a) The force on the straight portion of the loop is

$$\mathbf{F}_a = \int_{\mathcal{C}} I d\mathbf{l}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) \tag{2.18}$$

where $\mathcal{C}$ is the path followed by the current (in this case, in the $+\hat{\mathbf{z}}$ direction along this segment of the loop) and $\mathbf{r}$ parameterizes the position of points along this path. Thus:

$$d\mathbf{l}(\mathbf{r}) = \hat{\mathbf{l}}(\mathbf{r})\ dl = +\hat{\mathbf{z}}dz$$

Next, note:

$$d\mathbf{l}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) = +\hat{\mathbf{z}}dz \times \hat{\mathbf{x}}B_0 = \hat{\mathbf{y}}B_0 dz \tag{2.19}$$

Thus:

$$\mathbf{F}_a = \int_{z=-a}^{+a} I\hat{\mathbf{y}}B_0 dz = +\hat{\mathbf{y}}2IB_0 a = \boxed{+\hat{\mathbf{y}}0.84 \text{ N}} \quad \text{(a)} \tag{2.20}$$

(b) The force on the semi-circular portion of the loop is

$$\mathbf{F}_b = \int_{\mathcal{C}} I d\mathbf{l}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) \tag{2.21}$$

where $\mathcal{C}$ in this case is the $+\hat{\theta}$ direction along this segment of the loop. Thus:

$$d\mathbf{l}(\mathbf{r}) = \hat{\mathbf{l}}(\mathbf{r})\ dl = \hat{\theta}\ a\ d\theta$$

Next, note:

$$d\mathbf{l}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) = \hat{\theta}\ a\ d\theta \times \hat{\mathbf{x}}B_0 = (-\hat{\mathbf{y}}\sin\theta)\, B_0 a\ d\theta \tag{2.22}$$

Thus:

$$\mathbf{F}_b = \int_{\theta=0}^{\pi} I\,(-\hat{\mathbf{y}}\sin\theta)\, B_0 a\ d\theta \tag{2.23}$$

$$= -\hat{\mathbf{y}}IB_0 a \int_{\theta=0}^{\pi} \sin\theta\ d\theta \tag{2.24}$$

$$= -\hat{\mathbf{y}}IB_0 a \left[-\cos\theta|_0^{\pi}\right] \tag{2.25}$$

$$= -\hat{\mathbf{y}}2IB_0 a \tag{2.26}$$

$$= \boxed{-\hat{\mathbf{y}}0.84 \text{ N}} \quad \text{(b)} \tag{2.27}$$

(c) The net force on the loop is $\mathbf{F}_a + \mathbf{F}_b = \boxed{0}$ (c).

[m0017] [4]

2.2-4

From the problem statement:
$l = 5$ cm,
$I = 300$ mA in the $+\hat{\mathbf{x}}$ direction, and
$\mathbf{H}_0 = +\hat{\mathbf{z}}H_0$ where $H_0 = 800$ kA/m.

The force is:

$$\mathbf{F}_m = I\mathbf{l} \times \mathbf{B}_0 \tag{2.28}$$
$$= I\left(\hat{\mathbf{x}}l\right) \times \left(\hat{\mathbf{z}}\mu_0 H_0\right) \tag{2.29}$$
$$= Il\mu_0 H_0 \left(\hat{\mathbf{x}} \times \hat{\mathbf{z}}\right) \tag{2.30}$$
$$= Il\mu_0 H_0 \left(-\hat{\mathbf{y}}\right) \tag{2.31}$$
$$\cong \boxed{-\hat{\mathbf{y}}15.1 \text{ mN}} \tag{2.32}$$

[m0024] [1]

2.3-1

The total torque $\mathbf{T}$ is the sum of the torque on each of the four sides of the rectangular loop. The torque on side n of the loop is $\mathbf{T} = \mathbf{d}_n \times \mathbf{F}_n$ where $\mathbf{d}_n$ is the lever arm (shortest vector from axis of rotation to point of measurement) and $\mathbf{F}_n$ is the force on the point of measurement. $\mathbf{F}_n$ is given by

$$\mathbf{F}_n = I\mathbf{l}_n \times \mathbf{B} \tag{2.33}$$

In this equation, $\mathbf{l} = \hat{\mathbf{l}}_n l_n$, where $\hat{\mathbf{l}}_n$ is the reference direction for the flow of current on side n, and l_n is the length of side n. Thus:

$$\mathbf{F}_n = I\hat{\mathbf{l}}_n l_n \times \hat{\mathbf{y}} B_0 \tag{2.34}$$

$$= I l_n B_0 \left(\hat{\mathbf{l}}_n \times \hat{\mathbf{y}}\right) \tag{2.35}$$

Now considering the forces on each side:

- The force $\mathbf{F}_1$ on the part of the loop lying along the shaft ($\mathbf{l} = +\hat{\mathbf{z}}h$) is $IhB_0\,(+\hat{\mathbf{z}} \times \hat{\mathbf{y}})$, so $\mathbf{F}_1 = -IhB_0\hat{\mathbf{x}}$.
- The force $\mathbf{F}_2$ on the top part of the loop ($\mathbf{l} = +\hat{\rho}W$) is $IWB_0\,(\hat{\rho} \times \hat{\mathbf{y}})$, so $\mathbf{F}_2 = IWB_0\ \hat{\mathbf{z}}\cos\phi$.
- The force $\mathbf{F}_3$ on the part of the loop opposite the shaft ($\mathbf{l} = -\hat{\mathbf{z}}h$) is $IhB_0\,(-\hat{\mathbf{z}} \times \hat{\mathbf{y}})$, so $\mathbf{F}_3 = +IhB_0\ \hat{\mathbf{x}}$.
- The force $\mathbf{F}_4$ on the bottom part of the loop ($\mathbf{l} = -\hat{\rho}W$) is $IWB_0\,(-\hat{\rho} \times \hat{\mathbf{y}})$, so $\mathbf{F}_4 = -IWB_0\ \hat{\mathbf{z}}\cos\phi$.

Now considering the associated torques:

- The torque $\mathbf{T}_1$ on the part of the loop lying along the shaft is zero, because $\mathbf{d}_1 = 0$.
- The torque $\mathbf{T}_2$ on the top part of the loop cancels the torque $\mathbf{T}_4$ on the bottom part of the loop. This is because $\mathbf{d}_2 = \mathbf{d}_4$ (i.e., the lever arms are the same) but $\mathbf{F}_2 = -\mathbf{F}_4$ (i.e., the forces are equal and opposite).
- The torque $\mathbf{T}_3$ on the part of the loop opposite the shaft is $\hat{\rho}W \times IhB_0\ \hat{\mathbf{x}} = -\hat{\mathbf{z}}IhWB_0\ \sin\phi$.

Therefore the total torque $\mathbf{T} = \mathbf{T}_3 = \underline{-\hat{\mathbf{z}}IhWB_0\ \sin\phi}$. (a)

(b) Given $W = 20$ cm, $h = 40$ cm, $I = +500$ mA, $B_0 = +1.2$ T, and $\phi = 60°$, we find $\mathbf{T} = \underline{-\hat{\mathbf{z}}41.6 \text{ mN·m}}$. (b)

(c) Using the right-hand rule for torque, we see that this corresponds to motion in the $-\hat{\phi}$ direction, which is toward the observer. (c)

[m0024] [2]

2.3-2

From the problem statement:
Area = length times width = $LW = 7 \text{ mm}^2 = 7 \times 10^{-6} \text{ m}^2$
Current $I = 3$ mA
Magnetic field $B_0 = 0.3$ T.

The maximum torque is:

$$|\mathbf{T}| = |LWIB_0| \cong \boxed{6.30 \text{ nN·m (max.)}} \tag{2.36}$$

The minimum torque is simply $\boxed{0 \text{ (min.)}}$. The actual value of torque depends on the alignment of the loop relative to the direction of the magnetic field.

[m0066] [1]

2.4-1

The magnetic flux density due to the first wire segment is

$$\mathbf{B}_1 = \hat{\mathbf{y}}\frac{\mu_0 I_1}{2\pi d_1} \tag{2.37}$$

where $d_1 = 0.5$ m. The magnetic flux density due to the second wire segment is, using the right-hand rule,

$$\mathbf{B}_2 = \hat{\mathbf{y}}\frac{\mu_0 I_2}{2\pi d_2} \tag{2.38}$$

where $d_2 = 2.0 - 0.5 = 1.5$ m. The total magnetic field is

$$\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = \hat{\mathbf{y}}\frac{\mu_0}{2\pi}\left(\frac{I_1}{d_1} + \frac{I_2}{d_2}\right) \tag{2.39}$$

Therefore:

$$\mathbf{B} = \hat{\mathbf{y}}\frac{4\pi \times 10^{-7}\ \text{H/m}}{2\pi}\left(\frac{6\ \text{A}}{0.5\ \text{m}} + \frac{3\ \text{A}}{1.5\ \text{m}}\right) \tag{2.40}$$

$$= \boxed{\hat{\mathbf{y}}2.80\ \mu\text{T}} \tag{2.41}$$

[m0066] [2]

2.4-2

The Biot-Savart law is the solution to the differential form of Ampere's law for line current:

$$\mathbf{H}(\mathbf{r}) = \frac{I}{4\pi} \int_{\mathcal{C}} \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \tag{2.42}$$

where $\mathcal{C}$ is the path (i.e., the continuum of values of $\mathbf{r}'$) which the current follows and $\mathbf{R} = \hat{\mathbf{R}}R$ is the vector from the source point $\mathbf{r}'$ to the field point $\mathbf{r}$. In terms of the current given in the problem statement:

$$\boxed{\mathbf{H}(\mathbf{r}) = \frac{I}{4\pi} \int_{\mathcal{C}} \frac{\hat{\mathbf{l}}(\mathbf{r}')dl \times \hat{\mathbf{R}}}{R^2}} \tag{2.43}$$

[m0059] [1]

2.5-1

From the problem statement:
$\mathbf{l} = \hat{\mathbf{x}}l$ where $l = 5$ cm,
$\mathbf{v} = \hat{\mathbf{y}}v$ where $v = 3$ m/s, and
$\mathbf{B} = \hat{\mathbf{z}}B_0$ where $B_0 = 2$ T.

