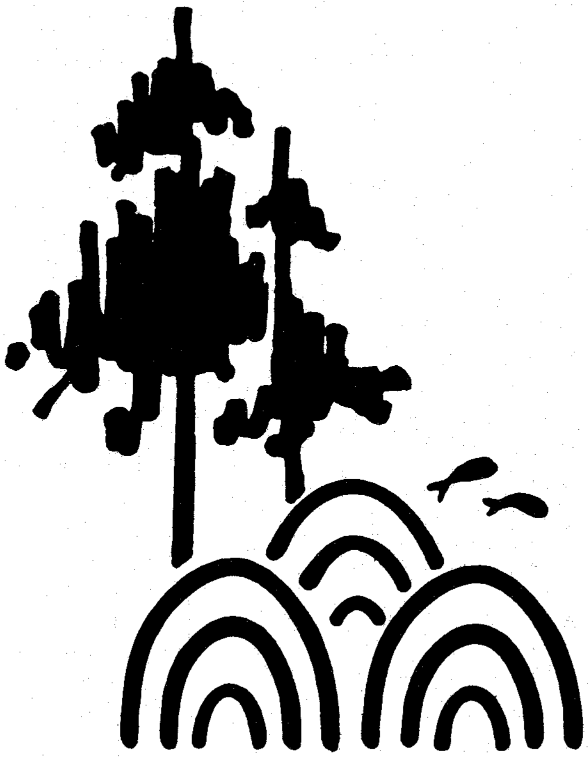
A Comparison of Growth and Yield Prediction Models for Loblolly Pine



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by

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AUTHORS

The authors are, respectively, Professor and Graduate Research Assistant in the Department of Forestry, Virginia Polytechnic Institute and State University, Blacksburg, VA, 24061, and Chief Mensurationist, USDA Forest Service, Southeastern Forest Experiment Station, Forestry Sciences Laboratory, Athens, GA, 30601.

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INTRODUCTION

Loblolly pine (Pinus taeda L.), a fast growing species suited to intensive management, is among the most important commercial tree species in the United States. As demand for forest products increases and acreage available for timber growing decreases, the need for efficient management of this valuable resource becomes acute. Efficient forest management requires accurate predictions of growth and yield.

Although a large number of growth and yield studies have been completed for loblolly pine, these studies vary widely in stand conditions sampled, analytical methods employed, and output options included. Managers are, consequently, faced with the task of sorting through and evaluating a sizeable body of material when selecting growth and yield prediction alternatives for loblolly pine. Consequently, we felt that a systematic evaluation of the various growth and yield alternatives available would be a valuable aid to those involved in applications of growth and yield systems. Further, we hoped that an evaluation of the state-of-the-art in growth and yield prediction for loblolly pine would serve as a useful guide to future research efforts for the species.

The objective of the study reported here was to analyze published growth and yield systems for loblolly pine, characterizing the nature of the data on which the study was based, specifying what input information is needed, and stating what output estimates and predictions are obtainable. Predicted values from various studies are also compared vis-a-vis those from other investigations, and, where possible, conclusions and recommendations are drawn.

In analyzing growth and yield systems for loblolly pine, some historical background and supporting information will be given. However, this report is not, nor was it intended to be, a comprehensive review of the growth and yield literature. An excellent bibliography on growth and yield of the four major southern pines (loblolly, shortleaf, slash, and longleaf) has been compiled by Williston (1975). In addition, review papers on growth and yield of the southern pines (including loblolly pine) have been published by Burkhart (1975, 1979), Farrar (1979), and others. These sources provide citations of literature that were not analyzed in the present study. In our analyses, emphasis was placed on growth and yield studies that were based on reasonably large sample sizes from essentially pure, even-aged stands in an area where loblolly pine is of commercial importance. Reports on the performance of individual stands, of small numbers of stands in a limited geographic area, and of stands outside the native range of loblolly pine are not included, unless they were useful as substantiating data. We analyzed only systems reported between 1960 and 1979 that were based on equations and readily programmable into a computer and for which published reports are readily available in the scientific literature.

CHARACTERIZATION OF THE GROWTH AND YIELD MODELS

Growth and yield models for both plantations and natural stands of loblolly pine were analyzed. In this section, the models selected for analysis will be identified and briefly characterized with regard to the types of stands to which they apply and the modeling methodology employed. The growth and yield models we evaluated, categorized by stand type and modeling approach, are:

Plantations	Natural Stands
Whole Stand Models	
Burkhart *et al.* (1972b)	Brender and Clutter (1970)
Coile and Schumacher (1964)	Burkhart *et al.* (1972a)
Goebel and Warner (1969)	Schumacher and Coile (1960)
	Sullivan and Clutter (1972)
Diameter Distribution Models	
Burkhart and Strub (1974)	
Feduccia *et al.* (1979)	
Lenhart (1972a)	
Lenhart and Clutter (1971)	
Smalley and Bailey (1974)	
Individual Tree Models	
Daniels and Burkhart (1975)	

Whole Stand Models

Yield prediction in the southern U. S. began with the development of normal yield tables for natural stands. Normal yield tables were developed using graphical techniques and the enduring "Miscellaneous Publication 50" (Anon. 1929) yield tables constructed in this manner are still being applied, to a limited extent, in the South.

A multiple regression approach to yield estimation, which also took into account stand density, was applied to loblolly pine stands by MacKinney and Chaiken (1939). This milestone study in quantitative analysis for growth and yield estimation is akin to methods still being used.

Many investigators have used multiple regression techniques to predict growth and/or yield for the total stand or for some merchantable portion of the stand. Under the whole stand approach, some specified aggregate stand volume is predicted from stand level variables (such as age, site index and basal area or number of trees per acre), but no information on volume distribution by size class is provided. Many of these multiple regression models are highly empirical "best fits to the data," but some work has been reported on biologically-based model forms (for example, Pienaar and Turnbull 1973). A major improvement in model specification methodology was suggested by Clutter (1963) when he derived compatible growth and yield models for loblolly pine. Clutter's (1963) definition of compatibility was that the yield model should be obtainable through mathematical integration of the growth model.

Diameter Distribution Models

There are several stand models which are based on a diameter distribution analysis procedure. In this approach, the number of trees per acre in each diameter class is estimated through the use of a probability density function giving the relative frequency of trees by diameters. Mean total tree heights are predicted for trees of given diameters growing under given stand conditions, and volume per diameter class is calculated by using the predicted mean tree heights and the midpoints of the diameter class intervals and substituting into tree volume equations. Per acre yield estimates are obtained by summing diameter classes of interest. Only overall stand values (such as age, site index, and number of trees per acre) are needed as input, but fairly detailed stand distribution information is obtainable as output. The various diameter distribution models differ chiefly in the function used to describe the diameter distribution. Initial applications of this technique to loblolly pine used the beta probability density function, whereas more recent applications have relied on the Weibull function.

Individual Tree Models

Approaches to predicting stand yields which use individual trees as the basic unit will be referred to as "individual tree models". The components of tree growth in these models are commonly linked together through a computer program which simulates the growth of each of the trees and then aggregates these to provide estimates of stand growth and yield. This approach, while receiving extensive attention and application in the Western and Lake States region of the U. S. as well as in Canada, has not been applied widely in the South.

The loblolly pine stand simulator published by Daniels and Burkhart (1975) is, to date, the only fully operational stand model for southern pine that uses individual trees as the basic modeling unit. More recently Daniels et al. (1979b) completed a publication on methods for modeling seeded loblolly pine stands by an individual tree approach. In Daniels and Burkhart's (1975) model, trees are assigned initial coordinate locations and sizes at the onset of competition. Subsequently annual growth, by diameter and height, is simulated as a function of size, site, age and an index reflecting competition from neighbors. Tree growth is adjusted by a random component representing genetic and/or microsite variability, and survival probability is controlled by tree size and competition. Per acre yield estimates are then obtained by summing the individual tree volumes (computed from tree volume equations) and multiplying by the appropriate expansion factor. Individual tree models provide detailed information about stand dynamics and structure, including the distribution of stand volume by size classes.

COMPARISONS

Users of growth and yield information need to know the characteristics of the data base used to estimate model coefficients in order to select the most appropriate alternatives for their situation. The input requirements and the outputs obtainable are also important considerations to potential users. Comparisons of yields from various models with comparable units of measure and comparable stand characteristics can also serve as a valuable aid to users faced with choosing among several alternatives. In this section we present results from our evaluations of the data base, input requirements, output options, and predicted yield and mean annual increment values for selected growth and yield models.

Table 1 presents the geographic location, stand treatment (thinned or unthinned), number of observations, plot size, and range in age, site index (base age 25 years), and trees per acre for the data sets used for the plantation models. The plantation models are further divided into those that apply to old-field, non-old-field, or both old-field and non-old-field sites. Similar information is shown in Table 2 for the natural stand projection systems.

The inputs required and outputs provided by each model were determined and tabulated in Table 3. This table is subdivided by model type (whole stand, diameter distribution, and individual tree) and by stand type (plantations, and natural stands). It should be noted that only the outputs provided (or easily computed from that publication or related publications from the same study) in the cited publications are listed. In diameter distribution and individual tree models, unlimited numbers of yield tables can be generated by computing complete diameter and height distributions and superimposing any selected threshold diameter and applying any chosen tree volume or weight equations. The equation forms used in constructing the stand models are presented for plantations (Table 4) and natural stands (Table 5). Only equation forms, and not specific coefficients, are shown for the various growth and yield models. In the original studies, some of the equation forms shown were repeatedly solved for different portions of the stand and inclusion of all of the coefficients would be prohibitively lengthy. These tables of equation forms should provide a ready comparison of similarities and differences between the models fitted by different analysts. Coefficients for specific applications are readily obtainable from the original sources.

When preparing tables of yield and mean annual increment values, it was necessary to select units of measure, threshold diameters, and top diameters that were most common and would allow direct comparison of figures for the majority of the systems. For plantations, cubic-foot volume inside bark to a 4-inch top outside bark was the quantity tabulated (Table 6). All publications, with the exception of Coile and Schumacher (1964), provide these inside-bark cubic-foot volumes. The Coile and Schumacher values in Table 6 are outside bark to a 4-inch top inside bark and thus are not directly comparable to the other yields, but are shown in the table in order that rough comparisons of trends can be made.

Yields of old-field plantations in terms of total cubic-foot volume outside bark given by the individual tree model are presented separately in Table 7. This table is based on number of trees planted rather than number of trees surviving as in Table 6. Due to the stochastic nature of mortality prediction in the Daniels and Burkhart (1975) model, trees surviving at any given age will vary from run to run with the same number of trees planted. Thus, the format chosen to present yields from this model involves averages from three runs with numbers of trees planted as the density variable.

Total cubic-foot volume inside bark and mean annual increment for all trees in the 1-inch class and above were tabulated for natural stands (Table 8). In this tabulation, the Brender and Clutter (1970) values differ from the others because they are outside bark volumes for trees 5.5 inches dbh and greater. In spite of this inconsistency, comparisons of general trends in the response surfaces can be made through examination of the tabled values.

Throughout our comparisons (Tables 6 and 8), we used site index curves, volume tables and other functions required for intermediate computations that were the same as those used in the cited publications. In some instances, different yield response surfaces from those shown can easily be generated through substitution of locally-applicable functions for some of the intermediate computations.

Considering variations in yields that are likely from the developers employing different volume tables, site index curves, and analytical techniques, the tabulated values and trends (Tables 6 and 8) are reasonably consistent. Plantation yields are very sensitive to site index, but less so to number of trees per acre (within the normal range of interest). Natural stand yields are sensitive to site index and to basal area levels.

Input Relationships

Predicted yields are influenced by the tree volume tables and site index curves used. Further, when projecting stands through time, some procedure must be employed for forecasting changes in stand density --

numbers of trees or basal area. As an additional aid in evaluating the yield surfaces generated by the various systems analyzed, we compared the volume tables, site index curves, and, where appropriate, mortality or basal area projection functions used.

Volume Tables

With the exception of Coile and Schumacher (1964) and Feduccia et al. (1979), the combined variable equation

$$V = b_0 + b_1D^2H$$

where V = tree volume

D = tree diameter at breast height

H = total tree height

was used to compute per tree cubic-foot volumes in the plantation yield studies. Coile and Schumacher used a somewhat more complex volume equation and Feduccia et al. (1979) integrated a taper equation (the model used was that of Bennett et al. 1978) to obtain per tree volumes (Table 9).

Figure 1 shows the trend of cubic-foot volume inside bark to 4-inch top (ob) for equations for old-field loblolly pine plantation trees. From this graph it is clear that per tree volume trends are very similar for the combined-variable models. The equation from Smalley and Bower (1968), based on data from the Tennessee, Alabama and Georgia highlands, is somewhat steeper than those for the Georgia piedmont (Bailey and Clutter 1970), the piedmont of Virginia and coastal plain of Virginia, North Carolina, Maryland, and Delaware (Burkhart et al. 1972b), and the Interior West Gulf Coastal Plain (Hasness and Lenhart 1972).

Relationships employed to estimate tree volumes in the yield systems for natural stands are more diverse in form than those used for plantations (Table 10 and Figure 2). The graphs of volume over D^2H values are, however, very similar for the equations developed by MacKinney and Chaiken (1939) and Burkhart et al. (1972a). Although shown in

Figure 2, the equation from Schumacher and Coile (1960) is not directly comparable to the others because the measure of height is that of the dominant stand rather than individual tree height.