The potential difference generated across this segment of wire is:

$$V_{21} = \int_{\mathcal{C}} [\mathbf{v} \times \mathbf{B}] \cdot d\mathbf{l} \tag{2.44}$$

$$= \int_0^l [\hat{\mathbf{y}}v \times \hat{\mathbf{z}}B_0] \cdot \hat{\mathbf{x}}dl \tag{2.45}$$

$$= \int_0^l \hat{\mathbf{x}}vB_0 \cdot \hat{\mathbf{x}}dl \tag{2.46}$$

$$= vB_0 \int_0^l dl \tag{2.47}$$

$$= vB_0 l \tag{2.48}$$

$$= \boxed{0.3 \text{ V}} \tag{2.49}$$

Chapter 3

Wave Propagation in General Media

[m0073] [1]

3.1-1

Poynting's theorem says that energy can be either stored in electric fields or stored in magnetic fields. The third process that should be considered is resistance; i.e., dissipation of power by conversion to heat. Resistance will make the energy available for later release less than the power that is captured.

[m0122] [1]

3.2-1

From the problem statement, $\eta \cong 376.7\,\Omega$ (free space), $\widetilde{\mathbf{E}} = \hat{\mathbf{z}}(4\,\mu\text{V/m})$ (peak), and $\widetilde{\mathbf{H}} = \hat{\mathbf{y}}H_0$ where H_0 is not indicated.

Power density is given by:

$$\frac{|\widetilde{\mathbf{E}}|^2}{2\eta_0} \cong \boxed{2.12 \times 10^{-14}\ \text{W/m}^2} \quad \text{(a)} \tag{3.1}$$

Power flows in the direction of $\widetilde{\mathbf{E}} \times \widetilde{\mathbf{H}}$; i.e., $\hat{\mathbf{z}} \times \hat{\mathbf{y}} = \boxed{-\hat{\mathbf{x}}}$ (b).

[m0128] [1]

3.3-1

From the problem statement, $\epsilon''/\epsilon' = \sqrt{8}$. The phase propagation constant is given by

$$\beta = \omega \left\{ \frac{\mu\epsilon'}{2} \left[\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1 \right] \right\}^{1/2} \tag{3.2}$$

One finds $\beta = \underline{\omega\sqrt{2\mu\epsilon'}}$.

[m0132] [1]

3.5-1

Complex permittivity is $\epsilon' - j\epsilon''$. Here:

$$\epsilon'' = \frac{\sigma}{\omega} = \frac{\sigma}{2\pi f} = \frac{5 \text{ nS/m}}{2\pi \, (2 \text{ MHz})} \cong 4.00 \times 10^{-16} \text{ F/m} \tag{3.3}$$

Since loss tangent is ϵ''/ϵ',

$$\epsilon' = \frac{\epsilon''}{2 \times 10^{-6}} \cong 2.00 \times 10^{-10} \text{ F/m} \tag{3.4}$$

Relative permittivity is $\epsilon'/\epsilon_0 \cong \boxed{22.5}$.

[m0132] [2]

3.5-2

From the problem statement, $\epsilon''/\epsilon' = 0.05$ at $f_0 \triangleq 2$ GHz. So:

$$\frac{\epsilon''}{\epsilon'} \triangleq \frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2\pi f_0 \epsilon} = 0.05 \tag{3.5}$$

Since conductivity σ is constant with frequency, the loss tangent at the new frequency $f = 2.5$ GHz is:

$$0.05 \cdot \frac{f_0}{f} = \boxed{0.04} \tag{3.6}$$

[m0130] [1]

3.6-1

The complex wave impedance is

$$\eta_c \triangleq \frac{\widetilde{E}}{\widetilde{H}} \tag{3.7}$$

where $\widetilde{E}$ and $\widetilde{H}$ are the scalar components the electric field intensity and magnetic field intensity vectors, respectively. Therefore the phase difference between $\widetilde{E}$ and $\widetilde{H}$ is equal the phase of η_c. Separately we know that

$$\eta_c = \sqrt{\frac{\mu}{\epsilon'}} \cdot \left[1 - j\frac{\epsilon''}{\epsilon'}\right]^{-1/2} \tag{3.8}$$

and

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon'} \tag{3.9}$$

From this, we see that the phase of η_c is zero when conductivity $\sigma = 0$, and that the magnitude of the phase must increase with increasing σ. Therefore the difference in phase between the electric and magnetic fields *increases* as the ice melts.

[m0130] [2]

3.6-2

From the problem statement:
$|E_0| = 100\ \mu\text{V/m}$,
$\alpha = 0.2\ \text{m}^{-1}$, and
$l = 7$ cm.

The electric field intensity after propagation over this additional distance is:

$$|E_0|\, e^{-\alpha l} \cong \boxed{98.6\ \mu\text{V/m}} \tag{3.10}$$

[m0133] [1]

3.7-1

Let α be the attenuation constant and l be the distance of propagation. From the problem statement, the attenuation is:

$$e^{-2\alpha l} = -3 \text{ dB} \tag{3.11}$$

which corresponds to a factor of $\cong 0.502$. Therefore the attenuation of the magnitude of the electric field intensity is:

$$\sqrt{e^{-2\alpha l}} = e^{-\alpha l} \cong \sqrt{0.502} \cong 0.708 \tag{3.12}$$

The reduction of the electric field intensity is

$$1 - e^{-\alpha l} \cong 0.292 = \boxed{29.9\%} \tag{3.13}$$

[m0155] [1]

3.9-1

From the problem statement, $\alpha = 0.03 \text{ m}^{-1}$ and $l = 20$ m. Using the concept of attenuation rate, the attenuation can be computed directly in dB as follows:

$$8.69\alpha l \cong \boxed{5.2 \text{ dB}} \tag{3.14}$$

[m0155] [2]

3.9-2

The relevant facts are that $\alpha = 0.2$ m^{-1} and $l = 7$ cm. There are two equally-good ways to solve this problem.

Method 1:

$$-20 \log_{10} e^{-\alpha l} \cong \boxed{+0.12 \text{ dB}} \tag{3.15}$$

Method 2: The attenuation rate is 8.69α dB/m. Therefore the attenuation is:

$$8.69\alpha l \cong \boxed{+0.12 \text{ dB}} \tag{3.16}$$

[m0156] [1]

3.10-1

Since the loss tangent is $\ll 1$, this material may be considered a "poor conductor", and therefore

$$\alpha \approx \omega\sqrt{\mu\epsilon'}\frac{\epsilon''}{2\epsilon'}$$

where
$\omega = 2\pi f = 2\pi\,(2.4\text{ GHz}) \cong 1.508 \times 10^{10}$ rad/s;
$\mu \approx \mu_0 \cong 4\pi \times 10^{-7}$ H/m, since this is a dielectric;
$\epsilon' = \epsilon_r\epsilon_0 = 3.0 \cdot (8.854 \times 10^{-12}\text{ F/m}) \cong 26.6$ pF/m; and
$\epsilon''/\epsilon' = 0.02$ (the loss tangent).
Therefore $\underline{\alpha \approx 0.871\text{ m}^{-1}}$.

[m0157] [1]

3.11-1

The phase velocity of lossless media is $c/\sqrt{\mu_r \epsilon_r}$, which would be approximately constant with frequency in this case.

The phase velocity of a poor conductor is *approximately* $c/\sqrt{\mu_r \epsilon_r}$, which again would be approximately constant with frequency in this case.

The phase velocity of a good conductor is

$$\sqrt{\frac{4\pi f}{\mu\sigma}} \tag{3.17}$$

which is proportional to the square root of frequency. This is the behavior described in the problem statement. We conclude that the material is a $\boxed{\text{good conductor}}$.

[m0158] [1]

3.12-1

The expression will have the form

$$\boxed{\widetilde{\mathbf{E}} = \hat{\mathbf{y}} E_0 e^{-\alpha z} e^{-j\beta z}} \quad (3.18)$$

The attenuation constant α for *any* homogeneous medium is

$$\alpha = \frac{1}{\delta_s} = \frac{1}{2\ \mu\text{m}} = \boxed{5.00 \times 10^5\ \text{m}^{-1} = \alpha} \quad (3.19)$$

In a good conductor, $\beta \approx \alpha$, so $\boxed{\beta = 5.00 \times 10^5\ \text{m}^{-1}}$.

[m0158] [2]

3.12-2

From the problem statement, $\delta_s = 2.5\ \mu$m. For a good conductor,

$$\delta_s \approx \frac{1}{\sqrt{\pi\sigma\mu f}} \tag{3.20}$$

Decreasing f by a factor of 4 gives us:

$$\delta_s \approx (2.5\ \mu\text{m})\,\frac{\sqrt{f}}{\sqrt{f/4}} = \boxed{5.0\ \mu\text{m}} \tag{3.21}$$

[m0158] [3]
3.12-3

From the problem statement, the distance of propagation $l = 2\delta_s$, where δ_s is skin depth. Therefore the power density is reduced by a factor of:

$$e^{-2\alpha l} = e^{-2(1/\delta_s)(2\delta_s)} = e^{-4} \tag{3.22}$$

In dB:

$$10 \log_{10} e^{-4} \cong -17.4 \text{ dB} \tag{3.23}$$

So, we would say the power density has been reduced by $\boxed{17.4 \text{ dB}}$.

Chapter 4

Current Flow in Imperfect Conductors

[m0159] [1]

4.2-1

From the problem statement we define the following:
length $l = 1$ cm,
side length $d = 0.1$ cm,
permeability $\mu = \mu_0$ since the material is non-magnetic,
conductivity $\sigma = 5 \times 10^6$ S/m,
frequency $f = 10$ MHz.
Since this is a good conductor, the skin depth of the material can be calculated as follows:

$$\delta_s \approx \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi\,(10\text{ MHz})\,(4\pi \times 10^{-7}\text{ H/m})\,(5 \times 10^6\text{ S/m})}} \cong 71.2\ \mu\text{m} \tag{4.1}$$

Note that $\delta_s \ll d$. Therefore the current is, to a good approximation, limited to a very thin region around the perimeter of the cross section. This allows us to calculate the resistance R as follows:

$$R \approx \frac{l}{\sigma\,(\delta_s W)} \tag{4.2}$$

where in this case $W \approx 4d$. Thus:

$$R \approx \frac{1}{\sigma\,(\delta_s 4d)} \cong \boxed{7.02\text{ m}\Omega} \tag{4.3}$$

[m0159] [2]

4.2-2

(a) Since this wire is comprised of a good conductor, and since the skin depth is small compared to the cross-section, reactance is equal to resistance. Therefore $X = \underline{7.02\ \mathrm{m}\Omega}$.
(b) Since $L = X/\omega$, the equivalent inductance is $\underline{0.112\ \mathrm{nH}}$.

[m0159] [3]

4.2-3

(a) From Ohm's law, the current density is $\underline{\sigma E_0}$, where σ is conductivity.

(b) As explained in the book,

$$I \approx \sigma E_0 W \frac{\delta_s}{1+j}$$

where δ_s is skin depth and W is the cross-sectional dimension of the wire. In this case $W = 2\pi a$, so:

$$I \approx \boxed{\sigma E_0 2\pi a \frac{\delta_s}{1+j}}$$

(c) Units check:

$$\left[\frac{\text{S}}{\text{m}}\right]\left[\frac{\text{V}}{\text{m}}\right][\text{m}]\frac{[\text{m}]}{(\text{unitless})} = [\text{S}]\,[\text{V}] = \left[\frac{1}{\Omega}\right][\text{V}] = [\text{A}] \quad , \tag{4.4}$$

as expected.

[m0159] [4]

4.2-4

From the problem statement:
$l = 2$ mm,
$\sigma = 10^8$ S/m, and
$f = 10$ GHz.

(a) The resistance of the pin is:

$$R \approx \frac{l}{\sigma \delta_s W_{eff}} \tag{4.5}$$

where W_{eff} is the "effective width" of the pin. This parameter is the simply the perimeter of the cross-section, so $W_{eff} = 0.2 + 0.1 + 0.2 + 0.1 = 0.6$ mm. Since this material is a non-magnetic good conductor, the skin depth is

$$\delta_s \approx \frac{1}{\sqrt{\pi f \mu_0 \sigma}} \cong 5.03 \times 10^{-7} \text{ m} \tag{4.6}$$

Therefore $R \approx \underline{66.2 \text{ m}\Omega}$.

(b) Reactance equals resistance for a good conductor, so

$$L_{eq} = \frac{X}{\omega} = \frac{R}{2\pi f} \cong \underline{1.05 \text{ pH}} \tag{4.7}$$

[m0159] [5]

4.2-5

From the problem statement, $R = R_{10} \triangleq 9$ mΩ at 10 MHz. Since the skin depth is much less than the radius of the wire, the resistance of a wire is proportional to $\sqrt{f}$ where f is frequency. Therefore the resistance at 5 MHz is

$$R_5 \approx R_{10}\sqrt{\frac{5 \text{ MHz}}{10 \text{ MHz}}} \cong 6.36 \text{ m}\Omega \tag{4.8}$$

Given the same assumption about the skin depth, it is known that the reactance $X_5 \approx R_5$. Therefore the impedance is:

$$Z_5 = R_5 + jX_5 = \boxed{6.36 + j6.36 \text{ m}\Omega} \tag{4.9}$$

Chapter 5

Wave Reflection and Transmission

[m0161] [1]

5.1-1

From the problem statement:
Region 1 has $\epsilon_{r1} \approx 1$ and $\mu_{r1} \approx 1$.
Region 2 has $\epsilon_{r2} = 36$ and $\mu_{r1} \approx 1$.
$|E_0^i| = 30$ V/m.
$f = 50$ MHz.

(a) The reflection coefficient is:

$$\Gamma_{12} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

where

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_{r1}\mu_0}{\epsilon_{r1}\epsilon_0}} \cong 376.7\ \Omega$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_{r2}\mu_0}{\epsilon_{r2}\epsilon_0}} \cong 62.8\ \Omega$$

so $\Gamma_{12} \cong \underline{-0.714}$.

(b) The time-average power density of the incident wave is:

$$\mathcal{S}_{av}^i = \frac{|E_0^i|^2}{2\eta_1} \cong \boxed{1.19\ \frac{\text{W}}{\text{m}^2}}$$

(c) The time-average power density of the reflected wave is:

$$\mathcal{S}_{av}^r = |\Gamma_{12}|^2\, \mathcal{S}_{av}^i \cong \boxed{0.61\ \frac{\text{W}}{\text{m}^2}}$$

(d) The fraction of incident power transmitted into the dielectric is:

$$1 - |\Gamma_{12}|^2 \cong 0.49 = \boxed{49\%}$$

[m0161] [2]

5.1-2

From the problem statement:
Region 1 (air): $\epsilon_{r,1} \approx 1$, $\mu_1 \approx \mu_0$.
Reqion 2: $\epsilon_{r,2} = 16$, $\mu_2 \approx \mu_0$.

The reflection coefficient is

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0/\sqrt{\epsilon_{r,2}} - \eta_0/\sqrt{\epsilon_{r,1}}}{\eta_0/\sqrt{\epsilon_{r,2}} + \eta_0/\sqrt{\epsilon_{r,1}}} = \frac{1/\sqrt{\epsilon_{r,2}} - 1/\sqrt{\epsilon_{r,1}}}{1/\sqrt{\epsilon_{r,2}} + 1/\sqrt{\epsilon_{r,1}}} \approx \frac{1/4 - 1}{1/4 + 1} \cong -\frac{3}{5} \tag{5.1}$$

The fraction of power transmitted into Region 2 is:

$$1 - |\Gamma|^2 \approx 0.64 = \boxed{64\%} \tag{5.2}$$

[m0161] [3]

5.1-3

Let η_1 be the wave impedance in Region 1 and let η_2 be the wave impedance in Region 2. Note:

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} \tag{5.3}$$

and

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \tag{5.4}$$

From the problem statement, $\mu_2 = 2\mu_1$ and $\epsilon_2 = 2\epsilon_1$. Therefore:

$$\eta_2 = \sqrt{\frac{2\mu_1}{2\epsilon_1}} = \eta_1 \tag{5.5}$$

The reflection coefficient for normal incidence is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \tag{5.6}$$

Since in this case $\eta_2 = \eta_1$, $\Gamma = 0$ and therefore the fraction of power reflected is $\boxed{0}$.

[m0163] [1]
5.3-1

Interpreting the problem statement in terms of the theory presented in the textbook, this is a planar slab problem with Region 1 being the unknown material, Region 2 being the radome with $\epsilon_{r2} = 6$, and Region 3 being air, which may approximate as free space. Since the materials in Regions 1 and 3 are different, half-wave matching is not available; instead, quarter-wave matching is required.