The volume equations used in most growth and yield studies in the past provide volume in selected units to fixed top diameter limits. With more detailed diameter distribution and individual-tree-based stand models, it is possible to develop yields for any selected portion of the stand and for any portion of the tree boles if sufficiently flexible tree volume estimation methods are available. One means of incorporating this flexibility into yield predictions is to use tree taper equations to develop tree profiles from dbh and height predictions and to integrate these taper curves to obtain volumes for any specified portion of the bole. Of the growth and yield systems evaluated here, only Feduccia _et al_. (1979) incorporated taper functions into their yield model. However, other taper equations for loblolly pine are available from Max and Burkhart (1976) and Liu and Keister (1978). Volume for any top limit can also be calculated through the application of volume ratio equations (Burkhart 1977). Any of these approaches can be used with a stand yield model that provides information on diameter distribution and total height. Cao _et al_. (1980) found from their evaluation of various alternatives for cubic-volume prediction of loblolly pine to any merchantable limit that no one form of taper or volume ratio function is consistently best for all the objectives for which they are used. Results from their paper should aid users in selecting an approach that will best satisfy specific objectives.

Site Index Equations

Yield predictions are very sensitive to site index values, thus it is extremely important to employ site index curves that are appropriate in a given stand projection situation. Except for the polymorphic curves applied by Lenhart and Clutter (1971), the various site index equations used in the plantation yield systems are anamorphic and generally employ a regression of the logarithm of height on the reciprocal of age (Table 11).

Site index equations for natural stands are listed in Table 12. Figures 3 (a,b,c) and 4 are graphical comparisons of the old-field plantation and natural stand site index curves, respectively.

Projection Functions for Mortality and Basal Area

To predict future stand yields the user must first predict future stand density. All of the plantation yield models considered here rely on number of trees per acre as the measure of stand density. A wide variety of functions showing numbers of trees surviving at given ages as a function of initial number of trees and, in some case, site index has been developed for loblolly pine plantations (Table 13). These "survival curves" vary markedly for given sets of conditions (Figure 5a,b,c). This wide variation probably stems from at least three sources: (1) survival is highly variable in both space and time, (2) the data bases available for analyses (relatively small plots sometimes observed on only one occasion) were not ideal for survival prediction, and (3) the analytical methods and models employed were quite variable.

Two basal area projection equations (Schumacher and Coile 1960 and Sullivan and Clutter 1972) developed in conjunction with yield studies in natural stands are listed in Table 14 and graphed in Figure 6a,b,c. Basal area trends through time are much steeper for the Sullivan and Clutter (1972) equation than for the Schumacher and Coile (1960) equation. When comparing these projections, one must keep in mind that the equations resulted from two different data sets that are not directly comparable and that different analytical approaches were used. The appropriate basal area projection to apply will depend on the types of stands involved, and the choice must be based on a careful study of the description of the data that were the basis for the reported growth and yield prediction system.

Effect of Spacing on Plantation Yields

Planting spacing is within the control of forest managers and economic considerations dictate that one strive for the "optimal" number of trees. This optimum will, of course, vary widely depending on the management objective, with wider spacings being used for sawtimber production and closer spacings for pulpwood. The plantation yield relationships show wide variation with regard to effects of numbers of trees per acre. This variation is presumed to be mainly from differences in the sample data used for fitting and by variation in the techniques and models employed in the analyses. Furthermore, for these models it is not, strictly speaking, legitimate to hold all factors constant and vary only numbers of trees to solve for an optimum density because numbers of trees were not controlled in the sample data used to solve for the coefficients. The sample data for these yield equations came from surveys of existing plantations rather than from designed experiments with density controlled at specified levels. Consequently, it is not legitimate, from a statistical standpoint, to treat density as a controlled variable in subsequent analyses of the response surfaces. Unfortunately, analyses of existing yield model predictions provide only limited insight into the important question "How many trees per acre should be planted?"

Studies designed with controlled spacing give insight into optimal spacing for loblolly pine plantations.[1] The oldest stand records found for three or more studies at different locations were for age 21 years. The three spacing studies at age 21 that were summarized and reported by Shepard (1974) show variable relationships between merchantable cubic-foot volume (inside bark to a 4-inch top) and growing space (expressed as square feet per tree) (Figure 7). There is considerable variability from study to study and it is assumed that the widest spacing at the Homer, Louisiana site (the Hill Farm) must be on a better site than the other spacings to cause the abrupt upturn in yield for that spacing. If this assumption is reasonable then the density that produces maximum merchantable cubic volume at age 21 on such sites seems to be around 70 to 75 square feet of growing space per tree or approximately 580 to 620 surviving trees per acre. The optimal number of trees to actually plant depends, of course, on product objective, site quality, proposed treatments and harvest age, expected survival and other factors, but these results from three spacing studies give some clues to relationships of spacing and merchantable cubic volume production of the conventional pulpwood portion of stands.

Determining Harvest Age

Managers use yield models to establish the ages at which stands should be harvested. "Optimum" harvest ages depend on whether some "physical" criterion is used, such as maximization of mean annual increment, or some financial criterion is employed, such as

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[1] We gratefully acknowledge the assistance of David D. Reed with the literature search and examination of spacing studies.

maximization of present net worth. Harvest ages are obviously influenced by product objective, site quality, stand density, and, when economic criteria are used, by costs, returns and interest rates. Thus no general guidelines or conclusions can be made about optimal harvest ages for all users. We can report only that the age of mean annual increment culmination did show reasonable trends for most of the yield prediction systems evaluated here.

Choosing An Appropriate Stand Model

Decisions must be made for individual stands, for entire forests, and for broad regional planning -- the projection period and the level of stand detail required may vary in each case. In choosing appropriate stand models one must be concerned with the reliability of estimates, the flexibility to reproduce desired management alternatives, the ability to provide sufficient detail for decisionmaking, and the efficiency in providing this information. Users must also pay particular attention to details such as definitions of variables and basic assumptions. These details can be obtained from the reports of the developers of the systems which are only summarized and cited here.

Daniels *et al*. (1979a) compared three stand models for loblolly pine plantations -- the whole stand model of Burkhart *et al*. (1972b), the diameter distribution model of Burkhart and Strub (1974), and the individual tree model of Daniels and Burkhart (1975) -- with independent data on the basis of merchantable cubic-foot yield estimates. Analysis of deviations of estimated from observed yields revealed that all three models provided reasonably accurate yield estimates. Thus, selection of an appropriate model depends on the level of stand detail desired and the management practices to be evaluated. Stand models which provide large amounts of stand detail are, of course, more expensive to apply than those which do not.

Although "advantages" and "disadvantages" cannot be ascribed to different modeling approaches except in the context of specific uses, general characteristics of the various alternatives can be briefly described. Whole stand models can generally be applied with existing inventory data and they are computationally efficient. However, whole stand models do not provide size-class information that is needed to evaluate various utilization options and product breakdowns and they are usually inflexible for analyzing a wide range of stand treatments.

Diameter distribution models require only overall stand values as input but they provide fairly detailed size-class information as output. Thus alternative utilization options can be evaluated. Computationally these models are somewhat more expensive to apply than whole stand approaches, and they are not highly flexible for evaluating a broad range of stand treatments.

Individual tree models provide maximum detail and flexibility for evaluating alternative utilization options and stand treatments. However, they are more expensive to develop, requiring a more detailed data base, and much more expensive to apply, requiring more sophisticated computing equipment and greater execution time for comparable stand estimates, than the whole stand or diameter distribution models.

CONCLUSION

In conclusion, the following general points can be made about the status of growth and yield projection for loblolly pine:

1. A wide variety of modeling approaches -- ranging across whole stand models to diameter distribution models to individual-tree-based models -- have been employed for loblolly pine plantations but only whole stand models are available for natural stands.

2. Most plantation models are for unthinned stands on old-field sites, with the majority of the loblolly pine growing region being represented by published models for such plantations. Coile and Schumacher (1964) included yields for non-old-field sites. Recently, yields have been published for unthinned plantations on cutover sites in the West Gulf region (Feduccia *et al*. 1979).

3. Little information is available, however, for thinned plantations. Coile and Schumacher (1964) presented yields for thinned plantations and Daniels and Burkhart (1975) included a thinning subroutine in their plantation stand simulator. There is some additional information on thinned stand yields from studies of restricted area and site conditions. However, we lack comprehensive systems that estimate growth and yield under different types and intensities of thinning.

Table 1. Nature of data used in old-field and non-old-field plantation models.

Model	Geographic Location	Stand Treatments	Number of obs.	Plot size	RANGE OF DATA		
					Age[1] (yrs)	SI 25 (feet)	Number of trees/acre
OLD-FIELD PLANTATION MODELS							
Burkhart and Strub (1974)	Piedmont and Coastal Plain Virginia. Coastal Plain regions of Delaware, Maryland and North Carolina	unthinned	186[2]	0.10 acre	10-35	47-84	300-2900
Burkhart *et al.* (1972b)	Piedmont and Coastal Plain Virginia. Coastal Plain regions of Delaware, Maryland and North Carolina	unthinned	189[2]	0.10 acre	9-35	47-84	300-2900
Goebel and Warner (1969)	Piedmont regions in South Carolina	unthinned	220	64 original planting spaces	10-25	40-75	500-1400
Lenhart (1972a)	Interior West Gulf Coastal Plain	unthinned	219	64 original planting spaces	10-30	40-70	500-1200
Lenhart and Clutter (1971)	Georgia Piedmont	unthinned	226	64 original planting spaces	9-33	40-80	750-1650

Table 1. (cont.1)

Model	Geographic Location	Stand Treatments	Number of obs.	Plot size	RANGE OF DATA		
					Age[1] (yrs)	SI 25 (feet)	Number of trees/acre
Smalley and Bailey (1974)	Highland regions of Tennessee, Alabama and Georgia	unthinned	267 (to fit models) 32 (randomly withheld) 3 (discarded)	0.05 acre	10-31[3]	31-89	202-2240
NON-OLD-FIELD PLANTATION MODELS							
Feduccia *et al.* (1979)	East Texas, Louisiana, southern Arkansas, southern Mississippi	Cutover sites, unthinned, no site prep, site prep not needed	409	varied but greater than 0.1 acre	3-45	22-78	250-1500
BOTH OLD-FIELD AND NON-OLD-FIELD PLANTATION MODELS							
Coile and Schumacher (1964)	North and South Carolina, Georgia, Florida, Alabama, Mississippi, Louisiana, and Texas	Half of the plots were thinned one or more times	370 (old-field) 28 (non-old-field)	0.10 acre (6-10 yrs) 0.20 acre (over 10 yrs)	5-35	35-80	--------
Daniels and Burkhart (1975)	Piedmont and Coastal Plain Virginia, Coastal Plain regions of Delaware, Maryland and North Carolina	unthinned	189[2] (old-field) 51 (non-old-field)	0.10 acre	8-35	47-84	300-2900

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[1] Age from planting except where otherwise noted.

[2] Data sets for the Burkhart and Strub (1974), Burkhart *et al.* (1972b) and Daniels and Burkhart (1975) studies are subsets of each other.

[3] Age from seed.

Table 2. Nature of data used in natural stand models.

Model	Geographic Location	Stand Treatments	Number of obs.	Plot size	RANGE OF DATA		
					Age (yrs)	SI 50 (feet)	Basal area (sq.ft./acre)
Brender and Clutter (1970)	Hitchiti Experimental Forest in Georgia Piedmont	light thinning or improvement cut	179	variable	15-70	50-100	10-120
Burkhart *et al.* (1972a)	Coastal Plains of North Carolina and Virginia. Piedmont Virginia.	unthinned	121	0.10 acre	13-77	53-92	35-217
Schumacher and Coile (1960)	Coastal Plain from Chesapeake Bay to Mobile Bay		420	0.20 acre	20-80[1]	60-120[1]	96-198[1]
Sullivan and Clutter (1972)	Georgia, Virginia and South Carolina	all plots were thinned	102	0.25 acre	21-69	53-110	30-154

[1] Range of data from yield tables (range of observed field data not explicitly given).

Table 3. Inputs and outputs for growth and yield models.

Model	Inputs	Outputs[1]
WHOLE STAND MODELS		
	PLANTATIONS	
Burkhart *et al.* (1972b)	- Age A - Number of surviving trees/acre at age A. - Site index (base age 25).	- Average height of dominants and codominants. - Cuft volumes (ob and ib): total, to 3- and 4-inch tops ob. - Cord volumes to 3- and 4-inch tops ob. - Green weights (ob and ib): total, to 3- and 4-inch tops ob. - Dry weights (ib): total, to 3- and 4-inch tops ob. - Bdft volume to 6-inch top ib.[3] - Pulpwood volumes (cuft and cords) in addition to sawtimber volume. - Topwood volumes ob and ib (cuft and cords) to 3- and 4-inch tops ob.
Coile and Schumacher (1964)	- Age A. - Number of trees/acre planted or surviving. - Site index (base age 25).	- Number of surviving trees/acre - Average height of dominants and codominants. - Basal area and average dbh. - Total cuft volume ib. - Cord volume to 4-inch top ib. - Feasible to compute cuft volume ob to 4-inch top ib.
Goebel and Warner (1969)	- Age A. - Number of surviving trees/acre at age A. - Site index (base age 25).	- Average height of dominants and codominants. - Cuft volume ib: total[2] to 3- and 4-inch tops ob.