(a) For quarter-wave matching,

$$\eta_2 = \sqrt{\eta_1 \eta_3}. \tag{5.7}$$

In this problem, $\eta_3 = \eta_0$; i.e., the wave impedance of free space. Solving for η_1:

$$\eta_1 = \frac{\eta_2^2}{\eta_0} \tag{5.8}$$

Recall $\eta = \sqrt{\mu/\epsilon}$ in general. Also recall that dielectrics are non-magnetic materials, so $\mu \approx \mu_0$ in all three regions. Therefore factors of $\sqrt{\mu_0}$ cancel out and we are left with

$$\frac{1}{\sqrt{\epsilon_{r1}}} = \frac{\left(1/\sqrt{\epsilon_{r2}}\right)^2}{1/\sqrt{1}} = \frac{1}{\epsilon_{r2}} \tag{5.9}$$

Subsequently

$$\epsilon_{r1} = \epsilon_{r2}^2 = 6^2 = \boxed{36} \quad \text{(a)} \tag{5.10}$$

(b) The minimum thickness is one-quarter wavelength in the dielectric of the radome. Let the wavelength in the radome be λ_2, and let the wavelength in free space be $\lambda 0$. Then:

$$\lambda_2 = \frac{\lambda_0}{\sqrt{\epsilon_{r2}}} = \frac{c/f}{\sqrt{\epsilon_{r2}}} \cong 12.25 \text{ mm} \tag{5.11}$$

Subsequently

$$\frac{\lambda_2}{4} \cong \boxed{3.06 \text{ mm}} \quad \text{(b)} \tag{5.12}$$

(c) Other thicknesses at which perfect transmission is possible are greater by an integer number of half-wavelengths. Therefore the second-smallest thickness is:

$$\frac{\lambda_2}{4} + \frac{\lambda_2}{2} \cong \boxed{9.19 \text{ mm}} \quad \text{(c)} \tag{5.13}$$

[m0163] [2]

5.3-2

In this problem, the three material regions are air, radome ($\epsilon_{r,2} = 8$), and air. Since the exterior regions are identical, half-wave matching must be used. Therefore the minimum thickness is $\lambda/2$ in the radome material.

The free space wavelength is:

$$\lambda_0 = \frac{c}{f} \cong 3.896 \text{ mm} \tag{5.14}$$

where $f = 77$ GHz from the problem statement. The wavelength in the radome is:

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_{r,2}}} \cong 1.377 \text{ mm} \tag{5.15}$$

Therefore the minimum thickness of the radome is $\lambda/2 \cong \underline{0.689 \text{ mm}}$.

The 30 cm distance between antenna and radome is irrelevant in this problem.

[m0165] [1]

5.4-1

The position vector $\mathbf{r}$ written in Cartesian coordinates is:

$$\mathbf{r} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z \tag{5.16}$$

The vector $\mathbf{k} \triangleq \beta\hat{\mathbf{k}}$ where β is the free-space phase propagation constant evaluated at $f = 600$ MHz. Thus:

$$\beta = \omega\sqrt{\mu\epsilon} = 2\pi f\sqrt{\mu_0\epsilon_0} \cong \boxed{12.575 \text{ rad/m}} \tag{5.17}$$

The unit vector $\hat{\mathbf{k}}$ is the direction of propagation, which points away from the origin. In this problem, it is simply the basis vector $\hat{\mathbf{r}}$ in the spherical coordinate system. Thus:

$$\hat{\mathbf{k}} = \hat{\mathbf{x}}\cos\phi_0\sin\theta_0 + \hat{\mathbf{y}}\sin\phi_0\sin\theta_0 + \hat{\mathbf{z}}\cos\theta_0 \tag{5.18}$$

Subsequently:

$$\mathbf{k}\cdot\mathbf{r} = (\beta\cos\phi_0\sin\theta_0)x + (\beta\sin\phi_0\sin\theta_0)y + (\beta\cos\theta_0)z \tag{5.19}$$

and

$$\widetilde{\mathbf{E}} = \hat{\mathbf{e}}E_0e^{-j\mathbf{k}\cdot\mathbf{r}} = \boxed{\hat{\mathbf{e}}E_0e^{-j\beta x\cos\phi_0\sin\theta_0}e^{-j\beta y\sin\phi_0\sin\theta_0}e^{-j\beta z\cos\theta_0}} \tag{5.20}$$

A great way to check your work is to evaluate the expression for various values of θ_0 and ϕ_0. For example, $\theta_0 = 0$ should correspond to propagation in the $+\hat{\mathbf{z}}$ direction. (It does.)

[m0166] [1]

5.5-1

TM ("transverse magnetic") means the magnetic field is transverse (perpendicular to) the plane of incidence. The electric field is perpendicular to the magnetic field. Therefore the electric field vector lies in the plane of incidence. **Option (a).**

[m0166] [2]

5.5-2

The magnetic field intensity **H** is in the plane of incidence. The electric field intensity **E** is perpendicular to **H**, so **E** is perpendicular to the plane of incidence. Therefore this situation is TE (a). This situation is not TEM since that requires both **E** and **H** to be parallel to the surface, which occurs only if the angle of incidence is 0°.

[m0167] [1]

5.6-1

The reflected field at the point of reflection is

$$\hat{\mathbf{e}}E_0\Gamma_{TE} = \boxed{\hat{\mathbf{e}}E_0\left(\frac{\eta_2\cos\psi^i - \eta_1\cos\psi^t}{\eta_2\cos\psi^i + \eta_1\cos\psi^t}\right)} \tag{5.21}$$

[m0164] [1]

5.7-1

From the textbook,

$$\Gamma_{TM} = \frac{-\eta_1 \cos\psi^i + \eta_2 \cos\psi^t}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} \tag{5.22}$$

Therefore

$$1 - \Gamma_{TM} = 1 - \frac{-\eta_1 \cos\psi^i + \eta_2 \cos\psi^t}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} \tag{5.23}$$

$$= \frac{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} - \frac{-\eta_1 \cos\psi^i + \eta_2 \cos\psi^t}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} \tag{5.24}$$

$$= \frac{+2\eta_1 \cos\psi^i}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} \tag{5.25}$$

and therefore

$$(1 - \Gamma_{TM}) \frac{\eta_2}{\eta_1} = \frac{+2\eta_2 \cos\psi^i}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} \tag{5.26}$$

But we could also try this:

$$1 + \Gamma_{TM} = 1 + \frac{-\eta_1 \cos\psi^i + \eta_2 \cos\psi^t}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} \tag{5.27}$$

$$= \frac{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} + \frac{-\eta_1 \cos\psi^i + \eta_2 \cos\psi^t}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} \tag{5.28}$$

$$= \frac{+2\eta_2 \cos\psi^t}{+\eta_1 \cos\psi^i + \eta_2 \cos\psi^t} \tag{5.29}$$

Comparing Equations 5.26 and 5.29, we see

$$(1 - \Gamma_{TM}) \frac{\eta_2}{\eta_1} = \boxed{(1 + \Gamma_{TM}) \frac{\cos\psi^i}{\cos\psi^t}} \tag{5.30}$$

[m0168] [1]

5.8-1

From the problem statement: $n_1 = 1$, $n_2 = 1.7$, angle of incidence $\psi^i = 23°$, and we seek the angle of transmission ψ^t. From Snell's law of transmission:

$$\sin \psi^t = \frac{\beta_1}{\beta_2} \sin \psi^i \tag{5.31}$$

Since both materials are non-magnetic, $\mu_1 \approx \mu_2 \approx \mu_0$. Therefore:

$$\frac{\beta_1}{\beta_2} = \frac{\omega\sqrt{\mu_1\epsilon_1}}{\omega\sqrt{\mu_2\epsilon_2}} \approx \frac{\omega\sqrt{\mu_0\epsilon_{r1}\epsilon_0}}{\omega\sqrt{\mu_0\epsilon_{r2}\epsilon_0}} = \frac{\sqrt{\epsilon_{r1}}}{\sqrt{\epsilon_{r2}}} = \frac{n_1}{n_2} = \frac{1}{1.7} \tag{5.32}$$

Subsequently:

$$\psi^t = \arcsin\left(\frac{1}{1.7}\sin 23°\right) \cong \boxed{13.3°} \tag{5.33}$$

[m0168] [2]

5.8-2

Snell's law of transmission states:

$$\sin \psi^t = \frac{\beta_1}{\beta_2} \sin \psi^i \tag{5.34}$$

Since the media are non-magnetic:

$$\frac{\beta_1}{\beta_2} = \frac{\omega\sqrt{\mu_1\epsilon_1}}{\omega\sqrt{\mu_2\epsilon_2}} \approx \frac{\omega\sqrt{\mu_0\epsilon_1}}{\omega\sqrt{\mu_0\epsilon_2}} = \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_2}} \tag{5.35}$$

Also, from the problem statement, we know $\eta_2 > \eta_1$. Since $\eta = \sqrt{\mu/\epsilon}$, this means

$$\sqrt{\frac{\mu_0}{\epsilon_2}} > \sqrt{\frac{\mu_0}{\epsilon_1}} \tag{5.36}$$

and subsequently $\epsilon_1 > \epsilon_2$. Returning to Equation 5.35, we see that β_1/β_2 must therefore be > 1, and subsequently

$$\sin \psi^t > \sin \psi^i \tag{5.37}$$

The angles ψ^i and ψ^t are limited to the range 0 to 2π rad; therefore $\psi^t > \psi^i$. The answer is therefore $\boxed{\text{(b)}}$; the transmitted wave bends away from the surface normal.

[m0168] [3]

5.8-3

From the problem statement:
Region 1: ϵ_{r1} to be determined, $\mu_{r1} \approx 1$.
Region 2: $\epsilon_{r2} = 3$ $\mu_{r2} \approx 1$.
$\psi^i = 30°$.
$\psi^t = 20°$.

Snell's law of transmission for non-magnetic media is:

$$\sin\psi^t = \frac{\sqrt{\epsilon_{r1}}}{\sqrt{\epsilon_{r2}}}\sin\psi^i \tag{5.38}$$

Solving for ϵ_{r1}:

$$\epsilon_{r1} = \left(\frac{\sqrt{\epsilon_{r2}}\sin\psi^t}{\sin\psi^i}\right)^2 \cong \boxed{1.40} \tag{5.39}$$

[m0172] [1]

5.10-1

"100% transmission" implies that the reflection coefficient is zero.

(a) Considering the transverse electric (TE) case: Γ_{TE} is never equal to zero for non-magnetic material, so the answer in this case is that no angles meet this criterion; i.e., this doesn't happen.

(b) Considering the transverse magnetic (TM) case: $\Gamma_{TM} = 0$ implies that the angle of incidence ψ^i is equal to the Brewster angle ψ^i_B. Therefore:

$$\tan\psi^i = \tan\psi^i_B = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \tag{5.40}$$

From the problem statement, $\epsilon_{r1} = 1$ and $\epsilon_{r2} = 5.0$. Therefore $\tan\psi^i = \sqrt{5.0}$. Since ψ^i is limited to the range 0 to $\pi/2$ rad, $\psi^i \cong \boxed{65.9^\circ}$.

[m0172] [2]

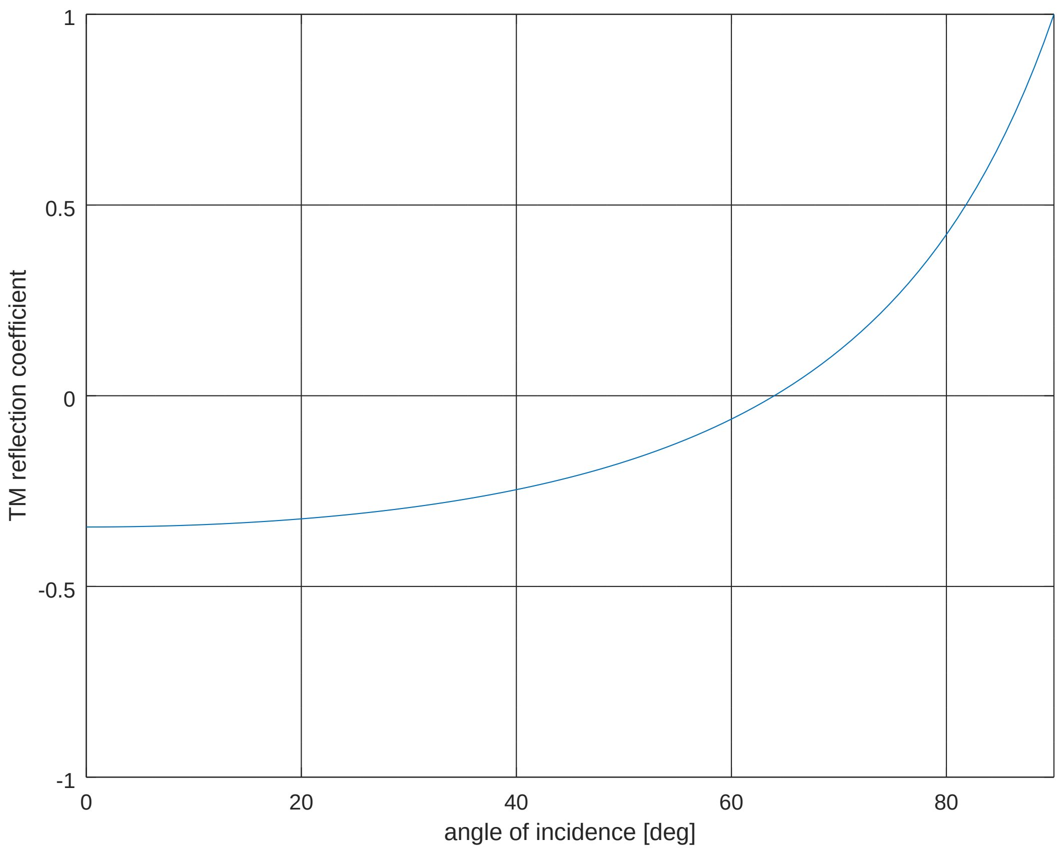
5.10-2

Dielectrics are non-magnetic, so

$$\Gamma_{TM} = \frac{-(\epsilon_{r2}/\epsilon_{r1})\cos\psi^i + \sqrt{\epsilon_{r2}/\epsilon_{r1} - \sin^2\psi^i}}{+(\epsilon_{r2}/\epsilon_{r1})\cos\psi^i + \sqrt{\epsilon_{r2}/\epsilon_{r1} - \sin^2\psi^i}} \tag{5.41}$$

From the problem statement, $\epsilon_{r1} = 1.5$ and $\epsilon_{r2} = 6.3$, so $\epsilon_{r2}/\epsilon_{r1} = 4.2$.

(a) The plot follows:



(b) Brewster's angle ψ_B^i in this case is given by

$$\tan\psi_B^i = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \cong 2.05 \tag{5.42}$$

Therefore $\underline{\psi_B^i \cong 64.0^\circ}$. This indeed is the point in which the curve intersects the $\Gamma_{TM} = 0$ line.

[m0172] [3]

5.10-3

TEM would mean that the angle of incidence is zero, so this possibility is ruled out. The TE reflection coefficient Γ_{TE} always has the same sign regardless of ψ^i, so this possibility is ruled out. The TM reflection coefficient Γ_{TM}, however, *does* change sign as ψ^i is increased from zero to 90°. Therefore this is TM (b) (only).

[m0169] [1]

5.11-1

Total internal reflection (TIR) requires $\psi^i \geq$ the critical angle ψ_c. Since the media are non-magnetic:

$$\sin\psi_c = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} = \sqrt{\frac{1}{2.1}} \cong 0.690 \tag{5.43}$$

Therefore $\psi_c \cong 43.6°$, and the range of angles of incidence over which TIR occurs is $\underline{\psi^i \geq 43.6°}$.

[m0169] [2]

5.11-2

From the problem statement, $\epsilon_{r1} = 2.9$ and $\epsilon_{r2} \approx 1$. Total internal reflection occurs when the angle of incidence ψ^i is greater than the critical angle ψ_c. Since the media are non-magnetic:

$$\psi_c = \arcsin \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \cong 36.0^\circ \tag{5.44}$$

Therefore there is non-zero transmission across the interface for $\underline{0 \leq \psi^i \leq 36.0^\circ}$.

[m0169] [3]

5.11-3

If the wave is arriving at an oblique angle of incidence and there is no transmission of power across the interface, then the wave is experiencing total internal reflection. Since the media are non-magnetic, $\boxed{\epsilon_2 \leq \epsilon_1}$.

Chapter 6

Waveguides

[m0174] [1]

6.3-1

The cutoff frequency for mode m is

$$f_c^{(m)} = \frac{m}{2a\sqrt{\mu\epsilon}} \tag{6.1}$$

where, from the problem statement, $a = 2$ cm. Mode $m = 0$ (TE_0) does not propagate in a parallel plate waveguide. Mode $m = 1$ (TE_1) has the lowest cutoff frequency of any propagating TE mode; therefore if we design the waveguide to have $f_c^{(1)} = 4$ GHz, then no propagating mode can have a frequency less than 4 GHz. Thus:

$$\frac{1}{2a\sqrt{\mu\epsilon}} = 4 \text{ GHz} \tag{6.2}$$

For a dielectric $\mu \approx \mu_0$ and $\epsilon = \epsilon_r\epsilon_0$. Thus:

$$\frac{1}{2a\sqrt{\mu_0\epsilon_r\epsilon_0}} = 4 \text{ GHz} \tag{6.3}$$

Solving for ϵ_r:

$$\epsilon_r = \left(\frac{1}{2a\sqrt{\mu_0\epsilon_0}\cdot(4 \text{ GHz})}\right)^2 \cong \boxed{3.51} \tag{6.4}$$

[m0174] [2]

6.3-2

The cutoff frequency for the $m = 2$ mode is

$$f_c^{(2)} = \frac{m}{2a\sqrt{\mu\epsilon}} \approx \frac{2}{2(15\text{ mm})\sqrt{\mu_0\epsilon_0}} \cong 20.0\text{ GHz} \tag{6.5}$$

Therefore $f = 1.5 f_c^{(2)} \cong 30.0$ GHz.