Table 3. (cont.1)

Model	Inputs	Outputs[1/]
	NATURAL STANDS	
Brender and Clutter (1970)	- Ages A1 and A2. - Basal area at age A1. - Site index (base age 50).	- Cuft volumes to 4-inch top[4/] at age A1 and age A2. - Bdft volumes to 8-inch top[3/] at age A1 and age A2.
Burkhart et al. (1972a)	- Age A. - Basal areas at age A of loblolly pine and of all species. - Site index (base age 50).	Same outputs as in Burkhart et al. (1972b)
Schumacher and Coile (1960)	- Ages A1 and A2. - Basal area at age A1. - Site index (base age 50).	- Basal area at age A2. - At age A1 and age A2: * Total cuft volume ib. * Cord volume to 4-inch top ob. * Bdft. volume to 6-inch top ib.[3/] * Average height of dominants and codominants. *Number of trees/acre.
Sullivan and Clutter (1972)	- Ages A1 and A2. - Basal area at age A1. - Site index (base age 50).	- Basal area at age A2. - Total cuft volumes ib at age A1 and age A2.
DIAMETER DISTRIBUTION MODELS		For each dbh class:
Burkhart and Strub (1974)	- Age A. - Number of surviving trees/acre at age A. - Average height of dominants and codominants.	- Average height. - Number of surviving trees/acre. - Feasible to compute cuft volumes (ob and ib): total, to 3- and 4-inch tops ob, using volume equations from Burkhart et al. (1972b)
Feduccia et al. (1979)	- Age A. - Number of trees/acre planted or surviving. - Site index (base age 25)	- Average height and crown ratio. - Number of surviving trees/acre. - Cuft volumes (ob and ib): total, to 2-, 3-, and 4-inch tops ob.

Table 3. (cont.2)

Model	Inputs	Outputs[1]
		For each dbh class:
Lenhart (1972a)	- Age A. - Number of surviving trees/acre at age A. - Site index (base age 25)	- Average height. - Number of surviving trees/acre. - Cuft volumes (ob and ib): total,[2] to 2-, 3- and 4-inch tops ob.
Lenhart and Clutter (1971)	- Age A. - Number of trees/acre planted or surviving. - Site index (base age 25).	- Average height. - Number of surviving trees/acre. - Cuft volumes (ob and ib): total,[2] to 3- and 4-inch tops ob.
Smalley and Bailey (1974)	- Age A. - Number of trees/acre planted or surviving. - Site index (base age 25).	- Average height. - Number of surviving trees/acre. - Cuft volumes (ob and ib): total, to 2-, 3- and 4-inch tops ob.
INDIVIDUAL TREE MODEL		
Daniels and Burkhart (1975)	- Age A. - Number of trees/acre planted or surviving. - Site index (base age 25). - Management choices: * Spacing pattern. * Site preparation. * Thinning. * Fertilization	- Number of trees/acre planted and surviving. - Average height of dominants and codominants. - Mean dbh. - Basal area, total cuft volume ob, total above ground dry weight: yield, increment, PAI, MAI. - For each dbh class: * Trees surviving * Trees died * Trees thinned } - Number of trees/acre. - Mean height.

1/ Total volume includes all trees 1 inch dbh and above; Merchantable volume includes trees above 4.5 inches dbh.

2/ Includes trees above 4.5 inches dbh.

3/ Includes trees above 7.5 inches dbh.

4/ Includes trees above 5.5 inches dbh.

Table 4. Basic equation forms used in plantation models.

Model	Equation form[1/]
MULTIPLE REGRESSION MODELS	
Burkhart *et al.* (1972b)	$\log V = b_0 + b_1/A + b_2(H/A) + b_3N + b_4(A)(\log N)$
Coile and Schumacher (1964)	$\log B = b_0 + b_1\log S + b_2/A + b_3\log N + b_4OF$ where OF = 1, old-field, = −1, non-old-field $V = b_0 + b_1N + b_2H + b_3B + b_4\ (B)\ (H)$
Goebel and Warner (1969)	$\log V = b_0 + b_1N + b_2/S + b_3(A)(\log N)$ $+ b_4(A/\log N) + b_5/A$
DIAMETER DISTRIBUTION MODELS	
	Beta distribution
Burkhart and Strub (1974)	$Dmin = b_0 + b_1H + b_2(A)(N) + b_3(H/N)$ $Dmax = b_0 + b_1H + b_2(A)(N) + b_3(H/N)$ $\hat{\alpha} = b_0 + b_1(A/N) + b_2(A)(H)$ $\hat{\beta} = b_0 + b_1(A/N) + b_2(N)(H)$ $\log H_i = b_0 + b_1\log H + b_2/A + b_3(\log N)/D_i$ $+ b_4/(AD_i) + b_5/D_i$

Table 4. (cont.1)

Model	Equation form[1/]
Lenhart (1972)	$\mathrm{Dmin} = b_0 + b_1 \log A + b_2 \log N + b_3/H$
	$\mathrm{Dmax} = b_0 + b_1 \log H + b_2 \log N + b_3/A$
	$\hat{\alpha} = b_0 + b_1/A + b_2/H$
	$\hat{\beta} = b_0 + b_1/H + b_2/N$
	$n_i = N \left[\frac{P_i/B_i}{\sum_j P_j/B_j} \right]$
	$\log H_i = \log H + b_0 + (1/D_i - 1/\mathrm{Dmax})$ $(b_1 + b_2/A + b_3 \log N)$
Lenhart and Clutter (1971)	$\mathrm{Dmin} = b_0 + b_1 A + b_2 H + b_3/N$
	$\mathrm{Dmax} = b_0 + b_1 H + b_2 \log N$
	$\hat{\alpha} = b_0 + b_1 \log A$
	$\hat{\beta} = b_0 + b_1 A$
	$n_i = N \left[\frac{P_i/B_i}{\sum_j P_j/B_j} \right]$
	$\log H_i = \log H + b_0 + (1/D_i - 1/\mathrm{Dmax})$ $(b_1 + b_2/A + b_3 \log N)$
	Weibull distribution
Smalley and Bailey (1974)	$a = b_0 + b_1 H$
	$a+b = b_0 + b_1 N + b_2 \log H + b_3/N$

Table 4. (cont.2)

Model	Equation form[1]
	$c = b_0 + b_1A + b_2\log N$
	$\log(H/H_i) = b_0 + (1/D_i - 1/Dmax)\left[b_1 + b_2(A)(N) + b_3(N/A) + b_4\log(N/A) + b_5\log(H/A)\right]$
Feduccia *et al.* (1979)	$a = b_0 + b_1\log H + b_2\log A + b_3\log N$
	$a+b = b_0 + b_1\log H + b_2\log A + b_3\log N$
	$c = b_0 + b_1\log H + b_2\log A + b_3\log N$
	$\log(H/H_i) = b_0 + (1/D_i - 1/Dmax)\left[b_1 + b_2(A)(N) + b_3(N/A) + b_4\log(N/A) + b_5\log(H/A)\right]$
INDIVIDUAL TREE SIMULATION MODEL	
Daniels and Burkhart (1975)	$HIN = PHIN\ (b_0 + b_1CR^{b_2}e^{b_3CI+b_4CR})$
	$DIN = PDIN\ (b_0 + b_1CL^{b_2}e^{b_3CI})$
	$PLIVE = b_1CR^{b_2}e^{b_3CI^{b_4}}$

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[1] Notation:

A = Age of the stand,

B = Basal area per acre of the stand

Table 4. (cont.)

H = Average height of the dominants and codominants,

N = Number of trees per acre surviving,

S = Site index (base age 25),

V = Stand volume,

Dmin = Minimum dbh,

Dmax = Maximum dbh,

$\hat{\alpha}$, $\hat{\beta}$ = Shape parameters of the betadistribution,

D_i = Midpoint of the ith dbh class,

B_i = Basal area of a tree having dbh D_i,

H_i = Total height corresponding to D_i,

n_i = Number of trees per acre surviving in the ith dbh class,

P_i = Proportion of the basal area per acre contained in the ith dbh class,

a,b,c = Parameters of the Weibull distribution,

PHIN = Potential height increment (annual basis),

HIN = Actual height increment (annual basis),

PDIN = Potential dbh increment (annual basis),

DIN = Actual dbh increment (annual basis),

CI = Competition index,

CR = Crown ratio,

CL = Crown length,

PLIVE = Survival probability (annual basis),

logX = Logarithm base 10 of X

$b_0, b_1 \ldots b_k$ = Constants estimated by regression techniques.

Table 5. Basic equation forms used in natural stand models.

Model	Equation form[1]
Brender and Clutter (1970)	$\log V_2 = b_0 + b_1 S + b_2/A_2 + b_3(1 - A_1/A_2) + b_4(\log B_1)(A_1/A_2)$ $\log V = b_0 + b_1 S + b_2/A + b_4 \log B$
Burkhart *et al.* (1972a)	$\log V = b_0 + b_1/A + b_2(H/A) + b_3 \log B_t + b_4(A)(\log B_t) + b_5(B_1/B_t)$
Schumacher and Coile (1960)	$\log N = b_0 + b_1/A + b_2 \log H + b_3 \log B$ $D = \left[183.35(B/N)\right]^{1/2}$ $\log(V/N) = b_0 + b_1 \log D + b_2 \log H$
Sullivan and Clutter (1972)	$\ln V_2 = b_0 + b_1 S + b_2/A + b_3(A_1/A_2)(\ln B_1) + b_4(1 - A_1/A_2) + b_5(1 - A_1/A_2)(S)$ $\ln V = b_0 + b_1 S + b_2/A + b_3 \ln B$ $\ln B_2 = (A_1/A_2)(\ln B_1) + b_1(1 - A_1/A_2) + b_2(1 - A_1/A_2)(S)$

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[1] Notation:

A = Age of the stand,

B = Basal area per acre of the stand,

Table 5. (cont.1)

B_1 = Basal area per acre of loblolly pine

B_t = Total basal area per acre,

D = Quadratic mean diameter of the stand,

H = Average height of the dominants and codominants,

S = Site index (base age 50),

V = Stand volume,

V_i, B_i = Stand volume and basal area per acre at age A_i, $i=1,2$,

$\log X$ = Logarithm base 10 of X,

$\ln X$ = Natural logarithm of X.

Table 6. Cubic-foot volume inside bark to 4-inch top outside bark and mean annual increment given by plantation models.

Site index in feet (base age 25)	50			60			70		
Number of surviving trees/acre	400	600	800	400	600	800	400	600	800
				AGE 10[2]					
OLD-FIELD PLANTATIONS									
Burkhart and Strub (1974)	155	147	133	306	305	299	505	528	532
	16	15	13	31	31	30	51	53	53
Burkhart *et al.* (1972b)	259	247	233	412	392	370	654	622	588
	26	25	23	41	39	37	65	62	59
Coile and[3] Schumacher (1964)	209	159	91	429	420	378	718	769	771
	21	16	9	43	42	38	72	77	77
Goebel and Warner (1969)	145[1]	149	143	410	422	404	811	835	800
	15[1]	15	14	41	42	40	81	83	80
Lenhart (1972a)	564	537	481	939	986	981	1351	1483	1547
	56	54	48	94	99	98	135	148	155
Lenhart and Clutter (1971)	205	164	129	482	465	436	908	985	1026
	21	16	13	48	46	44	91	99	103
Smalley and [4] Bailey (1974)	569	491	397	916	902	871	1341	1400	1452
	57	49	40	92	90	87	134	140	145

Table 6. (cont.1)

Site index in feet (base age 25)		50			60			70	
Number of surviving trees/acre	400	600	800	400	600	800	400	600	800
NON-OLD-FIELD PLANTATIONS									
Coile and [3]	80	3	0	237	181	101	447	429	374
Schumacher (1964)	8	0	0	24	18	10	45	43	37
Feduccia *et al.*	288	201	115	506	445	349	786	777	683
(1979)	29	20	12	51	45	35	79	78	68
					AGE 15				
OLD-FIELD PLANTATIONS									
Burkhart and	702	725	722	1184	1273	1313	1799	1967	2078
Strub (1974)	47	48	48	79	85	88	120	131	139
Burkhart *et al.*	907	875	834	1470	1418	1352	2384	2300	2193
(1972b)	60	58	56	98	95	90	159	153	146
Coile and [3]	834	907	928	1430	1636	1761	2192	2573	2842
Schumacher (1964)	56	60	62	95	109	117	146	172	189
Goebel and	642	735	756	1352	1548	1592	2261	2589	2662
Warner (1969)	43	49	50	90	103	106	151	173	177
Lenhart (1972a)	1229	1322	1390	1792	2007	2169	2397	2754	3016
	82	88	93	119	134	145	160	184	201

Table 6. (cont.2)

Site index in feet (base age 25)	50			60			70		
Number of surviving trees/acre	400	600	800	400	600	800	400	600	800
Lenhart and	666	678	673	1216	1365	1474	1969	2349	2667
Clutter (1971)	44	45	45	81	91	98	131	157	178
Smalley[4/]	1192	1203	1224	1801	1979	2132	2495	2786	3126
and Bailey (1974)	79	80	82	120	132	142	166	186	208
NON-OLD-FIELD PLANTATIONS									
Coile and[3/]	530	527	481	973	1062	1087	1548	1764	1890
Schumacher (1964)	35	35	32	65	71	72	103	118	126
Feduccia *et al.*	978	1083	1060	1488	1769	1805	2154	2558	2722
(1979)	65	72	71	99	118	120	144	171	181
					AGE 20				
OLD-FIELD PLANTATIONS									
Burkhart and	1295	1360	1374	2114	2286	2382	3169	3451	3659
Strub (1974)	65	68	69	106	114	119	158	173	183
Burkhart *et al.*	1532	1497	1441	2412	2357	2269	3798	3712	3573
(1972b)	77	75	72	121	118	113	190	186	179
Coile and[3/]	1472	1686	1819	2432	2869	3182	3652	4374	4925
Schumacher (1964)	74	84	91	122	143	159	183	219	246

Table 6. (cont.3)