The phase velocity in the waveguide is (note $\omega \triangleq 2\pi f$):

$$v_p \triangleq \frac{\omega}{k_z^{(2)}} = \frac{\omega}{\sqrt{\omega^2\mu_0\epsilon_0 - (2\pi/a)^2}} \cong \boxed{4.02 \times 10^8\text{ m/s}}\text{ (a)} \tag{6.6}$$

Note that we could obtain the same result using the alternative equation:

$$v_p = \frac{1}{\sqrt{\mu_0\epsilon_0}} \cdot \frac{1}{\sqrt{1 - \left(f_c^{(2)}/f\right)^2}} \tag{6.7}$$

The group velocity in the waveguide is:

$$v_g \triangleq \frac{\partial\omega}{\partial k_z^{(2)}} = \left(\frac{\partial k_z^{(2)}}{\partial\omega}\right)^{-1} = \left(\frac{\partial}{\partial\omega}\sqrt{\omega^2\mu_0\epsilon_0 - (2\pi/a)^2}\right)^{-1} \tag{6.8}$$

$$= \frac{1}{\sqrt{\mu_0\epsilon_0}} \cdot \sqrt{1 - \left(\frac{2\pi}{a\omega}\right)^2 \cdot \frac{1}{\mu_0\epsilon_0}} \tag{6.9}$$

$$\cong \boxed{2.23 \times 10^8\text{ m/s}}\text{ (b)} \tag{6.10}$$

Note that we could obtain the same result using the alternative equation:

$$v_g = \frac{1}{\sqrt{\mu_0\epsilon_0}} \cdot \sqrt{1 - \left(f_c^{(2)}/f\right)^2} \tag{6.11}$$

(c) The phase velocity in unbounded free space is simply $1/\sqrt{\mu_0\epsilon_0} = c \cong 3 \times 10^8$ m/s. In unbounded free space, group velocity equals phase velocity, so this is also the group velocity. We find:
phase velocity in the waveguide is greater than the phase velocity in unbounded free space, but that
group velocity in the waveguide is less than the group velocity in unbounded free space.

[m0174] [3]

6.3-3

From the problem statement, $a = 3$ cm, $\epsilon_r = 2.4$, and $\mu_r \approx 1$. Recall that TE_0 cannot propagate, so the lowest-order TE mode that can propagate is TE_1. The cutoff frequency for this mode is

$$f_c^{(1)} = \frac{1}{2a\sqrt{\mu_0 \epsilon_r \epsilon_0}} \cong 3.23 \text{ GHz} \tag{6.12}$$

The cutoff frequency for TE_2 is $f_c^{(2)} = 2f_c^{(1)}$. Above this frequency, more than one TE mode (i.e., TE_1 and TE_2) can propagate. Therefore the range of frequencies for which only one TE mode can propagate is $\underline{3.23 \text{ GHz} \leq f < 6.45 \text{ GHz}}$.

[m0174] [4]

6.3-4

Since the waveguide is air-filled, $\epsilon \approx \epsilon_0$ and $\mu \approx \mu_0$. "Free of mode dispersion" indicates that no more than one mode is able to propagate. This requires that only the lowest-order mode can propagate, which is TM_0. (Recall TE_0 cannot propagate regardless of frequency, so need not be considered.) Therefore we set the cutoff frequency for the next-higher mode ($m = 1$) as follows:

$$f_c^{(1)} = \left.\frac{m}{2a\sqrt{\mu\epsilon}}\right|_{m=1} = \frac{1}{2a\sqrt{\mu_0\epsilon_0}} = 94 \text{ GHz} \tag{6.13}$$

Solving for a, we find $a \cong \underline{1.59 \text{ mm}}$. Any larger value of a will result in the cutoff frequency for the next-higher order mode being less than 94 GHz.

[m0177] [1]

6.5-1

TM_0 has a cutoff frequency of zero and is always available. TE_0, on the other hand, is precluded by boundary conditions. Therefore $\boxed{\text{TM}}$ can always support one more mode (specifically, $m = 0$) than TE.

[m0220] [1]

6.6-1

From the problem statement:
$E_0 \triangleq \left|\widetilde{\mathbf{E}}\right| = 1.5$ mV/m at the midpoint between plates,
$\mu \approx \mu_0$, and
$\epsilon_r = 2.7$.

The TM_0 mode propagates just like a uniform plane wave in unbounded space. The intrinsic impedance of the medium in this case is

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \approx \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} = \frac{\eta_0}{\sqrt{\epsilon_r}} \cong \frac{376.7\ \Omega}{\sqrt{2.7}} \cong 229.3\ \Omega$$

The magnetic flux density B_0 at the same location that E_0 is measured is:

$$B_0 = \mu_0 H_0 = \mu_0 \frac{E_0}{\eta} \cong \boxed{8.22\ \text{pT}}$$

[m0220] [2]
6.6-2

The answer is $\boxed{\text{no}}$, because such a mode will violate the boundary condition that the tangent component of $\widetilde{\mathbf{E}}$ is zero on conducting surfaces. In the case of the parallel plate waveguide, TE_0 is ruled out because $m = 0$ permits no variation in the field over the cross-section, yet the component of $\widetilde{\mathbf{E}}$ tangent to the conducting surfaces must be zero. TM_0 exists because the electric field for that mode is perpendicular to the conducting surface, and therefore may be non-zero. In a rectangular waveguide, there are conducting surfaces in both directions, thus *both* possible orientations of $\widetilde{\mathbf{E}}$ are forced to zero by the associated boundary conditions.

[m0220] [3]

6.6-3

The $m = 0$ mode propagates like a plane wave in unbounded space. For such a wave, $v_p = c/\sqrt{\epsilon_r}$. Since $\epsilon_r > 1$ here, $\underline{v_p < c}$. (Note that for higher-order modes, v_p might be greater than c.)

Since the $m = 0$ mode propagates like a plane wave in unbounded space, $\underline{v_g = v_p}$.

Chapter 7

Transmission Lines Redux

[m0188] [1]

7.1-1

The characteristic impedance of parallel wire line embedded in a non-magnetic material having relative permittivity ϵ_r is:

$$Z_0 \approx \frac{1}{\pi}\frac{\eta_0}{\sqrt{\epsilon_r}}\ln\left(\frac{2D}{d}\right) \tag{7.1}$$

From the problem statement, $Z_0 = 300\ \Omega$ in air, for which $\epsilon_r \approx 1$. That is:

$$Z_0 \approx \frac{1}{\pi}\frac{\eta_0}{\sqrt{1}}\ln\left(\frac{2D}{d}\right) = 300\ \Omega \tag{7.2}$$

Once buried, ϵ_r increases to 3. Therefore the characteristic impedance becomes:

$$(300\ \Omega)\,\frac{\sqrt{1}}{\sqrt{3}} \cong \boxed{173\ \Omega} \tag{7.3}$$

[m0186] [1]

7.2-1

From the problem statement, $h/W = 0.001$, which meets the criterion for "wide" microstrip. For "wide" microstrip, the characteristic impedance is

$$Z_0 \approx \frac{\eta_0}{\sqrt{\epsilon_r}} \cdot \frac{h}{W}$$

Note that if h/W is doubled, then so too is Z_0. Therefore we expect the new value of Z_0 is about $\boxed{100\ \Omega}$.

[m0186] [2]

7.2-2

From the problem statement, $\epsilon_r = 3$, $h = 2$ mm, $W = 0.1$ mm, and trace thickness t can be neglected.

(a) Using the Wheeler 1977 formula with $W' = W$ (since trace thickness is negligible), we find $\boxed{209.3\ \Omega}$.

(b) Here, $h/W = 20$, so this best approximated as the $\boxed{\text{“narrow” case}}$.

(c) The appropriate formula for the narrow case is:

$$Z_0 \approx \frac{1}{\pi} \frac{\eta_0}{\sqrt{\epsilon_r}} \ln\left(4\frac{h}{W}\right) \cong \boxed{303.4\ \Omega} \tag{7.4}$$

Note that this is about 45% larger than predicted by the Wheeler 1977 formula. As discussed in the text, this is expected.

(d) The value of W that reduces Z_0 from 303.4 Ω to $0.8 \cdot 303.4 \cong 242.7$ Ω is obtained by solving

$$\frac{1}{\pi} \frac{\eta_0}{\sqrt{\epsilon_r}} \ln\left(4\frac{h}{W}\right) \cong \boxed{242.7\ \Omega} \tag{7.5}$$

for W, keeping the other parameters constant. We find $\boxed{W \cong 0.240 \text{ mm}}$.

(e) Using the Wheeler 1977 formula with $W' = W$ and the new value of W, we find the new value of Z_0 is about $\boxed{172.2\ \Omega}$. Note that the estimate from the “narrow” approximation was about 41% higher in this case.

This problem demonstrates that the “narrow” approximation is roughly 40% higher than the accurate Wheeler 1977 approximation over the range for which $h/W \gg 1$.