Site index in feet (base age 25)	50			60			70		
Number of surviving trees/acre	400	600	800	400	600	800	400	600	800
Goebel and	1172	1493	1648	2155	2745	3029	3311	4218	4655
Warner (1969)	59	75	82	108	137	151	166	211	233
Lenhart (1972a)	1663	1860	2005	2340	2686	2961	3063	3547	3962
	83	93	100	117	134	148	153	177	198
Lenhart and	1211	1343	1443	2044	2438	2767	3161	3912	4588
Clutter (1971)	61	67	72	102	122	138	158	196	229
Smalley and 4/	1769	1936	2085	2602	2951	3264	3510	4148	4705
Bailey (1974)	88	97	104	130	148	163	175	207	235
NON-OLD-FIELD PLANTATIONS									
Coile and 3/	1003	1098	1127	1729	1985	2143	2658	3128	3461
Schumacher (1964)	50	55	56	86	99	107	133	156	173
Feduccia *et al.*	1673	1895	1960	2448	2849	3095	3322	3984	4350
(1979)	84	95	98	122	142	155	166	199	218
					AGE 25				
OLD-FIELD PLANTATIONS									
Burkhart and	1792	1888	1894	2904	3126	3238	4330	4664	4906
Strub (1974)	72	76	76	116	125	130	173	187	196
Burkhart *et al.*	2046	2026	1969	3100	3071	2983	4698	4653	4521
(1972b)	82	81	79	124	123	119	188	186	181

Table 6. (cont.4)

Site index in feet (base age 25)	50			60			70		
Number of surviving trees/acre	400	600	800	400	600	800	400	600	800
Coile and Schumacher (1964)[3/]	2030 81	2373 95	2609 104	3305 132	3944 158	4426 177	4916 197	5937 237	6737 269
Goebel and Warner (1969)	1418 57	2009 80	2380 95	2420 97	3429 137	4061 162	3544 142	5022 201	5948 238
Lenhart (1972a)	1977 79	2229 89	2445 98	2723 109	3131 125	3493 140	3493 140	4120 165	4590 184
Lenhart and Clutter (1971)	1730 69	2001 80	2227 89	2842 114	3470 139	4032 161	4277 171	5400 216	6434 258
Smalley and Bailey (1974)[4/]	2289 92	2553 102	2809 112	3286 131	3867 155	4380 175	4426 177	5228 209	6053 242
NON-OLD-FIELD PLANTATIONS									
Coile and Schumacher (1964)[3/]	1423 57	1611 64	1712 68	2392 96	2800 112	3081 123	3626 145	4321 173	4840 194
Feduccia *et al.* (1979)	2282 91	2657 106	2884 115	3263 131	3934 157	4379 175	4389 176	5398 216	6053 242

Table 6. (cont.5)

1/ For each model, the first line presents yields and the second line mean annual increments, all in terms of cubic-foot volumes inside bark to 4-inch top outside bark.

2/ Age used in this table is age from planting.

3/ For Coile and Schumacher (1964) model, yields and mean annual increments presented here are in terms of cubic-foot volumes outside bark to 4-inch top inside bark.

4/ For Smalley and Bailey (1974) model, assume: age from planting = age from seed + 1.

Table 7. Total cubic-foot volumes outside bark and mean annual increment given by Daniels and Burkhart's (1975) individual tree simulation model for old-field plantations (average of three runs).

Site index in feet (base age 25)	50			60			70		
Number of trees/acre planted	600	1200	1800	600	1200	1800	600	1200	1800
AGE									
10	579	792	907	770	1021	1136	994	1223	1407
	58	79	91	77	102	114	99	122	141
15	1329	1519	1543	1874	2005	1938	2463	2476	2457
	89	101	103	125	134	129	164	165	164
20	2165	2168	2050	3062	2841	2670	3971	3540	3115
	108	108	102	153	142	133	199	177	156
25	2871	2690	2501	3910	3384	2968	5033	4189	3356
	144	135	125	195	169	148	252	209	168

Table 8. Total cubic-foot volumes inside bark and mean annual increments for site indices 70, 80, and 90 (base age 50) given by natural stand models.

Basal area at age 20	40				60				80			
Age A	20	30	40	50	20	30	40	50	20	30	40	50
Basal area at age A [1]	40	69	90	106	60	90	110	124	80	109	127	139
SITE INDEX 70												
Brender and [3]	821	1727	2489	3112	1197	2212	3001	3601	1565	2644	3430	4006
Clutter (1970)	41	58	62	62	60	74	75	72	78	88	86	80
Burkhart *et al.*	633	1405	2068	2665	934	1821	2526	3125	1230	2196	2914	3509
(1972a)	32	47	52	53	47	61	63	63	62	73	73	70
Schumacher and	743[2]	1554	2232	2785	1061	1962	2662	3196	1366	2321	3020	3533
Coile (1960)	37[2]	52	56	56	53	65	67	64	68	77	76	71
Sullivan and	564	1344	2074	2691	829	1737	2515	3140	1090	2085	2883	3502
Clutter (1972)	28	45	52	54	41	58	63	63	55	70	72	70

Table 8. (cont.1)

Basal area at age 20	40				60				80			
Age A	20	30	40	50	20	30	40	50	20	30	40	50
Basal area at age A[1]	40	75	103	124	60	98	126	146	80	119	145	164
SITE INDEX 80												
Brender and Clutter (1970)[3]	877 44	1994 66	3016 75	3848 77	1279 64	2558 85	3638 91	4480 90	1672 84	3065 102	4147 104	4992 100
Burkhart *et al.* (1972a)	725 36	1714 57	2614 65	3407 68	1069 53	2225 74	3195 80	4021 80	1408 70	2690 90	3675 92	4525 91
Schumacher and Coile (1960)	869[2] 43[2]	1954 65	2938 73	3736 75	1240 62	2471 82	3506 88	4312 86	1597 80	2930 98	3966 99	4775 96
Sullivan and Clutter (1972)	652 33	1690 56	2721 68	3622 72	958 48	2185 73	3299 82	4225 85	1259 63	2622 87	3783 95	4714 94

Table 8. (cont.2)

Basal area at age 20	40				60				80			
Age A	20	30	40	50	20	30	40	50	20	30	40	50
Basal area at age A[1]	40	82	117	146	60	107	144	171	80	130	166	192
						SITE INDEX 90						
Brender and[3] Clutter (1970)	937	2315	3628	4787	1367	2966	4402	5546	1787	3556	5025	6177
	47	77	91	96	68	99	110	111	89	119	126	124
Burkhart *et al.* (1972a)	898	2165	3311	4385	1324	2808	4072	5147	1744	3396	4691	5790
	45	72	83	87	66	94	102	103	87	113	117	116
Schumacher and Coile (1960)	997[2]	2425	3770	4949	1424	3063	4524	5685	1833	3634	5126	6294
	50[2]	81	94	99	71	102	113	114	92	121	128	126
Sullivan and Clutter (1972)	753	2125	3570	4874	1107	2748	4329	5686	1455	3297	4963	6344
	38	71	89	97	55	92	108	114	73	110	124	127

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[1] Basal area at age A is predicted from basal area at age 20 using Sullivan and Clutter (1972) model.

[2] For each model, the first line presents yields and the second line mean annual increments, all in terms of total cubic-foot volumes inside bark, including trees of 1 inch dbh and above.

[3] For Brender and Clutter (1970) model, yields and mean annual increments presented here are in terms of total cubic-foot volumes outside bark, including trees above 5.5 inches dbh.

Table 9. Tree volume equations used in plantation models.

Reference	Equations[1]
Burkhart and Strub (1974)	Same as Burkhart *et al.* (1972b)
Burkhart *et al.* (1972b)	V_{Tob}[2] $= 0.34864 + 0.00232\ (D^2H)$ $V_{Tib} = 0.11691 + 0.00185\ (D^2H)$ V_{3ob}[3] $= 0.14346 + 0.00231\ (D^2H)$ $V_{3ib} = -0.05729 + 0.00184\ (D^2H)$ V_{4ob}[4] $= -0.37097 + 0.00233\ (D^{2H})$ $V_{4ib} = -0.46236 + 0.00185\ (D^2H)$
Coile and Schumacher (1964)	$V_{Tib} = D^2\ [0.00186\ (H) - 0.00093]$ V'_{4ob}[5] $= (H-0.5)\ [0.001323\ (D^2) + 0.019184\ (D) - 0.09423]$
Daniels and Burkhart (1975)	Same as Burkhart *et al.* (1972b)
Feduccia *et al.* (1979)	Integrated taper equations.
Goebel and Warner (1969)	Not listed.
Lenhart (1972a) [equations from Hasness and Lenhart 1972]	$V_{Tob} = 0.13698 + 0.0023035\ (D^2H)$ $V_{Tib} = -0.08461 + 0.0019571\ (D^2H)$ V_{2ob}[6] $= 0.06159 + 0.0023050\ (D^2H)$ $V_{2ib} = -0.14520 + 0.0019584\ (D^2H)$ $V_{3ob} = -0.15438 + 0.0023164\ (D^2H)$ $V_{3ib} = -0.32395 + 0.0019680\ (D^2H)$ $V_{4ob} = -0.70594 + 0.0023521\ (D^2H)$ $V_{4ib} = -0.78752 + 0.0019973\ (D^2H)$

Table 9. (cont.1)

Reference	Equations [1/]
Lenhart and Clutter (1971) [equations from Bailey and Clutter 1970]	$V_{Tob} = 0.12680 + 0.0024700\ (D^2H)$
	$V_{Tib} = 0.00914 + 0.0019281\ (D^2H)$
	$V_{3ob} = -0.13681 + 0.0024700\ (D^2H)$
	$V_{3ib} = -0.21617 + 0.0019281\ (D^2H)$
	$V_{4ob} = -0.65542 + 0.0024700\ (D^2H)$
	$V_{4ib} = -0.62770 + 0.0019281\ (D^2H)$
Smalley and Bailey (1974) [equations from Smalley and Bower 1968]	$V_{Tob} = 0.1683 + 0.0026109\ (D^2H)$
	$V_{Tib} = -0.0709 + 0.0020695\ (D^2H)$
	$V_{2ob} = 0.1034 + 0.0026118\ (D^2H)$
	$V_{2ib} = -0.1201 + 0.0020702\ (D^2H)$
	$V_{3ob} = -0.1051 + 0.0026271\ (D^2H)$
	$V_{3ib} = -0.2811 + 0.0020821\ (D^2H)$
	$V_{4ob} = -0.6054 + 0.0026638\ (D^2H)$
	$V_{4ib} = -0.6586 + 0.0021059\ (D^2H)$

1/ Throughout this table D denotes diameter at breast height in inches and H denotes total tree height in feet.

2/ V_{Tob} (or V_{Tib}) = Total cubic-foot volume outside (or inside) bark.

3/ V_{3ob} (or V_{3ib}) = Cubic-foot volume outside (or inside) bark to 3" top outside bark.

4/ V_{4ob} (or V_{4ib}) = Cubic-foot volume outside (or inside) bark to 4" top outside bark.

5/ V'_{4ob} = Cubic-foot volume outside bark to 4" top inside bark.

6/ V_{2ob} (or V_{2ib}) = Cubic-foot volume outside (or inside) bark to 2" top outside bark.

Table 10. Tree volume equations used in natural stand models.

Reference	Equations[1]
Brender and Clutter (1970)	Previously constructed volume equations from Hitchiti data.
Burkhart *et al.* (1972a)	$V_{Tob} = 0.27611 + 0.00253\ (D^2H)$ $V_{Tib} = 0.00828 + 0.00205\ (D^2H)$ V_{3ob}[4] $= 0.03767 + 0.00253\ (D^2H)$ $V_{3ib} = -0.35192 + 0.00205\ (D^2H)$ V_{4ob}[5] $= -0.56843 + 0.00253\ (D^2H)$ $V_{4ib} = -0.84210 + 0.00205\ (D^2H)$
Schumacher and Coile (1960)[2]	V_{Tib}[3] $= 0.8170\ (D/10) + (22.7872\ (D/10)^2 - 6.4042\ (D/10) + 0.4237)\ H/100$
Sullivan and Clutter (1972) (from MacKinney and Chaiken 1939)	$\log(V_{Tib}) = -2.8209 + 1.9557 \log(D) + 1.0971 \log(H)$

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[1] Throughout this table D denotes diameter at breast height in inches, and H denotes total tree height in feet, unless noted otherwise.

[2] H in this equation denotes height of dominant stand.

[3] V_{Tob} (or V_{Tib}) = Total cubic-foot volume outside (or inside) bark.

[4] V_{3ob} (or V_{3ib}) = Cubic-foot volume outside (or inside) bark to 3" top outside bark.

[5] V_{4ob} (or V_{4ib}) = Cubic-foot volume outside (or inside) bark to 4" top outside bark.

Table 11. Site index equations used in plantation models.

Reference	Equations[1]
Burkhart and Strub (1974)	Same as Burkhart *et al*. (1972b)
Burkhart *et al*. (1972b)	$\log H = \log SI - 5.86537\,(1/A - 1/25)$
Coile and Schumacher (1964)	$\log H = \log SI + b\,(1/A - 1/25)$ where $b = -6.449$ for Coastal Plain, Gulf, and Sandhill regions (poorly to well drained), $b = -5.343$ for Coastal Plain, Gulf, and Sandhill regions (excessivly drained), $b = -8.193$ for Savana regions, $b = -5.190$ for Piedmont regions.
Feduccia et al. (1979)(from Popham *et al*. (1979)	$\log H = \log SI - 21.0977(1/A - 1/I)$ $+ 316.282\,(1/A^2 - 1/I^2)$ $- 2443.85\,(1/A^3 - 1/I^3)$ $+ 6318.86\,(1/A^4 - 1/I^4)$ where I = index age = 25
Goebel and Warner (1969)	Same as Lenhart and Clutter (1971)
Lenhart (1972a) (from Lenhart 1971)	$\log H = \log SI - 3.72183\,(1/A - 1/25)$
Lenhart and Clutter (1971)	$\log H = 1.5469 - 11.406/A + (0.76481 \log SI - 0.83419)\,10^{2.9110/A}$

Table 11. (cont.1)

Reference	Equations [1]
Smalley and Bailey (1974) [from Smalley and Bower 1971]	$\log H = \log SI - 2.460976\ (1/\sqrt{A} - 1/\sqrt{25})$

---

[1] Throughout this table H denotes the average height of dominants and codominants at age A, and SI denotes site index (base age 25 years).

Table 12. Site index equations used in natural stand models.

Reference	Equations [1]
Brender and [2] Clutter (1970)	stands below 50 years of age: Coile and Schumacher (1953), older stands: MacKinney (1936)
Burkhart et al. (1972a)	for SI < 75: $\log H = \log SI - 6.93220\ (1/A - 1/50)$ for $75 \leq SI \leq 85$: $\log H = \log SI - 6.91444\ (1/A - 1/50)$ for SI > 85: $\log H = \log SI - 5.98935\ (1/A - 1/50)$
Schumacher and Coile (1960)	$\log H = \log SI - 6.528\ (1/A - 1/50)$
Sullivan and [2] Clutter (1972)	from Coile (1952)

---

[1] Throughout this table H denotes the average height of dominants and codominants at age A, and SI denotes site index (base age 50 years).

[2] Site index equation is not needed when the yield and basal area projection equations are used, since site index is an independent variable instead of height of dominants and codominants.

Table 13. Mortality equations resulting from data used to construct the plantation models.

Reference	Equations [1]
Coile and Schumacher (1964)	$\log N = \log N_o + [2.1346 - 1.1103 \log N_o + 0.1384\ OF]\ A/100$ where $OF = 1$ = old field, $= -1$ = non-old-field
Feduccia *et al.* (1979)	$\log (N_o/N) = A\ [0.01348 \log N_o + 0.00060783\ H - 0.0084124 \sqrt{H}\ ]$
Lenhart (1972b)	$\text{Probit}\ (N/N_o) = 10.48246 - 1.290061 \log A - 1.136441 \log N_o$
Lenhart and Clutter (1971)	$\text{Probit}\ (N/N_o) = 9.3745 - 0.67637 \log A - 0.96269 \log N_o$
Smalley and Bailey (1974)	$\log (N_o/N) = A\ [0.0130 \log N_o + 0.0009\ H - 0.0109 \sqrt{H}\ ]$

---

[1] Throughout this table N_o denotes the number of trees per acre planted, N denotes the number of trees per acre surviving at age A, and H denotes the average height of dominants and codominants. The probit transformation is defined as:

$$\text{Probit}\ (x) = 5.0 + Z_x (0 < x < 1)$$

when Z_x is the value of the standard normal variable "Z" such that Probability $(Z \leq Z_x) = X$.

Table 14. Basal area projection equations used in natural stand models.

Reference	Equations [1]
Schumacher and Coile (1960)	$SP = B\,[0.8409 - 0.1707\,(H/100) + 0.1062\,(100/A) - 0.1408\,(H/A)]$ $\log SP_2 = 2 + (\log SP_1 - 2)(A_1/A_2)$
Sullivan and Clutter (1972)	$\ln B_2 = (A_1/A_2)\ln B_1 + 3.4344\,(1 - A_1/A_2) + 0.026748\,S\,(1 - A_1/A_2)$

[1]
A = Age of the stand
B = Basal area per acre,
H = Height of the dominants and codominants,
S = Site index (base age 50),
SP = Stocking percent,
B_1, B_2 = Basal area per acre at ages A_1 and A_2, respectively,
SP_1, SP_2 = Stocking percent at ages A_1 and A_2, respectively.

Figure 1. Plot of cubic-foot volumes inside bark to 4-inch top outside bark versus D^2H for the plantation tree volume equations.

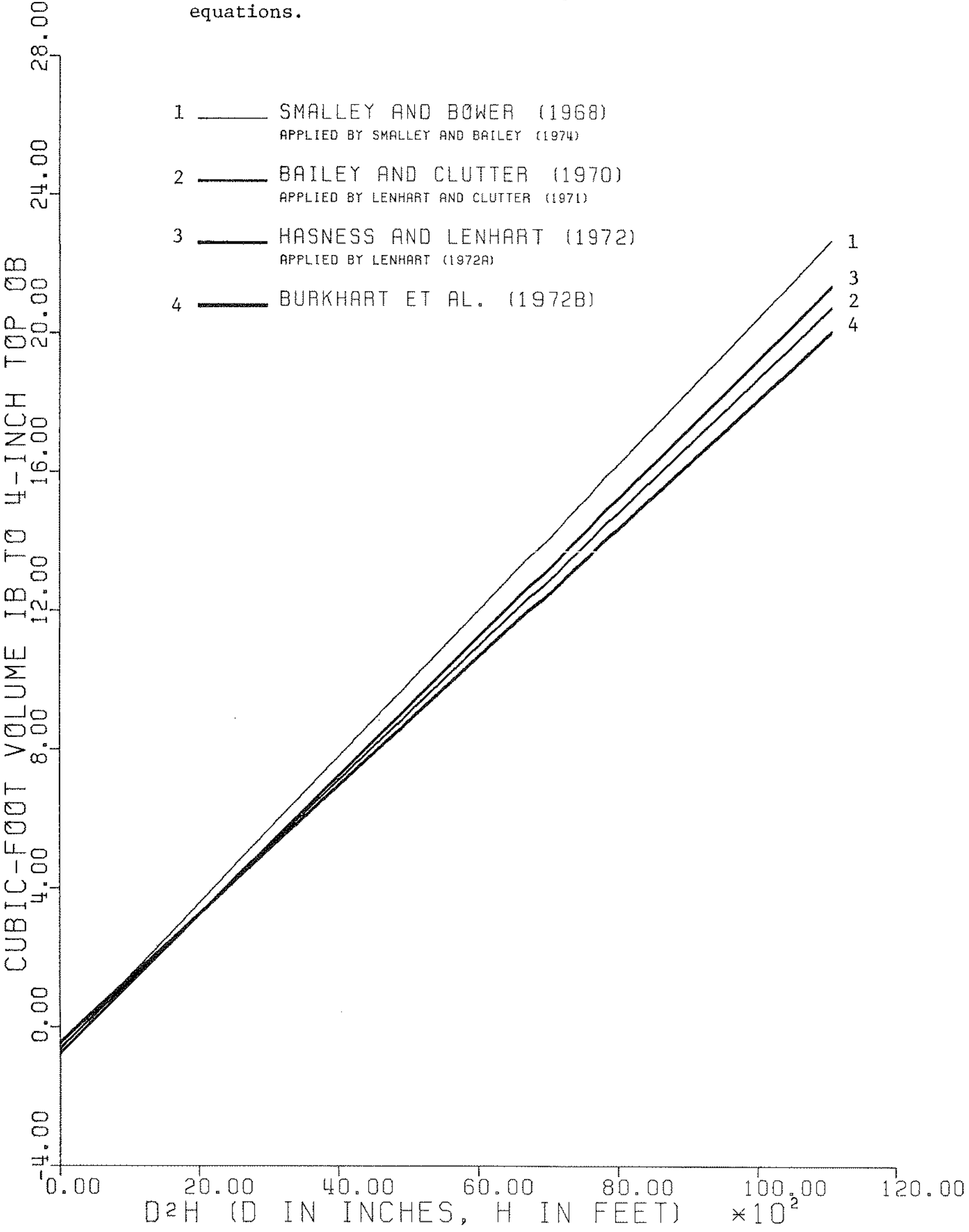


Figure 2. Plot of total cubic-foot volume inside bark versus D^2H for natural stand tree volume equations.

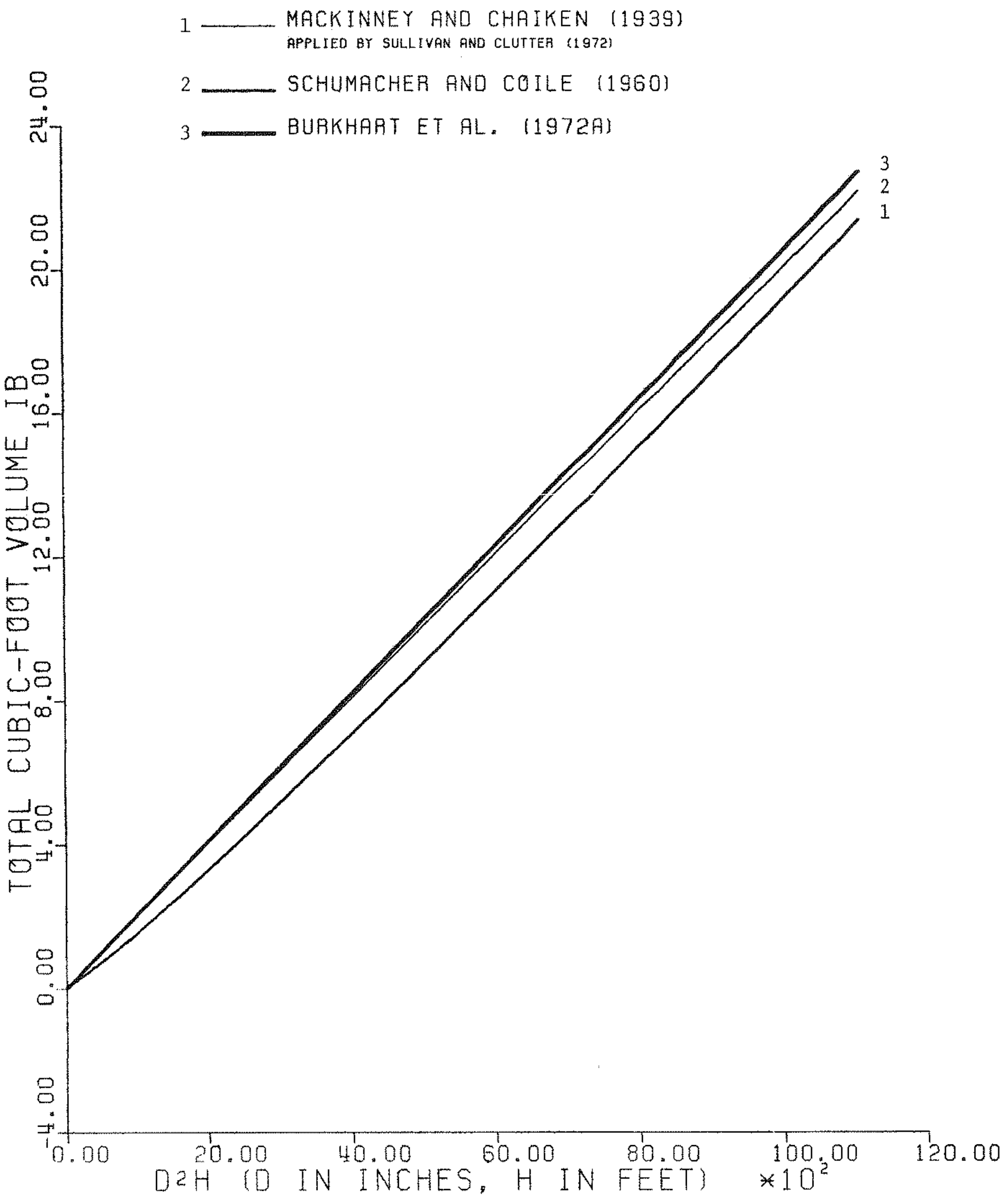


Figure 3a. Height-age curves used in plantation models - site index 50 (base age 25 years).