[m0186] [3]

7.2-3

The suggested strategy here is to assume that this is the wide case, and then check at the end to confirm. (If it turns out not to be the wide case, then you could check the slightly harder-to-analyze narrow case and would not have wasted much time. Similarly if the narrow case assumption doesn't check, then you would need to use a more complicated method such as Wheeler 1977 and again you wouldn't have wasted much time.) Assuming the wide case, the characteristic impedance is:

$$Z_0 \approx \frac{\eta_0}{\sqrt{\epsilon_r}} \left(\frac{h}{W} \right) \tag{7.6}$$

Solving for W:

$$W \approx \frac{\eta_0}{\sqrt{\epsilon_r}} \left(\frac{h}{Z_0} \right) \cong \frac{376.7\ \Omega}{\sqrt{3}} \left(\frac{1.52\ \text{mm}}{22\ \Omega} \right) \cong 15.03\ \text{mm} \tag{7.7}$$

Now we check: This value of W gives $h/W \cong 0.10$, so the narrow assumption is justified. Therefore $W \approx \underline{15.03\ \text{mm}}$.

[m0186] [4]

7.2-4

From the problem statement, we at first have: $h = 2$ mm, $W = W_1 \triangleq 0.10$ mm, and $Z_0 = Z_{0,1} \triangleq K$. Note that $h/W_1 = 20$, so this microstrip line may be considered to be "narrow". Thus:

$$Z_{0,1} \sim \frac{1}{\pi}\frac{\eta_0}{\sqrt{\epsilon_r}} \ln\left(\frac{4h}{W_1}\right) \sim K \tag{7.8}$$

The second microstrip line has $W = W_2 \triangleq 0.05$ mm. Since it is on the same printed circuit board, ϵ_r and h remain unchanged. Note $h/W_2 = 40$ so this line too may be considered "narrow", and the characteristic impedance is estimated as:

$$Z_{0,2} \sim \frac{1}{\pi}\frac{\eta_0}{\sqrt{\epsilon_r}} \ln\left(\frac{4h}{W_2}\right) \tag{7.9}$$

$$= \frac{1}{\pi}\frac{\eta_0}{\sqrt{\epsilon_r}} \ln\left(\frac{4h}{W_1}\right)\left[\ln\left(\frac{4h}{W_1}\right)\right]^{-1} \ln\left(\frac{4h}{W_2}\right) \tag{7.10}$$

$$= K\left[\ln\left(\frac{4h}{W_1}\right)\right]^{-1} \ln\left(\frac{4h}{W_2}\right) \tag{7.11}$$

$$= \boxed{1.16K} \tag{7.12}$$

Despite the fact that our expression for Z_0 is a very rough approximation, the way that Z_0 depends on W is correct. Thus this answer is expected to be quite accurate.

[m0189] [1]

7.3-1

Interpreting the problem statement:
$Z_0 = 75\ \Omega$,
$b = 1$ mm,
$a = 0.1$ mm,
$R' = 0.1\ \Omega$/m, and
$\sigma_s = 0.1$ mS/m.

(a) The attenuation constant α can be approximated as

$$\alpha \approx \alpha_R + \alpha_G \tag{7.13}$$

where the contribution α_R due to conductor resistance is

$$\alpha_R \triangleq \frac{1}{2}\frac{R'}{Z_0} \cong 6.67 \times 10^{-4}\ \text{m}^{-1} \tag{7.14}$$

and the contribution α_G due to current leaking through the dielectric is

$$\alpha_G \triangleq \frac{1}{2} G' Z_0 \tag{7.15}$$

where G' is the conductance per length due to current leakage:

$$G' = \frac{2\pi\sigma_s}{\ln(b/a)} \cong 0.2729\ \text{mS/m} \tag{7.16}$$

Therefore, $\alpha_G \cong 1.02 \times 10^{-2}\ \text{m}^{-1}$ and subsequently $\alpha \approx \boxed{0.0109\ \text{m}^{-1}}$.

(b) Since $\alpha_G \gg \alpha_R$, current leakage is dominating over conductor resistance as the principal contribution to attenuation in this case.

[m0189] [2]

7.3-2

From the problem statement, $l = 100$ m, $Z_0 = 50\ \Omega$ and $R' = 125\ \text{m}\Omega/\text{m}$. Since the conductivity of the spacer material is negligible, $\alpha_G \approx 0$. Therefore:

$$\alpha \approx \alpha_R = \frac{1}{2}\frac{R'}{Z_0} \cong 0.00125\ \text{m}^{-1} \tag{7.17}$$

The power reduction in this cable is a factor of

$$e^{-2\alpha l} \cong 0.779 \tag{7.18}$$

The value in dB is obtained by taking $10\log_{10}$, yielding $\cong -1.1$ dB. Therefore the power lost in this cable is $\underline{\cong 1.1\ \text{dB}}$.

Chapter 8

Optical Fiber

[m0192] [1]

8.2-1

NA is defined through the acceptance angle θ_a as $\sin\theta_a = \text{NA}/n_0$, where n_0 is the index of refraction of the media from which the light is offered. Note that θ_a is maximized if $n_0 = 1$ (free space), since this is as small as n_0 can be. Thus we have $\sin\theta_a = \text{NA}$; i.e., $\theta_a = \boxed{14.5^\circ}$.

[m0192] [2]

8.2-2

The internal half-angle of the cone of acceptance, θ_a, is related to numerical aperture NA by the relationship $\text{NA} = \sin\theta_a$. The chance of stray light being into the fiber is reduced by reducing θ_a. Therefore NA should be minimized (a).

[m0193] [1]

8.3-1

The delay spread τ is given by:

$$\tau = l\ \frac{n_f}{c}\left(\frac{n_f}{n_c} - 1\right) \tag{8.1}$$

where l is length (here, 45 m), $c = 3 \times 10^8$ m/s is the speed of light in free space, and n_f and n_c are the indices of refraction of the fiber and the cladding, respectively. From the problem statement we have $n_f = \sqrt{2.2} = 1.483$. We can solve for n_c through the numerical aperture:

$$\text{NA} = \sqrt{n_f^2 - n_c^2} \tag{8.2}$$

Giving us

$$n_c = \sqrt{n_f^2 - (\text{NA})^2} = \sqrt{1.483^2 - 0.25^2} = 1.462 \tag{8.3}$$

Using Equation 8.1, we obtain the delay spread $\boxed{\tau = 3.23 \text{ ns}}$.

[m0193] [2]

8.3-2

From the problem statement: $t_{on} = 1$ ns, $l = 10$ m, $n_f = 1.55$, and $n_c = 1.50$. The delay spread is

$$\tau = l\frac{n_f}{c}\left(\frac{n_f}{n_c} - 1\right) \cong 1.72 \text{ ns} \tag{8.4}$$

The total width of the pulse at the opposite end of the fiber, including both rising and falling edges, is

$$t_{on} + \tau \cong \boxed{2.72 \text{ ns}} \tag{8.5}$$

Chapter 9

Radiation

[m0194] [1]

9.1-1

First, take a moment to see how this works for the $\hat{\mathbf{z}}$-directed current moment. The reference direction of the electric field in this case is $\hat{\theta}$. In the $z = 0$ plane, $\hat{\theta} = -\hat{\mathbf{z}}$; that is, opposite the direction of the current moment. The distance to the field point is irrelevant.

In this problem, the current moment is $\hat{\mathbf{y}}$-directed, and we are evaluating the direction of the electric field in the $y = 0$ plane. Thus, the reference direction of the electric field in this case is $\underline{-\hat{\mathbf{y}}}$.

[m0195] [1]

9.2-1

The defining relationship for the magnetic vector potential $\widetilde{\mathbf{A}}$ is

$$\widetilde{\mathbf{B}} \triangleq \nabla \times \widetilde{\mathbf{A}} \tag{9.1}$$

The SI base units of $\widetilde{\mathbf{B}}$ in terms of webers are Wb/m^2, and in terms of teslas are simply T. The curl operation ($\nabla\times$) is differentiation with respect to position, and so contributes SI base units of m^{-1}.

Therefore the units of $\widetilde{\mathbf{A}}$ in terms of webers are $\underline{\text{Wb/m}}$ (a).

The units of $\widetilde{\mathbf{A}}$ in terms of teslas are $\underline{\text{T}\cdot\text{m}}$ (b).

[m0196] [1]

9.3-1

(a) The magnetic vector potential is defined by the relationship $\widetilde{\mathbf{B}} \triangleq \nabla \times \widetilde{\mathbf{A}}$ where $\widetilde{\mathbf{B}}$ is magnetic flux density. Magnetic flux density has units of Wb/m^2 (alternatively, T; however we shall find Wb/m^2 to be easier to use in this case). Curl is differentiation with respect to spatial coordinates, and so contributes units of m^{-1}. Therefore $\widetilde{\mathbf{A}}$ has units of $\boxed{\text{Wb/m}}$.