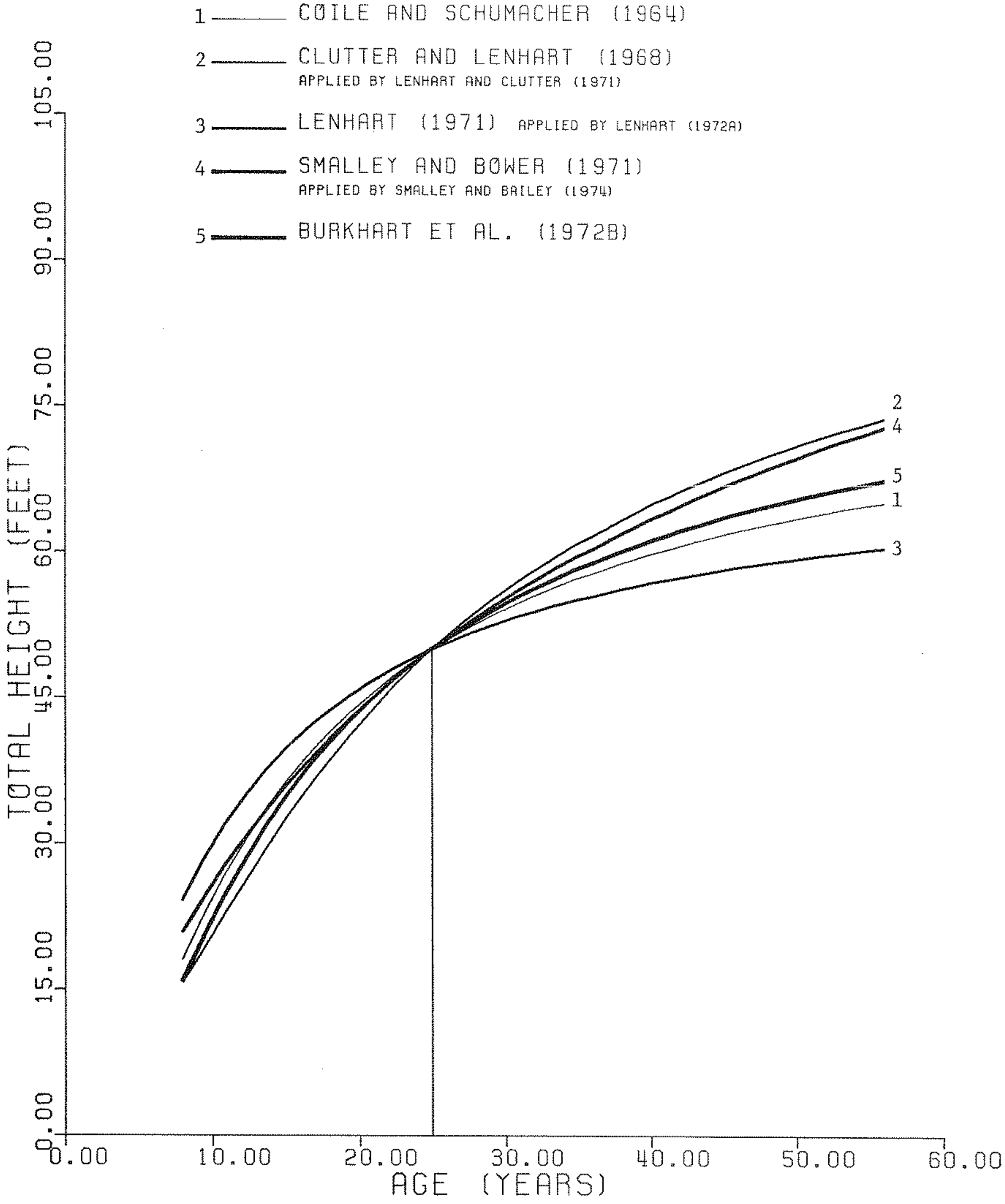


Figure 3b. Height-age curves used in plantation models - site index 60 (base age 25 years).

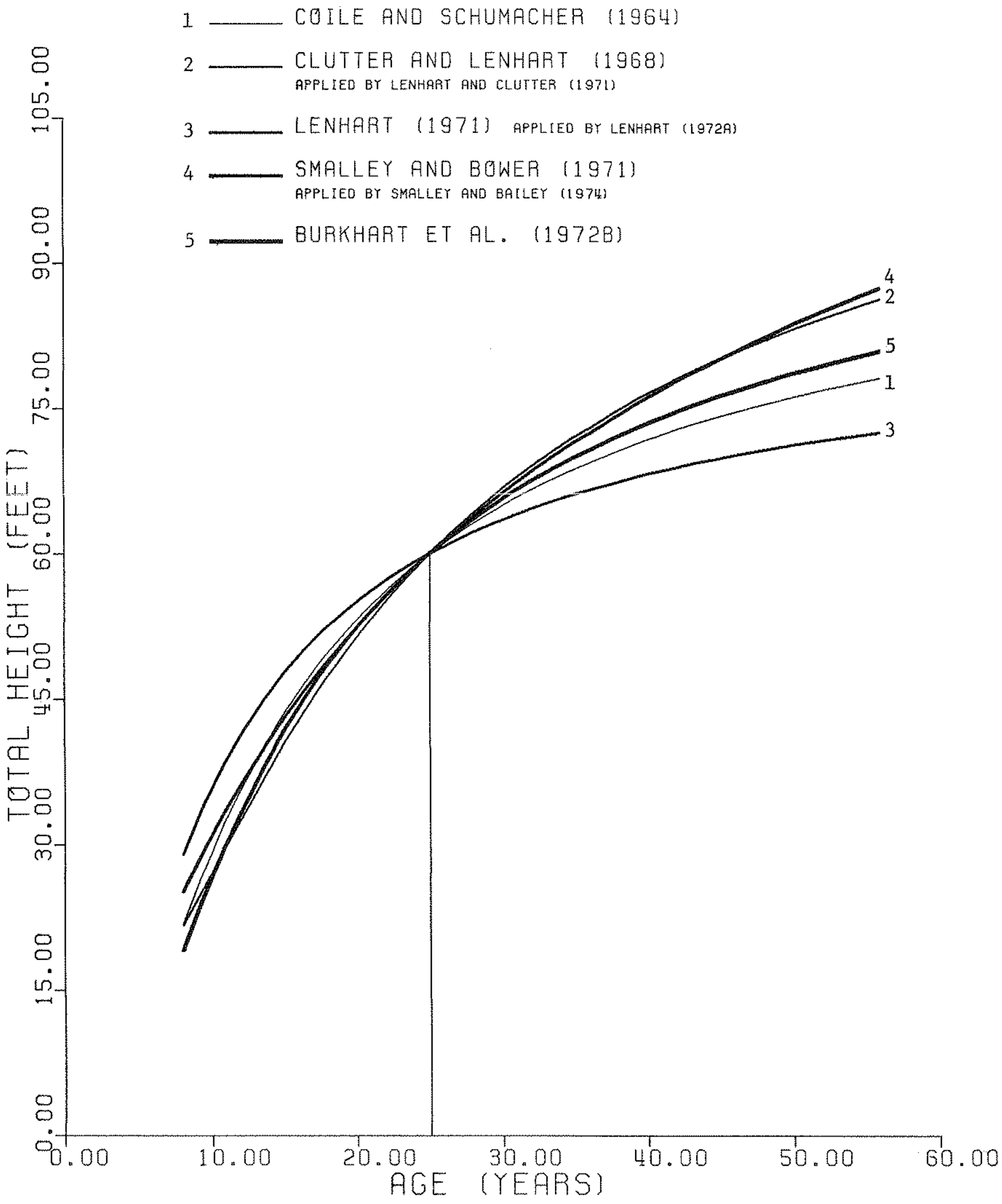


Figure 3c. Height-age curves used in plantation models - site index 70 (base age 25 years).

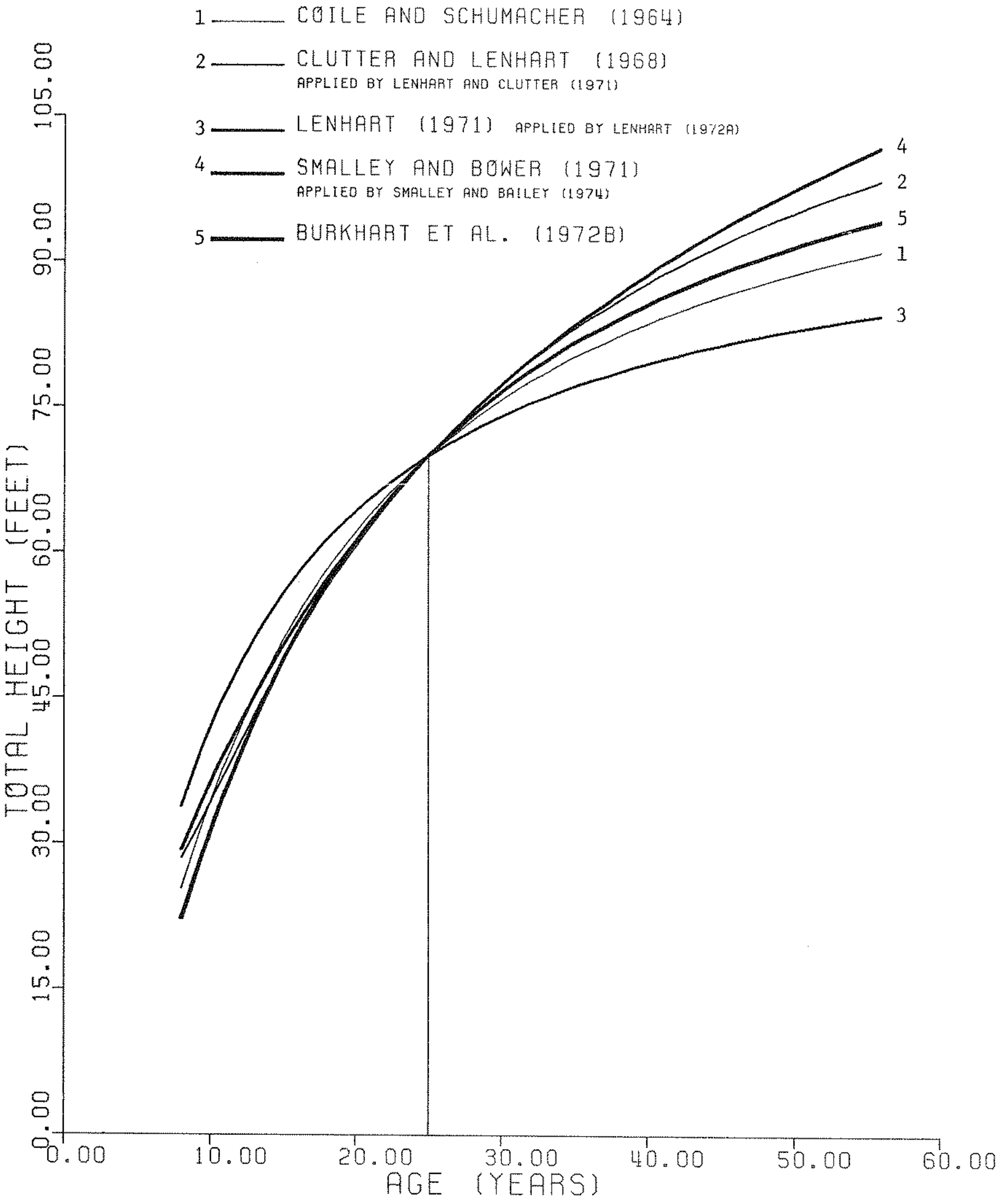


Figure 4. Height-age curves used in natural stand models.

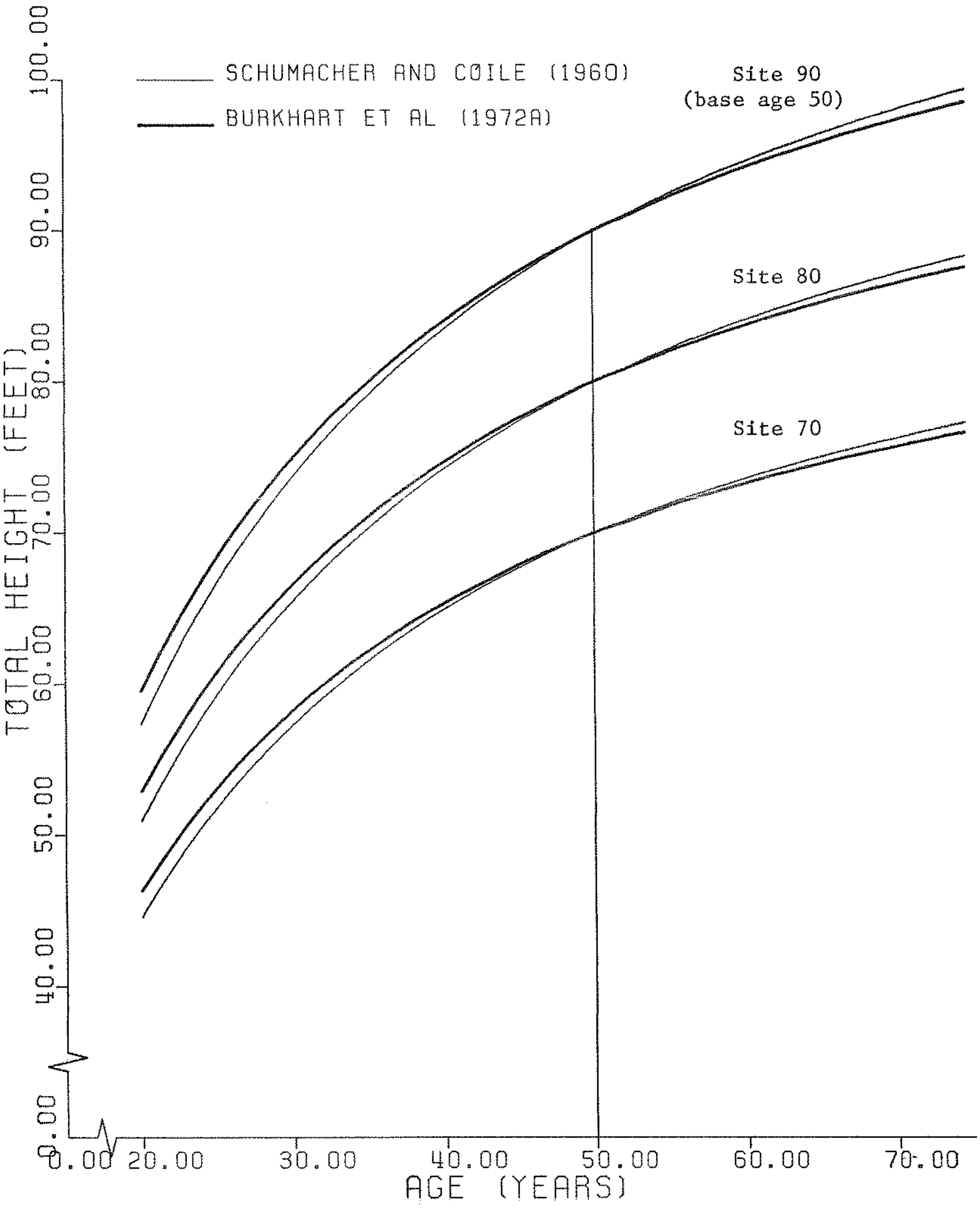


Figure 5a. Survival curves used in old-field plantation models - site index 50 (base age 25 years).

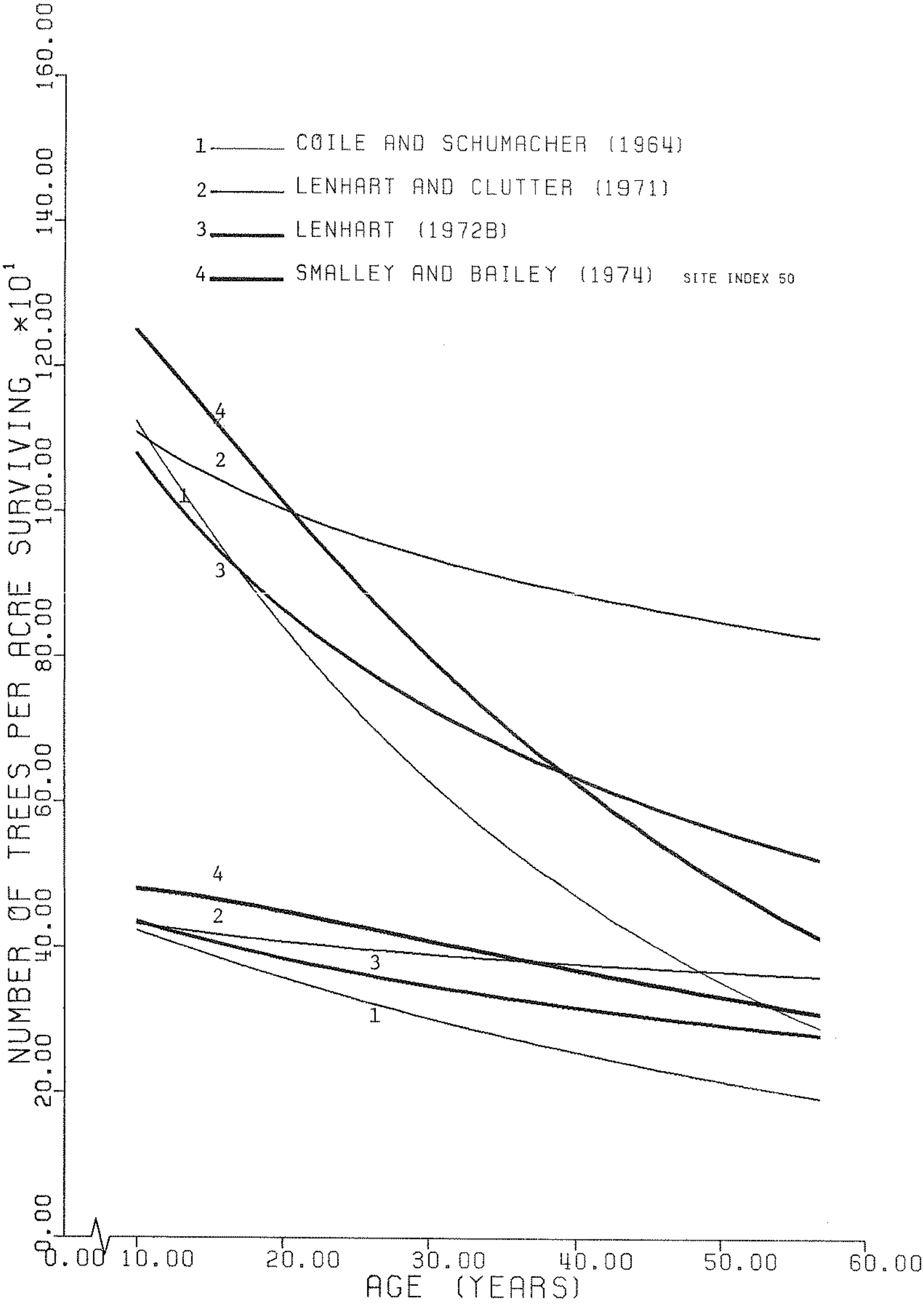


Figure 5b. Survival curves used in old-field plantation models - site index 60 (base age 25 years).

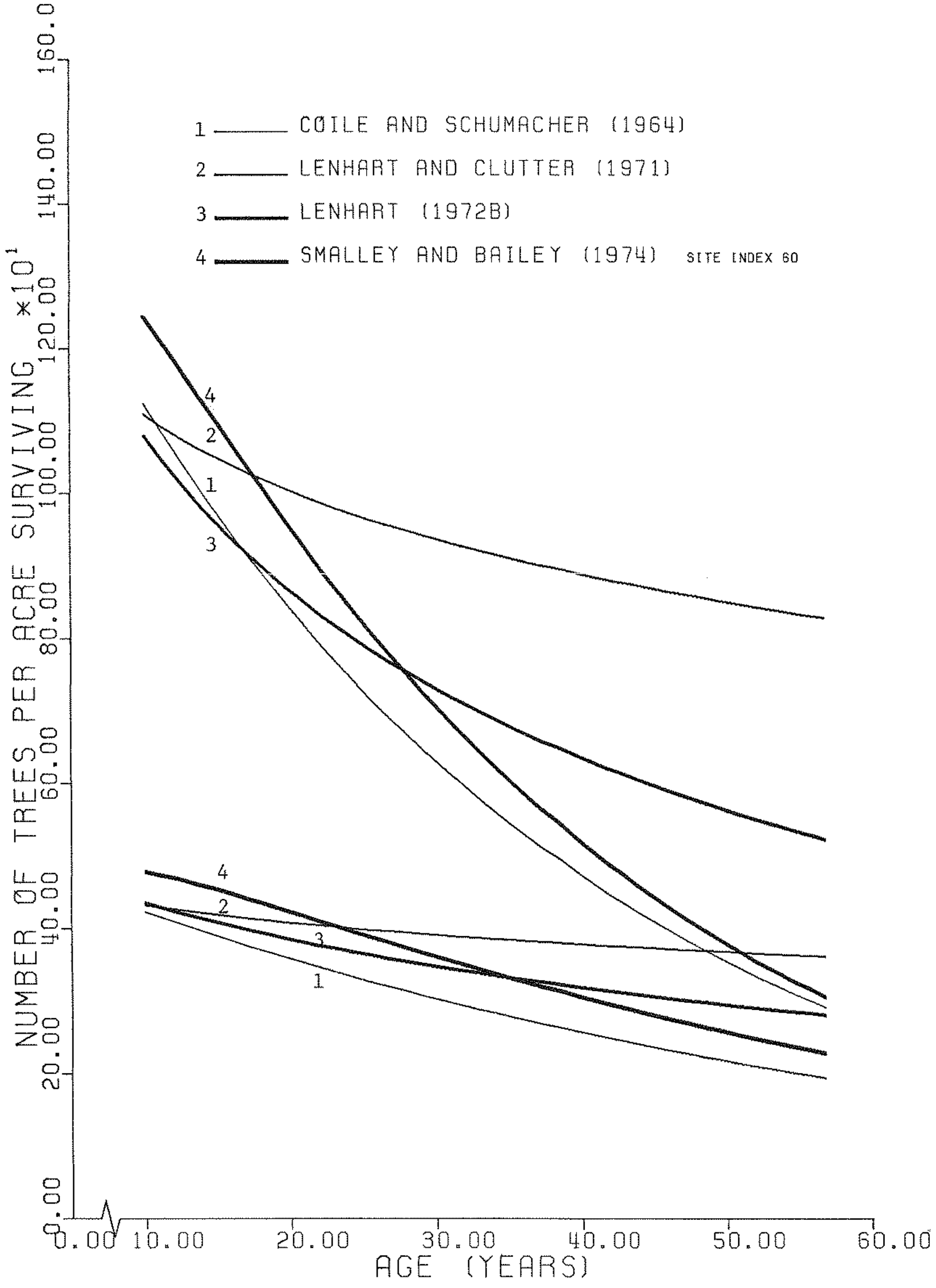


Figure 5c. Survival curves used in old-field plantation models - site index 70 (base age 25 years).

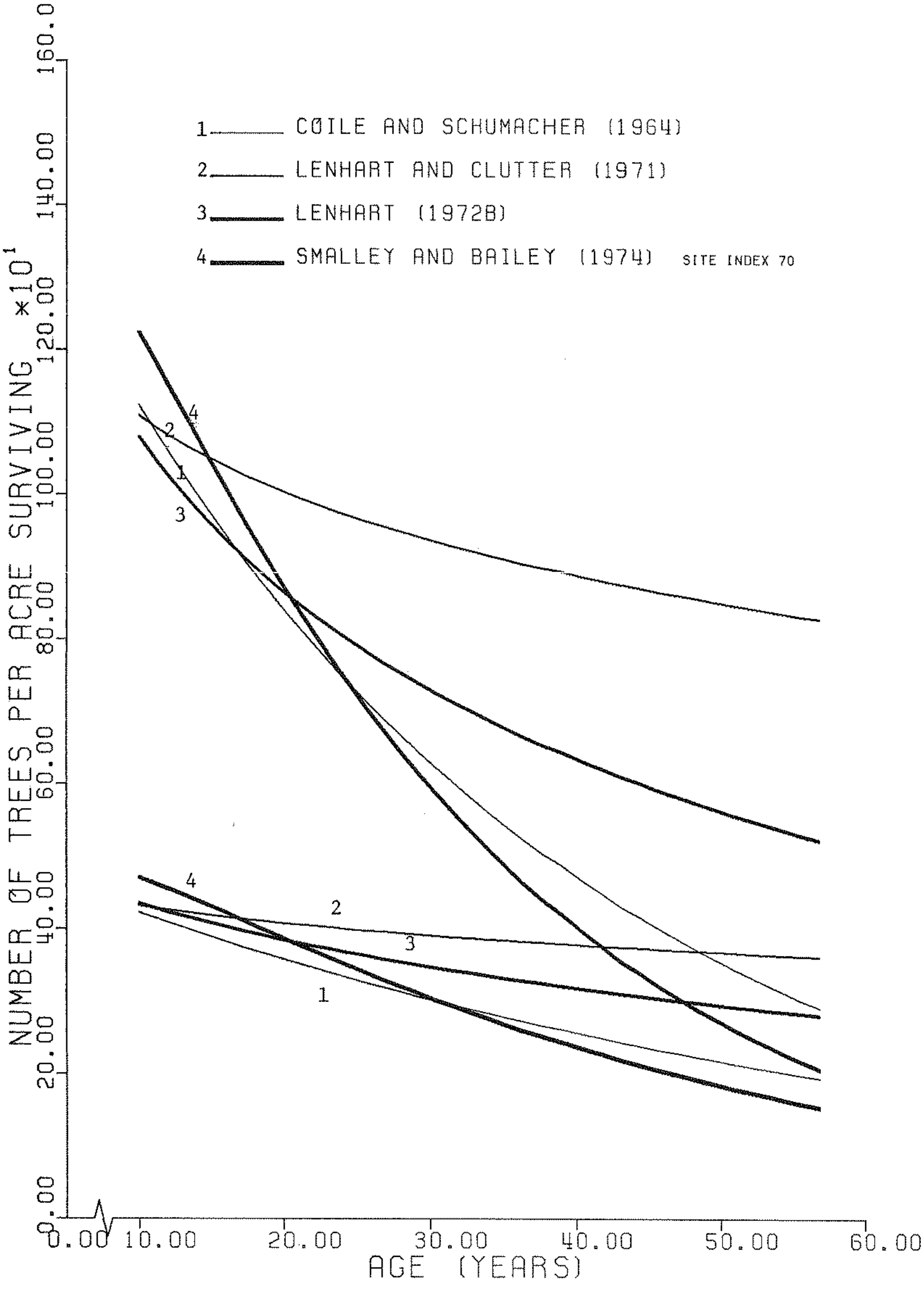


Figure 6a. Basal area projections for natural stands for site index 70 (base age 50 years).

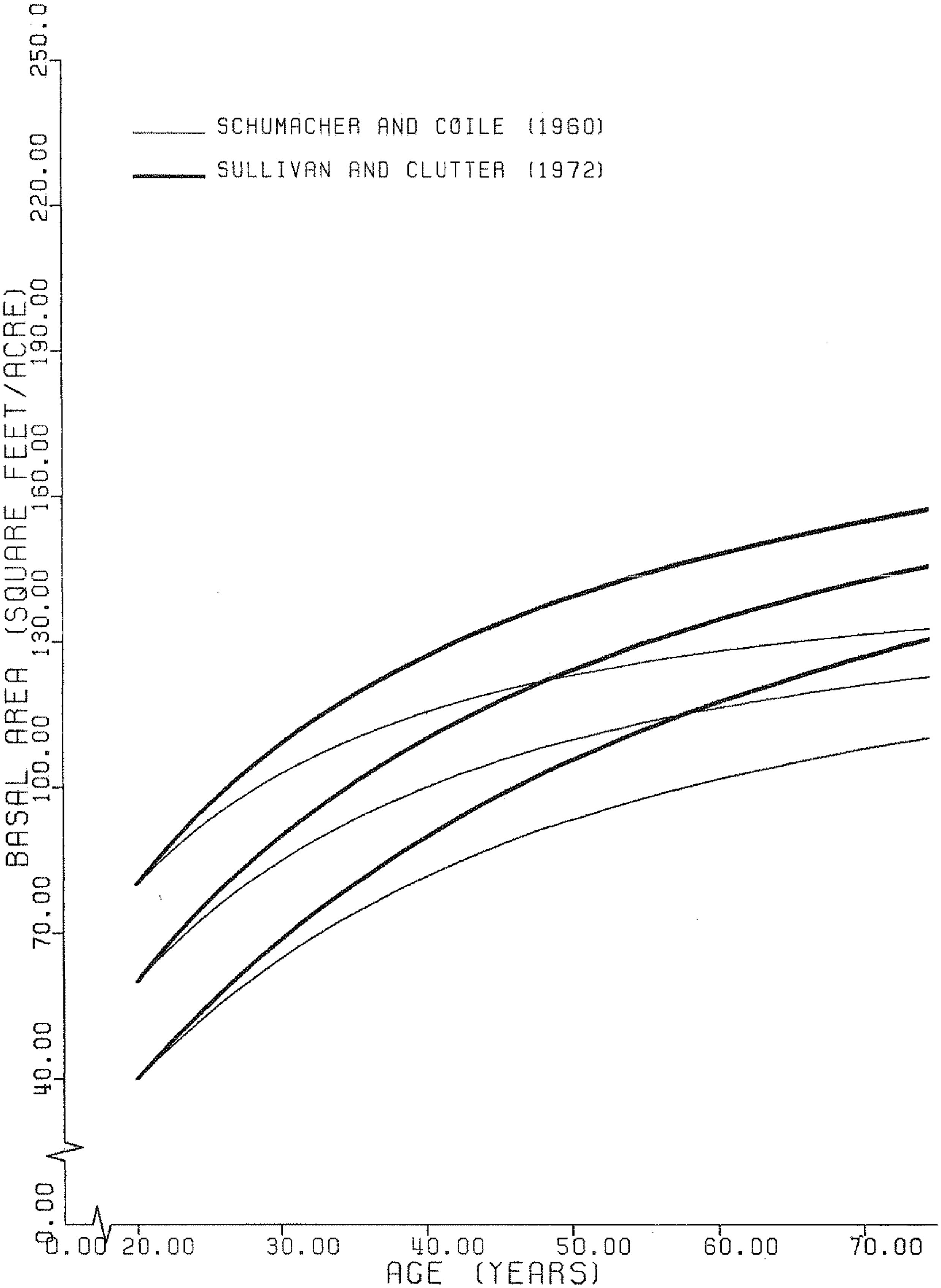


Figure 6b. Basal area projections for natural stands for site index 80 (base age 50 years).