(b) The equation of interest is:

$$\widetilde{\mathbf{A}} = \hat{\mathbf{l}}\ \mu \widetilde{I} \Delta l\ \frac{e^{-\gamma r}}{r} \tag{9.2}$$

Note that $\hat{\mathbf{l}}$ is unitless (since it is a unit vector) as is $e^{-\gamma r}$. Recall μ has units of H/m, and $\widetilde{I}\Delta l$ has units of A·m. Therefore the previous equation implies:

$$\left[\frac{\text{Wb}}{\text{m}}\right] = \left[\frac{\text{H}}{\text{m}}\right]\ [\text{A·m}]\ \frac{1}{[\text{m}]} \tag{9.3}$$

$$= \left[\frac{\text{H} \cdot \text{A}}{\text{m}}\right] \tag{9.4}$$

To make progress here, we need to express inductance L (units of H) in terms of other quantities present in the equation. Recall that inductance is magnetic flux (units of Wb) divided by current (units of A); therefore 1 H is 1 Wb/A. Subsequently:

$$\left[\frac{\text{Wb}}{\text{m}}\right] = \left[\frac{\text{Wb} \cdot \text{A}}{\text{A·m}}\right] \tag{9.5}$$

Units of A cancel, yielding the same units on both sides of the equation.

[m0198] [1]

9.5-1

Interpreting the problem statement:
$L = 0.3$ m
$a = 0.1$ mm
$I_0 = 0.5$ mA (peak)
$f = 10$ MHz
$r = 1000$ m

Note $a/L \sim 0.0003$, so this dipole can certainly be considered "thin." Also, the wavelength is $\lambda = c/f \cong 30$ m. Therefore, $L/\lambda \cong 0.01$, and so this dipole can certainly be considered "electrically-short." Furthermore, $r \gg \lambda$, so we may use "far field" expressions for the field radiated by a thin electrically-short dipole.

The magnitude of the electric field intensity for a z-directed dipole in this case is:

$$\left|\widetilde{\mathbf{E}}\right| \approx \left|\hat{\theta} j\eta \frac{I_0 \cdot \beta L}{8\pi} (\sin\theta) \frac{e^{-j\beta r}}{r}\right| \tag{9.6}$$

$$= \eta \frac{I_0 \cdot \beta L}{8\pi} (\sin\theta) \frac{1}{r} \tag{9.7}$$

This expression indicates that for r held constant, the minimum is achieved when $\sin\theta = 0$, and the corresponding field intensity is $\boxed{0}$ (at $\theta = 0$).

The maximum value is achieved when $\sin\theta = 1$ ($\theta = \pi/2$), where:

$$\left|\widetilde{\mathbf{E}}(\theta = \pi/2)\right| \approx \eta \frac{I_0 \cdot \beta L}{8\pi} (1) \frac{1}{r} \tag{9.8}$$

Here, $\eta \cong 376.7\ \Omega$ is the wave impedance of free space, and

$$\beta L = \frac{2\pi}{\lambda} \cdot L \cong 0.06283 \text{ rad} \tag{9.9}$$

Therefore the maximum value is $\boxed{471 \text{ nV/m (peak)}}$ (at $\theta = \pi/2$).

[m0198] [2]

9.5-2

From the problem statement:
$I_0 = 3$ mA (peak),
$L = 0.01\lambda$ (so the dipole is electrically-short),
$\eta = \eta_0 \cong 376.7\ \Omega$ (free space),
the distance r to the field point is 1000 m, and
$\lambda \ll r$.

In this case the magnitude of the electric field intensity $\widetilde{\mathbf{E}}$ is given by

$$\left|\widetilde{\mathbf{E}}\right| \approx \left|\hat{\theta} j \frac{\eta_0 I_0}{4} \frac{L}{\lambda} (\sin\theta) \frac{e^{-j\beta r}}{r}\right| \tag{9.10}$$

$$= \frac{\eta I_0}{4} \frac{L}{\lambda} (\sin\theta) \frac{1}{r} \tag{9.11}$$

The minimum value of $\left|\widetilde{\mathbf{E}}\right|$ occurs for $\theta = 0$, yielding $\boxed{0}$.

The maximum value of $\left|\widetilde{\mathbf{E}}\right|$ occurs for $\theta = \pi/2$, yielding $\boxed{2.83\ \mu\text{V/m}}$.

Chapter 10

Antennas

[m0207] [1]

10.2-1

From the problem statement:
Electrically-short dipole in free space,
$P_{rad} = -140$ dBW,
$f = 100$ MHz, and
$|I_0| = 1\ \mu$A rms.

The power radiated by an electrically-short dipole is:

$$P_{rad} \approx \eta \frac{|I_0|^2 (\beta L)^2}{48\pi} \tag{10.1}$$

where $\eta \approx \eta_0 \cong 376.7\ \Omega$ in free space and β is the free-space phase propagation constant at 100 MHz. To use this equation, we require P_{rad} in linear units:

$$P_{rad} = 10^{-140/10} = 1 \times 10^{-14} \text{ W} \tag{10.2}$$

Also, the equation presumes $|I_0|$ in peak units:

$$|I_0| = 1\ \mu\text{A (rms)} = \sqrt{2}\ \mu\text{A (peak)} \tag{10.3}$$

Solving first for βL:

$$\beta L \approx \frac{1}{|I_0|} \sqrt{48\pi \frac{P_{rad}}{\eta_0}} \tag{10.4}$$

Next, note $\beta = 2\pi/\lambda$, so:

$$L \approx \frac{\lambda}{|I_0|} \sqrt{\frac{12}{\pi} \frac{P_{rad}}{\eta_0}} \cong \boxed{0.00712\lambda} \tag{10.5}$$

Wavelength $\lambda = c/f \cong 3$ m, so $L \cong \boxed{21.4 \text{ cm}}$.

[m0207] [2]

10.2-2

From the problem statement:
$L = 0.01\lambda$ (so, electrically short),
$\eta = \eta_0 \cong 376.7~\Omega$ (free space), and
radiated power $P_{rad} = 30$ mW.

Since the dipole is electrically-short and in free space:

$$P_{rad} \approx \eta_0 \frac{|I_0|^2 (\beta L)^2}{48\pi} \tag{10.6}$$

where I_0 is, for the moment, in peak (not RMS) units. Solving for the current:

$$|I_0| \approx \sqrt{48\pi \frac{P_{rad}}{\eta_0}} \cdot \frac{1}{\beta L} \tag{10.7}$$

Note $\beta = 2\pi/\lambda$, so $\beta L = 2\pi L/\lambda$. Thus:

$$|I_0| \approx \sqrt{48\pi \frac{P_{rad}}{\eta_0}} \cdot \frac{1}{2\pi (L/\lambda)} \cong 1.74 \text{ A} \tag{10.8}$$

The RMS value is obtained by dividing by $\sqrt{2}$, yielding <u>1.23 A</u>.

[m0202] [1]

10.5-1

The fact that radiated power is maximized when the source impedance $R_S + jX_S = 10 + j2500\ \Omega$ means that $R_A = 10\ \Omega$ and $X_A = -2500\ \Omega$; we know this from basic circuit theory (i.e., maximum power transfer requires conjugate-matching of the load impedance to the source impedance). Since $R_A \triangleq R_{rad} + R_{loss} = 10\ \Omega$, and $R_{loss} = 25\ \text{m}\Omega$, $R_{rad} = 9.975\ \Omega$. Finally,

$$e_{rad} = \frac{R_{rad}}{R_A} = 0.9975 \tag{10.9}$$

Summarizing, $\boxed{R_{rad} = 9.975\ \Omega}$, $\boxed{X_A = -2500\ \Omega}$, and $\boxed{e_{rad} = 99.75\%}$.

[m0202] [2]

10.5-2

The output impedance of the source is 50 Ω, and note that this impedance has no imaginary component. The impedance of any antenna is $Z_A = R_A + jX_A$. The power delivered to the load (in this case, the antenna preceded by the proposed impedance matching circuit) is maximized when load impedance is $50 + j0$ Ω. So, ideally, the impedance-matching circuit should cancel the reactance X_A of the antenna, and then match the real part R_A of the antenna to 50 Ω.

For an electrically-short dipole, $|X_A| \gg R_A$ and $|X_A| \gg 50$ Ω, so the biggest benefit will come from canceling the reactance. Since $X_A < 0$ for an electrically-short dipole, you can do this by adding an inductor in series with the antenna. Therefore the single best option from the choices offered is (c) add inductor(s).

[m0204] [1]

10.6-1

From the problem statement, $L = 1.5$ m and $f \sim 1$ MHz. At this frequency, the wavelength $\lambda = c/f \sim 300$ m. Note that $L/\lambda \sim 0.005$; that is, this dipole is electrically-short. Subsequently,

$$R_{rad} \approx 20\pi^2 \left(\frac{L}{\lambda}\right)^2 \sim \boxed{4.93 \text{ m}\Omega} \tag{10.10}$$

[m0204] [2]

10.6-2

The radiation resistance of an electrically-short dipole is:

$$R_{rad} \approx 20\ \pi^2 \left(\frac{L}{\lambda}\right)^2 \quad \Omega \tag{10.11}$$

where L is the length of the dipole and λ is wavelength. Recall that $\lambda \propto 1/f$ where f is frequency. So, if f increases by a factor of 2, then λ decreases by a factor of 2 and subsequently R_{rad} increases by a factor of 4. If $R_{rad} = 1\ \Omega$ originally, then R_{rad} becomes $\underline{4\ \Omega}$.