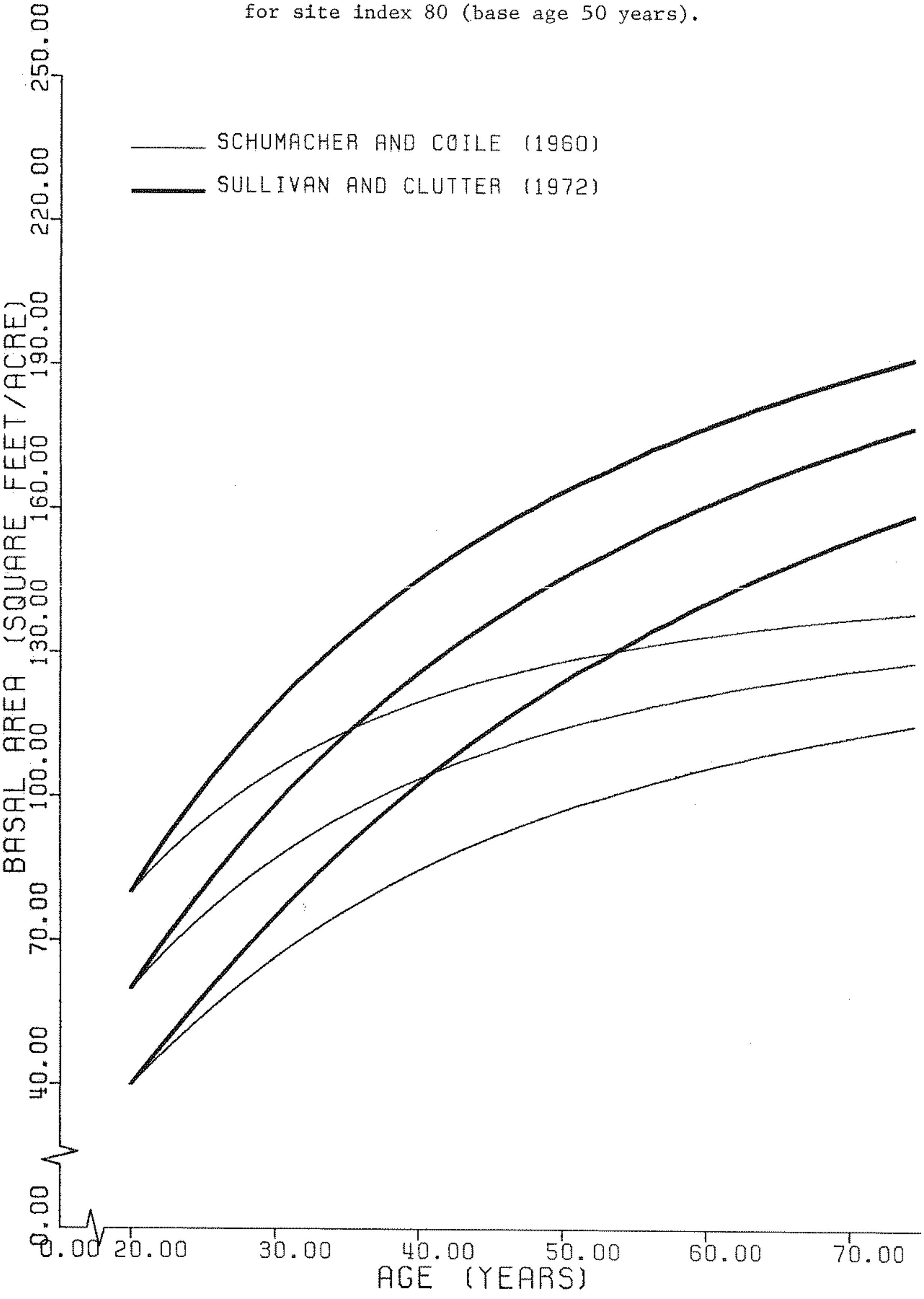


Figure 6c. Basal area projections for natural stands for site index 90 (base age 50 years).

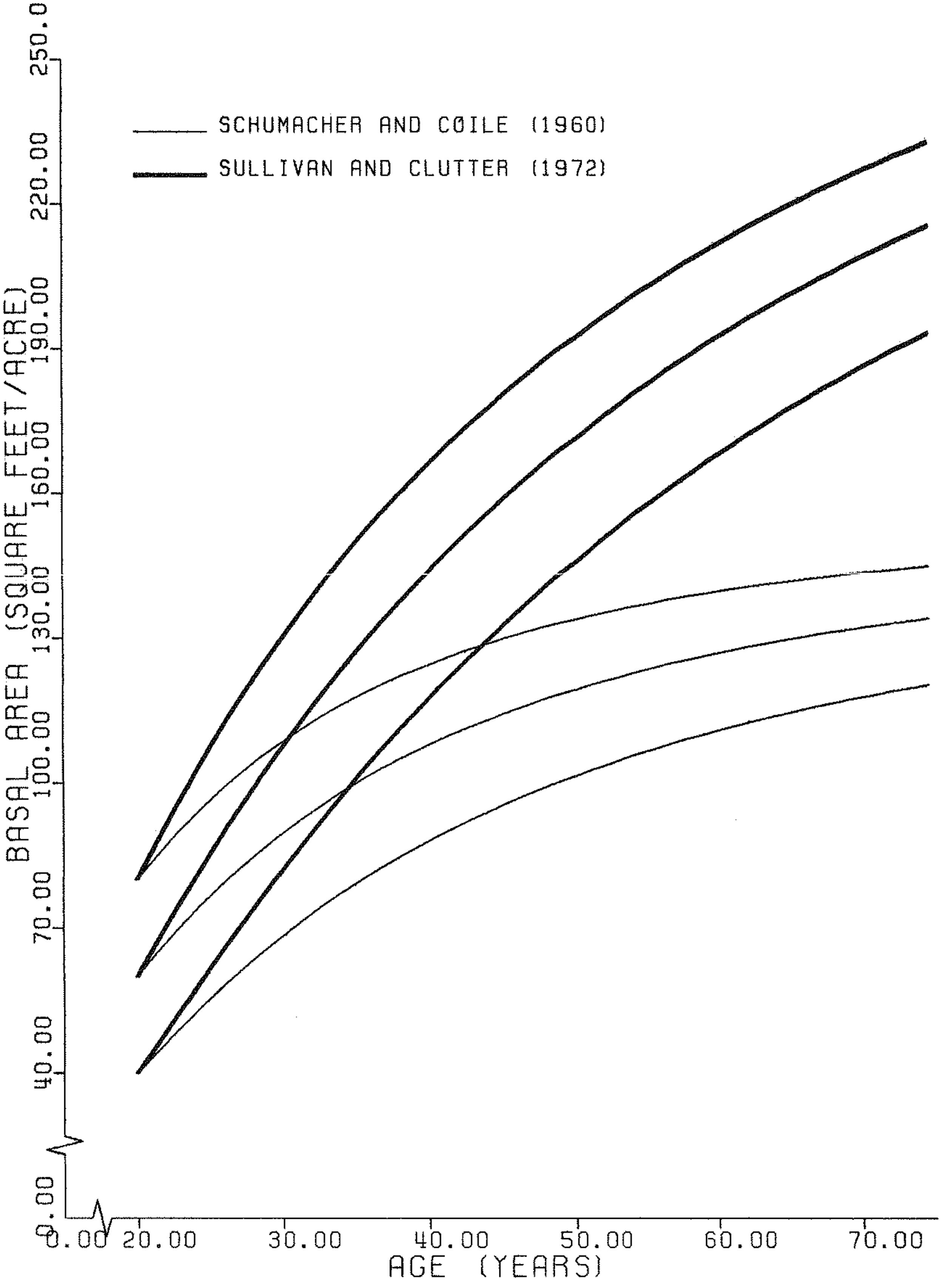
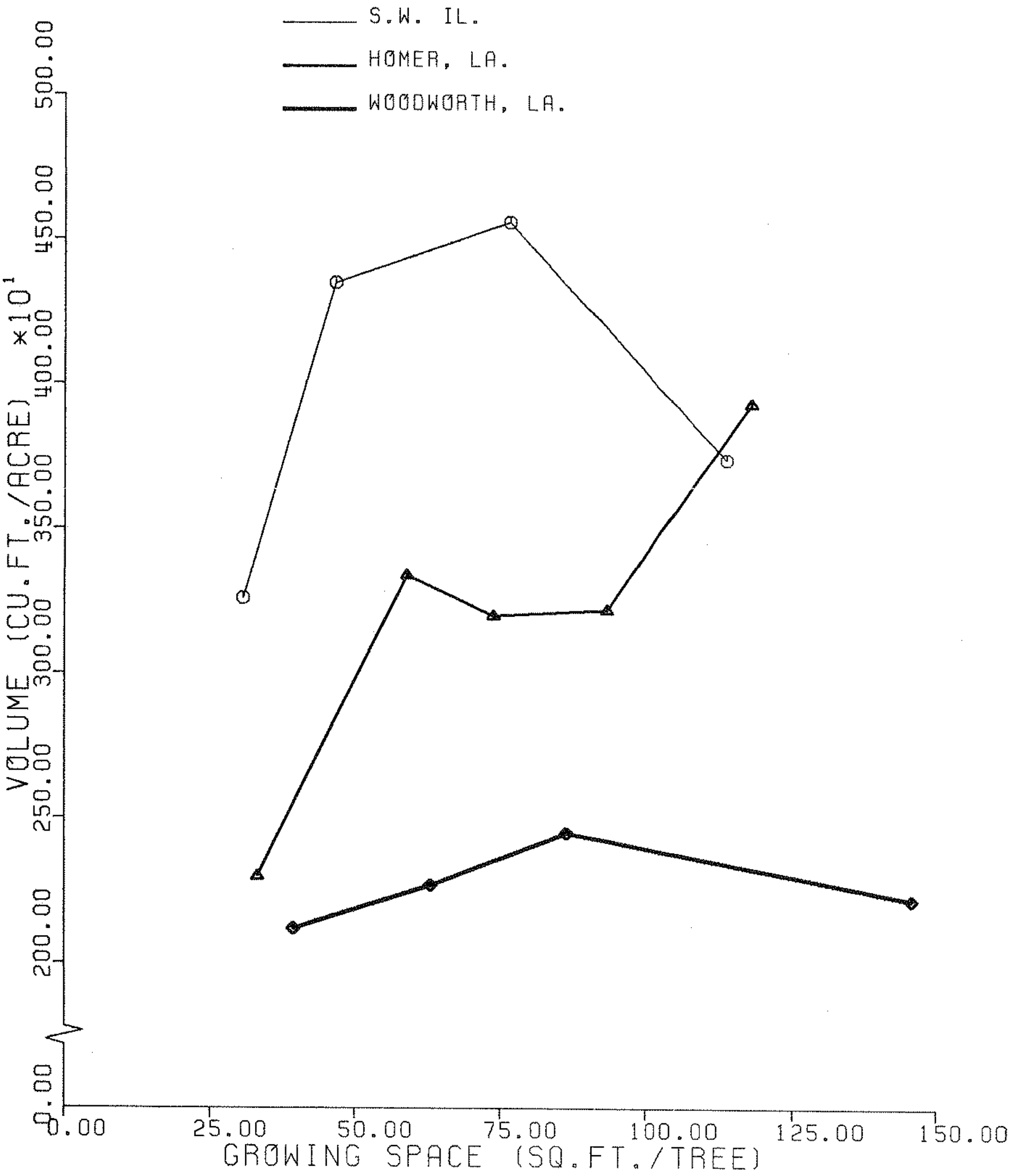


Figure 7. Relationship of growing space per tree to cubic-foot volume inside bark to a 4-inch top for 21-year-old loblolly pine in three spacing studies.



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